Consumer-Centric Community Electricity Markets with Agents' Utility Maximization: Robust Approaches and Analysis of Ancillary Services

Ifeoma Uchenna Onugha

Submitted in accordance with the requirements for the degree of Doctor of Philosophy

The University of Leeds

School of Electronic and Electrical Engineering

February 14, 2024

Declaration of Academic Integrity

The candidate confirms that the work submitted is his/her own, except where work that has formed part of jointly-authored publications has been included. The contribution of the candidate and the other authors to this work has been explicitly indicated in the report. The candidate confirms that appropriate credit has been given within the report where reference has been made to the work of others.

Chapter 3

I. Onugha, S. Dehghan, and P. Aristidou, "Rethinking consumer-centric markets under uncertainty: A robust approach to community-based energy trades," inProc. of the 2020 IEEE General Meeting, 2020.

The supervisor, Dr. Aristidou suggested the review of community-based market structure. The postdoctoral student Dr. Dehgan worked with the Ph.D. student I. Onugha on the development of a general robust model for the market and analysis of results. The PhD student developed a robust model of the market involving all community agents and analysed three microgrid communities of different agents and the results.

This copy has been supplied on the understanding that no quotation from the report may be published without proper acknowledgment.

Signed

Ifeoma Uchenna Onugha

Acknowlegement

My sincere gratitude towards the inception of my PhD research work goes to my supervisor Dr Petros Aristidou for all his inputs, advice, and wide base knowledge towards this research and to my co-supervisor Dr. Li X Zhang for her contributions, advice, and encouragement towards realising the objectives of this work.

I appreciate the head of the School of Electronic and Electrical for providing all the resources and support necessary to complete my PhD in record time.

I acknowledge the postgraduate research mentoring team for all the inductions and assistance that helped me settle in as an international student of the University.

I remain indebted to the Petroleum Technology Development Fund (PTDF) for funding my Ph.D. study, the Funds for Women Graduates (FFWG) for the grants towards my living expenses, and the University of Leeds for the Doctoral Grant.

Finally, I express my utmost gratitude to my family, my parents, siblings, husband, and children for their emotional support and encouragement throughout my PhD program.

Abstract

The flexibility of the end-users in the electricity markets is becoming more pertinent with the evolution of market mechanisms allowing consumers to participate actively. The advent of Distributed Energy Resources (DERs) and energy storage systems is gradually and continuously changing the roles of market operators. In the emerging consumer-centric markets, the consumers are equipped with DERs and can participate actively as prosumers, trading their energy resources with neighbours. The other community agents are mainly consumers without DERs and producers without load demands. The impact of the uncertainty of DERs and load demands on community-based electricity market (CBEM) structures has not been fully investigated. In this thesis, we propose a robust solution to CBEM operations under uncertainty and compare the optimal decisions on energy trades with deterministic, and opportunistic models. Also, we employ Taguchi's orthogonal array testing (TOAT) to generate proficient scenarios from uncertain parameters of prosumers', producers', and consumers' resources.

While the optimality of the solution provided by the CBEM mechanisms has been analyzed extensively, the ability to address the individual user preferences that would maximize their utility has been hard to incorporate. We also, extend the traditional, community-based, centralized electricity market to incorporate the consumer and producer preferences relating to economic aspects rather than technical constraints. This is achieved with the use of indifference curves for standard utility functions used in exchange economy (such as Cobb-Douglas utility, perfect substitutes, perfect complements, etc.)

This thesis further proposes a single-stage robust formulation of the traditional, community-based, centralized electricity market incorporating the agents' preferences relating to economic aspects rather than technical constraints. A single-stage robust optimization model of the CBEM with utility maximization is formulated by integrating uncertainty constraints defined within polyhedral uncertainty sets representing variations in agents' resources from forecasted/expected values. The proposed approach ensures the robustness of the market with the uncertainty of agents' generation and load resources. It allows community agents the ability to adjust their budgets to the given robust scenario. Thereafter the proposed methodology can control the degree of robustness as regards the uncertainty parameters of agents' resources.

With consideration to the emerging consumer-centric markets, the possibilities of the of-

fer of ancillary services besides demand-side responses, energy management, and peak shaving/shifting/leveling functions are being explored in providing flexibility and scalability in the market mechanism. The offer of these services within several market mechanisms and entities has been researched widely in literature, in our work we propose the formulation of the CBEM in a joint day ahead market model offering ancillary services of reserve and regulation while minimizing the total traded costs of agents' and the community manager and also maximizing the individual utility functions of agents.

Finally, a single-stage robust mixed-integer linear problem is presented which models the joint market over the worst-case realisations of uncertain parameters of agents' resources and reserve/regulation prices represented within polyhedral uncertainty sets.

In this work, the performance of the proposed CBEM market is implemented in three case studies with consideration to different market participants the prosumers, producers, and consumers to analyse the impact of uncertainty in CBEM with and without agent utility maximization and also in a joint day-ahead market offering ancillary services. The first case study presents 7 prosumers equipped with PV generation and load consumption, the second case study presents 15 agents with 7 producers equipped with PV generation and 8 consumers and the third case study presents 5 prosumers equipped with PV generation, 20 consumers, and three producers one with a wind production and the other two with conventional generation. Simulation results demonstrate the costs of robustness as a result of the impact of uncertainty, the agents' preference relations and utility maximization, and the total profits in the offer of ancillary services.

Contents

List of Figures

List of Tables

Nomenclature

ACRONYMS

$2G, 3G, 4G,$ *and* $5G$ Second, third, fourth, and fifth generation mobile networks.

$ADMM$ Alternating direction method of multipliers.

$ADRO$ Adaptive distributionally robust optimization.

AMI Advanced metering infrastructure.

AMR Automated meter reading.

ARO Adaptive robust optimization.

$ARSO$ Adaptive robust stochastic optimization.

AS Ancillary services.

$BCET_t$ Opportunistic community energy trade.

BM Start-Up Balancing mechanism start-up.

BS Bill sharing.

$CBEM$ Community based electricity markets.

CBM Community based markets.

CES Community energy storage.

CET_w Community energy trade in scenario w.

$CETM_{p\&c/cr}/DCETM_{p\&c/cr}$ Community energy trade with utility maximization representing total traded costs of producers and consumers with community-ownedgeneration resource.

$CETM_{p\&c}/DCETM_{p\&c}$ Community energy trade with utility maximization representing total traded costs of producers and consumers.

$CETM_{pr/cr}/DCETM_{pr}$ Community energy trade with utility maximization representing total traded costs of prosumers with community-owned generation resource.

$CETM_{prp\&c/cr}/DCETM_{prp\&c/cr}$ Community energy trade with utility maximization representing total traded costs of prosumers, producers, and consumers with community-owned generation resource.

$CETM_{pr}/DCETM_{pr}$ Community energy trade with utility maximization representing total traded costs of prosumers.

$DCET_t$ Deterministic community energy trade.

$DERs$ Distributed energy resources.

DFR Dynamic Frequency Response.

DGs Diesel generator.

DGs Distributed generation systems.

DRO Double-stage robust optimization.

DSM Demand side management .

EC European commission.

ESS Energy storage systems.

EV Electrical vehicle.

ICT Information and communication technology.

$IGDT$ Information-gap decision theory.

ImS Import share.

LTE Long Term Evolution.

MMR Mid-market rate.

MRS Marginal rate of substitution.

MU Marginal utility.

$NIST$ National Institute of Standards and Technology.

OA Orthogonal array.

$P2P$ Peer-to-peer.

PLC Power line communication.

PMU Phasor measurement unit.

PV Photovoltaic.

QoC Quality of cost.

QoT Quality of trade.

$RCET_t$ Robust community energy trade.

$RCETM_{p\&c/cr}, BCETM_{p\&c/cr}$ Robust and opportunistic community energy trade with utility maximization representing total traded costs of producers and consumers with community-owned resource.

$RCETM_{p\&c}, BCETM_{p\&c}$ Robust and opportunistic community energy trade with utility maximization representing total traded costs of producers and consumers.

$RCETM_{pr/cr}, BCETM_{pr}$ Robust and opportunistic community energy trade with utility maximization representing total traded costs of prosumers with community-owned resource.

$RCETM_{prp\&c/cr}, BCETM_{prp\&c/cr}$ Robust and opportunistic community energy trade with utility maximization representing total traded costs of prosumers, producers, and consumers with community-owned resource.

$RCETM_{prp\&c/cr}, BCETM_{prp\&c/cr}$ Robust community energy trade with utility maximization representing total profits of prosumers, producers, and consumers from offering reserve and regulation AS.

$RCETM_{pr}, BCETM_{pr}$ Robust and opportunistic community energy trade with utility maximization representing total traded costs of prosumers.

RES Renewable energy sources.

RO Robust Optimization.

$RRG/CETM_{p\&c}/RRG/DCETM_{p\&c}$ Deterministic community energy trade with utility maximization representing total profits of producers and consumers from offering reserve and regulation AS.

$RRG/CETM_{prp\&c}/RRG/DCETM_{prp\&c}$ Deterministic community energy trade with utility maximization representing total profits of prosumers, producers, and consumers from offering reserve and regulation AS.

$RRG/CETM_{pr}/RRG/DCETM_{pr}$ Deterministic community energy trade with utility maximization representing total profits of prosumers from offering reserve and regulation AS.

$RRG/RCETM_{p\&c}$ Robust community energy trade with utility maximization representing total profits of producers and consumers from offering reserve and regulation AS.

$RRG/RCETM_{pr}$ Robust community energy trade with utility maximization representing total profits of prosumers from offering reserve and regulation AS.

SDR Supply-demand ratio.

$SGETP$ Smart grid European technology platform.

SO System operator.

SRO Single-stage robust optimization.

$STOR$ Short term operating reserve.

$TOAT$ Taguchi's orthogonal array testing.

$WIFI$ Wireless fidelity.

Functions

β_{0r}^{d} Protection function for consumption budget constraint of prosumer.

β_{0r}^{g} Protection function for generation budget constraint of prosumer.

β_{0}^{d} Protection function for consumption budget constraint of consumer.

β_{0}^{g} Protection function for generation budget constraint of producer.

Indices

i Index of community-owned DG units.

j Index of prosumer agent resources.

k Index of blocks for community-owned DG units.

l Index of producer agent resources.

n_d Index of consumers in the community.

n_g Index of producers in the community.

n_r Index of prosumers in the community.

t Index of hours (from 1 to T).

w Index of scenarios (from 1 to W).

Parameters

a_{jn_r} Quadratic coefficient of φ_{n_r}.

a_{ln_g} Quadratic coefficient of φ_{n_g}.

ψ_i Cost function for community-owned DG unit i.

ϕ_{n_r} Cost function for prosumer agent n_r consumption resource.

φ_{jn_r} Cost function for prosumer agent n_r production resource.

φ_{ln_g} Cost function for producer agent n_g resource.

η^c, η^d The efficiency of storage unit s during charging and discharging respectively.

$\underline{p}^c_{itk}, \overline{p}^c_{itk}$ Minimum and maximum power of block k of community DG unit i at hour t

$\underline{p}^c_{it}, \overline{p}^c_{it}$ Minimum and maximum power of community DG unit i at hour t

p_c^{min}, p_d^{max} Maximum charging and discharging rates of CES at hour t

e_t^{min}, e_t^{max} The minimum and maximum capacity of CES at hour t.

$\Gamma^g_{jn_r}, \Gamma^d_{n_r}$ A parameter for the control of robustness for a prosumer agent n_r production and consumption resource.

$\Gamma^g_{ln_g}, \Gamma^d_{n_d}$ A parameter for the control of robustness for a producer n_g and consumer agent n_d production and consumption resource.

b_{jn_r} Linear coefficient of φ_{jn_r}.

b_{ln_g} Linear coefficient of φ_{ln_g}.

σ_{n_gt} A coefficient of the energy exported to the grid by producer agent n_g at hour t.

σ_{n_rt} A coefficient of the energy exported to the grid by prosumer agent n_r at hour t.

ρ_{n_gt} A coefficient of the energy exported within the community by producer agent n_g at hour t.

ρ_{n_rt} A coefficient of the energy exported within the community by prosumer n_r at hour t.

β_{n_dt} Positive parameter attributed to energy imported by consumer agent n_d from the grid at hour t.

β_{n_rt} Positive parameter attributed to energy imported by prosumer agent n_r from the grid at hour t.

α_{n_dt} Positive parameter attributed to energy imported by consumer agent n_d within the community at hour t.

$\alpha_{n_r t}$ Positive parameter attributed to energy imported by prosumer agent n_r within the community at hour t.

$\overline{p}^g_{jn_r t}$ Deterministic production of prosumer agent n_r resource j from the grid at hour t.

$\hat{p}^g_{jn_r t}$ Symmetric deviation of prosumer agent n_r resource j from expected value at hour t.

$\overline{p}^d_{n_r t}$ Deterministic consumption of prosumer agent resource from the grid at hour t.

$\hat{p}^d_{n_r t}$ Symmetric deviation of prosumer agent resource from the expected value at hour t.

$\overline{p}^g_{ln_g t}$ Deterministic production of producer agent n_g resource l from the grid at hour t.

$\hat{p}^g_{ln_g t}$ Symmetric deviation of producer agent n_g resource l from expected value at hour t.

$\overline{p}^d_{n_d t}$ Deterministic consumption of consumer agent resource from the grid at hour t.

βt Average energy consumed in up/down regulation at time period h^r for a 1kW regulation bidding.

f_c A factor used for the regulation capacity/performance prices of CES.

f_s A smoothing factor for offer of regulation.

$\hat{p}^d_{n_d t}$ Symmetric deviation of consumer agent resource from the expected value at hour t.

ρ^R_t Price of reserve market.

g Cost function for the community manager.

$s_{\lambda expt}$ single rate export cost by agents to the main grid at hour t.

$s_{\lambda impt}$ single rate import price by agents from the main grid at hour t.

ξ_{com} Transaction cost set by the community manager for trading within the community.

ϕ_{n_d} Cost function for consumer agent n_d.

Sets

$\Omega^{US_{grt}}, \Omega^{US_{drt}}$ Set of uncertain parameters of generation/consumption resource of a prosumer at hour t

$\Omega^{US_{gt}}, \Omega^{US_{dt}}$ Set of uncertain parameters of generation and consumption resource of a producer and consumer at hour t

Ω^{Z_t} Feasible space of continuous variables of the community manager at hour t.

N_d Set of consumers(Subset of N).

N_p Set of producers(Subset of N).

N_{pr} Set of prosumers(Subset of N).

R_g Set of prosumer agent resources.

R_p Set of producer agent resources.

$\Omega^{USP_t^R}, \Omega^{USP_t^{Rg}}$ Set of uncertain parameters for pay off costs for reserve and regulation ancillary services.

$\Omega^{W_{n_g t}}$ Feasible space of decision variables of producer n_g at hour t.

$\Omega^{X_{n_r t}}$ Feasible space of decision variables of prosumer n_r at hour t.

$\Omega^{Y_{n_d t}}$ Feasible space of decision variables of consumer n_d at hour t.

Variables

α Auxiliary variable.

δ_{exptw} Total energy exported to the main grid at hour t in scenario w.

δ_{expt} Total energy exported to the main grid at hour t.

$\delta_{n_d tm}$ Energy exported by consumer agent n_d to the grid at hour t with utility maximization.

$\delta^{res}_{n_g tm}$ Energy reserve bid from producer agent n_g resource at hour t in day-ahead market.

$\delta_{n_g tw}$ Energy exported by producer n_g to the grid at hour t in scenario w.

$\delta_{n_r tm}$ Energy exported by prosumer agent n_r to the grid at hour t with utility maximization.

$\delta^{res}_{n_r tm}$ Energy reserve bid from prosumer agent n_r resource at hour t in day-ahead market.

$\delta_{n_r tw}$ Energy exported by prosumer n_r to the grid at hour t in scenario w.

γ_{imptw} Total energy imported from the main grid at hour t in scenario w.

γ_{impt} Total energy imported from the main grid at hour t.

$\gamma_{n_d tm}$ Energy imported by consumer agent n_d from the main grid at hour t with utility maximization.

$\gamma_{n_d tw}$ Energy imported by consumer n_d from the main grid at hour t in scenario w.

$\gamma_{n_r tm}$ Energy imported by prosumer agent n_r from the main grid at hour t with utility maximization.

$\gamma_{n_r tw}$ Energy imported by prosumer n_r from the grid at hour t in scenario w

$\tilde{p}^g_{jn_r t}$ Uncertain production of prosumer agent n_r resource j from the grid at hour t.

$\tilde{p}^g_{ln_g t}$ Uncertain production of producer agent n_g resource l from the grid at hour t.

$\tilde{p}^d_{n_d t}$ Uncertain consumption of consumer agent resource from the grid at hour t.

$\tilde{p}^d_{n_r t}$ Uncertain consumption of prosumer agent resource from the grid at hour t.

$a^g_{n_d t}, b^d_t$ Dual variable for constraint on the consumption control parameter of consumer n_d at hour t.

$a^g_{n_g t}, b^g_t$ Dual variable for constraint on the production control parameter of producer n_g at hour t.

$a^d_{n_r t}, b^d_{rt}$ Dual variable for constraint on the consumption control parameter of prosumer n_r at hour t.

$a^g_{n_r t}, b^g_{rt}$ Dual variable for constraint on the production control parameter of prosumer n_r at hour t.

e_t The amount of stored energy in storage unit s at hour t.

$G_{n_r t}$ The unserved energy from utility maximization of prosumer agent n_r imported from the grid at hour t.

p_{ct}, p_{dt} Charging and discharging power of storage s at hour t.

p_{dt}^{res} Energy reserve bid from CES at hour t.

p_{dt}^{ur}, p_{ct}^{dr} Power reserved for upward/downward regulation by CES at hour t.

p_{gr}, p_{dr} Auxillary variables for generation and consumption uncertainty of prosumers resources.

p_g, p_d Auxillary variables for generation and consumption uncertainty of producer and consumer agents resources.

p_{itk}^{res} Energy reserve bid from Community DG unit i for block k at hour t.

p_{it}^{c}, p_{it}^{d} Generation of DG unit i at hour t with and without ancillary.

$p_{jn_rtw}^{g}$ Production of prosumer n_r from resource j at hour t in scenario w.

$p_{jn_rt}^{g}$ Production of prosumer agent n_r from resource j at hour t.

$p_{ln_gtw}^{g}$ Production of producer n_g from resource l at hour t in scenario w.

$p_{ln_gt}^{g}$ Production of producer agent n_g from resource l at hour t.

$p_{n_dtw}^{d}$ Consumption of consumer n_d at hour t in scenario w.

$p_{n_dt}^{d}$ Consumption of consumer agent n_d at hour t.

p_{n_rtw} Sum of Production and Consumption of prosumer n_r at hour t in scenario w.

$p_{n_rtw}^{d}$ Consumption of prosumer n_r at hour t in scenario w.

p_{n_rt} Sum of Production and Consumption of prosumer n_r at hour t.

$p_{n_rt}^{d}$ Consumption of prosumer agent n_r at hour t.

P_t^{bid} Energy reserve bid for the reserve market at hour t.

P_t^{reg} Energy reserve for regulation at hour t.

q_{n_gtm} Energy exported by producer agent n_g to the community at hour t with utility maximization.

q_{n_gtw} Energy exported by producer n_g to the community at hour t in scenario w.

q_{n_rtm} Energy exported by prosumer agent n_r to the community at hour t with utility maximization.

q_{n_rtw} Energy exported by prosumer n_r to the community at hour t in scenario w.

r_{n_dtm} Energy imported by consumer agent n_d from the community at hour t with utility maximization.

$r^c_{n_dtm}$ Actual energy imported by consumer agent n_d from the community at hour t with utility maximization.

r_{n_dtw} Energy imported by consumer n_d from the community at hour t in scenario w.

R_{n_dt} The unserved energy from utility maximization of consumer agent n imported from the grid at hour t.

r_{n_rtm} Energy imported by prosumer agent n_r from the community at hour t with utility maximization.

$r^c_{n_rtm}$ Actual energy imported by prosumer agent n_r from the community at hour t with utility maximization.

r_{n_rtw} Energy imported by prosumer n_r from the community at hour t in scenario w.

z_{ct}, z_{dt} Binary variable that allows for independent charging and discharging of storage unit s at hour t.

$z^g_{jn_r}, z^d_{n_r}$ Optimization variables for the protection function and control parameters for production and consumption of prosumer resources.

$z^g_{ln_g}, z^d_{n_d}$ Optimization variables for the protection function and control parameters for production and consumption of producer and consumer resources.

Vectors

$\mathbf{U_{grt}}/\mathbf{U_{drt}}$ vector of uncertain production/ consumption of prosumer agents.

$\mathbf{U_{gt}}/\mathbf{U_{dt}}$ vector of uncertain production/ consumption of producer/ consumer agents.

$\mathbf{W_{n_gt}}$ vector of decision variables of producer agents.

$\mathbf{X_{n_r t}}$ vector of decision variables of prosumer agents.

$\mathbf{Y_{n_d t}}$ vector of decision variables of consumer agents.

$\mathbf{Z_t}$ vector of continuous variables of the community manager.

Chapter 1

Introduction

The large-scale deployment of Distributed Generation systems (DGs) has resulted in a dynamic proliferation in the ownership of small-scale Distributed Energy Resources (DERs) by end-users who participate actively in electricity markets. Consumers have become producers equipped with renewable generation including storage in the form of electric vehicles and batteries. The end users are tending towards a more proactive role with the evolution of consumer-centric markets that allow for energy and information exchanges amongst prosumers. This advancement within the smart grid paradigm has to a considerable extent diversified the traditional power network from a hierarchical centralized system to a more decentralized structure. The evolving electrical power network is getting smarter and involves a bi-directional flow of information, autonomous control, and automation among the units involved. The future smart grid will integrate renewable energy generators, energy storage systems, electric vehicles, information and communication technology, energy management systems, automation and control infrastructures, energy protection systems, etc.

The energy management system with emphasis on the demand side response schemes initiated at the consumer end has before now sensitized consumers on the optimal management of their resources with market structures and electricity market tariffs which may not enhance their flexibility and scalability given the increased installed capacity of consumers. The real-time electricity tariffs and day-ahead electricity market prices continually fluctuate. Consumers are mandated to shift higher-rated loads to off-peak periods to smoothen the demand profile curve and allow for peak load shaving. These approaches may limit the active participation of the end-users in the electricity markets, as a result, the need to conform to the changing market prices by managing consumer demands becomes imminent.

The need for this research arises in the study of the evolution of consumer-centric markets where consumers are the suppliers as well as demanders of electricity and a community electricity market mechanism can be initiated among agents willing to trade their energy resources with neighbours in the form of imports and exports within and outside the community. Reference [1] proposed a novel framework for modelling the energy consumption of households connected to the grid within an inter-temporal trading economy. With the deployment of demand-side response and energy trading amongst agents in the microgrid network, optimal use of their resources is actualized. The electricity market structure is not as dynamic as expected with the fast-evolving decentralized power system. The flexibility of the end-users in the existing market structures is limited to the management of their consumption levels at the stipulated time-varying electricity market tariffs. The rise of many demand-side management techniques and energy management methods has sensitized consumers to the optimal use of their resources, even concerning increased installed capacity [2].

In the traditional markets, the System Operator (SO) calculates the equilibrium from the generation and demand function bids and decides on a generator supply schedule that meets the demand, [3]. In the proposed consumer-centric markets in literature, consumers will be sensitized to schedule their appliances based on real-time market prices to minimize their cost of energy consumption [4]. There is an increased awareness on the part of the consumers which entails that the optimal use of resources will be more beneficial to them if they participate actively in the electricity markets. The recent trend in this current market mechanism includes community-based markets and peer-to-peer markets where agents trade their energy resources within a community. References [2] [5] discussed the market structures for consumer-centric markets, allowing for a peer-to-peer electricity market among consumers through a community-based energy collective market and a decentralized peer-to-peer market trade. They exercise increased flexibility and preferences towards the type of distributed generation systems to meet their varied load demands. Consumers have evolved from the restrictions related to the supply from conventional generators, fixed electricity tariffs, energy management strategies, and demand-side management initiatives. The choice of moving towards low-carbon energy becomes beneficial in optimizing their resources. There are several energy management methods for demand-side response discussed in [6] for multi-agent electricity-based markets where consumers trade energy through auction and bid reverse models [7] and game theory [8].

The study in [2] developed a distributed optimization algorithm allowing prosumers to choose their price preferences in comparison with the utility tariffs and formulated a fairness index to allow for seamless trading between the prosumers and the grid. The study in [4] allowed energy sharing between PV prosumer nodes and the grid where the internal trading prices were formulated based on the Supply-Demand Ratio (SDR) at given time slots within the period in the day. The internal prices were formulated as a piecewise function of the SDR at the given time slots.

The advent of transactive energy which is one of the highly researched areas of the NIST [9] smart grid conceptual platforms and the US Department of Energy Gridwise Architecture Council enables the transactive exchanges of energy resources amongst prosumers and consumers based on energy value prices. The study on transactive energy is currently carried out in its modeling and simulation of developed transactive energy models and control approaches that will ensure grid resilience, stability, and reliability when connected to the grid. Energy sharing and trading are used interchangeably to refer to the trading of surplus amounts of energy amongst consumers in a microgrid. References [4] and [10] studied energy sharing implementing techniques such as SDR, Mid-market Rate (MDR), and Bill Sharing (BS), while references [11] and [12] studied energy trading. Reference [5] gives a comprehensive review on peer-to-peer and community-based energy trading.

The impact of uncertainty and robustness of community-based energy trades in the event of a worst-case scenario in renewable energy production and consumption has remained a little investigated area in the evolving market platforms. Most of the previous methods outlined are implemented in deterministic-based problems using forecasted values of load generation and consumption. The impact of uncertainty on import prices remains imminent as there is an increase in the price of purchasing an additional unit of energy on the spot markets. Before now, stochastic programming has been used to model forecast values in day-ahead and forward markets, [13], price adjustment and bidding strategies have been proposed in literature [14], [15], to model price variations. There are techniques in literature that model uncertainties in these independent variables and they include Stochastic Optimization [16], Robust Optimization [17], [18], Taguchi Orthogonal Array Testing (TOAT) [19], and Information-Gap Decision Theory (IGDT) [20].

The generic formulation of a community-based market problem involves the prosumers and a community manager who coordinates the given activities of a particular trade in the community microgrid. This model considers the trades of the prosumers within and outside the community. The community manager interacts with the prosumers within the community and the system/market operators outside the community or grid thereby controlling the import and export of energy in and out of the community. A robust formulation of the community-based market results in a solution that can withstand the worst-case realisations of uncertain variables. In this work, we employ the TOAT, a statistical tool that provides a reduced number of possible combinations of testing scenarios in an experimental design to achieve the desired robustness [19]. The TOAT is used in this research to propose a robust/opportunistic formulation of the community-based market under the uncertainty of DERs and the load demands of prosumers, producers, and consumers. The TOAT is used to characterise the best and worst-case scenarios of the uncertainty sources. We further evaluate the impact of uncertainty on the total energy traded costs within the community giving rise to the cost of robustness.

Based on consumer theory from the concepts of standard macroeconomics consumers may consume a bundle of goods and services from which they derive the most satisfaction [21] [22]. This satisfaction can be measured as a level of utility derived by the consumers from these number of commodities subject to their income and the prices of those goods/services referred to as budget constraints. The work carried out in [23] models a household's choice to invest in DERs with anticipation of how DERs affect derived electricity demand. Here the derived demand refers to demand derived from households' demands for goods and services that require electricity as an input. This concept from macroeconomics and consumer theory can be extended to model agents' behaviour within the community-based market platform to formulate appropriate utility functions of agents that reflect their preference relations in their production/consumption levels from their desired DERs. Reference [1] used consumer theory to determine the consumption space of households by formulating appropriate utility functions that reflected these households' preference relations in their consumption over different periods. The utility functions (Cobb-

Douglas $u(c_1, c_2)$) as a function of consumption were visualised as isoquants or contours (just as in economic theory) in a two-dimensional consumption space with each consumption period (in economic theory this will be non-identical commodities) on each axis and contour lines linking points of equal utility. Reference [24] used consumer theory in determining the joint production possibility of two mini-grids producing solar and wind energy. Here their preference relations are defined in a continuous energy space (either production or consumption) that is monotonic (non-decreasing) (terms used in economic theory). The utility functions (Cobb-Douglas u(s,w)) are formulated with regard to solar and wind energy reflecting the preferences of the utility grid. In this work, we have proposed a deterministic centralized community-based market optimization model which minimises the total costs of agents and the community manager while maximising the individual utility functions of the agents given their production and consumption preferences. A community energy storage (CES) is also integrated in the formulation to provide flexibility and energy balance in the agent's imports and exports within and outside the community/grid.

With regards to the utility functions which reflect the preference relations of agents to various DERs mix to satisfy their utility levels, the need also arises to ensure the availability of energy when required. Owing to the stochastic nature of renewable sources, the output levels realised may be insufficient to satisfy the agents' preferences. Given that these sources are not dispatchable compared to the conventional counterparts, the need arises for the proposal of computationally tractable models [25] that are less conservative and realises an optimal solution in itself such as the Robust Optimization (RO) [17]. In comparison to stochastic programming which is defined over probability distribution functions representing a certain number of scenarios, RO models are defined over polyhedral uncertainty sets and require no scenarios. The robust solution from these models should be feasible and an optimal solution should be obtainable from the worst-case realization of uncertain parameters defined within the uncertainty sets. The uncertainty sets are made up of a set of uncertain parameters with predetermined deviations from the expected values of uncertain elements, that can be defined within the dimensions of a polyhedron. Reference [26] presents a comprehensive survey on approaches to robust design optimization giving a detailed explanation of how to account for design uncertainties and how to measure robustness with a focus on Taguchi's robust design methodology. In this work, we propose a single-stage robust formulation of the community-based market optimization problem which minimises the community's cost while maximising the individual's utility functions. A polyhedral uncertainty set is used to represent all the uncertain parameters in agents' resources. An opportunistic formulation is used to compare the results realised from the robust and deterministic counterparts.

With the emerging consumer-centric markets, the possibilities of offering services such as ancillary are explored given the high penetration of DERs by community agents. In [27] an overview of the European electricity markets was presented with insight into the various metrics specific to each European country which include: Power markets (day-ahead and intraday markets), international power trading, and reserve products and remunerations. In this work, a community-based market model with the offer of reserve and regulation services is proposed through a joint market model framework while maximising the individual utility functions of community agents. In [28], a joint market model of energy and spinning reserve service was said to address the bidding problem of a virtual power plant. Another market mechanism the sequential method [29] is still used to design a large number of systems and modelling approaches such as equilibrium (Cournot and linear supply functions) and single-agent optimization models are proposed for the solution methodology.

Finally, robust approaches can be extended to analyse the impact of uncertainty on community-based markets offering energy and ancillary services. The work in [25] proposes the adaptive robust optimization model in a joint determination of day-ahead energy and reserve dispatch to yield a minimum system's costs which accounts for the cost of the redispatch decisions in the real-time stage in the worst-case realization of uncertain production within the uncertainty set. Reference [30] applies a robust mixed-integer linear programming technique to build hourly offering curves for a producer who is a price taker participating in a pool with consideration to price confidence intervals as input data. Concerning the modelling approach, [31] proposes a robust supply function equilibrium for a two-stage electricity market where each producer is equipped with a renewable and a conventional generator. In this work, the linear supply function equilibrium modelling approach is employed within the joint market model to propose a single-stage robust mixed-integer linear problem formulation of the joint market model with uncertainty in DERs and loads of agents which maximizes the joint market model as well as the individual utility functions of community agents over the worst-case realisations of uncertain parameters in agents' resources.

1.1 Research Objectives

The main aim of the work carried out in this thesis is to study the evolution of consumer-centric markets with consideration to the utility maximization of agents' utility functions. The other objectives include:

- Investigate the impact of uncertainty on community-based markets by applying robust optimization techniques.
- Evaluate the effects of uncertainty on total traded costs in the CBEM against the best and worst-case uncertainty scenarios generated by TOAT.
- Integrate the concepts of macroeconomics based on consumer theory in the proposal of a deterministic centralised CBEM problem that minimises the total traded costs of the community while maximising individual utility functions.
- Investigate the role of CES and conventional generators in providing flexibility and balance in periods of insufficiency.
- Develop a single-stage robust formulation of the CBEM market under uncertainty with consideration to the agents' utility maximization.
- Analyse the possibility of offering ancillary services within a joint market model of energy reserve, regulation markets, and the CBEM framework, optimizing the market model as well as the agents' utility functions.
- Develop a single-stage robust mixed-integer linear problem formulation of reserve and regulation markets within the CBEM as a joint market model optimizing the market model as well as the individual utility functions against the worst-case realisation of uncertain parameters.

1.2 Thesis Structure and Contributions

The main contributions and thesis structure are presented as follows:

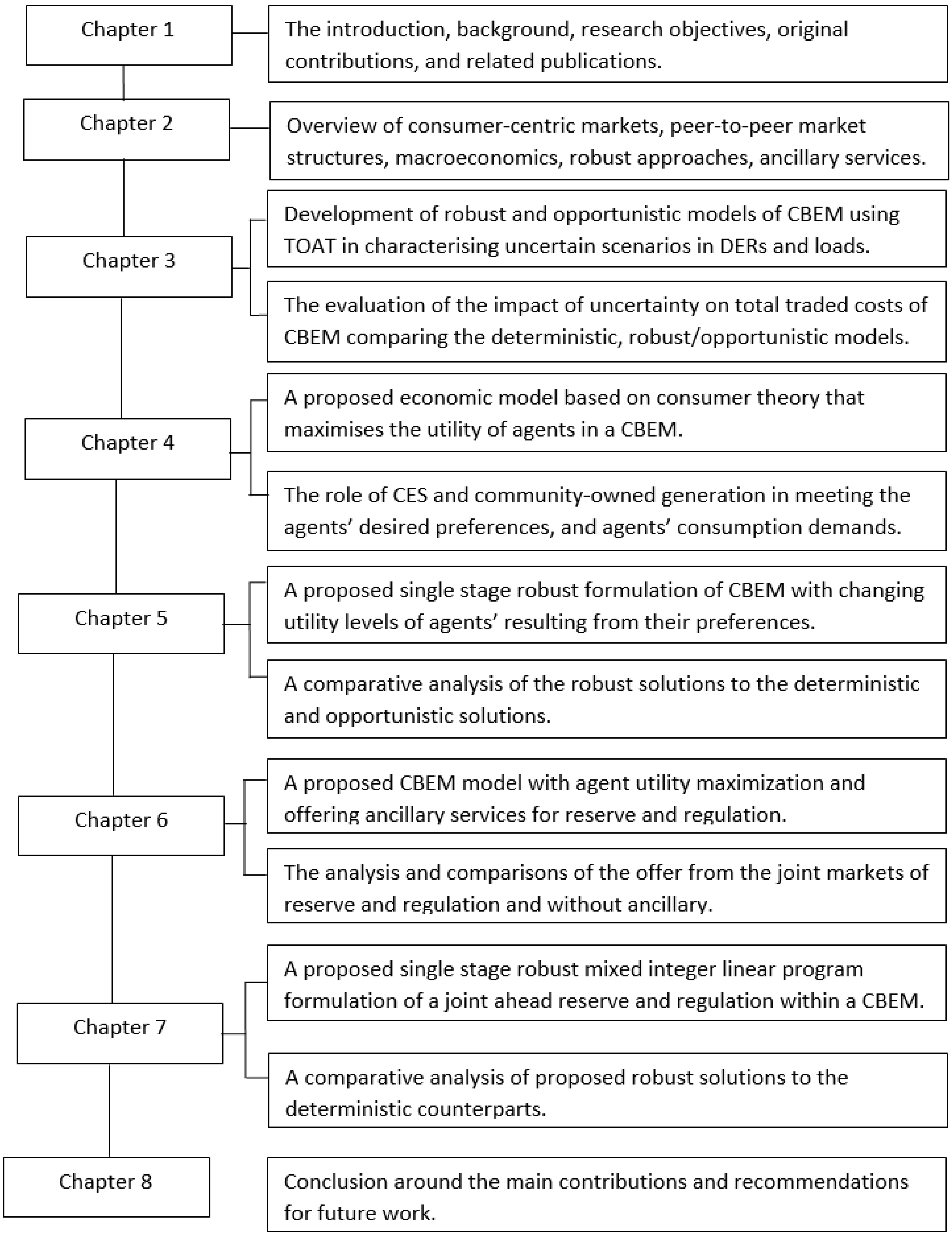


Figure 1.1: Thesis structure and main contributions.

In Chapter 1, the introduction, background study, research objectives, original contributions, and related publications are presented.

In Chapter 2, the overview of the main topics considered in the thesis is presented. A review of the traditional electricity power market structures is discussed. On this basis, the evolution of consumer-centric markets, the different peer-to-peer structures, and the solution methodology of P2P markets are discussed. Several robust approaches for uncertainty studies are discussed, and the uncertainty of agents' DER and load demands that make up uncertainty sets are introduced. The concepts of consumer theory and macroeconomics are discussed as a basis for the proposed community-based markets with agents' utility maximization. An overview of ancillary services application in markets, the development and construction of bidding models, and uncertainty modelling with ancillary services are discussed.

In Chapter 3, a robust approach to community-based markets under uncertainty is introduced, with a focus on the TOAT as the tool used for modelling the robust approach. TOAT is used in generating robust and opportunistic scenarios over the worst-case realisations of uncertain parameters in agent resources. The robust and opportunistic solutions are compared with the deterministic counterparts.

In Chapter 4, the community-based electricity market problem formulation is extended to integrate the economic utility maximization of agents' resources based on the concepts of consumer theory and macroeconomics.

In Chapter 5, a single-stage robust optimization model of the CBEM with agents' utility maximization is proposed to maximize the uncertainty against the worst-case realisations of uncertain parameters in agents' resources represented within a polyhedral uncertainty set.

In Chapter 6, a joint market model of energy and ancillary services for reserve and regulation markets and also the community-based market is implemented using the linear supply function equilibrium as the modelling approach in maximizing the profits/revenues realised from these markets while minimising the total traded costs in the community and also maximising the individual utility functions of agents.

In Chapter 7, a single-stage robust mixed-inter linear optimization model of the joint market model discussed in Chapter 6 is proposed to maximize the uncertainty against the worst-case realisation of the market. The results obtained are compared against the deterministic solutions.

Finally, Chapter 8 discusses the conclusions around the main contributions and provides recommendations for future work.

1.3 Related Publications

The original contributions of this thesis are outlined in the following publications:

- **Conference** I. Onugha, S. Dehghan, and P. Aristidou, "Rethinking consumer-centric markets under uncertainty: A robust approach to community-based energy trades," inProc. of the 2020 IEEE General Meeting, 2020.

Chapter 2

Overview of Consumer-centric Markets

The increase in energy demand around the world will result in more innovative solutions deployed through emerging energy technologies in response to the global rise in generation and demand variability [32]. Recent decarbonisation strategies relating to the proliferation of demand drivers such as electric vehicles and heat pumps towards a greener low carbon future have called for smarter approaches to ensuring reliability, efficiency, and security of electricity supply. The future smart grid will integrate a plethora of resources which include distributed energy resources, information and communication technologies, automation and control infrastructure, and smart devices allowing for the bi-directional flow of energy and information between the end users and the grid. As part of their contribution towards the smart grid evolution, the Smart Grid European Technology Platform (SG ETP) has initiated directives and policies for the European smart grid towards ensuring a reliable, flexible, and economical network [33]. According to the SG ETP, smart grids are defined as electricity networks that can intelligently integrate the behavior and actions of all users connected to it-generators, consumers, and those that do both- to efficiently deliver sustainable, economic, and secure electricity supplies. This definition recognises new key players the prosumers, initiating a proactive role in the smart grid paradigm.

Electricity markets and platforms enabling energy trading amongst end users are an aspect of the smart grid conceptual benefits of transactive energy. This has given rise to the evolution of consumer-centric markets in the realisation of emerging trading mechanisms that allow consumers to initiate a more active role. In comparison to the centralised hierarchal nature of existing traditional markets which is unidirectional in nature, consumer-centric markets allow the consumers to choose the sources of energy resources to meet their consumption preferences with consideration to a bottom-to-top approach [5]. These markets have transformed the outlook of the consumer from an end user managing his demands to one equipped with distributed power generation and storage. The resulting group of consumers is referred to as prosumers having the ability to trade their resources in recent electricity market design concepts known as the peer-to-peer markets. This enables a platform of emerging prosumer communities allowing for energy sharing amongst end users and the utility grid [34].

Energy trading amongst prosumers will require enabling technologies such as the integration of DERs which include distributed generation sources, energy storage, and flexible loads as well as information and communication control infrastructure. The active participants in these forms of markets the consumers, producers, and prosumers are driven by the motivation which may be one either to minimise costs of energy use, efficiency in energy management, show more dependency on their energy consumption within the community market as compared to the main grid [35, 36]. The structure required to perform the trading activities needs to be considered. This has led to the concept of peer-to-peer markets which allows agents to either trade their resources in a centralized [37–41] or distributed [40, 42, 43] manner. Based on references [35, 36] the different aspects of energy trades that are covered in literature encompassing these areas are shown in Figure 2.1

In this chapter energy trading resources and the different peer-to-peer structures with particular focus on community-based markets, and the solution methodology of P2P markets are discussed. Also, several robust approaches for uncertainty studies are discussed, and the uncertainty of agents' DER and load demands that make up uncertainty sets are introduced. The concepts of consumer theory and macroeconomics are discussed as a basis for the proposed community-based markets with agents' utility maximization. Finally, an overview of ancillary services application in markets, the development and construction of bidding models, and uncertainty modelling with ancillary services are discussed.

2.1 Energy Trading Resources

The concept of energy trading before now has assumed a uni-directional structure including all major players from large producers, to market operators and utility companies. More recently

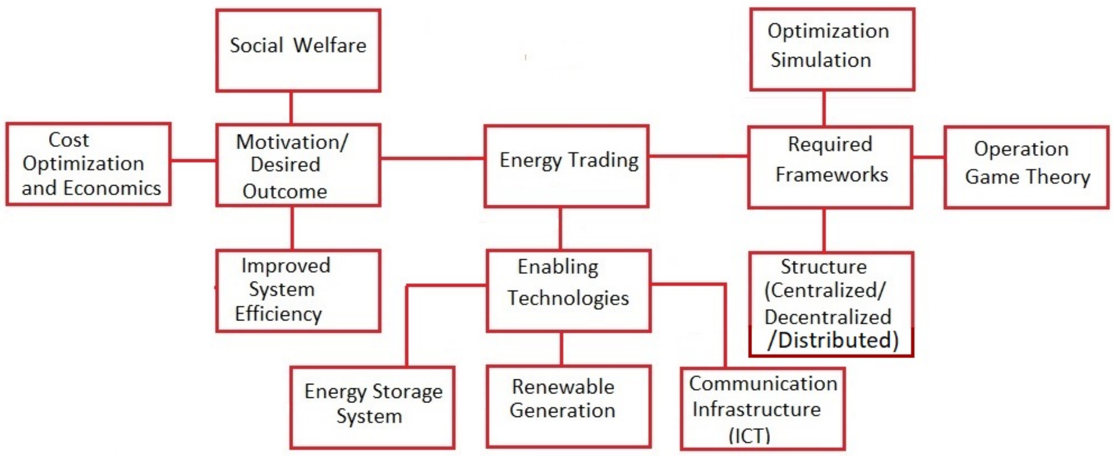


Figure 2.1: The overview of energy trading components.
[36]

market mechanisms involving prosumers have emerged in the establishment of energy exchange economies through agent-owned resources. The following are energy trading resources deployed within the smart grid that allow for a bidirectional flow of energy production, storage, and trading information among the prosumers.

2.1.1 Distributed Energy Resources

The integration of DERs increases the reliability of the power grid while providing a point-of-use connection for microgrids. In the given microgrids, energy is produced and consumed in either grid-connected or islanded modes. The deployment of DERs for energy trading in smart grids helps in the improvement of the overall generation capacity in meeting the demands of the connected system loads. Renewable energy sources such as solar, wind, hydro, biomass, tidal, and ocean waves are harnessed through the use of distributed generation systems and either connected to the electric grid or used as a microgrid in an islanded mode. The renewable energy systems used to harness these resources such as the photovoltaic systems and wind turbines, hydro turbines, digesters, etc. are designed and modelled to meet specified load demands. The excess energy from these systems can either be stored in islanded conditions or fed back to the grid, in grid-connected cases.

The flexibility of end users is improved by generating their energy and sharing amongst themselves with a higher installed capacity of their generating resources. The excess energy is either exported to the grid or sold to another consumer within the microgrid, while the redundant energy is either purchased from the grid or another consumer.

2.1.2 Energy Storage Systems

Energy storage systems are an essential resource in providing energy balance from distributed generation sources. They are capable of storing excess energy and making it available when needed. The excess energy generated by the grid or the consumer will be a waste if proper storage mechanisms through the use of designated energy storage systems are not put into consideration in DER microgrids.

Energy management techniques allow a consumer to optimize his available resources by storing his excess energy in a storage system, to meet his time-varying loads at each time slot. The excess or redundant energy can be exported to the grid or sold to another consumer in the microgrid. Energy storage is implemented to provide some of the ancillary services used to support the smart grid to balance generation and demand, such services include meeting peak load demands, load frequency control and voltage and reactive power control, power factor control transient stability, integration of numerous renewable energy sources in the future, etc.

The energy storage systems include batteries, pumped storage for hydroelectric systems, flywheels, supercapacitors, superconductors, and fuel cells. The Battery Energy storage system BESS and community energy storage CES discussed in [11] [44] are used to store the energy from renewable sources integrated into the grid. These storage systems are being implemented as a result of the fluctuating nature of renewable energy, to store energy, mitigate against waste, and have supply when needed at peak periods.

2.1.3 Electrical Loads

The connected electrical loads of each end user are time-varying and can either consume more or less of the available supply or storage. The loads range from fixed or non-shiftable loads such as the lighting appliances, refrigerator which are satisfied always by their connected DG's, and the controllable or shiftable loads such as water heaters, EV chargers, heat pumps, etc that can be satisfied at a later specified time when supply is higher than demand and electricity prices are lower. The fixed loads or inflexible loads must be on supply when needed, while the flexible/controllable loads can be used to participate in demand-side response programs to shift electricity demands by appliances to balance grid energy at peak periods [45].

In [2] the flexible and inflexible load demands are used within bounded intervals to participate in the community-based market problem. The inflexible/fixed loads are taken as lower bounds while the flexible loads in addition to twice the inflexible loads are taken as upper bounds for the community agents' load demands.

2.1.4 Information and Communication Technology

The interconnection between the electrical power network and communication systems forms the roadmap for actualizing the future intelligent smart grid. The dynamic nature of the communication infrastructure will continue to constitute an overlap with the grid network in the sense that future power networks depend on existing and future communication infrastructure. This is to further buttress that the smart grid will continue getting smarter with more recent communication, power, and energy technologies. The developed smart grid system should be scalable allowing the integration of new Information and Communication Technologies (ICT) and services. The communication infrastructure involves a two-way flow of information from the generation to the end users and vice versa.

With the advent of advanced metering systems, Phasor Measurement Units (PMU) the smart meters for the new smart grids which improved from the Automated Meter Readings (AMRs) to the Advanced Metering Infrastructure (AMI) [6] is the new wave in the smart grid paradigm. Owing to the ubiquitous nature of the communication infrastructure, consumer energy use can be measured through smart meters which can not only interpret the energy in real-time but also create a consciousness in both consumers and utility on how to reduce load demands and maximize utility capacity. These have led to the realization of several demand-side response schemes constantly researched as means of reducing load demands art peak periods.

The communication infrastructure can further be divided into wireless sensor networks and wired networks. The wireless sensor networks make use of the wireless sensors in all devices in the smart grid to communicate with each other. These sensors can be used in the smart appliances in smart homes, in automation and control devices in power electronic devices, etc. The wireless communication networks include cellular networks 2G, 3G, 4G LTE, 5G, Bluetooth, WIFI, satellite communications, cognitive radio, microwave communications, IEEE 802.15.4 and IEEE 802.11 networks. Wired networks are communication networks in which the flow of information is through large network cables and communication lines such as fiber optic cables and the power line communication PLC. The Information management system of the smart grid enables the numerous data collected from sensors, smart meters, and phasor management to be managed efficiently allowing for data and information latency and scalability. These techniques will involve data modelling, information analysis, integration, and optimization. The concept of big data analysis allows data mining, privacy, and latency of large valuable data within the control network that deals with volumes of data and information. The high volume of data gathered in smart grids is similar in size and characteristics to the big data concept [46] [47]. Another aspect of information management technology is outsourcing the basic information control network to the cloud interface. The electric utilities can decide to outsource the basic information management, storage systems, and assets to a fog or cloud network for either private or public computing purposes.

This smart interface also helps in networking end users' energy devices in peer-to-peer networks within a home area network or a neighborhood area towards the buying and selling of the energy produced. This forms the main focus of this research. Peer-to-peer networks are a decentralised network structure that allows interconnected nodes to share information amongst themselves directly without a supervisory control node. The information on power mismatches on supply and demand from each node is updated from their smart meters and their communication devices route this information to other nodes in the network. The rate of each session is a function of the sum of the rates of all sessions that enabled an optimal energy trade amongst consumers [48]. The work in [49] presented a heterogeneous network architecture to solve the economic dispatch problem for the optimal decision of DG's and electrical loads using a decentralized approach. Heterogeneous networks are wireless networks of interconnected nodes comprising different types of devices. Communication time delays are an important aspect of transactive energy integrating information and communication technology because of its ubiquitous nature in heterogeneous communication networks. The communication time delay effect is

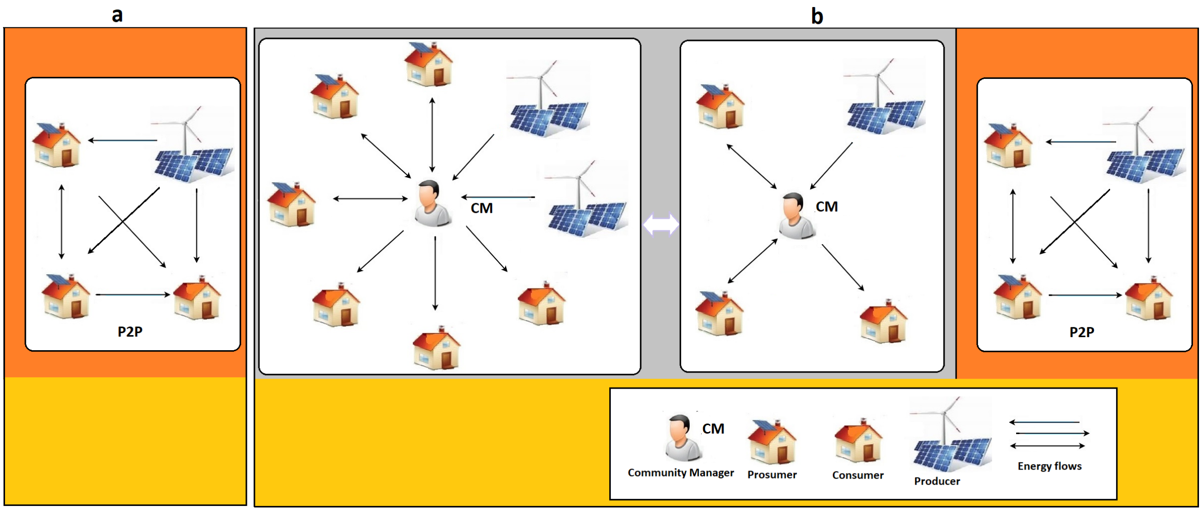


Figure 2.2: Peer-to-peer market structures: a) Peer-to-peer, b) Community-based.

modeled as part of the distributed economic dispatch algorithm for the rate of each session. [50] investigated the effect of time delay on the dispatch performance of their distributed economic dispatch algorithm through a well-developed consensus-based protocol and established that no matter how large the uniform finite delay could be there always exists a small learning gain parameter which ensures the convergence of the dispatch algorithm.

2.2 Introduction to Peer-to-Peer Markets

The concepts of peer-to-peer market structures are proposed based on potential end users seen as peers participating actively in trading their energy resources. More recently consumers are empowered to share their resources in a collaborative manner having flexibility in the decisions made towards the usage of energy. The resulting market mechanisms are analysed in the forms of centralised and decentralised market structures as the conceptual basis for consumer-centric markets. Figure 2.2 shows the centralised and decentralised structures of the market. Given this basis, consumer-centric markets can be classified into three structures: decentralised Peer-to-Peer markets, centralised markets, and distributed markets [5].

2.2.1 Decentralised Peer-to-Peer Markets

Decentralized markets are more distributed forms of the market where agents trade their energy resources directly with their neighbours without the coordination of supervisory control. There is privacy in the production and consumption of information and a negotiation mechanism is carried out iteratively in a distributed fashion until convergence. Reference [51] classifies peer-to-peer networks in relation to their architecture, analysing the structures on the control of power grid components while providing a basis for comparisons. A peer-to-peer market structure based on multi-bilateral economic dispatch which allows consumers to choose their preferences through product differentiation is proposed in [52]. Due to the impacts, P2P markets may have on the grid, reference [53] analysis of the impacts of attributed costs through exogenous network charges on energy trades and the grid. Other works have been proposed in distributed P2P markets such as P2P trading among EVs in [54], simulation of P2P trading using game theory [55], DSM integration with P2P energy trading [56], cost optimization [5], [2].

The concept of blockchains is another important aspect of solving decentralized P2P energy trades in a microgrid network of prosumers. A blockchain is a decentralised technology that allows the sharing of information and record of transactions amongst a network of participants. Reference [57] has proposed the use of blockchain as an important technology in the operation of microgrid markets, and the Brooklyn microgrid project has been used to analyse different market mechanisms. Blockchains and smart contracts are usually implemented side by side to facilitate P2P trading amongst prosumers.

2.2.2 Centralised Peer-to-Peer Markets

Centralised markets are monitored by a supervisory controller who coordinates all trading activities. The privacy trading information amongst peers is communicated with the coordinator who then decides the energy imports/exports of agents [36]. The aim of the supervisory control can either be in maximizing the social welfare [39], traded cost minimization [2, 5, 37]. A community can be formed among peers, equipped with DER resources and engaging in centralised energy trades with their neighbours. The common interest among agents can be in sharing the investment on DERs and revenues realised in the energy trades [5]. Reference [58] proposed an energy-sharing model with price-based demand response for a microgrid of PV prosumers. A community-based market of prosumers is proposed in [2] based on the concept of energy collectives allowing prosumers to share energy in a collaborative manner that reflects their preferences. The work in [59] proposes a market structure for prosumers with heterogeneous preferences based on the concept of multi-class energy management. The main advantage of the community-based market structure is the common interests and coordination of the agents involved in the community energy trades, and the sharing of the revenues amongst participants.

2.2.3 Distributed Peer-to-Peer Markets

The distributed peer-to-peer market involves some levels of both centralisation and decentralization. These markets are coordinated by a supervisory controller with minimal privacy of energy information exchanges as compared to the decentralised structure. A distributed approach can be proposed as a solution to the centralised or decentralised market model. Reference [2] proposed a distributed solution through the Alternating Direct Method of Multipliers (ADMM) to a community-based market of prosumers. ADMM is a distributed optimization algorithm that decomposes the main objective into smaller individual problems, and the solution is realised through an iterative process of the singular problems with information exchanges till convergence. A distributed optimization was used in [59] to solve the decentralised peer-to-peer energy trade amongst prosumers with heterogeneous preferences. The decentralised problem formulated in [37] was solved through a distributed convex optimisation method developed for energy trading between islanded microgrids. Game theory can also be applied in distributed solutions of centralised and decentralised market structures. An energy bidding platform Eclebay was proposed using game theory in [60] for peer-to-peer trading in a microgrid. Reference [43] proposes a game theoretic approach based on a multi-leader and multi-follower Stackelberg game for maximizing the economic benefits of participating microgrids.

The research carried out in consumer-centric markets usually concerns energy trading, energy sharing, and energy transportation. There are several techniques have been carried out effectively to model the sharing, trading, or transportation of energy as the case may be. The definitions of these energy marketing terms are closely interrelated while energy sharing refers to consumers in a multi-agent microgrid sharing their excess and redundant energy with the grid at specified internal electricity prices, energy trading involves centralized and decentralized peer-to-peer electricity markets within the microgrid where consumers tend to trade their excess or redundant energy to each other and with the grid. Energy transportation refers to the transportation of the traded or shared energy from one prosumer node to the other or from one prosumer node to the grid and vice versa. [58] [34] discussed energy sharing for techniques such as supply-demand ratio where a piecewise model was used to formulate the energy sharing model based on the supply-demand ratio at each given time slot within a period in a day, while [12] compared the supply-demand ratio technique to mid-market rate and bill sharing techniques and deduced that the SDR performs better than the other techniques. In [61] a peer-to-peer energy sharing amongst neighbouring microgrids was proposed for improving the utilization of local DERs and saving energy bills for all microgrids. In [58] an energy sharing model on a price-based demand response was proposed. [10] proposed a systematic index system to evaluate the performance of various peer-to-peer energy sharing mechanisms based on a multi-agent-based simulation framework. The step length control and learning process were developed to facilitate the convergence of the simulation.

References [2] and [62] discussed energy trading for solving the economic dispatch problem in community-based markets and decentralized peer-to-peer markets using techniques such as consensus plus relaxation [62] and distributed ADMM optimization and economic dispatch problems using consensus algorithms and Lagrangian multipliers [63] [64] [65] [66] [50] [67]. References [11] and [44] studied energy trading within the Internet of Energy platform. Reference [68] studied the feasibility of peer-to-peer energy trading in low voltage distribution networks by studying their demand profile patterns and optimal planning and selection of DG's to maximize the local demand and supply balancing. In [69] peer-to-peer trading was integrated with demand side management amongst households in a smart grid to minimize the cost of consumption. Reference [70] gives a comprehensive review of typical economic incentive approaches adopted in energy trading control mechanism, and [8] discussed energy trading using the game theory where the stackberg's game theory was used for agents in the microgrid network and [71] simulated the proposed bidding rates using the Eclebay trading platform and game theory. Auction bids were studied in [12] and [72] as a non-cooperative game approach in which players make decisions independently. Reference [73] proposed a model for energy transportation and storage amongst distribution networks in a microgrid using the Internet of Energy platform.

2.3 Robust Techniques for Uncertainty Analysis

Robust approaches to optimization problems have been studied extensively [17,26,74–78] through theoretical and practical applications as a means of handling uncertainty in system data. In comparison to the stochastic optimization that deals with the probability distribution of uncertain element [16, 79, 80], Robust Optimization (RO) assumes the uncertain data is bounded within an uncertainty set. The basic properties of RO include computational tractability, conservativeness, and flexibility [17]. The robust solutions obtained are said to be *immune* to data uncertainty [17, 75, 76, 81] this means that the solution is feasible against all realisations of uncertain parameters within the uncertainty set.

Robust optimization techniques have been proposed in literature which include Taguchi Orthogonal Array Testing (TOAT) [19], Information gap decision theory (IGDT) [20], Single-stage robust optimization (SRO) [78], Two-stage robust Optimization (DRO)/ Adaptive robust optimization (ARO) [18], Adaptive robust stochastic optimization (ARSO) [78], Adaptive distributionally robust optimization (ADRO) [82], Data-driven robust optimization [83]. Robust optimization techniques in peer-to-peer energy trades are yet to be widely researched. Reference [84] proposes a vertex scenario-based RO using TOAT to minimise the total costs of prosumers within a transactive energy trading platform. An economic and robust energy management model based on TOAT is proposed in [85] to maximise the total exchange cost while minimising the social welfare costs.

2.3.1 Taguchi's orthogonal Array Testing

In accounting for data uncertainties in engineering designs, Taguchi proposed a robust design methodology comprising system design, parameter design, and tolerance design [19, 26]. The Taguchi method has been considered to produce optimal design processes under varying conditions. The highlight of the design methodology is in the parameter design where the process designs select a combination of parameter settings that produce the best levels of quality designs with reduced variations. Taguchi proposed the use of signal-to-noise S/N ratio as a quality measure for value as compared to the use of standard deviation which decreases with the mean value [86]. Taguchi also proposed the use of orthogonal arrays (OA) in mathematical modelling by optimizing the objective function of the manufacturing process subject to the reduced number

Number of Levels	Number of Parameters (P)														
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	L4	L4	L8	L8	L8	L8	L12	L12	L12	L12	L16	L16	L16	L16	L32
3	L9	L9	L9	L18	L18	L18	L18	L27	L27	L27	L27	L27	L36	L36	L36
4	L'16	L'16	L'16	L'16	L'32	L'32	L'32	L'32	L'32						
5	L25	L25	L25	L25	L25	L50	L50	L50	L50	L50	L50				

Number of Levels	Number of Parameters (P)														
	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
2	L32	L32	L32	L32	L32	L32	L32	L32	L32	L32	L32	L32	L32	L32	L32
3	L36	L36	L36	L36	L36	L36	L36								
4															
5															

Figure 2.3: Taguchi's orthogonal array selector.
[86]

of combinations of design and control parameters to achieve optimal results. Figure 2.3 shows a typical orthogonal array selector in which a suitable OA for a design process is selected based on the number of parameters and a number of levels.

The orthogonal array is a fractional design matrix representing the reduced number of combinations of multiple factors and levels of a given experimental design. This array comprises of a N x m matrix containing s levels with m factors and N level runs; $OA_N(s^m)$. The OA is replaced by the notation L as in $L_N(s^m)$ which refers to Latin squares. The OAs are used to design fractional combinations of elements in an experiment where the columns of the realised design experiment correspond to the factors, the elements of each column are the test levels and the rows refer to the test runs [87]. The possible number of combinations of the experimental design is defined mathematically as s^m test runs and the selected OA represents a fraction N/s^m of the possible combination of outcomes. The $L_4(2^3)$ and $L_8(2^7)$ are two examples of OA with 4 and 8 test runs respectively, 2 levels each and 3 and 7 factors respectively represented by fractions of $4/2^3$ and $8/2^7$ respectively of the total test runs of individual experimental designs under study. An orthogonal array can also be formed as a sub-matrix of an existing OA in defining the necessary design conditions of an experiment without an existing OA. The resulting OA is in itself an OA where other forms of combinations of a given experimental design can be realised.

2.3.2 Single-stage Robust Optimization

Single-stage RO is a form of robust optimization technique that provides ex-ante protection against the worst-case realisation of uncertain parameters within the uncertainty set. The problem is generally formulated as a *min max* problem, where the uncertain parameters are maximised against the realisation of the worst uncertainty. Given the objective function $g(x, u)$, SRO aims to minimize the objective function while maximising the uncertainty in realising an optimal x^* given the worst uncertainty parameter u. Mathematically:

$$\min_{x \in X} \max_{u \in U} \; g(x, u) \tag{2.1}$$

where the vector $x \in X$ represents the decision variables and $u \in U$ are the uncertain parameters. The uncertainty parameter can be represented as both an inner and an outer maximization operation. Single-stage RO was compared with Adaptive RO and Distributionally robust techniques in [78] with respect to their preventive and corrective actions. In [30] a single-stage robust mixed integer linear programming optimization technique was proposed in developing offering curves for a producer participating in a pool market. Reference [88] proposed a robust mixed integer linear program for solving uncertainties in estimated investment costs and forecasted demand.

2.3.3 Two-stage Robust Optimization/ Adaptive Robust Optimization

A two-stage and/or adaptive RO is a robust approach that provides a recourse action offering solutions that are less conservative compared to the single-stage robust optimization. ARO presents solutions that provide a corrective action to mitigate the impact of uncertainty realizations. The general formulation is comprised of three levels of problem:

$$\min_{x \in X} \max_{u \in U} \min_{y \in Y} \; g(x, y, u) \tag{2.2}$$

The model of the planning stage is defined in the first level before any uncertainty considerations and the aim is to minimize the objective function $g(x, y, u)$ with decision variable vector $x \in X$. The second level presents the uncertainty parameter within the uncertainty set $u \in U$, and the aim is to maximize the uncertainty in the given objective function value. The third level provides a corrective operation aimed at mitigating the impact of uncertainty in the event of occurrence. The model provides both preventive and corrective actions realising ex-ante protection as well as ex-post correction actions. A two-stage adaptive robust optimization model is proposed in [18] to solve a security-constrained unit commitment problem given an inherent uncertainty in the nodal net injection. Reference [89] proposes a column and constraint generation approach in solving a two-stage robust optimization problem in addressing optimality and feasibility. In tackling the inherent uncertainties associated with investment and operation costs as well as electricity demands, [90] proposed a mixed linear integer program based on single and two-stage robust optimization to solve a generation expansion planning problem.

Other robust optimization techniques include adaptive robust stochastic optimization which provides a preventive protection action as well as stochastic scenario-based modelling. It is also a three-stage problem like the ARO involving a planning stage, a long-term uncertainty realization, a corrective action against the long-term uncertainty, and introducing short-term uncertain operating conditions. Another Robust optimisation technique is the adaptive distributionally robust optimization which defines a bounded and unknown distribution for each uncertain element. Here, the uncertain parameters are not defined within uncertainty sets. It also provides three-level protective and corrective actions as in the case of ARO and ARSO.

2.4 Utility Maximization

The concept of utility maximization arises when multi-agents or players such as in game theory want to maximize their economic benefits or social welfare within a given platform. They can either be identified as having common interests to share revenues [5] realised from a collective structure such as a P2P market structure [2] or a cooperative game theory [55], [60] or being more strategic in their approach such as in the non-cooperative game theory [91].

The approach of economic applications to utility maximization of agent's resources based on concepts of consumer theory macroeconomics has not been widely studied in literature [1], [24], [23]. Though there are studies that have related the concepts of consumer theory to P2P energy trading, such as in [23] where the electricity demand was referred to as a derived demand inferring services realised from electricity consuming appliances and DER capacity as goods/services to which a prosumer has an indirect utility to maximise. The services realised from electricity consumption from these appliances were assumed as the commodity of interest whereas all other services not related to prosumer demands were treated as composite goods as in consumer theory. In [24] a production equilibrium curve was developed in modelling the individuals and joint production possibilities of minigrids equipped with hybrid renewable energy technologies and a trading platform was initialised based on their production equilibrium which is a point on the joint possibility curves that maximises the production of the minigrids subject to constraints.

