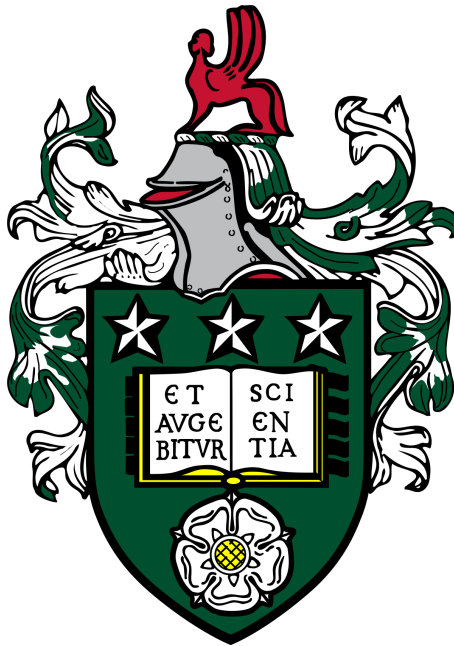


Remote Sensing, machine learning and numerical modelling approaches to understanding the role of fractures in Antarctic Ice Sheet dynamics

Trystan Surawy-Stepney

Submitted in accordance with the requirements for the degree of Doctor
of Philosophy



University of Leeds

School of Earth and Environment

April 2024

Publication Statement

I confirm that the work submitted is my own, except where work which has formed part of jointly authored publications has been included. My contribution and the other authors to this work has been explicitly indicated below. I confirm that appropriate credit has been given within the thesis where reference has been made to the work of others.

Chapter 2 is a reproduction of the paper “Episodic dynamic change linked to damage on the Thwaites Glacier Ice Tongue” by T. Surawy-Stepney, A. E. Hogg, S. L. Cornford and B. J. Davison; published in *Nature Geoscience* in January 2023.

TSS and AEH designed the work. TSS processed the ice velocity observations, generated fracture maps, calving front locations and performed the analysis. TSS and SLC performed the modelling and BJD analysed the ocean data. All authors contributed to scientific discussion, interpretation of the results and contributed to the manuscript.

Chapter 3 is a reproduction of the paper “Mapping Antarctic Crevasses and their Evolution with Deep Learning Applied to Satellite Radar Imagery” by T. Surawy-Stepney, A. E. Hogg, S. L. Cornford and D. C. Hogg; published in *The Cryosphere* in October 2023.

TSS and AEH designed the work. TSS designed the algorithms and processing chains described in the article, wrote the code that implements them, and wrote the manuscript. SLC and DCH respectively provided advice on numerical modelling and computer vision methods. All the authors contributed suggestions to the manuscript.

Chapter 4 is a reproduction of the paper “The effect of landfast sea ice buttressing on ice

dynamic speedup in the Larsen B embayment, Antarctica” by T. Surawy-Stepney, A. E. Hogg, S. L. Cornford, B. J. Wallis, B. J. Davison, H. L. Selley, R. A. W. Slater, E. K. Lie, L. Jakob, A. Ridout, N. Gourmelen, B. I. D. Freer, S. F. Wilson, and A. Shepherd; published in *The Cryosphere* in March 2024.

TSS and AEH designed the work. TSS led the modelling with contributions from SLC. TSS and AEH led the observations with contributions from the remaining authors. BJD processed ice speed data, analysed the ice speed data along with RAWs and SFW, analysed the atmospheric reanalysis data, and wrote the paper. BJW analysed the speed data and processed and provided grounding line locations. HLS and EKL provided calving front locations and fast sea ice extents. BIDE processed and analysed altimetry data across the glaciers. AR processed CryoSat-2 altimetry data, along with AS. LJ and NG produced and basal melt rate data over the Scar Inlet. All authors contributed to the scientific discussion, interpretation of results, and suggestions to the manuscript.

Chapter 3 is a reproduction of the paper “Using observations of surface fracture to address ill-posed ice softness estimation over Pine Island Glacier” by T. Surawy-Stepney, S. L. Cornford and A. E. Hogg; in preparation for submission to *The Cryosphere* in 2024.

TSS and SLC designed the work. TSS carried out the model development and simulations and wrote the manuscript. All authors contributed to discussion.

This copy has been supplied on the understanding that it is copyright material and that no quotation from the thesis may be published without proper acknowledgement.

Acknowledgements

Firstly, I would like to thank the University of Leeds and the European Space Agency for funding my time as a research student. Even more so, I would like to express my gratitude to Stephen Middleton and to Duncan and Jaynie Ford who personally donated money to my PhD fund. Without your generosity, I wouldn't have been able to carry out this research, or participate in the trips that have been such a fun and enriching part of PhD life - thank you so much.

On the whole, I have enjoyed my time as a PhD student which, given my general attitude, is no small statement. This is in large part due to my supervisors Anna Hogg, Steph Cornford and David Hogg. Thank you to Anna for giving me so much freedom, for supporting me in every possible way and being my greatest source of confidence. I have been so fortunate to have you as my primary supervisor. Thank you to Steph for being such a brilliant teacher. Our meetings were the thing I looked forward to most during my PhD. Thank you to David for being a constant source of insight and advice, and for being so generous with your time. Thank you also to everyone who has contributed to the papers that have formed the results chapters of this thesis. I am also hugely grateful to Rick Rickman - obviously. Finally, it isn't common to work in an environment as fun and supportive as our research group, so I would also like to express my gratitude to everyone in the Satellite Ice Dynamics group at Leeds and BAS.

Thank you to Mum and Dad - you have known that I have wanted to be a scientist for a long time now, and you have supported and helped me constantly throughout. I would not have been able to get close to this point without you.

Finally, thank you to Abbie for being the best person I've met. Thank you for your support and pride in me. This thesis is dedicated to you.

Abstract

The Antarctic Ice Sheet is currently losing mass. In large part, this is due to elevated flux of ice across grounding lines in West Antarctica, without the changes in accumulation over the ice sheet required to balance this. Over the last three decades, it has become apparent that the dynamics of these ice streams are highly sensitive to changes at their ocean margins, including to the floating ice shelves that extend out in front of many of them. So much so, in fact, that the dynamic imbalance of the ice sheet as a whole can be largely explained by a decrease in the amount of ice shelf buttressing in West Antarctica over recent decades, driven by enhanced sub-shelf melting due to incursions of warmer ocean waters onto the continental shelf. We have a good mechanical understanding of this process thanks to our treatment of marine ice sheet dynamics as a theory of viscous flow.

However, the processes of material failure, e.g. brittle fracturing, are not included in this theory but are important in determining boundary conditions and material properties at the ice sheet margin. For example, fracturing processes are responsible for the calving of ice, which determines the position of the margin itself, and the presence of fractures alters the ability of ice to bear and transmit membrane stresses.

This thesis deals with certain questions regarding the fracturing of ice, and, more generally, the sensitivity of ice stream dynamics to changes in material properties near the margin. I develop new methods for the extraction of crevasse data from satellite radar imagery, use the derived datasets to investigate changes to the structural integrity of ice shelves in the Amundsen Sea Embayment along with ice speed datasets from remote sensing imagery. I develop the capabilities of the BISICLES ice sheet model to solve inverse problems, and perform sensitivity analysis and diagnostic modelling, to investigate the sensitivity of ice dynamics to buttressing by land-fast sea ice, and investigate links between observations of fracturing and isotropic ‘damage’ -

a scalar field aiming to characterise how fractures change the large-scale material properties of the ice.

In chapter 2, I develop deep learning-based computer vision methods for the extraction of crevasses and calving fronts from Sentinel-1 synthetic aperture radar (SAR) imagery of the Thwaites Glacier Ice Tongue. I examine the concurrency of structural change and large fluctuations in ice speed in this area, measured using observations of ice speed derived from Sentinel-1 SAR data. By solving inverse problems for the softness of the ice shelf given observations of ice speed, I show that the quantity of rifts matches well the timeseries of ice softness in the shear margin - showing a link between the two and indicating that crevassing is dynamically important for the region. I show that the increases in crevassing and acceleration of the ice tongue reverse on timescales of years, indicating that any feedback between the two is not the dominant cause of change in the region.

In Chapter 3, I develop the crevasse-mapping procedure of chapter 2 into a process that can be used to generate pan-continental mosaics of crevasses on grounded and floating ice. This involves the use of deep learning methods similar to those of chapter 2, and the new ‘parallel structure filtering’ algorithm. I go on develop a method for quantifying structural change on ice shelves by measuring (roughly speaking) the local density of fractures. I show, in line with other literature, that there have been structural changes in parts of the Pine Island, Thwaites and Crosson Ice Shelves that contribute to ice shelf buttressing.

Chapter 4 is a bit of a detour from the theme of crevasses. Instead, through a case study on the Larsen-B Embayment, I look at whether landfast sea ice can provide buttressing to glaciers/ice shelves in the way that ice shelves provide buttressing to ice streams. I present satellite data showing the evacuation of landfast sea ice from the Larsen-B Embayment at the start of 2022, the subsequent disintegration of the Hektoria/Green/Evans and Crane Ice Shelves, and the acceleration by $\sim 100 \text{ m a}^{-1}$ of Hektoria, Green and Crane Glaciers. I use diagnostic modelling and sensitivity analysis, with a representation of the landfast sea ice as a thin meteoric ice shelf, to suggest that the observed changes to the glaciers are not due to a loss of direct mechanical buttressing by the landfast sea ice.

In Chapter 5, I focus on the process of solving inverse problems for the control fields representing slipperiness at the base of grounded ice, and the material softness of the ice from observations

of ice speed using BISICLES. This is an ill-posed problem suffering underdeterminedness and poor conditioning. I introduce two methods of using the crevasse data generated using the methods described in chapter 3, along with data on surface strain rates, as prior information to help with this, and apply it to a case study focused on Pine Island Glacier. I show that, by doing so, one can arrive at softness fields that resemble the crevasse patterns seen on floating ice without harming the solution misfit. However, I also show that the use of crevasse data on grounded ice does not result in more plausible-looking solutions, indicating that the crevasses here do not dominate our uncertainty in the rheology of ice.

Contents

1	Introduction	1
1.1	The Antarctic Ice Sheet	1
1.1.1	Recent changes to the AIS	3
1.2	A closer look at ice dynamics	5
1.2.1	Ice shelf buttressing and ocean-induced thinning	7
1.2.2	Dynamic instability in the WAIS	9
1.3	Fractures in the system	10
1.4	Modelling ice dynamics	14
1.5	Inverse problems and ill-posedness	16
1.6	Remote sensing and Sentinel-1 Data	21
1.6.1	How do the Sentinel-1 satellites make a SAR backscatter image?	23
1.7	Deep learning and neural networks	25
1.7.1	The structure of a neural network	27
1.7.2	Neural network optimisation	30
1.8	Thesis aim and objectives	34
1.9	Thesis structure	34
2	Episodic dynamic change linked to damage on the Thwaites Glacier Ice Tongue	46
2.1	Introduction	47
2.2	Results	48
2.2.1	Observed Changes in Ice Speed	48
2.2.2	Observed Changes in Fracture	50
2.2.3	Chronology of Change in Ice Speed and Structure	50

2.2.4	Model Inversions	53
2.3	Discussion	54
2.3.1	Long-term Context	54
2.3.2	Link Between Structural Change and Dynamic Variability	55
2.4	Conclusions	57
2.5	Methods	58
2.5.1	Ice Velocity Observations	58
2.5.2	Observations of Fracture and Ice Tongue Perimeter	59
2.5.3	Inverse Modelling	60
3	Mapping Antarctic Crevasses and their Evolution with Deep Learning Applied to Satellite Radar Imagery	66
3.1	Introduction	67
3.2	Identifying Crevasses	69
3.2.1	Methods	69
3.2.2	Results	75
3.2.3	Evaluation	78
3.3	Measuring Changes in Fracture Density	83
3.3.1	Methods	83
3.3.2	Results	84
3.3.3	Evaluation	87
3.4	Discussion	90
3.4.1	Crevasse development on Pine Island and Thwaites Eastern Ice Shelves .	90
3.4.2	Trends in Fracture Density as a Meaningful Measure of Structural Change	91
3.4.3	Future Work on Measuring Changes in Fracture Density	92
3.4.4	Representation of Damage in Numerical Modelling	93
3.4.5	The Use of SAR Backscatter Images	94
3.4.6	Future Improvements to the Crevasse Maps	96
3.4.7	A Comparison of Ice Shelf Crevasse Detection Methods	97
3.5	Conclusions	100
3.A	Parallel Structure Filtering	101
3.B	Measuring Trends in Fracture Density	103

3.C	Calculating the Buttressing Number	105
4	The effect of landfast sea ice buttressing on ice dynamic speedup in the Larsen-B Embayment, Antarctica	111
4.1	Introduction	112
4.2	Observations	113
4.2.1	Observational methods	113
4.2.2	Landfast sea ice area change	114
4.2.3	Ice dynamic and calving response	114
4.3	Landfast sea ice buttressing	116
4.3.1	Modelling methods	118
4.3.2	Modelling results	121
4.4	Environmental drivers	125
4.5	Discussion	128
4.5.1	The use of a viscous flow model	128
4.5.2	What impact does sea ice have on glacier ice?	130
4.5.3	Wider implications for the interaction of glaciers with landfast sea ice . .	131
4.5.4	Limitations and future work	132
4.6	Conclusions	133
4.A	Additional model details	134
4.A.1	L-curve analysis	134
4.A.2	Calculating gradients of ice speed using the model adjoint	136
4.A.3	Sensitivity to sliding physics	137
5	Using observations of surface fracture to address ill-posed ice softness esti- mation over Pine Island Glacier	144
5.1	Introduction	145
5.2	Methods	149
5.2.1	Fracture data assimilation in snapshot inverse problems	149
5.2.2	Fracture data assimilation through time	151
5.3	Results	153
5.3.1	Snapshot inverse problems	153
5.3.2	Inverse problems through time	156

5.4	Discussion	158
5.4.1	Grounded ice	158
5.4.2	Floating ice	159
5.4.3	A note on L-curves	160
5.5	Conclusions	161
5.A	Deriving a regularisation term from a prior distribution	162
5.B	Derivatives of $\mathcal{J}_\phi$	163
6	Discussion and conclusions	169
6.1	Summary of principal findings	170
6.1.1	A new dataset of crevasses over the Antarctic Ice Sheet margin	170
6.1.2	Quantifying structural change	170
6.1.3	A link between crevassing and short-term dynamic change on the Thwaites Glacier Ice Tongue	171
6.1.4	Landfast sea ice does not buttress glaciers like ice shelves do	172
6.1.5	Methods for the inclusion of fracture information in inverse problems . . .	173
6.1.6	Evidence for the link between crevasses and isotropic damage	174
6.2	Future work	175
6.2.1	Fractures as a routine, open-access product	175
6.2.2	Towards a data-informed continuum damage mechanics of glacier ice . . .	176
6.2.3	Assessing the relevance of ice shelf damage to grounded ice speed	177
6.2.4	Second-order derivatives in BISICLES	180
6.2.5	An estimate of error on ice speed derived via intensity feature-tracking . .	182
A	Deriving the shelfy-stream equations	189
A.1	First order Momentum Equations	189
A.1.1	Scaling Boundary Conditions	191
A.2	Vertical Integration	192
B	Supplementary materials for the article: Episodic dynamic change linked to damage on the Thwaites Glacier Ice Tongue	196
B.1	Observed Ice Speed and Error Measurements	197
B.2	Fracture and Calving Front Measurements	198

B.2.1	Deep Neural Networks - Background	198
B.2.2	Network Architecture	199
B.2.3	Neural Network Fracture Measurement	199
B.2.4	Neural Network Calving Front Delineation	201
B.3	The Temperature Component of Damage	204
B.4	External Forcing Mechanisms	205
B.4.1	Ocean Temperature Data	206
C	Supplementary figures for the article: The effect of landfast sea ice buttress-	
	ing on ice dynamic speedup in the Larsen-B Embayment, Antarctica	211
D	The first order adjoint model	217
D.1	A general adjoint system	217
D.2	The adjoint equation and gradient for the SSE	220

List of Figures

1.1	The Antarctic Ice Sheet	2
1.2	Antarctic bedrock elevation	2
1.3	Mass balance of the Earth’s ice sheets	4
1.4	25 years of Antarctic surface elevation change	5
1.5	Changes in flow speed in the Amundsen Sea Embayment between 1996 and 2013	6
1.6	The effect of thinning on ice shelf buttressing	7
1.7	The instantaneous effects of ice shelf thinning in the ASE	8
1.8	A Sno-Cat in trouble	11
1.9	SAR imaging.	24
1.10	Schematic of an artificial neuron and a fully connected neural network with one hidden layer	27
1.11	Schematic of a CNN operating on one-dimensional data	28
2.1	Ice speed anomalies on Thwaites Glacier (TG)	49
2.2	Observed fractures on the Thwaites Glacier Ice Tongue	51
2.3	Time series of Thwaites Glacier Ice Tongue (TGIT) flow speed, fracture and damage.	52
2.4	Damage over the Thwaites Glacier terminus from 2016 to 2021	54
3.1	The appearance of crevasses in SAR backscatter images	69
3.2	Processing chain for creating monthly fracture mosaics	71
3.3	Bootstrapping a neural network	73
3.4	Fractures in Antarctica: April 2022	76
3.5	Comparison between SAR images and a fracture map over the ASE	78

3.6	A visual evaluation of crevasse maps over Fimbul and Crosson Ice Shelves and their tributaries	81
3.7	Seven-and-a-half years of structural change over the ice shelves of the ASE . . .	85
3.8	Fracture density timeseries for selected points on the ice shelves of the ASE . . .	88
3.9	Changes in fracture density over the Brunt/Stancomb-Wills and Shackleton Ice Shelves	92
3.10	A comparison of ice shelf fracture mapping methods	98
3.11	The construction of a measure of fracture density change for a region on the Thwaites Eastern Ice Shelf	103
4.1	Speed and geometry changes over the glaciers of the Larsen-B Embayment	115
4.2	Modelled changes in speed with varying landfast sea ice thickness for Hektor, Green, Evans and Crane Glaciers	122
4.3	The sensitivity of ice speed to changes in effective ice thickness on Hektor and Crane Glaciers	123
4.4	The modelled stress and strain-rate response to the addition of a 10 m-thick ice shelf	124
4.5	Environmental forcing over the Larsen-B Embayment from 2000 to 2022	127
4.6	Resistive stresses in the modelled landfast sea ice compared to typical yield values for sea ice	128
4.7	L-curves for the choice of regularisation parameters	135
4.8	The sensitivity of our results to the choice of sliding speed characterising plastic basal traction	137
5.1	The different contributions to ξ	150
5.2	Solutions to the snapshot inverse problem with three methods of regularisation .	153
5.3	The conditioning of the snapshot inverse problem under three methods of regularisation	155
5.4	Softness solutions for the time-dependent inverse problem with three methods of regularisation	157
5.5	L-curve for the data-informed regularisation	160
6.1	Percentage difference in ice speed between ‘healed’ and ‘damaged’ ice shelves . .	178

6.2	The sensitivity of grounded ice speed to changes in ice shelf softness	179
6.3	Estimated uncertainty in ice speed measurements over Thwaites Glacier	183
B.1	Annual ice speed and error maps over the Thwaites Glacier Ice Tongue	197
B.2	U-net-like neural network architecture	198
B.3	Example training data for the fracture-mapping neural network	199
B.4	Neural network performance and output for an example Sentinel-1 backscatter image	200
B.5	Method for extracting errors in the calving front position from a neural network output	203
B.6	Damage over the Thwaites Glacier terminus	203
B.7	CTD profiles acquired close to the TGIT in 2014 and 2019	207
C.1	Annual sea ice front locations in the Larsen-B Embayment 2002-2021	212
C.2	Timeseries of speed on the Larsen-B Glaciers January 2015 to December 2022 . .	213
C.3	Calving front locations in the Larsen-B Embayment 2002-2022	213
C.4	Flow speed of landfast sea ice in the Larsen-B Embayment	214
C.5	Estimated landfast sea ice freeboard in the Larsen-B Embayment	215
C.6	Model setup at the end of the initialisation of the Larsen-B domain	215

Abbreviations

AD	Automatic differentiation
AIS	Antarctic Ice Sheet
AP	Antarctic Peninsula
ASE	Amundsen Sea Embayment
BSW	Brunt-Stancomb-Wills
CNN	Convolutional neural network
DEM	Digital elevation model
EAIS	East Antarctic Ice Sheet
EIS	Eastern Ice Shelf
ESM	Eastern shear margin
GFL	Glen’s flow law
HGE	Hektoria/Green/Evans
InSAR	Interferometric synthetic aperture radar
IW	Interferometric wide-swath
MISI	Marine ice sheet instability
NN	Neural network
NWPP	North-western Pinning Point
PIG	Pine Island Glacier
PIIS	Pine Island Ice Shelf
PIV	Particle image velocimetry
SAR	Synthetic aperture radar
SIA	Shallow ice approximation
SIE	Sea ice extent
SLC	Single look complex

SNR	Signal-to-noise ratio
SSA	Shallow/shelfy-stream approximation
SShA	Shallow-shelf approximation
SV	Singular value
TG	Thwaites Glacier
TGIT	Thwaites Glacier Ice Tongue
WAIS	West Antarctic Ice Sheet

Chapter 1

Introduction

1.1 The Antarctic Ice Sheet

The Antarctic Ice Sheet (AIS) is a 2.44×10^{15} -tonne layer of ice resting on the Antarctic continental bedrock. The ice is thickest in the interior of the continent and thinner towards the margins, with some variation due to deep channels in the bed. Though contiguous, the AIS is typically separated into three regions distinguished by their locations, climatology and physical characteristics. These are the large domed East Antarctic Ice Sheet (EAIS), the smaller *marine* West Antarctic Ice Sheet (WAIS) and the yet smaller Antarctic Peninsula (AP) (Fig. 1.1). Across each of these regions, the margins of the AIS are fringed by ice shelves, where the ice becomes thin enough (at the ‘grounding line’) to float in the denser ocean water surrounding the continent.

Perhaps more topically, the three regions of the AIS are distinguished by the mechanisms by which they might contribute to sea level rise in the near (~ 100 year) future. Climatology is a large factor, but also the geometry of the ice sheet. A significant proportion of the continental bedrock underlying the EAIS is submerged below sea-level due to isostatic depression of the lithosphere by the weight of ice above (Fig. 1.2). The WAIS rests on bedrock that is also submerged deep below sea-level, though this would be the case even without isostatic adjustment of the crust (Van der Veen, 1999). This makes the WAIS a *marine* ice sheet and distinguishes certain physical mechanisms that govern its evolution, perhaps increasing its likelihood of deglaciation under changes in climate forcing.

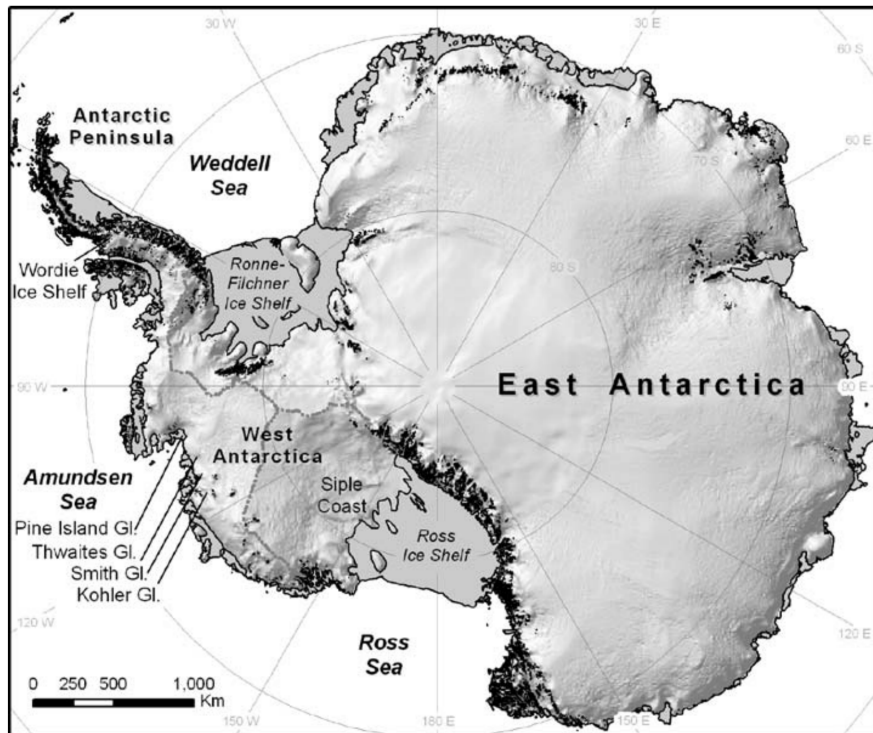


Figure 1.1: The Antarctic Ice Sheet (Vaughan, 2008).

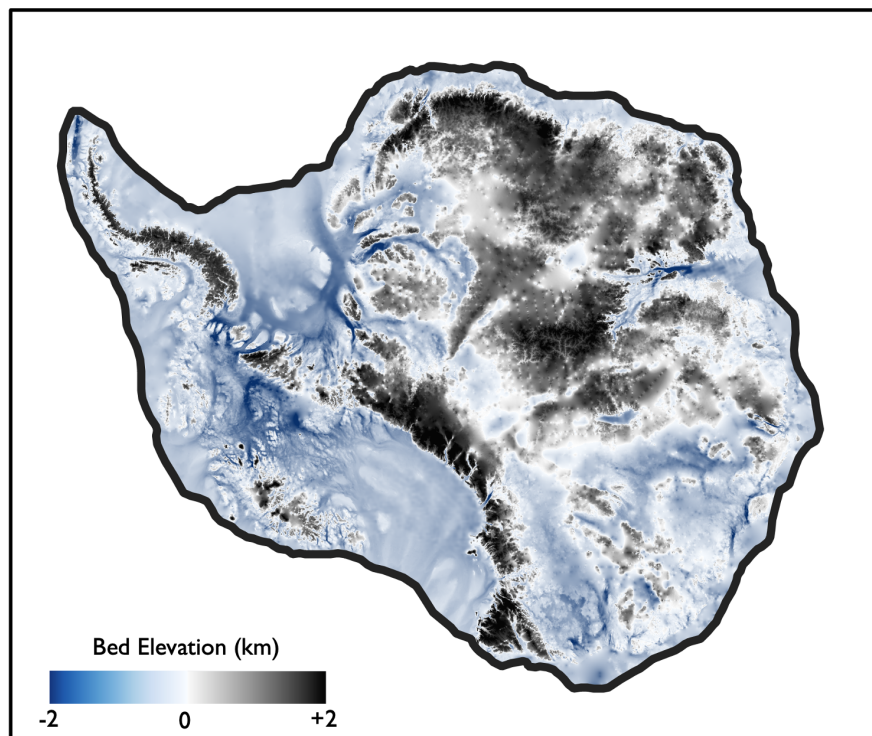


Figure 1.2: Bedrock elevation under the current Antarctic Ice Sheet according to BedMachine version 2 (Morlighem, 2020).

At present, the AIS holds enough ice to raise sea levels by around 58 m should it melt entirely (Fretwell et al., 2013). Approximately 88% of the mass is contained in the EAIS, which is likely

to have persisted continuously from ~ 14 -million years ago the present day (Sugden and Denton, 2004). Widespread glaciation occurred in Antarctica due to ocean and climatic changes during the Eocene-Oligocene transition ~ 34 -million years ago (Zachos et al., 2001), and its isolation brought about by the opening of the Drake passage (Kennett, 1977). The AP and WAIS Ice Sheets, on the other hand, are thought to have undergone periodic collapse and regrowth during glacial-interglacial cycles (Pollard and DeConto, 2009). The presence of warm ocean water on the continental shelf has likely been a strong control on the stability of the WAIS (Naish et al., 2009). This raises the question of whether rapid changes in the regions could occur as a result of human-induced changes to atmospheric and ocean forcing (Sutter et al., 2019; Uenzelmann-Neben et al., 2022). Though substantially smaller than the EAIS, the WAIS contains enough ice to raise global sea level by ~ 5 m on average, should it melt, making the investigation of the processes that govern its (de-) glaciation important for our understanding of the climate system as a whole.

Ice sheets are not static objects and change shape over time. This is due to changes in accumulation from falling snow, and ablation from melt at the ice surface or ice/ocean interface and the calving of ice from glacier termini. Additionally, ice sheets change shape under the action of gravity. This occurs for three principal reasons: 1) the surface of the ice sheet is not flat - leading to pressure gradients within the ice, 2) the ice sheet margin terminates in material with different density to the ice (air or ocean), leading to nonzero longitudinal stresses at the margin, and 3) despite being a solid, ice is deformable under the long-term application of external stress. This process is referred to as “ice dynamics”.

The interaction between accumulation, ablation and the transport of ice from the inner ice sheet to the margins over time determines the *mass balance* (the net gain in mass over a certain period of time) of the ice sheet. For those interested in how changes to the ice sheet might alter sea levels and the climate system, the current and future mass balance is the most significant metric.

1.1.1 Recent changes to the AIS

The large-scale evolution of ice sheets like the AIS occurs on timescales much longer than a human lifespan. As such, it is not possible to take the current presence of ice sheets as evi-

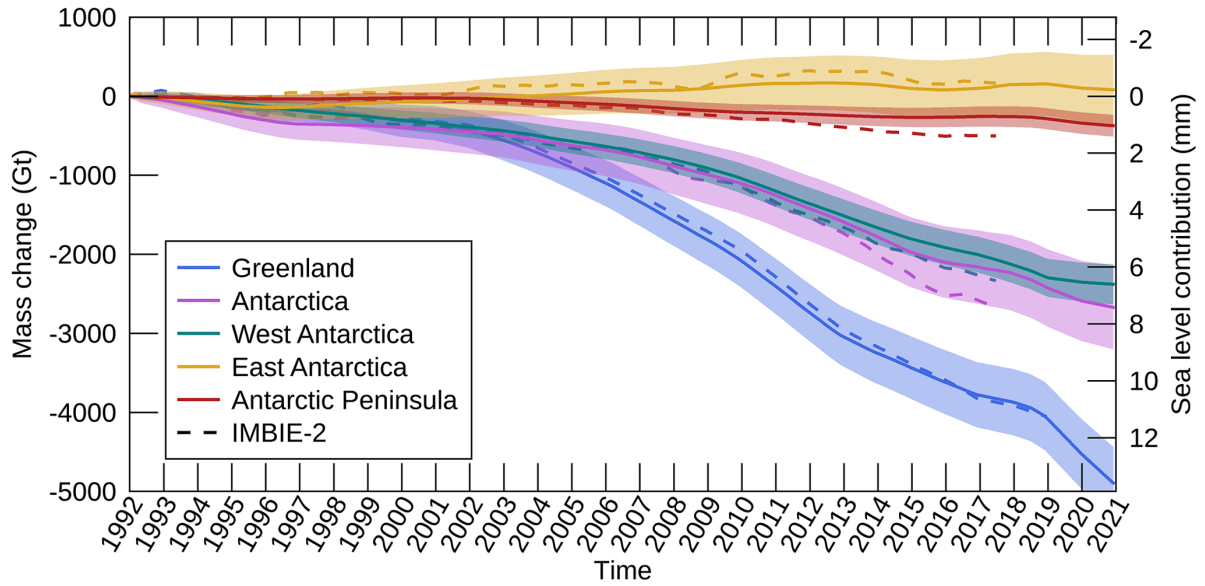


Figure 1.3: 40 years of mass balance on the Antarctic and Greenland Ice Sheets (Otosaka et al., 2023).

dence that they are in a state of balance. Indeed, by aggregating multiple estimates of mass change, derived in large part from satellite observations, the results of the Ice Sheet Mass Balance Intercomparison Exercise (Otosaka et al., 2023) have shown both the Greenland and Antarctic Ice Sheets to be losing mass. Between 1992 and 2020, the Antarctic Ice Sheet lost $2.671 \pm 0.53 \times 10^{12}$ tonnes of ice, raising sea global sea levels by an average of 7.4 ± 1.5 mm (Fig. 1.3). Furthermore, this rate of mass loss has grown larger over the last three decades - dominated by accelerated losses from the WAIS (Fig. 1.4). In Antarctica, where net changes to accumulation and ablation rates across the continent are subtle compared to, for example, the Greenland Ice Sheet or high mountain glaciers (e.g. Noël et al., 2019; Benn and Lehmkuhl, 2000), it is the evolving ice dynamic behaviour that is the key control on the ice sheet’s mass balance. (It is unclear whether this will remain true in the future under different possible changes to the climate.) Over the last half-century, the greatest changes to the dynamic behaviour seen in the WAIS have been in the Amundsen Sea Embayment (Fig. 1.5). Here, the discharge of ice into the sea by the Pine Island and Thwaites Glaciers, as well as those that feed the Crosson and Dotson Ice Shelves, has accelerated due to increased flow speeds (e.g. Mouginit et al., 2014a). As well as dominating current change, the dynamics of the WAIS is also believed to be the greatest source of uncertainty in future Antarctic mass-loss (Bamber et al., 2019).

The mechanism responsible for the mass imbalance of the WAIS is thought to be increases in

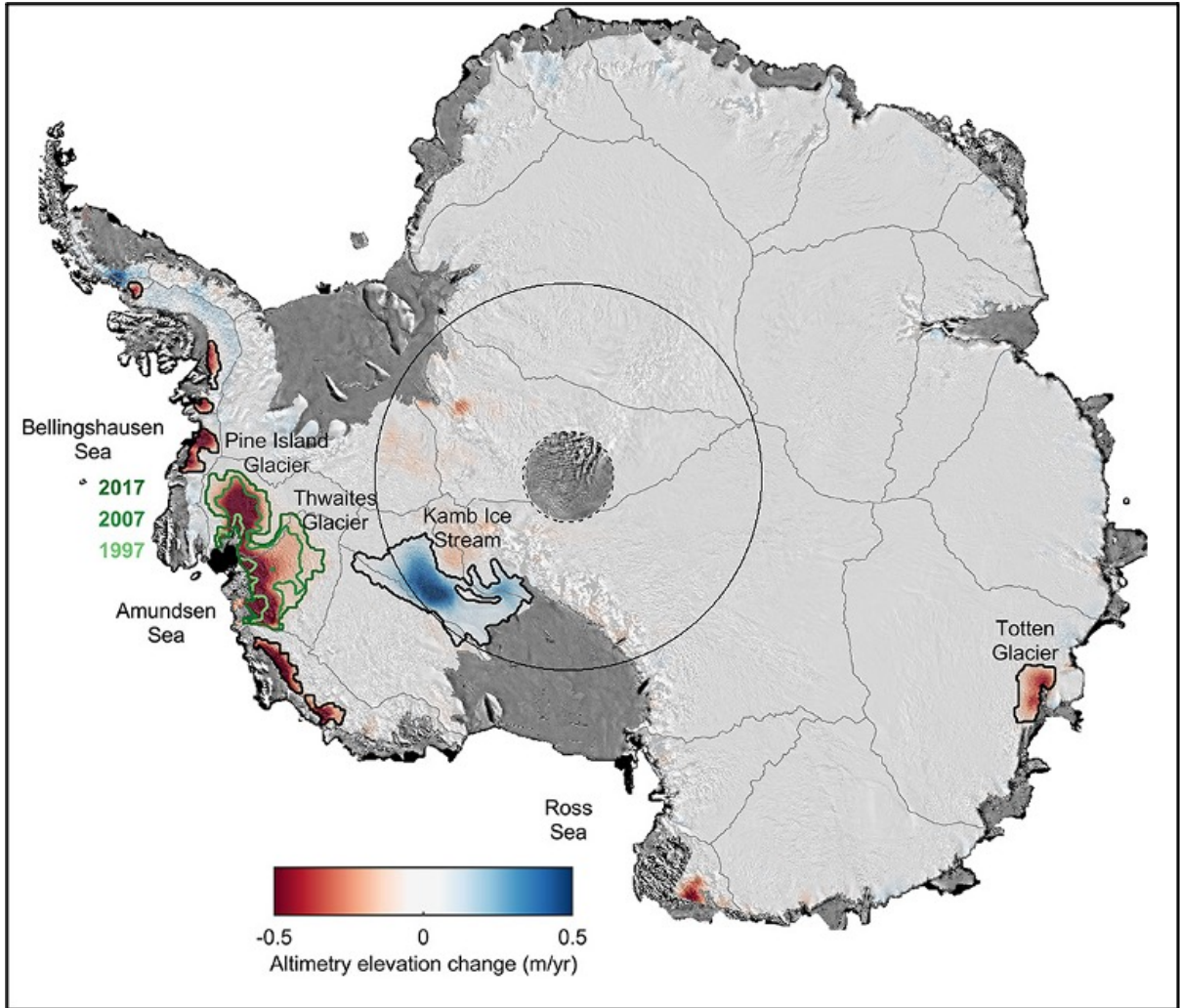


Figure 1.4: Surface elevation change between 1992 and 2017 (Shepherd et al., 2019)

ocean heat flux reaching the ice sheet margin in the ASE (e.g. Alley et al., 2015; Naughten et al., 2022). Over the last three decades, developments in our understanding marine ice sheet dynamics has shown why this might be the case.

1.2 A closer look at ice dynamics

The dynamics of ice sheets is best understood at large length- and time-scales as a theory of viscous flow. Ice is a shear-thinning material, where deformation largely occurs on the level of *grains* (mesoscopic, unevenly oriented crystals of ice) due to sliding at grain boundaries (grain boundary slip), or at smaller lengthscales due to the movement of crystal dislocations through the lattices of individual grains (dislocation creep). At very low stresses there is additional creep at the sub-grain level by the diffusion of molecules along crystals or grain boundaries - such as in parts of the Earth's mantle (Gordon, 1965). The governing principal of this kind of

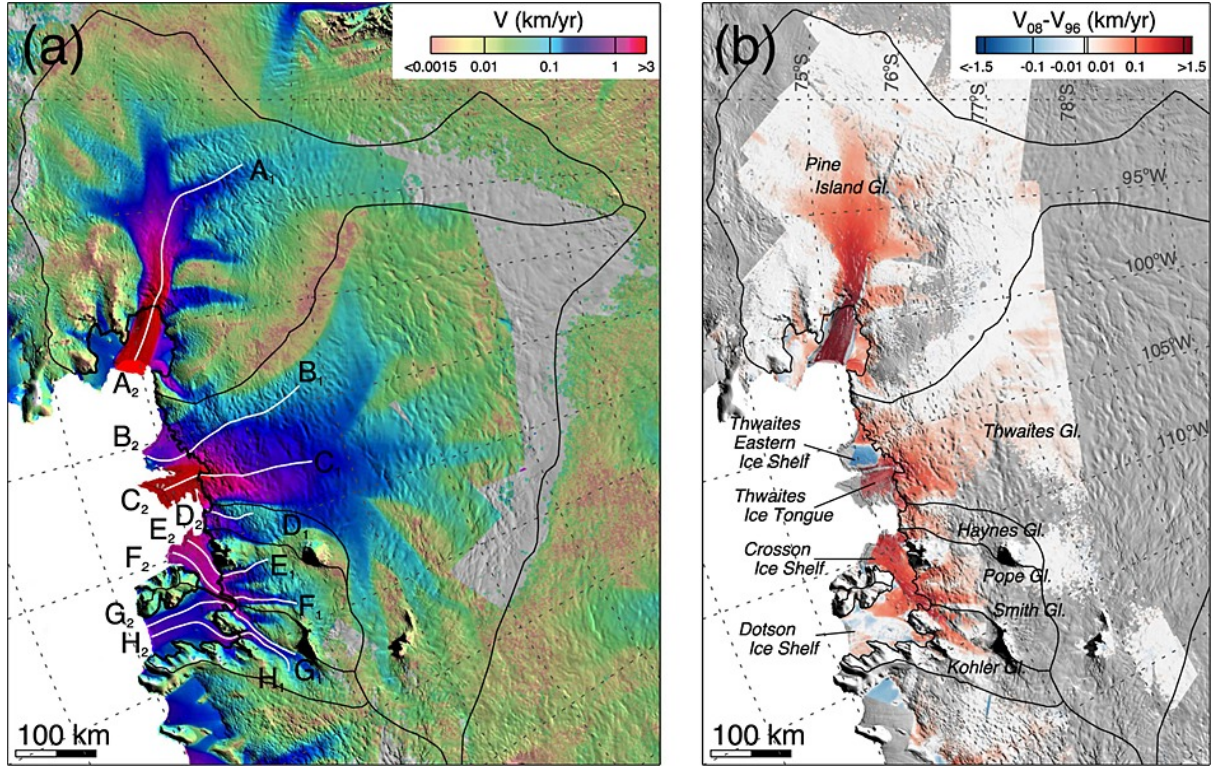


Figure 1.5: Changes in flow speed in the Amundsen Sea Embayment between 1996 and 2013 (Mouginot et al., 2014b).

continuum ice dynamics is that gravity/buoyancy apply an external stress that drives the flow of the ice, while viscous stresses associated with the creep behaviours described above and some combination of frictional stress, caused by the sliding of the ice over bedrock, and vertical shear stress within deformable sub-glacial till *resist* the flow of the ice.

Early conceptions of this balance assumed that resistive stresses were dominated by vertical shear within the ice column. In such a formulation, the speed of the glacier flow is determined by the local surface slope with the longitudinal transmission of stress, and sliding at the base, assumed to be negligible. This kind of approximation (referred to as the ‘shallow-ice approximation (SIA)’ when vertically-integrated) is appropriate to the slow motion of grounded continental ice masses over many millenia, where thickness changes are diffusive and respond slowly to changes in external forcing (e.g. Hutter, 1983). It became clear towards the end of the 20th century, however, that the transport of ice from the interior of Antarctica is not dominated by this kind of flow. Instead, the great majority (up to 90%) of ice transport from the interior ice sheet is associated with rapidly sliding ‘ice streams’ (Bamber et al., 2000; Bennett, 2003). These are thick, fast-flowing channels of ice that terminate at the ice sheet margin, such as the

Thwaites and Pine Island Glaciers, mentioned above.

Due to the dominance of ice streaming in mass-transport, especially near the coast, the study of ice sheet dynamics in the short term can be reasonably reduced to the study of ice *stream* dynamics. The SIA does not work for this kind of fast, channelised flow as the stress balance is dominated not by vertical shear but by *sliding* at the base of the ice and, the lateral transmission of horizontal shear stress. Furthermore, in the early 2000s it was becoming apparent that the ice shelves extending from the termini of many ice streams were dynamically coupled to the ice streams behind, and the grounding line was more than a shared, passive boundary. The interpretation of the stress balance in terms of sliding and horizontal shear enables a unified treatment of ice shelves and ice streams. Increased grounding line discharge is ultimately linked to the ocean-induced melting of ice shelves via the effect of *ice shelf buttressing*, which is understood through the lens of this coupled shelf-stream flow.

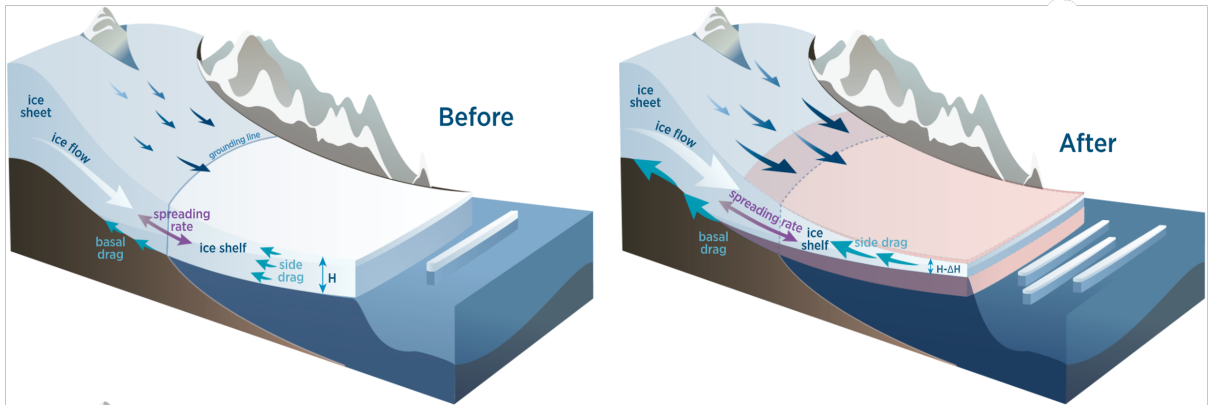


Figure 1.6: The effect of thinning on ice shelf buttressing, adapted from (Gudmundsson et al., 2019).

1.2.1 Ice shelf buttressing and ocean-induced thinning

At the turn of the 21st century, it was unclear whether ice shelves were able to influence grounded ice flow, in part because the dynamics of the two regions appeared to be governed by very different stress balance, with only a small region of overlap (Hindmarsh, 1996). However, the disintegration of the Larsen-B Ice Shelf in 2002 and the subsequent acceleration of its tributary glaciers (Scambos et al., 2004; Rignot et al., 2004) provided compelling evidence that ice shelves were important in regulating the dynamics of grounded ice. Schoof (2007a) showed that a finite-sized region around the grounding line in which basal shear stresses and

longitudinal stresses are both relevant to the momentum balance indeed couples the dynamics of ice shelves and slip-dominated grounded ice. However, this work also indicated that, in the approximation of no lateral spreading, no drag on pinning points and no horizontal shear, the presence or absence of ice shelves does nothing to alter the stress at the grounding line. In the context of Antarctic ice shelves, these approximations are rarely relevant. Instead, most ice shelves are confined to embayments, where drag of the ice shelf on sidewalls, or shearing at the margins does alter the stress balance of grounded ice. Similarly, many ice shelves are ‘pinned’ on bathymetric peaks that cause drag at the ice shelf base. Finally, ice shelves are not one dimensional and spread out due to buoyancy at the calving front, which exerts a pulling force on the grounded ice. These competing effects, when taken together, are known as *buttressing* (Fig. 1.6).

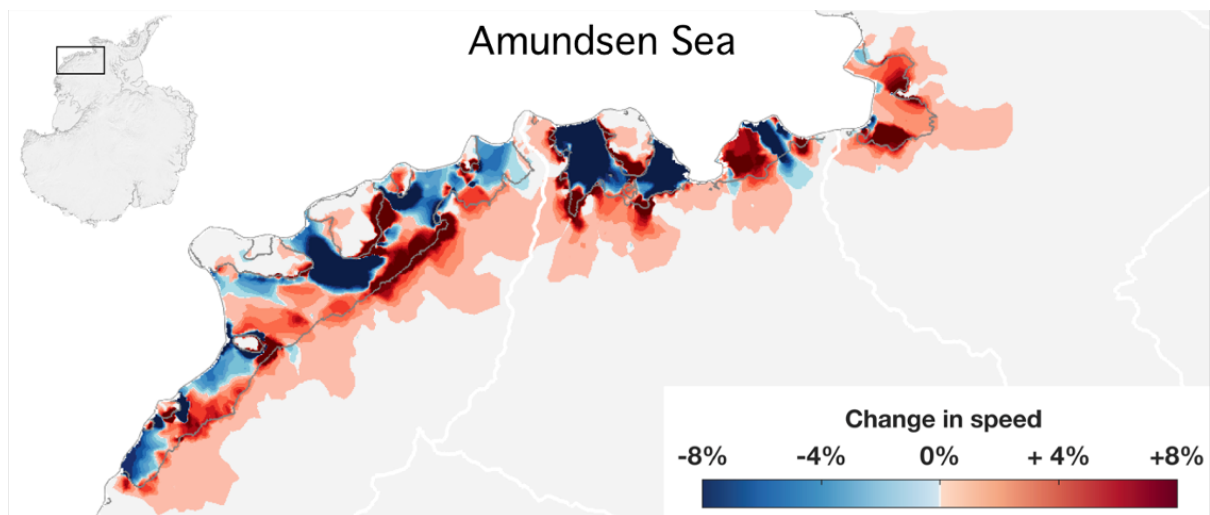


Figure 1.7: Computed instantaneous changes in speed given thinning between 1994 and 2014 (Gudmundsson et al., 2019); to be compared with figure 1.5 b.

Numerical studies in the 2010s demonstrated definitively that the buttressing of grounded ice streams by confined ice shelves in Antarctica is a pervasive and important phenomenon that reduces the flux of grounded ice into the ocean (e.g. De Rydt et al., 2015; Fürst et al., 2016). A combination of observational evidence (Pritchard et al., 2012) and numerical modelling studies (e.g. Gagliardini et al., 2010; Gudmundsson et al., 2019, Fig. 1.7) have built up a picture suggesting the dominant reason for the mass imbalance in the WAIS is due to the dynamic impact of reduced ice shelf buttressing caused by ocean-induced sub-shelf melting.

1.2.2 Dynamic instability in the WAIS

The particular reasons behind the propensity of the WAIS for rapid deglaciation might go beyond its sensitivity to ocean forcing, and include potential instabilities specific to the dynamics of marine ice sheets.

Over recent decades, a significant amount of work on the dynamics of the WAIS has involved understanding the Marine Ice Sheet Instability (MISI) hypothesis which states that grounding lines on submerged, retrograde beds (beds that slope down towards the interior of the ice sheet) exhibit no stable steady states and, hence, undergo rapid retreat. A heuristic proposition for this was made in the mid-1970s (Weertman, 1974; Thomas and Bentley, 1978) before being formalised for the case of rapidly-sliding, laterally unconfined ice streams and shelves in the mid-2000s (Schoof, 2007b; Schoof, 2007a). The essence of the argument is that if flux of ice over the grounding line increases with ice thickness and hence bed depth, then inland perturbations of grounding line position to deeper beds will cause accelerated thinning and further retreat - providing accumulation rates in the interior do not increase to counteract this thinning. Since 2007, certain assumptions in this model have been relaxed, showing MISI to be specific to a subset of marine-ice sheet geometries, namely those in which the buttressing effect of the ice shelves is small (Gudmundsson, 2013; Haselhoff and Sergienko, 2018a), where longitudinal stress gradients are small in the momentum balance compared to basal shear stresses (Sergienko and Wingham, 2019) and where bed topography varies on a large scale compared to the thickness of the ice at the grounding line (Sergienko and Wingham, 2022). However, numerical models have indicated that parts of the WAIS are susceptible to this phenomenon given our current knowledge of basal topography under continued forcing (Rosier et al., 2021; Reese et al., 2023). This is important for understanding the future evolution of the WAIS as, though its dynamics are currently tied closely to changes in ocean forcing, this kind of instability opens up the possibility of deglaciation even after a reversal of trends in such forcing. (It should be noted that, though MISI is considered a purely dynamic phenomenon, the presence of steady state grounding line positions and their stability is dependent on the accumulation/ablation rates across the ice sheet. For example, net ablation in the region of a retrograde bed could result in stable steady-state grounding line positions. Though this consideration is academic in the context of the WAIS which does not exhibit regions of negative surface mass balance (Mottram

et al., 2021), it could be relevant for marine glaciers in the Greenland Ice Sheet.)

1.3 Fractures in the system

Crevasses are some of the most dramatic and beautiful structures on the ice sheets. As well as providing eye candy, they play an important role in our understanding of the ice sheet system. A particularly dramatic example, as shown in in Fig. 1.8, is the potential hazard they pose for people and machinery carrying out fieldwork or exploration. In addition, surface crevasses, and surface depressions resulting from crevasses at the base, have implications for glacial surface hydrology by providing places in which water can pool and pathways for water to reach englacial aquifers or the glacier bed. Crevasses also alter the energy balance of the ice sheet by increasing the surface area for incident radiation and through their ability to trap some of that radiation. Fracturing processes are also responsible for the calving of ice from glacier termini, which is responsible for over half of the ablation from the ice sheet (Davison et al., 2023). This might have particular relevance to potential instabilities in the WAIS (Alley et al., 2023). Finally, and of greatest interest in this thesis, the presence of crevasses can induce a mechanical weakening of ice - particularly on ice shelves.

In a flow regime in which the momentum balance is dominated by horizontal gradients in longitudinal and shear stress, e.g. an ice shelf, the presence of crevasses can alter the dynamics significantly. This is because stresses cannot be effectively transmitted across regions of highly fractured ice and, for example, must be concentrated in the bands of intact ice below partial-thickness crevasses (or above partial-thickness basal crevasses). Similarly, the shear stress in the margins of a fast-flowing ice shelf is diminished when when these margins ‘soften’ due to the cumulative effect of microfracturing or the presence of macroscopic fractures.

As discussed, the dynamics of the AIS is highly sensitive to changes in ice shelf buttressing. For an ice shelf without ice rumples, the majority of this buttressing occurs in the shear margins (where gradients in viscous stress are large). Hence, material weakening in these regions due to the presence of fractures is likely to reduce the buttressing capacity of an ice shelf. In this way in particular, the fracturing of ice could have a distributed effect on the ice sheet’s evolution. This



Figure 1.8: A Sno-Cat stuck in a crevasse during the Commonwealth Trans Antarctic Expedition of 1957/58 (credit: *The Crossing of Antarctica*, G. Lowe and H. Lewis-Jones *Thames and Hudson*).

is an inherent nonlinearity in the dynamics of the ice sheet as crevasses form, in large part, due to changes in strain rates resulting from external stresses. Pertinent examples of this are, again, Pine Island and Thwaites Glaciers, where shear margins on their respective ice shelves have undergone a recent period of structural decline concurrent with increased flow speed across the ice shelves (Lhermitte et al., 2020). It has been shown in the context of the Larsen Ice Shelves that crevasses affect both the flow of the ice shelf and its buttressing capacity (Borstad et al., 2012; Borstad et al., 2013).

On the whole, ice sheet models do not do a good job at representing either the fracturing of ice, or the effect that fractures have on ice flow. Given that ice dynamics has largely developed as a continuum theory of viscous flow, certain compatibility issues arise when attempting to incorporate discontinuous processes such as material failure within the numerical models that implement this theory. There are ways of treating the ice sheet outside of this paradigm of continuum mechanics - for example, discrete element models are useful for the analysis of material failure under stress at the margins of particular glaciers of interest (Åström et al., 2013; Benn

et al., 2017). Additionally, brittle processes, governed by the elasto-plastic properties of the ice, occur on very short timescales compared to the large-scale flow of ice sheets and ice streams - making the two regimes difficult to reconcile within the same model.

A simple approach to dealing with these issues is that of continuum damage mechanics. In this perspective, it is assumed that the important effects of fracturing amount to the effect of material discontinuities on the bulk effective ice viscosity. This can be parameterised using a *damage* field that varies on spatial scales similar to the strain rate field, and evolves in time according to some function of stresses or strain rates in the system. Pralong and Funk (2005) introduced a continuum damage model to describe the mesoscopic ($\sim 10 - 100$ m) effect of the accumulation of microcracks/microvoids within the ice in this manner, and considered damage fields that act on the effective viscosity both isotropically and anisotropically. Later, similar considerations were applied to macroscopic damage amounting to the formation of rifts and calving of in the context of the Larsen B and C Ice Shelves (Borstad et al., 2012; Borstad et al., 2013). The former study made a link between damage and a control parameter that viscous flow models use to describe the softness of ice compared to that expected at a give temperature (more on this later). Following this, Albrecht and Levermann, 2014 developed an isotropic damage model representing the macroscopic effect of ‘fracture density’ on the flow of ice shelves. Since then, other continuum damage models have been developed along these lines. For example, that of Sun et al., 2017 which treats macroscopic fracturing as inherent to the constitutive relation of ice flow, such that the damage field and velocity field can be solved for concurrently - negating the difficulty surrounding the different timescales over which the damage field evolves compared to thickness/ice speed.

Such continuum approaches have obvious benefits, though their potential efficacy is uncertain in two ways:

1. It is not obvious whether a scalar damage field can suffice to represent the important effects of existing fractures, on a number of lengthscales, and what form that damage field should take.
2. It might be that there is be no consistent and sensible way evolve the damage field in a continuum model.

This latter point is due to the fact that in reality the evolution of fractures is not just governed by the bulk material properties of the ice and the far-field stress conditions, but also by stress concentrations brought about by the geometry of existing fractures.

In general, a greater amount and quality of data regarding fractures on the Antarctic Ice Sheet is required to understand the how fractures interact with ice dynamics. Similarly, the assimilation of these observations with existing numerical ice flow models is required to understand how to represent this interaction. These are both issues that, to some extent, this thesis aims to address.

The remainder of this introduction is dedicated to certain methods I have used to arrive at the results presented in this thesis: modelling ice dynamics, inverse problems, satellite imagery and deep learning. The results chapters are reproductions of papers either published or in draft, and so contain specific methods sections of their own. Hence, I have not written a detailed methods section for this thesis, instead the space below is used to review some fundamental aspects of these techniques.

1.4 Modelling ice dynamics

The continuum mechanics formulation of ice dynamics aims to characterise the different stresses acting on the ice sheet that balance the externally applied ‘driving’ stress. A full description of the stresses acting on a control volume of ice is given by the Cauchy stress tensor $\sigma(x)$. This is often partitioned into a traceless, deviatoric component $\tau(x)$ and a pressure field $P(x)$. The set of equations representing balance in angular and linear momentum in the ice can be written in terms of these quantities, respectively, as:

$$\sigma - \sigma^\top = 0, \quad (1.1)$$

$$\nabla \cdot \tau - \nabla P + \rho \mathbf{g} = 0. \quad (1.2)$$

The latter is the Cauchy momentum equation with the added assumption that inertia is negligible. Here, ρ is the density of ice and $\mathbf{g}$ is the acceleration due to gravity. The ice is also assumed to be incompressible, i.e. the velocity field $\mathbf{u}$ is divergence-free:

$$\nabla \cdot \mathbf{u} = 0. \quad (1.3)$$

Without making certain assumptions about how the components of the deviatoric stress and pressure relate to each other, the system described by these equations is underdetermined. It is common to take ice velocity as the state variable of interest and solve for this directly. This makes use of a *constitutive relation* that relates components of the deviatoric stress to gradients in the components of $\mathbf{u}$ (specified by the strain rate tensor $\dot{\epsilon}$, with components $\dot{\epsilon}_{ij} = \frac{1}{2}(\partial_i u_j + \partial_j u_i)$), and a *sliding law* that relates basal stress to ice velocity at the base. A common choice of isotropic constitutive relation which embeds the incompressibility condition is *Glen’s*

flow law (Nye and Mott, 1953; Glen and Perutz, 1955):

$$\dot{\varepsilon}_{ij} = A(T)\tau^{n-1}\tau_{ij}, \quad (1.4)$$

where $A(T)$ is a temperature-dependent rate factor and the exponent is often chosen to be $n \approx 3$, which describes creep at intermediate levels of stress. Equation (1.2), when written in terms of flow velocity is called the Stokes equation. Solving the Stokes equation (or an approximation there-of) is the aim of a continuum ice dynamics model. To determine how the thickness of the ice sheet evolves, this system is coupled with a conservation of mass equation:

$$\frac{\partial h}{\partial t} + \nabla \cdot (h\mathbf{u}) = a \quad (1.5)$$

where h is thickness and a is the net accumulation rate, derived from eq (1.3).

Some models solve the full Stokes equations in three dimensions (e.g. Larour et al., 2012; Gagliardini et al., 2013), while others make use of approximations regarding the dominant components of the momentum balance to solve vertically-integrated equations for the two horizontal components of velocity (e.g. Gudmundsson et al., 2012; Winkelmann et al., 2011). These assumptions generally reflect the “shallow ice” or “shelfy-stream” approximations referred to in section 1.2. These are considered ‘first-order’ approximations as only the very largest components of the Cauchy momentum equations are retained when expanded in terms of some length or stress scale. Finally, ‘higher order models’ sit somewhere between, for example by taking the shelfy-stream momentum balance, but including the contribution of vertical shear in the effective viscosity and flux (Schoof and Hindmarsh, 2010). (Many models are able to solve a number of different formulations of the momentum balance equations.) I use the BISICLES ice sheet model in this thesis which, though capable of solving such a higher-order system of equations, I choose to solve the first-order shelfy-stream system in the two-dimensional, vertically-integrated setting. This system is derived in Appendix A. The equations take the following two-dimensional form:

$$\nabla \cdot [\phi h \bar{\mu} (\nabla \mathbf{u} + (\nabla \mathbf{u})^\top + 2(\nabla \cdot \mathbf{u})\mathcal{I})] - Cf(\mathbf{u}) - \rho_i gh \nabla s = 0 \quad (1.6)$$

where C and ϕ are prescribed control fields representing, respectively, the slipperiness of the ice-bed interface (zero for ice shelves) and an enhanced stiffness of the ice, and $f(\mathbf{u})$ is a sliding

law which is given different forms at different points throughout this thesis. The field $\bar{\mu}$ is the strain-rate-dependent, vertically-integrated effective ice viscosity.

There are analytical solutions to various approximations of the momentum balance equations for marine ice sheets in certain simplified cases. For example, one-dimensional ‘flowline’ models have been used to great success to analyse the stability of grounding line locations (e.g. Schoof, 2007b; Haselhoff and Sergienko, 2018b). However, the simulation of realistic ice sheets requires the use of *numerical* models. As I have discussed, the models are differentiated by the particular approximation to the Stokes equations they solve and in how many dimensions. Additionally, they differ by the manner in which the equations are solved. In general, solutions are found by either discretising the equations (‘finite difference’ or ‘finite volume’ methods, such as BISICLES) or by discretising the space of solutions to the model equations (‘finite element methods’) and reducing the problem to one solvable by matrix-algebra methods. Additionally, certain models, including BISICLES, have the ability to change the resolution of the mesh on which the equations are solved in a spatially non-uniform way during timestepping. This is useful in resolving key dynamic boundaries like the grounding line while keeping the computational cost of the simulation low. I do not make use of this feature during the thesis, though it was one of the reasons BISICLES was created.

1.5 Inverse problems and ill-posedness

The velocity found by solving eq. (1.6) is highly sensitive to C and ϕ . In simulations of the ice sheet by models like BISICLES, *control fields* like these are prescribed *a priori* having been inferred from observations of ice speed by solving the ‘inverse problem’. A significant proportion of the modelling presented in this thesis concerns this process.

Inverse problems have been common feature of ice sheet modelling since the introduction of control methods for solving them in the early 1990s (MacAyeal, 1992; MacAyeal, 1993). (Different methods have been proposed since, e.g. that of Arthern and Gudmundsson, 2010, but variants on the framework outlined in MacAyeal, 1993 have been the most widespread, and include the approach normally taken by users of BISICLES.) A particular focus has been placed

on inferring basal shear stress from ice speed data derived from remote sensing observations, with the aim of initialising transient simulations of the ice sheet (e.g. Gillet-Chaulet et al., 2012; Cornford et al., 2015; Hill et al., 2021), or learning something about the physics of ice stream sliding/till deformation (Joughin et al., 2004; Joughin et al., 2009; Sergienko and Hindmarsh, 2013; Gillet-Chaulet et al., 2016) (perhaps making use of ice speed data at different times). It is also common to use the same method to infer properties of the ice rheology (either the ϕ of equation (1.6) or equivalent) (Borstad et al., 2012; Petra et al., 2012; Cornford et al., 2015).

Inverse problems such as this are not unique to glaciology and are ubiquitous throughout science and engineering. In general, if a forward model maps control parameters to model states:

$$\mathcal{M} : \mathbf{q} \mapsto u \tag{1.7}$$

then the inverse problem aims to map possibly sparse, possibly noisy data on the true state to estimates of the control parameters:

$$\mathcal{M}^{-1} : u_{\text{obs}} \mapsto \mathbf{q}_{\text{est}}. \tag{1.8}$$

Many problems like this are *ill-posed* - as defined in contrast to a *well-posed* problem. There are three conditions, usually attributed to Jacques Hadamard, that describe what it means for a problem to be well posed (Hadamard, 1902). These are:

1. the problem has a solution,
2. the solution is unique,
3. the solution depends continuously on the data.

The problem of estimating (C, ϕ) from observations of ice speed violates each of these conditions. Firstly, there are noisy observations of ice speed for which no set of control parameters satisfies equation (1.7). Secondly, we are attempting to find two control fields (C, ϕ) from one field of observations: ice flow speed. I.e. the problem is underdetermined. Finally, the solution to the inverse problem is highly dependent on noise in the input data, meaning that $\mathcal{M}^{-1}$ is discontinuous. In general, the source of this kind of discontinuity is related to the compactness

of the operator $\mathcal{M}$. In order to solve problems numerically, they must be discretised in some way, and multi-dimensional observational data is necessarily discrete. This last property of ill-posedness is, in the context of the discretised problem, determined by the *conditioning* of the discrete operator. Problems for which the conditioning is poor (and hence fail to be well-posed by the third of Hadamard’s conditions) are said to be *ill-conditioned*. (The fact that so many inverse problems in physics have the property of ill-posedness is a feature of how we construct theories describing macroscopic features of the world. For example, Laplace’s daemon does not differentiate forward and inverse problems and neither is more difficult to solve than the other. We do because many of the laws we construct to describe coarse-grained phenomena (such as temperature and the flow speed of a fluid), and attribute ‘forwardness’ to, are inherently information destroying.)

In order to deal with the first two issues, many inverse problems are cast as a least-squares optimisation of the form:

$$\mathbf{q}_{\text{est}} = \underset{\mathbf{q}'}{\operatorname{argmin}} \left\{ |u_{\text{obs}} - \mathcal{M}(\mathbf{q}')|^2 \right\}, \quad (1.9)$$

and solved using some sort of gradient-based method. In this thesis, the fields are optimised using the nonlinear *conjugate gradient method*. This is similar to the *steepest descent* method (gradient descent with an optimal step-size calculated on each iteration), except that directions along which minima are sought are always *conjugate* (i.e. orthogonal with respect to the inner product defined by a positive-semidefinite matrix) to those along which minima have been sought before. This leads to considerably faster convergence than steepest descent, even in the nonlinear case. (The reader is referred to Shewchuk, 1994 for more details.) Making the assumption that noise in the data occurs according to a Gaussian distribution, the solution to an optimisation of the form (1.9) coincides with the maximum a-posteriori estimate of the control parameters.

Poor conditioning is intrinsic to the problem and does not depend, for example, on the quality of the data u_{obs} . A significant amount of the theory regarding ill-conditioned problems deals with linear operators, but many of the insights extend to more general problems. In such cases, it is the spectrum of singular values (or eigenvalues if the problem admits them) that determines

the conditioning of the problem. An archetypal illustrative example is the following:

Consider the one-dimensional heat-conduction equation describing the evolution of the temperature θ in a material of thermal conductivity k and length L :

$$\frac{\partial \theta}{\partial t} = k \frac{\partial^2 \theta}{\partial x^2}, \quad (1.10)$$

subject to boundary and initial conditions:

$$\theta(x, 0) = f(x), \quad \theta(0, t) = \theta(L, t) = 0. \quad (1.11)$$

Writing the initial temperature profile as a linear combination of sinusoidal basis vectors:

$$f(x) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi}{L}x\right), \quad \text{where } A_n = \frac{2}{L} \int_0^L \sin\left(\frac{n\pi}{L}x\right) f(x) dx \quad (1.12)$$

leads to the well-known solution:

$$\theta(x, t) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi}{L}x\right) \exp\left(-k \left(\frac{n\pi}{L}\right)^2 t\right). \quad (1.13)$$

It can be seen that the operator $\mathcal{M}$ corresponding to the evolution in time of the temperature profile, subject to the particular dirichlet conditions specified, has eigenvectors $e_n(x) = A_n \sin\left(\frac{n\pi}{L}x\right)$ with corresponding eigenvalues $\lambda_n(t) = \exp\left(-k \left(\frac{n\pi}{L}\right)^2 t\right)$. Hence, the high-frequency (i.e. large n) components of the initial temperature profile are damped rapidly by the eigenvalues $\lambda_n(t)$. This is an example of the ‘information destroying’ nature of a macroscopic theory mentioned above - in the forward problem, we are happy for the information regarding highly oscillatory components of the temperature to diffuse away. The operator $\mathcal{M}$ in this example is equivalent to a diagonal matrix, so calculating its inverse $\mathcal{M}^{-1}$ is trivial. However, as we will see, the loss of this high-frequency information results in an ill-posed inverse problem.

Consider the inverse problem of estimating the initial temperature distribution $\theta(x, 0)$ from some data $\theta_{\text{obs}}(x, t)$ at time t subject to some noise ϵ . If we assume the noise obeys the same

dirichlet conditions, the data can be written:

$$\theta_{\text{obs}}(x, t) = \theta(x, 0) + \sum_{n=1}^{\infty} \epsilon_n e_n(x). \quad (1.14)$$

Then, given that the model is linear, the estimate of the initial state is:

$$\mathcal{M}^{-1}(\theta_{\text{obs}}(t)) = \theta(x, 0) + \sum_{n=1}^{\infty} \frac{\epsilon_n}{\lambda_n(t)} e_n(x) \quad (1.15)$$

$$= \theta(x, 0) + \sum_{n=1}^{\infty} \exp\left(\frac{k\pi^2}{L^2} n^2 t\right) \epsilon_n e_n(x). \quad (1.16)$$

Hence, high frequency components - with eigenvalues $\lambda_n(t)$ that become very small after a short period of time in the forward direction - can blow up if there are high frequency components in the noise (corresponding to non-vanishing coefficients ϵ_n with large n) in the reverse direction.

This example serves to show that the eigenspectrum of the forward model provides the information required to judge the conditioning of the inverse problem. In general, this is true of the spectrum of singular values of a linear operator. The condition number of such an operator is defined as the ratio of largest to smallest singular value, and low condition number corresponds to good conditioning.

There are similarities between this argument and that of Gudmundsson, 2003 showing that information regarding variations in basal shear stress is not preserved at the ice sheet surface. This embeds poor conditioning in the process of inferring C from surface speed data.

For a nonlinear problem such as ours - where we solve the optimisation problem of equation (1.9), it is the spectrum of the *Hessian matrix* (with components $\mathcal{H}_{ij}(\mathbf{q}'_{\text{est}}) = \frac{\partial^2}{\partial q_i \partial q_j} |u_{\text{obs}} - \mathcal{M}(\mathbf{q}'_{\text{est}})|^2$) that determines the conditioning of the problem. Small-magnitude singular values correspond to directions of low curvature of the cost function. Hence, gradient-based optimisation procedures either take a long time to converge, or converge to one of a number of solutions. Second order methods that use the Hessian matrix as part of the optimisation (e.g. Newton's method) require approximations to the inverse of $\mathcal{H}$. Hence, poor conditioning of the Hessian matrix can affect these methods as well.

Typically, to deal with ill-conditioning, regularisation is introduced to a problem like eq (1.9) (e.g. Hansen, 1994). These methods are often heuristic; explicitly designed to suppress the contribution of small-singular value components of the state estimate (often assumed to be high-frequency noise). For example, in the truncated singular value decomposition (TSVD) method, small singular values are simply removed from the calculation (Aster et al., 2013, p. 67). In *Tikhonov regularisation* - widely used in the ice sheet modelling community (e.g. Morlighem et al., 2013; Habermann et al., 2013; Brinkerhoff and Johnson, 2013; Cornford et al., 2015) - an additional term is added to the cost function representing the action of some linear operator L on the solution, so that eq. (1.9) becomes:

$$\mathbf{q}_{\text{est}} = \underset{\mathbf{q}'_{\text{est}}}{\operatorname{argmin}} \{ |u_{\text{obs}} - \mathcal{M}(\mathbf{q}'_{\text{est}})|^2 + \alpha^2 |L(\mathbf{q}'_{\text{est}})|^2 \} \quad (1.17)$$

where α is a ‘regularisation parameter’. Popular choices for L include the identity or the gradient operator which, respectively, penalise high amplitude or high frequency components of the solution. (In the former case, if $\mathcal{M}$ is a linear operator, this can be shown to be equivalent to replacing the singular values σ_n with $\sigma_n + \alpha$ - bounding the reciprocal of the SV’s at the top by $1/\alpha$ rather than $1/\sigma$ (Aster et al., 2013).)

An alternative way of looking at regularisation methods is that they make use of available prior information regarding the control parameters. (This link is particularly explicit when considering Tikhonov regularisation.) Chapter 5 of this thesis considers an illustrative example of this.

1.6 Remote sensing and Sentinel-1 Data

The introduction to this thesis so far has largely been concerned with introducing our physical and numerical models of the Antarctic ice sheet. However, this thesis is equally concerned with observational data.

The vast majority of the observations presented here are derived from remote-sensing data collected by satellites in near-Earth orbit. Over the last half-century, the use of satellites has

enabled us to monitor an increasing number of ice sheet features and physical properties at a scale that would be impossible using in-situ measurements or airborne remote sensing techniques. This includes passive remote sensing in which satellites collect solar radiation reflected from the Earth’s surface or black body radiation emitted at the surface, and active remote sensing in which satellites emit radiation and measure the reflected return signal. Optical and radar imagery from satellites has enabled us to visually monitor the extent of ice masses across the world (e.g. Bayr et al., 1994; Swithinbank, 1988; Bindshadler et al., 2008; Earl and Gardner, 2016; Andreasen et al., 2023), as well as to look in detail at surface features and processes occurring on glaciers and ice sheets such as debris cover (e.g. Shukla et al., 2009; Nagai et al., 2013), surface water (e.g. Mote et al., 1993; Kingslake et al., 2017) and calving (e.g. Fricker et al., 2002; Hogg and Gudmundsson, 2017). A number of important physical parameters can be derived from optical and radar data, including ice sheet surface elevation change (e.g. Schröder et al., 2019; Nilsson et al., 2022), grounding line location (e.g. Scambos et al., 2007; Rignot et al., 2011a) and ice flow velocity (e.g. Rignot et al., 2011b; Joughin et al., 2010).

Over recent decades, Synthetic Aperture Radar (SAR) data has become arguably the most useful tool for the large-scale extraction of each of these three datasets (except from, perhaps, surface elevation change which has competition from laser altimetry). In April 2014 and 2016 respectively, the European Space Agency launched the Sentinel-1 A and B satellites as part of the European Union’s “Copernicus” Earth observation programme. The C-band SAR instruments carried by both have illuminated a vast area of Antarctica, including almost the complete coastline, at a maximum spatial resolution of 5 m in the ground range and 20 m in the azimuth directions. These acquisitions are weather- and solar-illumination-independent and have a repeat period of 12 days for each satellite. Taken together, prior to the failure of Sentinel-1 B in December 2021, the satellites offered a repeat period of 6 days over much of Antarctica. The majority of the observations in this thesis make use of this SAR data from the Sentinel-1 satellites. Specifically, for the extraction of glaciological features visible in images of surface backscatter, and the extraction of surface displacements from pairs of such images. It is useful to have an idea how backscatter images are acquired and processed by the satellite sensor, and I give a high-level summary of this process below.

