Essays on the Effects of Unconventional Monetary Policy in Economies with Frictions

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Abstract

In response to the financial crises of the 2000s, central banks implemented unconventional monetary policies (UMP) to restore liquidity and promote credit creation. This thesis investigates the effects of these policies, which involved using the composition and size of central bank balance sheets as instruments of policy. Specifically, this thesis examines the theoretical foundations and quantitative transmission mechanisms of UMP in scenarios where interest-rate-based monetary policy is constrained by the lower bound of the nominal policy rate.

The first two papers of this thesis present theoretical models of a three-period economy, where households, firms, and a consolidated government interact. In the context of a liquidity trap scenario and the presence of nominal rigidities, the first paper shows that UMP in the form of outright purchases of illiquid assets by the central bank have no real effects in a perfect risk-sharing environment but can provide partial insurance against idiosyncratic unemployment risk under imperfect risk-sharing. The second paper examines the effects of UMP when an uncertainty shock generates the liquidity trap scenario and shows that, in this context, UMP can improve welfare.

The third paper of the thesis provides a quantitative assessment of the effects of UMP. Using a dynamic model of a small open economy member of a monetary union, I show that UMP in the context of a negative shock to the price of sovereign bonds, has positive effects on output and unemployment. By combining theoretical and quantitative analyses, this thesis sheds light on the complex mechanisms through which central bank balance sheet policies affect the economy in different scenarios and provides insights for policymakers and researchers alike.

Declaration

I, Néstor Iván González-Quintero, declare that this thesis, which consists of an introduction and three chapters, is a presentation of original work and I am the sole author. This work has not previously been presented for a degree or other qualification at this University or elsewhere. All sources are acknowledged as references.

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Introduction

This thesis concerns the effects of what has been called by the literature and policy actors as unconventional monetary policy. In the heat of the great financial crisis of 2007–2008 and the sovereign debt crisis of the European economies in 2011-2012, the central banks of most of the advanced economies exhausted the available room for monetary policy based on interest rate management and had to resort to the use of the composition and size of their balance sheet as unconventional instruments of policy to: *i)* restore liquidity in highly impaired asset markets and, *ii)* give a needed boost to credit creation which was severely hit by the liquidity drought of financial markets.

The portfolio of policies implemented by central banks like the Fed in the US, the Bank of England in the UK, and the European Central Bank (ECB), included, for instance, the increase of the intensity of traditional short-term REPO financial operations, the increase of the term associated with these open market operations[1], and a significant expansion of the set of assets allowed to be used as collateral by the different actors dealing with the central banks. In the most critical times of these particular episodes, the composition and size of the balance sheets of these central banks were massively adjusted via outright purchases of different sorts of short and long-term assets, including sovereign bonds.

After this radical change in the implementation of monetary policy, the literature began to determine not only the effects of the practical implementation of this kind of policy but also its possible theoretical foundations. Until now, no clear consensus has been reached on any of the two fronts, but big steps have been taken

[1] This was the case of the long-term refinancing operations (LTRO) implemented by the ECB after 2011

to better understand the nature of the transmission mechanism of these policies. The basic reference in the theoretical side is the neutrality of open market operations result brought by Wallace (1981), which was later renewed by Eggertsson and Woodford (2003) in the context of the New Keynesian model of business cycle analysis. Using these works as a reference and starting point, the analysis provided by the more modern literature has tended to conclude that for unconventional monetary policy to have real effects, the theoretical foundations must rely on the presence of financial frictions.

This is where this thesis pretends to make a significant contribution. I present three papers in which the effects of unconventional monetary policies in the form of REPO or outright purchase operations are assessed from a theoretical and a quantitative point of view. Each chapter highlights a different and important aspect of the transmission mechanism of these policies, but what they share in common is the particular general context in which the assessment is carried out. In particular, I consider a context in which the interest-rate-based conventional monetary policy has no more room to exploit, because of the existence of a natural limit or lower bound on the policy nominal rate and the presence of nominal rigidities, which extend such bound to the real interest rate. This type of scenario has been called a *liquidity trap* by the literature (e.g. Korinek and Simsek, 2016; Krugman, 1998).

In this sense, the assessment provided by each of the chapters cannot be taken outside the described context. In other words, I consider the effects of unconventional monetary policy in a particular crisis scenario, which leaves absent any other sort of assessment regarding, for instance, the consequences of the continued expansion of the balance sheets of the central banks on inflation, the credibility of the monetary authority, or the effects of fiscal dominance on conventional monetary policy once the economy is off the lower bound.

The contribution of the first two chapters of this thesis is purely on the theoretical side of the analysis of the transmission mechanism of unconventional monetary policy. Both chapters share the same basic framework which is a tractable model of a three-period economy composed of households, firms and a consolidated government in charge of both monetary and fiscal policy. [2] At the beginning of their lives, households are endowed with a fixed supply of a *Lucas-tree* asset which is perfectly illiquid and the unique source of consumption in the final period.

In the first chapter the liquidity trap scenario is generated by an unexpected level shock on the value of the future dividend of the illiquid asset, which affects the equilibrium prices and allocation once the zero-lower-bound of the policy interest rate is reached. The bound on the nominal rate imposes a bound on the real interest rate because of the existence of a nominal rigidity, and a positive unemployment rate emerges to characterise the new equilibrium path. In the baseline scenario of the chapter, potential workers supplying inelastically a unit of indivisible work, gather into households to secure themselves either via savings o intertemporal transfers between household members. This is what I consider a perfect risk-sharing scenario. Unconventional monetary policy in the form of outright purchases of the illiquid asset by the central bank is completely neutral.

Once we abandon the perfect risk assumption, things turn quite interesting. Members of the households are no longer allowed to transfer income in the inter-period and the main mechanism of self-insurance is severed. Unconventional monetary policy then has room to offer partial insurance against idiosyncratic unemployment risk, and its implementation is welfare improving via the reduction in the equilibrium unemployment rate. This characterisation of the transmission mechanism via insurance in an imperfect risk-sharing environment has not been made openly explicit by the literature and in that sense the contribution is important.

In the second chapter, the nature of the shock that generates the liquidity trap scenario is rather different from the one in the first chapter. In this case the dividend of the *Lucas-tree* asset is intrinsically random but characterised by a known two-state distribution. The shock consists of an increase in the known probability of the lowest-dividend state which increases the perception of uncertainty. Without leaving the perfect risk-sharing scenario, two types of unconventional monetary

[2] A warning must be included here for the reader. To be presented as independent papers with a potential purpose of future publication, these two chapters might often repeat themselves, especially in the sections describing the model and its equilibrium. Nonetheless, the key contributions are importantly different.

policy are assessed here. The first one is a REPO operation based on the households' holdings of the illiquid asset, whereas the second is an outright purchase like the one modeled in the first chapter. Because of its pure Ricardian nature, the REPO policy is completely neutral but, even under perfect risk sharing, the outright purchase policy is welfare-improving. In this case, the central bank fully insures households from uncertainty via the purchase of the increasingly risky asset, by assuming the aggregate consequences of risk and giving back a certain source of income in the form of a fiscal transfer. In this case, the fiscal side of the unconventional policy is key in its transmission to the economy. This is a novel analysis of the transmission mechanism of unconventional monetary policy which has been barely studied by the literature, although it has been an important part of the practical discussions of policy.

Chapter 3 takes a rather different approach. It concentrates specifically on the assessment of unconventional monetary policy in the context of a sovereign debt crisis. The framework of the analysis is a model of a periphery small open economy member of a monetary union that takes monetary policy as an exogenous object established by an external central bank, which has already fixed the short-term policy interest rate. The model highlights the role of an exchange rate peg and the credit channel of unconventional monetary policy. Specifically, I model banks for which credit activity may be restricted by a lending constraint determined by its net worth, mainly composed by local random maturity sovereign debt subject to price shocks. In this context, the analysis numerically solves the dynamic model taking as reference the optimal debt problem of the sovereign in the presence of a set of correlated shocks which affects the lending constraint of the banks and its intermediation role.

The external central bank implements a policy similar to an outright purchase of sovereign bonds in the baseline scenario. By performing multiple simulations of the model under the presence of unconventional monetary policy I select those that can be identified as a sovereign debt crisis. The effects of the implementation of policy are indirectly measured by a counterfactual scenario in which unconventional monetary policy is *switched-off*. Qualitatively, unconventional monetary policy has real effects via the credit channel, but once the model is calibrated to the economy of Spain the effects seem to be quantitatively small.

Chapter 1

Risk sharing and the effects of unconventional monetary policy in a liquidity trap

1.1 Introduction

During the great financial crisis of 2007–2008 and the European sovereign crisis of 2011–2012, advanced economies went through a crisis scenario characterised by a significantly weakened aggregate demand accompanied by increasing unemployment rates. As a response, both the Fed and the ECB systematically decreased their nominal interest policy rates to the point they reached their lowest bound but, once there, the negative effects of the crisis were far from disappearing. As an alternative response, central banks used their balance sheets as an instrument of systematic policy to restore credit activity and liquidity in heavily distorted markets (Lenza et al., 2010; Reichlin, 2014a). Not only the terms of standard REPO refinancing operations were increased, but direct interventions in the form of outright purchases of different sorts of illiquid and toxic assets made part of the unconventional monetary policy toolkit of these institutions.

There has not been a great deal of consensus on the effects of this set of unconventional policies, though. The empirical literature has found significative effects of the real effects of unconventional monetary policy (e.g. Hachula et al., 2020), which clashed with the classical theoretical neutrality results regarding the effects of open market operations (Eggertsson and Woodford, 2003; Wallace, 1981). Nonetheless, recent theoretical approaches have introduced financial frictions into the standard models and have found a space for unconventional monetary policy to have real effects. Part of this line of research has focused its attention on the role of collateral or liquidity constraints as the base to understand the transmission of this kind of policies (Araújo et al., 2015; Del Negro et al., 2017b), whereas another has highlighted the role of financial frictions as the key to understand its transmission channel via financial intermediation (e.g Cui and Sterk, 2021; Cúrdia and Woodford, 2011; Gertler and Karadi, 2011)

The purpose of this paper is to add to the theoretical discussion on the effects of unconventional monetary policy by focusing the attention on a novel risk-sharing transmission channel of policy. The approach in this paper consists on the comparison of two extreme scenarios. First, in an economy with perfect risk sharing, a neutrality result on the effects of unconventional monetary policy is established out of the Ricardian nature of policy in this risk-sharing scenario. Then, an imperfect risk sharing scenario is considered. In this particular context, a particular form of unconventional monetary policy which consists on an outright purchase of illiquid assets, backed up by a fiscal transfer to households, has real and welfare-improving effects by being a means of idiosyncratic unemployment risk insurance.

The results come from the analysis of a stylised and tractable three-period horizon model in which, by following the approaches in Shi (2015) and Heathcote and Perri (2018), workers gather into households to fully or partially insure the consequences of unemployment idiosyncratic risk. In the initial period of their finite horizon of life, households are endowed with a fixed supply of a perfectly illiquid asset which yields a real return only in the final period of the economy. On their part, household members are potential workers that inelastically supply an indivisible unit of time to the labour market in the first two periods of their lives. In the second period of this economy, because of the existence of a nominal rigidity and a real low bound on the real interest rate, labour supply might be rationed, and households provide insurance via their intertemporal decision on consumption and savings into a perfectly liquid asset.

In the model, the central bank determines the value of the nominal interest rate on the liquid asset via the implementation of an interest-rate-based conventional monetary policy rule, which is constrained by the existence of a zero lower bound. The liquidity trap scenario in which the effects of conventional monetary policy are analysed, is generated by an unexpected shock on the future value of the dividend of the illiquid asset in hands of the households. Once the ZLB bounds because of the response of conventional monetary policy to the shock, the equilibrium of the stylised economy is characterised by a positive probability of unemployment risk regardless of the risk-sharing within households assumption. It is in this context that the central bank opens its toolkit and implements an outright purchase market operation based on the household's holdings of the illiquid asset.

Including this introduction, the paper is divided into five sections. The baseline model under perfect risk-sharing within households is presented in Section 1.2. Section 1.3 characterises conventional and unconventional monetary policy under the perfect risk-sharing assumption, whereas Section 1.4 introduces imperfect risk-sharing into the model and reassesses the role of the unconventional monetary policy implemented by the central bank. The final section concludes.

1.2 The model

The closed economy is set in a finite horizon of three periods $t = \{0,1,2\}$. There is a perishable consumption good in the economy which is produced in periods 0 and 1 by a competitive firm using a continuum of intermediate varieties, each produced by a monopolistically competitive firm using only labour. In the final period, the consumption good is the yield of a Lucas-tree asset in the hands of households. The economy is inhabited by a continuum of households and a consolidated central government in charge of monetary and fiscal policy. The domestic currency is the *numeraire* of all nominal prices, whilst the perishable consumption good plays the same role for all relative (real) prices.

1.2.1 Production sector

Production of the perishable consumption good. In periods $t = \{0,1\}$, a competitive representative firm produces an amount Y_t of the perishable final good using a set of intermediate varieties $j \in [0,1]$, with technology:

$$Y_t = \left[\int_0^1 y_{jt}^{\frac{\eta-1}{\eta}} \, \mathrm{d}j \right]^{\frac{\eta}{\eta-1}} \tag{1.1}$$

With elasticity of substitution of varieties $\eta > 1$.

Given the money price p_{jt} of each intermediate variety, and the nominal wage W_t paid to one unit of labour, the static problem of the competitive representative firm in each period is to minimise costs $\int_0^1 p_{jt} y_{jt} \, \mathrm{d}j$ subject to equation (1.1). The solution to this minimisation program determines a demand function for each intermediate variety j such that:

$$y_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\eta} Y_t, \tag{1.2}$$

where the money-price of an additional unit of the produced perishable good is given by

$$P_t = \left(\int_0^1 p_{jt}^{1-\eta} \, \mathrm{d}j \right)^{\frac{1}{1-\eta}}. \tag{1.3}$$

In period $t = 2$, production of the perishable final good depends only on the real dividends $D > 0$ of a fixed-supply $\bar{h} = 1$ of a Lucas-tree asset in hands of the households. Hence, the total supply of the perishable consumption good is:

$$Y_2 = D \tag{1.4}$$

Production of the intermediate varieties and aggregate outcomes. In every period $t = \{0, 1\}$, a size-one continuum of monopolistic competitive firms, each one indexed by $j \in [0, 1]$, is in charge of the production of the intermediate varieties. An amount y_{jt} of the intermediate variety j is produced using only specialised labour N_{jt} via the linear technology:

$$y_{jt} = N_{jt} \tag{1.5}$$

At the beginning of period 0, each intermediate-producer firm's objective is to maximise its discounted flow of profits along the first two periods of the economy. As in Korinek and Simsek (2016), I assume that all intermediate-producer firms optimally determine their price in $t = 0$ and decide to fix it for all the production horizon. Given the nominal price rigidity assumption, if the interest rate policy of the monetary authority is constrained by a zero-lower-bound (ZLB), the equilibrium allocation of the economy might be one in which both final production and labour are purely determined by demand. Labour might be rationed in the labour market in the sense of Bénassy (1993), namely $N_1^d = \int_0^1 N_{j1} \, \mathrm{d}j \leq 1$. As a consequence, at the beginning of period 0 intermediate firms maximise their intertemporal flow of profits such that there is full employment in period 0 and that possible rationing of labour implies their labour demand to be determined as $1 - u$, where u is the proportion of specialised workers let go as a consequence of rationing.

Let $\Gamma_{jt} = p_{jt} y_{jt} - W_t N_{jt}$ be the period flow of nominal profits of the producer of an intermediate variety. Subject to the nominal price rigidity, the demand for its production given by equation (1.2), and its technology of production summarised by equation (1.5), the problem of the monopolistic competitive firm at the beginning of period 0 is to choose its fixed relative price $\frac{p_{j0}}{P_0}$ in order to maximise the expected discounted sum of profits

$$\left(\frac{p_{j0}}{P_0}\right)^{-\eta} \left[\left(\frac{p_{j0}}{P_0} - \frac{W_0}{P_0}\right) Y_0 + \Lambda_{0,1} \left(\frac{p_{j0}}{P_0} - \frac{W_1}{P_0}\right) Y_1\right].$$

where $\Lambda_{0,1}$ is the discount factor of firms. From the first order condition of this problem, the optimal relative price set in periods 0 and 1 is:

$$\frac{p_{j0}}{P_0} = \frac{\eta}{\eta - 1}\left(\frac{1}{1+\beta}\right)\left(\frac{W_0}{P_0} + \beta \frac{W_1}{P_0}\right) \tag{1.6}$$

which implies that aggregate real profits in periods 0 and 1 are given by:

$$\frac{\Gamma_t}{P_t} = \int_0^1 \left(\frac{p_{j0}}{P_0} - \frac{W_t}{P_0}\right) N_{jt}\,\mathrm{d}j. \tag{1.7}$$

1.2.2 Households

There is a mass 1 continuum of three-period lived ex-ante identical households, each one indexed by $i \in [0,1]$. Households derive utility from the consumption of the perishable consumption good. Their utility function is:

$$\mathbb{E}_0\left[\ln(c_{i0}) + \beta \ln(c_{i1}) + \beta^2 \ln(c_{i2})\right] \tag{1.8}$$

where $\mathbb{E}_0$ is the conditional expectations operator as of period 0, and $\beta \in (0,1]$ is a subjective time-discount factor. Each household is composed of a size-one continuum of potential workers, each indexed by $j \in [0,1]$, and endowed with $n_j = 1$ units of specialised indivisible labour in periods 0 and 1, which they inelastically supply in the labour market. In the final period of the economy, household members do not work.

At the beginning of period 0, all households are endowed with $\bar{h} = 1$ units of a *Lucas-tree* asset, each yielding a random real dividend D measured in units of the perishable consumption good at the beginning of period 2. The Lucas-tree asset is totally *illiquid* in the sense that no household can trade it in a spot market, nor can borrow against its future realised dividend.

Because of an unexpected wealth shock described further below, the members of any household might face unemployment idiosyncratic risk in period 1, since they can be forced into unemployment given the indivisibility of their individual fixed supply of labour. Let N_1^d be the aggregate demand for labour and $u = 1 - N_1^d$ the aggregate unemployment rate. Because of the law of large numbers, any potential worker faces the idiosyncratic labour supply shock

$$\varepsilon_{j1} = \begin{cases} 0 & \text{with probability } u \\ 1 & \text{with probability } 1-u \end{cases}.$$

In other words, a potential worker might be randomly assigned to unemployment and his effective supply of labour in period 1 is equal to $n_{j1}\varepsilon_{j1}$.

Members of the household do not individually participate in the credit market. Nevertheless, we assume there is perfect risk sharing in the sense that we allow for inter-period transfers of income among household members and that the household unit can save into non-negative deposits directly into the Central Bank in periods $t = \{0, 1\}$. As a counterpart, the monetary authority creates an equivalent amount of reserves $m_{it+1} \geq 0$ which yield a one-period gross nominal return $R_{t+1}\frac{P_{t+1}}{P_t}$, where R_{t+1} denotes the real interest rate. Compared to the Lucas-tree asset, deposits are perfectly *liquid* in the sense that reserves that back them up are convertible to currency immediately and at no cost. Households are also the owners of all firms and receive an equal share of the total nominal profits Γ_t in periods 0 and 1. To finance the interest payments on reserves, the government levies a nominal lump-sum tax T_{t+1} on all households.

Under the perfect risk-sharing assumption, it is possible to focus on a representative household that faces the flow budget constraints:

$$P_0c_0 + m_1 = W_0 + \Gamma_0 \tag{1.9}$$

$$P_1c_1 + m_2 + T_1 = W_1(1-u) + \Gamma_1 + R_1\frac{P_1}{P_0}m_1 \tag{1.10}$$

$$P_2c_2 + T_2 = R_2\frac{P_2}{P_1}m_2 + P_2D \tag{1.11}$$

Solution to the household's problem. At the beginning of period 0, the representative household maximises its flow of discounted utility in (1.8) subject to the flow budget constraints (1.9) – (1.11) and the sequence of non-negativity constraints $\{m_{t+1} \geq 0\}_{t=0}^{1}$. Given the recursivity of the problem and its finite-horizon setting, it is possible to solve it backwards.

Period 2. Given $\{P_2, R_2, T_2\}$, the value of the real dividend D, and the value of the state variable m_2, which summarises the past representative household's saving decision, the representative household follows its flow budget constraint (1.11) and chooses consumption according to the policy function:

$$c_2(m_2) = R_2 \frac{m_2}{P_1} + D - \frac{T_2}{P_2}. \tag{1.12}$$

The optimal value of the problem of the households is simply $V_2 = \ln(c_2)$.

Period 1. At the beginning of period 1, idiosyncratic unemployment risk is realised and, because of the law of large numbers, a proportion u of potential workers, members of the representative household, are forced into unemployment. By substituting equation (1.10) into the logarithmic instantaneous utility function, and taking as given the value of the savings state m_1 and the values of $\{\Gamma_1, P_1, W_1, T_1\}$, as of period 1 the problem of this household can be represented by the Bellman equation:

$$V_1(m_1) = \max_{m_2} \ln\left(\frac{W_1}{P_1}(1-u) + \frac{\Gamma_1}{P_1} + R_1 \frac{m_1}{P_0} - \frac{m_2}{P_1} - \frac{T_1}{P_1}\right) + \beta\, \mathbb{E}_1\left[V_2(m_2)\right]$$
$$\text{s.t. } m_2 \geq 0$$

The first order and envelope conditions of this problem with respect to the deposits decision of the representative household, lead to the Euler equation:

$$\frac{1}{c_1(m_1)} = \beta R_2 \left(\frac{1}{c_2(m_2(m_1))}\right) + \lambda_1(m_1) \tag{1.13}$$

where, from the flow budget equation (1.10), $c_1(m_1)$ is determined by the policy function:

$$c_1(m_1) = \frac{W_1}{P_1}(1-u) + \frac{\Gamma_1}{P_1} + R_1 \frac{m_1}{P_0} - \frac{m_2(m_1)}{P_1} - \frac{T_1}{P_1} \tag{1.14}$$

and $\lambda_1(m_1)$ is the optimal value of the Lagrange multiplier associated with the non-negativity constraint on the deposits decision. Equations (1.13) and (1.14) completely characterise the optimal savings and consumption decisions of households in this period.

Period 0. In period 0 the representative household makes consumption and savings decisions. By substituting the flow budget equation (1.9) in the logarithmic instantaneous utility function of period 0, and taking as given the values of $\{\Gamma_0, P_0, W_0\}$, the problem of the representative household in period 0 can be represented by the Bellman equation:

$$V_0 = \max_{m_1} \ln\left(\frac{W_0}{P_0} + \frac{\Gamma_0}{P_0} - \frac{m_1}{P_0}\right) + \beta\,\mathbb{E}_0\left[V_1(m_1)\right]$$
$$\text{s. t. } m_1 \geq 0$$

In a similar way to the solution of the two-period problem as of period 1, the implicit policy function derived from the first order and envelope conditions of this problem with respect to the nominal deposits choice leads to the Euler equation:

$$\frac{1}{c_0} = \beta R_1 \mathbb{E}_0\left[\frac{1}{c_1}\right] + \lambda_0 \tag{1.15}$$

where λ_0 represents the Lagrange multiplier associated with the non-negativity constraint on the deposits decision of the household.

1.2.3 Conventional monetary policy

Both monetary and fiscal policy are determined by a consolidated Government-Central Bank, which in periods 0 and 1 determines the value of the nominal gross interest rates paid on reserves in periods 1 and 2, the corresponding lump-sum taxes to finance such payments, and the value of the money price of the consumption good in the final period of the economy.

The monetary authority side of the consolidated government sets the nominal gross interest rates $\{R_{t+1}\frac{P_{t+1}}{P_t}\}_{t=0}^{1}$ and P_2 by following the rules:

$$R_{t+1}\frac{P_{t+1}}{P_t} = \max\left\{1, R^*_{t+1}\frac{P_{t+1}}{P_t}\right\} \tag{1.16}$$

$$P_2 \text{ s.t. } \frac{P_2}{P_1} = 1. \tag{1.17}$$

As most of the literature on the role of monetary policy in a liquidity trap scenario (e.g. Korinek and Simsek, 2016; Krugman, 1998; Werning, 2012), the rule in (1.16) makes explicit the constraint imposed by the Zero-Lower-Bound (ZLB). Because of the extreme nominal price rigidity assumption in periods 0 and 1, the ZLB does not only affects the determination of the value of the nominal rate, but also the value of the gross real interest rate and, therefore, the ability of the central bank to manipulate aggregate demand. The rule is such that, whilst not constrained by the ZLB, the central bank sets the gross nominal (and real) interest rate such that its value is equal to the flexible price perfect foresight full-employment equilibrium —natural—real interest rate of the economy, R^*_{t+1}.

On the other hand, the monetary authority also sets the value of the money price of the perishable consumption good in the final period, since there is no production and its total output is given exogenously by equation (1.4). In the same spirit of the interest rule in equation (1.16), the objective is to set the value of the price of the consumption good, such that the inflation rate in period 2 is equal to the equilibrium inflation rate in the flexible price perfect foresight full-employment equilibrium of the economy.

The consolidated government assumption implies that any monetary policy implemented following rules (1.16) and (1.17) has to be backed up by fiscal policy in the form of lump sum taxes to finance the payment of interests on the reserves in the households' accounts at the Central Bank. Let $\{m_{t+1}\}_{t=0}^{1}$ represent the total amount of reserves that the Central Bank creates to back up the total amount of deposits of all households. Therefore, the consolidation of policies is summarised by the budget equations:

$$T_{t+1} = \left(R_{t+1} \frac{P_{t+1}}{P_t} - 1 \right) m_{t+1}; \; t = \{0, 1\} \tag{1.18}$$

Notice that because of the nominal rigidity assumption and the price determination policy in (1.17), we are ruling out a role for seigniorage in financing the consolidated government.

