Essays on The Economics of Platform Competition

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Abstract

This thesis consists of three essays on the economics of platform competition. Theoretical approaches are undertaken to analyse the functioning of a two-sided market from different perspectives. The first chapter studies the impacts of a bandwagon, snob or congestion effects and sellers' competition in the presence of multihoming. Results demonstrate that all market participants, including buyers, sellers, and platforms, exhibit a preference for multihoming over singlehoming scenarios, irrespective of whether only one side or both sides of the market are multihoming. Whilst multihoming equilibrium fees are comparatively higher, the resulting aggregate surpluses on both market sides are also higher. Furthermore, the increased number of participants in the market provides platforms with additional fees, further boosting their profits.

The second chapter presents a novel framework for studying platform competition by examining the mechanisms through which asymmetric platforms attract agents, particularly how they appeal to buyers. To capture the strategic interactions between platforms, a two-stage game is considered in which heterogeneous platforms simultaneously choose features on buyers' side in the first stage and membership fees in the second stage. Results show that buyers' decisions to join a platform are influenced not only by membership fees and cross-network effects but also by the range of functionalities offered by the platform.

The third chapter develops a two-period dynamic model of platform competition, in which buyers have imperfect information concerning the quality of the platforms. Consequently, buyers must first experience platforms' features before deciding to switch to a different intermediary. A general insight is developed into the strategies employed by platforms in setting their membership fees and the implications this has on determining market shares, considering the presence of cross-group network effects and quality uncertainty on buyers' side. This framework allows to endogenise buyers' switching decisions and have asymmetric platforms in equilibrium.

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Dedication

To my wife Edith and my daughter Agnes.

Romans 11.36. *For from Him and through Him and to Him are all things. To Him be the glory forever. Amen.*

Declaration

I, Roberto Cavazos Flores declare that this thesis titled “Essays on The Economics of Platform Competition” is an original work of mine, and I am the sole author of the entire thesis, except for Chapter 2, which is a joint work with Dr. Bipasa Datta.

Chapter 2, titled “How do platforms appeal to buyers?”, was presented at the PhD Symposium in Industrial Economics during the Network of Industrial Economists Conference 2022 and at the XXXVI Jornadas de Economia Industrial (JEI) 2022.

This thesis has not been submitted for any previous degree or qualification, except as mentioned above, at this University or any other institution. All sources utilised in this thesis have been acknowledged and referenced.

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Introduction

An increasing number of businesses are adopting a two-sided market model. According to Hagiu and Altman (2017), the five most valuable firms in the world in 2017 (*Microsoft*, *Apple*, *Amazon*, *Alphabet*, and *Meta*) operate through two-sided markets. Furthermore, these firms have a higher market value within their respective industries compared to one-sided markets (e.g., *Airbnb* is worth more than *Marriott*).[1] A two-sided market refers to a market where an intermediary or platform brings together two types of agents, such as buyers and sellers, to interact and create value. Interactions between two types of agents generate network effects, resulting in more value being created on one side when the number of agents on the other side increases. Two-sided markets can take various forms: such as a physical market like "*Shambles Market*" in York, or a virtual marketplace like "*Amazon.com*", which brings together sellers and customers for trading purposes.

While virtual platforms have appeared as a novel form of e-commerce, their underlying business model of connecting two or more agents to facilitate value-creating exchanges has a longstanding precedent in physical marketplaces, such as shopping centres or newspapers connecting consumers with merchants or advertisers. In the current economic landscape, almost every business has a hybrid model that incorporates both online and physical channels. These platforms have proliferated across diverse industries, ranging from the lodging market with *Airbnb* and *Vrbo*, connecting travellers seeking accommodations with homeowners who rent out their properties, to retail markets where online marketplaces such as *eBay* and *Amazon* join buyers and sellers for various goods and services. Similarly, ride-hailing platforms like *Uber* and *Didi* have revolutionised the transportation industry by providing riders with convenient and cost-effective access to transportation services.

Moreover, a variety of virtual platforms have emerged that bring together different types of agents to create value in diverse markets. For instance, online payment platforms such as *PayPal* and *ApplePay* link cardholders and merchants. Social media platforms

[1] According to money.usnews.com up to August 16^{th} 2019, they still were in the top five biggest tech companies in the world.

such as *Facebook*, *Twitter*, and *Instagram* integrate content creators with end users and advertisers. Other examples of online platforms are *Netflix*, *Spotify*, *YouTube*, *Google*, *iTunes* and *Android*, etc. These platforms have transformed the ways in which value is created, exchanged, and consumed in modern markets, and their impact continues to shape the future of e-commerce.

Two-sided markets exhibit positive indirect network effects when the growth or activity of one group of participants on a platform leads to increased benefits for the other group. This means that as the number of users on one side of the market increases, it positively influences the attractiveness and participation of the other side. For instance, as the number of customers increases, shopping malls become more valuable to stores, and as the number of stores increases, shopping malls become more valuable to buyers. However, indirect network effects can be negative for one side. For example, as the number of readers increases, magazines or newspapers become more valuable to advertisers, but the same increase in advertising may cause a decline in value for readers.[2]

Evans and Schmalensee (2005) argued that multihoming, which refers to the practice of participants connecting to more than one platform or intermediary, is a significant determinant of the size and structure of a two-sided market, in addition to indirect network effects. Singlehoming, on the contrary, occurs when agents choose to use only one platform, while multihoming allows them to use more than one platform simultaneously. A common example of multihoming is in network television, where viewers and advertisers often connect to multiple channels. Similarly, in payment cards and video game platforms, users may choose to connect to multiple providers.

The seminal literature on the analysis of two-sided markets includes works by Caillaud and Jullien (2003), Armstrong (2006), Rochet and Tirole (2002, 2003, 2006). These works primarily focus on determining optimal pricing structures by taking into account factors such as the relative magnitudes of indirect network externalities, demand elasticities, and coordination. The issue of coordination arises when agents on one side of the market are willing to join a platform or intermediary only if a critical mass of agents on the other side also connect. One key finding of these papers is that two-sided markets are subject to unique competitive dynamics that can lead to counterintuitive outcomes. Specifically, they show that firms in two-sided markets may choose to subsidise one group of users in order to attract users on the other side of the market. For example, a credit card company might offer low-interest rates to cardholders to attract more merchants to accept the card. For a complete literature review survey on two-sided markets see Weyl (2010), Belleflamme and Peitz (2019a), Hagiu and Wright (2015), Sanchez-Cartas and León (2021), Jullien et al. (2021).

[2] Assuming readers dislike more advertising than less.

This thesis aims to study the economic principles and phenomena related to platforms or two-sided markets. Our goal is to examine various aspects such as the business models, market dynamics, pricing strategies, network effects, and competition associated with these platforms through three different theoretical environments. In Chapter 1, we explore the impacts of direct network effects on two-sided markets when participants can engage in multihoming to capture maximum cross-group network benefits. In Chapter 2, we extend the two-sided market model by introducing the level of attributes offered on buyers' side as a strategic variable on the vertical dimension. This allows us to analyse how platforms appeal to agents and particularly to buyers side. In Chapter 3, we allow buyers to have imperfect information about platforms' quality and develop a two-period dynamic model of platforms' competition where buyers may choose to switch between platforms once the quality realisation is experienced.

According to Chapter 1, two-sided markets exhibit intra-group or direct network effects when participants on one side are concerned not only with the other side but also with those on their own side. On buyers' side, the presence of other consumers can have both positive and negative effects on their utility. A "bandwagon effect" may occur, as defined by Leibenstein (1950), whereby the utility of purchasing goods increases with the number of other consumers acquiring the same good. Conversely, a "congestion effect" may arise, where buyers may be worse off if they shop in a crowded location, such as a shopping mall, and prefer a less crowded environment. Additionally, buyers may experience what Leibenstein (1950) defined as a "snob effect" when they seek to purchase exclusive goods that have not yet been purchased by others. On the other hand, competition on sellers' side reduces their profits, resulting in a negative direct network effect. However, buyers may benefit from sellers' competition, as it can lead to lower prices and a wider range of products.

Rochet and Tirole (2002) were among the pioneers to investigate the impact of within-network external effects in a two-sided market setting. Specifically, they examined a monopoly platform in the context of a payment card association allowing competition among merchants to determine the optimal access charges. Belleflamme and Toulemonde (2009) compared positive cross-group effects with negative within-group effects to establish the possibility of a competing platform. Hagiu (2009) introduced seller competition to the model, given that consumers prefer product variety, and used this to derive the optimal platform pricing structures. Belleflamme and Toulemonde (2016) incorporated sellers' within-group external effects in a two-sided singlehoming environment and analysed the game's equilibrium based on buyer-seller relationship outcomes. Lastly, Belleflamme and Peitz (2019a) explored how seller competition affects platform decisions and market structure.

Chapter 1 is motivated by two key factors: first, the growing prevalence of multihoming in numerous two-sided markets, often resulting from decreasing joining costs; and second, the observation that direct network effects are commonly experienced by both sides of the market. Sellers face competition from one another, while many buyers take into account the purchasing behaviour of other buyers, either due to conformity with the masses or to extract value (such as information) from the crowd. However, buyers can also exhibit the opposite tendency, seeking out niche or exclusive markets, such that their demand negatively correlates with market demand.

The model builds upon the work of Armstrong (2006) and extends the model introduced by Belleflamme and Peitz (2019b) to incorporate intra-group externalities. The objective is to investigate the impact of four different market scenarios on the model's equilibrium, namely: (*i*) when both sides of the market are singlehoming, (*ii*) when buyers are singlehoming and sellers are multihoming, (*iii*) when buyers are multihoming and sellers are singlehoming, and (*iv*) when both sides choose to multihoming.

Our contribution to this chapter lies in providing a framework for analysing the impact of direct network effects on two-sided markets in the presence of multihoming. The study sheds light on the factors that influence the fee adjustment process when there is an interaction between cross-group and within network effects, which are critical in determining the equilibrium fee structure on both sides of the market. The framework can be applied to various industries, including e-commerce, online advertising, and sharing economy platforms, to provide insights into the strategies that platforms can employ to attract and retain participants. Moreover, we explain how platform strategies can be tailored to leverage the strengths of the bandwagon and congestion effects along with competition on sellers' side and thereby maximise platform profits. Additionally, we provide a deeper understanding of the drivers behind multihoming behaviour in two-sided markets, which has important implications for platform competition and economic welfare.

Our results show that a bandwagon effect is observed when the participation of buyers on a platform reinforces each other, buyers attract more buyers and more sellers given the cross-group network effect, leading to a positive feedback loop. On the other hand, when congestion or competition is experienced on buyers and sellers' sides respectively, it can have an adverse impact on platforms' performance. The reason for this is the negative intra-group network effect leads to a reduction in the number of agents joining the platforms, which in turn, reduces the platforms' value and the aggregate surpluses of agents on both sides of the market.

In addition, we find that the adjustment of membership fees for both sides of the market is influenced by various factors when both agents multihome, such as the platforms' desire to attract and retain buyers and sellers, the intra-group network impacts, the relative strength of cross-group network effects, and revenue growth potential. In the presence of a bandwagon effect, platforms attract more buyers, making it less expensive to appeal to sellers with buyers. Thus, platforms can reduce buyers' fees and increase sellers' fees to compensate. This approach is effective when buyers' cross-group network effect on sellers is stronger than the cross-group network effect sellers have on buyers. Conversely, a congestion effect and high competition among sellers lead to the opposite impact.

Moreover, based on our economic welfare analysis all market participants, including buyers, sellers, and platforms, exhibit a preference for multihome scenarios over single-home cases, irrespective of whether only one side or both sides of the market are multi-homing. Despite the fact that multihoming equilibrium fees are comparatively higher, the resulting aggregate surpluses on both sides of the market are also higher. Furthermore, the increased number of participants in the market provides platforms with additional fees, further boosting their profits.

Chapter 2 presents a framework for analysing how platforms appeal to agents, specifically on buyers' side. We argue that buyers' decisions to join a platform are not only based on membership fees and cross-network effects but also on other attributes platforms offer. The combination of these three elements determines which platform buyers find most appealing. Rather than attempting to capture all the possible features a platform may have, we aim to integrate them into a single variable representing buyers' motivation or perception of the platform. Buyers are more inclined to join a platform that has built a favourable reputation and brand image over time by offering a diverse range of features. As the platform's attributes increase, it enhances buyers' perception of the platform's benefits, leading to a stronger reputation and brand image. This, in turn, increases the likelihood of buyers choosing to join the platform.

By considering platform's attributes in this manner we make an important contribution to the literature (Jullien et al. (2021); Sanchez-Cartas and León (2021)) on two-sided markets with vertical differentiation. Our model builds on the framework of Armstrong (2006), where equilibrium membership fees depend on cross-group network effects, and the literature on vertical differentiation, including Mussa and Rosen (1978); Gabszewicz and Thisse (1979); Shaked and Sutton (1982, 1983), which identify consumer income as a source of differentiation. The provision of attributes by platforms creates a competitive advantage in attracting agents in a two-sided market. This competitive advantage can be understood as heterogeneity within a vertical differentiated product space, where agents prefer platforms offering more attributes compared to those offering fewer attributes.

The seminal works of Economides (1989); Neven and Thisse (1989) were the first to jointly examine both horizontal and vertical product differentiation spaces. Horizontal differentiation pertains to the range of products offered, while vertical differentiation refers to the quality of the products sold in the market. Both studies yield comparable results, showing that firms maximise one dimension (variety) while minimising the other characteristic (quality) to gain a larger market share and increase profits. Building on these findings, Irmen and Thisse (1998) extended the previous models to include multiple characteristics and report similar results, indicating that firms choose to maximise differentiation in the dominant characteristic and minimise the remaining attributes to reduce price competition.

In this chapter, we extend Armstrong (2006) model by introducing the level of features offered on buyers' side as a strategic variable on the vertical dimension. This allows for the existence of asymmetric platforms in equilibrium, as shown by Gabszewicz and Wauthy (2014). Our model consists of two stages, where agents only join one platform (singlehome) and platforms simultaneously determine the level of attributes they offer on buyers' side in the first stage, and then determine membership fees in the second stage. We find equilibrium membership fees are similar to Armstrong (2006) result, but are adjusted by the differences in attributes offered by platforms on buyers' side, and weighted by the cross-group network effect one side exercise on the other side.

Recent studies have explored the intersection of two-sided markets and vertical differentiation. For instance, Gabszewicz and Wauthy (2014) introduced heterogeneity among participants and found that platform competition with cross-group externalities and vertical differentiation can result in the equilibrium coexistence of asymmetric platforms. Zennyo (2016) investigated vertically differentiated two-sided markets and found that in a sequential game, both platforms charged the same per-transaction fee in equilibrium, even with quality asymmetries. Under certain conditions, a low-quality platform was found to have higher profits than a high-quality platform. Roger (2017) studied two-sided markets where platforms compete for agents on both sides of the market, and concluded that when cross-group externalities are too strong, pure-strategy equilibrium may not exist. Lastly, Etro (2021) considered the differences between device-funded and ad-funded platforms. His results showed that device-funded platforms are more aligned with consumers because they provide high-quality products and services, while ad-funded platforms offer products at competitive prices and free services.

One of our main results is that the difference in attributes on buyers' side between two competing platforms not only affects their behaviour but also has an impact on sellers' side as a result of the presence of cross-group network effects on both sides of the market.

We establish conditions for a max-min strategy to enhance profits, as demonstrated in the early works of Economides (1989) and Neven and Thisse (1989) and the generalised model of Irmen and Thisse (1998). Specifically, we identified two scenarios where such a strategy is effective: when the cross-group network effects on both sides of the market are equal, and when the cross-group network effect buyers have on sellers is greater than the impact sellers exert on buyers. In the former situation, platforms differentiate themselves as much as possible on attributes on buyers' side (vertical dimension) and as little as possible on the product differentiation cost (horizontal dimension). In the latter setting, platforms differentiate themselves as little as possible on attributes on buyers' side and as much as possible on the horizontal dimension to maximise profits. Furthermore, we find conditions for a max-max strategy to maximise profits, as seen in recent studies by Garella and Lambertini (2014); Barigozzi and Ma (2018). In particular, we find platforms differentiate as much as possible on both dimensions when the cross-group network effect exerted by sellers on buyers outweighs those exercised by buyers on sellers.

Chapter 3 investigates how introducing quality as a vertical differentiation catalyst in a two-sided market affects participants' decision to switch amongst intermediaries and the effects on pricing strategies.

This analysis is motivated by considering that buying products or services online requires additional considerations compared to traditional brick-and-mortar stores. Online shopping presents the initial challenge of selecting the appropriate digital marketplace. Choosing between *Uber* or *Cabify* for transportation, *Just Eat* or *Deliveroo* for meals, or *Booking* or *Skyscanner* for lodging reservations can be overwhelming. One practical approach to this issue is to experience various online stores or platforms to determine which best suits one's expectations and performance standards. If the platform is not intuitive and basic functions such as registration processes or loading times are cumbersome, consumers may opt to switch to an alternative platform or online marketplace for their shopping needs.

However, after developing a business relationship with a service provider, transitioning to an alternative supplier can prove to be a challenging task. This can be attributed to the notion of switching costs, which refers to the barriers incurred by customers during a shift from one service provider to another. In the switching cost literature it has been identified by Klemperer (1987a) three types of costs that customers may incur when deciding to switch brands or products and services. The first type of switching cost is transaction costs, which arise due to the time, effort, and expense involved in researching, evaluating, and purchasing a new product or service. The second type is learning costs, which result from the need to acquire new knowledge or skills in order to use the alternative product or service effectively. Finally, artificial costs may arise due to the contractual or technological barriers that firms may create to impede customers from switching to competitors.

The model consists of two platforms competing for members. Platforms offer attributes on buyers' side and buyers and sellers engage with a platform to facilitate their transactions. However, given the initial uncertainty surrounding the quality of each platform, buyers must first join and engage with a platform to gain a firsthand understanding of its respective features and characteristics before deciding to switch to a different intermediary. Therefore, we introduce quality uncertainty on buyers' side and consider a two-period setting where platforms choose simultaneously membership fees in the first period then buyers and sellers choose which platform to join. At the end of the first period, buyers correctly evaluate platforms' quality. In the second period, buyers decide whether to switch or not conditional on the switching cost.

Some of the first research to study the implications of consumer switching costs were von Weizsacker (1984), Klemperer (1987a,b) and Farrell and Shapiro (1988). These studies have revealed two opposing effects that firms experience with their pricing strategies. On the one hand, firms are incentivised to charge higher prices to customers who are locked into their products or services, while on the other hand, they aim to charge lower prices to attract new customers. The prevailing incentive, according to these studies, is to charge higher prices, which can result in anti-competitive outcomes when compared to markets that do not have switching costs. A comprehensive review of the literature on switching costs can be found in Klemperer (1995), Farrell and Klemperer (2007) and Villas-Boas (2015).

There has been limited research on the topic of switching costs in two-sided markets. However, recent contributions by Lam (2017) and Tremblay (2019) have provided insight into this issue. Lam (2017) has shown that in the presence of strong cross-group network effects, the result where fees fall in the first period and rise in the second period as switching costs increase does not hold. The reason is because of the interaction between cross-group network effects and switching costs. Instead, she finds that the first-period fee always decreases with increasing switching costs and increasing switching costs on one side leads to a decrease in fees on the other side. In contrast, Tremblay (2019) has identified a different pattern of results. He finds that endogenous switching costs can lead to platforms subsidising content provision in the first period, rather than discounting consumer prices. This is because having more content providers in the first period generates a larger consumer lock-in, which leads to higher markups for consumers in the second period.

We find that when buyers have higher expectations than the actual quality of service offered by a platform they initially visit, they may opt to switch to another platform, taking into account the associated switching costs. On the contrary, when buyers underestimate platform's quality, they might choose to stay with the same provider. In such cases, the platform adopts a pricing strategy to reward their loyalty decreasing buyers and increasing sellers membership fees. Additionally, the platform recognises the significant influence of buyers on sellers and leverages this effect to attract more sellers to join its platform.

In light of our analysis, we have observed that platforms, anticipating the impact of higher switching costs as a deterrent for buyers to switch, implement a pricing strategy that involves reducing fees on the side of the market where the cross-group network effect is more influential. As a result, buyers experience a decrease in their fees.

The platform's pricing strategy is strategically implemented to foster its growth potential by attracting a larger user base. By lowering membership fees for buyers, the platform creates incentives for more buyers and sellers (given the cross-group network effects) to join and engage with one another. This increased participation in turn amplifies the network effects, as a larger number of buyers and sellers connect and benefit from the platform's services.

Under certain conditions, our findings indicate that platforms can experience an increase in profits when buyers underestimate the quality of the platform they initially visit and when the influence buyers have on sellers outweighs the influence sellers have on buyers.

Furthermore, our analysis highlights that the impact of an increase in switching costs on equilibrium profits depends on the relative magnitude of its effects on membership fees and market shares. If the effect is greater on membership fees, we observe an increase in profits. Conversely, if the effect is larger on market shares, equilibrium profits decrease. This emphasises the complex relationship between switching costs, quality uncertainty and cross-group network effects in determining the overall performance of the platform's profits.

Chapter 1

Bandwagon, snob or congestion effects and sellers' competition in Two-Sided Markets

1.1 Introduction

An increasing number of businesses are adopting a two-sided market model. This new business model has two distinct user groups interacting and deriving value from each other's participation. In such a market, a platform acts as an intermediary, facilitating transactions, interactions, or exchanges between the two sides.

In certain two-sided markets, participants on one side are concerned not only with the other side but also with those on their own side. These markets exhibit intra-group or direct network effects. Negative intra-group effects can arise in various scenarios. For instance, in the real estate brokerage industry, where property sellers benefit from more potential buyers joining the broker, but they may experience a decline in sales opportunities when more properties are advertised, resulting in a reduced likelihood of selling their property. It is widely recognised that competition among sellers reduces their profits, resulting in a negative direct network effect on sellers' side. However, buyers may benefit from sellers' competition, as it can lead to lower prices and a wider range of products. On sellers' side, positive intra-group external effects may arise in situations such as charity donations, where a larger group of contributors can help a greater number of people, thus benefiting the contributors by increasing their satisfaction with the act of helping others.

From the perspective of buyers, the presence of other consumers in a two-sided market can have both positive and negative effects on their utility. A "bandwagon effect" may occur, as defined by Leibenstein (1950), whereby the utility of purchasing goods increases with the number of other consumers purchasing the same good. Conversely, a "congestion effect" may arise, where buyers may be worse off if they shop in a crowded location, such as a shopping mall, and prefer a less crowded environment.[1] Additionally, buyers may experience what Leibenstein (1950) defined as a "snob effect" when they seek to purchase exclusive goods that have not yet been purchased by others.

Rochet and Tirole (2002) were among the pioneers to investigate the impact of within-network external effects in a two-sided market setting. Specifically, they examined a monopoly platform in the context of a payment card association allowing competition among merchants to determine the optimal access charges. Belleflamme and Toulemonde (2009) compared positive cross-group effects with negative within-group effects to establish the possibility of a competing platform. Hagiu (2009) introduced seller competition to the model, given that consumers prefer product variety, and used this to derive the optimal platform pricing structures. Belleflamme and Toulemonde (2016) incorporated sellers' within-group external effects in a two-sided singlehoming environment and analysed the game's equilibrium based on buyer-seller relationship outcomes. Lastly, Belleflamme and Peitz (2019a) explored how seller competition affects platform decisions and market structure.

Evans and Schmalensee (2005) posited that multihoming, which refers to the practice of participants connecting to more than one platform or intermediary, is a significant determinant of the size and structure of a two-sided market, in addition to indirect network effects. Singlehoming occurs when agents choose to use only one platform, while multihoming allows them to use more than one platform. A common example of multihoming is in network television, where viewers and advertisers often connect to multiple channels. Similarly, in payment cards and video game platforms, users may choose to connect to multiple providers.

This chapter makes a significant contribution by presenting a unified framework that incorporates direct and cross-group network effects, as well as the presence of singlehoming and multihoming agents. The originality lies in the comprehensive analysis of these elements and their simultaneous interactions within a coherent model. We address the combined effects of different types of network effects occurring simultaneously on both sides of the market, while also considering the choices of agents to singlehome or multihome.

The study is motivated by two key factors: first, the growing prevalence of multihoming in numerous two-sided markets, often resulting from decreasing joining costs; and

[1] This is a negative direct externality, where more buyers make a product less valuable, often referred as congestion, as in examples like traffic congestion or network congestion.

second, the observation that direct network effects are commonly experienced by both sides of the market. Sellers face competition from one another, while many buyers take into account the purchasing behaviour of other buyers, either due to conformity with the masses or to extract value (such as information) from the crowd. However, buyers can also exhibit the opposite tendency, seeking out niche or exclusive markets, such that their demand negatively correlates with market demand.

To the best of our knowledge, no existing study has examined the intricate interactions between these various elements in such depth. By integrating direct and cross-group network effects, along with the consideration of singlehoming and multihoming behaviours, this chapter provides a complete perspective on platform dynamics. The findings and insights from this analysis can shed new light on the complex dynamics of two-sided markets, enabling a better understanding of the interplay between direct and indirect network effects and agents' choices of participation on single or multiple platforms.

The framework can be applied to various industries, including e-commerce, online advertising, and sharing economy platforms, to provide insights into the strategies that platforms can employ to attract and retain participants. Moreover, we give details on how platform strategies can be tailored to leverage the strengths of the bandwagon and congestion effects along with competition on sellers' side and thereby maximise platform profits. Additionally, the study provides a deeper understanding of the drivers behind multihoming behaviour in two-sided markets, which has important implications for platform competition and economic welfare.

We find in a scenario where only one side of the market engages in multihoming while the other side is singlehoming, the multihoming agents do not consider the direct network effect, whether positive or negative, in their equilibrium membership fee. This phenomenon can be explained by the fact that when both participants decide to multihome, platforms cease competing for their attention, unlike when they were singlehoming, thereby decreasing the influence of within-network effects on platforms' strategy of setting fees.

Furthermore, a bandwagon effect is observed when the participation of buyers and sellers on a platform reinforces each other, leading to a positive feedback loop. On the other hand, when congestion or competition is experienced on buyers' and sellers' sides respectively, it can have an adverse impact on platforms' performance. The reason for this is the negative intra-group network effect leads to a reduction in the proportion of participants joining the platforms, which in turn, reduces the platforms' value and the aggregate surpluses on both sides of the market.

In addition, we find that the adjustment of membership fees for both sides of the market is influenced by various factors when both agents multihome, such as the platforms' desire to attract and retain buyers and sellers, the intra-group network effect, the relative strength of cross-group network effects, and revenue growth potential. In the presence of a bandwagon effect, platforms attract more buyers, making it less difficult to appeal to sellers with buyers (now there are more buyers attracting sellers given the cross-group network effects). Thus, platforms can reduce buyers' fees and increase sellers' fees to compensate for their strategy to increase revenue. This approach is effective when buyers' cross-group network effect on sellers is stronger than the cross-group network effect sellers have on buyers. Conversely, a congestion effect and competition among sellers lead to the opposite impact.

Moreover, platforms also realise the potential for more revenue on buyers' side, leading to reduce sellers' fees to attract them and subsequently attract more buyers considering the cross-group network effects. Additional revenue can then be generated by increasing buyers' fees, which is effective when sellers have a stronger influence on attracting buyers than vice versa.

As an illustration, in the context of ride-hailing services, if all riders utilise both *Uber* and *Cabify*, then drivers would only need to partner with one platform to access the entire pool of potential customers. Consequently, *Uber* and *Cabify* would not need to compete for riders, but instead, they would have to compete more intensively to attract drivers. However, if drivers are more inclined to multihome than riders, their strategies would need to be different. Moreover, drivers face competition from each other, and the presence of additional drivers could lead to reduced earnings due to fewer available rides. At the same time, riders may be more likely to trust a ride-hailing service with a larger user base, resulting in a bandwagon effect on the demand side. Another instance is shopping centres, where both buyers and sellers can multihome and face competition among themselves.[2] Buyers may experience benefits from having more buyers up to a certain point, but this positive direct network effect could transform into a negative congestion effect. Similar dynamics play out in the realm of dating apps, where both buyers and sellers can multihome, resulting in competition among themselves.

This chapter builds upon the work of Armstrong (2006) and extends the model introduced by Belleflamme and Peitz (2019b) to incorporate intra-group externalities and multihoming on both sides of the market. The objective is to investigate the impact of three different market scenarios on the model's equilibrium and compare them with a benchmark scenario where both buyers and sellers singlehome, namely: (*i*) when both sides singlehome, (*ii*) when buyers singlehome and sellers multihome, (*iii*) when buyers multihome and sellers singlehome, and (*iv*) when both sides multihome.

We compare the characteristics of the equilibrium that arises within the four sce-

[2]Buyers can shop in both shopping centres if the distance between them is not too far to go.

narios. Our results are as follows. We find that participants on one side (e.g., buyers) prefer that the other side (e.g., sellers) multihome when they singlehome because platforms charge them a lower fee than when both sides singlehome. This implies a greater aggregate surplus on the multihome side. Conversely, the other side (e.g., sellers) prefers the opposite and under certain conditions, platforms obtain higher profits.

Moreover, all market participants, including buyers, sellers, and platforms, exhibit a preference for multihoming scenarios over singlehome scenarios. Despite the fact that multihoming equilibrium fees are comparatively higher, the resulting aggregate surpluses on both sides of the market are also higher because multihoming impacts the proportion of buyers and sellers interacting. Furthermore, the extra participants provide platforms with additional fees, further boosting their profits.

The remainder of the chapter is structured as follows. Section 1.2 outlines the model and provides definitions of participants. Section 1.3 presents the results obtained under the benchmark assumption of both sides singlehoming. Section 1.4 extends the analysis to consider situations where sellers multihoming and buyers singlehoming. Section 1.5 examines scenarios where buyers multihoming and sellers singlehome. In Section 1.6, the analysis is further extended to consider situations where both sides multihome. Section 1.7 provides a comparative analysis of the results obtained in the previous scenarios. Finally, in Section 1.8 we conclude with a summary of the main findings.

1.2 Model

This chapter presents a platform competition model that incorporates intra- and inter-group network effects. The model features three distinct players, namely platforms, buyers, and sellers. Building upon Armstrong (2006) and the extension of his work by Belleflamme and Peitz (2019b), we further incorporate seller competition as in Hagiu (2009). However, we adopt a different surplus structure and allow for direct network effects to operate on both buyers' and sellers' sides within a multihoming environment on both sides of the market.

The game players are:

1. **Platforms**.
   In this model, two platforms, platform 1 and platform 2 compete to attract buyers (b) and sellers (s) by setting membership fees, following the approach in Armstrong (2006). The platforms are located at the opposite ends of a unit interval and exhibit horizontal differentiation similar to Hotelling's model.[3] Platforms incur a cost of f_b

[3]Platforms strategically exhibit horizontal differentiation by positioning their service/product along a linear continuum to distinguish themselves from competitors. Imagine a line representing the product

for serving buyers and f_s for serving sellers.[4]

2. **Buyers**.
   Buyers are uniformly distributed on a unit interval. To visit a platform, buyers incur a transportation cost or disutility because of mismatched preferences of τ_b per unit of length. A buyer located at x_b incurs a transportation cost of $\tau_b x_b$ to go to platform 1 and a transportation cost of $\tau_b (1 - x_b)$ to go to platform 2. In case they multihome, where buyers visit both platforms, the transportation cost is $\tau_b x_b + \tau_b (1 - x_b) = \tau_b$, which remains independent of buyer location.

3. **Sellers**.
   Sellers are uniformly distributed along the same unit interval incurring a transportation cost or disutility because of mismatched preferences of τ_s per unit of length for visiting a platform. A seller located at x_s incurs a transportation cost of $\tau_s x_s$ to go to platform 1 and a transportation cost of $\tau_s (1 - x_s)$ to go to platform 2. In case they multihome, where sellers visit both platforms, the transportation cost is $\tau_s x_s + \tau_s (1 - x_s) = \tau_s$, independent of seller location.[5]

Buyers and sellers can only engage in trade by interacting on a platform. This implies that buyers can only purchase a product unit offered by each seller through a platform, and likewise, sellers can only sell their products to a buyer through a platform. There exists a cross-group network effect sellers exert on buyers, υ, and buyers exert on sellers, π, where $\upsilon, \pi > 0$ and $\upsilon \neq \pi$.[6]

In addition, there exists an intra-group externality or direct network effect among buyers, denoted by α. On buyers' side, this direct network effect can take two forms: a

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space, where buyers and sellers are distributed along this line. Platforms recognise that buyers and sellers tend to prefer services that are closer to them, reflecting their similarity or proximity. To gain a competitive edge, platforms strategically choose specific locations along this linear space. The strategic goal is to capture a share of the market by appealing to a specific segment of buyers and sellers with preferences aligned to their chosen service/product characteristics.

The competitive dynamics in Hotelling's model create a scenario of horizontal differentiation where services/products appear somewhat homogeneous, yet buyers and sellers make choices based on subtle distinctions and the proximity of offerings. This strategic positioning allows platforms to navigate buyers' and sellers' preferences and maximise market share within the limitations of a one-dimensional product space.

[4]For a textbook explanation please refer to Chapter 22.3 Intermediaries as two-sided platforms in Belleflamme and Peitz (2015).

[5]d'Aspremont et al. (1979) showed that with linear transportation costs the presence of a discontinuity in the demand function does not impact the existence of a pure-strategy price equilibrium, provided that firms are not situated near each other. Therefore, in our model where platforms are located at the extremes of the unit line, this problem does not arise.

[6]Chu and Manchanda (2016) and Milone (2022) estimated cross-group network effects on the e-commerce platform *Taobao* and *Airbnb*, respectively. Their findings reveal significant differences in the magnitudes of cross-group network effects. Specifically, on *Taobao*, the supply-to-demand cross-group network effects were found to be four times greater than the demand-to-supply impacts. On the other hand, in the case of *Airbnb*, the indirect network effect of supply-to-demand was discovered to be six times larger than the cross-group network effect exerted by the demand side on the supply side.

positive effect ($\alpha > 0$), which we refer to as a bandwagon effect, and a negative effect ($\alpha < 0$), which we label as a snob or congestion effect. The bandwagon effect describes a phenomenon where the popularity or adoption of a product or service increases as more individuals or users adopt it. Conversely, the snob or congestion effect pertains to individuals displaying a preference for unique, exclusive, or uncommon products or services as a means of differentiating themselves from others. The term "congestion" is used to signify a situation where a particular platform becomes overcrowded or blocked, hindering the ease of joining.[7]

Furthermore, there exists a within-group or direct network effect among sellers, represented by β, which we define as sellers' competition. Given that sellers' competition tends to lower prices, we assume that the direct network effect on sellers' side is negative ($\beta > 0$).

Buyers and sellers receive a stand-alone benefit of R_b and R_s, respectively, from interacting on either platform 1 or 2, and the benefits are equal across platforms.[8] Multihoming agents receive the sum of the stand-alone benefits, $2R_b$ on buyers' side and $2R_s$ on sellers' side. The fraction of buyers and sellers who choose to interact on each platform are denoted by η_b^i and η_s^i, $i = 1, 2$, respectively. Platform i, $i = 1, 2$ charges a membership fee of p_b^i and p_s^i on buyers' and sellers' side, and there is no transaction fee, which is a common assumption when tracing transaction volumes is difficult or expensive.

A buyer or a seller visiting platform i, $i = 1, 2$ obtains a surplus, ν_b^i and ν_s^i equal to the stand-alone benefit, R_b and R_s, the cross-group network effect the other side is exerting on this side times the fraction of participants on the other side, $\upsilon\eta_s^i$ and $\pi\eta_b^i$, the direct within-group externality times the fraction of participants on this side, $\alpha\eta_b^i$ and $\beta\eta_s^i$, and the membership fee, p_b^i and p_s^i.

$$\nu_b^i = R_b + \upsilon\eta_s^i + \alpha\eta_b^i - p_b^i \tag{1.1a}$$

$$\nu_s^i = R_s + \pi\eta_b^i - \beta\eta_s^i - p_s^i \tag{1.1b}$$

The cross-group network effects υ and π can be seen as gains from trade because they are the spill-over gains obtained by interacting with the other side. Specifically, in Equation 1.1a υ represents buyers' gain from trade with sellers η_s^i, while in Equation 1.1b π represents sellers' gain from trade with buyers η_b^i.

[7] Henceforth, the terms "snob" and "congestion" will be used interchangeably.

[8] Joining a platform offers standalone benefits beyond just interacting with the other side. In the case of online marketplaces, platforms prioritise trust and security through features like reviews, ratings, and payment protection. Buyers can use these features to confidently make transactions and reduce risks associated with online purchases. Additionally, joining specific platforms may grant access to exclusive deals and discounts. Similarly, smartphones offer various functionalities beyond third-party content, including making calls, checking email, and browsing the web.

The direction of the intra-group externality varies depending on the market side and can be either positive or negative. Specifically, when buyers' side experiences a positive direct network effect, commonly referred to as a bandwagon behaviour, the parameter α in Equation 1.1a takes on a positive value, indicating an increase in buyers' surplus. Conversely, when a negative direct network effect, known as a snob/congestion impact α becomes negative, indicating a decrease in sellers' surplus. On the other hand, sellers' surplus is captured in Equation 1.1b. Furthermore, considering $\beta > 0$, the direct network effect becomes a negative component in sellers' surplus, contrary to how a within-network influence is denoted in buyers' surplus.

When agents multihome the surplus is the sum of the singlehome surpluses,

$$\nu_b^{1,2} = 2R_b + \upsilon\left(\eta_s^1 + \eta_s^2\right) + \alpha\left(\eta_b^1 + \eta_b^2\right) - \left(p_b^1 + p_b^2\right) \tag{1.2a}$$

$$\nu_s^{1,2} = 2R_s + \pi\left(\eta_b^1 + \eta_b^2\right) - \beta\left(\eta_s^1 + \eta_s^2\right) - \left(p_s^1 + p_s^2\right) \tag{1.2b}$$

There are alternative methods for modelling surpluses when participants engage in simultaneous multihoming. For instance, Bakos and Halaburda (2020) examine a scenario where participants interacting on both platforms experience a cross-group network effect only once. However, we adhere to the convention established by Belleflamme and Peitz (2019b).

Consistent with the existing literature (Armstrong (2006); Rochet and Tirole (2003); Jullien et al. (2021)) on two-sided markets, we assume that the stand-alone benefit is sufficiently significant to ensure a positive surplus on both sides of the market.

Using more general specifications to model two-sided markets, such as a nonlinear specification as in Salop (1979), is unnecessary for our analysis because we are not examining the location of platforms in the market. Salop (1979) circular model addresses difficulties that arise when firms are located at the endpoints of the unit line with linear transportation costs. These difficulties can be avoided with quadratic transportation costs or by positioning the firms at the ends of the unit line, as mentioned by d'Aspremont et al. (1979).

Similarly, employing more general transportation costs, such as nonlinear costs would introduce problems with the existence of equilibrium as proposed by Matsumura et al. (2005). However, equilibrium exists when firms are positioned at the endpoints of the unit line, as analysed by d'Aspremont et al. (1979) and specifically identified for two-sided markets by Armstrong (2006).

Additionally, using a general consumer (buyers and sellers in our case) distribution instead of a uniform distribution would cause firms to concentrate in high-density regions and charge lower prices, as found by Shilony (1981). This would introduce an extra layer of complexity, as membership fees would be influenced by factors other than cross-group and direct network effects or homing decisions.

Therefore, using a simpler specification such as Hotelling linear model to analyse two-sided markets allows us to isolate and highlight the key mechanisms affecting membership fees and platforms' profits when direct network effects are introduced and when participants (buyers or sellers) are allowed to multihome, without the confounding effects of additional complexity. This can lead to more intuitive insights and clearer conclusions.

The parameters in the different scenarios that we are analysing must meet the following assumptions.[9]

Assumption 1.1. $4(\tau_b - \alpha)(\tau_s + \beta) > (\pi + \upsilon)^2$

Assumption 1.2. $3(\tau_b - \alpha) < 2(R_b - f_b) < 4(\tau_b - \alpha)$

Assumption 1.3. $3(\tau_s + \beta) < 2(R_s - f_s) < 4(\tau_s + \beta)$

Assumption 1.1 is developed on the second-order conditions for the concavity of the platform profits function. It is sufficient for the second-order conditions to be satisfied across the four scenarios we are analysing. This condition requires the transportation cost τ_b and τ_s and the direct network effects α and β, on buyers' and sellers' side, greater than the cross-group network effects υ and π on both sides of the market. This condition further guarantees that the number of buyers and sellers decreases not only with their respective side's membership fee but also with the fee on the opposite side of the platform. Failure to meet this condition would lead all buyers and sellers to prefer the same platform because the fraction of participants on one platform would be an increasing function of their membership fee and the market would tip.[10]

The lower bound of Assumption 1.2 and Assumption 1.3 are obtained from the net surplus[11], and the upper bound is obtained from the participation of all buyers and sellers when a fraction of them multihome.[12] These assumptions are sufficient for the indifferent buyer and seller to have a positive net surplus at equilibrium. This means that buyers and sellers don't have the option to choose not to participate actively in the market,

[9]For further details on the different assumptions see Appendix A.1.

[10]Obtain the partial derivative of Equation 1.5a and Equation 1.5b when both buyers and sellers singlehome, Equation 1.14a and Equation 1.14b when sellers multihome and buyers singlehome, Equation 1.24a and Equation 1.24b when buyers multihome and sellers singlehome, and Equation 1.33a and Equation 1.33b when both buyers and sellers multihome.

[11]Compute $\nu_b - \frac{\tau_b}{2}$ and $\nu_s - \frac{\tau_s}{2}$ and derive conditions when they are positive using Equation 1.1a and Equation 1.1b when buyers and sellers are singlehoming, and Equation 1.2a and Equation 1.2b when they are multihoming.

[12]Compute η_b and η_s and obtain conditions when $\frac{1}{2} < \eta_b < 1$ and $\frac{1}{2} < \eta_s < 1$ using Equation 1.17 when sellers multihome and buyers singlehome, Equation 1.27 when buyers multihome and sellers singlehome and Equation 1.36a and Equation 1.36b when both buyers and sellers multihome.

therefore the market is fully covered. Furthermore, as we impose that some buyers and sellers multihome at equilibrium, then all buyers and all sellers participate (i.e., the ones that do not multihome, singlehome). Assumption 1.2 establishes both a lower and upper limit on the difference between buyers' standalone benefit and the platform's cost of serving buyers, $R_b - f_b$. This limit is defined by the transportation cost and the direct network effect on the buyers' side $\tau_b - \alpha$. Similarly, Assumption 1.3 sets a lower and upper boundary on the difference between sellers' standalone benefit and the platform's cost of serving sellers $R_s - f_s$, determined by the transportation cost and the direct network effect on the sellers' side, $\tau_s + \beta$.

In the subsequent sections of this chapter, we represent equilibrium market structures where we use the following specific notations to indicate particular equilibrium configurations. In Section 1.3, we use the superscript "*sh*" to denote the equilibrium values when both buyers and sellers singlehome. Section 1.4, we use the superscript "*smh*" to denote equilibrium values for the situation where sellers multihome and buyers singlehome. Similarly, in Section 1.5, we utilise the superscript "*bmh*" to denote equilibrium values for the situation where sellers singlehome and buyers multihome. Lastly, in Section 1.6, we use the superscript "*mh*" to indicate equilibrium values for the situation where both buyers and sellers multihome.

In this chapter, we adopt a two-stage game framework. In the first stage, both platforms simultaneously set membership fees p_b, p_s for both sides of the market. In the second stage, buyers and sellers simultaneously determine which platform to visit for trading. The subgame-perfect Nash equilibrium is computed to solve the game.

1.3 Benchmark Scenario: Both buyers and sellers singlehome

In this section, we present a two-sided market model that incorporates intra-group network effects under the assumption that both sides of the market are singlehoming. The singlehome setting may arise due to various factors, such as the high cost of connecting to multiple platforms[13], indivisibility constraints, or contractual restrictions. For instance, limited resources could make it prohibitively expensive for agents to connect to more than one platform, while indivisibility constraints prevent buyers and sellers from physically locating in more than one market, such as in the case of a farmers' market. Furthermore, streaming platforms may enter into contractual agreements with renowned

[13]The surplus is positive when agents choose to singlehome, due to the significant stand-alone benefit. However, when agents join more than one platform or engage in multihoming, the surplus is not guaranteed to be positive.

film/TV directors to obtain exclusive content.[14]

In the case where an agent, denoted by $k = b, s$ chooses to singlehome, the agent is indifferent between two platforms at the location x_k. Specifically, this location satisfies:

$$\nu_k^1 - \tau_k x_k = \nu_b^2 - \tau_k (1 - x_k)$$
$$\nu_k^1 - \nu_k^2 + \tau_k = 2\tau_k x_k$$

$$x_k = \frac{1}{2} + \frac{\nu_k^1 - \nu_k^2}{2\tau_k} \tag{1.3}$$

Each group of market participants, divided into platforms 1 and 2, is characterised by the agents' location between 0 and 1, where agents located between 0 and x_k visit platform 1, and those located between x_k and 1 visit platform 2. Thus, η_k^1 and η_k^2 denote the fraction of agents on each side of the market, where $\eta_k^1 = x_k$, $\eta_k^2 = (1 - x_k)$, and $\eta_k^1 + \eta_k^2 = 1$, for $k = b, s$. Incorporating the surplus of agent k obtained from Equation 1.1a and Equation 1.1b within Equation 1.3 yields the proportion of buyers and sellers on platform i, with $i, j = 1, 2$, $i \neq j$, as follows:[15]

$$\eta_b^i = \frac{\tau_b + (2\eta_s^i - 1)\,\upsilon - \alpha + \left(p_b^j - p_b^i\right)}{2\,(\tau_b - \alpha)} \tag{1.4a}$$

$$\eta_s^i = \frac{\tau_s + (2\eta_b^i - 1)\,\pi + \beta + \left(p_s^j - p_s^i\right)}{2\,(\tau_s + \beta)} \tag{1.4b}$$

To obtain the market shares based on membership fees, it is necessary to solve the system of Equation 1.4a and Equation 1.4b, resulting in:

$$\eta_b^i\left(p_b^i, p_s^i, p_b^j, p_s^j\right) = \frac{1}{2} + \frac{\upsilon\left(p_s^j - p_s^i\right) + (\tau_s + \beta)\left(p_b^j - p_b^i\right)}{2\left[(\tau_s + \beta)(\tau_b - \alpha) - \pi\upsilon\right]} \tag{1.5a}$$

$$\eta_s^i\left(p_b^i, p_s^i, p_b^j, p_s^j\right) = \frac{1}{2} + \frac{\pi\left(p_b^j - p_b^i\right) + (\tau_b - \alpha)\left(p_s^j - p_s^i\right)}{2\left[(\tau_s + \beta)(\tau_b - \alpha) - \pi\upsilon\right]} \tag{1.5b}$$

Assumption 1.1 ensures both platforms are active. As we can see in Equation 1.5a and Equation 1.5b, this assumption ensures buyers' and sellers' market shares decrease not only when their own side's membership fee increases but also when the fee on the other side increases too.[16]

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[14] According to theverge.com American filmmaker J.J. Abrams signed an exclusivity deal contract with WarnerMedia.

[15] For further details on how to solve the system of Equation 1.4a and Equation 1.4b, and also the system of Equation 1.5a and Equation 1.5b see Appendix A.2.1

[16] Let us consider the partial derivatives of Equation 1.5a and Equation 1.5b with respect to p_k^i, where $k = b$ or s. For instance, we can compute $\partial\eta_b^i/\partial p_b^i = \frac{-(\tau_s+\beta)}{2[(\tau_s+\beta)(\tau_b-\alpha)-\pi\upsilon]}$. To check that the previous expression is negative, we examine the denominator, as $\tau_s + \beta$ is always positive. We can use Assumption

Market Equilibrium

In this subsection[17], we explore the pricing strategy employed by platforms in determining their membership fees. Our objective is to gain insights into how platforms strategically set their fees to leverage both the intra and inter-network effects. Furthermore, we aim to understand why platforms, in their equilibrium state, adopt a differentiated pricing approach for each side of the market.

Definition 1.1. *A symmetric equilibrium is a pair p_b^{sh}, p_s^{sh}, such that p_b^{sh} and p_s^{sh} solves platform maximisation problem* $\max_{\{p_b^i, p_s^i\}} \Pi^i = (p_b^i - f_b)\, \eta_b^i\, (p_b^i, p_s^i, p_b^j, p_s^j) + (p_s^i - f_s)\eta_s^i (p_b^i, p_s^i, p_b^j, p_s^j)$ *for each* $i, j = 1, 2,\ i \neq j$.

From the first-order conditions of a symmetric equilibrium $p_b^i = p_b^j = p_b^{sh}$ and $p_s^i = p_s^j = p_s^{sh}$, we obtain the following best response functions:[18]

$$p_b^{sh} = f_b + (\tau_b - \alpha) - \frac{\pi}{(\tau_s + \beta)} (\upsilon + p_s - f_s) \tag{1.6a}$$

$$p_s^{sh} = f_s + (\tau_s + \beta) - \frac{\upsilon}{(\tau_b - \alpha)} (\pi + p_b - f_b) \tag{1.6b}$$

In determining the best response fees on buyers' and sellers' side, various factors are taken into account, including the platform cost of serving buyers (f_b) or sellers (f_s), the mismatched preferences disutility on buyers' side (τ_b) and on sellers' side (τ_s), the direct network effect on buyers' side (α) and on sellers' side (β), and the value of an additional participant from the other side, on buyers' side $\frac{\pi}{(\tau_s+\beta)}(\upsilon + p_s - f_s)$ and on sellers' side $\frac{\upsilon}{(\tau_b-\alpha)}(\pi + p_b - f_b)$.

For instance, sellers' best response fee is adjusted downwards by the factor $\frac{\upsilon}{\tau_b-\alpha}\big(\pi + p_b - f_b\big)$. As we can see from Equation 1.4a an additional seller draws in $\partial\eta_b^i/\partial\eta_s^i = \frac{\upsilon}{\tau_b-\alpha}$ additional buyers generating π per sellers and yielding a profit margin of $(p_b - f_b)$. Thus, the value of an additional buyer to the platform is given by the term $\frac{\upsilon}{\tau_b-\alpha}(\pi + p_b - f_b)$.

There are two main distinctions between the best response functions obtained by Armstrong (2006) and those presented in our model in Equation 1.6a and Equation 1.6b. Specifically, it should be noted that the first-order condition of Armstrong (2006) using our parameters are:

$$p_b^{sh} = f_b + \tau_b - \frac{\pi}{\tau_s} (\upsilon + p_s - f_s) \tag{1.7a}$$

1.1 to show the denominator is positive. First, we make the left side of both inequalities equal to compare the right side and show that the right side of Assumption 1.1 is greater. That is $(\pi + \upsilon)^2 > 4\pi\upsilon$ turns to $(\pi - \upsilon)^2 > 0$ if $\pi \neq \upsilon$. Therefore, we can conclude that $\frac{\partial\eta_b^i}{\partial p_b^i} < 0$.

[17] The equilibrium values are denoted by the superscript "*sh*".

[18] Further details can be found in Appendix A.2.2.

$$p_s^{sh} = f_s + \tau_s - \frac{\upsilon}{\tau_b}(\pi + p_b - f_b) \tag{1.7b}$$

The first difference between the present and the seminal model by Armstrong (2006) is that the best response fees are adjusted based on the direct network effect, leading to both upward and downward adjustments. Specifically, platforms' market power[19] τ_b, τ_s is adjusted by the direct network effects, $(\tau_b - \alpha)$ and $(\tau_s + \beta)$.

In scenarios where a bandwagon effect influences buyers, the platform's market power is reduced compared to a situation without direct network effects. This reduction occurs because the platform attracts a larger number of buyers due to the bandwagon effect ($\alpha > 0$), thereby both platforms compete strongly for the same buyers given the platform less market power.In the absence of direct network effects, platforms can typically charge higher fees, given the nearby captive buyers (given τ_b can also be seen as the disutility cost associated with mismatched preferences).

On the other hand, a negative direct network effect leads to an opposite effect. Platforms market power τ_b and τ_s is magnified in the presence of a snob or congestion effect $\alpha < 0$, as well as sellers' competition β because this discourages agents from joining the platform, enabling them to exert greater control and influence over the market.

The second difference pertains to the magnitude of the extra proportion of buyers or sellers that are attracted to the platform, $\frac{\pi}{(\tau_s+\beta)}$ or $\frac{\upsilon}{(\tau_b-\alpha)}$ which is influenced by the impact of the direct network effect. Specifically, the extra proportion of agents is greater or lower than what it would be without the within-network externality. For instance, on buyers' side, an extra proportion of buyers attracts a smaller proportion of additional sellers, given by $\frac{\pi}{\tau_s+\beta}$ when there is a negative direct network effect. Considering there is sellers' competition, fewer sellers feel attracted given an extra buyer joined the platform.

Conversely, on sellers' side, if there is a bandwagon effect an extra seller attracts a larger number of additional buyers. A positive direct network impact creates a positive feedback loop boosting the number of buyers and sellers who join. However, when there is a snob or congestion effect, the extra proportion of buyers is lower, than what it would be without a direct network effect.

Equilibrium membership fees

The next step involves solving the best response functions system of Equation 1.6a and Equation 1.6b to obtain the equilibrium membership fees as a function of the model

[19] The differentiation between platforms for buyers and sellers becomes more evident as transportation costs increase. When τ_b or τ_s increases, both platforms compete less intensely for the same buyers and sellers. The nearby buyer and seller of a platform become more captive, giving the platform greater "market power" that allows it to increase its fees.

parameters.[20]

$$p_b^{sh} = f_b + (\tau_b - \alpha) - \pi \tag{1.8a}$$

$$p_s^{sh} = f_s + (\tau_s + \beta) - \upsilon \tag{1.8b}$$

Following the framework proposed by Armstrong (2006), the equilibrium membership fees for both buyers and sellers are determined by the cost associated with serving a buyer or seller, the transportation cost or disutility incurred by preferences mismatched, and the cross-group network effects exerted by this side on the other side. Additionally, these fees are adjusted by the direct network effects that buyers and sellers exert on themselves.

Specifically, for buyers, the equilibrium membership fee is lower than in the absence of direct network effects if a bandwagon effect is present, given $\alpha > 0$. Conversely, if there is a snob/congestion effect, the equilibrium fee is higher, given $\alpha < 0$. For sellers, the equilibrium membership fee is higher than in the absence of direct network effects, since $\beta > 0$. These results can be intuitively explained by the presence of positive feedback loops in the case of a bandwagon effect and the contrasting dynamics in the case of congestion impact or sellers' competition. When buyers exhibit a bandwagon effect, the increased attraction of buyers to the platform leads to a corresponding increase in the number of sellers joining the platform due to the cross-group network effects. On the other hand, in the case of a negative direct effect, fewer agents are attracted, prompting the platforms to respond by increasing their fees in order to mitigate the impact of reduced participation.

Platform's equilibrium profits

We determine the equilibrium platform profits by employing Equation 1.8a and Equation 1.8b, taking into account that because platforms established identical fees at equilibrium the indifferent buyer and seller are positioned at $\eta_b^{sh} = 1/2$ and $\eta_s^{sh} = 1/2$ respectively, as:

$$\Pi^{sh} \equiv \left(p_b^{sh} - f_b\right)\eta_b^{sh} + \left(p_s^{sh} - f_s\right)\eta_s^{sh}$$

$$\Pi^{sh} \equiv \frac{1}{2}\left[f_b + (\tau_b - \alpha) - \pi - f_b\right] + \frac{1}{2}\left[f_s + (\tau_s + \beta) - \upsilon - f_s\right]$$

$$\Pi^{sh} \equiv \frac{1}{2}\left[(\tau_b - \alpha) - \upsilon + (\tau_s + \beta) - \pi\right] \tag{1.9}$$

Equilibrium profits in our model resemble the structure outlined in Armstrong (2006), characterised by a positive disutility cost associated with mismatched preferences and negative cross-group network effects. However, in our model, the presence of a band-

[20] Further details on how to obtain the equilibrium membership fees can be found in Appendix A.2.2 and for the second-order conditions of the profit-maximisation in Appendix A.2.3.

wagon effect decreases platform profits due to lower membership fees imposed on buyers. Conversely, when a congestion effect is present, along with sellers' competition, equilibrium profits increase as fees are raised on both sides of the market. In the context of direct network effects, platforms adopt a pricing strategy where they decrease fees on the side experiencing a positive intra-network impact while increasing fees on the side with a negative within-network effect.

Welfare

The gross surplus for both agents in equilibrium can be obtained using Equation 1.1a, Equation 1.1b, Equation 1.8a, and Equation 1.8b and taking into account that because platforms established identical fees at equilibrium the indifferent buyer and seller are positioned at $\eta_b^{sh} = 1/2$ and $\eta_s^{sh} = 1/2$ respectively,

$$\nu_b^{sh} = R_b + \upsilon\eta_s^{sh} + \alpha\eta_b^{sh} - p_b^{sh} \qquad \nu_s^{sh} = R_s + \pi\eta_b^{sh} - \beta\eta_s^{sh} - p_s^{sh}$$
$$= R_b + \upsilon\eta_s^{sh} + \alpha\eta_b^{sh} - (f_b + \tau_b - \alpha - \pi) \qquad = R_s + \pi\eta_b^{sh} - \beta\eta_s^{sh} - (f_s + \tau_s + \beta - \upsilon)$$

$$\nu_b^{sh} = (R_b - f_b) - \tau_b + \frac{1}{2}\upsilon + \pi + \frac{3}{2}\alpha \tag{1.10a}$$

$$\nu_s^{sh} = (R_s - f_s) - \tau_s + \frac{1}{2}\pi + \upsilon - \frac{3}{2}\beta \tag{1.10b}$$

Next, to estimate buyers' and sellers' aggregate surpluses we need to consider the transportation costs they face. We compute the total transportation cost as the area under the unit interval of joining platforms 1 and 2. Given the indifferent buyer and seller are positioned at $\frac{1}{2}$ on the unit interval, for example for buyers this turns to $CS^{sh} = \nu_b^{sh} - \int_0^{1/2} \tau_b x_b dx_b + \int_{1/2}^1 \tau_b(1 - x_b)dx_b$ or $CS^{sh} = \nu_b^{sh} - 2\int_0^{1/2} \tau_b x_b dx_b$ given both platforms are symmetric. Then, we obtain:[21]

$$CS^{sh} = \nu_b^{sh} - 2\int_0^{1/2} \tau_b x_b dx_b \;=\; (R_b - f_b) - \frac{5}{4}\tau_b + \frac{1}{2}\upsilon + \pi + \frac{3}{2}\alpha \tag{1.11a}$$

$$PS^{sh} = \nu_s^{sh} - 2\int_0^{1/2} \tau_s x_s dx_s \;=\; (R_s - f_s) - \frac{5}{4}\tau_s + \frac{1}{2}\pi + \upsilon - \frac{3}{2}\beta \tag{1.11b}$$

Incorporating the direct network effect into the analysis α and β, we observe that the presence of within-group impacts leads to an increase in consumer surplus and a decrease in seller surplus compared to a scenario that does not have these effects. This outcome can be attributed to a bandwagon effect, which creates a positive feedback loop on the platforms. As more buyers are attracted to join them, a larger number of sellers also

[21] See Appendix A.2.4 for further details.

join, considering the cross-group network effects, thereby increasing the platform's value and consequently raising the net surplus for all participants. Conversely, in scenarios characterised by congestion among buyers and sellers facing intense competition, the effects are reversed, resulting in a decrease in both consumer surplus and seller surplus.

Comparative Statics

The next step in our analysis is to assess the impact of variations in the direct and indirect network effects on the equilibrium strategic variables. The comparative statics[22] are presented in Table 1.1, which illustrates how changes in exogenous variables (i.e., cross-group and within-network effects) affect the equilibrium of endogenous variables in our model, namely membership fees, aggregate surplus and profits.

Table 1.1: Comparative Statics. Both sides singlehoming.

Strategic Variables*/ Parameters	Direct Network Effects			Cross-group Network Effects	
	α		β	π	υ
	b[i]	s/c[ii]			
p_b	−	+	0[iii]	−	0
p_s	0	0	+	0	−
CS	+	−	0	+	+
PS	0	0	−	+	+
Π	−	+	+	−	−

[*] p_b and p_s are buyers and sellers membership fees, CS is buyers surplus, PS is sellers surplus and Π is platform' profits.
[i] b refers to a bandwagon effect, $\alpha > 0$.
[ii] s/c refers to a snob or congestion effect, $\alpha < 0$.
[iii] 0 means there is no effect.

The intuition of the comparative statics shown on Table 1.1 is as follows. When there is an increase in the bandwagon effect on buyers' side buyers are keener to join the platform than before. This leads the platform strategy to lower buyers' fees. On the other hand, in the presence of a snob or congestion effect, the impact is the opposite: buyers become less motivated to join the platform, leading to an increase in buyer fees to recover the income it obtained before buyers stopped joining. Similarly, in the case of an increase in sellers' competition, fewer sellers are willing to join, and as a result, the platform increases its fees.

The implications of an increase in a cross-group network effect on equilibrium membership fees are consistent with Armstrong (2006). Specifically, the side of the market

[22]Refer to Appendix A.2.5 for detailed information on the comparative statics analysis discussed in this section.

that exerts a stronger cross-group network effect on the other side enjoys a reduction in its fee. For instance, if there is an increase in the cross-group network effect buyers have on sellers (π), then the equilibrium membership fee for buyers will decrease.

A rise in cross-group network effects π and υ leads to an increase in both aggregate surpluses. The rationale behind this is that higher cross-group network effects stimulate greater demand for the platform's services on both sides of the market, thus enhancing the value proposition for both buyers and sellers. Moreover, in the presence of a positive direct network effect (bandwagon effect) on buyers' side, the increase in buyers joining the platform attracts more sellers, creating a virtuous cycle. Conversely, when a negative direct network effect (congestion effect and sellers' competition) is present, fewer buyers and sellers are persuaded, which reduces the platform's value and ultimately lowers aggregate surplus for both sides of the market.

Platforms' profits are negatively impacted by the cross-group network effects υ and π because platforms have to compete to attract more agents from each side. Conversely, platforms' profits are higher in the presence of snob or congestion effects or increased sellers' competition since they can charge a higher fee on both sides of the market. The opposite effect is observed in the presence of a bandwagon effect.

1.4 Scenario 1: Sellers multihome and buyers singlehome

In this section, we introduce a setting where only one side of the market (buyers), singlehome, while the other side (sellers), can multihome, that is, they can participate on both platforms simultaneously to benefit from the maximum of the cross-group network effects. This scenario can be illustrated in the operating system market, where developers (sellers) create applications for various operating systems (*Linux*, *macOS*, *Android*, *Windows*), and end-users (buyers) typically use only one operating system. Another example is the media market (newspaper, magazine, or radio), where advertisers (sellers) place ads, and the audience (buyers) usually, due to time constraints or preferences, read only one newspaper or magazine and listen to only one radio station.

Following the convention established by various authors, such as Choi (2010); Belleflamme and Peitz (2019b); Bakos and Halaburda (2020), when the market is fully covered, we categorise sellers within the unit interval who engage in multihoming into three groups: those exclusively connected to platform 1, those exclusively connected to platform 2, and those connected to both platforms. This classification is illustrated in Figure 1.1. An indifferent seller, considering singlehoming on platform 1 versus multihoming, is located at x_{20} and x_{10} respectively. Therefore, $0 < x_{20} < x_{10} < 1$, with $\eta_s^1 = x_{10}$ and $\eta_s^2 = (1 - x_{20})$.

This indicates that the multihoming seller's position lies between x_{20} and x_{10}.

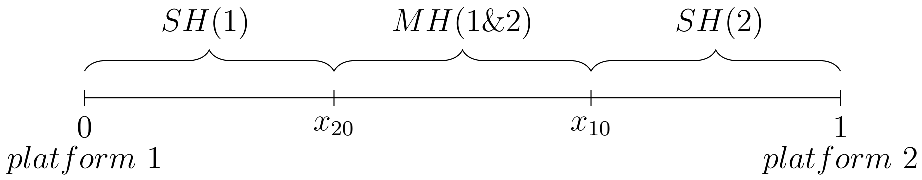

Figure 1.1: Buyers' and Sellers' Choice

We obtain the seller who is indifferent between singlehoming on platform 1 or 2 and multihoming on both platforms by:

$$\begin{aligned} \nu_s^1 - \tau_s x_{20} &= \nu_s^{1,2} - \tau_s x_{20} - \tau_s (1 - x_{20}) \\ \nu_s^1 &= \nu_s^{1,2} - \tau_s (1 - x_{20}) \\ (1 - x_{20}) &= \frac{\nu_s^{1,2} - \nu_s^1}{\tau_s} \end{aligned} \qquad \begin{aligned} \nu_s^2 - \tau_s (1 - x_{10}) &= \nu_s^{1,2} - \tau_s x_{10} - \tau_s (1 - x_{10}) \\ \nu_s^2 &= \nu_s^{1,2} - \tau_s x_{10} \\ x_{10} &= \frac{\nu_s^{1,2} - \nu_s^2}{\tau_s} \end{aligned}$$

Considering both platforms are symmetric, we use Equation 1.1b and Equation 1.2b to determine the fraction of sellers multihoming and buyers singlehoming at platform i, $i = 1, 2$.[23]

For sellers,

$$\eta_s^i = \frac{R_s + \pi \eta_b^i - p_s^i}{\tau_s + \beta} \tag{1.12}$$

The fraction of buyers that are singlehoming remains the same as in the previous Section 1.3, and using the fact that $\eta_s^i + \eta_s^j = 1$, $i, j = 1, 2$ and $i \neq j$ Equation 1.4a turns to:

$$\eta_b^i = \frac{\tau_b + \upsilon \left(\eta_s^i - \eta_s^j\right) - \alpha + \left(p_b^j - p_b^i\right)}{2 \left(\tau_b - \alpha\right)} \tag{1.13}$$

To obtain the fraction of buyers and sellers joining platform i as a function of membership fees, we need to solve the system of Equation 1.12 and Equation 1.13 simultaneously, resulting in the following expressions.[24]

$$\eta_b^i = \frac{1}{2} + \frac{\upsilon \left(p_s^j - p_s^i\right) + \left(\tau_s + \beta\right)\left(p_b^j - p_b^i\right)}{2 \left[\left(\tau_s + \beta\right)\left(\tau_b - \alpha\right) - \pi \upsilon\right]} \tag{1.14a}$$

$$\eta_s^i = \frac{R_s - p_s^i}{\left(\tau_s + \beta\right)} + \frac{\pi}{\left(\tau_s + \beta\right)} \left[\frac{1}{2} + \frac{\upsilon \left(p_s^j - p_s^i\right) + \left(\tau_s + \beta\right)\left(p_b^j - p_b^i\right)}{2 \left[\left(\tau_s + \beta\right)\left(\tau_b - \alpha\right) - \pi \upsilon\right]}\right] \tag{1.14b}$$

[23] See Appendix A.3.1 for more details.
[24] See Appendix A.3.1 for details.

Market Equilibrium

In this subsection, we determine platforms' pricing dynamics when sellers are multihoming and buyers singlehoming. We show the strategic considerations underlying platform fee-setting, particularly in light of the potential impact of intra and inter-network effects. We define equilibrium membership fees, market shares and platforms' profits.

Definition 1.2. *A symmetric equilibrium is a pair* p_b^{smh}, p_s^{smh}*, such that* p_b^{smh} *and* p_s^{smh} *solve the platform maximisation problem* $\max_{\{p_b^i, p_s^i\}} \Pi^i = (p_b^i - f_b)\, \eta_b^i \left(p_b^i, p_s^i, p_b^j, p_s^j\right) + \left(p_s^i - f_s\right) \eta_s^i \left(p_b^i, p_s^i, p_b^j, p_s^j\right)$ *for each* $i, j = 1, 2,\ i \neq j$.

From the first-order conditions of a symmetric equilibrium, $p_b^i = p_b^j = p_b$ and $p_s^i = p_s^j = p_s$, addressed in Appendix A.3.2, the following best response functions are obtained:

$$p_b = \frac{-\pi p_s - \pi\left(\upsilon - f_s\right) + \left(\tau_s + \beta\right)\left[\left(\tau_b - \alpha\right) + f_b\right]}{\left(\tau_s + \beta\right)} \tag{1.15a}$$

$$\begin{aligned} p_s = \frac{1}{4\left(\tau_s + \beta\right)\left(\tau_b - \alpha\right) - 3\pi\upsilon} \Big[& -\upsilon\left(\tau_s + \beta\right) p_b - \pi\upsilon\left(\pi + 2R_s + f_s\right) \\ & + \upsilon\left(\tau_s + \beta\right) f_b + \left(\tau_s + \beta\right)\left(\tau_b - \alpha\right)\left(\pi + 2\left(R_s + f_s\right)\right) \Big] \end{aligned} \tag{1.15b}$$

Equilibrium membership fees

Next, we solve[25] the set of Equation 1.15a and Equation 1.15b to determine the equilibrium membership fees as a function of the model's parameters:

$$p_b^{smh} = f_b + \left(\tau_b - \alpha\right) - \frac{\pi}{4\left(\tau_s + \beta\right)}\left[\left(\pi + 3\upsilon\right) + 2\left(R_s - f_s\right)\right] \tag{1.16a}$$

$$p_s^{smh} = \frac{1}{2}\left(R_s + f_s\right) + \frac{1}{4}\left(\pi - \upsilon\right) \tag{1.16b}$$

It is worth noting that sellers' equilibrium membership fee does not take into account the negative direct network effect (represented by sellers' competition β). The reason for this is that sellers have the flexibility to join both platforms simultaneously, which reduces competition amongst them. Additionally, platforms hold monopoly power over sellers' side as they have the ability to charge a premium fee for access to their exclusive pool of buyers.

It is important to mention that platforms still subsidise the side of the market that exerts a stronger cross-group network effect on the other side. Specifically, in the case

[25] Further details on how to obtain the equilibrium membership fees can be found in Appendix A.3.2 and Appendix A.3.3 for the second-order conditions of the profit-maximisation.

where the cross-group network effect sellers exert on buyers is stronger than the effect buyers exert on sellers $\upsilon > \pi$, sellers' fee decreases. Conversely, if the cross-group network effect exerted by buyers on sellers is greater than the effect sellers put on buyers ($\pi > \upsilon$), buyers' fee decreases while sellers' fee increases.

A crucial point to note is that buyers' equilibrium subscription fee accounts for both sides' within-network effects. Additionally, buyers' fee is adjusted downwards by a term $\frac{\pi}{4(\tau_s+\beta)}\left[\left(\pi + 3\upsilon\right) + 2\left(R_s - f_s\right)\right]$. As shown by partially differentiate η_s^i with respect to η_b^i in Equation 1.12, an extra buyer attracts $\frac{\pi}{(\tau_s+\beta)}$ sellers, allowing the platform to earn an extra profit of $[(\pi + 3\upsilon) + 2(R_s - f_s)]$. The presence of sellers' competition contributes to lowering buyers' equilibrium fees compared to what it would be in the absence of such competition. This effect arises because when sellers compete amongst themselves, fewer of them are drawn to the platform, leading to a reduced number of buyers joining given the cross-group network effect. Consequently, the platform's strategic response is to lower buyers' fees to directly attract them to participate.

Equilibrium market shares

At equilibrium, buyers' market share is equally divided between both platforms, whereas sellers' market share is obtained by substituting Equation 1.16b in Equation 1.12. That is

$$\eta_s^{smh} = \frac{R_s + \pi\eta_b^{smh} - p_s^{smh}}{\tau_s + \beta} \quad \text{then,} \quad (\tau_s + \beta)\,\eta_s^{smh} = R_s + \frac{1}{2}\pi - \left[\frac{2\left(R_s + f_s\right) + (\pi - \upsilon)}{4}\right]$$

$$\eta_s^{smh} = \frac{2\left(R_s - f_s\right) + (\pi + \upsilon)}{4\left(\tau_s + \beta\right)} \tag{1.17}$$

The market share of sellers who participate in both platforms is determined by a fraction that is influenced by the magnitude of cross-group network effects and the parameters that characterise sellers' side. As all sellers participate in the market and some choose to singlehome while others opt to multihome, Equation 1.17 dictates that sellers' market share must satisfy condition $\eta_s^{sh} < \eta_s^{smh} < 1$[26]. This provides sellers with an opportunity to increase their market share by participating in both platforms simultaneously.

Platform's equilibrium profits

Next, we determine the equilibrium platform profits by employing Equation 1.16a, Equation 1.16b and Equation 1.17, taking into account that because platforms established

[26]This condition turns to $2\left(\tau_s + \beta\right) < 2\left(R_s - f_s\right) + (\pi + \upsilon) < 4\left(\tau_s + \beta\right)$ which is satisfied if Assumption 1.3 holds. For more details see Appendix A.1 subsection market shares, sellers multihome and buyers singlehome

identical fees at equilibrium the indifferent buyer is located at $\eta_b^{smh} = 1/2$, as:[27]

$$\Pi^{smh} \equiv \left(p_b^{smh} - f_b\right)\eta_b^{smh} + \left(p_s^{smh} - f_s\right)\eta_s^{smh}$$

$$\Pi^{smh} \equiv \frac{8\left(\tau_s+\beta\right)\left(\tau_b-\alpha\right) - \left[\left(\pi+\upsilon\right)^2 + 4\pi\upsilon\right] + 4\left(R_s - f_s\right)^2}{16\left(\tau_s+\beta\right)} \tag{1.18}$$

It is important to emphasise that equilibrium profits are positive, considering the standalone benefit is sufficiently large such that $R_s - f_s > 0$, and as long as $8\left(\tau_s+\beta\right)\left(\tau_b-\alpha\right) - \left(\pi+\upsilon\right)^2 - 4\pi\upsilon > 0$.[28] Similar to the previous section, the presence of a bandwagon effect decreases platform profits, while a congestion effect has the opposite effect. This phenomenon occurs because a positive direct network effect decreases buyers' fees, whereas a negative within-network effect increases.

Welfare

Subsequently, we obtain the surplus for both agents in equilibrium using Equation 1.1a, Equation 1.1b, Equation 1.16a and Equation 1.16b where the indifferent buyer is located at 1/2 and sellers' market-share is given by Equation 1.17.[29]

$$\nu_b^{smh} = R_b + \upsilon\eta_s^{smh} + \alpha\eta_b^{smh} - p_b^{smh} \qquad\qquad \nu_s^{smh} = R_s + \pi\eta_b^{smh} - \beta\eta_s^{smh} - p_s^{smh}$$

$$\nu_b^{smh} = \left(R_b - f_b\right) - \tau_b + \frac{3}{2}\alpha + \frac{2\left(\pi+\upsilon\right)\left(R_s - f_s\right) + \left(\pi+\upsilon\right)^2 + 2\pi\upsilon}{4\left(\tau_s+\beta\right)} \tag{1.19a}$$

$$\nu_s^{smh} = \frac{\tau_s\left[2\left(R_s - f_s\right) + \left(\pi+\upsilon\right)\right]}{4\left(\tau_s+\beta\right)} \tag{1.19b}$$

Similar to the singlehome scenario, a bandwagon effect (congestion effect) causes buyers' surplus to increase (decrease) because of the positive loop it creates between both sides. Additionally, buyers' surplus is influenced by the difference between the standalone benefit and the cost of serving participants on sellers' side, as well as the cross-group network effect on both sides. Conversely, sellers' surplus decreases when sellers face more competition, which discourages them from joining the market and ultimately reduces platform's overall value. It is important to note sellers' equilibrium surplus is directly related to sellers' market share in equilibrium given by $\nu_s^{smh} = \tau_s\eta_s^{smh}$ as it can be seen in Equation 1.17 and Equation 1.19b.

[27]See Appendix A.3.4 for details.

[28]Considering Assumption 1.1 holds, this condition is satisfied. First, we make the left side of both inequalities equal to compare the right side and identify which is larger. That is $\frac{(\pi+\upsilon)^2}{4} > \frac{(\pi+\upsilon)^2+4\pi\upsilon}{8}$ turns to $2\left(\pi+\upsilon\right)^2 > \left(\pi+\upsilon\right)^2 4\pi\upsilon$ and then to $\pi^2 + 2\pi\upsilon + \upsilon^2 > 4\pi\upsilon$ as long as $\left(\pi-\upsilon\right)^2 > 0$ when $\pi \neq \upsilon$.

[29]See Appendix A.3.5 for details.

Buyers' surplus is impacted by an additional factor, namely sellers' competition parameter β. Although the direct network effect does not influence sellers' equilibrium membership fee, it does change their market share, which in turn affects sellers' surplus.

The aggregate surpluses are:[30]

$$CS^{smh} = \nu_b^{smh} - 2\int_0^{1/2} \tau_b x_b dx_b$$

$$CS^{smh} = (R_b - f_b) - \frac{5}{4}\tau_b + \frac{3}{2}\alpha + \frac{\left[2(\pi+\upsilon)(R_s - f_s) + (\pi+\upsilon)^2 + 2\pi\upsilon\right]}{4(\tau_s+\beta)} \tag{1.20}$$

To compute the Producer Surplus we refer to Figure 1.1 to determine how to measure the transportation cost associated with joining one platform versus joining both platforms simultaneously, considering the choice of some sellers to singlehome and others to multihome.

$$PS^{smh} = \int_0^{1-\eta_s^{smh}} \left(\nu_s^{smh} - \tau_s x_s\right) dx_s + \int_{1-\eta_s^{smh}}^{\eta_s^{smh}} \left(2\nu_s^{smh} - \tau_s\right) dx_s + \int_{\eta_s^{smh}}^{1} \left(\nu_s^{smh} - \tau_s(1-x_s)\right) dx_s$$

The first integral calculates the producer surplus from joining platform 1, denoted as $SH(1)$ on Figure 1.1. The second integral represents the producer surplus from simultaneously joining both platforms, labelled $MH(1\&2)$. Lastly, the third integral measures the producer surplus from joining platform 2, identified as $SH(2)$.

$$PS^{smh} = \frac{\left(\nu_s^{smh}\right)^2}{\tau_s} = \frac{\tau_s\left[2(R_s - f_s) + (\pi+\upsilon)\right]^2}{16(\tau_s+\beta)^2} \tag{1.21}$$

In contrast to the previous section, buyers' aggregate surplus is influenced by parameters related to sellers' side, such as π, τ_s and β. Specifically, sellers' competition (β) plays a significant role in determining the proportion of participants, which in turn affects the fraction of buyers attracted as a result of the cross-group network effect. As sellers face competition, fewer choose to join, resulting in fewer buyers connecting as well. Consequently, this downward trend adversely impacts the overall value of the platform, leading to a reduction in consumer surplus.

Sellers' aggregate surplus is directly related to sellers' equilibrium market share represented by $PS^{smh} = \tau_s\left(\eta_s^{smh}\right)^2$ as it can be seen in Equation 1.17 and Equation 1.21. The rationale behind this finding suggests that as more sellers engage in multihoming, the generated surplus increases because the presence of more sellers attracts more buyers, thereby increasing the overall platform value. This, in turn, attracts additional buyers and sellers due to the cross-group network effect, ultimately increasing sellers' aggregate

[30]See Appendix A.3.5 for details.

surplus.

Comparative Statics

As in Section 1.3, we are interested in the impacts of the cross-group and direct network effects on the equilibrium strategic variables, which can be seen in Table 1.2.

Table 1.2: Comparative Statics. Sellers multihoming and buyers singlehoming.

Variables*/ Parameters	Direct Network Effects			Cross-group Network Effects	
	α		β	π	υ
	b[i]	s/c[ii]			
p_b	−	+	+	−	−
p_s	0	0	0[iii]	+	−
η_s	0	0	−	+	+
CS	+	−	−	+	+
PS	0	0	−	+	+
Π	−	+	+[iv]	−	−

* p_b and p_s are buyers and sellers membership fees, η_s is sellers market shares, CS is buyers surplus, PS is sellers surplus and Π is platform' profits.

[i] b refers to a bandwagon effect, $\alpha > 0$.

[ii] s/c refers to a snob or congestion effect, $\alpha < 0$.

[iii] 0 means there is no effect.

[iv] As long as $2(R_s - f_s) < \sqrt{(\pi+\upsilon)^2 + 4\pi\upsilon}$.

Next, we explain the intuition of the comparative statics[31] shown on Table 1.2 that contrast with Table 1.1, as follows.

As sellers face increasing competition $\beta \uparrow$, fewer of them are inclined to join the platform, as evidenced by the decline in sellers' market share as described in Equation 1.17. In response to this trend, the platform adjusts its strategy by lowering buyers' fee to directly attract them, rather than relying entirely on the indirect impact of cross-group network effects.

The impacts of cross-group network effects on equilibrium membership fees are aligned with the existing literature on two-sided markets, such as Armstrong (2006); Rochet and

[31]Refer to Appendix A.3.6 for detailed information on the comparative statics analysis discussed in this section.

Tirole (2003); Belleflamme and Peitz (2019a). Specifically, platforms typically subsidise the side of the market exerting a stronger cross-group network effect on the other side. However, when sellers engage in multihoming, the impact of sellers' cross-group network effect on buyers affects buyers' membership fees. When this effect is stronger, the platform's strategy shifts towards reducing its fee. To understand this result intuitively, consider that υ represents buyers' gain from trade with sellers. As υ increases, the platform opts to lower its fee to attract more buyers and, consequently, more sellers because of the cross-group network effect. This strategy aims to enhance the platform's overall value by increasing participant engagement.

Conversely, when the cross-group network effect exerted by buyers on sellers increases (which is sellers' gain from trade with buyers), the platform chooses to raise its fee instead of lowering it. This decision is based on the platform's monopoly-like control over access to buyers. Sellers seeking access to these buyers are willing to pay higher fees, reinforcing the platform's fee-raising strategy.

Sellers' market share and aggregate surpluses increase when intra (positive) and inter-network effects are stronger. This is due to the increased attractiveness of the platform, which draws in more participants on both sides and leads to further participation on both sides. Conversely, these metrics decrease when there is more competition amongst sellers and congestion on buyers' side, as previously discussed.

Finally, equilibrium platform profits behave as in Section 1.3, i.e., they decrease on both cross-group network effects (υ, π), bandwagon behaviour (α), and increase in sellers' competition (β) as long as $2(R_s - f_s) < \sqrt{(\pi + \upsilon)^2 + 4\pi\upsilon}$ and increase in congestion effect on buyers' side (α).

1.5 Scenario 2: Buyers multihome and sellers singlehome

In this section, we allow for multihoming on buyers' side, while sellers' side is limited to singlehoming. This means buyers decide to participate on both platforms simultaneously to take advantage of the benefits from interactions across both markets. An illustration of this concept can be observed in the scenario of two shopping centres situated near each other, where buyers have the option to visit both of them while stores are limited to operating in only one of them due to exclusive agreements, as Steele (1978) mentioned. Similarly, in a ride-hailing market, buyers can use multiple apps to find a ride, but drivers are restricted to working for only one company due to apps incompatibility.[32]

[32] According to wired.com Mystro App lets drivers juggle competing *Uber* and *Lyft* rides.

Buyers are classified into three groups within the unit interval based on their multihoming status, as explained in the previous section. The first group connects only to platform 1, the second group only to platform 2, and the third group connects to both platforms simultaneously (refer to Figure 1.1). An indifferent buyer who is deciding whether to singlehome on platform 1 or multihome on both platforms is situated at x_{20}, while an indifferent buyer who is deciding whether to connect to platform 2 or multihome on both platforms is situated at x_{10}. Then $0 < x_{20} < x_{10} < 1$, where $\eta_b^1 = x_{10}$ and $\eta_b^2 = (1 - x_{20})$. This indicates that the multihoming buyer's position lies between x_{20} and x_{10}.

Considering both platforms are symmetric, we use Equation 1.1a and Equation 1.2a to determine the fraction of buyers multihoming and sellers singlehoming at platform i, $i = 1, 2$.[33]

For buyers,

$$\eta_b^i = \frac{R_b + \upsilon\eta_s^i - p_b^i}{\tau_b - \alpha} \tag{1.22}$$

The fraction of sellers that are singlehoming, remains the same as in Section 1.3, and using the fact that $\eta_b^i + \eta_b^j = 1$, $i, j = 1, 2$ and $i \neq j$ Equation 1.4b turns to:

$$\eta_s^i = \frac{\tau_s + \pi\left(\eta_b^i - \eta_b^j\right) + \beta + \left(p_s^j - p_s^i\right)}{2\left(\tau_s + \beta\right)} \tag{1.23}$$

To obtain buyers' and sellers' joining platform i as a function of membership fees, we need to solve the system of Equation 1.22 and Equation 1.23 simultaneously.[34]

$$\eta_b^i = \frac{R_b - p_b^i}{(\tau_b - \alpha)} + \frac{\upsilon}{(\tau_b - \alpha)}\left[\frac{1}{2} + \frac{\pi\left(p_b^j - p_b^i\right) + (\tau_b - \alpha)\left(p_s^j - p_s^i\right)}{2\left[(\tau_s + \beta)(\tau_b - \alpha) - \pi\upsilon\right]}\right] \tag{1.24a}$$

$$\eta_s^i = \frac{1}{2} + \frac{\pi\left(p_b^j - p_b^i\right) + (\tau_b - \alpha)\left(p_s^j - p_s^i\right)}{2\left[(\tau_s + \beta)(\tau_b - \alpha) - \pi\upsilon\right]} \tag{1.24b}$$

Market Equilibrium

In this subsection[35], we define equilibrium membership fees on both sides of the market, market shares and platforms profits when buyers multihoming and sellers singlehoming. Then, we provide some intuition for platforms' pricing strategy.

Definition 1.3. *A symmetric equilibrium is a pair* p_b^{bmh}, p_s^{bmh}*, such that* p_b^{bmh} *and* p_s^{bmh}

[33] See Appendix A.4.1 for details.
[34] See Appendix A.4.1 for details.
[35] The equilibrium values are denoted by the superscript "*bmh*".

solve the platform maximisation problem $\max_{\{p_b^i, p_s^i\}} \Pi^i = (p_b^i - f_b)\, \eta_b^i \left(p_b^i, p_s^i, p_b^j, p_s^j\right) + \left(p_s^i - f_s\right) \eta_s^i \left(p_b^i, p_s^i, p_b^j, p_s^j\right)$ *for each* $i, j = 1, 2,\ i \neq j$.

From the first-order conditions of a symmetric equilibrium, $p_b^i = p_b^j = p_b$ and $p_s^i = p_s^j = p_s$, addressed in Appendix A.4.2 the following best responses functions are obtained:

$$p_b = \frac{1}{4\left(\tau_s + \beta\right)\left(\tau_b - \alpha\right) - 3\pi\upsilon} \Big[-\pi\left(\tau_b - \alpha\right) p_s - \pi\upsilon\left(\upsilon + 2R_b + f_b\right) - \pi\left(\tau_b - \alpha\right) f_s + \left(\tau_s + \beta\right)\left(\tau_b - \alpha\right)\left(\upsilon + 2\left(R_b + f_b\right)\right) \Big] \tag{1.25a}$$

$$p_s = \frac{-\upsilon p_b - \upsilon\left(\pi - f_b\right) + \left(\tau_b - \alpha\right)\left[\left(\tau_s + \beta\right) + f_s\right]}{\left(\tau_b - \alpha\right)} \tag{1.25b}$$

Equilibrium membership fees

To obtain the membership fees as a function of the model's parameters, we need to solve[36] the best response functions in Equation 1.25a and Equation 1.25b simultaneously.

$$p_b^{bmh} = \frac{1}{2}\left(R_b + f_b\right) + \frac{1}{4}\left(\upsilon - \pi\right) \tag{1.26a}$$

$$p_s^{bmh} = f_s + \left(\tau_s + \beta\right) - \frac{\upsilon}{4\left(\tau_b - \alpha\right)}\left[\left(\upsilon + 3\pi\right) + 2\left(R_b - fb\right)\right] \tag{1.26b}$$

Equilibrium membership fee on buyers' side does not incorporate the bandwagon or congestion effect, which refers to a positive or negative direct network effect, denoted by α. The direct effect is missing because buyers have the option to join both platforms, which relaxes the bandwagon effect and reduces the congestion effect of joining just one platform. Furthermore, platforms possess monopoly power over buyers, as they can impose a premium fee for access to their exclusive group of sellers.

Platforms continue to subsidise the market side which has a stronger cross-group network effect on the other side. To be specific, when the cross-group network effect exerted by sellers on buyers is stronger than the effect exerted by buyers on sellers $\upsilon > \pi$, sellers' fee decreases and buyers' fees increase. On the other hand, when $\pi > \upsilon$ buyers' fee decreases.

Sellers' equilibrium membership fee considers the intra-network effects on both sides of the market. Moreover, sellers' fee is adjusted by the term $\frac{\upsilon}{4(\tau_b - \alpha)}\left[\left(\upsilon + 3\pi\right) + 2\left(R_b - f_b\right)\right]$. As shown by partially differentiating η_b^i with respect to η_s^i in Equation 1.22, an extra sellers attracts $\frac{\upsilon}{(\tau_b - \alpha)}$ buyers, allowing the platform to earn an extra profit of $\left[\left(\upsilon + 3\pi\right) +\right.$

[36]Further details on how to obtain the equilibrium membership fees can be found in Appendixes Appendix A.4.2 and Appendix A.4.3 for the second-order conditions of the profit-maximisation.

$2\,(R_b - f_b)\,\big]$. When there is a bandwagon effect an extra seller attracts a larger number of additional buyers because there is a positive feedback loop boosting the number of buyers and sellers joining both platforms. However, when there is a congestion effect, the extra proportion of buyers is lower, than what it would be without a direct network effect.