1.6.1 How do the Sentinel-1 satellites make a SAR backscatter image?

Each of Sentinel-1 A/B is an active microwave remote sensor. As the satellite travels along its flight path, it transmits pulses of microwave radiation towards the Earth in a ‘side-looking’ orientation, and collects the reflected radiation (figure 1.9). The intensity and phase of the returned signals are recorded along with their total travel time. The relative intensity of the return signal is dependent on the properties of the scattering surface within each resolution cell, and constitutes the ‘backscatter’ measurement, while the travel time, phase and frequency information are used to locate the return signal in radar geometry.

The Sentinel-1 satellites use a phased array of individual transmit-receive (T/R) modules along the antenna to steer the pulsed microwave beam in a particular direction. This works by applying a phase offset to the T/R modules such that constructive interference occurs along the intended direction. For simple real aperture radars (RAR) that operate in this way, the range resolution is proportional to the pulse duration while the azimuth resolution (an angular resolution analogous to that of a photographic camera) is determined by the along-track width of the beam footprint. This is a function of the beamwidth (the angle separating the full-width half maxima of the beam) and the range. Even with powerful beam steering, the beamwidth is limited by Fraunhofer diffraction that occurs due to the finite size of the antenna, leading to a wide footprint from orbital altitude. Synthetic aperture radar is a method of dealing with the associated poor azimuth resolution by synthesizing coherent, repeated echos from the same location on the ground.

The essence of this process is that the distance from a particular location on the ground to the antenna changes between pulses - imparting a phase shift to the returned signal that depends on the azimuth angle of the pulse (defined with respect to some reference, e.g. the zero-doppler-shift location). For a given look angle, the sequence of phase shifts over the each section of the ground being imaged by the sensor (‘phase history’) is unique to each azimuth angle. This angle can then be discerned, for example, with a matched-filtering method given a model of the phase history at different angles.

In addition to the aperture synthesis used to increase azimuth resolution, many operational

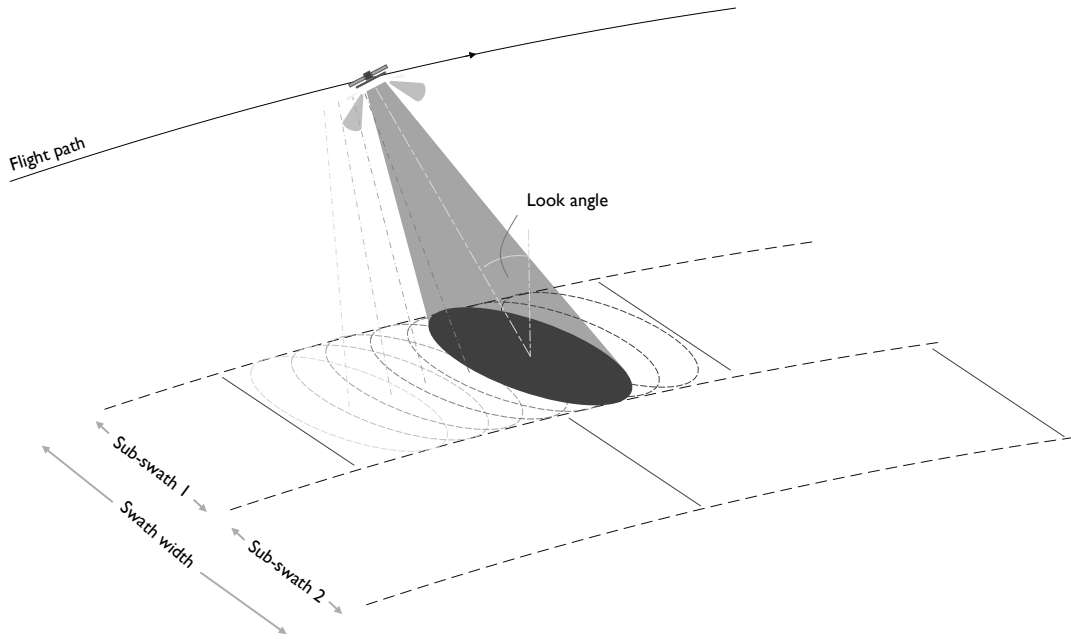


Figure 1.9: SAR imaging.

SAR satellites like Sentinel-1 A/B use longer, ‘chirped’ pulses to boost range resolution. In such a process, frequency is swept over some bandwidth during a pulse and match-filtering is applied to the return signal using a time-reversed version of the original pulse. This picks out individual echos in the return signal that might not be discernible in a single-frequency pulse.

The way in which the beam is steered to produce overlapping acquisitions has implications for the image size and azimuth resolution as well as the specific algorithms used to synthesize the data. Typical ‘imaging modes’ include emitting the beam at fixed look angle ensuring the footprints overlap on the ground (“stripmap mode”) and steering the emitted beam in the azimuth to illuminate the same region on the ground (“spotlight mode”). To increase the size of the swath (the total width of the area illuminated by the beam as the sensor passes, measured in the ground-range direction), smaller ‘subswaths’, separated in beam elevation, are often combined. A method that achieves this is “ScanSAR” where small bursts of pulses are emitted to image a section of a subswath before the beam elevation is changed to image a section of an adjacent swath. The ‘interferometric wide swath’ images acquired by the Sentinel-1 satellites that we use in this thesis use a variant of this imaging mode called Terrain Observation by Progressive Scans (TOPS) where the beam is steered in azimuth during each burst as well as elevation.

The uses of focussed SAR data in this thesis also require the projection of the backscatter from slant-range (the distance between antenna and target) to ground-range geometry using a digital elevation model of the ice sheet surface. This ‘geocoding’ process results in the backscatter images used for in the feature tracking algorithms which look for displacements between sequential images, and - after some resampling to equate range and azimuth resolution - the crevasse-mapping algorithms described in chapter 2 and 3.

1.7 Deep learning and neural networks

The early chapters of this thesis employ machine learning techniques developed largely in the field of computer vision. Essentially, this is to accelerate the extraction of data from satellite imagery compared to the rate at which a human could perform such tasks. The particular machine learning techniques employed come from a generation of methods termed ‘deep learning’ which use neural network structures to approximate complex maps between input data and representations of that data we might have use for. The two key elements that enable the enormous adaptability and utility of neural networks are: 1) their universal ability to approximate smooth functions given the small set of operations they employ, and 2) the ability to calculate gradients of performance in the desired task with respect to the parameters of the model. This second feature enables the use of simple gradient-based methods to optimise the parameters in these complex models. Without this capacity for ‘self-design’, the universality of the space of neural networks is of no greater use than, for example, the knowledge that smooth functions can be approximated by polynomials, discovered in the late 19th century. In sections 1.7.1 and 1.7.2, I will briefly describe the structure of neural networks that enables this universality, and the process by which gradients are calculated - a method which has broader application in ice sheet modelling.

Deep learning has its origins in the work of computer scientists exploring the computational processes of the human brain in the first half of the 20th century. It has proven a remarkably flexible tool, capable of finding elaborate structures in high-dimensional data that can take a variety of forms. Consequently, it has been well utilised in a number of different domains including medical image analysis, perception by autonomous vehicles and the currently notable

field of natural language processing.

Glaciology and remote sensing have not traditionally been at the forefront of usage and development of DL tools unlike, for example, the field of medical imaging or natural language processing driven by commercial application. However, in the last two decades, the employment of neural networks within glaciology has accelerated dramatically. This is concurrent with an increase in the computational resources at the disposal of research institutions, the increasing availability of large amounts of routinely acquired data, the proliferation of open source libraries with which deep learning methods can be implemented, and the demonstration of techniques in adjacent fields. Early studies included Steiner et al., 2005 who used a neural network to reconstruct the mass balance of an alpine glacier from temperature and precipitation data, and Reusch et al., 2005 who experimented with using neural networks and self-organising maps to reconstruct synoptic climate patterns from ice-core data from central-West Antarctica. At present, neural networks are most often used in glaciology in one of two ways: 1) to emulate numerical models, or particular physical processes within numerical models or 2) to extract data from remote-sensing observations. Examples of the former application include the emulation of Stokes/approximations-thereof solvers using convolutional neural networks, operator neural networks and physics-informed neural networks (e.g. Jouvét et al., 2022; Wang et al., 2022; He et al., 2023; Jouvét and Cordonnier, 2023), the creation of fully parameterized models of the evolution of mountain glaciers (Bolibar et al., 2020), solving complex inverse problems in different contexts (e.g. Brinkerhoff et al., 2021; Jouvét, 2023) and parametrizing complex sub-grid level processes such as ice shelf basal melting (Rosier et al., 2023).

Efforts to extract data from remote-sensing observations have largely been concerned with the segmentation of optical or radar satellite images to extract features visible to the human eye. For example, the classification of the surface on and around particular glaciers of interest, and mapping their extent using multispectral satellite data (e.g. Karimi et al., 2012; Haireti Alifu and Johnson, 2015; Shukla and Yousuf, 2017). A task that became a focus of interest, and ultimately demonstrated the capacity for deep learning to aid in generating continent-scale datasets, was the extraction of glacier calving front locations using UNet-like network architectures (Mohajerani et al., 2019; Zhang et al., 2019; Baumhoer et al., 2019). Following

this, these kinds of convolutional architectures have been applied to a number of problems including the extraction of grounding lines from interferograms over the margin of the Antarctic Ice Sheet (Mohajerani et al., 2021), the detection of surface melt on Antarctic ice shelves (Hu et al., 2021; de Roda Husman et al., 2024), the delineation of supraglacial lakes on the Antarctic Ice Sheet (Dirscherl et al., 2021) and measuring the sizes of large icebergs (Braakmann-Folgmann et al., 2023). It is this kind of task, using satellite data and convolutional encoder-decoder network architectures, that is the concern of chapters 2 and 3 of this thesis.

1.7.1 The structure of a neural network

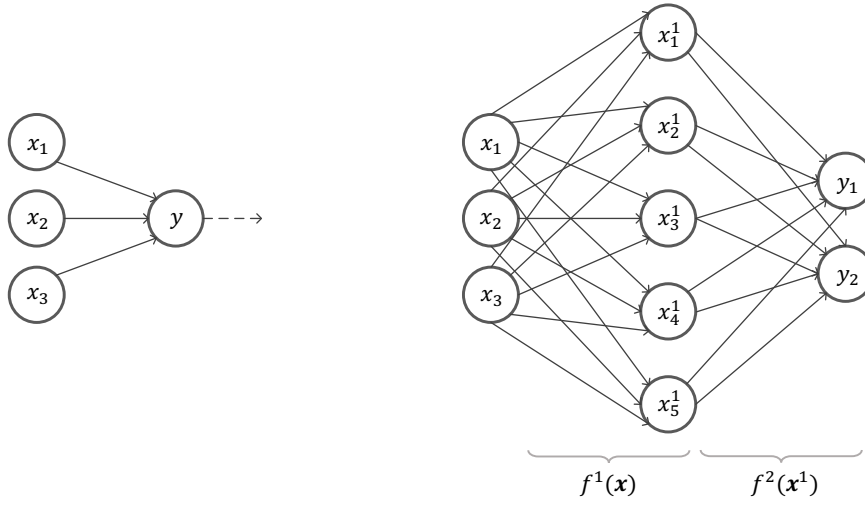


Figure 1.10: Schematic of an artificial neuron and a fully connected neural network with one hidden layer.

A neural network gets its name from the conceptual analogue between the models' basic unit of computation and the neurons of a biological brain. This is based on the 'perceptron' model by which a neuron responds to optical stimulation (Rosenblatt, 1958). In this model, impulses (excitatory and inhibitory) are sent to a node of 'association cells' which, if the net input reaches a certain threshold, themselves send an impulse along random connections to further collections of cells. (Incidentally, recent work, e.g. Beniaguev et al., 2021, has shown that you cannot approximate the complex input/output mapping of a biological neuron using a single artificial neuron, but can with a neural network with thousands of parameters - suggesting this analogy is fairly loose!) This kind of structure can be represented by an artificial neuron drawn schematically in figure 1.10 a. Here, the value y depends on three inputs $x_{\{1,2,3\}}$ and takes the form $y = a(\mathbf{w}^\top \mathbf{x} + b)$ where $\mathbf{w}$ is a vector of 'weights', b is a 'bias' and $a(z)$ is a non-linear, monotonic

cally non-decreasing function of z called an ‘activation function’. Common activation functions include the sigmoid function, which bears greatest resemblance to the original conception of the perceptron, and the ‘rectified linear unit’ (or ReLu) that is zero below some threshold (often 0) and linear in z above it - e.g. $a(z) = \max(0, z)$.

A neural network is a composition of these computational units, arranged into ‘layers’ of nodes with connections between them. For example, figure 1.10 b shows a schematic of a two-layer fully connected neural network (often described as having one ‘hidden’ layer). The function associated with the hidden layer is $\mathbf{x}^1 = f^1(\mathbf{x}) = a(W^1\mathbf{x} + \mathbf{b}^1)$ where the activation function a acts componentwise on $\mathbf{z}^1 = W^1\mathbf{x} + \mathbf{b}^1$ where W^1 is a 5×3 matrix of weights and $\mathbf{b}^1$ is a 5-component vector of biases. It turns out that simple constructions like this are enough to approximate any continuous function to arbitrary accuracy - making neural networks *universal* (e.g. Hornik et al., 1989; Cybenko, 1989; Zhou, 2020; Siegel and Xu, 2020).

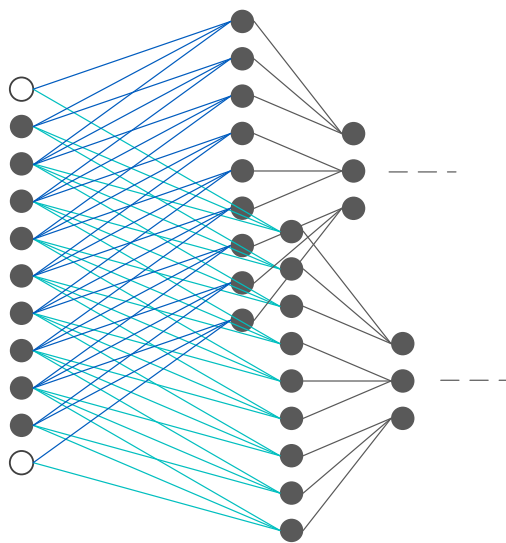


Figure 1.11: Schematic of a convolutional neural network (CNN) operating on one-dimensional data. The CNN has two branches and starts with the application of a three-pixel-wide filter with a stride of one to data padded by one datapoint at each end, before pooling is applied to each set of three nodes in the first hidden layers of each branch.

The field of computer vision has not found its greatest success with the use of fully-connected

neural networks but with *convolutional* neural networks (CNNs), which we focus on here, and more recently *visual transformers*. CNNs replace the dense weight matrices depicted in fig. 1.10 b with larger but sparse matrices representing small filters applied across the input data (e.g. LeCun et al., 1990), encoding the assumption that datapoints are semantically connected to others nearby. A schematic representing certain aspects of the function of a CNN (operating on one-dimensional data, rather than the two-dimensional images of concern in this thesis) is shown in figure 1.11. The figure shows two sets of 3-pixel wide filters applied to the input data (padded by one datapoint at each end) and then ‘pooling’ that downsamples these outputs by a factor of three before continuing. The two sets of filters are distinguished by the colour of the connecting lines between the layers. As well as the reduced number of connections, a key difference from the fully connected case is that the weight matrices only have a few independent trainable parameters each - i.e. within a branch of the network depicted in figure 1.11, each function that maps a set of three datapoints to the next layer is the same. This ‘weight sharing’ allows convolutional neural networks to *learn* sets of filters that extract useful information from input data, replacing the use of pre-selected filter banks (e.g. Gabor or Sobel filters; Fogel and Sagi, 1989; Kanopoulos et al., 1988) already used in image analysis.

The neural networks used in this thesis are of a particular form described first as ‘U-Net’ in Ronneberger et al., 2015. This extends the earlier notion of fully convolutional neural networks (Long et al., 2015) which use convolutions, activations and down/up-sampling to produce image-to-image translations, dispensing with the fully-connected layers that contribute unnecessarily large numbers of free parameters. These often contain a central ‘contraction’ portion that compress the spatial extent of images while building up many feature layers corresponding to combinations of different convolution filters, before this representation is ‘expanded’ to the original image dimension. The extent of this spatial compression, and the number of stages of convolution and down-sampling is sometimes referred to as the *depth* of the CNN. U-Net is recognisable by its symmetric contraction and expansion sections, and, more importantly, the *skip connections* that link feature layers in the contraction and expansion parts of the network. In some way, these skip connections allow the network to decide its own depth, depending on the task for which it is being trained, and encourage the network to take account of information from different levels of spatial compression. A diagram of this kind of network architecture can be found in appendix B (Fig. B.2.)

1.7.2 Neural network optimisation

There are many facets to the field of deep learning that have developed over the last few decades, though the number of fundamental concepts required to understand the basic functionality and training of neural networks is small. Aside from the implementation of the ideas in code and hardware, these are limited to the aforementioned universal approximation capacity of feedforward neural networks, and an understanding of the methods by which we can calculate gradients with respect to model parameters. I will give here a brief description of adjoint-mode automatic differentiation (AD) (also called reverse-mode AD or *backpropagation*) that underpins essentially all gradient-based optimisation methods used to train neural networks. Though not often stated explicitly, the calculation of gradients through complex models underpins a lot of the work presented in this thesis. Though, for the calculation of gradients in BISICLES, this is not done using AD. This is an interesting point of comparison given the growing list of ice sheet and other models that use these techniques (Goldberg and Heimbach, 2013; Goldberg et al., 2016; Koziol et al., 2021; Logan et al., 2020; Morlighem et al., 2021).

A neural network is a function $F_{\mathbf{p}}$ specified by parameters $\mathbf{p}$, mapping inputs $\mathbf{x} \in \mathbb{R}^n$ to outputs $\mathbf{y} \in \mathbb{R}^m$. A simple feed-forward neural network can be written as the composition of N functions representing the action of the N ‘layers’ of the network:

$$\mathbf{y} = F_{\mathbf{p}}(\mathbf{x}) = f_{\mathbf{p}^N}^N \circ f_{\mathbf{p}^{N-1}}^{N-1} \circ \cdots \circ f_{\mathbf{p}^1}^1(\mathbf{x}). \quad (1.18)$$

Here, each function $f_{\mathbf{p}^n}^n$ depends on a set of parameters $\mathbf{p}^n \subset \mathbf{p}$. We denote the output of function $f_{\mathbf{p}^n}^n$ as $\mathbf{x}^{n+1}$.

The Jacobian $\partial_{\mathbf{p}^n} \mathbf{y}$ of partial derivatives of the output vector $\mathbf{y}$ with respect to the vector of parameters $\mathbf{p}^n$ can be written, using the chain rule, as:

$$\partial_{\mathbf{p}^n} F_{\mathbf{p}}(\mathbf{x}) = \left(\partial_{\mathbf{x}^N} f_{\mathbf{p}^N}^N \Big|_{\mathbf{x}^N} \right) \left(\partial_{\mathbf{x}^{N-1}} f_{\mathbf{p}^{N-1}}^{N-1} \Big|_{\mathbf{x}^{N-1}} \right) \cdots \left(\partial_{\mathbf{p}^n} f_{\mathbf{p}^n}^n \Big|_{\mathbf{x}^n} \right) \quad (1.19)$$

Calculating this Jacobian therefore involves a series of $N - n$ matrix multiplications. Performing this calculation constitutes *tangent* mode AD (also called *forward* mode AD). Adjoint mode AD takes advantage of the fact that for training a neural network (or finding the sensitivity of a scalar functional of a model state for that matter) we are rarely interested in the Jacobian

(1.19), rather we seek the product of a vector with that Jacobian. For example, the gradient of a scalar cost function $\mathcal{C}(\mathbf{y})$ with respect to the parameters $\mathbf{p}^n$ takes the form:

$$\begin{aligned}\partial_{\mathbf{p}^n}\mathcal{C}(\mathbf{x}) &= (\partial_{\mathbf{y}}\mathcal{C}(\mathbf{y}))^* (\partial_{\mathbf{p}^n}F_{\mathbf{p}}(\mathbf{x})) \\ &= (\partial_{\mathbf{y}}\mathcal{C}(\mathbf{y}))^* \left(\partial_{\mathbf{x}^N} f_{\mathbf{p}^N}^N \Big|_{\mathbf{x}^N} \right) \left(\partial_{\mathbf{x}^{N-1}} f_{\mathbf{p}^{N-1}}^{N-1} \Big|_{\mathbf{x}^{N-1}} \right) \cdots \left(\partial_{\mathbf{p}^n} f_{\mathbf{p}^n}^n \Big|_{\mathbf{x}^n} \right)\end{aligned}\quad (1.20)$$

or, alternatively:

$$(\partial_{\mathbf{p}^n}\mathcal{C}(\mathbf{x}))^* = \left(\partial_{\mathbf{p}^n} f_{\mathbf{p}^n}^n \Big|_{\mathbf{x}^n} \right)^* \cdots \left(\partial_{\mathbf{x}^{N-1}} f_{\mathbf{p}^{N-1}}^{N-1} \Big|_{\mathbf{x}^{N-1}} \right)^* \left(\partial_{\mathbf{x}^N} f_{\mathbf{p}^N}^N \Big|_{\mathbf{x}^N} \right)^* \partial_{\mathbf{y}}\mathcal{C}(\mathbf{y}) \quad (1.21)$$

From right to left, this can be evaluated as a series of matrix-vector products. The matrices involved are the adjoints to the linearisations of each function evaluated at $\mathbf{x}^m$. For neural networks, the runtime calculation of such matrix-vector products during training is a task of considerably lower computational complexity than the equivalent matrix-matrix products for tangent mode AD. This saving in time comes at a cost in space. Specifically, in tangent mode we can see that as each function in eq. (1.18) is called, we only need to store the current $\mathbf{x}^m$ and the matrix $\left(\partial_{\mathbf{p}^m} f_{\mathbf{p}^m}^m \Big|_{\mathbf{x}^m} \right) \cdots \left(\partial_{\mathbf{p}^n} f_{\mathbf{p}^n}^n \Big|_{\mathbf{x}^n} \right)$. In adjoint mode, however, each $\mathbf{x}^m$ and matrix has to be stored until the end of the calculation of eq. (1.18) at which time eq. (1.21) can be evaluated right to left.

As described in section 1.7.1, in the case of simple feed-forward neural networks the functions $f_{\mathbf{p}^m}^m$ take the simple form:

$$f_{\mathbf{p}^m}^m(\mathbf{x}^m) = a(W\mathbf{x}^m + \mathbf{b}^m) \quad (1.22)$$

where W is a matrix of weights, $\mathbf{b}$ is a vector of biases, and a is a nonlinear activation function that acts component-wise on the vector $\mathbf{z}^m = W\mathbf{x}^m + \mathbf{b}^m$. The parameters $\mathbf{p}^m$ consist of W^m and $\mathbf{b}^m$. Let us consider the example of finding the gradient of the cost function $\mathcal{C}$ with respect to the weights matrix on the n^{th} layer of a neural network. We need to evaluate three kinds of derivatives. The first is $\partial_{\mathbf{y}}\mathcal{C}(\mathbf{y})$ which is generally simple by design. The second is the generic

derivative of the m^{th} layer with respect to its input. This takes the form:

$$\left. \partial_{\mathbf{x}^m} f_{\mathbf{p}^m}^m \right|_{\mathbf{x}^m} = (\partial_{\mathbf{z}^m} a(\mathbf{z}^m))^* W \Big|_{\mathbf{x}^m}. \quad (1.23)$$

Finally, the derivative of the function representing the n^{th} layer with respect to the weights matrix:

$$\left. \partial_{W^n} f_{\mathbf{p}^n}^n \right|_{\mathbf{x}^n} = (\partial_{\mathbf{z}^n} a(\mathbf{z}^n)) (\partial_{W^n} \mathbf{z}^n) \Big|_{\mathbf{x}^n}. \quad (1.24)$$

The second term on the right-hand side of this equation is a third-order tensor with components:

$$(\partial_{W^n} \mathbf{z}^n)_{ijk} = \delta_{ij} x_k \quad (1.25)$$

where δ_{ij} is the Kronecker delta. This has the potential to be unwieldy in the tangent-mode calculation but simplifies in the adjoint-mode where we only have to consider the product of this with a vector. Considering the product with an arbitrary vector $\mathbf{u}$, we see that:

$$(\partial_{W^n} \mathbf{z}^n)^* \mathbf{u} = \mathbf{x} \mathbf{u}^*. \quad (1.26)$$

Given that the derivatives $\partial_{\mathbf{z}^n} a(\mathbf{z}^n)$ are known, all that needs to be stored in the forward pass are $\mathbf{x}^m$, $\mathbf{z}^m$ and W^m (which are stored anyway).

The expression in eq. (1.18) of a complex function as a composition of intermediate functions of the same form (or with derivatives that are easy to compute), is quite generic and encompasses many neural networks. (It is worth noting that the *computational graph* of most contemporary neural network designs, such as the U-Nets developed as part of this thesis, cannot be written so simply and contain many branches. The most general form of adjoint mode AD records the forward computation on a linear tape before running through the tape backwards and summing up gradients at nodes at which the forward computation branches.) The expression in eq. (1.18), however, extends to numerical models that solve and integrate forward the equations of ice sheet dynamics. Here, the functions $f_{\mathbf{p}^m}^m$ represent solving the momentum balance equations for ice speed and evolving the geometry by one timestep according the conservation of mass. Hence, the kind of adjoint-mode AD described here can be used, for example, to find gradients of model

states with respect to initial conditions, or to find the sensitivity of metrics like net ice-discharge to uncertainties in initial conditions or control parameters.

As stated above, the calculation of gradients in this way is central to neural network training because of the reliance on gradient-based algorithms for optimising the network parameters. Though, a priori, it might seem slow and unwieldy, the dominant method of doing this is *stochastic gradient descent* (and the various small developments to this idea made over recent years, such as the Adam optimisation used in this thesis (Kingma and Ba, 2014)). This works by making small adjustments to the network parameters in the direction opposite to the gradient direction. For example, taking the notation from above, the parameters $\mathbf{p}^n$ will be adjusted so that:

$$\mathbf{p}^{n'} = \mathbf{p}^n - \alpha \times \frac{1}{n} \sum_{i=1}^n \partial_{\mathbf{p}^n} \mathcal{C}(\mathbf{x}_i). \quad (1.27)$$

where α is known as the *step size* and n is known as the *batch size*, normally significantly smaller than the size of the training dataset. The use of only a subset of the training examples in defining the gradient direction for each step is what differentiates stochastic gradient descent from gradient descent. This decreases the computational demands of calculating the gradients on each iteration, while potentially mitigating the effect of local minima caused by non-convexity of the optimisation problem and improving the *generalisation* of the network (how it performs on unseen real-world data).

When training a neural network, generally, it is not desirable to use the full gradient and train until the network parameters converge. This is because one is not seeking the set of network parameters that best fit the training data, but that which generalises the best. This is also why the performance of a neural network during training is not often measured with respect to the training data, but to a set of *validation* data which do not contribute to the calculation of gradients. Training might be stopped where performance on the validation reaches a maximum. (A further set of *test* data is often also used to give an unbiased measure of performance after training is completed.) In this thesis, due to the lack of available ground-truth data, less formal methods are employed to train the networks.

1.8 Thesis aim and objectives

The weakening of ice through material failure - particularly crevassing - is not well represented in continuum models of the AIS, which could be an important omission. Consequently, crevasses and the effect they have on ice dynamics are key themes of this thesis. This is a problem of modelling and observations, and this thesis reflects this by making extensive use of both. A second theme is the sensitivity of ice dynamics to changes at the ice sheet margin. Over the last two decades, it has become clear that the dynamics of the West Antarctic Ice Sheet is sensitive to changes at its terminus, as discussed above in the context of buttressing and ocean forcing. I look here at both crevassing as a factor that might change this, but also the mechanical strength of landfast sea ice. This ‘sensitivity’ perspective has led to the use of inverse methods and the computation of gradients as diagnostic tools throughout this thesis, and also makes chapter 4 look slightly less anomalous in the context of the other work presented here.

With these broad themes in mind, the aims of this thesis can be summarised as:

1. Improving observations of ice fracture,
2. Investigating links between fracture observations and ‘damage’ that influences ice flow,
3. Investigating how the inverse problem of inferring ice rheology and basal slipperiness might benefit from the inclusion of observations of fracture,
4. Assessing the buttressing capacity of landfast sea ice.

1.9 Thesis structure

The results of this thesis are presented in the four following chapters. These are exact reproductions of manuscripts addressing the research themes outlined above. The first three manuscripts have been published, while the last is in draft form.

Bibliography

- Albrecht, T. and Levermann, A. (2014). “Fracture-induced softening for large-scale ice dynamics”. In: *The Cryosphere* 8.2, pp. 587–605. DOI: 10.5194/tc-8-587-2014.
- Alley, R., Cuffey, K., Bassis, J., Alley, K., Wang, S., Parizek, B., Anandakrishnan, S., Christianson, K., and DeConto, R. (2023). “Iceberg Calving: Regimes and Transitions”. In: *Annual Review of Earth and Planetary Sciences* 51.1, pp. 189–215. DOI: 10.1146/annurev-earth-032320-110916.
- Alley, R. B., Anandakrishnan, S., Christianson, K., Horgan, H. J., Muto, A., Parizek, B. R., Pollard, D., and Walker, R. T. (2015). “Oceanic Forcing of Ice-Sheet Retreat: West Antarctica and More”. In: *Annual Review of Earth and Planetary Sciences* 43.1, pp. 207–231. DOI: 10.1146/annurev-earth-060614-105344.
- Andreasen, J. R., Hogg, A. E., and Selley, H. L. (2023). “Change in Antarctic ice shelf area from 2009 to 2019”. In: *The Cryosphere* 17.5, pp. 2059–2072. DOI: 10.5194/tc-17-2059-2023.
- Arthern, R. J. and Gudmundsson, G. H. (2010). “Initialization of ice-sheet forecasts viewed as an inverse Robin problem”. In: *Journal of Glaciology* 56.197, pp. 527–533. DOI: 10.3189/002214310792447699.
- Aster, R. C., Borchers, B., and Thurber, C. H. (2013). *Parameter estimation and inverse problems*. Elsevier. ISBN: 978-0-12-385048-5. DOI: 10.1016/C2009-0-61134-X.
- Åström, J. A., Riikilä, T. I., Tallinen, T., Zwinger, T., Benn, D., Moore, J. C., and Timonen, J. (2013). “A particle based simulation model for glacier dynamics”. In: *The Cryosphere* 7.5, pp. 1591–1602. DOI: 10.5194/tc-7-1591-2013.
- Bamber, J. L., Oppenheimer, M., Kopp, R. E., Aspinall, W. P., and Cooke, R. M. (2019). “Ice sheet contributions to future sea-level rise from structured expert judgment”. In: *Proceedings of the National Academy of Sciences* 116.23, pp. 11195–11200. ISSN: 0027-8424. DOI: 10.1073/pnas.1817205116.
- Bamber, J. L., Vaughan, D. G., and Joughin, I. (2000). “Widespread Complex Flow in the Interior of the Antarctic Ice Sheet”. In: *Science* 287.5456, pp. 1248–1250. DOI: 10.1126/science.287.5456.1248.
- Baumhoer, C. A., Dietz, A. J., Kneisel, C., and Kuenzer, C. (2019). “Automated Extraction of Antarctic Glacier and Ice Shelf Fronts from Sentinel-1 Imagery Using Deep Learning”. In: *Remote Sensing* 11.21. ISSN: 2072-4292. DOI: 10.3390/rs11212529.
- Bayr, K. J., Hall, D., and Kovalick, W. M. (1994). “Observations on glaciers in the eastern Austrian Alps using satellite data”. In: *International Journal of Remote Sensing* 15.9, pp. 1733–1742. DOI: 10.1080/01431169408954205.

- Beniaguev, D., Segev, I., and London, M. (2021). “Single cortical neurons as deep artificial neural networks”. In: *Neuron* 109.17, 2727–2739.e3. ISSN: 0896-6273. DOI: <https://doi.org/10.1016/j.neuron.2021.07.002>.
- Benn, D. I., Åström, J., Zwinger, T., Todd, J., Nick, F. M., Cook, S., Hulton, N. R. J., and Luckman, A. (2017). “Melt-under-cutting and buoyancy-driven calving from tidewater glaciers: new insights from discrete element and continuum model simulations”. In: *Journal of Glaciology* 63.240, pp. 691–702. DOI: [10.1017/jog.2017.41](https://doi.org/10.1017/jog.2017.41).
- Benn, D. I. and Lehmkuhl, F. (2000). “Mass balance and equilibrium-line altitudes of glaciers in high-mountain environments”. In: *Quaternary International* 65-66, pp. 15–29. ISSN: 1040-6182. DOI: [https://doi.org/10.1016/S1040-6182\(99\)00034-8](https://doi.org/10.1016/S1040-6182(99)00034-8).
- Bennett, M. R. (2003). “Ice streams as the arteries of an ice sheet: their mechanics, stability and significance”. In: *Earth-Science Reviews* 61.3, pp. 309–339. ISSN: 0012-8252. DOI: [https://doi.org/10.1016/S0012-8252\(02\)00130-7](https://doi.org/10.1016/S0012-8252(02)00130-7).
- Bindschadler, R., Vornberger, P., Fleming, A., Fox, A., Mullins, J., Binnie, D., Paulsen, S. J., Granneman, B., and Gorodetzky, D. (2008). “The Landsat Image Mosaic of Antarctica”. In: *Remote Sensing of Environment* 112.12, pp. 4214–4226. ISSN: 0034-4257. DOI: <https://doi.org/10.1016/j.rse.2008.07.006>.
- Bolibar, J., Rabatel, A., Gouttevin, I., Galiez, C., Condom, T., and Sauquet, E. (2020). “Deep learning applied to glacier evolution modelling”. In: *The Cryosphere* 14.2, pp. 565–584. DOI: [10.5194/tc-14-565-2020](https://doi.org/10.5194/tc-14-565-2020).
- Borstad, C. P., Khazendar, A., Larour, E., Morlighem, M., Rignot, E., Schodlok, M. P., and Seroussi, H. (2012). “A damage mechanics assessment of the Larsen B ice shelf prior to collapse: Toward a physically-based calving law”. In: *Geophysical Research Letters* 39.18. DOI: [10.1029/2012GL053317](https://doi.org/10.1029/2012GL053317).
- Borstad, C. P., Rignot, E., Mouginot, J., and Schodlok, M. P. (2013). “Creep deformation and buttressing capacity of damaged ice shelves: theory and application to Larsen C ice shelf”. In: *The Cryosphere* 7.6, pp. 1931–1947. DOI: [10.5194/tc-7-1931-2013](https://doi.org/10.5194/tc-7-1931-2013).
- Braakmann-Folgmann, A., Shepherd, A., Hogg, D., and Redmond, E. (2023). “Mapping the extent of giant Antarctic icebergs with deep learning”. In: *The Cryosphere* 17.11, pp. 4675–4690. DOI: [10.5194/tc-17-4675-2023](https://doi.org/10.5194/tc-17-4675-2023).
- Brinkerhoff, D. J. and Johnson, J. V. (2013). “Data assimilation and prognostic whole ice sheet modelling with the variationally derived, higher order, open source, and fully parallel ice sheet model VarGlaS”. In: *The Cryosphere* 7.4, pp. 1161–1184. DOI: [10.5194/tc-7-1161-2013](https://doi.org/10.5194/tc-7-1161-2013).
- Brinkerhoff, D., Aschwanden, A., and Fahnestock, M. (2021). “Constraining subglacial processes from surface velocity observations using surrogate-based Bayesian inference”. In: *Journal of Glaciology* 67.263, pp. 385–403. DOI: [10.1017/jog.2020.112](https://doi.org/10.1017/jog.2020.112).
- Cornford, S. L., Martin, D. F., Payne, A. J., Ng, E. G., Le Brocq, A. M., Gladstone, R. M., Edwards, T. L., Shannon, S. R., Agosta, C., Broeke, M. R. van den, et al. (2015). “Century-scale simulations of the response of the West Antarctic Ice Sheet to a warming climate”. In: *The Cryosphere* 9.4, pp. 1579–1600.
- Cybenko, G. (1989). “Approximation by superpositions of a sigmoidal function”. In: *Mathematics of control, signals and systems* 2.4, pp. 303–314. DOI: [10.1007/BF02551274](https://doi.org/10.1007/BF02551274).

- Davison, B. J., Hogg, A. E., Gourmelen, N., Jakob, L., Wuite, J., Nagler, T., Greene, C. A., Andreasen, J., and Engdahl, M. E. (2023). “Annual mass budget of Antarctic ice shelves from 1997 to 2021”. In: *Science Advances* 9.41, eadi0186. DOI: [10.1126/sciadv.adi0186](https://doi.org/10.1126/sciadv.adi0186).
- de Roda Husman, S., Lhermitte, S., Bolibar, J., Izeboud, M., Hu, Z., Shukla, S., van der Meer, M., Long, D., and Wouters, B. (2024). “A high-resolution record of surface melt on Antarctic ice shelves using multi-source remote sensing data and deep learning”. In: *Remote Sensing of Environment* 301, p. 113950. ISSN: 0034-4257. DOI: <https://doi.org/10.1016/j.rse.2023.113950>.
- De Rydt, J., Gudmundsson, G. H., Rott, H., and Bamber, J. L. (2015). “Modeling the instantaneous response of glaciers after the collapse of the Larsen B Ice Shelf”. In: *Geophysical Research Letters* 42.13, pp. 5355–5363. DOI: <https://doi.org/10.1002/2015GL064355>.
- Dirscherl, M., Dietz, A. J., Kneisel, C., and Kuenzer, C. (2021). “A Novel Method for Automated Supraglacial Lake Mapping in Antarctica Using Sentinel-1 SAR Imagery and Deep Learning”. In: *Remote Sensing* 13.2. ISSN: 2072-4292. DOI: [10.3390/rs13020197](https://doi.org/10.3390/rs13020197).
- Earl, L. and Gardner, A. (2016). “A satellite-derived glacier inventory for North Asia”. In: *Annals of Glaciology* 57.71, pp. 50–60. DOI: [10.3189/2016AoG71A008](https://doi.org/10.3189/2016AoG71A008).
- Fogel, I. Y. and Sagi, D. (1989). “Gabor filters as texture discriminator”. In: *Biological Cybernetics* 61, pp. 103–113. DOI: [10.1007/BF00204594](https://doi.org/10.1007/BF00204594).
- Fretwell, P. et al. (2013). “Bedmap2: improved ice bed, surface and thickness datasets for Antarctica”. In: *The Cryosphere* 7.1, pp. 375–393. DOI: [10.5194/tc-7-375-2013](https://doi.org/10.5194/tc-7-375-2013).
- Fricker, H. A., Young, N. W., Allison, I., and Coleman, R. (2002). “Iceberg calving from the Amery Ice Shelf, East Antarctica”. In: *Annals of Glaciology* 34, pp. 241–246. DOI: [10.3189/172756402781817581](https://doi.org/10.3189/172756402781817581).
- Fürst, J. J., Durand, G., Gillet-Chaulet, F., Tavard, L., Rankl, M., Braun, M., and Gagliardini, O. (2016). “The safety band of Antarctic ice shelves”. In: *Nature Climate Change* 6.5, pp. 479–482.
- Gagliardini, O., Durand, G., Zwinger, T., Hindmarsh, R. C. A., and Le Meur, E. (2010). “Coupling of ice-shelf melting and buttressing is a key process in ice-sheets dynamics”. In: *Geophysical Research Letters* 37.14. DOI: <https://doi.org/10.1029/2010GL043334>.
- Gagliardini, O., Zwinger, T., Gillet-Chaulet, F., Durand, G., Favier, L., Fleurian, B. de, Greve, R., Malinen, M., Martín, C., Råback, P., Ruokolainen, J., Sacchetti, M., Schäfer, M., Seddik, H., and Thies, J. (2013). “Capabilities and performance of Elmer/Ice, a new-generation ice sheet model”. In: *Geoscientific Model Development* 6.4, pp. 1299–1318. DOI: [10.5194/gmd-6-1299-2013](https://doi.org/10.5194/gmd-6-1299-2013).
- Gillet-Chaulet, F., Durand, G., Gagliardini, O., Mosbeux, C., Mouginot, J., Rémy, F., and Ritz, C. (2016). “Assimilation of surface velocities acquired between 1996 and 2010 to constrain the form of the basal friction law under Pine Island Glacier”. In: *Geophysical Research Letters* 43.19, pp. 10, 311–10, 321. DOI: <https://doi.org/10.1002/2016GL069937>.
- Gillet-Chaulet, F., Gagliardini, O., Seddik, H., Nodet, M., Durand, G., Ritz, C., Zwinger, T., Greve, R., and Vaughan, D. G. (2012). “Greenland ice sheet contribution to sea-level rise from a new-generation ice-sheet model”. In: *The Cryosphere* 6.6, pp. 1561–1576. DOI: [10.5194/tc-6-1561-2012](https://doi.org/10.5194/tc-6-1561-2012).

- Glen, J. W. and Perutz, M. F. (1955). “The creep of polycrystalline ice”. In: *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences* 228.1175, pp. 519–538. DOI: 10.1098/rspa.1955.0066.
- Goldberg, D. N. and Heimbach, P. (2013). “Parameter and state estimation with a time-dependent adjoint marine ice sheet model”. In: *The Cryosphere* 7.6, pp. 1659–1678. DOI: 10.5194/tc-7-1659-2013.
- Goldberg, D. N., Narayanan, S. H. K., Hascoet, L., and Utke, J. (2016). “An optimized treatment for algorithmic differentiation of an important glaciological fixed-point problem”. In: *Geoscientific Model Development* 9.5, pp. 1891–1904. DOI: 10.5194/gmd-9-1891-2016.
- Gordon, R. B. (1965). “Diffusion creep in the Earth’s mantle”. In: *Journal of Geophysical Research (1896-1977)* 70.10, pp. 2413–2418. DOI: 10.1029/JZ070i010p02413.
- Gudmundsson, G. H. (2013). “Ice-shelf buttressing and the stability of marine ice sheets”. In: *The Cryosphere* 7.2, pp. 647–655. DOI: 10.5194/tc-7-647-2013.
- Gudmundsson, G. H., Krug, J., Durand, G., Favier, L., and Gagliardini, O. (2012). “The stability of grounding lines on retrograde slopes”. In: *The Cryosphere* 6.6, pp. 1497–1505. DOI: 10.5194/tc-6-1497-2012.
- Gudmundsson, G. H. (2003). “Transmission of basal variability to a glacier surface”. In: *Journal of Geophysical Research: Solid Earth* 108.B5. DOI: <https://doi.org/10.1029/2002JB002107>.
- Gudmundsson, G. H., Paolo, F. S., Adusumilli, S., and Fricker, H. A. (2019). “Instantaneous Antarctic ice sheet mass loss driven by thinning ice shelves”. In: *Geophysical Research Letters* 46.23, pp. 13903–13909. DOI: <https://doi.org/10.1029/2019GL085027>.
- Habermann, M., Truffer, M., and Maxwell, D. (2013). “Changing basal conditions during the speed-up of Jakobshavn Isbræ, Greenland”. In: *The Cryosphere* 7.6, pp. 1679–1692. DOI: 10.5194/tc-7-1679-2013.
- Hadamard, J. (1902). “Sur les problèmes aux dérivées partielles et leur signification physique”. In: *Princeton university bulletin*, pp. 49–52.
- Haireti Alifu, R. T. and Johnson, B. (2015). “A new band ratio technique for mapping debris-covered glaciers using Landsat imagery and a digital elevation model”. In: *International Journal of Remote Sensing* 36.8, pp. 2063–2075. DOI: 10.1080/2150704X.2015.1034886.
- Hansen, P. C. (1994). “Regularization tools: A Matlab package for analysis and solution of discrete ill-posed problems”. In: *Numerical algorithms* 6.1, pp. 1–35. DOI: 10.1007/BF02149761.
- Haselhoff, M. and Sergienko, O. V. (2018a). “The effect of buttressing on grounding line dynamics”. In: *Journal of Glaciology* 64.245, pp. 417–431. DOI: 10.1017/jog.2018.30.
- Haselhoff, M. and Sergienko, O. V. (2018b). “The effect of buttressing on grounding line dynamics”. In: *Journal of Glaciology* 64.245, pp. 417–431. DOI: 10.1017/jog.2018.30.
- He, Q., Perego, M., Howard, A. A., Karniadakis, G. E., and Stinis, P. (2023). “A hybrid deep neural operator/finite element method for ice-sheet modeling”. In: *Journal of Computational Physics* 492, p. 112428. ISSN: 0021-9991. DOI: <https://doi.org/10.1016/j.jcp.2023.112428>.

- Hill, E. A., Rosier, S. H. R., Gudmundsson, G. H., and Collins, M. (2021). “Quantifying the potential future contribution to global mean sea level from the Filchner–Ronne basin, Antarctica”. In: *The Cryosphere* 15.10, pp. 4675–4702. DOI: 10.5194/tc-15-4675-2021.
- Hindmarsh, R. C. A. (1996). “Stability of ice rises and uncoupled marine ice sheets”. In: *Annals of Glaciology* 23, pp. 105–115. DOI: 10.3189/S0260305500013318.
- Hogg, A. E. and Gudmundsson, G. H. (2017). “Impacts of the Larsen-C Ice Shelf calving event”. In: *Nature Climate Change* 7.8, pp. 540–542. DOI: 10.1038/nclimate3359.
- Hornik, K., Stinchcombe, M., and White, H. (1989). “Multilayer feedforward networks are universal approximators”. In: *Neural Networks* 2.5, pp. 359–366. ISSN: 0893-6080. DOI: [https://doi.org/10.1016/0893-6080\(89\)90020-8](https://doi.org/10.1016/0893-6080(89)90020-8).
- Hu, Z., Kuipers Munneke, P., Lhermitte, S., Izeboud, M., and Broeke, M. van den (2021). “Improving surface melt estimation over the Antarctic Ice Sheet using deep learning: a proof of concept over the Larsen Ice Shelf”. In: *The Cryosphere* 15.12, pp. 5639–5658. DOI: 10.5194/tc-15-5639-2021.
- Hutter, K. (1983). “The Application of the Shallow-Ice Approximation”. In: *Theoretical Glaciology: Material Science of Ice and the Mechanics of Glaciers and Ice Sheets*. Dordrecht: Springer Netherlands, pp. 256–332. ISBN: 978-94-015-1167-4. DOI: 10.1007/978-94-015-1167-4_5.
- Joughin, I., MacAyeal, D. R., and Tulaczyk, S. (2004). “Basal shear stress of the Ross ice streams from control method inversions”. In: *Journal of Geophysical Research: Solid Earth* 109.B9. DOI: <https://doi.org/10.1029/2003JB002960>.
- Joughin, I., Smith, B. E., Howat, I. M., Scambos, T., and Moon, T. (2010). “Greenland flow variability from ice-sheet-wide velocity mapping”. In: *Journal of Glaciology* 56.197, pp. 415–430.
- Joughin, I., Tulaczyk, S., Bamber, J. L., Blankenship, D., Holt, J. W., Scambos, T., and Vaughan, D. G. (2009). “Basal conditions for Pine Island and Thwaites Glaciers, West Antarctica, determined using satellite and airborne data”. In: *Journal of Glaciology* 55.190, pp. 245–257. DOI: 10.3189/002214309788608705.
- Jouvet, G. (2023). “Inversion of a Stokes glacier flow model emulated by deep learning”. In: *Journal of Glaciology* 69.273, pp. 13–26. DOI: 10.1017/jog.2022.41.
- Jouvet, G. and Cordonnier, G. (2023). “Ice-flow model emulator based on physics-informed deep learning”. In: *Journal of Glaciology*, pp. 1–15. DOI: 10.1017/jog.2023.73.
- Jouvet, G., Cordonnier, G., Kim, B., Lüthi, M., Vieli, A., and Aschwanden, A. (2022). “Deep learning speeds up ice flow modelling by several orders of magnitude”. In: *Journal of Glaciology* 68.270, pp. 651–664. DOI: 10.1017/jog.2021.120.
- Kanopoulos, N., Vasanthavada, N., and Baker, R. (1988). “Design of an image edge detection filter using the Sobel operator”. In: *IEEE Journal of Solid-State Circuits* 23.2, pp. 358–367. DOI: 10.1109/4.996.
- Karimi, N., Farokhnia, A., Karimi, L., Eftekhari, M., and Ghalkhani, H. (2012). “Combining optical and thermal remote sensing data for mapping debris-covered glaciers (Alamkouh Glaciers, Iran)”. In: *Cold Regions Science and Technology* 71, pp. 73–83. ISSN: 0165-232X. DOI: <https://doi.org/10.1016/j.coldregions.2011.10.004>.

- Kennett, J. P. (1977). “Cenozoic evolution of Antarctic glaciation, the circum-Antarctic Ocean, and their impact on global paleoceanography”. In: *Journal of Geophysical Research (1896-1977)* 82.27, pp. 3843–3860. DOI: <https://doi.org/10.1029/JC082i027p03843>.
- Kingma, D. P. and Ba, J. (2014). “Adam: A method for stochastic optimization”. In: *arXiv preprint arXiv:1412.6980*.
- Kingslake, J., Ely, J. C., Das, I., and Bell, R. E. (2017). “Widespread movement of meltwater onto and across Antarctic ice shelves”. In: *Nature* 544.7650, pp. 349–352. DOI: 10.1038/nature22049.
- Koziol, C. P., Todd, J. A., Goldberg, D. N., and Maddison, J. R. (2021). “fenics_ice 1.0: a framework for quantifying initialization uncertainty for time-dependent ice sheet models”. In: *Geoscientific Model Development* 14.9, pp. 5843–5861. DOI: 10.5194/gmd-14-5843-2021.
- Larour, E., Seroussi, H., Morlighem, M., and Rignot, E. (2012). “Continental scale, high order, high spatial resolution, ice sheet modeling using the Ice Sheet System Model (ISSM)”. In: *Journal of Geophysical Research: Earth Surface* 117.F1. DOI: <https://doi.org/10.1029/2011JF002140>.
- LeCun, Y., Boser, B., Denker, J. S., Howard, R. E., Hubbard, W., Jackel, L. D., and Henderson, D. (1990). “Handwritten Digit Recognition with a Back-Propagation Network”. In: *Advances in Neural Information Processing Systems 2*. San Francisco, CA, USA: Morgan Kaufmann Publishers Inc., pp. 396–404. ISBN: 1558601007. DOI: 10.5555/109230.109279.
- Lhermitte, S., Sun, S., Shuman, C., Wouters, B., Pattyn, F., Wuite, J., Berthier, E., and Nagler, T. (2020). “Damage accelerates ice shelf instability and mass loss in Amundsen Sea Embayment”. In: *Proceedings of the National Academy of Sciences* 117.40, pp. 24735–24741. DOI: 10.1073/pnas.1912890117.
- Logan, L. C., Narayanan, S. H. K., Greve, R., and Heimbach, P. (2020). “SICOPOLIS-AD v1: an open-source adjoint modeling framework for ice sheet simulation enabled by the algorithmic differentiation tool OpenAD”. In: *Geoscientific Model Development* 13.4, pp. 1845–1864. DOI: 10.5194/gmd-13-1845-2020.
- Long, J., Shelhamer, E., and Darrell, T. (June 2015). “Fully Convolutional Networks for Semantic Segmentation”. In: *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*.
- MacAyeal, D. R. (1993). “A tutorial on the use of control methods in ice-sheet modeling”. In: *Journal of Glaciology* 39.131, pp. 91–98. DOI: 10.3189/S0022143000015744.
- (1992). “The basal stress distribution of Ice Stream E, Antarctica, inferred by control methods”. In: *Journal of Geophysical Research: Solid Earth* 97.B1, pp. 595–603. DOI: <https://doi.org/10.1029/91JB02454>.
- Mohajerani, Y., Jeong, S., Scheuchl, B., Velicogna, I., Rignot, E., and Milillo, P. (2021). “Automatic delineation of glacier grounding lines in differential interferometric synthetic-aperture radar data using deep learning”. In: *Scientific reports* 11.1, p. 4992. DOI: 10.1038/s41598-021-84309-3.
- Mohajerani, Y., Wood, M., Velicogna, I., and Rignot, E. (2019). “Detection of Glacier Calving Margins with Convolutional Neural Networks: A Case Study”. In: *Remote Sensing* 11.1. ISSN: 2072-4292. DOI: 10.3390/rs11010074.

- Morlighem, M. (2020). *MEaSURES BedMachine Antarctica, Version 2 (bed, thickness)*. Accessed on 05/10/2021. DOI: <https://doi.org/10.5067/E1QL9HFQ7A8M>.
- Morlighem, M., Seroussi, H., Larour, E., and Rignot, E. (2013). “Inversion of basal friction in Antarctica using exact and incomplete adjoints of a higher-order model”. In: *Journal of Geophysical Research: Earth Surface* 118.3, pp. 1746–1753. DOI: <https://doi.org/10.1002/jgrf.20125>.
- Morlighem, M., Goldberg, D., Dias dos Santos, T., Lee, J., and Sagebaum, M. (2021). “Mapping the Sensitivity of the Amundsen Sea Embayment to Changes in External Forcings Using Automatic Differentiation”. In: *Geophysical Research Letters* 48.23, e2021GL095440. DOI: <https://doi.org/10.1029/2021GL095440>.
- Mote, T. L., Anderson, M. R., Kuivinen, K. C., and Rowe, C. M. (1993). “Passive microwave-derived spatial and temporal variations of summer melt on the Greenland ice sheet”. In: *Annals of Glaciology* 17, pp. 233–238. DOI: 10.3189/S0260305500012891.
- Mottram, R., Hansen, N., Kittel, C., Wessem, J. M. van, Agosta, C., Amory, C., Boberg, F., Berg, W. J. van de, Fettweis, X., Gossart, A., Lipzig, N. P. M. van, Meijgaard, E. van, Orr, A., Phillips, T., Webster, S., Simonsen, S. B., and Souverijns, N. (2021). “What is the surface mass balance of Antarctica? An intercomparison of regional climate model estimates”. In: *The Cryosphere* 15.8, pp. 3751–3784. DOI: 10.5194/tc-15-3751-2021.
- Mouginot, J., Rignot, E., and Scheuchl, B. (2014a). “Sustained increase in ice discharge from the Amundsen Sea Embayment, West Antarctica, from 1973 to 2013”. In: *Geophysical Research Letters* 41.5, pp. 1576–1584.
- (2014b). “Sustained increase in ice discharge from the Amundsen Sea Embayment, West Antarctica, from 1973 to 2013”. In: *Geophysical Research Letters* 41.5, pp. 1576–1584. DOI: <https://doi.org/10.1002/2013GL059069>.
- Nagai, H., Fujita, K., Nuimura, T., and Sakai, A. (2013). “Southwest-facing slopes control the formation of debris-covered glaciers in the Bhutan Himalaya”. In: *The Cryosphere* 7.4, pp. 1303–1314. DOI: 10.5194/tc-7-1303-2013.
- Naish, T., Powell, R., Levy, R., Wilson, G., Scherer, R., Talarico, F., Krissek, L., Niessen, F., Pompilio, M., Wilson, T., et al. (2009). “Obliquity-paced Pliocene West Antarctic ice sheet oscillations”. In: *Nature* 458.7236, pp. 322–328. DOI: 10.1038/nature07867.
- Naughten, K. A., Holland, P. R., Dutrieux, P., Kimura, S., Bett, D. T., and Jenkins, A. (2022). “Simulated Twentieth-Century Ocean Warming in the Amundsen Sea, West Antarctica”. In: *Geophysical Research Letters* 49.5. e2021GL094566 2021GL094566, e2021GL094566. DOI: <https://doi.org/10.1029/2021GL094566>.
- Nilsson, J., Gardner, A. S., and Paolo, F. S. (2022). “Elevation change of the Antarctic Ice Sheet: 1985 to 2020”. In: *Earth System Science Data* 14.8, pp. 3573–3598. DOI: 10.5194/essd-14-3573-2022.
- Noël, B., Berg, W. J. van de, Lhermitte, S., and Broeke, M. R. van den (2019). “Rapid ablation zone expansion amplifies north Greenland mass loss”. In: *Science Advances* 5.9, eaaw0123. DOI: 10.1126/sciadv.aaw0123.
- Nye, J. F. and Mott, N. F. (1953). “The flow law of ice from measurements in glacier tunnels, laboratory experiments and the Jungfrau firn borehole experiment”. In: *Proceedings of the*

- Royal Society of London. Series A. Mathematical and Physical Sciences* 219.1139, pp. 477–489. DOI: 10.1098/rspa.1953.0161.
- Otosaka, I. N. et al. (2023). “Mass balance of the Greenland and Antarctic ice sheets from 1992 to 2020”. In: *Earth System Science Data* 15.4, pp. 1597–1616. DOI: 10.5194/essd-15-1597-2023.
- Petra, N., Zhu, H., Stadler, G., Hughes, T. J., and Ghattas, O. (2012). “An inexact Gauss-Newton method for inversion of basal sliding and rheology parameters in a nonlinear Stokes ice sheet model”. In: *Journal of Glaciology* 58.211, pp. 889–903. DOI: 10.3189/2012JoG11J182.
- Pollard, D. and DeConto, R. M. (2009). “Modelling West Antarctic ice sheet growth and collapse through the past five million years”. In: *Nature* 458.7236, pp. 329–332. DOI: 10.1038/nature07809.
- Pralong, A. and Funk, M. (2005). “Dynamic damage model of crevasse opening and application to glacier calving”. In: *Journal of Geophysical Research: Solid Earth* 110.B1. DOI: 10.1029/2004JB003104.
- Pritchard, H., Ligtenberg, S. R., Fricker, H. A., Vaughan, D. G., Broeke, M. R. van den, and Padman, L. (2012). “Antarctic ice-sheet loss driven by basal melting of ice shelves”. In: *Nature* 484.7395, pp. 502–505. DOI: 10.1038/nature10968.
- Reese, R., Garbe, J., Hill, E. A., Urruty, B., Naughten, K. A., Gagliardini, O., Durand, G., Gillet-Chaulet, F., Gudmundsson, G. H., Chandler, D., Langebroek, P. M., and Winkelmann, R. (2023). “The stability of present-day Antarctic grounding lines – Part 2: Onset of irreversible retreat of Amundsen Sea glaciers under current climate on centennial timescales cannot be excluded”. In: *The Cryosphere* 17.9, pp. 3761–3783. DOI: 10.5194/tc-17-3761-2023.
- Reusch, D. B., Hewitson, B. C., and Alley, R. B. (2005). “Towards ice-core-based synoptic reconstructions of west antarctic climate with artificial neural networks”. In: *International Journal of Climatology* 25.5, pp. 581–610. DOI: <https://doi.org/10.1002/joc.1143>.
- Rignot, E., Casassa, G., Gogineni, P., Krabill, W., Rivera, A., and Thomas, R. (2004). “Accelerated ice discharge from the Antarctic Peninsula following the collapse of Larsen B ice shelf”. In: *Geophysical Research Letters* 31.18. DOI: <https://doi.org/10.1029/2004GL020697>.
- Rignot, E., Mouginot, J., and Scheuchl, B. (2011a). “Antarctic grounding line mapping from differential satellite radar interferometry”. In: *Geophysical Research Letters* 38.10. DOI: <https://doi.org/10.1029/2011GL047109>.
- (2011b). “Ice Flow of the Antarctic Ice Sheet”. In: *Science* 333.6048, pp. 1427–1430. DOI: 10.1126/science.1208336.
- Ronneberger, O., Fischer, P., and Brox, T. (2015). *U-Net: Convolutional Networks for Biomedical Image Segmentation*.
- Rosenblatt, F. (1958). “The perceptron: a probabilistic model for information storage and organization in the brain.” In: *Psychological review* 65.6, p. 386. DOI: 10.1037/h0042519.
- Rosier, S. H. R., Bull, C. Y. S., Woo, W. L., and Gudmundsson, G. H. (2023). “Predicting ocean-induced ice-shelf melt rates using deep learning”. In: *The Cryosphere* 17.2, pp. 499–518. DOI: 10.5194/tc-17-499-2023.

- Rosier, S. H. R., Reese, R., Donges, J. F., De Rydt, J., Gudmundsson, G. H., and Winkelmann, R. (2021). “The tipping points and early warning indicators for Pine Island Glacier, West Antarctica”. In: *The Cryosphere* 15.3, pp. 1501–1516. DOI: 10.5194/tc-15-1501-2021.
- Scambos, T. A., Bohlander, J. A., Shuman, C. A., and Skvarca, P. (2004). “Glacier acceleration and thinning after ice shelf collapse in the Larsen B embayment, Antarctica”. In: *Geophysical Research Letters* 31.18. DOI: <https://doi.org/10.1029/2004GL020670>.
- Scambos, T., Haran, T., Fahnestock, M., Painter, T., and Bohlander, J. (2007). “MODIS-based Mosaic of Antarctica (MOA) data sets: Continent-wide surface morphology and snow grain size”. In: *Remote Sensing of Environment* 111.2. Remote Sensing of the Cryosphere Special Issue, pp. 242–257. ISSN: 0034-4257. DOI: <https://doi.org/10.1016/j.rse.2006.12.020>.
- Schoof, C. (2007a). “Ice sheet grounding line dynamics: Steady states, stability, and hysteresis”. In: *Journal of Geophysical Research: Earth Surface* 112.F3. DOI: <https://doi.org/10.1029/2006JF000664>.
- (2007b). “Marine ice-sheet dynamics. Part 1. The case of rapid sliding”. In: *Journal of Fluid Mechanics* 573, pp. 27–55. DOI: 10.1017/S0022112006003570.
- Schoof, C. and Hindmarsh, R. C. A. (2010). “Thin-Film Flows with Wall Slip: An Asymptotic Analysis of Higher Order Glacier Flow Models”. In: *Quarterly Journal of Mechanics and Applied Mathematics* 63.1, pp. 73–114. DOI: 10.1093/qjmam/hbp025.
- Schröder, L., Horwath, M., Dietrich, R., Helm, V., Broeke, M. R. van den, and Ligtenberg, S. R. M. (2019). “Four decades of Antarctic surface elevation changes from multi-mission satellite altimetry”. In: *The Cryosphere* 13.2, pp. 427–449. DOI: 10.5194/tc-13-427-2019.
- Sergienko, O. V. and Wingham, D. J. (2019). “Grounding line stability in a regime of low driving and basal stresses”. In: *Journal of Glaciology* 65.253, pp. 833–849. DOI: 10.1017/jog.2019.53.
- Sergienko, O. V. and Hindmarsh, R. C. A. (2013). “Regular Patterns in Frictional Resistance of Ice-Stream Beds Seen by Surface Data Inversion”. In: *Science* 342.6162, pp. 1086–1089. DOI: 10.1126/science.1243903.
- Sergienko, O. V. and Wingham, D. J. (2022). “Bed topography and marine ice-sheet stability”. In: *Journal of Glaciology* 68.267, pp. 124–138. DOI: 10.1017/jog.2021.79.
- Shepherd, A., Gilbert, L., Muir, A. S., Konrad, H., McMillan, M., Slater, T., Briggs, K. H., Sundal, A. V., Hogg, A. E., and Engdahl, M. E. (2019). “Trends in Antarctic Ice Sheet Elevation and Mass”. In: *Geophysical Research Letters* 46.14, pp. 8174–8183. DOI: <https://doi.org/10.1029/2019GL082182>.
- Shewchuk, J. R. (1994). *An Introduction to the Conjugate Gradient Method Without the Agonizing Pain*. Tech. rep. USA.
- Shukla, A., Gupta, R., and Arora, M. (2009). “Estimation of debris cover and its temporal variation using optical satellite sensor data: a case study in Chenab basin, Himalaya”. In: *Journal of Glaciology* 55.191, pp. 444–452. DOI: 10.3189/002214309788816632.
- Shukla, A. and Yousuf, B. (2017). “Evaluation of multisource data for glacier terrain mapping: a neural net approach”. In: *Geocarto International* 32.5, pp. 569–587. DOI: 10.1080/10106049.2016.1161078.

- Siegel, J. W. and Xu, J. (2020). “Approximation rates for neural networks with general activation functions”. In: *Neural Networks* 128, pp. 313–321. ISSN: 0893-6080. DOI: <https://doi.org/10.1016/j.neunet.2020.05.019>.
- Steiner, D., Walter, A., and Zumbühl, H. (2005). “The application of a non-linear back-propagation neural network to study the mass balance of Grosse Aletschgletscher, Switzerland”. In: *Journal of Glaciology* 51.173, pp. 313–323. DOI: 10.3189/172756505781829421.
- Sugden, D. and Denton, G. (July 2004). “Cenozoic landscape evolution of the Convoy Range to Mackay Glacier area, Transantarctic Mountains: Onshore to offshore synthesis”. In: *GSA Bulletin* 116.7-8, pp. 840–857. ISSN: 0016-7606. DOI: 10.1130/B25356.1.
- Sun, S., Cornford, S. L., Moore, J. C., Gladstone, R., and Zhao, L. (2017). “Ice shelf fracture parameterization in an ice sheet model”. In: *The Cryosphere* 11.6, pp. 2543–2554. DOI: 10.5194/tc-11-2543-2017.
- Sutter, J., Fischer, H., Grosfeld, K., Karlsson, N. B., Kleiner, T., Van Liefferinge, B., and Eisen, O. (2019). “Modelling the Antarctic Ice Sheet across the mid-Pleistocene transition – implications for Oldest Ice”. In: *The Cryosphere* 13.7, pp. 2023–2041. DOI: 10.5194/tc-13-2023-2019.
- Swithinbank, C. (1988). “Satellite image atlas of glaciers of the world-Antarctica”. In: DOI: 10.3133/pp1386B.
- Thomas, R. H. and Bentley, C. R. (1978). “A Model for Holocene Retreat of the West Antarctic Ice Sheet”. In: *Quaternary Research* 10.2, pp. 150–170. DOI: 10.1016/0033-5894(78)90098-4.
- Uenzelmann-Neben, G., Gohl, K., Hochmuth, K., Salzmann, U., Larter, R. D., Hillenbrand, C.-D., and Klages, J. P. (2022). “Deep water inflow slowed offshore expansion of the West Antarctic Ice Sheet at the Eocene-Oligocene transition”. In: *Communications Earth & Environment* 3.1, p. 36. DOI: 10.1038/s43247-022-00369-x.
- Van der Veen, C. J. (1999). *Fundamentals of glacier dynamics*. A.A. Balkema, Rotterdam, Brookfield.
- Vaughan, D. G. (2008). “West Antarctic Ice Sheet collapse—the fall and rise of a paradigm”. In: *Climatic Change* 91.1-2, pp. 65–79. DOI: 10.1007/s10584-008-9448-3.
- Wang, Y., Lai, C.-Y., and Cowen-Breen, C. (2022). “Discovering the rheology of Antarctic Ice Shelves via physics-informed deep learning”. In: *Preprint*. DOI: 10.21203/rs.3.rs-2135795/v1.
- Weertman, J. (1974). “Stability of the Junction of an Ice Sheet and an Ice Shelf”. In: *Journal of Glaciology* 13.67, pp. 3–11. DOI: 10.3189/S0022143000023327.
- Winkelmann, R., Martin, M. A., Haseloff, M., Albrecht, T., Bueler, E., Khroulev, C., and Levermann, A. (2011). “The Potsdam Parallel Ice Sheet Model (PISM-PIK) – Part 1: Model description”. In: *The Cryosphere* 5.3, pp. 715–726. DOI: 10.5194/tc-5-715-2011.
- Zachos, J., Pagani, M., Sloan, L., Thomas, E., and Billups, K. (2001). “Trends, Rhythms, and Aberrations in Global Climate 65 Ma to Present”. In: *Science* 292.5517, pp. 686–693. DOI: 10.1126/science.1059412.

- Zhang, E., Liu, L., and Huang, L. (2019). “Automatically delineating the calving front of Jakobshavn Isbræ from multitemporal TerraSAR-X images: a deep learning approach”. In: *The Cryosphere* 13.6, pp. 1729–1741. DOI: 10.5194/tc-13-1729-2019.
- Zhou, D.-X. (2020). “Universality of deep convolutional neural networks”. In: *Applied and Computational Harmonic Analysis* 48.2, pp. 787–794. ISSN: 1063-5203. DOI: <https://doi.org/10.1016/j.acha.2019.06.004>.

Chapter 2

Episodic dynamic change linked to damage on the Thwaites Glacier Ice Tongue

This chapter was published in 2023 in *Nature Geoscience* as the following article:

T. Surawy-Stepney, A. E. Hogg, S. L. Cornford and B. J. Davison: “*Episodic dynamic change linked to damage on the Thwaites Glacier Ice Tongue*”

Abstract

The stability and dynamics of Thwaites Glacier depend on the structural properties of its marine terminus; however, the relationship between these variables on the floating ice tongue is poorly understood. Here, we present a 6-year record of ice speed, derived from satellite observations starting in 2015, showing two large-magnitude ($\sim 30\%–45\%$) and prolonged ($\sim 1\text{--}2$ years) cycles of speed variation across the ice tongue. Using an automated, deep-learning-based method of extracting high resolution fracture maps from satellite imagery, we detail periods of increasing fracture development and subsequent reconsolidation in the ice tongue shear margin which coincide with the observed speed changes. Inverse modelling using the BISICLES ice sheet model indicates that the variation in ice speed can be accounted for by these observed changes to the spatial pattern of fracturing. This study provides further evidence of direct coupling between fracturing and dynamic variability in West Antarctica, but indicates that increased fracturing and associated speed changes are reversible on 1 to 2 year timescales. We suggest that fracturing does not necessarily lead to positive feedback with glacier acceleration

on these timescales and that damage process modelling is important for accurately predicting the evolution of the Antarctic Ice Sheet.

2.1 Introduction

Thwaites Glacier (TG) is the largest ice stream in West Antarctica, draining a 210,600km² catchment that holds enough ice to raise sea levels by 0.59 m alone (Holt et al., 2006). Satellite observations show that over the past 30-years, TG has been thinning (Shepherd et al., 2019; McMillan et al., 2014), accelerating (Mouginot et al., 2014) and retreating at the grounding line (Milillo et al., 2019). These decadal-scale dynamic changes are thought to be driven by incursions of warm circumpolar deep water, which likely strengthened in the mid 2000's (Walker et al., 2007; Dutrieux et al., 2014; Jacobs et al., 2011; Holland et al., 2019).

TG terminates in two distinct floating units: the Eastern Ice Shelf (EIS) and the TG Ice Tongue (TGIT). Observations show that the slower flowing EIS has weakened since the speed-up of TG intensified in ~ 2006 (Benn et al., 2021) and may be susceptible to disintegrate on a decadal timescale (Alley et al., 2021). The TGIT, which is, as of 2021, ~ 20 km wide, ~ 50 km long and ~ 300 m thick, is a remnant of the once much larger western ice shelf (Miles et al., 2020). The TGIT is heavily crevassed, is intermittently bonded to the EIS by a mixture of icebergs and sea ice, and pinned on a small bathymetric peak to the north-west (NW Pinning Point, NWPP). Historically, the TGIT has been a control on the dynamic behaviour of the EIS via the transmission of shear stress through the formerly robust eastern shear margin (ESM) (Alley et al., 2021). However, since the recent rapid structural weakening of the ESM and its propagation up glacier, the EIS and TGIT have largely decoupled (Mouginot et al., 2014; Miles et al., 2020).

Ice fracture - the presence of rifts or surface/basal crevasses - is an important component of the Antarctic ice sheet system, as modifications to the stress balance caused by regions of fractured ice have a distributed effect on ice-dynamics (Borstad et al., 2013; Khazendar et al., 2009). A feedback mechanism between the acceleration of fast-flowing ice shelves in the ASE and structural deterioration of their shear margins has recently been posited (Lhermitte et al., 2020). Given that the vast majority of west Antarctica's grounded ice mass loss comes from grounding line discharge (Rignot et al., 2019), increased understanding of this potential feedback mechanism (Lhermitte et al., 2020) and the wider impact of brittle processes on the stability of the region is essential for accurately predicting Antarctica's future contribution to sea level

rise. However, though progress has been made (Pralong and Funk, 2005; Plate et al., 2012; Åström et al., 2013; Bassis and Ma, 2015; Sun et al., 2017; Crawford et al., 2021), damage processes are not well integrated into realistic (as opposed to highly idealised) ice sheet models and large-scale quantitative observations of ice-fracture are sparse (Lai et al., 2020).

After a period of relatively steady flow since 2011, a $\sim 30\%$ increase in ice speed between late 2016 and 2017 was observed across the TGIT and the ESM (Miles et al., 2020). Here, we present a 6-year record of ice speed data from January 2015 to March 2021, which shows that this speed-up was the start of a period of high-magnitude ($\sim 30 - 45\%$) and short-term ($\sim 1 - 2$ yr) episodic ice-dynamic activity on the TG ice tongue (Sect. 2.2.1). We use a deep neural network to detect surface crevassing across western TG (Sect. 2.2.2), which details the inland propagation of fractured regions of the ESM and the TGIT during each period of ice acceleration (extending observations of Lhermitte et al., 2020; Miles et al., 2020) followed by the reconsolidation of the shear margin during periods of deceleration. Numerical modelling, carried out using the BISICLES ice sheet model, indicates that the observed speed changes can be largely accounted for by the changes in structural integrity of TGIT that we observe.

2.2 Results

2.2.1 Observed Changes in Ice Speed

After a period of stable flow ($3.4 - 4.1 \text{ km a}^{-1}$) between 2015 and 2016, TGIT accelerated in late-June 2017, reaching peak speeds in December 2018 that were 45% ($+ 1.5 - 1.9 \text{ km a}^{-1}$) greater than the 2015/2016 mean (Fig. 2.1a). Although the 2017/18 acceleration event was extensive across the majority of the structurally weak ice tongue, we observe spatial variability in the relative speed change, with a $20 - 40\%$ acceleration observed in the southern part of the ESM and a $20 - 60\%$ deceleration observed on the central part of the ESM (Fig. 2.1a). Between January and October 2019 the whole TGIT decelerated to its mean speed over the 2015-2021 period ($4 - 4.5 \text{ km a}^{-1}$). Whilst the TGIT decelerated, grounded ice speeds continued to steadily increase. Our observations on the TGIT show that a second speed-up event initiated in November 2019, during which the TGIT increased in speed by $25 - 30\%$ (Fig. 2.1b) for a period of 6-months. In September 2020, the ice tongue decelerated back to its 2015-2021 mean speed (Fig. 2.1c), with a slower reduction in speed at the grounding line.