1.2.4 Equilibrium

The focus is on the perfect foresight symmetric equilibrium of this economy, in which all agents as of period 0, share the same information set containing all present and future values of prices, wages, interest rates and tax rates, as well as the values of the unemployment rate, and the real dividend D. The perfect foresight equilibrium is symmetric because all producers of intermediate varieties face the same demand function when maximising profits at date 0, which implies that all monopolistic firms: *i)* set and fix the same relative price $\frac{p_{j0}}{P_0} = \frac{p_0}{P_0}$, *ii)* demand the same amount of specific labour $N_{jt} = N_t$, *iii)* produce the same amount of the intermediate variety output $y_{jt} = y_t$ and, *iv)* gain the same amount of nominal profits $\Gamma_{jt} = \Gamma_t$.

In equilibrium, the perishable good market must clear in all periods, i.e.

$$c_t = Y_t, \; \text{ for } t = \{0, 1\} \tag{1.19}$$

$$c_2 = Y_2 = D. \tag{1.20}$$

To pin down the value of the wage in period 1 for any perfect foresight equilibrium characterised by an allocation off-full-employment, we assume that

$$W_1 = W_0 \tag{1.21}$$

Equation (1.21) could be justified in a similar way to the price rigidity assumption above, but it is only included as a device to determine the level of the equilibrium wage in period one when the intermediate variety producers are able to reoptimize after the realisation of a future wealth related shock. We now proceed to the definition of equilibrium.

Definition 1. *Given the known value of the real dividend D, the perfect foresight symmetric general equilibrium of the economy consists of the allocation* $[\{Y_t\}_{t=0}^2, \{y_t\}_{t=0}^1, \{N_t\}_{t=0}^1, \{N_t^d\}_{t=0}^1, \{\Gamma_t\}_{t=0}^1, \{c_t\}_{t=0}^2, \{m_{t+1}\}_{t=0}^1, \{\lambda_t\}_{t=0}^1]$*, the price vector* $[\{p_t\}_{t=0}^1, \{P_t\}_{t=0}^1, \{W_t\}_{t=0}^1]$*, the policy vector* $[\{R_{t+1}\}_{t=0}^1, P_2, \{T_{t+1}\}_{t=0}^1]$ *and the unemployment rate* $u \in [0,1)$*, such that:*

1. *Households maximise utility in equation* (1.8)*, i.e. equations* (1.9) *and* (1.12) – (1.15) *hold, given* $[\{P_t\}_{t=0}^2, \{W_t\}_{t=0}^1, \{R_{t+1}\}_{t=0}^1, \{T_{t+1}\}_{t=0}^1]$*, u, and D.*

2. *The competitive representative firm producing the final perishable good, maximises its profits at each date, i.e.* (1.1)*,* (1.2)*, and* (1.3) *hold given* $[\{p_t\}_{t=0}^1]$.

3. *Given D, the supply of the perishable consumption good in period 2 is determined by* (1.4)*.*

4. *Each intermediate-variety producer firm maximises its discounted flow of profits in periods 0 and 1, given* $[\{P_t\}_{t=0}^1, \{W_t\}_{t=0}^1]$*, i.e. equations* (1.6) *and* (1.7) *hold given the nominal rigidity assumption* $p_1 = p_0$*, and the nominal wage rigidity given by equation* (1.21)*.*

5. *The labour market clears in period 0:*

$$N_0 = N_0^d = 1 \tag{1.22}$$

   and in period 1 labour supply might be rationed, i.e.:

$$N_1 = N_1^d \leq 1 \tag{1.23}$$

$$u = 1 - N_1^d \tag{1.24}$$

6. *In periods 0 and 1 the value of the gross nominal interest rate on reserves* $R_{t+1}\frac{P_{t+1}}{P_t}$ *is determined by equation* (1.16)*.*

7. *P_2 is set according to equation* (1.17).

8. *The lump sum tax rates* $\{T_{t+1}\}_{t=0}^{1}$ *are set by the consolidated Government such that the budget equations in* (1.18) *holds.*

9. *The market for the perishable good clears in all periods, i.e. the equilibrium conditions* (1.19) *and* (1.20) *hold.*

1.3 Conventional and unconventional monetary policy under perfect risk sharing

The analysis is set in a scenario in which at the beginning of period 1, whilst being on the full-employment (FE) perfect foresight equilibrium of our stylised three-period economy, households, firms and the consolidated government realise an unexpected change $\Delta = D' - D < 0$ in the future value of the real dividend of the *Lucas-tree* asset. In the context of this scenario, we assess the extent to which both interest-rate-based conventional monetary policy and a particular sort of unconventional monetary policy to be described below, are able to offset the spread of idiosyncratic unemployment risk among members of the representative household. Nonetheless, we need first to characterise the FE perfect foresight equilibrium of our model economy, based on the following assumption:

Assumption 1. $D \geq \beta$

The FE perfect foresight equilibrium. We characterise the full employment perfect foresight equilibrium by the value of the unemployment rate $u^{FE} = 0$. From Definition 1 it is then straightforward to fully determine the equilibrium allocation, prices and policy variables. For all $t = \{0,1\}$: *i)* equilibrium in the labour and perishable good markets implies $N_t^{d,FE} = N_t^{FE} = y_t^{FE} = Y_t^{FE} = c_t^{FE} = 1$; *ii)* from the intermediate firms optimal price condition (1.6), since the $P_0 = 1$ (numeraire), full employment implies that $p_t^{FE} = P_t^{FE} = 1$ and $\frac{W_t}{P_t}^{FE} = \frac{\eta-1}{\eta}$; *iii)* equation (1.7) imply $\Gamma_t^{FE} = \frac{1}{\eta}$; *iv)* from the household's optimal conditions (1.15) and

(1.13) $m_{t+1}^{FE} = \lambda_t^{FE} = 0$, $R_1^* = \frac{1}{\beta}$ and $R_2^* = \frac{D}{\beta}$; *v)* the conventional monetary policy rule in equation (1.16) then implies $R_1^{FE} = R_1^*$ and $R_2^{FE} = R_2^*$, whereas the price rule (1.17) implies $P_2^{FE} = 1$, and *iv)* the government budget constraints in (1.18) determine $T_{t+1}^{FE} = 0$. In period 2 the consumption good equilibrium market clearing condition implies $c_2^{FE} = Y_2^{FE} = D$.

1.3.1 Conventional monetary policy

Under Assumption 1, the monetary authority is not constrained by the ZLB in the FE equilibrium path. However, after the realisation of the unexpected shock, households and firms re-optimise since they might be affected by the consequences of the shock on the realisation of idiosyncratic unemployment risk. The extent to which conventional monetary policy is able to isolate households from the potential spreading of unemployment risk, depends on the relative size of the dividend shock, as established by Proposition 1.

Proposition 1. *Given Assumption 1, if the condition on the relative size of the wealth shock*

$$\frac{\Delta}{D-\beta} \leq 1 \tag{1.25}$$

holds, then the central bank is able to rule out positive unemployment risk as an equilibrium outcome in period 1 by implementing the rule in equation (1.16).

Proof. From the characterisation of the FE equilibrium path, as long as the condition in equation (1.25) holds $D' < \beta$ and the implementation of the rule $R_2 = R_2^{*\prime} = \frac{D'}{\beta}$ is not constrained by the ZLB. From condition (1.13) $c_1' = c_0^{FE} = c_1^{FE} = Y_1^{FE} = y_1^{FE}$, which implies $u' = u^{FE} = 0$. □

As long as the absolute value of the unexpected shock does not surpass the value of the difference between the initial value of the real dividend and the intertemporal discount factor of households, the central bank is not constrained by the ZLB when implementing policy. Despite the negative shock, the implementation of conventional monetary policy manages to completely rule out the realisation of positive idiosyncratic unemployment risk and keeps the economy on its full employment path.

However, if the initial value of the real dividend is close enough to the value of β, a sufficiently small shock might make the ZLB "to bind" eliminated + "binding" includedbinding as condition (1.25) is violated. At this point, the central bank is no longer able to impede the spread of positive unemployment idiosyncratic risk to households by decreasing the value of its policy rate. Lemma 1 establishes the positive equilibrium value of the unemployment rate which characterises a new equilibrium path away from FE.

Lemma 1. *Given Assumption 1, let the initial value of D to be low enough such that after the realisation of the shock* Δ *equation* (1.25) *does not hold. Hence, the central bank is constrained by the ZLB when implementing monetary policy according to the conventional monetary policy rule* (1.16)*, and the value of the nominal (real) interest rate is equal to* $R_2^{PF} = 1$*. Then, as of period 1, the new and unique incentive compatible perfect foresight equilibrium path of the three-period stylised economy is characterised by the positive unemployment rate:*

$$u^{PF} = 1 - \frac{D'}{\beta}. \tag{1.26}$$

Proof. See the proof in Appendix 1.A.1. □

1.3.2 Neutrality of unconventional monetary policy

Let $\mathbb{1}\left(R_2\frac{P_2}{P_1} = 1\right)$ be an indicator function that takes a value of 1 if the ZLB is binding and 0 otherwise. Then, consider a policy scenario in which the central bank of the consolidated government is willing to make an outright purchase of a fraction $\mathbb{1}\left(R_2\frac{P_2}{P_1} = 1\right) \times \phi$ of the total household's holdings of the illiquid asset at a price per unit equivalent to the *fair price* $q^{CB} = D'$ which represents the discounted value of its expected dividend. As a counterpart to the outright purchase operation, the central bank issues the amount of reserves

$$m_1^{CB} = P_1 q^{CB} \mathbb{1}\left(R_2 \frac{P_2}{P_1} = 1\right) \phi \bar{h} \tag{1.27}$$

into the household's account at the central bank.

The implementation of this particular kind of unconventional monetary policy implies that, as of period 1, the flow budget constraints of the representative household can be rewritten as:

$$P_1 c_1 + m_2 = W_1(1-u) + \Gamma_1 + R_1 \frac{P_1}{P_0} m_1 + P_1 q^{CB} \mathbb{1}\left(R_2 \frac{P_2}{P_1} = 1\right) \phi \bar{h} - T_1 \tag{1.28}$$

$$P_2 c_2 = R_2 \frac{P_2}{P_1} m_2 + P_2 \left(1 - \mathbb{1}\left(R_2 \frac{P_2}{P_1} = 1\right) \phi\right) D' - T_2. \tag{1.29}$$

This outright purchase operation has a fiscal implication in the last period of the economy. For whatever fraction of the *Lucas-tree* asset the household agrees to sell, the central bank transfers the realised dividends to the consolidated government which then transfers back such dividends to the representative household. The adjusted flow budget constraint of the consolidated government at the end of period 2 is now given by

$$\left(R_2 \frac{P_2}{P_1} - 1\right) m_2 = T_2 + P_2 \mathbb{1}\left(R_2 \frac{P_2}{P_1} = 1\right) \phi D'. \tag{1.30}$$

Once the shock Δ is realised at the beginning of period 1, the re-optimisation problem of the representative household is summarised by the value function:

$$\begin{aligned} V_1(m_1) = & \max_{c_1, m_2, \mathbb{1}\left(R_2 \frac{P_2}{P_1}=1\right)\phi} \ln(c_1) + \beta V_2\left(m_2, \mathbb{1}\left(R_2 \frac{P_2}{P_1} = 1\right) \phi\right) \\ & \text{s.t. } (1.28),(1.29) \\ & m_2 \geq 0;\ 0 \leq \mathbb{1}\left(R_2 \frac{P_2}{P_1} = 1\right) \phi \leq 1 \end{aligned} \tag{1.31}$$

The effects of unconventional monetary policy under perfect risk sharing among household members can be fully characterised by the solution to problem

(1.31). Proposition 2 establishes such effects.

Proposition 2. *Given assumption 1, assume that after the realisation of the shock* Δ *condition* (1.25) *does not hold and the central bank implements the outright purchase policy summarised in equation* (1.27). *Following Definition 1, the solution to the representative household in* (1.31) *implies that the perfect foresight equilibrium allocation is such that the positive value of the unemployment rate is still determined by equation* (1.26) *in Lemma 1. The implementation of the unconventional monetary policy is then completely neutral.*

Proof. See proof in Appendix 1.A.2. □

As in Wallace (1981) and Eggertsson and Woodford (2003), we find a neutrality result regarding the effects of the implementation of a conventional monetary policy. Under the perfect risk-sharing assumption, the household unit secures its members to the most possible extent given the excess supply equilibrium generated by a binding ZLB. Once the inter-period transfers between the members of the representative household have been realised, the household does not wish to change its borrowing decision in the incentive-compatible perfect foresight equilibrium and its optimal net savings decision resembles that in the scenario with no unconventional monetary policy.

1.4 Unconventional monetary policy under imperfect risk sharing

The neutrality result presented in subsection 1.3.2 depends mostly on the perfect risk-sharing assumption among members of the representative household of the three-period economy. In an extension of the model presented in Section 1.2, this assumption is abandoned and changed by one in which there is imperfect risk-sharing within the household. Under this new extension of the model, we put to test the neutrality result in Proposition 2.

1.4.1 Households with imperfect risk sharing

To introduce the notion of imperfect risk sharing, it is enough to rule out the possibility of inter-period transfers of wage income among members of the representative household in period 1 of the three-period economy. As in Heathcote and Perri (2018), preferences of the household now have to take into account that the consumption of unemployed and employed members of the household might be different in period 1, which implies that the new utility function of the representative household is

$$\mathbb{E}_0\left[\ln(c_0)+\beta\left((1-u)\ln(c_{E1})+u\ln(c_{U1})\right)+\beta^2\ln(c_2)\right] \tag{1.32}$$

In periods 0 and 2, the representative household still faces the flow budget constraints in equations (1.9) and (1.11). Nonetheless, in the inter-period of period 1 the consumption of employed ($E\subset[0,1]$) members and unemployed ($U\subset[0,1]$) members is constrained in such a way that

$$P_1c_{E1}\leq W_1+\Gamma_1+R_1\frac{P_1}{P_0}m_1 \tag{1.33}$$

$$P_1c_{U1}\leq \Gamma_1+R_1\frac{P_1}{P_0}m_1 \tag{1.34}$$

is consistent with the new flow budget constrained faced by the household in this period, which is

$$P_1((1-u)c_{E1}+uc_{U1})+m_2=W_1(1-u)+\Gamma_1+R_1\frac{P_1}{P_0}m_1-T_1, \tag{1.35}$$

and the non-negativity constraint on savings $m_2\geq 0$. Then, as of period 1, the problem of the household is now

$$\begin{aligned}V_1(m_1)=\max_{c_{E1},c_{U1},m_2}\ &(1-u)\ln(c_{E1})+u\ln(c_{U1})+\beta V_2(m_2)\\ \text{s.t. }&(1.33)\text{–}(1.35),\ (1.11)\\ &m_2\geq 0.\end{aligned} \tag{1.36}$$

The first order conditions of problem (1.36) can be summarised by the Euler equations

$$\frac{1}{c_{E1}} = \beta R_2 \mathbb{E}_1 \left[\frac{1}{c_2}\right] + \lambda_1 + \frac{\mu_1}{1-u} \tag{1.37}$$

$$\frac{1}{c_{U1}} = \beta R_2 \mathbb{E}_1 \left[\frac{1}{c_2}\right] + \lambda_1 + \frac{\omega_1}{u}, \tag{1.38}$$

where λ_1, μ_1 and ω_1 are the Lagrangian multipliers on the budget constraint in (1.35) and the interperiod constraints in (1.33) and (1.34), respectively. In period 0, the Euler equation characterising optimal savings and intertemporal consumption between periods 0 and 1 is now

$$\frac{1}{c_0} = \beta R_1 \mathbb{E}_0 \left[\frac{1-u}{c_{E1}} + \frac{u}{c_{U1}}\right] + \lambda_0 \tag{1.39}$$

1.4.2 Equilibrium under imperfect risk sharing and no unconventional monetary policy

In a scenario without the implementation of unconventional monetary policy ($\phi \equiv 0$). The definition of equilibrium under imperfect risk sharing coincides with the one presented in Definition 2 in Appendix 1.B. Notice that the characterisation of the perfect FE does not change since when the perfect foresight unemployment rate is equal to zero, imperfect risk sharing summarised in the inter-period conditions (1.33) and (1.34) does not play an active role, and the FE equilibrium path is equivalent to the one characterised in Section 1.3.

If the ZLB binds after the realisation of the unexpected shock Δ, the perfect foresight equilibrium can be characterised not only by a positive unemployment rate but also by consumption inequality between employed and unemployed workers. This result is established by Lemma 2 below.

Assumption 2. $1 < \eta \leq \frac{\beta}{D'}$.

Lemma 2. *Given assumptions 1 and 2, let the initial value of D be low enough such that after the realisation of the shock Δ equation* (1.25) *does not hold. Hence,* *the central bank is constrained by the ZLB when implementing monetary policy according to the policy rule* (1.16)*, and the value of the nominal (real) interest rate is equal to* $R_2^{PF} = 1$*. Then, as of period 1, under the assumption of imperfect risk sharing embedded in Definition 2, the new and unique incentive compatible perfect foresight equilibrium path of the three-period stylised economy is characterised by the positive unemployment rate:*

$$\hat{u}^{PF} = \eta \left(1 - \frac{D'}{\beta}\right) > u^{PF}, \tag{1.40}$$

and the equilibrium allocation is such that, in period 1, the consumption of workers inside the representative household is given by

$$\hat{c}_{E1}^{PF} = \frac{\eta - \hat{u}^{PF}}{\eta}$$

$$\hat{c}_{U1}^{PF} = \frac{1 - \hat{u}^{PF}}{\eta} < \hat{c}_{E1}^{PF}$$

Proof. See the proof in Appendix 1.A.3. □

With imperfect risk sharing, the representative household insurance capabilities are reduced and the excess supply situation created by the binding ZLB has even more negative consequences on the welfare of households and its members. It is time then to reassess the role of unconventional monetary policy in the context of this new equilibrium scenario.

1.4.3 The effects of unconventional monetary policy revisited

As in subsection 1.3.2, the central bank of the consolidated government implements its outright purchases policy once the ZLB binds and there is no more room to implement interest-rate-based conventional monetary policy. Apart from its fiscal implication in period 2, a key aspect of the policy is that the household can use the resources obtained from this market operation to offer further insurance to all its members, i.e. the inter-period constraints on the consumption of the unemployed members of the representative household can be rewritten as:

$$P_1 c_{E1} \leq W_1 + \Gamma_1 + R_1 \frac{P_1}{P_0} m_1 + m_1^{CB} \tag{1.41}$$

$$P_1 c_{U1} \leq \Gamma_1 + R_1 \frac{P_1}{P_0} m_1 + m_1^{CB} \tag{1.42}$$

where m_1^{CB} is determined by equation (1.27). Therefore, the new flow budget constraint of the representative household in period 1 is:

$$P_1((1-u)c_{E1} + uc_{U1}) + m_2 = W_1(1-u) + \Gamma_1 + R_1 \frac{P_1}{P_0} m_1 + m_1^{CB} - T_1, \tag{1.43}$$

and the problem of the household, consistent with the equilibrium defined in Definition 2, is

$$\begin{aligned} V_1(m_1) = & \max_{c_{E1}, c_{U1}, m_2} (1-u)\ln(c_{E1}) + u\ln(c_{U1}) + \beta V_2(m_2) \\ & \text{s.t. (1.41), (1.42), (1.43) and (1.11)} \\ & \quad m_2 \geq 0. \end{aligned} \tag{1.44}$$

Once again, the key to determining the effects of unconventional monetary policy in a scenario with imperfect risk sharing, is hidden in the solution to the problem in (1.44) and its relation to the perfect FE equilibrium path. In general, the shapes of the optimal conditions of the problem established in equations (1.37) and (1.38) do not change, apart from the fact that the Lagrange multipliers ω_1 and λ_1 are now related to the new inter-period constraint in (1.42) and the new flow budget constraint in (1.43). Proposition 3 characterises the effects of unconventional monetary policy under imperfect risk sharing

Proposition 3. *Given assumptions 1 and 2, assume that after the realisation of the shock Δ condition* (1.25) *does not hold and the central bank implements the outright purchase policy summarised in equation* (1.27)*. Following Definition 2, the solution to the representative household in* (1.31) *implies that the perfect foresight equilibrium allocation is such that the value of the equilibrium unemployment rate* *is*

$$\tilde{u}^{PF} = \eta \left(1 - (1-\beta)\frac{D'}{\beta}\right) < \hat{u}^{PF} \tag{1.45}$$

Unconventional monetary policy is welfare improving and unconventional monetary policy manages to partially insure households with the implementation of this outright purchases policy.

Proof. See the proof in Appendix 1.A.4. □

The result in Proposition 3 tells the story that a situation of perfect risk-sharing is the limit of an economy where financial frictions impede individuals to insure themselves; a situation which makes the negative effects of shocks more pervasive. It is when these frictions are in play that unconventional monetary policy offers both an insurance and a stabilisation role.

1.5 Conclusions

An extensive analysis of the conditions under which unconventional monetary policy has or has not real effects on the economy, leads to the conclusion that unconventional monetary policy in the form of outright purchases of illiquid assets has a stabilisation and insurance role when the economy faces financial restrictions that impede the individuals of the economy to self-insure. Imperfect risk sharing might be associated with particular situations in which individuals are restricted from participating in the credit or insurance markets or are particularly sensible to be liquidity constrained when facing recession episodes.

Using a tractable model with some initial financial constraints, the analysis carefully establishes the role of unconventional monetary policy starting from a scenario where policy is neutral because of the perfect risk-sharing assumption. Then, via an extension of the model in which households are no longer able to fully extend their insurance capabilities, the neutrality result is broken and unconventional monetary policy manages to step in to help insure households in a recession prone by the binding ZLB.

Nonetheless, given the structure of the stylised model presented here, as long as the ZLB is non-binding, conventional monetary policy perfectly insures households and their members from the realisation of unemployment idiosyncratic risk. In that sense, our analysis of the effects of unconventional monetary policy is only valid in a context where the economy has entered a liquidity trap recession scenario like the one observed during the great financial crisis of 2007–2008.

Appendix

1.A Proofs of lemmas and propositions

1.A.1 Proof of Lemma 1

Given assumption 1, assume that condition (1.25) in Proposition 1 does not hold. Let $u^{PF} \in [0,1)$ be the conjectured perfect foresight equilibrium value of the unemployment rate in period 1 for a given value of the nominal (real) interest rate $\bar{R}$ and the new value of the real dividend D'. Following Definition 1, equations (1.1) and (1.5), together with the profit share in (1.7), imply that the conjectured equilibrium values of production of the intermediate varieties and of the perishable consumption good are $y_1^{PF} = Y_1^{PF} = 1 - u^{PF}$ which, from the equilibrium condition in (1.19), must be equal to c_1^{PF}. Then, by substituting equation (1.12), (1.17) and (1.18) into the Euler equation (1.13), it is possible to characterise the fixed-R equilibrium by the condition:

$$\frac{1}{1-u^{PF}} = \beta\mathscr{R}\left(\frac{1}{\mathscr{R}m_2^{PF} + D'}\right) + \lambda_1^{PF} \tag{1.46}$$

1. *Proof that $R_2^{PF} = 1$ and $u^{PF} \neq u^{FE}$.* Assume $u^{PF} = u^{FE} = 0$. Then, following the characterisation of the FE equilibrium, the non-negativity constraint on deposits is non-binding and $\lambda_1^{PF} = 0$. Moreover, in the full employment equilibrium path, the representative household does not have incentives to save and $m_2^{PF} = 0$. Then, from equation (1.46)

$$\mathscr{R} = R_2^{*} = \frac{D'}{\beta}$$

Since condition (1.25) does not hold, $R_2^* < 1$. Hence, the central bank is constrained by the ZLB and the monetary policy rule (1.16) implies:

$$R_2^{PF} = 1 > R_2^*$$

which leads to a contradiction, since for $u^{PF} = u^{FE}$ to hold it must be that $R_2^{PF} = R_2^*$ holds. Then it must be that $u^{PF} \neq u^{FE}$.

