



University of  
Sheffield

DOCTORAL THESIS

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**The Quickest Evacuation Location Problem in  
Humanitarian Operations: A Multi-Objective  
Model Formulation and a Matheuristic Solution  
Approach**

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## *Abstract*

Disasters, both natural and man-made, affect millions of people every year. Recently, the frequency and severity of these disasters have been rising, emphasising the significance of humanitarian operations. Facility location problems and network flow problems are among the most important topics in humanitarian operations.

This research introduces the Quickest Evacuation Location Problem (QELP), a novel optimisation problem aimed at supporting humanitarian operations by combining the quickest flow problem and the discrete facility location problem. Its scope falls into the field of evacuation planning and design, intending to enhance evacuation network design and planning by identifying, among a finite set of candidates, the set of shelters that would allow the quickest possible evacuation process.

The QELP is first modelled by developing an ad-hoc network tool referred to as QELP-Time Expanded Network (QELP-TEN), which accounts for the lack of a predetermined set of sink nodes - as these need to be selected among the candidate sinks as part of the optimisation problem. To secure flexible and realistic decision support, a multi-objective mixed integer programming model is developed, aiming at minimising the evacuation makespan and the total budget required to install and operate the shelters while balancing the load of evacuees directed to each activated shelter.

The Robust Augmented  $\epsilon$ -constraint method (AUGMECON-R) is adopted as a solution scheme, and it is successfully combined with a novel Matheuristic approach to boost its performance while exploring the Pareto Set on increasing size networks. Despite the challenging complexity deriving from the use of time-expanded networks, experiments on realistic instances show scalable performance and the presence of regular trade-offs among the three objective functions (evacuation makespan, budget, and maximum load on shelters), thus confirming the suitability of the QELP to provide decision-makers with valuable support for real-world planning processes in humanitarian operations.

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## List of Abbreviations

<b>OCHA</b>	The United Nations <b>O</b> ffice for <b>C</b> oordination of <b>H</b> umanitarian <b>A</b> ffairs
<b>IFRC</b>	<b>I</b> nternational <b>F</b> ederation of <b>R</b> ed <b>C</b> ross and <b>R</b> ed <b>C</b> rescent <b>S</b> ocieties
<b>ICRC</b>	The <b>I</b> nternational <b>C</b> ommittee of the <b>R</b> ed <b>C</b> ross
<b>NGO</b>	<b>N</b> on-governmental <b>o</b> rganisations
<b>SDG</b>	<b>S</b> ustainable <b>D</b> evelopment <b>G</b> oals
<b>UNICEF</b>	<b>U</b> nited <b>N</b> ations <b>I</b> nternational <b>C</b> hildren's <b>E</b> mergency <b>F</b> und
<b>UNHCR</b>	<b>U</b> nited <b>N</b> ations <b>H</b> igh <b>C</b> ommissioner for <b>R</b> efugees
<b>IRC</b>	<b>I</b> nternational <b>R</b> escue <b>C</b> ommittee
<b>QELP</b>	<b>Q</b> uickest <b>E</b> vacuation <b>L</b> ocation <b>P</b> roblem
<b>TEN</b>	<b>T</b> ime <b>E</b> xpanded <b>N</b> etwork
<b>IHL</b>	<b>I</b> nternational <b>H</b> umanitarian <b>L</b> aw
<b>HO</b>	<b>H</b> umanitarian <b>O</b> perations
<b>HSC</b>	<b>H</b> umanitarian <b>S</b> upply <b>C</b> hain
<b>AUGMECON-R</b>	<b>R</b> obust <b>A</b> ugmented $\epsilon$ -constraint method

# Chapter 1

## Introduction

### 1.1 Background

"Leaving No One Behind", as articulated by the International Federation of Red Cross and Red Crescent Societies (IFRC) (IFRC, 2018a), calls for the necessity to give careful attention to the needs of those who are most vulnerable to disasters, while also highlighting the negative effects that disasters may have on the entire world. A disaster is defined as

*"serious disruptions to the functioning of a community that exceeds its capacity to cope using its resources. Disasters can be caused by natural, man-made, and technological hazards, as well as various factors that influence the exposure and vulnerability of a community."* (IFRC, 2022).

The difference between a *disaster* and a *hazard* is that a hazard is a dangerous phenomenon or condition. In contrast, a disaster is a significant disruption to the community's ability to function that goes beyond what it can handle, which is caused by hazards. Disasters, therefore, can and should be mitigated. We can prevent hazards from becoming disasters by assisting communities in being prepared, lowering their risks, and strengthening their resilience. This leads to the need for humanitarian operations and disaster management.

Disasters can be classified (Van Wassenhove, 2006) into *natural disasters* (e.g. earthquake, hurricane, flooding, etc.) or *man-made disasters* (e.g. war, chemical explosion, etc.) based on the cause of the disaster; and *sudden-onset* (e.g. earthquake, volcanic eruption, chemical explosion, etc.) or *slow-onset disasters* (e.g. drought, sea-level rise, climate change, etc.) based on the predictability and rapidity of the disaster. These classifications bring different opportunities and focus on humanitarian operations and disaster management. For example, in response to slow-onset disasters, the importance of a well-established preparedness and early warning system becomes predominant, whereas, for

sudden-onset disasters, immediate humanitarian response and efficient evacuation are crucial. Similarly, man-made disasters may offer more opportunities for preparedness and mitigation actions, while for natural disasters, since we can not naturally prevent hazards, how to deliver a quick response is fundamental.

Disasters have affected many lives and resulted in substantial economic damages. During the past 20 years since 2003, 12484 disasters have been recorded worldwide, including natural disasters, technology disasters, and other complex crises (EM-DAT, 2023), which affected billions of people's lives and caused trillions of dollars of economic losses. The figure 1.1 published by United Nations office for Disaster Risk Reduction (2020) shows the increasing effects of disasters with more reported disasters, deaths, total affected people and more economic losses.

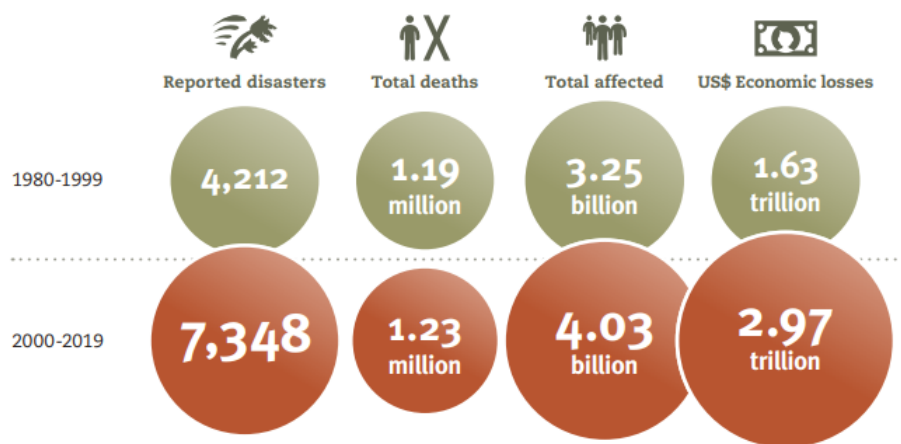


FIGURE 1.1: Disaster impact comparison

Natural disasters have inflicted catastrophic damage on human lives, resulting in a significant loss of life and extensive destruction, especially from those sudden-onset disasters. 2010 Haiti earthquakes resulted in a 30,000 death toll. 2011 Tōhoku earthquake and tsunami led to 19,749 deaths, and more recently, three devastating earthquakes struck Turkey and Syria, resulting in the loss of over 50,000 lives and leaving millions in need of aid (BritishRedCross, 2023). Man-made disasters have also caused massive damage to human lives. For example, the Beirut explosion in 2020 caused extensive damage: more than 300,000 people were left homeless, and more than 8 billion pounds of losses were incurred (BBCNews, 2020). Similarly, the refugee crisis caused by the Syrian civil war is internationally considered the most significant refugee crisis of recent times. Almost 11.1 million people needed humanitarian aid, half

of them were children (World Vision, 2021b). Given the large number of disasters and the severe damage they cause, it is imperative to prepare communities and increase their resilience in order to reduce the disruptions associated with disasters. This highlights the importance of humanitarian operations for emergency management.

## 1.2 General definitions and concepts

As mentioned, there are two disaster types: natural disasters and man-made disasters. Within the category of natural disasters, there are numerous sub-categories, such as floods, earthquakes, hurricanes, tornadoes, volcanic eruptions, and other geologic processes. In contrast, man-made disasters are the consequence of technological or human hazards, for example, wars, nuclear radiation, explosions, and fires.

Humanitarian operations is a term widely used to describe any activities to minimise human and economic losses before and after disasters. In this regard, Humanitarian Supply Chain Management (HLSCM) has been attracting increasing attention from researchers because of the increasing frequency of disasters. HLSCM, in a broader context of disaster management, consists essentially of Humanitarian Supply Chains and Humanitarian Logistics (Jabbour et al., 2019). Logistics deals with the delivery of supplies to people in need, and there are many differences between commercial logistics and humanitarian logistics and other operations (Kunz et al., 2017). The key differences are that there are many uncertainties in most disaster contexts, such as the timing, location, and size of disasters, which are difficult to forecast in most disaster scenarios. Another critical difference is that humanitarian and disaster relief is highly time-sensitive, where quicker disaster response can save many lives. These factors collectively highlight the importance of humanitarian operations in addressing the complex demands of disaster management, which can also be seen from the increasing number of papers on disaster operation management published in OR mainstream journals in the past decades (Altay and Green, 2006; Esposito Amideo et al., 2019; Dönmez et al., 2021).

Disaster response and development programs are two broad parts of humanitarian operations in general; where disaster response has the characteristics of short duration, high urgency and high unpredictability, development programs consider improving the life of the poorest people with long-term,

moderate urgency and relatively low uncertainty (Pedraza-Martinez and Van Wassenhove, 2016). Among those, the facility location problem can be both in disaster response and development programs depending on the timing that decisions are made in the disaster life-cycle and have a wide range of applications in both the public and private sectors. The success or failure of facilities is based primarily on their locations (Daskin, 2013). Facility location problems often deal with the optimal placement of facilities to achieve operational goals based on the context and objectives of the location problem. A brief introduction of fundamental facility location problems and a detailed review of facility location problems in the humanitarian operation context are presented in chapter 3.

The evacuation problem is another critical element of humanitarian operations, which deals with the challenge of safely and efficiently relocating populations from affected areas to safe places. In the literature, network flows are frequently applied to model evacuation problems. It is a problem domain with a long history and a wide range of applications, including engineering, management, operational research, and so on (Ahuja et al., 2014). It is a solid toolbox for assisting managers in their decision-making process in many different areas. By formulating evacuation as a network flow problem, efficient evacuation design and plan can be made, which can help decision-makers make informed decisions to ensure safe and effective evacuation operations during disasters. The introduction of fundamental network flow theories and a thorough literature review on network flows in evacuation modelling can be found in the chapter 4.

### **1.3 Research rationale**

In light of the increasing frequency and severity of disasters, we must figure out the most effective way to assist humanitarian operations in order to meet the urgent need to prepare and respond to disasters. One major challenge in this context is represented by facility location problems arising in this field to secure seamless and effective evacuation processes for humanitarian emergencies.

From the academic and theoretical perspective, despite an increasing number of contributions in the field of location analysis in humanitarian operations, the study of the literature exposes critical scientific gaps which need to be



addressed. Notably, few literature contributions in this field have simultaneously addressed the facility location and network flow problems. Moreover, most of the current methods fail to address time aspects in modelling disaster relief and evacuation while taking into account the location decisions. Among these, characteristics and features of the facilities required to support evacuees, one of the most critical areas of evacuation design and planning, along with the location of facilities, are rarely discussed in the existing literature. Therefore, this study aims to develop the Quickest Evacuation Location Problem (QELP), a novel optimisation problem aimed at supporting humanitarian operations by combining the quickest flow problem and the discrete facility location problem, in order to assist decision-makers in finding the best solutions for facility locations in humanitarian operation. The particular emphasis of QELP is to enhance evacuation network design and planning by identifying, among a finite set of candidates, the set of shelters that would allow the quickest possible evacuation process. In particular, the QELP combines facility location problems with the quickest flow problems in a general network using dynamic network flow modelling techniques.

From the managerial perspective, the QELP proves to be able to reduce the makespan of the evacuation process, to enable quicker rescue, and to reduce human suffering in humanitarian operations and emergency management. The QELP also reduces the total budget, which will help humanitarian organisations save money. Even though reducing the budget is not often the main goal in humanitarian operations, it is better to save money while taking into account other primary humanitarian operation goals. Furthermore, the QELP is able to minimise the maximum load for each shelter, which can spread the risk of each shelter. In reality, some shelters may have a higher risk than others, and balancing the load of these activated shelters can spread the risk and result in overall resilience in the whole system. Moreover, the QELP can support the decision-making process as the results indicate a clear trade-off between three objective functions (makespan, budget, and maximum load), which brings in significant managerial applications from a real-world perspective. Setting different budget levels and a maximum load plan can effectively reduce the makespan, which strongly indicates that QELP can find the most convenient solutions based on appropriate policy-making considerations. Also, the QELP can provide different options (set of Pareto-optimal solutions) to the decision-makers, which enable decision-makers to make the most suitable decisions based on the resources they have as well as the needs

of the policy-making. Overall, the QELP aims for better health and well-being, aligning with the Sustainable Development Goals (SDG) ([United Nations office for Disaster Risk Reduction, 2020](#)), especially *Goal 3: Good Health and Well-being* and *Goal 11: Sustainable Cities and Communities*.

## 1.4 Research objectives

The research objectives of this study are as follows:

- to critically review the literature in the field of shelter location and evacuation modelling in humanitarian operations from an Operational Research (OR) and, in particular, the optimisation perspective;
- to identify and analyse the main characteristics of existing models to assist the decision-making process in the field of shelter location and evacuation modelling in humanitarian operations in order to overcome current limitations and develop more efficient and impactful approaches;
- to identify the gaps in the existing literature based on the critical literature review of location analysis in humanitarian operations and network flow in evacuation modelling;
- to develop a novel optimisation model aimed at supporting shelter location and evacuation modelling in humanitarian operations, so as:
  - to address the operational needs in the disaster contexts;
  - to improve the effectiveness of disaster relief, particularly in enhancing the shelter location in evacuation design and planning;
- to provide high-quality solutions quickly and efficiently using an effective solution method for the developed model;
- to test the developed model on synthetic, computer-generated (still realistic) instances, inspired, when appropriate and possible, by real-world situations through secondary data to increase potential support to prospective users (decision-makers) to solve the combination problems of facility location problem and network flow problem arising in humanitarian operations.

## 1.5 Conclusions

This chapter has introduced the background of the current situations of disasters, supported by the introduction of general definitions and concepts in humanitarian operations with an emphasis on facility location problems and network flow problems. Building upon the background and gaps in the literature, this chapter has also presented the research rationale and objectives, forming the foundations of this research.

The structure of this research is shown as follows: the introduction of concepts in humanitarian operations is presented in chapter 2, followed by a detailed discussion of existing literature on facility location problems and network flow problems in chapter 3 and chapter 4. Then, a critical review of the combinations of facility location problems and network flow problems is shown. The gaps and motivations for conducting this study are thoroughly discussed in chapter 5. After that, a detailed problem description of QELP and the novel QELP-Time Expanded Network modelling techniques, along with the multi-objective mixed-integer programming model of QELP, are presented in chapters 6 and 7. In order to solve the QELP, we first applied a MIP method called AUGMECON-R, and we analysed its solutions and findings in chapter 8. From the results and findings obtained from AUGMECON-R, we found the computational time dramatically increases as the size of the instance increases. Therefore, a novel Matheuristic approach is developed and introduced, which can be found in chapter 9, where results show that the Matheuristic method can guarantee relatively high-quality solutions within a reasonable time period. Finally, chapter 10 provides a comprehensive summary of this research and points out the potential directions that future research can build on.

## Chapter 2

# Humanitarian Operations

### 2.1 Introduction

This chapter provides a comprehensive analysis of humanitarian operations, starting with introducing its definitions given by core humanitarian operations players to establish a robust foundation for the application context of this research. Then, the history of humanitarian operations is introduced by tracking the development over the past decades and the establishment of humanitarian laws and their impacts. After that, the key players in humanitarian operations are presented to enhance the understanding of the context. Furthermore, different humanitarian principles and standards are explored, which form the guidelines for every action in humanitarian operations. Finally, the critical components of humanitarian operations are discussed, which contribute to the conceptual development of the quickest evacuation location problem.

### 2.2 General concepts of humanitarian operations

This section is divided into two parts: the definitions of humanitarian operations and the relevance and significance of humanitarian operations. By understanding the different definitions of humanitarian operation provided by various core humanitarian players, we can gain a comprehensive idea of what humanitarian operation is with a clear view of the contexts in which the quickest evacuation location problem is being addressed. Furthermore, the relevance and significance of humanitarian operations motivate the introduction of the QELP.

### 2.2.1 Definitions of humanitarian operations

Humanitarian operations, humanitarian aid, or humanitarian assistance are all synonyms, and they all refer to helping those suffering from disasters. Humanitarian aid is the term used to describe the supply or other medical support given to those in need during disasters, while humanitarian operations are the broader term that includes the logistics, planning, and coordination needed to deliver relief and save lives efficiently. There are various definitions for humanitarian operations depending on the contexts in which they are used or the organisation that uses them. Here are some key players/organisations in providing humanitarian operations and their definitions of humanitarian operations.

- **The United Nations Office for Coordination of Humanitarian Affairs (OCHA)** is a department under the United Nations, which was established in 1991 with the aim of "coordinating the global emergency response to save lives and protect people in humanitarian crises" (OCHA, 2023). It defines humanitarian operations as the activities and initiatives carried out to respond to crises that result in suffering and displacement, as well as to improve the ability and capacity of impacted communities to overcome shocks and recover from them;
- **Sphere** is an initiative founded in 1997, seeking to "improve the quality and accountability of the humanitarian sector" (Sphere, 2023). They introduce the Sphere standards that provide a set of guidelines and bottom-line humanitarian standards in four technical areas: water supply, food security and nutrition, shelter and settlement, and health. Their standards define humanitarian operations as providing protection and assistance to the affected people and making sure that the affected people have the essential requirements for a dignified life;
- **International Federation of Red Cross and Red Crescent Societies (IFRC)** can be seen as an essential humanitarian aid organisation around the world, which was founded in 1919. It defines humanitarian operations as actions that "prevent and alleviate human suffering wherever it may be found", and its main code of conduct for humanitarian operations and disaster relief is to "recognise our obligation to provide humanitarian assistance wherever it is needed" (IFRC, 2018b).

In general, any actions taken to reduce the economic and human losses both

before and after disasters are referred to as humanitarian operations to support humanitarian aid. All these definitions highlight the importance of humanitarian operations, including providing humanitarian assistance, preserving human dignity, preventing people from suffering, and providing massive support for affected populations not only to get ready before or during the disaster but also to recover from disasters.

### **2.2.2 Relevance and significance of humanitarian operations**

Regarding humanitarian operations, it is widely recognised as a crucial part of achieving a better world for everyone. As highlighted in the introduction chapter, disasters can lead to catastrophic harm to individuals and impose substantial economic burdens on the community and society. Preventing people from disasters can reduce both human suffering and economic losses to the whole society. These all brought the importance of humanitarian operations.

Humanitarian operations are crucial for the following reasons. First, humanitarian operations help save lives and reduce people's suffering by providing people with confidence and capabilities to prepare, respond, and recover from crises such as natural disasters and man-made disasters. Actions such as providing food, water, shelter, and other types of protection to people in need all belong to humanitarian operations to ensure a timely rescue and assistance to save lives and reduce suffering.

Furthermore, protecting and supporting the most vulnerable populations is also part of humanitarian operations. For example, some humanitarian organisations, such as the United Nations International Children's Emergency Fund (UNICEF), were established to focus on providing humanitarian and developmental aid for children all over the world. In this case, when a disaster happens, these organisations will protect the vulnerable populations, which helps to achieve the SDGs.

Moreover, humanitarian operations play a central role in assisting communities to enhance their resilience, enabling them to recover from disasters and proactively prepare for future disasters. These operations encompass more than just immediate rescue efforts. Instead, they extend support across the entire disaster life-cycle, which includes various operations through four phases: mitigation, preparedness, response, and recovery stages. Mitigation and preparedness belong to the pre-disaster phases, with the aim of preparing and getting ready for potential disasters. At the same time, response and recovery

are post-disaster phases to provide disaster relief and recovery from disasters. By engaging in these comprehensive activities, humanitarian operations empower communities to navigate the challenges posed by disasters effectively. Further elaboration on the various stages of the disaster life-cycle will be provided in subsequent sections of this chapter.

## **2.3 History of humanitarian operations**

This section discusses the history of humanitarian operations with a brief introduction to the history of core humanitarian players. Then, the evolution of International Humanitarian Law and its impacts are also covered in this section.

### **2.3.1 The development of humanitarian operations**

The formalised humanitarian operations/aid/assistance can be traced back to the 19th century during the time of wars. The modern humanitarian movement was initiated by Henry Dunant, who saw the suffering of injured soldiers during the Battle of Solferino in 1859. As a result, the International Committee of the Red Cross (ICRC) was founded. In 1868, the Turkish Red Crescent was the first national organisation to use the Red Crescent logo ([The University of Oxford, 2023](#)).

In the late 19th and early 20th centuries, the establishment of many humanitarian organisations broadened the scope of humanitarian operations, aiming to provide adequate responses to natural disasters and health crises beyond conflicts and wars. For example, the British Red Cross was established in 1870 and renamed in 1905. Doctors Without Borders was founded in 1971.

In the mid-20th century, the decolonisation movements in lots of countries resulted in a greater emphasis on humanitarian efforts in newly independent nations. During this time, many non-governmental organisations (NGOs) such as Oxfam (1942), CARE International (1945), and World Vision International (1950) were established, aiming to provide humanitarian aid to those in need all over the world beyond the government level.

In the late 20th and early 21st centuries, the increase in complex crises brought new challenges to humanitarian organisations, for instance, the access constraints and the conflicts between different countries due to political issues, which calls for the need for coordination among all sectors. In recent years,

humanitarian organisations have shifted their focus from responding to disasters to addressing the underlying causes of those disasters with the aim of enhancing long-term resilience. This emphasises the importance of integrating humanitarian operations with developmental approaches and filling the gap between emergency aid and long-term development.

### **2.3.2 The International Humanitarian Law and its impacts**

The International Humanitarian Law (IHL) is also called "the law of war" or "the law of armed conflict" (ICRC, 2023), which is a set of rules to reduce the effects caused by armed conflicts for humanitarian reasons. It protects the people who are no longer taking part in the battles.

Even though the IHL does not directly set the guidelines for humanitarian operations and aid, the main objective is to protect civilians, the people no longer participating in the hostilities, and other persons affected by these conflicts. In particular, IHL acknowledges the significance of humanitarian operations and aid to the affected populations. Even in times of armed conflict, based on IHL, humanitarian assistance should be delivered quickly to the people in need. Furthermore, targeting facilities like hospitals, schools, and water supply systems that are essential for the life of the civilian population is forbidden by the IHL, which, in some cases, makes the conduction of humanitarian operations easier. Moreover, the IHL prioritises the treatment of wounded people by outlining the responsibility to offer medical treatments and protect the critical facilities, staff, and vehicles to guarantee the delivery of medical assistance to the people in need. This ensures that humanitarian operations can be delivered smoothly, particularly in conflict areas.

In all, the IHL provides a legal requirement that can be seen as the guidelines for humanitarian operations, and the actions of humanitarian operations demonstrate the practical implementation of the IHL. The development of the IHL not only provides legal standards for humanitarian operations but also ensures that humanitarian operations can occur whenever needed to reduce suffering, protect vulnerable populations and deliver humanitarian aid even in the context of armed conflicts.



## 2.4 Key players in humanitarian operations

There are many key players in humanitarian operations, and each of them is essential to the success of humanitarian operations. Therefore, it is crucial to acknowledge that no single player can successfully achieve effective and efficient humanitarian operations on its own, where the coordination and collaboration of multiple stakeholders are needed. Here are some core players in humanitarian operations, which can be categorised into the United Nations and its agencies, International organisations, NGOs, Local governments and communities, and finally, academic and research institutions.

### 2.4.1 The United Nations and its agencies

The United Nations plays a central role in coordinating and supporting humanitarian operations. It has an office for Coordination of Humanitarian Affairs (OCHA), as discussed in previous sections, which functions as the core entity of the UN, taking charge of coordinating humanitarian response, facilitating cooperation among different sectors, and gathering resources for emergencies. There are other agencies in the UN with special focuses and expertise to provide humanitarian assistance to people in need. For example, UNICEF aims to provide humanitarian and development aid and support to children all over the world. World Food Programme is also an agency within the UN providing food assistance to places in need. United Nations High Commissioner for Refugees (UNHCR) works to guarantee that everyone has the right to apply for asylum and protect refugees.

### 2.4.2 International organisations

The International Committee of the Red Cross (ICRC) and the International Federation of Red Cross and Red Crescent Societies (IFRC) are well-known international organisations with a long history of providing humanitarian operations and assistance. They recognised themselves as "neither governmental institutions nor wholly separate NGOs" (IFRC, 2023). They work with the public based on national and international laws, and they are "auxiliaries" to the public to provide humanitarian support and aid. They are usually formed by volunteers from diverse communities and provide tailored support in different countries to solve the needs in various contexts.

### 2.4.3 Non-Governmental Organisations

There are many NGOs around the world providing humanitarian operations and assistance. They work in various fields of humanitarian operations, for instance, disaster relief, food security, medical support and so on. They often work with local communities, governments, and other stakeholders to provide help and support to vulnerable populations affected by situations like disasters and other emergencies. Here are some examples of well-known NGOs that are active in providing humanitarian operations.

- **CARE International:** it was founded in 1945 and is one of the oldest and largest international humanitarian organisations. CARE works in over 100 countries around the world to "fight poverty and injustice to help create a more equal and gender-just world" (CARE, 2023);
- **Médecins Sans Frontières/Doctors without borders:** as its name shows, founded in 1971, it aims to provide medical assistance to people affected by natural or man-made disasters and to victims of armed conflicts (Médecins Sans Frontières, 2023);
- **Save the Children:** it is a UK-based NGO aiming to make sure children have food, healthcare, shelter and education. It specifically works with local communities in less-developing countries to prioritise children and help them "achieve their full potential" by ensuring they have a healthy and safe condition and a good education (Save The Children, 2023);
- **International Rescue Committee (IRC):** it was founded in 1933 and has a long history of helping people who are influenced by humanitarian crises, including disasters, climate change, and armed conflicts. IRC aims to help affected populations "survive, recover, and rebuild their lives" (IRC, 2023);
- **World Vision:** similar to Save The Children, World Vision is an international humanitarian aid provider with the aim of "ending violence against children " (World Vision, 2021a). It was founded in 1950 and has been working in 100 countries around the world.

There are many other NGOs around the world which are not detailed introduced above, such as **International Medical Corps (IMC)**, **Mercy Corps**, **Action Contre la Faim (Action Against Hunger)**, and many other national and regional organisations. They all play an important role in supporting people

in need and working in different areas of humanitarian operations and assistance. All of them are crucial in handling humanitarian crises and advancing efforts for a more sustainable and resilient system in order to prepare for future disasters.

#### **2.4.4 Local governments and communities**

Local governments and communities play the core role in humanitarian operations, especially in preparing and responding to disasters. First of all, local governments usually set guidance in building the infrastructure and preparing for disasters. After the disaster, local governments and communities typically take the lead in disaster relief, such as coordinating rescue teams and allocating local resources. Usually, the army will be assigned to help with disaster relief. In this case, an effective and well-coordinated humanitarian response will be achieved.

Second, local governments and communities have more contextual knowledge and expertise in responding to different emergencies so that they know better about their societies from every perspective. Therefore, the expertise of local governments and communities can provide more tailored responses and help along with international relief and humanitarian organisations to help with the special needs of the community.

Third, local governments and communities can support humanitarian operations from a strategic level. For example, they can allocate funds from their budget, coordinate local businesses for donations, and develop rules to guarantee smooth transportation to the affected area, saving time in disaster relief and saving lives. Finally, local governments and communities can help with long-term recovery from the disasters, like reconstruction of the buildings and infrastructure to prepare and mitigate the impact of future disasters.

To sum up, local governments and communities are more knowledgeable in the situation of affected situations. They can contribute to humanitarian operations through their leadership in disaster preparedness and response, allocating rescue teams and resources, and participating in long-term recovery, which is essential for sustainable humanitarian operations. They can ensure more effective and efficient humanitarian operations with the help of other players.

### 2.4.5 Academic and research institutions

During the past decade, there has been a significant increase in the number of researchers working in humanitarian operations, especially in the Operations Research field. The importance of effective humanitarian logistics in finding optimal solutions to respond to disasters effectively is witnessed by the growing number of literary contributions on Operational Research (OR) focusing on humanitarian operations (Boonmee et al., 2017). Similarly, Altay and Green (2006) highlighted the large number of papers on disaster operation management published in OR mainstream journals in the past decade. Furthermore, most of the models in the research are developed to tackle real-world problems, which can be used by stakeholders to support humanitarian operations. Moreover, some researchers in OR collaborate with humanitarian organisations, NGOs, and the government. These collaborations can encourage multi-disciplinary thinking and provide chances for academic works to be used in real-world humanitarian contexts, making it easier for theoretical models to be transformed into solving practical cases.

## 2.5 Humanitarian principles and standards

This section introduces several widely-recognised principles and standards for humanitarian operations. First, OCHA's humanitarian principles set the fundamental conduct for activities in humanitarian operations as *Humanity, Neutrality, Impartiality, and Independence*. In addition to these four principles, IFRC added another three principles: *Voluntary service, Unity, and Universality*. Furthermore, Sphere also introduces its Sphere Standards, which are widely used in humanitarian operations.

### 2.5.1 Principles of humanitarian operations by OCHA

The humanitarian principles (OCHA, 2022) introduced by OCHA are a set of guiding principles that shape and govern humanitarian operations and aid. These principles are widely accepted and supported by humanitarian organisations, governments, and non-governmental organisations (NGOs). The four core humanitarian principles are Humanity, Neutrality, Impartiality and Independence. These principles are essential for creating and maintaining access to the affected areas as well as for providing humanitarian relief to those people in need.

**Humanity**

The principle of humanity emphasises the importance of keeping and protecting human dignity and reducing human suffering. All humanitarian operations and aid should prioritise people's needs, and they should be provided in time, wherever it happens. The fundamental requirement of humanitarian operations and assistance is to protect life and respect all human beings.

**Neutrality**

This principle requires humanitarian operations and aid providers to be neutral and not to take sides or favour any particular political, racial, religious or ideological nature. All humanitarian operations and aid should solely be provided based on the needs of people, without any bias.

**Impartiality**

This principle aligns with the previous one, focusing on the requirement of fairness and the priority of urgent needs, which makes sure that humanitarian operations and aid reach those who are most vulnerable and in urgent need, regardless of people's background, race, nationality, or other political affiliation.

**Independence**

This principle refers to the autonomy and freedom of humanitarian operations and aid providers to take actions based on the best interests and urgent needs of affected people, which will not be influenced by political parties, economic problems/situations, military issues or other pressure. The humanitarian operation providers should have the ability to make their own decisions in providing aid and rescue independently.

These four principles provide a guidance foundation for humanitarian operation providers, ensuring all humanitarian operations and aid are made based on humanitarian needs, independently from external pressures such as political, military, race and nationalities. Implementing these four core humanitarian principles can achieve more adequate, dependable and timely humanitarian operations.

## 2.5.2 Additional principles of IFRC

In addition to these four principles, [IFRC \(2023\)](#) extended these into seven fundamental principles. The additional three principles are voluntary service, unity, and universality. These three principles extend the original ones and make them more comprehensive and meaningful for everyone in the humanitarian sector, not only the humanitarian provider but also for the educational purpose for everyone.

### **Voluntary service**

This principle represents the common motivation of doing humanitarian operations, not for financial gain but for the desire to help others. This principle of doing voluntary services brings in the most significant and critical difference between commercial and humanitarian operations, where the number one objective of humanitarian operations will not be profit maximisation.

### **Unity**

This principle aligns with impartiality, asking for fairness in delivering humanitarian operations and aid and also in recruiting volunteers. Recruiting volunteers from every area to ensure a range of people across the population so that humanitarian assistance and operations can be delivered to all people in need by many volunteers. In this case, the requirements of the population in each area might be identified and met. This principle aligns with IFRC and serves as a guiding framework for recruitment practices within a broader humanitarian operations community.

### **Universality**

"The universality of suffering requires a universal response." ([IFRC, 2023](#)). This principle calls for united support from all parts of the world to respond to disasters. It means that humanitarian operations and aid will never be the sole response and reaction towards the disaster by one country or only by IFRC. Instead, humanitarian operations require collective support and collaboration before, during, and after disasters for the benefit of all.

## 2.5.3 Sphere Standards

As discussed, the Sphere Standards set the minimum humanitarian standards for humanitarian operations and assistance. In other words, it shows the basic

needs of people in order to survive. People who are affected by a disaster have the right to get the necessities for life with dignity. Sphere Standards were introduced to describe the set of actions that should be taken in order to protect people's rights.

All standards are based on four protection principles and nine core humanitarian standards (Sphere, 2023). **The four protection principles** are: i) first of all, humanitarian operations should improve people's rights, dignity, and safety while preventing them from additional harm; ii) then, humanitarian operations should ensure that everyone has access to fair assistance and help that are provided without bias. The assistance and support should be provided according to the needs; iii) furthermore, humanitarian operations should assist those who are suffering from the physical and mental impacts of assault, compulsion, or deprivation, which means mental assistance is also the key to helping people; iv) finally, humanitarian operations should encourage people to stand up for their rights. These are four principles which provide clear guidance to protect people's rights. The **core humanitarian standard** introduced by (Sphere, 2023) are:

- humanitarian operations should be tailored and appropriate to people's needs;
- humanitarian operations should be delivered efficiently and on time;
- humanitarian operations should be taken to build local capacity and ability to prevent the negative effects of disasters;
- communication, involvement, and feedback are the foundations of humanitarian operations;
- complaints should also be welcomed and managed in humanitarian operations;
- coordination and collaboration are crucial in humanitarian operations;
- humanitarian operations should always learn from the experience and keep improving;
- volunteers and staff should be treated fairly while being supported to do their job;
- resources and donations should be carefully handled and put to use for their intended purposes.