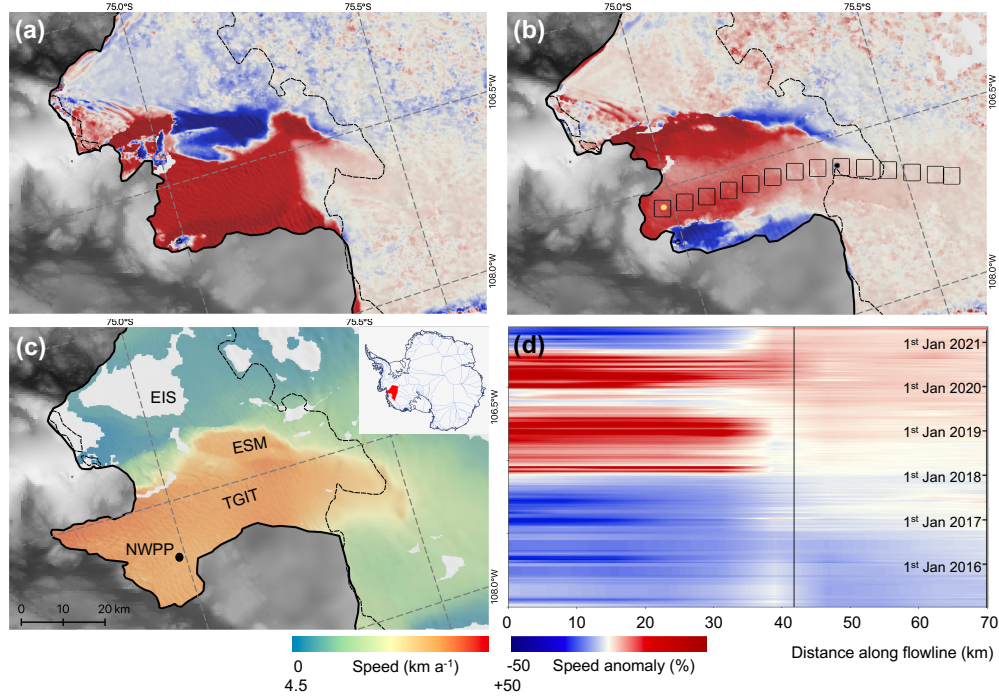


Figure 2.1: Ice speed anomalies on Thwaites Glacier (TG). (a) Spatial pattern of ice speed anomaly observed between October 2018 and April 2019 (i.e. peak of first speed-up event) compared to the 2015-2021 mean. (b) Spatial pattern of ice speed anomaly observed between November 2019 and May 2020 (i.e. peak of second speed-up event) compared to the 2015-2021 mean. (c) Mean ice speed on TG in 2016 in kilometres per year (km a^{-1}), prior to the two ice-dynamic episodes, and locations of the Eastern Ice Shelf (EIS), the Eastern Shear Margin (ESM), the Thwaites Glacier Ice Tongue (TGIT) and the North-West Pinning Point (NWPP). (c - inset) MEaSUREs grounding line, coastline and ice boundaries (Rignot et al., 2016), with the basin and ice shelf of TG highlighted in red. Velocity data are superimposed on a 125 m resolution MODIS mosaic (Haran et al., 2013) and BedMachine V2 bed data (Morlighem et al., 2020a) (grey shading). (d) Hovmöller diagram showing the ice speed anomaly relative to the 2015-2021 mean in a set of 14 equally spaced 4×4 km bounding boxes along a central flow-line (black box outlines Fig. 2.1b), with distance measured from the north (yellow point, 0 km) to the south. Linear interpolation is used in both the distance and time axes for missing data. The 2011 grounding line location is shown on Figs. 2.1a-c (dashed black line) and Fig. 2.1d (vertical black line) (Rignot et al., 2016).

The spatial pattern of the first acceleration episode displays a well-defined boundary between the structurally weak ice tongue where a large speedup was observed, compared to the upstream region of more consolidated floating ice where ice speeds remained relatively stable (Fig. 2.1a). Conversely, the second acceleration episode is characterised by a softening of this boundary, with speed up observed in both regions (Fig. 2.1b, d), and a $+ \sim 6\%$ increase in speed measured at the grounding line in 2021 compared to the pre-2016 mean.

2.2.2 Observed Changes in Fracture

Using a deep neural network, we segment SAR backscatter images into binary maps representing observable surface fractures. These data show that between March 2016 and March 2021 there was a broad increase in the amount of severe crevassing across the surface of the TGIT (Fig. 2.2a-f), indicating a continued decline in its structural integrity (Miles et al., 2020). The deterioration of the TGIT has several key characteristics. Firstly, in the ESM and along the central portion of the TGIT we observe an overall increase in the density of crevassing throughout the study period (Fig. 2.2) and the upstream propagation of the region of fractured ice towards the grounding line in the central TGIT and near the ESM (Lhermitte et al., 2020). Inland of the ESM, the increases in fracturing are due to new crevasses opening up in-situ, some of which caused the detachment of a ~ 10 km section of ice in March 2017 (Fig. 2.2b). This process also occurred on the portion of the TGIT close to the grounding line, though here, new fractures also appeared due across-flow extension of crevasses originating in the lateral margins of the TGIT (Fig. 2.2c-f).

The observed upstream propagation of fractured ice south of the ESM was not permanent. By March 2021, much of the fractured ice had been advected downstream, and maximum positive strain rates had decreased below the threshold required to promote the development of new crevasses (Fig. 2.2).

Extended Data Figure 1 shows the segmentations used in the production of Fig. 2.2 along with the SAR images from which the data were extracted.

2.2.3 Chronology of Change in Ice Speed and Structure

In order to assess the relationship between the two ice-dynamic episodes and observed changes in crevassing, we generated time-series of speed, fracture and ice shelf length on the TGIT (Fig. 2.3). We define “fracture density” as the fracture maps integrated over a particular region. The “relative” timeseries shown in Fig. 2.3c, d are created by dividing the fracture densities by the lowest value through time. The timeseries of ice shelf length was created using a neural network trained to segment SAR backscatter images of the TGIT into the classes “glacier” and “sea”.

Two cycles of speed change are clearly visible during the 6-year study period (Fig. 2.3a), characterised by two periods of elevated speed from June 2017 to April 2019 and January to September 2020, separated by a deceleration back to 2015-2021 mean speeds.

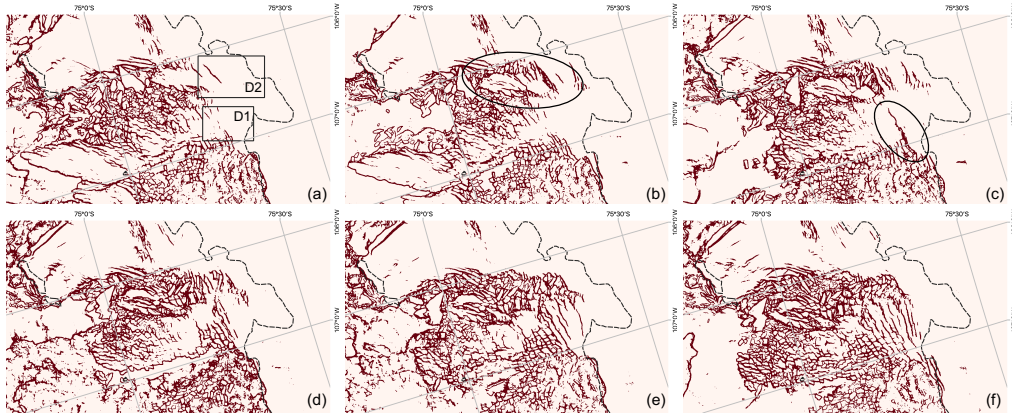


Figure 2.2: Observed fractures on the Thwaites Glacier Ice Tongue. Annual fracture maps generated by applying a trained neural network to Sentinel-1 SAR images acquired in March of each year during 2016 to 2021. Panels a to f show snapshots of mapped fractures (Section 2.2) on the respective dates: 29/03/2016, 12/03/2017, 25/03/2018, 26/03/2019, 08/03/2020 and 27/03/2021. Time-series of relative fracture density (Section 2.2) shown in Fig. 2.3c and d are extracted from bounding boxes D1 and D2. We highlight the location of a large section of ice that detached from the upstream shear margin in 2017 (black oval, Fig. 2.2b) and a ~ 14 km long transverse crack (black oval, Fig. 2.2c), with the 2011 grounding line location also shown on all maps (dashed black line) (Rignot et al., 2016).

Two key structural changes on the TGIT occurred prior to the first acceleration/deceleration event. Firstly, there was a significant increase in crevassing (a factor of ~ 5 in fracture density) inland of the ESM beginning in November 2015 (D2; Fig. 2.3d), which closely followed a loosening of the sea-ice/icebergs in the ESM (Miles et al., 2020). Secondly, we observed three calving events, starting in March 2016 when $\sim 180 \text{ km}^2$ of ice detached from the western edge of the TGIT (Fig. 2.3e), in March 2017 where the northern-most 15 km of the ice tongue detached creating a $\sim 235 \text{ km}^2$ iceberg (Miles et al., 2020), and in June 2017 where 35 km^2 of ice was lost (Fig. 2.3b, e). These latter two calving events caused the temporary unpinning of the TGIT from the NWPP for the first time on record, before it re-pinned a month later.

Following this temporary unpinning in June 2017, the first speed cycle initiated. This is characterised by a progressive increase in speed and relative fracture density until December 2018 (Fig. 2.3). During this speed cycle, the TGIT became entirely unpinned from the NWPP in January 2018. Following this, we measured a five-fold increase in fracture density on the central trunk of TG (D1), which peaked by early November 2018 (Fig. 2.3c), and a continued acceleration of TGIT. Crevassing inland of the ESM (D2) continued to increase until December 2018, though at a modest rate compared to that observed during 2015-2016 (Fig. 2.3d). The ice tongue grew back to within 5 km of its 2015 length (Fig. 2.3b), though it did not re-pin on the NWPP. From

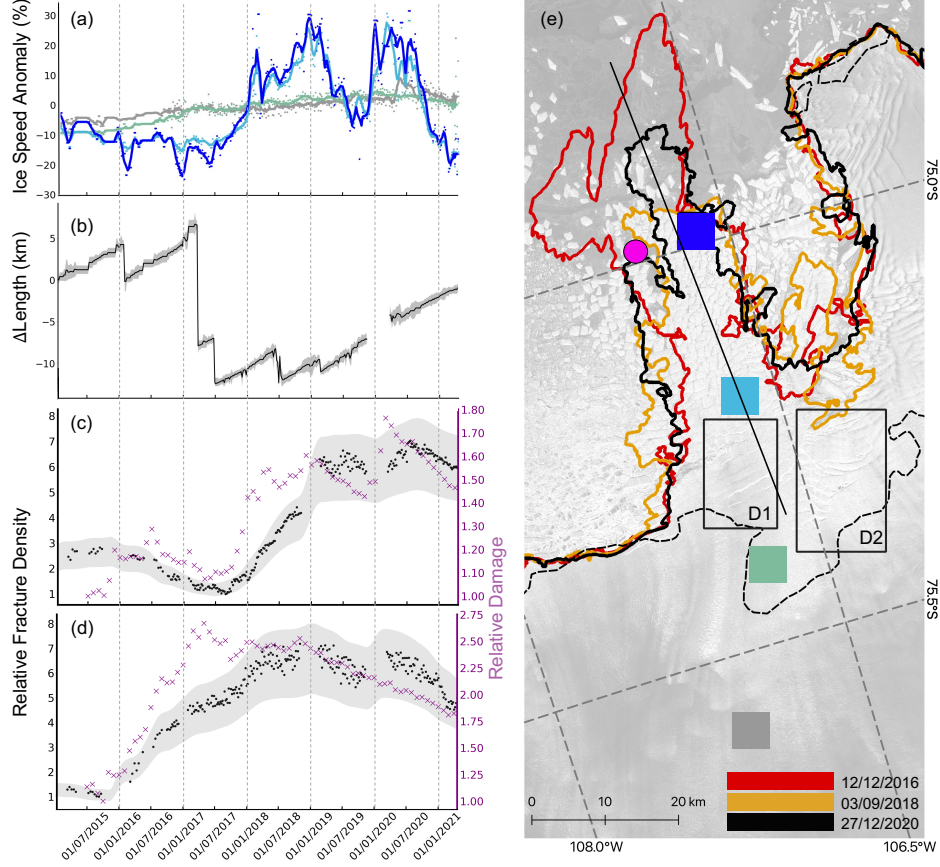


Figure 2.3: Time-series of Thwaites Glacier Ice Tongue (TGIT) flow speed, fracture and damage. Time-series from 2015 to 2021 of: (a) the ice speed anomaly relative to the 2015–2021 mean, extracted from four 4×4 km bounding boxes along a central flow-line, where dark blue corresponds to the most seaward and grey corresponds to the most inland positions (Fig. 2.3e); (b) change in length of the TGIT, measured along the transect shown in (e). The uncertainty band in light-grey indicates $1\text{-}\sigma$, where σ is the width of a sigmoid-fit to the transect profile near the calving front (Supplementary Materials B.2.4); (c) and (d) observed fracture density (black dots) and model-inferred damage (purple crosses) extracted from bounding boxes D1 (Fig. 2.3c) and D2 (Fig. 2.3d) respectively. The uncertainty band in light grey is bounded at the top by $f \times (2 - S)$ and at the bottom by $f \times P$, where f is the fracture density and S and P are respectively the *sensitivity* and *precision* of the binary segmentations calculated from a manually annotated reference image (Supplementary Materials B.2.3). Vertical dashed lines show the first of January of each year from 2016 to 2021. (e) Map showing the calving front location (red, orange and black lines) from 2016 to 2020 respectively on a Sentinel-1 amplitude image, the transect along which change in tongue length was calculated (straight black line), the location where velocity anomaly time-series were extracted (dark blue, light blue, turquoise and grey coloured boxes), and the locations where the fracture/damage measurements were extracted (black bounding boxes D1 and D2). The 2011 grounding line location (dashed black line) and North-West Pinning Point (NWPP) (pink dot) is also shown (Rignot et al., 2016).

December 2018 onwards, we observe a deceleration across the TGIT and a decrease in relative fracture density (Fig. 2.3). The decrease in fracture density reflects a pause in the development of new crevasses south of the ESM, and the advection of existing crevasses downstream.

Following an 8-month period of lower ice speeds ($\sim 4.5\text{km a}^{-1}$) from March to October 2019, we observed a more modest increase in crevassing in early 2020 on both the TGIT and ESM (Figs. 3c and 3d) which coincided with the second period of acceleration (Fig. 2.3a). Contact with the pinning point was remade in December 2019, though, rather than stabilising the ice tongue, this led to significant fracturing along its western edge. This re-pinning and the associated sudden structural change coincided closely with the onset of the second period of acceleration and led to a decrease in width of the TGIT (Fig. 2.3e), particularly at the seaward end, which was still apparent at the end of the study period in 2021.

Extended Data Figure 2 shows the segmentations used in the production of Fig. 2.3e along with the SAR images from which the data were extracted.

2.2.4 Model Inversions

We used the BISICLES ice sheet model (Cornford et al., 2015) in combination with satellite observations to assess the link between the changing speed of the ice tongue and its local structural properties during the two major ice-dynamic episodes (Fig. 2.3). We do so by inferring a scalar damage parameter D (henceforth referred to as “damage”) from ice velocity observations at monthly intervals that represents an isotropic decrease in the load bearing capacity of the ice. The time-series of “relative damage” was generated by summing D over the bounding boxes (D1 and D2, Fig. 2.3e) and dividing by the lowest value in the timeseries. A geometrical interpretation of damage is that it represents the ratio of the vertical area of crevasses to the total area for each vertical slice, integrated over the region. This parallels our definition of observed fracture density. We assessed the quality of the model optimisation by comparing the modelled and observed velocity fields, which agreed to within $2.7 \pm 1.4\%$ on average, over the observational period.

The spatial distribution of damage and its evolution through the 6-year study period broadly resemble the observed changes in fracture at the western TG terminus (Figs. 2a-f and 3d-e). In particular, the model coarsely reproduces the observed pattern of fracture south of the ESM, with damage intensifying and propagating south-west between January 2016 and June 2019 (Fig. 2.4b, d), followed by the spread of severe damage north-west onto TGIT through the rest of the study period (Fig. 2.4d-f).

A comparison of the time-series shows that the onset and pace of change in damage exhibits sig-

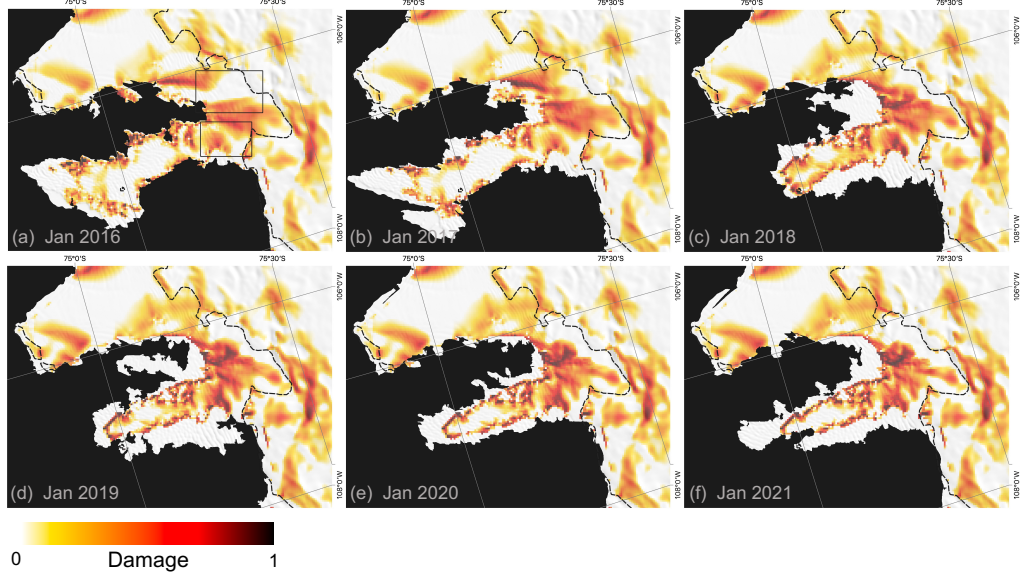


Figure 2.4: Annual maps of damage inferred using the BISICLES ice sheet model in January from 2016 to 2021. Panels (a) to (f) show a snapshot of damage, with the seaward limit of the model domain bounded by the observed glacier perimeter for the respective month, superimposed on a 125 m resolution MODIS mosaic (Haran et al., 2013). Time-series of relative damage are extracted from bounding boxes (D1 and D2, black outline) (Fig. 2.4a). We also highlight the location of a large section of ice that calved in 2017 (black oval, Fig. 2.4b). The grounding line and pinning point locations are also shown on all maps (dashed black line) (Rignot et al., 2016).

nificant congruity with the observed change in fracture density (Fig. 2.3c, d). This is especially true in D1, in which damage increases during the first acceleration event, and increases/decreases in line with fracture density thereafter, though changes appear to lead those of fracture density by 2-4 months. In D2, peak damage is reached in May 2017, prior to the start of the first period of acceleration, and two years before the maximum observed fracture density, potentially indicating that fractures formed at the ice base before they manifested as surface (observable) crevasses.

2.3 Discussion

2.3.1 Long-term Context

Significant dynamic changes have occurred on TG over the last three decades, with the speed of the central trunk increasing by 50% between 1992 and 2013 (Mouginot et al., 2014), doubling TG’s sea-level contribution between 1994-6 and 2010 (Medley et al., 2014). Our results extend the ~ 30 -year ice speed record on Thwaites Glacier to 2021, showing that the pattern of long-

term speed up with 5-10 year fluctuations on the TGIT (Miles et al., 2020; Mouginot et al., 2014), likely driven by decadal variability in ocean temperature (Jenkins et al., 2016; Holland et al., 2019), has seen the addition of shorter-term, higher magnitude acceleration and deceleration events. This indicates that TGIT has an increasing sensitivity to external forcing and internal feed-backs. We note that historical satellite observations are of insufficient temporal resolution to resolve velocity events on annual timescales or less, therefore it is not possible to determine whether the episodic ice-dynamic behaviour that we observe represents a departure from TG's historical behaviour.

Our measurements show a detailed picture of fracturing on the ice tongue that has increased and migrated inland since 2015. Since observations of fracturing were last made on the TGIT (Lhermitte et al., 2020; Miles et al., 2020) we observe that the region south of the ESM has undergone a period of fracture development and partial re-consolidation, showing that the previously identified acceleration-damage feedback of Lhermitte et al., 2020 is not closed. At least, our results suggest that the ‘sign’ of the feedback reversed in the shear margin at the peak of the first speed up in December 2018, did not initiate in the second acceleration event, and did not re-emerge before the end of the observational period. This suggests a feedback that is highly sensitive to external influences. We hypothesise that the sensitivity comes from fluctuating ice thickness in the region caused by changes to basal melt rates and changing ice flux from south-east of the ESM, with thicker ice reducing the likelihood of fracturing. Though we cannot rule out the influence of other processes, such as periodic changes to the grounding line position. It is likely, therefore, that this reconsolidated section of shear margin will go through more periods of fracturing. However, our observations suggest that the decline of the TGIT via a mechanism of weakening in its shear margin and acceleration causing increased shear strain rates will not progress monotonically.

2.3.2 Link Between Structural Change and Dynamic Variability

At the large-scale of structural change, our observations suggest that, after acceleration and fracturing in the ESM throughout 2016, the unpinning of the TGIT from the NWPP, caused by the detachment of a large section of the ice tongue in two parts between March and June 2017, provided the conditions for the whole ice tongue to respond rapidly and dynamically to external forcing. This triggered the first acceleration event in June 2017, which was compounded by a significant decrease in the structural integrity of the ice tongue, the upstream expansion of the

fractured ESM, and the propagation of damage inland along the central glacier trunk in the form of long transverse crevasses forming increasingly close to the grounding line. The weakened ice tongue was then susceptible to a further dynamic response, which manifested as the second ice-dynamic episode, where the prior inland expansion of the damaged ice likely contributed to the upstream propagation of the ice speedup and its lateral confinement to the central portion of the ice tongue. We identify the fracturing and acceleration of the ESM in 2016 to be a likely initiator of the dynamic changes seen throughout the observational period.

Previous studies have suggested that hydrofracture (Scambos et al., 2000; Scambos et al., 2003) and unpinning (Holland et al., 2015) are the two primary mechanisms for ice shelf destabilisation, while other work proposes a third mechanism, termed backstress-triggered failure, where backstress from a pinning point can cause ice-shelf fragmentation (Vaughan, 1993; Benn et al., 2021). Our measurements show that the fragility of the TGIT rendered it unable to form a stable configuration upon re-pinning on the NWPP in late 2019. Instead, it fractured along most of its western edge, with the affected area penetrating further into the bulk of the ice tongue as the compressive stress propagated south. This coincided with and, therefore, may have triggered the onset of the second period of acceleration in late 2019. Further studies are needed to test the impact of such a mechanism of failure on other non-steady state ice shelf systems, as it provides a mechanism for ice shelf destabilisation in the absence of an external climatological forcing mechanism.

By showing a strong correspondence between observed fracture patterns and damage inferred by the model, we have shown that the observed dynamic changes can be accounted for with changes in the spatial distribution of damage that are comparable to the changes in fracturing that we observe. An important implication of this is that a prognostic ice sheet model without new, more sophisticated parametrisations of damage processes would not be able to simulate the behaviour of highly fractured ice-shelves. Given that forward modelling of the Antarctic Ice Sheet largely relies on fixed fields for ice rheology, and that ice shelves are of such importance to the future evolution of the ice sheet, this study demonstrates in an un-idealised setting the existence of an important missing ingredient in the simulation of future sea-level rise. We have, however, provided further evidence that the impact of real fractures on ice dynamics can be well-represented in continuum ice sheet modelling with a simple, scalar damage parameter. This provides motivation for the development of process modelling methods that treat damage as a

local, isotropic change in effective ice viscosity.

While our results show that there is a continued, strong link between structural change and dynamic variability and that backstress-triggered failure likely played a causal role in the second (2020) dynamic episode, internal feedbacks cannot alone account for the initiation of the first ice-dynamic episode or the deceleration of TGIT. Instead, an external forcing mechanism is required to drive the initial damage development and calving front retreat, and, at least in part, the two deceleration events in 2019 and in the autumn of 2020 (Supplementary Materials B.4).

2.4 Conclusions

Our 6-year-long record of ice speed shows the onset and abatement of two major ice-dynamic episodes on TGIT between 2015 and 2021, which caused a 30%– $\sim$ 45% change in ice speed relative to the 2015-21 mean over monthly to annual timescales. We developed and applied deep learning methods to map surface crevasses and calving front location on TG, and used these measurements to quantify the decrease in structural integrity of the floating ice tongue. To the best of our knowledge, this is the first automated method for the extraction of dense, high-resolution fracture time-series, and can be used for future AIS-wide analyses of the effect of damage on ice-dynamics. The fracture mapping results show that the crevassed area on the central trunk of TG expanded throughout the study period, with pronounced fracturing occurring at the time of both the first and second ice-dynamic episodes. Fracturing in the shear margin, where a positive acceleration-damage feedback has been posited, increased prior to and during the first dynamic speedup, but decreased thereafter - suggesting such a feedback has a high degree of sensitivity to external conditions. A quantitative comparison between the observed fracture and model-inferred damage indicates that the observed changes in the structural integrity of the ice tongue has played a significant role in its dynamic behaviour. The intricate coupling between ice dynamics, calving processes, and fracture demonstrates that accurate projections of the short-term behaviour of marine terminating ice streams and their longer-term stability cannot be made without accounting for changes in the structural integrity of the ice. In the future, we must extend the observational record of ice-fracture and its change over time across the Antarctic Ice Sheet using data from both current and historical satellite missions. Ice-flow models must evolve to physically represent damage processes in forward-modelling, and must increase their capability to integrate larger volumes of observational data

at fine temporal resolution in order to capture short-term behaviour.

2.5 Methods

We measured ice speed (Sect. 2.5.1), fracture (Sect. 2.5.2) and ice tongue perimeter (Sect. 2.5.2) on TG over the last 6-years, using synthetic aperture radar (SAR) data from the Copernicus Sentinel-1a and -1b satellites. Single Look Complex images were acquired in Interferometric Wide swath mode between January 2015 and March 2021, with a temporal baseline of 12 days between image pairs until September 2016 and 6 days thereafter.

2.5.1 Ice Velocity Observations

Ice velocity was calculated using the intensity feature tracking method (Bindschadler and Scambos, 1991; Strozzi et al., 2002) (Fig. 2.1c), where displacement is measured by finding peaks in local normalised cross-correlation between pairs of sequential Sentinel-1 images. Tracked features include both radar speckle caused by unresolved scatterers and resolvable physical features such as crevasses. Measurements are made in windows of 350×76 pixels (covering a ground area of $\sim 2 \text{ km}^2$), using a 80×16 pixel grid resolution (corresponding to ground separations of $\sim 320 \text{ m}$). The velocity maps are resampled into ground range using a digital elevation model (DEM) (Bamber et al., 2009) and geocoded at 100 m resolution, with outlying values removed using a maximum velocity threshold (6 km a^{-1}) and a low-pass filter to ensure consistency between neighbouring regions (Selley et al., 2021). Sources of error include radar delay caused by ionospheric dispersion, uncertainty in the DEM surface, uncertainty in the position of the cross-correlation peak and inaccuracy in image geocoding and co-registration. We estimate a spatially varying error for each ground pixel by scaling the reciprocal cross correlation signal-to-noise ratio (SNR) by the ice velocity in that region (Lemos et al., 2018b; Lemos et al., 2018a). We define the SNR as the ratio between the amplitude of the cross-correlation peak and the average of the cross-correlation field excluding the primary peak. The largest absolute velocity errors are found at the highly crevassed ice front and in the shear margins, and we find that the mean velocity error is 12.7% across the TG region during the 6-year study period.

2.5.2 Observations of Fracture and Ice Tongue Perimeter

A fully convolutional neural network based on U-Net (Ronneberger et al., 2015) was employed to make $\sim$ weekly measurements of fracture across TG using the 6-year time-series of radiometrically terrain corrected SAR backscatter images at 50 m spatial resolution. Sentinel-1 SAR data is acquired at C-band frequencies that can penetrate up to ~ 10 m into the snow pack, depending on the snow density and water content at the time of image acquisition. This allows crevasses bridged by snow to be observed in the backscatter signal (Marsh et al., 2021), however, it is not possible to observe deep sub-surface or basal crevasses, we acknowledge that this imposes certain limitations on our results. We also note that small (sub-pixel scale) crevasses will not be detected in our processing. This additional source of error leads to fracture maps (e.g. Fig. 2.2) and fracture density time-series (e.g. Fig. 2.1c, d) that are conservative.

The neural network was trained to classify images into *fractured* and *unfractured* binary categories at the pixel-level, using a bootstrapping approach to supervised learning (Lai et al., 2020). Training data was produced from 20 SAR backscatter images from 10 locations in Antarctica, along with corresponding manually delineated *calving front* locations at 50 m spatial resolution (Supplementary Materials B.2). A fracture map was constructed from each SAR backscatter image by dividing the region into 256×256 -pixel patches, which were fed through the neural network and segmented. The resulting fracture maps were tiled together with an overlap of 128-pixels, and a Gaussian weighting was applied away from the centre of each map tile to reduce edge effects. Fracture time-series were generated by counting the number of pixels classified as fracture within a certain region (normalised by the lowest value in the timeseries). We refer to this quantity as the *relative fracture density*. Error on the fracture density time-series was computed from the precision and sensitivity of the network output for a selected input scene and corresponding ground-truth image (Supplementary Materials B.2.3).

We used the same convolutional neural network to measure the TGIT perimeter in the 6-year time-series of SAR backscatter images, following similar successful applications of deep learning for this task (Baumhoer et al., 2019; Zhang et al., 2019; Zhang et al., 2021; Mohajerani et al., 2019; Heidler et al., 2021). Adjustments were made to the loss function and training data (Supplementary Materials B.2.4), with training data now consisting of binary masks representing the desired classes of “glacier” and “sea”. Edge detection was applied to the output classification maps to produce the final calving front perimeters. The perimeter data was used

to make weekly measurements of the change in ice tongue length throughout the study period, defined along a transect through the centre of the ice tongue (Fig. 2.3e), and to bound the ice flow model domain (Sect. 2.2.4).

2.5.3 Inverse Modelling

The speed at which a glacier flows is a function of its boundary conditions and material properties. Given complete 2-dimensional fields for the glacier basal traction $C \equiv C(x, y)$ and viscosity enhancement factor $\phi \equiv \phi(x, y)$ (a coefficient which multiplies the effective viscosity of the ice and defines deviations from the value expected given Glen’s flow law) the BISICLES forward model solves a 2D stress-balance equation to compute the ice velocity for a glacier with given geometry (BedMachine v2 Morlighem et al., 2020b). In this study, we solve the inverse problem by using gradient-based optimisation to concurrently infer C and ϕ from the velocity observations (Lee et al., 2015).

We define the model-inferred damage (from now on referred to as “damage”) as the scalar variable $D \equiv D(x, y) = 1 - \phi \in (-0.2, 1)$, such that an increase in D indicates an isotropic decrease in the load-bearing capacity of the ice (Lemaitre, 2012). When calculated in this way, damage may also indicate uncertainty in the temperature field, regional changes in the temperature dependence of Glen’s flow law, and deviations from the assumed isotropy of creep deformation.

Changes to the validity of the assumption of isotropy are likely to occur over larger temporal scales than the macroscopic damage identified by the model, so we believe such contributions to D to be secondary. Additionally, by looking at the distribution of negative damage values, we conclude that errors in the temperature field only contribute an error to D of ~ 0.05 (Supplementary Materials B.3).

An inverse problem such as this is ill-posed, meaning that the resulting parameter values at each time-step are dependent on a choice of regularisation term added to the cost function to ensure the inversion is not under-determined or ill-conditioned. Some caution is therefore required when drawing conclusions about glacial processes from the evolution of the model parameters. I.e. the damage field resulting from this calculation is one possible explanation for the observed changes in speed, but not the only possible explanation. A consequence of the regularisation we choose, which penalises high spatial frequency components of ϕ , is that the resulting damage

fields are smoother than the fracture maps and rarely resolve individual crevasse-like features. Monthly median ice-velocity mosaics (Sect. 2.5.1) were used to define the misfit of the modelled velocity and derive monthly fields of D . The initial inversion was carried out using MEaSUREs Antarctic ice velocity (Rignot et al., 2011), due to its complete coverage over the region, and regularisation was incorporated to ensure consistency between the D fields inferred from sequential velocity observations (Selley et al., 2021). We take the first six months of inversions to be a “spin-up” period before looking at the output fields starting from July 2015. The seaward limit of the model domain was bounded using $\sim$ monthly observed perimeters (Sect. 2.5.2). This prevents the model from attributing velocity gradients at the calving front due to changes in its location, to changes in the effective viscosity, increasing the model sensitivity to real damage.

Bibliography

- Alley, K. E., Wild, C. T., Luckman, A., Scambos, T. A., Truffer, M., Pettit, E. C., Muto, A., Wallin, B., Klinger, M., Sutterley, T., et al. (2021). “Two decades of dynamic change and progressive destabilization on the Thwaites Eastern Ice Shelf”. In: *The Cryosphere Discussions*, pp. 1–31. DOI: doi.org/10.5194/tc-2021-76.
- Åström, J., Riikilä, T., Tallinen, T., Zwinger, T., Benn, D., Moore, J. C., and Timonen, J. (2013). “A particle based simulation model for glacier dynamics”. In: *The Cryosphere* 7.5, pp. 1591–1602.
- Bamber, J., Gomez-Dans, J., and Griggs, J. (2009). “A new 1 km digital elevation model of the Antarctic derived from combined satellite radar and laser data—Part 1: Data and methods”. In: *The Cryosphere* 3.1, pp. 101–111.
- Bassis, J. N. and Ma, Y. (2015). “Evolution of basal crevasses links ice shelf stability to ocean forcing”. In: *Earth and Planetary Science Letters* 409, pp. 203–211.
- Baumhoer, C. A., Dietz, A. J., Kneisel, C., and Kuenzer, C. (2019). “Automated extraction of antarctic glacier and ice shelf fronts from sentinel-1 imagery using deep learning”. In: *Remote Sensing* 11.21, p. 2529.
- Benn, D. I., Luckman, A., Åström, J. A., Crawford, A., Cornford, S. L., Bevan, S. L., Gladstone, R., Zwinger, T., Alley, K., Pettit, E., and Bassis, J. (2021). “Rapid fragmentation of Thwaites Eastern Ice Shelf, West Antarctica”. In: *The Cryosphere Discussions* 2021, pp. 1–25. DOI: [10.5194/tc-2021-288](https://doi.org/10.5194/tc-2021-288).
- Bindschadler, R. A. and Scambos, T. A. (1991). “Satellite-Image-Derived Velocity Field of an Antarctic Ice Stream”. In: *Science* 252.5003, pp. 242–246. ISSN: 0036-8075. DOI: [10.1126/science.252.5003.242](https://doi.org/10.1126/science.252.5003.242).
- Borstad, C. P., Rignot, E., Mouginot, J., and Schodlok, M. P. (2013). “Creep deformation and buttressing capacity of damaged ice shelves: theory and application to Larsen C ice shelf”. In: *The Cryosphere* 7.6, pp. 1931–1947. DOI: [10.5194/tc-7-1931-2013](https://doi.org/10.5194/tc-7-1931-2013).
- Cornford, S. L., Martin, D. F., Payne, A. J., Ng, E. G., Le Brocq, A. M., Gladstone, R. M., Edwards, T. L., Shannon, S. R., Agosta, C., Broeke, M. R. van den, et al. (2015). “Century-scale simulations of the response of the West Antarctic Ice Sheet to a warming climate”. In: *The Cryosphere* 9.4, pp. 1579–1600.
- Crawford, A. J., Benn, D. I., Todd, J., Åström, J. A., Bassis, J. N., and Zwinger, T. (2021). “Marine ice-cliff instability modeling shows mixed-mode ice-cliff failure and yields calving rate parameterization”. In: *Nature communications* 12.1, pp. 1–9.

- Dutrieux, P., De Rydt, J., Jenkins, A., Holland, P. R., Ha, H. K., Lee, S. H., Steig, E. J., Ding, Q., Abrahamsen, E. P., and Schröder, M. (2014). “Strong sensitivity of Pine Island ice-shelf melting to climatic variability”. In: *Science* 343.6167, pp. 174–178.
- Haran, T., Bohlander, J., Scambos, T., Painter, T., and Fahnestock, M. (2013). *2005, updated 2013. MODIS Mosaic of Antarctica 2003-2004 (MOA2004) Image Map, Version 1. moa125_r1_hp1.img.gz*. Boulder, Colorado USA. NASA National Snow and Ice Data Center Distributed Active Archive Center. doi: <http://dx.doi.org/10.7265/N5ZK5DM5>. Accessed 14/06/2021.
- Heidler, K., Mou, L., Baumhoer, C., Dietz, A., and Zhu, X. X. (2021). “HED-UNet: Combined Segmentation and Edge Detection for Monitoring the Antarctic Coastline”. In: *IEEE Transactions on Geoscience and Remote Sensing*, pp. 1–14. ISSN: 1558-0644. DOI: 10.1109/tgrs.2021.3064606.
- Holland, P. R., Bracegirdle, T. J., Dutrieux, P., Jenkins, A., and Steig, E. J. (2019). “West Antarctic ice loss influenced by internal climate variability and anthropogenic forcing”. In: *Nature Geoscience* 12.9, pp. 718–724.
- Holland, P., Brisbourne, A., Corr, H., Mcgrath, D., Purdon, K., Paden, J., Fricker, H., Paolo, F., and Fleming, A. (2015). “Oceanic and atmospheric forcing of Larsen C Ice-Shelf thinning”. In: *The Cryosphere* 9.3, pp. 1005–1024.
- Holt, J. W., Blankenship, D. D., Morse, D. L., Young, D. A., Peters, M. E., Kempf, S. D., Richter, T. G., Vaughan, D. G., and Corr, H. F. (2006). “New boundary conditions for the West Antarctic Ice Sheet: Subglacial topography of the Thwaites and Smith glacier catchments”. In: *Geophysical Research Letters* 33.9.
- Jacobs, S. S., Jenkins, A., Giulivi, C. F., and Dutrieux, P. (2011). “Stronger ocean circulation and increased melting under Pine Island Glacier ice shelf”. In: *Nature Geoscience* 4.8, pp. 519–523.
- Jenkins, A., Dutrieux, P., Jacobs, S., Steig, E. J., Gudmundsson, G. H., Smith, J., and Heywood, K. J. (2016). “Decadal ocean forcing and Antarctic ice sheet response: Lessons from the Amundsen Sea”. In: *Oceanography* 29.4, pp. 106–117.
- Khazendar, A., Rignot, E., and Larour, E. (2009). “Roles of marine ice, rheology, and fracture in the flow and stability of the Brunt/Stancomb-Wills Ice Shelf”. In: *Journal of Geophysical Research: Earth Surface* 114.F4. DOI: <https://doi.org/10.1029/2008JF001124>.
- Lai, C.-Y., Kingslake, J., Wearing, M. G., Chen, P.-H. C., Gentine, P., Li, H., Spergel, J. J., and Wessem, J. M. van (2020). “Vulnerability of Antarctica’s ice shelves to meltwater-driven fracture”. In: *Nature* 584.7822, pp. 574–578.
- Lee, V., Cornford, S. L., and Payne, A. J. (2015). “Initialization of an ice-sheet model for present-day Greenland”. In: *Annals of Glaciology* 56.70, pp. 129–140.
- Lemaitre, J. (2012). *A course on damage mechanics*. Springer Science & Business Media.
- Lemos, A., Shepherd, A., McMillan, M., Hogg, A. E., Hatton, E., and Joughin, I. (2018a). “Ice velocity of Jakobshavn Isbræ, Petermann Glacier, Nioghalvfjærdsfjorden, and Zachariæ Isstrøm, 2015–2017, from Sentinel 1-a/b SAR imagery”. In: *The Cryosphere* 12.6, pp. 2087–2097.

- Lemos, A., Shepherd, A., McMillan, M., and Hogg, A. E. (2018b). “Seasonal Variations in the Flow of Land-Terminating Glaciers in Central-West Greenland Using Sentinel-1 Imagery”. In: *Remote Sensing* 10.12. ISSN: 2072-4292. DOI: 10.3390/rs10121878.
- Lhermitte, S., Sun, S., Shuman, C., Wouters, B., Pattyn, F., Wuite, J., Berthier, E., and Nagler, T. (2020). “Damage accelerates ice shelf instability and mass loss in Amundsen Sea Embayment”. In: *Proceedings of the National Academy of Sciences* 117.40, pp. 24735–24741. ISSN: 0027-8424. DOI: 10.1073/pnas.1912890117.
- Marsh, O., Price, D., Courville, Z., and Floricioiu, D. (2021). “Crevasse and rift detection in Antarctica from TerraSAR-X satellite imagery”. In: *Cold Regions Science and Technology* 187, p. 103284. ISSN: 0165-232X. DOI: <https://doi.org/10.1016/j.coldregions.2021.103284>.
- McMillan, M., Shepherd, A., Sundal, A., Briggs, K., Muir, A., Ridout, A., Hogg, A., and Wingham, D. (2014). “Increased ice losses from Antarctica detected by CryoSat-2”. In: *Geophysical Research Letters* 41.11, pp. 3899–3905.
- Medley, B., Joughin, I., Smith, B., Das, S. B., Steig, E. J., Conway, H., Gogineni, S., Lewis, C., Criscitiello, A. S., McConnell, J. R., et al. (2014). “Constraining the recent mass balance of Pine Island and Thwaites glaciers, West Antarctica, with airborne observations of snow accumulation”. In: *The Cryosphere* 8.4, pp. 1375–1392.
- Miles, B. W. J., Stokes, C. R., Jenkins, A., Jordan, J. R., Jamieson, S. S. R., and Gudmundsson, G. H. (2020). “Intermittent structural weakening and acceleration of the Thwaites Glacier Tongue between 2000 and 2018”. In: *Journal of Glaciology* 66.257, pp. 485–495. DOI: 10.1017/jog.2020.20.
- Milillo, P., Rignot, E., Rizzoli, P., Scheuchl, B., Mouginot, J., Bueso-Bello, J., and Prats-Iraola, P. (2019). “Heterogeneous retreat and ice melt of Thwaites Glacier, West Antarctica”. In: *Science Advances* 5.1. DOI: 10.1126/sciadv.aau3433.
- Mohajerani, Y., Wood, M., Velicogna, I., and Rignot, E. (2019). “Detection of glacier calving margins with convolutional neural networks: A case study”. In: *Remote Sensing* 11.1, p. 74.
- Morlighem, M., Rignot, E., Binder, T., Blankenship, D., Drews, R., Eagles, G., Eisen, O., Ferraccioli, F., Forsberg, R., Fretwell, P., et al. (2020a). “Deep glacial troughs and stabilizing ridges unveiled beneath the margins of the Antarctic ice sheet”. In: *Nature Geoscience* 13.2, pp. 132–137.
- (2020b). “Deep glacial troughs and stabilizing ridges unveiled beneath the margins of the Antarctic ice sheet”. In: *Nature geoscience* 13.2, pp. 132–137. DOI: <https://doi.org/10.1038/s41561-019-0510-8>.
- Mouginot, J., Rignot, E., and Scheuchl, B. (2014). “Sustained increase in ice discharge from the Amundsen Sea Embayment, West Antarctica, from 1973 to 2013”. In: *Geophysical Research Letters* 41.5, pp. 1576–1584.
- Plate, C., Müller, R., Humbert, A., and Gross, D. (2012). “Evaluation of the criticality of cracks in ice shelves using finite element simulations”. In: *The Cryosphere* 6.5, pp. 973–984.
- Pralong, A. and Funk, M. (2005). “Dynamic damage model of crevasse opening and application to glacier calving”. In: *Journal of Geophysical Research: Solid Earth* 110.B1.

- Rignot, E., Mouginot, J., and Scheuchl, B. (2016). *MEaSUREs Antarctic Grounding Line from Differential Satellite Radar Interferometry, Version 2*. Boulder, Colorado USA. NASA National Snow and Ice Data Center Distributed Active Archive Center. doi: <https://doi.org/10.5067/IKBWW4RYHF1Q>. Accessed 15/01/2021.
- Rignot, E., Mouginot, J., and Scheuchl, B. (2011). “Ice flow of the Antarctic ice sheet”. In: *Science* 333.6048, pp. 1427–1430.
- Rignot, E., Mouginot, J., Scheuchl, B., Broeke, M. van den, Wessem, M. J. van, and Morlighem, M. (2019). “Four decades of Antarctic Ice Sheet mass balance from 1979–2017”. In: *Proceedings of the National Academy of Sciences* 116.4, pp. 1095–1103. DOI: 10.1073/pnas.1812883116.
- Ronneberger, O., Fischer, P., and Brox, T. (2015). *U-Net: Convolutional Networks for Biomedical Image Segmentation*.
- Scambos, T., Hulbe, C., and Fahnestock, M. (2003). “Climate-induced ice shelf disintegration in the Antarctic Peninsula”. In: *Antarctic Peninsula Climate Variability: Historical and Paleoenvironmental Perspectives, Antarct. Res. Ser* 79, pp. 79–92.
- Scambos, T. A., Hulbe, C., Fahnestock, M., and Bohlander, J. (2000). “The link between climate warming and break-up of ice shelves in the Antarctic Peninsula”. In: *Journal of Glaciology* 46.154, pp. 516–530. DOI: 10.3189/172756500781833043.
- Selley, H. L., Hogg, A. E., Cornford, S., Dutrieux, P., Shepherd, A., Wuite, J., Floricioiu, D., Kusk, A., Nagler, T., Gilbert, L., Slater, T., and Kim, T.-W. (Feb. 2021). “Widespread increase in dynamic imbalance in the Getz region of Antarctica from 1994 to 2018”. In: *Nature Communications* 12.1, p. 1133. ISSN: 2041-1723. DOI: 10.1038/s41467-021-21321-1.
- Shepherd, A., Gilbert, L., Muir, A. S., Konrad, H., McMillan, M., Slater, T., Briggs, K. H., Sundal, A. V., Hogg, A. E., and Engdahl, M. E. (2019). “Trends in Antarctic Ice Sheet elevation and mass”. In: *Geophysical Research Letters* 46.14, pp. 8174–8183.
- Strozzi, T., Luckman, A., Murray, T., Wegmuller, U., and Werner, C. L. (2002). “Glacier motion estimation using SAR offset-tracking procedures”. In: *IEEE Transactions on Geoscience and Remote Sensing* 40.11, pp. 2384–2391.
- Sun, S., Cornford, S. L., Moore, J. C., Gladstone, R., and Zhao, L. (2017). “Ice shelf fracture parameterization in an ice sheet model”. In: *The Cryosphere* 11.6, pp. 2543–2554.
- Vaughan, D. G. (1993). “Implications of the break-up of Wordie Ice Shelf, Antarctica for sea level”. In: *Antarctic Science* 5.4, pp. 403–408. DOI: 10.1017/S0954102093000537.
- Walker, D. P., Brandon, M. A., Jenkins, A., Allen, J. T., Dowdeswell, J. A., and Evans, J. (2007). “Oceanic heat transport onto the Amundsen Sea shelf through a submarine glacial trough”. In: *Geophysical Research Letters* 34.2. DOI: <https://doi.org/10.1029/2006GL028154>.
- Zhang, E., Liu, L., and Huang, L. (2019). “Automatically delineating the calving front of Jakobshavn Isbræ from multitemporal TerraSAR-X images: a deep learning approach”. In: *The Cryosphere* 13.6, pp. 1729–1741.
- Zhang, E., Liu, L., Huang, L., and Ng, K. S. (2021). “An automated, generalized, deep-learning-based method for delineating the calving fronts of Greenland glaciers from multi-sensor remote sensing imagery”. In: *Remote Sensing of Environment* 254, p. 112265.

Chapter 3

Mapping Antarctic Crevasses and their Evolution with Deep Learning Applied to Satellite Radar Imagery

This chapter was published in 2023 in *The Cryosphere* as the following article :

T. Surawy-Stepney, A. E. Hogg, S. L. Cornford and D. C. Hogg : “ *Mapping Antarctic Crevasses and their Evolution with Deep Learning Applied to Satellite Radar Imagery* ”

Abstract

The fracturing of glaciers and ice shelves in Antarctica influences their dynamics and stability. Hence, data on the evolving distribution of crevasses is required to better understand the evolution of the ice sheet, though such data has traditionally been difficult and time consuming to generate. Here, we present an automated method of mapping crevasses on grounded and floating ice with the application of convolutional neural networks to Sentinel-1 synthetic aperture radar backscatter data. We apply this method across Antarctica to images acquired between 2015 and 2022, producing a 7-and-a-half year record of composite fracture maps at monthly intervals and 50 m spatial resolution, showing the distribution of crevasses around the majority of the ice sheet margin. We develop a method of quantifying changes to the density of ice shelf fractures using the timeseries of crevasse maps, and show increases in crevassing on the Thwaites and Pine Island Ice Shelves over the observational period, with observed changes elsewhere in the Amundsen Sea dominated by the advection of existing crevasses. Using stress

fields computed using the BISICLES ice sheet model, we show that much of this structural change has occurred in buttressing regions of these ice shelves, indicating a recent and ongoing link between fracturing and the developing dynamics of the Amundsen Sea Sector.

3.1 Introduction

The dynamics of the Antarctic Ice Sheet is governed by its geometry, conditions at the ice-bedrock interface and the material properties of the ice. The geometry is influenced by calving due to fracture processes and, at a macroscopic level, the material properties are altered by the presence of crevasses (Pralong and Funk, 2005; Borstad et al., 2012). Additionally, surface crevasses can precondition ice shelves for disintegration via hydrofracture (Hughes, 1983; Rott et al., 1996; Scambos et al., 2009; Alley et al., 2018; Lai et al., 2020), can influence the surface energy balance of the ice sheet (Pfeffer and Bretherton, 1987; Purdie et al., 2022) and are a source of surface-to-bed hydrological pathways on grounded ice. Over the last decade, evidence has emerged that crevassing in the shear margins of fast-flowing ice shelves and ice streams can be of particular importance to the dynamics of the glacier (MacGregor et al., 2012; Lhermitte et al., 2020; Surawy-Stepney et al., 2023). In order to constrain theories regarding the role of fracturing in the evolution of the Antarctic Ice Sheet, a greater quantity of observational data is required.

Historically, the process of mapping fractures remotely has been achieved by the manual annotation of aerial or satellite images. Often, this has been in aid of studies focusing on particular glaciers, ice shelves or individual crevasses of interest (Hambrey and Müller, 1978; De Rydt et al., 2018), though there have been more sustained efforts covering multiple ice shelves (Hulbe et al., 2010). More recently, interferometric synthetic aperture radar (InSAR) data has been used to study individual crevasses (Hogg and Gudmundsson, 2017) and proposed as a basis for widespread analysis of crack growth (Libert et al., 2022). The remarkable sensitivity of interferograms to resolve crack tips makes this an advantageous method, however, the requirement for a high base level of interferometric coherence lessens its practicality for continent-wide analysis.

Satellite-acquired synthetic-aperture radar (SAR) backscatter amplitude data has great potential for crevasse mapping in Antarctica as its year-round, all-weather imaging capability suits Polar conditions. Sub-pixel sized and snow-bridged crevasses are often visible due to the co-

herence of scattered microwaves and the ~ 10 m penetration depth of microwave radiation into the snowpack (Thompson et al., 2020; Marsh et al., 2021).

Though few in number, there are methods for the automatic extraction of crevasse location data from satellite images, though these have been restricted to ice shelves. Work by Lai et al., 2020 included the pan-continental extraction of ice-shelf crevasse locations from optical satellite data with the application of a convolutional neural network. A similar neural network was used by Zhao et al., 2022 and applied to Sentinel-1 SAR data for the extraction of ice shelf crevasses at higher resolution. More recently, Izeboud and Lhermitte, 2023 showed the efficacy of a method of ice shelf fracture and orientation detection based on the application of radon-transforms to satellite images. Finally, previous work by Surawy-Stepney et al., 2023 presented quantitative analysis of the structural properties of the Thwaites Glacier Ice Tongue using crevasse timeseries generated from Sentinel-1 SAR data using a neural network. This previous work forms the basis of the methods presented here.

Here, we extract crevasse data for floating ice shelves and grounded ice in parallel from Sentinel-1 SAR backscatter imagery using a combination of computer vision techniques including the application of a convolutional neural network. We produce pan-continental maps of fracture at monthly intervals and 50 m spatial resolution, over the full Sentinel-1 acquisition area. This substantially increases the temporal coverage of previous large-scale automated crevasse mapping efforts, includes the provision of maps of grounded ice crevasses, and does so at high spatiotemporal resolution. Additionally, the use of Sentinel-1 data allows us to build up a dense timeseries of fracture maps that can be used to observe the development of crevasses. We present a method of quantifying changes in the density of fractures over time that can be used in quantitative analyses; improving on previous methods of assessing structural change by visual analysis of satellite images or crevasse maps.

In this article, we first describe the methods used to map crevasses, before presenting the results and discussing the distributions of ice-shelf and grounded crevasses around Antarctica. Using the timeseries of composite crevasse maps, we then describe how structural change can be measured, and show results focused on the ice shelves Amundsen Sea Embayment (ASE), including the observation of crevasse development in the buttressing regions of Pine Island Ice

Shelf and Thwaites Eastern Ice Shelf between 2015 and 2022, which evolves visibly on monthly-to-annual timescales.

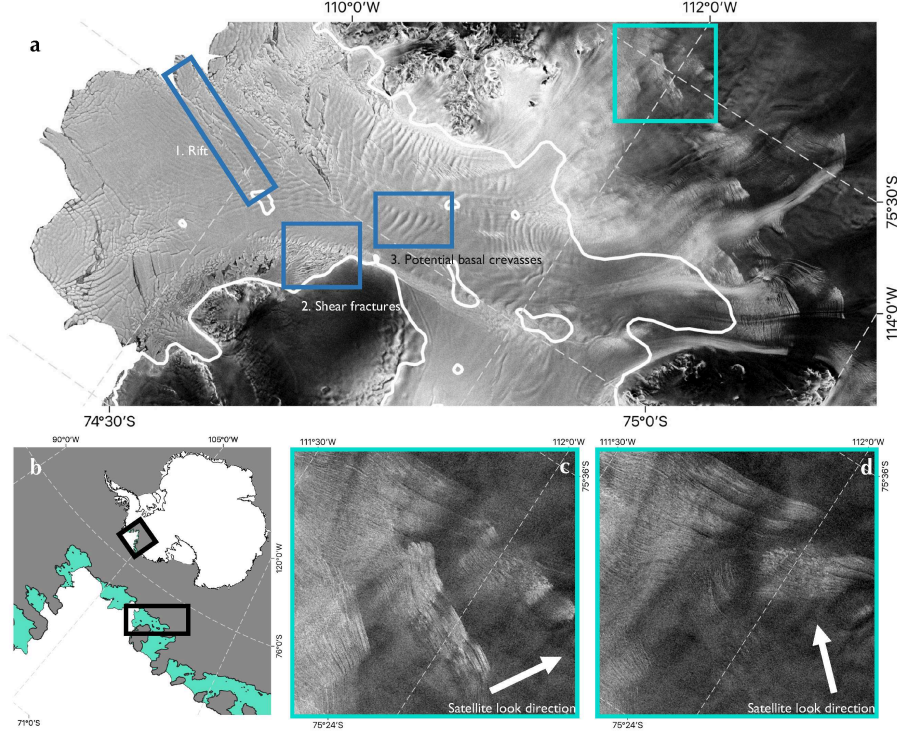


Figure 3.1: Crevasses visible in SAR data covering the Crosson Ice Shelf, West Antarctica. A Sentinel-1 SAR image acquired on 01/06/2021 covering the Crosson Ice Shelf is shown in (a). Blue boxes show examples of type-A crevasses on the floating ice: 1: rift; 2: shear fractures; 3: smooth depressions potentially resulting from basal crevasses. The green box shows type-B crevasses on grounded ice - shown larger in (c). The white line shows the MEaSUREs grounding line (Rignot et al., 2016). (b) Shows the location of the Crosson Ice Shelf region within the Amundsen Sea Embayment (ASE) and, in turn, the location of the ASE within Antarctica. Grey represents grounded ice and green represents floating ice - according to the MEaSUREs grounding line. (c) shows a blown up part of the large SAR image shown in (a) covering a patch of heavily crevassed grounded ice. (d) shows this same patch for a SAR image taken on the same day at a near-perpendicular angle to that shown in (c). The satellite look-angles are shown in (c) and (d) by the white arrows. The visibility of type-B features changes dramatically between the images taken at different acquisition angles.

3.2 Identifying Crevasses

3.2.1 Methods

Brittle fracture occurs mechanically in three modes, which are commonly denoted I, II and III (Irwin, 1957). Mode-I represents cracks opening in the direction of applied tensile stress, while modes-II and -III represent cracks caused by in-plane and out-of-plane shear stresses respectively

(Benn and Evans, 2014; Colgan et al., 2016). Additionally, due to the viscoplastic properties of ice, ductile processes can augment these brittle failure modes and produce complicated crevasse patterns. Mode-I failure tends to result in parallel, sharp sided surface crevasses or rifts clearly visible in the Sentinel-1 backscatter signal, and basal crevasses which can result in visible large-scale depressions in the surface (Vaughan et al., 2012; Luckman et al., 2012; McGrath et al., 2012), especially if subject to subsequent ductile deformation (“necking”) above the crack tip (Bassis and Ma, 2015). However, it is difficult to discern whether such depressions are an indicator of basal crevasses or other processes such as channelised melting of the subshelf, so we focus our methods on the detection of surface crevasses in the knowledge that the sharpest-looking basal crevasses (i.e. where there are large-magnitude intensity gradients perpendicular to the crevasse walls) will be detected as well. Shear failure, ubiquitous in the margins of fast flowing ice streams and shelves, can result in macroscopic crevasses or rifts when severe. Often, however, it results in interacting networks of microfractures, particularly on grounded ice streams. Though regions of high backscatter can indicate that the ice is rougher in the shear margins than elsewhere on the glacier, the “micro” nature of these fractures means we cannot rely on seeing them in the surface signal, so we do not attempt to map this type of diffuse fracture.

In general, we restrict our attention to ‘sharp-sided’ features that appear crevasse-like in isolation; largely surface fractures from mode-I and shear failure. We classify these features into two sets: type-A and type-B, based on their qualitatively different expressions in the backscatter data (Fig. 3.1). Different methods are required for the extraction of these different classes given the disparity in their visual appearance, and the resulting datasets are useful for different purposes. Type-A features are large, multiple pixels in width, and are visible from many look-angles of the satellite. Type-B features appear as fine, bright lines in the backscatter images. They can be pixel-scale in width and are most visible when the horizontal component of the satellite acquisition angle is perpendicular to the crevasse walls. Hence, these features need to be recovered from data covering multiple acquisition angles. Happily, in many places, the Sentinel-1 acquisition tracks overlap obliquely and, often, at near perpendicular angles (Fig. 3.1 c-d). Broadly, these two categories distinguish crevasses on floating and grounded ice: type-A features include ice-shelf surface crevasses, rifts and any basal crevasses that cause narrow surface depressions, while type-B features include grounded surface crevasses, and, to a far lesser extent, narrow ice-shelf surface crevasses bridged by snow.

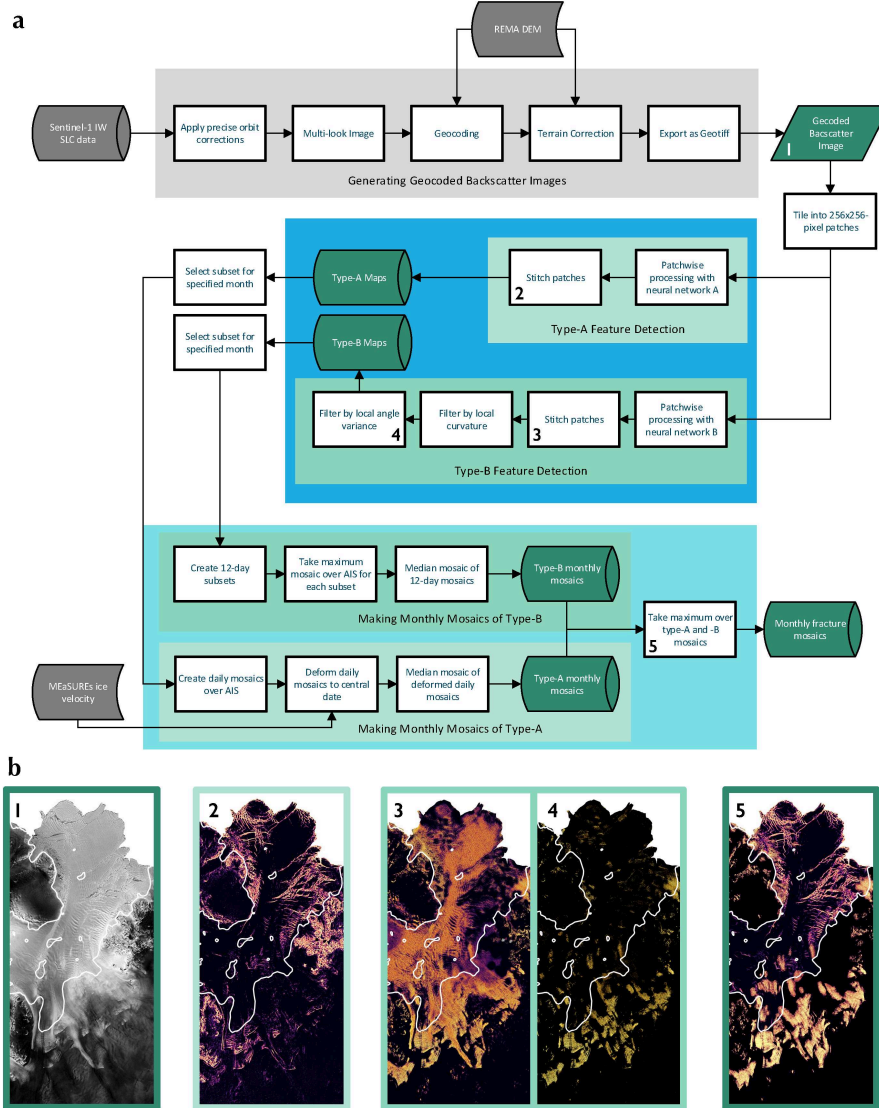


Figure 3.2: Outline of the processing chain that takes a month of Sentinel-1 IW SLC data and produces a fracture mosaic. (a) shows a flow diagram representing the process. (b1-5) show different stages of processing for an example SAR image over the Crosson Ice Shelf. In each panel b2-5, features that appear brighter are more likely to be crevasses according to the latest processing step. Numbers in the top-left corner correspond to the stages of processing that match the numbers in the flow diagram (a). (b1) A 50 m resolution SAR backscatter image from 01/06/2021. (b2) The image after processing with the neural network N_A ; showing type-A features. (b3) The image after processing with the neural network N_B . This displays type-B features along with a considerable noise on the floating ice. (b4) The result of applying the type-B filtering algorithm to (b3). Most of the noise is seen to be removed, leaving type-B features visible from that particular look angle. (b5) A mosaic for the month of June 2021, from images like those displayed in (b2) and (b4). The superimposed white lines show the MEaSURES grounding line (Rignot et al., 2016).

We developed two neural networks and additional filtering techniques to identify these separate features from individual geocoded Single Look Complex (SLC) amplitude images, acquired using

the interferometric-wideswath (IW) mode of Sentinel-1, and methods for constructing combined monthly mosaics of type-A and -B crevasses. Figure 3.1 shows example type-A and -B crevasses visible in Sentinel-1 SAR data over the Crosson Ice Shelf. Figure 3.2 shows an overview of the procedures involved in constructing monthly fracture mosaics, such as that shown in Fig. 3.4.

Bootstrapping Neural Networks

The networks used for the extraction of type-A and -B crevasses share a similar architecture, but were trained separately, resulting in networks we call N_A and N_B . The network architectures are essentially U-Net (Ronneberger et al., 2015), similar to those used in Lai et al., 2020 and Zhao et al., 2022, though shallower and lower-dimensional. This is because many of the features we are interested in, such as type-B crevasse fields and rifting in ice shelf shear margins are textural in nature. In order to avoid the laborious process of creating a training dataset by the manual annotation of satellite images, we employed a bootstrapping technique of the same kind detailed in Surawy-Stepney et al., 2023. In short, we initially trained the networks on a small training dataset consisting of pairs of SAR backscatter images and manually annotated *calving front positions*, the same as those used to train the calving front network in Surawy-Stepney et al., 2023 (Fig. 3.3 a). Training was stopped considerably before convergence of the network parameters, resulting in networks capable of removing speckle from SAR images, and highlighting semantic edges. The intuition behind this is that calving fronts represent a subset of linear, textural discontinuities in the SAR images which is ultimately defined by larger-scale contextual/semantic information. Early on in training, due to the hierarchical nature of the U-Net, along with its skip connections, the cost function can be reduced quickly using activations from the shallower layers which correspond to low-level textural information such as the presence of spatial intensity gradients, i.e. edges, at the pixel-level. The deeper layers might contribute semantic information about the length of linear features, though not that which differentiates the calving front edge from crevasse walls.

We then applied these partially trained networks to unseen data and manually selected images for which the network assigned relatively large values to the locations of crevasses (Fig. 3.3 b-c). A scaling was applied to these outputs to enhance the crevasse features and the scaled outputs were added, along with the corresponding input image to an updated training dataset (either for type-A or -B). We then retrained the networks on these larger datasets. This constitutes one round of a “bootstrapping” procedure that, after 3-4 iterations, led to a training dataset of

$\sim 10^3$ 256×256 -pixel, 64-bit images and networks that perform well in the desired task. Each time the networks were trained, their parameters were initialised to the values at the end of the last round of training. Hence, scaling the output images before adding them to the new training datasets was necessary to induce non-zero gradients of the cost function with respect to the network parameters.

By separately applying the networks N_A and N_B to input SAR images, we create type-A and intermediate type-B crevasse maps for each Sentinel-1 acquisition frame individually. To do this, SAR images are tiled into 256×256 -pixel patches, overlapping by half, processed by the neural networks N_A and N_B respectively and pieced together. For each network, a softmax function is applied to the output and the channel corresponding to “crevasse” was selected so that the outputs are normalised to the range $[0, 1]$ for each pixel, where 1 represents a high confidence of a crevasse, and 0 a low confidence.

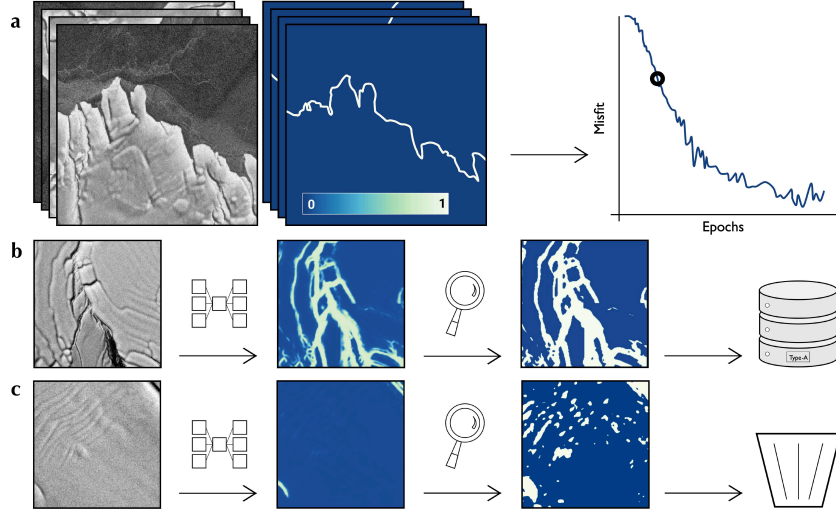


Figure 3.3: Bootstrapping partially trained neural network N_A . (a) Example input/target pairs for the initial phase of training of the randomly initialised network. Training was stopped well before parameter convergence. (b-c) Bootstrapping. The network was applied to unseen images and the outputs were visually inspected. If the outputs could be thresholded to look like a plausible target image (b), that threshold was applied before the input/output pair was added to a new training dataset. If not, (c) that input/output pair was discarded.

Type-A

We directly use the outputs from the neural network N_A as our map of type-A crevasses. If required, a threshold can be applied to produce binary maps, with optimal values that depend on the features of interest and the desired balance between performance metrics.

Type-B

The outputs of the neural network N_B highlight many of the fractures visible in the input images, but often contain spurious collections of randomly aligned features (Fig. 3.2 b.3). Visual assessment of the SAR backscatter images shows type-B crevasses to be linear on kilometre scales, and exist in patches of crevasses that are locally parallel. We have developed a filtering algorithm, which we call “parallel structure filtering” (PSF), that we apply to the network outputs to remove features that fail to conform to these conditions (Fig. 3.2 b.4). This starts by calculating the Hessian matrix local to each datapoint using Gaussian derivatives. The likelihood of each pixel being part of a linear structure is subsequently calculated from the Hessian eigenvalues (Frangi et al., 1998; Jerman et al., 2016) and those with likelihood above a certain threshold are kept. The angles of the structure on which these datapoints lie are extracted from the Hessian eigenvectors, before a local distribution of angles is calculated with a set of box-kernel convolutions. Datapoints are removed if the local angle variance exceeds a threshold of 0.71, tuned to best fit a small set of example manual annotations. Further details on the algorithm are provided in Appendix 3.A.

Making Monthly Mosaics

The methods described above allow us to produce separate type-A and type-B crevasse maps for individual Sentinel-1 acquisition frames. We create individual, pan-continental type-A and -B crevasse maps each month by combining these individual frames, before stitching these together to create combined type-A and -B mosaics. For type-A features, we make the approximation that crevasses move and deform according to the surface ice velocity, and compensate for this with a Lagrangian correction in which we post the maps to a common date at the middle of the time period using a remapping defined by the MEaSUREs Antarctic ice velocity dataset (Mouginot et al., 2012; Rignot et al., 2017). We then take a simple median mosaic over the time window of choice.

As mentioned above, type-B mosaics are complicated by the fact that the visibility of the features is dependent on the look angle of the satellite. Hence, before taking a median mosaic, we break the time period into 12-day windows that capture all look angles of the satellite and take maximum mosaics over each. We do not provide a Lagrangian correction for the type-B

mosaics, because their visibility often does not persist as they are advected downstream, and the locations where they are produced do not appear to change on monthly timescales. The final crevasse map is produced by masking the grounded ice sheet areas in the type-A mosaics, before taking a maximum composite with the type-B fracture mosaic. This results in a normalised score $D \in [0, 1]$ with 1 indicating ‘fracture’ and 0 indicating ‘no-fracture’ for each pixel in the map. We note that we are unable to map the fractures on large parts of the Ronne-Filchner and Ross Ice Shelves, and much of the interior of the Antarctic Ice Sheet, due to the latitudinal limits of the Sentinel-1 data acquisition plan.

3.2.2 Results

We applied the method described above to every Sentinel-1 acquisition between January 2015 and July 2022 and produced composite crevasse maps covering the Antarctic Ice Sheet for each month. Fig. 3.4 shows an example map made using Sentinel-1 acquisitions from April 2022. The results show crevasses to be a feature of a large and varied set of ice streams and shelves across the continent - from rifts in the shear margins of the fast-moving ice shelves of the Amundsen Sea (Fig. 3.4 h), to the fine surface fractures fringing the grounded ice streams of the Amery basin (Fig. 3.4 d).

The type-A fractures identified by the network take a wide variety of forms and are exclusive to floating ice shelves. Of those observed, the brightest and most identifiable are large ice-shelf rifts, such as Chasm-1 on the Brunt Ice Shelf and those penetrating into the bulk of Shackleton and Larsen D Ice Shelves (highlighted with grey arrows in Figs. 3.4 b, e, a respectively). Many fast-flowing ice shelves exhibit severe crevassing in shear margins that connect them to slower-flowing parts of the ice shelf, for example on: Stancomb-Wills Ice Tongue, Fimbul Ice Shelf, Shackleton Ice Shelf and Pine Island Ice Shelf (highlighted with blue boxes in Figs 3.4 b, c, e, h respectively). We also see fractures resulting from the interaction of ice shelves and ice rises around the coastline - for example on the Larsen D Ice Shelf (Fig. 3.4 a).