2. *Proof of unique incentive-compatible $u^{PF} > 0$.* Given $R_2^{PF} = 1$, the non-negativity constraint on the representative household's deposits implies that either $m_2^{PF} \geq 0$ and $\lambda_1^{PF} = 0$, or $m_2^{PF} = 0$ and $\lambda_1^{PF} > 0$. In the first case, from equation (1.46) we have:

$$\frac{1}{1-u^{PF}} = \beta \left(\frac{1}{m_2^{PF} + D'} \right) \tag{1.47}$$

On the other hand, from the clearing market condition (1.20) we also have:

$$m_2^{PF} + D' = D'$$

so it must be that $m_2^{PF} = 0$. Therefore, from (1.47) one possible equilibrium value of the positive unemployment rate is:

$$u^{PF} = 1 - \frac{D'}{\beta}. \tag{1.48}$$

On the other hand, given the nominal price rigidity assumption, re-optimisation of the intermediate-variety firms after the shock implies the profit maximisation problem:

$$\begin{aligned} &\max_{N_1} \left(1 - W_1^{PF}\right) N_1 \\ &\text{s.t. } N_1 \leq 1 - u^{PF} \end{aligned} \tag{1.49}$$

Given W_1^{PF} intermediate variety firms solve the problem in (1.49) by choosing $N_1^{PF} = 1 - u^{PF}$ which confirms the conjecture that in equilibrium

$$y_1^{PF} = Y_1^{PF} = 1 - u^{PF} \tag{1.50}$$

In the second case, with $\lambda_1^{PF} > 0$, let $u^{PF'} \neq u^{PF}$ denote an alternative conjectured unemployment equilibrium rate. Then, using the result in equation (1.48), equation (1.46) can be rewritten as:

$$\frac{1}{1-u^{PF'}} = \frac{1}{1-u^{PF}} + \lambda_1^{PF}$$

from which is possible to solve for $u^{PF'}$ as:

$$u^{PF'} = 1 - \frac{1-u^{PF}}{1+\lambda_1^{PF}(1-u^{PF})} > u^{PF}. \tag{1.51}$$

Equation (1.51) opens the door to an economy with (infinite) multiple sunspot equilibria where any value $\lambda_1^{PF} > 0$ would imply all households to believe the unemployment equilibrium rate to be higher and, therefore, feeling they are in an implicit *excess savings* situation given $R_2^{PF} = 1$. However, from the value function as of period 1 it is possible to see that:

$$\begin{aligned} V_1(u^{PF}) = \ln(1-u^{PF}) + \beta \ln(D') > \\ V_1(u^{PF'}) = \ln(1-u^{PF'}) + \beta \ln(D'), \end{aligned} \tag{1.52}$$

meaning all households being worse-off for any $\{u^{PF'}, \lambda_1^{PF'}\} \in (u^{PF}, 1) \times \mathbb{R}_+ \equiv S$. In other words, once all households' beliefs are revealed in period 1, they are all worse off because of such beliefs. Then, equilibria in S is not *incentive compatible*. The unique equilibria in which the representative household has no incentives to deviate from its beliefs must be that in which u^{PF} is determined by (1.48).

1.A.2 Proof of Proposition 2

We start from the solution to the utility maximisation problem of the representative household. By substituting (1.27) into the flow budget equations (1.28) and (1.29), it is possible to rewrite the problem of the representative household in (1.31) as:

$$V_1(m_1) = \max_{m_2, m_1^{CB}} \ln\left(\frac{W_1}{P_1}(1-u) + \frac{\Gamma_1}{P_1} + \frac{R_1 m_1}{P_0} + \frac{m_1^{CB}}{P_1} - \frac{m_2}{P_1} - \frac{T_1}{P_1}\right) + \beta V_2(m_2, m_1^{CB})$$
$$\text{s.t. } m_2 \geq 0;\ 0 \leq m_1^{CB} \leq P_1 q^{CB} \bar{h}$$

Let us define the alternative state variable $\tilde{a}_2 \equiv m_2 - m_1^{CB}$. Then, the problem can be rewritten once again as:

$$V_1(m_1) = \max_{\tilde{a}_2} \ln\left(\frac{W_1}{P_1}(1-u) + \frac{\Gamma_1}{P_1} + \frac{R_1 m_1}{P_0} - \frac{\tilde{a}_2}{P_1} - \frac{T_1}{P_1}\right) + \beta\, \mathbb{E}_1[V_2(\tilde{a}_2)]$$
$$\text{s.t. } \tilde{a}_2 \geq -P_1 q^{CB} \bar{h} \tag{1.53}$$

After substituting (1.30) into the flow budget constraint (1.29), we can follow the proof of Lemma 1 in Appendix 1.A.1 to determine that, as of period 1, the solution to problem (1.53) is characterised in equilibrium by the Euler equation:

$$\frac{1}{1-\tilde{u}^{PF}} = \beta\left(\frac{1}{\tilde{a}_2^{PF} + D'}\right) + \tilde{\lambda}_1^{PF}, \tag{1.54}$$

for which the perishable good market's clearing condition implies that $\tilde{a}_2^{PF} = 0$. The unique *incentive compatible* equilibrium unemployment rate is then equal to that in equation (1.26) which is implied by the equilibrium value $\tilde{\lambda}_1^{PF} = 0$.

1.A.3 Proof of Lemma 2

We start from the equilibrium versions of the optimal conditions (1.37) and (1.38) in a no unconventional monetary policy scenario, i.e. $\phi \equiv 0$. Given $\hat{R}_2^{PF} = 1$ the equilibrium condition (1.61) implies $\hat{m}_2^{PF} = 0$ which is consistent with any $\lambda_1^{\hat{P}F} \geq 0$.

Hence, we have

$$\frac{1}{\hat{c}_{E1}^{PF}} = \frac{\beta}{D'} + \hat{\lambda}_1^{PF} + \frac{\hat{\mu}_1^{PF}}{1-\hat{u}^{PF}} \tag{1.55}$$

$$\frac{1}{\hat{c}_{U1}^{PF}} = \frac{\beta}{D'} + \hat{\lambda}_1^{PF} + \frac{\hat{\omega}_1^{PF}}{\hat{u}^{PF}} \tag{1.56}$$

where $\hat{u}^{PF}$ is the conjectured equilibrium unemployment rate under imperfect risk sharing and no unconventional monetary policy. Notice that from the inter-period conditions (1.33) and (1.34) it is possible to say that:

i. If $\hat{\mu}_1^{PF} > 0$ then $c_{E1}^{PF} = \frac{\eta-\hat{u}^{PF}}{\eta}$, and if $\hat{\mu}_1^{PF} = 0$ then $c_{E1}^{PF} \leq \frac{\eta-\hat{u}^{PF}}{\eta}$

ii. If $\hat{\omega}_1^{PF} > 0$ then $c_{U1}^{PF} = \frac{1-\hat{u}^{PF}}{\eta}$, and if $\hat{\omega}_1^{PF} = 0$ then $c_{U1}^{PF} \leq \frac{1-\hat{u}^{PF}}{\eta}$.

1. Let us start assuming that $\hat{\mu}_1^{PF} = \hat{\omega}_1^{PF} = 0$. Since $m_2^{PF} = 0$ any allocation such that $c_{E1}^{PF} < \frac{\eta-\hat{u}^{PF}}{\eta}$ or $c_{U1}^{PF} < \frac{1-\hat{u}^{PF}}{\eta}$ goes against the principle of utility maximisation, then it must be that $c_{E1}^{PF} = \frac{\eta-\hat{u}^{PF}}{\eta}$ and $c_{U1}^{PF} = \frac{1-\hat{u}^{PF}}{\eta}$. Under this consideration, we can rewrite the equilibrium conditions as:

$$\frac{1}{\frac{\eta-\hat{u}^{PF}}{\eta}} = \frac{\beta}{D'} + \hat{\lambda}_1^{PF}$$

$$\frac{1}{\frac{1-\hat{u}^{PF}}{\eta}} = \frac{\beta}{D'} + \hat{\lambda}_1^{PF}$$

   From this, we can conclude that there is no positive $\hat{u}^{PF}$ consistent with both types of workers not being liquidity constrained at the same time in the inter-period.

2. Now, let us consider a situation in which $\hat{\mu}_1^{PF} = 0$ and $\hat{\omega}_1^{PF} > 0$. The equilibrium conditions would now be:

$$\frac{1}{\frac{\eta-\hat{u}^{PF}}{\eta}} = \frac{\beta}{D'} + \hat{\lambda}_1^{PF}$$

$$\frac{1}{\frac{1-\hat{u}^{PF}}{\eta}} = \frac{\beta}{D'} + \hat{\lambda}_1^{PF} + \frac{\hat{\omega}_1^{PF}}{\hat{u}^{PF}}$$

Given the value of λ_1^{PF}, we can solve for $\hat{u}^{PF}$ from the first condition to obtain

$$\hat{u}^{PF} = \eta \left(1 - \frac{D'}{\beta + \hat{\lambda}_1^{PF} D'}\right) \tag{1.57}$$

As an intermediate step for our next consideration, we can compute the minimum value of the unemployment rate for which $\hat{\mu}_1 = 0$ and the employed worker will not be constrained. From equation (1.56)

$$\hat{\mu}_1 > 0 \Rightarrow \hat{u}^{PF} > \eta \left(1 - \frac{D'}{\beta + \hat{\lambda}_1^{PF} D'}\right) \tag{1.58}$$

3. Finally, now consider the situation $\hat{\mu}_1^{PF} > 0$ and $\hat{\omega}_1^{PF} = 0$. The equilibrium conditions would now be:

$$\frac{1}{\frac{\eta - \hat{u}^{PF}}{\eta}} = \frac{\beta}{D'} + \hat{\lambda}_1^{PF} + \frac{\hat{\mu}_1^{PF}}{1 - \hat{u}^{PF}}$$

$$\frac{1}{\frac{1 - \hat{u}^{PF}}{\eta}} = \frac{\beta}{D'} + \hat{\lambda}_1^{PF}$$

Given the value of λ_1^{PF}, we can solve for $\hat{u}^{PF}$ from the second condition to obtain

$$\hat{u}^{PF} = 1 - \eta \left(\frac{D'}{\beta + \hat{\lambda}_1^{PF} D'}\right)$$

But condition (1.58) tells us we arrived at a contradiction since we were assuming $\hat{\mu}_1^{PF} > 0$ and this value of the unemployment rate is clearly below $\eta \left(1 - \frac{D'}{\beta + \hat{\lambda}_1^{PF} D'}\right)$, and the employed member should be not constrained instead. The only consistent equilibrium rate would be one given by (1.57) for any given value of $\hat{\lambda}_1^{PF}$.

4. Following the argumentation in the proof of Lemma 1, we conclude that $\hat{\lambda}_1^{PF} = 0$, and the only perfect foresight equilibrium rate is the one given by equation (1.40).

1.A.4 Proof of Proposition 3

Following the same steps as the proof of Lemma 2, the unique equilibrium rate associated with constrained employed households comes from the condition

$$\frac{\eta}{\eta(1+m_1^{CB})-\tilde{u}^{PF}}=\frac{\beta}{D'}, \tag{1.59}$$

together with the fact that the fiscal counterpart of the policy gives the incentive to the household to choose $\phi^{CB}=1$, which implies that in the new equilibrium $\tilde{m}_1^{CB}=D'$. Substituting this equilibrium outcome in equation (1.59) gives us the result in (1.40).

1.B Definition of equilibrium under imperfect risk sharing

Definition 2. *Given the known value of the real dividend D and the fair price q^{CB}, the perfect foresight symmetric general equilibrium of the economy is the allocation* $[\{Y_t\}_{t=0}^2, \{y_t\}_{t=0}^1, \{N_t\}_{t=0}^1, \{N_t^d\}_{t=0}^1, \{\Gamma_t\}_{t=0}^1, c_0, c_{E1}, c_{U1}, \phi, c_2, \{m_{t+1}\}_{t=0}^1, \{\lambda_t\}_{t=0}^1, \mu_1, \omega_1]$, *the price vector* $[\{p_t\}_{t=0}^1, \{P_t\}_{t=0}^1, \{W_t\}_{t=0}^1]$, *the policy vector* $[\{R_{t+1}\}_{t=0}^1, P_2, m_1^{CB}, \{T_{t+1}\}_{t=0}^1]$ *and the unemployment rate* $u\in[0,1)$, *such that:*

1. *Households maximise utility in equation* (1.32), *i.e. equations* (1.9), (1.12), (1.37), (1.38) *and* (1.39) *hold, given* $[\{P_t\}_{t=0}^2, \{W_t\}_{t=0}^1, \{R_{t+1}\}_{t=0}^1, \{T_{t+1}\}_{t=0}^1]$, *u, and D.*

2. *The competitive representative firm producing the final perishable good, maximises its profits at each date, i.e.* (1.1), (1.2), *and* (1.3) *hold given* $[\{p_t\}_{t=0}^1]$.

3. *Given D, the supply of the perishable consumption good in period 2 is determined by* (1.4).

4. *Each intermediate-variety producer firm maximises its discounted flow of profits in periods 0 and 1, given* $[\{P_t\}_{t=0}^1, \{W_t\}_{t=0}^1]$, *i.e. equations* (1.6) *and*

(1.7) *hold given the nominal rigidity assumption* $p_1 = p_0$*, and the nominal wage rigidity given by equation* (1.21).

5. *The labour market clears in period 0:*

$$N_0 = N_0^d = 1$$

and in period 1 labour supply might be rationed, i.e.:

$$N_1 = N_1^d \leq 1$$
$$u = 1 - N_1^d$$

6. *In periods 0 and 1 the value of the gross nominal interest rate on reserves* $R_{t+1}\frac{P_{t+1}}{P_t}$ *is determined by equation* (1.16).

7. *Given* ϕ*,* m_1^{CB} *is determined by equation* (1.27).

8. P_2 *is set according to equation* (1.17).

9. *The lump sum tax rates* $\{T_{t+1}\}_{t=0}^{1}$ *are set by the consolidated Government such that the budget equations in* (1.18) *holds.*

10. *The market for the perishable good clears in all periods, i.e. the equilibrium conditions*

$$c_0 = Y_0$$
$$(1-u)c_{E1} + uc_{U1} = Y_1 \tag{1.60}$$
$$c_2 = Y_2 \tag{1.61}$$

hold.

Chapter 2

The uncertainty channel of unconventional monetary policy

2.1 Introduction

By the spring of 2009, central banks in Europe, the United States and the United Kingdom had almost exhausted any room available concerning interest-rate-based monetary policy, in their efforts to counteract the pervasive effects of the great financial crisis on the functioning of credit markets. Money market rates were at their lowest and it came the time to retort with more aggressive forms of balance sheet management policies that had already been implemented since the beginning of the crisis. Central banks not only increased the overall size of their balance sheets but radically changed the asset composition by enlarging the set of counterparties to engage in open market operations, along with a widening of the set of financial assets which could be, for instance, used as collateral or traded in outright purchases.[1]

In this paper, we present a tractable model of a finite-horizon economy with one liquid asset and one illiquid asset to assess the potential counterfactual effects of this kind of unconventional monetary policy. In the presence of increased uncertainty regarding the fair valuation of the illiquid asset, and a binding zero lower

[1] A careful and detailed narrative of the particular aspects of the great financial crisis and the policies implemented by the central banks at each stage is presented in Lenza et al. (2010).

bound constraint on the monetary policy interest rate, the economy enters into a liquidity trap scenario similar to the one observed during the great financial crisis. The structure of the model allows to provide a novel theoretical mechanism for the transmission of unconventional monetary policy, namely the increase of welfare of risk-averse agents which are insured by a policy that withdraws risk from their portfolio and provides a source of risk-free income.

The focus on increased uncertainty as the main source of shocks and aggregate instability, is supported by the fact that all major recessions in history have been characterised by microeconomic uncertainty shocks with negative mean and positive variance, which ended up affecting negatively stock returns (Bloom et al., 2018) and widening the left tail of large negative outcomes among firms (Salgado et al., 2020). In this context, agents of the economy are highly exposed to the idiosyncratic uncertainty risk affecting their portfolios and future sources of income.

The transmission mechanism of unconventional monetary policy that we highlight in this paper, has been widely present in policy discussions but has not been properly discussed by the literature from a theoretical perspective. By insisting on the role of increased uncertainty as a critical source of shocks to the economy, it is possible to isolate the *uncertainty channel of unconventional monetary policy*. The analysis presented in this paper provides a characterisation of how, through this channel, this kind of monetary policy counterfactually helps to stabilise the economy by increasing overall welfare.

The analysis is built upon a finite-horizon model of a closed economy, which is inhabited by a continuum of three-period lived households, a representative competitive firm which produces a perishable final good, a continuum of monopolistic competitive producers of intermediate goods, and a consolidated government which plays the role of both the central bank in charge of monetary policy and the fiscal authority. Production takes place in the first two periods of the economy. In the final period, there is no production and the supply of the perishable final good is equal to the aggregate dividend of the fixed supply of a risky *Lucas-tree* asset.

Each household is comprised of a continuum of potential workers, each one inelastically supplying one indivisible unit of specialised labour for the production of intermediate goods. In the second period of this economy, workers might face idiosyncratic unemployment risk since their individual labour supply might be rationed in the labour market. The household provides partial insurance to workers via inter-period transfers among members of the household, which imply all households are identical even in the face of idiosyncratic unemployment risk. As in e.g. Shi (2015) and Heathcote and Perri (2018) this mechanism avoids concerns related to the distribution of wealth/income without eliminating the sources of idiosyncratic risk.

At the beginning of the first period of the economy, the representative household is endowed with one unit of the *Lucas-tree* asset which generates a random dividend of units of the consumption good in the last period of the economy. The distribution of the realisation of dividends is bounded by a set of two mutually exclusive different random outcomes. Given the random nature of dividends, the household also faces an idiosyncratic dividend risk in this final period of the economy. The risky asset is perfectly illiquid in the sense that it cannot be traded in a spot market nor households can borrow against its future dividends before it matures. As a single unit, the representative household is also allowed to save into deposits which yield a nominal gross interest rate determined by the central bank.[2]

The monopolistic competitive firms producing intermediate goods use specialised labour as Typo: its$\rightarrow$ theirtheir only input and, at the beginning of the initial period of the economy, they set the relative price of their product such that they maximise their discounted flow of profits. We assume they keep this relative price fixed during the two periods of production. Because of this nominal price rigidity, the economy might be forced into a demand-determined equilibrium path in which specialised labour might be rationed. On its part, the representative competitive firm generates the production of the perishable final good via a CES aggregator of the intermediate goods.

[2]This two-asset structure can be associated to households not being able to participate in the credit market, as in Weil (1992), or to households simply having different sources of wealth, one which is attached to a highly illiquid asset, as in Kaplan et al. (2014).

The central bank of the consolidated government issues reserves to back up households' deposits and determines its nominal gross yield via a policy rule. Conventional interest rate-based monetary policy determines the nominal interest rate to be such that, given nominal prices, the real interest rate is equal to the natural real interest rate in the perfect foresight full employment equilibrium path. Nonetheless, the rule is constrained by a zero lower bound (ZLB) on the nominal interest rate. The fiscal authority side of the consolidated government imposes a lump-sum tax on households to fund the interest payments on reserves. On the other hand, since there is no production in the last period of the economy, the central bank also sets the price of the perishable consumption good such that inflation in this final period is equal to zero.

The analysis is focused on the perfect foresight equilibrium path of the model economy presented above. In particular, by taking as reference its full-employment equilibrium path, an uncertainty shock is imposed in the second period of the finite-horizon economy. The shock consists of an increase in the probability of realisation of the worst possible value of the real dividend of the risky asset, which not only reduces the mean value of the random dividend but increases its variance. One first result shows that, by reducing the nominal interest rate, conventional monetary policy can keep the economy in its full employment equilibrium path as long as the ZLB constraint does not bind, insuring households from the realisation of the unemployment idiosyncratic risk.

However, if the ZLB binds as a consequence of the shock, the economy enters a liquidity-trap path in which the equilibrium allocation is demand-determined and characterised by a positive unemployment rate. In this particular context, the analysis includes the possibility for the central bank to implement two alternative unconventional monetary policies which temporarily or permanently expands its asset side of the balance sheet. In the second period of the economy, the representative household is now allowed to trade any chosen fraction of their holdings of the risky and illiquid asset with the monetary authority, either by agreeing to a REPO contract with repurchase commitment in the last period of the economy, or via an outright purchase of such fraction of the illiquid asset.

The impact of such policies is assessed via two propositions. The first one establishes the REPO-based policy to be completely neutral given its Ricardian nature. A result in the same spirit of the neutrality effects of open market operations in Wallace (1981) and Eggertsson and Woodford (2003). In the portrayed liquidity trap scenario, this policy temporarily increases the availability of liquid resources to the household to potentially increase its spending, but the fall in aggregate expenditure is generated by excess savings generated by the fall of the natural interest rate. Any additional liquid resources available will be saved since the household has to repurchase the risky asset whilst facing the realisation of the increased uncertainty regarding its dividends.

The second proposition, associated with the impact of the outright purchases policy, highlights the uncertainty transmission channel of unconventional monetary policy. The key for this policy to generate counterfactual changes in the equilibrium allocation of the liquidity trap scenario lies in the purpose of the policy of completely withdrawing the source of uncertainty from the household's portfolio, and assuming the lighter aggregate consequences of idiosyncratic risk. The fiscal side of the policy gives a transfer of certain income to the representative household. Given the risk-averse features of the household's preferences, Jensen's inequality ensures the new allocation to be welfare improving via a smaller value of the implicit real interest rate which leads to a reduction in the unemployment rate, and the higher expected utility obtained from certain income.

Related literature. This paper is directly related to the literature on the macroeconomic effects of unconventional monetary policy. Cúrdia and Woodford (2011) and Gertler and Karadi (2011) include the central bank's balance sheet into DSGE models with New Keynesian features and private financial intermediation. Imperfections in the provision of financial intermediation related to exogenous or endogenous agency-related problems, make unconventional monetary policy non-neutral. In the presence of strong financial disruption in the provision of credit, the intervention of the central bank by means of expanding the size of its balance sheet supports the flow of credit, especially if the ZLB on the policy interest rate is binding.

In a direct attempt to quantitatively measure the impact of the Fed's liquidity facilities during the great financial crisis, Del Negro et al. (2017b) include liquidity and resaleabilty constraints to investment funding of firms based on illiquid assets in their balance sheets. By making both the government bonds and money not subject to the resaleability constraint, any change in the composition of the balance sheet of the central bank that increases its holdings of illiquid assets, will loosen the constraints on investment funding and enhance credit activity.

The analysis presented in this paper also builds on a fundamental financial disruption, since households are banned from either selling or borrowing against their holdings of an illiquid asset. However, because of the particular assumptions behind the decision-making of agents in the final period of the economy, if the central bank expands its balance sheet to simply increase liquidity, unconventional monetary policy might not have real effects. In this particular sense, the focus of this paper is to associate the impact of the central bank's balance sheet policy with the increase in uncertainty instead.

Cui and Sterk (2021) build a heterogeneous agents model subject to exogenous idiosyncratic unemployment risk and incomplete markets to quantitatively measure the counterfactual role of quantitative easing during the great financial crisis in the US. Households' portfolio consists of shares of partially illiquid equity issued by mutual funds and savings into deposits. Mutual funds' portfolio consists of firms' profits and long-term government debt. In a scenario where interest rates reach the ZLB and the central bank issues reserves in exchange for long-term government debt sold by mutual funds, increased reserves end up in the hands of households with a high marginal propensity to consume out of liquid wealth via fiscal policy, increasing aggregate spending. Despite being a quantitative model, their approach is more similar to ours. Nonetheless, we model unemployment idiosyncratic risk as endogenous and our focus is not on the liquidity channel of policy. We also find that the real effects of unconventional monetary policy critically depend on its fiscal counterpart, but in the particular case of our model, the implicit reduction of the levels of uncertainty faced by households is the channel to exploit.

Because of its purely theoretical insight, Araújo et al. (2015) is the closest to the analysis presented in this paper. They also build a model of a finite horizon economy with flexible prices and an uncertain state in the final period in which households can use a durable good as collateral on private loans. In their case, the effects of a policy in which the central bank issues reserves in exchange for a risky durable good depends on the way and degree to which the collateral constraint binds. If the collateral constraint does not bind at all the policy is irrelevant. Although the structure of their model is somehow similar to the one presented here, they do not consider an alternative with nominal rigidities, as we do, and more critically it is the liquidity aspect of the policy that matters the most to their argument. In contrast, the key of the approach presented here is to isolate the uncertainty channel to establish how, by withdrawing the source of risk and replacing it with a certain source of income, a similar policy can be effective by offering further insurance to households.

The paper is divided into six sections including this introduction. Section 2.2 presents the model and defines the symmetric perfect foresight equilibrium path. Section 2.3 characterises the reference full-employment equilibrium path of our model economy, establishes the extent to which conventional interest-rate-based monetary policy can keep the economy in its full-employment equilibrium path after an unexpected uncertainty shock, and presents the liquidity-trap scenario once the ZLB constraint on conventional monetary policy binds. The counterfactual assessment of the two alternative unconventional monetary policies is presented in Section 2.4. Conclusions are presented in Section 2.5.

2.2 The Model

The closed economy is set in a finite horizon of three periods $t = \{0, 1, 2\}$. There is a perishable consumption good in the economy which is produced in periods 0 and 1 by a competitive firm using a continuum of intermediate varieties, each produced by a monopolistically competitive firm using only labour. In the final period, the consumption good is the yield of a Lucas-tree asset in hands of the households. The economy is inhabited by a continuum of households and a consolidated central government in charge of monetary and fiscal policy. The domestic currency is the *numeraire* of all nominal prices, whilst the perishable consumption good plays the same role for all relative (real) prices.

2.2.1 The private sector

2.2.1.1 Households

There is a continuum of mass 1 of three-period lived identical households, each one indexed by $i \in [0,1]$. Households are risk-averse and derive utility from the consumption of the produced perishable consumption good. Their time-additive utility function is:

$$\mathbb{E}_0 \left[\sum_{t=0}^{2} \beta^t \ln(c_{it}) \right] \tag{2.1}$$

where $\mathbb{E}_0$ is the conditional expectations operator as of period 0, and $\beta \in (0,1]$ is the subjective time-discount factor. For the model to be tractable I assume instantaneous utility to be logarithmic, which implies an intertemporal constant elasticity of substitution of consumption equal to 1.

Each household is composed of a continuum of potential workers, each indexed by $j \in [0,1]$ and endowed with $n_j = 1$ units of specialised indivisible labour in periods 0 and 1, which they inelastically supply in the labour market. In the final period of the economy, household members do not work.

During their lifetime, households and their members might face two types of idiosyncratic risk. In period 1, because of the co-existence of a nominal price rigidity and a low bound on the nominal interest rate, workers might be forced into unemployment given the indivisibility of their individual fixed supply of labour. Let N_1^d be the aggregate demand for labour and $u = 1 - N_1^d$ the aggregate unemployment rate. Because of the law of large numbers, any potential worker member of any household i faces the idiosyncratic labour supply shock

$$\varepsilon_{j1} = \begin{cases} 0 & \text{with probability } u \\ 1 & \text{with probability } 1-u \end{cases}.$$

In other words, a potential worker might be randomly assigned to unemployment and his effective supply of labour in period 1 is equal to $n_{j1}\varepsilon_{j1}$.

