

Modelling the Contact Behaviour in the Presence of Viscoelasticity

by

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The candidate confirms that the work submitted is his own and that appropriate credit has been given where reference has been made to the work of others.

Publication Statement

The candidate confirms that the work submitted is his own, except where work which has formed part of jointly authored publications has been included. The contribution of the candidate and other authors to this work has been explicitly indicated below. The candidate confirms that appropriate credit has been given within the thesis where reference has been made to the work of others.

The work in Chapter 2 of this thesis has appeared in publication as follows:

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The candidate and A.G. conceived the original idea independently. As the lead author, the candidate conducted the literature collection, formal analysis and drafting, while all co-authors contribute to the review and revision of the manuscript.

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Abstract

This project addresses critical knowledge gaps in frictional viscoelastic contact mechanics, focusing on the partial slip phenomenon, where the contacting area is divided into stick and slip zones under insufficient tangential loads. The transient evolution of these zones and their influence on subsequent gross sliding behaviour remain underexplored for viscoelastic materials. To fill in these gaps, a numerical framework was developed, integrating time-dependent viscoelastic effects and coupling mechanisms arising from the frictional contact of dissimilar materials.

The research is structured around four objectives. As the foundation of the numerical framework, an elastic partial slip contact model was first developed to investigate effects of surface roughness on partial slip solutions. For rough contact of similar materials (nominally-flat against flat), the stick ratio is unaffected by roughness parameters but follows a linear relationship with applied tangential load. However, the relationship becomes nonlinear when dissimilar materials are involved due to the coupling effects.

The elastic models were then converted into transient viscoelastic partial slip contact models using the elastic-viscoelastic correspondence principle. Different from elastic materials, the separation pattern of stick and slip regions for viscoelastic materials varies with time when different phenomena (creep induced by a constant load or stress relaxation induced by a constant displacement) are encountered in lateral and normal directions, even though the stick ratio remains constant under constant contact inputs.

A transient viscoelastic sliding contact model was further developed to explore effects of early partial slip period on sliding contact solutions, including the delay of the attainment of steady-state sliding and slight decrease of the peak pressure compared to frictionless sliding. These findings justify the use of a frictionless assumption in viscoelastic sliding contact analysis, where qualitative predictions of steady-state behaviour are the primary research concern, such as the assessment of long-term tribofilm performance.

Finally, the framework was extended to model viscoelastic layered contact, focusing on zinc dialkyl dithiophosphate (ZDDP)-derived tribofilms. The layer-substrate system exhibits viscoelastic or elastic-dominated behaviour depending on loading and sliding conditions. In practical applications, the tribofilm performs primarily as a soft elastic layer. This study bridges nanoscale viscoelastic findings with large-scale observations, offering insights for optimizing additive performance in tribological systems.

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List of Abbreviations

2D	two-dimensional
3D	three-dimensional
AFM	atomic force microscope
BEM	boundary element method
CGM	conjugate gradient method
DC-FFT	discrete convolution and fast Fourier transform
DCD-FFT	DC-FFT algorithm modified with duplicated padding
DCS-FFT	DC-FFT algorithm modified with summation
DEM	discrete element method
DLC	diamond-like-carbon
DMT	Derjaguin-Muller-Toporov
DS	direct summation
EHL	elastohydrodynamic lubrication
FEM	finite element method
FFT	fast Fourier transform
GFMD	Green's function molecular dynamics
GW	Greenwood-Williamson
IFFT	inverse fast Fourier transfer
JKR	Johnson-Kendall-Roberts
MD	molecular modelling
MEMS	microelectromechanical
MIM	matrix inversion method
MLMI	multi-level-multi-integration
PDMS	polydimethylsiloxane
PMMA	polymethylmethacrylate

PSD	power spectral density function
SAM	semi-analytical method
SLS	standard linear solid
TENG	triboelectric nanogenerator
THP	total hip replacement
TIM	two-scale iterative method
TTS	time-temperature superposition
TVE	thermoviscoelastic
TVEHL	thermal-viscoelasto-hydrodynamic lubrication
UHMWPE	ultra-high-molecular-weight polyethylene
VEHL	viscoelastic-hydrodynamic lubrication
ZDDP	zinc dialkyl dithiophosphate

1 Introduction

Viscoelastic materials, which exhibit a combination of elastic solid and viscous fluid behaviours under deformation, offer a unique set of advantages, including low friction, lightweight, minimal noise during operation, cost-effectiveness, dimensional stability, biocompatibility and long-term load-bearing capacity. Consequently, viscoelastic materials have been widely applied across diverse fields, making viscoelastic contact problems commonly encountered in both natural and industrial contexts. Examples include the optimization of mouthfeel in chocolate production within the food industry [1], the interaction between human skin and surgical scalpels in clinical applications [2], the flow of basaltic lava under high temperature in geothermal research [3], the tire-road interactions in automotive engineering [4], and contact mechanics in polymer-based knee replacements [5] in biomedical applications.

For accurate design or tribological analysis of these systems, it is crucial to have a comprehensive understanding of the interfacial mechanics, where viscoelastic characteristics, such as temperature-dependent rheological behaviour, energy dissipation, material creep and stress relaxation, can play a critical role in determining the overall contact performance. Enhanced time-dependent analysis on viscoelastic contact problems can drive the development of more efficient and safe product designs, leading to significant energy savings and improvements in quality of life.

Currently, the growing demands for higher efficiency, reduced material usage, lower environmental impact, and precise microscale manufacturing, particularly for polymer components used in microelectromechanical systems (MEMS) [6, 7], further highlight the requirement of robust analytical tools tailored to contact mechanics of viscoelastic materials. Developing such tools necessitates a fundamental understanding of contact mechanics, which forms the foundation for analysing the contact behaviour of any material and is described in the following section.

1.1 Background of Contact Mechanics

Given that contact mechanics is the study of deformation and stress of solids that interact at one or more contact points, this section starts with an overview of the types of contact interactions, followed by a concise review of classical contact mechanics. Since classical theories usually serve as preliminary analytical tool for solving idealised or simplified contact problems, modern numerical approaches have been developed to address more complicated and practical scenarios. These advancements are crucial for extending the

applicability of contact mechanics to real-world systems involving viscoelastic materials and are briefly introduced here.

1.1.1 Contact Conformity and its Role in Tribological Analysis

The surface geometry of contacting bodies plays a critical role in determining contact conformity, making it a fundamental consideration in tribological analysis. Before exploring the theories of contact mechanics, it is crucial to distinguish between conformal and non-conformal contacts based on well-defined criteria.

Conformal contact occurs when the curvatures of two contacting surfaces closely match, as illustrated in Figure 1-1 (a). It usually results in a large interaction region comparable to the size of the contacting bodies. Typical examples include clutches and brakes. In contrast, non-conformal contact arises when the contacting surfaces have significantly different radii or when one surface exhibits a high degree of curvature relative to the other (e.g. a sphere on a flat surface as shown in Figure 1-1 (b)). This leads to a smaller contact zone with high localized pressure, as seen in gears, ball bearings and cam-follower systems [8, 9].

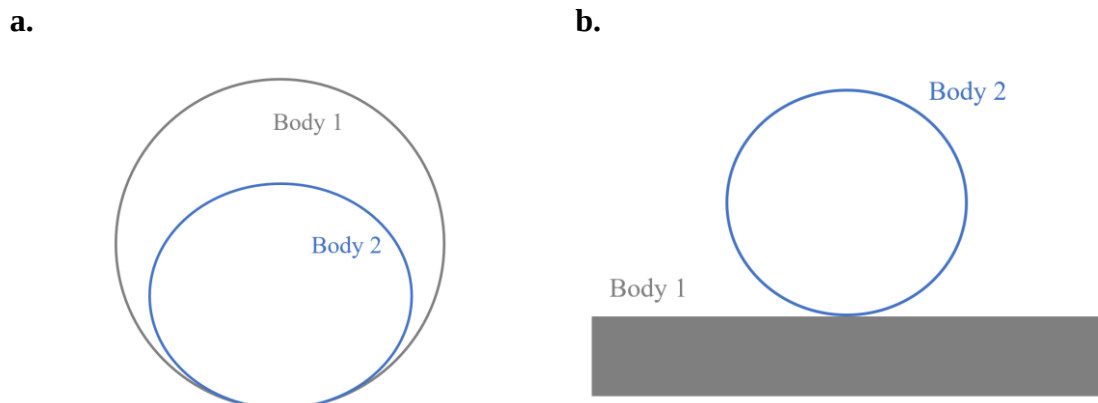


Figure 1-1 Illustration of contact configurations in different levels of conformity: (a) conformal contact and (b) non-conformal contact

Contact problems need to be analysed in different approaches depending on the conformity level. For conformal contact, the deformation of the macrostructure significantly affects the contact geometry, interaction region, and tribological behaviours. Given the diverse designs of contacting components in practice, conformal contact problems often require case-specific investigations that integrate the analysis of the interface and structure [8, 9].

In non-conformal contact, the disparity between the stresses within the localized contact zone and those distributed in the broader macrostructure enables simplifications.

Specifically, non-conformal contact problems can be reduced to a localized analysis near the contact area by neglecting the influences of the overall component structure. This allows the application of the half-space approximation, treating the contacting bodies as semi-infinite solids [8-11]. This approximation forms the foundation for many analytical and numerical solutions to non-conformal contact problems [12].

This project focuses exclusively on non-conformal contact problem. The rationale is that strictly conformal contacts are less encountered in practice due to surface roughness. Surface asperities cause highly localized points of interaction, even when the macroscopic geometries appear conformal, making non-conformal contact analysis more relevant to real-world applications.

1.1.2 Linearity and Homogeneity in Material Behaviour

In addition to surface geometries, material behaviour significantly influences tribological analysis. When the mechanical response of a material is described linear, its contact response is directly proportional to applied forces or displacements. This linear behaviour is typically characterised by equations such as Hooke's law, which relates stress and strain of materials in the linear elastic region as illustrated in Figure 1-2. The simplicity and predictability of linear relationships makes them a widely used assumption in preliminary contact analysis [10, 11, 13].

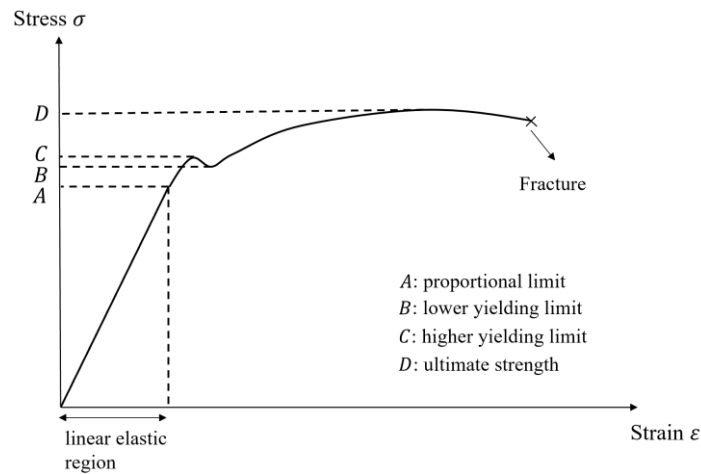


Figure 1-2 Simplified stress-strain relationship for a typical ductile material under continuously increasing load

On the other hand, nonlinear material responses occur under conditions such as high contact pressures exceeding the elastic limit of materials. The material undergoes plastic deformation, and the relationship between stress and strain becomes nonlinear under this circumstance [10, 11, 13] as illustrated in Figure 1-2. Accurately capturing such behaviour usually requires advanced modelling techniques. However, in many

engineering problems, deformations involved are often sufficiently small that the errors introduced by assuming linearity are negligible, justifying the usage of simpler linear models.

Viscoelastic materials present further complexity due to their time-dependent behaviour, influenced by both instantaneous loading and previous loading history. Their contact response can be treated as linear under conditions of low loads or short loading durations, in which condition the stress-strain relationship is expressed with linear differential equations. This linear framework allows the usage of linear operators and theories, including the Boltzmann superposition principle and concepts from linear elasticity, to facilitate the analysis of viscoelastic contact problems [14-16]. When contact stresses become high or surface deformations are significant, the material behaviour can transition to a nonlinear regime, necessitating more sophisticated nonlinear viscoelastic models.

This project focuses on the study of linear viscoelastic contact problems, as they offer practical and widely applicable engineering approximation in many scenarios. Even in cases where nonlinear models are required, linear viscoelasticity serves as a foundational framework for understanding basic contact behaviours, enabling efficient analysis and informing the development of more sophisticated models. Its balance of simplicity and relevance makes it an essential starting point for exploring contact problems in viscoelastic materials. A detailed introduction to the theories of linear viscoelasticity is provided in Chapter 2.

Beyond linearity, homogeneity is another important aspect of material behaviour. Homogeneous materials exhibit uniform properties, such as elasticity and hardness, across their volume or surface, which can simplify the contact analysis owing to the consistent behaviour through of the contacting bodies [10]. In contrast, inhomogeneous materials, such as composites or graded materials, exhibit spatially varying properties, complicating the contact analysis due to the need for localized property characterization [17, 18]. This project focuses on the contact analysis of homogeneous materials, with the incorporation of material inhomogeneity considered as a direction for future work.

1.1.3 Review of Classical Theories of Contact Mechanics

Although this project does not primarily focus on classical theories, they are fundamental to contact mechanics and play a critical role in developing numerical models. While limited to solving elastic contact problems under simplified conditions, these analytical solutions provide essential benchmarks for constructing and validating numerical

algorithms, forming the basis for addressing the complicated viscoelastic contact problems in this project.

The groundwork of smooth surface analysis was laid by Hertz [19], who investigated the non-conformal and frictionless contact between two homogeneous and linearly elastic bodies with simple geometries. He suggested that such a contact problem can be regarded as that of one composite elastic body pressing against a rigid plane. Despite the idealized assumptions rarely being met in practice, Hertzian theory remains invaluable for estimating contact stresses and deformations in scenarios where the effects of lubrication, friction, surface roughness, and material inelasticity and inhomogeneity are insignificant. Hertzian analysis has now been widely recognized as the cornerstone of contact mechanics, and it was used for model validation in this research. When Hertzian assumptions are violated, the resulting complexities are referred to as non-Hertzian contact problems.

Based on the Hertzian theory, the Johnson-Kendall-Roberts (JKR) [20] and Derjaguin-Muller-Toporov (DMT) [21] models were developed to address adhesive forces between contacting surfaces. The key difference between these two approaches is that the JKR model considers the effects of contact pressure and adhesion only within the Hertzian contacting area, while the DMT model assumes that additional adhesive forces exist outside the Hertzian contacting area while the contact field remains the same as the Hertzian solution. Tabor [22] and Maugis [23] parameters were raised later to determine which model is a better fit for specific adhesive contact scenarios.

The classical solutions to the surface deformation and stress distribution of an elastic half-space induced by prescribed normal and tangential tractions were proposed by Boussinesq [24] and Cerruti [25] respectively using the potential theory. Their theories, which were later advanced by Love [26] and generalized by Johnson [11], play a critical role in the development of contact models in this project.

Regarding contact problems in layer-substrate systems (the system includes an indenter, a substrate and a thin layer with finite thickness bonded to the substrate), where the half-space approximation can hardly be applied, pioneering studies were conducted by Burmister [27] and Chen [28] using the Fourier transform. Over the past decades, a number of researchers have proposed different analytical solutions to contact problems with single-layered materials [29, 30] or multi-layer materials [31] in the form of frequency response functions. These solutions have been adopted in the development of layered contact models in this research.

The significance of surface roughness in contact mechanics was not emphasized until the study of Bowden and Tabor [32], who demonstrated a discrepancy between real and apparent contact areas. Archard [33] contributed to the pioneering works of rough surface contact by summarizing a proportional relationship between the real contact area and normal force.

The Greenwood-Williamson (GW) theory [34], which approximates rough surfaces as ensembles of independent asperities with Gaussian-distributed heights and Hertzian contact behaviour, was frequently used for solving elastic rough surface contact problems. Based on this theory, various multi-asperity models were developed [35-39]. While effective for small contact areas, due to the neglect of interaction effects of surface asperities, these models break down when the contact evolves towards full-contact conditions [40]. Recent efforts have made process in addressing this limitation [41-43], but the correction work can hardly solve all the issues identified [40].

Opposite to the multi-asperity model, Persson's theory provides the exact solution for the full contact conditions as it is developed based on the contact of two flat surfaces [44] [40]. This theory assumes that the power spectral density (PSD) of the deformed surface of the elastic half-space can be approximated properly by that of the underlying rigid substrate. This assumption is valid under full-contact conditions but tends to become inaccurate for small contact areas under a small load [45]. The quantitative reliability of Persson's theory for rough contact problems remains a topic of ongoing debate.

These analytical solutions discussed above become inadequate when contact problems deviate heavily from their underlying assumptions, such as linear elasticity, material homogeneity and simplified surface geometry. In such cases, numerical models employing iterative approaches are indispensable for addressing the complexities of practical contact problems, paving the way for more accurate contact analysis.

1.1.4 Common Numerical Approaches for Contact Modelling

This section provides a brief overview of several numerical approaches commonly employed in the development of contact models, with an emphasis on their strengths, limitations and applicability to different contact scenarios.

To start with the finite element method (FEM), it is a well-known and robust method for studying continuum mechanics [40, 46]. FEM has been applied to various contact problems, such as spherical indentation of viscoelastic materials [47] and two-dimensional (2D) elastic-plastic contact [48]. Although FEM can be implemented

straightforwardly to solve contact problems with arbitrary surface geometries, a large simulation domain is always required to discretize the volumes of contacting bodies with a proper treatment of boundary conditions. Usually, the domain would have to be extended laterally and vertically at least several times the contact size in each direction. A typical contact configuration established in the FEM-based simulation work of Koumi, et al [49] is illustrated in Figure 1-3. The discretisation requirement for the entire domain can result in significant computational demands, especially for three-dimensional (3D) problems with rough surfaces, where small-size elements are needed to characterise tiny surface asperities [8]. This discourages the wide application of FEM to research on rough contact problems.

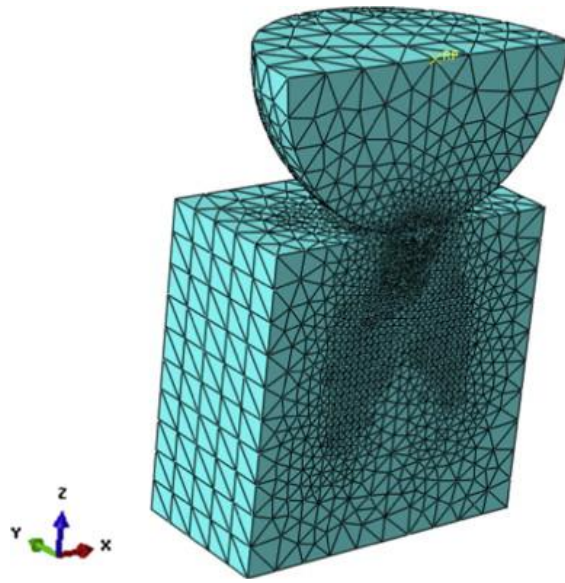


Figure 1-3 Finite element configuration of two contacting bodies (sphere on flat), reproduced with permission from the work of Koumi, et al. [49], © Elsevier B. V., 2025.

Remaining in the realm of continuum mechanics, the Boundary element method (BEM) is a more computationally efficient alternative for surface contact simulation. Unlike FEM, BEM requires meshing only for the boundary of the contacting area, as illustrated in Figure 1-4, reducing the 3D computational domain to a 2D one when body load can be neglected [8, 40]. The improvement in computational efficiency makes BEM particularly suitable for problems requiring detailed surface characterisation, such as the wear of materials [50-52].

Historically, BEM-based contact models were referred to by various names, such as finite surface element model [53], moving grid method [54], Fast-Fourier-Transform (FFT)-based method [55]. Following the work of Peng and Bhushan [56], these methods have been collectively recognized as BEM. By implementing advanced computational

techniques including discrete convolution fast Fourier transform (DC-FFT) and conjugate gradient method (CGM) in BEM, a more efficient numerical method known as the semi-analytical method (SAM) was developed, which is a special form of BEM as noted by Xu and Jackson [57].

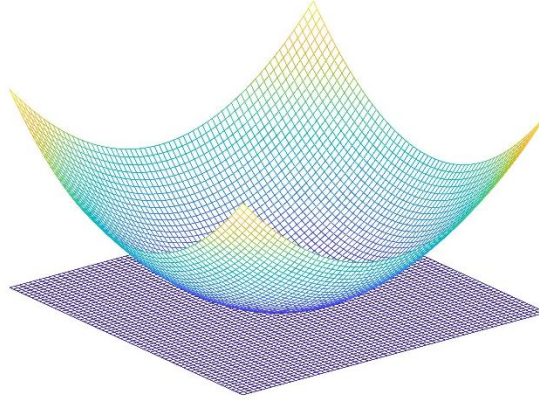


Figure 1-4 Discretised surface of two contacting bodies (sphere on flat) in BEM-based simulation

While BEM exhibits notable computational advantages over FEM [58], its applications is argued to be constrained by assumptions such as small-strain deformation, half-space approximation and linear material behaviour [40]. These limitations arise from the regular utilization of Green's functions in BEM, which are well-defined and mathematical tractable for problems satisfying the above assumptions but become difficult to derive and apply in nonlinear scenarios. Hence, for contact problems involving significant material nonlinearities or multi-physical couplings, FEM is often preferred despite its higher computational cost.

Over the past decades, advances in computational contact mechanics and numerical algorithms have led to the development of more sophisticated treatments for nonlinear problems within the framework of BEM [17, 18, 49, 59-74]. These adaptations extend the applicability of BEM at the expense of reduced computational efficiency. A detailed discussion on the capacity of BEM to accommodate nonlinearities, along with its advantages and disadvantages in the context of this project, is provided in section 1.4.1.

Given that FEM and BEM are usually applied to simulate contact problems on the component scale or mesoscale [40, 58], when investigating problems on smaller scales where continuum mechanics no longer applies, methods such as Molecular dynamics (MD) and Discrete element method (DEM) are more appropriate.

To introduce the two methods briefly, MD simulates the motion of atoms and molecules by solving Newtonian equations of motion, with interatomic and intermolecular force

derived from molecular mechanics [40, 75]. This approach is particularly effective for studying molecular interactions, such as those between lubricants and substrates on the nanoscale. On the other hand, DEM models the macroscopic behaviour of bulk materials based on the microscopic interactions of individual particles [76]. Similar to MD, DEM applies Newtonian law to determine the kinetics of particles. In DEM, particle velocities are assumed constant within each time step, allowing their positions to be iteratively updated. This allows DEM-based models to effectively capture changes in surface geometry and simulate material wear by tracking particle displacement during sliding contacts, such as the work of Capozza and Hanley [77].

1.2 Background of Viscoelastic Contact Modelling

The study of viscoelastic contact problems has evolved significantly through extensions of classical contact mechanics theories and implementation of numerical modelling approaches. This section outlines key solution developments, methodological advancements, and persistent challenges in viscoelastic contact analysis, while a detailed literature review and introduction of fundamental theories of viscoelasticity are presented in Chapter 2.

1.2.1 Overview of the Progress in Viscoelastic Contact Analysis

Early analytical efforts began with the work by Lee and Radok [78-80], who applied the elastic-viscoelastic correspondence principle [81, 82] to analyse spherical indentation problems. The correspondence principle remains fundamental to modern analytical and numerical models. It also serves as a cornerstone of the methodologies applied in this project. Extensions to the solution of Lee and Radok was conducted by subsequent researchers, including Huner [83], Graham [84], Yang [85], Ting [86, 87] and Greenwood [88], to address the limitations related to the surface geometry and assumptions of monotonic increase in contacting area. More recently, Persson [89] proposed an analytical solution to quantify the real contact area of viscoelastic solids interacting with rough surfaces, which is built upon his theory of contact mechanics [44]. Despite these advancements, analytical approaches are inherently limited in addressing practical problems involving surface roughness, arbitrary loading histories, and complicated rheological behaviour of viscoelastic materials.

To overcome these challenges, numerical approaches have become increasingly prominent with BEM-based models gaining significant recognition over the past decade. Chen, et al. [90] developed a robust point-contact model for the indentation contact between a spherical indenter and a viscoelastic half-space, which was later extended to

incorporate surface adhesion [91], material inhomogeneity [71] and line contact configurations [92]. For dynamic sliding and rolling contacts, Carbone and Putignano [93] introduced a novel correction factor to account for thickness effects of materials [94-97] in viscoelastic deformation, enabling steady-state simulations of rough viscoelastic contact at varying sliding speeds. In transient analyses, Koumi, et al. [49] modelled rolling contact with the incorporation of material inhomogeneity. Wallace, et al. [98] developed a layered contact model that allow for distinct viscoelastic or elastic properties in the layer and substrate, and Wang, et al. [99] explored the effects of interfacial imperfections on layered viscoelastic contact.

The literature mentioned above primarily focuses on addressing dry contact problems of linear viscoelastic materials under different operating conditions. Through applying different enrichment techniques, where the enrichment fields from solutions to adhesive contact (e.g., [20, 21]), thermal-elastic contact (e.g., [100]), elastohydrodynamic lubrication (e.g., [101]), are superimposed to foundational viscoelastic models like that of Chen, et al. [90] or Carbone and Putignano [93], models capable of solving multi-field viscoelastic contact problems, such as viscoelasto-hydrodynamic lubricated contact [102, 103] and thermoviscoelastic contact [104-106], have been developed. These efforts have paved the way for a more comprehensive numerical framework aiming to tackle complex real-world contact problems [107], as illustrated in Figure 1-5. Without the aim of detailing these studies in this overview, a detailed discussion on the modelling work and gradual development of the general numerical framework can be found in Chapter 2.

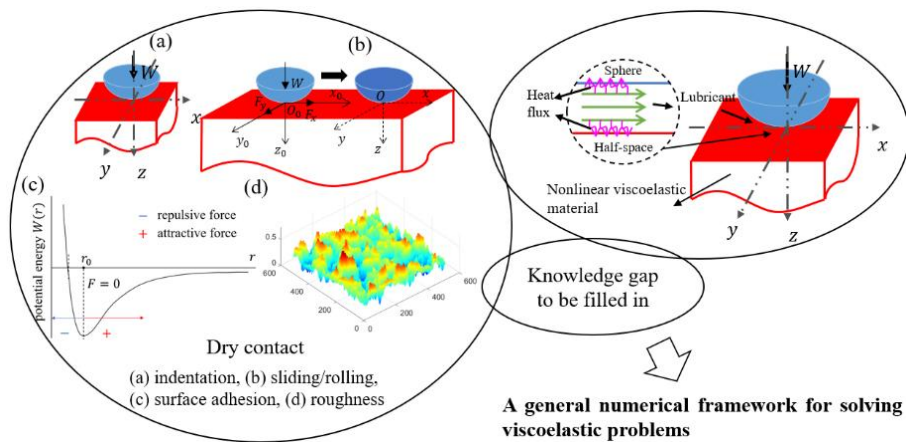


Figure 1-5 A general numerical framework for solving viscoelastic problems [107]

1.2.2 Knowledge Gap and Research Significance

The study of viscoelastic contact problems has made substantial progress, particularly in the development of numerical models for linear viscoelastic contact as mentioned above. However, most existing studies assume frictionless contact [49, 93-97, 99], which

simplifies the analysis but neglects the critical coupling effects between normal pressures and shear tractions that can be essential in real-world applications. This limitation of frictionless assumption is particularly pronounced in transient contact scenarios, where stick-slip behaviour is crucial for understanding how contacting surfaces transition from full stick to gross sliding.

The partial slip phenomenon, where the contact area is divided into stick zones (in which no relative motion happens) and slip zones (in which local relative movement exists) under insufficient tangential loads, remains underexplored for viscoelastic materials. Existing partial slip models, such as the work of Kaller [108] and Goryacheva and Sadeghi [109], generally fail to account for the coupling between normal and tangential forces. These limitations arise from the assumption of identical mechanical properties for contacting materials or the application of the Goodman approximation [110], which considers only the effects of pressure on tangential tractions. These simplifications undermine the reliability of simulations, particularly in engineering systems involving interfaces between hard and soft materials.

In recently developed viscoelastic sliding and rolling contact models, dry contact friction is often neglected, attributing frictional losses exclusively to viscoelastic hysteresis. Even when dry friction is considered, it merely exists in the form of dynamic friction proportional to the contact pressure, as carried out in the work of Goryacheva, et al. [111, 112], leaving the intricate coupling between shear and normal tractions unsolved. Many studies [93-95, 99, 113] also prioritise steady-state responses when simulating viscoelastic sliding contact, where the contact deformation and stress field remain stable over time. While the frictionless assumption may suffice for the steady-state analysis, it becomes unsuitable for transient sliding contact analysis, where the contacting bodies start the sliding motion from a fully-stick state. During the transient phases, stick and slip regions evolve dynamically before reaching the gross sliding state, the effects of which on the gross sliding behaviour remains poorly understood in viscoelastic contexts.

To date, only limited work has shed light on the stick-slip transitions in viscoelastic materials under the effects of dry friction. Wallace, et al [98] extended the partial slip model of Gallego, et al. [114] to a viscoelastic layered half-space. However, their work focused primarily on the effects of layer and substrate properties, providing limited insights into the role of viscoelasticity in partial slip dynamics and gross sliding transitions.

Incorporating partial slip and coupling effects into viscoelastic contact analysis can provide improved predictions of performance and durability for systems such as tire-road interactions and fretting contact in biomedical devices. Additionally, these models can provide valuable insights into the behaviour of viscoelastic components in engineering applications, such as the tribofilms derived from Zinc dialkyl dithiophosphate (ZDDP).

1.3 Research Aims and Objectives

The aim of this project is to develop a comprehensive and robust numerical framework that addresses the identified knowledge gap about dry friction in existing viscoelastic models. By integrating the time-dependent effects inherent to viscoelastic materials and coupling mechanisms arising from frictional contact of dissimilar materials, this project aims to advance the understanding of stick-slip dynamics for viscoelastic materials and provide insights into the influence of this stick-slip period on the overall sliding contact behaviour.

This framework is designed to capture both transient and steady-state contact behaviours accurately. It is expected to provide reliable solutions for various contact problems involving viscoelastic and elastic materials with different properties, surface geometries and loading conditions. By addressing theoretical challenges and meeting practical requirements, this project seeks to enhance understanding in the contact mechanics about partial slip, and drive innovation in the design and optimization of systems, where contact friction is crucial, and hard-on-soft material combinations are commonly used.

To achieve these research aims, the project is structured around four core objectives, referred to as OBJ, with a logical progression as illustrated in Figure 1-6. Each objective is accompanied by a focused research question, referred to as RQ, that guides the numerical investigation, ensuring the modelling work closely aligned with the overarching research aims.

The research objectives and their corresponding research questions are outlined as follows:

1. OBJ1. Develop an accurate elastic partial slip contact model. The model should account for the contact of either similar or dissimilar materials and provide accurate solutions to problems with arbitrary surface geometries.

While elastic partial slip modelling is well-documented by analytical [11, 115, 116] and numerical solutions [117, 118]), the influence of several specific surface roughness parameters on elastic partial slip solutions remains underexplored.

Addressing this knowledge gap can enhance understanding of partial slip mechanics and validate the capability of the numerical framework for rough surface simulations, thereby leading to the following research question:

RQ1: What are the effects of surface roughness on partial slip solutions?

2. OBJ2. Develop a transient viscoelastic partial slip contact model. The model should reflect the time-dependent characteristics of viscoelastic materials that are significantly different from ordinary elastic solids.

After incorporating viscoelasticity in partial slip simulations, the knowledge gap about the stick-slip transitions in viscoelastic materials can be filled by answering the following research question:

RQ2: What are the effects of viscoelastic materials on the partial slip solutions?

3. OBJ3. Develop a transient viscoelastic sliding contact model. The model should demonstrate how contacting surfaces move dynamically and reveal the transition of contacting surfaces from partial slip to gross sliding.

By taking into account the dynamic sliding contact, the knowledge gap about the role played by dry contact friction in transient sliding contact of viscoelastic materials can be filled by answering the following research question:

RQ3: What are the effects of the early partial slip period on subsequent sliding contact solutions compared to frictionless sliding?

4. OBJ4: Develop a transient viscoelastic layered contact model. This model should take a viscoelastic layer with arbitrary thickness into consideration and reflect how the layer properties and thickness influence the contact mechanics.

This model allows an engineering system with layered structure to be investigated by the numerical framework. Given that numerical studies on the viscoelasticity of ZDDP-derived tribofilm were rarely reported, despite their known viscoelastic behaviour and potential relation with the anti-wear properties [119, 120], the following research question is raised, which can demonstrate the applicability of the developed numerical framework while validating the layered contact aspect:

RQ4: What are the effects of ZDDP-derived tribofilm on the contact of two ordinary elastic solids when the viscoelastic layer is bonded to one elastic substrate?

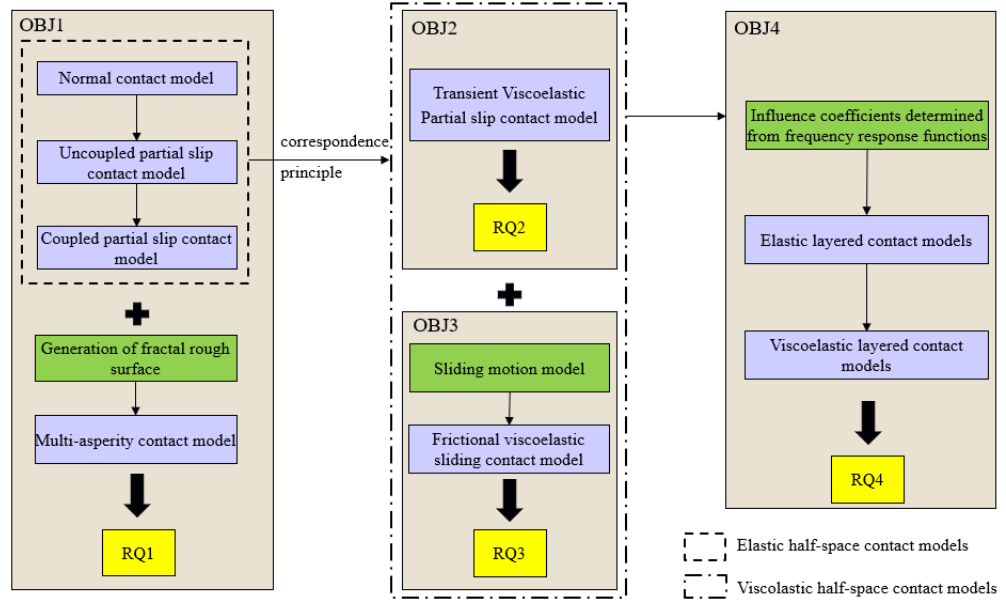


Figure 1-6 Numerical framework to be accomplished in the project

1.4 Research Methodologies

Instead of introducing how each contact model aligned with OBJ is developed based on the corresponding formulations and algorithms (details of which is given in each chapter solving RQ), this section focuses on describing the numerical approaches and computational techniques employed in the model development. Additionally, the rationale for selecting these computational methods and techniques is provided.

1.4.1 BEM vs FEM

As mentioned in section 1.1.4, compared with FEM, BEM is an efficient and accurate approach for simulating rough surface contacts. The modelling of remote boundaries away from the contacting area is not required in the context of BEM, simplifying analysis and reducing computational costs for linear and homogeneous problems. Although advanced FEM models, after incorporating techniques such as the adaptive mesh approach [121, 122], exhibit improved computational efficiency, BEM remains more cost-effective generally for such problems.

However, compared with the versatile FEM where non-linear and heterogeneous material properties can be readily accommodated, simulating contact problems with complexities such as plasticity and heterogeneous materials can negate the intrinsic simplicity of BEM. To describe this issue from the perspective of the detailed application of BEM, it frequently utilizes Green's functions or fundamental solutions for linear problems, which are well-defined and are mathematically tractable (e.g., integrals of Boussinesq [24] and Cerruti [25] for point loading of ideally elastic contact problems that are used in this

project) to streamline the contact analysis [57]. Introducing nonlinearity or material heterogeneity complicates the derivation and utilisation of the Green's functions significantly. Hence, BEM is argued to be typically suitable for problems where Green's functions can be computed, typically involving fields in linear homogenous media [40].

Over the previous decades, with the development of computational contact mechanics and numerical algorithms, more elaborate treatments have been introduced to solve contact problems involving material inhomogeneity and plasticity within the framework of BEM. These advanced approaches often incorporate iterative procedures, adaptive mesh refinement, or other strategies. However, such extensions often come at the expense of increased computational demands. Several examples of the BEM-based models that overcome the limitation of homogeneous linear elastic material behaviour are given in Table 1-1, where the works focusing on plasticity or/and inhomogeneity are arranged in the chronological order [17, 18, 49, 59-74].

The elastoplastic contact analysis is employed as an example to demonstrate how the computational efficiency is undermined when considering nonlinear contact behaviour in the framework of BEM. It is mentionable that plasticity is a vast area of studies and beyond the scope of this project. This section only presents a brief discussion on the efforts made by the researchers to simulate elastoplastic contact based on BEM.

As practiced by Lee and Ren [123] and Peng and Bhushan [124], the simplest approach is to set the threshold value of the maximum nodal pressure to be the material hardness by assuming an elastic-perfectly-plastic material behaviour. However, important factors like residual stress and strain are neglected in this method, leading to a pressure distribution that is simply flattened without any consideration of plastic flow. To achieve more accurate solutions, the permanent displacement caused by plastic deformation (i.e. residual displacement) needs to be accounted for when determining the surface gap between contacting surfaces during the contact interface analysis.

To efficiently calculate the residual displacements and stresses, the influence coefficients relating plastic strain to residual displacement need to be used. These solutions were first presented by Jacq, et al. [59], who used Chiu's cuboidal eigenstrain solutions [125, 126] as analytical influence coefficients to simulate the elastic field induced by plastic strain. Betti's reciprocal theorem was used to deduce the expression of the residual displacement. The plastic strain domain beneath the surface is divided into small cubic elements in which the plastic strain is considered constant. Similar with the way in which surface deformation is calculated, by assuming that the plastic strain in each element would cause

a certain amount of plastic deformation at surface, the total residual displacement, can be obtained by adding the contributions from every cubic element. 2D-FFT method is then used to solve the residual stresses layer by layer.

Table 1-1 Examples of numerical works that solves plastic and inhomogeneous problems based on BEM

Author	Plasticity	Inhomogeneity
Jacq, et al. 2002 [59]	Yes	No
Wang and Keer 2005 [60]	Yes	No
Nelias, et al. 2007 [61]	Yes	No
Chen, et al. 2008 [62, 63], 2010 [64]	Yes	No
Wang, et al., 2010 [65], 2013[66]	Yes	No
Fullerenger and Nelias, 2010 [67]	Yes	No
Leroux, et al. 2010 [18], 2011 [68]	No	Yes
Chaise and Nelias, 2011 [69]	Yes	No
Zhou, et al. 2011 [17]	No	Yes
Wang, et al. 2013 [70]	No	Yes
Koumi, et al. 2014 [71], 2015 [49]	No	Yes
Dong, et al. 2016 [72]	Yes	Yes
Shi, et al. 2019 [73]	No	Yes
Li, et al. 2022 [74]	No	Yes

Compared with the purely elastic contact analysis, the computational efficiency of BEM is degraded significantly, simply due to the fact that plastic deformation makes the contact pressure and subsurface stresses dependent on each other so that they have to be solved simultaneously through an iterative routine. Contact analysis needs to be performed before the plasticity analysis, while the latter has effects on the former. Pressure, residual displacements and residual stresses are needed in this module. The contact analysis of surface and subsurface is coupled under this circumstance, which implies that the advantage of BEM (e.g., only the contact interface needs to be meshed) is undermined under the circumstances.

Nevertheless, the computational efficiency of the BEM-based model is still better than that of FEM, especially for the contacts between rough surfaces. According to the numerical study conducted by Wang, et al. [65] in 2010, who improved the model of Jacq, et al. [59] by integrating the CGM algorithm with the FFT technique, given that computation for the FEM solutions on rough surfaces was beyond the capability of the PC, it took almost a day on a 2.8-GHz personal computer to get the results for a frictionless elastic-plastic contact problem of rough surfaces with a discretised computational domain with $128 \times 128 \times 20$ elements. The calculation of residual stress was found to take up most of the computing time. Although the computing power has advanced significantly since 2010, the computational time of a BEM-based model is still increased considerably after taking the plastic strain into consideration, given that a purely elastic contact problem can be solved within several minutes when employing the same size of computational domain.

Without the aim to provide an exhaustive review on the numerical simulation of elastoplastic contact problems, this section is to argue that although the historical appeal of BEM lies in the assumption of linearity and homogeneity for ease of implementation, it is not inherently limited to simulate these types of problems. Although BEM cannot readily accommodate material heterogeneity and nonlinear material behaviours within the volume of the contacting bodies, as FEM is capable, it can be adapted through certain modification [59-67]. Also, it must be acknowledged that such an extension is achieved at the expense of some of the critical advantages of BEM, such as its notably high computational efficiency.

Considering the feature of the contact problems aimed to be investigated in the project, BEM is identified as the optimal method for this project. While the current contact model is developed to simulate linear and homogeneous contact problems, it has great potential to be expanded for more intricate operating environments. The model is expected to serve as a robust foundation upon which advanced computational techniques can be implemented to address more challenging contact scenarios effectively and accurately in the future.

1.4.2 Acceleration Method for Deformation Calculation (DC-FFT vs MLMI)

The determination of surface deformation, which is usually expressed in the form of summation of the products of influence coefficient and nodal contact pressure, is a computationally demanding aspect of contact analysis, requiring iterative execution in BEM-based contact models. The solution sought through the direct summation (DS)

method often requires an enormous amount of computational time, as it requires N^2 multiplication operations to calculate deformation for N nodes in one iteration. For a 3D contact problem of rough surfaces, the required densely discretised grid with a very large N further increases the computation burden.

To accelerate deformation calculations, Brandt and Lubrecht [127] and Lubrecht and Ioannides [128] proposed the Multi-Level Multi-Integration (MLMI) method. The key idea of this approach is to start the computation only for the contact region with a coarse mesh, and finer meshes are interpolated level-by-level with corrections for the singularity of kernel functions. MLMI method greatly reduces the computational complexity from $O(N^2)$ to $O(N\log N)$. While efficient, MLMI requires significant development effort due to its hierarchical grid structure and correction procedures.

Another approach is to utilize the fast Fourier transform (FFT) to speed up deformation calculation in the entire computational domain. The basic idea of the FFT-based approaches is to transform the convolution operation for surface deformation calculation from the spatial domain into the frequency domain, where the surface deformations become simple point-to-point products. The computational complexity can also be reduced to $O(N\log N)$ by FFT. However, the assumption of periodicity makes FFT unsuitable to address finite and non-periodic problems like concentrated contact scenarios. Applying FFT straightforward to these problems can result in significant periodic errors, particularly at the boundaries of the computation domain [129].

Various techniques have been developed to mitigate periodicity errors, such as domain extension (e.g., work of Ju and Farris [130] and Nogi and Kato [30]) or decomposing the pressure matrix (e.g., work of Ai and Sawamiphakdi [131]). but they can compromise computational efficiency or introduce additional complexities. A major breakthrough was achieved by Liu, et al [129], who demonstrated that when the influence coefficients are obtained accurately, the discrete cyclic convolution with the processing of zero padding and wrap-around order can help avoid the periodic error associated with the FFT at a cost of only doubling the computational domain.

According to a comparative study on algorithms for accelerating surface deformation calculation conducted by Wang, et al [132], DC-FFT was found to be the most efficient tool while providing a comparable numerical accuracy among the investigated methods including DS, DC-FFT and MLMI. On the other hand, Sainsot and Lubrecht [133]

reported that the MLMI-based method excels in terms of computing speed and memory requirement while the FFT-based algorithm excels in robustness and implementation time. In this project, the DC-FFT approach is employed to accelerate the calculation of surface deformation due to its demonstrated superiority in computational efficiency, accuracy and versatility [134, 135]. Besides, DC-FFT method is more easily implemented from the perspective of computer programming owing to the straightforward operation of FFT in the context of MATLAB. However, it must be acknowledged that the utilization of DC-FFT is confined by the requirement of uniform grid. To date, the development of FFT algorithms with non-uniform mesh is still in the early stages, with few trial studies reported [134].

The detailed steps to utilize the circular convolution in the DC-FFT algorithm are described as follows:

1. Calculate the influence coefficient matrix $[C]_{2N_1 \times 2N_2}$, which relates the surface deformation and contact tractions for the contact problem, from $-N_1$ to $N_1 - 1$ in the x direction and $-N_2$ to $N_2 - 1$ in the y direction. $[]$ denotes the form of a matrix and the subscript $2N_1 \times 2N_2$ denotes the size of the matrix, which is the product of the number of rows and columns;
2. As illustrated in Figure 1-7, conduct the wrap-around and zero-padding (if necessary) to transform the influence coefficient matrix $[C]_{2N_1 \times 2N_2}$ into a cyclic matrix $[\langle C \rangle]_{2N_1 \times 2N_2}$ in the calculation domain denoted as $[0 \sim 2N_1 - 1, 0 \sim 2N_2 - 1]$ in the x and y directions, respectively;
3. Apply 2D FFT to the extended influence coefficient matrix to obtain $\text{FFT2}([\langle C \rangle]_{2N_1 \times 2N_2})$, where FFT2 denotes the 2D FFT operation;
4. Find the matrix of the pressure distribution $[P]_{N_1 \times N_2}$;
5. Apply zero-padding only to extend the pressure matrix to $[P_0]_{2N_1 \times 2N_2}$;
6. Apply 2D FFT to the extended pressure matrix to obtain $\text{FFT2}([P_0]_{2N_1 \times 2N_2})$;
7. Calculate the temporary frequency series through element-by-element production of the two variables $\text{FFT2}([\langle C \rangle]_{2N_1 \times 2N_2})$ and $\text{FFT2}([P_0]_{2N_1 \times 2N_2})$;
8. Apply 2D inverse fast Fourier transform (IFFT) to the temporary frequency series and obtain the surface deformation by only keeping the data within the original computational domain.

It is of note that although the periodic error inherent in Fourier-based methods can be effectively limited by using DC-FFT after careful management of domain size and

operations of wrap-around and zero-padding, the by-products of Fourier-transform-based methods, such as the aliasing and Gibbs phenomena, still need to be addressed carefully during the contact analysis for various problems [136]. To explain these two phenomena briefly, aliasing phenomenon arises from insufficient sampling in the frequency domain. The high-frequency components fold into lower frequencies, leading to distortion when reconstructing the original function in the spatial domain [137, 138]. This issue is particularly critical when converting frequency response functions into influence coefficients (e.g., solutions to layered contact problems) [136, 139]. On the other hand, Gibbs phenomenon occurs when a function with sharp gradient or discontinuities (e.g., a step function) is approximated by a finite number of Fourier series terms. Given that an infinite number of terms is required to perfectly represent such a function, oscillations near the discontinuities are generated after truncating the series to a finite number of terms in practical computations [138]. To avoid the artifacts of sampling and series truncation, additional pre-processing steps need to be employed in the DC-FFT algorithm, such as domain extension in the frequency domain and introduction of an aliasing control parameter when transforming the Fourier series to its FFT equivalence [136].

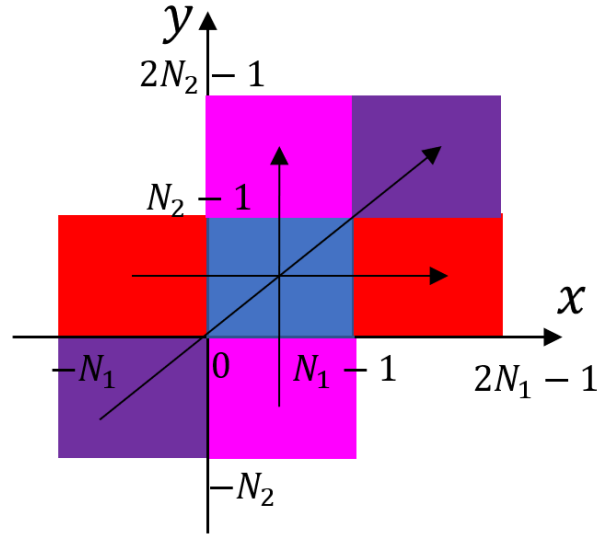


Figure 1-7 Wrap-around order in both x and y directions for the influence coefficient $[C]_{2N_1 \times 2N_2}$

1.4.3 Conjugate Gradient Method

One of the main difficulties in solving contact problems is that neither the contact area nor the pressure distribution is known in advance. As a result, an iterative approach or the so-called trial-and-error method is frequently applied. For the achievement of high computational efficiency along with the guarantee of proper numerical accuracy for BEM-based models, a good iterative scheme is essential. Given that algorithms always

start from an initial guess and generate the following guess for the next iteration by a series of relaxation methods such as the Jacob method and Gauss-Seidel method [140], how the adjustment for the guess value is done and how the new guess approaches the real solution affects the convergence rate of the algorithm. Among the potential methods, the conjugate gradient method (CGM) is the one that has been widely applied in the numerical modelling of contact problems [8]. To describe it briefly, CGM is one of the linear search methods for numerical optimisation problems, which can be expressed with the following equations [140-142]:

$$x_{k+1} = x_k + \alpha_k d_k, \quad \text{Equation 1-1}$$

$$d_k = -r_k + \beta_k d_{k-1}, \quad \text{Equation 1-2}$$

Equation 1-1 is the general form of the linear search methods showing how the new guess x_{k+1} is calculated on the basis of the previous one x_k , where d is the descent direction, α is the descent step and subscript k is the index number of iteration. Equation 1-2 shows how the search direction (d_k) in the current iteration is calculated on the basis of the one in the previous iteration (d_{k-1}), where r is the residual of a linear system ' $Ax = b$ ' and β is the scalar determined by the requirement that d_{k-1} and d_k must be in conjugate with respect to the symmetric and positive definite matrix A .

Given a linear system expressed as " $Ax = b$ ", the basic CGM procedure to achieve the correct solution is illustrated in Figure 1-8 and described as follows:

1. Initialize the algorithm input x_0 and determine the initial system residual r_0 and search direction d_0 according to the following equation:

$$r_0 = Ax_0 - b, d_0 = -r_0, \quad \text{Equation 1-3}$$

2. To check if the residual vanishes or is smaller than a prescribed value;
3. If condition 2 is not satisfied, update x with the determined search step α using the following equation:

$$\alpha_k = \frac{r_k^T r_k}{d_k^T A d_k}, x_{k+1} = x_k + \alpha_k d_k, \quad \text{Equation 1-4}$$

where the superscript T denotes transpose operation;

4. Determine the new residual and search direction for the next iteration according to the following equation:

$$r_{k+1} = r_k + \alpha_k A d_k, \beta_{k+1} = \frac{r_{k+1}^T r_{k+1}}{r_k^T r_k}, \quad \text{Equation 1-5}$$

$$d_{k+1} = -r_{k+1} + \beta_{k+1}d_k,$$

5. If the new system residual r_{k+1} meets condition 2, end the loop. Otherwise, repeat steps 3 and 4 until the condition 2 is satisfied.

Polonsky and Keer [143] proposed a similar single-loop CGM algorithm to solve the elastic contact problems of rough surfaces along with the implementation of MLMI, which now served as a good reference tool for many numerical models developed in the recent decade [8]. CGM is also applied when designing the algorithms of models in this project considering its following benefits:

- 1) Due to the superlinear rate of convergence [8], it usually requires a few iterations to obtain the solution to contact problems with a prescribed accuracy. The fast computational speed of CGM is verified by Allwood [144], who conducted a comparative study on different contact algorithms;
- 2) A rigorous proof of the CGM convergence for the quadratic optimisation problems with inequality constraints was already provided by Hestenes [145]. Therefore, its convergence is ensured when analysing contact problems through the minimisation of the total complementary potential energy [146].

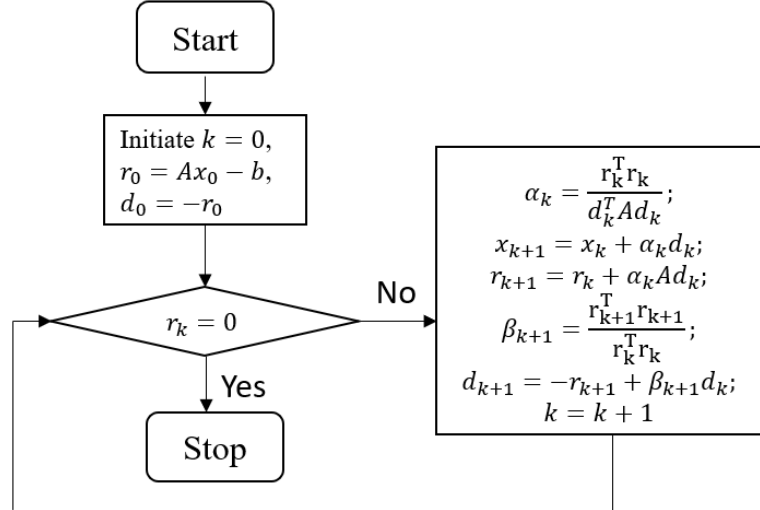


Figure 1-8 Flow chart of the basic CGM procedure

1.4.4 Generation of Rough Surfaces

To study the contact of rough surfaces deterministically, digitized surfaces are required as inputs in contact models. These surfaces can be obtained through two methods. One approach is to use sophisticated surface microscope techniques, such as the atomic force microscope (AFM) and white light interferometry, to generate a 3D topography map of a physical surface [147, 148]. It offers a direct visualization of the actual characteristics of

surfaces, accurately capturing the real-world roughness features within the measurement capabilities of instruments.

Alternatively, surfaces can be generated through mathematical and numerical techniques. These artificial surfaces can display 3D random topography that are characterized by predefined statistical roughness properties, such as the height distribution, which is often Gaussian. The approach excels in its ability to precisely control the statistical features of the surfaces, making it especially useful for parametric studies.

In this project, the rough surfaces are generated using a MATLAB code available on the MATLAB File Exchange Platform [149]. By means of fractal theory [150-152], the code can generate randomly rough isotropic surfaces. It uses the Fourier concept (specifically, the power spectral density PSD) for surface generation. Notably, PSD is an essential surface roughness parameter representing the distribution of surface roughness across spatial frequencies and serves as a key input for advanced contact mechanics models. The PSD of a typical fractal rough surface exhibits a characteristic power-law relation that holds in a limited wavevector region, as illustrated in Figure 1-9 and expressed by the following equation:

$$C(q) = C(q_0) \times \begin{cases} 1 & q_L < q \leq q_0 \\ (q/q_0)^{-2(1+H)} & q_0 < q \leq q_1 \\ 0 & \text{else} \end{cases} \quad \text{Equation 1-6}$$

where H is the Hurst exponent linked to the fractal dimension D_f ($H = 3 - D_f$) and q_L , q_0 , and q_1 are known as the smallest possible wavevector, short-distance roll-off wavevector and long-distance cut-off wavevector respectively. Closely related to the statistically self-affine of surfaces, the fractal dimension D_f can be determined by the slope of the $\log C - \log q$ plot for the part where $q > q_0$ as shown in Figure 1-9, which is typically a non-integer value quantifying the surface complexity and roughness across scales [153].

To describe the role played by the three wavevectors, including q_L , q_0 and q_1 , in the characterisation of rough surfaces, the smallest possible wavevector q_L represents the lower limit of spatial frequencies included in PSD. It corresponds to the largest wavelength ($\lambda = \frac{2\pi}{q_L}$) of surface features, determined by the overall size of the surface or domain being analyzed. Features larger than this wavelength cannot be captured in PSD. The short-distance roll-off wavevector q_0 defines the critical point in PSD where the power-law relation changes. As the name implies, it indicates a roll-off in the frequency

content. This parameter can be used to control how quickly the PSD decays at higher frequencies and reflects the limit of self-similarity in the fractal description of the surface. The long-distance cut-off wavevector q_1 corresponds to the highest spatial frequency included in PSD, representing the smallest wavelength ($\lambda = \frac{2\pi}{q_1}$) of surface features. It is determined by the pixel width of the mesh system. Features smaller than this wavelength cannot be captured in the PSD.

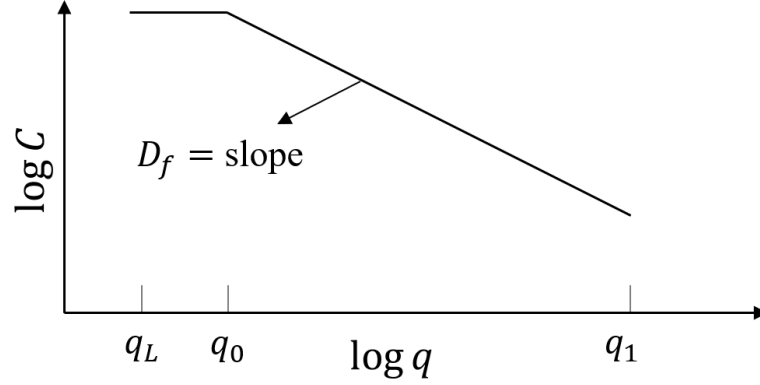


Figure 1-9 The PSD of a typical fractal rough surface which is self-affine for $q_0 < q < q_1$

By making use of the characteristic power-law relation, the surface can be designed by specifying the parameters including the root mean square roughness R_q (standard deviation of surface height), Hurst exponent H , the length of surface topography L and pixel width in x and y directions. Additionally, the roll-off wavevector q_0 for the PSD can be adjusted to tailor the frequency content of surface.

This method offers significant flexibility, allowing surfaces with varying degrees of roughness to be generated and analysed. By manipulating these parameters, the relationship between the interfacial characteristics and statistical properties of surfaces under different operating conditions can be found.

1.5 Thesis Layout

A systematic literature review of the modelling work about viscoelastic contact problems done by different researchers is given in Chapter 2. Besides, fundamentals of the modelling of viscoelastic contacts, such as the basic terminologies and theories of linear viscoelasticity are described briefly in this chapter.

RQ1 is explained in two separate chapters, where Chapter 3 discusses the effects of the surface roughness on the partial slip contact of two identically elastic materials. This

chapter also introduces how the partial slip contact model for two similar elastic materials is developed.

On the other hand, Chapter 4 presents a follow-up study showing the coupling effects derived from the material discrepancy on the partial slip solutions for smooth-surface and rough-surface contact problems. The way the normal and tangential contact problems are coupled with each other is detailed in this chapter.

RQ2 is answered in detail in Chapter 5. Apart from the introduction of how the viscoelastic model is developed based on the elastic models by applying elastic-viscoelastic correspondence principle, effects of viscoelasticity on partial slip solutions are investigated and presented.

RQ3 is answered in Chapter 6, which describes how the model for frictional sliding contact of viscoelastic materials is developed. The effects of the partial slip period preceding the sliding motion on the sliding solutions are discussed here.

RQ4 is answered in Chapter 7, where the development of the layered viscoelastic contact model is described. The effects of a viscoelastic ZDDP-derived tribofilm on the contact of two elastic solids are discussed in this chapter.

Chapter 8 presents an extended discussion on the results of all separated studies. Besides, the limitations of developed models are identified to indicate future directions for further development of versatile viscoelastic models.

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2 A Review on Modelling of Viscoelastic Contact Problems

2.1 Abstract

Approaches to solving viscoelastic problems have received extensive attention in recent decades as viscoelastic materials have been widely applied in various fields. An overview of relevant modelling approaches is provided in the paper. The review starts with a brief introduction of some basic terminologies and theories that are commonly used to describe the contact behaviour of viscoelastic materials. By building up the complexity of contact problems, including dry contact, lubricated contact, thermoviscoelastic contact and non-linear viscoelastic contact, tentative analytical solutions are first introduced as essential milestones. Afterwards, a series of numerical models for the various types of contact problems with and without surface roughness are presented and discussed. Examples, in which computational tools were employed to assist the analysis of viscoelastic components in different fields, are given as case studies to demonstrate that a comprehensive numerical framework is currently being developed to address complex viscoelastic contact problems that are prevalent in real life.

2.2 Introduction

Viscoelastic materials such as polymers have been extensively applied in many fields owing to their combined advantages, including low friction and weight, slight rubbing noise, inexpensive cost, dimensional stability, biocompatibility and capacity to sustain load in the long run. As a result, viscoelastic contact problems are prevailing, for example, the optimization of mouthfeel for chocolate in the food industry [1], the contact between human skins and scalpels in the biomedical industry [2], the tire-road contact in the automotive engineering industry [3] and the movement of basaltic lavas under high temperature in the geothermal field [4]. When designing and analyzing the engineering or natural products related to viscoelastic materials, the corresponding interfacial mechanics system must be understood, particularly where the roles played by the viscoelastic deformation, viscoelastic lubrication condition, energy dissipation and frictional temperature need to be considered. Taking a knee replacement made of polymers as an example, its long-term service life is determined by the time-dependent response of the prosthesis under different loading and lubrication conditions affected by human activities [5]. More efficient product designs based on the enhanced understanding of relevant viscoelastic contact problems can have prominent impacts in terms of energy savings and quality of life improvement. Currently, the increasingly strict efficiency

requirements and demands to minimize material use, as well as environmental impact, have been driving the development of analytical tools for viscoelastic materials.

Great efforts have been made in the mathematical analysis and numerical simulations of viscoelastic contact problems in the past decades [6, 7]. Due to the structural complexity and strong time-varying constitutive laws governing the behaviour of viscoelastic materials, a closed-form mathematical solution to viscoelastic contact problems can hardly be developed within the framework of classical contact mechanics. Although a series of analytical solutions based on the proposed assumptions have been developed [7, 8], they are subject to many limitations, including the simplified contact geometries, mono-tonic loading history and ideal rheological behaviour of viscoelastic materials (usually characterised by merely one relaxation time). Considering the existence of surface roughness and complicated interaction between contacting bodies in practical contact problems, it seems tougher to find the time-varying solution to viscoelastic contact problems solely on the basis of existing analytical theories. The numerical model presents itself as a suitable alternative tool in this dilemma. Various simulation tools have been advanced to assist the analysis of the products made from viscoelastic materials by predicting the time-dependent contact response in different contact systems [6]. Through applying different enrichment techniques, where the enrichment fields from the solutions to adhesive contact (e.g., [9, 10]), thermal-elastic contact problems (e.g., [7]), etc. are superimposed to basic viscoelastic models, more generalized numerical tools are currently being built up to tackle the complicated viscoelastic problems that are commonly encountered in our daily life.

To date, the extraordinary interest in the modelling of viscoelastic contact problems has been witnessed by hundreds of papers published on the topic. This review paper is intended to provide a systematic overview of these modelling attempts in a way where the complexity of contacting problems is built up. It is structured as follows: some basic terminology and theories of viscoelasticity are described briefly in section 2.3. The modelling approaches to dry contact modelling (under non-lubricated and isothermal conditions) of linear viscoelastic materials are introduced in section 2.4, where the indentation, frictionless or frictional sliding and rolling, rough surface contact and adhesive contact problems are included. The review on the modelling of the lubricated contact of viscoelastic materials (viscoelastic-hydrodynamic lubrication (VEHL)) is given in section 2.5, while sections 2.6 and 2.7 provide a brief review of the modelling of temperature effects and material nonlinearity in viscoelastic contact problems,

respectively. Section 2.8 provides some examples of numerical studies, where various numerical tools were applied to analyse viscoelastic con-tact problems in different fields. In section 2.9, the existing knowledge gaps in viscoelastic modelling are summarized, which remain to be addressed in the future for a more accurate understanding of tribological systems that involve viscoelastic materials.

2.3 Linear and Non-linear Viscoelastic Materials

In this section, some basic terminologies and relationships used to describe the contact behaviour of viscoelastic materials are introduced. It also presents a brief review of some fundamental physical laws including the theory of linear viscoelasticity and the Boltzmann superposition principle. The contents of this section are mainly a summary of materials from several mechanics books [11-18].

2.3.1 Characteristics of Viscoelastic Materials

Materials deform under an applied load. The behaviour of an ideal elastic solid follows the law of Hooke, which indicates that there is a linear relationship between stress and strain. Ideal elastic solids deform instantaneously when a load is applied and recover completely when unloaded. In comparison, an ideal viscous Newtonian fluid has no definite shape and will deform irreversibly and gradually under the action of external forces exhibiting a linear relationship between stress and strain rate. The materials exhibiting the behaviour of both elastic and viscous properties, as shown in Figure 2-1 (a), are the so-called viscoelastic materials. Among them, those that exhibit a linear relationship between stress and strain at any instant are known as linear viscoelastic materials. As illustrated in Figure 2-1 (b), the strain of a linear viscoelastic material is proportional to the stress for any specific time point (e.g., strain under $3\sigma_0$ is three times that under $1\sigma_0$). On the other hand, materials, which exhibit a time- and stress-dependent characteristics and thus hardly follow the linear relation, are named non-linear viscoelastic materials. It is of note that the linear response of a viscoelastic material is not its inherent property since the limit of linearity is determined by the experienced strain. For example, concrete and steel under small deformation can be assumed to exhibit linear viscoelastic behaviour, where infinitesimal strain theory can be applied. However, when considering elastomers and some biological soft tissues that experience large deformations, finite strain theory needs to be employed to characterise their non-linear contact response. Generally, the performance of viscoelastic materials is influenced by the four parameters named load, strain, temperature and time respectively. When investigating viscoelastic materials, two of the parameters are usually set to be constant

to investigate the relation between the other two parameters. Three characteristic phenomena are widely found for viscoelastic materials named creep, stress relaxation and hysteresis.

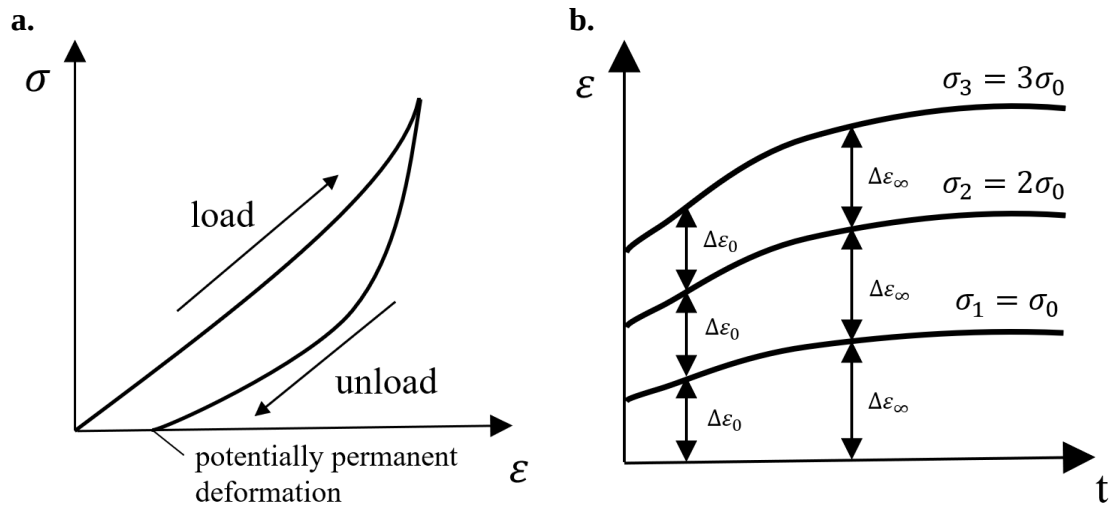


Figure 2-1 (a) Stress-strain curve of viscoelastic materials and (b) Strain-time curve of linear viscoelastic materials with varying stress

To explain them briefly, creep is the phenomenon that the strain keeps increasing with time as shown in Figure 2-2. Under a constant load, the instantaneous elastic strain first appears. It is followed by an ever-increasing strain over time known as creep strain. The creep strain normally would increase with an ever-decreasing strain rate, which eventually leads to a constant-strain steady state. When the surface is unloaded, the elastic recovery is experienced immediately, which is followed by anelastic recovery. A permanent strain might be experienced arising from viscosity. The creep performance of viscoelastic materials reveals their dimensional stability and their capacity to sustain the load in the long run.

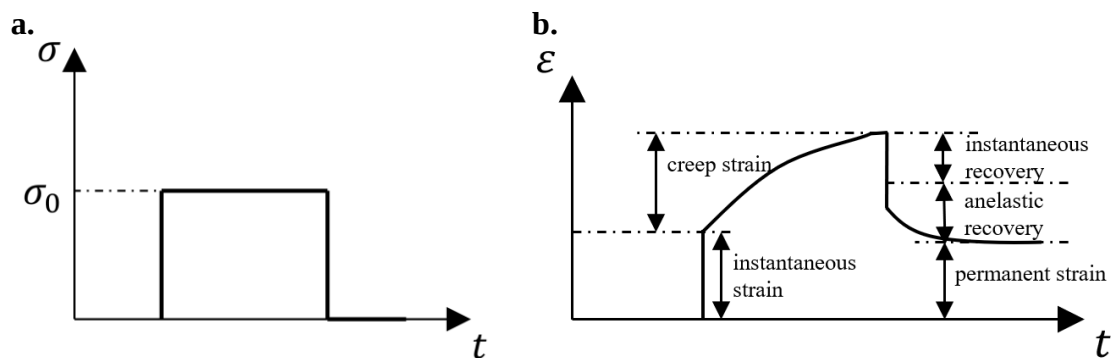


Figure 2-2 Input and output in a creep test for viscoelastic materials: (a) load input and (b) response of strain with time

As to stress relaxation, it is the phenomenon that the internal stress of viscoelastic materials keeps decreasing with time under a constant temperature and strain as shown in

Figure 2-3. Physically, stress relaxation results from the re-arrangement of the viscoelastic material on the molecular or micro-scale.

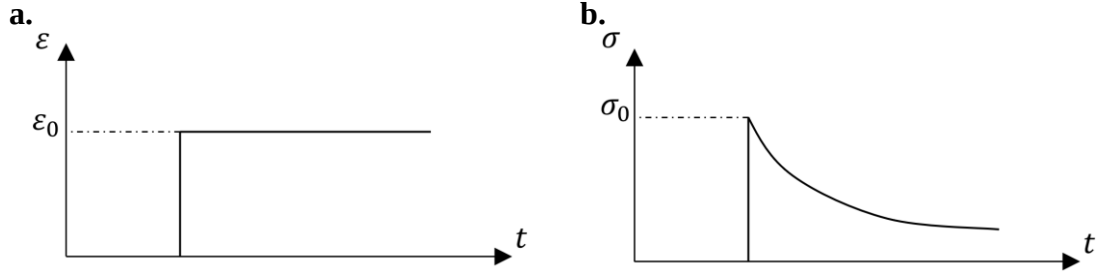


Figure 2-3 Input and output in a stress relaxation test for viscoelastic materials: (a) strain input and (b) response of stress with time

The hysteresis is the lag of the strain (denoted as vector ε) change to the stress (denoted as vector σ) change under a certain temperature and cyclic stress. A typical hysteresis phenomenon is illustrated in Figure 2-4. To explain it briefly, under the sinusoidal stress (solid line) expressed as:

$$\sigma(t) = \hat{\sigma} \sin \omega t, \quad \text{Equation 2-1}$$

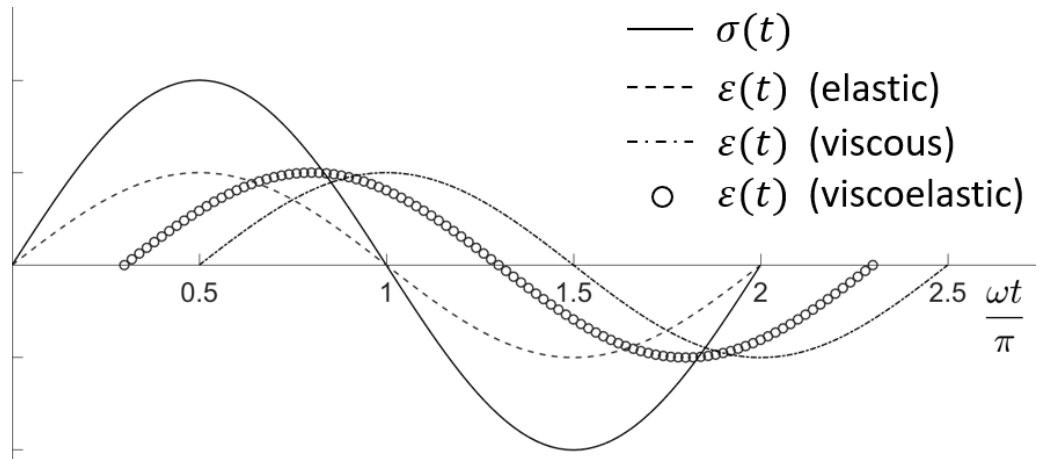


Figure 2-4 The strain response to sinusoidal stress (solid line) of different materials: elastic (dash line), viscous (dash-dot line) and viscoelastic (scatter)

The response of an ideal elastic material (dash line in Figure 2-4) should be a corresponding sinusoidal strain expressed as:

$$\varepsilon(t) = \hat{\varepsilon} \sin \omega t, \quad \text{Equation 2-2}$$

which implies that the work done by the external force in the period of loading is stored in the form of elastic energy and then released completely to return the material to its initial state. In other words, there is no energy loss during this process.

The response of an ideal viscous material (dash-dot line in Figure 2-4) is represented as follows:

$$\varepsilon(t) = \hat{\varepsilon} \sin\left(\omega t - \frac{\pi}{2}\right), \quad \text{Equation 2-3}$$

The phase difference of $\pi/2$ radians indicates that the energy deriving from the external force is completely transformed into heat.

Since viscoelasticity incorporates aspects of both solid behaviour (elastic) and fluid behaviour (viscous), the response of a viscoelastic material (scatter in Figure 2-4) is indicated by the following equation:

$$\varepsilon(t) = \hat{\varepsilon} \sin(\omega t - \xi), \quad \text{Equation 2-4}$$

where ξ is the phase difference between the stress and strain ranging from 0 to $\pi/2$ radians. This demonstrates that part of the energy is lost in every cycle of deformation (stress). As shown in Figure 2-1 (a), the region under the unloading curve is smaller than that under the loading curve in the hysteresis loop. The loss of energy caused by the hysteresis can be determined by the difference between the two regions (the area enclosed by the two curves).

2.3.2 Constitutive Law and Relevant Rheological Models

The theory of small-strain linear viscoelasticity plays an essential role in the modelling of viscoelastic materials. Although this material model framework was derived based on linear elasticity, it can always function as a useful starting point even in instances that require more elaborate treatment (e.g., non-linear viscoelastic contact problems). Following the theory of linear viscoelasticity, for materials exhibiting the linear relation, the response of stress to successive strain is cumulative and vice versa. Two time-varying material properties, namely creep compliance ($\Phi(t)$) and relaxation modulus ($\Psi(t)$), are always used to characterise the constitutive law of linear viscoelastic materials. As the name implies, creep compliance reveals the creep phenomenon of viscoelastic materials. Physically, the property describes the response of viscoelastic strain to a unit stress change. On the other hand, the relaxation modulus reveals the stress relaxation phenomenon. Physically, it describes the viscoelastic stress response to a unit change in strain. These two properties play an essential role when analysing the transient contact response of viscoelastic materials.

To mathematically describe the contact behaviour of linear viscoelastic materials, the Boltzmann superposition principle is commonly applied, where the response to successful excitations can be the sum of the response generated by applying each excitation individually. To determine stress $\sigma_1(t)$ at time t under the action of a certain strain ε_1

that is applied at the time t_1' before t , it can be expressed based on the definition of the relaxation modulus:

$$\sigma_1(t) = \Psi(t - t_1')\varepsilon_1 H(t - t_1'), \quad \text{Equation 2-5}$$

where $H(t)$ is the Heaviside step function.

Identically, stress $\sigma_2(t)$ at the same time t under the action of a strain ε_2 that is applied at another time t_2' in advance of t can be determined as followed:

$$\sigma_2(t) = \Psi(t - t_2')\varepsilon_2 H(t - t_2') \quad \text{Equation 2-6}$$

Following the analogous way, the response of the strain to a sequence of strain increments, which are arbitrary but can be considered as a continuous distribution, can be determined:

$$\sigma(t) = \int_0^t \Psi(t - t') \frac{d\varepsilon(t')}{dt'} dt' \quad \text{Equation 2-7}$$

The response of strain to stress can be obtained by a similar derivation by switching the input and output:

$$\varepsilon(t) = \int_0^t \Phi(t - t') \frac{d\sigma(t')}{dt'} dt' \quad \text{Equation 2-8}$$

Different from the case where the compliance and elastic modulus are reciprocal of each other in ideal elastic contact, the relationship between the creep compliance and relaxation modulus of linear viscoelastic materials in the time domain is expressed in the following form:

$$\int_0^t \Phi(t - t')\Psi(t)dt' = t \quad \text{Equation 2-9}$$

There exists the following essential mathematical relationship between these two material properties in the Laplace transform domain:

$$\bar{\Phi}(s) \bar{\Psi}(s) = \frac{1}{s^2}, \quad \text{Equation 2-10}$$

where hat ‘-’ denotes Laplace functional transformation and s is a complex number frequency parameter in the Laplace transform domain ($s = a + jb$).

To characterize the time-dependent material property of linear viscoelastic materials, rheological models, which are established with linear springs (representing perfectly elastic solid as the stress is proportional to the strain) and dashpots (representing ideal Newtonian fluid as the stress is proportional to the rate of strain), are built. If the spring

and dashpot are arranged in series as presented in Figure 2-5 (a), such a rheological model is called the Maxwell model. A Kelvin-Voigt model is established when the two elements are arranged in parallel as illustrated in Figure 2-5 (b). According to theoretical analysis [13], a Maxwell model can appropriately demonstrate the stress relaxation phenomenon of linear viscoelastic materials. However, it fails to account for their creep and recovery characteristics. In contrast, the Kelvin-Voigt model behaves in the opposite way. Moreover, it does not exhibit any instantaneous elastic response. A detailed description of these two models can be found in the work of Popov [13]. Since these two-element models can only provide qualitative descriptions, through coupling several numbers of Maxwell or Kelvin-Voigt units, more sophisticated models, such as the Maxwell-Wiechert model as shown in Figure 2-5 (c), are commonly employed to characterize real-life viscoelastic materials with more than one relaxation time. The number of elements required in a generalized Wiechert model depends on the naturally occurring spectrum of relaxation times of materials. The relaxation time is usually denoted by τ and determined by parameters of the rheological models in the following way:

$$\tau_i = \frac{\eta_i}{G_i}, \quad \text{Equation 2-11}$$

where η_i denotes the viscosity of the i -th dashpot and G_i denotes the modulus of the i -th spring.

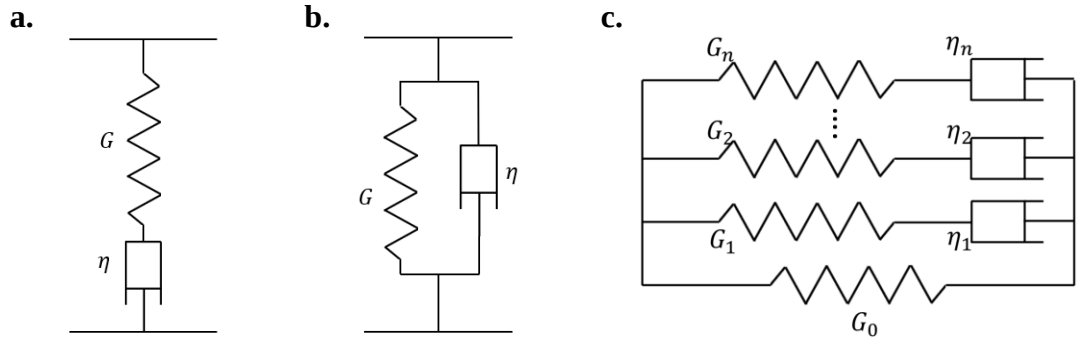


Figure 2-5 Rheological models describing the behaviour of linear viscoelastic materials: (a) Maxwell model, (b) Kelvin-Voigt model and (c) Maxwell-Wiechert model

By using the Maxwell-Wiechert model, the relaxation modulus of any linear viscoelastic material can be characterised by fitting the experimental data from standard relaxation tests to the following equation (Prony series):

$$\Psi(t) = G_0 + \sum_{i=1}^n G_i \exp\left(-\frac{t}{\tau_i}\right) \quad \text{Equation 2-12}$$

Notably, the spectrum of relaxation times of linear viscoelastic materials could be characterized by introducing as many exponential terms as required to obtain the desired accuracy of curve fitting. Once the relaxation modulus function is available, the corresponding creep compliance function can be easily determined by applying their mathematical relationship in the Laplace domain (Equation 2-12). A typical example can be found in the work of Chen, et al. [8]. By fitting a two-term Prony series to the relaxation curve from the standard relaxation test by Kumar and Narasimhan [19], the relaxation modulus and creep compliance functions of a thermoplastic polymer known as polymethyl methacrylate (PMMA) were determined. When a Maxwell-Wiechert model only contains one relaxation time, such a three-element model is known as the Zener model or standard linear solid (SLS) model. SLS models are frequently employed in numerical studies about viscoelastic contact problems owing to their capacity to capture the creep as well as stress relaxation phenomena.

The way to correctly reproduce the viscoelastic response of a half-space with the available rheological models can be found in the work of Bugnicourt et al. [20, 21]. They argued that the Maxwell–Wiechert model (known as the generalized Zener model in their study) should be used for rubber to capture its contact behaviour over a range of frequencies. It is also of note that the choice of models should be expressed by a function of the wavelength at small scales.

With the rheological models presented above, the complex modulus of viscoelastic materials E^* can be determined by applying the Fourier transform to the discrete form of the creep function with respect to the time variable t :

$$E^*(\omega) = [i\omega\tilde{\Phi}(\omega)]^{-1}, \quad \text{Equation 2-13}$$

where hat ‘ $\sim$ ’ denotes Fourier functional transformation and ω is the time-related frequency. The viscoelastic complex modulus is commonly employed when it comes to the steady-state contact analysis of viscoelastic materials.

The nonlinear viscoelastic response is usually encountered when a viscoelastic surface experiences large deformation or the material exhibits specific creep and relaxation functions, which are not only dependent on time but also on the experienced stress or strain. Compared with literature regarding linear viscoelastic response, relatively limited work sheds light on the characterization of non-linear viscoelastic behaviours as the Boltzmann superposition principle is unable to describe the creep and relaxation phenomena. Without the aim of providing an exhausting literature review on this topic

here, credit to the contributors to the development of a generalized constitutive equation for nonlinear viscoelastic materials can be given to Green [22], Rivlin [23], Spencer [24], Pipkin and Rogers [25], who adopted multiple integral representations by using multivariable relaxation functions as kernels, and Leaderman [26], Brueeller [27, 28] and Schapery [29], who used single integral representation based on the nonlinearization of stress and strain measurements.

2.4 Dry Contact Modelling of Viscoelastic Materials

The review of the methods to simulate viscoelastic contact problems starts with dry contact problems, where lubricants are not involved, temperature effects are neglected and the material behaviour is assumed to be linear viscoelastic. The indentation problem, which tends to be the simplest among contact problems, is first discussed. Afterwards, the numerical solutions to tangential contact problems including the quasi-static partial slip contact (e.g., Figure 2-6 (a)), and dynamic sliding and rolling contact problems (e.g., Figure 2-6 (b)) are presented. Apart from the simplified single-asperity contact problems, models that simulate multi-asperity problems, which closely resemble real-life contact problems, are discussed. Effects of surface roughness on the viscoelastic contact solutions are highlighted in special. The models investigating the effects of surface adhesion on viscoelastic contact are also presented in this section.

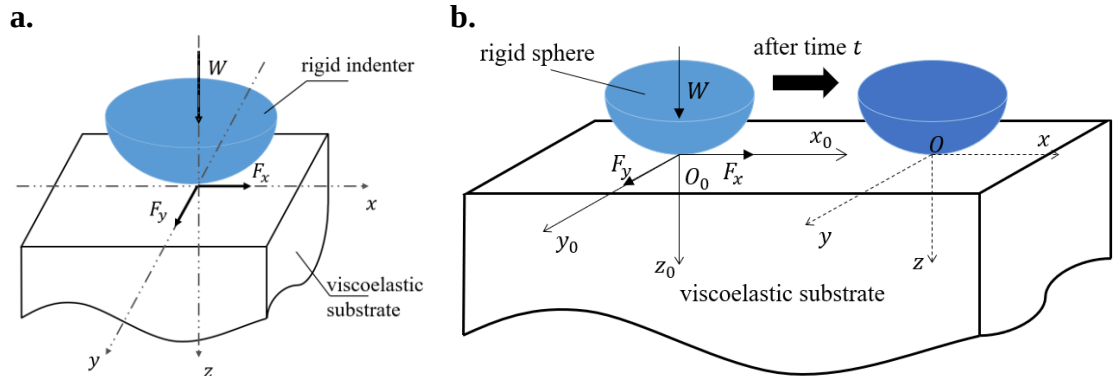


Figure 2-6 Typical geometries of dry viscoelastic contact problems: (a) quasi-static contact and (b) dynamic sliding contact

2.4.1 Indentation

By applying the Laplace transform to reduce viscoelastic problems to elastic ones, the problem of the time-dependent constitutive laws of viscoelastic materials was found avoidable mathematically by Lee [30]. The application of this convenient approach was later extended by Radok [31] to establish the independence of the Laplace method from the elastic problem. Based on the assumption that the contact area increases monotonically with time without surface adhesion (pressure $p < 0$), solutions to

spherical indentation problems of linear viscoelastic materials were developed by Lee and Radok [32]. An essential concept known as the ‘elastic-viscoelastic correspondence principle’ later, which was proposed by Alfrey [33] and generalized by Tsien [34], was first implemented in the analysis of Lee and Radok [30-32]. Instead of deducing solutions from scratch, small-strain linearly viscoelastic contact problems can be solved by manipulating the well-established theory of elastic contact through the elastic-viscoelastic correspondence principle (e.g., replacing the elastic compliance with the viscoelastic creep compliance in the Hertzian solution). Owing to this advantage, the principle has now become the foundation for many analytical and numerical models of viscoelastic contact problems recently developed. A detailed description of the principle can be found in the mechanics book by Christensen [18]. Limitations of the tentative analytical approach by Lee and Radok [32], including ideal contact geometry and monotony of contact radius, were later partially released by different researchers. Among them, Hunter [35] and Graham [36] obtained solutions to a similar problem in the case of monotonically increasing contact radius or when the radius attains a single maximum value. Yang [37] improved the viscoelastic contact solution by taking arbitrary quadratic contact geometry into account. The limitation regarding the monotony of contact radius was overcome by Ting [38, 39], who proposed a viscoelastic solution with further increased generality by describing contact radii with arbitrary functions of time and multiple connected contact regions.

