Towards A Welfare Model of Trade and FDI with Oligopolistic Competition

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Declaration

I hereby declare that except where specific reference is made to the work of others, the contents of this dissertation are original and have not been submitted in whole or in part for consideration for any other degree or qualification in this, or any other university. This dissertation is my own work and contains nothing which is the outcome of work done in collaboration with others, except as specified in the text and Acknowledgements. The copyright of this thesis rests with the author.

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Abstract

The thesis investigates the welfare effects of international trade and horizontal FDI with firm heterogeneity and variable markups using an oligopolistic competition framework. Generally, it contributes to three strands of literature concerning 1) the firm heterogeneity of international trade and FDI, 2) variable markups stemming from oligopolistic competition, and 3) the welfare effects of trade and FDI. No papers exist which consider these three perspectives together. The thesis finds the largest 'pro-competitive' effect and 'selection' effect for multinational firms exist compared to exporters and non-exporters, which results in the most extensive welfare gains for multinational firms. The first main chapter constructs a theoretical model with two symmetric countries and studies the properties of the model under three scenarios: autarky, trade openness and multinational production, separately to examine the welfare gains from each case. This chapter shows a critical source of welfare gains from trade liberalisation, the pro-competitive effect, which has attracted limited attention in the FDI literature. Multinational firms generate the most significant welfare gains through the largest selection effect and the lowest markups, increasing aggregate productivity and suffering the fiercest competition. The second main chapter extends the first by incorporating innovation, which enlarges the welfare gains via the innovation effect of cost reduction, and this is the third channel of the most significant welfare gains from multinational production. Finally, the third main chapter considers an economy where non-exporters, exporters and multinational firms can co-exist in the same industry. It introduces a different condition for endogenous variety compared to the first and second main chapters but is similar to the approach of Melitz (2003), to determine the economy's equilibrium. It examines the welfare gains of trade liberalisation because of the selection effect for non-exporters, the pro-competitive effect for exporters and the efficiency of engagement of horizontal FDI through the proximity-concentration effect between exporters and multinational firms, which is the new source of the welfare effects.

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Chapter 1

Introduction

1.1 Background and Motivation

The welfare implications of international trade and, more recently, multinational firms have been one of the central questions in economics. In recent decades, the volume of international trade has risen substantially among developed countries during the 1980s and spread to developing countries around the 1990s, (e.g., see Hsu et al., 2020; Navas and Licandro, 2011). Over the same period, the growth in multinational firms,[1] as a central element of globalisation, played an increasingly significant role in the global economy, (e.g., see Bombarda and Marcassa, 2020; Irarrazabal et al., 2013). This thesis studies the welfare effects of international trade and multinational production.

Foreign direct investment (FDI) is the volume of investment created by multinational firms and a capital account category in the Balance of Payments of a country Ramondo (2014). According to the nature of the investment,[2] there are two modes of FDI: horizontal FDI, in which an affiliate replicates the production process of the parent company elsewhere in the world. In this type of investment, firms aim to save on transportation costs when serving the host market, although this comes with an increase in the fixed cost so that two production plants can be maintained. It implies the multinational firm faces a trade-off between a lower price due to being close to the consumers and higher fixed costs due to less concentration across plants. This has been known in the literature as the proximity-concentration trade-off. Helpman et al. (2004) extend this proximity-concentration trade-off in an environment with heterogeneous firms and illustrate that only the most productive firms will become multinationals, which is in line with the analysis of my thesis. The other mode is vertical FDI, when the production chain is broken up, and parts of the production process are transferred to the affiliate plants. It takes advantage of the different costs of production factors in different locations. The difference between the two types of FDI is that horizontal FDI often happens between two developed countries. Here the firm locates production near large customer bases as the firm avoids trade and transport costs while paying the production costs in the foreign market for FDI decisions. Horizontal FDI is the largest component of the two modes, (e.g., see Brainard, 1997); therefore, in my thesis, I include only horizontal FDI[3] to construct

[1] A multinational firm (aka multinational enterprise (MNE)), is defined as 'an enterprise that controls and manages production establishments and plants located in at least two countries. It is one subspecies of a multi-plant firm Caves (1996), p1'. Multinational production (MP) is the activity of parents and affiliates, for example, sales or employment.

[2] There is another way of classification. According to the type of investment, FDI can be classified into greenfield, when a company builds a new production facility aboard and brownfield (cross-border mergers and acquisitions), when a domestic firm buys a controlling stake in a foreign firm.

[3] Throughout the thesis, I use the terms multinational firms, multinational production (MP), horizontal FDI and FDI interchangeably to refer to the activity of affiliate plants and multinational firms.

my theoretical models for simplicity to investigate the welfare effect of trade and FDI with variable markups and firm heterogeneity.

A vital aspect of the modelling environment in this thesis that is employed is innovation. Innovation, according to Rogers (1998), indicates the application of new ideas for firms to increase their performance, including products (goods and services), processes or any other activities. The two categories of innovation are technological product innovation and technological process innovation. The first is the type in which the new or improved product has different characteristics from the previous product by increasing the quality and variety of goods. In contrast, process innovation indicates applying a new or significantly improved production method, leading to improvements in the efficiency of production of the goods and lowering their costs and decreasing labour requirements (e.g., Calvino and Virgillito, 2018; OECD, 1997).