On the other hand, at the beginning of period 0, every household is endowed with $\bar{h} = 1$ units of a *Lucas-tree* asset, each yielding a random real dividend D_i measured in units of the perishable consumption good which is realised at the beginning of period 2. The value of the dividend is unknown at the beginning of periods 0 and 1, but its distribution is known and determined by the set of probabilities and values $\{(\mathsf{p}, \underline{D}), (1-\mathsf{p}, \overline{D})\} \in (0,1) \times \mathbb{R}_{+}$, where $\overline{D} > \underline{D}$. The Lucas-tree asset is totally *illiquid* in the sense that no household can trade it in a spot market, nor can borrow against its future realised dividend.

Members of a household do not individually participate in the credit market. Nevertheless, we assume there is perfect risk sharing in the sense that we allow for inter-period transfers of income among household members and that as a unit, they can save into non-negative deposits directly into the Central Bank in periods $t = \{0,1\}$. As a counterpart, the monetary authority creates an equivalent amount of reserves $m_{it+1} \geq 0$ which yield a one-period gross nominal return $R_{t+1}\frac{P_{t+1}}{P_t}$, where R_{t+1} denotes the real interest rate. Compared to the Lucas-tree asset, deposits are perfectly *liquid* in the sense that reserves that back them up are convertible to currency immediately and at no cost. Households are also the owners of all firms and receive an equal share of the total nominal profits Γ_t in periods 0 and 1. To finance the interest payments on reserves, the government levies a nominal lump-sum tax T_{t+1} on all households.

The flow budget constraints that a household faces during its life-span are:

$$P_0 c_{i0} + m_{i1} = W_0 + \Gamma_0 \tag{2.2}$$

$$P_1 c_{i1} + m_{i2} + T_1 = W_1 \int_0^1 n_{j1} \varepsilon_{j1} \, \mathrm{d}j + \Gamma_1 + R_1 \frac{P_1}{P_0} m_{i1} \tag{2.3}$$

$$P_2 c_{i2} + T_2 = R_2 \frac{P_2}{P_1} m_{i2} + P_2 D_i \tag{2.4}$$

Solution to the household's problem. At the beginning of period 0, all households are identical and they share the same information set when maximising their flow of discounted utility in (2.1) subject to the flow budget constraints (2.2) – (2.4) and the sequence of non-negativity constraints $\{m_{it+1} \geq 0\}_{t=0}^{1}$. Moreover, they know that in every household a proportion $u \in [0,1)$ of its working members will be randomly forced into unemployment in period 1, but because there is perfect risk sharing among household members, the *per-capita* consumption will be the same in every household as well. The only source of heterogeneity is the realisation of the real dividend of the *Lucas-tree* asset in the final period of the economy.

Given the recursivity of the problem and its finite-horizon setting, it is possible to solve it backwards knowing that: *i*) there will be two groups of households in period 2 fully characterised by the probability distribution of the real dividend D_i, and *ii*) as of period 0 and 1, we can summarise the decisions of all households via the determination of the solution of the maximisation problem of a representative household.

Period 2. Given the distribution of the real dividend D_i, a proportion p of households observes a realisation $\underline{D}$ of their holdings of the *Lucas-tree* asset, whereas the complementary proportion $1-\mathsf{p}$ observes a realisation $\overline{D}$. Taking as given the state variable m_2, which summarises the past representative household's savings decision, and the values of $\{P_2, R_2, T_2\}$, two groups of households indexed by $L \subset [0, p]$ and $H \subset (p, 1]$, follow their flow budget constraint (2.4) and choose consumption according to the policy functions:

$$c_{L2}(m_2) = R_2 \frac{m_2}{P_1} + \underline{D} - \frac{T_2}{P_2} \tag{2.5}$$

$$c_{H2}(m_2) = R_2 \frac{m_2}{P_1} + \overline{D} - \frac{T_2}{P_2}. \tag{2.6}$$

The optimal value of the problems of these groups of households is simply $V_{L2} = \ln(c_{L2})$ and $V_{H2} = \ln(c_{H2})$.

Period 1. At the beginning of period 1, the idiosyncratic unemployment risk is realised and, because of the law of large numbers, a proportion u of potential workers, members of the representative household, are forced into unemployment. By substituting equation (2.3) into the logarithmic instantaneous utility function, and taking as given the value of the savings state m_1 and the values of $\{\Gamma_1, P_1, W_1, T_1\}$, the problem of the representative household as of period 1 is represented by the Bellman equation:

$$V_1(m_1) = \max_{m_2} \ln\left(\frac{W_1}{P_1}(1-u) + \frac{\Gamma_1}{P_1} + R_1\frac{m_1}{P_0} - \frac{m_2}{P_1} - \frac{T_1}{P_1}\right) + \beta\,\mathbb{E}_1\left[V_2(m_2)\right]$$
$$\text{s.t. } m_2 \geq 0$$

The first order and envelope conditions of this problem with respect to the deposits decision of the representative household, leads to the Euler equation:

$$\frac{1}{c_1(m_1)} = \beta R_2 \left(\frac{\mathsf{p}}{c_{L2}(m_2(m_1))} + \frac{1-\mathsf{p}}{c_{H2}(m_2(m_1))}\right) + \lambda_1(m_1) \tag{2.7}$$

where, from the flow budget equation (2.3), $c_1(m_1)$ is determined by the policy function:

$$c_1(m_1) = \frac{W_1}{P_1}(1-u) + \frac{\Gamma_1}{P_1} + R_1\frac{m_1}{P_0} - \frac{m_2(m_1)}{P_1} - \frac{T_1}{P_1} \tag{2.8}$$

and $\lambda_1(m_1)$ is the optimal value of the Lagrange multiplier associated with the non-negativity constraint on the deposits decision. Equations (2.7) and (2.8) completely characterise the optimal savings and consumption decisions of the representative household in this period.

Period 0. In period 0 the representative household makes consumption and savings decisions whilst dealing with the uncertainty regarding the value of the unemployment rate in period 1 and the realisation of the dividend of the *Lucas-tree* asset. By substituting the flow budget equation (2.2) in the logarithmic instantaneous utility function of period 0, and taking as given the values of $\{\Gamma_0, P_0, W_0\}$, the problem of the representative household in period 0 can be represented by the Bellman equation:

$$V_0 = \max_{m_1} \ln\left(\frac{W_0}{P_0} + \frac{\Gamma_0}{P_0} - \frac{m_1}{P_0}\right) + \beta\,\mathbb{E}_0\left[V_1(m_1)\right]$$
$$\text{s. t. } m_1 \geq 0$$

In a similar way to the solution of the two-period problem as of period 1, the implicit policy function derived from the first order and envelope conditions of this problem with respect to the nominal deposits choice, leads to the Euler equation:

$$\frac{1}{c_0} = \beta R_1 \mathbb{E}_0\left[\frac{1}{c_{i1}}\right] + \lambda_0 \tag{2.9}$$

where λ_0 represents the Lagrange multiplier associated with the non-negativity constraint on the deposits decision of the household.

2.2.1.2 Production

Production of the perishable consumption good. In periods $t = \{0,1\}$, a competitive representative firm produces an amount Y_t of the perishable final good using imperfectly substitutable intermediate varieties, each indexed by j in the continuum $[0,1]$, such that:

$$Y_t = \left[\int_0^1 y_{jt}^{\frac{\eta-1}{\eta}}\,\mathrm{d}j\right]^{\frac{\eta}{\eta-1}} \tag{2.10}$$

where $\eta > 1$ is the elasticity of substitution of varieties.

Given the nominal price of each intermediate variety p_{jt}, and the nominal wage paid to one unit of labour W_t, the static problem of the competitive representative firm is to minimise costs $\int_0^1 p_{jt} y_{jt}\,\mathrm{d}j$ subject to equation (2.10). For each intermediate variety, the solution to this minimisation program determines the demand function:

$$y_{jt} = \left(\frac{p_{jt}}{P_t}\right)^{-\eta} Y_t \tag{2.11}$$

where the nominal price value of a unit of the produced perishable good is

$$P_t = \left(\int_0^1 p_{jt}^{1-\eta}\,\mathrm{d}j\right)^{\frac{1}{1-\eta}} \tag{2.12}$$

Production of intermediate varieties. A continuum of size one of monopolistic competitive firms is in charge of the production of intermediate varieties. An amount y_{jt} of the intermediate variety $j \in [0,1]$ is produced using specialised labour N_{jt} with the linear technology:

$$y_{jt} = N_{jt} \tag{2.13}$$

At the beginning of period 0, each intermediate-producer firm's objective is to maximise its discounted flow of profits along the first two periods of the economy. As in Korinek and Simsek (2016), I assume an extreme form of nominal price rigidity, where all intermediate-producer firms optimally determine their price in $t = 0$ and decide to fix it for all the production horizon. Given the nominal price rigidity assumption, if the interest rate policy of the monetary authority is constrained by a zero-lower-bound (ZLB), the equilibrium allocation of the economy might be one in which both final production and labour are purely determined by demand. Labour might be rationed in the labour market in the sense of Bénassy (1993), namely $N_1^d = \int_0^1 N_{j1}\,\mathrm{d}j \leq 1$. As a consequence, at the beginning of period 0 intermediate firms maximise their intertemporal flow of profits such that: *i)* there is full employment in period 0, and *ii)* rationing of labour supply in period 1 implies their labour demand to be determined as $1-u$.

Let $\Gamma_{jt} = p_{jt} y_{jt} - W_t N_{jt}$ be the flow of nominal profits of the producer of an intermediate variety in $t = \{0,1\}$. Subject to the nominal price rigidity, the demand for its production given by equation (2.11), and its technology of production summarised by equation (2.13), the problem of the monopolistic competitive firm at the beginning of period 0 is to choose its fixed relative price $\frac{p_{j0}}{P_0}$ to maximise the expected discounted sum of profits

$$\left(\frac{p_{j0}}{P_0}\right)^{-\eta}\left[\left(\frac{p_{j0}}{P_0}-\frac{W_0}{P_0}\right)Y_0+\Lambda_{0,1}\left(\frac{p_{j0}}{P_0}-\frac{W_1}{P_0}\right)Y_1\right].$$

Since firms are owned by households, in any equilibrium the firms' discount factor is $\Lambda_{0,1}=\beta\frac{Y_0}{Y_1}$, where $Y_0=1$. Hence, the first order condition of the maximisation problem implies that any monopolistic competitive firm sets its relative price such that

$$\frac{p_{j0}}{P_0}=\frac{\eta}{\eta-1}\left(\frac{1}{1+\beta}\right)\left(\frac{W_0}{P_0}+\beta\frac{W_1}{P_0}\right), \tag{2.14}$$

which implies that aggregate real profits in periods 0 and 1 are given by:

$$\frac{\Gamma_t}{P_t}=\int_0^1\left(\frac{p_{j0}}{P_0}-\frac{W_t}{P_0}\right)N_{jt}\,\mathrm{d}j \tag{2.15}$$

In the final period, the production of the perishable final good is totally determined by the aggregate real dividend D of the supply of Lucas-tree assets in hands of the households. Therefore:

$$Y_2=\mathsf{p}\underline{D}+(1-\mathsf{p})\bar{D}\equiv D \tag{2.16}$$

2.2.2 Consolidated government and conventional monetary policy

Both monetary and fiscal policy are determined by a consolidated Government-Central Bank, which determines the value of the nominal gross interest rates paid on reserves in periods 1 and 2, the corresponding lump-sum taxes to finance such payments, and the value of the money price of the consumption good in the final period of the economy.

In particular, the monetary authority side of the consolidated government sets

$\{R_{t+1}\frac{P_{t+1}}{P_t}\}_{t=0}^{1}$ and P_2 by following the rules:

$$R_{t+1}\frac{P_{t+1}}{P_t} = \max\left\{1, R^*_{t+1}\frac{P_{t+1}}{P_t}\right\} \tag{2.17}$$

$$P_2 \text{ s.t. } \frac{P_2}{P_1} = 1. \tag{2.18}$$

As most of the literature on the role of monetary policy in a liquidity trap scenario (e.g. Korinek and Simsek, 2016; Krugman, 1998; Werning, 2012), the rule in (2.17) makes explicit the constraint imposed by the Zero-Lower-Bound (ZLB). Because of the extreme nominal rigidity assumption in periods 0 and 1, the ZLB does not only affects the determination of the value of the nominal rate, but also the value of the gross real interest rate and, therefore, the ability of the central bank to manipulate aggregate demand. However, whilst not constrained, the monetary authority sets the gross nominal (and real) interest rate such that its value is equal to the flexible price perfect foresight full-employment equilibrium —natural—real interest rate of the economy, R^*_{t+1}.

On the other hand, the monetary authority also sets the value of the money price of the perishable consumption good in the final period, since there is no production and its total output is given exogenously by equation (2.16). In the same spirit of the interest rule in equation (2.17), the objective is to set the value of the price of the consumption good, such that the inflation rate in period 2 is equal to the equilibrium inflation rate in the flexible price perfect foresight full-employment equilibrium of the economy.[3]

The consolidated government assumption implies that any monetary policy implemented following rules (2.17) and (2.18) has to be backed up by fiscal policy in the form of lump sum taxes to finance the payment of interests on the reserves held by the representative household. Let $\{m_{t+1}\}_{t=0}^{1}$ represent the total amount of

[3] In the traditional analysis of optimal monetary policy when the ZLB is not binding, the optimal interest rate condition is precisely to set the value of the nominal rate equal to the natural rate when inflation is zero. This is needed to obtain the first best allocation (Werning, 2012). In the particular case of this finite horizon stylised model of the economy, the flexible price allocation is not efficient given the monopolistic competition structure of the production of the consumption good. However, this allocation corresponds to the full employment perfect foresight equilibrium, in which all households obtain the maximum level of utility given the inefficient equilibrium outcome.

reserves that the Central Bank creates to back up the total amount of deposits of the representative household. Therefore, the consolidation of policies is summarised by the budget equation:

$$T_{t+1} = \left(R_{t+1} \frac{P_{t+1}}{P_t} - 1 \right) m_{t+1}; \ t = \{0, 1\} \tag{2.19}$$

Notice that because of the nominal rigidity assumption and the price determination policy in (2.18), we are ruling out a role for seigniorage in financing the consolidated government.

2.2.3 Equilibrium

The focus is on the perfect foresight symmetric equilibrium of this economy, in which all agents as of period 0, share the same information set containing all present and future values of prices, wages, interest rates and tax rates, as well as the values of the unemployment rate, the possible values of the dividend of the Lucas-tree asset, and the probability p. The perfect foresight equilibrium is symmetric because all producers of intermediate varieties face the same demand function when maximising profits at date 0, which implies that all monopolistic firms: *i)* set and fix the same relative price $\frac{p_{j0}}{P_0} = \frac{p_0}{P_0}$, *ii)* demand the same amount of specific labour $N_{jt} = N_t$, *iii)* produce the same amount of the intermediate variety output $y_{jt} = y_t$ and, *iv)* gain the same amount of nominal profits $\Gamma_{jt} = \Gamma_t$.

Before properly defining the equilibrium of our stylised economy, we include an equation to pin down the value of the wage in period 1 for any perfect foresight equilibrium characterised by an allocation *off-full-employment*, namely

$$W_1 = W_0 \tag{2.20}$$

Equation (2.20) could be justified in a similar way to the price rigidity assumption above. In particular, it could be possible for specialised workers to organise into unions which have market power in the labour market and are able to set the wage (e.g. Erceg et al., 2000). Nevertheless, in our stylised economy, it is only included as a device to determine the level of the equilibrium wage in period one when the intermediate variety producers are able to reoptimize after the realisation of a future wealth-related shock. We now proceed to the definition of equilibrium.

Definition 1. *Given the possible values of the real dividend* $\{\underline{D},\overline{D}\}$ *and the known value of the probability* p*, the perfect foresight symmetric general equilibrium of the economy consists of the allocation* $[\{Y_t\}_{t=0}^{2},\{y_t\}_{t=0}^{1},\{N_t\}_{t=0}^{1},\{N_t^d\}_{t=0}^{1},\{\Gamma_t\}_{t=0}^{1},\{c_t\}_{t=0}^{1},c_{L2},c_{H2},c_2,\{m_{t+1}\}_{t=0}^{1},\{\lambda_t\}_{t=0}^{1}]$*, the price vector* $[\{p_t\}_{t=0}^{1},\{P_t\}_{t=0}^{1},\{W_t\}_{t=0}^{1}]$*, the policy vector* $[\{R_{t+1}\}_{t=0}^{1},P_2,\{T_{t+1}\}_{t=0}^{1}]$ *and the unemployment rate* $u \in [0,1)$*, such that:*

1. *Households maximise utility in equation* (2.1)*, i.e. equations* (2.2) *and* (2.5) – (2.9) *hold, given* $[\{P_t\}_{t=0}^{2},\{W_t\}_{t=0}^{1},\{R_{t+1}\}_{t=0}^{1},\{T_{t+1}\}_{t=0}^{1}]$*, u,* $\{\underline{D},\overline{D}\}$ *and* p.

2. *The competitive representative firm producing the final perishable good maximises its profits at each date, i.e.* (2.10)*,* (2.11)*, and* (2.12) *hold given* $[\{p_t\}_{t=0}^{1}]$.

3. *Given* $\{\underline{D},\overline{D}\}$ *and* p*, the supply of the perishable consumption good in period 2 is determined by* (2.16).

4. *Each intermediate-variety producer firm maximises its discounted flow of profits in periods 0 and 1, given* $[\{P_t\}_{t=0}^{1},\{W_t\}_{t=0}^{1}]$*, i.e. equations* (2.14) *and* (2.15) *hold given the nominal rigidity assumption* $p_1 = p_0$*, and the nominal wage rigidity given by equation* (2.20).

5. *The labour market clears in period 0:*

$$N_0 = N_0^d = 1 \tag{2.21}$$

and in period 1 labour supply might be rationed, i.e.:

$$N_1 = N_1^d \leq 1 \tag{2.22}$$

$$u = 1 - N_1^d \tag{2.23}$$

6. *In periods 0 and 1 the value of the gross nominal interest rate on reserves* $R_{t+1}\frac{P_{t+1}}{P_t}$ *is determined by equation* (2.17).

7. P_2 *is set according to equation* (2.18).

8. *The lump sum tax rates* $\{T_{t+1}\}_{t=0}^{1}$ *are set by the consolidated Government such that the budget equation in* (2.19) *holds.*

9. *The market for the perishable good clears in all periods, i.e. for* $t = \{0,1\}$*:*

$$c_t = Y_t \tag{2.24}$$

and in period 2:

$$\mathsf{p}c_{L2} + (1-\mathsf{p})c_{H2} = c_2 = Y_2 \tag{2.25}$$

2.3 Uncertainty shocks and the role of conventional monetary policy

Even economies in a full employment state might be more or less vulnerable to economic fluctuations depending on their prospects of future wealth. Increasing uncertainty regarding future wealth can give rise to scenarios in which households might not be able to properly self-insure against idiosyncratic shocks, ending up in a self-fulfilling unemployment trap equilibrium (Heathcote and Perri, 2018). To assess the extent of the role of conventional monetary policy in offsetting the spread of idiosyncratic risks faced by households, we consider the effects of a one-time unexpected change in the exogenous value of the probability p on the perfect foresight full-employment equilibrium path of the stylised economy in Section 2.2.

Before proceeding to characterise this reference equilibrium path, we need to make some additional assumptions on the distribution of the random real dividend of the *Lucas-tree* asset to make our analysis cleaner:

Assumption 1. $\underline{D} < \beta$ *and* $\overline{D} > \beta$.

Assumption 2. $\mathrm{p} < \frac{\underline{D}}{\beta}\left(\frac{\overline{D}-\beta}{\overline{D}-\underline{D}}\right)$.

2.3.1 The perfect foresight full-employment equilibrium

The perfect foresight full-employment (FE) equilibrium is completely characterised by a situation in the labour market in which in period 1 labour is not rationed and there are no unemployed workers.[4] Specifically, the FE equilibrium of the economy is characterised by an equilibrium level of labour demand in period 1 equal to

$$\left(N_1^d\right)^{FE} = 1, \tag{FE.1}$$

which implies that the equilibrium unemployment rate is equal to

$$u^{FE} = 0 \tag{FE.2}$$

Remark 1. *Given assumptions 1 and 2, Definition 1 determines that in the FE equilibrium path:*

1. *For* $t = \{0,1\}$*:*

$$N_t^{FE} = \left(N_1^d\right)^{FE} \tag{FE.3}$$

$$y_t^{FE} = Y_t^{FE} = 1 \tag{FE.4}$$

$$p_t^{FE} = P_t^{FE} = 1 \tag{FE.5}$$

$$W_t^{FE} = \frac{\eta - 1}{\eta} \tag{FE.6}$$

$$\Gamma_t^{FE} = \frac{1}{\eta} \tag{FE.7}$$

$$\lambda_t^{FE} = 0 \text{ and } m_{t+1}^{FE} = 0 \tag{FE.8}$$

$$c_t^{FE} = 1 \tag{FE.9}$$

$$T_{t+1}^{FE} = 0 \tag{FE.10}$$

[4]Notice that there is no need to characterise the situation of the labour market in period 0, since by construction the model economy is always in full-employment in this initial date (see equation (2.21)).

2. *Conventional monetary policy implementation implies:*

$$R_1^{FE} = R_1^* = \frac{1}{\beta} \tag{FE.11}$$

$$R_2^{FE} = R_2^* = \frac{1}{\beta}\left(\frac{\mathsf{p}}{\underline{D}} + \frac{1-\mathsf{p}}{\overline{D}}\right)^{-1} \tag{FE.12}$$

$$P_2^{FE} = 1 \tag{FE.13}$$

3. *In the final period:*

$$c_{L2}^{FE} = \underline{D} \tag{FE.14}$$

$$c_{H2}^{FE} = \bar{D} \tag{FE.15}$$

$$c_2^{FE} = Y_2^{FE} = \mathsf{p}\underline{D} + (1-\mathsf{p})\bar{D} \equiv D. \tag{FE.16}$$

4. *Overal welfare is given by*

$$V_0^{FE} = \beta^2\left[\mathsf{p}\ln(\underline{D}) + (1-\mathsf{p})\ln(\overline{D})\right] \tag{FE.17}$$

Given the non-negativity constraint on deposits, in the FE equilibrium the representative household optimally chooses not to make deposits in periods 0 and 1 (see equation (FE.8)), a decision driven by the perishable good equilibrium conditions in (2.24), the price rigidity assumption which implicitly makes the real interest rate in period 0 equal to the nominal interest rate in equilibrium, and the monetary policy rules (2.17) and (2.18). Since optimal savings in both periods 0 and 1 are equal to zero, fiscal consolidation determined by equation (2.19) implies lump-sum taxes to be equal to zero as well (see equation (FE.10)).

Because of Assumption 2, the implementation of conventional monetary policy is not constrained by the ZLB (see equation (FE.12)). In other words, by following the conventional monetary policy rule, the monetary authority sets a value for the real interest rate such that the representative household has no incentive to neither reduce nor increase present consumption to increase or decrease future consumption simply because, by consuming all what is produced of the perishable consumption good in the current period, they obtain the maximum possible welfare given the realisation of the real dividend of the *Lucas-tree* asset (see equations (FE.9), (FE.14) (FE.15) and (FE.17)).

On the other hand, equations (FE.1) and (FE.2) establish that production and demand for labour in period 1 must be exactly equal to those in period 0. Therefore, because of the nominal price rigidity assumption, the equilibrium nominal wages must also be the same, and its value in equation (FE.6) is pinned down by the optimal pricing condition in equation (2.14). In periods 0 and 1 full employment implies that in equilibrium the value of both the production of the intermediate varieties and the production of the perishable consumption good is equal to one (see equation (FE.4)). From equation (2.15) it is straightforward to determine the equilibrium value of real profits in equation (FE.7). In the final period, the clearing equilibrium condition in (2.25) implies that aggregate consumption is equal to the value of the aggregate real dividend of the Lucas-tree asset (see equation (FE.16)).

2.3.2 On the role of conventional monetary policy

We now consider the following scenario. At the beginning of period 1, whilst on the perfect foresight FE equilibrium path described in Remark 1, households, firms and the consolidated government perceive an unexpected increase in the probability of the realisation of the lowest value of the real dividend of the *Lucas-tree* asset $\Delta \mathsf{p} > 0$. To better characterise the shock, we make the following assumption on the initial value of p in the FE equilibrium:

Assumption 3. $\mathsf{p} \in [0, \varepsilon]$ *for any* $\varepsilon \to 0$.

Assumption 3 states that in the FE equilibrium, the known distribution of the real dividend of the *Lucas-tree* asset is such that the probability of realisation of the lowest outcome is very small. This implies that at the beginning of both periods 0 and 1, all agents foresee a mean value D close to the highest outcome $\bar{D}$. Then, the shock at the beginning of period one has two different direct effects. First, it lowers the mean value of the realisation of the dividend which implies that households in particular perceive a reduction in the value of their future wealth and, second, it increases the variance of the realisation of the dividend, increasing the general perception of uncertainty.[5]

Once this uncertainty shock is realised at the beginning of period 1, households and firms are forced to re-optimise. The extent to which conventional monetary policy is able to completely isolate households from the potential spreading of both unemployment risk and uncertainty, depends on the size of the shock, as stated below by Proposition 1.