In an economic utility maximization model, an agent is faced with choices within a set X of goods n where x_i represents the amount of each good $X(x_i, ..., x_n)$. The agent preferences is reflected in the choices between two goods x and y where $x \geq y$; and $y \geq x$ show their preferences for each good x and y over y and z respectively. These preferences can be termed as complete when comparing two goods and transitive if in relation to a third commodity, the preferences are consistent i.e if $x \geq y$; $y \geq z$ then $x \geq z$. The preferences are incomplete and intransitive whereas the agent preferential relations become unclear and ambiguous [92]. Utilities are referred to as quantifiers for agent preferences which makes it easier to identify goods with higher value/number than the others. A utility function is used to represent the utility relating to each commodity $x_i \in X$ denoted by $u(x_i) \in R$. The utility function represents the agents' choices of goods such that $u(x) \geq u(y)$ which is the same as when $x \geq y$

2.4.1 Characteristics of Preferences

The two important characteristics relating to preferences include monotonicity and convexity [21,92]. Regarding the agent's utility, an *indifference* is asserted between x and y if $u(x) \geq u(y)$.

Monotonicity

If for any two bundles of commodities $x = (x_1, ...x_n)$ and $y = (y_1, ...y_n)$, preferences as well as utilities are monotonous if $x_i \geq y_i$ for each i and for some i, implies that $x \geq y$ and $u(x) \geq u(y)$. This explains that indifference curves according to monotonicity assumptions are downward sloping and decreasing, and subject to any given budget constraint an agent will exhaust their budget. Based on this, agent utility maximization is implemented.

Convexity

The concept of convexity indicates that preferences are convex whenever $x \geq y$ then $rx + (1 - r)y \geq y \; \forall r \in [0, 1]$. As a result of the binary variable r, the agent preferences lie in the extremes, and given the agent's preferences for each of x and y is indifferent, then the agent prefers the average of $rx + (1-r)y \geq y$ to either $x$or y. Rewriting this to reflect the utility gives:

$$u(rx + (1-r)y) \geq u(y) \forall r \in [0, 1] \tag{2.3}$$

The utility function which satisfies (2.3) is referred to as *quasi-concave*.

In analysing the agent's utility maximization problem, both convexity and monotonicity concerning the agent's first-order conditions are a solution to the agent's problem.

2.4.2 Indifference Curves

The indifference curve of an agent refers to its preference relations which are represented in the utility function, in the form of isoquants or contours linking points of equal utility. These contours are indifference curves indicating the set of commodities that gives a constant level of utility [92]. An agent may have indifferent curves for different levels of utility. Given that an agent's utility function

$$u(x_1, x_2) = x_1 x_2 \tag{2.4}$$

If the utility function is replaced by a constant value k, then the indifference curve satisfies the equation $x_1 x_2 = k$. This results in the equation of a hyperbola. An indifference curve between x_1 and x_2 is shown in figure 2.4

2.4.3 Marginal Rate of Substitution

The Marginal Rate of Substitution (MRS) refers to an agent's willingness to forgo a good for another at the same level of utility. The slope of the indifference curve of the agent's utility is called the marginal rate of substitution. Mathematically:

$$MRS(x_1, x_2) = \frac{-dx_1}{dx_2} \tag{2.5}$$

MRS defined for the utility function of agents represents the marginal utility of each good over the other. Mathematically given the utility function $u(x_1, x_2)$, mathematically:

$$MRS(x_1, x_2) = \frac{MU_{x_1}}{MU_{x_2}} \tag{2.6}$$

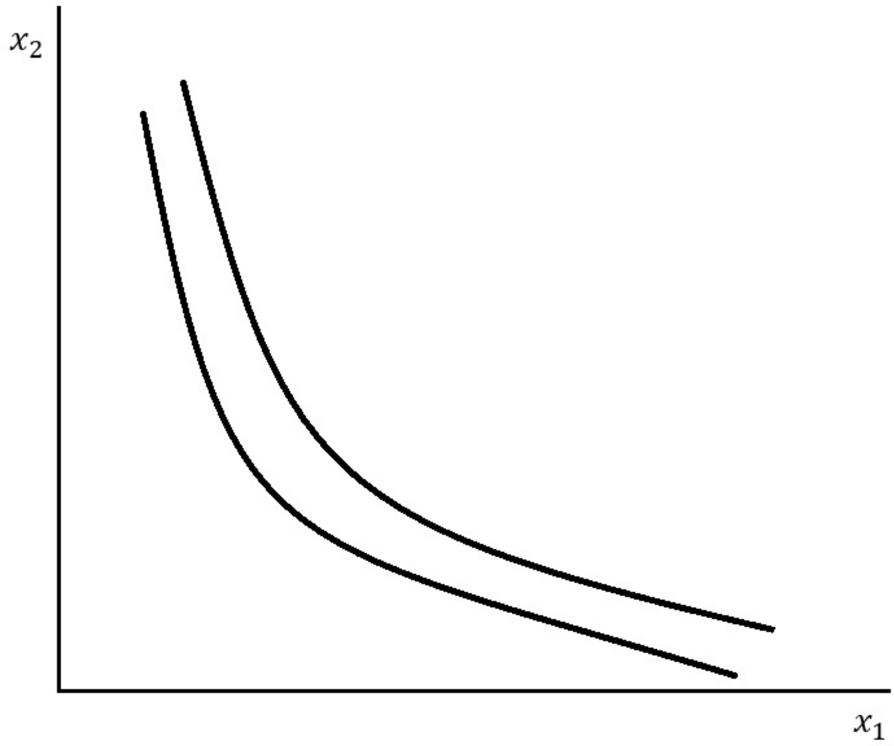


Figure 2.4: Indifference curves with constant utility.

The marginal utility of x_1 and x_2 can be calculated through implicit partial differentiation as follows:

$$MU_{x_1} = \frac{\partial u}{\partial x_1};\ MU_{x_2} = \frac{\partial u}{\partial x_2} \tag{2.7}$$

The marginal utility given a budget constraint becomes the slope of the budget line tangential to the indifference curve of the agent. Given the costs of c_1 and c_2 for goods x_1 and x_2, the marginal rate of substitution and the marginal utility for each good becomes:

$$MRS(x_1, x_2) = \frac{c_1}{c_2} \tag{2.8}$$

$$\frac{MU_{x_1}}{c_1} = \frac{MU_{x_2}}{c_2} \tag{2.9}$$

This implies that any constrained budget is spent such that utility is maximized when the marginal utility per unit of the amount spent on each good is equal. Figure 2.5 shows the MRS

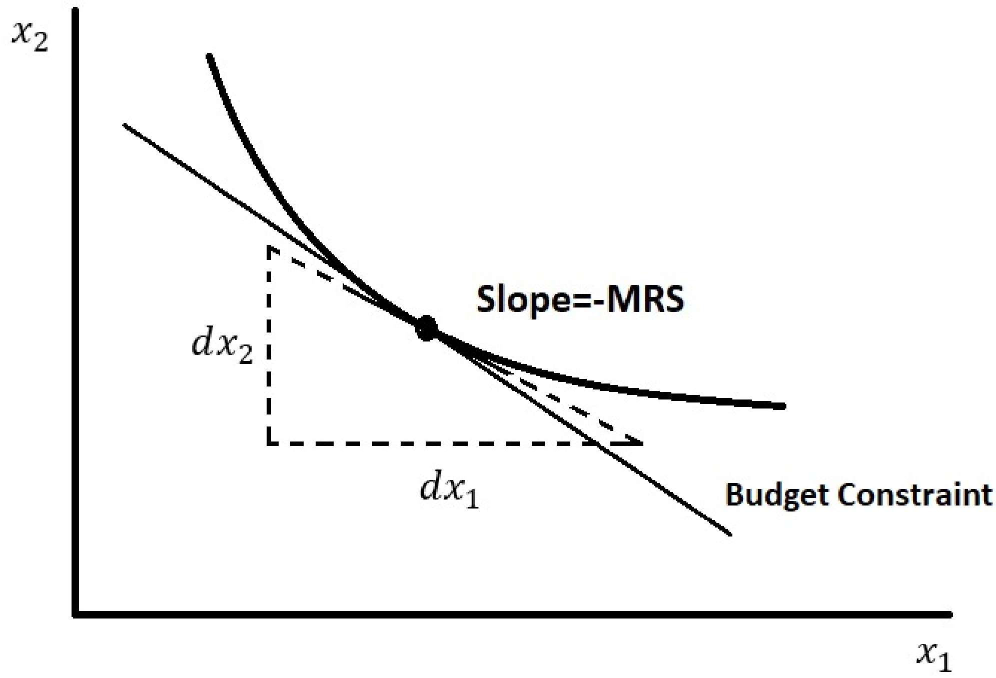


Figure 2.5: Marginal rate of substitution with budget constraint.

between x_1 and x_2

2.4.4 Common Utility Functions

The following utility functions are commonly used in consumer theory in expressing the preference relations of agents.

Cobb-Douglas

A utility function is defined according to Cobb Douglas such that:

$$u(x_1, x_2) = x_1^a x_2^b \ ; \ a > 0 \ b > 0 \tag{2.10}$$

The marginal utilities are defined thus:

$$MU_{x_1} = a x_1^{a-1} x_2^b; \ MU_{x_2} = b x_1^a x_2^{b-1} \tag{2.11}$$

The marginal rate of substitution becomes:

$$MRS = \frac{MU_{x_1}}{MU_{x_2}} = \frac{ax_2}{bx_1} \tag{2.12}$$

Given an income budget constraint such that

$$c_1x_1 + c_2x_2 = I \tag{2.13}$$

The maximizer must satisfy the equation (2.8) such that:

$$\frac{ax_2}{bx_1} = \frac{c_1}{c_2};\ ac_2x_2 = bc_1x_1 \tag{2.14}$$

Perfect Complements

A perfect complement utility function representing an agent's preferences between two goods is given below:

$$u(x_1, x_2) = \min ax_1, bx_2 \tag{2.15}$$

The goods are perfect complements of each other, if the preference for each good is in proportion to the other good. The indifference curve is L-shaped and at the points where the lines meet $ax_1 = bx_2$ as shown in figure 2.6. The MRS at this point is not defined as a result of the non-differentiable nature of the indifference curve.

Perfect Substitutes

The perfect substitute utility function of two goods x_1 and x_2 is represented in the form

$$u(x_1, x_2) = ax_1 + bx_2 \tag{2.16}$$

The goods are perfect substitutes for each other as one good can be replaced by the other. The preferences are weakly convex. the MRS is presented thus:

$$MRS = \frac{MU_{x_1}}{MU_{x_2}} = \frac{a}{b} \tag{2.17}$$

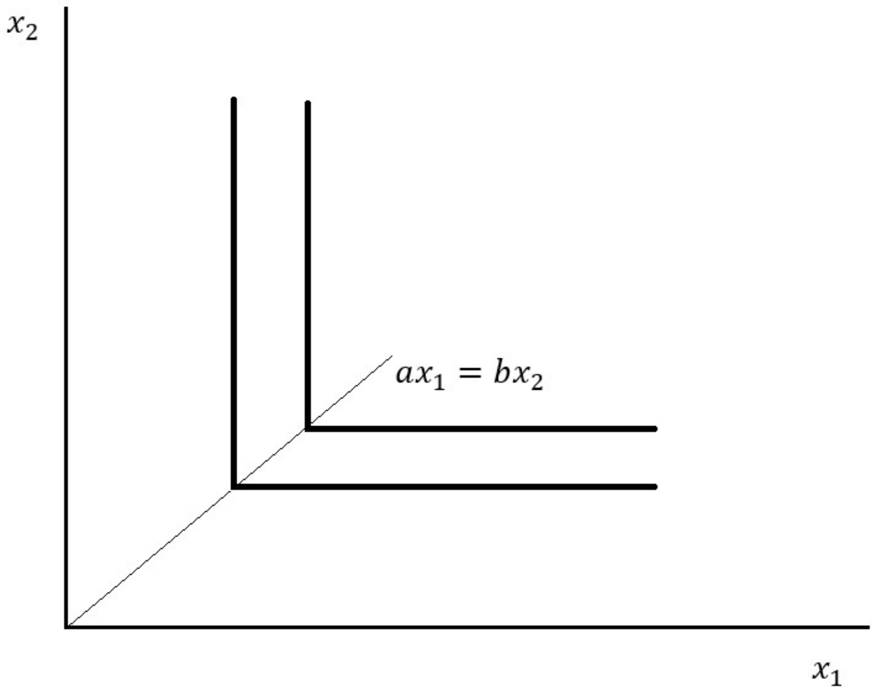


Figure 2.6: Perfect complements indifference curve.

In perfect substitutes, the marginal utilities are not dependent on the number of goods consumed, as a result, the indifference curves are straight lines as shown in figure 2.7.

The concepts of macroeconomics can be applied within a multi-agent framework integrating energy storage systems ESS to maximize the economic welfare of agents in realising an optimal demand profile. Using economic utility functions from consumer theory, the work in [1] proposes an economic consumption model using inter-temporal trades for balancing/leveling while proposing another utility function for the consumer cost minimization problem.

2.5 Ancillary Services

The advent of the deregulation of the electric power industry towards the restructuring of the vertically integrated utility has simultaneously liberalized the acquisition of ancillary services by the system operator in balancing the generation supply variability [93–96]. With the increase in distributed generation sources into the utility grid, there becomes a need for more reliability

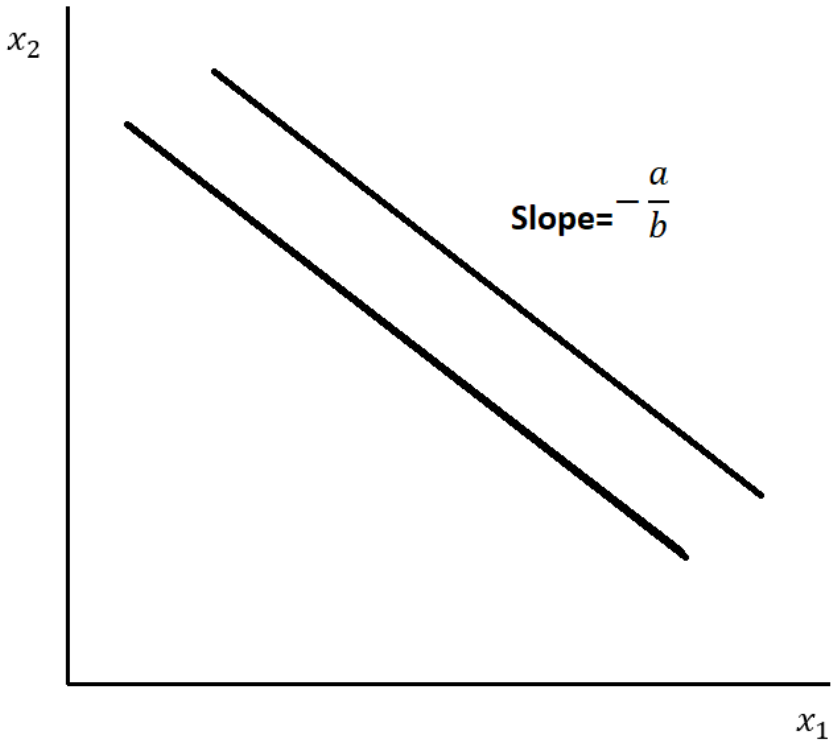


Figure 2.7: Perfect substitute indifference curve.

of supply towards ensuring a safe and secure power system operation. Ancillary services refer to capacity control services procured by the system operator to ensure the reliability of supply in maintaining the instantaneous and continuous balance between generation and supply [95]. These services can be provided through the control of generation, transmission, distribution, and sometimes demand equipment to realise the required level of power quality in the interconnected power system. This may be operational within a competitive market framework [93, 97] aside from its mandatory services requirement.

The development of bidding strategies for generation companies (gencos) towards the provision of AS is one aspect that projects the competitive nature of the market for such services. Usually, a market clearing model is initiated by the system operator to enable the gencos maximise their profits in AS markets. In [28, 98, 99] bidding strategies were developed for gencos in maximizing their profits in a joint energy and reserve market. The future demand for operating reserve and regulation services is expected to increase and the AS market will integrate more

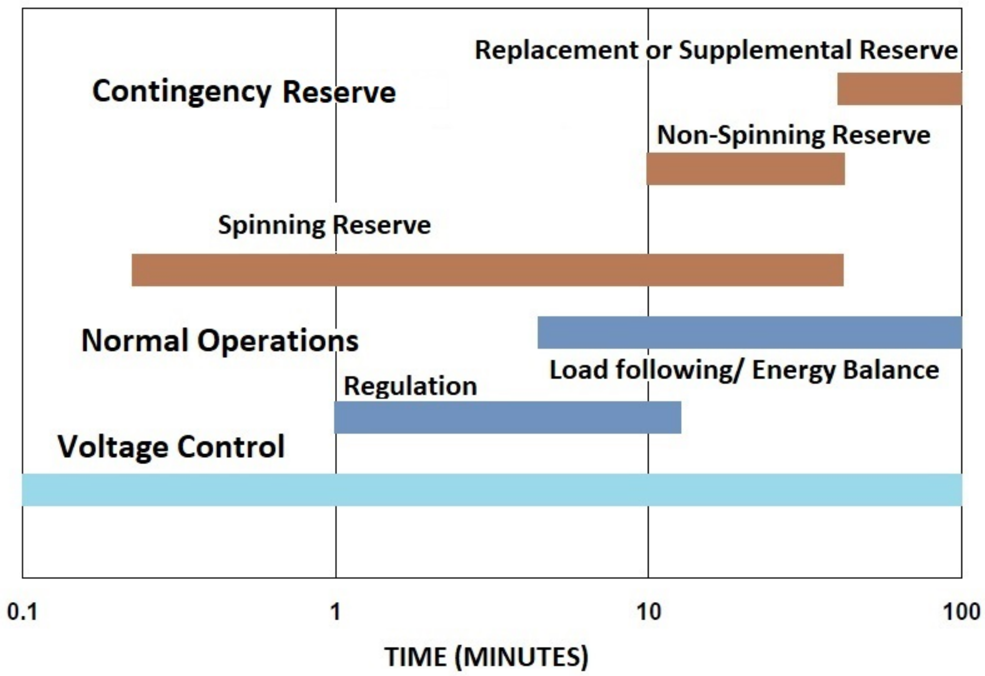


Figure 2.8: Ancillary services characterization based on response time and duration. [95]

entities aside from the gencos such as virtual power plants [28] and microgrid markets [100,101].

The following AS can be procured by the SO based on response time, response duration, and response frequency. Figure 2.8 shows how the services are distinguished by response time and duration.

2.5.1 Regulation

Regulation is one of the AS required to continuously balance generation and demand under normal conditions, the other being load following. [95]. It is the application of automatic generation control in rapid response to the SO requests for up and down reserve thereby tracking instantaneous fluctuations in demand for every minute in system load and subsequent resolution for unplanned fluctuations in generator output. AS can be renumerated as a regulated price, pay as a bid price, or a clearing price paid to the providers of the service [102].

2.5.2 Contingency Reserve

These reserves include operating reserves such as spinning, non-spinning also referred to as primary and secondary reserves, and supplemental reserves designed to restore the balance in generation and load following an unexpected loss of a major generator or transmission line. In spinning reserve, response time is immediate within 10 minutes subject to no load and no losses, and this typically can be sustained for about two hours. This reserve when called upon is readily available in response to a major loss in generation and transmission. The non-spinning reserve is similar to the spinning reserve in response time and duration asides the generation response time does not have to be immediate or online or in response to frequency. The supplemental reserve can be provided by both on and offline generation and responsive load. The generators deployed in these reserves must have telecommunications in response to the SO instructions.

2.5.3 Frequency Control

Frequency control aims to maintain the power network frequency within permissible limits by controlling the active power output in the system in maintaining the generation and load balance. A frequency control reserve in the form of an active power reserve is used to compensate for the deviation in frequency. The three levels of control used to maintain the frequency of the network include primary, secondary, and tertiary frequency control. While the primary frequency control adjusts the active power generation of the generating units, the secondary frequency control adjusts the active power generation and also restores the frequency back to the control limits. The tertiary frequency control restores the primary and secondary frequency control reserves, in restoring frequencies and tie line exchanges to their specified limits in the event the secondary control fails.

2.5.4 Voltage Control

The voltage control service aims to control the reactive power generated or consumed while maintaining the voltage of the system within permissible limits. While the aforementioned ancillary services control the active power generated, the voltage control service controls the reactive power. There are reactive power control devices available to compensate for losses that may arise as a result of voltage collapse and contingencies these include: Synchronous generators

Table 2.1: Capacities and Timescales for Balancing Services

Ancillary Services	Minimum Energy (MW)	Delivery Rate (MW/min),(secs)	Duration (hours,mins,secs)
Fast Reserve (Primary)	25	25	15 mins
STOR (Secondary)	3	20	2 hours
BM Start-Up (Tertiary)	-	-	89 mins
DFR (Primary)	1	10 sec	20 sec
DFR (Secondary)	1	30 sec	30 mins
DFR (Tertiary)	1	10 sec	indefinite

which supply reactive power in controlling the voltages during contingencies and can operate and a synchronous condenser.

Other AS include: black start (BS) service which occurs during an unplanned event such as outages and blackouts, and the power system is restored without any external source of electricity, Remote automatic generation control (RG) regulates the network frequency through an automatic centralised area control network, Grid loss compensation (GL) which allows for transmission loss compensations between generators and load and finally the Emergency control actions (EC) which integrate the use of special devices in maintaining a safe and secure operating network [93].

2.6 Ancillary Services Requirements

The ancillary services requirements are set by the electricity system operator (ESO), such as the National Grid ESO towards balancing generation and demand on the network. The services that are offered include reserve services, Frequency response services, demand flexibility services, etc. The energy capacities and timescales required for these balancing services vary depending on the type of service offered and the duration of service. Table 2.1 gives a summary of ancillary services, capacities, and timescales required by National Grid ESO [103].

2.7 Solvers

The existing solvers that can be used to solve Linear Programming/Quadratic programming problems on the basis for which the proposed community-based market problem and robust approaches are developed, are Gurobi and Mosek. Mosek is adapted more to solving general problems, but for mixed integer linear programs and general linear programming problems, Gurobi is highly recommended.

In this work, the Gurobi solver was used within the Yalmip Matlab code environment to solve the proposed community-based market case studies. The Yalmip program was downloaded and installed as a zip file from Github, and an academic license for Gurobi was downloaded from the website. This was run as a program on an Intel core i5 CPU, 64-bit operating system, and 8GB RAM size. The paths for Gurobi and Yalmip were added and saved, and a Yalmip test initiated confirmed the successful installation of the solver.

Chapter 3

A Robust Approach to Community-based Markets Under Uncertainty

The continuous proliferation of Distributed Energy Resources (DERs) in electricity grids is gradually changing the traditional roles of stakeholders. Consumers become producers, equipped with renewable generation, as well as storage in the form of electric vehicles or wall-mounted batteries. Along with the shift in production and consumption patterns, came the necessity for new, citizen-centered, market mechanisms to allow for the transparent and mutually beneficial exchange of energy between *prosumers*. The concept of auction theory enables interactions and energy exchanges among participants. In fostering engaging interactions among consumers and producers, the work in [38] allowed agents to place their orders within time slots and specified closing times.

The most promising mechanisms in this area are community-based and peer-to-peer (P2P) energy trading markets, where users can own small-scale DERs and actively trade their energy resources within a community [104]. Such markets offer novel designs for the restructuring of current electricity markets with regards to prosumers trading their excess/deficit energy with other end-users. The benefits and opportunities arising from such mechanisms are explored in [2]. Moreover, community-based markets can give rise to the sharing of energy resources amongst participating agents in the microgrid [4]. The evolution of these consumer-centric markets has initiated a more proactive role amongst prosumers. They can increase their flexibility and preferences towards the type of DER to meet their load demands. Consumers are no longer restricted to conventional generation, fixed electricity tariffs, and demand-side management initiatives to manage their consumption levels.

Opportunities have been sought to further decentralize the community-based market to full P2P market mechanisms [52,53,62,104–106]. There are many methods proposed in the literature for solving an economic dispatch problem within a network through community-based, P2P, or hybrid methods. The majority of these methods rely on formulating an optimization-based market design, including constraints relating to the peers' production and consumption, as well as financial limits. The solution of this problem is then performed, to obtain the optimal bilateral trades and prices [62]. The concept of blockchains is another important aspect of solving decentralized P2P energy trades in a microgrid network of prosumers. Reference [57] has proposed the use of blockchain as an important technology in the operation of microgrid markets, and the Brooklyn microgrid project has been used to analyse different market mechanisms. Blockchains and smart contracts are usually implemented side by side to facilitate P2P trading amongst prosumers.

There is uncertainty associated with renewable production and electricity consumption due to fluctuations in expected power output. While uncertainty has been tackled in many other optimization problems in power systems [107], it remains a little investigated area for P2P market mechanisms. The majority of the previous methods consider deterministic problems, using forecasted values of renewable generation and load demand. Several techniques have been introduced in the literature which models the uncertainties in renewable generations, load demands, and market prices, these include Stochastic Optimization [16], Robust Optimization [18], Taguchi's Orthogonal Array Testing (TOAT) [19], Information-Gap Decision Theory (IGDT) [20].

TOAT [19] is a statistical tool that provides representative testing scenarios that offer possible combinations to a given experimental design or analysis towards achieving the desired outcome. The main benefit of TOAT in comparison with Monte Carlo simulation is that TOAT is capable of characterizing different types of uncertain parameters with less deterministic scenarios and computation time. This is explicit from its capability to produce robust solutions to an optimization problem from combinations of different levels of uncertain elements, while Monte Carlo simulations are focused mainly on producing scenarios around deterministic solutions. In this work, TOAT was used to generate scenarios from the worst-case uncertain parameters of community agents' DER and load demands in solving the community market problem. TOAT has been previously used to solve different non-deterministic problems in power systems. Reference [108] has presented a transmission expansion planning problem under uncertainty where the optimal solution is robust against the worst-case scenarios generated by TOAT. Also, reference [109] has used TOAT to solve the economic dispatch problem with a non-smooth cost function under uncertainty. Reference [110] has introduced a TOAT-based probabilistic load flow model under the uncertainty of renewables and loads. However, to the best of our knowledge, there is no TOAT-based community-based energy trading model under uncertainty in the literature.

In this chapter, the following main contributions are as follows:

- An opportunistic (risk-seeker) and robust (risk-averse) community-based market mechanism under the uncertainty of renewable DERs and load demands is proposed. In the proposed approach, uncertainty sources are characterized by the best-case and worst-case scenarios generated by TOAT.
- An evaluation of the impact of uncertainties on total costs of community-based energy trading is carried out, by comparing the deterministic, and opportunistic/robust models with best-case/worst-case scenarios.

The rest of this chapter is organized as follows. In Section 3.1, the proposed scenario generation technique based on the notion of TOAT is presented. In Section 3.2, the mathematical formulations are introduced. In Section 3.3, the proposed approaches are tested on three microgrids as a community. Finally, Section 3.4 summarises the chapter.

3.1 Generating Robust Scenarios Using TOAT

In this section, TOAT is used to characterize the uncertainty of renewable DERs and load demands by generating appropriate best-case and worst-case scenarios.

Let $Y(X)$ be a function of the vector of uncertain parameters $X(a_1, ..., a_m, ...a_M)$ with M entries. Given N likely future realizations of each uncertain parameter a_m for $m = 1, ..., M$, there are N^M likely scenarios for future realizations of all M uncertain parameters belonging

Table 3.1: Orthogonal Array $L_4(2^3)$ with Four Scenarios and Three Uncertain Parameters

Number of Scenarios	α_1	α_2	α_3
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1

to the vector of uncertain parameters X. Needless to say, including all likely realizations of uncertain parameters in a specific problem may lead to intractability. Therefore, TOAT is used to reduce/increase the complexity/tractability of implementing all likely realizations of uncertain parameters.

In principle, Taguchi's method is formed based on orthogonal arrays (OA) represented by a general form $L_T(N^M)$, where L represents Latin squares based on TOAT terminology and T represents the number of reduced scenarios. For instance, suppose that there are three uncertain parameters (i.e., $M = 3$, a_1, a_2, and a_3) with two likely future realizations each (i.e., $N = 2$ where 1 and 2 represent the worst and best realization of each uncertain parameter, respectively). In this illustrative example, there are $N^M = 2^3 = 8$ different scenarios including all combinations of likely future realizations. To reduce the number of scenarios, a basic OA with $L_T(N^M) = L_4(2^3)$ can be used as depicted in Table 3.1. Accordingly, the number of scenarios can be reduced from 8 to 4. It is noteworthy to mention that these scenarios are generated in a way that each realization of every uncertain parameter appears an identical number of times in each column of the array in Table 3.1. Therefore, in this $L_4(2^3)$ array, realizations 1 and 2 occur two times [108], [109], and in general, T/N^M times for a generic $L_T(N^M)$ OA. There are available libraries with different OAs to characterize the uncertainty spectrum through the TOAT approach. Note that a particular OA may be modified for a specific problem by reducing the number of its uncertain parameters as compared to its original ones in the library.

3.2 Robust Community-Based Electricity Market

The community-based market structure consists of all agents (prosumers, producers and consumers) involved in energy trading inside and outside the community and a community manager who coordinates the activities of the trade. Figure 3.1 shows the structure of the community-based market with scenarios and mathematical formulations for three agents which include: a

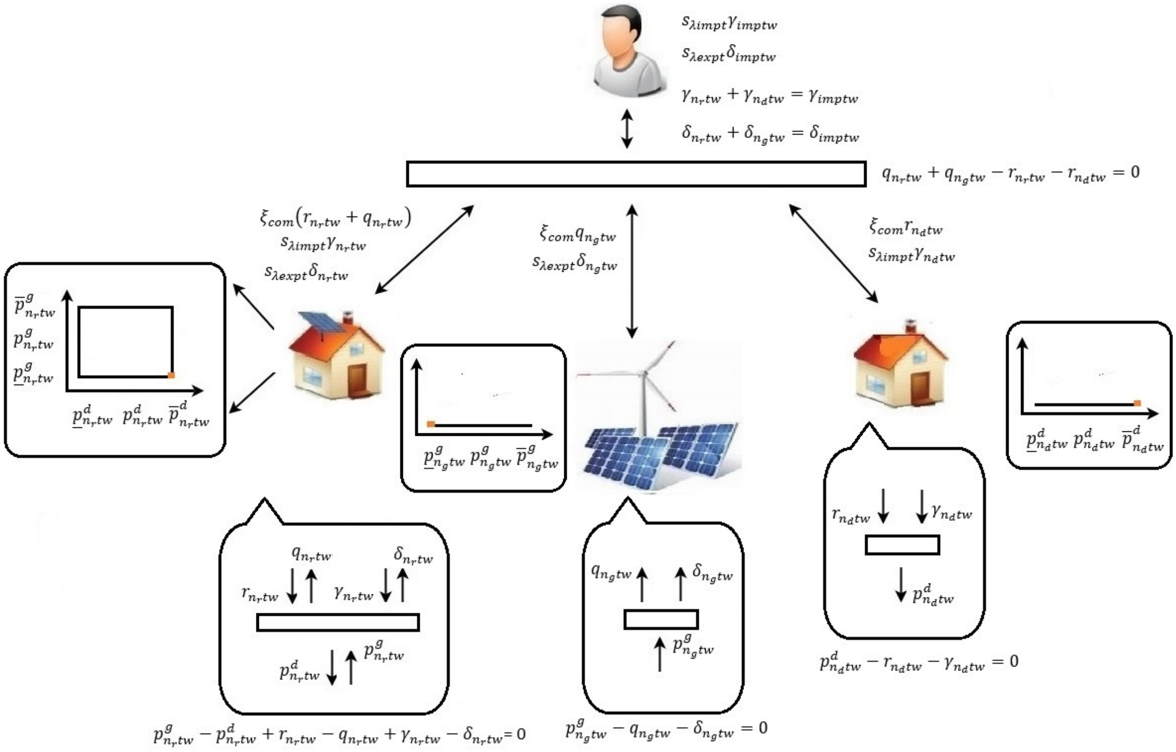


Figure 3.1: Schematic representation of a community-based market structure with scenarios.

prosumer, a producer, and a consumer single resource. In this section, a robust formulation of the community-based market is presented to obtain robust solutions for energy trades withstanding the worst-case realization of uncertain parameters.

The TOAT approach is used to capture the uncertainty of renewable DERs and load demands using appropriate scenarios. In this study, a symmetric bounded interval is considered for each uncertain parameter. Also, extreme points of each bounded interval are considered as the likely best-case or worst-case future realizations of each uncertain parameter. Clearly, the upper and lower bounds of the bounded intervals pertaining to the uncertain renewable DERs (i.e., $\tilde{p}^g_{jn_rt} = \left[p^g_{jn_rt} + \hat{p}^g_{jn_rt}, p^g_{jn_rt} - \hat{p}^g_{jn_rt}\right]$, where $p^g_{jn_rt}$ and $\hat{p}^g_{jn_rt}$ stand for expected value and deviations of renewable DEr of prosumer n_r at hour t) represent the best-case and worst-case scenarios, respectively, while the lower and upper bounds of the symmetric bounded intervals pertaining to uncertain load demands of prosumer n_r (i.e., $\tilde{p}^d_{n_rt} = \left[p^d_{n_rt} - \hat{p}^d_{n_rt}, p^d_{n_rt} + \hat{p}^d_{n_rt}\right]$, where $p^d_{n_rt}$ and $\hat{p}^d_{n_rt}$ stand for expected value and deviations of load demands at hour t) represent the best-case and worst-case scenarios, respectively. Mainly, the multi-period robust problem can be written as:

$$\max_{U_{g_rt}\in\Omega^{USgr_t},U_{d_rt}\in\Omega^{USdr_t}}\min_{X_{n_rt}\in\Omega^{X_{n_rt}},Z_t\in\Omega^{Z_t}}\left(\sum_{n_r=1}^{N_{pr}}\sum_{t=1}^{T}\varphi_{n_r}(X_{n_rt},U_{g_rt},U_{d_rt})+\sum_{t=1}^{T}g\left(Z_t,U_{g_rt},U_{d_rt}\right)\right) \tag{3.1}$$

where $\Omega^{USgr_t}/\Omega^{USdr_t}$ represents the set of uncertain parameters of the prosumer generation/consumption resources at hour t, $\Omega^{X_{n_rt}}$ represents the feasible space of decision variables of prosumer n_r at hour t, and Ω^{Z_t} represents the feasible space of continuous variables of the community manager at hour t. The proposed min-max problem in (3.1) minimizes the total costs of prosumers, i.e., $\varphi_{n_r}(X_{n_rt},U_{g_rt},U_{d_rt})$ where:
$X_{n_rt}=\{p^g_{n_rt},p^d_{n_rt},r_{n_rt},q_{n_rt},\gamma_{n_rt},\delta_{n_rt}\}$, and the total costs of the community manager in importing/exporting electricity inside/outside the community, i.e., $g(Z_t,U_{g_rt},U_{d_rt})$ where $Z_t=\{\gamma_{impt},\delta_{expt}\}$, under the worst-case realization of uncertain parameters, i.e., $U_{g_rt},U_{d_rt}=\{\tilde{p}^g_{n_rt},\tilde{p}^d_{n_rt}\}$. Needless to say, this problem cannot be solved for all realizations of uncertain parameters. To obtain a tractable counterpart, first, the orders of minimization and maximization problems are changed based on the classical minimax theorem (Proposition 5.5.4 in [111]), and then, the TOAT approach is used to generate appropriate scenarios capturing the uncertainty spectrum.

In Figure 3.1, $p^g_{n_rtw}/p^d_{n_rtw},p^g_{n_gtw}$ and $p^d_{n_dtw}$ corresponds to the production/consumption of a prosumer n_r, a producer n_g and a consumer n_d in the hour t and scenario w. The energy trading within the community by all agents is modelled in q_{n_rtw} and r_{n_rtw} for the prosumer's production and consumption resources, q_{n_gtw} for the producer's resources and r_{n_dtw} for the consumer's resources in the hour t and scenario w. The trades in the community are balanced by the community manager such that $q_{n_rtw}+q_{n_gtw}-r_{n_rtw}-r_{n_dtw}=0$ at the hour t and scenario w. Each agent can also trade energy with the outside community or the grid through the community manager $\gamma_{n_rtw}/\delta_{n_rtw}$, is the power imports/exports by the prosumer, γ_{n_dtw} are power imports by the consumers and δ_{n_gtw} are power exports for the producers at each hour t and scenario w. The sum of these trades is equal to the total imports γ_{imptw} and exports δ_{imptw} traded by the community manager in the hour t and scenario w and the import and export costs $s_{\lambda impt}$ and $s_{\lambda expt}$ are the costs set by the wholesale market for imports and exports from and to the grid. A quadratic cost function can be used to calculate the productions of different agents depending on the nature of their resources, and a transaction cost ξ_{com} is set by the community manager for the energy trades within the community.

Therefore, the multi-period robust mathematical formulation can be written as follows:

$$\min \ \alpha(p^g_{n_rtw}, p^d_{n_rtw}, r_{n_rtw}, q_{n_rtw}, \gamma_{n_rtw}, \delta_{n_rtw}, \gamma_{imptw}, \delta_{exptw}) \tag{3.2a}$$

$$\begin{aligned} \text{s.t.} \ \ \alpha \geq \sum_{j=1}^{R_g}\sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}} \varphi_{jn_r}(p^g_{jn_rtw}) + \sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}} \phi_{n_c}(p^d_{n_rtw}) + \\ \xi_{com}\sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}} r_{n_rtw} + \xi_{com}\sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}} q_{n_rtw} + \\ s_{\lambda impt}\sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}} \gamma_{n_rtw} + \\ s_{\lambda expt}\sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}} \delta_{n_rtw} + \sum_{t=1}^{T} g(\gamma_{imptw}, \delta_{exptw}), \ \forall w \in W \end{aligned} \tag{3.2b}$$

$$p^g_{n_rtw} - q_{n_rtw} - \delta_{n_rtw} = 0 \ \forall n_r \in N_{pr}, \forall t, \forall w \tag{3.2c}$$

$$p^d_{n_rtw} - r_{n_rtw} - \gamma_{n_rtw} = 0 \ \ \forall n_r \in N_{pr}, \forall t, \forall w \tag{3.2d}$$

$$p_{n_rtw} - p^g_{n_rtw} + p^d_{n_rtw} = 0 \ \ \forall n_r \in N_{pr}, \forall t, \forall w \tag{3.2e}$$

$$\sum_{j=1}^{R_j} p^g_{jn_rtw} = p^g_{n_rtw} \ \ \forall n_r \in N_{pr}, \forall t, \forall w \tag{3.2f}$$

$$p_{n_rtw} + r_{n_rtw} - q_{n_rtw} + \gamma_{n_rtw} - \delta_{n_rtw} = 0 \ \ \forall n_r \in N_{pr}, \forall t, \forall w \tag{3.2g}$$

$$\sum_{n_r=1}^{N_{pr}} q_{n_rtw} - \sum_{n_r=1}^{N_{pr}} r_{n_rtw} = 0 \ \ \forall t, \ \forall w \tag{3.2h}$$

$$\sum_{n_r=1}^{N_{pr}} \gamma_{n_rtw} = \gamma_{imptw} \ \ \forall t, \forall w \tag{3.2i}$$

$$\sum_{n_r=1}^{N_{pr}} \delta_{n_rtw} = \delta_{exptw} \ \ \forall t, \forall w \tag{3.2j}$$

$$\underline{p}^g_{jn_rtw} \leq p^g_{jn_rtw} \leq \overline{p}^g_{jn_rtw} \ \ \forall n_r \in N_{pr}, \forall t, \forall w \tag{3.2k}$$

$$\underline{p}^d_{n_rtw} \leq p^d_{n_rtw} \leq \overline{p}^d_{n_rtw} \ \ \forall n_r \in N_{pr}, \forall t, \forall w \tag{3.2l}$$

$$\gamma_{n_rtw} \geq 0, \delta_{n_rtw} \geq 0, \ \ \forall n_r \in N_{pr}, \forall t, \forall w \tag{3.2m}$$

where the objective function equation (3.2a) minimizes the total costs of prosumers, the total transactions costs, and the total costs of the community manager in importing/exporting electricity inside/outside the community under the worst scenario for uncertain parameters obtained from the TOAT approach where the auxiliary variable α is used to find the optimal total cost pertaining to the worst-case scenario. In this study, it is assumed that each agent as a prosumer is equipped with renewable DERs and optimizes its energy production/consumption. Concerning the energy produced/consumed by the prosumer for export/import outside/inside the community, the quadratic cost function $\varphi_{jn_r} = a_{jn_r} \cdot p^g_{jn_rtw}2 + b_{jn_r} \cdot p^g_{jn_rtw}$ is considered here. If the net production of the prosumer is from either renewable or non-renewable DERs, $a_{jn_r} = 0$, $b_{jn_r} = 0$ or $a_{jn_r} > 0$, $b_{jn_r} > 0$, respectively.

Constraint (3.2b) ensures that α is greater than or equal to the total costs of the worst scenario. Constraint (3.2c)/(3.2d) expresses the energy balance for each prosumer's production and consumption respectively. Constraint (3.2e) gives the sum of the production and consumption of each prosumer at hour t and scenario w. Constraint (3.2f) is the net production from all the generation assets of each prosumer at hour t and scenario w. Constraint (3.2g) represents the combined energy balance of each prosumer's production and consumption at hour t and scenario w. Constraint (3.2h) represents the total energy traded between prosumers within the community where its net value is equal zero. Also, constraint (3.2i)/(3.2j) stands for the total energy imported/exported by the community manager inside/outside the community and consumed/produced by different prosumers. Constraint (3.2k)/(3.2l) limits the power production/consumption of each prosumer resource where the lower-bound (i.e., $\underline{p}^g_{jn_rtw}$, $\underline{p}^d_{n_rtw}$) and upper-bound (i.e., $\overline{p}^g_{jn_rtw}$, $\overline{p}^d_{n_rtw}$) parameters for each scenario are obtained from the TOAT approach. Also, constraint (3.2m) ensures the non-negativity of variables indicating imported and exported energy by each prosumer. It is noteworthy to mention that the proposed model in (3.2a)-(3.2m) can find the robust solution through the worst-case scenarios and the opportunistic solution through the best-case scenarios generated by the TOAT approach. It can also find the deterministic solution by only including the forecast values of uncertain parameters.

Likewise, the robust model for prosumers is developed for other community agents such as the producers and consumers equipped with DERs and load respectively as follows:

$$\min \ \alpha(p^g_{n_g tw}, p^d_{n_d tw}, r_{n_d tw}, q_{n_g tw}, \gamma_{n_d tw}, \delta_{n_g tw}, \gamma_{imptw}, \delta_{exptw}) \tag{3.3a}$$

$$\begin{aligned} \text{s.t.} \ \ \alpha \geq \sum_{l=1}^{R_p}\sum_{t=1}^{T}\sum_{n_g=1}^{N_p} \varphi_{ln_g}(p^g_{ln_g tw}) + \sum_{t=1}^{T}\sum_{n_d=1}^{N_c} \phi_{n_d}(p^d_{n_d tw}) + \\ \xi_{com}\sum_{t=1}^{T}\sum_{n_d=1}^{N_c} r_{n_d tw} + \xi_{com}\sum_{t=1}^{T}\sum_{n_g=1}^{N_p} q_{n_g tw} + \\ s_{\lambda impt}\sum_{t=1}^{T}\sum_{n_d=1}^{N_c} \gamma_{n_d tw} + \\ s_{\lambda expt}\sum_{t=1}^{T}\sum_{n_g=1}^{N_p} \delta_{n_g tw} + \sum_{t=1}^{T} g(\gamma_{imptw}, \delta_{exptw}), \ \forall w \in W \end{aligned} \tag{3.3b}$$

$$p^g_{n_g tw} - q_{n_g tw} - \delta_{n_g tw} = 0 \ \forall n_g \in N_p, \forall t, \forall w \tag{3.3c}$$

$$p^d_{n_d tw} - r_{n_d tw} - \gamma_{n_d tw} = 0 \ \ \forall n_d \in N_c, \forall t, \forall w \tag{3.3d}$$

$$\sum_{l=1}^{R_p} p^g_{ln_g tw} = p^g_{n_g tw} \ \ \forall n_g \in N_p, \forall t, \forall w \tag{3.3e}$$

$$\sum_{n_g=1}^{N_p} q_{n_g tw} - \sum_{n_d=1}^{N_c} r_{n_d tw} = 0 \ \ \forall t, \ \forall w \tag{3.3f}$$

$$\sum_{n_d=1}^{N_c} \gamma_{n_d tw} = \gamma_{imptw} \ \ \forall t, \forall w \tag{3.3g}$$

$$\sum_{n_d=1}^{N_c} \delta_{n_d tw} = \delta_{exptw} \ \ \forall t, \forall w \tag{3.3h}$$

$$\underline{p}^g_{ln_g tw} \leq p^g_{ln_g tw} \leq \overline{p}^g_{ln_g tw} \ \ \forall n_g \in N_p, \forall t, \forall w \tag{3.3i}$$

$$\underline{p}^d_{n_d tw} \leq p^d_{n_d tw} \leq \overline{p}^d_{n_d tw} \ \ \forall n_d \in N_c, \forall t, \forall w \tag{3.3j}$$

$$\gamma^d_{n_d tw} \geq 0, \delta^g_{n_g tw} \geq 0 \ \ \forall n_d \in N_c, \forall n_g \in N_p \forall t, \forall w \tag{3.3k}$$

where the objective function equation (3.3a) aims to minimise the total energy traded costs of producer and consumer agents and the community manager in importing and exporting energy in and out of the community. Constraints (3.3b)-(3.3d),(3.3e), and (3.3f)-(3.3k) serve the same functions as (3.2b)-(3.2d), (3.2f) and (3.2h)-(3.2m) respectively. Finally, the robust model of the

CBM of all community agents prosumers, producers and consumers is presented as follows:

$$\min \ \alpha(p^g_{n_rtw}, p^d_{n_rtw}, r_{n_rtw}, q_{n_rtw}, \gamma_{n_rtw}, \delta_{n_rtw}, p^g_{n_gtw}, p^d_{n_dtw}, r_{n_dtw}, q_{n_gtw}, \gamma_{n_dtw}, \delta_{n_gtw}, \gamma_{imptw}, \delta_{exptw}) \tag{3.4a}$$

$$\begin{aligned} \text{s.t.} \ \ \alpha \geq & \sum_{j=1}^{R_g}\sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}} \varphi_{jn_r}(p^g_{jn_rtw}) + \sum_{l=1}^{R_p}\sum_{t=1}^{T}\sum_{n_g=1}^{N_p} \varphi_{ln_g}(p^g_{ln_gtw}) + \sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}} \phi_{n_r}(p^d_{n_rtw}) + \\ & \sum_{t=1}^{T}\sum_{n_d=1}^{N_c} \phi_{n_d}(p^d_{n_dtw}) + \xi_{com}\sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}}\sum_{n_d=1}^{N_c}(r_{n_rtw} + r_{n_dtw}) + \xi_{com}\sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}}\sum_{n_g=1}^{N_d}(q_{n_rtw} + q_{n_gtw}) + \\ & s_{\lambda impt}\sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}}\sum_{n_d=1}^{N_c}(\gamma_{n_rtw} + \gamma_{n_dtw}) + \\ & s_{\lambda expt}\sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}}\sum_{n_d=1}^{N_c}(\delta_{n_rtw} + \delta_{n_dtw}) + \sum_{t=1}^{T} g(\gamma_{imptw}, \delta_{exptw}), \ \forall w \in W \end{aligned} \tag{3.4b}$$

$$(3.2c) - (3.2g) \tag{3.4c}$$

$$\sum_{n_r=1}^{N_{pr}}\sum_{n_g=1}^{N_p}(q_{n_rtw} + q_{n_gtw}) - \sum_{n_r=1}^{N_{pr}}\sum_{n_d=1}^{N_c}(r_{n_rtw} + r_{n_dtw}) = 0 \ \ \forall t, \ \forall w \tag{3.4d}$$

$$\sum_{n_r=1}^{N_{pr}} \gamma_{n_rtw} + \sum_{n_d=1}^{N_c} \gamma_{n_dtw} = \gamma_{imptw} \ \ \forall t, \forall w \tag{3.4e}$$

$$\sum_{n_r=1}^{N_{pr}} \delta_{n_rtw} + \sum_{n_g=1}^{N_p} \delta_{n_gtw} = \delta_{exptw} \ \ \forall t, \forall w \tag{3.4f}$$

$$\underline{p}^g_{jn_rtw} \leq p^g_{jn_rtw} \leq \overline{p}^g_{jn_rtw} \ \ \forall n_r, \forall t, \forall w \tag{3.4g}$$

$$\underline{p}^d_{n_rtw} \leq p^d_{n_rtw} \leq \overline{p}^d_{n_rtw} \ \ \forall n_c, \forall t, \forall w \tag{3.4h}$$

$$\underline{p}^g_{ln_gtw} \leq p^g_{ln_gtw} \leq \overline{p}^g_{ln_gtw} \ \ \forall n_g, \forall t, \forall w \tag{3.4i}$$

$$\underline{p}^d_{n_dtw} \leq p^d_{n_dtw} \leq \overline{p}^d_{n_dtw} \ \ \forall n_d, \forall t, \forall w \tag{3.4j}$$

$$\gamma_{n_rtw} \geq 0, \gamma_{n_dtw} \geq 0, \delta_{n_rtw} \geq 0, \delta_{n_gtw} \geq 0 \ \ \forall n, \forall t, \forall w \tag{3.4k}$$

where the objective function equation (3.4a) aims to minimise the total energy traded costs of all agents including prosumers, producers and consumers, and the community manager in importing and exporting energy within and outside of the community. Constraints (3.4b)/(3.4c) and (3.2b)-(3.2g), (3.4d)-(3.4f) and (3.2b)-(3.2g) serve the same function. Constraints (3.4g)-(3.4j) limit the production/consumption of all agents resources within a lower and an upper bound while constraints (3.4k) gives the non-negativity constraints of all agent's imports and exports outside the community.

Hereafter, the optimal solutions of deterministic, robust, and opportunistic community-based energy trading models at each hour of the scheduling period are indicated by $DCET_t$, $RCET_t$, and $BCET_t$, respectively. It is worthwhile to note that the robust model solves the problem for different realizations of the uncertain parameters and characterizes different deviations from the nominal estimates of the uncertain parameters while the deterministic model only solves the problem for the nominal estimates of the uncertain parameters. Accordingly, the cost of the robust model is higher than the cost of the deterministic counterpart. The difference between the total costs of the robust and deterministic models is designated as the cost of robustness. The cost of robustness is the cost incurred or price forgone in obtaining a robust solution.

3.2.1 The Community Manager's Model

The community manager is responsible for coordinating the activities of the trade in realising an optimal solution. With regards to the robust nature of the community-based market problem, the model of g is considered in a centralised manner where the community manager minimizes the costs of importing energy into the community while maximizing the revenues realised from exporting the energy outside the community/ or the grid. The community manager's model in a given scenario w is presented according to [2], [5] as:

$$g(\gamma_{imptw}, \delta_{exptw}) = (\lambda_m + \tau)\gamma_{imptw} - (\lambda_m - \tau)\delta_{exptw} \tag{3.5}$$

Here it can be considered based on regulatory policies that the cost of imports is generally higher than the costs of exports [2]. The community manager's model can be further expressed as:

$$g(\gamma_{imptw}, \delta_{exptw}) = s_{\lambda impt}\gamma_{imptw} - s_{\lambda expt}\delta_{exptw} \tag{3.6}$$

where λ_m is the market price for importing energy from the main grid, and τ is a parameter representing the spread between import and export prices. This is given with consideration that the purchase price from the grid is generally higher than the selling price to the grid. $s_{\lambda impt} = \lambda_m + \tau$ and $s_{\lambda expt} = -\lambda_m + \tau$. This represents the price the community pays for import

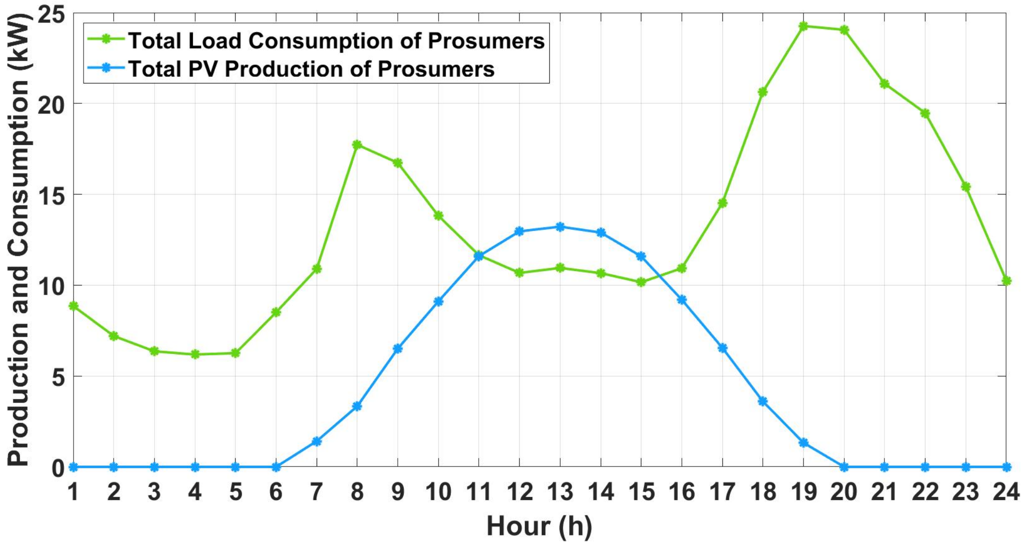


Figure 3.2: Total PV production and load consumption Case 1.

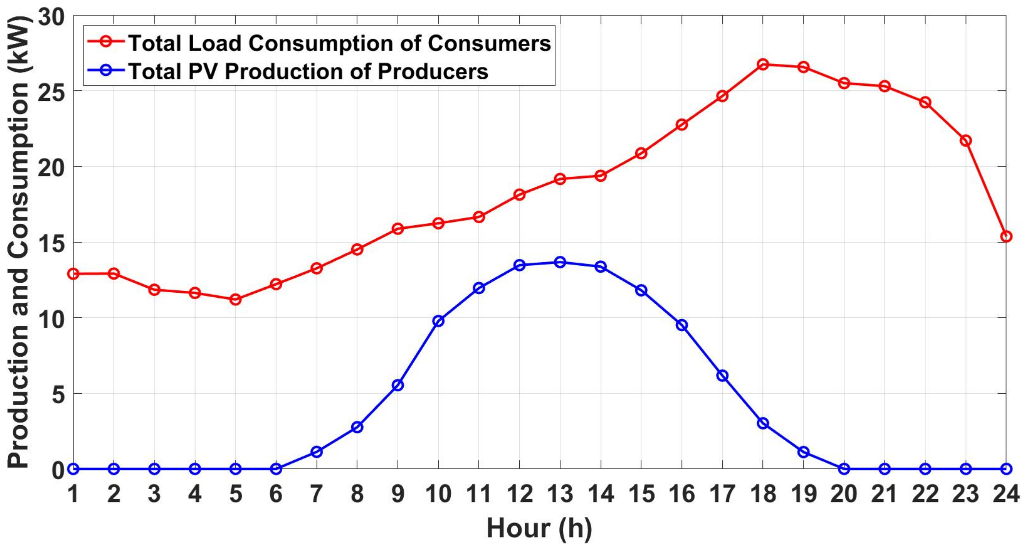


Figure 3.3: Total PV production and load consumption Case 2.

which is positive for cost minimization and negative for export representing revenue profit.

3.3 Case Study

In this work, three different case studies applying the robust community-based market models for prosumers only, producers and consumers, and all agents (prosumers, producers, and consumers) are analysed in realising the costs of robustness within community microgrids. The first case study involves a community microgrid of 7 prosumers with each equipped with a PV resource,

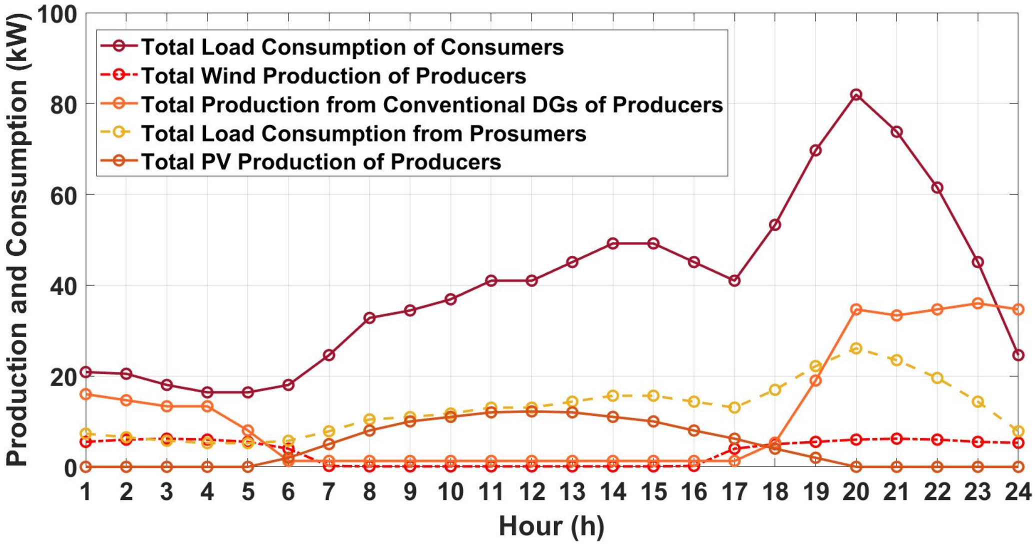


Figure 3.4: Total production and load consumption Case 3.

the second case study is a community microgrid with 7 producers and 8 consumers finally the third case study is the community microgrid of 5 prosumers, equipped with PV production, 3 producers, two of which have conventional resources (micro-turbine and fuel cell) one wind resource, and 20 consumers. The first and second case studies are obtained from the clean data set of the Australian distribution network Ausgrid dataset [112] while the third case study was obtained from an example case of the Cigre LV benchmark model [113]. Figures 3.5, 3.6, and 3.7 show the schematic representation of the case studies. The next section describes the contents of the data sets of each case study.

3.3.1 Data Set Case Studies 1 & 2

In this study, the proposed deterministic, opportunistic, and robust models are tested on a microgrid as a community with 7 prosumers for the first case study, and 7 producers and 8 consumers for the second case study, obtained from the *clean* dataset of 54 customers in the Ausgrid network aggregated load and generation data for summer and winter seasons [112]. The Ausgrid peak-tariff in 2019 [114] is considered as market prices (i.e., $\lambda_m = 23.5336$ c/kWh) set by the market operator, and a price is set at $\tau = 10$ c/kWh for using the grid. The import price becomes the market price plus the grid tariff (i.e., $\lambda_m + \tau = 33.5336$ c/kWh) and the export price is (i.e., $\lambda_m - \tau = 13.5336$ c/kWh). All agents' cost functions reflect their preferences for costs set for trading their resources. These values are obtained as normalised costs of energy

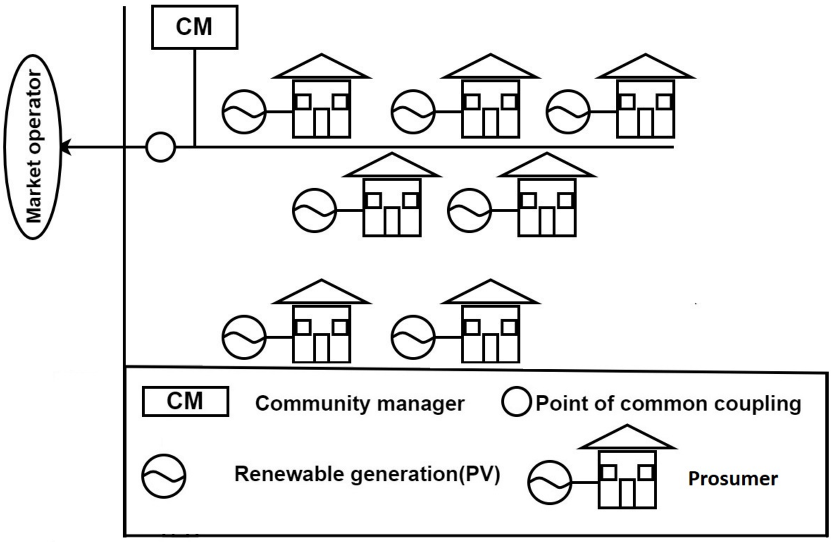


Figure 3.5: Schematic representation of test Case 1.

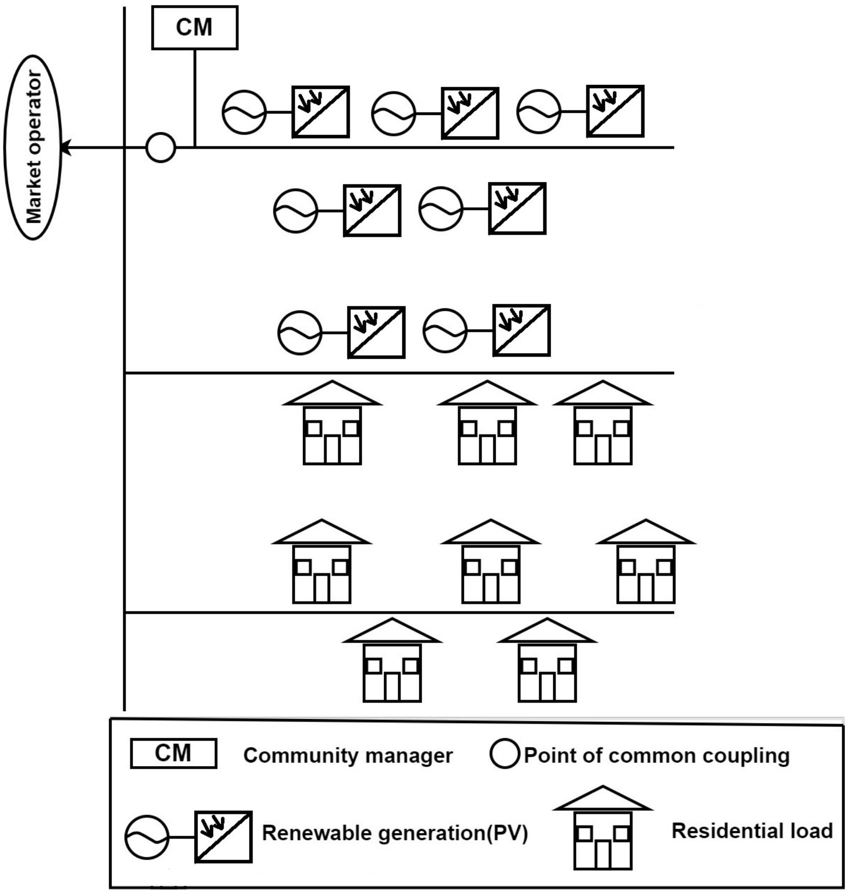


Figure 3.6: Schematic representation of test Case 2.

prices A and scaling it to the capacity of agent resources as used in [2]. The transaction cost of the community manager is fixed at 1 c/kWh. The total load consumption of prosumers and

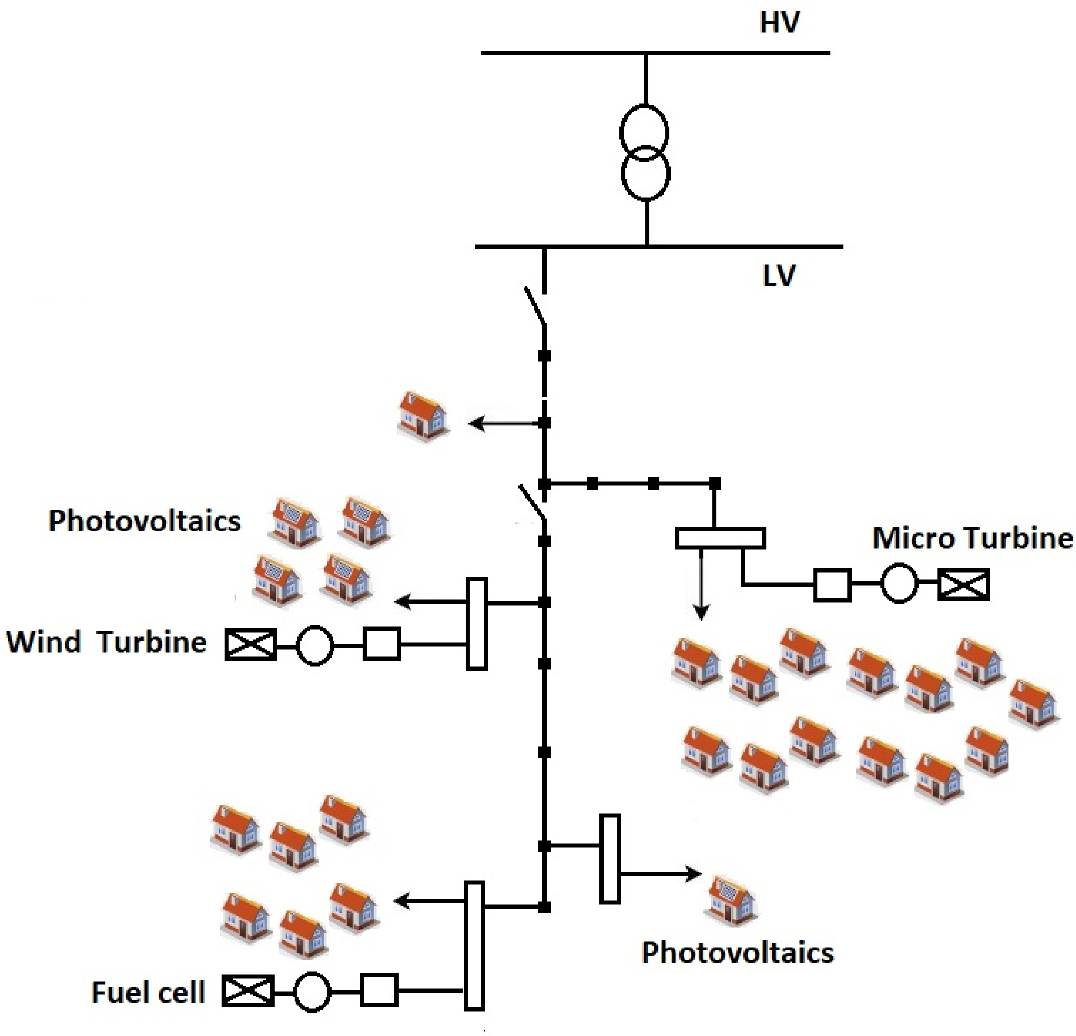


Figure 3.7: Schematic representation of test Case 3.

consumers and total PV production of prosumers and producers are illustrated in Fig. 3.2 and 3.3 respectively. The upper and lower bounds of agents' renewable production resources are considered the same (i.e. $\underline{p}^{g}_{jn_rtw} = \overline{p}^{g}_{jn_rtw}$) according to [5]. The consumption resource is a fixed/flexible load profile at a lower bound and an upper bound allowing a 10% increase. Additionally, 10% symmetric deviations from the expected values of the uncertain parameters are considered for all 24 hours of the scheduling horizon.