Why do I consider international trade and multinational production together? Recent empirical evidence from a trade perspective, (e.g., see Bernard et al., 2009), demonstrates that by 2000, more than 50% of the firms that import and export in the US represented almost 90% of US trade. Some empirical studies suggest that trade decisions closely interact with activities of multinational production, (e.g., Ramondo, 2008, 2014; Rodrigue, 2010). For instance, Ramondo (2008) describes that total subsidiary sales of multinational enterprises accounted for approximately 60% of world GDP, more than double the share of exports by 2001. Furthermore, Ramondo (2014) illustrates that world sales of multinational production subsidiaries were around twice as high as world exports. Besides, subsidy sales rose by a factor of seven while exports rose by a factor of five over the last two decades, according to UNCTAD (United Nations Conference on Trade and Development, 2009). In addition, multinational firms also play a significant role in innovation, controlling the majority of R&D investment, although they account for a smaller proportion of firms. For example, in 1985, 72% of the world FDI was from the US, UK, Japan and Germany, which accounted for 55% of the world's R&D investment, (e.g., see Dunning, 1994). To better understand the empirical evidence, several researchers have worked on theoretical models combining trade and multinational firms to explain the relationship between R&D investment and multinational firms. Some pioneering studies are as follows. Arkolakis et al. (2018) build a framework to capture the increased specialisation in innovation and production and examine the implications of openness to trade and multinational production. They find that comparative advantage results in the specialisation of innovation for some countries and the relegation of production to other countries via multinational firms. Lind and Ramondo (2018) review the recent theoretical literature on the nexus between the creation and diffusion of technology with openness. They extend the Eaton and

Kortum (2002) model to innovation, diffusion and multinational firms, which captures firms' productivities with a Freḉhet distribution, illustrating that 'international trade' and 'multinational firms' are two channels to spread new ideas and explore the welfare implications of trade. However, in my thesis, I complement Lind and Ramondo (2018) by including oligopolistic competition rather than perfect competition as in theirs, as the importance of oligopolistic competition can be shown as follows.

Compared to perfect or monopolistic competition, oligopolistic competition better characterises many industries by featuring a few large firms performing strategically. It has significant application in industrial organisation, (e.g., see Etro, 2014). However, in trade theory, considering oligopoly in industrial organisation, is a road less travelled because of its complications. The cornerstone model of oligopolistic trade was built by Brander (1981), focusing on identical commodities because of strategic interaction among firms. Brander and Krugman (1983) extend this to free entry, showing that the reciprocal dumping of two-way trade in the same product is unambiguously welfare-gaining. Neary (2003) introduces the assumption of large firms in their own market but small in the economy to capture the tractability of quadratic preferences, which is used in multiple applications in international trade and macroeconomics, among other fields when setting utility functions (e.g., see Impullitti and Licandro, 2018; Impullitti et al., 2022; Neary, 2010). In addition, under oligopoly, Neary (2007) suggests that trade liberalisation can incentivise countries to specialise and increase trade according to comparative advantage and predicts that the low-cost firms buy out the higher-cost rivals and that bilateral mergers are profitable. Neary (2016) provides a new oligopoly model in general equilibrium with implications for international trade, assuming different continuum–quadratic preferences. He explains how the gains from trade will rise as comparative advantage increases and the competition effect via the dispersion of markups occurs. My thesis is motivated by this dimension and also considers oligopolistic competition for international trade, more importantly, extending the model to multinational firms with firm heterogeneity.

The thesis is closely related to the recent literature developed by Impullitti and Licandro (2018), Impullitti et al. (2018), and Impullitti et al. (2022). These studies introduce variable markups stemming from oligopolistic competition in a trade model with heterogeneous firms. Impullitti and Licandro (2018) incorporate endogenous growth into the model with cost-reducing innovation and consider a simple version of the model and a general version. The difference is that the simple model considers exporters only, while the general version includes non-exporters and exporters simultaneously. They find welfare gains generated through the selection and pro-competitive effects and demonstrate that growth can magnify the gains from selection, as market share reallocations can affect the productivity level and its growth rate. Impullitti et al. (2018) devise a more sophisticated entry structure, which allows markups to vary with firm productivity through an endogenous number of firms, associated more closely with empirical evidence of large markup dispersion across firms. They adopt innovation without knowledge spillovers and demonstrate that innovation still has a non-negligible contribution to the gains from trade. Impullitti et al. (2022) build a similar economic environment but assume a discrete number of firms compared to the real number of firms of Impullitti et al. (2018) and find that trade induces an increase in market concentration, which is a significant source of gains from trade. The gains in welfare through the increase in market concentration are conducted through two mechanisms: increasing returns of manufacture and a scale effect on innovation. In the numerical analysis, market concentration is the main driver of welfare gains, specifically, less from increasing returns and more from the scale effect on innovation. My thesis is built with a similar economic environment as theirs, including heterogeneous firms under oligopolistic competition but extends to multinational firms.