The contact between a viscoelastic material and a pyramidal or spherical indenter under a triangle or trapezoidal loading history was investigated by Oyen [40] both experimentally and mathematically. Based on the solution of Ting [38], a theoretical model solving the contact problem between an axisymmetric indenter and a viscoelastic half-space was proposed by Greenwood [41]. For the contact problem where the whole indenter surface is in contact with the viscoelastic half-space, a theoretical solution was proposed by Fu [42], who related the indentation load to the penetration depth. By applying the correspondence principle, Yakovenko and Goryacheva [43] developed the analytical solution to periodic contact problems of multiple identical spherical indenters against a viscoelastic half-space, where the mutual influence of indenters was investigated. Recently, based on the solution of Lee and Radok [32], the cyclic repetitive indentation problem of a rigid sphere against a viscoelastic half-space and the involved viscoelastic dissipation were investigated by Papangelo and Ciavarella [44]. Apart from these half-space contacts, Argatov and Mishuris developed analytical solutions to

elliptical [45] and rebound spherical [46] and cylindrical [47] indentation contact problems between thin compressible and incompressible layers of arbitrary viscoelastic materials. However, these solutions are only applicable to the indentation problem with an arbitrary monotonic loading. Using the Hankel transform, a more general solution to indentation problems for a viscoelastic layer on an elastic half-space was developed by Chen, et al. [48].

As mentioned before, the application of these partially or fully analytical solutions is still limited considering the existence of surface roughness, arbitrary loading history and complicated rheological behaviour of viscoelastic materials (usually more than one relaxation time) in practical contact problems. The mathematical complexity of these solutions, which are frequently encountered during applications, also discourages their practical usage. Based on the matrix inversion method (MIM), the indentation problem of a rigid body with an arbitrary shape against a viscoelastic half-space was solved by Kalker [49], who later adopted an alternative scheme named the two-scale iterative method (TIM) to improve the computational efficiency of the BEM-based model [50]. By making use of the elastic-viscoelastic correspondence principle, the displacement response of viscoelastic solids under any arbitrary distribution of surface tractions was derived by Chen, et al. [8] based on the Boussinesq integral for elastic surfaces [51]. A robust model for solving the point contact problem between a rigid indenter and a homogeneous viscoelastic half-space was then developed by Chen using the Fast Fourier Transform (FFT)-based BEM model (also known as the semi-analytical method (SAM) in some literature), where the technique named the discrete convolution fast Fourier transform (DC-FFT) [52] was implemented. Although it was initially developed to simulate the non-conformal and normal contact of polymer-based materials characterised by a wide spectrum of relaxation times and complicated surface topographies, the model was later extended to investigate more complicated contact problems by different researchers. Among them, the effects of inhomogeneities in viscoelastic indentation problems were investigated by Koumi, et al. [53], where isotropic or anisotropic elastic inhomogeneity of any orientation is embedded into a viscoelastic half-space. They reported that the effects of the inhomogeneity, such as the decline of the normal and shear stresses at the interface between the elastic inhomogeneity and viscoelastic matrix, become more significant with time. A similar model for the indentation problem of viscoelastic composite materials was developed by Liu, et al. [54], who focused on the effects of the location of the inhomogeneity in relative to the indenter and its volume

fraction on the modulus of the composite material. Based on the model of Chen [8], the line contact configuration in viscoelastic indentation problems was simulated by Spinu [55].

Recently, the indentation problem for a viscoelastic half-space was revisited by Zhao, et al. [56], who used the derivative of the creep compliance function instead of the pressure to determine the surface deformation in the formulation of surface deformation caused by contact tractions. They claimed that the new equation allows more efficient computation while providing valid solutions simultaneously. A more complicated indentation problem, where anisotropic viscoelastic solids are involved, was simulated by Nguyen and Hwu [57, 58] using BEM-based models based on the correspondence principle and time-stepping method. In addition to the BEM, by making use of the discrete element method (DEM), a normal contact model for viscoelastic particles was developed by Olsson and Jelagin [59]. However, their model can only provide approximate solutions when the contacting viscoelastic surfaces are unloaded as it is based on the analytical solution of Lee and Radok [8, 35].

It is of note that two different shapes of pressure distribution, including the Hertzian-type contact pressure (e.g., Figure 2-7 (a) [60]) and pressure with spikes on the contacting edges (e.g., Figure 2-7 (b) [60]), were reported extensively for the spherical indentation problem of viscoelastic bodies [8, 32, 56]. To explain such a phenomenon, a numerical study was recently conducted by Wang, et al. [60] using a SAM-based model. The effect of the material parameter (e.g., the ratio of retardation time to relaxation time) in an SLS model on the shape of pressure distribution was reported in their work. Another argument was proposed by Koumi, et al. [61], who argued that the shape of contact pressure is determined by the contact configuration. According to them, a Hertzian-type pressure distribution will be obtained for a viscoelastic sphere indenting a rigid plane. The pressure spikes will only be obtained for a rigid sphere indenting a viscoelastic half-space.

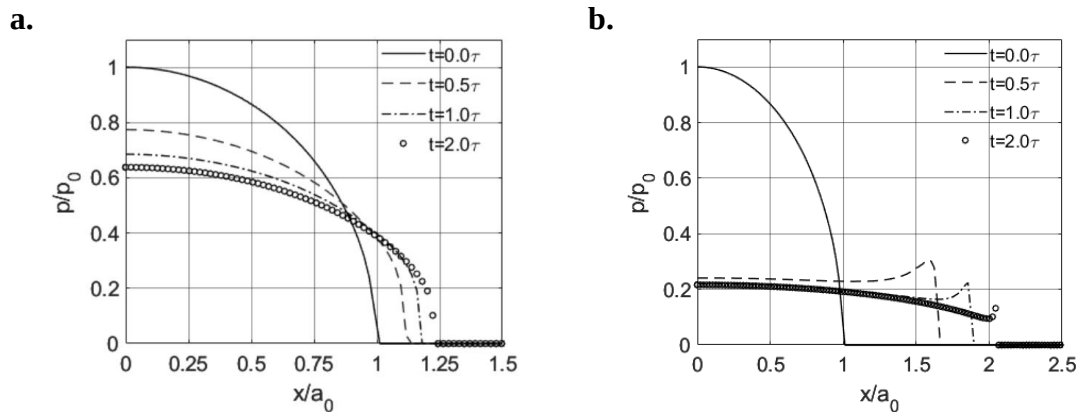


Figure 2-7 Different shapes of pressure profiles for spherical indentation problems of viscoelastic bodies (modified from the work of Wang, et al. [60]): (a) Hertzian-type pressure and (b) pressure profile with spikes on the contacting edges. a_0 and p_0 here represent the Hertz solution of contact radius and peak pressure respectively using the instantaneous modulus of the viscoelastic material

2.4.2 Frictionless or Frictional Sliding/Rolling Contact

Relative motion between contacting surfaces, including sliding and rolling, is commonly encountered in practice such as the type-road contact. Concerning the analysis of relative motion between viscoelastic surfaces, Hunter [62] first studied the two-dimensional (2D) frictionless rolling of a rigid cylinder over a viscoelastic half-space. At a low or medium speed, the pressure distribution in viscoelastic rolling contact was found to be significantly different from that in an elastic contact, where an asymmetric pressure profile can be observed exhibiting a spike at the leading edge in the moving direction. This consequently leads to asymmetric surface formation. By extending the solution to a similar problem [63], the line contact between two viscoelastic cylinders with different radii and mechanical responses was analyzed by Morland [64]. The steady-state analysis of Hunter [62] was later extended by Panek and Kaller [65] to three-dimensional (3D) problems by adopting an approximation on the basis of elastic line integral theory. Apart from these plane strain problems, Aleksandrov, et al. [66] simulated the frictionless contact between a smooth sphere sliding on a viscoelastic half-space. The application of these approaches is limited to ideal viscoelastic materials characterized by one relaxation time. Based on the assumption that the viscoelastic material stiffens when the rolling speed increases (e.g., higher frequency) while the pressure distribution remains of a Hertzian type, a novel analytical theory was proposed by Persson [67] to solve the rolling contact of a rigid cylinder or a sphere over a flat viscoelastic substrate. The rolling friction coefficient can be estimated reasonably by the theory of Persson [67]. However, the information on pressure distribution especially its non-Hertzian profile in a force-driven problem is not available within the framework of this theory.

The FEM was frequently employed in early attempts [68-73] to simulate viscoelastic rolling and sliding contact problems based on Lagrangian formulations. However, the computational complexity, which is related to the extremely fine mesh of both the interface and bulk requested to capture the extremely irregular contact geometries of rough bodies in real life, impedes the further application of FEM-based models. Regarding the models developed based on the alternative method (e.g., BEM), Carbone and Putignano [74] developed a model to generate steady-state solutions to sliding or rolling contact problems, where the involved viscoelastic material can either exist in the

form of a half-space [75] or a layer [76-78] by adjusting the thickness of the contacting body in their novel equation. The viscoelasticity was found to induce shrinkage of the contact area when the sliding speed increases [74]. The effects of the surface layer in sliding contact of viscoelastic materials were investigated by Torskaya and Stepanov [79], where the material combinations including a viscoelastic layer bonded with a rigid half-space and a viscoelastic half-space covered by a rigid layer were simulated. For the combination of a viscoelastic layer on a rigid substrate, it was found that the hysteretic losses increase with the layer thickness while the opposite effect was obtained for the combination of a rigid layer on a viscoelastic substrate. They also investigated the internal stress states for the sliding contact problem between a viscoelastic half-space and a smooth indenter [80].

Based on a novel formulation of the surface deformation, the frictionless sliding and rolling contact was efficiently simulated by Zhao, et al. [56]. Regarding the transient viscoelastic analysis, the frictionless rolling contact of a rigid smooth indenter against a flat viscoelastic half-space was simulated by Koumi, et al. [61], where the material inhomogeneity was considered. Based on Green's function molecular dynamics (GFMD), Dokum and Nicola [81] developed a model to provide transient as well as steady-state solutions to frictionless indentation and rolling contact problems of a rigid cylinder against a viscoelastic half-plane. Recently, Wallace, et al. [82] proposed a model which provides transient analysis of frictionless rolling and sliding viscoelastic problems, in which the involved viscoelastic layer and substrate can exhibit distinct properties. Meanwhile, the effects of imperfect bonding between layer and substrate on the transient as well as steady-state frictionless viscoelastic sliding contact problems were recently investigated by Zhang, et al. [83]. They reported that the subsurface stresses become lower when the contact is subjected to an imperfect bonding interface.

It is of note that there exist different friction laws in the theory of contact mechanics. The word 'frictionless' in the above contact analysis means that no explicit friction at the interface between the solids (e.g., Coulomb friction) was taken into account. Therefore, tangential contact problems (e.g., tangential shear stresses due to Coulomb friction) were not investigated since the surface can be in the sliding state without any input in the tangential direction. However, the friction derived from the viscoelastic loss was always considered, which is quantified as the product of the apparent friction coefficient and normal load in the literature. In the presence of dry contact friction, surfaces might experience the partial slip state before the frictional sliding motion when the input lateral

loads shown in Figure 2-6 (b) are not sufficient to induce gross sliding (e.g., shear tractions are lower than the local static friction). To briefly explain the two concepts, gross sliding is a dynamic process in which the contacting bodies continuously move relative to each other over time. On the other hand, the partial slip contact is a quasi-static process where there is no actual global relative sliding between surfaces. In this case, the contacting area is separated into the stick region, in which no relative motion occurs, and the slip region, in which local relative movement does happen.

Based on SAM, the conformal partial slip contact between a rigid pin and a viscoelastic plate was simulated by Dayalan and Sundaram [84], where fretting contact analysis was provided. A FEM-based model was developed by Bonari and Paggi [85] to investigate the periodic partial slip contact between a sinusoidal indenter and a viscoelastic layer with finite thickness. By using the time-stepping method, the stick-slip solutions to the contact of anisotropic viscoelastic solids were proposed by Nguyen and Hwu [86]. Regarding the dry contact friction in sliding or rolling contact, it was analyzed by Goryacheva [87] in a cylindrical rolling problem. However, the pressure solution was not affected by the dry contact friction (shear traction) as the two contacting viscoelastic bodies were assumed to exhibit the same mechanical properties (thus no mutual effects between contact tractions) in the study. Later, the Goodman approximation [88], which neglects the effect of shear tractions on normal pressure but considers that of pressure on shear tractions, was adopted by Goryacheva and Sadeghi [89] to analyse the role of Coulomb friction in the cylindrical rolling or sliding problem of an elastic substrate covered by a viscoelastic layer. Excluding the partial slip period, the frictional sliding of a sphere against a viscoelastic half-space was simulated by Goryacheva, et al. [90], where the effects of dry friction on sliding solutions were investigated [91]. Their results show that the existence of shear stress (Coulomb friction) would increase the peak pressure and shift the contacting region in the direction of the sliding motion. In addition to the half-space contact, Goryacheva and Miftakhova [92] analysed the effect of viscoelastic layers in the frictional rolling contact. The range of the relative slippage, which occurs in frictional rolling contact, was found to be significantly affected by the viscoelastic layer.

Beyond the unidirectional sliding or rolling contact, there are several numerical attempts to simulate reciprocating contacts. A BEM-based model was recently developed by Putignano, Carbone and Dini [93-95] to investigate the oscillating contact between a sinusoidal indenter and a viscoelastic half-space. In the meantime, a novel boundary element methodology was developed by Santeramo, et al. [96] to simulate conformal

contact problems of viscoelastic materials. Although the study only investigated frictionless and steady-state contact of 2D circular surfaces, it filled in the gap about viscoelastic circular contact solutions for conformal geometries based on BEM.

2.4.3 Viscoelastic Rough Contact

The above modelling work focuses mainly on single-asperity contact problems. However, it is known that surfaces encountered in real life are usually irregular showing complicated topography, and many of interest often exhibit a self-affine fractal feature [97, 98]. The power-law relation that holds in a limited wavevector region ($q_0 < q < q_1$) (illustrated in Figure 2-8) for the power spectral density of a typical fractal rough surface can be expressed with the following equation [97-99]:

$$C(q) = C(q_0) \times \begin{cases} 1 & q_L < q \leq q_0 \\ (q/q_0)^{-2(1+H)} & q_0 < q \leq q_1 \\ 0 & \text{else} \end{cases} \quad \text{Equation 2-14}$$

where H is the Hurst exponent linked to the fractal dimension D_f ($H = 3 - D_f$) and q_L , q_0 , and q_1 are known as the smallest possible wavevector, short-distance roll-off wavevector and long-distance cut-off wavevector respectively. The fractal dimension D_f can be determined by the slope of the $\log C - \log q$ plot for the part where $q > q_0$ as shown in Figure 2-8. The smallest possible wavevector q_L can be determined from the lateral size L of the rough surface ($q_L = 2\pi/L$) and it can be related to the long-distance cut-off wavevector with the number of scales N ($q_1 = Nq_L$).

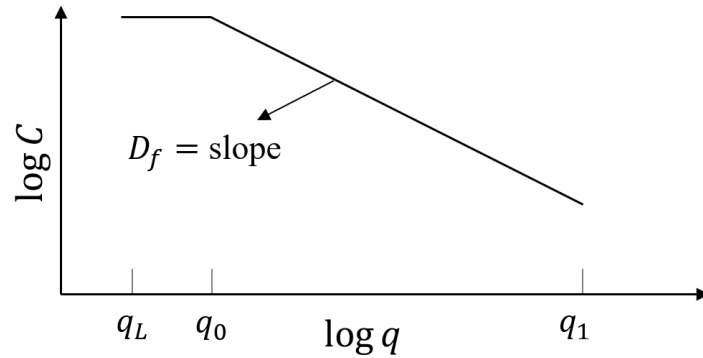


Figure 2-8 The PSD of a typical fractal rough surface which is self-affine for $q_0 < q < q_1$

The requirement to resolve interactions between surface asperities exacerbates the complexity of viscoelastic contact problems. The solution of the real contact area for a viscoelastic solid squeezed into a fractal rough surface was proposed by Persson, et al. [100] using a contact-mechanics-based theory [101]. Based on his theory of contact mechanics and rubber friction [101], the sliding contact of viscoelastic surfaces with

anisotropic surface roughness was analyzed by Carbone, et al. [102] and Afferrante, et al. [103]. The contribution of the fractal roughness to the rubber friction was analyzed by Scaraggi and Persson [101, 104]. Their theories were later applied by Scaraggi and Comingio [105], who developed a model underpinned by a Fourier-based residual molecular dynamics formulation to investigate the rough contact of viscoelastic graded materials. The role of small-scale wavelengths, which is related to the largest roughness frequency q_1 , on rubber friction was highlighted in their study. The sliding contact between a rigid sinusoid against a viscoelastic half-space was solved by Menga, et al. [75] analytically, where the role of viscoelastic interaction between asperities in periodic contact (e.g., viscoelastic stiffening and periodicity of contact) was investigated. The analytical solution to indentation problems of nominally flat viscoelastic surfaces was proposed by Papangelo and Ciavarella [106] by extending the contact theories of Persson [101] using the elastic-viscoelastic correspondence principle. The solution was then employed to analyze the viscoelastic bulk dissipation under a full cycle of harmonic loading and unloading. According to their numerical study [106], the maximum energy dissipation can be achieved when the loading period is close to the relaxation time of the viscoelastic materials and it can be increased by increasing the mean normal pressure and the ratio of retardation time to the relaxation time of the material.

By using an adaptive nonuniform mesh to address the huge computational complexity [107], Carbone and Putignano conducted a numerical study to simulate the sliding contact of rough viscoelastic materials in different forms (half-space [108] and layer [76, 78]) based on their BEM-based model [74]. The roughness power spectrum cut-off, which indicates the number of scales of rough surfaces N , was found by Putignano [109] to have significant effects on the viscoelastic friction such that the friction tends to increase with the number of scales. The anisotropy in terms of contact areas and displacement distribution was confirmed to be caused by the viscoelasticity in rough contact according to their later numerical investigation [110]. In the presence of Coulomb friction, the sliding contact of viscoelastic thin layers on a rigid substrate was simulated by Menga, et al. [111], where the coupling effects between the normal and tangential tractions on the enhanced viscoelastic friction and energy dissipation were highlighted. The transient frictionless contact of an incompressible viscoelastic semi-infinite body rolling on a rough rigid surface was analyzed by Bugnicourt, et al. [21] numerically. The apparent friction coefficient derived from dissipative losses inside the viscoelastic bulk in tyre-road contact was evaluated in their study. A numerical study was conducted by Liu, et al.

[54] to study the effect of microstructure and surface roughness on the indentation creep of viscoelastic composite materials using a SAM-based model. It was found that although the modulus of the composite materials increases with the surface roughness, the indentation creep is barely influenced by the surface roughness. Recently, a new BEM-based model was developed by Putignano and Carbone [112] to simulate the one-dimensional indentation problem between a rigid rough surface and a viscoelastic half-plane. The effects of surface roughness parameters including root mean square roughness, root mean square slope and Hurst exponent on the viscoelastic dissipation were revealed in the indentation problem. The root mean square roughness was found to play a leading role, which suggests that it is sufficient to estimate the viscoelastic dissipation by a simple measure of this roughness parameter. By using a BEM-based model to simulate the configuration of a rigid finite-size rough punch against a viscoelastic rough surface, the peculiar role played by surface roughness in viscoelastic reciprocating contact solutions was investigated by Putignano and Carbone [95].

2.4.4 Viscoelastic-adhesive Contact

A common limitation among the models presented above is the neglecting of adhesive force. Practically, such interaction force always exists between two contacting bodies. It decreases appreciably with the increasing distance between the surfaces. Although the adhesive force is usually relatively weak, it plays a significant role as long as one of the following conditions fits [13]:

1. The contacting surface is extremely smooth;
2. One of the contacting components is much softer or compliant;
3. The surface contact is considered on a micro-scale.

The adhesive force in contact mechanics can be explained by the Lennard-Jones potential [13, 113, 114], which is a model used conventionally to describe simple interactions between two particles (atoms). The model describes the inter-atomic van der Waals forces while neglecting the electrostatic and capillary force, the effects of which on adhesion can be found in the work of Israelachvili [114]. Based on physics, when the distance between the electrically neutral atoms is less than or larger than the size of the atoms (δ), the mutual interaction between them occurs under the influence of the van der Waals forces. As illustrated in Figure 2-9 (a), when the distance between two neutral atoms is over the equilibrium distance (r_0), the adhesive (attractive) force starts to perform. But once the two atoms are moving close to each other (smaller than the equilibrium distance), repulsive force tends to be generated [13, 113, 114].

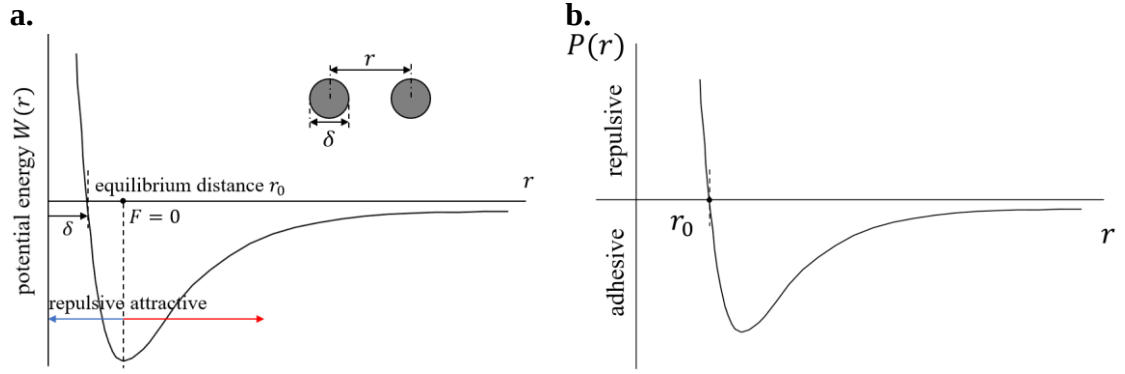


Figure 2-9 (a) Lennard-Jones potential representation, r_0 here represents the equilibrium distance where the surface force is zero, and δ here represents the size of the interacting particles (atoms) and (b) Force (P)-separation (r) for the parallel surface representation of the Lennard-Jones potential.

It is of note that the Lennard-Jones is limited in the reflection of interatomic potential in reality. Without the aim of giving an exhausting discussion on this topic in this review, readers can refer to the latest work by Muser, et al. [115] for a detailed description of interatomic potentials, where different functional forms for potentials (e.g., Buckingham potential and Axilrod-Teller-Muto potential) and potential classes (e.g., two-body potentials and multi-body potentials) were introduced from a physical perspective.

Given that the maximum adhesive force has a fundamental limit as shown in Figure 2-9 (b), the effects of the adhesion in contact are often neglected in contact analysis. It would not highly affect the modelling result when the contact pressure induced by the load applied to the surface is significantly higher than the maximum adhesive force. Hence, it is a common practice to ignore the adhesion when it comes to the contact problems of metallic materials in which occasion the effect of adhesion is usually insignificant. However, since viscoelastic materials usually have low contact compliance and exhibit the soft feature corresponding to the second condition shown above, they would have considerable deformation when having contact with hard materials. This would induce a relatively large contact area, thus narrowing the gap between the contact pressure and the adhesive force. As a result, the reliability of the simulation results derived from the viscoelastic contact models discussed above can be undermined due to the neglect of adhesion.

The development of the contact model integrating the adhesion and viscoelasticity is still in its early stage, where the available solutions are usually developed by applying an enrichment technique where the enrichment fields from adhesive contact solutions (such as the Johnson-Kendall-Roberts (JKR) theory [9] and Derjaguin-Muller-Toporov (DMT)

theory [10]) are superimposed to the ordinary elastic and viscoelastic problem solutions. Both JKR and DMT models studied the adhesive contact between a smooth sphere and a flat body but with different assumptions and methods, which leads to different results. It was suggested by Tabor [116] that both solutions are correct depending on the contact type, which could be characterised by the Tabor parameter expressed as:

$$\mu = \left(\frac{R\gamma^2}{E^{*2}r_0^3} \right)^{1/3}, \quad \text{Equation 2-15}$$

where μ here represents the Tabor parameter, R and E^* are the combined radius and composite modulus of contacting surfaces respectively, and γ is the work of adhesion. A more common summary of the two models states that the JKR model is effective for solving larger and more compliant contact problems while the DMT model is suitable to handle stiffer and smaller contact problems. It is mentionable that neither of the two models can provide a comprehensive understanding of the adhesion in contact mechanics although they could offer a decent approximate solution to a certain range of contact problems.

Classical theories of fracture mechanics also played an essential role in the early studies of viscoelastic-adhesive contact [117-119]. An attempt was conducted by Hui, et al. [120], where the JKR model and theory of viscoelastic fracture mechanics were implemented to simulate the viscoelastic-adhesive contact of spheres. By integrating the solution of Ting [38] and the restricted self-consistent adhesive model by Tabor [116], Haiat, et al. [121] developed a solution to the adhesive contact problems of viscoelastic spheres. A FEM-based model was developed by Lin and Hui [122] to simulate the spherical contact, where the adhesion was accounted for by the Dugdale-Barenblatt model [123, 124]. Based on the JKR model, an analytical solution to the adhesive contact of elastomer was proposed by Barthel and Fretigny [125]. The role played by viscoelasticity in the effective adhesion energy was quantified as a function of the contact edge velocity in the approximate analytical expressions. Yu, et al. [126] developed a model for the contact of viscoelastic materials with the adhesive interaction, where the viscoelastic aspect of the model is based on the model of Chen [8] for the indentation contact of viscoelastic materials while the adhesive aspect of the model is based on the theory of fracture mechanics by Schapery [127] and the theory of JKR [9], DMT [10] and Maugis [128]. By using the Maugis-Dugdale model [128] to characterise the dependence of the adhesive force on the gap between surfaces, a model was developed by Goryacheva and

Makhovskaya [129] to investigate the adhesion effect in the sliding contact of a spherical indenter [130] or a periodic surface [131] against a viscoelastic substrate.

A fully deterministic model for the normal contact of viscoelastic-adhesive contact was recently developed by Afferrante and Violano [132] using FEM. Inspired by Derjaguin approximation [133], the adhesive interaction between surfaces was characterised by non-linear springs following Lennard-Jones traction-gap law in their model. The pull-off load in smooth and adhesive viscoelastic contact problems was analysed by Afferrante and Violano [132] and they reported that the viscous dissipation is confined to the contact edges when short-ranged adhesion is considered. Together with the study of Afferrante and Violano [132], Muser and Persson [134] conducted a numerical work that supported the model of Persson and Brener [135], which was developed based on viscoelastic crack propagation. In their simulation of the adhesive contact between a rigid cylinder and a viscoelastic half-space, a maximum in the work of adhesion was found at intermediate pull-off velocities. Besides, they reported that realistic dissipation can hardly be achieved without correct short-range adhesion. The interplay between adhesion and viscoelasticity in steady sliding contact was studied by Carbone, et al. [136] numerically. Compared with the adhesiveless viscoelastic contact, the curve of hysteretic friction and sliding velocity is significantly changed presenting a more pronounced friction peak. By simulating the viscoelastic adhesive contact between a sphere and a 2D nano-wavy surface, Yang, et al. [137] reported that the effects of viscoelasticity can be neglected when the approach speed is high. However, viscoelasticity would bring significant change under low approach speed such as the increases of the maximum contact area and the pull-off load when compared with the elastic case. Besides, the pull-off force was found to change monotonically with the increasing wavy parameter while the maximum contact area responds in a nonmonotonically way. The effects of viscoelasticity on the adhesive hysteresis, was investigated in the latest numerical work of Perez-Rafols, et al. [138]. Apart from the finding that the viscoelasticity results in an increased effective work of adhesion due to the stiffening of contact, they argued that the contributions of surface roughness and viscoelasticity to the hysteresis in soft adhesive contacts are linearly additive in the regime where viscoelasticity is confined to the edges of the wavy contact. This novel opinion remains up to further investigations as they considered a single wavelength roughness instead of a realistic rough surface.

2.5 Lubricated Contact of Viscoelastic Bodies

As lubricants are frequently applied in practice for the control of friction and wear, appropriate lubrication analysis plays a vital role as it can not only evaluate the lubrication regimes but also helps to determine the design and manufacturing parameters of components working in a lubricated environment.

Regarding the effect of viscoelasticity on the lubrication performance, it was reported by Yoo [139] and Kaneko, et al. [140] that the solution, in which the viscoelastic effect is accounted for in the squeeze film lubrication model, can be significantly different from that under elastohydrodynamic lubrication (EHL) environment. Compared with the elastic counterpart, the film thickness as well as the bearing volume in the viscoelastic cases exhibit lower values [139]. By using an iterative Newton-Raphson scheme, Elsharkawy [141] proposed a numerical solution to the line contact lubrication problem of viscoelastic half-space. Similar contact problems were also analysed by Hooke and Huang [142], Scaraggi, et al. [143] and Pandey, et al. [144]. All of them reported remarkably different results in terms of the asymmetric pressure profiles and shrunk film thickness at the flow outlet when compared with the classical EHL regime response, which is derived from viscoelasticity. Apart from EHL, by studying the lubricating performance of a smooth cylinder sliding against a randomly rough nominally flat viscoelastic solid, Scaraggi, et al. [143] reported that the minimum (average) film separation does not monotonically increase with the sliding velocity from the boundary lubrication regime to the ordinary EHL regime. A Stribeck curve, which exhibits different structures when compared with that of elastic solids, was reported in their study. They argued that the common procedure to attain a Stribeck curve for elastic solids (e.g., using different fluid viscosities and plotting the friction coefficient as a function of the product of the kinetic viscosity and sliding speed) can be invalid for viscoelastic solids [143].

The newly defined viscoelasto-hydrodynamic lubrication (VEHL) has now been recognized as a peculiar regime that needs to be carefully considered when viscoelastic materials come into lubricated contact. A point contact lubrication model was developed by Putignano and Dini [145], where the effects of coupling between the fluid flow and the solid hysteretic response were considered. According to their simulation results, the friction in the VEHL system was found to be affected by the viscous loss as well as the viscoelastic hysteresis. The numerical approach was later generalized by Putignano [146] to take into account different contact configurations and to investigate the effects of viscoelasticity in each case. The latest work of Putignano and Campanale [147] extended the study of VEHL contact to non-steady-state conditions, where a strong coupling

between the time-dependent features of the fluid-dynamics problems and the viscoelastic relaxation of the material was found. A point VEHL contact model was recently developed by Zhao, et al [148]. A novel equation for determining the viscoelastic deformation was employed in their model to release the computational burden. A more complicated VEHL contact situation, in which layered materials with imperfect layer-substrate bonding are involved, was recently simulated by He, et al. [149] using SAM. Meanwhile, Li, et al. [150] developed a model to investigate the VEHL contact of viscoelastic materials containing arbitrary inhomogeneities.

It is of note that the role of viscoelasticity in the contact mechanics of lubricated solids has not been investigated theoretically and numerically to the same extent as for dry contact. To date, the role of surface roughness was barely investigated as the fluid film was fully formed in VEHL. However, when it comes to boundary lubrication or mixed lubrication, the effects of surface roughness can be significant on the lubricating performance, which is worthwhile to investigate in future numerical studies.

2.6 Thermoviscoelastic (TVE) Contact

Due to the time and temperature dependency, the properties of viscoelastic materials, including creep compliance and relaxation modulus, can be characterised as a function of temperature as well as time. However, the numerical studies discussed above were conducted with the assumption of isothermal conditions to simplify the contact problems. The viscoelastic contact problem, in which the mutual influences of material properties and temperature rise are involved, has attracted the attention of several studies in the few decades. Persson [151] analysed the sliding contact of a rubber indenter against a rigid rough substrate, where the effects of the flash temperature derived from friction were taken into account. As highlighted in their later numerical analysis [152, 153], the temperature rise due to relative motion during the contact of surfaces is of essential concern when viscoelastic materials are applied to a high-speed working environment. In addition to friction-induced heating, the heating derived from energy dissipation due to hysteresis, which tends to be unavoidable in the contact of viscoelastic materials, was considered in the formulation of Persson [151].

The development of the numerical model for thermoviscoelastic contact problems is still in its early period. By using an approximated approach to adapt the viscoelastic modulus to thermal effects in their BEM-based model [74], Putignano, et al. [154] provided qualitative solutions to thermos-viscoelastic rolling contact problems at high speeds. By extending the SAM-based VEHL model [149], He, et al. [155] developed a thermal-

viscoelasto-hydrodynamic lubrication model (TVEHL) model to simulate the contact behaviour of an elastic sphere against a viscoelastic half-space, where the temperature dependency of the surface displacement was determined using the frequency-temperature superposition [156, 157]. The thermos-viscoelastic contact of layered materials was simulated by Zhang, et al. [158], where the fully coupled velocity-frequency-temperature relationships were taken into account. The contact configurations including rubber coating against elastic substrate and elastic layer against rubber substrate were investigated in their study. To date, the effects of surface roughness in thermoviscoelastic contact were barely reported in the numerical studies based on these models, which is worthwhile to investigate in the future.

2.7 Non-linear Viscoelastic Contact

Although the models based on linear viscoelastic theory provide a great knowledge of the contact behaviour of viscoelastic materials, the simulation results are still an ideal abstraction of some real material behaviours to some degree. The nonlinearity of viscoelastic materials tends to be the biggest bottleneck problem for researchers. Efforts have been made in developing a suitable rheological (mathematical) model to characterise the nonlinear viscoelastic behaviour [159-161]. So far, a few models based on finite element analysis have been raised. By using the single-integral creep model with stress-dependent properties of Schapery [29], Henriksen [162] and Roy and Reddy [163] developed their own incremental iterative algorithms to solve the contact problems of nonlinear viscoelastic bodies. The work of Henriksen [162] was then extended by Lai and Bakker [164], who included the nonlinear effects derived from temperature and physical ageing in the constitutive model. By decoupling the deviatoric and volumetric response and subdividing the strain vector into instantaneous and hereditary parts, a finite element formulation was developed by Haj-Ali and Muliana [165] to model the nonlinear viscoelastic behaviour of isotropic materials. By characterising the material nonlinearity with strain-softening function and employing updated Lagrangian formulations, another formulation was developed by Shen, et al. [166] to analyse the three-dimensional contact of nonlinear viscoelastic problems with large deformations.

Apart from these early attempts, Mahmoud, et al. [167] recently developed a FEM-based model to simulate two-dimensional quasistatic frictionless contact problems, where nonlinear viscoelastic behaviour and large deformations were taken into account. The model was later extended to simulate the quasistatic frictional contact [168] and steady-state frictional rolling contact [169] of non-linear viscoelastic solids. As the modelling of

nonlinear viscoelastic contact is still in its early stage, the role played by surface roughness was barely investigated in the numerical studies, which needs to be investigated in the future.

2.8 Case Studies

The numerical tools discussed above can be readily extended to practical applications by specifying the contact input used in real life, including load, material properties, relative motion, surface geometry, lubricating conditions and operating temperature. Of course, additional expertise may be required for effective numerical analysis in specialized disciplines. In this chapter, numerical studies based on various viscoelastic models in different fields, including contact of ultra-high molecular weight polyethylene (UHMWPE) hip replacement in the biomedical field, contact of polydimethylsiloxane (PDMS) surface in the nanotechnology field and contact of rocks in the geological field, are presented and discussed to demonstrate how numerical models can help analyse and improve the design of various components in different contact systems.

2.8.1 Lubrication Analysis of UHMWPE Hip Replacements

In the biomedical industry, total hip replacement (THP) is a medical treatment that is usually implemented when the hip joint is worn or damaged to a serious extent that the personal mobility is significantly reduced and the patient is experiencing pain from the hip even during the resting time. Due to the existence of synovial fluid in human bodies, hip prostheses may work under different types of lubrication regimes depending on human activities. Based on the linear elastic theory, the squeeze-film lubrication problem for hip replacements, including an acetabular cup made of UHMWPE and a femoral head made of metal or ceramic materials, has been investigated numerically by several researchers [170-172]. The reported viscoelasticity of UHMWPE [173] undermines the reliability of these numerical studies. The essential role played by the viscoelasticity of UHMWPE in the lubrication performance of hip prostheses was reported by Lu, et al. [174] using a SAM-based transient lubrication model. By establishing the ball-in-socket lubrication model of the hip joint and specifying the contact inputs including load and angular velocity during the gait cycle, the viscoelasticity of the UHMWPE cup was found to have significant effects on the pressure profile such as lower peak pressure, despite barely influencing the lubricating film thickness. Recently, with the developed transient viscoelastic squeeze-film lubrication model, Lu, et al. [175] furthered the study on the viscoelastic effects of UHMWPE on the squeeze-film lubrication action of artificial hip replacements. Although the central film thickness increased, the minimum film thickness

was found to be decreased by the viscoelasticity, which leads to early direct contact. Besides, the viscoelastic lubricating performance was found to be enhanced by decreasing the relaxation time and mechanical loss factor of the material. The enhanced lubrication analysis helps to develop appropriate methods to ensure the long-term successful functioning of artificial joints by reducing wear. The parametric analysis in these numerical studies, including the relaxation time and the ratio of storage and loss modulus of the UHMWPE cup, provides essential guidance for the future design of UHMWPE hip implants. The contact performance of a hip replacement, such as the pressure and lubrication film thickness, can be manipulated by tuning these material parameters.

2.8.2 Viscoelastic-adhesive Contact Analysis of PDMS in Nanotechnology

In terms of the nanotechnology field, the triboelectric nanogenerator (TENG) is an evolving mechanical energy harvesting technology, which can be applied to collect mechanical energy from the surrounding environment and convert it into electrical energy for powering small devices based on the principles of contact charging and electrostatic induction [176]. A typical viscoelastic material known as PDMS is frequently applied. Micro- or nano-size textures are commonly fabricated on the PDMS surface in order to enhance the electrical performance of TENG, on which occasion the role of van der Waals interaction becomes dominant. By assuming that PDMS behaves as a linear elastic material, adhesive contact models were developed by Jin, et al. [177] and Yang, et al. [178] to investigate the contact of the textured film with the metal electrode. Recently, a viscoelastic-adhesive model was developed by Wang, et al. [179] to investigate the effects of viscoelasticity and texture sizes on the contact and electrical behaviour of TENG. By specifying the contact inputs of the TENG that works in a repeated contact-separation mode, the simulation results from the model were found to be closer to the experimental results when compared with those from an elastic-adhesive model. The effects of surface adhesion (e.g., the increase of the pull-off force with the loading cycle) and viscoelasticity (e.g., a larger contact area ratio and higher electrical outputs including open circuit voltage, short circuit current, transferred charge and load voltage) were reported in their numerical study. Under the application of a higher load, those with a smaller pitch and a larger width were found to produce higher electrical performance among the tested pyramid textures. These simulation results can serve as a useful reference when designing the viscoelastic surface texture of TENG in the future.

Apart from TENG, PDMS is also widely applied in micro-transfer printing, which is an advanced technology that makes use of surface adhesion to integrate functional devices

by transferring pre-fabricated micro- or nano-elements from the growth substrate to the receiver substrate through a viscoelastic stamp [180]. A FEM-based viscoelastic-adhesive model was recently developed by Jiang, et al. [181] to investigate the adhesive contact between a PDMS stamp and a spherical transferred element, where the Lennard-Jones surface force law was applied to simulate van der Waals forces. After specifying a loading-dwelling-unloading history, which is usually employed during the process of micro-transfer printing, a parametric study, including the effects of unloading velocity, preload and the thermodynamic work of adhesion on the pull-off force (e.g., adhesion strength) was conducted. These numerical results revealing relations between these parameters and the pull-off force provide a theoretical reference to control strategies of adhesive force between the viscoelastic stamp and the transferred element in the transfer printing process, which determines the capability of the sophisticated technique.

2.8.3 Failure and Instability Analysis of Viscoelastic Rocks

Applications of viscoelastic models can also be found in the geological field. Considering that rocks exhibit a pronounced time-dependent mechanical behaviour (e.g., deformation and failure of rocks evolve gradually over time instead of occurring instantaneously.), which is supported by evidence from several field studies [182-184], the rheology of rocks has become one of the essential topics in geomechanics. Recently, within the framework of irreversible thermodynamics, a finite-strain viscoelastic-damage model was developed by Cheng, et al. [185] using FEM to investigate the long-term strength and time-dependent failure and instability of rocks. Within the model, the heterogeneity of materials was considered on the mesoscale and a continuum damage constitutive relationship was employed to characterize the material damage process. In addition to the time-dependent property, the buckling instability mechanism of rocks was taken into account during modelling. After calibration with experimental results from creep and stress relaxation tests, the model was used to simulate the collapse of the rock slab. It was found that the collapse of rock slabs is caused by the interaction effect between the buckling instability mechanism and the time-dependent failure mechanism. The numerical analysis from the model shall provide meaningful guidance in terms of engineering design, safe construction and disaster prevention or mitigation, where rocks are involved.

2.9 Conclusion and Future Research

Numerical research over the last six decades has brought forward numerous fundamental insights into the contact analysis of viscoelastic bodies. The implications for enhanced

comprehension of the mechanical and tribological behaviour of viscoelastic materials under different operating conditions are profound by providing us with a better view of how these time-dependent materials can be made full use of in a more scientific way. Through applying efficient modelling techniques, along with the comprehensive theories that mathematically describe the contact response of viscoelastic materials under dry, lubricated or non-isothermal environments, various numerical tools have been developed to simulate the corresponding contact problems. The contact analysis derived from these models shall benefit all walks of life given the ubiquitous viscoelasticity problems to be addressed. Although a comprehensive numerical framework is being built up owing to the considerable efforts dedicated by the model developers shown in this review paper, the following knowledge gaps remain to be filled in for the further development of an ideal panacea that can tackle intractable viscoelastic contact problems in practice:

1. As discussed in section 2.4.2, the simulation of sliding or rolling contact was commonly assumed to be frictionless. The reason behind such practice is relevant to the intrinsic property of some specific polymers. It is known that elastomers are incompressible. For the problem of an incompressible half-space sliding on a rigid surface, there exists no coupling between the normal pressure and shear tractions. As the normal and tangential contact problems are independent of each other, the simplest way to simulate the sliding or rolling contact problems of viscoelastic materials is to neglect shear tractions in the lateral direction, together with their related coupling effects affecting the pressure profile, and to quantify the friction force exclusively derived from the viscoelastic losses. However, the case where uncoupled conditions can be applied is not ubiquitous. When addressing the common material combination for engineering products such as the knee or hip prosthesis (usually a hard metal against a soft and compressible polymer), the inclusion of the coupling effect seems inevitable. A fully coupled model, which provides information on both normal and tangential fields in frictional viscoelastic sliding contact problems, is worth studying for fine precision engineering applications. To date, there is no exhaustive numerical analysis in terms of the effects of the coupled partial slip regime on the later sliding contact solutions, which is worthwhile to investigate.
2. As mentioned in section 2.4.4, the development of viscoelastic-adhesive contact models is still in the early stage as the developed models can only solve indentation problems. Since the additional adhesive force makes it difficult to obtain closed-form contact solutions based on energy approaches and extremely careful consideration of energy terms is required, the surface adhesion in coupled normal and tangential contact of viscoelastic surfaces seems to be a knowledge gap.

3. Few BEM-based models simulating the nonlinear viscoelastic model were reported as BEM-based algorithms are limited by the assumptions of linear viscoelasticity. However, the extension of BEM-based models to contact problems, where nonlinear interface constitutive response is involved, may be possible by applying specific computational techniques or approximation approaches. Considering the higher computational efficiency of BEM compared with FEM, it is worthwhile to develop a general BEM-based nonlinear viscoelastic model.
4. Although the effect of surface roughness on the dry contact of linear viscoelastic materials was investigated and reported in several numerical studies as mentioned in section 2.4.3, it was barely included when it comes to the modelling of lubricated contact, thermoviscoelastic contact and nonlinear viscoelastic contact. The synergy of the surface roughness and other variables can be essential, which needs to be analyzed in future modelling work.

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2.10 Supplementary Notes for Chapter 2

In this section, additional discussion is presented, focusing on the contexts that are not fully covered in the literature review presented above.

2.10.1 Physics Behind Characteristics of Viscoelastic Materials

Viscoelastic materials exhibit unique properties combining characteristic of both elastic solids and viscous fluids when subjected to deformation. This duality is derived from the microstructural dynamics of these materials. Under external load, atomic bonds stretch elastically, but over time, atomic or molecular rearrangements occurs, leading to stress relaxation [186-188].

Memory effect of viscoelastic materials can be attributed to their dual energy storage and dissipation mechanisms. Due to the fact that only a portion of the deformational energy is elastically stored while the remainder is consistently dissipated through viscous mechanism in any deformation of viscoelastic materials, the response of viscoelastic materials lags behind the excitation, exhibiting a certain level of time dependency. The extent of this delay is dependent on the duration of the excitation. The molecules of the material cannot rearrange rapidly enough to fully accommodate the deformation in a short excitation. Conversely, in an excitation of long duration there is sufficient time for molecular rearrangements to take place, which explains why most viscoelastic contact problems can reach a steady state after a certain time [187]. Due to the lagged response, it always takes a certain amount of time for a viscoelastic material to reach its ultimate configuration under an excitation. The initial stress relaxes when the excitation is a strain and this process is known as stress relaxation, where the material molecules gradually accommodate the strain by conformational extension. Likewise, the final material strain is retarded if the excitation is a stress and this process is known as creep, where material molecules gradually accommodate the strain by bond distortion [186].

The molecular behaviour of viscoelastic materials is governed by the temperature. In this context, a term known as glass transition temperature (T_g) plays a crucial role, below which the material tends to behave like glass being hard and rigid. At temperatures significantly higher than the T_g , the material acts in a rubbery manner such that it exhibits large, instantaneous and fully recoverable deformation when being loaded. In the range near T_g , the material is in between the glassy and rubbery regimes [16, 186]. Its response is a combination of viscous fluidity and elastic solidity, and this region is termed viscoelastic regime [186].

2.10.2 Experimental Methodologies for Characterising Viscoelastic Materials

Accurate characterisation of viscoelastic behaviour is critical for valid simulation of viscoelastic contact problems. Mechanical characterization of viscoelastic materials often involves uniaxial tensile tests similar to those used for elastic solids but modified to reveal the time dependency response. Among various testing approaches, the following three are commonly encountered, namely, creep tests, stress relaxation tests, and dynamic mechanical analysis (DMA). These methods provide valuable insights into the time- and frequency-dependent behaviour of viscoelastic materials in their linear viscoelastic region.

As outlined in section 2.3, a constant stress σ_0 is applied to viscoelastic materials in creep tests to observe the time-dependent strain response $\varepsilon(t)$ as illustrated in Figure 2-2. The creep compliance can then be determined as the ratio of the strain response to the applied stress as expressed in Equation 2-16. Similarly, as illustrated in Figure 2-3, a constant strain ε_0 is imposed in stress relaxation tests to measure the decay of the stress $\sigma(t)$ over time and the relaxation modulus is determined by the ratio of the stress history to the strain as expressed in Equation 2-17.

$$\Phi(t) = \frac{\varepsilon(t)}{\sigma_0} \quad \text{Equation 2-16}$$

$$\Psi(t) = \frac{\sigma(t)}{\varepsilon_0} \quad \text{Equation 2-17}$$

These two experiments capture the transient response of viscoelastic materials under suddenly applied or sustained loads. As it calls for constant and suddenly applied stress or strain while the contact inputs have to ramp over a short duration in practical experiments, special care is always needed to address the artefacts derived from ramp effects (e.g., the work of Lee [189]). Additionally, environmental fluctuations (e.g., temperature and humidity) during the long-duration tests can undermine the reliability of test outcome.

While creep and stress relaxation tests are effective for studying long-term material response (minutes to days), they struggle to capture short-term viscoelastic behaviour (seconds or less) [186]. To address these limitations, dynamic tests such as DMA are employed, where the stress (or strain) resulting from a sinusoidal strain (or stress) is measured. As illustrated in Figure 2-4, when a viscoelastic material experiences a sinusoidally varying stress history, it will reach a steady state after a certain time, where the resulting strain lags the input stress by a phase angle that ranges from 0 to $\pi/2$ radians.

For the dynamic loading case, if the time axis origin is reset to the point where the strain reaches its maximum, the strain and stress functions can be rewritten as:

$$\varepsilon(t) = \varepsilon_0 \cos(\omega t), \quad \text{Equation 2-18}$$

$$\sigma(t) = \sigma_0 \cos(\omega t + \delta), \quad \text{Equation 2-19}$$

where the phase angle δ is commonly referred to as the loss angle.

Expanding Equation 2-19 using the trigonometric relations yields the following equation:

$$\sigma(t) = \sigma_0 \cos(\omega t) \cos(\delta) - \sigma_0 \sin(\omega t) \sin(\delta). \quad \text{Equation 2-20}$$

By defining the storage modulus E' and loss modulus E'' as follows:

$$E' = \frac{\sigma_0}{\varepsilon_0} \cos(\delta), E'' = \frac{\sigma_0}{\varepsilon_0} \sin(\delta), \quad \text{Equation 2-21}$$

Equation 2-20 can then be rewritten as:

$$\sigma(t) = \varepsilon_0 (E' \cos(\omega t) - E'' \sin(\omega t)). \quad \text{Equation 2-22}$$

Here, the storage modulus E' reveals the ratio of the in-phase stress to the strain, quantifying the elastic energy stored and recovered during the dynamic deformation. On the other hand, the loss modulus E'' reveals the ratio of out-of-phase stress to the strain, quantifying the energy dissipated as heat due to the internal friction. Together, these two properties are often written as the components of the complex modulus E^* :

$$E^* = E' + iE'', \quad \text{Equation 2-23}$$

where $i = \sqrt{-1}$ and the damping factor (also known as loss tangent) $\tan \delta$ can be determined as follows:

$$\tan \delta = E'/E''. \quad \text{Equation 2-24}$$

Alternatively, the harmonic stress and strain can be represented using complex exponentials as follows:

$$\sigma = \sigma_0^* e^{i\omega t}, \quad \text{Equation 2-25}$$

$$\varepsilon = \varepsilon_0^* e^{i\omega t}, \quad \text{Equation 2-26}$$

where the $e^{i\omega t}$ factor follows the Euler's formula expressed by Equation 2-27.

$$e^{i\theta} = \cos \theta + i \sin \theta \quad \text{Equation 2-27}$$

This approach simplifies the calculation of complex modulus, allowing it to be expressed as Equation 2-28 without the need to replace the origin at a time of maximum strain as was done above.

$$E^* = \sigma_0^* / \varepsilon_0^* \quad \text{Equation 2-28}$$

2.10.3 Representative Application of Viscoelastic Contact Model

An expanded discussion on practical applications of viscoelastic contact models is given here. Asphalt shows complicated contact responses, which often shift from elastic to viscoelastic when the temperature increases [190]. A FEM-based model was employed by Bai, et al. [191] to investigate the viscoelastic response and temperature sensitivity of asphalt pavements under realistic tire-pavement contact pressures. During the contact simulation, the effects of temperature variation, layer thickness of pavement and bonding conditions of layer interface were investigated. It is found that the viscoelasticity of asphalt pavement decreases tensile stresses at lower layers, transitioning the dominant failure mode from tensile cracking (predicted by elastic models) to shear damage on the surface at higher temperatures. Additionally, higher temperatures are found to reduce tensile and shear stresses at the bottom layers, decreasing bottom-up cracking risks. Besides, shear stresses on the surface are increased under higher temperatures, heightening the likelihood of surface shear failure. Thicker asphalt layers help reduce overall stress magnitudes, but their influence is less significant compared with the temperature effects. By integrating viscoelastic modelling, this work enhances the understanding of pavement deformation and failure mechanisms, paving the way for more durable infrastructure solutions.

2.10.4 Advanced rheological models

A discussion on advanced rheological model for characterising viscoelastic materials is provided in this section. The most straightforward method to integrate effects of time dependency into a mathematical model is to introduce viscous dashpot elements alongside elastic spring elements (e.g. Maxwell model, Kelvin-Voigt model, SLS model, Burgers model and Maxwell-Wiechert model). Theoretically, this approach allows for the representation of an arbitrary stress-strain time relation by incorporating an increasing number of spring-dashpot elements.

These linear viscoelastic models can be generalized using fractional derivatives, introducing fractional-order operators into rheological models. An extra element known as springpot (or Scott-Blair element) is integrated into the fractional viscoelastic models.

It is a fractional-order component that combines the behaviour of a spring and a dashpot. The springpot element expresses the following fractional derivative relationship between stress and strain [192]:

$$\sigma(t) = E\tau^\alpha \frac{d^\alpha \varepsilon(t)}{dt^\alpha}, \quad \text{Equation 2-29}$$

where E and τ are the elastic modulus and relaxation time, respectively, α ($0 \ll \alpha \ll 1$) is the fractional order that determines the balance between elastic and viscous behaviour of the springpot, and d^α/dt^α is the fractional derivative operator. Notably, this stress-strain relationship has different variants in literature [193-195].

As illustrated in Figure 2-10, the springpot behaves as an elastic spring when $\alpha = 0$ while it performs like a pure dashpot when $\alpha = 1$. For $0 < \alpha < 1$, the springpot exhibits a combined elastic-viscous behaviour.

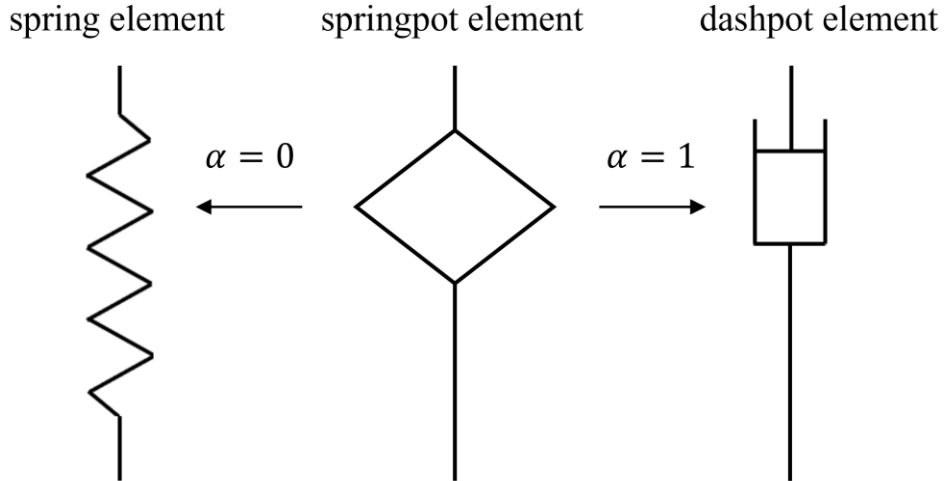


Figure 2-10 Schematic of a springpot element, which behaves as a spring when $\alpha = 0$ and as a dashpot when $\alpha = 1$

Compared with the traditional integer-order rheological models, the incorporation of springpot elements allows few parameters to be required for the fractional models to represent the viscoelastic behaviour of materials, allowing rheological parameters to be varied over a wide range. Additionally, a better agreement between experimental data and fractional models is reported [194]. Applications of these models have been explored in biomedical engineering [196-198], food science [199] and geotechnical engineering [200, 201]. However, their wide application is limited due to the uncommon notation and complex characteristics of fractional operators [194].

Fractional viscoelastic models are beyond the scope of this project. Hence, only a brief introduction is given in this section. A detailed description of the historical development

of various rheological models for characterising viscoelastic materials can be found in the work of Shitikova and Krusser [202].

2.10.5 Further Discussion on the Literature about Nonlinear Viscoelastic Materials

Simulation of nonlinear viscoelastic materials is beyond the scope of this project. Hence, a brief discussion on the characterisation of nonlinear viscoelastic behaviour is presented in this section, summarizing the work of Green [22], Rivlin [23], Spencer [24], Pipkin and Rogers [25], Leadermann [26], Brueller [27, 28] and Schapery [29]. A detailed description of these models and relevant formulations can be found in the work of Papanicolaou and Zaoutsos [16].

These models characterising the nonlinear viscoelastic behaviour are based on the integral form. Based on the extension of the Boltzmann superposition principle to finite strain theory [203], a nonlinear constitutive model incorporating the strain memory effect was developed by Green, Rivlin and Spencer [22-24], which is commonly referred to as the Green-Rivlin models in the literature. These models describe the stress response through multiple integral constitutive equations, also known as the Volterra integral series. While the Green-Rivlin models established a theoretical foundation for nonlinear viscoelastic behaviour, they are highly inefficient for modelling real materials since a large number of terms are required to match the experimental data accurately. This limitation was later addressed by Pipkin and Rogers [25], who introduced an approximation for multi-step strain histories by a linear combination of single step strain functions through a nonlinear hereditary integral [203]. The Pipkin and Rogers model is often referred to as the nonlinear superposition theory (NLSP) in literature.

On the other hand, by introducing a stress-dependent function to include the nonlinear effects in the time-dependent response, Leadermann [26] proposed a single integral representation for mapping the nonlinear viscoelastic via the generalization of Boltzmann's linear theory. By introducing two stress-dependent nonlinearity functions attributing the nonlinearity in the instantaneous response and transient creep, Brueller's models were developed for characterising time-dependent behaviour of glass and carbon fabric reinforced thermoplastics under creep loading [27, 28]. Similar to the work of Brueller, by incorporating stress and temperature-dependent factors reflecting instantaneous elastic compliance, transient creep compliance, load rate influence and a time scale shift factor, Schapery [29] developed a novel constitutive equation that can capture material nonlinearities under varying hygrothermal conditions. This model has

been widely employed to polymer and polymer matrix composites [204, 205] due to its outstanding ability to predict experimental data.

2.10.6 Critique of Literature on Analysis of Rough Viscoelastic Contacts

In this section, a further discussion on the analysis of rough viscoelastic contact done by various researchers is given, including the work of Persson, et al. [100], Carbone, et al. [102, 103], Scaraggi [101, 104] and Comingio [105].

Persson, et al. [100] investigated the time-dependent contact area between viscoelastic solids and rough substrate based on the contact mechanics theory for self-affine fractal surfaces [101]. The study analysed the effects of substrate roughness, material properties and contact magnification, with applications in tack, rubber friction and sealing. However, the applicability of this method is restricted to contact scenarios where the contact area increases monotonically and where the surface adhesion can be neglected.

Carbone, et al. [102] extended Persson's theory [101] to anisotropic rough surfaces, investigating the directional dependence of rubber friction. While their designed experiments qualitatively validate the theory, the neglect of surface adhesion leads to discrepancies in quantitative predictions. This study was further advanced by Afferrante, et al. [103], who focused on the force dynamics and energy loss in viscoelastic materials. A transverse force perpendicular to the sliding direction resulting from the asymmetry in normal contact pressure distribution was reported in their study, the role of which is to ensure system equilibrium without dissipating energy. Although their contact analysis aligns well with numerical simulations based on the model of Putignano, et al. [206], it fails to capture changes in anisotropy after surface deformation since the PSD of the deformed surface is determined using a simplified scaling approach that neglects the detailed distribution of contact patches.

A universal contact area law for the sliding contact between viscoelastic solids and hard, randomly rough substrates was proposed by Scaraggi and Persson [104] by combining numerical simulations (Residual Molecular Dynamics (RMD)) and Persson's theory. However, their study assumes isotropic surfaces and neglects critical factors such as adhesion and frictional heating. This work was extended later by Scaraggi and Cominio [105], who introduced a Fourier-based RMD framework to model rough contact mechanics in graded viscoelastic materials. This model incorporated multilayered structures and small-scale roughness effects, offering practical insights for tire design and biomedical applications, though it similarly neglects adhesion effects.

2.10.7 Extra Discussion on Adhesive Contact Analysis

Given that the maximum adhesive force is fundamentally limited in magnitude, as described by the Lennard-Jones potential, effects of the surface adhesion are often neglected in contact analysis. It would not highly affect the modelling result when the contact pressure induced by the load applied to the surface is significantly higher than the maximum adhesive force. To determine the threshold at which adhesive forces can be neglected in contact analysis, a brief explanation is provided here.

A criterion for neglecting adhesion effects in contact mechanics is provided in the adhesion map for the contact of elastic spheres proposed by Johnson and Greenwood [207] shown in Figure 2-11. In this map, different contact theories and models, including the work of Bradley [208], and JKR [9], DMT [10] and Maugis-Dugdale (M-D) [128] models, were suggested for the contact of elastic spheres based on the elasticity parameter (1.16μ , μ is the Tabor parameter) and nondimensional load ($\frac{P}{\pi\Delta\gamma R}$, P is the load, $\Delta\gamma$ is the work of adhesion and R is the combined radius of contacting surfaces). For contacts under extremely large compressive loads ($\frac{P}{\pi\Delta\gamma R} \gg 1000$) or in conditions where the Tabor parameter is considerably low and the applied load is relatively high ($1.16\mu \ll 0.1$ and $\frac{P}{\pi\Delta\gamma R} > 100$), adhesion effects become negligible, and Hertz theory can already provide a good approximation [207, 209].

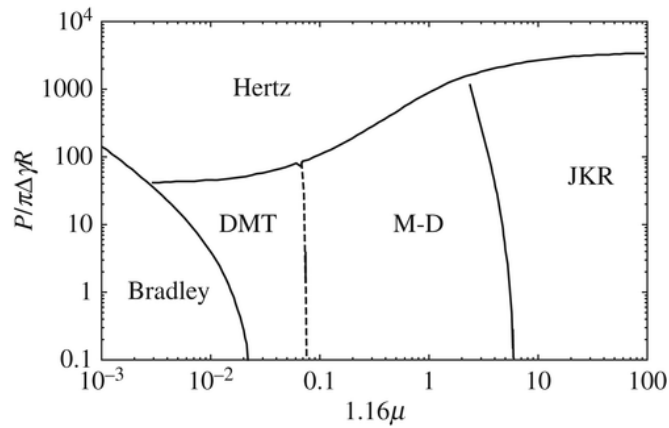


Figure 2-11 Adhesion map originally proposed by Johnson and Greenwood [207] and reproduced by Ciavarella, et al. [209], reused with permission from the work of Ciavarella, et al. [209], © Royal Society, 2025.

Notably, the Tabor parameter is determined as the ratio of the elastic displacement of the surface at the point of separation (pull-off) to the effective range of surface forces characterised by the equilibrium distance, at which the intermolecular forces vanish [210]. This parameter serves as a bridge between the JKR [9] and DMT [10] adhesive

contact models, describing their domains of applicability. As illustrated in the adhesion map in Figure 2-11, the JKR model was found to be valid only for the contact problem with a high Tabor parameter while the DMT model is applicable when the Tabor parameter is low [211]. The Tabor parameter has now been commonly used in adhesive contact analysis (e.g., work of Bazrafshan, et al. [212], Ciavarella, et al. [209], Ghanbarzadeh [213], Muser [214]), as it characterises the ratio of the elastic deformation to the range of action of the adhesive forces [215].

2.10.8 Extra Discussion on the Study of VEHL by Putignano and Campanale

Putignano and Campanale [147] conducted an innovative study on visco-elastohydrodynamic lubrication (VEHL) by integrating the finite difference method for solving fluid dynamics problems and BEM for solving viscoelastic solid deformation problems. They reported that there exists a strong coupling between the two time-dependent process including flow squeeze and viscoelastic relaxation in VEHL. The fluid-dynamics problem in their work centres on the squeeze lubrication process, where an isoviscous lubricant is confined between a cylindrical rigid punch and a viscoelastic half-plane. Different from the classical elasto-hydrodynamic lubrication (EHL), where the system is not dependent on the deformation history experienced by the material, the viscoelastic relaxation of the material is found to significantly influence the lubricant film thickness and pressure distribution over time, which subsequently alter the material deformation. This interplay makes the VEHL problem fundamentally dynamic.

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3 A New Numerical Model for Investigating the Effect of Surface Roughness on the Stick and Slip of Contacting Surfaces with Identical Materials

3.1 Abstract

A numerical model replicating the stick-slip contact of two elastically similar materials is presented in this paper. Based on the Coulomb's law described in the partial-slip theory of Ciavarella, the model adopts a complementary condition that facilitates the design of a novel algorithm to separate the stick and slip regions. Via the comparison between simulation results and analytical solutions, the developed model is validated and applied to study the role played by the roughness of realistic surfaces in the separation of stick and slip regions. It is found that a higher root mean square (RMS) gradient leads to smaller contacting area under constant load while the root mean square (RMS) roughness has insignificant influence if the RMS gradient is kept constant, which is in line with current literature. Under the effects of the RMS gradient, the relationship between varying skewness or kurtosis and contacting regions is irregular. Furthermore, stick regions are found to respond identically to those varying roughness factors. However, the ratio of stick regions to the total contacting region is revealed to be unaffected by the surface roughness parameters when a tangential load is applied on the contacting surfaces following a linear behaviour.

3.2 Introduction

The problem of the combined normal and tangential loading has always been an area of interest for determining the frictional behaviour of materials in contact. In this condition, stick and slip zones may coexist presenting a 'partial slip' contact interface. The partial slip phenomenon can be commonly found in engineering practices such as fretting. The relative movement of oscillations under alternating dynamic loads is small in amplitude compared to the size of the contact area under these conditions which eventually leads to surface failure [1, 2]. In addition to fretting wear, stick-slip analysis plays an essential role when it comes to the study of rubber friction [3, 4], powder tribology [5-7] and polymer tribology [8]. Pioneering contact analysis on the stick-slip behaviour was conducted by Cattaneo [9] and Mindlin [10], who independently proposed the partial slip solution for a generally elliptical or circular contact problem. Based on the Cattaneo-Mindlin solution, Johnson [11] developed the closed-form solution for the sliding (partial or gross) contact between two identical materials under tangential loading.

Given that the analytical solutions based on proposed assumptions are always subjected to various limitations including the simplified contact geometries and ideal loading history (usually constant), a numerical model presents itself as a suitable alternative in this dilemma. Based on finite element method (FEM) together with the domain decomposition method and boundary element method (BEM), Kosior et al. [12, 13] developed the numerical model studying the frictional elastic contact following Coulomb's law to investigate the effects of friction on the contact parameters including contact radius and normal pressure. The contact between an elastic-plastic sphere and a rigid plane in the gross slip or full stick state under the combined normal and tangential loading was simulated by Brizmer et al. [14, 15] using FEM. Notably, a large simulation domain is always required in the finite element analysis to approximate the half-space along with proper addressing of boundary conditions. This causes an enormous computational burden when analysing the contact of real rough surfaces where fine spatial discretisation is needed.

On the other hand, BEM is a more efficient method as only the boundary of the bodies needs to be discretised to characterize the surfaces. Using BEM and the conjugate gradient method (CGM), Polonsky and Keer [16] proposed a single-loop iterative CGM algorithm to solve the contact problems of rough surfaces, which served as a good reference tool for many numerical models developed in the last decade [17]. Pohrt and Li [18] proposed a displacement-controlled algorithm using CGM and BEM to solve the tangential contact problem. They converted the contact problem to an algebraic matrix equation relating the surface traction and displacement. This method identified the nodes within the non-contact region as well as stick and slip regions. Deformation for the stick nodes was always the prescribed tangential rigid body translation and shear stress for the slip nodes was always the critical value determined from the Coulomb's friction law. However, the normal and tangential contact problems are decoupled in their model, which makes it only valid for incompressible materials where Poisson's ratio is 0.5. A fully coupled model was developed by Chen and Wang using the semi-analytical method (SAM) [19], in which the algorithm is in a similar form to that of the frictionless normal contact model developed by Polonsky and Keer [16]. In their algorithm, the estimation of the tangential rigid body translation relies on analytical equations. However, the static force equilibrium was not enforced in their case and instead, they adjusted the rigid body translation according to the difference between the computed tangential force and the prescribed one.

Their model was later extended to investigate the partial slip of an elastic layered half-space [20].

Inspired by the fretting model developed by Spinu and Amarandei [21, 22], a stick-slip contact model, with the same form of algorithm for both the independent normal and tangential components, is advanced in this paper by considering the similarity between the normal contact and tangential contact problems for two elastically identical materials. With the implementation of discrete convolution, fast Fourier transform (DC-FFT), the normal contact model is developed on the basis of the CGM algorithm for contacting elastic rough surfaces by Keer and Polonsky [16]. Although the literature is continuously evolving in modelling of the stick and slip of rough surface, the criterion to separate the stick and slip regions is still ambiguous. In this paper, the Coulomb's law described in the stick-slip contact theory proposed by Ciavarella [23-25], which requires the shear traction in the stick region to always be less than the Coulomb friction, is followed to generate the complementary condition that separates the stick and slip regions for the individual tangential contact model. It should be noted that the same condition was also implemented in the partial slip contact modelling by Kosior [12, 13], Gallego [1, 26], Botto [27], Dong [28] and Bazrafshan [29]. Since a computational node exhibiting the shear stress equal to the local Coulomb friction can only happen in slip region according to the above-mentioned theory, a novel adjustment method is introduced in our algorithm of the tangential contact problem to ensure that the shear stress in the stick region is less than the critical value. Furthermore, the capability of the model to study the influence of important features of surface roughness including root mean square roughness (RMS roughness), root mean square gradient (RMS gradient), skewness and kurtosis on the stick and slip area ratios has been investigated in this paper.

3.3 Theory and Algorithm Description

Based on the explicit mathematical relation between the strain and stress for the contacting bodies, the BEM is implemented to develop the fully deterministic stick-slip contact model.

3.3.1 Problem Formulation

To start with numerical formulation, the quadratic programming approach [30] is applied to find the contact pressure and surface deformation via minimising the total complementary energy using the variational principle. For a normal contact problem of two elastic surfaces, the total complementary energy V^* is expressed as:

$$V^* = U_E^* - \int_{\Omega} p(u_{z0}^* + u_{z1}^*) d\Omega = U_E^* - \int_{\Omega} p \bar{u}_z^* d\Omega, \quad \text{Equation 3-1}$$

where U_E^* is the internal complementary energy of the stressed elastic solids, Ω is the assumed contacting area where the contact pressure acts, p is the contact pressure, u_{z0}^* and u_{z1}^* are the prescribed displacement of the two contacting solids respectively and $\bar{u}_z^*$ is the total prescribed displacement of the two contacting solids inside the assumed contact domain. Therefore, the term ' $\int_{\Omega} p \bar{u}_z^* d\Omega$ ' is the work done by the normal force.

For ideal linear elastic materials, the internal complementary energy U_E^* is numerically equal to the elastic strain energy due to the linear relation between the stress and strain as indicated in Figure 3-1, which is expressed as:

$$U_E^* = U_E = \frac{1}{2} \int_{\Omega} p (u_{z0} + u_{z1}) d\Omega = \frac{1}{2} \int_{\Omega} p \bar{u}_z d\Omega \quad \text{Equation 3-2}$$

where $\bar{u}_z$ is the composite surface displacement inside the contact zone and equals total contact deformation of two contacting solids (denoted as u_{z0} and u_{z1} correspondingly).

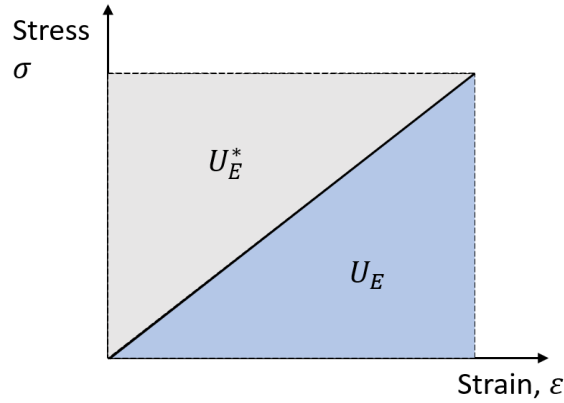


Figure 3-1 Relation between U_E^* and U_E for contact between linear elastic materials without plastic deformation

To express U_E^* with Equation 3-2 in Equation 3-1, the complementary potential energy for the normal contact problem is then given by:

$$V^* = \frac{1}{2} \int_{\Omega} p \bar{u}_z d\Omega - \int_{\Omega} p \bar{u}_z^* d\Omega, \quad \text{Equation 3-3}$$

Similarly, the total complementary energy can be minimized for coupled normal and tangential contact problems in the case of linear elastic materials and is expressed as follows:

$$V^* = \frac{1}{2} \iint t \bar{u} dx dy - \iint t \bar{u}^* dx dy, \quad \text{Equation 3-4}$$

where t is the complete surface stress vector, $\bar{u}^*$ is the prescribed displacement and $\bar{u}$ is the complete surface deformation vector determined by the following equations:

$$t = q_x e_x + q_y e_y + p e_z, \quad \text{Equation 3-5}$$

$$\bar{u} = u_x e_x + u_y e_y + u_z e_z, \quad \text{Equation 3-6}$$

where q_x , q_y and p are the traction force, u_x , u_y and u_z are the surface deformation and e_x , e_y and e_z are the Cartesian unit basis vectors in x , y and z directions respectively.

After discretisation, the fully coupled relationship between the contact deformation and loading can be expressed in the following matrix form:

$$\begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} = \begin{bmatrix} C_{xx} & C_{xy} & C_{xz} \\ C_{yx} & C_{yy} & C_{yz} \\ C_{zx} & C_{zy} & C_{zz} \end{bmatrix} \begin{bmatrix} q_x \\ q_y \\ p \end{bmatrix}, \quad \text{Equation 3-7}$$

where C_{ij} is the influence matrix of the deflection in i direction due to the load applied in j direction ($i = x, y, z; j = x, y, z$). The closed-form solutions of these influence coefficient matrices are provided by Ghanbarzadeh [31] and shown in Appendix 3A.

To simplify the stick-slip contact model, the following assumptions are made:

- Only the contact between two elastic surfaces with identical material properties is accounted for within the model. Thus, the composite shear modulus becomes zero and the Equation 3-7 relating the surface deformation to the traction force reduces to:

$$\begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} = \begin{bmatrix} C_{xx} & C_{xy} & 0 \\ C_{yx} & C_{yy} & 0 \\ 0 & 0 & C_{zz} \end{bmatrix} \begin{bmatrix} q_x \\ q_y \\ p \end{bmatrix}, \quad \text{Equation 3-8}$$

The solution of the normal contact problem is then decoupled from that of the tangential contact problem. However, the normal contact pressure is still required in advance to execute the algorithm of the tangential contact model as the Coulomb's law is followed within it to determine the critical shear traction (local Coulomb friction).

- Only the tangential force in the x -direction is considered within the model. The effect this has on the deformation in the y -direction is neglected considering the relatively large difference in the order of magnitude between the two relevant influence coefficient matrices (C_{xx} and C_{yx}) shown in Appendix 3B. Neglecting of the effects of stress in one specific direction on the deflections in two other directions is a common practice for many researchers when it comes to the preliminary modelling of tangential contact problems [18, 19]. Therefore, Equation 3-8 is further simplified to

$$\begin{bmatrix} u_x \\ 0 \\ u_z \end{bmatrix} = \begin{bmatrix} C_{xx} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & C_{zz} \end{bmatrix} \begin{bmatrix} q_x \\ 0 \\ p \end{bmatrix}, \quad \text{Equation 3-9}$$

To minimise the complementary potential energy, the contact problem is to find the contact stresses and contact deformations correlated by Equation 3-9 which meet a series of restriction conditions. It is noteworthy that after transforming continuous functions into piecewise constant functions that are uniform within the elementary cell of the equally spaced rectangular established in the contact interface, control points for the set grid are necessary. These points are the centroids of the elements in the developed model as shown in Figure 3-2. Therefore, the shear traction and deformation of each element becomes the focus of the model.

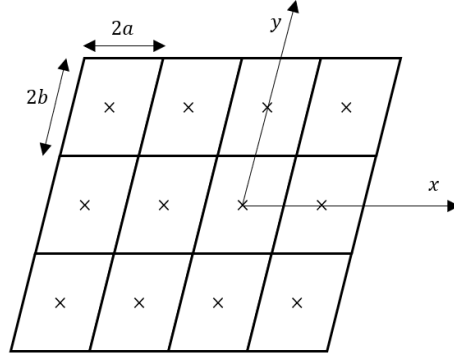


Figure 3-2 Discretised elements: ‘×’ is the centroid of each element, and $2a$ and $2b$ are the side lengths of each element in x and y directions respectively

For a discretised normal contact problem, the following contact equations and inequalities should be satisfied to find those pressure and deformation:

1. Static force equilibrium: The sum of the pressure distribution at the contact interface should be equal to the applied force (W):

$$W = \Delta \sum_{(i,j) \in I_c} p(i,j), \quad \text{Equation 3-10}$$

where Δ is the area of the element in the established mesh and equals $4ab$ indicated in Figure 3-2, I_c is the contact domain and i and j denote the indices of the mesh element (i,j) with $1 \leq i \leq N_1$ and $1 \leq j \leq N_2$ (N_1 and N_2 are total numbers of elements along x and y directions respectively).

2. Geometrical condition of deformation: The deformation of the surface in the normal direction should meet this geometrical condition:

$$h(i,j) = h_t(i,j) + u(i,j) - \delta, (i,j) \in I_p; \quad \text{Equation 3-11}$$

where I_p denotes the simulation domain and h_i , h , and δ are the gap between undeformed surfaces, the gap between deformed surfaces and the normal rigid body displacement respectively as shown in Figure 3-3.

3. Complementary conditions should be met over the contacting surfaces:

$$p(i, j) > 0 \text{ \& } h(i, j) = 0, (i, j) \in I_c;$$

$$p(i, j) = 0 \text{ \& } h(i, j) > 0, (i, j) \in I_p - I_c.$$

Equation 3-12

where $I_p - I_c$ denotes the non-contact domain.

As illustrated in Figure 3-3, the gap h for Point 1 in the range of contact radius (a_0) is 0 under the effect of the normal load. As Point 2 outside the contact radius is implying a vanishing contact pressure, there is a positive surface gap labelled as h^2 . Thus, the product of the contact pressure and surface gap is always zero over the contacting surfaces no matter which position of the interface is being referred to.

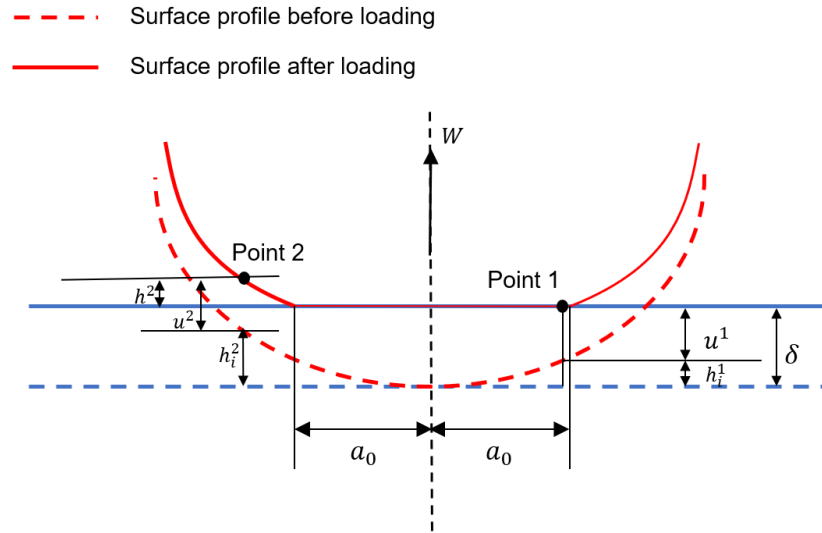


Figure 3-3 Displacement condition of deformation in the normal direction: h and h_i are the gap between the deformed surface and the initial surface gap respectively, u is the elastic deformation, δ is the rigid body indentation, a_0 is the contact radius, W is the applied normal load, and the superscripts 1 and 2 denote the points 1 and 2 being referred respectively.

The complementary conditions shown in Equation 3-12 imply that the adhesion effect ($p < 0$) is not considered in this normal contact problem and the contacting surfaces are impenetrable. Inclusion of adhesion in such scenario will be complex since the introduction of non-conservative forces makes it difficult to obtain closed-form solutions based on energy approaches. However, the inclusion of the surface adhesion may become feasible by directly using the Lennard-Jones potential at the separated computational nodes. Based on a non-adhesive normal contact algorithm, a deterministic and well-

validated normal contact model with adhesion was recently developed by Ghanbarzadeh [32] via the implementation of Lennard-Jones potential fields with the incorporation of a new surface integration method. This problem needs very careful consideration of energy terms as reported by Ciavarella [33, 34]. It is out of the scope of the current paper and will be the subject of future work.

Regarding the constraint conditions for the tangential contact problem, the following equations and inequalities must be satisfied:

1. Static force equilibrium equation:

$$\Delta \sum_{(i,j) \in I_c} q(i,j) = F_x, \quad \text{Equation 3-13}$$

where F_x is the applied tangential force.

2. Geometrical condition of deformation:

$$s(i,j) = u_x(i,j) - \delta_x, \quad (i,j) \in I_c \quad \text{Equation 3-14}$$

where s is the slip distance and δ_x is the right body displacement in the horizontal direction.

3. Complementary conditions should be satisfied over the contacting surfaces:

$$\begin{aligned} &|q(i,j)| < \mu p(i,j), \quad s(i,j) = 0, \quad (i,j) \in I_s; \\ &|q(i,j)| = \mu p(i,j), \quad |s(i,j)| > 0, \quad q(i,j) \cdot s(i,j) < 0, \quad (i,j) \in I_c - I_s; \end{aligned} \quad \text{Equation 3-15}$$

where μ is the coefficient of friction, I_s denotes the stick domain and $I_c - I_s$ denotes the slip domain.

As shown in Figure 3-4, there is no slip for the interaction region in stick state such as the Point O . When it comes to the slip region (Point A_1 and A_2), the direction of the slip (s_{x1} and s_{x2}) is always opposite to that of the applied load (F_{x1} and F_{x2}) as labelled.

Although the Coulomb's friction law is universally applied in frictional contact modelling, there are different complementary conditions applied in the partial slip modelling which result in the presence of different algorithms as listed in Table 3-1. For example, some of the recent literature [19-21, 35] in Table 3-1 used the following criteria expressed by:

$$\begin{aligned} &|q(i,j)| \leq \mu p(i,j), \quad s(i,j) = 0, \quad (i,j) \in I_s; \\ &|q(i,j)| = \mu p(i,j), \quad |s(i,j)| > 0, \quad q(i,j) \cdot s(i,j) < 0, \quad (i,j) \in I_c - I_s; \end{aligned} \quad \text{Equation 3-16}$$

In such case, the shear stress of the node in stick could be equal to the local critical value ' $\mu p(i,j)$ ' when the node exhibits a vanishing slip distance according to the Equation 3-16.

In this paper, we have used Equation 3-15 as the complementary conditions. The reason why the equal sign is discarded in the current developed model is related to the design of the relevant algorithm for the tangential contact problem:

- For the node presenting the shear stress of ' $\mu p(i, j)$ ', it could be either in stick or slip following the Equation 3-16, which depends on the corresponding nodal slip distance;
- However, the slip distance in the CGM algorithm is mathematically the residual of the linear system deriving from Equation 3-14, where the estimation of the rigid body indentation in tangential direction δ_x needs to be conducted in advance and it requires the identification of the stick regions;
- The complementary condition (Equation 3-16) leads to conflict in the algorithm since the estimation of δ_x requires the information of the system residual, where the latter is determined from the former.

This potential conflict derived from Equation 3-16 would be iterated in the description of the algorithm for the tangential contact model. To avoid the problem mentioned above, Chen and Wang [19] estimated δ_x with the analytical formula (Equation 3-30). Besides, they adjusted the rigid body translation with the difference between the computed tangential load and the prescribed load.

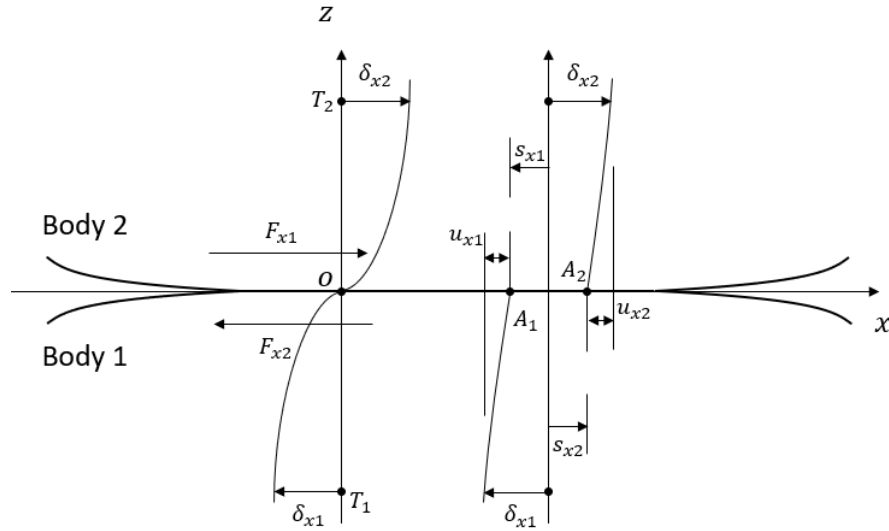


Figure 3-4 Displacement condition of deformation in the tangential direction: F_x is the tangential force applied to the contacting body, δ_x is the rigid body displacement in the horizontal direction, u_x is the elastic deformation in the horizontal direction of the body, s_x is the slip distance of body, and the subscripts 1 and 2 denote the contacting bodies 1 and 2 being referred respectively.