Table 3.2: The $L_{16}(2^{15})$ Orthogonal Array with 14 Uncertain Parameters

Number of Scenarios	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15^*}
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2
3	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2
4	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1
5	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2
6	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1
7	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1
8	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2
9	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
10	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1
11	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1
12	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2
13	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1
14	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2
15	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2
16	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1

3.3.2 Data Sets Case Study 3

The third case study involves a community microgrid of 5 prosumers, 3 producers, and 20 consumers obtained from an example case of the Cigre LV benchmark model. The European Commission (EC) import tariff in 2021 [115] is considered as market prices (i.e., $\lambda_m = 21.92$ c/kWh) set by the grid, while the price τ is also considered as a grid tariff. Therefore the import prices from the community manager becomes $\lambda_m + \tau = 31.92$ c/kWh while the export price is $\lambda_m - \tau = 11.92$ c/kWh. All agents' cost functions also reflect their cost preferences and are realised as normalised costs of energy prices scaled to the capacity of their resources. The transaction cost of the community manager is fixed at 1 c/kWh. The marginal costs for the PV and wind turbines are taken as zero ($a_{jn_r} = b_{jn_r} = 0,\ a_{ln_g} = b_{ln_g} = 0$). The total PV production and load consumption of prosumers, total production of producers, and consumption from consumers are illustrated in Fig. 3.4. Also, for the renewable production resources of agents the upper and lower bounds are the same while the conventional resources assume upper and lower bounds from the minimum and maximum generation/consumption outputs of the Cigre benchmark model. The consumption resources are taken as fixed/flexible load profile at lower bound and an upper bound with an increase by 10% . Likewise, 10% symmetric deviations from expected values of the uncertain parameters are considered for all 24 hours of the scheduling horizon.

In the community of 7 prosumers, there are 14 uncertain parameters representing 7 generation

Table 3.3: The $L_{16}(2^{15})$ Orthogonal Array

Number of Scenarios	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2
3	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2
4	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1
5	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2
6	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1
7	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1
8	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2
9	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
10	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1
11	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1
12	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2
13	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1
14	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2
15	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2
16	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1

Table 3.4: The $L_{32}(2^{31})$ Orthogonal Array

N/s	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{19}	a_{20}	a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}	a_{30}	a_{31}
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
3	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2
4	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1
5	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2
6	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1
7	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1
8	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	1	1	1	1	2	2	2	2
9	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2
10	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1
11	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1
12	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	1	1	2	2	1	1	2	2
13	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1
14	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2
15	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2
16	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	1	1	2	2	2	2	1	1
17	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
18	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1
19	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1
20	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	1	2	1	2	1	2	1	2
21	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1
22	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2
23	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2
24	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	1	2	1	2	2	1	2	1
25	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1
26	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2
27	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2
28	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	1	2	2	1	1	2	2	1
29	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2
30	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1
31	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1
31	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	1	2	2	1	2	1	1	2

and 7 consumption resources, while the community of 7 producers and 8 consumers has 15 uncertain parameters with the best and worst realizations at each hour of the scheduling problem

Table 3.5: Highest Energy Trading Costs for All Models in the First Community

	Highest Energy Trading Costs (c)		
Time Periods	$RCET_t$	$DCET_t$	$BCET_t$
12am-6am	653.2	593.8	534.4
6am-12pm	1161.4	1025.8	890.2
12pm-6pm	1366.2	1209.8	1053.3
6pm-12am	1774.8	1613.5	1452.1

Table 3.6: Highest Energy Trading Costs for All Models in the Second Community

	Highest Energy Trading Costs (c)		
Time Periods	$RCET_t$	$DCET_t$	$BCET_t$
12am-6am	1169.1	1062.8	956.6
6am-12pm	1044.7	927.7	820.3
12pm-6pm	2043.2	1821.8	1600.3
6pm-12am	2132.2	1917.0	1721.3

Table 3.7: Highest Energy Trading Costs for All Models in the Third Community

	Highest Energy Trading Costs (c)		
Time Periods	$RCET_t$	$DCET_t$	$BCET_t$
12am-6am	1332.6	1187.9	1043.2
6am-12pm	2397.6	2126.2	1854.7
12pm-6pm	3795.5	3402.7	3009.9
6pm-12am	4417.0	3961.6	3506.2

pertaining to all agent's resources. In the community of 5 prosumers, 3 producers, and 20 consumers there are 31 uncertain parameters which include: 10 uncertain parameters of prosumer resources, 1 uncertain parameter of the wind resource, and 20 uncertain parameters of consumer's consumption resource. In other words, there are $N^M = 2^{14} = 16384, 2^{15} = 32768$, $2^{31} = 2,147,483,648$ different scenarios at each hour of the scheduling horizon in all three case studies. Accordingly, the $L_{16}(2^{15})$ OA based on the TOAT approach is used to decrease the number of scenarios from 32768 to 16 and generate appropriate best-case and worst-case scenarios for the opportunistic and robust models, respectively for the first and second case studies as shown in Tables 3.2 and 3.3. The two case studies use the same orthogonal array except the $15th$ uncertain parameter is removed for the first case study to generate scenarios for 14 uncertain parameters. The $L_{32}(2^{31})$ OA is used to reduce the number of scenarios from $2,147,483,648$ to 32 to be able to generate the necessary scenarios for both proposed models as shown in Table 3.4

3.3.3 Discussion

According to the generation and demand profiles presented in Figures 3.2 and 3.3, from 12am to 6am, there are low production and consumption in the community. Between 6am-12pm, the load demands of the prosumers increase steadily and then decrease, and that of the producers increases, with an increase in the PV productions of prosumers and producers. Between 12pm-6pm, the load demands of producers further increase in figure 3.3 and that of prosumers remains stable in figure 3.2, with a reduction in the PV productions of producers and prosumers. Finally, from 6pm to 12am, the load demands of prosumers decrease with the minimum PV productions of prosumers. The energy trading costs of the deterministic, opportunistic, and robust models during different hours of the scheduling horizon are depicted in Figures 3.8 and 3.9 for the first and second case studies respectively. The energy trading costs of 16 scenarios pertaining to the robust model are also indicated by CET_w in Figures 3.8 and 3.9. According to the third case study in figure 3.4 from 12am to 6am, there is low production and consumption in prosumers' PV and consumption resources, producer PV resources and consumer's consumption resources while there is an increase in the producer's wind and conventional production. Between 6am-6pm, there is an increase in prosumers and consumers' consumption resources, and a decrease in the producers' production resources, and between 12pm-6pm there is a reduction in PV generation of prosumers. Between 6pm-6am there is a steady increase and subsequent decrease of prosumers and consumers consumption, and an increase in the producers' production and flattened PV generation of prosumers.

The total energy imported/exported from the community and the grid are shown in Figures 3.11, 3.12, and 3.13. In the first and second cases with mainly renewable generation as shown in Figures 3.11 and 3.12 respectively, between periods 12am to 6am and 6pm to 12am all the generation was imported from the grid and none from the community. Between 6am to 6pm more was imported from the grid in the second case with no exports as compared to the first case with minimal export. In the third case study, between 12am-6am more energy is imported/exported from the community as compared to the grid, and between 6am to 9pm more energy is imported from the grid as compared to the community. Between 9pm to 12am more energy is imported/exported from the community as compared to the grid. Additionally, the highest energy trading costs of these models at different hours are illustrated in Tables 3.5,

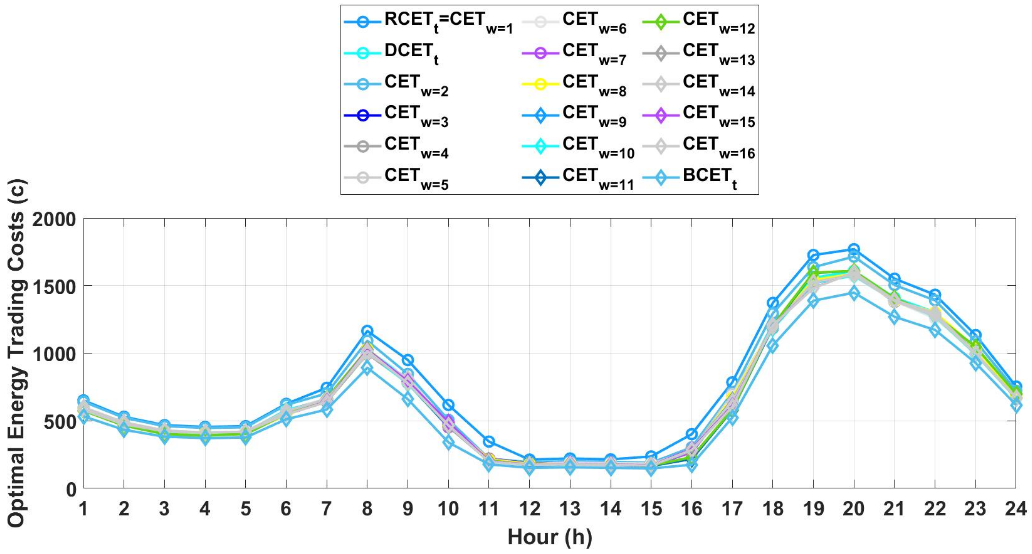


Figure 3.8: Total energy trading costs in the first community.

3.6, and 3.7.

According to Tables 3.5, 3.6 and 3.7, from 12am to 6am, the energy trading costs are the lowest in all models. As production and load consumption gradually increase from 6am to 12pm, the energy trading costs also increase. From 12pm to 6pm, as the PV, wind, and conventional production, decline and the load consumption increases, the energy trading costs remain high. Finally, from 6pm to 12am, although there is a reduction in the load demands of agents, the energy trading costs increase further due to the minimum PV productions, increased wind and conventional generation of agents during the entire scheduling horizon leading to more energy imports.

The highest total cost belongs to the energy trading model with the worst-case (robust) TOAT scenarios while the lowest total cost belongs to the energy trading model with the best-case (opportunistic) TOAT scenarios during a 24-hour scheduling period as depicted in Table 3.5, 3.6 and 3.7. Additionally, the total cost of the deterministic model is lower/higher than that of the worst-case/best-case model. Accordingly, as compared to the deterministic model, the opportunistic model is risk-seeker while the robust models are risk-averse. However, the conservatism of the robust model is higher than the opportunistic, and deterministic models. Consequently, its total costs and the cost of its robustness are higher than that of other models, including the deterministic model, as demonstrated in Figures 3.14, 3.15 and 3.16

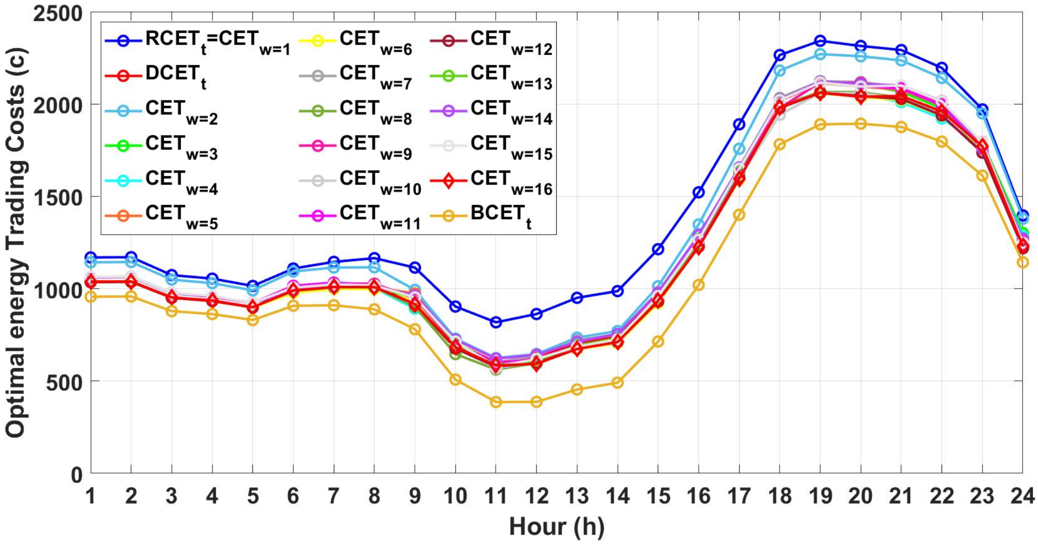


Figure 3.9: Total energy trading costs in the second community.

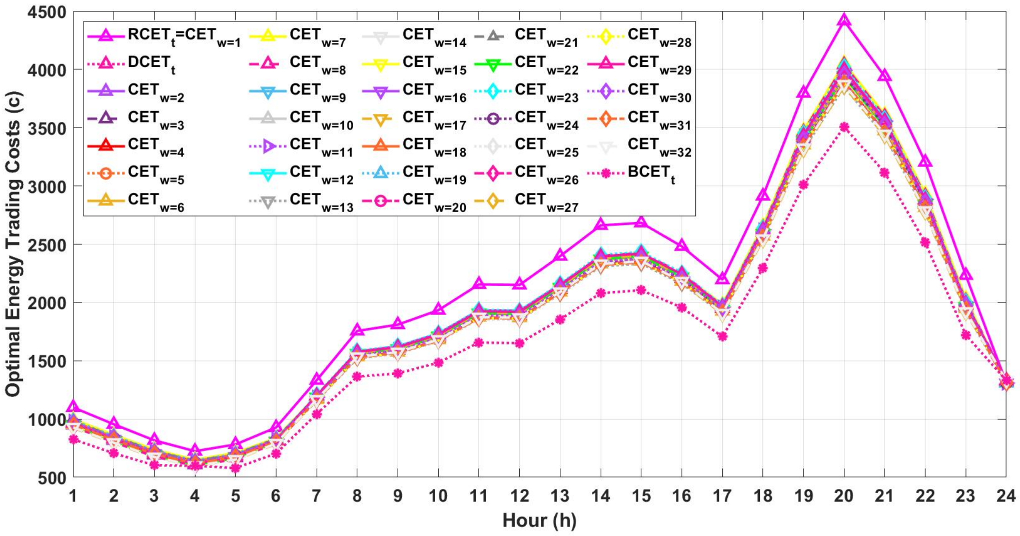


Figure 3.10: Total energy trading costs in the third community.

3.4 Summary

Given the recent trends in current electricity market structures, the evolving consumer-centric market requires more scalability in its integration with the existing markets. Since uncertain parameters can significantly affect consumer-centric energy trades, it is vital to appropriately characterize different types of uncertain parameters in these types of electricity markets. The risk-seeker and risk-averse performances of the consumer-centric markets have been studied here through the TOAT approach generating appropriate best-case and worst-case scenarios. In

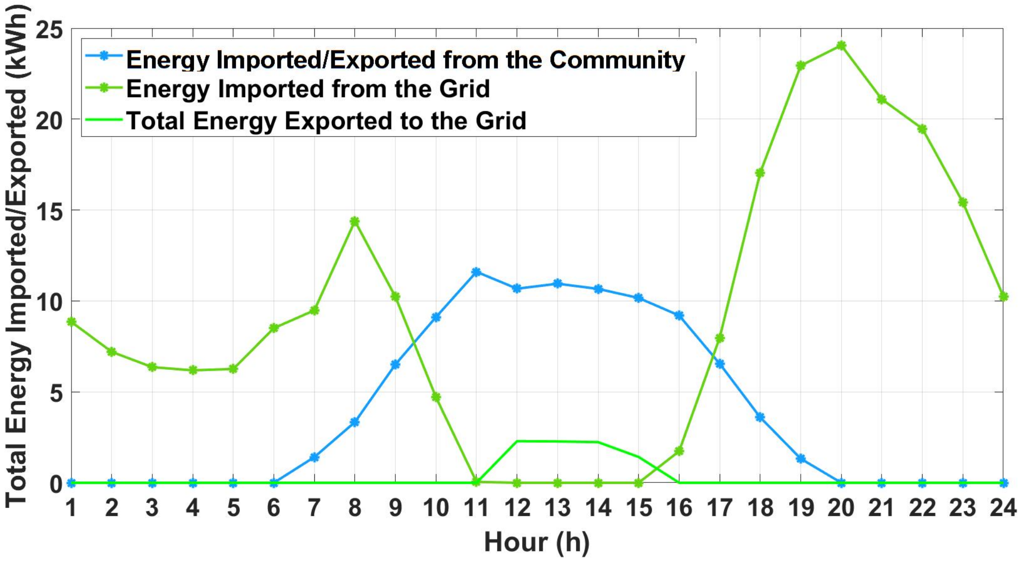


Figure 3.11: Total energy imported and exported in the first community.

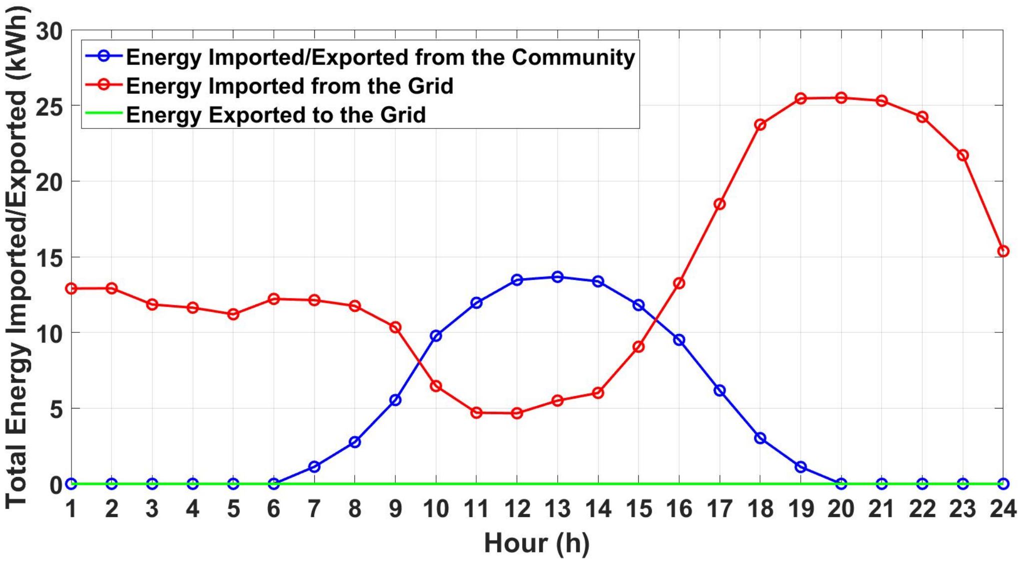


Figure 3.12: Total energy imported and exported in the second community.

addition, the total trading costs of these risk-seeker and risk-averse models have been compared with the deterministic ones. The case studies presented demonstrate that the risk-averse model has the highest robustness and total costs while the risk-seeker model has the lowest robustness and total costs. In future research works, the proposed approach can be extended and solved by proficient decomposition approaches to reduce its computation time for a large number of agent resources.

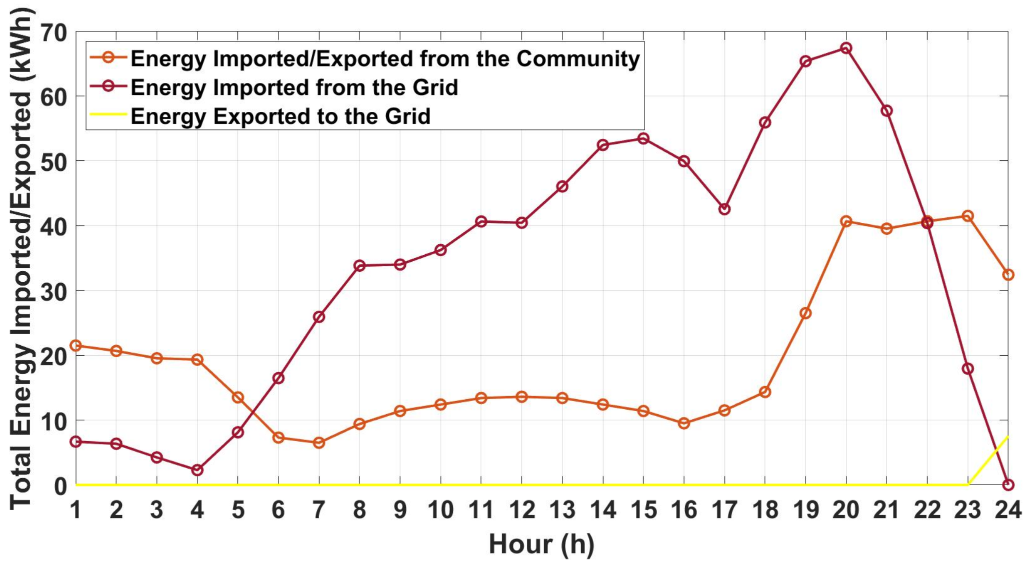


Figure 3.13: Total energy imported and exported in the third community.

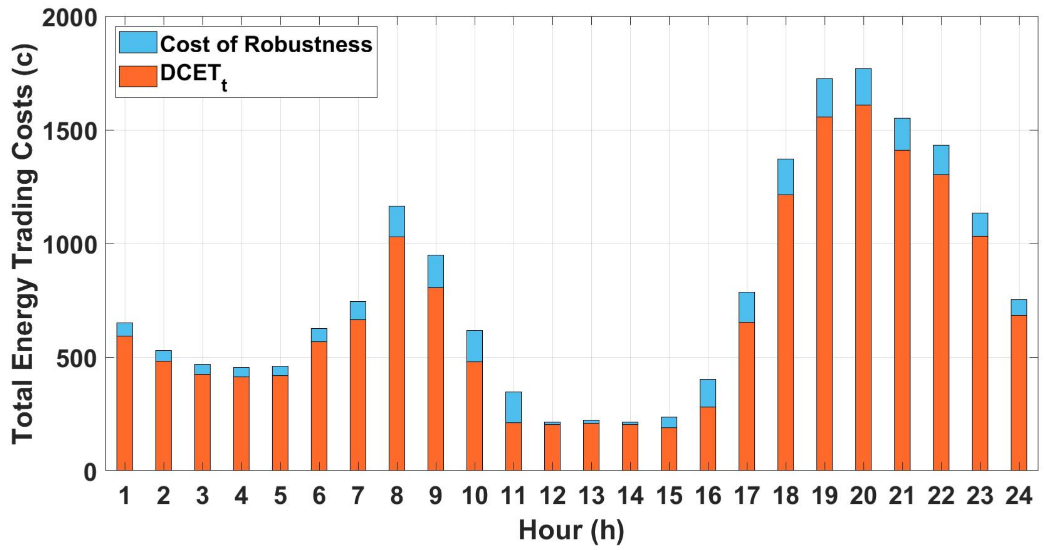


Figure 3.14: Cost of robustness in the first community.

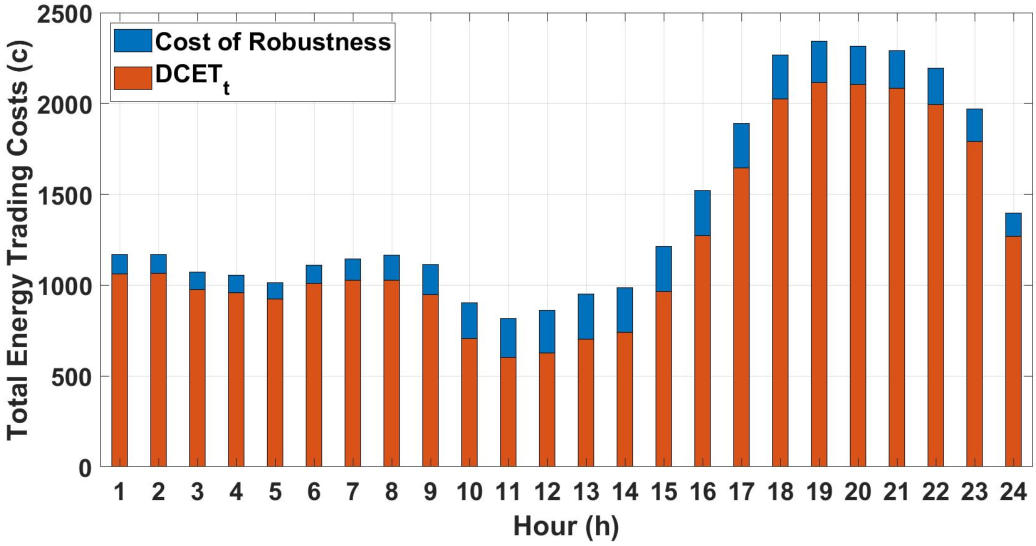


Figure 3.15: Cost of robustness in the second community.

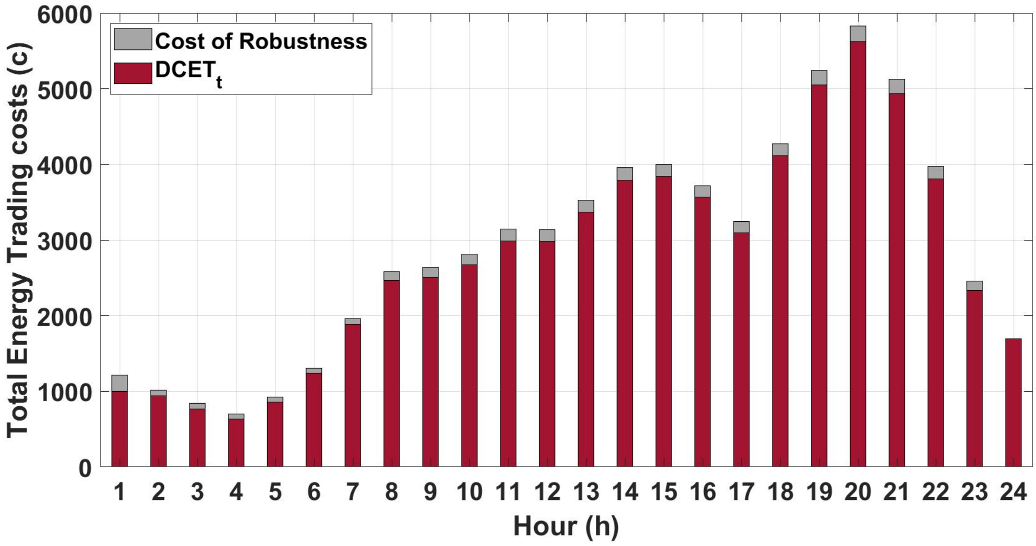


Figure 3.16: Cost of robustness in the third community.

Chapter 4

Community-based Market with Agent Utility Maximization

The increasing deployment of Distributed Energy Resources (DERs) by consumers (such as solar/wind generation, battery storage, etc.) has given rise to the concept of *prosumers*. To increase their flexibility and achieve their goals, prosumers and other agents (producers and consumers) can participate in local energy trading through Community-Based Electricity Markets (CBEMs) or even peer-to-peer (p2p) trading. Several market mechanisms have been proposed in the literature and a review focused on the description and technical aspects of the methods can be found in [2, 104]. In these papers, references were made with regards to heterogeneous preferences in the prosumers model which were stated as extra penalization on the imports/exports from outside the community that may reflect the will of prosumers to have more autonomy from the market and system operator

In reality, prosumers might have different preferences in terms of the goods traded. The consumer theory can be used to model the behaviour of individual consumers, showing their willingness to purchase certain goods of their choice. This behaviour is seen in the manner they measure their utility as a level of satisfaction derived from purchasing a number of commodities subject to their constrained budgets. The individual preferences produce indifference curves peculiar to each consumer and a budget line based on their budget constraint which is comprised of utility functions they want to maximize. Concerning consumer preferences in selecting the energy mix for their varied consumption patterns, they can also decide the amount of satisfaction to be derived from these mixes based on underlying economic models of consumer theory. These preferences can arise from the need to minimize costs subject to budgets, consumption/production quantities, storage units, etc, prioritizing renewable generation through dependence on local generation, weather variations giving rise to hybrid renewable schemes, costs of production units, and energy prices with cases involving energy trading. The factors that affect the consumers' differing utility functions are mostly price-based as the demand for a particular good decreases at increased costs of the product, hence utility reduces while utility increases at lower prices of products and hence an increase in demand [116, 117]. Other factors include their budgets, personal preferences, the influence of other consumers, availability of goods, etc. The preferences are usually expressed through their utility functions [22]. For example, in the specific case of CBEMs, the preference might be between the use of locally generated electricity (often renewables) against energy imported from the grid, between energy and flexibility, or between energy and resilience. Common utility functions are the perfect substitutes, perfect complements, and Cobb-Douglas utility. In perfect substitutes utility, one traded commodity might be substituted by another. The value associated with trading the first commodity may also be obtained by trading another as long as a certain total value is achieved. In the perfect complements utility, the value associated with trading one commodity does not increase without a certain amount of the other commodity. In other words, the value for one commodity is derived by *complementing* it with the other commodity.

If a consumer has a preference for a particular energy source, a higher attribute is attached to the utility function of the purchase of RES. The preference of the consumers for a specific energy source is affected by the availability of that resource. This is particularly of interest when the preference concerns Renewable Energy Sources (RESs). Thus, a trade-off is introduced between the reliability of supply and utility maximization based on consumer choices [116]. To alleviate this problem, Energy Storage Systems (ESSs) can be incorporated to enhance flexibility and the ability to satisfy the constraints in an inter-temporal manner. ESSs can offer both profit maximization capabilities and the support of ancillary services [118,119], thus maintaining the reliability in supply and market profits [120]. In CBEM, this is achieved by prosumers storing energy in off-peak periods and selling the surplus to their neighbours in peak periods [1, 121]. In [122], the economic and environmental consequences of ESS are assessed through a complementarity model of a Western European power system with market power, representation of the transmission grid, and uncertainty in RESs output.

Finally, there are inherent socio-economic factors that may affect consumer choices in achieving their desired level of satisfaction for their preferences. These factors include low probability, high impact events such as natural disasters and outages [123, 124], demographics, education, employment, and housing expenditures [125]. In [126], a sensitivity analysis on economic metrics was carried out to determine consumer welfare against certain statistical measurement standards.

There are limited studies on the modelling of consumer behaviours in local CBEMs using utility functions from the exchange economy. Reference [127] proposes a custom quadratic utility function to model the different behaviors of appliances within a stochastic load scheduling problem. Introducing intertemporal decisions is unavoidable when battery storage or flexible loads are introduced. In [1], two utility functions were introduced to define the preference between consuming to meet their current energy requirements and/or storing energy for future consumption or spending energy stored in the past.

In this Chapter, the deterministic, centralized, CBEM model proposed in [128] is extended to incorporate the preferences of consumers and producers concerning trading within the community against trading with the bulk grid. Two separate utility functions are used, maximized within the CBEM problem. The Cobb-Douglas is utilised for the consumers, while the perfect complement and perfect substitute are employed for the producers. A deterministic, centralized, CBEM optimization model is proposed that minimises the community cost while maximising the individual utility functions. A Community Energy Storage (CES) is used to provide flexibility and balance between imports/exports within the community and the grid.

The main contributions of this chapter are outlined as follows:

- An economic model based on consumer theory that maximises the utility of agents in a community-based electricity market is proposed as a function of their imports and exports from the DERs within the community and the conventional generation outside the community/grid.

- The role of the community energy storage system in meeting the agent's desired preferences is analysed.

- The contribution of community-owned conventional generation resources in supplying the agents' consumption demands at periods with low or no renewable generation is analysed.

The rest of this chapter is organized as follows: The basic utility functions used in this work and their solution is analyzed in Section 4.1. In Section 4.2, the proposed CBEM problem is introduced. In Section 4.3, the contribution of community-owned generation resources is analysed in the problem formulation. In Section 4.5, the test case studies are presented. Finally, Section 4.6 concludes the chapter.

4.1 Utility Functions in Consumer Theory

The rational choices of consumers are initiated based on their perceived preferences between two (or more) commodities [22]. In our case, we assume an energy consumption/production space for community agents to formulate appropriate utility functions that reflect [1] their preferences which relate to the preferences of community agents in trading within and outside the energy community (both buying or selling energy). Thus, it becomes necessary to formulate the utility functions for agents that align their preferences with their satisfaction levels. Some common utility functions used in consumer theory as shown in Figure 4.1 for two commodity model are given as follow:

$$u(x_1, x_2) = x_1 + x_2 \text{ (\textit{Perfect Substitutes})} \tag{4.1}$$

$$u(x_1, x_2) = \min(x_1, x_2) \text{ (\textit{Perfect Compliments})} \tag{4.2}$$

$$u(x_1, x_2) = x_1^a x_2^b \text{ (\textit{Cobb-Douglas})} \tag{4.3}$$

The utility $u(x_1, x_2)$ as a function of the amounts of commodities is represented as isoquants or contours with each commodity laying on each axis. These utility functions may serve as objective functions for any microgrid agents in a given optimization problem. [1] and depending on the motivation for optimization agents can select appropriate utility functions. Given an agent's objective is to minimize its overall consumption irrespective of the source of production/consumption then the perfect substitute models the preferences of the agent appropriately. Reference [24] illustrates the perfect substitute utility function by referring to a minigrid with two sources of energy production solar PV and wind, to maximise the production from the two sources not having any preferences for the amount of production from either of them. If the

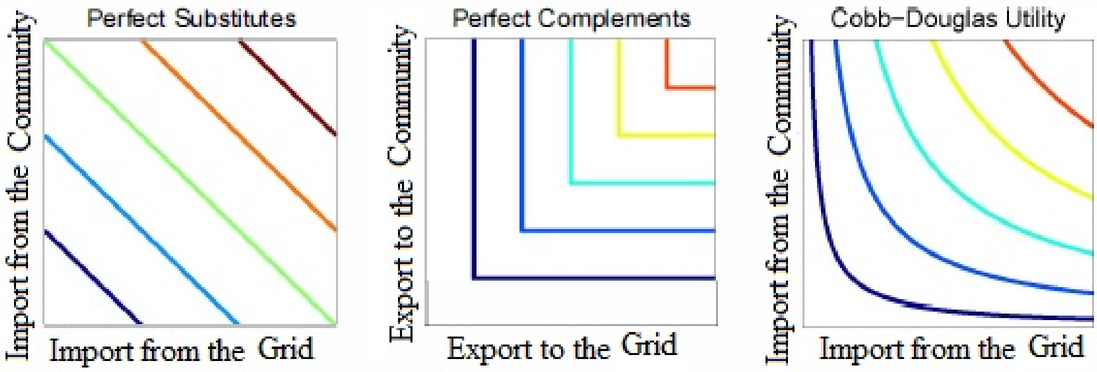


Figure 4.1: Common Utility functions: perfect substitutes, perfect complements, and Cobb-Douglas function. [1]

minigrid values one type of energy source with a certain minimum constraint on the other source, then the perfect complement is appropriate to model the preferences. However if the minigrid values a certain share of solar energy as compared to wind energy, then the Cobb-Douglas utility function becomes useful in modelling these preferences as it has certain desirable qualities such as monotonicity and continuity ensuring equilibrium of the market.

In this section, we introduce a compact form of an economic utility maximization problem and apply the same concept to the Cobb-Douglas utility function and the perfect complements.

An economic utility maximization problem is expressed as:

$$\begin{aligned} \max_{x} \quad & U(x_1, \ldots, x_k) \\ s.t \quad & c_1 x_1 + \ldots + c_n x_k \leq I \\ & x_1 \geq 0, \ldots, x_k \geq 0 \end{aligned} \tag{4.4}$$

where k represents a number of commodities in an economy, x_i denotes the amount of commodity i and $U(x_1, \ldots, x_k)$ represents the consumer's utility function which is a measure of the consumer's satisfaction with commodities $x = \{x_1, \ldots, x_k\}$. c_i is the price of commodity i and $I \geq 0$ represents the income the consumer budgets to spend on the k commodities.

In the case of the Cobb-Douglas utility function [21], the standard form is formulated as a production function. The general form for two commodities x_1, x_2 is thus expressed:

$$\begin{aligned} U(x_1, x_2) &= x_1^a x_2^b \\ s.t \;\; c_1x_1 + c_2x_2 &= I \end{aligned} \tag{4.5}$$

where $a > 0$ and $b > 0$ are positive parameters with $a + b = 1$ and reflect the preference of the consumer to each of the two commodities. It can be shown (see Chapter 22 in [21]), that the maximized form of the problem needs to satisfy:

$$\frac{ax_1^{a-1}x_2^b}{bx_1^a x_2^{b-1}} = \frac{c_1}{c_2} \tag{4.6}$$

From (4.6), c_2x_2 is expressed in terms of c_1x_1 in the budget constraint of (4.5) and c_1x_1 is expressed in terms of c_2x_2 in the same budget constraint equation to give:

$$c_1x_1 + \frac{bc_1x_1}{a} = I, \quad \frac{ac_2x_2}{b} + c_2x_2 = I \tag{4.7}$$

which gives the solution of the problem for each commodity x_1, x_2:

$$x_1 = \frac{aI}{c_1}, \quad x_2 = \frac{bI}{c_2}, \quad \lambda = \frac{a^a b^b}{c_1^a c_2^b} \tag{4.8}$$

if $c_1 = c_2 = 1$, it implies $x_1 = aI$ and $x_2 = bI$. The multiplier λ gives the sensitivity of the optimal utility to changes in the budget constraint I.

Similarly, the perfect complements [22] measures the maximum utility obtainable in complementary goods. Preferences are perfect complements when represented by:

$$U(x_1, x_2) = \min_{x_1, x_2}\{ax_1, bx_2\} \tag{4.9}$$

Given that x_1 and x_2 are perfect complements with fixed proportions (e.g., according to their price), then $ax_1 = bx_2$. Substituting in the budget constraint (4.5) gives:

$$x_1 = \frac{bI}{bc_1 + ac_2}, \quad x_2 = \frac{aI}{bc_1 + ac_2} \tag{4.10}$$

If $c_1 = c_2 = 1$, it implies $x_1 + x_2 = I$.

4.2 Proposed Community-based Electricity Market Formulation

Consider a community of agents equipped with renewable energy resources and are engaged in energy trading with their neighbours. The energy trades are supervised by a central agent known as the community manager. The community manager sets a transaction cost for trading amongst the agents within the community. The excess/deficient energy that is not sold/purchased within the community is exported/imported from outside the community (the grid) at the costs set by the grid. In the general formulation of the CBEM [2], the optimal energy costs are obtained by minimizing the total traded costs within/outside the community.

In the proposed CBEM, agents attribute a level of satisfaction to the amount of energy traded within and outside the community based on the resources. The optimal energy traded cost is obtained when the agents' utility levels are maximized with respect to their consumption/generation budgets. The agents define their preferences for the consumption from renewable energy sources within the community to the conventional energy source from the grid. The prosumers (consumption) and consumers maximize their utility using the Cobb-Douglas utility maximization in (4.5) where x_1 becomes the energy imported from the DERs within the community, x_2 is the energy imported from the conventional grid resource, a and b are positive attributes measuring their desired consumption preferences of prosumer and consumer agents and I is their consumption budget constraint.

For the prosumers (production) and producers, the energy is shared between the exports within and outside the community treating the two as complementary "goods". This is modeled using the *perfect complements* utility function in (4.9) where x_1 becomes the energy exported from the DERs within the community, x_2 is the energy export from to conventional grid source, a and b are positive coefficients of x_1 and x_2 respectively that keep them in fixed proportions equal to a generation budget I. The total traded costs of agents are then minimized at the community level including the prosumer, consumer, and producer preferences as constraints. Figures 4.2 show an illustration of the proposed community-based market with agent's utility functions.

This work proposes a heterogenous preference scheme for community agents which reflects agents' desire to trade energy within the community rather than outside the community. With regards to an economic perspective, this projects a better representation of agents' preferences

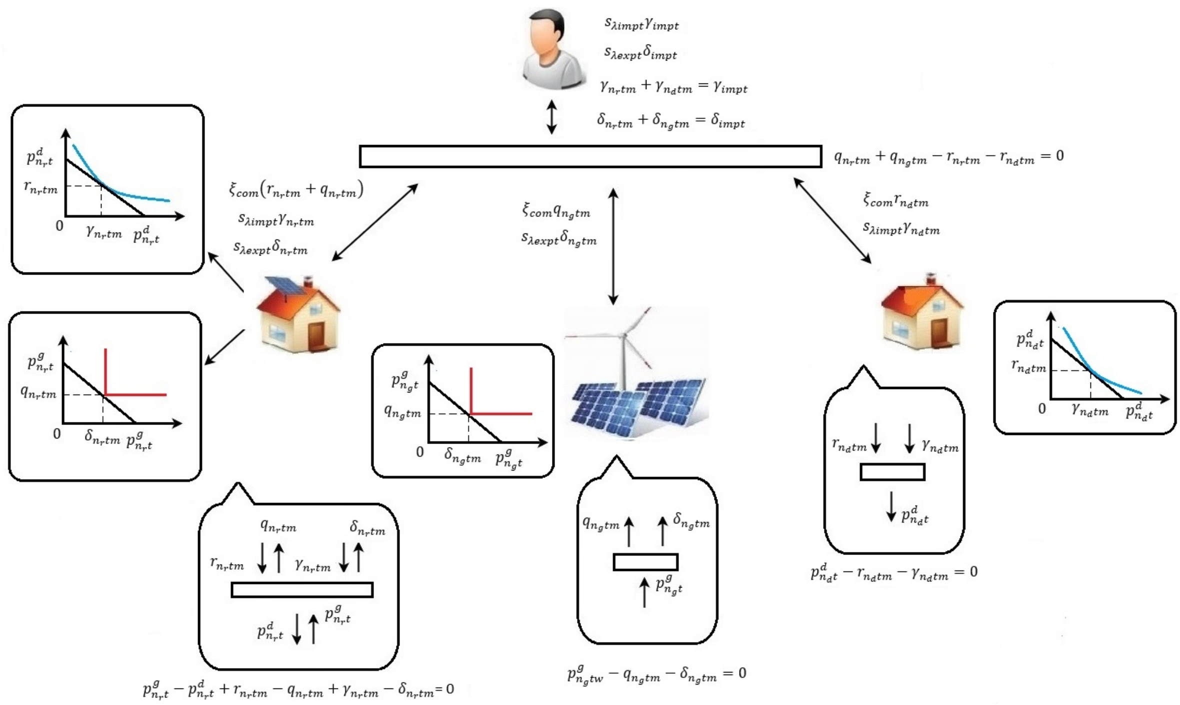


Figure 4.2: Proposed community based market structure.

which has evolved from when agents are limited to substitute choices rather than preferential choices. This approach promises a better comparison of output levels and a rethinking of their preference relations to reflect more autonomy. The Cobb Douglas and Perfect's complements utility function utilised to reflect agents' preferences are turned into linear problems by integrating them as constraints in the community-based market problem. A CES is integrated to balance the surplus/deficient energy from the agent's preferences and allow for inter-temporal trading. Figures 4.3, 4.4, and 4.5 illustrate the schematic representation of consumer, producer, and prosumer agent preferences as indifferent curves tangential to their consumption/generation budgets. It shows that if there is no generation within the community then all energy imports will come from the grid as there will be no consumption within the community, and consequently no exports out of the community. Whereas at the point where the indifference curves of utility functions of agents meet their budget constraints, there exists an optimal consumption/production from both sources by the agents.

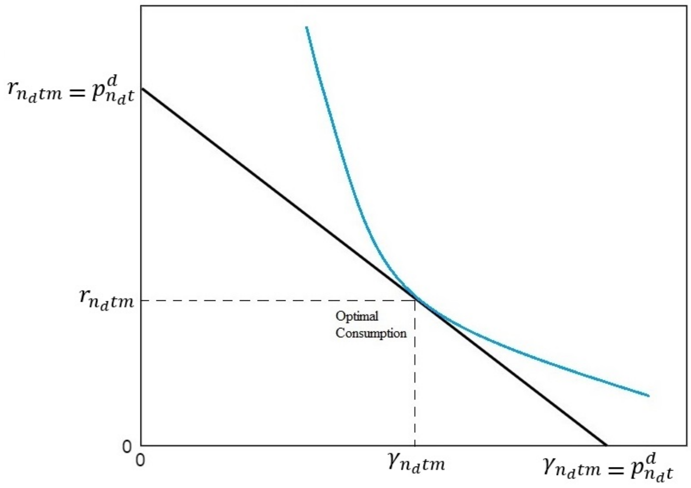


Figure 4.3: Schematic representation of consumption preferences of consumer agents.

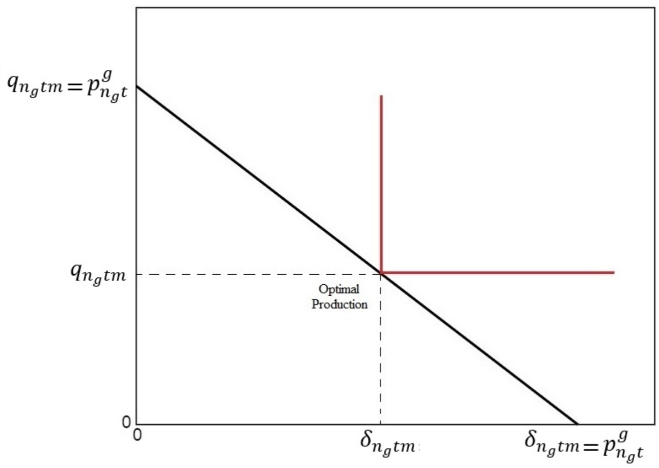
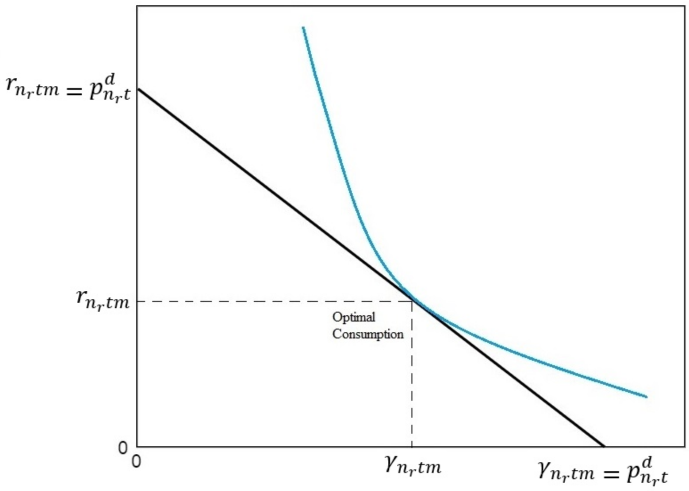


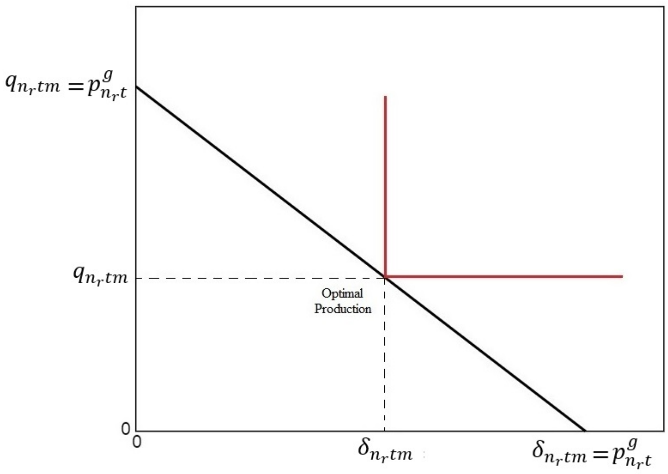
Figure 4.4: Schematic representation of production preferences of producer agents.

4.2.1 Agent's Utility Maximization Problem

An agent who is a consumer, a producer, or a prosumer models its preferences with the given economic utility function suited to their needs, attaching a higher attribute to the energy mix desired for more import/export respectively. The agents before the initialization of each energy trade, can solve their utility maximization problems subject to their consumption and generation budget constraints. The prosumer's utility maximization problem considers the production and consumption capabilities of the prosumer and models the preferences as separate utility functions while that of consumers and producers reflect their consumption and production utility functions



(a) Schematic representation of consumption preferences of prosumer agents.



(b) Schematic representation of production preferences of prosumer agents.

Figure 4.5: Schematic representation of consumption/production preferences of prosumer agents.

as presented in equations (4.11) and (4.16).

Consumer's Utility function

Given a consumer n_d values a certain share $\alpha_{n_d t}$ of consumption from within the community to consumption $\beta_{n_d t}$ from outside the community in period t the Cobb-Douglas function of (4.11a) is used to model the preferences. The maximum utility obtainable with consumption budget

$p^d_{n_dt}$ for each consumer n_d at hour t can be expressed using: (B.1)

$$\max_{r_{n_dtm},\gamma_{n_dtm}} \; r_{n_dtm}^{\alpha_{n_dt}} \gamma_{n_dtm}^{\beta_{n_dt}}, \;\; \forall n_d \in N_c, \forall t \in T \tag{4.11a}$$

$$r_{n_dtm} + \gamma_{n_dtm} = p^d_{n_dt}, \;\; \forall n_d \in N_c, \forall t \in T \tag{4.11b}$$

$$r_{n_dtm} \geq 0, \gamma_{n_dtm} \geq 0, \;\; \forall n_d \in N_c, \forall t \in T \tag{4.11c}$$

where $\alpha_{n_dt} + \beta_{n_dt} = 1 \; \forall n_d \in N_c, \forall t \in T$. (4.11a) maximizes the utility of consumers as a function of their imports from within the community and the grid, (4.11b) ensures that at each time t that the consumers budget is obeyed, and (4.11c) ensures the non-negativity of variables. The marginal utility of the consumer is defined as the partial derivative of the utility function with respect to each of his consumption preferences. It measures the rate at which the utility increases with respect to its respective preferences.

$$mu_{r_{n_dtm}} = \frac{\delta u(r_{n_dtm}, \gamma_{n_dtm})}{\delta_{r_{n_dtm}}} \tag{4.12}$$

$$mu_{\gamma_{n_dtm}} = \frac{\delta u(r_{n_dtm}, \gamma_{n_dtm})}{\delta_{\gamma_{n_dtm}}} \tag{4.13}$$

The marginal rate of substitution (MRS) of each of the consumer preferences for the other is defined as the rate at which r_{n_dtm} is traded for γ_{n_dtm}. The MRS is equal to the ratio of the marginal utilities of each preference type which is the same as the slope of the consumer's indifference curve.

$$MRS(r_{n_dtm}, \gamma_{n_dtm}) = \frac{\frac{\delta u(r_{n_dtm}, \gamma_{n_dtm})}{\delta_{r_{n_dtm}}}}{\frac{\delta u(r_{n_dtm}, \gamma_{n_dtm})}{\delta_{\gamma_{n_dtm}}}} \tag{4.14}$$

With MRS$(r_{n_dtm}, \gamma_{n_dtm}) = 1$, The consumer's maximum utility variables r_{n_dtm} and γ_{n_dtm} can be obtained directly by applying (4.8) (see Appendix B.1) as follows :

$$r_{n_dtm} = \alpha_{n_dt} p^d_{n_dt}; \quad \gamma_{n_dtm} = \beta_{n_dt} p^d_{n_dt} \tag{4.15}$$

and integrated directly into the CBEM problem.

Producer's Utility function

As with the maximum utility of the consumer, the producer may also value a certain share of his resource exported within the community than outside the community, in this case, if the energy exported within the community balances the energy imported, and there is a surplus, the excess energy can be stored for future use. The producer instead of a consumption budget; has a generation budget $p^g_{n_g t}$ and maximises profit based on this budget and their preferences. Likewise, the indirect utility is the minimum utility obtainable by producer n_g for energy exported within and outside the community which are proportional at each time t and can be expressed using perfect complements:

$$\min_{q_{n_g tm}, \delta_{n_g tm}} U(\rho_{nt} q_{n_g tm}, \sigma_{n_g t} \delta_{n_g tm}), \forall n_g \in N_p, \forall t \in T \tag{4.16a}$$

$$q_{n_g tm} + \delta_{n_g tm} = p^g_{n_g t}, \quad \forall n_g \in N_p, \forall t \in T \tag{4.16b}$$

$$\rho_{n_g t} q_{ntm} - \sigma_{n_g t} \delta_{n_g tm} = 0, \ \forall n_g \in N_p, \forall t \in T \tag{4.16c}$$

$$q_{n_g tm} \geq 0, \delta_{n_g tm} \geq 0, \quad \forall n_g \in N_p, \forall t \in T \tag{4.16d}$$

where (4.16a) maximizes the utility of producers as a minimized function of their exports to the community and to the grid, (4.16b) gives the energy balance of the producer, and (4.16c) ensures that $q_{n_g tm}$ and $\delta_{n_g tm}$ are in fixed proportions of the coefficients which reflects their preferences. Finally, (4.16d) ensures the non-negativity of variables. The producers maximum utility variables $q_{n_g tm}$ and $\delta_{n_g tm}$ can be obtained directly by applying (4.10) (see Appendix B.2) as follows :

$$q_{n_g tm} = \frac{\sigma_{n_g t} p^g_{n_g t}}{\sigma_{n_g t} + \rho_{n_g t}}; \delta_{n_g tm} = \frac{\rho_{n_g t} p^g_{n_g t}}{\sigma_{n_g t} + \rho_{n_g t}} \tag{4.17}$$

and integrated directly into the CBEM problem.

Prosumer's Utility function

The prosumer can engage in bi-directional energy trading within and outside the community. However, at each time t the prosumer is either producing or consuming energy [104]. The prosumer's utility functions are modeled separately for both consumption and production reflecting their preferences with respect to their consumption and generation budget constraints respectively. The prosumer's maximum utility is obtainable with consumption budget $p^d_{n_rt}$, and is formulated according to (4.11) by replacing the consumer agent n_d with the prosumer agent n_r. The prosumer's maximum utility from the import variables r_{n_rtm} and γ_{n_rtm} from consumption are obtained by applying (4.15)

$$r_{n_rtm} = \alpha_{n_rt} p^d_{n_rt}; \quad \gamma_{n_rtm} = \beta_{n_rt} p^d_{n_rt} \tag{4.18}$$

The prosumer's maximum utility is obtainable with production budget $p^g_{n_rt}$, and is formulated according to (4.16) by replacing the producer agent n_g with the prosumer agent n_r. The prosumer's maximum utility from the export variables q_{n_rtm} and δ_{n_rtm} from production are obtained by applying (4.17) as follows:

$$q_{n_rtm} = \frac{\sigma_{n_rt} p^g_{n_rt}}{\sigma_{n_rt} + \rho_{n_rt}}; \delta_{n_rtm} = \frac{\rho_{n_rt} p^g_{n_rt}}{\sigma_{n_rt} + \rho_{n_rt}} \tag{4.19}$$

4.2.2 CBEM with Utility Maximization

The CBEM problem subject to constraints which include the agent preference relations that maximises their utility is introduced in this section. These constraints ensure that the maximum utility derived from the preferences of agents is satisfied. In the event the generation budget of the producers/prosumers (producing) is not sufficient to meet the consumer/prosumer (consuming) agents preferred consumption, the unserved energy R_{n_dt}/G_{n_rt} of consumers/prosumers (consuming) after utility maximization is imported from the grid.

A CES is introduced in the formulation to provide a balance from possible energy mismatch due to the preferences of agents. This proposed CBEM market model will be solved when the total energy traded costs of the agents, and the total costs of the community manager in importing/exporting energy within/outside the community is minimized. The proposed community-based market model for prosumers integrating a CES and the constraint equations of (4.18) and (4.19) is presented as follows:

$$\min\ \alpha(p^g_{n_rt}, p^d_{n_rt}, r^c_{n_rtm}, q_{n_rtm}, \gamma_{n_rtm}, G_{n_rt}, \delta_{n_rtm}, \gamma_{impt}, \delta_{expt}) \tag{4.20a}$$

$$\begin{aligned}\text{s.t. } \alpha \geq \sum_{j=1}^{R_g}\sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}} \varphi_{jn_r}(p^g_{jn_rt}) + \sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}} \phi_{n_r}(p^d_{n_rt}) + \\ \xi_{com}\sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}} r^c_{n_rtm} + \xi_{com}\sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}} q_{n_rtm} + \\ s_{\lambda impt}\sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}}(\gamma_{n_rtm} + G_{n_rt}) + \\ s_{\lambda expt}\sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}} \delta_{n_rtm} + \sum_{t=1}^{T} g(\gamma_{impt}, \delta_{expt}),\end{aligned} \tag{4.20b}$$

$$p^g_{n_rt} - q_{n_rtm} - \delta_{n_rtm} = 0,\ \forall n_r \in N_{pr}, \forall t \in T \tag{4.20c}$$

$$p^d_{n_rt} - r_{n_rtm} - \gamma_{n_rtm} = 0,\ \forall n_r \in N_{pr}, \forall t \in T \tag{4.20d}$$

$$r_{n_rtm} - r^c_{n_rtm} - G_{n_rt} = 0,\ \forall n_r \in N_{pr}, \forall t \in T \tag{4.20e}$$

$$p_{n_rt} - p^g_{n_rt} + p^d_{n_rt} = 0,\ \forall n_r \in N_{pr}, \forall t \in T \tag{4.20f}$$

$$p_{n_rt} + r_{n_rtm} - q_{n_rtm} + \gamma_{n_rtm} - \delta_{n_rtm} = 0,\ \forall n_r \in N_{pr}, \forall t \in T \tag{4.20g}$$

$$\sum_{j=1}^{R_g} p^g_{jn_rt} = p^g_{n_rt},\ \forall n_r \in N_{pr}, \forall t \in T \tag{4.20h}$$

$$\sum_{n=1}^{N_{pr}} q_{n_rtm} - \sum_{n=1}^{N_{pr}} r^c_{n_rtm} + p_{dt} - p_{ct} = 0,\quad \forall t \in T \tag{4.20i}$$

$$e_t = e_{t-1} - 1/n^d p_{dt} + n^c p_{ct},\quad \forall t \in T \tag{4.20j}$$

$$\sum_{t=1}^{T}(n^c p_{ct} - 1/n^d p_{dt}) = 0, \tag{4.20k}$$

$$\sum_{n_r=1}^{N_{pr}}(\gamma_{n_rtm} + G_{n_rt}) = \gamma_{impt},\quad \forall t \in T \tag{4.20l}$$

$$\sum_{n_r=1}^{N_{pr}} \delta_{n_rtm} = \delta_{expt},\quad \forall t \in T \tag{4.20m}$$

$$\underline{p}^g_{jn_rt} \leq p^g_{jn_rt} \leq \overline{p}^g_{jn_rt},\quad \forall n_r \in N_{pr}, \forall t \in T \tag{4.20n}$$

$$\underline{p}^d_{n_rt} \leq p^d_{n_rt} \leq \overline{p}^d_{n_rt},\quad \forall n_r \in N_{pr}, \forall t \in T \tag{4.20o}$$

$$e^{min} \leq e_t \leq e^{max},\quad \forall t \in T \tag{4.20p}$$

$$0 \leq P_{ct} \leq P^{max}_c z_{tc},\quad \forall t \in T \tag{4.20q}$$

$$0 \leq P_{dt} \leq P^{max}_d z_{td},\quad \forall t \in T \tag{4.20r}$$

$$z_{tc} + z_{td} \leq 1,\quad \forall t \in T \tag{4.20s}$$

$$\gamma_{n_rtm} \geq 0,\ \delta_{n_rtm} \geq 0, G_{n_rt} \geq 0\ \ \forall n_r \in N_{pr} \forall t \in T \tag{4.20t}$$

$$(4.18)\ \&\ (4.19) \tag{4.20u}$$

where $X_{n_r t} = \left\{p^g_{n_r t}, q_{n_r tm}, \delta_{n_r tm}\right\}$, $Y_{n_r t} = \left\{p^d_{n_r t}, r_{n_r tm}, \gamma_{n_r tm}\right\}$, $Z_t = \{\gamma_{impt}, \delta_{expt}\}$, and the total costs of the community manager in importing/exporting electricity inside/outside the community is $g(Z_t)$. Constraint (4.20b) minimizes the total traded energy costs of prosumers, the total transaction costs, and the total costs of the community manager in importing/exporting energy inside/outside the community following the utility maximization of prosumers' production and consumption. (4.20c) and (4.20d) represent the energy balance for each prosumer's production and consumption respectively, (4.20e) shows the energy balance of each prosumer consumption within the community allowing for a "penalised" deviation $G_{n_r t}$, (4.20f) sums the production and consumption of each prosumer, (4.20g) represents the power balance from each prosumers' production and consumption combined, and (4.20h) shows the combined energy balance of each prosumer's production and consumption. In (4.20i) the community power balance includes the charged/discharged energy of the CES.

Constraint (4.20j) gives the storage level at hour t based on past charging/discharging while (4.20k) ensures the same stored energy at the beginning and end of the day. (4.20l)-(4.20m) gives the relationship between the total imported/exported energy and the community manager imports/exports. (4.20n)-(4.20o) give the boundary constraint for the power set point for prosumers' production and consumption resources, respectively. (4.20p) defines the limits of the stored energy in the CES. (4.20q)-(4.20r) give the limits of charging and discharging rates with binary variable z_{tc}, z_{td} showing that charging and discharging actions occur independently according to (4.20s). (4.20t) ensures the non-negativity of variables which indicates the imports and exports of agents. Finally, (4.20u) integrates the solution of each prosumer's utility maximization of production and consumption resources.

Likewise, the proposed community-based market model for producers and consumers integrating a CES and the constraint equations of (4.15) and (4.17) is presented as follows:

$$\min \ \alpha(p^g_{ln_gt}, p^d_{n_dt}, r^c_{n_dtm}, q_{n_gtm}, \gamma_{n_dtm}, R_{n_dt}, \delta_{n_gtm}, \gamma_{impt}, \delta_{expt}) \tag{4.21a}$$

$$\begin{aligned} \text{s.t. } \alpha \geq \sum_{l=1}^{R_p}\sum_{t=1}^{T}\sum_{n_g=1}^{N_p} \varphi_{ln_g}(p^g_{ln_gt}) + \sum_{t=1}^{T}\sum_{n_d=1}^{N_c} \phi_{n_d}(p^d_{n_dt}) + \\ \xi_{com}\sum_{t=1}^{T}\sum_{n_d=1}^{N_c} r^c_{n_dtm} + \xi_{com}\sum_{t=1}^{T}\sum_{n_g=1}^{N_p} q_{n_gtm} + \\ s_{\lambda impt}\sum_{t=1}^{T}\sum_{n_d=1}^{N_c}(\gamma_{n_dtm} + R_{n_dt}) + \\ s_{\lambda expt}\sum_{t=1}^{T}\sum_{n_g=1}^{N_p}(\delta_{n_gtm} + \sum_{t=1}^{T} g(\gamma_{imptw}, \delta_{exptw}), \end{aligned} \tag{4.21b}$$

$$p^g_{n_gt} - q_{n_gtm} - \delta_{n_gtm} = 0, \ \forall n_g \in N_p, \forall t \in T \tag{4.21c}$$

$$p^d_{n_dt} - r_{n_dtm} - \gamma_{n_dtm} = 0, \ \forall n_d \in N_c, \forall t \in T \tag{4.21d}$$

$$r_{n_dtm} - r^c_{n_dtm} - R_{n_dt} = 0, \ \forall n_d \in N_c, \forall t \in T \tag{4.21e}$$

$$\sum_{l=1}^{R_p} p^g_{ln_gt} = p^g_{n_gt}, \ \forall n_g \in N_p, \forall t \in T \tag{4.21f}$$

$$\sum_{n_g=1}^{N_p} q_{n_gtm} - \sum_{n_d=1}^{N_c} r^c_{n_dtm} + p_{dt} - p_{ct} = 0, \quad \forall t \in T \tag{4.21g}$$

$$(4.20j) \ \& \ (4.20k) \tag{4.21h}$$

$$\sum_{n_d=1}^{N_c}(\gamma_{n_dtm} + R_{n_dt}) = \gamma_{impt}, \quad \forall t \in T \tag{4.21i}$$

$$\sum_{n=g}^{N_p} \delta_{n_gtm} = \delta_{expt}, \quad \forall t \in T \tag{4.21j}$$

$$\underline{p}^g_{ln_rt} \leq p^g_{ln_rt} \leq \overline{p}^g_{ln_rt}, \quad \forall n_g \in N_p, \forall t \in T \tag{4.21k}$$

$$\underline{p}^d_{n_dt} \leq p^d_{n_dt} \leq \overline{p}^d_{n_dt}, \quad \forall n_d \in N_c, \forall t \in T \tag{4.21l}$$

$$(4.20p) - (4.20s) \tag{4.21m}$$

$$\gamma_{n_dtm} \geq 0, \ \delta_{n_gtm} \geq 0, R_{n_dt} \geq 0 \ \ \forall n_g \in N_p, \forall n_d \in N_c, \forall t \in T \tag{4.21n}$$

$$(4.15) \ \& \ (4.17) \tag{4.21o}$$

where the objective equation (4.21a) aims to minimise the total energy traded costs of the producer and consumer agents, total transaction costs, and the total costs of the community manager in importing and exporting energy in and out of the community following the utility maximization of agent's production and consumption. Constraints (4.21c)-(4.21e),(4.21f),(4.21g), (4.21i)-(4.21l), and (4.21n) serve the same functions as (4.20c)-(4.20e), (4.20h),(4.20i), (4.20l)-(4.20m), and (4.20t) respectively.