My thesis aims to answer the critical question in trade of what the welfare effect of trade and FDI will be and how large, within an economy including trade and horizontal FDI with heterogeneous firms under oligopolistic competition. My contributions are across several dimensions. Firstly, in my first main chapter, I extend the pro-competitive effect within the model of Impullitti and Licandro (2018) to multinational firms and derive a closed function of their productivity threshold at equilibrium, which is further compared with the scenarios of exporters and non-exporters[4]. Here, the scenarios are that: all are non-exporters, exporters and multinational firms separately in the global economy. The pro-competitive effect arises from oligopolistic competition from Cournot games, which allows firms to interact strategically with their direct rivals. That is to say; I find that a novel channel, variable markups, will increase welfare gains from trade openness and multinational firms. Secondly, the most significant welfare effect of multinational firms is found compared to exporters and non-exporters and it can be identified from the closed solutions of the productivity thresholds at equilibrium through the theoretical framework. Thirdly, considering firm heterogeneity like Melitz (2003) and oligopolistic competition, the selection effect potentially generates positive welfare gains for multinational firms, exporters and non-exporters, which is the largest for multinational firms compared to exporters and non-exporters. Fourthly, in my second main chapter, process innovation as an economic engine, combined with variable markups and the selection effect, is the third mechanism to enhance the welfare of trade and FDI. Lastly, in the third main chapter, considering non-exporters,

[4]In my thesis, the non-exporters in the three main chapters represent the domestic firms which are not productive enough to serve the foreign market; therefore, they only serve the domestic market. In addition, they are compared to the case of exporters and multinational firms. That is, the definition of non-exporters and domestic firms in autarky are interchangeable in the thesis.

exporters, and multinational firms coexist in an industry, the model is built in a complex environment to theoretically measure the welfare effect of trade liberalisation with trade and FDI, including numerical simulation analysis.

1.2 Overview of the Thesis

This thesis analyses the world economy with two perfectly symmetric countries in three main chapters to investigate the welfare effects of international trade and multinational firms via horizontal FDI with variable markups under oligopolistic competition and firm heterogeneity. My first chapter briefly introduces the thesis, including the context and motivation of my research, followed by an overview of my thesis. My second, third and fourth chapters are the main body of my thesis, providing a critical analysis of the research question. Finally, chapter 5 concludes and presents limitations and future research.[5]

My theoretical models in the three main chapters all feature firm heterogeneity and oligopolistic competition. My first main chapter provides intuition using a basic framework to study the mechanism of welfare effects of trade and FDI. I consider three separate scenarios: all firms operate only domestically[6], all are exporters, and all are multinational firms serving the foreign market via horizontal FDI. Furthermore, the framework provides the fundamental set-up, which allows me to further develop the models by incorporating process innovation in the second main chapter and the economy with non-exporters, exporters and multinational firms coexisting in my third main chapter.

In the first main chapter, the basic model suggests that the scenario of all multinational firms creates the largest welfare gains compared to the case of all exporters and all non-exporters. It highlights two mechanisms, via ‘pro-competitive’ and ‘selection’ effects, that arise from multinational firms with the lowest variable markups and the highest productivity threshold for the toughest selection effects. In contrast, the case of all exporters generates larger welfare gains than the case of all non-exporters through the same channels. My second main chapter complements the first one by considering process innovation. This leads to the conclusion that innovation, as the third channel, will enhance the welfare gains for multinational firms through the highest profits and quantities and lowest prices compared to exporters and non-exporters. Finally, the third main chapter extends the first by including the global economy with non-exporters, exporters and multinational firms being allowed to coexist in each industry and addresses how trade openness affects

[5] Notice that I prefer to refer directly to each of the main chapters, as the first, the second and the third, for simplicity, with a general introduction and conclusion.

[6] This means no firm can be capable of serving the foreign market, which the economy would also be called an autarkic case or autarky in the following context

welfare. Through numerical simulation analysis, I find that trade liberalisation will increase welfare gains by around 0.12% when the economy moves from autarky to free trade through three channels: selection effect for non-exporters, pro-competitive effect for exporters and efficiency from the switch from FDI to exporters via the proximity-concentration effect between exporters and multinational firms.

Chapter 2

Firms in Autarky, Trade Openness and Multinational Production

Abstract

This chapter constructs a multi-sector general equilibrium model in a world with two symmetric countries. It explains welfare gains from international trade and horizontal foreign direct investment (FDI) in a global economy with firm heterogeneity and variable markups stemming from oligopolistic competition. My model shows that the pro-competitive and selection effects of trade happen because trade openness induces the increase in product market competition via reducing markups and in aggregate productivity through firms' increasing productivity thresholds. The most significant contribution of the chapter is that I incorporate horizontal FDI with higher fixed sunk costs but no variable transportation costs compared to exporters, like Helpman et al. (2004), as an alternative way for firms to serve a foreign market in order to explore their effect on welfare. There are three scenarios of a world economy analysed: where all are non-exporters, all are exporters, and all are multinational firms. I find that all multinational production generates the largest welfare gains, all exporters produce a medium-large welfare effect, and the scenario with all non-exporters creates the lowest welfare. These findings arise from two channels: the pro-competitive and selection effects. Multinational firms, via horizontal FDI, create the largest selection and lowest markups from the strongest oligopolistic competition within product lines compared to exporters and non-exporters.

Keywords: firm heterogeneity; horizontal FDI; oligopolistic competition; welfare gains
JEL Classification: D60, F12, F13

2.1 Introduction

It has been a long-standing challenge for economists to identify the size and sources of gains from international trade and Foreign Direct Investment (FDI) (e.g., Helpman et al., 2004; Melitz and Ottaviano, 2008). Whether the welfare effects of trade are significant is a contentious argument in the trade literature. However, in recent decades, a line of research incorporating firm heterogeneity into trade models has uncovered a new source of welfare gains. Rich data sets have emerged, which allow us to look at firm heterogeneity and from which we have begun to see the potential importance of intra-industry differences. Firm heterogeneity arises empirically in the form where there are a small number of large firms who tend to be exporters, and they export a small fraction of their production within an industry (e.g., Aw et al., 2000; Bernard et al., 1995; Clerides et al., 1998). These models with firm heterogeneity (e.g., Bernard et al., 2003; Melitz, 2003) show that international trade, through an increase in competition expelling the least efficient firms from the market, can increase an industry's aggregate productivity and enhance welfare.