Table 3-1 List of some of work on the partial slip modelling based on Coulomb's law

Author	Roughness	Method	Coulomb's friction law
Guyot et al. [13]	No	FEM+BEM	Equation 3-15
Chen and Wang [19]	No	SAM	Equation 3-16
Gallego et al. [1]	No	SAM	Equation 3-15
Wang et al. [20]	No	SAM	Equation 3-16
Spinu and Amarandei [36]	No	SAM	Equation 3-16
Botto et al. [27]	No	FEM	Equation 3-15
Dong et al. [28]	Yes	SAM	Equation 3-15
Bazrafshan et al. [29]	Yes	BEM	Equation 3-15
Alakhrasing et al. [35]	Yes	FEM+BEM	Equation 3-16

3.3.2 Algorithm Description

One of the main difficulties in solving contact problems is that neither the contact area nor the pressure distribution is known in advance. As a result, an iterative approach or the so-called trial-and-error method is frequently applied, which starts with an assumption of a contact region and the computation of the pressure distribution and surface displacement based on the initial guess. If all the constraint conditions (Equation 3-10-Equation 3-12 for normal contact problems, Equation 3-13-Equation 3-15 for tangential contact problems) are verified by the obtained solution, contact problems are solved. Otherwise, the contact region must be adjusted, and the new pressure distribution and surface displacement need to be computed again with the new guess. Hence, a suitable iterative scheme is essential to achieve high computational efficiency as well as accuracy. Among the potential relaxation methods, CGM was implemented in the following algorithms considering it has been widely applied to the numerical modelling of contact problems owing to its superlinear rate of convergence and ensured convergence for quadratic optimization problems [17].

Apart from the iteration, the most time-consuming part during the numerical simulation of contact problems is the calculation of surface deformation (Equation 3-9), which always needs to be executed repeatedly in the iterative approach mentioned above. To explain it mathematically, for the contact problem discretised with a total number of N nodes, it takes N^2 multiplication operations to obtain the surface deformation in each iteration of the contact model using direct summation (DS) method. Furthermore, several iterations are often necessary for a converged solution. Therefore, the DS method

becomes extremely inefficient when considering about rough surface contact as a dense grid with a fairly large number of N is required to characterize the surface roughness. According to a comparative study of numerical methods to the surface deformation calculation conducted by Wang et al. [37], DC-FFT was found to be the most efficient tool while providing a comparable numerical accuracy among the investigated methods including the DS method, DC-FFT and Multi-Level-Multi-Integration (MLMI) method, where the MLMI is another well-established technique applied in the contact solver by Keer and Polonsky [16] and the main alternative to DC-FFT advanced by Lubrecht [38]. Considering that DC-FFT can be easily implemented from the perspective of computer programming, it was applied in this model to release the computation burden when it comes to surface deformation calculation and relevant convolution processing.

The algorithms of developed models with the implementation of CGM and DC-FFT can be separated into two sections including the normal contact algorithm and the tangential contact algorithm. To simplify the description, only the tangential part is shown below as the algorithm for the normal contact is developed referring to the classical model of Polonsky and Keer [16].

The CGM algorithm for the tangential contact model is integrated with the one for normal contact model and they have a similar form shown in the flow chart in Figure 3-5. The highlighted sections in the flow chart are the ones that need to be modified based on the algorithm for normal elastic contact model [16]. Especially, those in blue colour are the novel parts of the developed algorithm. The detailed steps of these modified sections are described as follows:

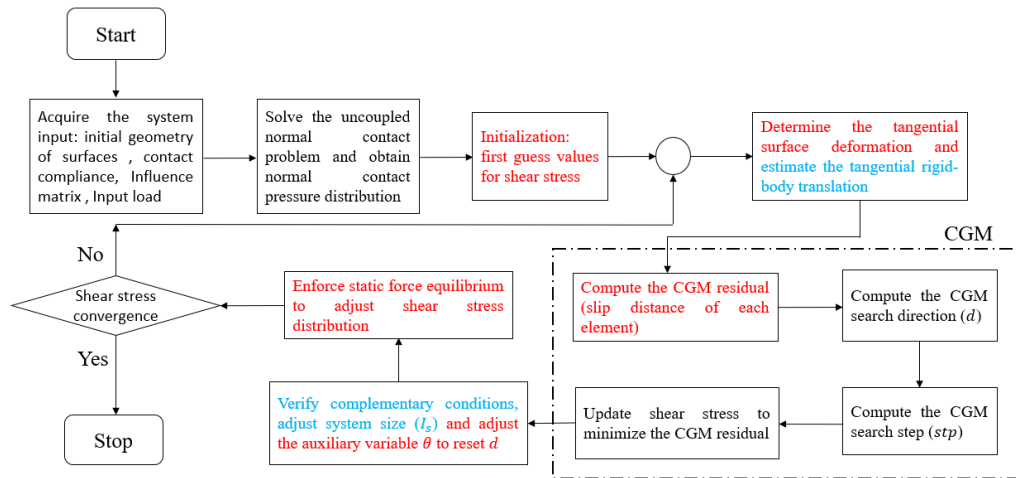


Figure 3-5 Flow chart of the load-controlled algorithm for the stick-slip elastic contact model

1. The initialization of the shear stress is based on the input tangential load in x direction:

$$q(i, j) = \frac{F_x}{\Delta \cdot I_c}, \quad \text{Equation 3-17}$$

Where I_c here denotes the number of nodes in the contacting region determined from the normal contact algorithm.

2. The tangential surface deformation is calculated from Equation 3-9 with DC-FFT and the estimation of the rigid body approach is calculated by:

$$\delta_x = \frac{1}{N_s} \sum_{(i,j) \in I_s} (u_x(i, j)), \quad \text{Equation 3-18}$$

where N_s is the number of nodes in the stick region.

3. The residual of the linear system and the square Euclidean norm for the set of cells in the stick domain for the tangential contact problem is calculated as follows:

$$r_x(i, j) = u_x(i, j) - \delta_x, (i, j) \in I_c, \quad \text{Equation 3-19}$$

$$R_x = \sum_{(i,j) \in I_s} (r_x(i, j))^2, \quad \text{Equation 3-20}$$

where mathematically the residual r_x is the slip distance of the node in the contact region s . The slip distance of the node in the stick region vanishes following the complementary condition. Therefore, the square Euclidean norm R_x should be approximated to zero when the algorithm converges.

It is noteworthy that the reason why the equal sign in the complementary condition is discarded is mainly related to the definition of the residual of the linear system. If the complementary condition in the literature (Equation 3-16) [19-21, 35] is applied here, the former estimation of the tangential rigid body approach is challenging to achieve as in the step 2 the identification of stick nodes is required where the information about slip distance of the node is necessary. However, it is also the residual of the linear system being calculated based on the estimated rigid body approach. That is why the designed algorithm is stated to be conflicting and can be hardly executed when the equal sign is included.

4. Regarding the adjustment for I_s and $I_c - I_s$ based on the complementary conditions, the following operations are conducted:
On one hand, the contact model allows no shear stress over the critical value ' μp '. Consequently, the shear stress of the node in slip state exceeding the value is decreased to μp . On the other hand, if the node in the slip region exhibits a shear

stress with the magnitude of μp but the product of the slip distance and the shear stress is positive, it is moved from the slip region to stick region by recalculating the nodal stress with the negative term ' $-stp \cdot r_x(i, j)$ '. This ensures that the stress in a stick region is always less than the critical value.

As a result, the shear stress of each cell meeting the conditions above should be adjusted as follows:

$$\text{For } I_l = \{(i, j) \in (I_c - I_s): q(i, j) > \mu p\},$$

$$q(i, j) = \mu p, \quad (i, j) \in I_l \quad \text{Equation 3-21}$$

$$\text{For } I_k = \{(i, j) \in (I_c - I_s): q(i, j) = \mu p, q(i, j) \cdot r_x(i, j) > 0\},$$

$$q(i, j) = q(i, j) - stp \cdot r_x(i, j), \quad (i, j) \in I_k \quad \text{Equation 3-22}$$

The stick domain I_s should then be modified as follows:

$$I_s = I_s \cup I_k \quad \text{Equation 3-23}$$

When any element in the slip region is moved to stick region, a new search of the descent direction for the node must be conducted from the beginning. This operation can be achieved by the adjustment of an auxiliary variable θ in the equation of the descent direction calculation as follows:

$$d(i, j) = r_x(i, j) - d(i, j) \cdot \theta \cdot \left(\frac{R_x}{R_{old}} \right), \quad \text{Equation 3-24}$$

where d is the search direction and R_{old} is the square Euclidean norm in the last iteration.

If I_k is empty, θ is set to one. Otherwise, it is set to zero.

5. Given that not only the shear stress of nodes in stick domain but also those in slip domain is included in the parameter q and the stress of nodes in slip region needs to remain constant (μp) without any modification, a new method is designed to enforce the static force equilibrium:

$$q(i, j) = q(i, j) + fac, (i, j) \in I_s, \quad \text{Equation 3-25}$$

where fac is the adjusting factor determined by the following equation:

$$fac = \frac{\frac{F_x}{\Delta} - \sum_{(i, j) \in I_c} q(i, j)}{N_s}. \quad \text{Equation 3-26}$$

A displacement-controlled algorithm is also developed for the stick-slip elastic contact model based on the one designed for the normal elastic contact model. The computational

complexity of the two algorithms (load-controlled and displacement-controlled) of the tangential contact model is $O(N_1 N_2 \log(N_1 N_2))$, which is the same with the individual normal contact model as no additional factor is accounted for in this case. However, they would take more time to converge compared to the algorithm for the normal elastic contact model due to the additional integration.

3.4 Model Validations and Results

The developed model was validated by comparing the simulation results with the analytical solutions. After the validation, the fully deterministic model developed here was used to study the influence of surface roughness parameters on the stick-slip separation.

3.4.1 Results and Validation of the Stick-Slip Contact Model

The contact of two elastic bodies made of carbon steel and discretised by 512×512 elements with the relevant parameters shown in Table 3-2 under the normal load of 100 N and the tangential load of 20 N was simulated.

Table 3-2 Parameters used in the numerical simulation

Parameter	Value	Description
R	3500	Radius of the sphere (μm)
E_1, E_2	213	Elastic modulus of two bodies (GPa)
μ_f	0.5	Coefficient of friction
ν_1, ν_2	0.29	Poisson's ratio of two bodies
W_x	20	Input tangential load
W	100	Input normal load

The shear stress distribution is shown in Figure 3-6 (a) with the stick and slip regions labelled. A three-dimensional (3D) plot of the shear stress distribution is also presented in Figure 3-6 (b). It can be found that the contacting region is separated into a central region and a surrounding slip annulus in the case of the simplified single-asperity contact.

The evolution of the circular stick region to slip region among the contacting area under the increasing tangential load was modelled as well. The non-linear response of the ratio of the stick region to the contacting region with respect to the monotonically increasing load is shown in Figure 3-7. The following analytical solution exhibiting a good

agreement with the numerical one shown in Figure 3-7 was reported by Johnson [11] based on Cattaneo's theory [9]:

$$\frac{c}{a_0} = \left(1 - \frac{F_x}{\mu_f W}\right)^{\frac{1}{3}} \quad \text{Equation 3-27}$$

$$\frac{A_{stick}}{A_{contact}} = \frac{\pi c^2}{\pi a_0^2} = \left(1 - \frac{F_x}{\mu_f W}\right)^{\frac{2}{3}} \quad \text{Equation 3-28}$$

where A_{stick} denotes the stick area, $A_{contact}$ denotes the contacting area, c denotes the stick radius and a_0 denotes the contact radius that can be determined from the equation [11] as follows:

$$a_0 = \left(\frac{3WR}{4E^*}\right)^{\frac{1}{3}}, \quad \text{Equation 3-29}$$

where E^* is the composite elastic modulus of the two contacting surfaces.

As observed from Figure 3-8 (a)-(e), the central stick region keeps shrinking with the rising tangential load while the slip annulus responds oppositely.

The computed tangential rigid body translation from the developed model was compared with the one derived from the analytical formulas [11] as follows:

$$\delta_x = \delta_0 \left[1 - \left(1 - \frac{F_x}{\mu_f W}\right)^{2/3}\right], \quad \text{Equation 3-30}$$

$$\delta_0 = 3\mu_f W \frac{\left[\frac{(2 - \nu_1)(1 + \nu_1)}{E_1} + \frac{(2 - \nu_2)(1 + \nu_2)}{E_2}\right]}{8a_0},$$

where δ_0 is the tangential rigid body translation when the contacting is at gross sliding ($F_x = \mu_f W$).

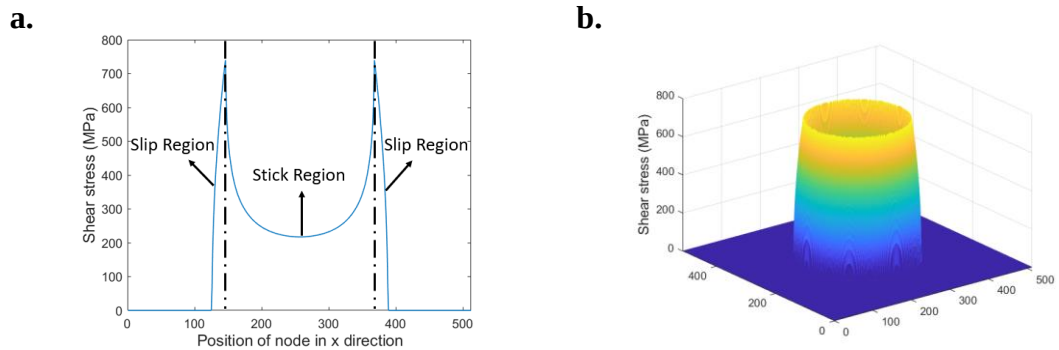


Figure 3-6 Simulation result under the specified input: (a) 2-D distribution of shear stress, (b) 3-D distribution of shear stress

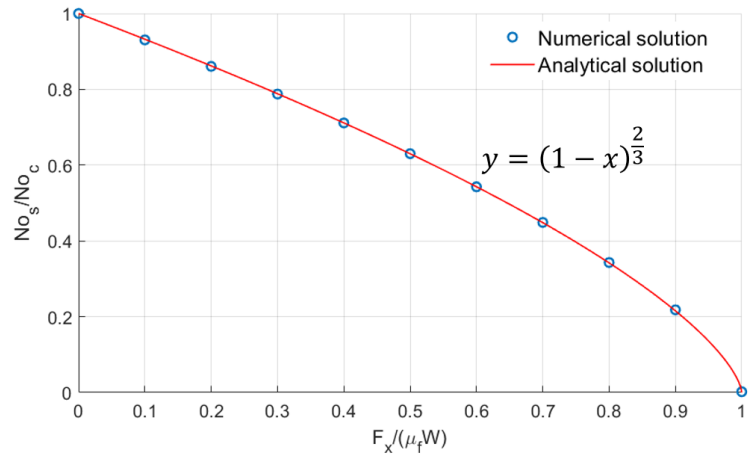
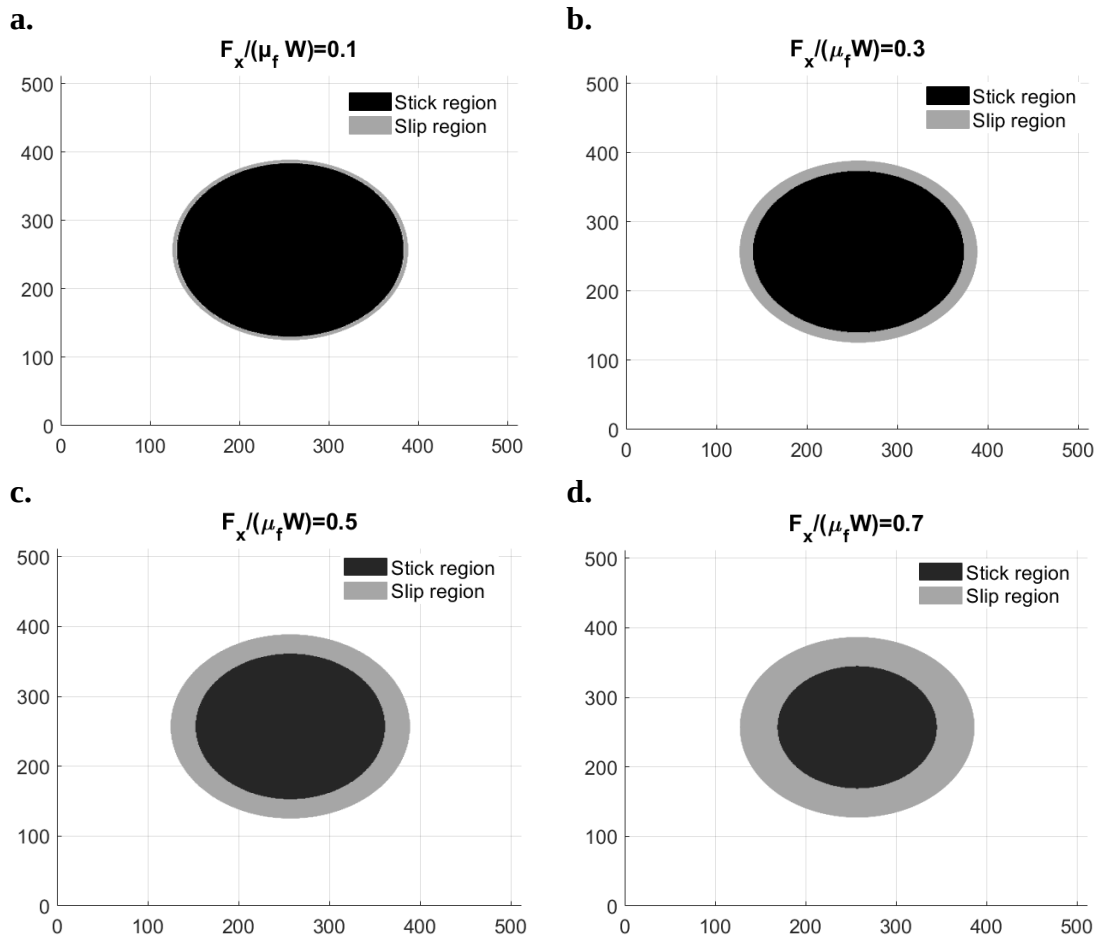


Figure 3-7 Evolution of ratio of the stick region to the contacting area

The comparison of the increasing tangential rigid body translation under varying tangential loads deriving from the current model and the analytical formula is shown in Figure 3-8 (f), where a good agreement is found.



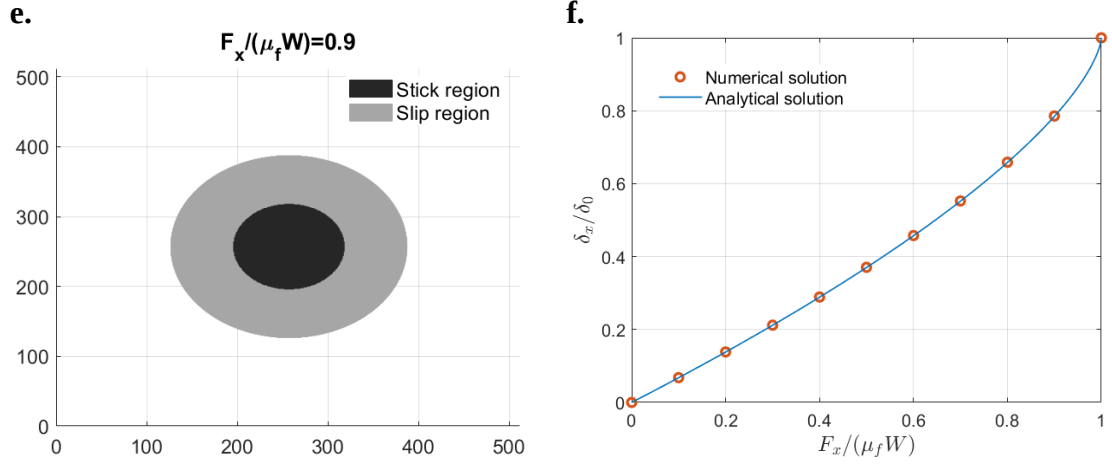


Figure 3-8 Results with increasing tangential loads: (a)-(e) simulated separation of stick and slip regions with increasing tangential load, (f) comparison of the growing tangential rigid body displacement with increasing tangential load deriving the developed model and the analytical method

To further validate the stick-slip model, the comparison in terms of shear stress distribution q_x was conducted as well. The analytical formula is shown as follows [11]:

$$q_x = \mu_f p_0 \left[\sqrt{1 - \left(\frac{x}{a_0}\right)^2} - H(c - x) \cdot \frac{c}{a_0} \sqrt{1 - \left(\frac{x}{c}\right)^2} \right], \quad \text{Equation 3-31}$$

Additionally, to compare the results of the shear traction derived from the current model with the one derived from the simplified stick-slip elastic model developed by Chen and Wang [19], the μ_f was changed to 0.2 and the result is presented in Figure 3-9. The analytical and the numerical solutions shown in Figure 3-9 exhibit a great match and the similarity between Figure 3-9 and the plot by Chen and Wang (Fig. 4a) in [19] indicates a good agreement between the two models.

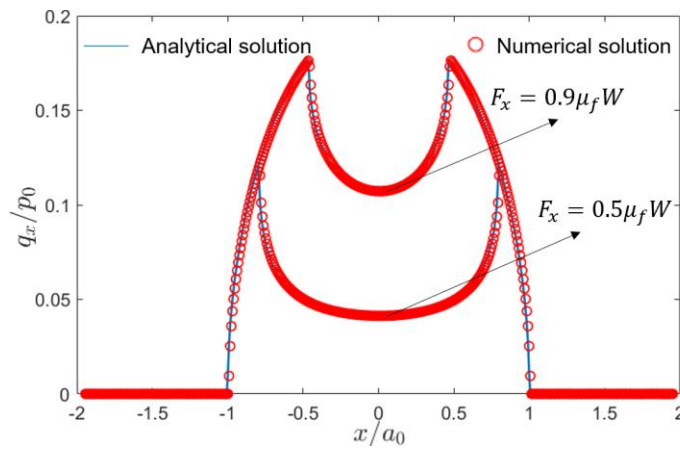


Figure 3-9 Comparison in terms of shear stress distribution between the numerical and analytical solutions

3.4.2 Simulation Results of Multi-asperity Contacts

Estimation of the real area of contact plays a central role in the tribological study in terms of adhesion, friction, lubrication and wear. Although simulation results of the contact between surfaces with simple geometries presented above are validated against the relevant analytical solutions, surfaces are always rough and irregular in practical applications. Their stochastic nature determines the area and geometry of the contacting region and subsequently affects the stiffness, electrical and thermal conductivity of the interface, friction and wear. Therefore, this surface study was extended from the single-asperity contact to multi-asperity contact, which approximates realistic rough surfaces.

3.4.2.1 Normal Contact of Rough Spheres

The contact between an elastic rough sphere and a smooth plane was first considered. Pastewka and Robbins [39] proposed an analytical solution that relates the variation of the real contact area to the normal load for this type of contact problem. Their parameter-free equations were later investigated by Muser [40] and were reported to have good accuracy in terms of the prediction of the real contact area (with less than 10% error). An adjusted formula was then developed for the relative contact area A_r by Muser [40] via abandoning the mean-field approximation and is given as follows:

$$A_r = \frac{A_{re}}{A_H} = \left(1 - \frac{1}{2k^2\tilde{p}^2}\right) \text{erf}(k\tilde{p}) + \frac{\exp(-k^2\tilde{p}^2)}{\sqrt{\pi}k\tilde{p}}, \quad \text{Equation 3-32}$$

where A_{re} denotes the real contact area, A_H denotes the Hertzian contact area (πa_0^2), k denotes a constant which turns out close to two for typical engineering surfaces and $\tilde{p}$ denotes the physical representation of the average contact pressure and is determined by the following equation:

$$\tilde{p} = \frac{3W}{4\sqrt{\pi}E^*\bar{g}a_0^2}, \quad \text{Equation 3-33}$$

where $\bar{g}$ is the root mean square gradient of the rough surface that is often denoted as Sdq. Assume $\frac{dz(x)}{dx}$ is the slope at a specified location on the surface, then the root mean square slope of a rough surface is given by:

$$\bar{g} = \sqrt{\frac{1}{L} \int_0^L \left(\frac{d}{dx} z(x) \right)^2 dx}, \quad \text{Equation 3-34}$$

where L is the sampling length.

To numerically generate the surface of the rough sphere, the smooth sphere and the micro rough surface are superimposed as shown in Figure 3-10. It is noted that many surfaces of interest in practice often have a self-affine fractal feature, for which each asperity has a smaller asperity on top of it at all scales. Therefore, the statistical properties of the self-affine fractal surfaces remain with a scale transformation [41, 42].

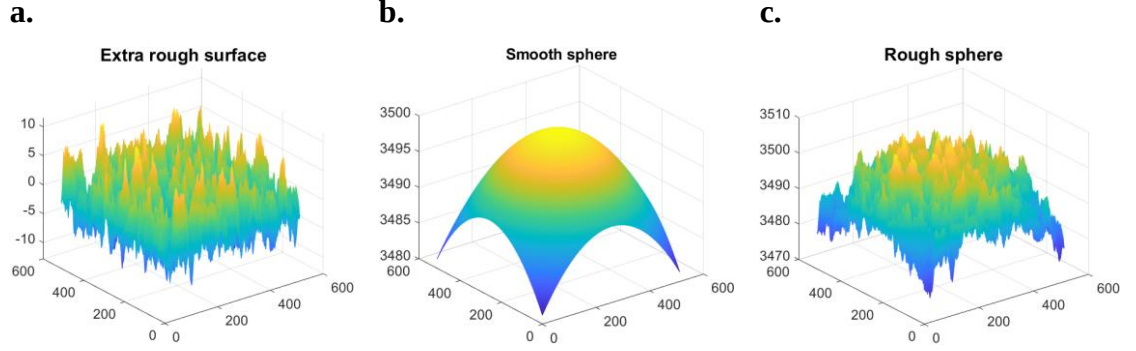


Figure 3-10 Generation of a rough sphere: (a) micro rough surface, (b) smooth sphere and (c) the final produced rough sphere by the combination of (a) and (b)

The micro rough surfaces with the self-affine properties in this study are generated through the power spectral density (PSD), which has the following power-law relation [43]:

$$C(q) \sim q^{-2(1+H)} \quad \text{Equation 3-35}$$

where H here is the Hurst exponent related to the fractal dimension D_f ($H = 3 - D_f$).

Since the surface cannot be self-affine over all length scale in reality, the above relation only holds in a limited wavevector region ($q_0 < q < q_1$) resulting in a form shown in Figure 3-11, where q_0 and q_1 are the short-distance roll-off wavevector and the long-distance cut-off wavevector respectively and depend on the relevant system. The fractal dimension is determined by the slope of the $\log C - \log q$ plot for the section where $q > q_0$. Besides, the smallest possible wavevector q_L depends on the lateral size L of the surface under consideration ($q_L = 2\pi/L$) [41, 43].

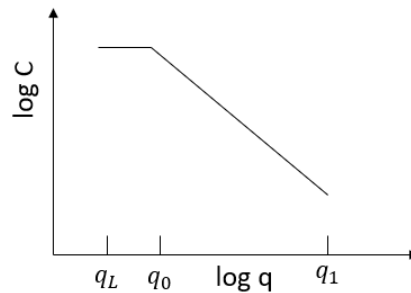


Figure 3-11 The PSD of a rough surface which is self-affine for $q_0 < q < q_1$

By using random numbers for the Fourier transform of the height profiles ($\tilde{h}(q)$), the height spectrum (the second moment of $\tilde{h}(q)$ on average) can be defined as [43, 44]:

$$C(q) \equiv \langle |\tilde{h}(q)|^2 \rangle = C(q_0) \times \begin{cases} 1 & q_L < q \leq q_0 \\ (q/q_0)^{-2(1+H)} & q_0 < q \leq q_1 \\ 0 & \text{else} \end{cases} \quad \begin{array}{l} \text{Equation} \\ \text{3-36} \end{array}$$

The operator $\langle \dots \rangle$ in Equation 2-14 denotes the average over different random-surface realization. It should be noted that all the surfaces generated following this method have a mean of zero in terms of the height profile ($\tilde{h}(q)$).

By defining related parameters including the root mean square roughness R_q , the Hurst exponent H , the length of topography in x direction L (assuming the length of topography in y direction is the same with that in x direction), number of nodes in x and y direction and the roll-off wavevector q_0 , the artificial randomly rough surfaces could be generated.

As Equation 3-32 and Equation 3-33 suggest, the variation of the contact area with the normal load is affected by the RMS gradient of rough surfaces. Therefore, three extra rough surfaces with the same $\bar{g}$ (0.7518) but different R_q values were generated as shown in Figure 3-12 to validate this relation. The relevant input contact parameters are given in Table 3-3.

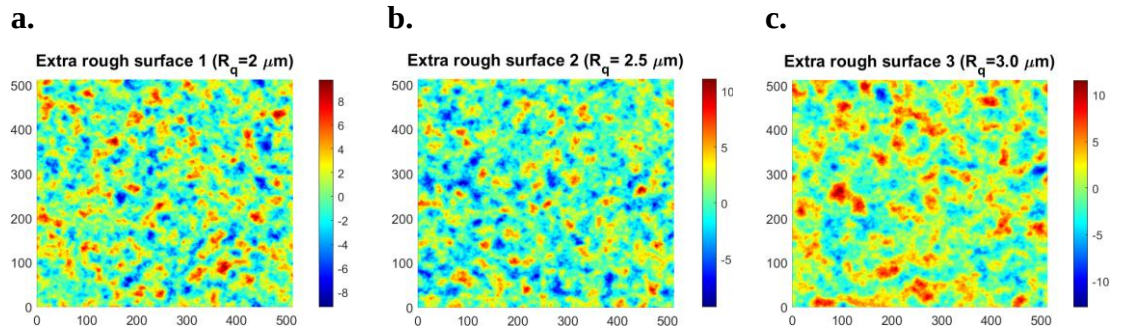


Figure 3-12 Generated micro rough surface with the same $\bar{g}$ but different R_q values

The relation between the relative contact area A_r and the reduced contact pressure P was frequently reported by many tribologists [45, 46], where A_r is determined from the ratio of the real contact area to the Hertzian contact area and P is determined from the following equation:

$$P = \frac{W/A_H}{E^* \bar{g}}, \quad \text{Equation 3-37}$$

where the term ' W/A_H ' is known as the macroscopic pressure.

Table 3-3 Input parameters in the contact between the rough sphere and the smooth plane

Parameter	Value	Description
R	3500	Radius of the smooth sphere (μm)
E_1	18.75	Elastic modulus of the sphere (MPa)
ν_1	0.5	Poisson's ratio of the rough sphere
W	0.00015-0.15	Range of applied normal load (N)
L	512	Length of the surface in x and y direction (μm)

In order to show the influence of the micro-roughness added to the smooth sphere, the comparison between the Hertzian contact region and the real contact region for the rough sphere (with extra rough surface 1) under the increasing reduced pressure P is presented in Figure 3-13. It can be observed that there exist contacting spots outside the perimeter of the Hertzian contacting area. The reason for it is that usually only the tip of the smooth sphere comes into contact while the added roughness allows other zones out of the Hertzian area to interact with the opposite plane.

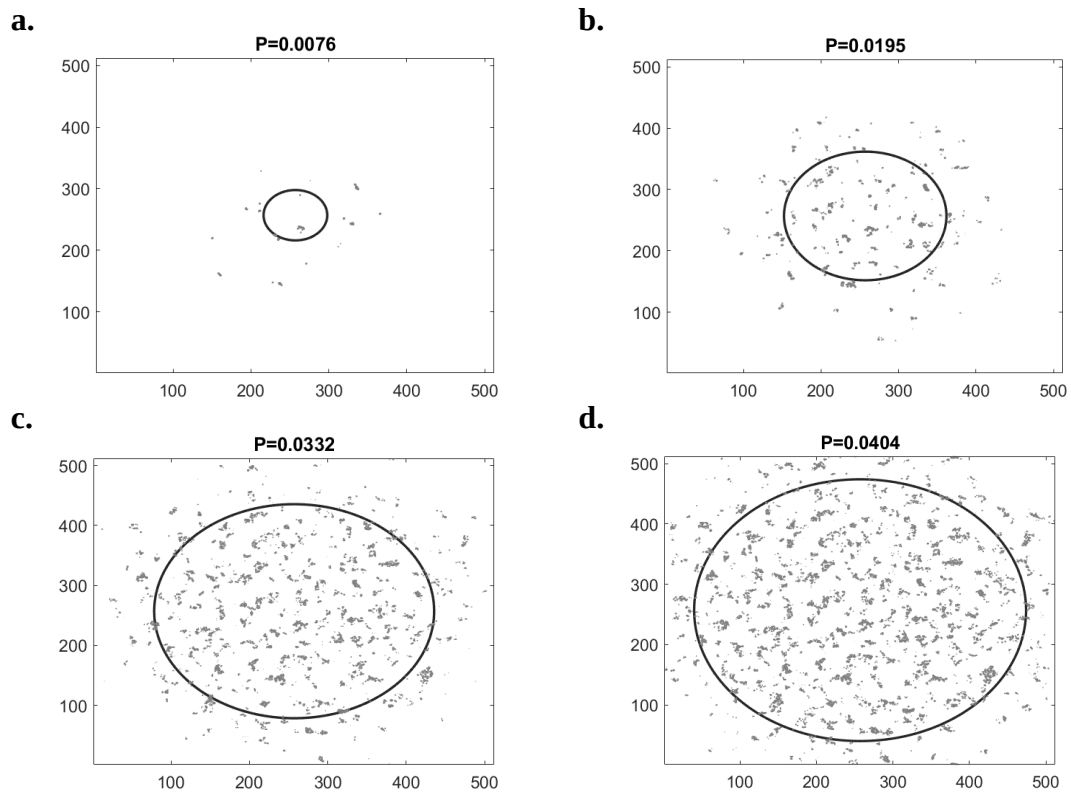


Figure 3-13 Comparison between the Hertzian contact area (area bounded by the solid black line) and the real contact area (grey dots) for the contact of the rough sphere (integration of the smooth sphere and the extra rough surface 1) against a smooth plane under increasing reduced pressure

The variation of the relative contact area A_r with the reduced pressure P is shown in Figure 3-14 for those three different rough surfaces with identical RMS gradient. The numerical solutions of the three rough sphere contacts were compared against Muser's solution derived from Equation 3-32 and Equation 3-33 [40] and a good agreement can be found. This shows that our numerical prediction is close to the solution proposed by Muser for the real contact area in rough surface contact and suggests that RMS roughness has minimal effect in this case.

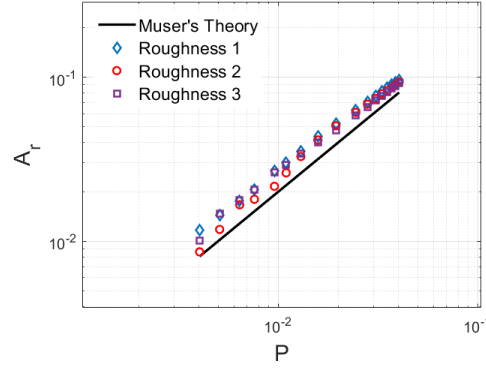


Figure 3-14 log-log plot of the variation of A_r with P

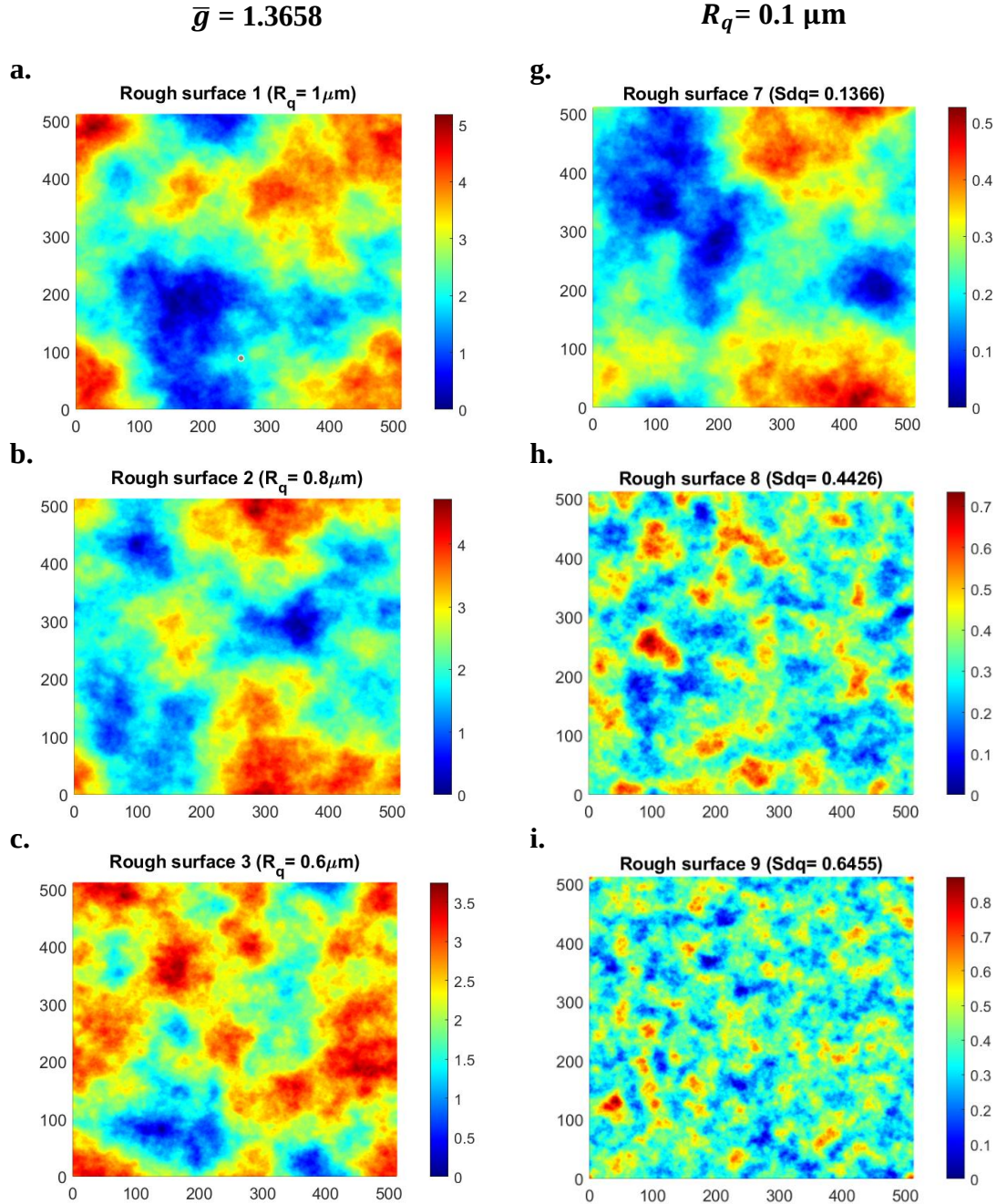
3.4.2.2 Influence of Roughness Parameters

The surface topography affects the real area of contact, which consists of both the stick and slip regions in the partial slip contact problem. To investigate the effects of the surface roughness on the separation of the stick and slip regions, the roughness parameters need to be investigated systematically. Since the RMS roughness is one of the commonly quoted measures of surface roughness and there are numerous studies and theories reflecting on the effect of the RMS roughness on the area of contact [47, 48], adhesion [32, 49], friction [29, 50, 51] and wear [52], it was first taken into account in this paper. Besides, the role of the RMS gradient $\bar{g}$ was analysed considering its importance in determining the real contact area according to the previous studies and the analytical solutions proposed by Pastewka and Robbins [39] and Muser [40]. To increase the range of investigated surface roughness parameters, the skewness R_{sk} and kurtosis R_{ku} of rough surfaces were considered as well, where the former provides a measure of surface asymmetry and can be determined from Equation 3-38 while the latter offers a measure of sharpness of profile peak and can be calculated from Equation 3-39.

$$R_{sk} = \frac{1}{R_q^3} \frac{1}{L} \int_0^L z(x)^3 dx \quad \text{Equation 3-38}$$

$$R_{ku} = \frac{1}{R_q^4} \frac{1}{L} \int_0^L z(x)^4 dx \quad \text{Equation 3-39}$$

Starting with the study of effects of RMS roughness, six nominally flat rough surfaces with different R_q values but same $\bar{g}$ as shown in Figure 3-15 (a)-(f) were generated. The relevant material properties and input load are shown in Table 3-4. The ratios of the stick and contact regions to the whole simulation domain for these contact problems under the specified inputs are shown in Figure 3-16 (a), where the nodes in stick or contact state are found to fluctuate around a corresponding certain value with increasing RMS roughness. Likewise, the ratio of the nodes in stick to the nodes in contact under the input RMS roughness fluctuates around 0.5 according to Figure 3-16 (b). Compared with that in the former case of smooth sphere contact (dash line), the proportion of the contacting region in stick decreases slightly within the range of R_q as shown in Figure 3-16 (b).



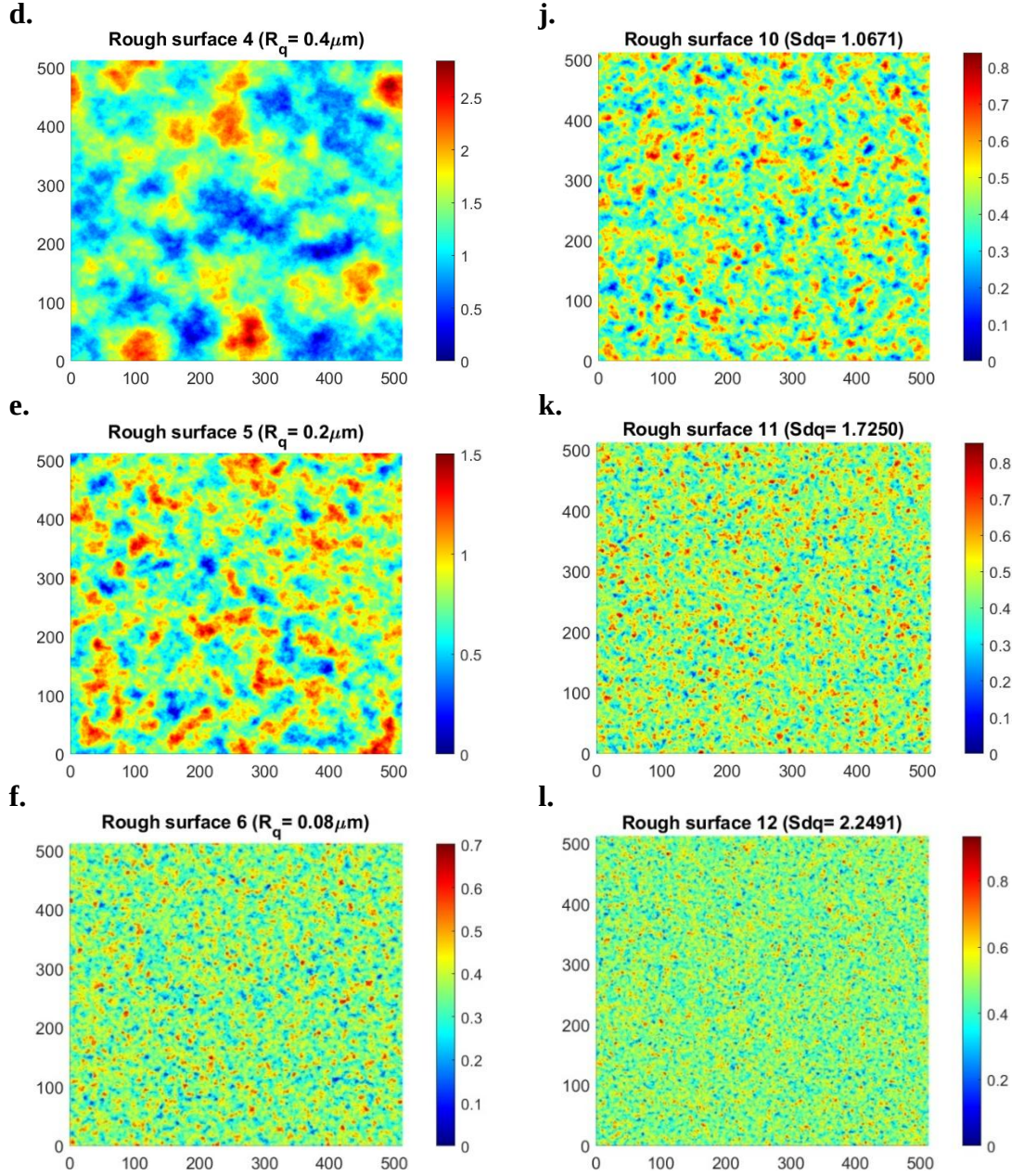


Figure 3-15 Generated nominally flat rough surface: (a)-(f) rough surfaces with different R_q value but the same $\bar{g}$ (1.3658) and (g)-(l) rough surfaces with different $\bar{g}$ values but the same R_q (0.1 μm)

To investigate the influence of RMS gradient, six rough surfaces with different $\bar{g}$ values but the same R_q were generated as shown in Figure 3-15 (g)-(l). The same material properties and input load given in Table 3-4 were used for these rough contact problems. The ratios of the stick and contact regions to the whole simulation domain were both found to decrease significantly with increasing RMS gradient as shown in Figure 3-17 (a). The relationship between the logarithm of RMS gradient and the logarithm of the two ratios under the constant load is found to be quadratic as indicated in Figure 3-17 (a), where the trend lines using quadratic fit perfectly match the plotted data according to the

theory of least-square regression ($R^2 = 1$). Notably, the decline of the contacting region with higher RMS gradient under the certain load agrees with that conclusion of the study conducted by McGhee et al. with similar surface roughness features [53]. Concerning the ratio of the nodes in stick to the nodes in contact, it fluctuates around 0.5 as shown in Figure 3-17 (b) and the proportion is lower than the one in the smooth sphere contact (dash line), which has similar trend as predicted by the former test with varying RMS roughness.

Table 3-4 Specified material properties and input load

Parameter	Value	Description
L	100	Length of surface in x and y direction (μm)
E	18.75	Elastic modulus of the rough surface (MPa)
ν	0.5	Poisson's ratio
μ_f	0.2	Coefficient of friction
W	0.0003	Applied normal load (N)
F_x	$0.5\mu_f W$	Applied tangential load (N)

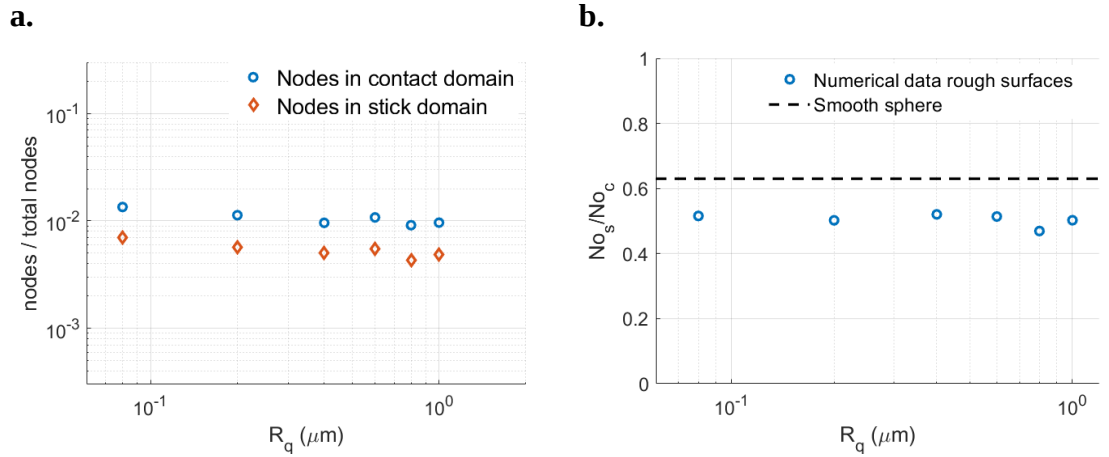


Figure 3-16 Simulation results under the specified R_q : (a) ratios of nodes in contact and stick regions to the whole simulation domain and (b) ratio of nodes in stick to nodes in contact

The skewness (R_{sk}) and kurtosis (R_{ku}) of the generated surfaces shown in Figure 3-15 were calculated to study their effects on the separation of stick and slip regions in the contact area. As shown in Figure 3-18 (a) and Figure 3-19 (a), the relationship between the ratio of the stick or contacting regions to the whole computational domain and the skewness (R_{sk}) or kurtosis (R_{ku}) seems to be not following any trend as expected.

Besides, the ratio of the stick area to the contacting area tends to remain around 0.5 regardless of the change of these two parameters shown in Figure 3-18 (b) and Figure 3-19 (b).

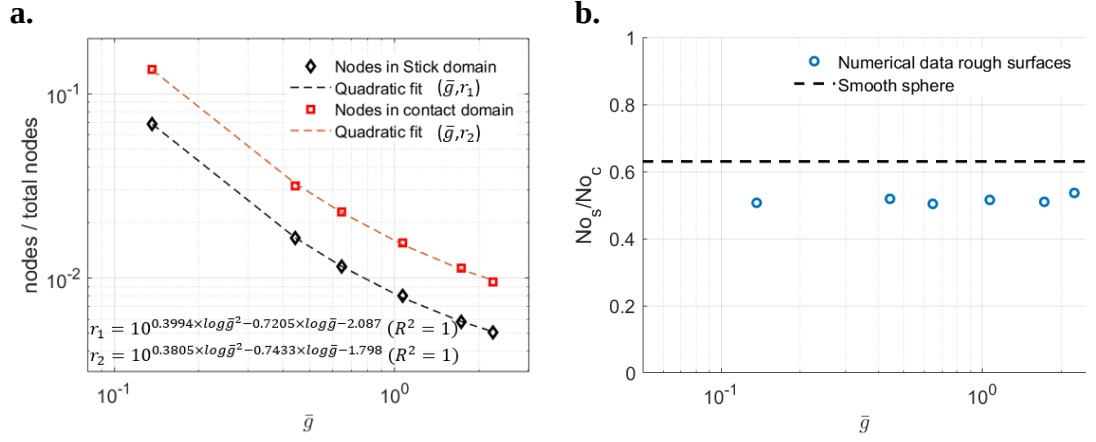


Figure 3-17 Simulation results under the specified $\bar{g}$: (a) ratios of nodes in contact and stick regions to the whole simulation domain and (b) ratio of nodes in stick to nodes in contact

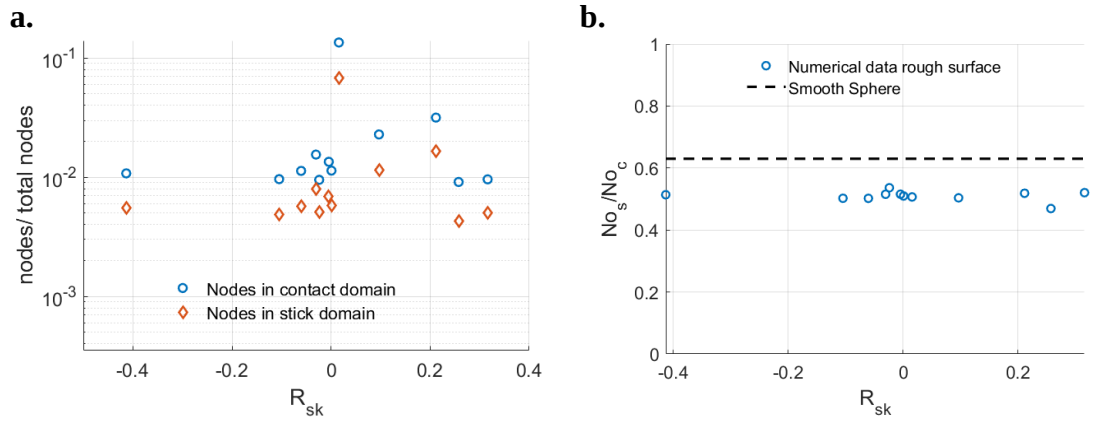


Figure 3-18 Simulation results with varying R_{sk} : (a) ratios of nodes in contact and stick regions to the whole simulation domain and (b) ratio of nodes in stick to nodes in contact

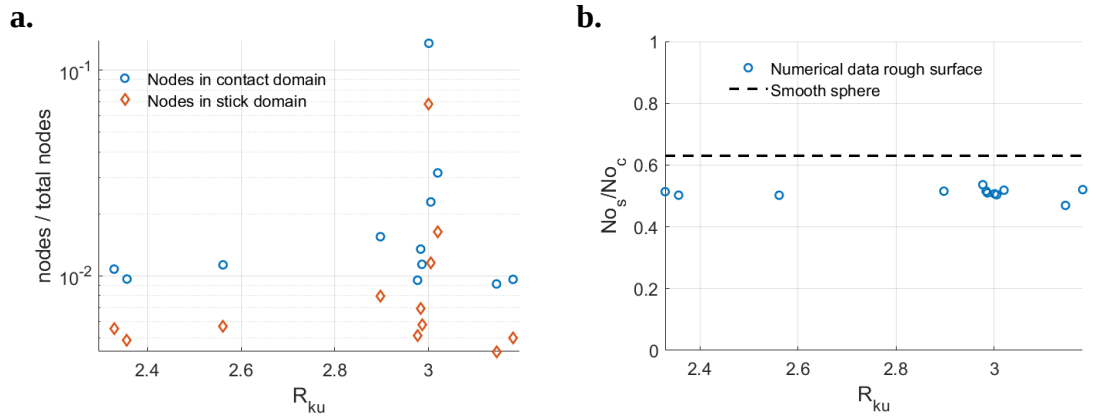


Figure 3-19 Simulation results with varying R_{ku} : (a) ratios of nodes in contact and stick regions to the whole simulation domain and (b) ratio of nodes in stick to nodes in contact

To explain the effect of the RMS roughness on the separation of the stick and slip regions, the simulation results of the rough surfaces 1 and 5 with RMS roughness of $1\mu\text{m}$ and $0.2\mu\text{m}$ respectively were compared and results are presented in Figure 3-20 and Figure 3-21 correspondingly. It was observed clearly that the contacting patches become smaller and more widely distributed with decreasing R_q from the amplified figures. This results from the fact that only the highest asperities have the capacity to interact with the opposite plane for the rough surface contact under small loads, which could be corresponded to the relevant rough surface profile. To achieve the same RMS gradient value for the two surfaces, the short cut-off wavevector of the rough surface 5 was adjusted to a high value bringing about the surface profile with small-size and wide-distributed asperities. Although it leads to more contacting asperities, the size of each asperity becomes smaller. As a result, the size of the total contacting region remains almost constant due to the balancing of these effects. Therefore, it can be drawn that what affects the contacting/stick region is the synergy of the short cut-off wavevector and the RMS roughness rather than RMS roughness itself, which was also highlighted in some recent theoretical work by Ciavarella et al. [49] and Violano et al. [54].

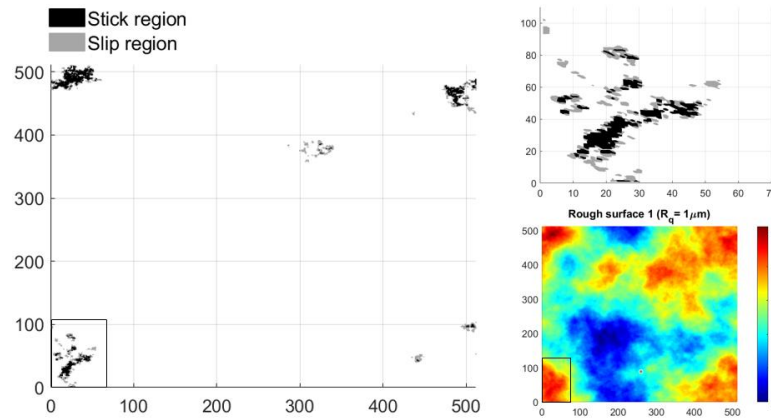


Figure 3-20 Simulation results of the rough surface 1 with R_q of $1\mu\text{m}$

Regarding the role played by the RMS gradient, the surface asperities were found to become smaller and were distributed more intensively when the gradient increases through the comparison between the simulation results of rough surfaces 7 and 10 shown in Figure 3-22 and Figure 3-23 respectively. With the increased RMS gradient, it was found that there tend to be more individual contact spots for surface 10 as shown in Figure 3-23 compared to the relatively large contact islands for the surface 7 with smaller RMS gradient shown in Figure 3-22. However, the height of asperities in rough surface 10 was observed to become higher according to the colour bars of the two rough surface profiles, which causes a decline in the number of asperities coming into contact. Under the

significant effects of the RMS gradient, there is no clear relation between the contact/stick regions and the skewness/kurtosis of the rough surface shown in Figure 3-18 (a) and Figure 3-19 (b) since the RMS gradient of these rough surfaces with increasing skewness/kurtosis changes in an irregular way.

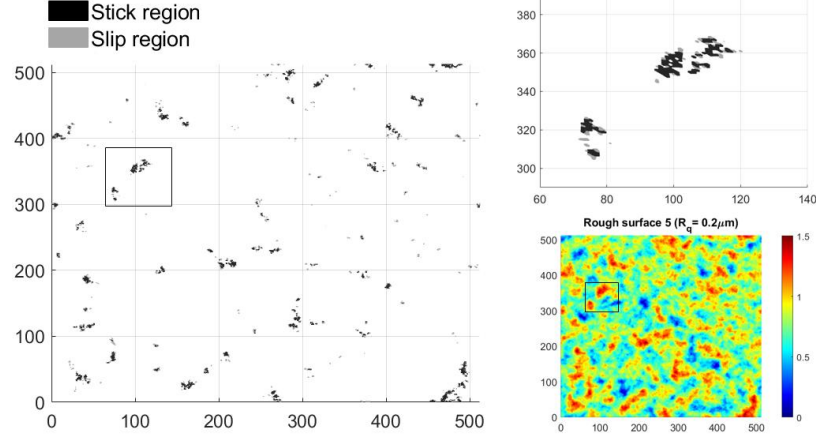


Figure 3-21 Simulation results of the rough surface 5 with R_q of $0.2\mu\text{m}$

Regardless of the response of the contacting and the stick regions to the variation of the four investigated surface roughness parameters, the ratio of the stick region to contacting region fluctuates around 0.5. Besides, the distribution of the stick and slip areas within the contact zone was found to always follow this similar pattern that for each contact spot, the slip region exists around the stick region as observed from the amplified figures shown in Figure 3-20-Figure 3-23. Inspired by the same value shared by the ratio of stick region to contact region and the ratio of the input tangential force to the friction force ($\mu_f W$), it is deduced that the input tangential load plays a dominant role in terms of the varying ratio of the stick region to contacting region for rough surface contacts irrespective of the surface roughness. To validate this opinion, the following study about the evolution of contacting region in stick with increasing tangential load was conducted.

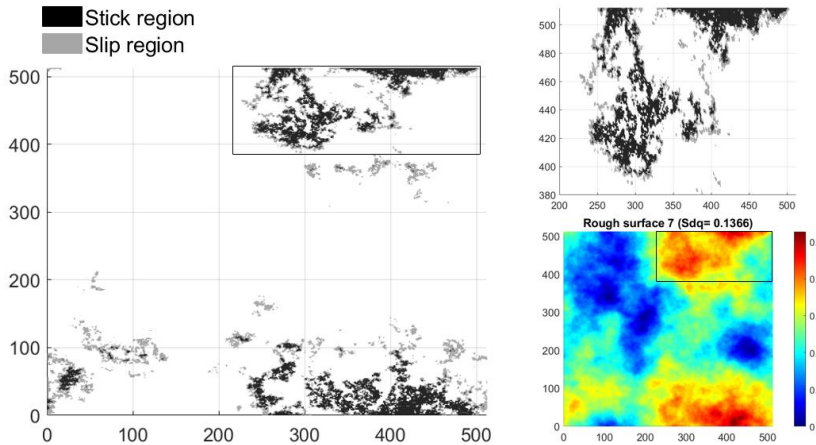


Figure 3-22 Simulation results of the rough surface 7 with $\bar{g}$ of 0.1366

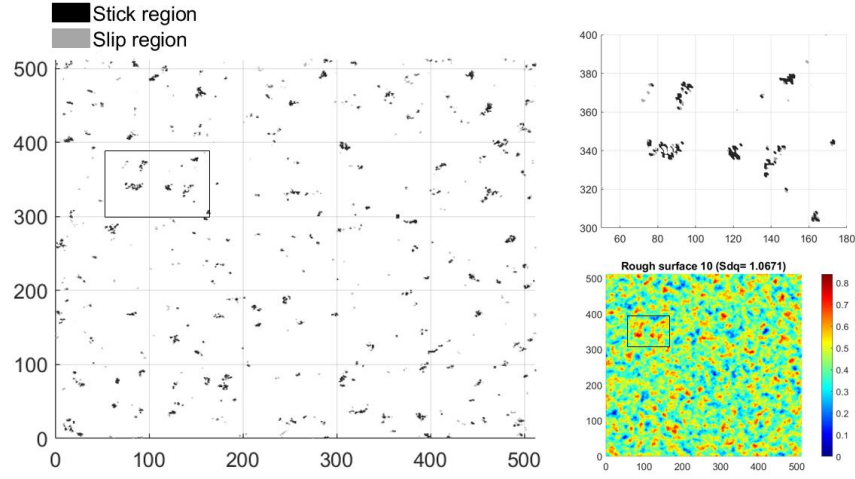
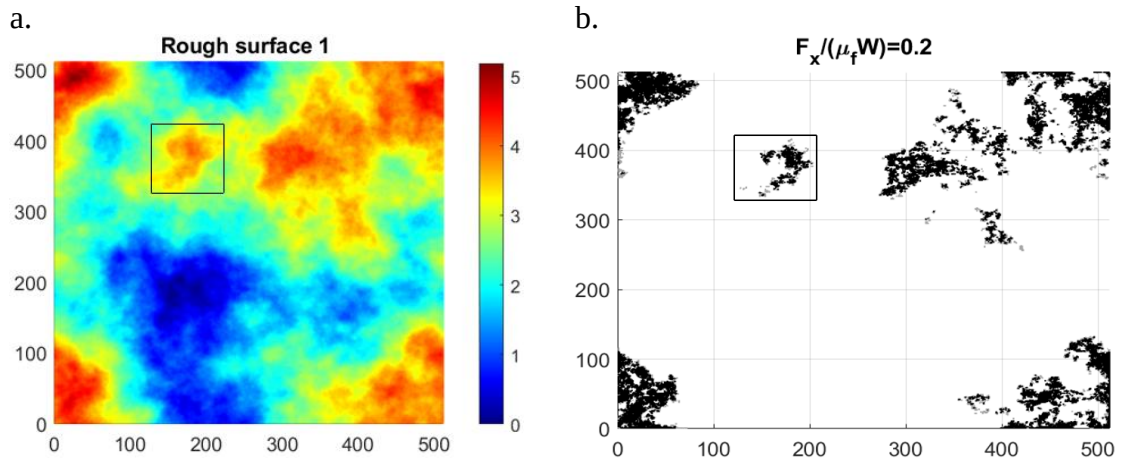


Figure 3-23 Simulation results of the rough surface 10 with $\bar{g}$ of 1.0671

3.4.2.3 Evolution of Stick Regions of Rough Surfaces

The contact between the rough surface 1 shown in Figure 3-15 (a) against a smooth plane under the specified material properties shown in Table 3-4 was first simulated while the input normal load was adjusted to 0.0025N. To study the effects of increasing tangential load on the separation of stick and slip regions for the contacting rough surfaces, the tangential load was specified to vary from 0 to $1.0\mu_f W$ with a constant interval of $0.1\mu_f W$. The separation of the stick and slip region under the tangential force of $0.2\mu_f W$ is shown in Figure 3-24 (b), where the contact regions consisting of the stick and slip regions increase due to the higher normal load compared with Figure 3-20. The slip regions appear at the border of the stick regions as expected and are shown in the amplified Figure 3-24 (d) of the selected asperities, which is consistent with findings in the case of the single-asperity tangential contact and multi-asperity studies discussed in subsections 3.4.1 and 3.4.2.2. This common phenomenon suggests that the stick-slip contact problem between a rough surface against a plane can be treated as that for numerous individual asperities against a rigid plane. As a result, for each asperity in the contact state, there is a separation between the stick region and slip region, where the latter is always around the periphery of the former.



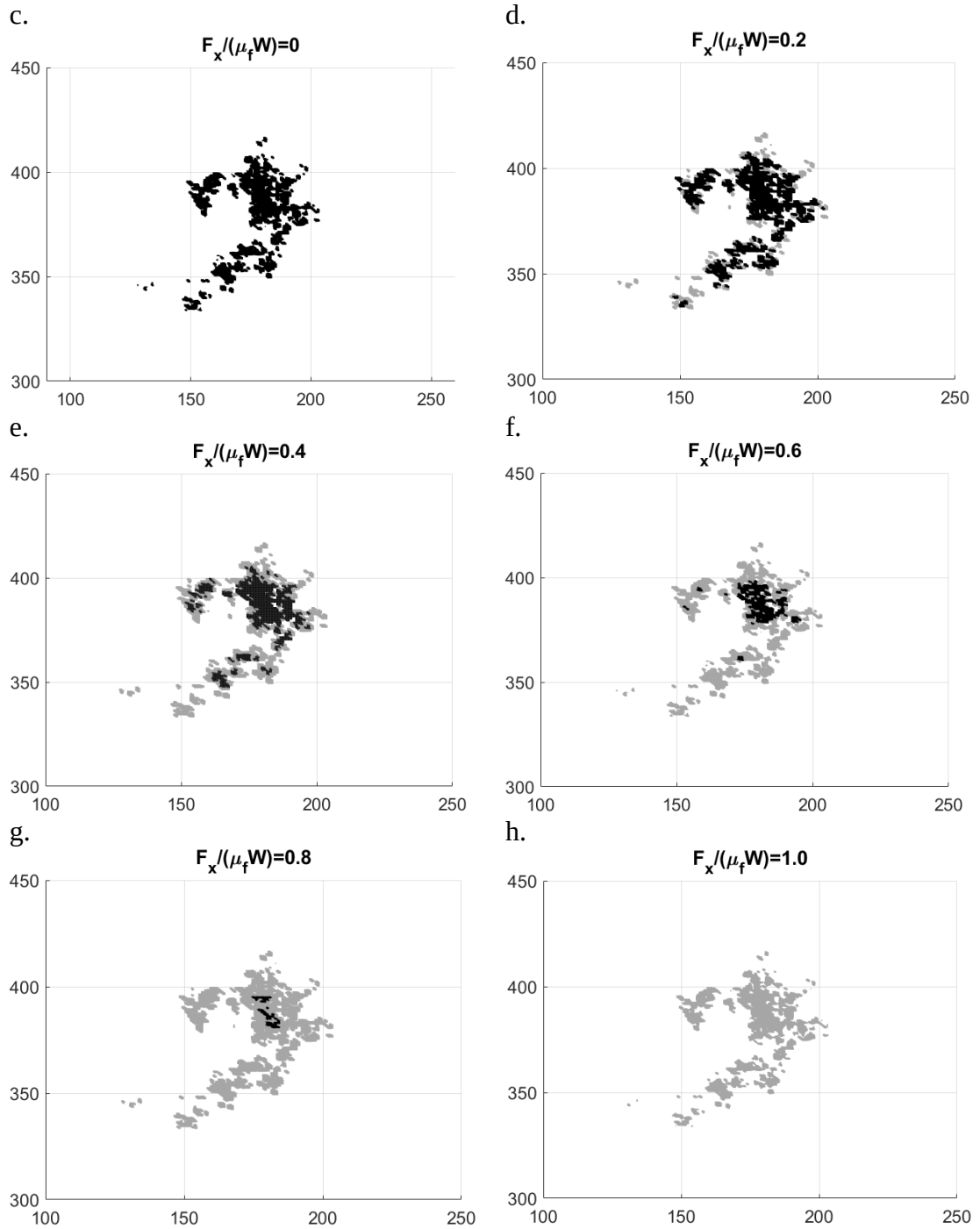


Figure 3-24: (a) Generate rough surface 1, (b) Simulation result under the tangential force of $0.2\mu_f W$ and (c)-(h) Separations of the stick and slip regions for the amplified sections under the increasing tangential loads

When the input tangential load increases, the separation of the stick and slip regions within the contacting area changes. The transition of the stick region to the slip region with increasing tangential load for the selected asperities was observed in those amplified figures (Figure 3-24 (c)-(h)). The evolution of the ratio of the nodes in stick to the nodes in contact with increasing tangential load is shown in Figure 3-25, where the stick region was found to decrease with increasing tangential load for this specified range. The decline

was found to be linear according to the trend line using linear fitting ($\frac{NO_s}{NO_c} = 1 - \frac{F_x}{\mu_f W}$).

For the purpose of robustness, the rough surface 3, 7 and 9 were also investigated to check if they have the same response where the input normal loads for these studies were modified to 0.0015N, 0.0003N and 0.0002N respectively to obtain different contacting areas. A good agreement between the simulation results of the four conducted cases could be observed from Figure 3-26. Besides, the ratio of the stick region to contacting region for the rough surface contacts was found to be always lower than the one for the smooth sphere contact as the relationship between the ratio and the applied tangential load changes from non-linear (smooth) to linear (rough) as shown in Figure 3-26. This explains the former finding of the decreased ratio of the stick regions to contact regions for the rough surface contact compared to the smooth sphere contact in the subsection 3.4.2.2.

It is noteworthy that the linearity between the ratio of the stick region to the contacting region and the tangential force for multi-asperity contacts was also found and validated by Paggi et al. [55] through their analytical analysis based on the Greenwood and Williamson model and the numerical simulation developed by Pohrt and Li [18]. Instead of the four investigated roughness parameters in the paper, the Hurst exponent of the rough surface was varied in their study. However, there was no physical interpretation presented about this transition (from non-linear to linear) for the relationship between the ratio of the stick region to contacting region and the tangential load.

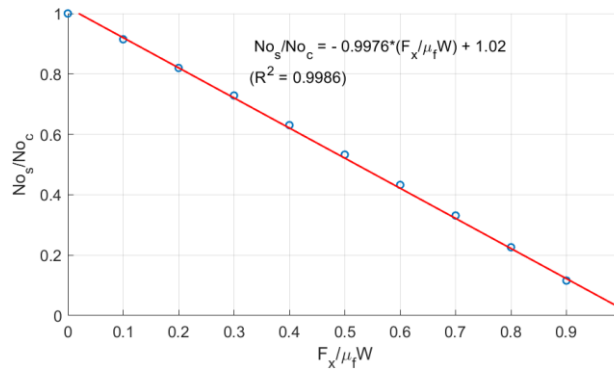


Figure 3-25 Evolution of the ratio of nodes in stick to nodes in contact with increasing tangential load

It must be noted that although the developed stick-slip model that follows the local Coulomb's friction law to determine the critical shear stress could provide valid simulation results, there exist several drawbacks of the principle. According to Chen and Wang [50], the tangential traction exceeding the shear strength of the material is unrealistic in practical problem. Besides, the constant coefficient of friction in the applied Coulomb's law fails to explain the variation trend of μ_f with the normal load and the

surface roughness such as the Euler's interlocking asperity theory stating that the friction increases with the gradient of the surface asperity. As Chen and Wang suggested [50], the local shear stress deriving from the Tabor equation could be used as the alternative cut-off limit of the shear traction and this step would be taken in the future.

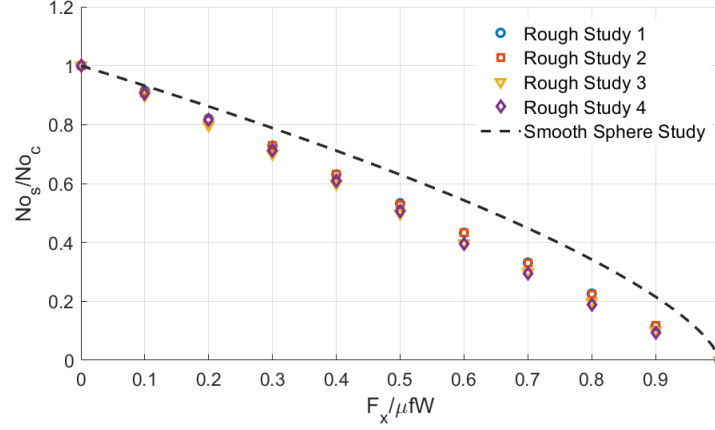


Figure 3-26 Evolution of the ratio of nodes in stick to nodes in contact with increasing tangential load for different rough surfaces under different normal loads

In addition, the validation of the developed model is all based on the well-established analytical solutions. The single-asperity contact was validated by the Hertzian-type (sphere-contact) stick-slip solutions. As to its reliability for multi-asperity contact problems, only the normal contact component of the model was validated by Muser's solution to rough sphere contact problems for lack of any analytical solutions and available experimental data for this type of partial slip contact problems. Although theoretically there should be no restrictions on the shape or configuration of contacting bodies, the validation work needs to be extended to consider different contact configuration such as cylinder contact and include several experimental work in the future to verify the capability of the model strictly.

Another future work is to further the developed stick-slip model from purely elastic contact to viscoelastic contact considering that viscoelastic materials have been extensively used in many engineering fields. When it comes to viscoelastic contact problems, the adhesive force plays a non-trivial role owing to the low contact compliance and soft feature of viscoelastic materials. Therefore, the effects of adhesion must be taken into consideration during the viscoelastic contact modelling. A viscoelastic normal contact model has been already developed by following the elastic-viscoelastic correspondence principle and validated by relevant analytical solutions. Its extension to a viscoelastic partial slip problem is the subject of current ongoing work.

3.5 Conclusion

In this work, a numerical model without any dependence on analytical contact formulas was developed to simulate the stick-slip contact of two elastically similar surfaces. By following the Coulomb's law described in the stick-slip theory of Ciavarella, a new algorithm to accommodate the complementary condition was developed to identify the stick and slip regions within the contacting area. Although only results of the load-controlled algorithm are presented in the paper, it is quite convenient to achieve the displacement-controlled ones for rough surface contact problems. Owing to the application of well-estimated computational techniques including DC-FFT and CGM, this model can provide valid solutions to the stick-slip contact problems in a short time (normally 40 seconds for smooth sphere contact problems and 140 seconds for rough surface contact problems with 512×512 nodes using a desktop PC).

Regarding the study of rough surfaces based on the developed partial slip model, the following conclusions are drawn:

- The contacting region and the stick region are unaffected by the RMS roughness of rough surfaces under certain loads (normal and tangential) while the RMS gradient is kept constant. This can be achieved by altering the short cut-off wavevector and RMS roughness simultaneously;
- Unlike the role of the RMS roughness, the RMS gradient evidently affects the contact region and stick region such that a higher RMS gradient leads to the decline of the contact area and the stick regions under constant load; an equation is proposed to identify the region of stick and the region of total contact with respect to RMS slope.
- Under the significant effects of the RMS gradient, the relationship between the skewness or kurtosis and the contacting/stick region is irregular;
- The ratio of the stick area to the total contacting area depends on the applied tangential load and is insensitive to the four roughness parameters (RMS roughness, RMS slope, skewness and kurtosis). Instead of the non-linear trend in the smooth Hertzian-type partial slip contact, the relationship between the ratio of the stick to contacting regions and the tangential load becomes linear when it comes to rough surface contacts. This is an important finding since identifying the ratio of stick and slip is important in determining friction and wear in many engineering applications.

Acknowledgement

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Appendix 3A. A Full Solution of the Boussinesq-Cerruti integral

In this appendix, the influence coefficient matrices were obtained via the full solution of the Boussinesq-Cerruti integral equations. The discretized form of solutions was achieved through integrating over the small rectangular mesh area of $2a \times 2b$ as follows:

$$C^{zz} = \frac{1-\nu}{2\pi G} \left\{ (x+a) \ln \left[\frac{(y+b)+\sqrt{(y+b)^2+(x+a)^2}}{(y-b)+\sqrt{(y-b)^2+(x+a)^2}} \right] + (y+b) \ln \left[\frac{(x+b)+\sqrt{(y+b)^2+(x+a)^2}}{(x-a)+\sqrt{(y+b)^2+(x-a)^2}} \right] + (x-a) \ln \left[\frac{(y-b)+\sqrt{(y-b)^2+(x-a)^2}}{(y+b)+\sqrt{(y+b)^2+(x-a)^2}} \right] + (y-b) \ln \left[\frac{(x-a)+\sqrt{(y-b)^2+(x-a)^2}}{(x+a)+\sqrt{(y-b)^2+(x+a)^2}} \right] \right\};$$

$$C^{yy} = \frac{1}{2\pi G} \left\{ (x+a) \ln \left[\frac{(y+b)+\sqrt{(y+b)^2+(x+a)^2}}{(y-b)+\sqrt{(y-b)^2+(x+a)^2}} \right] + (1-\nu)(y+b) \ln \left[\frac{(x+b)+\sqrt{(y+b)^2+(x+a)^2}}{(x-a)+\sqrt{(y+b)^2+(x-a)^2}} \right] + (x-a) \ln \left[\frac{(y-b)+\sqrt{(y-b)^2+(x-a)^2}}{(y+b)+\sqrt{(y+b)^2+(x-a)^2}} \right] + (1-\nu)(y-b) \ln \left[\frac{(x-a)+\sqrt{(y-b)^2+(x-a)^2}}{(x+a)+\sqrt{(y-b)^2+(x+a)^2}} \right] \right\};$$

$$C^{xx} = \frac{1}{2\pi G} \left\{ (1-\nu)(x+a) \ln \left[\frac{(y+b)+\sqrt{(y+b)^2+(x+a)^2}}{(y-b)+\sqrt{(y-b)^2+(x+a)^2}} \right] + (y+b) \ln \left[\frac{(x+b)+\sqrt{(y+b)^2+(x+a)^2}}{(x-a)+\sqrt{(y+b)^2+(x-a)^2}} \right] + (1-\nu)(x-a) \ln \left[\frac{(y-b)+\sqrt{(y-b)^2+(x-a)^2}}{(y+b)+\sqrt{(y+b)^2+(x-a)^2}} \right] + (y-b) \ln \left[\frac{(x-a)+\sqrt{(y-b)^2+(x-a)^2}}{(x+a)+\sqrt{(y-b)^2+(x+a)^2}} \right] \right\};$$

$$C^{zx} = \frac{1-2\nu}{4\pi G} \left\{ (y+b) \ln \left[\frac{\sqrt{(y+b)^2+(x+a)^2}}{\sqrt{(y-b)^2+(x-a)^2}} \right] + (y-b) \ln \left[\frac{\sqrt{(y-b)^2+(x-a)^2}}{\sqrt{(y+b)^2+(x+a)^2}} \right] + (x+a) \left[\tan^{-1} \frac{y+b}{x+a} - \tan^{-1} \frac{y-b}{x-a} \right] + (x-a) \left[\tan^{-1} \frac{y-b}{x-a} - \tan^{-1} \frac{y+b}{x-a} \right] \right\};$$

$$C^{xz} = -C^{zx};$$

$$C^{zy} = \frac{1-2\nu}{4\pi G} \left\{ (x+a) \ln \left[\frac{\sqrt{(y+b)^2+(x+a)^2}}{\sqrt{(y-b)^2+(x-a)^2}} \right] + (x-a) \ln \left[\frac{\sqrt{(y-b)^2+(x-a)^2}}{\sqrt{(y+b)^2+(x+a)^2}} \right] + (y+b) \left[\tan^{-1} \frac{x+a}{y+b} - \tan^{-1} \frac{x-a}{y+b} \right] + (y-b) \left[\tan^{-1} \frac{x-a}{y-b} - \tan^{-1} \frac{x+a}{y-b} \right] \right\};$$

$$C^{yz} = -C^{zy};$$

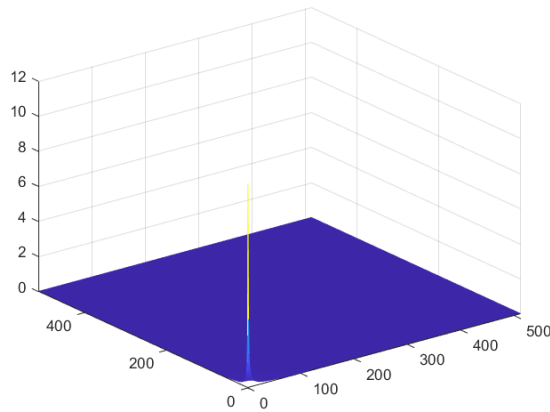
$$C^{xy} = \frac{\nu}{2\pi G} \left[\sqrt{(y-b)^2+(x+a)^2} - \sqrt{(y-b)^2+(x-a)^2} + \sqrt{(y+b)^2+(x-a)^2} - \sqrt{(y+b)^2+(x+a)^2} \right];$$

$$C^{yx} = C^{xy}.$$

Appendix 3B Comparison between Influence Coefficients

In this appendix, the material properties including the elastic modulus and Poisson's ratio given in Table 3-2 were used to determine the influence matrix C_{xx} and C_{yx} . The relatively large difference in the order of magnitude between these two is shown in Figure 3-27.

a.



b.

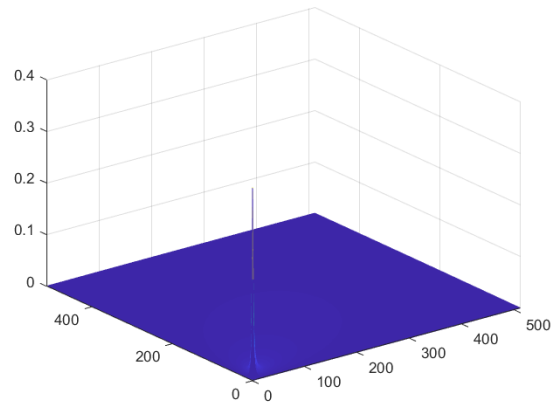


Figure 3-27 Comparison of the influence matrices: (a) C_{xx} and (b) C_{yx}

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4 A Fully Coupled Normal and Tangential Contact Model to Investigate the Effect of Surface Roughness on the Partial Slip of Dissimilar Elastic Materials

4.1 Abstract

By adopting a global search method for all shear tractions in the contacting area and using an alternative convergence criterion regarding load balance in the lateral direction, the newly developed algorithm provides stable solutions to partial slip problems of elastically dissimilar materials. The model is validated via the comparison between the simulation and literature results for a sphere-on-flat problem under fully coupled conditions. It is then employed to investigate the influence of surface roughness parameters including the root mean square (RMS) roughness and RMS slope on partial slip solutions under coupled conditions. Since the gross sliding condition is modified under coupling effects, the relationship between the ratio of the stick area to the contacting area (stick ratio) and tangential load in the coupled case, unlike that in the uncoupled one, becomes non-linear for all tested rough surfaces. Under low or medium tangential loads, the surface with a higher RMS gradient or a lower RMS roughness experiences more stick regions within the contacting area. This trend then becomes irregular at higher tangential loads.

4.2 Introduction

Frictional contact has always been an area of concern within the framework of contact mechanics since the phenomenon of the relative motion being impeded by friction is commonly encountered in practice when two contacting surfaces are subjected to tangential loads. Common examples of such a scenario are the rubber friction in automotive vehicles [1] and fretting wear in orthopaedic implants [2]. When the tangential load is not high enough to induce gross sliding, the interaction zone between surfaces is separated into localised stick and slip regions. The solution to this so-called ‘partial slip’ problem depends on the properties of both materials in contact. When they are identical as assumed in the well-known Cattaneo-Mindlin solution [3, 4] or where the two materials are incompressible, the normal and tangential contact problems are decoupled. In this case, the normal pressure causes no relative tangential displacement and correspondingly the shear traction does not impose any normal displacement. However, when it comes to the contact of two dissimilar and compressible materials, there exists mutual interaction between the normal pressures and shear tractions. As the corresponding parts of the two

surfaces experience different tangential displacements arising from the mismatch of their elastic responses, a normal load could solely cause a certain tangential displacement, which is resisted by friction and consequently results in shear tractions and micro-slip within the contacting areas.

The development of a closed-form analytical solution to such a problem can be troublesome because of the interrelation of the normal and tangential contact problems. A method known as the Goodman approximation [5] was frequently applied in early stick-slip analysis, which neglects the effect of shear tractions on normal pressure but takes that of pressure on tangential tractions into account. The normal contact problem is uncoupled from the tangential one in this way. Due to this limitation, the solution based on such an approximation is only valid for plane (cylindrical) contact problems in which the material Poisson's ratios are large, as according to Hills' later analysis [6]. Spence [7] adopted an iterative scheme to analyse the normal indentation contact of dissimilar materials in detail, where the indenter was limited to be a rigid and flat-ended punch. The solution to a spherical indentation problem was later developed by Spence [8] through a transformation technique. Hills [9] derived a similar coupled solution to elastic cylindrical contact problems and analysed the relevant fretting contact problem under oscillating tangential loading conditions.