Finally, the proposed community-based market model for all community agents prosumers, producers, and consumers integrating a CES and the constraint equations of (4.15), (4.17), (4.18), and (4.19) is presented as follows:

$$\min\ \alpha(p^g_{n_rt}, p^d_{n_rt}, r^c_{n_rtm}, q_{n_rtm}, \gamma_{n_rtm}, G_{n_rt}, \delta_{n_rtm}, p^g_{ln_gt}, p^d_{n_dt}, r^c_{n_dtm}, q_{n_gtm}, \gamma_{n_dtm}, R_{n_dt}, \delta_{n_gtm}) \tag{4.22a}$$

$$\text{s.t. } \alpha \geq \sum_{j=1}^{R_g}\sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}} \varphi_{jn_r}(p^g_{jn_rt}) + \sum_{l=1}^{R_p}\sum_{t=1}^{T}\sum_{n_g=1}^{N_p} \varphi_{ln_g}(p^g_{ln_gt}) + \sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}} \phi_{n_r}(p^d_{n_rt}) +$$
$$\sum_{t=1}^{T}\sum_{n_d=1}^{N_c} \phi_{n_d}(p^d_{n_dt}) + \xi_{com}\sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}}\sum_{n_d=1}^{N_c}(r_{n_rtm} + r_{n_dtm}) + \xi_{com}\sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}}\sum_{n_g=1}^{N_d}(q_{n_rtm} + q_{n_gtm}) +$$
$$s_{\lambda impt}\sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}}\sum_{n_d=1}^{N_c}(\gamma_{n_rtm} + \gamma_{n_dtm}) +$$
$$s_{\lambda expt}\sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}}\sum_{n_d=1}^{N_c}(\delta_{n_rtm} + \delta_{n_gtm}) + \sum_{t=1}^{T} g(\gamma_{impt}, \delta_{expt}), \tag{4.22b}$$

$$(4.20c) - (4.20h) \tag{4.22c}$$

$$(4.21c) - (4.21f) \tag{4.22d}$$

$$\sum_{n_r=1}^{N_{pr}}\sum_{n_g=1}^{N_p}(q_{n_rtm} + q_{n_gtm}) - \sum_{n=1}^{N_{pr}}\sum_{n=1}^{N_c}(r^c_{n_rtm} + r^c_{n_dtm}) + p_{dt} - p_{ct} = 0, \quad \forall t \in T \tag{4.22e}$$

$$(4.20j)\&(4.20k) \tag{4.22f}$$

$$\sum_{n_r=1}^{N_{pr}}(\gamma_{n_rtm} + G_{n_rt}) + \sum_{n_d=1}^{N_c}(\gamma_{n_dtm} + R_{n_dt}) = \gamma_{impt}, \quad \forall t \in T \tag{4.22g}$$

$$\sum_{n_r=1}^{N_{pr}}\delta_{n_rtm} + \sum_{n_g=1}^{N_p}\delta_{n_gtm} = \delta_{expt}, \quad \forall t \in T \tag{4.22h}$$

$$(4.20n)\&(4.20o) \tag{4.22i}$$

$$(4.21k)\&(4.21l) \tag{4.22j}$$

$$(4.20p) - (4.20s) \tag{4.22k}$$

$$\begin{aligned} \gamma_{n_rtm} \geq 0,\ \delta_{n_rtm} \geq 0,\ \gamma_{n_dtm} \geq 0,\ \delta_{n_gtm} \geq 0, \\ G_{n_rt} \geq 0\ R_{n_dt} \geq 0\ \ \forall n_r \in N_{pr}, n_g \in N_p, n_d \in N_c, \forall t \in T \end{aligned} \tag{4.22l}$$

$$(4.18)\ \&\ (4.19) \tag{4.22m}$$

$$(4.15)\ \&\ (4.17) \tag{4.22n}$$

where the objective equation (4.22a) aims to minimise the total energy traded costs of all agents including prosumers, producers, and consumers, total transaction costs, and total costs of the community manager in importing and exporting energy within and outside of the community following the utility maximization of all agents. Constraints (4.22e), (4.22g), (4.22h), and (4.22l) serve the same function as (4.20i)/(4.21g), (4.20l)/(4.21i), (4.20m)/(4.21j), and (4.20t)/(4.21n) respectively for all agents.

4.3 Integrating Community Owned Generation Resources

Community-owned generation resources in the form of conventional diesel generators are integrated within the proposed community-based electricity market formulation. The integration aims to analyse the contribution of these resources at periods when agents' generation sources produce insufficient or no supply in the hours before 6am and after 6pm for PV resources, and between 6am and 6pm for wind resources. With the implementation of quadratic cost function $a_i p_{it}^{c2} + b_i p_{it}^{c}$ of a conventional generation resource where $a_i > 0$, and $b_i > 0$, whereas with regards to renewable energy sources of agents $a_i = b_i = 0$. The proposed CBEM market problem for prosumers' energy trades from equations (4.20a)-(4.20u) are reformulated to integrate the community-owned generation resources as follows:

$$\min\ \alpha(p^{g}_{n_rt}, p^{d}_{n_rt}, p^{c}_{it}, r^{c}_{n_rtm}, q_{n_rtm}, q^{c}_{it}, \gamma_{n_rtm}, G_{n_rt}, \delta_{n_rtm}, \delta^{c}_{it}, \gamma_{impt}, \delta_{expt}) \tag{4.23a}$$

$$\begin{aligned} \text{s.t. } \alpha \geq & \sum_{j=1}^{R_g}\sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}} \varphi_{jn_r}(p^{g}_{jn_rt}) + \sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}} \phi_{n_r}(p^{d}_{n_rt}) + \sum_{i=1}^{C_g}\sum_{t=1}^{T} \varphi_i(p^{c}_{it}) \\ & \xi_{com}\sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}} r_{n_rtm} + \xi_{com}\sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}} q_{n_rtm} + \xi_{com}\sum_{t=1}^{T}\sum_{i=1}^{C_g} q^{c}_{it} + \\ & s_{\lambda impt}\sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}}(\gamma_{n_rtm} + G_{n_rt}) + \\ & s_{\lambda expt}\sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}}\sum_{i=1}^{C_g}(\delta_{n_rtm} + \delta^{c}_{it}) + \sum_{t=1}^{T} g(\gamma_{impt}, \delta_{expt}), \end{aligned} \tag{4.23b}$$

$$\text{(4.20c)\&(4.20d)},\ \text{(4.20f)} - \text{(4.20h)} \tag{4.23c}$$

$$p^{c}_{t} - q^{c}_{it} - \delta^{c}_{it} = 0,\ \forall I \in C_g, t \in T, \tag{4.23d}$$

$$\sum_{i=1}^{C_g} p^{c}_{it} = p^{c}_{t},\ \forall t \in T \tag{4.23e}$$

$$\sum_{n=1}^{N_{pr}} q_{n_rtm} + \sum_{i=1}^{C_g} q^{c}_{it} - \sum_{n=1}^{N_{pr}} r_{n_rtm} + p_{dt} - p_{ct} = 0, \quad \forall t \in T \tag{4.23f}$$

$$\text{(4.20j)} - \text{(4.20l)} \tag{4.23g}$$

$$\sum_{n_r=1}^{N_{pr}} \delta_{n_rtm} + \sum_{i=1}^{C_g} \delta^{c}_{it} = \delta_{expt}, \quad \forall t \in T \tag{4.23h}$$

$$\text{(4.20n)} - \text{(4.20s)} \tag{4.23i}$$

$$\underline{p}^{c}_{it} \leq p^{c}_{it} \leq \overline{p}^{c}_{it}, \quad \forall i \in C_g, \forall t \in T \tag{4.23j}$$

$$\gamma_{n_rtm} \geq 0,\ \delta_{n_rtm} \geq 0,\ \delta^{c}_{it} \geq 0\ \ \forall n_r \in N_{pr},\ \forall t \in T \tag{4.23k}$$

$$\text{(4.18) \& (4.19)} \tag{4.23l}$$

In addition to the objective (4.20a), the objective of (4.23a) also minimizes the total traded costs of community-owned resources in importing/exporting energy within/outside the community. Constraint (4.23d) shows the energy balance of the community generation with exports within and outside the community. (4.23e) sums the generation from all generation sources of the community. (4.23f) represents the community power balance including community generation, prosumer's generation/consumption, and charged/discharged energy of CES. (4.23h) shows the community manager's exports from both prosumers' and community-owned generation. (4.23j) gives the bound for each community-owned resource. (4.23k) shows the non-negativity constraint from the prosumers' and community imports/exports.

Likewise, the proposed CBEM market problem for producers and consumers' energy trades from (4.21a)-(4.21o) is reformulated to integrate the community-owned generation resources as follows:

$$\min\ \alpha(p^g_{l n_g t}, p^d_{n_d t}, p^c_{it}, r^c_{n_d tm}, q_{n_g tm}, q^c_{it}, \gamma_{n_d tm}, R_{n_d t}, \delta_{n_g tm}, \delta^c_{it}, \gamma_{impt}, \delta_{expt}) \tag{4.24a}$$

$$\begin{aligned}\text{s.t. } \alpha \geq & \sum_{l=1}^{R_p}\sum_{t=1}^{T}\sum_{n_g=1}^{N_p} \varphi_{l n_g}(p^g_{l n_g t}) + \sum_{t=1}^{T}\sum_{n_d=1}^{N_c} \phi_{n_d}(p^d_{n_d t}) + \sum_{i=1}^{C_g}\sum_{t=1}^{T} \varphi_i(p^c_{it}) + \\ & \xi_{com}\sum_{t=1}^{T}\sum_{n_d=1}^{N_c} r_{n_d tm} + \xi_{com}\sum_{t=1}^{T}\sum_{n_g=1}^{N_p} q_{n_g tm} + \xi_{com}\sum_{t=1}^{T}\sum_{i=1}^{C_g} q^c_{it} + \\ & s_{\lambda impt}\sum_{t=1}^{T}\sum_{n_d=1}^{N_c}(\gamma_{n_d tm} + R_{n_d t}) + \\ & s_{\lambda expt}\sum_{t=1}^{T}\sum_{n_g=1}^{N_p}\sum_{i=1}^{C_g}(\delta_{n_g tm} + \delta^c_{it}) + \sum_{t=1}^{T} g(\gamma_{impt}, \delta_{expt}),\end{aligned} \tag{4.24b}$$

$$(4.21c),\ (4.21d)\&(4.21f) \tag{4.24c}$$

$$(4.23c)\&(4.23d) \tag{4.24d}$$

$$\sum_{n_g=1}^{N_p} q_{n_g tm} + \sum_{i=1}^{C_g} q^c_{it} - \sum_{n_d=1}^{N_c} r_{n_d tm} + p_{dt} - p_{ct} = 0, \quad \forall t \in T \tag{4.24e}$$

$$(4.20j)\ \&\ (4.20k) \tag{4.24f}$$

$$(4.21g)\&(4.21i) \tag{4.24g}$$

$$\sum_{n_g=1}^{N_p} \delta_{n_g tm} + \sum_{i=1}^{C_g} \delta^c_{it} = \delta_{expt}, \quad \forall t \in T \tag{4.24h}$$

$$(4.21k)\&(4.21l) \tag{4.24i}$$

$$(4.20p) - (4.20s) \tag{4.24j}$$

$$\gamma_{n_d tm} \geq 0,\ \delta_{n_g tm} \geq 0,\ \delta^c_{it} \geq 0,\ \forall n_g \in N_p, \forall n_d \in N_c, \forall t \in T \tag{4.24k}$$

$$(4.15)\ \&\ (4.17) \tag{4.24l}$$

where the objective equation (4.24a) minimizes the total traded costs from the community-owned resources while satisfying the objective of equation (4.21a). Constraints (4.24e), (4.22h), and (4.22k) serve the same function as (4.21g), (4.21k), and (4.21n) respectively in addition to variables relating to community resources.

Finally, the proposed CBEM market problem for all agents' energy trades from equations (4.22a)-(4.22n) are reformulated to integrate the community-owned generation resources as follows:

$$\min\ \alpha(p^g_{n_rt}, p^d_{n_rt}, p^c_{it}, r^c_{n_rtm}, q_{n_rtm}, \gamma_{n_rtm}, G_{n_rt}, \delta_{n_rtm}, p^g_{ln_gt}, p^d_{n_dt}, r^c_{n_dtm}, q_{n_gtm}, q^c_{it}, \gamma_{n_dtm}, R_{n_dt}, \delta_{n_gtm}, \delta^c_{it}) \tag{4.25a}$$

$$\begin{aligned}\text{s.t. } \alpha \geq & \sum_{j=1}^{R_g}\sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}} \varphi_{jn_r}(p^g_{jn_rt}) + \sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}} \phi_{n_r}(p^d_{n_rt}) + \sum_{l=1}^{R_p}\sum_{t=1}^{T}\sum_{n_g=1}^{N_p} \varphi_{ln_g}(p^g_{ln_gt}) + \\ & \sum_{t=1}^{T}\sum_{n_d=1}^{N_c} \phi_{n_d}(p^d_{n_dt}) + \sum_{i=1}^{C_g}\sum_{t=1}^{T} \varphi_i(p^c_{it}) + \\ & \xi_{com}\sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}}\sum_{n_d=1}^{N_c}(r_{n_rtm} + r_{n_dtm}) + \xi_{com}\sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}}\sum_{n_g=1}^{N_p}(q_{n_rtm} + q_{n_gtm}) + \\ & \xi_{com}\sum_{t=1}^{T}\sum_{i=1}^{C_g} q^c_{it} + s_{\lambda impt}\sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}}\sum_{n_d=1}^{N_c}(\gamma_{n_rtm} + \gamma_{n_dtm} + G_{n_rt} + R_{n_dt}) + \\ & s_{\lambda expt}\sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}}\sum_{n_g=1}^{N_p}\sum_{i=1}^{C_g}(\delta_{n_rtm} + \delta_{n_gtm} + \delta^c_{it}) + \sum_{t=1}^{T} g(\gamma_{impt}, \delta_{expt}),\end{aligned} \tag{4.25b}$$

$$(4.20c)\&(4.20e),\ (4.20f) - (4.20h) \tag{4.25c}$$

$$(4.21c),\ (4.21d)\&(4.21f) \tag{4.25d}$$

$$(4.23d)\&(4.23e) \tag{4.25e}$$

$$\sum_{n_r=1}^{N_{pr}} q_{n_rtm} + \sum_{n_g=1}^{N_p} q_{n_gtm} + \sum_{i=1}^{C_g} q^c_{it} - \sum_{n_r=1}^{N_{pr}} r_{n_rtm} - \sum_{n_d=1}^{N_c} r_{n_dtm} + p_{dt} - p_{ct} = 0, \quad \forall t \in T \tag{4.25f}$$

$$(4.20j)\&(4.20k) \tag{4.25g}$$

$$(4.22g) \tag{4.25h}$$

$$\sum_{n_r=1}^{N_{pr}}\sum_{n_g=1}^{N_p}\sum_{i=1}^{C_g}(\delta_{n_rtm} + \delta_{n_gtm} + \delta^c_{it}) = \delta_{expt}, \quad \forall t \in T \tag{4.25i}$$

$$(4.20n) - (4.20s) \tag{4.25j}$$

$$(4.21k) - (4.21l) \tag{4.25k}$$

$$(4.23j) \tag{4.25l}$$

$$\gamma_{n_rtm} \geq 0, \gamma_{n_dtm} \geq 0\ \delta_{n_rtm} \geq 0, \delta_{n_gtm} \geq 0\ \delta^c_t \geq 0\ \ \forall n_r \in N_{pr}, n_g \in N_p, n_c \in N_c\ \forall t \in T \tag{4.25m}$$

$$(4.18)\ \&\ (4.19) \tag{4.25n}$$

$$(4.15)\ \&\ (4.17) \tag{4.25o}$$

The objective of (4.25a) minimizes the total traded costs from the community-owned resources while satisfying the objective of (4.22) for all agents. Constraints (4.25f), (4.25i), and (4.25m) serve the same function as (4.22g), (4.22h), and (4.22l) respectively in addition to variables relating to community resources.

4.4 Community Fairness Indicators

In this section, some community fairness indicators are presented and their impacts on community energy trades are analysed and compared with and without utility maximization of agents' resources. The impact of the community manager on individual prosumer behaviour using fairness indicators was analysed in [2]. In this work, the fairness indicators relating to Quality of Trade (QoT), Quality of Costs (QoC), and Import Share (ImS) are discussed and analysed based on the work in [2]. The QoT indicators calculate the volume of energy trades within the community as a percentage of the total generated/consumed energy capacity. The mathematical expression for QoT for a community-based market of prosumers' production and consumption volumes is given in (4.26).

$$QoT = \frac{\sum_{n_r=1}^{N_{pr}} q_{n_r tm}}{\sum_{n_r=1}^{N_{pr}} p^{g}_{n_r tm}};\ QoT = \frac{\sum_{n_r=1}^{N_{pr}} r_{n_r tm}}{\sum_{n_r=1}^{N_{pr}} p^{d}_{n_r tm}}\ \forall t \in T \tag{4.26}$$

The community energy trades are 100% fair when the community agents trade all their resources within the community with equal volumes of energy amongst the agents. The lower QoT values are indications that the community energy traded volumes are unequal with varying impacts. The QoC indicators calculate the perceived costs of energy trades by the agents within the community with respect to maximum price deviations of costs of imports/exports from outside the community. Mathematically, QoC is given in:

$$QoC = 1 - \frac{\sigma_{\lambda}}{\sigma_{\lambda max}} \tag{4.27}$$

Where σ_{λ} is the standard deviations of the linear costs of agents for trading within the community and $\sigma_{\lambda max}$ is the maximum price deviations of $(s_{\lambda impt} - s_{\lambda expt})$. The QoC is used to reflect more on a community agent's view of the trades as compared to the community's view in the case of the QoT. For instance, when QoC values range between 0.01 to 0.99, it implies a move from the agent's perspective to the community energy market framework. The ImS indicators calculate the minimum energy import with respect to the maximum energy import. Mathematically ImS for a community-based market of prosumers' imports is expressed in:

$$ImS = \frac{\min \gamma_{n_r tm}}{\max \gamma_{n_r tm}} \tag{4.28}$$

An ImS value of 1.0 indicates that the import shares are always nearly equal. The prosumers can import a certain amount of energy based on their preference relations which can impact community fairness. This means that prosumer or consumer agents with little or no need for energy imports from the grid must import energy and this compared to a higher import, will impact the community fairness resulting in a low import share. However, the import share is improved when energy imports are almost equal.

4.5 Case Study

In this work, the proposed economic model is tested on three community microgrids from Chapter 3. The first microgrid consists of 7 prosumers equipped with PV resources, the second microgrid consists of 7 producers and 8 consumers, and finally, the third case study is the community microgrid of 5 prosumers, equipped with PV production, 3 producers, two of which have conventional resources (micro-turbine and fuel cell) and one wind production, and 20 consumers. The community-owned generators are integrated to analyse the contribution of these resources in maximising the agents' utility. In the first community microgrid, 3 diesel generators are considered as community resources, while in the second community, 2 diesel generators are integrated, for the third community, the two conventional generators are used as community generation resources.

In the community microgrid case studies, three cases are analysed to test the sensitivity of the proposed CBEM model to changing preferences of agents as well as the community fairness indicators shown in Tables 4.1-4.5. In the first case, agents' preferences are dependent on the cost functions of their resources which were generated as normalised energy market prices [2]. In the second case, the weather variations as a result of the renewable technology used by agents are used to analyse agents' preferences for imports and exports. The third case limits agents' preferences (prosumers and consumers) to resource availability within the community. In [1], the Cobb-Douglas parameter was selected such that it provided a normalised cost of consumption thereby smoothing the peaks in demand. The Cobb Douglas positive parameters in the case of energy prices for prosumers (consumption) and consumers are used in the same manner such that $\alpha_{n_r t}$, and $\beta_{n_r t}$ for prosumers is presented thus: $\alpha_{n_r t} = \frac{s_{\lambda impt}}{b_{jn_r t}+s_{\lambda impt}}$ and $\beta_{n_r t} = \frac{b_{jn_r t}}{b_{jn_r t}+s_{\lambda impt}}$ while $\alpha_{n_d t}$, and $\beta_{n_d t}$ for consumers is presented thus: $\alpha_{n_d t} = \frac{s_{\lambda impt}}{b_{n_d t}+s_{\lambda impt}}$ and $\beta_{n_d t} = \frac{b_{n_d t}}{b_{n_d t}+s_{\lambda impt}}$. In the case of prosumers (production) and producers with renewable resources, the marginal costs are equal to 0, therefore the highest marginal costs $b_{jn_r t}/b_{n_d t}$ of prosumer/consumer resources are used to reflect preferences for more exports within the community as compared to outside the community as follows $\sigma_{n_r t} = \frac{s_{\lambda expt}}{b_{jn_r t}+s_{\lambda expt}}$ and $\rho_{n_r t} = \frac{b_{jn_r t}}{b_{jn_r t}+s_{\lambda expt}}$ for prosumers and $\sigma_{n_g t} = \frac{s_{\lambda expt}}{b_{n_d t}+s_{\lambda expt}}$ and $\rho_{n_g t} = \frac{b_{n_d t}}{b_{n_d t}+s_{\lambda expt}}$ for producers. The producers with conventional generation are taken as community generation and the preferences are only reflected in the cost functions of community resources and not in exports within or outside the community. With regards to weather variations, the type of renewable technology and periods of resource availability determine the preferences of agents with a higher preference for solar/wind energy during sunny/windy periods and a lower preference for grid energy and vice versa. In the third community microgrid where the production is significantly lesser than consumption, the production resources are oversized to accommodate the agents' preferences [24].

In the third case, the agent preferences are limited to resource availability within the community. During periods where the generation in the community is less than the demand, the prosumer and consumer agents' utility parameters are at or within certain prescribed deviations from the $generation - demand$ ratio. For periods where generation is greater than the demand, the $generation - demand$ ratio becomes greater than 1, in this case, the agents' preferences indicate more willingness to be autonomous from the grid.

For prosumer (production) and producer agents, the proportional coefficients $\rho_{n_r t}$ and $\sigma_{n_r t}$ for prosumers and $\rho_{n_g t}$ and $\sigma_{n_g t}$ for producers, are determined such that the producer prefers no export outside the grid in the case of weather variation and limited resource availability.

In total, three cases are analysed for each community microgrid with and without community generation. In the third case study, the CBEM model is tested with community generation. In the first and second cases, the agent's preference relations show that more attributes are given to the imports/exports within the community than the grid, reflecting their desire for either more local energy or to increase autonomy from the grid, while in the third case, their preferences for greener energy within the community are limited by the availability of the resource.

4.5.1 Data Set Case Studies 1 & 2

In the first and second community microgrid case studies, A CES is considered with $e^{max} = 40$ kWh, $e^{min} = 10$ kWh, $n^d = n^c = 0.9$, $P_c^{max} = P_c^{min} = 20$ kW, and $e_0 = 10$ kWh. The Ausgrid peak-tariff in 2019 [114] is considered (i.e., 23.5336 c/kWh) in addition to a cost set at $\tau = 10$ c/kWh) is used for importing energy from outside the community while the export is obtained by subtracting the cost from the peak tariff. The transaction cost of the community manager is fixed at 1 c/kWh. The marginal costs of agents' resources are used to obtain the preference parameters for all agents in the first case. In the second case of weather variations, the positive parameter is assumed within $\pm 1\%$ of the 70% for PV renewable resource of agents for sunny periods, and for periods with no sun, 0% is used indicating all imports will come from the grid. In the case of prosumer(production) and producers the preferences for export to the grid $\rho_{n_r t}$ and $\rho_{n_g t}$ for prosumers (production) and producers is equal to zero in the second and third cases. In the third case, the sum of all generation resources, fixed for renewable producers and at lower bounds for community generation is obtained and divided by the sum of the fixed consumption of prosumer/consumer agents. A deviation of $\pm 5\%$ from the $generation - demand$ ratio is assumed as the agent's preferences. 3 diesel generators rated at 5 kW each are considered in the first case study, while 2 diesel generators rated at 5 kW and 10 kW are considered in the second case study.

4.5.2 Data Set Case Study 3

In the third community microgrid an example case of the Cigre LV benchmark model, A CES is considered with $e^{max} = 50$ kWh, $e^{min} = 10$ kWh, $n^d = n^c = 0.9$, $P_c^{max} = P_c^{min} = 25$ kW, and $e_0 = 10$ kWh. The European Commission (EC) import tariff in 2021 [115] in addition to a grid tariff τ=10 c/kWh is considered as import prices of all agents for importing energy from outside the community while subtracting the grid tariff from the import tariff is used as the export price. The transaction cost of the community manager is fixed at 1 c/kWh. The marginal

Table 4.1: CBEM Trading Model for all Cases in the First Community without Community-owned Generation

CBEM Trading Model	Trading Results			QoT		QoC		ImS	
Model	Total Cost ($)	Total Import (kWh)	Total export (kWh)	Mean	Std dev.	Mean	Std dev.	Mean	Std dev.
CET Trading	165.65	222.20	8.22	0.42	0.132	0.99	0.135	0.00	0.00
CETM Case 1	193.96	261.96	47.97	0.242	0.069	0.99	0.135	0.27	0.156
CETM Case 2	166.09	219.060	0.00	0.42	0.171	0.99	0.135	0.26	0.148
CETM Case 3	163.78	215.56	0.00	0.44	0.137	0.99	0.135	0.20	0.201

Table 4.2: CBEM Trading Model for all Cases in the First Community with Community-owned Generation

CBEM Trading Model	Trading Results			QoT		QoC		ImS	
Model	Total Cost ($)	Total Import (kWh)	Total export (kWh)	Mean	Std dev.	Mean	Std dev.	Mean	Std dev.
CET Trading	113.81	120.21	13.63	0.45	0.28	0.99	0.130	0.00	0.00
CETM Case 1	113.69	100.09	59.41	0.63	0.32	0.99	0.130	0.30	0.177
CETM Case 2	106.90	95.10	26.37	0.64	0.084	0.99	0.130	0.32	0.185
CETM Case 3	117.16	120.58	12.80	0.51	0.078	0.99	0.130	0.26	0.229

Table 4.3: CBEM Trading Model for all Cases in the Second Community without Community-owned Generation

CBEM Trading Model	Trading Results			QoT		QoC		ImS	
Model	Total Cost ($)	Total Import (kWh)	Total export (kWh)	Mean	Std dev.	Mean	Std dev.	Mean	Std dev.
CET Trading	253.25	282.07	0.00	0.41	0.186	0.98	0.206	0.00	0.00
CETM Case 1	296.05	344.77	47.20	0.22	0.111	0.98	0.206	0.51	0.268
CETM Case 2	265.94	298.55	0.00	0.39	0.210	0.98	0.206	0.55	0.303
CETM Case 3	265.31	297.57	0.00	0.39	0.206	0.98	0.206	0.64	0.201

costs of agents for imports/exports are also used in determining the preference parameters. In the case of weather variations prosumers equipped with PV and wind resources, agents prefer 70% imports from the grid for periods between 1am-6am and 8am-12am because of no PV supply and more wind supply with little dependence on community resource. At other periods their preference is determined within ±1% of 70% of the PV, low wind, and low community generation. The third case preferences for the third community are applied as in the first and second cases. For prosumers (production) and producers, the preferred choices of agents are the same as in Community microgrids 1 and 2. The two conventional generators are modelled as community-owned generation resources and are rated 20 kW and 30 kW respectively. Therefore in the third community mocrogrid, the case studies are considered only with community-owned generation.

Table 4.4: CBEM Trading Model for all Cases in the Second Community with Community-owned Generation

CBEM Trading Model	Trading Results			QoT		QoC		ImS	
Model	Total Cost ($)	Total Import (kWh)	Total export (kWh)	Mean	Std dev.	Mean	Std dev.	Mean	Std dev.
CET Trading	211.32	190.47	0.00	0.53	0.015	0.98	0.188	0.00	0.00
CETM Case 1	189.68	126.40	60.41	0.64	0.328	0.98	0.188	0.62	0.176
CETM Case 2	175.64	115.08	8.53	0.69	0.136	0.98	0.188	0.53	0.386
CETM Case 3	213.09	187.89	16.16	0.61	0.124	0.98	0.188	0.63	0.216

Table 4.5: CBEM Trading Model for all Cases in the Third Community with Community-owned Generation

CBEM Trading Model	Trading Results			QoT		QoC		ImS	
Model	Total Cost ($)	Total Import (kWh)	Total export (kWh)	Mean	Std dev.	Mean	Std dev.	Mean	Std dev.
CET Trading	639.75	655.09	21.72	0.73	0.33	0.99	0.159	0.00	0.00
CETM Case 1	514.63	353.14	317.20	0.61	0.166	0.99	0.159	0.137	0.0002
CETM Case 2	550.98	352.83	540.99	0.68	0.207	0.99	0.159	0.15	0.003
CETM Case 3	545.52	487.81	99.00	0.71	0.166	0.99	0.159	0.00	0.00

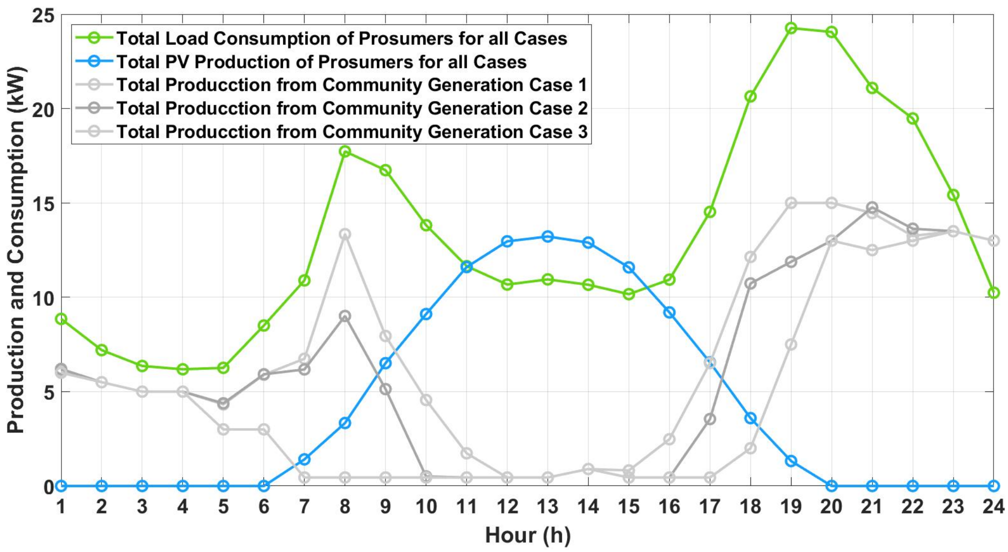


Figure 4.6: Total production and consumption in the first community.

4.5.3 Discussion

The total production and consumption for each community microgrid is presented in Figures 4.6, 4.7, and 4.8. The results show that for a given consumption and production budgets of prosumers/consumers and prosumers/producers respectively the community-owned generators will supply the amounts of energy shown for exports to/outside the community in satisfying agents' preferences. In the first and second community microgrids, with no PV supply and with

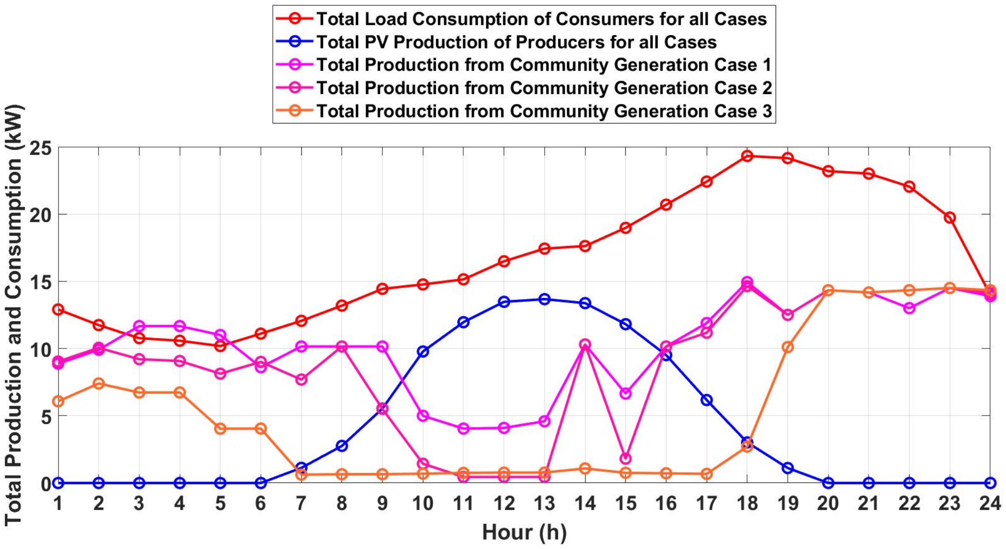


Figure 4.7: Total production and consumption in the second community.

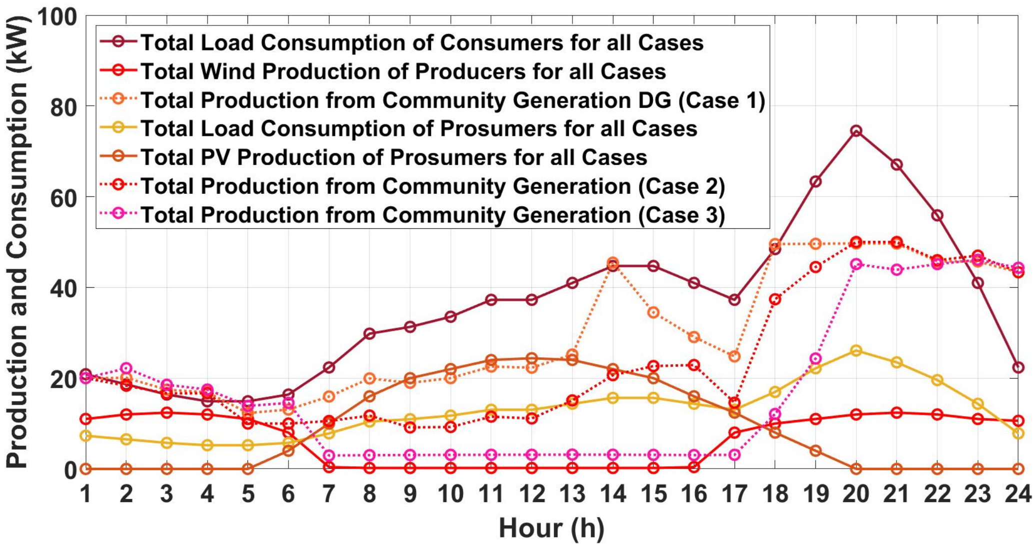


Figure 4.8: Total production and consumption in the third community.

community-owned generation for periods before 6am and after 7pm, agents' preferences based on weather variations will reflect all imports from the grid. With consideration to community-owned generation, the community resource charges the CES at those periods while supplying the agents with preferred lower demands. The third community microgrid is considered with community-owned generation, for Case 2 the weather variations for all periods are based on both wind and PV generation of the producer and prosumers respectively as well as community-owned generation.

The results of community energy trades and community fairness indicators analysed for

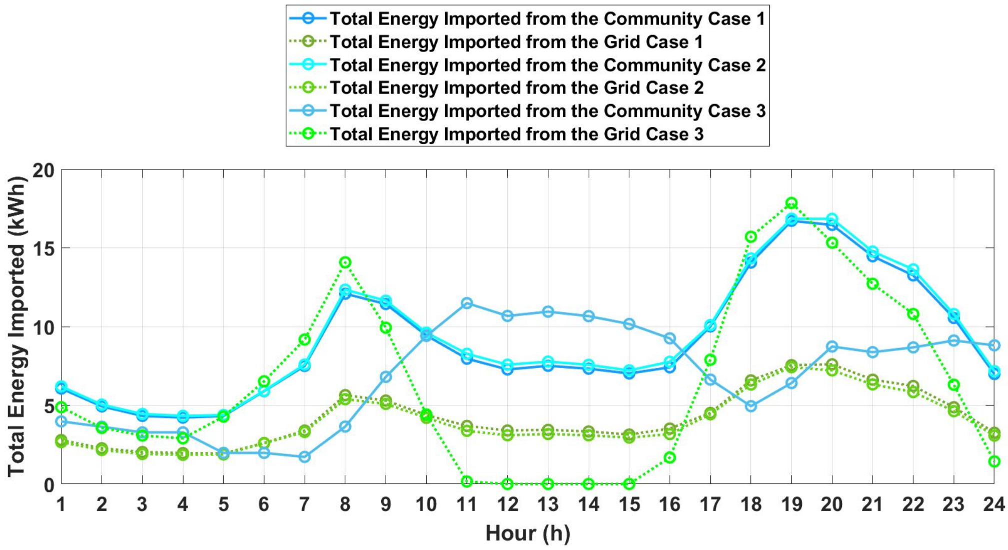


Figure 4.9: Total energy imported with community-owned generation for all cases in the first community.

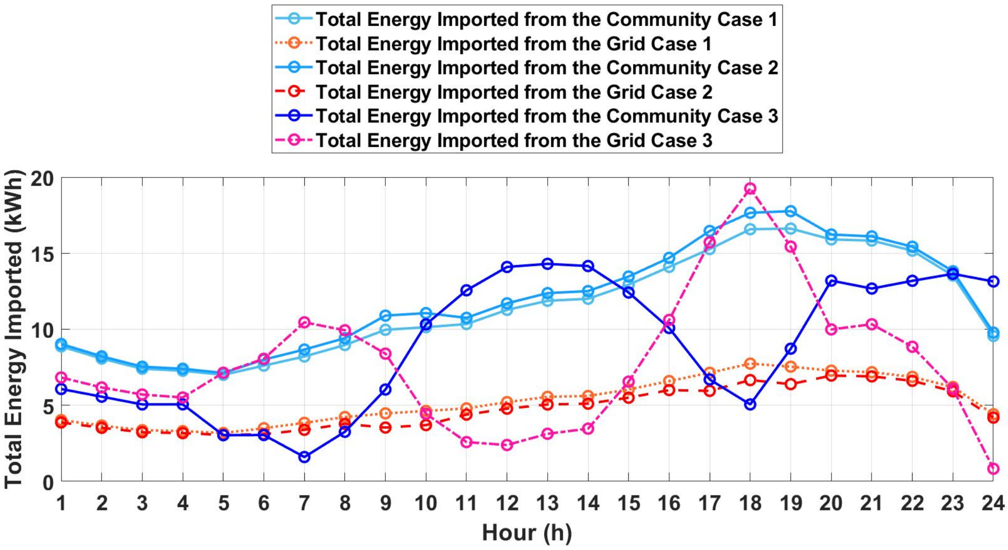


Figure 4.10: Total energy imported with community-owned generation for all cases in the second community.

the three cases in all community microgrids, with and without the agent's utility maximization are shown in Tables 4.1- 4.5. The results show that the QoT index for the first community is about 42% and 45% fair in the community energy trades without utility maximization, and without/with community-owned generation respectively, and 24%, 42%, *and* 44% and 63%, 64%, *and* 51% for Cases 1, 2 and 3 respectively in the community with utility maximization, and without/ with community owned generation. This shows that the QoT index

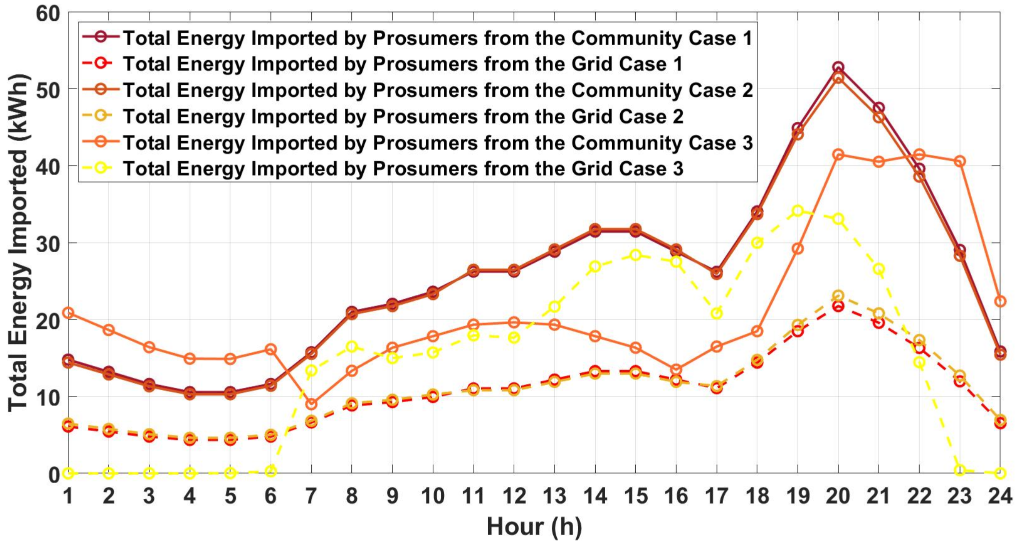


Figure 4.11: Total energy imported with community-owned generation for all cases in the third community.

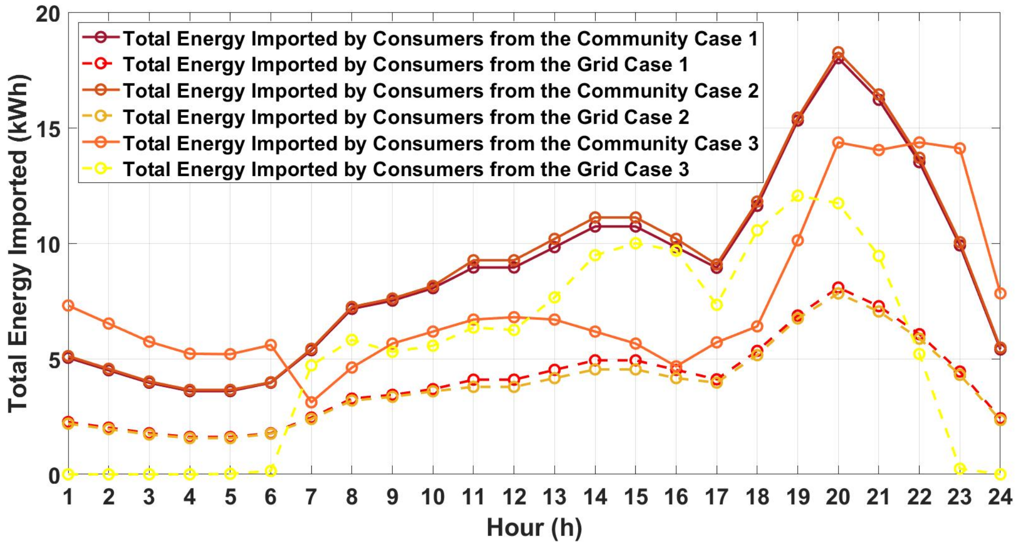


Figure 4.12: Total energy imported with community-owned generation for all cases in the third community.

has higher community fairness when the community provides a generation backup as compared to when the prosumer agents are trading with their resources. The QoC index shows that the prosumer agent's costs of energy trades reflect more community energy trade framework at 0.99 for the first and third communities and 0.98 for the second community, for community trading with and without utility maximization. These values reflect more of the community agent's view of the energy trades towards the community-based market rather than individual trading which will result in lower QoC indices. The ImS index shows that for the community energy

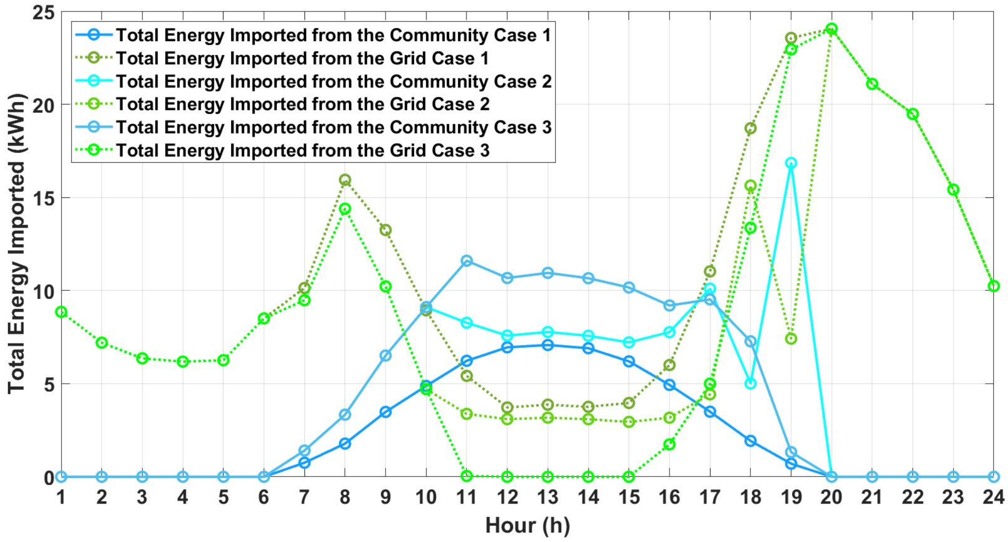


Figure 4.13: Total energy imported without community-owned generation for all cases in the first community.

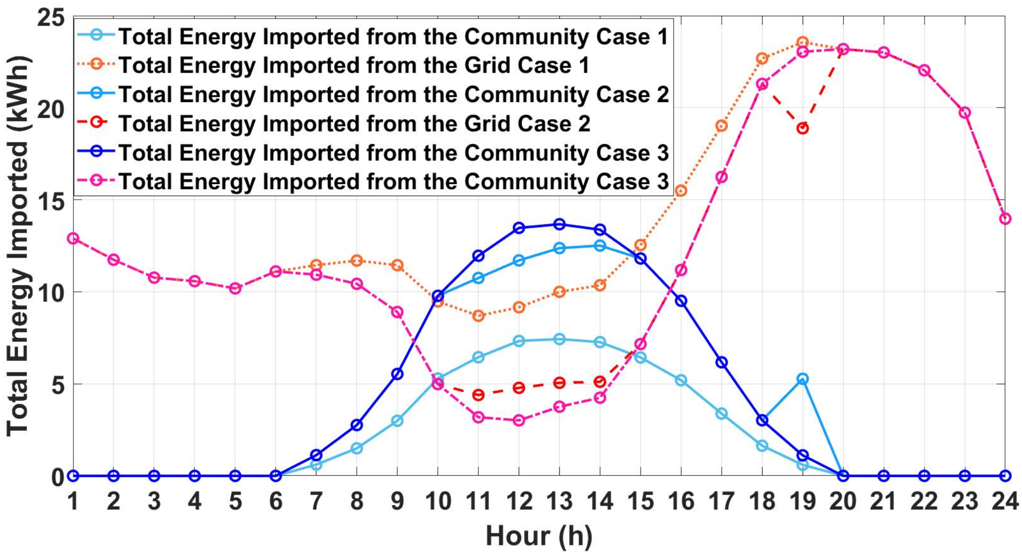


Figure 4.14: Total energy imported without community-owned generation for all cases in the second community.

trades the index is 0.00 because not all prosumer agents need energy imports and therefore the minimum import will be 0 divided by whatever the value of the maximum import. With regards to the ImS index for Cases 1, 2, and 3 for the first community, the prosumers' preferences reflect an amount of energy imports from the grid based on the cases analysed and therefore give a min-max ImS value of $0.27,\ 0.26,\ and\ 0.20$ and $0.30,\ 0.32,\ and\ 0.26$ for Cases 1, 2 and 3 respectively with utility maxmization and without/with community-owned generation. This

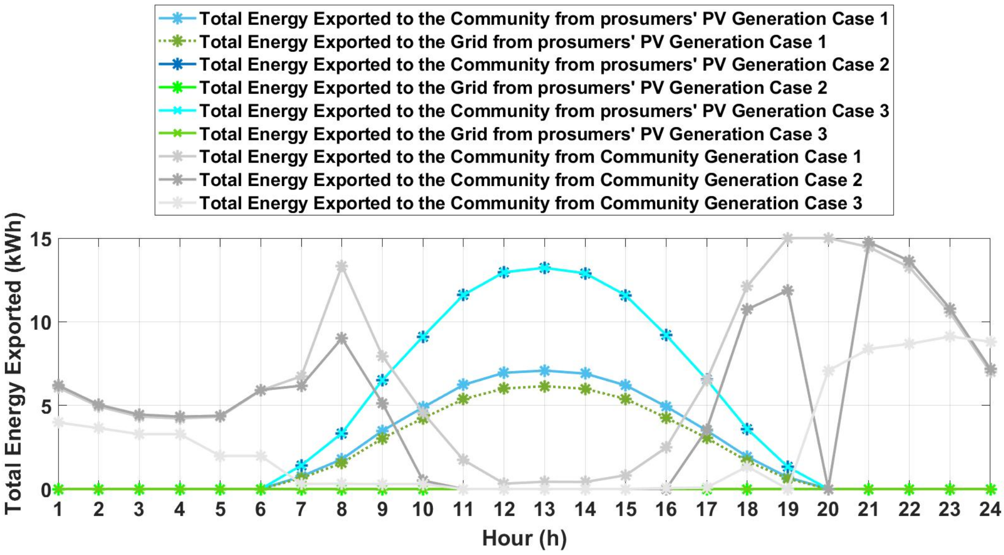


Figure 4.15: Total energy exported for all cases in the first community.

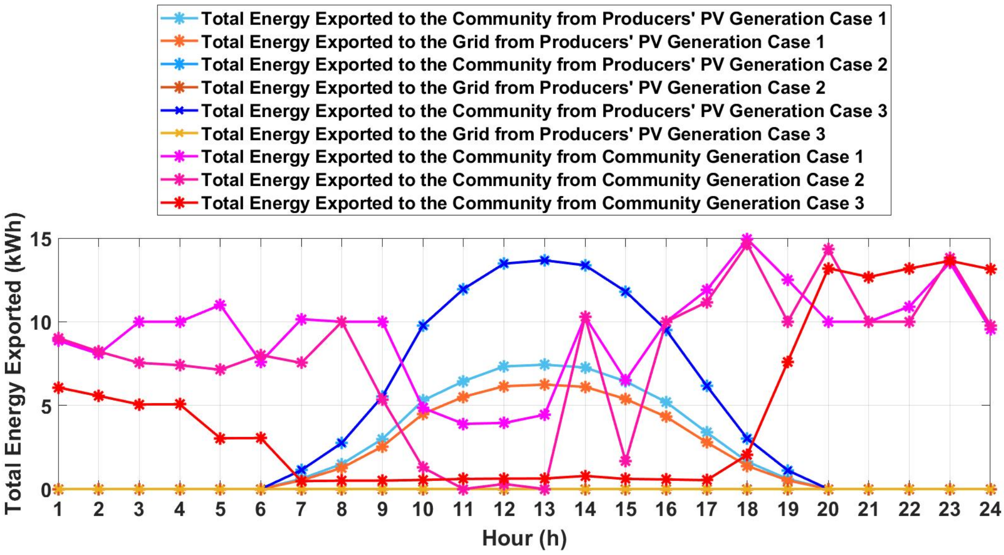


Figure 4.16: Total energy exported for all cases in the second community.

leads to lower community trade fairness as the ratios show some prosumer imports are lower by the stated amounts than others. The QoT value for the second community is about 41% and 53% fair in the community energy trades without utility maximization, and without/with community-owned generation respectively, and 22%, 39%, *and* 39% and 64%, 69%, *and* 61% for cases 1, 2 and 3 respectively in the community with utility maximization, and without/ with community owned generation. The QoT values are higher also for the community with community-owned generation as compared to those without community-owned generation. The ImS values for the second community are 0.51, 0.55, *and* 0.64 and 0.62, 0.53, *and* 0.63 for Cases

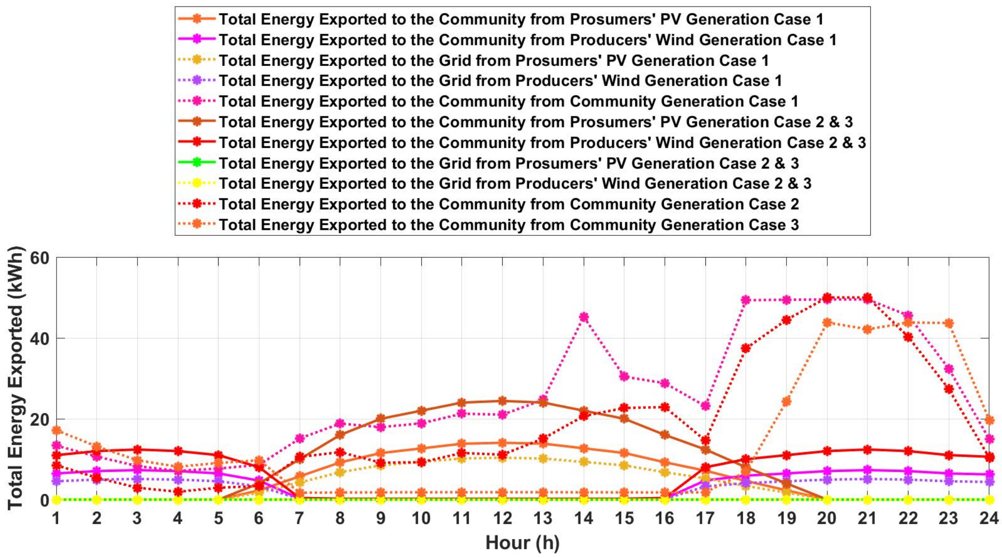
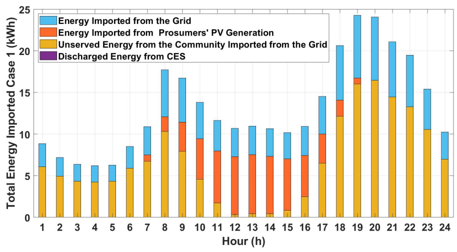
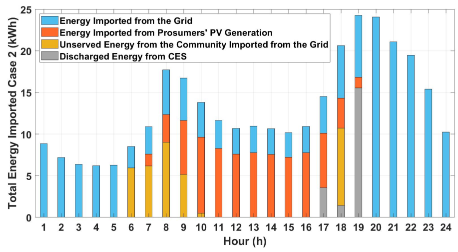


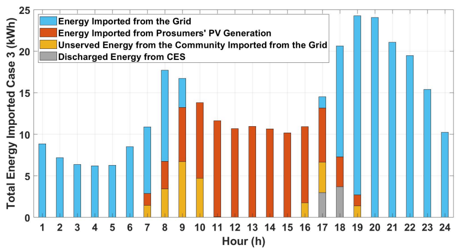
Figure 4.17: Total energy exported for all cases in the third community.



(a) Total energy imported without community-owned generation in the first community for Case 1.



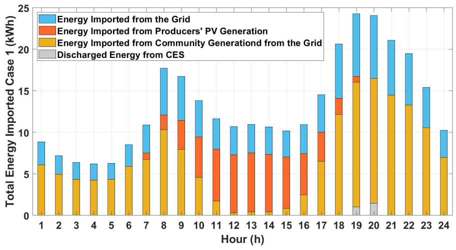
(b) total energy imported without community-owned generation in the first community for Case 2.



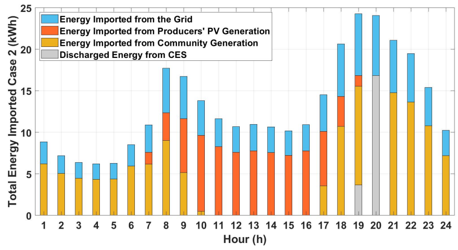
(c) Total energy imported without community-owned generation in the first community for Case 3.

Figure 4.18: Total energy imported without community-owned generation in all cases in the first community.

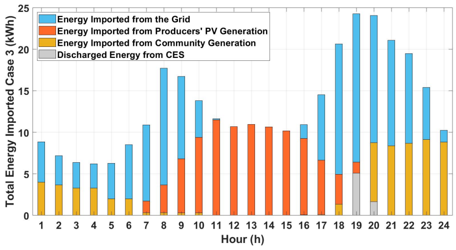
1, 2, and 3 respectively with utility maxmization and without/with community-owned generation. This shows a higher community trade fairness in the second community compared to the first community. The QoT value for the third community is about 73% fair in the community energy trades without utility maximization, and with community-owned generation respectively,



(a) Total energy imported with community-owned generation in the first community for Case 1.

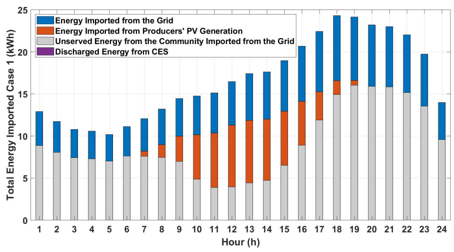


(b) Total energy imported with community-owned generation in the first community for Case 2.

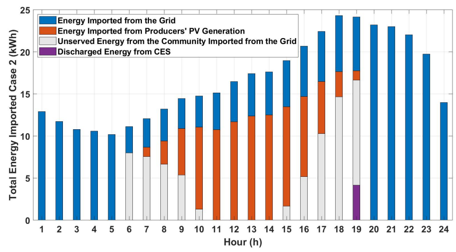


(c) Total energy imported with community-owned generation in the first community for Case 3.

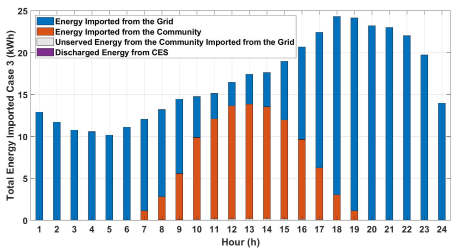
Figure 4.19: Total energy imported with community-owned generation in all cases in the first community.



(a) Total energy imported without community-owned generation in the second community for Case 1.

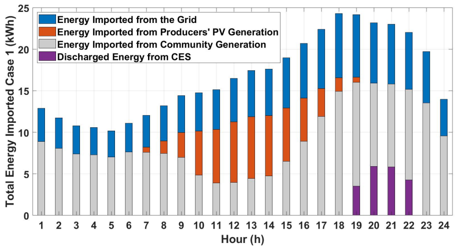


(b) Total energy imported without community-owned generation in the second community for Case 2.

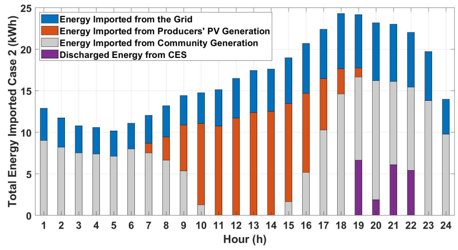


(c) Total energy imported without community-owned generation in the second community for Case 3.

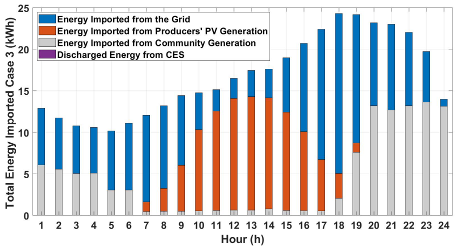
Figure 4.20: Total energy imported without community-owned generation in all cases in the second community.



(a) Total energy imported with community-owned generation in the second community for Case 1.

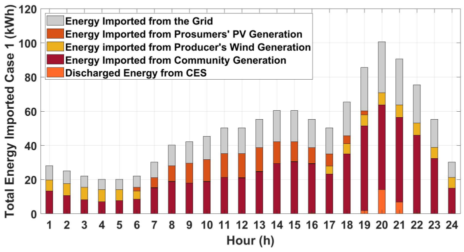


(b) Total energy imported with community-owned generation in the second community for Case 2.

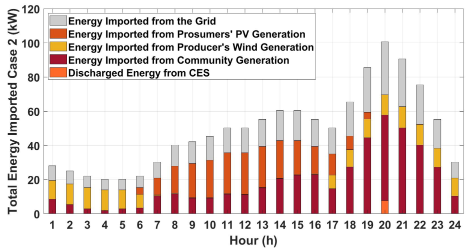


(c) Total energy imported with community-owned generation in the second community for Case 3.

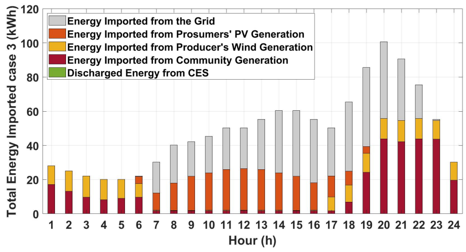
Figure 4.21: Total energy imported with community-owned generation in all cases in the second community.



(a) Total energy imported with community-owned generation in the third community for Case 1.

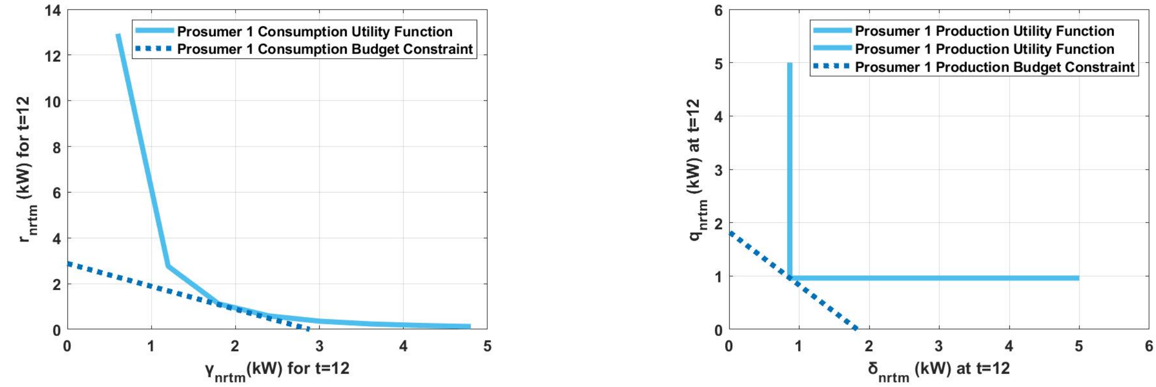


(b) Total energy imported with community-owned generation in the third community for Case 2.



(c) Total energy imported with community-owned generation in the third community for Case 3.

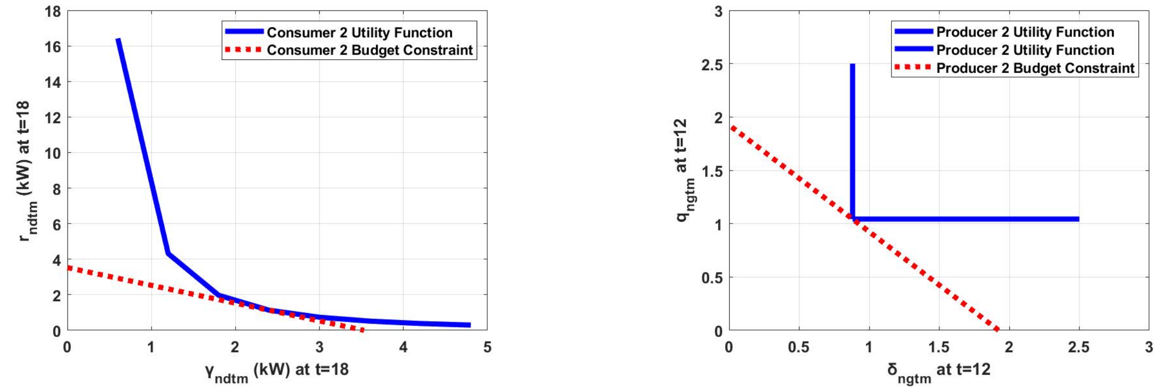
Figure 4.22: Total energy imported with community-owned generation in all cases in the third community.



(a) Indifference curve of prosumer 1 consumption at t=12 in the first community.

(b) Indifference curve of prosumer 1 production at t=12 in the first community.

Figure 4.23: Indifference curve of prosumer 1 consumption and production at t=12 in the first community.



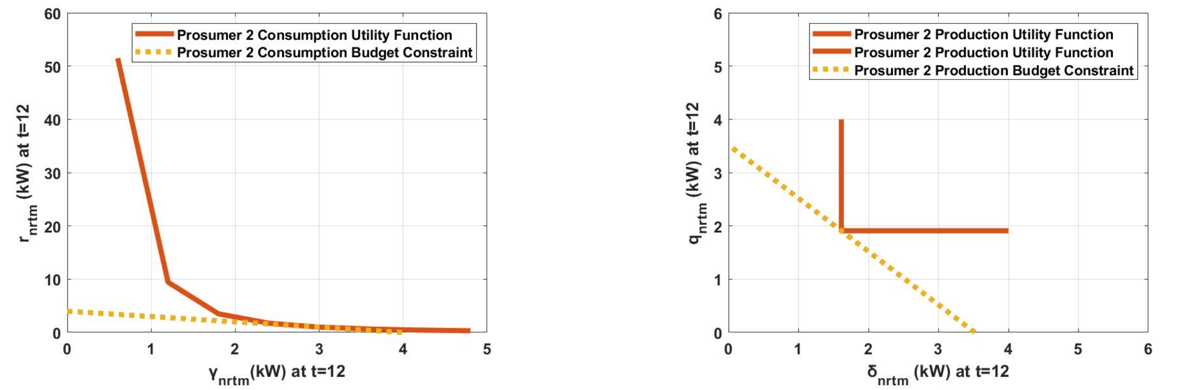
(a) Indifference curve of consumer 2 consumption at t=18 in the second community.

(b) Indifference curve of producer 2 production at t=12 in the second community.

Figure 4.24: Indifference curve of consumer 2 consumption and production at t=18, and t=12 respectively in the first community.

and 61%, 68%, *and* 71% for Cases 1, 2, and 3 respectively in the community with utility maximization, and with community owned generation. This shows that QoT values are generally higher when the community provides a generation resource to balance the community energy trades. The ImS values for the third community are $0.14, 0.15,$ *and* 0.00 for Cases 1, 2 and 3 respectively with utility maxmization and with community-owned generation. This shows a low community fairness in the third community as compared to the first and second communities.

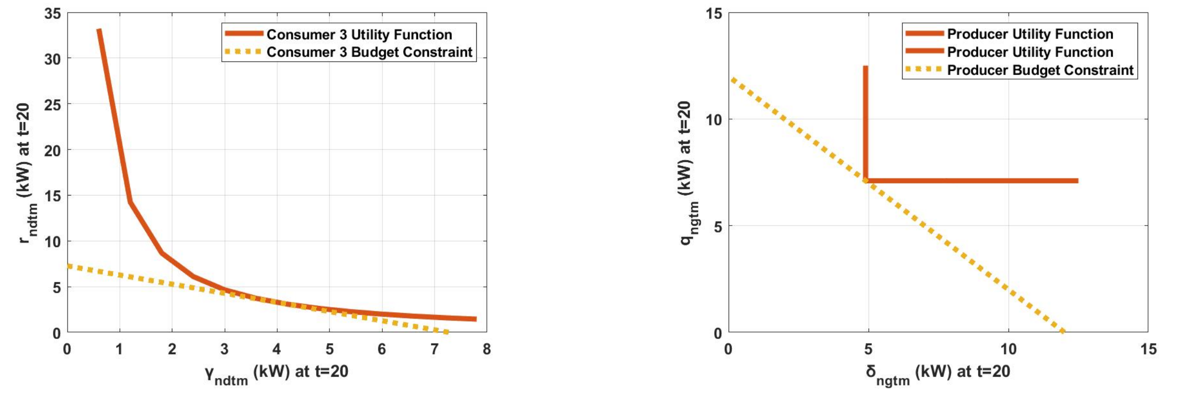
The imports/ exports of agents within and outside the community for all community microgrids with community-owned generation reflect agents' preference relations as shown in Figures 4.9, 4.10, 4.11, 4.12 for imports. In the community microgrid without community-owned gen-



(a) Indifference curve of prosumer 2 consumption at t=12 in the third community.

(b) Indifference curve of prosumer 2 production at t=12 in the third community.

Figure 4.25: Indifference curve of prosumer 2 consumption and production at t=12 in the third community.



(a) Indifference curve of consumer 3 consumption at t=20 in the third community.

(b) Indifference curve of the wind producer's production at t=20 in the third community.