Our observations show that type-B crevasses, though a less varied set of features, are as prevalent across the continent as type-A crevasses, and occur in every major basin of the Antarctic Ice Sheet, generally in dense patches. Approximately 85% of type-B crevassing appears on grounded

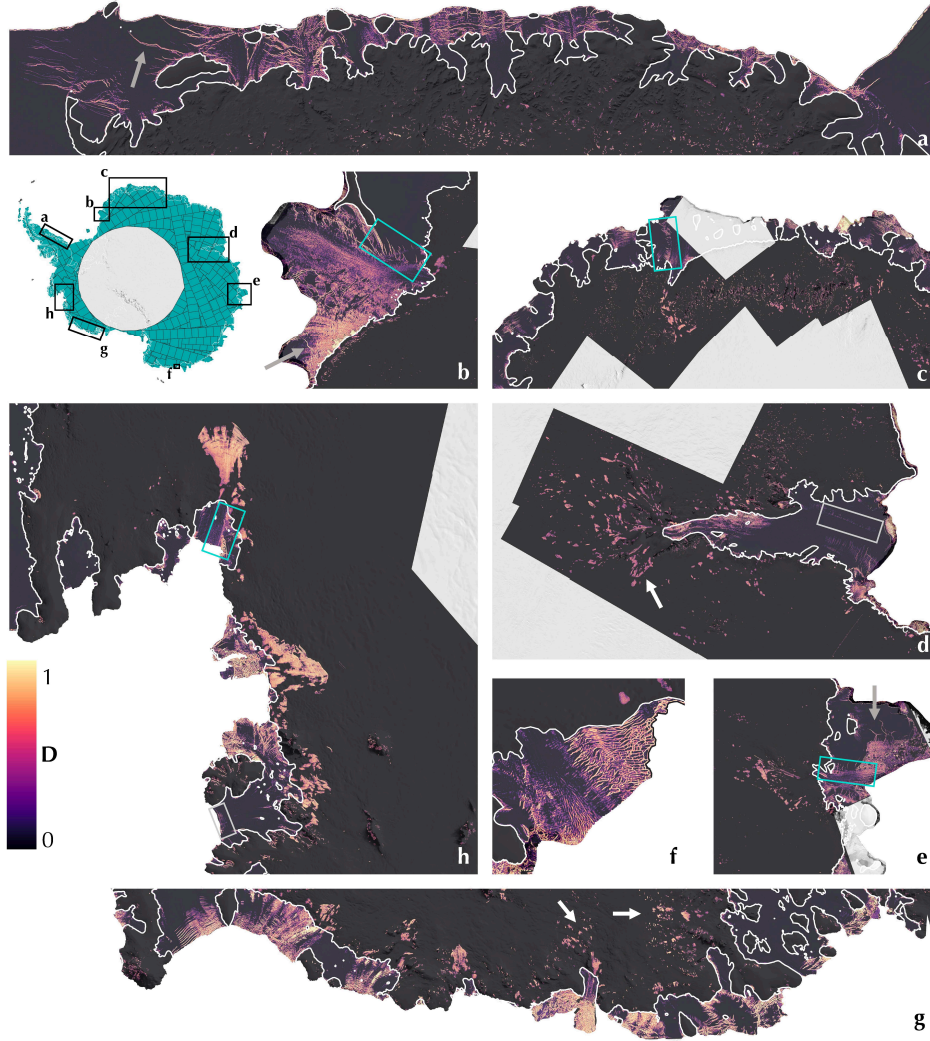


Figure 3.4: Fractures in Antarctica: April 2022. Results at different scales from a pan-continental fracture mosaic. $D \in [0, 1]$ is a normalised score with 1 indicating ‘fracture’ and 0 indicating ‘no-fracture’ for each pixel. The labelling of subfigures corresponds to a clockwise ordering around the Antarctic coastline starting from the Eastern Antarctic Peninsula. (a) The Larsen-D Ice Shelf. (b) The Brunt-Stancomb-Wills Ice Shelf. (c) Dronning-Maud Land including Fimbul Glacier and Ice Shelf. (d) The Amery Ice Shelf and its basin - including Lambert Glacier. (e) Shackleton Ice Shelf and Denman Glacier. (f) Cook Ice Shelf. (g) The coastline between West Getz (left) and Salzburger (right) Ice Shelves, with the Land Ice Tongue in the centre. (h) The Amundsen Sea Embayment, including, from top to bottom, Pine Island Glacier, Thwaites Glacier, Crosson Ice Shelf and its tributary glaciers, and Dotson Ice Shelf. Boxes and arrows overlaying the enlarged parts of the crevasse map highlight features mentioned in the main text: blue boxes show fractures in ice shelf shear margins, grey boxes show where crevasses that are known to exist appear only faintly in the crevasse map, white arrows show type-B fractures, grey arrows show large rifts. The top-left inset shows the Antarctic Ice Sheet with locations of the images (a-h) identified with black boxes. These give a sense of the different scales at which crevasses can be seen. In teal, we show the total extent of all Sentinel-1 acquisitions over the AIS so far. The white line shows the MEaSUREs grounding line (Rignot et al., 2016).

ice. Our observations show that type-B crevasses are particularly dense and widespread in the ice streams of the Amundsen Sea Embayment (Fig. 3.4 h). In a composite crevasse map of the grounded part of the ice sheet from June 2021, this region accounted for around 10% of grounded crevasses despite covering only 3.5% of the imaged surface area. These ice streams have undergone significant dynamic change over the last few decades, with ice flux across the grounding line increasing by 40 to 100% since the early 1990’s (Shepherd et al., 2004; Mouginot et al., 2014; Joughin et al., 2014; Konrad et al., 2017; Davison et al., 2023). The large number of grounded crevasses observed in this region may be a consequence of the sudden increase in longitudinal strain rates which will accompany the observed speedup. The effect of crevasse fields like this could be to decrease the effective viscosity of the ice. Hence, fracturing can provide a mechanism for the sudden manifestation and persistence of ice dynamic changes, beyond that which can be accounted for by dynamic thinning. In many grounded areas, however, crevasse depth is likely to be only a small fraction of ice thickness due to large overburden pressures (Benn and Evans, 2014).

Elsewhere around Antarctica, surface crevasses on grounded ice appear sporadically in patches. In some instances, they contour the edges of ice streams, such as on Lambert Glacier in East Antarctica (white arrow Fig. 3.4 d). Elsewhere, the locations of patches of crevasses appear dependent on vertical shear stresses, for example mode-I crevasses forming due to abrupt changes in basal slip or mixed-mode crevasses caused by sharp changes in bed topography (white arrows, Fig. 3.4 g). The size of these patches of crevassed ice are on the order of $\sim 10 - 100$ km in the along-flow direction, showing the crevasses can be healed when stress conditions change. Crevasse visibility is also influenced by spatial variability in the depth of drifting snow, and, in regions for which image acquisition angles vary little, changing crevasse orientation. In future, we can hope to constrain these additional factors using additional sensors to better bound the regions in which type-B crevasses appear on grounded ice.

We finally note that, due to the shallow networks, the bootstrapping procedure, and the absence of training data seaward of the Antarctic coastline, the networks are sensitive to local textural deviations at, and beyond, the calving front. For example, leads and fractures in sea ice, dense ice mélange, iceberg boundaries and calving fronts exhibit strong signals in the fracture mosaics when unmasked.

3.2.3 Evaluation

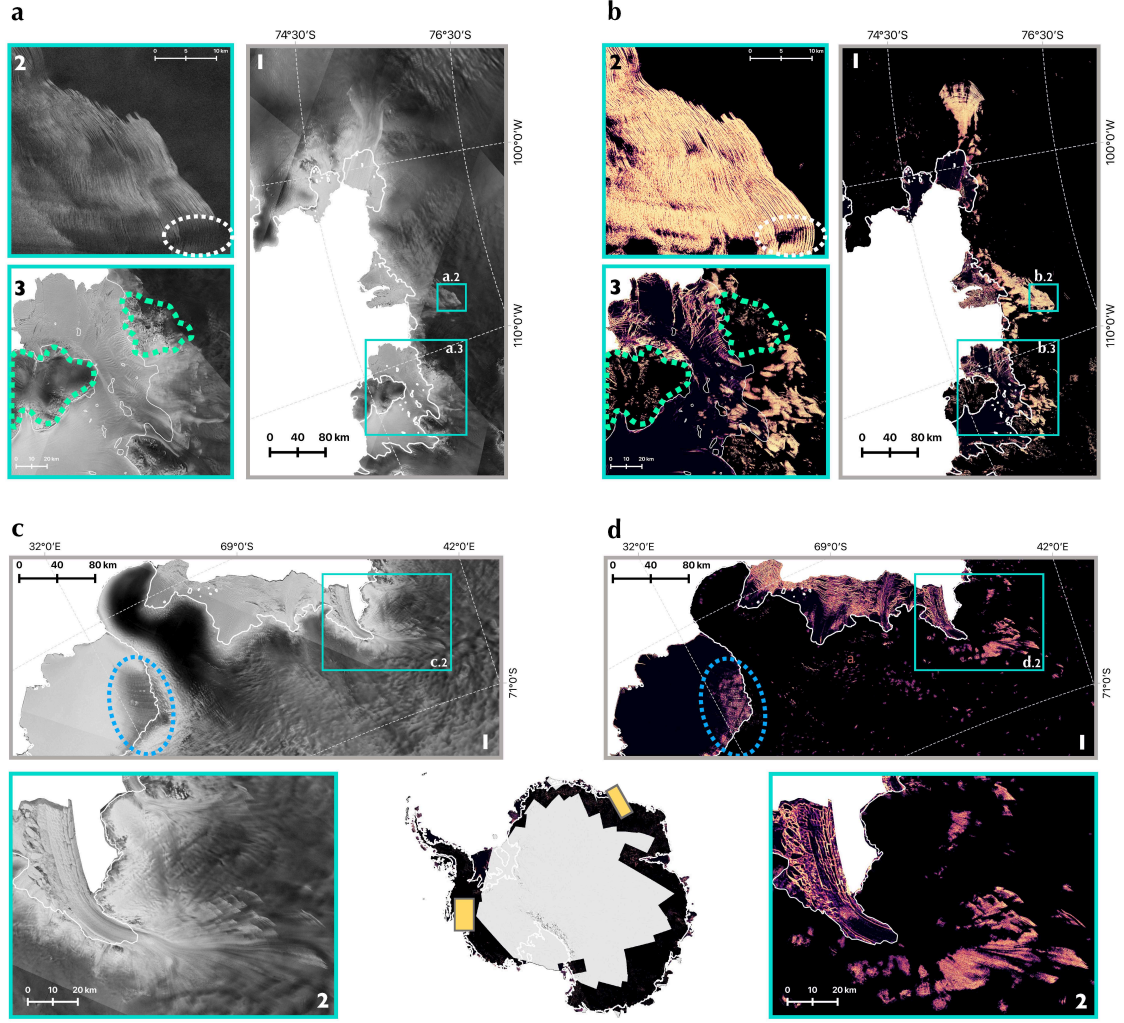


Figure 3.5: A comparison between a composite SAR image and crevasse map (June 2021). The left-hand side of the figure shows parts of the SAR backscatter composite (a simple mosaic, with later frames overlaying earlier ones), while the right-hand side shows corresponding parts of the fracture map. (a) SAR images covering the Amundsen Sea Embayment: (a.1) the glaciers of the ASE and locations of the enlarged regions (a.2) and (a.3) shown as cyan boxes, (a.2) Enlarged region over the grounded crevasses fringing the Thwaites Glacier ice stream, (a.3) Enlarged region over the Crosson Ice Shelf and surrounding glaciers. (b.1-3) show corresponding crevasse maps. White dashed ovals show where crevasses are not visible in the SAR image or the crevasse map, despite likely connecting crevasses either side. Green dashed boundaries show regions of steep topography on Bear Island and Mount Murphy, where type-B crevasses appear erroneously in the fracture map. (c) SAR images over a part of eastern Dronning-Maud Land. (c.1) The region as a whole and location of the enlarged region (c.2) shown as cyan box. (c.2) Enlarged region over Shirase Glacier and ice shelf. (d.1-2) show corresponding crevasse maps. Blue dashed ovals show the location of surface meltwater on Baudouin Ice Shelf, where crevasses appear erroneously in the fracture maps. (b) and (d) inherit the colourmap from Fig. 3.4. In each figure, the white line shows the MEaSUREs grounding line (Rignot et al., 2016). The bottom-middle shows a map of Antarctica with the full June 2021 crevasse map overlaid on the MODIS map of Antarctica (Haran et al., 2021); yellow boxes show the boundaries of the locations of the parts shown in (b.1) and (d.1).

Overall, we consider the Antarctic fracture maps we produce to be an accurate representation of crevasses across the continent that are linear on scales large compared to the resolution of the data, with a few exceptions. There are two components that inform this conclusion. Firstly, previous studies suggest that the majority of such crevasses are visible as type-A or -B features in the Sentinel-1 SAR backscatter images (Moctezuma-Flores and Parmiggiani, 2016; Thompson et al., 2020; Marsh et al., 2021). Secondly, the methods we have developed extract a majority of features in the backscatter images while highlighting few features erroneously. The fracture maps covering Dronning-Maud Land, the Amundsen Sea Sector and the eastern Antarctic Peninsula (Figs. 3.4 b, g, and h, respectively) show the lack of noise present in the monthly mosaics at different scales - indicating a high specificity for both type-A and type-B crevasses. In large part, this can be attributed to the ability of the neural networks to deal with radar speckle and the efficacy of our parallel structure filtering algorithm. These figures also show how mosaicking over monthly windows results in maps that do not display visible processing artefacts, such as the edges of SAR frames or boundaries between regions of different acquisition geometry. In part this is a consequence of using raw, un-normalised SAR data. This provides confidence that large features can be mapped across image acquisition boundaries, and therefore provide a good representation of the ice sheet surface.

The few locations in which type-B features are present in the crevasse maps without obvious fractures visible in the backscatter images are in regions of steep topography, such as the ice rises and mountains around Crosson Ice Shelf (Fig. 3.5 a.3, b.3, circled in white). As the topography of these regions is essentially unchanging over the time period of this study, these features could be dealt with by defining a mask based on the gradients of a digital elevation model or by retraining the neural networks with additional data covering such areas of steep terrain. However, as these regions are distinct from those with the kind of large-scale dynamic behaviour we are interested in, the misattribution of crevasses to these areas is incidental.

The specificity of crevasse detection is also low in regions where there is a large amount of surface meltwater - for example near the grounding line on Baudoin Ice Shelf in eastern Dronning-Maud Land (Fig. 3.5 c.1, d.1, circled in blue). Values of type-A crevasse probability are high in regions of surface melt due to the presence of features that are not usually visible, such as sharp contrasts in backscatter at the boundaries of meltponds, fluvial features originating from the

persistent pooling and flow of surface water (e.g. near the grounding line of Amery Ice Shelf - Fig. 3.4 c), as well as a greater sharpness of shallow surface depressions due to the decrease in microwave penetration depth. In static crevasse maps such regions appear to have a greater amount of crevassing compared to nearby areas with comparable surface structure. However, there are only few locations for which this is a persistent issue (e.g. Amery and George VI Ice Shelves), with other locations only experiencing melt events in the Austral summer (e.g. eastern Dronning-Maud Land). However, the effect of meltwater, even if intermittent, means caution is required when comparing crevasse maps covering ice shelves known to be affected by surface meltwater at different locations or times. If surface melt in Antarctica increases in the future in response to a changing climate, then methods will need to be developed to remove these features – or conversely to isolate and use the information as a proxy dataset for melt water extent.

The method appears sensitive to the great majority of crevasse-like features visible in the backscatter images across the continent. Unsurprisingly, it is most sensitive to sharp-sided ice shelf surface crevasses and rifts. However, some smooth features - likely to be basal crevasses - are under-represented in the type-A maps. For example, those near the calving front of Dotson Ice Shelf and those forming a track along the central-western part of the Amery Ice Shelf (shown by grey boxes in Fig. 3.4 h, d respectively) which appear only faintly.

The type-B mapping identifies the bulk of crevasses visible in the images. The exception is at the boundary of patches of grounded crevassing where the edges are sometimes cut-off by the low-resolution filtering. There are also crevasse fields that include small regions, on the order of 5 to 10 km in width, in which crevasses likely exist, but are less visible in the backscatter images (e.g. on the Thwaites ice stream - Fig. 3.5 a.2). These regions are blank in the crevasse maps, despite the fact that crevasses very likely propagate through them, linking the identified crevasses on either side.

To further assess the performance of our type-A and -B fracture maps, and quantify statements regarding the specificity and sensitivity of our crevasse maps, we compared the monthly mosaics with 3 manually annotated Sentinel-1 IW SAR frames, created independently, without reference to the fracture mosaics. The European Space Agency provides in-house references for the relative orbit and frame numbers for the image acquisitions which we shall use to distinguish

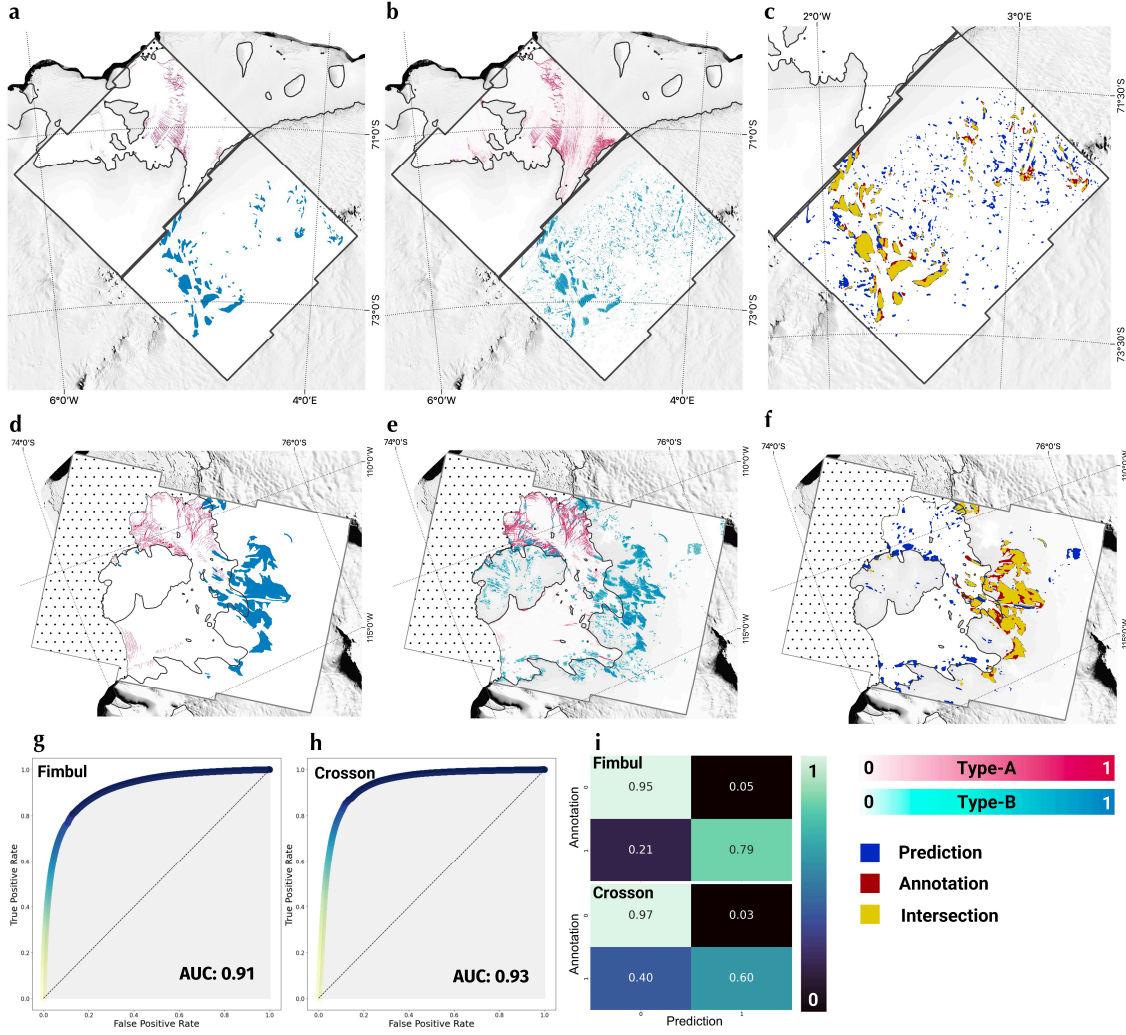


Figure 3.6: Evaluation of type-A and -B crevasse maps against manual annotations for Sentinel-1 frames covering the Crosson and Fimbul Ice Shelves and their tributary ice streams. (a) Annotated type-A crevasses and type-B crevasse fields covering two Sentinel-1 SAR frames over Fimbul Glacier (PF: 2/931 and 2/925, dated: 20180815). (b) Corresponding monthly mosaic of type-A and -B features over SAR frames PF: 2/931 and 2/925. (c) The intersection of the type-B field annotations and the smoothed and thresholded type-B mosaic for PF: 2/925. (d) Annotated type-A (red) crevasses and type-B crevasse fields for a Sentinel-1 SAR frame covering the Crosson Ice Shelf and its tributaries (PF: 7/913, dated: 20210607). (e) Corresponding monthly mosaic of type-A and -B features for PF: 7/913. (f) The intersection of the type-B field annotations and the smoothed and thresholded type-B mosaic for SAR frame PF: 7/913. (g) ROC plot for type-A features for Sentinel-1 PF: 7/913 (labelled “Fimbul”). (h) ROC plot for type-A features for Sentinel-1 PF: 7/913 (labelled “Crosson”). (i) Confusion matrices for type-B type-B field annotations and the smoothed and thresholded type-B mosaic for Sentinel-1 PF: 2/925 (labelled “Fimbul”) and 7/913 (labelled “Crosson”). The background on which the SAR frames are overlaid is the MODIS MOA (Haran et al., 2021; Greene et al., 2017). The grounded ice within the SAR frames is shaded using the REMA 1 km DEM (Porter et al., 2018). The grounding line shown is according to MEaSURES (Rignot et al., 2016) (black line). The spotted regions indicate the sea.

the SAR images here and in the rest of the article. We will refer to the relative orbit number as “path” and abbreviate the path/frame that identifies a specific acquisition as “PF”. The three SAR images chosen cover the Crosson and Dotson Ice Shelves and their tributary glaciers (PF: 7/913, dated: 20210607), the Fimbul Ice Shelf (PF: 2/931, dated: 20180815) and the grounded Fimbul Glacier (PF: 2/925, dated: 20180815). We annotated these in their entirety in order to help facilitate future inter-comparison with other methods. We selected these frames to represent a challenge to the method, as the full variety of crevasse features are present, as well as large regions of steep topography and persistent surface melt. For type-A features, we annotated individual crevasses at the pixel level in single SAR images corresponding to the chosen frame. However, for type-B features, it is all but impossible to pick out individual crevasses by hand. Instead, we delineated the boundaries of *crevasse fields*, which we then compared with versions of the type-B mosaics that were smoothed and thresholded to produce binary maps of contiguous crevasse fields. For the case of the SAR image covering Crosson Ice Shelf, we combined annotations from the given frame and another with a relatively oblique acquisition angle (PF: 10/855, dated: 20210607) to reflect the different crevasses that were visible from the different angles. Given the continuous output and class imbalance inherent in the type-A mosaics, we produced Receiver-Operator Characteristic (ROC) curves for the evaluation of the type-A maps, while we report confusion matrices for the evaluation of the type-B processing. Results are shown in Fig. 3.6.

The ROC curves quantify the sensitivity/specificity of the type-A mosaics over the full range of threshold values [0,1]. Respectively, the areas under the curves for the frames covering Crosson & Dotson and Fimbul Ice Shelves are 0.93 and 0.91 showing high discriminatory power (Fig. 3.6 g-h). This is despite the failure of the network to extract a large portion of the likely basal crevasse impressions on Dotson Ice Shelf (Fig. 3.6 d-e), those west of the Fimbul western shear margin (Fig. 3.6 a-b), and the misattribution of flow features on the central trunk of Fimbul Shelf and fluvial features in its eastern shear margin as crevasses (Fig. 3.6 a-b). The intersections of the annotated and predicted crevasse fields show a great deal of overlap in the crevasse fields on Fimbul (Fig. 3.6 c), Pope, Smith and Kohler (Fig. 3.6 f) Glaciers, with the largest fields sharing the most overlap. There are a number of smaller features appear in the type-B mosaics that do not in the annotations - especially in the case of Fimbul Glacier and surroundings. This is where mountains and steep rocky features have been identified by the

model as crevasses. However, the confusion matrices (Fig. 3.6 i) show that these areas cover under 5% of the total “undamaged” ice according to the manual annotations.

3.3 Measuring Changes in Fracture Density

3.3.1 Methods

After establishing the above method of reliably extracting fracture-location data, we moved onto the natural question of whether it is possible to use this dataset to measure changes to ice shelf ‘damage’ through time. At the outset it was unclear whether the 7.5-year study period was long enough for real structural change to evolve in response to ice dynamic processes in Antarctica, or whether the impact of radar interactions with the ice-surface properties may cause crevasse mapping to vary too much in response to environmental factors unrelated to crevasse evolution, such as weather-induced surface melt. It is certainly clear that variability in the properties of the ice surface renders the direct comparison of fracture maps at different times an unreliable method of assessing change.

The method we introduce here measures trends in the local density of fractures using the full timeseries of fracture maps, producing a scalar “fracture density change” value that can be used to compare different locations.

A brief analysis of the fracture maps shows minor changes to crevassing on the grounded ice sheet over the last 7.5 years. We focussed then on ice shelves, where crevasse patterns change on shorter timescales due to elevated flow speeds, interaction with rapidly changing ocean conditions and higher amplitude loading/unloading cycles. More specifically, we considered the ice shelves of the Amundsen Sea Embayment, given that a large amount of the ice-dynamic change in Antarctica has been observed in that region between 2015 and 2022.

We define *fracture density* as the fracture maps integrated over an area of interest (Albrecht and Levermann, 2012; Surawy-Stepney et al., 2023) and use this as a heuristic measure of how crevassed that region is. We looked for linear trends in this parameter over the observational period. To analyse the spatial pattern of change, we first defined a fixed $2.5\text{km} \times 2.5\text{km}$ grid of points over the ice shelves. Timeseries of fracture density and backscatter standard deviation were extracted from daily mosaics over a $10\text{km} \times 10\text{km}$ square buffer around each point, before

the fracture density change and error were found. Given that we were not interested in the development of individual crevasses, we did not apply Lagrangian corrections to the crevasse maps.

As mentioned above, we had to account for the dependency of our fracture maps on the surface properties of the ice before calculating trends in the timeseries. For example, seasonal changes to crevasse visibility due to changing firn-water content or thickness of the snowpack can dominate the signal over changes in crevasse length or concentration (Marsh et al., 2021). It was necessary, therefore, to separate the parts of the signal due to real changes in the crevasse pattern, and those due to changes in the surface expression of the crevasses resulting from such unknown environmental factors. This is only possible given the large number of crevasse maps, and hence fracture density datapoints, in the timeseries given the short repeat period of the Sentinel-1 satellites. Firstly, we discarded measurements made between December and March each year, due to the prevalence of surface melt during those months. We then made use of the observation that the surface properties of the ice seem to be discernible from the local standard deviation of the backscatter signal. We used this as a proxy to define sets of dates for which ice surface conditions were similar and constructed an ensemble of fracture density timeseries for each region. By taking a weighted mean and standard deviation of the trends and multiplying by the time span, we defined an estimated fracture density change over the time period (Fig. 3.7 a, b, f, j) and provided an uncertainty (Fig. 3.7 c, j, k). See Appendix 3.B for more details.

3.3.2 Results

By applying the method defined above to daily fracture map mosaics, we derived an estimated change in fracture density for the ice shelves of the ASE (Fig. 3.7 a). For the most part, our results show that notable changes in fracture density over the observational period were confined to the Pine Island, Thwaites and Crosson Ice Shelves (Figs. 3.7 b, f, j, respectively), with change elsewhere attributable to modest calving events or terminus advance (Fig. 3.7 a).

On Pine Island (Fig. 3.7 b-e), a large region of fracture density change in the interval $0.08 - 0.3$ shows a significant deterioration of the southern shear margin over the observational period

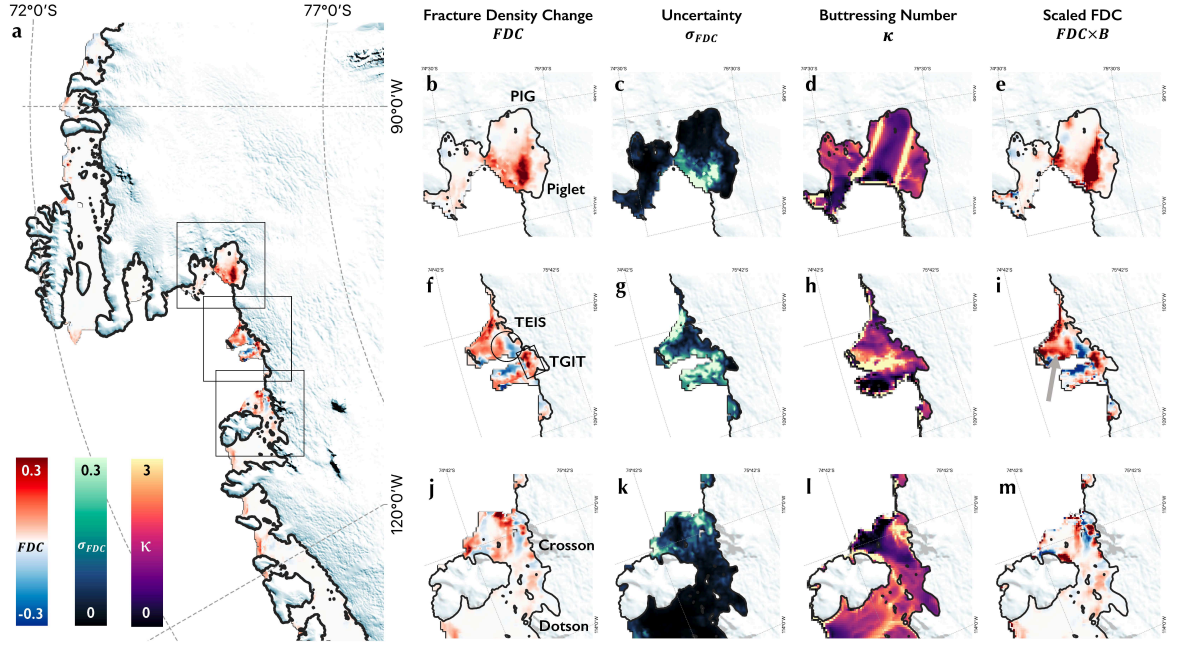


Figure 3.7: Structural change on the ice shelves of the ASE. (a) An overview showing the change in fracture density from January 2015 to July 2022 for the ASE as a whole. Grounded ice, according to the MEaSUREs grounding line (Rignot et al., 2016), is represented with the REMA 2 km DEM (Porter et al., 2018). The table of figures (b-m) show expanded maps of fracture density change (b, f, j), uncertainty in fracture density change (c, g, k), buttressing number (d, h, i) and fracture density change scaled by buttressing number (e, i, m) for Pine Island (b-e), Thwaites (f-i) and Crosson (j-m) Ice Shelves.

(Lhermitte et al., 2020), continuing a decades-long pattern of structural decline on this ice shelf (MacGregor et al., 2012). The intense fracture density change at the seaward-most part of the shear margin reflects its disintegration over the course of 2020 (Fig. 3.7 b, Fig. 3.8 b, c), completely decoupling the ice shelves of Pine Island Glacier and Piglet Glacier which, until 2018, flowed north-east into the southern shear margin of the Pine Island Ice Shelf, rendering it unbuttressed and marine terminating. We also see a general region of increased fracture density at the seaward limit of the ice shelf, which can be attributed to a series of propagating rifts forming downstream of an ephemeral grounding point near the centre of the ice shelf, that led to calving events between 2015 and 2020 (Joughin et al., 2021).

Moving around the coast of West Antarctica to the ice shelves of Thwaites Glacier (Fig. 3.7 f-i), a more complicated picture emerges. On the Eastern Ice Shelf, the data suggest a highly variable pattern of structural change. A patch of decreasing fracture density (black circle, Fig. 3.7 f) indicates where sharp-sided crevasses have travelled further onto the shelf followed by smoother (likely shallower surface or basal) crevasses (Fig. 3.8 d). Around the pinning point to the north of the ice shelf, we see decreased fracturing around its western end, and increased

fracturing to the south and east. This is due to crevasses/rifts opening up in-situ, perpendicular to the orientation of the pinning point on the main body of the shelf in response to increased shear stress in the latter half of the 2010's (Benn et al., 2021).

Thwaites Glacier's western ice tongue displays increased fracture density on the floating section of the main trunk closest to the grounding line and in the ice to the south of the eastern shear margin (black box, Fig. 3.7 f). Fracturing on the central trunk has been increasing steadily over the observational period, while that in the eastern shear margin increases and decreases with acceleration of the main trunk (Surawy-Stepney et al., 2023). Changes to fracturing in the regions closest to the grounding line have been periodic on timescales of a few years (Fig. 3.8 b, d), so our linear trends do not capture the behaviour of this region in full. The protruding ice tongue has been degrading steadily over the observational period (Miles et al., 2020; Surawy-Stepney et al., 2023), however, while our data shows a large region of positive change in fracture density, negative trends in fracture density are shown on the eastern side of the ice tongue. This reflects a reduction in the density of the *mélange* separating the ice tongue from the Eastern Ice Shelf, where, at some point, our fracture maps stop recognising the spaces between icebergs as fractures.

Our observations show changes in fracture density on the main body of Crosson Ice Shelf and in its shear margins (Fig. 3.7 j-m). On the main body, these are largely due to the advection of rifts towards the ocean, as can be seen by the stripes of increasing and decreasing fracture density (Fig. 3.7 j), though fracturing has progressed and increased slightly in density in the shear margins. Finally, we note that, despite our observations showing fracture density change outside of the Amundsen Sea Sector to be limited (Fig. 3.7 a, Fig. 3.8 b), some ice shelf crevasses which may be linked to interesting glacial processes - such as those perpendicular to the direction of ice flow that follow the sub-shelf melt channel on the western part of the Dotson Ice Shelf (Gourmelen et al., 2017) - are not visible in our fracture maps (Fig. 3.4 h, Fig. 3.8 f). This is because they have a distinctive visual representation in comparison to type-a and -b crevasses, which the two neural networks used in this study are not tuned to detect. Future studies should seek to evolve the existing method to identify and map additional classes of crevasses across Antarctica, which will be a useful dataset for assessing different glaciological conditions.

After measuring the spatial pattern of fracture density change in the Amundsen Sea Sector we investigated where these changes might be important for the dynamics of the glaciers. To evaluate how changes to the local stress distribution might impact the glacier at large, we employed the fractional difference between the second principal component of the vertically-integrated viscous stress tensor e_2 and the vertically-integrated hydrostatic pressure as a local notion of “buttressing” (Gudmundsson, 2013; Fürst et al., 2016; Appendix 3.C):

$$\kappa = 1 - e_2 \times \left(\frac{1}{2} \tilde{\rho} g h^2 \right)^{-1}$$

where $\tilde{\rho} = \rho_i(1 - \frac{\rho_i}{\rho_w})$, ρ_i is the density of ice, ρ_w is the density of sea water, g is the acceleration due to gravity and h is the ice thickness.

By weighting the fracture density change by this buttressing number as it was at the start of the observational period, we assessed where observed changes to fracturing might have a meaningful impact on the ice dynamics (Fig. 3.7 e, i, m). For example, where a major increase or decrease in fracture density had occurred in a buttressing region of an ice shelf, we expect to also observe a dynamic change. Our observations show the large increase in fracturing in the southern shear margin occurred in a strongly buttressing part of the ice shelf (Fig. 3.7 e). Similarly, those changes to crevassing on the Thwaites Eastern Ice Shelf are amplified around the offshore pinning point when buttressing is taken into consideration, while changes on the main body of Crosson Ice Shelf are diminished.

3.3.3 Evaluation

Our results show that our method of assessing structural change (Sec. 3.3.1) produces data that reflects the qualitative changes that are known to have occurred (Fig. 3.7), like the deterioration of the southern Pine Island Ice Shelf shear margin (MacGregor et al., 2012; Lhermitte et al., 2020) and the advection of rifts on Crosson Ice Shelf. The results also show limited change where they are not thought to have occurred, for example on Cosgrove Ice Shelf (Fig. 3.7 a). This inspires confidence in the data as a whole, including the additional unreported results such as the degradation of parts of the shear margins of Crosson Ice Shelf, and the limited changes on its central trunk.

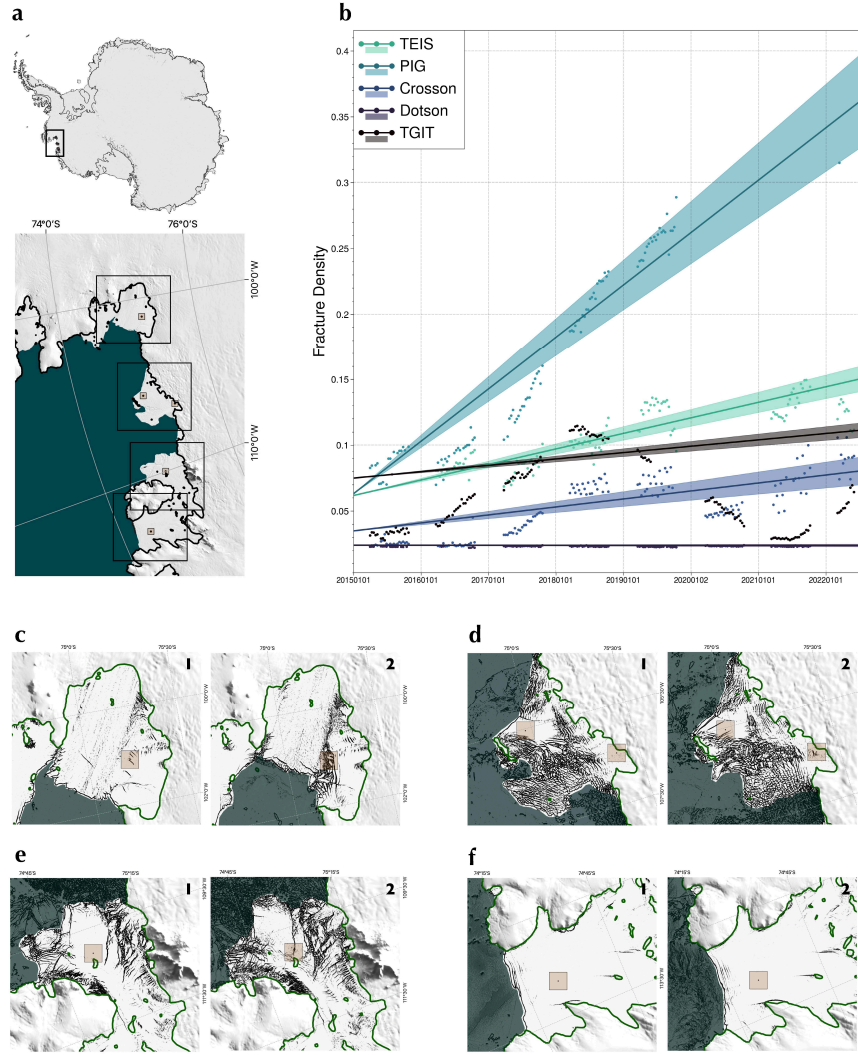


Figure 3.8: Timeseries of fracture density at specified points on the ice shelves of the Amundsen Sea Embayment between 01/01/2015 and 01/07/2022. (a) Image of the Antarctic Ice Sheet (MODIS MOA Haran et al., 2021; Greene et al., 2017) with Amundsen Sea study area identified with a black box. The study area is shown in the larger image, with regions for which timeseries of fracture density were extracted shown with transparent brown boxes, and locations for images shown in c-f shown with black boxes. (b) Timeseries of fracture density extracted from the transparent brown boxes shown in b-f over the Thwaites Eastern Ice Shelf (TEIS), Pine Island Glacier (PIG), Crosson Ice Shelf, Dotson Ice Shelf and the Thwaites Glacier Ice Tongue (TGIT). Trends are calculated from ensembles of trends as described in Sec. 3.3.1. Y-intercepts are the mean y-intercepts of the ensemble, and the error region includes one standard deviation above and below the mean slope. (c-f) Close-ups of PIG, TEIS & TGIT, Crosson Ice Shelf and Dotson Ice Shelf with fracture maps from January 2015 (c.1-f.1) and June 2022 (c.2-f.2) shown in greyscale. Regions over which the fracture density timeseries were extracted are shown with transparent brown boxes. The green line represents the grounding line, and the transparent green overlay shows open sea. Grounded ice is masked with the MODIS MOA (Haran et al., 2021; Greene et al., 2017). Dates of the fracture maps are: 10/01/2015 (e.1, f.1), 22/01/2015 (d.1), 26/01/2015 (c.1), 05/06/2022 (c.2), 14/06/2022 (d.2, e.2, f.2). Grounding lines are according to the MEaSUREs grounding line (Rignot et al., 2016).

Generally, the uncertainties associated with our estimates of fracture density change (Fig. 3.7 c, g, k) are similar in magnitude to the results. Of the ice shelves on which fracture patterns have changed, we have greatest confidence in the results covering the central part of the Thwaites Eastern Ice Shelf, the southern shear margin of Pine Island Ice Shelf, and the central body and northern shear margin of Crosson Ice Shelf.

The quantitative estimates of fracture density change are likely to be dependent on the size of the region over which the fracture densities were calculated and, to a lesser extent, on the resolution of the grid on which measurements were made. The parameters were chosen such that the grid spacing was not larger than the spatial extent of particular features of interest - such as the Pine Island Ice Shelf southern shear margin. We took care to ensure that the size of the regions over which fracture densities were measured were not smaller than the distances over which crevasses could have been advected during the observational period. Future studies should intercompare different fracture observation products, and investigate the sensitivity of the results to parameter choices and change estimates.

The usefulness of these results also depends on the linearity of changes in fracture density over the time period. For the most part, the assumption of slowly varying crevasse patterns allows us to see the most important structural changes in the region in our data. However, there are cases where important structural changes occur on short timescales and are not accurately captured when considering linear trends over longer, decadal periods. For example, a near flat signal is observed in fracture density near the grounding line of Thwaites Glacier Ice Tongue despite the large oscillatory changes observed over the last decade (Fig. 3.8 b), (Surawy-Stepney et al., 2023). Similarly, the method is insufficient when regions undergo rapid fragmentation, for example during the disintegration of the seaward part of the Pine Island southern shear margin in 2020. Here, the rapid changes in backscatter standard deviation led to a section of the fracture density timeseries being automatically discarded, despite the observed jump in fracture density reflecting a real event (Fig. 3.8 b, c).

Finally, due to the way we construct an ensemble of trend estimates, the results are likely to be biased towards small and negative changes in fracture density. This is because there is a component of the backscatter standard deviation timeseries due to the changing fracture

pattern on the ice surface, with a greater number of high contrast fractures generally increasing the standard deviation. Though our observations show this effect to be small compared to that of changing firn water content and the dependence of fracture density timeseries on fracturing on the ice surface, future work should aim to quantify this component and remove its effect from estimates of fracture density change.

3.4 Discussion

3.4.1 Crevasse development on Pine Island and Thwaites Eastern Ice Shelves

Though the main aim of this article is to present the methods we have developed and some of the data generated, there are some immediately interesting features in the fracture density change maps (Sec. 3.3.2) that merit further discussion; specifically regarding the Pine Island and Thwaites Eastern Ice Shelves.

Firstly, it is likely that the fracturing in the shear margins of Pine Island Ice Shelf (Fig. 3.7) are due to increased shear strain rates as the ice shelf accelerated over the last two decades (Mouginot et al., 2014; Lhermitte et al., 2020) and a thickness deficit in the shear zone partially maintained by channelised melting of the sub-shelf (Vaughan et al., 2012; Alley et al., 2019). By invoking a local measure of buttressing (Fig. 3.7), we have shown this crevassing to likely be, in turn, dynamically important for upstream ice. The relationship between fracture development and ice speed change is highly non-linear as crevasses caused by elevated strain rates directly impact the flow field by changing the constitutive ice rheology. Hence, our observations suggest that this crevassing in the shear margin is likely to be required to fully explain recent changes to the dynamics of this ice shelf (Joughin et al., 2021). Though Joughin et al., 2021 convincingly attributed the majority of the recent speed change to the calving of large tabular icebergs, we believe the full picture of dynamic change on the shelf will include the degradation of the southern shear margin.

The buttressing number we chose to consider can be thought of as a local measure of how the ice shelf differs from the archetypal freely-floating, one-dimensional case (MacAyeal, 1989). However, the net buttressing of an ice shelf *on the grounding line* is a highly non-local effect that depends on many factors that alter the transverse and longitudinal transmission of stresses

across the boundary. It is difficult to link the two notions in most cases, however, it is clear that a pattern of increased crevassing across the entire lateral shear margin of a confined ice shelf will decrease the net buttressing of the grounding line. Hence, in addition to the dynamics of the shelf itself, we believe the observed structural changes in the southern shear margin are also required to fully understand changes to grounding line flux over the last decade (Davison et al., 2023).

An analysis of satellite images over the Thwaites Glacier terminus shows the Eastern Ice Shelf loosening from the pinning point and a shear plane developing between the two (Benn et al., 2021). The fracture density change map we see is consistent with a picture of an ice shelf transitioning from one that is slow-moving and heavily pinned, where fracturing occurs due to compressive stresses originating from the pinning point, to a freely-floating ice shelf with crevasses forming and propagating perpendicular to flow by the action of internal stresses. This can be seen by the greater amount of fracturing at the Eastern calving front, and more regular fracturing perpendicular to the ice flow direction. Weighting by the buttressing number shows a stripe of increased fracture density close to the pinning point to be an important feature of structural change (grey arrow, Fig. 3.7 i). This closely mirrors a feature that was identified by Benn et al., 2021, using an ice sheet model to infer changes to material properties of the ice as the aforementioned slip plane developed.

3.4.2 Trends in Fracture Density as a Meaningful Measure of Structural Change

The benefit of using crevasse maps to assess structural change is that they can be used to derive quantitative information beyond that which can be gained by looking directly at satellite images. Our method achieves this, providing a scalar measure of structural change; we discuss in Sec. 3.4.3 a subset of the ways such a dataset could be useful in future work. Fundamentally, the use of the data relies on “fracture density” being a meaningful measure of structural weakness. Though it is not possible to find an exact mapping between fracture density and, for example, the continuum mechanics notion of *damage* (Lemaitre, 2012), this is likely to be a good assumption in most cases. Though the metric is degenerate with respect to orientation

and size or number of crevasses, it seems unlikely that crevassed regions would naturally evolve between these degenerate states in a way that changes the dynamics of the glacier. Additionally, there may be cases in which dynamically unimportant changes in crevassing lead to changes in fracture density. This is a consideration, for example, in regions dominated by uniform longitudinal stress such as near the calving fronts of wide ice shelves. Here, the net impact of a field of parallel crevasses of equal depth is equivalent over large distances to a single crevasse of that depth. However, such regions are not highly buttressed, so structural changes are less likely to matter to the dynamics of the glacier as a whole. For example, the fracture density changes at the terminus of Pine Island disappear when weighted by buttressing number (Fig. 3.7 b, e). Finally we note that, as our fracture maps primarily locate crevasse walls, the widening of crevasses, which could have conceivable dynamic implications, is not measured as a change in fracture density.

3.4.3 Future Work on Measuring Changes in Fracture Density

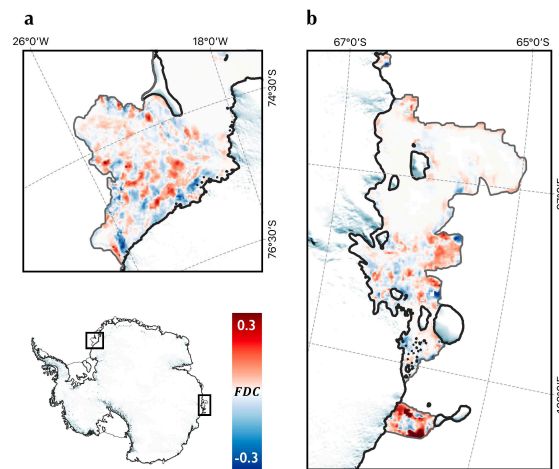


Figure 3.9: Fracture density change on the Brunt/Stancomb-Wills and Shackleton Ice Shelves. (a) shows fracture density change on the Brunt/Stancomb-Wills Ice Shelf. (b) shows that for the Shackleton Ice Shelf and surroundings. Locations are shown on the schematic of the Antarctic Ice Sheet in the bottom left corner. Black lines show the MEaSUREs grounding line (Rignot et al., 2016), and grey lines delineate the edges of the ice shelves. The interior of the ice sheet is masked with the REMA 2 km DEM (Porter et al., 2018).

Though our method of measuring the evolution of ice shelf crevassing is capable of resolving severe structural changes in fast-changing regions, few places in Antarctica are changing as rapidly as the Amundsen Sea Embayment (Mouginot et al., 2014; Shepherd et al., 2019). The results of applying this technique to the Antarctic Ice Sheet as a whole would likely be dom-

inated by noise. In some places, such as Crosson Ice Shelf (Fig. 3.7 j), the method may not be particularly useful where change is dominated by the advection of existing crevasses, and ‘interesting’ changes are subtle in comparison. For example, Fig. 3.9 shows fracture density changes on the Brunt-Stancomb-Wills (BSW) and Shackleton Ice Shelves. Here, the stripes of positive and negative trends in fracture-density show advection on BSW and in the Shackleton shear margin, though the steady propagation of Chasm-1 (Libert et al., 2022), for example, is difficult to discern. In future, we could apply Lagrangian corrections to fracture maps, before looking at fracture density change, or use a method that does not involve averaging over large regions and monitor the size of isolated rifts.

Finally, future studies should take place investigating alternative means of reducing the component of fracture density timeseries due to the dependence of crevasse visibility on surface water conditions. Our method uses backscatter standard deviation to alias consistent timeseries and build an ensemble of trends for each location. This is based on the observation that the fracture density and backscatter standard deviation signals appear to respond in the same way to changing firn/ice-surface properties. It ought to be possible, for example using a regional climate model or reanalysis data, to better constrain the most salient factors in the environmental contribution to the fracture density signal. These can then be isolated using additional datasets to perform the procedure outlined here more accurately.

3.4.4 Representation of Damage in Numerical Modelling

The physics of brittle and ductile failure is not well integrated into many large-scale ice sheet models but, through its distributed influence on stress conditions, this omission may have a large impact in predictions of the evolution of the ice sheet. Both the crevasse maps and fracture-density change maps introduced in this article have conceivable use in numerical modelling studies. Firstly, static, pan-continental mosaics of crevasses can be assimilated into numerical models to better constrain initial control parameters for simulations. Specifically, models often require a control parameter specifying changes to the effective ice viscosity from that provided by the ice constitutive relation (Cuffey and Paterson, 2010; Cornford et al., 2015). Given that the presence of fractures is assumed to change the effective ice viscosity, the fracture data can be used to constrain an inverse problem aiming to infer such a field from ice speed data. On grounded ice, these maps could be particularly useful in reducing the underdeterminedness of

inversions for both ice softness and basal friction.

It is also conceivable that the smooth, scalar fracture-density change maps we produce could be used more directly in diagnostic modelling. In continuum mechanics, a scalar “damage” parameter is often used to represent the reduction in effective ice viscosity caused by the presence of fractures (Lemaitre, 2012; Borstad et al., 2012). The dynamic impact of structural change for glaciers of interest, e.g. Pine Island Glacier, could be investigated by using fracture-density changed fields as a proxy for damage change.

Additionally, data on the location of crevasses can be used to inform our understanding of the physical processes which lead to crevasse development. In particular, the healing of grounded surface fractures suggests their presence is a function of instantaneous stress conditions, rather than a complicated and intractable stress history. This suggests the data could be used to constrain models seeking damage as a function of stress invariants, using stress fields inferred from coincident ice velocity observations. At a more basic level, such data, in combination with accurate vertical stress profiles, could be used to study the fracture toughness of meteoric glacier ice in the interior of the Antarctic Ice Sheet.

3.4.5 The Use of SAR Backscatter Images

We turn now to a discussion of synthetic-aperture radar data as the base dataset for this study. We have shown that large variety of crevasse-like features are visible in the SAR backscatter images acquired by Sentinel-1 (Fig. 3.5) and, using parallel processing of the images for type-A and -B features, that most of these can be automatically extracted in a way reliable enough to promote discussion of existing crevasse patterns and to measure important structural changes on ice shelves. However, there could be choices other than SAR as the base dataset for this work that have the desired spatial coverage. In particular, the use of optical satellite data or a high-resolution digital elevation model (DEM). The further need for a timeseries of fracture maps leaves optical data as the only appropriate alternative candidate, although this would preclude year-round monitoring during the Polar night.

Many, but not all of the features we are interested in can be seen in optical imagery of comparable resolution to our SAR data. Crucially, crevasses corresponding to type-B features can rarely be seen reliably in optical imagery, except from in the most high-resolution (< 10 m) cases. Even then, they are often bridged by snow. Hence, the use of SAR data allows for the mapping

of crevasses on grounded ice, which are essentially exclusively of type-B. Additionally, optical imagery, in contrast to SAR data, is hampered by the presence of clouds and cannot acquire images at night or during large parts of the Austral winter. SAR data is therefore preferable for generating a consistent, reliable timeseries which can be used to measure changes to crevassing over relatively short timescales. Of the SAR satellites available, we consider Sentinel-1 to be the best tool due to its pan-continental large scale acquisition plan which acquires new images every 6 to 12-days. Though it does not have as fine a spatial resolution as some other SAR satellites, such as TerraSAR-X, its short repeat period and extensive spatial coverage allows a consistent timeseries to be generated, covering the whole Antarctic margin.

There are, however, certain disadvantages to the use of SAR. Some ice shelf crevasses appear smoother on the surface than in optical imagery. For example, the crevasses on Dotson Ice Shelf, that appear faintly in SAR backscatter images at 50 m resolution, can be seen more clearly in the MODIS MOA image over the same region (Haran et al., 2021; Greene et al., 2017). Due to layover and shadowing effects, stemming from the fact that SAR images are reconstructed from range distances rather than incidence angles of received radiation, there are additional issues in geolocating crevasses. This is most important for type-B features where the geocoding errors induced by these effects can be on the order of a crevasse width. However, these are likely to be small in comparison to errors induced by deviations in the surface height through time from those given by the digital elevation model. Most importantly, optical data often comes with multi-band information such as different colours in the visible spectrum. We use single-band SAR data due to constraints in quantity of throughput data required to generate pan-continental mosaics. However, as discussed above, the type-A processing fails in the presence of meltponds. Optical data, in which the water appears blue, or multiband SAR with dual polarisation, would be useful input to a neural network trying to discern fractures from linear boundaries of these pools of water. Finally, the use of a DEM might allow for the simultaneous extraction crevasse-location and crevasse-depth data, the latter of which can only be estimated from SAR data.

The detection of crevasses using maps of interferometric fringes (Libert et al., 2022), phase coherence (Hogg and Gudmundsson, 2017) or strain rate fields (De Rydt et al., 2018) comes

with additional valuable information on the “activity” of crevasses - where their presence induces discontinuities in certain properties of the ice (e.g. its flow speed). Similarly, the use of precision altimetry data from ICESat-2 provides some information about the depth of crevasses (Herzfeld et al., 2021). The method presented in this work does not provide any such information, but, as a consequence, provides more extensive maps of crevasses with much greater coverage. As such, this data is useful for large-scale analyses, studies in regions with low interferometric coherence or imperfect velocity observations, and studies where the activity of the crevasses is of secondary importance - for example as a source of surface-to-bed hydrological pathways. Additionally, a dataset of crevasse locations such as that provided here, can be used in combination with such methods to learn more about the importance of different crevasses across the continent. For example, it would be simple to locate ICESat-2 observations coincident with identified crevasses to assess some measure of depth, with the understanding that a reliable estimate of true depth is unattainable in many cases due to the sub-resolution width of the crack tip, or to look for discontinuities in ice-shelf flow speed at crevasse locations in coincident velocity observations.

3.4.6 Future Improvements to the Crevasse Maps

The method of mapping crevasses presented in this work includes different processing chains for detecting the features we call type-A and type-B. This is necessary because of their different appearance in the backscatter images, but has the added benefit of providing an independent dataset for each fracture type, which allows for almost independent analysis of crevasses on grounded and floating ice. In order to constrain models of fracture development on ice shelves, it would be useful to further partition the type-A crevasses into basal and surface crevasses. This could be done by tuning the existing type-A network with small dataset of manually annotated basal and surface crevasses separately, and running them in parallel. This would also help solve the current insensitivity of the data to those features which might correspond to basal crevasses.

Improvements to the accuracy of the crevasse mapping can in all likelihood be gained without a great deal of work with the use of SAR data at multiple resolutions. We have been able to present accurate crevasse maps with large spatio-temporal coverage, in part, by limiting processing to the use of data at a single spatial resolution, with our choice of 50 m based on a trade-off between the detail in which crevasses can be seen in the data and the finite capacity for computational throughput at our disposal. However, a greater number of crevasses can be

seen at 10 m resolution, beyond the limit of what can be achieved with Sentinel-1 backscatter data which, for example, would allow for more accurate bounding of regions of type-B crevasses.

Our discussions (Sec. 3.4.5) suggest improvements can be made to the detection of basal crevasses and in discriminating between crevasses and the boundaries of surface meltponds using multi-band input. We believe the greatest improvements could be seen by combining input SAR data with any available coincident optical data. This could be used to bolster static maps, where timeseries are not required. Additionally, we note that the sensitivity of the crevasse maps is not enough to capture subtle changes to crevasse density and length. This can be tackled in two ways: increase the sensitivity, or increase the timescale of processes of interest. A potential method for the first is to work directly with timeseries data. If a neural network were designed to receive as input a sequence of images and to segment the central one, the persistence of features of interest could be learned by the model leading to more consistent timeseries. To tackle the second problem, it is feasible to train the network to segment older satellite imagery, e.g. from RADARSat, ENIVSAT or ERS1/2. This would enable a more extensive analysis of the longer-term (decadal) patterns of change in crevassing over the Antarctic continent.

3.4.7 A Comparison of Ice Shelf Crevasse Detection Methods

The crevasse maps presented in this work provide unified coverage over the extent of floating and grounded ice in Antarctica imaged by Sentinel-1. As far as the authors are aware, there are no existing methods with publicly available datasets or code for the large-scale detection of Antarctic surface crevasses on grounded ice. However, methods for crevasse detection on floating ice shelves do exist. We conduct a brief comparison, for a single Sentinel-1 image frame, between the results from the method presented in this work and two publicly available existing datasets/methods, namely those of Lai et al., 2020 and Izeboud and Lhermitte, 2023. We refer to these respectively as “L20” and “I23”, and to the method presented in this article as “S23”. L20 used a U-Net to extract ice shelf crevasses from optical data covering the AIS at 125 m resolution, while I23 use a method based on the normalised radon transform that can be applied to data from different sensors at different resolutions. We applied I23 to a Sentinel-1 backscatter image (Sentinel-1 PF: 7/913, dated: 20210607) at 50 m resolution using a window size of 10 pixels and a normalisation range of -30 to 0 dB. These are compared with the June 2021

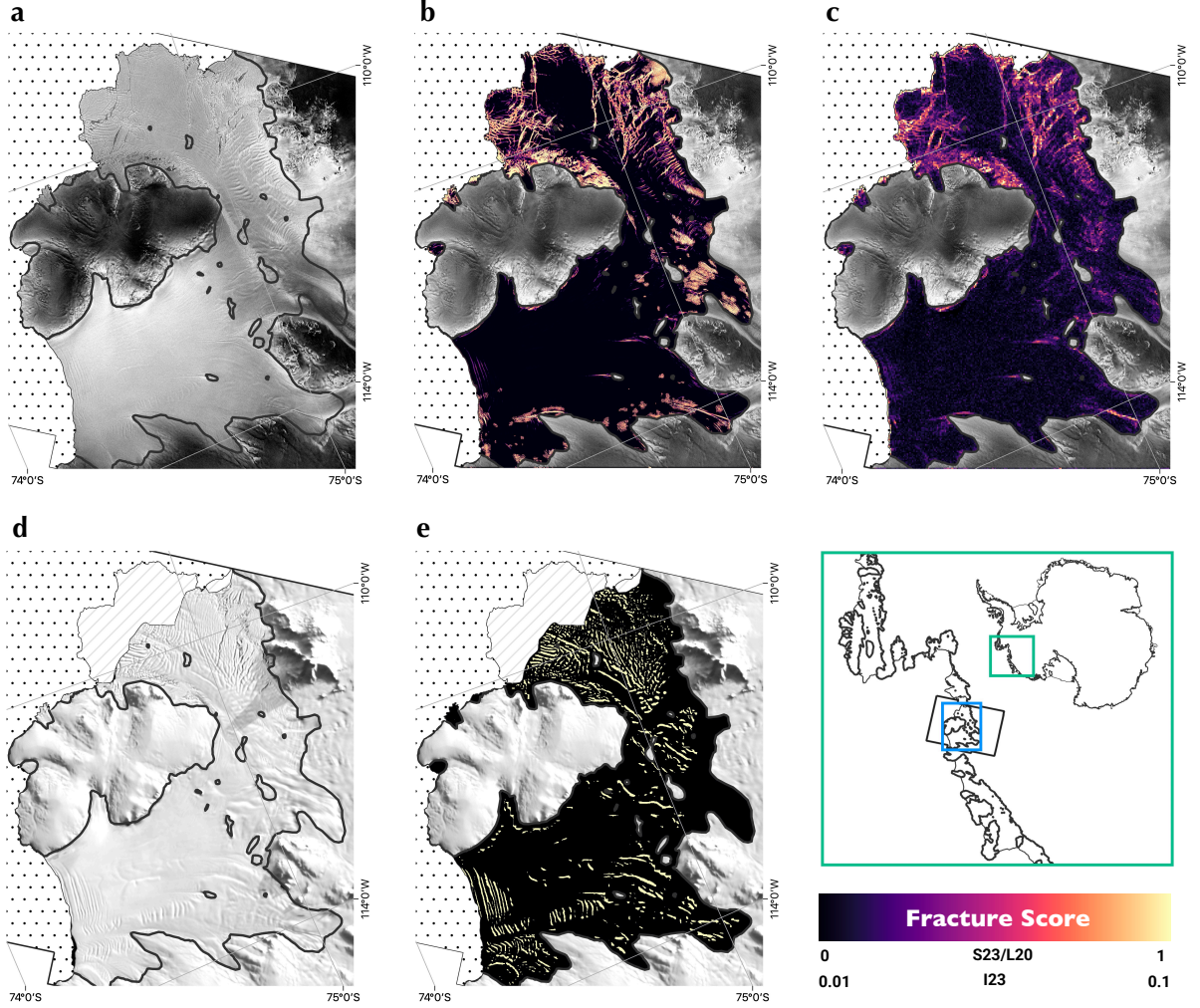


Figure 3.10: A comparison of fracture maps generated using our method (“S23”) versus two existing methods, Lai et al., 2020 “L20” and Izeboud and Lhermitte, 2023 “I23”, over the Crosson and Dotson Ice Shelves. (a) Sentinel-1 backscatter image at 50 m resolution, ESA PF: 7/913, dated 20210607, used for the S23 and I23 crevasse maps. (b) June 2021 crevasse mosaic using S23 restricted to the floating ice. (c) Result of application of I23 to the SAR backscatter image shown in (a); with the colourmap showing the range 0.01 to 0.1. Grounded ice in (b-c) is masked by the SAR backscatter image shown in (a). (d) MODIS Mosaic of Antarctica (Haran et al., 2014) restricted to the geometry of the SAR backscatter image shown in (a). (e) Crevasse map from L20, with the grounded ice masked by the MODIS MOA shown in (d). The area outside of the bounds of the SAR backscatter image shown in (a) are masked in (a-e). Grounding lines are as given by (Rignot et al., 2016), shown with a black line. The difference between the June 2021 Crosson Ice Shelf extent and the edge of the L20 dataset is shown in (d-e) as the striped region. The sea is shown as the spotted region. The blue box on the map in the bottom right shows the extent of the region shown in (a-e). This lies on a black outline showing the bounds of the SAR backscatter image shown in (a). Inset is a map of Antarctica showing the location of the blown-up region. For S23 and L20, the colourmap displays the range 0-1, while for I23, it displays the range 0.01-0.1.

mosaic over the extent of the aforementioned SAR backscatter image. This location (covering the Crosson and Dotson Ice Shelves of the Amundsen Sea Sector of West Antarctica) was chosen

again for the variety of crevasse features in the image. Results are shown in Fig. 3.10. Satellite images from which the data were generated are shown in Fig. 3.10 a and d, and the derived data are shown in b, c and e. The SAR image (a) corresponding to data shown in b-c is from the 7th of June 2021, while the MODIS image (d) from which e was derived, is constructed from images dating between 2008 and 2009. The outputs corresponding to L20 and S23 are in the range $[0,1]$ while the range for I23 is a parameter that is chosen based on the windows size and image resolution to tune the output to best represent fractures visible in the input image. In this case, we set the bounds to $[0.01, 0.1]$. We show in Fig. 3.10 a “fracture score” that uses the same colourmap for the outputs of each method, but with different bounds.

Overall, with bounds for the range of the I23 data set to $[0.01, 0.1]$, we see a relatively good agreement between S23 and I23. Both pick up the large-scale patterns of fracture on the Crosson Ice Shelf and display relatively little on Dotson Ice Shelf (Fig. 3.10 a-c), though the features in S23 are higher contrast, with lower background noise. The resolution of the I23 output is defined by the window size over which radon transforms are calculated, and, with a choice of 10-pixels, S23 improves on this resolution by an order of magnitude, though the comparison might not be fair given the possibility of using I23 with different, higher-resolution sensors. The S23 output is dominated by type-A crevasses, though closer to the grounding line there are fields of type-B fractures in the input SAR backscatter image that can be seen in S23 but not in I23. Finally, we note that the time taken to process this SAR frame was significantly higher in the case of I23 than our method when running with an equal number of processes and the same CPU hardware.

The differences between S23 and L20 are partly due to the evolution of the Crosson Ice Shelf in the period 2009 to 2021 (visible in Fig. 3.10 a and d) such as the advection and extension of a large central rift in the ice shelf and the degradation of the northern shear margin that borders Bear Island. However, there are also fundamental differences between these datasets that will influence their future application. Firstly, there is a preference for L20 to extract wider, smoother features over sharp rifts or deep, disordered fractures in the shear margins, while the reverse is true for S23. This leads to a near reversal in the features that are detected between the two methods. On Crosson Ice Shelf, the large central rifts visible in Fig. 3.10 d do not appear in L20 (e), while similar rifts are prominent features in (a-c). Additionally, the

flowband features in L20 (many of which appear not to be crevasses) do not appear in S23. On Dotson Ice Shelf, however, L20 contains a great deal of basal-crevasse-like features visible in d which appear only faintly if at all in S23.

Overall, considering ice shelves as well as grounded ice, we would advocate for the use of the data/method presented in this article, or a combination of the data presented here and those of Lai et al., 2020, when a single fully comprehensive map of crevasse features is required. This is especially true for continent-wide studies as our data do not cover the majority of the Ronne-Filchner and Ross Ice Shelves, unlike those of Lai et al., 2020. However, we acknowledge that the comparison presented here is largely qualitative, covers only a small area and includes only three datasets. Hence, future work should aim to conduct a more exhaustive comparison of existing crevasse-mapping products over a large and varied set of times and ice shelves to explore more completely the benefits and disadvantages of different methods.

3.5 Conclusions

We have developed and applied a method of crevasse detection from Sentinel-1 SAR backscatter imagery using convolutional neural networks in combination with parallel structure filtering. Our results show that crevasses are a major feature of most ice shelves around the Antarctic coastline, and much of the grounded ice in shear- and slip-dominated flow regimes. We have developed a method for measuring the change in fracturing on ice shelves over relatively short timescale, which requires careful consideration of environmental factors changing the surface expression of crevasses. These results, combined with analysis of the stress conditions using a numerical model, showed changes in the density of crevasses in buttressing regions of ice shelves in the Amundsen Sea Embayment within the 7.5-year study period from 2015 to 2022. We suggest that recent ice dynamic changes to these ice streams likely cannot be fully accounted for without the observed structural changes. On Pine Island Glacier, the changes in fracture density are particularly severe, and have occurred largely in a region of high shear stress. As a result, it is likely that change in fracturing has decreased the buttressing capacity of the ice shelf at the grounding line on this major west Antarctic ice stream, and is hence also required to explain recent change in grounding line ice mass flux and grounding line location. In the future, we will continue to develop the methods introduced here to differentiate crevasse types and to reduce the uncertainty on the timeseries of structural change. Future studies must develop an

improved physical representation of damage in ice flow models as it is clear that the ice dynamic response cannot be fully reproduced without accounting for change in damage.

Appendix

3.A Parallel Structure Filtering

The method of extracting locally parallel linear features from the type-B network uses as a starting point the method proposed by Frangi et al., 1998 for finding the likelihood of a pixel lying on a linear structures, using the Hessian matrix local to each pixel. We include the additional step of using the second eigenvector of the Hessian to calculate whether the linear structures we find are near-parallel to others nearby. We call this process “parallel structure filtering”.

Given a network output $\mathcal{D}(x, y)$, with values at each point between 0 and 1, the Hessian $\mathcal{H}(x, y)$ defines the second derivatives at each point. We find the components of $\mathcal{H}(x, y)$ by convolving $\mathcal{D}$ with a full set of Gaussian derivative kernels:

$$\mathcal{H}_{ij}(x, y) = \mathcal{G}_{ij}(x, y) * \mathcal{D}(x, y),$$

where i and j denote the coordinate directions x or y . The components of $\mathcal{G}(x, y)$ are given by the second derivatives of the two-dimensional Gaussian:

$$\begin{aligned}\mathcal{G}_{xx}(x, y) &= \frac{(-1 + x^2/\sigma^2)}{2\pi\sigma^2} \times \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right), \\ \mathcal{G}_{yy}(x, y) &= \frac{(-1 + y^2/\sigma^2)}{2\pi\sigma^2} \times \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right), \\ \mathcal{G}_{xy}(x, y) &= \frac{xy}{2\pi\sigma^6} \times \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right) \\ &= -\mathcal{G}_{yx}(x, y),\end{aligned}$$

where σ is chosen based on the width of the structures we see in the network outputs.

Frangi et al., 1998 describe an intuitive method of judging the “vesselness” $\mathcal{V}$ of each pixel (how

likely it is to be on part of a linear, tube-like feature of the image) based on the eigenvalues $\lambda_{1,2}(x, y)$ of $\mathcal{H}(x, y)$. We use a modification of this described in Jerman et al., 2016. At a particular location, we define the functions B and S of the pixel Hessian eigenvalues as:

$$B = \frac{|\lambda_2|}{|\lambda_1|}, \quad S = \sqrt{\lambda_1^2 + \lambda_2^2},$$

where $|\lambda_1| > |\lambda_2|$.

Because the locations of crevasses are brighter in $\mathcal{D}$ than the background, when $\lambda_2 > 0$ we take the likelihood of the pixel being part of a crevasse feature to be zero. Otherwise, we take:

$$\mathcal{V} = \left(1 - \exp\left(-\frac{B^2}{2\alpha^2}\right)\right) \times \left(1 - \exp\left(-\frac{S^2}{2\beta^2}\right)\right),$$

with $\alpha = 0.5$ and $\beta = \max\{S(x, y)\}$.

Setting pixels with $\mathcal{V} < 0.1$ to zero, we then calculate the angle of the tubular structure local to each pixel using the eigenvector $e_2 = (e_{2x}, e_{2y})^\top$ (corresponding to eigenvalue λ_2) that defines the direction along any identified structure:

$$\mathcal{A}(x, y) = \arctan\left(\frac{-e_{2y}(x, y)}{e_{2x}(x, y)}\right).$$

Finally, we calculate the local variance of the angles:

$$\Sigma_A^2(x, y) = K_{71} * (\mathcal{A}^2(x, y)) - (K_{71} * \mathcal{A}(x, y))^2,$$

where $K_{71}(x', y')$ is a box-kernel of width 71. We mask pixels (x, y) in the original image where $\mathcal{V}(x, y) < 0.1$ or $\Sigma_A(x, y) < 0.71$.

This method has been shown to be useful for the detection of type-B crevasses from network outputs, and, due to its intuitive nature and use of freely available tools, it could be utilised in any situation in which one is attempting to extract linear features that are locally parallel from relatively noise-free data.

We also note that this method provides us with smooth fields for the orientation of type-B crevasses. At present, this information is not used, but could be utilised in future, for example,

to assess strain rates using measurements at different times.

3.B Measuring Trends in Fracture Density

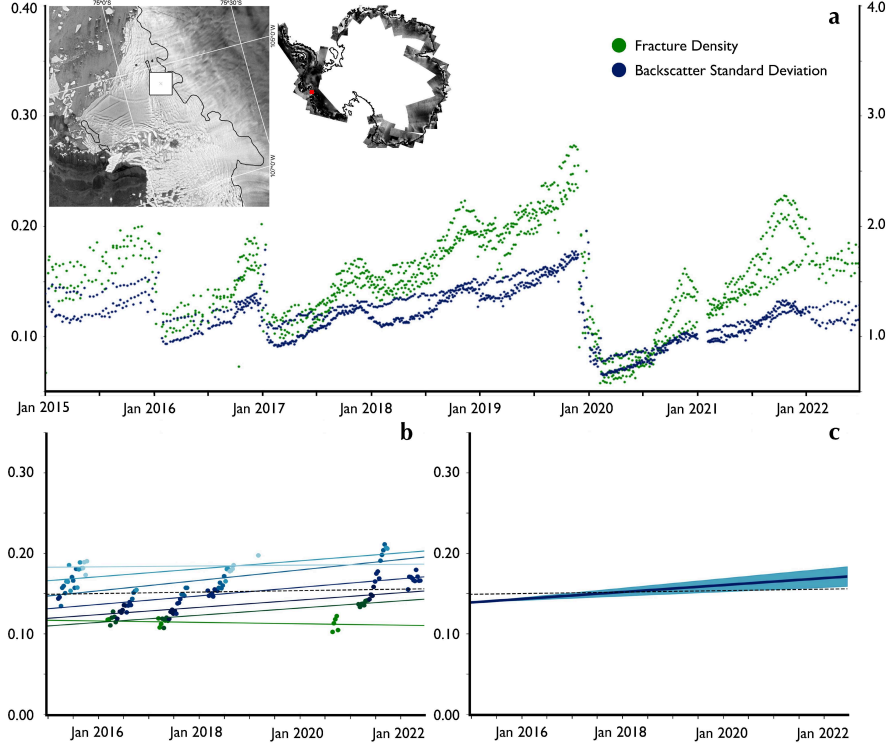


Figure 3.11: Evaluating fracture density change for a point on the Thwaites Eastern Ice Shelf. (a) A timeseries of fracture density (green) and backscatter standard deviation (blue) for a $10 \text{ km} \times 10 \text{ km}$ region over the Thwaites Eastern Ice Shelf. Inset: the left shows a SAR backscatter image of the Thwaites terminus with the region over which the timeseries were extracted highlighted with a white box. To the right is a SAR mosaic from April 2022 with the location of the Thwaites terminus image shown in red. Black lines show the MEaSUREs grounding line (Rignot et al., 2016). (b) Linear trends through an ensemble of data points corresponding to monthly median fracture densities for dates with backscatter standard deviations within 0.1 of each other. Different colours represent different collections of data points. The black dashed line shows the fit through the whole collection of datapoints, without partitioning using backscatter standard deviation. (c) shows the ensemble mean and uncertainty in the fits in (b) that result in a fracture density change measurement shown for this location in Fig. 3.7. The black dashed line shows the same as in (b). The figure shows how a non-zero trend is recovered when we apply correction for surface conditions, that is not visible in the original timeseries.

The approach we take to decouple the part of the signal resulting from real changes in the crevasse pattern, and those due to changes in the surface expression of the crevasses resulting from unknown environmental factors is to only compare fracture density datapoints generated from images showing ice with similar surface properties. We do so by looking directly at the backscatter images of the ice shelves, to find sets of dates where the standard deviation of the

backscatter signal over the region of interest are the comparable. The assumption is that the backscatter standard-deviation time series are dependent almost entirely on the same surface properties that dominate the fracture density time series, with the component due to changing fracture pattern being small, and that the correspondence between a set of surface properties and a particular standard deviation is close. A heuristic verification of this can be seen in the similarity of the standard deviation and fracture density timeseries for Dotson Ice Shelf, where the crevasses are known to be relatively unchanging. Comparing the standard-deviation and fracture density timeseries for Pine Island Glacier, where fracturing is increasing, shows the standard deviation mirroring the seasonal features of the fracture density timeseries without the accompanying trend.

For a given region, we generate fracture density and backscatter standard deviation timeseries. For a particular value of standard-deviation x_i , we find the dates in the timeseries with standard-deviation within a small neighbourhood of x_i . This gives us a set of dates in which the ice had similar surface properties. We then find the fracture densities for this set of dates. We perform a linear fit through this set of datapoints which gives us a y-intercept, a gradient and their associated errors: $(c_i, \delta c_i)$ and $(m_i, \delta m_i)$. By stepping through standard deviations we can, by following the above procedure, build a set of such coefficients.