Proposition 1. *Given assumptions 1, 2 and 3, if the uncertainty shock is such that*

$$\Delta \mathsf{p} \leq \frac{\underline{D}}{\beta}\left(\frac{\overline{D}-\beta}{\overline{D}-\underline{D}}\right)-\mathsf{p} \tag{2.26}$$

holds, then the central bank is able to rule out positive unemployment risk as an equilibrium outcome in period 1 by implementing conventional monetary policy following the rule in equation (2.17)*, i.e. by setting* $R_2 = R_2^* = \frac{1}{\beta}\left(\frac{\mathsf{p}+\Delta\mathsf{p}}{\underline{D}}+\frac{1-(\mathsf{p}+\Delta\mathsf{p})}{\overline{D}}\right)^{-1} < R_2^{FE}$*. Nonetheless, increased uncertainty has a negative impact by reducing the mean consumption of the representative household in the last period of the economy, i.e.*

$$V_0^{FE'} = \beta^2\left[(\mathsf{p}+\Delta\mathsf{p})\ln(\underline{D})+(1-(\mathsf{p}+\Delta\mathsf{p}))\ln(\overline{D})\right] < V_0^{FE}. \tag{2.27}$$

Proof. The perfect foresight FE equilibrium is defined for any given level of the value of the real dividend that complies with Assumption 2. As long as condition (2.26) holds, $\frac{(\mathsf{p}+\Delta\mathsf{p})}{\underline{D}}+\frac{(1-(\mathsf{p}+\Delta\mathsf{p}))}{\bar{D}} \geq \beta$ and the monetary policy rule in equation (2.17) implies $R_2 = R_2^*$, which makes the representative household to optimally choose $m_2 = m_2^{FE}$ and $c_1 = c_1^{FE}$. Hence, the demand-determined equilibrium level of output is Y_1^{FE} and the equilibrium unemployment rate is equal to zero. Welfare measured by the discounted expected flow of instantaneous utilities in all three

[5]In particular, under Assumption 3, the mean value of the real dividend of the Lucas-tree asset decreases by $\Delta\mathsf{p}(\bar{D}-\underline{\mathrm{D}})$, whereas its variance increases by $(1-2\mathsf{p})\Delta\mathsf{p}(\bar{D}-\underline{\mathrm{D}})^2$.

periods is only affected because of the new probability of realisation of $\underline{D}$. □

Proposition 1 highlights the importance of the size of the uncertainty shock, to understand the extent to which the central bank is able to perfectly rule out the spread of unemployment idiosyncratic risk to households by following the conventional monetary policy rule. As long as (2.26) holds, the central bank is not constrained by the ZLB when implementing policy. Despite the negative shock, the implementation of conventional monetary policy manages to completely rule out the realisation of positive idiosyncratic unemployment risk and keeps the economy in its full employment path in periods 0 and 1. In other words, interest-rate-based conventional monetary policy impedes increased uncertainty to trigger the spread of idiosyncratic unemployment risk.

Nevertheless, if the highest realisation of future wealth is low enough, a sufficiently small uncertainty shock might lead the economy into a scenario in which the ZLB is binding, and the central bank's conventional monetary policy is no longer able to impede the spread of positive unemployment idiosyncratic risk to households. The characterisation of the equilibrium path of the stylised economy with a positive unemployment rate, in which conventional monetary policy is constrained by the ZLB, is summarised in Lemma 1.

Lemma 1. *Let the value of $\bar{D}$ be low enough such that equation* (2.26) *does not hold once a sufficiently small uncertainty shock is realised. Hence, the central bank is constrained by the ZLB when implementing conventional monetary policy according to* (2.17) *and the value of the nominal (real) interest rate is equal to:*

$$R_2^{PF} = 1. \tag{2.28}$$

As of period 1, the new unique incentive-compatible perfect foresight equilibrium path of the stylised economy is characterised by

$$\lambda_1^{PF} = 0, \tag{2.29}$$

and the positive unemployment rate

$$u^{PF} = 1 - \frac{1}{\beta}\left(\frac{\mathsf{p}+\Delta\mathsf{p}}{\underline{D}} + \frac{1-(\mathsf{p}+\Delta\mathsf{p})}{\overline{D}}\right)^{-1} > 0. \tag{2.30}$$

Hence, in period 1 the representative household optimally decides to make deposits $m_2^{PF} = 0$ *and to consume:*

$$c_1^{PF} = \frac{1}{\beta}\left(\frac{\mathsf{p}+\Delta\mathsf{p}}{\underline{D}} + \frac{1-(\mathsf{p}+\Delta\mathsf{p})}{\overline{D}}\right)^{-1} = Y_1^{PF} < 1.$$

As of period 1, maximum expected utility is given by:

$$V_1(u^{PF}) = \ln(1-u^{PF}) + \beta\left[(\mathsf{p}+\Delta\mathsf{p})\ln(\underline{D}) + (1-(\mathsf{p}+\Delta\mathsf{p}))\ln(\overline{D})\right] \tag{2.31}$$

Proof. See proof in Appendix 2.A.1. □

Once the nominal policy interest rate is constrained by the ZLB, and the nominal price rigidity implies the real interest rate to be constrained as well, the economy enters a path in which the equilibrium allocation is purely demand determined. Implicitly, the uncertainty shock induces the real interest rate to be low enough such that its value is below the ZLB constraining the conventional monetary policy rule. This leads to a reduction in the aggregate demand for the perishable good, which is optimally confirmed by the production sector via a decrease in the aggregate demand for labour which rations workers' labour supply and generates involuntary unemployment as shown by equation (2.30). Uncertainty turns now into idiosyncratic unemployment risk.

2.4 On the role of unconventional monetary policy

The liquidity trap scenario portrayed in Lemma 1 is one in which conventional monetary policy is no longer effective because the nominal interest rate has reached the ZLB after responding to an uncertainty shock. Furthermore, the inflation rate is zero because of the nominal price rigidity assumption imposed on the price-setting behaviour of intermediate-variety producers, aggregate demand has been depressed and positive involuntary unemployment arises as an equilibrium outcome. The particular characteristics of this scenario are not far from what was observed during the great financial crisis of 2007/2008 and have been previously characterised by the literature (e.g. Korinek and Simsek, 2016; Krugman, 1998).

It is in this particular context that we consider two alternative unconventional policies which can be implemented by the monetary authority and that resemble a major component of the large asset purchase scheme program implemented in the United States, and the fixed rate/full allotment tender procedure implemented by the ECB in response to the 2008 financial crisis, as documented with great detail in e.g. Lenza et al. (2010). These two policies are introduced in the model of our stylised economy, by taking advantage of the two-asset structure of the portfolio of the representative household and the assumption that its endowment of the Lucas-tree asset cannot be traded nor used to borrow against its future dividends.

2.4.1 Liquidity provision using REPO contracts

Let $\mathbb{1}\left(R_2\frac{P_2}{P_1}=1\right)$ be an indicator function that takes a value of 1 if the ZLB is binding in $t=1$ and 0 otherwise. The representative household has now the possibility of engaging with the central bank of the consolidated government through a very simple REPO contract under which the monetary authority temporarily buys a fraction $\mathbb{1}\left(R_2\frac{P_2}{P_1}=1\right)\times\phi\in[0,1]$ of the household's holdings of the Lucas-tree asset, by issuing a certain amount of reserves m_1^{CB} into the household's account at the central bank. As an essential part of the two-party agreement, the representative household fully commits to repurchase the collateral in the final period of the economy.

To make the repurchase commitment operational we need to add a timing consideration regarding the final period of our stylised economy. In particular, we assume that all units of the *Lucas-tree* asset mature at the end of period 2, leaving consumption decisions of the two groups of households to be made at the end of this period as well. Households must then repurchase the collateral at the beginning of such period.

Since there is no market to price the non-traded *Lucas-tree* asset, we assume the central bank to value each unit of this asset by its *fair price* q^{CB}, namely the discounted value of its expected dividend:

$$q^{CB} = (\mathsf{p}+\Delta\mathsf{p})\underline{D} + (1-(\mathsf{p}+\Delta\mathsf{p}))\overline{D},$$

therefore, the corresponding amount of reserves issued by the central bank must be equal to

$$m_1^{CB} = P_1 q^{CB} \mathbb{1}\left(R_2 \frac{P_2}{P_1} = 1\right)\phi\bar{h}. \tag{2.32}$$

The REPO contract is a perfectly balanced operation and does not affect the flow budget constraints in (2.19).[6]

In periods 1 and 2, the possibility of agreeing on a REPO contract with the consolidated government affects the nature of the problem of the representative household by including a new variable of choice, which is the fraction of the illiquid asset to be sold to the central bank. Therefore, the implementation of this particular unconventional monetary policy implies that, as of period 1, the flow budget constraints of the re-optimiser representative household can be rewritten as:

$$P_1c_1 + m_2 = W_1(1-u) + \Gamma_1 + R_1\frac{P_1}{P_0}m_1 + P_1 q^{CB}\mathbb{1}\left(R_2\frac{P_2}{P_1} = 1\right)\phi\bar{h} - T_1 \tag{2.33}$$

$$P_2c_{i2} + P_2q^{CB}\mathbb{1}\left(R_2\frac{P_2}{P_1} = 1\right)\phi\bar{h} = R_2\frac{P_2}{P_1}m_2 + P_2D_i - T_2;\ \ i \in \{L,H\}. \tag{2.34}$$

Given prices $\{P_t\}_{t=1}^2$; the nominal wage W_1; the nominal profit Γ_1; the nominal interest rate R_2; the value of the indicator function $\mathbb{1}(R_2 = 1)$; lump-sum taxes

[6] Notice that in period 2, the explicit balance of flow budget operations is:

$$\left(R_2\frac{P_2}{P_1} - 1\right)m_2 + \frac{P_2}{P_1}m_1^{CB} = T_2 + P_2q^{CB}\mathbb{1}\left(R_2\frac{P_2}{P_1} = 1\right)\phi\bar{h},$$

which implies that the repurchase operation entangles a real cost for the consolidated government equal to the initial real value of the reserves created in the first period. In other words, since in period 2 the only source of real wealth is the fixed supply of the *Lucas-tree* asset, the central bank incurs a cost when "destroying" the reserves it created in period 1.

$\{T_{t+1}\}_{t=0}^{1}$; the *fair price* q^{CB}; the unemployment rate u; the probability p, and the uncertainty shock Δp, the reoptimising representative household solves the problem:

$$\begin{aligned} V_1(m_1) = & \max_{c_1, m_2, \mathbb{1}\left(R_2 \frac{P_2}{P_1}=1\right)\phi} \ln(c_1) + \beta\,\mathbb{E}_1\left[V_2\left(m_2, \mathbb{1}\left(R_2\frac{P_2}{P_1}=1\right)\phi\right)\right] \\ & \text{s.t. (2.33)–(2.34)} \\ & m_2 \geq 0;\ 0 \leq \mathbb{1}\left(R_2\frac{P_2}{P_1}=1\right)\phi \leq 1 \end{aligned} \tag{2.35}$$

2.4.2 A neutrality result

The definition of equilibrium under the implementation of the REPO-based unconventional monetary policy is slightly different to the one provided in Definition 1, and it is established by Definition 2 in Appendix 2.B.1. Under this definition, once the unexpected uncertainty shock is realised, the key to assessing the effects of the implementation of unconventional monetary policy on the equilibrium allocation lies in the characterisation of the solution of the representative household problem summarised in (2.35). Proposition 2 below states the effects of the REPO-based policy on the equilibrium allocation of our stylised economy.

Proposition 2. *Given assumptions 1, 2 and 3, and following Definition 2, the solution to the representative household in* (2.35) *implies that the perfect foresight equilibrium allocation is such that* $m_2^{PF} = m_1^{CB,PF}$ *and that, as of period 1, regardless of the equilibrium values* $\{\phi^{PF}, m_1^{CB,PF}, m_2^{PF}\} \in [0,1] \times [0, P_1^{PF} q^{CB} \phi \bar{h}] \times [0, P_1^{PF} q^{CB} \phi \bar{h}]$*, the perfect equilibrium allocation of the stylised economy is no different than the equilibrium allocation established by Lemma 1. The implementation of the unconventional monetary policy based on REPO contracts to provide liquidity to the representative household after the uncertainty shock is completely neutral.*

Proof. See proof in Appendix 2.A.2. □

In the spirit of the results in Wallace (1981) and Eggertsson and Woodford (2003), the implementation of the REPO-based policy is neutral regarding its real effects on the economy. Proposition 2 shows how the policy is Ricardian in nature, and how whatever potential increase in the liquid resources available to the household through the implementation of policy is optimally saved by the representative household, ruling out any change in aggregate demand.

Given the structure of the model of the finite horizon economy, this neutrality result can be explained by the ill–design features of this particular policy to counteract the sources of the shock. The perfect foresight allocation in Lemma 1 shows that involuntary unemployment is totally determined by the negative mean wealth effect of the shock, which leads the representative household to precautionarily save –optimally not to incur in debt–. A policy designed to temporarily increase the liquidity available by suddenly making an illiquid asset liquid does not reduce the negative impact on mean wealth and the representative household has no incentive to use the extra liquidity.

Nevertheless, the neutrality result in Proposition 2 serves as a reference to understand why an alternative policy, which deals with the source of the shock, might be better suited not only to reduce the spread of idiosyncratic unemployment risk but to reduce the negative impact of increased uncertainty on future mean wealth.

2.4.3 Outright purchases

We now slightly change the nature of the market operation behind the unconventional monetary policy implemented in period 1. Instead of agreeing on a REPO contract with the representative household, the central bank of the consolidated government is willing to make an outright purchase of a fraction $\mathbb{1}\left(R_2\frac{P_2}{P_1}=1\right)\times\phi$ of the total household's holdings of the risky and illiquid asset at a price per unit equivalent to the *fair price* q^{CB}. Under the possibility of this market operation, the flow budget constraint of the representative household in equation (2.33) still holds, but since there is no longer a repurchase agreement in period 2, the new set of households' budget constraints in this final period is

$$P_2c_{i2} = R_2\frac{P_2}{P_1}m_2 + P_2(1-\phi)D_i - T_2;\ \ i\in\{L,H\}. \tag{2.36}$$

Then, after the uncertainty shock, the problem of the household can be summarised by the value function

$$\begin{aligned} V_1(m_1) = & \max_{c_1, m_2, \mathbb{1}\left(R_2 \frac{P_2}{P_1}=1\right)\phi} \ln(c_1) + \beta\, \mathbb{E}_1\left[V_2\left(m_2, \mathbb{1}\left(R_2 \frac{P_2}{P_1} = 1\right)\phi\right)\right] \\ & \text{s.t. } (2.33),(2.36) \\ & \quad m_2 \geq 0;\ 0 \leq \mathbb{1}\left(R_2 \frac{P_2}{P_1} = 1\right)\phi \leq 1 \end{aligned} \tag{2.37}$$

As a counterpart to the purchase of the risky asset, the central bank creates an equivalent nominal value of reserves following equation (2.32). In contrast to the case of the REPO-based policy, this outright purchase operation has a fiscal implication in the last period of the economy. For whatever fraction of the *Lucas-tree* asset the household agrees to sell, the central bank bears the realisation of the wealth risk and, at the end of period 2, it transfers the realised dividends to the consolidated government which then transfers back such dividends to the representative household. The adjusted flow budget constraint of the consolidated government at the end of period 2 is now given by

$$\begin{aligned} \left(R_2 \frac{P_2}{P_1} - 1\right) m_2 = & T_2 + \left(P_2 \mathbb{1}\left(R_2 \frac{P_2}{P_1} = 1\right)\right. \\ & \left. \times\, \phi\left((\mathsf{p}+\Delta\mathsf{p})\underline{D} + (1-(\mathsf{p}+\Delta\mathsf{p}))\overline{D}\right)\right). \end{aligned} \tag{2.38}$$

The definition of equilibrium barely changes with respect to the one presented in Appendix 2.B.1. Maximisation of the discounted flow of utility in (2.1) now implies the representative household to solve problem (2.37) as of period 1, whilst T_2 has now to adjust for (2.38) to hold. The following proposition summarises the effects of the implementation of the central bank's outright purchases policy.

Proposition 3. *Given assumptions 1, 2 and 3, the implementation of the central bank's outright purchases policy implies that, as of period 1, the perfect foresight* *equilibrium allocation is such that:*

$$\hat{\phi}^{PF} = 1$$
$$\hat{m}_2^{PF} = 0.$$

Furthermore, the equilibrium unemployment rate is

$$\hat{u}^{PF} = 1 - \frac{(\mathrm{p} + \Delta p)\underline{D} + (1 - (\mathrm{p} + \Delta p))\overline{D}}{\beta} < u^{PF}, \tag{2.39}$$

whereas the consumption path is such that in period 1

$$\hat{c}_1^{PF} = 1 - \hat{u}^{PF} > c_1^{PF},$$

and in period 2

$$c_{L2} = c_{H2} = \hat{c}_2^{PF} = (\mathrm{p} + \Delta p)\underline{D} + (1 - (\mathrm{p} + \Delta p))\overline{D}$$

which implies that, as of period 1, the expected maximum utility of the representative household is

$$V_1(\hat{u}^{PF}) = \ln(1 - \hat{u}^{PF}) + \beta \ln((\mathrm{p} + \Delta \mathrm{p})\underline{D} + (1 - (\mathrm{p} + \Delta \mathrm{p}))\overline{D}) > V_1(u^{PF}) \tag{2.40}$$

Proof. See proof in Appendix 2.A.3. □

Proposition 3 establishes that once the central bank opens the possibility of acquiring any fraction of the *Lucas-tree* asset in hands of the household in the context of a liquidity trap scenario, the optimal response of the representative household is to sell all of its holdings of the risky asset. As a consequence of the implementation of this outright purchases policy, the central bank fully insures the representative risk-averse household by extracting all the sources of uncertainty from its portfolio. Furthermore, full insurance from uncertainty also reduces the exposure of the members of the household to idiosyncratic unemployment risk, as shown by equation (2.39).

Overall, the implementation of the outright purchases policy increases welfare with respect to the counterfactual scenario in Lemma 1. The proof in Appendix 2.A.3 shows that this increase of relative welfare is a consequence of the risk-averse feature of the preferences of the representative household and Jensen's inequality:

$$\mathbb{E}_1\left[\ln(1-u^{PF})+\beta\ln(D_i)\right] < \ln(1-\hat{u}^{PF})+\beta\ln(\underbrace{\mathbb{E}_1[D_i]}_{\equiv\hat{T}_2^{PF}}). \tag{2.41}$$

The right-hand side of (2.41) shows how after bearing all uncertainty from the realisation of the dividends of the risky asset, and by alternatively offering a certain income, the consolidated government makes the risk-averse representative household better-off. This is what I call the uncertainty channel of the transmission of unconventional monetary policy.

2.5 Conclusions

In a stylised and tractable model of a three-period economy in which there is a nominal price rigidity and households are endowed with a perfectly illiquid risky asset which matures in the final period, an uncertainty shock reduces the real natural interest, generating a scenario in which the economy might deviate from its full-employment equilibrium path. By reducing the nominal interest rate to match the real interest rate with the natural interest rate, conventional monetary policy is able to impede uncertainty risk to translate into idiosyncratic unemployment risk as long as the ZLB on the policy rate is non-binding, keeping the economy in its full-employment path.

However, if the prospects of the realisation of future wealth are low enough, the uncertainty shock might make the ZLB binding, leading the economy into a liquidity trap equilibrium path in which conventional monetary policy has reached its limit and it is unable to deter increased uncertainty from translating into idiosyncratic unemployment risk. In the context of this alternative equilibrium path, this paper assesses the counterfactual role of two alternative forms of unconventional monetary policy. In the case of a pure REPO-based policy in which the central bank temporarily issues reserves in exchange for a fraction of the illiquid risky asset in hands of the household, the main result is that the policy implementation does not change the equilibrium allocation of the liquidity-trap scenario. The policy is neutral because it is Ricardian in nature and does not offer real insurance against the uncertainty shock.

In contrast, an outright purchases policy in which the central bank permanently buys the illiquid risky asset holdings of households and fully bears the negative effects of increased uncertainty, has positive counterfactual effects with respect to the no-policy liquidity trap scenario. The positive effects of this outright purchases policy are deeply related to its fiscal counterpart in which the consolidated government makes a lump-sum transfer of the realised aggregate dividends of the assets now in hands of the central bank. This particular unconventional monetary policy exploits the uncertainty channel of transmission of the shock in two ways. First, it fully insures households from aggregate uncertainty by increasing their mean income in the final period of the economy and, second, it increases the natural real interest and reduces the spillover effect of uncertainty on unemployment-risk, making all households better off.

It is worth to emphasize here that the results of this assessment are strongly attached to the particular context in which unconventional monetary policy is implemented. We leave aside considerations regarding the consequences of the continuous rollover effects of this policy, like for instance fiscal dominance and inflation. As such, they have to be carefully considered as the short-term counterfactual effects of the implementation of unconventional monetary policy in a liquidity trap scenario.

Appendix

2.A Proofs of lemmas and propositions

2.A.1 Proof of Lemma 1

Given assumptions 1, 2 and 3, assume that condition (2.26) in Proposition 1 does not hold, i.e. $\Delta \mathsf{p} > \frac{\underline{D}}{\beta}\left(\frac{\overline{D}-\beta}{\overline{D}-\underline{D}}\right) - \mathsf{p}$. Let $u^{PF} \in [0,1)$ be the conjectured perfect foresight equilibrium value of the unemployment rate in period 1 for a given value of the nominal (real) interest rate $\mathscr{R}$ and the shock $\Delta \mathsf{p}$. Following Definition 1, equations (2.10) and (2.13), together with the profit share in (2.15), imply that the conjectured equilibrium values of production of the intermediate varieties and of the perishable consumption good are $y_1^{PF} = Y_1^{PF} = 1 - u^{PF}$, which from the equilibrium condition in (2.24) must be equal to c_1^{PF}. Then, substituting equations (2.5), (2.6), (2.18) and (2.19) into the Euler equation (2.7), the fixed-R equilibrium is characterised by:

$$\frac{1}{1-u^{PF}} = \beta \mathscr{R} \left(\frac{\mathsf{p}+\Delta \mathsf{p}}{\underline{D}+m_2^{PF}} + \frac{1-(\mathsf{p}+\Delta \mathsf{p})}{\overline{D}+m_2^{PF}} \right) + \lambda_1^{PF} \tag{2.42}$$

1. *Proof that $R_2^{PF} = 1$ and $u^{PF} \neq u^{FE}$.* Assume $u^{PF} = u^{FE} = 0$. Then, following Remark 1, the non-negativity constraint on deposits is non-binding and $\lambda_1^{PF} = 0$. Moreover, in the full employment equilibrium path, the representative household does not have incentives to save and $m_2^{PF} = 0$. Then, from equation (2.42)

$$\mathscr{R} = R_2^* = \frac{1}{\beta} \left(\frac{\mathsf{p}+\Delta \mathsf{p}}{\underline{D}} + \frac{1-(\mathsf{p}+\Delta \mathsf{p})}{\overline{D}} \right)^{-1}$$

Since condition (2.26) does not hold, $R_2^* < 1$. Hence, the central bank is constrained by the ZLB and the monetary policy rule (2.17) implies:

$$R_2^{PF} = 1 > R_2^*$$

which leads to a contradiction, since for $u^{PF} = u^{FE}$ to hold, it has to be that $R_2^{PF} = R_2^*$. Then it must be that $u^{PF} \neq u^{FE}$.

2. *Proof of unique incentive-compatible* $u^{PF} > 0$. Given $R_2^{PF} = 1$, the non-negativity constraint on the representative household's deposits implies that either $m_2^{PF} \geq 0$ and $\lambda_1^{PF} = 0$, or $m_2^{PF} = 0$ and $\lambda_1^{PF} > 0$. In the first case, from equation (2.42) we have:

$$\frac{1}{1-u^{PF}} = \beta\left(\frac{\mathsf{p}+\Delta\mathsf{p}}{\underline{D}+m_2^{PF}} + \frac{1-(\mathsf{p}+\Delta\mathsf{p})}{\overline{D}+m_2^{PF}}\right) \tag{2.43}$$

On the other hand, from the clearing market condition (2.25) we also have:

$$\begin{aligned}\mathsf{p}+\Delta\mathsf{p}(\underline{D}+m_2^{PF}) &+ (1-(\mathsf{p}+\Delta\mathsf{p}))(\overline{D}+m_2^{PF}) \\ &= \mathsf{p}+\Delta\mathsf{p}(\underline{D}) + (1-(\mathsf{p}+\Delta\mathsf{p}))(\overline{D})\end{aligned}$$

so it must be that $m_2^{PF} = 0$. Therefore, from (2.43) one possible equilibrium value of the positive unemployment rate is:

$$u^{PF} = 1 - \frac{1}{\beta}\left(\frac{\mathsf{p}+\Delta\mathsf{p}}{\underline{D}} + \frac{1-(\mathsf{p}+\Delta\mathsf{p})}{\overline{D}}\right)^{-1}. \tag{2.44}$$

On the other hand, given the nominal price rigidity assumption, re-optimisation of the intermediate-variety firms after the shock implies the profit maximisation problem:

$$\begin{aligned}&\max_{N_1} \left(1-W_1^{PF}\right)N_1 \\ &\text{s.t. } N_1 \leq 1-u^{PF}\end{aligned} \tag{2.45}$$

Given W_1^{PF} intermediate variety firms solve the problem in (2.45) by choosing $N_1^{PF} = 1 - u^{PF}$ which confirms the conjecture that in equilibrium

$$y_1^{PF} = Y_1^{PF} = 1 - u^{PF} \tag{2.46}$$

In the second case, with $\lambda_1^{PF} > 0$, let $u^{PF'} \neq u^{PF}$ denote an alternative conjectured unemployment equilibrium rate. Then, by using the result in equation (2.44), equation (2.42) can be rewritten as:

$$\frac{1}{1 - u^{PF'}} = \frac{1}{1 - u^{PF}} + \lambda_1^{PF}$$

from which is possible to solve for $u^{PF'}$ as:

$$u^{PF'} = 1 - \frac{1 - u^{PF}}{1 + \lambda_1^{PF}(1 - u^{PF})} > u^{PF}. \tag{2.47}$$

Equation (2.47) opens the door to an economy with (infinite) multiple sunspot equilibria where any value $\lambda_1^{PF} > 0$ would imply all households to believe that the unemployment equilibrium rate is higher and, therefore, feeling they are in an implicit *excess savings* situation given $R_2^{PF} = 1$. However, from the value function as of period 1, it is possible to see that:

$$\begin{aligned} V_1(u^{PF}) = \ln(1 - u^{PF}) + \beta\left[(\mathsf{p} + \Delta\mathsf{p})\ln(\underline{D}) + (1 - (\mathsf{p} + \Delta\mathsf{p}))\ln(\overline{D})\right] > \\ V_1(u^{PF'}) = \ln(1 - u^{PF'}) + \beta\left[(\mathsf{p} + \Delta\mathsf{p})\ln(\underline{D}) + (1 - (\mathsf{p} + \Delta\mathsf{p}))\ln(\overline{D})\right], \end{aligned} \tag{2.48}$$

meaning all households being worse off for any $\{u^{PF'}, \lambda_1^{PF'}\} \in (u^{PF}, 1) \times \mathbb{R}_+ \equiv S$. In other words, once all identical households' beliefs are revealed in period 1, all households end up being worse off because of such beliefs. Then, equilibria in S is not *incentive compatible*. The unique equilibria in which the representative household has no incentives to deviate from its beliefs must be that in which u^{PF} is determined by (2.44).