Figure 4.26: Indifference curve of consumer 3 and the wind producer at t=20 in the third community.

eration, as shown in 4.13 and 4.14 for the first and second community imports, the results show the prosumer/consumer preferences which maximise their utility, are not satisfied as the prosumers/producers generation is insufficient to meet the prosumers/consumers consumption. Therefore the community market problem is solved following the agents' preference relations with the unserved energy of prosumers/consumers from the community imported from the grid. The prosumer/producer exports reflect the preferences for exports within and outside the community with and without community-owned generation as shown in Figures 4.15, 4.16, and 4.17. The first and second case results show that the solution of the community market problem is solved with regards to the agent preference relation which is independent of the resource avail-

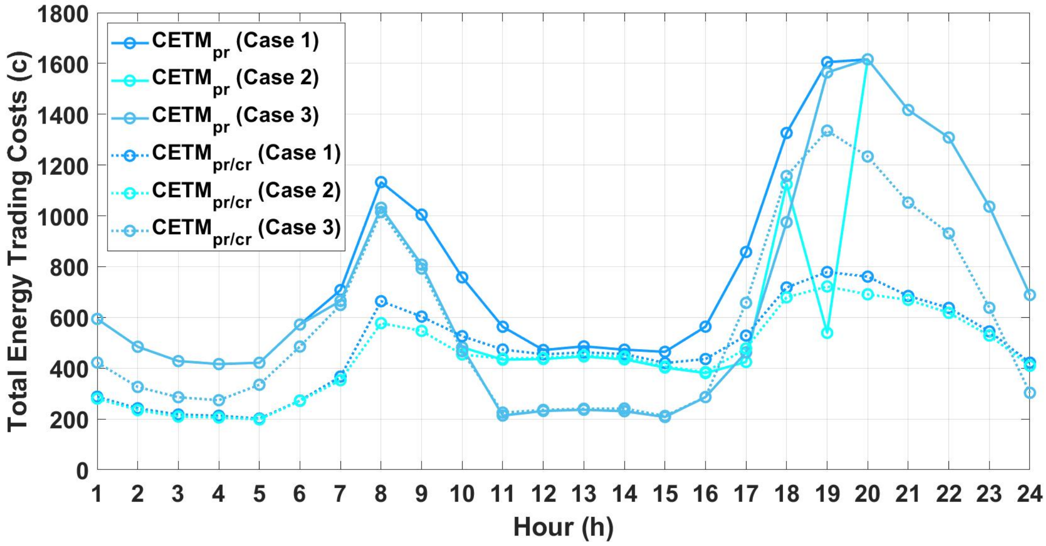


Figure 4.27: Total energy trading costs for all cases in the first community.

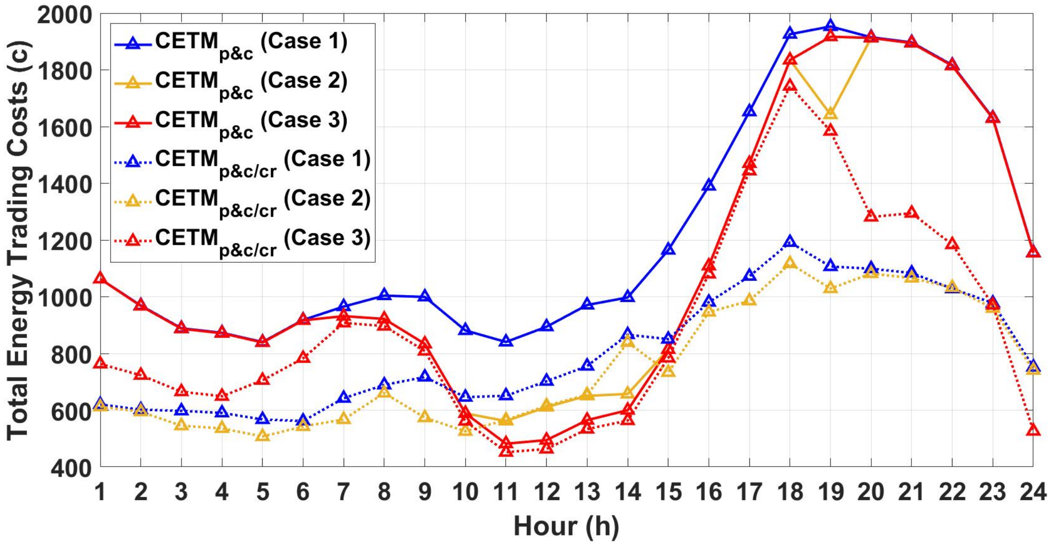


Figure 4.28: Total energy trading Costs for all cases in the second community.

ability whereas the third case results relate more to the general community market problem as the agents' preference relations reflect the community resource availability. The indifference curves of some community agents showing the maximised utility subject to their consumption and production budget constraints in the given community microgrids are presented in Figures 4.23,4.24, 4.25, and 4.26. The results show preference relations of community agents for all cases, where the first and second communities are presented based on cost preferences, and the third community is presented based on weather variations/cost preferences and resource availability/ cost preferences. Furthermore, Figures 4.18 and 4.20 show the unserved energy from

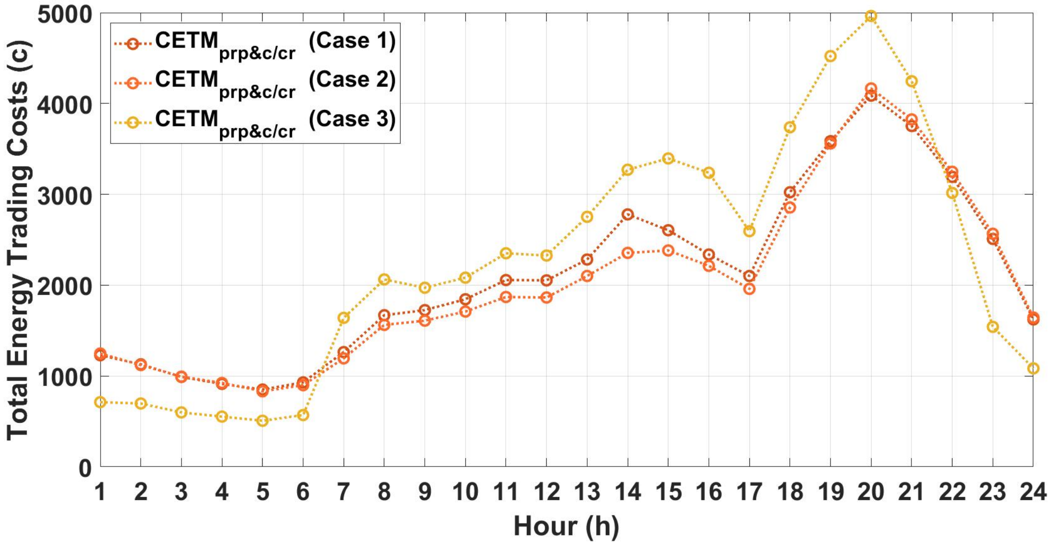


Figure 4.29: Total energy trading costs all cases in the third community.

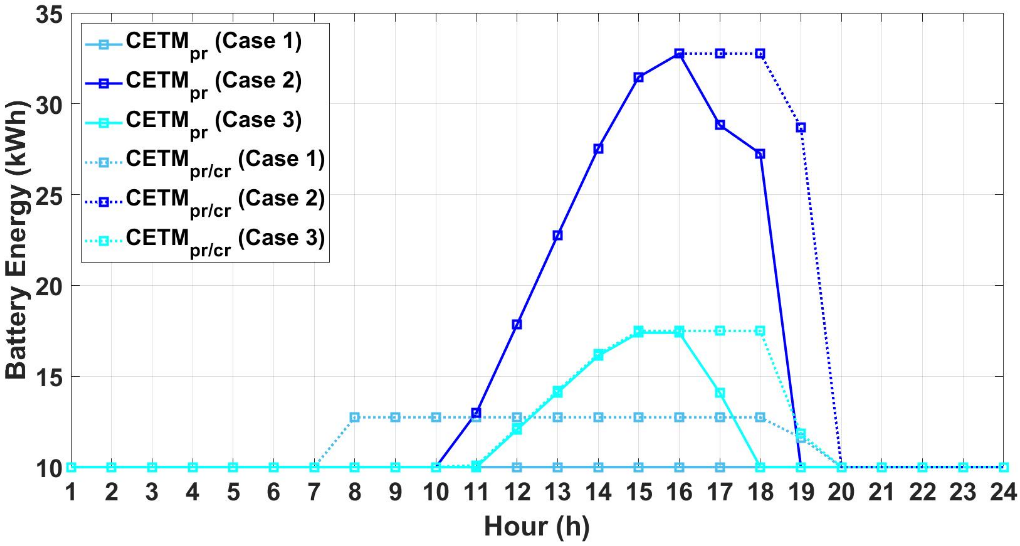


Figure 4.30: Battery energy profile for all cases in the first community.

the community imported from the grid by the prosumer/consumer agents in the first and second communities without community-owned generation. Figures 4.19, 4.21, 4.22 show the unserved energy from the community supplied from the community-owned generation in all the communities with a community-owned generation resource. The results show that the unserved energy from the community agents' preferences for imports within the community, is imported from the community-owned generation and the grid.

The total energy traded costs for all cases in each community microgrid with and without community-owned generation are shown in Figures 4.27, 4.28, and 4.29. The total energy traded

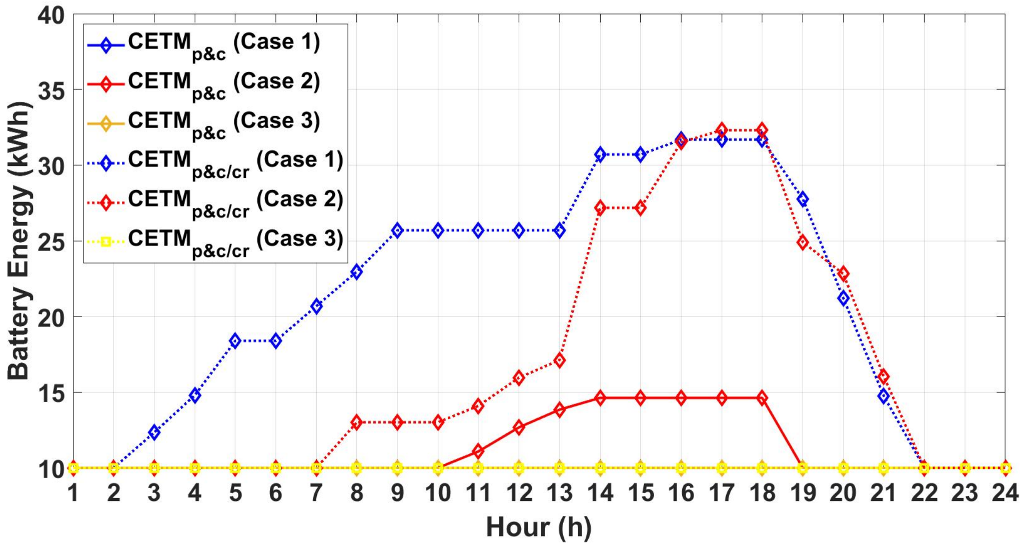


Figure 4.31: Battery energy profile for all cases in the second community.

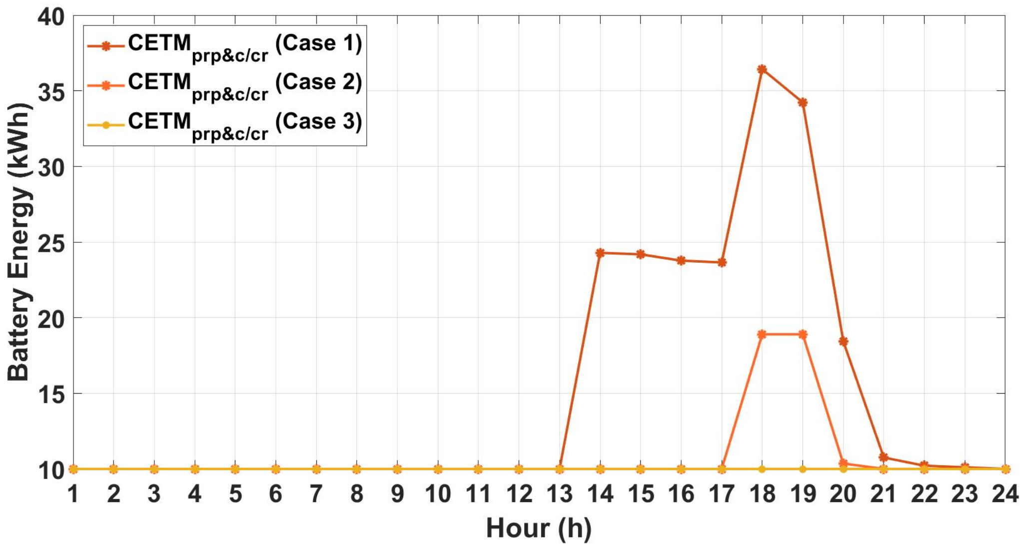


Figure 4.32: Battery energy profile for all cases in the third community.

costs for prosumers in the first community from Figure 4.27 is $CETM_{pr/cr}$ and $CETM_{pr}$ with and without community-owned generation respectively, the total energy traded costs for producers and consumers in the second community from Figure 4.28 is $CETM_{p\&c/cr}$ and $CETM_{p\&c}$ with and without community-owned generation respectively, and the total traded costs for all agents in the third community from Figure 4.29 is $CETM_{prp\&c/cr}$ with community-owned generation. The results reflect the agent preferences on the total energy traded costs for all cases analysed in the community microgrids. The total traded costs are lower with community-owned generation as it is cheaper to import from the community as compared to outside the community.

However, the total energy traded costs vary within the communities with/without utility maximization and with/without community-owned generation. The results of the total energy traded costs in the day, and total imports and exports are shown in Tables 4.1-4.5. The total traded costs of the community without utility maximization are higher at $165.65 and $253.25 without community-owned generation as compared to with community-owned generation at $113.81 and $211.32 in the first and second community microgrids. This is because it is cheaper to import from the community with community-owned generation than importing from the grid. In comparison to the total traded costs with utility maximization, the total traded costs are higher than the total costs without utility maximization for Cases 1 and 2 in the first community and all cases in the second community without community-owned generation. With community-owned generation, the total costs are lower for Cases 1 and 2 in the first and second communities. In the third community, the total costs are higher at $639.75 without utility maximization and with community-owned generation compared to Cases 1, 2, and 3 at $514.63, $550.63, and $545.52 respectively. These results show that in all communities with community-owned generation, and utility maximization, community agents' can set their preferences for more imports from the community than the grid resulting in lower energy traded costs, while their preferences for more imports from the community are impacted when there is no community-owned generation, resulting to more imports from the grid and therefore to higher traded costs.

The battery energy for all community microgrids is shown in Figures 4.30, 4.31, and 4.32. In all cases, in the community microgrid, the battery is used to satisfy the prosumers' and consumers' budgets and preferences by charging during the DER peak times as well as community-owned generation and discharging later in the day when there is no PV generation. In the first and second communities without community-owned generation, the battery stores lower levels of charge, due to the PV generation profile, but with the community-owned generation, the storage is charged more and supplied later in the day. In the third community with community generation, the battery storage is charged from both agents DERs and community generation in meeting the prosumer/consumer preferences.

4.6 Summary

The future power network within the smart grid paradigm is fast evolving and the dynamic changes occurring within the network concerning the system load and generation will result in expansions of the overall network. With the widespread acceptance of consumer-centric markets, this development should be scalable to preference relations of community agents engaged in energy trading concerning their consumption/production choices. To minimize imports/exports from/to the grid, policies can be initiated to limit agents' preferences depending on the choice of technology, production/consumption levels, energy prices, and weather variations given renewable technologies.

In this work, the impact of agent preferences on marginal costs of agents resources, weather variations, and resource availability is studied, to understand how energy trading in CBEMs as well as community fairness is affected. The results show agents' preferences in supporting more local energy trades within the community and therefore align their preferences in energy trading more within the community than outside the community/ grid. Furthermore, it is shown that in such markets, skewed by personal preference, the CES and community generation are necessary to satisfy the preferences through intertemporal energy transfer.

Chapter 5

A Robust Optimization Model of a Community-based Market with Agent Utility Maximization

In the future, the deployment of renewable energy systems into the existing traditional hierarchical power network will see accelerated growth as more entities within the network are acquiring assets of several distributed energy resources DERs to support the power supply and demand balance. The output levels from such power sources are usually stochastic with minimal assurance of the expected availability when needed. Given that these sources are non-dispatchable and differ from the known conventional sources, the need arises for the proposal of computationally tractable models [25] that are less conservative and realises an optimal solution in itself such as the Robust optimization (RO) [17]. RO models are defined over uncertainty sets with no scenarios as compared to the stochastic programming approach that deals with the probability distribution functions over a number of scenarios [16]. The solution realised from these models must be feasible across all realisation and should give an optimal solution against the worst-case realization of the uncertain parameters within the uncertainty sets. In [18, 129–132], the studies are focused on the application of RO to unit commitment problems in electricity markets. The application of a trilevel RO known as adaptive robust optimization is proposed where a decomposition technique such as the cutting plane approach is employed in solving the first-level objective function. In [90] stage and two-stage RO was applied to a generation expansion planning (GEP) problem using the mixed integer linear programming approach to model uncertainties related to the planning, forecasted generation, and load parameters as well as approximated investment and operation costs resulting to polyhedral uncertainty sets. References [108, 133–137] apply the RO in transmission planning to maximize the social welfare needed in making necessary investment decisions at the first level, and the worst case uncertainty realisation at the second stage and to minimize the overall operational costs of the system in the final stage.

An economic utility maximization approach was implemented in chapter four, to model agents' preferences in the choice of energy resource mix to satisfy their consumption/production levels. These preferences were subject to the budget constraints of the agents, which include: the consumption budget for the prosumers (consuming) and consumers and generation budget constraints for the prosumers (producing) and producers. RO methods have been applied to generation/transmission planning problems that involve minimizing some investment decision costs, unit commitment, and economic dispatch problems. The robust approach applied in [128] is updated to a single-stage robust optimization problem with consideration to the changing demand levels of prosumer/consumer agents and stochastic nature of generation resource of prosumer/producer agents which can result in changes in allocated budgets. Since the utility levels are constantly changing, and the agents' indifference curve at each instant of importing/exporting energy within or outside the community may vary, the need to model the uncertainties concerning the changing consumption patterns and intermittent renewable generation resources arises. Taguchi's orthogonal array testing (TOAT) technique was implemented in [128] to determine the worst-case uncertainty realization of uncertain parameter, in which the highest total energy traded costs and cost incurred due to robustness was realized for the robust scenario making it averse to risk. In this work, a single-stage robust optimization is implemented to model the uncertainty regarding the changing preferences of agents. The consumption and production budgets of the agents are taken as their reference values which is their expected demand/generation based on their budget.

The main contributions of this paper are presented as follows:

- A single-stage robust formulation of a community-based market is proposed, which models the changing utility levels of agents resulting from their preferences at any period t, in which all uncertain parameters are represented in a polyhedral uncertainty set.

- A comparative analysis of the robust solutions to the deterministic and opportunistic counterparts.

The rest of this chapter is organized as follows: The proposed solution algorithm and problem statement are presented in Section 5.1. In Section 5.2, the robust community-based market with utility maximization is analysed In Section 5.3, the test case study on the community microgrids is presented. Finally, Section 5.4 summarises the chapter.

5.1 Proposed Solution Algorithm

The uncertainty in the renewable generation of prosumer (producing)/ producer agents and the expected demand of prosumer (consuming)/consumer agents arise due to deviations from the expected value in the generation and consumption respectively. The generation budget for the prosumers'/producers' renewable generation resource and the consumption budget of the prosumers/ consumers are taken as the reference values for each agent's resources. The proposed CBEM problem subsequently minimises the total costs of energy to be imported and exported within/outside the community from the bounds $\tilde{p}^d_{n_rt} \leq p^d_{n_rt} \leq \tilde{p}^d_{n_rt}$ and $\tilde{p}^d_{n_dt} \leq p^d_{n_dt} \leq \tilde{p}^d_{n_dt}$ for prosumer/ consumer imports, and $\tilde{p}^g_{jn_rt} \leq p^g_{jn_rt} \leq \tilde{p}^g_{jn_rt}$ and $\tilde{p}^g_{ln_gt} \leq p^g_{ln_gt} \leq \tilde{p}^g_{ln_gt}$ for prosumer/ producer exports in representing agents' generation and consumption budgets, given the constrained budgets and uncertainty related to changes in preferences are within same bounds. The problem is formulated to maximize the uncertainty against the worst-case realisations of uncertain parameters within a defined polyhedral uncertainty set while minimizing the total energy traded costs of agents and the community manager. The robust approach is implemented by modelling a protection function to offer an ante rather than ex-ante protection for the agents' resources against risk. This protection model is proposed due to the budget constraints of agents which could allow them to change decisions on energy trades taken abruptly. For this purpose, control parameters $\Gamma^g_{jn_r}$ and $\Gamma^d_{n_r}$ for a prosumer's production and consumption resources, $\Gamma^g_{ln_g}$ for a producer, and $\Gamma^d_{n_d}$ for a consumer's resources, are modelled as part of the protection function to control the level of desired robustness by agents in trading their energy resources within and outside the community. Opportunistic and deterministic model results are used to compare the results from the proposed algorithm.

5.2 Robust Community-based Market with Utility Maximization Formulation

The robust counterpart of the CBEM market model integrating the utility maximization levels of agents with a community energy storage (CES) is presented. With consideration to the uncertainties concerning the consumption/generation budget, we represent the uncertain parameter as a polyhedron uncertainty set with deviation around the reference value. The uncertainty parameter $\tilde{p}^g_{jn_rt}, \tilde{p}^d_{n_rt}, \tilde{p}^g_{ln_gt}, \tilde{p}^d_{n_dt} \in R$ for vector $\tilde{p}^g_{jn_rt}, \tilde{p}^d_{n_rt}, \tilde{p}^g_{ln_gt}, \tilde{p}^d_{n_dt} \in \Omega^{USgr_t}, \Omega^{USdr_t}, \Omega^{USg_t}$ and Ω^{USd_t} respectively is expressed according to [129] as follows:

$$\Omega^{USgr_t} = [\overline{p}^g_{jn_rt} - \hat{p}^g_{jn_rt} \leq \tilde{p}^g_{jn_rt} \leq \overline{p}^g_{jn_rt} + \hat{p}^g_{jn_rt}] \quad \forall j \in R_g, \forall n_r \in N_{pr}, \forall t \in T \tag{5.1}$$

$$\Omega^{USdr_t} = [\overline{p}^d_{n_rt} - \hat{p}^d_{n_rt} \leq \tilde{p}^d_{n_rt} \leq \overline{p}^d_{n_rt} + \hat{p}^d_{n_rt}] \quad \forall n_r \in N_{pr}, \forall t \in T \tag{5.2}$$

$$\Omega^{USg_t} = [\overline{p}^g_{ln_gt} - \hat{p}^g_{ln_gt} \leq \tilde{p}^g_{ln_gt} \leq \overline{p}^g_{ln_gt} + \hat{p}^g_{ln_gt}] \quad \forall l \in R_p, \forall n_g \in N_p, \forall t \in T \tag{5.3}$$

$$\Omega^{USd_t} = [\overline{p}^d_{n_dt} - \hat{p}^d_{n_dt} \leq \tilde{p}^d_{n_dt} \leq \overline{p}^d_{n_dt} + \hat{p}^d_{n_dt}] \quad \forall n_d \in N_c, \forall t \in T \tag{5.4}$$

$$\sum_{n_r \in N_{pr}} \frac{|\tilde{p}^g_{jn_rt} - \overline{p}^g_{jn_rt}|}{\hat{p}^g_{jn_rt}} \leq \Gamma^g_{jn_r} \quad \forall j \in R_g, \forall n_r \in N_{pr} \forall t \in T \tag{5.5}$$

$$\sum_{n_r \in N_{pr}} \frac{|\tilde{p}^d_{n_rt} - \overline{p}^d_{n_rt}|}{\hat{p}^d_{n_rt}} \leq \Gamma^d_{n_r} \quad \forall n_r \in N_{pr} \forall t \in T \tag{5.6}$$

$$\sum_{n_g \in N_p} \frac{|\tilde{p}^g_{ln_gt} - \overline{p}^g_{ln_gt}|}{\hat{p}^g_{ln_gt}} \leq \Gamma^g_{ln_g} \quad \forall l \in R_p, \forall n_g \in N_p \forall t \in T \tag{5.7}$$

$$\sum_{n_d \in N} \frac{|\tilde{p}^d_{n_dt} - \overline{p}^d_{n_dt}|}{\hat{p}^d_{n_dt}} \leq \Gamma^d_{n_d} \quad \forall n_d \in N_c \forall t \in T \tag{5.8}$$

$$\tilde{p}^g_{jn_rt} = \overline{p}^g_{jn_rt} + z^g_{jn_r} \hat{p}^g_{jn_rt} \quad \forall j \in R_g, \forall n_r \in N_{pr} \forall t \in T \tag{5.9}$$

$$\tilde{p}^{d}_{n_rt} = \overline{p}^{d}_{n_rt} + z^{d}_{n_r}\hat{p}^{d}_{n_rt} \quad \forall n_r \in N_{pr} \forall t \in T \tag{5.10}$$

$$\tilde{p}^{g}_{ln_gt} = \overline{p}^{g}_{ln_gt} + z^{g}_{ln_g}\hat{p}^{g}_{ln_gt} \quad \forall l \in R_p, \forall n_g \in N_p \forall t \in T \tag{5.11}$$

$$\tilde{p}^{d}_{n_dt} = \overline{p}^{d}_{n_dt} + z^{d}_{n_d}\hat{p}^{d}_{n_dt} \quad \forall n_d \in N_c \forall t \in T \tag{5.12}$$

The size of the uncertainty sets for the agents resources Ω^{USgr_t}, Ω^{USdr_t}, Ω^{USg_t} and Ω^{USd_t} can be controlled based on different values of $\Gamma^{g}_{jn_r}$, $\Gamma^{d}_{n_r}$, $\Gamma^{g}_{ln_g}$, and $\Gamma^{d}_{n_d}$ respectively. The control parameter for each agent resource adjusts the degree of robustness or level of conservativeness from 0 to the respective range (symmetric deviation) provided for each nominal estimate of the agent resource. When $\Gamma^{g}_{jn_r}$, $\Gamma^{d}_{n_r}$, $\Gamma^{g}_{ln_g}$ and $\Gamma^{d}_{n_d}$ are equal to 0, it means the uncertainty variables of agents resources in Ω^{USgr_t}, Ω^{USdr_t}, Ω^{USg_t} and Ω^{USd_t} assume their reference values ($\overline{p}^{g}_{jn_rt}, \overline{p}^{d}_{n_rt}, \overline{p}^{g}_{lnt}$, and $\overline{p}^{d}_{n_dt}$) respectively and when equal to $\hat{p}^{g}_{jn_rt}, \hat{p}^{d}_{n_rt}, \hat{p}^{g}_{ln_gt}$ and $\hat{p}^{d}_{n_dt}$ the uncertain variables assume the upper estimates ($\overline{p}^{g}_{jn_rt}+\hat{p}^{g}_{jn_rt}, \overline{p}^{d}_{n_rt}+\hat{p}^{d}_{n_rt}\overline{p}^{g}_{ln_gt}+\hat{p}^{g}_{ln_gt}$, and $\overline{p}^{d}_{nt}+\hat{p}^{d}_{nt}$) [90] for the prosumers (producing/consuming), producers and consumer agents respectively. $z^{g}_{jn_r}, z^{d}_{n_r}, z^{g}_{ln_g}$, and $z^{d}_{n_d}$ are modelled as variables of the protection function in which the functions must satisfy the variable bounds.

5.2.1 Robust Problem Formulation

The proposed robust model of the CBEM with utility maximization is a single-stage max-min model as follows:

$$\max_{U_{gr_t}\in\Omega^{USgr_t}, U_{dr_t}\in\Omega^{USdr_t}, U_{gt}\in\Omega^{USg_t}, U_{dt}\in\Omega^{USd_t}} \; \min_{X_{n_rt}\in\Omega^{X_{n_rt}}, W_{n_gt}\in\Omega^{W_{n_gt}} Y_{n_dt}\in\Omega^{Y_{n_dt}}, Z_t\in\Omega^{Z_t}}$$
$$\sum_{j=1}^{R_g}\sum_{n_r=1}^{N_{pr}}\sum_{t=1}^{T}\varphi_{n_r}(X_{n_rt}, U_{gr_t}, U_{d_rt}) + \sum_{l=1}^{R_p}\sum_{n_g=1}^{N_p}\sum_{t=1}^{T}\phi_{n_g}(W_{n_gt}, U_{gt}) + \tag{5.13}$$
$$\sum_{n_d=1}^{N_c}\sum_{t=1}^{T}\phi_n(Y_{n_dt}, U_{dt}) + \sum_{t=1}^{T} g(Z_t, U_{g_rt}, U_{d_rt}, U_{gt}, U_{dt})$$

The robust model aims to maximize the worst-case uncertainty realization of agents' DER resources while minimizing the total energy traded costs within/ outside the community. The robust problem formulation for prosumer agents is presented as follows:

$$\min \ \alpha(p^g_{n_rt}, p^d_{n_rt}, r^c_{n_rtm}, q_{n_rtm}, \gamma_{n_rtm}, G_{n_rt}, \delta_{n_rtm}, \gamma_{impt}, \delta_{expt}) \tag{5.14a}$$

$$\text{s.t. } \alpha \geq \sum_{j=1}^{R_g}\sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}} \varphi_{jn_r}(p^g_{jn_rt}) + \sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}} \phi_{n_r}(p^d_{n_rt}) +$$
$$\xi_{com}\sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}} r^c_{n_rtm} + \xi_{com}\sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}} q_{n_rtm} + s_{\lambda impt}\sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}} (\gamma_{n_rtm} + G_{n_rt}) + \tag{5.14b}$$
$$s_{\lambda expt}\sum_{t=1}^{T}\sum_{n_r=1}^{N_{pr}} \delta_{n_rtm} + \sum_{t=1}^{T} g(\gamma_{impt}, \delta_{expt}),$$

$$(4.20\text{c}) - (4.20\text{m}) \tag{5.14c}$$

$$p^g_{jn_rt} - \overline{p}^g_{jn_rt} + \underbrace{\max_{z^g_{n_r}}[(z^g_{jn_r}\hat{p}^g_{jn_rt})p_{gr}]}_{\beta^g_{0r}} \leq 0 \quad \forall n \in N_{pr}, \forall j \in R_g, \forall t \in T \tag{5.14d}$$

$$-p^d_{n_rt} + \overline{p}^d_{n_rt} + \underbrace{\max_{z^d_{n_r}}[(z^d_{n_r}\hat{p}^d_{n_rt})p_{dr}]}_{\beta^d_{0r}} \leq 0, \quad \forall n \in N_c, \forall t \in T \tag{5.14e}$$

$$p^g_{jn_rt} - \tilde{p}^g_{jn_rt} \leq 0, \quad \forall n_r \in N_{pr}, \forall t \in T \tag{5.14f}$$

$$p^d_{n_rt} - \tilde{p}^d_{n_rt} \leq 0, \quad \forall n_r \in N_{pr}, \forall t \in T \tag{5.14g}$$

$$\overline{p}^g_{jn_rt} - \hat{p}^g_{jn_rt} \leq \tilde{p}^g_{jn_rt} \leq \overline{p}^g_{jn_rt} + \hat{p}^g_{jn_rt}, \quad \forall n_r \in N_{pr}, \forall j \in R_g, \forall t \in T \tag{5.14h}$$

$$\overline{p}^d_{n_rt} - \hat{p}^d_{n_rt} \leq \tilde{p}^d_{n_rt} \leq \overline{p}^d_{n_rt} + \hat{p}^d_{n_rt}, \quad \forall n_r \in N_{pr}, \forall j \in R_g, \forall t \in T \tag{5.14i}$$

$$\tilde{p}^g_{jn_rt} \leq p^g_{jn_rt} \leq \tilde{p}^g_{jn_rt}, \quad \forall n_r \in N_{pr}, \forall j \in R_g, \forall t \in T \tag{5.14j}$$

$$\tilde{p}^d_{n_rt} \leq p^d_{n_rt} \leq \tilde{p}^d_{n_rt}, \quad \forall n_r \in N_{pr}, \forall j \in R_g, \forall t \in T \tag{5.14k}$$

$$(4.20\text{p}) - (4.20\text{t}) \tag{5.14l}$$

$$(4.18) \ \& \ (4.19) \tag{5.14m}$$

The proposed min-max problem of (5.14) minimizes the total traded costs of the prosumers and community manager relating to the worst-case uncertainty realizations of prosumers' production and consumption. Constraints (5.14d)/(5.14e) integrates a protection function β^g_{0r} and β^d_{0r} for prosumers' production and consumption respectively to offer controlled protection against the worst case uncertain parameter realisations of prosumer agents. Constraints (5.14f)/(5.15g) gives the relationship between the energy production/consumption of prosumer agents' resources and the uncertainty of the resource. Constraints (5.14h)/(5.14i) gives the bounds of uncertain parameters of prosumers' production and consumption resources respectively, similar to the polyhedron uncertainty set of equations (5.1) and (5.2). Constraints (5.14j)/(5.14k) give the bounds for the net production/consumption of prosumer resources with the uncertain parameter as lower and upper bounds. Likewise, the robust community market formulation for producers and consumers is given as follows:

$$\min \ \alpha(p^g_{ln_gt}, p^d_{n_dt}, r^c_{n_dtm}, q_{n_gtm}, \gamma_{n_dtm}, R_{n_dt}, \delta_{n_gtm}, \gamma_{impt}, \delta_{expt}) \tag{5.15a}$$

$$\text{s.t. } \alpha \geq \sum_{l=1}^{R_p}\sum_{t=1}^{T}\sum_{n_g=1}^{N_p} \varphi_{ln_g}(p^g_{ln_gt}) + \sum_{t=1}^{T}\sum_{n_d=1}^{N_c} \phi_{n_d}(p^d_{n_dt}) +$$
$$\xi_{com}\sum_{t=1}^{T}\sum_{n_d=1}^{N_c} r^c_{n_dtm} + \xi_{com}\sum_{t=1}^{T}\sum_{n_g=1}^{N_p} q_{n_gtm} + s_{\lambda impt}\sum_{t=1}^{T}\sum_{n_d=1}^{N_c}(\gamma_{n_dtm} + R_{n_dt}) + \tag{5.15b}$$
$$s_{\lambda expt}\sum_{t=1}^{T}\sum_{n_g=1}^{N_p}(\delta_{n_gtm} + \sum_{t=1}^{T} g(\gamma_{impt}, \delta_{expt}),$$

$$(4.21c) \ - \ (4.21j) \tag{5.15c}$$

$$p^g_{ln_gt} - \overline{p}^g_{ln_gt} + \underbrace{\max_{z^g_{n_g}}[(z^g_{ln_g}\hat{p}^g_{ln_gt})p_g]}_{\beta^g_0} \leq 0 \quad \forall n_g \in N_p, \forall l \in R_p, \forall t \in T \tag{5.15d}$$

$$-p^d_{n_dt} + \overline{p}^d_{n_dt} + \underbrace{\max_{z^d_{n_d}}[(z^d_{n_d}\hat{p}^d_{n_dt})p_d]}_{\beta^d_0} \leq 0, \quad \forall n_d \in N_c, \forall t \in T \tag{5.15e}$$

$$p^g_{ln_gt} - \tilde{p}^g_{ln_gt} \leq 0, \quad \forall n_g \in N_p, \forall t \in T \tag{5.15f}$$

$$p^d_{n_dt} - \tilde{p}^d_{n_dt} \leq 0, \quad \forall n_d \in N_c, \forall t \in T \tag{5.15g}$$

$$\overline{p}^g_{ln_gt} - \hat{p}^g_{ln_gt} \leq \tilde{p}^g_{ln_gt} \leq \overline{p}^g_{ln_gt} + \hat{p}^g_{ln_gt}, \quad \forall n_g \in N_p, \forall l \in R_p, \forall t \in T \tag{5.15h}$$

$$\overline{p}^d_{n_dt} - \hat{p}^d_{n_dt} \leq \tilde{p}^d_{n_dt} \leq \overline{p}^d_{n_dt} + \hat{p}^d_{n_dt}, \quad \forall n_d \in N_c, \forall t \in T \tag{5.15i}$$

$$\tilde{p}^d_{n_rt} \leq p^g_{ln_gt} \leq \tilde{p}^g_{ln_gt}, \quad \forall n_g \in N_p, \forall l \in R_p, \forall t \in T \tag{5.15j}$$

$$\tilde{p}^d_{n_dt} \leq p^d_{n_dt} \leq \tilde{p}^d_{n_dt}, \quad \forall n_d \in N_c, \forall t \in T \tag{5.15k}$$

$$(4.20p) - (4.20s) \tag{5.15l}$$

$$\gamma_{n_dtm} \geq 0, \ \delta_{n_gtm} \geq 0, R_{n_dt} \geq 0 \quad \forall n_g \in N_p, \forall n_d \in N_c, \forall t \in T \tag{5.15m}$$

$$(4.15) \ \& \ (4.17) \tag{5.15n}$$

The objective of (5.15) minimizes the total traded costs of producers and consumers with relation to the worst-case uncertainty realisations of the agents' resources. Constraints (5.15d) to (5.15l) for producer and consumer agents serves the same functions as (5.14d) to (5.14l) for prosumer agents. The robust community-based market problem is also applied to a community of all agents including prosumers, producers, and consumers. The objective function and constraints of (4.22) in addition to constraint equations (5.14d)-(5.14k) for prosumers and (5.15d)-(5.15k) for producers and consumers are implemented. Likewise, in integrating the community generation, equation (4.23) in addition to constraint equations (5.14d)-(5.14k) is implemented for a community of prosumers, equation (4.24) in addition to (5.15d)-(5.15k) is implemented for a community of producers and consumers and (4.23) in addition to constraint equations (5.14d)-(5.14k) and (5.15d)-(5.15k) is implemented for a community of all agents which includes prosumers, producers and consumers.

The optimization variables $z^g_{jn_r}$ and $z^d_{n_r}$ for prosumers, and $z^g_{ln_g}$ and $z^d_{n_d}$ for producers and consumers from the proposed protection function are expressed in relation to the uncertain generation and consumption parameters of equations (5.9) to (5.12). Every protection function β^g_{0r}, β^d_{0r}, β^g_0 and β^d_0 should satisfy the bounds for the variables providing the needed robustness at different values of uncertainty budget $\Gamma^g_{jn_r}$, $\Gamma^d_{n_r}$, $\Gamma^g_{ln_g}$, and $\Gamma^d_{n_d}$ respectively [90]

$$0 \leq z^g_{jn_r} \leq 1; \sum_{j \in Rg} z^g_{jn_r} \leq \Gamma^g_{jn_r} \ \forall n_r \in N_{pr} \tag{5.16}$$

$$0 \leq z^d_{n_r} \leq 1; \sum_{n_r \in N_{pr}} z^d_{n_r} \leq \Gamma^d_{n_r} \tag{5.17}$$

$$0 \leq z^g_{ln_g} \leq 1; \sum_{l \in R_p} z^g_{ln_g} \leq \Gamma^g_{ln_g} \ \forall n_g \in N_p \tag{5.18}$$

$$0 \leq z^d_{n_d} \leq 1; \sum_{n_d \in N_c} z^d_{n_d} \leq \Gamma^d_{n_d} \tag{5.19}$$

The protection functions β^{gr}_0, β^{dr}_0, β^g_0, and β^d_0 can be formulated into minimization problems by applying duality theory as follows:

$$\beta_{0r}^{g} = \min_{b_{rt}^{g} \forall a_{n_r t}^{g}} \sum_{n_r \in N_{pr}} a_{n_r t}^{g} + \Gamma_{jn_r}^{g} b_{rt}^{g} \quad \forall j \in R_g \forall n_r \in N_{pr} \forall t \in T \tag{5.20}$$

$$\text{s.t} \quad a_{n_r t}^{g} + b_{rt}^{g} \geq \hat{p}_{n_r t}^{g} p_{gr} \quad \forall n_r \in N_{pr}, \forall t \in T \tag{5.21}$$

$$\beta_{0r}^{d} = \min_{b_{rt}^{d} \forall a_{n_r t}^{d}} \sum_{n_r \in N_{pr}} a_{n_r t}^{d} + \Gamma_{n_r}^{d} b_{rt}^{d} \quad \forall n_r \in N_{pr} \forall t \in T \tag{5.22}$$

$$\text{s.t} \quad a_{n_r t}^{d} + b_{rt}^{d} \geq \hat{p}_{n_r t}^{d} p_{dr} \quad \forall n_r \in N_{pr}, \forall t \in T \tag{5.23}$$

$$\beta_{0}^{g} = \min_{b_{t}^{g} \forall a_{n_g t}^{g}} \sum_{n_g \in N_p} a_{n_g t}^{g} + \Gamma_{ln_g}^{g} b_{t}^{g} \quad \forall l \in R_p \; n_g \in N_p \forall t \in T \tag{5.24}$$

$$\text{s.t} \quad a_{n_g t}^{g} + b_{t}^{g} \geq \hat{p}_{n_g t}^{g} p_{g} \quad \forall n_g \in N_p, \forall t \in T \tag{5.25}$$

$$\beta_{0}^{d} = \min_{b_{t}^{d} \forall a_{n_d t}^{d}} \sum_{n_d \in N_c} a_{n_d t}^{d} + \Gamma_{n_d}^{d} b_{t}^{d} \quad \forall t \in T \tag{5.26}$$

$$\text{s.t} \quad a_{n_d t}^{d} + b_{t}^{d} \geq \hat{p}_{n_d t}^{d} p_{d} \quad \forall n_d \in N_c, \forall t \in T \tag{5.27}$$

$$\begin{gathered} a_{n_r t}^{g} \geq 0 \; b_{rt}^{g} \geq 0; a_{n_r t}^{d} \geq 0 \; b_{rt}^{d} \geq 0; \\ p_{gr} \geq 0 \; p_{dr} \geq 0 \; \forall n_r \in N_{pr}, \; \forall t \in T \end{gathered} \tag{5.28}$$

$$\begin{gathered} a_{n_g t}^{g} \geq 0 \; b_{t}^{g} \geq 0; a_{n_d t}^{d} \geq 0 \; b_{t}^{d} \geq 0; \\ p_{g} \geq 0 \; p_{d} \geq 0 \quad \forall n_g \in N_p, \forall n_d \in N_c \; \forall t \in T \end{gathered} \tag{5.29}$$

Equation (5.14d), (5.14e), (5.15d) and (5.15e) is rewritten by integrating the updated protection function from duality theory as follows:

$$\tilde{p}_{jn_r t}^{g} - \underline{p}_{jn_r t}^{g} + \underbrace{a_{n_r t}^{g} + \Gamma_{jn_r}^{g} b_{rt}^{g}}_{\beta_{0r}^{g}} \leq 0 \quad \forall j \in R_g, \forall n_r \in N_{pr}, \forall t \in T \tag{5.30}$$

$$-\tilde{p}_{n_r t}^{d} + \underline{p}_{n_r t}^{d} + \underbrace{a_{n_r t}^{d} + \Gamma_{n_r}^{d} b_{rt}^{d}}_{\beta_{0r}^{d}} \leq 0 \quad \forall n_r \in N_{pr}, \forall t \in T \tag{5.31}$$

$$\tilde{p}^g_{ln_gt} - \underline{p}^g_{ln_gt} + \underbrace{a^g_{n_gt} + \Gamma^g_{ln_g} b^g_t}_{\beta^g_0} \leq 0 \quad \forall l \in R_p, \forall n_g \in N_p, \forall t \in T \tag{5.32}$$

$$-\tilde{p}^d_{n_dt} + \underline{p}^d_{n_dt} + \underbrace{a^d_{n_dt} + \Gamma^d_{n_d} b^d_t}_{\beta^d_0} \leq 0 \quad \forall n_d \in N_c, \forall t \in T \tag{5.33}$$

The equations (5.30) and (5.31) substituting equations (5.14d) and (5.14e), in addition to constraint equations (5.21), (5.23), and (5.28) are integrated to the equation (5.14) to solve the robust community market model of prosumers as well as the model allowing for the integration of community-owned generation. Also, the equations (5.32) and (5.33) substituting equations (5.15d) and (5.15e), in addition to constraint equations (5.25), (5.27) and (5.29) are integrated to the equation (5.15) to solve the robust community market model of producers and consumers together with the model allowing for community integration. Likewise, the community market problem of all agents implements all constraint equations for prosumers, producers and consumers as well as with community integration.

5.2.2 Opportunistic model Formulation

In [128], the opportunistic model was determined using the scenarios generated by TOAT. In this work, this model which is the opportunistic solution given the best-case realisation of uncertain parameters is obtained by modifying the robust formulation. By modifying constraints (5.14d) and (5.14e), (5.15d) and (5.15e), and (5.30)-(5.33), the robust CBEM market formulation of (5.14) and (5.15) becomes opportunistic models as follows:

$$p^g_{n_rt} - \underline{p}^g_{n_rt} - \underbrace{\max_{z^g_{jn_r}}[(z^g_{jn_r}\hat{p}^g_{jn_rt})p_{gr}]}_{\beta^g_{0r}} = 0 \quad \forall j \in R_g \forall n_r \in N_p, \forall t \in T \tag{5.34}$$

$$p^d_{n_rt} - \underline{p}^d_{n_rt} + \underbrace{\max_{z^d_{n_r}}[(z^d_{n_r}\hat{p}^d_{n_rt})p_{dr}]}_{\beta^d_{0r}} \leq 0 \quad \forall n_r \in N_p, \forall t \in T \tag{5.35}$$

$$p^g_{ln_gt} - \underline{p}^g_{ln_gt} - \underbrace{\max_{z^g_{ln_g}}[(z^g_{ln_g}\hat{p}^g_{ln_gt})p_g]}_{\beta^g_0} \leq 0 \quad \forall l \in R_p, \forall n_g \in N_p, \forall t \in T \tag{5.36}$$

$$p^d_{n_dt} - \underline{p}^d_{n_dt} + \underbrace{\max_{z^d_{n_d}}[(z^d_{n_d}\hat{p}^d_{n_dt})p_d]}_{\beta^d_0} \leq 0 \quad \forall n_d \in N_c, \forall t \in T \tag{5.37}$$

Equations (5.34) to (5.37) is substituted with the protection function reflecting the opportunistic scenario as follows:

$$\tilde{p}^g_{jn_rt} - \underline{p}^g_{jn_rt} - \underbrace{a^g_{n_rt} - \Gamma^g_{jn_r} b^g_{rt}}_{\beta^g_{0r}} \leq 0 \quad \forall j \in R_g \forall n_r \in N_{pr}, \forall t \in T \tag{5.38}$$

$$\tilde{p}^d_{n_rt} - \underline{p}^d_{n_rt} + \underbrace{a^d_{n_rt} + \Gamma^d_{n_r} b^d_{rt}}_{\beta^d_{0r}} \leq 0 \quad \forall n \in N_{pr}, \forall t \in T \tag{5.39}$$

$$\tilde{p}^g_{n_gt} - \underline{p}^g_{n_gt} - \underbrace{a^g_{n_gt} - \Gamma^g_{ln_g} b^g_t}_{\beta^g_0} \leq 0 \quad \forall l \in R_p, \forall n_g \in N_p, \forall t \in T \tag{5.40}$$

$$\tilde{p}^d_{n_dt} - \underline{p}^d_{n_dt} + \underbrace{a^d_{n_dt} + \Gamma^d_{n_d} b^d_t}_{\beta^d_0} \leq 0 \quad \forall n_d \in N_c, \forall t \in T \tag{5.41}$$

5.3 Numerical Case Study

The proposed robust community-based market model is tested on three community microgrids from chapter four. The three cases analysed for testing the sensitivity of the proposed economic utility maximization model are also implemented for the robust model. All parameter values already in use are implemented in addition to a symmetric deviation of 10% from deterministic values. The uncertainty budgets of agents' resources $\Gamma^g_{jn_r}$, $\Gamma^d_{n_r}$, $\Gamma^g_{ln_g}$, and $\Gamma^d_{n_d}$ can assume values between 0 and 1, in the control of robustness of the optimal solutions with regards uncertainty in agent's generation and demand. The results are simulated when $\Gamma^g_{jn_r} = \Gamma^d_{n_r} = \Gamma^g_{ln_g} = \Gamma^d_{n_d} = 1$. The maximum robust solution is realised at this value (i.e.$= 1$), and this provides the total energy traded costs immunized completely against generation and demand uncertainty. Given lower levels of the uncertainty budgets,(1.e ≤ 0.1) the symmetric deviation has little or no effect on the total energy traded costs resulting in deterministic or near deterministic values [88]. However, the robust results are more visible when the uncertainty budgets exceed 0.1 with symmetric deviations more than 5%. The results of the total energy traded costs for all cases analysed for the robust, deterministic, and opportunistic models for $\Gamma_{jn_r} = \Gamma_{n_r} = \Gamma_{ln_g} = \Gamma_{n_d} = 0.1$, 0.9

Table 5.1: Robust CBEM Trading Model for all Cases in the First Community without Community-owned Generation

CBEM Trading Model		Total Traded Costs ($) at different Γ levels			Detm. Total costs	Imports/Exports (kWh) at $\Gamma = 1$		COR
Model	Cases	$\Gamma = 0.1$	$\Gamma = 0.9$	$\Gamma = 1$	Total Cost ($)	Total Import	Total Export	at $\Gamma = 1$
$RCETM_{pr}$	Case 1	212.21	221.10	217.33	-	299.22	43.18	23.37
	Case 2	185.53	192.70	190.96	-	258.66	0.00	24.87
	Case 3	182.80	190.35	189.23	-	256.04	0.00	25.45
$DCETM_{pr}$	Case 1	-	-	-	193.96	-	-	-
	Case 2	-	-	-	166.09	-	-	-
	Case 3	-	-	-	163.78	-	-	-
$BCETM_{pr}$	Case 1	182.03	178.01	173.87	-	233.94	44.93	-
	Case 2	154.82	149.28	149.11	-	196.46	0.00	-
	Case 3	151.90	147.39	146.67	-	192.53	0.00	-

Table 5.2: Robust CBEM Trading Model for all Cases in the First Community with Community-owned Generation

CBEM Trading Model		Total Traded Costs ($) at different Γ levels			Detm. Total costs	Imports/Exports (kWh) at $\Gamma = 1$		COR
Model	Cases	$\Gamma = 0.1$	$\Gamma = 0.9$	$\Gamma = 1.0$	Total Cost ($)	Total Import	Total Export	at $\Gamma = 1$
$RCETM_{pr}$	Case 1	117.19	124.61	123.03	-	110.10	51.07	9.34
	Case 2	109.55	115.79	114.48	-	104.61	11.54	7.58
	Case 3	121.43	173.06	126.86	-	132.64	3.19	9.70
$DCETM_{pr}$	Case 1	-	-	-	113.69	-	-	-
	Case 2	-	-	-	106.90	-	-	-
	Case 3	-	-	-	117.16	-	-	-
$BCETM_{pr}$	Case 1	109.39	106.75	104.34	-	90.08	67.87	-
	Case 2	101.36	99.28	97.49	-	85.58	31.60	-
	Case 3	109.87	108.24	107.20	-	108.52	23.69	-

and 1 in the three communities are shown in Tables 5.1-5.5. The tables also present the total imports and exports and the cost of robustness when $\Gamma^g_{jn_r} = \Gamma^d_{n_r} = \Gamma^g_{ln_g} = \Gamma^d_{n_d} = 1$, which is the difference in total energy traded costs of the robust models and the deterministic models. These results are analysed with and without consideration of the community generation.

5.3.1 Discussions

The robust, deterministic, and opportunistic solutions for the three cases analysed for each community microgrid are presented. In all the communities the results from the figures for each community presented include the total production and consumption, the total energy imported, the total energy traded costs, the battery energy profiles, and the cost of robustness. These are compared with and without community-owned generation for the first and second communities,

Table 5.3: Robust CBEM Trading Model for all Cases in the Second Community without Community-owned Generation

CBEM Trading Model		Total Traded Costs ($) at different Γ levels			Detm. Total costs	Imports/Exports (kWh) at Γ = 1		COR
Model	Cases	Γ = 0.1	Γ = 0.9	Γ = 1.0	Total Cost ($)	Total Import	Total Export	at Γ = 1
$RCETM_{p\&c}$	Case 1	321.15	335.59	333.01	-	390.46	42.48	36.96
	Case 2	294.58	307.10	305.30	-	347.99	0.00	39.36
	Case 3	297.27	306.56	305.30	-	347.99	0.00	39.99
$DCETM_{p\&c}$	Case 1	-	-	-	296.05	-	-	-
	Case 2	-	-	-	265.94	-	-	-
	Case 3	-	-	-	265.31	-	-	-
$BCETM_{p\&c}$	Case 1	271.28	266.82	264.52	-	307.40	45.00	-
	Case 2	248.12	241.29	239.18	-	268.47	0.00	-
	Case 3	244.96	240.80	238.59	-	267.58	0.00	-

Table 5.4: Robust CBEM Trading Model for all Cases in the Second Community with Community-owned Generation

CBEM Trading Model		Total Traded Costs ($) at different Γ levels			Detm. Total costs	Imports/Exports (kWh) at Γ = 1		COR
Model	Cases	Γ = 0.1	Γ = 0.9	Γ = 1.0	Total Cost ($)	Total Import	Total Export	at Γ = 1
$RCETM_{p\&c}$	Case 1	192.53	207.78	211.25	-	139.04	53.65	21.57
	Case 2	178.97	195.59	196.80	-	126.51	4.35	21.16
	Case 3	216.28	281.33	237.27	-	206.68	18.11	24.1
$DCETM_{p\&c}$	Case 1	-	-	-	189.68	-	-	-
	Case 2	-	-	-	175.64	-	-	-
	Case 3	-	-	-	213.09	-	-	-
$BCETM_{p\&c}$	Case 1	169.56	169.75	168.53	-	113.76	64.95	-
	Case 2	159.34	159.62	158.05	-	103.51	14.82	-
	Case 3	193.31	193.32	193.48	-	169.10	33.60	-

Table 5.5: Robust CBEM Trading Model for all Cases in the Third Community with Community-owned Generation

CBEM Trading Model		Total Traded Costs ($) at different Γ levels			Detm. Total costs	Imports/Exports(kWh) at Γ = 1		COR
Model	Cases	Γ = 0.1	Γ = 0.9	Γ = 1.0	Total Cost ($)	Total Import	Total Export	at Γ = 1
$RCETM_{prp\&c}$	Case 1	569.24	589.16	562.21	-	388.45	226.25	47.58
	Case 2	568.95	575.79	619.52	-	388.11	346.14	95.58
	Case 3	553.61	688.55	608.66	-	536.594	84.57	63.14
$DCETM_{prp\&c}$	Case 1	-	-	-	514.63	-	-	-
	Case 2	-	-	-	523.94	-	-	-
	Case 3	-	-	-	545.52	-	-	-
$BCETM_{prp\&c}$	Case 1	503.36	498.37	505.09	-	317.82	592.49	-
	Case 2	511.73	488.99	434.50	-	317.55	167.46	-
	Case 3	491.66	490.77	492.77	-	439.03	152.79	-

and with community-owned generation for the third community.

In the first community the figures include Figures 5.1 and 5.2 without and with community-owned generation respectively for Cases 1. The costs of robustness for each case in the first community are presented in figure 5.3 without community-owned generation and figure 5.4 with community-owned generation. For the second community, the figures include Figures 5.5 and 5.6 without and with community-owned generation respectively for Cases 2. Also, the costs of robustness for each case in the second community are presented in 5.7 without community-owned generation and 5.8 with community-owned generation. For the third community, the figure 5.9 is shown with community-owned generation for case 3. Also, the costs of robustness for each case in the third community are presented in figure 5.10 with community-owned generation.

The results for the total production and consumption for all three communities with and without community-owned generation reflect the worst and best-case uncertainty realisations of the generation and consumption resources of community agents. In the robust scenario, the agent's generation resources is within the lower bounds of the uncertainty generation sets while the consumption is within the upper bounds of the uncertainty sets. In the deterministic scenario, the generation/consumption are the expected values of generation and consumption from the agents' resources, while in the opportunistic scenario generation resources are within the upper bounds of the uncertainty generation sets and the consumption is within the lower bounds of the uncertainty sets.

The results for the total energy imported and exported for all three communities with and without community-owned generation reflect the preference relations that maximizes the utility of community agents in trading their energy resources within the community and the grid. The total energy imported/exported to the community is lower for the robust scenarios as compared to the deterministic and opportunistic scenarios. The total energy imported from the grid is higher for the robust scenario compared to the deterministic and opportunistic scenarios for cases without community-owned generation and higher for the robust scenario for total energy imported from the community and the grid for cases with community generation. The total energy exported to the community is lower for the robust scenarios in the communities with and without community-owned generation.

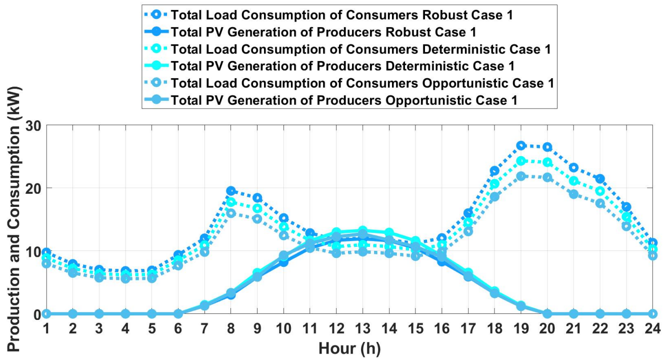
From Tables 5.1-5.5, the total energy traded costs for the robust models in the three communities $RCETM_{pr}$, $RCETM_{p\&c}$, and $RCETM_{prp\&c}$ are higher when compared to the deterministic models $DCETM_{pr}$, $DCETM_{p\&c}$, and $DCETM_{prp\&c}$, while the total energy traded costs for the deterministic models are higher than the costs obtained for the opportunistic models $BCETM_{pr}$, $BCETM_{p\&c}$, and $BCETM_{prp\&c}$. These results of the robust models are higher with and without community-owned generation ($RCETM_{pr/cr}$, $RCETM_{p\&c/cr}$, and $DCETM_{prp\&c/cr}$,$DCETM_{pr/cr}$, $DCETM_{p\&c/cr}$, and $DCETM_{prp\&c/cr}$, $BCETM_{pr/cr}$, $BCETM_{p\&c/cr}$, and $BCETM_{prp\&c/cr}$) at varying values of the control parameter at 0.1, 0.9, and 1 of all agents resources. The total imports from the grid are higher for the robust models as compared to the deterministic and opportunistic models. The cost of robustness at $\Gamma^g_{jn_r} = \Gamma^d_{jn_r} = \Gamma^g_{ln_g}, \Gamma^d_{n_d} = 1$ is \$23.37, \$24.87, *and* \$25.45 and \$9.34, \$7.58, *and* \$9.70 for the first community with and without community-owned generation, \$36.96, \$39.36, *and* \$39.99 and \$21.57, \$21.16, *and* \$24.10 for the second community with and without community-owned generation and \$47.58, \$95.58, *and* \$63.14 for the third community with community-owned generation.

In all three communities, the results of the battery profiles show the state of charge and discharge in balancing the energy trades that maximises the utility of all community agents for all cases and trading models analysed.

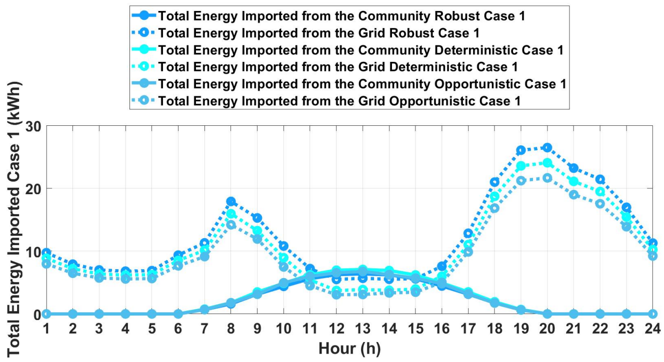
The robust solutions are obtained at the worst-case uncertainty realisation of agents' generation (lower) and demand (higher), the deterministic solutions were obtained from nominal/expected values of generation/demand while the opportunistic solutions are obtained from the best-case uncertainty realisations of agents' generation (higher) and demand(lower). The results of agents' imports and exports show their utility was maximised for their imports/exports within and outside the community.

5.4 Summary

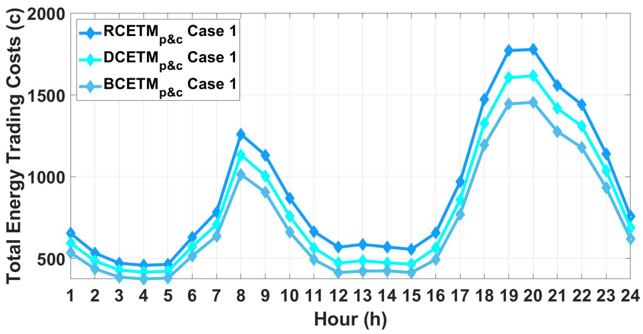
This chapter presents a robust approach to a community-based market of agents having preference relations and determining the amount of energy imports/exports in/out of the community, within constrained generation/consumption budgets subject to uncertainty. The results of the three cases analysed for three community microgrids reflect higher costs and costs of robustness for the proposed robust model as compared to the deterministic and opportunistic models at the same degree of robustness. The agents' preferences for imports and exports given their budgets were also satisfied with community-owned generation.



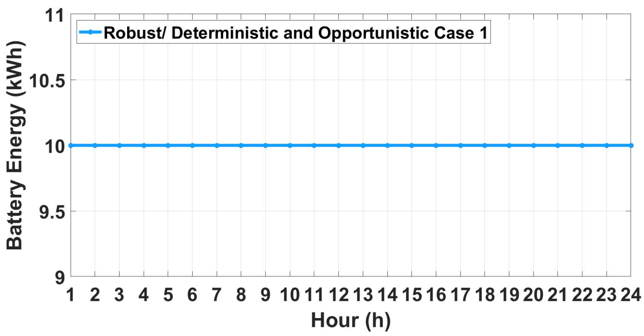
(a) Total production and consumption without community-owned generation in the first community for Case 1.



(b) Total energy imported without community-owned generation in the first community for Case 1.

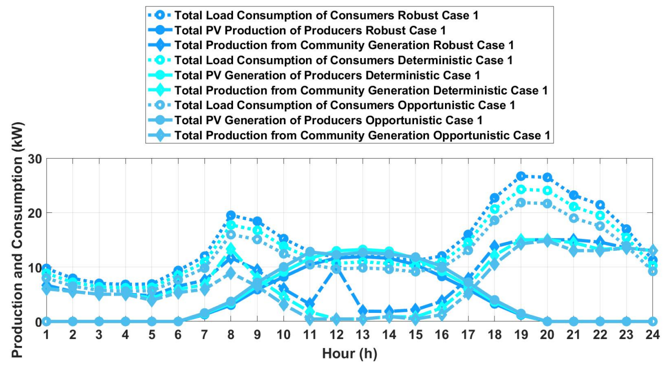


(c) Total energy traded costs without community-owned generation in the first community for Case 1.

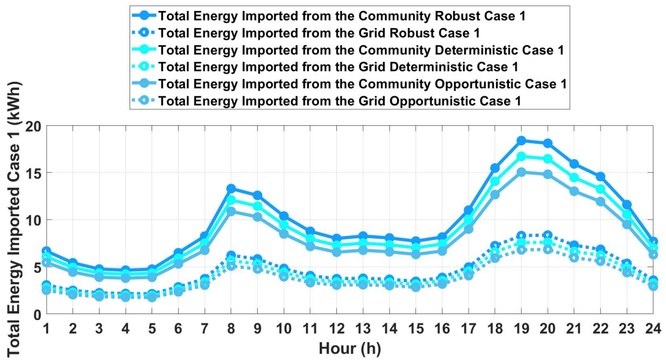


(d) Battery energy profile of CES without community-owned generation in the first community for Case 1.

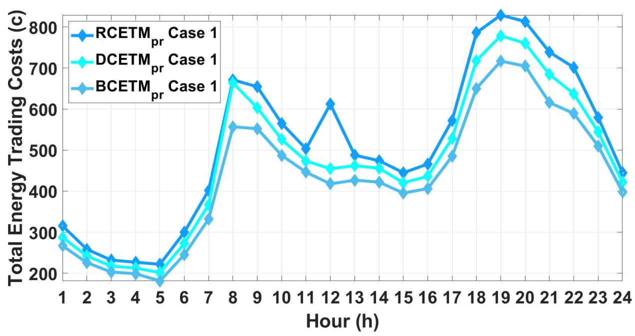
Figure 5.1: Robust, Deterministic and Opportunistic solutions without community-owned generation for the first community for Case 1.



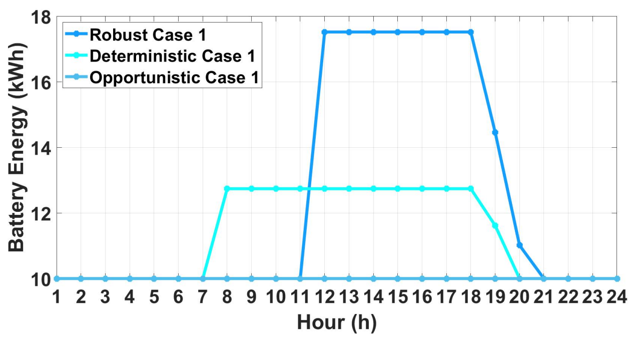
(a) Total production and consumption with community-owned generation in the first community for Case 1.



(b) Total energy imported with community-owned generation in the first community for Case 1.

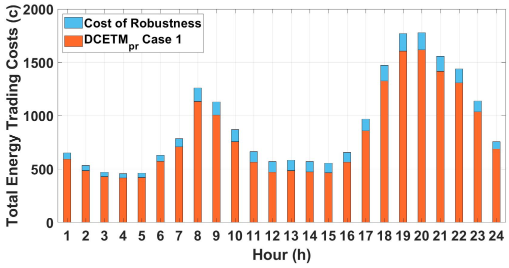


(c) Total energy traded costs with community-owned generation in the first community for Case 1.

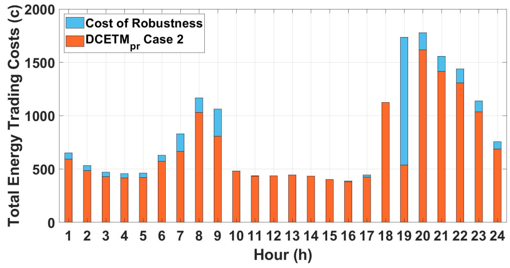


(d) Battery energy profile of CES with community-owned generation in the first community for Case 1.

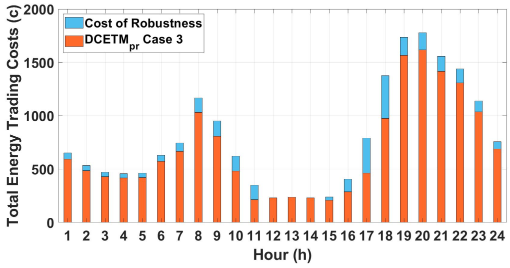
Figure 5.2: Robust, Deterministic, and Opportunistic solutions without community-owned generation for the first community for Case 1.



(a) Cost of robustness without community-owned generation in the first community for Case 1.