These basic humanitarian standards can be grouped into four technical areas: water supply, sanitation and hygiene promotion (WASH), food security and nutrition, shelter and settlement, and health (Sphere, 2023). These four technical areas cover the basic needs of people in order to survive disasters. More information can be found in the Sphere handbook (Sphere, 2023), which aligns with other humanitarian principles and standards introduced by OCHA and IFRC.

In all, the Sphere Standards, the additional IFRC principles for recruiting volunteers, and the main OCHA principles of humanitarian operations all provide a very comprehensive set of guidelines on how to protect people's fundamental rights and to ensure more effective, timely, and efficient humanitarian operations and disaster relief. All the research in humanitarian operations should try to achieve as many principles as possible in order to make it impactful in real-world cases.

## 2.6 Key components of humanitarian operations

In the previous sections, the background, history, and importance of humanitarian operations have been discussed and emphasised. This section introduces the essential components and concepts underlying humanitarian operations.

The main differences between humanitarian operations and other commercial operations are: i) the objectives of humanitarian operations are not for profit, ii) there are more uncertainties in humanitarian operations depending on the situation of the disaster and crisis, iii) the funders of humanitarian operations usually are governments and NGOs. The review paper of Kovács and Moshtari (2019) identifies several critical challenges in humanitarian operations (HO) compared with other commercial operations. The first is to recognise humanitarian contexts, such as the type of disasters, the working pattern of the humanitarian organisations, and the local environment. The second is to identify the uncertainties in HO, for example, the demand uncertainty, infrastructural damages, or beneficiaries' behaviour (Caunhye et al., 2012; Bayram, 2016) etc. Then, choosing the most appropriate method to solve the problems in HO is another critical challenge considering the data availability and uncertainty. In addition, complex communication and coordination and limited resources are other challenges identified in Caunhye et al. (2012).



### 2.6.1 Disaster life-cycle/management cycle

Activities within humanitarian operations are generally classified based on the four life-cycle stages of the disaster (Erbeyoglu and Bilge, 2020): mitigation, preparedness, response, and recovery. The mitigation and preparedness phases belong to the pre-disaster stage and aim to avoid or minimise the effects of a disaster (Anaya-Arenas et al., 2014; Seifert et al., 2018). On the other hand, the response and recovery phases are in the post-disaster stage, seeking to support the affected community back to the pre-disaster condition or even better (Seifert et al., 2018).

#### Mitigation

Mitigation is defined as the application of measures that prevent disasters or reduce the chance of their happening, as well as lessen the potential effects of disasters. Mitigation is usually applied before the disaster to reduce vulnerabilities and build society's resilience in responding to disasters (McLoughlin, 1985; FEMA, 2004; Haddow et al., 2008; Hoyos et al., 2015). Mitigation strategies include actions like: i) risk assessment to identify potential risks and resolve some problems in advance, ii) building codes to help set the requirement for the buildings and infrastructure to make sure they can survive during disasters, iii) buying disaster insurance to lessen the economic impact, to get some financial help after the disaster, and to construct barriers to prevent hazards.

The main goal of the mitigation stage is to reduce the vulnerabilities of both people and infrastructure and build upon the resilience of the whole community to lessen the long-term effects and possible costs of disasters. Mitigation strategies can also reduce the pressure of other stages, such as response and recovery, and help reduce the response time during a disaster.

#### Preparedness

Preparedness is the "state of readiness" (Haddow et al., 2008) to develop operational capabilities for handling an emergency before it happens. Compared with mitigation strategies, preparedness strategies focus more on improving the readiness and capacity to respond effectively to approaching disasters that are very likely to happen (McLoughlin, 1985; FEMA, 2004; Hoyos et al., 2015). Preparedness actions involve planning, organising, and coordinating different

stakeholders to reduce the likelihood of loss of life and increase the effectiveness of response efforts.

Preparedness actions include: i) emergency evacuation plans where the roles, responsibilities, and procedures for different stakeholders are outlined to make sure everyone knows what to do or how to evacuate when the disaster happens, ii) establishing and maintaining an effective warning system is beneficial in alerting people about potential or upcoming disasters, iii) pre-positioning relief resources and pre-locating facilities are key in responding to the disaster so that evacuees can have a safe place to go when disaster happens. In addition, pre-positioning necessary resources such as water, food, and medical equipment can also increase the readiness for disasters

The preparedness stage aims to get well-prepared, trained and informed so that people can respond more effectively, quickly and confidently when disasters come. These actions can also reduce the effects of disasters and improve the resilience of society, as well as build the foundation for the response stage.

## **Response**

Response is implemented immediately during or directly after the disaster to minimise the effects of the disaster and prevent further loss. Coordination is key in disaster relief in the response phase, where coordinating efforts between various stakeholders such as government, humanitarian organisations, and other parties to provide effective and efficient disaster response. In particular, information, resources and duty sharing are necessary for increasing the effectiveness of humanitarian aid and to avoid duplication and waste (Anaya-Arenas et al., 2014; Hoyos et al., 2015). Other actions that often take place in the response phase are:

- emergency plan activation: when a disaster occurs, the activation of the emergency plan can provide guidance on how to respond to the disaster, ensuring quick disaster relief;
- relief and evacuation: providing the basic necessities for the affected population, such as food, water, shelter, and medical care, is critical to meeting the basic needs of people. The direction and rescue of the affected population to a safe place are also significant in saving lives and reducing human suffering;
- humanitarian logistics: in order to coordinate the timely and effective storage and distribution of relief resources such as food, water, and other

medical aid, an effective humanitarian logistic network is needed, consisting of warehouses used to store the supplies and vehicles used to deliver the resources. In this case, the right amount of resources can be delivered to the right place after the disaster.

The response phase is a real challenge as so many uncertainties occur after the disaster. In order to reduce human suffering and restore dignity to the affected population, all the stakeholders should collaborate and cooperate together to guarantee efficient and effective disaster relief and response.

### Recovery

Recovery is implemented in the post-disaster stage to help affected areas recover from the disaster and return to normal. The recovery function begins directly after the disaster and lasts for months or even years. It includes both short-term activities, such as temporary housing, repairing and providing shelters, and long-term actions, like the reconstruction of infrastructure or providing financial assistance to pay for repairs. In addition, recovery also builds upon the ability of the affected community to mitigate and prepare for future disasters (McLoughlin, 1985; Anaya-Arenas et al., 2014; Hoyos et al., 2015).

Generally speaking, the preparedness and response phases have received the most significant attention among other phases in the literature Galindo and Batta (2013). At the same time, mitigation is seldom studied in the literature (Trivedi and Singh, 2018), which has been changed since Altay and Green (2006) where they found around 44% of paper they reviewed studied mitigation. In comparison, only 11% of the paper addressed recovery. This means that most research focuses on disaster preparedness and response stages. Goldschmidt and Kumar (2016) identify the disaster life-cycle as preparedness, response, rehabilitation, and mitigation. The definitions of preparedness and response are the same as mentioned above. The rehabilitation phase is similar to the recovery phase but with different terminology. The mitigation is different from what was mentioned before; in their work, they define mitigation activities as part of rehabilitation, such as strengthening buildings and infrastructure as structural activities, along with the legislation and insurance as non-structural measures. They claim that the mitigation process can be done in the post-disaster phases after the disaster.

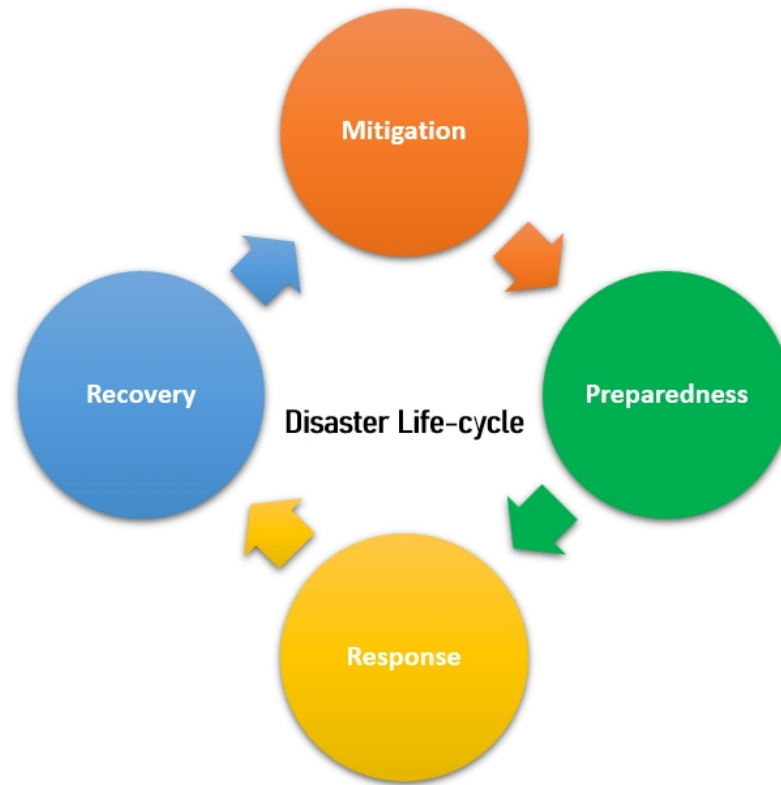


FIGURE 2.1: Disaster life-cycle/management cycle

To sum up, even though each phrase has different terminologies, the whole disaster life-cycle or disaster management cycle demonstrates a cyclical pattern just as 2.1 shows. It starts with the pre-disaster phases, focusing on preparing for the disaster and reducing the risks of potential disasters. This is followed by the post-disaster phases, involving quick emergency response and relief efforts. Then, the recovery phase focuses on the long-term rebuilding and restoration of the affected area. Finally, the cycle turned to the mitigation and preparedness phases, where the recovery phase can also help improve the mitigation ability to reduce future risks. By recognising the cyclical pattern of whole disaster operations management, each community can learn from past disasters, make practical plans for future disasters and improve the resilience of the entire system to decrease the effects of upcoming disasters.

### 2.6.2 Humanitarian logistics

Humanitarian logistics is another important concept in humanitarian operations, and it is defined as "the process of planning, implementing and controlling the efficient, cost-effective flow of and storage of goods and materials as

well as related information, from the point of origin to the point of consumption to alleviate the suffering of vulnerable people." (Thomas and Mizushima, 2005). In other words, it deals not only with the storage and flow of relief supplies, such as water, tents, food, and shelters (Trivedi and Singh, 2018) but also with the related information communication process throughout all the phases of a disaster. The importance of effective humanitarian logistics in finding optimal solutions to respond to disasters effectively is witnessed by the growing number of literary contributions on Operational Research (OR) focusing on humanitarian logistics (Boonmee et al., 2017). Humanitarian logistics is a very challenging process due to the complex nature of disaster relief. Overstreet et al. (2011) identified six essential elements which made humanitarian logistics very difficult.

- *Unknowns*: the primary obstacles for humanitarian logistics are caused by uncertainties, such as the unpredictability of the timing and location of disasters. Additionally, disaster relief is challenged by events that change quickly, such as aftershocks. Therefore, the effectiveness and efficiency of humanitarian logistics are heavily impacted by the adequacy of infrastructure and availability of resources;
- *Time*: timely disaster relief is very important in humanitarian logistics, which will reduce lots of human suffering. However, in the context of humanitarian logistics, getting relief resources to arrive timely is very difficult where the road and links might be terribly destroyed, and this increases the complexity of the humanitarian logistics;
- *Trained logisticians*: it is challenging to find logisticians who can organise, evaluate and manage both people and material resources for disaster relief. Overstreet et al. (2011) also argue that retaining experienced logisticians to support humanitarian logistics is also very difficult, and the loss of these knowledgeable logisticians will have a severely detrimental impact on the efficiency of humanitarian logistics;
- *The media and funding*: the development of social media changed the way of collecting donations. In Overstreet et al. (2011), they mentioned that donors usually respond generously when a disaster is well-publicised. Still, when a disaster is not covered massively by the media, donors tend to lose interest. However, the development of social media solves this kind of problem where the spread of the news is so fast, and hundreds of donations are from online appeals;

- *Equipment and information technology*: having multiple and incompatible information systems is common in many humanitarian organisations, making sharing and communicating information effectively difficult. In order to enable smooth exchange of information, practical information system management is needed;
- *Interference*: in disaster relief and humanitarian operations, there are many disruptions which are caused by the human side. For example, political conflicts, corruption in the distribution of supplies, and inadequate coordination among stakeholders (Overstreet et al., 2011). This interference significantly increases the complexity of humanitarian logistics, leading to barriers to the timely and effective delivery of relief aid.

These challenges make humanitarian logistics and operations complicated to conduct and operate, leading to the need to build a comprehensive humanitarian logistics research framework.

### Humanitarian logistics research framework

Overstreet et al. (2011) proposed an insightful research framework to describe the process of humanitarian logistics. This framework not only highlights the existing contributions in humanitarian logistics but also suggests a clear path for future advancements. As shown in Figure 2.2, the research framework demonstrates the interconnected nature of various components in the logistics process. When a humanitarian crisis arises, critical activities such as inventory management, equipment and infrastructure maintenance, information technology and communication management, and transportation planning collaborate, driven by the available organisational resources. Consequently, an effective plan for delivering humanitarian relief is developed. Moreover, the decision-making process within this interrelated system allows for a comprehensive review of the relationships between each function, supported by learning from past experience.

### 2.6.3 Humanitarian supply chain

The humanitarian supply chain, also called the humanitarian relief chain, aims to quickly provide necessary supplies to those affected by disasters in order to reduce suffering and losses. Humanitarian supply chain is similar to the commercial supply chain, where they all try to deliver the right amount of supplies to the right place at the right time. As is shown in figure 2.3 from Balçık

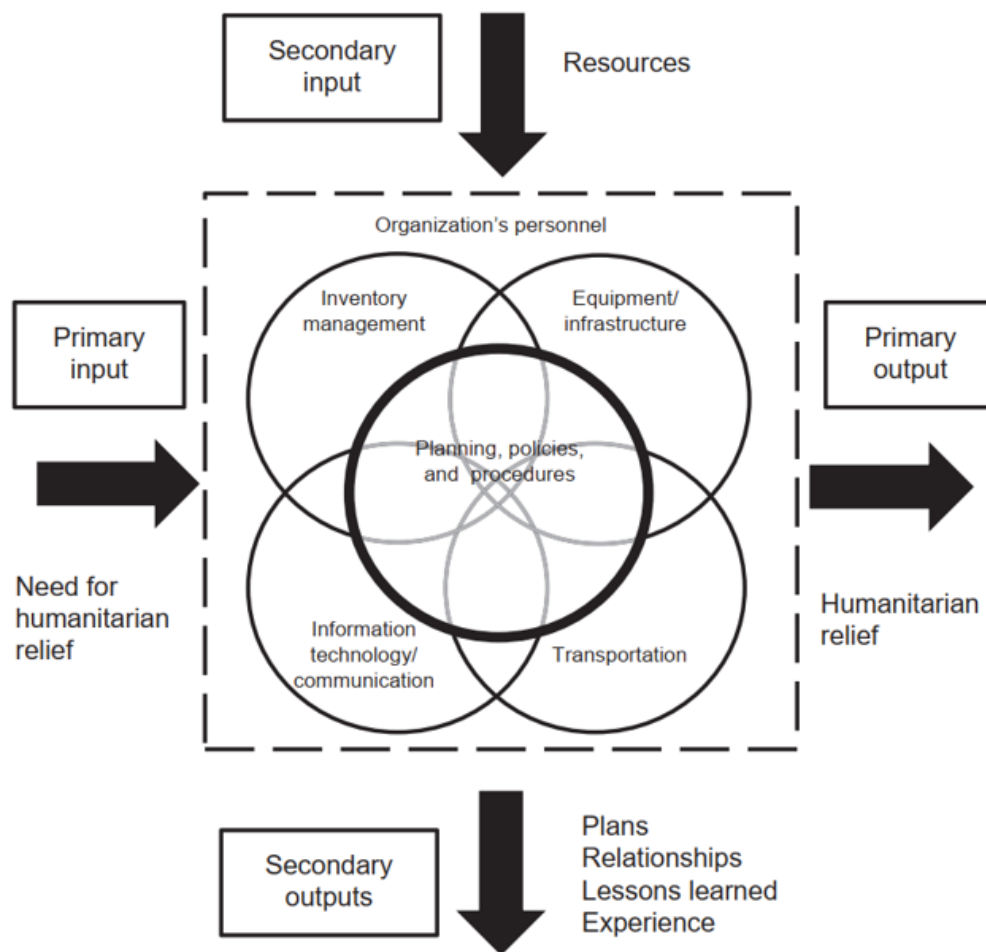


FIGURE 2.2: Humanitarian logistics research framework (Overstreet et al., 2011)

et al. (2010), the whole humanitarian supply (relief) chain is complicated and involves many vital components such as supply part, including acquisition and procurement, preparedness activities like pre-positioning and warehousing, transportation is also the key and finally the last mile distribution of relief resources. The complexity of the humanitarian relief chain results in the challenging nature of coordination.

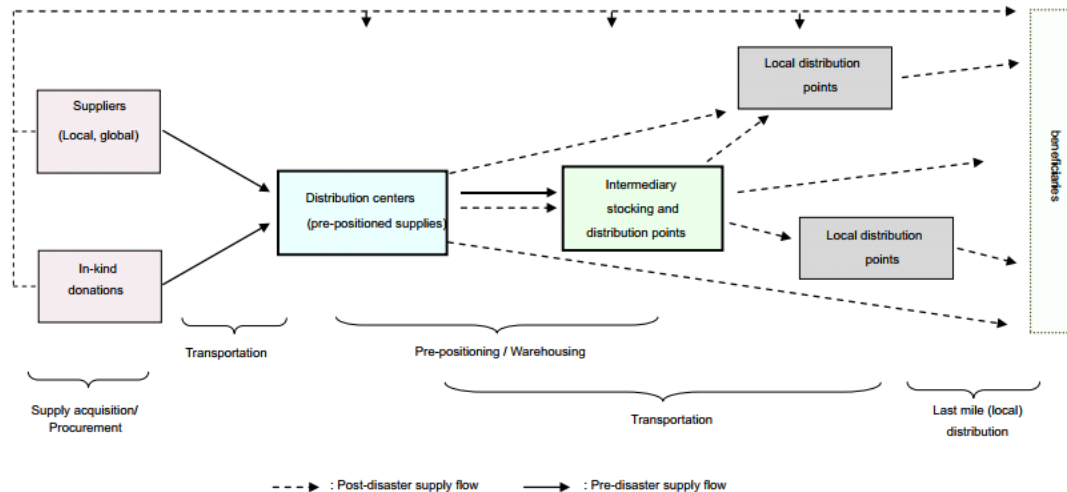


FIGURE 2.3: Humanitarian Supply Chain (Balçık et al., 2010)

### Challenges in humanitarian supply chain

The challenges due to the nature of disasters bring in the importance of coordination. In Balçık et al. (2010), they point out the six factors which significantly influence the coordination in the humanitarian supply chain: i) various types of stakeholders, ii) expectations of donors and funding arrangements, iii) impacts of the media and financing competition, iv) unpredictable nature during disasters, v) resource reduction or overproduction, vi) costs of coordination. The need for coordination can be found in many parts of the humanitarian supply chain. For example, the procurement coordination in the alliances between suppliers and buyers and the collaboration in procurement can reduce lots of waste and delay in supply. Moreover, warehousing and inventory coordination are also essential in reducing time and costs to find the best location for the warehouse and the inventory needed to be pre-positioned inside the warehouse. In most cases, warehousing functions are outsourced to other businesses and organisations, which brings in the need for coordination between all of the parties. Moreover, within the whole humanitarian supply chain, coordination between transportation parties is crucial because transportation expenditures make up a major portion of supply chain expenses,



and it is vital to achieving on-time delivery and reducing the lead time. Better transportation coordination improves the whole humanitarian supply chain's success. Coordinating is frequently done by outsourcing transportation to a logistics company, and the decision is based on the operational size of shippers, ability, and willingness to maintain internal logistical operations.

Similarly, Kovács and Spens (2009) also identify several challenges which make the humanitarian supply chain more challenging, where: i) the unpredictable nature of demand, ii) the unforeseen timing of the disaster's occurrence, iii) and the lack of resources are indications of the difficulties faced by humanitarian logisticians. They developed a conceptual model to identify challenges in humanitarian logistics, as figure 2.4 shows. Firstly, based on the disaster types, the relief challenges come from the different warning times. For example, the warning time for sudden-onset disasters is very short, while for those slow-onset disasters can be long. This also brings in the possible preparations that can be done to get ready for different disasters. Secondly, based on the location of the humanitarian organisations, different kinds of humanitarian organisations have different focuses, and in turn, result in different priorities and functions when disaster comes, such as regional presence and dependence on the declaration of a state of emergency. Finally, the appearance of various stakeholders can also bring in many challenges, like which one should be the priority rather than others and what should be the requirements of making this decision.

## 2.7 Conclusions

In this chapter, we have introduced the core aspects of humanitarian operations, which provides a comprehensive overview of humanitarian operations. This chapter has first introduced the definitions of humanitarian operations introduced by core players in humanitarian operations. Then, this chapter has discussed the significance of humanitarian operations and the history of humanitarian operations. After that, international humanitarian law has been introduced to provide guidance on humanitarian activities. Key players from the UN agencies and international NGOs to local government and academic institutions have been introduced and discussed to show their importance in the development of humanitarian operations. Furthermore, the humanitarian principles and standards from different authorities are introduced, and they

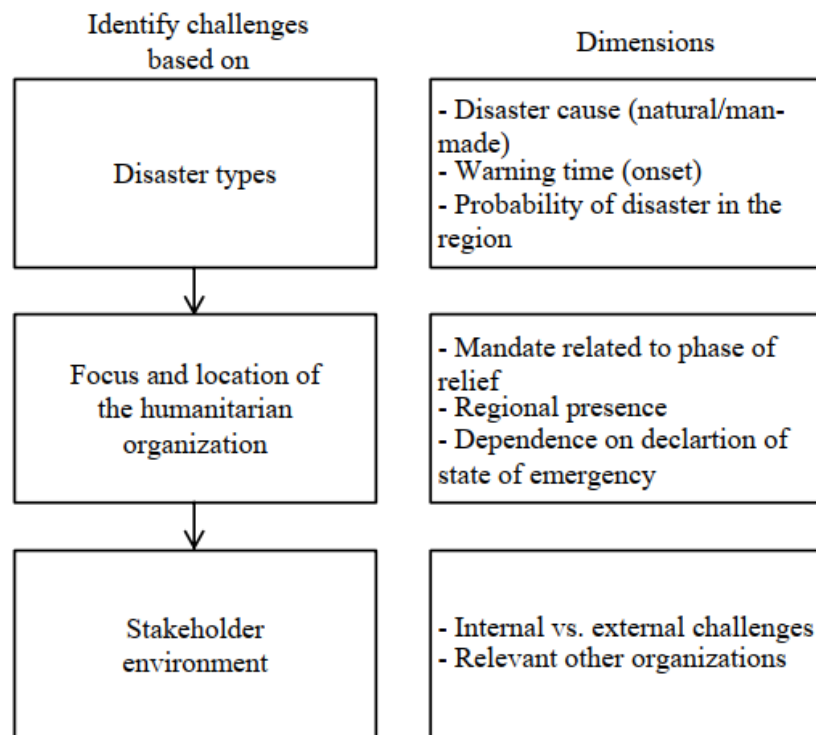


FIGURE 2.4: Conceptual model of identifying challenges  
(Kovács and Spens, 2009)

all provide explicit standards to every stakeholder in the field of humanitarian operations. Moreover, key components of humanitarian operations such as disaster life-cycle, humanitarian logistics, and humanitarian supply chain management are discussed to draw a picture of existing contributions that have been made in these areas. These all form the conceptual background of the development of the quickest evacuation location problem. The next chapter will specifically introduce facility location problems and their applications in humanitarian operations.

## Chapter 3

# Facility Location Problems

### 3.1 Introduction

Facility location problems, commonly called location problems, are very close to everyday life in both private and public sectors. For example, big corporations need to find the optimal locations for constructing factories and establishing offices. Individuals make location decisions like where to buy a house, etc. It is clear that the effectiveness of facilities in both private and public sectors significantly depends on the location of the facility (Daskin, 2013), emphasising the importance of location problems.

This chapter aims to provide a comprehensive analysis of facility location problems by first providing an overview of facility location problems and discussing the main questions the facility location problems are trying to answer. Then, four basic categories of facility location problems are introduced, and a comprehensive comparison and summary of these four fundamental facility location problems is provided. Moreover, this chapter offers a detailed discussion of the applications of facility location problems in humanitarian operations, further enriching the conceptual foundation of this research.

### 3.2 An overview of facility location problems

Facility location problems, or location problems, are a set of optimisation problems aiming to find the optimal placement of facilities and allocations of facilities within a specific geographic area. The main objective of the facility location problems is usually to identify the optimal locations for facilities in order to satisfy particular criteria based on different contexts. The standard objective functions of facility location problems can be minimising the cost

and maximising the coverage. The key questions the facility location problems are trying to answer include (Daskin, 2013):

- How many facilities should be located?
- Where is the optimal location for each facility?
- How large should each facility be?
- How should demand be allocated to each activated facility?

Some other questions can be (Daskin and Maass, 2018):

- What level of service is currently being offered to the customers in the supply chain in the current configuration?
- In the circumstance of the facility failure, what backup plans are in place?

The decisions on the location of facilities are often long-term decisions which will not change in the short term because it is usually not easy to change the location of facilities, and it is expensive to move them once they have been established. In this case, the facility location problems usually need to be strategically planned to align with the long-term goals of organisations. The location decisions are generally made from a strategic level.

### 3.3 Model categories for facility location problems

Generally, models for location problems can be divided into four categories (Daskin and Maass, 2018):

- **Analytic model** makes assumptions regarding the information they have, and they can be easily solved in closed form. Analytic models help understand the structure and provide insights into the issue and potential solutions, but they do not directly pinpoint the location of the facilities;
- **Continuous location model** is a problem domain where facilities can be located anywhere within the plane. In this case, facilities can serve demands from different magnitudes in the plane. The continuous location models are often used in situations where the location decisions require a certain level of flexibility in choosing the exact location within a continuous space;
- **Network location model** is the problem that considers the location of facilities at pre-determined nodes or links within a network. Instead of

locating facilities at arbitrary locations in the continuous space, network location models find the location of facilities on the nodes or arcs on the network. The network within the network location model can be any network that represents the connectivity of infrastructures or other subjects. Arcs represent the links between these subjects like roads, highways or telecommunication lines. The network location models usually focus on "finding polynomial time algorithms for specially structured instances of various problems" (Daskin and Maass, 2018);

- **Discrete location model** is a general term to describe the location problem with a discrete set of candidate nodes and a discrete set of demand nodes. The discrete location model does not always assume there is a network structure underneath, and it can be modelled into different structures based on the problem contexts. In this case, all location models with a discrete set of possibilities can be referred to as discrete location models, and all location models that consider the placement of facilities within a network while taking connections and distances between locations into account can be referred to as network location models.

The discrete location model is among the most used models in many fields. It is a very broad area where the decisions of facility locations are limited to a discrete set of candidate locations, and the set of demand nodes is also discrete. The discrete location model usually has a set of pre-determined candidate locations. It aims to find the optimal choice among these candidates to achieve objective functions.

### 3.4 Basic discrete location problems

The context in which the location problem is being solved and the underlying objective functions of the facility location problems define the answers to the questions we discussed before. Based on different contexts, location problems are influenced by different constraints like capacity constraints, the number of constraints of facilities, and other limitations on resources. The differences in contexts lead to different types of discrete location problems. There are four core types of location problems:

1. Covering problems
2. Center problems
3. Median problems

#### 4. Fixed charge facility location problems

In the following sections, we will provide a detailed discussion of these four core discrete location problems and their variants. The in-depth discussions of these four core problems will provide a thorough understanding of facility location challenges and their applications in real-world cases.

### 3.5 Covering problems

The covering problem was first introduced by [Hakimi \(1965\)](#), aiming to identify the minimum number of police required to cover nodes on a roadway network. The concept of coverage means that the distance between the demand node and the service provider (supply node) is within the service radius of the facility, and demand nodes can be seen as being properly serviced when located within the facility's coverage range. In contrast, if the demand node is outside the coverage distance, it means that the demand cannot be served. Consequently, the objective of the covering problem is to minimise the total cost or distance linked to establishing facilities to ensure the coverage of either all or a substantial portion of the demand nodes. Within the field of covering problems, there are two problems: the set covering facility location problem (set covering problem) and the maximum covering facility location problem (maximum covering problem), which will be introduced in the following sections.

#### General input for the covering problem

$a_{ij} = 1$  if demand node  $i$  is covered by candidate facility  $j$ , otherwise 0

#### General decision variables for the covering problem

$x_j = 1$  if a candidate facility  $j$  is activated to be located, otherwise 0

#### 3.5.1 The set covering facility location problem

The set covering problem aims to find the optimal location of the facility among a set of candidate facilities so that each demand node can be covered by at least one facility. In contrast, the total cost of establishing facilities is minimised ([Farahani et al., 2012](#)). In this case, the objective function of the set covering problem is to minimise the total cost needed to establish the activated facilities. The constraints are for each demand node and should be

covered by at least one facility. The mathematical model for the set covering problem is as follows:

#### Additional input

$f_j$  = costs related to locating a candidate facility  $j$

With this notation, the set covering location model is as follows:

$$\min \sum_{j \in J} f_j \cdot x_j \quad (3.1)$$

s.t.

$$\sum_{j \in J} a_{ij} \cdot x_j \geq 1 \quad i \in I \quad (3.2)$$

$$x_j \in \{0, 1\} \quad j \in J \quad (3.3)$$

The objective function 3.1 is to minimise the total cost of locating the facilities, which is a fixed cost related to the activation of the facility. Constraint 3.2 ensures each demand node must be covered by at least one facility, and constraint 3.3 defines the decision variables as binary variables.

### 3.5.2 The maximum covering facility location problem

The maximum covering problem addresses the first two limitations of the set covering problem. It deals with the situation where the number of facilities required to serve every demand node is likely to exceed the number that can be created (due to financial constraints and other factors) (Church and ReVelle, 1974). In order to address this situation, the maximum covering problem sets limitations for the number of facilities that can be located as  $p$ . In the maximum covering problem, the objective function is to find the optimal locations of facilities from a set of candidate facilities so that the maximum number of demand nodes is covered. Therefore, a set of demand nodes, a set of candidate facilities, and the number of facilities to be located ( $p$ ) are given. Different from the set covering problem, some demand nodes might not be covered in the maximum covering problem. In some real-world cases, a coverage requirement can be introduced, showing the minimal number of demand nodes that each activated facility should cover or, overall, a certain percentage of demand nodes that should be covered, and these all depend on the context where the maximum covering problem is used.

**Additional input** $h_i$  = demand at node  $i \in I$  $P$  = number of facilities to locate**Additional decision variable** $z_i = 1$  if node  $i \in I$  is covered, otherwise 0

With this notation, the maximum covering location model is as follows:

$$\max \sum_{i \in I} h_i \cdot z_i \quad (3.4)$$

s.t.

$$z_i \leq \sum_{j \in J} a_{ij} \cdot x_j \quad i \in I \quad (3.5)$$

$$\sum_{j \in J} x_j \leq P \quad (3.6)$$

$$x_j \in \{0, 1\} \quad j \in J \quad (3.7)$$

$$z_i \in \{0, 1\} \quad i \in I \quad (3.8)$$

The objective of the maximum covering facility problem 3.4 is to maximise the number of covered demands. Constraint 3.5 makes sure that the demand node is covered by at least one facility only if that demand node is selected to be covered by an activated facility. Constraint 3.6 sets the maximum number of facilities that can be located in all. Constraint 3.7 - 3.8 set the binary decision variables.

### 3.6 Center problems

The center problems overcome the shortness of the set covering problem. Instead of using an "exogenously specified coverage distance" (Daskin, 2013) and trying to minimise the number of facilities needed to cover all the demand, the center problem aims at finding the location of  $p$  facilities such that all the demand nodes are covered, and the coverage distance is minimised. In this case,  $p$  can be any constant value depending on the context, and it is usually called the  $p$ -center problem or *minmax* problem because it minimises the maximum distance between activated facilities and demand nodes (Garfinkel et al., 1977). In the  $p$ -center problem, a set of demand nodes, a set of candidate facilities and the distance between demand nodes and candidate facilities are



given. The demand at each demand node and the number of facilities to be located ( $p$ ) are also provided as inputs. By offering these inputs, the  $p$ -center problem tries to find the optimal locations of facilities among the set of candidate locations such that the maximum distance between a demand node and the nearest facility is minimised. All the demand nodes are covered within the "endogenously determined distance" (Daskin, 2013) by one of the facilities. The mathematical model for the  $p$ -center problem is shown as follows:

#### Input and parameters

$I$  = set of demand nodes

$J$  = set of candidate facilities

$h_i$  = demand at node  $i \in I$

$P$  = number of facilities to locate

$d_{ij}$  = distance from demand node  $i$  to candidate facility  $j$

#### Decision variables

$y_{ij} = 1$  the demand node  $i$  served by a facility  $j$

$x_j = 1$  if a candidate facility  $j$  is activated to be located, otherwise 0

$W$  = maximum distance between a demand node and the nearest facility

With these notation, the  $p$ -center location model is as follows:

$$\min W \quad (3.9)$$

s.t.

$$\sum_{j \in J} y_{ij} = 1 \quad i \in I \quad (3.10)$$

$$\sum_{j \in J} x_j = P \quad (3.11)$$

$$y_{ij} \leq x_j \quad i \in I, j \in J \quad (3.12)$$

$$w \geq \sum_{j \in J} d_{ij} \cdot y_{ij} \quad i \in I \quad (3.13)$$

$$x_j \in \{0, 1\} \quad j \in J \quad (3.14)$$

$$y_{ij} \in \{0, 1\} \quad i \in I, j \in J \quad (3.15)$$

The objective of the  $p$ -center problem is to minimise the maximum distance between the demand node and the nearest facility. Constraint 3.10 indicates that all demand nodes should be assigned to a facility. Constraint 3.11 sets the total number of located facilities as the constant value  $p$ . Constraint 3.12

ensures that the demand cannot be assigned to the facility unless the facility is open. Constraint 3.13 defines the maximum distance between the demand node and facility, which is used in the objective function to be minimised. Finally, constraints 3.14 and 3.15 indicate the domain of decision variables.