Melitz (2003), and Bernard et al. (2003) found welfare-enhancing properties of trade when they introduced firm heterogeneity and imperfect competition into the economic environment in their theoretical models. Bernard et al. (2003) applied Bertrand competition in a Ricardian framework as a technique to explain the same basic kind of trade-induced reallocations, where market shares are reallocated from low to highly productive firms so that sectoral efficiency is increased. The framework delivers an endogenous distribution of markups through competition between domestic and foreign firms of the same variety. A fundamental assumption in the model is an exogenously fixed total number of varieties produced and consumed. Melitz (2003) develops a dynamic industry model to investigate the welfare gains from trade. His framework, based on the work of Dixit and Stiglitz (1977), features monopolistic competition, increasing returns to scale and a love for variety. He found a fundamental mechanism, the 'competition channel', through which trade positively impacts aggregate productivity and welfare. One limitation in his model is that firms' markups are exogenously fixed since a symmetric elasticity of substitution between varieties holds.

Recent work in the literature on the welfare effects of trade suggests, however, that welfare gains from trade are similar across different types of models and the selection effect may not be so relevant, and their gains could be relatively small (Arkolakis et al., 2012). In my model, I introduce firm heterogeneity like Melitz (2003) with a constant elasticity of substitution (CES) utility function, assuming a continuum of imperfectly substitutable varieties, or product lines, introduced by firms with different productivities. However, a significant distinction from Melitz (2003) is that my model focuses on oligopolistic competition with several perfectly substitutable producers in each product line. In contrast, Melitz (2003) has only one firm within each variety in the monopolistic market. That is to say, the critical assumption of my model is that: I assume there is a continuum of varieties and several potential firms for each variety. The firms perform as Cournot games, which allows them to interact strategically with others by incorporating firm heterogeneity, while there is Bertrand competition across homogeneous products in Bernard et al. (2003).

In addition to the studies focused on welfare effects from trade, some studies are devoted to gains from multinational production (e.g., Ahn, 2014; Ramondo, 2014; Ramondo and Rodríguez-Clare, 2010, 2013). Ramondo and Rodríguez-Clare (2010) point out that many possible channels exist for gains from a country's openness: international trade, multinational production (MP), and diffused technology. Research dealing with welfare effects has given much attention to trade but less to the activity of MP. By 2007, sales of multinational firms were almost twice as high as exporters in the global world, according to UNCTAD (2007), which means that multinational firms have increasingly become significant channels for countries to exchange goods, capital, ideas and technology. Ramondo and Rodríguez-Clare (2013) investigate the welfare effects of FDI, building a model with a setting in which trade and MP are competitive and complementary methods to serve a foreign market. Specifically, there are three ways: 1) 'horizontal' FDI, in which trade and FDI are competitive methods serving the foreign market, resulting in the substitutability between trade and MP; 2) 'vertical' FDI, in which foreign affiliates import intermediate goods from the home country, resulting in complementarity between trade and MP, as MP facilitates trade and vice versa; 3) and an export platform, as known as bridge MP (BMP), in which firms choose another country as a platform to serve a particular market, resulting in complementarity between trade and MP, as BMP flows involve both trade and MP simultaneously. They quantify the overall gains from the country's openness and conclude that the welfare gains from trade in their model are twice as high as in trade-only models due to the domination of complementarity forces through 'vertical' FDI and BMP. In contrast, the gains from MP in their model are slightly lower than gains in MP-only models due to the domination of substitution forces through 'horizontal' FDI. It is significant as my results also indicate that the scenario of all multinational firms gains the most welfare. Ramondo (2014) studies the gains from multinational firms from another point of view. She evaluates the importance of welfare gains from only-FDI with foreign firms as the only way to serve a foreign market by building a model combined with Lucas (1978) and Eaton and Kortum (2002). Ramondo and Rodríguez-Clare (2013) and Ramondo (2014) both consider horizontal FDI as a substitutable way to serve the foreign market, similar to mine. Garetto (2013) investigates the gains from FDI applied to the Eaton and Kortum

(2002) type of model, emphasizing only vertical FDI and quantifying the welfare gains from intra-firm trade at approximately 0.23% of consumption per capita.

However, other literature exploring the welfare effect of multinational activities draws the opposite conclusion. Reis (2001) builds a theoretical framework with asymmetric countries to explain that foreign investment by multinational firms may decrease national welfare since the returns may be repatriated, which indicates that the transfer of capital returns to foreigners would reduce the national welfare. In addition, Yang (2015) builds a theoretical model with domestic oligopolies and suggests that if foreign firms switch production strategy from FDI to exporting, it leads to a fall in domestic welfare because of the decrease in overall cost efficiency. My model focuses on 'horizontal FDI' like Ethier (1986), which offers a competitive (substitutable) way to serve a foreign market with exporters like Helpman et al. (2004). I examine the welfare effect of horizontal FDI through a simple and analytically tractable version of the model in which the number of oligopolistic firms per product line is exogenously fixed. Furthermore, I assume all operating producers are multinational firms, compared with the scenarios where all potential firms are non-exporters and all operative firms are exporters with non-zero export fixed costs.