Considering the simplified contact geometries in analytical solutions along with their mathematical complexity during applications, recently great efforts have been made to solve the coupled partial slip problem numerically, for which the boundary element method (BEM) has been widely applied. Based on the Gauss-Seidel method, Willner [10] developed a fully coupled model for the point contact of dissimilar elastic materials. A similar coupled model was developed by Chen and Wang [11] using the conjugate gradient method (CGM). Their model was later extended to analyse multi-layered fretting contact problems with the addition of twisting moments by Wang et al. [12-14]. They also investigated the effects of inhomogeneous materials [15] and elastoplasticity [16] on the stick-slip behaviour. Different CGM-based algorithms were proposed by Gallego [17] and Spinu [18, 19] to investigate the fretting contact of distinct elastic materials. The former was then further developed by Leroux and Nelias [20] to analyse the stick-slip between a rigid sphere and an inhomogeneous cylinder.

The studies described above each investigated the partial slip of the corresponding smooth elastic surfaces only. Nevertheless, it has long been recognized that surface roughness plays an essential role in contact problems of real engineering surfaces. In terms of

uncoupled rough surface contact, fretting wear analysis of sinusoidal rough surfaces was initiated by Ciavarella [21], in which a closed-form analytical solution to the shear traction distribution was proposed. Kasarekar et al. [22] investigated how fretting wear evolves in a rough circular stick-slip contact, where the effect of surface roughness was highlighted in their wear calculation. A similar uncoupled stick-slip contact analysis was conducted by Dini and Hills [23], who focused on the relationship between the roughness and frictional energy dissipation in a rough Hertzian-type contact problem. The tangential stiffness and energy loss in a rough fretting contact were investigated by Eriten et al. [24] using a physics-based frictional contact model, for which the effectiveness of provided solutions is limited to small loads. A study concerning the fretting wear of rough point contacts was conducted by Lehtovaara and Lonnqvist [25], who reported that the ratio of the tangential force to the static friction controls the separation of stick and slip regions under uncoupled partial slip conditions. A similar finding of the linear relationship between the ratio of the stick area to the contacting area and the ratio of the applied tangential load to the static frictional load was reported in the studies of Paggi et al. [26] and Wang et al. [27]. Based on their respective BEM models, the effect of the Hurst exponent of rough surfaces on the transition of the contact state (from full-stick to full-slip) was analysed by Paggi et al. [26] while Wang et al. [27] conducted similar research focusing on the role of the root mean square (RMS) roughness, RMS slope, skewness and kurtosis. Effects of the RMS roughness on the preliminary displacement (the displacement at which full slip starts) in the frictional contact of self-affine fractal surfaces have also been analysed by Grzemba et al [28], where the preliminary slip of surfaces with a longer cut-off wavelength was found to have an order of magnitude of the RMS roughness multiplied with the coefficient of friction.

The reported research on the coupled partial slip contact of rough surfaces is relatively limited compared to the uncoupled approaches as discussed. Chen and Wang [29] simulated the partial slip of a rough sphere against a flat half-space, where the rough surfaces were generated using a digital filtration algorithm. The relationship between the surface roughness and the static friction coefficient was analysed using their coupled stick-slip algorithm. Similar to Chen and Wang [29], Bazrafshan et al. [30] included surface adhesion in the partial slip contact problem and reported the effects of adhesion and surface roughness on the contact solution. It was reported in their study that the surface adhesion increases the pre-sliding distance and the static friction while the surface roughness only affects the former factor. To date, the contact of two dissimilar elastic

rough surfaces remains an under-exploited field of study. Few studies shed light on the role of surface roughness in the separation of stick and slip regions under coupling circumstances. This is the problem we intend to address in more detail in the current paper.

Inspired by the coupled partial-slip algorithm of Chen and Wang [11], a three-level-iteration model is developed to solve non-conformal contact problems of two dissimilar elastic bodies with arbitrary geometries and surface roughness. Within our framework, the shear traction in the stick region must be less than the local Coulomb friction. CGM is used as the relaxation method in the model considering its guaranteed convergence for quadratic optimization problems. A global adjustment method for all shear tractions within the contacting area is adopted during the design of the CGM-based algorithm. This novel approach to updating contact tractions helps to avoid oscillating results at the interface between stick and slip regions. The Discretised Convolution Fast Fourier Transform (DC-FFT) is used in the algorithm to accelerate the relevant convolution operation. The combination of these two computational techniques (DC-FFT and CGM) has been widely applied in many numerical modelling of contact problems in the past few decades [31]. To avoid convergence issues arising from the lack of enforcement of load balance in the tangential direction, which could be encountered in traditional models [11, 12, 17, 32], a new alternative criterion was proposed here to improve the stability of the algorithm. The model is well validated by simulating a sphere-on-flat problem under coupled partial slip conditions, to which the relevant solutions have been reported previously by Chen and Wang [11]. It is then extended to simulate the coupled partial slip of rough surfaces. By comparing the modelling results with our former uncoupled study [27], the influence of surface roughness on the contact solutions is investigated when coupling effects are included.

4.3 Methods

On the basis of the coupled relationship between surface displacements and contact tractions, the following fully deterministic partial slip contact model has been developed, which refers to Chen and Wang's [11] CGM algorithm for the point contact of dissimilar materials.

4.3.1 Problem Formulation

A typical partial-slip contact problem of a smooth sphere (Body 1) against a flat elastic half-space (Body 2) is illustrated in Figure 4-1 (a), in which the material properties of the two bodies (E_1, E_2, ν_1, ν_2) are dissimilar. The x and y axes are positioned laterally on the

surface while the z axis is perpendicular to those and normal to the surfaces. The sphere behaves as an indenter to compress the half-space under a normal force (W). Meanwhile, the tangential loads (F_x, F_y), which are parallel to the x and y axes respectively, are applied in the plane of contact aiming to generate a relative movement between the two bodies. The surface interaction resulting from the input load brings about contact tractions (q_x, q_y, p) and surface displacements (u_x, u_y, u_z). The relations between these tractions and displacements at any point of the surface are summarised by Wang and Zhu [31] as follows:

$$\begin{aligned} u_x(x, y) &= G_{xx} * q_x + G_{xy} * q_y + G_{xz} * q_z, \\ u_y(x, y) &= G_{yx} * q_x + G_{yy} * q_y + G_{yz} * q_z, \\ u_z(x, y) &= G_{zx} * q_x + G_{zy} * q_y + G_{zz} * q_z, \end{aligned} \quad \text{Equation 4-1}$$

in which the operation ‘ $*$ ’ denotes the continuous convolution and G_{ij} ($i, j = x, y, z$) denotes the Green function. Physically, the Green function G_{ij} characterises the displacement on the i axis caused by the unit traction on the j axis and mathematically, it is affected by the material properties of the two contacting bodies and the position of the surface point being considered. The continuous formulas to determine all the relevant Green functions are given by Wang and Zhu [31] and shown in Appendix 4A.

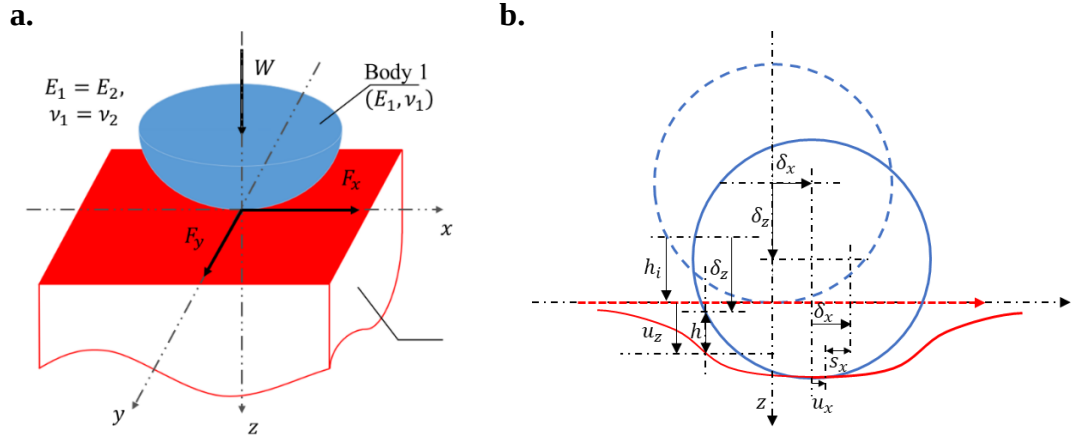


Figure 4-1 (a) Geometry of the contacting bodies: W, F_x, F_y are the input normal load and tangential loads in x and y directions respectively, E and ν are the elastic modulus and Poisson's ratio of the material respectively with subscripts denoting the independent bodies, and **(b) Displacement condition on the $x - z$ plane:** h_i, h denote the gaps between the unloaded surfaces and deformed surfaces respectively, s_x is the slip distance in the x direction, δ is the rigid body displacement, and u is the surface deformation with subscripts denoting the direction of relevant vectors.

For the sake of simplicity, contact is assumed to be purely elastic. Besides, Coulomb's friction law is applied and the coefficient of friction μ_f is assumed to be constant in the

following formulation and modelling work. According to this friction law, every point within the contacting area is either in a stick state, in which no relative motion occurs for the corresponding parts of the surface and the shear traction is of magnitude less than $'\mu_f p'$, or in a slip state, in which there exists relative tangential motion and the shear traction opposes the instantaneous direction of such slip motion with magnitude $'\mu_f p'$. It should be noted that partial slip is intrinsically an incremental process, to which the solution depends on the loading history. The separation of the stick and slip regions is expected to be identified based on the change in magnitude and direction of tangential loads. In this way, the state determined for any point within the contacting area on a loading curve relies on the previous solution despite a purely elastic analysis being conducted. In such context, one of the prerequisites for the effectiveness of the partial-slip solution provided in the manuscript is that the load (normal or tangential) is applied at a sufficiently slow speed such that the stick-slip contact problem can be treated as a quasi-static process in which inertial effects are negligible. Hence, the system experiences a series of equilibrium states and the contact problem can be reduced to an equivalent static problem. In this case, the linear theory of elasticity can be applied and the final state of the surface points being assessed can be determined by applying the load in one step. Such an assumption is valid when the loading is monotonic and proportional. As the conducted stick-slip analysis is not load-path dependent, the problem of the dissipative friction and its irreversibility, which requires an incremental application of the oscillating tangential load that occurs in fretting contacts, is not addressed in the paper. It is noted that although the assumptions including a purely elastic contact and a constant coefficient of friction are commonly implemented in numerical studies [10-12, 17-19, 33], they may not be suitable when addressing more realistic contact problems such as the junction growth derived from the elastic-plastic contact of surfaces [34, 35].

To solve the contact problem based on those assumptions described above by using BEM, the potential contact area between the two bodies is meshed into equally spaced rectangular elements with the size of $2a \times 2b$ and number of N_1 and N_2 in x and y directions respectively. The control points for the established grid are the centroids of the elements. To assume the element size is small enough that the contact traction and surface deformation are constant inside any element, the fully-coupled relation between the tractions and displacements can now be expressed in the following matrix form:

$$\begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} = \begin{bmatrix} C_{xx} & C_{xy} & C_{xz} \\ C_{yx} & C_{yy} & C_{yz} \\ C_{zx} & C_{zy} & C_{zz} \end{bmatrix} \begin{bmatrix} q_x \\ q_y \\ p \end{bmatrix}, \quad \text{Equation 4-2}$$

where C_{ij} ($i, j = x, y, z$) is an influence matrix and has a similar physical meaning as that of the Green function but in a discretised form. Via the full solution of the Boussinesq-Cerruti integral equations [36, 37], the closed-form solutions of these influence coefficient matrices are provided by Ghanbarzadeh et al. [38] and given in Appendix 4B.

For the discretised partial-slip contact problems under coupled conditions, a sequence of equations and inequalities summarized by Johnson [39] must be satisfied to obtain valid contact solutions:

1. Static force equilibrium:

Sum of contact tractions must be equal to the specified force in the corresponding direction:

$$W = \Delta \sum_{(i,j) \in I_c} p(i,j), F_x = \Delta \sum_{(i,j) \in I_c} q_x(i,j), F_y = \Delta \sum_{(i,j) \in I_c} q_y(i,j), \quad \text{Equation 4-3}$$

where Δ ($\Delta = 4ab$) is the area of each mesh element, I_c is the domain of contacting area and i and j here are the indices of the element (i,j) ($1 \leq i \leq N_1$ and $1 \leq j \leq N_2$).

2. Geometrical condition of surface displacement:

In the normal direction, the following equation should be satisfied:

$$h(i,j) = h_i(i,j) + u_z(i,j) - \delta_z, (i,j) \in I_p, \quad \text{Equation 4-4}$$

where I_p is the whole computational domain and h_i , h , and δ_z are the initial surface gap before loading, the gap between contacting surfaces under loading and the rigid body indentation respectively as illustrated in Figure 4-1 (b).

In lateral directions, the following equation should be satisfied:

$$\begin{aligned} s_x(i,j) &= u_x(i,j) - \delta_x, (i,j) \in I_c; \\ s_y(i,j) &= u_y(i,j) - \delta_y, (i,j) \in I_c, \end{aligned} \quad \text{Equation 4-5}$$

where s_x and s_y denote the slip distances in x and y directions respectively while δ_x and δ_y denote the tangential displacements in x and y directions respectively as shown in Figure 4-1 (b).

3. The following contact conditions must be met for the two surfaces:

In the normal direction, the Kuhn-Tucker complementary conditions should be satisfied:

$$p(i, j) > 0, h(i, j) = 0, (i, j) \in I_c;$$

$$p(i, j) = 0, h(i, j) > 0, (i, j) \in I_p - I_c, \quad \text{Equation 4-6}$$

here $I_p - I_c$ is the non-contacting region.

The boundary conditions (Equation 6) indicate that problems of surface adhesion ($p < 0$) and penetrable surfaces ($h < 0$) are not accounted for in the current study. In tangential directions, the norm of shear tractions in the stick zone is limited to always be less than the local friction (product of the coefficient of friction μ_f and nodal pressure) and there should exist no slip distance as specified by Coulomb's friction law. As to shear tractions in the slip zone, the norm should be equal to the local friction. In addition, the directions of the shear stress and slip distance should be opposite. Any point within the contacting domain I_c must be in one of the two states (stick or slip) and such a condition is expressed by the following equation:

$$\sqrt{q_x(i, j)^2 + q_y(i, j)^2} < \mu_f p(i, j), |s_x(i, j)^2 + s_y(i, j)^2| = 0, (i, j) \in I_s;$$

$$q_x(i, j) \cdot s_x(i, j) + q_y(i, j) \cdot s_y(i, j) < 0, (i, j) \in I_c - I_s;$$

$$\text{Equation 4-7}$$

$$\sqrt{q_x(i, j)^2 + q_y(i, j)^2} = \mu p(i, j), |s_x(i, j)^2 + s_y(i, j)^2| \neq 0, (i, j) \in I_c - I_s,$$

here I_s is the stick domain and $I_c - I_s$ is the slip domain.

It is noted that although Coulomb's friction law is applied in our model as well as Chen and Wang's model, the actual formulas of the complementary conditions used in the two algorithms are different. The norm of shear tractions in the stick zone is allowed to be equal to the local friction as long as the nodal slip distance vanishes in Chen and Wang's model [11]. A discussion on the different complementary conditions and the potential conflicts caused by the equal sign was given in our former uncoupled partial slip study [27]. To facilitate the design of the later algorithm, we have maintained the same complementary condition applied in our former research.

4.3.2 Algorithm Description

To determine the contact traction distribution and surface deflection in the contacting area, of which the location is not known a priori, iterative methods are frequently applied during the development of contact models. Considering that the mathematical form of the established linear system deriving from Equation 4-4 and Equation 4-5 tends to be a

numerical optimization problem, where the surface gap h and slip distance s are the system residual to be minimized, the linear search method CGM is adopted to develop the partial slip contact model. An overview of the developed 3-level-iteration algorithm based on the mutual adjustment between normal pressure and shear traction is shown in Figure 4-2. The normal pressures and shear tractions are determined in two separate algorithms (normal contact solver and tangential contact solver).

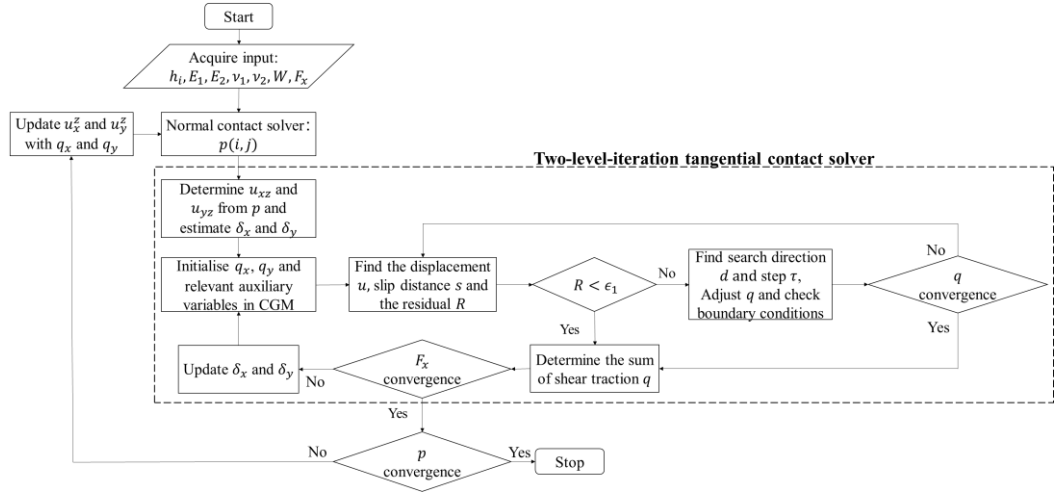


Figure 4-2 Overview of the algorithm for the coupled partial slip solver

As illustrated in Figure 4-2, the search for contact solutions starts with the determination of the normal pressure, where the shear traction vanishes during the normal contact analysis in the first iteration of the outmost loop. Hence, the contact problem being addressed here becomes decoupled temporarily. The CGM-based algorithm employed to search for the pressure solution is that given by the classic model of Polonsky and Keer [40]. Similar to the Goodman approximation [5], the obtained normal pressure is then used as the input to determine the local friction ' $\mu_f p$ ' at each node and to distinguish the existence of slip from that of stick following Coulomb's friction law. The search of shear tractions within the stick and slip regions is executed by the two-level-iteration tangential contact algorithm, and it is detailed as follows:

1. Determine the components of surface displacements in the x and y directions arising from the normal pressure:

$$u_{xz} = IC_{xz} * p, u_{yz} = IC_{yz} * p,$$

Equation 4-8

Estimate the rigid body displacement in the x and y directions based on Johnson's analytical solution [39]:

$$\delta_i = \delta_0 \left[1 - \left(1 - \frac{F_i}{\mu_f W} \right)^{2/3} \right], \quad i = x, y;$$

$$\delta_0 = 3\mu_f W \frac{\left[\frac{(2 - \nu_1)(1 + \nu_1)}{E_1} + \frac{(2 - \nu_2)(1 + \nu_2)}{E_2} \right]}{8a_0}, \quad \text{Equation 4-9}$$

where δ_0 is the tangential rigid body translation when the uncoupled contact is under gross sliding conditions ($F_x = \mu_f W$).

2. Initialise the shear traction q , the search direction d , and the square Euclidean norm of the system residual R :

$$q_k(i, j) = 0, d_k(i, j) = 0, R = 1, \quad k = x, y, (i, j) \in I_p \quad \text{Equation 4-10}$$

It is noted that the initialisation for nodal shear traction is necessary for each iteration of the outmost loop to obtain pressure convergence. The coupling effect changes the contact area and the range of the shear tractions covered must correspond to this change.

3. Determine the complete surface displacements in the x and y directions, the slip distance for each node, and the square Euclidean norm:

$$u_x = u_{xz} + IC_{xx} * q_x + IC_{xy} * q_y, \quad \text{Equation 4-11}$$

$$u_y = u_{yz} + IC_{yx} * q_x + IC_{yy} * q_y,$$

$$s_x = u_x - \delta_x, \quad \text{Equation 4-12}$$

$$s_y = u_y - \delta_y,$$

where the slip distance in the stick region is the system residual, and its square Euclidean norm is to be minimized (approximated to zero) by the CGM algorithm. This is expressed as:

$$R = \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} |s_x(i, j)^2 + s_y(i, j)^2|, \quad (i, j) \in I_s \quad \text{Equation 4-13}$$

If $R < \varepsilon_1$, the inner loop of the tangential algorithm stops and then proceeds to step 7.

4. Determine the search directions (d_x and d_y) as well as search step (τ) for the shear tractions, modify the shear traction using these two variables.

Two different methods are commonly used to search for valid shear tractions. The first approach is to only adjust shear tractions in the stick region and maintain those in the slip region (search direction for the nodes in slip region is 0), such as the algorithms of Spinu [18] and Chen and Wang [11]. The second one is to

conduct a global adjustment for all the shear tractions in the contacting region, for example, see the algorithms of Gallego et al. [17] and Wang et al. [12]. It was reported by Wang et al.[12] and verified by the authors of this manuscript that the latter method provides a more stable solution. A smoother transition from the stick to slip regions was achieved in this way.

Inspired by the method used in Wang's layered contact model [12], the global search direction and step for the shear tractions are calculated as follows:

$$d_k = s_k + d_k \cdot \frac{R}{R_{old}}, k = x, y \quad \text{Equation 4-14}$$

$$\tau = \frac{[s_x \ s_y] \cdot [d_x \ d_y]^T}{[d_x \ d_y] \begin{bmatrix} C_{xx} & C_{yx} \\ C_{xy} & C_{yy} \end{bmatrix} [d_x \ d_y]^T}, \quad \text{Equation 4-15}$$

Shear tractions within the contacting area are adjusted in the following way:

$$q_i = q_i - \tau \cdot d_i, i = x, y \quad \text{Equation 4-16}$$

5. Check the complementary conditions:

If $\sqrt{q_x(i,j)^2 + q_y(i,j)^2} > \mu_f p(i,j)$, the node (i,j) is moved from the stick domain to the slip domain. The nodal shear traction is updated to enforce the norm of the shear traction to be equal to the local friction:

$$q_k(i,j) = \frac{\mu_f p(i,j)}{\sqrt{q_x(i,j)^2 + q_y(i,j)^2}} q_k(i,j), k = x, y \quad \text{Equation 4-17}$$

6. Check the convergence criterion for the shear traction:

If $\Delta \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} (|q_x(i,j) - q_{x_{old}}(i,j)| + |q_y(i,j) - q_{y_{old}}(i,j)|) < \varepsilon_1$, go to step 7.

Or else, move back to step 3.

7. Determine the sum of the shear tractions and check the convergence criterion for the loads in the tangential direction:

$$F'_x = \Delta \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} q_x(i,j), F'_y = \Delta \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} q_y(i,j), (i,j) \in I_p \quad \text{Equation 4-18}$$

If $|F_x - F'_x| + |F_y - F'_y| < \varepsilon_3$, the iteration stops, or else, move to the following step.

8. Update the rigid body displacements in the x and y direction as follows and move back to step 2:

$$\delta_x = \delta_x \frac{F_x}{F_x'}, \delta_y = \delta_y \frac{F_y}{F_y'} \quad \text{Equation 4-19}$$

It is noted that in some contact problems the convergence for the outer loop regarding the load may be difficult to achieve as the load balance is not enforced in this tangential contact algorithm. The unguaranteed convergence speed of this loop relies heavily on how the updated shear tractions and rigid body displacements affect each other. An alternative to the convergence criterion for the load balance is to compare the shear tractions after the updated rigid body displacement (q_x^1, q_y^1) and those before the relevant adjustment (q_x^2, q_y^2) , expressed as:

$$\Delta \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} (|q_x^1(i, j) - q_x^2(i, j)| + |q_y^1(i, j) - q_y^2(i, j)|) < \varepsilon_4 \quad \text{Equation 4-20}$$

By employing Equation 4-20 as the substitute convergence criterion, the convergence speed for the outer loop of the tangential contact solver is improved and the load balance is satisfied.

As indicated in Figure 4-2, pressure convergence is checked as follows after the determination of the shear tractions:

$$\sum_{i=1}^{N_1} \sum_{j=1}^{N_2} |p_{new}(i, j) - p_{old}(i, j)| < \varepsilon_5 \quad \text{Equation 4-21}$$

In the first iteration of the outmost loop, the old pressure p_{old} is the pressure initialised during the normal contact analysis, in which the whole simulation domain is assumed to be in contact with a uniform pressure of $\frac{W}{N_1 \times N_2 \times \Delta}$. And the updated pressure p_{new} is the output of the normal contact analysis. Hence, the pressure convergence can hardly be satisfied. When it comes to the second iteration of the loop, the variable p_{old} becomes the p_{new} in the last iteration. Shear tractions obtained from the tangential contact analysis in the last iteration are then used to determine the resulting component of the normal displacement, a new contact pressure profile p_{new} is subsequently determined in the normal contact analysis with the updated surface displacement. Afterwards, a new tangential contact analysis is conducted with the updated pressure p_{new} . This whole

process keeps running until the convergence criterion of the outmost loop (Equation 4-21) is satisfied.

4.4 Results and Discussion

The non-conformal contact model based on the algorithm described above was validated by comparing the output solutions with the available literature in the case of a smooth contact problem (a sphere against a flat half-surface) under coupled partial slip conditions. It was then extended to multi-asperity contact problems (a nominally flat surface against a flat half-space) to analyse the effects of surface roughness on the separation of stick and slip regions.

4.4.1 Model Validation

To validate the developed model, the partial slip contact between a rigid smooth sphere and a flat elastic half-space (carbon steel) was simulated here, to which the solution is compared to that derived from Chen and Wang's model [11] using the identical material parameters as shown in Table 4-1. The whole computational domain ($3a_0 \times 3a_0$) is discretised by a mesh system with 256×256 uniformly distributed elements. The coefficient of friction μ_f is set to be the value of the Dundurs constant β , which is a parameter characterizing the level of dissimilarity of two contacting materials as given by [11]:

$$\beta = \frac{\frac{(1 + \nu_1)(1 - 2\nu_1)}{2E_1} - \frac{(1 + \nu_2)(1 - 2\nu_2)}{2E_2}}{\frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2}}. \quad \text{Equation 4-22}$$

It is of note that the ratio of the frictional coefficient to the Dundurs constant (μ_f/β) plays an essential role in partial slip analysis. However, in the following contact simulation this parameter is maintained to be unity as the study of its effects is beyond the scope of the current paper.

To simply the contact problem, only a tangential load in the x direction was applied to the two contacting bodies. The simulation results presented are nondimensionalized by the Hertzian solutions to the sphere indentation problem, where the contact tractions and nodal coordinates are normalized by the peak pressure (p_0) and contacting radius (a_0) respectively.

The evolution of stick and slip regions with increasing tangential load is visualized in Figure 4-3. The distribution of the stick region and slip region under no tangential load is similar to that of an uncoupled case, on which occasion the central stick region is

surrounded by a slip annulus. Nevertheless, the stick region starts to recede and it tends to shift to the rear part of the contacting area with increasing tangential load. This transition is peculiar, in that the decreasing stick region keeps moving in the direction opposite to the input load when the load is relatively low and up until it reaches the edge of the contacting area when the load is approximately $0.6\mu_f W$. Once the input tangential load exceeds this value ($0.6\mu_f W$) the stick region shrinks dramatically in the opposing direction until gross sliding is reached.

Table 4-1 Parameters used in the partial slip simulation of smooth sphere contact

Parameter	Value	Description (Unit)
R	18	Radius of sphere (mm)
E_1/E_2	210/ ∞	Elastic modulus of half-space/sphere (GPa)
ν_1/ν_2	0.3	Poisson's ratio of materials
μ_f	0.2857	Coefficient of friction
W	20	Input normal load (N)
a_0	105.373	Hertzian contacting radius (μm)
p_0	860	Hertzian peak normal pressure (MPa)

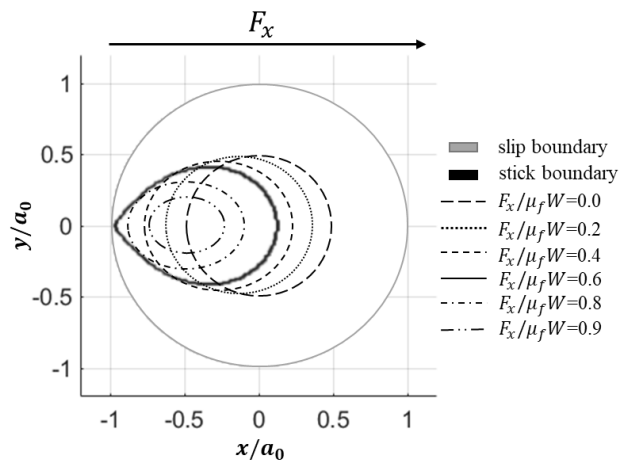


Figure 4-3 Separations of the stick and slip zones with increasing tangential load: the area between the grey line and the dark line is the slip region and the areas bounded by the dark lines are stick regions

Such separation patterns of the stick and slip regions can be expected based on the corresponding distribution of normal pressure shown in Figure 4-4 (a). The area in which comparatively low pressure exists tends to slip as it is easy for the shear traction to reach the limit of local friction following Coulomb's law of friction. The distribution of stick

and slip regions is still of the uncoupled type when only normal load is applied as the pressure distribution in this case is symmetric about x and y axes). However, the increase of the specified tangential load breaks down this symmetry such that it reduces the pressure at the contact edge in the force-leading direction but enhances that in the opposite direction. Hence, shrinking stick regions tend to appear at the contacting edge in the friction trailing direction before vanishing.

Regarding the distribution of shear tractions in the x direction shown in Figure 4-4 (b), apart from the same loss of axial symmetry, there exist both positive and negative shear tractions under smaller tangential loads ($F_x \leq 0.6\mu_f W$) as shown in Figure 4-4 (b). Globally positive shear tractions are obtained when the load is higher than $0.6\mu_f W$.

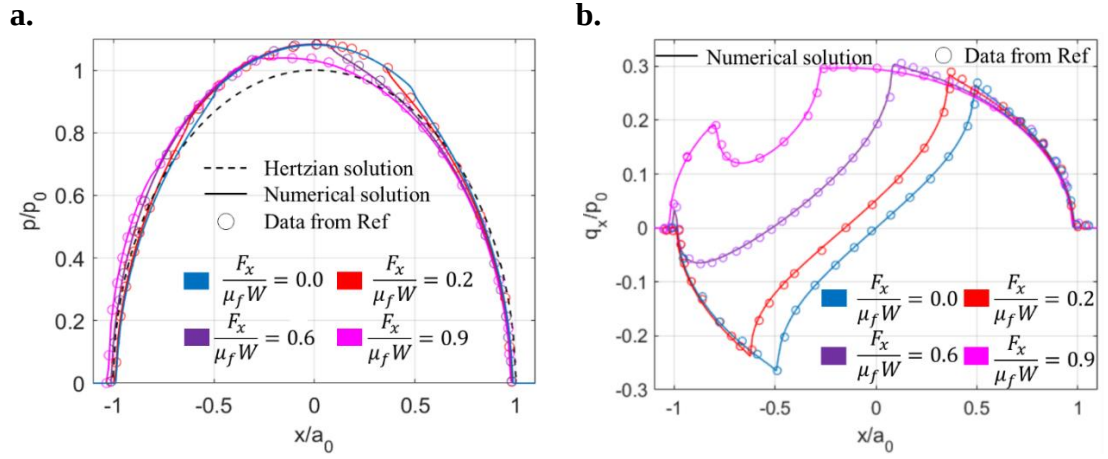


Figure 4-4 Comparison of the non-dimensional contact solutions from the present model (solid lines) with the results from Chen and Wang (scatters) [11]: a. Pressure distribution p (Hertzian solution (dotted line) is also plotted for comparison) and b. Shear traction distribution q_x

The dynamic response of the stick ratio ($\frac{A_s}{A_c}$) to increasing tangential load is shown in Figure 4-5. The load required for a vanishing stick area is found to be lower ($0.9463\mu_f W$) when compared with the uncoupled case. This mainly results from the addition of shear traction in the y direction when identifying the stick and slip region. The gross slip obtained in advance (before $\mu_f W$) and the existing micro-slip under normal load alone illustrated in Figure 4-5 are two essential contact phenomena arising from the coupling between shear traction and normal pressure. It is noted that the difference between uncoupled and coupled solutions to smooth sphere problems depends on the level of the dissimilarity of the two contacting materials (Dundurs constant). As labelled in Figure 4-5, the nonlinear relationship between the stick ratio and ratio of the tangential load to static friction is following an analytical solution under uncoupled circumstances. Assuming the same coefficient of friction, a material combination with a higher Dundurs

constant will exhibit a more significant difference between uncoupled and coupled solutions. Good agreement can be found between the contact solutions including the pressure, shear traction and stick ratio derived from the present model and those from Chen and Wang [11], as indicated in Figure 4-4 (a), Figure 4-4 (b) and Figure 4-5 respectively.

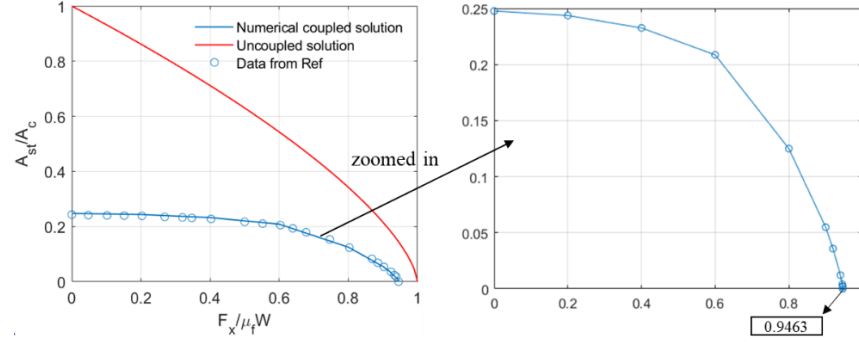


Figure 4-5 Comparison of the stick ratio solutions from the present model (blue solid line) with the results from Chen and Wang (blue scatters) [11]: the uncoupled solution (red solid line) is also plotted for comparison

4.4.2 Rough Surface Study

The current study on the effects of surface roughness on the separation of stick and slip regions under coupled circumstances is an important extension work to our former study which concerned uncoupled situations [27]. Among the known roughness parameters, RMS roughness (R_q) is firstly investigated considering that it is one of the factors frequently employed to quantify the surface roughness and there exist several studies reporting the essential role of RMS roughness in determining the contact area [41, 42], surface adhesion [43, 44], contact friction [29, 30, 45] and material wear [46]. Besides, as reported in our former uncoupled partial slip study [27], the RMS gradient ($\bar{g}$) was found to influence the contact region and stick region, where explicit mathematical formulas were given. Analytical solutions linking the real area of contact to the RMS gradient were also proposed by Pastewka and Robbins [47] and Muser [48] respectively. Thus, the effect of the RMS slope on the coupled partial slip solutions is investigated here as well.

To facilitate the analysis of additional coupling effects in nominally flat-flat contact problems, the rough surfaces used in the following simulations are identical to those that were used previously. The roughness parameters of the twelve surfaces analysed in this study are given in Table 4-2. The relevant surface profiles are shown in Appendix 4C. The reader could refer to reference [27] to review the detail on the fractal method used to generate them.

Table 4-2 Roughness parameters of the twelve investigated rough surfaces

No.	1	2	3	4	5	6	7	8	9	10	11	12
R_q (μm)	1.0	0.8	0.6	0.4	0.2	0.08	0.1	0.1	0.1	0.1	0.1	0.1
$\bar{g}$	1.366	1.366	1.366	1.366	1.366	1.366	0.137	0.443	0.646	1.067	1.725	2.249

Note that in our former uncoupled surface study [27] the nominally-flat rough surfaces are assumed to be incompressible to decouple the normal and tangential contact problems. The coefficient of friction used in the former uncoupled case is different from that used in the current study, as a coefficient of friction ($\mu_f = 0.2857$) related to the Dundurs constant is intended to be used here to facilitate the later comparison between smooth and rough contact problems. Therefore, instead of comparing the magnitudes of simulation results, the trend difference of those contact responses with respect to surface roughness parameters is highlighted when comparing solutions to contact problems of rough surfaces under coupled and uncoupled conditions.

4.4.2.1 Contact of Rough Surfaces under Constant Loads

To investigate the role played by RMS roughness, partial slip problems of rough surfaces with the same RMS slope but different values of RMS roughness (rough surfaces 1-6 given in Table 4-2) were simulated. The material properties and load input are given in Table 4-3.

Table 4-3 Parameters used in the rough partial slip simulation under constant tangential load

Parameter	Value	Description (Unit)
E_1/E_2	18.75/ ∞	Elastic modulus of rough surface/flat plane (MPa)
ν_1/ν_2	0.3	Poisson's ratio of materials
μ_f	0.2857	Coefficient of friction
W	0.0003	Input normal load (N)
F_x	$0.5\mu_f W$	Input tangential load in x direction (N)
L	100	Length of rough surfaces in x and y directions (μm)
$N_1 \times N_2$	512×512	Number of nodes in x and y direction

As shown in Figure 4-6 (a), the contacting areas tend to remain almost unchanged with varying RMS roughness while the relationships between the stick zone and RMS

roughness are different in the coupled and uncoupled cases. Compared with the uncoupled solution, the stick region in the coupled case decreases with the increasing RMS roughness significantly, which leads to the decline of the stick ratio ($\frac{A_s}{A_c}$) with the increasing RMS roughness as shown in Figure 4-6 (b). It must be highlighted that those surface parameters including the RMS slope, RMS roughness and cut-off frequency (q_0) are symbiotic. When the RMS slopes of the rough surfaces 1-6 were adjusted to be identical, the cut-off frequency q_0 of each surface was changed simultaneously. Therefore, it is the synergy of the RMS roughness and the cut-off frequency that really determines the contact area. A similar conclusion was reached in the recent theoretical work regarding rough surface contact by Violano et al. [49] and Ciavarella [44]. Although the interaction between shear traction and normal pressure is considered in the current coupled partial slip simulation, the coupling effects on the contacting area tend to be trivial, which can also be observed in the former study on the smooth sphere contact (Figure 4-3 and Figure 4-4). This explains the insignificant variation of contacting area with the increasing RMS roughness under coupled conditions. On the occasion when the coupling effect is absent, the variation of the contacting region in stick is insensitive to the change of the surface roughness as shown in Figure 4-6 (b), where the stick ratio fluctuates around 0.5. Different from the response of the contacting area, the separation of stick and stick regions is more vulnerable to the coupling effects since the contact condition of gross sliding and the distribution pattern as well as the magnitude of shear tractions are now altered significantly.

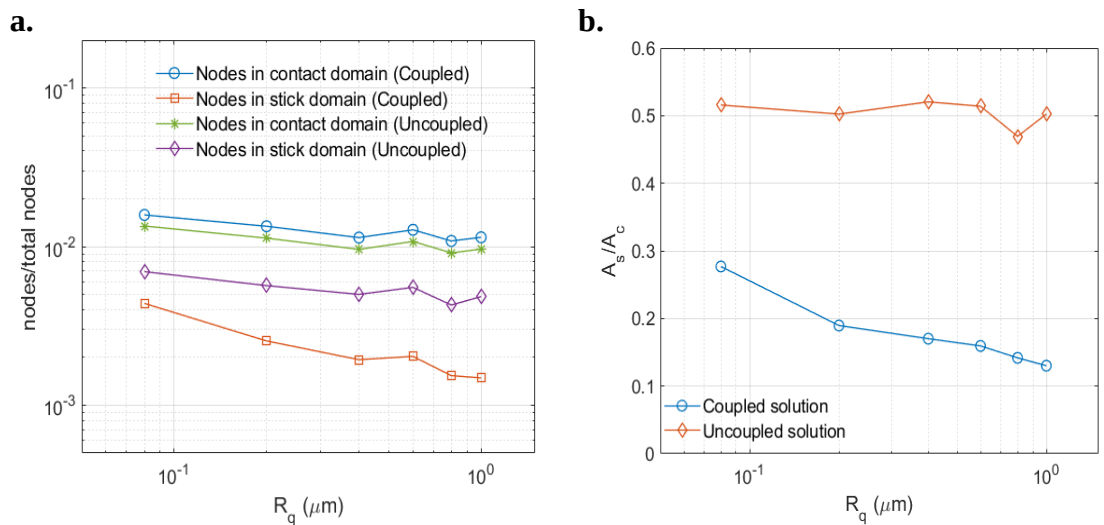


Figure 4-6 Contact solutions of rough surfaces with different R_q : (a) Ratio of the contacting/stick area to the computational domain, (b) Stick ratio ($\frac{A_s}{A_c}$)

Regarding the study on the effect of RMS slope ($\bar{g}$) on coupled partial slip solutions, the contacts of these surfaces (rough surfaces 7-12 as given in Table 4-2) with the identical RMS roughness but different values of RMS slope were simulated under the inputs given in Table 4-3. The relationship between the ratio of the contacting or stick area to the computational domain and the RMS slope is observed to be an exponential decay as indicated in Figure 4-7 (a). The trend lines using exponential decay were found to perfectly fit the plotted data following the theory of least-squares regression, where r_1 and r_2 denote the ratio of the contacting nodes and stick nodes to the whole computational domain respectively, and R^2 is the coefficient of determination characterising how close the data is to the fitted regression line globally. It is of note that trend lines, which were generated using quadratic fit and perfectly fit the uncoupled data, were also reported in our former uncoupled rough surface study concerning the RMS slope. The reader could refer to reference [27] to review the relevant equations. As mentioned above, concerning the explicit mathematical relationship between the contacting area and RMS slope, a well-validated formula for uncoupled rough sphere indentation problems was proposed by Muser [48] based on the parameter-free equations developed by Pastewka and Robbins [47]. The increasing RMS slope of rough surfaces employed here is achieved by increasing the cut-off wavevector q_0 while keeping the RMS roughness constant, which again explains that the change of contacting area results from the aforementioned synergy of the RMS roughness and cut-off frequency. As to the stick ratio, it increases significantly with the growing RMS slope in the coupled case compared with the relatively insensitive response in the uncoupled case as shown in Figure 4-7 (b).

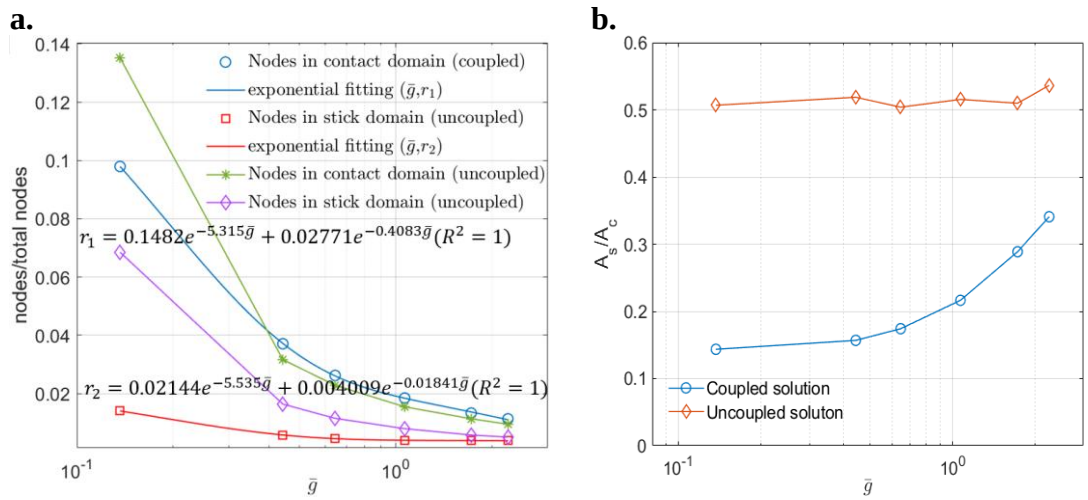


Figure 4-7 Contact solutions of rough surfaces with different RMS slope $\bar{g}$: (a) Ratio of the contacting/stick area to the computational domain, (b) Stick ratio ($\frac{A_s}{A_c}$)

These results suggest that rough surfaces with different RMS roughness or RMS slopes can have different responses to the coupling condition, leading to different stick ratios under the same loading circumstances. To visualize such behaviours, partial slip solutions of rough surfaces 1, 5, 7, and 10 are presented here to discuss in detail (note that surfaces 1 and 5 have the same RMS slope but different RMS roughness while surfaces 7 and 10 have the same RMS roughness but different RMS slopes).

As shown in Figure 4-8, lateral sizes of asperities of the surfaces 5 and 10 are significantly smaller compared with that of surfaces 1 and 7. As the highest asperities of rough surfaces always come into contact first when the normal load is imposed, relatively large contacting islands are observed for surfaces 1 and 7 (Figure 4-9 (a) and (c) respectively). On the contrary, the height distributions of surfaces 5 and 10 lead to smaller and more uniformly distributed contacting spots as illustrated in Figure 4-9 (b) and (d) respectively.

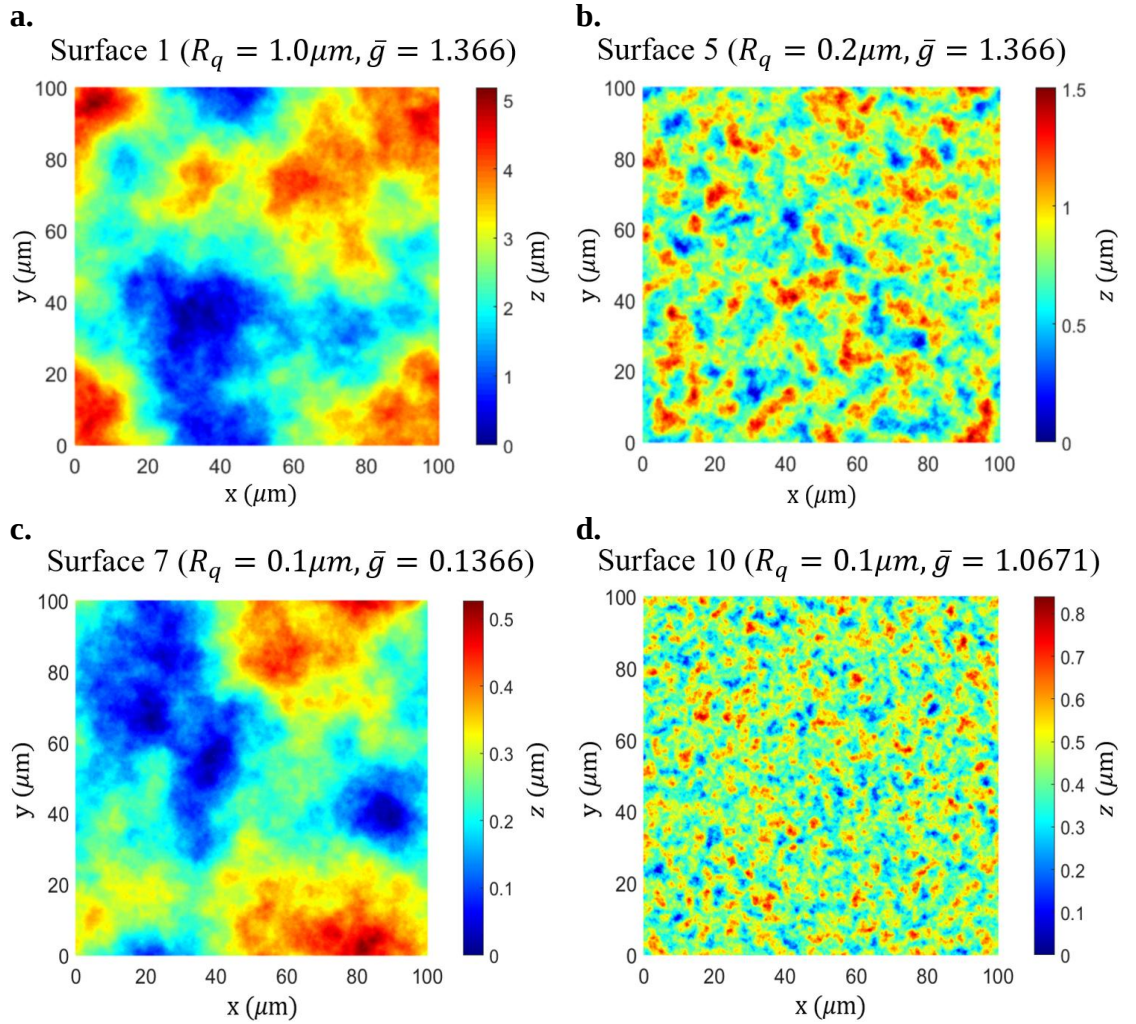


Figure 4-8 Profiles of analysed rough surfaces: (a) Surface 1, (b) Surface 5, (c) Surface 7 and (d) Surface 10

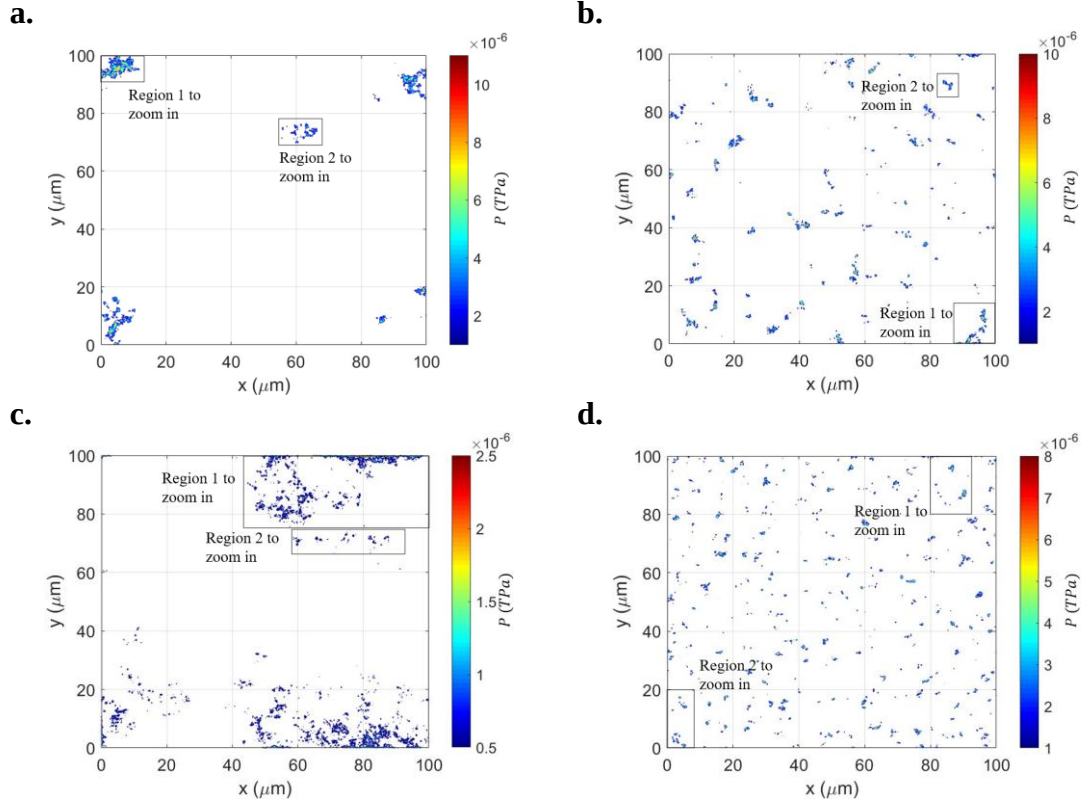


Figure 4-9 Pressure distributions for different rough surfaces: (a) Surface 1, (b) Surface 5, (c) Surface 7 and (d) Surface 10

Under the same tangential load, the separations of stick and slip regions within the four different contacting areas are shown in Figure 4-10. Although uneven pressure profiles are usually observed for rough contact problems, surfaces 1 and 7 exhibit more pronounced non-uniform pressure distributions compared with those of surfaces 5 and 10. Two regions are selected on each surface to zoom in and to have a detailed analysis. For surface 1, the pressure in region 1 is significantly higher than that in region 2 shown in Figure 4-11 (a) and (b) respectively. Since the slip tends to occur in the area where a lower local pressure profile is experienced, region 1 is still in a partial slip state (stick ratio is 0.1728 in region 1) while region 2 tends to almost reach gross sliding (stick ratio is 0.0131 in region 2) as indicated in Figure 4-11 (c) and (d) respectively. On the other hand, for surface 5 which exhibits the asperities that are comparatively smaller and distributed more intensively as shown in Figure 4-8 (b), the discrepancy between the pressures of the two zoomed-in regions is less significant as shown in Figure 4-12 (a) and (b). This results in similar partial slip states experienced by the two regions in surface 5, where stick ratios in regions 1 and 2 are 0.1784 and 0.1868 as labelled in Figure 4-12 (c) and (d) respectively. Regarding surfaces 7 and 10 that show the same RMS roughness but different RMS slopes, for similar reasons, the same contact phenomena are observed for the selected regions as demonstrated in Figure 4-13 and Figure 4-14 respectively.

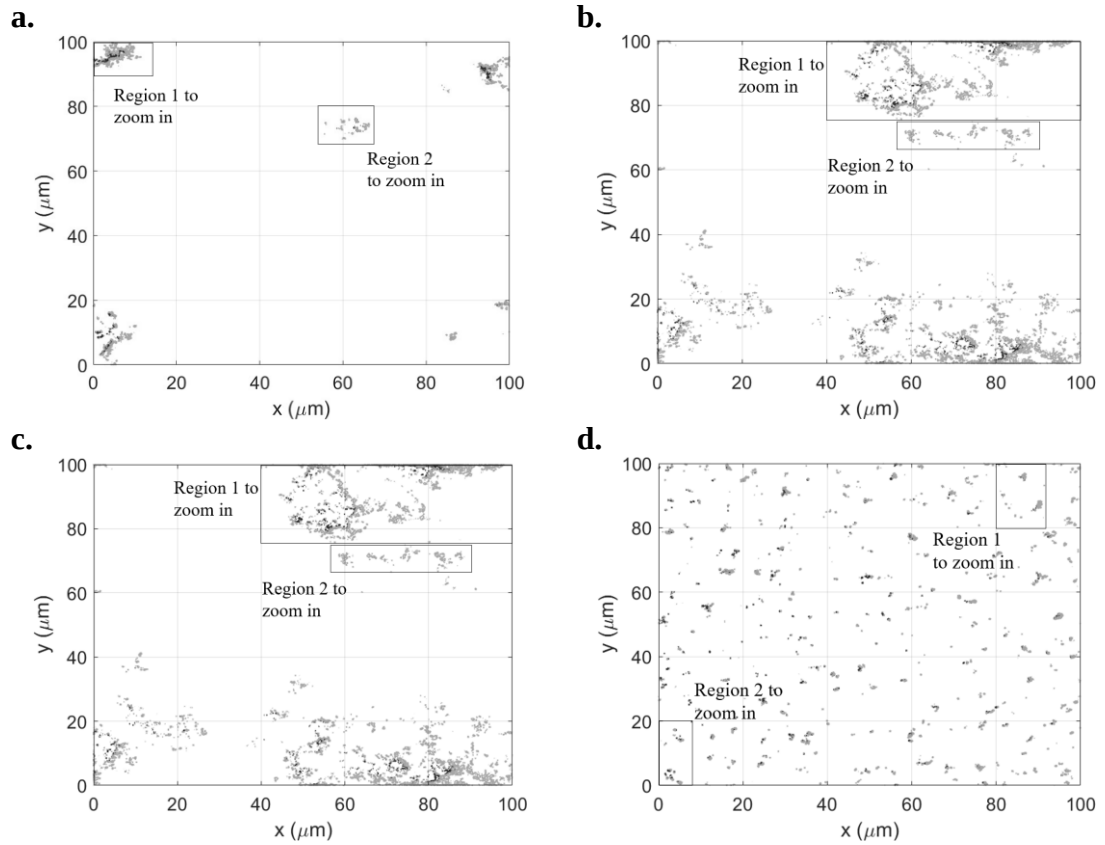


Figure 4-10 Separations of stick (dark) and slip (grey) regions for different rough surfaces: (a) Surface 1, (b) Surface 5, Surface 7 and Surface 10

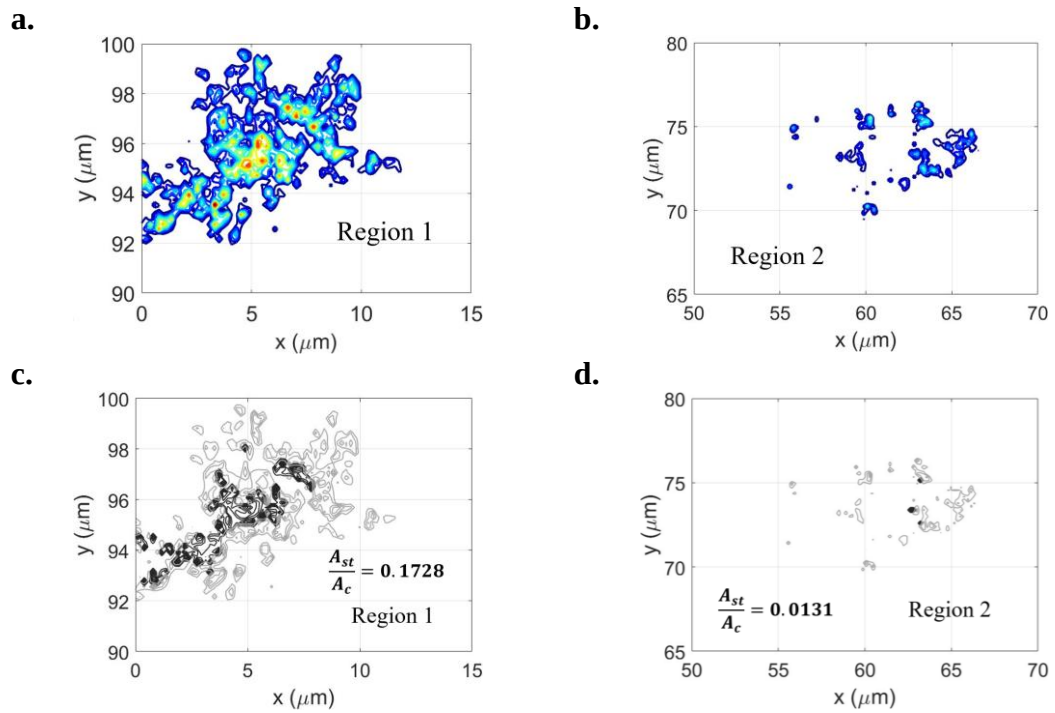


Figure 4-11 Contact solutions of regions selected to zoom in for Surface 1: (a) and (b): Pressure profiles of regions 1 and 2 respectively and (c) and (d): Separations of stick (dark) and slip (grey) zones for regions 1 and 2 respectively

The root cause of these results is the introduction of the coupling between shear tractions and pressure, which changes the contact condition of gross sliding to make the surface easier to reach the sliding state with a lower tangential load. When comparing the contact solutions of two surfaces with the same RMS slope or RMS roughness under the current medium loading condition ($F_x = 0.5\mu_f W$), a more unevenly distributed pressure profile is often obtained for the surface with the higher RMS roughness or lower RMS slope. This leads to early slip for the contacting island where lower pressure is experienced and consequently brings about a lower stick ratio. It is noted that gross sliding should be more difficult to achieve for such a surface, in that higher loads are required to be in the slip state for the areas exhibiting higher pressure. Hence, a different response of the stick ratio to the surface roughness parameters shall be expected under a higher loading condition.

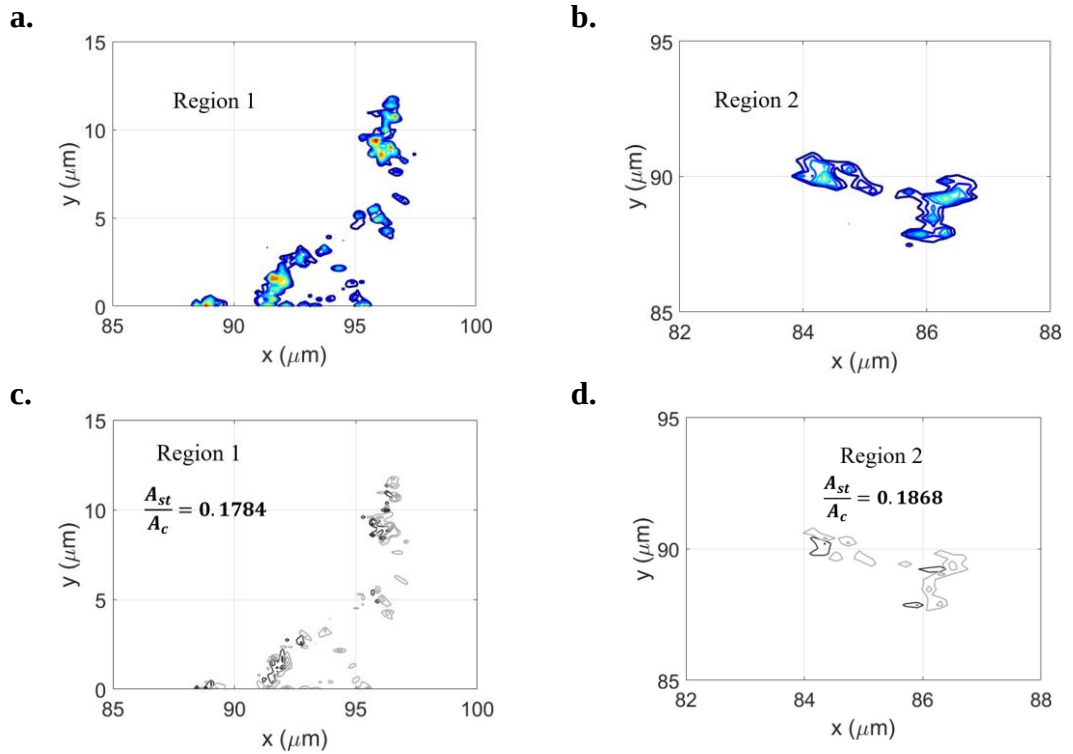
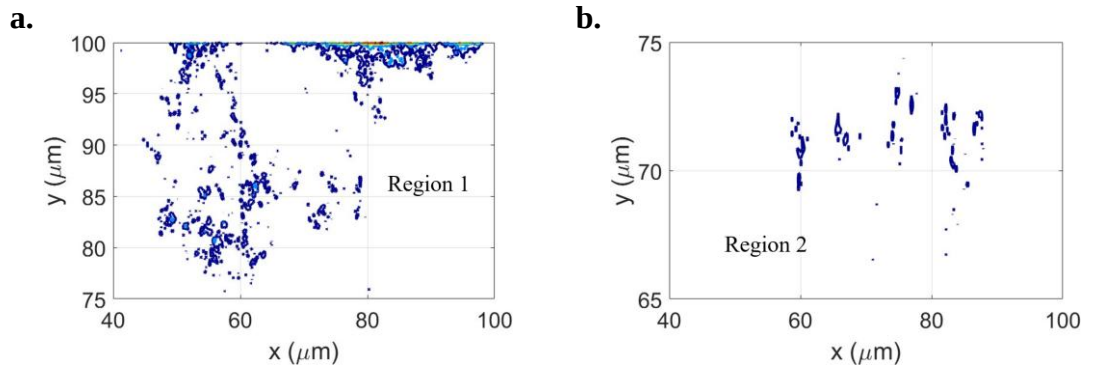


Figure 4-12 Contact solutions of regions selected to zoom in for Surface 5: (a) and (b): Pressure profiles of regions 1 and 2 respectively and (c) and (d): Separations of stick (dark) and slip (grey) zones for regions 1 and 2 respectively



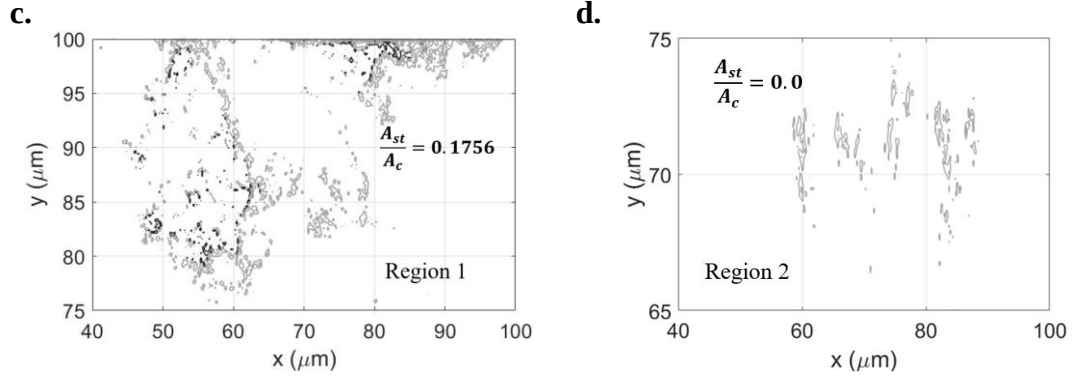


Figure 4-13 Contact solutions of regions selected to zoom in for Surface 7: (a) and (b): Pressure profiles of regions 1 and 2 respectively and (c) and (d): Separations of stick (dark) and slip (grey) zones for regions 1 and 2 respectively

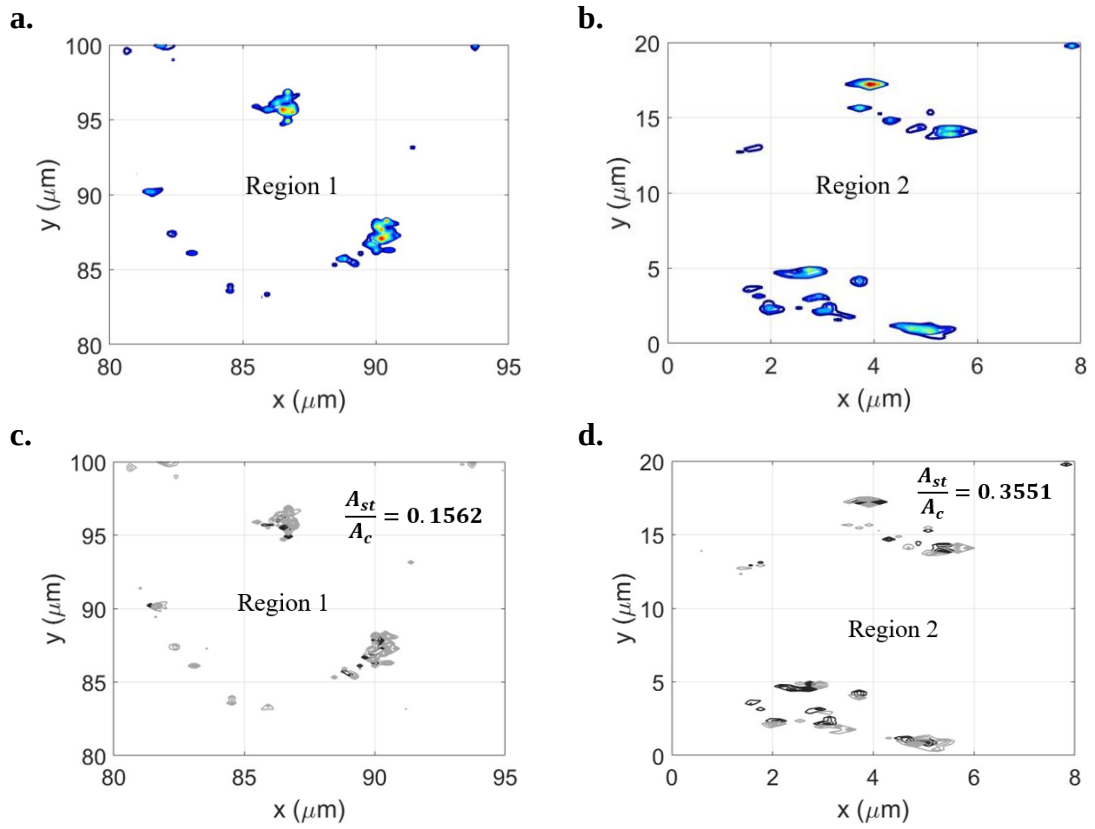
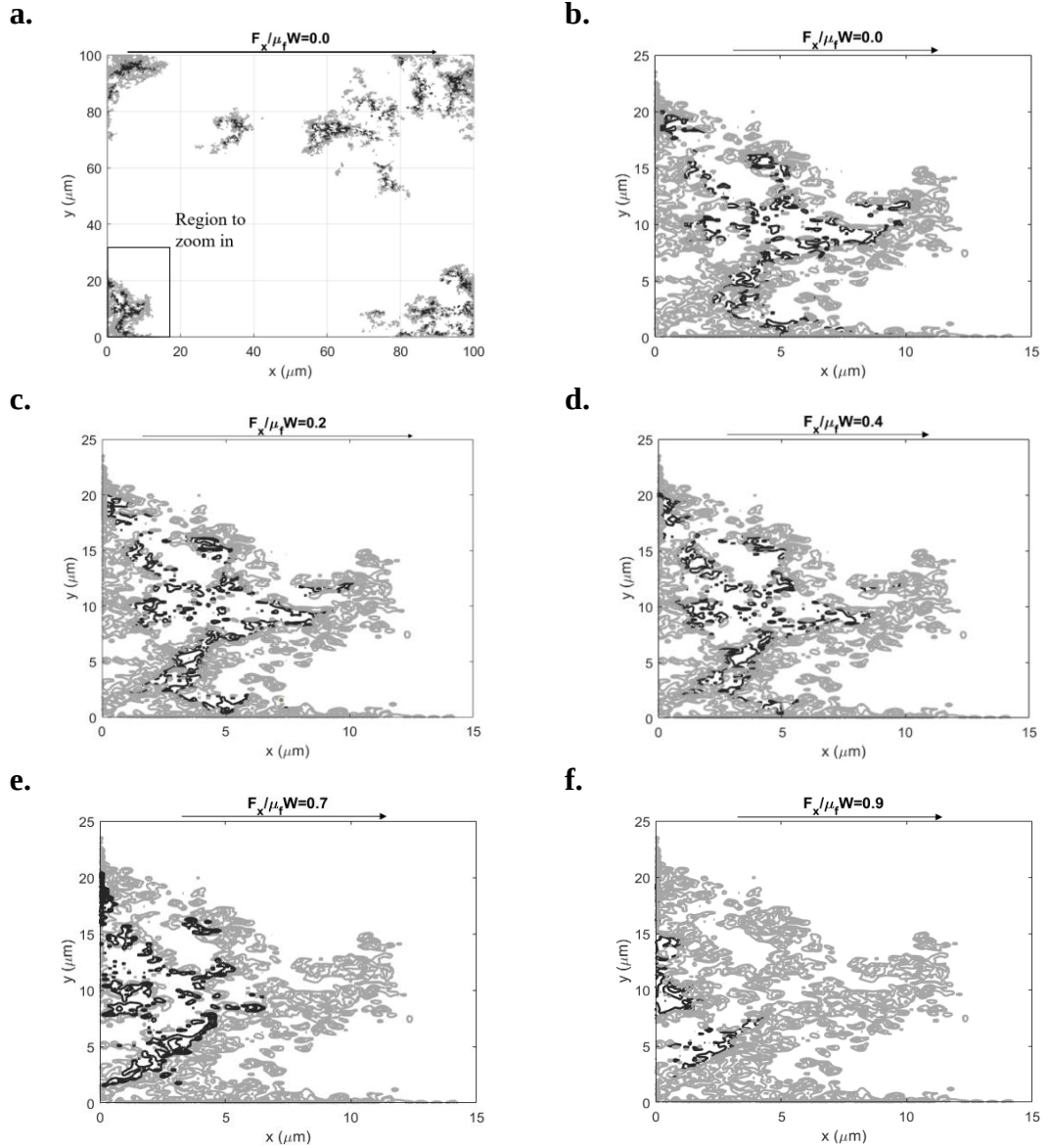


Figure 4-14 Contact solutions of regions selected to zoom in for Surface 10: (a) and (b): Pressure profiles of regions 1 and 2 respectively and (c) and (d): Separations of stick (dark) and slip (grey) zones for regions 1 and 2 respectively

4.4.2.2 Contact of Rough Surfaces under Increasing Loads

A linear relationship between the stick ratio and ratio of input tangential load to static friction was reported in our former uncoupled rough surface study [27]. To verify if such a relation still holds under coupled conditions, the evolution of stick and slip regions in rough surface contact with increasing tangential load was simulated. The rough surface 1 presented in Figure 4-8 (a) and a higher normal load of 0.0025 N were used to facilitate the observation of the dynamic transition from partial slip to gross slip within the

contacting area. The separation of the stick (dark) and slip (grey) zones under no tangential load is given in Figure 4-15 (a), and an area was chosen and zoomed in to analyse. The stick zone within this area is found to keep decreasing and shifting to the rear part of the contacting region with increasing tangential load as shown in Figure 4-15 (b)-(g), where such a change is relatively slow and steadily when the load is low ($F_x \leq 0.7\mu_f W$) compared with the significant shrink of the stick zone at a higher load ($F_x > 0.7\mu_f W$). The evolution of stick ratio with the increasing tangential load is shown in Figure 4-15 (h), where the approximated linear relationship reported in our former uncoupled study [27] is apparently lost due to the coupling effects. Compared with the solution to the coupled smooth sphere contact problem investigated in section 4.4.1, the rough surface experiences a lower portion of contacting area in stick when the normal load is applied alone but it requires a higher tangential load to reach the gross sliding state as labelled in Figure 4-15 (h).



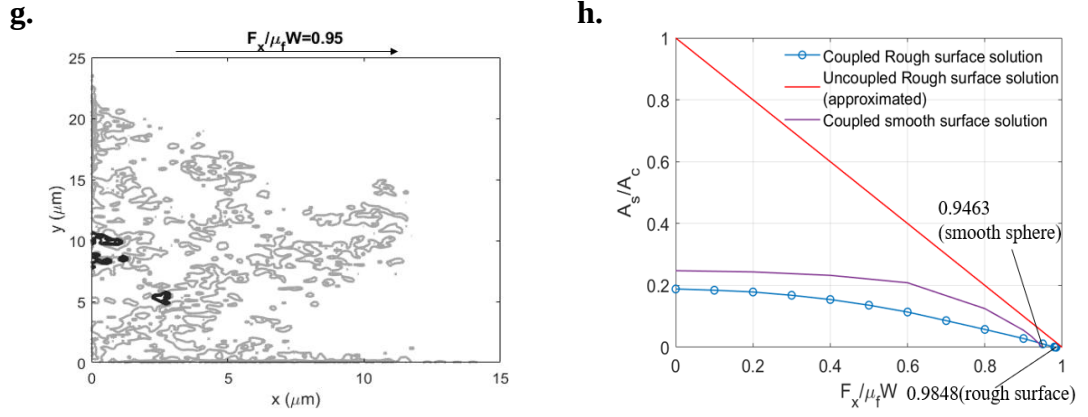


Figure 4-15: (a) Contact solution under no tangential force, (b)-(g) Separation of the stick (dark) and slip (grey) regions for the zoomed-in area under increasing tangential load, and (h) Evolution of the stick ratio with increasing tangential load

To analyse the dynamic transition further, the partial slip contacts of different rough surfaces (Rough surfaces 1, 3, 5, 9, 10, 11 given in Table 4-2) were simulated under the same normal load of 0.0003 N but increasing tangential load. The relevant contact solutions are given in Figure 4-16. The input normal load is found to affect the separation of the stick and slip regions since the contact of rough surface 1 ($R_q = 1.0 \mu\text{m}$) provides dissimilar solutions under normal loads of 0.0025 N (blue line in Figure 4-15 (h)) and 0.0003 N (orange line in Figure 4-16). This is different from what was reported in our former uncoupled rough surface study [27]. Our interpretation of this is that a higher normal load brings about higher normal pressures due to the load balance, which in turn leads to more significant coupling effects on the shear traction (e.g., the magnitude of the surface displacement u_{xz} increases). The tangential loads required to achieve gross sliding in all the simulated rough-surface contact problems were found to always be higher than that in of an equivalent smooth sphere contact problem ($F_x = 0.9463\mu_f W$).

Regarding the trend difference for surfaces with different roughness parameters under the increasing tangential load, one with the higher RMS roughness tends to always have less proportion of contacting area in stick when the load is low or medium, as illustrated in Figure 4-16. This is consistent with the results obtained from the previous rough surface study under constant loading conditions (section 4.4.2.1).

However, when the input tangential load is high (approximately 0.76-0.9725 as shown in zoomed view 1 in Figure 4-16), the surface with the highest RMS roughness ($R_q = 1.0 \mu\text{m}$) exhibits an intermediate value of stick ratio. The trend then becomes similar to that under small and medium loading conditions once the load exceeds this range. The solutions presented regarding RMS slope are irregular as well when it comes to a high

loading condition. As shown in Figure 4-16, the surface with a higher RMS slope always experiences more stick ratio in the early period of the dynamic loading process. However, the surface with the lowest RMS slope ($\bar{g} = 0.6455$) turns out to require the highest tangential load to induce gross sliding. On the other hand, the surface with the highest RMS slope ($\bar{g} = 1.7250$) only requires an intermediate value of tangential load for global slip as labelled in the zoomed view 2 in Figure 4-16. Such inconsistent trends can be related to the more non-uniformly distributed pressure experienced by some rough surfaces along with the coupling effects as mentioned in section 4.4.2.1. On one hand, when the tangential load is low or medium, it is easier for the area where pressure is relatively low to be in the slip state and this leads to a smaller stick ratio. On the other hand, it is more difficult for the entire surface to reach gross sliding since the area exhibiting higher pressure now requires a higher load to slip.

Although the trend between the surface roughness parameters (RMS roughness and RMS slope) and the stick ratio turns out to depend on the loading condition, these results tend to prove that the effects of the coupling between normal pressure and shear traction on the separation of the stick and slip regions depend on the roughness parameters of surfaces.

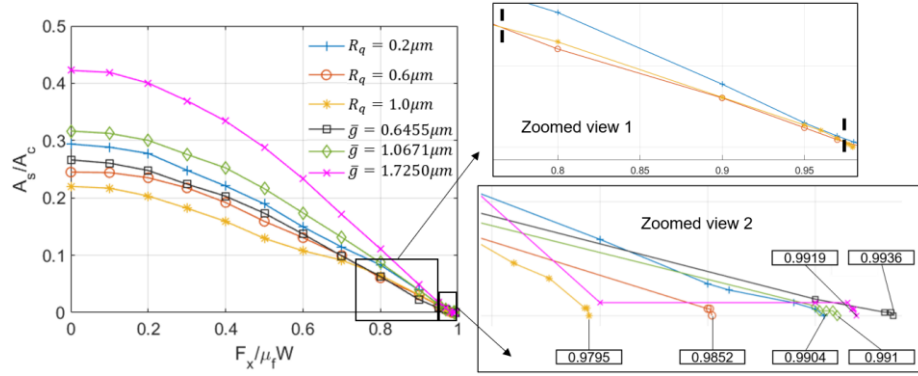


Figure 4-16 Evolution of the stick ratio with increasing tangential load for different rough surfaces: the two extended views (Zoomed views 1 and 2) show different trends of stick ratio with respect to the surface roughness parameters under high loading conditions (when contact is close to the sliding inception) when compared with those under low or medium loads

4.5 Concluding Remarks

By adjusting the shear tractions within the contacting region globally and adopting an alternative convergence criterion regarding the load balance in the tangential direction, the partial slip contact of two elastically dissimilar materials was successfully reproduced by the newly developed model. The identification of stick and slip regions within the contacting area could be accomplished in a short time. (Usually, 120 seconds for smooth

sphere contact problems with 256×256 nodes and 300 seconds for rough surface contact problems with 512×512 nodes using a desktop PC with 4 cores).

The numerical results of rough stick-slip contacts under coupled partial slip conditions lead to the following conclusions:

- Regardless of the roughness parameters, the tangential load needed for a nominally flat surface to reach gross sliding is higher than that needed for a smooth sphere.
- With an identical RMS slope, the contacting region is insensitive to the change of RMS roughness under constant loads (low or medium). However, a higher RMS roughness leads to the decrease of stick region.
- The contact area or the stick area is found to decay exponentially with the RMS slope under constant loads (low or medium); equations are proposed to determine the proportion of stick and contacting area to the whole simulation domain.
- The linear response of the stick ratio to the input tangential load that occurs in the rough surface contact under uncoupled partial slip conditions is now lost due to the additional interaction between shear tractions and normal pressures. Such coupling effects on the separation of stick and slip regions are found to vary with the surface roughness parameters. During the transition from partial slip to gross sliding, there are multiple observations. Under low and medium tangential loads, the surface with a higher RMS gradient or a lower RMS roughness exhibits more ratio of contacting area in stick as mentioned above. However, the trend becomes irregular when the load keeps increasing to the final loading point needed to achieve the gross sliding.

Acknowledgement

The authors are thankful to the school of mechanical engineering at the University of Leeds for supporting this study by funding the PhD studentship of Dongze Wang. This work is supported by the Engineering and Physical Sciences Research Council (grant no. EP/T024542/1) as part of ‘STOP fibrous microplastic pollution from textiles by elucidating fibre damage and manufacturing novel textiles’ standard research.

Appendix 4A Formulations of Green Functions for Elastic Contact Problems

In this appendix, the Green functions $G_{ij}(i, j = x, y, z)$ of a specific point are determined by the material properties of the two contacting bodies (E_1, ν_1, E_2, ν_2) and its local pointwise coordinates (x, y) . They are summarized by Wang and Zhu [31] as follows:

$$G_{xx} = \frac{1}{\pi r^3} \left(\frac{x^2}{E'} + \frac{y^2}{E^*} \right), G_{xy} = \frac{xy}{\pi \mu^* r^3}, G_{xz} = -\frac{x}{\pi \mu' r^2},$$

$$G_{yy} = \frac{1}{\pi r^3} \left(\frac{x^2}{E^*} + \frac{y^2}{E'} \right), G_{yx} = G_{xy}, G_{yz} = -\frac{y}{\pi \mu' r^2},$$

$$G_{zz} = \frac{1}{\pi E^* r^3}, G_{zx} = -G_{xz}, G_{zy} = -G_{yz},$$

in which

$$r = \sqrt{x^2 + y^2}, \frac{1}{E'} = \frac{1 + \nu_1}{E_1} + \frac{1 + \nu_2}{E_2}, \frac{1}{E^*} = \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2},$$

$$\frac{1}{\mu'} = \frac{(1 + \nu_1)(1 - 2\nu_1)}{2E_1} - \frac{(1 + \nu_2)(1 - 2\nu_2)}{2E_2}, \frac{1}{\mu^*} = \frac{\nu_1(1 + \nu_1)}{E_1} + \frac{\nu_2(1 + \nu_2)}{E_2}.$$

Appendix 4B

The contents here is repeated. It retains in the thesis as it is part of the publication following the requirement of school for thesis by publication style.

In this appendix, the solutions of all the influence coefficient matrices in the discretised form are achieved through integrating over the small rectangular mesh area of $2a \times 2b$.