### 3.7 Median Problems

The covering problem and center problem, as discussed in previous sections, all use the concept of coverage, assuming that a demand node can be fully served by a facility if the demand node is within the coverage distance. If not, the demand node cannot be served. Furthermore, in some cases, the quality of services each demand node can get depends on the distance between the demand node and the nearest facility. For example, the benefits (cost) can gradually decrease (increase) as the distance increases (Daskin and Maass, 2018).

On the other hand, the median problem takes into account the cost functions intending to find the optimal locations of facilities so that the total cost is minimised. In particular, the cost is related to the demand and the distance between the demand node and the facility, which is also called the  $p$ -median problem where  $p$  facilities are located. Considering the distance factor when computing total costs can overcome the issue where the distance between the demand node and the nearest facility can significantly influence the costs. Therefore, the  $p$ -median problem overcomes the limitations of the covering problem and the center problem. The  $p$ -median problem is developed to find optimal locations of  $p$  facilities among the candidates to cover all the demand and minimise the total demand-weighted distance. The mathematical formulation of the  $p$ -median problem is as follows:

#### Input and parameters

$I$  = set of demand nodes

$J$  = set of candidate facilities

$h_i$  = demand at node  $i \in I$

$P$  = number of facilities to locate

$d_{ij}$  = distance from demand node  $i$  to candidate facility  $j$

#### Decision variables

$y_{ij}$  = 1 if the demand node  $i$  is served by a facility  $j$

$x_j$  = 1 if a candidate facility  $j$  is activated to be located, otherwise 0

With these notation, the  $p$ -median location model is as follows:

$$\min \sum_{i \in I} \sum_{j \in J} h_i \cdot d_{ij} \cdot y_{ij} \quad (3.16)$$

s.t.

$$\sum_{j \in J} y_{ij} = 1 \quad i \in I \quad (3.17)$$

$$\sum_{j \in J} x_j = P \quad (3.18)$$

$$y_{ij} \leq x_j \quad i \in I, j \in J \quad (3.19)$$

$$x_j \in \{0, 1\} \quad j \in J \quad (3.20)$$

$$y_{ij} \in \{0, 1\} \quad i \in I, j \in J \quad (3.21)$$

The objective function of the  $p$ -median problem is to minimise the total demand-weighted distance between each demand node and the facilities. Constraint 3.17 ensures that each demand node is only assigned to one facility, and constraint 3.19 indicates the demand nodes can only be assigned to the activated facilities. Constraint 3.18 sets the maximum number of located facilities. Constraints 3.20 and 3.21 are for decision variables.

### 3.8 Fixed charge facility location problems

Recall what has been discussed in the previous sections: in the  $p$ -center and  $p$ -median problems, the number of facilities to be located is fixed as a constant value  $p$ . Similarly, in the maximum covering problem, the number of facilities is also fixed. The set covering problem is a bit different as it tries to minimise the number of facilities (costs). In all these facility location problems, the number of facilities is "determined endogenously" (Daskin, 2013). In this case, they do not consider the costs of establishing the facilities, which cannot reflect real-world situations.

The fixed charge location problem addresses this concern from the other problems. The fixed charge facility location problem (in short, fixed charge location problem) is similar to the previous problems except for the fixed number of facilities. Instead, the fixed charge problem considers the fixed costs associated with the establishment of facilities and the cost related to the distance in computing the total costs, and its objective function is to minimise the total costs. In this case, the fixed charge problem can find the optimal number

and location of facilities by balancing the fixed costs of locating facilities and the transportation costs related to the distance between demand and facilities (Fernández and Landete, 2015). There are two variants in the domain of fixed charge location problems: uncapacitated fixed charge facility location problems and capacitated fixed charge facility location problems.

### General input and parameters for the fixed charge facility location problem

$I$  = set of demand nodes

$J$  = set of candidate facilities

$h_i$  = demand at node  $i \in I$

$d_{ij}$  = distance from demand node  $i$  to candidate facility  $j$

$f_j$  = fixed cost of locating a facility  $j$

$c$  = cost per unit distance related to each demand

### General decision variables for the fixed charge facility location problem

$y_{ij} = 1$  the demand node  $i$  served by a facility  $j$

$x_j = 1$  if a candidate facility  $j$  is activated to be located, otherwise 0

## 3.8.1 The uncapacitated fixed charge facility location problem

The uncapacitated fixed charge facility location problems address scenarios where the capacities of facilities are not limited. In this context, the objective of the uncapacitated fixed charge facility location problems is to identify the optimal facility locations that minimise both the fixed costs of establishing the facility and the variable costs related to transportation or routing costs, which is associated with the distance between the demand node and the facility. Therefore, the uncapacitated fixed charge problem aims to find the optimal locations for facilities among the set of candidate facilities to cover all the demands such that the total costs are minimised. The mathematical formulation of the uncapacitated fixed charge location model is shown as follows:

The uncapacitated fixed charge facility location model is as follows:

$$\min \sum_{j \in J} f_j \cdot x_j + c \cdot \sum_{i \in I} \sum_{j \in J} h_i \cdot d_{ij} \cdot y_{ij} \quad (3.22)$$

s.t.

$$\sum_{j \in J} y_{ij} = 1 \quad i \in I \quad (3.23)$$

$$y_{ij} \leq x_j \quad i \in I, j \in J \quad (3.24)$$

$$x_j \in \{0, 1\} \quad j \in J \quad (3.25)$$

$$y_{ij} \in \{0, 1\} \quad i \in I, j \in J \quad (3.26)$$

The objective function 3.22 minimises the fixed cost related to locating the facility and the variable cost associated with the number of demands and the distance between the demand node and the facility. Constraint 3.23 makes sure that each demand will be served and only be served by an activated facility (constraint 3.24). Constraint 3.25 and constraint 3.26 set the domain for decision variables. Since the capacity of this problem is unlimited, the demand node will be assigned to the nearest facility as the objective function is trying to minimise the total cost.

### 3.8.2 The capacitated fixed charge facility location problem

The only difference between a capacitated fixed charge facility location problem and an uncapacitated fixed charge facility location problem is that there is a capacity constraint imposed on each facility, which will totally change the solutions in most cases. In reality, capacity is so important that it can decide the success or failure of the facility and the service it provides. For example, if the beds in the hospital are 100, and 200 patients are allocated to this hospital and require treatment simultaneously, the hospital will not function well and, in turn, put all patients' lives at risk. Therefore, taking into account the capacity can solve lots of practical issues, which leads to the introduction of the capacitated fixed charge location problem.

The capacitated fixed charge location problem aims to find the optimal number and location of facilities among the set of candidate facilities to meet the total demand requirement, where each candidate facility has a capacity limit such that the total costs (consisting of fixed costs and variable costs of transportation) are minimised.

#### Additional input and parameters

$k_j$  = capacity of a facility  $j$

With this notation, the capacitated fixed charge facility location model is as follows:

$$\min \sum_{j \in J} f_j \cdot x_j + c \cdot \sum_{i \in I} \sum_{j \in J} h_i \cdot d_{ij} \cdot y_{ij} \quad (3.27)$$

s.t.

$$\sum_{j \in J} y_{ij} = 1 \quad i \in I \quad (3.28)$$

$$y_{ij} \leq x_j \quad i \in I, j \in J \quad (3.29)$$

$$\sum_{i \in I} h_i \cdot y_{ij} \leq k_j \cdot x_j \quad j \in J \quad (3.30)$$

$$x_j \in \{0, 1\} \quad j \in J \quad (3.31)$$

$$y_{ij} \in \{0, 1\} \quad i \in I, j \in J \quad (3.32)$$

The objective function 3.27 and constraints 3.28 - 3.32 are the same as those in uncapacitated fixed charge facility location problem. The capacitated fixed charge facility location problems are trying to minimise the total costs, which consist of the fixed cost of facility establishments and the various costs related to the transportation subject to the constraints where each demand node should be served and only by the activated facility. The only difference is that constraint 3.30 sets the maximum number of demands each facility can serve.

### 3.9 Critical analysis of facility location problems: comparisons, applications and limitations

There are four main facility location problems discussed in this chapter so far. They are: i) Covering problems (including the set covering problem and the maximum covering problem), ii) Center problems (the  $p$ -center problem), iii) Median problems (the  $p$ -median problem), iv) Fixed charge location problems (including the uncapacitated and the capacitated fix charge location problems).

#### The covering problem

The set covering facility location problem is widely applied and modified based on real-world cases. There are many applications of the set covering problem. For example, it can be used in service planning, such as finding the optimal locations of warehouses, distribution centres or schools to provide services or products to the set of demand nodes. It is also applied in emergency service planning, like deciding the locations of fire stations and ambulance stations in order to make sure every demand node can be covered. Other applications can be in the telecommunication field to determine the optimal locations of base stations such that the costs are minimised and everyone is covered. These are only a few examples of applications of the set covering facility location problem in real-world cases. The set covering facility location

problem can be modified according to different contexts of the situation, and it can be applied to many areas of supply chain management, transportation network design, healthcare, and telecommunications.

There are some limitations of the set covering problem (Daskin and Maass, 2018). First, more facilities are frequently required than can be deployed to meet all requests. In the set covering problem, there is no requirement on the number of facilities that can be located, in which case, the solution of the set covering problem usually deploys as many facilities, as they can to meet all the demand, which may go beyond the number of resources they have. Second, all the demand nodes are treated as equally important, no matter how large the demand is, in which sense, both small and large demand nodes are treated equally and covered at least once. This can lead to significant problems where the large demand nodes are under-treated, and there may not have enough resources for them. Third, the set covering problem does not consider the situation where the facility is overloaded or that facility has failed. If the facility is overloaded or fails to function properly, this will lead to a terrible failure in providing necessary services to the people in need. For example, if a fire station is overloaded when an emergency happens, there may be a lack of staff or services that can be provided for the emergency, and this will lead to a large number of human losses. Finally, there are frequently alternative optima, and the model does not offer a mechanism to distinguish between these solutions, which is due to its nature of mathematical formulations.

The maximum covering problem is often used in situations where resources are limited, and a limited number of facilities can be located, so it aims to find the optimal locations of facilities such that the coverage of potential customers can be achieved. In real-world cases, those facilities are often strategically located in the area where the demand is high. For example, the maximum covering problem is usually applied in deciding the location of supermarkets or shopping malls. In this situation, the business often has the budget/resources for only one facility. In this case, the maximum covering problem is usually applied to find the optimal location where the largest number of customers can be covered. The maximum covering problem is also used in the healthcare sector, such as finding the best location of medical facilities such as hospitals or GP clinics. It is clear that the establishment of hospitals or GP clinics requires a large amount of budget, resources, and types of equipment, and there are only one or two big hospitals within a particular area. Therefore, how to make the most of the hospital is crucial, requiring finding the optimal

location of hospitals which cover the maximum amount of people. In all, the maximum covering problem is used to find the limited number of facilities such that the maximum amount of demand is covered, which has very wide applications in real-world cases, especially in situations where the resources are limited, and there is a strict limitation in the number of facilities that can be located.

The maximum covering problem can be modified based on the contexts of the problem, but it still has some limitations. First, the maximum covering problem is NP-hard, indicating that it is challenging to solve it computationally within a reasonable amount of time, especially as the problem size increases. Second, the maximum covering problem aims to maximise the total coverage, which is not realistic in real-world cases where it does not consider the capacity of the facility. Furthermore, the maximum covering problem assumes that each demand node can be fully covered by a single facility. Still, in reality, some demand nodes can be served by multiple facilities, or partial coverage also happens a lot.

### **The $p$ -center problem**

The  $p$ -center problem can be adapted to various contexts in real-world scenarios where the decision maker aims to find the optimal locations for facilities that are really close to each demand node. It can be applied to the healthcare field where medical facilities such as medical centres and clinics should be strategically located close to the demand, ensuring accessibility for people in need and improving the delivery of services. It can also be applied in determining the locations for retail stores, especially for those convenience stores where close to the demand nodes are required. Other applications, such as locations for emergency services, warehouses, or distribution centres, can also be modelled as the  $p$ -center problem based on the contexts.

In all, the limitations of the  $p$ -center problem are pretty similar to those in covering problems. As the models discussed in this chapter are fundamental, they do not take into account various elements such as the capacity of each facility, the cost of travelling or the priority of different demand nodes. These issues can be solved when modelling real-world contexts by taking into account these factors for better modelling of different scenarios. The main challenge of the  $p$ -center problem is the complexity of the computational process, especially when the size of the instance increases. It takes much longer to get the optimal solutions.



### **The $p$ -median problem**

Similar to what has been discussed in previous sections, the  $p$ -median problem can be modified according to different contexts of real-world cases, and it can also be applied in many areas in healthcare, logistics and supply chain, and other public services. The main difference between the  $p$ -median problem and other discussed location problems is that it takes into account the costs brought by the distance between demand nodes and nearest facilities, which is more appropriate in real-world cases. For example, the decision to find the optimal location for medical clinics not only needs to consider covering all the demand in this area but also needs to take into account the efficiency and costs that demand need to pay to get the service where the  $p$ -median problem can achieve that. In this case, the location decisions of the  $p$ -median problem consider the efficiency and costs from the demand side, which made it more practical in modelling real-world situations.

Similar limitations also apply to the  $p$ -median problem, which is computationally complex to solve this problem, and the assumptions of the  $p$ -median problem are too simple to catch up with the complex situations in real-world scenarios. Furthermore, the distance between the demand node and the candidate facility is used, which requires lots of effort to provide an accurate distance in order to feed the model and get more reliable solutions. Moreover, the  $p$ -median problem does not take into account the capacity issue of the candidate facility, which may lead to overloaded or underloaded facilities, and in turn, either overloading or underloading can result in the failure of the operation of facilities.

### **The fixed charge facility location problem**

The uncapacitated fixed charge location problem is often applied in the context where the costs are crucial in making the decision and also in the case where the value of the distance from the demand node and candidate facility is known. The uncapacitated fixed charge location problem aims to find the optimal location of facilities while considering the fixed costs related to establishing the facilities and the variable costs related to the distance between the demand node and the facility. It can be applied and modified in many areas, such as supply chain management, retail store network design, telecommunications planning and so on. Unlike the previous problems, the uncapacitated fixed charge location problem considers the fixed costs of establishing the facilities, making it more applicable in various sectors.

Compared with the uncapacitated fixed charge location problem, the capacitated fixed charge location problem considers the capacity of the facility, which makes it more representative of real-world scenarios. In this case, it can be applied to different areas, such as manufacturing and production planning, where the capacity constraints and the fixed costs are usually very tight in the location decision of the factory. Therefore, using the capacitated fixed charge location problem can help the decision-maker find the optimal locations of factories such that the total costs are minimised. At the same time, capacity constraints are also considered—similar application fields such as retail stores and other public service locations.

Overall, similar limitations are shared within all types of facility location problems where the complexity of finding the optimal solutions are high, and it takes a long time to find the best solutions. In this case, heuristics are often used to overcome this issue. Furthermore, it is important to acknowledge that these four basic discrete facility location problems have many assumptions which may not be applicable in real-world circumstances. Therefore, these facility location models are often modified based on the contexts in order to account for the complexities in real-world scenarios.

### Comparisons

A comparison is conducted based on the original assumptions, parameters, and objective functions of these four problems. The comparison between these four basic types of discrete facility location problems is conducted through six different aspects: i) the objective function each problem is trying to achieve, ii) if all the demands are covered or not, iii) if there is a fixed number of facilities that can be located ( $p$ ) or not, iv) types of costs that each problem considers, v) types of the distance between each demand node and facility, vi) if the capacity of the facility is considered or not.

The comparison is conducted as follows:

- **Objective function:** the objective of the set covering problem is to minimise the total costs; the objective of the maximum covering problem is to maximise the total covered demand; the objective of  $p$ -center problem is to minimise the maximum distance between demand node and the facility; the objective of the  $p$ -median problem is to minimise the total demand weighted distance; the objective of both uncapacitated and capacitated facility location problems is to minimise the total costs;

- **All demand covered:** in the maximum covering problem, the demand may not be fully covered. While in other facility location problems discussed in this chapter, the demand is fully covered;
- **Fixed number of facilities ( $p$ ):** in the maximum covering problem,  $p$ -median problem, and  $p$ -center problem, they introduce the fixed number constant  $p$  to determine the number of facilities that will be located. On the other hand, there is no fixed number set for facilities in the set covering problem, uncapacitated and capacitated fixed charge location problems;
- **Costs considered:** in terms of costs, the set covering problem only considers the fixed cost of establishing the facilities, while both uncapacitated and capacitated fixed charge location problems consider fixed costs related to the facility establishments and the variable costs associated to transportation. In other problems like the maximum covering problem,  $p$ -center problem, and  $p$ -median problem, they do not consider the costs at all. This is actually related to the constant number of facilities. In the maximum covering problem,  $p$ -center problem, and  $p$ -median problem, they use the constraint of the number of facilities to tackle the costs, which means the constant value  $p$  of facility number and the costs function the same, and they are interchangeable;
- **Distance considered:** this category discusses how each facility location problem addresses the separation between candidate and demand nodes. In the covering problems, there are no specific distance values. Instead, they introduce the coverage concept, where a set of binary variables are introduced, indicating whether the demand node is within the coverage distance or not. Other problems in the center problems, median problems, and fixed charge location problems all use the specific distance between the demand node and the nearest facility;
- **Capacity considered:** among all these six facility location problems, only the capacitated fixed charge problem considers the capacity of the facility, while others do not.

These are the comparisons and discussions made between these six basic discrete facility location problems in order to provide a comprehensive outstanding of facility location problems, and all these aspects are summarised in the table 3.1.

TABLE 3.1: Summary of all facility location problems discussed

	Covering problem		Center problem	Median problem	Fixed charge location problem	
	Set covering	Maximum covering	$p$ -center	$p$ -median	Uncapacitated fixed charge	Capacitated fixed charge
Objective function	Min costs	Max covered demand	Min maximum distance	Min demand weighted distance	Min costs	Min costs
All demand covered	Yes	No	Yes	Yes	Yes	Yes
$p$ facilities	No	Yes	Yes	Yes	No	No
Costs considered	Fixed costs	None	None	None	Fixed cost & transportation cost	Fixed cost & transportation cost
Distance considered	Coverage	Coverage	Specific distance	Specific distance	Specific distance	Specific distance
Capacity considered	No	No	No	No	No	Yes

To sum up, the applications of these four types of facility location problems are similar, and they can be modified and combined based on real-world contexts. There are other types of facility location problems besides these four basic models, and facility location problems can be combined with other problems such as location-allocation problems, location-routing problems and so on. These all make facility location problems more practical in tackling real-world cases.

### 3.10 Facility location problems with applications in humanitarian operations

Facility location problems have an extensive range of applications in humanitarian operations, and according to the review of [Roh et al. \(2015\)](#), a large portion of research in humanitarian operations studies facility location problems (41.03%). As discussed in the last chapter, the nature of the disaster is full of uncertainties, and good preparation can significantly improve the efficiency and effectiveness of humanitarian operations and disaster relief. In order to achieve timely and efficient disaster relief, facility location problems are often applied in strategic plans, and the location decisions are usually planned and located in both pre- and post-disaster phases.

#### 3.10.1 Types of facilities addressed in humanitarian operations

Facility location is a crucial topic in humanitarian operations and fundamental to achieving humanitarian relief goals successfully and potentially accelerating the disaster recovery process ([Trivedi and Singh, 2018](#)). Facility location problems in humanitarian operations often deal with the determination of facilities and the allocation of the demand to the selected facilities. There

are many terminologies regarding different facility types in the literature, but they can mainly be classified into four groups. Types of facilities discussed in journal articles include:

- *Warehouse and stock facilities*: they are used for storage where critical supplies are pre-positioned in these warehouses and stock facilities to tackle the needs of affected populations. Besides the storing function, the warehouse also involves inventory management by deciding what types of supplies will be stored and keeping track of how inventory goes, which enables efficient and effective humanitarian operations and disaster relief;
- *Hospital or medical centres*: these types of facilities are used for providing medical aid to people suffering from bad injuries during disasters. In this case, hospitals are also called field hospitals, which differs from the typical hospitals in the city. Field hospitals and other medical centres play a crucial part in emergency response after a disaster by providing emergency medical services, prioritising patients, and treating injured people. In addition, they also try to control the diseases that disasters might bring;
- *Humanitarian aid distribution centres and logistics hubs*: in the humanitarian context, the distribution centre is where disaster relief products or supplies can be stored to ensure quicker disaster relief. A distribution centre is similar to a warehouse, apart from offering some value-added services such as packaging and reassembling. In contrast, a warehouse may just be used for storing supplies or donations. The humanitarian aid distribution centres can also act as operational hubs for organising, storing, and delivering disaster relief supplies. They also coordinate the distribution of humanitarian aid supplies to the demand points, which helps to achieve effective disaster relief;
- *Temporary emergency facilities*: these types of facilities are used for disaster relief, including shelters, places of safety and assembling points. They provide a safe environment and protection to evacuees from further harm. These facilities are usually strategically planned in the pre-disaster phases and implemented during the post-disaster operations to increase the efficiency and effectiveness of the evacuation process. Temporary emergency facilities provide short-term housing for people who are affected by the disaster, in which they not only provide the space for evacuees but also provide other supplies or medical services.

When there is a disaster or emergency, these facilities usually coordinate and cooperate as a cohesive system to provide support, supplies, safe places and medical treatments to the people in need. [Dönmez et al. \(2021\)](#) further classified the facilities into six categories based on their functions: i) supplies, ii) distribution centres, iii) points of distribution, iv) shelters, v) field hospitals, vi) and blood centres. In particular, the distribution centre is used to deliver relief supplies to people in need. It has other names, such as local depots, transfer depots, etc. Points of distribution are often used as a place for affected people to get relief supplies. Shelters are temporary places to rest and protect people from further danger. Field hospitals often have medical equipment on-site to provide medical aid, and the blood centres are used for collecting, processing and distributing blood. In the literature, the distribution centre is the most studied facility type in the location problem in the humanitarian operations context ([Dönmez et al., 2021](#)).

However, research about locating temporary emergency facilities, such as the shelter location problem, is less addressed in the existing literature compared to other types of facilities in the humanitarian context (only 24% of studies according to the review of [Trivedi and Singh, 2018](#)) and this work focuses on the shelter location problem. The shelter location problem deals with strategically placing shelters to provide temporary protection and aid to people in need. It is usually involved in the pre-disaster phase for evacuation design and planning in terms of preparing for future disasters, including the optimal selection of suitable shelters among the candidates and each with specific requirements, such as schools or stadiums, to ensure the effectiveness and efficiency of disaster response and relief.

[Pan \(2010\)](#) proposed two deterministic models based on the maximum set-covering problem to assist in the decision of shelter locations and to help the government and the public prepare for a typhoon. [Kılıcı et al. \(2015\)](#) developed a mixed-integer linear programming model to find the location of shelter sites and improve the disaster preparation process. [Bayram et al. \(2015\)](#) developed a constrained system optimal model to find optimal shelter locations and assign evacuees to the shortest paths to their nearest shelter sites, shortest and nearest with a degree of tolerance to minimise the total evacuation time for both pre- and post-disaster stages. [Bayram and Yaman \(2018a\)](#) then introduced a scenario-based two-stage stochastic evacuation planning model that can optimally locate shelter sites and assign evacuees to the nearest shelters

and to the shortest paths with a degree of tolerance where the total evacuation time is minimised. In particular, they consider the uncertainty of the disruption of roads and shelters and the demand for evacuation. Based on this, [Bayram and Yaman \(2018b\)](#) further developed an exact algorithm utilising Benders decomposition to solve their scenario-based two-stage stochastic evacuation planning model with many scenarios. Decisions about shelter locations along with other facilities are considered in the preparedness model developed by [Rodríguez-Espíndola et al. \(2018\)](#), combining multi-objective optimisation with Geographical Information Systems (GIS) to minimise the costs and maximum unfulfillments of products and services in shelters from an equity perspective. [Gu et al. \(2018\)](#) developed a mixed-integer programming formulation to determine medical relief shelters before the disaster impact, tackling natural and anthropogenic disasters and considering the severity and distribution of patients. [Kinay et al. \(2018\)](#) introduced a new modelling framework in which a chance-constrained model was proposed for the shelter site problem in the preparedness phase for disaster relief to capture the uncertainty in demand. Another modelling framework was developed by [Kinay et al. \(2019\)](#) for multi-criteria chance-constrained discrete facility location problems with single sourcing, where they consider vectorial optimisation and goal programming paradigms in multi-criteria decision-making. [Eriskin and Karatas \(2022\)](#) developed a robust shelter location-allocation model to plan and decide the location of shelters for an earthquake to improve disaster preparedness.

Generally, four main facility types appeared in the context of humanitarian operations, namely warehouses/stock facilities, hospital/health centres/medical services, humanitarian aid distribution centres or logistics hubs, and temporary emergency facilities such as shelters or places of safety, as discussed previously. The timing of locating the facility varies across different contexts. However, there exists a general pattern wherein specific types of facilities are typically situated during distinct phases of the disaster life-cycle ([Trivedi and Singh, 2018](#)).

- *Warehouse/stock facilities* are usually planned in the pre-disaster phase in order to be ready before the disasters to prepare for the coming disaster. In general, the size of the warehouse is big, with a large amount of supplies and resources which might be used to support disaster relief. Warehouse location problem usually involves the pre-positioning of

supplies and the delivery problem related to the delivery of those supplies. Therefore, warehouse location problem usually happens in pre-disaster phases;

- *Hospital or other medical facilities* which provide medical services are usually planned for the post-disaster phase. It is also quite apparent that the decisions about where to locate medical assistance should be based on the places where the disaster happened. In the context of humanitarian operations, the hospital is not a normal hospital. It is a medical facility that can provide medical assistance to people affected by disasters. Those hospitals are temporary, and they can be mobile. Therefore, decisions for the locations of medical facilities are usually made in post-disaster situations;
- *Humanitarian aid distribution centres/logistics hubs* are also planned in the pre-disaster phase and located in the post-disaster to ensure quick and on-time relief distribution. The humanitarian aid distribution centres can range in size depending on different contexts, and they are usually located close to the area near disaster in order to provide in-time delivery and support. Therefore, they are often located after knowing the location of the disaster;
- *Temporary emergency facilities* are often planned and located before and after the disaster. This complies with the logic of real-world scenarios where some temporary emergency facilities are located before disasters to get ready to respond to the disasters. At the same time, some temporary shelter sites need to be located based on the disaster location to provide quick and on-time relief.

The location decisions for different types of facilities depend on their specific characteristics in the context of disasters and real-world problems. There are no universal rules that can apply to all situations. Furthermore, the location decisions also need to take into account the amount of resources the decision maker/organisation has in order to find the most appropriate solutions. The attributes of the facilities that were previously addressed are summed up in the following table 3.2.

Due to the uncertain nature of disasters, facility location models are extended



General types of facilities in HO		
	Usage	Disaster life-cycle stages
Stock facilities	storing supplies	pre-disaster
Medical facilities	providing medical aid	post-disaster
Distribution centres	distributing supplies	pre- and post-disaster
Temporary emergency facilities	shelters and places of safety	pre- and post-disaster

TABLE 3.2: Summary of facilities in humanitarian operations contexts

to dynamic, stochastic, and robust models to tackle the uncertainties. [Boonmee et al. \(2017\)](#) performed a survey on facility location challenges in humanitarian logistics based on modelling types and problem categories. They suggest that future facility location modelling can apply more dynamic or robust optimisation to solve real cases and consider the uncertainty in time periods, environments, demands, disruptions or other disaster risks. In particular, dynamic and robust facility location models can be applied in safety area planning to prepare and transfer residents further away from the risk area. They also agree that finding the optimal locations of facilities such as warehouses, shelters, temporary or permanent distribution centres and medical centres in pre-disaster phases can better prepare for potential disasters and improve the probability of survival and reduce financial and human losses. Furthermore, they point out that the key to successful disaster relief is to quickly find the optimal location for shelters and medical services that can handle the demand and guarantee that the injured people can be evacuated to a safe place and get treatment in time. [Caunhye et al. \(2012\)](#) agree with this aspect, and they also suggest that considering facility location problems in post-disaster phases is also very important as this can make sure the right resources and supports can be placed at the right place at the right time.

### 3.10.2 Objective functions for facility location problems

The objective functions in facility location problems in humanitarian operations are based on the context and the goals of the decision-makers. Many different objective functions have emerged in the literature. There are, however, several main categories of objectives used in the formulation of mathematical modelling:

- *Cost minimisation*: this objective function is one of the most frequently used objective function goals, trying to minimise the total cost incurred

during the humanitarian operation process. However, it does not usually serve as the primary goal because the main focus of humanitarian operations is to reduce human suffering and save lives;

- *Non-covered demand minimisation*: this objective function is also frequently used in the literature in humanitarian operations, with the aim of covering all people in need, which is crucial in the evacuation process;
- *Transportation duration minimisation and response time minimisation*: these two objective functions aim at shortening the time cost in different sections to increase the efficiency of evacuation operations;
- *Facility-opened minimisation*: this objective function is another commonly used objective goal in the location problem where the main reason for minimising the opened facility is to minimise the costs and expenses incurred. The minimisation of the number of facilities opened and the cost minimisation are often interchangeable as they achieve the same goal.

In the literature, these four main categories are the most used objective functions in the facility location problems. [Boonmee et al. \(2017\)](#) suggest that the future objective in facility location problems could focus more on the environmental effect, reliability, risk and ease of access and, during the past several years, more and more research has taken into account these factors due to more coordination and collaborations between stakeholders have achieved in humanitarian operations.

### 3.11 Conclusions

This chapter has provided a comprehensive discussion of facility location problems, which begins with an overview and an introduction to the fundamental questions addressed by facility location problems. Then, this chapter has explored four distinct categories of location models: analytic models, continuous location models, network location models, and discrete location models. Moreover, this chapter has discussed and compared the concepts, applications, and limitations of four core discrete facility location problems, namely the covering problems,  $p$ -center problems,  $p$ -median problems, and fixed charge

facility location problems. Finally, in the last section of this chapter, the significance and applications of facility location problems in the context of humanitarian operations are evaluated, emphasising effective modelling of facility location problems can increase the efficiency of humanitarian operations, which, in turn, will lead to a reduction in human suffering.

## Chapter 4

# Network Flow Problems

### 4.1 Introduction

Network flow theory has been acknowledged as a powerful decision-making support tool, which consists of a branch of operations research and graph theory to analyse the flows through a network. This chapter first provides an overview of network flow theory. Then, it focuses on discussing and comparing two essential elements of network flows, which are the static network flows and the dynamic network flows. After that, the chapter further investigates the applications of network flow problems in the context of evacuation scenarios, emphasising the advantages of dynamic network flow problems in evacuation design and planning. Finally, the commonly utilised modelling techniques employed in network flow theories are introduced and discussed at the end of this chapter.

### 4.2 An overview of network flow theory

The network flow theory deals with the optimisation of the movement of flows on the network. It can be applied in various areas, including those that, by nature, have a network structure, such as distribution networks, logistics, and transportation networks where flows of people or products travel through networks of roads, ships, or planes. Telecommunication and data networks are also examples of networks where the data (e.g., messages and videos) are sent through the server based on network structures. The delivery of products can also be modelled as a network flow problem, where the destinations of each customer and the flow of delivery drivers create a network. In addition, financial transactions can also be modelled as a network where the payer and receiver, along with the flow of money, build the structure of a network.

Networks are everywhere in our daily lives, in which network flow theories are widely applied.

Directed networks are often utilised in the network flow theory, where nodes usually stand in for sources, destinations, or intermediate locations. At the same time, the links and roads are represented by the arcs. Each arc has a capacity, which is the largest amount of flow that can be on that arc. Therefore, a flow network is defined as a digraph  $G = (N, A)$ , which has a set of  $N$  nodes and a set of  $A$  directed arcs. Each arc  $(i, j)$ , where the tail is node  $i$ , and the head is node  $j$ , has a capacity constraint  $u_{ij}$ , and that is the largest amount of flow that can traverse that arc  $(i, j)$ . Here is an example to explain this further. Figure 4.1 is a toy example of a flow network. In this toy example, there are four nodes: node 1, 2, 3, 4, and four directed arcs: arc  $(1, 2)$ ,  $(2, 4)$ ,  $(1, 3)$ ,  $(3, 4)$ . Each arc is associated with a capacity constraint; for example,  $u_{12} = 2$  means that the largest amount of flow that can enter arc  $(1, 2)$  is 2. Similarly,  $u_{24} = 1$  means that only 1 flow can enter arc  $(2, 4)$ . The largest amount of flow that can enter  $(1, 3)$  is 3, and the largest amount of flow can enter arc  $(3, 4)$  is 4 as  $u_{13} = 3$  and  $u_{34} = 4$ .