We report the error-weighted mean and standard deviation of the estimates of the trend, multiplied by the time span:

$$\mu = \frac{\sum_i \frac{m_i}{\delta m_i^2}}{\sum_i \frac{1}{\delta m_i^2}} \quad (3.1)$$

and

$$\sigma = \sqrt{\frac{\sum_i \frac{1}{\delta m_i^2} (m_i - \mu)^2}{\frac{N-1}{N} \sum_i \frac{1}{\delta m_i^2}}} \quad (3.2)$$

where N is the number of estimates.

3.C Calculating the Buttrressing Number

We use the following scalar field to represent how ‘buttressed’ a region is:

$$\kappa = 1 - \frac{e_2}{N}$$

where e_2 is the smallest eigenvalue of the vertically-integrated viscous stress tensor R , N is the value of the vertically-integrated hydrostatic pressure at that location:

$$N = \frac{\rho_i}{2} \left(1 - \frac{\rho_i}{\rho_w} \right) gh^2,$$

ρ_i is the density of water, ρ_w is the density of sea water, g is the acceleration due to gravity, and h is the ice thickness (Gudmundsson, 2013; Fürst et al., 2016).

The comparison of N with the second principal viscous stress e_2 rather than, for example, the viscous stress in the direction of flow or the first principal viscous stress creates a maximum valued κ (Fürst et al., 2016), as the eigenvector with eigenvalue e_2 defines the direction along which viscous stresses are most compressive. The use of R , rather than, for example, the full Cauchy stress, is convenient as we use a two-dimensional approximation to the ice shelf that boils down to the shallow-shelf approximation (MacAyeal, 1989).

We use the BISICLES ice sheet model (Cornford et al., 2013) to generate fields representing the components of R from observations of ice velocity (Rignot et al., 2017) and ice sheet geometry (Morlighem, 2020; Morlighem et al., 2020). This was done by first solving the inverse problem for a basal drag and ice stiffness parameter on the ice shelves (Cornford et al., 2015) and retrieving the viscous stress tensor given those fields as a by-product. We take the buttressing number at the start of the observational period, in order to assess changes to fracture density in regions that had a distributed effect on stresses within the ice shelf before additional crevasses developed.

Bibliography

- Albrecht, T. and Levermann, A. (2012). “Fracture field for large-scale ice dynamics”. In: *Journal of Glaciology* 58.207, pp. 165–176. DOI: 10.3189/2012JoG11J191.
- Alley, K., Scambos, T., Miller, J., Long, D., and MacFerrin, M. (2018). “Quantifying vulnerability of Antarctic ice shelves to hydrofracture using microwave scattering properties”. In: *Remote Sensing of Environment* 210, pp. 297–306. ISSN: 0034-4257. DOI: 10.1016/j.rse.2018.03.025.
- Alley, K. E., Scambos, T. A., Alley, R. B., and Holschuh, N. (2019). “Troughs developed in ice-stream shear margins precondition ice shelves for ocean-driven breakup”. In: *Science Advances* 5.10, eaax2215. DOI: 10.1126/sciadv.aax2215.
- Bassis, J. and Ma, Y. (2015). “Evolution of basal crevasses links ice shelf stability to ocean forcing”. In: *Earth and Planetary Science Letters* 409, pp. 203–211. ISSN: 0012-821X. DOI: 10.1016/j.epsl.2014.11.003.
- Benn, D. I., Luckman, A., Åström, J. A., Crawford, A., Cornford, S. L., Bevan, S. L., Gladstone, R., Zwinger, T., Alley, K., Pettit, E., and Bassis, J. (2021). “Rapid fragmentation of Thwaites Eastern Ice Shelf, West Antarctica”. In: *The Cryosphere Discussions* 2021, pp. 1–25. DOI: 10.5194/tc-2021-288.
- Benn, D. I. and Evans, D. J. (2014). *Glaciers & glaciation*. Routledge.
- Borstad, C. P., Khazendar, A., Larour, E., Morlighem, M., Rignot, E., Schodlok, M. P., and Seroussi, H. (2012). “A damage mechanics assessment of the Larsen B ice shelf prior to collapse: Toward a physically-based calving law”. In: *Geophysical Research Letters* 39.18. DOI: 10.1029/2012GL053317.
- Colgan, W., Rajaram, H., Abdalati, W., McCutchan, C., Mottram, R., Moussavi, M. S., and Grigsby, S. (2016). “Glacier crevasses: Observations, models, and mass balance implications”. In: *Reviews of Geophysics* 54.1, pp. 119–161. DOI: 10.1002/2015RG000504.
- Cornford, S. L., Martin, D. F., Graves, D. T., Ranken, D. F., Le Brocq, A. M., Gladstone, R. M., Payne, A. J., Ng, E. G., and Lipscomb, W. H. (2013). “Adaptive mesh, finite volume modeling of marine ice sheets”. In: *Journal of Computational Physics* 232.1, pp. 529–549. ISSN: 0021-9991. DOI: 10.1016/j.jcp.2012.08.037.
- Cornford, S. L., Martin, D. F., Payne, A. J., Ng, E. G., Le Brocq, A. M., Gladstone, R. M., Edwards, T. L., Shannon, S. R., Agosta, C., Broeke, M. R. van den, et al. (2015). “Century-scale simulations of the response of the West Antarctic Ice Sheet to a warming climate”. In: *The Cryosphere* 9.4, pp. 1579–1600.
- Cuffey, K. M. and Paterson, W. S. B. (2010). *The physics of glaciers*. Academic Press.

- Davison, B. J., Hogg, A. E., Rigby, R., S, E., Wessem, J. M. van, Broke, M. van den, Holland, P. R., Selley, H. L., and Dutrieux, P. (2023). “Sea level rise from West Antarctic mass loss significantly modified by large snowfall anomalies”. In: *Nature Communications* 14.1, p. 1479. DOI: 10.1038/s41467-023-36990-3.
- De Rydt, J., Gudmundsson, G. H., Nagler, T., Wuite, J., and King, E. C. (2018). “Recent rift formation and impact on the structural integrity of the Brunt Ice Shelf, East Antarctica”. In: *The Cryosphere* 12.2, pp. 505–520. DOI: 10.5194/tc-12-505-2018.
- Frangi, A. F., Niessen, W. J., Vincken, K. L., and Viergever, M. A. (1998). “Multiscale vessel enhancement filtering”. In: *International conference on medical image computing and computer-assisted intervention*. Springer, pp. 130–137.
- Fürst, J. J., Durand, G., Gillet-Chaulet, F., Tavard, L., Rankl, M., Braun, M., and Gagliardini, O. (2016). “The safety band of Antarctic ice shelves”. In: *Nature Climate Change* 6.5, pp. 479–482.
- Gourmelen, N., Goldberg, D. N., Snow, K., Henley, S. F., Bingham, R. G., Kimura, S., Hogg, A. E., Shepherd, A., Mouginot, J., Lenaerts, J. T. M., Ligtenberg, S. R. M., and Berg, W. J. van de (2017). “Channelized Melting Drives Thinning Under a Rapidly Melting Antarctic Ice Shelf”. In: *Geophysical Research Letters* 44.19, pp. 9796–9804. DOI: 10.1002/2017GL074929.
- Greene, C. A., Gwyther, D. E., and Blankenship, D. D. (2017). “Antarctic Mapping Tools for Matlab”. In: *Computers & Geosciences* 104, pp. 151–157. ISSN: 0098-3004. DOI: 10.1016/j.cageo.2016.08.003.
- Gudmundsson, G. H. (2013). “Ice-shelf buttressing and the stability of marine ice sheets”. In: *The Cryosphere* 7.2, pp. 647–655. DOI: 10.5194/tc-7-647-2013.
- Hambrey, M. and Müller, F. (1978). “Structures and Ice Deformation in the White Glacier, Axel Heiberg Island, Northwest Territories, Canada”. In: *Journal of Glaciology* 20.82, pp. 41–66. DOI: 10.3189/S0022143000021213.
- Haran, T., Bohlander, J., Scambos, T., Painter, T., and Fahnestock, M. (2014). *MODIS Mosaic of Antarctica 2008-2009 (MOA2009) Image Map, Version 1*. DOI: 10.7265/N5KP8037.
- Haran, T. M., Bohlander, J., Scambos, T. A., Painter, T. H., and Fahnestock, M. A. (2021). *MODIS Mosaic of Antarctica 2003-2004 (MOA2004) Image Map, Version 2*. DOI: 10.5067/68TBT0CGJS0J.
- Herzfeld, U. C., Trantow, T., Lawson, M., Hans, J., and Medley, G. (2021). “Surface heights and crevasse morphologies of surging and fast-moving glaciers from ICESat-2 laser altimeter data - Application of the density-dimension algorithm (DDA-ice) and evaluation using airborne altimeter and Planet SkySat data”. In: *Science of Remote Sensing* 3, p. 100013. ISSN: 2666-0172. DOI: 10.1016/j.srs.2020.100013.
- Hogg, A. E. and Gudmundsson, G. H. (2017). “Impacts of the Larsen-C Ice Shelf calving event”. In: *Nature Climate Change* 7.8, pp. 540–542. DOI: 10.1038/nclimate3359.
- Hughes, T. (1983). “On the Disintegration of Ice Shelves: The Role of Fracture”. In: *Journal of Glaciology* 29.101, pp. 98–117. DOI: 10.3189/S0022143000005177.

- Hulbe, C. L., LeDoux, C., and Cruikshank, K. (2010). “Propagation of long fractures in the Ronne Ice Shelf, Antarctica, investigated using a numerical model of fracture propagation”. In: *Journal of Glaciology* 56.197, pp. 459–472. DOI: 10.3189/002214310792447743.
- Irwin, G. R. (1957). “Analysis of stresses and strains near the end of a crack traversing a plate”. In: 24, pp. 361–364. DOI: 10.1115/1.4011547.
- Izeboud, M. and Lhermitte, S. (2023). “Damage detection on antarctic ice shelves using the normalised radon transform”. In: *Remote Sensing of Environment* 284, p. 113359. ISSN: 0034-4257. DOI: 10.1016/j.rse.2022.113359.
- Jerman, T., Pernuš, F., Likar, B., and Špiclin, Ž. (2016). “Enhancement of Vascular Structures in 3D and 2D Angiographic Images”. In: *IEEE Transactions on Medical Imaging* 35.9, pp. 2107–2118. DOI: 10.1109/TMI.2016.2550102.
- Joughin, I., Shapero, D., Smith, B., Dutrieux, P., and Barham, M. (2021). “Ice-shelf retreat drives recent Pine Island Glacier speedup”. In: *Science Advances* 7.24, eabg3080. DOI: 10.1126/sciadv.abg3080.
- Joughin, I., Smith, B. E., and Medley, B. (2014). “Marine ice sheet collapse potentially under way for the Thwaites Glacier Basin, West Antarctica”. In: *Science* 344.6185, pp. 735–738.
- Konrad, H., Gilbert, L., Cornford, S. L., Payne, A., Hogg, A., Muir, A., and Shepherd, A. (2017). “Uneven onset and pace of ice-dynamical imbalance in the Amundsen Sea Embayment, West Antarctica”. In: *Geophysical Research Letters* 44.2, pp. 910–918. DOI: 10.1002/2016GL070733.
- Lai, C.-Y., Kingslake, J., Wearing, M. G., Chen, P.-H. C., Gentine, P., Li, H., Spergel, J. J., and Wessem, J. M. van (2020). “Vulnerability of Antarctica’s ice shelves to meltwater-driven fracture”. In: *Nature* 584.7822, pp. 574–578.
- Lemaitre, J. (2012). *A course on damage mechanics*. Springer Science & Business Media.
- Lhermitte, S., Sun, S., Shuman, C., Wouters, B., Pattyn, F., Wuite, J., Berthier, E., and Nagler, T. (2020). “Damage accelerates ice shelf instability and mass loss in Amundsen Sea Embayment”. In: *Proceedings of the National Academy of Sciences* 117.40, pp. 24735–24741. DOI: 10.1073/pnas.1912890117.
- Libert, L., Wuite, J., and Nagler, T. (2022). “Automatic delineation of cracks with Sentinel-1 interferometry for monitoring ice shelf damage and calving”. In: *The Cryosphere* 16.4, pp. 1523–1542. DOI: 10.5194/tc-16-1523-2022.
- Luckman, A., Jansen, D., Kulessa, B., King, E. C., Sammonds, P., and Benn, D. I. (2012). “Basal crevasses in Larsen C Ice Shelf and implications for their global abundance”. In: *The Cryosphere* 6.1, pp. 113–123. DOI: 10.5194/tc-6-113-2012.
- MacAyeal, D. R. (1989). “Large-scale ice flow over a viscous basal sediment: Theory and application to ice stream B, Antarctica”. In: *Journal of Geophysical Research: Solid Earth* 94.B4, pp. 4071–4087. DOI: 10.1029/JB094iB04p04071.
- MacGregor, J. A., Catania, G. A., Markowski, M. S., and Andrews, A. G. (2012). “Widespread rifting and retreat of ice-shelf margins in the eastern Amundsen Sea Embayment between 1972 and 2011”. In: *Journal of Glaciology* 58.209, pp. 458–466. DOI: 10.3189/2012JoG11J262.

- Marsh, O., Price, D., Courville, Z., and Floricioiu, D. (2021). “Crevasse and rift detection in Antarctica from TerraSAR-X satellite imagery”. In: *Cold Regions Science and Technology* 187, p. 103284. ISSN: 0165-232X. DOI: 10.1016/j.coldregions.2021.103284.
- McGrath, D., Steffen, K., Scambos, T., Rajaram, H., Casassa, G., and Rodriguez Lagos, J. L. (2012). “Basal crevasses and associated surface crevassing on the Larsen C ice shelf, Antarctica, and their role in ice-shelf instability”. In: *Annals of Glaciology* 53.60, pp. 10–18. DOI: 10.3189/2012AoG60A005.
- Miles, B. W. J., Stokes, C. R., Jenkins, A., Jordan, J. R., Jamieson, S. S. R., and Gudmundsson, G. H. (2020). “Intermittent structural weakening and acceleration of the Thwaites Glacier Tongue between 2000 and 2018”. In: *Journal of Glaciology* 66.257, pp. 485–495. DOI: 10.1017/jog.2020.20.
- Moctezuma-Flores, M. and Parmiggiani, F. (2016). “SAR observations of the Nansen Ice Shelf fracture”. In: *The European Physical Journal Plus* 131.11, pp. 1–5. DOI: 10.1140/epjp/i2016-16384-y.
- Morlighem, M. (2020). *MEaSURES BedMachine Antarctica, Version 2 (bed, thickness)*. Accessed on 05/10/2021. DOI: <https://doi.org/10.5067/E1QL9HFQ7A8M>.
- Morlighem, M., Rignot, E., Binder, T., Blankenship, D., Drews, R., Eagles, G., Eisen, O., Ferraccioli, F., Forsberg, R., Fretwell, P., et al. (2020). “Deep glacial troughs and stabilizing ridges unveiled beneath the margins of the Antarctic ice sheet”. In: *Nature Geoscience* 13.2, pp. 132–137.
- Mouginot, J., Rignot, E., and Scheuchl, B. (2014). “Sustained increase in ice discharge from the Amundsen Sea Embayment, West Antarctica, from 1973 to 2013”. In: *Geophysical Research Letters* 41.5, pp. 1576–1584.
- Mouginot, J., Scheuchl, B., and Rignot, E. (2012). “Mapping of Ice Motion in Antarctica Using Synthetic-Aperture Radar Data”. In: *Remote Sensing* 4.9, pp. 2753–2767. ISSN: 2072-4292. DOI: 10.3390/rs4092753.
- Pfeffer, W. T. and Bretherton, C. S. (1987). “The effect of crevasses on the solar heating of a glacier surface”. In: *IAHS Publ* 170, pp. 191–205.
- Porter, C. et al. (2018). *ArcticDEM, Version 3*. Version V1. DOI: 10.7910/DVN/OHHUKH.
- Pralong, A. and Funk, M. (2005). “Dynamic damage model of crevasse opening and application to glacier calving”. In: *Journal of Geophysical Research: Solid Earth* 110.B1. DOI: 10.1029/2004JB003104.
- Purdie, H., Zavar-Reza, P., Katurji, M., Schumacher, B., Kerr, T., and Bealing, P. (2022). “Variability in the vertical temperature profile within crevasses at an alpine glacier”. In: *Journal of Glaciology*, pp. 1–15. DOI: 10.1017/jog.2022.73.
- Rignot, E., Mouginot, J., and Scheuchl, B. (2016). *MEaSURES Antarctic Grounding Line from Differential Satellite Radar Interferometry, Version 2*. Boulder, Colorado USA. NASA National Snow and Ice Data Center Distributed Active Archive Center. doi: <https://doi.org/10.5067/IKBWW4RYHF1Q>. Accessed 15/01/2021.
- (2017). *MEaSURES InSAR-Based Antarctica Ice Velocity Map, Version 2*. DOI: 10.5067/D7GK8F5J8M8R.

- Ronneberger, O., Fischer, P., and Brox, T. (2015). “U-net: Convolutional networks for biomedical image segmentation”. In: *International Conference on Medical image computing and computer-assisted intervention*. Springer, pp. 234–241.
- Rott, H., Skvarca, P., and Nagler, T. (1996). “Rapid Collapse of Northern Larsen Ice Shelf, Antarctica”. In: *Science* 271.5250, pp. 788–792. DOI: 10.1126/science.271.5250.788.
- Scambos, T., Fricker, H. A., Liu, C.-C., Bohlander, J., Fastook, J., Sargent, A., Massom, R., and Wu, A.-M. (2009). “Ice shelf disintegration by plate bending and hydro-fracture: Satellite observations and model results of the 2008 Wilkins ice shelf break-ups”. In: *Earth and Planetary Science Letters* 280.1, pp. 51–60. ISSN: 0012-821X. DOI: 10.1016/j.epsl.2008.12.027.
- Shepherd, A., Gilbert, L., Muir, A. S., Konrad, H., McMillan, M., Slater, T., Briggs, K. H., Sundal, A. V., Hogg, A. E., and Engdahl, M. E. (2019). “Trends in Antarctic Ice Sheet Elevation and Mass”. In: *Geophysical Research Letters* 46.14, pp. 8174–8183. DOI: 10.1029/2019GL082182.
- Shepherd, A., Wingham, D., and Rignot, E. (2004). “Warm ocean is eroding West Antarctic Ice Sheet”. In: *Geophysical Research Letters* 31.23. DOI: 10.1029/2004GL021106.
- Surawy-Stepney, T., Hogg, A. E., Cornford, S. L., and Davison, B. J. (Jan. 2023). “Episodic dynamic change linked to damage on the thwaites glacier ice tongue”. In: *Nature Geoscience*. ISSN: 1752-0908. DOI: 10.1038/s41561-022-01097-9.
- Thompson, S., Cook, S., Kulesa, B., Winberry, J., Fraser, A., and Galton-Fenzi, B. (2020). “Comparing satellite and helicopter-based methods for observing crevasses, application in East Antarctica”. In: *Cold Regions Science and Technology* 178, p. 103128. ISSN: 0165-232X. DOI: 10.1016/j.coldregions.2020.103128.
- Vaughan, D. G., Corr, H. F. J., Bindshadler, R. A., Dutrieux, P., Gudmundsson, G. H., Jenkins, A., Newman, T., Vornberger, P., and Wingham, D. J. (2012). “Subglacial melt channels and fracture in the floating part of Pine Island Glacier, Antarctica”. In: *Journal of Geophysical Research: Earth Surface* 117.F3. DOI: 10.1029/2012JF002360.
- Zhao, J., Liang, S., Li, X., Duan, Y., and Liang, L. (2022). “Detection of Surface Crevasses over Antarctic Ice Shelves Using SAR Imagery and Deep Learning Method”. In: *Remote Sensing* 14.3. ISSN: 2072-4292. DOI: 10.3390/rs14030487.

Chapter 4

The effect of landfast sea ice buttressing on ice dynamic speedup in the Larsen-B Embayment, Antarctica

This chapter was published in 2024 in *The Cryosphere* as the following article :

T. Surawy-Stepney, A. E. Hogg, S. L. Cornford, B. J. Wallis, B. J. Davison, H. L. Selley, R. A. W. Slater, E. K. Lie, L. Jakob, A. Ridout, N. Gourmelen, B. I. D. Freer, S. F. Wilson and A. Shepherd : “ *The effect of landfast sea ice buttressing on ice dynamic speedup in the Larsen-B Embayment, Antarctica* ”

Abstract

We observe the evacuation of 11-year-old landfast sea ice in the Larsen B embayment on the East Antarctic Peninsula in January 2022, which was in part triggered by warm atmospheric conditions and strong offshore winds. This evacuation of sea ice was closely followed by major changes in the calving behaviour and dynamics of a subset of the ocean-terminating glaciers in the region. We show using satellite measurements that, following a decade of gradual slow-down, Hektoria, Green, and Crane glaciers sped up by approximately 20%–50% between February and the end of 2022, each increasing in speed by more than 100 m a^{-1} . Circumstantially, this is attributable to their transition into tidewater glaciers following the loss of their ice shelves after the landfast sea ice evacuation. However, a question remains as to whether the landfast sea ice could have influenced the dynamics of these glaciers, or the stability of their ice shelves, through

a buttressing effect akin to that of confined ice shelves on grounded ice streams. We show, with a series of diagnostic modelling experiments, that direct landfast sea ice buttressing had a negligible impact on the dynamics of the grounded ice streams. Furthermore, we suggest that the loss of landfast sea ice buttressing could have impacted the dynamics of the rheologically weak ice shelves, in turn diminishing their stability over time; however, the accompanying shifts in the distributions of resistive stress within the ice shelves would have been minor. This indicates that this loss of buttressing by landfast sea ice is likely to have been a secondary process in the ice shelf disaggregation compared to, for example, increased ocean swell or the drivers of the initial landfast sea ice disintegration.

4.1 Introduction

The Antarctic Ice Sheet lost 2,671 Gt of ice mass between 1992 and 2020 contributing 7.4 mm towards global sea level rise (Otosaka et al., 2023), with almost all of this loss attributed to ocean-driven ice dynamic processes (Slater et al., 2021). The Antarctic Peninsula (AP) is one of the most rapidly changing parts of the Antarctic Ice Sheet due to its exposure to increasing air temperatures relative to a warm baseline (Trusel et al., 2015; Banwell et al., 2021), increased ocean forcing (Smith et al., 2020), decrease in ice shelf area (Cook and Vaughan, 2010), and changes in wind and sea ice conditions (Christie et al., 2022; Fraser et al., 2023).

The Larsen-B Embayment, located on the East AP - bordered by Seal Nunataks in the north and Jason Peninsula in the south - contains 12 major glaciers that flow into the Weddell Sea. Over the last 30 years, satellites have observed major changes in this region. Following a period of relative stability in the 1990's, the collapse of the Larsen-B Ice Shelf in 2002 (Scambos et al., 2004) caused an immediate eight-fold increase in speed (1.0 to 2.8 km a^{-1}) on the Jorum, Crane and Hektor, Green, Evans Glaciers which were previously buttressed by the ice shelf (Rignot et al., 2004). Though continued buttressing by the Larsen-B remnant in Scar Inlet prevented the neighbouring Flask and Leppard Glaciers from speeding up in 2002/3, their ice discharge was 42% higher by 2013 relative to 1995 (Wuite et al., 2015).

Following the ice shelf collapse in 2002, first-year sea ice formed annually during the austral winter in the Larsen-B Embayment, however, it remained ice free each year during the summer months. In 2011, the winter sea ice became landfast, and persisted continuously for 11-years (Christie et al., 2022) before it suddenly disintegrated in January 2022 and was evacuated from

the embayment (Ochwat et al., 2023). During the weeks and months following the landfast sea ice evacuation, the floating ice shelves that had developed on the Hektoria/Green/Evans (HGE) glacier system and Crane Glacier (Rott et al., 2018) disintegrated, and their tributary glaciers increased in flow speed (Ochwat et al., 2023). There are clear visual parallels between the clearance of landfast sea ice in 2022 and the disintegration of the Larsen-B Ice Shelf 20 years earlier, and the subsequent dynamic responses of these glaciers. However, uncertainty remains about the mechanisms driving the increased ice speeds on HGE and Crane in the latter case, particularly regarding the “buttressing” role that the landfast sea ice was able to provide the glaciers prior to collapse (Sun et al., 2023; Ochwat et al., 2023).

Here, we present observations of the spatial pattern of ice speed change before and after the landfast sea ice disintegration in January 2022, along with measurements of glacier calving front location, landfast sea ice extent and thickness amongst others. These results complement recent work by Sun et al., 2023 and Ochwat et al., 2023, which document many similar observations to those presented here, though we focus on the glaciers that showed the greatest dynamic response after 2022 - namely Crane and Hektoria/Green/Evans. These observations form the background and motivation for an investigation into the buttressing capacity of the landfast sea ice in which we perform simple diagnostic modelling experiments aiming to quantify the redistribution of stress as landfast sea ice of varying thickness is added to the Larsen-B Embayment.

4.2 Observations

4.2.1 Observational methods

We collected satellite-derived datasets over the period 2002-2023 to assess changes in landfast sea ice extent, ice shelf extent and ice flow speed across the Larsen-B Embayment and the glaciers that terminate there. We used multi-spectral optical Landsat-8 imagery to visually identify and delineate the landfast sea ice edge in the Larsen-B Embayment in November each year from 2002 to 2022. We used this Landsat-8 data, and additional Sentinel-1 Synthetic Aperture Radar (SAR) backscatter images, to manually delineate the calving front location of each glacier feeding the Larsen-B Embayment (Fig. 4.1 f, supplementary figure C.3). For much of the period prior to the first calving events of 2022 the transition between consolidated ice shelf to landfast sea ice appeared smooth in satellite images, encompassing a region of ice mélange, making the calving fronts difficult to define precisely. However, this does not impact

our understanding of the timings of the calving events in 2022. We applied feature tracking techniques to single-look complex Sentinel-1 SAR data collected in Interferometric Wide-swath (IW) mode, to produce a 8-year record of ice speed over the 12 outlet glaciers in the study region from January 2015 to April 2023 (Davison et al., 2023a). We extracted timeseries of ice speed averaged over 1 km-long flowline segments on 12 glaciers flowing into the Larsen-B Embayment, and used a Kalman smoother with an identity transition matrix to filter the results (4.1 a-c) (Wallis et al., 2023). These locations were chosen to be in the centre of the grounded ice streams, close to the grounding line but with enough room for a 1 km buffer along the flowlines that pass through them. Uncertainty indicators in these speeds were calculated by scaling the reciprocal signal-to-noise ratio in the cross-correlation field with the ice speed (Lemos et al., 2018).

4.2.2 Landfast sea ice area change

Our data show that the landfast sea ice went through phases of growth and decay during the period 2002-2011, with sea ice covering the entire embayment at times, and only the smaller pro-glacial embayments at others (supplementary figure C.1). After its formation in winter 2011, the landfast sea ice was retained throughout the summer months until 2022. Its seaward margin retreated and advanced consistently throughout the period 2011-2022, though these oscillations reached a higher amplitude from 2017 onwards (Fig. 4.1 a, supplementary figure C.1). Between the 18th and 23rd of January 2022, the multi-year landfast sea ice disintegrated and was evacuated out of the Larsen-B Embayment (Ochwat et al., 2023), leaving open ocean.

4.2.3 Ice dynamic and calving response

Our velocity measurements show that between October 2014 and January 2022 Hektoria, Green and Crane Glaciers slowed by approximately 100 m a^{-1} (Fig. 4.1 b, d), with smaller decreases in speed on Evans (Fig. 4.1 c, d), Punchbowl, Jorum and Melville Glaciers (4.1 b, supplementary figure C.2). Over the same period Flask Glacier sped-up slightly by 5% (30 m a^{-1}), and the remaining 4 glaciers exhibited fairly stable speeds on annual timescales (4.1 c, supplementary figure C.2).

During the period of steady or declining speeds between 2014 and 2021 we observe progressive advance of the calving front on the majority of glaciers in the study region. Hektoria and Crane Glaciers advanced the furthest, with approximately 12 km and 7 km of growth observed between

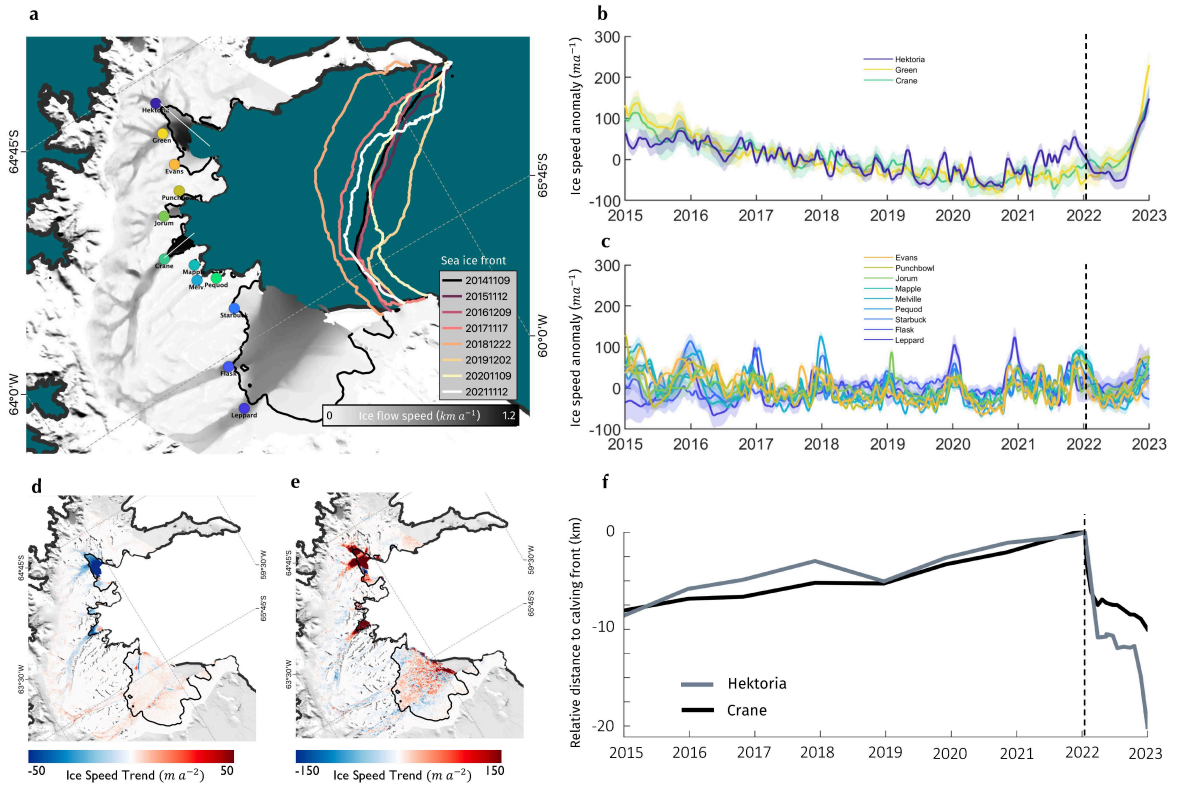


Figure 4.1: Ice Speed and speed change map of the Larsen-B Glaciers. a) Inverse-error-weighted mean ice speed of glaciers flowing into the Larsen-B Embayment on the East Antarctic Peninsula, measured between October 2014 and April 2023 (greyscale map). Grounding line location (that of Wallis et al., 2024 in HGE and Crane, and the MEaSURES InSAR-derived grounding line elsewhere (Mouginot et al., 2017)) is shown with the solid black line. Coloured lines show the landfast sea ice fronts, measured annually between November 2014 and November 2021. b) Ice speed anomaly (signal minus the timeseries mean) from January 2015 to January 2023 on Hektoria, Green and Crane Glaciers. c) Ice speed anomaly on the other glaciers shown in (a). For the plots in (b-c), data was extracted over 1 km long segments of flowlines with centrepoinets shown by the circles in (a). The uncertainties shown are 1σ either side of the mean. d) A map of the observed rate of change in ice speed between October 2015 and October 2021. e) A map of the observed rate of change in ice speed between October 2021 and April 2023, spanning the disintegration event in February 2022. f) Timeseries of calving front distance on Hektoria Glacier (grey line) and Crane Glacier (black line) from January 2015 to January 2023. These distances are measured along the white lines shown in (a). Datapoints are annual between 2015 and 2022, and monthly thereafter. Black dashed lines in panels (b, c, f) show the 18th January 2022, the start of the break up of the landfast sea ice in the Larsen-B Embayment. The basemap in (a, d, e) is the MODIS Mosaic of Antarctica (Haran et al., 2021).

2015 and 2021 respectively, though this advance was not monotonic (Fig. 4.1 f, supplementary figure C.3 b). The Larsen-B remnant in Scar Inlet is the only calving front to have continually advanced between 2015 and 2021, growing by 6 km. The remaining glaciers experienced changes in calving front position of 3 km or less between 2014 and 2022.

Following the disintegration of the landfast sea ice, we observe a large speed-up on Green and Crane Glaciers beginning in early-to-mid 2022 and accelerating from June/July 2022 (Fig. 4.1 b, e), followed by a speedup on Hektoria Glacier beginning in July 2022 (Fig. 4.1 b, e), though this glacier exhibits a more varied signal. At the grounded ice locations chosen for extraction of speed timeseries, we see changes in speed between January and December 2022 of $35.5 \pm 10.4\%$ on Hektoria Glacier, $46.9 \pm 7.0\%$ on Green Glacier and $17.8 \pm 5.5\%$ on Crane Glacier. We also see a potential sign change in the ice speed trend on Evans and Jorum (Fig. 4.1 c, e) Glaciers in early 2022 from negative to positive, though the changes in speed are comparable to historical variability in the ice speed data. On Hektoria, Green and Crane Glaciers, where the speed-up is most pronounced, speed changes extend up to 10 km upstream of the 2021 grounding line. Our velocity measurements show that there was no pronounced change in speed of Leppard, Flask, Starbuck, Pequod, Melville, Mapple or Punchbowl Glaciers discernible from the variability in ice speed over the preceding decade (Fig. 4.1 c, e, supplementary figure C.2).

The large dynamic changes on Crane and Green Glaciers were preceded immediately by a period of terminus retreat in February 2022 of 6 km and 12 km on the Crane and HGE Ice Shelves respectively (Fig. 4.1 f). Crane Ice Shelf continued to calve during the period of acceleration and, by the end of our study period, had retreated 12 km relative to its maximum position in December 2022. The HGE Ice Shelf retreated by a further 9 km between September and December 2022, decoupling the ice shelves of Hektoria and Green Glaciers (Fig. 4.1 f, supplementary figure C.3 b).

4.3 Landfast sea ice buttressing

Previous studies focused on the area have shown ice speed changes on HGE and Crane Glaciers to be concurrent with changes in terminus position (Wuite et al., 2015; Rott et al., 2018) prior to 2011. In the case of HGE these changes fluctuated, while on Crane steady terminus advance accompanied steadily decreasing glacier speeds. Following the growth of persistent landfast sea ice in 2011, we see persistent terminus advance and decreasing speed - show here and elsewhere (Wuite et al., 2015; Rott et al., 2018; Ochwat et al., 2023). These observations, along with that of ice shelf disintegration after the sea ice evacuation in 2022, shown here and in Ochwat et al., 2023, suggests a coupling of landfast sea ice to glacier dynamics in which landfast sea ice

permitted the growth of the ice shelves in front of HGE and Crane Glaciers prior to 2022 which acted as a control on the upstream flow. However, it is unclear the extent to which the landfast sea ice could have itself acted to buttress the upstream glaciers, and whether the growth of the ice shelves is attributable to the *buttressing* effect of landfast sea ice as opposed to other mechanisms by which landfast sea ice can confer stability to regions of ice mélange.

Recent observational reports of the January 2022 evacuation of landfast sea ice from the Larsen-B Embayment provide conflicting accounts of the possible buttressing effect the landfast sea ice could have had. Ochwat et al., 2023 suggest that the growth of the ice shelves during the residency of the landfast sea ice, the potential dampening of ice speed in Scar Inlet and the immediate speed up of certain ice shelves following the collapse of the landfast sea ice is evidence of its buttressing effect. However, Sun et al., 2023 suggest that the limited immediate response of the glaciers to the landfast sea ice evacuation, and the potential plastic rheological response of the landfast sea ice to sudden changes in upstream stress are reason to believe any buttressing was minimal.

In the context of ice shelves, “buttressing” refers to the hypothetical difference in englacial stress with and without the ice shelf (Gudmundsson, 2013; Fürst et al., 2016). To be consistent, we take “buttressing” to have the same meaning in the context of landfast sea ice. Consequently, there are two ways that landfast sea ice buttressing could have contributed to the observed speed changes on the HGE and Crane Glaciers: 1) directly influencing the stress distribution in the glaciers such that the disintegration of the landfast sea ice caused an instantaneous speed change on the grounded ice, and 2) reducing stresses in the ice shelves which would have otherwise been too great for the ice shelves to withstand. This latter mechanism is a second order effect of buttressing on grounded ice speed, the implication being that the disintegration of the landfast sea ice in turn caused the disintegration of the ice shelves via loss of buttressing, and hence the loss of the ice shelves as a control on the upstream dynamics. This is to be contrasted with other non-buttressing mechanisms by which the landfast sea ice could influence the stability of the ice shelves, such as by bonding fragments of mélange together, preventing small calving events at the glacier terminus, stopping the export of icebergs, and dampening swell-induced loading cycles.

Here, we use the BISICLES ice sheet model (Cornford et al., 2013) to directly investigate these possible effects for the glaciers that exhibited the most pronounced changes in dynamic behaviour after January 2022 – namely the HGE system of glaciers and Crane Glacier.

4.3.1 Modelling methods

BISICLES is a finite-volume, adaptive mesh model that solves a discretised form of the shallow-stream approximation to the momentum balance equations:

$$\nabla \cdot [\phi h \bar{\mu} (\nabla \mathbf{u} + (\nabla \mathbf{u})^\top + 2(\nabla \cdot \mathbf{u}) \mathcal{I})] - C f(u) \mathbf{u} - \rho_i g h \nabla s = 0, \quad (4.1)$$

where $\mathbf{u}$ is the ice velocity, h is the ice thickness, s is the ice surface elevation, $\mathcal{I}$ is the identity operator, $f(u)$ is a function parametrising our sliding law, C is a scalar ‘basal slipperiness’ field, ρ_i is the density of ice, g is the acceleration due to gravity, $\bar{\mu}$ is the vertically-averaged effective ice viscosity and the scalar field ϕ is an ‘stiffness’ that scales $\bar{\mu}$. We use Glen’s flow law with an exponent of 3, a rate factor according to Cuffey and Paterson, 2010 and an internal energy field generated from a continent-wide thermomechanical spin-up in the calculation of the effective ice viscosity. Simulations were carried out at a maximum of 125 m or 250 m resolution.

Initially, we set up a model domain with geometry approximately reflecting the HGE and Crane basins of the Larsen-B Embayment, using a combination of smoothed bedrock elevations according to Huss and Farinotti, 2014, grounding line positions timestamped for the year 2019/20 (Wallis et al., 2024) and surface elevations from the Reference Elevation Model of Antarctica (REMA) digital elevation model (DEM) (Howat et al., 2019) which is timestamped to May 2015. Contemporary grounding line locations are required as there has been significant grounding line retreat on these glaciers since the MEASURES InSAR grounding line datasets were produced (Wallis et al., 2024). The REMA DEM does not reach the edge of the HGE and Crane Ice Shelves as they were in 2021, so we filled this gap by extrapolating the DEM along flowlines. We performed an inversion for basal slipperiness (C) and stiffness (ϕ) fields using observations of ice speed across the HGE and Crane basins averaged over 2021 (Cornford et al., 2015). At this point we did not include the landfast sea ice in the model geometry, so the glaciers terminate in open sea. The choice to do this presupposes that the inclusion of the landfast sea ice will do little to change the solution to the inverse problem and is necessary as

the thickness of the landfast sea ice is not well constrained. We shall see that this assumption is validated by the results. We used regularisation with a Tikhonov operator that approximates the gradients of the control fields to improve the conditioning of the problem. L-curve analysis (Hansen, 1994) was used to select an appropriate level of regularisation (Fig. 4.7). The gaps in time between the surface elevation, grounding lines and ice speed data used for the inversions meant that a certain amount of ‘relaxation’ of the geometry was required. We ran 5 inverse problems separated by a year of thickness evolution. Supplementary figure C.6 shows geometry and control fields at the end of the model initialisation. We note that, as the thickness and bedrock data in this region of Antarctica is very poorly constrained, the geometry we construct should be considered plausible as opposed to fully accurate. As such, our conclusions are subject to change under replication using different glacier geometries. Given the potential influence of unknown deviations in the real geometry from the data available to us, we avoid performing transient simulations which risk amplifying the impact of these uncertainties which, for the basic mechanical arguments made here, are not likely to be important.

To simulate the effect of the landfast sea ice in the embayment, we assume that it can be treated as a thin ice shelf with the same constitutive ice rheology as the upstream glaciers. Considering a range of length- and time-scales, sea ice is typically treated as a visco-plastic (e.g. Hibler, 1979) or elasto-plastic material (e.g. Hunke and Dukowicz, 1997). This captures a material that is strong in compression and weak in extension and shear, dominated by plastic deformation in thin-ice-covered leads or pressure ridges. Regarding the landfast sea ice that inhabited the Larsen-B Embayment prior to 2022, satellite images show a fairly uniform, unbroken ice coverage with a smooth deformation field (supplementary figure C.4). This suggests incompressible flow with stress continuity between the landfast sea ice edge and the glacier calving fronts and largely smooth deformation over its decade of residency, though indications of larger-lengthscale plastic response to sudden loading have been observed (Sun et al., 2023). Any sub-critical viscous deformation internal to the sea ice is controlled by a rheology which depends on the relative abundances of meteoric and congelation ice, with differences in crystal structure and the presence of brine-inclusion leading to a lower effective viscosity in the latter case. Similarly, surface melt-induced porosity in the meteoric ice would lower its effective viscosity relative to glacier ice. The use of the same formula for the effective viscosity of land and sea ice, along with the assumption of viscous deformation, means that our treatment is to likely

provide an upper bound to the buttressing strength that unbroken landfast sea ice could exert on the upstream glaciers. We provide additional evidence in support of this claim in Sec. 4.5.1.

After setting up the model domain in the way described above, we add such modelled landfast sea ice between the glacier calving fronts and the observed seaward limit of the multi-year landfast ice (Fig. 4.2 a). We then recalculate ice speed over the domain by solving the stress-balance equations in this new configuration. We compare these along flowline transects with the speed in the absence of the landfast sea ice (Fig. 4.2 b-d) and also with flowline transects of quarterly-averaged speed observations from before and after the landfast sea-ice evacuation. These speed observations are from the second quarter of 2021 (the last before the sea ice evacuation - excluding the months October-December 2021 where high surface melt rendered the data unreliable) and the last quarter of 2022, and are smoothed with a 5 km window. The transects of observations are cut off at the most land-ward calving front position observed during the quarter, or at the edge of our speed observations (which coincide with the edge of the REMA DEM). This gives us the results of Sec. 4.3.2.

For the simulations, we use a regularised-Coulomb sliding law (Schoof, 2005; Joughin et al., 2019) with a threshold ice speed of $u_o = 300 \text{ m a}^{-1}$ so that sliding is plastic on much of the grounded ice. This ensures that basal stresses remain relatively unchanged as landfast sea ice is introduced resulting in enhanced changes to the resistive stress and greater ice speed change. Our choice of $u_o = 300 \text{ m a}^{-1}$ is based on the speeds of tributary glaciers flowing into HGE and Crane Glaciers, though a choice of an even lower threshold makes little difference to the main results (Sec. 4.A.3). This was carried out for landfast sea ice thicknesses of 1 m, 2 m, 5 m and from 10 m to 50 m in increments of 10 m – a range that extends beyond what one might expect to be realistic landfast sea ice thicknesses in the study region. CryoSat-2 radar altimetry observations show the landfast sea ice had a mean freeboard under 1 m over the period 2013 to 2020 (supplementary figure C.5), implying an approximate thickness under 10 m. We chose to model the landfast sea ice as uniform in thickness, despite these data suggesting otherwise, as the data do not extend to the critical zone at the glacier calving front. This should do little to change the conclusions of the modelling results. The sea ice was assumed to be of vertically and horizontally uniform temperature of $-5 \text{ }^{\circ}\text{C}$. This is likely to lead to stiffer ice with greater buttressing strength than in reality as observations of surface melt (Ochwat et al., 2023) indicate

that the sea ice might be better modelled as temperate.

Additionally, we assess the sensitivity of grounded ice speed to changes in the thickness of the landfast sea ice compared to the thickness of the glaciers themselves (Sec. 4.3.2). This gives us some intuition as to where any changes in geometry that occurred in 2022 might have led to changes in glacier speed. We treat the stiffness ϕ as a proxy for ice thickness (h) as, to first order, perturbations in these quantities have the same effect on vertically-integrated effective viscosity. We consider two 1 km-radius circular regions on the grounded ice of Hektor and Crane Glaciers (Ω_H and Ω_C respectively) and define the function $\mathcal{J}$ as the mean ice speed over these regions. We then calculate the magnitude of the gradient of $\mathcal{J}$ with respect to ϕ using standard adjoint-based methods (Sec. 4.A.2). To ensure that we are not aliasing an atypical part of the solution space, we find these sensitivities for 6 realisations of the control variables C and ϕ corresponding to solutions of the inverse problem with different amounts of regularisation.

Finally, to examine the component of ice shelf stability due to the buttressing of landfast sea ice (Sec. 4.3.2) we further study the case in which 10 m of landfast sea ice is added to the embayment. We look at how strain rates and stresses near the glacier termini change. Principal strain rates ε_1 and stresses σ_1 (the largest eigenvalues of the strain rate and resistive stress tensors local to each parcel of ice) were calculated and compared for the HGE and Crane ice shelves for 10 m of landfast sea ice vs 0 m.

4.3.2 Modelling results

Direct buttressing of grounded glaciers

Transects located approximately along flowlines of four glaciers that accelerated in 2022 show that speeds along the glaciers change smoothly as a function of landfast sea ice thickness in the embayment (Fig. 4.2 b-e). However, these changes in speed are strongly attenuated upstream of the calving front. At the grounded-ice locations used to produce the timeseries in Fig. 4.1 b, we see instantaneous ice speed changes on the order of 0-1% with the addition of a realistic upper limit of 10 m thick landfast sea ice, and a speed change of 0-9% with the highly unlikely thickness of 50 m (Fig. 4.2 b-f). This is considerably below the 15-50% speed change observed on these glaciers (Fig 4.1 b, Fig. 4.2 b-e) after January 2022. These modelled changes in grounded

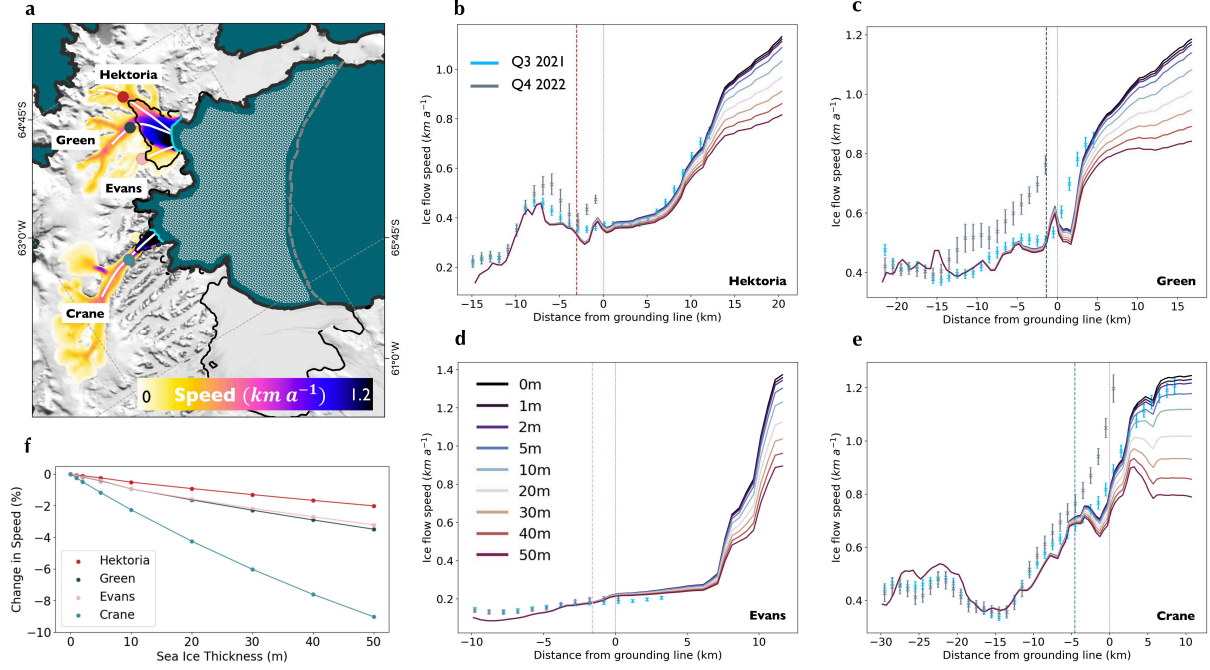


Figure 4.2: Modelled changes in speed with varying landfast sea ice thickness. (a) The Larsen-B Embayment. Flow speeds following the inversions for stiffness and basal slip coefficient fields over the Hektoria and Crane basins. The patterned area bound by the embayment walls and the grey dashed line indicates where landfast sea ice was added during the simulations. Coloured circles on the glaciers show where timeseries of speed were extracted in Fig. 4.1. Flowlines used to extract speeds for different landfast sea ice thicknesses (b-e) are shown in white. The basemap is the MODIS Mosaic of Antarctica (Haran et al., 2021). (b-e) Modelled ice speeds for different landfast sea ice thicknesses, where dark blue indicates 0 m and dark red indicates 50 m, along the flowlines shown in (a): Hektoria (b), Green (c), Evans (d) and Crane (e). Thin, vertical, grey dashed lines show the positions of the grounding lines, and coloured vertical lines show the positions of the corresponding circles in (a). Points plotted in cyan show the ice speed measured along these transects in the third quarter of 2021 - smoothed with a 5 km window. Points plotted in grey show the equivalent for the last quarter of 2022. (f) Percentage change in modelled ice speed for different landfast sea ice thicknesses at the locations with colours shown in (a).

ice speed are to be contrasted with the much larger 2 – 10% changes in speed seen at the calving fronts and on the floating ice shelves in the simulations with landfast sea ice thickness of 10 m (Fig. 4.2 b-e). These modelled percentage changes in speed are similar in magnitude on all glaciers including Evans, where we do not observe a substantial dynamic response (Fig. 4.1 g). This is an indication in its own right that the buttressing effect of landfast sea ice was not its primary control on the dynamics of the glaciers of the Larsen B Embayment.

In the range 0 – 50 m, the addition of ice in the Larsen-B Embayment produces changes in speed that vary approximately linearly with thickness (Fig. 4.2 f). We invert this relationship

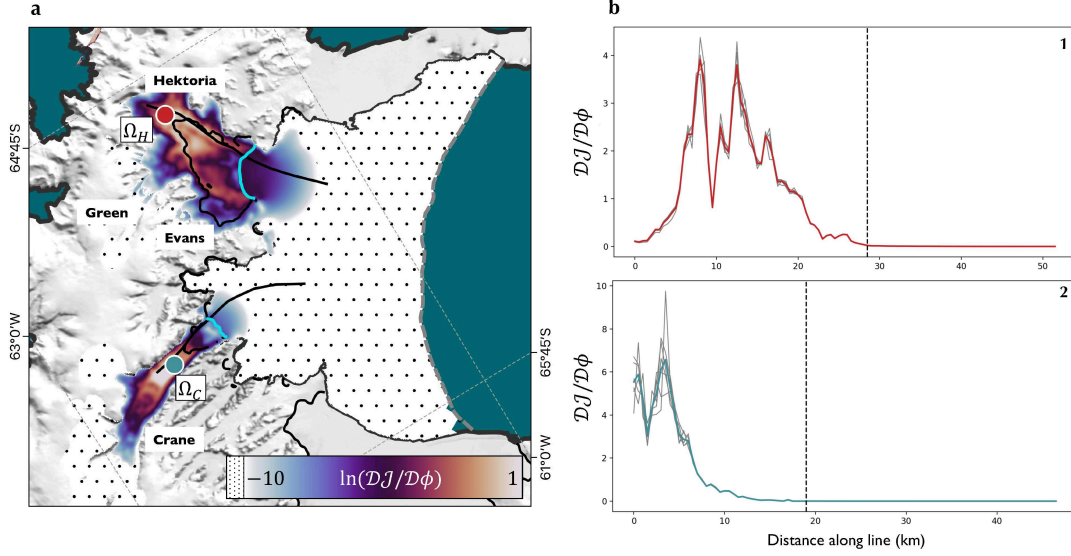


Figure 4.3: Sensitivity of ice speed to changes in effective ice thickness. (a) Magnitudes of differential sensitivities of ice speed in the locations marked by the coloured circles to change in stiffness across the domain. The black spotted area shows where the sensitivity is under e^{-10} . Cyan lines show the boundaries between the glacier termini and the start of the landfast sea ice according to the model geometry. Black lines show where transects of modelled speeds were collected to produce the graphs in (b)-(c). The basemap is the MODIS Mosaic of Antarctica (Haran et al., 2021). (b) Magnitudes of sensitivities along the transects shown in (a). Black dashed lines show the location along the transect of the glacier terminus: Hektoria (b.1) and Crane (b.2). Grey lines indicate the different realisations of the control variables C and ϕ , while coloured lines show the mean sensitivities. Note: data is presented logarithmically in (a) and linearly in (b).

and look, in Fig. 4.3, at the magnitudes of gradients of ice speed at two locations (Fig. 4.3 a), with respect to effective thickness, assuming 10 m of landfast sea ice. Though Fig. 4.2 shows that changes in landfast sea ice thickness result in non-zero changes to ice speed along the whole glacier length, we now see that the sensitivity of grounded ice speed to changes in the effective thickness of landfast sea ice is minute in comparison to its sensitivity to changes in the effective thickness of glacier ice. This can be seen especially clearly in transects taken along shear margins of Hektoria/Crane Glaciers and out into the landfast sea ice (Fig 4.3 b.1, b.2). The map of logarithmic sensitivity (Fig 4.3 a) shows the landfast sea ice in the pro-glacial embayments to have an impact on the grounded ice, though this diminishes quickly further out to sea. These results further suggest that direct landfast sea ice buttressing of the parts of the glaciers that showed speed increase in 2022 is likely to have been negligible.

Buttressing of the floating ice shelves

Despite the fact that speeds in these regions change by a mean of under 4% except very close to the calving fronts when 10 m of landfast sea ice is added to the embayment (Fig. 4.2), there are changes in principal strain rates across parts of the HGE and Crane Ice Shelves on the order of 10% (Fig. 4.4 a-b), with changes largest in the seaward-most parts of the shear margins. Together with Fig. 4.2, this indicates that the buttressing effect of 10 m of landfast sea ice in the embayment is enough to produce some dynamic response on the weak floating ice. The important question for the Larsen-B glacier system that remains is whether this change could have destabilised the ice shelves of HGE and Crane Glaciers.

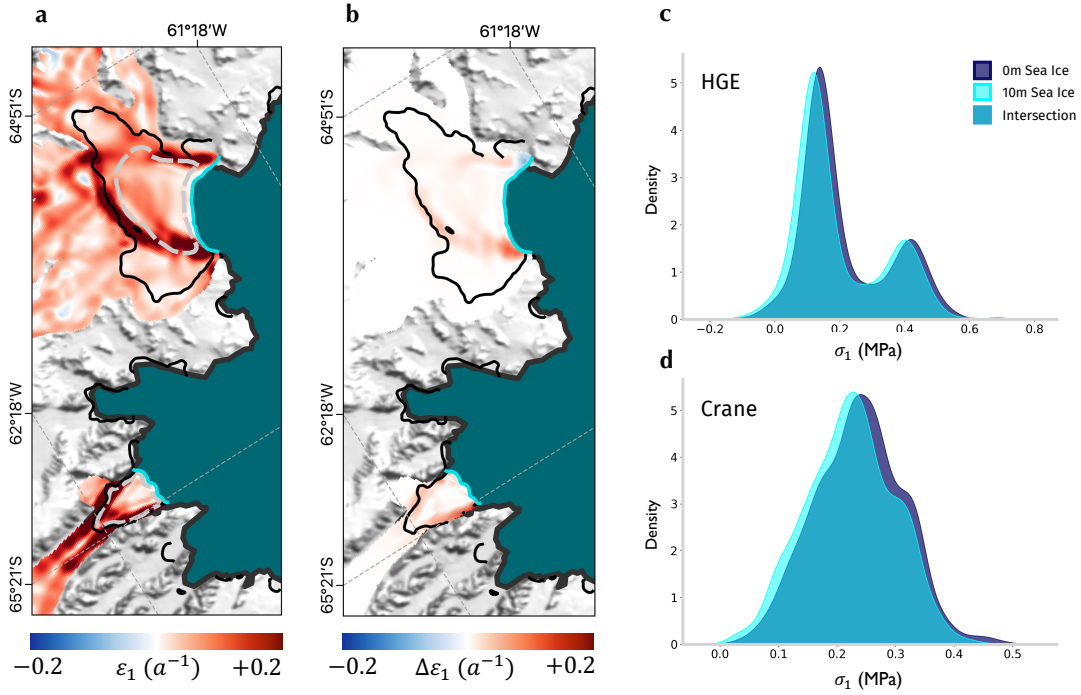


Figure 4.4: Change in the modelled strain rates and stresses in response to the addition of 10 m thick landfast sea ice. (a) Principal strain rate across the HGE and Crane Ice Shelves with 10 m thick landfast sea ice. (b) Difference in principal strain rate across the HGE and Crane Ice Shelves with no sea ice compared to 10 m. The basemap in (a-b) is the MODIS Mosaic of Antarctica (Haran et al., 2021). (c-d) Histograms of principal stress for 10 m thick landfast sea ice (cyan) compared with 0 m (navy) and their overlap over the HGE and Crane Ice Shelves respectively. These graphs are produced for the floating ice regions bounded with the grey dashed lines in (a).

The simulations exhibit a smooth redistribution of stress when the landfast sea ice is added, however this is unrealistic for highly crevassed ice shelves where thicknesses can vary considerably from place to place. Continuity instead ensures that the vertically-integrated stress is

concentrated in thinner areas. It is possible that small changes in the this stress would surpass the load-bearing capacity of these thinner sections of glacier ice, causing the ice shelf to break up. To assess how plausible this is, we compare the distributions of vertically-averaged principal resistive stresses across the ice shelves with and without 10 m thick landfast sea ice (Fig. 4.4 c-d). We see on both HGE and Crane Glaciers that the addition of the landfast sea ice reduces the mean principal stress, however this shift is small compared to the variance of stresses within the ice shelves. Assuming that weaknesses in the ice shelves are not concentrated in a small region, it is improbable, therefore, that the ice shelves were stable prior to the landfast sea ice removal and that this removal caused a large enough perturbation in resistive stress to account for the spatially extensive ice shelf break-up that was observed.

The above analysis relates to instantaneous changes in the stresses and strain rates within the floating ice that result from a loss of landfast sea ice buttressing. We have suggested that the distributions of principal resistive stresses within the floating ice change, but by an amount that is unlikely to have led to the rapid collapse of the ice shelves such as seen on the Crane and HGE Glaciers. However, changes in strain rate (Fig. 4.4 a-b) can have implications for ice shelf stability on longer timescales. For example, increased ice shelf thinning rates, resulting from enhanced velocity gradients, can lessen its robustness to fracturing. Additionally, elevated strain rates can lead to faster changes to the glacier geometry. We cannot rule out the hypothesis that such processes were in part responsible for the elevated calving rate on Crane and HGE Ice Shelves starting in September 2022. However, if this were the case, it seems likely that the calving events themselves would have had a greater impact on subsequent calving rate than the loss of sea ice.

4.4 Environmental drivers

The aim of this article is to address the potential buttressing capacity of landfast sea ice in light of its evident relationship to glacier dynamics shown by recent events in the Larsen-B Embayment. However, we briefly diverge here to note a couple of interesting climatic factors that may have played a part in the 2022 landfast sea ice fragmentation and the subsequent dynamic response of the glaciers. To examine these, we looked at air temperature and wind velocity data from ERA5 reanalysis over the Larsen-B Embayment between 2000 and 2022 and

estimates of basal melt-rate from swath mode CryoSat-2 radar altimetry data acquired between November 2010 and January 2022 (Gourmelen et al., 2017; Davison et al., 2023b).

ERA-5 reanalysis data suggest that annual mean surface air temperatures over the 11 years of landfast sea ice residence had been steadily increasing at a rate of $0.25^{\circ}\text{C a}^{-1}$ over the Vaughan and Exasperation Inlets (Fig. 4.5 b). By 2022, the air temperature had increased to 2°C above the 2000-2022 mean in the Larsen-B Embayment. ERA-5 temperature data shows an even more pronounced localised peak in the air temperature anomaly over the Larsen-B Embayment in the months prior to the landfast sea ice disintegration. These data suggest the possibility that the landfast sea ice in the Larsen-B Embayment was unable to persist through a longer and more intense melt season than it had encountered in previous years, brought about by trends in atmospheric conditions. However, work by Ochwat et al., 2023 using estimates of melt from passive microwave data shows a peak in melt days in the summer of 2019/20. Additionally, ERA-5 wind velocity data suggest there were anomalously strong north-westerlies over the Antarctic Peninsula in 2022 compared to the 2000-2022 mean (Fig 4.5 c). These strong offshore winds could well have contributed to the landfast sea ice disintegration, and would have aided its evacuation from the embayment before it could re-freeze as pack-ice.

Estimates of ice shelf basal melt rates over Scar Inlet, the last remaining remnant of the original Larsen-B Ice Shelf, show that the highest melt rates are located at the grounding line (Fig. 4.5 d). A timeseries of the mean basal melt rate from this region shows that the rates were fairly constant at $0 \pm 4 \text{ m a}^{-1}$ for the majority of the period from 2010 to 2018, after which basal melt rates generally increased, up to a maximum of $6 \pm 4 \text{ m a}^{-1}$ in January 2023 (Fig. 4.5 e). While there is an absence of direct ocean temperature measurements during the period around 2021, it is possible that the strong winds shown by the ERA5 data (Fig. 4.5 a) drove an upwelling in ocean circulation bringing warm water up from depth, in addition to blowing out the disintegrated sea ice. If this proxy is representative of changes in ocean temperature across the Larsen-B Embayment then it indicates that increased grounding line ablation could have had a role to play in the ice dynamic changes seen on HGE and Crane Glaciers. Further work is required to establish whether the deeply grounded glaciers of the Larsen-B Embayment, and perhaps beyond, exhibited a dynamic signal before the landfast sea ice evacuation that could

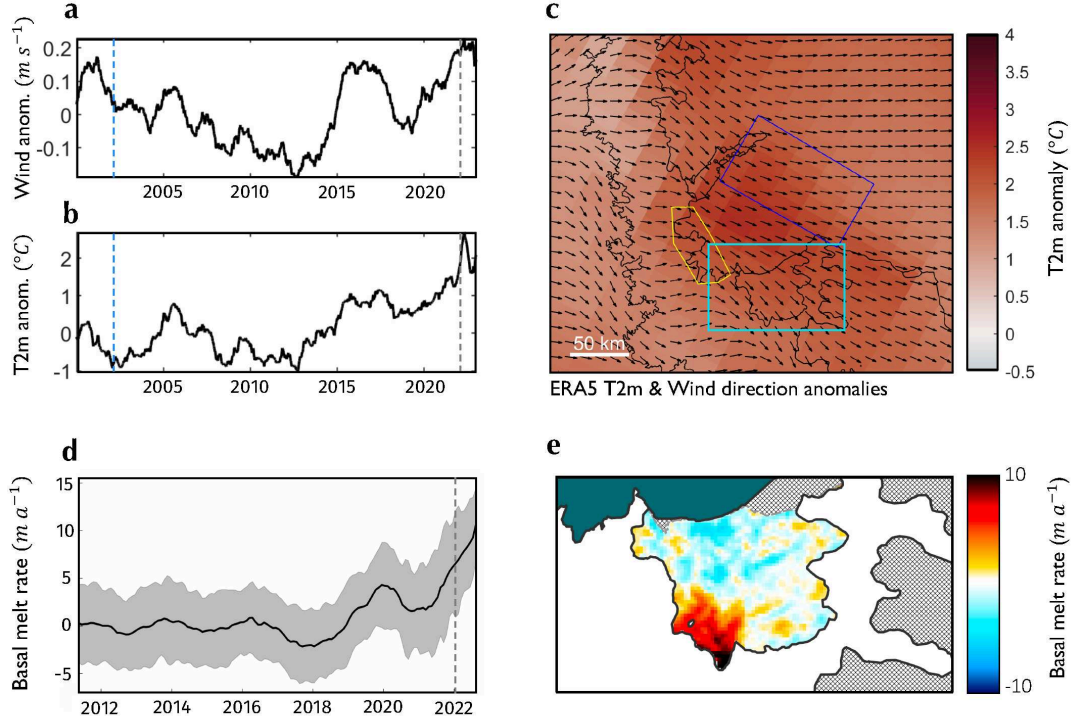


Figure 4.5: Environmental forcing over the Larsen-B Embayment. (a-c) Wind velocity and air temperature data over the period 2000-2022. (a) 2022 wind speed anomaly (signal compared to timeseries mean) extracted from the purple box shown in (c). (b) 2 m air temperature anomaly extracted from the yellow box shown in (c). (c) Normalised wind direction anomaly (vector field) and 2 m air temperature anomaly (colour) over the Larsen-B Embayment in 2022 compared to the 2000-2022 mean. Vertical blue dashed lines indicate March 2002 - when the Larsen-B Ice Shelf disintegrated, and vertical grey dashed lines show January 2022 when the landfast sea ice disintegrated. (d-e) CryoSat-2 swath-mode ice shelf basal melt rate observations from November 2010 to January 2022 over Scar Inlet in the Larsen-B Embayment. (d) Timeseries of mean monthly basal melt rate in Scar Inlet (indicated with a blue box in (c)). The vertical grey dashed line shows January 2022. (e) Mean basal melt rate over the Scar Inlet (shown by the blue box in (c)) between November 2010 and January 2022.

point to the influence of enhanced sub-shelf or grounding line ablation. While the basal melt rates observed on Scar Inlet since 2020 of up to $10 m a^{-1}$ (Fig. 4.5 d) have not yet caused a notable speedup on Flask and Leppard Glaciers, which may be in part explained because they remain buttressed by the laterally constrained ice shelf remnant, these basal melt rates are comparable to those observed on ice streams flowing into the Amundsen Sea Sector of West Antarctica (Shean et al., 2019). If sustained or even increased in the future, these basal melt rates may suggest that a dynamic response on these glaciers could be expected in the longer-term.

4.5 Discussion

Given the abundance of landfast sea ice fringing the fast-flowing margin of the Antarctic Ice Sheet (Fraser et al., 2021), and the potentially large contribution of ice dynamics to future changes to Antarctic mass balance (Joughin and Alley, 2011; Pattyn and Morlighem, 2020), understanding how changes in landfast sea ice extent and thickness alters the dynamic behaviour of glaciers is evidently valuable. The sudden evacuation of the landfast sea ice from the Larsen-B Embayment, and the changes to the dynamics of HGE and Crane Glaciers that followed, has provided us with a natural opportunity investigate these relationships. The concurrency of the evacuation of the landfast sea ice and the observed changes in calving behaviour and dynamics of the upstream glaciers demonstrates the crucial impact of landfast sea ice in the region, however our modelling results suggest that the component of this due to buttressing, as it is understood in the context of ice shelves, is likely to have been minimal.

4.5.1 The use of a viscous flow model

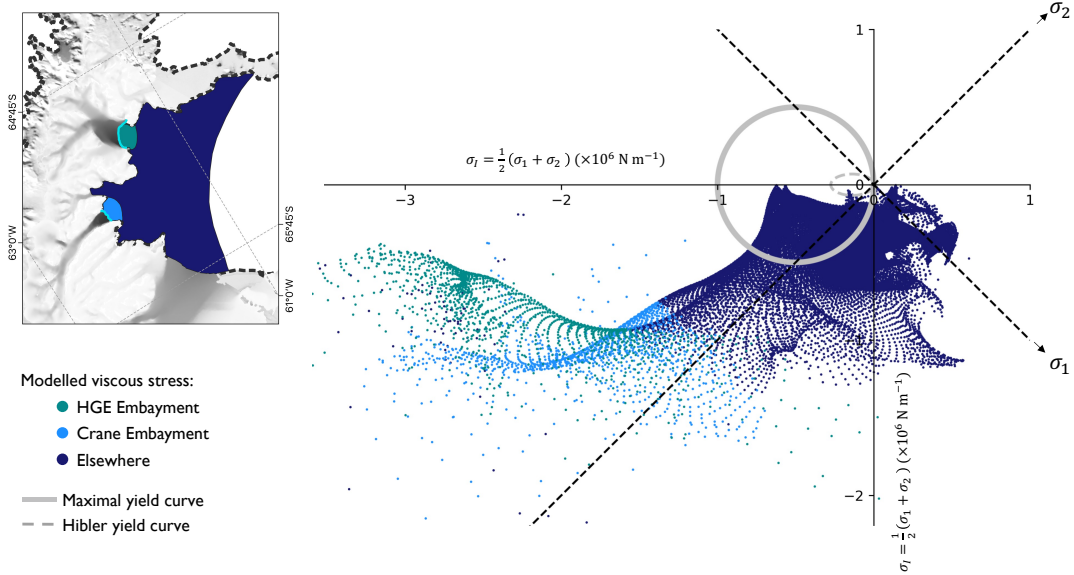


Figure 4.6: A comparison of resistive stresses in the modelled landfast sea ice and a possible yield curve for sea ice. The graph shows resistive stresses in the modelled landfast sea ice plotted as a function of “negative pressure” σ_I and “maximum shear stress” σ_{II} (Feltham, 2008). The colour of the points indicates where in the domain the modelled resistive stress is extracted - corresponding to regions on the inset map. The grey circle on the graph indicates a yield curve for a possible sea ice rheology of the form of eq. (4.3) with $P = 10^6 \text{ N m}^{-1}$ and eccentricity of 0. The grey dashed ellipse shows a yield curve with eccentricity of 2 and $P = 2.75 \times 10^5 \text{ N m}^{-1}$ as suggested in Hibler, 1979. The basemap in the inset map is the MODIS Mosaic of Antarctica (Haran et al., 2021).

The conclusions of this article rely on the assertion that model we chose to use, which treats

landfast sea ice in the same way as the land ice, gives an upper bound on the buttressing that the landfast sea ice could provide. We discuss in Sec. 4.3.1 some of the reasons why this is likely to be true. Here, we provide additional evidence for this by comparing stresses in the modelled landfast sea ice to plausible yield stresses in a widely used visco-plastic sea ice model; namely that of Hibler, 1979. In general, sea ice rheology is modelled as being plastic in the case of large deformation due to the opening of cracks, raising of pressure ridges and shearing along crack boundaries. In the case of small-scale deformation, the rheology is sometimes argued to be elastic, reflecting the interaction of floes as they bump into each other (Coon et al., 1974) or viscous, as an approximation to the random jostling of small floes together (Hibler, 1977). The continuity of speed across the glacier/landfast sea ice boundary, as well as the smooth deformation field we observe (supplementary figure C.4) indicates that the deformation of the landfast sea ice in the Larsen-B Embayment, if not dominated by internal viscous deformation, is unlikely to have been in such a sub-critical regime. The model of Hibler, 1979 uses a rheology of the form:

$$\sigma_{ij} = 2\eta\dot{\epsilon}_{ij} + [(\zeta - \eta)\dot{\epsilon}_{kk} - \frac{P}{2}]\delta_{ij} \quad (4.2)$$

where P parametrises the strength of the ice and $\zeta \equiv \zeta(\dot{\epsilon}_{ij}, P)$ and $\eta \equiv \eta(\dot{\epsilon}_{ij}, P)$ are bulk and shear viscosities respectively. These are decreasing functions of strain rate invariants such that the stress states for typical strain rates lie on an elliptical yield curve. We define $\sigma_I = \frac{1}{2}(\sigma_1 + \sigma_2)$ and $\sigma_{II} = \frac{1}{2}(\sigma_2 - \sigma_1)$, where $\sigma_{1,2}$ are the principal stresses. Assuming isotropic ice, the yield curve then can be plotted as a function of σ_I and σ_{II} (Feltham, 2008). This curve passes through the origin, has major axis width of P and centre at $(-P/2, 0)$ and an eccentricity that depends on the relative strength of the ice in shear and compression.

The ice strength itself is approximated as:

$$P = P^* h A c^{-c^*(1-A)} \quad (4.3)$$

where $P^* = 2.75 \times 10^4 \text{ N m}^{-2}$, $c^* = 20$, h is the sea ice thickness and A is its concentration. P^* is sometimes treated as a tunable parameter but P is rarely greater than a factor of 10 away from 10^5 N m^{-1} (Feltham, 2008). Regardless of the precise sub-critical rheology, the strength parameter P is the key scale for stresses that can be maintained within the landfast sea ice.

We consider a yield curve for the landfast sea ice in the Larsen-B Embayment with $P = 10^6 \text{ N m}^{-1}$ and eccentricity of 0. We consider this to be a ‘maximal’ yield curve as P is likely to be smaller in reality, and sea ice is generally far weaker in shear than in compression. (We also plot a more realistic yield curve with eccentricity of 2 and $P = 2.75 \times 10^5 \text{ N m}^{-1}$ suggested in Hibler, 1979.) We compare the resistive stresses in the modelled landfast sea ice to this yield curve, paying special interest to the pro-glacial embayments in front of the HGE and Crane Ice Shelves (Fig. 4.6). In large part, the resistive stresses lie far outside the yield curve, and those in the proglacial embayments all do. This indicates that the stresses born by the landfast sea ice in our model are substantially larger than would be expected of real landfast sea ice. Hence, our modelled landfast sea ice has a considerably greater buttressing capacity than could be expected from a more realistic and well-known model.