They are summarised by Ghanbarzadeh et al. [38] as follows:

$$C_{zz} = \frac{1-\nu}{2\pi G} \left\{ (x+a) \ln \left[\frac{(y+b)+\sqrt{(y+b)^2+(x+a)^2}}{(y-b)+\sqrt{(y-b)^2+(x+a)^2}} \right] + (y+b) \ln \left[\frac{(x+b)+\sqrt{(y+b)^2+(x+a)^2}}{(x-a)+\sqrt{(y+b)^2+(x-a)^2}} \right] + \right. \\ \left. (x-a) \ln \left[\frac{(y-b)+\sqrt{(y-b)^2+(x-a)^2}}{(y+b)+\sqrt{(y+b)^2+(x-a)^2}} \right] + (y-b) \ln \left[\frac{(x-a)+\sqrt{(y-b)^2+(x-a)^2}}{(x+a)+\sqrt{(y-b)^2+(x+a)^2}} \right] \right\};$$

$$C_{yy} = \frac{1}{2\pi G} \left\{ (x+a) \ln \left[\frac{(y+b)+\sqrt{(y+b)^2+(x+a)^2}}{(y-b)+\sqrt{(y-b)^2+(x+a)^2}} \right] + (1-\nu)(y+b) \ln \left[\frac{(x+b)+\sqrt{(y+b)^2+(x+a)^2}}{(x-a)+\sqrt{(y+b)^2+(x-a)^2}} \right] + \right. \\ \left. (x-a) \ln \left[\frac{(y-b)+\sqrt{(y-b)^2+(x-a)^2}}{(y+b)+\sqrt{(y+b)^2+(x-a)^2}} \right] + (1-\nu)(y-b) \ln \left[\frac{(x-a)+\sqrt{(y-b)^2+(x-a)^2}}{(x+a)+\sqrt{(y-b)^2+(x+a)^2}} \right] \right\};$$

$$C_{xx} = \frac{1}{2\pi G} \left\{ (1-\nu)(x+a) \ln \left[\frac{(y+b)+\sqrt{(y+b)^2+(x+a)^2}}{(y-b)+\sqrt{(y-b)^2+(x+a)^2}} \right] + (y+b) \ln \left[\frac{(x+b)+\sqrt{(y+b)^2+(x+a)^2}}{(x-a)+\sqrt{(y+b)^2+(x-a)^2}} \right] + \right. \\ \left. (1-\nu)(x-a) \ln \left[\frac{(y-b)+\sqrt{(y-b)^2+(x-a)^2}}{(y+b)+\sqrt{(y+b)^2+(x-a)^2}} \right] + (y-b) \ln \left[\frac{(x-a)+\sqrt{(y-b)^2+(x-a)^2}}{(x+a)+\sqrt{(y-b)^2+(x+a)^2}} \right] \right\};$$

$$C_{zx} = \frac{1-2\nu}{2\pi G} \left\{ (y+b) \ln \left[\frac{\sqrt{(y+b)^2+(x+a)^2}}{\sqrt{(y-b)^2+(x-a)^2}} \right] + (y-b) \ln \left[\frac{\sqrt{(y-b)^2+(x-a)^2}}{\sqrt{(y+b)^2+(x+a)^2}} \right] + (x+a) \right. \\ \left. \left[\tan^{-1} \frac{y+b}{x+a} - \tan^{-1} \frac{y-b}{x-a} \right] + (x-a) \left[\tan^{-1} \frac{y-b}{x-a} - \tan^{-1} \frac{y+b}{x-a} \right] \right\};$$

$$C_{xz} = -C_{zx};$$

$$C_{zy} = \frac{1-2\nu}{2\pi G} \left\{ (x+a) \ln \left[\frac{\sqrt{(y+b)^2+(x+a)^2}}{\sqrt{(y-b)^2+(x-a)^2}} \right] + (x-a) \ln \left[\frac{\sqrt{(y-b)^2+(x-a)^2}}{\sqrt{(y+b)^2+(x+a)^2}} \right] + (y+b) \right. \\ \left. \left[\tan^{-1} \frac{x+a}{y+b} - \tan^{-1} \frac{x-a}{y+b} \right] + (y-b) \left[\tan^{-1} \frac{x-a}{y-b} - \tan^{-1} \frac{x+a}{y-b} \right] \right\};$$

$$C_{yz} = -C_{zy};$$

$$C_{xy} = \frac{\nu}{2\pi G} \left[\sqrt{(y-b)^2 + (x+a)^2} - \sqrt{(y-b)^2 + (x-a)^2} + \sqrt{(y+b)^2 + (x-a)^2} - \sqrt{(y+b)^2 + (x+a)^2} \right];$$

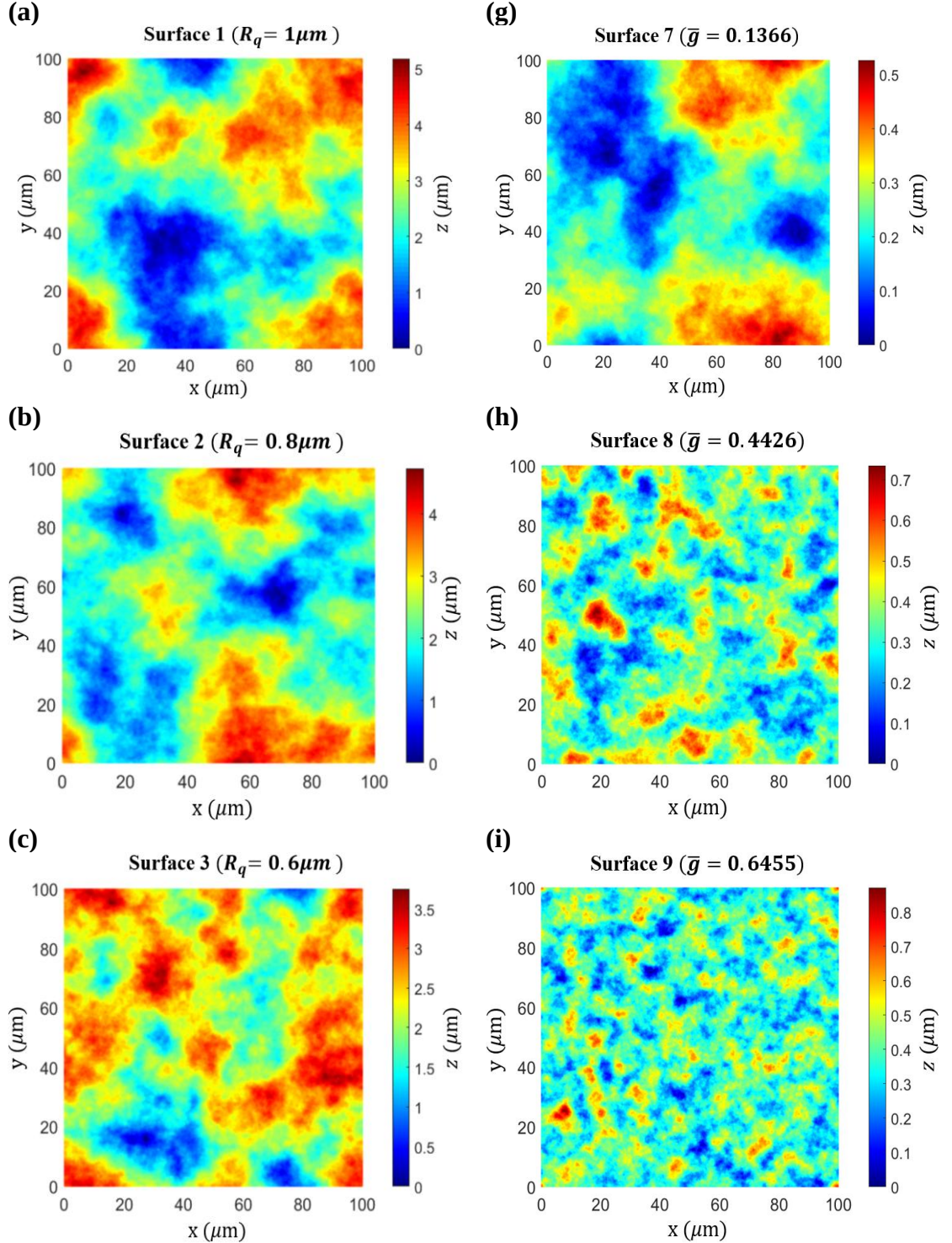
$$C_{yx} = C_{xy}.$$

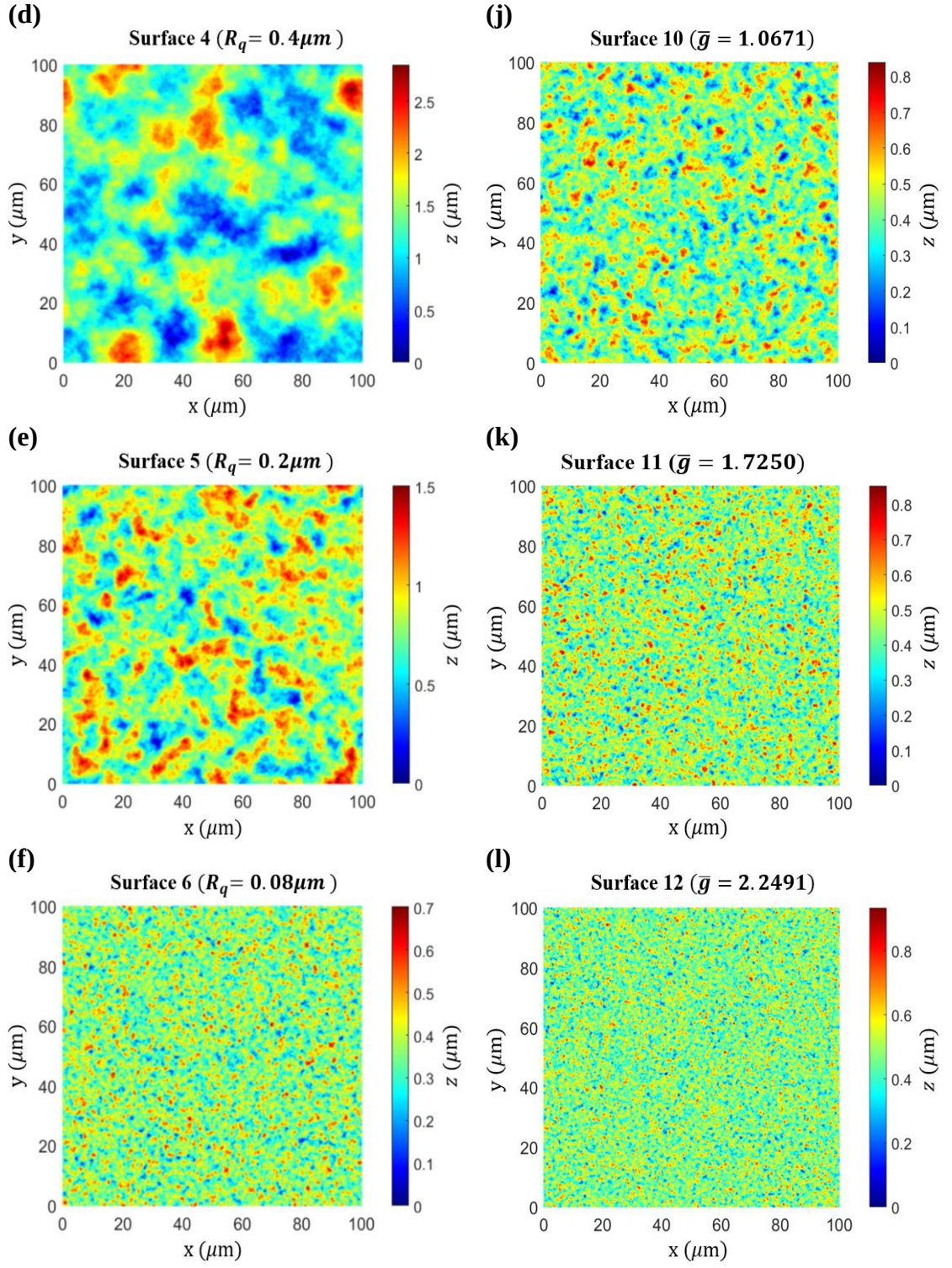
Appendix 4C

In this appendix, the following are the investigated nominally flat rough surfaces, where rough surfaces (1)-(6) exhibit different R_q values but the same $\bar{g}$ (1.3658) while rough surfaces (7)-(12) exhibit different $\bar{g}$ values but the same R_q (0.1 μm)

$$\bar{g} = 1.3658$$

$$R_q = 0.1 \mu\text{m}$$





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5 A Numerical Model for Investigating the Effect of Viscoelasticity on the Partial Slip Solution

5.1 Abstract

To investigate the effects of viscoelasticity on the stick-slip behaviour, a new model reproducing the partial slip of viscoelastic materials under fully coupled conditions is developed in this paper. The ratio of retardation time to relaxation time is employed to characterize the rheological property of a viscoelastic material. It is found that materials with higher ratios exhibit more fluid-like behaviours while those with lower ratios perform more like solid. As long as the contact input (load or displacement) is constant, the stick ratio (ratio of stick area to contacting area) is found to be insensitive to the viscoelasticity of materials. However, the separation pattern of the stick and slip regions varies with time when different contact phenomena (creep or stress relaxation) are encountered in the lateral and normal directions. The transition process from partial slip to gross sliding of viscoelastic materials, unlike the elastic response, tends to be abrupt when fully coupled conditions between shear tractions and pressure are introduced. When identical contact parameters are specified for different viscoelastic materials, the more fluid-like material always experiences a quicker transition from partial slip to gross sliding.

5.2 Introduction

Owing to the dimensional stability and capacity to sustain load over long periods of time, viscoelastic materials have been extensively applied in many engineering fields such as rubbers in automotive [1] and polymers in prosthetic joints [2]. When it comes to the design optimization and tribological analysis of these engineering components, the mechanical response of viscoelastic materials must be considered. However, a closed-form mathematical solution to viscoelastic contact problems is a significant challenge within the framework of classical contact mechanics as the structural complexity and time-dependent properties of these materials impede its development.

Great efforts have been made over the past few decades providing solutions to various viscoelastic contact problems. Based on the assumption that the contact area increases monotonically without surface adhesion, the solution to a spherical indentation problem of linear viscoelastic materials was developed by Lee and Radok [3]. An essential concept known as the ‘elastic-viscoelastic correspondence principle’ was proposed in this study, in which the theory of Boltzmann hereditary integral is applied. This principle has now

become the foundation for many recently developed analytical and numerical models of viscoelastic contact problems. The application of this tentative analytical method, including limited contact geometry and monotony of contact radius, was later somewhat extended by many researchers including Hunter [4], Graham [5], Yang [6], Ting [7, 8] and Greenwood [9] using different approaches. The solution of the real contact area for a viscoelastic solid squeezed into a randomly rough surface was proposed by Persson et al. [10] using a contact-mechanics-based theory. However, these updated analytical solutions are still limited in the applications where they could be employed. Considering the existence of surface roughness, arbitrary loading history and complicated rheological behaviour of viscoelastic materials in practical contact problems, it is difficult to obtain a valid time-varying solution based on existing analytical theories. A numerical model presents itself as a suitable alternative in this dilemma, for which, several simulation tools have been developed to assist in the analysis of viscoelastic components. A robust numerical model solving the point contact problem between a rigid indenter and a homogeneous viscoelastic half-space was presented by Chen et al. [11] using the semi-analytical method (SAM). Although the model was initially developed to simulate the normal and non-conformal contact of polymer-based materials characterised by a wide spectrum of relaxation times and complicated surface topographies, it was extended to investigate more complicated contact problems. Yu et al. [12] included surface adhesion, Koumi et al. [13] considered heterogeneous viscoelastic materials and Spinu [14] analysed the line contact configuration.

Apart from indentation problems, the sliding or rolling contact of viscoelastic materials has been one of the most investigated topics in the last few decades. Based on the correspondence principle, the rolling contact of a rigid cylinder on a viscoelastic half-space was analyzed by Hunter [15]. The solution to the rolling friction between a hard cylinder and a viscoelastic sphere was provided by Persson [16], and the sliding contact of a rigid wavy surface against a viscoelastic half-space was investigated by Menga et al. [17]. The geometrical limitation of the contacting bodies in these analytical solutions was overcome by numerical models developed later. Carbone and Putignano [18] proposed a novel method to analyse steady-state viscoelastic contact, in which a correction factor was introduced that takes into account the thickness of the contacting viscoelastic surface as well as the sliding speed. The model was employed to simulate the frictionless sliding contact of rough viscoelastic materials in different applications (half-space [19] and layer [20]). To further understand the complexity of viscoelastic sliding contact problems in

practice, an ellipsoidal inhomogeneity was included by Koumi et al. [21] The rolling and sliding contact of a layered viscoelastic half-space, in which the viscoelastic layer and substrate exhibit distinct properties, was simulated by Wallace et al. [22]. Meanwhile, the effects of interfacial imperfections between the layer and substrate on the transient and steady-state solutions were investigated by Zhang et al. [23].

Compared to the above-mentioned models of frictionless sliding or rolling contact of viscoelastic surfaces, less attention has been paid to the partial slip aspect of tangential contact problems. Under such a contact condition a global relative sliding does not occur. Instead, the contacting area is separated into stick zones, in which no relative motion happens, and slip zones, in which local relative movement exists. This type of contact phenomenon is commonly encountered in practice when two contacting surfaces are subjected to a tangential load that is not enough to induce gross sliding. An early viscoelastic partial slip solution was obtained by Goryacheva [24], where a viscoelastic cylinder rolling on a half-space of the same material was studied. Based on Coulomb's friction law, the partial slip analysis on the rolling contact of viscoelastic multi-layered cylinders was provided by Kaller [25] while Goryacheva and Sadeghi [26] considered a different contact configuration (a cylinder sliding or rolling on a viscoelastic layer bonded to an elastic half-space) using the Goodman approximation [27].

The assumptions and approximations implemented in these tentative viscoelastic partial slip solutions become inapplicable when it comes to the contact of two dissimilar materials due to the significant interactions between normal pressures and shear tractions. Considering that a metal-polymer contact is one of the most common interfacial material combinations in engineering practice, such as medical devices [28], the coupling effects must be considered to obtain a valid solution. Numerical modelling of the coupled partial slip contact of viscoelastic materials was attempted by Spinu and Cerlinca [29], who considered the fretting of a steel sphere against a viscoelastic half-space under oscillating tangential loads. Based on the partial slip model of elastic materials by Gallego et al. [30], the stick-slip analysis in a frictional sliding contact of a rigid sphere on a viscoelastic layered half-space was presented in the work of Wallace et al. [22]. The partial slip periodic contact between a sinusoidal surface and a viscoelastic layer of finite thickness was analysed by Bonari and Jacopo [31] using the finite element method (FEM), in which the coupling between the shear tractions in lateral directions was neglected. To date, the number of studies reporting the influence of viscoelasticity on the separation of stick and slip regions under fully coupled conditions is rather limited, and there is still a lack of

understanding of the transition from partial slip to gross sliding in the field of viscoelastic contact. This is the problem we intend to address in the current paper.

By applying the elastic-viscoelastic correspondence principle, the novel conjugate gradient method (CGM)-based algorithm for the coupled normal and tangential contacts of dissimilar elastic materials, developed by the authors of the current manuscript [32], was extended here to obtain solutions to stick-slip contact problems of viscoelastic surfaces. The numerical technique known as Discretised Convolution Fast Fourier Transform (DC-FFT) was implemented to improve the computational efficiency of the algorithm. The well-validated model was extended to investigate the effects of time-dependent properties of viscoelastic materials on stick-slip solutions, where materials characterized by different rheological models were tested. A novel generalized view to understand the responses of viscoelastic materials exhibiting different rheological properties is provided to explain why the shape of contact tractions of a viscoelastic contact problem may vary significantly from the elastic solutions and why the unconventional transition process from partial slip to gross sliding occurs for viscoelastic materials.

5.3 Theory and Algorithm Description

To explain how the viscoelastic model is developed, some of the basic principles about viscoelasticity are reviewed here for clarity. Viscoelastic materials exhibiting a linear relationship between stress and strain at any time point are known as linear viscoelastic materials. The viscoelastic surfaces being addressed in this work are assumed to behave linearly and the simulation of the non-linear viscoelastic contact problems is beyond the scope of the current paper. Considering that viscoelastic materials are usually soft, they can hardly deform plastically. Therefore, plastic contact is not considered in the modelling work. In addition, the temperature during contact is assumed to be constant.

5.3.1 Theory of Linear Viscoelasticity

Within the framework of linear viscoelasticity, the responses of stress/strain to successive strain /stress stimuli are cumulative. The constitutive law of linear viscoelastic materials can be explained by two Heaviside step response functions to excitations known as creep compliance function ($\Phi(t)$) and relaxation modulus function ($\Psi(t)$). As their names suggest, the creep compliance function reveals the creep phenomenon and mathematically, it describes the viscoelastic strain response to a unit change in stress. On the other hand, the relaxation modulus function characterises the stress relaxation of viscoelastic materials and mathematically, it represents the viscoelastic stress response to

a unit change in strain. The Boltzmann hereditary integral is applied to characterize such a contact behaviour, where the response to sequential excitations can be the summation of the responses that would have been generated by each excitation applied alone.

To consider the stress $\sigma_1(t)$ at time t under the acting of a certain strain $\Delta\varepsilon_1$ applied at the time t'_1 , it can be expressed as:

$$\sigma_1(t) = \Psi(t - t'_1)\Delta\varepsilon_1 \quad \text{Equation 5-1}$$

Likewise, the stress $\sigma_2(t)$ at time t under the performance of a strain $\Delta\varepsilon_2$ applied at t'_2 is determined by:

$$\sigma_2(t) = \Psi(t - t'_2)\Delta\varepsilon_2 \quad \text{Equation 5-2}$$

Given a number of input strain increments that could be treated as a continuous distribution, the response of the strain to such an arbitrary sequence of strain becomes [33]:

$$\sigma(t) = \int_0^t \Psi(t - t') \frac{d\varepsilon(t')}{dt'} dt' \quad \text{Equation 5-3}$$

When the input quantity is the stress instead of the strain, the following relationship is obtained via an analogous derivation:

$$\varepsilon(t) = \int_0^t \Phi(t - t') \frac{d\sigma(t')}{dt'} dt' \quad \text{Equation 5-4}$$

Unlike the case of an ideal elastic contact problem where the compliance and elastic modulus are mutual reciprocal, the relationship between the creep compliance and the relaxation modulus of viscoelastic materials is expressed as follows [34]:

$$\int_0^t \Phi(t - t') \psi(t) dt' = t \quad \text{Equation 5-5}$$

In the Laplace transform domain, there exists the following essential mathematical relationship [34]:

$$\bar{\Phi}(s) \bar{\Psi}(s) = \frac{1}{s^2}, \quad \text{Equation 5-6}$$

where s is the variable in the Laplace transform domain ($s = a + jb$).

Such time-dependent behaviours of linear viscoelastic materials can be expressed in terms of rheological models established with linear springs (perfectly elastic body) and dashpots (ideal Newtonian fluid). When a spring and a dashpot are arranged in series as presented

in Figure 5-1 (a), such a rheological model is known as the Maxwell model. The Kelvin-Voigt model is established if the two elements are arranged in parallel as illustrated in Figure 5-1 (b). It is acknowledged that a Maxwell model demonstrates the stress relaxation of linear viscoelastic materials appropriately, but it fails to account for their creep and recovery characteristics. On the contrary, the Kelvin-Voigt model performs oppositely. Furthermore, it exhibits no instantaneous elastic response. A detailed analysis of these two models can be found in the work of Popov [33]. Considering that these two-element models can only provide qualitative descriptions, a more sophisticated model like the Generalized Weichert model comprising more elements as shown in Figure 5-1 (c) is usually used to characterize the linear viscoelastic materials with precise quantitative information. The number of elements in the model is determined by the naturally occurring spectrum of relaxation times, where the relaxation time is denoted by τ and determined by the properties of elements as follows:

$$\tau = \frac{\eta_i}{G_i}, \quad \text{Equation 5-7}$$

here η_i is the viscosity of the i -th dashpot and G_i is the modulus of the i -th spring.

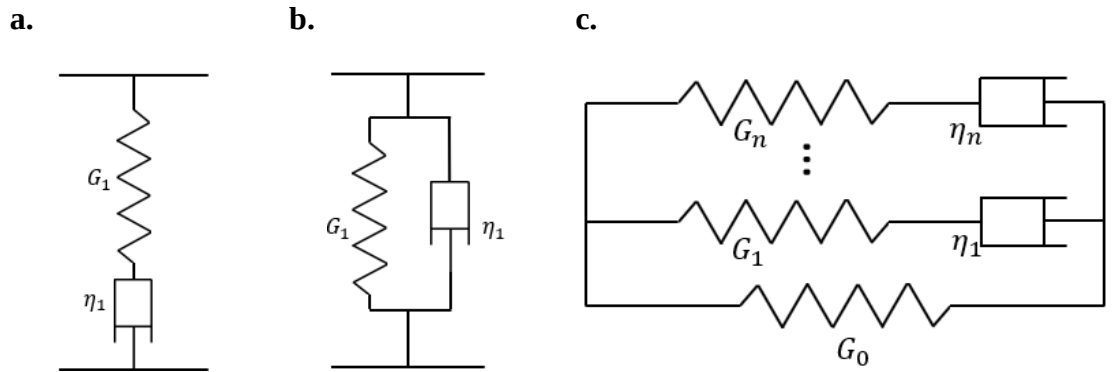


Figure 5-1 Rheological models describing the behaviour of linear viscoelastic materials: (a) Maxwell model, (b) Kelvin-Voigt model and (c) Generalized Weichert model: G is the spring stiffness and η is the dashpot viscosity

With the Generalized Weichert model, the relaxation modulus function of any linear viscoelastic material can be expressed mathematically by fitting the experimental data collected from the conducted relaxation test to the following equation (Prony series [33]) and adjusting the parameters:

$$\Psi(t) = G_0 + \sum_{i=1}^n G_i \exp\left(-\frac{t}{\tau_i}\right) \quad \text{Equation 5-8}$$

It is noted that the spectrum of relaxation times of linear viscoelastic materials can be characterized by introducing as many exponential terms as needed to attain the expected accuracy of curve fitting. Once the relaxation modulus function is available, the corresponding creep compliance function can be easily determined as well utilizing their mathematical relationship in the Laplace domain (Equation 5-6). If the material is characterized by one relaxation time in this case, such a three-element model is known as the Zener model or standard linear solid model.

5.3.2 Problem Formulation

Although the time derivatives appearing in the governing equations (i.e. Equation 5-3 and Equation 5-4) add the third argument to problem parameters and subsequently bring about complications when addressing the viscoelastic contact problems, the field equations still possess certain mathematical features. It makes the viscoelastic solution (i.e. the displacement response of linear viscoelastic materials to an arbitrary distribution of contact tractions) readily accessible instead of deriving it from scratch. A common practice is to adapt the existing well-developed elastic solutions to make them applicable to linear viscoelastic materials following the elastic-viscoelastic correspondence principle. Steps to take for the transition from elastic solutions to the viscoelastic counterparts when dealing with a typical half-space contact problem (a rigid sphere against a viscoelastic half-space shown in Figure 5-2 (a)) are detailed as follows:

1. replace the constant elastic contact compliance $(1/2G)$ with the time-dependent creep compliance $\Phi(t)$;
2. subdivide the pressure history in the simulation time $p(t)$ into infinitesimal intervals $(\frac{\partial p}{\partial t})$;
3. superpose the contributions of pressures in all subdivided time intervals by making use of the hereditary integral $(\int_0^t \dots dt')$.

Based on the corresponding solution for an elastic material (the Boussinesq solution [35]), the normal displacement of the viscoelastic surface caused by the normal traction at the time t can be determined as follows:

$$u_{zz}^e(x_1, x_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{(1-\nu)p(x'_1, x'_2)dx'_1dx'_2}{2\pi G\sqrt{(x_1-x'_1)^2 + (x_2-x'_2)^2}};$$

$$u_{zz}^e(x_1, x_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G^e(x_1-x'_1, x_2-x'_2)p(x'_1, x'_2) dx'_1dx'_2,$$

Equation 5-9

$$u_{zz}^v(x_1, x_2, t) = \int_0^t \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{(1-\nu)\Phi(t-t')}{\pi\sqrt{(x_1-x'_1)^2 + (x_2-x'_2)^2}} \frac{\partial p(x'_1, x'_2, t')}{\partial t'} dx'_1dx'_2dt';$$

$$u_{zz}^v(x_1, x_2, t) = \int_0^t \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G_{zz}^v(x_1-x'_1, x_2-x'_2, t-t') \frac{\partial p(x'_1, x'_2, t')}{\partial t'} dx'_1dx'_2dt',$$

Equation 5-10

where Equation 5-9 is the Boussinesq integral for elastic surfaces and Equation 5-10 is the adapted viscoelastic solution based on the corresponding principle, in which the Green's function $G_{zz}^v(x_1, x_2, t)$ characterizes the normal deformation at the point of coordinates (x_1, x_2) and at the time point of t induced by a unit concentrated and normal force and it is determined as follows:

$$G_{zz}^v(x_1, x_2, t) = \frac{(1 - \nu)\Phi(t)}{\pi\sqrt{x_1^2 + x_2^2}} \quad \text{Equation 5-11}$$

Through the manipulation of elastic solutions, all the continuous equations accounting for the effects of loading history on the surface displacements during the viscoelastic contact period can be obtained.

A proper discretisation is a prerequisite for the development of a valid viscoelastic contact model. Apart from the spatial discretisation, where the area of interest between two contacting bodies is meshed into equally spaced rectangular elements with the size of $2a \times 2b$ and number of N_1 and N_2 in x and y directions respectively, supplementary temporal discretization is necessary to model the viscoelastic contact due to the aforementioned effects of loading history on the viscoelastic deformation. The total simulation time T is thus discretised into N_t time points. The time interval is Δt which is uniform and assumed to be sufficiently short so that the elemental pressure is assumed to be constant. Based on the assumption, the partial derivative $\frac{\partial p(x'_1, x'_2, t) dt'}{\partial t'}$ in Equation 5-10 can be substituted by the finite pressure difference $'p(i, j, k) - p(i, j, k - 1)'$, where $p(i, j, k)$ is the discretised counterpart of $p(x, y, t)$ with $i = 1 \cdots N_1$, $j = 1 \cdots N_2$, $t = k\Delta t$ and $k = 1 \cdots N_t$.

As a result of the spatial and temporal discretisation, the continuous form of Equation 5-10 is modified into the following piecewise definition:

$$u_{zz}^v(i, j, k) = \sum_{n=1}^{N_t} \sum_{l=1}^{N_1} \sum_{m=1}^{N_2} IC_{zz}^v(i - l, j - m, k - n) (p(l, m, n) - p(l, m, n - 1)), \quad \text{Equation 5-12}$$

where $IC_{zz}^v(i - l, j - m, k - n)$ is known as the viscoelastic influence coefficient characterizing the normal displacement observed after k time steps in the node (i, j) of the spatial mesh under the effect of a uniform pressure of $1/(4ab)$ Pa acting on the node (l, m) in the n -th time step after the reference time, with $n \leq k$.

After determining the closed-form discretized solutions of all influence coefficient matrices relating the surface displacements and contact tractions in x , y and z directions,

nodal displacements of a linear viscoelastic surface induced by an arbitrary history of nodal tractions can then be expressed as follows:

$$\begin{aligned} \begin{bmatrix} u_x(i, j, k) \\ u_y(i, j, k) \\ u_z(i, j, k) \end{bmatrix} &= \sum_{n=1}^{N_t} \sum_{l=1}^{N_1} \sum_{m=1}^{N_2} \begin{bmatrix} IC_{xx}^v(i-l, j-m, k-n) & IC_{xy}^v(i-l, j-m, k-n) & IC_{xz}^v(i-l, j-m, k-n) \\ IC_{yx}^v(i-l, j-m, k-n) & IC_{yy}^v(i-l, j-m, k-n) & IC_{yz}^v(i-l, j-m, k-n) \\ IC_{zx}^v(i-l, j-m, k-n) & IC_{zy}^v(i-l, j-m, k-n) & IC_{zz}^v(i-l, j-m, k-n) \end{bmatrix} \\ &\cdot \begin{bmatrix} q_x(l, m, n) - q_x(l, m, n-1) \\ q_y(l, m, n) - q_y(l, m, n-1) \\ p(l, m, n) - p(l, m, n-1) \end{bmatrix}, \end{aligned} \quad \begin{array}{l} \text{Equation} \\ \text{5-13} \end{array}$$

where DC-FFT is applied to accelerate the convolution operations. For the sake of simplicity, Coulomb's friction law is implemented here to identify the stick and slip regions within the contacting area based on a constant and uniform coefficient of friction μ_f . A quasi-static process is assumed to avoid the problem of dissipative friction and its irreversibility related to the load-path dependency when addressing the frictional contact problems. It is important to note that the Poisson's ratio of a viscoelastic material employed in practice is usually time-dependent [36], but it is assumed to be constant here for simplicity.

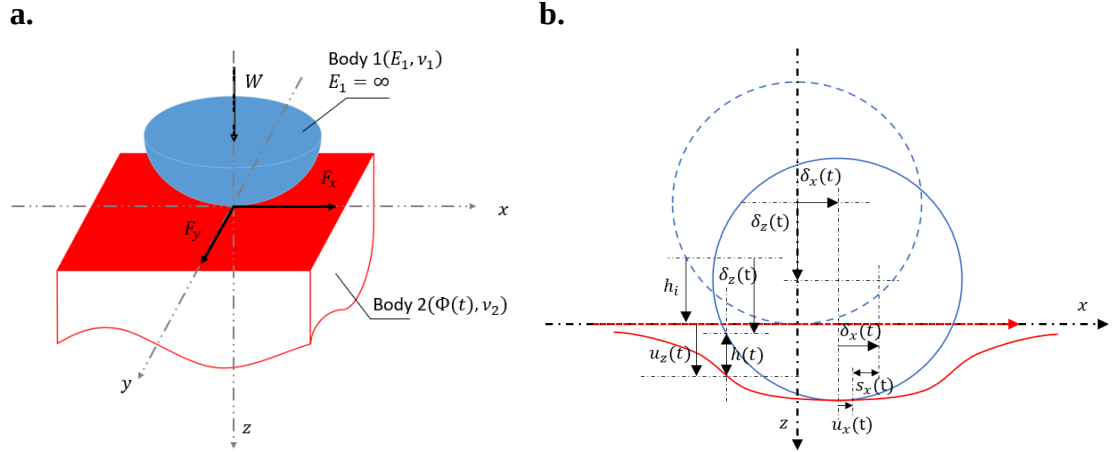


Figure 5-2 (a) Geometry of the contacting bodies: W, F_x, F_y are the input normal load and the tangential loads in x and y directions respectively, E_1 ($E_1 = \infty$) and ν_1 are the elastic modulus and Poisson's ratio of the rigid sphere respectively, and $\Phi(t)$ and ν_2 are the time-dependent creep compliance and Poisson's ratio of the viscoelastic half-space, and **(b) Transient displacement condition on the $x - z$ plane:** h_i denotes the gap between surfaces before loading, $h(t), s_x(t), \delta(t), u(t)$ are the gap between deformed surfaces, the slip distance in the x direction, the rigid body displacement, and the surface deformation after a certain time t respectively with subscript denoting the direction of relevant vectors.

With the equations and assumptions described above, a sequence of discretised contact problems with complementary conditions can be constructed to search for the solution that accurately reproduces the history of the viscoelastic contact process. The boundary conditions in discretised viscoelastic contact problems are in an analogous form to the elastic ones summarized by Johnson [37] except that the time is now accounted for:

1. Load balance:

The sum of traction distribution at the contact interface should be strictly equal to the input load in the corresponding direction at any specific time point during the discretised simulation period:

$$W(t) = \Delta \sum_{(i,j) \in I_c(k)} p(i,j,k), F_x(t) = \Delta \sum_{(i,j) \in I_c(k)} q_x(i,j,k),$$

$$F_y(t) = \Delta \sum_{(i,j) \in I_c(k)} q_y(i,j,k), k = 1 \cdots N_t,$$

Equation 5-14

where Δ ($2a \times 2b$) is the area of each element in the set mesh and $I_c(k)$ is the contact domain at the k -th time point.

2. The deformation of viscoelastic surfaces must meet the following geometrical conditions at any specific time:

In the normal direction, the following condition of surface separation should be met:

$$h(i,j,k) = h_i(i,j) + u_z(i,j,k) - \delta_z(k), k = 1 \cdots N_t, (i,j) \in I_p$$

Equation 5-15

where I_p denotes the whole simulation domain, h_i is the gap between undeformed surface, and h , and δ_z are the gap between deformed surfaces and the normal rigid body displacement at the specified time point k respectively as illustrated in Figure 5-2 (b).

In the two tangential directions, the following displacement equation should be met:

$$s_x(i,j,k) = u_x(i,j,k) - \delta_x(k), (i,j) \in I_c(k);$$

$$s_y(i,j,k) = u_y(i,j,k) - \delta_y(k), (i,j) \in I_c(k),$$

Equation 5-16

where s_x and s_y are respectively the slip distances in x and y directions at a specific time point while δ_x and δ_y denote the time-dependent rigid body displacements in x and y directions respectively as shown in Figure 5-2 (b).

3. Complementary conditions should be satisfied at any specific time over the contacting surfaces:

In the normal direction, the product of pressure and surface gap must always be zero over the whole computational domain I_p expressed as:

$$p(i,j,k) > 0 \ \& \ h(i,j,k) = 0, (i,j) \in I_c(k);$$

$$p(i,j,k) = 0 \ \& \ h(i,j) > 0, (i,j) \in I_p - I_c(k),$$

Equation 5-17

where $I_p - I_c(k)$ denotes the non-contacting domain at the time step k .

Equation 5-17 implies that the investigated surfaces are impenetrable and the surface adhesion is not considered in the given contact analysis.

In the lateral directions, the shear traction magnitude must be lower than the local friction and no slip should exist in the time-dependent stick zone according to Coulomb's friction law. As to the slip region, the norm of shear tractions must be equal to the local friction. In addition, the direction of the shear stress should be opposite to that of the slip distance. Such contact conditions are expressed as follows:

$$\begin{aligned}
 & \sqrt{q_x(i, j, k)^2 + q_y(i, j, k)^2} < \mu_f p(i, j, k), \quad |s_x(i, j, k)^2 + s_y(i, j, k)^2| = 0, \quad (i, j) \\
 & \quad \in I_s(k); \\
 & q_x(i, j, k) \cdot s_x(i, j, k) + q_y(i, j, k) \cdot s_y(i, j, k) < 0, \quad (i, j) \in I_c(k) - I_s(k); \\
 & \sqrt{q_x(i, j, k)^2 + q_y(i, j, k)^2} = \mu p(i, j, k), \quad |s_x(i, j, k)^2 + s_y(i, j, k)^2| \neq 0, \quad (i, j) \\
 & \quad \in I_c(k) - I_s(k).
 \end{aligned}
 \tag{Equation 5-18}$$

5.3.3 Algorithm Description

In order to search the time-dependent contact tractions and surface deflections meeting the conditions described above, CGM is adopted considering its guaranteed convergence for quadratic optimization problems with inequality constraints. For each time step, the corresponding surface gap g in the contacting area, and slip distance s in stick regions, are the system residuals to be minimized in established numerical optimization problems. The normal and tangential contact problems are addressed in two separate solvers, in which the contribution of contact tractions in all directions is considered when determining the surface displacement.

When the loads or displacements are just applied at the first time point, there are no effects of loading history. Hence, the viscoelastic contact problem being addressed becomes a time-independent coupled elastic one temporarily, for which the constant material property is the instantaneous modulus $\Psi(0)$. The iterative process to obtain a coupled elastic solution is detailed in reference [32], after which, the loading history is accounted for to obtain the contact solutions of the following time steps. The overall numerical approach to viscoelastic partial slip contact problems is shown in Figure 5-3. The effects of loading history are included via considering the contribution of the past loading history to the surface displacement. To consider the surface displacement in the normal direction derived from normal pressures u_{zz} at time step α , the additional displacement component from the pressure history can be determined as follows:

$$u_{incre}(i, j, \alpha) = \sum_{n=1}^{\alpha-1} \sum_{l=1}^{N_1} \sum_{m=1}^{N_2} (IC_{zz}^v(i-l, j-m, \alpha+1-n) \cdot p(l, m, n) - IC_{zz}^v(i-l, j-m, \alpha-n) \cdot p(l, m, n))$$

Equation 5-19

According to Equation 5-19, the extra deformation for u_{zz} at the second time point should be ' $IC_{zz}^v(:, :, 2) * p(:, :, 1) - IC_{zz}^v(:, :, 1) * p(:, :, 1)$ '. The incremental deformations at the rest time points, as well as those arising from contact tractions in other directions, can be determined in an identical fashion, which is suggested to be executed outside the iteration process of the contact solvers to reduce the computational time.

After adjusting the geometrical conditions of deformation (Equation 5-15 and Equation 4-5) by converting the surface deformation u into the following form,

$$u = u_{incre} + u_t$$

Equation 5-20

$$u_t = IC(:, :, 1) * t(:, :, \alpha)$$

Equation 5-21

the unknown terms in the new equations are reduced to the contact tractions at the time point being considered $t(:, :, \alpha)$ and the resulting displacement components u_t :

The viscoelastic stick-slip contact problem is thus transformed into a sequence of instantaneous elastic counterparts, for which an algorithm similar to that given in the work of Wang et al. [32] is employed to search for the solutions until the end of the simulation time is reached.

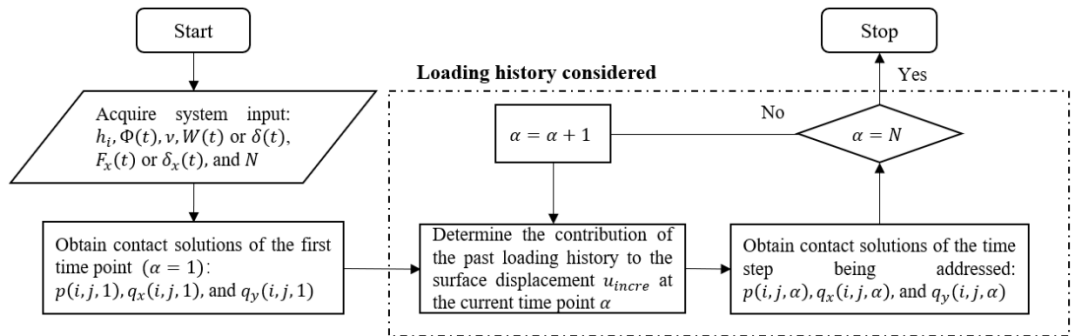


Figure 5-3 Flow chart of the algorithm for the viscoelastic partial slip contact problems

5.4 Model Validation

Due to limited literature concerning the coupled partial slip of viscoelastic materials, the validation work for the developed model is separated into two separate sections, in which reference solutions are given. Firstly, the coupled stick-slip aspect of the model has been validated by simulating the partial slip contact between a rigid sphere and an elastic half-

space, of which a detailed description is given in the work of Wang et al. [32]. Secondly, the viscoelastic aspect of the model is validated by simulating the contact of a rigid sphere indenting a viscoelastic material, and it is detailed as follows.

To simulate the normal contact problem of a rigid sphere indenting an incompressible viscoelastic half-space, the time-dependent behaviour of the viscoelastic material is simplified and described by a Maxwell model, in which the creep compliance and relaxation modulus are characterized as follows:

$$\Phi(t) = \frac{1}{G} + \frac{t}{\eta} \quad \text{Equation 5-22}$$

$$\Psi(t) = G \cdot \exp\left(-\frac{t}{\tau}\right) \quad \text{Equation 5-23}$$

The geometry and material properties of the contacting surfaces are given in Table 5-1.

Table 5-1 Parameters used in the simulation of a rigid sphere indenting a viscoelastic half-space (Maxwell model) under constant normal load

Parameter	Value	Description (Unit)
R	3.5	Radius of the sphere (mm)
G	81.92	Shear modulus of the spring in Maxwell model (GPa)
η	81.92	Viscosity of the dashpot in Maxwell model (GPa. s)
τ	1	Relaxation time of the viscoelastic material (s)
ν	0.5	Poisson's ratio of the viscoelastic material
a_0	117	Hertzian contacting radius (μm)
p_0	3487	Hertzian peak normal pressure (MPa)

A step load of 100 N is applied instantaneously at the beginning of the simulation window ($t = 0$ s) and kept throughout the whole contact period. The simulation results derived from the model are nondimensionalized by the Hertzian solutions of a normal sphere contact, where the instantaneous modulus $\Psi(0)$ is the input material property. The time-dependent contact tractions and nodal coordinates are normalized by the peak normal pressure p_0 and contact radius a_0 (shown in Table 5-1) respectively. The contact solutions given hereinafter are all normalized in the same method if it is not mentioned

otherwise. The potential contact area ($512 \mu\text{m} \times 512 \mu\text{m}$) is discretized by a mesh system with 256×256 uniformly distributed elements, and the whole viscoelastic contact period ($T = 2\tau$) is subdivided into 101 time points.

The instantaneous geometrical change of the two contacting bodies under the constant load is shown in Figure 5-4, where the interaction region (i.e. contacting area) is observed to keep increasing with time. This growing area tends not to diminish over time as the Maxwell model has no so-called steady state and thus cannot fully characterize the creep phenomenon. The normalized pressure distribution at different times during the viscoelastic contact process is shown in Figure 5-5, where the peak pressure keeps decreasing while the contacting area increases with time. Under constant normal load, the position of peak pressure is found to change with time and a spike on the contacting edge is observed after a relatively long time ($t = 2.0\tau$). To validate the simulation result, the numerical solutions (normal pressure distribution $p(t)$) are compared with Lee and Radok's [3] analytical solutions expressed as follows:

$$p(r, t) = \frac{2}{\pi R(1-\nu)} \int_0^t \Psi(t-t') \frac{d}{dt'} (Re\sqrt{a^2(t') - r^2}) dt' \quad \text{Equation 5-24}$$

where the operator ' $Re()$ ' denotes taking the real part of the complex quantity and the time-dependent contact radius $a(t)$ is determined as follows [3]:

$$a^3(t) = \frac{3(1-\nu)}{4} RW\Phi(t) \quad \text{Equation 5-25}$$

A good agreement between the analytical solutions (scatter plots) and simulation results (solid lines) can be found in Figure 5-5.

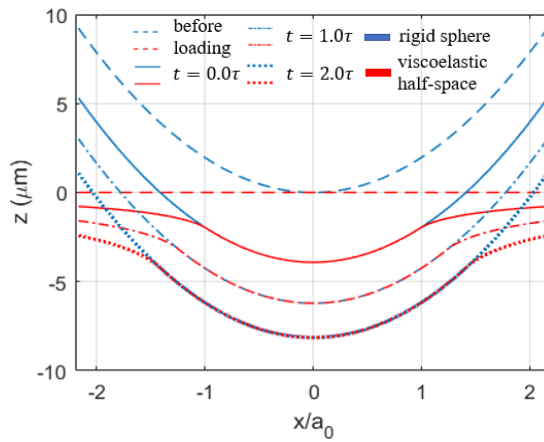


Figure 5-4 Creep of surface profile at different time points under the input normal load

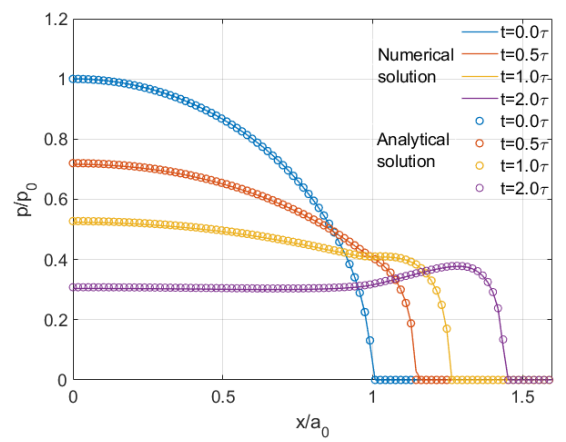


Figure 5-5 Normalized pressure p for the viscoelastic sphere contact (Maxwell model)

To account for an arbitrary history of loading conditions, contact of a rigid sphere ($R = 3.5 \text{ mm}$) indenting a real viscoelastic material exhibiting more than one relaxation time under dynamic loading conditions was considered. The contact behaviour of a thermoplastic polymer known as polymethyl methacrylate (PMMA), which satisfies the assumption of linear viscoelastic material, is simulated under a triangle-shape loading condition expressed as follows:

$$W(t) = 10t \cdot W(t) - 20(t - 10) \cdot H(t - 10), \quad (\text{N}) \quad \text{Equation 5-26}$$

where $H(t)$ denotes the Heaviside step function.

Based on the results of a relevant standard relaxation test conducted by Kumar and Narasimhan [38], a generalized Weichert model was established by Chen et al. [11] to characterize the mechanical response of PMMA. The relaxation modulus is indicated by a two-term Prony series (two relaxation times):

$$\begin{aligned} \Psi(t) = & 1429.71 + 184.62 \cdot \exp\left(-\frac{t}{8.93}\right) + 191.06 \\ & \cdot \exp\left(-\frac{t}{117.96}\right), \quad (\text{MPa}) \end{aligned} \quad \text{Equation 5-27}$$

The creep compliance function for PMMA is then determined based on the relation between the relaxation modulus and creep compliance of linear viscoelastic materials in the Laplace domain (Equation 5-6):

$$\begin{aligned} \Phi(t) = & 0.699 - 0.0838 \cdot \exp\left(-\frac{t}{133.869}\right) - 0.06174 \\ & \cdot \exp\left(-\frac{t}{9.9404}\right), \quad \left(\frac{1}{\text{GPa}}\right) \end{aligned} \quad \text{Equation 5-28}$$

A time-independent Poisson's ratio ($\nu = 0.38$) is assumed here referring to Chen et al.'s work [11]. The area of interest ($1280 \mu\text{m} \times 1280 \mu\text{m}$) is discretized by a mesh system with 256×256 uniformly distributed elements, and the whole viscoelastic contact period ($T = 20 \text{ s}$) is subdivided into 41 time steps. For validation reasons, the coupling between the normal pressures and shear tractions derived from the dissimilar material properties of the PMMA half-space and rigid plane is neglected.

The pressure distributions at different time points are presented in Figure 5-6 (a), which are normalized by the Hertzian solution based on the instantaneous modulus $\Psi(0)$ and the maximum input load W_{max} . The pressures at the loading period and unloading period are found to be different even though the magnitudes of the indentation load are identical. Pressure at the unloading period has a lower peak value but a wider covering range

indicating a more conforming contact. Such a difference shows the effects of loading history on solutions to viscoelastic contact problems. Good agreement can be found between our simulation results and those derived from Chen et al's model [11] as shown in Figure 5-6 (a).

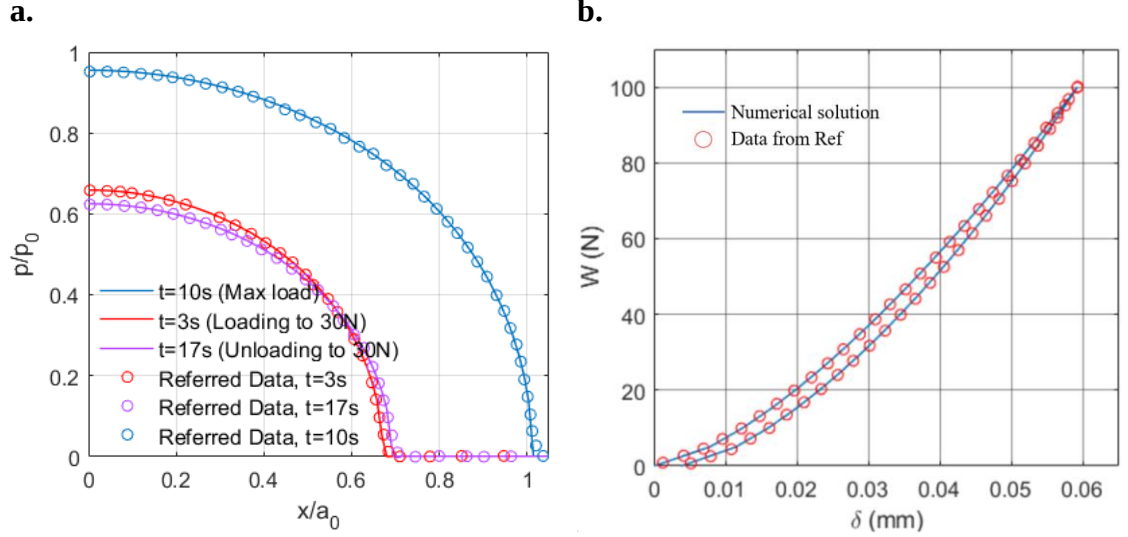


Figure 5-6 Comparison of the contact solutions from the present model (solid lines) with the results from Chen et al. [11] (scatter) under the triangular-shape loading conditions: (a) Normalized pressure p at different times and (b) Evolution of rigid body indentation δ with the varying normal load

Another validation in terms of the evolution of rigid body indentation with the varying indentation load is given in Figure 5-6 (b), where the simulation result from our model agrees with the referred solution of Chen, et al [11]. The remaining indentation corresponds to the unrecovered surface deformation arising from the viscosity of PMMA. The out-of-phase response of the displacement to input load is known as hysteresis which results from the strain energy lost or dissipated as heat between loading and unloading periods due to internal friction in the viscoelastic material. To describe it in detail, it arises from the viscous component of the material properties. When a force is applied to a viscoelastic material, there exists a certain resistance to this input force leading to more energy (larger area under loading curve) to be exerted than that would have been expected to extend a similar purely elastic material. Such a resistance still performs during the unloading period leading to a less amount of energy returned (smaller area under unloading curve). Owing to this specific capacity to dissipate energy, viscoelastic materials are commonly used to produce protective products in practice.

5.5 Results and Discussion

The validated model is employed to simulate how the separation of stick and slip regions within the contacting area evolves in a viscoelastic frictional contact problem. Based on a material parameter that characterizes the rheological property of materials, a new unified explanation of the viscoelastic contact behaviour under different conditions is provided here. These simulation results are somehow counter-intuitive from the perspective of the traditional theory of elastic solid contact.

Before the stick-slip simulation study, we want to raise an argument regarding an intrinsic behaviour affecting the normal pressure distribution in viscoelastic contacts.

5.5.1 What Affects the Shape of Pressure Distribution?

In a few studies concerning viscoelastic contact analysis [3, 39], the spikes appearing on the edges in the normal pressure distribution as shown in Figure 5-5 were argued to be one of the features of viscoelastic materials. Such an opinion is not rigorously validated since the Hertzian-type contact pressure was reported by Spinu and Cerlinca [40] while the spikes on the edges were found by Bugnicourt [39]. Both studies used Zener models to characterize the viscoelastic response of simulated materials in their study. It was also proposed by Koumi et al. [21] that the configuration of the two contacting bodies determines the shape of pressure distribution. In other words, the shape of contact tractions depends on which one of the two contacting bodies is viscoelastic whereas the other could be considered rigid. According to Koumi, et al. [21], when it comes to the contact between a sphere and a half-space, if the sphere is rigid and the half-space is viscoelastic, the normal pressure will contain spikes on the edge. Contrarily, the Hertzian-type contact pressure is obtained. However, the authors of the current manuscript have tested both cases using a Maxwell model with the properties given in Table 5-1. The spikes on the contacting edges appear in the normal pressure profiles after a relatively long time ($t \geq 1.0\tau$) in both cases, which contradicts the claims of Koumi et al. [21]. The solutions to the surface displacement and normal pressure are identical as that presented in Figure 5-5 no matter which one of the contacting bodies is viscoelastic. Compared with the solutions shown in the section 5.4 using the Maxwell model, the only dissimilarity arising from an opposite contact configuration is the geometry change of the two contacting surfaces at different time points as shown in Figure 5-7. In this case, identical magnitudes of deformation and indentation are now experienced by the sphere and flat rigid plane respectively. It is of note that in Koumi et al.'s study [21], the normal displacement is specified while the indentation load is specified in the current study.

Considering the limitation of Maxwell models to quantify the viscoelastic response of materials, several simulation attempts using Zener models were conducted. The relaxation modulus and creep compliance of a Zener model are usually defined in the following general form in several numerical studies [18-23]:

$$\Psi(t) = \mu_0 + (\mu_\infty - \mu_0) \exp\left(-\frac{t}{\frac{\mu_0}{\mu_\infty} \tau}\right), \quad \text{Equation 5-29}$$

$$\Phi(t) = \frac{1}{\mu_\infty} + \frac{1}{\mu_1} \left(1 - \exp\left(-\frac{t}{\tau}\right)\right),$$

$$\mu_1 = \frac{1}{\frac{1}{\mu_0} - \frac{1}{\mu_\infty}}, \quad \text{Equation 5-30}$$

where μ_∞ is the initial shear modulus, μ_0 is the modulus after infinite time, the ratio (μ_∞/μ_0) is known as the ratio of the retardation time (characteristic time for creep) to the relaxation time and it was commonly specified to be 10 in different numerical simulations of viscoelastic contact problems conducted by many researchers [18, 21-23].

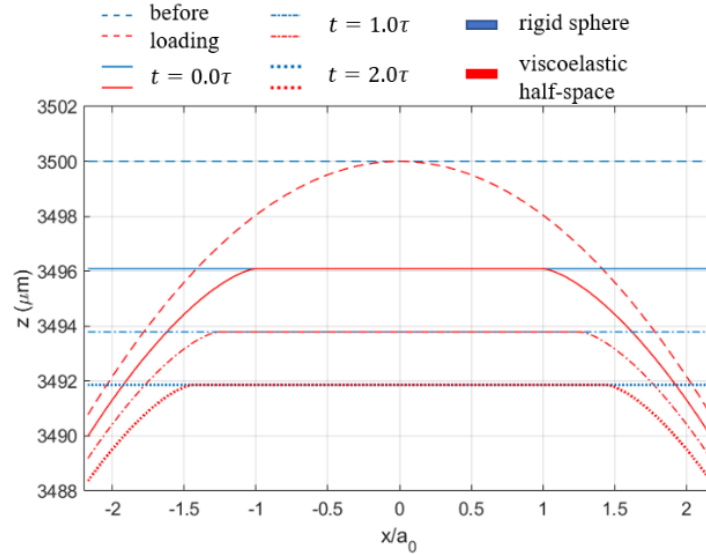


Figure 5-7 Geometry changes of the surface profiles at different times for the contact problem of a rigid plane indenting a viscoelastic sphere (Maxwell model) under constant normal load

To investigate the physical meaning of the ratio (μ_∞/μ_0) and show its effects on the contact solutions, some of the material properties including the instantaneous modulus μ_∞ , the relaxation time τ , the Poisson's ratio ν , the geometry of contacting bodies, and the indentation load were set to be identical to the study of Wallace [22]. In addition, the ratio (μ_∞/μ_0) is specified to vary within a certain range shown in Table 5-2. The potential contact area $(5.5a_0 \times 5.5a_0)$ is discretized by a mesh system with 256×256 uniformly

distributed elements, and the whole viscoelastic contact period ($T = 3\tau$) is subdivided into a number of time points depending on the ratio value.

Table 5-2 Parameters used in the contact simulation of a rigid sphere indenting a viscoelastic half-space (Zener model) under constant normal load

Parameter	Value	Description (Unit)
W	0.15	Input indentation load (N)
R	10	Radius of the sphere (mm)
μ_∞	3.86	Initial shear modulus of material (MPa)
$\frac{\mu_\infty}{\mu_0}$	2,4,8,10	Ratio of retardation time to relaxation time (ratio of initial shear modulus to modulus after infinite time)
τ	0.01	Relaxation time of the viscoelastic material (s)
ν	0.3	Poisson's ratio of the viscoelastic material
a_0	588.7	Hertzian contacting radius for nondimensionalization (μm)
p_0	0.2067	Hertzian peak normal pressure for nondimensionalization (MPa)

Normalized results presented in Figure 5-8 (a)-(e) show that the shape of normal pressure in a viscoelastic indentation problem is significantly affected by the ratio (μ_∞/μ_0) of a material. The spikes on the contacting edge are only observed for viscoelastic materials with a relatively high ratio (Figure 5-8 (c) for $\mu_\infty/\mu_0 = 8$ and Figure 5-8 (e) for $\mu_\infty/\mu_0 = 10$) while a Hertzian-type normal pressure distribution is observed for the materials with a lower ratio (Figure 5-8 (a) for $\mu_\infty/\mu_0 = 2$ and Figure 5-8 (b) for $\mu_\infty/\mu_0 = 4$). It is of note that for the simulation of viscoelastic materials with higher ratio (μ_∞/μ_0), more time steps are usually required to discretize the specified time domain to avoid the oscillating results around the contacting edge during the early period of simulation as labelled in Figure 5-8 (d). The distinct shapes of pressure distribution for viscoelastic materials characterized by standard linear solids with different ratios (μ_∞/μ_0) were also reported by Yakovanko and Goryacheva [41], but there exists no detailed interpretation of the outcome.

The spikes on the contacting edges do not exist permanently. After extending the simulation time from 3τ to 6τ for the contact simulation of the viscoelastic material with the ratio $\mu_\infty/\mu_0 = 10$, the spikes disappear and the pressure distribution becomes the

Hertzian-type at the end of simulation time as shown in Figure 5-8 (f). In addition, the solutions are found to be close to each other after a relatively long time ($t \geq 5.0\tau$), which implies that a steady state is almost reached for the viscoelastic contact.

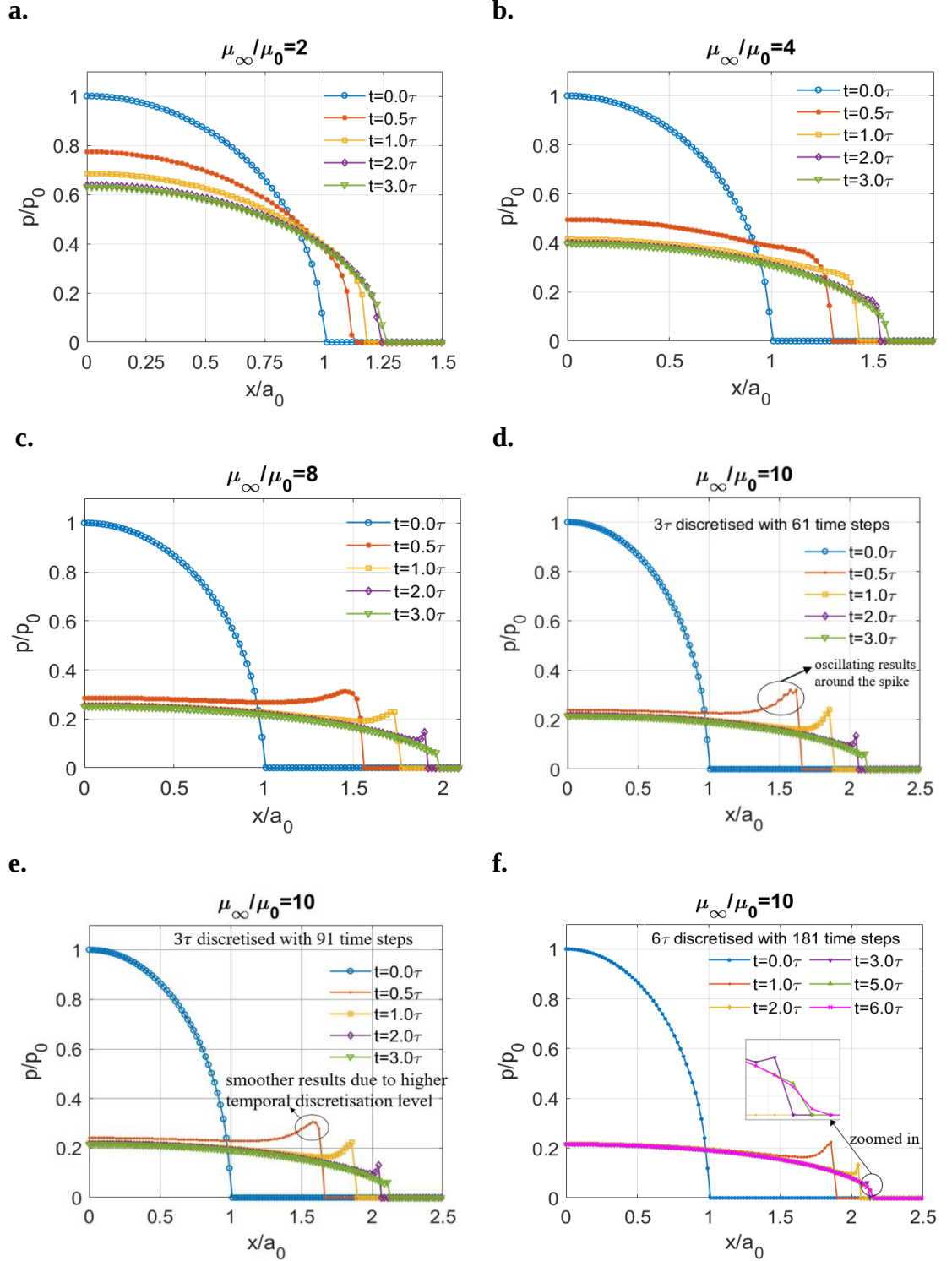


Figure 5-8 Normalized normal pressure distribution at different moments for different viscoelastic materials (Zener model) within the simulation time

To explain the observed phenomena, the changes of surface geometry for the materials with the lowest and highest ratio of μ_∞/μ_0 under the constant indentation load within the

same simulation time (Figure 5-9 (a) and (b) respectively) are compared. The viscoelastic surface with the higher ratio of μ_{∞}/μ_0 experiences dramatically increasing deformation with time, during which, the contact becomes significantly more conformal. In other words, the creep phenomenon is more intense for the surface with a higher ratio. To assume that a viscoelastic body experiences such a weak creep that the change of surface deformation per time is extremely insignificant, it could be approximated as an ordinary elastic solid. One exhibiting a converse contact behaviour could then be treated as a fluid. Based on the assumption, the ratio of the retardation time to relaxation time determines whether a viscoelastic is more fluid-like or solid-like. For materials with lower ratios of μ_{∞}/μ_0 , the elasticity plays a dominant role leading to a Hertzian-type pressure distribution. In this case, the viscosity of materials has relatively trivial effects causing the increase of contacting area with a rapidly decreasing strain rate. Eventually, the steady state of the viscoelastic contact is quickly reached. For materials with higher ratios of μ_{∞}/μ_0 , their responses to the indentation load are similar to that of a pack of liquid once squeezed. Under this condition, the load is distributed to contacting edges when the material is squeezed after a period of time. However, due to a decreasing strain rate, a steady state would be reached for any viscoelastic contact using a Zener model. The pressure distribution eventually evolves into the Hertzian type arising from the elasticity of materials. The time needed to reach this state is also determined by the ratio value so that a higher ratio leads to a longer time.

It is of note that for viscoelastic solids the retardation time must always be greater than the relaxation time, which means that the ratio μ_{∞}/μ_0 should always be more than 1. A ratio that is less than 1 is normally valid for viscoelastic fluids that can be characterized with an anti-Zener model [42].

Due to the linearly increasing creep compliance, the Maxwell model shows no decreasing strain rate as shown in Figure 5-10. According to Chen et al [11], a Maxwell model is usually employed to characterize the evident viscoelastic response exhibited by soft thermoplastic polymers in the vicinity of their melting temperature. Even though the simulation time for the contact problem of the Maxwell model investigated in section 5.4 is extended, normalized results (Figure 5-11) show that the spikes on the edges always exist for the normal pressure distribution. This indicates that the load keeps being distributed on the contact edges because of its extremely strong fluid-like property.

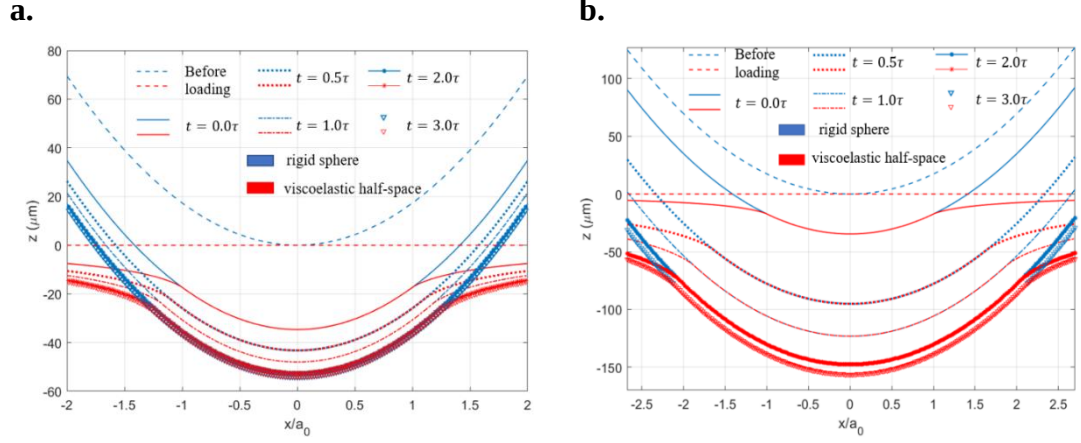


Figure 5-9 Creep of surface profiles at different time points for different viscoelastic materials: (a) $\frac{\mu_{\infty}}{\mu_0} = 2$ and (b) $\frac{\mu_{\infty}}{\mu_0} = 10$

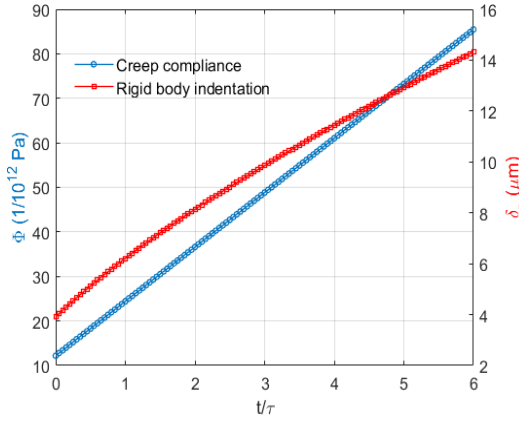


Figure 5-10 Changes of the creep compliance Φ (y axis right) and the resulting rigid body indentation δ (y axis right) with time

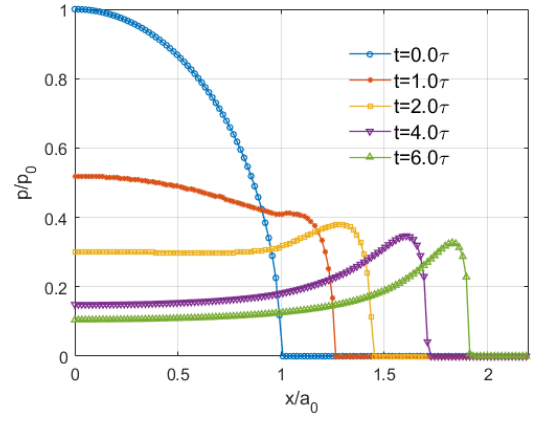


Figure 5-11 Distribution of normal pressure at different time points within the simulation window ($t = 6.0\tau$)

For viscoelastic materials characterized by the Generalized Weichert models exhibiting more than one relaxation time, there exist multiple ratios of μ_{∞}/μ_0 . To find a general rule in this case, the normal indentation problems of PMMA and a fictitious viscoelastic material were simulated. The Poisson's ratio of the fictitious material is the same as PMMA ($\nu = 0.38$) but the creep compliance is adjusted based on that of PMMA (Equation 5-28) as follows:

$$\Phi(t) = 0.699 - 0.5595 \cdot \exp\left(-\frac{t}{133.869}\right) - 0.02 \cdot \exp\left(-\frac{t}{9.9404}\right), \quad \left(\frac{1}{\text{GPa}}\right) \quad \text{Equation 5-31}$$

The two ratios (μ_{∞}/μ_0) of the materials characterized by a two-element Generalized Weichert model are evaluated approximately by transforming the creep compliance

function into the forms in two Zener models. To take the PMMA material as an example, its creep compliance (Equation 5-28) is converted into the following forms:

$$\Phi(t)_1 = 0.699 - 0.0838 \cdot \exp\left(-\frac{t}{133.869}\right), \quad \left(\frac{1}{\text{GPa}}\right) \quad \text{Equation 5-32}$$

$$\Phi(t)_2 = 0.699 - 0.06174 \cdot \exp\left(-\frac{t}{9.9404}\right), \quad \left(\frac{1}{\text{GPa}}\right) \quad \text{Equation 5-33}$$

which leads to two relatively low ratios ($\mu_\infty/\mu_{01} \approx 1.136$ and $\mu_\infty/\mu_{02} \approx 1.097$). Using the same method, the ratios of the fictitious viscoelastic material are determined as well ($\mu_\infty/\mu_{01} \approx 5$ and $\mu_\infty/\mu_{02} \approx 1.03$).

The surface geometry, contact configuration and spatial discretization used for the indentation simulation are identical to that specified for the dynamic loading contact problem of PMMA in section 5.4. A step load of 10 N is specified at the beginning of the simulation and held constant for 600 seconds, which are discretized into 61 and 121 time steps for the contacts of PMMA and fictitious material respectively.

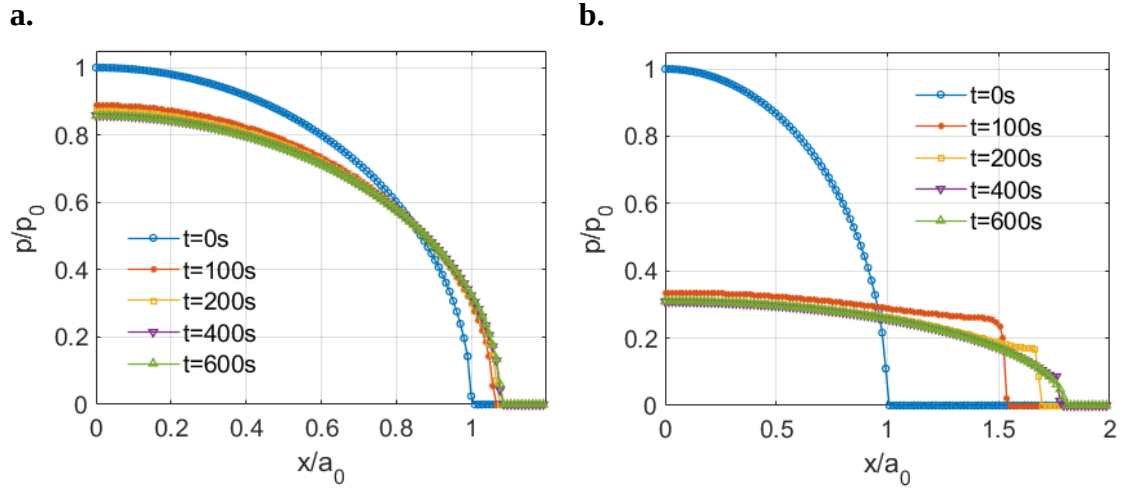


Figure 5-12 Pressure distributions for different viscoelastic materials (Generalized Weichert model) under normal loading: (a) PMMA and (b) Fictitious viscoelastic material

Under these contact inputs, the PMMA is found to experience relatively insignificant creep deformations that the steady-state contact is quickly reached as shown in Figure 5-12 (a). Since the two ratios of PMMA are extremely small, the PMMA surface should perform more like a solid. On the contrary, spikes on the edges could be observed in the normalized pressure profiles of the fictitious material (Figure 5-12 (b)) although a Hertzian-type pressure appears after a relatively long simulation time. A short-term conclusion regarding the Generalized Weichert model can be drawn that as long as one

of the ratios is high enough that the material can be considered more fluid-like, spikes on the edges would be observed for the normal pressure distribution at a limited period.

5.5.2 Does the Viscoelasticity Affect the Stick and Slip Separation?

5.5.2.1 Stick-Slip under Constant Inputs

A normal load alone can result in partial slip contact of two dissimilar materials when the coupling between normal pressure and shear tractions is considered. The study of viscoelastic partial slip contact can be performed without any tangential load. Based on our understanding of the effects of material ratio (μ_∞/μ_0) on the contact solutions, Zener models with two different ratio values ($\mu_\infty/\mu_{0_1} = 3$ and $\mu_\infty/\mu_{0_2} = 10$) are employed here to simulate the partial slip contacts of different viscoelastic materials. Some of the contact inputs including the surface geometry, contact configuration and spatial discretization, and normal load are specified as those given in Table 5-2 while the total simulation time ($T = 2\tau$) is discretized with 81 time steps. A constant coefficient of friction μ_f is assumed to be 0.30.

The normalized contact solutions for these two different viscoelastic materials are given in Figure 5-13. Since tangential load is not applied yet, the normal pressure distributions of the two surfaces still exhibit axial symmetry. For simplicity, only the pressures on the front part of the contacting areas are presented in Figure 5-13 (a) and (b), where the peak pressure at the beginning is higher than the Hertzian pressure due to the coupling effects. The normal pressure profile of the viscoelastic surface with the smaller ratio shows no significant change related to the coupling effects. In comparison, the extra spikes in the middle of the contacting region as indicated in Figure 5-13 (b) are found for the viscoelastic surface with a higher ratio due to the coupling. To explain such results in detail, the solutions of the normal pressure and shear tractions are interdependent in a coupled case. For the more solid-like viscoelastic surface ($\mu_\infty/\mu_0 = 3$), a Hertzian-type pressure distribution is expected in the uncoupled case arising from the dominant performance of its elasticity. The resulting shear tractions are expected to be similar to what could be achieved from a purely elastic surface. Although its peak value decreases while the contacting area increases with time, shear traction results of the more solid-like material (Figure 5-13 (c)) agree with this hypothesis. For the more fluid-like viscoelastic surface ($\mu_\infty/\mu_0 = 10$), pressure spikes on the contacting edges are expected before the steady-state time point under uncoupled conditions. As a result, additional irregular features appear on its shear traction profile as highlighted in Figure 5-13 (d) when

coupling effects are included. These irregularities in turn affect the pressure distribution leading to those extra spikes.

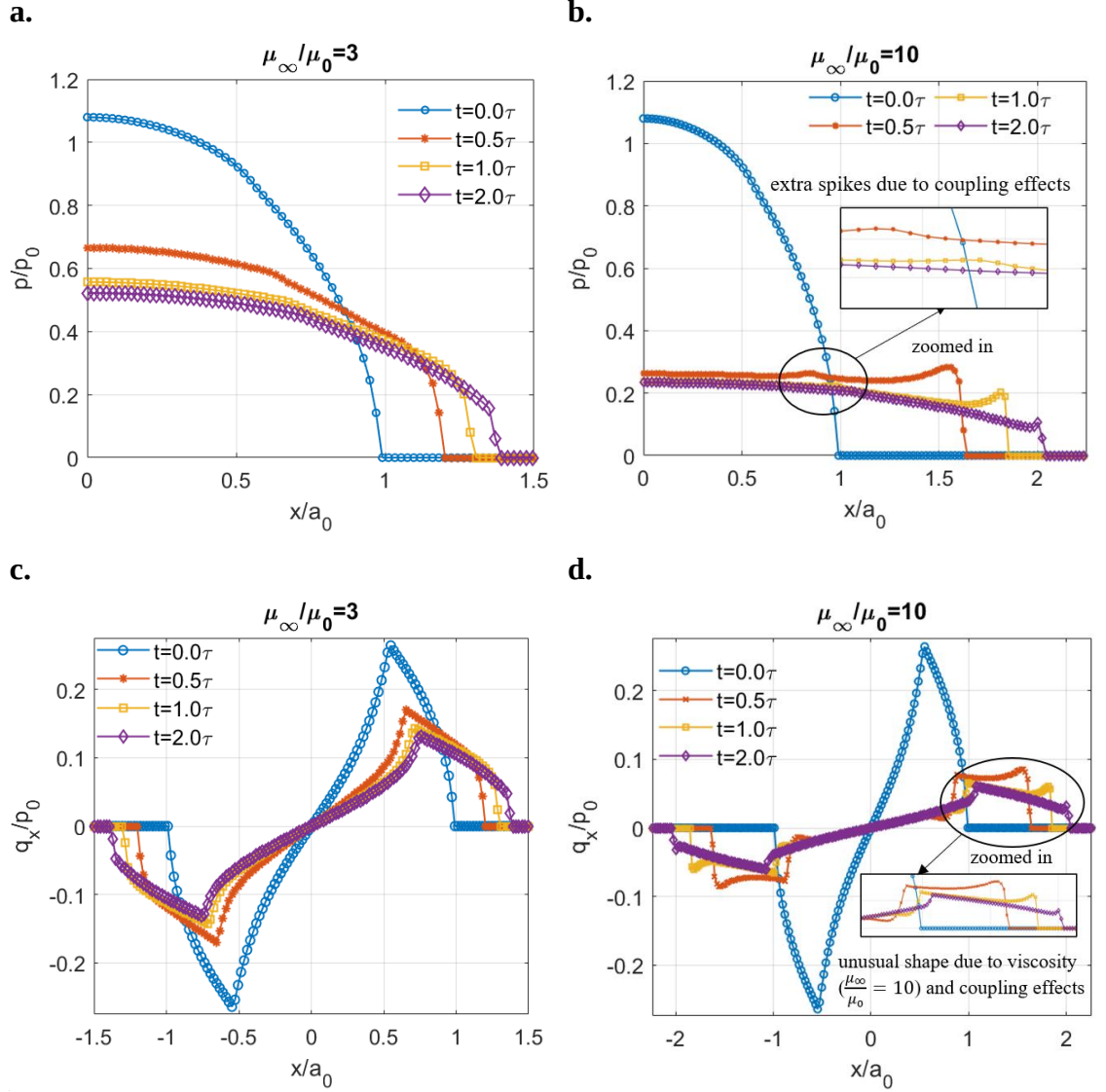


Figure 5-13 Coupled contact solutions (normalized pressure and shear traction) of different viscoelastic materials under normal load alone

Despite of the distinct profiles of contact tractions, the stick-slip separations of the two viscoelastic surfaces show similar shapes as shown in Figure 5-14. In both cases, the contacting area is separated into a central stick region (region surrounded by the blue line) and a surrounding annulus (the region between the red line and blue line) regardless of the time. The areas of both the stick (A_s) and contacting zones (A_c) are found to increase with the ratio (μ_∞/μ_0) of viscoelastic materials (Figure 5-15 (a)). However, as illustrated in Figure 5-15 (b), the ratios of the stick region to the contacting region (stick ratio hereinafter) do not vary much between the two investigated viscoelastic materials. This

suggests that the stick ratio is insensitive to the viscoelasticity of materials under current loading conditions.

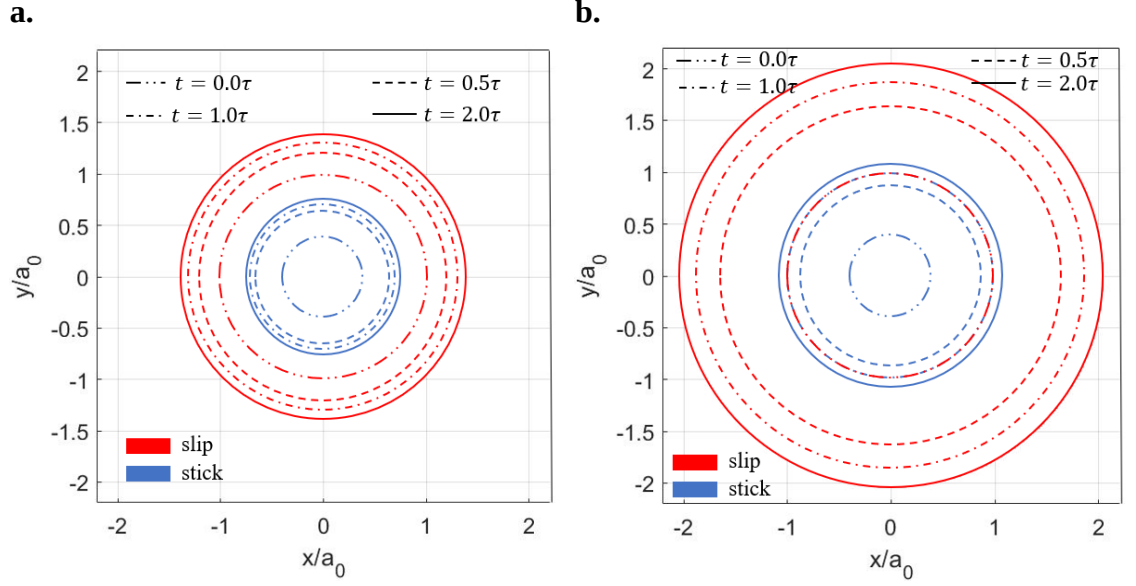


Figure 5-14 Separation of stick and slip regions with time for different viscoelastic materials: (a) $\frac{\mu_\infty}{\mu_0} = 3$ and (b) $\frac{\mu_\infty}{\mu_0} = 10$ (the blue lines are the boundaries of stick regions and the regions between the blue lines and red lines are the slip regions)

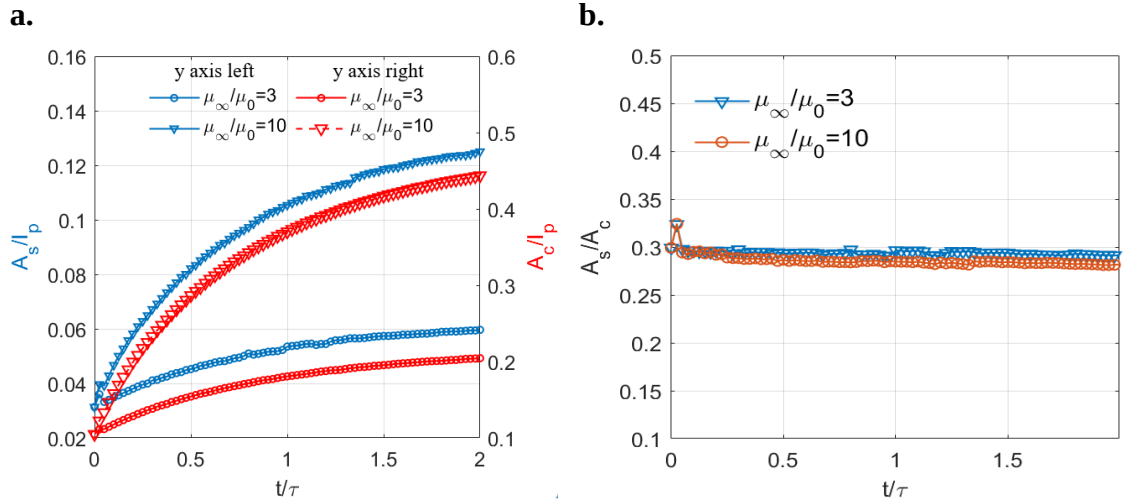


Figure 5-15 Simulation solutions of different viscoelastic materials under normal load alone: (a) Variations of ratios of stick region (A_s/I_p , left y axis) and contacting region to simulation domain (A_c/I_p , right y axis) with time for different viscoelastic materials and (b) Variation of ratios of stick region to contacting region (short for stick ratio: A_s/A_c) with time for different viscoelastic materials

To further investigate the stick-slip contact for viscoelastic materials under constant inputs, a Zener model with an intermediate ratio value ($\mu_\infty/\mu_0 = 4$) is employed. It is of note that solutions to viscoelastic contact problems depend on the way the contact is imposed. To visualize the specific feature of viscoelastic materials, two separate cases are

first studied here: constant load and constant displacement in the lateral direction. Most of the simulation inputs remain the same with the former case (no input in the lateral direction) except that the input in the x direction is now changed to be a constant load ($F_x = 0.5\mu_f W$) or a constant displacement ($\delta_x = 6.1987$). In other words, two different contact phenomena in the x direction including creep and stress relaxation are simulated here. It is of note that a load-controlled algorithm in the tangential direction could fail to converge when the contact of a viscoelastic material with a relatively high ratio is considered. A flow chart showing the search method of contact solutions in the tangential direction is given in the Appendix 5A. In the algorithm, the convergence speed of the outer loop regarding the load balance relies heavily on how the updated shear tractions adjust the tangential displacements (δ_x and δ_y) to make them close to the correct solutions. When it comes to the contact of a more fluid-like viscoelastic material, the incremental deformation derived from the previous tractions u_{incre} determined from Equation 5-19 could be large after a relatively long time. Therefore, the contribution of the deformation derived from the shear traction at the current time step u_t determined from Equation 5-21 does not significantly affect the total deformation. As a result, the change of the current-time-step shear tractions fails to search for the right tangential displacement and results in an endless loop regarding the load balance in the tangential direction. To obtain the partial slip solution of a viscoelastic surface under constant loading conditions, based on an extrapolation approach, a dynamic rigid body displacement in x direction is specified in the displacement-controlled algorithm to obtain results with constant tangential loading. The partial slip simulation test under this input condition is named the load-constant (LC) test to facilitate the following description while the one under the input of constant displacement in x direction is named the displacement-constant (DC) test.