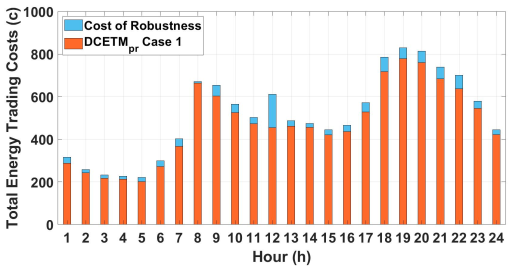


(b) Cost of robustness without community-owned generation in the first community for Case 2.

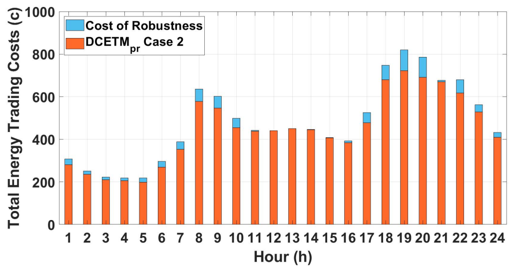


(c) Cost of Robustness without community-owned generation in the first community for Case 3.

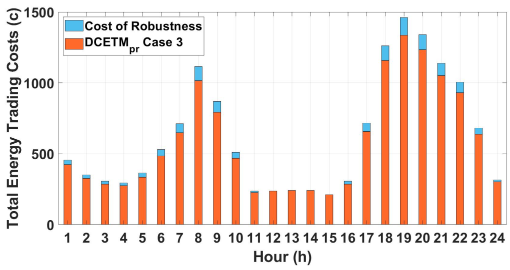
Figure 5.3: Cost of robustness without community-owned generation for all cases in the first community.



(a) Cost of robustness with community-owned generation in the first community for Case 1.

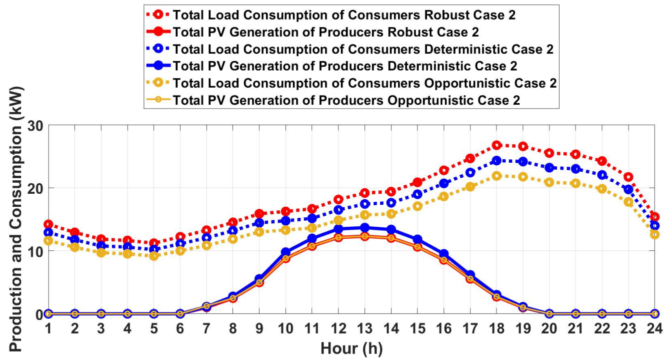


(b) Cost of robustness with community-owned generation in the first community for Case 2.

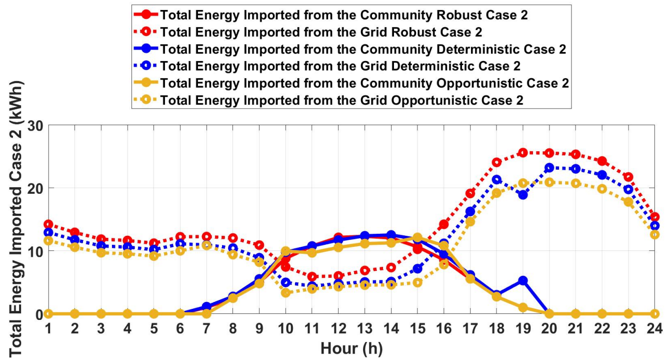


(c) Cost of robustness with community-owned generation in the first community for Case 3.

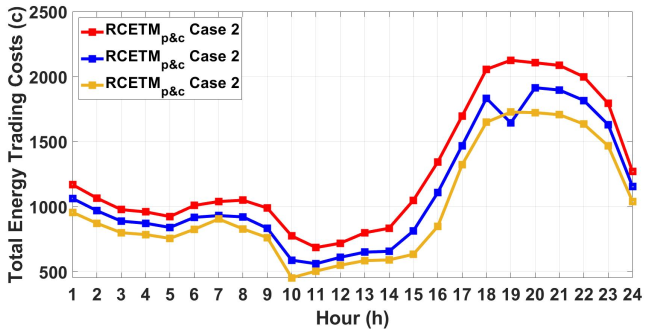
Figure 5.4: Cost of Robustness with community generation for all cases in the first community.



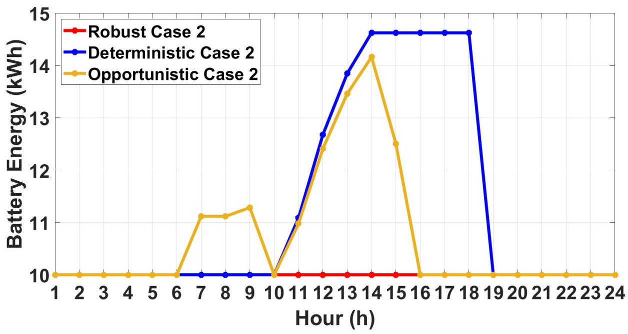
(a) Total production and consumption without community-owned generation in the second community for Case 2.



(b) Total energy imported without community-owned generation in the second community for Case 2.

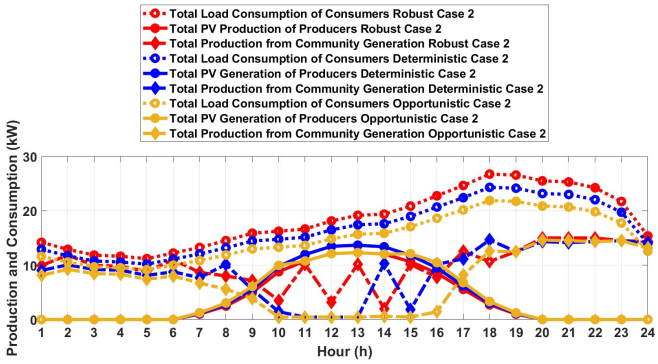


(c) Total energy traded costs without community-owned generation in the second community for Case 2.

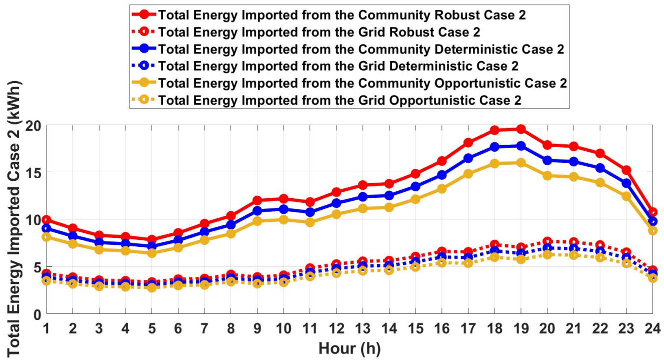


(d) Battery energy profile of CES without community-owned generation in the second community for Case 2.

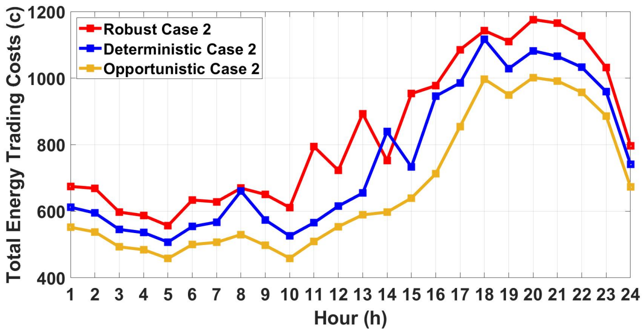
Figure 5.5: Robust, Deterministic, and Opportunistic solutions without community-owned generation for the second community for Case 2



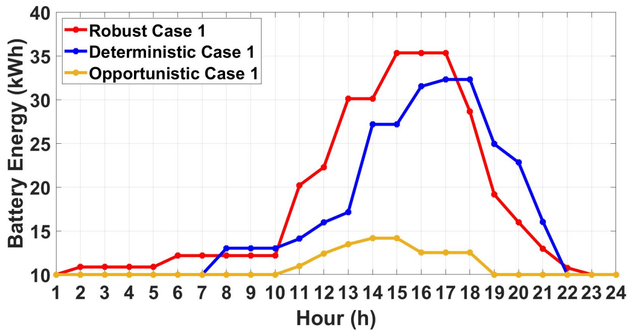
(a) Total production and consumption with community-owned generation in the second community for Case 2.



(b) Total energy imported with community-owned generation in the second community for Case 2.

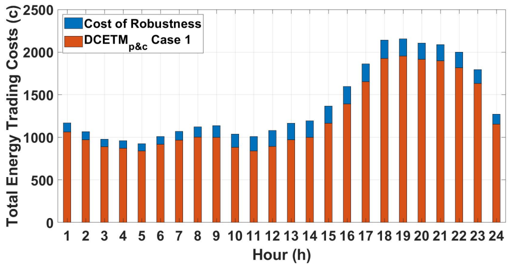


(c) Total energy traded costs with community-owned generation in the second community for Case 2.

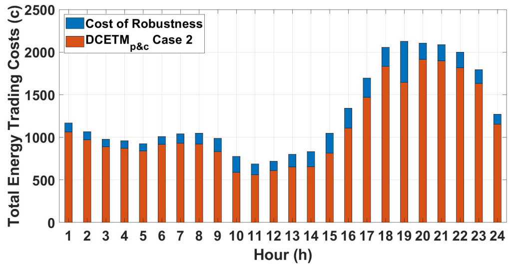


(d) Battery energy profile of CES with community-owned generation in the second community for Case 2.

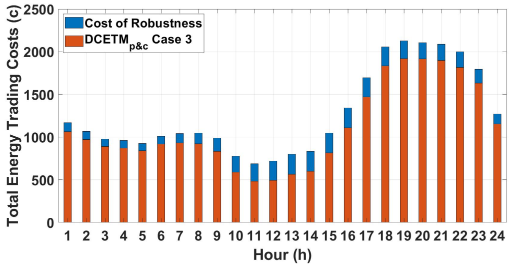
Figure 5.6: Robust, Deterministic, and Opportunistic solutions without community-owned generation for the second community for Case 2.



(a) Cost of robustness without community-owned generation in the second community for Case 1.

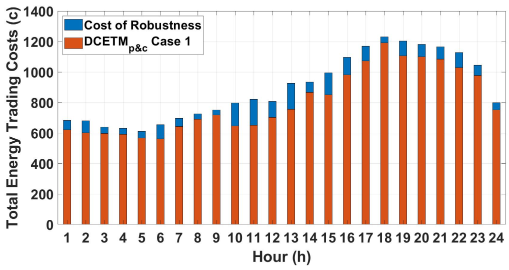


(b) Cost of robustness without community-owned generation in the second community for Case 2.

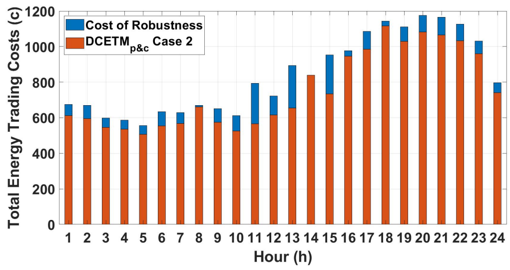


(c) Cost of robustness without community generation in the second community for Case 3.

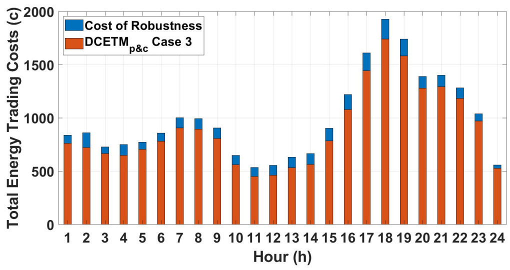
Figure 5.7: Cost of robustness without community-owned generation for all cases in the second community.



(a) Cost of Robustness with community-owned generation in the second community for Case 1.

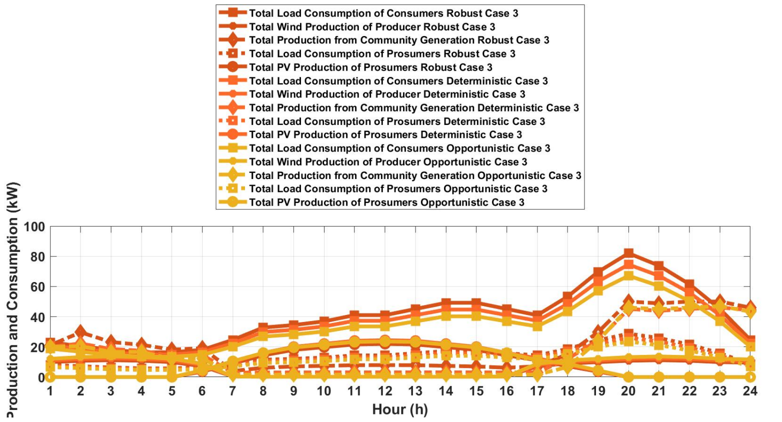


(b) Cost of Robustness with community-owned generation in the second community for Case 2.

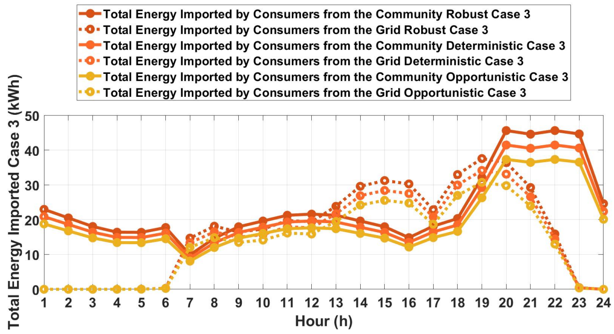


(c) Cost of robustness with community-owned generation in the second community for Case 3.

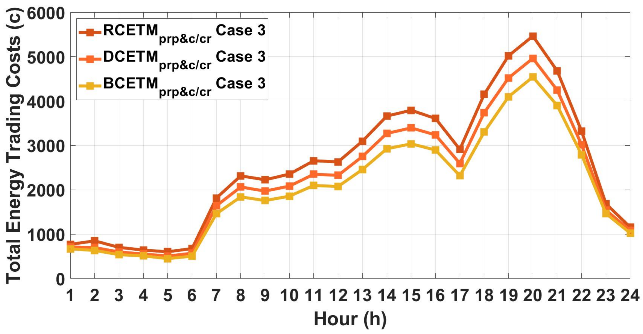
Figure 5.8: Cost of robustness with community-owned generation for all cases in the second community.



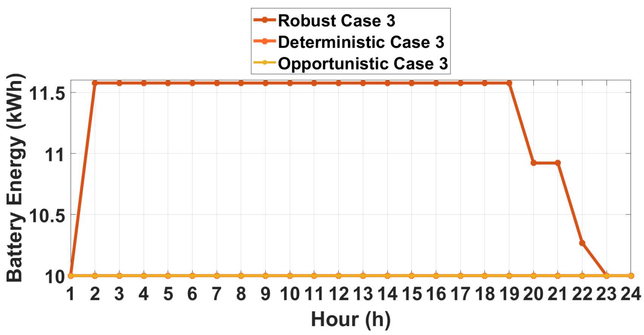
(a) Total production and consumption with community-owned generation in the third community for Case 3.



(b) Total energy imported with community-owned generation in the third community for Case 3.

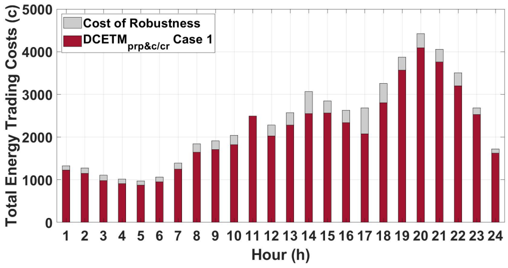


(c) Total energy traded costs with community-owned generation in the third community for Case 3.

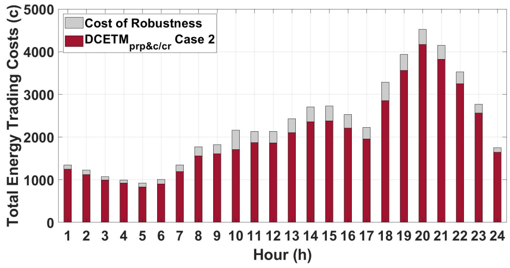


(d) Battery energy profile of CES with community-owned generation in the third community for Case 3.

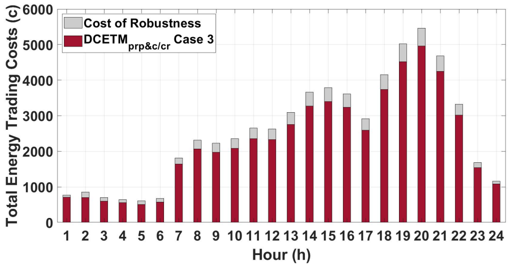
Figure 5.9: Robust, Deterministic and Opportunistic Solutions without community-owned generation for the third community for Case 3.



(a) Cost of robustness with community-owned generation in the third community for Case 1.



(b) Cost of robustness with community-owned generation in the third community for Case 2.



(c) Cost of robustness with community-owned generation in the third community for Case 3.

Figure 5.10: Cost of robustness with community-owned generation for all cases in the third community.

Chapter 6

Community-based Market with Agent Utility Maximization Offering Ancillary Service.

6.1 Introduction

Consumer-centric markets in the future will require an active contribution of a share of their resources arising from the deployment of Distributed generators and distributed energy resources; These include Electric vehicles, battery energy storage BESS, and Flexible loads (HVAcs and CHP). Before now ancillary services in the form of frequency regulation [138] and voltage control [139] have been offered in regulation and reserve markets to ensure the reliability and resilience of the electrical grid network. These have resulted in markets where such services are traded to address short-term imbalances between power supply and demand leading to transmission line failures and impact on power plants [140].

The mode of operation for several ancillary service providers around the world is distinguishable based on factors that include, the response times, remuneration schemes, performance-based regulation for batteries, and the control mechanisms (manual or automatic generation control (AGC)) [29]. In the North American regulation markets, four types of services are distinguishable which include: Regulation, spinning reserve, non-spinning reserve, and replacement reserve. In Europe on the other hand, the three types of ancillary regulation offered are primary, secondary, and tertiary reserves.

The approach commonly used and has been explored in literature is the optimization of overall operational costs of the grid in both grid-connected and islanded modes and costs of offering the ancillary services [100, 101]. Here the ancillary services are offered by distributed generation [141,142] and in a grid-connected mode where the connected Battery Energy Storage System (BESS) injects power back to the grid as an ancillary support for an impending frequency deviation from permissible limits. Reference [143] explores how ancillary market designs are implemented and the changes that may occur given a high penetration of variable renewable energy sources.

Reference [144] proposed a non-linear stochastic method based on mixed integer linear programming to manage optimally an increased number of PV-battery systems for the provision of up and down regulation in the ancillary services market. The method proposed minimizes the cost of energy imported from the grid by the aggregator of residential prosumers equipped with the PV-battery system assessing the up and down flexibility curves against the offer prices.

The construction of an optimal bidding strategy ensures the profit maximization of the retail/generation company or microgrid offering the bid. This usually entails the development of strategic bidding models such as the equilibrium models, non-equilibrium models, and single agent optimization models. Equilibrium models such as the Cournot and linear supply functions are widely used in developing step-wise bidding curves with approximations and the rational analysis of market power [99,145]. Non-equilibrium models have also been utilised in developing bidding strategies such as the approximate model developed in [146] for analysing the impact of the bidding strategy of a GENCO on the market clearing price. Reference [28] utilises a non-equilibrium model based on a price-based unit commitment for the development of the bidding strategy of a virtual power plant (VPP) [28]. A VPP is a group of interconnected DERs that can be used to trade electrical energy in the wholesale market or offer support services to the system operator. The single-agent optimization models are used to optimize the unique bidding curves of the specific generators under study for the maximization of their profits. This approach was used in [29] to model the competitor's behaviour using the residual demand curve of the company.

Distributed Batteries in the form of BESS can be integrated within a network of DERs or used solely as aggregator-owned units to provide energy and ancillary services. In [147] an optimal decision model formulated by an aggregator is proposed to determine the operation and bidding strategy for distributed batteries with consideration to the battery characteristics, including charging capacity, efficiency, and degradation costs. An optimal bidding strategy considering the profitability of batteries in participating in ASM is explored in [148]. By incorporating a battery life cycle model which limits its operational strategy to some extent, into the profit maximization model, the optimal bids in day-ahead, spinning reserves, and regulation markets are determined. Reference [149] investigated the use of Lithium-ion batteries in providing secondary reserve and showed how the cost savings realised could be increased through the use of model-based optimization techniques.

ASM has also been deployed in demand-side response programs to ascertain its importance in providing a more efficient and faster response from the curtailment of demand-side resources as opposed to the ramping of thermal and hydro plants [150]. In making demand side response valuable in the participation in the ancillary services market, reference [151] explores the works of EnerNoc, a curtailment service provider in providing reliable reductions in these markets through actively bidding demand response resources into reserve markets.

Ancillary services markets for ensuring power systems security are also being developed. In [96] the ancillary market operations based on some transparent market rules is used as a basis to formulate an algorithm that determines which offers are to be accepted to optimize the system security at reduced overall costs. Regarding the uncertainties relating to wind power, reference [152] proposes a DC security-constrained optimal power flow subject to some probabilistic constraints. The ancillary services type/products are explored in [138, 140, 153] as primary frequency control/regulating reserves, secondary frequency control/contingency reserves, and tertiary frequency control products can be offered in these reserve markets within given time frames which are accounted for.

The restricting of power system networks has led to many countries around the world [93] developing a market framework for the offer of ancillary services. Reference [94] surveys the frequency and voltage regulation ancillary services in power systems in various parts of the world. The survey is carried out in two parts comparing the technical requirements and economic features [102] in these countries.

In this work, we explore the possibilities of offering ancillary services within a community-based market framework equipped with community energy storage (CES) and agents engaged in trading their energy resources within/outside the community. The services include reserve and regulation. A joint model of the CBEM market framework and the reserve and regulation markets are considered. The aim of the community-based market is in maximizing the profits realised from its bidding strategies in the reserve and regulation markets while minimizing the overall energy traded costs of agents, and the community manager subject to constraints relating to agents' and community manager resources, ancillary provisions and the maximization of agent's utility functions.

The main contributions of this work are as follows:

- A community-based market model with agents' utility maximization is proposed to achieve maximum profits for ancillary in the form of reserve and regulation services, from the community-owned conventional generators, renewable generators of agents, and community-energy storage CES, while minimizing the total energy trading costs of agents and the community manager.

- The analysis and comparisons of the offer from the joint markets of reserve and regulation and without ancillary.

The rest of this chapter is organized as follows: The proposed community-based market with the offer for ancillary services is presented in Section 6.2. In Section 6.3 the bidding model for the reserve markets by CBM is presented. In Section 6.4, the joint market model formulation is presented. The case study on a community microgrid is presented in Section 6.5. Finally, Section 6.6 summarises the chapter.

6.2 Ancillary Support in Community-based Market with Agents' Utility Maximization

The offer of ancillary services is investigated as a joint market model within the community-based market framework to analyse the amounts of energy for the provision of reserve and regulation services from agents' DERs and community-owned resources (CES and DG's) which can be used for the support of ancillary. These supports take the form of bids and offers from community-owned generation and agent resources agreed by the System's operator (SO) in the day-ahead market as market participants in the reserve and regulation markets. These bids offered by the market participant are usually used for the restoration of known grid parameters such as frequency and voltage within their permissible limits. Before now there were committed conventional generators in traditional electricity markets, which can give these sorts of services either by reducing their outputs giving rise to a downward regulation, or increasing the output resulting in an upward regulation for the control of frequency and voltage. In recent times the advent of distributed generators (DGs) and battery energy storage systems (BESS) within active distribution networks are capable of supporting ancillary services. The following ancillary services for reserve and regulation are analysed with consideration to remunerations offered for both upward and downward regulations for the given reserve and regulation services.

6.2.1 Ancillary services products

In this section, some ancillary services products are presented as technical constraints within the community-based market problem. The aim is to model the capabilities of offering reserve and regulation in a joint market model within the community-based market framework. To provide these products; energy reserve that can be delivered at any time required for both reserve and upward/ downward regulation is offered by the community-owned generation units, the agents' renewable resources, and the CES.

Reserve market

The bids submitted by the CBM for the offer of the the reserve is constructed in power (KW) and price (c/kWh) pairs formulated within bidding blocks for each generating unit owned by the community. As in the case of a generating company's Genco, the supply function equilibrium model is employed to develop the bidding strategies for the community-owned resources. The producer agent's renewable generation resources and the CES are also used for reserve bid offers and jointly for energy trading within/outside the community. In the reserve market, the reserve for bids is only considered without consideration of any remuneration schemes. The prosumer and consumer agents' load bids are not considered though they can easily be modelled as negative generation sources. They are considered a constant source based on the economic utility models.

Reserve and Regulation markets

In the market model for the offer of reserve and regulation, remuneration costs are paid for the offer of both upward and downward reserves asides from the offer for the main reserve bids [27]. These reserves are offered from the community-owned generation units, agents' DERs, and the CES. The joint energy market model incorporates the remunerations per unit of energy offered for both reserves in the objective function subject to constraints for the upward and downward regulations. The market model incorporates both reserve and regulation, the markets are initiated by introducing the model for the bidding strategy in the next section.

6.3 Bidding Strategy Model for Community-based Markets in Ancillary Markets

As with the case of Gencos submitting bid offers in the day ahead market, It is considered in this work that the reserve bids are developed by the community for each of the conventional generation units. Suppose all community-owned conventional units have a quadratic generation cost function as follows:

$$C_i = C(P_i) = a_i P_i^2 + b_i P_i + e_i \tag{6.1}$$

where P_i= Output generation of generator i, and a_i, b_i, and e_i are generation cost coefficients. The marginal cost of each generator i is calculated as:

$$\lambda_i = \frac{\delta C(P_i)}{\delta P_i} = 2a_i P_i + b_i \tag{6.2}$$

This represents a linear function of its generation P_i. In this work, a_i is taken to be non-negative, and b_i can be changed.

In practice, an hourly non-decreasing price c_{ik} and quantity pairs p_{ik} for each unit i and k from $(1, ..., K)$ representing the number of blocks for each unit [99, 145], is assumed as the bids submitted for each unit. Genco submits each generator bid to the SOs using a piece-wise supply curve as shown in Figure 6.1. Each Genco will select their generation bid blocks for submission

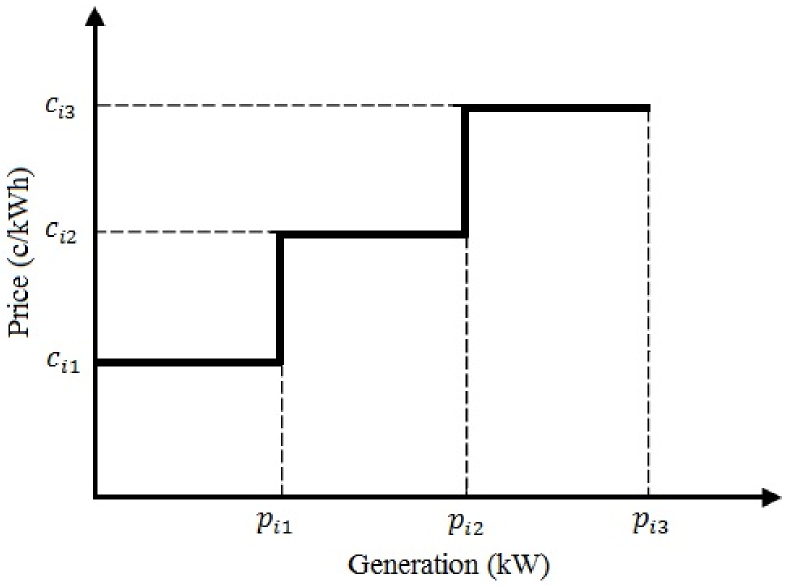


Figure 6.1: Generation bidding curve for a Unit i.
[145]

to the SO according to the following linear supply function for each generator P_i.

$$c_i = e_i \frac{\delta C(P_i)}{\delta P_i} = d_i(2a_iP_i + b_i) \tag{6.3}$$

where c_i is the bidding price, e_i is the bidding strategy (a real number), which is equal to one for price takers and non-strategic bidders. The bid pairs for a generator i and blocks $K = 3$ appear as follows: $(p_{i1}, c_{i1}), (p_{i2} - p_{i1}, c_{i2})$ and $(p_{i3} - p_{i2}, c_{i3})$.

Gencos utilizing multi-blocks for their strategic bids have to deal with more complicated models than compared to others that submit a single block for each generator. The case of the multi-block requires more decision variables, a bidding strategy for the individual blocks, and power for each block. For simplicity, an individual block is bid for by each community-owned conventional unit. An initial energy block k_0 is proposed from community generation that allows for community energy trading as shown in 6.2. The generation for this block is bounded between the minimum for the community asset i and a maximum corresponding to the minimum generation for submitted bids for the first block $k = 1$. The bid pairs for a community generation asset i for blocks $K = 3$ appear as follows: $(p_{i1m} - p_{i0}, c_{i1})$, $(p_{i2m} - p_{i1m}, c_{i2})$ and $(p_{i3m} - p_{i2m}, c_{i3})$. The maximum values of the blocks will be at p_{i1}, p_{i2}, and p_{i3} to the reference

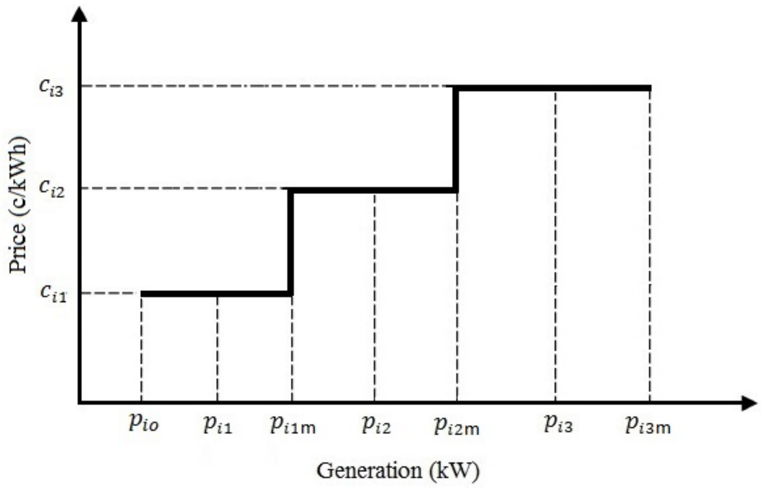


Figure 6.2: Proposed reserve bidding curve for unit i with capacity for community energy trades.

value p_{i0}. In the case of each of the community agents n equipped with renewable generation as well as the CES, we assume a linear cost function for their bids in a single block where $a_i = bi = 0$. [28] The clear differences between the bidding strategies of traditional Genco's and that proposed for the community-based market are as follows:

- The Gencos are the only producers in which the market clearance models are handled by the SO concerning constraints on generator and transmission line limits whereas the community microgrid considers a joint market of ancillary and energy trades amongst agents in the community.

- The supply-demand balance for Genco's considers the generator units quantity, prices, and the bids whereas that of the community will integrate community resources, DERs, and CES in the formulation.

- In the case of a security-constrained problem, the community microgrid considers both network and DER constraints for energy exchanges in its bidding, unlike Gencos which considers the generator and transmission line constraints for its market clearance.

6.4 Joint Market Model Formulation

The proposed model for the community-owned resources (conventional generator units, agents renewable generators, and CES) for bidding in the joint market is presented. In addition to the bid exchanges with the reserve market, it bids a part of its capacity to the regulation market (for upward and downward) regulation based on expected/forecasted prices, as well as determining the discharge and charge states of the CES. We assume the community resources, CES, and agents' resources are price takers/ non-strategic in their bidding.

6.4.1 Objective Function

The bidding problem of the community microgrid is to maximize the profits from both reserve and regulation markets while minimizing the total traded costs of agents and the community manager in trading their resources within/outside the community. In this objective considerations are made towards the type of services offered in this case reserve and regulation markets that offer remunerations on the type of regulation services offered. The objective function and constraints for the joint market model for reserve and regulation within the community-based market framework of prosumers are presented in (6.4)

$$
\begin{aligned}
\max \text{Profit}^{RG} = \rho_t^R \sum_{i=1}^{C_g} p_{itk}^{res} + \rho_t^R p_{dt}^{res} + \rho_t^R \sum_{n_r=1}^{N_{pr}} \delta_{n_r tm}^{res} - \sum_{i=1}^{C_g} \psi_i(p_{it}^c) - \sum_{i=1}^{C_g} \psi_i(p_{it}^d) + \\
f_s \rho_t^R \sum_{i=1}^{C_g} q_{it}^{dr} + f_s \rho_t^R \sum_{i=1}^{C_g} q_{it}^{ur} + f_c \rho_t^R p_{dt}^{ur} + f_c \rho_t^R p_{ct}^{dr} \\
- \sum_{j=1}^{R_g} \sum_{n_r=1}^{N_{pr}} \varphi_{jn_r}(p_{jn_r t}^g) - \sum_{n_r=1}^{N_{pr}} \phi_{n_c}(p_{n_r t}^d) - \xi_{com} \sum_{n_r=1}^{N_{pr}} r_{n_r tm}^c - \xi_{com} \sum_{n_r=1}^{N_{pr}} q_{n_r tm} \\
-\xi_{com} \sum_{i=1}^{C_g} q_{it} - s_{\lambda impt} \sum_{n_r=1}^{N_{pr}} (\gamma_{n_r tm} - G_{n_r t}) - s_{\lambda expt} \delta_t^c - g(\gamma_{impt}, \delta_{expt}) \ \forall t \in T, k \in K
\end{aligned}
\tag{6.4}
$$

The objective of the joint market model for other agents producers and consumers is realised by replacing the prosumer agents' DER and load resources with that of the producer and consumer agents' resources. Likewise, the objective for all agents prosumers producers, and consumers is integrated by expanding the model of prosumers to include all agents.

6.4.2 Constraints

The feasibility of engaging in energy exchanges within the community while offering bids for the reserve and regulation market is considered. The DERs of agents are taken to be non-dispatchable and some of its outputs are reserved for ancillary services as well as scheduled to satisfy the energy demands of agents. The CES is used for energy balance, the offer for upward/downward regulation, and energy reserve. The community-owned DG capacity is used for both reserve bids, regulation, and the community energy market. The following constraints concerning bids are stated thus:

Supply-Demand Balance Constraint

In the case of the reserve market, the power-balance constraints are considered to serve the reserve bids from community-owned DG's, agents' DERs, and CES as shown in Figures (6.5) and (6.6) for prosumers and both prosumers' and producers' DERs respectively. The power used to offer regulation services for the reserve and regulation market is expressed in Figure (6.7) while the relationship between the power offered for reserve and for upwards and downward regulation by the CES are presented in Figures (6.8) and (6.9). The reserve and regulation bid limits are presented in Figures (6.10) and (6.11)

$$\sum_{i=1}^{C_g} p_{itk}^{res} + \sum_{n=1}^{N_{pr}} \delta_{n_r tm}^{res} + p_{dt}^{res} = P_t^{bid} \quad \forall t \in T, k \in K \tag{6.5}$$

$$\sum_{i=1}^{C_g} p_{itk}^{res} + \sum_{n=1}^{N_{pr}} \delta_{n_r tm}^{res} + \sum_{n_g=1}^{N_p} \delta_{n_g tm}^{res} + p_{dt}^{res} = P_t^{bid} \quad \forall t \in T, k \in K \tag{6.6}$$

$$\sum_{i=1}^{C_g} q_{ikt}^{ur} + \sum_{i=1}^{C_g} q_{ikt}^{dr} + p_{dt}^{ur} + p_{ct}^{dr} = P_t^{reg} \quad \forall t \in T, \tag{6.7}$$

and;

$$p_{dt}^{ur} + p_{dt}^{res} \leq p_d^{max} - p_{dt} + p_{ct} \; \forall t \in T \tag{6.8}$$

$$p_{ct}^{dr} \leq p_{dt} - p_{ct} - p_c^{min} \; \forall t \in T \tag{6.9}$$

$$0 \leq P_t^{bid} \leq \overline{P}_t^{bid} \; \forall \; t \in T \tag{6.10}$$

$$0 \leq P_t^{reg} \leq \overline{P}_t^{reg} \;\forall\; t \in T \tag{6.11}$$

The community power balance ensures supply/demand balance of agents' resources, community-owned DG's capacity, and also the charging and discharging of the CES for prosumers (4.23f), producers and consumers (4.24e) and all agents (4.25f) are given in constraint (6.12).

$$(4.23\text{f}), (4.24\text{e}), (4.25\text{f}) \tag{6.12}$$

Agents' DER Constraints

These constraints are considered for the joint market model and are decided such that part of the agents' preferences are utilised in reserve bids and the rest are used for energy exchanges within/outside the community. Constraints (6.13) and (6.14) is the prosumer and producer agent's power balance allowing for DER bids.

$$p^g_{n_r t} - q_{n_r tm} - \delta^{res}_{n_r tm} = 0 \;\forall n_r \in N_{pr},\; t \in T \tag{6.13}$$

$$p^g_{n_g t} - q_{n_g tm} - \delta^{res}_{n_g tm} = 0 \;\forall n_g \in N_p,\; t \in T \tag{6.14}$$

$$(4.20\text{h})\&(4.20\text{n}) \tag{6.15}$$

$$(4.21\text{f})\&(4.21\text{k}), \tag{6.16}$$

Community-owned DG Constraints

These constraints are considered with reserve bid offers in the case of the reserve from equation (6.17) and for both reserve and regulation markets from equation (6.19)-(6.20).

$$p^c_{it} - p^{res}_{itk} = 0 \;\forall i \in C_g,\; t \in T,\; k \tag{6.17}$$

$$p^c_{itk_0} - q^c_{it} - \delta^c_{it} = 0 \;\forall i \in C_g,\; t \in T \tag{6.18}$$

$$p_{itk}^{res} + q_{it}^{ur} \le \overline{p}_{it}^{c} \ \forall i \in C_g,\ t \in T, \forall k \tag{6.19}$$

$$p_{itk}^{res} - q_{it}^{dr} \ge \underline{p}_{itk}^{c} \ \forall i \in C_g,\ t \in T, \forall k \tag{6.20}$$

$$\underline{p}_{itk}^{c} \le p_{it}^{c} \le \overline{p}_{it}^{c} \ \forall i \in C_g,\ t \in T, \forall k \tag{6.21}$$

$$\underline{p}_{it}^{c} \le p_{itk_0}^{c} \le \underline{p}_{itk}^{c} \ \forall i \in C_g,\ t \in T, \forall k \tag{6.22}$$

$$\underline{p}_{itk}^{c} \le p_{itk}^{res} \le \overline{p}_{itk}^{c} \ \forall i \in C_g,\ t \in T, \forall k \tag{6.23}$$

$$\underline{p}_{itk}^{c} \le q_{itk}^{ur} \le \overline{p}_{it}^{c} \ \forall i \in C_g,\ t \in T, \forall k \tag{6.24}$$

$$\underline{p}_{itk}^{c} \le q_{itk}^{dr} \le \overline{p}_{it}^{c} \ \forall i \in C_g,\ t \in T, \forall k \tag{6.25}$$

The energy reserved from the community-owned generation for each unit i at period t is expressed in (6.17). Constraint (6.18) represents the energy balance from the community generation reserved for energy trades. Constraints (6.19) and (6.20) limits the energy reserved from bids and the energy used for upward and downward regulation within bounds, with the lower bound as the minimum of the first block $k = 1$ of unit i to the upper bound as the maximum output of community generation unit i. Constraint (6.21) gives the bounds for community generation unit i. Constraint (6.22) gives the bounds of the proposed initial block k_0 reserved for community energy trades. Constraint (6.23) gives the bounds for the reserve bids within the first block $k = 1$ of unit i, while constraints (6.24) and (6.25) gives the bounds for upward and downward regulations within the $k = 1$ block minimum to the maximum output of generator i.

Agents' Demand Constraints

The agents' demand constraints are taken as a constant load rather than a negative generation and are not utilized in the bidding process. The constraints are given for prosumers and consumers in (6.26)-(6.29)(4.20).

$$(4.20d)\&(4.20e) \tag{6.26}$$

$$(4.21d)\&(4.21e) \tag{6.27}$$

$$(4.20o)\&(4.21l) \tag{6.28}$$

The power balance from each prosumer's generation and consumption is given in

$$(4.20g) \tag{6.29}$$

CES Constraints

The constraints considered for the CES are with the charging and discharging of the CES in the energy balance of trades within the community. The power reserve is for the reserve market which is considered on an hourly basis h and the regulation which is considered to be served within a $15min$ interval h^r. The change in the state of charge of the CES is dependent on the energy trades, power reserves, and regulation services offered. The models according to [147,148] are presented in (6.30)-(6.34) where (6.30) and (6.31) are the CES state of charge constraints for reserve and regulation respectively.

$$e_t = e_{t-1} - 1/n^d p_{dt} + n^c p_{ct} - 1/n^d p_{dt}^{res} h \ \forall t \in T \tag{6.30}$$

$$\begin{aligned} e_t = e_{t-1} - 1/n^d p_{dt} + n^c p_{ct} - \beta t/n^d p_{dt}^{ur} h^r \\ -1/n^d p_{dt}^{res} h + \beta t n^c p_{ct}^{dr} h^r \\ \forall t \in T \end{aligned} \tag{6.31}$$

$$0 \leq p_{dt}^{res} \leq p_d^{max} z_{tc} \ \forall t \in T \tag{6.32}$$

$$0 \leq p_{dt}^{ur} \leq p_d^{max} z_{td} \ \forall t \in T \tag{6.33}$$

$$0 \leq p_{dt}^{dr} \leq p_c^{max} z_{tc} \ \forall t \in T \tag{6.34}$$

$$(4.20p) - (4.20s) \tag{6.35}$$

Utility maximization Constraints

The constraints derived from the economic application of utility maximization for prosumers (production and consumption) (6.36) and producers and consumers (6.37) are presented here as follows:

$$(4.18)\&(4.19) \tag{6.36}$$

$$(4.15)\&(4.17) \tag{6.37}$$

Energy Imports/Exports Constraints

These constraints are considered concerning the relationship between the total energy imported/exported outside the community and the community manager's imports/exports from outside the community. They are presented in (6.38) for imports and (6.39) for exports for prosumers only, producers and consumers, and all agents (prosumers, producers, and consumers.

$$(4.20\text{l}), (4.21\text{i}), \&(4.22\text{g}) \tag{6.38}$$

$$(4.20\text{m}), (4.21\text{j}), \&(4.22\text{h}) \tag{6.39}$$

Non-negativity Constraints

These constraints ensure all imports/exports from outside the community are non-negative as seen in (6.39).

$$\gamma_{n_r tm} \geq 0, \gamma_{n_d tm} \geq 0, \delta^{res}_{n_r tm} \geq 0, \delta^{res}_{n_g tm} \geq 0, \delta^{c}_{it} \geq 0, R_{n_d t} \geq 0, G_{n_r t} \geq 0 \quad \forall t \in T \tag{6.40}$$

6.5 Case Study

In this section, the proposed joint market model of ancillary and community-based energy trades is tested on three community microgrids from chapter four. The first and second community microgrids each have two community generation units of 20 kW and 40 kW that can offer ancillary services. The third community microgrid has two community generation units of 50 kW and 60 kW that can also be used for ancillary services. For simplicity, a single block (k=1) is considered for each of the units, where the block for the two generating units ranges from $\underline{p}^{c}_{itk}$ = 5 kW, 10 kW and $\overline{p}^{c}_{itk}$ = 15 kW, 30 kW for the first and second communities respectively and $\underline{p}^{c}_{itk}$ = 20 kW, 30 kW and $\overline{p}^{c}_{itk}$ = 50 kW, 60 kW for the third community. The generation used for energy trades is bounded within a lower bound $\underline{p}^{c}_{itk_0}$ and a maximum equal to the minimum of the

Table 6.1: Total Profits, Reserve, and Regulation Bids in the First Community

CBEM with Ancillary		Total Profits	Avr. Hourly Bids (kW)		Avr. Hourly D ev. (kW)		Percentage bidding (%)	
Model	Cases	Profits ($)	Reserve	Regulation	Reserve	Regulation	Reserve	Regulation
$DCETM$	Case 1	-123.74	-	-	-	-	-	-
	Case 2	-114.63	-	-	-	-	-	-
	Case 3	-126.65	-	-	-	-	-	-
$RRG/CETM_{pr}$	Case 1	68.43	33.20	67.59	2.39	22.36	7.20	33.08
	Case 2	71.78	32.87	64.14	2.30	23.54	6.99	36.70
	Case 3	62.10	32.98	72.50	1.16	19.84	3.51	27.36

Table 6.2: Total Profits, Reserve, and Regulation Bids in the Second Community

CBEM with Ancillary		Total Profits	Avr. Hourly Bids (kW)		Avr. Hourly Dev. (kW)		Percentage bidding (%)	
Model	Cases	Profits ($)	Reserve	Regulation	Reserve	Regulation	Reserve	Regulation
$DCETM$	Case 1	-200.90	-	-	-	-	-	-
	Case 2	-182.97	-	-	-	-	-	-
	Case 3	-220.09	-	-	-	-	-	-
$RRG/CETM_{p\&c}$	Case 1	146.93	33.62	75.11	3.20	14.05	9.51	18.70
	Case 2	135.68	32.91	73.87	2.60	14.88	7.89	20.14
	Case 3	175.83	32.98	80.83	0.83	4.08	2.51	5.05

first block $\underline{p}^c_{itk}$, this is assumed as the initial block k_0. The case study is analysed using the European reserve market prices of 1st July 2021 [154] and the regulation prices are obtained as a smoothing factor $f_s = 0.9$ (for the DGs) and a regulation performance/capacity factor $f_c = 1$ (for CES) of these reserve prices. A CES is considered with $e^{max} = 100$ kWh, $e^{min} = 10$ kWh, $n^d = n^c = 0.9$, $P_c^{max} = P_c^{min} = 50$ kW, and $e_0 = 10$ kWh and $\beta t = 0.13$ is obtained from [148]. Energy prices for trading by agents within the community are set as normalised costs of the Ausgrid peak tariff of 2019 [114]. Here also, the three cases from chapter four are analysed to test the sensitivity of the proposed model with the preferences on the renewable energy exports of agents used as part of the energy reserve bids. For the case of energy reserve, an hourly reserve of $h = 1$ was considered for the offer of ancillary while for the reserve and regulation, a periodic offer $h^r = 15min$ was considered for the first four hours from $t = 1, 2, 3, 4$, and the hourly $h = 1$ for the rest of the day. The three case studies analysed in chapter four are considered here for each community microgrid including 10% of agents' DEr outputs towards reserve bids. The total energy traded costs of the community including imports and exports with and without the offer of ancillary services for the case studies analysed in the three communities are shown in Tables 6.1-6.3 with the respective cost savings.

Table 6.3: Total Profits, Reserve and Regulation Bids in the Third Community

CBEM with Ancillary		Total Profits	Avr. Hourly Bids (kW)		Avr. Hourly Dev. (kW)		Percentage bidding (%)	
Model	Cases	Profits ($)	Reserve	Regulation	Reserve	Regulation	Reserve	Regulation
$DCETM$	Case 1	-512.59	-	-	-	-	-	-
	Case 2	-497.43	-	-	-	-	-	-
	Case 3	-556.14	-	-	-	-	-	-
$RRG/CETM_{prp\&c}$	Case 1	351.57	48.28	90.42	12.44	2.04	25.78	2.26
	Case 2	334.09	47.62	90.42	13.85	2.04	29.08	2.26
	Case 3	385.73	47.23	90.42	1.60	2.04	3.39	2.26

Table 6.4: Total Profits from the Total Traded Costs from the Community, and Revenues from Reserve and Regulation Markets in the First Community

CBEM with Ancillary Trading Model		Total Profits Breakdown ($)			Percentage (%)		
Model	Cases	Community Costs	Reserve Revenue	Regulation Revenue	Community %	Reserve %	Regulation %
$RRG/CETM_{pr}$	Case 1	-70.45	47.45	91.44	-102.95	69.33	133.62
	Case 2	-61.56	46.92	86.42	-85.76	65.36	120.39
	Case 3	-84.05	47.12	99.03	-135.35	75.88	159.47

Table 6.5: Total Profits from the Total Traded Costs from the Community, and Revenues from Reserve, and Regulation Markets in the Second Community

CBEM with Ancillary Trading Model		Total Profits Breakdown ($)			Percentage (%)		
Model	Cases	Community Costs	Reserve Revenue	Regulation Revenue	Community %	Reserve %	Regulation %
$RRG/CETM_{p\&c}$	Case 1	-3.04	47.98	102.00	-2.07	32.65	69.42
	Case 2	-11.30	46.88	100.11	-8.33	34.55	73.78
	Case 3	17.79	47.10	110.94	10.12	26.79	63.10

Table 6.6: Total Profits from the Total Traded Costs from the Community, and Revenues from Reserve, and Regulation Markets in the Third Community

CBEM with Ancillary Trading Model		Total Profits Breakdown ($)			Percentage (%)		
Model	Cases	Community Costs	Reserve Revenue	Regulation Revenue	Community %	Reserve %	Regulation %
$RRG/CETM_{prp\&c}$	Case 1	158.84	69.49	123.24	45.18	19.77	35.05
	Case 2	142.53	68.23	123.24	42.66	20.45	36.89
	Case 3	194.12	67.52	123.24	50.44	17.54	32.02

6.5.1 Discussion

The results of the proposed joint market model are presented in Figures 6.3-6.6, showing the total production and consumption of agents, total energy imported, total energy exported, and the total daily profits under reserve and regulation ancillary in the first community respectively, presented in Figures 6.9-6.12 for the second community respectively, and Figures 6.15-6.18 for the third community respectively. The results show that in the three communities, the community agents' preference relations for imports/ exports of energy in the community and the grid are satisfied thereby maximizing their utility from the agents' DER resources, initial block k_0 of the community generation units and the CES. The total production and consumption satisfying the agents' preference relations reflect the variability of the community generation units and the energy balancing of the CES in meeting these preferences.

The results of the total daily profits ($) in the community with and without ancillary, the average hourly bids, and average hourly deviations of the reserve and regulation bids with percentages are summarised in Tables 6.1-6.3 for the three communities respectively. From these tables for the three communities, the total daily profits for the three cases analysed without the offer of ancillary services are −$123.74, −$114.63, −$126.65, and $68.43, $71.78, and $62.10 with the offering of ancillary services for the first community, −$200.90, −$182.97, −$220.09, and $146.93, $135.68, and $175.83 with and without ancillary for the second community, and −$512.59, −$497.43, −$556.14, and $351.57, $334.09, and $385.73 with and without ancillary for the third community. The negative values of the profits are a result of the cost minimization for the community energy trades, while the positive values are a result of the profits from the offer of ancillary services. The average hourly bids and deviations for the reserve and regulation bids are an indication that more energy is offered in the regulation market as compared to the reserve market for all three communities. The percentage bids are obtained in expressing the average hourly deviations of the regulation and reserve markets as a percentage of their average hourly bid values, the results show that in the first and second communities, the reserve market bids are closer in value giving reduced hourly deviations as compared to the regulation markets, while in the third community, the regulation market bids are closer in value as compared to the reserve market bids.

The breakdown of the total daily profits resulting from the total energy traded costs from the community energy market, and the revenues realised from the reserve, and regulation markets, in addition to the percentage costs are shown in Tables 6.4-6.6. From these tables for the first and second communities, the regulation market has the highest percentage of the revenue as compared to the reserve and the community market costs while for the third community, energy was traded more within the community as compared to the reserve and regulation markets. This shows that it is more profitable to purchase from the regulation markets as compared to the reserve markets in balancing the energy trades within the community in the first and second communities when compared to the third community.

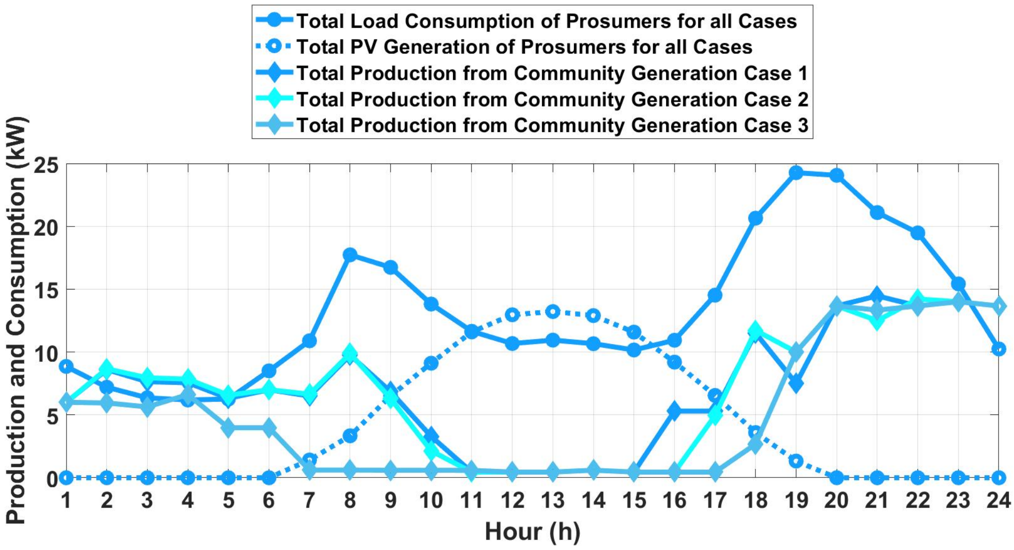


Figure 6.3: Total production and consumption under reserve and regulation ancillary services in the first community.

The reserve and regulation bids from all sources of Community generation, agents' DERs, and CES in all communities, as well as energy exports from agents' DErs and Community generation, are presented in Figures 6.7 and 6.13, 6.19. The results give the amounts of regulation, reserve, and energy exports for each hour. The regulation bids have the highest capacity for all three cases, which explains that more energy is purchased from the grid to balance energy loss during the provision of regulation ancillary on the spot market.

The reserve and regulation bids as well as charged energy for community trades and overall battery energy are shown in Figures 6.8 and 6.14, 6.20. The results show that most of the battery energy is utilised for regulation as compared to reserve markets.

6.6 Summary

The chapter presents a proposed joint market model of a community of agents offering ancillary services of reserve and regulation from their DER and community resources including generation units and CES while maximising their utility from community energy trading. The results for the three cases analysed for the community microgrids show that from the proposed bidding strategy the community agents' preference relations for energy imports and exports were satisfied from agents DER resources, an initial block k_0 allocated for community generation, and the CES while offering reserve and regulation ancillary services. The results show that the community agents

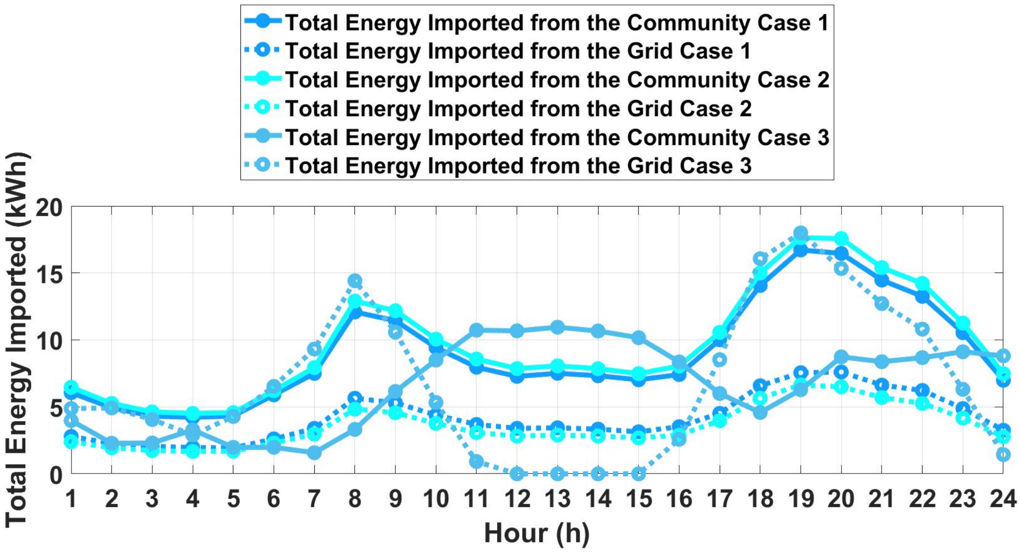


Figure 6.4: Total energy imported under reserve and regulation ancillary services in the first community.

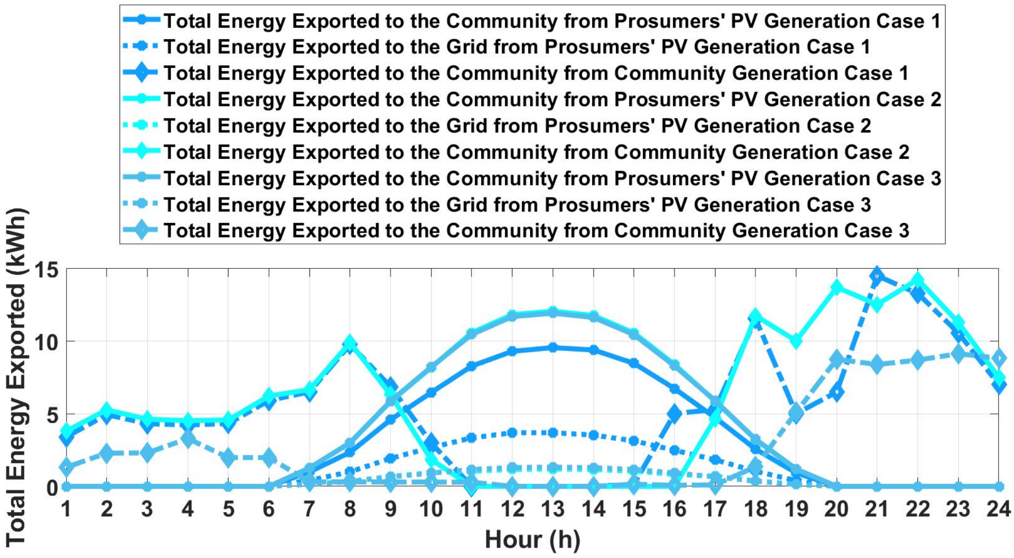


Figure 6.5: Total energy exported under reserve and regulation ancillary services in the first community.

maximise their profits and utility while participating in ancillary services as well as community energy trades as compared to cost minimization of their total energy traded costs when not participating in ancillary markets.

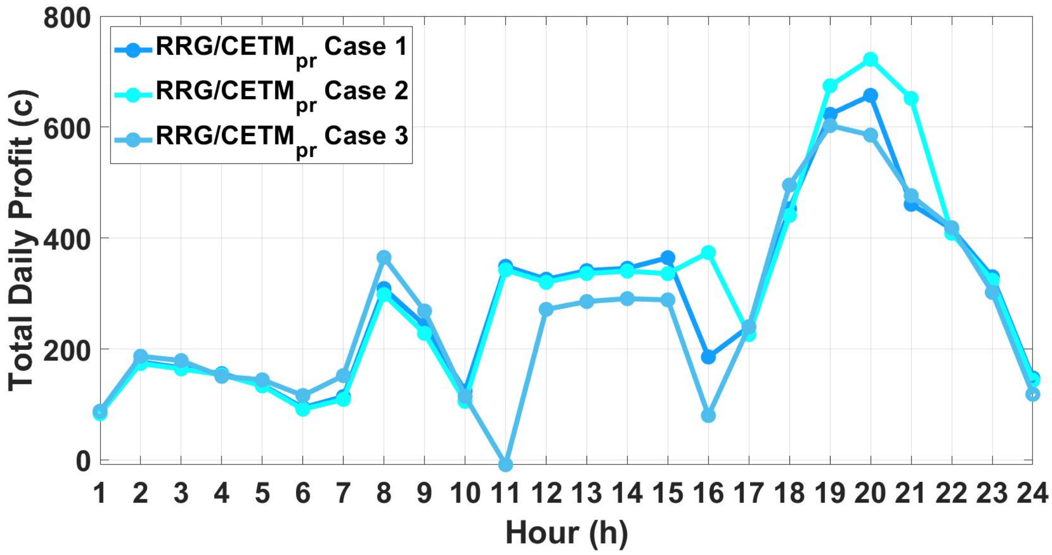
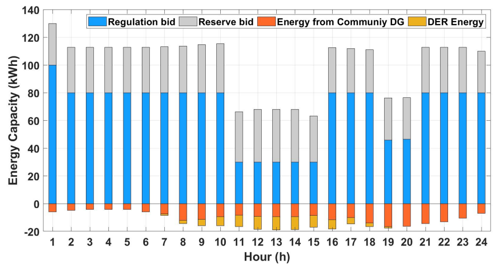
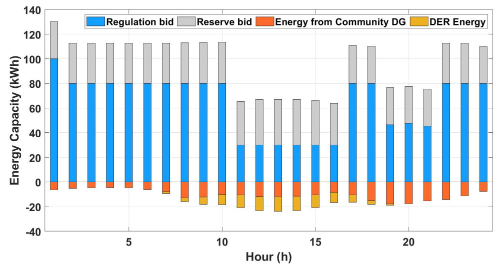


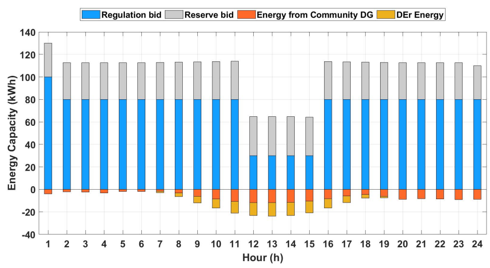
Figure 6.6: Total daily profits with reserve and regulation ancillary services in the first community.



(a) Energy reserve and regulation bids, and energy exported in the first community for Case 1.

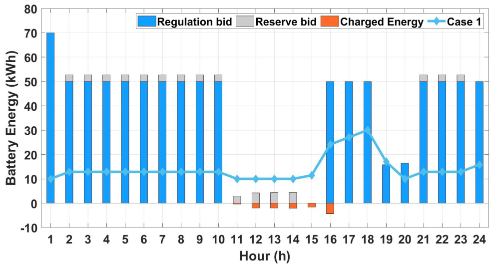


(b) Energy reserve and regulation bids, and energy exported in the first community for Case 2.

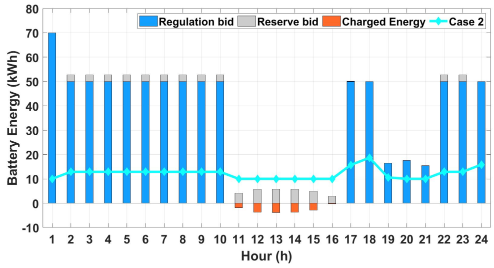


(c) Energy reserve and regulation bids, and energy exported in the first community for Case 3.

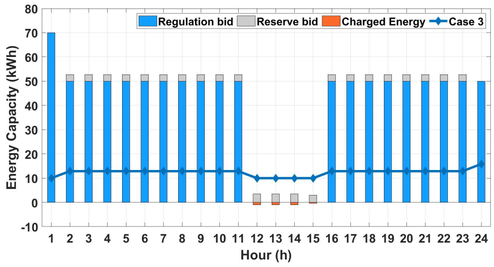
Figure 6.7: Energy reserve and regulation bids, and energy exported for all cases in the first community.



(a) Energy reserve and regulation bids, and energy charged from CES in the first community for Case 1.



(b) Energy reserve and regulation bids, and energy charged from CES in the first community for Case 2.



(c) Energy reserve and regulation bids, and Energy charged from CES in the first community for Case 3.

Figure 6.8: Energy reserve and regulation bids, and energy charged from CES for all cases in the first community.

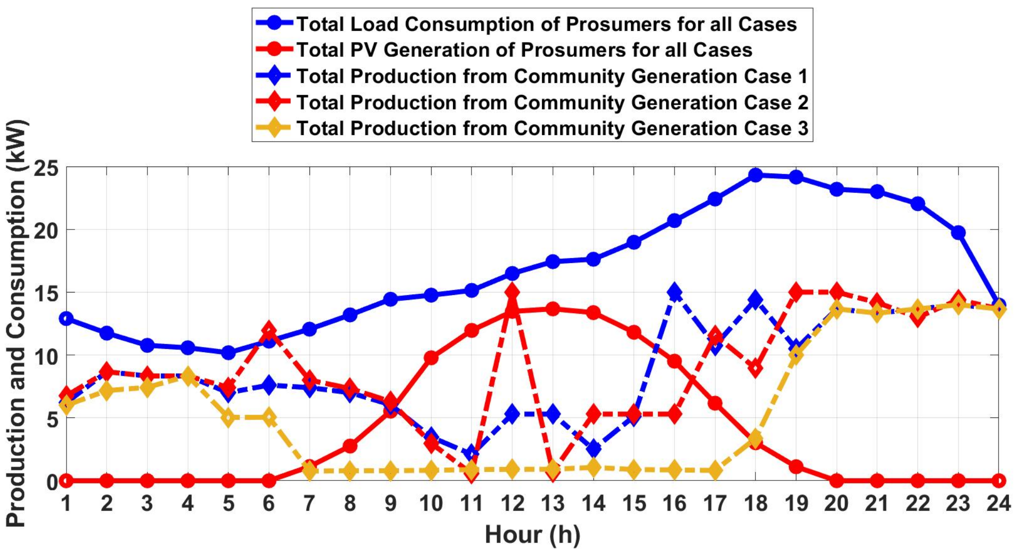


Figure 6.9: Total production and consumption under reserve and regulation ancillary services in the second community.

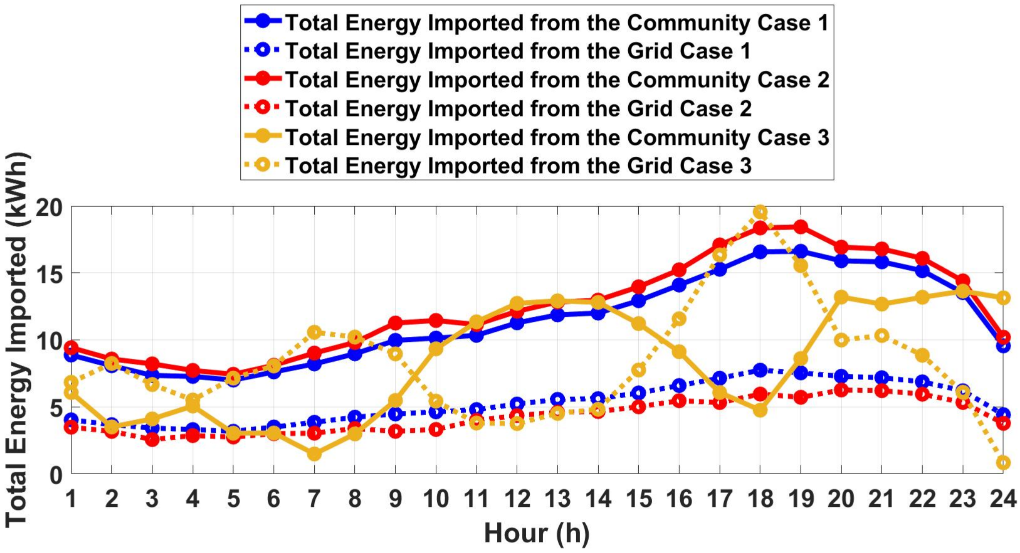


Figure 6.10: Total energy imported under reserve and regulation ancillary services in the second community.