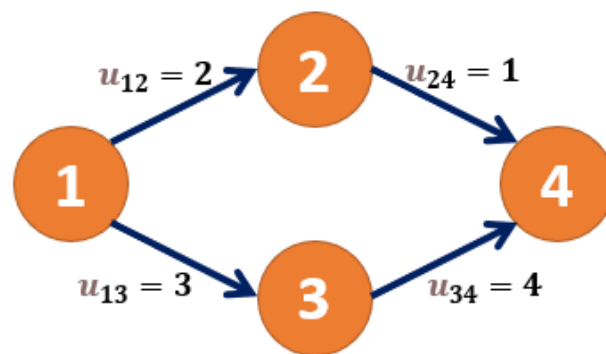


FIGURE 4.1: An example of flow network

These are essential components of a flow network, and by considering the capacity constraint on each arc, it can significantly reduce the possibility of arc collapse, which is important in modelling real-world scenarios. In general, there are two categories of network flow models: static network flows and dynamic network flows. The critical difference is that dynamic network flows, also known as flows over time, introduce a delay time or travel time on arcs by explicitly stating the amount of time needed to move one unit of flow from an arc's tail to its head. In the literature, dynamic network flows are among the most appropriate tools compared with static network flows

in terms of modelling the evacuation problems from a macroscopic perspective as it considers the time element in modelling, allowing for more precise monitoring and control throughout the entire flow process across a specific time period. In recent years, many pieces of research in evacuation design and planning use macroscopic models, which are presented as dynamic network flow models (see [Chalmet et al. \(1982\)](#), [Mamada et al. \(2004\)](#), [Hamacher et al. \(2013\)](#), [Schmidt and Skutella \(2014\)](#), [Melchiori and Sgalambro \(2015\)](#), [Shin et al. \(2019\)](#)) for evacuation problems. The detailed introduction and discussions of static network flows and dynamic network flows will be presented in the following sections.

### 4.3 Static network flow problems

The static network flow problem is a class of optimisation problems in network flow theories aiming to find an optimal static flow in the network, subject to various constraints and objective functions. Among all other network flow problems, the static network flow problem refers explicitly to the situation where the flow patterns remain constant over time. In other words, it means there are no changes or time elements in the network. The static network flow problems can often be formulated using the directed graph, where nodes are used to represent the sources, sinks, or other intermediate points. In contrast, arcs are used to show the connections between those nodes. Each arc has a capacity constraint, which represents the maximum flow that can pass through it, as shown in figure 4.1. Many static network flow models developed during the past years in literature, for instance, the minimum cost flow problem, maximum flow problem, and shortest path problem.

The notation of the Static network flows is shown as follows:

#### General input and parameters for the static network flow problem

$G = (N, A)$ : a directed network

$N$ : as the set of nodes

$A$ : as the set of directed arcs

$c_{ij}$ : the cost associated with each directed arc  $(i, j) \in A$

$u_{ij}$ : capacity of the arc  $(i, j) \in A$

$b_i$ : supply/demand on each node  $i \in N$ .

#### Decision variable

$x_{ij}$ : the flow on each arc  $(i, j) \in A$

### 4.3.1 The minimum cost flow Problem

The minimum cost flow problem is considered the most fundamental among network flow problems (Ahuja et al., 2014), aims to find the minimal cost flow on the directed networks subject to capacity constraints on the arcs from the source to the destination. In the minimum cost flow problem, sources and destinations are represented by nodes, and the arcs represent the links between each node. It introduces the cost parameter on each arc to compute the total cost where  $c_{ij}$  is the cost associated with each arc  $(i, j)$ . In addition, the supply or demand on each arc (the net flow) is also introduced as an input as  $b_i$ . The computation of the net flow for each node  $i$  ( $b_i$ ) is computed by the amount flow exiting from node  $i$  minus the amount of flow entering the node  $i$ , whose mathematical formulation is  $\sum_{j:(i,j) \in A} x_{ij} - \sum_{j:(j,i) \in A} x_{ji} = b_i$ . If  $b_i > 0$ , it means that the amount of flow exiting is bigger than the amount of flow entering, indicating that the node  $i$  in this situation is a *supply* node, which is also called the *source* node. In contrast, if  $b_i < 0$ , it shows that the amount of flow exiting is smaller than the amount of flow entering, meaning that the node  $i$  in this context is a *demand* node, which is also called the *sink* node. In addition, if  $b_i = 0$ , the flow entering the node  $i$  is exactly the same as the flow exiting, which suggests that the node  $i$  here is a *transshipment* node.

The minimum cost flow problem can be formulated as follows:

$$\min \sum_{(i,j) \in A} c_{ij} \cdot x_{ij} \quad (4.1)$$

s.t.

$$\sum_{j:(i,j) \in A} x_{ij} - \sum_{j:(j,i) \in A} x_{ji} = b_i \quad i \in N \quad (4.2)$$

$$0 \leq x_{ij} \leq u_{ij} \quad (i, j) \in A \quad (4.3)$$

The objective function 4.1 is to minimise the total cost of all the flow and to find all the flow movements in the network with the minimum costs. Constraint 4.2 is the flow conservation constraint by computing the net flow for each node. Constraint 4.3 sets the upper bound of flow that can traverse each arc in the network.

### Assumptions of the minimum cost flow problem

There are many assumptions in the minimum cost flow problem (Ahuja et al., 2014). First, the network is directed, which means the arc  $(i, j)$  and arc  $(j, i)$  are not the same. Second, the sum of the net flow at each node  $i$  is equal to 0 so that the minimum cost flow problem has feasible solutions. Otherwise, it becomes unfeasible. In this case, for each node  $i$ ,  $\sum_{i \in N} b_i = 0$ . Furthermore, all the costs of arcs are non-negative. This can become more representative of real-world situations. Finally, all data, such as capacity, cost, and supply and demand for each node, are all integers. This is not a hard requirement because integer values can be obtained by multiplying big numbers in the computational process.

### Applications of the minimum cost flow problem

The minimum cost flow problem is applied in many industries, such as health care, telecommunications, retailing, transportation, and so on, whose goal is to determine the optimal flow of any subject (i.e., resources or information). In contrast, the total transportation costs are minimised. For example, in some transportation and logistics problems, the minimum cost flow problem is used to find the best flow for the products, such as finding the best way to deliver products to the customers. By using the minimum cost flow problem, the efficiency of flow on the network is improved, and the total costs are minimised.

#### 4.3.2 The shortest path problem

The shortest path problem focuses on identifying the path with the lowest cost or length from a specific source node ( $s$ ) to a sink node ( $t$ ). The shortest path problem is a classic problem in graph theory, and it can be applied in many fields and attracts lots of interest from researchers and practitioners. There are several reasons: first, it is applicable to be used in settings where the objective is to find the optimal path between two points (i.e. from retailer to customers) to send the flow (i.e. materials, products) in a network with the lowest costs. Second, the shortest path problem is relatively easy to solve. Third, although the shortest path problem is easy, it captures all core ingredients of network flow problems, providing a benchmark for more complex network models. Finally, the shortest path problem can be used as subproblems in solving complex combinatorial and network optimisation problems (Ahuja et al., 2014).



### Notation and mathematical model

In the shortest path problem, each arc  $(i, j)$  in the network has an associated cost  $(c_{ij})$ . Unlike the minimum cost flow problem, the shortest path problem assigns  $b_s = 1$ ,  $b_t = -1$ , and  $b_i = 0$  to all other nodes in the network when the flow is one unit. By minimising the cost, the shortest path can be determined.

Based on the discussion above, the notation and the mathematical model of the shortest path problem are shown as follows (in the case the total flow is 1 unit):  $b_i$ : supply/demand on each node  $i \in N$ .  $b_s = 1$  as the source node,  $b_t = -1$  as the sink/destination node and  $b_i = 0$  as a transshipment node in the context where the flow is one unit

The shortest path problem can be formulated as follows:

$$\min \sum_{(i,j) \in A} c_{ij} \cdot x_{ij} \quad (4.4)$$

s.t.

$$\sum_{j:(i,j) \in A} x_{ij} - \sum_{j:(j,i) \in A} x_{ji} = b_i = \begin{cases} 1, & i = s \\ 0, & i \in N, i \neq s, i \neq t \\ -1, & i = t \end{cases} \quad (4.5)$$

The objective function 4.4 is to minimise the total cost of all the flow and to find the path in the network such that the total cost is minimised. Constraint 4.5 is the flow conservation constraint to state the source node, transshipment nodes, and sink node as discussed in the minimum cost flow problem.

### Assumptions of the shortest path problem

The shortest path problem lies behind several assumptions. First, similar to the minimum cost flow problem, the shortest path problem also assumes the network is directed. Second, there must be a directed path from the source node  $s$  to every other node in the network. Otherwise, there is no feasible solution to this problem. Furthermore, all the lengths of arcs are integers. This assumption is not a restrictive constraint as multiplying non-integers to a large number. It will become integer numbers. Finally, the shortest path problem requires no negative cycle in the network.

### Some extensions of the shortest path problem

Based on the simple shortest path problem, some extended versions are introduced in the literature. For example, some problems aim to find the shortest paths from one node to all other nodes where all the arc length is non-negative. Beyond that, some other problems try to find the shortest path from one node to all other nodes in which the network has any arc length. Moreover, some extended problems aim to find the shortest paths from any source node to any destination node. These are some examples of extensions of the shortest path problem, and they can be adjusted based on different real-world situations.

### Applications of the shortest path problem

As the fundamental problem in network flow problems, the shortest path problem can be applied in many areas, such as transportation and telecommunications, in the context where the decision maker tries to find the optimal path between two geographical locations with the least costs or time. For example, if a company needs to deliver a certain number of products to one specific customer, the shortest path problem can be used to find the best path with the least costs. Furthermore, the shortest path problem can also be used as the subproblem of other complex network problems to solve complex combinatorial problems as quickly as possible.

### 4.3.3 The maximum flow problem

The maximum flow problem and the shortest path problem are integral types of the minimum cost flow problem. The maximum flow problem deals with sending as many flows as possible from the specific source  $s$  to the sink  $t$ , considering the capacity constraint  $u_{ij}$ , which is the largest amount of flow that can travel on the arc  $(i, j) \in A$ . Recall that the shortest path problem is to find the shortest path in the network from two points, which has the lowest costs without any capacity constraints on each arc. The maximum flow problem aims to find the maximum flow in the network between two specific nodes by taking into account the capacity constraint on each node.

#### Notation and mathematical model

In the maximum flow problem, each arc  $(i, j)$  in the network has a capacity  $(u_{ij})$ . Unlike the shortest path problem, the maximum flow problem assigns

$b_s = v$ ,  $b_t = -v$ , and  $b_i = 0$  to all other nodes in the network and the amount of  $v$  is the total amount of flow it aims to maximise.

Based on the discussion above, the notation and the mathematical model of the maximum flow problem are shown as follows:

$$\max v \quad (4.6)$$

s.t.

$$\sum_{(i,j) \in A} x_{ij} - \sum_{(j,i) \in A} x_{ji} = \begin{cases} v, & i = s \\ 0, & i \in N, i \neq s, i \neq t \\ -v, & i = t \end{cases} \quad (4.7)$$

$$0 \leq x_{ij} \leq u_{ij} \quad (i, j) \in A \quad (4.8)$$

The objective function 4.6 is to maximise flow that can traverse through the network from the source node  $s$  and sink node  $t$ . Constraint 4.7 is the flow conservation constraint to state the source node, transshipment nodes, and sink node. Constraint 4.8 is the set of capacity constraints to set the limitations to the largest amount of flow that can enter each arc in the network.

### Assumptions of the maximum flow problem

Similar assumptions appear in the maximum flow problem. First, the network is directed, the same as the other two problems. Second, the capacity of each arc should be non-negative integers. Third, the network should not have any directed path from the source node  $s$  to the sink node  $t$  with infinitely capacity arcs. It is quite apparent that if there is any in the path with infinity capacities, infinity flow can be sent in that path. Therefore, the objective function is unbounded, making this problem meaningless. Furthermore, there should not exist any parallel arcs with the same tail and heads and finally, if there is an arc  $(i, j) \in A$ , the arc  $(j, i)$  should also belong to  $A$ .

### Applications of the maximum flow problem

The applications of the maximum flow problem are various in different fields, such as telecommunications, transportation and logistics, and network planning. For example, in the area of telecommunications, the maximum flow

problem can help to find the maximum amount of data that can be transmitted in the telecommunication networks, which will increase the utilisation of the resources. Another application in real-world cases can be determining the largest amount of flow that can travel in the network of pipes or channels with respect to the capacity of each pipe. By maximising the flow in the capacitated network, the efficiency and usage of the network can be significantly improved. Similar to other problems, the maximum flow problem can also be used as the subproblem of complex network flow problems.

#### 4.3.4 Critical analysis of static network flow problems

The static network flow problem, as the fundamental problem in network flow theories, aims to identify an optimal static flow that, among all feasible alternatives, optimises certain objective functions based on the context of the problem. There are many other extensions of the basic static network flow problems, such as multi-commodity flow problems that extend one type of commodity to multiple commodities. The static network flow problem can also be combined with other problems.

Although static network flows have great potential to support a wide range of real-world cases, some limitations restrict their applicability in broader scenarios. First, static network flows assume the fixed capacities for the arcs in the network, which disregards the variations in capacities caused by situations like road congestion or other issues. Furthermore, static network flow models do not consider other elements, such as robustness, rather than focusing only on flow optimisation. Moreover, the most critical limitation is that static network flows fail to accurately capture those real-world scenarios where the dynamics of flows over a given time period come into play. In particular, the static network flows fail to capture the movement of flows on the network from microscopic aspects like the amount of time it takes for each unit of flow to travel along the arcs in the network and the variation in the level of utilisation of the network over time. In addition, the flow movements in the static network flow problems are modelled as they occur instantly, which fails to reflect the concepts of delays and breaks. In all, the static network flows fail to model some real-world scenarios where the critical factor of their nature depends on the time elements, such as telecommunication networks or transportation networks, or disaster evacuations. This leads to the importance of dynamic network flows.

## 4.4 Dynamic network flow problems

Dynamic network flows, also called flows over time, overcome the critical limitation of static network flows by considering the time elements in modelling. The static network flows cannot capture the time used for flows moving along the network or the congestion levels in the network over time. On the contrary, dynamic network flows consider the time parameters in the modelling of the situation in the network, which is called *dynamic digraph* by introducing two parameters/labels on each arc: a delay/travel time and a capacity and also by stating the total time horizon for the problem that needs to be optimised. The delay time or travel time on each arc shows the amount of time required for each unit to traverse the arc at each time instant, and the capacity label indicates the largest amount of flow that can enter the arc. By considering these two elements together, the dynamic network flow problem can trace the precise location of flows at each time instant, enabling tight control and monitoring of flows throughout the analysed time period.

### 4.4.1 Characteristics of dynamic network flow problems

Dynamic network flows can be broadly categorised based on different types of time horizons in the network. The time horizons can be finite or infinite, and time instants can be discrete or continuous. The different settings of time elements will largely influence the network structure and the solutions in the end. Additionally, the capacity and delay time for each arc can be varied or fixed during the considered time period. Furthermore, the pace of flow in the network can be independent or dependent on the total amount of flow already entering the arc and the amount of flow entering the arc simultaneously, which is key in different dynamic network flow problems.

Another crucial characteristic of dynamic network flow problems is that if waiting is allowed, the flow can stay on the node until the next available time instant. In this case, the delay in the transshipment process can be modelled. In order to model this feature, additional sets of *holdover arcs* are introduced. Moreover, the dynamic network flow problems can also prevent the use of paths that traverse nodes more than once, forcing flows only to be assigned through loopless paths. This feature aligns with many real-world applications. For example, in humanitarian operations, by applying this feature,

the evacuees will not be directed to dangerous places again and again. Finally, the dynamic network flow problems can be modelled as single or multi-commodity problems based on different contexts behind the problem.

#### 4.4.2 An overview of dynamic network flow problems

The dynamic network flow problem, which is also called flows over time or dynamic (network) flows, was first introduced by [Ford and Fulkerson \(1958\)](#) as a dynamic digraph with capacity and transit time on each arc. The transit time (also called delay time or travel time) is the amount of time needed to travel from the tail to the head on each arc, and the capacity on the arc is the largest amount of flow that can enter into the arc at each time instant as discussed previously. In this case, the movement of flows can be monitored. It should be pointed out that the definition of the capacity provided on each arc does not mean the total amount of flow on the arc at a specific instant in time, which is not considered in this research.

The notation of the dynamic network flows is shown in the following:

##### General input and parameters for the static network flow problem

$G = (N, A, T)$ : the dynamic digraph

$N$ : as the set of nodes

$A$ : as the set of directed arcs

$c_{ij}$ : capacity of the arc  $(i, j) \in A$

$d_{ij}$ : the delay time associated with each arc  $(i, j)$  on each time instant

##### Decision variable

$x_{ij}(t)$ : the flow on each arc  $(i, j) \in A$  at time  $t$

In order to solve the problem, [Ford and Fulkerson \(1958\)](#) introduced a Time-Expanded Network (TEN), an essential tool for solving all ranges of discrete-time dynamic network flow problems. A time-expanded network (TEN) is defined to represent the time-dependent characteristics of the problem by replicating the sets of physical nodes for each period of a finite and discrete time horizon and connecting the time copies of the nodes with arcs according to the configuration of the delay time on the arcs (for instance, ([Crainic and Sgalambro, 2014](#))). If waiting on a node is permitted for a unit of flow (an evacuee in the humanitarian operations), additional *holdover arcs* are added between the  $i$ th and the  $(i + 1)$ th copy of the same physical node. In other words, evacuees can wait at the nodes for the next available time instant to be evacuated. A toy example (see figure 4.2, 4.3) is given to explain the concept of TEN.

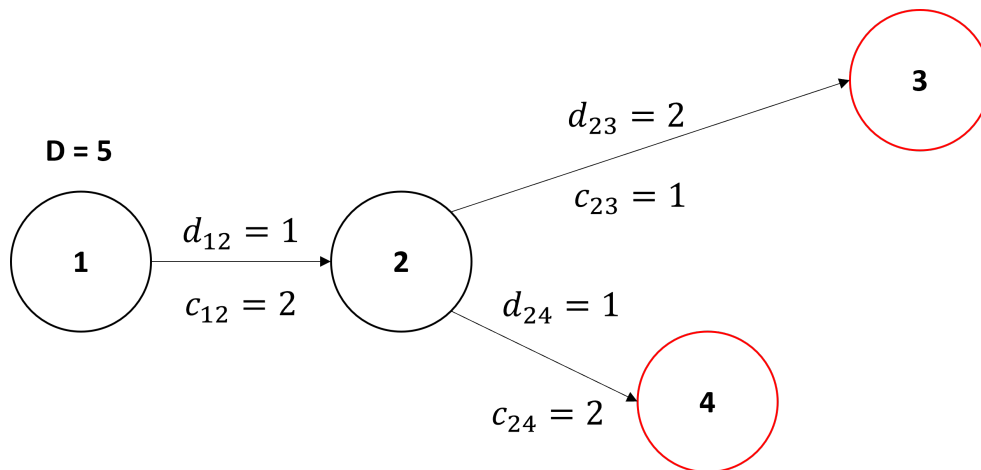


FIGURE 4.2: Dynamic digraph (example)

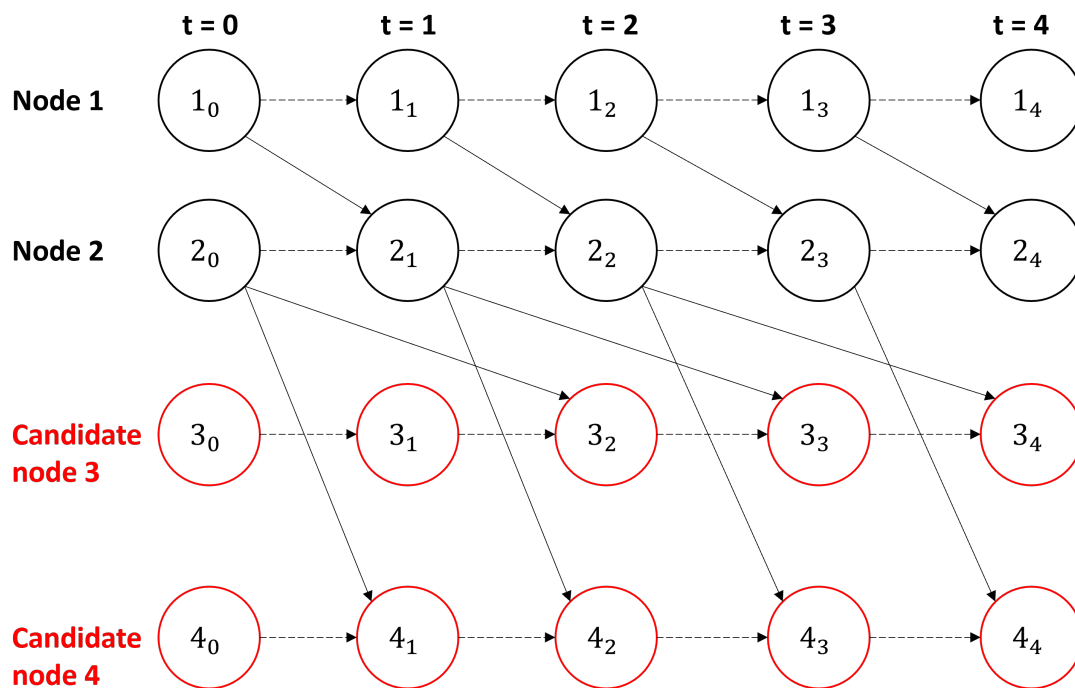


FIGURE 4.3: Time Expanded Network explained

This toy example shows how TEN works. As it is shown in the original dynamic digraph in figure 4.2, there are four physical nodes: node 1 to node 4, and there are three arcs: arc  $(1,2)$ ,  $(2,3)$ , and  $(2,4)$  where each arc has a delay time label and a capacity label. We then expanded this original dynamic digraph into the time-expanded network with the total time period is  $T = 4$ . Therefore, each physical node is duplicated four times with one physical node in each time instant. The holdover arcs are added between the same physical nodes with the one in the next time instant, showing that people can wait on the node until the next available time to move and the waiting time on the node can be traced and calculated by the holdover arcs on each time instant.

The physical arcs between each physical node are also introduced in the time-expanded network based on the delay time label. For example, in the original physical network, the delay time between node 1 and node 2 is one. Therefore, the arc  $(1_1, 2_2)$  indicates that it takes one unit of time to travel from node 1 to node 2. Similarly, the delay time between node 2 and node 3 is two, which is represented by the arc  $(2_1, 3_3)$ , showing that it takes two time instants to traverse that arc.

### 4.4.3 Key problems within the dynamic network flow problem

Many problems belong to the domain of dynamic network flows, including the Maximal Dynamic Flow problem constructed from static flows (Ford and Fulkerson, 1958), the Universal Maximal Flow problem generalised from the Ford-Fulkerson dynamic network where the capacity and transit time of arcs vary over time (Gale, 1959), the Quickest Transshipment Problem dealing with transshipping demand from multiple sources to multiple sinks in the quickest manner (Hoppe and Tardos, 2000), the Extended Universal Maximum Flow problem with time-varying vertex capacities (Cai et al., 2001), and the Earliest Arrival Flow problem maximising the population arriving at the destination at every point in time (Schmidt and Skutella, 2014). In this section, these key problems in the domain of dynamic network flows will be introduced in detail.

#### The maximal dynamic flow problem

The maximal dynamic flow problem introduced by Ford and Fulkerson (1958) builds the foundation of dynamic network flows, aiming to find the maximum amount of flows to be transferred from the source node to the destination within a specified time period. In order to solve this problem, they use the method called temporally repeated flow, which starts with the initial static network flow solution and then incrementally modifies the flow to handle changes to find the maximum flow as long as time is sufficient for all the flow to get to the destination node.

In the maximal dynamic flow problem, each arc  $(i, j)$  has two labels  $c_{ij}$  and  $d_{ij}$ , and each of these is a positive integer. The total time horizon  $T$  is also given, indicating the total number of time instants in the problem. Based on the



discussion above, the notation and the mathematical model of the maximum dynamic flow problem are shown as follows:

$$\max v(T) \quad (4.9)$$

s.t.

$$\sum_{j \in N} \sum_{t \in T} [x_{0j}(t) - x_{j0}(t - d_{j0})] - v(T) = 0 \quad (4.10)$$

$$\sum_{j \in N} [x_{ij}(t) - x_{ji}(t - d_{ji})] = 0 \quad i \in (1, n - 1), t \in T \quad (4.11)$$

$$\sum_{j \in (0, n-1)} \sum_{t \in T} [x_{nj}(t) - x_{jn}(t - d_{jn})] + v(T) = 0 \quad (4.12)$$

$$0 \leq x_{ij}(t) \leq c_{ij} \quad (i, j) \in A \quad (4.13)$$

The objective function of the maximum dynamic flow problem 4.9 is to maximise the total amount of flow within the given time period in the network and the  $T$  means over the entire time horizon. The constraints 4.10 to 4.12 are used to compute the total amount of flow in order to feed the objective function, which are also the flow conservation constraints. The last constraint 4.13 is used to set the bound of the largest amount of flow that can enter each arc at each time instant.

### The universal maximal dynamic flow and time-varying universal maximum flow problem

The universal maximal dynamic flow problem develops the original maximal flow problem by including the time-varying features where the capacities of arcs and nodes vary over time. Recall that in the maximum dynamic network flow problem, the capacity of each arc stays the same at each time instant, and there is no capacity constraint used to bound the largest amount of flow that can enter each node. Differently, according to the universal maximal dynamic flow concepts, Gale (1959) introduce a capacity constraint for each node, indicating the largest amount of flow can wait at the node. Meanwhile, both capacities of the nodes and arcs are time-varying, which means different capacities were given to each arc and node at different time instants. The aim of the universal maximal dynamic flow problem is to find the optimal maximum flow within a specific time period  $T$  in a *time-varying network*.

These specific features of universal maximal dynamic flow are helpful in some real-world cases. The research of [Cai et al. \(2001\)](#) provides a wonderful example to explain this point. For instance, in a transportation network problem, the source node represents a manufacturing company, and the sink node is the customers to whom the manufacturers of the products will be sent. Furthermore, the manufacturing company has an extensive product line, producing different products at different seasons, and they must deliver those products to the destination within a specific time window. In this case, the universal maximal dynamic flow is applicable in determining the maximum amount of the products at various periods of time, and these can be used in planning the routes and vehicles for the delivery of the products.

Based on the original universal maximal dynamic flows, [Cai et al. \(2001\)](#) present three variations of the universal maximum flow problem in time-varying networks where the delay time/transit time, capacity of each arc, and the capacity of each node are time-varying. Three variations involve examining scenarios with zero, arbitrary, and bounded waiting times at nodes.

### **The minimum cost dynamic network flow problem**

The minimum cost dynamic network flow problem is the dynamic version of the original minimum cost flow problem, aiming to find optimal flow in the network so that the total cost is minimised within a certain period. Unlike other dynamic network flow problems, despite the capacity and transit time labels on each arc, the minimum cost dynamic network flow problem also introduces a cost label on each arc. In all, the minimum cost dynamic network flow problem is defined in the research of [Klinz and Woeginger \(2004\)](#) as: in a dynamic network  $G = (N, A, T)$ , there is one source node  $s$  and a sink node  $t$ . On each arc  $(i, j)$  in the dynamic network, there are three labels:  $c_{ij}$  as the capacity,  $u_{ij}$  as the costs, and  $d_{ij}$  as the delay time/transit times. The minimum cost dynamic network flow problem is to find the optimal flows from source node  $s$  to sink node  $t$  within the time limit  $T$ ; in such a way, the total costs incurred are minimised.

### **The quickest transshipment problem**

The quickest transshipment problem is also defined in the dynamic network, where each arc has two labels: capacity and delay time. The difference between the quickest transshipment problem and problems discussed previously

is that there are multiple source and sink nodes where each source node has a certain amount of supply, and each sink node has a certain amount of demand.

The quickest transshipment problem aims to find the quickest way to transfer flows (the supply from each source node) to the sink nodes, in order to meet the demand on each sink node by taking into account the capacity constraint and the delay time constraints on each arc within a certain amount of time period.

### **The quickest flow problem**

The quickest flow problem is a particular variant of the quickest transshipment Flow problem, requiring sending the flow from a single source to a single sink such that the entire evacuation process is as quick as possible or, equivalently, the *makespan* of the process is minimised (Liu and Jaillet, 2015) within a given time period. It is also defined in the dynamic digraph  $G = (N, A, T)$ , consisting of a set of nodes, arcs, and a total time period and capacity and delay time labels on each arc.

Due to the nature of the quickest flow problem, it has wide applications in many areas, such as transportation, evacuation and humanitarian operations. In particular, in the context of humanitarian operations, the makespan depends on the time instant when the last evacuee reaches the safety facility and corresponds to the maximum amount of time needed to complete the whole evacuation process. This specific feature makes the Quickest Flow problem widely used in modelling evacuation processes, with prominent applications in transportation and humanitarian operations.

### **The earliest arrival flow problem**

The earliest arrival flow problem (also called the Earliest arrival time problem) is also a widely studied optimisation problem in network flows. It deals with the determination of the optimal flow of goods or resources in the dynamic network with the objective of maximising the number of people arriving at each node at each time instant. Unlike what has been discussed in the quickest flow problem, the objective function is to minimise the total makespan to achieve the quickest flow process in the network. Different from the maximum dynamic flow problem, which is trying to maximise the total flow within a given time period, the earliest arrival flow problem aims to maximise the amount of flow arriving at the destination at each time instant.

#### 4.4.4 Critical analysis of dynamic network flow problems

Some surveys on dynamic network flows/flows over time can be found in [Aronson \(1989\)](#), [Skutella \(2009\)](#), [Higashikawa and Katoh \(2019\)](#). Overall, dynamic network flow problems have distinct advantages over static network flow problems by introducing the capacity label and delay time label on each arc to overcome the limitations of the static network flow problems, enabling more precise and dynamic monitoring of how resources/products/items flow on the network. These lead to more accurate and practical solutions for modelling real-world cases and solving real-world problems where dynamic network flow problems can better reflect the scenarios due to uncertainties or other accidents, especially when used in humanitarian operations.

Additionally, dynamic network flow problems can allocate resources and direct flows in real-time, increasing effectiveness and efficiency by enabling the network to react quickly to changes or disturbances. Furthermore, the dynamic network flow problems can facilitate real-time decisions, which is crucial in situations that are time-sensitive, such as evacuation, humanitarian operations, and emergency management. Finally, the ability of the dynamic network flows to better monitor the flow process also supports a much easier decision-making process, increasing the resilience of the whole system.

### 4.5 Network flow problems with application in evacuation

Evacuation problems deal with moving people from danger to safety points. An evacuation network is used to represent and model the evacuation process, where evacuees who need to be transhipped (the demand) are usually modelled as located at nodes in the network. In contrast, streets, roads, or any connections are modelled using the arcs linking nodes. The models used to solve evacuation problems can be grouped into three models: the macroscopic model, the microscopic model and the mesoscopic model [Bayram \(2016\)](#). The macroscopic model treats flow together to determine the time needed to evacuate the population, often used in large-scale evacuation problems. On the

contrary, microscopic models treat flow individually in a more detailed manner. The mesoscopic models, as their name suggests, combine the characteristics of macroscopic and microscopic models, and they are formed by disaggregating more significant segments of macroscopic models into smaller segments and have properties of both macroscopic and microscopic models.

In the literature, dynamic network flows are among the most appropriate tools compared with static network flows in terms of modelling the evacuation problems from a macroscopic perspective as they consider the time element in modelling, allowing for more precise monitoring and control throughout the entire flow process over a specific time period. In recent years, many pieces of research in evacuation design and planning use macroscopic models, which are presented as dynamic network flow models (see [Chalmet et al. \(1982\)](#), [Mamada et al. \(2004\)](#), [Hamacher et al. \(2013\)](#), [Schmidt and Skutella \(2014\)](#), [Melchiori and Sgalambro \(2015\)](#), [Shin et al. \(2019\)](#)) for evacuation problems. Thanks to modelling features such as arc delay times and arc inflow capacities, flows-over-time models enable more accurate and reliable modelling of any step-by-step evolution in the flow allocation over the time horizon units, compared to static network flow, which is more appropriate to represent flows at steady-state. This explains the reasons why dynamic flows are increasingly adopted for evacuation modelling and also clarifies the importance of filling the identified gap in the literature by adopting a flows-over-time dynamic approach while combining evacuation modelling and facility location problems to guarantee better performance and enable a more accurate decision-making process.

Due to its unique property of considering time elements, dynamic network flows are frequently used in the literature to model and address the evacuation design and planning challenge. [Lim et al. \(2012\)](#) introduced their capacitated network flow problem on the time-expanded network, which expands the static network over the time horizon and applies them to three case studies from the map of the greater Houston area. The results show that the dynamic network flow model can produce accurate results and better support the decision-making process. [Shin et al. \(2019\)](#) introduced four optimisation models to find the optimal routes and entrances for evacuation and entrance planning. [Pyakurel et al. \(2022\)](#) further extended the abstract network with intermediate storage to support evacuation planning, where the abstract network is an expansion of the classical network that has a set of elements and

a linearly ordered subset of those elements called paths that meet the switching property. In Melchiori and Sgalambro (2018, 2020), the  $k$ -splittable flow variant of the Quickest Flow problem is studied, imposing constraints on the maximum number of supporting paths and proposing matheuristics and exact approaches, respectively. A dynamic flow model was developed by Tirado et al. (2014) for solving the aid distribution problem in the disaster while considering multiple criteria, which consider the quantity to be distributed, time, cost and equity-related goals. In Shin and Moon (2023), the crucial advantages of the dynamic network flow model have been emphasised. They developed a robust evacuation planning problem using dynamic network flow modelling to find the optimal routes for evacuees by considering the uncertainty of further collapse in the building. The case studies they use are based on the mega-mall Central City located in Seoul, and the results confirm that dynamic network flows can better model real-world cases and get reliable results.

Dynamic network flows can overall, better and more correctly reflect these changes in real-time than static network flows by considering all of the dynamics involved in the evacuation process. Dynamic network flow can also be combined with other problems, and more discussions will be presented in the next chapter.