Equilibrium market shares

At equilibrium, unlike the previous scenario Section 1.4, sellers' market share is equally divided between both platforms, whereas buyers' market share is obtained by substituting Equation 1.26a in Equation 1.22. That is:

$$\eta_b^{bmh} = \frac{R_b + \upsilon\eta_s^{bmh} - p_b^{bmh}}{\tau_b - \alpha} \quad \text{then,} \quad (\tau_b - \alpha)\,\eta_b = R_b + \upsilon\frac{1}{2} - \left[\frac{2\,(R_b + f_b) + (\upsilon - \pi)}{4}\right]$$

$$\eta_b^{bmh} = \frac{2\,(R_b - f_b) + (\upsilon + \pi)}{4\,(\tau_b - \alpha)} \tag{1.27}$$

The market share of buyers who engage in both platforms simultaneously is subject to a fraction that is impacted by the magnitude of cross-group network effects π, υ and the parameters that depict buyers' side, R_b, f_b, τ_b and α. Since all buyers engage in the platform, with some preferring to singlehome and others to multihome, Equation 1.27 specifies that buyers' market share must satisfy condition $\eta_b^{sh} < \eta_b^{bmh} < 1$.[37] This circumstance presents buyers with the possibility of increasing their market share by joining both platforms at the same time.

Platform's equilibrium profits

Next, we determine the equilibrium platform profits by using Equation 1.26a, Equation 1.26b and Equation 1.27, considering that because platforms established identical fees at equilibrium the indifferent seller is located at $\eta_s^{bmh} = \frac{1}{2}$, as:[38]

$$\Pi^{bmh} \equiv \left(p_b^{bmh} - f_b\right)\eta_b^{bmh} + \left(p_s^{bmh} - f_s\right)\eta_s^{bmh}$$

$$\Pi^{bmh} \equiv \frac{8\,(\tau_b - \alpha)\,(\tau_s + \beta) - (\upsilon + \pi)^2 - 4\upsilon\pi + 4\,(R_b - f_b)^2}{16\,(\tau_b - \alpha)} \tag{1.28}$$

As in the previous section, equilibrium profits are positive considering the stand-alone benefit is sufficiently large such that $R_b - f_b > 0$, and as long as $8\,(\tau_s + \beta)\,(\tau_b - \alpha) -$

[37] This condition turns to $2\,(\tau_b - \alpha) < 2\,(R_b - f_b) + (\pi + \upsilon) < 4\,(\tau_b - \alpha)$ which is satisfied if Assumption 1.2 holds. For more details see Appendix A.1 subsection market shares, buyers multihome and sellers singlehome.

[38] See Appendix A.4.4 for details.

$(\pi + \upsilon)^2 - 4\pi\upsilon > 0$.[39] The presence of a bandwagon or congestion effect and sellers' competition have the same implications as when sellers multihome and buyers singlehome are addressed in Section 1.4.

Welfare

We obtain the surplus for both buyers and sellers in equilibrium using Equation 1.1a, Equation 1.1b, Equation 1.26a and Equation 1.26b, where the indifferent seller is located at $\frac{1}{2}$ and buyers' market-share is given by Equation 1.27.[40]

$$\nu_b^{bmh} = R_b + \upsilon\eta_s^{bmh} - \alpha\eta_b^{bmh} - p_b^{bmh} \qquad \nu_s^{bmh} = R_s + \pi\eta_b^{bmh} - \beta\eta_s^{bmh} - p_s^{bmh}$$

$$\nu_b^{bmh} = \frac{\tau_b \left[2\left(R_b - f_b\right) + (\upsilon + \pi)\right]}{4\left(\tau_b - \alpha\right)} \tag{1.29a}$$

$$\nu_s^{bmh} = \left(R_s - f_s\right) - \tau_s - \frac{3}{2}\beta + \frac{\left(2\left(\pi + \upsilon\right)\left(R_b - f_b\right) + \left(\pi + \upsilon\right)^2 + 2\pi\upsilon\right)}{4\left(\tau_b - \alpha\right)} \tag{1.29b}$$

A bandwagon effect increases buyers' and sellers' surplus by generating a positive feedback loop within and across market sides. As more buyers join the platform due to, perhaps its popularity, the attraction for it increases, reinforcing the perception of its value and generating positive spillovers to other market participants. It is important to note buyers' equilibrium surplus is directly related to buyers' market share in equilibrium given by $\nu_b^{bmh} = \tau_b\eta_b^{bmh}$ as it can be seen in Equation 1.27 and Equation 1.29a.

Conversely, sellers' competition, which creates a negative direct network effect, decreases sellers' surplus and the overall value of the platform. As competition among sellers intensifies, it is less attractive to join the platform. This, in turn, impacts buyers' willingness to join, ultimately decreasing the value of the platform.

We follow the same process as in Section 1.4 to compute the aggregate surpluses.[41] To obtain the Consumer Surplus we refer to Figure 1.1 to determine how to measure the transportation cost associated with joining one platform versus joining both platforms simultaneously, considering some buyers choose to singlehome and others to multihome.

$$CS^{bmh} = \int_0^{1-\eta_b^{bmh}} \left(\nu_b^{bmh} - \tau_b x_b\right) dx_b + \int_{1-\eta_b^{bmh}}^{\eta_b^{bmh}} \left(2\nu_b^{bmh} - \tau_b\right) dx_b + \int_{\eta_b^{bmh}}^{1} \left(\nu_b^{bmh} - \tau_b\left(1 - x_b\right)\right) dx_b$$

The first integral calculates the consumer surplus from joining platform 1, denoted as $SH(1)$ on Figure 1.1. The second integral represents the consumer surplus from simulta-

[39] Considering Assumption 1.1 holds, this condition is satisfied as it was shown in footnote 28.
[40] See Appendix A.4.5 for details.
[41] See Appendix A.4.5 for details.

neously joining both platforms, labelled $MH(1\&2)$. Lastly, the third integral measures the consumer surplus from joining platform 2, identified as $SH(2)$.

$$CS^{bmh} = \frac{\left(\nu_b^{bmh}\right)^2}{\tau_b} = \frac{\tau_b\left[2\left(R_b - f_b\right) + \left(\upsilon + \pi\right)\right]^2}{16\left(\tau_b - \alpha\right)^2} \tag{1.30}$$

Sellers' aggregate surplus is:

$$PS^{bmh} = \nu_s^{bmh} - 2\int_0^{1/2} \tau_s x_s dx_s$$

$$PS^{bmh} = \left(R_s - f_s\right) - \frac{5}{4}\tau_s - \frac{3}{2}\beta + \frac{\left(2\left(\pi + \upsilon\right)\left(R_b - f_b\right) + \left(\pi + \upsilon\right)^2 + 2\pi\upsilon\right)}{4\left(\tau_b - \alpha\right)} \tag{1.31}$$

We note buyers' aggregate surplus in equilibrium is directly related to buyers' market share in equilibrium represented by $CS^{bmh} = \tau_b\left(\eta_b^{bmh}\right)^2$ as it can be seen in Equation 1.27 and Equation 1.30.

When a bandwagon effect is present on buyers' side, the aggregate surpluses of both buyers and sellers are higher than when a congestion effect is at work. This is because a bandwagon effect generates a positive direct network effect that attracts more buyers and, due to cross-group network effects, also more sellers. As a result, aggregate surpluses are higher. In contrast, when a congestion effect is at work on buyers' side, buyers feel less attracted to join and also discourages sellers from participating, leading to a decrease in aggregate surpluses for both groups.

Comparative Statics

As in previous sections we are interested in the impacts of the cross-group and direct network effects on the equilibrium strategic variables, which can be seen in Table 1.3.

Table 1.3: Comparative Statics. Buyers multihoming and sellers singlehoming.

Variables*/ Parameters	Direct Network Effects			Cross-group Network Effects	
	α		β	π	υ
	b[i]	s/c[ii]			
p_b	0[iii]	0	0	−	+
p_s	−	+	+	−	−
η_b	+	−	0	+	+
CS	+	−	0	+	+
PS	+	−	−	+	+
Π	+[iv]	+[v]	+	−	−

* p_b and p_s are buyers and sellers membership fees, η_b is buyers market shares, CS is buyers surplus, PS is sellers surplus and Π is platform' profits.

[i] b refers to a bandwagon effect, $\alpha > 0$.

[ii] s/c refers to a snob or congestion effect, $\alpha < 0$.

[iii] 0 means there is no effect.

[iv] As long as $2(R_b - f_b) > \sqrt{(\pi+\upsilon)^2 + 4\pi\upsilon}$.

[v] As long as $2(R_b - f_b) < \sqrt{(\pi+\upsilon)^2 + 4\pi\upsilon}$.

Next, we describe the intuition of the comparative statics[42] shown on Table 1.3 that contrast with Table 1.2, as follows.

The impact of a bandwagon effect (α) extends beyond buyers' side and also affects sellers' side. This is a consequence of the interdependence of both sides of the market, specifically given the existence of cross-group network effects. As the positive direct network effect strengthens, platforms opt to reduce sellers' fees. Conversely, if negative within-network effects, such as congestion are present, the effects on sellers' equilibrium fees are reversed.

A bandwagon effect, as previously discussed, can have a positive impact on both buyers' market share and aggregate surpluses. This effect creates a positive feedback loop between both sides of the market, resulting in increased attraction to the platform from both buyers and sellers. On the contrary, a congestion effect has detrimental impacts on these measures. Moreover, in cases where buyers participate in both platforms, an increase in seller competition (β) only adversely affects sellers' aggregate surplus. This occurs because increased competition leads to a reduction in the fraction of sellers joining

[42]Refer to Appendix A.4.6 for detailed information on the comparative statics analysis discussed in this section.

the platform, resulting in fewer buyers and a corresponding decrease in platforms' value, thereby reducing sellers' surplus.

The outcomes following an increase in the cross-group network effects align with the previous sections, wherein platforms provide subsidies to participants that have a greater influence on the other side. Finally, an interesting result arises regarding the equilibrium platform profits: when the bandwagon effect becomes more prominent (indicated by an increase in α), platforms' profits increase, provided that $2\left(R_b - f_b\right) > \sqrt{\left(\pi + \upsilon\right)^2 + 4\pi\upsilon}$. This outcome contrasts with the scenario where sellers multihome and buyers singlehome in Section 1.4. The rationale behind this result is that a positive direct network effect on buyers' side attracts more buyers and subsequently more sellers due to the cross-group network effect. This increase in participation allows the platform to charge additional fees, resulting in higher profits. The remaining impacts, including congestion effects, sellers' competition, and cross-group network effects, align consistently with those described in the preceding Section 1.4.

1.6 Scenario 3: Both buyers and sellers multihome

This section enables buyers and sellers to multihome, meaning that they can simultaneously participate in both platforms to reap the full benefits of cross-group network effects. For example, network television viewers can access multiple channels, while advertisers can place advertisements across different networks. Similarly, securities brokerage clients can hold accounts with various firms to trade different assets.[43]

As in the previous two sections, since both buyers and sellers can multihome, they can be classified into three groups based on their activity across the two platforms (as shown in Figure 1.1): those who only use platform 1, those who only use platform 2, and those who use both.

Then, the indifferent buyer and seller between multihoming on both platforms and singlehoming on platform i, $i = 1, 2$ can be taken from our previous analysis. On sellers' side from Equation 1.12 and on buyers' side from Equation 1.22. That is,

$$\text{For buyers} \quad \eta_b^i = \frac{R_b + \upsilon\eta_s^i - p_b^i}{\tau_b - \alpha} \tag{1.22}$$

$$\text{For sellers} \quad \eta_s^i = \frac{R_s + \pi\eta_b^i - p_s^i}{\tau_s + \beta} \tag{1.12}$$

To obtain buyers' and sellers' joining platform i as a function of membership fees, we

[43] According to Evans and Schmalensee (2005) "The average securities brokerage client has accounts at three firms".

need to solve equations Equation 1.12 and Equation 1.22 simultaneously.[44]

$$\eta_b^i = \frac{(\tau_s + \beta)\left(R_b - p_b^i\right) + \upsilon\left(R_s - p_s^i\right)}{(\tau_b - \alpha)(\tau_s + \beta) - \pi\upsilon} \tag{1.33a}$$

$$\eta_s^i = \frac{(\tau_b - \alpha)\left(R_s - p_s^i\right) + \pi\left(R_b - p_b^i\right)}{(\tau_b - \alpha)(\tau_s + \beta) - \pi\upsilon} \tag{1.33b}$$

In contrast to the previous sections, where buyers' and sellers' market shares were influenced by the membership fee differences between both platforms, when both sides multihome, market shares depend exclusively on the membership fees of each side within the analysed platform (p_b^i and p_s^i for platform i). Specifically, buyers' and sellers' market shares are determined by a margin on their respective sides, represented as $(R_b - p_b)$ on buyers' side and $(R_s - p_s)$ on seller's side, adjusted by the parameters of the model $(\tau_b, \tau_s, \pi, \upsilon)$.

Market Equilibrium

In this subsection[45], our focus lies on the determination of equilibrium membership fees, market shares, and platform profits in a scenario where buyers and sellers engage in multihoming. Additionally, we aim to provide insights into the underlying rationale behind platforms' pricing strategies, thereby establishing a clearer understanding of their decision-making process.

Definition 1.4. *A symmetric equilibrium is a pair p_b^{mh}, p_s^{mh}, such that p_b^{mh} and p_s^{mh} solve the platform maximisation problem $\max_{\{p_b^i, p_s^i\}} \Pi^i = \left(p_b^i - f_b\right)\eta_b^i\left(p_b^i, p_s^i, p_b^j, p_s^j\right) + \big(p_s^i - f_s\big)\eta_s^i\big(p_b^i, p_s^i, p_b^j, p_s^j\big)$ for each $i = 1, 2,\ i \neq j$.*

From the first-order conditions of a symmetric equilibrium, $p_b^i = p_b^j = p_b$ and $p_s^i = p_s^j = p_s$, addressed in Appendix A.5.2 the following best responses functions are obtained:

$$p_b^{mh} = \frac{(R_b + f_b)}{2} - \frac{\pi\left(p_s - f_s\right) - \upsilon\left(R_s - p_s\right)}{2\left(\tau_s + \beta\right)} \tag{1.34a}$$

$$p_s^{mh} = \frac{(R_s + f_s)}{2} - \frac{\upsilon\left(p_b - f_b\right) - \pi\left(R_b - p_b\right)}{(\tau_b - \alpha)} \tag{1.34b}$$

It is important to highlight the interplay between pricing strategies on both sides of the market, which is contingent on the strength of the cross-group network effects. When the platform increases the fee on one side, the best response is to decrease the other side's fee, which is called a strategic substitution relationship. In this case, an increase in sellers' fees leads to a decrease in buyers' fee response, as indicated by $\frac{\partial p_b^{mh}}{\partial p_s^{mh}} = -\frac{(\pi+\upsilon)}{2(\tau_s+\beta)}$.

[44] See Appendix A.5.1 for details.

[45] The equilibrium values are denoted by the superscript "mh".

Buyers' best response membership fee is determined by the stand-alone benefit and the cost of serving buyers $R_b + f_b$, adjusted by the relative difference of two profit margins on sellers' side $(p_s - f_s)$ weighted by the cross-group network effect buyers exert on sellers π and $(R_s - p_s)$ weighted by the cross-group network effect sellers exert on buyers υ, along with other parameters on sellers' side, τ_s and β. Sellers' best response function has a similar structure.

It is essential to recognise some distinctions between the singlehoming case discussed in Section 1.3 and the current multihoming scenario. When both agents are multihoming, no direct network impacts are involved on the same side's best response function. This is a consequence of joining both platforms simultaneously because the within-network effect is diluted.

For example on sellers' side, platform's best response is to charge a proportion of sellers' stand-alone benefit (R_s), along with a profit margin from the other side denoted by $p_s - f_s$ and $R_s - p_s$. An extra seller attracts $\frac{\upsilon}{\tau_b - \alpha}$ extra buyer, as indicated by partially differentiated Equation 1.22 with respect to η_s^i, an a profit of $\frac{1}{2}\left[p_b - f_b - \frac{\pi}{\upsilon}\left(R_b - p_b\right)\right]$.

Equilibrium membership fees

Next, we proceed by solving the best response functions presented in Equation 1.34a and Equation 1.34b to obtain the equilibrium membership fees as a function of the model's parameters.[46]

$$p_b^{mh} = \frac{\overbrace{\left(2\left(\tau_b - \alpha\right)\left(\tau_s + \beta\right) - \pi\upsilon\right)}^{a}\left(R_b + f_b\right) - \left(\pi^2 R_b + \upsilon^2 f_b\right) + \overbrace{\left(\tau_b - \alpha\right)\left(R_s - f_s\right)}^{b}\left(\upsilon - \pi\right)}{4\left(\tau_b - \alpha\right)\left(\tau_s + \beta\right) - \left(\pi + \upsilon\right)^2} \tag{1.35a}$$

$$p_s^{mh} = \frac{\overbrace{\left(2\left(\tau_b - \alpha\right)\left(\tau_s + \beta\right) - \pi\upsilon\right)}^{a}\left(R_s + f_s\right) - \left(\upsilon^2 R_s + \pi^2 f_s\right) + \overbrace{\left(\tau_s + \beta\right)\left(R_b - f_b\right)}^{b}\left(\pi - \upsilon\right)}{4\left(\tau_b - \alpha\right)\left(\tau_s + \beta\right) - \left(\pi + \upsilon\right)^2} \tag{1.35b}$$

Membership fees on both sides exhibit a similar structure to when only sellers or buyers multihome, as discussed in Section 1.4 and Section 1.5. Specifically, they comprise primarily of the stand-alone benefit and the cost of serving buyers or sellers. Furthermore, the difference between the cross-group network effects on both sides of the market is weighted by certain proportions.

The proportion "a" for both buyers and sellers is determined by a combination of the degree of product differentiation (disutility cost because of mismatched preferences), as well as the direct and indirect network effects on both sides of the market. Conversely, proportion "b" for buyers (sellers) is a composite of buyers' (sellers') transportation cost

[46]Further details on how to obtain the equilibrium membership fees can be found in Appendix A.5.2 and Appendix A.5.3 for the second-order conditions of the profit-maximisation.

and direct network effect, along with the difference between sellers' (buyers') stand-alone benefit and the cost of serving sellers (buyers).

Equilibrium market shares

Subsequently, we use the fraction of buyers Equation 1.33a and sellers Equation 1.33b and buyers and sellers equilibrium membership fees at Equation 1.35a and Equation 1.35b, respectively to compute buyers and sellers equilibrium market-shares.[47]

$$\eta_b^{mh} = \frac{2\left(\tau_s+\beta\right)\left(R_b-f_b\right)+\left(\pi+\upsilon\right)\left(R_s-f_s\right)}{4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)^2} \tag{1.36a}$$

$$\eta_s^{mh} = \frac{2\left(\tau_b-\alpha\right)\left(R_s-f_s\right)+\left(\pi+\upsilon\right)\left(R_b-f_b\right)}{4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)^2} \tag{1.36b}$$

When both buyers and sellers are multihoming, buyers' (sellers') market shares are determined by a profit margin on their side $R_b - f_b$ ($R_s - f_s$ for sellers), weighted by the disutility cost of mismatched preferences and direct network effect on the other side, $\tau_s+\beta$ ($\tau_b - \alpha$ for sellers); and a profit margin on the other side of the market $R_s - f_s$, ($R_b - f_b$ for sellers), weighted by the cross-group network effects on both sides, $(\pi + \upsilon)$. This result contrasts with buyers' and sellers' market shares when only one side multihomes. Therefore, when both sides multihome, they have the potential to increase their market share by benefiting from a profit margin on the other side of the market. Specifically, buyers' market share increases by $(R_s - f_s)$, while sellers' market share increases by $(R_b - f_b)$.

Since all buyers and sellers engage in the platform, with some preferring to singlehome and others to multihome, Equation 1.36a and Equation 1.36b must satisfy condition $\eta_b^{sh} < \eta_b^{mh} < 1$ and $\eta_s^{sh} < \eta_s^{mh} < 1$.[48]

Platform's equilibrium profits

Next, we determine equilibrium platform profits by using equilibrium membership fees Equation 1.35a and Equation 1.35b and equilibrium market shares Equation 1.36a and Equation 1.36b, as:[49]

$$\Pi^{mh} = \frac{\left(\tau_s+\beta\right)\left(R_b-f_b\right)^2+\left(\tau_b-\alpha\right)\left(R_s-f_s\right)^2+\left(\pi+\upsilon\right)\left(R_b-f_b\right)\left(R_s-f_s\right)}{4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)^2} \tag{1.37}$$

[47] See Appendix A.5.4 for details.

[48] This condition turns to $\frac{1}{2}\epsilon < 2\left(\tau_s+\beta\right)\left(R_b-f_b\right)+\left(\pi+\upsilon\right)\left(R_s-f_s\right) < \epsilon$ on buyers' side and $\frac{1}{2}\epsilon < 2\left(\tau_b-\alpha\right)\left(R_s-f_s\right)+\left(\pi+\upsilon\right)\left(R_b-f_b\right) < \epsilon$ on sellers' side, where $\epsilon \equiv 4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)^2$. Both conditions are satisfied if Assumption 1.2 and Assumption 1.3 hold. For more details see Appendix A.1 subsection market shares, buyers and sellers multihome.

[49] See Appendix A.5.5 for details.

In the scenario where both buyers and sellers choose to engage in multihoming, platforms generate a profit margin that accounts for the difference between the standalone benefits and the cost associated with serving both buyers and sellers. This profit margin is adjusted by factors such as the disutility cost arising from mismatched preferences, as well as the intra and inter-network effects that influence both sides of the market. Equilibrium profits are positive considering the stand-alone benefit is sufficiently large such that $R_b - f_b$ and $R_s - f_s$ are positive.

Welfare

We can now calculate the surplus for both buyers and sellers in equilibrium. using the equilibrium market shares on both sides from Equation 1.33a and Equation 1.33b, along with the equilibrium membership fees from Equation 1.35a and Equation 1.35b.[50]

$$\nu_b^{mh} = R_b + \upsilon\eta_s^{mh} + \alpha\eta_b^{mh} - p_b^{mh} \qquad \nu_s^{mh} = R_s + \pi\eta_b^{mh} - \beta\eta_s^{mh} - p_s^{mh}$$

$$\nu_b^{mh} = \frac{\tau_b\left[2\left(\tau_s+\beta\right)\left(R_b-f_b\right)+\left(\pi+\upsilon\right)\left(R_s-f_s\right)\right]}{4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)^2} \tag{1.38a}$$

$$\nu_s^{mh} = \frac{\tau_s\left[2\left(\tau_b-\alpha\right)\left(R_s-f_s\right)+\left(\pi+\upsilon\right)\left(R_b-f_b\right)\right]}{4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)^2} \tag{1.38b}$$

We can see that buyers' and sellers' surpluses in Equation 1.38a and Equation 1.38b are directly related to buyers' and sellers' market shares in Equation 1.36a and Equation 1.36b as it can be seen by $\nu_b^{mh} = \tau_b\eta_b^{mh}$ and $\nu_s^{mh} = \tau_s\eta_s^{mh}$.

We follow the same method as in Section 1.4 and Section 1.5 to compute the aggregate surpluses. As a result, we can employ the definitions of consumer and producer surplus outlined in the previous section, along with Equation 1.38a and Equation 1.38b to get:

$$CS^{mh} = \frac{\left(\nu_b^{mh}\right)^2}{\tau_b} = \frac{\tau_b\Big[2\left(\tau_s+\beta\right)\left(R_b-f_b\right)+\left(\pi+\upsilon\right)\left(R_s-f_s\right)\Big]^2}{\left[4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)^2\right]^2} \tag{1.39a}$$

$$PS^{mh} = \frac{\left(\nu_s^{mh}\right)^2}{\tau_s} = \frac{\tau_s\Big[2\left(\tau_b-\alpha\right)\left(R_s-f_s\right)+\left(\pi+\upsilon\right)\left(R_b-f_b\right)\Big]^2}{\left[4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)^2\right]^2} \tag{1.39b}$$

We can see that there is a direct relationship between the aggregate surpluses and market shares on both sides. As it can be seen that $CS^{mh} = \tau_b\left(\eta_b^{mh}\right)^2$ and $PS^{mh} = \tau_s\left(\eta_s^{mh}\right)^2$. Furthermore, the aggregate surplus of buyers and sellers indicates an increasing pattern due to a bandwagon effect, while it is impacted negatively by the presence of

[50] See Appendix A.5.6 for details.

congestion effects and sellers' competition. This trend follows the intuition observed in earlier sections, wherein the platform's value is amplified by a positive intra-network effect, leading to a favourable feedback loop on both sides of the market. Conversely, the implications are reversed when a negative within-network effect comes into play.

Comparative Statics

We are interested in the impacts of the cross-group and direct network effects on the equilibrium strategic variables, which can be seen in Table 1.4.

Table 1.4: Comparative Statics. Buyers and sellers multihoming.

Variables*/ Parameters	Direct Network Effects			Cross-group Network Effects	
	α		β	π	υ
	b[i]	s/c[ii]			
p_b	$+^{*}$	$-^{*}$	$+^{**}$	$-^{**}$	$+^{*}$
p_s	$+^{**}$	$-^{**}$	$+^{*}$	$+^{**}$	$-^{*}$
η_b	+	−	−	+	+
η_s	+	−	−	+	+
CS	+	−	−	+	+
PS	+	−	−	+	+
Π	+	−	−	+	+

* p_b and p_s are buyers and sellers membership fees, η_b and η_s are buyers and sellers market shares, CS is buyers surplus, PS is sellers surplus and Π is platform' profits.
[i] b refers to a bandwagon effect, $\alpha > 0$.
[ii] s/c refers to a snob or congestion effect, $\alpha < 0$.
* Considering $\upsilon > \pi$.
** Considering $\pi > \upsilon$.

We describe next the intuition of the comparative statics[51] shown on Table 1.4 that contrasts with the previous sections.

The impact of an increase in a bandwagon effect (congestion effect) on membership fees is increasing (decreasing). Essentially, as a platform gains greater appeal to buyers, more sellers join, prompting the platform to raise fees on both ends. However, this increase on buyers' side is contingent upon the cross-group network effect exerted by

[51] Refer to Appendix A.5.7 for detailed information on the comparative statics analysis discussed in this section.

sellers on buyers being greater than vice versa ($\upsilon > \pi$). Conversely, an increase in sellers' fees occurs whenever the cross-group effect exerted by buyers on sellers is higher than vice versa ($\pi > \upsilon$). The platform adjusts fees on the side that places a higher value on interaction with the other side; if buyers value interaction with sellers more ($\upsilon > \pi$), their fee increases; otherwise, sellers' fees rise.

Sellers' competition positively influences equilibrium membership fees on both sides of the market. On buyers' side, fee increments occur when the cross-group network effect exerted by buyers on sellers exceeds the reverse scenario ($\pi > \upsilon$); likewise, increases in sellers' fees happen when the cross-group network effect exerted by sellers on buyers exceeds vice versa ($\upsilon > \pi$). In instances where sellers value interaction with buyers more ($\pi > \upsilon$), a trade-off arises between platform participation and abstention due to the negative direct-network effect ($\beta \uparrow$). The platform chooses to increase fees on buyers' side, anticipating sellers will join despite an increase in competition among them ($\beta \uparrow$), thus attracting more buyers via the cross-group network effect and generating additional revenue for each additional participant. Conversely, when buyers value interaction with sellers more ($\upsilon > \pi$), platforms raise sellers' fees since increased buyer participation draws in more sellers despite intensified competition among them ($\beta \uparrow$).

The impacts of an increase in the cross-group network effect align with existing literature as in Armstrong (2006); Rochet and Tirole (2003), wherein platforms adjust fees on the side that exerts a greater influence on the other. If the cross-group network effect exerted by sellers on buyers exceeds vice versa ($\upsilon > \pi$), sellers' fees decrease while buyers' fees increase because of an increase in the cross-group network effect exerted by sellers on buyers (υ). Conversely, an increase in the cross-group network effect exerted by buyers on sellers (π) leads to the opposite outcome.

Equilibrium market shares and aggregate surpluses exhibit similar movements consistent with prior sections. They increase with a rise in both direct and indirect positive network effects (α, υ, π), and decline with negative direct network effects (congestion effect and sellers' competition). Bandwagon and cross-group network effects attract more participants, enhancing the platform's value, and thereby increasing its market share and participants' aggregate surpluses.

Equilibrium platform profits increase with a greater bandwagon effect and cross-group network effects, as they enable charging an additional fee to new participants on both sides, thereby increasing revenue and profits. Conversely, profits decrease with an increase in congestion effect or sellers' competition.

1.7 Comparison of market structure under different scenarios

This section conducts a comparative analysis of the strategic variables in equilibrium, namely fees, market shares, net surpluses, and platforms' profits, under various market conditions. Specifically, the contrast is made between the singlehome environment on both sides, as discussed in Section 1.3, the multihome environment on sellers' side, as described in Section 1.4, the situation where buyers engage with both platforms simultaneously, as explained in Section 1.5, and the scenario where both buyers and sellers engage with both platforms at the same time, as outlined in Section 1.6. Subsequently, several propositions are presented to determine the most favourable market condition for buyers, sellers, and platforms regarding the strategic variables.

1.7.1 Both buyers and sellers singlehome vs. Sellers multihome and buyers singlehome

This subsection presents a comparative analysis of the outcomes obtained in Section 1.3, where both agents singlehome and Section 1.4, where sellers multihome and buyers singlehome. As outlined previously, when sellers multihome and buyers singlehome, platforms possess a monopoly control on sellers' side. This is because platforms can charge a higher fee to enable sellers to interact with their exclusive buyers, while simultaneously reducing the fees paid by buyers. These measures typically result in higher buyers' aggregate surplus, and higher platforms' profits can vary depending on specific parameter values.

To formalise the previous intuition, we state the next proposition.

Proposition 1.1. *When sellers multihome and buyers singlehome compare to situations where both buyers and sellers singlehome:*

1. *Platforms charge a higher fee on sellers' side and a lower fee on buyers' side,* $p_s^{smh} > p_s^{sh}$ *and* $p_b^{smh} < p_b^{sh}$ *as long as* $(\tau_s + \beta) < \upsilon$.
2. *Buyers are always better off* $CS^{smh} > CS^{sh}$.
3. *Platforms have higher profits* $\Pi^{smh} > \Pi^{sh}$ *as long as* $(\tau_s + \beta) < \upsilon$ *and* $\upsilon > \pi$.

Proof: See Appendix A.6.1.

When sellers engage in multihoming while buyers adopt singlehoming, platforms possess a monopoly power over sellers by imposing higher fees for access to their exclusive buyers, while reducing fees for buyers. This platform strategy remains feasible provided that the cross-group network effect exerted by sellers on buyers exceeds the combined impact of transportation costs and direct network effects on sellers, represented by $\tau_s + \beta < \upsilon$.

Lower buyer fees attract a larger audience to the platform, regardless of whether buyers exhibit bandwagon or snob behaviour, thereby drawing more participants from sellers' side and subsequently from buyers' side due to the cross-group network effects. Consequently, platforms collect greater profits by charging additional fees per participant, independent of opposing movements in membership fees, as previously discussed. This outcome is sustained with higher values of the cross-group network effect sellers exert on buyers' side, particularly, above $\tau_s + \beta$ threshold, and the cross-group network effect exerted by sellers on buyers outweighs the reverse scenario, $\upsilon > \pi$.

Initially, this result may appear counterintuitive as the seminal model proposed by Armstrong (2006) typically indicates a reduction in membership fees for the side exerting a significant cross-group network effect on the other side. In this case, where $\upsilon > \pi$, one might expect a decrease in sellers' fees. However, the effect of the monopoly power applied by platforms over sellers overcomes the impact of the cross-group network effect, enabling platforms to raise fees on sellers' side and counterbalancing by decreasing buyers' fees.

Then, platforms have higher profits when sellers engage in multihoming while buyers stick to singlehoming, particularly when the fee on sellers' side when they singlehome is low, possibly limited to the serving cost (f_s). This is illustrated in Equation 1.8b where $\upsilon > \tau_s + \beta$. Consequently, the platform favours situations where sellers engage in multihoming while buyers remain singlehoming, as opposed to scenarios where both parties choose singlehoming.

Welfare comparisons

Consumers' aggregate surplus is greater when sellers engage in multihoming while buyers engage in singlehoming, compared to scenarios where both type of participants commit to singlehoming, primarily due to lower buyers' membership fees.

There are no conclusive results on sellers' side. However, sellers who engage in multihoming have access to all buyers, potentially enhancing their surplus despite incurring higher fees and transportation costs. The intuition behind this lies in the easing of the negative direct network effect on sellers' side when they engage in multihoming, now they have the option of joining both platforms decreasing the competition amongst them, thereby facilitating greater seller participation. Consequently, this encourages an increase in the number of buyers joining the platform, creating a positive feedback loop wherein more agents from both sides are encouraged to participate. As a result, the overall aggregate surplus may increase.

1.7.2 Both buyers and sellers singlehome vs. Buyers multihome and sellers singlehome

In this subsection, we aim to compare the findings obtained in Section 1.3, where both agents are singlehoming, with the ones obtained in Section 1.5, where buyers multihome and sellers singlehome. As previously discussed, in situations where buyers multihome and sellers singlehome, platforms hold a monopoly power on buyers' side, whereby they can charge a premium fee to buyers to interact with their exclusive sellers while reducing the fees charged to sellers. This could result in a higher aggregate surplus for sellers, it might also lead to an increase in buyers' aggregate surplus. Furthermore, platforms' profits are higher under specific parameter values. To formalise the previous intuition, we can state the next proposition.

Proposition 1.2. *When buyers multihome and sellers singlehome compare to situations where both buyers and sellers singlehome:*

1. *Platforms charge a higher fee on buyers' side and a lower fee on sellers' side, $p_b^{bmh} > p_b^{sh}$ and $p_s^{bmh} < p_s^{sh}$ as long as $(\tau_b - \alpha) < \pi$.*
2. *Sellers consistently benefit in terms of producer surplus, $PS^{bmh} > PS^{sh}$.*
3. *Platforms have higher profits $\Pi^{bmh} > \Pi^{sh}$ as long as $(\tau_b - \alpha) < \pi$ and $\pi > \upsilon$.*

Proof: See Appendix A.6.2.

In situations where buyers engage in multihoming while sellers choose to singlehome, buyers' membership fee is greater and sellers' fee is lower compared to situations where both type of participants singlehome as long as the cross-group network effect buyers exert on sellers (π) is large enough to exceed $(\tau_b - \alpha)$. This is because platforms can charge a higher membership fee to buyers for the privilege of interacting with exclusive sellers.

Platforms have higher profits when buyers multihome and sellers singlehome compared to a scenario where both singlehome as long as the cross-group network effect buyers exert on sellers (π) is large enough to exceed $(\tau_b - \alpha)$ and the cross-group network effect buyers exert on sellers is stronger than the cross-group network sellers exert on buyers. $(\pi > \upsilon)$. Platforms have higher profits when buyers engage in multihoming while sellers stick to singlehoming, particularly when the fee on buyers' side when they singlehome is low, possibly limited to the serving cost (f_b). This is illustrated in Equation 1.8a where $\pi > \tau_b - \alpha$. Consequently, the platform favours situations where buyers engage in multihoming while sellers remain singlehoming, as opposed to scenarios where both type of participants choose singlehoming.

Welfare comparisons

Producers' aggregate surplus is higher when buyers engage in multihoming and sellers singlehome compared to scenarios where both sides singlehome because their membership fee is lower.

There are no conclusive results on buyers' aggregate surplus. It could potentially be higher even when their fees are higher, as a result of the presence of a bandwagon effect that draws more buyers and consequently more sellers. This enhances the overall value of the platform and increases buyers' aggregate surplus. In contrast, when there is a congestion effect, participation is discouraged on both sides, reducing the platform's attractiveness and decreasing buyers' aggregate surplus.

1.7.3 Buyers multihome and sellers singlehome vs. Both buyers and sellers multihome

In the present subsection, we conduct a comparison between the outcomes obtained in Section 1.5, where buyers multihome and sellers singlehome, and those obtained in Section 1.6, where both agents engage in multihoming. As discussed in earlier sections, the dynamics of platform competition differ depending on whether buyers are multihoming and sellers singlehome or both sides engage in multihoming. Platforms possess monopoly power over the side multihoming when the other side joins only one provider, whereas multihoming on both sides eliminates this control. Despite higher fees, participants prefer multihoming to benefit from the cross-group network effects. Buyers prefer seller multihoming to increase their market share and obtain a higher surplus. Platforms strategy is to develop incentives for sellers to engage in multihoming to have higher profits. These insights are formalised in the following proposition.

Proposition 1.3. *When both buyers and sellers multihome compare to situations where buyers multihome and sellers singlehome:*

1. *Platforms charge a higher fee on buyers' side, $p_b^{mh} > p_b^{bmh}$ as long as $\upsilon > \pi$.*
2. *Buyers are consistently better in terms of consumer surplus, $CS^{mh} > CS^{bmh}$.*
3. *Platforms prefer both buyers and sellers multihoming because they have higher profits $\Pi^{mh} > \Pi^{bmh}$.*

Proof: See Appendix A.6.3.

When buyers shift from a situation where they are the only ones connecting to two platforms simultaneously, while sellers singlehome, to a scenario where both type participants engage in multihoming, platforms lose their monopoly power over buyers. Furthermore, platforms no longer have the option of reducing fees to attract participants on both sides of the market as this strategy becomes less effective in the multihoming scenario.

As long as the cross-group network effect sellers exert on buyers is stronger than the impact buyers exert on sellers, $\upsilon > \pi$ platforms prefer both buyers and sellers multihoming than just buyers joining both platforms simultaneously because the charge higher fees. However, when the relationship between the cross-group network effects is reversed, $\pi > \upsilon$ buyers membership fee is higher in a scenario where just buyers multihome and sellers singlehome than when both type of participants singlehome. This result is driven by the strategy platforms follow of reducing the fee on the side that is having a stronger impact on the other side.

When the influence of sellers on buyers outweighs that of buyers on sellers ($\upsilon > \pi$), platforms favour both buyers and sellers engaging with multiple platforms rather than just buyers joining both platforms simultaneously. This preference arises because platforms can charge higher fees in such scenarios. Conversely, when the balance of cross-group network effects shifts ($\pi > \upsilon$), the membership fee for buyers is higher when only buyers engage with multiple platforms while sellers stick to a single platform, compared to when both type of participants are multihoming. This outcome is a result of platforms strategically adjusting fees based on which side exerts a stronger influence on the other.

There are not conclusive results on sellers membership fees, however it's reasonable to infer that platforms exhibit similar behaviour on sellers' side as they do on buyers' side.

Platforms can achieve higher profits when both buyers and sellers engage in multihoming compared to the scenario where only buyers multihome and sellers singlehome. This is attributed to three factors. Firstly, as stated before, at least buyers membership fees are higher when participants engage in multihoming. Secondly, buyers' and sellers' market shares are greater when both agents multihome. Finally, as mentioned before, the bandwagon effect is strengthened while the negative direct network effect (congestion and sellers competition) are weakened, resulting in a positive feedback loop that attracts more participants on both sides of the market. Consequently, platforms can charge an additional fee per additional agent who chooses to multihome, thereby higher profits.

Welfare comparisons

Buyers surplus is higher when both buyers and sellers multihome, even though they have to pay higher fee compared to the scenario where only buyers multihome and sellers singlehome. This outcome can be primarily attributed to the fact that simultaneous participation on both platforms enhances the engagement of even more agents and platforms become more valuable increasing aggregate surplus when they engage in transactions on both providers.

The lack of conclusive results on sellers' side leaves open the possibility that joining both platforms simultaneously might be more profitable for sellers, even if it entails higher fees compared to joining just one platform. However, the negative direct-network effect could discourage their motivation to participate, making it less appealing to join both platforms. This may also affect the attractiveness of participation on the other side, potentially leading to a decrease in aggregate surplus.

1.7.4 Sellers multihome and buyers singlehome vs. Both buyers and sellers multihome

Now, we analyse and contrast the outcomes obtained in Section 1.4 where sellers multihome and buyers singlehome to Section 1.6 where both agents multihome. As discussed previously, the scenario where sellers multihome and buyers singlehome grants platforms monopoly power on sellers' side. However, in the scenario where both sides multihome, platforms lose their monopoly power. Despite the higher fee charged to participants when they multihome than when they singlehome (because they pay twice to join both platforms), they still prefer it because they obtain a larger cross-group network effect. Similarly, sellers prefer buyers to multihome, as it results in a higher surplus due to a greater market share than when buyers were singlehoming. To increase their profits, platforms can create the right incentives for buyers to be involved in multihoming and obtain higher profits. To formalise the previous intuition we state the next proposition.

Proposition 1.4. *When both buyers and sellers multihome compare to situations where sellers multihome and buyers singlehome:*

1. *Platforms charge a higher fee on sellers' side, $p_s^{mh} > p_s^{smh}$ as long as $\pi > \upsilon$.*
2. *Sellers are consistently better in terms of producer surplus, $PS^{mh} > PS^{smh}$.*
3. *Platforms prefer both buyers and sellers multihoming because they have higher profits $\Pi^{mh} > \Pi^{smh}$.*

Proof: See Appendix A.6.4.

When both buyers and sellers engage with multiple platforms, sellers' membership fee is higher than when only sellers engage with both platforms and buyers stick to a single platform, provided that the cross-group network effect exerted by buyers on sellers is stronger than vice versa ($\pi > \upsilon$). Conversely, if the strength of these cross-group network effects is reversed ($\upsilon > \pi$), sellers' membership fees are higher when only sellers engage with multiple platforms and buyers singlehome compared to when both type of participants engage with multiple platforms. This result is consistent with Proposition 1.3, as platforms tend to charge lower fees on the side exerting a stronger impact on the other side.

It is noteworthy that the results remain unaffected by direct network effects. This is because simultaneous participation on both platforms mitigates the influences of bandwagon effects, congestion effects, and seller competition, as the platforms no longer compete for the same participants.

Platforms achieve higher profits by enabling multihoming for both buyers and sellers, as opposed to allowing only sellers to multihome while buyers remain singlehoming. This can be attributed to three key factors as in Proposition 1.3. Firstly, as was established, membership fees increase when participants get involved in multihoming. Secondly, when both sides of the market engage in multihoming, their respective market shares are increased. Finally, more buyers and sellers joining results in a positive feedback loop that attracts more participants on both sides of the market. As a result, platforms charge an additional fee for each additional participant that engages in multihoming, ultimately increasing their profits.

Welfare comparisons

The intuition on why sellers aggregate surplus is higher when both buyers and sellers multihome, even though they have to pay higher fees compared to the scenario where only sellers multihome and buyers singlehome is the same as in Proposition 1.3.

Finally, it is important to highlight that Proposition 1.3 and 1.4 reveal that participants in the market, including buyers, sellers, and platforms, consistently obtain a better outcome when both sides of the market become involved in multihoming, rather than when only one side join both platforms simultaneously and the other side singlehome.

1.7.5 Both buyers and sellers singlehome vs. Both buyers and sellers multihome

In this subsection, we compare the results obtained in Section 1.3 where both buyers and sellers singlehome, with those presented in Section 1.6 where both sides engage in multihoming. In previous sections, we established that the market shares of both buyers and sellers increase when they engage in multihoming as opposed to singlehoming. This in turn leads to higher profits for platforms, as they can charge additional fees to a larger pool of participants. This intuition can be formalised in the following proposition.

Proposition 1.5. *When both buyers and sellers multihome compare to situations where both buyers and sellers singlehome:*

1. *Platforms charge a higher fee on buyers' side,* $p_b^{mh} > p_b^{sh}$*, as long as* $\upsilon > \pi$ *and* $\pi > \tau_b - \alpha$ *and a higher fee on sellers' side,* $p_s^{mh} > p_s^{sh}$*, as long as* $\pi > \upsilon$ *and* $\upsilon > \tau_s + \beta$*.*

2. *Platforms have higher profits* $\Pi^{mh} > \Pi^{sh}$ *as long as* $\pi > \tau_b - \alpha$ *and* $\upsilon > \tau_s + \beta$*, or as long as* $\upsilon > \tau_b - \alpha$ *and* $\pi > \tau_s + \beta$

Proof: See Appendix A.6.5.

Buyers face higher membership fees when both parties engage in multihoming, rather than when both singlehome, as long as the cross-group network effect exerted by sellers on buyers is stronger than vice versa ($\upsilon > \pi$) and the cross-group network effect exerted by buyers on sellers (π) is sufficiently strong to satisfy $\pi > \tau_b - \alpha$.

Similarly, sellers encounter higher membership fees when both parties engage in multihoming, rather than when both single-home, under the condition that the cross-group network effect exerted by buyers on sellers is stronger than vice versa ($\pi > \upsilon$) and the cross-group network effect exerted by sellers on buyers (υ) is strong enough to satisfy $\upsilon > \tau_s + \beta$.

As buyers and sellers shift from a singlehoming scenario to a multihoming one, their market shares increase. This increase attracts more participants to join both platforms, driven by the cross-group network effects regardless of the direct effects being positive or negative. Platforms revenue increases as a result, through the imposition of higher fees on at least one side of the market (because the strength of the cross-group network effects comes into play), in contrast to a scenario where the market is evenly split between two providers.

Platforms generate higher profits by enabling multihoming on both sides of the market, rather than when both agents singlehome, as long as the cross-group network effects are strong enough to satisfy either $\pi > \tau_b - \alpha$ and $\upsilon > \tau_s + \beta$, or $\upsilon > \tau_b - \alpha$ and $\pi > \tau_s + \beta$. The first-pair of conditions mirror those required for higher membership fees when both buyers and sellers engage in multihoming compared to single-homing.

The latter set of conditions[52], $\upsilon > \tau_b - \alpha$ and $\pi > \tau_s + \beta$, fosters a positive feedback loop between the two sides of the market. As an extra proportion of buyers join, more sellers are incentivised to participate, and vice versa. Consequently, platforms can charge additional fees for each additional participant engaging in multihoming, thereby having higher profits.

Welfare comparisons

There are no conclusive results on aggregate surpluses. However, it is reasonable to think that multihoming benefits both buyers and sellers by enabling them to take

[52] These conditions are based on Equation 1.22 and Equation 1.12, where $\frac{\partial \eta_b}{\partial \eta_s} = \frac{\upsilon}{\tau_b - \alpha}$ and $\frac{\partial \eta_s}{\partial \eta_b} = \frac{\pi}{\tau_s + \beta}$ which implies that an extra proportion of buyer and seller to the platform attracts even more sellers and buyers.

advantage of the cross-group network effect, despite the higher fees. As a result, transactions generate more platform value and attract more participants on both sides, creating stronger cross-group network effects and higher aggregate surpluses.

Proposition 1.5 share the same results as Proposition 1.3 and 1.4, regarding the optimal scenario for platforms is when both buyers and sellers multihome.

1.8 Conclusions

We have extended Armstrong (2006) and Belleflamme and Peitz (2019b) models to incorporate a direct network effect on *both* buyers' and sellers' sides. Furthermore, we have allowed *both* buyers and sellers to singlehome and multihome. In this framework, we examine the impact of a positive direct-network effect (also known as a bandwagon effect), as well as a negative direct externality (also known as a snob/congestion effect) on buyers' sides and (also known as sellers competition) on sellers' side. These interactions are analysed across distinct market structures, including singlehoming for both sides, multihoming for sellers and singlehoming for buyers, multihoming for buyers and singlehoming for sellers, and multihoming for both sides. We find that in all scenarios, the interplay between direct-network effects on both sides and cross-group network effects alters the primary results of both Armstrong (2006) and Belleflamme and Peitz (2019b).

This study makes a significant contribution to the existing literature such as Evans and Schmalensee (2005); Jullien et al. (2021); Sanchez-Cartas and León (2021) by integrating direct and indirect network effects with singlehoming and multihoming decisions made by both sides of the market within a unified framework. To the best of our knowledge, no previous study has simultaneously coherently examined these crucial factors. By incorporating different types of network effects operating concurrently on both sides and allowing agents to engage in singlehoming or multihoming, our research provides a more realistic and comprehensive model that extends the current understanding of platform competition. This unified framework enables policymakers to capture the complex interplay between network effects and agent behaviours, shedding new insight into the strategic interactions and equilibrium outcomes in two-sided markets.

We find that buyers exhibit a preference for sellers who engage in multihoming when they singlehome, since platforms charge lower fees in such instances, resulting in a higher aggregate surplus for consumers. On the other hand, sellers prefer to singlehome to avoid higher fees imposed by platforms. Platforms, in turn, prefer a scenario where sellers engage in multihoming, while buyers engage in singlehoming because profits are higher.

Sellers typically prefer buyers who engage in multihoming while they singlehome, as lower fees charged by platforms in such instances result in a higher aggregate surplus. Conversely, buyers prefer to singlehome to avoid higher fees imposed by platforms. Platforms prefer a scenario where buyers engage in multihoming, while sellers engage in singlehoming because they obtain higher profits.

Platform profits are higher when both buyers and sellers engage in multihoming, rather than when just buyers multihoming and sellers singlehoming. Sellers may still choose to join both platforms even if they have to pay a higher fee because they benefit by the cross-group network effects on both platforms. Consumers' aggregate surplus tends to be higher when both agents multihome compared to when only buyers multihome and sellers singlehome, provided that with both sides multihoming, market shares on both sides are greater boosting the value generated by the interactions between both types of participants. This outcome is drawn regardless of a positive or negative direct network effect.

Platforms can maximise their profits by encouraging both sellers and buyers to multihome, rather than having only sellers multihoming and buyers singlehoming. Sellers pay a higher fee to join both platforms if they value interaction more than buyers. Sellers' aggregate surplus is higher despite costly fees and negative direct network effects.

When buyers and sellers multihome simultaneously, platforms obtain higher profits by facilitating multihoming on both sides of the market, as compared to a situation where both types of participants are singlehoming as long as participants on one side highly respond to participants on the other side. This is attributed to three main elements. Firstly, when participants engage in multihoming, both membership fees are higher. Secondly, buyers' and sellers' market shares are greater when both agents multihome than when they singlehome. Thirdly, there is a positive feedback loop that attracts more participants on both sides of the market. As a result, platforms charge additional fees to the extra participants choosing to multihome, thereby increasing their profits.

Policymakers can use these findings to design regulations that encourage multihoming behaviours, fostering a more competitive and efficient platform market landscape. However, as we showed, there exists a dynamic relationship between engaging in multihoming or remaining singlehoming for both participants (buyers and sellers). The decision hinges on factors such as whether the other side is multihoming or not. For example, if sellers are already multihoming, buyers could consider multihoming despite potentially higher fees in exchange for a greater surplus. Consequently, public policy interventions concerning singlehoming or multihoming should not be unilaterally focused on one side alone; both sides of the market must be taken into account.

Chapter 2

How do platforms appeal to buyers?

2.1 Introduction

In recent times, there has been a significant surge in the volume of electronic commerce attributable to the widespread availability of the Internet. According to Euromonitor International's 2018 report, the proportion of retail sales conducted online accounted for 13.7% and 17% in the United States and the United Kingdom, respectively, while globally, it represented 11.5% of all retail sales. These figures translate into substantial revenue, with online retail sales reaching over $400 billion, $86 billion, and $1.7 trillion for the USA, UK, and worldwide, respectively.[1]

E-commerce typically involves buying and selling goods or services through online platforms, which is a business model connecting buyers and sellers, enabling them to engage in value-creating exchanges. It is common that a dominant platform is present in this type of market, such as *Amazon* in the online retailing sector[2], *Airbnb* in lodging services[3], and *Uber* in the ride-hailing industry[4], among others. The underlying factors that contribute to these platforms' successful attraction and retention of agents have generated significant scholarly and practical interest. One potential explanation for their success is the platform's ability to serve as an intermediary between agents and actively shape the business model. It is this active involvement that may give rise to heterogeneity among platforms, and may, in turn, affect agents' incentives and valuations regarding which platform to join.

In this study, we present a framework for analysing how platforms appeal to agents, specifically on buyers' side. Our argument is that buyers' decisions to join a platform are

[1] Data is taken from Exhibit 7 of Amazon.com, 2019 Case 716-402.
[2] Amazon.com, 2019 Case 716-402.
[3] World's Leading Online Travel Accommodation Marketplace 2020, accessed August 2021
[4] Global Top 100 Brands 2019

not only based on membership fees and cross-network effects but also on other attributes platforms offer.[5] The combination of these three elements determines which platform buyers find most appealing. Buyers are more inclined to join a platform that has built a favourable reputation and brand image over time by offering a diverse range of features. As the quality of platform's features increases, buyers' perception of the platform's benefits improves, resulting in a stronger reputation and brand image, thereby increasing the likelihood of buyers joining the platform.

A specific example is *Amazon*, which not only works as an intermediary between buyers and sellers but also has an active function adopting a customer-centric approach to generate attributes that create value. For buyers, the platform's benefits proposition transcends beyond product pricing. It extends to the ability to appeal to and initiate a loyal customer base, enhancing their browsing experience through the provision of flexible delivery options, an extensive product assortment, swift checkout processes, and a lenient refund and return policy. On the seller side, having their products affiliated with Amazon's brand name enhances their credibility with customers and leverages the platform's Prime audience. Wells et al. (2019) observed that the majority of attributes developed by *Amazon* are primarily buyer-oriented. *Amazon* strives to attract buyers to its site by developing various attributes to meet their needs.

This chapter makes a twofold novel contribution to the existing literature on two-sided markets. Firstly, we introduce the platform's features as a form of vertical product differentiation on buyers' side, shedding light on the importance of quality attributes in shaping market structure. Secondly, we analyse the intricate interactions between these quality attributes and cross-group network effects to gain insights into the resulting market configurations. By exploring these dimensions, our study expands the understanding of two-sided markets and offers valuable insights for market participants and policymakers alike.

Vertical differentiation refers to the differentiation of products or services offered by platforms based on their perceived quality, features, or attributes that cater to the distinct needs of both sides of the market. Platforms offer different levels of quality to enhance their features, functionality, user experience, or service level to attract and retain users on both sides of the market. Rather than attempting to capture all possible features a platform may have, we integrate them into a single variable representing buyers' perception of the quality of the platform.

Our model builds on the framework of Armstrong (2006), where equilibrium membership fees depend on cross-group network effects, and the literature on vertical differ-

[5]For simplification purposes attributes, features and characteristics are used interchangeably throughout the chapter.

entiation, including Mussa and Rosen (1978); Gabszewicz and Thisse (1979); Shaked and Sutton (1982, 1983), which identify consumer income as a source of differentiation. We extend Armstrong (2006) model by introducing the level of features offered on the buyers' side as a strategic variable on the vertical dimension. This allows for the existence of asymmetric platforms in equilibrium, as shown by Gabszewicz and Wauthy (2014).

The provision of attributes by platforms creates a competitive advantage in attracting agents in a two-sided market. This competitive advantage can be understood as heterogeneity within a vertically differentiated product space, where agents prefer platforms offering more attributes compared to those offering fewer attributes. The concept of vertical product differentiation space was first explored by scholars such as Mussa and Rosen (1978); Gabszewicz and Thisse (1979); Shaked and Sutton (1982, 1983). Mussa and Rosen (1978) investigated a monopoly pricing model for quality differentiated goods, and found that a monopolist cannot price discriminate in the usual way, but rather assigns a price-quality pair to customers to partially discriminate against them, thereby reducing the quality sold to customers compared to a competitive market. Gabszewicz and Thisse (1979) analysed a non-cooperative price equilibrium between firms, where consumers have different willingness to pay for quality improvements, and found that with less income disparity, the firm selling the lowest quality product will exit the market. Moreover, when consumers' tastes are less differentiated, Cournot's equilibrium price is near zero. Shaked and Sutton (1982, 1983) studied vertical differentiation in a competitive market and found that firms differentiate themselves by choosing distinct qualities to lower price competition and earn positive profits.

Subsequently, the seminal works of Economides (1989); Neven and Thisse (1989) were the first to jointly examine both horizontal and vertical product differentiation spaces. Horizontal differentiation pertains to the range of products offered, while vertical differentiation refers to the quality of the products sold in the market. Both studies yield comparable results, showing that firms maximise one dimension (variety) while minimising the other characteristic (quality) to gain a larger market share and increase profits. Building on these findings, Irmen and Thisse (1998) extended the previous models to include multiple characteristics and report similar results, indicating that firms choose to maximise differentiation in the dominant characteristic and minimise the remaining attributes to reduce price competition.

These models have undergone extensions to encompass a diverse range of sectors. Degryse (1996) explored banking services, Baake and Boom (2001) examined markets with network externalities. Inderst and Irmen (2005) focused on space and time as strategic variables in horizontal product differentiation, specifically in the retail markets, and Hansen and Nielsen (2011) investigated price as a proxy for quality in the trade between two countries. Garella and Lambertini (2014) identifies situations in which firms select maximum differentiation in both characteristics by studying economies of scope. Finally, Barigozzi and Ma (2018) developed a general specification model that allows for general consumer preference distributions, general production cost functions (increasing and convex), and firms selecting any arbitrary number of quality characteristics.

Recent studies have explored the intersection of two-sided markets and vertical differentiation. For instance, Gabszewicz and Wauthy (2014) introduced heterogeneity among participants and found that platform competition with cross-group externalities and vertical differentiation can result in the equilibrium coexistence of asymmetric platforms. Zennyo (2016) investigated vertically differentiated two-sided markets and found that in a sequential game, both platforms charged the same per-transaction fee in equilibrium, even with quality asymmetries. Under certain conditions, a low-quality platform was found to have higher profits than a high-quality platform. Roger (2017) studied two-sided markets where platforms compete for agents on both sides of the market, and concluded that when cross-group externalities are too strong, pure-strategy equilibrium may not exist. Lastly, Etro (2021) considered the differences between device-funded and ad-funded platforms. His results showed that device-funded platforms are more aligned with consumers because they provide high-quality products and services, while ad-funded platforms offer products at competitive prices and free services.

The seminal models of Caillaud and Jullien (2003); Armstrong (2006); Rochet and Tirole (2003, 2006) analysing two-sided markets have been extended in various directions by subsequent research. Belleflamme and Toulemonde (2009); Hagiu (2009); Belleflamme and Toulemonde (2016); Belleflamme and Peitz (2019b) introduced competition among sellers and investigate how pricing equilibrium, product variety, and the optimal number of platforms are affected in the presence of a monopolistic or duopolistic platform. Their findings indicate that while consumers and producers prefer product variety, platforms prefer to minimise differentiation among them. Weyl (2010) proposed a nonlinear tariff that is conditional on the participation of agents on both sides in order to address the problem of equilibrium multiplicity. Choi (2010); Choi et al. (2017) investigated the impact of tying in a two-sided market where agents can use multiple platforms. They find that allowing multihoming can improve welfare through tying. Gao (2018) analysed the effects of overlapping agents on both sides of a platform. Finally, Karle et al. (2020); Jeitschko and Tremblay (2020) examined how agents endogenously determine whether to singlehome or multihome.

Our model consists of two stages, where agents can join one platform (singlehome) only and platforms simultaneously determine the level of attributes they offer on buyers' side in the first stage, and then determine membership fees in the second stage. We find equilibrium membership fees follow Armstrong (2006) result, but are adjusted by the differences in attributes offered by platforms on buyers' side, and weighted by the cross-group network effect one side exercises on the other side.

Our first key finding is that the difference in attributes on buyers' side between two competing platforms not only affects their behaviour but also has an impact on sellers' side as a result of the presence of cross-group network effects on both sides of the market. We analyse two different scenarios based on the strength of these cross-group network effects. The first scenario establishes identical indirect network effects on both sides of the market. The second scenario analyses when the network effects are distinct. We find that when both cross-network effects are equal, sellers' equilibrium membership fee remains as Armstrong (2006) stated, indicating that the difference in attributes on buyers' side only impacts buyers' decisions.

We establish conditions for a max-min strategy to enhance profits, as demonstrated in the early works of Economides (1989) and Neven and Thisse (1989) and the generalized model of Irmen and Thisse (1998). Specifically, we identified two scenarios where such a strategy is effective: when the cross-group network effects on both sides of the market are equal, and when the cross-group network effect buyers have on sellers is greater than the impact sellers exert on buyers. In the former situation, platforms differentiate themselves as much as possible on attributes on buyers' side (vertical dimension) and as little as possible on the product differentiation cost (horizontal dimension). In the latter setting, platforms differentiate themselves as little as possible on attributes on buyers' side and as much as possible on the horizontal dimension to maximise profits. Furthermore, we find conditions for a max-max strategy to maximise profits, as seen in recent studies by Garella and Lambertini (2014); Barigozzi and Ma (2018). In particular, we find platforms differentiate as much as possible on both dimensions when the cross-group network effect exerted by sellers on buyers outweighs those exercised by buyers on sellers.

This chapter is structured as follows. Section 2.2 outlines the model primitives, while Section 2.3 presents the solution to stage 2 of the model to obtain equilibrium membership fee configurations. Section 2.4 provides the solution to stage 1 of the model, deriving equilibrium attribute configurations on buyers' side. In addition, Section 2.5 analyses and compares market structure where the cross-group network effects on both sides of the market are identical and opposite. In both cases, we express the strategic variables as a function of the model parameters and provide intuitive explanations for the results. The chapter concludes in Section 2.6.

2.2 Model

This chapter considers a model of platform competition with cross-group external effects and attributes on buyers' side. There are three different players: platforms, buyers and sellers. The model follows Armstrong (2006) considering two platforms that are horizontally differentiated and charge access fees to both sides of the market. Buyers and sellers whom we refer to as agents, make a decision to join a single platform, a scenario known as singlehoming. In this model we introduce the level of attributes q_b as a strategic variable capturing various platform features on buyers' side: the higher the value of q_b, the more attractive the platform is for buyers, given membership fees.

Two platforms engage in competition through membership fees and attributes offered on buyers' side. This setup is designed to facilitate interactions between a unit mass of sellers and buyers, generating positive cross-group network effects. Positioned at the extremes of a unit interval, the platforms exhibit horizontal differentiation à la Hotelling and bear a constant cost of f_b and f_s for serving buyers and sellers, respectively. Buyers and sellers, uniformly distributed across this interval, face a cost of visiting a platform that increases linearly in distance, τ_b and τ_s, respectively. This cost can be interpreted as a potential mismatch with buyers' and sellers' preferences. Considering our focal point is the relationship between the cross-group network effects and the attributes a platform offers on buyers' side, we assume, that the cost associated with visiting a platform is homogeneous across both platforms and both sides. This means that both buyers and sellers face the same disutility cost when their preferences are mismatched, and we defined it as $\tau_b = \tau_s = \tau$.

Buyers, upon joining the platform, purchase one unit of product from each active seller on the same platform. For each trade, buyers and sellers obtain a cross-group network effect of υ and π, respectively; which can also be seen as gains from trade. Additionally, there exists a stand-alone benefit of R_b for buyers and R_s for sellers when they visit the platform, a benefit uniform across both platforms. We define η_b^i and η_s^i as the mass of buyers and sellers joining platform i, $i = 1, 2$. The membership fees charged to buyers and sellers on platform i are denoted as p_b^i and p_s^i, respectively.

In addition, buyers receive q_b^i for the attributes platform i offers. Platform i's for $i = 1, 2$ production cost of providing these attributes on buyers' side is set as $C^i\left(q_b^i\right) = \frac{1}{2}\alpha^i\left(q_b^i\right)^2$. The parameter α^i captures the efficiency of platform i developing characteristics on buyers' side. We assume $0 < \alpha^1 < \alpha^2$, meaning platform 1 is more efficient in developing these attributes compared to platform 2. This is possible, either because it can produce more features with the same inputs or deliver the same level of features at a lower cost. As a result, platforms are heterogeneous in terms of both product differentiation and the characteristics they offer on buyers' side.

Therefore, buyers and sellers, respectively, obtain a surplus of visiting platform i,

$i = 1, 2$, of:[6]

$$\nu_b^i = R_b + q_b^i + \upsilon\eta_s^i - p_b^i \tag{2.1a}$$

$$\nu_s^i = R_s + \pi\eta_b^i - p_s^i \tag{2.1b}$$

The model consists of two stages. In the first stage platforms simultaneously choose characteristics on buyer's side and in the second stage choose simultaneously membership fees and then buyers and sellers choose simultaneously which platform to join. We analyse different cases using the previous framework in the next sections.

The model parameters must meet the following assumptions.[7]

Assumption 2.1. $\tau > \frac{\pi+\upsilon}{2}$ if $\pi > \upsilon$ or $\frac{\pi+\upsilon}{2} < \tau < \frac{\pi+2\upsilon}{3}$ if $\upsilon > \pi$

Assumption 2.2. $\alpha^i > \frac{2\tau}{\Sigma}$, $i = 1, 2$ where $\Sigma \equiv 9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$

Assumption 2.1 is developed on the second-order conditions of the platform maximisation problem at stage 2 of the model and from the conditions to guarantee equilibrium market shares are restricted to a unit interval. This condition stipulates that the degree of product differentiation must fall within a range defined by the cross-group network effects. This condition also is needed to have positive equilibrium attributes[8].

Assumption 2.2 is established on the conditions to have positive equilibrium attributes. This condition means that the parameter measuring platform i efficiency in developing attributes on buyers' side is not negligible. This condition guarantees the second-order conditions of the platform maximisation problem at stage 1 of the model to be satisfied.

2.3 Equilibrium membership fees

We develop a two-stage model of two-sided markets with vertical differentiation where agents singlehome. We solve our model using backward induction. In this section, we solve the second stage of the game where platforms choose simultaneously membership fees and then agents choose simultaneously which platform to join, assuming the level of attributes on buyers' side as given. We then obtain market shares and platform profits at equilibrium, offering some insights into the results.

We identify a buyer (b) and a seller (s) positioned at locations x_b and x_s within a

[6]We assume that the market is fully covered, implying that buyers and sellers do not have an outside option to interact. This assumption is standard in the literature, as evidenced by Choi (2010); Hagiu (2009).

[7]For further details on both assumptions see Appendix B.1.1.

[8]Equilibrium attributes are defined in Definition 2.2 and equilibrium market shares are defined in Equations (2.5a) and (2.5b)

unit interval, respectively, who are indifferent between joining platform 1 and 2, such that $\nu_k^1 - \tau x_k = \nu_k^2 - \tau(1 - x_k)$ where $k = b, s$. Buyers and sellers located between 0 and x_b or x_s visit platform 1, while those positioned between x_b or x_s and 1 visit platform 2. Consequently, we have $\eta_b^1 = x_b$, $\eta_b^2 = (1 - x_b)$, $\eta_s^1 = x_s$ and $\eta_s^2 = (1 - x_s)$, with the total number of buyers and sellers on both platforms being $\eta_b^1 + \eta_b^2 = \eta_s^1 + \eta_s^2 = 1$. We then determine the proportion of buyers and sellers for platform i, $i = 1, 2$ using the expressions for the indifferent buyer and seller along with expressions for x_b and x_s and the surpluses given by Equations (2.1a) and (2.1b):[9]

$$\eta_b^i = \frac{1}{2} + \frac{\tau\left(q_b^i - q_b^j\right) + \tau\left(p_b^j - p_b^i\right) + \upsilon\left(p_s^j - p_s^i\right)}{2\left(\tau^2 - \pi\upsilon\right)} \tag{2.2a}$$

$$\eta_s^i = \frac{1}{2} + \frac{\pi\left(q_b^i - q_b^j\right) + \tau\left(p_s^j - p_s^i\right) + \pi\left(p_b^j - p_b^i\right)}{2\left(\tau^2 - \pi\upsilon\right)} \tag{2.2b}$$

We are interested in a solution where both platforms remain active. This implies that not only do buyers' and sellers' market shares decrease when their own side's membership fee increases, but also when the membership fee of the other side increases.[10] In other words, the market shares of both sides are influenced by changes in fees on either side of the market.[11]

Definition 2.1. *An equilibrium at stage two of the model is a pair p_b^i, p_s^i such that p_b^i and p_s^i solves the platform maximisation problem* $\max_{\{p_b^i, p_s^i\}} \Pi^i = \left(p_b^i - f_b\right)\eta_b^i\left(p_b^i, p_b^j, p_s^i, p_s^j, q_b^i, q_b^j\right) + \left(p_s^i - f_s\right)\eta_s^i\left(p_b^i, p_b^j, p_s^i, p_s^j, q_b^i, q_b^j\right) - \frac{\alpha^i\left(q_b^i\right)^2}{2}$ *for each $i, j = 1, 2$, $i \neq j$*

From the first-order conditions for platform i's maximisation problem, the following best response functions are obtained:[12]

$$p_b^i = \frac{f_b + \tau + p_b^j}{2} + \frac{\left(q_b^i - q_b^j\right)}{2} - \frac{\upsilon\left(p_s^i - p_s^j\right)}{2\tau} - \frac{\pi\left(\upsilon + p_s^i - f_s\right)}{2\tau} \tag{2.3a}$$

$$p_s^i = \frac{f_s + \tau + p_s^j}{2} + \frac{\pi\left(q_b^i - q_b^j\right)}{2\tau} - \frac{\pi\left(p_b^i - p_b^j\right)}{2\tau} - \frac{\upsilon\left(\pi + p_b^i - f_b\right)}{2\tau} \tag{2.3b}$$

The best strategy for platform i when the difference in characteristics $q_b^i - q_b^j$ on

[9] See Appendix B.1.2 for further details on how market shares are determined.

[10] Alternatively, if the cross-group network effects outweigh the opportunity cost associated with mismatched preferences on both sides of the market, i.e., $\tau^2 < \pi\upsilon$, both sides' market shares would become an increasing function of their membership fee. Consequently, both buyers and sellers would opt for the same platform, leading to a tipping point in the market.

[11] The partial derivative of Equations (2.2a) and (2.2b) concerning both membership fees are negative, as long as $\tau > \sqrt{\pi\upsilon}$. Considering Assumption 2.1, expressed as $\tau - \frac{\pi+\upsilon}{2} > 0$, we can show that $\tau - \sqrt{\pi\upsilon} > 0$. This can be derived from the inequality $(\pi+\upsilon)^2 > 4\pi\upsilon$ which simplifies to $(\pi-\upsilon)^2 > 0$ if $\pi \neq \upsilon$.

[12] See Appendix B.1.3 for details.

buyers' side is positive[13] is to increase the membership fee on both sides of the market. At the same time, platform i's best response is to increase the membership fee on both sides when the other platform increases its fees on either side $(\partial p_b^i/\partial p_b^j \equiv \partial p_b^i/\partial p_s^j > 0)$. However, platform i decreases the membership fee on one side when the membership fee on the other side increases, $(\partial p_b^i/\partial p_s^i \equiv \partial p_s^i/\partial p_b^i < 0)$. Following Bulow et al. (1985), membership fees' best responses are strategic substitutes amongst platforms but strategic complements between sides.

Although platform i's best response is to increase both sides' membership fee when the difference in attributes is positive, on sellers' side the best response is boosted when the cross-group network effect buyers exert on sellers π increases. This behaviour is common in two-sided markets where sellers benefit as more buyers join the platform. Platform i developed attributes on buyers' side appealing to more buyers because they can enjoy more features, but also appealing to more sellers given the cross-group network effect.

The next step is to solve the best response functions in Equations (2.3a) and (2.3b) to obtain the equilibrium membership fees as a function of the model parameters and the difference in attributes on buyers' side:[14]

$$p_b^i = f_b + \tau - \pi + \left[\frac{3\tau^2 - \pi(\pi + 2\upsilon)}{9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)}\right]\left(q_b^i - q_b^j\right) \tag{2.4a}$$

$$p_s^i = f_s + \tau - \upsilon + \left[\frac{\tau(\pi - \upsilon)}{9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)}\right]\left(q_b^i - q_b^j\right) \tag{2.4b}$$

First, notice the difference in attributes $q_b^i - q_b^j$ is part of the equilibrium membership fees on both sides of the market, even though they were developed only on buyers' side. Sellers' side is affected by the difference in characteristics on the other side because of the cross-group network effects one side exerts on the other side. Therefore platforms adjust sellers' membership fees taking into account the difference in features on buyers' side.

Both agents' equilibrium membership fees on platform i are a function of two terms. The first term is Armstrong (2006) result, the cost of serving buyers and sellers f_b and f_s, the disutility for mismatch preference τ, and the cross-group network effect this side exerts on the other side, π for buyers and υ for sellers. The second term captures the difference in attributes developed on buyers' side $q_b^i - q_b^j$. This extra markup could be positive or negative depending on which side exerts a stronger cross-network effect on the

[13] When we refer to the difference of a strategic variable: *membership fees*, *market-shares*, *attributes* and *platforms' profits*, it is always between both platforms.

[14] We are interested in obtaining an equilibrium where both platforms are active. Therefore, Assumption 2.1 guarantees, platform i profit function is concave and the second-order conditions of the maximisation problem are satisfied. See Appendix B.1.4 for more details.

other side.

In a one-sided market, a firm typically increases its prices as it offers more attributes to customers. However, in a two-sided market, pricing dynamics are influenced by the interplay of cross-group network effects on both sides of the market. As a result, membership fees on one side may actually decrease despite platforms offering additional features, as they can offset this decrease by charging a higher fee on the other side, using the indirect network effects present in the market.

We summarise our discussion in the next proposition:

Proposition 2.1. *For $\left(q_b^i - q_b^j\right) > 0$, whenever this difference in attributes increases, platform i,*

(i) Increases buyers' and decreases sellers' equilibrium membership fees, whenever the cross-group network effect experienced by buyers is higher than the one experienced by sellers $(i.e., \upsilon > \pi)$;

(ii) Increases sellers' and decreases buyers' equilibrium membership fees, whenever the influence exerted on sellers by buyers outweighs the impact on buyers by sellers $(i.e., \pi > \upsilon)$.

Proof: See Appendix B.1.5

Platform i appeals to more agents by increasing the features on buyers' side, attracting more buyers directly and more sellers indirectly since the cross-group network effect. This creates a positive loop considering more agents are attracted on both sides, i.e buyers join platform i given there are more features developed for them, sellers join as well because more buyers joined, then more buyers,..., and this behaviour continues.

Platform i, decides to charge a lower fee on the side that exerts a stronger cross-group network effect on the other side. On the one hand, platform i decreases buyers' fee if the influence buyers exert on sellers is higher than sellers on buyers, $(\pi > \upsilon)$. On the contrary, platform i decreases sellers' fee if the cross-group network effect sellers exert on buyers is higher than the impact buyers exert on sellers $(\upsilon > \pi)$.

Equilibrium market shares and profits (stage 2)

At equilibrium, buyers' and sellers' market shares for platform i where $i, j = 1, 2$, $i \neq j$, are:[15]

$$\eta_b^i = \frac{1}{2} + \left[\frac{3\tau}{2\left[9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)\right]}\right]\left(q_b^i - q_b^j\right) \tag{2.5a}$$

[15]For more details on how to derive market shares see Appendix B.1.6.

$$\eta_s^i = \frac{1}{2} + \left[\frac{(\pi + 2\upsilon)}{2\left[9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)\right]} \right] \left(q_b^i - q_b^j\right) \tag{2.5b}$$

As with equilibrium membership fees, we find that even when platform features are exclusively developed on buyers' side, the difference in attributes impacts both sides' market shares. Sellers join platform i even in the absence of tailored attributes for them, through the influence of cross-group network effects. Furthermore, buyers' and sellers' market shares experience an increase when there is a positive difference in attributes developed on the buyers' side $(q_b^i - q_b^j)$, regardless of which side places a higher value on interaction with the other side.[16]

Platform i can increase its position in the market by developing more attributes on buyers' side. Buyers and sellers will be drawn to join platform i, buyers will join to enjoy more features developed for them and sellers will join because they can interact with more buyers (cross-group network effects).

As we already have the equilibrium membership fees and market shares on both sides of the market, we can compute equilibrium profits for platform i as:

$$\Pi^i = \tau - \frac{(\pi + \upsilon)}{2} + \frac{\tau\left(q_b^i - q_b^j\right)^2 + \Omega\left(q_b^i - q_b^j\right)}{2\left[9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)\right]} - \frac{\alpha^i\left(q_b^i\right)^2}{2} \tag{2.6}$$

where $\Omega \equiv 6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)$.

Equilibrium profits are equal to the degree of product differentiation on both sides of the market (τ), adjusted downwards by the cross-group network effects, π and υ as in Armstrong (2006) main result. Furthermore, profits are adjusted by two additional elements. The first term, $\frac{1}{2\Sigma}\left[\tau\left(q_b^i - q_b^j\right)^2 + \Omega\left(q_b^i - q_b^j\right)\right]$, where $\Sigma \equiv 9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$ is an extra markup associated with the difference in attributes on buyers' side and the second component $\frac{\alpha^i\left(q_b^i\right)^2}{2}$ is the cost of developing these attributes.

We can see from Equation (2.6) that platform i's equilibrium profits increase when additional attributes on buyers' side are developed.[17] When platforms offer new and innovative features, they can appeal to more agents (buyers and sellers) and increase

[16] Partially differentiate equilibrium market-shares in Equations (2.5a) and (2.5b) respect the difference in attributes on buyers' side. The numerator is always positive and to show the denominator $9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$ is positive we use Assumption 2.1 by making the left side of both inequalities equal to compare the right side, showing that Assumption 2.1 right side is greater and therefore the condition is positive. That is, Assumption 2.1 can be transform to be $\tau^2 > \frac{(\pi+\upsilon)^2}{4}$, then we have $\frac{(\pi+\upsilon)^2}{4} > \frac{(2\pi+\upsilon)(\pi+2\upsilon)}{9}$ which simplifies to $9\left(\pi^2 + 2\pi\upsilon + \upsilon^2\right) - 4\left(2\pi^2 + 5\pi\upsilon + 2\upsilon^2\right) > 0$ and then simplifies to $(\pi - \upsilon)^2 > 0$ if $\pi \neq \upsilon$.

[17] This can be seen by partially differentiate Equation (2.6) with respect to q_b^i.That is $\frac{\partial \Pi^i}{\partial q_b^i} = \frac{2q_b^i\left(\tau - \alpha^i\Sigma\right) - 2\tau q_b^j + \Omega}{2\Sigma} > 0$ if $q_b^i > \frac{2\tau q_b^j - \Omega}{2(\tau - \alpha^i\Sigma)}$, where $\Sigma \equiv 9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$.

customer satisfaction leading to higher profits.

2.4 Equilibrium attributes

In this section, we find the equilibrium values of attributes on buyers' side at stage 1 of the model. Platform i differentiates by the features offered on buyers' side, measured by q_b^i. There is a cost of providing q_b^i of $C^i\left(q_b^i\right) = \frac{1}{2}\alpha^i\left(q_b^i\right)^2$, where $i = 1, 2$ and $\alpha^2 > \alpha^1 > 0$. The parameter α^i measures the efficiency platform i has in developing attributes on buyers' side. The fact that platform 1 is more efficient in developing attributes can be related to specialisation in certain technology, experience in having a better understanding of buyers' needs, or innovation by investing more in research and development.

In stage 1 platforms simultaneously choose the characteristics' levels on buyers' side q_b^i, $i = 1, 2$. We can state the next definition:[18]

Definition 2.2. *An equilibrium at stage one of the model is q_b^i such that q_b^i solves the platform maximisation problem $\max_{\{q_b^i\}} \Pi^i \equiv \left(p_b^i - f_b\right)\eta_b^i\left(q_b^i\right) + \left(p_s^i - f_s\right)\eta_s^i\left(q_b^i\right) - \frac{\alpha^i\left(q_b^i\right)^2}{2}$ for each $i = 1, 2$.*

From the first-order conditions for platform i's maximisation problem, we obtained the following best response function:

$$q_b^i = \frac{-\tau q_b^j}{\left(\alpha^i\Sigma - \tau\right)} + \frac{6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)}{2\left(\alpha^i\Sigma - \tau\right)} \text{ for each } i, j = 1, 2,\ i \neq j \tag{2.7}$$

where $\Sigma \equiv 9\tau^2 - (\pi + 2\upsilon)(2\pi + \upsilon)$

Note that attributes are strategic substitutes considering the best response function in Equation (2.7). Platform i's employs a strategy of increasing attributes on buyers' side whenever its competitor takes the opposite approach.[19]

Solving the best response function in Equation (2.7) for $i = 1, 2$ we obtain the equilibrium attributes on buyers' side as a function of the model parameters, that is:[20]

$$q_b^i = \frac{\left(\alpha^j\Sigma - 2\tau\right)\left[6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)\right]}{2\Sigma\left[\alpha^i\alpha^j\Sigma - \left(\alpha^i + \alpha^j\right)\tau\right]} \text{ for each } i, j = 1, 2,\ i \neq j \tag{2.8}$$

where $\Sigma = 9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$.

[18] See Appendix B.1.7 for more details.

[19] The partial derivative of Equation (2.7) respect to q_b^j is negative. $\partial q_b^i / \partial q_b^j = -\frac{\tau}{\left(\alpha^i\Sigma - \tau\right)}$, where $\alpha^i\Sigma - \tau$ is positive as long as Assumption 2.2 holds.

[20] Assumption 2.2 guarantees, platform i profit function is concave and the second-order conditions of the maximisation problem at stage 1 of the model are satisfied. See Appendices B.1.1 and B.1.8 for more details.

We observe from Equation (2.8) that the rivals efficiency parameter in developing attributes is what differentiates equilibrium attributes on buyers' side between both platforms. Platform 1 increases attributes when platform 2 becomes less efficient in developing characteristics on buyers' side (higher α^{2})[21], as long as Assumption 2.1 and Assumption 2.2 hold.[22] Platform 1 enhances attributes offered on buyers' side to appeal buyers and sellers, establishing itself as a leading intermediary in the industry.

Finally, we define the difference in attributes on buyers' side, using Equation (2.8) as:

$$\Delta q_b^i \equiv q_b^i - q_b^j = \frac{\left(\alpha^i - \alpha^j\right)\left[6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)\right]}{2\left[\alpha^i \alpha^j \Sigma - \left(\alpha^i + \alpha^j\right)\tau\right]} \tag{2.9}$$

for each $i, j = 1, 2$, $i \neq 2$ and $\Sigma = 9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$.

The equilibrium difference in attributes on buyers' side in Equation (2.9) is positive for platform 1 and negative for platform 2 considering platform 1 is more efficient in developing features compared to platform 2 ($\alpha^{\mathit{2}} > \alpha^{\mathit{1}}$). [23]

Therefore, recognising the significance of the impacts that the difference in cross-group network effects has on platform attributes and overall market equilibrium (fees, market shares and profits), our focus now shifts towards a comprehensive analysis of these effects in the subsequent section.

2.5 Analysis of cross-group network effects on market configurations

In this section, we study how cross-group network effects shape the structure and dynamics of the market. We explore two distinct scenarios to gain insights into the interactions between platform's attributes and cross-group network effects. Firstly, we consider a benchmark case where cross-group network effects are identical on both sides of the market. Secondly, we explore a scenario where the cross-side network impacts are allowed to differ.

[21] To avoid confusion between squared parameters and parameters of platform *2*, italic numbers will be used instead of normal numerals 1 and 2 when referring to platforms *1* and *2* in the mathematical expressions.

[22] Partially differentiate q_b^{1} in Equation (2.8) respect to α^{2}. $\partial q_b^{\mathit{1}} / \partial \alpha^{\mathit{2}} = \frac{\Omega\tau}{2\Sigma A^2}\left(\alpha^{\mathit{1}}\Sigma - 2\tau\right) > 0$, where $\Omega \equiv 6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)$, $\Sigma = 9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$ and $A \equiv \left(\alpha^{\mathit{1}}\alpha^{\mathit{2}}\Sigma - \left(\alpha^{\mathit{1}} + \alpha^{\mathit{2}}\right)\tau\right)$

[23] Appendix B.1.9 shows conditions for positive equilibrium attributes, which applies to the difference in attributes in Equation (2.9).

2.5.1 Benchmark scenario: Identical cross-group network effects, $\pi = \upsilon$

In this section, we develop a benchmark scenario where the cross-group network effects are identical on both sides of the market, ($\pi = \upsilon$). We use the game's solution at stage 1 to obtain the strategic variables as a function of the model's parameters. We use the superscript "*bs*" to denote the equilibrium market structures. Furthermore, we provide some intuition for the results that are going to help us to examine asymmetric network effects in the next section.

Using equilibrium attributes at Equation (2.8) and the fact that $\pi = \upsilon$, benchmark equilibrium attributes on buyers' side are:

$$\left(q_b^i\right)^{bs} = \frac{9\alpha^j\left(\tau^2 - \pi^2\right) - 2\tau}{3\left[9\alpha^i\alpha^j\left(\tau^2 - \pi^2\right) - \left(\alpha^i + \alpha^j\right)\tau\right]} \text{ for each } i, j = 1, 2,\ i \neq j \tag{2.10}$$

Equilibrium attributes on buyers' side on Equation (2.10) is positive as long as Assumption 2.1 and Assumption 2.2 holds[24], and considering identical cross-group network effects on both sides of the market we can state the next proposition:

Proposition 2.2. *Equilibrium attributes on buyers' side decrease in the product differentiation parameter τ and increase in the cross-group network effect ($\pi = \upsilon$). Moreover, an increase in the cross-group network effect is stronger in the platform that is more efficient in developing attributes.*

Proof: See Appendix B.2.1

Based on Proposition 2.2, when platforms prioritise increasing their product differentiation parameter in the horizontal dimension (τ), they simultaneously reduce attributes on buyers' side, thereby differentiating less on the vertical dimension. As platform 1 increases its product differentiation parameter across both sides of the market, it no longer has incentives to further enhance attributes on buyers' side. This is due to the cost associated with simultaneously differentiating on both the horizontal and vertical dimensions.

Instead, to gain a competitive advantage, platform 1 opts for a broader degree of product differentiation, catering to a wide range of preferences from both buyers and sellers. Rather than focusing on increasing the level of features on buyers' side for a specific set of preferences, platform 1 engages in less intense competition for the same pool of agents as the degree of product differentiation expands. Consequently, agents become more captive and there is reduced pressure to develop additional attributes on

[24] When $\upsilon = \pi$ Assumption 2.1 turns to $\tau > \pi$ and Assumption 2.2 turns to $\alpha^i > \frac{2\tau}{9\sigma}$, where $\sigma \equiv \tau^2 - \pi^2$.

the buyers' side.

We notice also from Proposition 2.2 that platform 1 increases the attributes on buyers' side whenever the cross-group network effects increase because this attracts directly more buyers and more sellers, given the cross-side network effects. This creates a positive loop where the more agents use platform 1, the more valuable it becomes to buyers and sellers, which in turn attracts even more agents. Considering platform 1 is more efficient in developing attributes than platform 2, $\alpha^2 > \alpha^1$, this outcome is more pronounced on platform 1.

Corollary 2.1. *The difference in attributes on buyers' side decreases when there is a higher product differentiation on both sides of the market and increases when the cross-group network effects become stronger.*

Proof: See Appendix B.2.1

Corollary 2.1 extends the proven arguments on Proposition 2.2 to the difference in attributes on buyers' side. For this reason, the intuition is the same as in Proposition 2.2.

Equilibrium membership fees

We now obtain equilibrium membership fees, market shares and platform profits as a function of the model parameters.

For the equilibrium membership fees we have:

$$\left(p_b^i\right)^{bs} = f_b + \tau - \pi + \frac{\left(\alpha^j - \alpha^i\right)\sigma}{9\alpha^i\alpha^j\sigma - \left(\alpha^i + \alpha^j\right)\tau} \quad = \quad f_b + \tau - \pi + \frac{1}{3}\left(\Delta q_b^i\right)^{bs} \tag{2.11a}$$

$$\left(p_s^i\right)^{bs} = f_s + \tau - \upsilon; \quad \upsilon = \pi \tag{2.11b}$$

for each $i, j = 1, 2$, $i \neq 2$ and where $\sigma \equiv \tau^2 - \pi^2$.

When the cross-group network effects are identical on both sides of the market $\pi = \upsilon$, platforms charge symmetric fees on sellers' side. This is a consequence that the difference in attributes on buyers' side does not influence sellers' fees when the cross-network effects are the same. Both platforms charge sellers the same fee as in Armstrong (2006) seminal model.

However, buyers' equilibrium membership fee is higher on platform 1 than it would have been without the development of specific features for them. This is due to the extra markup denoted by $\frac{1}{3}\left(\Delta q_b^i\right)^{bs}$, which is positive for platform 1 considering $(\alpha^2 > \alpha^1)$. Consequently, platform 1 lacks the option to discern which side values interaction more, and thus, cannot adjust the fee accordingly when the cross-group network effects are identical on both sides of the market.

We examine the effects of the model parameters on the difference in equilibrium fees between the two platforms, under the assumption that platform 1 is more efficient in developing attributes compared to platform 2, ($\alpha^2 > \alpha^1$). Hence, we set the following:

Proposition 2.3. *The difference in equilibrium fees buyers pay decreases when there is a greater heterogeneity between platforms (higher τ) and increases when platforms become more valuable for both groups (stronger $\pi = \upsilon$). In addition, buyers' fees are more expensive in the platform which is more efficient in developing attributes whenever the cross-group network effect is stronger.*

Proof: See Appendix B.2.2

Proposition 2.3 reveals that as the product differentiation parameter increases ($\tau \uparrow$), platform 1 reduces buyers' fees because the difference in attributes between platforms decreases. This fee reduction serves as an incentive to attract more buyers. Then, it raises sellers' fees, as indicated in Equation (2.11b), to compensate for the decrease in buyers' fees. Conversely, when the cross-group network effect ($\pi = \upsilon$) increases, it raises buyers' fees as it has developed more attributes to enhance their experience. Simultaneously, it lowers sellers' fees to encourage greater participation from sellers, as observed in Equation (2.11b).[25]

An increase in the cross-group network effect has a greater impact on buyers' equilibrium membership fee in platform 1. This is because platform 1 is more proficient in developing features, which attracts a larger number of buyers. Consequently, it exploits this by charging buyers a higher fee, allowing it to extract a greater portion of buyers' surplus.