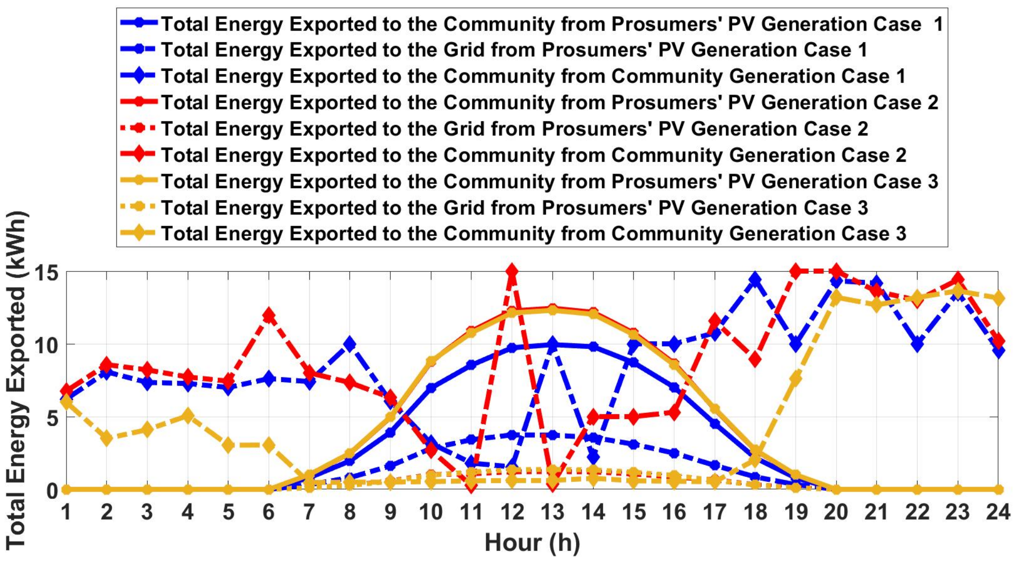


Figure 6.11: Total energy exported under reserve and regulation ancillary services in the second community.

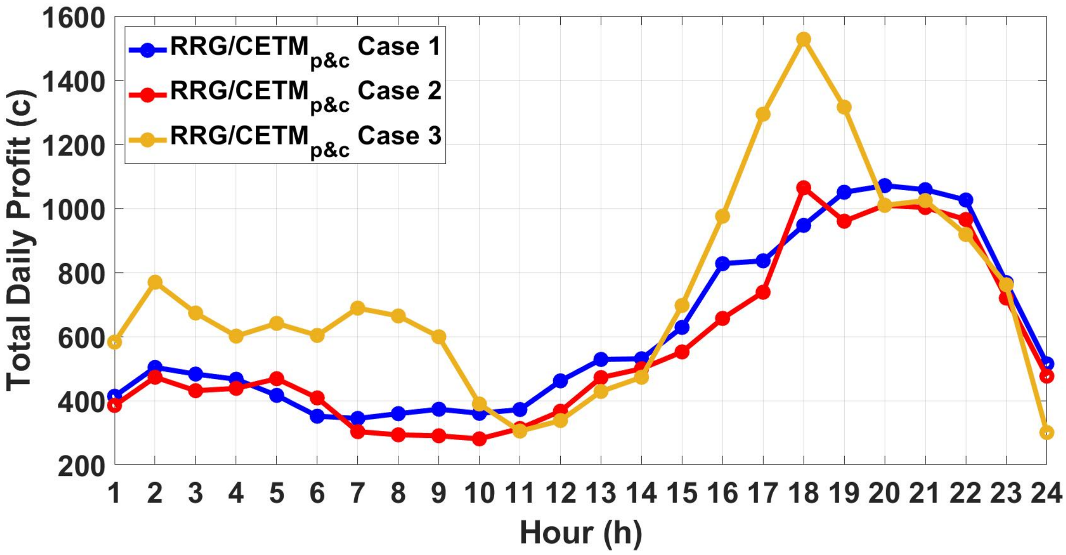
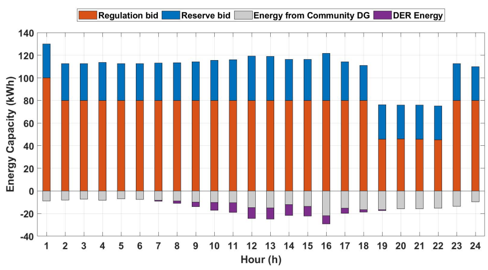
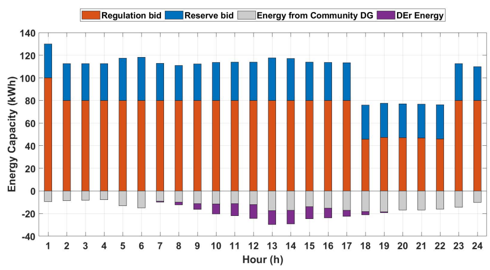


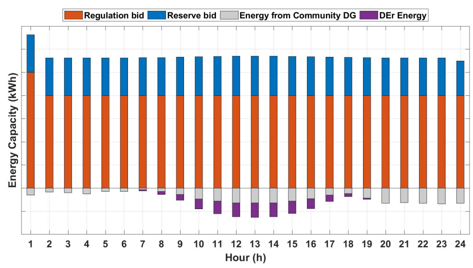
Figure 6.12: Total daily profits with reserve and regulation ancillary services in the second community.



(a) Energy reserve and regulation bids, and energy exported in the second community for Case 1.

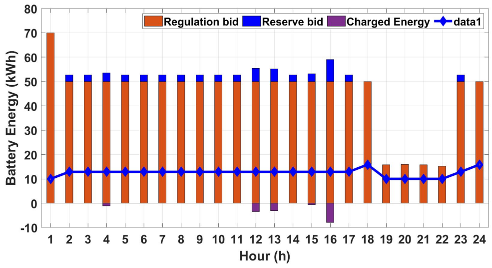


(b) Energy reserve and regulation bids, and energy exported in the second community for Case 2.

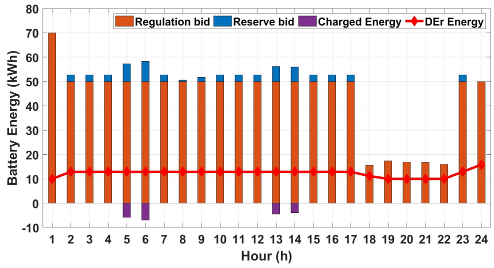


(c) Energy reserve and regulation bids, and energy exported in the second community for Case 3.

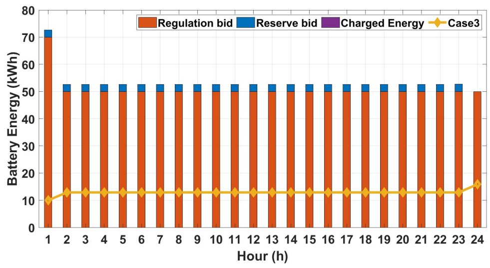
Figure 6.13: Energy reserve and regulation bids, and energy exported for all cases in the second community.



(a) Energy reserve and regulation bids, and energy charged from CES in the second community for Case 1.



(b) Energy reserve and regulation bids, and energy charged from CES in the second community for Case 2.



(c) Energy reserve and regulation bids, and Energy charged from CES in the second community for Case 3.

Figure 6.14: Energy reserve and regulation bids, and energy charged from CES for all cases in the second community.

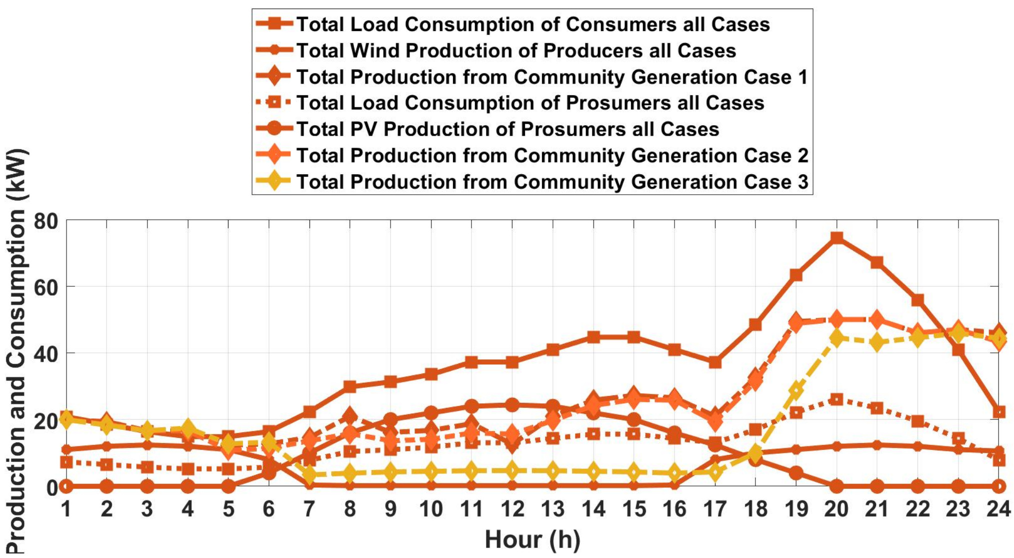


Figure 6.15: Total production and consumption under reserve and regulation ancillary services in the third community.

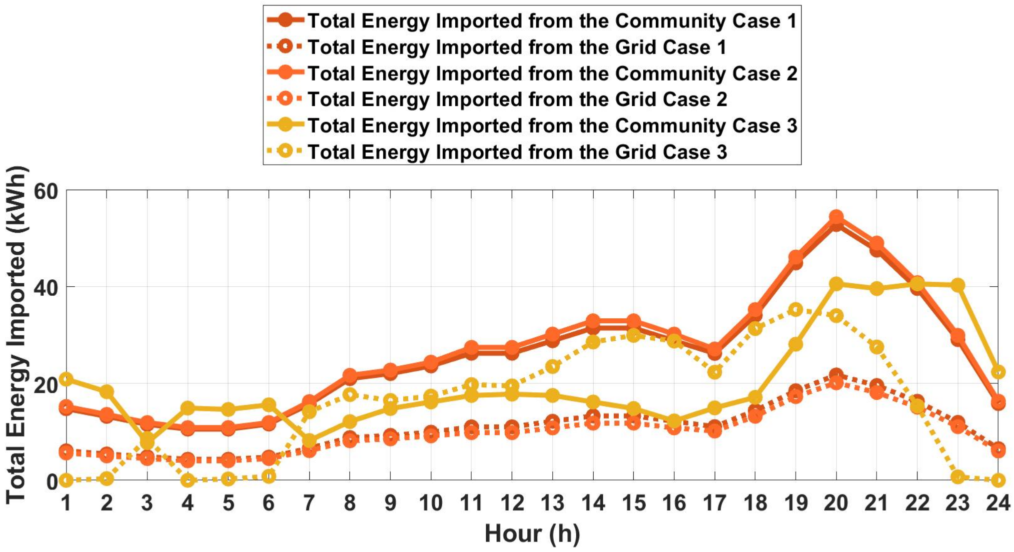


Figure 6.16: Total energy imported under reserve and regulation ancillary services in the third community.

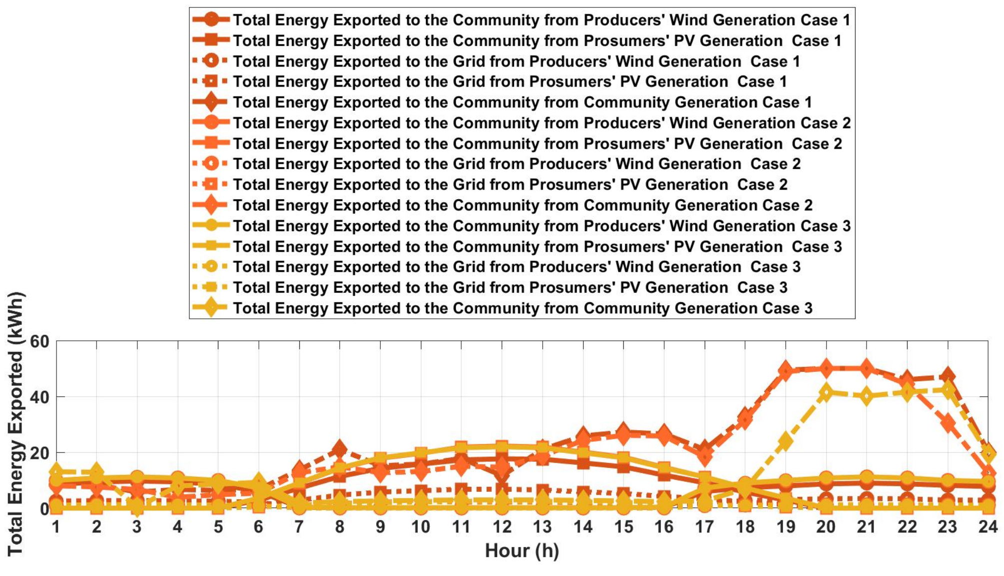


Figure 6.17: Total energy exported under reserve and regulation ancillary services in the third community.

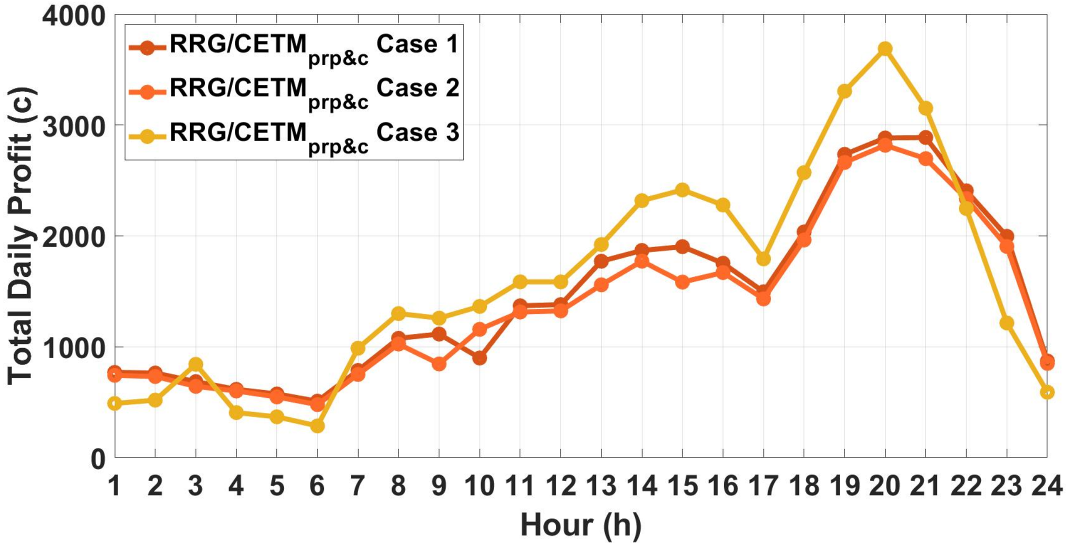
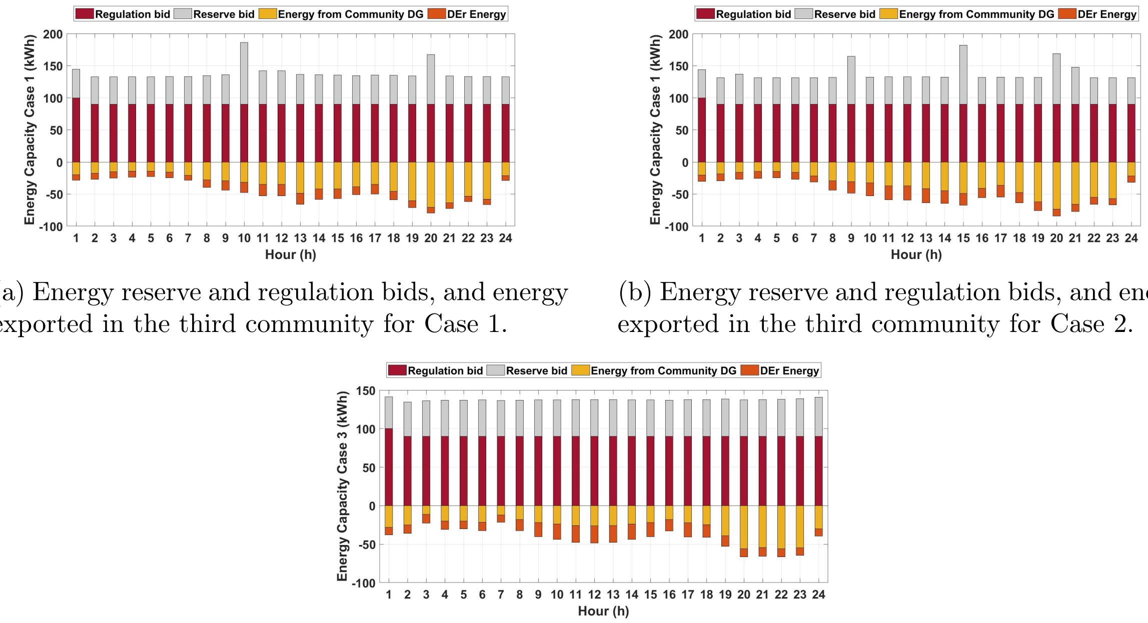


Figure 6.18: Total daily profits with reserve and regulation ancillary services in the third Community.

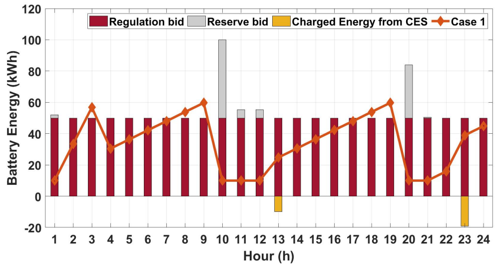


(a) Energy reserve and regulation bids, and energy exported in the third community for Case 1.

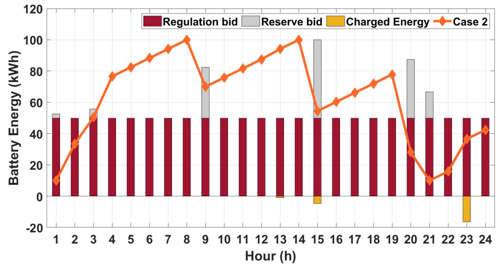
(b) Energy reserve and regulation bids, and energy exported in the third community for Case 2.

(c) Energy reserve and regulation bids, and energy exported in the third community for Case 3.

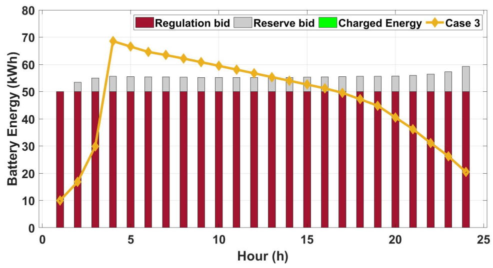
Figure 6.19: Energy reserve and regulation bids, and energy exported for all cases in the third community.



(a) Energy reserve and regulation bids, and energy charged from CES in the third community for Case 1.



(b) Energy reserve and regulation bids, and energy charged from CES in the third community for Case 2.



(c) Energy reserve and regulation bids, and energy charged from CES in the third community for Case 3.

Figure 6.20: Energy reserve and regulation bids, and energy charged from CES for all cases in the third community.

Chapter 7

A Robust Approach to Ancillary Services Offer in Community-based Markets under Uncertainty

In recent times, entities such as Gencos, LV and MV distribution, community microgrids, and virtual power plants have experienced a large integration of distributed energy resources (DERs), Distributed generation systems (DGs), flexible loads (HVAcs and CHPs), and Battery Energy Storage Systems (BESS) in the energy mix. This has resulted in the contribution of a share of the resources to the provision of ancillary services, peak load shaving, and demand response programs.

With a focus on ancillary services offer in reserve markets [28, 29, 99, 142] and regulation markets [144, 147, 148], it becomes imperative that the availability of some of these resources such as DERs, and actual predictions from forecasts in load demand and prices are impacted due to their stochastic and intermittent nature. Deterministic approaches [28, 142, 145] may not be adequate in realising the inherent technical, and financial constraints needed in the modelling of reserve and regulation markets offered by these entities as a result of the uncertainties in price, demand, generation bidding strategies. Non-deterministic approaches such as probabilistic [138, 152] and stochastic models have been explored in literature [29,97,99,144,147,148] in countering these inherent uncertainties. In [152] uncertainties in wind power are analysed and a DC security-constrained optimal power flow model subject to some probabilistic constraints is proposed. The impact of uncertainty on generation is studied in [138] through a chance-constrained optimal power flow problem for the analysis of active distribution grids offering ancillary services in grid-connected as well as islanded mode using tightened probabilistic constraints. [97] analyses the use of stochastic modelling technique in the determination of reserve and regulation market price behaviours. The uncertainties in prices are modelled [29] through the development of scenarios using the residual demand curves of other competitors to maximize the profit of a company considering a day-ahead opportunity cost for the reserve cost curve. Reference [144] develops a novel non-linear stochastic approach that enables an aggregator to optimize a high concentration of PV systems for the provision of up and down regulation in ancillary markets. Uncertainties associated with the bidding strategies, wind generation, and load demand were modelled in [155] through a bilevel stochastic optimization model for generating optimal bidding strategies for wind power producers as price makers. The work in [156] proposes a stochastic optimization model to obtain an optimal hourly schedule of a group of hydropower plants n realising the operation within the week that maximises the expected profit from a joint energy and regulation reserves market. In [157] the joint day-ahead energy, regulation, and reserve market are analysed to model the stochastic uncertainties in prices and the robust solar energy uncertainties of concentrating solar power (CSP) plant.

Robust optimization [17, 26, 76, 77] approaches are a class of non-deterministic models that offer more tractable and less conservative solutions in the optimization of deterministic problems with data uncertainties. Besides the application of robust techniques in distribution [158], transmission [88, 108, 134], generation planning [90, 159] problems, unit commitment problems [18, 129, 130], they have found wide implementation in energy and ancillary services markets to mitigate against uncertainties arising from energy prices, load, generation, and bidding uncertainties. Reference [25] proposes an adaptive robust optimization model in day-ahead energy and reserve market to minimize the overall system cost while accounting for redispatch decisions in the real-time market in the event of the worst-case uncertain production within the predefined uncertainty-set. The robust approach resulted in an improved solution to risk when compared to the stochastic counterpart. The work in [30] proposes a single-stage robust mixed integer linear programming approach in developing hourly offering curves for a producer who is a price-taker. In [31] a two-stage electricity market utilising a robust supply function bidding with incomplete information about other producers is presented, to determine the day-ahead market clearing prices and energy commitments by each producer while trying to meet the energy commitment bids from the day-ahead market in the real-time market.

Based on the work carried out in [30, 81, 88], this work proposes a single-stage robust mixed linear integer programming to model the inherent uncertainties associated with price uncertainties, agents' DERs, and load demands in a joint day-ahead market for reserve, regulation and community-based market trades with consideration to maximizing the utility with regards the preference relations of agents under uncertainty of their differing resources. The impact of uncertainty on their desired preferences is analysed as the stochastic nature of generation and demand resources may result in deviations reflecting changes in consumption and generation patterns given the consumption and generation budgets of agents.

- A single-stage robust mixed linear integer program formulation of a joint day-ahead reserve, regulation ancillary within a community-based market framework is proposed, which maximizes the economic utility of agents in which all uncertainty in energy balance prices, agents DERs, and load demands are represented within a polyhedral uncertainty set.
- A comparative analysis of proposed robust solutions to the deterministic counterparts.

The rest of this paper is organized as follows: The description of the proposed approach is presented in Section 7.1. In Section 7.2, the robust joint market model is analysed as well as the stochastic counterpart. In Section 7.3, the test case study on a community microgrid is presented. Finally, Section 7.4 concludes the chapter.

7.1 Problem Description

Given the joint market model framework under consideration, the impacts of uncertainty are inherent concerning the uncertain parameters to be analysed. The day-ahead market structure ensures that the day-ahead reserve/ regulation bids are accounted for in advance and as a result of the stochastic nature of DERs which serve as part of these bids and the load demands that are supplied by DERs and community-owned conventional generating units, there is uncertainty in the real-time market prices scheduled to pay-off these bids. In this work, 24-hour reserve market prices and agents' generation and load demand profiles are modeled using polyhedral uncertainty sets. The robust approach in [81, 88] is employed in defining the elements of the polyhedral set which is aimed at making the robust optimal solution against the worst-case realisation of uncertain parameters, less conservative. A protection function is likewise modelled to provide an ante instead of ex-ante protection against risks from data uncertainties. In addition, control parameters Γ_p, $\Gamma^g_{jn_r}/\Gamma^g_{ln_g}$, and $\Gamma^d_{n_r}/\Gamma^d_{n_d}$ modelled with the protection function to control the conservation level of the uncertain parameters in energy balance prices for reserve/regulation, production of prosumers/producers and load demand of prosumers/consumers.

7.2 Robust Joint Market Model Formulation

With regards to the RO approach whose aim is to realise an optimal solution against the worst-case realisations of uncertain parameters under study, a robust mixed linear integer program is proposed for the solution to the joint market model of reserve and regulation within the community-based market framework. A control parameter Γ_m $(m = 0, 1, ...M)$ [88] is employed as a varying number taking several values within the set $[0, |J_m|]$ to control the level of conservation of the robust solution with uncertain parameters. Where J_m represents a set of uncertain parameters of either the objective function $(m = 0)$ $J_0 = [j|\hat{c}_j > 0]$ or the m^{th} constraint $(m = 1, ...M)$ $J_m = [j|\hat{a}_{mj} > 0]$. Given the inherent uncertainties in the community-based market model under study, the control range for the reserve market prices becomes $J_p = [\hat{\rho}^R_t > 0]$, $J_{gr} = [\hat{p}^g_{n_r t} > 0]/J_g = [\hat{p}^g_{n_g t} > 0]$ for prosumer/producer agents' DERs and $J_{dr} = [\hat{p}^d_{n_r t} > 0]/J_d = [\hat{p}^d_{n_d t} > 0]$ for prosumer/consumer agents load demands. Given that not all values in the respective ranges of uncertain parameters can change at the same time, we employ according to [81, 88] that up to the value $\lfloor\Gamma_p\rfloor$, $\lfloor\Gamma_{gr}\rfloor/\lfloor\Gamma_g\rfloor$ and $\lfloor\Gamma_{dr}\rfloor/\lfloor\Gamma_d\rfloor$ can vary within their respective ranges. The uncertainty parameter $\tilde{\rho}^R_t, \tilde{p}^g_{n_r t}\ \tilde{p}^g_{n_g t}, \tilde{p}^d_{n_r t}\ \tilde{p}^d_{n_d t} \in R$ for vector $\tilde{\rho}^R_t, \tilde{p}^g_{n_r t}\ \tilde{p}^g_{n_g t}, \tilde{p}^d_{n_d t}\ \tilde{p}^d_{nt} \in \Omega^{USp_t}, \Omega^{USgr_t}\ \Omega^{USg_t}$, and $\Omega^{USdr_t}\ \Omega^{USd_t}$ respectively is expressed within their respective ranges in bounded intervals as follows:

$$\Omega^{USp_t} = [\overline{\rho}^R_t - \hat{\rho}^R_t \leq \tilde{\rho}^R_t \leq \overline{\rho}^R_t + \hat{\rho}^R_t] \quad \forall t \in T \tag{7.1}$$

$$(5.1)\&(5.2) \tag{7.2}$$

$$(5.3)\&(5.4) \tag{7.3}$$

These ranges can vary within a more truncated set as not all parameters can change simultaneously as follows:

$$\Omega^{USp_t} = [\overline{\rho}_t^R - (\Gamma_p - \lfloor\Gamma_p\rfloor)\hat{\rho}_t^R \leq \tilde{P}_t^R \leq \overline{\rho}_t^R + (\Gamma_p - \lfloor\Gamma_p\rfloor)\hat{\rho}_t^R] \quad \forall t \in T \tag{7.4}$$

$$\Omega^{USgr_t} = [\overline{p}_{jn_rt}^g - (\Gamma_{jn_r}^g - \lfloor\Gamma_{jn_r}^g\rfloor)\hat{p}_{jn_rt}^g \leq \tilde{p}_{jn_rt}^g \leq \overline{p}_{jn_rt}^g + (\Gamma_{jn_r}^g - \lfloor\Gamma_{jn_r}^g\rfloor)\hat{p}_{jn_rt}^g] \quad \forall j \in R_g n_r \in N_{pr}, \forall t \in T \tag{7.5}$$

$$\Omega^{USg_t} = [\overline{p}_{ln_gt}^g - (\Gamma_{ln_g}^g - \lfloor\Gamma_{ln_g}^g\rfloor)\hat{p}_{ln_gt}^g \leq \tilde{p}_{ln_gt}^g \leq \overline{p}_{ln_gt}^g + (\Gamma_{ln_g}^g - \lfloor\Gamma_{ln_g}^g\rfloor)\hat{p}_{ln_gt}^g] \quad \forall l \in R_p n_g \in N_p, \forall t \in T \tag{7.6}$$

$$\Omega^{USdr_t} = [\overline{p}_{n_rt}^d - (\Gamma_{dr} - \lfloor\Gamma_{dr}\rfloor)\hat{p}_{n_rt}^d \leq \tilde{p}_{n_rt}^d \leq \overline{p}_{n_rt}^d + (\Gamma_{dr} - \lfloor\Gamma_{dr}\rfloor)\hat{p}_{n_rt}^d] \quad \forall n_r \in N_{pr}, \forall t \in T \tag{7.7}$$

$$\Omega^{USd_t} = [\overline{p}_{n_dt}^d - (\Gamma_d - \lfloor\Gamma_d\rfloor)\hat{p}_{n_dt}^d \leq \tilde{p}_{n_dt}^d \leq \overline{p}_{n_dt}^d + (\Gamma_d - \lfloor\Gamma_d\rfloor)\hat{p}_{n_dt}^d] \quad \forall n_d \in N_c, \forall t \in T \tag{7.8}$$

7.2.1 Formulation of Robust Problem

With consideration to the community resources, CES, and prosumer/producer agents being price takers, the general formulation for the profit maximization model for reserve and regulation ancillary are given in equation (7.9)

$$\max_{U_{g_r t}\in\Omega^{USg_r t},U_{d_r t}\in\Omega^{USd_r t},U_{gt}\in\Omega^{USg_t},U_{dt}\in\Omega^{USd_t},U_{pt}\in\Omega^{USp_t^R},U_{rt}\in\Omega^{USp_t^{Rg}}}\ \min_{X_{n_r t}\in\Omega^{X_{n_r t}},W_{n_g t}\in\Omega^{W_{n_g t}}Y_{n_d t}\in\Omega^{Y_{n_d t}},Z_t\in\Omega^{Z_t}}$$

$$-\varphi_R(P_t^R,U_{pt})-\varphi_{Rg}(P_t^{Rg},U_{rt})+\sum_{j=1}^{R_g}\sum_{n_r=1}^{N_{pr}}\sum_{t=1}^{T}\varphi_{n_r}(X_{n_r t},U_{g_r t},U_{d_r t})+\sum_{l=1}^{R_p}\sum_{n_g=1}^{N_p}\sum_{t=1}^{T}\phi_{n_g}(W_{n_g t},U_{gt})+$$
$$\sum_{n_d=1}^{N_c}\sum_{t=1}^{T}\phi_{n_d}(Y_{n_d t},U_{dt})+\sum_{t=1}^{T}g(Z_t,U_{g_r t},U_{d_r t},U_{gt},U_{dt}) \tag{7.9}$$

A general formulation of the joint market model for reserve and regulation provided by the community-based market is presented in equation (7.9) given that the reserve/regulation prices are expected/forecasted values. However given that these prices are uncertain parameters and can take on random variables [30, 88], then the objective function is substituted by a function that represents all profit distributions for reserve and regulation prices. A compact form of the problem of equation (7.9) can be rewritten as a MILP problem as follows:

$$\min_{x_j,\forall j}\sum_{j=1}^{J}c_j x_j \tag{7.10}$$

s.t

$$\sum_{j=1}^{J}a_{mj}x_j\le b_m \quad m=1,2....M \tag{7.11}$$

$$\underline{x}_j\le x_j\le\overline{x}_j \quad j=1,2....J \tag{7.12}$$

$$x_j\in\{0,1\} \quad j=1,2....J \tag{7.13}$$

An RMILP problem is formulated from equations (7.10) to (7.13) for the objective function coefficients c_j being within bounded intervals. The coefficients c_j and a_{mj} of the objective function and constraint are uncertain elements and can be represented using the nominal value and range as follows:

$$\tilde{c}_j=[\overline{c}_j-\hat{c}_j,\overline{c}_j+\hat{c}_j] \tag{7.14}$$

$$\tilde{a}_{mj}=[\overline{a}_{mj}-\hat{a}_{mj},\overline{a}_{mj}+\hat{a}_{mj}] \tag{7.15}$$

In formulating the RMILP problem, it is important to define the control parameters Γ_0 and

Γ_m to control the conservative level of the objective function and m^{th} constraint respectively with uncertainties. According to [88] elements that amount to $\lfloor \Gamma_m \rfloor$ can take values within their respective ranges as given in (7.14) and (7.15) and the remaining uncertain element can take values within a truncated range as follows:

$$\begin{aligned} \tilde{c}_{t_0} = [\overline{c}_{t_0} - (\Gamma_0 - \lfloor \Gamma_0 \rfloor)\hat{c}_{t_0}, \overline{c}_{t_0} + (\Gamma_0 - \lfloor \Gamma_0 \rfloor)\hat{c}_{t_0}] \\ \tilde{c}_{t_0} \in J_0 \ m = 0 \end{aligned} \tag{7.16}$$

$$\begin{aligned} \tilde{a}_{mt_m} = [\overline{a}_{mt_m} - (\Gamma_m - \lfloor \Gamma_m \rfloor)\hat{a}_{mt_m}, \overline{a}_{mt_m} + (\Gamma_m - \lfloor \Gamma_m \rfloor)\hat{a}_{mt_m}] \\ \tilde{a}_{mt_m} \in J_m \ m = 1, 2, ..., M \end{aligned} \tag{7.17}$$

Therefore the robust form of (7.10) can be rewritten as follows:

$$\begin{aligned} \min_{x_j, \forall j} \sum_{j=1}^{J} \overline{c}_j x_j + \\ \max_{\{S_0 \cup t_0 S_0 \subseteq J_0, |S_0| = \lfloor \Gamma_0 \rfloor, t_0 \in J_0 \setminus S_0\}} \left(\sum_{j \in S_0} \hat{c}_j |x_j| + (\Gamma_0 - \lfloor \Gamma_0 \rfloor)\hat{c}_{t_0} |x_{t_0}| \right) \end{aligned} \tag{7.18}$$

$s.t$

$$\begin{aligned} \sum_{j=1}^{J} \overline{a}_{mj} x_j + \\ \max_{\{S_m \cup t_m S_m \subseteq J_m, |S_m| = \lfloor \Gamma_m \rfloor, t_m \in J_m \setminus S_m\}} \left(\sum_{j \in S_m} \hat{a}_{mj} |x_j| + (\Gamma_m - \lfloor \Gamma_m \rfloor)\hat{a}_{mt_m} |x_{t_m}| \right) \leq b_m \end{aligned} \tag{7.19}$$

$$(7.12) \ \& \ (7.13) \tag{7.20}$$

The RMILP of (7.18) is rewritten in more linear terms by applying the duality theorem as follows:

$$\min_{x_j, \forall j} \sum_{j=1}^{J} c_j x_j + z_0 \Gamma_0 + \sum_{j=1}^{J} \rho_{0j} \tag{7.21}$$

$s.t$

$$\sum_{j=1}^{J} a_{mj} x_j + z_m \Gamma_m + \sum_{j=1}^{J} \rho_{mj} \leq b_m \ m = 1, 2, ..., M \tag{7.22}$$

$$z_0 + \rho_{0j} \geq \hat{c}_j y_j \forall j \in J \tag{7.23}$$

$$z_m + \rho_{mj} \geq \hat{a}_{mj} y_j \;\forall j \in J,\; m = 1, 2, ..., M \tag{7.24}$$

$$-y_j \leq x_j \leq y_j \quad j = 1, 2....J \tag{7.25}$$

$$\rho_{0j} \geq 0;\; \rho_{mj} \geq 0 \;\forall j \in J \tag{7.26}$$

$$y_j \geq 0 \;\forall j \in J \tag{7.27}$$

$$z_0 \geq 0;\; z_m \geq 0 \;\forall j \in J \tag{7.28}$$

Given the bounded intervals of the uncertain parameter defined in equation (7.15), the solution of equation (7.21) is optimal for a given set of values of the control parameters Γ_0 and Γ_m. The variables z_0, ρ_{0j}, z_m, ρ_{mj} are dual variables arising from the robust problem of (7.10)-(7.13) and y_j is an auxiliary variable associated with the uncertain parameter [160]. With regards to the general robust formulation discussed, the reformulation of the robust form of the joint market model for reserve and regulation within the CBM market platform with inherent uncertainty within bounded intervals in equations (7.1)-(7.3) is presented thus:

$$\begin{aligned}
\min_{p_i t, z_0, \rho_{0j} \forall t} & -\rho_t^R \sum_{i=1}^{C_g} p_{itk}^{res} - \rho_t^R p_{dt}^{res} - \rho_t^R \sum_{n_r=1}^{N_{pr}} \delta_{n_r tm}^{res} + \sum_{i=1}^{C_g} \psi_i(p_{it}^c) + \sum_{i=1}^{C_g} \psi_i(p_{it}^d) - \\
& f_s \rho_t^R \sum_{i=1}^{C_g} q_{it}^{dr} - f_s \rho_t^R \sum_{i=1}^{C_g} q_{it}^{ur} - f_c \rho_t^R p_{dt}^{ur} - f_c \rho_t^R p_{ct}^{dr} \\
& + \sum_{j=1}^{R_g} \sum_{n_r=1}^{N_{pr}} \varphi_{jn_r}(p_{jn_r tw}^g) + \sum_{n_r=1}^{N_{pr}} \phi_{n_c}(p_{n_r tw}^d) + \xi_{com} \sum_{n_r=1}^{N_{pr}} r_{n_r tm}^c + \xi_{com} \sum_{n_r=1}^{N_{pr}} q_{n_r tm} \\
& + \xi_{com} \sum_{i=1}^{C_g} q_{it} - s_{\lambda impt} \sum_{n_r=1}^{N_{pr}} (\gamma_{n_r tm} - G_{n_r t}) + s_{\lambda expt} \delta_t^c + g(\gamma_{impt}, \delta_{expt}) + \\
& z_0 \Gamma_0 + \sum_{j=1}^{R_g} \rho_{0jt} \;\forall t \in T, k \in K
\end{aligned} \tag{7.29}$$

Additional Constraints

In addition to the constraints associated with the deterministic formulation of the joint market model offering ancillary services of reserve and regulation and agent utility maximization within the CBM market framework, already defined in equations (6.6)-(6.40), the following robust constraints with regards the uncertain elements in agents DERs and load demands are presented:

$$\tilde{p}^g_{jn_rt} - \underline{p}^g_{jn_rt} + z^g_{n_rt}\Gamma^g_{jn_r} + \rho^g_{jt} \leq 0 \;\; \forall j \in R_g, \forall n_r \in N_{pr}, \forall t \in T \tag{7.30}$$

$$-\tilde{p}^d_{n_rt} + \underline{p}^d_{n_rt} + z^d_{n_rt}\Gamma^d_{n_r} + \rho^d_{rt} \leq 0 \;\; \forall n_r \in N_{pr}, \forall t \in T \tag{7.31}$$

$$\tilde{p}^g_{ln_gt} - \underline{p}^g_{ln_gt} + z^g_{n_gt}\Gamma^g_{ln_g} + \rho^g_{lt} \leq 0 \;\; \forall l \in R_p, \forall n_g \in N_p, \forall t \in T \tag{7.32}$$

$$-\tilde{p}^d_{n_dt} + \underline{p}^d_{n_dt} + z^d_{n_dt}\Gamma^d_{n_d} + \rho^d_{t} \leq 0 \;\; \forall n_d \in N_c, \forall t \in T \tag{7.33}$$

$$z_0 + \rho_{0t} \geq \hat{\rho}^R_t y_{\rho^R_t} \;\forall t \in T \tag{7.34}$$

$$z^g_{n_rt} + \rho^g_{jt} \geq \hat{p}^g_{jn_rt} y_{p^g_{n_rt}} \;\; \forall n_r \in N_{pr}, t \in T \tag{7.35}$$

$$z^d_{n_rt} + \rho^d_{rt} \geq \hat{p}^d_{n_rt} y_{p^d_{n_rt}} \;\; \forall n_r \in N_{pr}, t \in T \tag{7.36}$$

$$z^g_{n_g} + \rho^g_{lt} \geq \hat{p}^g_{n_gt} y_{p^g_{n_gt}} \;\; \forall n_g \in N_p, t \in T \tag{7.37}$$

$$z^d_{n_d} + \rho^d_{t} \geq \hat{p}^d_{n_dt} y_{p^d_{n_dt}} \;\; \forall n_d \in N_c, t \in T \tag{7.38}$$

$$\rho_{0t} \geq 0;\;\; \rho_g \geq 0 \; \rho_d \geq 0 \;\forall t \in T \tag{7.39}$$

$$y_{\rho^R_t} \geq 0;\;\; y_{p^g_{n_rt}} \geq 0;\;\; y_{p^d_{n_rt}} \geq 0;\;\; y_{p^g_{n_gt}} \geq 0;\;\; y_{p^d_{n_dt}} \geq 0 \;\forall n \in N_p, n \in N_c, t \in T \tag{7.40}$$

$$z_0 \geq 0;\;\; z^g_{n_rt} \geq 0;\;\; z^d_{n_rt} \geq 0; z^g_{n_g} \geq 0;\;\; z^d_{n_d} \geq 0 \;\forall n_r \in N_{pr}, n_g \in N_p, n_d \in N_c \tag{7.41}$$

$$(6.6) - (6.10);\;\; (6.12 - 6.40) \tag{7.42}$$

Constraints (7.30) to (7.33) provides ante protection against the worst case realisation of the uncertain parameters of agents' resources through the dual variables $z^{g}_{n_r t}$, $z^{d}_{n_r t}$,$z^{g}_{n_g}$, and $z^{d}_{n_d}$ modelled in the protection function by duality theorem. Constraints (7.34) to (7.38) limits the respective range for agents' uncertain parameters to within the bounds provided by protection variables and control parameters. Constraints (7.39)-(7.41) are non-negative constraints of all variables under consideration.

7.3 Case Study

The proposed robust joint community-based market model with an integrated ancillary market is tested on three community microgrids from Chapter 6. The three cases implemented for analysing the varying impacts of the joint markets are also implemented on the joint robust model. The uncertain parameters of the agents' resources are expected to impact the robust joint market model when compared to the deterministic counterpart. Therefore the uncertain generation and demand of agents' resources are within bounded intervals such as $p^{gmin}_{jn_r t} = (1 - \alpha_0)\, \underline{p}^{g}_{jn_r t}$; $p^{gmax}_{jn_r t} = (1+\alpha_0)\, \underline{p}^{g}_{jn_r t}$ and $p^{dmin}_{n_r t} = (1-\alpha_0)\, \underline{p}^{d}_{n_r t}$; $p^{dmax}_{n_r t} = (1+\alpha_0)\, \underline{p}^{d}_{dn_r t}$ for prosumers respectively, and $p^{gmin}_{ln_g t} = (1-\alpha_0)\, \underline{p}^{g}_{ln_g t}$; $p^{gmax}_{ln_g t} = (1+\alpha_0)\, \underline{p}^{g}_{ln_g t}$ and $p^{dmin}_{n_d t} = (1-\alpha_0)\, \underline{p}^{d}_{n_d t}$; $p^{dmax}_{n_d t} = (1 + \alpha_0)\, \underline{p}^{d}_{n_d t}$ for producers' generation and consumers' consumption.

The parameter values used from the deterministic joint market model are implemented in addition to a symmetric deviation of $\alpha_0 = 0.1$. The uncertain reserve market prices can assume values from 0 to 10 for the first and second communities and 0 to 9 for the third community (This represents the number of generation resources contributing to the reserve). The actual energy balance prices and price bounds set within 10% deviation used as reserve prices are shown in 7.1. $\Gamma^{g}_{jn_r}$, $\Gamma^{d}_{n_r}\Gamma^{g}_{ln_g}$, and $\Gamma^{d}_{n_d}$ can take different values from 0 to 1 to control the robustness of the optimal solution relating to the uncertainty of agent's generation and consumption resources. The control parameter for the uncertain reserve market prices is assumed as $\Gamma_\rho = 10/2 = 5$ for the first and second community, and $\Gamma_\rho = 9/2 = 4.5$ for the third community. The results are simulated when the control parameter for the robustness of agents generation and consumption resources $\Gamma^{g}_{jn_r} =, \Gamma^{d}_{n_r} = \Gamma^{g}_{ln_g} = \Gamma^{d}_{n_d} = 1$.

7.3.1 Discussion

The results of the RMILP model of the joint market framework of the community offering ancillary services in comparison with the deterministic solution for all cases for the three community microgrids are presented in figure 7.2 for Case 1 in the first community, figure 7.4 for Case 2 in the second community, and 7.6 for Case 3

in the third community respectively.

For the three cases implemented in the three community microgrids, the figures show the total production and consumption under reserve and regulation ancillary, in figure 7.2a, for Case 1 in the first community, figure 7.4a for Case 2 in the second community, and figure 7.6a for Case 3 in the third community. The results show that for the robust case, in the three communities, the agents' generation resources are within the lower bounds of the uncertainty generation sets, while their consumption resources are within the upper bounds of the uncertainty demand set and the deterministic case are the expected values of agents' resources. This justifies that community agents will trade their energy resources in the worst-case realisations of uncertain parameters of the resources to produce a robust solution to their energy market problems.

The figures for the total energy imported under reserve and regulation ancillary are shown in figure 7.2b, for Case 1 in the first community, figure 7.4b for Case 2 in the second community, and figure 7.6b for Case 3 in the third community. The results show that the agents maximize their utility from energy imports/exports from the community and the grid for their preference relations. The total energy imported/exported to the community is lower for the robust scenario as compared to the deterministic scenario. The total energy imported from the grid is higher for the robust scenario compared to the deterministic scenarios while the total energy exported to the grid is higher for the robust scenario compared to the deterministic scenarios.

The figures for the total daily profits with reserve and regulation ancillary services are shown in figure 7.2c for Case 1 in the first community, figure 7.4c for Case 2, in the second community, and figure 7.6c for Cases 3 in the third community. The results of the total daily profits in the community for the robust and deterministic scenarios with ancillary services, the average hourly bids, and the average hourly deviations of the reserve and regulation bids with percentages are summarised in tables 7.1-7.3 for the three communities respectively. The results show that the total daily profits for the three cases analysed for each community with the offer of ancillary services for the robust scenarios are \$77.29, \$80.12, \$65.92 for the first community, \$184.00, −\$170.95, −\$220.20, for the second community, and \$146.93, \$135.68, \$175.83, for the third community. This shows a higher daily profit is realised when compared to the deterministic scenarios, and therefore results in the following costs of robustness in profits \$8.86, \$8.34, \$3.82 for the first community, \$37.07, \$35.27, \$44.37 for the second community, and \$44.44, \$50.17, \$54.94 for the third community. In the robust and deterministic scenarios, the average hourly bids and deviations for the reserve and regulation bids show that more energy is offered in the regulation market as compared to the reserve market for all three communities. The results of the percentage bids for the robust and deterministic scenarios show that in the first and second communities, the reserve market bids are closer in value giving a reduced average hourly deviation as compared to the regulation markets, while in the third community, the regulation market bids are closer in value as compared to the reserve market bids.

The breakdown of the total daily profits resulting from the total traded costs from community trades, and the revenues realised from the reserve and regulation markets, in addition to the percentage costs for both robust and deterministic scenarios is shown in Tables 7.4-7.6. The results show the contributions of each of the market frameworks analysed, i.e., the community market, the reserve, and the regulation markets, to the total daily profits of the joint market model. The results show that in the first community, the regulation market contributed mostly to the robust profit, while for the second community, both community markets and regulation markets contributed the most. In the third market the community market, contributed the most to the robust profit. In the robust and deterministic scenarios for the first and second communities, the regulation market has the highest percentage of the total profit as compared to the reserve and the community market, while for the third community, energy was traded more within the community as compared to the reserve and regulation markets.

The figures for the reserve and regulation bid, the charged energy for the community trades, and the battery profile from the CES are shown in Figures 7.3a, 7.3b, and 7.3c for Cases 1, 2, and 3 in the first community, 7.5a, 7.5b, and 7.5c for Cases 1, 2, and 3 in the second community, and 7.7a, 7.7b, and 7.7c for Cases 1, 2, and 3 in the third community. The results show that in both the robust and deterministic scenarios most of the battery energy is utilised in the regulation market as compared to the reserve and community markets.

Table 7.1: Robust Total Profits, Reserve and Regulation Bids in the First Community

Robust CBEM with Ancillary		Total Profits	Avr. Hourly Bids (kW)		Avr. Hourly Dev. (kWh)		Percentage bidding (%)		COR($)
Model	Cases	Profits ($)	Reserve	Regulation	Reserve	Regulation	Reserve	Regulation	
$RRG/RCETM_{pr}$	Case 1	77.29	33.22	73.25	2.07	18.45	6.24	25.18	8.86
	Case 2	80.12	32.15	69.84	1.78	20.81	5.48	29.79	8.34
	Case 3	65.92	32.20	80.83	1.54	4.08	4.77	4.05	3.82
$RRG/DCETM_{pr}$	Case 1	68.43	33.20	67.59	2.39	22.36	7.20	33.08	-
	Case 2	71.78	32.87	64.14	2.30	23.54	6.99	36.70	-
	Case 3	62.10	32.98	72.50	1.16	19.84	3.51	27.36	-

Table 7.2: Robust Total Profits, Reserve and Regulation Bids in the Second Community

Robust CBEM with Ancillary		Total Profits	Avr. Hourly Bids (kW)		Avr. Hourly Dev. (kW)		Percentage bidding (%)		COR($)
Model	Cases	Profits ($)	Reserve	Regulation	Reserve	Regulation	Reserve	Regulation	
$RRG/RCETM_{p\&c}$	Case 1	184.00	33.11	80.83	2.49	4.08	7.52	5.05	37.07
	Case 2	170.95	32.21	80.83	2.13	4.08	6.61	5.05	35.27
	Case 3	220.20	32.91	80.83	0.81	4.08	2.46	5.05	44.37
$RRG/DCETM_{p\&c}$	Case 1	146.93	33.62	75.11	3.20	14.05	9.51	18.70	-
	Case 2	135.68	32.91	73.87	2.60	14.88	7.89	20.14	-
	Case 3	175.83	32.98	80.83	0.83	4.08	2.51	5.05	-

Table 7.3: Robust Total Profits, Reserve and Regulation Bids in the Third Community

Robust CBEM with Ancillary		Total Profits	Avr. Hourly Bids (kW)		Avr. Hourly Dev. (kW)		Percentage bidding (%)		COR ($
Model	Cases	Profits ($)	Reserve	Regulation	Reserve	Regulation	Reserve	Regulation	
$RRG/RCETM_{prp\&c}$	Case 1	396.01	48.68	90.42	10.61	2.04	21.79	2.26	44.44
	Case 2	384.26	46.89	90.42	11.35	2.04	24.20	2.26	50.17
	Case 3	440.67	46.35	90.42	12.37	2.04	26.69	2.26	54.94
$RRG/DCETM_{prp\&c}$	Case 1	351.57	48.28	90.42	12.44	2.04	25.78	2.26	-
	Case 2	334.09	47.62	90.42	13.85	2.04	29.08	2.26	-
	Case 3	385.73	47.23	90.42	1.60	2.04	3.39	2.26	-

The figures for the reserve and regulation bid from the community generation sources, the agents' DERs, and CES are shown in Figures 7.3a, 7.3b, and 7.3c for Cases 1, 2, and 3 in the first community, Figures 7.5a, 7.5b, and 7.5c for Cases 1, 2, and 3 in the second community, and Figures 7.7a, 7.7b, and 7.7c for Cases 1, 2, and 3 in the third community. The results show that in the robust and deterministic scenarios, more energy is utilised from the grid in balancing the provision for regulation as compared to the reserve and community markets.

Table 7.4: Robust Total Profits from the Total Traded Costs in the Community, and Revenues from Reserve and Regulation Markets in the First Community

Robust CBEM with Ancillary Trading Model		Total Profit Breakdown ($)			Percentage Costs (%)		
Model	Cases	Community Costs	Reserve Revenues	Regulation Revenues	Community %	Reserve %	Regulation %
$RRG/RCETM_{pr}$	Case 1	-69.98	47.45	99.82	-90.55	61.39	129.15
	Case 2	-61.15	46.39	94.88	-76.32	57.90	118.42
	Case 3	-90.96	45.94	110.94	-137.98	69.69	168.28
$RRG/DCETM_{pr}$	Case 1	-70.45	47.45	91.44	-102.95	69.33	133.62
	Case 2	-61.56	46.92	86.42	-85.76	65.36	120.39
	Case 3	-84.05	47.12	99.03	-135.35	75.88	159.47

Table 7.5: Robust Total Profits from the Total Traded Costs in the Community, and Revenues from Reserve and Regulation Markets in the Second Community

Robust CBEM with Ancillary Trading Model		Total Profit Breakdown ($)			Percentage Costs (%)		
Model	Cases	Community Costs	Reserve Revenues	Regulation Revenues	Community %	Reserve %	Regulation %
$RRG/RCETM_{p\&c}$	Case 1	25.79	47.26	110.94	14.02	25.69	60.30
	Case 2	14.02	45.99	110.94	8.20	26.90	64.90
	Case 3	62.25	47.01	110.94	28.27	21.35	50.38
$RRG/DCETM_{p\&c}$	Case 1	-3.04	47.98	102.00	-2.07	32.65	69.42
	Case 2	-11.30	46.88	100.11	-8.33	34.55	73.78
	Case 3	17.79	47.10	110.94	10.12	26.79	63.10

Table 7.6: Robust Total Profits from the Total Traded Costs in the Community, and Revenues from Reserve and Regulation Markets in the Third Community

Robust CBEM with Ancillary Trading Model		Total Profit Breakdown ($)			Percentage Costs (%)		
Model	Cases	Community Costs	Reserve Revenues	Regulation Revenues	Community %	Reserve %	Regulation %
$RRG/RCETM_{prp\&c}$	Case 1	203.05	69.73	123.24	51.27	17.61	31.12
	Case 2	194.05	66.97	123.24	50.50	17.43	32.07
	Case 3	251.06	66.38	123.24	56.97	15.06	27.97
$RRG/DCETM_{prp\&c}$	Case 1	158.84	69.49	123.24	45.18	19.77	35.05
	Case 2	142.53	68.23	123.24	42.66	20.45	36.89
	Case 3	194.12	67.52	123.24	50.44	17.54	32.02

7.4 Summary

In this chapter, a robust approach to the joint market model of a community of agents trading their DER resources, maximising their utility from their DERs and community resources including generation units and CES while offering ancillary services of reserve and regulation is proposed. The results show that in the three case studies analysed for the three community microgrids, the robust solution resulted in higher optimal traded profits when compared to the deterministic scenario and associated costs of robustness. In all three communities, more energy was utilised in the regulation market than in the reserve and community markets. Also, in comparing the contributions of each of the markets analysed to the total profits for the robust solutions, the regulation market contributed the most in the first community, the community

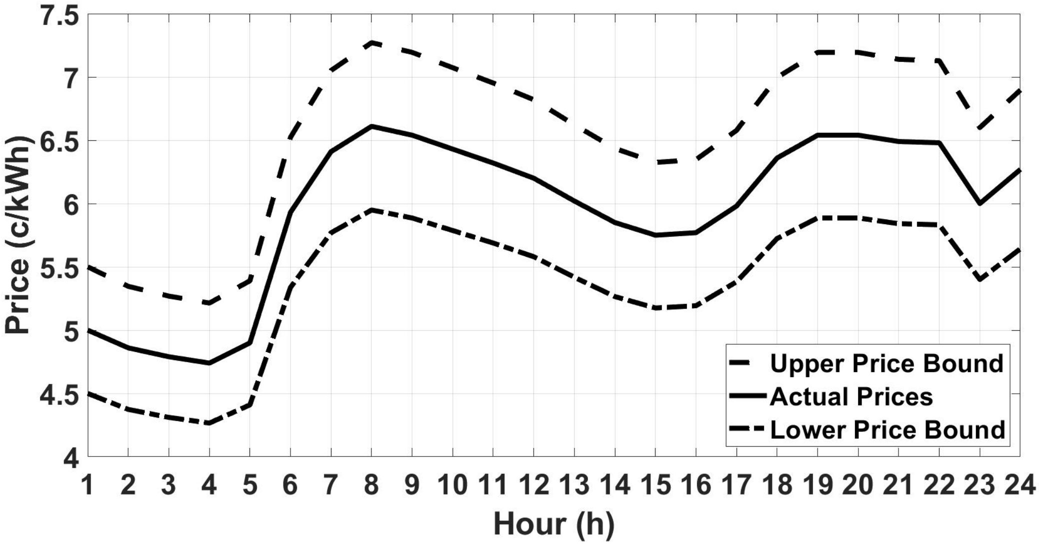
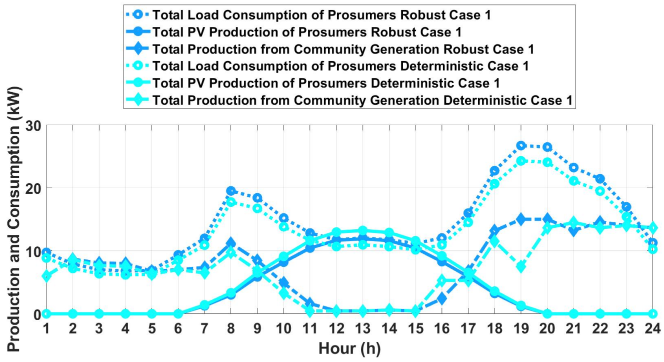
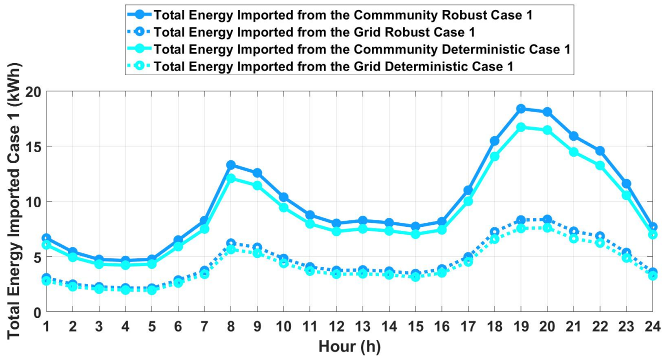


Figure 7.1: Actual energy balance prices and price bounds.

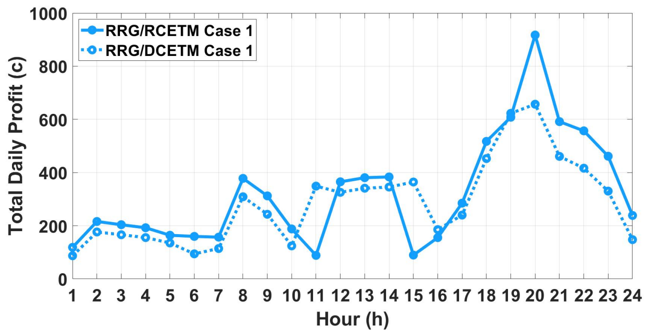
market and regulation market contributed more in the second community while the community market contributed the most in the third community.



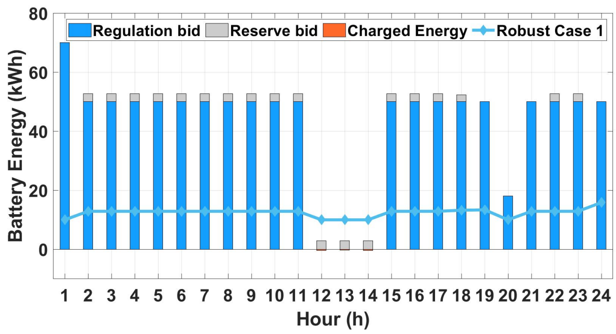
(a) Total production and consumption under reserve and regulation ancillary in the first community for robust Case 1.



(b) Total energy imported under reserve and regulation ancillary in the first community for robust Case 1.

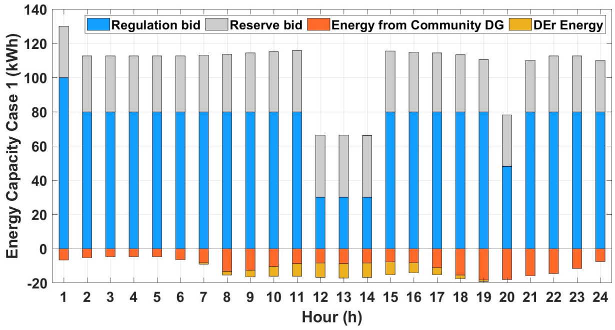


(c) Total daily profits with reserve and regulation ancillary services in the first community for robust Case 1.

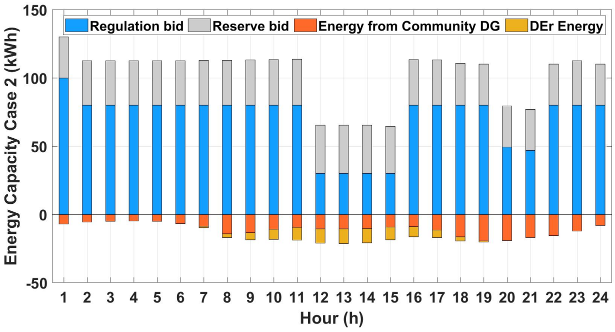


(d) Energy reserve and regulation bids, and energy charged from CES in the first community for robust Case 1.

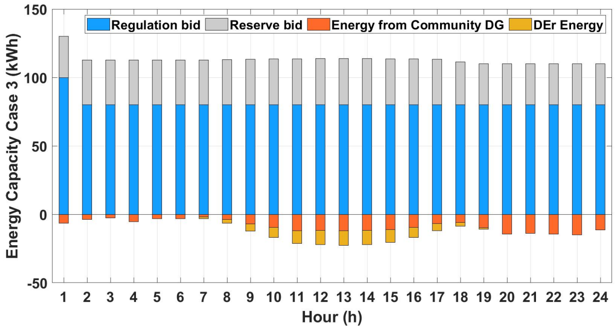
Figure 7.2: Robust and Deterministic solutions with reserve and regulation ancillary for the first community for Case 1.



(a) Energy reserve and regulation bids, and energy exported in the first community for robust Case 1.

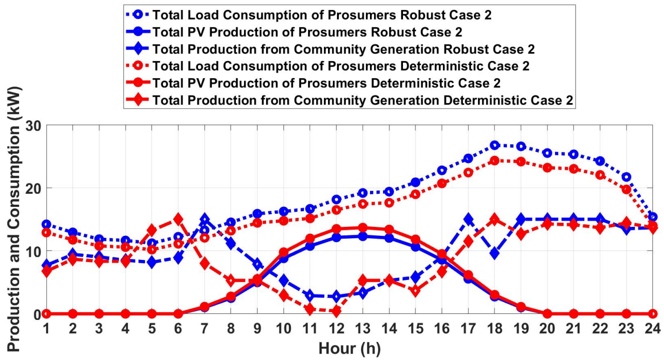


(b) Energy reserve and regulation bids, and energy exported in the first community for robust Case 2.

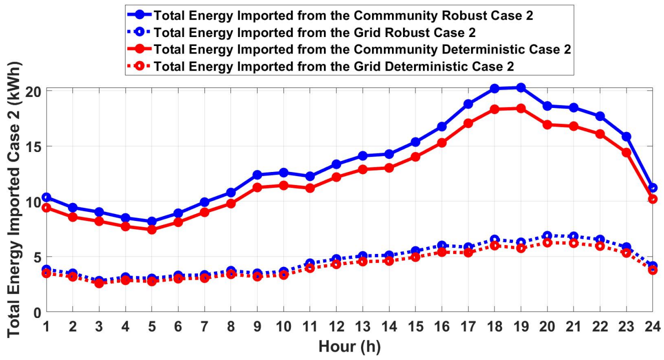


(c) Energy reserve and regulation bids, and energy exported in the first community for robust Case 3.

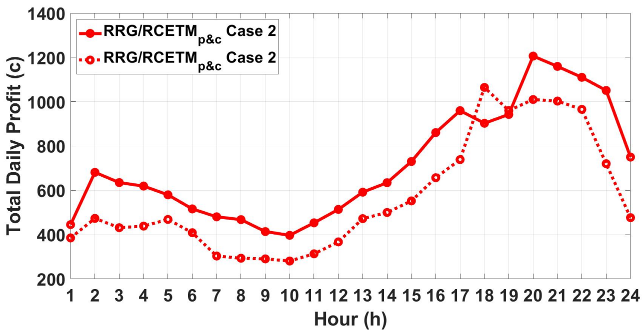
Figure 7.3: Energy reserve and regulation bids, and energy exported for all robust cases in the first community.



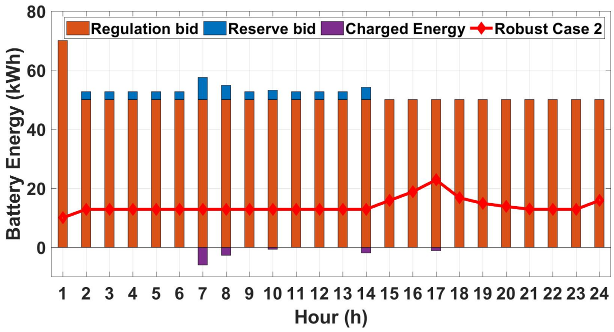
(a) Total production and consumption under reserve and regulation ancillary in the second community for robust Case 2.



(b) Total energy imported under reserve and regulation ancillary in the second community for robust Case 2.