## 4.6 Modelling techniques: dummy nodes and arcs

In terms of modelling the evacuation problem in the network, dummy sink nodes and dummy arcs are widely used in modelling and solving emergency evacuation models. Han et al. (2006) proposed a one-destination method to find the optimal solutions for the emergency evacuation model. They introduced dummy arcs that connect each original real-world destination to a final single common node called the dummy destination point by assuming that all the dummy links and the final dummy destination point have infinite capacity and zero delay time. In this case, they transferred the original two-step decision-making process for the evacuation assignment problem with  $m$  sources and  $n$  sinks ( $m$ -to- $n$  assignment) into a traffic-assignment problem with  $m$  sources and one destination ( $m$ -to-1 assignment). This proposed model substantially reduces the number of flow conservation constraints required to find the global optimisation, as the reduction in flow conservation

constraints will not change the mathematical properties. Instead, it will arrive at the optimal solution more quickly and efficiently. Therefore, the one-destination method in modelling has been proven to improve the efficiency of evacuation planning and operations. [Lujak and Giordani \(2018\)](#) also used the dummy/fictitious sink and the dummy arcs in the modelling to find the shortest agile evacuation routes. Their work imposed a fictitious sink node alongside all the destination nodes linked by fictitious arcs with infinite capacity, and it is easier and time-saving to find the agile routes towards the safe exits, which considerably reduces the model's complexity by reducing the flow conservation constraints. Additionally, the use of dummy sink nodes and arcs in the dynamic network flows, especially in the quickest flow problem, is instrumental in monitoring and controlling the makespan. The alternative modelling techniques described in the literature differ from this approach, as they do not consider makespan but instead focus on optimising the sum of all the arc flows. More examples of using dummy arcs as a modelling tool for evacuation processes can be found in [Faturechi et al. \(2018\)](#) and [Pyakurel et al. \(2022\)](#).

## 4.7 Conclusions

In this chapter, two fundamental domains of network flow theory have been introduced and discussed: static network flows and dynamic network flows. The static network flow problems deal with the problem where the parameters are not time-varying, and there are only fixed cost and capacity labels on the arc in the static network. Some classical problems belonging to the domain of static network flows are discussed and evacuated, such as the maximum flow problem, minimum cost flow problem, and shortest path problem.

On the other hand, the dynamic network flows capture the dynamic features into modelling by introducing two labels: both capacity and delay time labels on each arc. The main problems in the dynamic network flows have also been presented: the maximal dynamic flow problem, universal maximal dynamic flow problem, minimum cost dynamic network flow problem, quickest transshipment problem, quickest flow problem and earliest arrival flow problem.

Furthermore, a detailed discussion of the applications of network flow problems has also been presented. From the discussion, it is clear that dynamic network flows provide more real-time and accurate monitoring of flow traversing

the network, which allows for a more effective decision-making process, adaptive resource allocation, and improved network resilience. These all make dynamic network flow problems applicable in many real-world scenarios. The next chapter will introduce the combination of facility location and network flow problems to build the foundations for this research.



## Chapter 5

# Combinations of Facility Location and Network Flow Problems

### 5.1 Introduction

The previous chapters have discussed the concepts and main problem categories of facility location problems and network flow problems. This chapter explores the combinations that result from the intersection of these two domains, building on the concepts and problem categories addressed in chapters 3 and 4, in order to form the theoretical background of the research rationale and aim introduced in chapter 1. First, a discussion around the motivations underlying the combination of facility location problems and network flow problems is presented by investigating the advantages of such combinations, especially stating the reasons behind combining facility location problems with evacuation problems to support the introduction of the new problem. Then, this chapter further introduces and analyses the existing combinations of these two problems by thoroughly evaluating the existing combinations in the literature. Finally, the remaining gaps in the existing literature are identified based on the previous comprehensive analysis of the literature. By doing so, this chapter aims to provide the motivations for filling these gaps, which initiates the necessity of this research and the introduction of a new QELP.

### 5.2 Motivations behind combining facility location and network flow problems

The facility location problem aims to find the optimal locations of facilities, and the network flow problem is to find the optimal flow movement within

the network such that their objective functions are achieved. Combining these two problems means considering the location decisions when designing the flow network, which is important in various domains, from disaster management and humanitarian operations to daily transportation network designs and urban planning.

Coupling facility location decisions with network flows into evacuation modelling is critical in disaster evacuation design and planning. By considering both factors together, we can make decisions on the locations of facilities based on the impact that they have on the dynamics of evacuation. In particular, considering locations for evacuation facilities, such as shelters, medical facilities, and warehouses, in evacuation planning can make it easier to find the optimal locations for those facilities which match the overall strategy of the evacuation process. Therefore, it can better prepare for potential disasters, increase the mobility of the evacuation, and ensure a more efficient relief and rescue process.

In addition, when dealing with these two problems separately, it ignores the interactive effects raising in the evacuation process. The consideration of additional elements that affect the effectiveness of the evacuation is made possible by merging facility location decisions with evacuation difficulties, such as transportation networks, traffic conditions, road capacity, or other limitations in the resource. These elements will influence the efficiency of the evacuation process. Considering the location of the facility along with these critical issues in evacuation network flows can better address key bottlenecks during the evacuation and improve the efficiency of the evacuation process.

Furthermore, combining the facility location and network flow problems takes into account the interdependencies between these two decisions, making it possible to plan ahead and get ready for various situations, especially in humanitarian operations and emergency evacuations. Similarly, considering the interdependencies of facility location decisions and network flow decisions enables the integration of the decision support systems, enabling real-time monitoring, timely adaptations and better decisions.

To sum up, the combination of facility location problems and network flow problems is beneficial and can enhance the effectiveness and efficiency of the whole evacuation process. By optimising the locations of the facilities and considering the critical issues in evacuation network design and planning, such as transportation networks and road capacities, this integration supports the decision-making process comprehensively. It makes evacuation design and

planning more robust and resilient, reducing the time needed for the evacuation process and saving human lives.

### 5.3 Existing combinations of facility location and evacuation problems

It is important to study the problem, which combines the facility location and the network flow problems, as discussed in the previous section, where the network flow can provide many advantages that other evacuation problems do not. Even though many problems in the literature consider the facility location decisions and evacuation planning together, their network settings and modelling techniques differ from the network flows. They do not share the same advantages as network flow problems, especially dynamic network flows. For example, location-allocation problems and location-routing problems. Before discussing the existing combinations of facility location problems and network flow problems, we first introduce the existing combinations of facility location and evacuation problems in order to provide a comparison of the differences in the network settings.

[Coutinho-Rodrigues et al. \(2012\)](#) developed a multi-objective mix-integer linear programming model for a location-routing problem. In their model, they aim to find the specific evacuation plan for each evacuation location, which includes the identification of the plan, the specific plan associated with each evacuation, the primary evacuation path, and the secondary evacuation path, in order to optimise the six objective functions: the minimisation of total travel distance from population to shelter, the minimisation of total risk of primary path, the minimisation of total travel distance of using backup path, the minimisation of total risk at the shelters, the minimisation of the total time used for people to transfer from shelter to hospital, and the minimisation of the total number of shelters.

[Li et al. \(2012\)](#) introduced a stochastic bi-level approach for shelter location decisions by taking into account the impacts of location decisions on the behaviour of drivers in making choices for the path in order to capture the unpredictable character of hurricanes. The upper level of this model is a shelter location-allocation model where they try to find the optimal location for shelter sites and allocate the population to the shelters. The lower level of this model aims to find the dynamic user equilibrium by using the Stackelberg

leader-follower game to investigate the choice of drivers based on the shelter site they were allocated to.

**Hamacher et al. (2013)** introduced the FlowLoc problem, which tries to find the optimal location of facilities while finding the optimal flows in the network. They introduced a tool to predict and evaluate evacuation plans, the aim of which is to select arcs where new facilities can be located on a dynamic network in such a way as to minimise its impact on the increase in the time required for its evacuation, in order to locate the facilities on the edges while maximising flow and retaining as many source-sink paths as possible.

**Goerigk et al. (2014b)** introduced the Integrated Bus Evacuation Problem (IBEP) by considering the location decisions with the bus evacuation problem in order to minimise the total evacuation time. In particular, the bus evacuation problem deals with fixed location and assignment decisions. Therefore, their integrated bus evacuation problem is a location-allocation and bus scheduling problem.

**Bayram et al. (2015)** introduced a problem in finding the optimal locations of shelter sites and the optimal assignment of evacuees to the nearest shelters. In their model, they try to find the location of  $p$  shelters to minimise the total evacuation time by achieving the system optimum. In particular, they use BPR to compute the travel time, where the travel time related to the practical capacity, base travel time, and assigned volume can travel on the link. There are many differences between this problem and network flows.

**Kilci et al. (2015)** developed a mixed-integer linear programming model for a location-allocation problem, aiming to maximise the minimum weight of open shelter areas while determining the optimal location for shelter sites and assigning populations to each open shelter. By doing so, they control the utilisation of open shelter areas. This problem is a particular example of a location-allocation problem where the decision variables here are all binary variables to determine the location of shelters and whether the affected population is assigned to the shelter or not.

**Higashikawa et al. (2015)** also proposed a model considering the  $k$ -sink location problem in a dynamic path network to find the optimal locations for the  $k$ -sink on the path to ensure that all evacuees can arrive at one sink while minimising the maximum evacuation time. In particular, the dynamic path network means that those networks have “undirected paths with positive

edge lengths", "uniform edge capacity", and "positive vertex supplies" (Higashikawa et al., 2015). Based on this, they further developed the minimax regret 1-median problem in the dynamic path network such that the maximum regret for all scenarios is minimised (Higashikawa et al., 2018).

Based on Higashikawa et al. (2018), Luo et al. (2021) introduced the k-sink location problem in the path network, aiming to minimise the combination of the maximum completion time and the total completion time. As discussed in the previous paragraph, their model is based on a path network where the capacity of the edge is uniform, and there is no delay time on the arc, no waiting on the node, and all the supplies from each supply node should be evacuated to the same sink node using the same routes.

Gama et al. (2016) introduced a multi-period capacitated location-allocation model for the flood in order to minimise the total travelling time across the given time period between demand nodes and shelters. In their model, the travelling time between each node and shelter is time-varying, and the availability of the shelter is also time-varying. In addition, their model considers that people do not evacuate simultaneously, and people can only be evacuated to the shelter if they are assigned to it.

Shahparvari et al. (2016) introduced a bi-objective integer-programming model for late evacuation in the context of bushfires, aiming to maximise the number of persons who are evacuated using the most reliable routes to the closest activated shelters within a given clearance time and minimise the allocation of used resources at the same time. This problem is a location-allocation-routing problem where they find the optimal location of shelters and allocate the evacuees to the nearest active shelters while considering the different vehicle types to get to the shelters.

Pyakurel and Dhamala (2017) proposed a mathematical model for the continuous dynamic contraflow problem and presented different algorithms to solve the problems in a continuous-time model. Bayram and Yaman (2018a) developed a scenario-based two-stage stochastic evacuation planning model to find the optimal locations for  $p$  shelter sites and the evacuation route assignments such that the expected total evacuation time is minimised. They also consider the uncertainties in evacuation demand and the disruption in the road and shelters.

Similarly, Dhungana and Dhamala (2019) introduced three problems based on the research of Hamacher et al. (2013): the maximum FlowLoc problem

over continuous time, the maximum static ContraFlowLoc problem, and the maximum dynamic ContraFlowLoc problems, which aim to maximise the total flow within a given time period. They claim that these problems are the combinations of the facility location problems with the maximum flow problem and the maximum dynamic flow problem. They try to find the optimal locations of facilities on the arc, and they modelled the time setting here as continuous. They include contraflow (lane reversal strategy), where two-way capacities of arcs are added, and both directions are allowed with symmetric capacity and transit times.

Nath et al. (2020) introduced the Quickest ContraFlowLoc problem to find the optimal locations for the introduction of facilities on arcs, considering two-way flows such that the quickest time is minimised. None of these researches is modelled using the general dynamic network flows, which leaves the gap of combining location problem with the general dynamic network flow problem.

Jiang et al. (2023) introduced a reliable location and routing model, including the backup service plans to find the location of pickup stations and the route for the service vehicles to pick up evacuees at these stations to minimise the total cost. In this case, this problem belongs to the location-allocation and vehicle-routing problems.

## **5.4 Existing combinations of facility location and network flow problems**

Recall that the facility location problem deals with finding the optimal locations for facilities. The network flow problem, which has two domains: static network flows and dynamic network flows, deals with the optimal flow in the network to optimise the objective functions, subject to the capacity constraint on each arc. In particular, the population is usually modelled as the nodes, and the links (i.e., roads and rails) are modelled as links. The critical difference between static network flows, and dynamic network flows is that dynamic network flows consider the time elements in modelling by introducing the delay time/ transit time on each arc along with the capacity so that the dynamic flow process can be clearly monitored.

The existing combinations of network flow problem and facility location problem models for the evacuation process in the literature are developed based on the static version of network flows (static network flow problems) and a

dynamic network flow with various problem settings. Heßler and Hamacher (2016) combined the sink location problem with the static network problem to determine the optimal location of shelters such that the opening costs of shelters are minimised. They also considered the capacity for both edge and shelter in the modelling, and the edges are undirected.

Farahani et al. (2018) proposed a mixed-integer linear programming model that combines locational decisions with the static maximum flow problem in order to find one or more locations in the capacitated network while maximising the number of dispatched people. Similarly, Liang et al. (2019) developed a risk-averse shelter location and evacuation routing assignment problem by taking into account the uncertainty in demand, in order to minimise the total evacuation time spent by evacuees in the network. They also consider private evacuation and traffic flow in the network, and each arc has a capacity label. In addition, they consider the evacuation time as an overall formulation related to the flow.

A static network flow model is also applied by (Jin et al., 2021) to address the underground emergency shelter location and pedestrian evacuation routing problem, aiming to maximise total satisfied evacuation demand and to minimise the evacuation distance.

Moreover, Esposito Amideo et al. (2021) introduced the Scenario-Indexed Shelter Location and Evacuation Routing (SISLER) model, combining the bus evacuation problem, the capacitated facility location problem, and the multicommodity flow problem to find the optimal locations for shelters and the best routes for both self-evacuation and the bus evacuation, in order to minimise the expected bus-based evacuation maximum completion time. The SISLER model is a static model. In addition, this problem uses the location-allocation problem for self-evacuation by imposing a travelling time threshold to tackle the evacuation time so that the objective is to minimise the completion time for bus evacuation.

In all, the problems discussed above all consider the location decisions with static network flow problems. While there are a few problems that combine the location decisions with the dynamic network flow problems, the problem settings in terms of network structures, modelling the delay times, and evacuation process are different from the typical modelling settings in the dynamic network flow problems.

Goerigk et al. (2014a) introduced a Comprehensive Evacuation Problem (CEP),

which combines the shelter location problem with private traffic evacuation and also with public traffic evacuation in order to minimise the time, the number of shelters, and the risks. In particular, the private traffic evacuation is modelled as a dynamic network flow, and the public traffic evacuation is modelled as a dynamic multicommodity network flow.

Similarly, [Idoudi et al. \(2022\)](#) developed an agent-based dynamic framework to find the optimal locations for the shelters and the evacuation path by combining the shelter location allocation problem and dynamic traffic simulation such that the total evacuation time is minimised. In this research, they model the dynamics in the evacuation in a way where the travel time from the source node to the sink node is time-dependent. In addition, there are no capacity and delay time labels on each arc, where this work was not modelled as a dynamic network flow problem but as a dynamic traffic assignment problem.

To sum up, the nature of the disaster is uncertain and unpredictable, where effective and efficient evacuation design and planning are crucial. In this case, dynamic network flows, also called flows over time, represent one of the most effective and suitable approaches to deal with evacuation problems because they achieve better approximation results within an acceptable computational time. Moreover, the discretisation of the model better models the real-life solutions.

## 5.5 Gaps in the literature

By exploring the literature on applications of OR to disaster operation management and humanitarian operations, dynamic network flows are often considered more appropriate in modelling evacuation in the context of humanitarian operations than static network flows because they can capture the dynamics of the network in evacuation. Finding the optimal location of temporary emergency facilities based on the impacts that will have on the dynamics of evacuation is crucial, where the existing combinations of facility location problems and static network flow fail to achieve that. Only a few existing studies on the combination of location decision and network flows are in the literature; specifically, they only consider facility location problems with static network flows, which are based on a static approximation of the evacuation while locating the facilities or a different network modelling to capture the dynamics in the network. Thanks to modelling features such as arc delay times and arc inflow capacities, flows-over-time models enable more accurate and



reliable modelling of any step-by-step evolution in the flow allocation over the time horizon units, compared to static network flow, which is more appropriate to represent flows at steady-state. This explains why dynamic flows are increasingly adopted for evacuation modelling and also clarifies the importance of filling the identified gap in the literature by adopting a flows-over-time dynamic approach while combining evacuation modelling and facility location problems to guarantee better performance and enable a more accurate decision-making process.

In particular, among dynamic network flow problems, the quickest flow problem seeks to minimise the makespan and hence focuses on finding the quickest way to move flows, such as people or products, through the network towards a safe destination. It is often used to model and optimise the evacuation process, ensuring flows can be moved to safety as quickly and efficiently as possible. Compared with static network flow approaches, the quickest flow problem introduces both capacities and delay time on each arc in the network, which can be adapted to the dynamic changes during the evacuation process in real-time, such as road congestion. Furthermore, the evacuation process is time-sensitive and moving evacuees to the shelters or places of safety as quickly as possible is key. In short, the quickest flow problem deals with finding the quickest way to move people to safety, and this is more appropriate in modelling the evacuation process.

By thoroughly reviewing the existing literature in the field, it appears that only the pioneering work in [Goerigk et al. \(2014a\)](#) proposed a first attempt to combine facility location problems and dynamic network flow problems to model evacuation by integrated public (bus) and private transport, whilst looking at the minimisation of evacuation makespan and the number of opened shelters. Given the high potential relevance of the topic for practical purposes, this motivates the need to further expand the studies in this direction by exploring ways combined quickest flows and facility location could be actually utilised for delivering a principled and effective humanitarian response. This work focuses on shelter location decisions from a decision support standpoint, aiming at securing real-world decision support systems to assist humanitarian stakeholders in making decisions most effectively.

In order to contribute to bridging the gap in the literature, we introduce the Quickest Evacuation Location problem (QELP), combining the quickest flows and facility location problem while focusing on private, in particular, the pedestrian evacuation, to better capture the dynamics of the evacuation process,

ensuring a more efficient evacuation process in humanitarian operations and enhancing evacuation networks design. This is obtained by defining and combining a range of operational-oriented optimisation goals, namely, the evacuation time makespan, the expected budget to set up and operate the set of selected shelters, and the robustness and quality of the shelter network configuration, with a view to boosting the usability of the arising modelling tools for saving lives and minimising the impact of disasters. Notably, unlike public evacuation, private evacuation refers to individuals or groups evacuating a particular, typically private area like a house using their vehicles rather than waiting for public transport to be evacuated. Pedestrian evacuation is a specific type of private evacuation where people evacuate themselves by foot, typically on streets and walkways. This work focuses on pedestrian evacuation, which can be modified into private evacuation, allowing for thorough and explicit configuration of evacuees without any restrictions because when a disaster occurs, people usually self-evacuate to a safe place, and in some cases, public transportation cannot reach disaster areas.

## **5.6 Conclusions**

This chapter is devoted to discussing the existing combinations of facility location problems and network flow problems. First, the motivations behind combining facility location and network flow problems have been provided by stating the advantages brought by the combinations, which can support the decision-making process and enhance the effectiveness and efficiency of the whole evacuation process. Then, the existing combinations of facility location problems with various evacuation problems have been introduced, and the differences between these problems and the normal network flow problems have also been evaluated by stating the different modelling techniques used in these works compared with the network flow theories.

In general, many problems in the literature consider the location decisions in evacuation planning, such as location-allocation problems, location-routing problems, and FlowLoc problems. However, the modelling tools used in these problems differ from those in the standard network flow problems. After that, the existing combinations of facility location problems and static network flow problems are discussed, which fail to present dynamic monitoring in the whole process, thus motivating the need for further research in this direction, as presented in the following chapters of this thesis.

Furthermore, the existing combinations of facility location problems and dynamic network flow problems are evaluated and discussed, which leads to the need to bridge the gap in the literature to have a better problem setting in order to support the decision-making process by taking into account both makespan, budget, and the potential risk associated with the shelters. For example, although the CEP of [Goerigk et al. \(2014a\)](#) combines the facility location problem with the dynamic network flow problem to limit the total evacuation time, this is done by focusing on the overall number of located facilities, without considering the actual resource effort needed to set-up and operate the selected shelters, hence not capturing some of the main drivers of the decision-making process in evacuation design and planning. Furthermore, the quality of the design is assessed by considering the risk associated with the evacuation flows instead of evaluating the robustness of the shelter network configuration against potential risks and disruptions. Similarly [Goerigk et al. \(2014b\)](#) considers the location decisions with the maximum dynamic flow problems, the assumption of undirected edges differs from the directed graph features in the normal dynamic network flow problems. Also, it does not reflect the real-world cases where an undirected graph may cause severe congestion. These all lead to the gap where combining the facility location problem with the general dynamic network flows is necessary and important, particularly the quickest flow problem, to better support the decision-making process. More explanations of the difference between QELP and the CEP will be shown in the following chapters (chapters 6 and 7).

## Chapter 6

# Introducing the Quickest Evacuation Location problem (QELP)

### 6.1 Introduction

In the previous chapter, a comprehensive analysis of existing literature is conducted and unveils the gaps remaining that need to be addressed. In particular, it emphasises the need and importance of bridging the gap by combining the facility location problem and the quickest flow problem using the modelling tool of a general dynamic network flow digraph.

This chapter aims to provide an introduction to the Quickest Evacuation Location Problem (QELP) by first stating the research aim and questions that guide the development of the QELP model. Then, the research methodology and research philosophy used in this work are thoroughly discussed, providing solid support for the development of QELP and its model. The QELP is fully explained in this chapter, focusing on its conceptual foundations and the problem description. By doing so, the necessity of introducing QELP can be carefully explained.

### 6.2 Research aims and questions

#### 6.2.1 Research aims

This work aims to bridge the gap in the literature by introducing the QELP, a novel optimisation problem combining the discrete Facility Location problem with the Quickest Flow problem in the context of humanitarian operations.

The QELP can be defined on a standard dynamic network as the location problem of selecting one or more destinations from a set of candidate nodes for an evacuation problem, given a set of source nodes with associated demand. The principal goal of any evacuation planning process is to minimise the overall evacuation time, also referred to as *makespan*. The purpose of QELP is to take into account the dynamics within the evacuation process into the location decisions of facilities in the context of humanitarian operations, to better monitor and control the evacuation process and to select the optimal locations for shelters. As a result, a smooth and effective evacuation process can be achieved, which in turn, reduces the economic losses and human suffering.

### 6.2.2 Research questions

Based on the research aim discussed above, research questions cover the critical elements in formulating an effective mathematical model to assist the decision-making process for a smooth and efficient evacuation design and planning for QELP and address the gaps identified from the critical literature review by combining the shelter location problem with the quickest flow problem. The research questions are shown as follows:

- **RQ1:** What are the most appropriate objective functions to be modelled and adopted? What are the benefits of a multi-objective model?
- **RQ2:** What are the key specific modelling features that should be considered when formulating the optimisation model for QELP? Besides objective functions in **RQ1**, What are the constraints, parameters, and decision variables?
- **RQ3:** How to solve the QELP optimisation model in **RQ2** in such a way that it will generate good solutions in a reasonable computational time?

These are the research aims and research questions embedded in the development of the QELP model. Detailed answers will be discussed in the last section of this chapter to show the conceptual foundations of QELP by stating the problem it will tackle, the objective functions, constraints, and the parameters of its mathematical model.

Notably, the research aim and questions align with the research objectives introduced in chapter 1 where the introduction of the QELP as the research aim fills the gap in the existing literature based on the critical literature review of shelter location and evacuation modelling in humanitarian operations. Those

critical modelling features of QELP covered by the research questions fulfil the research objectives of developing a novel optimisation model to address the operational needs in the disaster contexts and to improve the effectiveness of disaster relief, particularly in enhancing the shelter location in evacuation design and planning. Moreover, the answers to **RQ 1-3** accomplish the research objectives mentioned in chapter 1 to develop a tailored mathematical model for QELP and an efficient solution method to increase the potential support to prospective decision-makers.

## 6.3 Research methodology and research philosophy

This study falls into the discipline of Operational/Operations Research (OR). According to The Institute for Operations Research and the Management Sciences (INFORMS, 2023), Operational/Operations Research (OR) is defined as "the scientific process of transforming data into insights to making better decisions". It is a task involving data analysis and a decision-making process where OR techniques are employed to find the best (optimal or near-optimal) solutions to the decision-making problems. The Operational Research Society (The Operational Research Society, 2023) defines OR as "a scientific approach to the solution of problems in the management of complex systems that enables decision makers to make better decisions". These two highly-recognised definitions confirm that OR can support the decision-making process by providing optimal and better decisions based on complex situations, involving lots of data analysis and other OR techniques.

### 6.3.1 Science versus technology debate

In the existing literature, there has been a lively debate on the nature of OR. The debate concerns whether OR can be seen as science or technology. Some early OR researchers and practitioners viewed OR as a science (Larnder, 1984), because OR was driven initially by the practices in World War 2, and the early OR practitioners were scientists who conducted research to support the war. The view that OR is a science is supported by Miser (1991). Based on the viewpoint of OR as science, many researchers argue that OR could be seen as coming under the Social Sciences (Checkland and Haynes, 1994) as OR solves real-life problems. Hindrichs (1953), for example, states that OR applies the same rigour and accurate techniques and approaches to solve real-world or social science problems; in short, it is a science. On the contrary, there are

increasing arguments claiming that OR is a technology because of its use of mathematical techniques to solve decision-making problems (Ormerod, 1996; Keys, 1998).

In the past several decades, OR has gained more attention and is widely applied in various situations in real-world cases rather than wartime. The main goal of OR has become more focused on the development of mathematical models or other OR techniques, which aims to help people solve issues in the best possible ways and make wise decisions. It has been applied in various fields such as business, industry, engineering, transportation, logistics, finance, medicine and many other sectors. Therefore, Monks (2016) argues that operational research is implementation science, which is beyond the debate of science vs technology. Similarly, Utley et al. (2022) suggest that operational research is better described as an approach to framing and addressing problems that involve working with critical issue owners and subject matter experts to apply scientific methods and modelling approaches in order to better understand and modify the complex operations of organisations such that specific goals are achieved.

### 6.3.2 Philosophy of this research

This sub-section discusses the philosophical assumptions of this research. Research philosophy is related to the nature of knowledge and the way to develop knowledge (Saunders et al., 2016).

There is limited literature about research philosophy relating to OR. Even though OR involves some philosophical considerations, there is no consensus on philosophy in this field. Hindrichs (1953) claims that realism is a suitable epistemology for OR, stating that the objects in the problem exist independent of the human mind. This kind of knowledge can be generalised into a broader context. Mingers (2000) builds on this viewpoint and argues that realism is not suitable for social science since intransitive objects cannot be investigated by social science. Instead, he proposed a new research philosophy for OR, which is called "critical realism (CR)". He argues that people experience things as sensations rather than experiencing them directly. He believes critical realism considers both natural and social problems simultaneously, and the reproductive methodology of critical realism can be used in the practical OR. Critical realism also goes against the idea of empiricism and positivism, with Mingers (2000) arguing that human beings have emotional feelings about the world, resulting in differences in the ways each person sees the world. Moreover, he

believed that scientists decide which kind of experimental activities they wish to conduct, leading to particular results from those activities. Papoulias (1984) suggested a similar opinion that the problems are eventually solved by methods which were initially intended to find the solution because the approach employed to solve the problem takes into account the nature of the problem. It considers the historical development of this type of problem. In other words, Papoulias (1984) believed that the solutions to the problems are influenced by the historical approaches to this type of problem and are decided by the nature of the problem.

Meredith (2001) provides an opposing viewpoint to Mingers (2000). Meredith (2001) argues that the attention to the research philosophy in OR should shift from realism to relativism. Meredith (2001) believes that OR focuses more on modelling rather than models, with the modelling highly dependent on the researchers. This is supported by Ulrich (2007) where the claim is that OR is not objective, existing independently of human knowledge. Ulrich (2007) raised the point that the research philosophy should shift towards critical pragmatism. Critical pragmatism is proposed based on the original pragmatism. Ulrich (2007) argues that the original pragmatism does not take into account the social aspects of knowledge, and it dismisses the influence of ethics on social problems. The critical pragmatism proposed by Ulrich (2007) considers the classical pragmatist concepts such as truth and inquiry jointly with ethics for social problems, which is central to the work of critical systems heuristic and boundary critiques.

Design research is another research paradigm for OR. Design research is defined as “a set of analytical techniques and perspectives (complementing the Positivist and Interpretive perspectives) for performing research in information systems.” (Kuechler and Vaishnavi, 2012). Kuechler and Vaishnavi (2012) suggest that design research is a paradigm used to solve problems that consistently analyses and evaluates the performance of designed artefacts and, in turn, improves the whole information system in the end. Table 6.1 compares the positivist, interpretive and design research paradigms.

The discussion of different paradigms from ontology (the researcher’s view of reality), epistemology (the researcher’s view of how to develop knowledge), methodology, and axiology (the researcher’s view of values in research) suggests that design research is the most suitable research paradigm for this study.

As mentioned in the research aim, the new QELP is introduced based on real-world situations in humanitarian operations and the gaps in the literature,



Basic Belief	Research Perspective		
	Positivist	Interpretive	Design
Ontology	A single reality; knowable, probabilistic	Multiple realities, socially constructed	Multiple, contextually situated alternative world-states. Socio-technologically enabled
Epistemology	Objective; dispassionate. Detached observer of truth	Subjective, i.e. values and knowledge emerge from the researcher-participant interaction.	<i>Knowing through making</i> : objectively constrained construction within a context. <u>Iterative circumscription</u> reveals meaning.
Methodology	Observation; quantitative, statistical	Participation; qualitative. Hermeneutical, dialectical.	Developmental. Measure artifactual impacts on the composite system.
<u>Axiology</u>	Truth: universal and beautiful; prediction	Understanding: situated and description	Control; creation; progress (i.e. improvement); understanding

FIGURE 6.1: Comparison of three main research paradigms (adapted from Kuechler and Vaishnavi (2012))

aiming to improve the efficiency and effectiveness of the evacuation process by taking into account the location decisions for shelters. Based on the QELP, a mathematical model is developed to support the decision-making process in evacuation in humanitarian operations by combining facility location problems for disasters and the quickest flow problem for evacuation network design and planning, which are related to management problems. In this study, OR is accepted as a social science methodology. The QELP involves a certain level of interpretation and intervention of the decision-maker based on the particular situation. This description matches the ontology and epistemology of design research as design research introduces novel artefacts to adapt to the multiple states of the world, and design research aims to develop knowledge through “an iterative process of construction and circumscription” (Manson, 2006).

## 6.4 Problem description of the Quickest Evacuation Location Problem

The QELP is developed as an optimisation tool to bridge the gap in the literature, combining the modelling features of the quickest flow problem and the discrete facility location problem to identify the optimal configuration of sinks on a network to favour a seamless and quick evacuation process. The scope of the problem falls into the field of evacuation planning and design. It is meant

to identify those nodes among a set of candidates that, if selected as sinks or destinations, would allow the quickest possible evacuation process.

A multi-objective mixed-integer mathematical model is introduced here to find the optimal locations for the temporary emergency facilities such as shelters, places of safety and assembly points such that the *makespan* is minimised. Namely, the time measured from the start of the evacuation process until the last evacuee reaches a shelter. As the installation of each shelter presents a required setup cost and the quality of the evacuation plan depends on an even distribution of evacuee flows over the network, we consider the required budget and the maximum load for each shelter as two further - yet conflicting - objectives to be considered for minimisation besides the makespan. In this case, the QELP model is formulated as a min-max multi-objective problem.

#### **6.4.1 Multiple objective functions: answering RQ1**

Drawing upon the characteristics of the inspiring application field, we consider here the presence of multiple, conflicting objectives for this problem, including, in addition to the *makespan*, the total budget required to set up the shelter facilities and the maximum load of evacuees that each located shelter will be facing throughout the evacuation process to make sure each activated shelter is balanced-loaded.

Considering concurrently these three objective functions together secures increased applicability of the QELP model as a decision support tool for managerial applications. Minimising the makespan means completing the evacuation process as quickly as possible. It is, therefore, the primary driver in humanitarian operations, as it enables quicker disaster relief and saves human suffering. In the literature, most of the research uses a constant value  $p$  to set the fixed number of facilities to tackle the presence of a limited budget. In particular, in the literature, it is clear that the constant number of facilities and the cost function are interchangeable. Therefore, in the analysis, I emphasise how the reality of humanitarian operations management is more complex, as the required budget encompasses fixed and variable costs to open and operate facilities, which are, in turn, impacted by features such as candidate capacities, among other factors. Including the expected budget for shelter location is therefore instrumental in counterbalancing the goal of minimising the makespan, and adopting a multi-objective approach where the budget is explicitly considered as a major goal will allow for identifying cost-effective network configurations while exploring Pareto-optimal options.

Furthermore, in the context of humanitarian operations, it is crucial to make sure the shelter can provide a stable and safe environment to the people in need, and one approach to achieve this is to have a balanced load of evacuees among shelters. A balanced load means the evacuees are more evenly directed towards the active shelters during the evacuation process. It largely contributes to diversifying the risk and ensuring the safety of people reaching and living in the shelters. In emergencies, there may be unexpected events that impact one shelter more severely than others. By balancing the number of evacuees directed to each activated shelter, the overall system becomes more resilient. Therefore, whilst exploring sets of Pareto-optimal solutions associated with certain ranges of budget and makespan, it becomes paramount to be able to prioritise those solutions which make use of shelters in a more distributed way as a decision support tool to enhance risk diversification, increase system robustness, and mitigate potential congestion during the evacuation process. Thus, the QELP is defined as an inherently multi-objective problem, which can secure a balanced level of risk in terms of the availability of the shelter and the robustness of the evacuation plan by considering all these three objective functions together.