Equilibrium market shares and profits

Using equations Equations (2.5a), (2.5b) and (2.10) we obtain the following equilibrium market shares:[26]

$$\left(\eta_b^i\right)^{bs} = \frac{1}{2} + \frac{\left(\alpha^j - \alpha^i\right)\tau}{2\left[9\alpha^i\alpha^j\sigma - \left(\alpha^i + \alpha^j\right)\tau\right]} \tag{2.12a}$$

$$\left(\eta_s^i\right)^{bs} = \frac{1}{2} + \frac{\left(\alpha^j - \alpha^i\right)\pi}{2\left[9\alpha^i\alpha^j\sigma - \left(\alpha^i + \alpha^j\right)\tau\right]} \tag{2.12b}$$

for each $i, j = 1, 2$, $i \neq 2$ and where $\sigma \equiv \tau^2 - \pi^2$.

Platform 1 gains a larger market share among both buyers and sellers considering it

[25] The proof for Proposition 2.3 is straightforward, partially differentiate Equation (2.11a) with respect to the model parameters. For details see Appendix B.2.2.

[26] Condition for buyers' and sellers' market shares distributed in the unit interval is $\alpha^i > \frac{2\tau}{9\sigma}$. For details see Appendix B.2.3.

is more efficient in developing attributes on buyers' side, ($\alpha^2 > \alpha^1$). Equilibrium market shares on both sides increase when the cross-group network effect is stronger ($\pi = \upsilon$). Platform 1 becomes more valuable to both buyers and sellers as the cross-group network effects strengthen, resulting in the development of more attributes for buyers. This positive feedback loop contributes to a rapid expansion of its market share, potentially leading to its dominance in the market.[27]

Using equilibrium membership fees in Equations (2.11a) and (2.11b) and equilibrium market shares Equations (2.12a) and (2.12b) we obtain equilibrium profits as a function of the equilibrium features configurations:

$$\left(\Pi^i\right)^{bs} = \tau - \pi + \frac{9\sigma\left(\alpha^j - \alpha^i\right)\left[9\alpha^i\alpha^j\sigma - \left(\alpha^i + \alpha^j\right)\tau\right] - \alpha^i\left(9\alpha^j\sigma - 2\tau\right)\left(9\alpha^i\sigma - 2\tau\right)}{18\left[9\alpha^i\alpha^j\sigma - \left(\alpha^i + \alpha^j\right)\tau\right]^2} \tag{2.13}$$

for each $i, j = 1, 2$, $i \neq 2$ and where $\sigma = \tau^2 - \pi^2$.

Platform i's equilibrium profits are a function of two terms. The first term $(\tau - \pi)$ is similar to Armstrong (2006) having product differentiation on both sides of the market ($\tau_b = \tau_s = \tau$) and cross-group network effects ($\pi = \upsilon$). The second term is an extra markup related to the difference in attributes on buyers' side between both platforms, which is positive for platform 1 because is more efficient in developing attributes and as long as Assumption 2.2 holds.[28]

We obtain some insights into platforms' strategy to maximise profits, under the assumption that platform 1 is more efficient in developing attributes compared to platform 2, ($\alpha^2 > \alpha^1$), in the following proposition:

Proposition 2.4. *The difference in equilibrium profits decreases as the degree of product differentiation intensifies (higher τ) and increases the more valuable it becomes for both buyers and sellers since the cross-group network effect ($\pi = \upsilon$) turns stronger.*

Proof: See Appendix B.2.6

Proposition 2.4 contrasts with Armstrong (2006) where equilibrium platforms' profits are increasing on the degree of product differentiation (τ) and decreasing on cross-group network effects ($\pi = \upsilon$). In our benchmark scenario, the effects in equilibrium profits are the opposite.

As platform 1 becomes more horizontally differentiated ($\tau \uparrow$), there is a decrease in the development of attributes on buyers' side. Consequently, the number of buyers

[27] Partially differentiate Equations (2.12a) and (2.12b) respect the model parameters. For details see Appendix B.2.4.

[28] See Appendix B.2.5 for details.

joining the platform decreases, along with the number of sellers, considering the cross-group network effect. As a result, platform 1 has a smaller pool of agents to charge additional fees to, leading to a decline in the difference in equilibrium profits.

Conversely, an increase in cross-group network effects leads to an increase in attributes on buyers' side. This attracts a larger number of buyers and sellers, taking into account the cross-effect of the networks. In response, platform 1 charges a higher fee on buyers' side and a lower fee on the sellers' side, as indicated in Proposition 2.3. Accordingly, it charges an additional fee per additional agent, resulting in higher profits.

These findings align with the early work conducted by Economides (1989) and Neven and Thisse (1989) and the generalised model by Irmen and Thisse (1998). These studies suggest that platforms' profit-maximising strategy involves maximising differentiation on one dimension while minimising differentiation on the other dimension. In the current scenario, platform i increases the vertical dimension by developing attributes on buyers' side when the horizontal dimension, representing the product differentiation parameter on both sides of the market, decreases.

2.5.2 Non-Identical cross-group network effects, $\pi \neq \upsilon$

In this section, our objective is to analyse the presence of asymmetric cross-group network effects. To ensure that the analysis remains tractable without sacrificing its essence, we simplify the model by setting the side that exerts a weaker network effect on the other side to zero.[29]

The first case we consider is when buyers value interactions more than sellers or when the cross-group network effect sellers exert on buyers is greater than vice versa ($\upsilon > \pi$). To keep our analysis tractable, we normalise the value of π to zero. The second case we examine is when sellers value interaction more than buyers or when the cross-group network effect buyers exert on sellers is greater than vice versa ($\pi > \upsilon$). Again, for simplicity, we normalise the value of υ to zero. By using the game's solution at stage 1, we obtain the strategic variables as functions of the model's parameters and gain insights into the results.

Equilibrium attributes

Using equilibrium attributes in Equation (2.8) we obtain platforms equilibrium attributes on buyers' side for two different scenarios:

$$\text{When } \upsilon > \pi \ \ (\pi = 0)\,, \quad \left(q_b^i\right)\Big|_{\upsilon>\pi} = \frac{\left(\alpha^j\sigma_\upsilon - 2\tau\right)\left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right)}{2\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]} \tag{2.14a}$$

[29] Ideally, what we mean is that the network effect exerted by this side is negligible compared to the magnitude of the network effect originating from the other side.

$$\text{When } \pi > \upsilon \ \ (\upsilon = 0)\,, \quad \left(q_b^i\right)\Big|_{\pi > \upsilon} = \frac{\left(\alpha^j \sigma_\pi - 2\tau\right)\left(3\tau + \pi\right)\left(2\tau - \pi\right)}{2\sigma_\pi \left[\alpha^i \alpha^j \sigma_\pi - \left(\alpha^i + \alpha^j\right)\tau\right]} \tag{2.14b}$$

for each $i, j = 1, 2$, $i \neq 2$ and where $\sigma_\upsilon \equiv 9\tau^2 - 2\upsilon^2$ and $\sigma_\pi \equiv 9\tau^2 - 2\pi^2$.

We can observe in Equations (2.14a) and (2.14b) that equilibrium attributes on buyers' side $(q_b^i)\Big|_{\upsilon>\pi}$ and $(q_b^i)\Big|_{\pi>\upsilon}$ are positive if Assumption 2.1 and Assumption 2.2 holds.[30] Then we can state the next proposition:

Proposition 2.5. *The difference in equilibrium attributes on buyers' side decreases as the degree of product differentiation increases and rises with a stronger cross-group network effect, as long as $\tau > 4\upsilon$ when the cross-group network effect sellers exert on buyers is greater than the effect exerted by buyers on sellers, $\upsilon > \pi$, $\pi = 0$.*

Proof: See Appendix B.3.1

Propositions Proposition 2.2 and Proposition 2.5 provide similar insights regarding equilibrium attributes on buyers' side. Regardless of whether the cross-group network effects are identical or if one side exerts a stronger network effect on the other, these propositions establish that equilibrium attributes on buyers' side unambiguously decrease with a higher degree of product differentiation ($\tau \uparrow$) and increase with stronger cross-group network effects ($\pi, \upsilon \uparrow$).

Proposition 2.5 is based on the observation that as the degree of product differentiation (τ) increases, platform 1 engages in less aggressive competition for both agents. This is because the unique and distinct nature of its services reduces the need to develop additional attributes on buyers' side to attract them. Conversely, when there is a stronger relationship between the two groups, characterised by increased features on buyers' side, given higher cross-group network effects, the platform becomes more valuable to both agents. The growth of one group enhances the value of the other group, resulting in mutual growth. When the effect sellers exert on buyers is stronger than vice versa, $\upsilon > \pi$, $\pi = 0$, the degree of product differentiation has to exceed a certain threshold ($\tau > 4\upsilon$), for an increase in attributes on buyers' side to attract more participants, as it becomes more costly (τ was $\frac{\upsilon}{2}$ and now is 4υ) for them to join and can feel discouraged. Therefore, platform 1 starts developing more attributes to appeal to more buyers and eventually more sellers given the cross-group network effect.

[30] Assumption 2.1 and Assumption 2.2 turn to $\tau > \frac{\pi}{2}$ and $\alpha^i > \frac{2\tau}{\sigma_\pi}$ respectively, when $\pi > \upsilon$ $(\upsilon = 0)$. Conversely, they turn to $\frac{\upsilon}{2} < \tau < \frac{2\upsilon}{3}$ and $\alpha^i > \frac{2\tau}{\sigma_\upsilon}$ respectively, when $\upsilon > \pi$ $(\pi = 0)$. Where $\sigma_\upsilon \equiv 9\tau^2 - 2\upsilon^2$ and $\sigma_\pi \equiv 9\tau^2 - 2\pi^2$.

Equilibrium market shares and profits

We proceed to obtain the equilibrium market shares on both sides of the market using Equations (2.5a) and (2.5b) and equilibrium attributes in Equation (2.8)[31]

$$\eta_b^i = \frac{1}{2} + \frac{(\alpha^j - \alpha^i)\, 3\tau \left[6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)\right]}{4\Sigma\left[\alpha^i \alpha^j \Sigma - (\alpha^i + \alpha^j)\tau\right]} \tag{2.15a}$$

$$\eta_s^i = \frac{1}{2} + \frac{(\alpha^j - \alpha^i)(\pi + 2\upsilon)\left[6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)\right]}{4\Sigma\left[\alpha^i \alpha^j \Sigma - (\alpha^i + \alpha^j)\tau\right]} \tag{2.15b}$$

for each $i, j = 1, 2$, $i \neq 2$ and where $\Sigma \equiv 9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$.

Based on the equilibrium market shares in Equations (2.15a) and (2.15b), we can conclude that platform 1 gains a competitive advantage over its rival by being more efficient in developing attributes on buyers' side ($\alpha^2 > \alpha^1$). This advantage remains regardless of whether the cross-group network effects are identical ($\pi = \upsilon$), as mentioned in Section 2.5.1, or if the indirect network effect exerted by sellers on buyers is larger ($\upsilon > \pi, \pi = 0$), or if the cross-group network effect exerted by buyers on sellers is stronger ($\pi > \upsilon, \upsilon = 0$) in Section 2.5.2. Platform 1 outperforms platform 2 because it is capable of producing more features on buyers' side with fewer resources and/or in less time.

The following claim captures the impact of model parameters τ and π, υ, on buyers' and sellers' equilibrium market shares under the assumption that platform 1 is more efficient in developing attributes compared to platform 2, ($\alpha^2 > \alpha^1$).[32]

Claim 2.1. *Buyers' and sellers' equilibrium market shares decrease when platform* 1 *is more heterogeneous in the horizontal dimension (higher* τ*) and increase when the cross-group network effects become stronger (higher* υ, π*).*

Proof: See Appendix B.3.3

The claim states that as platform 1 becomes more heterogeneous in terms of the degree of product differentiation ($\tau \uparrow$), the number of attributes on buyers' side decreases. This reduction diminishes the incentives for buyers and sellers to join the platform. Conversely, as the cross-group network effects increase, platform 1 becomes more valuable, attracting more participants on both sides of the market.[33]

The next step is to obtain platform i equilibrium profits as a function of the equilib-

[31] Buyers' and sellers' market shares are distributed in the unit interval as long as Assumption 2.1 and Assumption 2.2 hold. For more details see Appendix B.3.2.

[32] As we observe equilibrium market shares on buyers' side is $\eta_b^i = \frac{1}{2} + \frac{3\tau}{2\Sigma}\Delta q_b i$ and on sellers' side is $\eta_s^i = \frac{1}{2} + \frac{(\pi+2\upsilon)}{2\Sigma}\Delta q_b i$

[33] The detailed derivation of these results can be found in Appendix B.3.3, where Equations (2.15a) and (2.15b) are partially differentiated with respect the model parameters.

rium features in Equation (2.8):

$$\Pi^i = \tau - \frac{\pi + \upsilon}{2} + \left[\frac{\left(\alpha^j - \alpha^i\right) \Sigma \left[\alpha^i \alpha^j \Sigma - \left(\alpha^i + \alpha^j\right) \tau\right] - \alpha^i \left(\alpha^j \Sigma - 2\tau\right) \left(\alpha^i \Sigma - 2\tau\right)}{8\Sigma^2 \left[\alpha^i \alpha^j \Sigma - \left(\alpha^i + \alpha^j\right) \tau\right]^2} \right] \Omega^2 \quad (2.16)$$

for each $i, j = 1, 2$, $i \neq 2$ and where $\Sigma \equiv 9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$ and $\Omega \equiv 6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)$.

Platform i's equilibrium profits are a function of two terms. The first term is similar to Armstrong (2006), product differentiation cost and cross-side network effects on both sides of the market $\tau - \frac{\pi+\upsilon}{2}$. The second term is a markup related to the difference in attributes on buyers' side, which is positive for platform 1 because $\alpha^2 > \alpha^1$ and as long as Assumption 2.2 holds.[34]

Case 1: When sellers exert a stronger influence on buyers: $\upsilon > \pi \, (\pi = 0)$.

Equilibrium membership fees and Platform Profits

In this case, we have:

$$\left(p_b^i\right)\Big|_{\upsilon > \pi,\ \pi = 0} = f_b + \tau + \frac{3\left(\alpha^j - \alpha^i\right)\tau^2 (3\tau + 2\upsilon)(2\tau - \upsilon)}{2\sigma_\upsilon \left[\alpha^i \alpha^j \sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]} \quad (2.17a)$$

$$\left(p_s^i\right)\Big|_{\upsilon > \pi,\ \pi = 0} = f_s + \tau - \upsilon - \frac{\left(\alpha^j - \alpha^i\right)\tau\upsilon (3\tau + 2\upsilon)(2\tau - \upsilon)}{2\sigma_\upsilon \left[\alpha^i \alpha^j \sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]} \quad (2.17b)$$

for each $i, j = 1, 2$, $i \neq 2$ and where $\sigma_\upsilon \equiv 9\tau^2 - 2\upsilon^2$.

Note that the extra markup on Equations (2.17a) and (2.17b) is positive in platform 1 considering $\alpha^2 > \alpha^1$ and as long as Assumption 2.1 and Assumption 2.2 hold. Therefore, when the cross-group network effect sellers exert on buyers outweighs the effect buyers exert on sellers ($\upsilon > \pi$), platform 1 implements a pricing strategy that deviates from the seminal results by Armstrong (2006). Specifically, platform 1 charges on buyers' side an additional markup while reducing sellers' subscription fees. That is $\left(p_b^1\right)\Big|_{\upsilon > \pi,\ \pi = 0} > \left(p_b^1\right)^{Armstrong}$ and $\left(p_s^1\right)\Big|_{\upsilon > \pi,\ \pi = 0} < \left(p_s^1\right)^{Armstrong}$.

Next, we characterise the impacts on the difference in equilibrium fees considering platform 1 is more efficient in developing attributes compared to platform 2, ($\alpha^2 > \alpha^1$)

Proposition 2.6a. *For $\upsilon > \pi \, (\pi = 0)$, the difference in equilibrium membership fees*

(i) On buyers' side decreases and on sellers' side increases when τ increases.

(ii) On buyers' side increases and sellers' side decreases as the cross-group network effect becomes stronger (i.e., when υ increases).

[34] See Appendix B.3.4 for details.

Proof: See Appendix B.3.5.

According to Proposition 2.6a, as the degree of product differentiation increases ($\tau \uparrow$), there is no need for platform 1 to develop additional attributes on buyers' side. Platform 1 is perceived as offering unique and distinct services compared to the other platform. As a result, the features on buyers' side decrease, discouraging buyers from joining it.

To counteract this potential decrease in buyer participation, the platform adjusts its pricing strategy by charging a lower fee on buyers' side. This lower fee is aimed at attracting and retaining buyers. To compensate for the revenue loss from lower buyer fees, the platform charges a higher fee on sellers' side. The higher fee is justified by the increased participation of sellers due to the positive cross-group network effect.

This finding contrasts with the results of Armstrong (2006), where membership fees on both sides of the market increase as the degree of product differentiation increases. The difference arises from the fact that in our model, platforms adjust their pricing strategies indirectly by manipulating the features developed on buyers' side, rather than directly adjusting the membership fees.

Furthermore, when the cross-group network effect exerted by sellers on buyers is stronger ($\upsilon > \pi$), platform 1 increases the attributes on buyers' side. This strategy aims to appeal to more buyers and incentivise their participation in the platform. Consequently, it charges a higher fee to buyers, reflecting the additional value provided through the developed attributes. Additionally, the stronger cross-group network effect encourages more sellers to join the platform, as they benefit from the increased buyer participation. To attract and retain sellers, platform 1 charges them a lower fee.

This result aligns with existing findings in the literature on two-sided markets as in Armstrong (2006); Jullien et al. (2021), where platforms often adjust their pricing strategies by charging a lower subscription fee on the side that exerts a more substantial influence on the other side. In this particular scenario, sellers have a more prominent effect on buyers. By charging a lower fee to sellers, platform i promotes their participation, which, in turn, attracts more agents on both sides of the market.

The next step is to obtain the difference in platforms' equilibrium profits using Equation (2.16):

$$\Delta\Pi^{i}_{\upsilon>\pi} = \left(\alpha^{j} - \alpha^{i}\right) \left[\frac{2\sigma_\upsilon \left[\alpha^{i}\alpha^{j}\sigma_\upsilon - \left(\alpha^{i} + \alpha^{j}\right)\tau\right] + \left(\alpha^{j}\sigma_\upsilon - 2\tau\right)\left(\alpha^{i}\sigma_\upsilon - 2\tau\right)}{8\sigma_\upsilon^2 \left[\alpha^{i}\alpha^{j}\sigma_\upsilon - \left(\alpha^{i} + \alpha^{j}\right)\tau\right]^2}\right] \Omega_\upsilon^2 \quad (2.18)$$

for each $i, j = 1, 2$, $i \neq 2$ and where $\sigma_\upsilon \equiv 9\tau^2 - 2\upsilon^2$ and $\Omega_\upsilon \equiv (3\tau + 2\upsilon)(2\tau - \upsilon)$.

Next, we characterise the impacts on the difference in equilibrium profits considering platform 1 is more efficient in developing attributes compared to platform 2, ($\alpha^{2} > \alpha^{1}$)

Proposition 2.7a. *For $\upsilon > \pi$ (i.e., sellers exert a stronger cross-group network effect on buyers' side) the difference in equilibrium profits increases as the degree of product differentiation and the indirect network effect grow. The impact of the cross-group network effect holds as long as $\tau > 4\upsilon$.*

Proof: See Appendix B.3.6

Proposition 2.7a shows that when the cross-group network effect exerted by sellers on buyers is stronger, $\upsilon > \pi, (\pi = 0)$, the difference in equilibrium profits increases. This is because as platforms become more valuable to buyers (indicated by higher υ), the profit-increasing strategy involves developing additional attributes if the degree of product differentiation τ is big enough as 4υ. The intuition on why platform 1 develop more attributes is the same as in Proposition 2.5. This prompts participants from both sides of the market to join, resulting in an additional fee per buyer and seller and ultimately leading to an increase in the platform's profits.

On the contrary, when the degree of product differentiation is below 4υ the difference in equilibrium profits decreases as the cross-group network effect exerted by sellers on buyers increases. This occurs because fewer attributes are developed, discouraging both buyers and sellers (given the cross-group network effect) from joining the platform. Consequently, this behaviour impacts platform revenue by reducing the number of participants available to charge fees, ultimately decreasing its profits.

The result on Proposition 2.7a aligns with more recent research by Garella and Lambertini (2014) and Barigozzi and Ma (2018), which suggests that platforms strive to differentiate themselves on both dimensions to maximise profits. Specifically, platforms aim to increase the degree of product differentiation in the horizontal dimension by becoming more heterogeneous, and in the vertical dimension by enhancing features on buyers' side, as buyers are highly valued by platforms. By pursuing these strategies, platforms can effectively increase their profits in the market.

Case 2: When buyers exert a stronger influence on sellers, $\pi > \upsilon \, (\upsilon = 0)$.

Equilibrium membership fees and Platform Profits

In this case, we have:

$$\left(p_b^i\right)\Big|_{\pi>\upsilon,\ \upsilon=0} = f_b + \tau - \pi + \frac{\left(\alpha^j - \alpha^i\right)\left(3\tau^2 - \pi^2\right)\left(3\tau + \pi\right)\left(2\tau - \pi\right)}{2\sigma_\pi\left[\alpha^i\alpha^j\sigma_\pi - \left(\alpha^i + \alpha^j\right)\tau\right]} \tag{2.19a}$$

$$\left(p_s^i\right)\Big|_{\pi>\upsilon,\ \upsilon=0} = f_s + \tau + \frac{\left(\alpha^j - \alpha^i\right)\tau\pi\left(3\tau + \pi\right)\left(2\tau - \pi\right)}{2\sigma_\pi\left[\alpha^i\alpha^j\sigma_\pi - \left(\alpha^i + \alpha^j\right)\tau\right]} \tag{2.19b}$$

for each $i, j = 1, 2,\ i \neq 2$ and where $\sigma_\pi \equiv 9\tau^2 - 2\pi^2$.

Note that the additional markup on Equation (2.19b) is positive in platform 1 considering $\alpha^2 > \alpha^1$ and as long as Assumption 2.1 and Assumption 2.2 hold. However, on Equation (2.19a), it turns negative when $3\tau^2 - \pi^2 < 0$ holds true, provided that $\tau < \frac{\pi}{\sqrt{3}}$.[35] When the cross-group network effect exerted by buyers on sellers is stronger than the effect sellers have on buyers ($\pi > \upsilon$), platform 1 also adopts a pricing strategy that deviates from the seminal results presented in Armstrong (2006) as in case 1. Specifically, platform 1 charges a lower subscription fee for buyers, $(p_b^1)\Big|_{\pi>\upsilon} < (p_b^1)^{Armstrong}$. Additionally, platform 1 applies an extra markup on sellers' side, $(p_s^1)\Big|_{\pi>\upsilon} > (p_s^1)^{Armstrong}$. This sets the stage to develop the following proposition:

Proposition 2.6b. *For $\pi > \upsilon\,(\upsilon = 0)$, the difference in equilibrium membership fees*

(i) On buyers' side increases and sellers' side decreases when τ increases.

(ii) On buyers' side decreases and on sellers' side increases as the cross-group network effect becomes stronger (i.e., when π increases).

Proof: See Appendix B.3.5.

It is noteworthy that platform 1's pricing strategy in Proposition 2.6b is the opposite of Proposition 2.6a. The reason is as a consequence of the reversal in the strength of the cross-group network effects, form $\upsilon > \pi$, $\pi = 0$ to $\pi > \upsilon$, $\upsilon = 0$.

According to Proposition 2.6b, platform 1 adjusts its pricing strategy by lowering the equilibrium fee for sellers, acknowledging their higher valuation of interaction with the other side of the market, ($\pi > \upsilon$). This adjustment is in response to a reduction in features on buyers' side, given an increase in the degree of product differentiation (τ).

On the one hand, this strategy discourages buyers from joining the platform, and as a result, it also affects the sellers' participation due to the cross-group network effect. On the other hand, sellers fee reduction attracts more of them and, in turn, encourages buyers to join the platform due to the positive cross-group network effect. However, to compensate for the fee decrease on sellers' side, platform 1 charges a higher fee to buyers.

Furthermore, when the cross-group network effect exerted by buyers on sellers is stronger ($\pi > \upsilon$), platform 1 develops more attributes on buyers' side to appeal to a larger number of participants. This increased attractiveness of the platform to sellers, who value interaction more, leads to a higher equilibrium fee charged to them. At the same time, the platform adopts a pricing policy of lowering buyers' subscription fees. This strategy creates a positive feedback loop, as the lower fees attract more buyers, which in turn further enhances the benefits of platform 1.

[35] This condition is compatible with Assumption 2.1 since $\frac{\pi}{2} < \tau < \frac{\pi}{\sqrt{3}}$.

The next step is to obtain the difference in platforms' equilibrium profits using Equation (2.16):

$$\Delta\Pi^{i}_{\pi>\upsilon} = \left(\alpha^{j} - \alpha^{i}\right)\left[\frac{2\sigma_{\pi}\left[\alpha^{i}\alpha^{j}\sigma_{\pi} - \left(\alpha^{i}+\alpha^{j}\right)\tau\right] + \left(\alpha^{j}\sigma_{\pi} - 2\tau\right)\left(\alpha^{i}\sigma_{\pi} - 2\tau\right)}{8\sigma_{\pi}^{2}\left[\alpha^{i}\alpha^{j}\sigma_{\pi} - \left(\alpha^{i}+\alpha^{j}\right)\tau\right]^{2}}\right]\Omega_{\pi}^{2} \quad (2.20)$$

for each $i, j = 1, 2$, $i \neq 2$ and where $\sigma_{\pi} \equiv 9\tau^{2} - 2\pi^{2}$ and $\Omega_{\pi} \equiv (3\tau + \pi)(2\tau - \pi)$.

Next, we characterise the impacts on the difference in equilibrium profits considering platform 1 is more efficient in developing attributes compared to platform 2, ($\alpha^{2} > \alpha^{1}$)

Proposition 2.7b. *For $\pi > \upsilon$ (i.e., buyers exert a stronger cross-group network effect on sellers' side) the difference in equilibrium profits increases as the degree of product differentiation grows and decreases as the cross-group network effect rises.*

Proof: See Appendix B.3.6

It is important to notice that contrary to the previous scenario where the cross-group network effects on both sides are identical when the indirect network effects on both sides of the market are different, the difference in equilibrium profits increase in the degree of product differentiation τ as in the seminal model of Armstrong (2006).

Proposition 2.7a and Proposition 2.7b specify that when platforms are more heterogeneous (higher τ) the difference in equilibrium profits increases whether one side influences the other more or vice versa. The mechanism by which this occurs is as follows:

- Platform 1 offers unique and differentiated services compared to the other platform, there is no obligation to develop additional attributes on buyers' side. Consequently, the features available to buyers decrease, which can lead to a decrease in their motivation to continue using or joining platform 1 on both sides of the market.

- If buyers value interaction more than sellers ($\upsilon > \pi$), the platform charges them a lower fee. To balance this, charges a higher fee on sellers' side, as more sellers are expected to join due to the cross-group network effect. This combination of pricing strategies leads to an increase in the difference in equilibrium profits.

- Conversely, when the cross-group network effect exerted by buyers on sellers is stronger ($\pi > \upsilon$), platform 1 adjusts its pricing strategy by lowering sellers' equilibrium fees. This strategy encourages more buyers to join, driven by the cross-group network effect. To offset the fee decrease on the sellers' side, it charges buyers a higher fee.

As seen in Proposition 2.7b the result driven from the cross-group network effect may seem counterintuitive. As platform 1 becomes more valuable for both agents (higher π), it develops more features on buyers' side, attracting more participants and generating additional fees per agent. However, the increase in sellers' cross-group network effect enhances their value, leading platforms to compete more intensely to attract sellers. This intensified competition prompts platforms to develop even more attributes on buyers' side (an increase in π increases the difference in equilibrium attributes), escalating competition further. Finally, this results in a decrease in the difference in equilibrium profits.

As in the scenario where the cross-group network effects on both sides of the market are identical, Proposition 2.7b aligns with the earlier work of Economides (1989) and Neven and Thisse (1989), as well as the generalised model of Irmen and Thisse (1998). This result suggests that platforms strive to maximise their differentiation on one dimension while minimising it on the other to increase profits. Specifically, platforms focus on increasing differentiation in the horizontal dimension by becoming more heterogeneous, while reducing differentiation in the vertical dimension by developing fewer features on buyers' side when the cross-group network effect exerted by sellers decreases.

2.6 Conclusion

We have introduced a novel two-stage model for a two-sided market that incorporates the concept of vertical differentiation. By analysing the intricate interplay between quality attributes and cross-group network effects, our research provides valuable insights into various market configurations. This study enables us to explore the relation of price competition, cross-group network effects and platform's quality between two-sided platforms that are differentiated both horizontally and vertically, thus extending the seminal findings of Armstrong (2006); Rochet and Tirole (2002, 2006).

We introduced platform attributes on buyers' side to account for the vertical dimension. In the first stage of the model, platforms selected the level of attributes they offer to buyers simultaneously. In the second stage, platforms simultaneously chose membership fees. The equilibrium membership fees, market shares, and profits were determined by the difference in attributes on buyers' side. Although the features were developed only on buyers' side, they also influenced decisions on sellers' side. As a result, we demonstrate that vertical differentiation allows for the existence of asymmetric platforms in equilibrium. Overall, our contribution is to provide a comprehensive model that captures the dynamics of competition in two-sided markets with vertical differentiation.

Our study examines two scenarios depending on the strength of the cross-group network effects. Specifically, we consider the following scenarios: Firstly, we explore a case where the indirect network effects on both sides of the market are identical. Secondly, we centre our attention where sellers' cross-group network effect on buyers is stronger than buyers' impact on sellers, normalising sellers' network effect to zero. Then, we analyse where buyers' cross-group network effect on sellers is stronger than sellers' impact on buyers, normalising buyers' network effect to zero. By examining these scenarios, we contribute to the existing literature on two-sided markets by offering insights into the influence of cross-group network effects and attributes as a vertical differentiation variable on platform competition. This knowledge can be leveraged to devise effective strategies that enhance platform performance and support overall market welfare.

Our analysis shows platforms use attributes on buyers' side as the main trigger to adjust their strategies to appeal to agents and boost profits. We find that the more heterogeneous platforms are (measured by the degree of product differentiation), the fewer attributes they develop on buyers' side. Whereas the more valuable platforms become given a stronger cross-group network effect, the more attributes are offered on buyers' side. This mechanism drives platforms to adjust equilibrium membership fees and profits. Our analysis also uncovers interesting insights into the impact of model parameters on equilibrium membership fees, which are contingent on the relative strength of cross-group network effects between the two sides of the market. By providing such granular insights, platforms design optimal pricing strategies in two-sided markets with attributes on buyers' side.

We also identify the optimal conditions for platforms to maximise their profits by strategically balancing the degree of product differentiation on the horizontal dimension and attributes on buyers' side on the vertical dimension. This finding aligns with previous research conducted by Garella and Lambertini (2014) and Barigozzi and Ma (2018). Specifically, we observe that this optimal strategy occurs when the cross-group network effect exerted by sellers on buyers is stronger than the impact buyers have on sellers. Moreover, we establish the conditions under which it is optimal to maximise one dimension while minimising the other dimension to enhance profitability. This pattern is consistent with earlier studies, including Economides (1989) and Neven and Thisse (1989), as well as the generalised model proposed by Irmen and Thisse (1998). Particularly, we observe that this optimal strategy occurs when the cross-group network effect exerted by buyers on sellers is stronger than the effect that sellers have on buyers.

Our findings shed light on the strategic trade-offs platforms face in two-sided markets with vertical differentiation seen as attributes on buyers' side, and provide important insights for platform managers and policymakers seeking to optimise their pricing strategies. By understanding the optimal conditions for maximising profits, platforms can enhance their performance and contribute to the overall welfare of the market.

Furthermore, our findings can provide valuable insights for regulators seeking to establish minimum quality standards to identify opportunities to enhance social welfare. However, it is crucial to consider the influence of cross-group network effects on price competition and, consequently, on the welfare of participants. This entails understanding how interactions between buyers and sellers across horizontal and vertical differentiation affects two-sided market dynamics and overall welfare.

One potential extension of the study involves incorporating features on sellers' side, which would contribute to a more comprehensive model that better reflects real-world dynamics. Additionally, enabling both buyers and sellers to engage in multihoming would provide valuable insights into how platforms define their pricing strategies. In addition, a welfare analysis can be included by comparing the aggregate surpluses of buyers and sellers across the different scenarios. By including these additional features, a more thorough understanding of the platform's decision-making processes can be attained.

Chapter 3

Switching decisions in two-sided markets under quality uncertainty

3.1 Introduction

Customers often exhibit a strong preference for a particular brand or service provider for a multitude of reasons. One pivotal element that can impact their decision-making process is the reputation of the provider. Customers are inclined towards providers with a solid reputation for delivering quality, reliability, and exceptional customer service. Convenience can also play a significant role, as customers may opt for providers that are in close proximity to their home or workplace, or offer the convenience of online ordering or delivery. Additionally, pricing can be a determining factor, as customers often seek providers that offer competitive pricing and greater value for their money.

Purchasing products or services online requires additional considerations compared to traditional brick-and-mortar stores. When buying clothing or services such as hotel accommodations or taxi rides in physical stores, customer service is readily available to provide information and facilitate returns. Conversely, online shopping presents the initial challenge of selecting the appropriate digital marketplace. Choosing between *Uber* or *Cabify* for transportation, *Just Eat* or *Deliveroo* for meals, or *Booking* or *Skyscanner* for lodging reservations can be overwhelming. Additionally, deciding whether to purchase apparel from *Amazon* or *Wish* can be a daunting task.

One effective approach to this issue is to experience various online stores or platforms to determine which best suits one's expectations and performance standards. Key factors to consider include ease of navigation, transaction processing, and customer service. If the platform is not intuitive and basic functions such as registration processes or loading times are cumbersome, consumers may opt to switch to an alternative platform or online marketplace for their shopping needs.

However, transitioning to an alternative supplier can prove to be a challenging task after developing a business relationship with a service provider. This difficulty can be attributed to the notion of switching costs, which encompass temporal, physical, and financial expenses incurred by customers during a shift from one service provider to another. For instance, customers may need to expend significant amounts of time and effort researching alternative providers, comparing prices and services, and assessing the quality of the offerings. In addition, there may be financial costs such as termination fees or expenses associated with returning products to the original provider. Moreover, customers may experience psychological costs such as anxiety or uncertainty related to exploring a new service provider.

In the switching cost literature it has been identified by Klemperer (1987a) three types of costs that customers may incur when deciding to switch brands or products and services. The first type of switching cost is transaction costs, which arise due to the time, effort, and expense involved in researching, evaluating, and purchasing a new product or service. The second type is learning costs, which results from the need to acquire new knowledge or skills in order to use the alternative product or service effectively. Finally, artificial costs may arise due to the contractual or technological barriers that firms may create to impede customers from switching to competitors.

In this chapter, we investigate how introducing quality as a vertical differentiation catalyst in a two-sided market affects participants' decision to switch amongst intermediaries and the effects on pricing strategies. Specifically, we investigate the phenomenon whereby buyers assess the functionalities or quality offered by a platform and determine whether to persist in patronising it or switch to an alternative one considering they did not like it or felt disappointed about the service. As was mentioned previously, this situation can be exemplified by shoppers moving to a different shopping centre that has a larger car parking or more lifts or more toilets than the previous shopping centre they were visiting. Riders moving from $Lyft$ to *Uber* App since *Uber* App is more rider-oriented allowing them to schedule rides in advance and pay in cash.

The model consists of two platforms competing for members. Platforms offer attributes on buyers' side and buyers and sellers engage with a platform to facilitate their transactions. These attributes can be viewed as a reflection of the quality standards that a platform implements to attract a more extensive user base, and it is assumed that higher levels of quality confer an enhanced status of utility to users. However, given the initial uncertainty surrounding the quality of each platform, buyers must first join and engage with a platform to gain a firsthand understanding of its respective features and characteristics.

While buyers may be able to conduct web searches to identify some of a platform's attributes, they would not have access to precise information regarding its reliability until after an extended period of use. Consequently, they will visit the platform a pre-determined number of times to gain a better understanding of its quality and make an informed decision about whether to continue using that platform or switch to a different one.

Given its complexity or information asymmetry, agents can have imperfect information about an intermediary or platform's quality. For example, a customer may not fully understand the technical specifications of an app or game console making it hard to assess its quality. A platform may have incentives to misrepresent its quality to attract more customers, and this lead to imperfect information as buyers may not be aware of the true quality of the service being provided until they join it and experience it. After joining the platform and becoming familiar with its specifications, buyers gain a precise understanding of the actual quality being delivered. In other words, their firsthand experience confirms or not that the platform's quality aligns with their expectations.

In this study, our focus is on investigating the circumstances under which buyers decide to switch to a different platform after visiting and evaluating their quality. We specifically examine the strategies employed by platforms in setting their membership fees and the implications this has on determining market shares, considering the presence of cross-group network effects and quality uncertainty on buyers' side. We aim to understand how platforms navigate the interplay between quality uncertainty and the possibility of buyers switching platforms, particularly in the context of cross-group network effects. By analysing these two factors and their interactions, we can gain insights into the pricing strategies adopted by platforms and the impact it has on their market positioning.

There exist two strands of theoretical literature relevant to this study. The first is the literature on switching costs which have been widely studied in the field of economics with notable contributions from authors such as Klemperer (1987a,b) and Farrell and Shapiro (1988). These studies have revealed two opposing effects that firms experience concerning their pricing strategies. On the one hand, firms are incentivised to charge higher prices to customers who are locked into their products or services, while on the other hand, they aim to charge lower prices to attract new customers. The prevailing incentive, according to these studies, is to charge higher prices, which can result in anti-competitive outcomes when compared to markets that do not have switching costs. A comprehensive review of the literature on switching costs can be found in Klemperer (1995), Farrell and Klemperer (2007) and Villas-Boas (2015).

The second is the literature on two-sided markets where the seminal studies by Armstrong (2006) and Rochet and Tirole (2003, 2006) deal with the optimal pricing structure, which is contingent on the price elasticities exhibited by both sides of the market, the cross-group network effects, and the cost associated with the addition of a new agent to the platform. One key finding of these papers is that two-sided markets are subject to unique competitive dynamics that can lead to counterintuitive outcomes. Specifically, they show that firms in two-sided markets may choose to subsidise one group of users in order to attract users on the other side of the market. For example, a credit card company might offer low-interest rates to cardholders to attract more merchants to accept the card. For a complete literature review survey on two-sided markets see Weyl (2010), Belleflamme and Peitz (2019a), Hagiu and Wright (2015), Sanchez-Cartas and León (2021), Jullien et al. (2021).

In the present chapter, we develop a two-period dynamic model of platforms' competition in two-sided markets, in which it is assumed that buyers have imperfect information concerning the quality of the platforms. Consequently, buyers must first experience a platform's features before deciding to switch to a different intermediary. We introduce quality uncertainty on buyers' side and consider a two-period setting where platforms are able to set membership fees at the onset of the first period, which remain constant throughout both periods. We assume sellers do not switch since platform attributes are developed solely on buyers' side. These assumptions enable us to investigate how buyers' imperfect information regarding platform quality can lead to switching behaviour between platforms while avoiding analytical challenges that arise when platforms adjust fees over time to extract rent from locked-in members.

We find that buyers who initially underestimated the quality of a platform they visited with their observed quality may choose not to switch. However, if a buyer previously overestimated the quality of the selected platform, they may consider switching to another platform. The switching decision is contingent on the magnitude of switching costs. Accordingly, platforms consider cross-group network effect interplay to determine their fee strategy when buyers may decide to switch to a different platform taking into account the size of the switching cost.

Furthermore, in scenarios where the cross-group network effect exerted by buyers on sellers is stronger than the effect received by buyers from sellers, the platform's pricing strategy involves reducing fees on buyers' and increasing on sellers' side if buyers underestimate the quality of the initially visited platform.

When buyers' expectations exceed the actual quality of service provided by the platform visited during the first period, they may choose to switch to an alternative platform, considering the magnitude of the switching cost. In response, the platform implements a pricing policy that involves increasing its fees, to generate additional revenue from each new buyer joining the platform. Moreover, due to the stronger influence of buyers on sellers, more sellers are incentivised to join the platform. As a result, the platform also raises sellers' fees, as each new seller contributes to its revenue growth.

Our analysis also reveals that in anticipation of higher switching costs acting as a deterrent for buyers to switch to other providers, platforms adopt a pricing strategy aimed at lowering fees on the side of the market that exerts a stronger cross-group network effect. This adjustment results in a decrease in buyers' fees and an increase in sellers' fees.

The platform's pricing strategy is strategically implemented to foster its growth potential by attracting a larger user base. By lowering membership fees on buyers' side, the platform creates incentives for more participants to join and engage with one another. This increased participation in turn amplifies the network effects, as a larger proportion of buyers and sellers (considering the cross-group network effects) connect and benefit from the platform's services.

To gain insights into how platform profits are influenced by buyers' perceptions of platform quality and switching costs, we analyse a simplified version of the profit function. This approach allows us to isolate and evaluate the effects of two critical factors: the direct effect, which captures shifts in market shares, and the strategic effect, which represents fee adjustments. By comparing the magnitudes of these effects, we can determine which factor dominates and, consequently, ascertain the overall impact on platform's profit.

When buyers underestimate the quality of the platform visited in the first period and the cross-group network effect exerted by buyers on sellers is stronger than the influence sellers have on buyers. We find that an increase in buyers' perception of the quality of the platform they visited in the first period, increases equilibrium profits when the direct effect dominates the strategic effect on buyers' side. On the contrary, when the strategic effect dominates the direct effect on buyers' side and the combined direct and strategic effects on sellers' side, platform profits decrease.

Finally, our analysis reveals that when the cross-group network effect buyers exert on sellers is stronger than the influence sellers have on buyers. An increase in switching costs results in higher equilibrium profits when the direct effect dominates the strategic effect on buyers' side. However, when the strategic effect dominates the direct effect on buyers' side and the combined direct and strategic effects on sellers' side, platform profits decrease.

There has been limited research on the topic of switching costs in two-sided markets. However, recent contributions by Lam (2017) and Tremblay (2019) have shed light on this issue. Lam (2017) has shown that in the presence of strong cross-group network effects, the standard U-shaped result for fees, where fees fall in the first period and rise in the second period as switching costs increase, does not hold. This is due to the interaction between cross-group network effects and switching costs. Instead, she finds that the first-period fee always decreases with increasing switching costs and increasing switching costs on one side lead to a decrease in fees on the other side. In contrast, Tremblay (2019) has identified a different pattern of results. He finds that endogenous switching costs can lead to platforms subsidising content provision in the first period, rather than discounting consumer prices. This is in view of the fact that having more content providers in the first period generates a larger consumer lock-in, which leads to higher markups for consumers in the second period.

Our research provides insight into how platforms define their membership fees by carefully considering the dynamics between quality uncertainty and the potential for buyer switching in the presence of cross-group network effects. By examining this relationship, we contribute to a deeper understanding of the pricing strategies employed by platforms in two-sided markets under the previously mentioned characteristics. This study fills a gap in the existing literature by exploring the interplay between quality uncertainty, cross-group network effects, and endogenous switching decisions.

The paper is organised as follows. The model setup is described in Section 3.2. Section 3.3 presents the second-period analysis, followed by the first-period analysis in Section 3.4. The equilibrium membership fees analysis is discussed in Section 3.5. Finally, Section 3.6 offers some conclusions and final comments.

3.2 Model

We study competition between two platforms over two periods. The model we use builds on Lam (2017) and Cavazos and Datta (2023). We extend the standard two-sided markets model of horizontal differentiation and cross-group network effects by introducing quality uncertainty on buyers' side which enables us to account for endogenous switching decisions and heterogeneous platforms in equilibrium.

Consider two platforms 1 and 2 located at the endpoints of a unit line competing to enable transactions between members, e.g. buyers and sellers. Platforms face constant costs by serving each additional member which we assume are equal to zero. Membership fees are established by platforms for two periods, meaning that they are chosen only at the beginning of period one and remain fixed for the duration of both periods.[1]

There are various reasons why platforms may choose to establish fees only at the beginning of the first period and not adjust them in the second period. One reason is that setting fees for an extended period enables them to plan and allocate resources more

[1]A similar assumption has been made by von Weizsacker (1984) in his continuous time model. He introduces a concept of a "reputation equilibrium" wherein firms are bound to uphold a consistent pricing strategy across all periods.

efficiently. It reduces the costs associated with continuous monitoring of market conditions, which would be necessary if fees were adjusted in each period. Additionally, fixed fees can help platforms establish their brand and reputation as stable and trustworthy online businesses, which can attract loyal customers over time.

Another reason for setting fees only at the beginning of the first period could be to avoid negative perceptions from participants who might feel that fee adjustments are manipulative or unfair. Fixed fees can lead to greater transparency and predictability for buyers and sellers, enhancing customer satisfaction and loyalty. Furthermore, frequently adjusting prices can create additional uncertainty among agents, which may deter them from joining the platform.

Platform i, $i = 1, 2$ offers a service of quality q_b^i on buyers' side, which is exogenously determined for both periods.[2] These qualities were developed through prior research and development processes by their respective platforms and remain constant throughout the two periods. Platforms know perfectly the quality they are offering and also know the quality of their competition, i.e., quality is common knowledge for platforms.[3] The existence of platforms with different qualities is possible due to differentiation along their individual characteristics or features, reflecting the reality that buyers may have varying preferences for different platform attributes. As a result, platforms are differentiated both horizontally and vertically. Horizontal differentiation refers to differences in platforms' services that cater to different preferences of buyers and sellers, without being inherently better or worse. This is illustrated by the location of platforms along the unit line. Buyers and sellers are spread along this line and prefer services or products closer to their location due to transportation costs or mismatch preferences. Vertical differentiation, on the other hand, involves differences in the quality or features of products/services that make one objectively better or worse than another. Platforms offer services or products of varying quality, and buyers and sellers make their choices based on their willingness to pay for higher quality. Both platforms discount the second-period profit at rate δ_p, where $\delta_p \in (0, 1)$.

Buyers (b) and sellers (s) are situated on opposite sides of the market, both uniformly distributed along the same unit line and experience a cost of visiting a platform that increases linearly in distance of τ_b and τ_s, respectively. Considering our focal point is the relationship between the cross-group network effects, the quality attributes a platform offers on buyers' side and the switching cost, we assume a linear disutility cost $\tau_b = \tau_s = 1$, in our analysis. Moreover, we assume buyers and sellers singlehome, which means they

[2]Determination of endogenous quality is considered in Cavazos and Datta (2023).

[3]Platforms can obtain knowledge of their rivals' product or service quality through various means. For example, consumer reports, which conduct extensive testing and customer satisfaction surveys across different industries. Additionally, supply chain relationships can provide insights into the quality of components or materials used by rivals.

exclusively choose to participate and transact on a single platform within the market.

Our assumption on how buyers perceive and engage with a platform closely aligns with the method employed by Datta and Fraser (2017). Prior to joining a platform in period $t = 1$, buyers possess imperfect information regarding its quality. Consequently, they must enrol in a platform to familiarise themselves with its features, such as user-friendliness, and accurately gauge its quality. While buyers may utilise web searches to identify some platform characteristics, they are unable to obtain precise information about reliability until they have managed the platform for a certain period of time, i.e., the first period. Therefore, buyers visit a platform a predetermined number of times (normalised to unity) during period one to acquire knowledge regarding its quality.

In period $t = 1$, buyers possess imperfect information about the quality of platforms, which may be characterised by incorrect or uncertain knowledge. Consequently, buyers are limited to observations of the distribution of platform quality rather than the exact quality itself. We denote this quality as q_b^i, $i = 1, 2$ and assume it is uniformly and symmetrically distributed within an interval $[0, \bar{q}_b]$.[4] The platforms' quality can also be viewed as attributes developed to attract a broader range of buyers and potentially increase market share and profitability. Specifically, higher quality attributes are more desirable. Examples of such attributes may include software features in a platform app, backwards compatibility and storage capacity in a video game console, and processing power and camera quality in a smartphone.

Upon their initial visit in period $t = 1$, buyers promptly and accurately ascertain the quality of the platform visited, signifying that buyers do not exhibit differences in their appraisals of platform quality. Buyers realise the platform's quality at the end of the first period. At the beginning of period $t = 2$, buyers having evaluated the quality decide whether to switch platforms or not. If they decide to switch they incur a cost of s_b. Consequently, platform i, $i = 1, 2$ and buyers who joined in the first period have an identical and precise knowledge of q_b^i.

Buyers remain exclusive to a sole platform during each period (i.e., singlehoming). For tractability we adopt the assumption buyers have myopic behaviour, prioritising their utility solely in the current period. In other words, buyers' decisions are driven solely by immediate considerations without accounting for the potential influence of future periods on their utility. Specifically, buyers discount their second-period utility at a rate of $\delta_b = 0$.

Buyers' myopic behaviour, characterised by their decision-making process being un-

[4] One common approach to model imperfect information relating to platforms' quality is to introduce a probability distribution over buyers' beliefs concerning the quality of such platforms. Such a distribution can be illustrated through a density function, where the density assigned to each point along the line reflects the proportion of buyers subscribing to the notion that the quality of the platform is situated at that particular point.

affected by future consequences, can be attributed to several factors. One plausible explanation is the limited information buyers possess regarding the platforms' quality. As a result, buyers make decisions on a period-by-period basis, concentrating on the available knowledge and their immediate needs rather than considering potential future quality states.

Moreover, buyers may exhibit myopic behaviour due to the complexity and uncertainty surrounding long-term implications. Predicting future quality accurately can be challenging, especially in dynamic markets where various factors can influence quality fluctuations. In such cases, buyers might find it more rational to base their decisions on the existing information and current circumstances rather than attempting to anticipate future dynamics.

Additionally, transaction costs and search costs associated with gathering and evaluating information about platforms' quality could discourage buyers from considering long-term effects. The effort and time required to research and assess multiple platforms might outweigh the perceived benefits of long-term decision-making, prompting buyers to adopt myopic behaviour.

Sellers, on the other hand, are attracted to platform i, $i = 1, 2$ by the presence of buyers on the other side of the market, which increases the likelihood of selling their product or service. They remain exclusively affiliated with a particular platform throughout both periods (i.e., through singlehoming), as they are not required to assess platform quality when selecting or transitioning between platforms. Our assumption is established on the notion that platforms solely possess a vertical dimension on buyers' side. As such, sellers maintain their status quo by remaining with the same platform across both periods of the game.

Sellers may not switch between platforms as frequently as buyers do because, unlike buyers, sellers invest a significant amount of time and resources in building their reputation, establishing relationships with customers, and creating a network of connections within the platform ecosystem. Therefore, switching between platforms can be costly and may result in a loss of these valuable assets. Moreover, sellers often have more stable preferences regarding the platform they choose to join, as they are primarily concerned with maximising their profits over the long term. In contrast, buyers' preferences are influenced by factors such as price, quality, and convenience. Therefore, sellers are less likely to switch between platforms in response to short-term changes in market conditions.

Finally, platforms may use various strategies to incentivise sellers to remain loyal to them. For instance, they may offer sellers exclusive benefits or discounts, provide tools and resources to help sellers grow their business, or invest in marketing and advertising efforts that attract more buyers to the platform. By doing so, platforms can create a sense of loyalty and trust among their sellers, reducing the likelihood of them switching to a competitor platform.

The structure of the game is as follows and is illustrated in Figure 3.1:

At the end of the period t=1 buyers having currently evaluated the quality experienced in t=1, decide at the beginning of period 2 whether to switch or not conditional on the magnitude of the switching cost.

1. Period $t = 1$
   - Platforms simultaneously choose membership fees
   - Buyers and sellers simultaneously choose which platform to join. Initially, buyers have imperfect information about the quality of both platforms.
   - Buyers experience the platform's quality and realise the true quality at the end of the first period.
2. Period $t = 2$
   - Buyers having currently evaluated the quality experienced in $t = 1$, decide at the beginning of period 2 whether to switch or not conditional on the magnitude of the switching cost. If they opt to switch, they do so without prior knowledge of the actual quality offered by the alternative platform, and they only discover it upon joining and experiencing the new provider.

The timeline of the game is described in Figure 3.1.

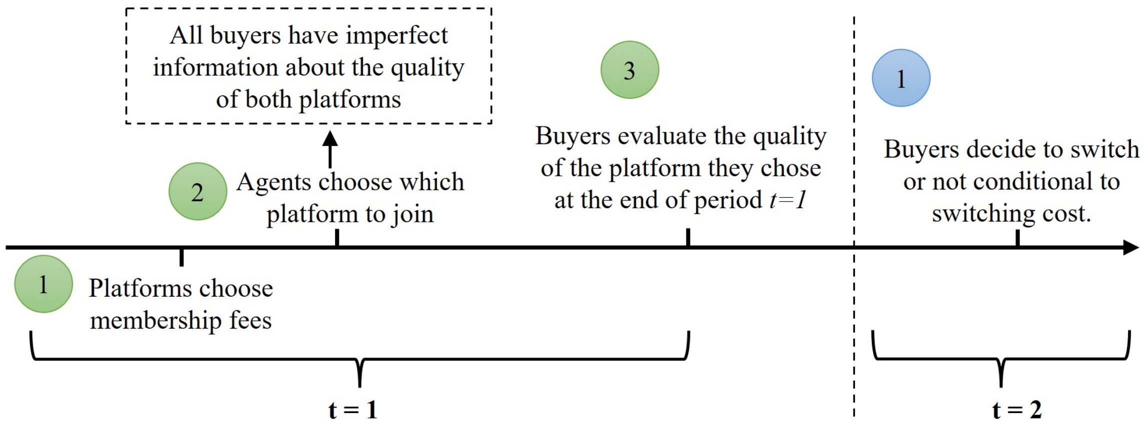

Figure 3.1: Timeline

The solution concept is subgame perfect Nash equilibrium.

Since platforms' quality is a random variable for buyers' side, the utility they obtain by joining a specific platform is also random, although once they have joined, buyers have complete knowledge of the platform's quality. Buyers' expected utility (gross of transportation costs) on platform i, $i = 1, 2$ at time t, is:

$$\mathbb{E}[U^i_{b,t}] = R_b + \mathbb{E}[q^i_b] + \upsilon\eta^i_{s,t} - p^i_b \tag{3.1}$$

Furthermore, sellers' utility (gross of transportation costs) on platform i, $i = 1, 2$, facing no uncertainty is:

$$U^i_s = R_s + \pi\eta^i_{b,t} - p^i_s \tag{3.2}$$

Buyers' utility increase linearly with platform i quality level, q^i_b, $i = 1, 2$, R_b and R_s are buyers' and sellers' stand-alone benefits from visiting a platform and are symmetric across both platforms. We assume R_k, $k = b, s$ is large enough to ensure complete market coverage. This means all buyers and sellers choose to participate rather than opting out. υ and π capture the sizes of the cross-group network effects that each group exerts on the other. $\eta^i_{b,t}$ and $\eta^i_{s,t}$ denote the proportion of active buyers and sellers, respectively, on platform i, $i = 1, 2$ at time $t = 1, 2$. Finally, p^i_k, $k = b, s$ is the membership fees platform i, $i = 1, 2$ charges buyers and sellers.

The model parameters must meet the following assumptions.[5]

Assumption 3.1. $4 > (\pi + \upsilon)^2$

Assumption 3.2. $1 > \frac{\delta_p(\pi-\upsilon)^2|\Omega|}{2(1-\pi\upsilon)(1+\delta_p)\left[4-(\pi+\upsilon)^2\right]}$

where $\Omega \equiv \Delta q^i_b + \Delta q^j_b + 2s_b$, $\Delta q^i_b = q^i_b - \frac{\bar{q}_b}{2}$, for $i = 1, 2$.

Assumption 3.1 and Assumption 3.2 are developed on the second-order conditions of the platform maximisation problem specified in Definition 3.1. Assumption 3.1 requires the cross-group network effects to remain below a specific threshold (4). This condition guarantees that the proportion of buyers and sellers in both periods decreases as membership fees increase, thus preventing the market from tipping.[6] If this condition is not satisfied, all buyers and sellers would prefer the same platform, as the fraction of participants on one platform would increase with higher membership fees.[7]

We do not impose a constraint on whether buyers underestimate $q^i_b > \frac{\bar{q}_b}{2}$ or overestimate $q^i_b < \frac{\bar{q}_b}{2}$ the platform's quality. Therefore, we consider $\Omega \equiv \Delta q^i_b + \Delta q^j_b + 2s_b$, $\Delta q^i_b = q^i_b - \frac{\bar{q}_b}{2}$, for $i = 1, 2$ in absolute terms, $|\Omega|$.

We solve our model using backward induction.

[5]For further details on both assumptions see Appendix C.1.

[6]In 2022, the European Commission released the Digital Markets Act, a comprehensive regulatory initiative that specifically targets markets with tipping behaviour as potential subjects for intervention.

[7]This can be demonstrated by taking the partial derivatives of Equations (3.10a) and (3.10b) and Equations (3.11a) and (3.11b).

3.3 Second period analysis

In the second period, buyers accurately evaluate the quality of the platform they initially selected, while remaining uninformed about the quality of the alternative platform. Consequently, if buyers who initially chose platform i in the first period decide to switch to platform j, they must confront both the uncertainty surrounding its quality and the associated switching cost denoted as s_b.

Buyers' switching decision

In our model, buyers make a decision to join either platform i or platform j in the first period. At the end of period one, they experience and evaluate the quality of the platform they have chosen, leading to a realised utility denoted as $\tilde{U}^i_{b,1}$ for platform i and $\tilde{U}^j_{b,1}$ for platform j. If buyers decide to remain with their initial choice in the second period, their utility remains unchanged. The realised utility in period two for buyers who stay with platform i is $U^i_{b,2}$, which is equal to $\tilde{U}^i_{b,1}$.

The decision of whether buyers continue with platform i or switch to platform j depends on the comparison between the utility of staying with platform i and the expected utility from switching to platform j conditional on the magnitude of switching cost. Specifically, when buyers have chosen platform i in period $t = 1$ and are aware of the realised quality q^i_b, their decision to switch to platform j is contingent upon whether:

$$U^i_{b,2} \geq \mathbb{E}[U^j_{b,2}] - s_b \tag{3.3a}$$

Conversely, if buyers have selected platform j in period $t = 1$ and are aware of the realised quality q^j_b, they will choose to switch to platform i if:

$$\mathbb{E}[U^i_{b,2}] - s_b \leq U^j_{b,2} \tag{3.3b}$$

Then, buyers who initially underestimated the quality of platform i, with their observed quality (q^i_b) being greater than their expected quality $\mathbb{E}[q^i_b]$, may choose not to switch conditional to the size of the switching cost. However, if a buyer previously overestimated the quality of the selected platform in period $t = 1$, they may consider switching to another platform conditional on the magnitude of the switching cost.

In such cases, buyers who overestimated the quality of their chosen platform, e.g., platform j, the perceived gain from platform j compared with platform i decreases by the difference between the expected quality of platform i, the quality of platform j and the switching cost, $\mathbb{E}[q^i_b] - q^j_b - s_b$. Buyers revise their estimate of platform j's quality downwards by the difference $\mathbb{E}[q^i_b] - q^j_b$, and if decide to move to platform i they incur the switching cost s_b.

Buyers' joining decision

Considering buyers' decision-making process regarding whether to remain or switch between platforms, the next step involves determining the market shares in the second period. A buyer located at $b^i_{t=2}$ within the unit interval, who joined platform i in the first period is indifferent between continuing to remain attached to platform i and switching to platform j, in the second period if:[8]

$$U^i_{b,2} - b^i_{t=2} = \mathbb{E}[U^j_{b,2}] - \left(1 - b^i_{t=2}\right) - s_b \tag{3.4a}$$

On the other hand, a buyer located at $b^j_{t=2}$ on the unit interval, who joined platform j in the first period is indifferent between continuing to be attached to platform j and switching in the second period to platform i if

$$\mathbb{E}[U^i_{b,2}] - b^j_{t=2} - s_b = U^j_{b,2} - \left(1 - b^j_{t=2}\right) \tag{3.4b}$$

Using buyers' expected utility in Equation (3.1) along with Equations (3.4a) and (3.4b) and the fact that the total proportion of sellers on both platforms adds up to one, $\eta^i_{s,t} + \eta^j_{s,t} = 1$, the indifferent buyer who began using platform i in the first period is equally willing to stick with platform i or switch to platform j in the second period is:[9]

$$b^i_{t=2} = \frac{1}{2} + \frac{q^i_b - \mathbb{E}[q^j_b] + \upsilon\left(2\eta^i_{s,s} - 1\right) + p^j_b - p^i_b + s_b}{2} \tag{3.5a}$$

On the contrary, the indifferent buyer between patronising platform j in the first period and switching to platform i in the second period is:

$$b^j_{t=2} = \frac{1}{2} + \frac{\mathbb{E}[q^i_b] - q^j_b + \upsilon\left(2\eta^i_{s,s} - 1\right) + p^j_b - p^i_b - s_b}{2} \tag{3.5b}$$

At this point, both platforms have already acquired a share of members on both sides of the market. Therefore, considering first-period market shares $\eta^i_{b,1}$ and $\eta^i_{s,1}$, buyers' market share on period $t = 2$ on platform i is given by

$$\eta^i_{b,2} = \eta^i_{b,1} b^i_{t=2} + \left(1 - \eta^i_{b,1}\right) b^j_{t=2} \tag{3.6}$$

[8] The superscript in $b^i_{t=2}$ denotes whether buyers were on platform i or platform j in period $t = 1$.

[9] This interpretation considers the expected values at this stage, as the right-hand side represents the expected quality. However, once the quality is realised, the expression becomes deterministic, as there is no longer uncertainty or variability associated with it.

The right-hand side comprises the initial buyers of platform i who remain loyal in the second period (the first term) and platform j's buyers from the first period who switch to platform i in the second period (the second term).

Sellers do not switch between platforms therefore a seller located at x_s within the unit interval is indifferent between joining platform i and j, such that $U^i_{s,2} - x_s = U^j_{s,2} - (1 - x_s)$. Sellers located between 0 and x_s visit platform i, while those positioned between x_s and 1 visit platform j. Consequently, we have $\eta^i_{s,t} = x_s$ and $\eta^j_{s,t} = 1 - x_s$, with the total proportion of buyers and sellers on both platforms being $\eta^i_{s,t} + \eta^j_{s,t} = 1$ and $\eta^i_{b,t} + \eta^j_{b,t} = 1$. We then determine the proportion of sellers on platform i using the expressions for the indifferent seller $\eta^i_{s,2} = x_s = \frac{1}{2} + \frac{U^i_{s,2} - U^j_{s,2}}{2}$ along with sellers' utility in Equation (3.2) to obtain:

$$\eta^i_{s,2} = \frac{1}{2} + \frac{\pi\left(2\eta^i_{b,2} - 1\right) + p^j_s - p^i_s}{2} \tag{3.7}$$

Next, as q^i_b is uniformly and symmetrically distributed within an interval $[0, \bar{q}_b]$, we obtain:[10]

$$\mathbb{E}[q^i_b] = \int_0^{\bar{q}_b} q^i_b f\left(q^i_b\right)\, dq^i_b \quad = \quad \frac{\bar{q}_b}{2}$$

Where the term $f\left(q^i_b\right)$ is the probability density function of the continuous uniform distribution.

Solving the system of Equations (3.6) and (3.7) we can obtain buyers' and sellers' market shares at period $t = 2$.

$$\eta^i_{b,2} = \frac{1}{2} + \frac{\left(2\eta^i_{b,1} - 1\right) s_b + \Delta q^i_b \eta^i_{b,1} - \Delta q^j_b\left(1 - \eta^i_{b,1}\right) + \left(p^j_b - p^i_b\right) + \left(p^j_s - p^i_s\right)\upsilon}{2\left(1 - \pi\upsilon\right)} \tag{3.8a}$$

$$\eta^i_{s,2} = \frac{1}{2} + \frac{\pi\left(2\eta^i_{b,1} - 1\right) s_b + \pi\Delta q^i_b \eta^i_{b,1} - \Delta q^j_b\left(1 - \eta^i_{b,1}\right)\pi + \left(p^j_b - p^i_b\right)\pi + \left(p^j_s - p^i_s\right)}{2\left(1 - \pi\upsilon\right)} \tag{3.8b}$$

where $\Delta q^i_b = q^i_b - \frac{\bar{q}_b}{2}$, for $i = 1, 2$

The market shares in the second period are conditional on the market shares in the first period due to the existing customer base served by the platforms. Considering the presence of cross-group network effects that impact both sides of the market, sellers' market share is also affected by the platforms' quality, despite being primarily developed for buyers' side.

[10] The expected value can be defined as the weighted average of outcomes for the platforms' quality, which are independently selected. In this context, $f\left(q^i_b\right)$, $i = 1, 2$, represents the probability density function associated with q^i_b. For a continuous uniform distribution, the probability density function is denoted as $f(x) = \frac{1}{b-a}$ for $a \le x \le b$ and 0 otherwise.

It is crucial to note, as depicted in Equations (3.8a) and (3.8b), that both sides' market shares in period $t = 2$ are adjusted proportionally by a factor Δq_b^i, $i = 1, 2$. This factor represents the difference between the quality realisation and the expected quality. Depending on whether buyers underestimated or overestimated their platform's quality expectations upon joining, this difference can be positive or negative.

From platform i's perspective if buyers underestimated the quality $q_b^i > \frac{\bar{q}_b}{2}$, then $\Delta q_b^i > 0$ those who initially chose platform i in period $t = 1$ will continue using the same platform in period $t = 2$. Conversely, if they overestimated the quality $q_b^j < \frac{\bar{q}_b}{2}$, then $\Delta q_b^j < 0$ and buyers who were initially on platform j in the first period will switch to platform i in the second period dependent on the size of switching cost.[11]

Furthermore, the magnitude of switching costs also impacts the composition of both sides' market shares in the second period. Higher switching costs tend to stimulate a greater proportion of buyers and sellers who initially chose platform i to continue using that platform in the following periods.[12]

3.4 First period analysis

In the initial period, buyers make decisions to join platform i without considering future switching choices due to their limited information regarding the quality of both platforms. Their decision-making process follows a myopic intertemporal preference, characterised by a discount rate of $\delta_b = 0$. Consequently, buyers select platform i over platform j if the expected utility of joining platform i exceeds the expected utility of joining platform j, $\mathbb{E}[U_{b,1}^i] > \mathbb{E}[U_{b,1}^j]$.

As a result, buyers and sellers can obtain the following utility from joining platform i

$$\mathbb{E}[U_{b,1}^i] = R_b + \upsilon\eta_{s,1}^i + \mathbb{E}[q_b^i] - p_b^i$$

$$U_{s,1}^i = R_s + \pi\eta_{b,1}^i - p_s^i$$

Assuming a buyer and a seller are indifferent between using platform i or j, their locations can be denoted as x_b and x_s respectively. This is expressed as $\mathbb{E}[U_{b,1}^i] - x_b = \mathbb{E}[U_{b,1}^j] - (1 - x_b)$ for buyers, and $U_{s,1}^i - x_s = U_{s,1}^j - (1 - x_s)$ for seller, where i and j represent the platforms. As in the second period, it follows that $\eta_{k,1}^i = x_k$ and $\eta_{k,1}^j = 1 - x_k$,

[11]We observe these results by partially differentiating buyers' and sellers' market shares on Equations (3.8a) and (3.8b) by Δq_b^i and Δq_b^j. Specifically, on buyers' side $\partial\eta_{b,2}^i/\partial\Delta q_b^i = \eta_{b,1}^i/2\,(1 - \pi\upsilon) > 0$ if $\Delta q_b^i > 0$. In contrast $\partial\eta_{b,2}^i/\partial\Delta q_b^j = \eta_{b,1}^j/2\,(1 - \pi\upsilon) > 0$ if $\Delta q_b^j < 0$. Assumption 3.1 confirms $1 > \pi\upsilon$ as was demonstrated in Appendix C.1. The same reasoning applies on sellers' side.

[12]This relationship can be seen by partially differentiating buyers' market share in Equation (3.8a), turning into $\partial\eta_{b,2}^i/\partial s_b = \left(2\eta_{b,1}^i - 1\right)/2\,(1 - \pi\upsilon) > 0$ if $\eta_{b,1}^i > 1/2$.

where $k = b, s$. Additionally, the total proportion of buyers and sellers on both platforms adds up to one, i.e., $\eta^i_{k,1} + \eta^j_{k,1} = 1$. Given these conditions, buyers and sellers who choose to join platform i, $i = 1, 2$ can be determined as:

$$\eta^i_{b,1} = \frac{1}{2} + \frac{\upsilon\left(2\eta^i_{s,1} - 1\right) + \left(p^j_b - p^i_b\right)}{2} \tag{3.9a}$$

$$\eta^i_{s,1} = \frac{1}{2} + \frac{\pi\left(2\eta^i_{b,1} - 1\right) + \left(p^j_s - p^i_s\right)}{2} \tag{3.9b}$$

Solving the previous system of equations, we can determine the market shares of buyers and sellers as a function of the model parameters and the membership fees on both sides of the market.

$$\eta^i_{b,1} = \frac{1}{2} + \frac{(p^j_b - p^i_b) + \upsilon(p^j_s - p^i_s)}{2\left(1 - \pi\upsilon\right)} \tag{3.10a}$$

$$\eta^i_{s,1} = \frac{1}{2} + \frac{\pi(p^j_b - p^i_b) + (p^j_s - p^i_s)}{2\left(1 - \pi\upsilon\right)} \tag{3.10b}$$

It is important to recognise that under the assumption of a symmetrical distribution of quality for each platform, the magnitude of the difference in expected quality between both platforms does not have an impact on the allocation of market shares among buyers.

3.5 Equilibrium Membership Fees

Before establishing platform i's equilibrium membership fees, we derive second-period market shares as functions of membership fees and model parameters, using first-period market shares in Equations (3.10a) and (3.10b) and second-period market shares in Equations (3.8a) and (3.8b) by denoting them as a function of the model parameters and both sides membership fees. This can be expressed as follows:

$$\eta^i_{b,2} = \frac{1}{2} + \frac{q^i_b - q^j_b}{4\left(1 - \pi\upsilon\right)} - \frac{\Omega\left(p_b + \upsilon p_s\right)}{4\left(1 - \pi\upsilon\right)^2} - \frac{p_b + \upsilon p_s}{2\left(1 - \pi\upsilon\right)} \tag{3.11a}$$

$$\eta^i_{s,2} = \frac{1}{2} + \frac{\pi\left(q^i_b - q^j_b\right)}{4\left(1 - \pi\upsilon\right)} - \frac{\pi\Omega\left(p_b + \upsilon p_s\right)}{4\left(1 - \pi\upsilon\right)^2} - \frac{\pi p_b + p_s}{2\left(1 - \pi\upsilon\right)} \tag{3.11b}$$

where $\Omega \equiv \Delta q^i_b + \Delta q^j_b + 2s_b$, $\Delta q^i_b = q^i_b - \frac{\bar{q}_b}{2}$, for $i = 1, 2$ and $p_k \equiv p^i_k - p^j_k$, for $k = b, s$.

Buyers' and sellers' market shares are defined by the observed quality difference and the difference in membership fees between the two platforms. Subscription payments are adjusted based on the variation between the quality realisation and the expected quality between platforms, as well as the associated switching cost. Having obtained buyers' and sellers' market shares as a function of membership fees and model parameters, we state the following definition.

Definition 3.1. *An equilibrium of the game is a pair* p_b^i, p_s^i*, such that* p_b^i *and* p_s^i *solve the platform i's maximisation problem* $\max_{\{p_b^i, p_s^i\}} \Pi^i \equiv p_b^i \eta_{b,1}^i + p_s^i \eta_{s,1}^i + \delta_p \left(p_b^i \eta_{b,2}^i + p_s^i \eta_{s,2}^i \right)$*, for each* $i, j = 1, 2,\ i \neq j$*.*

Platform's i, $i = 1, 2$ first-order conditions are:[13]

$$\frac{\partial \Pi^i}{\partial p_b^i} = \frac{1}{2}\left(1+\delta_p\right) + \frac{\delta_p \left(q_b^i - q_b^j\right)}{4\left(1-\pi\upsilon\right)} + \frac{\left[2\left(1-\pi\upsilon\right)\left(1+\delta_p\right) + \delta_p \Omega\right]\left[p_b^j - 2p_b^i\right]}{4\left(1-\pi\upsilon\right)^2} + \frac{\left[2\left(1-\pi\upsilon\right)\left(1+\delta_p\right) + \delta_p \Omega\right]\left[\upsilon p_s^j - \left(\pi+\upsilon\right) p_s^i\right]}{4\left(1-\pi\upsilon\right)^2} = 0$$

$$\frac{\partial \Pi^i}{\partial p_s^i} = \frac{1}{2}\left(1+\delta_p\right) + \frac{\pi\delta_p \left(q_b^i - q_b^j\right)}{4\left(1-\pi\upsilon\right)} + \frac{\left[2\left(1-\pi\upsilon\right)\left(1+\delta_p\right) + \delta_p \Omega\right]\left[\pi p_b^j - \left(\pi+\upsilon\right) p_b^i\right]}{4\left(1-\pi\upsilon\right)^2} + \frac{\left[2\left(1-\pi\upsilon\right)\left(1+\delta_p\right) + \pi\upsilon\delta_p \Omega\right]\left[p_s^j - 2p_s^i\right]}{4\left(1-\pi\upsilon\right)^2} = 0$$

where $\Omega \equiv \Delta q_b^i + \Delta q_b^j + 2s_b$, $\Delta q_b^i = q_b^i - \frac{\bar{q}_b}{2}$, for $i = 1, 2$

Next, we solve the previous first-order conditions to obtain the equilibrium membership fees on both sides of the market, described as a function of the model parameters. The equilibrium participation fees can be determined based on the market shares for buyers and sellers in both the first and second periods.

$$p_b^i = 1 - \pi - \frac{\delta_p}{\Phi}\left(1-\pi\upsilon\right)\Omega + \frac{\delta_p\left(1-\pi\upsilon\right)}{2\Phi\Psi}\left[2\left(1-\pi\upsilon\right)\left(1+\delta_p\right)\left[3-\pi\left(\pi+2\upsilon\right)\right] + \delta_p \pi\left(\upsilon-\pi\right)\Omega\right]\left(q_b^i - q_b^j\right) \tag{3.12a}$$

$$p_s^i = 1 - \upsilon + \frac{\delta_p\left(1-\pi\upsilon\right)}{2\Psi}\left(\pi-\upsilon\right)\left(q_b^i - q_b^j\right) \tag{3.12b}$$

where $\Omega \equiv \Delta q_b^i + \Delta q_b^j + 2s_b$, $\Delta q_b^i = q_b^i - \frac{\bar{q}_b}{2}$, for $i = 1, 2$ and $\sigma \equiv 9 - \left(2\pi+\upsilon\right)\left(\pi+2\upsilon\right)$ $\Phi \equiv 2\left(1-\pi\upsilon\right)\left(1+\delta_p\right) + \delta_p\Omega$ and $\Psi \equiv \left(1-\pi\upsilon\right)\left(1+\delta_p\right)\sigma - \delta_p\left(\pi-\upsilon\right)^2\Omega$.

The transaction fee imposed by platform i on buyers' side consists of three components. The first component is $1-\pi$, which is similar to Armstrong (2006) results under the assumption that $\tau_b = 1$. The second component $\frac{\delta_p}{\Phi}\left(1-\pi\upsilon\right)\Omega$ is associated with buyers' switching decisions, while the third component $\frac{\delta_p(1-\pi\upsilon)}{2\Phi\Psi}\Big[2\left(1-\pi\upsilon\right)\left(1+\delta_p\right)\big[3 -$

[13] See Appendix C.2 for more details.

$\pi(\pi+2\upsilon)\big] + \delta_p \pi(\upsilon-\pi)\Omega\Big]\left(q_b^i - q_b^j\right)$ is linked to the difference between the observed quality on both platforms, adjusted by the model parameters.

In contrast, the access fee charged by platform i on sellers' side is composed of two distinct elements. As mentioned earlier, the first component, denoted as $1-\upsilon$, is similar to Armstrong (2006) results under the assumption that $\tau_s = 1$. The second element $\frac{\delta_p(1-\pi\upsilon)}{2\Psi}(\pi-\upsilon)\left(q_b^i - q_b^j\right)$ covers the difference in the observed quality between the two platforms on buyers' side adjusted by the model parameters. Notably, even though there is no quality developed on sellers' side, the variation in quality experienced by buyers is incorporated into sellers' equilibrium fees because of the impact the cross-group network effects have in the market.

To analyse the pricing strategies adopted by platforms in the presence of quality uncertainty on buyers' side and switching costs, we present the next two propositions.

Proposition 3.1. *Whenever the difference between the quality realisation and the expected quality increase and platform i has a higher quality than platform j, $q_b^i > q_b^j$ and the cross-group network effect buyers exert on sellers is stronger than the effect buyers receive from sellers, $\pi > \upsilon$, platform i:*

(i) Charges a lower fee on buyers' side as long as $1 < \frac{\pi(\pi+2\upsilon)}{3}$ whenever buyers underestimate platform i's quality.

(ii) Charges a higher fee on sellers' side whenever buyers underestimate platform i's quality.

Proof: See Appendix C.3

Proposition 3.1 shows that when buyers underestimated platform i's quality, meaning their expectations do not exceed the actual quality of the service, $(q_b^i > \mathbb{E}[q_b^i])$, they have a tendency to stay with the same provider. In response, platform i decreases its fees, to reward their loyalty. Additionally, as the influence of buyers on sellers outweighs the impact of sellers on buyers, more sellers are encouraged to join the platform. Consequently, the platform raises sellers' fees because each new seller increases its revenue.

There are no conclusive results when buyers overestimated platform i's quality in the first period, meaning their expectations exceeded the actual quality of the service, $(q_b^i < \mathbb{E}[q_b^i])$. Buyers are more likely to switch to a different platform, depending on the size of the switching cost. This behaviour prompts the platform to increase its fees to generate additional revenue from each new buyer joining. As a result, more sellers are encouraged to join the platform given the cross-group network effect, and the platform might also increase sellers' fees to further boost its revenue.

Corollary 3.1. *When the cross-group network effect sellers exert on buyers is stronger* *than the impact buyers have on sellers, ($\upsilon > \pi$), platform i charges a lower fee on sellers' side when buyers underestimate platform i's quality.*

Proof: See Appendix C.3

Conversely, Corollary 3.1 shows that when buyers underestimate platform's quality but the cross-group network effect exerted by sellers on buyers is stronger than the impact buyers have on sellers, the pricing policy aligns with the traditional findings in the two-sided markets literature. In this case, platform i subsidises the side of the market that exercises a greater influence on the other side. Consequently, sellers' membership fee decreases. This shift in pricing strategy reflects the platform's recognition of the changing dynamics of cross-group network effects. The scenario changes and buyers are more attracted to join (given $\upsilon > \pi$), creating a positive feedback loop that expands the participant base on both sides of the market increasing the platform's attractiveness and benefits to its users.

The following proposition examines how the pricing strategy of platform i is influenced by switching costs.

Proposition 3.2. *When the cross-group network effect buyers exert on sellers is stronger than the effect buyers receive from sellers, $\pi > \upsilon$, platform i equilibrium membership fees decrease on buyers' side as long as $1 < \frac{\pi(\pi+2\upsilon)}{3}$ and increase on sellers' side when switching cost increases. On sellers' side the impact is reversed when sellers exert a stronger cross-group network effect on buyers compared to the network effect buyers exert on sellers, $\upsilon > \pi$.*

Proof: See Appendix C.4

Proposition 3.2 states that platform i strategically adjusts its pricing based on the expectation that a higher switching cost will discourage buyers from switching to another platform. As a result, platform i reduces the fee on the side of the market that exerts a stronger cross-group network effect, which results in lower fees for buyers and higher fees for sellers, when $\pi > \upsilon$. Conversely, if the cross-group network effects impact are reversed, $\upsilon > \pi$ sellers' fees decrease.

Platform i's pricing strategy is designed to increase its growth potential by attracting a larger user base. By lowering membership fees for buyers, platform i can encourage more participants to join and interact with each other. This increased participation enhances the network effects, as more buyers and sellers connect and derive greater benefits from the interactions. Platform i creates a positive feedback loop, where the growing user base attracts more participants who are eager to engage with a larger crowd.

This, in turn, makes the platform more appealing to both buyers and sellers. The increased user base and enhanced value provide platform i with more opportunities to generate revenue, which can be reinvested to improve platform features and further expand the base of buyers' and sellers' side.

Equilibrium market-shares

As a result of platform i, $i = 1, 2$ equilibrium fees, expressed in Equations (3.12a) and (3.12b), we can develop the corresponding equilibrium market shares for both buyers and sellers in both the first and second periods.[14]

First Period Market Shares

$$\eta_{b,1}^{i} = \frac{1}{2} - \frac{\delta_p}{2\Phi\Psi}\Big[\Psi - 3\left(1 - \pi\upsilon\right)^2\left(1 + \delta_p\right)\Big]\left(q_b^i - q_b^j\right) \tag{3.13a}$$

$$\eta_{s,1}^{i} = \frac{1}{2} - \frac{\delta_p}{2\Phi\Psi}\Big[\pi\left[\Psi - 3\left(1 - \pi\upsilon\right)^2\left(1 + \delta_p\right)\right] + \left(1 - \pi\upsilon\right)\left(\pi - \upsilon\right)\Phi\Big]\left(q_b^i - q_b^j\right) \tag{3.13b}$$

where $\Omega \equiv \Delta q_b^i + \Delta q_b^j + 2s_b$, $\Delta q_b^i = q_b^i - \frac{\bar{q}_b}{2}$, for $i = 1, 2$ and $\sigma \equiv 9 - (2\pi + \upsilon)(\pi + 2\upsilon)$ $\Phi \equiv 2(1 - \pi\upsilon)(1 + \delta_p) + \delta_p\Omega$ and $\Psi \equiv (1 - \pi\upsilon)(1 + \delta_p)\sigma - \delta_p(\pi - \upsilon)^2\Omega$

In equilibrium, first-period market shares are not evenly divided between the two platforms because there is an extra proportion depending on the difference between platforms' quality $\left(q_b^i - q_b^j\right)$.

Second Period Market Shares

$$\eta_{b,2}^{i} = \frac{1}{2} + \frac{1}{4\Phi\Psi}\Big[2\big[\Psi - 3\big(1 - \pi\upsilon\big)^2\big(1 + \delta_p\big)\big] + 3\left(1 - \pi\upsilon\right)\left(1 + \delta_p\right)\Phi\Big]\left(q_b^i - q_b^j\right) \tag{3.14a}$$

$$\eta_{s,2}^{i} = \frac{1}{2} + \frac{1}{4\Phi\Psi}\left[\pi\Big[2\Psi + 3\delta_p\left(1 - \pi\upsilon\right)\Omega\Big] + \delta_p\left(1 - \pi\upsilon\right)\left(\pi + 2\upsilon\right)\Phi\right]\left(q_b^i - q_b^j\right) \tag{3.14b}$$

where $\Omega \equiv \Delta q_b^i + \Delta q_b^j + 2s_b$, $\Delta q_b^i = q_b^i - \frac{\bar{q}_b}{2}$, for $i = 1, 2$ and $\sigma \equiv 9 - (2\pi + \upsilon)(\pi + 2\upsilon)$ $\Phi \equiv 2(1 - \pi\upsilon)(1 + \delta_p) + \delta_p\Omega$ and $\Psi \equiv (1 - \pi\upsilon)(1 + \delta_p)\sigma - \delta_p(\pi - \upsilon)^2\Omega$.

Considering buyers make decisions on a period-by-period basis (i.e., myopic behaviour), buyers' and sellers' market shares in the second period are independent of the intertemporal preferences that buyers may exhibit between the initial and subsequent periods.

It is essential to mention that, while sellers are presumed to remain static across platforms over time, their market share can fluctuate in response to changes in the proportion of buyers between both platforms, as a result of cross-group network effects.

To get some insight into how buyers' decision to switch between platforms and switch-

[14]See Appendix C.5 for more details.

ing cost affects the market shares' composition, we specify the following observations considering platforms i's quality is higher, $\left(q_b^i > q_b^j\right)$.

Observation 3.1. *When buyers come to realise that their quality realisation is above their initial estimation of the platform's quality, i.e.,* $\left(q_b^i > \mathbb{E}[q_b^j]\right)$*, they may choose to stay in the same platform in the second period conditional to the magnitude of the switching cost. From platform i's perspective, this leads to an increase in the proportion of buyers in the second period compared to what they have in the first period and there is an increase in buyers' proportion on platform i in the second period relative to their first period.*[15]

Proof: See Appendix C.6

Observation 3.2. *In a scenario where the cost of switching from one provider to another increases, buyers tend to remain on the platform they initially chose. As a result, from the perspective of platform i, the proportion of buyers increases in the second period relative to the proportion they have in the first period and there is a decrease in buyers' proportion on platform j in the second period relative to their first period.*[16]

Proof: See Appendix C.7

Therefore, when buyers decide to switch from platform i to platform j in the second period, it has an impact on platform i in terms of the proportion of buyers. Specifically, if the proportion of incoming buyers to platform i is less than the proportion of outgoing buyers, platform i experiences a decline.

This means that when buyers leave platform i to join platform j, platform i loses a portion of its customer base. If the proportion of new buyers joining platform i is not sufficient to compensate for the loss of outgoing buyers, the overall proportion of buyers on platform i decreases.

Platform's equilibrium profits

Subsequently, we examine the effects when buyers underestimate $q_b^i > \mathbb{E}[q_b^j]$ and overestimate $q_b^i < \mathbb{E}[q_b^j]$ platforms i's, $i = 1, 2$ quality, along with the impact of switching costs in platform's equilibrium profits.

Firstly, we modify platforms profits $\Pi^i \equiv p_b^i \eta_{b,1}^i + p_s^i \eta_{s,1}^i + \delta_p \left[p_b^i \eta_{b,2}^i + p_s^i \eta_{s,2}^i\right]$ set in Definition 1.3 as $\Pi^i \equiv p_b^i \left[\eta_{b,1}^i + \delta_p \eta_{b,2}^i\right] + p_s^i \left[\eta_{s,1}^i + \delta_p \eta_{s,2}^i\right]$. Then we, define N_b^i and N_s^i as the present value of buyers' and sellers' market share in periods one and two. Namely, $N_b^i \equiv \eta_{b,1}^i + \delta_p \eta_{b,2}^i$ and $N_s^i \equiv \eta_{s,1}^i + \delta_p \eta_{s,2}^i$. Therefore, we obtain the following reduced-form

[15] This observation holds as long as $1 < \frac{(\pi+\upsilon)^2 - \pi\upsilon}{3}$, $\upsilon > \pi$ and $\eta_{b,1}^i > \frac{1}{2}$. See Appendix C.6 for more details.

[16] This observation holds by the same conditions in Footnote 15. See Appendix C.7 for more details.

profit function.

$$\Pi^i\left(\Delta q_b^i, \Delta q_b^j, s_b\right) = p_b^i N_b^i + p_s^i N_s^i \tag{3.15}$$

where equilibrium membership fees are given by Equations (3.12a) and (3.12b) and first-period buyers and sellers equilibrium market shares are given by Equations (3.13a) and (3.13b) and second-period equilibrium on Equations (3.14a) and (3.14b).

Impacts of buyers' perception of platform i's quality

In consequence, we maximise Equation (3.15) with respect to buyers perception of platform i's quality[17] Δq_b^i and obtain:

$$\frac{\partial \Pi^i}{\partial \Delta q_b^i} = p_b^i \frac{\partial N_b^i}{\partial \Delta q_b^i} + N_b^i \frac{\partial p_b^i}{\partial \Delta q_b^i} + p_s^i \frac{\partial N_s^i}{\partial \Delta q_b^i} + N_s^i \frac{\partial p_s^i}{\partial \Delta q_b^i} \tag{3.16}$$

We observe two distinct effects on each side of the market. Firstly, we refer to the direct effect as $\frac{\partial N_k^i}{\partial \Delta q_b^i}$ for $k = b, s$, which quantifies the direct influence of a change in buyers' perception of platform i's quality on market shares. This effect highlights how the difference between the quality realisation and the expected quality impacts the distribution of market participation. Secondly, we refer to the strategic effect as $\frac{\partial p_k^i}{\partial \Delta q_b^i}$ for $k = b, s$, which captures the strategic response of platform i to changes in buyers' perception of its quality. This effect examines how alterations in perceived quality influence the pricing decisions made by platform i, thereby impacting the fees charged to participants.

To gain deeper insights into the dominance of one effect over the other and their respective implications for profits, we present the following propositions.

Proposition 3.3. *When buyers underestimate platform i's quality (quality realisation is above the expected quality) and the cross-group network effect exerted by buyers on sellers is stronger than the influence sellers have on buyers* ($\pi > \upsilon$) *and* $1 < \frac{\pi(\pi+2\upsilon)}{3}$, *an increase in buyers perception of platform i's quality:*

1. *Increases platform i's equilibrium profits when the direct effect dominates the strategic effect on buyers' sides of the market.*

2. *Decreases platform i's equilibrium profits when the strategic effect dominates the direct effect on buyers' side and the combined direct and strategic effects on sellers' side.*

Proof: See Appendix C.8

[17] We refer to the difference between "the quality realisation and the expected quality" as the perception of platform i's quality. Considering "perception" is defined as a belief or opinion based on appearances, This description captures the difference between buyers' expectations and their actual experience. This perception can be underestimated when $\Delta q_b^i > 0$ and overestimated when $\Delta q_b^i < 0$.