4.5.2 What impact does sea ice have on glacier ice?

We have seen that, given a definition of buttressing analogous to that of ice shelves, landfast sea ice has limited ability to buttress glaciers - essentially due to its relative thinness. However, there is a clear link between the landfast sea ice and the stability of the glaciers in the region. As mentioned in Sec. 4.3, one way in which this could occur is through the promotion of ice shelf growth, for example by the following mechanism. Landfast sea ice allows ice shelves to grow initially by providing a barrier to icebergs that, having calved from the glacier terminus, would otherwise drift away. Sea ice formation between these icebergs bonds them together, and to the glacier calving front, forming a rigid ice mélange with material properties determined by the size and density of the icebergs, and the strength of the sea ice bonds. These proto-ice shelves get progressively more robust over time as icebergs calving upstream are more tightly constrained and sea ice thickens. Such a mechanism would lead to ice shelves with no clear boundary in thickness or rigidity separating them from the landfast sea ice - such as those in the HGE and Crane Glaciers prior to 2022 - rather a gradient in those properties. Hence, landfast sea ice confined to an embayment could stabilise ice shelves through the twin mechanisms of inhibiting the export of icebergs and increasing the strength of bonds between them. If this landfast sea ice disintegrates, the visible calving of mélange at the seaward end of the ice shelf would increase through the first mechanism and, if ocean or atmospheric conditions led to the break-up of sea ice within the ice shelf itself, some level of fragmentation of the mélange would also be expected. This is perhaps the process responsible for the partial fragmentation of the

Crane and HGE ice shelves concurrently with the landfast sea ice evacuation, which led to the speed-up of Crane and Green Glaciers.

Perhaps more importantly for the additional calving events seen on the Crane Ice Shelf in the months following the landfast sea ice evacuation, and those on Hektor and Green Glaciers in September 2022, sea ice can act to attenuate ocean swell that originates outside of the embayment (e.g. Voermans et al., 2021). This occurs through scattering of ocean waves from heterogeneities in the ice cover, and the dissipation of wave energy, for example by the elastic plate-bending of the sea ice at different lengthscales and the short wavelength fracturing of ice near its margin (Squire, 2020). Without the sea ice, ocean swells cause higher-amplitude flexural loading cycles that can lead to super- and sub-critical brittle failure (Holdsworth and Glynn, 1978; Massom et al., 2018). The observations of immediate fragmentation of weak seaward parts of the HGE and Crane Ice Shelves in January 2022 and the continuation of enhanced calving on the HGE and Crane Ice Shelves months after the evacuation of the landfast sea ice is compatible with the idea that the ice shelves were not capable of withstanding persistent elevated swell after the break up, or even large swell events leading to critical failure such as that proposed by Ochwat et al., 2023 as a cause of the landfast sea ice disintegration itself.

4.5.3 Wider implications for the interaction of glaciers with landfast sea ice

The results presented here are relevant to a certain limit of landfast sea ice and glacier conditions. Namely: i) the landfast sea ice existed in a relatively enclosed embayment, ii) it was likely to have been relatively thick due to its multi-year persistence over the previous 11-years in a relatively cold ocean, iii) it appeared spatially coherent and deformed smoothly and iv) the inflowing ice shelves were rheologically weak. How well the specific results generalise depends on how well these variables apply to the situation under consideration. However, the conclusion that unbroken landfast sea ice has limited capacity to directly buttress glaciers is likely to hold in general as the conditions explored here are consistent with a ‘maximum buttressing’ example. (The exception is in the rheology of the ice shelves, where stiffer ice may be more capable of transmitting stress upstream.)

In recent decades, the general picture of Antarctic sea ice has been one of regional fluctuation

and relative continental stability, with sea ice extent (SIE) increasing slightly between the early 1970s and the mid 2010s (Turner et al., 2022). 2014 saw the beginning of a fall in SIE, culminating in a record Antarctic sea ice low in 2022 (Turner et al., 2022), which already looks set to be exceeded by even lower sea ice extent in 2023.

Several studies have indicated the importance of sea ice in maintaining glacier stability in Antarctica and Greenland (Arthur et al., 2021; Christoffersen et al., 2012), and the results of this study do little to suggest otherwise - merely highlighting that this importance is unlikely to stem from its ability to buttress glaciers in the way an ice shelf can buttress a grounded ice stream. However, to more accurately judge the extent to which landfast sea ice stabilises ice shelves, a greater understanding of the mechanisms by which this happens is required. To help with observational and modelling studies that aim to do this, work should be carried out to close the gaps in landfast sea ice extent, concentration, and altimetry measurements at the critical zone near glacier calving fronts. For example, knowledge of sea ice thickness at the point of contact with the calving front, along with calving front morphology would help better our understanding of processes in which sea ice might inhibit iceberg calving via its influence on torques at the glacier front.

4.5.4 Limitations and future work

The geometry used in our modelling experiments was one in which glacier ice gave way suddenly to sea ice. In reality, in early January 2022, the boundaries between the HGE and Crane Ice Shelves and the landfast sea ice in front of them were less obvious. The transition between glacier and sea ice involved stages of increasingly rarefied *mélange* including icebergs of a range of sizes. It is quite possible that this transition zone has the dual properties of being able to supply meaningful buttressing to the upstream ice shelf through interactions between icebergs, and being itself vulnerable to the loss of landfast sea ice. Idealised configurations not dissimilar to this were investigated in Robel, 2017. It is very possible that the dynamics of these regions played a role in the destabilisation of the parts of the HGE and Crane Ice Shelves that disintegrated immediately after the landfast sea ice evacuation. That these regions of *mélange* disaggregated at the time of the sea ice disintegration could suggest a buttressing of them by the landfast sea ice and hence a second order buttressing effect of the landfast sea ice on the glaciers. However, it seems more likely, though more mundane, that the more rarefied parts of the *mélange* would be susceptible to the same forcing as the landfast sea ice so disintegrated

in January 2022 for the same reasons. Future work should look in greater detail at the mechanisms by which landfast sea ice can interact with glaciers through such transitional zones of ice mélange, as these might be key to the coupling in embayment and fjord-like geometries.

The sensitivities presented in Fig. 4.3 are reported for a number of realisations of the control fields C and ϕ . This is because, by looking at solutions corresponding to different amounts of regularisation, we hope to show that the spatial pattern of ice speed sensitivities is typical for solutions near the misfit minimum. A more complete picture of the sensitivities might be obtained in future by looking at the curvature of ice speed around the solution, i.e. the principal components of the Hessian matrix. At present, such analysis is difficult to achieve in BISICLES, but is possible for models employing automatic differentiation (e.g. Recinos et al., 2023). Additionally, this would enable a more exact computation of the gradient, rather than the linear approximations used here (Goldberg and Sergienko, 2011).

Additional observations will help to further our understanding of the relative importance of the components of the effect of landfast sea ice on the stability of floating glacier ice. We see in the sensitivity maps (Fig. 4.3) a focussing of the effect of landfast sea ice thickness close to the glacier termini. This suggests that the buttressing component of the effect of landfast sea ice on the ice shelves is concentrated close to the glaciers. However, the attenuation of ocean swell might rely on the full extent of sea ice in the embayment and beyond (Ochwat et al., 2023). Future observations of sea ice growth in the small embayments, e.g. seasonally, vs the Larsen-B Embayment as a whole, and the impacts on the calving behaviour of Hektor and Crane Glaciers can help shed more light on the relative importance of these processes on the growth of ice shelves.

4.6 Conclusions

Our results show that multi-year landfast sea ice which has been present in the Larsen-B Embayment for the last 11-years, following the collapse of the Larsen-B Ice Shelf in 2002, completely disintegrated between the 18th and 23rd of January 2022. This was followed in February with the onset of major ice dynamic speedup events and changes in the calving behaviour of glaciers flowing into the Larsen-B Embayment. Hektor, Green and Crane Glaciers sped up by approx-

imately 15-50% between February and December 2022, with the most pronounced increase of approximately 240 m a^{-1} on Green Glacier upstream of the grounding line. These glaciers lost the majority of their floating ice shelves, that had built up over the preceding decade, by the end of 2022, with the largest retreats of 12 km and 6 km on HGE and Crane Glaciers immediately following the loss of the landfast sea ice.

Model simulations suggest that the increases in speed on the now tidewater parts of Hektor, Green and Crane Glaciers are not due to the loss of direct mechanical buttressing supplied by the landfast sea ice that formerly covered the Larsen-B Embayment. However, the landfast sea ice undoubtedly had an effect on the floating parts of these glacier systems. This effect can be partitioned into the bonding of mélange by sea ice in the ice shelves, the dampening of ocean swell that would otherwise cause high amplitude stress cycles in the ice shelves, and the buttressing that reduces internal stresses. The modelling and observations presented here suggest that direct buttressing of the landfast sea ice could have been large enough to have had a dynamic impact on the floating ice, but that the disintegration of the ice shelves is unlikely to have been related to the associated small changes in resistive stress. This leads us to suggest that the term “buttressing” should not be used in the context of sea ice in the way it is understood when applied to ice shelves. However, a more complete model of the glaciers, ice shelves and landfast sea ice (including the transition zone of ice mélange) is required to fully quantify the relative importance of the effects of sea ice on floating glacier termini, along with further observations of sea ice/ice shelf interactions in the Larsen-B Embayment and elsewhere.

Appendix

4.A Additional model details

4.A.1 L-curve analysis

As discussed briefly in Sec. 4.3.1, we use a form of Tikhonov regularisation to replace the ill-posed inverse problem with a ‘nearby’ well-posed one, with an operator that calculates spatial

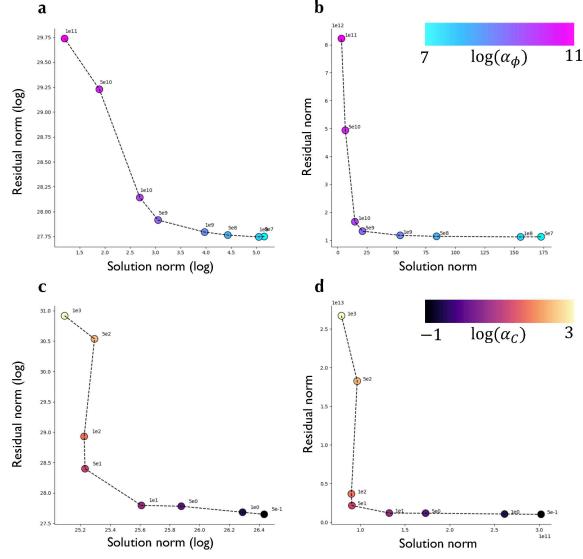


Figure 4.7: L-curves for the choice of regularisation parameters. (a) L-curve for α_ϕ . (b) modified L-curve for α_ϕ . (c) L-curve for α_C . (d) modified L-curve for α_C .

gradients of C and ϕ . The inverse problem can be written:

$$\begin{aligned} \operatorname{argmin}_{C, \phi} & \left\{ \int_{\Omega} |u - u_o|^2 d\Omega + \int_{\Omega} (\alpha_\phi |\nabla \phi|^2 + \alpha_C |\nabla C|^2) d\Omega \right\}, \\ \text{s.t. } & G(u, C, \phi) = 0 \end{aligned} \quad (4.4)$$

where u is the modelled ice speed, u_o is the observed ice speed, and $G(u, C, \phi) = 0$ are the shallow-stream momentum balance equations (4.1). This is approximately solved in BISICLES using a non-linear conjugate gradient method.

We use L-curve analysis to find optimal values of α_C and α_ϕ (Fig. 4.7). This is a heuristic method that posits that the optimal values of the regularisation parameters lead to a solution that balances sensitivities of the misfit and regularisation parts of the cost function to changes in their relative weights (Hansen, 1994). Given the tendency for L-curve analysis to over-regularise, we take the values to the immediate right of the position of maximum curvature in the L-curves, namely $\alpha_C = 1$, $\alpha_\phi = 10^9$.

4.A.2 Calculating gradients of ice speed using the model adjoint

Fig. 4.3 displays the gradient of the functional:

$$J(u(C, \phi)) = \int |u| d\Omega_{HC}$$

with respect to the field ϕ . Here, the domain Ω_{HC} is a union between the neighbourhoods marked by coloured circles (Ω_H and Ω_C) in Fig. 4.3 a. We write $J(u(C, \phi))$ as $\tilde{J}(C, \phi)$, a pure functional of C and ϕ . The Gâteaux derivative of $\tilde{J}(C, \phi)$ with respect to ϕ in the direction $\delta\phi$ can be written:

$$\langle D\tilde{J}, \delta\phi \rangle = \lim_{\epsilon \rightarrow 0} \frac{\tilde{J}(C, \phi + \epsilon\delta\phi) - \tilde{J}(C, \phi)}{\epsilon}.$$

We approximate this as

$$\langle D\tilde{J}, \delta\phi \rangle \approx \int \delta\phi \times \bar{\mu}h\nabla\boldsymbol{\lambda} \left(\nabla\mathbf{u} + (\nabla\mathbf{u})^\top + 2(\nabla \cdot \mathbf{u})\mathcal{I} \right) d\Omega \quad (4.5)$$

where $\boldsymbol{\lambda}$ is a vector field of lagrange multipliers that solves the adjoint equation:

$$-\nabla \cdot [\phi h \bar{\mu} (\nabla\boldsymbol{\lambda} + (\nabla\boldsymbol{\lambda})^\top + 2(\nabla \cdot \boldsymbol{\lambda})\mathcal{I})] + C\boldsymbol{\lambda} = \begin{cases} \hat{\mathbf{u}}, & \text{in } \Omega_{HC} \\ 0, & \text{elsewhere} \end{cases} \quad (4.6)$$

with reflection boundary conditions on the domain boundary:

$$\begin{aligned} \hat{\mathbf{n}} \cdot \boldsymbol{\lambda} &= 0, \\ \hat{\mathbf{t}} \cdot \nabla\boldsymbol{\lambda} \cdot \hat{\mathbf{n}} &= 0 \end{aligned}$$

(where $\hat{\mathbf{n}}$ and $\hat{\mathbf{t}}$ are normal and tangent vectors to the boundary respectively).

A more detailed exposition of this kind of procedure is given in Morlighem et al., 2013. To construct (4.6), we have neglected non-linearities in the dependence of $\bar{\mu}$ on $\mathbf{u}$, and in the sliding law (Goldberg and Sergienko, 2011). The field we show in Fig. 4.3 comes from interpreting eq (4.5) as the projection of the functional gradient along the direction $\delta\phi$ with the standard L_2 inner product. Hence, the gradient shown in Fig. 4.3 is the field:

$$\bar{\mu}h\nabla\boldsymbol{\lambda} \left(\nabla\mathbf{u} + (\nabla\mathbf{u})^\top + 2(\nabla \cdot \mathbf{u})\mathcal{I} \right)$$

4.A.3 Sensitivity to sliding physics

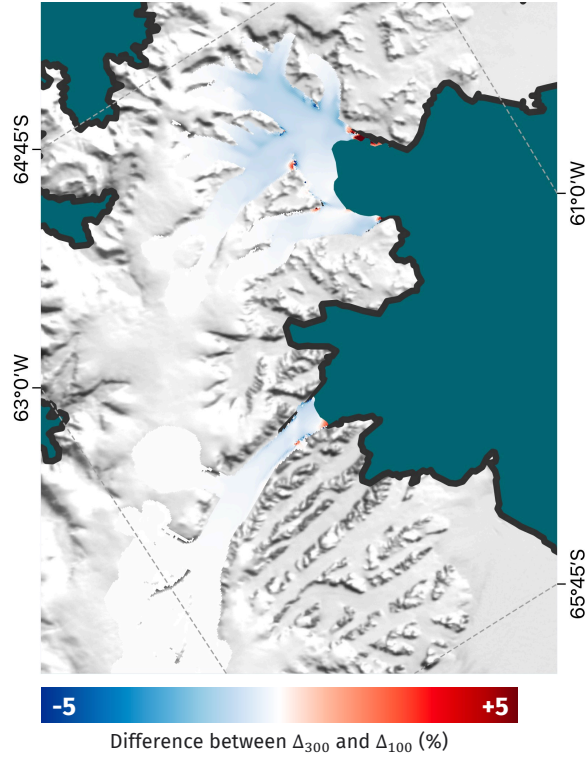


Figure 4.8: Difference between Δ_{100} and Δ_{300} - the percentage change in ice speed with the addition of 50 m of landfast sea ice for $u_o = 100 \text{ m a}^{-1}$ and $u_o = 300 \text{ m a}^{-1}$ respectively. The background image is the MODIS MOA (Haran et al., 2021), and the black line is the glacier boundary according to Mouginot et al., 2017.

For the simulations presented in this article, we used a Regularised-Coulomb sliding law for basal stress τ_b in terms of basal ice velocity $\mathbf{u}_b$ of the form:

$$\tau_b = -C \left(\frac{|\mathbf{u}_b|}{|\mathbf{u}_b|/u_o + 1} \right)^m \frac{\mathbf{u}_b}{|\mathbf{u}_b|} \quad (4.7)$$

with a Weertman-like exponent of $m = 1/3$ and a threshold ice speed of $u_o = 300 \text{ m a}^{-1}$ that represents a transition between viscous and plastic sliding (Joughin et al., 2019). This sliding law is physically plausible for the fast-flowing glaciers under consideration, and allows for the greatest change in grounded ice speed with the small changes to resistive stress at the calving front brought about by the addition of sea ice. The value of u_o is the main control on how far these speed changes propagate upstream of the grounding line.

To ensure the conclusions of this study are independent of the chosen u_o , we also consider a value of $u_o = 100 \text{ m a}^{-1}$. For values of both 100 m a^{-1} and 300 m a^{-1} , the percentage difference in speed between the cases of no sea ice and 50 m of sea ice was calculated. We call these Δ_{100}

and Δ_{300} respectively. Fig. 4.8 shows the difference between these quantities.

We see that the difference is below 1% across the HGE and Crane basins. The transects shown in Fig. 4.2 f indicate changes in grounded ice speed with the addition of 50 m of landfast sea ice and a threshold ice speed of 300 m a^{-1} is on the order of 5% where speed measurements were made.

Bibliography

- Arthur, J. F., Stokes, C. R., Jamieson, S. S. R., Miles, B. W. J., Carr, J. R., and Leeson, A. A. (2021). “The triggers of the disaggregation of Voyeykov Ice Shelf (2007), Wilkes Land, East Antarctica, and its subsequent evolution”. In: *Journal of Glaciology* 67.265, pp. 933–951. DOI: 10.1017/jog.2021.45.
- Banwell, A. F., Datta, R. T., Dell, R. L., Moussavi, M., Brucker, L., Picard, G., Shuman, C. A., and Stevens, L. A. (2021). “The 32-year record-high surface melt in 2019/2020 on the northern George VI Ice Shelf, Antarctic Peninsula”. In: *The Cryosphere* 15.2, pp. 909–925. DOI: 10.5194/tc-15-909-2021.
- Christie, F. D. W., Benha, T. J., Batchelor, C. L., Rack, W., Montelli, A., and Dowdeswell, J. A. (2022). “Antarctic ice-shelf advance driven by anomalous atmospheric and sea-ice circulation”. In: *Nature Geoscience* 15.5, pp. 356–362. DOI: 10.1038/s41561-022-00938-x.
- Christoffersen, P., O’Leary, M., Van Angelen, J. H., and Van Den Broeke, M. (2012). “Partitioning effects from ocean and atmosphere on the calving stability of Kangerdlugssuaq Glacier, East Greenland”. In: *Annals of Glaciology* 53.60, pp. 249–256. DOI: 10.3189/2012AoG60A087.
- Cook, A. J. and Vaughan, D. G. (2010). “Overview of areal changes of the ice shelves on the Antarctic Peninsula over the past 50 years”. In: *The Cryosphere* 4.1, pp. 77–98. DOI: 10.5194/tc-4-77-2010.
- Coon, M. D., Maykut, G. A., and Pritchard, R. S. (1974). “Modeling the pack ice as an elastic-plastic material”. In: *AIDJEX Bull.* 24, pp. 0-105.
- Cornford, S. L., Martin, D. F., Graves, D. T., Ranken, D. F., Le Brocq, A. M., Gladstone, R. M., Payne, A. J., Ng, E. G., and Lipscomb, W. H. (2013). “Adaptive mesh, finite volume modeling of marine ice sheets”. In: *Journal of Computational Physics* 232.1, pp. 529–549. ISSN: 0021-9991. DOI: 10.1016/j.jcp.2012.08.037.
- Cornford, S. L., Martin, D. F., Payne, A. J., Ng, E. G., Le Brocq, A. M., Gladstone, R. M., Edwards, T. L., Shannon, S. R., Agosta, C., Broeke, M. R. van den, et al. (2015). “Century-scale simulations of the response of the West Antarctic Ice Sheet to a warming climate”. In: *The Cryosphere* 9.4, pp. 1579–1600.
- Cuffey, K. M. and Paterson, W. S. B. (2010). *The physics of glaciers*. Academic Press.
- Davison, B. J., Hogg, A. E., Rigby, R., Veldhuijsen, S., Wessem, J. M. van, Broeke, M. R. van den, Holland, P. R., Selley, H. L., and Dutrieux, P. (2023a). “Sea level rise from West Antarctic mass loss significantly modified by large snowfall anomalies”. In: *Nature Communications* 14.1, p. 1479. DOI: 10.1038/s41467-023-36990-3.

- Davison, B. J., Hogg, A. E., Gourmelen, N., Jakob, L., Wuite, J., Nagler, T., Greene, C. A., Andreasen, J., and Engdahl, M. E. (2023b). “Annual mass budget of Antarctic ice shelves from 1997 to 2021”. In: *Science Advances* 9.41, eadi0186. DOI: 10.1126/sciadv.adi0186.
- Feltham, D. L. (2008). “Sea Ice Rheology”. In: *Annual Review of Fluid Mechanics* 40.1, pp. 91–112. DOI: 10.1146/annurev.fluid.40.111406.102151.
- Fraser, A. D., Massom, R. A., Handcock, M. S., Reid, P., Ohshima, K. I., Raphael, M. N., Cartwright, J., Klekociuk, A. R., Wang, Z., and Porter-Smith, R. (2021). “Eighteen-year record of circum-Antarctic landfast-sea-ice distribution allows detailed baseline characterisation and reveals trends and variability”. In: *The Cryosphere* 15.11, pp. 5061–5077. DOI: 10.5194/tc-15-5061-2021.
- Fraser, A. D. et al. (2023). “Antarctic Landfast Sea Ice: A Review of Its Physics, Biogeochemistry and Ecology”. In: *Reviews of Geophysics* 61.2, e2022RG000770. DOI: <https://doi.org/10.1029/2022RG000770>.
- Fürst, J. J., Durand, G., Gillet-Chaulet, F., Tavard, L., Rankl, M., Braun, M., and Gagliardini, O. (2016). “The safety band of Antarctic ice shelves”. In: *Nature Climate Change* 6.5, pp. 479–482.
- Goldberg, D. N. and Sergienko, O. V. (2011). “Data assimilation using a hybrid ice flow model”. In: *The Cryosphere* 5.2, pp. 315–327. DOI: 10.5194/tc-5-315-2011.
- Gourmelen, N., Goldberg, D. N., Snow, K., Henley, S. F., Bingham, R. G., Kimura, S., Hogg, A. E., Shepherd, A., Mouginot, J., Lenaerts, J. T. M., Ligtenberg, S. R. M., and Berg, W. J. van de (2017). “Channelized Melting Drives Thinning Under a Rapidly Melting Antarctic Ice Shelf”. In: *Geophysical Research Letters* 44.19, pp. 9796–9804. DOI: 10.1002/2017GL074929.
- Gudmundsson, G. H. (2013). “Ice-shelf buttressing and the stability of marine ice sheets”. In: *The Cryosphere* 7.2, pp. 647–655. DOI: 10.5194/tc-7-647-2013.
- Hansen, P. C. (1994). “Regularization tools: A Matlab package for analysis and solution of discrete ill-posed problems”. In: *Numerical algorithms* 6.1, pp. 1–35. DOI: 10.1007/BF02149761.
- Haran, T. M., Bohlander, J., Scambos, T. A., Painter, T. H., and Fahnestock, M. A. (2021). *MODIS Mosaic of Antarctica 2003-2004 (MOA2004) Image Map, Version 2*. DOI: 10.5067/68TBT0CGJS0J.
- Hibler, W. D. I. (1977). “A viscous sea ice law as a stochastic average of plasticity”. In: *Journal of Geophysical Research (1896-1977)* 82.27, pp. 3932–3938. DOI: 10.1029/JC082i027p03932.
- (1979). “A dynamic thermodynamic sea ice model”. In: *Journal of physical oceanography* 9.4, pp. 815–846. DOI: 10.1175/1520-0485(1979)009<0815:ADTSIM>2.0.CO;2.
- Holdsworth, G. and Glynn, J. (1978). “Iceberg calving from floating glaciers by a vibrating mechanism”. In: *Nature* 274.5670, pp. 464–466. DOI: 10.1038/274464a0.
- Howat, I. M., Porter, C., Smith, B. E., Noh, M.-J., and Morin, P. (2019). “The Reference Elevation Model of Antarctica”. In: *The Cryosphere* 13.2, pp. 665–674. DOI: 10.5194/tc-13-665-2019.

- Hunke, E. C. and Dukowicz, J. K. (1997). “An elastic–viscous–plastic model for sea ice dynamics”. In: *Journal of Physical Oceanography* 27.9, pp. 1849–1867. DOI: 10.1175/1520-0485(1997)027<1849:AEVPMF>2.0.CO;2.
- Huss, M. and Farinotti, D. (2014). “A high-resolution bedrock map for the Antarctic Peninsula”. In: *The Cryosphere* 8.4, pp. 1261–1273. DOI: 10.5194/tc-8-1261-2014.
- Joughin, I. and Alley, R. B. (2011). “Stability of the West Antarctic ice sheet in a warming world”. In: *Nature Geoscience* 4.8, pp. 506–513. DOI: 10.1038/ngeo1194.
- Joughin, I., Smith, B. E., and Schoof, C. G. (2019). “Regularized Coulomb Friction Laws for Ice Sheet Sliding: Application to Pine Island Glacier, Antarctica”. In: *Geophysical Research Letters* 46.9, pp. 4764–4771. DOI: <https://doi.org/10.1029/2019GL082526>.
- Lemos, A., Shepherd, A., McMillan, M., and Hogg, A. E. (2018). “Seasonal Variations in the Flow of Land-Terminating Glaciers in Central-West Greenland Using Sentinel-1 Imagery”. In: *Remote Sensing* 10.12. ISSN: 2072-4292. DOI: 10.3390/rs10121878.
- Massom, R. A., Scambos, T. A., Bennetts, L. G., Reid, P., Squire, V. A., and Stammerjohn, S. E. (2018). “Antarctic ice shelf disintegration triggered by sea ice loss and ocean swell”. In: *Nature* 558.7710, pp. 383–389.
- Morlighem, M., Seroussi, H., Larour, E., and Rignot, E. (2013). “Inversion of basal friction in Antarctica using exact and incomplete adjoints of a higher-order model”. en. In: *Journal of Geophysical Research: Earth Surface* 118.3, pp. 1746–1753. ISSN: 2169-9011. DOI: 10.1002/jgrf.20125.
- Mouginot, J., Scheuchl, B., and Rignot, E. (2017). *MEaSUREs Antarctic Boundaries for IPY 2007-2009 from Satellite Radar, Version 2*. Boulder, Colorado USA. NASA National Snow and Ice Data Center Distributed Active Archive Center. doi: <http://dx.doi.org/10.5067/AXE4121732AD>. Accessed 14/02/2023.
- Ochwat, N. E., Scambos, T. A., Banwell, A. F., Anderson, R. S., MacLennan, M. L., Picard, G., Shates, J. A., Marinsek, S., Margonari, L., Truffer, M., and Pettit, E. C. (2023). “Triggers of the 2022 Larsen B multi-year landfast sea ice break-out and initial glacier response”. In: *The Cryosphere Discussions* 2023. In review, pp. 1–34. DOI: 10.5194/tc-2023-88.
- Otosaka, I. N. et al. (2023). “Mass balance of the Greenland and Antarctic ice sheets from 1992 to 2020”. In: *Earth System Science Data* 15.4, pp. 1597–1616. DOI: 10.5194/essd-15-1597-2023.
- Pattyn, F. and Morlighem, M. (2020). “The uncertain future of the Antarctic Ice Sheet”. In: *Science* 367.6484, pp. 1331–1335. DOI: 10.1126/science.aaz5487.
- Recinos, B., Goldberg, D., Maddison, J. R., and Todd, J. (2023). “A framework for time-dependent Ice Sheet Uncertainty Quantification, applied to three West Antarctic ice streams”. In: *The Cryosphere Discussions* 2023, pp. 1–46. DOI: 10.5194/tc-2023-27.
- Rignot, E., Casassa, G., Gogineni, P., Krabill, W., Rivera, A., and Thomas, R. (2004). “Accelerated ice discharge from the Antarctic Peninsula following the collapse of Larsen B ice shelf”. In: *Geophysical Research Letters* 31.18. DOI: <https://doi.org/10.1029/2004GL020697>.
- Robel, A. A. (2017). “Thinning sea ice weakens buttressing force of iceberg mélange and promotes calving”. In: *Nature Communications* 8.1, pp. 1–7.

- Rott, H., Abdel Jaber, W., Wuite, J., Scheiblauer, S., Floricioiu, D., Wessem, J. M. van, Nagler, T., Miranda, N., and Broeke, M. R. van den (2018). “Changing pattern of ice flow and mass balance for glaciers discharging into the Larsen A and B embayments, Antarctic Peninsula, 2011 to 2016”. In: *The Cryosphere* 12.4, pp. 1273–1291. DOI: 10.5194/tc-12-1273-2018.
- Scambos, T. A., Bohlander, J. A., Shuman, C. A., and Skvarca, P. (2004). “Glacier acceleration and thinning after ice shelf collapse in the Larsen B embayment, Antarctica”. In: *Geophysical Research Letters* 31.18. DOI: <https://doi.org/10.1029/2004GL020670>.
- Schoof, C. (2005). “The effect of cavitation on glacier sliding”. In: *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences* 461.2055, pp. 609–627. DOI: 10.1098/rspa.2004.1350.
- Shean, D. E., Joughin, I. R., Dutrieux, P., Smith, B. E., and Berthier, E. (2019). “Ice shelf basal melt rates from a high-resolution digital elevation model (DEM) record for Pine Island Glacier, Antarctica”. In: *The Cryosphere* 13.10, pp. 2633–2656. DOI: 10.5194/tc-13-2633-2019.
- Slater, T., Lawrence, I. R., Otosaka, I. N., Shepherd, A., Gourmelen, N., Jakob, L., Tepes, P., Gilbert, L., and Nienow, P. (2021). “Review article: Earth’s ice imbalance”. In: *The Cryosphere* 15.1, pp. 233–246. DOI: 10.5194/tc-15-233-2021.
- Smith, B., Fricker, H. A., Gardner, A. S., Medley, B., Nilsson, J., Paolo, F. S., Holschuh, N., Adusumilli, S., Brunt, K., Csatho, B., Harbeck, K., Markus, T., Neumann, T., Siegfried, M. R., and Zwally, H. J. (2020). “Pervasive ice sheet mass loss reflects competing ocean and atmosphere processes”. In: *Science* 368.6496, pp. 1239–1242. DOI: 10.1126/science.aaz5845.
- Squire, V. A. (2020). “Ocean Wave Interactions with Sea Ice: A Reappraisal”. In: *Annual Review of Fluid Mechanics* 52.1, pp. 37–60. DOI: 10.1146/annurev-fluid-010719-060301.
- Sun, Y., Riel, B., and Minchew, B. (2023). “Disintegration and Buttressing Effect of the Landfast Sea Ice in the Larsen B Embayment, Antarctic Peninsula”. In: *Geophysical Research Letters* 50.16, e2023GL104066. DOI: <https://doi.org/10.1029/2023GL104066>.
- Trusel, L. D., Frey, K. E., Das, S. B., Karnauskas, K. B., Kuipers Munneke, P., Van Meijgaard, E., and Van Den Broeke, M. R. (2015). “Divergent trajectories of Antarctic surface melt under two twenty-first-century climate scenarios”. In: *Nature Geoscience* 8.12, pp. 927–932. DOI: 10.1038/ngeo2563.
- Turner, J., Holmes, C., Caton Harrison, T., Phillips, T., Jena, B., Reeves-Francois, T., Fogt, R., Thomas, E. R., and Bajish, C. C. (2022). “Record Low Antarctic Sea Ice Cover in February 2022”. In: *Geophysical Research Letters* 49.12, e2022GL098904. DOI: <https://doi.org/10.1029/2022GL098904>.
- Voermans, J. J., Liu, Q., Marchenko, A., Rabault, J., Filchuk, K., Ryzhov, I., Heil, P., Waseda, T., Nose, T., Kodaira, T., Li, J., and Babanin, A. V. (2021). “Wave dispersion and dissipation in landfast ice: comparison of observations against models”. In: *The Cryosphere* 15.12, pp. 5557–5575. DOI: 10.5194/tc-15-5557-2021.
- Wallis, B. J., Hogg, A. E., Zhu, Y., and Hooper, A. (2024). “Change in grounding line location on the Antarctic Peninsula measured using a tidal motion offset correlation method”. In: *EGU sphere* 2024, pp. 1–32. DOI: 10.5194/egusphere-2023-2874.

- Wallis, B. J., Hogg, A. E., Wessem, J. M. van, Davison, B. J., and Broeke, M. R. van den (2023). “Widespread seasonal speed-up of west Antarctic Peninsula glaciers from 2014 to 2021”. In: *Nature Geoscience* 16.3, pp. 231–237. DOI: 10.1038/s41561-023-01131-4.
- Wuite, J., Rott, H., Hetzenecker, M., Floricioiu, D., De Rydt, J., Gudmundsson, G. H., Nagler, T., and Kern, M. (2015). “Evolution of surface velocities and ice discharge of Larsen B outlet glaciers from 1995 to 2013”. In: *The Cryosphere* 9.3, pp. 957–969. DOI: 10.5194/tc-9-957-2015.

Chapter 5

Using observations of surface fracture to address ill-posed ice softness estimation over Pine Island Glacier

This chapter is in prep for submission in 2024 as the following article :

T. Surawy-Stepney, S. L. Cornford and A. E. Hogg : “ *Using observations of surface fracture to address ill-posed ice softness estimation over Pine Island Glacier* ”

Abstract

Numerical models that use a viscous approximation to internal glacier dynamics, such as those used to simulate the evolution of the Antarctic Ice Sheet, require the specification of basal boundary conditions on stress and local deviations in the assumed rheological properties of the ice. Scalar fields representing these unknown components of the system are often found by solving an inverse problem given observations of model state variables - typically ice flow speed. However, these optimisation problems are generally ill posed, resulting in degenerate solutions and poor conditioning. In this study, we propose the use of data on the locations of surface fractures and high shear strain rates to provide prior information to the inverse problem, in an effort to better constrain the ice softness field compared with methods using more heuristic regularisation techniques. Specifically, the prior distribution we consider encodes the assumption that values of ice softness significantly different from our initial guess are, for the most part, due to the fractures present in the ice with subsidiary effects in the shear margins of fast flowing

ice streams such as shear heating and the development of macroscopic fabrics or atypical grain sizes. Using Pine Island Glacier as a case study, we investigate the solutions and conditioning of the data-informed inverse problem versus alternatives. We go on to investigate the use of timeseries of fracture maps over the floating ice shelf to constrain the evolution of the softness field through time, making the assumption that changes to ice softness occurring on annual timescales are dominated by the fracturing of ice. We show that this method can result in softness fields that visually mimic fracture patterns without significantly affecting the quality of the misfit.

5.1 Introduction

Large-scale ice sheet models commonly treat ice within the paradigm of continuum mechanics - as a shear thinning viscous fluid; an approach that has been successful in modelling the behaviour of large ice masses relatively cheaply (e.g. Seroussi et al., 2020). Within this framework, the flow of the ice can be accounted for in large part by a balance between gravity, viscous stress due to internal deformation and frictional stress at ice/bedrock interfaces. To close the system and allow the model to solve for ice speed, equations relating viscous and frictional stresses to ice speed are specified, informed by laboratory data and physical arguments.

The former ‘constitutive relation’ takes the form of Glen’s flow law:

$$\tau_{ij} = 2\eta\dot{\epsilon}_{ij}, \quad \text{where } \eta = \frac{1}{2}A(T)^{-\frac{1}{n}}\epsilon^{\frac{1}{n}-1} \quad (5.1)$$

where τ_{ij} is the deviatoric stress tensor, $\dot{\epsilon}_{ij}$ is the strain rate tensor, η is the strain-rate-dependent effective ice viscosity, ϵ is the second invariant of the strain rate tensor, $A(T)$ is a temperature-dependent rate factor and the exponent $n \approx 3$. It is possible to treat $A(T)$ and/or n as free parameters that can be fitted to observations, given the uncertainties involved in both and the different physical mechanisms that distinguish them. Here, we consider the approach in which these are prescribed *a priori* and a ‘stiffness’ field $\phi(x)$, that scales the effective ice viscosity η , is defined over the domain to account for unknown deviations in the expected ice rheology. Used in this way, ϕ approximates the effect of uncertainties in the temperature and thickness fields, regional changes in the temperature dependence of Glen’s flow law, deviations from the assumed isotropy of creep deformation and, of particular interest to this study, fractures in the

ice at different lengthscales.

The relation between frictional stress and basal sliding speed is known as a ‘sliding law’, and has a functional form that depends on a number of often poorly constrained factors such as the expected amount of deformation around topographic features in the bed, sliding over smooth bedrock, and shearing of the sub-glacial till. A single sliding law is often combined with a spatially varying ‘basal slip’ parameter $C(x)$ to approximate this stress:

$$\boldsymbol{\tau}_b = Cf(\mathbf{u}). \quad (5.2)$$

Taken together, the equations considered here take the form of the shallow-stream approximation to the Cauchy momentum equations:

$$\nabla \cdot [\phi h \bar{\eta} (\nabla \mathbf{u} + (\nabla \mathbf{u})^\top + 2(\nabla \cdot \mathbf{u})\mathcal{I})] - Cf(\mathbf{u}) - \rho_i g h \nabla s = 0, \quad (5.3)$$

where $\mathbf{u} = (u_x, u_y)^\top$ is the horizontal velocity, $\bar{\eta}$ is the vertically-integrated effective ice viscosity, ρ_i is the density of ice, h is the ice thickness and s is the ice surface. In this study we use a linear sliding law $f(\mathbf{u}) = \mathbf{u}$ for ease of computing the adjoint velocities during the inverse problem. In this article, we sometimes refer to the “softness” field φ - related to the stiffness by $\varphi = (1 - \phi)$.

In order to simulate real ice masses accurately, the fields C and ϕ are inferred simultaneously from observations of ice speed using inverse methods - a suite of techniques for inferring model control parameters from observed state variables (MacAyeal, 1992) - (e.g. Petra et al., 2012; Arthern et al., 2015; Cornford et al., 2015; Gudmundsson et al., 2019). Unfortunately, this inverse problem is ill-posed: the two fields (C, ϕ) are calculated from the single observed field u (the problem is “underdetermined”) and the results are highly dependent on noise in the input data (the problem, at least in its discrete form, is “ill-conditioned”). To obtain reliable control fields, it is beneficial to replace this ill-posed problem by a nearby well-posed one before solving it. The problem is sometimes simplified by solving for C only on grounded ice, and ϕ on floating ice, thereby separating the two fields spatially removing a portion of the degeneracy that arises from the mixing of C and ϕ (e.g. Goldberg et al., 2019). However, though you would expect C to be the dominant control on grounded ice speed, there is no reason to be especially confident in the guess of $\phi = 1$. Another approach, and one that shall be taken here, is to regularise the

solution by providing additional constraints on the control fields. Such a regularised inverse problem takes the form of the following optimisation:

$$(C, \phi) = \underset{C, \phi}{\operatorname{argmin}} \{ \mathcal{J}_m(u, u_o) + \alpha_C \mathcal{J}_C(C) + \alpha_\phi \mathcal{J}_\phi(\phi) \}, \quad \text{s.t.} \quad G(u, C, \phi) = 0 \quad (5.4)$$

where $\mathcal{J}_m(u, u_o) = \|u - u_o\|_2^2$ is a misfit function calculating the distance of the modelled ice speed u from the observed ice speed u_o , $\mathcal{J}_C$ and $\mathcal{J}_\phi$ are regularisation functions for the C and ϕ fields, with strengths controlled by the parameters α_C and α_ϕ respectively, and $G(u, C, \phi) = 0$ are the momentum balance equations (5.3).

A popular approach, aimed at improving the conditioning of the problem by suppressing the amplification of high-frequency components of the input data, is to use Tikhonov regularisation in a form that favours either low spatial frequency or low amplitude components of the solution (e.g. Morlighem et al., 2013; Habermann et al., 2013; Brinkerhoff and Johnson, 2013; Cornford et al., 2015), e.g.:

$$\alpha_\phi \mathcal{J}_\phi(\phi) = \alpha_\phi \int_{\Omega} |\nabla \phi|^2 d^2 \mathbf{x}. \quad (5.5)$$

However, this kind of regularisation is entirely heuristic and, when it comes to distinguishing C and ϕ , relies on assumed differences in the lengthscales over which changes in the control fields can influence strain rates. Generally, in regions without significant shear, these lengthscales are not easily distinguished, and degeneracies between solutions for C and ϕ proliferate. Additional difficulties arise when a control field contains distinct contributions with different spatial frequencies. For example, uncertainty in englacial temperature can vary on the scales of long-term atmospheric or geothermal heat sources, or over the width of a shear margin. Often, an imperfect but acceptable lengthscale is found by searching parameter space informed by heuristics such as l-curve analysis (Hansen, 1994).

The aim of this study is to investigate whether the introduction of genuine prior information into the inverse problem can result in substantively different solutions, and whether these solutions are more appealing than those found using other, heuristic regularisation methods.

Previous studies have investigated instances in which softness fields found through solving in-

verse problems have mirrored observed fracture features (Borstad et al., 2013; Surawy-Stepney et al., 2023b) - suggesting that the presence of fractures has the potential to dominate ϕ . With recent advancements in observational methods for locating fractures in remote sensing data (Lai et al., 2020; Izeboud and Lhermitte, 2023; Zhao et al., 2022; Surawy-Stepney et al., 2023a), we are moving towards reliable data that can be used to inform us at least about this specific component of the softness field. It has also been shown previously that the use of strain rate data to inform the solution of inverse problems has the potential to reduce a certain amount of mixing between C and ϕ (Ranganathan et al., 2021).

Here, we investigate two ways in which fracture and strain-rate observations can be used to inform the inverse problem to replace or complement existing heuristic methods. The first is to use fracture maps (obtained from Sentinel-1 imagery - described in Surawy-Stepney et al., 2023a) along with estimates of surface strain-rates to construct a prior distribution for ϕ for use in snapshot inverse problems (single optimisations carried out for a set of geometry and speed data collected at a specific instant in time). This prior simply says that we expect $\phi \approx 1$ away from regions of observed fracture, or where there are high shear strain rates (which can contribute the effects of enhanced anisotropy, shear heating and microfracturing to ϕ). In practise, this is equivalent to a form of Tikhonov regularisation using a diagonal Tikhonov matrix with entries weighted away from where we expect soft ice.

The use of fracture maps as a prior makes assumptions about the relative contributions of different uncertainties to ϕ . For example, for this to be at all appropriate, we have to have a certain amount of faith in the 3D temperature field we use. A more easily justified belief is that *changes* to ϕ on monthly-to-annual timescales are dominated by the fracturing of ice, as other contributions to ϕ are likely to vary on significantly longer timescales. Hence, we also investigate the use of timeseries of fracture maps in constraining the solutions to inverse problems carried out over multiple timesteps. Specifically, we initialise the inverse problem with heuristic regularisation, before imposing a regularisation that penalises the changes to ϕ except where we have seen the evolution of fractures in the observational data. We show, with this method, that one can generate softness fields that mimic, in certain ways, the changing fracture patterns on the Pine Island Ice Shelf between 2016 and 2021, without substantially affecting the

misfit of the problem. This may have potential uses in constraining models that aim to evolve softness fields in response to englacial stresses.

5.2 Methods

The simulations presented in this article were performed using the BISICLES ice sheet model (Cornford et al., 2013). This is an adaptive mesh, finite volume model which we choose here to solve discretized versions of the two-dimensional shallow-stream equations (5.3). Each simulation is carried out over Pine Island Glacier in the Amundsen Sea Sector of West Antarctica with a domain encompassing the whole present-day drainage basin (Zwally et al., 2012). This region was chosen as it represents a potentially strong correspondence between fracturing and ice softness, given the abundant crevasses in the shear margins, upstream of the grounding line and the regular formation of rifts near the terminus, as well as the established dynamic impact of some of this fracturing (Joughin et al., 2021; Sun and Gudmundsson, 2023). Across the rest of Antarctica, we expect the link between the dynamics of ice and the extent of fracturing to be weaker in general. We use a form of the rate factor $A(T)$ described in Cuffey and Paterson, 2010, with an internal energy field generated using a 100 000 year calculation in which surface temperature, thickness and velocity are held at present day values and the combined ice temperature and moisture fraction field $E = CT + Lw$ evolves toward equilibrium. We used geometry defined by BedMachine-v3 (Morlighem, 2022), with prescribed calving front positions extracted from Sentinel-1 backscatter images. Each simulation used velocity and fracture data from within the five-year period between November 2016 and November 2021. We used 200 m resolution, monthly-averaged ice velocity observations made using feature tracking applied to Sentinel-1 image pairs (Wuite et al., 2021) (<https://cryoportal.enveo.at/data/>) as the input data to the cost function and to estimate shear strain rates. Crevasse data was generated according to the methods described in Surawy-Stepney et al., 2023a. The inverse problem we consider at each stage takes the form of eq. (5.4) and is solved in BISICLES using a non-linear conjugate gradient method (Cornford et al., 2015).

5.2.1 Fracture data assimilation in snapshot inverse problems

The snapshot problem we consider is the joint estimation of C and ϕ over Pine Island Glacier in May 2019. We use mean velocities over the month and median composite fracture maps.

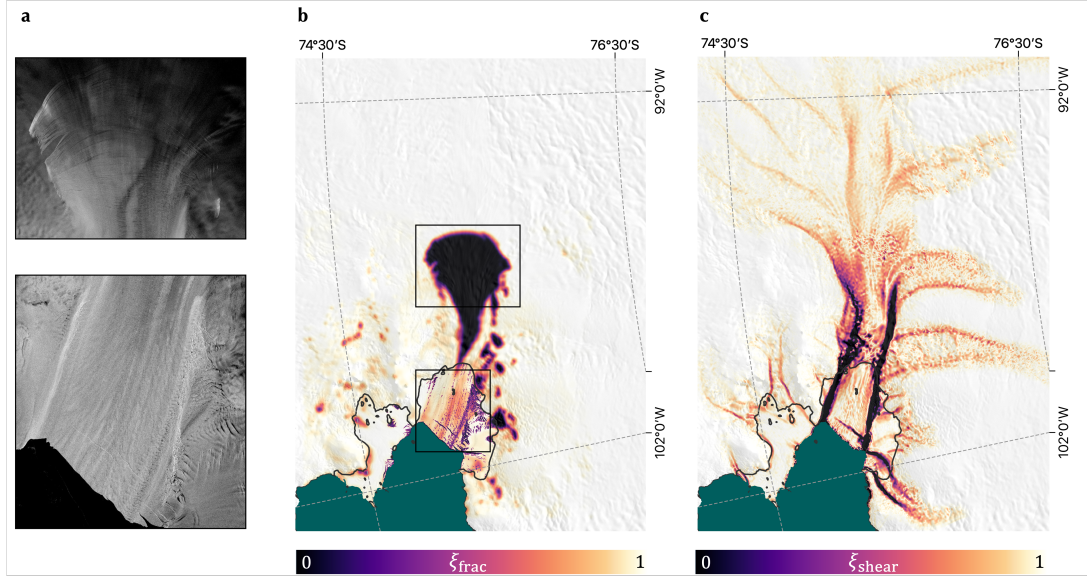


Figure 5.1: Contributions to the field ξ , representing, in our prior for the softness field, where we have observations of surface fracture or high shear strain rates. a) SAR backscatter images over grounded and floating parts of Pine Island Glacier from May 2019 showing regions visible crevassing. b) The component of ξ due to the observation of crevasse features, made from fracture maps developed in Surawy-Stepney et al., 2023a. c) The component of ξ due to the presence of high shear strain rates. Background images are the MODIS Mosaic of Antarctica (Haran et al., 2021), and grounding lines (shown in black) are according to Rignot et al., 2016.

To construct a prior for ϕ , we first form a field ξ which is 0 in regions which have high shear strain rates or where fractures have been observed and 1 elsewhere. For the surface fracture contribution to ξ , we use monthly mosaics of fracture maps covering grounded and floating ice - slightly smoothed to produce contiguous fracture fields on the grounded ice. We call these fracture maps f_i . The fracture-map contribution to ξ is simply $\xi_{\text{frac}} = 1 - f_i$ (Fig. 5.1 b). We create the strain-rate contribution to ξ using the same velocity data that we use in our misfit function. To estimate the derivatives $\partial_i u_j$, we differentiated the velocity components using a method described in Chartrand, 2017, using Tikhonov regularisation to promote smoothness (regularisation parameters were chosen with some trial-and-error, where preference was given to solutions in which regions of high shear varied smoothly over lengthscales comparable to the widths of visible shear margins). Aligning the x -coordinate with local flow direction, we define regions of *high shear* to be those in which $|\dot{\epsilon}_{xy}| > 0.1 \text{ a}^{-1}$. This threshold is a bit discretionary, though it corresponds to stresses within the range 90 – 320 kPa of tensile strength suggested in Vaughan, 1993 for a wide range of englacial temperatures. Then $\xi_{\text{shear}} = \max\{0, 1 - 10|\dot{\epsilon}_{xy}|\}$ (Fig. 5.1 c) and $\xi = \min\{\xi_{\text{frac}}, \xi_{\text{shear}}\}$.

The assumption we wish to encode is that $\phi \rightarrow 1 + \epsilon$ as $\xi \rightarrow 1$, where $\epsilon \sim \mathcal{N}(0, \gamma^2)$ and γ is a small number. Such a prior over the ϕ field can be written:

$$p_{\Phi}(\phi) \propto \exp\left(-\frac{1}{2\gamma^2} \int_{\Omega} (1 - \phi)^2 \xi \, d\Omega\right). \quad (5.6)$$

Assuming the distribution of measurement errors is isotropic, with covariance $\sigma^2 \mathcal{I}$, this translates to a regularisation term:

$$\alpha_{\phi} = \frac{\sigma^2}{\gamma^2}, \quad \mathcal{J}_{\phi}(\phi) = \int_{\Omega} (1 - \phi)^2 \xi \, d\Omega. \quad (5.7)$$

A greater exposition of this link between priors and regularisation parameters is given in appendix 5.A.

We solve the inverse problem for the regularisation term shown in (5.7), as well as the heuristic regularisation (5.5) and no regularisation.

The initial guess for the control fields can have a large influence on the optimisation problem. For the ϕ field, we use an initial guess of 1 everywhere. For C , we take the view that the initial guess should be the field required to reproduce the observations on grounded ice as closely as possible with a uniform $\phi = 1$. Hence, before carrying out the full optimisation including both control fields, we solve an inverse problem for C with fixed $\phi = 1$, matching speeds only on grounded ice and use this as the initial guess. This has the effect of considerably reducing the deviation of ϕ from 1 in the solution. This has the added bonus of allowing us to search independently for the regularisation parameters α_C and α_{ϕ} .

5.2.2 Fracture data assimilation through time

As previously noted, the field ϕ contains contributions from sources that cannot easily be distinguished by the spatial scales on which they vary. However, it seems likely that the contribution to ice softness due to fracturing varies on a shorter *temporal* scale than any other contribution. Hence, while attributing ice softness to the presence of fractures requires a large number of

assumptions, we can reasonably attribute changes in ice softness required by the model to fit observations over monthly-annual timescales to the fracturing or ‘healing’ of ice.

Given a series of timesteps with times $\{t_i | i = 1, \dots, n\}$, separated by Δt (e.g. one month), we solve the following inverse problem for the control parameters (C_i, ϕ_i) at each timestep:

$$(C_i, \phi_i) = \underset{C_i, \phi_i}{\operatorname{argmin}} \{ \mathcal{J}_m(u_i, u_{o_i}) + \alpha_C \mathcal{J}_C(C_i) + \alpha_\phi \mathcal{J}_\phi(\phi_i) + \frac{\alpha_t}{\Delta t} \mathcal{J}_t(\phi_i, \phi_{i-1}) \}, \quad (5.8)$$

where we have introduced the regularisation through time $\mathcal{J}_t(\phi_i, \phi_{i-1})$ relating the softness in the current timestep to that of the previous timestep. Though not particularly sophisticated, a method such as described by Eq. (5.8) is immediately amenable to the introduction of fracture data in the form of $\mathcal{J}_t$. Previous studies (Hogg et al., 2017; Selley et al., 2021) have used such a method with $\mathcal{J}_t = \int_{\Omega} |\phi_i - \phi_{i-1}|^2 d\Omega$ and we modify this only slightly here. We propose the regularisation function:

$$\mathcal{J}_t = \int_{\Omega} (1 - |f_i - f_{i-1}|) \times |\phi_i - \phi_{i-1}|^2 d\Omega \quad (5.9)$$

where f_i is the map showing the locations of fractures over the domain at time t_i . Hence, changes to the softness field are preferred in regions in which the fracture pattern has changed, with a strength that depends on the length of the timestep and the regularisation parameter α_t . For these problems, we also set $\alpha_\phi = 0$.

We carry out such a procedure on Pine Island Glacier with 5 years of speed and fracture observations from December 2016 to December 2021, and timesteps of one month. This captures three calving events and the major disintegration of the southern shear margin of the ice shelf, and that of the calving front of Piglet Glacier (Joughin et al., 2021; Surawy-Stepney et al., 2023a). For each month, we use the mean speeds measured over that month as our observed speeds, and median fracture map composites.

5.3 Results

5.3.1 Snapshot inverse problems

We begin with the results of both methods of fracture data assimilation applied to a snapshot inverse problem on Pine Island Ice Shelf. We consider how using the data-informed regularisation alters the problem compared to a case of no regularisation, and the heuristic regularisation of eq. (5.5). We refer to optimisations in which ϕ is unregularised as ‘case 1’, those in which we apply heuristic Tikhonov regularisation as ‘case 2’ and those in which we apply the data-informed regularisation given by eq. (5.7) as ‘case 3’. We look at the misfits, the output control fields and changes to the problem conditioning.

Softness fields

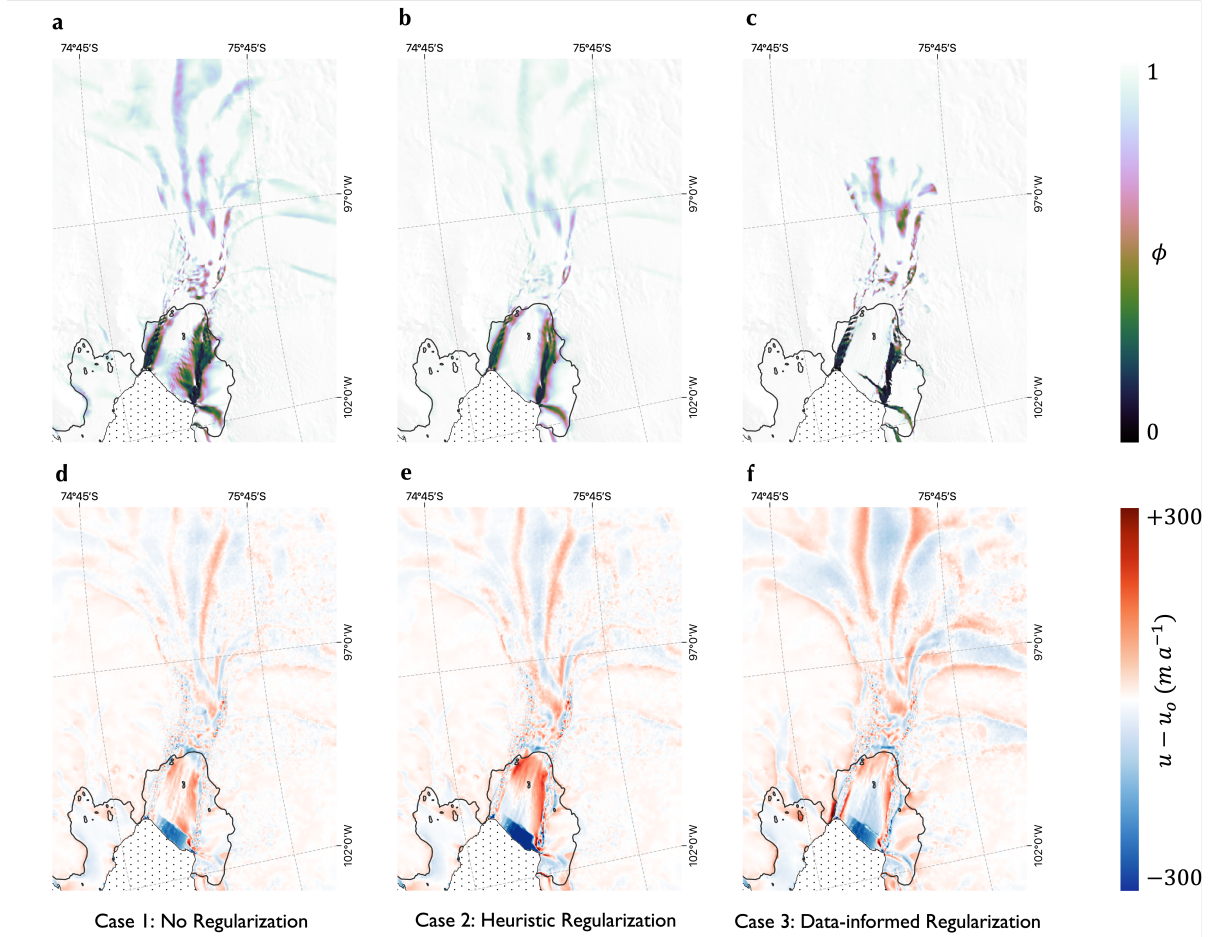


Figure 5.2: Solutions to the inverse problem with three methods of regularisation. a-c) Stiffness fields for the unregularised, heuristically regularised and data-informed inverse problems respectively. d-f) Misfits for the unregularised, heuristically regularised and data-informed inverse problems respectively. Background images are the MODIS Mosaic of Antarctica (Haran et al., 2021), and grounding lines (shown in black) are according to Rignot et al., 2016.

The ϕ fields in each case are substantively different on Pine Island Glacier for this set of geometry and speed data (Fig. 5.2). This is true for both the grounded and floating ice. Firstly, in both cases 1 and 2 there are large deviations of ϕ from 1 far upstream of the grounding line including substantial softening in the shear margins of even slow-flowing ice streams (Fig. 5.2 a, b). This is completely absent in the solution to case 3 (Fig. 5.2 c). Given the lower misfits in these regions (Fig. 5.2 e, f) compared to case 3 (Fig. 5.2 g), it appears that the model finds it difficult to compensate for the velocity gradients at the margins of the tributary ice streams by enhancing gradients in C where it is encouraged not to alter ϕ . In the large fractured region upstream of the grounding line (Fig. 5.1 a, b), the solution for case 3 shows higher amplitude deviations of ϕ from 1 than in cases 1 and 2.

The differences in ϕ between the different forms of regularisation are just as pronounced on the floating ice shelf. In cases 1 and 2, softnesses on the ice shelf are smooth and spread to large distances either side of the shear margins. In contrast, in the solution to case 3, softness is concentrated in the shear margin with larger amplitude deviations of ϕ from 1 confined to a smaller area. Some of the degeneracy in the solution for ϕ on Pine Island Glacier occurs because the central shelf moves almost entirely by pure advection. In the absence of any real strain rates, many solutions for ϕ in this region fit the data equally well, and the explicit prior appears to help with this.

The rift that propagated across the ice shelf at the time the speed data was collected caused a discontinuity in the data. The feature is much more clearly resolved in the solution to case 3 than case 2, and even case 1. Hence, it appears difficult for the model to assign low values of ϕ to a region very local to the rift unless encouraged to do so. This is perhaps due to the distributed influence of the ice at the terminus on the dynamics of the ice shelf as a whole (Joughin et al., 2021; Bevan et al., 2023). On the floating ice, the misfit for case 3 is considerably better than case 2 (Fig. 5.2 e-f).

The effect on problem conditioning

A well conditioned problem damps the contribution of oscillatory, high frequency components of the input data, such as uncorrelated noise in the speed data, while an ill-conditioned problem

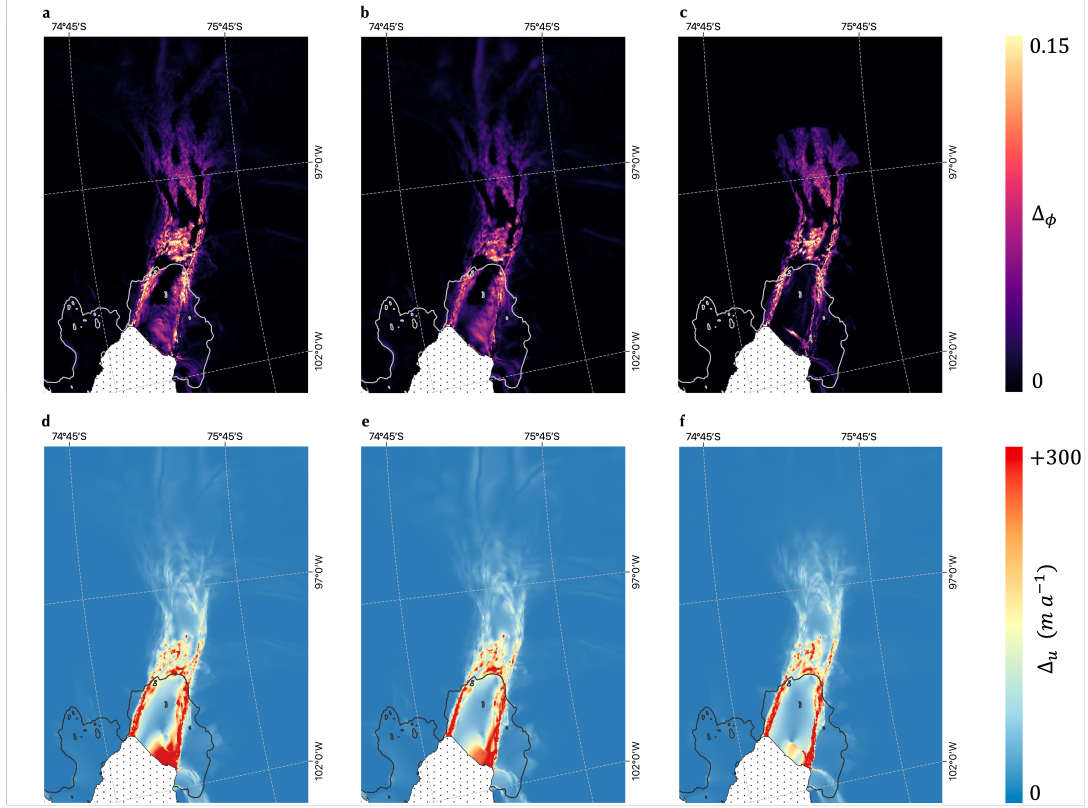


Figure 5.3: Variation in the solutions for the three methods of regularisation. a-c) Standard deviation in the softness fields between 10 optimisations with Gaussian noise added to the speed data for the unregularised, heuristically regularised and data-informed inverse problems respectively. d-f) Associated standard deviations in the modelled speed for the unregularised, heuristically regularised and data-informed inverse problems respectively.

is highly sensitive to it. Bringing prior information into the inverse problem has the potential to change the conditioning by enhancing gradients in previously flat regions of the cost landscape. In order to test this change in conditioning, we investigated the impact of perturbations in the input velocity data on the spread of resulting ϕ and u fields.

We performed 10 inverse problems with the addition of uncorrelated Gaussian noise to the input data for the case of data-informed regularisation, heuristic regularisation and no regularisation. Noise was added with a mean of zero and standard deviation of 10% of the local speed. In each case, we measured the cell-wise standard deviation over the 10 C , ϕ and u output fields (Fig. 5.3).

Unsurprisingly, the regularised problems show a smaller spread in the solutions for the control fields - suggesting improved conditioning (Fig. 5.3 a-c). The spread of solutions for ϕ is

confined in the case of the data-informed regularisation to the regions of very low ξ , while in those regions, the standard deviations are of similar magnitude to the unregularised case. This is expected because in essence, the data-informed regularisation separate regions where high-amplitude deviations of ϕ from 1 are penalised (where $\xi \rightarrow 1$) from regions that are entirely unregularised. The heuristic regularisation, case 2, that is explicitly devised to improve the problem conditioning indeed looks to result in the most well-conditioned problem on grounded ice. However, this is not the case on the central ice shelf, where the degeneracy described above leads to a larger solution variance than in the data-informed case. The spreads of speed (Fig. 5.3 d-f) reflect the spreads of the control fields.

5.3.2 Inverse problems through time

We consider two instances of temporal regularisation of the type described in eq. (5.8): the ‘data-informed’ case:

$$\mathcal{J}_t = \int_{\Omega} (1 - |f_i - f_{i-1}|) \times |\phi_i - \phi_{i-1}|^2 d\Omega \quad \text{and } \alpha_{\phi} = 0, \alpha_t = 5 \times 10^6, \quad (5.10)$$

and the ‘heuristic’ case:

$$\mathcal{J}_t = \int_{\Omega} |\phi_i - \phi_{i-1}|^2 d\Omega \quad \text{and } \alpha_{\phi} = 1.5 \times 10^9, \alpha_t = 10^4, \quad (5.11)$$

equivalent to that used in Selley et al., 2021.

Using fracture data in successive timesteps to weight the temporal regularisation has a significant effect on the softness fields over the five years of observations compared with the simpler approach (Fig. 5.4 a, b). The data-informed case shows features of low ϕ resembling crevasses starting to appear in the southern shear margins after ~ 18 months. Rifts that led to the calving of large icebergs in October 2018 and February 2020 are visible as highly linear features of soft ice in the solutions to the data-informed problem (Fig. 5.4 b). These features are visible in Fig. 5.4 a, though are less easily discernible as rifts. The softness fields in the two cases appear similar by May 2021, with that of the heuristic regularisation looking essentially like a blurred out version of the data-informed case. Both show the southerly migration of the seaward end of the southern shear margin through the time period, and a stripe of soft ice connecting the

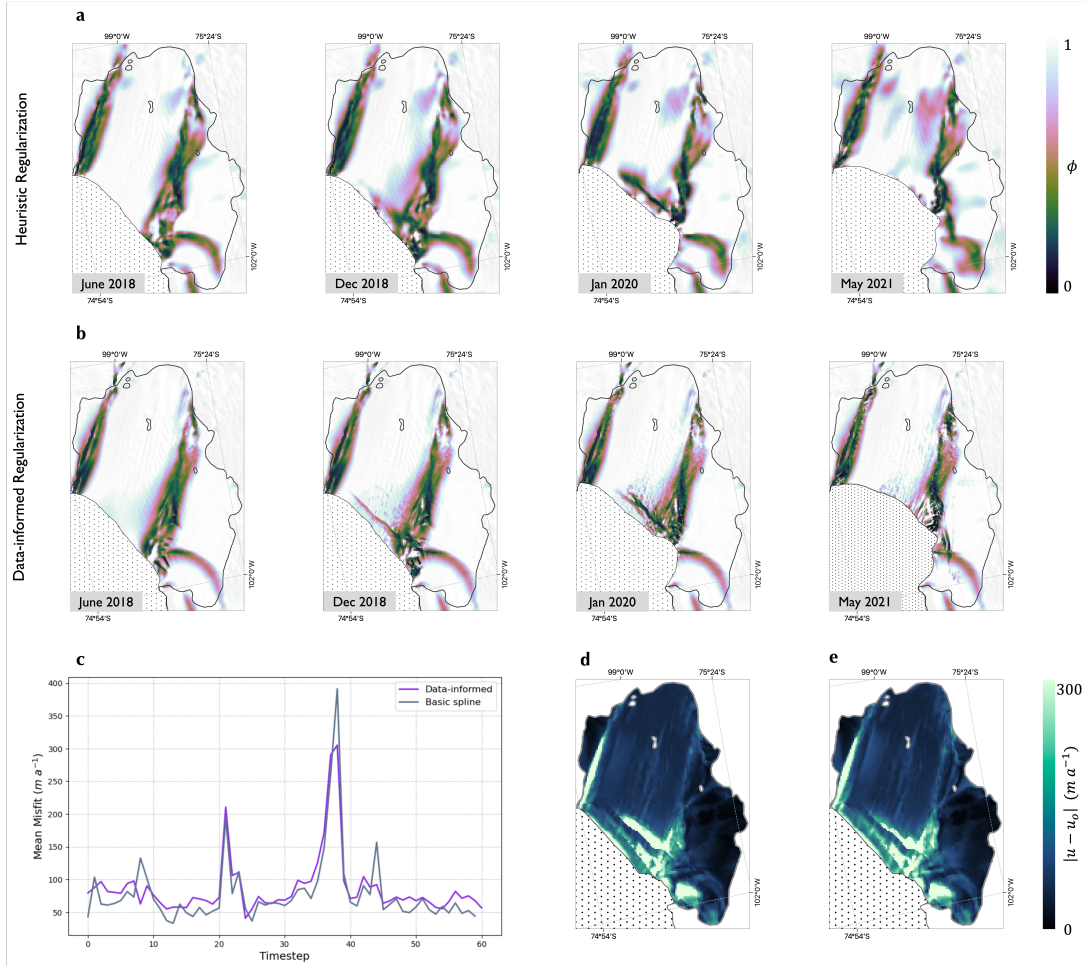


Figure 5.4: The evolution of the stiffness on Pine Island Ice Shelf between June 2018 and May 2021 for heuristic (a) and data-informed (b) regularisation. c) Mean misfit over the ice shelf for the two cases through time. d) Mean misfit over the ice shelf for the heuristically-regularised problem. e) Mean misfit over the ice shelf for the data-informed problem. Background images are the MODIS Mosaic of Antarctica (Haran et al., 2021), and grounding lines (shown in black) are according to Rignot et al., 2016.

shear margins of Pine Island and Piglet ice shelves by the end. It is only clear in Fig. 5.4 b that this stripe of soft ice corresponds to a number of long, parallel rifts. Diffuse blobs of softness can be seen on the central ice shelf in Fig. 5.4 a (May 2021) which are not present in the data-informed case. As the simulation contains no thickness advection and no accumulation rate is specified, these, which are missed in the case in which we encourage ϕ not to change arbitrarily, could be the result of localised thinning. Otherwise they could once more be the result of under-determinedness.

Throughout the simulation period, the misfits associated with each case are very similar, with generally slightly larger mean misfits over the region in the data-informed case (Fig. 5.4 c, d). The exceptions to this are in the months in which calving events occur - where the misfit is

generally elevated as the model struggles to deal with the sudden appearance of large velocity gradients near the glacier terminus. At these times, the data-informed case does slightly better as the observations of rift growth nudge the model towards the right pattern of softening near the terminus.

5.4 Discussion

The problem of accurately estimating ice softness and basal slip fields from observations of ice speed is dogged by the specter of ill-posedness. In an effort to improve this, we have presented two simple ways of assimilating fracture data into the inverse problem for a marine-terminating ice stream, as a way of providing the problem with genuine prior information. The effect of these methods, their success and what we learn from the experiments depends on whether we are interested in grounded or floating ice.

5.4.1 Grounded ice

Fractures are only a contributing factor to ϕ , and the efficacy of the methods presented here depend on the extent to which we apportion softness to fracturing. We have seen in our example of snapshot problems over Pine Island that softness fields on grounded ice found using the data-informed regularisation vary considerably within contiguous areas of observed fracture (Fig. 5.2 c). If fracturing in these regions were truly the main contribution of changes in softness from 1, one would expect consistency in ϕ within this region. This suggests that here at least, the dominant contribution to ice softness is not fracturing but some combination of other factors. This is consistent with the fact that prescribing the ξ field on the grounded ice dampens the softness away from these regions of fracture but does not change the shape of the solution greatly otherwise. This suggests that observations of surface fracture on grounded ice have limited use in reducing the degeneracy between solutions caused by overlapping C and ϕ fields. This directly answers in the negative the suggestion made in Surawy-Stepney et al., 2023a.

Grounded crevasses don't influence the dynamics

This also constitutes substantial evidence that this kind of grounded surface crevasse has a limited impact on ice dynamics, despite the very low basal frictions we find in this part of Pine Island Glacier (described in Joughin et al., 2009) and the compensating enhanced membrane

stresses. This is consistent with previous assumptions made about the depth of these crevasses (Benn and Evans, 2014).

Further use of strain rates

The softness fields on grounded ice (and also substantially on floating ice) found using heuristic regularisation (Fig. 5.2 b) mimic many of the features of the strain rate map in Fig. 5.1 c. This suggests great potential for this data to be used to constrain the softness but the prior we are currently using doesn't capture our assumptions fully. A better prior might, for example, be to assume softness is linear in principal strain rate. Future work should look to investigate different priors that better utilise the strain rate data at our disposal.

5.4.2 Floating ice

We have shown in both snapshot inverse problems and time-dependent inverse problems that the resulting softness fields on the floating ice appear more like what we would expect if the softening were due to fracturing/shearing of ice compared to more heuristic methods. The model is happy to concentrate softness in regions of observed fracture or high shear without suffering a worse misfit. This results in softness fields that appear more likely to accurately represent the material properties of the ice shelf at the time the ice speed data was collected. Unfortunately, the ill-posedness of the problem means that methods of evaluating the efficacy of these methods do not extend far beyond a visual assessment of whether the solutions 'look right'. However, this is a technique that should not be ignored!

Should these methods be used?

The example we have chosen for the snapshot inverse problem, where a large rift can be seen on the central trunk of PIIS along with an associated discontinuity in u_o , is somewhat contrived to show the differences between the regularisation methods discussed. It is unlikely that a model-user looking to initialise a century-long simulation would choose such data, and would more likely choose a more representative initial state. To some extent, this is also true for fractured shear margins. Without a method of sensibly evolving the softness field through time, it is reasonable to initialise a model with a smoother solution for (C, ϕ) that might be less representative of the true initial state, but is also less *specific* to it. Hence, softness fields acquired

with the use of fracture data are more likely to be useful in diagnostic simulations, or transient simulations with timescales on the order of years.

Use in constraining damage models

A major motivation for investigating these methods of constraining the inverse problem is that the time-varying solutions have potential use in evaluating models that take a continuum damage mechanics approach to parameterising the effect of fractures on large-scale ice rheology (e.g. Sun et al., 2017). In particular, the softness fields could be used to constrain the way in which a scalar damage field, that acts isotropically on the rheology, is evolved by such a model (Borstad et al., 2016).