#### 6.4.2 Modelling tools: answering RQ2

We model this problem using a dynamic digraph (dynamic flow network), which considers two sets of arc labels: the arc capacity ( $c_{ij}$ ) and the delay (or travel) time ( $d_{ij}$ ). The capacity of the arc is the maximum amount of flow that can enter into that arc on each time unit, whereas the delay time for each arc is the amount of time required by each unit of flow to travel from the tail to the head of the arc. Dynamic network flows can be modelled through discrete or continuous approaches based on the way that the time horizon is modelled. Here, we consider a discrete-time horizon ( $T$ ) with a finite set of time instants. A set of source nodes ( $N_D$ ), also referred to as demand nodes, are defined within the set of nodes ( $N$ ) in the network, each associated with a given demand, expressed as an integer number of units that need to be evacuated towards any of the activated shelters in the quickest possible time. A given set of candidate sinks (or shelters or destinations,  $N_c$ ) is also defined as a subset of network nodes, each associated with a capacity value ( $C_i$ ), representing the maximum number of flow units that can reach the destination during the time horizon. This is meant to reproduce the characteristics of each candidate sink in terms of size and limited capacity to shelter the evacuees

and is clearly a parameter which predominantly affects the quality of an evacuation process. In fact, while installing a few shelters with large capacities can lead, in principle, to contained makespan values, it could also produce evacuation plans where a substantial amount of evacuee flows are allocated to a concentrated area, all sharing the same connections and facilities, thus leading to increased risks of congestion, delays and collapses, and possibly to less robust evacuation plans in case of unforeseen disruptions. Recall that the value of a makespan when adopting any flow optimisation model needs to be considered as a lower bound for the evacuation process rather than a realistic value. Additionally, to resemble the characteristics of an actual planning process, we do not assume here that the number of sinks to be located needs to be fixed in advance as equal to any constant value  $p$ . Instead, we consider it more realistic to encompass an overall financial budget ( $B$ ) to be allocated for the implementation of the evacuation plan.

The arising evacuation network design and planning problem presents multiple conflicting goals to be analysed, and the role of an optimisation-based decision support system will be, therefore, giving a range of possible Pareto-optimal plans to the decision-maker, who will then evaluate the appropriate trade-off between different objective values to identify the most valuable plan to be implemented. Accordingly, the QELP is formulated as a multi-objective optimisation problem, aiming at minimising the makespan, the total budget required to set up the shelter facilities, and the maximum load of evacuees that each located shelter will be facing throughout the evacuation process.

Besides the objective functions, the full mathematical model will be introduced in the next chapter, which will provide a detailed explanation of the constraints introduced and the parameters modified from the real-world cases to form the QELP. In all, a detailed answer to **RQ2** will be presented in the next chapter.

### **6.4.3 An overview of solution methods: answering RQ3**

By exploring the Pareto-optimal solutions, decision-makers can gain a comprehensive understanding of the trade-offs between the three objectives, thereby enabling them to make informed decisions that best align with their priorities and optimise the utilisation of available resources. For example, organisations often have limited resources, and budget-cutting measures can negatively impact the time that people can arrive at the shelter and receive protection. Moreover, in some cases, a very limited increase in the budget can substantially

reduce the makespan. Compared to single-objective approaches, exploring Pareto-optimal solutions can provide decision-makers with more informed options and help them focus on a restricted number of high-quality options and make the best decisions based on awareness of available resources and an informed evaluation of political priorities. Therefore, a tailored Matheuristic approach is developed, exploiting linear relaxations and approximations of the original MIP model to identify high-quality solutions in reasonable computational times. In order to explore the Pareto Set efficiently, this is framed within a multi-objective scheme based on the Robust Augmented  $\varepsilon$ -constraint method (AUGMECON-R) (Nikas et al., 2020).

This is an overview of the solution method used in solving the QELP model. Detailed explanation and discussion will be presented in chapters 8 and 9, which will provide a thorough answer to the **RQ3**.

## 6.5 Conclusions

This chapter has provided an introduction to the new Quickest Evacuation Location Problem. The QELP aims to find the optimal locations for shelters and the optimal assignments of flow to the shelters such that the makespan, budget, and maximum load for each shelter are minimised by combining the quickest flow problem and the shelter location problem. The QELP is developed based on the real-world problem and the gaps in humanitarian operations, aiming at increasing the efficiency and effectiveness of humanitarian operations and enhancing evacuation network design and planning. Three research questions are devised to achieve the research aim: **RQ1**: What are the most appropriate objective function goals to be modelled and adopted as multiple objective functions? What are the benefits of the multi-objective model? **RQ2**: What are the key specific modelling features that should be considered when formulating the optimisation model for QELP? Besides objective functions in **RQ1**, What are the constraints and parameters? **RQ3**: How to solve the QELP optimisation model in **RQ2** in such a way that it will generate good solutions in a reasonable computation time?

In addition, the research philosophy of this research has also been discussed, where the QELP falls into the domain of OR, which is an implementation science and approach to addressing complex real-world problems and providing optimal solutions to support the decision-making process. This research also matches the research paradigms where the QELP is introduced from the

real-world problem. The QELP model is developed as a multiple objective optimisation problem, subject to various constraints. Different solution methods are used to get the optimal solutions to support the decision-makers, which provide answers to those research questions.

Furthermore, detailed explanations and motivations of the multiple objective functions used in the model have been provided to answer the **RQ1**. An overview of modelling tools and solution methods have also been provided in this chapter to answer **RQ2** and **RQ3**. A detailed introduction of the mathematical model of the QELP will be presented in the next chapter (chapter 7), and the thorough explanation and implementations of the solution methods will be provided in chapter 8.

## Chapter 7

# The Quickest Evacuation Location Problem model

### 7.1 Introduction

This chapter comprehensively explains the QELP model formulation, addressing the **RQ2**. It is structured into three sections. The first section outlines the fundamental assumptions underlying the quickest evacuation location problem and the QELP model. The second section introduces the ad-hoc modelling concept: QELP-Time Expanded Network (QELP-TEN), providing a toy example demonstration to understand better how the optimisation model is constructed. Building upon the QELP-TEN, we then present the formulation of the QELP mathematical model. Finally, the chapter concludes by summarising the key points discussed.

### 7.2 Model assumptions

There are some assumptions underlying the quickest evacuation location problem and its modelling tools. First, in the network, populations and demands are modelled as the nodes, and the links (i.e., roads and rails) are modelled as arcs. The decisions on the locations of shelters are made by selecting from a set of candidates which is located in the nodes. Therefore, the QELP is modelled as the discrete facility location problem, and all the facilities are located on the nodes. Second, each arc has two arc labels: capacity and delay/transit/travel time. They are fixed for each arc, which means they will not change as time changes (time-varying). Third, the QELP is modelled in a directed graph, meaning that all the arcs in the network have directions. Finally, we assume that once the evacuation process starts, the population in each demand node

will start evacuation simultaneously. These assumptions are widely used in the literature and represent the evacuation scenario reasonably.

### 7.3 Modelling concept: QELP-Time Expanded Network

The QELP is modelled here on the Time Expanded Networks (TEN), which are important tools for solving all ranges of discrete-time dynamic network flow problems. A time-expanded network (TEN) is defined to represent the time-dependent characteristics of the problem by replicating the sets of physical nodes for each period of a finite and discrete time horizon and connecting the time copies of the nodes with arcs according to the configuration of the delay time on the arcs (Lim et al., 2012; Crainic and Sgalambro, 2014). If waiting on a node is permitted for a unit of flow (an evacuee in this case), additional holdover arcs are added between the  $i$ th and the  $(i + 1)$ th copy of the same physical node. In other words, evacuees can wait at the nodes for the next available time instant to be evacuated. By utilising the TEN as a modelling tool, the discrete dynamic network flow problem can be represented as a static source-to-sink flow in the time-expanded network, and the maximum dynamic flow is presented as the maximal flow in the time-expanded network.

Differently from any approach based on the Quickest Flows and TEN modelling in the literature, in the QELP, we do not have a predetermined set of sink nodes, as these need to be selected among the candidate sinks as part of the QELP optimisation problem. To cope with this peculiar feature in the QELP model, we elaborate a modified time-expanded network, which we refer to as QELP-TEN. First, we include all candidate sinks. Then we introduce a set of *dummy time sinks*, that is, one dummy sink repeated for each time instant in the time horizon, plus one *final dummy super sink*. We also introduce all the *dummy arcs* needed to link the representation of the candidate sinks in the relevant time period with the respective *dummy time sinks*, and link each *dummy time sink* to the *final dummy super sink*. In this way, we can utilise the dummy arcs from the candidate nodes to *dummy time sinks* to model the decision on activation of the candidate sinks. Specifically, the activation of a candidate sink is determined by the presence of flow traversing the dummy arc connecting it to the corresponding dummy time sink. In addition, *dummy time sinks* and the *final dummy super sink* are introduced to track the completion time of the evacuation process for each flow. As long as a unit of flow arrives at the *final*



*dummy super sink*, it means that the evacuation for that particular unit of flow is finished. In this case, by tracking which *dummy time sinks* unit of flow comes from, we can know the exact completion time of the evacuation for that flow unit. In order to better detail the QELP-TEN, a toy example is provided in the following and is depicted in Figs. 7.1-7.3 and the following section goes into further detail about this toy example.

### 7.3.1 A toy example of QELP-Time expanded network

As shown in figure 7.1, nodes 1, 2, 3, 4 are the original physical nodes in the dynamic digraph in the Quickest Evacuation Location problem. Node 1 is the source node where we assume that five units of demand (or people) need to be evacuated. While node 2 represents a transshipment node in the example, nodes 3 and 4 are the candidate nodes for the shelters. We introduce a set of *dummy time sinks* and the *final dummy super sink* to help us build the model and solve the problem. Figure 7.2 shows the procedure used to solve the problem in dynamic digraph by applying the QELP-TEN, and figure 7.3 demonstrates the solution process in the physical network at each time instant. For this small instance, the total time instant is set as  $T = 4$ . Then, we expand the static network into four time instants. The green nodes  $S_1$  to  $S_4$  are the *dummy time sinks* on each time instant of the time horizon, while the blue node  $S_d$  is the *final dummy super sink* representing the entire evacuation completion. Meanwhile, the set of dashed arcs consists of the arcs from the candidate shelter node 3 and node 4 to *dummy time sinks* and the arcs linking *dummy time sinks* and *final dummy super sink* are *dummy arcs*. In particular, the dashed arcs that connect the physical nodes are *holdover arcs*. The arcs coloured purple show how flows are allocated in a solution of the QELP.

First, at  $t = 0$ , there are *five* people waiting for evacuation. Since the delay time from node 1 to node 2 is equal to one and the capacity for the arc is two, at  $t = 1$ , *two* people (group A) moved from node 1 to node 2 and *three* people are still waiting at node 1. At the second time instant ( $t = 2$ ), group A moved from node 2 to node 4 and then moved through the dummy time sink  $S_2$  to the final dummy super sink  $S_d$ , which means that evacuation for group A is completed at  $t = 2$ .

Meanwhile, another group of *two* people (group B) moved from node 1 to node 2, and only *one* person (group C) remained at node 1 waiting for evacuation. Then, at  $t = 3$ , group B moved from node 2 to node 4 and travelled through the dummy time sink  $S_3$  to the final dummy super sink  $S_d$ , which

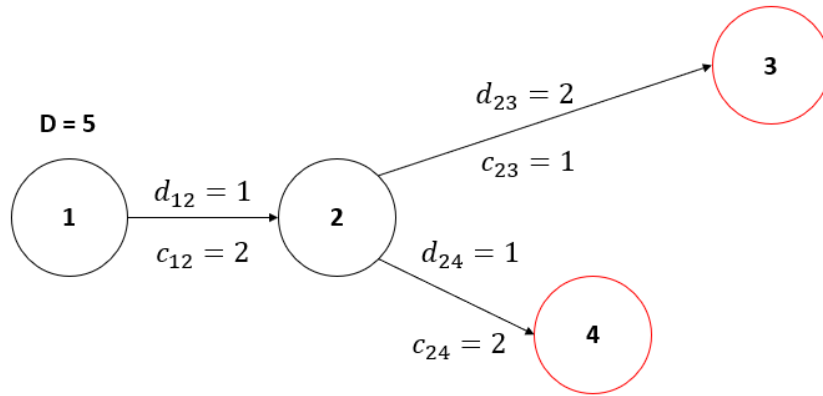


FIGURE 7.1: Dynamic digraph

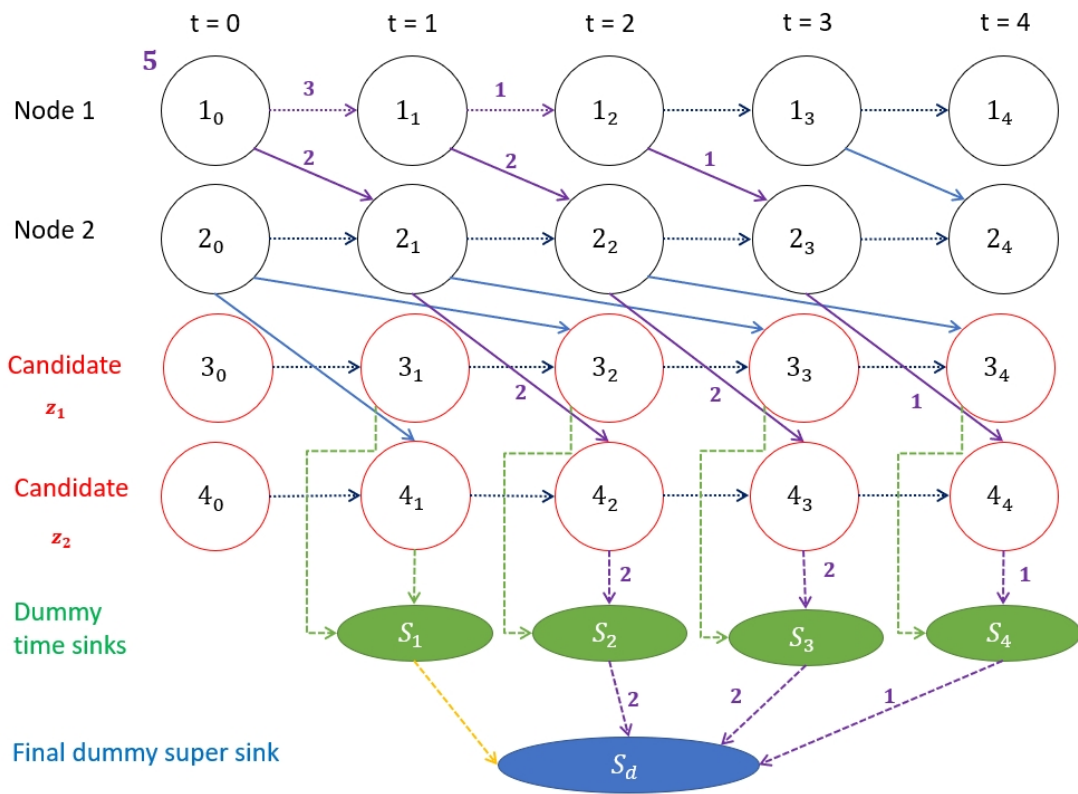


FIGURE 7.2: QELP-Time Expanded Network

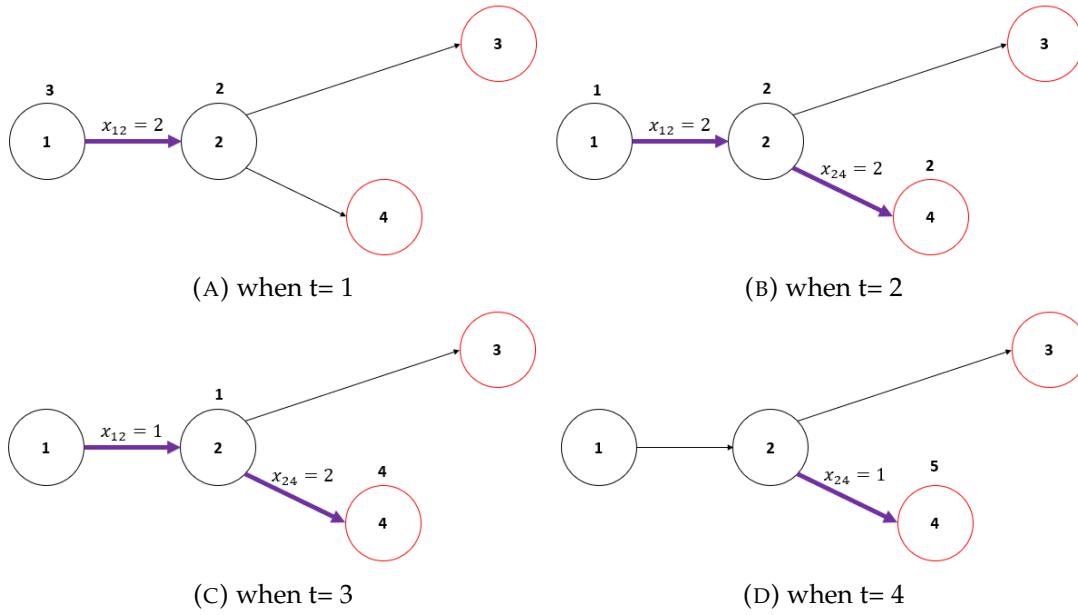


FIGURE 7.3: Solution process

means that group B has been successfully evacuated. At the same time, group C, who was waiting at node 1, moved to node 2 at  $t = 3$ . Finally, at  $t = 4$ , group C moved from node 2 to node 4 and travelled to the final dummy super sink  $S_d$  through the dummy time sink  $S_4$ . In this case, the last person (group C) arrived at the final dummy super sink at  $t = 4$ , which means that the entire evacuation process is finished and the makespan of this toy example, in this case, is equal to four.

This toy example is presented to explain the logic behind the QELP-TEN better. In all, the QELP-TEN is developed based on the original TEN by taking into account the location decisions, and by introducing the dummy arcs and nodes, we can better monitor the flow of evacuees in the network while finding the optimal locations of shelters. In this case, the QELP-TEN also can better compute the total makespan of the whole evacuation process. In the next section, we introduce a detailed and formal mixed-integer programming formulation for QELP.

## 7.4 Model formulation

Here, we introduce the notation used in modelling the QELP as a QELP-TEN and its respective mathematical formulation. A graph  $G = (N, A, T)$  is given, where  $N$  is a set that contains  $n$  nodes, among which  $N_c \subseteq N$  is the set of candidate nodes where a shelter can be located,  $N_D \subseteq N$  is the set of source

or demand nodes,  $D$  is the set of demand associated with each demand node, namely the number of people who need to be evacuated at each source node. Then  $A$  is the set of  $m$  arcs, and each arc has two labels, namely capacity and delay time, both defined as time-independent w.l.o.g. for the purposes of this paper. For each arc  $(i, j) \in A$ ,  $h_{ij}$  is the non-negative capacity associated with this arc, showing that the largest amount of flow can enter into the arc and  $t_{ij}$  is the delay/travel time, representing the amount of time instants needed to travel from the tail node  $i$  to the head node  $j$ .  $T$  is the set of time instants, representing the discrete-time horizons.  $S_t$  is the set of dummy nodes that represent a super-sink for each time instant, which are also called dummy time sinks.  $S_d$ , in this case, is the final dummy super sink. Both  $S_t$  and  $S_d$  are dummy arcs introduced to efficiently monitor the evacuation process and compute the makespan, which does not belong to  $N$ . As long as the flow arrives at  $S_d$ , it means the whole evacuation process is completed. For each candidate shelter,  $C_i$  is the associated capacity,  $k$  is the multiplicative cost coefficient associated with the capacity, and  $f$  is the fixed cost to open one shelter (independent of its capacity).

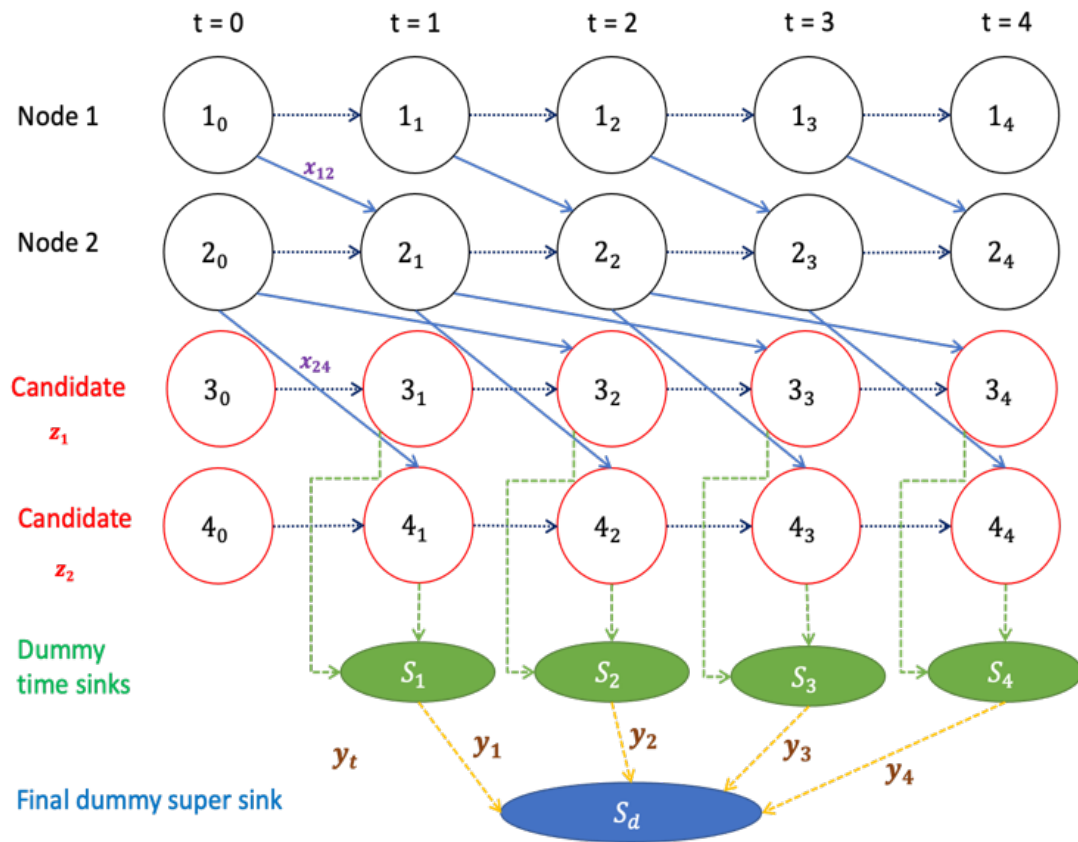


FIGURE 7.4: Decision variables in QELP-TEN

TABLE 7.1: Summary of notation for the description of the QELP

$G = (N, A, T)$ $N$ : set of $n$ nodes $N_c$ : candidate nodes—subset of nodes where a shelter can be located $N_D$ : source nodes—subset of nodes where the demand is located $A$ : set of $m$ arcs $T$ : set of time instants, indexed with $t$ $h_{ij}$ : capacity of arc $(i, j)$ $t_{ij}$ : delay / travel time of arc $(i, j)$ $S_t$ : set of dummy nodes representing a super sink for each time instant $S_d$ : the final dummy super sink representing the whole evacuation completion $D_i$ : demand of evacuees on each source node $M$ : a big constant value $C_i$ : the capacity of each candidate shelter $k$ : cost value associated with the capacity of each candidate shelter $f$ : fixed cost associated with the opening of the candidate shelter
<b>Decision variables:</b> $x_{ij}$ : the amount of flow on each arc of the QELP-TEN $y_t$ : binary variables associated with the dummy arcs $z_i$ : binary variables associated with the activation of selected candidate shelters $\lambda$ : makespan $B$ : budget $ML$ : maximum load $l_i$ : the sum of exiting flows from each candidate shelter

We define decision variables for the QELP MIP model as follows:  $x_{ij}$  as non-negative flow variables defined on each arc of the QELP-TEN;  $y_t$  as binary variables on the dummy arcs linking dummy time sinks to the final dummy super sink; and  $z_i$  as binary variables associated with the activation of the candidate nodes. Additional decision variables to represent objective functions are: i)  $\lambda$  as the makespan, which is the time elapsed between the start and end time of the evacuation process and can be measured here as the time instant where the last unit of flow reaches a shelter, meaning that the entire evacuation process is completed, ii)  $B$  as the budget, which is the total cost related to the opening of the shelters, iii) and  $ML$  as the maximum load of flows reaching each activated shelter throughout the entire time horizon. Figure 7.4 presents the decision variables in the QELP-TEN, and all the notation is summarised in table 7.1.

### 7.4.1 Mathematical formulation

We present here a mixed-integer programming formulation for the above-described multi-objective optimisation problem:

$$\text{QELP – MIP : } \min \lambda \quad (7.1)$$

$$\min B \quad (7.2)$$

$$\min ML \quad (7.3)$$

s.t.

$$t \cdot y_t \leq \lambda \quad t \in T \quad (7.4)$$

$$x_{S_t S_d} \leq M \cdot y_t \quad t \in T \quad (7.5)$$

$$\sum_{(i,j) \in A} x_{ij} - \sum_{(j,i) \in A} x_{ji} = \begin{cases} D_i, & i \in N_D \\ 0, & i \in N, i \notin N_D, i \neq S_d \\ -\sum_{i \in N_D} D_i, & i = S_d \end{cases} \quad (7.6)$$

$$\sum_{i \in N_c} (k \cdot C_i + f) \cdot z_i \leq B \quad (7.7)$$

$$\sum_{(i,j) \in A: j \in S_t} x_{ij} \leq C_i \cdot z_i \quad i \in N_c \quad (7.8)$$

$$\sum_{(i,j) \in A: j \in S_t} x_{ij} \leq ML \quad i \in N_c \quad (7.9)$$

$$l_i = \sum_{(i,j) \in A: j \notin S_t} x_{ij} \quad i \in N_c \quad (7.10)$$

$$z_i \cdot l_i = 0 \quad i \in N_c \quad (7.11)$$

$$0 \leq x_{ij} \leq h_{ij} \quad (i,j) \in A \quad (7.12)$$

$$y_t \in \{0,1\} \quad t \in T \quad (7.13)$$

$$z_i \in \{0,1\} \quad i \in N_c \quad (7.14)$$

The objective (7.1) aims to minimise the makespan, the objective (7.2) refers to the minimisation of the total budget, and the objective (7.3) is used to minimise the maximum load of each activated shelter. Constraint (7.4) is the set of min-max constraints used to calculate the makespan and minimise the maximum evacuation time. Constraint (7.5) is used to activate binary variables associated with dummy arcs. It allows one to keep track of the makespan, where as long as there is a flow on the dummy arc, that dummy arc will be activated. Constraint (7.6) is the set of flow conservation constraints where, for demand nodes, the net flow is equal to demand on each demand node. For intermediate nodes, the net flow equals 0, where the flow exiting equals the flow entering. For the final dummy super sink, the flow entering should be the total demand, which means all the people have been evacuated to a safe place. Constraint (7.7) is used to calculate the budget, where the total budget is the sum of the fixed cost of opening a shelter and the variable cost related to holding the capacity. Constraint (7.8) is used to ensure that a candidate is activated if any unit of flow reaches it as a destination of the evacuation process and also imposes a limit on the amount of such flow throughout the entire evacuation process to make sure the capacity constraint for each destination is imposed. Constraint (7.9) is used to keep track of this amount of total entering flow arriving in the active shelters in such a way as to allow minimisation of this quantity as a third objective function. Constraints (7.10) - (7.11) are used to forbid any flow from an activated shelter to exit at any time, except for reaching a dummy time sink. It has important real-world applications in that there is no intention for evacuated people to move anywhere else once they arrive at an activated shelter. In the QELP-TEN, this means that when evacuees arrive at the activated shelter, they will immediately move to the dummy time sinks and then get to the final dummy super sink without any waiting. In this instance, we ensure a more accurate modelling of the situation in which the evacuation process is completed if evacuees arrive at an activated shelter. To achieve this, for each candidate shelter  $i \in N_c$ , we define the sum of exiting flows from each candidate shelter as constraint (7.10). Then, the quadratic constraint (7.11) is added to ensure that there is no flow leaving the activated shelter, except for the dummy time sinks. To linearise the quadratic constraint (7.11), it can be substituted with:

$$l_i \leq U \cdot (1 - z_i) \quad i \in N_c \quad (7.15)$$

where the parameter  $U$  is an upper bound on the value of  $l_i$ , defined as the

total demand multiplied by the number of time instants. Constraints (7.12)-(7.14) set the domain for decision variables.

## 7.5 Conclusions

This chapter has focused on providing a comprehensive and rigorous explanation of the quickest evacuation location problem model formulation, emphasising the development of QELP mathematical formulation that can effectively address the quickest evacuation location problem and answers the **RQ2**. At the beginning of this chapter, we have established the basis of the QELP and its model by carefully explaining the assumptions used in the modelling. These assumptions are carefully identified and justified as the foundation for the quickest evacuation location problem. These assumptions are devised from real-world cases and are widely used in the literature. Nevertheless, while these assumptions are fundamental to shelter location and network flow problems, there are still some limitations. Chapter 10 offers a comprehensive exploration of their strengths and opportunities for future enhancements and extensions.

After the identification and justification of assumptions embedded in the problem and model, we have introduced the QELP-Time Expanded Network (QELP-TEN), the ad-hoc modelling tool to model the QELP. The QELP-TEN is developed based on the original TEN by taking into account the location decisions and introducing the sets of dummy arcs and dummy nodes to better monitor the flow in the network for achieving the objectives of this problem and to support the formulation of the model. A toy example is also provided to explain the basis of QELP-TEN better.

Finally, the multi-objective mixed-integer programming model of the quickest evacuation location problem has been introduced based on the QELP-TEN, aiming to minimise the makespan, total budget, and the maximum load in each activated shelter for a more efficient and effective evacuation design and planning, which is also subject to various constraints mentioned above. In the next chapter, the methods used to solve the QELP model will be introduced and evaluated.

After introducing the problem description and the mathematical model of the QELP, it is clear that the QELP is quite different from the Comprehensive Evacuation Problem (CEP) model (Goerigk et al., 2014a). From the problem-setting perspective, first, the Quickest Evacuation Location Problem (QELP)



does not consider public evacuation (which CEP considers the bus evacuation). Instead, we focus on private evacuation, particularly pedestrian evacuation, which can also be modified into car evacuation. This is more reasonable in the evacuation where after the disaster occurs, people tend to self-evacuate to safe places. In some cases, public transportation may not get access to the disaster area. Second, we model the QELP as a multi-objective mixed-integer programming model where we consider the total budget and try to save the total costs rather than only minimising the number of shelters. In this case, we pay attention to the economic resources required in designing and planning the evacuation process. In this way, we can provide decision-makers with an actionable decision support system. Furthermore, we try to reduce the risks of the shelters and the coefficients in the formulation can be changed from different scenarios, where the QELP accounts for the risks and reliability when planning for the location of shelters.

Regarding the modelling perspective, we introduced the tailored QELP-TEN, which can produce a more compact formulation by introducing the dummy nodes and arcs. By doing so, we reduce the complexity of the model, especially through introducing the dummy time sinks to monitor the finish time of each flow. This allows us to produce the original problem in tailored QELP-TEN. Without loss of generality, the evacuation time is modelled to respond to the disaster where all the evacuees start evacuation at the beginning of the time period. In this case, we also include the holdover arcs in the tailored QELP-TEN to make it compact as an opportunity that we can start the evacuation process at any time in the evacuation. Overall, the QELP can bridge the gap in the literature by providing a decision support system for evacuation design and planning in real-world scenarios.

## Chapter 8

# Solving the Quickest Evacuation Location Problem: AUGMECON-R

### 8.1 Introduction

As discussed in the previous chapter, the QELP is developed as a multi-objective mixed-integer programming model. We used two methods to test the QELP model: AUGMECON-R and a Matheuristic method embedded within the AUGMECON-R scheme to solve the problem as network size increases quickly. This chapter starts with a brief overview of the fundamental methods that are frequently used to solve multi-objective optimisation problems. After that, the AUGMECON-R is introduced and selected to test the QELP model. The solutions obtained from the AUGMECON-R are also discussed in this chapter. The computational time increases dramatically as the network size increases, which leads to the need to introduce the novel matheuristic method, which will be presented in the next chapter.

### 8.2 Classic multi-objective optimisation methods

The multi-objective optimisation problem, suggested by its name, contains more than one objective function that is needed to be minimised or maximised. In most cases, the objective functions in the multi-objective optimisation problem usually conflict with each other. In this case, the solutions to a multi-objective optimisation problem do not typically have only one optimal solution satisfying every objective function. Instead, the solutions are usually a set of options that specify the optimal trade-offs between these objective functions.

This leads to the concept of Pareto-optimal solutions, which is the set of solutions that are not dominated by other solutions. In other words, the Pareto-optimal solutions are those solutions that can not be improved in one objective without at least one of the others being worsened. In multi-objective optimisation, it is important to identify the Pareto-optimal solutions and the whole non-dominated set of solutions is called *Pareto-optimal set*. Therefore, the multi-objective optimisation methods are used to find the Pareto-optimal set for the problem they studied. In this section, three classic multi-objective optimisation methods are presented: The weighted sum method, the lexicographic method, and the  $\varepsilon$ -constraint method, along with its improved variants.

### 8.2.1 Weighted sum method

The weighted sum method is among one of the easiest methods in multi-objective optimisation, where it scalarises the set of objective functions into one single objective by multiplying each objective with a different weight. By putting different weights on each objective function, the weighted sum method allows decision-makers to prioritise the most important objective as the sum of the weight equals 100% (Marler and Arora, 2010).

By applying the weighted sum method, the original multi-objective optimisation problem can be transformed into (an example):

$$\min \sum_{i \in [1, n]} w_i \cdot f_i(x) \quad (8.1)$$

s.t.

$$\sum_{i \in [1, n]} w_i = 1 \quad (8.2)$$

$$0 \leq w_i \leq 1 \quad i = 1, \dots, n \quad (8.3)$$

$$x \in X \quad (8.4)$$

The advantage of the weighted sum method is that it is straightforward to understand. However, setting the weight vectors to get the Pareto-optimal solution in a desired area of the objective space is challenging, and it is not applicable in the context of a nonconvex objective space (Deb, 2001).