The variations of tangential load with time in these two cases are shown in Figure 5-16 (a). The load is kept constant for the LC case (creep) while the stress relaxation phenomenon is simulated for the DC case. Due to the relaxed load in the DC case, its stick ratio is lower than that in the LC case after a certain time, but the difference tends to be insignificant. Figure 5-16 (b) shows the variations of contacting area and stick areas with time under the two different input systems. The growth rate of the stick region in the DC case is higher under the effect of stress relaxation. Although a constant normal load is applied in both tests, the growth rate of contacting area in the LC case is higher. To explain this, apart from the contribution of surface creep under normal loading, the higher

tangential load in the LC case leads to slight additional contacting areas under the coupling effects for each time step.

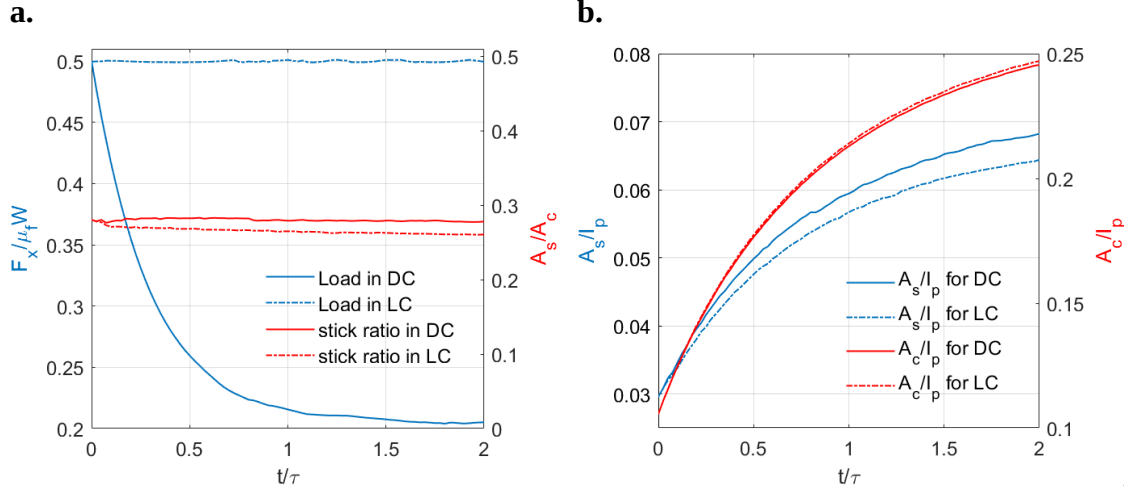


Figure 5-16 Simulation solutions of viscoelastic partial slip contact under different input conditions: (a) Variation of tangential loads (y axis left) and stick ratios (y axis right) with time in the DC and LC cases and (b) Variation of the ratio of stick region to simulation domain (y axis left) and ratio of contacting area to simulation domain with time (y axis right) in the DC and LC cases

The creep (LC) and stress relaxation (DC) encountered in the x direction result in different separation patterns of stick and slip regions with time. As shown in Figure 5-17, the shape of stick regions in the LC case does not change with time (always a raindrop shape) while that in the DC case varies (from a raindrop shape to a circular shape). Furthermore, the position of the stick region changes with time in the DC case, which starts from almost the rearrest zone of the contacting area and then shifts to the front. Such a separation of stick and slip regions is expected according to the distribution of contact tractions. The normal pressures are shown in Figure 5-18. Compared with the solutions of the LC case, it is evident that the DC case shows a higher peak pressure, and its position is closer to the front part of contacting area. Also, the pressures on the rear section of the contacting area are found to be lower for the DC case while those on the front section are higher. Figure 5-19 (a) and (b) show the normalized shear tractions (q_x and q_y respectively) for the two cases, where the DC case tends to show higher q_x at the rear zone of contacting area and lower q_x at the front zone. The area in which relatively low normal pressure or high shear traction exists is more prone to slip as it is easier for the shear tractions to reach the boundary of local friction following Coulomb's law. As a result, there exist more slip regions at the rear part of contacting area for the DC case.

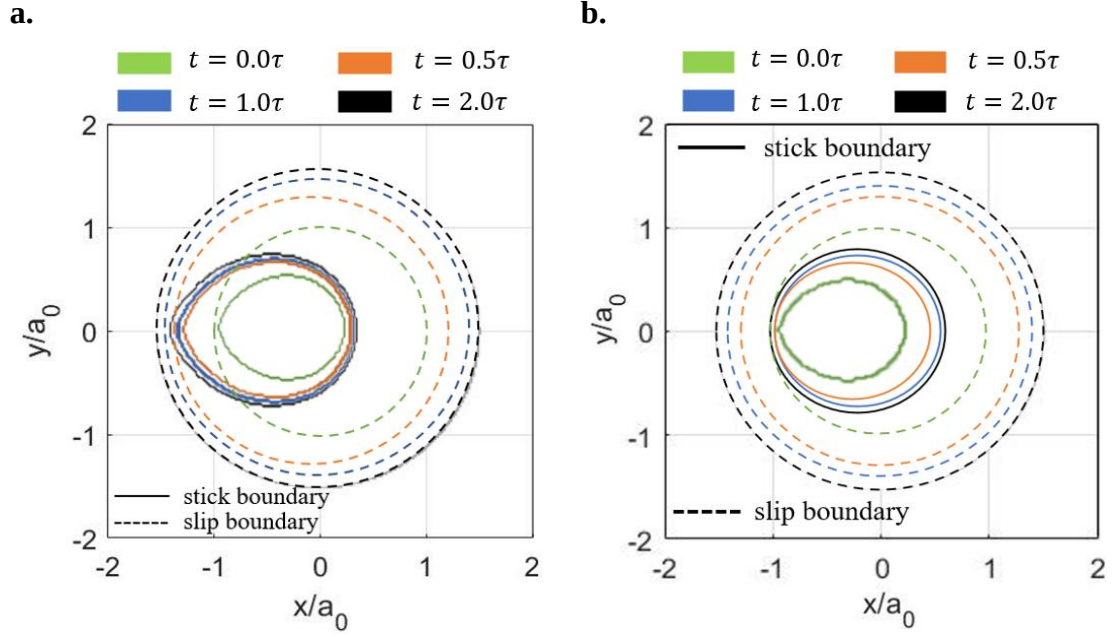


Figure 5-17 Separations of stick and slip regions under constant inputs in x directions: (a) LC case and (b) DC case (the solid lines are the boundaries of stick regions and the regions between the solid lines and dash lines are the slip regions)

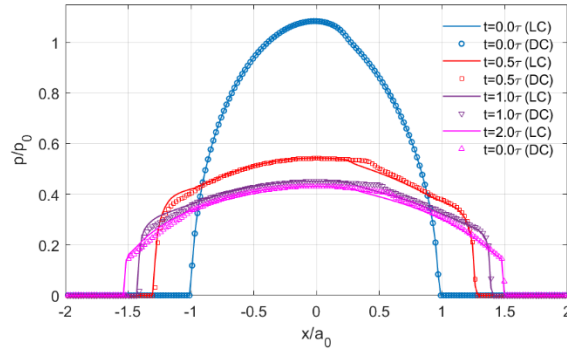


Figure 5-18 Changes of Normal pressure distribution p along the x axis with time in the LC (solid line) and DC (scatter plot) cases

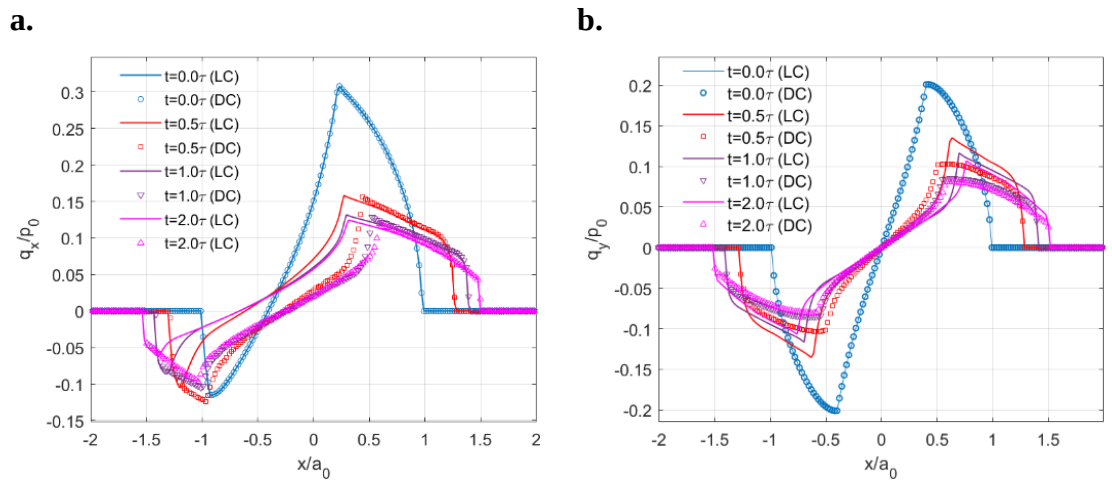


Figure 5-19 Changes of shear traction distribution with time in the LC (solid line) and DC (scatter plot) cases: (a) q_x along the x axis, and (b) q_y along the y axis

To study the effect of material properties (μ_∞/μ_0) on the stick and slip separation within contacting area under constant displacement input in the tangential direction (DC condition), Zener models with three different ratio values ($\mu_\infty/\mu_{01}=3$, $\mu_\infty/\mu_{02}=5$, $\mu_\infty/\mu_{03}=10$) were employed. As expected, different growth rates of stick and contacting areas are experienced by these materials shown in Figure 5-20 (a). A lower stick ratio is observed in Figure 5-20 (b) for the viscoelastic material with a higher ratio (μ_∞/μ_0). However, the differences between the stick ratios of the three cases are not significant despite of a more notably stress relaxation phenomenon exhibited by the material with a higher ratio.

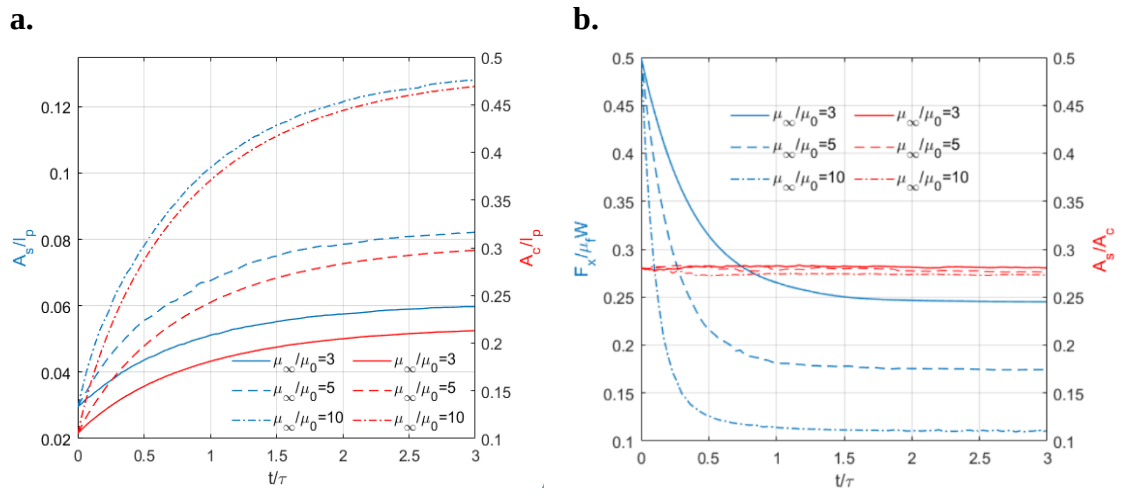


Figure 5-20 Solutions to the stick-slip contact of different viscoelastic materials (Zener model) under constant displacement in x direction: (a) Variation of the ratio of stick region to simulation domain (y axis left) and ratio of contacting area to simulation domain (y axis right) with time and (b) Variation of the resulting tangential loads (y axis left) and stick ratios (y axis right) with time for different viscoelastic materials (Zener model)

Now to vary the contact input in the normal direction, the Zener model ($\mu_\infty/\mu_0 = 4$) is tested again but with constant displacement in both normal and tangential directions ($\delta_z = 34.358 \mu\text{m}$ and $\delta_x = 6.1987 \mu\text{m}$). In other words, the stress relaxation phenomenon is encountered in both normal and tangential directions. To facilitate subsequent clarification, the current test is named the relaxation case while the former test (DC case) with the constant displacement in the tangential direction but constant load in the normal direction is now named the creep case. As shown in Figure 5-21 (a), the ratio of the tangential load to the static friction remains with time for the relaxation case while it drops for the former relaxation case. This consequently leads to a slightly lower stick ratio for the relaxation case as indicated in Figure 5-21 (a). Different from the results of the former creep case (Figure 5-17 (b)), the shape and size of stick and slip regions for the current

relaxation case are found to be time-independent, which follows a certain separation pattern shown in Figure 5-21 (b). Such results can be expected based on the contact tractions of the corresponding case. As shown in Figure 5-22, the normal pressure and shear tractions in the relaxation case vary insignificantly with time when comparing those in the former creep case.

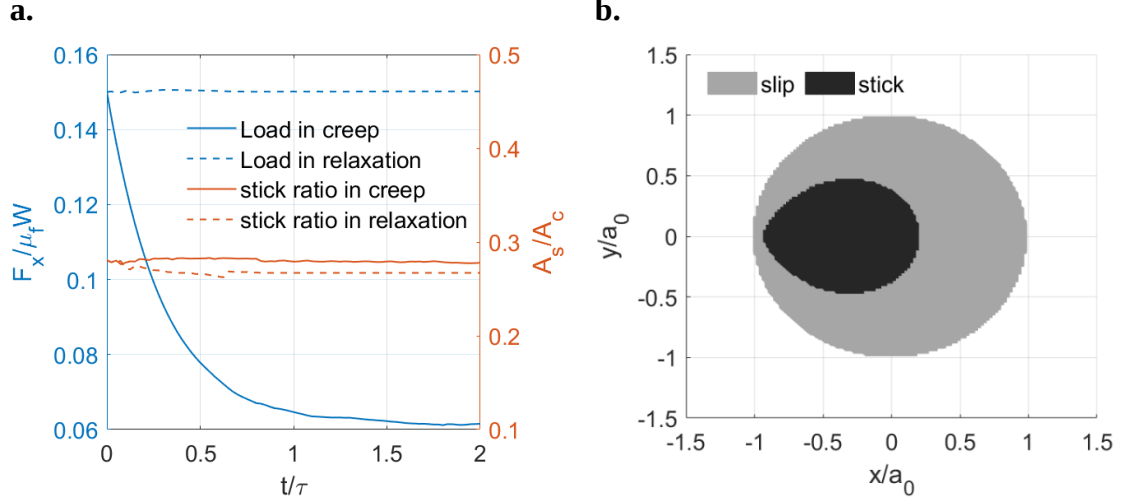


Figure 5-21 (a) Variation of the resulting tangential loads (y axis left) and stick ratios (y axis right) with time in the relaxation (solid line) and creep (scatter) cases and (b) Time-independent separation pattern of stick (dark) and slip (grey) regions for the relaxation case

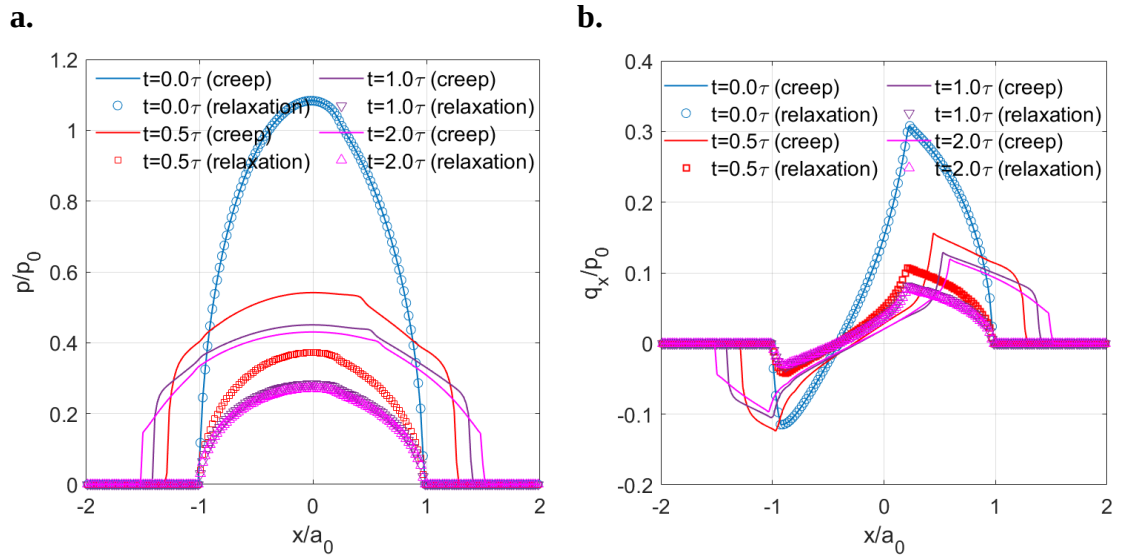


Figure 5-22 Change of contact tractions with time for the creep (solid line) and relaxation (scatter) cases: (a) pressure along the x axis and (b) shear traction q_x along the x axis

Combining the results of the above partial slip tests, a short-term conclusion can be drawn. The stick ratio is insensitive to the time-varying property of viscoelastic materials as long as the specified contact inputs (displacement or load) in the normal and tangential

directions are constant. However, the separation pattern of stick and slip regions depends on the contact phenomena experienced by the viscoelastic surface in the normal and tangential directions. When different contact phenomena are encountered (different types of contact inputs are specified) in the normal and tangential directions, the separation pattern of stick and slip regions varies with time. Otherwise, it tends to be time-independent.

5.5.2.2 Evolution of Stick-Slip with Increasing Displacement

To investigate the transition from partial slip to gross sliding for different viscoelastic materials, a dynamic loading profile is specified in the lateral direction. The simulation starts with the semi-coupled condition, where only the coupling between pressure and shear traction in x direction is considered. The contact inputs employed here are a constant displacement in the normal direction and a linearly increasing displacement in the x direction. The surface geometry, contact configuration, coefficient of friction, spatial discretization, and input normal displacement are maintained to be those used in the former partial slip simulations in section 5.5.2.1. By varying the ratio (μ_∞/μ_0) of materials and also the increasing rate of tangential displacement v in the x direction, the evolutions of the stick ratio with time for different materials under the different increasing rates of tangential displacement is shown in Figure 5-23 (a). Due to the discrepancy between the linear input of tangential displacement and the non-linear mechanical response of viscoelastic materials, oscillating results are experienced in the early simulation period as labelled in Figure 5-23 (a). The variations of tangential loads resulting from the specified tangential displacement are shown in Figure 5-23 (b). As expected, a higher increasing rate of tangential displacement leads to a quicker transition from partial slip to gross sliding, in which state the tangential load is equal to the static friction ($\mu_f W$). Besides, under the same increasing rate of displacement, the more fluid-like material ($\mu_\infty/\mu_0=10$) can always reach the gross sliding state within a shorter time owing to its stronger flowability.

To switch to the fully-coupled condition, under the same increasing rate of displacement, the tested more fluid-like material ($\frac{\mu_\infty}{\mu_0} = 10$) can now reach gross sliding within a shorter time when compared with the case under semi-coupled conditions, as illustrated in Figure 5-24 (a). This brings about a lower tangential load required to reach gross sliding shown in Figure 5-24 (a). Apart from the ordinary full coupling effects, the transition from partial slip to gross sliding now tends to be abrupt as indicated in Figure 5-24 (a). The stick ratio

jumps from a relatively high value to zero. To obtain a smoother transition, trial tests with smaller time intervals were conducted using the same contact inputs. However, the difference between those partial slip solutions is insignificant when the time intervals used are considerably small as shown in Figure 5-24 (b). This suggests that the abrupt jump is inevitable when the fully coupled condition is introduced. Mathematically, such an abrupt transition can be caused by the unstable and time-varying properties of viscoelastic materials and the modified condition of gross sliding when fully coupled conditions are introduced.

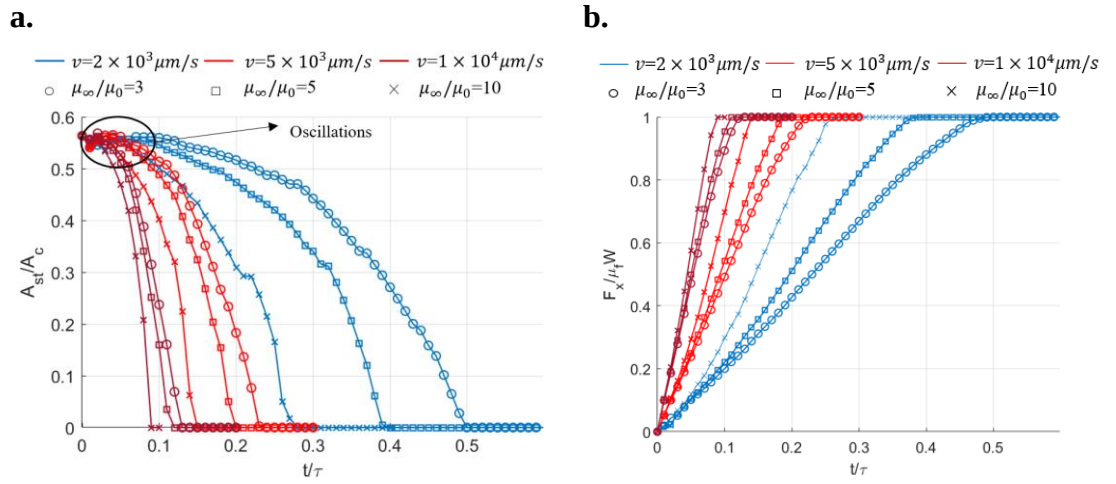


Figure 5-23 Simulation results under semi-coupled conditions: (a) Evolution of stick ratio with time for different viscoelastic materials and (b) Evolution of resulting tangential load with time for different viscoelastic materials (v is the increasing rate of tangential displacement)

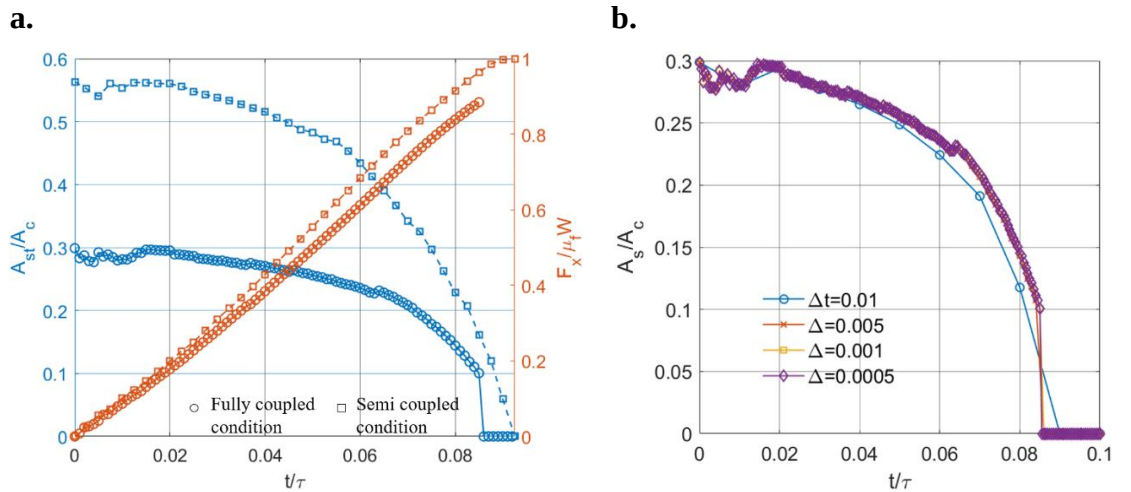


Figure 5-24 (a) Comparison of partial slip solutions under fully-coupled and semi-coupled conditions and (b) Comparison of partial slip solutions using different time intervals (Δ) under fully coupled conditions

To validate if such a relation holds, the fluid-like material (μ_∞/μ_0) was further tested, for which the gross sliding was obtained at different times under fully coupled conditions by

varying the increasing rate of surface displacement. Results (Figure 5-25) show that the variation of the stick ratio just before sliding (short for critical stick ratio hereinafter) with the time when the gross sliding is achieved is relevant to the evolution of the relaxation modulus of the material within the time. As shown in Figure 5-25 (a), the critical stick ratio first increases when a longer time is required for the contact to be in the sliding state. Meanwhile, the relaxation modulus drops dramatically shown in Figure 5-25 (b). However, the critical stick ratio starts to decrease when the changing rate of relaxation modulus becomes lower, as indicated in Figure 5-25. For the purple line ($v = 1 \times 10^3 \mu\text{m/s}$) where the gross sliding happens when the material property is steady, the transition from partial slip to gross sliding tends to be less abrupt.

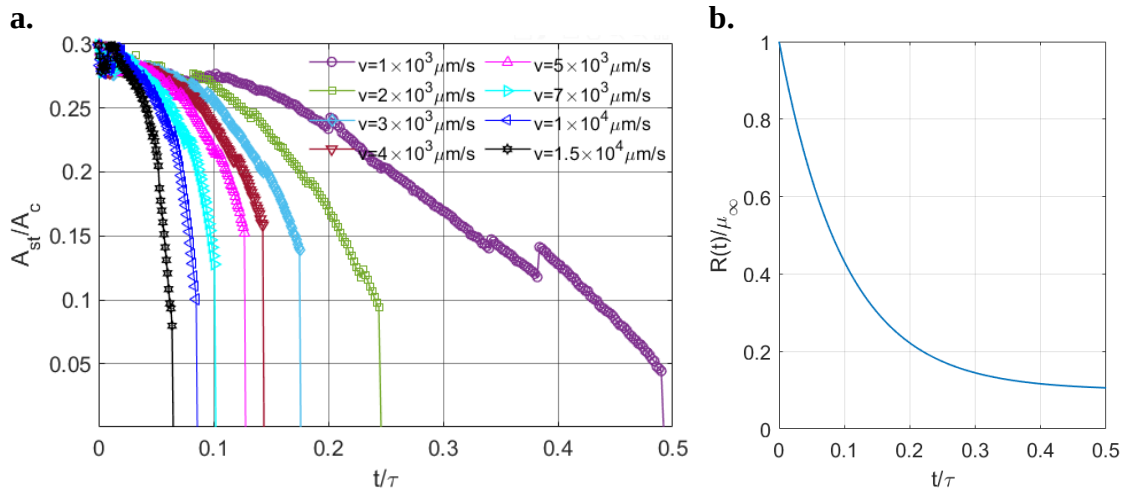


Figure 5-25 (a) Fully coupled solution regarding the variations of the stick ratio with time under different increasing rates of displacement and (b) Variation of relaxation modulus $R(t)$ of the material with time

Such an abrupt transition is counter-intuitive if analysing the result within the framework of classical contact mechanics. A brand-new view concerning the rheological behaviour of viscoelastic materials is provided here to explain such results. It is well-recognized that there is a threshold concerning the tangential load needed to be reached in order to induce sliding for any elastic solid. On the other hand, fluids will flow as long as there is a shear in any lateral direction since there exists no threshold to generate relative motion. In other words, the gross sliding is attained immediately when a tangential load is applied to a fluid. A viscoelastic material, as its name implies, should exhibit a response incorporating both the contact characteristics of solids and fluids. From this perspective, our coupled model shows a reasonable outcome resulting from such integrated contact behaviour. What can be observed from our results is that during the early period of transition from partial slip to gross sliding, viscoelastic surfaces tend to require the tangential displacement or the equivalent load to reach a threshold value. This is identical to the

elastic case. But the final transition tends to be an abrupt process, which is expected for a fluid, rather than a smooth one that is commonly encountered when addressing an analogous elastic stick-slip problem under monotonically increasing tangential load or displacement.

The partial slip problems discussed above were all simulated when a fixed displacement is specified in the normal direction. To build up the complication, the transition from partial slip to gross sliding for the fluid-like material ($\frac{\mu_\infty}{\mu_0} = 10$) was simulated again but with a constant normal load. Compared with the current case where a creep phenomenon is encountered in the normal direction, results (Figure 5-26) show that it is always quicker for the former stress relaxation case to reach gross sliding under the increasing rate of tangential displacement. Since the load keeps decreasing with time when a stress relaxation is experienced, the sliding condition is easier to be satisfied. Arising from such difference, two different evolution forms of stick and slip regions within different times are obtained for the creep and relaxation cases as shown in Figure 5-27.

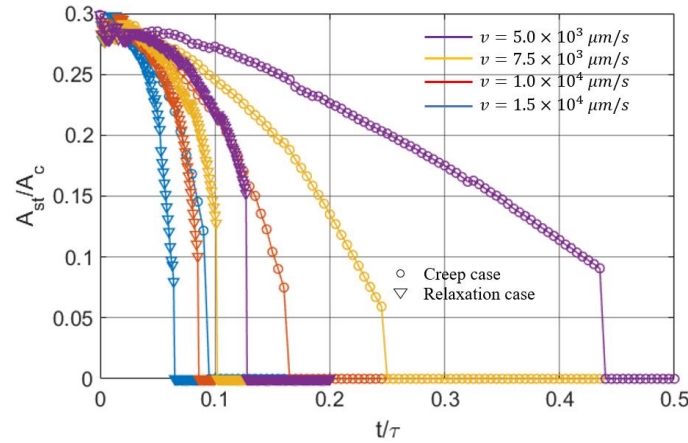
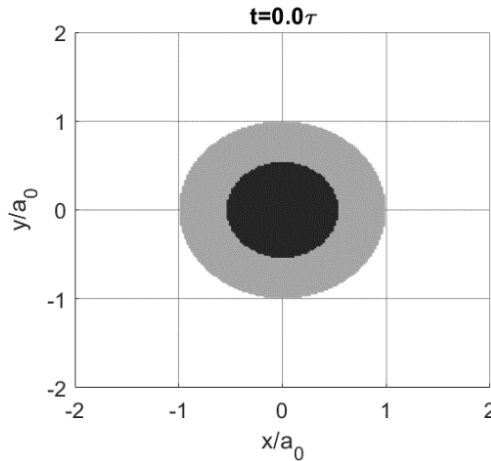
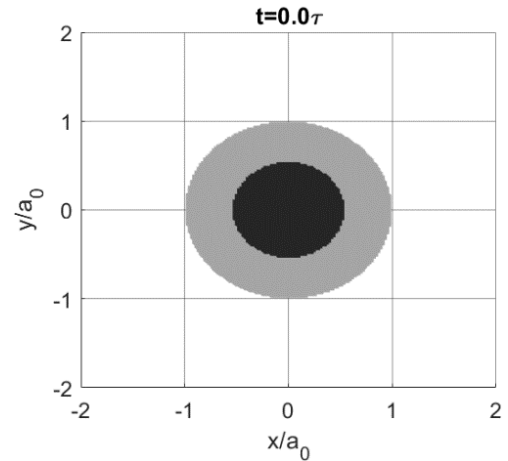


Figure 5-26 Fully coupled solution regarding the variations of the stick ratio with time under different inputs (creep or stress relaxation) in the normal direction

a.



e.



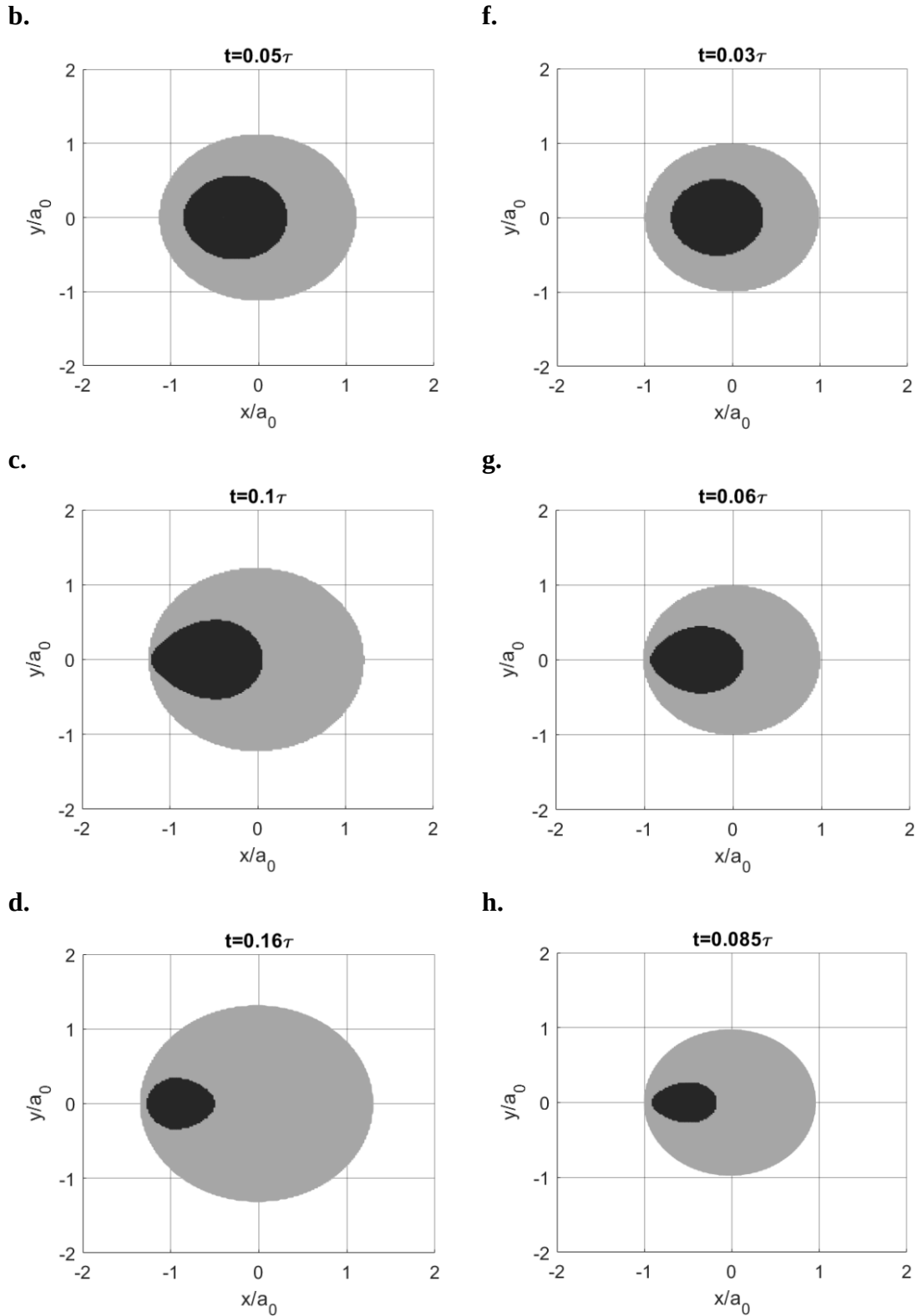


Figure 5-27 Fully coupled solution regarding the evolution of stick (dark) and slip (grey) regions for the creep case ((a)-(d)) and relaxation case ((e)-(h))

5.6 Concluding Remarks

The three main contact phenomena of viscoelastic materials including creep, stress relaxation and hysteresis are reproduced by the newly developed viscoelastic contact

model. The ratio of retardation time to relaxation time (μ_∞/μ_0) is employed to characterize the rheological property of viscoelastic materials. Although the exact boundary value is not known, materials with high ratios are observed to exhibit more fluid-like contact behaviours while those with low ratios behave more like solid. No matter what rheological model is employed to characterize the viscoelastic material, it is the ratio (μ_∞/μ_0) that determines the shape of contact tractions. Concerning the stick-slip contact analysis of different viscoelastic materials, the following conclusions can be drawn:

- For the creep contact of a more fluid-like viscoelastic material, the spike on the edge of the normal pressure distribution is observed before the steady state, which results in irregular shapes of shear tractions due to the coupling between pressure and shear stress. The pressure spikes also influence the subsurface stress. The position of the peak stress can be shifted to the edge of contacting area depending on how skewed the pressure profile is.
- Different contact solutions including normal pressures and shear tractions can be obtained depending on whether a constant load or displacement is specified in the normal and tangential directions. The evolution of the stick ratio is insensitive to the time-varying modulus and the rheological property of viscoelastic materials as long as the contact input (load or displacement) is constant.
- However, the separation pattern of stick and slip regions within the contacting area can be different even though the contact inputs are constant. This is determined by the contact phenomena encountered by the viscoelastic surface. When the same contact phenomenon (creep or stress relaxation) is experienced in both normal and tangential directions, the evolution of the stick region follows a certain pattern. Otherwise, the way the stick and slip regions are separated would vary with time.
- Unlike that in an elastic solution, the transition from partial slip to gross sliding of viscoelastic materials tends to be abrupt under fully coupled conditions. Under the same contact inputs, a quicker transition can be achieved for a more fluid-like material. Compared with the case where a stress relaxation phenomenon is encountered in the normal direction, it requires a longer time for the viscoelastic surface to be in the gross sliding state when a creep phenomenon is encountered. Our findings suggest for the first time that a fully-coupled contact condition leads to significantly different results than the semi-coupled one and the future analysis of

the stick-slip or sliding of viscoelastic materials need to be developed with consideration of fully-coupled normal and tangential loads.

Although valid partial slip solutions can be achieved from the developed viscoelastic half-space model, films with finite thickness, rather than a half-space, are the most common applications of viscoelastic materials, as stated by Zhang et al [43]. A frictional contact model taking into account viscoelastic materials in the form of coating or layer can provide helpful insights when designing a layer-substrate system. Besides, the modelling of frictional sliding or rolling contact of viscoelastic surfaces remains a field that has not been fully exploited. These are the subjects of our current ongoing work.

Acknowledgement

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Appendix 5A

In this appendix, the flow chart of the tangential contact solver for the elastic stick-slip problem is shown in

Figure 5-28.

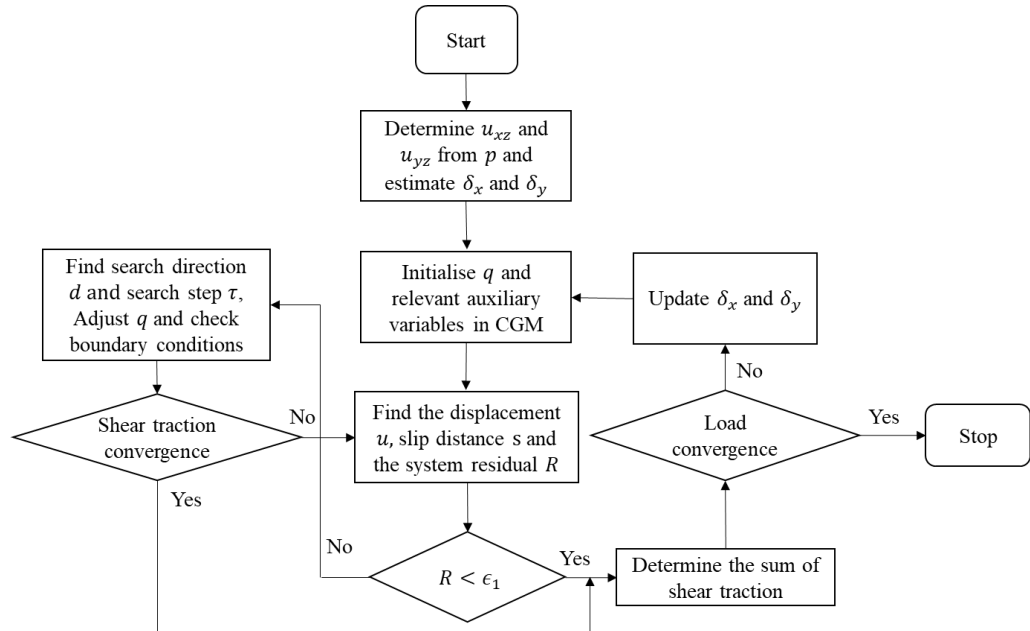


Figure 5-28 Flow chart of the algorithm for the tangential contact solver

5.7 Supplementary Notes for Chapter 5

5.7.1 Correction for Explanation of Results from the Work of Koumi

In section 5.5.1, the work of Koumi, et al [21] was discussed, who focused on the simulation of viscoelastic sliding and rolling contact between a sphere and a flat half-space. Their simulation results demonstrate that the shape of pressure profiles depends on which one of the contacting bodies (sphere or half-space) is modelled as viscoelastic, whereas the other is considered rigid. Specifically, they reported that pressure spikes on the contacting edge can emerge in steady-state sliding contact when the flat half-space is viscoelastic and sphere is rigid, while a Hertzian-type pressure distribution arises when the sphere is viscoelastic and half-space is rigid. The underlying mechanisms governing these distinct pressure profiles are discussed in detail in section 6.7.1 within the context of transient viscoelastic sliding contact.

It must be highlighted that their findings do not apply to indentation scenarios, which is the focus of the current section. A misunderstanding was made in our previous interpretations of their numerical solutions to indentation problems under certain displacement as follows: if the sphere is rigid and the half-space is viscoelastic, the steady-state pressure solution will contain spikes on the edges, while the Hertzian-type contact pressure is observed for the opposite configuration.

Actually, the pressure distribution is identical for both contact configurations, and it corresponds to the Hertzian-type pressure due to the stress relaxation phenomena that are experienced in both cases. What is really causing the difference (our misunderstanding) here is the way the pressure is plotted such that the pressure is always normal to the deformed contacting interface. Therefore, as illustrated in Figure 5-29 (a), the plotted pressure vectors are always vertical for viscoelastic sphere against rigid flat. For the opposite configuration, the plotted pressure vectors point toward the centre of the sphere exhibiting pressure spikes visually as shown in Figure 5-29 (b). It is of note that for both figures there is a magnification factor along the vertical direction. As a matter of fact, the magnitudes of the pressure vectors (the distance between a point of the contact interface up to the pressure profile distribution) at the same contacting spots for the two configurations are identical as indicated in Figure 5-29.

The above discussion has been kindly confirmed after contacting with the corresponding author (Prof. Nelias) of the work of Koumi, et al. [21].

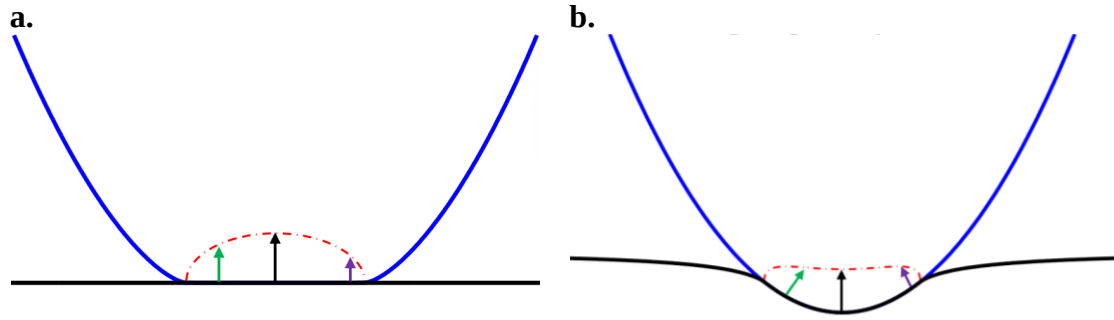


Figure 5-29 frictionless indentation problem between a sphere and a flat with same imposed normal displacement: (a) viscoelastic sphere against rigid flat and (b) rigid sphere against viscoelastic half-space (the marked arrows are pressure vectors with different colours indicating different contacting spots). Reproduced with permission from the work of Koumi, et al. [21], © Elsevier B. V., 2025.

5.7.2 Discussion on the Source of the Oscillations near the Pressure Spikes

To further discuss the oscillating results for the pressure distributions of more fluid-like viscoelastic materials, they usually occur when sharp pressure spikes exist on the contacting edges. As the resolution of computational grid determines how well spatial or temporal variations can be captured in the DC-FFT algorithm, to minimize the numerical artefacts due to the under-sampling of these sharp gradient, the resolution of the computational mesh needs to be increased, and smaller time intervals shall be used in this transient contact analysis.

From the perspective of signal analysis, these oscillations around the pressure spikes can be interpreted as a manifestation of the Gibbs phenomenon as mentioned in section 1.4.2. To review the concept, the Gibbs phenomenon refers to oscillations that arise near discontinuities or sharp transitions. Theoretically, the pressure distribution function is approximated by a sum of sinusoidal functions through the Fourier transform, where sharp transitions or discontinuities require inclusion of high-frequency components for accurate representation. However, in the numerical implementation of the DC-FFT algorithm, these high-frequency components are inherently truncated because only a finite number of Fourier series terms can be employed. Additionally, the highest frequency in the numerical implementation is limited by the sampling rate (Nyquist frequency). A low spatial resolution can result in aliasing, where the high-frequency components fold into the lower frequencies. This can further distort the solution and amplify the oscillations. By adopting a finer spatial discretization, the Nyquist frequency is increased, allowing higher-frequency components to be included during the Fourier transform, thereby alleviate the oscillatory artifacts.

Due to the time dependency of viscoelastic materials, the temporal resolution also plays an essential role such that large time steps can lead to sharp change of pressure distribution between consecutive time points. To avoid this temporal sharpness (existing as extra artifacts in the spatial domain) that can amplify the oscillations, small time steps need to be used to help accurately capture the gradual change of pressure. This can be evidenced by the improved approximation of the sharp pressure transitions shown in Figure 5-8 (e) after adopting a small time interval.

Beyond the currently investigated creep contact, these oscillations are commonly encountered in the analysis of frictionless sliding contact of viscoelastic materials, where a sharp pressure is observed at the leading edge of the contact area (e.g., Figure 6-4 (a) and (b) in Chapter 6). Notably, oscillating results have been widely reported in numerical studies on transient viscoelastic contact problems employing the DC-FFT approach (e.g., work of Koumi, et al. [21], Wallace, et al. [22] and Zhang, et al [23]). As mentioned in section 1.4.2, these oscillations stem from the intrinsic limitations of Fourier transform of DC-FFT while it provides benefits such as implementation simplicity.

5.7.3 Physical Interpretations of Pressure Spikes in Viscoelastic Indentation Contact

As discussed in section 5.5.1, pressure spikes are frequently encountered when viscoelastic materials exhibit higher values of μ_{∞}/μ_0 . A robust explanation on reasons behind the occurrence of these pressure spikes is still lacking and is given in this section.

Due to the time-dependent increase of contact area, those in the centre of contact area have been in the deformation state since the contact initializes. This allows molecular chains to have more time to realign and slides under the constant loading history, leading to stress relaxation over time. On the other hand, the surface points on the contact edges which just enters the contact due to the creep motion, does not relax for the time as experienced by those in the contact centre. Therefore, when the viscoelastic material allows continuous deformation at the centre of contact under the action of viscous flow for a relatively long period of time, the pressure profile in this specific contacting zone relaxes significantly such that it tends to be flattened, while the surface points on the contacting edge just enters the contact. This difference in differential loading history between the centre and edges of the contacting area can create a "M-shaped" pressure distribution with pressure spikes on both contacting edges, rather than the classical Hertzian bell-shaped distribution seen in elastic materials.

As in a pure stress relaxation scenario with step displacement, the surface deformation is applied instantly and held constant. Hence, all surface points within the contact area experience their deformation simultaneously. In other word, the relaxation process starts at the same time for all points, leading to the uniform decrease of the pressure with the peak pressure stays in the centre of the contact area as illustrated in Figure 5-30. The pressure distribution in the stress relaxation case typically starts as a classical elastic-like distribution, gradually decreases in magnitude while maintaining similar shape with time as the relaxation process is uniform across the contact area.

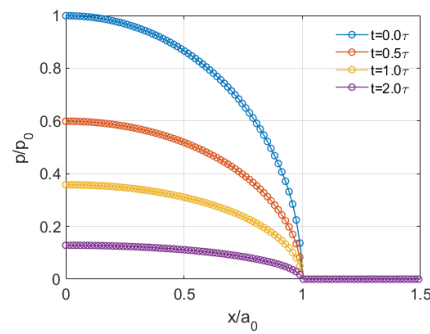


Figure 5-30 Variation of pressure under stress relaxation phenomenon

The idea of characterising the rheological behaviour based on the ratio of retardation to relaxation time, can be justified based on the physical interpretation above and the mechanism of the rheological model. To review the structure of the Zener model, it is constructed by a spring in parallel with a Maxwell element. The instantaneous contact response is dominated by properties of both springs characterised by μ_∞ . In the intermediate period, the dashpot in the Maxwell elements starts to deform, causing non-uniform relaxation. With time keeps going to a relatively high value, the Maxwell element fully relaxes. The time needed for the dashpot to fully relaxes is relevant to the relaxation time τ . The Zener system then behaves like a single spring with the contact response characterised by μ_0 . At this time point, the viscoelastic material returns to a classical elastic-like behaviour, resulting in a Hertzian-type pressure distribution no matter what shape of pressure profile observed in the intermediate period (e.g., Figure 5-8 (f)). The reason for observing no steady state for the Maxwell model (a single spring in series with a dashpot) shown in Figure 5-11 is that the model has no equilibrium state such that the equilibrium modulus would keep approaching zero with time, leading to continuous creep of contacting area, with the pressure at the centre of the contact area keeps being relaxed while the pressure spikes on the edges being more and more significant.

The difference between μ_∞ and μ_0 determines the level of the stress relaxation experienced by the materials. When the ratio μ_∞/μ_0 is large enough such that the

pressures in the centre of contact area can experience significant stress relaxation under a certain duration of constant load, the pressure spikes would be observed (e.g., Figure 5-8 (e)). On the other hand, a low ratio limits the extent of the stress relaxation, retaining the stress reduction on the centre of the contact area being insignificant. Although the magnitude of pressure changes with time, the shape of pressure profiles stays similar to Hertzian. For these types of materials characterised by low μ_{∞}/μ_0 , the time dependency manifests more in the overall contact area and pressure magnitude rather than the pressure distribution shape.

Notably, apart from the ratio of retardation time to relaxation time, the rheological behaviour of viscoelastic material also has a relation with the relaxation time. The relaxation time in the study assumed identical relaxation time for all the fictitious materials characterised by Zener model. But for the more solid-like viscoelastic material in practice, similar with the behaviour of traditional solids, they usually have shorter relaxation times so that the material can redistribute stresses more quickly.

The relativeness of the relaxation time of viscoelastic materials and loading duration also plays a significant role in the occurrence of the pressure spikes. Notably, in the study presented in section 5.5.1, the loading time is nondimensionalized by the relaxation time, so this characteristic is not exhibited. In practice, for materials characterised with Zener model with very short relaxation time, the dashpot fully relaxes very quickly. If the loading time is too large compared with the relaxation time, only the steady-state Hertzian-type pressure distribution can be captured. On the other hand, if the relaxation time is considerably long, the dashpot relaxes in an extremely slow rate such that the significant stress relaxation cannot be captured when the loading time is not long enough. Hence, for materials exhibiting a high μ_{∞}/μ_0 but significantly long or short relaxation time, the pressure spikes may not be observed.

The above analysis is based on the Zener model. The viscoelastic behaviour will not be as straightforward when considering complex viscoelastic models having multiple relaxation times.

5.7.4 Critique of Parameters Used in the Prony Series for PMMA

In section 5.3.3 a two-term Prony series shown in Equation 5-27 was employed to characterise the relaxation modulus of PMMA based on the two-hour relaxation test conducted by Kumar and Narasimhan [38] at room temperature, and curve fitting by Chen, et al. ($R^2 = 0.99957$) [11].

The two-term Prony series seems to express the material modulus with reasonable accuracy, based on which Chen, et al. [11] reported a fairly good agreement between the numerical simulation outcome and experimental data from the work of Kumar and Narasimhan [38] for the indentation problem of a sphere against a PMMA half-space subjected to a triangular-shape loading history. The same relaxation function was also used by Wallace, et al. [22] when simulating the contact of a rigid sphere against the PMMA layer bonded to a fictitious viscoelastic steel for validation. Notably, the simulation outcome is compared with that derived from a FEM model instead of experimental results in their tests.

As discussed in the work of Xu and Engquist [44], Prony series is an empirical tool primarily utilized for fitting experimental data rather than predicting relaxation modulus outside of the finite experimental window. For realistic viscoelastic materials, the spectrum of relaxation times can cover more than eight to ten decades from very fast molecular motions to slow structural relaxations, where a huge number of terms are usually required to correctly characterize the entire response by Prony series [18]. As a matter of fact, the full relaxation spectrum of PMMA extends far beyond the two-hour experimental window conducted by Kumar and Narasimhan [38], which exhibits complicated behaviour across wide ranges of frequencies and temperatures as captured by the DMA test conducted by Putignano and Dini [45]. As a result, although the two-term Prony series can provide some approximated results when the operating condition closely match the experimental set-up of Kumar and Narasimhan [38], it oversimplifies the multi-relaxation-time material behaviour, leading to inaccuracies when it comes to simulation involving multi-timescale loading (e.g., rapid contact or extremely long-term loading).

Instead of fitting the Prony series to the relaxation modulus curve determined from a limited-duration relaxation test, more accurate rheological properties of viscoelastic materials can be determined by DMA, where the spectrum of relaxation times can be determined by curve fitting the master curve for the complex viscoelastic modulus (both storage and loss moduli) over a wide frequency range built based on the time-temperature superposition (TTS) principle [46].

5.7.5 Effects of Loading Rates on Contact Solutions

As shown in Figure 5-6, a hysteresis phenomenon can be experienced by viscoelastic materials when a triangular-shape loading history is specified as shown in Figure 5-31, on which occasion the loading rate can play an essential role.

A relevant study was conducted by Chen, et al [11], who varied the loading rates of the triangular-shape loading history. Under a high loading rate, the viscoelastic material was found to demonstrate a stronger contact resistance behaviour, indicated by a lower peak indentation depth as illustrated in Figure 5-32. For the simulated indentation contact, the loading and unloading curves (red lines) almost coincide with each other under the highest loading rate of 100 N/s, indicating insignificant energy dissipation. This implies that within such a short period of time, the elastic deformation of the viscoelastic material plays the dominant role. Hence, different from the significant hysteresis phenomenon reported in Figure 5-6 and illustrated in Figure 5-32 under the loading rate of 10 N/s, where there exists a residual displacement after the loading-unloading process, the viscoelastic material can almost fully recover once the indentation load is released under an extremely high loading rate.

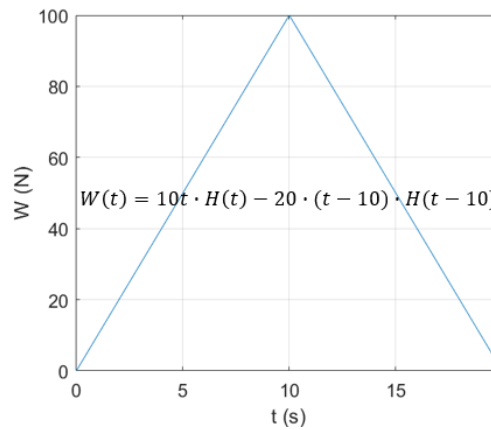


Figure 5-31 The triangular-shape loading history applied to induce hysteresis phenomenon (loading rate is 10 N/s)

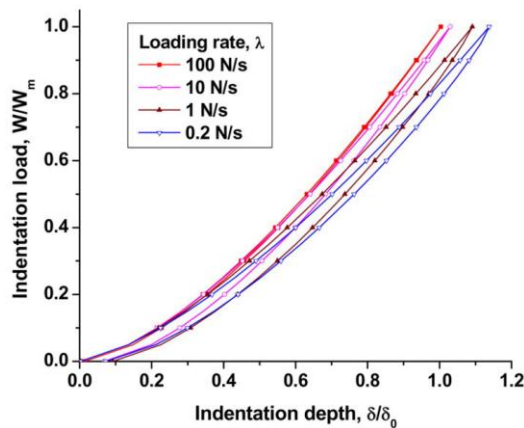


Figure 5-32 Evolution of nondimensionalized rigid body indentation with the nondimensionalized load under different loading rates, δ_0 is the Hertz solution of rigid body indentation at the peak load $W_m = 100$ N, reproduced with permission from the work of Chen, et al. [11], © ASME INTERNATIONAL, 2025.

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6 Numerical Model to Simulate the Transient Frictional Viscoelastic Sliding Contact

6.1 Abstract

Sliding motion has always been one of the major concerns when it comes to the analysis of viscoelastic contact problems. A new model simulating the transient sliding contact of smooth viscoelastic surfaces is developed in this paper. By taking the dry contact friction and the coupling between shear tractions and normal pressure into account, the effect of the early partial slip period, which is often neglected in the study of viscoelastic sliding contact problems, is investigated numerically. Compared with solutions based on the frictionless assumption, the steady-state pressure profile is found to be slightly different under the effect of the partial slip regime, including a lower peak pressure and the shift of the contacting region in the direction opposite to the sliding motion. Furthermore, the time required for the viscoelastic contact to reach its steady state is delayed owing to the partial slip period preceding the global sliding motion.

6.2 Introduction

Beyond natural selection, viscoelastic materials have been universally applied in practice owing to their numerous advantages such as low friction, slight rubbing noise, light weight, dimensional stability and inexpensive cost. A more efficient design of engineering products involving viscoelasticity requires a clear understanding of the relevant interfacial mechanical system, which is determined by the viscoelastic deformation, viscoelastic dissipation, frictional temperature rise, time-dependent material property and lubrication condition. In the meantime, the prevailing relative motion between the contacting bodies including sliding and rolling can be critical during the viscoelastic contact analysis. Typical examples of such a scenario are the tire-road contact [1] for vehicles, the cutting of soft tissues with scalpels during operations [2], the contact between underlying bones and articular cartilages in human bodies [3], the interaction between the polymer liner component and metal femoral component observed for hip implants [4] and the delightful mouthfeel when tasting chocolate [5].

In practice, a wide range of length scales and time scales is required to characterize wavelengths of rough surfaces and relaxation times of viscoelastic materials, respectively. Meanwhile, the time-dependent constitutive laws make it more challenging to address real-life viscoelastic sliding and rolling contact problems theoretically and experimentally [6]. Nevertheless, a number of early attempts have been made by different researchers.

Hunter [7] studied the two-dimensional (2D) frictionless problem of a rigid cylinder rolling over a viscoelastic half-space. This steady-state solution was later extended by Panek and Kaller [8] to three-dimensional (3D) problems by adopting an approximation on the basis of elastic line integral theory. The contact friction was taken into account in the line contact problem by Goriacheva [9]. However, the pressure solution was not affected by the dry contact friction (shear traction) as the two contacting viscoelastic bodies were assumed to exhibit the same mechanical properties (e.g., no coupling between shear tractions and pressures). Apart from these plane strain problems, Aleksandrov, et al. [10] simulated the frictionless contact between a smooth sphere sliding on a viscoelastic half-space. The application of these approaches is limited to ideal viscoelastic materials with one relaxation time. Based on the assumption that the viscoelastic material stiffens when the rolling speed increases (e.g., higher frequency) while the pressure distribution remains of a Hertzian type, a novel analytical theory was proposed by Persson [11] to solve the rolling contact of a rigid cylinder or a sphere over a flat viscoelastic substrate. The rolling friction coefficient can be estimated reasonably by the theory of Persson. However, the information on pressure distribution especially its non-Hertzian profile in a force-driven problem is not available within the framework of this theory.

To overcome the drawbacks of these analytical solutions, great efforts have been made in the past decades in developing numerical models that can take into account any viscoelastic material and any geometry of contacting bodies [12]. One of the most common modelling tools, e.g., the Finite Element Method (FEM), was frequently employed in early attempts [13-18] to simulate viscoelastic rolling and sliding contact problems based on Lagrangian formulations. However, the computational cost can be relatively high for models based on FEM when addressing rough contact problems since they require an extremely fine mesh to capture complex surface irregularities. In this case, the Fast Fourier Transform (FFT) assisted Boundary Element Method (BEM), also referred to as the semi-analytical method (SAM), is widely applied as an alternative. Carbone and Putignano [19] developed a BEM-based model to generate steady-state solutions to rough sliding or rolling contact problems [20], where the involved viscoelastic material can either exist as the form of a half-space [21] or a layer [22-24] with finite thickness. Recently, Zhao, et al. [25] adopted a novel formulation, which relates the surface deformation to contact tractions, to simulate the frictionless sliding or rolling contact of viscoelastic surfaces. They claimed that the new equation helps to

improve the computational efficiency of their BEM-based model using computational techniques including multi-grid and multilevel multi-integration (MLMI) methods [26, 27].

Regarding the transient viscoelastic analysis using BEM, the frictionless rolling contact of a rigid smooth indenter against a flat viscoelastic half-space was simulated by Koumi, et al. [28], where the material inhomogeneity is considered. The frictionless contact of an incompressible viscoelastic semi-infinite body rolling on a rough rigid surface was analyzed by Bugnicourt, et al. [29]. Wallace, et al. [30] proposed a model which provides transient analysis of frictionless rolling and sliding viscoelastic problems, in which the involved viscoelastic layer and substrate can exhibit distinct properties. The effects of imperfect bonding between layer and substrate on the transient as well as steady-state frictionless viscoelastic sliding contact problems were recently investigated by Zhang, et al. [31].

It is noted that considering the transient response is computationally demanding as all the time scales that describe the behaviour of viscoelastic materials need to be tracked. Additionally, the numerical time interval must be sufficiently small to capture the occurrence of sudden events. This explains why several researchers [19-22, 31] focused on the steady-state response of the contact problems instead of the transient response by making use of the constant complex modulus of viscoelastic materials. Such analysis shall work well for frictionless viscoelastic sliding or rolling contact problems as precise knowledge of the tangential field is not needed. However, during the transient part of frictional sliding, the contact area can be separated into stick and slip regions when the transient tangential load is not enough to induce gross sliding. This separation may play an important role in determining the later gross sliding behaviour. There exists some literature reporting the effects of dry contact friction in the sliding contact of viscoelastic surfaces such as the work of Goryacheva, et al. [32, 33]. To date, the effects of friction, especially the evolution from partial slip to gross sliding in transient viscoelastic sliding contact problems, are barely investigated numerically [12], which is the subject of the current study. Furthermore, few reports have elucidated the effect of surface roughness on the transient sliding of viscoelastic materials in the presence of contact friction.

A frictional sliding contact model for viscoelastic materials is developed in this study, where transient contact solutions are available. The steady-state solution can be achieved by extending the simulation time. The modelling starts from the quasi-static state to the dynamic state, where the complete transition from partial slip to gross sliding and the

coupling effects between shear traction and pressure are considered. A novel strategy to simulate the surface movement during the global slip stage is proposed, which is convenient to be implemented compared with other approaches in the literature [30, 34]. The effects of the partial slip period on the later sliding contact solutions are investigated with the model, including the role of the shear traction and the time when the partial slip ends (before or after the time when the material property becomes steady).

6.3 Theory and Algorithm Description

To explain how the model of frictional viscoelastic sliding contact is developed, some basic terminology and relationships commonly used to describe the behaviour of viscoelastic materials are first presented in this section.

6.3.1 Theory of Linear Viscoelasticity and Friction

Among viscoelastic materials, those exhibiting a linear relationship between stress and strain at any time are known as linear viscoelastic materials. The materials simulated in the current study are assumed to behave linearly following the infinitesimal strain theory. As viscoelastic materials are usually soft, they can hardly experience plastic deformation. Therefore, the plastic contact is neglected in the following modelling work. In addition, all the analyses presented here are under isothermal conditions. In other words, the temperature effects on the mechanical behaviour of materials are not considered during the simulation.

Following the theory of linear viscoelasticity, the responses of stress (denoted as vector σ) to successive strain (denoted as vector ε) stimuli are cumulative and vice versa, which can be described as follows by using the Boltzmann hereditary integral:

$$\sigma(t) = \int_0^t \psi(t - t') \frac{d\varepsilon(t')}{dt'} dt', \quad \text{Equation 6-1}$$

$$\varepsilon(t) = \int_0^t \phi(t - t') \frac{d\sigma(t')}{dt'} dt', \quad \text{Equation 6-2}$$

where $\psi(t)$ and $\phi(t)$ are known as the relaxation modulus function and creep compliance function respectively.

Different from an ideal elastic contact material where the compliance and modulus are mutually reciprocal, there exists the following essential mathematical relationship between the two material properties for linear viscoelastic materials in the Laplace transform domain:

$$\bar{\phi}(s) \bar{\psi}(s) = \frac{1}{s^2}, \quad \text{Equation 6-3}$$

where s is the variable in the Laplace transform domain ($s = a + jb$).

The time-dependent mechanical response of linear viscoelastic materials can be characterised in a discretised form by building rheological models. By arranging several springs and dashpots in series or parallel, different rheological models can be constructed. Among them, the Generalized Weichert model shown in Figure 6-1 is usually employed to characterise the complicated response of realistic materials such as polymethyl methacrylate (PMMA) [35]. With the Generalized Weichert model, the relaxation modulus function of any linear viscoelastic material can be expressed appropriately by fitting the experimental data collected from the conducted relaxation test to the following equation (Prony series) and adjusting the parameters:

$$\psi(t) = G_0 + \sum_{i=1}^n G_i \exp\left(-\frac{t}{\tau_i}\right), \quad \text{Equation 6-4}$$

where the subscript “ i ” is the index notation of the spring and dashpot in series ($i = 1 \dots n$) as shown in Figure 6-1. G is the spring modulus and τ is the relaxation time ($\tau_i = \eta_i/G_i$) with η denoting the viscosity of the dashpot.

Once the relaxation modulus function is available, the corresponding creep compliance function can be easily determined as well by utilizing their mathematical relationship in the Laplace transform domain (Equation 5-6). If the material is characterized by one relaxation time in this case (only one spring in series with one dashpot), such a three-element model is known as the Zener model or standard linear solid model. Zener models are commonly employed to build fictitious viscoelastic materials for the numerical study of the properties of different viscoelastic materials [30, 36, 37].

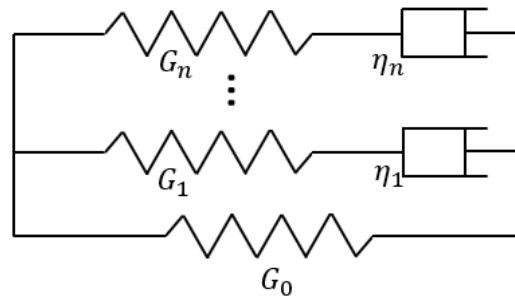


Figure 6-1 Structure of the Generalized Weichert model: G is spring modulus and η is viscosity of dashpot

To avoid confusion about concepts about friction in the following modelling work, different friction laws commonly employed in contact analysis are stated here in advance.

The first is the Amontons-Coulomb friction law (short for Coulomb friction law hereinafter). It describes the macroscale phenomenon that when a solid is under sliding, it opposes a tangential force proportional to the normal force. It is widely applied in many studies [32, 33, 36, 38, 39] due to its simplicity and versatility. If this approximation is not satisfactory enough to characterise the contact phenomena occurring at different scales, it is always easy to modify the law to make the friction coefficient dependent on different parameters such as pressure and velocity. The second is the constant shear stress friction law, which claims that friction is affected by contact conditions. This law is usually employed to characterise the contact phenomena occurring on the microscale, where the role of van der Waals bonds and physisorption becomes essential. Besides, the surface roughness tends to affect the friction force in this case. Apart from these two laws, the viscoelastic losses that happened in the bulk of a viscoelastic material can result in some friction force. The apparent friction coefficient analysed in the frictionless viscoelastic sliding or rolling contact modelling work is related to the friction derived from the viscoelasticity of materials.

When it comes to the viscoelastic sliding contact analysis, friction is always deduced from viscoelastic losses such as these frictionless simulation studies [7, 19, 28-31]. The word ‘frictionless’ in these works means that the dry contact friction is not included in the sliding or rolling analysis. However, the frictional force derived from the hysteresis losses in the deformation cycles of viscoelastic materials is taken into account. The reason behind such practice is relevant to the intrinsic property of some specific polymers. It is known that elastomers are incompressible. For the problem of an incompressible half-space sliding on a rigid surface, there exists no coupling between the normal pressure and shear tractions. A good representation of this type of frequently investigated problem is the rubber tyre sliding on the hard road surface on the mesoscale, as investigated by Bugnicourt, et al. [29]. As the normal and tangential contact problems are independent of each other, the simplest way to simulate the sliding or rolling contact problems of viscoelastic materials is to neglect shear tractions in the lateral direction, together with their related coupling effects affecting the pressure profile, and to quantify the friction force exclusively derived from the viscoelastic losses. However, the case where uncoupled conditions can be applied is not ubiquitous. When addressing the common material combination for engineering products such as the knee or hip prosthesis (usually a hard metal against a soft and compressible polymer), the inclusion of the coupling effect seems inevitable. Furthermore, evidence can be found that as the modulus of viscoelastic

materials changes with time, so does the Poisson's ratio [40, 41]. Because the latter quantity governs the coupling between normal and tangential tractions, a fully coupled model, which provides information on both normal and tangential fields in viscoelastic contact problems, is worth studying for fine precision engineering applications. It is noteworthy that the applications of first and third friction laws do not conflict with each other, although frictional forces are derived from different media. For example, in the experimental study on the frictional behaviour of sliding viscoelastic materials by Carbone and Putignano [19], the Coulomb friction coefficient (characterising dry contact friction) and apparent friction coefficient (characterising viscoelastic losses) were applied in an additive manner (e.g., the total friction force is the summation of the dry contact friction and viscoelastic friction).

6.3.2 Problem Formulation and Algorithm Development

The 3D problem of a rigid sphere sliding on a compressible viscoelastic half-space with contact friction is illustrated in Figure 6-2. The formulation for the frictional sliding contact problem can be divided into two parts including the partial slip and fully sliding problems.

To discretize the problem based on BEM, the potential contact region between the two contacting surfaces is meshed into equally spaced rectangular elements with the size of $\Delta_1 \times \Delta_2$ and number of $N_1 \times N_2$ in x and y directions respectively. Temporal discretization is necessary to model transient viscoelastic contact problems. The simulation time T is discretized into N_t time steps with a time interval that is uniform and short enough that the element pressure can be assumed to be constant for each time interval.

Coulomb friction law is employed here to determine the dry contact friction and to identify the stick or slip state of each surface node before gross sliding. A constant coefficient of friction μ_f is assumed for the sake of simplicity. To further facilitate the determination of contact tractions, semi-coupled conditions (coupling between pressure p and the shear traction in x direction q_x) are implemented during the simulation.

The surface movement after the partial slip period needs to be included in the dynamic sliding problem. This can be achieved by changing the coordinates of the contact problem correspondingly with the sliding motion. The Cartesian coordinate system used during the simulation process either maintains or keeps being updated according to the sliding state of the surface being experienced. During the partial slip period, a fixed coordinate

system $R_0(O_0, X_0, Y_0, Z_0)$, which corresponds to the initial contact set of axes at the initial simulation time ($t = 0$), is employed. Once the surfaces are in the gross sliding stage, the coordinate system $R(O, X, Y, Z)$ must follow the movement of the rigid sphere to search for the transient contact solutions in the updated contact fields. The contact conditions and equations are formulated in a new coordinate system, which is centred on the projection of the centre of the indenter on the flat half-space.

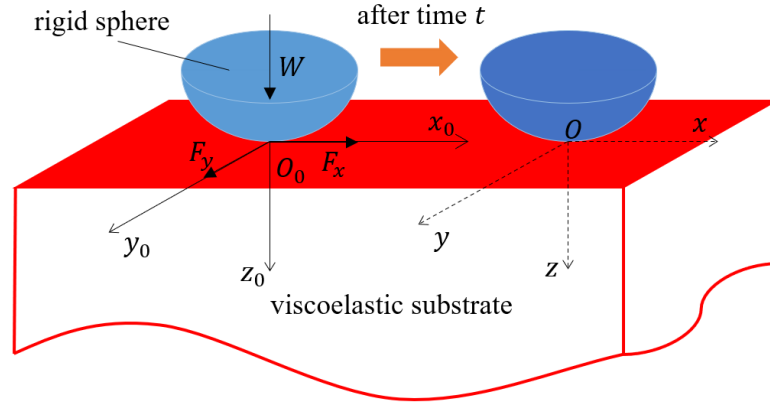


Figure 6-2 Geometrical description of the frictional contact of a rigid sphere against a viscoelastic half-space under input loads (the sphere slides with a constant velocity v once the contact reaches the gross sliding state)

To assume that the sliding velocity of viscoelastic surfaces remains constant after reaching the fully slip state, by relating the sliding velocity (v) to the pixel width (Δ_1, Δ_2), the coordinates of the rigid sphere are translated by one mesh element in the sliding direction at each time step. This approach can avoid updating the surface geometry by the interpolation of surfaces according to the slip distance at each time step [31, 34], which could be time-consuming and also undermine the accuracy of simulation results.

However, since the size of the computational domain is intended to remain unchanged during the simulation, information on past pressure history might be partially lost due to the update (movement) of the system coordinate. As the size of the mesh element is affected by the constant sliding velocity, the number of nodes must be large enough that the computational domain can characterize the creep of the contacting area when the velocity is relatively low. A considerably large number of nodes (e.g., 512×512 or 1024×1024) is needed to keep the whole past pressure history available when the total simulation time is long and the time step is extremely small. Therefore, a compromise might need to be made between computational efficiency and accuracy when this approach to updating surface movement is adopted. A detailed discussion on the effect of the lost past pressure history is given in section 5.4.

To apply the elastic-viscoelastic correspondence principle, the surface displacement at any position caused by the contact tractions at any time can be determined as follows:

$$u_{\alpha\beta}(i, j, k) = \sum_{n=1}^{N_t} \sum_{l=1}^{N_1} \sum_{m=1}^{N_2} IC_{\alpha\beta}(i-l, j-m, k-n) \left(t_\gamma(l, m, n) - t_\gamma(l, m, n-1) \right), \quad \text{Equation 6-5}$$

where $u_{\alpha\beta}$ is the displacement in α ($\alpha = x, y, z$) direction caused by the traction in β ($\beta = x, y, z$) direction and t_γ is the contact traction in the γ ($\gamma = x, y, z$) direction. $IC_{\alpha\beta}(i-l, j-m, k-n)$ is known as the viscoelastic influence coefficient characterizing the displacement in α direction observed after k time steps in the node (i, j) of the spatial mesh under the effect of uniform traction in the β direction of acting on the node (l, m) in the n -th time step after the reference time, with $n \leq k$ ($i = 1 \cdots N_1, j = 1 \cdots N_2, t = k\Delta_t$ and $k = 1 \cdots N_t$). For a detailed description of how to determine the influence coefficient based on Boussinesq solutions [42] using the correspondence principle, readers can refer to the work by Wang, et al. [36].

In the current study, only the tangential problem in the x direction is considered. The nodal displacements of the viscoelastic half-space induced by an arbitrary history of nodal tractions under semi-coupled conditions can then be expressed as follows:

$$\begin{bmatrix} u_x \\ u_z \end{bmatrix} = \sum_{n=1}^{N_t} \sum_{l=1}^{N_1} \sum_{m=1}^{N_2} \begin{bmatrix} IC_{xx}(i-l, j-m, k-n) & IC_{xz}(i-l, j-m, k-n) \\ IC_{zx}(i-l, j-m, k-n) & IC_{zz}(i-l, j-m, k-n) \end{bmatrix} \begin{bmatrix} q_x(l, m, n) - q_x(l, m, n-1) \\ p_z(l, m, n) - p_z(l, m, n-1) \end{bmatrix}, \quad \text{Equation 6-6}$$

where discrete convolution fast Fourier Transform (DC-FFT) is applied to accelerate the convolution operations.

A quasi-static process is assumed to avoid the problem of dissipative friction and its irreversibility related to the load-path dependency when addressing the partial slip problem. As mentioned before, the Poisson's ratio of a realistic viscoelastic material employed in practice is usually time-dependent [40, 41], but it is assumed to be constant here for simplicity.

The frictional sliding contact problem can be solved by searching for solutions that can satisfy the following boundary conditions:

1. Load balance: The sum of the pressure within the contact area should always be equal to the specified normal load at any time step. This can be expressed as follows:

$$W(t) = \Delta \sum_{(i,j) \in I_c(k)} p(i,j,k), \quad \text{Equation 6-7}$$

where I_c denotes the time-varying contact region.

2. The surface displacement of the contacting surfaces should meet the following geometrical condition in the normal direction:

$$h(i,j,k) = h_i(i,j) + u_z(i,j,k) - \delta_z(k), (i,j) \in I_p, \quad \text{Equation 6-8}$$

where I_p denotes the computational domain.

In the lateral direction, the following condition must be satisfied:

$$s_x(i,j,k) = u_x(i,j,k) - \delta_x(k), (i,j) \in I_c(k); \quad \text{Equation 6-9}$$

3. The following complementary condition must be met to obtain valid contact solutions:

The Kuhn-Tucker complementary conditions should be satisfied in the normal direction at any time point, which also implies that there exists no surface adhesion and the simulated surfaces are impenetrable:

$$\begin{aligned} p(i,j,k) > 0 \ \& \ h(i,j,k) = 0, (i,j) \in I_c(k); \\ p(i,j,k) = 0 \ \& \ h(i,j,k) > 0, (i,j) \in I_p - I_c(k), \end{aligned} \quad \text{Equation 6-10}$$

where $I_p - I_c$ denotes the non-contacting region within the computational domain.

During the partial slip period, the stick or slip state of each surface node within the contacting region I_c can be distinguished by the following boundary conditions in the tangential direction:

$$\begin{aligned} |q_x(i,j,k)| < \mu_f p(i,j,k), \ s_x(i,j,k) = 0, \ (i,j) \in I_{st}(k), \\ q_x(i,j,k) \cdot s_x(i,j,k) < 0, \ (i,j) \in I_c(k) - I_{st}(k), \\ |q_x(i,j,k)| = \mu p(i,j,k), \ s_x(i,j,k) \neq 0, \ (i,j) \in I_c(k) - I_{st}(k), \end{aligned} \quad \text{Equation 6-11}$$

where I_{st} denotes the stick region and $I_c - I_{st}$ denotes the slip region.

Once the surface starts to slide globally, the shear traction q_x is always equal to the local friction expressed as follows:

$$q_x(i,j,k) = \mu_f \cdot p(i,j,k), (i,j) \in I_c(k); \quad \text{Equation 6-12}$$

The algorithm for the frictional sliding contact problems is shown in Figure 6-3, which can be separated into two sections including the partial slip and fully sliding solvers. As illustrated, the contact solutions to the semi-coupled partial slip problem keep being searched until the ratio of the stick region to the contacting area (short for stick ratio hereinafter) vanishes. A detailed description of the partial slip solver can be found in our previous work on the partial slip modelling of viscoelastic surfaces [36]. Once the surface starts to slide, only the normal contact problem needs to be addressed as the shear traction is always equal to the local friction. During the sliding phase, the progress to search for solutions to the frictional sliding problem is similar to that for a frictionless problem. The difference is the time when the coordinate system of the contact field needs to be updated. This is determined by the time when the partial slip period ends, the past pressure history affected by the coupling effects with shear traction, and the extra normal displacement derived from the shear traction.

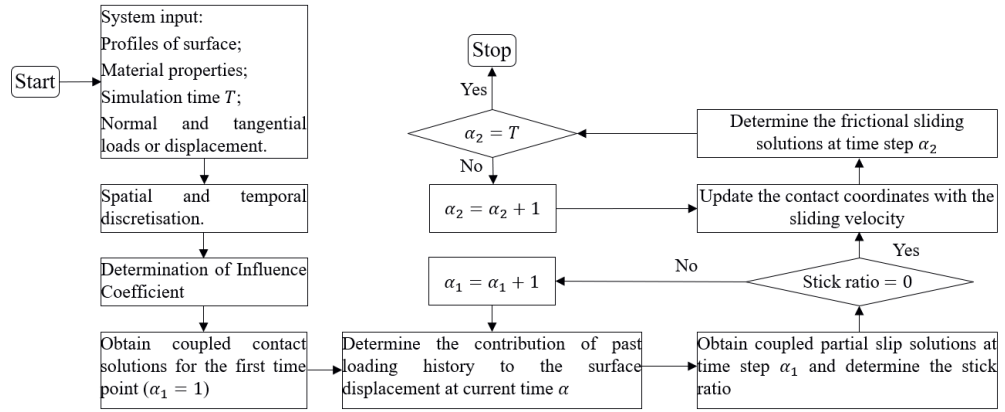


Figure 6-3 Flow chart of the algorithm for the frictional viscoelastic problem

It is of note that the reason why fully coupled conditions between shear tractions and pressure are not applied is relevant to the way the algorithm produces contact solutions to partial slip solutions. When normal and tangential problems are fully coupled, the state of gross sliding is identified by checking if the algorithm of the partial slip solver fails to separate the stick and slip regions and outputs NaN (Not a number) results in MATLAB. Therefore, the exact contact tractions are not accessible when surfaces are in the sliding state under fully coupled conditions.

6.4 Model Validation

Since the coupled partial slip aspect of the model has been validated and described in detail in our previous work [36, 39], only the validation work of its fully slip field is shown in this section. By simulating a frictionless sliding contact problem of a rigid sphere against a viscoelastic half-space, the sliding aspect is validated by comparing the

simulation results from the degenerated forms of our developed model with the corresponding results reported by Koumi, et al. [28]. The contact input specified during the simulation is given in Table 6-1. To capture the creep phenomena of the contacting area under different sliding speeds, the computational domain is set to be $-3.84a_0 \times 3.84a_0$ for the case with the highest speed ($v = 1.2a_0/\tau$) while a smaller computational domain ($-2.56a_0 \times 2.56a_0$) is employed for the cases with the lowest and medium speeds ($v = 0.4a_0/\tau$ and $v = 0.8a_0/\tau$ respectively). The computational domain is discretized by 256×256 nodes.

Table 6-1 Contact parameters employed in the validation test

Parameter	Value	Description (Unit)
W	1.48	Input indentation load (N)
R	10	Radius of the sphere (mm)
μ_∞	3.86	Initial shear modulus of material (MPa)
μ_∞/μ_0	10	Ratio of retardation time to relaxation time
τ	0.01	Relaxation time of the viscoelastic material (s)
T	4τ	Total simulation time (s)
ν	0.3	Poisson's ratio of the viscoelastic material
a_0	588.7	Hertzian contacting radius for nondimensionalization (μm)
p_0	0.2067	Hertzian peak pressure for nondimensionalization (MPa)
v	$0.4a_0/\tau$	Constant frictionless sliding speed of the surface (Normalized)
	$0.8a_0/\tau$	
	$1.2a_0/\tau$	

A Zener model is employed to characterize the mechanical response of the viscoelastic half-space, where the relaxation modulus and creep compliance are usually described in the following form:

$$\psi(t) = \mu_0 + (\mu_\infty - \mu_0) \exp\left(-\frac{t}{\frac{\mu_0}{\mu_\infty} \tau}\right), \quad \text{Equation 6-13}$$

$$\phi(t) = \frac{1}{\mu_\infty} + \frac{1}{\mu_1} \left(1 - \exp\left(-\frac{t}{\tau}\right) \right), \mu_1 = \frac{1}{\frac{1}{\mu_0} - \frac{1}{\mu_\infty}}, \quad \text{Equation 6-14}$$

where μ_∞ is the initial shear modulus and μ_0 is the modulus after infinite time. It is noted that the parameter $\frac{\mu_\infty}{\mu_0}$, which is known as the ratio of retardation time to relaxation time, determines the rheological behaviour of viscoelastic material characterized by a Zener model [36].

The simulation results under the given contact inputs are normalized by the Hertzian solution based on the instantaneous modulus $\Psi(0)$ and the input load W . Since the pressure profile tends to become steady after the time of 2τ , transient pressure distributions at four different time points within the simulation of 2τ are plotted. As a fixed normal load is applied, the creep of contacting area with time can be observed in Figure 6-4 (a)-(c) for the three tests with different sliding velocities. Due to the resistance force arising from the viscoelasticity of the material during sliding, the pressure is distributed non-symmetrically. Compared with the pressure at the trailing edge of the contact area, that at the leading edge tends to be higher showing a sharp spike. Besides, the contact region shifts in the sliding direction with time even though the effects of shear stress on the pressure are neglected in the current frictionless tests. Meanwhile, the increase in the sliding velocity is found to cause an increase in the peak pressure, a decrease in the contacting area and a more pronounced contact shift phenomenon. One can see from Figure 6-4 (a)-(c) that the back side of the contacting area becomes closer to the centre of the indenter when the sliding velocity increases.

To quantify the friction derived from viscoelastic loss, the apparent friction coefficient μ_{app} is usually determined. A time-dependent tangential force F_T can be induced by the hysteresis loss within a viscoelastic material, which can be calculated as follows:

$$F_T(t) = \int \int_{I_c(t)} \frac{\partial u_z(x, y, t)}{\partial x} p(x, y, t) dx dy \quad \text{Equation 6-15}$$

As a common practice, the finite difference method is used to calculate the factor $\frac{\partial u_z(x, y, t)}{\partial x}$, and the rectangle rule is applied to evaluate the integral for $F_T(t)$. The apparent friction coefficient μ_{app} can then be calculated as the ratio of the resulting tangential force F_T to the applied normal load W .