Theoretical frameworks and quantitative analysis in the trade field dealing with welfare effects are characterised mainly by perfect or monopolistic competition (e.g., Arkolakis et al., 2012; Eaton and Kortum, 2002; Ramondo, 2014; Ramondo and Rodríguez-Clare, 2013). However, some evidence shows that large, highly productive firms with substantial market power prevail in modern economies, as shown in Impullitti et al. (2018). For example, empirical work by Bernard et al. (2007) documents that international trade is a rare activity: just 4 percent of 5.5 million firms operated as exporters in the United States in 2000 and around 96 percent of the total US exporters account for the top 10 percent of exporting firms. Regarding trade models, standard frameworks with firm heterogeneity cannot embody strategic competition among large firms. So in my chapter, I explore the welfare effect of trade and FDI in an economy with heterogeneous firms and oligopolistic competition featuring variable markups. I focus on the welfare gains from international trade and multinational production when the economy consists of oligopolistic firms with heterogeneity in productivity and market power. The welfare analysis of trade and horizontal FDI comes from the response of the market structure to lowering trade barriers; this is a new channel of welfare gains from variable markups, as previous models focused on monopolistic competition with invariant markups.

I assume a global economy which consists of two entirely symmetric countries with the same set of operative product lines and productivity distribution. My model's economy has a continuum of potential imperfectly substitutable product lines, known as varieties, with different productivities. Moreover, in each variety, a small number of identical firms produce perfectly substitutable goods competing in an oligopolistic market and generating variable markups, contrasting with Melitz (2003). So individual firms are characterised by 'large in the small but small in the large', which means firms are infinitesimal compared to the whole economy, but large within their variety (e.g., Neary, 2003, 2010). The firms interact strategically with a small number of identical direct competitors, domestic and foreign rivals, in a Cournot competition game (e.g., Brander and Krugman, 1983) and confront indirect competitors among firms within other varieties. The model's framework is similar to Impullitti and Licandro (2018), and a small number of identical firms within each variety enter a market and draw their productivity from the Pareto distribution conditional on paying sunk costs. If a firm enters successfully, it plays Cournot games with its direct competitors within each variety. Notice that, in order to investigate the welfare effect of multinational firms, firstly, I begin with a simple and basic framework in which the number of firms n within each variety is exogenous and consider three scenarios where all potential firms are non-exporters, all operating firms are exporters with a zero and non-zero fixed cost of exporting, and all active producers are multinational firms. Then I compare welfare gains from these three cases and find that multinational firms arrive at the highest gains caused by the 'pro-competitive effect with lowest markups' and 'largest selection effect with highest productivity cutoff', leading less productive firms to exit the market.

This chapter is structured in four sections after the introduction. In section 2, I conduct a review of the literature. In section 3, I provide the assumptions and set out the basics of my model in order to present and solve a simple version of three scenarios. They include all non-exporters, all are exporters with zero export fixed cost, and all are multinational firms as a new competitive way to serve a foreign market separately. Notice that I consider an extra case where all are exporters with a non-zero fixed cost of exporting to conduct a comparison with multinational firms. In section 4, I investigate the welfare gains from three scenarios, represented by the case in which all are exporters with a zero export fixed cost. Then I compare the welfare effect from three scenarios and explain the gains when the country opens to trade and the welfare effect of international trade. Finally, section 6 concludes that horizontal FDI in the open economy will produce the highest welfare gains under certain conditions.

2.2 Literature Review

My work brings together and develops the literature on international trade (e.g., Impullitti and Licandro, 2018; Impullitti et al., 2018, 2022; Krugman, 1980; Melitz, 2003) and MP (e.g., Helpman et al., 2004; Ramondo, 2014; Ramondo and Rodríguez-

Clare, 2013). It closely relates to three significant strands of the literature. Firstly, the literature on multinational firms via horizontal FDI with firm heterogeneity and constant markups in the monopolistic market comprised the first strand of literature. That is, how they expand international trade to multinational firms in a similar economic environment. Then, I review the welfare effect of trade in an environment that considers oligopolistic competition with variable markups. My aim for the chapter is to extend those papers (e.g., Edmond et al., 2015; Impullitti and Licandro, 2018; Impullitti et al., 2022; Li and Miao, 2020) and investigate the welfare gains from multinational firms, following the way introduced by Helpman et al. (2004). Thirdly, I compare my chapter regarding the welfare effects of multinational firms with related papers (e.g., Ramondo, 2014; Ramondo and Rodríguez-Clare, 2013).

The observations which informed the first branch of trade literature since the 1970s were that world trade flows were predominantly between developed countries with similar technologies or factor endowments. At the forefront of this literature is Krugman (1980), the 'new trade theory', which considers firms within an international industry under monopolistic competition. He assumes that consumers love variety and that an economy exhibits internal economies of scale. Welfare gains from trade in this model come purely from the variety channel since consumers love variety, and there would be more of it available to consumers when a country opens to trade. Melitz (2003)'s 'new new trade theory', extends Krugman (1980) to take into account firm heterogeneity. He develops a dynamic model of entry and exit based on Hopenhayn (1992), with constant elasticity of substitution (CES) utility, increasing returns to scale (IRS), and monopolistic competition featuring constant markups. He finds that welfare gains will increase as trade results in a reallocation of market share from less to more productive firms by expelling the least productive firms from the market.