The mechanism to explain Proposition 3.3 can be provided by combining the insights from Proposition 3.1 and Observation 3.1. When buyers underestimate platform i's quality, they choose to remain with their current provider, conditional on the size of the switching cost. In order to reward this loyalty, platform i refrains from raising its fees. This creates a positive feedback loop that attracts more buyers and sellers (given $\pi > \upsilon$), leading to the expansion of the user base. Now, platform i obtains an additional fee for each extra participant on both sides of the market, as long as the direct effect dominates the strategic effect on buyers' side.

Conversely, when the strategic effect dominates both the direct effect on buyers' side and the combined direct and strategic effects on sellers' side, an increase in buyers' perception of platform i's quality leads to a decrease in the platform's profits. This outcome is the consequence of intensified competition between both platforms. Since buyers prefer to stick with the provider they initially chose, platforms lower their fees to try to attract these buyers and encourage them to switch. This escalates the competition and ultimately reduces the platforms' profits.

Corollary 3.2. *When the cross-group network effect exerted by sellers on buyers is stronger than the influence buyers have on sellers ($\upsilon > \pi$), an increase in buyers perception of platform i's quality decreases platform i's equilibrium profits when the strategic effect dominates the direct effect on sellers' side and the combined direct and strategic effects on buyers' side.*

Proof: See Appendix C.8

The intuition of Corollary 3.2 is similar to the second result of Proposition 3.3. When the cross-group network effect exerted by sellers on buyers is stronger than the influence buyers have on sellers ($\upsilon > \pi$), an increase in buyers' perception of platform i's quality leads to a decrease in sellers' membership fees. This intensifies competition between the platforms as they strive to attract sellers before they commit to either platform, ultimately reducing the platforms' profits.

Impacts of switching cost

Moreover, we maximise Equation (3.15) with respect to switching cost s_b and obtain:

$$\frac{\partial \Pi^i}{\partial s_b} = p_b^i \frac{\partial N_b^i}{\partial s_b} + N_b^i \frac{\partial p_b^i}{\partial s_b} + p_s^i \frac{\partial N_s^i}{\partial s_b} + N_s^i \frac{\partial p_s^i}{\partial s_b} \tag{3.17}$$

As in the impacts of buyers' perception of platform i's quality, we also observe two distinct effects on each side of the market. Firstly, we refer to the direct effect as $\frac{\partial N_k^i}{\partial s_b}$ for $k = b, s$, which quantifies the direct influence of a change in switching cost on market shares. Secondly, we refer to the strategic effect as $\frac{\partial p_k^i}{\partial s_b}$ for $k = b, s$, which captures the strategic response of platform i to changes in buyers' switching cost on membership fees.

To gain deeper insights into the dominance of one effect over the other and their respective implications for profits, we present the following propositions.

Proposition 3.4. *When buyers underestimate platform i's quality (quality realisation is above the expected quality) and the cross-group network effect exerted by buyers on sellers is stronger than the influence sellers have on buyers ($\pi > \upsilon$) and $1 < \frac{\pi(\pi+2\upsilon)}{3}$, an increase in increase in switching cost:*

1. *Increases platform i's equilibrium profits when the direct effect dominates the strategic effect on buyers' sides of the market.*

2. *Decreases platform i's equilibrium profits when the strategic effect dominates the direct effect on buyers' side and the combined direct and strategic effects on sellers' side.*

Proof: See Appendix C.8

We find on Proposition 3.4 that an increase in switching costs has a significant impact on the composition of market shares from the perspective of platform i. Initially, there is an increase in the proportion of participants, but this is followed by a subsequent decrease in the next period. In this scenario, buyers tend to remain loyal to their chosen platform, leading the platform to implement a pricing strategy that rewards this loyalty. This strategy involves reducing fees on buyers side. As a result, it incentivises more buyers and sellers (given cross-group network effects) to join the platform, leading to an expansion of the user base. This produces an increase in equilibrium profits when the direct effect dominates the strategic effect on buyers' side. However, when the strategic effect dominates the direct effect on buyers' side and the combined direct and strategic effects on sellers' side, platform i experiences a decrease in equilibrium profits. The intuition for this result is similar as in Proposition 3.3.

Corollary 3.3. *When the cross-group network effect exerted by sellers on buyers is stronger than the influence buyers have on sellers ($\upsilon > \pi$), an increase in switching cost decreases platform i's equilibrium profits when the strategic effect dominates the direct effect on sellers' side and the combined direct and strategic effects on buyers' side.*

Proof: See Appendix C.8

Conversely, when the cross-group network effect exerted by sellers on buyers is stronger than the influence buyers have on sellers ($\upsilon > \pi$), platform i adjusts its strategy accordingly. As platform i primarily focuses on providing quality on buyers' side, it decides to lower sellers' fees to attract them directly, creating an intensified competition between platforms because they strive to attract sellers before they commit to either platform, ultimately reducing the platforms' profits.

3.6 Conclusions

Building upon Armstrong (2006) in this study we make a significant contribution to the literature on two-sided markets (Armstrong (2006); Lam (2017)) by introducing several novel elements. Firstly, our study incorporates the interplay between cross-group network effects, quality uncertainty, and switching costs, which has been relatively underexplored in previous research. By considering these factors within a dynamic platform competition setting, we provide a more comprehensive understanding of the dynamics at play in two-sided markets.

Furthermore, our findings give insight into the determinants of buyers' switching decisions. When buyers have higher expectations of the platform's quality compared to the actual quality realisation, they are more likely to switch to an alternative provider, taking into account the size of switching costs involved. On the contrary, when buyers underestimate the quality of the platform they joined in the first period, they prefer to stay with the same provider in the second period.

The interplay between cross-group network effects, quality uncertainty, and switching costs plays a crucial role in shaping buyers' preferences and their propensity to switch between platforms. Our model captures this intricate relationship and demonstrates how these factors collectively influence the platforms' strategy to determine their fees.

When buyers underestimate the platform quality, they choose to remain with their current provider, conditional on the size of the switching cost. To reward this loyalty, the platform refrains from raising its fees. This creates a positive feedback loop that attracts more buyers and sellers (given the cross-group network effects), leading to an expansion of its user base.

Conversely, when the influence of sellers on buyers surpasses the impact of buyers on sellers, the pricing policy aligns with the conventional findings in the two-sided market literature, such as Armstrong (2006); Jullien et al. (2021). In such cases, the platform adopts a subsidy approach, providing reduced membership fees for the side of the market that holds a greater impact over the other side. This adjustment in pricing strategy makes the platform charge a lower fee on sellers' side.

To understand how platform profits are affected by buyers' perceptions of platform quality and switching costs, we analyse a simplified profit function. This approach isolates and evaluates two key factors: the direct effect, which captures changes in market shares, and the strategic effect, which represents fee adjustments. By comparing the magnitudes of these effects, we can determine which factor is more influential and thereby assess the overall impact on the platform's profit.

When buyers underestimate the quality of the platform visited in the first period and the cross-group network effect exerted by buyers on sellers is stronger than the influence sellers have on buyers. We find that an increase in buyers' perception of the quality of the platform they visited in the first period or an increase in the switching cost, increases equilibrium profits when the direct effect dominates the strategic effect on buyers' side. On the contrary, when the strategic effect dominates the direct effect on buyers' side and the combined direct and strategic effects on sellers' side, platform profits decrease.

This outcome is the consequence of intensified competition between both platforms. Since buyers prefer to stick with the provider they initially chose, platforms lower their fees to try to attract these buyers and encourage them to switch. This escalates the competition and ultimately reduces the platforms' profits.

Finally, an increase in switching costs has a significant impact on the composition of market shares from the perspective of the platform. Initially, there is an increase in the proportion of participants, but this is followed by a subsequent decrease in the next period. In this scenario, buyers tend to remain loyal to their chosen platform, leading the platform to implement a pricing strategy that rewards this loyalty. This strategy involves reducing fees on buyers side. As a result, it incentivises more buyers and sellers (given cross-group network effects) to join the platform, leading to an expansion of the user base. This produces an increase in equilibrium profits when the direct effect dominates the strategic effect on buyers' side. However, when the strategic effect dominates the direct effect on buyers' side and the combined direct and strategic effects on sellers' side, the platform experiences a decrease in equilibrium profits.

We have established that buyers might switch to a different platform if they overestimate the quality of the platform they used in the first period, depending on the magnitude of switching costs. Conversely, they might stay with the same provider if they underestimate the quality. This scenario can disincentivise platforms from improving quality to attract new buyers, as switching costs might be the primary factor driving buyer decisions rather than perceived quality. Policymakers need to consider the interplay between switching costs and quality uncertainty when regulating these markets.

Additionally, policymakers often aim to determine whether switching costs raise or lower firms' profits to decide on regulatory measures (See Section 2.9 in Farrell and Klemperer (2007)). Our findings indicate that platform profits decrease when the strategic effect outweighs the direct effect on the buyers' side and the combined direct and strategic effects on the sellers' side. Moreover, when the cross-group network effect exerted by sellers on buyers is stronger than the influence buyers have on sellers, an increase in switching costs also decreases platform profits if the strategic effect dominates the direct effect on sellers' side and the combined direct and strategic effects on buyers' side. Therefore, policymakers must consider the magnitude of cross-group network effects in markets with switching costs when analysing two-sided markets, before deciding whether or not to regulate them.

In this study, the focus is primarily on analysing the effects of buyers' switching decisions and their impact on platform dynamics. However, a potential path for future research would be to extend the analysis to include the possibility of sellers switching platforms in response to buyers' movements. By allowing sellers to switch, we could explore how their decisions and market shares are affected, providing a more comprehensive understanding of the interactions and dynamics within the two-sided market.

Additionally, another possible extension may look into defining a framework where platforms possess incomplete information about buyers' switching decisions, leading to an asymmetry of information between platforms and buyers. This scenario would introduce an additional layer of complexity, as platforms would need to make pricing decisions without complete knowledge of how buyers' switching decisions are taken. Exploring the implications of this information asymmetry on platform behaviour and outcomes could provide valuable insights into real-world market dynamics.

Moreover, an interesting approach would also be to develop a model where platforms learn about buyers' switching decisions over time and can dynamically adjust their pricing strategies accordingly. This adaptive learning framework would capture the evolving dynamics between switching decisions and pricing strategies and enhance platform performance in two-sided markets.

Conclusion

This thesis provides novel frameworks to address three different scenarios in two-sided markets. In Chapter 1 we have developed a model to incorporate a direct network effect on both buyers' and sellers' sides. We examine the impact of a positive direct-network effect or bandwagon behaviour, as well as a negative direct externality or snob/congestion conduct on buyers' sides. Furthermore, we include the impact of seller competition on the other side of the market. These interactions are analysed across four distinct environments, including singlehoming for both sides, multihoming for sellers and singlehoming for buyers, multihoming for buyers and singlehoming for sellers, and multihoming for both sides. We find that in all scenarios, the interplay between direct-network effects on both sides and cross-group network effects alters the primary results of both Armstrong (2006) and Belleflamme and Peitz (2019b).

We found that when sellers engage in multihoming and buyers choose to singlehome, buyers benefit more than they would in a single platform environment. This is due to the bandwagon effect that creates a positive feedback loop leading to lower fees for buyers and an increase in their overall surplus. Platforms prefer sellers multihome and buyers singlehome because their profits are higher as long as the cross-group network effect sellers exert on buyers is larger than the effect buyers impact on sellers.

Furthermore, when buyers adopt a multihoming strategy while sellers choose to singlehome, sellers enjoy the benefits of a lower fee and a higher aggregate surplus compared to when both parties singlehome. Platforms prefer buyers multihome and sellers singlehome because their profits are higher as long as the cross-group network effect buyers exert on sellers is larger than the effect sellers impact on buyers.

Moreover, when both buyers and sellers engage in multihoming, the aggregate surpluses are larger compared to scenarios where only buyers multihome while sellers singlehome or vice versa, despite the higher membership fees.

Platforms can achieve higher profits when both buyers and sellers engage in multihoming compared to the scenario where only one side of the market is multihoming or when both sides are singlehoming. This is attributed to three key factors. Firstly, both membership fees increase when participants engage in multihoming. Secondly, buyers' and sellers' market shares are greater when both agents multihome. Finally, there is a positive feedback loop that attracts more participants on both sides of the market. Consequently, platforms can charge an additional fee per additional agent who chooses to multihoming, thereby increasing their profits.

In Chapter 2 we have developed a novel two-stage model that incorporates vertical differentiation. We introduced platform attributes on buyers' side to account for the vertical dimension. In the first stage of the model, platforms selected the level of attributes they offer to buyers simultaneously. In the second stage, platforms simultaneously chose membership fees. The equilibrium membership fees, market shares, and profits were determined by the difference in attributes on buyers' side. Although the features were developed only on buyers' side, they also influenced decisions on sellers' side. As a result, we demonstrate that vertical differentiation allows for the existence of asymmetric platforms in equilibrium. Overall, our contribution is to provide a comprehensive model that captures the dynamics of competition in two-sided markets with vertical differentiation.

Our analysis showed platforms used attributes on buyers' side as the main trigger to adjust their strategies to appeal to agents and boost profits. We found that the more heterogeneous platforms are, the fewer attributes they develop on buyers' side. Whereas the more valuable platforms become given a stronger cross-group network effect, the more attributes are offered on buyers' side. This mechanism drives platforms to adjust equilibrium membership fees and profits. Our analysis also uncovers interesting insights into the impact of model parameters on equilibrium membership fees, which are contingent on the relative strength of cross-group network effects between the two sides of the market. By providing such granular insights, platforms design optimal pricing strategies in two-sided markets with attributes on buyers' side.

We identified optimal conditions under which platforms can maximise profits by simultaneously optimising product differentiation on the horizontal dimension and attributes on buyers' side on the vertical dimension, as observed in prior research by Garella and Lambertini (2014); Barigozzi and Ma (2018). Specifically, we find this strategy is optimal where the cross-group network effect exerted by sellers on buyers is more prominent than the impact buyers have on sellers. Furthermore, we derived conditions under which the optimal strategy is to maximise one dimension while minimising the other dimension to increase profits. This is observed in cases where the cross-group network effect buyers have on sellers outweighs the effect sellers have on buyers. These results are consistent with previous studies such as Economides (1989); Neven and Thisse (1989), and the generalised model of Irmen and Thisse (1998).

In Chapter 3 we significantly contributed to the existing literature such as Jullien et al.

(2021); Sanchez-Cartas and León (2021) on two-sided markets by introducing several innovative elements. Notably, we examine the intricate interplay between cross-group network effects, quality uncertainty, and switching costs, which have received limited attention in previous research. By incorporating these factors into a dynamic platform competition framework, we enhance our understanding of the underlying dynamics that drive two-sided markets.

Additionally, our analysis stressed the strategic implications for platforms aiming to maximise their profits. Given the interdependence of cross-group network effects, quality uncertainty, and switching costs, platforms must carefully consider these factors when devising their pricing strategies.

Our analysis showed the impact of changes in buyers' quality valuation and switching costs on equilibrium fees. Our findings, among others, revealed that when buyers have higher expectations of platform i's quality compared to the actual quality realisation, they are more likely to switch to an alternative provider, considering the size of switching costs involved. Conversely, when buyers underestimate the quality of the platform they visited in the first period, they are more likely to stay with it in the second period. In response to this behaviour, the platform adjusts its pricing strategy by rewarding their loyalty with a decreased membership fee, compensating for this by increasing the fees charged to sellers.

Furthermore, when the influence of sellers on buyers surpasses the impact of buyers on sellers, the pricing policy aligns with the conventional findings in the two-sided market literature. In such cases, the platform adopts a subsidy approach, providing reduced membership fees for the side of the market that holds a greater impact over the other side. This adjustment in pricing strategy reflects the platform's acknowledgement of the evolving dynamics of cross-group network effects. As a result, more sellers are attracted to the platform, which in turn draws more buyers as a result of the cross-group network effect. This initiates a positive feedback loop that boosts participation on both sides of the market. This growth enhances the platform's attractiveness and benefits to its users.

Finally, we observe two distinct effects on the platform's profits based on buyers' perceptions of platform quality and changes in switching costs. The first is a direct effect, which quantifies the direct impact on market shares. The second is a strategic effect, which captures the influence on membership fees. There is an increase in the platform's profits when the direct effect dominates the strategic effect on buyers' sides of the market, whenever there is an increase in buyers' perception of the platform's quality. Contrary, there is a decrease when the strategic effect dominates the direct effect on buyers' side and the combined direct and strategic effects on sellers' side. The effects of an increase in switching costs have the same impacts on the platform's profits.

Appendix A

Appendix: Chapter 1

A.1 Model Assumptions

In this section, we show how the model assumptions are defined.

Second-order conditions

The second-order conditions of the platform maximisation problem guarantee a unique equilibrium in which both platforms are active.

Considering we have four different scenarios the second-order conditions change according to the setting we are analysing. When both buyers and sellers singlehome, as in Section 1.3 the conditions developed at Appendix A.2.3 for the second order conditions of the platform maximisation problem to be satisfied are (i) $\tau_b - \alpha > 0$, (ii) $(\tau_b - \alpha)(\tau_s + \beta) > \pi\upsilon$ and (iii) $4(\tau_b - \alpha)(\tau_s + \beta) > (\pi + \upsilon)^2$.

We can compare the stringency of the three conditions as follows. First, we compare conditions (i) and (ii), making the left side of both inequalities equal to compare the right side and identify which is larger. That is, (i) $\tau_b > \alpha$ and (ii) $\tau_b > \alpha + \frac{\pi\upsilon}{(\tau_s+\beta)}$, then since $\alpha + \frac{\pi\upsilon}{(\tau_s+\beta)} > \alpha$ because $\frac{\pi\upsilon}{(\tau_s+\beta)} > 0$ condition (ii) is more restrictive than condition (i). Now we compare conditions (ii) and (iii) doing the same process as before, (ii) $(\tau_b - \alpha)(\tau_s + \beta) > \pi\upsilon$ and (iii) $(\tau_b - \alpha)(\tau_s + \beta) > \frac{(\pi+\upsilon)^2}{4}$, then comparing the right side $\frac{(\pi+\upsilon)^2}{4} > \pi\upsilon$ which simplifies to $\pi^2 + 2\pi\upsilon + \upsilon^2 > 4\pi\upsilon$ and then to $(\pi - \upsilon)^2 > 0$ if $\pi \neq \upsilon$. Condition (iii) is more restrictive than condition (ii). Therefore if condition (iii) holds, conditions (i) and (ii) are satisfied.

Next, when sellers multihome and buyers singlehome, as in Section 1.4 the conditions developed in Appendix A.3.3 for the second order conditions of the platform maximisation problem to be satisfied are (i) $2(\tau_b - \alpha)(\tau_s + \beta) > \pi\upsilon$, (ii) $(\tau_b - \alpha)(\tau_s + \beta) > \pi\upsilon$ and

$(iii)\ 8\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)>\left(\pi+\upsilon\right)^2+4\pi\upsilon$.

We can compare which condition is more restrictive by following the same method as before. First, we compare conditions (i) and (ii) by making the left side of both inequalities equal to compare the right side and identify which is larger. That is (i) $\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)>\frac{\pi\upsilon}{2}$ and (ii) $\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)>\pi\upsilon$. Since $\pi\upsilon>\frac{\pi\upsilon}{2}$, then condition (ii) is more stringent than condition (i). Additionally, condition (iii) is more stringent than condition (ii) because $\frac{(\pi+\upsilon)^2+4\pi\upsilon}{8}>\pi\upsilon$ turns to $\pi^2+2\pi\upsilon+\upsilon^2+4\pi\upsilon>8\pi\upsilon$ and then simplifies to $\left(\pi-\upsilon\right)^2>0$ when $\pi\neq\upsilon$. Therefore if condition (iii) holds, conditions (i) and (ii) are satisfied.

Next, when buyers multihome and sellers singlehome, as in Section 1.5 the conditions developed in Appendix A.4.3 for the second order conditions of the platform maximisation problem to be satisfied are (i) $\tau_b-\alpha>0$, (ii) $2\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)>\pi\upsilon$, (iii) $\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)>\pi\upsilon$ and (iv) $8\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)>\left(\pi+\upsilon\right)^2+4\pi\upsilon$.

As was seen previously, when we compare the stringency of the four conditions we observed that condition (i) is less restrictive than condition (ii), which is in turn less restrictive than condition (iii), which is in turn less restrictive than condition (iv). Therefore if condition (iv) holds, conditions (i), (ii) and (iii) are satisfied.

Finally, when buyers and sellers multihome, as in Section 1.6 the conditions developed in Appendix A.5.3 for the second order conditions of the platform maximisation problem to be satisfied are (i) $\tau_b>\alpha$ for $\alpha>0$, (ii) $\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)>\pi\upsilon$ and (iii) $4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)>\left(\pi+\upsilon\right)^2$. These conditions are the same as when both buyers and sellers singlehome. Then, if condition (iii) holds, conditions (i) and (ii) are satisfied.

Now, we can compare which condition is more stringent between (i) $4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)>\left(\pi+\upsilon\right)^2$ and (ii) $8\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)>\left(\pi+\upsilon\right)^2+4\pi\upsilon$ by making the left side of both inequalities equal to compare the right side and identify which is larger. That is (i) $\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)>\frac{(\pi+\upsilon)^2}{4}$ and (ii) $\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)>\frac{(\pi+\upsilon)^2+4\pi\upsilon}{8}$. Comparing the right side, $\frac{(\pi+\upsilon)^2}{4}>\frac{(\pi+\upsilon)^2+4\pi\upsilon}{8}$ which turns to $2\left(\pi+\upsilon\right)^2>\left(\pi+\upsilon\right)^2+4\pi\upsilon$ and then simplifies to $\pi^2+2\pi\upsilon+\upsilon^2>4\pi\upsilon$ and finally to $\left(\pi-\upsilon\right)^2>0$ when $\pi\neq\upsilon$, condition (i) is more stringent than condition (ii).

$\therefore$ Therefore, by establishing assumption $4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)>\left(\pi+\upsilon\right)^2$ we guarantee that all previous conditions are met.

Full participation on both sides of the market

Full participation on both sides of the market means the indifferent buyer and seller must have a positive net surplus at equilibrium.

Both buyers and sellers singlehome

On buyers' side, we have condition $\nu_b - \frac{\tau_b}{2} > 0$. By using Equation 1.1a, we derive $R_b + \upsilon\eta_s + \alpha\eta_b - p_b - \frac{\tau_b}{2} > 0$. Using equilibrium membership fees as defined in Equation 1.8a, and given that platforms set identical fees at equilibrium, the indifferent participants on both sides are located at $\eta_b = \eta_s = \frac{1}{2}$. Consequently, we obtain: $R_b + \frac{\upsilon}{2} + \frac{\alpha}{2} - [f_b + (\tau_b - \alpha) - \pi] - \frac{\tau_b}{2} > 0$ which turns to $2(R_b - f_b) > 3(\tau_b - \alpha) - (\upsilon + 2\pi)$.

On sellers' side, we have condition $\nu_s - \frac{\tau_s}{2} > 0$. By using Equation 1.1b, we derive $R_s + \pi\eta_b - \beta\eta_s - p_s - \frac{\tau_s}{2} > 0$. Using equilibrium membership fees as defined in Equation 1.8b, and given that platforms set identical fees at equilibrium, the indifferent participants on both sides are located at $\eta_b = \eta_s = \frac{1}{2}$. Consequently, we obtain: $R_s + \frac{\pi}{2} - \frac{\beta}{2} - [f_s + (\tau_s + \beta) - \pi] - \frac{\tau_s}{2} > 0$ which turns to $2(R_s - f_s) > 3(\tau_s + \beta) - (2\upsilon + \pi)$.

Now, we can establish conditions $2(R_b - f_b) > 3(\tau_b - \alpha)$ and $2(R_s - f_s) > 3(\tau_s + \beta)$, which are more general than the preceding ones. This implies that if $2(R_b - f_b) > 3(\tau_b - \alpha)$ is met, then $2(R_b - f_b) > 3(\tau_b - \alpha) - (\upsilon + 2\pi)$ is satisfied. Furthermore, should $2(R_s - f_s) > 3(\tau_s + \beta)$ be satisfied, then $2(R_s - f_s) > 3(\tau_s + \beta) - (2\upsilon + \pi)$ holds true.

Sellers multihome and buyers singlehome

On buyers' side, we have condition $\nu_b - \frac{\tau_b}{2} > 0$. By using Equation 1.1a, we derive $R_b + \upsilon\eta_s + \alpha\eta_b - p_b - \frac{\tau_b}{2} > 0$. Using equilibrium membership fees as defined in Equation 1.16a, and buyers and sellers participation defined as $\eta_b = \frac{1}{2}$ and η_s in Equation 1.17. Consequently, we obtain: $R_b + \frac{\upsilon}{4(\tau_s+\beta)}[2(R_s - f_s) + (\pi + \upsilon)] + \frac{\alpha}{2} - \Big[f_b + (\tau_b - \alpha) - \frac{\pi}{4(\tau_s+\beta)}[(\pi + 3\upsilon) + 2(R_s - f_s)]\Big] - \frac{\tau_b}{2} > 0$ which turns to $4(R_b - f_b)(\tau_s + \beta) + 2(R_s - f_s)(\upsilon + \pi) > 6(\tau_b - \alpha)(\tau_s + \beta) - \big[(\upsilon + \pi)^2 + 2\pi\upsilon\big]$.

On sellers' side, we have condition $\nu_s - \tau_s > 0$. By using Equation 1.2b, we derive $2[R_s + \pi\eta_b - \beta\eta_s - p_s] - \tau_s > 0$. Using equilibrium membership fees as defined in Equation 1.16b, and buyers and sellers participation defined as $\eta_b = \frac{1}{2}$ and η_s in Equation 1.17. Consequently, we obtain: $2\Big[R_s + \frac{\pi}{2} - \frac{\beta}{4(\tau_s+\beta)}[2(R_s - f_s) + (\pi + \upsilon)] - \big[\frac{1}{2}(R_s + f_s) + \frac{1}{4}(\pi - \upsilon)\big]\Big] - \tau_s > 0$ which turns to $\frac{2}{4(\tau_s+\beta)}\Big[4(\tau_s + \beta)R_s + 2(\tau_s + \beta)\pi - 2(R_s - f_s)\beta - \beta(\pi + \upsilon) - 2(\tau_s + \beta)(R_s + f_s) - (\tau_s + \beta)(\pi - \upsilon)\Big] - \tau_s > 0$ and turns to $\frac{2}{4(\tau_s+\beta)}\big[2(\tau_s + \beta)(R_s - f_s) - 2(R_s - f_s)\beta + (\tau_s + \beta)(\pi + \upsilon) - \beta(\pi + \upsilon)\big] - \tau_s > 0$ which converts to $\frac{2\tau_s}{4(\tau_s+\beta)}\big[2(R_s - f_s) + (\pi + \upsilon)\big] > \tau_s$ and finally turns to $2(R_s - f_s) > 2(\tau_s + \beta) - (\pi + \upsilon)$

Now, we can establish conditions $4(R_b - f_b)(\tau_s + \beta) + 2(R_s - f_s)(\upsilon + \pi) > 6(\tau_b - \alpha)(\tau_s + \beta)$ and $(R_s - f_s) > (\tau_s + \beta)$, which are more general than the preceding ones. This implies that if $4(R_b - f_b)(\tau_s + \beta) + 2(R_s - f_s)(\upsilon + \pi) > 6(\tau_b - \alpha)(\tau_s + \beta)$ is met, then $4(R_b-f_b)(\tau_s+\beta)+2(R_s-f_s)(\upsilon+\pi)>6(\tau_b-\alpha)(\tau_s+\beta)-\left[(\upsilon+\pi)^2+2\pi\upsilon\right]$ is satisfied. Furthermore, if $(R_s-f_s)>(\tau_s+\beta)$ holds, $2(R_s-f_s)>2(\tau_s+\beta)-(\pi+\upsilon)$ is satisfied.

Buyers multihome and sellers singlehome

On buyers' side, we have condition $\nu_b-\tau_b>0$. By using Equation 1.2a, we derive $2[R_b+\upsilon\eta_s+\alpha\eta_b-p_b]-\tau_b>0$. Using equilibrium membership fees as defined in Equation 1.26a, and buyers and sellers participation defined as $\eta_s=\frac{1}{2}$ and η_b in Equation 1.27. Consequently, we obtain: $2\Big[R_b+\frac{\upsilon}{2}-\frac{\alpha}{4(\tau_b-\alpha)}[2(R_b-f_b)+(\pi+\upsilon)]-\left[\frac{1}{2}(R_b+f_b)+\frac{1}{4}(\upsilon-\pi)\right]\Big]-\tau_b>0$ which turns to $\frac{2}{4(\tau_b-\alpha)}\Big[4(\tau_b-\alpha)R_b+2(\tau_b-\alpha)\upsilon+2(R_b-f_b)\alpha+\alpha(\pi+\upsilon)-2(\tau_b-\alpha)(R_b+f_b)-(\tau_b-\alpha)(\upsilon-\pi)\Big]-\tau_b>0$ and turns to $\frac{2}{4(\tau_b-\alpha)}\big[2(\tau_b-\alpha)(R_b-f_b)+2(R_b-f_b)\alpha+(\tau_b-\alpha)(\pi+\upsilon)+\alpha(\pi+\upsilon)\big]-\tau_b>0$ and then converts to $\frac{2\tau_b}{4(\tau_b-\alpha)}\big[2(R_b-f_b)+(\pi+\upsilon)\big]>\tau_b$ and finally turns to $2(R_b-f_b)>2(\tau_b-\alpha)-(\pi+\upsilon)$

On sellers' side, we have condition $\nu_s-\frac{\tau_s}{2}>0$. By using Equation 1.1b, we derive $R_s+\pi\eta_b-\beta\eta_s-p_s-\frac{\tau_s}{2}>0$. Using equilibrium membership fees as defined in Equation 1.26b, and buyers and sellers participation defined as $\eta_s=\frac{1}{2}$ and η_s in Equation 1.27. Consequently, we obtain: $R_s+\frac{\pi}{4(\tau_b-\alpha)}[2(R_b-f_b)+(\pi+\upsilon)]+\frac{\beta}{2}-\Big[f_s+(\tau_s+\beta)-\frac{\upsilon}{4(\tau_b-\alpha)}[(\upsilon+3\pi)+2(R_b-f_b)]\Big]-\frac{\tau_s}{2}>0$ which turns to $4(R_s-f_s)(\tau_b-\alpha)+2(R_b-f_b)(\upsilon+\pi)>6(\tau_b-\alpha)(\tau_s+\beta)-\left[(\upsilon+\pi)^2+2\pi\upsilon\right]$.

Now, we can establish conditions $(R_b-f_b)>(\tau_b-\alpha)$ and $4(R_s-f_s)(\tau_b-\alpha)+2(R_b-f_b)(\upsilon+\pi)>6(\tau_b-\alpha)(\tau_s+\beta)$, which are more general than the preceding ones. This implies that if $(R_b-f_b)>(\tau_b-\alpha)$ holds $2(R_b-f_b)>2(\tau_b-\alpha)-(\pi+\upsilon)$ is satisfied. Additionally if $4(R_s-f_s)(\tau_b-\alpha)+2(R_b-f_b)(\upsilon+\pi)>6(\tau_b-\alpha)(\tau_s+\beta)$ holds $4(R_s-f_s)(\tau_b-\alpha)+2(R_b-f_b)(\upsilon+\pi)>6(\tau_b-\alpha)(\tau_s+\beta)-\left[(\upsilon+\pi)^2+2\pi\upsilon\right]$ is satisfied.

Buyers and sellers multihome

On buyers' side, we have condition $\nu_b-\tau_b>0$. By using Equation 1.2a, we derive $2[R_b+\upsilon\eta_s+\alpha\eta_b-p_b]-\tau_b>0$. Using equilibrium membership fees as defined in Equation 1.35a, and buyers and sellers participation defined in Equation 1.36a and Equation 1.36b. Consequently, we obtain: $\frac{2}{\gamma}\Big[R_b\gamma+\alpha\big[2(\tau_s+\beta)(R_b-f_b)+(\pi+\upsilon)(R_s-f_s)\big]+\upsilon\big[2(\tau_b-\alpha)(R_s-f_s)+(\pi+\upsilon)(R_b-f_b)\big]-\big[\left[2(\tau_b-\alpha)(\tau_s+\beta)-\pi\upsilon\right](R_b+f_b)-\left(\pi^2R_b+\upsilon^2f_b\right)+(\tau_b-\alpha)(R_s-f_s)(\upsilon-\pi)\big]\Big]-\tau_b>0$, where $\gamma\equiv4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2$ which turns to $2(\tau_b-\alpha)(\tau_s+\beta)(R_b-f_b)+2(\tau_s+\beta)(R_b-f_b)\alpha+(\pi+\upsilon)(R_s-f_s)\alpha+2(\tau_b-\alpha)(R_s-f_s)\upsilon-(\tau_b-\alpha)(R_s-f_s)(\upsilon-\pi)>\frac{\gamma\tau_b}{2}$ and the turns to $\tau_b(R_s-f_s)\big(\pi+$

$\upsilon) + 2\tau_b (\tau_s + \beta)(R_b - f_b) > \frac{\gamma\tau_b}{2}$ and then turns to $2(\tau_s + \beta)(R_b - f_b) + (R_s - f_s)(\pi + \upsilon) > \frac{\gamma}{2}$.

On sellers' side, we have condition $\nu_s - \tau_s > 0$. By using Equation 1.2b, we derive $2[R_s + \pi\eta_b - \beta\eta_s - p_s] - \tau_s > 0$. Using equilibrium membership fees as defined in Equation 1.35b, and buyers and sellers participation defined in Equation 1.36a and Equation 1.36b. Consequently, we obtain: $\frac{2}{\gamma}\big[R_s\gamma - \beta\big[2(\tau_b - \alpha)(R_s - f_s) + (\pi + \upsilon)(R_b - f_b)\big] + \pi\big[2(\tau_s + \beta)(R_b - f_b) + (\pi + \upsilon)(R_s - f_s)\big] - \big[\big[2(\tau_b - \alpha)(\tau_s + \beta) - \pi\upsilon\big](R_s + f_s) - \big(\upsilon^2 R_s + \pi^2 f_s\big) + (\tau_s + \beta)(R_b - f_b)(\pi - \upsilon)\big]\big] - \tau_s > 0$, where $\gamma \equiv 4(\tau_b - \alpha)(\tau_s + \beta) - (\pi + \upsilon)^2$ which turns to $2(\tau_b - \alpha)(\tau_s + \beta)(R_s - f_s) - 2(\tau_b - \alpha)(R_s - f_s)\beta - (\pi + \upsilon)(R_b - f_b)\beta + 2(\tau_s + \beta)(R_b - f_b)\pi - (\tau_s + \beta)(R_b - f_b)(\pi - \upsilon) > \frac{\gamma\tau_s}{2}$ and turns to $\tau_s(R_b - f_b)(\pi + \upsilon) + 2\tau_s(\tau_b - \alpha)(R_s - f_s) > \frac{\gamma\tau_s}{2}$ and finally turns to $2(\tau_b - \alpha)(R_s - f_s) + (R_b - f_b)(\pi + \upsilon) > \frac{\gamma}{2}$.

Now, we can establish conditions $4(\tau_s + \beta)(R_b - f_b) + 2(R_s - f_s)(\pi + \upsilon) > 4\big(\tau_b - \alpha\big)\big(\tau_s + \beta\big)$ and $4(\tau_b - \alpha)(R_s - f_s) + 2(R_b - f_b)(\pi + \upsilon) > 4(\tau_b - \alpha)(\tau_s + \beta)$ which are more general than the preceding ones. This implies that if $4(\tau_s + \beta)(R_b - f_b) + 2\big(R_s - f_s\big)\big(\pi + \upsilon\big) > 4(\tau_b - \alpha)(\tau_s + \beta)$ holds $2\big(\tau_s + \beta\big)(R_b - f_b) + (R_s - f_s)(\pi + \upsilon) > \frac{\gamma}{2}$ is satisfied. Additionally, if $4(\tau_b - \alpha)(R_s - f_s) + 2(R_b - f_b)(\pi + \upsilon) > 4(\tau_b - \alpha)(\tau_s + \beta)$ holds then $2(\tau_b - \alpha)(R_s - f_s) + (R_b - f_b)(\pi + \upsilon) > \frac{\gamma}{2}$ is satisfied.

The subsequent step involves defining a set of assumptions that capture all the previous conditions. This can be achieved by selecting the more stringent ones. Firstly, we compare conditions when both sides singlehome against conditions when sellers multihome and buyers singlehome.

On buyers' side $2(R_b - f_b) > 3(\tau_b - \alpha)$ vs $4(R_b - f_b)(\tau_s + \beta) + 2(R_s - f_s)(\upsilon + \pi) > 6(\tau_b - \alpha)(\tau_s + \beta)$. We can reorder the second condition to have $2(R_b - f_b) > \frac{1}{2(\tau_s + \beta)}\big[6\big(\tau_b - \alpha\big)(\tau_s + \beta) - 2(R_s - f_s)(\upsilon + \pi)\big]$. Now, given the right side of both conditions are the same, we compare the left side on both conditions to see which one is greater, where $3(\tau_b - \alpha) > \frac{6(\tau_b - \alpha)(\tau_s + \beta) - 2(R_s - f_s)(\upsilon + \pi)}{2(\tau_s + \beta)}$, turns to $6(\tau_b - \alpha)(\tau_s + \beta) > 6(\tau_b - \alpha)(\tau_s + \beta) - 2(R_s - f_s)\ \big(\upsilon + \pi\big)$ which simplifies to $2(R_s - f_s)\big(\upsilon + \pi\big) > 0$. Then if $2(R_b - f_b) > 3(\tau_b - \alpha)$ holds $4(R_b - f_b)\big(\tau_s + \beta\big) + 2(R_s - f_s)(\upsilon + \pi) > 6(\tau_b - \alpha)(\tau_s + \beta)$ is satisfied.

On sellers' side $2(R_s - f_s) > 3(\tau_s + \beta)$ vs $(R_s - f_s) > (\tau_s + \beta)$. We can reorder the second condition to have $2(R_s - f_s) > 2(\tau_s + \beta)$. Now we follow the same method to compare the right side on both conditions and see which is greater, where $3(\tau_s + \beta) > 2(\tau_s + \beta)$. Then if $2(R_s - f_s) > 3(\tau_s + \beta)$ holds $(R_s - f_s) > (\tau_s + \beta)$ is satisfied.

Secondly, we compare conditions when both sides singlehome against conditions when buyers multihome and sellers singlehome.

On buyers' side $2(R_b - f_b) > 3(\tau_b - \alpha)$ vs $(R_b - f_b) > (\tau_b - \alpha)$. We can reorder the second condition to have $2\left(R_b-f_b\right)>2\left(\tau_b-\alpha\right)$. Now we follow the same previous method to compare the right side on both conditions and see which is greater, where $3\left(\tau_b-\alpha\right)>2\left(\tau_b-\alpha\right)$. Then if $2\left(R_b-f_b\right)>3\left(\tau_b-\alpha\right)$ holds $\left(R_b-f_b\right)>\left(\tau_b-\alpha\right)$ is satisfied.

On sellers' side $2\left(R_s-f_s\right)>3\left(\tau_s+\beta\right)$ vs $4\left(R_s-f_s\right)\left(\tau_b-\alpha\right)+2\left(R_b-f_b\right)\left(\upsilon+\pi\right)>6\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)$. We can reorder the second condition to have $2\left(R_s-f_s\right)>\frac{1}{2(\tau_b-\alpha)}\Big[6\Big(\tau_b-\alpha\Big)\left(\tau_s+\beta\right)-2\left(R_b-f_b\right)\left(\upsilon+\pi\right)\Big]$. Now we compare the right side on both conditions, following the same method, where $3\left(\tau_s+\beta\right)>\frac{6(\tau_b-\alpha)(\tau_s+\beta)-2(R_b-f_b)(\upsilon+\pi)}{2(\tau_b-\alpha)}$ turns to $6\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)>6\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-2\left(R_b-f_b\right)\left(\upsilon+\pi\right)$ and simplifies to $2\Big(R_b-f_b\Big)\Big(\upsilon+\pi\Big)>0$. Then if $2\left(R_s-f_s\right)>3\left(\tau_s+\beta\right)$ holds $4\Big(R_s-f_s\Big)\Big(\tau_b-\alpha\Big)+2\left(R_b-f_b\right)\Big(\upsilon+\pi\Big)>6\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)$ is satisfied.

Finally, we compare conditions when both sides singlehome against conditions when both sides multihome.

On buyers' side $2\left(R_b-f_b\right)>3\left(\tau_b-\alpha\right)$ vs $4\left(\tau_s+\beta\right)\left(R_b-f_b\right)+2\left(R_s-f_s\right)\left(\pi+\upsilon\right)>4\Big(\tau_b-\alpha\Big)\Big(\tau_s+\beta\Big)$. We can reorder the second condition to have $2\left(R_b-f_b\right)>\frac{1}{2(\tau_s+\beta)}\Big[4\Big(\tau_b-\alpha\Big)\left(\tau_s+\beta\right)-2\left(R_s-f_s\right)\left(\pi+\upsilon\right)\Big]$. Now we compare the right side of both conditions, where $3\left(\tau_b-\alpha\right)>\frac{4(\tau_b-\alpha)(\tau_s+\beta)-2(R_s-f_s)(\pi+\upsilon)}{2(\tau_s+\beta)}$ which turns to $\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)+\Big(R_s-f_s\Big)\left(\pi+\upsilon\right)>0$. Then if $2\left(R_b-f_b\right)>3\left(\tau_b-\alpha\right)$ holds $4\left(\tau_s+\beta\right)\left(R_b-f_b\right)+2\Big(R_s-f_s\Big)\Big(\pi+\upsilon\Big)>4\Big(\tau_b-\alpha\Big)\Big(\tau_s+\beta\Big)$ is satisfied.

On sellers' side $2\left(R_s-f_s\right)>3\left(\tau_s+\beta\right)$ vs $4\left(\tau_b-\alpha\right)\left(R_s-f_s\right)+2\left(R_b-f_b\right)\left(\pi+\upsilon\right)>4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)$. We can reorder the second condition to have $2\left(R_s-f_s\right)>\frac{1}{2(\tau_b-\alpha)}\Big[4\Big(\tau_b-\alpha\Big)\left(\tau_s+\beta\right)-2\left(R_b-f_b\right)\left(\pi+\upsilon\right)\Big]$. Now we compare the right side on both conditions, similar as what we have been doing previously, where $3\left(\tau_s+\beta\right)>\frac{4(\tau_b-\alpha)(\tau_s+\beta)-2(R_b-f_b)(\pi+\upsilon)}{2(\tau_b-\alpha)}$ turns to $\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)+\left(R_b-f_b\right)\left(\pi+\upsilon\right)>0$. Then if $2\left(R_s-f_s\right)>3\left(\tau_s+\beta\right)$ holds $4\left(\tau_b-\alpha\right)\left(R_s-f_s\right)+2\left(R_b-f_b\right)\left(\pi+\upsilon\right)>4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)$ is satisfied.

$\therefore$ Therefore, by establishing assumptions $2\left(R_b-f_b\right)>3\left(\tau_b-\alpha\right)$ and $2\left(R_s-f_s\right)>3\left(\tau_s+\beta\right)$, we guarantee that all previous conditions are met.

Market shares

We impose that some buyers and sellers multihome at equilibrium, which implies that if some buyers and sellers multihome, then all buyers and all sellers participate (i.e., the ones that do not multihome, singlehome). This is the case for multihoming if $\frac{1}{2}<\eta_b<1$ and $\frac{1}{2}<\eta_s<1$.

Sellers multihome and buyers singlehome

We impose $\frac{1}{2} < \eta_s^{smh} < 1$. Using Equation 1.17 we have $\eta_s^{smh} = \frac{2(R_s - f_s) + (\pi + \upsilon)}{4(\tau_s + \beta)} > \frac{1}{2}$ turns to $2(R_s - f_s) > 2(\tau_s + \beta) - (\pi + \upsilon)$. For $\eta_s^{smh} = \frac{2(R_s - f_s) + (\pi + \upsilon)}{4(\tau_s + \beta)} < 1$ turns to $2(R_s - f_s) < 4(\tau_s + \beta) - (\pi + \upsilon)$. Then we can set a more general condition such as $(R_s - f_s) > (\tau_s + \beta)$ and $(R_s - f_s) < 2(\tau_s + \beta)$. If these more general conditions hold, then the previous ones are satisfied.

Next, we compare these conditions with the one established for sellers' side previously, which is $2(R_s - f_s) > 3(\tau_s + \beta)$ to define which one is more stringent.

We can redefine condition $(R_s - f_s) > (\tau_s + \beta)$ to be $2(R_s - f_s) > 2(\tau_s + \beta)$, then it is straightforward to notice that $3(\tau_s + \beta) > 2(\tau_s + \beta)$. Therefore if $2(R_s - f_s) > 3(\tau_s + \beta)$ holds $(R_s - f_s) > (\tau_s + \beta)$ is satisfied.

Now redefining $(R_s - f_s) < 2(\tau_s + \beta)$ to be $2(R_s - f_s) < 4(\tau_s + \beta)$ we can place $3(\tau_s + \beta) < 2(R_s - f_s) < 4(\tau_s + \beta)$ as our assumption on sellers' side.

Buyers multihome and sellers singlehome

We impose $\frac{1}{2} < \eta_b^{smh} < 1$. Using Equation 1.27 we have $\eta_b^{bmh} = \frac{2(R_b - f_b) + (\pi + \upsilon)}{4(\tau_b - \alpha)} > \frac{1}{2}$ turns to $2(R_b - f_b) > 2(\tau_b - \alpha) - (\pi + \upsilon)$. For $\eta_b^{bmh} = \frac{2(R_b - f_b) + (\pi + \upsilon)}{4(\tau_b - \alpha)} < 1$ turns to $2(R_b - f_b) < 4(\tau_b - \alpha) - (\pi + \upsilon)$. Then we can set a more general condition such as $(R_b - f_b) > (\tau_b - \alpha)$ and $(R_b - f_b) < 2(\tau_b - \alpha)$. If these more general conditions hold, then the previous ones are satisfied.

Next, we compare these conditions with the one established for buyers' side previously, which is $2(R_b - f_b) > 3(\tau_b - \alpha)$ to define which is more stringent.

We can redefine condition $(R_b - f_b) > (\tau_b - \alpha)$ to be $2(R_b - f_b) > 2(\tau_b - \alpha)$, then it is straightforward to notice that $3(\tau_b - \alpha) > 2(\tau_b - \alpha)$. Therefore if $2(R_b - f_b) > 3(\tau_b - \alpha)$ holds $(R_b - f_b) > (\tau_b - \alpha)$ is satisfied.

Now redefining $(R_b - f_b) < 2(\tau_b - \alpha)$ to be $2(R_b - f_b) < 4(\tau_b - \alpha)$ we can place $3(\tau_b - \alpha) < 2(R_b - f_b) < 4(\tau_b - \alpha)$ as our assumption on buyers' side.

Buyers and sellers multihome

For buyers' side we impose $\frac{1}{2} < \eta_b^{mh} < 1$. Using Equation 1.36a we have $\eta_b^{mh} = \frac{1}{\gamma}\left[2(\tau_s + \beta)\left(R_b - f_b\right) + (\pi + \upsilon)(R_s - f_s)\right] > \frac{1}{2}$ where $\gamma \equiv 4(\tau_b - \alpha)(\tau_s + \beta) - (\pi + \upsilon)^2$ turns to $2(\tau_s + \beta)\left(R_b - f_b\right) + (\pi + \upsilon)(R_s - f_s) > \frac{\gamma}{2}$. For $\eta_b^{mh} = \frac{1}{\gamma}\left[2(\tau_s + \beta)\left(R_b - f_b\right) + (\pi + \upsilon)(R_s - f_s)\right] < 1$ turns to $2(\tau_s + \beta)\left(R_b - f_b\right) + (\pi + \upsilon)(R_s - f_s) < \gamma$. Then we can set a more general condition such as $2(\tau_s + \beta)\left(R_b - f_b\right) + (\pi + \upsilon)(R_s - f_s) >$

$2(\tau_b-\alpha)(\tau_s+\beta)$ and $2(\tau_s+\beta)\big(R_b-f_b\big)+(\pi+\upsilon)(R_s-f_s)<4(\tau_b-\alpha)(\tau_s+\beta)$. If these more general conditions hold, then the previous ones are satisfied.

Next, we compare these conditions to the one set previously for buyers' side, which is $3(\tau_b-\alpha)<2(R_b-f_b)<4(\tau_b-\alpha)$.

First, we reorder condition $2(\tau_s+\beta)\big(R_b-f_b\big)+(\pi+\upsilon)(R_s-f_s)>2(\tau_b-\alpha)(\tau_s+\beta)$ to be $2(R_b-f_b)>\frac{1}{2(\tau_s+\beta)}\big[4(\tau_b-\alpha)(\tau_s+\beta)-2(\pi+\upsilon)(R_s-f_s)\big]$. Comparing the right side on both conditions, we can show that $3(\tau_b-\alpha)>\frac{4(\tau_b-\alpha)(\tau_s+\beta)-2(\pi+\upsilon)(R_s-f_s)}{2(\tau_s+\beta)}$, this turns to $6(\tau_b-\alpha)(\tau_s+\beta)>4(\tau_b-\alpha)(\tau_s+\beta)-2(\pi+\upsilon)(R_s-f_s)$ which is always positive because $2(\tau_b-\alpha)(\tau_s+\beta)+2(\pi+\upsilon)\big(R_s-f_s\big)>0$. Then if $2(R_b-f_b)>3(\tau_b-\alpha)$ holds, $2(\tau_s+\beta)\big(R_b-f_b\big)+(\pi+\upsilon)(R_s-f_s)>2(\tau_b-\alpha)(\tau_s+\beta)$ is satisfied.

Following, we reorder condition $2(\tau_s+\beta)\big(R_b-f_b\big)+(\pi+\upsilon)(R_s-f_s)<4(\tau_b-\alpha)(\tau_s+\beta)$ to be $2(R_b-f_b)<\frac{1}{(\tau_s+\beta)}\big[4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)(R_s-f_s)\big]$. We can show that $4(\tau_b-\alpha)>\frac{4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)(R_s-f_s)}{(\tau_s+\beta)}$, this turns to $4(\tau_b-\alpha)(\tau_s+\beta)>4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)(R_s-f_s)$ which is always positive because $(\pi+\upsilon)\big(R_s-f_s\big)>0$. Then if $2(R_b-f_b)<4(\tau_b-\alpha)$ holds, $2(\tau_s+\beta)\big(R_b-f_b\big)+(\pi+\upsilon)(R_s-f_s)<4(\tau_b-\alpha)(\tau_s+\beta)$ is satisfied.

For sellers' side we impose $\frac{1}{2}<\eta_s^{mh}<1$. Using Equation 1.36b we have $\eta_s^{mh}=\frac{1}{\gamma}\big[2(\tau_b-\alpha)\big(R_s-f_s\big)+(\pi+\upsilon)(R_b-f_b)\big]>\frac{1}{2}$ where $\gamma\equiv4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2$ turns to $2(\tau_b-\alpha)\big(R_s-f_s\big)+(\pi+\upsilon)(R_b-f_b)>\frac{\gamma}{2}$. For $\eta_s^{mh}=\frac{1}{\gamma}\big[2(\tau_b-\alpha)\big(R_s-f_s\big)+(\pi+\upsilon)(R_b-f_b)\big]<1$ turns to $2(\tau_b-\alpha)\big(R_s-f_s\big)+(\pi+\upsilon)(R_b-f_b)<\gamma$. Then we can set a more general condition such as $2(\tau_b-\alpha)\big(R_s-f_s\big)+(\pi+\upsilon)(R_b-f_b)>2(\tau_b-\alpha)(\tau_s+\beta)$ and $2(\tau_b-\alpha)\big(R_s-f_s\big)+(\pi+\upsilon)(R_b-f_b)<4(\tau_b-\alpha)(\tau_s+\beta)$. If these more general conditions hold, then the previous ones are satisfied.

Next, we compare these conditions to the one set previously for sellers' side, which is $3(\tau_s+\beta)<2(R_s-f_s)<4(\tau_s+\beta)$.

First, we reorder condition $2(\tau_b-\alpha)\big(R_s-f_s\big)+(\pi+\upsilon)(R_b-f_b)>2(\tau_b-\alpha)(\tau_s+\beta)$ to be $2(R_s-f_s)>\frac{1}{2(\tau_b-\alpha)}\big[4(\tau_b-\alpha)(\tau_s+\beta)-2(\pi+\upsilon)(R_b-f_b)\big]$. We can show that $3(\tau_s+\beta)>\frac{4(\tau_b-\alpha)(\tau_s+\beta)-2(\pi+\upsilon)(R_b-f_b)}{2(\tau_b-\alpha)}$, this turns to $6(\tau_b-\alpha)(\tau_s+\beta)>4(\tau_b-\alpha)(\tau_s+\beta)-2(\pi+\upsilon)(R_b-f_b)$ which is always positive because $2(\tau_b-\alpha)(\tau_s+\beta)+2(\pi+\upsilon)\big(R_b-f_b\big)>0$. Then if $2(R_s-f_s)>3(\tau_s+\beta)$ holds, $2(\tau_b-\alpha)\big(R_s-f_s\big)+(\pi+\upsilon)(R_b-f_b)>2(\tau_b-\alpha)(\tau_s+\beta)$ is satisfied.

Lastly, we reorder condition $2(\tau_b-\alpha)\big(R_s-f_s\big)+(\pi+\upsilon)(R_b-f_b)<4(\tau_b-\alpha)(\tau_s+\beta)$ to be $2(R_s-f_s)<\frac{1}{(\tau_b-\alpha)}\big[4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)(R_b-f_b)\big]$. We can show that $4(\tau_s+\beta)>\frac{4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)(R_b-f_b)}{(\tau_b-\alpha)}$, this turns to $4(\tau_b-\alpha)(\tau_s+\beta)>4(\tau_b-\alpha)(\tau_s+\beta)-2(\pi+\upsilon)(R_b-f_b)$ which is always positive because $(\pi+\upsilon)\big(R_b-f_b\big)>0$. Then if $2(R_s-f_s)<4(\tau_s+\beta)$ holds, $2(\tau_b-\alpha)\big(R_s-f_s\big)+(\pi+\upsilon)(R_b-f_b)<4(\tau_b-\alpha)(\tau_s+\beta)$

is satisfied.

$\therefore$ Therefore, by establishing assumptions $3(\tau_b - \alpha) < 2(R_b - f_b) < 4(\tau_b - \alpha)$ and $3(\tau_s + \beta) < 2(R_s - f_s) < 4(\tau_s + \beta)$, we guarantee that all previous conditions are met.

A.2 Two-sided Singlehome

A.2.1 Market's Shares

We compute buyers' and sellers' participation at Equation 1.3 from Equation 1.1a and Equation 1.1b. For buyers $\eta_b^i = \frac{1}{2} + \frac{\nu_b^i - \nu_b^j}{2\tau_b}$ turns to $\eta_b^i = \frac{1}{2} + \frac{1}{2\tau_b}\left[R_b + \upsilon\eta_s^i + \alpha\eta_b^i - p_b^i - \left(R_b + \upsilon\eta_s^j + \alpha\eta_b^j - p_b^j\right)\right]$ turns to $2\tau_b\eta_b^i = \tau_b + \upsilon\left(\eta_s^i - \eta_s^j\right) + \alpha\left(\eta_b^i - \eta_b^j\right) + \left(p_b^j - p_b^i\right)$. Then, since $\eta_b^i + \eta_b^j = 1$ and $\eta_s^i + \eta_s^j = 1$ we have $2\tau_b\eta_b^i = \tau_b + \upsilon\left(2\eta_s^i - 1\right) + \alpha\left(2\eta_b^i - 1\right) + \left(p_b^j - p_b^i\right)$ and turns to $\eta_b^i = \frac{\tau_b + \left(2\eta_s^i - 1\right)\upsilon - \alpha + \left(p_b^j - p_b^i\right)}{2(\tau_b - \alpha)}$.

For sellers $\eta_s^i = \frac{1}{2} + \frac{\nu_s^i - \nu_s^j}{2\tau_s}$ turns to $\eta_s^i = \frac{1}{2} + \frac{1}{2\tau_s}\left[R_s + \pi\eta_b^i - \beta\eta_s^i - p_s^i - \left(R_s + \pi\eta_b^j - \beta\eta_s^j - p_s^j\right)\right]$ turns to $2\tau_s\eta_s^i = \tau_s + \pi\left(\eta_b^i - \eta_b^j\right) - \beta\left(\eta_s^i - \eta_s^j\right) + \left(p_s^j - p_s^i\right)$. Then, since $\eta_b^i + \eta_b^j = 1$ and $\eta_s^i + \eta_s^j = 1$ we have $2\tau_s\eta_s^i = \tau_s + \pi\left(2\eta_b^i - 1\right) - \beta\left(2\eta_s^i - 1\right) + \left(p_s^j - p_s^i\right)$ and turns to $\eta_s^i = \frac{\tau_s + \left(2\eta_b^i - 1\right)\pi - \beta + \left(p_s^j - p_s^i\right)}{2(\tau_s + \beta)}$. Then we have:

$$\eta_b^i = \frac{\tau_b + \left(2\eta_s^i - 1\right)\upsilon - \alpha + \left(p_b^j - p_b^i\right)}{2(\tau_b - \alpha)} \tag{1}$$

$$\eta_s^i = \frac{\tau_s + \left(2\eta_b^i - 1\right)\pi + \beta + \left(p_s^j - p_s^i\right)}{2(\tau_s + \beta)} \tag{2}$$

We solve the previous system of equations to obtain η_b^i and η_s^i as a function of membership fees. First, we find the value of $(2\eta_s^i - 1)$ from equation (2) and substitute this value into equation 1 and then solve for η_b^i. That is, from equation (2) we have $2\eta_s^i - 1 = \frac{1}{(\tau_s + \beta)}\left[\left(2\eta_b^i - 1\right)\pi + \left(p_s^j - p_s^i\right)\right]$, then we substitute it in equation (1) $2(\tau_b - \alpha)\eta_b^i = (\tau_b - \alpha) + \left(p_b^j - p_b^i\right) + \frac{\upsilon}{\tau_s + \beta}\left[\left(2\eta_b^i - 1\right)\pi + \left(p_s^j - p_s^i\right)\right]$ turns to $2\left[(\tau_b - \alpha)(\tau_s + \beta) - \pi\upsilon\right]\eta_b^i = \left[(\tau_b - \alpha)(\tau_s + \beta) - \pi\upsilon\right] + (\tau_s + \beta)\left(p_b^j - p_b^i\right) + \upsilon\left(p_s^j - p_s^i\right)$. Then it turns to $\eta_b^i = \frac{1}{2} + \frac{\upsilon\left(p_s^j - p_s^i\right) + (\tau_s + \beta)\left(p_b^j - p_b^i\right)}{2[(\tau_s + \beta)(\tau_b - \alpha) - \pi\upsilon]}$. Then we substitute the previous result into equation (2) to get $\eta_s^i = \frac{1}{2} + \frac{\pi\left(p_b^j - p_b^i\right) + (\tau_b - \alpha)\left(p_s^j - p_s^i\right)}{2[(\tau_s + \beta)(\tau_b - \alpha) - \pi\upsilon]}$. The solution for the system of equations (1) and (2) are:

$$\eta_b^i = \frac{1}{2} + \frac{\upsilon\left(p_s^j - p_s^i\right) + (\tau_s + \beta)\left(p_b^j - p_b^i\right)}{2\left[(\tau_s + \beta)(\tau_b - \alpha) - \pi\upsilon\right]}$$
$$\eta_s^i = \frac{1}{2} + \frac{\pi\left(p_b^j - p_b^i\right) + (\tau_b - \alpha)\left(p_s^j - p_s^i\right)}{2\left[(\tau_s + \beta)(\tau_b - \alpha) - \pi\upsilon\right]}$$

A.2.2 Platform profit Maximisation

Platforms maximise the next expression concerning both sides' membership fees:

$$\max_{\{p_b^i, p_s^i\}} \Pi^i \equiv \left(p_b^i - f_b\right) \eta_b^i \left(p_b^i, p_s^i, p_b^j, p_s^j\right) + \left(p_s^i - f_s\right) \eta_s^i \left(p_b^i, p_s^i, p_b^j, p_s^j\right)$$

The first-order conditions are:

$$\frac{\partial \Pi^i}{\partial p_b^i} = \eta_b^i + \frac{\partial \eta_b^i}{\partial p_b^i}\left(p_b^i - f_b\right) + \frac{\partial \eta_s^i}{\partial p_b^i}\left(p_s^i - f_s\right) = 0$$

$$\frac{\partial \Pi^i}{\partial p_s^i} = \frac{\partial \eta_b^i}{\partial p_s^i}\left(p_b^i - f_b\right) + \eta_s^i + \frac{\partial \eta_s^i}{\partial p_s^i}\left(p_s^i - f_s\right) = 0$$

Using Equation 1.5a and Equation 1.5b the first-order conditions turn to:

$$\frac{\partial \Pi^i}{\partial p_b^i} = \frac{1}{2} + \frac{1}{2\Omega}\left[\upsilon\left(p_s^j - p_s^i\right) + \left(\tau_s + \beta\right)\left(p_b^j - p_b^i\right)\right] - \frac{\left(\tau_s + \beta\right)}{2\Omega}\left(p_b^i - f_b\right) - \frac{\pi}{2\Omega}\left(p_s^i - f_s\right) = 0$$
$$\frac{\partial \Pi^i}{\partial p_s^i} = \frac{1}{2} + \frac{1}{2\Omega}\left[\pi\left(p_b^j - p_b^i\right) + \left(\tau_b - \alpha\right)\left(p_s^j - p_s^i\right)\right] - \frac{\left(\tau_b - \alpha\right)}{2\Omega}\left(p_s^i - f_s\right) - \frac{\upsilon}{2\Omega}\left(p_b^i - f_b\right) = 0$$

Where $\Omega = \left[\left(\tau_s + \beta\right)\left(\tau_b - \alpha\right) - \pi\upsilon\right]$

From the first-order conditions for a symmetric equilibrium $p_b^i = p_b^j = p_b$ and $p_s^i = p_s^j = p_s$ we obtain:

$$= \frac{1}{2} - \frac{\left(\tau_s + \beta\right)}{2\left[\left(\tau_s + \beta\right)\left(\tau_b - \alpha\right) - \pi\upsilon\right]}\left(p_b - f_b\right) - \frac{\pi}{2\left[\left(\tau_s + \beta\right)\left(\tau_b - \alpha\right) - \pi\upsilon\right]}\left(p_s - f_s\right) = 0$$
$$= \frac{1}{2} - \frac{\left(\tau_b - \alpha\right)}{2\left[\left(\tau_s + \beta\right)\left(\tau_b - \alpha\right) - \pi\upsilon\right]}\left(p_s - f_s\right) - \frac{\upsilon}{2\left[\left(\tau_s + \beta\right)\left(\tau_b - \alpha\right) - \pi\upsilon\right]}\left(p_b - f_b\right) = 0$$

By solving the first equation for p_b and the subsequent equation for p_s, we establish the corresponding best response functions as:

$$p_b^{sh} = f_b + \tau_b - \alpha - \frac{\pi}{\tau_s + \beta}\left(\upsilon + p_s - f_s\right) \tag{3}$$

$$p_s^{sh} = f_s + \tau_s + \beta - \frac{\upsilon}{\tau_b - \alpha}\left(\pi + p_b - f_b\right) \tag{4}$$

The next step is to solve the previous system of equations (3) and (4) to have explicit expressions for the membership fees. First, we substitute p_s^{sh} from equation (4) into equation (3) and solve for p_b^{sh}. That is $\left(\tau_s + \beta\right) p_b^{sh} = \left(\tau_s + \beta\right) f_b + \left(\tau_b - \alpha\right)\left(\tau_s + \beta\right) - \pi\upsilon + \pi f_s - \pi\big[f_s + \tau_s + \beta - \frac{\upsilon}{\tau_b - \alpha}\left(\pi + p_b - f_b\right)\left(\tau_s + \beta\right)\big]$ and turns to $\big[\left(\tau_b - \alpha\right)\left(\tau_s + \beta\right) - \pi\upsilon\big] p_b = \big[\left(\tau_b - \alpha\right)\left(\tau_s + \beta\right) - \pi\upsilon\big] f_b + \left(\tau_b - \alpha\right)\big[\left(\tau_b - \alpha\right)\left(\tau_s + \beta\right) - \pi\upsilon\big] - \pi\big[\left(\tau_b - \alpha\right)\left(\tau_s + \beta\right) - \pi\upsilon\big]$

and the to $p_b^{sh} = f_b + \tau_b - \alpha - \pi$. Next, we substitute the previous result into equation (4) to have $p_s^{sh} = f_s + \tau_s + \beta - \upsilon$. Therefore, the solution for the system of equations (3) and (4) are:

$$p_b^{sh} = f_b + \tau_b - \alpha - \pi$$
$$p_s^{sh} = f_s + \tau_s + \beta - \upsilon$$

A.2.3 Second order conditions - Two-sided Singlehome

We obtain the following second-order conditions from the profit-maximisation problem of Appendix A.2.2 to define the Hessian matrix:

$$\mathrm{H} = \begin{pmatrix} \Pi^i_{p_b p_b} \equiv -\frac{(\tau_s+\beta)}{[(\tau_s+\beta)(\tau_b-\alpha)-\pi\upsilon]} & \Pi^i_{p_b p_s} \equiv -\frac{(\pi+\upsilon)}{2[(\tau_s+\beta)(\tau_b-\alpha)-\pi\upsilon]} \\ \Pi^i_{p_s p_b} \equiv -\frac{(\pi+\upsilon)}{2[(\tau_s+\beta)(\tau_b-\alpha)-\pi\upsilon]} & \Pi^i_{p_s p_s} \equiv -\frac{(\tau_b-\alpha)}{[(\tau_s+\beta)(\tau_b-\alpha)-\pi\upsilon]} \end{pmatrix}$$

In order to guarantee that platforms' profits reach a maximum with equilibrium fees in Equation 1.8a and Equation 1.8b a sufficient condition is having H negative definite, indicating that $|H| > 0$, and either $\Pi^i_{p_b p_b} < 0$ or $\Pi^i_{p_s p_s} < 0$. To show $\Pi^i_{p_b p_b}$ is negative, the denominator must be positive since the numerator is always positive. Thus, we have $(\tau_s + \beta)(\tau_b - \alpha) > \pi\upsilon$. To show $\Pi^i_{p_s p_s}$ is negative, the numerator must be positive as previously has been shown that for $\Pi^i_{p_b p_b} < 0$, $(\tau_s + \beta)(\tau_b - \alpha) - \pi\upsilon > 0$, then $\tau_b - \alpha > 0$. Finally, to show $|H| > 0$, we have $\frac{(\tau_s+\beta)(\tau_b-\alpha)}{[(\tau_s+\beta)(\tau_b-\alpha)-\pi\upsilon]^2} - \frac{(\pi+\upsilon)^2}{4[(\tau_s+\beta)(\tau_b-\alpha)-\pi\upsilon]^2} > 0$, and turns to $\frac{4(\tau_s+\beta)(\tau_b-\alpha)-(\pi+\upsilon)^2}{4[(\tau_s+\beta)(\tau_b-\alpha)-\pi\upsilon]^2} > 0$. The previous expression to be positive, given the denominator is always positive is to have the numerator positive. That is $4(\tau_b - \alpha)(\tau_s + \beta) > (\pi + \upsilon)^2$

In summary, for the second-order conditions defined by the Hessian matrix to be negative definite, the following conditions must hold: (i) $\tau_b - \alpha > 0$, (ii) $(\tau_b - \alpha)(\tau_s + \beta) > \pi\upsilon$, and (iii) $4(\tau_b - \alpha)(\tau_s + \beta) > (\pi + \upsilon)^2$.

Now, we show which of the three conditions is the most stringent, ensuring that the rest of the conditions are also satisfied as long as it is met. First, we compare conditions (i) and (ii), making the left side of both inequalities equal to compare the right side and identify which is larger. That is, (i) $\tau_b > \alpha$ and (ii) $\tau_b > \alpha + \frac{\pi\upsilon}{(\tau_s+\beta)}$, then since $\alpha + \frac{\pi\upsilon}{(\tau_s+\beta)} > \alpha$ because $\frac{\pi\upsilon}{(\tau_s+\beta)} > 0$ condition (ii) is more restrictive than condition (i). Now we compare conditions (ii) and (iii) using the same method as comparing conditions (i) and (ii). Next, we compare the right side of conditions (ii) $(\tau_b - \alpha)(\tau_s + \beta) > \pi\upsilon$ and (iii) $(\tau_b - \alpha)(\tau_s + \beta) > \frac{(\pi+\upsilon)^2}{4}$ obtaining $\frac{(\pi+\upsilon)^2}{4} > \pi\upsilon$, which simplifies to $\pi^2 + 2\pi\upsilon + \upsilon^2 > 4\pi\upsilon$ and finally to $(\pi - \upsilon)^2 > 0$ when $\pi \neq \upsilon$. Then, condition (iii) is more restrictive than condition (ii). Therefore if condition (iii) holds, conditions (i) and (ii) are satisfied.

A.2.4 Aggregate surpluses

We compute Consumer and Producer Surpluses by considering buyers and seller surplus at Equation 1.10a and Equation 1.10b and the transportation cost both participants face. The transportation cost is the area under the unit interval of joining platform one or two. Given both platforms are symmetric, we get:

$$
\begin{aligned}
CS^{sh} = \nu_b^{sh} - 2\int_0^{1/2} \tau_b x_b dx_b &= \nu_b^{sh} - 2\tau_b \left(\frac{1}{2} x_b^2 \Big|_0^{1/2} \right) = \nu_b^{sh} - \frac{1}{4}\tau_b \\
PS^{sh} = \nu_s^{sh} - 2\int_0^{1/2} \tau_s x_s dx_s &= \nu_s^{sh} - 2\tau_s \left(\frac{1}{2} x_s^2 \Big|_0^{1/2} \right) = \nu_s^{sh} - \frac{1}{4}\tau_s
\end{aligned}
$$

Using Equation 1.10a and Equation 1.10b we get:

$$
\begin{aligned}
CS^{sh} &= (R_b - f_b) - \frac{5}{4}\tau_b + \frac{1}{2}\upsilon + \pi + \frac{3}{2}\alpha \\
PS^{sh} &= (R_s - f_s) - \frac{5}{4}\tau_s + \frac{1}{2}\pi + \upsilon - \frac{3}{2}\beta
\end{aligned}
$$

A.2.5 Comparative Statics

Membership fees

Partially differentiate equilibrium membership fees at Equation 1.8a and Equation 1.8b regarding the parameters of the model. On buyers' side, when there is a bandwagon effect $\partial p_b^{sh}/\partial\alpha = -1 < 0$ and when there is a congestion effect $\partial p_b^{sh}/\partial\alpha = 1 > 0$. $\partial p_b^{sh}/\partial\pi = -1 < 0$. On sellers' side, $\partial p_s^{sh}/\partial\beta = 1 > 0$, $\partial p_s^{sh}/\partial\upsilon = -1 < 0$.

Aggregate Surplus

Partially differentiate equilibrium aggregate surpluses at Equation 1.11a and Equation 1.11b regarding the parameters of the model. $\partial CS^{sh}/\partial\alpha = \frac{3}{2} > 0$ when there is a bandwagon effect and $\partial CS^{sh}/\partial\alpha = -\frac{3}{2} < 0$ when there is a congestion effect, $\partial CS^{sh}/\partial\upsilon = \frac{1}{2} > 0$, $\partial CS^{sh}/\partial\pi = 1 > 0$, $\partial PS^{sh}/\partial\pi = \frac{1}{2} > 0$, $\partial PS^{sh}/\partial\upsilon = 1 > 0$ and $\partial PS^{sh}/\partial\beta = -\frac{3}{2} < 0$

Platform Profits

Partially differentiate equilibrium platform's profits at Equation 1.9 regarding the parameters of the model. For a bandwagon effect $\partial\Pi^{sh}/\partial\alpha = -\frac{1}{2} < 0$ and for a congestion effect $\partial\Pi^{sh}/\partial\alpha = \frac{1}{2} > 0$, $\frac{\partial\Pi^{sh}}{\partial\upsilon} \equiv \frac{\partial\Pi^{sh}}{\partial\pi} = -\frac{1}{2} < 0$ and $\partial\Pi^{sh}/\partial\beta = \frac{1}{2} > 0$.

A.3 Multihome on sellers' side

A.3.1 Market's Shares

The indifferent seller between singlehome in platform 1 or multihome on both platforms is determined by $\eta_s^1 = \frac{\nu_s^{1,2} - \nu_s^2}{\tau_s}$. By substituting the surpluses from Equation 1.1b and Equation 1.2b, we obtain:[1]

$$\eta_s^1 = \frac{1}{\tau_s}\left[R_s + \pi\left(\eta_b^1 + \eta_b^2\right) - \beta\left(\eta_s^1 + \eta_s^2\right) - \left(p_s^1 - p_s^2\right) - \left[R_s + \pi\eta_b^2 - \beta\eta_s^2 - p_s^2\right]\right]$$

$$\eta_s^1 = \frac{R_s + \pi\eta_b^1 - p_s^1}{\tau_s + \beta} \tag{5}$$

On buyers' side we use Equation 1.4a $\eta_b^1 = \frac{\tau_b + \upsilon\left(2\eta_s^1 - 1\right) - \alpha + \left(p_b^2 - p_b^1\right)}{2(\tau_b - \alpha)}$ and the fact that $\eta_s^1 + \eta_s^2 = 1$ to have

$$\eta_b^1 = \frac{\tau_b + \upsilon\left(\eta_s^1 - \eta_s^2\right) - \alpha + \left(p_b^2 - p_b^1\right)}{2\left(\tau_b - \alpha\right)} \tag{6}$$

We solve the previous system of equations(5) and (6) to obtain η_s^1 and η_b^1 as a function of the membership fees. Equation (5) represents the fraction of indifferent sellers between joining platform 1 and joining both platforms at the same time. Then $\eta_s^2 = \frac{R_s + \pi\eta_b^2 - p_s^2}{\tau_s + \beta}$ represents the fraction of indifferent sellers between joining platform 2 and joining both platforms at the same time. Now we can obtain $(\eta_s^1 - \eta_s^2)$ on equation (6) using the fact that $\eta_b^1 + \eta_b^2 = 1$

$$\eta_s^1 - \eta_s^2 = \frac{R_s + \pi\eta_b^1 - p_s^1}{\tau_s + \beta} - \frac{R_s + \pi\eta_b^2 - p_s^2}{\tau_s + \beta} = \frac{\pi\left[\eta_b^1 - \left(1 - \eta_b^1\right)\right] + p_s^2 - p_s^1}{\tau_s + \beta}$$

$$\eta_s^1 - \eta_s^2 = \frac{\pi\left(2\eta_b^1 - 1\right) - \left(p_s^2 - p_s^1\right)}{\tau_s + \beta} \quad \text{substitute the previous expression in equation (6)}$$

$$2\eta_b^1\left(\tau_b - \alpha\right) = \left(\tau_b - \alpha\right) + \frac{\upsilon}{\left(\tau_s + \beta\right)}\left[\pi\left(2\eta_b^1 - 1\right) + \left(p_s^2 - p_s^1\right)\right] + \left(p_b^2 - p_b^1\right)$$

$$2\left[\left(\tau_s + \beta\right)\left(\tau_b - \alpha\right) - \pi\upsilon\right]\eta_b^1 = \left[\left(\tau_s + \beta\right)\left(\tau_b - \alpha\right) - \pi\upsilon\right] + \upsilon\left(p_s^2 - p_s^1\right) + \left(\tau_s + \beta\right)\left(p_b^2 - p_b^1\right)$$

$$\eta_b^1 = \frac{1}{2} + \frac{\upsilon\left(p_s^2 - p_s^1\right) + \left(\tau_s + \beta\right)\left(p_b^2 - p_b^1\right)}{2\left[\left(\tau_s + \beta\right)\left(\tau_b - \alpha\right) - \pi\upsilon\right]}$$

Next, we substitute the previous expression in equation (5)

$$\eta_s^1 = \frac{R_s - p_s^1}{\left(\tau_s + \beta\right)} + \frac{\pi}{\left(\tau_s + \beta\right)}\left[\frac{1}{2} + \frac{\upsilon\left(p_s^2 - p_s^1\right) + \left(\tau_s + \beta\right)\left(p_b^2 - p_b^1\right)}{2\left[\left(\tau_s + \beta\right)\left(\tau_b - \alpha\right) - \pi\upsilon\right]}\right]$$

[1] Follow the same method for the indifferent seller between singlehome in platform 2 or multihome on both platforms.

A.3.2 Platform profit Maximisation

The first-order conditions of the platform maximisation problem set in Definition 1.2 using Equation 1.14a and Equation 1.14b are:

$$\frac{\partial \Pi}{\partial p_b^i} = \frac{1}{2} + \frac{\upsilon\left(p_s^j - p_s^i\right) + \left(\tau_s + \beta\right)\left(p_b^j - p_b^i\right)}{2\Omega} - \frac{\left(\tau_s + \beta\right)\left(p_b^i - f_b\right)}{2\Omega} - \frac{\pi\left(p_s^i - f_s\right)}{2\Omega} = 0$$

$$\frac{\partial \Pi}{\partial p_s^i} = \frac{R_s - p_s^i}{\tau_s + \beta} + \frac{\pi}{\tau_s + \beta}\left[\frac{1}{2} + \frac{\upsilon\left(p_s^j - p_s^i\right) + \left(\tau_s + \beta\right)\left(p_b^j - p_b^i\right)}{2\Omega}\right] - \frac{p_s^i - f_s}{\tau_s + \beta} - \frac{\pi\upsilon\left(p_s^i - f_s\right)}{2\Omega\left(\tau_s + \beta\right)} - \frac{\upsilon\left(p_b^i - f_b\right)}{2\Omega} = 0$$

Where $\Omega \equiv \left[\left(\tau_s + \beta\right)\left(\tau_b - \alpha\right) - \pi\upsilon\right]$

From the first-order conditions for a symmetric equilibrium $p_b^i = p_b^j = p_b$ and $p_s^i = p_s^j = p_s$ we obtain

$$= \frac{1}{2} - \frac{\left(\tau_s + \beta\right)\left(p_b - f_b\right)}{2\Omega} - \frac{\pi\left(p_s - f_s\right)}{2\Omega} = 0$$

$$= \frac{R_s - p_s}{\tau_s + \beta} + \frac{\pi}{2\left(\tau_s + \beta\right)} - \frac{p_s - f_s}{\tau_s + \beta} - \frac{\pi\upsilon\left(p_s - f_s\right)}{2\Omega\left(\tau_s + \beta\right)} - \frac{\upsilon\left(p_b - f_b\right)}{2\Omega} = 0$$

By solving the first equation for p_b and the subsequent equation for p_s, we establish the corresponding best response functions as:

$$p_b^{smh} = \frac{-\pi p_s - \pi\left(\upsilon - f_s\right) + \left(\tau_s + \beta\right)\left[\left(\tau_b - \alpha\right) + f_b\right]}{\left(\tau_s + \beta\right)} \tag{7}$$

$$p_s^{smh} = \frac{-\upsilon\left(\tau_s + \beta\right)p_b - \pi\upsilon\left(\pi + 2R_s + f_s\right) + \upsilon\left(\tau_s + \beta\right)f_b + \left(\tau_s + \beta\right)\left(\tau_b - \alpha\right)\left(\pi + 2\left(R_s + f_s\right)\right)}{4\left(\tau_s + \beta\right)\left(\tau_b - \alpha\right) - 3\pi\upsilon} \tag{8}$$

The next step is to solve the previous system of equations (7) and (8) to have explicit expressions for the membership fees. First, we substitute p_b^{smh} from equation (7) into equation (8) and solve for p_s^{smh}. That is, first we can define as $W \equiv 4\left(\tau_s + \beta\right)\left(\tau_b - \alpha\right) - 3\pi\upsilon$, then we have $Wp_s^{smh} = \upsilon\left(\tau_s + \beta\right)f_b + \left(\tau_s + \beta\right)\left(\tau_b - \alpha\right)\left(\pi + 2\left(R_s + f_s\right)\right) - \pi\upsilon\big(\pi + 2R_s + f_s\big) - \frac{\upsilon(\tau_s+\beta)}{(\tau_s+\beta)}\left[-\pi p_s - \pi\left(\upsilon - f_s\right) + \left(\tau_s + \beta\right)\left[\left(\tau_b - \alpha\right) + f_b\right]\right]$, turns to $\big[4\left(\tau_s + \beta\right)\big(\tau_b - \alpha\big) - 3\pi\upsilon - \pi\upsilon\big]p_s^{smh} = \left(\tau_s + \beta\right)\left(\tau_b - \alpha\right)\left(\pi - \upsilon\right) - \pi\upsilon\left(\pi - \upsilon\right) + 2\left(R_s + f_s\right)\big[\left(\tau_s + \beta\right)\big(\tau_b - \alpha\big) - \pi\upsilon\big]$ and then $p_s^{smh} = \frac{1}{2}\left(R_s + f_s\right) + \frac{1}{4}\left(\pi - \upsilon\right)$.

Next, we substitute the previous result into equation (7) to have $\left(\tau_s + \beta\right)p_b^{smh} = -\pi\left(\upsilon - f_s\right) + \left(\tau_s + \beta\right)\left[\left(\tau_b - \alpha\right) + f_b\right] - \pi\left[\frac{1}{2}\left(R_s + f_s\right) + \frac{1}{4}\left(\pi - \upsilon\right)\right]$ turns to $p_b^{smh} = f_b + \left(\tau_b - \alpha\right) - \frac{\pi}{4(\tau_s+\beta)}\left[2\left(R_s - f_s\right) + \left(\pi + 3\upsilon\right)\right]$. Therefore, the solution for the system of equations (7) and (8) are:

$$p_s^{smh} = \frac{1}{2}\left(R_s + f_s\right) + \frac{1}{4}\left(\pi - \upsilon\right)$$
$$p_b^{smh} = f_b + \tau_b - \alpha - \frac{\pi}{4\left(\tau_s + \beta\right)}\left[\left(\pi + 3\upsilon\right) + 2\left(R_s - f_s\right)\right]$$

A.3.3 Second order conditions - Multihome on sellers' side

We obtain the following second-order conditions from the profit-maximisation problem of Appendix A.3.2 to define the Hessian matrix:

$$\mathrm{H} = \begin{pmatrix} \Pi^i_{p_b p_b} \equiv -\frac{(\tau_s+\beta)}{[(\tau_s+\beta)(\tau_b-\alpha)-\pi\upsilon]} & \Pi^i_{p_b p_s} \equiv -\frac{(\pi+\upsilon)}{2[(\tau_s+\beta)(\tau_b-\alpha)-\pi\upsilon]} \\ \Pi^i_{p_s p_b} \equiv -\frac{(\pi+\upsilon)}{2[(\tau_s+\beta)(\tau_b-\alpha)-\pi\upsilon]} & \Pi^i_{p_s p_s} \equiv \frac{-[2(\tau_s+\beta)(\tau_b-\alpha)-\pi\upsilon]}{(\tau_s+\beta)[(\tau_s+\beta)(\tau_b-\alpha)-\pi\upsilon]} \end{pmatrix}$$

In order to guarantee that platforms' profits reach a maximum with equilibrium fees in Equation 1.16a and Equation 1.16b a sufficient condition is having H negative definite, indicating that $|H| > 0$, and either $\Pi^i_{p_b p_b} < 0$ or $\Pi^i_{p_s p_s} < 0$. To show $\Pi^i_{p_b p_b}$ is negative, the denominator must be positive since the numerator is always positive. Thus, we have $\left(\tau_s + \beta\right)\left(\tau_b - \alpha\right) > \pi\upsilon$. To show $\Pi^i_{p_s p_s}$ is negative, the numerator must be positive as previously has been shown that for $\Pi^i_{p_b p_b} < 0$, $\left(\tau_s + \beta\right)\left(\tau_b - \alpha\right) - \pi\upsilon > 0$, then $2\left(\tau_s + \beta\right)\left(\tau_b - \alpha\right) > \pi\upsilon$. Finally, to show $|H| > 0$, we have $\frac{2(\tau_s+\beta)(\tau_b-\alpha)-\pi\upsilon}{[(\tau_s+\beta)(\tau_b-\alpha)-\pi\upsilon]^2} - \frac{(\pi+\upsilon)^2}{4[(\tau_s+\beta)(\tau_b-\alpha)-\pi\upsilon]^2} > 0$ turns to $\frac{8(\tau_s+\beta)(\tau_b-\alpha)-4\pi\upsilon-(\pi+\upsilon)^2}{4[(\tau_s+\beta)(\tau_b-\alpha)-\pi\upsilon]^2} > 0$. The previous expression to be positive, given the denominator is always positive is having the numerator positive. That is $8\left(\tau_s + \beta\right)\left(\tau_b - \alpha\right) > \left(\pi + \upsilon\right)^2 + 4\pi\upsilon$.

In summary, for the second-order conditions defined by the Hessian matrix to be negative definite, the following conditions must hold: (i) $2\left(\tau_b - \alpha\right)\left(\tau_s + \beta\right) > \pi\upsilon$, (ii) $\left(\tau_b - \alpha\right)\left(\tau_s + \beta\right) > \pi\upsilon$, and (iii) $8\left(\tau_b - \alpha\right)\left(\tau_s + \beta\right) > \left(\pi + \upsilon\right)^2 + 4\pi\upsilon$.

Now, we show which of the three conditions is the most stringent, ensuring that the rest of the conditions are also satisfied as long as it is met. First, we compare conditions (i) and (ii) by making the left side of both inequalities equal to compare the right side and identify which is larger. That is (i) $\left(\tau_b - \alpha\right)\left(\tau_s + \beta\right) > \frac{\pi\upsilon}{2}$ and (ii) $\left(\tau_b - \alpha\right)\left(\tau_s + \beta\right) > \pi\upsilon$. Since $\pi\upsilon > \frac{\pi\upsilon}{2}$, then condition (ii) is more stringent than condition (i). Additionally, condition (iii) is more stringent than condition (ii) because $\frac{(\pi+\upsilon)^2+4\pi\upsilon}{8} > \pi\upsilon$ which simplifies to $\pi^2 + 2\pi\upsilon + \upsilon^2 + 4\pi\upsilon > 8\pi\upsilon$ and finally turns to $\left(\pi - \upsilon\right)^2 > 0$ when $\pi \neq \upsilon$. Therefore if condition (iii) holds, conditions (i) and (ii) are satisfied.