### 8.2.2 Lexicographic method

The lexicographic method sets the priority for each objective function in lexicographic order. The optimisation is carried out by keeping the already optimised objectives constant while successively optimising the objective functions in a predetermined order (Fishburn, 1974). This means that the decision-makers set the order to each objective function first, then optimise each objective function according to that order while keeping the already optimised objective as a constant for a constraint in other optimisations.

By applying the lexicographic method, the original multi-objective optimisation problem can be transformed into (an example):

$$\min f_i(x) \tag{8.5}$$

*s.t.*

$$f_j(x) \leq y_j \quad j = 1, \dots, i - 1 \tag{8.6}$$

$$x \in X \tag{8.7}$$

The lexicographic method is also relatively simple to understand and implement, and it allows for setting priority to each objective function, which can be very practical in real-world cases. In this case, the lexicographic method can generate a set of solutions, each tailored to a particular objective function and presented in the predetermined order. First, the first objective with the highest priority is optimised as shown in the objective function 8.5 and the remaining objective functions are treated as constraints as constraint 8.6. Then, optimise the second objective function with the second-highest priority while fixing the solution for the first objective and setting the remaining constraints as constraints. This process continues until all the objective functions have been optimised. This will make the decision-making process easier for decision-makers to compare different options and make the most suitable choice based on the context. However, there are two main limitations of the lexicographic method, despite the fact that the lexicographic method can generate solutions that are ideal for each objective function in the predetermined order. The total set of solutions may not always accurately be the Pareto-optimal set. In this case, the solutions obtained from the lexicographic method may not truly reflect the trade-offs between objective functions. Furthermore, the priority and order of objective function have a significant impact on the quality of the solutions obtained. The solutions obtained might not be helpful if the order of

objective function does not sufficiently reflect the choices of decision-makers or the features of the situation, and this may lead to ineffective solutions.

### 8.2.3 Epsilon ( $\varepsilon$ )-constraint method

The  $\varepsilon$ -constraint method is one of the most used methods to solve multi-objective optimisation problems. It directly transfers a multi-objective optimisation problem into a single objective optimisation problem by selecting one objective function as the primary objective function and transferring the remaining objective functions into constraints using the predetermined bounds ( $\varepsilon$ ). Then, the primary objective function is optimised within these confines of constraints. By varying the  $\varepsilon$  values for each transferred objective constraint, the set of Pareto-optimal solutions can be obtained (Mavrotas, 2009).

By applying the  $\varepsilon$ -constraint method, the original multi-objective optimisation problem can be transformed into (an example):

$$\min f_i(x) \tag{8.8}$$

s.t.

$$f_j(x) \leq \varepsilon_j \quad j = 1, \dots, n, j \neq i \tag{8.9}$$

$$x \in X \tag{8.10}$$

The  $\varepsilon$ -constraint method allows decision-makers to explore the Pareto-optimal set by setting different  $\varepsilon$  values. Large  $\varepsilon$  values can lead to a comprehensive set of solutions. In contrast, small  $\varepsilon$  values can obtain a more focused Pareto-optimal set, which leaves the flexibility for decision-makers to choose from. However, the solution set is also highly impacted by the subjectivity in the selection of  $\varepsilon$  values, and in this case, the primary objective function might be overemphasised. In addition, the increase in the number of objective functions will lead to an increase in the transferred constraints and, finally, will increase the computational complexity.

These are three widely used methods to solve multi-objective optimisation problems. Each of them has both advantages and limitations. To sum up, the weighted sum method uses weights to combine the multiple objective functions into a single objective function, but it may not be able to capture the

entire Pareto-optimal set, and the weights used are very subjective. The Lexicographic method uses a simple strategy to set the priorities in a predetermined order, which is very easy to understand and enables the priority set by the decision-makers. But it may not cover all the Pareto-optimal solutions. Finally, the  $\varepsilon$ -constraint method is very useful in solving linear problems, and it allows an explicit exploration of trade-offs between objective functions by varying  $\varepsilon$  values.

In order to solve the quickest evacuation location problem, we applied the improved version of the  $\varepsilon$ -constraint method, which is called the Robust Augmented  $\varepsilon$ -constraint (AUGMECON-R) method. In the next section, we will give a clear introduction to AUGMECON-R and its advantages compared with the original  $\varepsilon$ -constraint method. Then, we will introduce the testbed used to apply the AUGMECON-R and discuss the solutions obtained from AUGMECON-R.

### 8.3 Exploring Pareto Set: AUGMECON-R

Robust Augmented  $\varepsilon$ -constraint (AUGMECON-R) (Nikas et al., 2020) is utilised as an exact method to efficiently explore the Pareto Set of QELP and is an improved version of AUGMECON2 that is developed based on  $\varepsilon$ -constraint.  $\varepsilon$ -constraint is one of the most popular methods used to solve multi-objective mathematical programming (MOMP) problems and to explore the whole Pareto Set (Keller, 2017). In  $\varepsilon$ -constraint methods, one of the multiple objectives is chosen as the main objective to optimise, and the remaining objectives are converted into constraints by setting the upper bounds as  $\varepsilon$ -vectors where the exact Pareto front can be generated by varying the  $\varepsilon$ -vectors. However, the  $\varepsilon$ -constraint method has some ambiguities: the first is that the range of objective functions needed to be calculated, and the second is that even though the efficient Pareto-optimal is obtained, the solution cannot be guaranteed to be not weak; the third is that if there are more than two objective functions, it is very time consuming to obtain solutions (Bababeik et al., 2018; Nikas et al., 2020). These weaknesses of the  $\varepsilon$ -constraint method drive the development of the augmented  $\varepsilon$ -constraint (AUGMECON) method (Mavrotas, 2009) and its variations, such as AUGMECON2 (Mavrotas and Florios, 2013) and AUGMECON-R (Nikas et al., 2020).

AUGMECON is a novel version of the conventional  $\varepsilon$ -constraint method developed to remedy the pitfalls of the original  $\varepsilon$ -constraint method (Mavrotas,

2009). First, lexicographic optimisation is applied to obtain the pay-off table, ensuring the Pareto optimality of the obtained solution. Second, non-negative slack or surplus variables are introduced to transform objective functions into equalities. Meanwhile, the slack or surplus variables are also used in the second/third term in the objective functions where the objective function is augmented with the weighted sum of slack or surplus variables (Zhang and Reimann, 2014). All of these force the program to produce only efficient solutions. AUGMECON2 is an improved version of AUGMECON developed by Mavrotas and Florios (2013). It can ensure that the exact Pareto-optimal solutions are produced by introducing a bypass coefficient of the innermost loop, leading to fewer sub-problems needing to be solved. More specifically, the bypass coefficient is used to show the number of consecutive iterations to bypass (Mavrotas and Florios, 2013). AUGMECON and AUGMECON2 are widely used in solving MOMP problems in network design and supply chain management (Tanksale et al., 2021; Rabbani et al., 2020; Khorshidian et al., 2016; Caglayan and Satoglu, 2021).

The robust augmented  $\varepsilon$ -constraint method (AUGMECON-R) was introduced by Nikas et al. (2020) and is an improved robust variant of AUGMECON2. Similarly to AUGMECON2, in the case of unitary steps and integer coefficients, AUGMECON-R explores all exact Pareto-optimal solutions. In the case of large-scale problems with more than two objective functions, the computational time of AUGMECON2 is extremely long. Therefore, AUGMECON-R introduces the bypass coefficient to every objective function in each outer loop instead of only one bypass coefficient in the innermost loop in AUGMECON2. Meanwhile, the flag array and the notion of pure optimisations are used as indicators of jumps in the innermost loop. These significantly reduce time by skipping unnecessary optimisations because of infeasibilities and can solve problems whose nadir points are unknown by introducing very low and zero-value lower bounds to overcome the weaknesses of AUGMECON2. In addition, grid points are introduced to solve the problem step-by-step, which brings in the main advantage that the number of efficient solutions can be controlled by appropriately adjusting the number of grid points on which each optimisation is solved, along with the range of each objective function. Overall, AUGMECON-R significantly reduces the number of models solved; in turn, less time is needed. Furthermore, AUGMECON-R can solve the problem without extra time, even when the nadir points are unknown.

The QELP can now be formulated by adopting the following transformed

objective function and additional constraints based on the AUGMECON-R method:

$$\text{(QELP-AUGMECON-R) } \min \left[ \lambda - \delta * \left( \frac{s_2}{r_2} + 10^{-1} \frac{s_3}{r_3} \right) \right] \quad (8.11)$$

subject to

Constraints (7.4) – (7.14)

$$f_2(x) + s_2 = e_2 \quad (8.12)$$

$$f_3(x) + s_3 = e_3 \quad (8.13)$$

The parameter  $\delta$  is set as equal to  $10^{-3}$ , while  $r_2$  and  $r_3$  are the ranges for the second and third objective functions, computed by performing a lexicographic optimisation approach to obtain the pay-off table before running the algorithm which helps to find exact Pareto solutions. The bypass coefficient  $b_i$  is incorporated into the model as much as the objective function  $i$ , which is the integer part of the result of the slack variable divided by the step, allowing for an accelerated solution by avoiding unnecessary iterations. For further details on the AUGMECON-R method, the reader is referred to [Nikas et al. \(2020\)](#).

$$b_i = \text{int}\left(\frac{s_i}{\text{step}_i}\right) \quad (8.14)$$

$$\text{step}_i = \frac{r_i}{q_i} \quad (8.15)$$

## 8.4 AUGMECON-R: computational experiments setup

There are two main parts in this section. The instances used to test the QELP model are first introduced, along with a thorough discussion of the settings employed. We then discuss the experiment setup and the solver we used to implement the experiments.

### 8.4.1 Instance description

QELP model was applied to six different networks of various sizes, from 37 nodes to 398 nodes, to evaluate the performance of the model. The first set of networks consists of three Swain networks, including the Small Swain network (37 nodes), the original Swain network (55 nodes), and the Extended Swain network (150 nodes). The Small Swain network and the original Swain



network which were first introduced by Swain (1974) and were used in many studies such as Church and ReVelle (1974), Daskin (1983), and Van Den Berg and Van Essen (2019). The Extended Swain network with 150 nodes and 321 arcs, introduced by Ruiz-Hernández et al. (2016), was also used to test the model. These three networks, which have a complexity range of 37 to 150 nodes and are commonly used as testbeds in the literature, are utilised as a starting point to evaluate the effectiveness of the QELP model because they have relatively smaller sizes and require less computational time.

The second set of networks consists of three different transportation networks, consisting of networks constructed over the map of three regions of Berlin, including the Berlin Friedrichshain network, the Berlin Tiergarten network, and the Berlin Mitte center network (Transportation Networks for Research Core Team, 2020), which originally appeared in Jahn et al. (2005) and were also used in Farahani et al. (2018) as testbeds. These are real-world cases constructed from various regions of Berlin with numerous practical applications where we can assess QELP's capability for dealing with real-world issues by considering these real-world examples. Since the QELP model was created to assist in the decision-making process for evacuation design and planning for actual disasters, using actual instances as testbeds can demonstrate the usefulness of the model. Furthermore, these three networks have larger sizes compared to the Swain networks, ranging from 224 nodes to 398 nodes. In this case, we can use these instances to better test the performance of the QELP model and get a more precise comparison of the performance of the Matheuristic approach with the exact method.

In total, there are six networks, and each network has five instances with the same candidate nodes but five different sets of demand nodes; therefore, in total, we have 30 different instances used to test the model. The Small Swain network is a subset of nodes of the original Swain network, containing 37 nodes and 170 arcs. The original Swain network has 55 nodes and 268 arcs. The Extended Swain network is an extended version of the original Swain network, which has 150 nodes and 642 arcs. The second set of networks we used to test the model are three Berlin region networks, where the Berlin Friedrichshain network has 224 nodes and 568 arcs, the Berlin Tiergarten network has 361 nodes and 984 arcs, and the Berlin Mitte center network has 398 nodes and 1000 arcs. Table 8.1 summarises the characteristics of each network used as testbeds.

The procedure for selecting demand nodes and candidate nodes, as well as

setting up their demand and capacity, is the same across all instances in the setup process.

- *Candidate nodes*: candidate nodes are chosen and fixed throughout different instances in each network, which can represent the real-world case where candidate shelters are often decided before disasters and usually have specific characteristics, such as schools or stadiums, where they can provide a large place for evacuees. The number of candidate nodes is set as 2% of the total number of nodes with a minimum number of 5. The locations of the candidate nodes are fixed and spread throughout the network, which better simulates real-world scenarios. The capacity of the candidate nodes is randomly chosen between  $\pm 20\%$  of the four times of total demand divided by the number of candidate nodes. In this case, we can ensure that the total capacity can cover all the demand, and we can also have different capacities of different candidate nodes, which better represent real-world scenarios;
- *Demand nodes*: the number of demand nodes is set as 5% of the total number of nodes with a minimum number of 10 with the same demand of 200 for each demand node. The locations of the demand nodes are randomly selected throughout the network, and for each network, five different demand scenarios are generated.

TABLE 8.1: Description of the testbed

Network name	Nodes number	Arcs number	Demand nodes number	Total demand	Candidate nodes number	Total capacity
Small Swain Network	37	170	10	2000	5	7853
Original Swain Network	55	268	10	2000	5	7926
Extended Swain Network	150	642	10	2000	5	8098
Berlin Friedrichshain	224	568	12	2400	5	10395
Berlin Tiergarten	361	984	19	3800	8	16494
Berlin Mitte center	398	1000	20	4000	8	17201

## 8.4.2 Experiment setup

All instances used to test QELP are modelled as dynamic network flows (flows over time), introducing delay/travel time ( $d_{ij}$ ) and capacity ( $c_{ij}$ ) on each arc. Meanwhile, we assume that each arc has two directions, which aims to better simulate real-world situations. For example, from node  $i$  to node  $j$ , there are two arcs: Arc  $a_{ij}$  and Arc  $a_{ji}$  that connect the nodes  $i$  and  $j$ . We assume that they have the same capacity and delay time for the benefit of simplicity. The

values of capacity and delay time in each network are set in the original networks and modified in the experiments. The delay/travel time is the units of time required to travel through the arcs. They are helpful in keeping track of the time needed in the evacuation process and providing a clear and precise reflection of real-world cases by considering the arc length. Meanwhile, the capacity of the arc represents the largest amount of flow that can enter the arc at each time instant. It is necessary to consider the capacity of the arcs to set the upper bound of the flow that can be evacuated on each arc to avoid any congestion or additional casualties in the evacuation process. Moreover, QELP is introduced to enhance the efficiency of evacuation design and planning in the pre-disaster stage. Therefore, considering the capacity and delay time on each arc allows for a better reflection of real-world evacuation situations in design and planning, as well as more precise monitoring and controlling in the evacuation process.

*Time-horizon setup:* the length of the time horizon ( $T$ ) has a significant impact on the dimension of the problem. Hence, on the solution time, and therefore, with the aim of containing its value, it is defined for each instance as a characteristic parameter using an approximation method, as follows. Since QELP is a multi-objective problem, we assume first that only the shelter with the lowest associated cost is opened. Then, using this fixing for the shelter location variables, we compute the optimal value of the makespan as an objective function. The latter is used as an upper bound and assigned to the time instant parameter  $T$  of the considered instance. We repeat such a procedure for each instance to define the associated  $T$  parameter.

*Dummy-arcs parameters setup:* the value of  $M$  was used to activate the set of dummy arcs ( $y_t$ ) and to bound the maximum value of the flows on the dummy arcs, which varies from different networks. The maximum value of flows on the dummy arcs is bounded by the minimum value between the total demand and the capacity of the inflow arcs. Therefore, we set different values for  $M$  based on instances by selecting the minimum value between the total demand ( $\sum_{i \in N_D} D_i$ ) and the total inflow capacity of the arcs entering the dummy time sinks, which is also equal to the total inflow capacity of the arcs entering the candidate nodes ( $\sum_{(i,j) \in A: j \in N_c} c_{ij}$ ). The total inflow capacity of the arcs entering the candidate nodes is the maximum amount of inflow that can enter the time sinks for each time instant. Hence, it indicates the upper bound on the amount of flow that can be allocated to dummy arcs at one time instant. Accordingly, the value of  $M$  is set as the minimum value between the total demand and the

inflow capacity of arcs ( $M = \min(\sum_{i \in N_D} D_i, \sum_{(i,j) \in A: j \in N_c} c_{ij})$ ). Therefore, the value of  $M$  is set as needed, allowing more accurate and less time-consuming performance.

*AUGMECON-R grid-points setup:* as it is introduced in the section 8.3, using unitary steps and integer coefficients as in the original paper [Nikas et al. \(2020\)](#) would lead to an exhaustive exploration of the Pareto Set. Still, it would easily take too much computational time. Therefore, grid-points are introduced, following the original paper, to efficiently explore the Pareto-optimal solutions. In this work, AUGMECON-R is applied to solve QELP, where we test experiments at varying grid-points to explore the Pareto Set:  $10 \times 10 = 100$  grid-points,  $20 \times 20 = 400$  grid-points,  $30 \times 30 = 900$  grid-points, and  $40 \times 40 = 1600$  grid-points. In total, we have 120 instances used to test this model.

The QELP model was coded in Python, and the tests were run using DoCplex from IBM® ILOG® CPLEX® Interactive Optimizer 12.10.0.0 in Python. All tests are run on a server equipped with Intel Xeon Gold 6246R 3.4ghz CPUs, 512GB Ram and Ubuntu Server 20.04. LTS.

## 8.5 AUGMECON-R: solution discussion

Table 8.2 summarises the average values of the computational process using the AUGMECON-R method. We compute the average values of the computational time, the number of Pareto-optimal solutions found, and the feasibility and bypass jumps for each network under various grid-points. From the tables, the data show that an increase in grid-points is associated with a growth in the number of feasibility jumps and bypass jumps and leads to a rise in the number of Pareto-optimal solutions found. The impact of the growth in the number of grid-points on the computational time appears relatively contained compared to the improvement in accuracy. Moreover, the results show that AUGMECON-R performs well in solving the QELP model, where a good number of Pareto-optimal solutions are identified within a reasonable amount of time, and it quite regularly increases with the density of the grid.

The values for each objective function and the number of shelters located when the grid-point equals 1600 are shown in the table 8.3. First, we observe various ranges in the values of three objective functions, especially in the middle size of a network like Extended Swain Network. It is obvious that the larger the grid-points, the more Pareto-optimal solutions are obtained, which is in line with the findings in the table 8.2. Second, the ranges of the number

of activated shelters associated with the Pareto-optimal solutions indicate that not all of the shelters will be activated, even though one of the objectives is the makespan minimisation. This is an important feature that confirms the suitability of QELP and its MIP model for finding realistic optimal shelter configurations, as a realistic number of shelters is associated with the Pareto-optimal solutions to be evaluated by the decision-maker. For example, although there are several candidate locations, the number of activated shelters inefficient solutions ranges between 5 and 7 for the Berlin Mitte center network at most, which is reasonable and achievable in real life from an operations and supply chain management perspective in consideration of the size of such networks.

Finally, from the two largest examples of Pareto curves of the Berlin Tiergarten and Berlin Mitte center networks in figure 8.1, and along with the extended ranges of the values of three objectives in table 8.3, there is an apparent regular conflict between the three considered objectives (makespan, budget, and maximum load), confirming the presence of a trade-off among these three goals from a managerial and practical perspective. For example, in the Berlin Mitte center network, the makespan for the whole evacuation process will range between 50 and 60 time instants, based on the choice made by the decision-maker on the basis of the budget to be allocated and the capacity of shelters. This aspect emphasises the importance of setting different budget levels, and the decision-maker should consider this when designing and planning the evacuation process. A bigger budget will substantially reduce the makespan. Therefore, it is crucial to consider the trade-off between makespan, budget, and maximum load in the design and planning phases to achieve less human suffering.

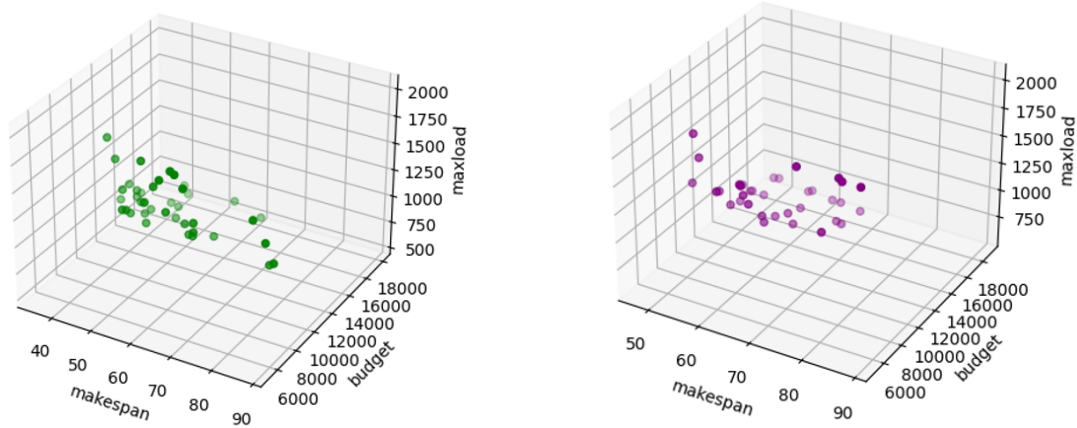
To sum up, it is clear that the increase in grid-points leads to an increase in the number of Pareto-optimal solutions. At the same time, the increase in the grid-points results in a significant rise in the computational time. Furthermore, as we can see from the results, as the network expands from the Small Swan network to the Berlin Mitte Center network, the computational time increases significantly, which raises the need for a Matheuristic method to solve the problem and get good solutions within a reasonable amount of time. The next chapter will introduce the Matheuristic approach to solve the quickest evacuation location problem.

TABLE 8.2: Average values on the computational process of AUGMECON-R

Network name	# grid-point	Feasibility jumps	Bypass jumps	Computational time (s)	# PO solutions
Small Swain Network	100	49	5	12	5
	400	191	75	31	7
	900	436	244	49	8
	1600	770	490	68	8
Original Swain Network	100	38	5	86	9
	400	150	92	340	13
	900	332	270	340	12
	1600	597	603	586	12
Extended Swain Network	100	41	7	371	5
	400	171	93	909	8
	900	380	270	1494	12
	1600	685	556	2056	11
Berlin Friedrichshain	100	49	6	605	4
	400	197	84	1328	5
	900	436	261	2333	6
	1600	802	523	3046	6
Berlin Tiergarten	100	29	1	4076	13
	400	117	75	11977	23
	900	255	285	18149	31
	1600	460	572	30583	39
Berlin Mitte center	100	29	4	3729	10
	400	117	86	9344	15
	900	280	278	17104	17
	1600	474	631	23818	26

TABLE 8.3: Average statistics for objective functions and shelter location results for the testbed considered at grid-point=1600

Network name	Shelters			Makespan			Budget			MaxLoad		
	min	median	max	min	mean	max	min	mean	max	min	mean	max
Small Swain Network	2.6	3.4	4.0	15.8	18.0	21.4	5043.2	6785.6	8244.8	500.0	670.0	874.6
Original Swain Network	2.0	3.1	4.0	17.0	22.6	30.4	3676.0	6205.8	8324.0	500.0	769.5	1107.4
Extended Swain Network	2.6	3.0	4.0	60.2	80.0	108.4	5227.2	6703.6	8477.2	500.0	697.8	960.0
Berlin Friedrichshain	2.6	3.5	4.0	50.0	64.0	75.6	6380.8	8537.0	10179.8	601.4	819.2	1040.0
Berlin Tiergarten	5.2	6.3	7.0	36.8	42.0	50.4	13484.8	16052.5	18081.0	544.2	737.9	1131.6
Berlin Mitte center	5.2	6.5	7.0	50.8	53.3	59.0	13736.0	16745.0	18599.2	582.8	680.9	864.0



(A) Pareto-optimal solutions of Berlin Tiergarten

(B) Pareto-optimal solutions of Berlin Mitte center

FIGURE 8.1: Examples of Pareto-optimal solutions

## 8.6 Conclusions

This chapter aims to introduce the mixed-integer programming (MIP) model we used to solve the QELP model: AUGMECON-R. First, we have discussed that it is important to find the Pareto-optimal set for a multi-objective optimisation problem as there is no single solution that can fit all. Then, we have discussed three classical methods in solving multi-objective optimisation problems: the weighted sum method, the lexicographic method and the  $\varepsilon$ -constraint method. Although these three classic approaches have many benefits in terms of simplicity, adaptability, and decision-maker engagement, we also critically analysed their limitations.

Furthermore, we have introduced the MIP we used to solve this quickest evacuation location problem: AUGMECON-R. It is the improved version of the original  $\varepsilon$ -constraint method, and by varying grid-points, AUGMECON-R can explore the Pareto-optimal solutions in a more robust way. The solutions obtained from the AUGMECON-R clearly show that there is an apparent regular conflict between three objective functions where different levels of budget and maximum load in each active shelter can significantly impact the makespan. Additionally, we also observe that the increase in the grid-points leads to an increase in the number of Pareto-optimal solutions. Meanwhile, it also increases the computational time.

Moreover, the results indicate that the computational time increases significantly as the network expands, emphasising the need to apply Matheuristics.

Matheuristics, combining mathematical programming with heuristic insights, has gained much attention and emerged as an attractive solution when the needs of real-world applications expand to incorporate more complex, large-scale multi-objective optimisation problems. In the next chapter, we will introduce a novel Matheuristics, which is applied to solve the QELP. A discussion and comparison of solutions obtained between the Matheuristic method and AUGMECON-R will also be presented in the next chapter.



## Chapter 9

# Solving the Quickest Evacuation Location Problem: A Novel Matheuristic Approach

### 9.1 Introduction

It is clear from the results of AUGMECON-R from the previous chapter that when the size of the network grows, the computing time dramatically increases. As a result, finding a quicker method to obtain relatively good quality solutions becomes vital. Matheuristics has gained much attention for its effectiveness and quicker computing time in solving large-scale instances. Therefore, this chapter starts with a brief overview of the fundamental concepts of Matheuristics. Then, the main focus of this chapter is on explaining the tailored Matheuristic method, which was developed to address the difficulty in solving large-scale networks in the quickest evacuation location problem. Moreover, a discussion and comparison of solutions obtained from the tailored Matheuristic method and AUGMECON-R are also presented.

### 9.2 An overview of Matheuristics

The Matheuristic method belongs to the broad idea of heuristics, which integrates traditional mathematical programming concepts such as linear programming, integer programming or bender decomposition (Boschetti et al., 2023). It is a hybrid optimisation methodology that combines the advantages of both mathematical programming (exact method) and heuristics, seeking to effectively tackle complex and large-scale optimisation problems by combining the strengths of these two methods.

There are many advantages of using Matheuristics. First, Matheuristics are well-known for their suitability and capability to solve large-scale and intricate optimisation problems with plenty of decision variables, constraints, and objectives. The use of mathematical programming helps to find the global optimality or near-optimality. Meanwhile, the use of heuristics can help to navigate massive solution spaces, which are very competent at solving large-scale, real-world problems. Therefore, the Matheuristic method is very suitable to be applied in those large-scale real-world problems. Furthermore, Matheuristics can generate high-quality solutions where the exact method used will provide high precision, and the heuristics will explore diverse solutions, which can improve the insights of the decision-making process.

To sum up, the exact method is guaranteed to get the optimal solutions, given sufficient computational time and memory. Heuristics do not always produce the best results, but they are often faster in terms of computational time and produce solutions that are "acceptable" in quality (Boschetti et al., 2023). By combining both of them, Matheuristics can achieve the balance between solution quality and computational effectiveness, leading to a faster computing process with relatively high-quality solutions. Therefore, we developed the tailored Matheuristic method to solve the quickest evacuation location problem, which will be thoroughly explained in the following sections.

### **9.3 A Matheuristic approach for approximating the Pareto Set**

Based on the results of the proposed original MIP model embedded in the AUGMECON-R scheme, and nevertheless, the use of grid-points can reduce the computational burden, it is clear that computational times increase dramatically as the network expands as a result of a concurrent growth in the number of nodes and arcs and in the time horizon, which poses the need for more efficient methods to approximate the Pareto Set on large size instances. In this section, a tailored multi-objective Matheuristic approach is designed and implemented to solve the QELP model, combining Mathematical Programming with a heuristic procedure to obtain good quality feasible solutions quickly on big instances.

In this case, the Matheuristic approach is framed within the AUGMECON-R scheme as a quicker alternative to solving the exact MIP model whilst exploring the Pareto Set. The main steps of the Matheuristic method can be described

as follows.

For each combination of epsilon values for budget ( $\epsilon_{budget}$ ) and maximum load ( $\epsilon_{maxload}$ ) within AUGMECON-R:

1. introduce a modified model (referred to as *Relaxed – QELP*) obtained as follows:
  - (a) perform a linear relaxation of the binary location variable  $z_i \in \{0, 1\}$  by substituting these with new location variables  $zr_i \in [0, 1]$ ;
  - (b) associate strongly increasing cost labels ( $k_t$ ) to the dummy arcs, which link the dummy time sinks to the final dummy super sink. In order to define such costs, we start by considering a very large value, say  $M_g$ , which can still be correctly coded within the modelling and computing environment;
  - (c) we then define each dummy arc label  $k_t$  based on the time instant  $t$  associated with the dummy arc to which it refers, as follows:  
substitute the original objective function with the following one:

$$\min \sum_{t \in T} k_t \cdot x_{S_t S_d} \quad (9.1a)$$

The original makespan objective function was substituted by this minimum cost objective function, in order to get rid of the binary variable  $y_t$ . Instead, this aims to minimise the total evacuation cost, thus acting as a linear proxy for the makespan minimisation and, in this way, avoiding the need for further binary variables associated with the dummy arcs;

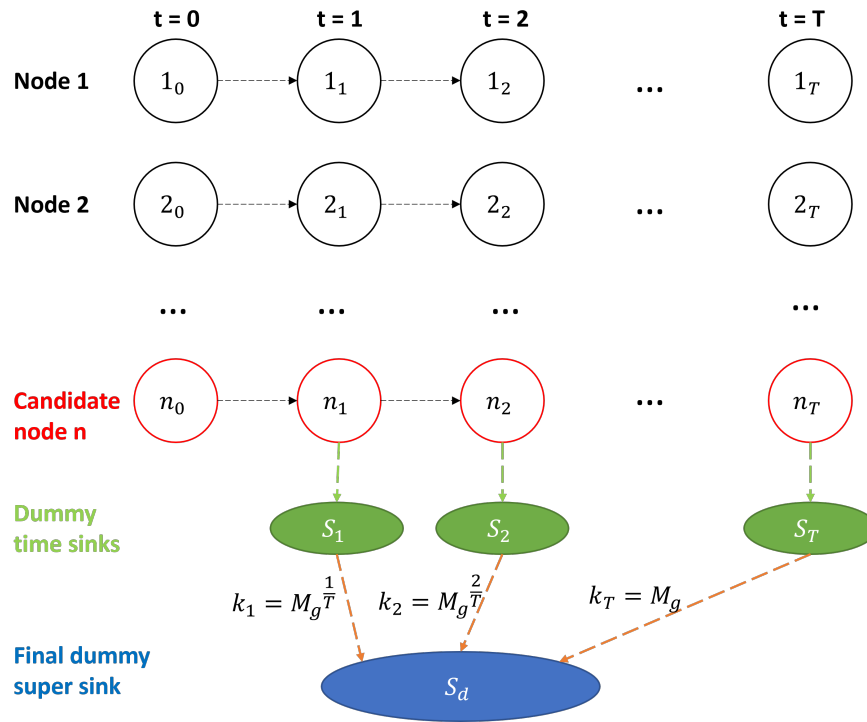


FIGURE 9.1: Minimum cost objective function of the Matheuristic approach

2. solve the *Relaxed – QELP* at the optimum. If there are no feasible solutions, skip the current combination of epsilon values (and go back to AUGMECON-R), while if an optimal solution is obtained for the *Relaxed – QELP*, go to step 3;
3. suppose that an optimal solution for *Relaxed – QELP* is obtained from step 2. If all the relaxed location variables are integers, then the solution is saved, being feasible for the original problem. If not, location variables  $zr_i$  (which could be fractional) are rounded up with a sequential randomised ordering based on the use of fractional variables as a probability until the epsilon (budget) is filled in. For example, if the value obtained equals 0.8, the possibility of the location variable rounded to 1 will be set as 0.8. In this case, the variable who gets 0.8 will have more chance to round up to 1 compared with the variable that gets the value of 0.4. Therefore, we can ensure a more fair and respective value for the location variable. Then, location variables are fixed;
4. finally, solve *Relaxed – QELP* again with the fixed location variables  $zr_i$ , and the solutions are obtained. If the arising solution is optimal and feasible, feed this as a solution for the pool.

The last two steps can be repeated for a given number of rounds in order to obtain multiple feasible location variable fixings (Step 3) and, in turn, multiple feasible solutions for the original problem (Step 4).