In terms of introducing multinational firms, Helpman et al. (2004) extend Melitz (2003) by introducing horizontal FDI: a firm's affiliate replicates the production process of its parent firm in its domestic facilities elsewhere in the world. It is assumed that firms can serve the foreign market either by export or FDI, and there are different fixed costs for both exporters and multinational firms. They are the fixed cost of exporting f_X, and the fixed cost of creating a new plant in the foreign country f_I, separately. In addition to those fixed costs, a variable iceberg cost τ exists for the exporters only, which means $\tau > 1$ units of goods should be produced in order to deliver per unit of the good at the destination. Under particular conditions, among different costs, they find that the most productive firms will serve the foreign markets through FDI; the middle productivity firms will serve the foreign markets via exports, and the less productive firms will remain domestic only. In this chapter, I include horizontal FDI and introduce related fixed costs of planting a new company in a foreign country as Helpman et al. (2004). I combine horizontal FDI with a simplified version of Impullitti and Licandro (2018), which embraces oligopolistic competition within each variety and assumes the number of firms in each product line is exogenous, including all operating firms as exporters with a zero fixed cost of exporting. Moreover, I set up three scenarios where all operating firms are domestic, all potential firms export and all operative firms are with FDI activities. Then, I compare the welfare gains in these three cases. While different from Helpman et al. (2004), my contribution is that I incorporate multinational firms in an environment with variable markups generated by oligopolistic competition. This is significant because incorporating oligopolistic competition in the model allows multinational firms via horizontal FDI to interact strategically with others in Cournot games with firm heterogeneity, which has not been explored in the existing literature.

Another strand of literature focuses on variable markups stemming from strategic interaction between firms with oligopolistic competition, represented by Impullitti and Licandro (2018) and Impullitti et al. (2018). Impullitti and Licandro (2018) construct an endogenous growth model with cost-reducing innovation, oligopolistic competition and firm heterogeneity and find trade reduces markups through pro-competitive effects, which are absent in Melitz (2003) where markups are constant because of only one firm within each variety. From the perspective of model solving technique, another difference from Melitz (2003), Impullitti and Licandro (2018) consider firms' exit conditions and labour market clearing conditions to constrain the equilibrium productivity cutoff. They did not consider the free entry condition in the simplified version of the model but assumed the number of varieties is endogenous depending on firms' productivity thresholds. In contrast, Melitz (2003) combines a zero cutoff profit condition and free entry condition to derive the productivity threshold at equilibrium. Besides, he assumes the mass of entrants can respond endogenously to changes in trade costs, potentially taming or even offsetting the loss of varieties due to selection. In Impullitti and Licandro (2018), selection has a starker negative welfare effect through the loss of varieties. They conclude that trade liberalisation generates larger firm selection, increasing average firm productivity, as in Melitz (2003). Moreover, selection incentivises firms to innovate, leading to a higher innovation-driven growth rate. This can potentially boost the gains from selection, leading to further welfare improvements since market share reallocations can affect both the productivity level and its growth rate.

Impullitti et al. (2018) is closely related to the above, introducing oligopolistic competition, innovation and firm heterogeneity in a static model with two symmetric countries. In each country, an endogenous number of firms in the same product line play a symmetric Cournot game: each firm chooses its strategies, taking its competitors' strategies as given. They conclude that oligopolistic competition and the free entry condition make markups responsive to firm heterogeneity and trade costs; a reduction in variable trade costs reduces markups on domestic sales but increases markups on export sales since firms do not pass the entire reduction in trade costs onto foreign consumers. They conclude that the overall effect of trade liberalisation is pro-competitive, and it is a crucial channel of the welfare gains since the increase in export's foreign markups is never sufficiently strong to offset the pro-competitive effect on domestic markups. My model introduces oligopolistic competition as in Impullitti and Licandro (2018) and Impullitti et al. (2018) and extends it with horizontal FDI to the framework with firm heterogeneity as in Helpman et al. (2004), aiming to explore the welfare gains from multinational production.

The third strand of the literature shows some results regarding the welfare effect of multinational firms (e.g., Ahn, 2014; Ramondo, 2014; Ramondo and Rodríguez-Clare, 2013). Ramondo and Rodríguez-Clare (2013) explore the welfare effects of trade and FDI and capture key dimensions of the interaction between these two flows. One way of their interaction is that international trade and MP via horizontal FDI behave as competing methods to serve a foreign market. (e.g., Helpman et al., 2004; Markusen and Venables, 2000). Much attention has been paid to quantifying the gains from single mechanisms in isolation, especially trade in goods and, to a lesser extent, foreign direct investment. They build on the Ricardian model of international trade developed by Eaton and Kortum (2002) and incorporate MP into the model by allowing a country's technologies to be used for production abroad. They quantify the overall gains from openness and conclude that the welfare from trade is twice as high as in trade-only models, while the gains from MP are slightly lower than gains in MP-only models. Compared to their model, I apply the constant elasticity of substitution (CES) utility function in a monopolistic market, abandon the constant returns to scale in the perfectly competitive market and introduce oligopolistic competition for several firms within each variety. This is motivated by the significance of strategic interaction among firms through oligopolistic competition (e.g., Edmond et al., 2015; Impullitti and Licandro, 2018; Impullitti et al., 2022). However, these papers do not consider multinational firms via horizontal FDI with firm heterogeneity, while my thesis does. Another deviation is that I compare the welfare gains from three individual scenarios in which all firms are non-exporters, all operating firms export, and all potential firms serve the foreign market via horizontal FDI. I do not consider their interactions in this chapter, which will be explored in the third chapter. Ahn (2014) builds a theoretical model and examines the pro-competitive channel of horizontal FDI on welfare gains with firm heterogeneity. The paper finds welfare gains for the source country and welfare decreases for the host country due to the production reallocation leading to the increase of domestic firms in the home country and a decrease in the host country. Like this paper, I consider firm heterogeneity and find the largest pro-competitive effect of horizontal FDI while including oligopolistic competition.