2.A.2 Proof of Proposition 2

We start from the solution to the utility maximisation problem of the representative household. By substituting (2.32) into the flow budget equations (2.33) and (2.34), it is possible to rewrite the problem of the representative household in (2.35) as:

$$V_1(m_1) = \max_{m_2, m_1^{CB}} \ln\left(\frac{W_1}{P_1}(1-u) + \frac{\Gamma_1}{P_1} + \frac{R_1 m_1}{P_0} + \frac{m_1^{CB}}{P_1} - \frac{m_2}{P_1} - \frac{T_1}{P_1}\right) + \beta\,\mathbb{E}_1[V_2(m_2, m_1^{CB})]$$

$$\text{s.t. } m_2 \geq 0;\, 0 \leq m_1^{CB} \leq P_1 q^{CB}\bar{h}$$

Let us define the alternative state variable $\tilde{a}_2 \equiv m_2 - m_1^{CB}$. Then, the problem can be rewritten once again as:

$$V_1(m_1) = \max_{\tilde{a}_2} \ln\left(\frac{W_1}{P_1}(1-u) + \frac{\Gamma_1}{P_1} + \frac{R_1 m_1}{P_0} - \frac{\tilde{a}_2}{P_1} - \frac{T_1}{P_1}\right) + \beta\,\mathbb{E}_1[V_2(\tilde{a}_2)] \qquad \text{s.t. } \tilde{a}_2 \geq -P_1 q^{CB}\bar{h} \tag{2.49}$$

As in the proof of Lemma 1 in Appendix 2.A.1, as of period 1 the solution to problem (2.49) is characterised in equilibrium by the euler equation:

$$\frac{1}{1-\tilde{u}^{PF}} = \beta\left(\frac{\mathsf{p}+\Delta\mathsf{p}}{\underline{D}+\tilde{a}_2^{PF}} + \frac{1-(\mathsf{p}+\Delta\mathsf{p})}{\overline{D}+\tilde{a}_2^{PF}}\right) + \tilde{\lambda}_1^{PF}, \tag{2.50}$$

for which the clearing perishable good market condition implies that $\tilde{a}_2^{PF} = 0$. The unique *incentive compatible* equilibrium unemployment rate is then equal to that in equation (2.30) which is implied by the equilibrium value $\tilde{\lambda}_1^{PF} = 0$. Furthermore, $\tilde{a}_2^{PF} = 0$ does not allow to determine neither the value of m_2^{PF}, ϕ^{PF} nor m_1^{CB}, but $\tilde{\lambda}_1^{PF} = 0$ is consistent with any ordered tuple $\{\phi^{PF}, m_1^{CB}, m_2\} \in [0,1] \times (0, q^{CB}\phi\bar{h}, q^{CB}\phi\bar{h})$.

2.A.3 Proof of Proposition 3

As of period 1, the only intrinsic uncertainty along the perfect foresight equilibrium is the realisation of D_i in period 2. As long as $\phi < 1$, the portfolio of the households is subject to this intrinsic uncertainty. However, if $\phi = 1$, the representative household no longer has holdings of the risky asset and from the budget constraint in (2.36) its unique certain nominal income in period 2 is equal to $\left(R_2 \frac{P_2}{P_1} - 1\right) m_2 - T_2$. Then, by using equation (2.32), the value function in (2.37) can be rewritten as

$$V_1(m_1) = \max\left\{V_1^U(m_1), V_1^C(m_1)\right\} \tag{2.51}$$

where, by using (2.32),

$$\begin{aligned} V_1^U(m_1) = \max_{m_2, m_1^{CB}} \ln\left(\frac{W_1}{P_1}(1-u) + \frac{R_1 m_1}{P_0} + \frac{m_1^{CB}}{P_1} - \frac{m_2}{P_1} - \frac{T_1}{P_1}\right) + \\ \beta\left[(\mathsf{p}+\Delta\mathsf{p})\ln\left(\frac{R_2 m_2}{P_1} + \left(1 - \frac{m_1^{CB}}{P_1 q^{CB}}\right)\underline{D} - \frac{T_2}{P_2}\right) + \right. \\ \left.(1-(\mathsf{p}+\Delta\mathsf{p}))\ln\left(\frac{R_2 m_2}{P_1} + \left(1 - \frac{m_1^{CB}}{P_1 q^{CB}}\right)\overline{D} - \frac{T_2}{P_2}\right)\right] \\ \text{s.t. } m_2 \geq 0;\ m_1^{CB} < P_1 q^{CB} \end{aligned} \tag{2.52}$$

and

$$\begin{aligned} V_1^C(m_1) = \max_{m_2} \ln\left(\frac{W_1}{P_1}(1-u) + \frac{R_1 m_1}{P_0} + q^{CB} - \frac{m_2}{P_1} - \frac{T_1}{P_1}\right) + \beta \ln\left(\frac{R_2 m_2}{P_1} - \frac{T_2}{P_2}\right) \\ \text{s.t. } m_2 \geq 0 \end{aligned} \tag{2.53}$$

Given that in period 0 there is full employment, in equilibrium:

1. under the conjecture that $\tilde{u}^{PF} > 0$, the first order conditions which characterise the solution to problem (2.52) can be written as:

$$\frac{1}{1-\tilde{u}^{PF}} = \beta \left(\frac{\mathsf{p}+\Delta\mathsf{p}}{\tilde{m}_2^{PF} + \underline{D} + \tilde{\phi}^{PF}(1-(\mathsf{p}+\Delta\mathsf{p}))(\overline{D}-\underline{D})} + \frac{1-(\mathsf{p}+\Delta\mathsf{p})}{\tilde{m}_2^{PF} + \overline{D} - \tilde{\phi}^{PF}(\mathsf{p}+\Delta\mathsf{p})(\overline{D}-\underline{D})} \right) + \tilde{\lambda}_1^{PF} \tag{2.54}$$

$$\frac{1}{1-\tilde{u}^{PF}} = \beta \left(\frac{(\mathsf{p}+\Delta\mathsf{p})\underline{D}}{\tilde{m}_2^{PF} + \underline{D} + \tilde{\phi}^{PF}(1-(\mathsf{p}+\Delta\mathsf{p}))(\overline{D}-\underline{D})} + \frac{(1-(\mathsf{p}+\Delta\mathsf{p}))\overline{D}}{\tilde{m}_2^{PF} + \overline{D} - \tilde{\phi}^{PF}(\mathsf{p}+\Delta\mathsf{p})(\overline{D}-\underline{D})} \right) + \tilde{\mu}_1^{PF} \tag{2.55}$$

where $\tilde{\phi}^{PF} = \frac{m_1^{CB,PF}}{q^{CB}}$ is the conjectured equilibrium value of the fraction of the *Lucas-tree* asset holdings sold by the representative household, $\tilde{\lambda}_1^{PF}$ is the conjectured equilibrium value of the Lagrange multiplier associated with the non-negativity constraint on savings, and $\tilde{\mu}_1^{PF}$ is the conjectured equilibrium value of the Lagrange multiplier associated with the constraint on the maximum possible amount of reserves to receive from the outright purchase operation.

First, notice that if $\tilde{\lambda}_1^{PF} = 0$ then the representative household is optimally saving a non-negative amount of reserves, so it must be that it is optimally not selling any fraction of its holdings of the risky asset, i.e. $\tilde{\phi}^{PF} = 0$ and the equilibrium allocation converges to the equilibrium allocation characterised by Lemma 1 and $\tilde{u}^{PF} = u^{PF}$. Assume this is not true and that whilst $\tilde{m}_2^{PF} = 0$ and $\tilde{\lambda}_1^{PF} = 0$ then $\tilde{0} < \phi^{PF} < 1$ and $\mu_1^{PF} = 0$. This would imply the right-hand side (RHS) of condition (2.54) to be equal to the RHS of condition (2.55), which is a contradiction given the known distribution of the dividends of the risky asset from which $\overline{D} > \underline{D}$. For $\tilde{u}^{PF} \in [0,1)$ to hold, the equilibrium unemployment rate has to be fully determined by condition (2.54) given $\tilde{\lambda}_1^{PF} = 0, m_2^{PF} = 0$ and $\phi^{PF} = 0$.

Second, for any other positive $\tilde{\phi}^{PF} < 1$, the household is optimally selling a non-zero fraction of its risky asset holdings, which means that $m_2^{PF} = 0$ and $\lambda_1^{PF} > 0$, i.e. the nonnegativity constraint is binding and the household is willing to obtain further free-risk resources from selling any positive fraction of its holdings of the risky asset to the central bank. To prove this statement assume that given $\tilde{\phi}^{PF} > 0$ and $\tilde{\mu}_1^{PF} = 0$ then $\tilde{\lambda}_1^{PF} = 0$. This would imply the RHS of condition (2.54) to be equal to the RHS of condition (2.55) which is not true since $\overline{D} > \underline{D}$. Furthermore, the term within the round brackets in condition (2.55) is greater than the term within round brackets in condition (2.54), then it must be that $\tilde{\lambda}_1^{PF} > 0$.

From (2.55)

$$\tilde{u}^{PF} = 1 - \frac{1}{\beta}\left(\frac{(\mathsf{p}+\Delta\mathsf{p})\underline{D}}{\underline{D}+\tilde{\phi}^{PF}(1-(\mathsf{p}+\Delta\mathsf{p}))(\overline{D}-\underline{D})} + \frac{(1-(\mathsf{p}+\Delta\mathsf{p}))\overline{D}}{\overline{D}-\tilde{\phi}^{PF}(\mathsf{p}+\Delta\mathsf{p})(\overline{D}-\underline{D})}\right)^{-1} < u^{PF}$$

However, the allocation characterised by the unemployment rate $\tilde{u}^{PF}$ implies that in period 2:

$$\tilde{c}_{L2}^{PF} = \underline{D} + \tilde{\phi}^{PF}(1-(\mathsf{p}+\Delta\mathsf{p}))(\overline{D}-\underline{D})$$
$$\tilde{c}_{H2}^{PF} = \overline{D} - \tilde{\phi}^{PF}(\mathsf{p}+\Delta\mathsf{p})(\overline{D}-\underline{D})$$

and the perishable good market clearing condition

$$((\mathsf{p}+\Delta\mathsf{p})\tilde{c}_{L2}^{PF} + (1-(\mathsf{p}+\Delta\mathsf{p}))\tilde{c}_{H2}^{PF}) = ((\mathsf{p}+\Delta\mathsf{p})\underline{D} + (1-(\mathsf{p}+\Delta\mathsf{p}))\overline{D})$$

only holds if $\tilde{\phi}^{CB} = 0$, which is a contradiction. Then, the equilibrium allocation cannot be characterised by $0 < \tilde{\phi}^{CB} < 1$ and $\tilde{u}^{PF} < u^{PF}$. Then, it must be that $\tilde{\lambda}_1^{PF} = 0$, $\tilde{m}_2^{PF} = 0$, $\tilde{\phi}^{PF} = 0$, and that the maximum value of utility as of period 1 is given by equation (2.31), i,e.

$$V_1^U(\tilde{u}^{PF}) = \ln(1-u^{PF}) + \beta\left[(\mathsf{p}+\Delta\mathsf{p})\ln(\underline{D}) + (1-(\mathsf{p}+\Delta\mathsf{p}))\ln(\overline{D})\right] \quad (2.56)$$

2. under the conjecture $\hat{u}^{PF} > 0$, the first order condition of problem (2.53) can be written as:

$$\frac{1}{1-\hat{u}^{PF}} = \beta\left(\frac{1}{(\mathsf{p}+\Delta\mathsf{p})\underline{D} + (1-(\mathsf{p}+\Delta\mathsf{p}))\overline{D}}\right) + \lambda_1^{\hat{P}F} \quad (2.57)$$

using the same line of arguments as the one presented in the proof of Lemma (1) in Appendix 2.A.1, $\hat{\lambda}_1^{PF} = 0$ and the unique incentive compatible unemployment rate is

$$\hat{u}^{PF} = 1 - \frac{(\mathsf{p}+\Delta\mathsf{p})\underline{D} + (1-(\mathsf{p}+\Delta\mathsf{p}))\overline{D}}{\beta}$$

which is less than the value of u^{PF} in equation (2.44) of Lemma 1 since risk aversion expressed by the convex instantaneous utility $\ln(\cdot)$ implies that Jensen's inequality holds and

$$\frac{\mathsf{p}+\Delta\mathsf{p}}{\underline{D}} + \frac{1-(\mathsf{p}+\Delta\mathsf{p})}{\overline{D}} < \frac{1}{(\mathsf{p}+\Delta\mathsf{p})\underline{D} + (1-(\mathsf{p}+\Delta\mathsf{p}))\overline{D}}.$$

The allocation is such that there is a unique representative household at the end of period 2 which consumes:

$$\hat{c}_2^{PF} = (\mathsf{p}+\Delta\mathsf{p})\underline{D} + (1-(\mathsf{p}+\Delta\mathsf{p}))\overline{D} = Y_2$$

As of period 1, the maximum expected utility of the representative household determined by this allocation is

$$V_1^C(\hat{u}^{PF}) = \ln(1-\hat{u}^{PF}) + \beta\ln((\mathsf{p}+\Delta\mathsf{p})\underline{D} + (1-(\mathsf{p}+\Delta\mathsf{p}))\overline{D}) \quad (2.58)$$

3. Since $\hat{u}^{PF} < u^{PF}$, Jensen's inequality implies that

$$(\mathsf{p}+\Delta\mathsf{p})\ln(\underline{D}) + (1-(\mathsf{p}+\Delta\mathsf{p}))\ln(\overline{D}) < \ln((\mathsf{p}+\Delta\mathsf{p})\underline{D} + (1-(\mathsf{p}+\Delta\mathsf{p}))\overline{D}).$$

Therefore, $V_1^C(\hat{u}^{PF}) > V_1^U(u^{PF})$ and under the outright purchases policy the equilibrium allocation is such that $\hat{\phi}^{PF} = 1$, $\hat{m}_1^{CB} = q^{CB}$, $\hat{m}_2^{PF} = 0$ and $\hat{u}^{PF} < u^{PF}$.

2.B Definition of equilibrium under unconventional monetary policy

2.B.1 Definition of equilibrium under the REPO contract

Definition 2. *Given the possible values of the real dividend* $\{\underline{D}, \overline{D}\}$*, the known value of the probability* p*, and the fair price* q^{CB}*, the perfect foresight symmetric general equilibrium of the economy consists of the allocation* $[\{Y_t\}_{t=0}^2, \{y_t\}_{t=0}^1, \{N_t\}_{t=0}^1, \{N_t^d\}_{t=0}^1, \{\Gamma_t\}_{t=0}^1, \{c_t\}_{t=0}^1, c_{L2}, c_{H2}, c_2, \{m_{t+1}\}_{t=0}^1, \{\lambda_t\}_{t=0}^1, \phi]$*, the price vector* $[\{p_t\}_{t=0}^1, \{P_t\}_{t=0}^1, \{W_t\}_{t=0}^1]$*, the policy vector* $[\{R_{t+1}\}_{t=0}^1, \mathbb{1}(R_2 = 1), m_1^{CB}, P_2, \{T_{t+1}\}_{t=0}^1]$ *and the unemployment rate* $u \in [0,1)$*, such that:*

1. *Households maximise utility in equation* (2.1)*, subject to equations* (2.2)*,* (2.33)*,* (2.34) *and the inequality* $0 \leq \phi \leq 1$ *given* $[\{P_t\}_{t=0}^2, \{W_t\}_{t=0}^1, \{R_{t+1}\}_{t=0}^1, \mathbb{1}(R_2 = 1), q^{CB}, \{T_{t+1}\}_{t=0}^1]$*,* u*,* $\{\underline{D}, \overline{D}\}$ *and* p.

2. *The competitive representative firm producing the final perishable good maximises its profits at each date, i.e.* (2.10)*,* (2.11)*, and* (2.12) *hold given* $[\{p_t\}_{t=0}^1]$.

3. *Given* $\{\underline{D}, \overline{D}\}$ *and* p*, the supply of the perishable consumption good in period 2 is determined by* (2.16).

4. *Each intermediate-variety producer firm maximises its discounted flow of*

profits in periods 0 and 1, given $[\{P_t\}_{t=0}^{1}, \{W_t\}_{t=0}^{1}]$*, i.e. equations* (2.14) *and* (2.15) *hold given the nominal rigidity assumption* $p_1 = p_0$*, and the nominal wage rigidity given by equation* (2.20)*.*

5. *The labour market clears in period 0:*

$$N_0 = N_0^d = 1$$

and in period 1 labour supply might be rationed, i.e.:

$$N_1 = N_1^d \leq 1$$
$$u = 1 - N_1^d$$

6. *In periods 0 and 1 the value of the gross nominal interest rate on reserves* $R_{t+1}\frac{P_{t+1}}{P_t}$ *and the indicator function* $\mathbb{1}(R_2 = 1)$ *are determined by equation* (2.17)*.*

7. P_2 *is set according to equation* (2.18)*.*

8. *The lump sum tax rates* $\{T_{t+1}\}_{t=0}^{1}$ *are set by the consolidated government such that the budget equations in* (2.19) *holds.*

9. *The value of reserves* m_1^{CB} *is determined by equation* (2.32) *given* $\{P_2, q^{CB}, \mathbb{1}(R_2 = 1), \phi\}$*.*

10. *The market for the perishable good clears in all periods, i.e. for* $t = \{0, 1\}$*:*

$$c_t = Y_t$$

and in period 2:

$$\mathsf{p}c_{L2} + (1 - \mathsf{p})c_{H2} = c_2 = Y_2$$

Chapter 3

The stabilisation role of unconventional monetary policy in the periphery of a monetary union: a quantitative assessment

3.1 Introduction

In the aftermath of the great financial recession of 2007–2008, the economies of the European monetary union went into another recession in 2011. This time, pressures on the market of sovereign bonds quickly mounted up, generating solvency problems and restrictions in the credit markets of the union (Reichlin, 2014b). Specifically, banks in the periphery of the monetary union (e.g. Ireland, Italy, Spain and Portugal), which had a strongly home-biased portfolio of sovereign bonds, went through serious solvency problems because of the rapid increase in the yields of their countries' long-term sovereign bonds, a situation that translated into severe credit restrictions and a recession characterised by increasing unemployment rates (see Figure 3.1.1).

By this time, the European Central Bank (ECB) had almost exhausted any available space regarding conventional interest-rate monetary policy and responded by further increasing the intensity of their unconventional monetary policy measures. In particular, not only the term of the long-term refinancing operations (LTRO) with banks was increased to up to 3 years, but even outright purchases in the sovereign bonds market started to make part of the monetary policy toolkit. In this context, the purpose of this paper is to provide a measure of the pure stabilisation (i.e. recession smoothing) effects of this kind of unconventional monetary policy tools in the periphery economies of the European monetary union.

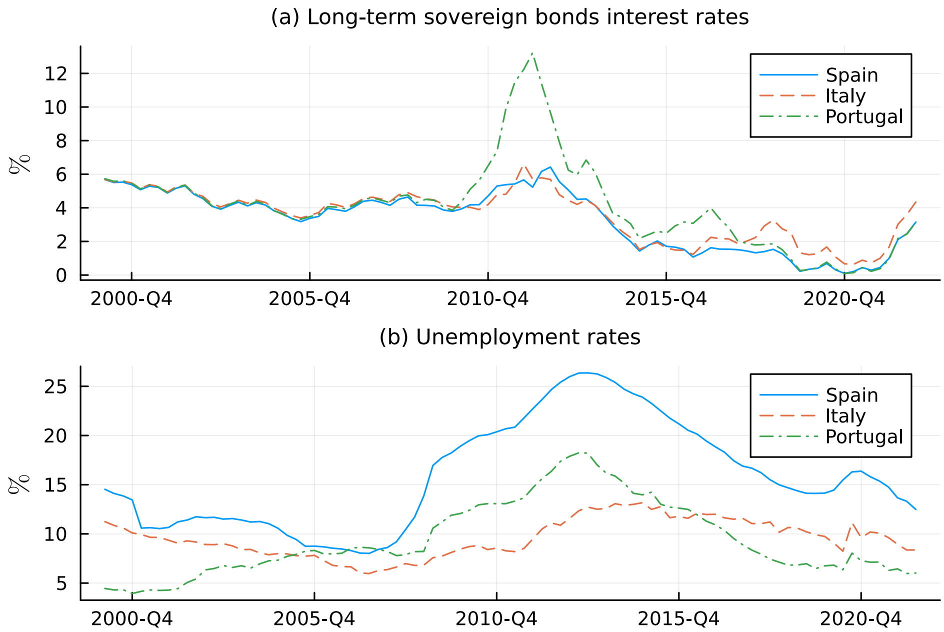

Figure 3.1.1: Long-term government bonds yields and unemployment rates in the periphery of the European monetary union

The quantitative measure is generated via simulations of a dynamic model that embeds the intermediation of international capital flows by the banking system into the structure of a small open economy with a currency peg, which is a *periphery* member of a monetary union, and a downward nominal rigidity of wages (e.g. Bianchi and Mondragon, 2021; Bianchi et al., 2021; Schmitt-Grohé and Uribe, 2016). The portfolio of banks in the model is home biased in the sense that its major source of net worth comes from claims on a proportion of the total outstanding long-term sovereign debt issued by the local government. The small open economy constantly faces two correlated exogenous shocks, one of them being a sovereign bond price shock which directly affects the lending restriction of banks.

In the baseline scenario, the central bank is an external agent which has already fixed the risk-free interest rate and implements unconventional monetary policy by making an outright purchase of a fixed proportion of the outstanding long-term sovereign bonds in the balance sheet of banks. After simulating multiple equilibriums of the model under the active unconventional monetary policy rule, those sets of simulations which resemble the sovereign debt crisis in the periphery economies of Europe are identified. The quantitative measure of the stabilisation effect comes from the comparison of the selected simulations of the baseline scenario, with a counterfactual scenario consisting of another set of similarly identified simulations in which the unconventional monetary policy rule is *switched-off* inside the model.

The model is calibrated to Spain, an important referent of the European periphery economies. Its numerical solution is based on a value function iteration algorithm which solves the optimal debt problem of the sovereign. Two main conclusions can be derived from the quantitative exercise. First, unconventional monetary has real effects and is welfare improving. By relaxing the lending constraint of banks, outright purchases increase credit availability in a scenario of difficult solvency conditions for both banks and the sovereign government of the periphery economy. Credit expansion has a direct positive effect on the labour market through a systematic reduction of the equilibrium unemployment rate within the simulated recession time window. Second, the pure stabilisation effects are rather small. The counterfactual increase of the unemployment rate in the no-policy scenario is always less than one percentage point, and the consumption loss is not enough to produce changes in the optimal path of long-term government debt.

The analysis in this paper intentionally abstracts from the role of default risk on the price of long-term sovereign bonds (e.g. Bianchi and Mondragon, 2021; Hatchondo and Martínez, 2009), since its main purpose is to isolate the pure stabilisation effect of unconventional monetary policy. Nonetheless, both the purely empirical approach in Hachula et al. (2020) and the more theoretically driven quantitative exercise in Constain et al. (2022) have documented the positive effect that unconventional monetary policy has had on the yield of long-term sovereign bonds in the European Union. In terms of the model in this paper, this could relax even more the lending constraint of banks, and the quantitative measure presented here can be read as the lowest possible positive effect of unconventional monetary policy in the context of a sovereign crisis.

Related literature. This work is related to various strands of the literature. First, this paper is related to the literature on business cycle policy identification in the context of nominal wage rigidities and pegs (Bianchi and Mondragon, 2021; Bianchi and Ottonello, 2023; Na et al., 2018; Schmitt-Grohé and Uribe, 2016). However, the analysis presented here includes the active role of banks as the driver of the credit channel of monetary policy, adapting the approach in Sosa-Padilla (2018).