The whole procedure of the tailored multi-objective Matheuristic approach is further detailed and summarised in this paragraph. First, compute the payoff table using the lexicographic method for the total budget ( $B$ ) and maxload ( $ML$ ) that are going to be used as constraints and then compute the range for budget ( $r_{budget}$ ) and maxload ( $r_{maxload}$ ) based on the payoff table. After that, divide  $r_{budget}$  and  $r_{maxload}$  into  $g$  equal intervals. In this case, the total grid-points that are used to parametrically vary the RHS ( $e$ ) for the total budget objective function and the max load objective function, and the total number of runs becomes  $(g + 1) \times (g + 1)$ . One of the desirable characteristics of this method is that we can control the density of an efficient set by assigning different values to the grid-points. The more dense the grid-points, the more solutions we can get, and in turn, the longer the computation time it will be. In the experiments, we test experiments at varying grid-points: 100, 400, 900, and 1600 total grid-points in order to obtain comprehensive solutions. After defining the grid-point, we define the *Relaxed – QELP* problem as  $\hat{z}$  with relaxed location decision variables  $zr_{N_c}$ . Then, we initialise  $i$  and  $j$  and set them as 0. In particular,  $i$  is looping for the epsilon constraint for budget ( $\epsilon_{budget}$ ), and  $j$  is looping for the epsilon constraint for maxload ( $\epsilon_{maxload}$ ). For  $i$  in the range 1 to  $g + 1$  and for  $j$  in the range 1 to  $g + 1$ , the  $flag[i][j]$  is introduced, if  $flag[i][j] = 0$ , we compute  $\epsilon_{budget}$  and  $\epsilon_{maxload}$  and use them to solve  $\hat{z}$ . If the solution from  $\hat{z}$  is feasible and all integers, we save these solutions as they are the exact optimal solutions. If the solution is feasible but not all the solutions are integers, then we introduce a while loop to find as many solutions as possible. Within the while loop, we fix the location variables  $zr_{N_c}$  using a sequential randomised ordering until the epsilon (budget) is filled in. Then, solve  $\hat{z}$  again using the fixed location variables and compute the bypass coefficient for both the budget objective function ( $b_{budget}$ ) and the maxload objective function ( $b_{maxload}$ ). After that, update the flag matrix using the bypass coefficients. If  $flag[i][j] \neq 0$ , set  $i = g + 1$ , if  $i < g + 1$ , go back to the previous steps and solve  $\hat{z}$  again. If  $i > g + 1$ , set  $i = 0$ . In this case, if  $j < g + 1$ , set  $j = j + 1$  and go back to previous steps and solve  $\hat{z}$  again until it reaches  $g + 1$ . If there are no feasible solutions from  $\hat{z}$ , set  $i = i + flag[i][j]$  and repeat to solve  $\hat{z}$ . Loop until  $i$  and  $j$  all reach  $g + 1$ . Then, the solutions are saved, and the algorithm is completed.

---

**Algorithm 1** QELP-Matheuristic Procedure

---

```

1: Compute the payoff table
2: Set the upper bound for budget ( $Max\_budget$ ) and maxload
   ( $Max\_maxload$ )
3: Compute ranges  $r_{budget}$  and  $r_{maxload}$ 
4: Divide  $r_{budget}$  and  $r_{maxload}$  into  $g$  intervals (so the number of grid-points =
    $(g + 1) \times (g + 1)$ )
5:  $\hat{z} = \text{Relaxed-QELP}()$ 
6: Initialise  $i = 0$  and  $j = 0$ 
7: for  $j \leq g + 1$  do
8:   for  $i \leq g + 1$  do
9:     if  $flag[i][j] = 0$  then
10:       $\epsilon_{budget} = Max\_budget - i * \frac{r_{budget}}{g_{budget}}$ 
11:       $\epsilon_{maxload} = Max\_maxload - j * \frac{r_{maxload}}{g_{maxload}}$ 
12:      Solve  $\hat{z}$ 
13:      if  $\hat{z}$  is feasible and solutions are integers then
14:        Save the solutions
15:      else if  $\hat{z}$  is feasible but not all the solutions are integers then
16:        while  $Countattempts \leq Roundattempts$  do
17:          Fix the location variables  $zr_{N_c}$  at random
18:          Solve  $\hat{z}$  with fixed  $zr_{N_c}$ 
19:          Compute bypass coefficient  $b_{budget} = int(\frac{s_{budget}}{step_{budget}})$ 
20:          Compute bypass coefficient  $b_{maxload} = int(\frac{s_{maxload}}{step_{maxload}})$ 
21:          update  $flag[i][j]$  using bypass coefficients
22:        end while
23:         $i = i + 1$ 
24:        Repeat 9 - 23
25:      else
26:         $i = g + 1$ 
27:        if  $i < g + 1$  then
28:          Repeat to 9 - 26
29:        else
30:           $i = 0$ 
31:          if  $j < g + 1$  then
32:             $j = j + 1$  (until reach to  $g + 1$ )
33:          end if
34:        end if
35:      else
36:         $i = i + flag[i][j]$ 
37:        Repeat to 27 - 36
38:      end if
39: The solutions are saved and the algorithm is completed

```

---

## 9.4 The Matheuristic method: solution discussion and comparison

In order to evaluate the quality of the Matheuristic approach to increasing size networks, we adopted an additional network resembling the city of Barcelona, which consists of 1020 nodes and 3018 arcs ([Transportation Networks for Research Core Team, 2020](#)). The same approach is applied in developing instances where 21 candidate nodes were selected and fixed five sets of 51 demand nodes were randomly selected. To achieve a fair comparison, we set four hours of CPU time on each experiment for both the AUGMECON-R and the Matheuristic approach. Given that QELP aims to enhance evacuation design and planning and is applied during the pre-disaster phase. A CPU time of four hours is reasonable for the model to produce insightful solutions, considering the complexity of the problem and the large-scale networks we used to test the model.

This section compares and discusses the computational results of AUGMECON-R and Matheuristic approaches. Table 9.1 shows the average values on the computational process of the tailored Matheuristic approach by varying repetitions ranging from 3 to 11 in steps 3 to 4. As introduced above, step 3 and step 4 are repeated for several rounds. By using the different location variables fixings in step 3 and feeding them into step 4, we can explore a wider range of solutions and increase the likelihood of finding the optimal solution under each circumstance. In this work, we repeat for three, five, seven, nine, and eleven rounds. The algorithm is based on repetitions. Hence, three is considered as the minimum value to exploit this approach. In contrast, from the preliminary tests, repetition values up to 11 are values where the trade-off between quality and time increase appears particularly balanced. The growth in the number of repetitions can improve the quality and number of the solutions obtained but also increase the computational time, especially for a large-scale network like the Barcelona network. When the repetition value goes beyond 11, the increase in the computational time is far too high compared with the rise in the quality of the results. Therefore, we start with a small number of repetitions (e.g., 3) and gradually increase it until 11 repetitions to obtain the Pareto-optimal solutions. This ensures that the results obtained are robust and also allows for a comprehensive analysis of the performance of the matheuristic method.

The results in table 9.1 indicate that more repetitions lead to fewer feasibility

jumps and bypass jumps. This suggests that a higher number of high-quality solutions are obtained with fewer unnecessary computational jumps. Additionally, we can observe empirically that the number of Pareto-optimal solutions obtained on large networks also increases with an increase in the number of repetitions, highlighting the importance of considering different numbers of repetitions in order to obtain high-quality solutions. Overall, the variants with more repetitions in the Matheuristic approach seem preferable.

TABLE 9.1: Average values on the computational process of the Matheuristic approach

Network name	# Repetitions	Feasibility jumps	Bypass jumps	Cpu time (s)	# PO solutions
Small Swain Network	MH-3rep	780.6	96.4	601.6	8.4
	MH-5rep	777.6	84.2	873.8	8.4
	MH-7rep	770.6	69.4	1172.3	7.0
	MH-9rep	770.0	61.6	1433.5	7.6
	MH-11rep	770.0	55.2	1732.2	8.6
Original Swain Network	MH-3rep	603.4	210.2	2705.3	13.0
	MH-5rep	602.2	161.8	3733.0	13.2
	MH-7rep	593.8	192.8	5079.3	14.4
	MH-9rep	595.0	153.0	5966.0	14.2
	MH-11rep	593.6	191.4	7272.1	12.8
Extended Swain Network	MH-3rep	111.0	459.8	14416.6	12.4
	MH-5rep	61.8	334.4	14437.6	12.2
	MH-7rep	50.8	285.4	14427.1	11.0
	MH-9rep	36.8	289.6	14453.1	11.8
	MH-11rep	28.0	303.0	14447.1	12.2
Berlin Friedrichshain	MH-3rep	119.8	535.6	14420.6	8.0
	MH-5rep	61.6	343.2	14449.8	8.0
	MH-7rep	46.0	341.6	14460.9	7.6
	MH-9rep	35.8	294.6	14441.4	7.4
	MH-11rep	30.0	284.4	14429.2	7.6
Berlin Tiergarten	MH-3rep	4.2	266.4	14437.4	16.8
	MH-5rep	0	111.4	14490.3	19.6
	MH-7rep	0	98.6	14470.4	19.2
	MH-9rep	0	63.2	14542.8	17.8
	MH-11rep	0	75.6	14575.7	20.4
Berlin Mitte center	MH-3rep	8.2	207.8	14474.9	14.2
	MH-5rep	4.2	103.4	14446.9	14.8
	MH-7rep	2.6	97.4	14466.8	15.4
	MH-9rep	2.2	84.0	14549.7	14.2
	MH-11rep	1.4	47.2	14532.6	16.2
Barcelona Network	MH-3rep	14.6	575.8	16155.9	3.6
	MH-5rep	15.0	585.0	15959.5	3.8
	MH-7rep	15.2	592.8	17023.7	3.4
	MH-9rep	7.6	296.4	16219.5	3.4
	MH-11rep	7.6	296.4	19313.7	4.2

Table 9.2 shows the average statistics for the objective functions and the number of shelters located using the Matheuristic approach. First of all, from the maximum and minimum values of the objective functions, we can observe a regular range appears despite the number of repetitions because the increase in the number of repetitions usually mainly results in finding more intermediate points in the Pareto Set and only in a few cases the extreme points are



TABLE 9.2: Average statistics for objective functions and shelter location results for the testbed considered of the Matheuristic approach

Network name	# Repetitions	Shelters			Makespan			Budget			MaxLoad		
		min	median	max	min	mean	max	min	mean	max	min	mean	max
Small Swain Network	MH-3rep	2.8	3.3	4.0	15.6	17.6	20.6	5489.4	6991.3	8235.2	504.0	660.7	860.0
	MH-5rep	2.8	3.4	4.0	15.6	17.7	20.6	5489.4	7059.7	8258.8	504.0	662.6	881.0
	MH-7rep	2.8	3.3	4.0	15.6	17.8	20.6	5489.4	6892.9	8229.4	504.0	650.9	807.0
	MH-9rep	2.8	3.3	4.0	15.6	17.7	20.6	5489.4	6906.1	8229.4	504.0	642.8	792.0
	MH-11rep	2.8	3.5	4.0	14.8	17.3	20.6	5489.4	7060.7	8294.6	504.0	642.6	830.0
Original Swain Network	MH-3rep	2.0	3.0	4.0	17.4	22.8	29.6	3785.6	6207.1	8462.0	500.4	770.3	1123.6
	MH-5rep	2.0	3.0	4.0	17.6	23.0	30.0	3676.0	6101.1	8350.0	500.4	791.0	1174.4
	MH-7rep	2.0	3.0	4.0	17.0	23.2	30.2	3676.0	6071.2	8462.0	500.4	792.0	1152.8
	MH-9rep	2.0	3.2	4.0	17.2	23.1	30.4	3676.0	6172.9	8663.6	500.4	796.6	1132.4
	MH-11rep	2.0	3.0	4.0	17.2	22.9	29.6	3730.8	6154.7	8324.0	500.4	783.9	1128.4
Extended Swain Network	MH-3rep	2.6	3.5	4.0	60.2	79.8	101.8	5247.0	7019.2	8477.2	503.0	695.8	1001.0
	MH-5rep	2.6	3.5	4.0	60.2	78.6	97.8	5277.2	7106.4	8477.2	503.0	697.1	986.0
	MH-7rep	2.6	3.4	4.0	60.2	77.2	94.0	5297.0	7019.2	8477.2	503.0	704.2	956.0
	MH-9rep	2.6	3.5	4.0	60.2	78.7	95.0	5297.0	7038.6	8477.2	503.0	696.2	982.0
	MH-11rep	2.6	3.5	4.0	60.2	79.1	100.6	5297.0	6997.1	8477.2	503.0	706.0	986.0
Berlin Friedrichshain	MH-3rep	2.8	3.7	4.0	50.0	64.9	82.4	6936.6	8958.4	10277.4	608.8	795.1	1026.2
	MH-5rep	3.0	3.8	4.0	50.0	64.9	79.8	7532.4	9189.8	10277.4	608.8	767.3	907.6
	MH-7rep	3.0	4.0	4.0	50.0	64.5	79.8	7532.4	9298.2	10277.4	608.8	763.8	904.0
	MH-9rep	3.0	4.0	4.0	50.0	64.9	79.8	7532.4	9333.4	10277.4	608.8	763.3	904.0
	MH-11rep	3.0	4.0	4.0	50.0	65.3	79.8	7532.4	9209.8	10277.4	608.8	784.1	983.6
Berlin Tiergarten	MH-3rep	3.4	5.6	7.0	36.8	48.8	68.2	8850.8	14298.1	18169.8	545.0	885.0	1639.6
	MH-5rep	3.6	5.4	7.0	36.8	48.2	68.8	9100.8	14084.9	18169.8	545.0	904.6	1667.2
	MH-7rep	3.6	5.6	7.0	36.8	48.5	71.4	9222.8	13978.9	18169.8	545.0	893.3	1488.8
	MH-9rep	3.6	5.6	7.0	36.8	48.3	68.4	9182.6	14269.1	18172.2	545.0	889.9	1379.6
	MH-11rep	3.4	5.4	7.0	36.8	47.7	64.2	8887.6	14019.5	18223.4	545.0	948.8	1622.8
Berlin Mitte center	MH-3rep	4.0	5.5	7.0	50.8	61.9	78.0	10411.6	14719.8	18795.0	590.0	889.5	1475.6
	MH-5rep	3.4	5.2	7.0	50.8	62.7	85.2	9052.0	14111.1	18508.0	590.0	946.0	1519.0
	MH-7rep	3.8	5.4	7.0	50.8	62.8	84.8	9719.6	14126.7	18508.0	590.0	940.7	1669.2
	MH-9rep	3.4	5.6	7.0	50.8	62.1	87.2	8854.6	14143.4	18556.2	590.0	960.0	1646.4
	MH-11rep	3.4	5.3	7.0	50.8	63.6	87.2	8817.4	13758.0	18508.0	590.0	1068.9	1974.6
Barcelona Network	MH-3rep	10.2	11.9	13.8	75.6	78.4	81.6	25877.8	29951.2	34693.8	1550.4	1792.9	2054.8
	MH-5rep	11.0	12.0	13.4	75.6	79.6	88.8	27398.2	30476.7	33977.2	1547.2	1768.2	2063.2
	MH-7rep	8.8	11.3	12.8	75.6	78.4	81.6	21877.2	27635.9	32062.4	1547.2	1725.9	1960.0
	MH-9rep	9.0	11.0	12.4	75.6	78.5	81.6	22545.4	27212.5	31141.6	1547.2	1851.7	2214.4
	MH-11rep	8.8	10.5	12.4	75.6	78.4	84.0	22011.2	26541.8	31322.4	1547.2	1772.4	2099.8

affected. Second, similar minimum values of each objective function were obtained using the Matheuristic approach to those in the AUGMECON-R approach, indicating that the use of the modified minimum-cost function as an approximation of the makespan works very well in the Matheuristic approach. Moreover, similar ranges of values of three objective functions are observed compared with those obtained from the AUGMECON-R approach in the small size of networks. On the contrary, wider ranges were obtained in the larger size of networks such as Berlin Tiergarten, Berlin Mitte center, and Barcelona network, where more widely-spread solutions were obtained from the Matheuristic approach, which aligns with the findings in table 9.1.

Table 9.3 and figure 9.2 show the percentages of Pareto-optimal solutions obtained from each approach to compare the performance of the matheuristic method and AUGMECON-R method. In particular, figure 9.2 is a stacked bar chart using the same data as the table 9.3, in order to see how each method contributes to the total PO solutions for a clear and more straightforward comparison. The results indicate that the quality and dominance of solutions obtained from the Matheuristic approach appear appropriate. In fact,

the Matheuristic approach always manages to find some solutions which even outperform other approaches when the times are comparable, which can be seen in figure 9.2. Moreover, as discussed, more repetitions lead to relatively more Pareto-optimal solutions. Therefore, we compared the dominance of solutions between AUGMECON-R and the 11 repetitions (MH-11rep) for each network to get an insight into the quality of solutions, which can be seen in table 9.4 and figure 9.3. In particular, table 9.4 shows the average percentage of PO solutions of AUGMECON-R and MH-11rep, which are visually represented by the stacked bar chart in the figure 9.3 for a clear comparison. The percentages of MH-11rep confirm that high-quality Pareto-optimal solutions were obtained from the Matheuristic approach, and this is more apparent in larger size of networks like Berlin Tiergarten, Berlin Mitte center, and Barcelona network where MH-11rep outperforms the AUGMECON-R approach. Overall, the Matheuristic technique secures remarkable quality results and is able to solve more extensive networks.

TABLE 9.3: Average percentage (%) of PO solutions of AUGMECON-R and the Matheuristic approach at grid-point = 1600

Network name	Size (# nodes)	AUGMECON-R (%)	MH-3rep (%)	MH-5rep (%)	MH-7rep (%)	MH-9rep (%)	MH-11rep (%)
Small Swain Network	37	100	10	10	12	12	10
Original Swain Network	55	91	27	28	31	33	30
Extended Swain network	150	81	36	38	39	33	38
Berlin Friedrichshain	224	75	29	24	28	24	20
Berlin Tiergarten	361	33	32	34	30	23	27
Berlin Mitte center	398	42	19	24	22	23	24
Barcelona network	1020	35	6	8	17	15	19

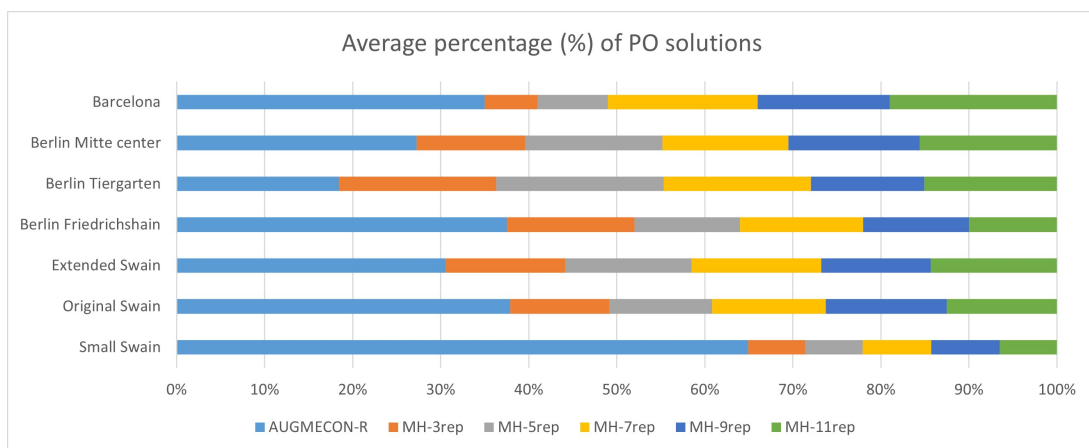


FIGURE 9.2: Average percentage of PO solutions

To sum up, the same results on conflicts and trade-offs of three objective functions are observed from the Matheuristic method. By comparing the dominance of the solutions obtained from the Matheuristic with the solutions from

TABLE 9.4: Average percentage (%) of PO solutions of AUGMECON-R and MH-11rep at grid-point = 1600

Network name	Size (# nodes)	AUGMECON-R (%)	MH-11rep (%)
Small Swain Network	37	100	10
Original Swain Network	55	94	32
Extended Swain network	150	83	43
Berlin Friedrichshain	224	75	32
Berlin Tiergarten	361	41	62
Berlin Mitte center	398	47	59
Barcelona network	1020	38	62

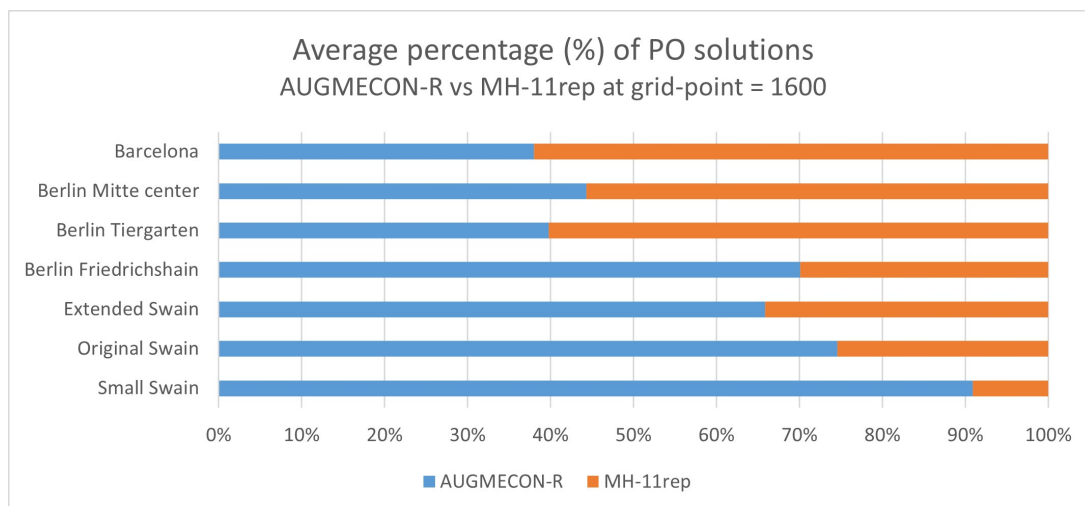


FIGURE 9.3: Average percentage of PO solutions: AUGMECON-R VS MH-11rep

AUGMECON-R, the results show that the dominance of Matheuristic solutions appears appropriate. In particular, in big-size networks like Barcelona Network, as figure 9.3 shows, the Matheuristic method generates more percentages of Pareto-optimal solutions, which means that in the same amount of computational time, the Matheuristic method can find higher percentages of high-quality Pareto-optimal solutions.

## 9.5 Conclusions

This chapter has provided an introduction to Matheuristics, a hybrid approach combining mathematical programming and heuristics to address the computational difficulty of large-scale problems. We first provided a brief overview of Matheuristics and its advantages compared with the exact method where Matheuristics can find relatively high-quality solutions in a timely manner.

Furthermore, this chapter has introduced and discussed a tailored Matheuristic method. From the comparison of solutions obtained from the Matheuristic method with the AUGMECON-R, it is clear that the Matheuristic can generate more percentages of high-quality Pareto-optimal solutions within the same computational time than the AUGMECON-R method. Therefore, the tailored Matheuristic method works well in solving the quickest evacuation location problem, especially in large-size networks.

## Chapter 10

# Conclusions and Future Directions

### 10.1 Introduction

This chapter presents a comprehensive summary of the thesis, beginning with an in-depth conclusion of the QELP, its associated model and the tailored Matheuristic method. Furthermore, it dives into the theoretical contributions and managerial applications derived from the QELP. Finally, this chapter discusses potential future directions aimed at improving the QELP and its model, building upon the underlying assumptions to enhance its further effectiveness.

### 10.2 A summary of this research

In this research, we introduced the novel Quickest Evacuation Location Problem (QELP), which combines the discrete facility location problem with the quickest flow problem, aiming at providing a reliable and realistic decision support system in increasing the efficiency of humanitarian operations and enhancing the evacuation network design and planning. The introduction of the QELP fulfil the research aim we proposed, where the goal of QELP is to take into account the dynamics within the evacuation process into the location decisions of facilities in the context of humanitarian operations, to better monitor and control the evacuation process and select the optimal locations for shelters and to better prepare for the upcoming disasters. As a result, a smooth and effective evacuation design and planning can be achieved, which in turn, reduces the economic losses and human suffering.

We also introduced a novel modelling tool, QELP-Time Expanded Network, to represent the QELP, which builds upon the techniques from the original Time Expanded Network by encompassing non-predetermined sink nodes,

dummy time sink, final dummy super sink, and sets of dummy arcs for securing a more straightforward and time-saving computational process while obtaining the decisions for locations. An original multi-objective mixed-integer programming model is proposed, considering facility location decisions while minimising evacuation makespan, the total budget required to install and operate the shelters, and balancing the load of evacuees directed to each activated shelter.

A tailored Matheuristic approach is also designed and implemented to solve the model with seven realistic networks ranging from a 37-node small network to a 1020-node extensive network with five different demand scenarios for each network. High-quality and practical solutions are obtained from the Matheuristic approach, which turns out to be suitable to be adopted in solving real-world cases. For example, the results show that only eight to twelve shelters are activated among the total 21 candidate shelters in the Barcelona network. This confirms the suitability of QELP and its MIP model for finding realistic optimal shelter configurations in such large networks. Meanwhile, the results show that the Matheuristic approach outperforms the original MIP model on large-scale networks by generating higher percentages of Pareto-optimal solutions within the same computational time, which offers more alternatives for the purposes of the decision-making process. Regarding computational perspective, the Matheuristic approach is therefore applicable and achievable, indicating that QELP can effectively and efficiently assist in the strategic decision-making process.

These together have provided promising answers to the research question proposed at the beginning of this research by covering the critical elements in formulating an effective mathematical model to achieve the research aim and addressing the gaps identified from the critical literature review:

- **RQ1:** What are the most appropriate objective functions to be modelled and adopted? What are the benefits of a multi-objective model?
- **RQ2:** What are the key specific modelling features that should be considered when formulating the optimisation model for QELP? Besides objective functions in **RQ1**, What are the constraints, parameters, and decision variables?
- **RQ3:** How to solve the QELP optimisation model in **RQ2** in such a way that it will generate good solutions in a reasonable computational time?

The introduction of QELP in the research aim and the steps used to answer the research questions are developed to achieve the research objectives introduced in chapter 1. Here is a detailed explanation.

- to critically review the literature in the field of shelter location and evacuation modelling in Humanitarian Operations. **This is achieved in the chapter 2, chapter 3, and chapter 4;**
- to identify and analyse the main characteristics of existing models to assist the decision-making process in the field of shelter location and evacuation modelling in Humanitarian Operations in order to overcome current limitations and develop more efficient and impactful approaches. **This is achieved in the chapter 2, chapter 3, and chapter 4;**
- to identify the gaps in the existing literature based on the critical literature review of location analysis in humanitarian operations and network flow in evacuation modelling. **This is achieved in the chapter 5;**
- to develop a novel optimisation model aimed at supporting shelter location and evacuation modelling in Humanitarian Operations, so as:
  - to address the operational needs in the disaster contexts;
  - to improve the effectiveness of disaster relief, particularly in enhancing the shelter location in evacuation design and planning;**These are achieved in chapters 6 and 7.**
- to test the developed model on synthetic, computer-generated (still realistic) instances, inspired, when appropriate and possible, by real-world situations through secondary data to increase potential support to prospective users (decision-makers) to solve the combination problems of facility location problem and network flow problem arising in humanitarian operations. **This is achieved in the chapters 8 and 9.**

### 10.3 Theoretical contributions

First of all, we introduced a new multi-objective mixed-integer programming model to define the optimal location for shelters while taking into account the dynamics in the evacuation design and planning by combining the discrete facility location problem with the quickest flow problem. The Quickest Evacuation Location Problem not only aims to reduce the makespan but also minimise the containing budget and balance the load across activated shelters,

which provides an actionable decision support system for evacuation design and planning.

Furthermore, we also introduced a tailored QELP-TEN tool to model the QELP features. The QELP-TEN was built upon the original TEN by adding a set of dummy time sinks, a set of dummy arcs, and a final dummy super sink without any predetermined sink nodes so that QELP-TEN can be more effective in tracing the makespan of every group of demands while finding the optimal location for shelter nodes at the same time. In this case, the QELP-TEN can be applied to other circumstances that ask for the location decisions and compute the makespan simultaneously.

Moreover, we answered the research questions carefully through the development of the multi-objective mixed-integer programming model to tackle the quickest evacuation location problem. In order to solve the large-scale instance more quickly and efficiently, we developed a tailored Matheuristic method, and the results indicate that the quality of the solutions is appropriate and can be obtained within a reasonable computational time.

## **10.4 Managerial impact for humanitarian applications**

The QELP proves to be able to reduce the makespan of the evacuation process, enabling quicker rescue and reducing human suffering in humanitarian operations and emergency management. The QELP also reduces the total budget, which will help humanitarian organisations save money. Even though reducing the budget is not often the main goal in humanitarian operations, it is better to save money while taking into account other primary humanitarian operation goals.

Furthermore, the QELP can minimise the maximum load for each shelter, which can spread the risk of each shelter. In reality, some shelters may suffer a higher risk of damage than others, and balancing the load of these activated shelters can spread the risk and result in overall resilience in the whole system.

Moreover, the QELP can support the decision-making process as the results indicate a clear trade-off between three objective functions (makespan, budget, and maximum load), which brings in significant managerial applications from a real-world perspective. Setting different budget levels and a maximum load plan can effectively reduce the makespan, which strongly indicates that



QELP can support decision-makers in finding the most convenient solutions based on appropriate policy-making considerations. Also, the QELP can provide different options (set of Pareto-optimal solutions) to the decision-makers, enabling them to make the most suitable decisions based on their resources and the needs of the policy-making. These align with the Sustainable Development Goals (SDG) (United Nations office for Disaster Risk Reduction, 2020) for better health and well-being.

## 10.5 Future research directions

This research introduces the novel Quickest Evacuation Location Problem, the QELP multi-objective mixed-integer programming model, and a proposed Matheuristic method. Many potential future directions can be followed to extend the QELP and make it more applicable in various real-world scenarios.

### **Towards multiple-commodity modelling**

This research treats the population from all demand nodes together. It aims to minimise the overall makespan for the purpose of efficient evacuation design and planning from a macroscopic point of view. In other words, we did not distinguish each demand (people) and treated them together in evacuation design and planning.

Future developments of this research might explore a multiple-commodity approach from a microscopic perspective, thus differentiating the makespan for populations associated with different demand nodes and making it easier to keep track of the evacuation process of each demand node. This may allow increased flexibility to adjust evacuation design and planning for different disaster situations, allowing, for instance, the introduction of priorities or further constraints related to operations features. For example, some seriously injured people or the elderly and children can be prioritised in evacuation, which can be considered using the multiple-commodity modelling.

### **From car-based evacuation to bus-based evacuation**

This research deals with self-evacuees, which means that the evacuees use their private vehicles for evacuation. In this case, we did not consider the cost of transportation in the quickest evacuation location problem as evacuees follow the guidelines and move to the shelters by themselves.

Future developments of this research can take into account the different evacuation modes, such as public mass evacuation (which is also called bus-based

evacuation) mode. According to [Esposito Amideo et al. \(2019\)](#), evacuees can be categorised into three different types: i) self-evacuees who move towards a shelter (SES), ii) self-evacuees who move towards other destinations (SED), iii) and supported evacuees who move towards shelters (SE). These different types of evacuees can be considered as a further development of the QELP. In this case, multi-model evacuation can be developed so that different types of evacuees can be considered in the problem. Furthermore, the problem considering public mass evacuation or assisted evacuation can be modelled using the location-routing problem techniques. Similar to what has been suggested by [Esposito Amideo et al. \(2019\)](#), the QELP can be extended by incorporating various modes of transportation for a better and more comprehensive representation of humanitarian operations.

### **Considering various evacuee behaviours**

In this research, the quickest evacuation location problem is developed based on the assumption that evacuees will strictly adhere to given instructions. In this way, we can optimise the flow for the quickest evacuation. However, it is essential to acknowledge that real-world humanitarian operations are quite different from this ideal scenario. Evacuees might have their own route preferences, leading them to diverge from planned routes. Additionally, some may choose to go to different places first to reunite with family and friends before evacuating together, which makes the problem more complex.

Several factors, such as the time of day, route diversion, demographics, route preferences, and warning signals, can significantly influence evacuee behaviours during the evacuation process ([Esposito Amideo et al., 2019](#)). Acknowledging these behavioural dynamics and incorporating them into our research would make QELP more capable of representing real-world cases. Therefore, future development of this study can take into account some of these evacuee behaviours.

### **Dealing with uncertainty concerns**

As introductory research, the quickest evacuation location problem did not consider any uncertainty in its modelling. However, uncertainty is one of the common and challenging issues in the context of humanitarian operations due to unpredictable disasters. In humanitarian operations, there are three common uncertainties: i) uncertainty in supply, ii) uncertainty in demand, iii) and uncertainty in network connectivity ([Dönmez et al., 2021](#)).

The uncertainty in supply within the context of humanitarian operations refers to the unpredictability of changes in the availability of humanitarian resources,

such as food, water, services, and other aids, which arises from the disruptions of disasters or the changing nature of donations. The uncertainty in supply, particularly the lack of supplies, has a considerable impact on the effectiveness of humanitarian operations. The uncertainty in demand is often referred to as the uncertain nature of the demand, which is the number of people who need aid in the disaster. Due to the predictable nature of disasters, the number of demands or what kind of demand each person needs is uncertain. Additionally, the different evacuee behaviours can also lead to uncertainty in demand. Finally, the uncertainty in network connectivity is related to situations where disaster-related traffic congestion or road damages influence the overall connectivity of the network, which further increases the difficulty in the logistical challenges of humanitarian operations.

Future developments can be made by considering these uncertainties in expanding the QELP, using Stochastic programming or Robust programming to tackle the uncertainties, which will lead to a more resilient and robust evacuation network design and planning.

#### **Strengthening the exact (MIP) solution methods**

Additional lines of research are related to the design of tailored exact algorithms for speeding up the solution of the QELP during the exploration of the Pareto Set. Although, the tailored Matheuristic method can efficiently generate high-quality solutions within a reasonable amount of time. Due to the nature of Matheuristics, as discussed, it may not cover all the Pareto-optimal solutions. Therefore, it is worthwhile to develop new exact algorithms to find the Pareto set of the Quickest Evacuation Location problem as well as reduce the time needed.

#### **Combining with other OR methods**

We only applied optimisation techniques in forming and solving the QELP. In the future, it is worth studying the integration between the optimisation methods as mentioned above and cutting-edge multiple criteria methods, for example, simulation, and agent-based optimisation modelling, in such a way to enable, in potential future developments and applications of the QELP, feeding of the models with parameters potentially extracted from direct experience of stakeholders. By considering the opinions of different stakeholders in expanding the QELP, the quickest evacuation location problem can be more reliable and practical in solving various real-world problems.

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