In this chapter, I do not consider the free entry condition, which will endogenise the number of oligopolistic firms competing in each product line for simplicity. However, like Melitz (2003) and its extension of Helpman et al. (2004), I set different types of fixed costs, for example, the fixed costs of production for non-exporters, exporters and multinational firms, separately. My model also applies variable iceberg transportation costs to trade via exports.

2.3 The Model

I now outline the theoretical model, beginning with a simple benchmark case with restricted entry and autarky. I then extend this to the case of exporting with no associated fixed cost and analyse the case where all viable firms export. Key to solving and analysing the model in Melitz (2003) are the zero cutoff profit condition (ZCP) and free entry condition (FE). He pins down these conditions assuming the mass of producing firms in equilibrium is $M > 1$. In contrast, here, entry is restricted, but there is no entry cost, a continuum of varieties has endogenous mass $M \in (0, 1)$ and the productivity cutoff threshold is pinned down with an exit condition (EC) and labour market clearing condition (MC), as in Impullitti and Licandro (2018). Notice that Melitz (2003) uses a price index to arrive at the welfare effect while Impullitti and Licandro (2018) do not since they apply a different inverse demand function to deal with the model including the differentiated good according to X (see Eq. (2.2), below).

2.3.1 Preferences and Demand

Consider an economy with two identical countries; each is composed of a continuum of individuals of measure 1. In each country, there are $H + 1$ final goods. Good 1 is a homogeneous good while each of the H other goods is differentiated. More precisely, in each sector $h \in H$, there is a continuum of varieties, $v \in V_h$, where V_h is the set of all potential varieties in sector h. Utility is given by a standard Cobb-Douglas utility function with homogeneous goods and composite goods characterized by a constant elasticity of substitution (CES) utility function. The following utility function shows preferences across goods:

$$U = \left(1 - \sum_{h=1}^{H} \gamma_h\right) \ln Y + \sum_{h=1}^{H} \frac{\gamma_h}{\alpha_h} \ln \left(\int_{v \in V_h} x_h(v)^{\alpha_h} dv\right) \tag{2.1}$$

where Y is consumption of the homogeneous good and $x_h(v)$ is consumption of heterogeneous variety v from sector h. Notice that equivalent to V_h, a continuum of variety of endogenous mass $M \in (0,1)$ is assumed in sector h. $\alpha_h = \frac{\sigma_h - 1}{\sigma_h}$ controls for the elasticity of substitution (σ_h) between any two varieties in sector h, where $\alpha_h \in (0,1)$.[1] Let:

$$X_h = \left(\int_0^M x_h(v)^{\alpha_h} dv \right)^{\frac{1}{\alpha_h}} \tag{2.2}$$

Consumers maximize utility subject to the following budget constraint:

$$Y + \sum_{h=1}^{H} \int_{v \in V_h} p_h(v) x_h(v) dv = E \tag{2.3}$$

where the homogeneous good, Y, is considered to be the numéraire, E denotes total expenditure and consumers are endowed with a unit flow of labour. The consumers spend a fraction γE of their income on composite goods and $(1-\gamma)E$ on homogeneous goods from sector h. Solving this utility maximization problem yields the following inverse demand function for each variety, v, from a particular sector h:

$$p_h(v) = \frac{\gamma_h E}{X_h^{\alpha}} x_h(v)^{\alpha - 1} \tag{2.4}$$

where $Y = (1 - \bar{\gamma})E$ and $\bar{\gamma} = \sum_{h=1}^{H} \gamma_h$. For simplicity, henceforth, we omit the subscript h.

2.3.2 Production and Firm Behaviour

There is a unique production factor, labour L. The homogeneous goods Y are treated as numéraire. It is produced in perfect competition using a linear technology that involves using one unit of labour to obtain one unit of output. Given that good Y is the numéraire (its price equals 1) and perfect competition, implying a unit wage: $w = 1$. Besides, the homogeneous goods Y are also assumed to be freely traded with no additional costs, which implies that wages are equal across countries in the presence of homogeneous goods in both countries. Each of the differentiated goods is also produced with a linear technology whose cost function is given by:

$$C(z) = l(z) = z^{\frac{\alpha - 1}{\alpha}} q(z) + \lambda_d \tag{2.5}$$

where, z denotes firm productivity (which varies across varieties and is assigned to the firm following the mechanism explained below), $q(z)$ is firm output, $C(z)$ is total

[1] Note that the utility function presented here, Eq. (2.1), is a monotonic transformation of the following utility function: $U = Y^{(1 - \sum_{h=1}^{H} \gamma_h)} \prod_{h=1}^{H} X_h^{\gamma_h}$.

cost and $l(z)$ is the total amount of labour used in production. In order to produce output, the firm must incur a fixed cost, λ_d.