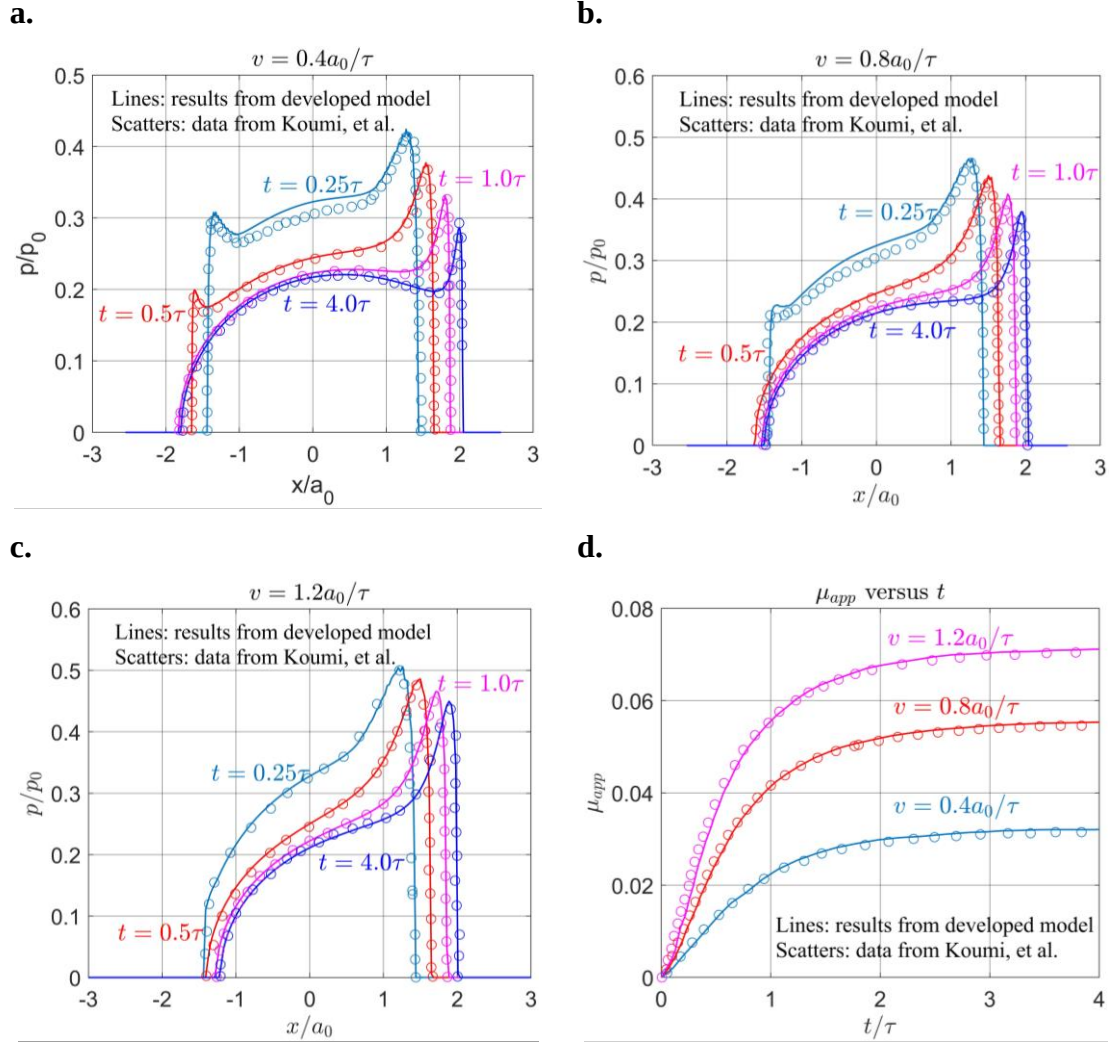


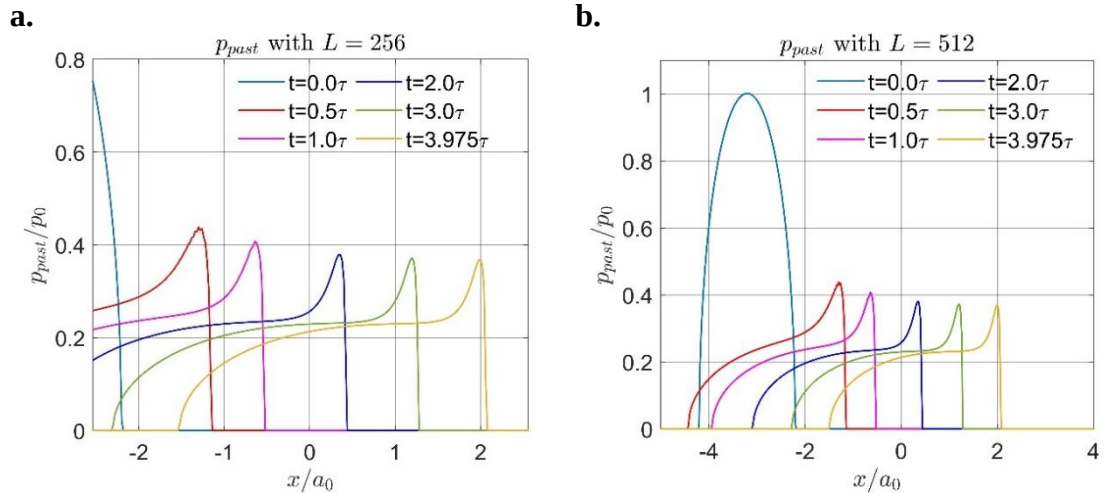
Figure 6-4 Comparison of the simulation results from our model (solid lines) with the results from Koumi, et al. [28] (scatters): (a)-(c): Pressure distribution with different dimensionless sliding velocities $\frac{v\tau}{a_0} = 0.4, 0.8, 1.2$ respectively, and (d) Evolution of the apparent friction coefficient μ_{app} with time at different sliding velocities

As shown in Figure 6-4 (d), for all the tests, the apparent friction coefficient tends to become steady when the simulation time reaches 3τ . This coefficient is affected by the sliding velocity such that a higher velocity leads to a higher apparent friction coefficient (more viscoelastic friction). It is noted that such a monotonic trend only holds for viscoelastic contacts with low or medium sliding velocity. When the sliding velocity is considerably high, the response of the apparent friction coefficient to the sliding velocity tends to be the opposite. A complete map showing the evolution of the apparent friction coefficient with the normalized sliding velocity, which is also known as the Deborah number, can be found in the steady viscoelastic contact analysis by Carbone and Putignano [19].

Good agreement can be found between the simulation results derived from our model and the referred data from Koumi, et al. [28] as shown in Figure 6-4 (a)-(d).

As mentioned in section 6.3.2, sliding motion is simulated by updating the coordinate of the contacting surface at any time point in the model. Some of the past pressure histories could be lost as a computational domain with a fixed size is applied during the simulation, where the pixel width is related to the sliding distance per time step. It is possible to keep the whole past pressure history in the record by extending the computational domain, but such an operation will impose a certain computational burden. To investigate the effect of the lost past pressure information on the sliding contact solution and to identify if it would become one of the limitations of our model, the following test, where two different numbers of nodes in x/y direction (denoted as L) were applied in a frictionless sliding contact simulation with the velocity of $0.8a_0/\tau$, was conducted. The computational domain is set to be $-2.56a_0 \times 2.56a_0$ for the case with 256×256 nodes ($L = 256$) while it is increased to $-5.12a_0 \times 5.12a_0$ for the case with 512×512 nodes ($L = 512$).

As shown in Figure 6-5 (a), some of the past pressure histories are lost at several time points, including 0.0τ , 0.5τ , 1.0τ , when 256×256 nodes are applied. On the other hand, past pressure information is recorded completely when 512×512 nodes are applied as illustrated in Figure 6-5 (b). Contact solutions of the two cases (solid line for $L = 256$ and scatter for $L = 512$) are given in Figure 6-5 (c). The two curves of contact pressure fit perfectly initially while some insignificant difference appears at the final time point ($t = 3.975\tau$) as presented in the zoomed-in view in Figure 6-5 (c). The globally good agreement between the simulation results with different numbers of nodes indicates that the lost past pressure information has micro effects on the contact solutions. Therefore, results derived from our model are reliable if an appropriate computational domain is specified to capture the creep of viscoelastic surfaces.



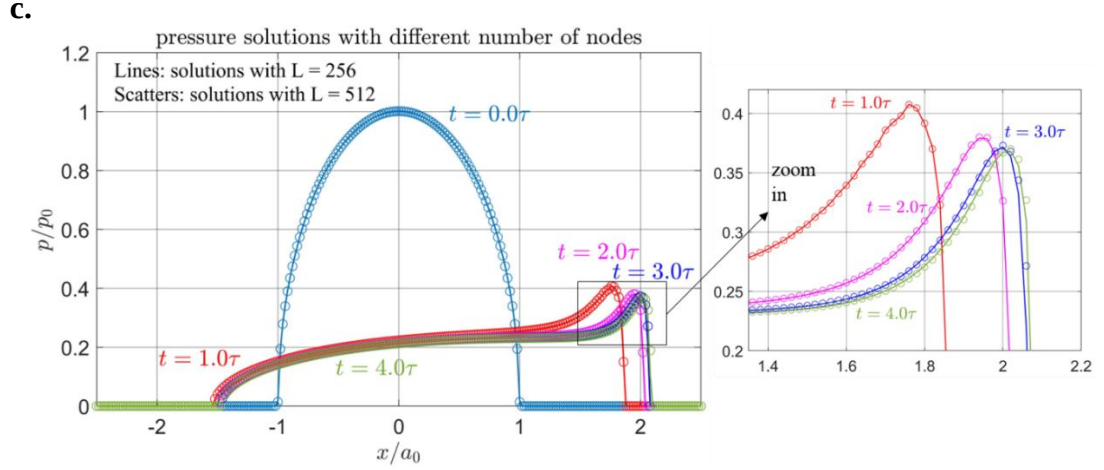


Figure 6-5 Simulation results with different numbers of nodes L : (a) and (b): Past pressure history at the updated coordinates within $T = 4\tau$ with $L = 256$ and $L = 512$ respectively and (c): Comparison of contact pressures at different times for the two cases with $L = 256$ and $L = 512$ respectively

6.5 Results and Discussion

Before moving to the frictional sliding contact modelling, it is noted that in our previous study on viscoelastic contacts [36], the parameter known as the ratio of retardation time to relaxation time (μ_∞/μ_0) is found to determine the rheological behaviour of viscoelastic materials characterized by Zener models. Although the boundary value is yet unknown, materials with higher ratios tend to behave more fluid-like while those with lower ratios exhibit a more solid-like contact response. As a follow-up study, here the dependence of the pressure distribution and apparent friction coefficient on the rheology of viscoelastic materials is first investigated.

6.5.1 Contact Solutions in Frictionless Sliding and the Effects of Rheological Properties

The frictionless sliding contacts of three different viscoelastic materials characterized by Zener models are simulated here. It is of note that contact solutions can be different depending on whether the contact is generated via a prescribed displacement or a fixed load since they represent distinct contact phenomena for viscoelastic materials. To describe it briefly, a creep phenomenon is encountered under a fixed normal load, where the contacting area keeps increasing with time. On the other hand, a stress relaxation phenomenon is encountered under a fixed normal displacement, where the contacting area remains while the normal pressure relaxes (decreases) with time. To complicate the study on the effects of rheological properties of viscoelastic materials on the sliding solutions, two different contact phenomena encountered in the normal direction are tested by varying the contact input (constant load or displacement) to investigate their effects on

sliding solutions. The detailed contact inputs specified in the tests are given in Table 6-2. The computational domain is set to be $-2.56a_0 \times 2.56a_0$ for all the tests, which is discretized by 256×256 nodes ($L = 256$).

As shown in Figure 6-6 (a)-(c), when a stress relation phenomenon is experienced in the normal direction, the rear part of the contact region keeps shrinking with time. The shrink of contact area and relaxation of pressure becomes more significant for the more fluid-like material (e.g., the material with the higher value of μ_∞/μ_0). For the more solid-like material ($\frac{\mu_\infty}{\mu_0} = 3$), the pressure profile is skewed less significantly compared with that of the more fluid-like material ($\frac{\mu_\infty}{\mu_0} = 10$). Besides, it seems to take more time for the frictionless viscoelastic contact to reach the steady state for the more solid-like material. This can be related to the relaxation modulus functions of the materials as shown in Figure 6-6 (d) such that the modulus of the more fluid-like material can reach a steady value within a shorter time.

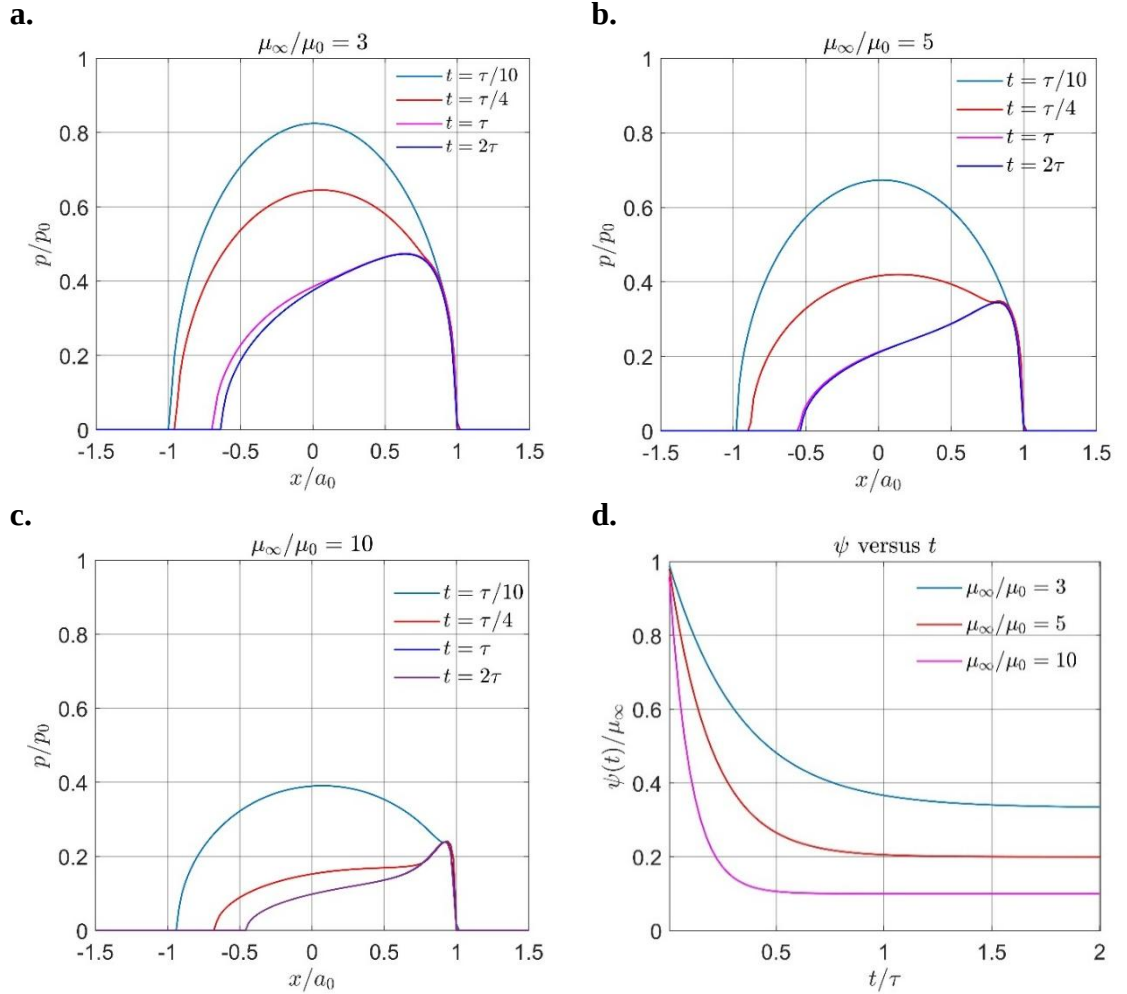


Figure 6-6 Results of tested frictionless sliding contact problems under a constant displacement: (a)-(c): Pressure distributions of different materials characterised by Zener models $\frac{\mu_\infty}{\mu_0} = 3, 5, 10$ respectively, and (d): Variations of relaxation modulus function with time for different materials

To switch to the load-controlled case, the more fluid-like material (the material with a higher value of μ_∞/μ_0) is found to exhibit a more significant creep phenomenon as shown in Figure 6-7 (a)-(c). Similar to the displacement-controlled case, the pressure profile is skewed more significantly for the more fluid-like material such that the peak pressure tends to appear on the rear edge of contacting area. However, the more fluid-like material tends to require more time to reach its steady contact state in the current creep case, which is related to the creep compliance of tested materials shown in Figure 6-7 (d). These simulation results are consistent with the finding of Zhao, et al. [25], who reported the effects of a similar material parameter, which is known as the elasticity ratio in their study, on the steady-state pressure distribution when a creep phenomenon is encountered in the normal direction.

Table 6-2 Contact parameters employed in the frictionless sliding test of different viscoelastic materials

Parameter	Value	Description (Unit)
W	1.50	Input indentation load (N)
δ	100	Input normal displacement (μm)
R	10	Radius of the sphere (mm)
μ_∞	3.86	Initial shear modulus of material (MPa)
$\frac{\mu_\infty}{\mu_0}$	3,5,10	Ratio of retardation time to relaxation time (ratio of initial shear modulus to modulus after infinite time)
τ	0.01	Relaxation time of the viscoelastic material (s)
ν	0.3	Poisson's ratio of the viscoelastic material
v	$0.8a_0/\tau$	Frictionless sliding speed of the surface (Normalized)

Regarding the pressure spikes in the pressure profile, the reason behind their occurrence shall be highlighted here. In our previous viscoelastic study regarding the effects of the rheological behaviour of materials on the shape of pressure in a normal indentation problem [36], the pressure spikes on both contacting edges can only be observed for the more fluid-like material for a certain time while the more solid-like always exhibits a

Hertzian-type pressure distribution. The role played by the viscosity of viscoelastic materials determines the shape of the pressure profile in indentation problems. In this case, the viscosity of the more solid-like viscoelastic material has relatively trivial effects, where the contact behaviour is similar to that of elastic materials. Regarding the fluid-like material, its mechanical response could be first assumed as a pack of liquid being squeezed, where the indentation load keeps being distributed to the contacting edges. However, the material is not a real fluid after all. Eventually, it will reach a steady state, where elasticity plays a dominant role instead of viscosity and leads to a Hertzian-type pressure profile. For the sliding problem investigated here, it is the synergistic effect of sliding motion and viscosity of material (e.g., the friction derived from viscoelasticity during sliding) that leads to the commonly observed asymmetric pressure distribution. According to the studies of Carbone [19] and Zhao [25], the pressure profile becomes symmetric at considerably low or extremely high sliding speeds.

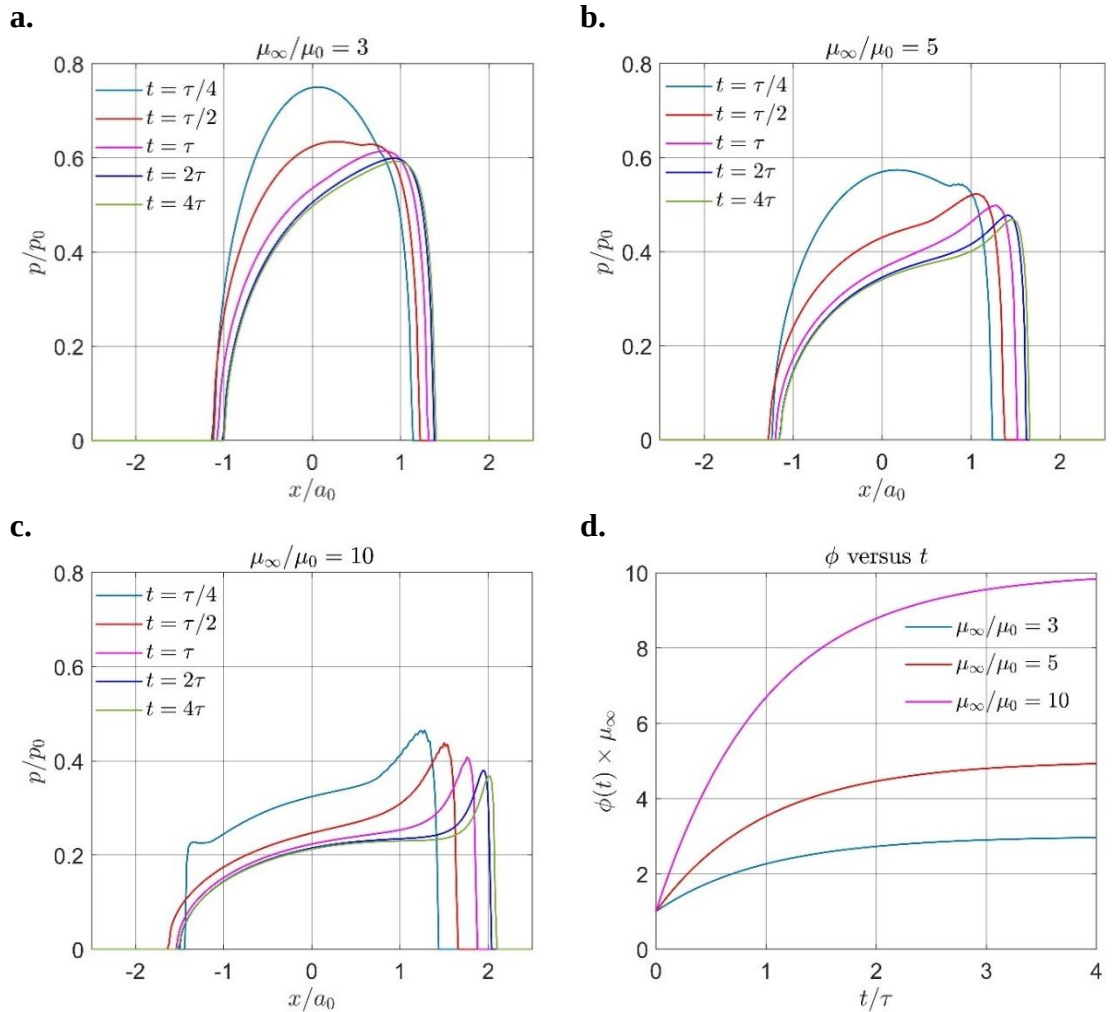


Figure 6-7 Results of tested frictionless sliding contact problems under a constant load: (a)-(c): Pressure distributions of different viscoelastic materials characterised by Zener models $\frac{\mu_\infty}{\mu_0} = 3, 5$ and 10 respectively, and (d): Variation of creep compliance with time for different materials

In terms of the frictional behaviour exhibited by different viscoelastic materials, as shown in Figure 6-8, the more fluid-like material always shows a higher apparent friction coefficient regardless of the time and the experienced contact phenomenon. This results from the more significant viscoelastic loss related to the dominant role played by the viscosity of the material. Compared with solutions under stress relaxation shown in Figure 6-8 (a), the apparent friction coefficient is always higher when the creep phenomenon is experienced by the same viscoelastic material as shown in Figure 6-8 (b). This suggests that a material can exhibit different levels of viscoelastic friction depending on whether the contact is generated by a specified penetration or load (experienced contact phenomenon).

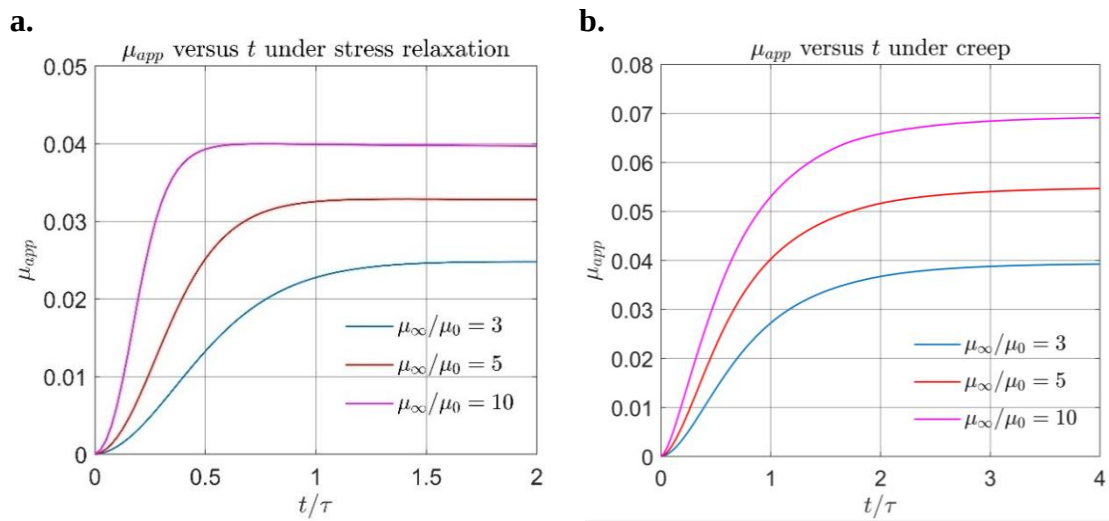


Figure 6-8 Response of apparent friction coefficient with time for tested viscoelastic materials under different contact phenomena: (a) stress relaxation and (b) creep

6.5.2 Effects of Partial Slip Period on the Sliding Solutions

To show the effects of the partial slip period, here four different cases of viscoelastic sliding are tested when a fixed displacement is specified in the normal direction. The first is the already conducted frictionless test. The second case is a frictional sliding test but without the initial partial slip period. In other words, the contact starts with gross slip and the effects of shear traction on the pressure profile are taken into account in this case, where the magnitude of shear traction is always equal to the local friction. The third case includes the partial slip period in the frictional sliding contact, where the gross slip happens before the material property reaches its steady state (gross sliding achieved at 0.1τ). The case where the gross slip happens relatively late (gross sliding achieved at 0.6τ , when the time-dependent material property becomes stable) is considered in the final test.

By specifying different linearly increasing rates of tangential displacement in x direction Δx in our displacement-controlled algorithm (e.g., the increasing displacement is the model input while the increasing load is the output), the occurrences of gross sliding at different times for cases 3 and 4 are achieved. Once the gross sliding state is reached for the viscoelastic contact, the same sliding velocity v is specified for all four cases. The difference between static friction and dynamic friction is neglected here since a unified coefficient of friction is intended to be applied to simplify the frictional sliding simulations. The detailed contact inputs used in the tests are given in Table 6-3. The computational domain is set to be $-3.07a_0 \times 3.07a_0$ for all the cases, which is discretized by 256×256 nodes ($L = 256$).

Table 6-3 Contact parameters employed in the frictional viscoelastic sliding test

Parameter	Value	Description (Unit)
δ_z	34.45	Input normal displacement (μm)
R	10	Radius of the sphere (mm)
μ_∞	3.86	Initial shear modulus of material (MPa)
μ_∞/μ_0	10	Ratio of retardation time to relaxation time (ratio of initial shear modulus to modulus after infinite time)
τ	0.01	Relaxation time of the viscoelastic material (s)
ν	0.3	Poisson's ratio of the viscoelastic material
μ_f	0.3	Coefficient of dry contact friction
$\Delta\delta_x$	$\frac{0.017a_0}{\tau}, \frac{0.17a_0}{\tau}$	Increment of tangential displacement per time step
v	$1.2a_0/\tau$	Constant sliding speed of the viscoelastic surface (normalized) when the surface is in the gross sliding

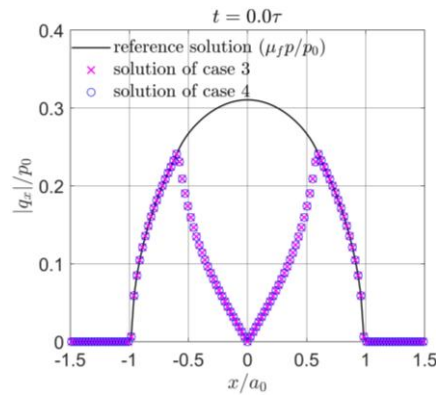
The contact solutions of the four cases when the contact just initializes ($t = 0.0\tau$) are shown in Figure 6-9 (a). Due to the presence of shear tractions, cases 2, 3 and 4 exhibit different pressure profiles as illustrated in Figure 6-9 (a), right. The shear traction q_x is equal to the static friction (μp) for case 2 while cases 3 and 4 experience the same partial slip state exhibiting identical distributions of pressure and shear tractions as shown in Figure 6-9 (a), left. Compared with the two cases without a partial slip period, cases 3 and 4 show higher peak pressures at this moment.

For the next investigated time interval ($t = 0.1\tau$), the gross sliding is just reached for case 3 as indicated in Figure 6-9 (b), left, where the curve of shear traction agrees with that of static friction. The difference between the pressure profiles of cases 1 and 2 exists but it is still not significant at this time point as shown in Figure 6-9 (b), right. The shift of the contacting area in the direction of the sliding motion can be observed for these two cases while the contacting region tends to remain for cases 3 and 4 due to the mainly experienced stress relaxation phenomena in static states. Since case 4 is still in the partial slip period at $t = 0.1\tau$ as demonstrated in Figure 6-9 (b), left, the pressure profiles of cases 3 and 4 are distinct currently.

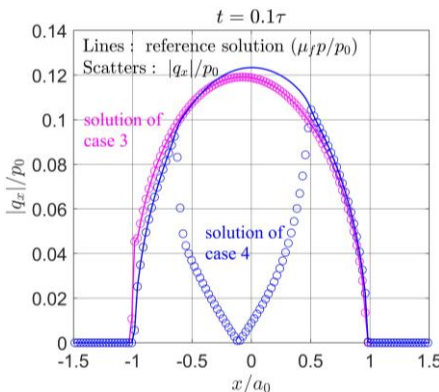
For the time interval when the material property becomes stable and the frictionless sliding solution becomes steady ($t = 0.5\tau$), since case 4 has yet not reached the gross sliding region as indicated in Figure 6-9 (c), left, its pressure distribution is significantly different from the other cases as shown in Figure 6-9 (c), right. Due to the viscoelastic sliding motion, contact areas of surfaces in cases 1,2 and 3 have shrunk, which leads to significantly higher peak pressures compared with the solution of case 4. Besides, after being in gross sliding for a certain time, the contact area of case 3 is found to shift in the same way as cases 1 and 2. However, this case now shows a lower peak pressure compared with the two cases without a partial slip period.

Shear tractions of cases 3 and 4

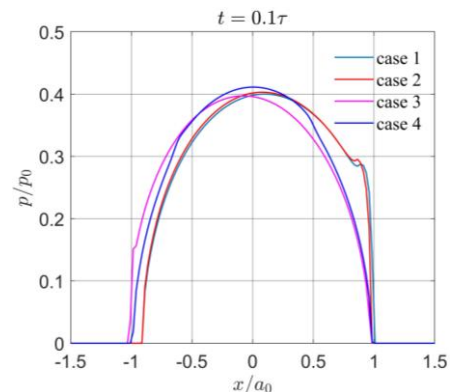
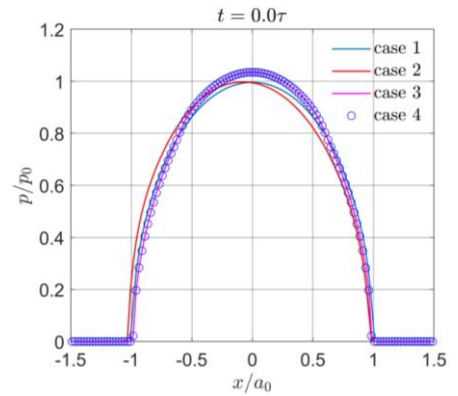
a.



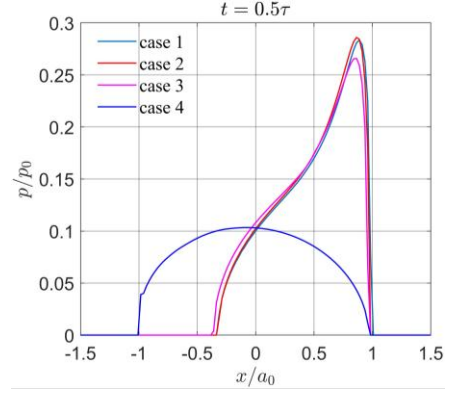
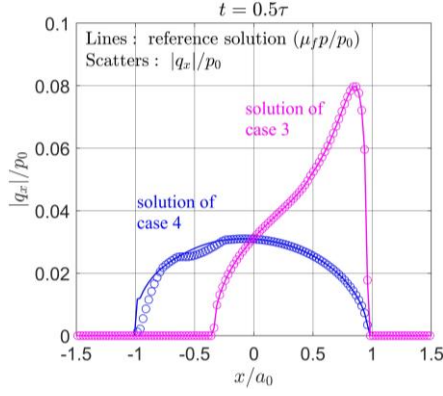
b.



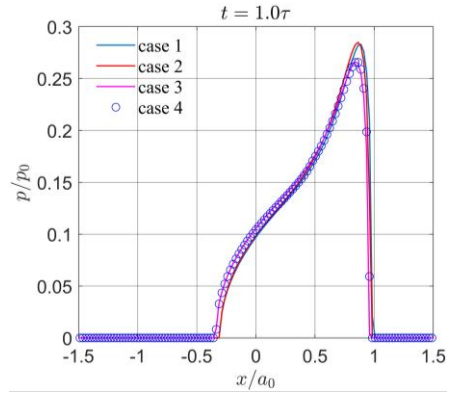
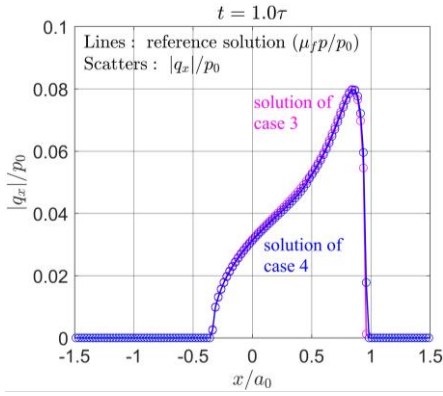
Normal pressure of all cases



c.



d.



e.

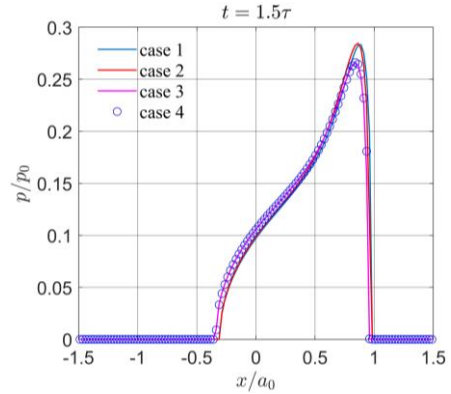
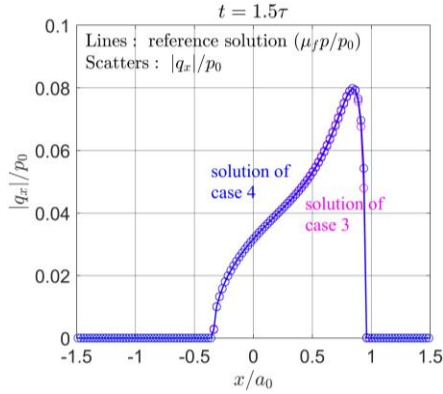


Figure 6-9 Nondimensionalized contact solutions including the shear tractions of cases 3 and 4 (left) and normal pressures of four different cases (right) at different times: (a) $t = 0.0\tau$, (b) $t = 0.1\tau$, (c) $t = 0.5\tau$, (d) $t = 1.0\tau$ and (e) $t = 1.5\tau$

Contact solutions of another later time point ($t = 1.0\tau$) are given in Figure 6-9 (d), where the surface in case 4 has already been in a global slip state for a certain time as indicated in Figure 6-9 (d), left. Eventually, the shift of the shrinking contacting area is observed for case 4. Besides, the pressure solutions of other cases seem to reach a steady state as the difference can hardly be found between Figure 6-9 (c), right and Figure 6-9 (d), right for cases 1,2 and 3.

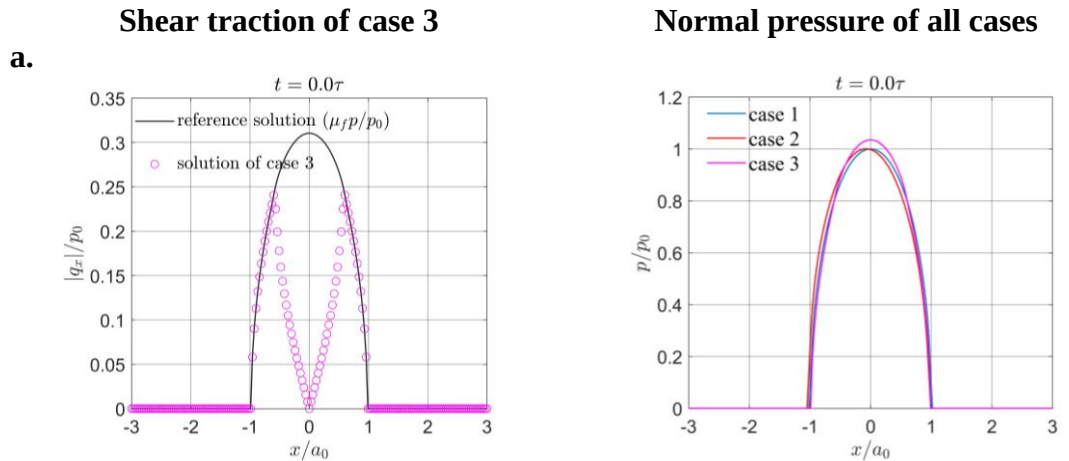
By extending the simulation time to 1.5τ , the contact becomes steadier for case 4 as the viscoelastic surface has been in the gross sliding state for a longer time as illustrated in

Figure 6-9 (e), left. The overlap of pressure distribution curves is found for cases 3 and 4 as shown in Figure 6-9 (e), right. This suggests that the same pressure profile can be obtained by extending the simulation time to the steady contact state no matter when the partial slip period ends in a frictional sliding problem.

Cases including the partial slip period show a minor variation with the frictionless sliding solution regarding the shape of pressure distribution when the contact becomes steady. When the contact is in the steady regime, a slight difference can be observed when comparing the pressure magnitude as a lower peak pressure is observed for the case where partial slip has been experienced. Besides, the contacting region is slightly shifted in the direction opposite to the sliding motion for those cases. The difference between the solutions of frictional sliding contact without partial slip period and frictionless sliding contact is relatively micro regardless of the time being referred to.

To switch the contact input in the normal direction to be a fixed load ($W = 0.15 \text{ N}$), since the time when the gross sliding happens barely causes any difference as long as the viscoelastic sliding contact reaches the steady state, only one frictional sliding contact with the presence of partial slip under creep input was tested with $\Delta\delta_x = 0.17a_0/\tau$. It was named case 3 while cases 1 and 2 stand for the frictionless test and frictional test without partial slip respectively. To take into account the creep of the contacting area under a constant normal load, the computational domain is increased to $-3.84a_0 \times 3.84a_0$ for the following tests.

The time-dependent contact solutions under the input of constant normal load are shown in Figure 6-10. Like the above relaxation tests, when the contact initializes ($t = 0.0\tau$), due to the addition of dry contact friction in cases 2 and 3 and the resulting partial slip phenomenon in case 3 shown in Figure 6-10 (a), left, the three cases exhibit different pressure profiles as shown in Figure 6-10 (a), right.



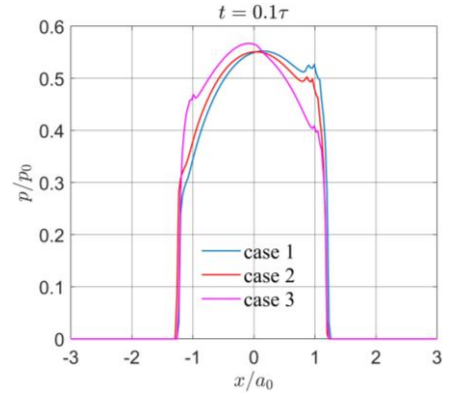
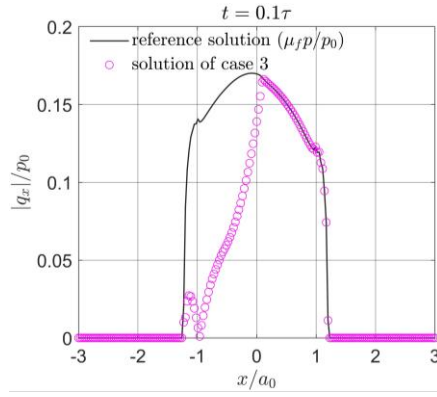
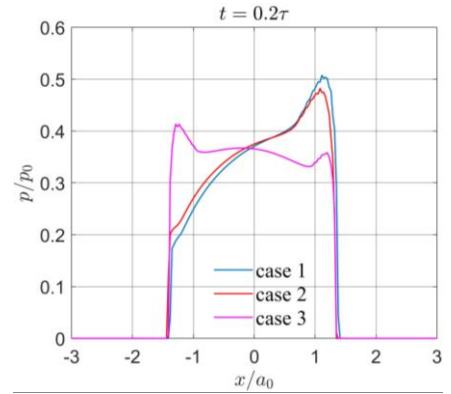
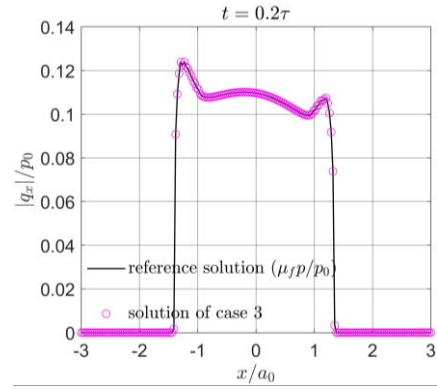
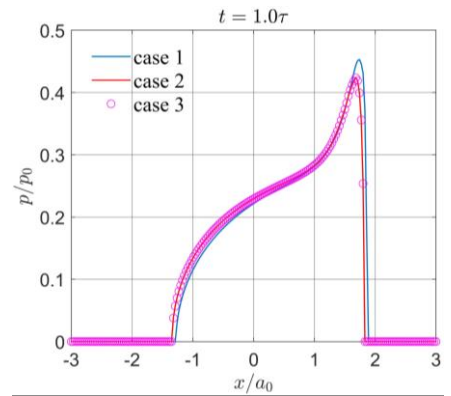
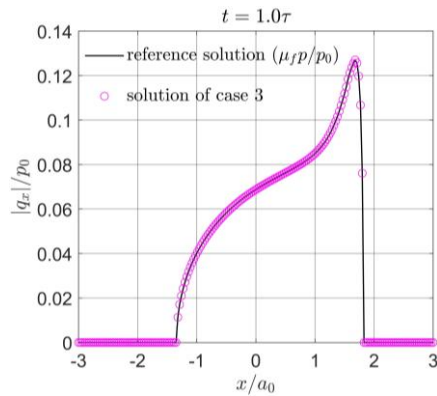
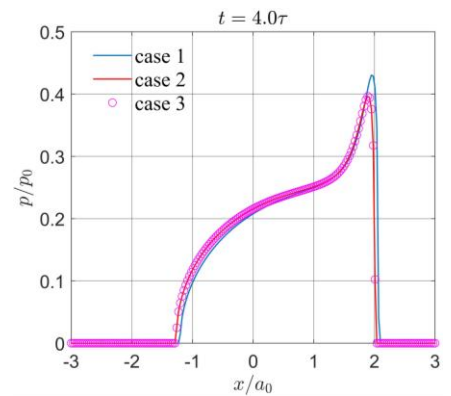
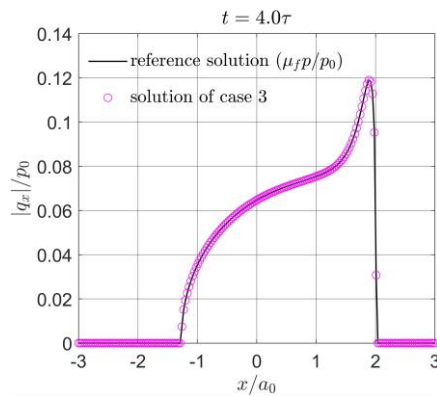
b.**c.****d.****e.**

Figure 6-10 Nondimensionalized solutions including the shear tractions of case 3 (left) and normal pressures of the three different cases (right) at different time points: (a) $t = 0.0\tau$, (b) $t = 0.1\tau$, (c) $t = 0.2\tau$, (d) $t = 1.0\tau$, and (e) $t = 4.0\tau$

For the following two time points including $t = 0.1\tau$ and $t = 0.2\tau$, the viscoelastic surface in case 3 is found to experience a partial slip state as shown in Figure 6-10 (b), left and a gross slip state that is just achieved as illustrated in Figure 6-10 (c), left, respectively. Hence, the pressure profiles of case 3 are significantly different from the other two fully-slip cases at these two time points as shown in Figure 6-10 (b), right and Figure 6-10 (c), right, respectively.

Similar to the finding in our former study about viscoelastic indentation problems [36], fluctuating results are observed on the contact solutions for all the cases in Figure 10 (b) and (c). Although such oscillations could be alleviated by adopting a smaller time interval, the computational domain must be increased to characterize the creep of the contacting area as the pixel width of the mesh system is determined by the sliding distance during each time interval.

For the rest investigated time points ($t = 1.0\tau$ and $t = 4.0\tau$) when the viscoelastic surface in case 3 is in the gross slip state after certain times as illustrated in Figure 6-10 (d), left and Figure 6-10 (e), left, respectively, the pressure profiles of case 3 tend to become closer to those of case 2 as shown in Figure 6-10 (d), right and Figure 6-10 (e), right, respectively. What is different from the former relaxation test is that the load in the current creep test does not relax with time. Besides, different from the relaxation test where the contacting area of the sliding surface starts to shrink while that of the surface in the partial slip state remains, the contacting areas of the three cases almost remain identical regardless of the sliding states that are experienced by the viscoelastic surface at any time point as illustrated in Figure 6-10. Together with the fact that viscoelastic surface exhibits different frictional resistance to sliding depending on whether the stress relaxation or creep phenomenon is experienced in the normal direction, the partial slip regime has different effects on the later sliding solutions as discussed. However, as shown in Figure 6-9 and Figure 6-10, the differences between the pressure profiles of all the tested cases are not significant but yet exist when the contacting viscoelastic surface is at a steady state.

6.6 Concluding Remarks

The sliding aspect of the developed model is validated against the simulation results of Koumi, et al. [28] for a spherical frictionless sliding contact problem. The dependence of the distribution of contact pressure and dissipative losses on the rheological property of viscoelastic materials is analysed based on the developed sliding contact model for viscoelastic materials. It was found that the more fluid viscoelastic material, for which

the pressure profile is skewed more remarkably, exhibits higher frictional resistance to sliding motion due to the more dissipative loss arising from the dominant role of its viscosity. As a result, the shrinkage of contacting area for the more fluid-like viscoelastic is more significant when the stress relaxation phenomenon is experienced in the normal direction.

The frictional viscoelastic sliding contact with the partial slip period is analysed in this paper. Beyond the fact that the partial slip regime caused a micro change in the normal pressure distribution when the contact becomes steady, it delays the time required by the viscoelastic sliding surface to reach its steady state. This suggests that the assumption of frictionless contact (no dry friction), which is adopted in most numerical and theoretical models, could induce quantitative differences when analysing the sliding contact of viscoelastic materials. Qualitative errors may even be encountered when the observation time is too small to capture the actual steady state of the contacting surfaces. We believe that our numerical study can serve as a good reference for the future modelling work about the sliding contact of viscoelastic materials. If the steady-state solution is the primary concern and of greater interest for researchers, they can simplify the contact problem under consideration (i.e., neglect the dry contact friction and conduct a frictionless sliding contact analysis instead) to obtain qualitative-correct results. However, in some cases such as a contact system subjected to dynamic loading, it is necessary to perform transient contact analysis on the sliding viscoelastic materials, where the role played by dry friction should be considered. Information on the contact traction history during the evolution period from partial slip to gross sliding and the time at which gross sliding motion occurs can provide a quantitatively-correct contact analysis, thereby improving the understanding of friction and wear of materials and enhancing the prediction of material performance.

Considering that the contact solutions presented in the study are limited to the half-space approximation while viscoelastic materials often exist in the form of a layer with a finite thickness in practice, the development of a model for the sliding contact of viscoelastic layers is necessary. This is the subject of our current ongoing work.

Apart from the single-asperity contact considered in the study, the transient multi-asperity (rough-surface) contact needs to be investigated numerically to shed light on the role played by the surface roughness in frictional sliding contact solutions of viscoelastic materials. Furthermore, the assumption of no adhesive force might undermine the validity of the simulation results especially when the viscoelastic material is significantly softer

or more compliant than the other contacting materials. A model, which integrates the effects of adhesive force and viscoelasticity, shall provide a more helpful contact analysis of viscoelastic materials. These are knowledge gaps that currently exist and need to be filled in the future.

Acknowledgements

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6.7 Supplementary Notes for Chapter 6

6.7.1 Effects of Contact Configurations on Pressure Distributions

The contact configuration can have significant effects on the pressure distribution, especially in the currently investigated dynamic case.

To review the reason why pressure spikes occur in the indentation contact of viscoelastic materials, it is caused by the different loading history experienced by the centre and edges of the contacting area as discussed in section 5.7.3. Similarly, under moderate sliding speed, the front and rear part of the contacting area for the viscoelastic material can also experience different loading history, under which circumstances the contact configuration can play a significant in determining the pressure distribution. Koumi, et al. [28] simulated the frictionless sliding and rolling contact problems of a sphere on flat, where the different contact configurations can lead to different contact solutions as illustrated in Figure 6-11. Notably, the pressure is plotted such that the pressure is always normal to the deformed contacting interface. Hence, the plotted pressure vectors always point toward the centre of the sphere for the rigid sphere against viscoelastic flat. On the other hand, the plotted pressure vectors are always vertical for viscoelastic sphere against rigid flat.

For the purely frictionless sliding contact of rigid sphere on viscoelastic flat under a constant displacement, when the sliding motion proceeds, surfaces that have not been deformed in the flat viscoelastic materials come into contact, leading to different loading history experienced by the front and rear parts of the contacting area such that the rear part allows the pressure to relax for a longer time leading to lower pressures as illustrated in Figure 6-11.

Notably, the above viscoelastic contact mechanism only applies to moderate sliding speeds. When it comes to extremely high-speed situation, the difference between the loading histories experienced by the front and rear parts of the contacting area becomes trivial as the viscoelastic sphere travels so quickly, leading to a symmetric pressure distribution, as reported by Carbone and Putignano [19].

In addition to the asymmetric pressure distribution, the contacting area shrinks in the direction opposite to the sliding motion, which can be related to the superimposed effects of the delayed recovery of viscoelastic materials. As illustrated in Figure 6-12, the time-dependent increase of the surface deformation at the rear part of the contacting area leads to the shrink of the contact area in this specific contacting zone.

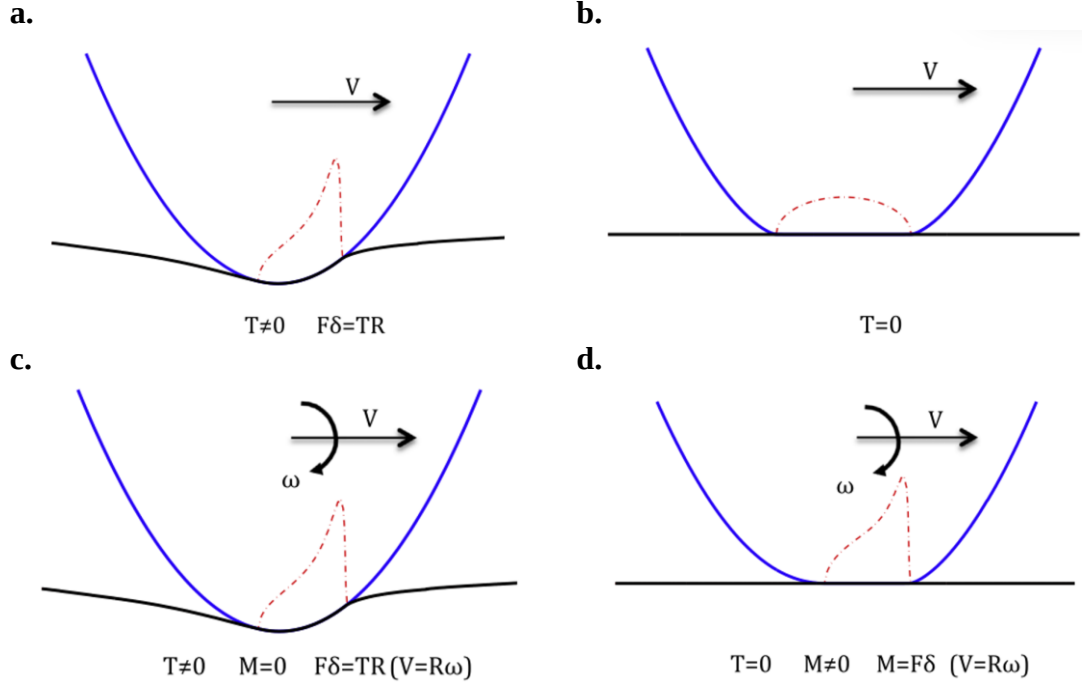


Figure 6-11 frictionless dynamic problem between a sphere and a flat with same imposed normal displacement: (a) rigid sphere sliding against viscoelastic flat, (b) viscoelastic sphere sliding against rigid half-space, (c) rigid sphere rolling against viscoelastic flat and (d) viscoelastic sphere rolling against rigid flat. Reproduced with permission from the work of Koumi, et al. [28], © Elsevier B. V., 2025.

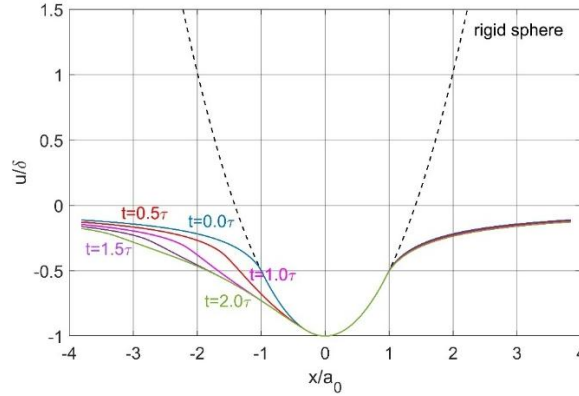


Figure 6-12 Time-dependent deformation of the viscoelastic half-space sliding against a rigid sphere under a prescribed normal displacement $\delta = 0.1$ mm and speed $v = 1.2a_0/\tau$

On the other hand, when it comes to the purely frictionless sliding contact of a viscoelastic sphere against a rigid flat under a constant displacement. When the sliding motion proceeds, the viscoelastic materials that come into contact remain with time such that the front and rear part of the contacting area experiences exactly same loading history. As a result, a symmetric pressure distribution is obtained, as illustrated in Figure 6-11 (b).

However, when it comes to the pure rolling contact of a viscoelastic sphere against a rigid flat, new materials of the viscoelastic sphere keep coming into contact with the rigid sphere due to the rolling motion, which causes different load history experienced by the

rear and front areas of the contacting area. Thus, as illustrated in Figure 6-11 (c) and (d), for the rolling contact, both contact configurations exhibit asymmetric pressure distribution with shrinking contacting area in the direction opposite to the sliding motion.

6.7.2 Display between Friction Forces Derived from Different Sources

As mentioned at the end of section 6.3.1, in the experiments conducted by Carbone and Putignano [19], the measured friction coefficient is the summation of two terms, including interfacial dry friction μ_f , which is governed by Coulomb's friction law, and apparent friction coefficient μ_{app} , which is associated with the viscoelastic losses that happened in the bulk of a viscoelastic material. Since the scale difference tends to be not significant between the macroscale Coulomb friction and bulk viscoelastic friction, it is a common practice to add the two together to obtain the total friction force. For example, in the thermo-viscoelastic contact modelling work of layered viscoelastic materials by Zhang, et al. [43], the total interfacial heat source distribution generated by the Coulomb friction and viscoelastic dissipation are simply added together.

However, friction in viscoelastic material contacts can span multiple scales. Microstructure friction can be the friction on the small-scale level, influenced by many factors such as surface adhesion. On the other hand, the macroscale friction is the friction observed in large-scale interactions such as the Coulomb's friction mentioned above. To date, there exists no universal law that links microstructure friction to macroscale friction although the complicated multiscale coupling between them were discussed by many researchers, such as the work of Temizer [44], Stoyanov and Chromik [45] and Broitman [46]. Some studies, such as Nettingsmeier and Wriggers [47], suggest incorporating microscale friction coefficients into high-level calculations. The approach of summing friction contributions derived from different sources is commonly adopted in literature (e.g., work of Emami, et al. [48] and Makhovskaya [49])

A recent study by Carbone, et al. [50] brings insights to the interplay between viscoelasticity and surface adhesion in frictional contacts. In their investigated adhesive sliding contact between viscoelastic materials and rough surfaces, friction forces derived from two different sources are analysed. To briefly them briefly, adhesive friction is caused by the adhesion hysteresis arising from molecular debonding and rebinding at asperity contacts, which modifies the energy release rates at the leading and trailing edges of the contact. On the larger scale, viscoelastic friction is derived from the bulk energy dissipation under cyclic deformation. To link the microstructure friction with the macroscale friction in this context, an energy-balance-based theory was developed by

equating the change of surface energy to the corresponding work of the internal stress. Based on this approach, it was found that the overall friction coefficient for the viscoelastic adhesive contact is always larger than the summation of adhesive friction coefficient and adhesiveless viscoelastic friction coefficient, implying a scale-dependent mechanisms of interaction between adhesion and viscoelasticity. They reported that the interplay of surface adhesion and viscoelasticity governs the velocity-dependent pull-off forces, hysteretic friction, and energy release rates at the contact edges. A good agreement between their theoretical prediction and the experimental work of Grosch [51] in terms of the trend of friction with sliding speed implies that energy-based approaches can be a good start for considering these multi-scale problems.

6.7.3 Effects of Sliding Speeds on Viscoelastic Friction

Due to the limited range of sliding speeds considered during simulation, the apparent friction coefficient μ_{app} was found to increase monotonically with the sliding speed, as illustrated in Figure 6-4 (d). However, as discussed by Carbone and Putignano [19], this trends only holds for a certain range. As illustrated Figure 6-13, the sliding speed specified in our simulations is limited to a relatively small range while the viscoelastic friction tends to decrease with the sliding speed once it reaches its maximum value when $\frac{v\tau}{a_0} \approx 1$. Notably, Figure 6-13 was obtained based on fixed displacement while Figure 6-4 (d) was obtained based on fixed load. However, the bell-shaped relationship between the viscoelastic friction and dimensionless sliding speed still holds when fixed normal load is specified. Such a relation can be relevant to the variation of the loss tangent with frequency. Similarly, the value of loss tangent is only large in the intermediate frequency range but becomes significantly lower at low or high frequencies.

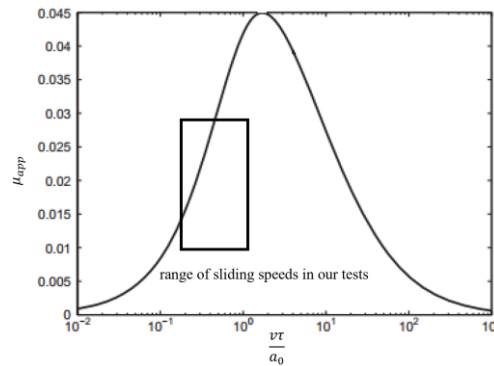


Figure 6-13 Evolution of apparent friction coefficient with the non-dimensional sliding speed for a viscoelastic material with one relaxation time $\tau = 0.01s$, $\mu_\infty = 10^7$ Pa, and $\mu_\infty/\mu_0 = 10$, results are shown for fixed penetration $\delta = 0.1$ mm, this relation also holds for fixed load, reproduced with permission from the work of Carbone and Putignano [19], © Elsevier B. V., 2025.

6.7.4 Significance of Viscoelastic Losses Compared with Dry Contact Friction in Sliding Contact

The magnitude of viscoelastic losses is highly material- and system-dependent, influenced by many factors, including material properties, sliding speed, working temperature, loading conditions, contact geometry, and so on. So, it is difficult to exactly quantify how significant the viscoelastic losses can be compared with the dry contact friction. To take the sliding speed mentioned above for example, generally viscoelastic friction is most significant at moderate sliding speed ($\frac{v\tau}{a_0} \approx 1$) while it can be insignificant when the sliding speed is extremely high or low as reported by Carbone and Putignano [19]. Besides, different viscoelastic materials can exhibit distinct degrees of viscoelastic losses depending on their rheological behaviour as reported in the study. For the more solid-like materials such as the rigid polymers at room temperatures, the viscoelastic losses can be negligible. On the other hand, when considering the viscoelastic loss of materials that are specifically designed for damping, which usually have large values of μ_∞/μ_0 and loss tangent indicating great ability to dissipate energy, the apparent friction coefficient μ_{app} can be comparable to μ_f under specific contact conditions such as high load and intermediate sliding speed, especially when the material itself possesses a very low μ_f .

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7 Time-Dependent Contact Behaviour of ZDDP-Derived Tribofilms: A Viscoelastic Layered Model Approach

7.1 Abstract

The ZDDP-derived tribofilm was recently reported to be viscoelastic based on a creep experiment, where a Burgers material model mathematically represents its creep compliance. This study develops a contact model for layered materials by extending a previously established viscoelastic half-space contact model. The approach involves converting analytical frequency response functions into influence coefficients, enabling the investigation of the viscoelastic behavior of ZDDP-derived tribofilms. The results reveal that the tribofilm exhibits a highly fluid-like response when modeled as a half-space body being in contact with a carbon steel ball during indentation or sliding. When bonded to an elastic substrate in its typical thin-film form (on the nanometer scale), the contact behaviour can still exhibit time-dependent characteristics, depending on the operating conditions. Creep and stress relaxation are observed during indentation, particularly under low loads, while high loads result in a more pronounced viscoelastic response in extremely slow-speed contacts. However, under moderate sliding speeds ranging from millimeters to meters per second, time-dependent effects become negligible, regardless of the applied load. These findings indicate that although ZDDP-derived tribofilms exhibit significant viscoelasticity, their behaviour in practical applications generally resembles that of a soft elastic layer, as typical sliding speeds fall outside the range where pronounced time-dependent effects occur.

7.2 Introduction

Zinc dialkyl dithiophosphate (ZDDP) is one of the most common anti-wear additives applied in various industries considering its outstanding ability to generate protective films known as ZDDP-derived tribofilms on contacting surfaces [1, 2]. Apart from providing anti-wear properties, ZDDP can also work as a multifunctional additive showing anti-oxidant and anti-corrosion performance. Although its usage brings certain benefits to the system, ZDDP presents several drawbacks. The generated tribofilms have been argued to aggravate the micro pitting in rolling contacts due to the consequent high friction and low wear rate [3, 4]. The running-in (smoothing) of rough surfaces was found to be hindered due to this protective tribofilm, which leads to plastic deformations and stress concentrations at surface asperities. This intensifies subsurface stress fields, increases the likelihood of micro-crack formation and propagation, and elevates the risk

of surface fatigue. The severity degree of micro pitting increases with the thickness of tribofilm, which leads to the reduction of bearing service life [5]. In addition, due to the presence of sulphur and phosphorus, when ZDDP is applied to systems equipped with catalytic converters such as the exhaust system of vehicles, it can poison the catalyst producing harmful emissions [6, 7], although its performance is highly dependent on the two chemical elements. Furthermore, ZDDP additives are considered unsuitable for electrical vehicle (EV) and hybrid transmission systems [8]. While they provide anti-wear benefits, their sulphur content can corrode copper components, undermining motor functionality [9]. Additionally, ZDDP-derived tribofilm can increase electric resistance at contact points [8], leading to local overheating, oxidation, and wear under bearing currents [10, 11]. To address these issues, e-fluids tailored for EVs are being developed to meet lubrication needs without compromising component integrity [12, 13]. With EVs gaining popularity and stricter limits on ZDDP usage, it is crucial to develop sulphur-free and eco-friendly alternatives with comparable performance. To completely replace ZDDP, knowledge about ZDDP-derived tribofilms in terms of their mechanical properties, kinetics, morphology, and relevant rheological study is necessary.

Although ZDDP-derived tribofilms have been regarded as a solid-like surface film that reacts elastically under normal load for a long time [14], the viscoelasticity of tribofilms has been developed from a proposed assumption to an experimental finding. It was first proposed by Heinike [15] that the film inside the tribological contact should be in a magma state (an extremely viscous high-temperature liquid) and likely in the plasma state (a high-temperature ionized gas). A breakthrough was achieved by Pidduck and Smith [16], who found the existence of a soft and viscous overlayer covering the solid-like surface film. The presence of the viscous layer was further proved by the relevant chemical analysis on tribofilms [2, 17, 18]. From the pin-on-plate test by Minfray, et al. [19], the pads of tribofilms were found to elongate in the direction of sliding implying that the flow of the tribofilm is similar to fluid. Numerical results of their developed molecular dynamic model showed a particular rheological behaviour of the zinc polyphosphate under friction conditions suggesting a zinc phosphate layer modification from a solid-like to a liquid-like phase under extreme conditions. Dorgham [20] recently reported that ZDDP-derived tribofilms behave as viscoelastic interfaces between rubbing surfaces, exhibiting fluid-like flow and spreading under shear. Through creep and squeeze-flow experiments, Dorgham, et al. [21] quantified the viscoelastic properties of these tribofilms. They found that the high viscosity of tribofilms can contribute to their

superior anti-wear performance by allowing them to flow during formation (under shear) and to maintain nanoscale organization through adaptive movement of tribofilm pads at the contact interface.

In the meantime, most solutions within the framework of classical contact mechanics [22, 23] are heavily dependent on the half-space approximation, where the thickness of contacting solids is considerably larger than the contact radius. However, owing to the nature of viscoelastic materials (e.g., low contact compliance) and the time-varying material creep, the viscoelastic contacting area is usually larger than the ordinary elastic solid. Indeed, a viscoelastic surface layer (e.g., the ZDDP-derived tribofilm investigated in the current study) has often to be accounted for many engineering surfaces. In these cases, the surface layer or coating always exhibits characteristics that are significantly different when compared with the remaining bulk region of the contacting body. Therefore, the reliability of contact analysis based on the half-space approximation is undermined on this occasion. As argued by Pauk and Wozniak [24], Chen and Chen [25], and Zhang, et al. [26], viscoelastic materials such as polymers mainly exist as a finite film instead of a half-space when it comes to their application. The thin layers, or a more general layer-substrate system, are widely employed in engineering systems, such as gears with viscoelastic coatings, seals of finite thickness, and bearings with anti-wear layers. A model simulating the contact behaviour of these surface films is of fundamental importance in tribological applications, which shall be helpful for the analysis or design of relevant engineering components.

Over the past decades, great efforts have been made in solving the indentation, sliding, or rolling contact problems of viscoelastic layers. The first attempt was made by Batra and Ling [27], who investigated the surface deformation, friction dissipation, and contact stresses of a viscoelastic-elastic layered system under the influence of a moving load. Later, Naghieh, et al. [28, 29] proposed analytical solutions to the frictionless indentation problem of a rigid smooth indenter against a single-layered viscoelastic material bonded to a rigid substrate. The key roles played by the viscoelasticity and sliding velocity in the distributions of contact pressure and internal stress (e.g., non-symmetrical pressure profile caused by a viscoelastic layer) were reported by Goryacheva, et al. [30] in their study about the contact of a rough body against a viscoelastic layered semi-infinite plane. The different indentation problems between layers of arbitrary viscoelastic materials, including elliptical [31], rebound spherical [32] and cylindrical contact [33], were investigated analytically by Argatov and Mishuris. However, these proposed solutions

are limited by the assumption of monotonic loading. A more general solution to indentation problems of viscoelastic layered materials was developed by Chen, et al. [34] using the Hankel transform. By extending Persson's theory of contact mechanics to layered materials [35], Scaraggi and Persson investigated the behaviour of a viscoelastic layer in rough surface contact. Effects of finite roughness size and rubber thickness were studied in their study.

Regarding the numerical attempts for layered contact of viscoelastic materials, a Boundary Element Method (BEM)-based model was developed by Carbone and Putignano [36, 37] to simulate steady-state sliding contact problems. A correction factor, which takes the thickness of the viscoelastic surface and sliding speed into account, was introduced in their novel formulation of Green's function. The model was then extended to investigate the effect of the thickness of viscoelastic layer in rough surface contact [38, 39] and the problem of anisotropy induced by the viscoelasticity in rough sliding contact problems [40]. The two different material combinations, including a viscoelastic layered bonded with a rigid half-space and a viscoelastic half-space covered by a rigid surface layer, were simulated by Torskava and Stepanov [41]. The effects of the surface layer on the hysteretic losses during the viscoelastic sliding contact were reported in the study. A more general transient analysis of frictionless sliding viscoelastic problem was recently proposed by Wallace, et al. [42]. The involved layer and substrate can exhibit different properties (either elastic or viscoelastic). The problem of imperfect bonding between layer and substrate in viscoelastic sliding contact was analysed numerically by Zhang et al. [43]. To date, the framework of numerical modelling for viscoelastic layered contact has been initially established, but the application of these models to real-life contact scenarios is rather limited.

In this study, by developing the elastic layered contact model first based on the well-established theory of elastic contact, a BEM-based model providing transient contact analysis of viscoelastic layered surfaces was developed after converting the elastic layered contact model through the elastic-viscoelastic correspondence principle. A Burgers material model was built to characterise the contact behaviour of the ZDDP-derived tribofilm by fitting the four-term equation with the creep compliance curve reported by Dorgham, et al. [21]. By using the specific properties of the ZDDP-derived tribofilm as the material input for the layer, the role played by the viscoelasticity of the tribofilm under different contact conditions, including normal indentation and sliding, was exposed numerically for the first time.

7.3 Theory and Algorithm Description

Before the formulation description of the contact problem, it is mentionable that a tribofilm is widely recognized as the thin reacted film on the substrate as a product of tribochemical reactions at the interface driven by the frictional heating and rubbing of contacting surfaces [44]. It can be stress-induced at a temperature that is much lower than that required for thermal films under the effects of rubbing on the reaction kinetics. These thermal effects are out of the scope of the study but shall be investigated in the future considering the essential role of tribochemistry in the performance of tribofilms. Besides, ZDDP-derived tribofilms were argued to exhibit a rough, patchy, and graded structure. Each layer was found to exhibit distinct rheological behaviour such that the top layer performs the most fluid-like while the bottom one close to the substrate performs the most solid-like. In this work the tribofilm was assumed to behave as a uniform, smooth and homogenous layer to simplify the following formulation as well as simulation.

7.3.1 Problem Formulation

The point contact problem of a spherical indenter against a smooth viscoelastic layer bonded to an elastic body is illustrated in Figure 7-1. The viscoelastic layer is assumed to be in a uniform thickness h and bonded perfectly to an elastic substrate. Regarding the z coordinate illustrated in Figure 7-1, the layer surface and the interface between the layer and substrate are defined as $z = 0$ and $z = h$ respectively.

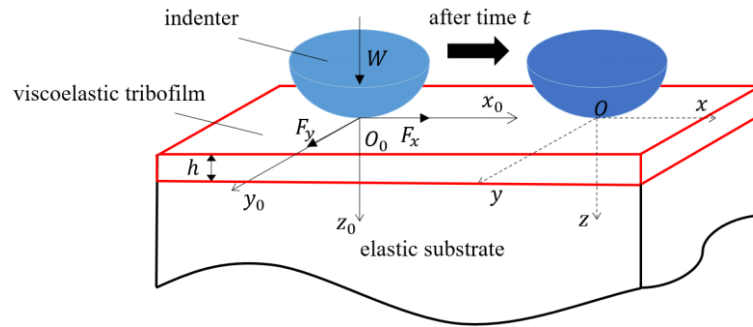


Figure 7-1 Geometrical description of the contact of a rigid sphere against a viscoelastic layer with uniform thickness h bonded to an elastic substrate under the normal load W and tangential loads F_x and F_y

For such layered contact problems, explicit expressions of influence coefficients relating the surface deformations with contact tractions can hardly be derived in a conventional way as those for homogeneous half-space (e.g., via Boussinesq integral [45] and Cerruti's solutions [46]). However, as suggested by Liu, et al. [47, 48], the problem can be solved by deducing the response functions in the frequency domain first and then using the reverse Fourier transform to obtain the corresponding influence coefficients in the spatial

domain. Considering that the correspondence principle was applied to the conversion between elastic and viscoelastic problems, only the derivation of the frequency response functions for elastic layered contact problems is described in detail here to avoid repeated contents.

For an elastic layered contact system, when considering the action of normal pressure p and shear traction in x direction q_x , there exist following boundary conditions regarding the stress σ at the upper layer surface ($z = 0$), the layer is denoted with superscript 1:

$$\sigma_{zz}^{(1)}(x, y, 0) = -p(x, y), \sigma_{xz}^{(1)}(x, y, 0) = q_x(x, y), \sigma_{yz}^{(1)}(x, y, 0) = 0 \quad \text{Equation 7-1}$$

Besides, the following continuous conditions in terms of the stresses and displacements shall be satisfied for the interface between the layer and substrate ($z = h$).

$$\begin{aligned} \sigma_{xz}^{(1)}(x, y, h) &= \sigma_{xz}^{(2)}(x, y, 0), \sigma_{yz}^{(1)}(x, y, h) = \sigma_{yz}^{(2)}(x, y, 0), \\ \sigma_{zz}^{(1)}(x, y, h) &= \sigma_{zz}^{(2)}(x, y, 0), \\ u_x^{(1)}(x, y, h) &= u_x^{(2)}(x, y, 0), u_y^{(1)}(x, y, h) = u_y^{(2)}(x, y, 0), \\ u_z^{(1)}(x, y, h) &= u_z^{(2)}(x, y, 0), \end{aligned} \quad \text{Equation 7-2}$$

For the substrate denoted with superscript 2, the stresses and displacements shall vanish at a considerably large distance from the contact surface expressed as:

$$\sigma^{(2)}(x, y, \infty) = 0, u^{(2)}(x, y, \infty) = 0 \quad \text{Equation 7-3}$$

The response functions for calculating the displacements derived from contact tractions can be determined in the frequency domain by employing the Papkovitch-Neuber potentials (results of the combination of Helmholtz representation and Navier equations) and double Fourier transform. For zero body force, the Papkovitch-Neuber potentials φ and $\psi(\psi_1, \psi_2, \psi_3)$ are harmonic functions of x, y, z in the spatial domain. The number of independent harmonic functions can be reduced to three by arbitrarily choosing one of $\psi(\psi_1, \psi_2, \psi_3)$ functions to be zero. As a common practice, here ψ_2 is taken to be zero.

The displacement and stress can be expressed in the following forms as functions of the Papkovitch-Neuber potentials:

$$\begin{aligned} 2G_k u_i^{(k)} &= \varphi_{,i}^{(k)} + x\psi_{1,i}^{(k)} + z_k\psi_{3,i}^{(k)} - (3 - 4\nu_k)\psi_i^{(k)}, \\ \sigma_{ij}^{(k)} &= \varphi_{,ij}^{(k)} - 2\nu_k(\psi_{1,1}^{(k)} + \psi_{3,3}^{(k)})\delta_{ij} - (1 - 2\nu_k)(\psi_{i,j}^{(k)} + \psi_{j,i}^{(k)}) + \\ &\quad x\psi_{1,ij}^{(k)} + z_k\psi_{3,ij}^{(k)}, \end{aligned} \quad \text{Equation 7-4}$$

where the index notations i and j have values of 1, 2 and 3 corresponding to x , y and z coordinates, respectively, the superscript k represents the layer ($k = 1$) or substrate ($k = 2$) being referred to, G_k denotes the shear modulus, ν_k denotes the Poisson's ratio and δ_{ij} denotes the Kronecker delta ($\delta_{ij} = 1$ if $i = j$, or $\delta_{ij} = 0$ if $i \neq j$).

By applying double Fourier transform, Equation 7-4 is transformed into the following form:

$$\begin{aligned}\tilde{u}_i^{(k)} &= \frac{1}{2G_k} FT_{xy}[\varphi_{,i}^{(k)} + x\psi_{1,i}^{(k)} + z\psi_{3,i}^{(k)} - (3 - 4\nu_k)\psi_i^{(k)}], \\ \tilde{\sigma}_{ij}^{(k)} &= FT_{xy}[\varphi_{,ij}^{(k)} - 2\nu_k(\psi_{1,1}^{(k)} + \psi_{3,3}^{(k)})\delta_{ij} - (1 - 2\nu_k)(\psi_{i,j}^{(k)} + \psi_{j,i}^{(k)}) + x\psi_{1,ij}^{(k)} + z\psi_{3,ij}^{(k)}],\end{aligned}\quad \text{Equation 7-5}$$

where the hat “ $\approx$ ” and symbol FT_{xy} indicate the double Fourier transform operation with respect to x and y directions.

In the frequency domain, the double Fourier transformed forms of Papkovitch-Neuber potentials are expressed as:

$$\begin{aligned}\tilde{\phi}^{(k)} &= A^{(k)}e^{-\alpha z_k} + \bar{A}^{(k)}e^{\alpha z_k}, \quad \tilde{\psi}_1^{(k)} = B^{(k)}e^{-\alpha z_k} + \bar{B}^{(k)}e^{\alpha z_k}, \\ \tilde{\psi}_3^{(k)} &= C^{(k)}e^{-\alpha z_k} + \bar{C}^{(k)}e^{\alpha z_k},\end{aligned}\quad \text{Equation 7-6}$$

Considering that the stresses and displacements at a large distance from the substrate surface can be assumed to be zero (expressed as Equation 7-3), $\bar{A}^{(2)} = \bar{B}^{(2)} = \bar{C}^{(2)} = 0$. Thus, there are nine unknown parameters in total. The parameter α is determined by $\alpha = \sqrt{m^2 + n^2}$, where m and n are the transformed frequency variables with respect to x and y in the spatial domain, respectively.

To substitute Equation 7-6 into Equation 7-5, considering that the stresses and surface displacements are subjected to the nine boundary and interfacial continuity conditions (expressed as Equation 7-1, Equation 7-2, and Equation 7-3), the nine parameters, including $A^{(1)}$, $\bar{A}^{(1)}$, $A^{(2)}$, $B^{(1)}$, $\bar{B}^{(1)}$, $B^{(2)}$, $C^{(1)}$, $\bar{C}^{(1)}$, $C^{(2)}$ can be determined. The closed-form expressions of the influence coefficients in the frequency domain, which are constituted by these parameters, are given in the Appendix 7A. For the detailed derivation describing the double Fourier transform of all the terms, it can be found in the work of Wang, et al. [49] and Wang and Zhu [50].

It is of note that the computation of the frequency response functions in the frequency domain requires a refined mesh (e.g., a small sampling interval) for reducing the effect of

the aliasing phenomenon. For a detailed description of the aliasing and Gibbs phenomena in the Fourier analysis, it can be found in the work of Morrison [51] and Liu and Wang [48]. In this study, a refinement of 2^4 times the original mesh was applied to determine the frequency response functions. To avoid the singularity problem of the frequency response functions in the origin point ($m = 0, n = 0$), the 64-point Gaussian quadrature integration method was applied to the region around the origin to calculate the response functions.

After determining the frequency response functions for the elastic layered contact problems, those for viscoelastic counterparts can be derived readily by replacing the elastic properties ($\frac{1}{2G}$) with the viscoelastic creep compliance $\phi(t)$. The influence coefficients can then be determined by applying the inverse Fourier transform to the frequency response functions. Once the influence coefficients are obtained, contact tractions and surface displacements in normal and tangential layered contact problems can be determined following the same procedures taken to solve half-space contact problems.

7.3.2 Algorithm Description

Once the influence coefficients are obtained, contact tractions and surface displacements in normal and tangential layered contact problems can be determined following an algorithm that is similar with that taken to solve half-space contact problems developed previously [52, 53], as illustrated in Figure 7-2. Computational techniques, including the discrete convolution Fast Fourier Transform (DC-FFT) and conjugate gradient method (CGM), were applied to improve the computational efficiency of the algorithm. Hence, instead of describing the details of the algorithms for the layered contact model, here the computational procedures to address the common problems that are encountered in layered contact modelling are presented.

To assume that the target computational domain is in the size of $L_1 \times L_2$ in the spatial dimension, the grid constituted with $N_1 \times N_2$ elements in the uniform size of $\Delta_x \times \Delta_y$ is established to discretize the computational domain. A domain extension factor χ , which is usually the power of 2 is employed to extend the established mesh system. By employing number of $\chi N_1 \times \chi N_2$ elements to discretize the extended computational domain, the initial sampling interval in the spatial dimension ($\Delta_x = \frac{L_1}{N_1}, \Delta_y = \frac{L_2}{N_2}$) is preserved. On the other hand, after adopting an extended computational domain with more mesh elements, the sampling interval in the frequency dimension is reduced, thus

alleviating the effects of the aliasing phenomenon since the domain size in the frequency dimension remains as $\frac{2\pi}{\Delta_x} \times \frac{2\pi}{\Delta_y}$. The comparison between the computational parameters in the spatial and frequency dimensions is given in Table 7-1.

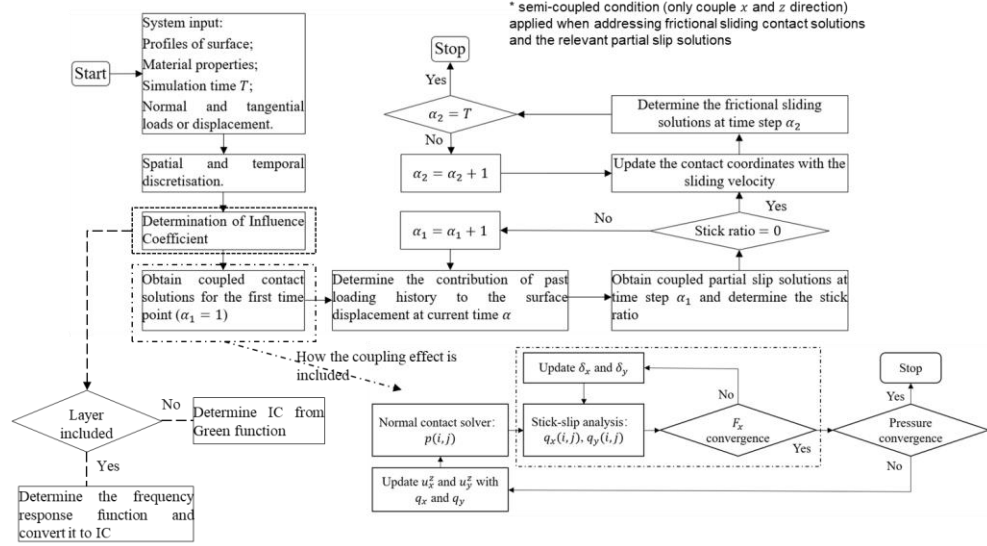


Figure 7-2 Overview of the algorithm for coupled viscoelastic contact problems (viscoelastic material can either exist as a half-space or layer)

Once the frequency response function for each surface node at each discretized time step $F(m, n, t)$ in the extended computational domain is known, instead of applying the inverse Fourier transform, the continuous Fourier-transformed influence coefficients $\tilde{\tilde{C}}$ needs to be determined first through the shape function as follows:

$$\tilde{\tilde{C}}(m, n, t) = F(m, n, t) \cdot \tilde{\tilde{Y}}(m, n), \quad \text{Equation 7-7}$$

where Y is the shape function and has the following forms in the spatial and frequency domains and t is the index for the discretized time steps ($t = 1 \dots N_{t-1}, N_t$, N_t is the total number of time points).

$$Y(x, y) = \begin{cases} 1, & |x| \leq \frac{\Delta_x}{2} \text{ and } |y| \leq \frac{\Delta_y}{2}, \\ 0, & \text{otherwise} \end{cases}$$

$$\tilde{\tilde{Y}}(m, n) = \frac{4 \sin\left(\frac{m\Delta_x}{2}\right) \sin\left(\frac{n\Delta_y}{2}\right)}{mn}. \quad \text{Equation 7-8}$$

Afterwards, the discrete transformed form of influence coefficient in the Fourier domain $\hat{\tilde{C}}$ needs to be determined from the continuous Fourier-transformed influence coefficients $\tilde{\tilde{C}}$ in the following way:

$$\hat{C}(m, n, t) = \frac{1}{\Delta_x \Delta_y} \sum_{rx=-AL}^{rx=AL} \sum_{ry=-AL}^{ry=AL} \tilde{C} \left(\frac{2\pi}{\chi N_1 \Delta_x} i - \frac{2\pi}{\Delta_x} r_x, \frac{2\pi}{\chi N_2 \Delta_y} i - \frac{2\pi}{\Delta_y} r_y, t \right), \quad \text{Equation 7-9}$$

where the hat “ $\hat{}$ ” represents double discrete Fourier transform operation with respect to x and y directions. AL is the level of aliasing control, and i and j here are the indices for the coordinates of surface nodes in the spatial dimension $(-\frac{\chi N_1}{2} < i \leq \frac{\chi N_1}{2})$, $(-\frac{\chi N_2}{2} < j \leq \frac{\chi N_2}{2})$.