5.4.3 A note on L-curves

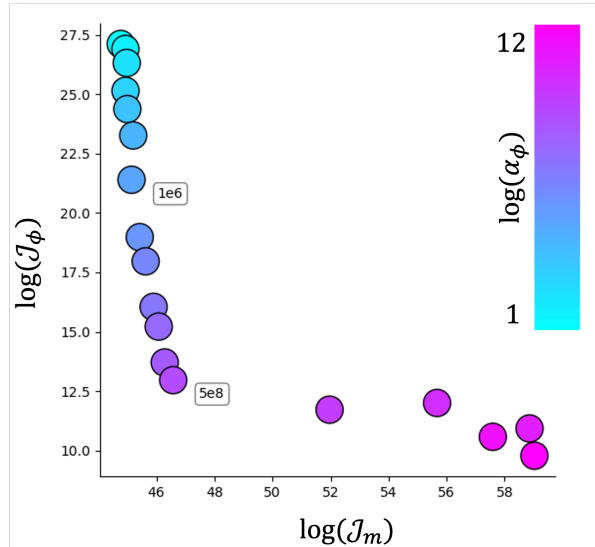


Figure 5.5: L-curve for the data-informed regularisation. Solution norm (y) and misfit (x) are plotted on a logarithmic scale for different choices of the regularisation parameter α_ϕ .

Fig. 5.5 shows, on a logarithmic scale, solution and misfit norms at convergence for a number of possible regularisation parameters α_ϕ , known as an L-curve (Hansen, 2000). The intuition is that the optimal position on the curve is where changes to the regularisation parameter cause balanced deviations in solution and misfit norm, i.e. at the corner of the curve. In our case this is $\alpha_\phi \approx 5 \times 10^8$. However, this choice of parameter results in solutions with fewer crevasse features than we expect to see - such as the rift near the ice shelf terminus (Fig. 5.4 b). Hence

we choose a parameter an order of magnitude smaller, where we are satisfied with the misfit (staying on the ‘vertical branch’ of the L-curve) but can see some of the detail we believe should be present in the softness field. Though very useful, L-curve analysis can be a blunt instrument and should always be used alongside other heuristics such as visual assessment of the control fields in deciding the regularisation parameter. Its use is based on the assertion that the preferred solution to an inverse problem is one that contains the least extraneous structure (Wolovick et al., 2023). However, for structure to be deemed ‘extraneous’, a cost function that precisely encodes the entirety of your prior knowledge is required. This tendency for L-curve analysis to produce over-regularized solutions has been noted previously (e.g. Chamorro-Servent et al., 2019; Milovic et al., 2021; Recinos et al., 2023).

5.5 Conclusions

We have introduced two ways in which fracture location data, and in one case strain rate data, can be used as prior information to inform the estimation of basal slip and ice softness fields from observations of ice speed. Applications of these methods to a marine-terminating ice stream in West Antarctica show little improvement on grounded ice compared to a popular heuristic regularisation method. However, on floating ice, we see the resolution of fracture features in the softness fields without impacting the misfit, suggesting that such methods can provide us with softness fields that better represent the true material properties of the ice shelf. Such softness fields have potential use in diagnostic modelling, and in constraining models seeking to evolve softness fields in time.

Appendix

5.A Deriving a regularisation term from a prior distribution

Consider a version of the inverse problem in which the C field is known. Then the forward model solves $u(x) = f(\phi(x))$. We can replace this with the stochastic model:

$$U = f(\Phi) + E \quad (5.12)$$

where U , Φ and E are random variables representing modelled speed, the stiffness ϕ and an additive error respectively (Calvetti and Somersalo, 2018). Let the error be governed by the probability distribution p_E . Eq. (5.12) then implies:

$$p_{U|\Phi}(u|\phi) = p_E(u - f(\phi)) \quad (5.13)$$

and Bayes' rule gives:

$$p_{\Phi|U}(\phi|u) \propto p_E(u - f(\phi))p_{\Phi}(\phi), \quad (5.14)$$

where $p_{\Phi}(\phi)$ constitutes our prior for the distribution of Φ .

We assume that as $\xi \rightarrow 1$, $\phi \rightarrow 1 + \epsilon$, where $\epsilon \sim \mathcal{N}(0, \gamma^2)$. We can encode this as the following relation for Φ :

$$\xi\Phi = \gamma W \quad (5.15)$$

where γ controls how much we allow Φ to vary, and W is a Gaussian random field with zero mean and identity covariance. Then

$$p_{\Phi}(\phi) \propto \exp\left(-\frac{1}{2\gamma^2} \int_{\Omega} (1 - \phi)^2 \xi \, d\Omega\right). \quad (5.16)$$

Assuming Gaussian error distribution with zero mean and isotropic covariance $\sigma^2\mathcal{I}$, gives

$$p_E(u - f(\phi)) \propto \exp\left(-\frac{1}{2\sigma^2} \int_{\Omega} (u - f(\phi))^2 \, d\Omega\right) \quad (5.17)$$

Hence, from eq. (5.14):

$$p_{\Phi|U}(\phi|u) \propto \exp\left\{-\frac{1}{2\sigma^2} \left(\int_{\Omega} (u - f(\phi))^2 \, d\Omega + \frac{\sigma^2}{\gamma^2} \int_{\Omega} (1 - \phi)^2 \xi \, d\Omega\right)\right\}. \quad (5.18)$$

A maximum a posteriori estimate for $\phi(x)$ given $u(x)$ is, therefore, the solution to:

$$\phi_{MAP} = \arg \min \left\{ \int_{\Omega} (u - f(\phi))^2 d\Omega + \frac{\sigma^2}{\gamma^2} \int_{\Omega} (1 - \phi)^2 \xi d\Omega \right\}, \quad (5.19)$$

i.e. a minimisation over our original cost function with:

$$\alpha_{\phi} = \frac{\sigma^2}{\gamma^2}, \quad \mathcal{J}_{\phi}(\phi) = \int_{\Omega} (1 - \phi)^2 \xi d\Omega. \quad (5.20)$$

A reasonable prior might be to allow ϕ to vary from 1 away from fractured areas ($\xi \rightarrow 1$) with a standard deviation of 0.1, corresponding to $\gamma^2 = 0.01$. Taking σ to be of order 100 m/y, this gives us a value of $\alpha_{\phi} \sim 10^6$ for the coefficient of the ϕ regularisation term in our initial cost function. Note, we have assumed in this analysis a spatially uniform estimate of uncertainty in our velocity observations. If a more reliable estimate of this uncertainty existed, it could be included as a modification to ξ .

5.B Derivatives of $\mathcal{J}_{\phi}$

The inverse problem is solved using a nonlinear conjugate gradient method. This requires the projection of the Jacobian $\nabla \mathcal{J}(\phi)$ along the direction of the residual $u - u_o$.

Let $\phi = \phi_0 e^q$ so that $\phi > 0$.

Define the stiffness part of the cost function as:

$$\mathcal{J}_{\phi} = \int_{\Omega} (1 - \phi)^2 \xi d\Omega \quad (5.21)$$

The Gâteaux differential is defined by the projection of the functional gradient onto the direction defined by a perturbation δq :

$$\langle \delta \mathcal{J}_{\phi}, \delta q \rangle = \lim_{\epsilon \rightarrow 0^+} \frac{\mathcal{J}_{\phi}(q + \epsilon \delta q) - \mathcal{J}_{\phi}(q)}{\epsilon}, \quad (5.22)$$

where the binary operator $\langle \cdot, \cdot \rangle$ is the inner product over the space of functions. In our case:

$$\langle \delta \mathcal{J}_\phi, \delta q \rangle = - \int_{\Omega} \delta q \phi (1 - \phi) \xi d\Omega \quad (5.23)$$

and we interpret $\phi(1 - \phi)\xi$ as the functional gradient. This is calculated in each iteration of the non-linear conjugate gradient method, and is used to update the value of q .

Bibliography

- Arthern, R. J., Hindmarsh, R. C. A., and Williams, C. R. (2015). “Flow speed within the Antarctic ice sheet and its controls inferred from satellite observations”. In: *Journal of Geophysical Research: Earth Surface* 120.7, pp. 1171–1188. DOI: <https://doi.org/10.1002/2014JF003239>.
- Benn, D. I. and Evans, D. J. (2014). *Glaciers & glaciation*. Routledge.
- Bevan, S., Cornford, S., Gilbert, L., Otosaka, I., Martin, D., and Surawy-Stepney, T. (2023). “Amundsen Sea Embayment ice-sheet mass-loss predictions to 2050 calibrated using observations of velocity and elevation change”. In: *Journal of Glaciology*, pp. 1–11. DOI: [10.1017/jog.2023.57](https://doi.org/10.1017/jog.2023.57).
- Borstad, C. P., Rignot, E., Mouginot, J., and Schodlok, M. P. (2013). “Creep deformation and buttressing capacity of damaged ice shelves: theory and application to Larsen C ice shelf”. In: *The Cryosphere* 7.6, pp. 1931–1947. DOI: [10.5194/tc-7-1931-2013](https://doi.org/10.5194/tc-7-1931-2013).
- Borstad, C., Khazendar, A., Scheuchl, B., Morlighem, M., Larour, E., and Rignot, E. (2016). “A constitutive framework for predicting weakening and reduced buttressing of ice shelves based on observations of the progressive deterioration of the remnant Larsen B Ice Shelf”. In: *Geophysical Research Letters* 43.5, pp. 2027–2035. DOI: <https://doi.org/10.1002/2015GL067365>.
- Brinkerhoff, D. J. and Johnson, J. V. (2013). “Data assimilation and prognostic whole ice sheet modelling with the variationally derived, higher order, open source, and fully parallel ice sheet model VarGlaS”. In: *The Cryosphere* 7.4, pp. 1161–1184. DOI: [10.5194/tc-7-1161-2013](https://doi.org/10.5194/tc-7-1161-2013).
- Calvetti, D. and Somersalo, E. (2018). “Inverse problems: From regularization to Bayesian inference”. In: *Wiley Interdisciplinary Reviews: Computational Statistics* 10.3, e1427.
- Chamorro-Servent, J., Dubois, R., and Coudière, Y. (2019). “Considering New Regularization Parameter-Choice Techniques for the Tikhonov Method to Improve the Accuracy of Electrocardiographic Imaging”. In: *Frontiers in Physiology* 10. ISSN: 1664-042X. DOI: [10.3389/fphys.2019.00273](https://doi.org/10.3389/fphys.2019.00273).
- Chartrand, R. (2017). “Numerical differentiation of noisy, nonsmooth, multidimensional data”. In: *2017 IEEE Global Conference on Signal and Information Processing (GlobalSIP)*, pp. 244–248. DOI: [10.1109/GlobalSIP.2017.8308641](https://doi.org/10.1109/GlobalSIP.2017.8308641).
- Cornford, S. L., Martin, D. F., Graves, D. T., Ranken, D. F., Le Brocq, A. M., Gladstone, R. M., Payne, A. J., Ng, E. G., and Lipscomb, W. H. (2013). “Adaptive mesh, finite volume modeling of marine ice sheets”. In: *Journal of Computational Physics* 232.1, pp. 529–549. ISSN: 0021-9991. DOI: [10.1016/j.jcp.2012.08.037](https://doi.org/10.1016/j.jcp.2012.08.037).

- Cornford, S. L., Martin, D. F., Payne, A. J., Ng, E. G., Le Brocq, A. M., Gladstone, R. M., Edwards, T. L., Shannon, S. R., Agosta, C., Broeke, M. R. van den, et al. (2015). “Century-scale simulations of the response of the West Antarctic Ice Sheet to a warming climate”. In: *The Cryosphere* 9.4, pp. 1579–1600.
- Cuffey, K. M. and Paterson, W. S. B. (2010). *The physics of glaciers*. Academic Press.
- Goldberg, D. N., Gourmelen, N., Kimura, S., Millan, R., and Snow, K. (2019). “How Accurately Should We Model Ice Shelf Melt Rates?”. In: *Geophysical Research Letters* 46.1, pp. 189–199. DOI: <https://doi.org/10.1029/2018GL080383>.
- Gudmundsson, G. H., Paolo, F. S., Adusumilli, S., and Fricker, H. A. (2019). “Instantaneous Antarctic ice sheet mass loss driven by thinning ice shelves”. In: *Geophysical Research Letters* 46.23, pp. 13903–13909. DOI: <https://doi.org/10.1029/2019GL085027>.
- Habermann, M., Truffer, M., and Maxwell, D. (2013). “Changing basal conditions during the speed-up of Jakobshavn Isbræ, Greenland”. In: *The Cryosphere* 7.6, pp. 1679–1692. DOI: [10.5194/tc-7-1679-2013](https://doi.org/10.5194/tc-7-1679-2013).
- Hansen, P. C. (1994). “Regularization tools: A Matlab package for analysis and solution of discrete ill-posed problems”. In: *Numerical algorithms* 6.1, pp. 1–35. DOI: [10.1007/BF02149761](https://doi.org/10.1007/BF02149761).
- (2000). “The L-curve and its use in the numerical treatment of inverse problems”. English. In: *InviteComputational Inverse Problems in Electrocardiology*. InviteComputational Inverse Problems in Electrocardiology ; Conference date: 01-01-2000. WIT Press.
- Haran, T. M., Bohlander, J., Scambos, T. A., Painter, T. H., and Fahnestock, M. A. (2021). *MODIS Mosaic of Antarctica 2003-2004 (MOA2004) Image Map, Version 2*. DOI: [10.5067/68TBT0CGJS0J](https://doi.org/10.5067/68TBT0CGJS0J).
- Hogg, A. E., Shepherd, A., Cornford, S. L., Briggs, K. H., Gourmelen, N., Graham, J. A., Joughin, I., Mouginot, J., Nagler, T., Payne, A. J., Rignot, E., and Wuite, J. (2017). “Increased ice flow in Western Palmer Land linked to ocean melting”. In: *Geophysical Research Letters* 44.9, pp. 4159–4167. DOI: <https://doi.org/10.1002/2016GL072110>.
- Izeboud, M. and Lhermitte, S. (2023). “Damage detection on antarctic ice shelves using the normalised radon transform”. In: *Remote Sensing of Environment* 284, p. 113359. ISSN: 0034-4257. DOI: [10.1016/j.rse.2022.113359](https://doi.org/10.1016/j.rse.2022.113359).
- Joughin, I., Shapero, D., Smith, B., Dutrieux, P., and Barham, M. (2021). “Ice-shelf retreat drives recent Pine Island Glacier speedup”. In: *Science Advances* 7.24, eabg3080. DOI: [10.1126/sciadv.abg3080](https://doi.org/10.1126/sciadv.abg3080).
- Joughin, I., Tulaczyk, S., Bamber, J. L., Blankenship, D., Holt, J. W., Scambos, T., and Vaughan, D. G. (2009). “Basal conditions for Pine Island and Thwaites Glaciers, West Antarctica, determined using satellite and airborne data”. In: *Journal of Glaciology* 55.190, pp. 245–257. DOI: [10.3189/002214309788608705](https://doi.org/10.3189/002214309788608705).
- Lai, C.-Y., Kingslake, J., Wearing, M. G., Chen, P.-H. C., Gentine, P., Li, H., Spergel, J. J., and Wessem, J. M. van (2020). “Vulnerability of Antarctica’s ice shelves to meltwater-driven fracture”. In: *Nature* 584.7822, pp. 574–578.
- MacAyeal, D. R. (1992). “The basal stress distribution of Ice Stream E, Antarctica, inferred by control methods”. In: *Journal of Geophysical Research: Solid Earth* 97.B1, pp. 595–603.

- Milovic, C., Prieto, C., Bilgic, B., Uribe, S., Acosta-Cabronero, J., Irarrazaval, P., and Tejos, C. (2021). “Comparison of parameter optimization methods for quantitative susceptibility mapping”. In: *Magnetic resonance in medicine* 85.1, pp. 480–494. DOI: 10.1002/mrm.28435.
- Morlighem, M. (2022). *MEaSURES BedMachine Antarctica, Version 3*. DOI: 10.5067/FPSUOV1MWUB6.
- Morlighem, M., Seroussi, H., Larour, E., and Rignot, E. (2013). “Inversion of basal friction in Antarctica using exact and incomplete adjoints of a higher-order model”. In: *Journal of Geophysical Research: Earth Surface* 118.3, pp. 1746–1753. DOI: <https://doi.org/10.1002/jgrf.20125>.
- Petra, N., Zhu, H., Stadler, G., Hughes, T. J., and Ghattas, O. (2012). “An inexact Gauss-Newton method for inversion of basal sliding and rheology parameters in a nonlinear Stokes ice sheet model”. In: *Journal of Glaciology* 58.211, pp. 889–903. DOI: 10.3189/2012JoG11J182.
- Ranganathan, M., Minchew, B., Meyer, C. R., and Gudmundsson, G. H. (2021). “A new approach to inferring basal drag and ice rheology in ice streams, with applications to West Antarctic Ice Streams”. In: *Journal of Glaciology* 67.262, pp. 229–242. DOI: 10.1017/jog.2020.95.
- Recinos, B., Goldberg, D., Maddison, J. R., and Todd, J. (2023). “A framework for time-dependent Ice Sheet Uncertainty Quantification, applied to three West Antarctic ice streams”. In: *The Cryosphere Discussions* 2023, pp. 1–46. DOI: 10.5194/tc-2023-27.
- Rignot, E., Mouginot, J., and Scheuchl, B. (2016). *MEaSURES Antarctic Grounding Line from Differential Satellite Radar Interferometry, Version 2*. Boulder, Colorado USA. NASA National Snow and Ice Data Center Distributed Active Archive Center. doi: <https://doi.org/10.5067/IKBWW4RYHF1Q>. Accessed 15/01/2021.
- Selley, H. L., Hogg, A. E., Cornford, S., Dutrieux, P., Shepherd, A., Wuite, J., Floricioiu, D., Kusk, A., Nagler, T., Gilbert, L., Slater, T., and Kim, T.-W. (Feb. 2021). “Widespread increase in dynamic imbalance in the Getz region of Antarctica from 1994 to 2018”. In: *Nature Communications* 12.1, p. 1133. ISSN: 2041-1723. DOI: 10.1038/s41467-021-21321-1.
- Seroussi, H. et al. (2020). “ISMIP6 Antarctica: a multi-model ensemble of the Antarctic ice sheet evolution over the 21st century”. In: *The Cryosphere* 14.9, pp. 3033–3070. DOI: 10.5194/tc-14-3033-2020.
- Sun, S., Cornford, S. L., Moore, J. C., Gladstone, R., and Zhao, L. (2017). “Ice shelf fracture parameterization in an ice sheet model”. In: *The Cryosphere* 11.6, pp. 2543–2554. DOI: 10.5194/tc-11-2543-2017.
- Sun, S. and Gudmundsson, G. H. (2023). “The speedup of Pine Island Ice Shelf between 2017 and 2020: reevaluating the importance of ice damage”. In: *Journal of Glaciology*, pp. 1–9. DOI: 10.1017/jog.2023.76.
- Surawy-Stepney, T., Hogg, A. E., Cornford, S. L., and Hogg, D. C. (2023a). “Mapping Antarctic crevasses and their evolution with deep learning applied to satellite radar imagery”. In: *The Cryosphere* 17.10, pp. 4421–4445. DOI: 10.5194/tc-17-4421-2023.
- Surawy-Stepney, T., Hogg, A. E., Cornford, S. L., and Davison, B. J. (Jan. 2023b). “Episodic dynamic change linked to damage on the thwaites glacier ice tongue”. In: *Nature Geoscience*. ISSN: 1752-0908. DOI: 10.1038/s41561-022-01097-9.

- Vaughan, D. G. (1993). “Relating the occurrence of crevasses to surface strain rates”. In: *Journal of Glaciology* 39.132, pp. 255–266. DOI: 10.3189/S0022143000015926.
- Wolovick, M., Humbert, A., Kleiner, T., and Rückamp, M. (2023). “Regularization and L-curves in ice sheet inverse models: a case study in the Filchner–Ronne catchment”. In: *The Cryosphere* 17.12, pp. 5027–5060. DOI: 10.5194/tc-17-5027-2023.
- Wuite, J., Hetzenecker, M., Nagler, T., and Scheiblauer, S. (2021). *ESA Antarctic Ice Sheet Climate Change Initiative (Antarctic_Ice_Sheet_cci): Antarctic Ice Sheet Monthly Velocity from 2017 to 2020, Derived from Sentinel-1, v1*. DOI: 10.5285/00fe090efc58446e8980992a617f632f.
- Zhao, J., Liang, S., Li, X., Duan, Y., and Liang, L. (2022). “Detection of Surface Crevasses over Antarctic Ice Shelves Using SAR Imagery and Deep Learning Method”. In: *Remote Sensing* 14.3. ISSN: 2072-4292. DOI: 10.3390/rs14030487.
- Zwally, H. J., Giovinetto, M. B., Beckley, M. A., and Saba, J. L. (2012). “Antarctic and Greenland drainage systems”. In: *GSFC Cryospheric Sciences Laboratory* 265.

Chapter 6

Discussion and conclusions

In this thesis, I have presented a set of papers linked by the themes of fracturing and the sensitivity of ice sheet dynamics to marginal ice conditions. This was in large part motivated by the difficulty that continuum ice sheet models have in representing the processes of material failure and its - likely significant - effects on ice flow. The greater part of the work has been in developing preliminary observational and modelling techniques (chapters 2, 3 and 5) that provide tools for investigating the links between fracturing and ice sheet dynamics. Chapters 3 and 5 also explore these links via the application of these tools to specific parts of the Amundsen Sea Embayment. Additionally, I have performed more in-depth case studies aimed at understanding the role of fracturing in short-term speed variation on the Thwaites Glacier Ice Tongue (chapter 2) and the nature of the control placed on glacier dynamics by landfast sea ice in the Larsen-B Embayment (chapter 4). Collectively, the work presented here has involved the processing of Sentinel-1 data, the development of computer vision methods - including the use of deep learning - to analyse these data, the development and application of the BISICLES ice sheet model for diagnostic experiments, sensitivity analysis and inverse problems.

The aim of this section is to briefly re-emphasise the key findings and methodological advances of each piece of work, expand on certain things that can be learned from the articles collectively, and discuss, within the context of the current literature, possible future avenues of work motivated by the studies presented here.

6.1 Summary of principal findings

6.1.1 A new dataset of crevasses over the Antarctic Ice Sheet margin

Expanding on methods developed in chapter 2, I established in chapter 3 a method for creating pan-continental maps of crevasses from Sentinel-1 backscatter data at 50 m resolution, using a mixture of well-established deep learning techniques and novel training and filtering methods. Arguably, this is the single biggest component of the work presented in this thesis and its most significant contribution. The paucity of data in this area has inhibited our progress in understanding the brittle failure modes of glacier ice and ultimately the inclusion of processes involving material failure (such as calving and fracturing in shear margins) in ice sheet models. The process outlined in chapter 3 represents the first example of the large-scale extraction of crevasses on grounded ice, which required the use of radar remote sensing to penetrate through snow-bridges. Though it is not the first dataset of ice-shelf fractures in Antarctica, with that of Lai et al., 2020 being the most widely used to-date, and others including those of Izeboud and Lhermitte, 2023 making useful contributions, this work represents the first viable method for the routine extraction of crevasse-location data across the Antarctic continent. The process unifies the extraction of crevasse features on grounded and floating ice, while making use of routinely acquired data with a long-term future acquisition plan. The data improves on the resolution and accuracy of Lai et al., 2020 and Izeboud and Lhermitte, 2023, and is considerably more scalable.

6.1.2 Quantifying structural change

Observations of the evolution of crevassing at the ice sheet margin can both aid in the understanding of material failure in the ice sheet, but also provide an indication of the ‘health’ of ice shelves. A number of studies in recent times have discussed such structural evolution of ice shelves around Antarctica (e.g. Miles et al., 2020; Lhermitte et al., 2020; Benn et al., 2021). However, these discussions have been qualitative in nature, with visual analysis of satellite images used to understand the changes. A possible use for the automated extraction of crevasse location data is to move towards quantitative analysis where appropriate. Chapter 2 introduces the first such attempt to quantify structural change by looking at the amount of crevassing found by an automated detection algorithm through time. I extend this analysis in chapter 3 to larger regions and produce maps of structural change, while attempting to mitigate the

effects of changing surface ice conditions on crevasse visibility. I applied this method to the ice shelves of the Amundsen Sea Embayment showing that it is possible to quantify changes that are visible in satellite images and reported elsewhere. Given the number of different features found by the crevasse-detection algorithm, with no indication of crevasse depth, it is not yet clear whether the ‘crevasse-density’ I consider is the most useful metric when considering how crevasses might influence ice dynamics, however, this is a first step in the right direction.

Also in chapter 3, I used the BISICLES ice sheet model (essentially as a way to smooth velocity data) to create maps showing where the buttressing takes place on the ice shelves of the ASE and compared these to the fracture density change maps. I made the claim that changes in structural integrity in regions with large buttressing number will likely be accompanied by a change in the dynamics of the ice shelf. This is not particularly original, but it offers a simple way of analysing where structural changes might impact dynamics. Since the publication of this article, work by Sun and Gudmundsson, 2023 has also indicated that the fracturing of the Pine Island southern shear margin is required to explain the dynamic changes seen on the ice shelf in the last decade.

6.1.3 A link between crevassing and short-term dynamic change on the Thwaites Glacier Ice Tongue

After large calving events in the 2000s, in which it lost the majority of its surface area, the Thwaites Glacier Ice Tongue maintained a state of rapid decline in structural integrity and changeable dynamic behaviour in the 2010s (Miles et al., 2020). This included the inland propagation of crevasses in the eastern shear margin - separating the ice tongue from the more intact Eastern Ice Shelf. Work by Lhermitte et al., 2020 suggested that this kind of correlation between crevassing in the shear margin and acceleration of the ice shelf (seen both here on Thwaites and on Pine Island Ice Shelf over the same time period) constituted a feedback which would render this kind of ice shelf unstable. The question of what the precise relationship is between fracturing and speed change on this part of Thwaites Glacier formed the principal research aim of the work described in chapter 2. The evidence from this case study suggests that though the presence of crevasses is linked to the dynamic change in a non-trivial way, the system is more sensitive to external forcings than this internal feedback mechanism. Using remote sensing data,

I showed that the crevassing and ice dynamic changes were highly concurrent. Further, using inverse modelling, I showed that the presence of crevasses indeed had a mechanical effect that is likely to be required to explain the observed changes in ice speed. However, the observations of ice speed and fracture showed the changes to be reversible on short timescales, suggesting that the nonlinearity between ice flow and crevassing is weaker than other controls on the dynamics.

The work also contained the first quantitative use of timeseries of fracture maps created automatically from remote-sensing data. Furthermore, the work explored the links between these crevasse data and ice rheology through time. The ‘relative damage’ (taken as a sum over the ice softness field in a region) found by solving inverse problems through time - linked by simple temporal regularisation, show good resemblance to the time series of observed fracture densities. Building on previous work (e.g. Vieli et al., 2007; Borstad et al., 2012), this shows that fracturing can be the dominant contributor to ice softness. (Or, at least, fracturing can be the dominant contributor to *changes* in ice softness.) Additionally, it suggests that inverse problems are able to capture the effect of crevasses in a scalar field that acts isotropically on the effective viscosity. We will discuss this further in section 6.1.6.

6.1.4 Landfast sea ice does not buttress glaciers like ice shelves do

Decade-old landfast sea ice in the Larsen-B Embayment disintegrated in January 2022 and was followed by considerable changes in the calving behaviour and dynamics of a subset of the glaciers that terminate in the embayment. I showed this in chapter 4, expanding upon observations by Ochwat et al., 2023; Sun et al., 2023. This shows a clear link between the presence of landfast sea ice and the stability of the glaciers in the region, and builds on evidence for this coupling noticed elsewhere (e.g. Massom et al., 2015; Miles et al., 2017; Aoki, 2017). The ubiquity of sea ice in contact with the Antarctic Ice Sheet margin, and the changes to its extent and seasonal longevity (Purich and Doddridge, 2023) mean that a good understanding of the sensitivity of marine glaciers to the presence of sea ice and the mechanisms responsible for this coupling is valuable. Through a series of diagnostic experiments showing the effect of the inclusion of a very thin ice shelf (representing landfast sea ice) in the Larsen-B Embayment on the stress balance, I showed in chapter 4 that the sea ice does not confer stability to the glaciers through a first-order buttressing effect analogous to ice shelf buttressing. This conclusion is fairly plain - sea ice is simply too thin to make a difference to the overall stress-balance of the

system. However, I made the argument explicit due to disagreements surrounding this point in the concurrent literature (Ochwat et al., 2023; Sun et al., 2023). There is a second-order buttressing effect that is not investigated but, as suggested in the study, could be quite important, in which landfast sea ice could provide buttressing to ice mélange which, in turn, can buttress the ice shelves behind - through a mechanism like that discussed in Robel, 2017. This study also allowed me to develop the adjoint-based sensitivity analysis that I hope to extend to different aspects of ice sheet dynamics (see future work below).

6.1.5 Methods for the inclusion of fracture information in inverse problems

The estimation of control fields C (basal slipperiness) and ϕ (ice softness) used by ice sheet models like BISICLES is a difficult process. This difficulty does not arise in the inversion of the viscous tensor operator (which is even self-adjoint in the linear approximation) or in the application of well-known optimisation techniques like the nonlinear conjugate gradient method, but in the inherent ill-posedness of the problem. The only way to improve this is to introduce more prior information regarding the control fields, though the large number of disparate and varied unknowns they describe makes this difficult. I developed in chapter 5 two methods of using the fracture maps described in chapter 3 as prior information regarding the control parameter ϕ - one for snapshot inverse problems and the other for repeated inverse problems through time. Both methods indicate that fracture features on the floating ice can be resolved in the softness field without damaging the solution misfit or problem conditioning. This provides an advancement in the methods we use to solve inverse problems, resulting in control fields that we might be more confident reflect real englacial properties. The results are also relevant to the discussion of the extent to which crevassing plays a role in the dynamics of Pine Island Glacier, and the extent to which we can model the effects of crevasses with a scalar field (section 6.1.6).

In particular, the method of using crevasse maps in snapshot inversions does not appear to be useful in reducing solution degeneracy on grounded ice, which indicates that uncertainties other than the amount of fracturing dominate the deviation of ϕ from 1 in these areas. This is consistent with recent work by Hoffman et al., 2024 showing that much of this type of surface crevassing on grounded ice is only firn-deep. If so, any expected changes to the constitutive relation brought about by these crevasses would be negligible. It also contradicts the statement

I made in chapter 3 that these crevasses could enable dynamic change on the fast flowing ice streams of the Amundsen Sea Sector, as well as being a symptom of this change. This leads me to conclude that crevasses on grounded ice are not likely to have an impact on vertically-integrated effective viscosity in this region.

The methods outlined in chapter 5 are part of a broader ongoing conversation regarding the extent to which crevasses contribute to ice softness. Recent work by Gerli et al., 2023b considers correlations between the fluidity (roughly $1/A(T)^{1/3}$ where $A(T)$ is the rate factor mentioned throughout this thesis) and crevasses in certain datasets on Pine Island, Ronne-Filchner and Ross Ice Shelves. They find minimal correlation and conclude that crevasse data is not likely to be useful as a prior for ice fluidity. (Implicitly, the assumption is made that the set of solutions for the fluidity field with acceptable misfit reflect the true state of the ice.) My results are evidence against this assertion as they show that you can form softness fields that *are* correlated with crevasse maps without influencing the misfit too greatly. It is unclear the extent to which inverting for the full fluidity rather than softness affects how comparable the conclusions of the studies are, but it will depend on the initial guess for the temperature field.

6.1.6 Evidence for the link between crevasses and isotropic damage

In the introduction to this thesis, I wrote that models of material failure based on continuum damage mechanics potentially suffer from the fact that the effect of fractures on ice flow might not be amenable to expression in terms of a damage/softness field, and that there could be difficulty in finding a way to evolve such a field through time. Elements of the work presented in this thesis are relevant to both of these questions. We shall consider here point 1, and discuss point 2 in the future work section below.

The results of chapters 2 and 5, as mentioned in the discussion of chapter 2, provide some evidence that an isotropic, scalar damage field that alters the effective ice viscosity through a multiplicative factor can broadly account for the effect of crevasses on ice dynamics. They show that ice softness fields resulting from solving inverse problems can look naturally (chapter 2), or be forced to look without major consequences to the solution misfit (chapter 5), like observations of the spatial pattern and density of ice fractures through time. In other words,

such softness fields are a candidate for the damage field used by a continuum damage mechanics model of macroscopic fracture. These are, of course, the simplest forms of such damage fields and have been considered for a long time - they ignore the complexities of anisotropic action of the damage field on, for example, horizontal and vertical components of the effective viscosity and act linearly on the single component of viscosity we consider.

6.2 Future work

There are a number of directions for future work motivated by the studies presented here. These include improving the fracture mapping and data-assimilation methods developed during this thesis, as well as the further use of these methods in answering questions of scientific interest. A number of these things have been mentioned in the discussions of each article, so I won't go over them again here. Instead, the space below lists certain other directions of work that could make use of the advances brought about in this thesis, or a where the work presented here has highlighted a deficiency in current ways of doing things.

6.2.1 Fractures as a routine, open-access product

So far there is no routinely-collected, publicly available repository of crevasse data for use by the glaciological community. After the work of Baumhoer et al., 2019; Heidler et al., 2021 on extracting calving front locations from Sentinel-1 imagery over Antarctic ice shelves, the authors dedicated time to developing an automatically updating repository of calving front locations across the AIS (Baumhoer et al., 2023). It would be beneficial to invest time in developing the fracture mapping process described in chapter 3 to do something similar. The viability of this was shown in chapter 3 where I created seven and a half years of monthly fracture mosaics across the continent. Meeting the computational and storage demands required to maintain and host such a repository does not represent much of a challenge. However, in order to make the product analysis-ready, and usable without good knowledge of the methods involved in generating the data, general improvements to the accuracy of the product would be required. Initially, in order to do this, I would aim to retrain the neural network involved in extracting type-A crevasses using a portion of manually-annotated data. Creating this dataset is an uninteresting task, but I have learnt during the development of the methods described in chapter 3 that the methods of

getting around this can be convoluted and perhaps limit the performance of a neural network. Others, e.g. Lai et al., 2020; Zhao et al., 2022; Hoffman et al., 2024 have been able to annotate satellite images without a great deal of difficulty.

6.2.2 Towards a data-informed continuum damage mechanics of glacier ice

The results from the time-dependent inverse problems of chapter 5 suggest a way of constraining the evolution equation in a continuum model of ice shelf damage (suggested in 6.1.6)

A basic way to evolve the damage field is through an advection-source equation of the form:

$$\frac{\partial(D)}{\partial t} + \nabla \cdot (\mathbf{u}D) = \mathcal{S}(\tau, \dots). \quad (6.1)$$

(Note that we do not necessarily want D to be a conserved quantity in two-dimensional flow if we want to represent the presence of fractures.) Such a model has been considered, for example, in Pralong and Funk, 2005; Jouvét et al., 2011; Albrecht and Levermann, 2014; Sun et al., 2017. The biggest differences between the uses of this formula, aside from specific advection schemes and numerical implementations etc., have been in the functional form of the source term. For example, Sun et al., 2017 used the Nye-zero stress model of crevasse penetration based on the first principal stress, while Pralong and Funk, 2005 consider more complicated, empirical relationships based on the full Cauchy stress.

By construction, the changes in the softness fields (along with whatever changes in C and ϕ occur on the grounded ice) found by solving sequential inverse problems (fig. 5.4) can account for dynamic changes over that time. The complex and ill-posed nature of the inverse problem means that many, possibly quite disparate-looking softness fields will yield similar misfits. The use of fracture maps as a prior in these inverse problems results in a particular subset of these possibilities in which the softness fields take on the appearance of fracture maps. It is plausible, though it is difficult to prove, that these kinds of softness fields are more likely to be amenable to description by an equation of the form (6.1) than most. According to the method described in chapter 5, we can generate a sequence of n damage fields $D_i = 1 - \phi_i$, $i \in \{1, \dots, n\}$. Assuming pure advection, we can then compute a series of $n - 1$ advected damage fields $D'_i = D_{i-1} - \nabla \cdot (\mathbf{u}_{i-1}D_{i-1})\Delta t$ and a series of residuals $R_i = D_i - D'_i$, $i \in \{n, \dots, n\}$.

If eq (6.1) is strictly obeyed and the timestep is not too large, these residuals should satisfy $R_i/\Delta t = \mathcal{S}(\tau_{i-1}, \dots)$. Hence, if, for example, we expand the source term as a function of invariants of the deviatoric stress tensor:

$$\mathcal{S} = c_1\tau_I + c_2\tau_I^2 + c_3\tau_{II} + c_4\tau_{II}^2 + c_5(\tau_I^2 + \tau_{II}^2 - \tau_I\tau_{II})^{\frac{1}{2}} + \dots \quad (6.2)$$

we can run multilinear regression on our set of residuals to determine the coefficients c_i . In this way, we can hope to learn equation (6.1) from data.

6.2.3 Assessing the relevance of ice shelf damage to grounded ice speed

The results of chapter 3 have shown us where the crevassing is across Antarctica, but it would also be nice to investigate further where the crevassing *matters*. Specifically, some initial questions that we could make progress on are: 1) where around Antarctica are current ice speeds at all attributable to the fact that the ice shelves are fractured, and 2) where on the ice shelves does fracturing make a difference to ice speed (particularly grounded ice speed). If a softness field of the kind considered in chapters 2 and 5 can be made a faithful representation of the effect of crevassing in the system, aspects of these questions can be investigated with diagnostic modelling experiments and sensitivity analysis like that described in chapter 3. It is plausible that the results of chapter 5 might allow us to make some progress with this.

On floating ice, the softness fields generated using the data-informed regularisation of chapter 5 are sparser than those found using heuristic regularisation. If we make the leap to suggesting these softness fields quantify the effect of crevasses alone on the effective ice viscosity, we can use them to provide an upper bound on the importance of fracturing to the dynamics of different ice shelves. A method of doing this is to ‘heal’ the ice on the ice shelves (setting ϕ to 1), and re-solve the stress-balance equations. Comparing the speed to the case with ‘damaged’ ice shelves, we can see how much the ice speed is instantaneously affected by the presence of crevasses.

Results of this kind of experiment are shown in figure 6.1. (The figures in this section are illustrative examples, so the inversions were not done particularly carefully. However, I suspect

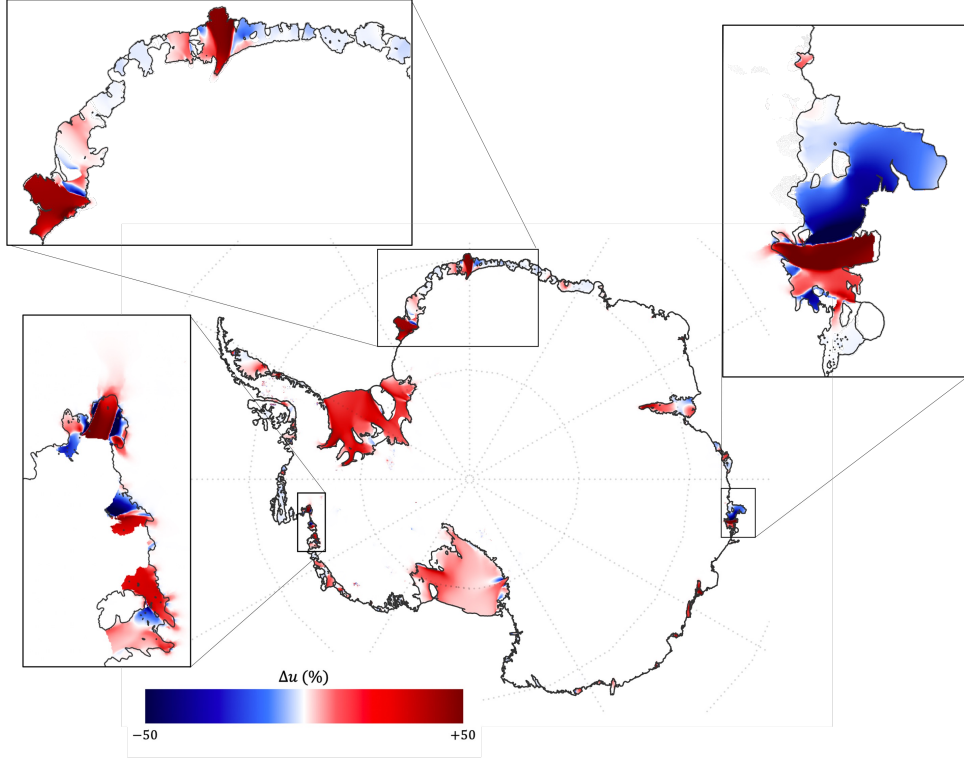


Figure 6.1: Percentage difference in ice speed between ‘healed’ and ‘damaged’ ice shelves ($\frac{100}{u_{\text{healed}}}(u_{\text{damaged}} - u_{\text{healed}})$).

that the conclusions are roughly correct.)

We see that damage ($\phi < 1$) has a marked effect on the speed of the floating ice across the continent, with a broad picture of increased speed when the ice shelves are damaged vs healed. Certain ice shelves are up to 50% faster flowing when damage is included in the model. Some of the locations that decrease in speed when the ice is healed border regions of significant speed *increase*. Areas of fractured ice act to decrease correlation lengthscales by dampening the transfer of membrane stresses between regions of more well consolidated ice. For example, the area south-west of the main trunk of the Pine Island ice shelf is relatively slow flowing due to the large speed gradients allowed across the heavily fractured southern shear margin. When this is healed the ice here is entrained into the faster flowing region due to the stiffer connecting ice. The same phenomenon can be seen for example around Fimbul Ice Shelf and, most markedly, east of the Shackleton ice shelf.

The speed difference between the damaged and undamaged ice shelves is largely confined to the ice-shelves themselves, with ice-speed increase decaying quickly upstream of the grounding

line. This can be seen in the case of Brunt-Stancomb-Wills, Fimbul, Totten and others. In contrast, for ice streams terminating in the Amundsen Sea Embayment, ice speed variations appear inland of the grounding line; most evidently in Piglet Glacier and Pine Island Glacier, where speed differences are evident up to 50 km inland of the grounding line.

Figure 6.1 appears quite different to recent results by Gerli et al., 2023a who investigated, in a diagnostic way similar to the suggestion here, that the ‘activation’ (full-thickness penetration) of surface crevasses on ice shelves found by Lai et al., 2020 does little to change the grounded ice speed in the Amundsen Sea Embayment, but changes that of the ice streams feeding the Ross and Ronne-Filchner ice shelves considerably. In fact, this difference is likely to be due to the slightly different questions being asked in the two experiments, and the different crevasse datasets being used.

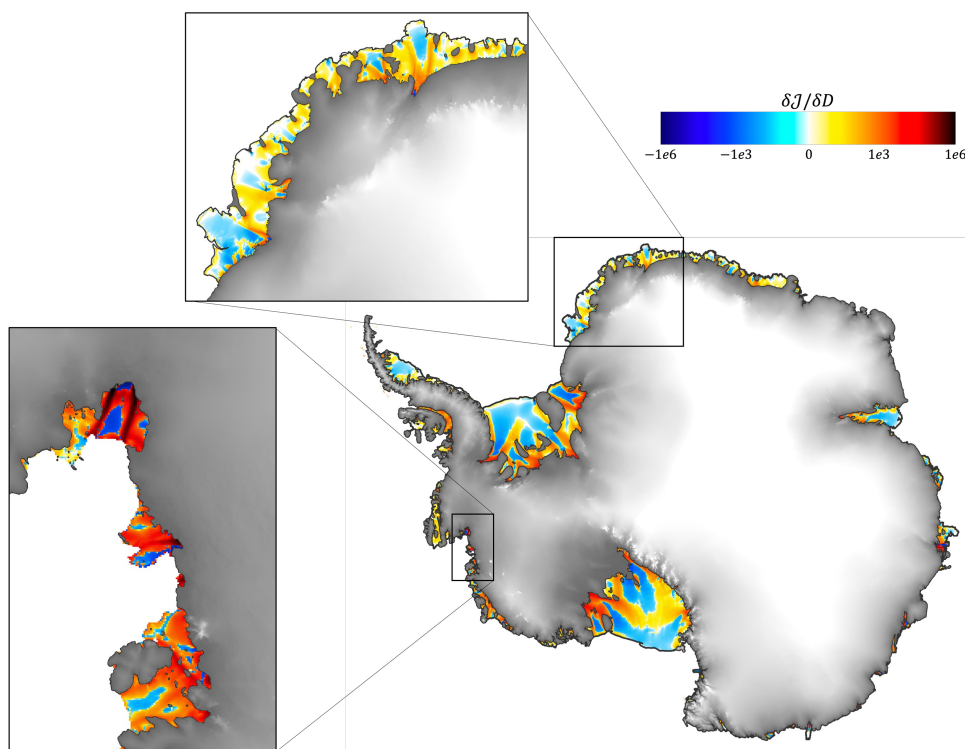


Figure 6.2: The sensitivity of grounded ice speed to changes in ice shelf softness.

We use of the kind of sensitivity analysis used in chapter 4 to look at the second question posed above, in a less sophisticated, but similar way to Morlighem et al., 2021 who analysed the sensitivity of transient mass loss across the ASE to changes in slipperiness and softness. For example, we show in Fig. 6.2 the gradient of integrated grounded ice speed with respect to ϕ at

the solution found using data-informed regularisation across the Antarctic Ice Sheet, computed in a way analogous to Fig 4.3. This shows where changes to the material strength of the ice, brought about by increased fracturing, would most affect the grounded ice speed. We see strong correspondence between regions of high shear and the effect that thinning or weakening would have on the grounding line flux - further establishing these as the regions ‘where the buttressing takes place’. Further, we see regions with negative sensitivity - where thinning or weakening of the ice would decrease discharge across the grounding line. As has become the expected, the largest magnitude effects are seen in the Amundsen Sea Embayment.

Further work is required to do these experiments properly and make sense of the results. In particular, the inversions used to find the softness fields for the experiments above were not carried out particularly carefully. Furthermore, it is not clear how close the softness field is to representing the effects of crevasses on the material strength of the ice. Though the use of fracture data in regularising the inverse problem is likely to push the softness field closer to this, there will still be other contributions. Additionally, it is the transient response of the system to perturbations in the softness/thickness that really matters, and it is not clear how these results translate to this.

6.2.4 Second-order derivatives in BISICLES

An alternative way of assessing the behaviour of the model around a certain state, to the type of sensitivity analysis discussed above, is to look at the Hessian matrix of the cost functional $\mathcal{J}$ (minimised during the inverse problem) at the solution, i.e. the matrix of second partial derivatives $\mathcal{H}_{ij} = \frac{\partial^2 \mathcal{J}}{\partial \phi_i \partial \phi_j}$. First order deviations in $\mathcal{J}$ due to perturbations in the control parameters are necessarily zero at the solution of the inverse problem, and the Hessian matrix gives you information about the curvature of the cost function at that point. Aside from the different array of optimisation methods available once the Hessian matrix (or a Hessian-vector product) can be calculated, there is scientific interest in finding the Hessian, especially its eigenspace. The eigenvectors with largest eigenvalue tell you the directions in which perturbations to the control parameters translate into the largest deviations in ice speed from the observations. If there are relatively few large eigenvalues, then it is feasible to compute all the associated eigenvectors and report these as the principal patterns of, for example, thinning or crevassing that would lead to greatest change in dynamic behaviour. The distribution of eigenvalues would also tell

you how generic those changes can be expected to be.

There is also interest in the small eigenvalues and eigenvectors, which characterise the null space of the inverse problem. In essence, these tell you where the speed data used in the inverse problem provide no information about the control fields. I.e. where uncertainties in the speed data translate into the largest uncertainties in the control fields. In general, the calculation of the Hessian matrix is required to propagate uncertainties in speed data into uncertainties in the model control parameters - something that is not new (e.g. Recinos et al., 2023) but ought to be implemented widely if the community is both confident in the model physics and concerned with characterising uncertainty in sea-level rise predictions. Finally, whether at the solution or not, the ability to calculate the eigenspectrum of the Hessian gives us the information we need to characterise the conditioning of the inverse problem. In chapter 5, we used arguments based on the translation of noise in the data to variance in the control fields by the inverse problem to argue that the conditioning was not impacted by the addition of the fracture data. A more complete picture of how introducing new data affects the well-posedness of the inverse problem would be gained by calculating the Hessian.

Though automatic differentiation is a useful tool for the calculation of second-order derivatives in many new ice sheet models (Logan et al., 2020; Recinos et al., 2023), I would propose to use a second-order adjoint method in BISICLES. Such a method takes a similar form to the first-order adjoint method. We start by solving the first-order adjoint system for the vector field of lagrange multipliers λ before taking perturbations of the first-order gradient, the first-order adjoint system and the model equations. By introducing two new lagrange multiplier fields, and solving two sets of second-order adjoint equations for them, we can find the Hessian-vector product. Assuming, again, a linear viscous tensor operator, most of the terms in the second-order adjoint system resemble the first-order system so can be dealt with using the PDE solvers we currently use. From the Hessian-vector product, the largest eigenvectors and eigenvalues can be computed using, for example, a scaled power method (Strang, 1986; Wang et al., 1992). (It remains to be seen how many ‘large’ eigenvalues there are. If the eigenspectrum does not fall off exponentially, it could be that this process is infeasible as it is expensive to perform many times.)

I have taken steps towards implementing this in BISICLES by deriving this second-order adjoint system for the shelfy-stream equations.

6.2.5 An estimate of error on ice speed derived via intensity feature-tracking

In chapter 5, I stated that the link between a regularisation term and the specification of a prior distribution for the softness field relies on certain assumptions about uncertainties in our ice speed data. Additionally, I have written above about using a second-order adjoint method to propagate uncertainties in the model state to the control fields found by solving the inverse problems. However, unfortunately, we do not have a particularly good understanding of how large these uncertainties are.

In the Gamma software I use in this thesis (GAMMA Remote Sensing AG: Gumligen, Switzerland, 2017. Version 1.7, 2017) (and other software packages that do the same job), ice surface displacements are found in the following way: two coregistered images are split into small windows centred on the points of a grid. Iterating over the points on the grid, each pair of subimages is then cross-correlated. The peak of the cross-correlation field is found and its position is recorded as the displacement between the two images. The uncertainty in the displacement comes from changes in the visibility of features between images, degeneracy in the mappings between the bright features in each image, the finite size of the features, non-uniform displacements across the windows and inaccuracies in the peak finding algorithm.

At present the community uses either a uniform estimate of uncertainty (e.g. representing the maximum accuracy of sub-pixel peak finding algorithms in particle image velocimetry (PIV), as in, for example, Rignot et al., 2011), a statistical measure of the uncertainty (e.g. Gardner et al., 2022), or an indicator of the measurement quality in its place. The latter ‘quality indicator’ approach was used, for example, in chapter 2, where we scaled the reciprocal signal to noise ratio of the cross-correlation field by the speed. This has been used a number of times before (e.g. Lemos et al., 2018; Selley et al., 2021). However, it is not truly an estimate of our uncertainty in the measurement. Rather, it is a statement about the uniqueness of the peak we find. Despite being important in the study of ice sheet dynamics through observation, there is not a great deal of literature that tries to address this issue.

What our quality indicator does recognise is that a significant amount of uncertainty information is contained within the cross-correlation field (excluding geocoding or coregistration errors). This has been noted elsewhere in the literature by Altena et al., 2022, which suggests treating the cross-correlation field as a probability density function for displacements and finding an uncertainty based on the covariance matrix of a two-dimensional gaussian fit across the correlation peak. They cite Bhattacharya et al., 2018 as a source for this, however, this paper does not quite make this claim. Instead, they state that under certain assumptions, the *generalised* cross correlation (found by considering only the phase contribution to the cross-correlation field) represents the probability density function for displacements in the context of PIV, which is a close analogue to speckle tracking. Using the algorithm of Bhattacharya et al., 2018, and a gaussian fit over the pdf peak results in errors that look like figure 6.3. This example, over the terminus of Thwaites Glacier (TG), shows certain features that we would expect to see. For example, large uncertainties in regions of high shear (including the shear margins of Piglet Glacier in the top left) and highly correlated uncertainties on the smooth, slow-moving ice.

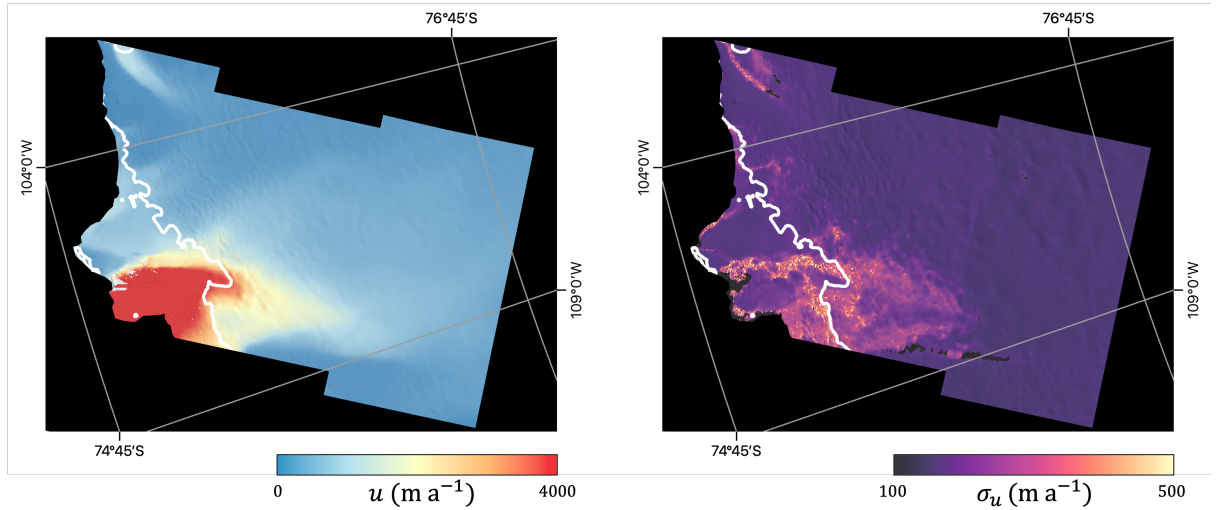


Figure 6.3: Estimated uncertainty in ice speed measurements over Thwaites Glacier. Left: MEaSUREs ice speed over TG (Mouginot et al., 2012). Right: Uncertainty in ice speed from two Sentinel-1 acquisitions separated by 6 days in 2019, using a multivariate Gaussian fit to the generalised cross-correlation field found in the way described in Bhattacharya et al., 2018. The white line is the measures grounding line (Rignot et al., 2016).

Though appealing, the assumptions that underlie this analysis do not hold for the majority of the cross-correlations performed when we calculate displacements between two Sentinel-1 images

of the ice sheet surface. A good estimate of the measurement uncertainty might be found as a function of certain metrics of the cross-correlation field e.g. the SNR, the width of the largest peak, the ratio of largest to second-largest peaks, etc. Future work could look to find such a function using synthetic data in which uncertainties are prescribed (e.g. as described in Sciacchitano, 2019).

Bibliography

- Albrecht, T. and Levermann, A. (2014). “Fracture-induced softening for large-scale ice dynamics”. In: *The Cryosphere* 8.2, pp. 587–605. DOI: 10.5194/tc-8-587-2014.
- Altena, B., Kääb, A., and Wouters, B. (2022). “Correlation dispersion as a measure to better estimate uncertainty in remotely sensed glacier displacements”. In: *The Cryosphere* 16.6, pp. 2285–2300. DOI: 10.5194/tc-16-2285-2022.
- Aoki, S. (2017). “Breakup of land-fast sea ice in Lützow-Holm Bay, East Antarctica, and its teleconnection to tropical Pacific sea surface temperatures”. In: *Geophysical Research Letters* 44.7, pp. 3219–3227. DOI: <https://doi.org/10.1002/2017GL072835>.
- Baumhoer, C. A., Dietz, A. J., Heidler, K., and Kuenzer, C. (2023). “IceLines—A new data set of Antarctic ice shelf front positions”. In: *Scientific Data* 10.1, p. 138. DOI: 10.1038/s41597-023-02045-x.
- Baumhoer, C. A., Dietz, A. J., Kneisel, C., and Kuenzer, C. (2019). “Automated extraction of antarctic glacier and ice shelf fronts from sentinel-1 imagery using deep learning”. In: *Remote Sensing* 11.21, p. 2529.
- Benn, D. I., Luckman, A., Åström, J. A., Crawford, A., Cornford, S. L., Bevan, S. L., Gladstone, R., Zwinger, T., Alley, K., Pettit, E., and Bassis, J. (2021). “Rapid fragmentation of Thwaites Eastern Ice Shelf, West Antarctica”. In: *The Cryosphere Discussions* 2021, pp. 1–25. DOI: 10.5194/tc-2021-288.
- Bhattacharya, S., Charonko, J. J., and Vlachos, P. P. (Oct. 2018). “Particle image velocimetry (PIV) uncertainty quantification using moment of correlation (MC) plane”. In: *Measurement Science and Technology* 29.11, p. 115301. DOI: 10.1088/1361-6501/aadfb4.
- Borstad, C. P., Khazendar, A., Larour, E., Morlighem, M., Rignot, E., Schodlok, M. P., and Seroussi, H. (2012). “A damage mechanics assessment of the Larsen B ice shelf prior to collapse: Toward a physically-based calving law”. In: *Geophysical Research Letters* 39.18. DOI: 10.1029/2012GL053317.
- GAMMA Remote Sensing AG: Gumligen, Switzerland, 2017. Version 1.7 (2017). *GAMMA REMOTE SENSING*. Version 1.7.
- Gardner, A., Fahnestock, M., and Scambos, T. (2022). *MEaSUREs ITS.LIVE Landsat Image-Pair Glacier and Ice Sheet Surface Velocities, Version 1*. DOI: 10.5067/IMR9D3PEI28U.
- Gerli, C., Rosier, S., and Gudmundsson, G. H. (2023a). “Activation of Existing Surface Crevasses Has Limited Impact on Grounding Line Flux of Antarctic Ice Streams”. In: *Geophysical Research Letters* 50.6, e2022GL101687. DOI: <https://doi.org/10.1029/2022GL101687>.

- Gerli, C., Rosier, S., Gudmundsson, H., and Sun, S. (2023b). “Weak relationship between remotely detected crevasses and inferred ice rheological parameters on Antarctic ice shelves”. In: *EGUsphere* 2023, pp. 1–20. DOI: 10.5194/egusphere-2023-2362.
- Heidler, K., Mou, L., Baumhoer, C., Dietz, A., and Zhu, X. X. (2021). *HED-UNet: Combined Segmentation and Edge Detection for Monitoring the Antarctic Coastline*.
- Hoffman, A. O., Christianson, K., Lai, C.-Y., Joughin, I., Holschuh, N., Case, E., Kingslake, J., and GHOST science team, the (2024). “Inland migration of near-surface crevasses in the Amundsen Sea Sector, West Antarctica”. In: *EGUsphere* 2024, pp. 1–29. DOI: 10.5194/egusphere-2023-2956.
- Izeboud, M. and Lhermitte, S. (2023). “Damage detection on antarctic ice shelves using the normalised radon transform”. In: *Remote Sensing of Environment* 284, p. 113359. ISSN: 0034-4257. DOI: 10.1016/j.rse.2022.113359.
- Jouvet, G., Picasso, M., Rappaz, J., Huss, M., and Funk, M. (2011). “Modelling and Numerical Simulation of the Dynamics of Glaciers Including Local Damage Effects”. In: *Math. Model. Nat. Phenom.* 6.5, pp. 263–280. DOI: 10.1051/mmnp/20116510.
- Lai, C.-Y., Kingslake, J., Wearing, M. G., Chen, P.-H. C., Gentine, P., Li, H., Spergel, J. J., and Wessem, J. M. van (2020). “Vulnerability of Antarctica’s ice shelves to meltwater-driven fracture”. In: *Nature* 584.7822, pp. 574–578.
- Lemos, A., Shepherd, A., McMillan, M., Hogg, A. E., Hatton, E., and Joughin, I. (2018). “Ice velocity of Jakobshavn Isbræ, Petermann Glacier, Nioghalvfjærdsfjorden, and Zachariæ Isstrøm, 2015–2017, from Sentinel 1-a/b SAR imagery”. In: *The Cryosphere* 12.6, pp. 2087–2097.
- Lhermitte, S., Sun, S., Shuman, C., Wouters, B., Pattyn, F., Wuite, J., Berthier, E., and Nagler, T. (2020). “Damage accelerates ice shelf instability and mass loss in Amundsen Sea Embayment”. In: *Proceedings of the National Academy of Sciences* 117.40, pp. 24735–24741. DOI: 10.1073/pnas.1912890117.
- Logan, L. C., Narayanan, S. H. K., Greve, R., and Heimbach, P. (2020). “SICOPOLIS-AD v1: an open-source adjoint modeling framework for ice sheet simulation enabled by the algorithmic differentiation tool OpenAD”. In: *Geoscientific Model Development* 13.4, pp. 1845–1864. DOI: 10.5194/gmd-13-1845-2020.
- Massom, R. A., Giles, A. B., Warner, R. C., Fricker, H. A., Legrésy, B., Hyland, G., Lescarmontier, L., and Young, N. (2015). “External influences on the Mertz Glacier Tongue (East Antarctica) in the decade leading up to its calving in 2010”. In: *Journal of Geophysical Research: Earth Surface* 120.3, pp. 490–506. DOI: <https://doi.org/10.1002/2014JF003223>.
- Miles, B. W. J., Stokes, C. R., and Jamieson, S. S. R. (2017). “Simultaneous disintegration of outlet glaciers in Porpoise Bay (Wilkes Land), East Antarctica, driven by sea ice break-up”. In: *The Cryosphere* 11.1, pp. 427–442. DOI: 10.5194/tc-11-427-2017.
- Miles, B. W. J., Stokes, C. R., Jenkins, A., Jordan, J. R., Jamieson, S. S. R., and Gudmundsson, G. H. (2020). “Intermittent structural weakening and acceleration of the Thwaites Glacier Tongue between 2000 and 2018”. In: *Journal of Glaciology* 66.257, pp. 485–495. DOI: 10.1017/jog.2020.20.

- Morlighem, M., Goldberg, D., Dias dos Santos, T., Lee, J., and Sagebaum, M. (2021). “Mapping the Sensitivity of the Amundsen Sea Embayment to Changes in External Forcings Using Automatic Differentiation”. In: *Geophysical Research Letters* 48.23, e2021GL095440. DOI: <https://doi.org/10.1029/2021GL095440>.
- Mouginot, J., Scheuchl, B., and Rignot, E. (2012). “Mapping of Ice Motion in Antarctica Using Synthetic-Aperture Radar Data”. In: *Remote Sensing* 4.9, pp. 2753–2767. ISSN: 2072-4292. DOI: 10.3390/rs4092753.
- Ochwat, N. E., Scambos, T. A., Banwell, A. F., Anderson, R. S., MacLennan, M. L., Picard, G., Shates, J. A., Marinsek, S., Margonari, L., Truffer, M., and Pettit, E. C. (2023). “Triggers of the 2022 Larsen B multi-year landfast sea ice break-out and initial glacier response”. In: *The Cryosphere Discussions* 2023. In review, pp. 1–34. DOI: 10.5194/tc-2023-88.
- Pralong, A. and Funk, M. (2005). “Dynamic damage model of crevasse opening and application to glacier calving”. In: *Journal of Geophysical Research: Solid Earth* 110.B1. DOI: 10.1029/2004JB003104.
- Purich, A. and Doddridge, E. W. (2023). “Record low Antarctic sea ice coverage indicates a new sea ice state”. In: *Communications Earth & Environment* 4.1, p. 314. DOI: 10.1038/s43247-023-00961-9.
- Recinos, B., Goldberg, D., Maddison, J. R., and Todd, J. (2023). “A framework for time-dependent Ice Sheet Uncertainty Quantification, applied to three West Antarctic ice streams”. In: *The Cryosphere Discussions* 2023, pp. 1–46. DOI: 10.5194/tc-2023-27.
- Rignot, E., Mouginot, J., and Scheuchl, B. (2016). *MEaSUREs Antarctic Grounding Line from Differential Satellite Radar Interferometry, Version 2*. Boulder, Colorado USA. NASA National Snow and Ice Data Center Distributed Active Archive Center. doi: <https://doi.org/10.5067/IKBWW4RYHF1Q>. Accessed 15/01/2021.
- (2011). “Ice Flow of the Antarctic Ice Sheet”. In: *Science* 333.6048, pp. 1427–1430. DOI: 10.1126/science.1208336.
- Robel, A. A. (2017). “Thinning sea ice weakens buttressing force of iceberg mélange and promotes calving”. In: *Nature Communications* 8.1, pp. 1–7.
- Sciacchitano, A. (July 2019). “Uncertainty quantification in particle image velocimetry”. In: *Measurement Science and Technology* 30.9, p. 092001. DOI: 10.1088/1361-6501/ab1db8.
- Selley, H. L., Hogg, A. E., Cornford, S., Dutrieux, P., Shepherd, A., Wuite, J., Floricioiu, D., Kusk, A., Nagler, T., Gilbert, L., Slater, T., and Kim, T.-W. (Feb. 2021). “Widespread increase in dynamic imbalance in the Getz region of Antarctica from 1994 to 2018”. In: *Nature Communications* 12.1, p. 1133. ISSN: 2041-1723. DOI: 10.1038/s41467-021-21321-1.
- Strang, G. (1986). *Introduction to Applied Mathematics*. Wellesley-Cambridge Press, p. 758.
- Sun, S., Cornford, S. L., Moore, J. C., Gladstone, R., and Zhao, L. (2017). “Ice shelf fracture parameterization in an ice sheet model”. In: *The Cryosphere* 11.6, pp. 2543–2554. DOI: 10.5194/tc-11-2543-2017.
- Sun, S. and Gudmundsson, G. H. (2023). “The speedup of Pine Island Ice Shelf between 2017 and 2020: reevaluating the importance of ice damage”. In: *Journal of Glaciology*, pp. 1–9. DOI: 10.1017/jog.2023.76.

- Sun, Y., Riel, B., and Minchew, B. (2023). “Disintegration and Buttreassing Effect of the Landfast Sea Ice in the Larsen B Embayment, Antarctic Peninsula”. In: *Authorea Preprints*. In review.
- Vieli, A., Payne, A., Shepherd, A., and Du, Z. (2007). “Causes of pre-collapse changes of the Larsen B ice shelf: Numerical modelling and assimilation of satellite observations”. In: *Earth and Planetary Science Letters* 259.3, pp. 297–306. ISSN: 0012-821X. DOI: <https://doi.org/10.1016/j.epsl.2007.04.050>.
- Wang, Z., Navon, I. M., Le Dimet, F.-X., and Zou, X. (1992). “The second order adjoint analysis: theory and applications”. In: *Meteorology and atmospheric physics* 50, pp. 3–20. DOI: 10.1007/BF01025501.
- Zhao, J., Liang, S., Li, X., Duan, Y., and Liang, L. (2022). “Detection of Surface Crevasses over Antarctic Ice Shelves Using SAR Imagery and Deep Learning Method”. In: *Remote Sensing* 14.3. ISSN: 2072-4292. DOI: 10.3390/rs14030487.

Appendix A

Deriving the shelfy-stream equations

Much of this is adapted from the Úa Compendium by Hilmar Gudmundsson.

A.1 First order Momentum Equations

We assume ice deformation is inertia-free, so that the Cauchy momentum equations read:

$$\partial_i \sigma_{ij} + \rho g_j = 0, \quad i, j \in \{x, y, z\} \quad (\text{A.1})$$

where $\sigma = \sigma^\top$ denotes the Cauchy stress, ρ denotes the density of ice, g is the acceleration due to gravity; and we have employed the Einstein summation convention. The *deviatoric* stress components are defined through $\tau_{ij} = \sigma_{ij} + p\delta_{ij}$ where $p = -\frac{1}{3}\sigma_{ij}\delta_{ij}$ is the pressure.

The essence of the shallow-stream equations is that gravitational driving stresses are ‘balanced’ by a mixture of membrane stresses acting in the x-y plane, and basal shear stress. We make a number of assumptions at the outset

1. Vertical scales are of order δ compared to horizontal scales, where $\delta \ll 1$. We assume a characteristic length scale $[x]$, so that:

$$(x, y, z) = [x](x^*, y^*, \delta z^*) \quad (\text{A.2})$$

2. Vertical shear stresses are of order δ compared to all other components of the Cauchy

stress. Given a characteristic scale for the stress of $[\sigma]$, we can write the Cauchy stress in terms of dimensionless quantities:

$$\sigma = [\sigma] \begin{pmatrix} \tau_{xx}^* + p^* & \tau_{xy}^* & \delta\tau_{xz}^* \\ \tau_{yx}^* & \tau_{yy}^* + p^* & \delta\tau_{yz}^* \\ \delta\tau_{zx}^* & \delta\tau_{zy}^* & \tau_{zz}^* + p^* \end{pmatrix} \quad (\text{A.3})$$

Written in this way, it is clear that normal deviatoric stresses, normal Cauchy stresses and the pressure field are assumed to have the same scale.

We align the z-axis with acceleration due to gravity, such that $\mathbf{g} = (0, 0, -g)^\top$ and parametrise the surface of the ice by $z_{surf} = s(x, y)$, the base of the ice by $z_{base} = b(x, y)$ and the thickness of the ice by $h(x, y) = s(x, y) - b(x, y)$.

Using the scalings defined in (A.2) and (A.3), we can re-write the components of eq. (A.1) as:

$$\frac{[\sigma]}{[x]} \left(\partial_{x^*} \sigma_{xx}^* + \partial_{y^*} \tau_{xy}^* + \frac{\delta}{\delta} \partial_{z^*} \tau_{xz}^* \right) = 0 \quad (\text{A.4})$$

$$\frac{[\sigma]}{[x]} \left(\partial_{y^*} \tau_{yx}^* + \partial_{y^*} \sigma_{yy}^* + \frac{\delta}{\delta} \partial_{z^*} \tau_{yz}^* \right) = 0 \quad (\text{A.5})$$

$$\frac{[\sigma]}{[x]} \left(\delta \partial_{x^*} \tau_{zx}^* + \delta \partial_{y^*} \tau_{zy}^* + \frac{1}{\delta} \partial_{z^*} \sigma_{zz}^* \right) = -\rho g. \quad (\text{A.6})$$

The characteristic vertical normal stress is of order ρgh , i.e.:

$$[\sigma] = \rho g \delta [x] \quad (\text{A.7})$$

Using this, eq. (A.6) can be written:

$$\delta^2 \partial_{x^*} \tau_{zx}^* + \delta^2 \partial_{y^*} \tau_{zy}^* + \partial_{z^*} \sigma_{zz}^* = -1. \quad (\text{A.8})$$

Hence, horizontal variations in vertical shear stresses can be ignored in the momentum equation to second order in δ . The zeroth (and first) order conservation of momentum equations are,

therefore:

$$\partial_x \sigma_{xx} + \partial_y \tau_{xy} + \partial_z \tau_{xz} = 0 \quad (\text{A.9})$$

$$\partial_y \tau_{yx} + \partial_y \sigma_{yy} + \partial_z \tau_{yz} = 0 \quad (\text{A.10})$$

$$\partial_z \sigma_{zz} = -\rho g. \quad (\text{A.11})$$

A.1.1 Scaling Boundary Conditions

To obtain the two-dimensional shallow-stream equations, we have to integrate the three-dimensional momentum equations (A.9)-(A.11) from $z = b(x, y)$ to $z = s(x, y)$. In order to do this, we need some knowledge of the stresses at the surface and base of the ice. We have a free-surface boundary condition stating that the air above provides no traction. Explicitly:

$$n_i \sigma_{ij} \Big|_s = 0, \quad i \in \{x, y, z\} \quad (\text{A.12})$$

where $n_i = \partial_i(z - s(x, y))$ are the components of the normal vector to the surface of the ice.

The components of this equation read:

$$(-\sigma_{xx} \partial_x s - \tau_{xy} \partial_y s + \tau_{xz}) \Big|_s = 0 \quad (\text{A.13})$$

$$(-\tau_{yx} \partial_y s - \sigma_{yy} \partial_y s + \tau_{yz}) \Big|_s = 0 \quad (\text{A.14})$$

$$(-\tau_{zx} \partial_x s - \tau_{zy} \partial_y s + \sigma_{zz}) \Big|_s = 0. \quad (\text{A.15})$$

After applying length and stress scales, we have:

$$(-\delta \sigma_{xx}^* \partial_x^* s^* - \delta \tau_{xy}^* \partial_y^* s^* + \delta \tau_{xz}^*) \Big|_s = 0 \quad (\text{A.16})$$

$$(-\delta \tau_{yx}^* \partial_y^* s^* - \delta \sigma_{yy}^* \partial_y^* s^* + \delta \tau_{yz}^*) \Big|_s = 0 \quad (\text{A.17})$$

$$(-\delta^2 \tau_{zx}^* \partial_x^* s^* - \delta^2 \tau_{zy}^* \partial_y^* s^* + \sigma_{zz}^*) \Big|_s = 0. \quad (\text{A.18})$$

Hence, to second order, the free-surface boundary conditions can be written:

$$(-\sigma_{xx}\partial_x s - \tau_{xy}\partial_y s + \tau_{xz}) \Big|_s = 0 \quad (\text{A.19})$$

$$(-\tau_{yx}\partial_y s - \sigma_{yy}\partial_y s + \tau_{yz}) \Big|_s = 0 \quad (\text{A.20})$$

$$\sigma_{zz} \Big|_s = 0. \quad (\text{A.21})$$

At the base of the ice, we would like to cast the stresses in terms of the “basal traction” - the stress acting tangentially to the bed. This has components:

$$t_j = n_i \sigma_{ij} - (n_i \sigma_{ik} n_k) n_j, \quad i, j, k \in \{x, y, z\} \quad (\text{A.22})$$

where n_i are the components of the normal vector to the bed.