A.3.4 Platform profits

We use Equation 1.16a, Equation 1.16b and Equation 1.17, and $\eta_b^{smh} = 1/2$ to compute equilibrium platform profits as:

$$\Pi^{smh} \equiv \left(p_b^{smh} - f_b\right) \eta_b^{smh} + \left(p_s^{smh} - f_s\right) \eta_s^{smh}$$

$$\Pi^{smh} = \left(f_b + \tau_b - \alpha - \frac{\pi}{4(\tau_s + \beta)} \left[(\pi + 3\upsilon) + 2(R_s - f_s)\right] - f_b\right) \frac{1}{2}$$

$$+ \left(\frac{2(R_s + f_s) + (\pi - \upsilon)}{4} - f_s\right) \left(\frac{2(R_s - f_s) + (\pi + \upsilon)}{4(\tau_s + \beta)}\right)$$

$$= \frac{4(\tau_b - \alpha)(\tau_s + \beta) - \pi(\pi + 3\upsilon) - 2\pi(R_s - f_s)}{8(\tau_s + \beta)}$$

$$+ \frac{(2(R_s - f_s) + (\pi - \upsilon))(2(R_s - f_s) + (\pi + \upsilon))}{16(\tau_s + \beta)}$$

$$= \Big(8(\tau_b - \alpha)(\tau_s + \beta) - 2\pi(\pi + 3\upsilon) - 4\pi(R_s - f_s) + 4(R_s - f_s)^2$$

$$+ 2(R_s - f_s)(\pi + \upsilon) + 2(R_s - f_s)(\pi - \upsilon) + (\pi - \upsilon)(\pi + \upsilon)\Big) \Big/ 16(\tau_s + \beta)$$

$$\Pi^{smh} = \frac{8(\tau_s + \beta)(\tau_b - \alpha) - (\pi + \upsilon)^2 - 4\pi\upsilon + 4(R_s - f_s)^2}{16(\tau_s + \beta)}$$

A.3.5 Surpluses

Gross surpluses

We use Equation 1.1a, Equation 1.16a and Equation 1.17, and $\eta_b^{smh} = 1/2$ to compute buyers' gross (from transportation cost) surplus as:

$$\nu_b^{smh} = R_b + \upsilon \eta_s^{smh} + \alpha \eta_b^{smh} - p_b^{smh}$$

$$\nu_b^{smh} = R_b + \upsilon \left(\frac{2(R_s - f_s) + (\pi + \upsilon)}{4(\tau_s + \beta)}\right) + \frac{\alpha}{2} - \left(f_b + \tau_b - \alpha - \frac{\pi\left[(\pi + 3\upsilon) + 2(R_s - f_s)\right]}{4(\tau_s + \beta)}\right)$$

$$= (R_b - f_b) - \tau_b + \frac{3}{2}\alpha + \frac{2\upsilon(R_s - f_s) + \upsilon(\pi + \upsilon) + \pi(\pi + 3\upsilon) + 2\pi(R_s - f_s)}{4(\tau_s + \beta)}$$

$$\nu_b^{smh} = (R_b - f_b) - \tau_b + \frac{3}{2}\alpha + \frac{2(\pi + \upsilon)(R_s - f_s) + (\pi + \upsilon)^2 + 2\pi\upsilon}{4(\tau_s + \beta)}$$

We use Equation 1.1b, Equation 1.16b and Equation 1.17, and $\eta_b^{smh} = 1/2$ to compute sellers' gross (from transportation cost) surplus when they join only one platform:

$$\nu_s^{smh} = R_s + \pi\eta_b^{smh} - \beta\eta_s^{smh} - p_s^{smh}$$
$$= R_s + \frac{\pi}{2} - \beta\left(\frac{2\left(R_s - f_s\right) + \left(\pi + \upsilon\right)}{4\left(\tau_s + \beta\right)}\right) - \left(\frac{1}{2}\left(R_s + f_s\right) + \frac{1}{4}\left(\pi - \upsilon\right)\right)$$
$$= \frac{1}{2}\left(R_s - f_s\right) + \frac{1}{4}\left(\pi + \upsilon\right) - \left(\frac{2\beta\left(R_s - f_s\right) + \beta\left(\pi + \upsilon\right)}{4\left(\tau_s + \beta\right)}\right)$$
$$= \frac{2\left(R_s - f_s\right)\left(\tau_s + \beta\right) + \left(\pi + \upsilon\right)\left(\tau_s + \beta\right) - 2\beta\left(R_s - f_s\right) - \beta\left(\pi + \upsilon\right)}{4\left(\tau_s + \beta\right)}$$
$$\nu_s^{smh} = \frac{\tau_s\left[2\left(R_s - f_s\right) + \left(\pi + \upsilon\right)\right]}{4\left(\tau_s + \beta\right)}$$

Aggregate surpluses

Using the same method as in the previous Section 1.3 where both buyers and sellers singlehome and Equation 1.19a we get:

$$CS^{smh} = \nu_b^{smh} - 2\int_0^{1/2} \tau_b x_b dx_b$$
$$CS^{smh} = \left(R_b - f_b\right) - \frac{5}{4}\tau_b + \frac{3}{2}\alpha + \frac{2\left(\pi + \upsilon\right)\left(R_s - f_s\right) + \left(\pi + \upsilon\right)^2 + 2\pi\upsilon}{4\left(\tau_s + \beta\right)}$$

To calculate the Producer Surplus we refer to Figure 1.1 to determine how to measure the transportation cost associated with joining one platform versus joining both platforms simultaneously, considering the choice of some sellers to singlehome and others to multihome.

$$PS^{smh} = \int_0^{1-\eta_s^{smh}} \left(\nu_s^{smh} - \tau_s x_s\right) dx_s + \int_{1-\eta_s^{smh}}^{\eta_s^{smh}} \left(2\nu_s^{smh} - \tau_s\right) dx_s + \int_{\eta_s^{smh}}^{1} \left(\nu_s^{smh} - \tau_s\left(1 - x_s\right)\right) dx_s$$
$$= \nu_s^{smh} x_s\Big|_0^{1-\eta_s^{smh}} - \frac{\tau_s}{2}x_s^2\Big|_0^{1-\eta_s^{smh}} + 2\nu_s^{smh} x_s\Big|_{1-\eta_s^{smh}}^{\eta_s^{smh}} - \tau_s x_s\Big|_{1-\eta_s^{smh}}^{\eta_s^{smh}} + \nu_s^{smh} x_s\Big|_{\eta_s^{smh}}^{1} - \tau_s x_s\Big|_{\eta_s^{smh}}^{1} + \frac{\tau_s}{2}x_s^2\Big|_{\eta_s^{smh}}^{1}$$
$$= \nu_s^{smh}\left(1 - \eta_s^{smh}\right) - \frac{\tau_s}{2}\left(1 - \eta_s^{smh}\right)^2 + 2\nu_s^{smh}\eta_s^{smh} - 2\nu_s^{smh}\left(1 - \eta_s^{smh}\right) - \tau_s\eta_s^{smh} + \tau_s\left(1 - \eta_s^{smh}\right) + \nu_s^{smh}\left(1 - \eta_s^{smh}\right) - \tau_s\left(1 - \eta_s^{smh}\right) + \frac{\tau_s}{2}\left(1 - \left(\eta_s^{smh}\right)^2\right)$$
$$= 2\nu_s^{smh}\eta_s^{smh} - \frac{\tau_s}{2}\left(1 - \eta_s^{smh}\right)^2 - \tau_s\eta_s^{smh} + \frac{\tau_s}{2}\left(1 - \left(\eta_s^{smh}\right)^2\right)$$

$$= 2\nu_s^{smh}\eta_s^{smh} - \tau_s\eta_s^{smh} - \frac{\tau_s}{2}\left(1-\eta_s^{smh}\right)\left(1-\eta_s^{smh}-1-\eta_s^{smh}\right)$$

$$= 2\nu_s^{smh}\eta_s^{smh} - \tau_s\left(\eta_s^{smh}\right)^2$$

Using the fact that $\frac{\nu_s^{smh}}{\tau_s} = \eta_s^{smh}$, then

$$= 2\nu_s^{smh}\left(\frac{\nu_s^{smh}}{\tau_s}\right) - \tau_s\left(\frac{\left(\nu_s^{smh}\right)^2}{\tau_s^2}\right) \quad \text{we obtain } PS^{smh} = \frac{\left(\nu_s^{smh}\right)^2}{\tau_s}$$

$$PS^{smh} = \frac{\tau_s\left[2\left(R_s-f_s\right)+(\pi+\upsilon)\right]^2}{16\left(\tau_s+\beta\right)^2}$$

A.3.6 Comparative Statics

Membership fees

Partially differentiate equilibrium membership fees at Equation 1.16a and Equation 1.16b regarding the parameters of the model. On buyers' side, when there is a bandwagon effect $\partial p_b^{smh}/\partial\alpha = -1 < 0$ and when there is congestion effect $\partial p_b^{smh}/\partial\alpha = 1 > 0$, $\partial p_b^{smh}/\partial\beta = \frac{4\pi[(\pi+3\upsilon)+2(R_s-f_s)]}{16(\tau_s+\beta)^2} > 0$, $\partial p_b^{smh}/\partial\upsilon = -\frac{3\pi}{4(\tau_s+\beta)} < 0$ and $\partial p_b^{smh}/\partial\pi = -\frac{(2\pi+3\upsilon)+2(R_s-f_s)}{4(\tau_s+\beta)} < 0$. On sellers' side $\partial p_s^{smh}/\partial\pi = 1/4 > 0$, $\partial p_s^{smh}/\partial\upsilon = -\frac{1}{4} < 0$.

Sellers' market-shares

Partially differentiate equilibrium sellers' market shares at Equation 1.17 regarding the parameters of the model. $\frac{\partial\eta_s^{smh}}{\partial\beta} = -\frac{2(R_s-f_s)+(\pi+\upsilon)}{4(\tau_s+\beta)^2} < 0$, $\frac{\partial\eta_s^{smh}}{\partial\pi} = \frac{\partial\eta_s^{smh}}{\partial\upsilon} = \frac{1}{4(\tau_s+\beta)} > 0$.

Aggregate Surplus

Partially differentiate equilibrium aggregate surplus at Equation 1.20 and Equation 1.21 regarding the parameters of the model. When there is a bandwagon effect $\partial CS^{smh}/\partial\alpha = \frac{3}{2} > 0$, when there is a congestion effect $\partial CS^{smh}/\partial\alpha = -\frac{3}{2} < 0$, $\partial CS^{smh}/\partial\beta = -\frac{\left[2(\pi+\upsilon)(R_s-f_s)+(\pi+\upsilon)^2+2\pi\upsilon\right]}{4(\tau_s+\beta)^2} < 0$, $\partial CS^{smh}/\partial\upsilon = \frac{(R_s-f_s)+(2\pi+\upsilon)}{2(\tau_s+\beta)} > 0$, $\frac{\partial CS^{smh}}{\partial\pi} = \frac{1}{2(\tau_s+\beta)}\left[\left(R_s-f_s\right)+(\pi+2\upsilon)\right] > 0$. On sellers' side, $\partial PS^{smh}/\partial\beta = -\frac{\tau_s[2(R_s-f_s)+(\pi+\upsilon)]^2}{8(\tau_s+\beta)^3} < 0$ and $\partial PS^{smh}/\partial\upsilon = \partial PS^{smh}/\partial\pi = \frac{\tau_s[2(R_s-f_s)+(\pi+\upsilon)]}{(\tau_s+\beta)^2} > 0$.

Platforms Profits

Partially differentiate equilibrium platforms profits at Equation 1.18 regarding the parameters of the model. When there is a bandwagon effect $\partial\Pi^{smh}/\partial\alpha = -\frac{1}{2} < 0$ and when there is a congestion effect $\partial\Pi^{smh}/\partial\alpha = \frac{1}{2} > 0$, $\partial\Pi^{smh}/\partial\upsilon = -\frac{3\pi+\upsilon}{8(\tau_s+\beta)} < 0$ and $\partial\Pi^{smh}/\partial\pi = -\frac{\pi+3\upsilon}{8(\tau_s+\beta)} < 0$, $\partial\Pi^{smh}/\partial\beta = -\frac{4(R_s-f_s)^2-\left[(\pi+\upsilon)^2+4\pi\upsilon\right]}{16(\tau_s+\beta)^2} > 0$ if $2\left(R_s-f_s\right) <$

$\sqrt{(\pi+\upsilon)^2+4\pi\upsilon}$. To show the previous expression is satisfied we use Assumption 1.3, where $2(R_s-f_s)<4(\tau_s+\beta)$, then comparing the right side of both expressions we need $4(\tau_s+\beta)>\sqrt{(\pi+\upsilon)^2+4\pi\upsilon}$. This condition is compatible with Assumption 1.1 as long as $\tau_b-\alpha<\frac{(\pi+\upsilon)^2}{\sqrt{(\pi+\upsilon)^2+4\pi\upsilon}}$.

A.4 Multihome on buyers' side

A.4.1 Market's Shares

The indifferent buyer between singlehome in platform 1 or multihome on both platforms is $\eta_b^1=\frac{\nu_b^{1,2}-\nu_b^2}{\tau_b}$, By substituting the surpluses at Equation 1.1a and Equation 1.2a we get:[2]

$$\eta_b^1=\frac{1}{\tau_b}\left[R_b+\upsilon\left(\eta_s^1+\eta_s^2\right)+\alpha\left(\eta_b^1+\eta_b^2\right)-\left(p_b^1-p_b^2\right)-\left[R_b+\upsilon\eta_s^2+\alpha\eta_b^2-p_b^2\right]\right]$$

$$\eta_b^1=\frac{R_b+\upsilon\eta_s^1-p_b^1}{\tau_b-\alpha} \tag{9}$$

On sellers' side we use Equation 1.4b $\eta_s^1=\frac{\tau_s+\pi\left(2\eta_b^1-1\right)+\beta+\left(p_s^2-p_s^1\right)}{2(\tau_s+\beta)}$ and the fact that $\eta_b^1+\eta_b^2=1$ to have

$$\eta_s^1=\frac{\tau_s+\pi\left(\eta_b^1-\eta_b^2\right)+\beta+\left(p_s^2-p_s^1\right)}{2\left(\tau_s+\beta\right)} \tag{10}$$

We solve the previous system of equations(9) and (10) to obtain η_b^1 and η_s^1 as a function of the membership fees in the same way as in Appendix A.3.1. Then substituting equation (9) into equation (10) we have

$$2\eta_s^1\left(\tau_s+\beta\right)=\left(\tau_s+\beta\right)+\frac{\pi}{\left(\tau_b-\alpha\right)}\left[\upsilon\left(2\eta_s^1-1\right)+\left(p_b^2-p_b^1\right)\right]+\left(p_s^2-p_s^1\right)$$

$$\eta_s^1=\frac{1}{2}+\frac{\pi\left(p_b^2-p_b^1\right)+\left(\tau_b-\alpha\right)\left(p_s^2-p_s^1\right)}{2\left[\left(\tau_s+\beta\right)\left(\tau_b-\alpha\right)-\pi\upsilon\right]}$$

Next, we substitute the previous expression in equation (9)

$$\eta_b^1=\frac{R_b-p_b^1}{\left(\tau_b-\alpha\right)}+\frac{\upsilon}{\left(\tau_b-\alpha\right)}\left[\frac{1}{2}+\frac{\pi\left(p_b^2-p_b^1\right)+\left(\tau_b-\alpha\right)\left(p_s^2-p_s^1\right)}{2\left[\left(\tau_s+\beta\right)\left(\tau_b-\alpha\right)-\pi\upsilon\right]}\right]$$

[2] Follow the same process for the indifferent buyer between singlehome in platform 2 or multihome on both platforms.

A.4.2 Platform profit Maximisation

The first-order conditions of the platform maximisation problem set in Definition 1.3 using Equation 1.24a and Equation 1.24b are:

$$\frac{\partial \Pi}{\partial p_b^i} = \frac{R_b - p_b^i}{\tau_b - \alpha} + \frac{\upsilon}{\tau_b - \alpha}\left[\frac{1}{2} + \frac{\pi\left(p_b^j - p_b^i\right) + (\tau_b - \alpha)\left(p_s^j - p_s^i\right)}{2\Omega}\right] - \frac{p_b^i - f_b}{\tau_b - \alpha} - \frac{\pi\upsilon\left(p_b^i - f_b\right)}{2\Omega\left(\tau_b - \alpha\right)} - \frac{\pi\left(p_s^i - f_s\right)}{2\Omega} = 0$$

$$\frac{\partial \Pi}{\partial p_s^i} = \frac{1}{2} + \frac{\pi\left(p_b^j - p_b^i\right) + (\tau_b - \alpha)\left(p_s^j - p_s^i\right)}{2\Omega} - \frac{(\tau_b - \alpha)\left(p_s^i - f_s\right)}{2\Omega} - \frac{\upsilon\left(p_b^i - f_b\right)}{2\Omega} = 0$$

Where $\Omega \equiv \left[(\tau_s + \beta)(\tau_b - \alpha) - \pi\upsilon\right]$

From the first-order conditions for a symmetric equilibrium $p_b^i = p_b^j = p_b$ and $p_s^i = p_s^j = p_s$, we obtain:

$$= \frac{R_b - p_b}{\tau_b - \alpha} + \frac{\upsilon}{2(\tau_b - \alpha)} - \frac{p_b - f_b}{\tau_b - \alpha} - \frac{\pi\upsilon\left(p_b - f_b\right)}{2\Omega\left(\tau_b - \alpha\right)} - \frac{\pi\left(p_s - f_s\right)}{2\Omega} = 0$$

$$= \frac{1}{2} - \frac{(\tau_b - \alpha)\left(p_s - f_s\right)}{2\Omega} - \frac{\upsilon\left(p_b - f_b\right)}{2\Omega} = 0$$

By solving the first equation for p_b and the subsequent equation for p_s, we establish the corresponding best response functions as:

$$p_b^{bmh} = \frac{-\pi(\tau_b - \alpha)p_s - \pi\upsilon\left(\upsilon + 2R_b + f_b\right) + \pi(\tau_b - \alpha)f_s + (\tau_s + \beta)(\tau_b - \alpha)\left(\upsilon + 2\left(R_b + f_b\right)\right)}{4(\tau_s + \beta)(\tau_b - \alpha) - 3\pi\upsilon} \tag{11}$$

$$p_s^{bmh} = \frac{-\upsilon p_b - \upsilon\left(\pi - f_b\right) + (\tau_b - \alpha)\left[(\tau_s + \beta) + f_s\right]}{(\tau_b - \alpha)} \tag{12}$$

The next step is to solve the previous system of equations (11) and (12) to have explicit expressions for the membership fees. First, we substitute p_s^{bmh} from equation (12) into equation (11) and solve for p_b^{bmh}. That is, first we can define as $W \equiv 4(\tau_s + \beta)(\tau_b - \alpha) - 3\pi\upsilon$, then we have $Wp_b^{bmh} = \pi(\tau_b - \alpha)f_s + (\tau_b - \alpha)(\tau_s + \beta)\left(\upsilon + 2\left(R_b + f_b\right)\right) - \pi\upsilon\big(\upsilon + 2R_b + f_b\big) - \frac{\pi(\tau_b - \alpha)}{(\tau_b - \alpha)}\left[-\upsilon p_b - \upsilon\left(\pi - f_b\right) + (\tau_b - \alpha)\left[(\tau_s + \beta) + f_s\right]\right]$, turns to $\big[4(\tau_s + \beta)\big(\tau_b - \alpha\big) - 3\pi\upsilon - \pi\upsilon\big]p_b^{bmh} = (\tau_s + \beta)(\tau_b - \alpha)(\upsilon - \pi) - \pi\upsilon(\upsilon - \pi) + 2\left(R_b + f_b\right)\big[(\tau_s + \beta)\big(\tau_b - \alpha\big) - \pi\upsilon\big]$ and then $p_b^{bmh} = \frac{1}{2}\left(R_b + f_b\right) + \frac{1}{4}(\upsilon - \pi)$.

Next, we substitute the previous result into equation (12) to have $(\tau_b - \alpha)p_s^{bmh} = -\upsilon\left(\pi - f_b\right) + (\tau_b - \alpha)\left[(\tau_s + \beta) + f_s\right] - \upsilon\left[\frac{1}{2}\left(R_b + f_b\right) + \frac{1}{4}(\upsilon - \pi)\right]$ turns to $p_s^{bmh} = f_s + (\tau_s + \beta) - \frac{\upsilon}{4(\tau_b - \alpha)}\left[2\left(R_b - f_b\right) + (\upsilon + 3\pi)\right]$. Therefore, the solution for the system of equations (11) and (12) are:

$$p_b^{bmh} = \frac{1}{2}(R_b + f_b) + \frac{1}{4}(\upsilon - \pi)$$
$$p_s^{bmh} = f_s + \tau_s + \beta - \frac{\upsilon}{4(\tau_b - \alpha)}[(\upsilon + 3\pi) + 2(R_b - f_b)]$$

A.4.3 Second order conditions - Multihome on buyers' side

We obtain the following second-order conditions from the profit-maximisation problem of Appendix A.4.2 to define the Hessian matrix:

$$\text{H} = \begin{pmatrix} \Pi^i_{p_bp_b} \equiv \frac{-[2(\tau_s+\beta)(\tau_b-\alpha)-\pi\upsilon]}{(\tau_b-\alpha)[(\tau_s+\beta)(\tau_b-\alpha)-\pi\upsilon]} & \Pi^i_{p_bp_s} \equiv -\frac{(\pi+\upsilon)}{2[(\tau_s+\beta)(\tau_b-\alpha)-\pi\upsilon]} \\ \Pi^i_{p_sp_b} \equiv -\frac{(\pi+\upsilon)}{2[(\tau_s+\beta)(\tau_b-\alpha)-\pi\upsilon]} & \Pi^i_{p_sp_s} \equiv -\frac{(\tau_b-\alpha)}{[(\tau_s+\beta)(\tau_b-\alpha)-\pi\upsilon]} \end{pmatrix}$$

In order to guarantee that platforms' profits reach a maximum with equilibrium fees in Equation 1.26a and Equation 1.26b a sufficient condition is having H negative definite, indicating that $|H| > 0$, and either $\Pi^i_{p_bp_b} < 0$ or $\Pi^i_{p_sp_s} < 0$. To show $\Pi^i_{p_sp_s}$ is negative, we need both numerator and denominator positive, meaning $\tau_b - \alpha > 0$, and $(\tau_s + \beta)(\tau_b - \alpha) > \pi\upsilon$. To show $\Pi^i_{p_bp_b}$ is negative, the numerator must be positive as previously has been shown that for $\Pi^i_{p_sp_s} < 0$, $(\tau_s + \beta)(\tau_b - \alpha) - \pi\upsilon > 0$ and $\tau_b - \alpha > 0$, then $2(\tau_s + \beta)(\tau_b - \alpha) > \pi\upsilon$. Finally, to show $|H| > 0$, we have $\frac{2(\tau_s+\beta)(\tau_b-\alpha)-\pi\upsilon}{[(\tau_s+\beta)(\tau_b-\alpha)-\pi\upsilon]^2} - \frac{(\pi+\upsilon)^2}{4[(\tau_s+\beta)(\tau_b-\alpha)-\pi\upsilon]^2} > 0$ and turns to $\frac{8(\tau_s+\beta)(\tau_b-\alpha)-4\pi\upsilon-(\pi+\upsilon)^2}{4[(\tau_s+\beta)(\tau_b-\alpha)-\pi\upsilon]^2} > 0$. The previous expression to be positive, given the denominator is always positive is having the numerator positive. That is $8(\tau_s + \beta)(\tau_b - \alpha) > (\pi + \upsilon)^2 + 4\pi\upsilon$.

In summary, for the second-order conditions defined by the Hessian matrix to be negative definite, the following conditions must hold: (i) $\tau_b - \alpha > 0$, (ii) $2(\tau_b - \alpha)(\tau_s + \beta) > \pi\upsilon$, (iii) $(\tau_b - \alpha)(\tau_s + \beta) > \pi\upsilon$, and (iv) $8(\tau_b - \alpha)(\tau_s + \beta) > (\pi + \upsilon)^2 + 4\pi\upsilon$.

Now, we show which of the four conditions is the most stringent, ensuring that the rest of the conditions are also satisfied as long as it is met. First, we compare conditions (i) and (ii) by making the left side of both inequalities equal to compare the right side and identify which is larger. That is $\frac{\pi\upsilon}{2(\tau_s+\beta)} > 0$, then condition (ii) is more stringent than condition (i). Now, we compare conditions (ii) and (iii) following the same method, that is (ii) turns to $(\tau_b - \alpha)(\tau_s + \beta) > \frac{\pi\upsilon}{2}$ and (iii) is $(\tau_b - \alpha)(\tau_s + \beta) > \pi\upsilon$. Since $\pi\upsilon > \frac{\pi\upsilon}{2}$, then condition (iii) is more stringent than condition (ii). Finally, condition (iv) is more stringent than condition (iii) because $\frac{(\pi+\upsilon)^2+4\pi\upsilon}{8} > \pi\upsilon$ which turns to $\pi^2 + 2\pi\upsilon + \upsilon^2 + 4\pi\upsilon > 8\pi\upsilon$ and finally simplifies to $(\pi - \upsilon)^2 > 0$ when $\pi \neq \upsilon$. Therefore if condition (iv) holds, conditions (i), (ii) and (iii) are satisfied.

A.4.4 Platform profits

We use Equation 1.26a, Equation 1.26b and Equation 1.27, and $\eta_s^{bmh} = 1/2$ to compute equilibrium platform profits as:

$$\Pi^{bmh} \equiv \left(p_b^{bmh} - f_b\right)\eta_b^{bmh} + \left(p_s^{bmh} - f_s\right)\eta_s^{bmh}$$

$$\begin{aligned}
\Pi^{bmh} &= \left(\frac{2(R_b + f_b) + (\upsilon - \pi)}{4} - f_b\right)\left(\frac{2(R_b - f_b) + (\pi + \upsilon)}{4(\tau_b - \alpha)}\right) \\
&\quad + \left(f_s + \tau_s + \beta - \frac{\upsilon}{4(\tau_b - \alpha)}\left[(\upsilon + 3\pi) + 2(R_b - f_b)\right] - f_s\right)\frac{1}{2} \\
&= \frac{(2(R_b - f_b) + (\upsilon - \pi))(2(R_b - f_b) + (\pi + \upsilon))}{16(\tau_b - \alpha)} \\
&\quad + \frac{4(\tau_b - \alpha)(\tau_s + \beta) - \upsilon(\upsilon + 3\pi) - 2\upsilon(R_b - f_b)}{8(\tau_b - \alpha)} \\
&= \Big(8(\tau_b - \alpha)(\tau_s + \beta) - 2\upsilon(\upsilon + 3\pi) - 4\upsilon(R_b - f_b) + 4(R_b - f_b)^2 \\
&\quad + 2(R_b - f_b)(\pi + \upsilon) + 2(R_b - f_b)(\upsilon - \pi) + (\upsilon - \pi)(\pi + \upsilon)\Big)\Big/ 16(\tau_b - \alpha) \\
\Pi^{bmh} &= \frac{8(\tau_s + \beta)(\tau_b - \alpha) - (\pi + \upsilon)^2 - 4\pi\upsilon + 4(R_b - f_b)^2}{16(\tau_b - \alpha)}
\end{aligned}$$

A.4.5 Surpluses

Gross surpluses

We use Equation 1.1a, Equation 1.26a and Equation 1.27, and $\eta_s^{bmh} = 1/2$ to compute buyers' gross (from transportation cost) surplus when they join only one platform:

$$\begin{aligned}
\nu_b^{bmh} &= R_b + \upsilon\eta_s^{bmh} + \alpha\eta_b^{bmh} - p_b^{bmh} \\
&= R_b + \frac{1}{2}\upsilon + \alpha\left(\frac{2(R_b - f_b) + (\upsilon + \pi)}{4(\tau_b - \alpha)}\right) - \left(\frac{1}{2}(R_b + f_b) + \frac{1}{4}(\upsilon - \pi)\right) \\
&= \frac{1}{2}(R_b - f_b) + \frac{1}{4}(\pi + \upsilon) + \left(\frac{2\alpha(R_b - f_b) + \alpha(\pi + \upsilon)}{4(\tau_b - \alpha)}\right) \\
&= \frac{2(R_b - f_b)(\tau_b - \alpha) + (\pi + \upsilon)(\tau_b - \alpha) + 2\alpha(R_b - f_b) + \alpha(\pi + \upsilon)}{4(\tau_b - \alpha)} \\
\nu_b^{bmh} &= \frac{\tau_b\left[2(R_b - f_b) + (\upsilon + \pi)\right]}{4(\tau_b - \alpha)}
\end{aligned}$$

Next, we use Equation 1.1b, Equation 1.26b and Equation 1.27, and $\eta_s^{bmh} = 1/2$ to compute sellers' gross (from transportation cost) surplus as:

$$\nu_s^{bmh} = R_s + \pi\eta_b^{bmh} - \beta\eta_s^{bmh} - p_s^{bmh}$$

$$\nu_s^{bmh} = R_s + \pi\left(\frac{2(R_b - f_b) + (\pi + \upsilon)}{4(\tau_b - \alpha)}\right) - \frac{\beta}{2} - \left(f_s + \tau_s + \beta - \frac{\upsilon\left[(\upsilon + 3\pi) + 2(R_b - f_b)\right]}{4(\tau_b - \alpha)}\right)$$

$$= (R_s - f_s) - \tau_s - \frac{3}{2}\beta + \frac{2\pi(R_b - f_b) + \pi(\pi + \upsilon) + \upsilon(\upsilon + 3\pi) + 2\upsilon(R_b - f_b)}{4(\tau_b - \alpha)}$$

$$\nu_s^{bmh} = (R_s - f_s) - \tau_s - \frac{3}{2}\beta + \frac{2(\pi + \upsilon)(R_b - f_b) + (\pi + \upsilon)^2 + 2\pi\upsilon}{4(\tau_b - \alpha)}$$

Aggregate surpluses

To compute the Consumer Surplus we refer to Figure 1.1 to determine how to measure the transportation cost associated with joining one platform versus joining both platforms simultaneously, considering the choice of some buyers to singlehome and others to multihome.

$$CS^{bmh} = \int_0^{1-\eta_b^{bmh}} \left(\nu_b^{bmh} - \tau_b x_b\right) dx_b + \int_{1-\eta_b^{bmh}}^{\eta_b^{bmh}} \left(2\nu_b^{bmh} - \tau_b\right) dx_b + \int_{\eta_b^{bmh}}^{1} \left(\nu_b^{bmh} - \tau_b(1 - x_b)\right) dx_b$$

$$= \nu_b^{bmh} x_b\Big|_0^{1-\eta_b^{bmh}} - \frac{\tau_b}{2}x_b^2\Big|_0^{1-\eta_b^{bmh}} + 2\nu_b^{bmh} x_b\Big|_{1-\eta_b^{bmh}}^{\eta_b^{bmh}} - \tau_b x_b\Big|_{1-\eta_b^{bmh}}^{\eta_b^{bmh}} + \nu_b^{bmh} x_b\Big|_{\eta_b^{bmh}}^{1} - \tau_b x_b\Big|_{\eta_b^{bmh}}^{1} + \frac{\tau_b}{2}x_b^2\Big|_{\eta_b^{bmh}}^{1}$$

$$= \nu_b^{bmh}\left(1 - \eta_b^{bmh}\right) - \frac{\tau_b}{2}\left(1 - \eta_b^{bmh}\right)^2 + 2\nu_b^{bmh}\eta_b^{bmh} - 2\nu_b^{bmh}\left(1 - \eta_b^{bmh}\right) - \tau_b\eta_b^{bmh} + \tau_b\left(1 - \eta_b^{bmh}\right) + \nu_b^{bmh}\left(1 - \eta_b^{bmh}\right) - \tau_b\left(1 - \eta_b^{bmh}\right) + \frac{\tau_b}{2}\left(1 - \left(\eta_b^{bmh}\right)^2\right)$$

$$= 2\nu_b^{bmh}\eta_b^{bmh} - \frac{\tau_b}{2}\left(1 - \eta_b^{bmh}\right)^2 - \tau_b\eta_b^{bmh} + \frac{\tau_b}{2}\left(1 - \left(\eta_b^{bmh}\right)^2\right)$$

$$= 2\nu_b^{bmh}\eta_b^{bmh} - \tau_b\eta_b^{bmh} - \frac{\tau_b}{2}\left(1 - \eta_b^{bmh}\right)\left(1 - \eta_b^{bmh} - 1 - \eta_b^{bmh}\right)$$

$$= 2\nu_b^{bmh}\eta_b^{bmh} - \tau_b\left(\eta_b^{bmh}\right)^2$$

Using the fact that $\frac{\nu_b^{bmh}}{\tau_b} = \eta_b^{bmh}$, then

$$= 2\nu_b^{bmh}\left(\frac{\nu_b^{bmh}}{\tau_b}\right) - \tau_b\left(\frac{\left(\nu_b^{bmh}\right)^2}{\tau_b^2}\right) \text{ we obtain } CS^{bmh} = \frac{\left(\nu_b^{bmh}\right)^2}{\tau_b}$$

$$CS^{bmh} = \frac{\tau_b\left[2(R_b - f_b) + (\upsilon + \pi)\right]^2}{16(\tau_b - \alpha)^2}$$

Using the same method as in the previous Section 1.4 where sellers multihome and buyers singlehome and Equation 1.29b we get:

$$PS^{bmh} = \nu_s^{bmh} - 2\int_0^{1/2} \tau_s x_s dx_s$$
$$PS^{bmh} = (R_s - f_s) - \frac{5}{4}\tau_s - \frac{3}{2}\beta + \frac{\left(2(\pi+\upsilon)(R_b - f_b) + (\pi+\upsilon)^2 + 2\pi\upsilon\right)}{4(\tau_b - \alpha)}$$

A.4.6 Comparative Statics:

Membership fees

Partially differentiate equilibrium membership fees at Equation 1.26a and Equation 1.26b regarding the parameters of the model. On buyers' side $\partial p_b^{bmh}/\partial\upsilon = \frac{1}{4} > 0$, $\partial p_b^{bmh}/\partial\pi = -\frac{1}{4} < 0$. On sellers' side when there is a bandwagon effect $\partial p_s^{bmh}/\partial\alpha = -\frac{\upsilon[(\upsilon+3\pi)+2(R_b-f_b)]}{4(\tau_b-\alpha)^2} < 0$ and when there is a congestion effect $\partial p_s^{bmh}/\partial\alpha = \frac{\upsilon[(\upsilon+3\pi)+2(R_b-f_b)]}{4(\tau_b-\alpha)^2} > 0$, $\partial p_s^{bmh}/\partial\beta = 1 > 0$, $\partial ps^{bmh}/\partial\pi = -\frac{3\upsilon}{4(\tau_b-\alpha)} < 0$ and $\partial p_s^{bmh}/\partial\upsilon = -\frac{(2\upsilon+3\pi)+2(R_b-f_b)}{4(\tau_b-\alpha)} < 0$

Buyers' market-shares

Partially differentiate equilibrium buyers' market shares at Equation 1.27 regarding the parameters of the model. When there is a bandwagon effect $\frac{\partial\eta_b^{bmh}}{\partial\alpha} = \frac{2(R_b-f_b)+(\pi+\upsilon)}{4(\tau_b-\alpha)^2} > 0$ and when there is a congestion effect $\frac{\partial\eta_b^{bmh}}{\partial\alpha} = -\frac{2(R_b-f_b)+(\pi+\upsilon)}{4(\tau_b-\alpha)^2} < 0$, $\frac{\partial\eta_b^{bmh}}{\partial\pi} = \frac{\partial\eta_b^{bmh}}{\partial\upsilon} = \frac{1}{4(\tau_b-\alpha)} > 0$.

Aggregate Surplus

Partially differentiate equilibrium aggregate surplus at Equation 1.30 and Equation 1.31 concerning the parameters of the model. When there is a bandwagon effect $\partial CS^{bmh}/\partial\alpha = \frac{\tau_b[2(R_b-f_b)+(\upsilon+\pi)]^2}{8(\tau_b-\alpha)^3} > 0$ and when there is a congestion effect $\partial CS^{bmh}/\partial\alpha = -\frac{\tau_b[2(R_b-f_b)+(\upsilon+\pi)]^2}{8(\tau_b-\alpha)^3} < 0$, $\partial CS^{smh}/\partial\upsilon \equiv \partial CS^{smh}/\partial\pi = \frac{\tau_b[2(R_b-f_b)+(\upsilon+\pi)]}{8(\tau_b-\alpha)^2} > 0$. On sellers' side when there is a bandwagon effect $\partial PS^{bmh}/\partial\alpha = \frac{2(\pi+\upsilon)(R_b-f_b)+\left[(\pi+\upsilon)^2+2\pi\upsilon\right]}{4(\tau_b-\alpha)^2} > 0$ and when there is congestion effect $\partial PS^{bmh}/\partial\alpha = -\frac{2(\pi+\upsilon)(R_b-f_b)+\left[(\pi+\upsilon)^2+2\pi\upsilon\right]}{4(\tau_b-\alpha)^2} < 0$, $\frac{\partial PS^{bmh}}{\partial\beta} = -\frac{3}{2} < 0$, $\frac{\partial PS^{bmh}}{\partial\upsilon} = \frac{(R_b-f_b)+(2\pi+\upsilon)}{2(\tau_b-\alpha)} > 0$ and $\frac{\partial PS^{bmh}}{\partial\pi} = \frac{(R_b-f_b)+(\pi+2\upsilon)}{2(\tau_b-\alpha)} > 0$

Platforms Profits

Partially differentiate equilibrium platform's profits at Equation 1.28 regarding the parameter of the model. When there is a bandwagon effect $\frac{\partial\Pi^{bmh}}{\partial\alpha} = \frac{4(R_b-f_b)^2-\left[(\pi+\upsilon)^2+4\pi\upsilon\right]}{16(\tau_b-\alpha)^2}$

> 0 if $2(R_b - f_b) > \sqrt{(\pi+\upsilon)^2 + 4\pi\upsilon}$. To show the previous expression is satisfied we use Assumption 1.2, where $2(R_b - f_b) > 3(\tau_b - \alpha)$, we compare both expressions by making the left side of both inequalities equal to compare the right side and identify which is larger. That is $3(\tau_b - \alpha) > \sqrt{(\pi+\upsilon)^2 + 4\pi\upsilon}$. This condition is compatible with Assumption 1.1 as long as $4(\tau_s + \beta) < 3(\pi+\upsilon)^2 - \sqrt{(\pi+\upsilon)^2 + 4\pi\upsilon}$. When there is a congestion effect $\partial\Pi^{bmh}/\partial\alpha = -\frac{4(R_b-f_b)^2 - \left[(\pi+\upsilon)^2+4\pi\upsilon\right]}{16(\tau_b-\alpha)^2} > 0$ if $2(R_b - f_b) < \sqrt{(\pi+\upsilon)^2 + 4\pi\upsilon}$. To show the previous expression is satisfied we follow the same steps as previously. That is using Assumption 1.2, we have $4(\tau_b - \alpha) > \sqrt{(\pi+\upsilon)^2 + 4\pi\upsilon}$. This condition is compatible with Assumption 1.1 as long as $(\tau_s + \beta) < \frac{(\pi+\upsilon)^2}{\sqrt{(\pi+\upsilon)^2+4\pi\upsilon}}$. $\partial\Pi^{bmh}/\partial\beta = \frac{1}{2} > 0$, $\partial\Pi^{bmh}/\partial\upsilon = -\frac{(3\pi+\upsilon)}{8(\tau_b-\alpha)} < 0$ and $\partial\Pi^{bmh}/\partial\pi = -\frac{(\pi+3\upsilon)}{8(\tau_b-\alpha)} < 0$.

A.5 Multihome on both sides

A.5.1 Market's Shares

We solve the system of Equation 1.12 and Equation 1.22 to obtain buyers, η_b^i and sellers, η_s^i participation as a function of the membership fees. First, we substitute Equation 1.12 into Equation 1.22 and then we do the opposite, substitute Equation 1.22 into Equation 1.12 to have:

$$(\tau_s + \beta)\,\eta_s^i = R_s + \pi\left(\frac{R_b + \upsilon\eta_s^i - p_b^i}{\tau_b - \alpha}\right) - p_s^i \text{ then, } \eta_s^i = \frac{(\tau_b - \alpha)(R_s - p_s^i) + \pi(R_b - p_b^i)}{(\tau_b - \alpha)(\tau_s + \beta) - \pi\upsilon}$$
$$(\tau_b - \alpha)\,\eta_b^i = R_b - p_b^i + \upsilon\left(\frac{R_s + \pi\eta_b^i - p_s^i}{\tau_s + \beta}\right) \text{ then, } \eta_b^i = \frac{(\tau_s + \beta)(R_b - p_b^i) + \upsilon(R_s - p_s^i)}{(\tau_b - \alpha)(\tau_s + \beta) - \pi\upsilon}$$

A.5.2 Platform profit Maximisation

The first-order conditions of the platform maximisation problem set in Definition 1.4 using equations Equation 1.33a and Equation 1.33b are:

$$\frac{\partial\Pi}{\partial p_b^i} = \frac{(\tau_s + \beta)(R_b - p_b^i) + \upsilon(R_s - p_s^i)}{\Omega} - \frac{(\tau_s + \beta)(p_b^i - f_b)}{\Omega} - \frac{\pi(p_s^i - f_s)}{\Omega} = 0$$
$$\frac{\partial\Pi}{\partial p_s^i} = \frac{(\tau_b - \alpha)(R_s - p_s^i) + \pi(R_b - p_b^i)}{\Omega} - \frac{(\tau_b - \alpha)(p_s^i - f_s)}{\Omega} - \frac{\upsilon(p_b^i - f_b)}{\Omega} = 0$$

Where $\Omega \equiv [(\tau_s + \beta)(\tau_b - \alpha) - \pi\upsilon]$

From the first-order conditions for a symmetric equilibrium $p_b^i = p_b^j = p_b$ and $p_s^i =$

$p_s^j = p_s$, we obtain:

$$p_b^{mh} = \frac{(R_b + f_b)}{2} - \frac{1}{2(\tau_s + \beta)}\Big[(\pi + \upsilon) p_s - (\upsilon R_s + \pi f_s)\Big] \tag{13}$$

$$p_s^{mh} = \frac{(R_s + f_s)}{2} - \frac{1}{(\tau_b - \alpha)}\Big[(\pi + \upsilon) p_b - (\pi R_b + \upsilon f_b)\Big] \tag{14}$$

The next step is to solve the previous system of equations (13) and (14) to have explicit expressions for the membership fees. First, we substitute p_s^{mh} from equation (14) into equation (13) and solve for p_b^{mh}.

$$\begin{aligned}
2(\tau_s + \beta) p_b^{mh} &= (\tau_s + \beta)(R_b + f_b) + \upsilon R_s + \pi f_s \\
&\quad - (\pi + \upsilon)\left(\frac{(\tau_b - \alpha)(R_s + f_s) - (\pi + \upsilon) p_b^{mh} + \pi R_b + \upsilon f_b}{2(\tau_b - \alpha)}\right) \\
4(\tau_b - \alpha)(\tau_s + \beta) p_b^{mh} &= 2(\tau_b - \alpha)(\tau_s + \beta)(R_b + f_b) + 2(\upsilon R_s + \pi f_s)(\tau_b - \alpha) \\
&\quad - (\pi + \upsilon)(\tau_b - \alpha)(R_s + f_s) + (\pi + \upsilon)^2 p_b^{mh} - (\pi + \upsilon)(\pi R_b + \upsilon f_b) \\
\left(4(\tau_b - \alpha)(\tau_s + \beta) - (\pi + \upsilon)^2\right) p_b^{mh} &= 2(\tau_b - \alpha)(\tau_s + \beta)(R_b + f_b) \\
&\quad + (\tau_b - \alpha)((\upsilon - \pi) R_s - (\upsilon - \pi) f_s) - \left(\pi^2 R_b + \pi\upsilon f_b + \upsilon\pi R_b + \upsilon^2 f_b\right) \\
p_b^{mh} &= \frac{(2(\tau_b - \alpha)(\tau_s + \beta) - \pi\upsilon)(R_b + f_b) - (\pi^2 R_b + \upsilon^2 f_b) + (\tau_b - \alpha)(\upsilon - \pi)(R_s - f_s)}{4(\tau_b - \alpha)(\tau_s + \beta) - (\pi + \upsilon)^2}
\end{aligned}$$

Then, we can substitute the previous result into equation (13) or we substitute p_b^{mh} from equation (13) into equation (14) and solve for p_s^{mh} which is straightforward.

$$\begin{aligned}
2(\tau_b - \alpha) p_s^{mh} &= (\tau_b - \alpha)(R_s + f_s) + \pi R_b + \upsilon f_b \\
&\quad - (\pi + \upsilon)\left(\frac{(\tau_s + \beta)(R_b + f_b) - (\pi + \upsilon) p_s^{mh} + \upsilon R_s + \pi f_s}{2(\tau_s + \beta)}\right) \\
4(\tau_b - \alpha)(\tau_s + \beta) p_s^{mh} &= 2(\tau_b - \alpha)(\tau_s + \beta)(R_s + f_s) + 2(\pi R_b + \upsilon f_b)(\tau_s + \beta) \\
&\quad - (\pi + \upsilon)(\tau_s + \beta)(R_b + f_b) + (\pi + \upsilon)^2 p_s^{mh} - (\pi + \upsilon)(\upsilon R_s + \pi f_s) \\
\left(4(\tau_b - \alpha)(\tau_s + \beta) - (\pi + \upsilon)^2\right) p_s^{mh} &= 2(\tau_b - \alpha)(\tau_s + \beta)(R_s + f_s) \\
&\quad + (\tau_s + \beta)((R_b - f_b)\pi - \upsilon(R_b - f_b)) - \pi\upsilon(R_s + f_s) - \left(\upsilon^2 R_s + \pi^2 f_s\right) \\
p_s^{mh} &= \frac{(2(\tau_b - \alpha)(\tau_s + \beta) - \pi\upsilon)(R_s + f_s) - (\upsilon^2 R_s + \pi^2 f_s) + (\tau_s + \beta)(\pi - \upsilon)(R_b - f_b)}{4(\tau_b - \alpha)(\tau_s + \beta) - (\pi + \upsilon)^2}
\end{aligned}$$

As a result, the solution for the system of equations (13) and (14) are:

$$\begin{aligned}
p_b^{mh} &= \frac{(2(\tau_b - \alpha)(\tau_s + \beta) - \pi\upsilon)(R_b + f_b) - (\pi^2 R_b + \upsilon^2 f_b) + (\tau_b - \alpha)(\upsilon - \pi)(R_s - f_s)}{4(\tau_b - \alpha)(\tau_s + \beta) - (\pi + \upsilon)^2} \\
p_s^{mh} &= \frac{(2(\tau_b - \alpha)(\tau_s + \beta) - \pi\upsilon)(R_s + f_s) - (\upsilon^2 R_s + \pi^2 f_s) + (\tau_s + \beta)(\pi - \upsilon)(R_b - f_b)}{4(\tau_b - \alpha)(\tau_s + \beta) - (\pi + \upsilon)^2}
\end{aligned}$$

A.5.3 Second order conditions - Two-sided Multihome

We obtain the following second-order conditions from the profit-maximisation problem of Appendix A.5.2 to define the Hessian matrix:

$$\text{H} = \begin{pmatrix} \Pi^i_{p_b p_b} \equiv -\frac{2(\tau_s+\beta)}{(\tau_s+\beta)(\tau_b-\alpha)-\pi\upsilon} & \Pi^i_{p_b p_s} \equiv -\frac{(\pi+\upsilon)}{(\tau_s+\beta)(\tau_b-\alpha)-\pi\upsilon} \\ \Pi^i_{p_s p_b} \equiv -\frac{(\pi+\upsilon)}{(\tau_s+\beta)(\tau_b-\alpha)-\pi\upsilon} & \Pi^i_{p_s p_s} \equiv -\frac{2(\tau_b-\alpha)}{(\tau_s+\beta)(\tau_b-\alpha)-\pi\upsilon} \end{pmatrix}$$

The Hessian matrix obtained previously resembles the one derived in Appendix A.2.3, thus the criteria for it to be negative definite remain unchanged. For detailed information on how these criteria are obtained refer to Appendix A.2.3.

A.5.4 Buyers' and Sellers' market shares

To obtain buyers' market share in Equation 1.33a, we use Equation 1.35a and Equation 1.35b.

$$\begin{aligned}
\eta_b^{mh} = \Bigg[& (\tau_s+\beta)\Big[R_b - \Big[(2(\tau_b-\alpha)(\tau_s+\beta)-\pi\upsilon)(R_b+f_b) - \big(\pi^2 R_b + \upsilon^2 f_b\big) + (\tau_b-\alpha) \\
& \big(\upsilon-\pi\big)(R_s-f_s)\Big] \Big/ \left[4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2\right]\Big] + \upsilon\Big[R_s - \Big[\big(2(\tau_b-\alpha)(\tau_s+\beta) \\
& -\pi\upsilon\big)(R_s+f_s) - \big(\upsilon^2 R_s + \pi^2 f_s\big) + (\tau_s+\beta)(\pi-\upsilon)(R_b-f_b)\Big] \Big/ \Big[4(\tau_b-\alpha)(\tau_s+\beta) - \\
& (\pi+\upsilon)^2\Big]\Big]\Bigg] \Big/ \left[(\tau_b-\alpha)(\tau_s+\beta)-\pi\upsilon\right] \\
= \Bigg[& (\tau_s+\beta)R_b\left[4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2\right] - (2(\tau_b-\alpha)(\tau_s+\beta)-\pi\upsilon)(R_b+f_b)\big(\tau_s \\
& +\beta\big) + \big(\pi^2 R_b+\upsilon^2 f_b\big)(\tau_s+\beta) - (\tau_b-\alpha)(\upsilon-\pi)(R_s-f_s)(\tau_s+\beta)\Bigg] \Big/ \Big[4(\tau_b-\alpha)(\tau_s+\beta) \\
& -(\pi+\upsilon)^2\Big]\left[(\tau_b-\alpha)(\tau_s+\beta)-\pi\upsilon\right] + \Bigg[\upsilon R_s\left[4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2\right] - \\
& \upsilon\big(2(\tau_b-\alpha)(\tau_s+\beta)-\pi\upsilon\big)(R_s+f_s) + \upsilon\big(\upsilon^2 R_s+\pi^2 f_s\big) - \upsilon(\tau_s+\beta)(\pi-\upsilon) \\
& (R_b-f_b)\Bigg] \Big/ \left[4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2\right]\left[(\tau_b-\alpha)(\tau_s+\beta)-\pi\upsilon\right] \\
= \Big[& 2(\tau_b-\alpha)(\tau_s+\beta)^2(R_b-f_b) + (\tau_s+\beta)\left[-\pi^2 R_b - 2\pi\upsilon R_b - \upsilon^2 R_b + \pi\upsilon f_b + \pi^2 R_b\right. \\
& \left.+\upsilon^2 R_b + \pi\upsilon f_b\right] + \upsilon(\tau_b-\alpha)(\tau_s+\beta)(R_s-f_s) + \pi(\tau_b-\alpha)(\tau_s+\beta)(R_s-f_s) \\
& +\upsilon\left[-\pi^2 R_s - 2\pi\upsilon R_s - \upsilon^2 R_s + \pi\upsilon R_s + \pi\upsilon f_s + \upsilon^2 R_s + \pi^2 f_s\right]\Big] \Big/ \\
& \left[4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2\right]\left[(\tau_b-\alpha)(\tau_s+\beta)-\pi\upsilon\right] \\
= \Big[& 2(\tau_s+\beta)(R_b-f_b)\left[(\tau_b-\alpha)(\tau_s+\beta)-\pi\upsilon\right] + (\tau_b-\alpha)(\tau_s+\beta)(R_s-f_s)(\pi+\upsilon) \\
& -\upsilon\pi(\pi+\upsilon)(R_s-f_s)\Big] \Big/ \left[4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2\right]\left[(\tau_b-\alpha)(\tau_s+\beta)-\pi\upsilon\right]
\end{aligned}$$

$$\eta_b^{mh} = \frac{2\left(\tau_s+\beta\right)\left(R_b-f_b\right)+\left(\pi+\upsilon\right)\left(R_s-f_s\right)}{4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)^2}$$

To obtain sellers' market share in Equation 1.33b, we use Equation 1.35a and Equation 1.35b.

$$\eta_s^{mh} = \Bigg[\left(\tau_b-\alpha\right)\Big[R_s - \Big[\left(2\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\pi\upsilon\right)\left(R_s+f_s\right)-\left(\upsilon^2R_s+\pi^2f_s\right)+\left(\tau_s+\beta\right)$$
$$\left(\pi-\upsilon\right)\left(R_b-f_b\right)\Big]\Big/\left[4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)^2\right]\Big]+\pi\Big[R_b-\Big[\big(2\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)$$
$$-\pi\upsilon\big)\left(R_b+f_b\right)-\left(\pi^2R_b+\upsilon^2f_b\right)+\left(\tau_b-\alpha\right)\left(\upsilon-\pi\right)\left(R_s-f_s\right)\Big]\Big/\big[4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-$$
$$\left(\pi+\upsilon\right)^2\big]\Big]\Bigg]\Bigg/\left[\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\pi\upsilon\right]$$
$$= \Big[R_s\left(\tau_b-\alpha\right)\left[4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)^2\right]-\left(2\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\pi\upsilon\right)\left(R_s+f_s\right)\big(\tau_b$$
$$-\alpha\big)+\left(\upsilon^2R_s+\pi^2f_s\right)\left(\tau_b-\alpha\right)-\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)\left(\pi-\upsilon\right)\left(R_b-f_b\right)\Big]\Big/\Big[4\left(\tau_b-\alpha\right)$$
$$\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)^2\Big]\left[\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\pi\upsilon\right]+\Big[\pi R_b\left[4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)^2\right]$$
$$-\pi\left(2\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\pi\upsilon\right)\left(R_b+f_b\right)+\pi\left(\pi^2R_b+\upsilon^2f_b\right)-\pi\left(\tau_b-\alpha\right)\left(\upsilon-\pi\right)$$
$$\left(R_s-f_s\right)\Big]\Big/\left[4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)^2\right]\left[\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\pi\upsilon\right]$$
$$= \Big[2\left(\tau_b-\alpha\right)^2\left(\tau_s+\beta\right)\left(R_s-f_s\right)+\left(\tau_b-\alpha\right)\big[-\pi^2R_s-2\pi\upsilon R_s-\upsilon^2R_s+\pi\upsilon f_s+\upsilon^2R_s$$
$$+\pi^2R_s+\pi\upsilon f_s\big]+\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)\left[\upsilon\left(R_b-f_b\right)+\pi\left(R_b-f_b\right)\right]+\pi\big[-\pi^2R_b-2\pi\upsilon R_b$$
$$-\upsilon^2R_b+\pi\upsilon R_b+\pi\upsilon f_b+\pi^2R_b+\upsilon^2f_b\big]\Big]$$
$$\Big/\left[4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)^2\right]\left[\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\pi\upsilon\right]$$
$$= \Big[2\left(\tau_b-\alpha\right)\left(R_s-f_s\right)\left[\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\pi\upsilon\right]+\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)\left(R_b-f_b\right)\left(\pi+\upsilon\right)$$
$$-\upsilon\pi\left(\pi+\upsilon\right)\left(R_b-f_b\right)\Big]\Big/\left[4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)^2\right]\left[\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\pi\upsilon\right]$$
$$\eta_s^{mh} = \frac{2\left(\tau_b-\alpha\right)\left(R_s-f_s\right)+\left(\pi+\upsilon\right)\left(R_b-f_b\right)}{4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)^2}$$

A.5.5 Platform profits

We use Equation 1.35a, Equation 1.35b, Equation 1.36a and Equation 1.36b to compute equilibrium platform profits as:

$$\Pi^{mh} = \left(p_b^{mh}-f_b\right)\eta_b^{mh}+\left(p_s^{mh}-f_s\right)\eta_s^{mh}$$

$$= \left[\frac{(2(\tau_b-\alpha)(\tau_s+\beta)-\pi\upsilon)(R_b+f_b)-\left(\pi^2R_b+\upsilon^2f_b\right)+(\tau_b-\alpha)(\upsilon-\pi)(R_s-f_s)}{4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2} - f_b\right]\left[\frac{2(\tau_s+\beta)(R_b-f_b)+(\pi+\upsilon)(R_s-f_s)}{4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2}\right] +$$
$$\left[\frac{(2(\tau_b-\alpha)(\tau_s+\beta)-\pi\upsilon)(R_s+f_s)-\left(\upsilon^2R_s+\pi^2f_s\right)+(\tau_s+\beta)(\pi-\upsilon)(R_b-f_b)}{4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2} - f_s\right]\left[\frac{2(\tau_b-\alpha)(R_s-f_s)+(\pi+\upsilon)(R_b-f_b)}{4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2}\right]$$
$$= \Big[2(\tau_b-\alpha)(\tau_s+\beta)[R_b+f_b-2f_b] - \left[\pi\upsilon R_b+\pi\upsilon f_b+\pi^2R_b+\upsilon^2f_b-\pi^2f_b-2\pi\upsilon f_b-\upsilon^2f_b\right]$$
$$+(\tau_b-\alpha)(\upsilon-\pi)(R_s-f_s)\Big]\left[\frac{2(\tau_s+\beta)(R_b-f_b)+(\pi+\upsilon)(R_s-f_s)}{\left[4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2\right]^2}\right] +$$
$$\Big[2(\tau_b-\alpha)(\tau_s+\beta)[R_s+f_s-2f_s] - \left[\pi\upsilon R_s+\pi\upsilon f_s+\upsilon^2R_s+\pi^2f_s-\pi^2f_s-2\pi\upsilon f_s-\upsilon^2f_s\right]$$
$$+(\tau_s+\beta)(\pi-\upsilon)(R_b-f_b)\Big]\left[\frac{2(\tau_b-\alpha)(R_s-f_s)+(\pi+\upsilon)(R_b-f_b)}{\left[4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2\right]^2}\right]$$
$$= \Big[4(\tau_b-\alpha)(\tau_s+\beta)^2(R_b-f_b)^2 + 2(\tau_b-\alpha)(\tau_s+\beta)(R_b-f_b)(\pi+\upsilon)(R_s-f_s) - 2\pi\big(\tau_s$$
$$+\beta\big)(\pi+\upsilon)(R_b-f_b)^2 - \pi(\pi+\upsilon)^2(R_b-f_b)(R_s-f_s) + 2(\tau_b-\alpha)(\tau_s+\beta)(\upsilon-\pi)\big(R_s$$
$$-f_s\big)(R_b-f_b) + (\tau_b-\alpha)(\upsilon-\pi)(\pi+\upsilon)(R_s-f_s)^2 + 4(\tau_b-\alpha)^2(\tau_s+\beta)(R_s-f_s)^2$$
$$+2(\tau_b-\alpha)(\tau_s+\beta)(R_s-f_s)(\pi+\upsilon)(R_b-f_b) - 2\upsilon(\tau_b-\alpha)\big(\pi+\upsilon\big)(R_s-f_s)^2$$
$$-\upsilon(\pi+\upsilon)^2(R_s-f_s)(R_b-f_b) - 2(\tau_b-\alpha)(\tau_s+\beta)(\upsilon-\pi)(R_b-f_b)(R_s-f_s) + \big(\tau_s+$$
$$\beta\big)(\pi-\upsilon)(\pi+\upsilon)(R_b-f_b)^2\Big] \Big/ \left[4(\tau_b/-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2\right]^2$$
$$= \Big[(\tau_s+\beta)(R_b-f_b)^2\left[4(\tau_b-\alpha)(\tau_s+\beta) - \left[2\pi^2+2\upsilon\pi-\pi^2-\pi\upsilon+\pi\upsilon+\upsilon^2\right]\right] - \big(\pi+$$
$$\upsilon\big)^3(R_b-f_b)(R_s-f_s) + (\tau_b-\alpha)(R_s-f_s)^2\left[4(\tau_b-\alpha)(\tau_s+\beta) - \left[2\upsilon\pi+2\upsilon^2-\upsilon\pi-\upsilon^2+\right.\right.$$
$$\left.\left.\pi^2+\pi\upsilon\right]\right] + 4(\tau_b-\alpha)(\tau_s+\beta)(R_b-f_b)(\pi+\upsilon)(R_s-f_s)\Big] \Big/ \left[4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2\right]^2$$
$$\Pi^{mh} \equiv \frac{(\tau_s+\beta)(R_b-f_b)^2 + (\tau_b-\alpha)(R_s-f_s)^2 + (\pi+\upsilon)(R_b-f_b)(R_s-f_s)}{4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2}$$

A.5.6 Surpluses

We use buyers surplus in Equation 1.1a, equilibrium membership fees in Equation 1.35a and equilibrium market shares in Equation 1.36a, and Equation 1.36b to compute buyers' gross (from transportation cost) surplus in equilibrium:

$$\nu_b^{mh} = R_b + \upsilon\eta_s^{mh} - \alpha\eta_b^{mh} - p_b^{mh}$$
$$= R_b + \upsilon\left[\frac{2(\tau_b-\alpha)(R_s-f_s)+(\pi+\upsilon)(R_b-f_b)}{4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2}\right] + \alpha\left[\frac{2(\tau_s+\beta)(R_b-f_b)+(\pi+\upsilon)(R_s-f_s)}{4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2}\right]$$
$$-\left[\frac{(2(\tau_b-\alpha)(\tau_s+\beta)-\pi\upsilon)(R_b+f_b)-(\pi^2R_b+\upsilon^2f_b)+(\tau_b-\alpha)(\upsilon-\pi)(R_s-f_s)}{4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2}\right]$$

$$= \Big[2(\tau_b - \alpha)(\tau_s + \beta)[2Rb - R_b - f_b] + (\tau_b - \alpha)(R_s - f_s)[2\upsilon - \upsilon + \pi] + (\pi + \upsilon)$$
$$[\upsilon R_b - \upsilon f_b - \pi R_b - \upsilon R_b] + 2\alpha(\tau_s + \beta)(R_b - f_b) + \alpha(\pi + \upsilon)(R_s - f_s) + \pi\upsilon(R_b + f_b)$$
$$+ \left(\pi^2 R_b + \upsilon^2 f_b\right)\Big] \Big/ \left[4(\tau_b - \alpha)(\tau_s + \beta) - (\pi + \upsilon)^2\right]$$
$$\nu_b^{mh} = \frac{\tau_b [2(\tau_s + \beta)(R_b - f_b) + (\pi + \upsilon)(R_s - f_s)]}{4(\tau_b - \alpha)(\tau_s + \beta) - (\pi + \upsilon)^2}$$

Next, we use sellers surplus in Equation 1.1b, equilibrium membership fees in Equation 1.35b and equilibrium market shares in Equation 1.36a, and Equation 1.36b to compute sellers' gross (from transportation cost) surplus in equilibrium:

$$\nu_s^{mh} = R_s + \pi\eta_b^{mh} - \beta\eta_s^{mh} - p_s^{mh}$$
$$= R_s + \pi\left[\frac{2(\tau_s+\beta)(R_b-f_b)+(\pi+\upsilon)(R_s-f_s)}{4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2}\right] - \beta\left[\frac{2(\tau_b-\alpha)(R_s-f_s)+(\pi+\upsilon)(R_b-f_b)}{4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2}\right]$$
$$-\left[\frac{(2(\tau_b - \alpha)(\tau_s + \beta) - \pi\upsilon)(R_s + f_s) - (\upsilon^2 R_s + \pi^2 f_s) + (\tau_s + \beta)(\pi - \upsilon)(R_b - f_b)}{4(\tau_b - \alpha)(\tau_s + \beta) - (\pi + \upsilon)^2}\right]$$
$$= \Big[2(\tau_b - \alpha)(\tau_s + \beta)[2R_s - R_s + f_s] + (\tau_s + \beta)(R_b - f_b)[2\pi - \pi + \upsilon] + (\pi + \upsilon)$$
$$[\pi R_s - \pi f_s - \pi R_s - \upsilon R_s] - 2\beta(\tau_b - \alpha)(R_s - f_s) - \beta(\pi + \upsilon)(R_b - f_b) + \pi\upsilon(R_s + f_s)$$
$$+ \left(\upsilon^2 R_s + \pi^2 f_s\right)\Big] \Big/ \left[4(\tau_b - \alpha)(\tau_s + \beta) - (\pi + \upsilon)^2\right]$$
$$\nu_s^{mh} = \frac{\tau_s [2(\tau_b - \alpha)(R_s - f_s) + (\pi + \upsilon)(R_b - f_b)]}{4(\tau_b - \alpha)(\tau_s + \beta) - (\pi + \upsilon)^2}$$

A.5.7 Comparative Statics

We are only showing the impacts of an increase in a bandwagon effect. Conversely, the impact of an increase in congestion effect produces the opposite outcome.

Membership fees

Partially differentiate equilibrium membership fees at Equation 1.35a and Equation 1.35b regarding the parameters of the model. On buyers' side when there is a bandwagon effect

$$\frac{\partial p_b^{mh}}{\partial \alpha} = \frac{1}{\epsilon^2}\Big[(-2(\tau_s + \beta)(R_b + f_b) - (\upsilon - \pi)(R_s - f_s))\left(4(\tau_b - \alpha)(\tau_s + \beta) - (\pi + \upsilon)^2\right)$$
$$+4(\tau_s + \beta)\Big((2(\tau_b - \alpha)(\tau_s + \beta) - \pi\upsilon)(R_b + f_b) + \left(\pi^2 R_b + \upsilon^2 f_b\right) - (\tau_b - \alpha)(\upsilon - \pi)$$
$$(R_s - f_s)\Big)\Big], \text{ where } \epsilon \equiv 4(\tau_b - \alpha)(\tau_s + \beta) - (\pi + \upsilon)^2$$

$$= \Big[-8(\tau_s+\beta)^2(\tau_b-\alpha)(R_b+f_b)+2(\tau_s+\beta)(R_b+f_b)(\pi+\upsilon)^2-4(\tau_b-\alpha)(\tau_s+\beta)$$
$$\big(\upsilon-\pi\big)(R_s-f_s)+(\upsilon-\pi)(R_s-f_s)(\pi+\upsilon)^2+8(\tau_s+\beta)^2(\tau_b-\alpha)(R_b+f_b)-4\big(\tau_s+$$
$$\beta\big)\pi\upsilon(R_b+f_b)-4(\tau_s+\beta)\big(\pi^2R_b+\upsilon^2 f_b\big)+4(\tau_b-\alpha)(\tau_s+\beta)(\upsilon-\pi)(R_s-f_s)\Big]\Big/\epsilon^2$$
$$= \frac{1}{\epsilon^2}\Big[2(\tau_s+\beta)\left[-2(R_b+f_b)\pi\upsilon-2\pi^2R_b+2\upsilon^2 f_b+(R_b+f_b)(\pi+\upsilon)^2\right]$$
$$+(\upsilon-\pi)(R_s-f_s)(\pi+\upsilon)^2\Big]$$
$$= \frac{1}{\epsilon^2}\Big[2(\tau_s+\beta)\big(\upsilon^2-\pi^2\big)(R_b-f_b)+(\upsilon-\pi)(R_s-f_s)(\pi+\upsilon)^2\Big]$$
$$\frac{\partial p_b^{mh}}{\partial\alpha} = \frac{(\upsilon-\pi)(\pi+\upsilon)\left[2(\tau_s+\beta)(R_b-f_b)+(\pi+\upsilon)(R_s-f_s)\right]}{\left[4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)\right]^2} > 0 \ \text{ if } \ \upsilon>\pi$$

$$\frac{\partial p_b^{mh}}{\partial\beta} = \frac{1}{\epsilon^2}\Big[2(\tau_b-\alpha)(R_b+f_b)\big(4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2\big)-4(\tau_b-\alpha)$$
$$\Big((2(\tau_b-\alpha)(\tau_s+\beta)-\pi\upsilon)(R_b+f_b)-\big(\pi^2R_b+\upsilon^2 f_b\big)+(\tau_b-\alpha)(\upsilon-\pi)(R_s-f_s)\Big)\Big]$$
$$= \frac{2(\tau_b-\alpha)}{\epsilon^2}\Big[4(\tau_b-\alpha)(\tau_s+\beta)(R_b+f_b)-(\pi+\upsilon)^2(R_b+f_b)-4(\tau_b-\alpha)(\tau_s+\beta)$$
$$(R_b+f_b)+2\pi\upsilon(R_b+f_b)+2\big(\pi^2R_b+\upsilon^2 f_b\big)-2(\tau_b-\alpha)(\upsilon-\pi)(R_s-f_s)\Big]$$
$$= \frac{2(\tau_b-\alpha)}{\epsilon^2}\Big[\big(\pi^2-\upsilon^2\big)(R_b-f_b)+2(\tau_b-\alpha)(\pi-\upsilon)(R_s-f_s)\Big]$$
$$\frac{\partial p_b^{mh}}{\partial\beta} = \frac{2(\tau_b-\alpha)(\pi-\upsilon)\big[2(\tau_b-\alpha)(R_s-f_s)+(\pi+\upsilon)\big(R_b-f_b\big)\big]}{\left[4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)\right]^2} > 0 \ \text{ if } \ \pi>\upsilon$$

$$\frac{\partial p_b^{mh}}{\partial\upsilon} = \frac{1}{\epsilon^2}\Big[(-\pi(R_b+f_b)-2\upsilon f_b+(\tau_b-\alpha)(R_s-f_s))\big(4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2\big)$$
$$+2(\pi+\upsilon)\Big((2(\tau_b-\alpha)(\tau_s+\beta)-\pi\upsilon)(R_b+f_b)-\big(\pi^2R_b+\upsilon^2 f_b\big)+(\tau_b-\alpha)(\upsilon-\pi)$$
$$(R_s-f_s)\Big)\Big], \text{ where } \epsilon\equiv 4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2$$
$$= \frac{1}{\epsilon^2}\Big[4(\tau_b-\alpha)(\tau_s+\beta)\Big[-\pi(R_b+f_b)-2\upsilon f_b+(\pi+\upsilon)(R_b+f_b)\Big]+(\pi+\upsilon)\Big[\pi\big(\pi+\upsilon\big)$$
$$\big(R_b+f_b\big)+2\upsilon f_b(\pi+\upsilon)-2\pi\upsilon(R_b+f_b)-2\pi^2R_b-2\upsilon^2 f_b\Big]+4(\tau_b-\alpha)^2(\tau_s+\beta)$$
$$(R_s-f_s)+(\tau_b-\alpha)(\pi+\upsilon)(R_s-f_s)\left[2\upsilon-2\pi-\pi-\upsilon\right]\Big]$$
$$= \frac{1}{\epsilon^2}\Big[4(\tau_b-\alpha)(\tau_s+\beta)(R_b-f_b)-\pi(\pi+\upsilon)^2(R_b-f_b)+(\tau_b-\alpha)(R_s-f_s)\big[4(\tau_b-\alpha)$$
$$(\tau_s+\beta)-(\pi+\upsilon)(3\pi+\upsilon)\big]\Big]$$
$$\frac{\partial p_b^{mh}}{\partial\upsilon} = \frac{1}{\epsilon^2}\Big[\left[4\upsilon(\tau_b-\alpha)(\tau_s+\beta)-\pi(\pi+\upsilon)^2\right](R_b-f_b)+(\tau_b-\alpha)(R_s-f_s)$$
$$\big[4\big(\tau_b-\alpha\big)(\tau_s+\beta)-(\pi+\upsilon)(3\pi-\upsilon)\big]\Big]$$

For the previous expression to be positive we need to have the numerator positive considering the denominator is always positive. To have a positive numerator, it is sufficient that the expressions $4\upsilon(\tau_b-\alpha)(\tau_s+\beta)-\pi(\pi+\upsilon)^2$ and $4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)(3\pi-\upsilon)$ are positive. We use Assumption 1.1 to establish it. First, we make the left side of both inequalities equal to the left side of Assumption 1.1 to compare that the right side of Assumption 1.1 is greater. Consequently, both expressions are confirmed to be positive. Then for the first expression we have $(\pi+\upsilon)^2>\frac{\pi}{\upsilon}(\pi+\upsilon)^2$ if $\upsilon>\pi$ and for the second expression we have $(\pi+\upsilon)^2>(\pi+\upsilon)(3\pi-\upsilon)$ turns to $\pi+\upsilon>3\pi-\upsilon$ if $\upsilon>\pi$. Therefore, $\partial p_b^{mh}/\partial\upsilon>0$ if $\upsilon>\pi$.

$$\begin{aligned}\frac{\partial p_b^{mh}}{\partial\pi}&=\frac{1}{\epsilon^2}\Big[\left(-\upsilon(R_b+f_b)-2\pi R_b-(\tau_b-\alpha)(R_s-f_s)\right)\big(4(\tau_b-\alpha)(\tau_s+\beta)\\&-(\pi+\upsilon)^2\big)+2(\pi+\upsilon)\Big(\left(2(\tau_b-\alpha)(\tau_s+\beta)-\pi\upsilon\right)(R_b+f_b)-\left(\pi^2R_b+\upsilon^2f_b\right)+\\&(\tau_b-\alpha)(\upsilon-\pi)(R_s-f_s)\Big)\Big]\text{ , where }\epsilon\equiv4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2\\&=-\frac{1}{\epsilon^2}\Big[4(\tau_b-\alpha)(\tau_s+\beta)\Big[-(\pi+\upsilon)(R_b+f_b)+\upsilon(R_b+f_b)+2\pi R_b\Big]-(\pi+\upsilon)\Big[-\upsilon\\&(\pi+\upsilon)(R_b+f_b)-2\pi(\pi+\upsilon)R_b+2\pi\upsilon(R_b+f_b)+2\pi^2R_b+2\upsilon^2f_b\Big]+(\tau_b-\alpha)\\&(R_s-f_s)\left[4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)\left[\pi+\upsilon+2\upsilon-2\pi\right]\right]\\\frac{\partial p_b^{mh}}{\partial\pi}&=-\frac{1}{\epsilon^2}\Big[\left[4\pi(\tau_b-\alpha)(\tau_s+\beta)-\upsilon(\pi+\upsilon)^2\right](R_b-f_b)+(\tau_b-\alpha)(R_s-f_s)\\&\left[4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)(3\upsilon-\pi)\right]\Big]\end{aligned}$$

To show that the previous expression is negative we follow the same method as with $\partial p_b^{mh}/\partial\upsilon$. Then, comparing the right-hand side of the first expression on the numerator, we have $(\pi+\upsilon)^2>\frac{\upsilon}{\pi}(\pi+\upsilon)^2$ if $\pi>\upsilon$ and for the second expression we have $(\pi+\upsilon)^2>(\pi+\upsilon)(3\upsilon-\pi)$ turns to $\pi+\upsilon>3\upsilon-\pi$ if $\pi>\upsilon$. Therefore, $\partial p_b^{mh}/\partial\pi<0$ if $\pi>\upsilon$.

On sellers' side when there is a bandwagon effect $\partial p_s^{mh}/\partial\alpha=\frac{1}{\epsilon^2}\left[(\tau_s+\beta)(\pi-\upsilon)\left[2(\tau_s+\beta)(R_b-f_b)+(\pi+\upsilon)(R_s-f_s)\right]\right]>0$ if $\pi>\upsilon$. Where $\epsilon\equiv4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2$. $\partial p_s^{mh}/\partial\beta=\frac{1}{\epsilon^2}\left[(\pi+\upsilon)(\upsilon-\pi)\left[2(\tau_b-\alpha)(R_s-f_s)+(\pi+\upsilon)(R_b-f_b)\right]\right]>0$ if $\upsilon>\pi$.

$$\begin{aligned}\frac{\partial p_s^{mh}}{\partial\upsilon}&=\frac{1}{\epsilon^2}\Big[\left(-\pi(R_s+f_s)-2\upsilon R_s-(\tau_s+\beta)(R_b-f_b)\right)\left(4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2\right)\\&+2(\pi+\upsilon)\Big(\left(2(\tau_b-\alpha)(\tau_s+\beta)-\pi\upsilon\right)(R_s+f_s)-\left(\upsilon^2R_s+\pi^2f_s\right)+(\tau_s+\beta)(\pi-\upsilon)\\&(R_b-f_b)\Big)\Big]\text{ , where }\epsilon\equiv4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2\end{aligned}$$

$$= -\frac{1}{\epsilon^2}\Big[4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)\big[\pi\left(R_s+f_s\right)+2\upsilon R_s-\left(\pi+\upsilon\right)\left(R_s+f_s\right)\big]-\left(\pi+\upsilon\right)\big[-\pi$$

$$\big(\pi+\upsilon\big)\big(R_s+f_s\big)-2\upsilon R_s\left(\pi+\upsilon\right)+2\pi\upsilon\left(R_s+f_s\right)+2\upsilon^2R_s+2\pi^2f_s\big]+\left(\tau_s+\beta\right)$$

$$\left(R_b-f_b\right)\big[4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)\big(\pi+\upsilon+2\pi-2\upsilon\big)\big]$$

$$\frac{\partial p_s^{mh}}{\partial \upsilon} = -\frac{1}{\epsilon^2}\Big[\big[4\upsilon\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\pi\left(\pi+\upsilon\right)^2\big]\left(R_s-f_s\right)+\left(\tau_s+\beta\right)\big(R_b-f_b\big)$$

$$\big[4\big(\tau_b-\alpha\big)\big(\tau_s+\beta\big)-\left(\pi+\upsilon\right)\left(3\pi-\upsilon\right)\big]\Big]$$

Following the same method used on the impacts on buyers' side, we need both conditions on the numerator to be positive for $\partial p_s^{mh}/\partial\upsilon<0$. Both conditions $4\upsilon\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\pi\left(\pi+\upsilon\right)^2$ and $4\big(\tau_b-\alpha\big)\big(\tau_s+\beta\big)-\left(\pi+\upsilon\right)\left(3\pi-\upsilon\right)$ are the same as in $\partial p_b^{mh}/\partial\upsilon$, therefore $\partial p_s^{mh}/\partial\upsilon<0$ if $\upsilon>\pi$.

$$\frac{\partial p_s^{mh}}{\partial \pi} = \frac{1}{\epsilon^2}\Big[\left(-\upsilon\left(R_s+f_s\right)-2\pi f_s+\left(\tau_s+\beta\right)\left(R_b-f_b\right)\right)\big(4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)$$

$$-\left(\pi+\upsilon\right)^2\big)+2\left(\pi+\upsilon\right)\big(\left(2\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\pi\upsilon\right)\left(R_s+f_s\right)-\big(\upsilon^2R_s+\pi^2f_s\big)+$$

$$\left(\tau_s+\beta\right)\left(\pi-\upsilon\right)\left(R_b-f_b\right)\big)\Big]\text{ , where }\epsilon\equiv4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)^2$$

$$= \frac{1}{\epsilon^2}\Big[4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)\Big[\left(\pi+\upsilon\right)\left(R_s+f_s\right)-\upsilon\left(R_s+f_s\right)-2\pi f_s\Big]-\left(\pi+\upsilon\right)\Big[-\upsilon$$

$$\left(\pi+\upsilon\right)\left(R_s+f_s\right)-2\pi\left(\pi+\upsilon\right)f_s+2\pi\upsilon\left(R_s+f_s\right)+2\upsilon^2R_s+2\pi^2f_s\Big]+\left(\tau_s+\beta\right)$$

$$\left(R_b-f_b\right)\big[4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)\big[\pi+\upsilon+2\upsilon-2\pi\big]\big]$$

$$\frac{\partial p_b^{mh}}{\partial \pi} = \frac{1}{\epsilon^2}\Big[\big[4\pi\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\upsilon\left(\pi+\upsilon\right)^2\big]\left(R_s-f_s\right)+\left(\tau_s+\beta\right)\left(R_b-f_b\right)$$

$$\big[4\big(\tau_b-\alpha\big)\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)\left(3\upsilon-\pi\right)\big]\Big]$$

Both conditions $4\pi\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\upsilon\left(\pi+\upsilon\right)^2$ and $4\big(\tau_b-\alpha\big)\big(\tau_s+\beta\big)-\left(\pi+\upsilon\right)\left(3\upsilon-\pi\right)$ are the same as in $\partial p_s^{mh}/\partial\upsilon$, therefore $\partial p_s^{mh}/\partial\pi>0$ if $\pi>\upsilon$.

Buyers and sellers market shares

Partially differentiate equilibrium market shares at Equation 1.36a and Equation 1.36b regarding the parameters of the model. On buyers' side $\partial\eta_b^{mh}/\partial\alpha=\frac{4(\tau_s+\beta)}{\epsilon^2}\big[2\left(\tau_s+\beta\right)\big(R_b-f_b\big)+\big(\pi+\upsilon\big)\big(R_s-f_s\big)\big]>0$, where $\epsilon\equiv4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)^2$. $\partial\eta_s^{mh}/\partial\beta=\frac{1}{\epsilon^2}\Big[2\left(R_b-f_b\right)\big[4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)^2\big]-4\big(\tau_b-\alpha\big)\big[2\big(\tau_s+\beta\big)\big(R_b-f_b\big)+\left(\pi+\upsilon\right)\big(R_s-f_s\big)\big]\Big]$ turns to $\partial\eta_s^{mh}/\partial\beta=-\frac{2(\pi+\upsilon)}{\epsilon^2}\Big[\big(\pi+\upsilon\big)\big(R_b-f_b\big)+2\left(\tau_b-\alpha\right)\big(R_s-f_s\big)\Big]<0$. $\partial\eta_b^{mh}/\partial\pi\equiv\partial\eta_b^{mh}/\partial\upsilon=\frac{1}{\epsilon^2}\Big[\left(R_s-f_s\right)\big[4\big(\tau_b-\alpha\big)\big(\tau_s+\beta\big)-\left(\pi+\upsilon\right)^2\big]+2\left(\pi+\upsilon\right)\big[2\left(\tau_s+\beta\right)\left(R_b-f_b\right)+\left(\pi+\upsilon\right)\left(R_s-f_s\right)\big]\Big]$ turns to $\partial\eta_b^{mh}/\partial\pi\equiv\partial\eta_b^{mh}/\partial\upsilon=\frac{1}{\epsilon^2}\Big[4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)\left(R_s-f_s\right)+\left(\pi+\upsilon\right)\big[4\left(\tau_s+\beta\right)\left(R_b-f_b\right)+\left(\pi+\upsilon\right)\big(R_s-f_s\big)\big]\Big]>0$.

On sellers' side $\partial\eta_s^{mh}/\partial\alpha = \frac{1}{\epsilon^2}\Big[-2\left(R_s - f_s\right)\left[4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)^2\right]+4\big(\tau_s+\beta\big)\big[2\left(\tau_b-\alpha\right)\left(R_s-f_s\right)+\left(\pi+\upsilon\right)\left(R_b-f_b\right)\big]\Big]$ turns to $\partial\eta_s^{mh}/\partial\alpha = \frac{2(\pi+\upsilon)}{\epsilon^2}\big[\left(\pi+\upsilon\right)\big(R_s-f_s\big)+2\left(\tau_s+\beta\right)\left(R_b-f_b\right)\big] > 0$. $\partial\eta_s^{mh}/\partial\beta = -\frac{4(\tau_b-\alpha)}{\epsilon^2}\Big[2\left(\tau_b-\alpha\right)\left(R_s-f_s\right)+\left(R_b-f_b\right)\big(\pi+\upsilon\big)\Big] < 0$. $\partial\eta_s^{mh}/\partial\pi \equiv \partial\eta_s^{mh}/\partial\upsilon = \frac{1}{\epsilon^2}\Big[\left(R_b-f_b\right)\left[4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)^2\right]+2\big(\pi+\upsilon\big)\big[2\left(\tau_b-\alpha\right)\left(R_s-f_s\right)+\left(R_b-f_b\right)\left(\pi+\upsilon\right)\big]\Big]$ turns to $\partial\eta_s^{mh}/\partial\pi \equiv \partial\eta_s^{mh}/\partial\upsilon = \frac{1}{\epsilon^2}\Big[4\big(\tau_b-\alpha\big)\left(\tau_s+\beta\right)\left(R_b-f_b\right)+\big(\pi+\upsilon\big)\big[4\left(\tau_b-\alpha\right)\left(R_s-f_s\right)+\big(\pi+\upsilon\big)\left(R_b-f_b\right)\big]\Big] > 0$. Where $\epsilon \equiv 4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)^2$

Aggregate Surpluses

Partially differentiate equilibrium aggregate surpluses at Equation 1.39a and Equation 1.39b regarding the model's parameters. Considering $CS^{mh} = \tau_b\left(\eta_b^{mh}\right)^2$ and $PS^{mh} = \tau_s\left(\eta_s^{mh}\right)^2$, Then $\partial CS^{mh}/\partial\alpha = 2\tau_b\left(\partial\eta_b^{mh}/\partial\alpha\right) > 0$. $\partial CS^{mh}/\partial\beta = 2\tau_b\left(\partial\eta_b^{mh}/\partial\beta\right) < 0$. $\partial CS^{mh}/\partial\upsilon = 2\tau_b\left(\partial\eta_b^{mh}/\partial\upsilon\right) > 0$ and $\partial CS^{mh}/\partial\pi = 2\tau_b\left(\partial\eta_b^{mh}/\partial\pi\right) > 0$. On seller's side, $\partial PS^{mh}/\partial\alpha = 2\tau_b\left(\partial\eta_s^{mh}/\partial\alpha\right) > 0$. $\partial PS^{mh}/\partial\beta = 2\tau_b\left(\partial\eta_s^{mh}/\partial\beta\right) < 0$. $\partial PS^{mh}/\partial\upsilon = 2\tau_b\left(\partial\eta_s^{mh}/\partial\upsilon\right) > 0$ and $\partial PS^{mh}/\partial\pi = 2\tau_b\left(\partial\eta_s^{mh}/\partial\pi\right) > 0$.

Platforms Profits

Partially differentiate equilibrium platform's profits at Equation 1.37 regarding the model's parameters.

$$\begin{aligned}
\frac{\partial\Pi^{mh}}{\partial\alpha} &= \frac{1}{\epsilon^2}\Big[-\left(R_s-f_s\right)^2\left[4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)^2\right]+4\left(\tau_s+\beta\right)\big[\left(\tau_s+\beta\right) \\
&\quad \left(R_b-f_b\right)^2+\left(\tau_b-\alpha\right)\left(R_s-f_s\right)^2+\left(\pi+\upsilon\right)\left(R_b-f_b\right)\left(R_s-f_s\right)\big]\Big] \\
&= \frac{1}{\epsilon^2}\Big[4\left(\tau_s+\beta\right)^2\left(R_b-f_b\right)^2+4\left(\tau_s+\beta\right)\left(\pi+\upsilon\right)\left(R_b-f_b\right)\left(R_s-f_s\right)+2\left(R_s-f_s\right)^2\left(\pi+\upsilon\right)^2\Big] \\
&= \frac{1}{\epsilon^2}\Big[2\left(\tau_s+\beta\right)\left(R_b-f_b\right)\left[2\left(\tau_s+\beta\right)\left(R_b-f_b\right)+\left(\pi+\upsilon\right)\left(R_s-f_s\right)\right] \\
&\quad +\left(R_s-f_s\right)\left(\pi+\upsilon\right)\left[2\left(\tau_s+\beta\right)\left(R_b-f_b\right)+\left(\pi+\upsilon\right)\left(R_s-f_s\right)\right]\Big] \\
\frac{\partial\Pi^{mh}}{\partial\alpha} &= \frac{\left[2\left(\tau_s+\beta\right)\left(R_b-f_b\right)+\left(\pi+\upsilon\right)\left(R_s-f_s\right)\right]^2}{\left[4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)^2\right]^2} > 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial\Pi^{mh}}{\partial\beta} &= \frac{1}{\epsilon^2}\Big[\left(R_b-f_b\right)^2\left[4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)^2\right]-4\left(\tau_b-\alpha\right)\big[\left(\tau_s+\beta\right)\left(R_b-f_b\right)^2 \\
&\quad +\left(\tau_b-\alpha\right)\left(R_s-f_s\right)^2+\left(\pi+\upsilon\right)\left(R_b-f_b\right)\left(R_s-f_s\right)\big]\Big] \\
&= -\frac{1}{\epsilon^2}\Big[4\left(\tau_b-\alpha\right)^2\left(R_s-f_s\right)^2+4\left(\tau_b-\alpha\right)\left(\pi+\upsilon\right)\left(R_b-f_b\right)\left(R_s-f_s\right)+\left(\pi+\upsilon\right)^2\left(R_b-f_b\right)^2\Big] \\
&= -\frac{1}{\epsilon^2}\Big[2\left(\tau_b-\alpha\right)\left(R_s-f_s\right)\left[2\left(\tau_b-\alpha\right)\left(R_s-f_s\right)+\left(\pi+\upsilon\right)\left(R_b-f_b\right)\right]
\end{aligned}$$

$$+\left(\pi+\upsilon\right)\left(R_b-f_b\right)\Big[2\left(\tau_b-\alpha\right)\left(R_s-f_s\right)+\left(\pi+\upsilon\right)\left(R_b-f_b\right)\Big]\Big]$$

$$\frac{\partial\Pi^{mh}}{\partial\beta}=-\frac{\left[2\left(\tau_b-\alpha\right)\left(R_s-f_s\right)+\left(R_b-f_b\right)\left(\pi+\upsilon\right)\right]^2}{\left[4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)^2\right]^2}<0$$

$$\frac{\partial\Pi^{mh}}{\partial\upsilon}\equiv\frac{\partial\Pi^{mh}}{\partial\pi}=\frac{1}{\epsilon^2}\Big[\left(R_b-f_b\right)\left(R_s-f_s\right)\left[4\left(\tau_b-\alpha\right)\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)^2\right]$$
$$+2\left(\pi+\upsilon\right)\left[\left(\tau_s+\beta\right)\left(R_b-f_b\right)^2+\left(\tau_b-\alpha\right)\left(R_s-f_s\right)^2+\left(\pi+\upsilon\right)\left(R_b-f_b\right)\left(R_s-f_s\right)\right]\Big]$$
$$=\frac{1}{\epsilon^2}\Big[2\left(\tau_s+\beta\right)\left(R_b-f_b\right)\left[2\left(\tau_b-\alpha\right)\left(R_s-f_s\right)+\left(\pi+\upsilon\right)\left(R_b-f_b\right)\right]$$
$$+\left(\pi+\upsilon\right)\left(R_s-f_s\right)\left[2\left(\tau_b-\alpha\right)\left(R_s-f_s\right)+\left(\pi+\upsilon\right)\left(R_b-f_b\right)\right]\Big]$$
$$\frac{\partial\Pi^{mh}}{\partial\upsilon}\equiv\frac{\partial\Pi^{mh}}{\partial\pi}=\frac{1}{\epsilon^2}\Big[2\left(\tau_b-\alpha\right)\left(R_s-f_s\right)+\left(\pi+\upsilon\right)\left(R_b-f_b\right)\Big]$$
$$\Big[2\left(\tau_s+\beta\right)\left(R_b-f_b\right)+\left(\pi+\upsilon\right)\left(R_s-f_s\right)\Big]>0$$

A.6 Comparing Results

A.6.1 Singlehome vs. Sellers Multihome

Proof of Proposition 1.1

Proof.

Membership fees

First, we compute from Equation 1.16b and Equation 1.8b $p_s^{smh}-p_s^{sh}=\frac{1}{2}\left(R_s+f_s\right)+\frac{1}{4}\left(\pi-\upsilon\right)-\big[f_s+\left(\tau_s+\beta\right)-\upsilon\big]$. We can rearrange the expression to be $p_s^{smh}-p_s^{sh}=\frac{1}{4}\Big[\big[2\left(R_s-f_s\right)+\left(\pi+\upsilon\right)\big]-\left[4\left(\tau_s+\beta\right)-2\upsilon\right]\Big]$. For $p_s^{smh}-p_s^{sh}>0$ we need $\frac{1}{4}\Big[\big[2\left(R_s-f_s\right)+\left(\pi+\upsilon\right)\big]-\left[4\left(\tau_s+\beta\right)-2\upsilon\right]\Big]>0$. Reordering the previous expression, we need $\Big[2\big(R_s-f_s\big)-\left[2\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)\right]\Big]-2\left[\left(\tau_s+\beta\right)-\upsilon\right]>0$.