The paper is also related to the literature in which financial frictions in the banking sector and the real sector collude with different sources of shocks to generate business cycle fluctuations in the presence of nominal rigidities (e.g Eggertsson et al., 2019; Ennis, 2018). As in most of this literature, the approach presented here emphasises the importance of these restrictions for the amplification of business cycle fluctuations.

Most importantly, this paper is directly related to the literature on the effects of unconventional monetary policy on the real side of the economy (e.g Chang and Velasco, 2017; Cui and Sterk, 2021; Cúrdia and Woodford, 2011; Del Negro et al., 2017a; Gertler and Karadi, 2011) which emphasize that this kind of monetary policy is non-neutral in the framework of financial and liquidity frictions. The work presented here is most closely related to Bletzinger and von Thadden (2021) which analyses unconventional monetary policy in a monetary union. However, their approach seeks to determine the optimal unconventional monetary policy whereas this paper's focus is on quantifying the effect of this kind of monetary policy in the context of the particular European sovereign crisis of 2011–2012.

The analysis presented in this paper is divided into four sections including this Introduction. Section 3.2 presents the model, establishes the decentralised equilibrium and presents the optimal debt problem of the government consistent with the equilibrium definition. In Section 3.3 the solution to the optimal debt problem calibrated to Spain is presented, and the quantification of the effects of unconventional monetary policy is calculated via the simulation approach. Finally, Section 3.4 concludes.

3.2 The Model

The small open economy is inhabited by four types of agents: households, firms, banks and the government. Households periodically receive a stochastic stream of a traded good, consume a basket of traded and non-traded goods, and inelastically supply labour to firms, but do not have access to any credit market. Firms produce the non-traded good using only labour and face a working capital constraint, which obliges them to incur in loan contracts with local banks to pay a fraction of their wage bill in advance (Neumeyer and Perri, 2005; Sosa-Padilla, 2018). To smooth the consumption of households, the government issues risky random-maturity debt bonds (Chatterjee and Eyigungor, 2012) into the international credit market, which are demanded by risk-neutral foreign investors.

On their part, foreign investors are also the main shareholders of the local banks and provide funding for their intermediation operation via short-term risk-free deposits and the transferred ownership of a fixed fraction of the sovereign funds bought from the local government, which can be potentially traded in an international secondary market. The central bank of the monetary union implements unconventional monetary policy by trading risky assets with banks in the secondary market as part of an open market operation.

Timing of the model. As in Neumeyer and Perri (2005), because of the friction embedded in the existence of the working capital constraint on firms and the provision of intra-period loans by banks, each period is divided into two subperiods; namely *beginning of period* and *end of period*. A summary of events happening in each subperiod is provided below:

1. *Beginning of period*:

- Banks receive deposits from foreign lenders and claim ownership of a fraction of outstanding sovereign bond holdings, which are sold either in a private secondary market at the realised stochastic price or as part of an open market operation with the central bank of the monetary union at a given price. Deposits and the proceeds from the sovereign bonds sale determine the supply of loans of banks.
- Firms demand loans from banks to pay in advance a fraction of their wage bill.
- Households receive the realised stochastic stream of the tradable good and an advance payment of a fraction of their wage income from firms.

2. *End of period*:

- Firms produce the non-tradable good which they sell at a competitive price. They also pay the remaining fraction of their wage bill, pay back loans to banks, and distribute profits to households.
- The government issues random-maturity bonds, which are traded with foreign lenders at the stochastic realised price, and receives income from a lump-sum tax from households.
- Households receive the remaining fraction of their wage income and a share of profits from firms. Then, they pay a lump-sum tax to the government and optimally choose consumption.
- Banks receive the gross return of loans from firms and transfer current dividends to foreign lenders.
- Foreign lenders claim ownership of any remaining bank holdings of sovereign bonds.

In the context of this timing of events, now I present the detailed structure of the model focusing on the role of each type of agent.

3.2.1 Households

The small open economy is populated by a continuum of identical households. Households have preferences over consumption given by

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\,\mathscr{U}(c_t)\right];\;\mathscr{U}(c)=\left(\frac{c^{1-\sigma}-1}{1-\sigma}\right), \tag{3.1}$$

where $\beta \in (0,1]$ is the intertemporal subjective discount factor of households, σ is both the inverse of the intertemporal elasticity of consumption, and the constant relative risk aversion factor, and $\mathbb{E}_t$ is the conditional expectations operator conditional on shocks realised as of period t.

Consumption c_t is a CES composite of the consumption of a tradable good c_t^T, and the consumption of a nontradable good c_t^{NT}:

$$c_t \equiv \mathscr{C}\left(c_t^T, c_t^{NT}\right)=\left(\omega(c_t^T)^{1-\frac{1}{\xi}}+(1-\omega)(c_t^{NT})^{1-\frac{1}{\xi}}\right)^{\frac{1}{1-\frac{1}{\xi}}}, \tag{3.2}$$

which depends on the intra-temporal elasticity of substitution between the consumption of the tradable and nontradable goods $\xi > 0$, and the share of the former on aggregate consumption $\omega \in (0,1)$. Let P_t^T and P_t^{NT} denote the domestic currency prices of the tradable and non-tradable goods, whereas e denotes the value of the nominal exchange rate of the small open economy. As is standard in the literature, I assume that the law of one price holds for the tradable good. Therefore, if $P_t^{*T} = 1$ is the foreign currency price of the tradable good, then the relative price of the non-tradable good is $p_t \equiv \frac{P_t^{NT}}{e}$.

In every period, households inelastically supply $\bar{h}$ hours of labour to firms. Because of the existence of a wage rigidity, the labour supply of any household might be rationed. Hence, a household's effective labour supply is

$$h_t \leq \bar{h}, \tag{3.3}$$

and its labour income measured in units of the tradable good is equal to $w_t h_t$, where

$w_t \equiv \frac{W_t}{e}$ and W_t is the wage per hour in units of the local currency of the small open economy.

Households do not have access to any credit market, but each period they are endowed with a stochastic flow of the tradable good y_t^T, which follows a first-order Markov process. They are also entitled to equal shares of profits π_t (denominated in terms of the tradable good) from firms producing the non-tradable good and face both a proportional tax (subsidy) on the consumption of the non-tradable good $\tau_t > 0 (< 0)$, and a lump-sum tax (transfer) of $tx_t > 0 (< 0)$ units of the tradable good. The flow budget constraint of a household in units of the tradable good is then given by

$$c_t^T + (1+\tau_t)p_t c_t^{NT} \leq y_t^T + w_t h_t + \pi_t - tx_t \tag{3.4}$$

Taking as given the sequences of: *i*) the relative price of the non-tradable good, $\{p_t\}_{t=0}^{\infty}$, *ii*) the wage in terms of the tradable good, $\{w_t\}_{t=0}^{\infty}$, *iii*) the stochastic endowment of the tradable good, $\{y_t^T\}_{t=0}^{\infty}$, *iv*) the effective labour supply, $\{h_t\}_{t=0}^{\infty}$, and *v*) fiscal policy variables, $\{\tau_t, tx_t\}_{t=0}^{\infty}$; households choose the sequences of consumption of the tradable and non-tradable goods $\{c_t^T, c_t^{NT}\}_{t=0}^{\infty}$ to maximise (3.1) subject to the consumption aggregator (3.2), and the sequence of budget constraints given by (3.4). The first order intra-temporal optimality condition of households is:

$$\frac{\mathscr{C}_{NT}\left(c_t^T, c_t^{NT}\right)}{\mathscr{C}_T\left(c_t^T, c_t^{NT}\right)} = \left(\frac{1-\omega}{\omega}\right)\left(\frac{c_t^T}{c_t^{NT}}\right)^{\frac{1}{\xi}} = (1+\tau_t)p_t, \tag{3.5}$$

which implies that for consumption to be optimal, the ratio of consumption between the tradable and non-tradable good must be uniquely determined by the gross relative price of the non-tradable good.

3.2.2 Firms

There is a continuum of identical competitive firms owned by households, which produce the nontradable good using only labour. A representative firm has access to the technology $\mathscr{F}(h_t^D) = \left(h_t^D\right)^{\alpha}$, where $\alpha \in (0,1)$ determines the degree of marginal diminishing returns when using an additional unit of labour and h_t^D represents aggregate labour demand. As in Neumeyer and Perri (2005) and Sosa-Padilla (2018), at the beginning of each period firms face a working capital constraint in the sense that they must pay in advance a fraction κ of their wage bill, which obliges them to demand loans l_t^D, measured in units of the tradable good, from local banks to be paid at the end of the period with interest rate i_t^l.

Every period the representative firm chooses the optimal values of aggregate labour demand and loans such that it maximises profits:

$$\pi_t = \max_{\{h_t^D, l_t^D\}} p_t \mathscr{F}(h_t^D) + l_t^D - w_t h_t^D - (1 + i_t^l) l_t^D$$
$$\text{s.t. } \kappa w_t h_t^D \leq l_t^D \tag{3.6}$$

In equilibrium, the working capital constraint (3.6) holds with equality since it is not optimal for the firm to demand loans beyond the wage bill to be paid in advance. Therefore, profits are given by

$$\pi_t = \max_{h_t^D} p_t \mathscr{F}(h_t^D) - (1 + \kappa i_t^l) w_t h_t^D \tag{3.7}$$

and the optimality condition to determine h_t^D is

$$p_t \mathscr{F}'(h_t^D) = p_t \alpha \left(h_t^D\right)^{\alpha - 1} = (1 + \kappa i_t^l) w_t \tag{3.8}$$

3.2.3 Government

Following Chatterjee and Eyigungor (2012), at the end of every period the government issues ι_t units of a real debt bond with random maturity. Each unit matures at the end of next period with probability $\theta \in (0, 1)$ and pays one unit of the tradable good. Any non-maturing sovereign bond yields a coupon of $\delta > 0$ units of the tradable good instead. Here I consider the polar case in Na et al. (2018) in which the small open economy, being a member of a monetary union, has a currency peg and its government has full commitment to repay its debt. Absent default risk, the average maturity period of a random-maturity bond is equal to $1/\theta$.

It is assumed that the realisation of maturity is independent between bonds, which implies that in any period a fraction θ of the outstanding stock of sovereign bonds matures with certainty. Hence, the evolution of the stock of debt of the government b_t is given by

$$b_t = \iota_t + (1-\theta)b_{t-1}.$$

Non-maturing bonds issued on different dates are indistinguishable and can be traded in any period at the same ex-coupon price q_t. Any proceeds from the participation of the government in the international credit market, are rebated to the households in the form of a lump-sum tax (transfer) of tx_t units of the tradable good. It is assumed that the government does not play any Ponzi games in the international credit market.

On the other hand, to eliminate the inefficiency in the labour market generated by a positive interest rate on intra-period loans to firms, the government imposes a proportional tax (subsidy) τ_t on the household's consumption of the non-tradable good measured in units of the tradable good. Let l_t denote the equilibrium value of intra-period loans. The value of the proportional tax is such that in equilibrium, every period the interest rate on loans i_t^l is equal to zero, i.e. the condition

$$1+\tau_t = \alpha\kappa\left(\frac{1-\omega}{\omega}\right)\frac{(c_t^T)^{\frac{1}{\xi}} h_t^{\alpha\left(1-\frac{1}{\xi}\right)}}{l_t}, \tag{3.9}$$

holds. All these flow transactions are summarised in the budget constraint of the government:

$$\tau_t p_t c_t^{NT} + tx_t + q_t(b_t - (1-\theta)b_{t-1}) = (\theta + \delta(1-\theta))b_{t-1} \tag{3.10}$$

3.2.4 Foreign lenders

A continuum of risk-neutral foreign lenders trade risk-free one-period bonds denominated in units of the tradable good, and random-maturity sovereign bonds in the international credit market. In every period a sovereign bond is priced by the non-arbitrage condition

$$1+i^* = \frac{\theta + \mathbb{E}_t[(\delta + q_{t+1})(1-\theta)]}{q_t}$$

in which the expected return of the random-maturity bond must be equal to the risk-free gross return on the one period asset $1+i^*$. Absent any sources of risk the long-run equilibrium price of the sovereign bond is given by

$$\bar{q} = \frac{\theta + \delta(1-\theta)}{i^* + \theta}. \tag{3.11}$$

Despite the assumption of full commitment of the government to repay its debt, which closes the door to the possibility of endogenous default risk, I introduce a notion of exogenous risk following Neumeyer and Perri (2005) and Na et al. (2018). In particular, let $1/q_t$ be the implied gross interest rate of one random-maturity sovereign bond with an average maturity of $1/\theta$. I assume that the spread or *country risk* factor

$$\ln(s_t) = \ln\left(\frac{1/q_t}{1/\bar{q}}\right) \tag{3.12}$$

follows a first-order Markov process.

Foreign lenders are also the shareholders of the local banks of the small open economy. At the beginning of each period, they deposit d^* units of the tradable good from their holdings of the risk-free asset and transfer ownership of a fraction ψ of their outstanding holdings of local sovereign random-maturity bonds to banks in the *periphery* economy. At the end of the period, foreign lenders are entitled to ϕ_t units of the tradable good in the form of bank dividends and claim ownership of the outstanding random-maturity sovereign bond holdings of banks.

3.2.5 Banks and unconventional monetary policy

There is a continuum of competitive and risk-neutral identical banks which, as in Chang and Velasco (2017), exist only for one period and maximise its current-period dividend ϕ_t measured in units of the tradable good. At the beginning of each period, the role of the bank is to intermediate resources in the form of intra-period loans to firms using external financing coming from deposits d^*, which yield a deposit interest rate $i_t^d = i^*$, and the proceeds from its participation in a private secondary market of random-maturity bonds where any bond is traded at the market stochastic price q_t.

Unconventional monetary policy and loan supply of banks. At the beginning of any period, the central bank of the monetary union might choose to buy a fraction χ_t^{CB} of the stock of non-maturing sovereign bonds in hands of the bank at unit price $q_t^{CB} > q_t$. The realisation of this open market operation is an autonomous and exogenous decision of the central bank. Therefore, in a no-unconventional policy scenario $\chi_t^{CB} = 0$. To be as close as possible to the scenario observed during the sovereign crisis of 2011–2012, a simple monetary policy rule is assumed:

$$\chi_t^{CB} = \begin{cases} \chi^* \in (0,1) & \text{if } \frac{q_t}{\bar{q}} - 1 < 0 \text{ and } \left|\frac{q_t}{\bar{q}} - 1\right| > threshold \\ 0 & \text{otherwise} \end{cases} \tag{3.13}$$

This targeted rule is implied by the nature of the implementation of the long-term refinancing operations which were not only increased in amount but its final term was also increased to up to 3 years once the yield rates of sovereign bonds started to increase (Reichlin, 2014b).

As in Sosa-Padilla (2018), the bank's loans supply l_t^S is limited to the available funding at the beginning of the period, which is determined by deposits and the realisation of the open market operation with the central bank of the monetary union. Since random-maturity bonds either mature or generate a coupon until the end of the period, the lending constraint affecting the bank is

$$l_t^S \leq d^* + q_t \iota_t^B + q_t^{CB} \chi_t^{CB} \psi b_{t-1} \tag{3.14}$$

The problem of the bank. The unique role of the representative bank is to intermediate resources to firms in the form of intra-period loans using available funding, and its ownership structure determines that the bank does not take any intertemporal decisions. The sequence of budget constraints of the bank is:

$$\phi_t = q_t \iota_t^B + q_t^{CB} \chi_t^{CB} \psi b_{t-1} + i_t^l l_t^S - i^* d^* \tag{3.15}$$

In every period, given: *i)* exogenous net deposits payments $i^* d^*$, *ii)* the stochastic price q_t, *iii)* the interest rate on loans i_t^l, and *v)* unconventional monetary policy summarised by $\{\chi_t^{CB}, q_t^{CB}\}$; the static problem of the bank is to maximise (3.15) subject to the lending constraint (3.14), by choosing its sales of random-maturity sovereign bonds ι_t^B and its loans supply l_t^S.

The first order conditions of the problem with respect to the supply of loans and the static Lagrange multiplier on the lending constraint (λ) are:

$$i_t^l = \lambda$$

$$l_t^S \leq d^* + (q_t(1-\chi_t^{CB}) + q_t^{CB} \chi_t^{CB}) \psi b_{t-1}$$

$$\lambda \left\{ l_t^S - \left[d^* + (q_t(1-\chi_t^{CB}) + q_t^{CB} \chi_t^{CB}) \psi b_{t-1} \right] \right\} = 0,$$

which imply the loan supply function

$$l_t^S = \begin{cases} d^* + (q_t(1-\chi_t^{CB}) + q_t^{CB} \chi_t^{CB}) \psi b_{t-1} & \text{if } i_t^l \geq 0 \\ 0 & \text{if } i_t^l < 0 \end{cases} \tag{3.16}$$

Notice that, given exogenous deposits, once shocks and unconventional monetary policy are realised at the beginning of the period, equation (3.16) implies that there is a fixed supply of loans from banks every period.

3.2.6 Nominal wage rigidity and currency peg

The small open economy is characterised by the existence of two nominal rigidities. First, as in Bianchi and Mondragon (2021) and Bianchi et al. (2021), the wage per hour measured in units of the local currency, W_t, is partially rigid in the sense that there exists a minimum wage value $\bar{W}$ such that $W_t \geq \bar{W}$ has to hold in every period. Second, because of its membership to a monetary union, the *periphery* economy has a *de facto* currency peg which implies that its nominal exchange rate is always fixed. Let e be the fixed value of the nominal exchange rate of the small open economy. The joint presence of these two nominal rigidities implies that

$$w_t \geq \bar{w}, \tag{3.17}$$

where $\bar{w} \equiv \frac{\bar{W}}{e}$. Hence, the real wage measured in units of the tradable good is also rigid and in the general equilibrium of the economy, the amount of hours supplied to the labour market by households might be rationed in the sense that

$$h_t = \min\{h_t^D, \bar{h}\}.$$

As a consequence, in every period the complementarity slackness condition

$$(\bar{w} - w_t)(\bar{h} - h_t) = 0 \tag{3.18}$$

must hold in equilibrium. If $w_t \geq \bar{w}$ the small open economy is in a full employment equilibrium, i.e. $h_t = \bar{h}$. Otherwise, if $w_t < \bar{w}$, the periphery economy is in an involuntary unemployment equilibrium ($h_t < \bar{h}$), and the real wage must be equal to the minimum wage $\bar{w}$.

3.2.7 General equilibrium

In equilibrium, the market for the non-tradable good must clear every period, i.e.

$$c_t^{NT} = \mathscr{F}(h_t). \tag{3.19}$$

Moreover, the equilibrium value of loans l_t must be determined by the equality between the loans supply of banks and the loans demand from firms. This equilibrium condition, together with condition (3.9), imply that the equilibrium value of the interest rate on loans i_t^l is equal to zero in every period. Hence, from equation (3.16), the equilibrium value of loans is given by

$$l_t = d^* + (q_t(1-\chi_t^{CB}) + q_t^{CB}\chi_t^{CB})\psi b_{t-1}. \tag{3.20}$$

The flow budget constraint of households (3.4), together with the equilibrium value of the interest rate on loans, maximum firms profits (3.7), the flow budget constraints of the government (3.10), and the equilibrium condition of the non-tradable good market (3.19), imply that the resource constraint of the tradable good is:

$$c_t^T = y_t^T + q_t(b_t - (1-\theta)b_{t-1}) - (\theta + \delta(1-\theta))b_{t-1}. \tag{3.21}$$

Now it is possible to define the general equilibrium of the small open *periphery* economy member of a monetary union.

Definition 1. *Given the sequences of fiscal and monetary policies* $\{b_t\}_{t=0}^{\infty}$, $\{q_t^{CB}, \chi_t^{CB}\}_{t=0}^{\infty}$, *the stochastic sequences* $\{y_t^T, s_t\}_{t=0}^{\infty}$, *the values of the number of hours supplied to the labour market by households* $\bar{h}$, *the fraction* κ *of the wage-bill to be paid in advance by firms, the probability* θ, *the coupon* δ, *the real international interest rate* i^*, *the amount of deposits* d^*, *the fraction* ψ *of the stock of sovereign random-maturity bonds in hands of the banks, the minimum wage* $\bar{w}$, *and the initial value* b_{-1}; *the general equilibrium of the small open economy, member of a monetary union, is the sequence of: allocations* $\left\{c_t^T, c_t^{NT}, h_t, h_t^D, l_t\right\}_{t=0}^{\infty}$, *relative prices* $\{p_t, w_t\}_{t=0}^{\infty}$, *interest rates* $\{i_t^l\}_{t=0}^{\infty}$, *and taxes* $\{\tau_t, tx_t\}_{t=0}^{\infty}$ *such that:*

1. *Households maximise utility, i.e* (3.4) *and* (3.5) *hold given* $\{p_t, w_t, y_t^T, h_t, \tau_t, tx_t\}_{t=0}^{\infty}$.

2. *Firms maximise profits, i.e* (3.6), (3.7) *and* (3.8) *hold given* $\{p_t, w_t, i_t^l\}_{t=0}^{\infty}$ *and* κ.

3. τ_t *is determined by* (3.9).

4. *The sequences of flow budget constraints of the government* (3.10) *hold given* $\{\tau_t, q_t\}_{t=0}^{\infty}$ *and* b_{-1}, θ, δ.

5. $i_t^l = 0$ *for all t given* τ_t.

6. *Banks maximise dividends, i.e.* (3.15) *and* (3.20) *holds given* $\{i_t^l, q_t, q_t^{CB}, \chi_t^{CB}\}_{t=0}^{\infty}$.

7. *The equilibrium condition of the non-tradable good market* (3.19) *holds.*

8. *Labour-market conditions* (3.17) *and* (3.18) *hold.*

9. *The resource constraint of the non-tradable good is given by* (3.21).

3.2.8 Shocks and unconventional monetary policy transmission in partial equilibrium

The issuing of new sovereign random-maturity bonds is the only (implicit) intertemporal decision in the model. By assuming as fixed the stock of sovereign debt b and the lump-sum tax tx, it is possible, from a partial equilibrium perspective, to characterise the transmission of shocks and unconventional monetary policy in the small open *periphery* economy.

Let $\zeta = [y^T, s]$ be the vector of correlated exogenous shocks and $\mathscr{M} = [q^{CB}, \chi^{CB}]$ the vector of unconventional monetary policy variables determined by the central bank of the monetary union. Once ζ, $\mathscr{M}$ and d^* are realised at the beginning of any period, the consumption of the tradable good consumption c^T and loans l are determined by the resource constraint (3.21) and equation (3.20), respectively. From here, the intra-period interaction of markets for the non-tradable good, domestic credit, and labour, determines the (partial) equilibrium values of the non-tradable good consumption, its relative price, the real wage, and hours worked.

Following Definition 1, the system of equations

$$p = \frac{l}{\alpha \kappa h^{\alpha}} \tag{PE.1}$$

$$w = \alpha p h^{\alpha - 1} \tag{PE.2}$$

$$w \geq \bar{w} \tag{PE.3}$$

$$(\bar{w} - w)(\bar{h} - h) = 0, \tag{PE.4}$$

determine p, w and h. Equation (PE.1) is the representation of the interaction between the domestic credit market and the non-tradable good market, and it is obtained by substituting the expression for τ in (3.9) and the equilibrium condition (3.19) into equation (3.5); whereas equation (PE.2) represents the interaction between the credit market and the labour market and it is obtained from the optimal demand for labour (3.8), after substituting for the equilibrium value of the interest rate on loans. On their part, equations (PE.3) and (PE.4) are just the partial equilibrium versions of the labour market equilibrium conditions (3.17) and (3.18).

The key component to understanding the transmission of shocks and unconventional monetary policy is the *in-equilibrium* labour demand function

$$h = \frac{l}{\kappa w} \tag{3.22}$$

which comes from the substitution of equation (PE.1) into equation (PE.2). Notice that (3.22) is just the equilibrium representation of the working capital constraint in (3.6). To fully grasp how both shocks and policy are transmitted, let the *periphery* economy start from a situation in which *i)* there is no intervention of the central bank of the monetary union ($\mathscr{M} = \mathbf{0}$), and *ii)* the initial (partial) equilibrium allocation of the small open economy is such that there is full employment ($h = \bar{h}$), and the equilibrium real wage is equal to the minimum real wage $\bar{w}$. Then, a new set of correlated shocks $\zeta^{-} \ll \zeta$ is realised, along with new values of consumption of the tradable good $c^{T-} < c^{T}$ and loans $l(\zeta^{-}) < l(\zeta)$.

Figure 3.2.1: Transmission of shocks and unconventional monetary policy in partial equilibrium

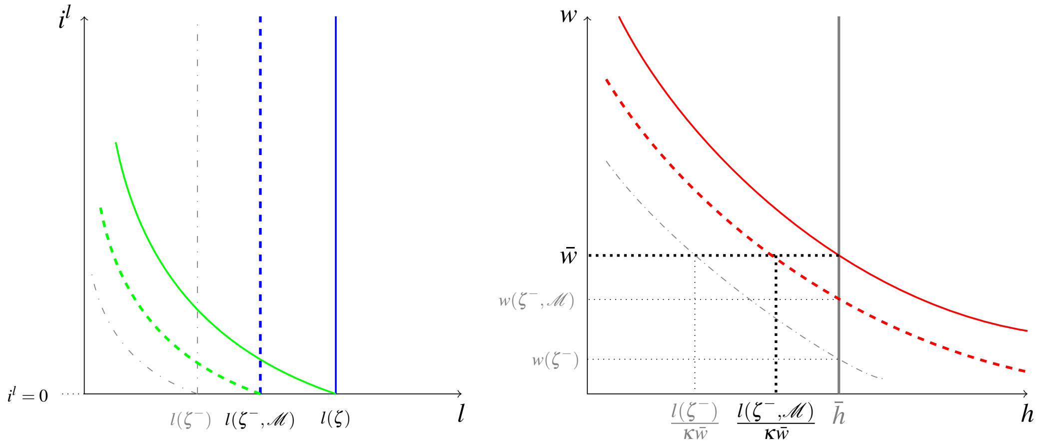

Figure 3.2.1 shows the interaction of the domestic credit and labour markets given shocks and unconventional monetary policy in a partial equilibrium setting. The left panel represents the equilibrium of the domestic credit market. In the initial situation of the economy, with no unconventional monetary policy, the thick vertical line shows the bank's supply of loans, whilst the thick negatively-sloped curve represents the firm's demand for loans given τ. The initial (partial) equilibrium amount of loans is $l(\zeta)$. On the other hand, the right panel shows the equilibrium in the labour market. The thick vertical line represents the inelastic supply of hours of work by households, whereas the thick negatively-sloped curve is the representation of the *in-equilibrium* demand function in (3.22). As assumed before, in the initial equilibrium situation $h = \bar{h}$ and $w = \bar{w}$.