After a certain amount of algebra, we find that the basal traction is of order δ and can be written:

$$\mathbf{t} = [\sigma] \begin{pmatrix} \delta \partial_{x^*} b^* (\sigma_{zz}^* - \sigma_{xx}^*) - \delta \partial_{y^*} b^* \tau_{xy}^* + \delta \tau_{xz}^* + \mathcal{O}(\delta^3) \\ \delta \partial_{y^*} b^* (\sigma_{zz}^* - \sigma_{yy}^*) - \delta \partial_{x^*} b^* \tau_{yx}^* + \delta \tau_{yz}^* + \mathcal{O}(\delta^3) \\ \mathcal{O}(\delta^2) + \mathcal{O}(\delta^4) \end{pmatrix} \quad (\text{A.23})$$

A.2 Vertical Integration

The vertical integration of eq. (A.11) is trivial and gives, after the application of eq. (A.21):

$$\sigma_{zz}(z) = \rho g(s - z) \quad (\text{A.24})$$

and, in particular:

$$\sigma_{zz} \Big|_b = \rho g(s - b). \quad (\text{A.25})$$

We now integrate the x-component of the momentum equation (A.9):

$$0 = \int_b^s (\partial_x \sigma_{xx} + \partial_y \tau_{xy} + \partial_z \tau_{xz}) dz \quad (\text{A.26})$$

$$= \int_b^s \partial_x \sigma_{xx} dz + \int_b^s \partial_y \tau_{xy} dz + \tau_{xz} \Big|_s - \tau_{xz} \Big|_b \quad (\text{A.27})$$

$$\begin{aligned} &= \partial_x \int_b^s \sigma_{xx} dz - (\sigma_{xx} \partial_x z) \Big|_s + (\sigma_{xx} \partial_x z) \Big|_b \\ &\quad + \partial_y \int_b^s \tau_{xy} dz - (\tau_{xy} \partial_y z) \Big|_s + (\tau_{xy} \partial_y z) \Big|_b \\ &\quad + \tau_{xz} \Big|_s - \tau_{xz} \Big|_b \end{aligned} \quad (\text{A.28})$$

where Leibnitz' rule has been used to take the partial derivatives outside the integrals between the second and third lines. Using (A.19), we are left with:

$$0 = \partial_x \int_b^s \sigma_{xx} dz + \partial_y \int_b^s \tau_{xy} dz + (\sigma_{xx} \partial_x z + \tau_{xy} \partial_y z - \tau_{xz}) \Big|_b. \quad (\text{A.29})$$

The scaled x-component of the basal traction vector shown in eq. (A.23) indicates that

$$(\sigma_{xx} \partial_x z + \tau_{xy} \partial_y z - \tau_{xz}) \Big|_b \approx -t_x + (\sigma_{zz} \partial_x z) \Big|_b \quad (\text{A.30})$$

so that eq. (A.29) becomes:

$$0 = \partial_x \int_b^s \sigma_{xx} dz + \partial_y \int_b^s \tau_{xy} dz - t_x + (\sigma_{zz} \partial_x z) \Big|_b. \quad (\text{A.31})$$

Using that the trace of the deviatoric stress is zero and the fact that $\sigma_{xx} = \tau_{xx} - p$, we write $\sigma_{xx} = 2\tau_{xx} + \tau_{yy} + \sigma_{zz}$ so that

$$0 = \partial_x \int_b^s \sigma_{zz} dz + \partial_x \int_b^s (2\tau_{xx} + \tau_{yy}) dz + \partial_y \int_b^s \tau_{xy} dz - t_x + (\sigma_{zz} \partial_x z) \Big|_b. \quad (\text{A.32})$$

We assume now that the ice is approximately *cryostatic*, such that the vertical variations in vertical-normal stress shown in eq. (A.24) are dominated by changes in pressure. I.e. $\partial_z \tau_{zz} \approx 0$. The deviatoric stress tensor is traceless, so $\partial_z \tau_{xx} + \partial_z \tau_{yy} = 0$. The system is invariant under coordinate rotations in the x-y plane, which suggests that $\partial_z \tau_{xx} \approx \partial_z \tau_{yy} \approx \partial_z \tau_{xy} \approx 0$ as well. Hence, we see that the only stresses that vary vertically in the system given this cryostatic assumption are vertical shear stresses, vertical normal stresses and pressure. This allows us to

trivially solve the second and third integrals in the above equation A.32, writing it as:

$$0 = \partial_x \int_b^s \sigma_{zz} dz + \partial_x(h(2\tau_{xx} + \tau_{yy})) + \partial_y(h\tau_{xy}) - t_x + \sigma_{zz} \Big|_b \partial_x b. \quad (\text{A.33})$$

Using eq. (A.24), we have:

$$= \rho gh \partial_x(s - b) + \partial_x(h(2\tau_{xx} + \tau_{yy})) + \partial_y(h\tau_{xy}) - t_x + \rho gh \partial_x b \quad (\text{A.34})$$

$$= \rho gh \partial_x s + \partial_x(h(2\tau_{xx} + \tau_{yy})) + \partial_y(h\tau_{xy}) - t_x. \quad (\text{A.35})$$

Carrying out the same procedure with the y-component of the zeroth-order momentum equations gives us the shallow-stream equations:

$$\partial_x(h(2\tau_{xx} + \tau_{yy})) + \partial_y(h\tau_{xy}) - t_x = \rho gh \partial_x s \quad (\text{A.36})$$

$$\partial_y(h(2\tau_{yy} + \tau_{xx})) + \partial_x(h\tau_{xy}) - t_y = \rho gh \partial_y s. \quad (\text{A.37})$$

Defining the symmetric “resistive stress tensor” as:

$$\Sigma = \begin{pmatrix} 2\tau_{xx} + \tau_{yy} & \tau_{xy} \\ \tau_{yx} & 2\tau_{yy} + \tau_{xx} \end{pmatrix} \quad (\text{A.38})$$

we can write the shallow stream equations as:

$$\partial_i(h\Sigma_{ij}) - t_j = \rho gh \partial_j s, \quad i, j \in \{x, y\}. \quad (\text{A.39})$$

We note that these equations have been derived from the Cauchy momentum equation for an inertia-free system without reference to the constitutive behaviour of the ice, except for the fairly general statement that the incompressibility condition relies on an isotropic constitutive relation. Even non-isotropy can be accommodated without a great deal of difficulty. This permits many different constitutive relations, the most interesting of which are viscous and plastic rheologies which can be seen to conform to these conditions. In the case of ice shelves, the basal traction, usually prescribed as a function of ice speed at the base, can be neglected, and the equations permit linear elastic constitutive relations as well.

Any relation specifying how the components of a two-dimensional field $\mathbf{u}$ relate to the three

unknown components of the deviatoric stress of the form:

$$\gamma_{ij}(u_x, u_y) = f(\tau, \dots)\tau_{ij} \quad (\text{A.40})$$

(where $f(\tau, \dots)$ indicates some function of the invariants of the deviatoric stress tensor as well as fields describing englacial properties such as temperature, and $u_{x,y}$ are the components of a velocity or displacement field) will result in an equation of the same form as that which we write throughout this thesis. They may have differing degrees of interpretability.

For example, for the power-law viscous deformation often considered throughout this thesis, γ is the strain rate tensor defined by

$$\gamma_{ij} = \dot{\varepsilon}_{ij} = \frac{1}{2}(\partial_i u_j + \partial_j u_i) \quad (\text{A.41})$$

and $u_{x,y}$ are the components of the two-dimensional velocity field. This gives the most common form of the shelfy-stream equations.

In the case of isotropic elastic material undergoing small strain, γ is the strain tensor defined by

$$\gamma_{ij} = \varepsilon_{ij} = \frac{1}{2}(\partial_i u_j + \partial_j u_i) \quad (\text{A.42})$$

and $u_{x,y}$ are the components of the two-dimensional displacement field, relative to some nearby reference.

Appendix B

Supplementary materials for the
article: Episodic dynamic change
linked to damage on the Thwaites
Glacier Ice Tongue

B.1 Observed Ice Speed and Error Measurements

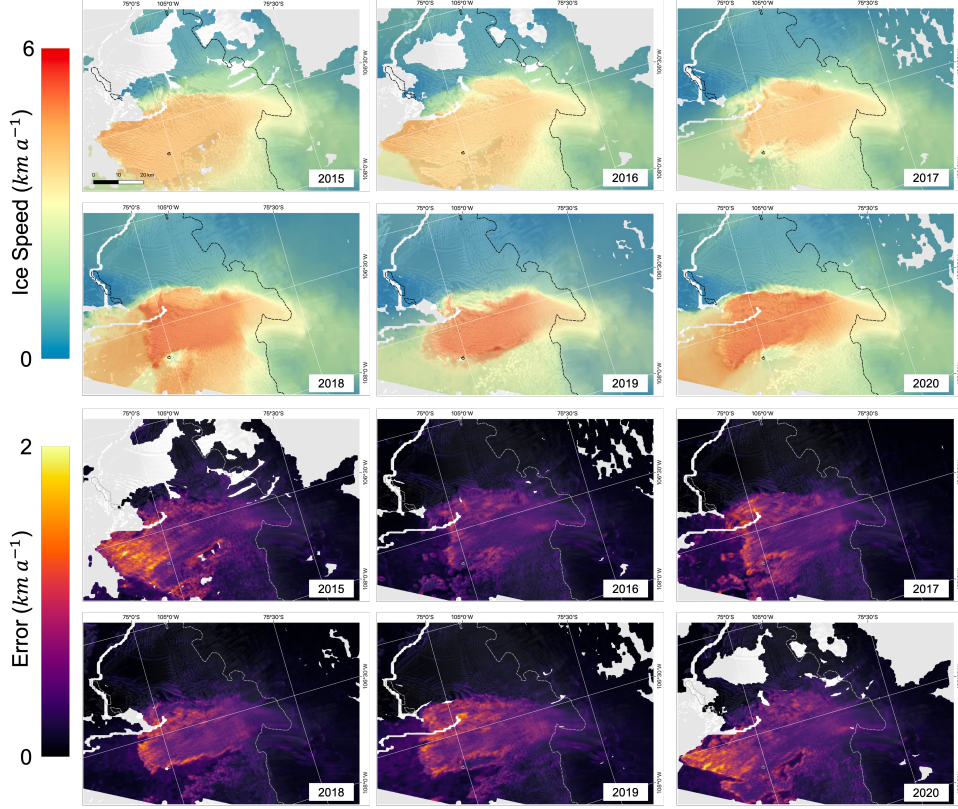


Figure B.1: Ice Speed and Error Maps. Annual median mosaics of ice speed (rows 1 and 2) and ice speed error (rows 3 and 4) over TG from 2015 to 2020, as measured using the intensity feature tracking method. The ice speed error is estimated as the reciprocal signal-to-noise ratio of the cross-correlation peak at each feature tracking window, scaled by the local measurement of ice velocity. We define the signal-to-noise ratio as the amplitude of the primary cross-correlation peak divided by the mean of all secondary peaks (as in Lange et al., 2007). Pixels with less than 20% data coverage during the year are excluded from the annual mosaics. The 2011 grounding line location is also shown as the dashed black lines in rows 1 and 2, and the dashed white lines in rows 3 and 4 (Rignot et al., 2016).

B.2 Fracture and Calving Front Measurements

B.2.1 Deep Neural Networks - Background

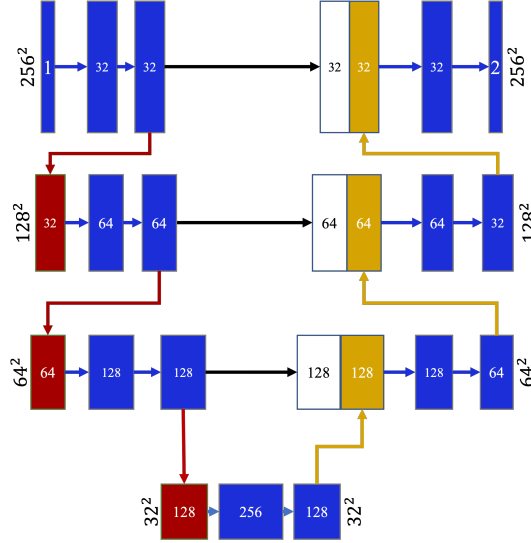


Figure B.2: Neural Network Architecture. Network architecture showing 2D convolutions (blue arrows), max-pooling (red arrows) with a kernel size of 3, stride of 2 and padding of 1, skip connections (black arrows), and bilinear up-sampling followed by 2D convolution (gold arrows). Numbers inside boxes indicate the number of channels at each stage, while numbers outside boxes indicate the number of pixels in each channel. All 2D convolutions are of kernel size 3, stride of 1 and padding of 1 except the first two which use kernel size of 7, stride of 1 and padding of 3. The number of trainable parameters in the model totals 534 230.

Over the last decade, deep learning in the field of computer vision has been dominated by convolutional neural networks. These models have been responsible for significant advances in areas such as image classification, segmentation and feature extraction (Krizhevsky et al., 2012; Long et al., 2014; Ronneberger et al., 2015; Szegedy et al., 2014). Such neural networks operate by applying to the elements x_i of a dataset $\mathcal{X} = \{x_i\}_{i=1}^N$ a series of successive convolutional transformations with trainable parameters θ , separated by non-linear transformations. In a supervised learning regime the output of this series of transformations $\hat{y}_i$ is compared to the desired output - or *target* - $y_i \in \mathcal{Y}$ using a “cost function”, $f : (y_i, \hat{y}_i) \mapsto f(y_i, \hat{y}_i) \in \mathbb{R}$, which defines a distance measure between model outputs and targets. By updating the parameters θ according to the minimisation of the cost function via stochastic gradient descent (where the gradient at each step is calculated only for a subset of the data $b \subset \mathcal{X}$ with a number of elements referred to as the “batch size”), the neural network is trained to approximate the desired mapping $m : \mathcal{X} \rightarrow \mathcal{Y}$.

B.2.2 Network Architecture

The architecture we use (Fig. B.2) for both fracture and calving front detection was based on the U-Net (Ronneberger et al., 2015), and is similar to that used by Kattenborn et al., 2019 for the segmentation of vegetation from remote sensing imagery. Unlike the original U-Net, we perform bilinear up-sampling followed by 2D convolution in the decoder section instead of transposed convolution, include batch normalisation after each 2D convolution, dropout with probability 0.1 after each Leaky-ReLU activation, and reduce the network depth considerably. The model was written in Pytorch (Paszke et al., 2019) and trained using an Nvidia Tesla K80 GPU.

B.2.3 Neural Network Fracture Measurement

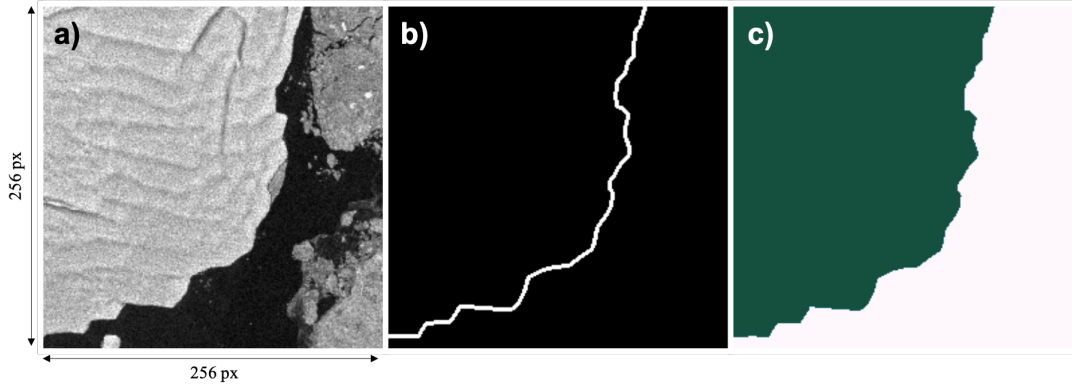


Figure B.3: Example Training Data. (a) An example 256×256 -pixel SAR backscatter image included in the training dataset for both fracture and calving fronts. (b) The manually delineated calving front for the corresponding SAR image (a) used during the training of the U-Net for fracture detection, note that no crevasses are identified. (c) The target image corresponding to (a) used to train the network in calving front delineation; this image is segmented into the classes glacier (green) and sea (white).

A large training dataset $(\mathcal{X}, \mathcal{Y})$ is required to train the U-Net to identify surface crevasses on TG in SAR images. Given the scale of the challenge presented by creating such a dataset manually, we opt instead to use a combination of manual annotation, bootstrapping and image augmentation to achieve this. First, we manually delineated the *calving front location* at 50 m spatial resolution in 20 Sentinel-1 interferometric-wideswath mode SAR backscatter images, from 10 glacier locations in Antarctica, ignoring the presence of crevasses in the images. These scenes were selected to represent the realistic variation of surface features seen in SAR images of Antarctica. The images were then tiled into 256×256 pixel patches, before random rigid transformations were applied - preferentially to image patches containing calving fronts. This

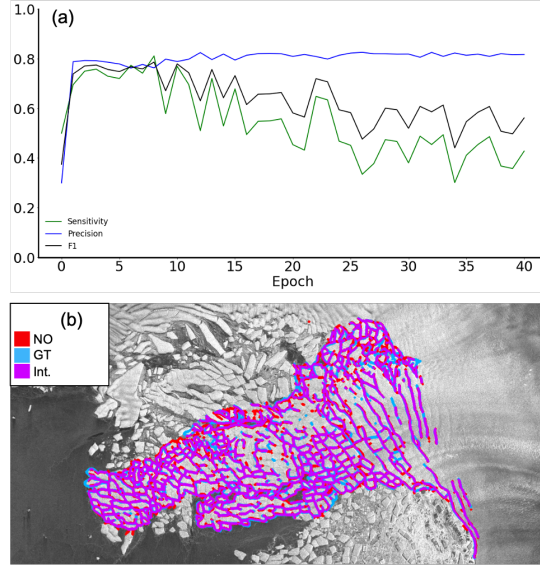


Figure B.4: Network Performance and Output. (a) Performance of the neural network as a detector of crevasses after training for 0-40 epochs. (b) Comparison between the network output (red) after 9 epochs and the ground truth (light blue), with the intersection (Int.) between the two data-sets also shown (purple).

resulted in 3000 images with associated segmentation maps, 85% of which contained sections of calving front (an example training pair is given in Fig. B.3a and b). The neural network was then trained on this calving front dataset using cross entropy loss, Adam optimisation (Kingma and Ba, 2014) with a learning rate of 0.0005, and a batch size of 32. The training was stopped after 9 epochs (44 minutes), before convergence of the network parameters. At this time, the network had become capable of discerning large linear features, including the calving fronts, on which it was trained, and crevasses.

We treat calving fronts as a subset of the set of features corresponding to large, linear textural discontinuities in the backscatter images. This subset is ultimately defined by high-level, semantic features e.g. the identification of sea on one side of the calving front, and glacier ice on the other. U-Net is a hierarchical network in which early layers learn to identify low-level spatial features such as texture and the presence of steep intensity gradients, while higher-level semantic features are learned by deeper layers. Skip connections combine early layers with those towards the end of the decoder section of the network, and enable the network to rapidly reduce the cost function early on in training, by using the activations of the shallow layers to inform its output. At this stage, basic semantic information is provided by the deeper layers, for example constraints on the size and linearity of features. Therefore, early on in training, the network learns to identify the superset of linear features consisting of calving fronts and crevasses.

We then applied this partially trained network to a number of unseen images, identified patches for which the network appeared to have performed well, and added those network outputs and corresponding inputs to the training data set, replacing a random sample of the calving front training images. The network was then retrained with the new data-set before this procedure was repeated. This formed the “bootstrapping” part of the training process.

Fracture Detection: Error Estimation and Performance

In order to assess the performance of the trained network for the detection of crevasses, the network output after 9 epochs, for a large test image covering the TGIT, was compared to a manually delineated ground truth. The resulting segmentation maps and their intersection clearly shows that there is significant agreement between the ground truth and the network output (Fig. B.4b), with both data-sets showing good visual representation of the surface crevasse features visible in the corresponding SAR image. We generated accuracy metrics (Fig. B.4a), where *Sensitivity* (S) is the proportion of true fracture locations to be correctly identified by the network, and *precision* (P) is the proportion of fracture locations identified by the network that is true according to the manual annotation. Based on these metrics we define error on the fracture density (ρ_f) in a region to be bounded at the top by $\rho_f \times (2 - S)$ and at the bottom by $\rho_f \times P$. The evolution of the accuracy metrics is shown in Fig. B.4b. The sensitivity (green line) is shown to increase to a maximum value of 0.81 after 9 epochs and decrease thereafter, as the network - trained on calving front locations - learns to ignore smaller crevasses. The precision (blue line) increases sharply and plateaus, showing that the network remains successful in the identification of true crevasses. The F1 score ($2PS/(P + S)$) (black line) is also shown, taken as an overall measure of performance. This peaks at 9 epochs, at which point we take the network to be at its optimal efficacy in identifying damage.

B.2.4 Neural Network Calving Front Delineation

The dataset used in training the U-Net for the segmentation of SAR images into the classes *glacier* and *sea* was split into two parts. The network architecture is the same as that used in the training of the network for crevasse detection, with the calving front target images replaced with binary segmentation maps (Fig. B.3a and c). After initial training with this dataset, the model was “tuned” using a smaller training set consisting of 500 patches generated from 5 SAR images of TG, and their manually segmented target images. In both the initial training and

the tuning, we used a binary cross-entropy loss function, Adam optimisation with a learning rate of 0.0001, and a batch size of 12. The model training time comprised of 6 hours of initial training and ~ 3 minutes of tuning.

Upon implementation of the trained network, a softmax function was applied to the network output to generate normalised images for each two-channel output. The channels in these normalised images can be interpreted as the probabilities that each pixel corresponds to the classes ‘sea’ and ‘glacier’ respectively. Though this association between a normalised output and a probability is naïve, we make use of it to define a notion of uncertainty in the segmentation (Sec. B.2.4). As with the fracture measurements, to reconstruct a larger image the 256×256 -pixel patches, which overlap by 128-pixels, are tiled together with a Gaussian weighting away from the centre pixels applied. We extract the calving front location from the probability maps using thresholding and canny edge detection. Unreliable calving front data-points, that don’t appear in Fig. 2.3e of the main text, were identified according to the procedure defined in the methods.

Neural Network Calving Front Error Estimation

We use probability maps, which represent the likelihood that each pixel in the input image is ‘glacier’ (P_g) or ‘sea’ (P_s), to calculate an uncertainty in the calving front position. If we define a transect $c(t)$ along which we measure the distance to the calving front, with $t_0 = 0$ in the sea, and $t_1 = 1$ upstream of the calving front, we can define the probability that each point along that line corresponds to glacier $p_g(t)$. If the value of t corresponds to a true point on the calving front (t_{cf}), then we can denote the probability that each point on the transect corresponds to a point on the calving front as $P_{cf}(t = t_{cf})$. This allows for the following correspondence: $p_g(t) = P_{cf}(t > t_{cf})$. Therefore, if we take $(t_0, t_1) \mapsto (-\infty, +\infty)$, we can view $p_g(t)$ as the cumulative probability distribution of $P_{cf}(t = t_{cf})$ (Fig. B.5). A cross-section through P_g approximates a sigmoid function near the calving front, which we can express using parameters μ and σ as:

$$p_g(t) = \frac{1}{1 + e^{-\frac{t-\mu}{q\sigma}}} \quad (\text{B.1})$$

where $q = \sqrt{3}/\pi$. Differentiating this, we find that $P_{cf}(t = t_{cf})$ follows a logistic distribution with mean μ and standard deviation σ .

Hence, by fitting a sigmoid curve ($s(t; \mu, \sigma)$) to the probabilities that pixels in a cross section of

an image P_g , defined by the line $c(t)$, we can estimate the uncertainty in our measured calving front position along that line, namely: σ .

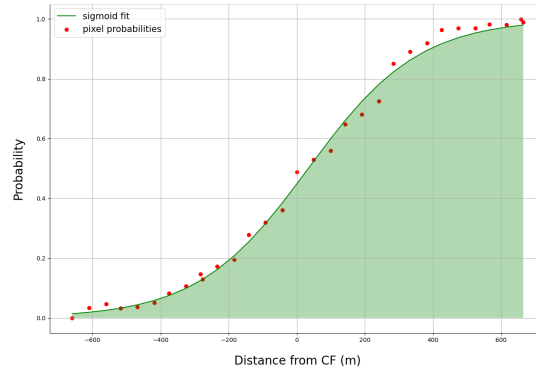


Figure B.5: Sigmoid fit to the probabilities that a pixel corresponds to the class ‘glacier’ for pixels close to the calving front along an example measuring line.

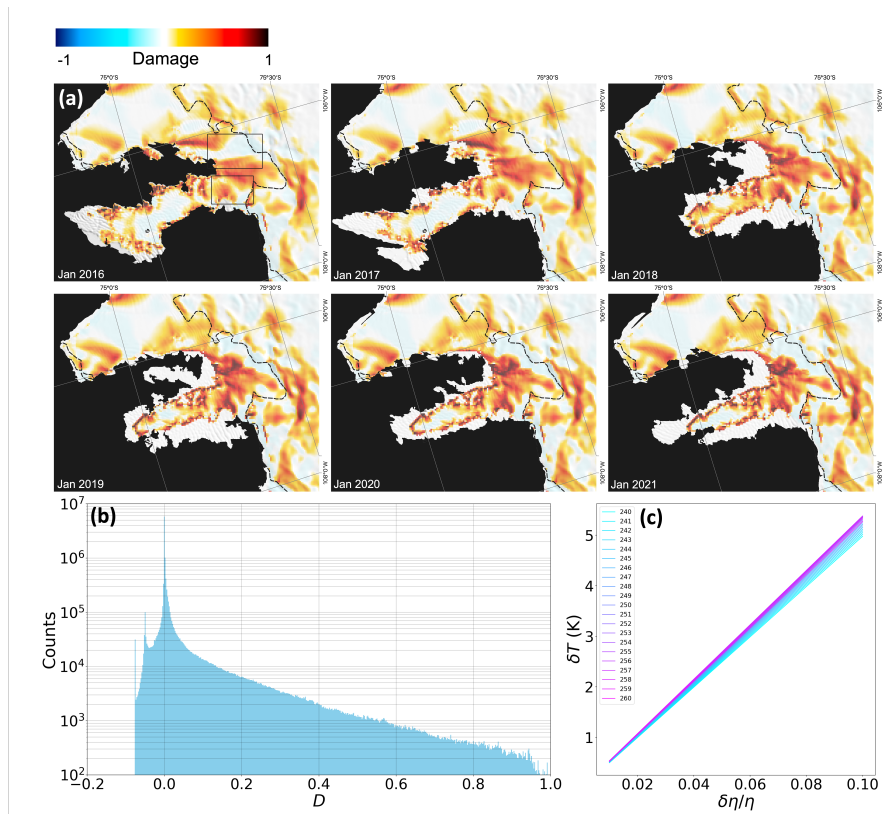


Figure B.6: Distribution of D over the TG terminus. (a) shows a reproduction of figure 4 with negative values of damage also shown in blue. The MEaSUREs grounding line is shown as the dashed black line Rignot et al., 2016. (b) the distribution of damage values over the whole domain and timeseries. (c) temperature fluctuations corresponding to changes in the effective viscosity in the linear regime, using eq. B.3, for values of T between 240 and 260 K.

B.3 The Temperature Component of Damage

The rate factor $A(T)$, in Glen's flow law depends exponentially on temperature. The effective viscosity of ice, η , defined through $\tau_{ij} = \eta \dot{\epsilon}_{ij}$, can be written

$$\eta = A(T)^{-1} \tau_{II}^{1-n}, \quad (\text{B.2})$$

where $\tau_{II} = (\tau_{ij}\tau_{ij})^{1/2}$ is the second invariant of the deviatoric stress tensor. Hence, the effective viscosity is strongly temperature dependent as well. Using Hooke's modification to the Arrhenius relation for $A(T)$ (LeB. Hooke, 1981), we find, at constant stress, and at temperature T , that for a small fluctuation in temperature δT :

$$\frac{\delta\eta}{\eta} = - \left(\frac{Q}{RT^2} + \frac{3Ck}{(T_0 - T)^{k+1}} \right) \delta T, \quad (\text{B.3})$$

where Q is the activation energy for creep, R is the universal gas constant, $T_0 = 273.39$ K, $C = 0.16612$ K ^{k} is the flow rate factor, and $k = 1.17$. The model inversions undertaken in this study assume a constant temperature field $T \equiv T(x, y)$ (Seroussi et al., 2019) when calculating ϕ , however, clearly inaccuracies in the temperature field that alter the value of η from eq. B.2 will cause changes in ϕ to compensate. We argue that the temperature field is accurate enough for us to ignore this contribution to ϕ and assume that deviations of ϕ from 1 are dominated by ice fracture.

Warmer temperatures in T will cause η in the model to be smaller than in reality, hence a ϕ greater than unity and a negative value of D . We, therefore, investigate errors in the temperature field by looking at negative damage in the inversion outputs.

In Fig. B.6 (a)-(b) we see that there are few areas of negative damage, with negative values anyway close to zero. Using the distribution of negative damage values (Fig. B.6 (b)), we define a rough error on ϕ induced by errors in the temperature field of $\Delta D \sim 0.05$. We also see, in Fig. B.6 (c) that, for a such an error in the effective viscosity, the temperature field would have to be accurate to $\sim 2K$, showing that T is likely to be a very good representation of the temperatures in this region of the WAIS.

B.4 External Forcing Mechanisms

In the discussion of the main text, we make clear that external influences beyond the interplay between fracture development and dynamic change are likely required to explain the observations. Here, we briefly discuss such forcing.

Environmental forcing mechanisms known to impact the West Antarctic Ice Sheet include incursions of warm ocean water onto the continental shelf (Jacobs et al., 2011; Jenkins et al., 2018; Dutrieux et al., 2014) or increased discharge of meltwater through the sub-glacial drainage system (Schroeder et al., 2013; Malczyk et al., 2020; Siegfried et al., 2016; Selley et al., 2021), both of which can drive higher rates of basal melt and lead to dynamic speedup. The known decadal variability in ocean forcing in the ASE (Jenkins et al., 2016) has likely been responsible for past dynamic events and structural changes on the ice tongue (Mouginot et al., 2014; Miles et al., 2020) and will continue to have an influence today. Examination of conductivity-temperature-depth (CTD) casts in the region (Wåhlin et al., 2021b) suggest that ocean temperatures in the 400 – 600 m depth range were 0.25 – 0.5 °C warmer in 2019 than in 2014 (Fig. B.7).

The absence of detailed knowledge on ocean temperature and circulation during the first ice dynamic episode makes it difficult to robustly link these observations. However, the strong El Nino event in 2016 (Paolo et al., 2018) could plausibly have increased oceanic heat delivery to the TGIT during 2017 and may therefore have played a role in the 2017-2018 acceleration episode and the continued deterioration of TGIT.

Increased sub-glacial discharge is caused by the drainage of sub-glacial lakes, and satellite observations show that two large sub-glacial lakes in the Thwaites catchment drained in March 2017 (Malczyk et al., 2020). These results indicate that the drainage events led to the filling of a downstream lake rather than routing water to the ice margin, which would seemingly preclude an influence of these lakes on TGIT dynamics. However, given the temporal coincidence of these events we do not rule out a potential link between the drainage event and the acceleration of TGIT.

Surface melting is not thought to play a significant role in Antarctic ice mass loss (Slater et al., 2021), however, observations in Greenland show that surface meltwater penetration to the bed can cause large and rapid speed variations (Davison et al., 2019). Despite observing a decrease in backscatter signal throughout the summer of 2018/19, we found no evidence of supra-glacial

melt ponds, but further work is required to understand if surface melt processes were linked to the observed dynamic ice speedup on TG.

Sea ice can act to inhibit iceberg calving even in unconfined Antarctic ice shelves via the bonding of small peripheral icebergs, and by attenuating ocean swell and reducing the associated ice-shelf flexure (Massom et al., 2018). A visual analysis of SAR backscatter images shows that sea ice retreated beyond the northern-most part of the TGIT in November 2016, and did not fully reform before July 2017 after the large calving events in March and June of that year, indicating that such a process contributed to the structural decline of the TGIT (Miles et al., 2020). Further analysis of SAR images over the full 6-year study period shows that the period of deceleration coincided with an almost complete removal of sea-ice cover in the ocean around TG in January to March 2019, before the return of sea-ice cover prior to the second period of acceleration. This indicates that, although it is responsible for maintaining the structural integrity of the northern sections of the TGIT, sea ice may not always play a decisive role in regulating the speed of the TGIT. Future studies are required to make observations of sea ice thickness in the ice shelf peripheral region, so that the buttressing strength of 1 – 5 m thick sea ice against the flow of the 300 m thick shelf ice can be quantitatively determined.

B.4.1 Ocean Temperature Data

To evaluate the possible effect of changing ocean conditions on the stability of TGIT, we examined conductivity, temperature, depth (CTD) profiles acquired near the TGIT in 2014 and 2019. The CTD profiles are from the Ocean2ice iSTAR cruise in 2014 (available at: <http://www.bodc.ac.uk/>) and from the 2019 Nathaniel B, Palmer cruise (available at: <https://doi.pangaea.de/10.1594/PANGAEA.860066>; Wåhlin et al., 2021a). All profiles were acquired within 60 km of the TGIT, but the majority of profiles were acquired within 30 km. These profiles (Fig. B.7) show that water column temperature in the 400 – 600 m depth range was approximately 0.25 – 0.5 °C warmer in 2019 compared to 2014. This difference in temperature was consistent - 85% of the profiles acquired in 2019 were warmer than the 2014 average in the 400 – 600 m depth range - implying a sustained increase in oceanic heat delivery to the TGIT in 2019 compared to 2014.

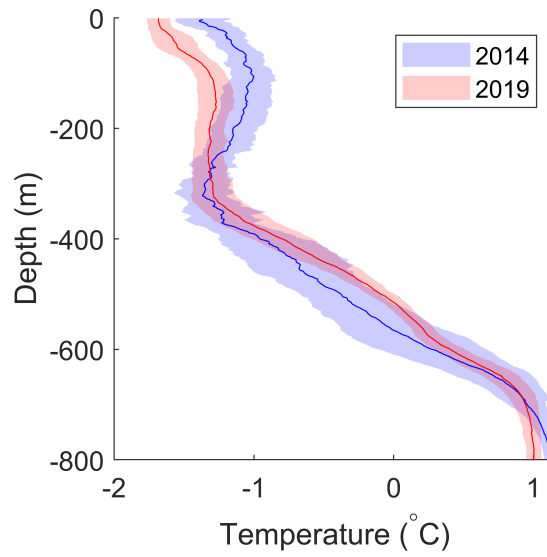


Figure B.7: CTD profiles acquired close to the TGIT in 2014 (blue) and 2019 (red). Shaded areas indicate the standard deviation of water column temperature from all profiles acquired within 50 km of the TGIT in each year.

Bibliography

- Davison, B. J., Sole, A. J., Livingstone, S. J., Cowton, T. R., and Nienow, P. W. (2019). “The influence of hydrology on the dynamics of land-terminating sectors of the Greenland ice sheet”. In: *Frontiers in Earth Science* 7, p. 10.
- Dutrieux, P., De Rydt, J., Jenkins, A., Holland, P. R., Ha, H. K., Lee, S. H., Steig, E. J., Ding, Q., Abrahamsen, E. P., and Schröder, M. (2014). “Strong sensitivity of Pine Island ice-shelf melting to climatic variability”. In: *Science* 343.6167, pp. 174–178.
- Jacobs, S. S., Jenkins, A., Giulivi, C. F., and Dutrieux, P. (2011). “Stronger ocean circulation and increased melting under Pine Island Glacier ice shelf”. In: *Nature Geoscience* 4.8, pp. 519–523.
- Jenkins, A., Dutrieux, P., Jacobs, S., Steig, E. J., Gudmundsson, G. H., Smith, J., and Heywood, K. J. (2016). “Decadal ocean forcing and Antarctic ice sheet response: Lessons from the Amundsen Sea”. In: *Oceanography* 29.4, pp. 106–117.
- Jenkins, A., Shoosmith, D., Dutrieux, P., Jacobs, S., Kim, T. W., Lee, S. H., Ha, H. K., and Stammerjohn, S. (2018). “West Antarctic Ice Sheet retreat in the Amundsen Sea driven by decadal oceanic variability”. In: *Nature Geoscience* 11.10, pp. 733–738.
- Kattenborn, T., Eichel, J., and Fassnacht, F. E. (2019). “Convolutional Neural Networks enable efficient, accurate and fine-grained segmentation of plant species and communities from high-resolution UAV imagery”. In: *Scientific reports* 9.1, pp. 1–9.
- Kingma, D. P. and Ba, J. (2014). “Adam: A method for stochastic optimization”. In: *arXiv preprint arXiv:1412.6980*.
- Krizhevsky, A., Sutskever, I., and Hinton, G. E. (2012). “Imagenet classification with deep convolutional neural networks”. In: *Advances in neural information processing systems* 25, pp. 1097–1105.
- Lange, R. de, Luckman, A., and Murray, T. (2007). “Improvement of satellite radar feature tracking for ice velocity derivation by spatial frequency filtering”. In: *IEEE Transactions on Geoscience and Remote Sensing* 45.7, pp. 2309–2318.
- LeB. Hooke, R. (1981). “Flow law for polycrystalline ice in glaciers: comparison of theoretical predictions, laboratory data, and field measurements”. In: *Reviews of Geophysics* 19.4, pp. 664–672.
- Long, J., Shelhamer, E., and Darrell, T. (2014). *Fully Convolutional Networks for Semantic Segmentation*. DOI: 10.48550/ARXIV.1411.4038.

- Malczyk, G., Gourmelen, N., Goldberg, D., Wuite, J., and Nagler, T. (2020). “Repeat Subglacial Lake Drainage and Filling Beneath Thwaites Glacier”. In: *Geophysical Research Letters* 47.23, e2020GL089658.
- Massom, R. A., Scambos, T. A., Bennetts, L. G., Reid, P., Squire, V. A., and Stammerjohn, S. E. (2018). “Antarctic ice shelf disintegration triggered by sea ice loss and ocean swell”. In: *Nature* 558.7710, pp. 383–389.
- Miles, B. W. J., Stokes, C. R., Jenkins, A., Jordan, J. R., Jamieson, S. S. R., and Gudmundsson, G. H. (2020). “Intermittent structural weakening and acceleration of the Thwaites Glacier Tongue between 2000 and 2018”. In: *Journal of Glaciology* 66.257, pp. 485–495. DOI: 10.1017/jog.2020.20.
- Mouginot, J., Rignot, E., and Scheuchl, B. (2014). “Sustained increase in ice discharge from the Amundsen Sea Embayment, West Antarctica, from 1973 to 2013”. In: *Geophysical Research Letters* 41.5, pp. 1576–1584.
- Paolo, F., Padman, L., Fricker, H., Adusumilli, S., Howard, S., and Siegfried, M. (2018). “Response of Pacific-sector Antarctic ice shelves to the El Niño/Southern oscillation”. In: *Nature geoscience* 11.2, pp. 121–126.
- Paszke, A. et al. (2019). *PyTorch: An Imperative Style, High-Performance Deep Learning Library*. DOI: 10.48550/ARXIV.1912.01703.
- Rignot, E., Mouginot, J., and Scheuchl, B. (2016). *MEaSURES Antarctic Grounding Line from Differential Satellite Radar Interferometry, Version 2*. Boulder, Colorado USA. NASA National Snow and Ice Data Center Distributed Active Archive Center. doi: <https://doi.org/10.5067/IKBWW4RYHF1Q>. Accessed 15/01/2021.
- Ronneberger, O., Fischer, P., and Brox, T. (2015). *U-Net: Convolutional Networks for Biomedical Image Segmentation*.
- Schroeder, D. M., Blankenship, D. D., and Young, D. A. (2013). “Evidence for a water system transition beneath Thwaites Glacier, West Antarctica”. In: *Proceedings of the National Academy of Sciences* 110.30, pp. 12225–12228. ISSN: 0027-8424. DOI: 10.1073/pnas.1302828110.
- Selley, H. L., Hogg, A. E., Cornford, S., Dutrieux, P., Shepherd, A., Wuite, J., Floricioiu, D., Kusk, A., Nagler, T., Gilbert, L., Slater, T., and Kim, T.-W. (Feb. 2021). “Widespread increase in dynamic imbalance in the Getz region of Antarctica from 1994 to 2018”. In: *Nature Communications* 12.1, p. 1133. ISSN: 2041-1723. DOI: 10.1038/s41467-021-21321-1.
- Seroussi, H., Nowicki, S., Simon, E., Abe-Ouchi, A., Albrecht, T., Brondex, J., Cornford, S., Dumas, C., Gillet-Chaulet, F., Goelzer, H., et al. (2019). “initMIP-Antarctica: an ice sheet model initialization experiment of ISMIP6”. In: *The Cryosphere* 13.5, pp. 1441–1471.
- Siegfried, M. R., Fricker, H. A., Carter, S. P., and Tulaczyk, S. (2016). “Episodic ice velocity fluctuations triggered by a subglacial flood in West Antarctica”. In: *Geophysical Research Letters* 43.6, pp. 2640–2648.
- Slater, T., Lawrence, I. R., Otosaka, I. N., Shepherd, A., Gourmelen, N., Jakob, L., Tepes, P., Gilbert, L., and Nienow, P. (2021). “Earth’s ice imbalance”. In: *The Cryosphere* 15.1, pp. 233–246.

- Szegedy, C., Liu, W., Jia, Y., Sermanet, P., Reed, S., Anguelov, D., Erhan, D., Vanhoucke, V., and Rabinovich, A. (2014). *Going Deeper with Convolutions*. DOI: 10.48550/ARXIV.1409.4842.
- Wåhlin, A. K., Graham, A. G. C., Hogan, K. A., Queste, B. Y., Boehme, L., Larter, R. D., Pettit, E. C., Wellner, J., and Heywood, K. J. (2021a). “Pathways and modification of warm water flowing beneath Thwaites Ice Shelf, West Antarctica”. In: *Science Advances* 7.15. DOI: 10.1126/sciadv.abd7254.
- Wåhlin, A., Graham, A., Hogan, K., Queste, B., Boehme, L., Larter, R., Pettit, E., Wellner, J., and Heywood, K. (2021b). “Pathways and modification of warm water flowing beneath Thwaites Ice Shelf, West Antarctica”. In: *Science Advances* 7.15, eabd7254.

Appendix C

Supplementary figures for the
article: The effect of landfast sea ice
buttressing on ice dynamic speedup
in the Larsen-B Embayment,
Antarctica

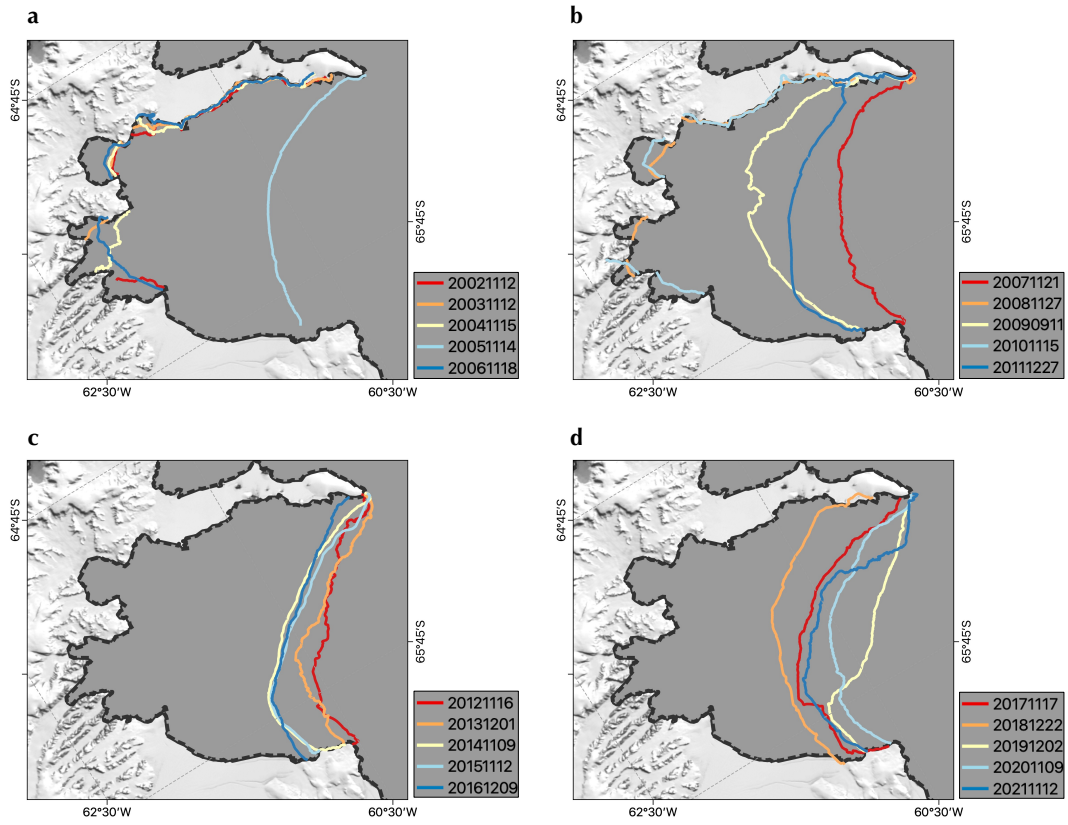


Figure C.1: Annual sea ice front locations in the Larsen-B Embayment delineated from multi-spectral optical Landsat-8 imagery and Sentinel-1 synthetic aperture radar backscatter imagery between November 2002 and November 2021. a) 2002-2006, b) 2007-2011, c) 2012-2016, d) 2017-2021. The basemap is the MODIS Mosaic of Antarctica (Haran et al., 2021) and the black dashed line is the coastline according to MEaSUREs (Mouginot et al., 2017).

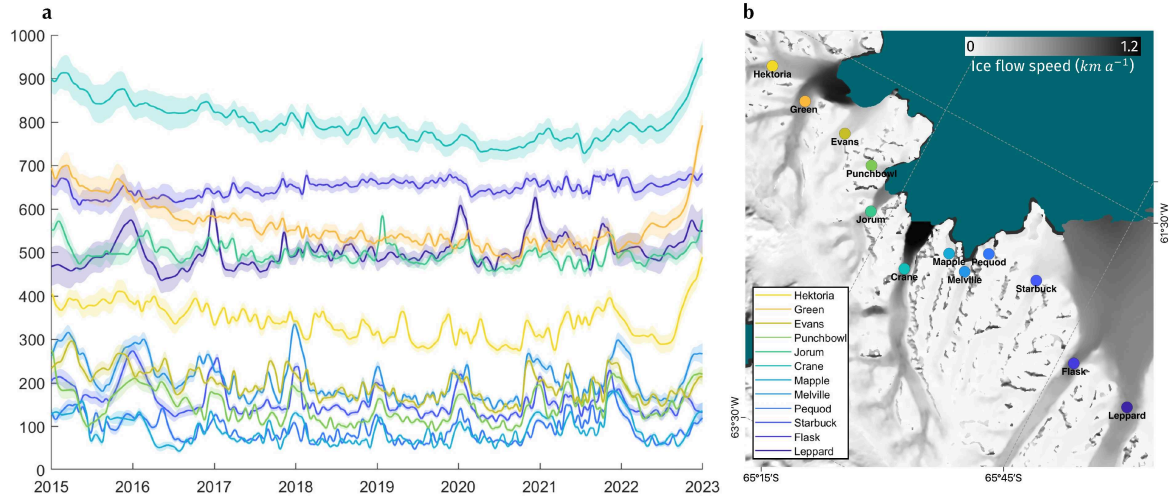


Figure C.2: Timeseries of speed on the Larsen-B Glaciers. a) Timeseries of ice flow speed for 12 glaciers flowing into the Larsen-B Embayment between January 2015 and December 2022 from data made using Interferometric Wide (IW) mode synthetic Aperture Radar (SAR) data acquired by the Sentinel-1a/b satellites. Each datapoint is the mean speed over a 1 km-long segment of a flowline located on the corresponding glacier. A Kalman smoother was used along with a 30-day moving average to smooth the data. The uncertainties shown are 1σ either side of the mean. b) Coloured circles represent the centre points of the 1 km-long flowline segments used to extract the timeseries in (a) with corresponding colour. Greyscale colourmap: Inverse-error-weighted mean ice speed over the region, measured between October 2014 and April 2023. The sea is coloured dark green.

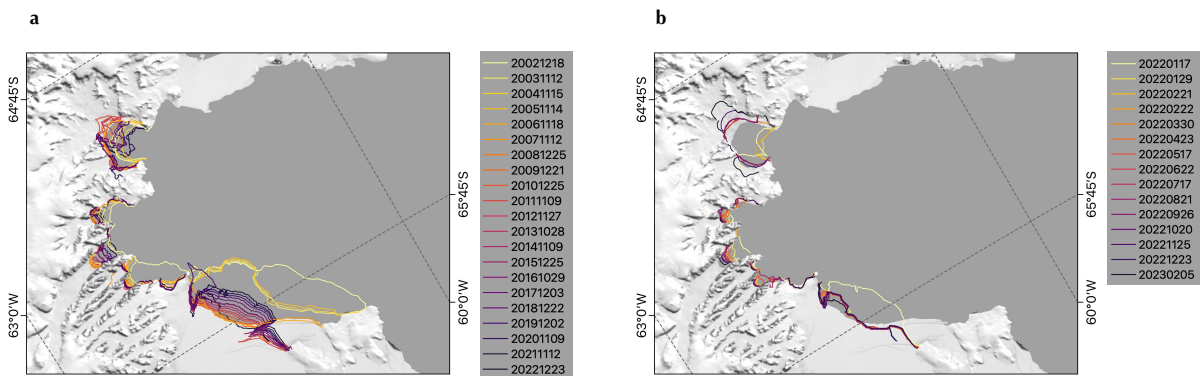


Figure C.3: Calving front locations in the Larsen-B Embayment delineated from multi-spectral optical Landsat-8 imagery and Sentinel-1 synthetic aperture radar backscatter imagery. (a) annual calving front locations between December 2002 and December 2022. (b) monthly calving front locations between January 2021 and February 2023. The basemap is the MODIS Mosaic of Antarctica (Haran et al., 2021)

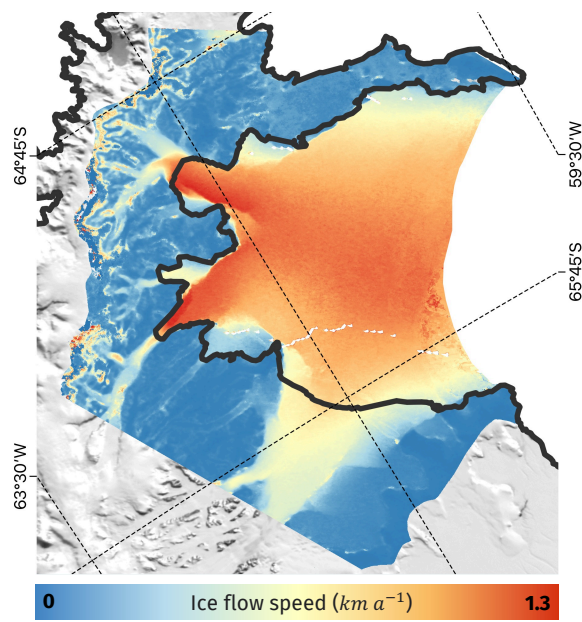


Figure C.4: Median ice speed over the landfast sea ice in the Larsen-B Embayment measured between November 2015 and July 2017 using Interferometric Wide (IW) mode synthetic Aperture Radar (SAR) data acquired by the Sentinel-1a/b satellites. The basemap is the MODIS Mosaic of Antarctica (Haran et al., 2021) and the black line is the MEaSUREs coastline according to Mouginot et al. (2017)

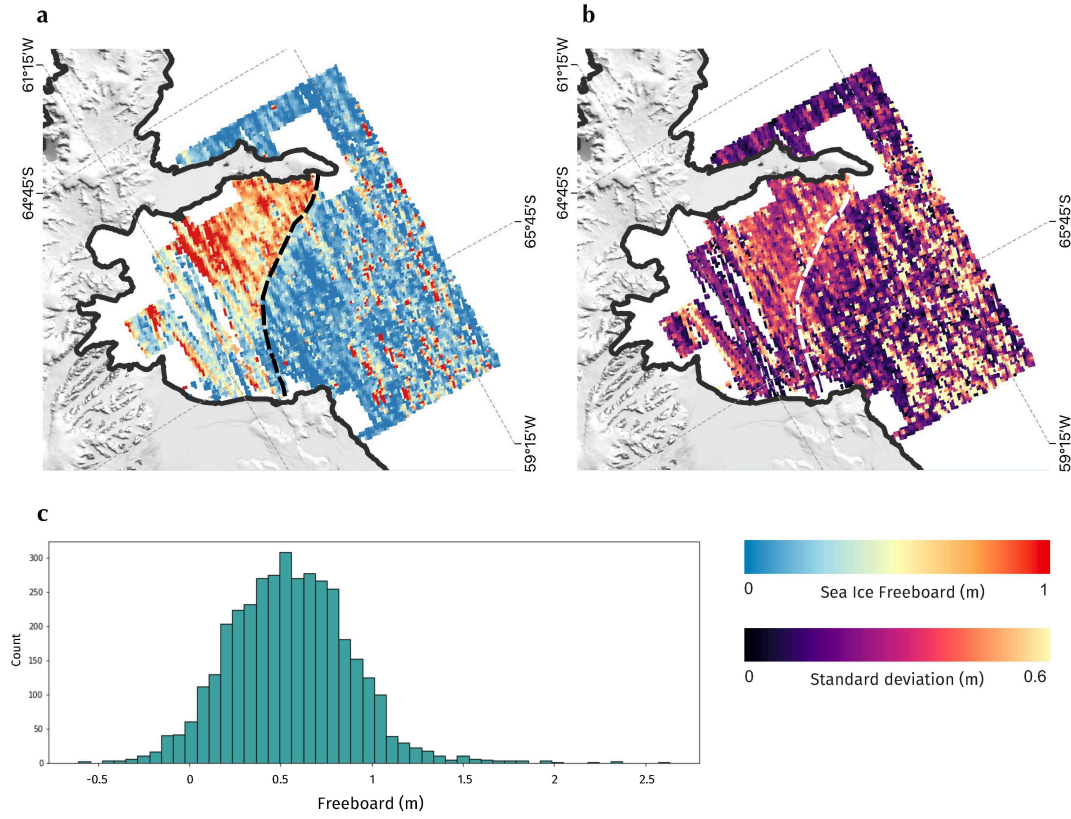


Figure C.5: CryoSat-2 derived measurements of sea ice freeboard (Tilling et al., 2018). (a) Mean freeboard heights between 2013 and 2020. (b) Standard deviation of measured freeboard heights between 2013 and 2020. (c) Histogram of mean freeboard heights (shown in (a)) in the Larsen-B Embayment, landward of the edge of the landfast sea ice (black dashed line in (a), and the white dashed line in (b)). Basemaps show the MODIS MOA (Haran et al., 2021). The black line is the coastline according to MEaSUREs (Mouginot et al., 2017).

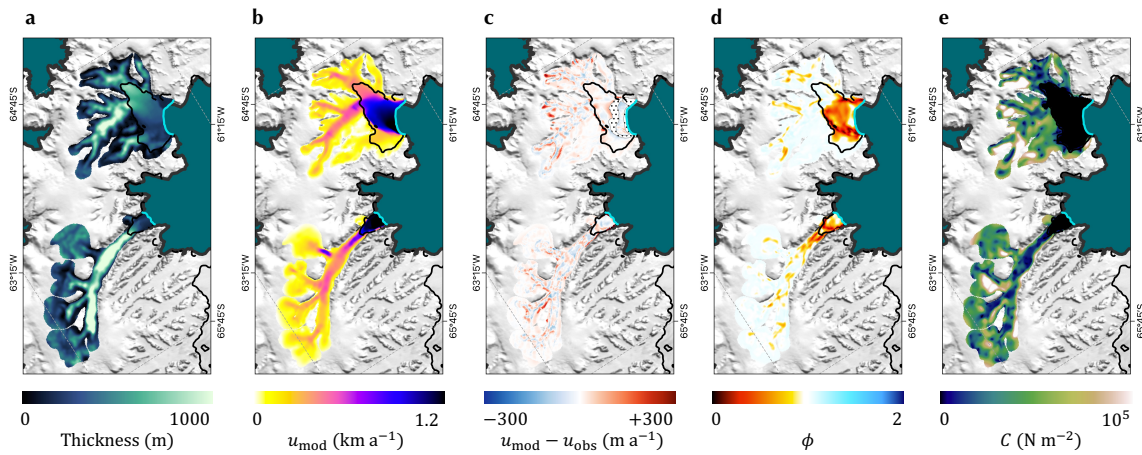


Figure C.6: Model setup at the end of the initialisation. a) Thickness of HGE and Crane Glaciers used in the model. b) Modelled ice speed. c) Misfit between the modelled and observed ice speed. d) Stiffness field, with values less than 1 indicating softer or thinner ice than assumed and values greater than 1 the opposite. e) Basal slip coefficient.

Bibliography

- Haran, T. M., Bohlander, J., Scambos, T. A., Painter, T. H., and Fahnestock, M. A. (2021). *MODIS Mosaic of Antarctica 2003-2004 (MOA2004) Image Map, Version 2*. DOI: 10.5067/68TBT0CGJS0J.
- Mouginot, J., Scheuchl, B., and Rignot, E. (2017). *MEaSUREs Antarctic Boundaries for IPY 2007-2009 from Satellite Radar, Version 2*. Boulder, Colorado USA. NASA National Snow and Ice Data Center Distributed Active Archive Center. doi: <http://dx.doi.org/10.5067/AXE4121732AD>. Accessed 14/02/2023.
- Tilling, R. L., Ridout, A., and Shepherd, A. (2018). “Estimating Arctic sea ice thickness and volume using CryoSat-2 radar altimeter data”. In: *Advances in Space Research* 62.6. The CryoSat Satellite Altimetry Mission: Eight Years of Scientific Exploitation, pp. 1203–1225. ISSN: 0273-1177. DOI: <https://doi.org/10.1016/j.asr.2017.10.051>.

Appendix D

The first order adjoint model

Throughout this thesis techniques have been employed to quantify the sensitivity of ice speed to changes in the control parameters C and ϕ - be it to estimate how changes in thickness across the domain affect grounded ice speed in the Larsen-B Embayment (chapter 4) or in the calculation of gradient directions for use in inverse problems solved using the conjugate gradient method. This is done using an adjoint method which is not described in detail. Nor is this method, especially when couched in the language of linear operators rather than vectors and matrices, really described thoroughly elsewhere in the literature - perhaps as it is considered standard. However, I have used the space below to derive the first-order adjoint system for the (linearised) shelfy-stream equations and show that the linear viscous tensor operator is self-adjoint.

D.1 A general adjoint system

We consider the sensitivities of a functional $\mathcal{J}$ of ice speed with respect to changes in the control parameter ϕ . An example is $\mathcal{J} = \int_{\Omega_{\text{gnd}}} |u| d\Omega$, the integral over grounded ice speed discussed in section 6.2.3. In general, our $\mathcal{J}$ is a function of ice speed, and this ice speed is a function of ϕ , as well as the basal traction coefficient, ice thickness and other variables we take to be fixed.

Taking a linear formulation of the sliding law, the stress balance equations solved by BISICLES take the form of the shelfy-stream equation (SSE):

$$\nabla \cdot [\phi h \bar{\mu} (\nabla \mathbf{u} + (\nabla \mathbf{u})^\top + 2(\nabla \cdot \mathbf{u}) \mathcal{I}_2)] - C \mathbf{u} - \rho_i g h \nabla s = 0. \quad (\text{D.1})$$

Setting $\phi = e^q$, to ensure positivity of the softness field, we write these equations simply as $G(q, \mathbf{u}) = 0$.

The ultimate quantity of interest in calculating the sensitivity of $\mathcal{J}$ changes in ϕ , is the Gâteaux derivative of $\mathcal{J}$ in the direction δq :

$$\delta \mathcal{J}(\mathbf{u}, q, \delta q) = \lim_{\varepsilon \rightarrow 0} \left\{ \frac{1}{\varepsilon} (\mathcal{J}(\mathbf{u}, q + \varepsilon \delta q) - \mathcal{J}(\mathbf{u}, q)) \right\}. \quad (\text{D.2})$$

We write this as the projection of the functional gradient of $\mathcal{J}$ onto the direction δq :

$$\langle \nabla_q \mathcal{J}, \delta q \rangle, \quad (\text{D.3})$$

where the operator $\langle \cdot, \cdot \rangle$ is the standard L_2 inner product. The first-order adjoint method is a way of finding the gradient $\nabla_q \mathcal{J}$ efficiently by solving a set of auxiliary ‘adjoint’ equations. We cast (D.3) in terms of a calculation of the derivatives of a Lagrangian formed of the objective function, the stress balance equations and the vector field $\boldsymbol{\lambda}$ representing freely chosen Lagrange multipliers.

Let:

$$\begin{aligned} \mathcal{L}(q, \boldsymbol{\lambda}, \mathbf{u}) &= \mathcal{J}(\mathbf{u}) + \int_{\Omega} \boldsymbol{\lambda} \cdot G(q, \mathbf{u}) d\Omega \\ &= \mathcal{J}(\mathbf{u}) + G_{\boldsymbol{\lambda}}(q, \boldsymbol{\lambda}, \mathbf{u}) \end{aligned} \quad (\text{D.4})$$

This Lagrangian is just $\mathcal{J}(\mathbf{u})$, when the velocities solve the model equations, as $G(q, \mathbf{u})$ is everywhere zero. The variation of $G(q, \mathbf{u})$ is also zero at the solution. We can hence equate gradients of $\mathcal{J}$ around the solution to those of the Lagrangian.

Differentiating the Lagrangian with respect to q and using the chain rule gives:

$$\begin{aligned} \langle \nabla_q \mathcal{L}, \delta q \rangle &= \langle \nabla_q \mathcal{J}, \delta q \rangle + \langle \nabla_q G_{\boldsymbol{\lambda}}, \delta q \rangle \\ &= \langle \partial_{\mathbf{u}} \mathcal{J}, (\partial_q \mathbf{u}, \delta q) \rangle + \langle \partial_{\boldsymbol{\lambda}} G_{\boldsymbol{\lambda}}, (\partial_q \boldsymbol{\lambda}, \delta q) \rangle + \langle \partial_{\mathbf{u}} G_{\boldsymbol{\lambda}}, (\partial_q \mathbf{u}, \delta q) \rangle + \langle \partial_q G_{\boldsymbol{\lambda}}, \delta q \rangle. \end{aligned} \quad (\text{D.5})$$

Where I have defined the notation (A, b) representing the action of a linear operator A on a field b defined over Ω , resulting in another field defined over Ω . The linear operators, and their

action on fields have the following properties:

$$(A, (B, c)) = (AB, c) \quad (\text{D.6})$$

$$\langle (A, b), c \rangle = \langle b, (A^*, c) \rangle \quad (\text{D.7})$$

$$(AB)^* = B^* A^*. \quad (\text{D.8})$$

Equation (D.5) contains a number of expressions of the form $\partial_q \mathbf{v}$ where $\mathbf{v}$ is a field over Ω . This is the *Jacobian* of $\mathbf{v}$ with respect to q , and is written as $J_{\mathbf{v}}$. These are difficult to calculate and the essence of the adjoint method is to choose Lagrange multipliers that ensure we never have to. (In the discrete case, $\mathbf{u}$, q , $\boldsymbol{\lambda}$ are all thought of as vectors instead of functions defined over the (x, y) -plane in Ω . This perspective makes the Jacobian, as well as equations (D.6)-(D.8), easy to interpret.)

We require $G(q, \mathbf{u}) = 0$ at the solution about which we are perturbing, so the second term in the above expansion is zero.

Then, collecting terms, we have:

$$\langle \nabla_q \mathcal{L}, \delta q \rangle = \langle \partial_{\mathbf{u}} \mathcal{J} + \partial_{\mathbf{u}} G_{\boldsymbol{\lambda}}, (J_{\mathbf{u}}, \delta q) \rangle + \langle \partial_q G_{\boldsymbol{\lambda}}, \delta q \rangle. \quad (\text{D.9})$$

The first term includes a calculation of the Jacobian $J_{\mathbf{u}}$, so we choose our Lagrange multipliers to solve the following equation relating the derivatives of the objective function and stress balance equations with respect to $\mathbf{u}$:

$$\partial_{\mathbf{u}} \mathcal{J} + \partial_{\mathbf{u}} G_{\boldsymbol{\lambda}} = 0. \quad (\text{D.10})$$

This is the *adjoint equation* for $\boldsymbol{\lambda}$. Solving it gives us $\boldsymbol{\lambda}$ and reduces Eq. D.9 to a simple calculation for the Gâteaux derivative of $\mathcal{J}$ with respect to q :

$$\langle \nabla_q \mathcal{L}, \delta q \rangle \equiv \langle \nabla_q \mathcal{J}, \delta q \rangle = \langle \partial_q G_{\boldsymbol{\lambda}}, \delta q \rangle, \quad (\text{D.11})$$

D.2 The adjoint equation and gradient for the SSE

Once, again the shelfy-stream equations are:

$$\nabla \cdot [\phi h \bar{\mu}(\nabla \mathbf{u} + (\nabla \mathbf{u})^\top + 2(\nabla \cdot \mathbf{u})\mathcal{I}_2)] - C\mathbf{u} - \rho_i g h \nabla s = 0. \quad (\text{D.12})$$

subject to boundary conditions:

$$\hat{\mathbf{n}} \cdot \mathbf{u} = 0, \quad \hat{\mathbf{t}} \cdot \nabla \mathbf{u} \cdot \hat{\mathbf{n}} = 0 \quad (\text{D.13})$$

(where $\hat{\mathbf{n}}$ and $\hat{\mathbf{t}}$ are normal and tangent vectors to the boundary respectively).

We start by writing eq. (D.10) for the SSE. It is much easier to solve if we ignore the non-linear dependence of ϕ on $\mathbf{u}$, as this gives the functional derivative $D_{\mathbf{u}}G_{\boldsymbol{\lambda}}$ a simple form (as we shall see below). This omission is discussed in Goldberg and Sergienko, 2011; Morlighem et al., 2013.

We start with the expression $D_{\mathbf{u}}G_{\boldsymbol{\lambda}}$ which, in full, reads:

$$\begin{aligned} & \nabla_{\mathbf{u}} \left(\int_{\Omega} \boldsymbol{\lambda} \cdot (\nabla \cdot [\phi h \bar{\mu}(\nabla \mathbf{u} + (\nabla \mathbf{u})^\top + 2(\nabla \cdot \mathbf{u})\mathcal{I}_2]) - C\mathbf{u} - \rho_i g h \nabla s) d\Omega \right) \\ &= \nabla_{\mathbf{u}} \left(\int_{\Omega} \boldsymbol{\lambda} \cdot (\nabla \cdot \gamma) d\Omega \right) + C\boldsymbol{\lambda}, \end{aligned}$$

where

$$\gamma = \phi h \bar{\mu}(\nabla \mathbf{u} + (\nabla \mathbf{u})^\top + 2(\nabla \cdot \mathbf{u})\mathcal{I}_2).$$

The component of the adjoint equation associated with the effect of basal friction in the stress balance equation is simple, so we focus on the first term and consider the integral in isolation.

We use the following generalisation of Green's first identity:

Lemma D.1. *For an n -dimensional vector field $\mathbf{v}$ and an $n \times n$ -dimensional, rank-2 tensor field T defined over the domain Ω :*

$$\int_{\Omega} \mathbf{v} \cdot (\nabla \cdot T) d\Omega = \int_{\partial\Omega} (T \cdot \hat{\mathbf{n}}) \mathbf{v} d\Sigma - \int_{\Omega} T : (\nabla \mathbf{v}) d\Omega,$$

where $\hat{\mathbf{n}}$ is the unit normal to the boundary $\partial\Omega$. The binary operator $(\cdot : \cdot)$ represents the double

dot product such that $A : B = A_{ij}B_{ji}$.

Proof. We start with the divergence theorem for vector field $\mathbf{u}$:

$$\int_{\Omega} \nabla \cdot \mathbf{u} \, d\Omega = \int_{\partial\Omega} \mathbf{u} \cdot \hat{\mathbf{n}} \, d\Sigma.$$

Multiplying the vector field $\mathbf{u}$ by a scalar field φ then gives

$$\int_{\Omega} \nabla \cdot (\varphi \mathbf{u}) \, d\Omega = \int_{\partial\Omega} (\varphi \mathbf{u}) \cdot \hat{\mathbf{n}} \, d\Sigma \quad (\text{D.14})$$

$$\implies \int_{\Omega} \varphi (\nabla \cdot \mathbf{u}) + (\nabla \varphi \cdot \mathbf{u}) \, d\Omega = \int_{\partial\Omega} (\varphi \mathbf{u}) \cdot \hat{\mathbf{n}} \, d\Sigma, \quad (\text{D.15})$$

which is a general version of Green's first identity.

Note that the tensor field T can be written $T = (\mathbf{T}_1, \dots, \mathbf{T}_n)$ where $\mathbf{T}_i$ are n -dimensional vector fields, so we can write the integral of interest as:

$$\int_{\Omega} \mathbf{v} \cdot (\nabla \cdot T) \, d\Omega = \int_{\Omega} v_i (\nabla \cdot \mathbf{T}_i) \, d\Omega,$$

where we have employed the Einstein summation convention in which repeated indices are summed over. Using equation (D.15), we see

$$\int_{\Omega} v_i (\nabla \cdot \mathbf{T}_i) \, d\Omega = \int_{\partial\Omega} (v_i \mathbf{T}_i) \cdot \hat{\mathbf{n}} \, d\Sigma - \int_{\Omega} \mathbf{T}_i \cdot (\nabla v_i) \, d\Omega$$

which can equivalently be written in the form shown in Lemma D.2 □

Using this, we find:

$$\int_{\Omega} \boldsymbol{\lambda} \cdot (\nabla \cdot \boldsymbol{\gamma}) \, d\Omega = \int_{\partial\Omega} (\boldsymbol{\gamma} \cdot \hat{\mathbf{n}}) \cdot \boldsymbol{\lambda} \, d\Sigma - \int_{\Omega} (\nabla \boldsymbol{\lambda}) : \boldsymbol{\gamma} \, d\Omega.$$

We choose the boundary conditions for the Lagrange multiplier field to be those of the velocity field $\mathbf{u}$ which, along with the fact that there is zero vertical velocity at the domain boundary (so $\nabla \cdot \mathbf{u} = 0$ in 2D) ensure the above boundary term is zero. Hence, after substituting in the expression for $\boldsymbol{\gamma}$ and rearranging, we have:

$$\int_{\Omega} \boldsymbol{\lambda} \cdot (\nabla \cdot \boldsymbol{\gamma}) \, d\Omega = - \int_{\Omega} [\nabla \mathbf{u} + (\nabla \mathbf{u})^\top + 2(\nabla \cdot \mathbf{u})\mathcal{I}_2] : (\phi h \bar{\mu} \nabla \boldsymbol{\lambda}) \, d\Omega$$

Using Lemma D.2 again, and noting that $(\nabla \boldsymbol{\lambda}) : (\nabla \cdot \mathbf{u} \mathcal{I}_2) = (\nabla \cdot \boldsymbol{\lambda} \mathcal{I}_2) : (\nabla \mathbf{u})$, we have:

$$\begin{aligned} \int_{\Omega} \boldsymbol{\lambda} \cdot (\nabla \cdot \boldsymbol{\gamma}) \, d\Omega &= \int_{\Omega} \mathbf{u} \cdot \left(\nabla \cdot [\phi h \bar{\mu} (\nabla \boldsymbol{\lambda} + (\nabla \boldsymbol{\lambda})^\top + 2(\nabla \cdot \boldsymbol{\lambda}) \mathcal{I}_2)] \right) \, d\Omega \\ &\quad - \int_{\partial\Omega} \phi h \bar{\mu} [(\nabla \boldsymbol{\lambda}) \cdot \hat{\mathbf{n}} + (\nabla \boldsymbol{\lambda})^\top \cdot \hat{\mathbf{n}} + (\mathbf{u} \cdot \hat{\mathbf{n}})(\nabla \cdot \boldsymbol{\lambda})] \, d\Sigma. \end{aligned}$$

Once again, each term in the boundary integral disappears when we take into account the boundary conditions for $\mathbf{u}$ and $\boldsymbol{\lambda}$.

(Note that this proves that the viscous tensor operator $\mathcal{H}$ is self-adjoint when the rheology is assumed to be linear. We have shown that $\langle \mathcal{H}(\mathbf{u}), \boldsymbol{\lambda} \rangle \stackrel{!}{=} \langle \mathbf{u}, \mathcal{H}^*(\boldsymbol{\lambda}) \rangle = \langle \mathbf{u}, \mathcal{H}(\boldsymbol{\lambda}) \rangle$)

Hence, the left hand side of the adjoint equation becomes:

$$\nabla_{\mathbf{u}} \int_{\Omega} \mathbf{u} \cdot \left(\nabla \cdot [\phi h \bar{\mu} (\nabla \boldsymbol{\lambda} + (\nabla \boldsymbol{\lambda})^\top + 2(\nabla \cdot \boldsymbol{\lambda}) \mathcal{I}_2)] \right) \, d\Omega + C \boldsymbol{\lambda} \quad (\text{D.16})$$

$$= \mathbf{u} \cdot \left(\nabla \cdot [\phi h \bar{\mu} (\nabla \boldsymbol{\lambda} + (\nabla \boldsymbol{\lambda})^\top + 2(\nabla \cdot \boldsymbol{\lambda}) \mathcal{I}_2)] \right) + C \boldsymbol{\lambda}. \quad (\text{D.17})$$

The adjoint equation for the SSE therefore reduces to the following boundary value problem:

$$\nabla \cdot [\phi h \bar{\mu} (\nabla \boldsymbol{\lambda} + (\nabla \boldsymbol{\lambda})^\top + 2(\nabla \cdot \boldsymbol{\lambda}) \mathcal{I}_2)] + C \boldsymbol{\lambda} = -\partial_{\mathbf{u}} \mathcal{J}, \quad (\text{D.18})$$

(and, presumably by construction, $\partial_{\mathbf{u}} \mathcal{J}$ is not too difficult to write down),

with reflection boundary conditions on the domain boundary:

$$\hat{\mathbf{n}} \cdot \boldsymbol{\lambda} = 0, \quad \hat{\mathbf{t}} \cdot \nabla \boldsymbol{\lambda} \cdot \hat{\mathbf{n}} = 0. \quad (\text{D.19})$$

Finally, the Gateaux derivative in (D.11) can be written:

$$\langle \nabla_q \mathcal{J}, \delta q \rangle \approx \int_{\Omega} \delta q \phi \bar{\mu} h \nabla \boldsymbol{\lambda} (\nabla \mathbf{u} + (\nabla \mathbf{u})^\top + 2(\nabla \cdot \mathbf{u}) \mathcal{I}_2) \, d\Omega, \quad (\text{D.20})$$

and the gradient as:

$$\nabla_q \mathcal{J} \approx \phi \bar{\mu} h \nabla \boldsymbol{\lambda} (\nabla \mathbf{u} + (\nabla \mathbf{u})^\top + 2(\nabla \cdot \mathbf{u}) \mathcal{I}_2), \quad (\text{D.21})$$

Bibliography

- Goldberg, D. N. and Sergienko, O. V. (2011). “Data assimilation using a hybrid ice flow model”. In: *The Cryosphere* 5.2, pp. 315–327. DOI: 10.5194/tc-5-315-2011.
- Morlighem, M., Seroussi, H., Larour, E., and Rignot, E. (2013). “Inversion of basal friction in Antarctica using exact and incomplete adjoints of a higher-order model”. en. In: *Journal of Geophysical Research: Earth Surface* 118.3, pp. 1746–1753. ISSN: 2169-9011. DOI: 10.1002/jgrf.20125.