Then, we compute from Equation 1.16a and Equation 1.8a $p_b^{sh}-p_b^{smh}=f_b+\left(\tau_b-\alpha\right)-\pi-\Big[f_b+\left(\tau_b-\alpha\right)-\frac{\pi}{4(\tau_s+\beta)}\Big[\left(\pi+3\upsilon\right)+2\left(R_s-f_s\right)\Big]\Big]$. We can rearrange the expression to be $p_b^{sh}-p_b^{smh}=\frac{\pi}{4(\tau_s+\beta)}\Big[\left[2\left(R_s-f_s\right)+\left(\pi+\upsilon\right)\right]-\left[4\left(\tau_s+\beta\right)-2\upsilon\right]\Big]$. For $p_b^{smh}-p_b^{sh}>0$ we need $\frac{\pi}{4(\tau_s+\beta)}\Big[\big[2\big(R_s-f_s\big)+\big(\pi+\upsilon\big)\big]-\big[4\big(\tau_s+\beta\big)-2\upsilon\big]\Big]>0$. Reordering the previous expression, we need $\Big[2\big(R_s-f_s\big)-\left[2\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)\right]\Big]-2\left[\left(\tau_s+\beta\right)-\upsilon\right]>0$.

Next, we break expression $\Big[2\big(R_s-f_s\big)-\left[2\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)\right]\Big]-2\left[\left(\tau_s+\beta\right)-\upsilon\right]$ down into two separate parts. Firs part as $2\big(R_s-f_s\big)-\left[2\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)\right]$ and part 2 as $-2\left[\left(\tau_s+\beta\right)-\upsilon\right]$. Afterwards, we use the lower bound of Assumption 1.3 to show that part 1 is always positive. First, we make the left sides of both expressions equivalent to compare the right sides. If the right side of Assumption 1.3 is greater than the right side of part 1, then part 1 is always positive. That is $3(\tau_s+\beta) > 2(\tau_s+\beta)-(\pi+\upsilon)$ given that $(\tau_s+\beta)+(\pi+\upsilon) > 0$. Now part 2 to be positive we need $(\tau_s+\beta)-\upsilon < 0$ which give us $\upsilon > \tau_s+\beta$. Therefore, if the lower bound of Assumption 1.3 and condition $\upsilon > \tau_s+\beta$ hold, then $\left[2\left(R_s-f_s\right)-\left[2\left(\tau_s+\beta\right)-\left(\pi+\upsilon\right)\right]\right]-2\left[\left(\tau_s+\beta\right)-\upsilon\right] > 0$ and $p_s^{smh} > p_s^{sh}$ and $p_b^{sh} > p_b^{smh}$.

Buyers Aggregate Surplus

Second, we compute from Equation 1.20 and Equation 1.11a $CS^{smh}-CS^{sh} = R_b - f_b - \frac{5}{4}\tau_b + \frac{3}{2}\alpha + \frac{2(\pi+\upsilon)(R_s-f_s)+(\pi+\upsilon)^2+2\pi\upsilon}{4(\tau_s+\beta)} - \left[R_b - f_b - \frac{5}{4}\tau_b + \frac{1}{2}\upsilon + \pi + \frac{3}{2}\alpha\right]$. This expression turns to $CS^{smh}-CS^{sh} = \left[\frac{1}{4(\tau_s+\beta)}\left[2\left(R_s-f_s\right)+\left(\pi+\upsilon\right)\right]-\frac{1}{2}\right]\upsilon + \frac{\pi}{4(\tau_s+\beta)}\left[\left[2\left(R_s-f_s\right)+\left(\pi+\upsilon\right)\right]-\left[4\left(\tau_s+\beta\right)-2\upsilon\right]\right]$. Note that the first part of the previous expression $\left[\frac{1}{4(\tau_s+\beta)}\left[2\left(R_s-f_s\right)+\left(\pi+\upsilon\right)\right]-\frac{1}{2}\right]\upsilon$ can be denote as $\left[\eta_s^{smh}-\frac{1}{2}\right]\upsilon$ and the second part $\frac{\pi}{4(\tau_s+\beta)}\left[\left[2\left(R_s-f_s\right)+\left(\pi+\upsilon\right)\right]-\left[4\left(\tau_s+\beta\right)-2\upsilon\right]\right]$ as $p_b^{sh}-p_b^{smh}$. Then $CS^{smh}-CS^{sh} = \left[\eta_s^{smh}-\frac{1}{2}\right]\upsilon + \left[p_b^{sh}-p_b^{smh}\right]$. The first part of the expression $\eta_s^{smh}-\frac{1}{2}$ is positive considering it was defined $\eta_s^{sh} < \eta_s^{smh} < 1$ to provide sellers with an opportunity to increase their market share by participating in both platforms simultaneously. Additionally, the second part of the expression $\left[p_b^{sh}-p_b^{smh}\right]$ is positive as was verified in the first step of this proof.

Platform Profits

Finally, we compute from Equation 1.18 and Equation 1.9 $\Pi^{smh}-\Pi^{sh} = \frac{1}{16(\tau_s+\beta)}\left[8\left(\tau_s+\beta\right)\left(\tau_b-\alpha\right)-\left[\left(\pi+\upsilon\right)^2+4\pi\upsilon\right]+4\left(R_s-f_s\right)^2\right]-\frac{1}{2}\left[\left(\tau_b-\alpha\right)-\upsilon+\left(\tau_s+\beta\right)-\pi\right]$. We can rearrange the expression to be $\Pi^{smh}-\Pi^{sh} = \frac{1}{16(\tau_s+\beta)}\left[4\left[\left(R_s-f_s\right)^2-\left(\tau_s+\beta\right)^2\right]+8\left(\tau_s+\beta\right)\left(\pi+\upsilon\right)-4\left(\tau_s+\beta\right)^2-\left[\left(\pi+\upsilon\right)^2+4\pi\upsilon\right]\right]$. Now, we separate the previous expression into 2 parts. Part 1 is $4\left[\left(R_s-f_s\right)^2-\left(\tau_s+\beta\right)^2\right]$. Subsequently, we use the lower bound of Assumption 1.3 to show that part 1 is always positive. We make the left side of both inequalities equal to compare the right side and identify which is larger. That is $3(\tau_s+\beta) > (\tau_s+\beta)$.

Now part 2 is $8\left(\tau_s+\beta\right)\left(\pi+\upsilon\right)-4\left(\tau_s+\beta\right)^2-\left[\left(\pi+\upsilon\right)^2+4\pi\upsilon\right]$. Next, to show the previous expression is positive we differentiate it with respect to the cross-group network effect sellers exert on buyers υ, and evaluate the result at $\upsilon = \tau_s+\beta$ which is the condition that ensures that $p_s^{smh} > p_s^{sh}$ and $p_b^{sh} > p_b^{smh}$. That is, $\left(\frac{\partial Part2}{\partial \upsilon}\right)_{\upsilon=\tau_s+\beta} = 8\left(\tau_s+\beta\right)-2\left(\pi+\upsilon\right)-4\pi$, then evaluating $\upsilon = \tau_s+\beta$ is $8\upsilon-2\left(\pi+\upsilon\right)-4\pi$ and then turns to $6\left(\upsilon-\pi\right)$. Then, expression $8\left(\tau_s+\beta\right)\left(\pi+\upsilon\right)-4\left(\tau_s+\beta\right)^2-\left[\left(\pi+\upsilon\right)^2+4\pi\upsilon\right]$ is always positive if $\upsilon > \pi$. Higher values of υ, above $\tau_s+\beta$ result in higher profits when sellers multihome and buyers singlehome compared to situations when both buyers and sellers singlehome. □

A.6.2 Singlehome vs. Buyers Multihome

Proof of Proposition 1.2

Proof.

Membership fees

First, we compute from Equation 1.26a and Equation 1.8a $p_b^{bmh} - p_b^{sh} = \frac{1}{2}(R_b + f_b) + \frac{1}{4}(\upsilon - \pi) - \big[f_b + (\tau_b - \alpha) - \pi\big]$. We can rearrange the expression to be $p_b^{bmh} - p_b^{sh} = \frac{1}{4}\big[\big[2(R_b - f_b) + (\pi + \upsilon)\big] - [4(\tau_b - \alpha) - 2\pi]\big]$. For $p_b^{bmh} - p_b^{sh} > 0$ we need $\frac{1}{4}\big[\big[2(R_b - f_b) + (\pi + \upsilon)\big] - [4(\tau_b - \alpha) - 2\pi]\big] > 0$. Reordering the previous expression, we need $\big[2\big(R_b - f_b\big) - [2(\tau_b - \alpha) - (\pi + \upsilon)]\big] - 2[(\tau_b - \alpha) - \pi] > 0$.

Then, we compute from Equation 1.26b and Equation 1.8b $p_s^{sh} - p_s^{bmh} = f_s + (\tau_s + \beta) - \upsilon - \big[f_s + (\tau_s + \beta) - \frac{\upsilon}{4(\tau_b - \alpha)}\big[(\upsilon + 3\pi) + 2(R_b - f_b)\big]\big]$. We can rearrange the expression to be $p_s^{sh} - p_s^{bmh} = \frac{\upsilon}{4(\tau_b - \upsilon)}\big[[2(R_b - f_b) + (\pi + \upsilon)] - [4(\tau_b - \alpha) - 2\pi]\big]$. For $p_s^{bmh} - p_s^{sh} > 0$ we need $\frac{\upsilon}{4(\tau_b - \beta)}\big[\big[2\big(R_b - f_b\big) + \big(\pi + \upsilon\big)\big] - \big[4\big(\tau_b - \alpha\big) - 2\pi\big]\big] > 0$. Reordering the previous expression, we need $\big[2\big(R_b - f_b\big) - [2(\tau_b - \alpha) - (\pi + \upsilon)]\big] - 2[(\tau_b - \alpha) - \pi] > 0$.

Next, we divide expression $\big[2\big(R_b - f_b\big) - [2(\tau_b - \alpha) - (\pi + \upsilon)]\big] - 2[(\tau_b - \alpha) - \pi]$ into two parts. Part 1 as $2\big(R_b - f_b\big) - [2(\tau_b - \alpha) - (\pi + \upsilon)]\big]$ and part 2 as $-2[(\tau_b - \alpha) - \pi]$. Afterwards, we use the lower bound of Assumption 1.2 to show that part 1 is always positive. Next, we make the left sides of both expressions equivalent to compare the right sides. If the right side of Assumption 1.2 is greater than the right side of part 1, then part 1 is always positive. That is $3(\tau_b - \alpha) > 2(\tau_b - \alpha) - (\pi + \upsilon)$ given that $(\tau_b - \alpha) + (\pi + \upsilon) > 0$. Now part 2 to be positive we need $(\tau_b - \alpha) - \pi < 0$ which give us $\pi > \tau_b - \alpha$. Therefore, if the lower bound of Assumption 1.2 and condition $\pi > \tau_b - \alpha$ hold, then $\big[2\big(R_b - f_b\big) - [2(\tau_b - \alpha) - (\pi + \upsilon)]\big] - 2[(\tau_b - \alpha) - \pi] > 0$ and $p_b^{bmh} > p_b^{sh}$ and $p_s^{sh} > p_s^{bmh}$.

Sellers Aggregate Surplus

Second, we compute from Equation 1.31 and Equation 1.11b $PS^{bmh} - PS^{sh} = R_s - f_s - \frac{5}{4}\tau_s - \frac{3}{2}\beta + \frac{2(\pi+\upsilon)(R_b - f_b) + (\pi+\upsilon)^2 + 2\pi\upsilon}{4(\tau_b - \alpha)} - \big[R_s - f_s - \frac{5}{4}\tau_s + \frac{1}{2}\pi + \upsilon - \frac{3}{2}\beta\big]$. This expression turns to $PS^{bmh} - PS^{sh} = \Big[\frac{1}{4(\tau_b - \alpha)}\big[2(R_b - f_b) + (\pi + \upsilon)\big] - \frac{1}{2}\Big]\pi + \frac{\upsilon}{4(\tau_b - \upsilon)}\big[\big[2(R_b - f_b) + (\pi + \upsilon)\big] - \big[4(\tau_b - \alpha) - 2\pi\big]\big]$. Note that the first part of the previous expression $\Big[\frac{1}{4(\tau_b - \alpha)}\big[2\big(R_b - f_b\big) + (\pi + \upsilon)\big] - \frac{1}{2}\Big]\pi$ can be denote as $\big[\eta_b^{bmh} - \frac{1}{2}\big]\pi$ and the second part $\frac{\upsilon}{4(\tau_b - \alpha)}\big[\big[2(R_b - f_b) + (\pi + \upsilon)\big] - \big[4(\tau_b - \alpha) - 2\pi\big]\big]$ as $p_s^{sh} - p_s^{bmh}$. Then $PS^{bmh} - PS^{sh} = \big[\eta_b^{bmh} - \frac{1}{2}\big]\pi + \big[p_s^{sh} - p_s^{bmh}\big]$. The first part of the expression $\eta_b^{bmh} - \frac{1}{2}$ is positive considering it was defined $\eta_b^{sh} < \eta_b^{bmh} < 1$ to provide buyers with an opportunity to increase their market share by participating in both platforms simultaneously. Additionally, the second part of the expression $\big[p_s^{sh} - p_s^{bmh}\big]$ is positive as was verified in the first step of this proof.

Platform Profits
Finally, we compute from Equation 1.28 and Equation 1.9 $\Pi^{bmh} - \Pi^{sh} = \frac{1}{16(\tau_b-\alpha)}\Big[8\big(\tau_s + \beta\big)\big(\tau_b - \alpha\big) - \big[(\pi+\upsilon)^2 + 4\pi\upsilon\big] + 4\big(R_b - f_b\big)^2\Big] - \frac{1}{2}\left[(\tau_b-\alpha) - \upsilon + (\tau_s+\beta) - \pi\right]$. We can rearrange the expression to be $\Pi^{bmh} - \Pi^{sh} = \frac{1}{16(\tau_b-\alpha)}\big[4\big[(R_b - f_b)^2 - (\tau_b-\alpha)^2\big] + 8(\tau_b-\alpha)(\pi+\upsilon) - 4(\tau_b-\alpha)^2 - \big[(\pi+\upsilon)^2 + 4\pi\upsilon\big]\big]$. Now, we separate the previous expression into two parts. Part one is $4\big[(R_b-f_b)^2 - (\tau_b-\alpha)^2\big]$. Subsequently, we use the lower bound of Assumption 1.2 to show that part one is always positive. We make the left side of both inequalities equal to compare the right side and identify which is larger. That is $3(\tau_b-\alpha) > (\tau_b-\alpha)$.

Now part two is $8(\tau_b-\alpha)(\pi+\upsilon) - 4(\tau_b+\alpha)^2 - \big[(\pi+\upsilon)^2 + 4\pi\upsilon\big]$. Next, to show the previous expression is positive we differentiate it with respect to the cross-group network effect buyers exert on sellers π, and evaluate the result at $\pi = \tau_b - \alpha$ which is the condition that ensures that $p_b^{bmh} > p_b^{sh}$ and $p_s^{sh} > p_s^{smh}$. That is, $\big(\frac{\partial Part2}{\partial\pi}\big)_{\pi=\tau_b-\alpha} = 8(\tau_b-\alpha) - 2(\pi+\upsilon) - 4\upsilon$, then evaluating $\pi = \tau_b - \alpha$ is $8\pi - 2(\pi+\upsilon) - 4\upsilon$ and then turns to $6(\pi-\upsilon)$. Then, expression $8(\tau_b-\alpha)(\pi+\upsilon) - 4(\tau_b-\alpha)^2 - \big[(\pi+\upsilon)^2 + 4\pi\upsilon\big]$ is always positive if $\pi > \upsilon$. Higher values of π, above $\tau_b - \alpha$ result in higher profits when buyers multihome and sellers singlehome compared to situations when both buyers and sellers singlehome. □

A.6.3 Buyers Multihome and Sellers Singlehome vs. Both Sides Multihome

Proof of Proposition 1.3

Proof.
Membership fees
First, we compute from Equation 1.26a and Equation 1.35a $p_b^{mh} - p_b^{bmh}$, that is

$$p_b^{mh} - p_b^{bmh} = \frac{(2(\tau_b-\alpha)(\tau_s+\beta)-\pi\upsilon)(R_b+f_b)-\big(\pi^2R_b+\upsilon^2f_b\big)+(\tau_b-\alpha)(\upsilon-\pi)(R_s-f_s)}{4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2} - \left[\frac{(R_b+f_b)}{2} + \frac{(\upsilon-\pi)}{4}\right]$$

$$= \frac{1}{\epsilon}\Big[8(\tau_b-\alpha)(\tau_s+\beta)(R_b+f_b) - 4\pi\upsilon(R_b+f_b) - 4\pi^2R_b - 4\upsilon^2f_b + 4(\tau_b-\alpha)(\upsilon-\pi)$$
$$(R_s-f_s) - 8(\tau_b-\alpha)(\tau_s+\beta)(R_b+f_b) + 2(\pi+\upsilon)^2(R_b+f_b) - 4(\tau_b-\alpha)(\tau_s+\beta)$$
$$(\upsilon-\pi) + (\pi+\upsilon)^2(\upsilon-\pi)\Big]$$

$$= \frac{1}{\epsilon}\Big[4(\tau_b-\alpha)(\upsilon-\pi)\left[(R_s-f_s) - (\tau_s+\beta)\right] + (\pi+\upsilon)^2(\upsilon-\pi) + 2\Big[\pi^2R_b + 2\pi\upsilon R_b$$
$$+\upsilon^2R_b + \pi^2f_b + 2\pi\upsilon f_b + \upsilon^2f_b - 2\pi\upsilon R_b - 2\pi\upsilon f_b - 2\pi^2R_b - 2\upsilon^2f_b\Big]\Big]$$

$$= \frac{(\upsilon-\pi)}{\epsilon}\Big[4(\tau_b-\alpha)\left[(R_s-f_s) - (\tau_s+\beta)\right] + (\upsilon+\pi)\left[2(R_b+f_b) + (\upsilon+\pi)\right]\Big]$$

Where $\epsilon \equiv 4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2$.

Then for $p_b^{mh}-p_b^{bmh}>0$ it is sufficient to have $\upsilon>\pi$ and $\big(R_s-f_s\big)-\big(\tau_s+\beta\big)>0$. We use the lower bound of Assumption 1.3 to show that the previous condition is satisfied. First, we make the left side on both conditions equal to compare the right side and show that the right side of Assumption 1.3 is larger than the right side of the previous conditions and therefore $\big(R_s-f_s\big)-\big(\tau_s+\beta\big)$ is positive. That is, $\frac{3}{2}\big(\tau_s+\beta\big)>\big(\tau_s+\beta\big)$, then $p_b^{mh}-p_b^{bmh}>0$ if $\upsilon>\pi$

Buyers Aggregate Surplus

Second, we compute from Equation 1.39a and Equation 1.30 $CS^{mh}-CS^{bmh}$ and using the fact that $CS^{mh}=\tau_b\left(\eta_b^{mh}\right)^2$ and $CS^{bmh}=\tau_b\left(\eta_b^{bmh}\right)^2$ we get $CS^{mh}-CS^{bmh}=\tau_b\left(\eta_b^{mh}\right)^2-\tau_b\left(\eta_b^{bmh}\right)^2$ turning to $\eta_b^{mh}-\eta_b^{bmh}$, that is

$$
\begin{aligned}
CS^{mh}-CS^{bmh} &= \frac{2(\tau_s+\beta)(R_b-f_b)+(\pi+\upsilon)(R_s-f_s)}{4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2}-\frac{2(R_b-f_b)+(\upsilon+\pi)}{4(\tau_b-\alpha)}\\
&= \frac{1}{4(\tau_b-\alpha)\epsilon}\Big[8(\tau_b-\alpha)(\tau_s+\beta)(R_b-f_b)+4(\tau_b-\alpha)(\pi+\upsilon)(R_s-f_s)-8(\tau_b-\alpha)\\
&\quad (\tau_s+\beta)(R_b-f_b)-4(\tau_b-\alpha)(\tau_s+\beta)(\upsilon+\pi)+2(\pi+\upsilon)^2(R_b-f_b)+(\pi+\upsilon)^3\Big]\\
&= \frac{(\pi+\upsilon)}{4(\tau_b-\alpha)\epsilon}\Big[4(\tau_b-\alpha)\big[(R_s-f_s)-(\tau_s+\beta)\big]+(\pi+\upsilon)\big[2(R_b-f_b)+(\pi+\upsilon)\big]\Big]
\end{aligned}
$$

Where $\epsilon \equiv 4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2$

Then for $CS^{mh}-CS^{bmh}>0$ it is sufficient to have $\big(R_s-f_s\big)-\big(\tau_s+\beta\big)>0$. We use the same method as in comparing membership fees. Using the lower bound of Assumption 1.3 to show that the previous condition is satisfied. That is $\frac{3}{2}\big(\tau_s+\beta\big)>\big(\tau_s+\beta\big)$, then $CS^{mh}-CS^{bmh}>0$

Platforms Profits

Finally, we compute from Equation 1.37 and Equation 1.28 $\Pi^{mh}-\Pi^{bmh}$ getting:

$$
\begin{aligned}
&\Pi^{mh}-\Pi^{bmh}=\\
&= \frac{(\tau_s+\beta)(R_b-f_b)^2+(\tau_b-\alpha)(R_s-f_s)^2+(\pi+\upsilon)(R_b-f_b)(R_s-f_s)}{4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2}-\frac{\big[8(\tau_b-\alpha)(\tau_s+\beta)-(\upsilon+\pi)^2-4\upsilon\pi\big]+4(R_b-f_b)^2}{16(\tau_b-\alpha)}\\
&= \frac{1}{16(\tau_b-\alpha)\epsilon}\Big[16(\tau_b-\alpha)(\tau_s+\beta)(R_b-f_b)^2+16(\tau_b-\alpha)\big[(\tau_b-\alpha)(R_s-f_s)^2\\
&\quad +(\pi+\upsilon)(R_b-f_b)(R_s-f_s)\big]-16(\tau_b-\alpha)(\tau_s+\beta)(R_b-f_b)^2-\big[8(\tau_b-\alpha)(\tau_s+\beta)\\
&\quad -(\upsilon+\pi)^2-4\upsilon\pi\big]\big[4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2\big]\Big]
\end{aligned}
$$

$$= \frac{1}{16(\tau_b-\alpha)\epsilon}\Big[16(\tau_b-\alpha)^2(R_s-f_s)^2+16(\tau_b-\alpha)(\pi+\upsilon)(R_b-f_b)(R_s-f_s)$$
$$-32(\tau_b-\alpha)^2(\tau_s+\beta)^2-(\pi+\upsilon)^2\left[(\pi+\upsilon)^2+4\upsilon\pi\right]+8(\tau_b-\alpha)(\tau_s+\beta)(\pi+\upsilon)^2$$
$$+4(\tau_b-\alpha)(\tau_s+\beta)\left[(\pi+\upsilon)^2+4\upsilon\pi\right]\Big]$$
$$\Pi^{mh}-\Pi^{bmh}=\frac{1}{16(\tau_b-\alpha)\epsilon}\Big[16(\tau_b-\alpha)^2\left[(R_s-f_s)^2-2(\tau_s+\beta)^2\right]+(\pi+\upsilon)^2\Big[8(\tau_b-\alpha)$$
$$(\tau_s+\beta)-\left[(\pi+\upsilon)^2+4\upsilon\pi\right]\Big]+4(\tau_b-\alpha)\Big[4(\pi+\upsilon)(R_b-f_b)(R_s-f_s)$$
$$+(\tau_s+\beta)\left[(\pi+\upsilon)^2+4\upsilon\pi\right]\Big]\Big] \text{ where } \epsilon\equiv 4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2$$

For $\Pi^{mh}-\Pi^{bmh}>0$ it is sufficient to have $(R_s-f_s)^2-2(\tau_s+\beta)^2$ and $8\big(\tau_b-\alpha\big)\big(\tau_s+\beta\big)-\left[(\pi+\upsilon)^2+4\upsilon\pi\right]$ positive considering the rest of the expression is positive. We use the lower bound of Assumption 1.3 to show that $(R_s-f_s)^2-2(\tau_s+\beta)^2$ is positive by using the same method as before. That is $\frac{3}{2}(\tau_s+\beta)>\sqrt{2}(\tau_s+\beta)$ which turns to $\frac{3}{2}-\sqrt{2}>0$. Then, we use Assumption 1.1 to show $8(\tau_b-\alpha)(\tau_s+\beta)-\left[(\pi+\upsilon)^2+4\upsilon\pi\right]$ is positive. First, we make the left-hand side of both expressions equal to compare the right-hand side and show that the right-hand side of Assumption 1.1 is greater. That is $(\tau_b-\alpha)(\tau_s+\beta)>\frac{(\pi+\upsilon)^2}{4}$ and $(\tau_b-\alpha)(\tau_s+\beta)>\frac{(\pi+\upsilon)^2+4\pi\upsilon}{8}$. Then, $\frac{(\pi+\upsilon)^2}{4}>\frac{(\pi+\upsilon)^2+4\pi\upsilon}{8}$ turns to $2(\pi+\upsilon)^2>(\pi+\upsilon)^2+4\pi\upsilon$ and then to $\pi^2+2\pi\upsilon+\upsilon^2>4\pi\upsilon$ then turns to $(\pi-\upsilon)^2>0$ if $\pi\neq\upsilon$. □

A.6.4 Sellers Multihome and Buyers Singlehome vs. Both Sides Multihome

Proof of Proposition 1.4

Proof.

Membership fees

First, we compute from Equation 1.16b and Equation 1.35b $p_s^{mh}-p_s^{bmh}$, that is

$$p_s^{mh}-p_s^{smh}=\frac{(2(\tau_b-\alpha)(\tau_s+\beta)-\pi\upsilon)(R_s+f_s)-\left(\upsilon^2R_s+\pi^2f_s\right)+(\tau_s+\beta)(\pi-\upsilon)(R_b-f_b)}{4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2}-\left[\frac{(R_s+f_s)}{2}+\frac{(\pi-\upsilon)}{4}\right]$$
$$=\frac{1}{\epsilon}\Big[8(\tau_b-\alpha)(\tau_s+\beta)(R_s+f_s)-4\pi\upsilon(R_s+f_s)-4\upsilon^2R_s-4\pi^2f_s+4(\tau_s+\beta)(\pi-\upsilon)$$
$$(R_b-f_b)-8(\tau_b-\alpha)(\tau_s+\beta)(R_s+f_s)-4(\tau_b-\alpha)(\tau_s+\beta)(\pi-\upsilon)$$
$$+2(\pi+\upsilon)^2(R_s+f_s)+(\pi+\upsilon)^2(\pi-\upsilon)\Big]$$
$$=\frac{1}{\epsilon}\Big[4(\tau_s+\beta)(\pi-\upsilon)\left[(R_b-f_b)-(\tau_b-\alpha)\right]+(\pi+\upsilon)^2(\pi-\upsilon)+2\Big[\pi^2R_s+2\pi\upsilon R_s$$
$$+\upsilon^2R_s+\pi^2f_s+2\pi\upsilon f_s+\upsilon^2f_s-2\pi\upsilon R_s-2\pi\upsilon f_s-2\upsilon^2R_s-2\pi^2f_s\Big]\Big]$$
$$=\frac{(\pi-\upsilon)}{\epsilon}\Big[4(\tau_s+\beta)\left[(R_b-f_b)-(\tau_b-\alpha)\right]+(\pi+\upsilon)\left[2(R_s+f_s)+(\pi+\upsilon)\right]\Big]$$

Where $\epsilon \equiv 4(\tau_b - \alpha)(\tau_s + \beta) - (\pi + \upsilon)^2$.

Then for $p_s^{mh} - p_s^{smh} > 0$ it is sufficient to have $\pi > \upsilon$ and $\big(R_b - f_b\big) - \big(\tau_b - \alpha\big) > 0$. We use the lower bound of Assumption 1.2 to show that the previous condition is satisfied. First, we make the left side on both conditions equal to compare the right side and show that the right side of Assumption 1.2 is greater than the right-hand side of the previous conditions and therefore $\big(R_b - f_b\big) - \big(\tau_b - \alpha\big)$ is positive. That is, $\frac{3}{2}\big(\tau_b - \alpha\big) > \big(\tau_b - \alpha\big)$, then $p_s^{mh} - p_s^{smh} > 0$ if $\pi > \upsilon$

Sellers Aggregate Surplus

Second, we compute from Equation 1.39b and Equation 1.21 $PS^{mh} - PS^{smh}$ and using the fact that $PS^{mh} = \tau_s\left(\eta_s^{mh}\right)^2$ and $PS^{smh} = \tau_s\left(\eta_s^{smh}\right)^2$ we get $PS^{mh} - PS^{smh} = \tau_s\left(\eta_s^{mh}\right)^2 - \tau_s\left(\eta_s^{smh}\right)^2$ turning to $\eta_s^{mh} - \eta_s^{smh}$, that is

$$PS^{mh} - PS^{smh} = \frac{2(\tau_b - \alpha)(R_s - f_s) + (\pi + \upsilon)(R_b - f_b)}{4(\tau_b - \alpha)(\tau_s + \beta) - (\pi + \upsilon)^2} - \frac{2(R_s - f_s) + (\pi + \upsilon)}{4(\tau_s + \beta)}$$

$$= \frac{1}{4(\tau_s + \beta)\epsilon}\Big[8(\tau_b - \alpha)(\tau_s + \beta)(R_s - f_s) + 4(\tau_s + \beta)(\pi + \upsilon)(R_b - f_b) - 8(\tau_b - \alpha)$$
$$(\tau_s + \beta)(R_s - f_s) - 4(\tau_b - \alpha)(\tau_s + \beta)(\upsilon + \pi) + 2(\pi + \upsilon)^2(R_s - f_s) + (\pi + \upsilon)^3\Big]$$
$$= \frac{(\pi + \upsilon)}{4(\tau_s + \beta)\epsilon}\Big[4(\tau_s + \beta)\big[(R_b - f_b) - (\tau_b - \alpha)\big] + (\pi + \upsilon)\big[2(R_s - f_s) + (\pi + \upsilon)\big]\Big]$$

Where $\epsilon \equiv 4(\tau_b - \alpha)(\tau_s + \beta) - (\pi + \upsilon)^2$

Then for $PS^{mh} - PS^{smh} > 0$ it is sufficient to have $\big(R_b - f_b\big) - \big(\tau_b - \alpha\big) > 0$. We use the same method as in comparing membership fees. Using the lower bound of Assumption 1.2 to show that the previous condition is satisfied. That is $\frac{3}{2}\big(\tau_b - \alpha\big) > \big(\tau_b - \alpha\big)$, then $PS^{mh} - PS^{smh} > 0$

Platforms profits

Finally, we compute from Equation 1.37 and Equation 1.18 $\Pi^{mh} - \Pi^{smh}$ getting:

$$\Pi^{mh} - \Pi^{smh} =$$
$$= \frac{(\tau_s+\beta)(R_b-f_b)^2 + (\tau_b-\alpha)(R_s-f_s)^2 + (\pi+\upsilon)(R_b-f_b)(R_s-f_s)}{4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2} - \frac{\big[8(\tau_b-\alpha)(\tau_s+\beta)-(\upsilon+\pi)^2-4\upsilon\pi\big]+4(R_s-f_s)^2}{16(\tau_s+\beta)}$$
$$= \frac{1}{16(\tau_s + \beta)\epsilon}\Big[16(\tau_b - \alpha)(\tau_s + \beta)(R_s - f_s)^2 + 16(\tau_s + \beta)\big[(\tau_s + \beta)(R_b - f_b)^2$$
$$+ (\pi + \upsilon)(R_b - f_b)(R_s - f_s)\big] - 16(\tau_b - \alpha)(\tau_s + \beta)(R_s - f_s)^2 - \big[8(\tau_b - \alpha)(\tau_s + \beta)$$
$$- (\upsilon + \pi)^2 - 4\upsilon\pi\big]\big[4(\tau_b - \alpha)(\tau_s + \beta) - (\pi + \upsilon)^2\big]\Big]$$
$$= \frac{1}{16(\tau_s + \beta)\epsilon}\Big[16(\tau_s + \beta)^2(R_b - f_b)^2 + 16(\tau_s + \beta)(\pi + \upsilon)(R_b - f_b)(R_s - f_s)$$
$$-32(\tau_b - \alpha)^2(\tau_s + \beta)^2 - (\pi + \upsilon)^2\big[(\pi + \upsilon)^2 + 4\upsilon\pi\big] + 8(\tau_b - \alpha)(\tau_s + \beta)(\pi + \upsilon)^2$$

$$+ 4(\tau_b-\alpha)(\tau_s+\beta)\left[(\pi+\upsilon)^2+4\upsilon\pi\right]\Big]$$

$$\Pi^{mh}-\Pi^{smh}=\frac{1}{16(\tau_s+\beta)\epsilon}\Big[16(\tau_s+\beta)^2\left[(R_b-f_b)^2-2(\tau_b-\alpha)^2\right]+(\pi+\upsilon)^2\Big[8(\tau_b-\alpha)(\tau_s+\beta)-\left[(\pi+\upsilon)^2+4\upsilon\pi\right]\Big]+4(\tau_s+\beta)\Big[4(\pi+\upsilon)(R_b-f_b)(R_s-f_s)+(\tau_b-\alpha)\left[(\pi+\upsilon)^2+4\upsilon\pi\right]\Big]\Big] \text{ where } \epsilon\equiv 4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2$$

For $\Pi^{mh}-\Pi^{smh}>0$ it is sufficient to have $(R_b-f_b)^2-2(\tau_b-\alpha)^2$ and $8(\tau_b-\alpha)(\tau_s+\beta)-\left[(\pi+\upsilon)^2+4\upsilon\pi\right]$ positive considering the rest of the expression is positive. We use the lower bound of Assumption 1.2 to show that $(R_b-f_b)^2-2(\tau_b-\alpha)^2$ is positive by using the same method as before. That is $\frac{3}{2}(\tau_b-\alpha)>\sqrt{2}(\tau_b-\alpha)$ which turns to $\frac{3}{2}-\sqrt{2}>0$. Then, we use Assumption 1.1 to show $8(\tau_b-\alpha)(\tau_s+\beta)-\left[(\pi+\upsilon)^2+4\upsilon\pi\right]$. First, we make the left-hand side of both expressions equal to compare the right-hand side and show that the right-hand side of Assumption 1.1 is greater. That is $(\tau_b-\alpha)(\tau_s+\beta)>\frac{(\pi+\upsilon)^2}{4}$ and $(\tau_b-\alpha)(\tau_s+\beta)>\frac{(\pi+\upsilon)^2+4\pi\upsilon}{8}$. Then, $\frac{(\pi+\upsilon)^2}{4}>\frac{(\pi+\upsilon)^2+4\pi\upsilon}{8}$ turns to $2(\pi+\upsilon)^2>(\pi+\upsilon)^2+4\pi\upsilon$ and then to $\pi^2+2\pi\upsilon+\upsilon^2>4\pi\upsilon$ an finally simplifies to $(\pi-\upsilon)^2>0$ when $\pi\neq\upsilon$. □

A.6.5 Singlehome vs. Multihome

Proof of Proposition 1.5

Proof.

Membership fees

First, we compute from Equation 1.35a and Equation 1.8a $p_b^{mh}-p_b^{sh}=\frac{1}{\epsilon}\Big[\big[\left[2(\tau_b-\alpha)(\tau_s+\beta)-\pi\upsilon\right](R_b+f_b)-(\pi^2R_b+\upsilon^2f_b)+(\tau_b-\alpha)(\upsilon-\pi)(R_s-f_s)\big]-\left[f_b+(\tau_b-\alpha)-\pi\right]>0$, where $\epsilon\equiv 4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2$. This turns to $2(\tau_b-\alpha)(\tau_s+\beta)(R_b+f_b)-4(\tau_b-\alpha)(\tau_s+\beta)f_b-\pi^2(R_b-f_b)-\pi\upsilon(R_b-f_b)+(\tau_b-\alpha)(\upsilon-\pi)(R_s-f_s)-\left[4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2\right]\left[(\tau_b-\alpha)-\pi\right]>0$ and then to $\left[2(\tau_b-\alpha)(\tau_s+\beta)-\pi(\pi+\upsilon)\right](R_b-f_b)+(\tau_b-\alpha)(\upsilon-\pi)(R_s-f_s)-\left[4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2\right]\left[(\tau_b-\alpha)-\pi\right]>0$.

Next, to show the previous expression is positive, we first break it down into three separate parts. First, $2(\tau_b-\alpha)(\tau_s+\beta)-\pi(\pi+\upsilon)$. Second, $(\tau_b-\alpha)(\upsilon-\pi)(R_s-f_s)$ and third $-\left[4(\tau_b-\alpha)(\tau_s+\beta)-(\pi+\upsilon)^2\right]\left[(\tau_b-\alpha)-\pi\right]$.

Afterwards, we reorder the first part to be $4(\tau_b-\alpha)(\tau_s+\beta)-2\pi(\pi+\upsilon)$ and use Assumption 1.1 $4(\tau_b-\alpha)(\tau_s+\beta)>(\pi+\upsilon)^2$ to show it is positive. Then, since the left side of both inequalities is equivalent we compare the right side. If the right side of Assumption 1.1 is greater than the right side of the condition, then the condition is satisfied. That is $(\pi+\upsilon)^2>2\pi(\pi+\upsilon)$ if $\upsilon>\pi$. Now, for the second part $(\tau_b-\alpha)(\upsilon-\pi)(R_s-f_s)$ to be positive, we need $\upsilon>\pi$. Finally, for part three to be positive we need

$(\tau_b - \alpha) - \pi < 0$, which turns to $\pi > \tau_b - \alpha$. Therefore, for $p_b^{mh} - p_b^{sh} > 0$ we need to have $\upsilon > \pi$ and $\pi > (\tau_b - \alpha)$.

In the next step, we compute from Equation 1.35b and Equation 1.8b $p_s^{mh} - p_s^{sh} = \frac{1}{\epsilon}\big[\big[2(\tau_b - \alpha)\big(\tau_s + \beta\big) - \pi\upsilon\big](R_s + f_s) - (\upsilon^2 R_s + \pi^2 f_s) + (\tau_s + \beta)\big(\pi - \upsilon\big)\big(R_b - f_b\big)\big] - [f_s + (\tau_s + \beta) - \upsilon] > 0$, where $\epsilon \equiv 4(\tau_b - \alpha)(\tau_s + \beta) - (\pi + \upsilon)^2$. This turns to $2(\tau_b - \alpha)\big(\tau_s + \beta\big)(R_s + f_s) - 4(\tau_b - \alpha)\big(\tau_s + \beta\big) f_s - \upsilon^2 (R_s - f_s) - \pi\upsilon(R_s - f_s) + (\tau_s + \beta)\big(\pi - \upsilon\big)\big(R_b - f_b\big) - \big[4(\tau_b - \alpha)\big(\tau_s + \beta\big) - (\pi + \upsilon)^2\big][(\tau_s + \beta) - \upsilon] > 0$ and then to $\big[2(\tau_b - \alpha)\big(\tau_s + \beta\big) - \upsilon(\pi + \upsilon)\big](R_s - f_s) + (\tau_s + \beta)\big(\pi - \upsilon\big)\big(R_b - f_b\big) - \big[4(\tau_b - \alpha)\big(\tau_s + \beta\big) - (\pi + \upsilon)^2\big]\big[(\tau_s + \beta) - \upsilon\big] > 0$.

Next, to show the previous expression is positive, we first break it down into three separate parts. First, $2(\tau_b - \alpha)\big(\tau_s + \beta\big) - \upsilon(\pi + \upsilon)$. Second, $(\tau_s + \beta)\big(\pi - \upsilon\big)\big(R_b - f_b\big)$ and third $-\big[4(\tau_b - \alpha)\big(\tau_s + \beta\big) - (\pi + \upsilon)^2\big][(\tau_s + \beta) - \upsilon]$.

Afterwards, we reorder the first part to be $4(\tau_b - \alpha)\big(\tau_s + \beta\big) - 2\upsilon(\pi + \upsilon)$ and use Assumption 1.1 to show it is positive. Then, since the left side of both inequalities is equal we compare the right side. If the right side of Assumption 1.1 is greater than the right side of the condition, then the condition is satisfied. That is $(\pi + \upsilon)^2 > 2\upsilon(\pi + \upsilon)$ if $\pi > \upsilon$. Now, for the second part $(\tau_s + \beta)\big(\pi - \upsilon\big)\big(R_b - f_b\big)$ to be positive, we need $\pi > \upsilon$. Finally, for part three to be positive we need $(\tau_s + \beta) - \upsilon < 0$, which turns to $\upsilon > \tau_s + \beta$. Therefore, for $p_s^{mh} - p_s^{sh} > 0$ we need to have $\pi > \upsilon$ and $\upsilon > (\tau_s + \beta)$.

Platform Profits

Finally, we compute from Equation 1.37 and Equation 1.9 $\Pi^{mh} - \Pi^{sh} = \frac{1}{\epsilon}\big[(\tau_s + \beta)\big(R_b - f_b\big)^2 + (\tau_b - \alpha)(R_s - f_s)^2 + (\pi + \upsilon)(R_b - f_b)\big(R_s - f_s\big)\big] - \frac{1}{2}[(\tau_b - \alpha) - \pi + (\tau_s + \beta) - \upsilon] > 0$, where $\epsilon \equiv 4(\tau_b - \alpha)(\tau_s + \beta) - (\pi + \upsilon)^2$. For the previous expression to be positive we need $\pi > \tau_b - \alpha$ and $\upsilon > \tau_s + \beta$. These conditions are the ones developed for $p_b^{mh} - p_b^{sh} > 0$ and $p_s^{mh} - p_s^{sh} > 0$ at the beginning of this proof.

In the same way, we can have conditions $\upsilon > \tau_b - \alpha$ and $\pi > \tau_s + \beta$ for $\Pi^{mh} - \Pi^{sh} > 0$. These conditions are also derived from Equation 1.22 and Equation 1.12, where $\frac{\partial \eta_b}{\partial \eta_s} = \frac{\upsilon}{\tau_b - \alpha}$ and $\frac{\partial \eta_s}{\partial \eta_b} = \frac{\pi}{\tau_s + \beta}$ which implies that these partial derivatives are greater than one, indicating that the addition of an extra proportion of buyer and seller to the platform attracts even more sellers and buyers. □

Appendix B

Appendix: Chapter 2

B.1 Model

B.1.1 Model Assumptions

In this section, we show how the model assumptions are defined.

Second-order conditions

First, to guarantee a unique equilibrium where both platforms remain active, the second-order conditions of the platform maximisation problem must be satisfied in both stages of the game. Specifically, the sufficient conditions required for the second-order conditions at stage two are detailed in Appendix B.1.4 and are (i) $\tau > \sqrt{\pi \upsilon}$ and (ii) $\tau > \frac{(\pi+\upsilon)}{2}$.

Now, we determine which of the two conditions is more stringent, ensuring the other is also met. Initially, since the left side of both inequalities is equal, we compare the right sides to identify the greater one. This yields $\frac{\pi+\upsilon}{2} > \sqrt{\pi \upsilon}$, which can be rewritten as $(\pi+\upsilon)^2 > 4\pi\upsilon$. Further simplification leads to $\pi^2 + 2\pi\upsilon + \upsilon^2 > 4\pi\upsilon$, which simplifies to $(\pi-\upsilon)^2 > 0$ if $\pi \neq \upsilon$. Therefore, if condition (ii) holds, condition (i) is satisfied. Thereby Assumption 2.1 is established.

Second, the sufficient condition that needs to be set for the second order conditions of the platform maximisation problem at stage one to be satisfied is obtained in Appendix B.1.8 and is (i) $\alpha^i > \frac{\tau}{\Sigma}$. This condition is satisfied as condition $\alpha^i > \frac{2\tau}{\Sigma}$ is more stringent (this condition guarantees positive equilibrium attributes and is going to be shown next).

Positive Equilibrium Attributes

Third, the conditions to have positive attributes in equilibrium obtained in Appendix B.1.9 are (i) $\alpha^j > \frac{2\tau}{\Sigma}$, (ii) $\tau > \frac{(\pi+\upsilon)}{2}$ and (iii) $\alpha^i > \frac{\alpha^j\tau}{\Sigma-\tau}$. Next, we show that these conditions are satisfied. For the first condition, we use the fact that platform 1 is more efficient in developing attributes than platform 2, that is $\alpha^2 > \alpha^1$, as was defined in Section 2.2. Therefore if $\alpha^2 > \alpha^1$ and $\alpha^2 > \frac{2\tau}{\Sigma}$ we derive $\alpha^1 > \frac{2\tau}{\Sigma}$. Then $\alpha^i > \frac{2\tau}{\Sigma}$ for $i = 1, 2$. Thereby Assumption 2.2 is established. The second condition (ii) $\tau > \frac{(\pi+\upsilon)}{2}$ is the same as Assumption 2.1. The third condition (iii) $\alpha^i > \frac{\alpha^j\tau}{\Sigma-\tau}$ is satisfied if Assumption 2.2 is more stringent. We show this by comparing the right side of both inequalities, then if the right side of Assumption 2.2 is greater, the condition is satisfied. Next, comparing the right side we have $\frac{2\tau}{\Sigma} > \frac{\alpha^j\tau}{\alpha^j\Sigma-\tau}$ which simplifies to $2\left(\alpha^j\Sigma - \tau\right) - \alpha^j\Sigma > 0$ and simplifies to $\alpha^j\Sigma - 2\tau > 0$ if $\alpha^j > \frac{2\tau}{\Sigma}$, which is the same Assumption 2.2. Therefore if Assumption 2.2 holds, condition $\alpha^i > \frac{\alpha^j\tau}{\alpha^j\Sigma-\tau}$ is satisfied.

Equilibrium market shares

Fourth, the conditions to have equilibrium market shares on both sides within the unit interval, $0 < \eta_b^i < 1$ and $0 < \eta_s^i < 1$, obtained in Appendix B.3.2 are (i) $\tau < \frac{\pi+2\upsilon}{3}$ if $\upsilon > \pi$ or $\tau > \frac{\pi+2\upsilon}{3}$ if $\pi > \upsilon$. Furthermore, (ii) $\tau > \sqrt{\frac{(\pi+\upsilon)(\pi+2\upsilon)}{6}}$ and (iii) $\tau > \frac{\pi+2\upsilon}{3}$. Now, we show these conditions are satisfied using Assumption 2.1. For (i) $\tau > \frac{\pi+2\upsilon}{3}$, we compare the right sides of the inequalities to show that the right side of Assumption 2.1 is more stringent and therefore condition (i) is met. That is $\frac{\pi+\upsilon}{2} > \frac{\pi+2\upsilon}{3}$ which simplifies to $3\left(\pi + \upsilon\right) > 2\left(\pi + 2\upsilon\right)$, which further simplifies to $\pi - \upsilon > 0$ if $\pi > \upsilon$. For (ii), we use the same method comparing the right side of both inequalities and showing the right side of Assumption 2.1 is more stringent and therefore condition (ii) is satisfied. That is $\frac{\pi+\upsilon}{2} > \sqrt{\frac{(\pi+\upsilon)(\pi+2\upsilon)}{6}}$ which simplifies to $3\left(\pi + \upsilon\right) - 2\left(\pi + 2\upsilon\right) > 0$ and further simplifies to $\pi - \upsilon > 0$ if $\pi > \upsilon$. For (iii) we have $\frac{\pi+\upsilon}{2} > \frac{(\pi+2\upsilon)}{3}$ which turns to $3\left(\pi + \upsilon\right) > 2\left(\pi + 2\upsilon\right)$ which simplifies to $\pi - \upsilon > 0$ if $\pi > \upsilon$.

To summarise, the assumptions we are establishing are (i) $\tau > \frac{\pi+\upsilon}{2}$ if $\pi > \upsilon$, and $\frac{\pi+\upsilon}{2} < \tau < \frac{\pi+2\upsilon}{3}$ if $\upsilon > \pi$, (ii) $\alpha^i > \frac{2\tau}{\Sigma}$.

B.1.2 Market's Shares

To get the proportion of buyers and sellers at Equations (2.2a) and (2.2b) we use Equations (2.1a) and (2.1b). For buyers $\eta_b^i = \frac{1}{2} + \frac{\nu_b^i - \nu_b^j}{2\tau}$ turns to $\eta_b^i = \frac{1}{2} + \frac{1}{2\tau}\left[R_b + q_b^i + \upsilon\eta_s^i - p_b^i - \left(R_b + q_b^j + \upsilon\eta_s^j - p_b^j\right)\right]$ turns to $2\tau\eta_b^i = \tau + \upsilon\left(\eta_s^i - \eta_s^j\right) + q_b^i - q_b^j + \left(p_b^j - p_b^i\right)$. Then, since $\eta_b^i + \eta_b^j = 1$ and $\eta_s^i + \eta_s^j = 1$ we have $2\tau\eta_b^i = \tau + \upsilon\left(2\eta_s^i - 1\right) + \left(q_b^i - q_b^j\right) + \left(p_b^j - p_b^i\right)$ and turns to $\eta_b^i = \frac{\tau + \left(2\eta_s^i - 1\right)\upsilon + \left(q_b^i - q_b^j\right) + \left(p_b^j - p_b^i\right)}{2\tau}$.

For sellers $\eta_s^i = \frac{1}{2} + \frac{\nu_s^i - \nu_s^j}{2\tau}$ turns to $\eta_s^i = \frac{1}{2} + \frac{1}{2\tau}\left[R_s + \pi\eta_b^i - p_s^i - \left(R_s + \pi\eta_b^j - p_s^j\right)\right]$ turns to $2\tau\eta_s^i = \tau + \pi\left(\eta_b^i - \eta_b^j\right) + \left(p_s^j - p_s^i\right)$. Then, since $\eta_b^i + \eta_b^j = 1$ and $\eta_s^i + \eta_s^j = 1$ we have $2\tau\eta_s^i = \tau + \pi\left(2\eta_b^i - 1\right) + \left(p_s^j - p_s^i\right)$ and turns to $\eta_s^i = \frac{\tau + \left(2\eta_b^i - 1\right)\pi + \left(p_s^j - p_s^i\right)}{2\tau}$. Then we have:

$$\eta_b^i = \frac{\tau + \left(2\eta_s^i - 1\right)\upsilon + \left(q_b^i - q_b^j\right) + \left(p_b^j - p_b^i\right)}{2\tau} \tag{1}$$

$$\eta_s^i = \frac{\tau + \left(2\eta_b^i - 1\right)\pi + \left(p_s^j - p_s^i\right)}{2\tau} \tag{2}$$

We solve the previous system of equations to obtain η_b^i and η_s^i as a function of membership fees. First, we find the value of $\left(2\eta_s^i - 1\right)$ from equation (2) and substitute this value into equation 1 and then solve for η_b^i. That is, from equation (2) we have $2\eta_s^i - 1 = \frac{1}{\tau}\left[\left(2\eta_b^i - 1\right)\pi + \left(p_s^j - p_s^i\right)\right]$, then we substitute it in equation (1) $2\tau\eta_b^i = \tau + \left(q_b^i - q_b^j\right) + \left(p_b^j - p_b^i\right) + \frac{\upsilon}{\tau}\left[\left(2\eta_b^i - 1\right)\pi + \left(p_s^j - p_s^i\right)\right]$ turns to $2\left(\tau^2 - \pi\upsilon\right)\eta_b^i = \left(\tau^2 - \pi\upsilon\right) - \pi\upsilon + \tau\left(q_b^i - q_b^j\right) + \tau\left(p_b^j - p_b^i\right) + \upsilon\left(p_s^j - p_s^i\right)$. Then it turns to $\eta_b^i = \frac{1}{2} + \frac{\tau\left(q_b^i - q_b^j\right) + \upsilon\left(p_s^j - p_s^i\right) + \tau\left(p_b^j - p_b^i\right)}{2\left(\tau^2 - \pi\upsilon\right)}$. Then we substitute the previous result into equation (2) to get $\eta_s^i = \frac{1}{2} + \frac{\pi\left(q_b^i - q_b^j\right) + \pi\left(p_b^j - p_b^i\right) + \tau\left(p_s^j - p_s^i\right)}{2\left(\tau^2 - \pi\upsilon\right)}$. The solution for the system of equations (1) and (2) are:

$$\eta_b^i = \frac{1}{2} + \frac{\tau\left(q_b^i - q_b^j\right) + \upsilon\left(p_s^j - p_s^i\right) + \tau\left(p_b^j - p_b^i\right)}{2\left(\tau^2 - \pi\upsilon\right)}$$
$$\eta_s^i = \frac{1}{2} + \frac{\pi\left(q_b^i - q_b^j\right) + \pi\left(p_b^j - p_b^i\right) + \tau\left(p_s^j - p_s^i\right)}{2\left(\tau^2 - \pi\upsilon\right)}$$

B.1.3 Maximisation Problem - stage 2

Platforms maximise the next expression concerning both sides' membership fees to have:

$$\max_{\{p_b^i, p_s^i\}} \Pi^i \equiv \left(p_b^i - f_b\right)\eta_b^i\left(p_b^i, p_s^i, p_b^j, p_s^j\right) + \left(p_s^i - f_s\right)\eta_s^i\left(p_b^i, p_s^i, p_b^j, p_s^j\right) - \frac{\alpha^i\left(q_b^i\right)^2}{2}$$

The first-order conditions for platform $i = 1, 2$:

$$\frac{\partial \Pi^i}{\partial p_b^i} = \eta_b^i + \frac{\partial \eta_b^i}{\partial p_b^i}\left(p_b^i - f_b\right) + \frac{\partial \eta_s^i}{\partial p_b^i}\left(p_s^i - f_s\right) = 0$$

$$\frac{\partial \Pi^i}{\partial p_s^i} = \frac{\partial \eta_b^i}{\partial p_s^i}\left(p_b^i - f_b\right) + \eta_s^i + \frac{\partial \eta_s^i}{\partial p_s^i}\left(p_s^i - f_s\right) = 0$$

Using Equations (2.2a) and (2.2b) the first-order conditions for platform i turn to

Platform 1 first-order conditions:

$$\frac{\partial \Pi^1}{\partial p_b^1} = \frac{1}{2} + \frac{\tau\left(q_b^1 - q_b^2\right) + \tau\left(p_b^2 - p_b^1\right) + \upsilon\left(p_s^2 - p_s^1\right)}{2\left(\tau^2 - \pi\upsilon\right)} - \frac{\tau\left(p_b^1 - f_b\right)}{2\left(\tau^2 - \pi\upsilon\right)} - \frac{\pi\left(p_s^1 - f_s\right)}{2\left(\tau^2 - \pi\upsilon\right)} = 0$$

$$\frac{\partial \Pi^1}{\partial p_s^1} = \frac{1}{2} + \frac{\pi\left(q_b^1 - q_b^2\right) + \tau\left(p_s^2 - p_s^1\right) + \pi\left(p_b^2 - p_b^1\right)}{2\left(\tau^2 - \pi\upsilon\right)} - \frac{\tau\left(p_s^1 - f_s\right)}{2\left(\tau^2 - \pi\upsilon\right)} - \frac{\upsilon\left(p_b^1 - f_b\right)}{2\left(\tau^2 - \pi\upsilon\right)} = 0$$

Platform 2 first-order conditions:

$$\frac{\partial \Pi^2}{\partial p_b^2} = \frac{1}{2} + \frac{\tau\left(q_b^2 - q_b^1\right) + \tau\left(p_b^1 - p_b^2\right) + \upsilon\left(p_s^1 - p_s^2\right)}{2\left(\tau^2 - \pi\upsilon\right)} - \frac{\tau\left(p_b^2 - f_b\right)}{2\left(\tau^2 - \pi\upsilon\right)} - \frac{\pi\left(p_s^2 - f_s\right)}{2\left(\tau^2 - \pi\upsilon\right)} = 0$$

$$\frac{\partial \Pi^2}{\partial p_s^2} = \frac{1}{2} + \frac{\pi\left(q_b^2 - q_b^1\right) + \tau\left(p_s^1 - p_s^2\right) + \pi\left(p_b^1 - p_b^2\right)}{2\left(\tau^2 - \pi\upsilon\right)} - \frac{\tau\left(p_s^2 - f_s\right)}{2\left(\tau^2 - \pi\upsilon\right)} - \frac{\upsilon\left(p_b^2 - f_b\right)}{2\left(\tau^2 - \pi\upsilon\right)} = 0$$

From the first-order conditions on both platforms, we obtain:

$$2\tau p_b^i + (\pi + \upsilon)\, p_s^i - \tau p_b^j - \upsilon p_s^j = \tau f_b + \pi f_s + \left(\tau^2 - \pi\upsilon\right) + \tau\left(q_b^i - q_b^j\right) \tag{b1}$$

$$(\pi + \upsilon)\, p_b^i + 2\tau p_s^i - \pi p_b^j - \tau p_s^j = \tau f_s + \upsilon f_b + \left(\tau^2 - \pi\upsilon\right) + \pi\left(q_b^i - q_b^j\right) \tag{b2}$$

$$-\tau p_b^i - \upsilon p_s^i + 2\tau p_b^j + (\pi + \upsilon)\, p_s^j = \tau f_b + \pi f_s + \left(\tau^2 - \pi\upsilon\right) + \tau\left(q_b^j - q_b^i\right) \tag{b3}$$

$$-\pi p_b^i - \tau p_s^i + (\pi + \upsilon)\, p_b^j + 2\tau p_s^j = \tau f_s + \upsilon f_b + \left(\tau^2 - \pi\upsilon\right) + \pi\left(q_b^j - q_b^i\right) \tag{b4}$$

Then, we solve for p_s^j in equation (b3) and then substitute it into equations (b1), (b2) and (b4) to obtain:

$$\tau(2\pi + \upsilon)\, p_b^i + \pi(\pi + 2\upsilon)\, p_s^i + \tau(\upsilon - \pi)\, p_b^j = \tau(\pi + 2\upsilon)\, f_b + \pi(\pi + 2\upsilon)\, f_s + \left(\tau^2 - \pi\upsilon\right)(\pi + 2\upsilon) + \tau\pi\left(q_b^i - q_b^j\right) \tag{b5}$$

$$-\left[\tau^2 - (\pi + \upsilon)^2\right] p_b^i + \tau(2\pi + \upsilon)\, p_s^i + \left[2\tau^2 - \pi(\pi + \upsilon)\right] p_b^j = \left[\tau^2 + \upsilon(\pi + \upsilon)\right] f_b + \tau(2\pi + \upsilon)\, f_s + (\tau + (\pi + \upsilon))\left(\tau^2 - \pi\upsilon\right) - \left(\tau^2 - \pi(\pi + \upsilon)\right)\left(q_b^i - q_b^j\right) \tag{b6}$$

$$\left[2\tau^2 - \pi(\pi + \upsilon)\right] p_b^i + \tau(\upsilon - \pi)\, p_s^i - \left[4\tau^2 - (\pi + \upsilon)^2\right] p_b^j = -\left[2\tau^2 - \upsilon(\pi + \upsilon)\right] f_b + \tau(\upsilon - \pi)\, f_s - (2\tau - (\pi + \upsilon))\left(\tau^2 - \pi\upsilon\right) + \left[2\tau^2 - \pi(\pi + \upsilon)\right]\left(q_b^i - q_b^j\right) \tag{b7}$$

Then, we solve for p_b^j in equation (b7) and substitute it into equation (b5) and (b6) to obtain:

$$\tau\left[6\tau^2-(\pi+\upsilon)^2-2\pi\upsilon\right]p_b^i+\left[\tau^2(5\pi+\upsilon)-\pi(\pi+\upsilon)(\pi+2\upsilon)\right]p_s^i=\tau\big[6\tau^2-(\pi+\upsilon)^2 \\ -2\pi\upsilon\big]f_b+\left[\tau^2(5\pi+\upsilon)-\pi(\pi+\upsilon)(\pi+2\upsilon)\right]f_s+\left[6\tau^2-(\pi+\upsilon)(\pi+2\upsilon)\right]\left(\tau^2-\pi\upsilon\right) \\ +\tau(\upsilon-\pi)\left(\tau^2-\pi\upsilon\right)+2\tau\left(\tau^2-\pi\upsilon\right)\left(q_b^i-q_b^j\right) \quad \text{(b8)}$$

$$\left[\tau^2(\pi+5\upsilon)-\upsilon(\pi+\upsilon)(2\pi+\upsilon)\right]p_b^i+\tau\left[6\tau^2-(\pi+\upsilon)^2-2\pi\upsilon\right]p_s^i=\big[\tau^2(\pi+5\upsilon) \\ -\upsilon(\pi+\upsilon)(2\pi+\upsilon)\big]f_b+\tau\left[6\tau^2-(\pi+\upsilon)^2-2\pi\upsilon\right]f_s+\left[6\tau^2-(\pi+\upsilon)(2\pi+\upsilon)\right]\left(\tau^2-\pi\upsilon\right) \\ +\tau(\pi-\upsilon)\left(\tau^2-\pi\upsilon\right)+(\pi+\upsilon)\left(\tau^2-\pi\upsilon\right)\left(q_b^i-q_b^j\right) \quad \text{(b9)}$$

Next, we solve for p_s^i in equation (b9) and then substitute it into equation (b8) to express p_b^i as a function of the model parameter and the attributes developed on buyers' side. Subsequently, we substitute this outcome into equation (b9) to obtain:

$$p_b^i=f_b+\tau-\pi+\left[\frac{3\tau^2-\pi(\pi+2\upsilon)}{9\tau^2-(2\pi+\upsilon)(\pi+2\upsilon)}\right]\left(q_b^i-q_b^j\right)$$
$$p_s^i=f_s+\tau-\upsilon-\left[\frac{\tau(\upsilon-\pi)}{9\tau^2-(2\pi+\upsilon)(\pi+2\upsilon)}\right]\left(q_b^i-q_b^j\right)$$

for $i,j=1,2, \quad i\neq j$.

B.1.4 Second-order conditions at stage 2

We obtain the following second-order conditions from the profit maximisation problem at stage 2 of the game in Appendix B.1.3, which define the Hessian matrix as:

$$\text{H}=\begin{pmatrix}\Pi^i_{p_b^ip_b^i}\equiv\frac{\partial^2\Pi^i}{\partial(p_b^i)^2}=-\frac{\tau}{(\tau^2-\pi\upsilon)} & \Pi^i_{p_b^ip_s^i}\equiv\frac{\partial^2\Pi^i}{\partial p_b^i\,\partial p_s^i}=-\frac{(\pi+\upsilon)}{2(\tau^2-\pi\upsilon)} \\ \Pi^i_{p_s^ip_b^i}\equiv\frac{\partial^2\Pi^i}{\partial p_s^i\,\partial p_b^i}=-\frac{(\pi+\upsilon)}{2(\tau^2-\pi\upsilon)} & \Pi^i_{p_s^ip_s^i}\equiv\frac{\partial^2\Pi^i}{\partial(p_s^i)^2}=-\frac{\tau}{(\tau^2-\pi\upsilon)}\end{pmatrix}$$

In order to guarantee that platforms' profits reach a maximum with equilibrium fees in Equations (2.4a) and (2.4b) a sufficient condition is having H negative definite, indicating that $|H|>0$, and either $\Pi^i_{p_bp_b}<0$ or $\Pi^i_{p_sp_s}<0$. To show $\Pi^i_{p_bp_b}$ and $\Pi^i_{p_sp_s}$ are negative, the denominator $\tau^2-\pi\upsilon$ must be positive because the numerator is always positive, then we get $\tau^2>\pi\upsilon$ that turns to $\tau>\sqrt{\pi\upsilon}$. To show $|H|>0$ we have $\frac{\tau^2}{(\tau^2-\pi\upsilon)^2}-\frac{(\pi+\upsilon)^2}{4(\tau^2-\pi\upsilon)^2}>0$ that turns to $4\tau^2-(\pi+\upsilon)^2>0$, that turns to $\tau>\frac{\pi+\upsilon}{2}$.

In summary, for the second-order conditions defined by the Hessian matrix to be negative definite, the following conditions must hold (i) $\tau>\sqrt{\pi\upsilon}$ and (ii) $\tau>\frac{\pi+\upsilon}{2}$.

Now, we determine which of the two conditions is more stringent, ensuring that the other condition is also met. Initially, since the left side of both inequalities is equal, we compare the right sides to identify the greater one. That is $\frac{\pi+\upsilon}{2} > \sqrt{\pi\upsilon}$, which turns to $(\pi+\upsilon)^2 > 4\pi\upsilon$. Further simplification leads to $\pi^2 + 2\pi\upsilon + \upsilon^2 > 4\pi\upsilon$, which simplifies to $(\pi-\upsilon)^2 > 0$ if $\pi \neq \upsilon$. Therefore, if condition (ii) holds, condition (i) is satisfied.

B.1.5 Proof of Proposition 2.1

Proof. Partially differentiate equilibrium membership fees at stage one of the game in Equation 2.4a and Equation 2.4b regarding the difference in attributes on buyers' side. First, we define $\Delta q_b^i \equiv q_b^i - q_b^j$. Now, on buyers' side we have $\frac{\partial p_b^i}{\partial \Delta q_b^i} = \frac{3\tau^2 - \pi(\pi+2\upsilon)}{9\tau^2 - (2\pi+\upsilon)(\pi+2\upsilon)}$. To demonstrate that the previous expression is positive is sufficient to show both the numerator and denominator are positive. The denominator is positive if this condition $9\tau^2 - (2\pi+\upsilon)(\pi+2\upsilon)$ is positive. We use Assumption 2.1 to show $9\tau^2 - (2\pi+\upsilon)(\pi+2\upsilon)$ is positive. First, we make the left side of both inequalities equivalent to compare the right side, showing that Assumption 2.1 right side is greater and therefore the condition is positive. Assumption 2.1 can be transform to be $\tau^2 > \frac{(\pi+\upsilon)^2}{4}$, then we have $\frac{(\pi+\upsilon)^2}{4} > \frac{(2\pi+\upsilon)(\pi+2\upsilon)}{9}$ which simplifies to $9(\pi^2 + 2\pi\upsilon + \upsilon^2) - 4(2\pi^2 + 5\pi\upsilon + 2\upsilon^2) > 0$ and then simplifies to $(\pi-\upsilon)^2 > 0$ if $\pi \neq \upsilon$. Therefore, $9\tau^2 - (2\pi+\upsilon)(\pi+2\upsilon)$ is positive.

Following, we use the same method to show $3\tau^2 - \pi(\pi+2\upsilon)$ is positive by comparing the right side of both inequalities and showing Assumption 2.1 right side is greater, so the condition is positive. That is $\frac{(\pi+\upsilon)^2}{4} > \frac{\pi(\pi+2\upsilon)}{3}$ which turns to $3\upsilon^2 - 2\pi\upsilon - \pi^2 > 0$ which simplifies to $(3\upsilon+\pi)(\upsilon-\pi) > 0$ if $\upsilon > \pi$. Therefore $\partial p_b^i / \partial \Delta q_b^i > 0$ if $\upsilon > \pi$.

On sellers' side we have $\frac{\partial p_s^i}{\partial \Delta q_b^i} = \frac{\tau_s(\pi-\upsilon)}{9\tau^2 - (2\pi+\upsilon)(\pi+2\upsilon)}$ which is positive if $\pi > \upsilon$, considering we showed the denominator $9\tau^2 - (2\pi+\upsilon)(\pi+2\upsilon)$ is always positive as long as Assumption 2.1 holds. Therefore $\partial p_s^i / \partial \Delta q_b^i > 0$ if $\pi > \upsilon$. □

B.1.6 Buyers and sellers Market-shares

We obtain equilibrium market shares at stage two of the model in Equations (2.5a) and (2.5b) using membership fees in Equations (2.4a) and (2.4b). First, we compute the difference in membership fees on both sides of the market, $p_b^j - p_b^i = -\frac{2}{\Sigma}\left[3\tau^2 - \pi(\pi+2\upsilon)\right]\left(q_b^i - q_b^j\right)$ and $p_s^j - p_s^i = -\frac{2\tau}{\Sigma}(\pi-\upsilon)\left(q_b^i - q_b^j\right)$. Where $\Sigma \equiv 9\tau^2 - (2\pi+\upsilon)\left(2\upsilon+\pi\right)$. Then we substitute these expressions into Equations (2.2a) and (2.2b) to get:

$$\begin{aligned}\eta_b^i &= \frac{1}{2} + \frac{\tau\left[9\tau^2 - (2\pi+\upsilon)(2\upsilon+\pi) - 2\left(3\tau^2 - \pi(\pi+2\upsilon)\right) - 2\upsilon(\pi-\upsilon)\right]\left(q_b^i - q_b^j\right)}{2\left(9\tau^2 - (2\pi+\upsilon)(2\upsilon+\pi)\right)\left(\tau^2 - \pi\upsilon\right)} \\ &= \frac{1}{2} + \frac{3\tau^2 - \upsilon(\pi+2\upsilon) - 2\upsilon(\pi-\upsilon)}{2\left(9\tau^2 - (2\pi+\upsilon)(2\upsilon+\pi)\right)\left(\tau^2 - \pi\upsilon\right)}\end{aligned}$$

$$\eta_b^i = \frac{1}{2} + \left[\frac{3\tau}{2\left[9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)\right]}\right]\left(q_b^i - q_b^j\right)$$

$$\eta_s^i = \frac{1}{2} + \frac{\left[\pi\left(9\tau^2 - (2\pi + \upsilon)(2\upsilon + \pi)\right) - 2\tau^2(\pi - \upsilon) - 2\pi\left(3\tau^2 - \pi(\pi + 2\upsilon)\right)\right]\left(q_b^i - q_b^j\right)}{2\left(9\tau^2 - (2\pi + \upsilon)(2\upsilon + \pi)\right)\left(\tau^2 - \pi\upsilon\right)}$$

$$= \frac{3\tau^2\pi - \pi\upsilon(\pi + 2\upsilon) - 2\tau^2(\pi - \upsilon)}{2\left(9\tau^2 - (2\pi + \upsilon)(2\upsilon + \pi)\right)\left(\tau^2 - \pi\upsilon\right)}$$

$$\eta_s^i = \frac{1}{2} + \left[\frac{(\pi + 2\upsilon)}{2\left[9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)\right]}\right]\left(q_b^i - q_b^j\right)$$

B.1.7 Attributes Maximisation Problem - Stage 1

The first-order conditions of the platform i, $i = 1, 2$ maximisation problem at stage 1 come from maximising Equation 2.6, that is:

$$\frac{\partial \Pi^1}{\partial q_b^1} = \frac{2\tau\left(q_b^1 - q_b^2\right) + \left[6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)\right]}{2\left[9\tau^2 - (2\pi - \upsilon)(\pi + 2\upsilon)\right]} - \alpha^1 q_b^1 = 0$$

$$\frac{\partial \Pi^2}{\partial q_b^2} = \frac{2\tau\left(q_b^2 - q_b^1\right) + \left[6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)\right]}{2\left[9\tau^2 - (2\pi - \upsilon)(\pi + 2\upsilon)\right]} - \alpha^2 q_b^2 = 0$$

From the first-order conditions on both platforms, we obtain:

$$2\left[\alpha^i\left[9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)\right] - \tau\right] q_b^i = -2\tau q_b^j + 6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi) \tag{b10}$$

$$2\left[\alpha^j\left[9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)\right] - \tau\right] q_b^j = -2\tau q_b^i + 6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi) \tag{b11}$$

Then, we solve for q_b^j on both equations (b10) and (b11), then we compare them to get q_b^i, that is:

$$\frac{1}{2\tau}\left[-2\left(\alpha^i\Sigma - \tau\right) q_b^i + \left[6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)\right]\right] = \frac{1}{2\left(\alpha^j\Sigma - \tau\right)}\left[-2\tau q_b^i + \left[6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)\right]\right]$$

Where $\Sigma \equiv 9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$

$$2\left(\alpha^i\Sigma - \tau\right)\left(\alpha^j\Sigma - \tau\right) q_b^i + \tau\left[6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)\right] = 2\tau^2 q_b^i + \left(\alpha^j\Sigma - \tau\right)\left[6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)\right]$$

$$2\Sigma\left[\alpha^i\alpha^j\Sigma - \left(\alpha^i + \alpha^j\right)\tau\right] q_b^i = \left(\alpha^j\Sigma - 2\tau\right)\left[6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)\right]$$

$$q_b^i = \frac{\left(\alpha^j\Sigma - 2\tau\right)\left[6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)\right]}{2\Sigma\left[\alpha^i\alpha^j\Sigma - \left(\alpha^i + \alpha^j\right)\tau\right]} \quad \text{for } i, j = 1, 2, \quad i \neq j$$

where $\Sigma \equiv 9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$

B.1.8 Second-order conditions at stage 1

We obtain the following second-order condition from the profit maximisation at stage 1 of the game in Appendix B.1.4 as:

$$\Pi^i_{q_b^i q_b^i} \equiv \frac{\partial^2 \Pi^i}{\partial (q_b^i)^2} = \frac{\tau}{9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)} - \alpha^i = 0$$

To guarantee that platforms' profits reach a maximum at stage 2 of the game with equilibrium attributes in Equation (2.8), a sufficient condition is to have the previous second partial derivative negative. To show $\Pi^i_{q_b^i q_b^i} < 0$ is sufficient to have $\alpha^i > \frac{\tau}{\Sigma}$ where $\Sigma \equiv 9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$

B.1.9 Positive Equilibrium Attributes

In order to achieve positive equilibrium attributes in Equation (2.8), we require the following: $q_b^i = \frac{\left(\alpha^j \Sigma - 2\tau\right)\left[6\tau^2 - (\pi+\upsilon)(\pi+2\upsilon) + \tau(\upsilon - \pi)\right]}{2\Sigma\left[\alpha^i \alpha^j \Sigma - (\alpha^i + \alpha^j)\tau\right]} > 0$. Where $\Sigma \equiv 9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$. Initially, we see that q_b^i is made up of three different elements. Let's call $(\alpha^j \Sigma - 2\tau)$ part one, $[6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)]$ part two and $2\Sigma\left[\alpha^i \alpha^j \Sigma - (\alpha^i + \alpha^j)\tau\right]$ part three. Then it is sufficient to show that all three parts are positive to confirm positive equilibrium attributes.

Firstly, element number one $(\alpha^j \Sigma - 2\tau)$ is positive if $\alpha^j > \frac{2\tau}{\Sigma}$. Now, we show it is satisfied using the fact that platform 1 is more efficient in developing attributes than platform 2, that is $\alpha^2 > \alpha^1$ as was defined in Section 2.2. Therefore if $\alpha^2 > \alpha^1$ and $\alpha^2 > \frac{2\tau}{\Sigma}$ we derive $\alpha^1 > \frac{2\tau}{\Sigma}$. Then we obtain $\alpha^i > \frac{2\tau}{\Sigma}$ for $i = 1, 2$ which is Assumption 2.2.

Secondly, for element number two $6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)$ to be positive, we determine a value for τ that ensures the entire expression is positive. We rearrange the expression as a quadratic polynomial in τ, that is $6\tau^2 - (\pi - \upsilon)\tau - (\pi + \upsilon)(\pi + 2\upsilon)$. Then, employing the quadratic formula to find the roots, we obtain $\tau = \frac{-[-(\pi-\upsilon)] \pm \sqrt{[-(\pi-\upsilon)]^2 - 4(\pi+\upsilon)(\pi+2\upsilon)(-6)}}{12}$ which simplifies to $\tau = \frac{(\pi-\upsilon) \pm (5\pi + 7\upsilon)}{12}$. The first root is $\tau_{r_1} = \frac{\pi+\upsilon}{2}$ and the second root is $\tau_{r_2} = \frac{-(\pi+2\upsilon)}{3}$. Since the square term of the polynomial in τ is positive, the expression $6\tau^2 - (\pi - \upsilon)\tau - (\pi + \upsilon)(\pi + 2\upsilon)$ is positive for values outside both roots, that is for $\tau > \frac{\pi+\upsilon}{2}$ and for $\tau < \frac{-(\pi+2\upsilon)}{3}$. Since transportation cost τ is positive by definition, values for $\tau < \frac{-(\pi+2\upsilon)}{3}$ are dismissed. Consequently, $6\tau^2 - (\pi - \upsilon)\tau - (\pi + \upsilon)(\pi + 2\upsilon) > 0$ if $\tau > \frac{\pi+\upsilon}{2}$, as stated in Assumption 2.1.

Finally, we show the third component is positive. First, we show $\Sigma \equiv 9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$ is positive by using Assumption 2.1. We make the left side of both inequalities equal to compare the right side, showing that Assumption 2.1 right side is greater and therefore proving $\Sigma > 0$. That is, Assumption 2.1 can be transform to be $\tau^2 > \frac{(\pi+\upsilon)^2}{4}$, then we have $\frac{(\pi+\upsilon)^2}{4} > \frac{(2\pi+\upsilon)(\pi+2\upsilon)}{9}$ which simplifies to $9(\pi^2 + 2\pi\upsilon + \upsilon^2) - 4(2\pi^2 + 5\pi\upsilon + 2\upsilon^2) > 0$ and simplifies to $(\pi - \upsilon)^2 > 0$ if $\pi \neq \upsilon$. So we have demonstrated Σ is positive. Now, for $[\alpha^i\alpha^j\Sigma - (\alpha^i + \alpha^j)\tau]$ to be positive it is sufficient to have $\alpha^i > \frac{\alpha^j\tau}{\alpha^j\Sigma-\tau}$. Then, we show it is satisfied if Assumption 2.2 is more stringent than the previous condition. We compare the right side of both inequalities, that is $\frac{2\tau}{\Sigma} > \frac{\alpha^j\tau}{\alpha^j\Sigma-\tau}$ which simplifies to $2(\alpha^j\Sigma - \tau) - \alpha^j\Sigma > 0$ and simplifies to $\alpha^j\Sigma - 2\tau > 0$ if $\alpha^j > \frac{2\tau}{\Sigma}$, which is Assumption 2.2. Then if Assumption 2.2 holds, condition $\alpha^i > \frac{\alpha^j\tau}{\alpha^j\Sigma-\tau}$ is satisfied, which leads us to have proven the third element of q_b^i to be positive.

Summarising, $q_b^i > 0$ if $\alpha^j > \frac{2\tau}{\Sigma}$, $\alpha^i > \frac{\alpha^j\tau}{\alpha^j\Sigma-\tau}$ and $\tau > \frac{(\pi+\upsilon)}{2}$ which are satisfied as long as Assumption 2.1 and Assumption 2.2 hold.

B.2 Benchmark Scenario: $\pi = \upsilon$

When $\upsilon = \pi$ Assumption 2.1 turns to $\tau > \pi$ and Assumption 2.2 turns to $\alpha^i > \frac{2\tau}{9\sigma}$, where $\sigma \equiv \tau^2 - \pi^2$.

B.2.1 Proof of Proposition 2.2 and Corollary 2.1

Proof. We prove Proposition 2.2 by partially differentiate Equation (2.10) with respect to τ and π under the assumption that platform 1 is more efficient in developing attributes on buyers' side than platform 2, $\alpha^2 > \alpha^1$.

$$\frac{\partial \left(q_b^i\right)^{bs}}{\partial \tau} = \frac{3\left[9\alpha^i\alpha^j\sigma - (\alpha^i + \alpha^j)\tau\right](18\alpha^j\tau - 2) - 3(9\alpha^j\sigma - 2\tau)\left[18\alpha^i\alpha^j\tau - (\alpha^i + \alpha^j)\right]}{9\left[9\alpha^i\alpha^j\sigma - (\alpha^i + \alpha^j)\tau\right]^2}$$

$$= \frac{18\alpha^i\alpha^j(9\alpha^j\tau - 1) - 2\tau(\alpha^i + \alpha^j)(9\alpha^j\tau - 1) - 18\alpha^i\alpha^j\tau(9\alpha^j\sigma - \tau) + (\alpha^i + \alpha^j)(9\alpha^j\sigma - \tau)}{3\left[9\alpha^i\alpha^j\sigma - (\alpha^i + \alpha^j)\tau\right]^2}$$

$$\frac{\partial \left(q_b^i\right)^{bs}}{\partial \tau} = \frac{18\alpha^i\alpha^j(2\tau^2 - \sigma) - 9\alpha^j(\alpha^i + \alpha^j)(2\tau^2 - \sigma)}{3\left[9\alpha^i\alpha^j\sigma - (\alpha^i + \alpha^j)\tau\right]^2} = \frac{-3\alpha^j(\alpha^j - \alpha^i)(\tau + \pi^2)}{\left[9\alpha^i\alpha^j\sigma - (\alpha^i + \alpha^j)\tau\right]^2}$$

$$\frac{\partial \left(q_b^i\right)^{bs}}{\partial \pi} = \frac{-54\alpha^j\pi\left[9\alpha^i\alpha^j\sigma - (\alpha^i + \alpha^j)\tau\right] + 54\alpha^i\alpha^j\pi(9\alpha^j\sigma - 2\tau)}{9\left[9\alpha^i\alpha^j\sigma - (\alpha^i + \alpha^j)\tau\right]^2}$$

$$\frac{\partial \left(q_b^i\right)^{bs}}{\partial \pi} = \frac{6\alpha^j\pi\left[-2\alpha^i\tau + (\alpha^i + \alpha^j)\tau\right]}{\left[9\alpha^i\alpha^j\sigma - (\alpha^i + \alpha^j)\tau\right]^2} = \frac{6\alpha^j\tau\pi(\alpha^j - \alpha^i)}{\left[9\alpha^i\alpha^j\sigma - (\alpha^i + \alpha^j)\tau\right]^2}$$

Now, to know the signs of both partial derivatives for platform 1, we need to find out the signs of their elements. The denominators are positive given they are squared. The elements on the numerators are positive considering platform 1 is more efficient in developing attributes compared to platform 2, ($\alpha^2 > \alpha^1$). Therefore, $\frac{\partial\left(q_b^i\right)^{bs}}{\partial\tau} = \frac{-3\alpha^j\left(\alpha^j-\alpha^i\right)\left(\tau+\pi^2\right)}{\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2} < 0$ and $\frac{\partial\left(q_b^i\right)^{bs}}{\partial\pi} = \frac{6\alpha^j\left(\alpha^j-\alpha^i\right)\tau\pi}{\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2} > 0$.

Furthermore, $\frac{\partial\left(q_b^i\right)^{bs}}{\partial\pi} - \frac{\partial\left(q_b^j\right)^{bs}}{\partial\pi} =$

$$\frac{6\alpha^j\left(\alpha^j-\alpha^i\right)\tau\pi}{\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2} - \frac{6\alpha^j\left(\alpha^i-\alpha^j\right)\tau\pi}{\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2} = \frac{6\left(\alpha^i+\alpha^j\right)\left(\alpha^j-\alpha^i\right)\tau\pi}{\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2} > 0$$

The difference between the partial derivatives of the equilibrium attributes with respect to the cross-group network effect on both platforms, $\frac{\partial\left(q_b^1\right)^{bs}}{\partial\pi}$ and $\frac{\partial\left(q_b^2\right)^{bs}}{\partial\pi}$ is positive given the same argument shown previously.

Next, we prove Corollary 2.1 by partially differentiate $\left(\Delta q_b^i\right)^{bs} = q_b^i - q_b^j$ with respect to τ and π, considering platform 1 is more efficient in developing attributes compared to platform 2, ($\alpha^2 > \alpha^1$). Firstly, We use Equation (2.10) to compute $\left(\Delta q_b^i\right)^{bs}$, which is $\left(\Delta q_b^i\right)^{bs} = \frac{\left(9\alpha^j\sigma-2\tau\right)-\left(9\alpha^i\sigma-2\tau\right)}{3\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]}$, which simplifies to $\left(\Delta q_b^i\right)^{bs} = \frac{3\sigma\left(\alpha^j-\alpha^i\right)}{9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau}$. Then, we have:

$$\frac{\partial\left(\Delta q_b^i\right)^{bs}}{\partial\tau} = \frac{6\tau\left(\alpha^j-\alpha^i\right)\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right] - 3\sigma\left(\alpha^j-\alpha^i\right)\left[18\alpha^i\alpha^j\tau-\left(\alpha^i+\alpha^j\right)\right]}{\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2}$$
$$= \frac{3\left(\alpha^j-\alpha^i\right)\left[-2\left(\alpha^i+\alpha^j\right)\tau^2+\sigma\left(\alpha^i+\alpha^j\right)\right]}{\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2} = \frac{-3\left(\alpha^i+\alpha^j\right)\left(\alpha^j-\alpha^i\right)\left(\tau+\pi^2\right)}{\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2}$$

$$\frac{\partial\left(\Delta q_b^i\right)^{bs}}{\partial\pi} = \frac{6\pi\left(\alpha^j-\alpha^i\right)\left[-9\alpha^i\alpha^j\sigma+\left(\alpha^i+\alpha^j\right)\tau+9\alpha^i\alpha^j\sigma\right]}{\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2} = \frac{6\left(\alpha^i+\alpha^j\right)\left(\alpha^j-\alpha^i\right)\tau\pi}{\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2}$$

The partial derivatives $\frac{\partial\left(\Delta q_b^i\right)^{bs}}{\partial\tau} < 0$ is negative and $\frac{\partial\left(\Delta q_b^i\right)^{bs}}{\partial\pi} > 0$ is positive as established using the same reasoning presented in the proof of Proposition 2.2 □

B.2.2 Proof of Proposition 2.3

Proof. We prove Proposition 2.3 by partially differentiating the difference in buyers' equilibrium membership fees with respect to τ and π using Equation (2.11a).

Firstly, we manipulate the expression for the difference in buyers' equilibrium membership fees in the following way: $\left(\Delta p_b^i\right)^{bs} = \left(p_b^i\right)^{bs} - \left(p_b^j\right)^{bs} = f_b + \tau - \pi + \frac{1}{3}\left(\Delta q_b^i\right)^{bs} - \left[f_b + \tau - \pi + \frac{1}{3}\left(\Delta q_b^i\right)^{bs}\right] = \frac{2\sigma\left(\alpha^j-\alpha^i\right)}{9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau}$. Then we see that $\left(\Delta p_b^i\right)^{bs} = \frac{2}{3}\left(\Delta q_b^i\right)^{bs}$.

Now, we obtain $\frac{\partial\left(\Delta p_b^i\right)^{bs}}{\partial\tau} = \frac{2}{3}\frac{\partial\left(\Delta q_b^i\right)^{bs}}{\partial\tau}$. We have shown that $\frac{\partial\left(\Delta q_b^i\right)^{bs}}{\partial\tau} < 0$ in the proof of Proposition 2.2, therefore $\frac{\partial\left(\Delta p_b^i\right)^{bs}}{\partial\tau} < 0$. Next, we compute $\frac{\partial\left(\Delta p_b^i\right)^{bs}}{\partial\pi} = \frac{2}{3}\frac{\partial\left(\Delta q_b^i\right)^{bs}}{\partial\pi}$. We have shown that $\frac{\partial\left(\Delta q_b^i\right)^{bs}}{\partial \pi} > 0$ in the proof of Proposition 2.2, therefore $\frac{\partial\left(\Delta p_b^i\right)^{bs}}{\partial \pi} > 0$.

Finally, we compute $\frac{\partial\left(p_b^i\right)^{bs}}{\partial \pi} - \frac{\partial\left(p_b^j\right)^{bs}}{\partial \pi} = -1 + \frac{6\left(\alpha^i+\alpha^j\right)\left(\alpha^j-\alpha^i\right)\tau\pi}{\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2} - \left[-1 + \frac{6\left(\alpha^i+\alpha^j\right)\left(\alpha^i-\alpha^j\right)\tau\pi}{\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2}\right]$ which simplifies to $\frac{6\left(\alpha^i+\alpha^j\right)\left(\alpha^i-\alpha^j\right)\tau\pi}{\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2}$. Then, considering platform 1 is more efficient in developing attributes compared to platform 2, ($\alpha^2 > \alpha^1$) we get $\frac{\partial\left(p_b^i\right)^{bs}}{\partial \pi} - \frac{\partial\left(p_b^j\right)^{bs}}{\partial \pi} > 0$. □

B.2.3 Market-shares conditions

We obtain conditions for buyers' and sellers' market shares to be distributed in the unit interval using Equations (2.12a) and (2.12b)

$0 < \left(\eta_b^i\right)^{bs} < 1$. For $\left(\eta_b^i\right)^{bs} > 0$ we have $\frac{1}{2} + \frac{\left(\alpha^j-\alpha^i\right)\tau}{2\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]} > 0$. This inequality turns into $\left(\alpha^j-\alpha^i\right)\tau > -9\alpha^i\alpha^j\sigma + \left(\alpha^i+\alpha^j\right)\tau$, which simplifies to $\alpha^i\left(9\alpha^j\sigma - 2\tau\right) > 0$ if $\alpha^j > \frac{2\tau}{9\sigma}$. This condition holds under Assumption 2.2 when $\pi = \upsilon$. For $\left(\eta_b^i\right)^{bs} < 1$ we get $9\alpha^i\alpha^j\sigma - \left(\alpha^i+\alpha^j\right)\tau - \left(\alpha^j-\alpha^i\right)\tau > 0$. This inequality simplifies to $\alpha^j\left(9\alpha^i\sigma - 2\tau\right) > 0$ if $\alpha^i > \frac{2\tau}{9\sigma}$. This condition holds under Assumption 2.2 when $\pi = \upsilon$.