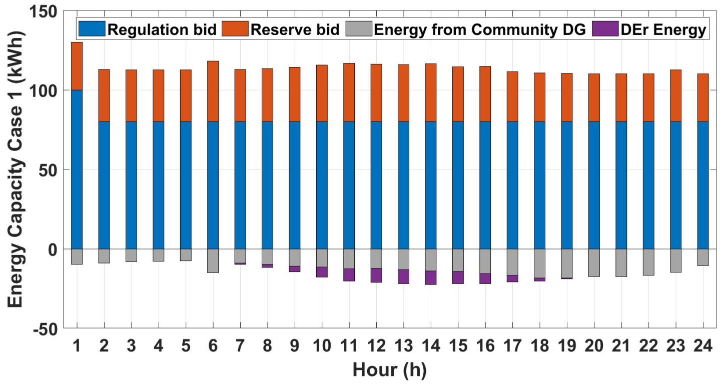


(c) Total daily profits with reserve and regulation ancillary services in the second community for robust Case 2.

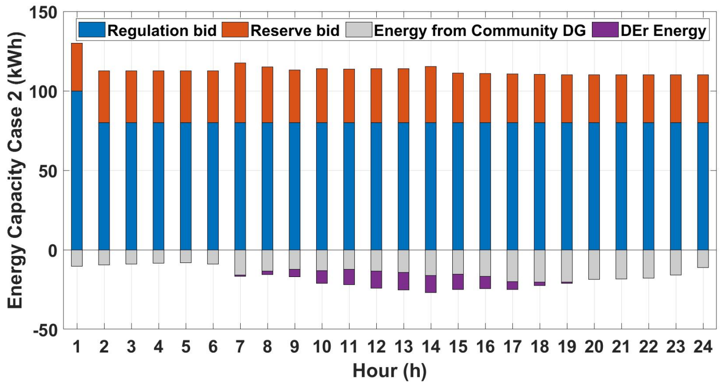


(d) Energy reserve and regulation bids, and energy charged from CES in the second community for robust Case 2.

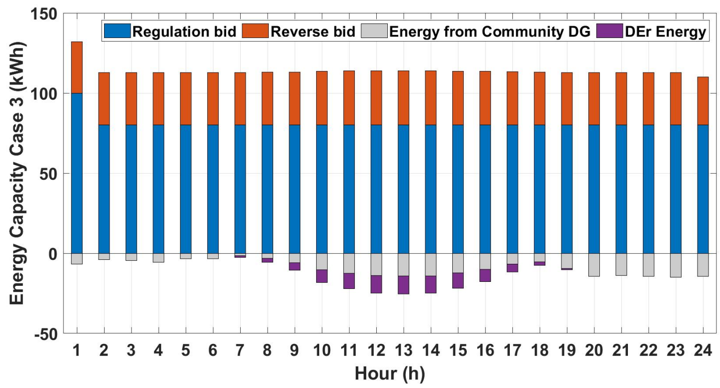
Figure 7.4: Robust and Deterministic solutions with community generation and ancillary for the second community for Case 2.



(a) Energy reserve and regulation bids, and energy exported in the second community for robust Case 1.

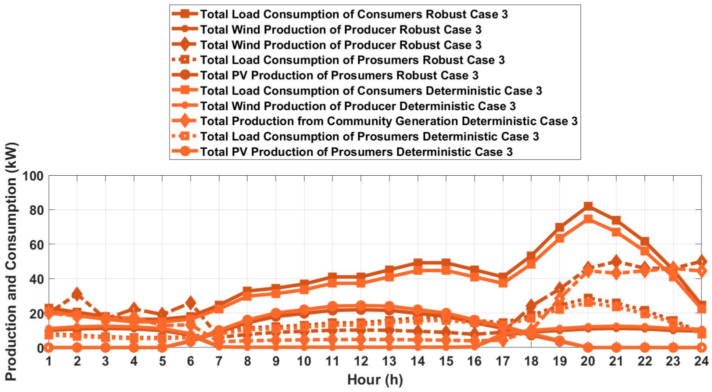


(b) Energy reserve and regulation bids, and energy exported in the second community for robust Case 2.

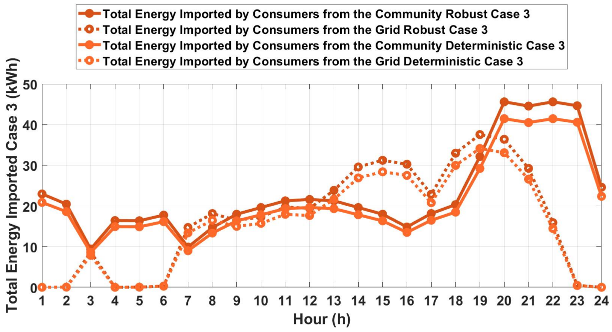


(c) Energy reserve and regulation bids, and energy exported in the second community for robust Case 3.

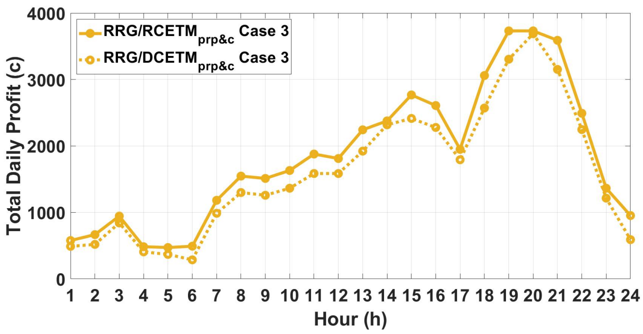
Figure 7.5: Energy reserve and regulation bids, and energy exported for all robust cases in the second community.



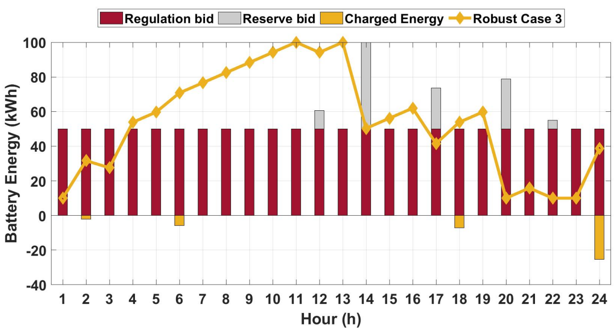
(a) Total production and consumption under reserve and regulation ancillary in the third community for robust Case 3.



(b) Total energy imported under reserve and regulation ancillary in the third Community for robust Case 3.

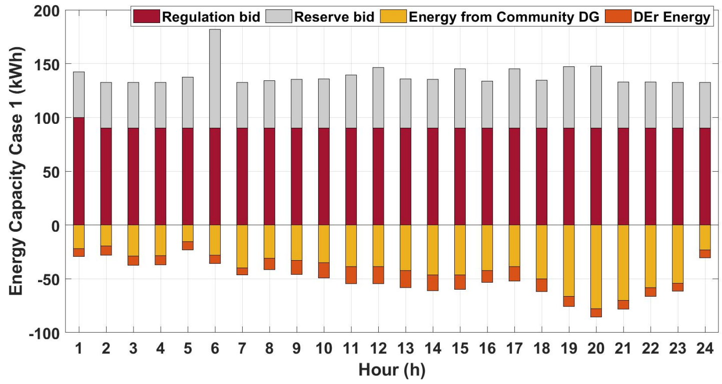


(c) Total daily profits with reserve and regulation ancillary services in the third community for robust Case 3.

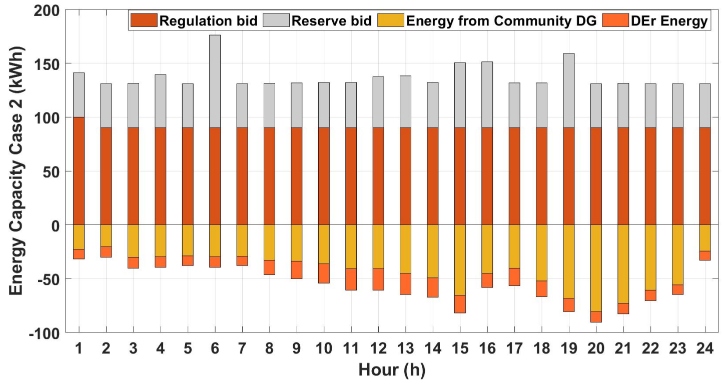


(d) Energy reserve and regulation bids, and energy charged from CES in the third community for robust Case 3.

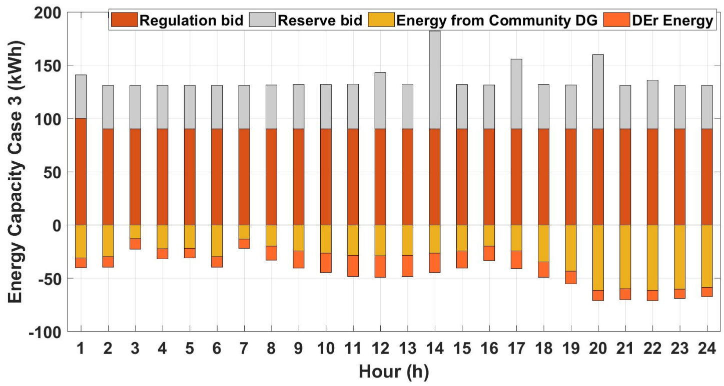
Figure 7.6: Robust and Deterministic solutions with community generation and ancillary for the third community for Case 3.



(a) Energy reserve and regulation bids, and energy exported in the third community for robust Case 1.



(b) Energy reserve and regulation bids, and energy exported in the second community for robust Case 2.



(c) Energy reserve and regulation bids, and energy exported in the third community for robust Case 3.

Figure 7.7: Energy reserve and regulation bids, and energy exported for all robust cases in the third community.

Chapter 8

Conclusion

With the recognition of the emerging consumer-centric markets, this research work proposes robust approaches to a community-based market platform of agents engaging in energy trading of their excess/deficit resources with their neighbours in a community microgrid under uncertainty. These forms of markets are centralised with a community manager playing a supervisory role in the control of energy trades between the agents in optimising the total traded costs and profits. The robust solution is realised against the worst-case scenario of uncertain parameters of agents' resources resulting in the highest traded costs, and an impact of uncertainty measured by the cost of robustness. The scenarios are generated by selecting an appropriate OA for the community-based market problem from the array selector. The TOAT array selector was used instead of the Monte Carlo simulation because of its ability in characterising uncertain parameters towards producing the worst and best case scenarios for robust optimization problems, while the Monte Carlo simulation is used to generate scenarios for more deterministic and stochastic problems in producing near-optimal solutions. The robust solutions obtained through TOAT are more conservative, hence control parameters are introduced in the robust CBEM to control the level of conservatives.

The work further proposes a community-based market model that integrates the economic utility of the agents' preference relations on imports/exports within and outside the community which is subject to constraints relating to the consumer's/prosumer's consumption budget and the producer's/prosumer's generation budget. Individual utility functions reflecting the agent's desired preferences are modeled as indifference curves within their energy consumption and generation spaces. Case studies for different community microgrids of agents were analysed and compared based on the perceived prices of agents, weather variability, and resource availability. The comparison extends to the use of fairness indicators in the analysis of the market results of community-based markets with and without economic utility maximation across all case studies. The results show that for each case study analysed in the community microgrid, the community-based market problem is solved to provide optimal trading costs from the preference relations of the agents, which changes as a result of the limited generation of agents, in the absence of a community-owned generation resource. With the integration of the community-owned generation resource and CES which can be scaled in capacity depending on agents' preference relations, the economic utility of agents is maximised. This additional capacity can provide more supply and storage for the increased demands of community agents' in assuming more autonomy from the grid. This improves the benefit of community-based market problems towards more locally traded energy in the community, compared to the peer-to-peer market which has no connection to the grid, and no supervisory control, and involves a negotiation process of available supply until convergence.

Furthermore, the provision of ancillary services by the community microgrid in the form of energy reserve and regulation is analysed in a joint market model which aims to offer these services while maximising the utility of agents. Given the proposed community-based market model is unable to supply the required capacity enough to participate in ancillary services as published by National grid [103], the community is assumed to be in a set of aggregated communities N, which can provide the necessary capacity needed for the provision of AS by calculating the total capacities from all the aggregated communities. The impact of uncertainty on this market is investigated to determine the cost of robustness as compared with the deterministic counterpart which provides comparisons with and without the offer of ancillary services while maximising the community agent's utility levels using robust optimisation. The results realised reflect the profits of the community when trading AS in comparison to not trading AS. The cost breakdown shows the revenues and percentage compositions realised from trading ancillary as well as CBEM trading costs. The robust solutions reflect the highest trading profits as more energy is purchased from the grid to balance the energy during the AS offering.

8.1 Future Work

In the future, many flexible energy market models that will enhance the reliability of the grid will be rolled out, therefore it is important to ensure seamless connectivity and communication when interfaced with the proposed community market structure. This may require information exchanges such as imports/exports from the community to the grid for the flexible markets to analyse accurately the imports/exports amounts in and out of the grid. This will involve economic and technical applications at the grid level in ensuring generation continuously balances demand. Therefore for future work, this thesis will be extended to integrate into flexible energy markets to enable energy import and export levels to be monitored towards ensuring grid reliability and resilience.

Another relevant aspect is in the parallel convergence of the negotiation process through distributed optimisation for the markets at community and grid levels. This may be impacted greatly by the scalability of the market, and integrating a large number of community agents may slow down the process of convergence. Therefore a move to reduce information exchanges between agent nodes will enhance the negotiation process. In the future, it is recommended to apply more distributed optimisation for community agents to solve their problems while improving the negotiation algorithm towards convergence.

Finally, the proposed community-based market will face more strategic behaviours from market participants and therefore it is important to utilise approaches within the market mechanism that enhances fairness amongst agents and ensures the grid constraints are within permissible limits. Sanctions in the form of penalties can be incurred by agents who do not follow the market rules at the community level, as well the grid may incur network access costs on maximum imports/exports in and out of the grid to ensure the flexibility and reliability of the network. In the future, the proposed models will be extended to penalise community agents' behaviours towards the community-based market problem and to monitor how this will improve community fairness.

Appendix A

Normalised Costs for Agents' Preferential Cost Curves

In this work, the Ausgrid peak tariff 2019 is considered as market prices for imports. Small deviations are applied to this tariff to realise a mean μ_M and standard deviation σ_M around which community agents set their preferred cost curves for energy trades within the community.

A normalised cost function is proposed according to [2] to model a price c_{jn_r}, c_{n_r}, c_{n_d}, c_{ln_g} community agents (prosumers, producers, consumers) are willing to pay or be paid for their consumption and production resources and an increase or decrease in the price of d_{jn_r}, d_{n_r}, d_{n_d}, d_{ln_g} corresponding to the minimum and maximum values of assets. This is expressed for each community agent as follows:

$$c_{jn_r} \approx \mathcal{N}(\mu_M, \sigma_M), d_{jn_r} \approx \mathcal{N}(0, \sigma_M) \ \forall j \in R_g \ n_r \in N_{pr} \tag{A.1}$$

$$c_{n_r} \approx \mathcal{N}(\mu_M, \sigma_M), d_{n_r} \approx \mathcal{N}(0, \sigma_M) \ \forall n_r \in N_{pr} \tag{A.2}$$

$$c_{n_d} \approx \mathcal{N}(\mu_M, \sigma_M), d_{n_d} \approx \mathcal{N}(0, \sigma_M) \ \forall n_d \in N_c \tag{A.3}$$

$$c_{n_g} \approx \mathcal{N}(\mu_M, \sigma_M), d_{ln_g} \approx \mathcal{N}(0, \sigma_M) \ \forall n_g \in N_p \tag{A.4}$$

For agents equipped with conventional generators and community-owned Dgs, the costs are scaled also to the market prices. Given an agent's conventional resource or community-owned generation unit, the willingness to be paid for the agent is represented as c_{ijn_r} for prosumers,

c_{iln_g} for producers, and the community set cost for its generation unit is c_i, the normalised costs is represented thus:

$$c_{ijn_r} \approx \mathcal{N}(\mu_M, \sigma_M), d_{ijn_r} \approx \mathcal{N}(0, \sigma_M)\ \forall n_r \in N_{pr} \forall i \in C_g \tag{A.5}$$

$$c_{iln_g} \approx \mathcal{N}(\mu_M, \sigma_M), d_{iln_g} \approx \mathcal{N}(0, \sigma_M) \forall n_g \in N_p \tag{A.6}$$

$$c_i \approx \mathcal{N}(\mu_M, \sigma_M), d_i \approx \mathcal{N}(0, \sigma_M) \forall i \in C_g \tag{A.7}$$

The mathematical expression for the Normalised costs for each agent resource becomes:

$$\mathcal{N}_{c_{jn_r}} = \frac{c_{jn_r} - \mu_M}{\sigma_M}, \mathcal{N}_{d_{jn_r}} = \frac{M_p - \mu_M}{\overline{M_p} - \underline{M_p}} \sigma_M\ \forall j \in R_g\ n_r \in N_{pr} \tag{A.8}$$

$$\mathcal{N}_{c_{n_r}} = \frac{c_{n_r} - \mu_M}{\sigma_M}, \mathcal{N}_{d_{n_r}} = \frac{M_p - \mu_M}{\overline{M_p} - \underline{M_p}} \sigma_M\ \forall\ n_r \in N_{pr} \tag{A.9}$$

$$\mathcal{N}_{c_{n_d}} = \frac{c_{n_d} - \mu_M}{\sigma_M}, \mathcal{N}_{d_{n_d}} = \frac{M_p - \mu_M}{\overline{M_p} - \underline{M_p}} \sigma_M\ \forall\ n_d \in N_c \tag{A.10}$$

$$\mathcal{N}_{c_{ln_g}} = \frac{c_{ln_g} - \mu_M}{\sigma_M}, \mathcal{N}_{d_{ln_g}} = \frac{M_p - \mu_M}{\overline{M_p} - \underline{M_p}} \sigma_M\ \forall\ n_d \in N_c \tag{A.11}$$

where M_p represents the market price in each period t following the small deviations applied. For the community agents and the community's conventional resources:

$$\mathcal{N}_{c_{in_r}} = \frac{c_{in_r} - \mu_M}{\sigma_M}, \mathcal{N}_{d_{in_r}} = \frac{M_p - \mu_M}{\overline{M_p} - \underline{M_p}} \sigma_M\ \forall i\ n_r \in N_{pr} \tag{A.12}$$

$$\mathcal{N}_{c_{iln_g}} = \frac{c_{iln_g} - \mu_M}{\sigma_M}, \mathcal{N}_{d_{iln_g}} = \frac{M_p - \mu_M}{\overline{M_p} - \underline{M_p}} \sigma_M\ \forall i\ n_r \in N_{pr} \tag{A.13}$$

Therefore from the normalised costs of the prices community agents are willing to pay, the marginal costs for their resources $a_{jn_r}, a_{n_r},\ a_{n_d}, a_{ln_g}$, and $b_{jn_r}, b_{n_r},\ b_{n_d}, b_{ln_g}$ and $a_{in_r},\ a_{in_g},\ a_i$, and $b_{in_r},\ b_{in_g},\ b_i$ are modelled as follows:

$$a_{jn_r} = 2\frac{\mathcal{N}_{c_{jn_r}}}{\overline{p}_{jn_r} - \underline{p}_{jn_r}}, b_{jn_r} = \mathcal{N}_{c_{jn_r}} + (\mathcal{N}_{d_{jn_r}} \frac{\overline{p}_{jn_r} - \underline{p}_{jn_r}}{\overline{p}_{jn_r} - \underline{p}_{jn_r}})\ \forall j \in R_g\ n_r \in N_{pr} \tag{A.14}$$

$$a_{n_r} = 2\frac{\mathcal{N}_{c_{n_r}}}{\overline{p}_{n_r} - \underline{p}_{n_r}}, b_{n_r} = \mathcal{N}_{c_{n_r}} + (\mathcal{N}_{d_{n_r}} \frac{\overline{p}_{n_r} - \underline{p}_{n_r}}{\overline{p}_{n_r} - \underline{p}_{n_r}})\ \forall\ n_r \in N_{pr} \tag{A.15}$$

$$a_{n_d} = 2\frac{\mathcal{N}_{c_{n_d}}}{\overline{p}_{n_d} - \underline{p}_{n_d}}, b_{n_d} = \mathcal{N}_{c_{n_d}} + (\mathcal{N}_{d_{n_d}} \frac{\overline{p}_{n_d} - \underline{p}_{n_d}}{\overline{p}_{n_d} - \underline{p}_{n_d}})\ \forall\ n_d \in N_c \tag{A.16}$$

$$a_{ln_g} = 2\frac{\mathcal{N}_{c_{ln_g}}}{\overline{p}_{ln_g} - \underline{p}_{ln_g}}, b_{ln_g} = \mathcal{N}_{c_{ln_g}} + (\mathcal{N}_{d_{ln_g}} \frac{\overline{p}_{ln_g} - \underline{p}_{ln_g}}{\overline{p}_{ln_g} - \underline{p}_{ln_g}})\ \forall l \in R_p\ n_g \in N_p \tag{A.17}$$

The agents' conventional resource and community generation unit, is expressed thus:

$$a_{in_r} = 2\frac{\mathcal{N}_{c_{in_r}}}{\overline{p}_{in_r} - \underline{p}_{in_r}}, b_{in_r} = \mathcal{N}_{c_{in_r}} + (\mathcal{N}_{d_{in_r}} \frac{\overline{p}_{in_r} - \underline{p}_{in_r}}{\overline{p}_{in_r} - \underline{p}_{in_r}})\ \forall i\ n_r \in N_{pr} \tag{A.18}$$

$$a_{in_g} = 2\frac{\mathcal{N}_{c_{in_r}}}{\overline{p}_{in_g} - \underline{p}_{in_g}}, b_{in_g} = \mathcal{N}_{c_{in_g}} + (\mathcal{N}_{d_{in_g}} \frac{\overline{p}_{in_g} - \underline{p}_{in_g}}{\overline{p}_{in_g} - \underline{p}_{in_g}})\ \forall i\ n_g \in N_p \tag{A.19}$$

$$a_i = 2\frac{\mathcal{N}_{c_i}}{\overline{p}_i^c - \underline{p}_i^c}, b_i = \mathcal{N}_{c_i} + (\mathcal{N}_{d_i} \frac{\overline{p}_i^c - \underline{p}_i^c}{\overline{p}_i^c - \underline{p}_i^c})\ \forall i \in C_g \tag{A.20}$$

The renewable generation resources of community agents have marginal costs $a_{jn_r}, a_{ln_g} = 0$ and $b_{jn_r}, b_{ln_g} = 0$ as a result of their lower and upper bounds being equal, so their generation outputs must be dispatchable.

Appendix B

Utility Maximization Formulation for Community Agents

B.1 Cobb Douglas

In this section, the economic model for prosumers' and consumers' consumption from within and outside the community is presented.

$$\max_{r_{n_dtm},\gamma_{n_dtm}} r_{n_dtm}^{\alpha_{n_dt}} \gamma_{n_dtm}^{\beta_{n_dt}},\ \forall n_d \in N_c, \forall t \in T \tag{B.1a}$$

$$r_{n_dtm} + \gamma_{n_dtm} = p_{n_dt}^{d},\ \forall n_d \in N_c, \forall t \in T \tag{B.1b}$$

$$r_{n_dtm} \geq 0, \gamma_{n_dtm} \geq 0,\ \forall n_d \in N_c, \forall t \in T \tag{B.1c}$$

Using Lagrangian multiplier λ

$$r_{n_dtm}^{\alpha_{n_dt}} \gamma_{n_dtm}^{\beta_{n_dt}} + \lambda(p_{n_dt}^{d} - r_{n_dtm} - \gamma_{n_dtm})\ \forall n_d \in N_c, \forall t \in T \tag{B.2}$$

$$\frac{\delta u(r_{n_dtm}, \gamma_{n_dtm})}{\delta_{r_{n_dtm}}} = \alpha_{n_dt} r_{n_dtm}^{\alpha_{n_dt}-1} \gamma_{n_dtm}^{\beta_{n_dt}} - \lambda\ \forall n_d \in N_c, \forall t \in T \tag{B.3}$$

$$\frac{\delta u(r_{n_dtm}, \gamma_{n_dtm})}{\delta_{\gamma_{n_dtm}}} = \beta_{n_dt} r_{n_dtm}^{\alpha_{n_dt}} \gamma_{n_dtm}^{\beta_{n_dt}-1} - \lambda\ \forall n_d \in N_c, \forall t \in T \tag{B.4}$$

Equations (B.3) and (B.4) are equal being both equal to λ. Dividing (B.3) by (B.4) gives the MRS

$$\frac{\alpha_{n_d t}\gamma_{n_d tm}}{\beta_{n_d t} r_{n_d tm}} = 1 \ \forall n_d \in N_c, \forall t \in T \tag{B.5}$$

$$\alpha_{n_d t}\gamma_{n_d tm} = \beta_{n_d t} r_{n_d tm} \ \forall n_d \in N_c, \forall t \in T \tag{B.6}$$

From equation (B.1b), representing each of $r_{n_d tm}$ and $\gamma_{n_d tm}$ explicitly gives:

$$r_{n_d tm} = p^d_{n_d t} - \gamma_{n_d tm}, \ \forall n_d \in N_c, \forall t \in T \tag{B.7}$$

$$\gamma_{n_d tm} = p^d_{n_d t} - r_{n_d tm}, \ \forall n_d \in N_c, \forall t \in T \tag{B.8}$$

solving each of equations (B.7) and (B.8) simultaneously with (B.6) gives:

$$\alpha_{n_d t}\gamma_{n_d tm} = \beta_{n_d t}(p^d_{n_d t} - \gamma_{n_d tm}) \ \forall n_d \in N_c, \forall t \in T \tag{B.9}$$

$$\gamma_{n_d tm}(\alpha_{n_d t} + \beta_{n_d t}) = \beta_{n_d t} p^d_{n_d t} \ \forall n_d \in N_c, \forall t \in T \tag{B.10}$$

$$\gamma_{n_d tm} = \frac{\beta_{n_d t}}{\alpha_{n_d t} + \beta_{n_d t}} p^d_{n_d t} \ \forall n_d \in N_c, \forall t \in T \tag{B.11}$$

Likewise solving for $r_{n_d tm}$ gives

$$r_{n_d tm} = \frac{\alpha_{n_d t}}{\alpha_{n_d t} + \beta_{n_d t}} p^d_{n_d t} \ \forall n_d \in N_c, \forall t \in T \tag{B.12}$$

Since $\alpha_{n_d t} + \beta_{n_d t} = 1$,

$$r_{n_d tm} = \alpha_{n_d t} p^d_{n_d t}; \quad \gamma_{n_d tm} = \beta_{n_d t} p^d_{n_d t} \tag{B.13}$$

These same models are used for prosumer consumption preferences by replacing a consumer n_d with the prosumer n_r.

B.2 Perfect Complements

In this section, the economic model for prosumers' and producers' production from within and outside the community is presented.

$$\min_{q_{n_gtm},\delta_{n_gtm}} U(\rho_{nt}q_{n_gtm}, \sigma_{n_gt}\delta_{n_gtm}), \forall n_g \in N_p, \forall t \in T \tag{B.14a}$$

$$q_{n_gtm} + \delta_{n_gtm} = p^g_{n_gt}, \quad \forall n_g \in N_p, \forall t \in T \tag{B.14b}$$

$$\rho_{nt}q_{n_gtm} - \sigma_{n_gt}\delta_{n_gtm} = 0, \ \forall n_g \in N_p, \forall t \in T \tag{B.14c}$$

$$q_{n_gtm} \geq 0, \delta_{n_gtm} \geq 0, \quad \forall n_g \in N_p, \forall t \in T \tag{B.14d}$$

From (B.14c)

$$q_{ntm} = \frac{\sigma_{n_gt}\delta_{n_gtm}}{\rho_{n_gt}}, \ \forall n_g \in N_p, \forall t \in T \tag{B.15}$$

$$\delta_{n_gtm} = \frac{\rho_{n_gt}q_{n_gtm}}{\sigma_{n_gt}}, \ \forall n_g \in N_p, \forall t \in T \tag{B.16}$$

Substituting each of equations (B.15) and (B.16) in (B.14b) gives:

$$\frac{\sigma_{n_gt}\delta_{n_gtm}}{\rho_{n_gt}} + \delta_{n_gtm} = p^g_{n_gt}, \quad \forall n_g \in N_p, \forall t \in T \tag{B.17}$$

Multiply through by ρ_{n_gt}.

$$\sigma_{n_gt}\delta_{n_gtm} + \rho_{n_gt}\delta_{n_gtm} = \rho_{n_gt}p^g_{n_gt}, \quad \forall n_g \in N_p, \forall t \in T \tag{B.18}$$

$$\delta_{n_gtm}(\sigma_{n_gt} + \rho_{n_gt}) = \rho_{n_gt}p^g_{n_gt}, \quad \forall n_g \in N_p, \forall t \in T \tag{B.19}$$

$$\delta_{n_gtm} = \frac{\rho_{n_gt}}{\sigma_{n_gt} + \rho_{n_gt}}p^g_{n_gt}, \quad \forall n_g \in N_p, \forall t \in T \tag{B.20}$$

Likewise solving for δ_{n_gtm} gives

$$q_{n_gtm} = \frac{\sigma_{n_gt}}{\sigma_{n_gt} + \rho_{n_gt}}p^g_{n_gt}, \quad \forall n_g \in N_p, \forall t \in T \tag{B.21}$$

These same models are used for prosumer consumption preferences by replacing a consumer n_g with the prosumer n_r.

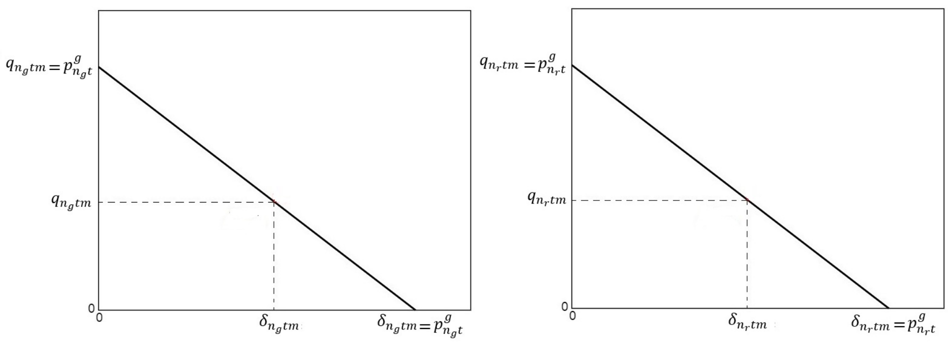


Figure B.1: Schematic representation of production preferences of producer agents: perfect substitute

B.3 Perfect Substitutes

In the second and third case studies for analysing weather variation and resource availability from Chapter 4, the producer and prosumer preferences are assumed, to export all the energy produced from their resources to the community allowing for zero export to the grid. This can be modeled using the perfect substitute utility function as follows:

$$\max_{q_{n_g tm}, \delta_{n_g tm}} \rho_{nt} q_{n_g tm} + \sigma_{n_g t} \delta_{n_g tm}, \forall n_g \in N_p, \forall t \in T \tag{B.22a}$$

$$\rho_{nt} q_{n_g tm} + \sigma_{n_g t} \delta_{n_g tm} = p^g_{n_g t}, \quad \forall n_g \in N_p, \forall t \in T \tag{B.22b}$$

$$q_{n_g tm} \geq 0, \delta_{n_g tm} \geq 0, \quad \forall n_g \in N_p, \forall t \in T \tag{B.22c}$$

Equation (B.22b) is expressed explicitly to give the equation of a line as follows:

$$\sigma_{n_g t} \delta_{n_g tm} = p^g_{n_g t} - \rho_{nt} q_{n_g tm}, \quad \forall n_g \in N_p, \forall t \in T \tag{B.23}$$

$$\delta_{n_g tm} = \frac{p^g_{n_g t}}{\sigma_{n_g t}} - \frac{\rho_{nt}}{\sigma_{n_g t}} q_{n_g tm}, \quad \forall n_g \in N_p, \forall t \in T \tag{B.24}$$

The slope at any point of the indifference curve should be equal to $\frac{\rho_{nt}}{\sigma_{n_g t}} = -1$ for (B.14b) to hold. The producer agent is indifferent and can decide to forgo all exports to the grid, and only export within the community and store the excess energy in the CES.

Bibliography

[1] J. Rajasekharan and V. Koivunen, “Optimal energy consumption model for smart grid households with energy storage,” *IEEE Journal of Selected Topics in Signal Processing*, vol. 8, no. 6, pp. 1154–1166, 2014.

[2] F. Moret and P. Pinson, “Energy collectives: A community and fairness based approach to future electricity markets,” *IEEE Transactions on Power Systems*, vol. 34, pp. 3994–4004, Sep. 2019.

[3] J. Qin, R. Rajagopal, and P. Varaiya, “Flexible market for smart grid: Coordinated trading of contingent contracts,” *IEEE Transactions on Control of Network Systems*, vol. 5, no. 4, pp. 1657–1667, 2017.

[4] Z. Liu, Q. Wu, S. Huang, and H. Zhao, “Transactive energy: A review of state of the art and implementation,” *2017 IEEE Manchester PowerTech*, pp. 1–6, 2017.

[5] T. Sousa, T. Soares, P. Pinson, F. Moret, T. Baroche, and E. Sorin, “Peer-to-peer and community-based markets: A comprehensive review,” *arXiv preprint arXiv:1810.09859*, 2018.

[6] X. Fang, S. Misra, G. Xue, and D. Yang, “Smart grid—the new and improved power grid: A survey,” *IEEE Communications Surveys & Tutorials*, vol. 14, no. 4, pp. 944–980, 2011.

[7] A. J. Rathnayaka, V. M. Potdar, O. Hussain, and T. Dillon, “Identifying prosumer’s energy sharing behaviours for forming optimal prosumer-communities,” in *Proceedings - 2011 International Conference on Cloud and Service Computing, CSC 2011*, 2011.

[8] L. Jia and L. Tong, “Dynamic pricing and distributed energy management for demand response,” *IEEE Transactions on Smart Grid*, vol. 7, no. 2, pp. 1128–1136, 2016.

[9] Communications Technology Laboratory/ Smart Connected Systems Division, "Transactive energy: An overview," *Smart Grid Group*, 2017. Available at https://www.nist.gov/el/smart-grid-menu/hot-topics/transactive-energy-overview.

[10] Y. Zhou, J. Wu, and C. Long, "Evaluation of peer-to-peer energy sharing mechanisms based on a multiagent simulation framework," *Applied Energy*, vol. 222, pp. 993–1022, 2018.

[11] C.-C. Lin, D.-J. Deng, W.-Y. Liu, and L. Chen, "Peak load shifting in the internet of energy with energy trading among end-users," *IEEE Access*, vol. 5, pp. 1967–1976, 2017.

[12] C. Zhang, J. Wu, Y. Zhou, M. Cheng, and C. Long, "Peer-to-peer energy trading in a microgrid," *Applied Energy*, vol. 220, pp. 1–12, 2018.

[13] A. Gholami, T. Shekari, F. Aminifar, and M. Shahidehpour, "Microgrid scheduling with uncertainty: The quest for resilience," *IEEE Transactions on Smart Grid*, vol. 7, pp. 2849–2858, Nov 2016.

[14] S. E. Fleten and E. Pettersen, "Constructing bidding curves for a price-taking retailer in the norwegian electricity market," *IEEE Transactions on Power Systems*, vol. 20, pp. 701–708, May 2005.

[15] T. Morstyn, A. Teytelboym, and M. D. Mcculloch, "Bilateral contract networks for peer-to-peer energy trading," *IEEE Transactions on Smart Grid*, vol. 10, pp. 2026–2035, March 2019.

[16] J. R. Birge and F. Louveaux, *Introduction to stochastic programming.* Springer Science & Business Media, 2011.

[17] D. Bertsimas, D. B. Brown, and C. Caramanis, "Theory and applications of robust optimization," *SIAM Review*, vol. 53, no. 3, pp. 464–501, 2011.

[18] D. Bertsimas, E. Litvinov, X. A. Sun, J. Zhao, and T. Zheng, "Adaptive robust optimization for the security constrained unit commitment problem," *IEEE Transactions on Power Systems*, vol. 28, no. 1, pp. 52–63, 2012.

[19] K.-L. Tsui, "An overview of taguchi method and newly developed statistical methods for robust design," *Iie Transactions*, vol. 24, no. 5, pp. 44–57, 1992.

[20] Y. Ben-Haim, *Info-gap decision theory: decisions under severe uncertainty.* Elsevier, 2006.

[21] C. P. Simon, L. Blume, *et al.*, *Mathematics for economists*, vol. 7. Norton New York, 1994.

[22] G. Renshaw, *Maths for economics.* Oxford University Press, 2012.

[23] R. Meade, "Measuring prosumer welfare: Modelling household demand for distributed energy resources and residual electricity supply," in *Twelfth Toulouse Conference on The Economics of Energy and Climate*, 2019.

[24] J. Rajasekharan and V. Koivunen, "Production equilibrium in cooperative smart hybrid renewable minigrids," in *2014 48th Annual Conference on Information Sciences and Systems (CISS)*, pp. 1–6, IEEE, 2014.

[25] M. Zugno and A. J. Conejo, "A robust optimization approach to energy and reserve dispatch in electricity markets," *European Journal of Operational Research*, vol. 247, no. 2, pp. 659–671, 2015.

[26] H.-G. Beyer and B. Sendhoff, "Robust optimization–a comprehensive survey," *Computer Methods in Applied Mechanics and Engineering*, vol. 196, no. 33-34, pp. 3190–3218, 2007.

[27] F. Economics, "Metis technical note t4: Overview of european electricity markets," *METIS Tech Notes*, vol. 67, 2016.

[28] E. Mashhour and S. M. Moghaddas-Tafreshi, "Bidding strategy of virtual power plant for participating in energy and spinning reserve markets—part i: Problem formulation," *IEEE Transactions on Power Systems*, vol. 26, no. 2, pp. 949–956, 2010.

[29] F. A. Campos, A. M. San Roque, E. F. Sanchez-Ubeda, and J. P. González, "Strategic bidding in secondary reserve markets," *IEEE Transactions on Power Systems*, vol. 31, no. 4, pp. 2847–2856, 2015.

[30] L. Baringo and A. J. Conejo, "Offering strategy via robust optimization," *IEEE Transactions on Power Systems*, vol. 26, no. 3, pp. 1418–1425, 2010.

[31] Y. Xiao, C. Bandi, and E. Wei, "Robust supply function bidding in electricity markets with renewables," in *2016 54th Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, pp. 243–247, IEEE, 2016.

[32] D. Xenias, C. J. Axon, L. Whitmarsh, P. M. Connor, N. Balta-Ozkan, and A. Spence, "Uk smart grid development: An expert assessment of the benefits, pitfalls and functions," *Renewable Energy*, vol. 81, pp. 89–102, 2015.

[33] N. IqtiyaniIlham, M. Hasanuzzaman, and M. Hosenuzzaman, "European smart grid prospects, policies, and challenges," *Renewable and Sustainable Energy Reviews*, vol. 67, pp. 776–790, 2017.

[34] A. J. Rathnayaka, V. M. Potdar, O. Hussain, and T. Dillon, "Identifying prosumer's energy sharing behaviours for forming optimal prosumer-communities," in *Proceedings - 2011 International Conference on Cloud and Service Computing, CSC 2011*, 2011.

[35] I. S. Bayram, M. Z. Shakir, M. Abdallah, and K. Qaraqe, "A survey on energy trading in smart grid," in *2014 IEEE Global Conference on Signal and Information Processing (GlobalSIP)*, pp. 258–262, IEEE, 2014.

[36] O. Jogunola, A. Ikpehai, K. Anoh, B. Adebisi, M. Hammoudeh, S.-Y. Son, and G. Harris, "State-of-the-art and prospects for peer-to-peer transaction-based energy system," *Energies*, vol. 10, no. 12, p. 2106, 2017.

[37] J. Matamoros, D. Gregoratti, and M. Dohler, "Microgrids energy trading in islanding mode," in *2012 IEEE Third International Conference on Smart Grid Communications (SmartGridComm)*, pp. 49–54, IEEE, 2012.

[38] D. Ilic, P. G. Da Silva, S. Karnouskos, and M. Griesemer, "An energy market for trading electricity in smart grid neighbourhoods," in *2012 6th IEEE International Conference on Digital Ecosystems and Technologies (DEST)*, pp. 1–6, IEEE, 2012.

[39] S. Nguyen, W. Peng, P. Sokolowski, D. Alahakoon, and X. Yu, "Optimizing rooftop photovoltaic distributed generation with battery storage for peer-to-peer energy trading," *Applied Energy*, vol. 228, pp. 2567–2580, 2018.

[40] J. Guerrero, A. Chapman, and G. Verbic, "A study of energy trading in a low-voltage network: Centralised and distributed approaches," in *2017 Australasian Universities Power Engineering Conference (AUPEC)*, pp. 1–6, IEEE, 2017.

[41] M. R. Alam, M. St-Hilaire, and T. Kunz, "Peer-to-peer energy trading among smart homes," *Applied Energy*, vol. 238, pp. 1434–1443, 2019.

[42] D. Gregoratti and J. Matamoros, "Distributed energy trading: The multiple-microgrid case," *IEEE Transactions on Industrial Electronics*, vol. 62, no. 4, pp. 2551–2559, 2014.

[43] J. Lee, J. Guo, J. K. Choi, and M. Zukerman, "Distributed energy trading in microgrids: A game-theoretic model and its equilibrium analysis," *IEEE Transactions on Industrial Electronics*, vol. 62, no. 6, pp. 3524–3533, 2015.

[44] C.-C. Lin, D.-J. Deng, C.-C. Kuo, and Y.-L. Liang, "Optimal Charging Control of Energy Storage and Electric Vehicle of an Individual in the Internet of Energy with Energy Trading," *IEEE Transactions on Industrial Informatics*, 2017.

[45] H. Karimi and S. Jadid, "Optimal energy management for multi-microgrid considering demand response programs: A stochastic multi-objective framework," *Energy*, vol. 195, p. 116992, 2020.

[46] M. Jaradat, M. Jarrah, A. Bousselham, Y. Jararweh, and M. Al-Ayyoub, "The internet of energy: Smart sensor networks and big data management for smart grid," in *Procedia Computer Science*, 2015.

[47] A. Al-Ali, "Internet of Things Role in the Renewable Energy Resources," *Energy Procedia*, 2016.

[48] P. Chakraborty, C. Mannweiler, and H. D. Schotten, "Evolutionary Approach for Multi-objective Optimization of Wireless Mesh Networks," tech. rep.

[49] H. Liang, B. J. Choi, A. Abdrabou, W. Zhuang, and X. Shen, "Decentralized economic dispatch in microgrids via heterogeneous wireless networks," *IEEE Journal on Selected Areas in Communications*, 2012.

[50] C. Zhao, X. Duan, and Y. Shi, "Analysis of Consensus-based Economic Dispatch Algorithm under Uniform Time Delays," tech. rep., 2016.

[51] H. Beitollahi and G. Deconinck, "Peer-to-peer networks applied to power grid," in *Proceedings of the International Conference on Risks and Security of Internet and Systems (CRiSIS)*, p. 8, Citeseer, 2007.

[52] E. Sorin, L. Bobo, and P. Pinson, "Consensus-based approach to peer-to-peer electricity markets with product differentiation," *IEEE Transactions on Power Systems*, vol. 34, no. 2, pp. 994–1004, 2018.

[53] T. Baroche, P. Pinson, R. L. G. Latimier, and H. B. Ahmed, "Exogenous cost allocation in peer-to-peer electricity markets," *IEEE Transactions on Power Systems*, vol. 34, pp. 2553–2564, July 2019.

[54] R. Alvaro-Hermana, J. Fraile-Ardanuy, P. J. Zufiria, L. Knapen, and D. Janssens, "Peer to peer energy trading with electric vehicles," *IEEE Intelligent Transportation Systems Magazine*, vol. 8, no. 3, pp. 33–44, 2016.

[55] C. Zhang, J. Wu, Y. Zhou, M. Cheng, and C. Long, "Peer-to-Peer energy trading in a Microgrid," *Applied Energy*, 2018.

[56] M. R. Alam, M. St-Hilaire, and T. Kunz, "An optimal P2P energy trading model for smart homes in the smart grid," *Energy Efficiency*, 2017.

[57] E. Mengelkamp, J. Gärttner, K. Rock, S. Kessler, L. Orsini, and C. Weinhardt, "Designing microgrid energy markets: A case study: The brooklyn microgrid," *Applied Energy*, vol. 210, pp. 870–880, 2018.

[58] N. Liu, X. Yu, C. Wang, C. Li, L. Ma, and J. Lei, "Energy-Sharing Model with Price-Based Demand Response for Microgrids of Peer-to-Peer Prosumers," *IEEE Transactions on Power Systems*, 2017.

[59] T. Morstyn and M. D. McCulloch, "Multiclass energy management for peer-to-peer energy trading driven by prosumer preferences," *IEEE Transactions on Power Systems*, vol. 34, no. 5, pp. 4005–4014, 2018.

[60] C. Zhang, J. Wu, M. Cheng, Y. Zhou, and C. Long, “ScienceDirect A Bidding System for Peer-to-Peer Energy Trading in a Grid-connected Microgrid,” pp. 1876–6102, 2016.

[61] T. Liu, X. Tan, B. Sun, Y. Wu, X. Guan, and D. H. Tsang, “Energy management of cooperative microgrids with P2P energy sharing in distribution networks,” in *2015 IEEE International Conference on Smart Grid Communications, SmartGridComm 2015*, 2016.

[62] F. Moret, T. Baroche, E. Sorin, and P. Pinson, “Negotiation algorithms for peer-to-peer electricity markets: Computational properties,” in *2018 Power Systems Computation Conference (PSCC)*, pp. 1–7, IEEE, 2018.

[63] H. Xing, Z. Lin, and M. Fu, “An ADMM + consensus based distributed algorithm for dynamic economic power dispatch in smart grid,” in *2015 34th Chinese Control Conference (CCC)*, 2015.

[64] M. Hamdi, M. Chaoui, L. Idoumghar, and A. Kachouri, “Coordinated consensus for smart grid economic environmental power dispatch with dynamic communication network,” *IET Generation, Transmission & Distribution*, vol. 12, no. 11, pp. 2603–2613, 2018.

[65] A. Cherukuri and J. Cortes, “Distributed Generator Coordination for Initialization and Anytime Optimization in Economic Dispatch,” *IEEE Transactions on Control of Network Systems*, 2015.

[66] S. Yang, S. Tan, and J.-X. Xu, “Consensus Based Approach for Economic Dispatch Problem in a Smart Grid,” *IEEE Transactions on Power Systems*, 2013.

[67] R. Olfati-Saber, J. A. Fax, and R. M. Murray, “Consensus and cooperation in networked multi-agent systems,” *Proceedings of the IEEE*, 2007.

[68] C. Long, J. Wu, C. Zhang, M. Cheng, and A. Al-Wakeel, “Feasibility of Peer-to-Peer Energy Trading in Low Voltage Electrical Distribution Networks,” in *Energy Procedia*, 2017.

[69] M. R. Alam, M. St-Hilaire, and T. Kunz, “An optimal P2P energy trading model for smart homes in the smart grid,” *Energy Efficiency*, 2017.

[70] K. Zhang, Y. Mao, S. Leng, S. Maharjan, Y. Zhang, A. Vinel, and M. Jonsson, "Incentive-Driven Energy Trading in the Smart Grid," *IEEE Access*, 2016.

[71] C. Zhang, J. Wu, M. Cheng, Y. Zhou, and C. Long, " A Bidding System for Peer-to-Peer Energy Trading in a Grid-connected Microgrid," pp. 1876–6102, 2016.

[72] M. Motalleb and R. Ghorbani, "Non-cooperative game-theoretic model of demand response aggregator competition for selling stored energy in storage devices," *Applied Energy*, 2017.

[73] W. Hou, G. Tian, L. Guo, X. Wang, X. Zhang, and Z. Ning, "Cooperative Mechanism for Energy Transportation and Storage in Internet of Energy," *IEEE Access*, 2017.

[74] A. Ben-Tal and A. Nemirovski, "Robust optimization–methodology and applications," *Mathematical programming*, vol. 92, no. 3, pp. 453–480, 2002.

[75] A. Ben-Tal and A. Nemirovski, "Selected topics in robust convex optimization," *Mathematical Programming*, vol. 112, no. 1, pp. 125–158, 2008.

[76] B. L. Gorissen, İ. Yanıkoğlu, and D. den Hertog, "A practical guide to robust optimization," *Omega*, vol. 53, pp. 124–137, 2015.

[77] V. Gabrel, C. Murat, and A. Thiele, "Recent advances in robust optimization: An overview," *European Journal of Operational Research*, vol. 235, no. 3, pp. 471–483, 2014.

[78] A. J. Conejo and X. Wu, "Robust optimization in power systems: a tutorial overview," *Optimization and Engineering*, pp. 1–23, 2021.

[79] A. Prékopa, *Stochastic programming*, vol. 324. Springer Science & Business Media, 2013.

[80] A. Ruszczyński and A. Shapiro, "Stochastic programming models," *Handbooks in Operations Research and Management Science*, vol. 10, pp. 1–64, 2003.

[81] D. Bertsimas and M. Sim, "The price of robustness," *Operations Research*, vol. 52, no. 1, pp. 35–53, 2004.

[82] H. Rahimian and S. Mehrotra, "Distributionally robust optimization: A review," *arXiv preprint arXiv:1908.05659*, 2019.

[83] D. Bertsimas, V. Gupta, and N. Kallus, "Data-driven robust optimization," *Mathematical Programming*, vol. 167, no. 2, pp. 235–292, 2018.

[84] X. Chang, Y. Xu, and H. Sun, "Vertex scenario-based robust peer-to-peer transactive energy trading in distribution networks," *International Journal of Electrical Power & Energy Systems*, vol. 138, p. 107903, 2022.

[85] Y. Xiang, J. Liu, and Y. Liu, "Robust energy management of microgrid with uncertain renewable generation and load," *IEEE Transactions on Smart Grid*, vol. 7, no. 2, pp. 1034–1043, 2015.

[86] S. K. Karna, R. Sahai, *et al.*, "An overview on taguchi method," *International Journal of Engineering and Mathematical Sciences*, vol. 1, no. 1, pp. 1–7, 2012.

[87] R. N. Kacker, E. S. Lagergren, and J. J. Filliben, "Taguchi's orthogonal arrays are classical designs of experiments," *Journal of Research of the National Institute of Standards and Technology*, vol. 96, no. 5, p. 577, 1991.

[88] B. Alizadeh, S. Dehghan, N. Amjady, S. Jadid, and A. Kazemi, "Robust transmission system expansion considering planning uncertainties," *IET Generation, Transmission & Distribution*, vol. 7, no. 11, pp. 1318–1331, 2013.

[89] B. Zeng and L. Zhao, "Solving two-stage robust optimization problems using a column-and-constraint generation method," *Operations Research Letters*, vol. 41, no. 5, pp. 457–461, 2013.

[90] S. Dehghan, N. Amjady, and A. Kazemi, "Two-stage robust generation expansion planning: A mixed integer linear programming model," *IEEE Transactions on Power Systems*, vol. 29, no. 2, pp. 584–597, 2013.

[91] M. Motalleb and R. Ghorbani, "Non-cooperative game-theoretic model of demand response aggregator competition for selling stored energy in storage devices," *Applied Energy*, 2017.

[92] S. Board, "Preferences and utility," *UCLA, Oct*, 2009.

[93] A. M. Pirbazari, "Ancillary services definitions, markets and practices in the world," in *2010 IEEE/PES Transmission and Distribution Conference and Exposition: Latin America (T&D-LA)*, pp. 32–36, IEEE, 2010.

[94] Y. G. Rebours, D. S. Kirschen, M. Trotignon, and S. Rossignol, "A survey of frequency and voltage control ancillary services—part i: Technical features," *IEEE Transactions on Power Systems*, vol. 22, no. 1, pp. 350–357, 2007.

[95] B. Kirby, "Ancillary services: Technical and commercial insights," *Retrieved October*, vol. 4, p. 2012, 2007.

[96] M. B. Zammit, D. J. Hill, and R. J. Kaye, "Designing ancillary services markets for power system security," *IEEE Transactions on Power Systems*, vol. 15, no. 2, pp. 675–680, 2000.

[97] P. Wang, H. Zareipour, and W. D. Rosehart, "Descriptive models for reserve and regulation prices in competitive electricity markets," *IEEE Transactions on Smart Grid*, vol. 5, no. 1, pp. 471–479, 2013.

[98] V. Vahidinasab and S. Jadid, "Stochastic multiobjective self-scheduling of a power producer in joint energy and reserves markets," *Electric Power Systems Research*, vol. 80, no. 7, pp. 760–769, 2010.

[99] T. Li, M. Shahidehpour, and Z. Li, "Risk-constrained bidding strategy with stochastic unit commitment," *IEEE Transactions on Power Systems*, vol. 22, no. 1, pp. 449–458, 2007.

[100] A. Majzoobi and A. Khodaei, "Application of microgrids in providing ancillary services to the utility grid," *Energy*, vol. 123, pp. 555–563, 2017.

[101] C. Yuen and A. Oudalov, "The feasibility and profitability of ancillary services provision from multi-microgrids," in *2007 IEEE Lausanne Power Tech*, pp. 598–603, IEEE, 2007.

[102] Y. G. Rebours, D. S. Kirschen, M. Trotignon, and S. Rossignol, "A survey of frequency and voltage control ancillary services—part ii: Economic features," *IEEE Transactions on Power Systems*, vol. 22, no. 1, pp. 358–366, 2007.

[103] N. G. ESO, "National grid eso balancing services," *Official Website of the European Union*, 2023. Available at:`https://www.nationalgrideso.com/industry-information/balancing-services`.

[104] T. Sousa, T. Soares, P. Pinson, F. Moret, T. Baroche, and E. Sorin, "Peer-to-peer and community-based markets: A comprehensive review," *Renewable & Sustainable Energy Reviews*, vol. 104, pp. 367–378, 2019.

[105] Y. Parag and B. K. Sovacool, "Electricity market design for the prosumer era," *Nature Energy*, vol. 1, no. 4, p. 16032, 2016.

[106] C. Long, J. Wu, C. Zhang, L. Thomas, M. Cheng, and N. Jenkins, "Peer-to-peer energy trading in a community microgrid," in *2017 IEEE Power & Energy Society General Meeting*, pp. 1–5, IEEE, 2017.

[107] S. S. Reddy, V. Sandeep, and C.-M. Jung, "Review of stochastic optimization methods for smart grid," *Frontiers in Energy*, vol. 11, no. 2, pp. 197–209, 2017.

[108] H. Yu, C. Chung, and K. Wong, "Robust transmission network expansion planning method with taguchi's orthogonal array testing," *IEEE Transactions on Power Systems*, vol. 26, no. 3, pp. 1573–1580, 2010.

[109] D. Liu and Y. Cai, "Taguchi method for solving the economic dispatch problem with nonsmooth cost functions," *IEEE Transactions on Power Systems*, vol. 20, no. 4, pp. 2006–2014, 2005.

[110] Y.-Y. Hong, F.-J. Lin, and T.-H. Yu, "Taguchi method-based probabilistic load flow studies considering uncertain renewables and loads," *IET Renewable Power Generation*, vol. 10, no. 2, pp. 221–227, 2016.

[111] D. Bertsekas, *Convex optimization theory*, vol. 1. Athena Scientific, 2009.

[112] E. Ratnam, S. Weller, C. Kellett, and A. Murray, "Residential load and rooftop pv generation: an australian distribution network dataset," *International Journal of Sustainable Energy*, vol. 36, no. 8, pp. 787–806, 2017.

[113] S. Papathanassiou, N. Hatziargyriou, K. Strunz, *et al.*, "A benchmark low voltage microgrid network," in *Proceedings of the CIGRE Symposium: Power Systems with Dispersed Generation*, pp. 1–8, CIGRE, 2005.

[114] Ausgrid's Regulator Proposal 2019-24, "Tariff structure statements-attachment 10.01," *Final Decision*, 2019. Available at: https://www.ausgrid.com.au/-/media/Documents/Regulation/Reports-plans/Ausgrid-approved-TSS-2019-24.pdf.

[115] E. Commission, "Electricity and gas prices in the first half of 2021," *Official website of the European Union*, 2021. Available at:https://ec.europa.eu/info/news/electricity-and-gas-prices-first-half-2021-2021-oct-20_en#:~:text=Eurostat%20reports%20that%20in%20the%20first%20half%20of,100%20kWh%20in%20the%20first%20half%20of%202021.

[116] J. Levin and P. Milgrom, "Consumer theory," *Available at:https://web.stanford.edu/~jdlevin/Econ%20202/Consumer%20Theory.pdf*, vol. 20202, 2004.

[117] T. Elrod, G. J. Russell, A. D. Shocker, R. L. Andrews, L. Bacon, B. L. Bayus, J. D. Carroll, R. M. Johnson, W. A. Kamakura, P. Lenk, *et al.*, "Inferring market structure from customer response to competing and complementary products," *Marketing Letters*, vol. 13, no. 3, pp. 221–232, 2002.

[118] P. Zou, Q. Chen, Q. Xia, G. He, and C. Kang, "Evaluating the contribution of energy storages to support large-scale renewable generation in joint energy and ancillary service markets," *IEEE Transactions on Sustainable Energy*, vol. 7, no. 2, pp. 808–818, 2015.

[119] B. Xu, Y. Wang, Y. Dvorkin, R. Fernández-Blanco, C. A. Silva-Monroy, J.-P. Watson, and D. S. Kirschen, "Scalable planning for energy storage in energy and reserve markets," *IEEE Transactions on Power systems*, vol. 32, no. 6, pp. 4515–4527, 2017.

[120] Z. Zhang, Y. Zhang, Q. Huang, and W.-J. Lee, "Market-oriented optimal dispatching strategy for a wind farm with a multiple stage hybrid energy storage system," *CSEE Journal of Power and Energy Systems*, vol. 4, no. 4, pp. 417–424, 2018.

[121] J. Rajasekharan, J. Lundén, and V. Koivunen, "Competitive equilibrium pricing and co-operation in smart grids with energy storage," in *2013 47th Annual Conference on Information Sciences and Systems (CISS)*, pp. 1–5, IEEE, 2013.

[122] V. Virasjoki, P. Rocha, A. S. Siddiqui, and A. Salo, "Market impacts of energy storage in a transmission-constrained power system," *IEEE Transactions on Power Systems*, vol. 31, no. 5, pp. 4108–4117, 2015.

[123] A. Gholami, F. Aminifar, and M. Shahidehpour, "Front lines against the darkness: Enhancing the resilience of the electricity grid through microgrid facilities," *IEEE Electrification Magazine*, 2016.

[124] J. Liu, M. Kazemi, A. Motamedi, H. Zareipour, and J. Rippon, "Security-constrained optimal scheduling of transmission outages with load curtailment," *IEEE Transactions on Power Systems*, vol. 33, no. 1, 2017.

[125] D. v. Walle, "On the use of the susenas for modelling consumer behaviour," *Bulletin of Indonesian Economic Studies*, 1988.

[126] M. Aklin, P. Bayer, S. Harish, and J. Urpelainen, "Does basic energy access generate socioeconomic benefits? a field experiment with off-grid solar power in india," *Science Advances*, vol. 3, no. 5, 2017.

[127] R. Deng, Z. Yang, J. Chen, and M.-Y. Chow, "Load scheduling with price uncertainty and temporally-coupled constraints in smart grids," *IEEE Transactions on Power Systems*, vol. 29, no. 6, pp. 2823–2834, 2014.

[128] I. Onugha, S. Dehghan, and P. Aristidou, "Rethinking consumer-centric markets under uncertainty: A robust approach to community-based energy trades," in *Proc. of the 2020 IEEE General Meeting*, 2020.

[129] N. Amjady, S. Dehghan, A. Attarha, and A. J. Conejo, "Adaptive robust network-constrained ac unit commitment," *IEEE Transactions on Power Systems*, vol. 32, no. 1, pp. 672–683, 2016.

[130] R. Jiang, J. Wang, M. Zhang, and Y. Guan, "Two-stage minimax regret robust unit commitment," *IEEE Transactions on Power Systems*, vol. 28, no. 3, pp. 2271–2282, 2013.

[131] Y. Guan and J. Wang, "Uncertainty sets for robust unit commitment," *IEEE Transactions on Power Systems*, vol. 29, no. 3, pp. 1439–1440, 2013.

[132] C. Zhao, J. Wang, J.-P. Watson, and Y. Guan, "Multi-stage robust unit commitment considering wind and demand response uncertainties," *IEEE Transactions on Power Systems*, vol. 28, no. 3, pp. 2708–2717, 2013.

[133] X. Zhang and A. J. Conejo, "Robust transmission expansion planning representing long- and short-term uncertainty," *IEEE Transactions on Power Systems*, vol. 33, no. 2, pp. 1329–1338, 2017.

[134] C. Ruiz and A. J. Conejo, "Robust transmission expansion planning," *European Journal of Operational Research*, vol. 242, no. 2, pp. 390–401, 2015.

[135] R. A. Jabr, "Robust transmission network expansion planning with uncertain renewable generation and loads," *IEEE Transactions on Power Systems*, vol. 28, no. 4, pp. 4558–4567, 2013.

[136] S. Dehghan, N. Amjady, and A. J. Conejo, "Adaptive robust transmission expansion planning using linear decision rules," *IEEE Transactions on Power Systems*, vol. 32, no. 5, pp. 4024–4034, 2017.

[137] Z. Wu, Y. Liu, W. Gu, Y. Wang, and C. Chen, "Contingency-constrained robust transmission expansion planning under uncertainty," *International Journal of Electrical Power & Energy Systems*, vol. 101, pp. 331–338, 2018.

[138] S. Karagiannopoulos, J. Gallmann, M. G. Vayá, P. Aristidou, and G. Hug, "Active distribution grids offering ancillary services in islanded and grid-connected mode," *IEEE Transactions on Smart Grid*, vol. 11, no. 1, pp. 623–633, 2019.

[139] H. Karbouj and Z. H. Rather, "Voltage control ancillary service from wind power plant," *IEEE Transactions on Sustainable Energy*, vol. 10, no. 2, pp. 759–767, 2018.

[140] D. Aikema, R. Simmonds, and H. Zareipour, "Data centres in the ancillary services market," in *2012 International Green Computing Conference (IGCC)*, pp. 1–10, IEEE, 2012.

[141] A. C. Rueda-Medina, A. Padilha-Feltrin, and J. Mantovani, "Capacity of active power reserve for frequency control enhanced by distributed generators," in *2013 IEEE Power & Energy Society General Meeting*, pp. 1–5, Ieee, 2013.

[142] F. D. Moya, G. D. Jannuzzi, and L. C. Da Silva, "Distributed generation for the provision of operating reserves," in *2008 IEEE/PES Transmission and Distribution Conference and Exposition: Latin America*, pp. 1–7, IEEE, 2008.

[143] E. Ela, B. Kirby, N. Navid, and J. C. Smith, "Effective ancillary services market designs on high wind power penetration systems," in *2012 IEEE Power and Energy Society General Meeting*, pp. 1–8, IEEE, 2012.

[144] P. Siano and D. Mohammad, "Milp optimization model for assessing the participation of distributed residential pv-battery systems in ancillary services market," *CSEE Journal of Power and Energy Systems*, vol. 7, no. 2, pp. 348–357, 2020.

[145] T. Li and M. Shahidehpour, "Strategic bidding of transmission-constrained gencos with incomplete information," *IEEE Transactions on Power Systems*, vol. 20, no. 1, pp. 437–447, 2005.

[146] X. Guan, Y.-C. Ho, and F. Lai, "An ordinal optimization based bidding strategy for electric power suppliers in the daily energy market," *IEEE Transactions on Power Systems*, vol. 16, no. 4, pp. 788–797, 2001.

[147] K. Liu, Q. Chen, C. Kang, W. Su, and G. Zhong, "Optimal operation strategy for distributed battery aggregator providing energy and ancillary services," *Journal of Modern Power Systems and Clean Energy*, vol. 6, no. 4, pp. 722–732, 2018.

[148] G. He, Q. Chen, C. Kang, P. Pinson, and Q. Xia, "Optimal bidding strategy of battery storage in power markets considering performance-based regulation and battery cycle life," *IEEE Transactions on Smart Grid*, vol. 7, no. 5, pp. 2359–2367, 2015.

[149] C. Goebel, H. Hesse, M. Schimpe, A. Jossen, and H.-A. Jacobsen, "Model-based dispatch strategies for lithium-ion battery energy storage applied to pay-as-bid markets for secondary reserve," *IEEE Transactions on Power Systems*, vol. 32, no. 4, pp. 2724–2734, 2016.

[150] O. Ma, N. Alkadi, P. Cappers, P. Denholm, J. Dudley, S. Goli, M. Hummon, S. Kiliccote, J. MacDonald, N. Matson, *et al.*, "Demand response for ancillary services," *IEEE Transactions on Smart Grid*, vol. 4, no. 4, pp. 1988–1995, 2013.

[151] K. Schisler, T. Sick, and K. Brief, "The role of demand response in ancillary services markets," in *2008 IEEE/PES Transmission and Distribution Conference and Exposition*, pp. 1–3, IEEE, 2008.

[152] L. Roald, F. Oldewurtel, T. Krause, and G. Andersson, "Analytical reformulation of security constrained optimal power flow with probabilistic constraints," in *2013 IEEE Grenoble Conference*, pp. 1–6, IEEE, 2013.

[153] J. Gallmann, S. Karagiannopoulos, M. González Vayá, and G. Hug, "On frequency control provision with a microgrid containing battery energy systems and renewable energy sources," 2018.

[154] . Swissgrid Balance Energy July 1st, "Swissgrid swiss transmission system operator-balance groups," *Swissgrid*, 2021. Available at: `https://www.swissgrid.ch/en/home/customers/balance-groups.html`.

[155] H. Huang and F. Li, "Bidding strategy for wind generation considering conventional generation and transmission constraints," *Journal of Modern Power Systems and Clean Energy*, vol. 3, no. 1, pp. 51–62, 2015.

[156] M. Chazarra, J. García-González, J. I. Pérez-Díaz, and M. Arteseros, "Stochastic optimization model for the weekly scheduling of a hydropower system in day-ahead and secondary regulation reserve markets," *Electric Power Systems Research*, vol. 130, pp. 67–77, 2016.

[157] G. He, Q. Chen, C. Kang, and Q. Xia, "Optimal offering strategy for concentrating solar power plants in joint energy, reserve and regulation markets," *IEEE Transactions on Sustainable Energy*, vol. 7, no. 3, pp. 1245–1254, 2016.

[158] N. Amjady, A. Attarha, S. Dehghan, and A. J. Conejo, "Adaptive robust expansion planning for a distribution network with ders," *IEEE Transactions on Power Systems*, vol. 33, no. 2, pp. 1698–1715, 2017.

[159] J. Zhu and M.-y. Chow, "A review of emerging techniques on generation expansion planning," *IEEE Transactions on Power Systems*, vol. 12, no. 4, pp. 1722–1728, 1997.

[160] D. Bertsimas and M. Sim, "Robust discrete optimization and network flows," *Mathematical Programming*, vol. 98, no. 1, pp. 49–71, 2003.