With no unconventional monetary policy, Figure 3.2.1 shows that the new realisation of shocks implies a contraction of both the supply of loans and demand for loans, which is represented by the dashed-dotted vertical and curve loci in the left panel. The reduction in the supply of loans is induced by the fall in the price of the sovereign random-maturity bonds, whilst the fall in demand is mainly determined by the adjustment of τ. Since demand for hours of labour is determined by loans (see equation (3.22)), the credit crunch in the domestic credit market induces a reduction in the demand for hours for any value of the real wage. This is shown by the dashed-dotted negatively-sloped curve in the right panel. Given the new demand for hours of labour demand, full employment would imply the real wage to be equal to $w(\zeta^-) < \bar{w}$, but the real wage rigidity implied by equations (PE.3) and (PE.4) set the new value of the real wage to $\bar{w}$, inducing a new equilibrium value of hours of labour equal to $\frac{l(\zeta^-)}{\kappa\bar{w}} < \bar{h}$. The new (partial) equilibrium allocation is one in which there is involuntary unemployment, less consumption of the non-tradable good and a reduction of its relative price.

It is now straightforward to characterise the transmission of policy in a counterfactual scenario where, apart from the negative and correlated shocks, unconventional monetary policy is active ($\mathscr{M} \gg \mathbf{0}$). From equation (3.20) it is easy to see that compared with a situation with no policy intervention, the open market operation implemented by the central bank of the monetary union implies a higher supply of loans by the bank. This situation is represented by the dashed-thick vertical line in the left panel of Figure 3.2.1 together with the accommodated dashed-thick curve representing loans demand by firms. The counterfactual (partial) equilibrium amount of loans is $l(\zeta^-, \mathscr{M}) > l(\zeta^-)$. As a consequence of this new alternative value of equilibrium loans, *in-equilibrium* labour demand does not fall as much as in the scenario with no policy, as it is represented by the dashed-thick negatively-sloped curve in the right panel. In the particular instance of Figure 3.2.1, unconventional monetary policy does not eliminate involuntary unemployment but reduces it compared to the no-policy scenario, going against the negative effects of the credit crunch generated by both the falling stream of income and the decreasing price of the random-maturity bonds.

From this characterisation, it is possible to see that equilibrium in the labour market can be summarised by the condition

$$h(\zeta, \mathscr{M}) = \min\left\{\frac{l(\zeta, \mathscr{M})}{\kappa\bar{w}}, \bar{h}\right\} \tag{3.23}$$

which will be key to characterising the optimal general equilibrium allocation under the optimal debt policy of a government with full commitment.

3.2.9 Equilibrium under optimal debt policy with commitment

Here I consider the case of the government of a *periphery* small open economy member of a monetary union, which optimally chooses how many random-maturity bonds to issue each period in order to maximise the welfare of the representative household as measured by the utility function in (3.1). The government is characterised by its full commitment to repay its liabilities in the international credit market in every period. This setting is close to the extreme case in Na et al. (2018), where commitment is associated with an implicit bailout of the agents of the economy.

Lemma 1 establishes the recursive problem of the government under full commitment. Its solution fully characterises the general equilibrium allocation described in Definition 1.

Lemma 1. *Given the vector of exogenous states ζ, and unconventional monetary policy summarised by $\mathscr{M}$. The solution to the optimal debt problem:*

$$V(\zeta, b; \mathscr{M}) = \max_{\{c^T, h, b'\}} \mathscr{U}(\mathscr{C}(c^T, \mathscr{F}(h))) + \beta \mathbb{E}[V(\zeta', b'; \mathscr{M}')]$$
$$s.t.\ c^T = y^T + qb' - (\theta + (1-\theta)(q+\delta))b \tag{3.24}$$
$$h \leq h(\zeta, b; \mathscr{M})$$

where

$$h(\zeta, b; \mathscr{M}) = \min \left\{ \frac{d^* + (q\eta(s) + q^{CB}\chi^{CB})\psi b}{\kappa \bar{w}}, \bar{h} \right\} \tag{3.25}$$

fully characterises the equilibrium allocation described in Definition 1.

Proof. See proof in Appendix 3.A. □

3.3 Quantitative assessment

A quantitative assessment of the possible effects of unconventional monetary policy in a periphery economy member of a monetary union, is presented based on the solution to the optimal debt problem in (3.24) calibrated to the economy of Spain.

3.3.1 Solution to the optimal debt problem

Following Schmitt-Grohé and Uribe (2016), I assume that the Markov processes of the tradable good endowment and the country-risk factor are correlated in such a way, that they can be estimated via the VAR(1) model specification:

$$\begin{bmatrix} \ln\left(y_t^T\right) \\ \\ \ln\left(s_t\right) \end{bmatrix} = \Gamma \begin{bmatrix} \ln\left(y_{t-1}^T\right) \\ \\ \ln\left(s_{t-1}\right) \end{bmatrix} + \varepsilon_t; \quad \hat{\Sigma} = \frac{\sum_{t=2}^{T} \hat{\varepsilon}_t \hat{\varepsilon}_t'}{T}.$$

To estimate the VAR(1) model, I constructed measures of y^T and s using data from Spain in a period covering the first quarter of 1995 and the second quarter of 2010. In the first case, using the chain linked volume aggregates of value added and aggregates of imports and exports by industry, downloaded from Eurostat, the measure of tradable output is the aggregate of the linked volume value added of industries for which their total amount of imports and exports accounted for at least 10% of its value added.[1] After applying the Hoddrick-Prescott filter on this measure of tradable output, the measure of y^T is the cyclical component of the cycle-trend decomposition.

For the case of the country-risk factor, a measure for q was constructed from the inverse of the quarterly real gross yield of the outstanding long-term sovereign debt of Spain. The nominal yield on government bonds was obtained from the IMF's international financial statistics, which was transformed into a real interest rate by dividing its gross value by an estimated forecast of the future quarterly inflation rate.[2] The measure of $\bar{q}$ is the inverse of the long-run average of the real gross yield on long-term government debt. Finally, s is computed as the ratio $q/\bar{q}$.

Once the estimated matrices $\{\hat{\Gamma}, \hat{\Sigma}\}$ are available, a finite-state approximation of the exogenous states in ζ is performed following Schmitt-Grohé and Uribe (2009). Based on an evenly spaced grid of 31 points for y^T, and 16 points for q, a discretisation of 469 non-empty states is obtained, along with its respective prob-

[1] Here I decided to adopt the threshold suggested by Bianchi and Mondragon (2021).

[2] As suggested by Na et al. (2018), the model to forecast inflation is based on the best AR specification determined by the Bayesian information criteria. Quarterly data on the CPI index from IMF-IFS was used to construct the inflation measure.

ability transition matrix. The optimal debt problem is then solved using a finite horizon value function iteration algorithm, based on the described finite-state approximation of the exogenous states, an evenly spaced grid of 200 points for the outstanding level of random maturity bonds b, and the Bellman equation (3.24).

Table 3.3.1: Parameters used for the calibration of the model

Parameter	Value	Source
β	0.985	Na et al. (2018)
σ	2.0	Bianchi and Ottonello (2023)
ξ	0.5	Bianchi and Ottonello (2023)
ω	0.26	Bianchi and Ottonello (2023)
$\bar{h}$	1	Normalisation
$\bar{w}$	0.75	Calibrated to reach average unemployment of 14%.
ψ	0.33	Balance sheet of the financial sector. Source: Bank of Spain.
κ	1.0	Assumption
$\bar{q}$	0.989	(Inverse) average gross real yield of Spain's government debt 1999Q1-2010Q2.
θ	0.05	Chatterjee and Eyigungor (2012)
δ	0.01	θ, $\bar{q}$ and equation (3.11).
χ^{CB}	0.2	Consolidated balance sheet of the Eurosystem and Spain's ECB capital key.

The values of the structural parameters used are listed in Table 3.3.1. The values for β, σ, ξ and ω are standard in the business cycle literature based on models for Spain or a close periphery country such as Greece. The inverse of the average random maturity of bonds, θ is taken from Chatterjee and Eyigungor (2012) and, as is also usual in the related literature, the maximum amount of labour hours is normalised to 1. Given a long-run value of outstanding government debt with respect to GDP of 0.5 during the aforementioned period of the economy of Spain, and a value of outstanding sovereign debt with respect to GDP of 0.16 in the balance sheet of the financial sector, the value of ψ is calibrated to 0.33.

Given the value of $\bar{q}$ used in the estimation of the VAR process and the value of θ, the coupon δ is calibrated following the steady state equation (3.11). From the ECB's capital key for Spain, which determines the shares of the ECB's asset purchases from country members, and the 2011–2012 average increase of the long-term refinancing operations carried out by the ECB, as registered in the consolidated balance sheet of the Eurosystem, the unconventional policy fraction χ^{CB} was approximated to a value of 0.2. Finally, the minimum wage $\bar{w}$ was calibrated such that average unemployment in the simulations including the unconventional monetary policy parameters, was approximately 14%.

3.3.2 Identifying a sovereign debt crisis

A sovereign crisis, as the one witnessed by the periphery countries of the European monetary union between 2011 and 2012, is identified via a 1,200,000 times simulation of the solved model in subsection 3.3.1. A special and unique feature of the European sovereign crises was the continuous increment in the yield on the periphery government debt bonds during this two-year period. Based on this fact, from the simulations of the model we select 16-quarter windows for which during the last 8 quarters q falls by 2 or more unconditional standard deviations with respect to $\bar{q}$. Following this strategy, a total of 1423 windows are identified. The mean value for the main outcome variables of the model within the windows are shown in Figure 3.3.1.

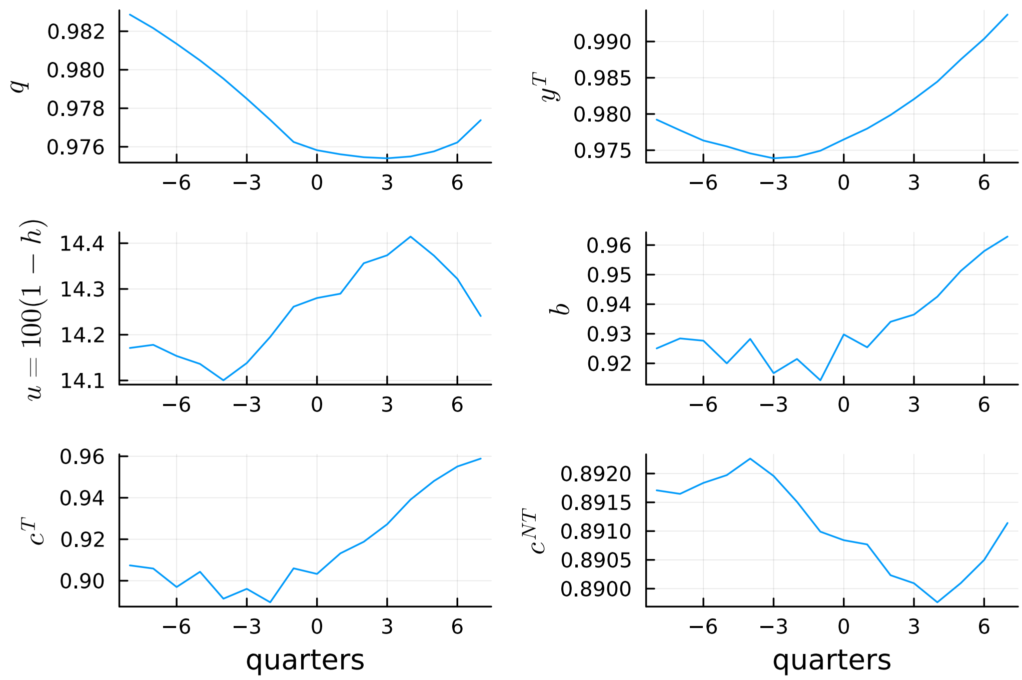

Figure 3.3.1: Identified patterns of selected variables after a negative shock on q

It is clear from the pattern of q that the lowest level of the sovereign bond price is reached in the second part of the 4-year window. As the Markov processes of q and y^T are correlated, it also can be seen that during most parts of the selected time windows, tradable output is below its steady-state level. However, between 2 and 3 quarters before the second 8-quarter period y^T, it shows an increasing pattern, which is inconsistent with what was observed in the sovereign crisis period. This can be explained mostly by the low estimated unconditional correlation of 0.33 between both Markov processes. Another rather inconsistent feature is the increasing pattern of the level of outstanding government debt b during the second part of the window period.

To better identify the sovereign crisis, a condition on the level of tradable output was also included on top of the condition for q. Namely, y^T must be at least one standard deviation below its steady-state level. The intersection of the two criteria left only 216 windows of 16 quarters, which gives a flavour of *extreme event* to the identified windows. Figure 3.3.2 presents the mean patterns of the main variables of the model along the 2-year window. The pattern of tradable output y^T is now decreasing during most of the period. More importantly, the pattern of outstanding sovereign debt b is decreasing in a subperiod that covers 3 quarters before the second part of the window period and 4 quarters within it, which closely resembles the solvency problem of the periphery European economies during the sovereign debt crisis.

Moreover, the patterns are consistent with the two-crisis scenario documented by Reichlin (2014b), i.e. the periphery European economies facing a sovereign debt crisis in the aftermath of the great financial crisis of 2007–2008. Hence, it is this last scenario the one that we identify as a sovereign debt crisis under an active implementation of unconventional monetary policy according to the rule in equation (3.13). To assess the impact of policy, we need to construct a counterfactual scenario under no implementation of unconventional monetary policy.

3.3.3 A counterfactual scenario for the assessment of the effects of unconventional monetary policy

In the same spirit of the approach in Cui and Sterk (2021), a counterfactual scenario under no implementation of unconventional monetary policy is provided by solving

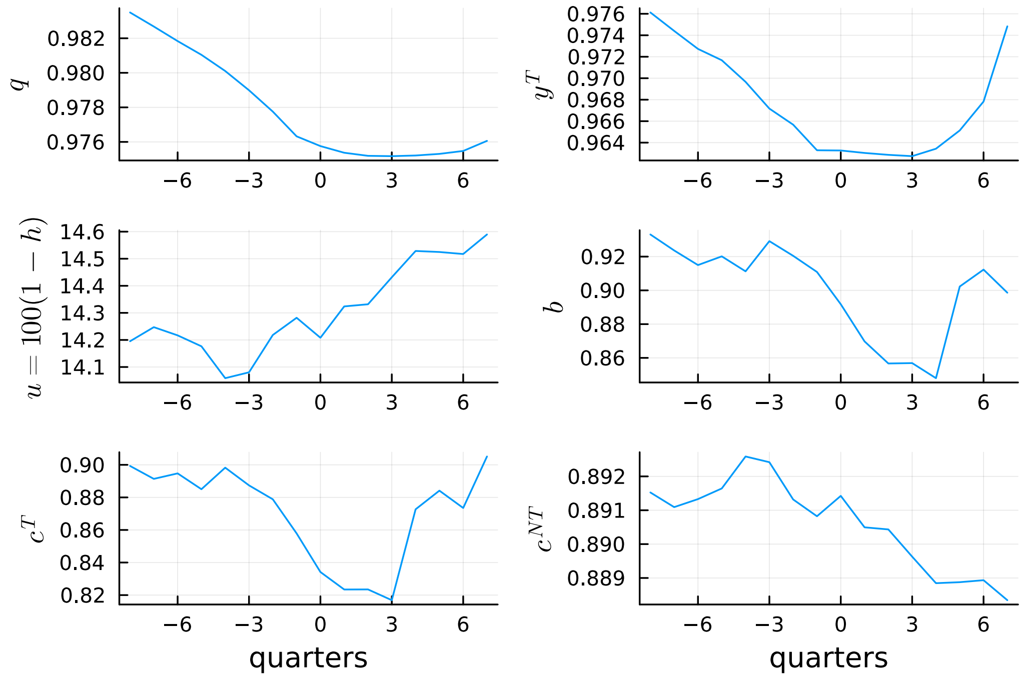

Figure 3.3.2: Identification of a sovereign debt crisis in a periphery economy

the optimal debt problem whilst completely switching off the role of the rule in equation (3.13), i.e. by imposing $\chi^{CB} = 0$ regardless of the value of $q/\bar{q}$. As in the baseline scenario with policy implementation, the solved model is simulated 1,200,000 times, and the same criteria used to identify the sovereign debt crisis in subsection 3.3.2, are imposed to obtain a new set of 16-quarter period windows. The contrast between the patterns under the baseline scenario and the no policy counterfactual scenario is shown by Figure 3.3.3

The contrast between the patterns of the unemployment rate u and non-tradable consumption c^{NT} is evidence of the credit channel of unconventional monetary policy portrayed in Figure 3.2.1. In the context of a systematic fall in the price of sovereign bonds, the implementation of unconventional monetary policy loosens the lending restriction of banks and indirectly the working capital restriction of firms. In contrast to the counterfactual scenario, unemployment is reduced which implies an increase in the consumption of the non-tradable good. This is observed all along the 4-year period given the targeted nature of the unconventional monetary policy rule.

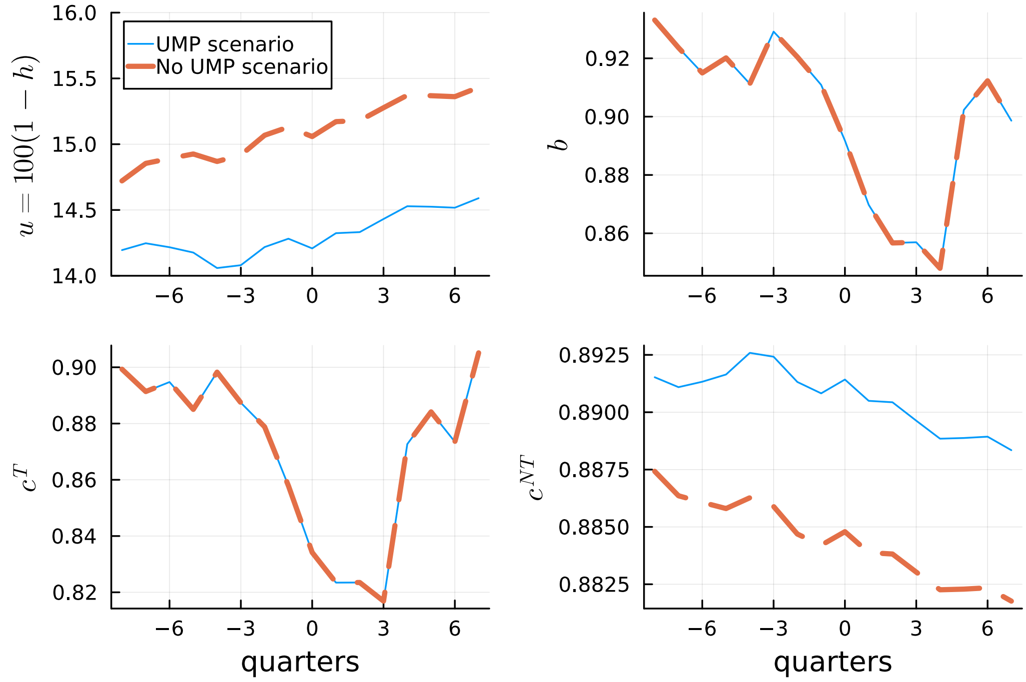

Figure 3.3.3: Assessment of the stabilisation role of unconventional monetary policy

Despite the positive effects, the optimal decisions regarding government debt and the consumption of tradable goods are practically unchanged, implying that the changes in employment and their effects on consumption are not big enough to produce optimal changes in the endogenous state of the model b. Notice that the counterfactual reductions in the unemployment rate are always less than one percentage point, and so are the falls in the consumption of the non-tradable good. Quantitatively, the stabilisation role of unconventional monetary policy is positive but small.

3.4 Conclusions

Based on a model for a small open economy which is a periphery member of a monetary union, the critical role of the credit channel of unconventional monetary policy is emphasised by imposing a lending restriction on banks which finance the working capital of firms producing a consumption non-tradable good. The context is one in which the government issues random-maturity bonds to smooth households' aggregate consumption by issuing random-maturity bonds in the international credit market. In this economy, a fixed nominal exchange rate and a downward rigidity on nominal wages end up imposing a real wage rigidity, which might generate a rationing of the fixed supply of labour of households.

The small open economy is subject to exogenous shocks on an endowment of a tradable good for consumption and on the exogenous price of the random maturity sovereign bonds. By summarising the general equilibrium of the model into an optimal debt problem of the government, we solve and calibrate the equilibrium path for the economy of Spain using quarterly data covering a period of 10 years previous to the sovereign debt crisis of 2011–2012. We then simulate the model to identify a scenario which resembles the crisis including the role of the implementation of an unconventional monetary policy by an external central bank with the unique role of buying a fraction of the sovereign bonds in the balance sheet of banks in case of a significant fall in the market price of such bonds.

A counterfactual scenario in which the role of the external central bank is completely shut down allows us to uncover and quantify the effects of the implementation of policy. The implementation of unconventional monetary policy has positive and welfare-improving effects summarised via a systematic reduction in the unemployment rate of households and a corresponding increase in the consumption of the non-tradable good. Nonetheless, the effects are small enough to not generate changes in the optimal decisions of debt issuing of the government. In particular, the reductions in the unemployment rate are less than 1 percentage point.

Appendix

3.A Proof of Lemma 1

Consider the optimal conditions

$$l_t^D = \kappa w_t h_t^D \tag{3.26}$$

$$l_t = d^* + (q_t(1-\chi_t^{CB}) + q_t^{CB}\chi_t^{CB})\psi b_{t-1} \tag{3.27}$$

$$w_t \geq \bar{w} \tag{3.28}$$

$$(\bar{w} - w_t)(\bar{h} - h_t) = 0. \tag{3.29}$$

Solving for h_t from the pair of equations (3.26) and (3.27) we obtain:

$$h_t = \frac{d^* + (q_t(1-\chi_t^{CB}) + q_t^{CB}\chi_t^{CB})\psi b_{t-1}}{\kappa w_t} \tag{3.30}$$

Given b_{t-1}, starting from (3.30) let us consider the object:

$$h_{\bar{w}} = \frac{d^* + (q_t(1-\chi_t^{CB}) + q_t^{CB}\chi_t^{CB})\psi b_{t-1}}{\kappa \bar{w}}. \tag{3.31}$$

and let w_t be the real wage that clears the labour market. From (3.28) and (3.29): *i)* if $\bar{w} \leq w_t$ then $h_{\bar{w}} \geq \bar{h}$, and h_t must be equal to $\bar{h}$, *ii)* if $\bar{w} > w_t$ then $h_{\bar{w}} < \bar{h} = h_t$. Then $w_t = \bar{w}$ and $h_t = h_{\bar{w}} < \bar{h}$. Hence

$$h_t = \min\{h_{\bar{w}}, \bar{h}\} \tag{3.32}$$

is consistent with equilibrium in both the credit market and the labour market. Once

h_t is determined by (3.32), equation (3.27) and the optimal conditions (3.8) and (3.9) determine $\{p_t, i_t^l, \tau_t\}$, and c_t^{NT} is also determined as $\mathscr{F}(h_t)$.

Now, given b_{t-1} and b_t equation (3.21) is consistent with the equilibrium in the non-tradable goods market, the maximum profits of firms and the flow budget constraint of the government, then any value of b_t which comes from the solution of problem (3.24) must characterise the equilibrium allocation in Definition 1.

Chapter 4

Conclusions

There are multiple takeaways from this thesis. First, it manages to highlight important and novel aspects of the transmission mechanism of unconventional monetary policies which can be easily related to policy design discussions. Chapters 1 and 2 put in the insurance aspect of the transmission of policy either against purely idiosyncratic unemployment risk that cannot be self-insured by individuals, or against intrinsic uncertainty which makes risk-averse individuals to be cautious in the face of shocks. This insurance aspect of the implementation of unconventional monetary policy, is especially important in recession contexts in which agents of the economy are more vulnerable to the presence of financial barriers or frictions and its aggregate consequences.

In the particular context of a monetary union, unconventional monetary policy serves as a stabilisation instrument in the presence of increasing sovereign risk. The small quantitative effects also remind us of other potential channels that are not being taken into account by our analysis but that can be affected by the implementation of policy. One of these channels can be the extraction of default risk that this kind of policy can also have in the context of increased sovereign risk.

In general, the thesis makes an important contribution to the understanding of the role of different frictions in the economy and its role in the transmission mechanism of unconventional monetary policy. The analysis presented here further strengthens the consensus that policies based in the change of composition and size of the balance sheet have real effects because of the presence of these frictions, but also because they have a particular fiscal side which can either annulate such effects or amplify them.

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