$0 < \left(\eta_s^i\right)^{bs} < 1$. For $\left(\eta_s^i\right)^{bs} > 0$ we have $\frac{1}{2} + \frac{\left(\alpha^j-\alpha^i\right)\pi}{2\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]} > 0$. This inequality turns into $\left(\alpha^j-\alpha^i\right)\pi > -9\alpha^i\alpha^j\sigma + \left(\alpha^i+\alpha^j\right)\tau$, which simplifies to $9\alpha^i\alpha^j\sigma - \alpha^i\left(\tau+\pi\right) - \alpha^j\left(\tau-\pi\right) > 0$ if $\alpha^i > \frac{\alpha^j(\tau-\pi)}{9\alpha^j\sigma-(\tau+\pi)}$. Now, if the right side of Assumption 2.2 is greater than the right side of $\alpha^i > \frac{\alpha^j(\tau-\pi)}{9\alpha^j\sigma-(\tau+\pi)}$ the condition is satisfied. That is $\frac{2\tau}{9\sigma} > \frac{\alpha^j(\tau-\pi)}{9\alpha^j\sigma-(\tau+\pi)}$ turns to $18\alpha^j\sigma\tau - 2\tau\left(\tau+\pi\right) - 9\alpha^j\sigma\left(\tau-\pi\right) > 0$. This inequality simplifies to $\left(\tau+\pi\right)\left(9\alpha^j\sigma - 2\tau\right) > 0$. This inequality is positive under Assumption 2.2 when $\pi = \upsilon$. For $\left(\eta_s^i\right)^{bs} < 1$ we get $9\alpha^i\alpha^j\sigma - \left(\alpha^i+\alpha^j\right)\tau - \left(\alpha^j+\alpha^i\right)\pi > 0$ if $\alpha^j > \frac{\alpha^i(\tau-\pi)}{9\alpha^i\sigma-(\tau+\pi)}$. We follow the same method comparing the right side of this condition and Assumption 2.2. That is $\frac{2\tau}{9\sigma} > \frac{\alpha^i(\tau-\pi)}{9\alpha^i\sigma-(\tau+\pi)}$ turns to $18\alpha^i\sigma\tau - 2\tau\left(\tau+\pi\right) - 9\alpha^i\sigma\left(\tau-\pi\right) > 0$. This inequality simplifies to $\left(\tau+\pi\right)\left(9\alpha^i\sigma - 2\tau\right) > 0$. This inequality is positive under Assumption 2.2 when $\pi = \upsilon$.

In summary, as long as $\alpha^i > \frac{2\tau}{9\sigma}$, for $i = 1, 2$, which is guaranteed by Assumption 2.2 when $\pi = \upsilon$, then the conditions $0 < \left(\eta_b^i\right)^{bs} < 1$ and $0 < \left(\eta_s^i\right)^{bs} < 1$ are satisfied.

B.2.4 Impacts on Equilibrium Market-shares

We compute the impacts on buyers' and sellers' equilibrium market shares respect parameters τ and π using Equations (2.12a) and (2.12b).

Firstly, we manipulate the expression for buyers' and sellers' equilibrium market shares in Equations (2.12a) and (2.12b) as follows: We know that $\left(\Delta q_b^i\right)^{bs} = \frac{3\sigma\left(\alpha^j-\alpha^i\right)}{9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau}$ then $\left(\eta_b^i\right)^{bs} = \frac{1}{2} + \frac{\left(\alpha^j-\alpha^i\right)\tau}{2\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]}$ can be rewritten as $\left(\eta_b^i\right)^{bs} = \frac{1}{2} + \frac{\tau}{6\sigma}\left(\Delta q_b^i\right)^{bs}$, and $\left(\eta_s^i\right)^{bs} =$

$\frac{1}{2} + \frac{(\alpha^j - \alpha^i)\pi}{2[9\alpha^i\alpha^j\sigma - (\alpha^i+\alpha^j)\tau]}$ can be rewritten as $(\eta_s^i)^{bs} = \frac{1}{2} + \frac{\pi}{6\sigma}(\Delta q_b^i)^{bs}$.

Next, we compute the partial derivatives on buyers' side, $\frac{\partial(\eta_b^i)^{bs}}{\partial\tau} = \frac{1}{6^2\sigma^2}\left[6(\Delta q_b^i)^{bs}(\sigma - 2\tau^2) + 6\sigma\partial(\Delta q_b^i)^{bs}/\partial\tau\right]$, which simplifies to $\frac{\partial(\eta_b^i)^{bs}}{\partial\tau} = \frac{1}{6\sigma^2}\left[\sigma\left(\frac{\partial(\Delta q_b^i)^{bs}}{\partial\tau}\right) - (\Delta q_b^i)^{bs}(\tau^2 + \pi^2) + \right]$. All the elements of the partial derivative are positive except $\frac{\partial(\Delta q_b^i)^{bs}}{\partial\tau}$, which we demonstrated in the proof of Proposition 2.2 that $\frac{\partial(\Delta q_b^i)^{bs}}{\partial\tau} < 0$, consequently, it follows that $\frac{\partial(\eta_b^i)^{bs}}{\partial\tau} < 0$. $\frac{\partial(\eta_b^i)^{bs}}{\partial\pi} = \frac{\tau}{18\sigma^2}\left[3\sigma\left(\frac{\partial(\Delta q_b^i)^{bs}}{\partial\pi}\right) + \pi(\Delta q_b^i)^{bs}\right]$. We demonstrated in the proof of Proposition 2.2 that $\frac{\partial(\Delta q_b^i)^{bs}}{\partial\pi} > 0$, and all elements of the partial derivative are positive, consequently, it follows that $\frac{\partial(\eta_b^i)^{bs}}{\partial\pi} > 0$.

Next, we compute the partial derivatives on sellers' side, $\frac{\partial(\eta_s^i)^{bs}}{\partial\tau} = \frac{\pi}{6\sigma^2}\left[\sigma\left(\frac{\partial(\Delta q_b^i)^{bs}}{\partial\tau}\right) - 2\tau(\Delta q_b^i)^{bs}\right]$. All the elements of the partial derivative are positive except $\frac{\partial(\Delta q_b^i)^{bs}}{\partial\tau}$, which we demonstrated in the proof of Proposition 2.2 that $\frac{\partial(\Delta q_b^i)^{bs}}{\partial\tau} < 0$, consequently, it follows that $\frac{\partial(\eta_s^i)^{bs}}{\partial\tau} < 0$. $\frac{\partial(\eta_s^i)^{bs}}{\partial\pi} = \frac{1}{6\sigma^2}\left[(\Delta q_b^i)^{bs}(\sigma + 2\pi^2) + \pi\sigma\left(\frac{\partial(\Delta q_b^i)^{bs}}{\partial\pi}\right)\right]$ which simplifies to $\frac{\partial(\eta_s^i)^{bs}}{\partial\pi} = \frac{1}{6\sigma^2}\left[(\Delta q_b^i)^{bs}(\tau^2 + \pi^2) + \pi\sigma\left(\frac{\partial(\Delta q_b^i)^{bs}}{\partial\pi}\right)\right]$. We demonstrated in the proof of Proposition 2.2 that $\frac{\partial(\Delta q_b^i)^{bs}}{\partial\pi} > 0$, and all elements of the partial derivative are positive, consequently, it follows that $\frac{\partial(\eta_s^i)^{bs}}{\partial\pi} > 0$.

B.2.5 Positive Equilibrium Profits

We show the conditions for Equilibrium profits in Equation (2.13) for platform 1 to be positive. We notice Equation (2.13) is compose of two elements, the first is $\tau - \pi$ and the second is $\frac{9\sigma(\alpha^j - \alpha^i)[9\alpha^i\alpha^j\sigma - (\alpha^i+\alpha^j)\tau] - \alpha^i(9\alpha^j\sigma - 2\tau)(9\alpha^i\sigma - 2\tau)}{18[9\alpha^i\alpha^j\sigma - (\alpha^i+\alpha^j)\tau]^2}$. The first component is positive under Assumption 2.1 when $\pi = \upsilon$. To determine if the second element is positive, we can partially differentiate it with respect to α^2 and evaluate the result when $\alpha^2 = \frac{2\tau}{9\sigma}$.

$$\begin{aligned}
\frac{\partial part2}{\partial\alpha^2} = 18\Big[& \left[9\alpha^1\alpha^2\sigma - (\alpha^1+\alpha^2)\tau\right]^2\left[9\sigma\left[9\alpha^1\alpha^2\sigma - (\alpha^1+\alpha^2)\tau\right] + 9\sigma(\alpha^2 - \alpha^1)\right. \\
& (9\alpha^1\sigma - \tau) - 9\alpha^1\sigma(9\alpha^1\sigma - 2\tau)\Big] - 2(9\alpha^1\sigma - \tau)\left[9\alpha^1\alpha^2\sigma - (\alpha^1+\alpha^2)\tau\right]\left[9\sigma(\alpha^2 - \alpha^1)\right. \\
& \left[9\alpha^1\alpha^2\sigma - (\alpha^1+\alpha^2)\tau\right] - \alpha^1(9\alpha^2\sigma - 2\tau)(9\alpha^1\sigma - 2\tau)\Big]\Big]\Big/18^2\left[9\alpha^1\alpha^2\sigma - (\alpha^1+\alpha^2)\tau\right]^4 \\
= \Big[& 18\sigma\left[9\alpha^1\alpha^2\sigma - (\alpha^1+\alpha^2)\tau\right]\left[9\alpha^1\alpha^2\sigma - 9(\alpha^1)^2\sigma - 2\tau(\alpha^2 - \alpha^1)\right] - 18\sigma(\alpha^2 - \alpha^1) \\
& (9\alpha^1\sigma - \tau)\left[9\alpha^1\alpha^2\sigma - (\alpha^1+\alpha^2)\tau\right] + 2\alpha^1(9\alpha^1\sigma - \tau)(9\alpha^1\sigma - 2\tau) \\
& (9\alpha^2\sigma - 2\tau)\Big]\Big/18\left[9\alpha^1\alpha^2\sigma - (\alpha^1+\alpha^2)\tau\right]^3
\end{aligned}$$

$$= \Big[18\sigma\left[9\alpha^1\alpha^2\sigma - \left(\alpha^1+\alpha^2\right)\tau\right]\left[\alpha^2\left(9\alpha^1\sigma-\tau\right) - \alpha^1\left(9\alpha^1\sigma-\tau\right)\right] - 18\sigma\left(\alpha^2-\alpha^1\right) \left(9\alpha^1\sigma-\tau\right)\left[9\alpha^1\alpha^2\sigma-\left(\alpha^1+\alpha^2\right)\tau\right] + 2\alpha^1\left(9\alpha^1\sigma-\tau\right)\left(9\alpha^1\sigma-2\tau\right) \left(9\alpha^2\sigma-2\tau\right)\Big]\Big/ 18\left[9\alpha^1\alpha^2\sigma-\left(\alpha^1+\alpha^2\right)\tau\right]^3 = \frac{\alpha^1\left(9\alpha^1\sigma-\tau\right)\left(9\alpha^1\sigma-2\tau\right)\left(9\alpha^2\sigma-2\tau\right)}{9\left[9\alpha^1\alpha^2\sigma-\left(\alpha^1+\alpha^2\right)\tau\right]^3}$$

$$\frac{\partial part2}{\partial\alpha^2} = \frac{\alpha^1\left(9\alpha^1\sigma-\tau\right)\left(9\alpha^1\sigma-2\tau\right)\left(9\alpha^2\sigma-2\tau\right)}{9\left[\alpha^2\left(9\alpha^1\sigma-2\tau\right)+\tau\left(\alpha^2-\alpha^1\right)\right]^3}$$

As it can be observed, for values of α^1 and α^2 greater than $\frac{2\tau}{9\sigma}$, the equilibrium profits on platform 1 in Equation (2.13) are always positive. This condition $\alpha^i > \frac{2\tau}{9\sigma}$, for $i = 1, 2$ is Assumption 2.2 when $\pi = \upsilon$.

B.2.6 Proof of Proposition 2.4

Proof. We prove Proposition 2.4 by partially differentiating the difference in equilibrium profits with respect to τ and π using Equation (2.13) under the assumption that platform 1 is more efficient in developing attributes on buyers' side than platform 2, $\alpha^2 > \alpha^1$.

$$\frac{\partial\left(\Delta\Pi^i\right)^{bs}}{\partial\tau} = 18\left(\alpha^j-\alpha^i\right)\Big[18\tau\left[27\alpha^i\alpha^j\sigma-4\left(\alpha^i+\alpha^j\right)\tau\right]+9\sigma\left[54\alpha^i\alpha^j\sigma-4\left(\alpha^i+\alpha^j\right)\right] +8\tau\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2 - 2\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]\left[18\alpha^i\alpha^j\tau-\left(\alpha^i+\alpha^j\right)\right] \left[9\sigma\left[27\alpha^i\alpha^j\sigma-4\left(\alpha^i+\alpha^j\right)\tau\right]+4\tau^2\right]\Big]\Big/18^2\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^4$$

$$= \left(\alpha^j-\alpha^i\right)\Big[9\big[54\alpha^i\alpha^j\sigma-2\left(\alpha^i+\alpha^j\right)\left(\sigma+2\tau^2\right)\big]\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]+4\tau\big[9\alpha^i\alpha^j\sigma -\left(\alpha^i+\alpha^j\right)\tau\big] - 9\sigma\left[18\alpha^i\alpha^j\tau-\left(\alpha^i+\alpha^j\right)\right]\left[27\alpha^i\alpha^j\sigma-4\left(\alpha^i+\alpha^j\right)\tau\right] - 4\tau^2\left[18\alpha^i\alpha^j\tau-\left(\alpha^i+\alpha^j\right)\right]\Big]\Big/9\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^3$$

$$= \frac{\left(\alpha^j-\alpha^i\right)\left(\sigma-2\tau^2\right)\left[9\alpha^i\alpha^j\left(\alpha^i+\alpha^j\right)\sigma-2\tau\left(\alpha^i+\alpha^j\right)^2+4\alpha^i\alpha^j\tau\right]}{\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^3}$$

$$\frac{\partial\left(\Delta\Pi^i\right)^{bs}}{\partial\tau} = \frac{-\left(\alpha^j-\alpha^i\right)\left(\tau^2+\pi^2\right)\left[\left(\alpha^i\right)^2\left(9\alpha^j\sigma-2\tau\right)+\left(\alpha^j\right)^2\left(9\alpha^i\sigma-2\tau\right)\right]}{\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^3}$$

To guarantee that $\frac{\partial\left(\Delta\Pi^i\right)^{bs}}{\partial\tau}$ is negative for platform 1, it is sufficient for all of its components to be positive. The denominator can be expressed as $\left[\alpha^j\left(9\alpha^j\sigma-2\tau\right)+\tau\left(\alpha^j-\alpha^i\right)\right]^3$, which is positive if Assumption 2.2 holds when $\pi = \upsilon$. The elements in the numerator are all positive, based on the same reasoning as for the denominator, provided that platform 1 is more efficient in developing attributes, $\alpha^2 > \alpha^1$ and Assumption 2.2 holds when $\pi = \upsilon$.

$$\frac{\partial\left(\Delta\Pi^i\right)^{bs}}{\partial\pi} = \left(\alpha^j-\alpha^i\right)\Big[-18\pi\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2\Big[\left[27\alpha^i\alpha^j\sigma-4\left(\alpha^i+\alpha^j\right)\tau\right]$$

$$+27\alpha^i\alpha^j\sigma\big] + 36\alpha^i\alpha^j\pi\left[9\alpha^i\alpha^j\sigma - \left(\alpha^i+\alpha^j\right)\tau\right]\left[9\sigma\left[27\alpha^i\alpha^j\sigma - 4\left(\alpha^i+\alpha^j\right)\tau\right] + 4\tau^2\right]\Big]$$

$$\Big/ 18\left[9\alpha^i\alpha^j\sigma - \left(\alpha^i+\alpha^j\right)\tau\right]^4$$

$$= 2\pi\left(\alpha^j-\alpha^i\right)\Big[-\left[9\alpha^i\alpha^j\sigma - \left(\alpha^i+\alpha^j\right)\tau\right]\left[27\alpha^i\alpha^j\sigma - 2\left(\alpha^i+\alpha^j\right)\tau\right]$$

$$+ 9\alpha^i\alpha^j\sigma\left[27\alpha^i\alpha^j\sigma - 4\left(\alpha^i+\alpha^j\right)\tau\right] + 4\alpha^i\alpha^j\tau^2\Big] \Big/ \left[9\alpha^i\alpha^j\sigma - \left(\alpha^i+\alpha^j\right)\tau\right]^3$$

$$= \frac{2\tau\pi\left(\alpha^j-\alpha^i\right)\left[9\alpha^i\alpha^j\left(\alpha^i+\alpha^j\right)\sigma - 2\left(\alpha^i\right)^2\tau - 2\left(\alpha^j\right)^2\tau\right]}{\left[9\alpha^i\alpha^j\sigma - \left(\alpha^i+\alpha^j\right)\tau\right]^3}$$

$$\frac{\partial\left(\Delta\Pi^i\right)^{bs}}{\partial\pi} = \frac{2\left(\alpha^j-\alpha^i\right)\tau\pi\left[\left(\alpha^i\right)^2\left(9\alpha^j\sigma-2\tau\right) + \left(\alpha^j\right)^2\left(9\alpha^i\sigma-2\tau\right)\right]}{\left[9\alpha^i\alpha^j\sigma - \left(\alpha^i+\alpha^j\right)\tau\right]^3}$$

To guarantee that $\frac{\partial\left(\Delta\Pi^i\right)^{bs}}{\partial\pi}$ is positive for platform 1, it is sufficient for all of its components to be positive. The denominator can be expressed as $\left[\alpha^j\left(9\alpha^j\sigma-2\tau\right)+\tau\left(\alpha^j-\alpha^i\right)\right]^3$, which is positive if Assumption 2.2 holds when $\pi=\upsilon$. The elements in the numerator are all positive, based on the same reasoning as for the denominator, provided that platform 1 is more efficient in developing attributes, $\alpha^2>\alpha^1$ and Assumption 2.2 holds when $\pi=\upsilon$. □

B.3 Scenario: $\upsilon \neq \pi$

When $\upsilon > \pi$ ($\pi = 0$) Assumption 2.1 turns to $\frac{\upsilon}{2} < \tau < \frac{2\upsilon}{3}$ and Assumption 2.2 turns to $\alpha^i > \frac{2\tau}{\sigma_\upsilon}$. Conversely, when $\pi > \upsilon$ ($\upsilon = 0$) Assumption 2.1 turns to $\tau > \frac{\pi}{2}$ and Assumption 2.2 turns to $\alpha^i > \frac{2\tau}{\sigma_\pi}$, where $\sigma_\upsilon \equiv 9\tau^2 - 2\upsilon^2$ and $\sigma_\pi \equiv 9\tau^2 - 2\pi^2$.

B.3.1 Proof of Proposition 2.5

Proof. We prove Proposition 2.5 by partially differentiating the difference in equilibrium attributes with respect to τ, υ and π using Equations (2.14a) and (2.14b) under the assumption that platform 1 is more efficient in developing attributes on buyers' side than platform 2, $\alpha^2 > \alpha^1$.

Case 1. $\upsilon > \pi$**,** $\pi = 0$

$$\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial\tau} = \frac{\left(\alpha^j-\alpha^i\right)\Big[\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i+\alpha^j\right)\tau\right]\left(12\tau+\upsilon\right) - \left[18\alpha^i\alpha^j\tau - \left(\alpha^i+\alpha^j\right)\right]\left(3\tau+2\upsilon\right)\left(2\tau-\upsilon\right)\Big]}{2\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i+\alpha^j\right)\tau\right]^2}$$

$$= \frac{\left(\alpha^j-\alpha^i\right)}{2\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i+\alpha^j\right)\tau\right]^2}\Bigg[\left[\alpha^i\left(\alpha^j\sigma_\upsilon-2\tau\right) - \tau\left(\alpha^j-\alpha^i\right)\right]\left(12\tau+\upsilon\right)$$

$$- (3\tau + 2\upsilon)(2\tau - \upsilon)\left[2\alpha^i\left(9\alpha^j\tau - 1\right) - \left(\alpha^j - \alpha^i\right)\right]\Bigg]$$

$$= \frac{-\left(\alpha^j - \alpha^i\right)}{2\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]^2}\Bigg[\alpha^i\left[2\left(9\alpha^j\tau - 1\right)\left(3\tau + 2\upsilon\right)(2\tau - \upsilon) - \left(\alpha^j\sigma_\upsilon - 2\tau\right)(12\tau + \upsilon)\right]$$

$$+ \left(\alpha^j - \alpha^i\right)\left[\tau(12\tau + \upsilon) - \left(3\tau + 2\upsilon\right)(2\tau - \upsilon)\right]\Bigg]$$

Next, $\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial\tau}$ is negative if $\alpha^i\left[2\left(9\alpha^j\tau - 1\right)\left(3\tau + 2\upsilon\right)(2\tau - \upsilon) - \left(\alpha^j\sigma_\upsilon - 2\tau\right)(12\tau + \upsilon)\right] + \left(\alpha^j - \alpha^i\right)\left[\tau(12\tau + \upsilon) - \left(3\tau + 2\upsilon\right)(2\tau - \upsilon)\right]$ is positive. The first part $\alpha^i\left[2\left(9\alpha^j\tau - 1\right)\left(3\tau + 2\upsilon\right)(2\tau - \upsilon) - \left(\alpha^j\sigma_\upsilon - 2\tau\right)(12\tau + \upsilon)\right]$ can be rearranged as $\alpha^i\left[\alpha^j\left[18\tau\left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right) - \sigma_\upsilon(12\tau + \upsilon)\right] - 2\left[(3\tau + 2\upsilon)(2\tau - \upsilon) - \tau\left(12\tau + \upsilon\right)\right]\right]$, and is positive if $\alpha^j > \frac{2[(3\tau+2\upsilon)(2\tau-\upsilon)-\tau(12\tau+\upsilon)]}{18\tau(3\tau+2\upsilon)(2\tau-\upsilon)-\sigma_\upsilon(12\tau+\upsilon)}$. We use Assumption 2.2 to show the condition is satisfied by making the left side on both inequalities equal and comparing the right side. Then showing that Assumption 2.2 right side is greater we guarantee the condition is positive. That is $\frac{2\tau}{\sigma_\upsilon} > \frac{2[(3\tau+2\upsilon)(2\tau-\upsilon)-\tau(12\tau+\upsilon)]}{18\tau(3\tau+2\upsilon)(2\tau-\upsilon)-\sigma_\upsilon(12\tau+\upsilon)}$, which simplifies to $\left(18\tau^2 - \sigma_\upsilon\right)(3\tau + 2\upsilon)(2\tau - \upsilon) > 0$ and finally turns to $\left(9\tau^2 + 2\upsilon^2\right)(3\tau + 2\upsilon)(2\tau - \upsilon) > 0$ if $\tau > \frac{\upsilon}{2}$. Then, the first part is negative. The second part $\left(\alpha^j - \alpha^i\right)\left[\tau(12\tau + \upsilon) - \left(3\tau + 2\upsilon\right)(2\tau - \upsilon)\right]$ simplifies to $2\left(\alpha^j - \alpha^i\right)\left(3\tau^2 + \upsilon^2\right)$, which is positive for platform 1 given $\alpha^2 > \alpha^1$

Therefore, $\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial\tau} < 0$ under Assumption 2.1 and Assumption 2.2 when $\upsilon > \pi$, $\pi = 0$.

$$\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial\upsilon} = \frac{\left(\alpha^j - \alpha^i\right)\left[\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right](\tau - 4\upsilon) + 4\alpha^i\alpha^j\upsilon(3\tau + 2\upsilon)(2\tau - \upsilon)\right]}{2\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]^2}$$

$$= \frac{\left(\alpha^j - \alpha^i\right)\left[\left[\alpha^j\left(\alpha^i\sigma_\upsilon - 2\tau\right) + \left(\alpha^j - \alpha^i\right)\tau\right](\tau - 4\upsilon) + 4\alpha^i\alpha^j\upsilon(3\tau + 2\upsilon)(2\tau - \upsilon)\right]}{2\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]^2}$$

Therefore, $\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial\upsilon}$ is positive under Assumption 2.2 when $\upsilon > \pi$, $\pi = 0$ and as long as $\tau > 4\upsilon$.

Case 2. $\pi > \upsilon$**,** $\upsilon = 0$

$$\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial\tau} = \frac{\left(\alpha^j - \alpha^i\right)\left[\left[\alpha^i\alpha^j\sigma_\pi - \left(\alpha^i + \alpha^j\right)\tau\right](12\tau - \pi) - \left[18\alpha^i\alpha^j\tau - \left(\alpha^i + \alpha^j\right)\right](3\tau + \pi)(2\tau - \pi)\right]}{2\left[\alpha^i\alpha^j\sigma_\pi - \left(\alpha^i + \alpha^j\right)\tau\right]^2}$$

$$= \frac{\left(\alpha^j - \alpha^i\right)}{2\left[\alpha^i\alpha^j\sigma_\pi - \left(\alpha^i + \alpha^j\right)\tau\right]^2}\Bigg[\left[\alpha^i\left(\alpha^j\sigma_\pi - 2\tau\right) - \tau\left(\alpha^j - \alpha^i\right)\right](12\tau - \pi)$$

$$- (3\tau + \pi)(2\tau - \pi)\left[2\alpha^i\left(9\alpha^j\tau - 1\right) - \left(\alpha^j - \alpha^i\right)\right]\Bigg]$$

$$= \frac{-\left(\alpha^j - \alpha^i\right)}{2\left[\alpha^i\alpha^j\sigma_\pi - \left(\alpha^i + \alpha^j\right)\tau\right]^2}\Bigg[\alpha^i\left[2\left(9\alpha^j\tau - 1\right)\left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right) - \left(\alpha^j\sigma_\pi - 2\tau\right)\left(12\tau - \pi\right)\right] + \left(\alpha^j - \alpha^i\right)\left[\tau\left(12\tau - \pi\right) - \left(3\tau + \pi\right)\left(2\tau - \pi\right)\right]\Bigg]$$

Next, $\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial\tau}$ is negative if $\alpha^i\left[2\left(9\alpha^j\tau - 1\right)\left(3\tau + \pi\right)\left(2\tau - \pi\right) - \left(\alpha^j\sigma_\pi - 2\tau\right)\left(12\tau - \pi\right)\right] + \left(\alpha^j - \alpha^i\right)\left[\tau\left(12\tau - \pi\right) - \left(3\tau + \pi\right)\left(2\tau - \pi\right)\right]$ is positive. The first part $\alpha^i\left[2\left(9\alpha^j\tau - 1\right)\left(3\tau + \pi\right)\left(2\tau - \pi\right) - \left(\alpha^j\sigma_\pi - 2\tau\right)\left(12\tau - \pi\right)\right]$ can be rearranged as $\alpha^i\left[\alpha^j\left[18\tau\left(3\tau + \pi\right)\left(2\tau - \pi\right) - \sigma_\pi\left(12\tau - \pi\right)\right] - 2\left[\left(3\tau + \pi\right)\left(2\tau - \pi\right) - \tau\left(12\tau - \pi\right)\right]\right]$, and is positive if $\alpha^j > \frac{2[(3\tau+\pi)(2\tau-\pi)-\tau(12\tau-\pi)]}{18\tau(3\tau+\pi)(2\tau-\pi)-\sigma_\pi(12\tau-\pi)}$. We use Assumption 2.2 to show the condition is satisfied by making the left side on both inequalities equal and comparing the right side. Then showing that Assumption 2.2 right side is greater we guarantee the condition is positive. That is $\frac{2\tau}{\sigma_\pi} > \frac{2[(3\tau+\pi)(2\tau-\pi)-\tau(12\tau-\pi)]}{18\tau(3\tau+\pi)(2\tau-\pi)-\sigma_\pi(12\tau-\pi)}$, which simplifies to $\left(18\tau^2 - \sigma_\pi\right)\left(3\tau + \pi\right)\left(2\tau - \pi\right) > 0$ and finally turns to $\left(9\tau^2 + 2\pi^2\right)\left(3\tau + \pi\right)\left(2\tau - \pi\right) > 0$ if $\tau > \frac{\pi}{2}$. Then, the first part is negative. The second part $\left(\alpha^j - \alpha^i\right)\left[\tau\left(12\tau - \pi\right) - \left(3\tau + \pi\right)\left(2\tau - \pi\right)\right]$ simplifies to $\left(\alpha^j - \alpha^i\right)\left(6\tau^2 + \pi^2\right)$, which is positive for platform 1 given $\alpha^2 > \alpha^1$

Therefore, $\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial\tau} < 0$ under Assumption 2.1 and Assumption 2.2 when $\pi > \upsilon$, $\upsilon = 0$.

$$\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial\pi} = \frac{\left(\alpha^j - \alpha^i\right)\left[-\left[\alpha^i\alpha^j\sigma_\pi - \left(\alpha^i + \alpha^j\right)\tau\right]\left(\tau + 2\pi\right) + 4\alpha^i\alpha^j\pi\left(3\tau + \pi\right)\left(2\tau - \pi\right)\right]}{2\left[\alpha^i\alpha^j\sigma_\pi - \left(\alpha^i + \alpha^j\right)\tau\right]^2}$$
$$= \frac{\left(\alpha^j - \alpha^i\right)\left[\alpha^i\left[\alpha^j\left[4\pi\left(3\tau + \pi\right)\left(2\tau - \pi\right) - \sigma_\pi\left(\tau + 2\pi\right)\right] + \tau\left(\tau + 2\pi\right)\right] + \alpha^j\tau\left(\tau + 2\pi\right)\right]}{2\left[\alpha^i\alpha^j\sigma_\pi - \left(\alpha^i + \alpha^j\right)\tau\right]^2}$$

To determine the sign of $\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial\pi}$ it is sufficient to find the sign of the following expression $\alpha^i\left[\alpha^j\left[4\pi\left(3\tau + \pi\right)\left(2\tau - \pi\right) - \sigma_\pi\left(\tau + 2\pi\right)\right] + \tau\left(\tau + 2\pi\right)\right] + \alpha^j\tau\left(\tau + 2\pi\right)$. We use Assumption 2.2 to show the condition is positive. We make the left side of both expressions equal and compare the right side. Then showing that Assumption 2.2 right side is greater we guarantee the condition is positive. That is $\frac{2\tau}{\sigma_\upsilon} > \frac{-\alpha^j\tau(\tau+2\pi)}{B}$, where $B \equiv \alpha^j\left[4\pi\left(3\tau + \pi\right)\left(2\tau - \pi\right) - \sigma_\pi\left(\tau + 2\pi\right)\right] + \tau\left(\tau + 2\pi\right)$. Then we have $\alpha^j\left[8\pi\left(3\tau + \pi\right)\left(2\tau - \pi\right) - \sigma_\pi\left(\tau + 2\pi\right)\right] + 2\tau\left(\tau + 2\pi\right) > 0$. Then we use the same method of comparing the right side of Assumption 2.2 and the previous inequality and show the condition is satisfied. That is $\frac{2\tau}{\sigma_\pi} > \frac{-2\tau(\tau+2\pi)}{8\pi(3\tau+\pi)(2\tau-\pi)-\sigma_\pi(\tau+2\pi)}$, which simplifies to $8\pi\left(3\tau + \pi\right)\left(2\tau - \pi\right) > 0$, this condition is satisfied under Assumption 2.1 when $\pi > \upsilon$, $\upsilon = 0$.

Therefore, $\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial\pi} > 0$ under Assumption 2.1 and Assumption 2.2 when $\pi > \upsilon$, $\upsilon = 0$. □

B.3.2 Market-shares conditions

Buyers and sellers equilibrium market shares in Equations (2.15a) and (2.15b) must satisfy conditions $0 < \eta_b^i < 1$ and $0 < \eta_s^i < 1$, respectively.

Firstly, for platform 1, both equilibrium market shares are positive because all of their elements are positive. Given platform 1 is more efficient in developing attributes, $\alpha^2 - \alpha^1 > 0$ and given Assumption 2.1 and Assumption 2.2 hold. That is $6\tau^2 - (\pi - \upsilon)\tau - (\pi + \upsilon)(\pi + 2\upsilon) > 0$ when $\tau > \frac{\pi+\upsilon}{2}$, as was demonstrated previously in Appendix B.1.9. Furthermore, $[\alpha^i\alpha^j\Sigma - (\alpha^i + \alpha^j)\,\tau]$ is positive, which can be observed when rewritten as $\alpha^j\,(\alpha^i\Sigma - 2\tau) + (\alpha^j - \alpha^i)\,\tau$, where $\Sigma \equiv 9\tau^2 - (2\pi + \upsilon)\,(\pi + 2\upsilon)$.

For $\eta_b^i < 1$ we have $\eta_b^i = \frac{1}{2} + \frac{3\tau\left(\alpha^j - \alpha^i\right)\left[6\tau^2 - (\pi-\upsilon)\tau - (\pi+\upsilon)(\pi+2\upsilon)\right]}{4\Sigma[\alpha^i\alpha^j\Sigma - (\alpha^i + \alpha^j)\tau]} < 1$. This inequality can be rewritten as $2\Sigma\left[\alpha^i\alpha^j\Sigma - (\alpha^i + \alpha^j)\,\tau\right] > 3\tau\left[6\tau^2 - (\pi - \upsilon)\,\tau - (\pi + \upsilon)\,(\pi + 2\upsilon)\right]\left(\alpha^j - \alpha^i\right)$, which simplifies to $2\alpha^j\Sigma\,(\alpha^i\Sigma - 2\tau) + (\alpha^j - \alpha^i)\,\tau\left[2\Sigma - 3\left[6\tau^2 - (\pi + \upsilon)\,(\pi + 2\upsilon) + \tau\,(\upsilon - \pi)\right]\right] > 0$. To show that the previous condition is positive, it is sufficient to demonstrate that $2\Sigma - 3\left[6\tau^2 - (\pi + \upsilon)\,(\pi + 2\upsilon) + \tau\,(\upsilon - \pi)\right]$ is positive, considering that the other elements are positive. This expression turns to $18\tau^2 - 2\,(2\pi + \upsilon)\,(\pi + 2\upsilon) - 18\tau^2 + 3\,(\pi + \upsilon)\,(\pi + 2\upsilon) - 3\tau\,(\upsilon - \pi) > 0$ which simplifies to $(\upsilon - \pi)\,[(\pi + 2\upsilon) - 3\tau] > 0$ if $\tau < \frac{\pi+2\upsilon}{3}$ and $\upsilon > \pi$ or $\tau > \frac{\pi+2\upsilon}{3}$ and $\pi > \upsilon$. We use Assumption 2.1 to show the previous condition is satisfied by comparing the right side of both inequalities. Therefore showing that if Assumption 2.1 right side is greater the condition is satisfied. That is $\frac{(\pi+\upsilon)}{2} > \frac{(\pi+2\upsilon)}{3}$ which simplifies to $\pi - \upsilon > 0$ if $\pi > \upsilon$.

For $\eta_s^i < 1$ we have $\eta_s^i = \frac{1}{2} + \frac{(\pi+2\upsilon)\left(\alpha^j - \alpha^i\right)\left[6\tau^2 - (\pi-\upsilon)\tau - (\pi+\upsilon)(\pi+2\upsilon)\right]}{4\Sigma[\alpha^i\alpha^j\Sigma - (\alpha^i + \alpha^j)\tau]} < 1$. This inequality can be rewritten as which turns to $2\Sigma\left[\alpha^i\alpha^j\Sigma - \left(\alpha^i + \alpha^j\right)\tau\right] - (\pi + 2\upsilon)\,(\alpha^j - \alpha^i)\left[6\tau^2 - (\pi - \upsilon)\,\tau - (\pi + \upsilon)\,(\pi + 2\upsilon)\right] > 0$, which simplifies to $2\alpha^j\Sigma\,(\alpha^i\Sigma - 2\tau) + (\alpha^j - \alpha^i)\left[2\tau\Sigma - (\pi + 2\upsilon)\left[6\tau^2 - (\pi - \upsilon)\,\tau - (\pi + \upsilon)\,(\pi + 2\upsilon)\right]\right] > 0$. To show that the previous condition is positive, it is sufficient to demonstrate that $2\tau\Sigma - (\pi + 2\upsilon)\left[6\tau^2 - (\pi - \upsilon)\,\tau - (\pi + \upsilon)\,(\pi + 2\upsilon)\right]$ is positive, considering that the other elements are positive. This expression turns to $18\tau^3 - 2\tau\,(2\pi + \upsilon)\,(\pi + 2\upsilon) - 6\tau^2\,(\pi + 2\upsilon) + (\pi + 2\upsilon)^2\,(\pi + \upsilon) - \tau\big(\upsilon - \pi\big)\big(\pi + 2\upsilon\big) > 0$ which simplifies to $6\tau^2\left[3\tau - (\pi + 2\upsilon)\right] - 3\tau\,(\pi + 2\upsilon)\,(\pi + \upsilon) + (\pi + 2\upsilon)^2\left(\pi + \upsilon\right) > 0$ and then simplifies to $\left[6\tau^2 - (\pi + \upsilon)\,(\pi + 2\upsilon)\right][3\tau - (\pi + 2\upsilon)] > 0$. For the previous condition to be positive, it is sufficient to have expressions $6\tau^2 - (\pi + \upsilon)\,(\pi + 2\upsilon) > 0$ and $3\tau - (\pi + 2\upsilon) > 0$. We use Assumption 2.1 to show the first condition is satisfied by making the left side on both inequalities equal and comparing the right side. Then showing that Assumption 2.1 right side is greater we guarantee the condition is positive. First, Assumption 2.1 can be expressed as $\tau^2 > \frac{(\pi+\upsilon)^2}{4}$, then comparing the right side we have $\frac{(\pi+\upsilon)^2}{4} > \frac{(\pi+\upsilon)(\pi+2\upsilon)}{6}$ which simplifies to $3\,(\pi + \upsilon) - 2\,(\pi + 2\upsilon) > 0$ an finally simplifies to $\pi - \upsilon > 0$ if $\pi > \upsilon$. Moreover, $3\tau - (\pi + 2\upsilon) > 0$ if $\tau > \frac{(\pi+2\upsilon)}{3}$. This condition is met if Assumption 2.1 holds as was previously shown for $\eta_b^i < 1$.

In summary conditions $0 < \eta_b^i < 1$ and $0 < \eta_s^i < 1$ are satisfied if $\tau < \frac{\pi+2\upsilon}{3}$ and $\upsilon > \pi$ or $\tau > \frac{\pi+\upsilon}{2}$ and $\pi > \upsilon$ which is stated in Assumption 2.1 and $\alpha^i > \frac{2\tau}{\Sigma}$ which is stated in Assumption 2.2.

B.3.3 Proof of Claim 2.1

Proof. We prove Claim 2.1 by partially differentiating the equilibrium market shares on both sides with respect to τ, υ and π using Equations (2.15a) and (2.15b) under the assumption that platform 1 is more efficient in developing attributes on buyers' side than platform 2, $\alpha^2 > \alpha^1$.

Case 1. $\upsilon > \pi$, $\pi = 0$

We use Equations (2.15a) and (2.15b) to compute the equilibrium market shares on buyers' and sellers' sides, and then we use Equation (2.14a) to express the market shares as a function of the difference in attributes in equilibrium.

$$\begin{aligned}
\left(\eta_b^i\right)_{\upsilon>\pi} &= \frac{1}{2} + \frac{3\tau\left(\alpha^j - \alpha^i\right)\left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right)}{4\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]} = \frac{1}{2} + \frac{3\tau}{2\sigma_\upsilon}\left(\Delta q_b^i\right)_{\upsilon>\pi} \\
\frac{\partial\left(\eta_b^i\right)_{\upsilon>\pi}}{\partial\tau} &= \frac{3}{2\sigma_\upsilon^2}\left[\sigma_\upsilon\left[\left(\Delta q_b^i\right)_{\upsilon>\pi} + \tau\left(\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial\tau}\right)\right] - 18\tau^2\left(\Delta q_b^i\right)_{\upsilon>\pi}\right] \\
&= -\frac{3}{2\sigma_\upsilon^2}\left[\left(9\tau^2 + 2\upsilon^2\right)\left(\Delta q_b^i\right)_{\upsilon>\pi} - \sigma_\upsilon\tau\left(\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial\tau}\right)\right]
\end{aligned}$$

According to Proposition 2.5 and its proof in Appendix B.3.1, we know that $\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial\tau}$ is negative. Therefore $\partial\left(\eta_b^i\right)_{\upsilon>\pi}/\partial\tau < 0$.

$$\begin{aligned}
\frac{\partial\left(\eta_b^i\right)_{\upsilon>\pi}}{\partial\upsilon} &= \frac{3\left(\alpha^j - \alpha^i\right)\tau}{4\sigma_\upsilon^2\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]^2}\Bigg[\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]\left(\tau - 4\upsilon\right) \\
&\qquad + \left[8\upsilon\alpha^i\alpha^j\sigma_\upsilon - 4\tau\upsilon\left(\alpha^i + \alpha^j\right)\right]\left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right)\Bigg] \\
&= \frac{3\left(\alpha^j - \alpha^i\right)\tau}{4\sigma_\upsilon^2\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]^2}\Bigg[\tau\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right] - 4\upsilon\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right] \\
&\qquad + 8\upsilon\alpha^i\alpha^j\sigma_\upsilon\left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right) - 4\tau\upsilon\left(\alpha^i + \alpha^j\right)\left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right)\Bigg] \\
&= \frac{3\left(\alpha^j - \alpha^i\right)\tau}{4\sigma_\upsilon^2\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]^2}\Bigg[\tau\sigma_\upsilon\left[\alpha^j\left(\alpha^i\sigma_\upsilon - 2\tau\right) + \left(\alpha^j - \alpha^i\right)\tau\right] + 4\alpha^i\alpha^j\sigma_\upsilon\upsilon\big[2\left(3\tau + 2\upsilon\right) \\
&\qquad \left(2\tau - \upsilon\right) - \sigma_\upsilon\big] + 4\left(\alpha^i + \alpha^j\right)\tau\upsilon\left[\sigma_\upsilon - \left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right)\right]\Bigg]
\end{aligned}$$

$$= \frac{3\left(\alpha^j-\alpha^i\right)\tau}{4\sigma_\upsilon^2\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]^2}\Bigg[\tau\sigma_\upsilon\left[\alpha^j\left(\alpha^i\sigma_\upsilon-2\tau\right)+\left(\alpha^j-\alpha^i\right)\tau\right] + 4\alpha^i\alpha^j\sigma_\upsilon\upsilon\left[3\tau^2+2\tau\upsilon-2\upsilon^2\right]+4\left(\alpha^i+\alpha^j\right)\tau^2\upsilon\left(3\tau-\upsilon\right)\Bigg]$$

To determine the sign of $\frac{\partial(\eta_b^i)_{\upsilon>\pi}}{\partial\upsilon}$ it is sufficient to examine the sign of $3\tau^2+2\tau\upsilon-2\upsilon^2$ and $3\tau-\upsilon$ as the remaining elements of the partial derivative are positive. Specifically, $\tau\sigma_\upsilon\left[\alpha^j\left(\alpha^i\sigma_\upsilon-2\tau\right)+\left(\alpha^j-\alpha^i\right)\tau\right]$ is positive under Assumption 2.2 when $\upsilon>\pi$, $\pi=0$, and given platform 1 is more efficient in developing attributes $\alpha^2>\alpha^1$.

Next, $3\tau^2+2\upsilon\tau-2\upsilon^2$ is a quadratic polynomial in τ. We use the quadratic formula to find the values of τ that make the expression positive. The roots are $\tau=\frac{1}{3}\left(-\upsilon\pm\upsilon\sqrt{6}\right)$. Thus, the first root is $\tau_{r_1}=\frac{\sqrt{6}-1}{3}\upsilon$ and the second root is $\tau_{r_2}=-\frac{\sqrt{6}+1}{3}\upsilon$. Since the square term of the polynomial in τ is positive, $3\tau^2+2\upsilon\tau-2\upsilon^2$ is positive for values outside both roots, meaning $\tau>\frac{\sqrt{6}-1}{3}\upsilon$ and $\tau<-\frac{\sqrt{6}+1}{3}\upsilon$. Given that transportation cost τ is positive by definition, we dismiss the negative root. Therefore, $3\tau^2+2\upsilon\tau-2\upsilon^2$ is positive if $\tau>\frac{\sqrt{6}-1}{3}\upsilon$. This condition is satisfied under Assumption 2.1 when $\upsilon>\pi$, $\pi=0$. Namely, if the right side of $\tau>\frac{\upsilon}{2}$ is greater than the right side of the previous condition, we guarantee it holds true. That is $\frac{\upsilon}{2}>\frac{\sqrt{6}-1}{3}\upsilon$, which results in $3>2.89$. Moreover, $3\tau-\upsilon$ is positive if $\tau>\frac{\upsilon}{3}$, which is also satisfied under Assumption 2.1 when $\upsilon>\pi$, $\pi=0$ similar to the the previous condition. That is $\frac{\upsilon}{2}>\frac{\upsilon}{3}$, which results in $3>2$.

Therefore, $\frac{\partial(\eta_b^i)_{\upsilon>\pi}}{\partial\upsilon}$ is positive under Assumption 2.1 and Assumption 2.2 when $\upsilon>\pi$, $\pi=0$.

$$\left(\eta_s^i\right)_{\upsilon>\pi}=\frac{1}{2}+\frac{\upsilon\left(\alpha^j-\alpha^i\right)\left(3\tau+2\upsilon\right)\left(2\tau-\upsilon\right)}{2\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]}=\frac{1}{2}+\frac{\upsilon}{\sigma_\upsilon}\left(\Delta q_b^i\right)_{\upsilon>\pi}$$
$$\frac{\partial\left(\eta_s^i\right)_{\upsilon>\pi}}{\partial\tau}=-\frac{\upsilon}{\sigma_\upsilon^2}\left[18\tau\left(\Delta q_b^i\right)_{\upsilon>\pi}-\sigma_\upsilon\left(\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial\tau}\right)\right]$$

According to Proposition 2.5 and its proof in Appendix B.3.1, we know that $\frac{\partial(\Delta q_b^i)_{\upsilon>\pi}}{\partial\tau}$ is negative. Therefore $\partial\left(\eta_s^i\right)_{\upsilon>\pi}/\partial\tau<0$.

$$\frac{\partial\left(\eta_s^i\right)_{\upsilon>\pi}}{\partial\upsilon}=\frac{\left(\alpha^j-\alpha^i\right)}{2\sigma_\upsilon^2\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]^2}\Bigg[\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]\big[\upsilon\left(\tau-4\upsilon\right) + \left(3\tau+2\upsilon\right)\left(2\tau-\upsilon\right)\big]+\upsilon\left(3\tau+2\upsilon\right)\left(2\tau-\upsilon\right)\left[8\upsilon\alpha^i\alpha^j\sigma_\upsilon-4\tau\upsilon\left(\alpha^i+\alpha^j\right)\right]\Bigg]$$
$$=\frac{\left(\alpha^j-\alpha^i\right)}{2\sigma_\upsilon^2\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]^2}\Bigg[\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]\left[\tau\upsilon+\left(3\tau+2\upsilon\right)\left(2\tau-\upsilon\right)\right]$$

$$+4\alpha^i\alpha^j\sigma_\upsilon\upsilon^2\left[2\left(3\tau+2\upsilon\right)\left(2\tau-\upsilon\right)-\sigma_\upsilon\right]+4\tau\upsilon^2\left(\alpha^i+\alpha^j\right)\left[\sigma_\upsilon-\left(3\tau+2\upsilon\right)\left(2\tau-\upsilon\right)\right]\Bigg]$$

$$=\frac{\left(\alpha^j-\alpha^i\right)}{2\sigma_\upsilon^2\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]^2}\Bigg[\sigma_\upsilon\left[\alpha^j\left(\alpha^i\sigma_\upsilon-2\tau\right)+\left(\alpha^j-\alpha^i\right)\tau\right]\left[\tau\upsilon\right.$$

$$\left.+\left(3\tau+2\upsilon\right)\left(2\tau-\upsilon\right)\right]+4\alpha^i\alpha^j\sigma_\upsilon\upsilon^2\left[3\tau^2+2\tau\upsilon-2\upsilon^2\right]+4\tau^2\upsilon^2\left(\alpha^i+\alpha^j\right)\left(3\tau-\upsilon\right)\Bigg]$$

To determine the sign of $\frac{\partial\left(\eta_s^i\right)_{\upsilon>\pi}}{\partial\upsilon}$ it is sufficient to examine the sign of $3\tau^2+2\tau\upsilon-2\upsilon^2$ and $3\tau-\upsilon$ as the remaining elements of the partial derivative are positive. Specifically, $\sigma_\upsilon\left[\alpha^j\left(\alpha^i\sigma_\upsilon-2\tau\right)+\left(\alpha^j-\alpha^i\right)\tau\right]$ is positive under Assumption 2.2 when $\upsilon>\pi$, $\pi=0$, and $\left(3\tau+2\upsilon\right)\left(2\tau-\upsilon\right)$ is positive under Assumption 2.1 when $\upsilon>\pi$, $\pi=0$ and given platform 1 is more efficient in developing attributes $\alpha^2>\alpha^1$.

As it has been shown in $\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial\upsilon}>0$ that both conditions $3\tau^2+2\tau\upsilon-2\upsilon^2$ and $3\tau-\upsilon$ are positive, we conclude that $\frac{\partial\left(\eta_s^i\right)_{\upsilon>\pi}}{\partial\upsilon}$ is also positive under Assumption 2.1 and Assumption 2.2 when $\upsilon>\pi$, $\pi=0$.

Case 2. $\pi>\upsilon$**,** $\upsilon=0$

We use Equations (2.15a) and (2.15b) to compute the equilibrium market shares on buyers' and sellers' sides, and then we use Equation (2.14b) to express the market shares as a function of the difference in attributes in equilibrium.

$$\left(\eta_b^i\right)_{\pi>\upsilon}=\frac{1}{2}+\frac{3\tau\left(\alpha^j-\alpha^i\right)\left(3\tau+\pi\right)\left(2\tau-\pi\right)}{4\sigma_\pi\left[\alpha^i\alpha^j\sigma_\pi-\left(\alpha^i+\alpha^j\right)\tau\right]}=\frac{1}{2}+\frac{3\tau}{2\sigma_\pi}\left(\Delta q_b^i\right)_{\pi>\upsilon}$$

$$\frac{\partial\left(\eta_b^i\right)_{\pi>\upsilon}}{\partial\tau}=\frac{3}{2\sigma_\pi^2}\left[\sigma_\pi\left[\left(\Delta q_b^i\right)_{\pi>\upsilon}+\tau\left(\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial\tau}\right)\right]-18\tau^2\left(\Delta q_b^i\right)_{\pi>\upsilon}\right]$$

$$=-\frac{3}{2\sigma_\pi^2}\left[\left(9\tau^2+2\pi^2\right)\left(\Delta q_b^i\right)_{\pi>\upsilon}-\sigma_\pi\tau\left(\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial\tau}\right)\right]$$

According to Proposition 2.5 and its proof in Appendix B.3.1, we know that $\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial\tau}$ is negative. Therefore $\partial\left(\eta_b^i\right)_{\pi>\upsilon}/\partial\tau<0$.

$$\frac{\partial\left(\eta_b^i\right)_{\pi>\upsilon}}{\partial\pi}=\frac{3\tau}{2\sigma_\pi^2}\left[\sigma_\pi\left(\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial\pi}\right)+4\pi\left(\Delta q_b^i\right)_{\pi>\upsilon}\right]$$

According to Proposition 2.5 and its proof in Appendix B.3.1, we know that $\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial\pi}$ is positive. Therefore $\partial\left(\eta_b^i\right)_{\pi>\upsilon}/\partial\pi>0$.

$$\left(\eta_s^i\right)_{\pi>\upsilon}=\frac{1}{2}+\frac{\pi\left(\alpha^j-\alpha^i\right)\left(3\tau+\pi\right)\left(2\tau-\pi\right)}{4\sigma_\pi\left[\alpha^i\alpha^j\sigma_\pi-\left(\alpha^i+\alpha^j\right)\tau\right]}=\frac{1}{2}+\frac{\pi}{2\sigma_\pi}\left(\Delta q_b^i\right)_{\pi>\upsilon}$$

$$\frac{\partial \left(\eta_s^i\right)_{\pi>\upsilon}}{\partial \tau} = -\frac{\pi}{2\sigma_\pi^2}\left[18\tau\left(\Delta q_b^i\right)_{\pi>\upsilon} - \sigma_\pi\left(\frac{\partial \left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial \tau}\right)\right]$$

According to Proposition 2.5 and its proof in Appendix B.3.1, we know that $\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial \tau}$ is negative. Therefore $\partial \left(\eta_s^i\right)_{\pi>\upsilon}/\partial \tau < 0$.

$$\begin{aligned}\frac{\partial \left(\eta_s^i\right)_{\pi>\upsilon}}{\partial \pi} &= \frac{1}{2\sigma_\pi^2}\left[\sigma_\pi\left[\left(\Delta q_b^i\right)_{\pi>\upsilon} + \pi\left(\frac{\partial \left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial \pi}\right)\right] + 4\pi^2\left(\Delta q_b^i\right)_{\pi>\upsilon}\right] \\ &= \frac{1}{2\sigma_\pi^2}\left[\left(9\tau^2 + 2\pi^2\right)\left(\Delta q_b^i\right)_{\pi>\upsilon} + \pi\sigma_\pi\left(\frac{\partial \left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial \pi}\right)\right]\end{aligned}$$

According to Proposition 2.5 and its proof in Appendix B.3.1, we know that $\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial \pi}$ is positive. Therefore $\partial \left(\eta_s^i\right)_{\pi>\upsilon}/\partial \pi > 0$. □

B.3.4 Positive Equilibrium Profits

We show the conditions for Equilibrium profits in Equation (2.16) for platform 1 to be positive. We notice Equation (2.16) is compose of two elements, the first is $\tau - \frac{\pi+\upsilon}{2}$ and the second is $\frac{\left[\Sigma\left(\alpha^j-\alpha^i\right)\left[\alpha^i\alpha^j\Sigma-\left(\alpha^i+\alpha^j\right)\tau\right]-\alpha^i\left(\alpha^j\Sigma-2\tau\right)\left(\alpha^i\Sigma-2\tau\right)\right]\Omega^2}{8\Sigma^2\left[\alpha^i\alpha^j\Sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2}$. The first component is positive under Assumption 2.1. To determine if the second element is positive, we can partially differentiate it with respect to α^2 and evaluate the result when $\alpha^2 = \frac{2\tau}{9\sigma}$.

$$\begin{aligned}\frac{\partial part2}{\partial \alpha^2} &= 8\Sigma^2\Big[\left[\alpha^1\alpha^2\Sigma - \left(\alpha^1+\alpha^2\right)\tau\right]^2\left[\Sigma\left[\alpha^1\alpha^2\Sigma - \left(\alpha^1+\alpha^2\right)\tau\right] + \Sigma\left(\alpha^2-\alpha^1\right)\right. \\ &\left(\alpha^1\Sigma-\tau\right) - \alpha^1\Sigma\left(\alpha^1\Sigma-2\tau\right)\big] - 2\left(\alpha^1\Sigma-\tau\right)\left[\alpha^1\alpha^2\Sigma - \left(\alpha^1+\alpha^2\right)\tau\right]\left[\Sigma\left(\alpha^2-\alpha^1\right)\right. \\ &\left[\alpha^1\alpha^2\Sigma - \left(\alpha^1+\alpha^2\right)\tau\right] - \alpha^1\left(\alpha^2\Sigma-2\tau\right)\left(\alpha^1\Sigma-2\tau\right)\big]\Big]\Big/18^2\left[\alpha^1\alpha^2\Sigma - \left(\alpha^1+\alpha^2\right)\tau\right]^4 \\ &= \Big[2\Sigma\left[\alpha^1\alpha^2\Sigma - \left(\alpha^1+\alpha^2\right)\tau\right]\left[\alpha^1\alpha^2\Sigma - \left(\alpha^1\right)^2\Sigma - \tau\left(\alpha^2-\alpha^1\right)\right] - 2\Sigma\left(\alpha^2-\alpha^1\right) \\ &\left(\alpha^1\Sigma-\tau\right)\left[\alpha^1\alpha^2\Sigma - \left(\alpha^1+\alpha^2\right)\tau\right] + 2\alpha^1\left(\alpha^1\Sigma-\tau\right)\left(\alpha^1\Sigma-2\tau\right) \\ &\left(\alpha^2\Sigma-2\tau\right)\Big]\Big/8\Sigma^2\left[\alpha^1\alpha^2\Sigma - \left(\alpha^1+\alpha^2\right)\tau\right]^3 \\ &= \Big[\Sigma\left[\alpha^1\alpha^2\Sigma - \left(\alpha^1+\alpha^2\right)\tau\right]\left[\alpha^2\left(\alpha^1\Sigma-\tau\right) - \alpha^1\left(\alpha^1\Sigma-\tau\right)\right] - \Sigma\left(\alpha^2-\alpha^1\right) \\ &\left(\alpha^1\Sigma-\tau\right)\left[\alpha^1\alpha^2\Sigma - \left(\alpha^1+\alpha^2\right)\tau\right] + \alpha^1\left(\alpha^1\Sigma-\tau\right)\left(\alpha^1\Sigma-2\tau\right) \\ &\left(\alpha^2\Sigma-2\tau\right)\Big]\Big/4\Sigma^2\left[\alpha^1\alpha^2\Sigma - \left(\alpha^1+\alpha^2\right)\tau\right]^3 = \frac{\alpha^1\left(\alpha^1\Sigma-\tau\right)\left(\alpha^1\Sigma-2\tau\right)\left(\alpha^2\Sigma-2\tau\right)}{4\Sigma^2\left[\alpha^1\alpha^2\Sigma - \left(\alpha^1+\alpha^2\right)\tau\right]^3} \\ \frac{\partial part2}{\partial \alpha^2} &= \frac{\alpha^1\left(\alpha^1\Sigma-\tau\right)\left(\alpha^1\Sigma-2\tau\right)\left(\alpha^2\Sigma-2\tau\right)}{4\Sigma^2\left[\alpha^2\left(\alpha^1\Sigma-2\tau\right) + \tau\left(\alpha^2-\alpha^1\right)\right]^3}\end{aligned}$$

As it can be observed, for values of α^2 greater than $\frac{2\tau}{\Sigma}$, the equilibrium profits on platform 1 in Equation (2.16) are always positive.

B.3.5 Proof of Propositions 2.6a and 2.6b

Proof. We prove Proposition 2.6a and Proposition 2.6b by partially differentiating the difference in equilibrium market shares with respect to the parameters of the model τ, υ and π using Equations (2.17a) and (2.17b) when the cross-group network effect sellers exert on buyers are stronger than vice versa $\upsilon > \pi$, $\pi = 0$; and Equations (2.19a) and (2.19b) when the cross-group network effect buyers exert on sellers are stronger than vice versa $\pi > \upsilon$, $\upsilon = 0$, under the assumption that platform 1 is more efficient in developing attributes on buyers' side than platform 2, $\alpha^2 > \alpha^1$.

Case 1. $\upsilon > \pi$, $\pi = 0$

We use Equation (2.17a) to compute the difference in equilibrium membership fees on buyers' side. Then, we use Equation (2.14a) to express this difference as a function of the difference in equilibrium attributes as

$$\Delta\left(p_b^i\right)_{\upsilon>\pi} = \frac{3\left(\alpha^j-\alpha^i\right)\tau^2\left(3\tau+2\upsilon\right)\left(2\tau-\upsilon\right)}{\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]} = \frac{6\tau^2}{\sigma_\upsilon}\left(\Delta q_b^i\right)_{\upsilon>\pi}$$

$$\begin{aligned}
\frac{\partial\Delta\left(p_b^i\right)_{\upsilon>\pi}}{\partial\tau} &= \frac{6}{\sigma_\upsilon^2}\left[\sigma_\upsilon\left[2\tau\left(\Delta q_b^i\right)_{\upsilon>\pi}+\tau^2\left(\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial\tau}\right)\right]-18\tau^3\left(\Delta q_b^i\right)_{\upsilon>\pi}\right]\\
&= \frac{6\tau}{\sigma_\upsilon^2}\left[-2\left(\Delta q_b^i\right)_{\upsilon>\pi}\left(18\tau^2-\sigma_\upsilon\right)+\sigma_\upsilon\tau\left(\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial\tau}\right)\right]\\
&= \frac{6\tau}{\sigma_\upsilon^2}\left[-2\left(\Delta q_b^i\right)_{\upsilon>\pi}\left(9\tau^2+2\upsilon^2\right)+\sigma_\upsilon\tau\left(\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial\tau}\right)\right]
\end{aligned}$$

According to Proposition 2.5 and its proof in Appendix B.3.1, we know that $\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial\tau}$ is negative. Therefore $\frac{\partial\Delta\left(p_b^i\right)_{\upsilon>\pi}}{\partial\tau} < 0$.

$$\begin{aligned}
\frac{\partial\Delta\left(p_b^i\right)_{\upsilon>\pi}}{\partial\upsilon} &= \frac{3\tau^2\left(\alpha^j-\alpha^i\right)}{\sigma_\upsilon^2\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]^2}\left[\left[\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]\left(\tau-4\upsilon\right)\right.\right.\\
&\qquad\left.+\left[8\upsilon\alpha^i\alpha^j\sigma_\upsilon-4\tau\upsilon\left(\alpha^i+\alpha^j\right)\right]\left(3\tau+2\upsilon\right)\left(2\tau-\upsilon\right)\right]\\
= \frac{3\tau^2\left(\alpha^j-\alpha^i\right)}{\sigma_\upsilon^2\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]^2}&\left[\tau\sigma_\upsilon\left[\alpha^j\left(\alpha^i\sigma_\upsilon-2\tau\right)+\left(\alpha^j-\alpha^i\right)\tau\right]+4\alpha^i\alpha^j\sigma_\upsilon\upsilon\left[2\left(3\tau+2\upsilon\right)\right.\right.\\
&\qquad\left.\left.\left(2\tau-\upsilon\right)-\sigma_\upsilon\right]+4\left(\alpha^i+\alpha^j\right)\tau\upsilon\left[\sigma_\upsilon-\left(3\tau+2\upsilon\right)\left(2\tau-\upsilon\right)\right]\right]\\
= \frac{3\tau^2\left(\alpha^j-\alpha^i\right)}{\sigma_\upsilon^2\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]^2}&\left[\tau\sigma_\upsilon\left[\alpha^j\left(\alpha^i\sigma_\upsilon-2\tau\right)+\left(\alpha^j-\alpha^i\right)\tau\right]\right.
\end{aligned}$$

$$+ 4\alpha^i\alpha^j\sigma_\upsilon\upsilon\left[3\tau^2 + 2\tau\upsilon - 2\upsilon^2\right] + 4\left(\alpha^i + \alpha^j\right)\tau^2\upsilon\left(3\tau - \upsilon\right)\Bigg]$$

To determine the sign of $\frac{\partial\Delta\left(p_b^i\right)_{\upsilon>\pi}}{\partial\upsilon}$ it is sufficient to examine the sign of $3\tau^2 + 2\tau\upsilon - 2\upsilon^2$ and $3\tau - \upsilon$ as the remaining elements of the partial derivative are positive. Specifically, $\sigma_\upsilon\left[\alpha^j\left(\alpha^i\sigma_\upsilon - 2\tau\right) + \left(\alpha^j - \alpha^i\right)\tau\right]$ is positive under Assumption 2.2 when $\upsilon > \pi$, $\pi = 0$, and $(3\tau + 2\upsilon)(2\tau - \upsilon)$ is positive under Assumption 2.1 when $\upsilon > \pi$, $\pi = 0$ and given platform 1 is more efficient in developing attributes $\alpha^2 > \alpha^1$.

As it has been shown in the proof of Claim 2.1 in Appendix B.3.3, specifically in $\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial\upsilon} > 0$ that both conditions $3\tau^2 + 2\tau\upsilon - 2\upsilon^2$ and $3\tau - \upsilon$ are positive, we conclude that $\frac{\partial\Delta\left(p_b^i\right)_{\upsilon>\pi}}{\partial\upsilon}$ is also positive under Assumption 2.1 and Assumption 2.2 when $\upsilon > \pi$, $\pi = 0$.

Next, we use Equation (2.17b) to compute the difference in equilibrium membership fees on sellers' side. Then, we use Equation (2.14a) to express this difference as a function of the difference in equilibrium attributes as

$$\Delta\left(p_s^i\right)_{\upsilon>\pi} = -\frac{\left(\alpha^j - \alpha^i\right)\tau\upsilon\left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right)}{\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]} = -\frac{2\tau\upsilon}{\sigma_\upsilon}\left(\Delta q_b^i\right)_{\upsilon>\pi}$$

$$\begin{aligned}
\frac{\partial\Delta\left(p_s^i\right)_{\upsilon>\pi}}{\partial\tau} &= \frac{-2\upsilon}{\sigma_\upsilon^2}\left[\sigma_\upsilon\left[\left(\Delta q_b^i\right)_{\upsilon>\pi} + \tau\left(\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial\tau}\right)\right] - 18\tau^2\left(\Delta q_b^i\right)_{\upsilon>\pi}\right] \\
&= \frac{-2\upsilon}{\sigma_\upsilon^2}\left[-\left(\Delta q_b^i\right)_{\upsilon>\pi}\left(18\tau^2 - \sigma_\upsilon\right) + \sigma_\upsilon\tau\left(\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial\tau}\right)\right] \\
&= \frac{2\upsilon}{\sigma_\upsilon^2}\left[\left(\Delta q_b^i\right)_{\upsilon>\pi}\left(9\tau^2 + 2\upsilon^2\right) - \sigma_\upsilon\tau\left(\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial\tau}\right)\right]
\end{aligned}$$

According to Proposition 2.5 and its proof in Appendix B.3.1, we know that $\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial\tau}$ is negative. Therefore $\frac{\partial\Delta\left(p_s^i\right)_{\upsilon>\pi}}{\partial\tau} > 0$.

$$\frac{\partial\Delta\left(p_s^i\right)_{\upsilon>\pi}}{\partial\upsilon} = -\frac{\left(\alpha^j - \alpha^i\right)\tau}{\sigma_\upsilon^2\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]^2}\Bigg[\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]\left[\upsilon\left(\tau - 4\upsilon\right)\right.$$

$$\left. + \left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right)\right] + \upsilon\left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right)\left[8\upsilon\alpha^i\alpha^j\sigma_\upsilon - 4\tau\upsilon\left(\alpha^i + \alpha^j\right)\right]\Bigg]$$

$$= -\frac{\left(\alpha^j - \alpha^i\right)\tau}{\sigma_\upsilon^2\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]^2}\Bigg[\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]\left[\tau\upsilon + \left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right)\right]$$

$$+4\alpha^i\alpha^j\sigma_\upsilon\upsilon^2\left[2\left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right) - \sigma_\upsilon\right] + 4\tau\upsilon^2\left(\alpha^i + \alpha^j\right)\left[\sigma_\upsilon - \left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right)\right]\Bigg]$$

$$= -\frac{(\alpha^j - \alpha^i)\,\tau}{\sigma_\upsilon^2\left[\alpha^i\alpha^j\sigma_\upsilon - (\alpha^i + \alpha^j)\,\tau\right]^2}\Bigg[\sigma_\upsilon\left[\alpha^j\left(\alpha^i\sigma_\upsilon - 2\tau\right) + \left(\alpha^j - \alpha^i\right)\tau\right]\left[\tau\upsilon\right.$$

$$\left. + (3\tau + 2\upsilon)(2\tau - \upsilon)\right] + 4\alpha^i\alpha^j\sigma_\upsilon\upsilon^2\left[3\tau^2 + 2\tau\upsilon - 2\upsilon^2\right] + 4\tau^2\upsilon^2\left(\alpha^i + \alpha^j\right)(3\tau - \upsilon)\Bigg]$$

To determine the sign of $\frac{\partial\Delta\left(p_s^i\right)_{\upsilon>\pi}}{\partial\upsilon}$ it is sufficient to examine the sign of $3\tau^2 + 2\tau\upsilon - 2\upsilon^2$ and $3\tau - \upsilon$ as the remaining elements of the partial derivative are positive. Specifically, $\sigma_\upsilon\left[\alpha^j\left(\alpha^i\sigma_\upsilon - 2\tau\right) + \left(\alpha^j - \alpha^i\right)\tau\right]$ is positive under Assumption 2.2 when $\upsilon > \pi$, $\pi = 0$, and $(3\tau + 2\upsilon)(2\tau - \upsilon)$ is positive under Assumption 2.1 when $\upsilon > \pi$, $\pi = 0$ and given platform 1 is more efficient in developing attributes $\alpha^2 > \alpha^1$.

As it has been shown in the proof of Claim 2.1 in Appendix B.3.3, specifically in $\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial\upsilon} > 0$ that both conditions $3\tau^2 + 2\tau\upsilon - 2\upsilon^2$ and $3\tau - \upsilon$ are positive, we conclude that $\frac{\partial\Delta\left(p_s^i\right)_{\upsilon>\pi}}{\partial\upsilon}$ is also negative under Assumption 2.1 and Assumption 2.2 when $\upsilon > \pi$, $\pi = 0$.

Case 2. $\pi > \upsilon$ **,** $\upsilon = 0$

We use Equation (2.19a) to compute the difference in equilibrium membership fees on buyers' side. Then, we use Equation (2.14b) to express this difference as a function of the difference in equilibrium attributes as

$$\Delta\left(p_b^i\right)_{\pi>\upsilon} = \frac{(\alpha^j - \alpha^i)(3\tau^2 - \pi^2)(3\tau + \pi)(2\tau - \pi)}{\sigma_\pi\left[\alpha^i\alpha^j\sigma_\pi - (\alpha^i + \alpha^j)\,\tau\right]} = \frac{2\,(3\tau^2 - \pi^2)}{\sigma_\pi}\left(\Delta q_b^i\right)_{\pi>\upsilon}$$

$$\frac{\partial\Delta\left(p_b^i\right)_{\pi>\upsilon}}{\partial\tau} = \frac{2}{\sigma_\pi^2}\left[\sigma_\pi\left[6\tau\left(\Delta q_b^i\right)_{\pi>\upsilon} + \left(3\tau^2 - \pi^2\right)\left(\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial\tau}\right)\right] - 18\tau\left(3\tau^2 - \pi^2\right)\left(\Delta q_b^i\right)_{\pi>\upsilon}\right]$$

$$= \frac{2}{\sigma_\pi}\left[6\tau\left(\Delta q_b^i\right)_{\pi>\upsilon}\left(\sigma_\pi - 9\tau^2 + 3\pi^2\right) + \sigma_\pi\left(3\tau^2 - \pi^2\right)\left(\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial\tau}\right)\right]$$

$$= \frac{2}{\sigma_\pi}\left[6\tau\pi^2\left(\Delta q_b^i\right)_{\pi>\upsilon} + \sigma_\pi\left(3\tau^2 - \pi^2\right)\left(\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial\tau}\right)\right]$$

According to Proposition 2.5 and its proof in Appendix B.3.1, we know that $\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial\tau}$ is negative. Furthermore, as was mentioned in the intuition of the equilibrium membership fees in Equations (2.19a) and (2.19b) $3\tau^2 - \pi^2$ is negative as long as $\tau < \frac{\pi}{\sqrt{3}}$, which is compatible with Assumption 2.1 when $\pi > \upsilon$, $\upsilon = 0$ since $\frac{\pi}{2} < \tau < \frac{\pi}{\sqrt{3}}$. Therefore $\frac{\partial\Delta\left(p_b^i\right)_{\pi>\upsilon}}{\partial\tau} > 0$.

$$\frac{\partial\Delta\left(p_b^i\right)_{\pi>\upsilon}}{\partial\pi} = \frac{2}{\sigma_\pi^2}\left[\sigma_\pi\left[-2\pi\left(\Delta q_b^i\right)_{\pi>\upsilon} + \left(3\tau^2 - \pi^2\right)\left(\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial\tau}\right)\right] - 4\pi\left(3\tau^2 - \pi^2\right)\left(\Delta q_b^i\right)_{\pi>\upsilon}\right]$$

$$= \frac{2}{\sigma_\pi}\left[2\pi\left(\Delta q_b^i\right)_{\pi>\upsilon}\left(-\sigma_\pi+6\tau^2-2\pi^2\right)+\sigma_\pi\left(3\tau^2-\pi^2\right)\left(\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial\pi}\right)\right]$$
$$= -\frac{2}{\sigma_\pi}\left[6\tau^2\pi\left(\Delta q_b^i\right)_{\pi>\upsilon}-\sigma_\pi\left(3\tau^2-\pi^2\right)\left(\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial\pi}\right)\right]$$

According to Proposition 2.5 and its proof in Appendix B.3.1, we know that $\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial\pi}$ is negative. Furthermore, as was mentioned previously $3\tau^2-\pi^2$ is negative as long as $\tau<\frac{\pi}{\sqrt{3}}$, which is compatible with Assumption 2.1 when $\pi>\upsilon$, $\upsilon=0$ since $\frac{\pi}{2}<\tau<\frac{\pi}{\sqrt{3}}$. Therefore $\frac{\partial\Delta\left(p_b^i\right)_{\pi>\upsilon}}{\partial\pi}<0$.

We use Equation (2.19b) to compute the difference in equilibrium membership fees on sellers' side. Then, we use Equation (2.14b) to express this difference as a function of the difference in equilibrium attributes as

$$\Delta\left(p_s^i\right)_{\pi>\upsilon} = \frac{\left(\alpha^j-\alpha^i\right)\tau\pi\left(3\tau+\pi\right)\left(2\tau-\pi\right)}{\sigma_\pi\left[\alpha^i\alpha^j\sigma_\pi-\left(\alpha^i+\alpha^j\right)\tau\right]} = \frac{2\tau\pi}{\sigma_\pi}\left(\Delta q_b^i\right)_{\pi>\upsilon}$$
$$\frac{\partial\Delta\left(p_s^i\right)_{\pi>\upsilon}}{\partial\tau} = \frac{2\pi}{\sigma_\pi^2}\left[\sigma_\pi\left[\left(\Delta q_b^i\right)_{\pi>\upsilon}+\tau\left(\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial\tau}\right)\right]-18\tau^2\left(\Delta q_b^i\right)_{\pi>\upsilon}\right]$$
$$= -\frac{2\pi}{\sigma_\pi^2}\left[\left(\Delta q_b^i\right)_{\pi>\upsilon}\left(9\tau^2+2\pi^2\right)-\sigma_\pi\tau\left(\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial\tau}\right)\right]$$

According to Proposition 2.5 and its proof in Appendix B.3.1, we know that $\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial\tau}$ is negative. Therefore $\frac{\partial\Delta\left(p_s^i\right)_{\pi>\upsilon}}{\partial\tau}<0$.

$$\frac{\partial\Delta\left(p_s^i\right)_{\pi>\upsilon}}{\partial\pi} = \frac{2\tau}{\sigma_\pi^2}\left[\sigma_\pi\left[\left(\Delta q_b^i\right)_{\pi>\upsilon}+\pi\left(\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial\pi}\right)\right]+4\pi^2\left(\Delta q_b^i\right)_{\pi>\upsilon}\right]$$
$$= \frac{2\tau}{\sigma_\pi^2}\left[\left(\Delta q_b^i\right)_{\pi>\upsilon}\left(9\tau^2+2\pi^2\right)+\pi\sigma_\pi\left(\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial\pi}\right)\right]$$

According to Proposition 2.5 and its proof in Appendix B.3.1, we know that $\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial\pi}$ is positive. Therefore $\frac{\partial\Delta\left(p_s^i\right)_{\pi>\upsilon}}{\partial\pi}>0$. □

B.3.6 Proof of Propositions 2.7a and 2.7b

Proof. We prove Proposition 2.7a and Proposition 2.7b by partially differentiating the difference in equilibrium profits with respect to the parameters of the model τ, υ and π when the cross-group network effect sellers exert on buyers are stronger than vice versa $\upsilon>\pi$, $\pi=0$; and when the cross-group network effect buyers exert on sellers are stronger than vice versa $\pi>\upsilon$, $\upsilon=0$, under the assumption that platform 1 is more efficient in developing attributes on buyers' side than platform 2, $\alpha^2>\alpha^1$.

Case 1. $\upsilon > \pi$**,** $\pi = 0$

We use Equations (2.17a) and (2.17b) and Equations (2.15a) and (2.15b) when $\upsilon > \pi$, $\pi = 0$ to compute the difference in equilibrium profits. Then we use Equation (2.14a) to express this difference as a function of the difference in equilibrium attributes as

$$\begin{aligned}
\Delta\Pi^i_{\upsilon>\pi} &= \left[(p_b)^i_{\upsilon>\pi} - f_b\right](\eta_b)^i_{\upsilon>\pi} + \left[(p_s)^i_{\upsilon>\pi} - f_s\right](\eta_s)^i_{\upsilon>\pi} - \frac{\alpha^i}{2}\left(q^i_b\right)^2_{\upsilon>\pi} \\
&\quad - \left[(p_b)^j_{\upsilon>\pi} - f_b\right](\eta_b)^j_{\upsilon>\pi} - \left[(p_s)^j_{\upsilon>\pi} - f_s\right](\eta_s)^j_{\upsilon>\pi} + \frac{\alpha^j}{2}\left(q^j_b\right)^2_{\upsilon>\pi} \\
&= \left[\tau + \frac{3\tau^2}{\sigma_\upsilon}\Delta\left(q^i_b\right)_{\upsilon>\pi}\right]\left[\frac{1}{2} + \frac{3\tau}{2\sigma_\upsilon}\Delta\left(q^i_b\right)_{\upsilon>\pi}\right] - \left[\tau + \frac{3\tau^2}{\sigma_\upsilon}\Delta\left(q^j_b\right)_{\upsilon>\pi}\right]\left[\frac{1}{2} + \frac{3\tau}{2\sigma_\upsilon}\Delta\left(q^j_b\right)_{\upsilon>\pi}\right] \\
&+ \left[\tau - \upsilon - \frac{\tau\upsilon}{\sigma_\upsilon}\Delta\left(q^i_b\right)_{\upsilon>\pi}\right]\left[\frac{1}{2} + \frac{\upsilon}{\sigma_\upsilon}\Delta\left(q^i_b\right)_{\upsilon>\pi}\right] - \left[\tau - \upsilon - \frac{\tau\upsilon}{\sigma_\upsilon}\Delta\left(q^j_b\right)_{\upsilon>\pi}\right]\left[\frac{1}{2} + \frac{\upsilon}{\sigma_\upsilon}\Delta\left(q^j_b\right)_{\upsilon>\pi}\right] \\
&+ \frac{\alpha^j}{2}\left[\frac{\left(\alpha^i\sigma_\upsilon - 2\tau\right)^2(3\tau + 2\upsilon)^2(2\tau - \upsilon)^2}{4\sigma^2_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]^2}\right] - \frac{\alpha^i}{2}\left[\frac{\left(\alpha^j\sigma_\upsilon - 2\tau\right)^2(3\tau + 2\upsilon)^2(2\tau - \upsilon)^2}{4\sigma^2_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]^2}\right] \\
&= \frac{3\tau^2}{\sigma_\upsilon}\left[\Delta\left(q^i_b\right)_{\upsilon>\pi} - \Delta\left(q^j_b\right)_{\upsilon>\pi}\right] + \frac{9\tau^3}{2\sigma^2_\upsilon}\left[\Delta\left(q^i_b\right)^2_{\upsilon>\pi} - \Delta\left(q^j_b\right)^2_{\upsilon>\pi}\right] \\
&\quad + \frac{\upsilon(\tau - 2\upsilon)}{2\sigma_\upsilon}\left[\Delta\left(q^i_b\right)_{\upsilon>\pi} - \Delta\left(q^j_b\right)_{\upsilon>\pi}\right] - \frac{\tau\upsilon^2}{\sigma^2_\upsilon}\left[\Delta\left(q^i_b\right)^2_{\upsilon>\pi} - \Delta\left(q^j_b\right)^2_{\upsilon>\pi}\right] \\
&\quad + \frac{(3\tau + 2\upsilon)^2(2\tau - \upsilon)^2}{8\sigma^2_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]^2}\left[\alpha^j\left(\alpha^i\sigma_\upsilon - 2\tau\right)^2 - \alpha^i\left(\alpha^j\sigma_\upsilon - 2\tau\right)^2\right] \\
&= \frac{6\tau^2\left(\alpha^j - \alpha^i\right)(3\tau + 2\upsilon)(2\tau - \upsilon)}{2\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]} + \frac{9\tau^3}{2\sigma^2_\upsilon}\left[\Delta\left(q^i_b\right)_{\upsilon>\pi} + \Delta\left(q^j_b\right)_{\upsilon>\pi}\right] \\
&\quad \left[\Delta\left(q^i_b\right)_{\upsilon>\pi} - \Delta\left(q^j_b\right)_{\upsilon>\pi}\right] + \frac{2\upsilon(\tau - 2\upsilon)\left(\alpha^j - \alpha^i\right)(3\tau + 2\upsilon)(2\tau - \upsilon)}{4\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]} \\
&\quad - \frac{\tau\upsilon^2}{\sigma^2_\upsilon}\left[\Delta\left(q^i_b\right)_{\upsilon>\pi} + \Delta\left(q^j_b\right)_{\upsilon>\pi}\right]\left[\Delta\left(q^i_b\right)_{\upsilon>\pi} - \Delta\left(q^j_b\right)_{\upsilon>\pi}\right] \\
&+ \frac{(3\tau + 2\upsilon)^2(2\tau - \upsilon)^2}{8\sigma^2_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]^2}\left[\alpha^j\left(\alpha^i\right)^2\sigma^2_\upsilon + 4\alpha^j\tau^2 - \alpha^i\left(\alpha^j\right)^2\sigma^2_\upsilon - 4\alpha^i\tau^2\right]
\end{aligned}$$

Since $\Delta\left(q^i_b\right)_{\upsilon>\pi} = \frac{\left(\alpha^j - \alpha^i\right)(3\tau + 2\upsilon)(2\tau - \upsilon)}{2\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]}$, we have $\Delta\left(q^i_b\right)_{\upsilon>\pi} + \Delta\left(q^j_b\right)_{\upsilon>\pi} = 0$, then

$$\begin{aligned}
&= \frac{6\tau^2}{\sigma_\upsilon}\Delta\left(q^i_b\right)_{\upsilon>\pi} + \frac{\upsilon(\tau - 2\upsilon)}{\sigma_\upsilon}\Delta\left(q^i_b\right)_{\upsilon>\pi} - \frac{\left[\alpha^i\alpha^j\sigma^2_\upsilon - 4\tau^2\right](3\tau + 2\upsilon)(2\tau - \upsilon)}{4\sigma^2_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]}\Delta\left(q^i_b\right)_{\upsilon>\pi} \\
\Delta\Pi^i_{\upsilon>\pi} &= \frac{\Delta\left(q^i_b\right)_{\upsilon>\pi}}{\sigma_\upsilon}(3\tau + 2\upsilon)(2\tau - \upsilon) - \frac{\left[\alpha^i\alpha^j\sigma^2_\upsilon - 4\tau^2\right](3\tau + 2\upsilon)(2\tau - \upsilon)}{4\sigma^2_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]}\Delta\left(q^i_b\right)_{\upsilon>\pi}
\end{aligned}$$

Next, we partially differentiate $\Delta\Pi^i_{\upsilon>\pi}$ respect τ and υ, obtaining:

$$\frac{\partial\Delta\Pi^i_{\upsilon>\pi}}{\partial\tau} = \frac{1}{\sigma^2_\upsilon}\left[\sigma_\upsilon\left[\frac{\partial\Delta\left(q^i_b\right)_{\upsilon>\pi}}{\partial\tau}(3\tau + 2\upsilon)(2\tau - \upsilon) + (12\tau + \upsilon)\Delta\left(q^i_b\right)_{\upsilon>\pi}\right]\right.$$

$$-18\tau\left(3\tau+2\upsilon\right)\left(2\tau-\upsilon\right)\Delta\left(q_b^i\right)_{\upsilon>\pi}\Bigg]-\frac{1}{4\sigma_\upsilon^4\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]^2}\Bigg[\sigma_\upsilon^2\Big[\frac{\partial\Delta\left(q_b^i\right)_{\upsilon>\pi}}{\partial\tau}$$

$$\left(3\tau+2\upsilon\right)\left(2\tau-\upsilon\right)\left[\alpha^i\alpha^j\sigma_\upsilon^2-4\tau^2\right]+\Delta\left(q_b^i\right)_{\upsilon>\pi}\left[\alpha^i\alpha^j\sigma_\upsilon^2-4\tau^2\right]\left(12\tau+\upsilon\right)+4\tau\Delta\left(q_b^i\right)_{\upsilon>\pi}$$

$$\left(3\tau+2\upsilon\right)\left(2\tau-\upsilon\right)\left(9\alpha^i\alpha^j\sigma_\upsilon-2\right)\Big]\Big[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\Big]-\Big[\left[\alpha^i\alpha^j\sigma_\upsilon^2-4\tau^2\right]\left(3\tau+2\upsilon\right)$$

$$\left(2\tau-\upsilon\right)\Delta\left(q_b^i\right)_{\upsilon>\pi}\Big]\Big[54\alpha^i\alpha^j\sigma_\upsilon^2\tau-\left(\alpha^i+\alpha^j\right)\sigma_\upsilon\left(36\tau^2+\sigma_\upsilon\right)\Big]\Bigg]$$

Next, we evaluate the partial derivative when $\tau=\frac{\upsilon}{2}$ getting:

$$\left.\frac{\partial\Delta\Pi_{\upsilon>\pi}^i}{\partial\tau}\right|_{\tau=\frac{\upsilon}{2}}=\frac{1}{\sigma_\upsilon}\left(12\tau+\upsilon\right)\Delta\left(q_b^i\right)_{\upsilon>\pi}-\frac{\Delta\left(q_b^i\right)_{\upsilon>\pi}\left[\alpha^i\alpha^j\sigma_\upsilon^2-4\tau^2\right]\left(12\tau+\upsilon\right)}{4\sigma_\upsilon^2\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]}$$

$$=\frac{\left(12\tau+\upsilon\right)\Delta\left(q_b^i\right)_{\upsilon>\pi}}{\sigma_\upsilon}\left[1-\frac{\left[\alpha^i\alpha^j\sigma_\upsilon^2-4\tau^2\right]}{4\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]}\right]$$

$$=\frac{\left(12\tau+\upsilon\right)\Delta\left(q_b^i\right)_{\upsilon>\pi}}{\sigma_\upsilon}\left[\frac{4\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]-\left[\alpha^i\alpha^j\sigma_\upsilon^2-4\tau^2\right]}{4\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]}\right]$$

$$=\frac{\left(12\tau+\upsilon\right)\Delta\left(q_b^i\right)_{\upsilon>\pi}}{\sigma_\upsilon}\left[\frac{3\alpha^i\alpha^j\sigma_\upsilon^2-4\sigma_\upsilon\left(\alpha^i+\alpha^j\right)\tau+4\tau^2}{4\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]}\right]$$

$$=\frac{\left(12\tau+\upsilon\right)\Delta\left(q_b^i\right)_{\upsilon>\pi}}{\sigma_\upsilon}\left[\frac{3\alpha^i\alpha^j\sigma_\upsilon^2-6\alpha^j\sigma_\upsilon\tau-2\alpha^i\sigma_\upsilon\tau+4\tau^2+2\alpha^j\sigma_\upsilon\tau-2\alpha^i\sigma_\upsilon\tau}{4\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]}\right]$$

$$=\frac{\left(12\tau+\upsilon\right)\Delta\left(q_b^i\right)_{\upsilon>\pi}}{\sigma_\upsilon}\left[\frac{3\alpha^j\sigma_\upsilon\left(\alpha^i\sigma_\upsilon-2\tau\right)-2\tau\left(\alpha^i\sigma_\upsilon-2\tau\right)+2\sigma_\upsilon\tau\left(\alpha^j-\alpha^i\right)}{4\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]}\right]$$

$$=\frac{\left(12\tau+\upsilon\right)\Delta\left(q_b^i\right)_{\upsilon>\pi}}{\sigma_\upsilon}\left[\frac{\left(3\alpha^j\sigma_\upsilon-2\tau\right)\left(\alpha^i\sigma_\upsilon-2\tau\right)+2\sigma_\upsilon\tau\left(\alpha^j-\alpha^i\right)}{4\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]}\right]$$

$$\left.\frac{\partial\Delta\Pi_{\upsilon>\pi}^i}{\partial\tau}\right|_{\tau=\frac{\upsilon}{2}}=\frac{7\upsilon^3\Delta\left(q_b^i\right)_{\upsilon>\pi}\left[\left(\frac{3}{4}\alpha^j\upsilon-1\right)\left(\frac{1}{4}\alpha^j\upsilon-1\right)+\frac{\upsilon}{4}\left(\alpha^j-\alpha^i\right)\right]}{\frac{\upsilon^5}{4}\left[\alpha^i\alpha^j\frac{u}{4}-\frac{1}{2}\left(\alpha^i+\alpha^j\right)\right]}$$

$$=\left[\frac{7\left[\left(3\alpha^j\upsilon-4\right)\left(\alpha^j\upsilon-4\right)+4\upsilon\left(\alpha^j-\alpha^i\right)\right]}{\upsilon^2\left[\alpha^j\left(\alpha^i\upsilon-4\right)+2\left(\alpha^j-\alpha^i\right)\right]}\right]\Delta\left(q_b^i\right)_{\upsilon>\pi}$$

The previous expression is positive under the assumption that platform 1 is more efficient in developing attributes on buyers' side than platform 2, $\alpha^2>\alpha^1$; and for τ values greater than $\frac{\upsilon}{2}$ (Assumption 2.1); and under Assumption 2.2 which turns to $\alpha^i>\frac{4}{\upsilon}$ when $\tau=\frac{\upsilon}{2}$. Therefore, $\frac{\partial\Delta\Pi_{\upsilon>\pi}^i}{\partial\tau}>0$.

$$\frac{\partial\Delta\Pi_{\upsilon>\pi}^i}{\partial\upsilon}=\frac{1}{\sigma_\upsilon^2}\Bigg[\sigma_\upsilon\left[\frac{\partial\Delta\left(q_b^i\right)_{\upsilon>\pi}}{\partial\upsilon}\left(3\tau+2\upsilon\right)\left(2\tau-\upsilon\right)+\left(\tau-4\upsilon\right)\Delta\left(q_b^i\right)_{\upsilon>\pi}\right]$$

$$+4\upsilon\left(3\tau+2\upsilon\right)\left(2\tau-\upsilon\right)\Delta\left(q_b^i\right)_{\upsilon>\pi}\Bigg]-\frac{1}{4\sigma_\upsilon^4\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]^2}\Bigg[\sigma_\upsilon^2\Big[\frac{\partial\Delta\left(q_b^i\right)_{\upsilon>\pi}}{\partial\upsilon}$$

$$\left(3\tau+2\upsilon\right)\left(2\tau-\upsilon\right)\left[\alpha^i\alpha^j\sigma_\upsilon^2-4\tau^2\right]+\Delta\left(q_b^i\right)_{\upsilon>\pi}\left[\alpha^i\alpha^j\sigma_\upsilon^2-4\tau^2\right]\left(\tau-4\upsilon\right)$$

$$-8\alpha^i\alpha^j\upsilon\sigma_\upsilon\tau\Delta\left(q_b^i\right)_{\upsilon>\pi}\left(3\tau+2\upsilon\right)\left(2\tau-\upsilon\right)\Big]\Big[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\Big]$$

$$+ 4\upsilon\sigma_\upsilon \Big[\left[\alpha^i\alpha^j\sigma_\upsilon^2 - 4\tau^2\right] (3\tau + 2\upsilon)(2\tau - \upsilon)\,\Delta\left(q_b^i\right)_{\upsilon>\pi} \Big] \Big[3\alpha^i\alpha^j\sigma_\upsilon - 2\left(\alpha^i + \alpha^j\right)\tau \Big] \Big]$$

Next, we evaluate the partial derivative when $\tau = \frac{\upsilon}{2}$ getting:

$$\begin{aligned}
\left.\frac{\partial \Delta\Pi^i_{\upsilon>\pi}}{\partial \upsilon}\right|_{\tau=\frac{\upsilon}{2}} &= \frac{1}{\sigma_\upsilon}(\tau - 4\upsilon)\,\Delta\left(q_b^i\right)_{\upsilon>\pi} - \frac{\Delta\left(q_b^i\right)_{\upsilon>\pi}\left[\alpha^i\alpha^j\sigma_\upsilon^2 - 4\tau^2\right](\tau - 4\upsilon)}{4\sigma_\upsilon^2\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]} \\
&= \frac{(\tau - 4\upsilon)\,\Delta\left(q_b^i\right)_{\upsilon>\pi}}{\sigma_\upsilon}\left[1 - \frac{\left[\alpha^i\alpha^j\sigma_\upsilon^2 - 4\tau^2\right]}{4\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]}\right]
\end{aligned}$$

The right side of the previous expression is the same as in $\frac{\partial \Delta\Pi^i_{\upsilon>\pi}}{\partial \tau}$, consequently we obtain:

$$\begin{aligned}
&= \frac{(\tau - 4\upsilon)\,\Delta\left(q_b^i\right)_{\upsilon>\pi}}{\sigma_\upsilon}\left[\frac{\left(3\alpha^j\sigma_\upsilon - 2\tau\right)\left(\alpha^i\sigma_\upsilon - 2\tau\right) + 2\sigma_\upsilon\tau\left(\alpha^j - \alpha^i\right)}{4\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]}\right] \\
\left.\frac{\partial \Delta\Pi^i_{\upsilon>\pi}}{\partial \upsilon}\right|_{\tau=\frac{\upsilon}{2}} &= \left[\frac{(\tau - 4\upsilon)\left[\left(3\alpha^j\upsilon - 4\right)\left(\alpha^j\upsilon - 4\right) + 4\upsilon\left(\alpha^j - \alpha^i\right)\right]}{\upsilon^2\left[\alpha^j\left(\alpha^i\upsilon - 4\right) + 2\left(\alpha^j - \alpha^i\right)\right]}\right]\Delta\left(q_b^i\right)_{\upsilon>\pi}
\end{aligned}$$

The previous expression is positive under the assumption that platform 1 is more efficient in developing attributes on buyers' side than platform 2, $\alpha^2 > \alpha^1$; and for τ values greater than 4υ; and under Assumption 2.2 which turns to $\alpha^i > \frac{4}{\upsilon}$ when $\tau = \frac{\upsilon}{2}$. Therefore, $\frac{\partial \Delta\Pi^i_{\upsilon>\pi}}{\partial \upsilon} > 0$.