Table 7-1 Computational parameters in spatial and frequency dimensions

Parameters	Spatial dimension	Frequency dimension
Length of domain before extension	$L_1 \times L_2$	$\frac{2\pi}{\Delta_x} \times \frac{2\pi}{\Delta_y}$
Length of domain after extension	$\chi L_1 \times \chi L_2$	$\frac{2\pi}{\Delta_x} \times \frac{2\pi}{\Delta_y}$
Number of elements before extension	$N_1 \times N_2$	$N_1 \times N_2$
Number of elements after extension	$\chi N_1 \times \chi N_2$	$\chi N_1 \times \chi N_2$
Sampling interval before extension	$\Delta_x = \frac{L_1}{N_1}, \Delta_y = \frac{L_2}{N_2}$	$\frac{2\pi}{\Delta_x} / N_1, \frac{2\pi}{\Delta_y} / N_2$
Sampling interval after extension	$\Delta_x = \frac{L_1}{N_1}, \Delta_y = \frac{L_2}{N_2}$	$\frac{2\pi}{\Delta_x} / (\chi \times N_1), \frac{2\pi}{\Delta_y} / (\chi \times N_2)$

After applying the inverse fast Fourier transform (*IFFT*) to the discretized series in the Fourier domain $\hat{C}$, attention needs to be taken when applying the DC-FFT method in terms of the wrap-around and anti-wrap-around order [48]. The anti-wrap-around order is first applied such that the terms corresponding to the negative frequencies are rearranged after the ones corresponding to positive frequencies. After extracting the real parts of the new series of discrete samples C_{temp} , a matrix IC_{temp} in the size of $2N_1 \times 2N_2$ can be obtained in the following way:

$$IC_{temp}(i, j) = Re \left(C_{temp} \left(i + \frac{\chi N_1}{2} - N_1, j + \frac{\chi N_2}{2} - N_2 \right) \right), \quad \text{Equation 7-10}$$

where $i = 1 \dots 2 \times N_1$ and $j = 1 \dots 2 \times N_2$.

To apply the wrap-around order to the matrix IC_{temp} , the conversion from the frequency response function to the influence coefficient is eventually completed. Through converting the discrete linear convolution into discrete cyclic convolution, this

transformed influence coefficient matrix is ready to be used for the determination of surface deformation in the DC-FFT algorithm [48] with the pressure matrix after the operation of zero-padding.

7.4 Model Validation

Considering that the algorithm developed for the viscoelastic layered contact problems is basically identical to the one that was developed for viscoelastic half-space contact problems, only the layer aspect of the developed model (i.e., the conversion process from frequency response function to influence coefficients) was validated in this section. For the detailed validation in terms of the viscoelastic, coupled partial slip and sliding aspect, readers can refer to the work by Wang, et al. [53] [54].

As shown in Figure 7-3, the first validation was done by comparing the solutions of the degenerated form of the developed model (i.e., elastic layered contact model) (solid line) with the analytical solutions by O’Sullivan and King [55] (scatter) to elastic layered indentation problems. Here the indentation contact problem between a rigid sphere against an elastic layer bonded to an elastic substrate was simulated. The contact inputs are given in Table 7-2.

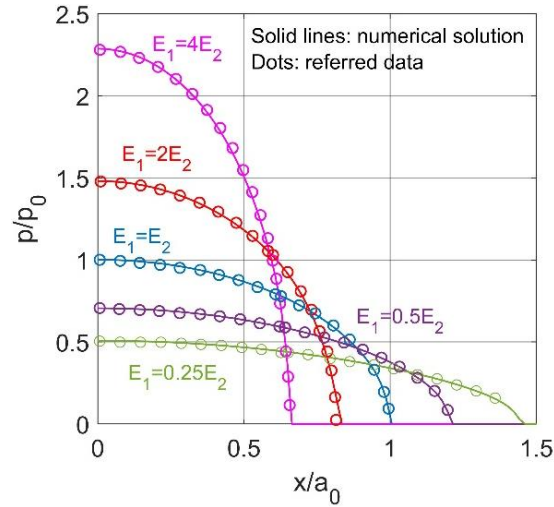


Figure 7-3 Comparison of nondimensionalized results for elastic layered indentation derived from our model (solid line) and analytical solutions by O’Sullivan and King (scatter). E_1 denotes the elastic modulus of layer, and E_2 denotes the elastic modulus of substrate. [55]

By varying the ratio of the layer modulus E_1 to substrate modulus E_2 as presented in Table 7-2, different pressure distributions are observed for elastic layered indentation problems. As shown in Figure 7-3, the peak pressure p increases with the modulus ratio while the contacting radius a_0 responds oppositely. Good agreement can be found between the solution derived from our model (solid lines) and analytical solutions (dots) proposed by O’Sullivan and King [55].

Table 7-2 Parameters used in validation of elastic layered indentation contact

Parameter	Value	Description (Unit)
R	18	Radius of sphere (mm)
E_1	52.5,105,210,420,840	Elastic modulus of layer (GPa)
E_2	210	Elastic modulus of substrate (GPa)
E_3	∞	Elastic modulus of indenter (GPa)
$\nu_1/\nu_2/\nu_3$	0.3	Poisson's ratio of layer/substrate/indenter
W	20	Input normal load (N)
a_0	105.373	Hertzian contacting radius (μm)
h/a_0	1	Nondimensionalized layer thickness
p_0	860	Hertzian peak normal pressure (MPa)

To validate the successful application of the elastic-viscoelastic correspondence principle to viscoelastic layered problems, another test was conducted. This test simulated the indentation of a rigid sphere against a viscoelastic layered bonded to an elastic substrate. Two extreme cases were considered by specifying significantly larger and small dimensionless layer thicknesses (h/a_0), respectively. Here, the parameter a_0 represents the Hertzian contacting radius for the indentation problem of the rigid sphere against the elastic substrate without the viscoelastic layer. The viscoelastic material was modelled using a Maxwell model, and the relevant contact inputs are given in Table 7-3.

As shown in Figure 7-4 (a), the contact solutions of the half-space contact (viscoelastic material as the half-space) and layered contact fit together when the viscoelastic layer is extremely thick ($\frac{h}{a_0} = 1 \times 10^4$). To highlight the time dependency of materials, the parameters used to nondimensionalize the solutions shown in Figure 7-4 (a), including a_{v0} and p_{v0} , are the contacting radius and the peak normal pressure respectively when the viscoelastic contact initialises ($t = 0$). On the other hand, there tends to be no time dependency for the contact solutions when the viscoelastic layer is considerably thin ($\frac{h}{a_0} = 1 \times 10^{-5}$) as illustrated in Figure 7-4 (b). Besides, the solution is extremely closer to the Hertzian half-space solution (elastic material as the half-space). The layered contact solutions are nondimensionalized by the Hertzian solutions (a_0 and p_0) to highlight the

effects of the layer. Unless otherwise specified, the half-space and layered contact solutions presented hereinafter are nondimensionalized in the same way, respectively.

Table 7-3 Parameters used in validation of viscoelastic layered indentation contact

Parameter	Value	Description (Unit)
R	3.5	Radius of sphere (mm)
G	80.77	Shear modulus of spring in Maxwell model (GPa)
η	80.77	Viscosity of dashpot in Maxwell model (GPa · s)
E_2	210	Elastic modulus of substrate (GPa)
E_3	∞	Elastic modulus of indenter (GPa)
$\nu_1/\nu_2/\nu_3$	0.3	Poisson's ratio of the layer/substrate/indenter
W	100	Input normal load (N)
a_0	104.388	Hertzian contacting radius (μm)
h/a_0	$1 \times 10^4, 1 \times 10^{-5}$	Nondimensionalized thickness of viscoelastic layer
a_{v0}	131.5206	Initial contacting radius for viscoelastic half-space contact (μm)
p_0	4.3817	Hertzian peak pressure (GPa)
p_{v0}	2.7603	Initial peak pressure for viscoelastic contact (GPa)

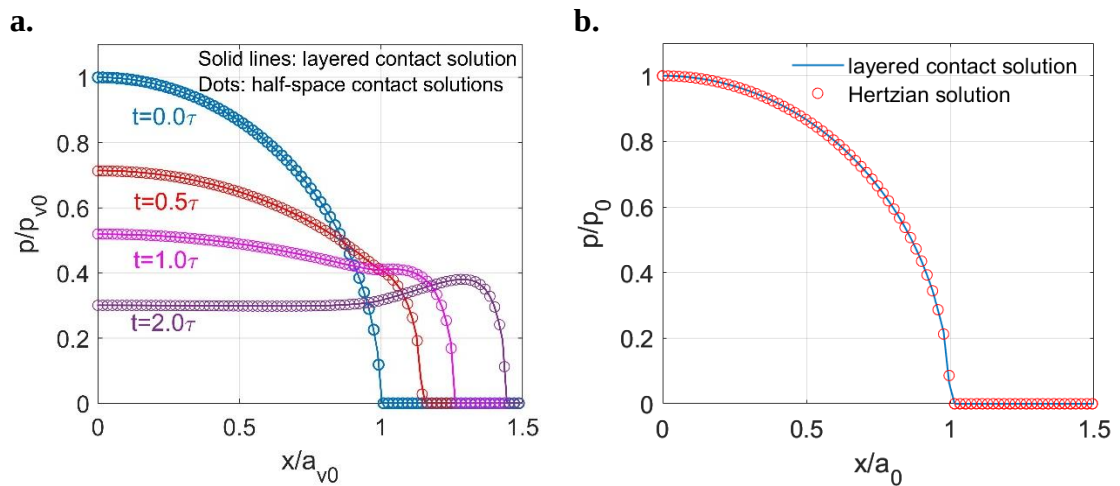


Figure 7-4 Comparison between the nondimensionalized solutions of viscoelastic layered indentation problem with different layer thicknesses (solid line) and half-space contact problems (scatter): (a) $\frac{h}{a_0} = 1 \times 10^4$ and (b) $\frac{h}{a_0} = 1 \times 10^{-5}$

The good agreement between the simulation results derived from our model and analytical solutions validates as best possible the viscoelastic layered aspect of the developed model.

7.5 Contact Analysis on the Effects of a Viscoelastic ZDDP-derived Tribofilm Layer

The validated model can now be extended to study the response of a ZDDP-derived tribofilm layer. To characterize the linear viscoelastic behaviour of the ZDDP-derived tribofilm, as proposed by Dorgham, et al. [21], a Burgers material model, which has a structure illustrated in Figure 7-5 and can be mathematically expressed by the four-term Equation 7-11, was employed.

To describe the experimental work by Dorgham, et al. [21] briefly, it utilized an in-situ atomic force microscopy (AFM) set-up in a high-temperature liquid cell to generated ZDDP-derived tribofilm. The films were formed by rubbing the AFM tip (Diameter $D \approx 150 \text{ nm}$) against a steel substrate in a liquid medium of poly- α -olefin (PAO) base oil containing ZDDP additive under controlled high-temperature and high-pressure conditions. The nanoscale viscosity of the formed tribofilms was then quantified using a creep method, where a constant stress was applied via the AFM tip. The creep compliance was calculated based on the ratio of the strain (normalized change in tribofilm area) and stress (shear stress).

According to Dorgham, et al. [21], it can be observed that after a relatively long time, the viscosity of tribofilm tends to play a dominant role in its creep behaviour as the creep compliance keeps increasing with time following a linear trend. This implies that the ZDDP-derived tribofilm is extremely fluid-like such that its creep compliance can hardly reach a steady state. According to the findings reported in our previous studies regarding the effects of the rheological behaviour of viscoelastic materials [53], pressure spikes on contacting edges shall be expected in the following tests, especially when the tribofilm exists as a half-space.

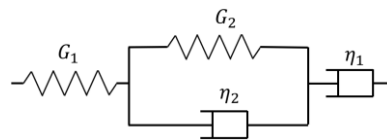


Figure 7-5 Structure of the Burgers material model used to characterise the creep compliance of the ZDDP-derived tribofilm: G is the modulus of the linear spring and η is the viscosity of the dashpot

$$\Phi(t) = \frac{1}{G_1} + \frac{1}{G_2} (1 - \exp(-\frac{G_2}{\eta_2} t)) + \frac{1}{\eta_1} t, \quad \text{Equation 7-11}$$

By applying curve fitting on several selected data points extracted from Figure 7-6 (a) of the work of Dorgham, et al [21], and incorporated additional data points proximal to the linear-fit trend lines originally generated by Dorgham to refine the curve fitting effects, a trend line, which is mathematically expressed as Equation 7-12, is obtained as depicted in Figure 7-6 (b). A close correlation can be found between the trend line and the referred data ($R^2 = 0.9808$) as observed in Figure 7-6 (b).

$$\phi(t) = \frac{1}{1.5} + \frac{1}{0.02778} [1 - \exp(-\frac{0.02778}{241.8} \times t)] + \frac{1}{6297.2} t, \left(\frac{1}{\text{GPa}}\right) \quad \text{Equation 7-12}$$

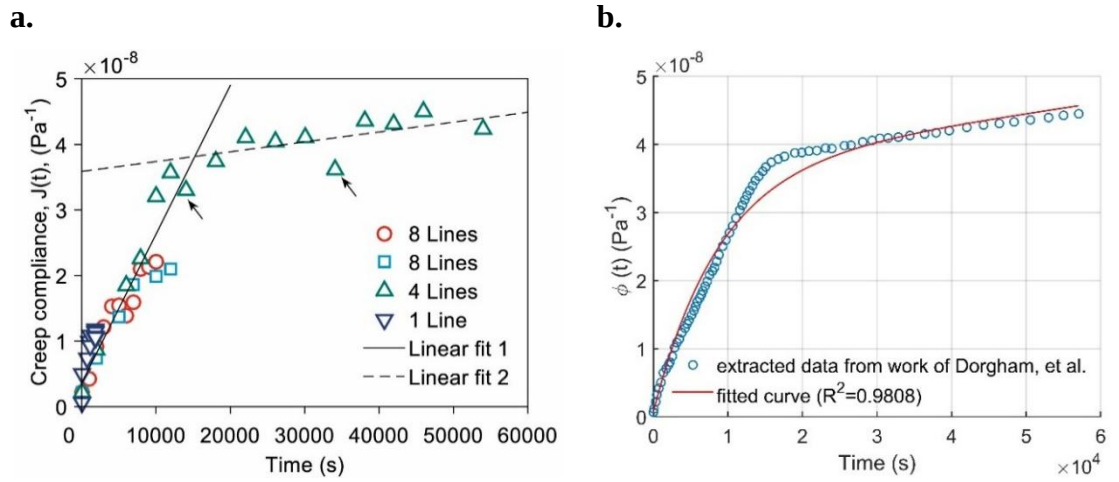


Figure 7-6 Creep compliance of the ZDDP-derived tribofilm reported by Dorgham, et al. [21]. Reproduced with permission from Ref. [21], © Elsevier B. V., 2024. and (b) Four-term trend line generated through curve fitting ($R^2 = 0.9808$)

To study the effects of ZDDP-derived tribofilms with the aforementioned viscoelastic property, it is crucial to specify the typical contact conditions encountered in realistic applications. While the developed model can handle rough frictional contact problems (e.g., partial slip or sliding contact), our previous study [52] demonstrated that for steady sliding contact, the role of dry contact friction can be neglected due to its minimal impact on the steady solutions. Additionally, including dry contact friction significantly increases the computational cost due to the coupling between shear tractions and pressures during partial slip analysis. For instance, on a desktop computer with 6 cores, solving an uncoupled partial slip contact problem of elastic rough surfaces with a resolution of 512×512 nodes takes approximately 140 seconds [56], while incorporating coupling effects increases the computational time to 300 seconds [54]. This extra computational cost will be significantly higher for viscoelastic contact problems.

In real-world applications, such as gears and bearings, tribofilms perform under sustained sliding conditions. These films form dynamically at contact interfaces, with their formation and removal rates balancing over time to eventually maintain a steady layer thickness. This mechanism ensures a continuous protective surface layer that mitigates direct contact between bodies in the long run [2, 20, 57]. By assuming frictionless contact, the balance between the computational efficiency and ability to capture the critical viscoelastic effects of the tribofilm under steady sliding conditions is achieved. To closely resemble the realistic scenarios, the applied load, geometry and materials of the two contacting bodies were selected based on the tribotest conducted by Ghanbarzadeh, et al. [58].

Regarding other properties of the tribofilm, considering that it is usually very thin as it works in the boundary lubrication regime, the film thickness should be in the order of nanometers. Here the tribofilm thickness is assumed to be time-independent ($h = 150 \text{ nm}$), which is a typical value according to the experimental study of Ghanbarzadeh, et al. [58] and Dorgham, et al. [21]. However, as mentioned before, the formation and removal of the tribofilm occur simultaneously in practice, the thickness of the tribofilm should vary with time before its generation process reaches a relatively steady state. The variation of the layer thickness as well as its material property may have synergistic effects on the contact solutions. To our best knowledge, the transient contact of a tribofilm with a time-dependent layer thickness cannot yet be simulated with the developed viscoelastic layered model alone as the time-dependent influence coefficient cannot describe the surface displacement induced by the unit pressure while the material property and layer thickness vary simultaneously. Thus, the following studies focus on the effects of the tribofilm when its thickness has already reached a steady state while the transient effects of tribofilm formation and removal are left for future work.

The tribofilm is assumed to have a constant Poisson's ratio ($\nu = 0.3$), which is within a reasonable range according to the study of Matori, et al. [59]. Given that the tribofilm itself has no steady state in terms of its creep compliance as mentioned above, the simulated contact time starts with 120 minutes in the following simulations for the sake of computational efficiency. Such a test duration is commonly adopted in experimental work about ZDDP-derived tribofilms (e.g., the study of Ghanbarzadeh, et al. [58, 60]). All these contact parameters are given in Table 7-4. To note, the simulated contact time was adjusted according to the time when the solution becomes steady for the problems under different operating conditions.

Table 7-4 Parameters used in the simulation of ZDDP-derived tribofilm

Parameters	Value	Description (Unit)
D	19.05	Diameter of sphere (mm)
E_2	210	Elastic modulus of substrate (GPa)
E_3	210	Elastic modulus of indenter (GPa)
$\nu_1/\nu_2/\nu_3$	0.3	Poisson's ratio of the layer/substrate/indenter
W	60	Input normal load (N)
a_0	155	Hertzian contacting radius (μm)
p_0	1.19	Hertzian peak pressure (GPa)
T	120	Simulation time (min)
h	150	Thickness of tribofilm (nm)

7.5.1 Indentation Contact

Before the sliding simulation, the role played by the tribofilm under indentation conditions was first investigated to evaluate its time-dependent viscoelastic behaviour under quasi-static conditions. As a reference, the first test was conducted by assuming that the tribofilm has an infinite thickness such that it behaves as a half-space. This half-space case highlights the intrinsic viscoelastic behaviour of the tribofilm without the interference from layer thickness effects or substrate properties, which helps the following analysis of simulation outcomes. For the contact between a steel sphere-shape indenter against the flat viscoelastic half-space under a normal load, the variation of the pressure distribution with time is shown in Figure 7-7 (a). As expected, when existing as a half-space body, the tribofilm behaves as an extremely fluid-like material. This is indicated by the phenomenon that the load keeps being distributed to both edges of the increasing contacting area, leading to sharp pressure spikes [53].

To switch to the layered indentation problem (the contact between a steel sphere indenter against the tribofilm bonded to a flat steel substrate) under the same normal load, as illustrated in Figure 7-7 (b), the compliant viscoelastic layer results in the decrease of the contact pressure and increase of contacting area. However, the pressure spikes cannot be observed in this case, which suggests that the combination of the tribofilm and carbon steel substrate now performs more solid-like compared with the tribofilm half-space.

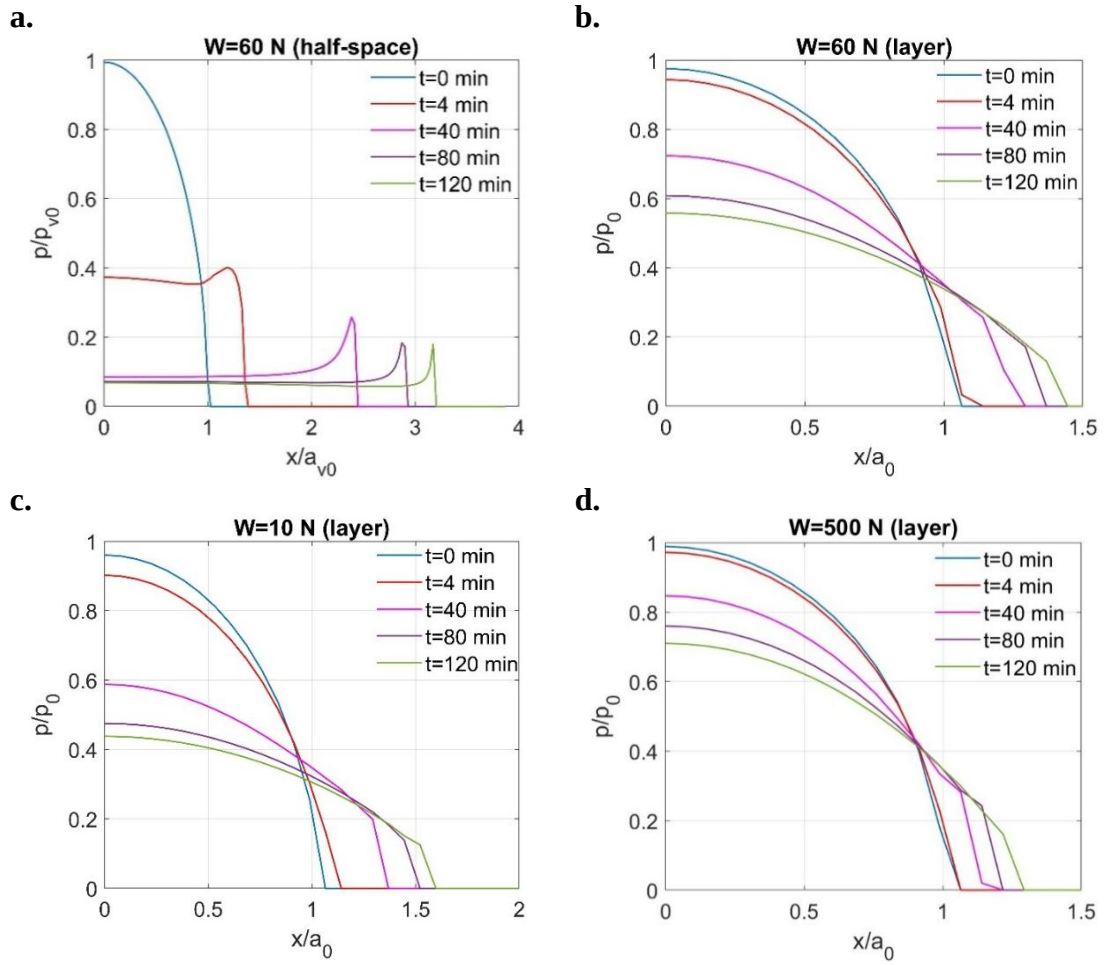


Figure 7-7 Pressure distributions for ZDDP tribofilm indentation problem under fixed normal loads: (a) half-space ($W = 60$ N), (b) layered ($W = 1$ N), (c) layered ($W = 10$ N), and (d) layered ($W = 500$ N),

Such rheological behaviour can be modified by varying the input load when the layer thickness remains with time. As illustrated in Figure 7-7 (c) and (d), more fluid-like contact behaviour (e.g., significantly reduced pressure profiles) is achieved for the current viscoelastic layered contact problem when the normal load decreases from 500 N to 10 N. This contact behaviour is related to the ratio of the layer thickness to the Hertzian contacting radius (h/a_0) given in Table 7-5. The contact response of the combined substance shall keep approaching that of a viscoelastic half-space when the dimensionless layer thickness increases.

Apart from the creep phenomenon under a constant normal load, the stress relaxation phenomenon under a constant displacement for the ZDDP-derived tribofilm is investigated here. The rigid body displacements (δ_z) of the first time point determined in the former four creep tests are used as the input for the following relaxation tests, correspondingly.

The half-space contact of the ZDDP-derived tribofilm under a fixed normal displacement ($\delta_z = 36.1380 \mu\text{m}$) is first simulated as a reference. As shown in Figure 7-8 (a), a typical stress relaxation phenomenon, including the constant contacting area and significant decrease of pressure with time, is observed when the tribofilm performs as a half-space.

Table 7-5 Variation of the dimensionless layer thickness h/a_0 with the constant contact input in layered indentation problem

Input in layered indentation problem	h/a_0	p_0 (GPa)
$W = 10 \text{ N}$	0.0018	0.657
$W = 60 \text{ N}$	9.6853×10^{-4}	1.2
$W = 500 \text{ N}$	4.7772×10^{-4}	2.42
$\delta_z = 0.7796 \mu\text{m}$	0.0096	0.665
$\delta_z = 2.5468 \mu\text{m}$	9.6308×10^{-4}	1.2
$\delta_z = 10.4002 \mu\text{m}$	0.2313	2.43

Notable results are observed as illustrated in Figure 7-8 (b) when the tribofilm is modelled as a finite-thickness layer under a specific displacement ($\delta_z = 2.5469 \mu\text{m}$). The contacting area increases over time although it tends to be steady eventually. By increasing the surface displacement, thereby reducing the dimensionless layer thickness as given in Table 7-5, the stress relaxation effect becomes less pronounced as depicted in Figure 7-8 (c) and (d), where the displacement increases from $0.7796 \mu\text{m}$ to $10.4002 \mu\text{m}$, respectively. In all layered cases, an increase in the contacting areas was observed. These findings suggest that the combination of the tribofilm and substrate can demonstrate unique contact behaviour under a constant displacement, differing from the typical responses of purely viscoelastic or elastic solids.

To investigate why the contact area increases over time under a constant surface displacement in the layered tribofilm contact, the surface deformations for the half-space case and layered case are plotted in Figure 7-9. These cases are simplified to the indentation of a rigid sphere against a flat half-space with equivalent material properties derived from all the contacting bodies. For the layered case, the contact becomes increasingly conformal over time, even though the surface displacement remains constant as illustrated in Figure 7-9 (a). In contrast, the contact geometry for the half-space case remains unchanged over time, as illustrated in Figure 7-9 (b). This difference can be

attributed to the confinement by the substrate. It limits vertical deformation of the viscoelastic tribofilm, causing the tribofilm to flow laterally over time. This lateral flow leads to an increase in contacting area.

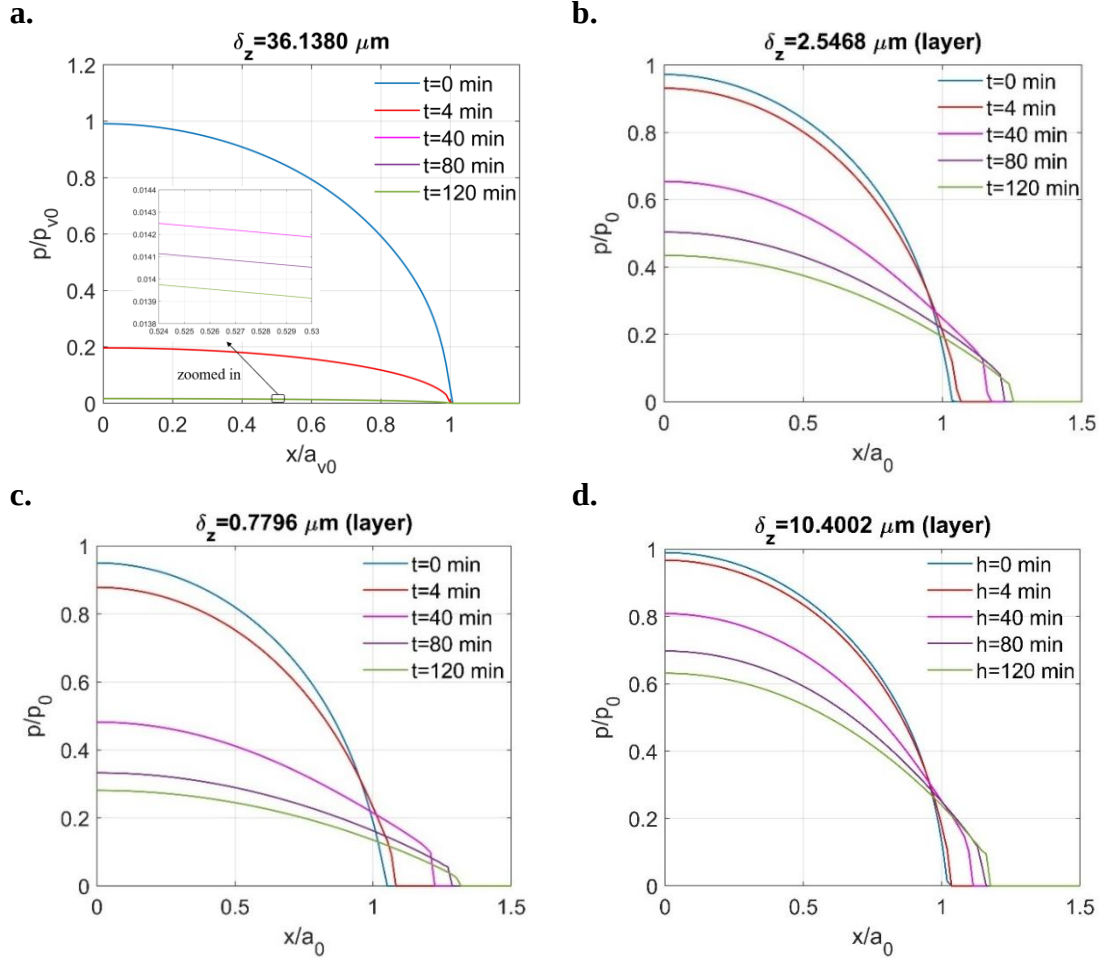


Figure 7-8 Solutions to the indentation problem of the ZDDP tribofilm in different forms under a fixed normal displacement: (a) half-space contact ($\delta_z = 36.1380 \mu\text{m}$), (b) layered contact ($\delta_z = 2.5468 \mu\text{m}$), (c) layered contact ($\delta_z = 0.7796 \mu\text{m}$), and (d) layered contact ($\delta_z = 10.4002 \mu\text{m}$)

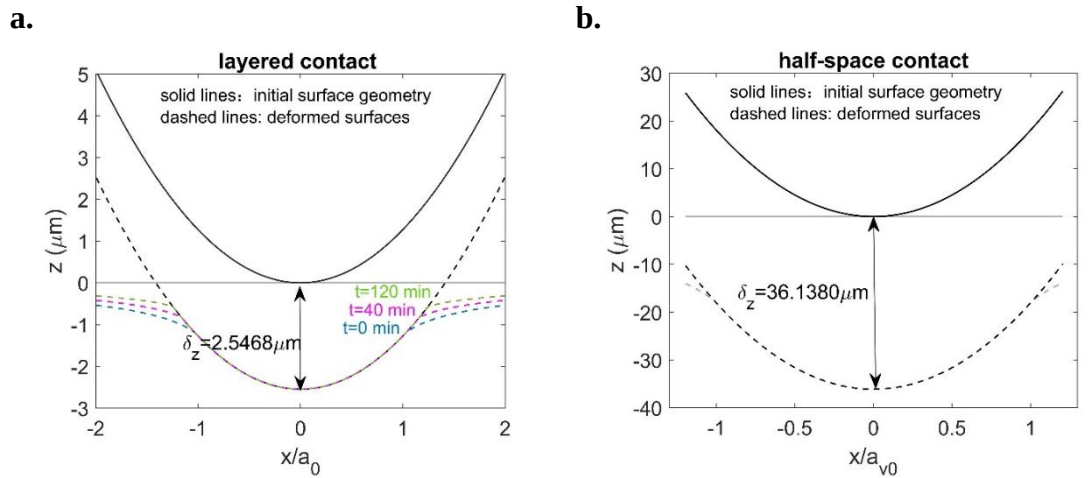


Figure 7-9 Surface deformation of the contacting bodies under different conditions (a) layered contact ($\delta_z = 2.5468 \mu\text{m}$), and (b) half-space contact ($\delta_z = 36.1380 \mu\text{m}$)

To validate this idea, two extreme cases under constant surface displacement ($\delta_{z1} = 50 \mu\text{m}$ and $\delta_{z2} = 0.01 \text{ nm}$) were tested. As shown in Figure 7-10 (a), when the displacement is extremely large ($\delta_{z1} = 50 \mu\text{m}$), the tribofilm initially has minimal effect on contact solutions, with the initial pressure closely matching the Hertzian solution. However, the contacting area starts to exhibit time dependency as time progresses, indicating the lateral flow of tribofilm. On the other hand, when the surface displacement is extremely small ($\delta_{z2} = 0.01 \text{ nm}$) even compared with the ultra-thin tribofilm, the contacting area tends to remain constant while the pressure relaxes and reaches the steady state within a short time, as illustrated in Figure 7-10 (b). This behaviour resembles a half-space contact, as the tribofilm tends to behave as if unbounded by the substrate under such a low displacement, which is in consistency with our prior understanding.

It is important to note that this phenomenon is influenced by several factors, including the rheological properties of the material, layer thickness, specified surface displacement, and substrate stiffness. When the layered material behaves more solid like (high resistance to flow), the layer thickness is much greater than the specified surface displacement (contact response similar to a bulk material), and the substrate exhibits mechanical properties similar to the layer (insignificant confinement effect), the increase in the contacting area becomes negligible. For instance, Wallace et al. [42] conducted a test simulating the indentation of a rigid sphere against a thick polymethyl methacrylate (PMMA) coating bonded to the carbon steel substate under a comparatively low surface displacement, where no increase in contacting area was observed.

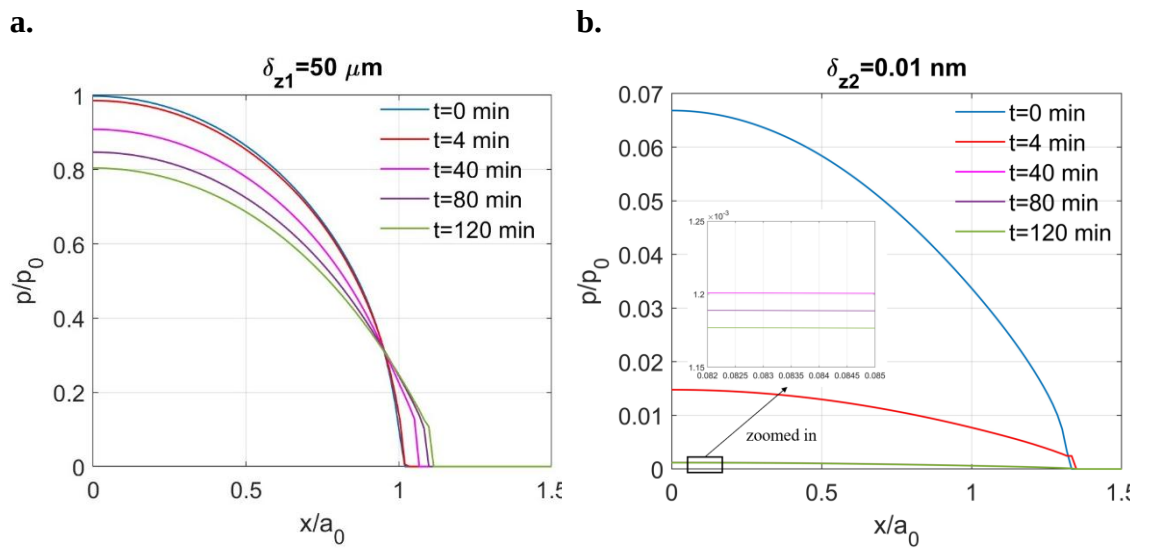


Figure 7-10 Pressure distribution under extreme displacement: (a) $\delta_{z1} = 50 \mu\text{m}$ and (b) $\delta_{z2} = 0.01 \text{ nm}$

7.5.2 Sliding Contact

Based on the previously developed algorithm for the sliding contact of viscoelastic half-space bodies [52], the sliding motion is achieved by keeping the indenter (sphere) static while moving the counter body (tribofilm and substrate) in the direction opposite to the sliding motion. It is noted that the sliding speed correlates with the pixel width and time interval of the established system such that for each time step, the counter body is moved in the distance of certain amount of pixel width. This leads to one limitation of the developed model such that when a high sliding speed is specified, an extremely large simulation domain needs to be generated, within which the actual contacting area may take up a small amount and this can lead to serious discretization error. On the other hand, an extremely small simulation domain needs to be specified for a considerably low-speed sliding contact, which struggles to accommodate the real contacting area. The issue of the discretization of the spatial domain can be avoided by adjusting the way the temporal domain is discretized. However, this action may undermine computational efficiency from another perspective when a great number of time steps are used. A compromise needs to be made to ensure a balance between computational accuracy and efficiency.

To investigate the effects of tribofilm in sliding contact, the simulation starts with the case when the tribofilm exists as a half-space while the carbon steel sphere slides against it for reference. The sliding speed is assumed to be constant during the contact. To facilitate the sliding simulation, the speed is set to be $0.4028 \mu\text{m/s}$ (extremely small speed in practice) as it allows a quick computation while providing a typical fluid-like contact response in the long run. As shown in Figure 7-11 (a), under a constant normal load ($W = 60 \text{ N}$), the expanded contacting area keeps shifting in the sliding direction with time. As a result, the pressure profile is skewed significantly such that pressure spikes are observed on the leading edge of the contacting area for the plotted time points. On the other hand, when a constant surface displacement ($\delta_z = 36.138 \mu\text{m}$) is specified in the normal direction, as illustrated in Figure 7-11 (b), apart from the phenomenon that the pressure relaxes with time, the rear part of the contacting area is found to keep decreasing with time. A sharp pressure spike can also be observed on the leading edge of the contacting area in this case.

When the tribofilm performs as a layer with an ultra-low thickness ($h = 150 \text{ nm}$), the combination of the tribofilm and substrate may still exhibit certain viscoelastic responses. In the following simulations, different load-controlled-based tests were conducted to

investigate the effects of tribofilm under different operating conditions, including loads and sliding speeds.

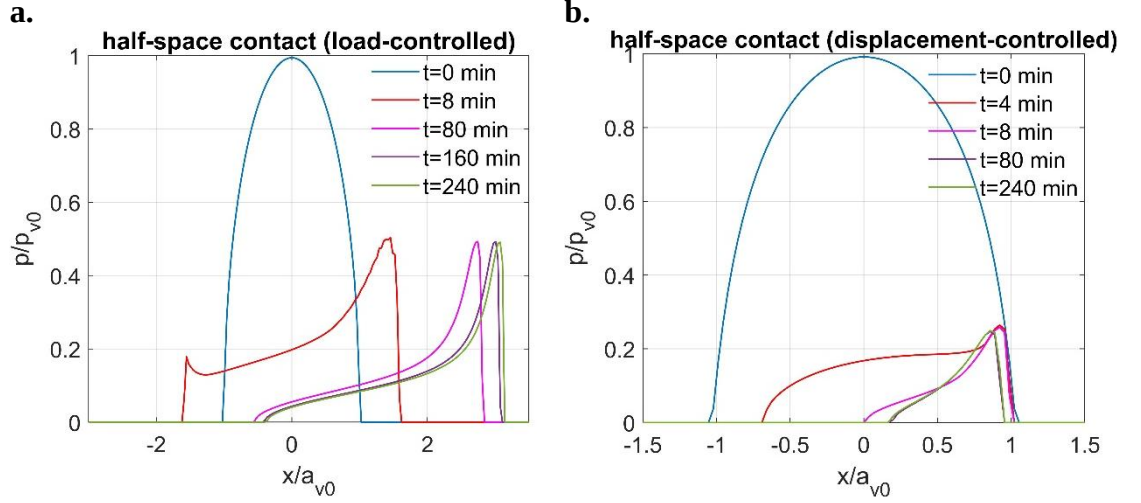


Figure 7-11 Contact solutions to frictionless sliding problems of ZDDP-derived tribofilm in the form of half-space under a fixed sliding speed ($v = 0.4028 \mu\text{m/s}$) with different contact inputs: (a) $W = 60 \text{ N}$ and (b) $\delta_z = 36.1380 \mu\text{m}$

Considering that the problem being simulated is on the length scale of micrometers (a_0 ranges from $40 \mu\text{m}$ to $160 \mu\text{m}$), an extremely large simulation domain along with a substantial number of time steps are needed if a speed on the scale of millimeters per second needs to be specified, making the simulation computationally intensive. For example, obtaining the simulation result for the high-speed tests ($v = 1.611 \text{ mm/s}$) under low or high loading conditions, as shown in Figure 7-13 (e) and (f) respectively in Appendix 7B, requires approximately 9 hours of computation on a desktop computer with 6 cores. Instead of providing a quantitative analysis on the behaviour of ZDDP tribofilm under an exact operating environment, the study aims to obtain a qualitative trend regarding the relationship between the sliding speed and viscoelastic response of tribofilm. To achieve decent computational efficiency and accuracy, the high speeds that are specified in the following simulations are still relatively low. As some of the tested cases exhibit similar features, limited results are shown and discussed below with more test results shown in Appendix 7B.

At an extremely low sliding speed ($v = 0.0336 \mu\text{m/s}$), the contact solutions exhibit pronounced viscoelastic characteristics under both loading conditions, such as pressure relaxation and shifting contacting areas shown in Figure 7-12 (a) and (b). However, these effects are less significant compared to the half-space case, where distinct pressure spikes are observed at a higher sliding speed. Notably, when the sliding speed in the layered test is increased to the same value specified in the half-space test ($v = 0.4028 \mu\text{m/s}$), the

viscoelastic effect of the tribofilm would be less pronounced, as illustrated in Figure 7-13 (a) and (b) in Appendix 7B.

When the sliding speed increases to $v = 0.8055 \mu\text{m/s}$, the viscoelastic responses of the two loading cases become much less significant, where only micro changes of pressure distributions can be observed as shown in Figure 7-12 (c) and (d) while the contacting area no longer shifts significantly with time.

After further increasing the sliding speed to $v = 6.441 \mu\text{m/s}$, time-dependent changes of the contact solutions can hardly be observed for both loading conditions, where micro changes of pressure distributions with time can only be observed in the zoomed-in views shown in Figure 7-12 (e) and (f). It is of note that it always takes longer time for the high-loading case ($W = 60 \text{ N}$) to reach a steady state for all three tests conducted at different sliding speeds.

The simulation outcome highlights the dual role of the tribofilm. On one hand, in ultra-low-speed sliding or indentation cases, the viscoelasticity of tribofilms has pronounced effects on contact solutions. It gradually reduces the contact pressure over time for metal-to-metal contact. The reduced pressure can lead to the decline of subsurface stress and subsequently contribute to mitigation of surface wear. Additionally, the damping mechanism provided by the viscosity of the tribofilm can absorb a portion of the energy that would otherwise contribute to material wear. Dorgham, et al. [20, 21] conducted nanoscale experiments with sliding speed varying on the scale of micrometer per second, effectively capturing and validating the role of viscoelasticity in enhancing wear resistance.

On the other hand, these potential benefits are speed-dependent (scale-dependent) and may diminish when operating conditions change. As mentioned before, the sliding speed considered in this study is relatively low, especially when compared to the typical operating conditions where ZDDP additives are used. For example, in cylinder liner piston ring assemblies, the average sliding speed is reported to range from 0.08 m/s to 0.32 m/s [61], and similar speeds ranging from millimeters to meters per second are employed in tribofilm formation experiments [62, 63]. At these high sliding speeds, the viscoelastic response of the layered contact becomes negligible. Tribofilms primarily behave as soft elastic layers, with pressure reduction occurring primarily during the initial contact and stabilizing shortly thereafter. Consequently, the potential impact of viscoelasticity on wear reduction shall be less significant.

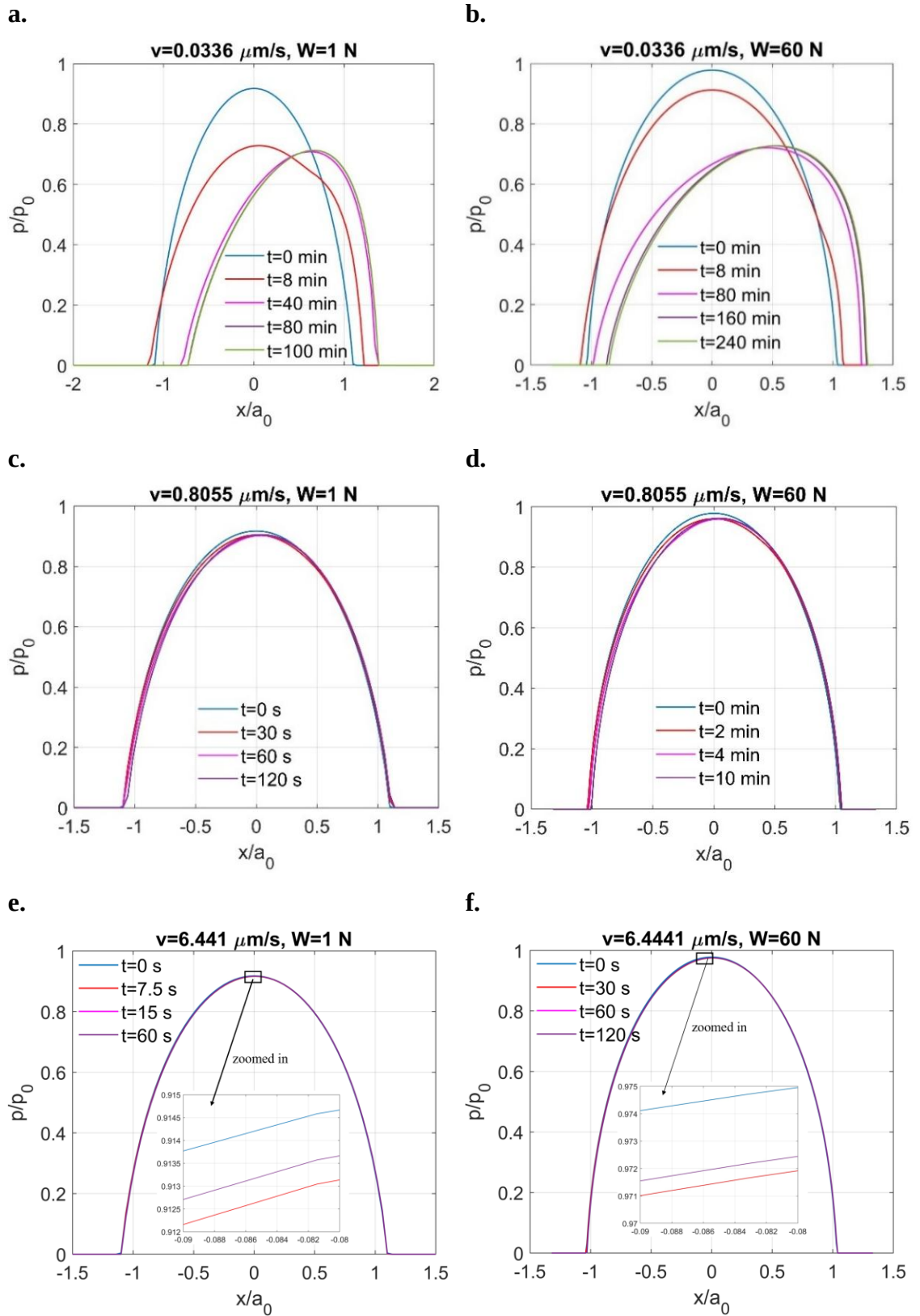


Figure 7-12 Contact solutions for frictionless sliding of ZDDP-derived tribofilm layer under varying loading conditions and sliding speeds: (a) $v = 0.0336 \mu\text{m/s}$, $W = 1 \text{ N}$ (b) $v = 0.0336 \mu\text{m/s}$, $W = 60 \text{ N}$ (c) $v = 0.8055 \mu\text{m/s}$, $W = 1 \text{ N}$, (d) $v = 0.8055 \mu\text{m/s}$, $W = 60 \text{ N}$, (e) $v = 6.441 \mu\text{m/s}$, $W = 1 \text{ N}$ and (f) $v = 6.441 \mu\text{m/s}$, $W = 60 \text{ N}$

7.6 Conclusion

By converting frequency response functions to influence coefficients, the model for viscoelastic layered contact is developed on the basis of the half-space contact model. The model is then extended to investigate the viscoelastic behaviour of a ZDDP-derived tribofilm, which is characterized by a Burgers material model.

The contact response of tribofilm is found to be extremely fluid-like when it exists as a half-space body. When the tribofilm is bonded to an elastic solid, for example a carbon steel substrate being considered in this study, viscoelastic contact responses, including creep, stress relaxation can be observed, while the extent of which is not as significant as that in the half-space case. In the layered indentation case, the contact exhibits more remarkable viscoelastic response (creep or stress relaxation) under low-loading conditions. Additionally, it is found that when a constant surface displacement, which is not significantly lower than the layer thickness, is applied, the contacting area increases gradually with time due to the flow of the tribofilm being bounded to the elastic substrate. When it comes to layered sliding contact problems, significant time-dependent solutions are only observed under extremely low sliding speeds ($v < 1 \mu\text{m/s}$), where the high loading cases exhibit more significant viscoelastic effects. On the other hand, trivial changes of pressure are observed under moderate sliding speeds ranging from millimeters to meters per second.

Given that ZDDP additives are typically applied in scenarios involving sliding or mixed sliding-rolling motion, which promote the formation of the protective tribofilm on metal surfaces through tribochemistry, the results suggest that the contact analysis of tribofilm can be simplified to a soft elastic layered contact problem depending on the sliding velocity where the ZDDP additive is applied.

Therefore, the effectiveness of ZDDP-derived tribofilms as protective layers is not only governed by their inherent material properties but also by the operating conditions. At higher speeds, where viscoelastic effects are negligible, the performance of tribofilm is dominated by its instant elastic response. Conversely, under extremely-low-speed or static conditions (e.g., the work of Dorgham, et al. [20, 21]), viscoelasticity can emerge as a critical factor influencing pressure distribution, and wear mitigation. By integrating numerical, experimental and practical perspectives, a clear distinction is established between low-speed, viscoelastic-dominated responses, and high-speed, elasticity-dominated tribofilm behaviours. This insight bridges the gap between the nanoscale findings of Dorgham on the viscoelastic nature of tribofilms and large-scale experimental

and practical observations, where tribofilms predominantly act as soft and elastic layers [2, 14].

Based on this, how the interplay of loading conditions, film thickness, and speed variations affect the transition between viscoelastic and elastic-dominated behaviour for the tribofilms derived from different sources could be explored in the future work to provide deeper insights into optimizing additive performance for specific applications. Additionally, exploring the transient effects of tribofilm formation and removal can enhance the understanding of contact mechanics and provide more accurate solutions for tribofilm-affected systems.

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Appendix 7A

In this appendix, frequency response functions of surface displacements in elastic layered half-space due to p are presented as follows:

$$\tilde{\tilde{C}}_p^{u_z}(m, n, 0) = \frac{1}{2G_c} [-\alpha(A^{(1)} - \bar{A}^{(1)}) - (3 - 4\nu_1)(C^{(1)} + \bar{C}^{(1)})],$$

or

$$\tilde{\tilde{C}}_p^{u_z}(m, n, 0) = \frac{\nu_c - 1}{G_c} (1 + 4\alpha h k \theta - \lambda k \theta^2) \alpha R,$$

$$\tilde{\tilde{C}}_p^{u_x}(m, n, 0) = \frac{1}{2G_c} [im(A^{(1)} + \bar{A}^{(1)})],$$

$$\tilde{\tilde{C}}_p^{u_y}(m, n, 0) = \frac{1}{2G_c} [in(A^{(1)} + \bar{A}^{(1)})],$$

$$\alpha = \sqrt{m^2 + n^2}, \theta = e^{-2\alpha h}, G = \frac{G_c}{G_s}, k = \frac{G - 1}{G + 3 - 4\nu_c},$$

$$\lambda = 1 - \frac{4(1 - \nu_c)}{1 + G(3 - 4\nu_s)}, R = -\frac{1}{[1 - (\lambda + k + 4k\alpha^2 h^2)\theta + \lambda k \theta^2]\alpha^2},$$

$$A^{(1)} = R \left\{ -(1 - 2\nu_c)[1 - (1 - 2\alpha h)k\theta] + \frac{1}{2}(k - \lambda - 4k\alpha^2 h^2)\theta \right\},$$

$$\bar{A}^{(1)} = R\theta \left\{ (1 - 2\nu_c)k(1 + 2\alpha h - \lambda\theta) + \frac{1}{2}(k - \lambda - 4k\alpha^2 h^2) \right\},$$

$$C^{(1)} = [1 - (1 - 2\alpha h)k\theta]\alpha R,$$

$$\bar{C}^{(1)} = (1 + 2\alpha h - \lambda\theta)k\theta\alpha R.$$

Frequency response function of surface displacements in elastic layered half-space due to q_x :

$$\tilde{\tilde{C}}_{q_x}^{u_z}(m, n, 0) = \frac{1}{2G_c} [-\alpha(A^{(1)} - \bar{A}^{(1)}) - (3 - 4\nu_1)(C^{(1)} + \bar{C}^{(1)})],$$

$$\tilde{\tilde{C}}_{q_x}^{u_x}(m, n, 0) = \frac{1}{2G_c} [im(A^{(1)} + \bar{A}^{(1)}) - 4(1 - \nu_1)(B^{(1)} + \bar{B}^{(1)}) - m(B_{,m}^{(1)} + \bar{B}_{,m}^{(1)})],$$

$$\tilde{\tilde{C}}_{q_x}^{u_y}(m, n, 0) = \frac{1}{2G_c} [in(A^{(1)} + \bar{A}^{(1)}) - n(B_{,m}^{(1)} + \bar{B}_{,m}^{(1)})],$$

$$B_{,m} = \partial B / \partial m,$$

$$\alpha = \sqrt{m^2 + n^2}, \theta = e^{-2\alpha h}, G = \frac{G_c}{G_s}, k = \frac{G-1}{G+3-4\nu_c}, \lambda = 1 - \frac{4(1-\nu_c)}{1+G(3-4\nu_s)},$$

$$S_0 = [G + (3 - 4\nu_c)](1 - k\theta),$$

$$\bar{B}^{(1)} = -\frac{(G-1)\theta}{(1+G) + (1-G)\theta} \cdot \frac{1}{2\alpha(1-\nu_c)},$$

$$B^{(1)} = \bar{B}^{(1)} - \frac{1}{2\alpha(1-\nu_c)},$$

$$B^{(2)} = -\frac{2(1-\nu_c)}{1-\nu_s} \cdot \frac{\sqrt{\theta}}{(1+G) + (1-G)\theta} \cdot \frac{1}{2\alpha(1-\nu_c)},$$

$$C^{(1)} = \frac{(1-\lambda)S_0R_c}{\{4(1-\nu_c)(G+3-4\nu_c)[1-(\lambda+k+4k\alpha^2h^2)\theta+\lambda k\theta^2]\}},$$

$$\bar{C}^{(1)} = \frac{[2(G-1)\alpha h\theta C^{(1)} + \theta R_a]}{S_0},$$

$$A^{(1)} = \frac{[-(3-4\nu_c)C^{(1)} + \bar{C}^{(1)} + \alpha(R_1 + R_2)]}{2\alpha},$$

$$\bar{A}^{(1)} = \frac{\{(1-\lambda)\theta C^{(1)} + [(3-4\nu_c)(1-\theta) - 2\alpha h]\bar{C}^{(1)} + \theta R_d\}}{[2\alpha(1-\theta)]},$$

$$C^{(2)} = \{[4(1-\nu_c)(1-\lambda)\sqrt{\theta}]C^{(1)} + (1-\lambda)\alpha\sqrt{\theta}(R_3 - R_4 + R_5 - R_6)\}/[4(1-\nu_c)],$$

$$A^{(2)} = \{2\alpha h\sqrt{\theta}[S_0 - (G-1)(1-\theta)]C^{(1)} - [(3-4\nu_s)S_0]C^{(2)} \\ + \alpha\sqrt{\theta}S_0(R_1 + R_2 - R_3 - R_4) - \sqrt{\theta}(1-\theta)R_a\}/(2\alpha S_0),$$

$$-\alpha^2 R_1 = im(B^{(1)} - \bar{B}^{(1)}) + i\alpha^2(B_m^{(1)} - \bar{B}_m^{(1)}),$$

$$-\alpha^2 R_2 = 2im(1-\nu_c)(B^{(1)} + \bar{B}^{(1)}) + i\alpha^2(B_m^{(1)} + \bar{B}_m^{(1)}),$$

$$-\alpha^2 R_3 = i(m - m\alpha h)B^{(1)} - i(m + m\alpha h)\theta^{-1}\bar{B}^{(1)} - im\theta^{-\frac{1}{2}}B^{(2)} + i\alpha^2 B_{,m}^{(1)} \\ - i\alpha^2 \theta^{-1}\bar{B}_{,m}^{(1)} - i\alpha^2 \theta^{-\frac{1}{2}}\bar{B}_{,m}^{(2)},$$

$$-\alpha^2 R_4 = [2i(1-\nu_1)m - im\alpha h]B^{(1)} + [2i(1-\nu_1)m + im\alpha h]\theta^{-1}\bar{B}^{(1)} \\ - [2i(1-\nu_2)m]\theta^{-\frac{1}{2}}B^{(2)} + i\alpha^2 B_{,m}^{(1)} + i\alpha^2 \theta^{-1}\bar{B}_{,m}^{(1)} - i\alpha^2 \theta^{-\frac{1}{2}}\bar{B}_{,m}^{(2)},$$

$$-\alpha^2 R_5 = -im\alpha h B^{(1)} + im\alpha h \theta^{-1}\bar{B}^{(1)} + i\alpha^2 B_{,m}^{(1)} + i\alpha^2 \theta^{-1}\bar{B}_{,m}^{(1)} - iG\alpha^2 \theta^{-\frac{1}{2}}\bar{B}_{,m}^{(2)},$$

$$\begin{aligned}
-\alpha^2 R_6 &= i(m - m\alpha h)B^{(1)} - i(m + m\alpha h)\theta^{-1}\bar{B}^{(1)} - iGm\theta^{-\frac{1}{2}}B^{(2)} + i\alpha^2 B_{,m}^{(1)} \\
&\quad - i\alpha^2\theta^{-1}\bar{B}_{,m}^{(1)} - iG\alpha^2\theta^{-\frac{1}{2}}\bar{B}_{,m}^{(2)},
\end{aligned}$$

$$R_a = (G - 1)\alpha(R_1 + R_2) - G\alpha(R_3 + R_4) + \alpha(R_5 + R_6),$$

$$R_b = \alpha(R_2 - R_1) + \alpha\theta(R_3 - R_4),$$

$$R_c = \frac{4(1 - \nu_1)}{1 - \lambda} \left(\frac{2\alpha h\theta}{S_0} R_a + R_b \right) - \alpha\theta(R_3 - R_4 + R_5 - R_6),$$

$$R_d = \alpha(R_1 - R_2 - R_3 + R_4) + \frac{(1 - \lambda)\alpha}{4(1 - \nu_1)} (R_3 - R_4 + R_5 - R_6).$$

Frequency response function of surface displacements in elastic layered half-space due to q_y can be determined from symmetrical characters:

$$\begin{aligned}
\widehat{C}_{q_y}^{\bar{u}_x}(m, n, 0) &= \widehat{C}_{q_x}^{\bar{u}_y}(n, m, 0), \widehat{C}_{q_y}^{\bar{u}_y}(m, n, 0) = \widehat{C}_{q_x}^{\bar{u}_x}(n, m, 0), \widehat{C}_{q_y}^{\bar{u}_z}(m, n, 0) \\
&= -\widehat{C}_p^{\bar{u}_x}(n, m, 0)
\end{aligned}$$

Appendix 7B

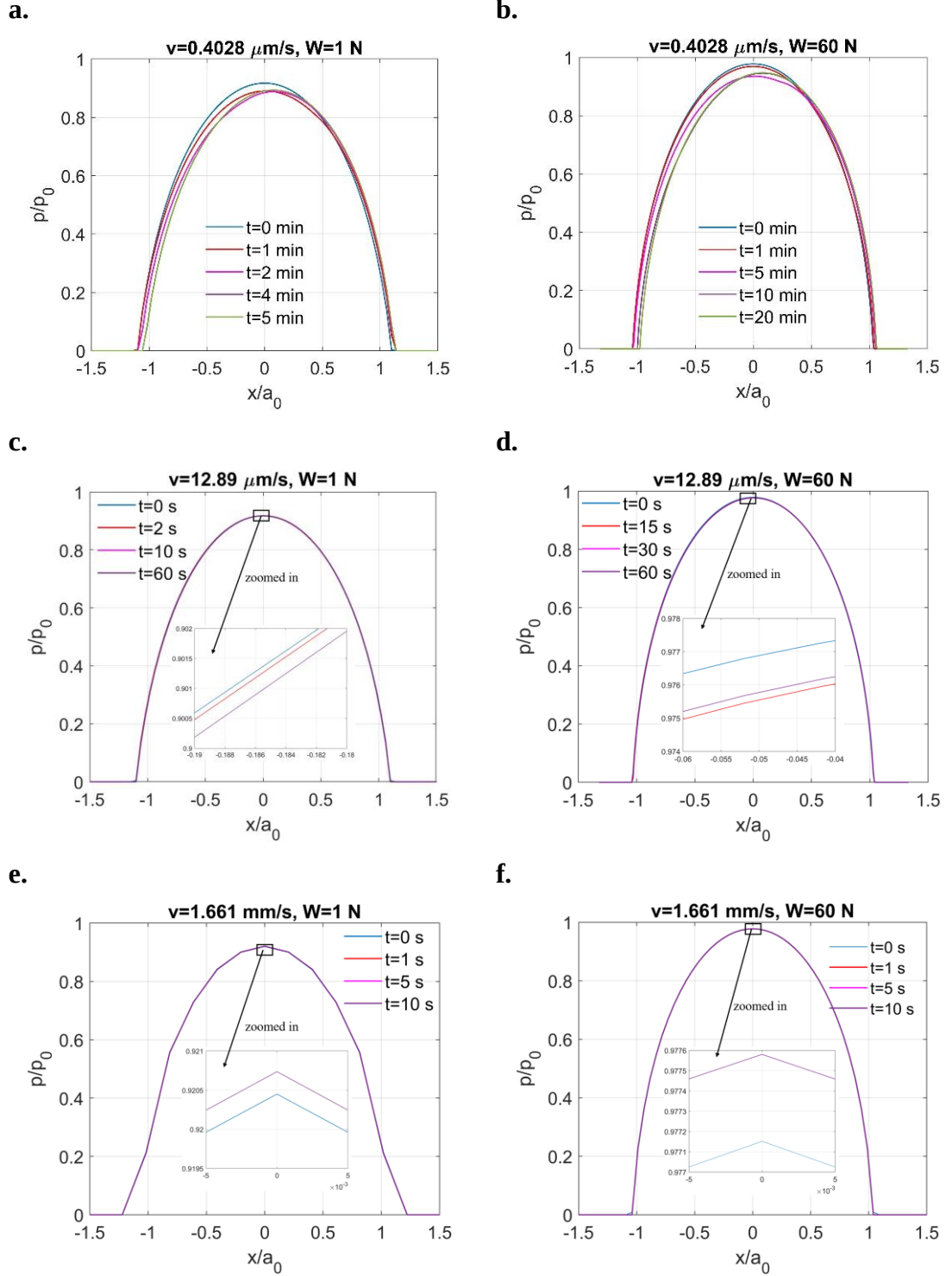


Figure 7-13 Extra sliding solutions for layered tribofilm at varying inputs: (a) $v = 0.4028 \mu\text{m/s}$, $W = 1 \text{ N}$ (b) $v = 0.4028 \mu\text{m/s}$, $W = 60 \text{ N}$ (c) $v = 12.89 \mu\text{m/s}$, $W = 1 \text{ N}$, (d) $v = 12.89 \mu\text{m/s}$, $W = 60 \text{ N}$, (e) $v = 1.661 \text{ mm/s}$, $W = 1 \text{ N}$ and (f) $v = 1.661 \text{ mm/s}$, $W = 60 \text{ N}$

Notably, for the high-speed case ($v = 1.661 \text{ mm/s}$), the sliding contact quickly reaches the steady state, showing consistent results at $t = 1 \text{ s}$, $t = 5 \text{ s}$, and $t = 10 \text{ s}$.

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8 Discussion and Conclusion

Through accomplishing the specified four research objectives and investigating the raised research questions aligned with these objectives, this project enhances the understanding of viscoelastic contact mechanics and partial slip phenomena experienced by various materials under different operating conditions. In this section, the main research outcome is summarized, along with the identification of open lines of investigation that shall be addressed in future.

8.1 Summary of Work

The main accomplishment of the project is the establishment of the target numerical framework that can solve dry frictional point contact problems for both elastic and viscoelastic materials. This framework accommodates various contact configurations, including half-space and layer-substrate systems. The implementation of computational techniques including DC-FFT and CGM in the BEM-based contact models ensures rapid convergence of solutions. Validation against analytical solutions (e.g., Hertzian solution [1], Muser's theory [2], generalized Cattaneo-Mindlin solutions by Johnson [3], solution of Lee and Radok [4], solutions of O'Sullivan and King [5]) and simulation outcome reported by other model developers (e.g., Chen and Wang [6], Chen, et al. [7], Koumi, et al. [8]) demonstrates the accuracy and versatility of the developed numerical framework.

Among all the contact models within the framework, the elastic partial slip models serve as the foundation for the project as all other models within the framework are their extensions. To enhance the understanding of partial slip mechanics of elastic materials, especially the partial slip contact problem of nominally flat rough surfaces, RQ1 was raised and investigated.

8.1.1 RQ1: Effects of Surface Roughness on Elastic Partial Slip Solutions

Different from the transition from fully stick to gross sliding for a smooth sphere on flat contact configuration, which exhibits a nonlinear relationship between the stick ratio ($\frac{A_{stick}}{A_{contact}}$) and dimensionless tangential load ($\frac{F_x}{\mu_f W}$) expressed as Equation 8-1, when it comes to the multi-asperity contact (nominally-flat against flat contact), the relationship between the stick ratio and dimensionless tangential load becomes linear (approximated as Equation 8-2) without any dependence on the investigated surface roughness parameters, including root mean square slope, root mean square gradient, skewness and kurtosis.

$$\frac{A_{stick}}{A_{contact}} = \left(1 - \frac{F_x}{\mu_f W}\right)^{\frac{2}{3}} \quad \text{Equation 8-1}$$

$$\frac{A_{stick}}{A_{contact}} = 1 - \frac{F_x}{\mu_f W} \quad \text{Equation 8-2}$$

This finding agrees well with what was reported in a similar study by Paggi, et. [9], who focused on effects of Hurst exponent of fractal rough surfaces instead. This linear relationship provides crucial information about contact stability in the scenario where control of slip is essential (e.g. brake and clutch systems). By shedding light on the role of surface topography in partial slip phenomena, the linear relationship suggests that the macroscopic frictional response can be predicted without detailed knowledge of the microscale roughness, reducing the requirement of exhaustive surface characterisation in engineering designs.

Notably, this linearity only applies for the contact of nominally flat on flat with elastically similar material properties. Whether this linear relationship still holds in partial slip contact of other rough contact geometries (e.g., a rough sphere against a flat, a rough sphere against nominally flat) needs further investigations.

The relationship between the stick ratio and tangential load in the rough surface contact becomes nonlinear when taking into account coupling effects of dissimilar materials, highlighting the interplay between the material properties and surface roughness in frictional systems. Due to the coupling between shear tractions and normal pressures, a normal load can induce partial slip phenomena alone. Besides, the contact can reach gross sliding state under a tangential load that is less than the static friction ($\mu_f p$) since the condition of global sliding is changed from Equation 8-3 to Equation 8-4.

$$q = \mu_f p \quad \text{Equation 8-3}$$

$$\sqrt{q_x^2 + q_y^2} = \mu_f p \quad \text{Equation 8-4}$$

Different from the uncoupled case where the separation of stick and slip regions always follow a certain pattern such that the central stick region is surrounded by the slip annulus before gross sliding motion, the separation pattern of stick and slip zones under coupled conditions can exhibit a load-dependent shape, the location of which varies. As the pressure tends to be asymmetric when a tangential load that is not enough to induce gross sliding is applied, the trailing edge of the contact area exhibits higher pressure than the

leading edge, causing the stick zone to appear in the rear part of contacting area as the sliding condition (Equation 8-4) is more difficult to reach.

This study highlights the importance of considering interfacial coupling effects in contact mechanics, which can be commonly encountered in the design and analysis of systems with hard-on-soft material combination. Understanding the coupling between normal and tangential stresses in these systems can be essential for optimizing their frictional performance.

The deviation of the coupled solution from the uncoupled solution (coupling effects) can be affected by many contact parameters such as the Dundurs constant β (expressed by Equation 4-22) characterising the dissimilarity level of contacting materials and Coulomb's friction coefficient μ_f . As reported by Chen and Wang [6], a significant increases of stick zone in the slightly decreased contact area can be observed as the value of β/μ_f increases for spherical partial slip contact problems. Based on this finding, it can be estimated that the relationship between the stick ratio and tangential load approaches linearity as the value β/μ_f tends to vanish for the contact of nominally flat against flat. But this needs further validation numerically and experimentally.

Together, these findings (insensitivity of stick ratio to roughness parameters in elastically similar materials and nonlinear trend stick ratio with tangential load in contact of dissimilar elastic materials) provide a comprehensive understanding of partial slip in rough surfaces contacts. They emphasize the distinct roles of material properties and surface topography in governing frictional behaviour and offer a unified framework for analysing both similar and dissimilar material contacts.

8.1.2 RQ2: Effects of Viscoelasticity on Partial Slip Solutions

Models for the coupled partial slip contact of viscoelastic surfaces are developed by converting the well-validated elastic models following the elastic-viscoelastic correspondence principle. The time-dependent property of viscoelastic materials brings new insights into the mechanism of partial slip contact.

During the numerical simulation, it is found that the rheological behaviour of a viscoelastic material can be correlated to its variation of creep compliance or relaxation modulus with time. Specially, for materials characterised by the SLS model, their rheological behaviour can be estimated by the parameter known as the ratio of retardation time to relaxation time μ_∞/μ_0 (sometimes referred to as elasticity ratio [10] or ratio of glassy modulus to rubbery modulus [11]).

Although the boundary value is not known clearly, materials with a higher ratio value tend to perform more fluid-like and vice versa. Similar to what was observed by Zhao, et al. [10], pressure spikes on the contacting edges are often observed for the more fluid-like materials under a constant normal loading history due to the significant stress relaxation and different loading history experienced by the centre and edges of the contacting area. As a result, different from the ordinary elastic solutions, the peak pressure can move from the centre of the contacting region to both contacting edges in a certain range of time for materials with higher values of μ_{∞}/μ_0 although they would eventually exhibit a Hertzian-type pressure distribution once the steady state is reached.

This empirical relation is limited to viscoelastic materials characterised with Zener (SLS) models. For realistic materials exhibiting broad relaxation spectrum, the relation will not be as straightforward. In practice, the loss tangent $\tan \delta$ (ratio of loss modulus to storage modulus) measured from DMA is often employed to provide the dominant behaviour of a viscoelastic material (damping capacity of viscoelastic materials) [12]. According to its mathematical definition, the material acts more like viscous fluid when its loss tangent is higher than 1 while it behaves more solid-like when the loss tangent is less than 1. However, the present model is not developed based on the usage of the frequency-dependent complex modulus but the transient creep compliance and relaxation modulus. The numerical studies about viscoelastic materials were conducted based on the SLS models with different values of ratio of retardation time to relaxation time.

Due to the time-dependency nature of viscoelastic materials and coupling effects between normal pressure and shear tractions, the contact traction distribution can be in irregular shapes as illustrated in Figure 5-13, especially for more fluid-like viscoelastic materials that exhibits pressure spikes during the contact duration. These unique contact tractions can influence the subsurface stress distribution, especially the position and magnitude of the peak stress.

Although the stick ratio is found to be not affected by the time-dependent nature of viscoelastic materials so long as the normal and tangential contact inputs (either load or displacement) are maintained within the contact period, since different contact phenomena including creep and relaxation can be experienced by the viscoelastic material depending on whether the contact is imposed through a step load or displacement, the separation pattern of the stick and slip regions in viscoelastic partial slip problems can vary with time even though contact inputs are constant. When the same contact phenomenon (stress relaxation or creep) is experienced in both normal and tangential

directions, the evolution of the stick region follows a certain pattern. Otherwise, the separation pattern of stick and slip regions will vary with time although the stick ratio remains under the constant contact inputs.

Additionally, when the same increasing rate of surface displacement is specified in the tangential direction, the viscoelastic surface experiencing a stress relaxation phenomenon in the normal direction can always reach the gross sliding state within a short time when compared to that experiencing a creep phenomenon in the normal direction. The reason behind it is that the pressure keeps relaxing in the former case such that the gross sliding condition can be satisfied with a lower tangential load (equivalent displacement).

When the fully coupled condition between normal pressures and shear tractions is considered, the unstable and time-dependent properties of viscoelastic materials, along with the altered condition of gross sliding ($\sqrt{q_x^2 + q_y^2} = \mu_f p$) can lead to a sudden transition from partial slip to gross sliding. Such a transition somehow demonstrates that viscoelastic materials indeed perform as a combination of solid and fluid under the specified contact inputs. Initially, the material behaves similarly to an elastic solid such that a threshold of tangential displacement (load) needs to be reached to induce global sliding. However, the final transition tends to be abrupt, like a fluid, which slides instantly once it is sheared. Theoretically, it shall become less abrupt for the more solid-like viscoelastic material, whose property changes less dramatically. Although the above explanation tends to be self-consistent, robust verification work needs to be done to verify the finding experimentally.

8.1.3 RQ3: Effects of the Early Partial Slip period on Sliding Contact Solutions

After incorporating sliding motion in the developed viscoelastic contact models, the role played by dry contact friction in transient sliding contact of viscoelastic materials is investigated. The simulation outcome shows that the early partial slip period has trivial effects on the sliding contact solutions once the system reaches a steady state, including a lower peak pressure, a shift of the contacting area in the direction opposite to the sliding motion and an extended transition time to the steady state. These findings justify the usage of frictionless contact assumption commonly employed in the study of Carbone and Putignano [13-15] and work of Koumi, et al [8], and Zhao, et al. [10] if the primary objective of numerical studies is to analyse the steady-state behaviour of viscoelastic materials during sliding, although certain qualitative errors may be introduced.

This study can serve as a reference for future modelling efforts on viscoelastic sliding contact problems. Researchers can simplify the sliding contact analysis by neglecting dry friction to obtain qualitatively accurate results effectively when their research focus is the steady-state solution. On the other hand, for contact systems subjected to irregular and dynamic loading histories, transient contact analysis, which takes into account the evolution from partial slip to gross sliding, can be essential to provide more precise insights into friction and wear of materials based on a detailed contact traction history and the derived time-dependent subsurface stress distributions.

In addition to the finding about the effects of dry contact friction on sliding contact solutions, the study reported that the SLS models with higher ratio values of retardation time to relaxation time μ_{∞}/μ_0 tends to exhibit higher viscoelastic friction, which further validates the relation between the rheological behaviour of viscoelastic materials and this specific parameter.

8.1.4 RQ4: Effects of Viscoelastic ZDDP-Derived Tribofilm on Contact of Elastic Solids

The viscoelastic layered contact model is developed based on the half-space contact models. Although it can simulate the partial slip and frictional sliding contact of a layer-substrate system, by extending the findings from RQ3 to RQ4, a degenerated form of the layered viscoelastic contact model was employed to study ZDDP-derived tribofilm contact. Given that the study mainly focuses on the role played by the tribofilm in the long run, during which its thickness stabilizes and provides sustained support for the contact of two elastic solids, the effects of dry contact friction during the transient sliding contact are neglected. This simplification significantly reduces the computational cost while preserving the qualitative accuracy of predictions for tribofilm layered contact.

The viscoelastic effects of tribofilms are found to be significant only at extremely low sliding speeds or under indentation conditions, where pronounced time-dependent effects can be observed. Given the critical role of shear forces in tribofilm formation [16] and its typical operating conditions, where sliding speeds range from hundreds of millimetres per second to metres per second [17-19], it is concluded that the ZDDP-derived tribofilm performs predominantly as a soft elastic layer, despite its inherently extremely fluid-like viscoelastic nature, as evidenced by sharp pressure spikes and significant stress relaxation observed when modelling it as a half-space body. In practical applications, its primary function is to prevent direct contact and to slightly reduce the contact pressure, while time-dependent effects remain negligible. These findings align with the prevailing

consensus on the behaviour of ZDDP-derived tribofilm [20, 21] and bridges the gap between the nanoscale viscoelasticity observation [22, 23] and the macroscale elastic-dominant performance observed in real-world applications [21].

This study suggests that exploring the interplay of loading conditions, film thickness and sliding speed in the transition between viscoelastic and elastic-dominated behaviour for tribofilms derived from different sources can provide valuable insights into optimizing the additive performance for specific applications.

8.2 Future Directions

The versatility of developed models in practical applications, as well as the reliability and robustness of the conducted numerical studies, can be further enhanced by a series of extension work.

8.2.1 Extension of Partial Slip Contact to Fretting Contact

Partial slip contact provides a foundational understanding of stick-slip mechanics, typically studied in the context of a single load that increases monotonically in this project. Fretting contact extends this concept to cyclic loading, where small-amplitude oscillatory motion between two contacting surfaces can lead to progressive surface degradation over time.

Fretting wear is a critical issue in practical applications, such as knee replacements, where the repetitive micro-motions between the femoral and tibial components result in material loss and a reduced prosthesis lifespan [24]. In addition to biomedical applications, fretting is prevalent in aerospace [25] and automotive components [26, 27], where viscoelastic materials are increasingly applied.

Despite extensive numerical modelling efforts in fretting of elastic materials [28-30], research on fretting contact in viscoelastic systems remains limited. The partial slip models developed in this project can be extended systematically to simulate fretting contact through an incremental application of the oscillating tangential load and additional adaptations to incorporate time-dependent viscoelastic effects. This extended fretting contact model can be employed to conduct parametric studies to mitigate fretting damage by exploring the effects of key factors such as contact load, cyclic displacement, material properties and other environmental inputs.

8.2.2 Inclusion of Line Contact Geometry

The developed numerical framework focuses primarily on point contact while line contact geometries are also frequently encountered in engineering applications such as gear teeth

interactions, cam-follower systems and contact of cylindrical roller bearings. To enhance the versatility of developed models, line contact can be incorporated by modifying the employed DC-FFT algorithm. Different from spherical contact problems, cylindrical contact exhibits periodicity along the length direction while remaining non-periodic in the width direction, necessitating tailored computational adaptations.

To date, several FFT-based algorithms have been proposed in the literature to solve 3D line contact problems [31], including DC-FFT algorithm modified with duplicated padding (DCD-FFT), DC-FFT algorithm modified with summation (DCS-FFT) and a hybrid algorithm combining DC-FFT and continuous convolution fast Fourier transform (CC-FFT). According to a recent study by Sun, et al. [32], the DCS-FFT algorithm demonstrates the highest efficiency and accuracy. Implementing this method within the existing point contact models requires only minor modifications. The computational performance can be optimized as the summation operation for calculating the influence coefficient matrices is performed once and it remains outside the iterative process of the CGM algorithm.

By integrating DCS-FFT algorithm into the contact models and introducing a flag parameter that switch between line and point contact algorithms, the developed numerical framework can be extended to a broader range of engineering applications.

8.2.3 Inclusion of Thermal Effects

An isothermal assumption was made in this project to simplify the simulation of viscoelastic contact problems by neglecting thermal effects during contact analysis. Referring to the thermo-viscoelastic contact modelling work by Zhang, et al. [33], thermal effects can be incorporated by taking into account the change of material properties caused by the viscoelastic heating due to internal energy dissipation and frictional heating at the interface. Notably, the frequency-dependent viscoelastic modulus is employed in their modelling while this project utilizes the time-dependent creep compliance and relaxation modulus for the viscoelastic materials. Hence, following the work of Zhang, et al. [33], modifications to the problem formulation and contact algorithms are needed when taking into account thermal effects based on the currently developed viscoelastic contact models.

In the work of Zhang, et al. [33], the viscoelastic properties are updated dynamically based on the local temperature variations, following a velocity-frequency-temperature relationship. The fully coupled thermo-viscoelastic problem, including the

interdependence of temperature, viscoelastic modulus and contact tractions, is solved in their DC-FFT-based contact model by integrating the contact mechanics, heat transfer, and change of temperature-dependent material properties into a unified framework. Similar with the coupling problem between normal pressures and shear tractions in partial slip problems, here the temperature variations affect viscoelastic modulus, which, in turn, alters the distribution of contact tractions and the heat generation. The problems of contact mechanics, heat partitioning and update of viscoelastic property are solved sequentially and iteratively until a series of convergence criteria is achieved in an iterative computational approach, details of which can be found in the work of Zhang, et al. [33].

8.2.4 Other Model Extensions

Apart from the work discussed above, the following extensions can be done based on the well-established theory and numerical work:

- Subsurface stress analysis

Current contact analysis primarily focuses on the contact interface, providing limited insights into the subsurface field, especially the subsurface stress distribution that can be critical when conducting failure prediction for the contacting bodies. The stress components vary in the x -, y – and z – directions (stress tensors) can be determined in a similar way with the surface deformation via the DC-FFT algorithm after introducing the well-documented influence coefficients for the half-space contact or frequency response functions for layered contact problems [34] relating stress tensors with contact tractions.

- Incorporation of plasticity and material inhomogeneity

The plastic contact is neglected during the model development. Although the project is intended to develop models for the contact of viscoelastic materials, which can hardly deform plastically considering their soft features, there exist some exceptions especially when it comes to rough contact of relatively hard viscoelastic material such as PMMA at room temperature. Including the plastic contact of elastic and viscoelastic materials in the developed model shall provide more precise simulation results when addressing real-life contact problems with highly loaded interfaces.

Based on the currently developed contact model, the plasticity can be incorporated by referring to the work of Wang, et al. [35], who introduced plasticity effects (e.g. residual stress fields and residual displacement) using eigenstrain method within the framework of DC-FFT-based contact algorithms.

Additionally, the present models assume homogeneous materials. Considering the increasing usage of composite materials in engineering applications, where viscoelastic matrices reinforced with anisotropic elastic fibres exhibit complex mechanical behaviour, it is necessary to incorporate material inhomogeneity for accurate modelling. The work of Koumi, et al. [8] can be referred, who unitized Eshelby's equivalent inclusion method (EIM) to characterise the effects of an ellipsoidal inhomogeneity embedded into a viscoelastic half-space in their DC-FFT-based contact model.

- Multi-layered contact

Multilayer coatings are widely applied as surface modifications in mechanical, electrical, and biomedical components to enhance performance, durability, and functionality for specific applications. Achieving an optimal design for these coatings requires a comprehensive understanding of their mechanical behaviour. The developed single-layered contact model can be extended to take the multi-layered contact into consideration, where the analytical frequency response functions for the contact of multi-layered elastic materials proposed by Yu, et al. [36] can be incorporated.

- Inclusion of surface adhesion

The surface adhesion between the contacting bodies is neglected in the problem and algorithm designs. As mentioned in section 2.4.4, it is reasonable to neglect the adhesive force when analysing the contact metallic materials as its effect is insignificant. However, the adhesive force can play a non-trivial role in viscoelastic contact problems depending on the contact compliance and soft features of viscoelastic materials. To date, the model for the viscoelastic-adhesive indentation contact within the framework of BEM is established (e.g., work of Hui, et al. [37]). However, a fully coupled (coupling between normal pressures and shear tractions) model for the viscoelastic-adhesive contact, which requires careful consideration of energy terms is still lacking. As discussed in 2.9, it is one of the knowledge gaps that remain to be filled in the field of viscoelastic contact mechanics modelling.

- Extension to Nonlinear Viscoelasticity

The viscoelastic material simulated is assumed to follow the theory of linear viscoelasticity. However, the validity of the linear viscoelastic behaviour of many real-life materials is very limited (strain limit for polymers in the glassy state is less than 1% [38]) while most of their applications are beyond this limit. As

discussed in section 2.9, the development of the BEM-based model for the contact of nonlinear viscoelastic materials is still in its early stage. Therefore, it is worthwhile to investigate if the nonlinear viscoelastic response can be simulated based on the developed BEM model in an approximation approach.

8.2.5 Further Numerical Investigations

On the other hand, the numerical studies conducted and shown in the thesis can be furthered by adding the following work:

- The dry frictional contact simulations conducted in the project adhere to Coulomb's friction law to determine the shear tractions. However, as noted by Chen and Wang [39], allowing the shear tractions to exceed the shear strength of materials is unrealistic in practical problem. Additionally, the constant coefficient of friction assumed in Coulomb's law fails to explain the variation trend of μ_f with the normal load and surface roughness such as the Euler's interlocking asperity theory. To address some of these limitations, Chen and Wang [39] suggested using the the local shear stress deriving from the Tabor equation as a more realistic cut-off limit of the shear traction. Implementing this approach in the future numerical simulation of frictional contacts could provide a more accurate representation of shear tractions and its dependence on contact conditions.
- Apart from the single-asperity contact considered in the study of viscoelastic contact, the transient multi-asperity (rough-surface) contact needs to be investigated numerically to shed light on the role played by the surface roughness in the indentation and frictional sliding contact solutions of viscoelastic materials.
- To further investigate the contact response of the ZDDP-derived tribofilm, the graded structure of the tribofilm needs to be taken into account. The continuous layers of the tribofilm can be discretised into several layers as shown in Figure 8-1, where the top one is most fluid-like while the bottom one close to the base layer is most solid-like following the description of Spikes [20]. The factors defining the relaxation modulus (Prony series) of these layers are expected to vary with the thickness of tribofilm following an exponential or linear relation.

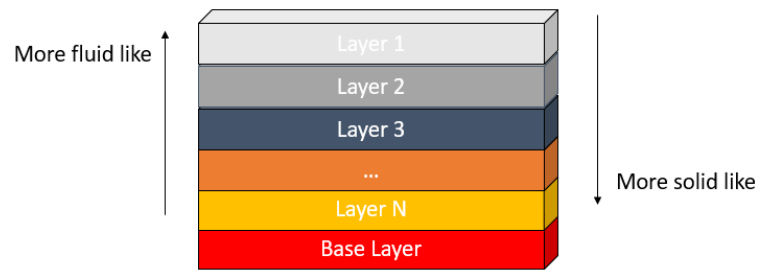


Figure 8-1 Subsection of the continuous layers of tribofilm

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