Case 2. $\pi > \upsilon$, $\upsilon = 0$

We use Equations (2.19a) and (2.19b) and Equations (2.15a) and (2.15b) when $\pi > \upsilon$, $\upsilon = 0$ to compute the difference in equilibrium profits. Then we use Equation (2.14b) to express this difference as a function of the difference in equilibrium attributes as:

$$\begin{aligned}
\Delta\Pi^i_{\pi>\upsilon} &= \left[(p_b)^i_{\pi>\upsilon} - f_b\right](\eta_b)^i_{\pi>\upsilon} + \left[(p_s)^i_{\pi>\upsilon} - f_s\right](\eta_s)^i_{\pi>\upsilon} - \frac{\alpha^i}{2}\left(q_b^i\right)^2_{\pi>\upsilon} \\
&\quad - \left[(p_b)^j_{\pi>\upsilon} - f_b\right](\eta_b)^j_{\pi>\upsilon} - \left[(p_s)^j_{\pi>\upsilon} - f_s\right](\eta_s)^j_{\pi>\upsilon} + \frac{\alpha^j}{2}\left(q_b^j\right)^2_{\pi>\upsilon} \\
&= \left[\tau - \pi + \frac{3\tau^2 - \pi^2}{\sigma_\pi}\Delta\left(q_b^i\right)_{\pi>\upsilon}\right]\left[\frac{1}{2} + \frac{3\tau}{2\sigma_\pi}\Delta\left(q_b^i\right)_{\pi>\upsilon}\right] \\
&\quad - \left[\tau - \pi + \frac{3\tau^2 - \pi^2}{\sigma_\pi}\Delta\left(q_b^j\right)_{\pi>\upsilon}\right]\left[\frac{1}{2} + \frac{3\tau}{2\sigma_\pi}\Delta\left(q_b^j\right)_{\pi>\upsilon}\right] \\
&\quad + \left[\tau + \frac{\tau\pi}{\sigma_\pi}\Delta\left(q_b^i\right)_{\pi>\upsilon}\right]\left[\frac{1}{2} + \frac{\pi}{2\sigma_\pi}\Delta\left(q_b^i\right)_{\pi>\upsilon}\right] - \left[\tau + \frac{\tau\pi}{\sigma_\pi}\Delta\left(q_b^j\right)_{\pi>\upsilon}\right]\left[\frac{1}{2} + \frac{\pi}{2\sigma_\pi}\Delta\left(q_b^j\right)_{\pi>\upsilon}\right] \\
&\quad + \frac{\alpha^j}{2}\left[\frac{\left(\alpha^i\sigma_\pi - 2\tau\right)^2(3\tau + \pi)^2(2\tau - \pi)^2}{4\sigma_\pi^2\left[\alpha^i\alpha^j\sigma_\pi - \left(\alpha^i + \alpha^j\right)\tau\right]^2}\right] - \frac{\alpha^i}{2}\left[\frac{\left(\alpha^j\sigma_\pi - 2\tau\right)^2(3\tau + \pi)^2(2\tau - \pi)^2}{4\sigma_\pi^2\left[\alpha^i\alpha^j\sigma_\pi - \left(\alpha^i + \alpha^j\right)\tau\right]^2}\right] \\
&= \frac{6\tau^2 - 3\tau\pi - \pi^2}{2\sigma_\pi}\left[\Delta\left(q_b^i\right)_{\pi>\upsilon} - \Delta\left(q_b^j\right)_{\pi>\upsilon}\right] + \frac{3\tau\left(3\tau^2 - \pi^2\right)}{2\sigma_\pi^2}\left[\Delta\left(q_b^i\right)^2_{\pi>\upsilon} - \Delta\left(q_b^j\right)^2_{\pi>\upsilon}\right] \\
&\quad + \frac{\tau\pi}{\sigma_\pi}\left[\Delta\left(q_b^i\right)_{\pi>\upsilon} - \Delta\left(q_b^j\right)_{\pi>\upsilon}\right] - \frac{\tau\pi^2}{2\sigma_\pi^2}\left[\Delta\left(q_b^i\right)^2_{\pi>\upsilon} - \Delta\left(q_b^j\right)^2_{\pi>\upsilon}\right]
\end{aligned}$$

$$+\frac{(3\tau+\pi)^2(2\tau-\pi)^2}{8\sigma_\pi^2\left[\alpha^i\alpha^j\sigma_\pi-(\alpha^i+\alpha^j)\tau\right]^2}\left[\alpha^j\left(\alpha^i\sigma_\pi-2\tau\right)^2-\alpha^i\left(\alpha^j\sigma_\pi-2\tau\right)^2\right]$$

As was mentioned in $\frac{\partial\Delta\Pi^i_{\pi>\upsilon}}{\partial\tau}$, $\Delta\left(q_b^i\right)_{\pi>\upsilon}+\Delta\left(q_b^j\right)_{\pi>\upsilon}=0$, then $\Delta\left(q_b^i\right)^2_{\pi>\upsilon}-\Delta\left(q_b^j\right)^2_{\pi>\upsilon}=\left[\Delta\left(q_b^i\right)_{\pi>\upsilon}+\Delta\left(q_b^j\right)_{\pi>\upsilon}\right]\left[\Delta\left(q_b^i\right)_{\pi>\upsilon}-\Delta\left(q_b^j\right)_{\pi>\upsilon}\right]=0$, Furthermore, $\Delta\left(q_b^i\right)_{\pi>\upsilon}-\Delta\left(q_b^j\right)_{\pi>\upsilon}=\frac{(3\tau+\pi)(2\tau-\pi)}{2[\alpha^i\alpha^j\sigma_\pi-(\alpha^i+\alpha^j)\tau]}\left[2\left(\alpha^j-\alpha^i\right)\right]$, then it turns to $\Delta\left(q_b^i\right)_{\pi>\upsilon}-\Delta\left(q_b^j\right)_{\pi>\upsilon}=2\Delta\left(q_b^i\right)_{\pi>\upsilon}$, therefore we obtain:

$$\Delta\Pi^i_{\pi>\upsilon}=\frac{(3\tau+\pi)(2\tau-\pi)}{\sigma_\pi}\Delta\left(q_b^i\right)_{\pi>\upsilon}-\frac{\left[\alpha^i\alpha^j\sigma_\pi^2-4\tau^2\right](3\tau+\pi)(2\tau-\pi)}{4\sigma_\pi^2\left[\alpha^i\alpha^j\sigma_\pi-(\alpha^i+\alpha^j)\tau\right]}\Delta\left(q_b^i\right)_{\pi>\upsilon}$$

Next, we partially differentiate $\Delta\Pi^i_{\pi>\upsilon}$ respect τ and π, obtaining:

$$\begin{aligned}\frac{\partial\Delta\Pi^i_{\pi>\upsilon}}{\partial\tau}&=\frac{1}{\sigma_\pi^2}\left[\sigma_\pi\left[\frac{\partial\Delta\left(q_b^i\right)_{\pi>\upsilon}}{\partial\tau}(3\tau+\pi)(2\tau-\pi)+(12\tau-\pi)\Delta\left(q_b^i\right)_{\pi>\upsilon}\right]\right.\\&\left.-18\tau(3\tau+\pi)(2\tau-\pi)\Delta\left(q_b^i\right)_{\pi>\upsilon}\right]-\frac{1}{4\sigma_\pi^4\left[\alpha^i\alpha^j\sigma_\pi-(\alpha^i+\alpha^j)\tau\right]^2}\left[\sigma_\pi^2\left[\frac{\partial\Delta\left(q_b^i\right)_{\pi>\upsilon}}{\partial\tau}\right.\right.\\&(3\tau+\pi)(2\tau-\pi)\left[\alpha^i\alpha^j\sigma_\pi^2-4\tau^2\right]+\Delta\left(q_b^i\right)_{\pi>\upsilon}\left[\alpha^i\alpha^j\sigma_\pi^2-4\tau^2\right](12\tau-\pi)+4\tau\Delta\left(q_b^i\right)_{\pi>\upsilon}\\&\left.(3\tau+\pi)(2\tau-\pi)\left(9\alpha^i\alpha^j\sigma_\pi-2\right)\right]\left[\alpha^i\alpha^j\sigma_\pi-\left(\alpha^i+\alpha^j\right)\tau\right]-\left[\left[\alpha^i\alpha^j\sigma_\pi^2-4\tau^2\right](3\tau+\pi)\right.\\&\left.\left.(2\tau-\pi)\Delta\left(q_b^i\right)_{\pi>\upsilon}\right]\left[54\alpha^i\alpha^j\sigma_\pi^2\tau-\left(\alpha^i+\alpha^j\right)\sigma_\pi\left(36\tau^2+\sigma_\pi\right)\right]\right]\end{aligned}$$

Next, we evaluate the partial derivative when $\tau=\frac{\pi}{2}$ and obtain:

$$\begin{aligned}\left.\frac{\partial\Delta\Pi^i_{\pi>\upsilon}}{\partial\tau}\right|_{\tau=\frac{\pi}{2}}&=\frac{1}{\sigma_\pi}(12\tau-\pi)\Delta\left(q_b^i\right)_{\pi>\upsilon}-\frac{\Delta\left(q_b^i\right)_{\pi>\upsilon}\left[\alpha^i\alpha^j\sigma_\pi^2-4\tau^2\right](12\tau-\pi)}{4\sigma_\pi^2\left[\alpha^i\alpha^j\sigma_\pi-(\alpha^i+\alpha^j)\tau\right]}\\&=\frac{(12\tau-\pi)\Delta\left(q_b^i\right)_{\pi>\upsilon}}{\sigma_\pi}\left[1-\frac{\left[\alpha^i\alpha^j\sigma_\pi^2-4\tau^2\right]}{4\sigma_\pi\left[\alpha^i\alpha^j\sigma_\pi-(\alpha^i+\alpha^j)\tau\right]}\right]\end{aligned}$$

The right side of the previous expression is the same as in $\frac{\partial\Delta\Pi^i_{\upsilon>\pi}}{\partial\tau}$, consequently we obtain:

$$\begin{aligned}&=\frac{(12\tau-\pi)\Delta\left(q_b^i\right)_{\pi>\upsilon}}{\sigma_\pi}\left[\frac{(3\alpha^j\sigma_\pi-2\tau)(\alpha^i\sigma_\pi-2\tau)+2\sigma_\pi\tau\left(\alpha^j-\alpha^i\right)}{4\sigma_\pi\left[\alpha^i\alpha^j\sigma_\pi-(\alpha^i+\alpha^j)\tau\right]}\right]\\\left.\frac{\partial\Delta\Pi^i_{\pi>\upsilon}}{\partial\tau}\right|_{\tau=\frac{\pi}{2}}&=\frac{5\pi^3\Delta\left(q_b^i\right)_{\pi>\upsilon}\left[\left(\frac{3}{4}\alpha^j\pi-1\right)\left(\frac{1}{4}\alpha^j\pi-1\right)+\frac{\pi}{4}\left(\alpha^j-\alpha^i\right)\right]}{\frac{\pi^5}{4}\left[\alpha^i\alpha^j\frac{\pi}{4}-\frac{1}{2}\left(\alpha^i+\alpha^j\right)\right]}\\&=\left[\frac{5\left[(3\alpha^j\pi-4)(\alpha^j\pi-4)+4\upsilon\left(\alpha^j-\alpha^i\right)\right]}{\pi^2\left[\alpha^j\left(\alpha^i\pi-4\right)+2\left(\alpha^j-\alpha^i\right)\right]}\right]\Delta\left(q_b^i\right)_{\pi>\upsilon}\end{aligned}$$

The previous expression is positive under the assumption that platform 1 is more efficient in developing attributes on buyers' side than platform 2, $\alpha^2>\alpha^1$; and for τ values greater than $\frac{\pi}{2}$ (Assumption 2.1); and under Assumption 2.2 which turns to $\alpha^i>\frac{4}{\pi}$ when $\tau=\frac{\pi}{2}$.

Therefore, $\frac{\partial \Delta \Pi^i_{\pi > \upsilon}}{\partial \tau} > 0$.

$$\frac{\partial \Delta \Pi^i_{\pi>\upsilon}}{\partial \pi} = \frac{1}{\sigma_\pi^2}\Bigg[\sigma_\pi\left[\frac{\partial \Delta\left(q_b^i\right)_{\pi>\upsilon}}{\partial \pi}\left(3\tau+\pi\right)\left(2\tau-\pi\right) - \left(\tau+2\pi\right)\Delta\left(q_b^i\right)_{\pi>\upsilon}\right]$$
$$+4\pi\left(3\tau+\pi\right)\left(2\tau-\pi\right)\Delta\left(q_b^i\right)_{\pi>\upsilon}\Bigg] - \frac{1}{4\sigma_\pi^4\left[\alpha^i\alpha^j\sigma_\pi - \left(\alpha^i+\alpha^j\right)\tau\right]^2}\Bigg[\sigma_\pi^2\Big[\frac{\partial \Delta\left(q_b^i\right)_{\pi>\upsilon}}{\partial \pi}$$
$$\left(3\tau+\pi\right)\left(2\tau-\pi\right)\left[\alpha^i\alpha^j\sigma_\pi^2 - 4\tau^2\right] - \Delta\left(q_b^i\right)_{\pi>\upsilon}\left[\alpha^i\alpha^j\sigma_\pi^2 - 4\tau^2\right]\left(\tau+2\pi\right)$$
$$-8\alpha^i\alpha^j\pi\sigma_\pi\tau\Delta\left(q_b^i\right)_{\pi>\upsilon}\left(3\tau+\pi\right)\left(2\tau-\pi\right)\Big]\left[\alpha^i\alpha^j\sigma_\pi - \left(\alpha^i+\alpha^j\right)\tau\right]$$
$$+4\pi\sigma_\pi\Big[\left[\alpha^i\alpha^j\sigma_\pi^2 - 4\tau^2\right]\left(3\tau+\pi\right)\left(2\tau-\pi\right)\Delta\left(q_b^i\right)_{\pi>\upsilon}\Big]\left[3\alpha^i\alpha^j\sigma_\pi - 2\left(\alpha^i+\alpha^j\right)\tau\right]\Bigg]$$

Next, we evaluate the partial derivative when $\tau = \frac{\pi}{2}$ getting:

$$\left.\frac{\partial \Delta \Pi^i_{\pi>\upsilon}}{\partial \pi}\right|_{\tau=\frac{\pi}{2}} = -\frac{1}{\sigma_\pi}\left(\tau+2\pi\right)\Delta\left(q_b^i\right)_{\upsilon>\pi} + \frac{\Delta\left(q_b^i\right)_{\pi>\upsilon}\left[\alpha^i\alpha^j\sigma_\pi^2 - 4\tau^2\right]\left(\tau+2\pi\right)}{4\sigma_\pi^2\left[\alpha^i\alpha^j\sigma_\pi - \left(\alpha^i+\alpha^j\right)\tau\right]}$$
$$= -\frac{\left(\tau+2\pi\right)\Delta\left(q_b^i\right)_{\pi>\upsilon}}{\sigma_\pi}\left[1 - \frac{\left[\alpha^i\alpha^j\sigma_\pi^2 - 4\tau^2\right]}{4\sigma_\pi\left[\alpha^i\alpha^j\sigma_\pi - \left(\alpha^i+\alpha^j\right)\tau\right]}\right]$$

The right side of the previous expression is the same as in $\frac{\partial \Delta \Pi^i_{\pi>\upsilon}}{\partial \tau}$, consequently we get

$$= -\frac{\left(\tau+2\pi\right)\Delta\left(q_b^i\right)_{\pi>\upsilon}}{\sigma_\pi}\left[\frac{\left(3\alpha^j\sigma_\pi - 2\tau\right)\left(\alpha^i\sigma_\pi - 2\tau\right) + 2\sigma_\pi\tau\left(\alpha^j - \alpha^i\right)}{4\sigma_\pi\left[\alpha^i\alpha^j\sigma_\pi - \left(\alpha^i+\alpha^j\right)\tau\right]}\right]$$
$$\left.\frac{\partial \Delta \Pi^i_{\pi>\upsilon}}{\partial \upsilon}\right|_{\tau=\frac{\pi}{2}} = -\left[\frac{5\left[\left(3\alpha^j\pi - 4\right)\left(\alpha^j\pi - 4\right) + 4\pi\left(\alpha^j - \alpha^i\right)\right]}{2\pi^2\left[\alpha^j\left(\alpha^i\pi - 4\right) + 2\left(\alpha^j - \alpha^i\right)\right]}\right]\Delta\left(q_b^i\right)_{\pi>\upsilon}$$

The previous expression is positive under the assumption that platform 1 is more efficient in developing attributes on buyers' side than platform 2, $\alpha^2 > \alpha^1$; and for τ values greater than $\frac{\pi}{2}$; and under Assumption 2.2 which turns to $\alpha^i > \frac{4}{\pi}$ when $\tau = \frac{\pi}{2}$. Therefore, $\frac{\partial \Delta \Pi^i_{\pi>\pi}}{\partial \pi} < 0$.

□

Appendix C

Appendix: Chapter 3

C.1 Model Assumptions

In this section, we show how the model assumptions are defined.

First, to ensure a unique equilibrium where both platforms remain active, the second-order conditions of the platform maximisation problem must be satisfied. Specifically, the sufficient conditions required for the second-order conditions at stage two are detailed in Appendix C.2 and are (i) $1 > \pi\upsilon$ and (ii) $1 > \frac{\delta_p(\pi-\upsilon)^2|\Omega|}{2(1-\pi\upsilon)(1+\delta_p)\left[4-(\pi+\upsilon)^2\right]}$ if $4 > (\pi+\upsilon)^2$.

Next, we determine which of the two conditions $1 > \pi\upsilon$ and $4 > (\pi+\upsilon)^2$ is more stringent, ensuring the other is also met. To do this, we first set the left side of both inequalities equal and then compare the right sides to identify which one is greater. Transforming $4 > (\pi+\upsilon)^2$ into $1 > \frac{(\pi+\upsilon)^2}{4}$ and comparing it with $1 > \pi\upsilon$, we get $\frac{(\pi+\upsilon)^2}{4} > \pi\upsilon$. Simplifying this, we find $\pi^2 + 2\pi\upsilon + \upsilon^2 - 4\pi\upsilon > 0$, which further reduces to $(\pi-\upsilon)^2 > 0$. This inequality is positive as long as $\pi \neq \upsilon$. Therefore, if $4 > (\pi+\upsilon)^2$ holds, then $1 > \pi\upsilon$ is also satisfied.

Tu summarise the assumptions we are establishing are (i) $4 > (\pi+\upsilon)^2$ and (ii) $1 > \frac{\delta_p(\pi-\upsilon)^2|\Omega|}{2(1-\pi\upsilon)(1+\delta_p)\left[4-(\pi+\upsilon)^2\right]}$.

C.2 Maximisation Problem

Platform i, $i = 1, 2$ maximises the following expression with respect to both sides membership fees:

$$\max_{\{p_b^i, p_s^i\}} \Pi^i \equiv p_b^i \eta_{b,1}^i + p_s^i \eta_{s,1}^i + \delta_p \left(p_b^i \eta_{b,2}^i + p_s^i \eta_{s,2}^i \right)$$

First-order conditions:

$$\frac{\partial \Pi^i}{\partial p_b^i} = \frac{1}{2}(1+\delta_p) + \frac{\delta_p\left(q_b^i - q_b^j\right)}{4(1-\pi\upsilon)} + \frac{\Phi\left[p_b^j - 2p_b^i + \upsilon p_s^j - (\pi+\upsilon)p_s^i\right]}{4(1-\pi\upsilon)^2} = 0$$

$$\frac{\partial \Pi^i}{\partial p_s^i} = \frac{1}{2}(1+\delta_p) + \frac{\pi\delta_p\left(q_b^i - q_b^j\right)}{4(1-\pi\upsilon)} + \frac{\Phi\left[\pi p_b^j - (\pi+\upsilon)p_b^i\right]}{4(1-\pi\upsilon)^2} + \frac{\left[\Phi - \delta_p(1-\pi\upsilon)\Omega\right]\left[p_s^j - 2p_s^i\right]}{4(1-\pi\upsilon)^2} = 0$$

where $\Omega \equiv \Delta q_b^i + \Delta q_b^j + 2s_b$, $\Delta q_b^i = q_b^i - \frac{\bar{q}_b}{2}$, for $i = 1, 2$ and $\Phi \equiv 2(1-\pi\upsilon)(1+\delta_p) + \delta_p\Omega$

Based on the first-order conditions for both platforms, we obtain:

$$2Mp_b^i + (\pi+\upsilon)Mp_s^i - Mp_b^j - \upsilon Mp_s^j = X + \delta_p(1-\pi\upsilon)\left(q_b^i - q_b^j\right) \quad \text{(c1)}$$
$$(\pi+\upsilon)Mp_b^i + 2Np_s^i - \pi Mp_b^j - Np_s^j = X + \delta_p\pi(1-\pi\upsilon)\left(q_b^i - q_b^j\right) \quad \text{(c2)}$$
$$-Mp_b^i - \upsilon Mp_s^i + 2Mp_b^j + (\pi+\upsilon)Mp_s^j = X - \delta_p(1-\pi\upsilon)\left(q_b^j - q_b^i\right) \quad \text{(c3)}$$
$$-\pi Mp_b^i - Np_s^i + (\pi+\upsilon)Mp_b^j + 2Np_s^j = X - \delta_p\pi(1-\pi\upsilon)\left(q_b^j - q_b^i\right) \quad \text{(c4)}$$

where $M \equiv 2(1-\pi\upsilon)(1+\delta_p)+\delta_p\Omega$, $N \equiv 2(1-\pi\upsilon)(1+\delta_p)+\delta_p\pi\upsilon\Omega$, $X \equiv 2(1-\pi\upsilon)^2\big(1+\delta_p\big)$ and $\Omega \equiv \Delta q_b^i + \Delta q_b^j + 2s_b$.

Then, we solve for p_s^j in equation (c4) and substitute this expression into equations (c1), (c2) and (c3) to obtain:

$$M(4N-\pi\upsilon M)p_b^i + MN(2\pi+\upsilon)p_s^i - M[2N-\upsilon(\pi+\upsilon)M]p_b^j = (2N+\upsilon M)X + \delta_p(1-\pi\upsilon)(2N-\pi\upsilon M)\left(q_b^i - q_b^j\right) \quad \text{(c5)}$$

$$(\pi+2\upsilon)Mp_b^i + 3Np_s^i + (\upsilon-\pi)Mp_b^j = 3X + \delta_p\pi(1-\pi\upsilon)\left(q_b^i - q_b^j\right) \quad \text{(c6)}$$

$$-M[2N-\pi(\pi+\upsilon)M]p_b^i + (\pi-\upsilon)MNp_s^i + M\left[4N-(\pi+\upsilon)^2M\right]p_b^j = [2N-(\pi+\upsilon)M]X - \delta_p(1-\pi\upsilon)[2N-\pi(\pi+\upsilon)M]\left(q_b^i - q_b^j\right) \quad \text{(c7)}$$

Next, we solve for p_s^i in equation (c6) and substitute this expression into equation (c5) and (c7) to obtain:

$$M\left[6N-\left[(\pi+\upsilon)^2+2\pi\upsilon\right]M\right]p_b^i - M\left[3N-\left[(\pi+\upsilon)^2-\pi\upsilon\right]M\right]p_b^j = 3(N-\pi M)X + \delta_p(1-\pi\upsilon)[3N-\pi(\pi+2\upsilon)M]\left(q_b^i - q_b^j\right) \quad \text{(c8)}$$

$$-M\left[3N-\left[(\pi+\upsilon)^2-\pi\upsilon\right]M\right]p_b^i + M\left[6N-\left[(\pi+\upsilon)^2+2\pi\upsilon\right]M\right]p_b^j = 3(N-\pi M)X - \delta_p(1-\pi\upsilon)[3N-\pi(\pi+2\upsilon)M]\left(q_b^i - q_b^j\right) \quad \text{(c9)}$$

Next, we solve for p_b^j in equation (c9) and substitute this expression into equation (c8) to obtain:

$$p_b^i = \frac{3\left(N-\pi M\right)X}{\left[\left[6N-\left[\left(\pi+\upsilon\right)^2+2\pi\upsilon\right]M\right]-\left[3N-\left[\left(\pi+\upsilon\right)^2-\pi\upsilon\right]M\right]\right]M} + \frac{\delta_p\left(1-\pi\upsilon\right)\left[3N-\pi\left(\pi+2\upsilon\right)M\right]\left(q_b^i-q_b^j\right)}{\left[\left[6N-\left[\left(\pi+\upsilon\right)^2+2\pi\upsilon\right]M\right]+\left[3N-\left[\left(\pi+\upsilon\right)^2-\pi\upsilon\right]M\right]\right]M}$$

We manipulate the previous expression for p_b^i using the definitions $M\equiv 2\left(1-\pi\upsilon\right)\left(1+\delta_p\right)+\delta_p\Omega$, $N\equiv 2\left(1-\pi\upsilon\right)\left(1+\delta_p\right)+\delta_p\pi\upsilon\Omega$, $X\equiv 2\left(1-\pi\upsilon\right)^2\left(1+\delta_p\right)$ and $\Omega\equiv\Delta q_b^i+\Delta q_b^j+2s_b$, to transform it into Equation (3.12a):

$$p_b^i = 1-\pi-\frac{\delta_p\left(1-\pi\upsilon\right)\Omega}{2\left(1-\pi\upsilon\right)\left(1+\delta_p\right)+\delta_p\Omega}+ \frac{\delta_p\left(1-\pi\upsilon\right)\left[2\left(1-\pi\upsilon\right)\left(1+\delta_p\right)\left[3-\pi\left(\pi+2\upsilon\right)\right]+\delta_p\pi\left(\upsilon-\pi\right)\Omega\right]\left(q_b^i-q_b^j\right)}{2\left[2\left(1-\pi\upsilon\right)\left(1+\delta_p\right)+\delta_p\Omega\right]\left[\left(1-\pi\upsilon\right)\left(1+\delta_p\right)\sigma-\delta_p\left(\pi-\upsilon\right)^2\Omega\right]}$$

where $\Omega\equiv\Delta q_b^i+\Delta q_b^j+2s_b$, $\Delta q_b^i=q_b^i-\frac{\bar{q}_b}{2}$, for $i=1,2$.

Next, we solve for p_b^j in equation (c3) and substitute this expression into equations (c1), (c2) and (c4) to obtain:

$$3Mp_b^i+\left(2\pi+\upsilon\right)Mp_s^i+\left(\pi-\upsilon\right)Mp_s^j=3X+\delta_p\left(1-\pi\upsilon\right)\left(q_b^i-q_b^j\right) \quad \text{(c5.1)}$$

$$\left(\pi+2\upsilon\right)Mp_b^i+\left(4N-\pi\upsilon M\right)p_s^i-\left[2N-\pi\left(\pi+\upsilon\right)M\right]p_s^j = \left(2+\pi\right)X+\delta_p\pi\left(1-\pi\upsilon\right)\left(q_b^i-q_b^j\right) \quad \text{(c6.1)}$$

$$\left(\upsilon-\pi\right)Mp_b^i-\left[2N-\upsilon\left(\pi+\upsilon\right)M\right]p_s^i+\left[4N-\left(\pi+\upsilon\right)^2M\right]p_s^j = \left[2-\left(\pi+\upsilon\right)\right]X+\delta_p\left(1-\pi\upsilon\right)\left(\upsilon-\pi\right)\left(q_b^i-q_b^j\right) \quad \text{(c7.1)}$$

Next, we solve for p_b^i in equation (c5.1) and substitute this expression into equation (c6.1) and (c7.1) to obtain:

$$\left[6N-\left[\left(\pi+\upsilon\right)^2+2\pi\upsilon\right]M\right]p_s^i-\left[3N-\left[\left(\pi+\upsilon\right)^2-\pi\upsilon\right]M\right]p_s^j = 3\left(1-\upsilon\right)X+\delta_p\left(1-\pi\upsilon\right)\left(\pi-\upsilon\right)\left(q_b^i-q_b^j\right) \quad \text{(c8.1)}$$

$$-\left[3N-\left[(\pi+\upsilon)^2-\pi\upsilon\right]M\right]p_s^i+\left[6N-\left[(\pi+\upsilon)^2+2\pi\upsilon\right]M\right]p_s^j=$$
$$3\left(1-\upsilon\right)X+\delta_p\left(1-\pi\upsilon\right)\left(\upsilon-\pi\right)\left(q_b^i-q_b^j\right) \quad \text{(c9.1)}$$

Next, we solve for p_s^j in equation (c9.1) and substitute this expression into equation (c8.1) to obtain:

$$p_s^i=\frac{3\left(1-\upsilon\right)X}{\left[6N-\left[(\pi+\upsilon)^2+2\pi\upsilon\right]M\right]-\left[3N-\left[(\pi+\upsilon)^2-\pi\upsilon\right]M\right]}+$$
$$\frac{\delta_p\left(1-\pi\upsilon\right)\left(\pi-\upsilon\right)\left(q_b^i-q_b^j\right)}{\left[6N-\left[(\pi+\upsilon)^2+2\pi\upsilon\right]M\right]+\left[3N-\left[(\pi+\upsilon)^2-\pi\upsilon\right]M\right]}$$

We manipulate the previous expression for p_s^i using the definitions $M\equiv 2\left(1-\pi\upsilon\right)\left(1+\delta_p\right)+\delta_p\Omega$, $N\equiv 2\left(1-\pi\upsilon\right)\left(1+\delta_p\right)+\delta_p\pi\upsilon\Omega$, $X\equiv 2\left(1-\pi\upsilon\right)^2\left(1+\delta_p\right)$ and $\Omega\equiv\Delta q_b^i+\Delta q_b^j+2s_b$, to transform it into Equation (3.12b):

$$p_s^i=1-\upsilon+\frac{\delta_p\left(1-\pi\upsilon\right)\left(\pi-\upsilon\right)\left(q_b^i-q_b^j\right)}{2\left[\left(1-\pi\upsilon\right)\left(1+\delta_p\right)\sigma-\delta_p\left(\pi-\upsilon\right)^2\Omega\right]}$$

where $\Omega\equiv\Delta q_b^i+\Delta q_b^j+2s_b$, $\Delta q_b^i=q_b^i-\frac{\bar{q}_b}{2}$, for $i=1,2$ and $\sigma\equiv 9-\left(2\pi+\upsilon\right)\left(\pi+2\upsilon\right)$.

Next, we obtain the second-order conditions which define the following Hessian matrix:

$$\mathrm{H}=\begin{pmatrix}\Pi^i_{p_b^ip_b^i}\equiv\frac{\partial^2\Pi^i}{\partial(p_b^i)^2}=-\frac{2\Phi}{4(1-\pi\upsilon)^2} & \Pi^i_{p_b^ip_s^i}\equiv\frac{\partial^2\Pi^i}{\partial p_b^i\partial p_s^i}=-\frac{(\pi+\upsilon)\Phi}{4(1-\pi\upsilon)^2}\\ \Pi^i_{p_s^ip_b^i}\equiv\frac{\partial^2\Pi^i}{\partial p_s^i\partial p_b^i}=-\frac{(\pi+\upsilon)\Phi}{4(1-\pi\upsilon)^2} & \Pi^i_{p_s^ip_s^i}\equiv\frac{\partial^2\Pi^i}{\partial(p_s^i)^2}=-\frac{2[\Phi-\delta_p(1-\pi\upsilon)\Omega]}{4(1-\pi\upsilon)^2}\end{pmatrix}$$

where $\Omega\equiv\Delta q_b^i+\Delta q_b^j+2s_b$, $\Delta q_b^i=q_b^i-\frac{\bar{q}_b}{2}$, for $i=1,2$ and $\Phi\equiv 2\left(1-\pi\upsilon\right)\left(1+\delta_p\right)+\delta_p\Omega$.

To guarantee that platforms' profits reach a maximum with the equilibrium fees in Equations (3.12a) and (3.12b) it is sufficient for H to be negative definite, indicating that $|H|>0$, and either $\Pi^i_{p_bp_b}<0$ or $\Pi^i_{p_sp_s}<0$. To show that $\Pi^i_{p_bp_b}$ and $\Pi^i_{p_sp_s}$ are negative, we need to examine the numerators 2Φ that becomes $2\left[2\left(1-\pi\upsilon\right)\left(1+\delta_p\right)+\delta_p\Omega\right]$ and $2\left[\Phi-\delta_p\left(1-\pi\upsilon\right)\Omega\right]$ that becomes $2\left[2\left(1-\pi\upsilon\right)\left(1+\delta_p\right)+\pi\upsilon\delta_p\Omega\right]$. Both expressions must be positive because their denominators are positive. Therefore, if $1>\pi\upsilon$ for $|\Omega|$ we can confirm that both expressions are positive, and we can guarantee both $\Pi^i_{p_bp_b}$ and $\Pi^i_{p_sp_s}$ to be negative. We are not imposing a constraint on platform's quality on buyers' side being either underestimated $q_b^i>\frac{\bar{q}_b}{2}$ or overestimated $q_b^i<\frac{\bar{q}_b}{2}$. Consequently, we set Ω to be in absolute value $|\Omega|$.

For $|H|>0$ we have $\frac{\Phi}{16(1-\pi\upsilon)^4}\left[4\left[\Phi-\delta_p\left(1-\pi\upsilon\right)\Omega\right]-\Phi\left(\pi+\upsilon\right)^2\right]>0$, which turns to $\frac{\Phi}{16(1-\pi\upsilon)^4}\left[2\left(1-\pi\upsilon\right)\left(1+\delta_p\right)\left[4-\left(\pi+\upsilon\right)^2\right]-\delta_p\left(\pi-\upsilon\right)^2\Omega\right]>0$ the previous expression is positive if $\left[2\left(1-\pi\upsilon\right)\left(1+\delta_p\right)\left[4-\left(\pi+\upsilon\right)^2\right]-\delta_p\left(\pi-\upsilon\right)^2\Omega\right]>0$, which is positive as long as $1 > \frac{\delta_p(\pi-\upsilon)^2|\Omega|}{2(1-\pi\upsilon)(1+\delta_p)\left[4-(\pi+\upsilon)^2\right]}$ and $4 > (\pi+\upsilon)^2$.

In summary, for the second-order conditions defined by the Hessian matrix to be negative definite, the following conditions must hold (i) $1 > \pi\upsilon$ and (ii) $1 > \frac{\delta_p(\pi-\upsilon)^2|\Omega|}{2(1-\pi\upsilon)(1+\delta_p)\left[4-(\pi+\upsilon)^2\right]}$ if $4 > (\pi+\upsilon)^2$.

C.3 Proof of Proposition 3.1 and Corollary 3.1

Proof. We prove Proposition 3.1 by partially differentiating buyers and sellers equilibrium membership fees with respect to Δq_b^i using Equations (3.12a) and (3.12b) under the assumption that $q_b^1 > q_b^2$.

$$\frac{\partial p_b^i}{\partial \Delta q_b^i} = -\frac{\delta_p(1-\pi\upsilon)\left[\Phi-\delta_p\Omega\right]}{\Phi^2} + \frac{\delta_p^2(1-\pi\upsilon)\left(q_b^i-q_b^j\right)}{2\Phi^2\Psi^2}\Bigg[\pi(\upsilon-\pi)\Phi\Psi - \left[2(1-\pi\upsilon)(1+\delta_p)\left[3-\pi(\pi+2\upsilon)\right]+\delta_p\pi(\upsilon-\pi)\Omega\right]\left[\Psi-(\pi-\upsilon)^2\Phi\right]\Bigg]$$

where $\Omega \equiv \Delta q_b^i + \Delta q_b^j + 2s_b$, $\Delta q_b^i = q_b^i - \frac{\bar{q}_b}{2}$, for $i = 1, 2$ and $\sigma \equiv 9 - (2\pi+\upsilon)(\pi+2\upsilon)$ $\Phi \equiv 2(1-\pi\upsilon)(1+\delta_p) + \delta_p\Omega$ and $\Psi \equiv (1-\pi\upsilon)(1+\delta_p)\sigma - \delta_p(\pi-\upsilon)^2\Omega$.

Firstly, we simplify the numerator of the first element of the partial derivative $-\delta_p\big(1-\pi\upsilon\big)\big[\Phi-\delta_p\Omega\big]$ using the definitions $\Phi \equiv 2(1-\pi\upsilon)(1+\delta_p)+\delta_p\Omega$ and $\Psi \equiv (1-\pi\upsilon)\big(1+\delta_p\big)\sigma - \delta_p(\pi-\upsilon)^2\Omega$ to transform it into $-2\delta_p(1-\pi\upsilon)^2(1+\delta_p)$.

Next, we simplify the second element of the partial derivative $\pi(\upsilon-\pi)\Phi\Psi - \big[2\big(1-\pi\upsilon\big)\big(1+\delta_p\big)\left[3-\pi(\pi+2\upsilon)\right]+\delta_p\pi(\upsilon-\pi)\Omega\big]\left[\Psi-(\pi-\upsilon)^2\Phi\right]$ using the same definitions of Φ and Ψ as before. It turns to $\pi(\upsilon-\pi)\Phi\Psi - 2(1-\pi\upsilon)(1+\delta_p)\left[3-\pi(\pi+2\upsilon)\right]\Psi$ $-\delta_p\pi\big(\upsilon-\pi\big)\Omega\Psi + 2(1-\pi\upsilon)(1+\delta_p)\left[3-\pi(\pi+2\upsilon)\right](\pi-\upsilon)^2\Phi + \delta_p\pi(\upsilon-\pi)(\pi-\upsilon)^2\Omega\Phi$ and then it simplifies to $\pi(\upsilon-\pi)\Psi\left[\Phi-\delta_p\Omega\right] - 2(1-\pi\upsilon)(1+\delta_p)\left[3-\pi(\pi+2\upsilon)\right]\Psi + 2\big(1-\pi\upsilon\big)(1+\delta_p)\left[3-\pi(\pi+2\upsilon)\right](\pi-\upsilon)^2\Phi + \delta_p\pi(\upsilon-\pi)(\pi-\upsilon)^2\Omega\Phi$, which simplifies to $-2(1-\pi\upsilon)(1+\delta_p)\Psi\left[3-\pi(\pi+2\upsilon)-\pi(\upsilon-\pi)\right] + 2(1-\pi\upsilon)(1+\delta_p)\big[3-\pi(\pi+2\upsilon)\big]\big(\pi-\upsilon\big)^2\Phi + \delta_p\pi(\upsilon-\pi)(\pi-\upsilon)^2\Omega\Phi$ and finally simplifies to $-6(1-\pi\upsilon)^2(1+\delta_p)\Psi + 2\big(1-\pi\upsilon\big)(1+\delta_p)\big[3-\pi(\pi+2\upsilon)\big]\big(\pi-\upsilon\big)^2\Phi + \delta_p\pi(\upsilon-\pi)(\pi-\upsilon)^2\Omega\Phi$. Therefore the partial derivative turns to:

$$\frac{\partial p_b^i}{\partial \Delta q_b^i} = -\frac{\delta_p(1-\pi\upsilon)^2(1+\delta_p)}{\Phi^2} - \frac{\delta_p^2(1-\pi\upsilon)}{2\Phi^2\Psi^2}\Bigg[6(1-\pi\upsilon)^2(1+\delta_p)\Psi - 2(1-\pi\upsilon)(1+\delta_p)\left[3-\pi(\pi+2\upsilon)\right]\big(\pi-\upsilon\big)^2\Phi + \delta_p\pi(\pi-\upsilon)^3\Omega\Phi\Bigg]\left(q_b^i-q_b^j\right)$$

Next, if both elements of the partial derivative are positive, then $\partial p_b^i / \partial \Delta q_b^i$ is negative. The first element $\frac{\delta_p(1-\pi\upsilon)^2(1+\delta_p)}{\Phi^2}$ is entirely positive. For the second element $\frac{\delta_p^2(1-\pi\upsilon)}{2\Phi^2\Psi^2}\Big[6(1-\pi\upsilon)^2(1+\delta_p)\Psi - 2(1-\pi\upsilon)(1+\delta_p)[3-\pi(\pi+2\upsilon)]\big(\pi-\upsilon\big)^2\Phi + \delta_p\pi\big(\pi-\upsilon\big)^3\Omega\Phi\Big]$ to be positive, it is sufficient to have $1 > \pi\upsilon$, $\Psi > 0$, $2(1-\pi\upsilon)(1+\delta_p)\big[3-\pi(\pi+2\upsilon)\big]\big(\pi-\upsilon\big)^2\Phi < 0$, $\pi > \upsilon$.

Condition $1 > \pi\upsilon$ is satisfied if Assumption 3.1 holds. This means that if the right side of Assumption 3.1 is greater than the right side of $1 > \pi\upsilon$, the condition is met. Specifically, $\frac{(\pi+\upsilon)^2}{4} > \pi\upsilon$, simplifies to $(\pi-\upsilon)^2 > 0$ which holds as long as $\pi \neq \upsilon$.

Condition $\Psi > 0$ turns to $(1-\pi\upsilon)(1+\delta_p)\sigma - \delta_p(\pi-\upsilon)^2\Omega > 0$. We use Assumption 3.2 to show it is satisfied. First, we make the left side on both inequalities equal and compare the right side. If the right side of Assumption 3.2 is greater, the condition is satisfied. This condition becomes $1 > \frac{\delta_p(\pi-\upsilon)^2\Omega}{(1-\pi\upsilon)(1+\delta_p)\sigma}$. Next, we compare the right side of both conditions and get: $\frac{\delta_p(\pi-\upsilon)^2|\Omega|}{2(1-\pi\upsilon)(1+\delta_p)\left[4-(\pi+\upsilon)^2\right]} > \frac{\delta_p(\pi-\upsilon)^2\Omega}{(1-\pi\upsilon)(1+\delta_p)\sigma}$, which turns to $\sigma - 2\left[4-(\pi+\upsilon)^2\right] > 0$ and then turns to $1 > 2\pi^2 + 5\pi\upsilon + 2\upsilon^2 - 2\pi^2 - 4\pi\upsilon - 2\upsilon^2 > 0$ and finally simplifies to $1 > \pi\upsilon$. As previously shown, this condition is satisfied.

Finally, condition $2(1-\pi\upsilon)(1+\delta_p)\big[3-\pi(\pi+2\upsilon)\big]\big(\pi-\upsilon\big)^2\Phi < 0$ is satisfied if $3-\pi(\pi+2\upsilon) < 0$, which we transform to $1 < \frac{\pi(\pi+2\upsilon)}{3}$. This condition is compatible with Assumption 3.1. Specifically, $\frac{\pi(\pi+\upsilon)^2}{4} < 1 < \frac{\pi(\pi+2\upsilon)}{3}$ holds as $\frac{\pi(\pi+2\upsilon)}{3} > \frac{\pi(\pi+\upsilon)^2}{4}$, which further turns to $\pi^2 + 2\pi\upsilon - 3\upsilon^2 > 0$ and finally simplifies to $(\pi+3\upsilon)(\pi-\upsilon) > 0$, which is positive whenever $\pi > \upsilon$.

Therefore, when buyers underestimate platform i's quality $\Delta q_b^i > 0$, $\frac{\partial p_b^i}{\partial \Delta q_b^i} < 0$ if $1 < \frac{\pi(\pi+2\upsilon)}{3}$ and $\pi > \upsilon$.

For sellers' equilibrium membership fee, we partially differentiate Equation (3.12b)

$$\frac{\partial p_s^i}{\partial \Delta q_b^i} = \frac{\delta_p^2(1-\pi\upsilon)(\pi-\upsilon)^3\left(q_b^i - q_b^j\right)}{2\Psi^2}$$

where $\Omega \equiv \Delta q_b^i + \Delta q_b^j + 2s_b$, $\Delta q_b^i = q_b^i - \frac{\bar{q}_b}{2}$, for $i = 1,2$ and $\sigma \equiv 9 - (2\pi+\upsilon)(\pi+2\upsilon)$ and $\Psi \equiv (1-\pi\upsilon)(1+\delta_p)\sigma - \delta_p(\pi-\upsilon)^2\Omega$.

As was shown previously in $\frac{\partial p_b^i}{\partial \Delta q_b^i}$, conditions $1 > \pi\upsilon$ and $\Psi > 0$ are satisfied as long as Assumption 3.1 and Assumption 3.2 hold.

Therefore, when buyers underestimate platform i's quality $\Delta q_b^i > 0$, $\frac{\partial p_s^i}{\partial \Delta q_b^i} > 0$ if $\pi > \upsilon$ and $\frac{\partial p_s^i}{\partial \Delta q_b^i} < 0$ if $\upsilon > \pi$. □

C.4 Proof of Proposition 3.2

Proof. We prove Proposition 3.2 by partially differentiating buyers and sellers equilibrium membership fees with respect to s_b using Equations (3.12a) and (3.12b) under the assumption that $q_b^1 > q_b^2$.

$$\frac{\partial p_b^i}{\partial s_b} = -\frac{4\delta_p (1-\pi\upsilon)[\Phi - \delta_p \Omega]}{\Phi^2} + \frac{\delta_p^2 (1-\pi\upsilon)\left(q_b^i - q_b^j\right)}{\Phi^2\Psi^2}\Bigg[\pi(\upsilon-\pi)\Phi\Psi - \\ [2(1-\pi\upsilon)(1+\delta_p)[3-\pi(\pi+2\upsilon)] + \delta_p\pi(\upsilon-\pi)\Omega]\left[\Psi - (\pi-\upsilon)^2\Phi\right]\Bigg]$$

where $\Omega \equiv \Delta q_b^i + \Delta q_b^j + 2s_b$, $\Delta q_b^i = q_b^i - \frac{\bar{q}_b}{2}$, for $i = 1, 2$ and $\sigma \equiv 9 - (2\pi+\upsilon)(\pi+2\upsilon)$ $\Phi \equiv 2(1-\pi\upsilon)(1+\delta_p) + \delta_p\Omega$ and $\Psi \equiv (1-\pi\upsilon)(1+\delta_p)\sigma - \delta_p(\pi-\upsilon)^2\Omega$.

The partial derivative is similar to $\frac{\partial p_b^i}{\partial \Delta q_b^i}$ which was computed in Appendix C.3 but with the first element multiplied by four and the second element multiplied by two. Thus, we have:

$$\frac{\partial p_b^i}{\partial s_b} = -\frac{4\delta_p (1-\pi\upsilon)^2(1+\delta_p)}{\Phi^2} - \frac{\delta_p^2(1-\pi\upsilon)}{\Phi^2\Psi^2}\Bigg[6(1-\pi\upsilon)^2(1+\delta_p)\Psi \\ -2(1-\pi\upsilon)(1+\delta_p)[3-\pi(\pi+2\upsilon)]\left(\pi-\upsilon\right)^2\Phi + \delta_p\pi(\pi-\upsilon)^3\Omega\Phi\Bigg]\left(q_b^i - q_b^j\right)$$

As demonstrated in Appendix C.3, the conditions $1 > \pi\upsilon$, $\Psi > 0$ and $3 - \pi(\pi+2\upsilon) < 0$ are satisfied. Therefore, when buyers underestimate platform i's quality $\Delta q_b^i > 0$, $\frac{\partial p_b^i}{\partial s_b} < 0$ if $1 < \frac{\pi(\pi+2\upsilon)}{3}$, $\pi > \upsilon$.

For sellers' equilibrium membership fee, we partially differentiate Equation (3.12b):

$$\frac{\partial p_s^i}{\partial s_b} = \frac{\delta_p^2(1-\pi\upsilon)(\pi-\upsilon)^3\left(q_b^i - q_b^j\right)}{\Psi^2} < 0, \quad \text{if} \quad \pi > \upsilon \quad \text{and} \quad \frac{\partial p_s^i}{\partial s_b} > 0, \quad \text{if} \quad \upsilon > \pi$$

where $\Omega \equiv \Delta q_b^i + \Delta q_b^j + 2s_b$, $\Delta q_b^i = q_b^i - \frac{\bar{q}_b}{2}$, for $i = 1, 2$ and $\sigma \equiv 9 - (2\pi+\upsilon)(\pi+2\upsilon)$ and $\Psi \equiv (1-\pi\upsilon)(1+\delta_p)\sigma - \delta_p(\pi-\upsilon)^2\Omega$.

As was shown in Appendix C.3 that $\frac{\partial p_b^i}{\partial \Delta q_b^i}$, conditions $1 > \pi\upsilon$ and $\Psi > 0$ are satisfied as long as Assumption 3.1 and Assumption 3.2 hold.

Therefore, when buyers underestimate platform i's quality $\Delta q_b^i > 0$, $\frac{\partial p_s^i}{\partial s_b} > 0$ if $\pi > \upsilon$ and $\frac{\partial p_s^i}{\partial s_b} < 0$ if $\upsilon > \pi$. □

C.5 Equilibrium Market-Shares

We obtain first-period equilibrium market shares in Equations (3.10a) and (3.10b) using equilibrium membership fees in Equations (3.12a) and (3.12b).

For buyers' side

$$\eta_{b,1}^i = \frac{1}{2} - \frac{\delta_p}{2\Phi\Psi}\Bigg[2\left(1-\pi\upsilon\right)\left(1+\delta_p\right)\left[3-\pi\left(\pi+2\upsilon\right)\right] + \delta_p \pi\left(\upsilon-\pi\right)\Omega + 2\upsilon\left(\pi-\upsilon\right)\left(1-\pi\upsilon\right)\left(1+\delta_p\right) + \delta_p\upsilon\left(\pi-\upsilon\right)\Omega\Bigg]\left(q_b^i - q_b^j\right)$$

where $\Omega \equiv \Delta q_b^i + \Delta q_b^j + 2s_b$, $\Delta q_b^i = q_b^i - \frac{\bar{q}_b}{2}$, for $i = 1,2$ and $\sigma \equiv 9 - (2\pi+\upsilon)(\pi+2\upsilon)$ $\Phi \equiv 2\left(1-\pi\upsilon\right)\left(1+\delta_p\right) + \delta_p\Omega$ and $\Psi \equiv \left(1-\pi\upsilon\right)\left(1+\delta_p\right)\sigma - \delta_p\left(\pi-\upsilon\right)^2\Omega$.

Next, we simplify expression $2\left(1-\pi\upsilon\right)\left(1+\delta_p\right)\left[3-\pi\left(\pi+2\upsilon\right)\right] + \delta_p\pi\left(\upsilon-\pi\right)\Omega + 2\upsilon\left(\pi-\upsilon\right)\left(1-\pi\upsilon\right)\left(1+\delta_p\right) + \delta_p\upsilon\left(\pi-\upsilon\right)\Omega$. This turns to $2\left(1-\pi\upsilon\right)\left(1+\delta_p\right)\big[3-\pi\big(\pi+2\upsilon\big)\big] + 2\upsilon\left(\pi-\upsilon\right)\left(1-\pi\upsilon\right)\left(1+\delta_p\right) - \delta_p\left(\pi-\upsilon\right)^2\Omega$. Then using definition $\Psi \equiv \big(1-\pi\upsilon\big)\big(1+\delta_p\big)\sigma - \delta_p\left(\pi-\upsilon\right)^2\Omega$ we get $\Psi + 2\left(1-\pi\upsilon\right)\left(1+\delta_p\right)\left[3-\pi\left(\pi+2\upsilon\right)\right] - \left(1-\pi\upsilon\right)\left(1+\delta_p\right)\sigma + 2\upsilon\left(\pi-\upsilon\right)\left(1-\pi\upsilon\right)\left(1+\delta_p\right)$. It then simplifies to $\Psi - \left(1-\pi\upsilon\right)\left(1+\delta_p\right)\big[\sigma - 6 + 2\pi\big(\pi+2\upsilon\big) - 2\upsilon\left(\pi-\upsilon\right)\big]$. Finally, it simplifies to $\Psi - 3\left(1-\pi\upsilon\right)^2\left(1+\delta_p\right)$. Then $\eta_{b,1}^i$ turns to:

$$\eta_{b,1}^i = \frac{1}{2} - \frac{\delta_p}{2\Phi\Psi}\left[\Psi - 3\left(1-\pi\upsilon\right)^2\left(1+\delta_p\right)\right]\left(q_b^i - q_b^j\right)$$

For sellers' side, we use Equation (3.9b) to get:

$$\eta_{s,1}^i = \frac{1}{2} + \frac{\pi\left(2\eta_{b,1}^i - 1\right) + \left(p_s^j - p_s^i\right)}{2}$$

$$= \frac{1}{2} + \pi\left[\frac{1}{2} - \frac{\delta_p\left[\Psi - 3\left(1-\pi\upsilon\right)^2\left(1+\delta_p\right)\right]\left(q_b^i - q_b^j\right)}{2\Phi\Psi}\right] - \frac{\pi}{2} - \frac{\delta_p\left(1-\pi\upsilon\right)\left(\pi-\upsilon\right)\left(q_b^i - q_b^j\right)}{2\Psi}$$

$$\eta_{s,1}^i = \frac{1}{2} - \frac{\delta_p}{2\Phi\Psi}\left[\pi\left[\Psi - 3\left(1-\pi\upsilon\right)^2\left(1+\delta_p\right)\right] + \left(1-\pi\upsilon\right)\left(\pi-\upsilon\right)\Phi\right]\left(q_b^i - q_b^j\right)$$

Next, we obtain second-period equilibrium market shares in Equations (3.11a) and (3.11b) using equilibrium membership fees in Equations (3.12a) and (3.12b).

For buyers' side, we first transform Equation (3.11a) to obtain:

$$\eta_{b,2}^i = \frac{1}{2} + \frac{q_b^i - q_b^j}{4\left(1-\pi\upsilon\right)} - \frac{\left[2\left(1-\pi\upsilon\right) + \Omega\right]}{2\left(1-\pi\upsilon\right)}\left[\frac{\left(p_b^i - p_b^j\right) + \upsilon\left(p_s^i - p_s^j\right)}{2\left(1-\pi\upsilon\right)}\right]$$

Using Equation (3.10a) $\eta_{b,1}^i = \frac{1}{2} + \frac{(p_b^j - p_b^i) + \upsilon(p_s^j - p_s^i)}{2(1-\pi\upsilon)}$ we obtain:

$$\eta_{b,2}^i = \frac{1}{2} + \frac{q_b^i - q_b^j}{4(1-\pi\upsilon)} + \frac{[2(1-\pi\upsilon)+\Omega]}{2(1-\pi\upsilon)}\left[\eta_{b,1}^i - \frac{1}{2}\right]$$

$$= \frac{1}{2} + \frac{q_b^i - q_b^j}{4(1-\pi\upsilon)} - \frac{[2(1-\pi\upsilon)+\Omega]}{2(1-\pi\upsilon)}\left[\frac{\delta_p}{2\Phi\Psi}\left[\Psi - 3(1-\pi\upsilon)^2(1+\delta_p)\right]\left(q_b^i - q_b^j\right)\right]$$

$$\eta_{b,2}^i = \frac{1}{2} + \frac{\left[\Phi\Psi - \delta_p[2(1-\pi\upsilon)+\Omega]\left[\Psi - 3(1-\pi\upsilon)^2(1+\delta_p)\right]\right]}{4(1-\pi\upsilon)\Phi\Psi}\left(q_b^i - q_b^j\right)$$

where $\Omega \equiv \Delta q_b^i + \Delta q_b^j + 2s_b$, $\Delta q_b^i = q_b^i - \frac{\bar{q}_b}{2}$, for $i = 1, 2$ and $\sigma \equiv 9 - (2\pi+\upsilon)(\pi+2\upsilon)$ $\Phi \equiv 2(1-\pi\upsilon)(1+\delta_p) + \delta_p\Omega$ and $\Psi \equiv (1-\pi\upsilon)(1+\delta_p)\sigma - \delta_p(\pi-\upsilon)^2\Omega$.

Next, we simplify expression $\Phi\Psi - \delta_p[2(1-\pi\upsilon)+\Omega]\left[\Psi - 3(1-\pi\upsilon)^2(1+\delta_p)\right]$. This turns to $\Phi\Psi - [2(1-\pi\upsilon)\delta_p + \delta_p\Omega]\left[\Psi - 3(1-\pi\upsilon)^2(1+\delta_p)\right]$. Then using definition $\Phi \equiv 2(1-\pi\upsilon)(1+\delta_p) + \delta_p\Omega$, we obtain $\Phi\Psi - \left[2(1-\pi\upsilon)\delta_p + 2(1-\pi\upsilon)(1+\delta_p) - 2(1-\pi\upsilon)\left(1+\delta_p\right) + \delta_p\Omega\right]\left[\Psi - 3(1-\pi\upsilon)^2(1+\delta_p)\right]$. It then simplifies to $\Phi\Psi - \left[\Phi - 2(1-\pi\upsilon)\right]\left[\Psi - 3(1-\pi\upsilon)^2(1+\delta_p)\right]$. Then, turns to $\Phi\Psi - \Phi\left[\Psi - 3(1-\pi\upsilon)^2(1+\delta_p)\right] + 2(1-\pi\upsilon)\left[\Psi - 3(1-\pi\upsilon)^2(1+\delta_p)\right]$. Finally, simplifies to $(1-\pi\upsilon)\left[2\left[\Psi - 3\left(1-\pi\upsilon\right)^2\left(1+\delta_p\right)\right] + 3(1-\pi\upsilon)(1+\delta_p)\Phi\right]$. Then $\eta_{b,2}^i$ turns to:

$$\eta_{b,2}^i = \frac{1}{2} + \frac{1}{4\Phi\Psi}\left[2\left[\Psi - 3\left(1-\pi\upsilon\right)^2\left(1+\delta_p\right)\right] + 3(1-\pi\upsilon)(1+\delta_p)\Phi\right]\left(q_b^i - q_b^j\right)$$

For sellers' side we use Equation (3.7) to obtain:

$$\eta_{s,2}^i = \frac{1}{2} + \frac{1}{4\Phi\Psi}\Big[2\pi\left[\Psi - 3\left(1-\pi\upsilon\right)^2\left(1+\delta_p\right)\right] + (1-\pi\upsilon)\Phi\big[3\pi(1+\delta_p) - 2\delta_p(\pi-\upsilon)\big]\Big]\left(q_b^i - q_b^j\right)$$

$$= \frac{1}{2} + \frac{1}{4\Phi\Psi}\left[2\pi\left[\Psi - 3\left(1-\pi\upsilon\right)^2\left(1+\delta_p\right)\right] + (1-\pi\upsilon)\Phi[3\pi + \delta_p(\pi+2\upsilon)]\right]\left(q_b^i - q_b^j\right)$$

$$= \frac{1}{2} + \frac{1}{4\Phi\Psi}\Big[\pi[2\Psi + 3(1-\pi\upsilon)[\Phi - 2(1-\pi\upsilon)(1+\delta_p)]] + \delta_p(1-\pi\upsilon)(\pi+2\upsilon)\Phi\Big]\left(q_b^i - q_b^j\right)$$

$$\eta_{s,2}^i = \frac{1}{2} + \frac{1}{4\Phi\Psi}\left[\pi\left[2\Psi + 3\delta_p(1-\pi\upsilon)\Omega\right] + \delta_p(1-\pi\upsilon)(\pi+2\upsilon)\Phi\right]\left(q_b^i - q_b^j\right)$$

C.6 Proof of Observation 3.1

Proof. We prove Observation 3.1 by partially differentiating buyers' and sellers' equilibrium market shares at periods 1 and 2 with respect to Δq_b^i under the assumption that

$q_b^1 > q_b^2$.

For buyers' first-period equilibrium market shares, we partially differentiate Equation (3.13a): First, we simplify $\Psi - 3\left(1-\pi\upsilon\right)^2\left(1+\delta_p\right)$ from $\eta_{b,1}^i = \frac{1}{2} - \frac{\delta_p}{2\Phi\Psi}\Big[\Psi - 3\big(1-\pi\upsilon\big)^2\big(1+\delta_p\big)\Big]\left(q_b^i - q_b^j\right)$ into: $\Psi - 3\left(1-\pi\upsilon\right)^2\left(1+\delta_p\right) = \left(1-\pi\upsilon\right)\left(1+\delta_p\right)\left[\sigma - 3\left(1-\pi\upsilon\right)\right] - \delta_p\left(\pi-\upsilon\right)^2\Omega = \left(1-\pi\upsilon\right)\left(1+\delta_p\right)\left[6\left(1-\pi\upsilon\right) - 2\left(\pi-\upsilon\right)^2\right] - \delta_p\left(\pi-\upsilon\right)^2\Omega = 6\big(1-\pi\upsilon\big)^2\big(1-\pi\upsilon\big) - \left(\pi-\upsilon\right)^2\left[2\left(1-\pi\upsilon\right)\left(1+\delta_p\right) + \delta_p\Omega\right] = 6\left(1-\pi\upsilon\right)^2\left(1-\pi\upsilon\right) - \left(\pi-\upsilon\right)^2\Phi$. Then $\eta_{b,1}^i$ turns to $\eta_{b,1}^i = \frac{1}{2} - \frac{\delta_p}{2\Phi\Psi}\Big[6\left(1-\pi\upsilon\right)^2\left(1-\pi\upsilon\right) - \left(\pi-\upsilon\right)^2\Phi\Big]\left(q_b^i - q_b^j\right)$

$$\frac{\partial \eta_{b,1}^i}{\partial \Delta q_b^i} = \frac{\delta_p^2}{2\Phi^2\Psi^2}\Bigg[\left(\pi-\upsilon\right)^2\Phi\Psi + \Big[6\left(1-\pi\upsilon\right)^2\left(1+\delta_p\right) - 2\left(1-\pi\upsilon\right)\left(1+\delta_p\right)\left(\pi-\upsilon\right)^2 - \delta_p\left(\pi-\upsilon\right)^2\Omega\Big]\left[\Psi - \left(\pi-\upsilon\right)^2\Phi\right]\Bigg]\left(q_b^i - q_b^j\right)$$

where $\Omega \equiv \Delta q_b^i + \Delta q_b^j + 2s_b$, $\Delta q_b^i = q_b^i - \frac{\bar{q}_b}{2}$, for $i = 1,2$ and $\sigma \equiv 9 - \left(2\pi+\upsilon\right)\left(\pi+2\upsilon\right)$ $\Phi \equiv 2\left(1-\pi\upsilon\right)\left(1+\delta_p\right) + \delta_p\Omega$ and $\Psi \equiv \left(1-\pi\upsilon\right)\left(1+\delta_p\right)\sigma - \delta_p\left(\pi-\upsilon\right)^2\Omega$.

We simplify expression $\left(\pi-\upsilon\right)^2\Phi\Psi + \left[6\left(1-\pi\upsilon\right)^2\left(1+\delta_p\right) - 2\left(1-\pi\upsilon\right)\big(1+\delta_p\big)\big(\pi-\upsilon\big)^2 - \delta_p\left(\pi-\upsilon\right)^2\Omega\right]\left[\Psi - \left(\pi-\upsilon\right)^2\Phi\right]$ using the definitions for Φ and Ψ to get:

$$\begin{aligned}
&= \left(\pi-\upsilon\right)^2\Psi\left[2\left(1-\pi\upsilon\right)\left(1+\delta_p\right) + \delta_p\Omega - 2\left(1-\pi\upsilon\right)\left(1+\delta_p\right) - \delta_p\Omega\right] + 6\left(1-\pi\upsilon\right)^2\left(1+\delta_p\right)\Psi \\
&\quad + 2\left(1-\pi\upsilon\right)\left(1+\delta_p\right)\left(\pi-\upsilon\right)^2\Phi\left[\left(\pi-\upsilon\right)^2 - 3\left(1-\pi\upsilon\right)\right] + \delta_p\left(\pi-\upsilon\right)^4\Phi\Omega \\
&\quad = 6\left(1-\pi\upsilon\right)^2\left(1+\delta_p\right)\Psi + \delta_p\left(\pi-\upsilon\right)^4\Phi\Omega \ + \\
&\qquad 2\left(1-\pi\upsilon\right)\left(1+\delta_p\right)\left(\pi-\upsilon\right)^2\Phi\left[-3 + \left(\pi^2 + \pi\upsilon + \upsilon^2\right)\right]
\end{aligned}$$

Next, for the expression $-3 + \left(\pi^2 + \pi\upsilon + \upsilon^2\right)$ to be positive we require $3 < \left(\pi^2 + \pi\upsilon + \upsilon^2\right)$. This condition is compatible with Assumption 3.1, as $\frac{\left(\pi+\upsilon\right)^2}{4} < 1 < \frac{\pi^2+\pi\upsilon+\upsilon^2}{3}$. Then $\frac{\partial \eta_{b,1}^i}{\partial \Delta q_b^i}$ turns to:

$$\frac{\partial \eta_{b,1}^i}{\partial \Delta q_b^i} = \frac{\delta_p^2}{2\Phi^2\Psi^2}\Bigg[6\left(1-\pi\upsilon\right)^2\left(1+\delta_p\right)\Psi + \delta_p\left(\pi-\upsilon\right)^4\Phi\Omega \ + 2\left(1-\pi\upsilon\right)\left(1+\delta_p\right)\left(\pi-\upsilon\right)^2\Phi\left[-3 + \left(\pi^2 + \pi\upsilon + \upsilon^2\right)\right]\Bigg]\left(q_b^i - q_b^j\right) > 0$$

For sellers' first-period equilibrium market shares, we partially differentiate Equation (3.9b): First, we use Equation (3.12b) to transform Equation (3.9b) into:

$$\eta_{s,1}^i = \frac{1}{2} + \frac{\pi\left(2\eta_{b,1}^i - 1\right)}{2} - \frac{\delta_p\left(1-\pi\upsilon\right)\left(\pi-\upsilon\right)}{2\Psi}\left(q_b^i - q_b^j\right)$$, then the partial derivative is:

$$\frac{\partial \eta^i_{s,1}}{\partial \Delta q^i_b} = \pi \frac{\partial \eta^i_{b,1}}{\partial \Delta q^i_b} + \frac{\delta_p^2 (1-\pi\upsilon)(\upsilon-\pi)^3}{2\Psi^2}\left(q^i_b - q^j_b\right)$$

where $\Omega \equiv \Delta q^i_b + \Delta q^j_b + 2s_b$, $\Delta q^i_b = q^i_b - \frac{\bar{q}_b}{2}$, for $i = 1,2$ and $\sigma \equiv 9 - (2\pi+\upsilon)(\pi+2\upsilon)$ and $\Psi \equiv (1-\pi\upsilon)(1+\delta_p)\sigma - \delta_p(\pi-\upsilon)^2\Omega$.

As previously demonstrated in Appendix C.6, $\frac{\partial \eta^i_{b,1}}{\partial \Delta q^i_b} > 0$ if $1 < \frac{\pi^2+\pi\upsilon+\upsilon^2}{3}$. Therefore, $\frac{\partial \eta^i_{s,1}}{\partial \Delta q^i_b} > 0$ under the same condition and as long as $\upsilon > \pi$.

For buyers' market share in equilibrium in the second period we use Equation (3.11a) which turns to:

$$\eta^i_{b,2} = \frac{1}{2} + \frac{q^i_b - q^j_b}{4(1-\pi\upsilon)} + \frac{[2(1-\pi\upsilon)+\Omega]}{2(1-\pi\upsilon)}\left[\eta^i_{b,1} - \frac{1}{2}\right]$$

$$\frac{\partial \eta^i_{b,2}}{\partial \Delta q^i_b} = \frac{1}{2(1-\pi\upsilon)}\left[\left[\eta^i_{b,1} - \frac{1}{2}\right] + \frac{\partial \eta^i_{b,1}}{\partial \Delta q^i_b}[2(1-\pi\upsilon)+\Omega]\right]$$

where $\Omega \equiv \Delta q^i_b + \Delta q^j_b + 2s_b$, $\Delta q^i_b = q^i_b - \frac{\bar{q}_b}{2}$, for $i = 1,2$.

Considering $\frac{\partial \eta^i_{b,1}}{\partial \Delta q^i_b} > 0$ if $1 < \frac{\pi^2+\pi\upsilon+\upsilon^2}{3}$, then if $\eta^i_{b,1} > \frac{1}{2}$, $\frac{\partial \eta^i_{b,2}}{\partial \Delta q^i_b} > 0$

For sellers' second-period equilibrium market shares, partially differentiate Equation (3.7):

$$\frac{\partial \eta^i_{s,2}}{\partial \Delta q^i_b} = \pi \frac{\partial \eta^i_{b,2}}{\partial \Delta q^i_b} - \frac{\delta_p (1-\pi\upsilon)(\pi-\upsilon)^3}{2\Psi^2}\left(q^i_b - q^j_b\right)$$

where $\Omega \equiv \Delta q^i_b + \Delta q^j_b + 2s_b$, $\Delta q^i_b = q^i_b - \frac{\bar{q}_b}{2}$, for $i = 1,2$ and $\sigma \equiv 9 - (2\pi+\upsilon)(\pi+2\upsilon)$ and $\Psi \equiv (1-\pi\upsilon)(1+\delta_p)\sigma - \delta_p(\pi-\upsilon)^2\Omega$.

Considering $\frac{\partial \eta^i_{b,2}}{\partial \Delta q^i_b} > 0$ if $\eta^i_{b,1} > \frac{1}{2}$. Then $\frac{\partial \eta^i_{s,2}}{\partial \Delta q^i_b} > 0$ if $\upsilon > \pi$. □

C.7 Proof of Observation 3.2

Proof. We prove Observation 3.2 by partially differentiating buyers' and sellers' equilibrium market shares at periods 1 and 2 with respect to s_b under the assumption that $q^1_b > q^2_b$.

To determine the impact on buyers' first-period equilibrium market shares, we partially differentiate the expression derived from Equation (3.13a) in the proof of Observation 1 in Appendix C.6, which is $\eta^i_{b,1} = \frac{1}{2} - \frac{\delta_p}{2\Phi\Psi}\left[6(1-\pi\upsilon)^2(1-\pi\upsilon) - (\pi-\upsilon)^2\Phi\right]\left(q^i_b - q^j_b\right)$.

$$\frac{\partial \eta^i_{b,1}}{\partial s_b} = \frac{\delta_p^2}{\Phi^2\Psi^2}\Bigg[(\pi-\upsilon)^2\Phi\Psi + \Big[6(1-\pi\upsilon)^2(1+\delta_p) - 2(1-\pi\upsilon)(1+\delta_p)(\pi-\upsilon)^2$$

$$-\delta_p (\pi - \upsilon)^2 \Omega\Big] \left[\Psi - (\pi - \upsilon)^2 \Phi\right] \Bigg] \left(q_b^i - q_b^j\right) = \frac{1}{2}\left[\frac{\partial \eta_{b,1}^i}{\partial \Delta q_b^i}\right]$$

where $\Omega \equiv \Delta q_b^i + \Delta q_b^j + 2s_b$, $\Delta q_b^i = q_b^i - \frac{\bar{q}_b}{2}$, for $i = 1,2$ and $\sigma \equiv 9 - (2\pi + \upsilon)(\pi + 2\upsilon)$ $\Phi \equiv 2(1 - \pi\upsilon)(1 + \delta_p) + \delta_p \Omega$ and $\Psi \equiv (1 - \pi\upsilon)(1 + \delta_p)\sigma - \delta_p (\pi - \upsilon)^2 \Omega$.

As it was shown in the proof of Observation 1 in Appendix C.6 $\frac{\partial \eta_{b,1}^i}{\partial \Delta q_b^i} > 0$ if $1 < \frac{\pi^2 + \pi\upsilon + \upsilon^2}{3}$. Therefore, as $\frac{\partial \eta_{b,1}^i}{\partial s_b} = \frac{1}{2}\left[\frac{\partial \eta_{b,1}^i}{\partial \Delta q_b^i}\right]$, then $\frac{\partial \eta_{b,1}^i}{\partial s_b} > 0$ if $1 < \frac{\pi^2 + \pi\upsilon + \upsilon^2}{3}$.

For sellers' first-period equilibrium market shares, we partially differentiate Equation (3.9b): First, we use Equation (3.12b) for $i = 1,2$ to transform Equation (3.9b) into:

$$\eta_{s,1}^i = \frac{1}{2} + \frac{\pi\left(2\eta_{b,1}^i - 1\right)}{2} - \frac{\delta_p (1 - \pi\upsilon)(\pi - \upsilon)}{2\Psi}\left(q_b^i - q_b^j\right) \text{ , then the partial derivative is:}$$

$$\frac{\partial \eta_{s,1}^i}{\partial s_b} = \pi \frac{\partial \eta_{b,1}^i}{\partial s_b} + \frac{\delta_p^2 (1 - \pi\upsilon)(\upsilon - \pi)^3}{\Psi^2}\left(q_b^i - q_b^j\right)$$

where $\Omega \equiv \Delta q_b^i + \Delta q_b^j + 2s_b$, $\Delta q_b^i = q_b^i - \frac{\bar{q}_b}{2}$, for $i = 1,2$ and $\sigma \equiv 9 - (2\pi + \upsilon)(\pi + 2\upsilon)$ and $\Psi \equiv (1 - \pi\upsilon)(1 + \delta_p)\sigma - \delta_p (\pi - \upsilon)^2 \Omega$.

As was shown previously, $\frac{\partial \eta_{b,1}^i}{\partial s_b} > 0$, then $\frac{\partial \eta_{s,1}^i}{\partial s_b} > 0$ if $\upsilon > \pi$.

For buyers' market share in equilibrium in the second period we use Equation (3.11a) which turns to:

$$\eta_{b,2}^i = \frac{1}{2} + \frac{q_b^i - q_b^j}{4(1 - \pi\upsilon)} + \frac{[2(1 - \pi\upsilon) + \Omega]}{2(1 - \pi\upsilon)}\left[\eta_{b,1}^i - \frac{1}{2}\right]$$

$$\frac{\partial \eta_{b,2}^i}{\partial s_b} = \frac{1}{2(1 - \pi\upsilon)}\left[2\left[\eta_{b,1}^i - \frac{1}{2}\right] + \frac{\partial \eta_{b,1}^i}{\partial s_b}[2(1 - \pi\upsilon) + \Omega]\right]$$

where $\Omega \equiv \Delta q_b^i + \Delta q_b^j + 2s_b$, $\Delta q_b^i = q_b^i - \frac{\bar{q}_b}{2}$, for $i = 1,2$.

Considering $\frac{\partial \eta_{b,1}^i}{\partial s_b} > 0$ if $1 < \frac{\pi^2 + \pi\upsilon + \upsilon^2}{3}$, then if $\eta_{b,1}^i > \frac{1}{2}$, $\frac{\partial \eta_{b,2}^i}{\partial s_b} > 0$

For sellers' second-period equilibrium market shares, partially differentiate Equation (3.7):

$$\frac{\partial \eta_{s,2}^i}{\partial s_b} = \pi \frac{\partial \eta_{b,2}^i}{\partial s_b} - \frac{\delta_p (1 - \pi\upsilon)(\pi - \upsilon)^3}{\Psi^2}\left(q_b^i - q_b^j\right)$$

where $\Omega \equiv \Delta q_b^i + \Delta q_b^j + 2s_b$, $\Delta q_b^i = q_b^i - \frac{\bar{q}_b}{2}$, for $i = 1,2$ and $\sigma \equiv 9 - (2\pi + \upsilon)(\pi + 2\upsilon)$ and $\Psi \equiv (1 - \pi\upsilon)(1 + \delta_p)\sigma - \delta_p (\pi - \upsilon)^2 \Omega$.

Considering $\frac{\partial \eta_{b,2}^i}{\partial s_b} > 0$ if $\eta_{b,1}^i > \frac{1}{2}$. Then $\frac{\partial \eta_{s,2}^i}{\partial s_b} > 0$ if $\upsilon > \pi$. □

C.8 Proof of Propositions 3.3 and 3.4 and Corollary 3.2 and 3.3

Proof. We show the impacts on platforms i's equilibrium profits using the results from Propositions 3.1 and 3.2. First, we use the equilibrium platform profits defined in Equation (3.15). Specifically, $\Pi^i = p_b^i N_b^i + p_s^i N_s^i$, where $N_b^i \equiv \eta_{b,1}^i + \delta_p \eta_{b,2}^i$ and $N_s^i \equiv \eta_{s,1}^i + \delta_p \eta_{s,2}^i$. By using the equilibrium market shares of buyers and sellers in the first period in Equations (3.13a) and (3.13b) and in the second period in Equations (3.14a) and (3.14b) we obtain:

$$\begin{aligned}
N_b^i &\equiv \eta_{b,1}^i + \delta_p \eta_{b,2}^i \\
&= \frac{1}{2}(1+\delta_p) + \frac{\delta_p}{4\Phi\Psi}\Big[2\left[\Psi - 3(1-\pi\upsilon)^2(1+\delta_p)\right] + 3(1-\pi\upsilon)(1+\delta_p)\Phi \\
&\qquad - 2\left[\Psi - 3(1-\pi\upsilon)^2(1+\delta_p)\right]\Big]\left(q_b^i - q_b^j\right) \\
N_b^i &= (1+\delta_p)\left[\frac{1}{2} + \frac{3\delta_p(1-\pi\upsilon)}{4\Psi}\left(q_b^i - q_b^j\right)\right]
\end{aligned}$$

$$\begin{aligned}
N_s^i &\equiv \eta_{s,1}^i + \delta_p \eta_{s,2}^i \\
&= \frac{1}{2}(1+\delta_p) + \frac{\delta_p}{4\Phi\Psi}\Big[\pi\left[2\Psi + 3\delta_p(1-\pi\upsilon)\Omega - 2\Psi + 6(1-\pi\upsilon)^2(1+\delta_p)\right] \\
&\qquad + (1-\pi\upsilon)\Phi\left[\delta_p(\pi+2\upsilon) - 2(\pi-\upsilon)\right]\Big]\left(q_b^i - q_b^j\right) \\
&= \frac{1}{2}(1+\delta_p) + \frac{\delta_p}{4\Phi\Psi}\Big[3\pi(1-\pi\upsilon)\left[2(1-\pi\upsilon)(1+\delta_p) + \delta_p\Omega\right] \\
&\qquad + (1-\pi\upsilon)\Phi\left[\delta_p(\pi+2\upsilon) - 2(\pi-\upsilon)\right]\Big]\left(q_b^i - q_b^j\right) \\
&= \frac{1}{2}(1+\delta_p) + \frac{\delta_p}{4\Phi\Psi}\Big[(1-\pi\upsilon)\Phi\left[3\pi + \delta_p(\pi+2\upsilon) - 2(\pi-\upsilon)\right]\Big]\left(q_b^i - q_b^j\right) \\
N_s^i &= (1+\delta_p)\left[\frac{1}{2} + \frac{\delta_p(1-\pi\upsilon)(\pi+2\upsilon)}{4\Psi}\left(q_b^i - q_b^j\right)\right]
\end{aligned}$$

where $\Omega \equiv \Delta q_b^i + \Delta q_b^j + 2s_b$, $\Delta q_b^i = q_b^i - \frac{\bar{q}_b}{2}$, for $i = 1, 2$ and $\Phi \equiv 2(1-\pi\upsilon)(1+\delta_p) + \delta_p\Omega$ and $\Psi \equiv (1-\pi\upsilon)(1+\delta_p)\sigma - \delta_p(\pi-\upsilon)^2\Omega$.

Next, we partially differentiate N_b^i and N_s^i with respect to Δq_b^i and s_b to obtain:

$$\frac{\partial N_b^i}{\partial \Delta q_b^i} = \frac{3\delta_p^2(1+\delta_p)(1-\pi\upsilon)(\pi-\upsilon)^2}{4\Psi^2}\left(q_b^i - q_b^j\right) > 0 \tag{C.6a}$$

$$\frac{\partial N_s^i}{\partial \Delta q_b^i} = \frac{\delta_p^2(1+\delta_p)^2(1-\pi\upsilon)(\pi-\upsilon)^2(\pi+2\upsilon)}{4\Psi^2}\left(q_b^i - q_b^j\right) > 0 \tag{C.6b}$$

$$\frac{\partial N_b^i}{\partial s_b} = \frac{3\delta_p^2 (1+\delta_p)(1-\pi\upsilon)(\pi-\upsilon)^2}{2\Psi^2}\left(q_b^i - q_b^j\right) > 0 \tag{C.6c}$$

$$\frac{\partial N_s^i}{\partial s_b} = \frac{\delta_p^2 (1+\delta_p)^2 (1-\pi\upsilon)(\pi-\upsilon)^2(\pi+2\upsilon)}{2\Psi^2}\left(q_b^i - q_b^j\right) > 0 \tag{C.6d}$$

where $\Omega \equiv \Delta q_b^i + \Delta q_b^j + 2s_b$, $\Delta q_b^i = q_b^i - \frac{\bar{q}_b}{2}$, for $i = 1, 2$ and $\sigma \equiv 9 - (2\pi+\upsilon)(\pi+2\upsilon)$ and $\Psi \equiv (1-\pi\upsilon)(1+\delta_p)\sigma - \delta_p(\pi-\upsilon)^2\Omega$.

Next, we partially differentiate Equation (3.15) with respect to buyers perception of platform i's quality Δq_b^i to obtain:

$$\frac{\partial \Pi^i}{\partial \Delta q_b^i} = p_b^i \frac{\partial N_b^i}{\partial \Delta q_b^i} + N_b^i \frac{\partial p_b^i}{\partial \Delta q_b^i} + p_s^i \frac{\partial N_s^i}{\partial \Delta q_b^i} + N_s^i \frac{\partial p_s^i}{\partial \Delta q_b^i}$$

Next, according to Proposition 3.1 $\frac{\partial p_b^i}{\partial \Delta q_b^i} < 0$ if $\pi > \upsilon$ and $1 < \frac{\pi(\pi+2\upsilon)}{3}$, and $\frac{\partial p_s^i}{\partial \Delta q_b^i} > 0$ if $\pi > \upsilon$. Furthermore, as shown in Equations (C.6a) and (C.6b) $\frac{\partial N_b^i}{\partial \Delta q_b^i} > 0$ and $\frac{\partial N_s^i}{\partial \Delta q_b^i} > 0$.Therefore, assuming positive membership fees on both sides of the market, we obtain $\frac{\partial \Pi^i}{\partial \Delta q_b^i} > 0$ if direct effect $\frac{\partial N_b^i}{\partial \Delta q_b^i}$ dominates the strategic effect $\frac{\partial p_b^i}{\partial \Delta q_b^i}$ on buyers side.

Conversely, if the strategic effect $\frac{\partial p_b^i}{\partial \Delta q_b^i}$ dominates the direct effect $\frac{\partial N_b^i}{\partial \Delta q_b^i}$ on buyers' side, $\frac{\partial \Pi^i}{\partial \Delta q_b^i} < 0$ as long as the strategic effect $\frac{\partial p_b^i}{\partial \Delta q_b^i}$ on buyers side dominates both, direct and strategic effect on sellers' side. Furthermore, when $\upsilon > \pi$ the strategic effect on sellers' side is negative $\frac{\partial p_s^i}{\partial \Delta q_b^i} < 0$, therefore if it dominates the direct effect on sellers' side and both, direct and strategic effects on buyers' side, platforms' profits decreases when buyers perception of platform i's quality increases, $\frac{\partial \Pi^i}{\partial \Delta q_b^i} < 0$.

Next, we partially differentiate Equation (3.15) with respect to switching cost s_b to obtain:

$$\frac{\partial \Pi^i}{\partial s_b} = p_b^i \frac{\partial N_b^i}{\partial s_b} + N_b^i \frac{\partial p_b^i}{\partial s_b} + p_s^i \frac{\partial N_s^i}{\partial s_b} + N_s^i \frac{\partial p_s^i}{\partial s_b}$$

Next, according to Proposition 3.2 $\frac{\partial p_b^i}{\partial s_b} < 0$ if $\pi > \upsilon$ and $1 < \frac{\pi(\pi+2\upsilon)}{3}$, and $\frac{\partial p_s^i}{\partial s_b} > 0$ if $\pi > \upsilon$. Furthermore, as shown in Equations (C.6c) and (C.6d) $\frac{\partial N_b^i}{\partial s_b} > 0$ and $\frac{\partial N_s^i}{\partial s_b} > 0$.Therefore, assuming positive membership fees on both sides of the market, we obtain $\frac{\partial \Pi^i}{\partial s_b} > 0$ if direct effect $\frac{\partial N_b^i}{\partial s_b}$ dominates the strategic effect $\frac{\partial p_b^i}{\partial s_b}$ on buyers side.

Conversely, if the strategic effect $\frac{\partial p_b^i}{\partial s_b}$ dominates the direct effect $\frac{\partial N_b^i}{\partial s_b}$ on buyers' side, $\frac{\partial \Pi^i}{\partial s_b} < 0$ as long as the strategic effect $\frac{\partial p_b^i}{\partial s_b}$ on buyers side dominates both, direct and strategic effect on sellers' side. Furthermore, when $\upsilon > \pi$ the strategic effect on sellers' side is negative $\frac{\partial p_s^i}{\partial s_b} < 0$, therefore if it dominates the direct effect on sellers' side and both, direct and strategic effects on buyers' side, platforms' profits decreases when the switching cost increases, $\frac{\partial \Pi^i}{\partial s_b} < 0$. □

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