In this simple set-up, I assume there is restricted entry. Before entry, firm productivity is unknown. More precisely, the productivity, z, for all firms producing a given variety in a sector is drawn from a continuous Pareto productivity distribution with a distribution function given by the following expression:

$$G(z) = 1 - \left(\frac{\underline{z}}{z}\right)^k = 1 - z^{-k}, \quad z \geq 1, \quad k \geq 1 \tag{2.6}$$

with lower productivity bound, $\underline{z} = 1$, and shape parameter, k, measures the inverse of the heterogeneity, which implies a high k is more homogeneous, like Chaney (2008).

In the restricted entry model, I assume that a firm generates a specific variety with their direct competitors participating in Cournot competition in each sector h. Before entry, the productivity of a variety with identical firms is unknown, and firms within each new variety enter until expected profits are zero, which means the number of firms within each variety n is the same for all of these varieties. It allows strategic interaction but abstracts from non-trivial complications associated with firm number endogeneity. These firms observe their variety's productivity draw and stay if expected profit is positive with $n-1$ rivals in their variety. Otherwise, the firm (and all $n-1$ rivals) will exit. Notice that n identical firms always exit simultaneously within a variety, which means a variety with n firms disappears if only one firm within the variety chooses to exit. Firms in a surviving variety compete *à la* Cournot.

Finally, I assume firms face the risk of death at each point in time, as in Melitz (2003). However, here, rather than firms facing this risk individually, I assume that variety v in sector h faces this risk collectively. Hence, all firms in variety v of sector h face a bad shock that leads to exit with probability, δ. Moreover, this probability is common across varieties in all sectors despite the different productivity of the variety.

The following subsections set the benchmark case where all viable varieties are traded only in a firm's domestic market. I then introduce the case where all firms export without fixed export costs. I finally generalise the model to include exporting fixed costs.

2.3.3 Autarky

In the domestic scenario, the firms will be able to serve only their domestic market, and I assume there are only non-exporters in a two-country economy with symmetry. A firm manufacturing a non-exported variety with productivity level z will maximize its profits subject to the inverse demand function in Eq. (2.4), taking the quantities their competitors produce as given since firms producing the same variety play a symmetric Cournot game and behave non-cooperatively. The firm's problem is:

$$\max_{q_a} \pi_a = \underbrace{\frac{\gamma E}{X^\alpha}(\hat{x}_a + q_a)^{\alpha-1}}_{p_a} q_a - z^{\frac{\alpha-1}{\alpha}} q_a - \lambda_d \tag{2.7}$$

where subscript a indicates that firms produce a domestic, non-exported variety, that is, firms only serve the domestic market. Here, q_a is the firm's production and $\hat{x}_a$ is the production of its direct competitors within a variety with the same productivity level, z. Total output or consumption for a variety in the domestic country is therefore $x_a = q_a + \hat{x}_a$. Since labour is the numéraire, the first order condition for a firm is:

$$\frac{\gamma E}{X^\alpha}((\alpha - 1)(\hat{x}_a + q_a)^{\alpha-2} q_a + (\hat{x}_a + q_a)^{\alpha-1}) = z^{\frac{\alpha-1}{\alpha}} \tag{2.8}$$

Given symmetry, the equilibrium is such that $x_a = nq_a$, hence Eq. (2.8) can be written:

$$\underbrace{\frac{\gamma E}{X^\alpha} x_a^{\alpha-1}}_{p_a} \left(\frac{(\alpha - 1) q_a}{n q_a} + 1 \right) = z^{\frac{\alpha-1}{\alpha}} \tag{2.9}$$

Simplifying and rearranging Eq. (2.9), I have:

$$p_a(z, n) = \frac{z^{\frac{\alpha-1}{\alpha}}}{\theta_a} \tag{2.10}$$

and

$$x_a(z, n) = \left(\frac{\gamma E}{X^\alpha} \right)^{\frac{1}{1-\alpha}} \theta_a^{\frac{1}{1-\alpha}} z^{\frac{1}{\alpha}} \tag{2.11}$$

where, θ_a represents the inverse of the markup of a domestic firm that cannot export:

$$\theta_a \equiv \frac{n - 1 + \alpha}{n} \tag{2.12}$$

Let $\bar{z}$ and e denote, respectively, the average productivity and the expenditure per differentiated firm for sector h:

$$\bar{z} = (1/M) \int_0^M z_v dv \tag{2.13a}$$

$$e \equiv \frac{\gamma E}{nM} \tag{2.13b}$$

The following expression shows a non-exporter's variable production cost:[2]

$$l_a(z,n) - \lambda_d = z^{\frac{\alpha-1}{\alpha}} q_a(z,n) = e\theta_a \left(\frac{z}{\bar{z}}\right) \tag{2.14}$$

where l_a is the labour of a non-exporter devoted to producing goods for their market.

Entry and Exit

There is a unit mass of potential varieties $M \in (0,1)$. That is to say, non-operative new varieties $1 - M$ can be identified as produced by n firms trying to enter the economy at zero cost, where associated productivity z is jointly drawn for each of them from a Pareto distribution $G(z)$ with lower productivity bound $z = 1$. Moreover, an exogenous death shock δ can cause all firms to exit the market.

In order to establish the productivity cutoff point, I begin with equilibrium profit for a non-exporter, with productivity z given average productivity $\bar{z}$. Using Eqs. (2.10), (2.11) and (2.14), firm profit can be expressed as:

$$\pi_a\left(z/\bar{z}\right) = p_a q_a - l_a = e(z/\bar{z})(1-\theta_a) - \lambda_d \tag{2.15}$$

With restricted entry and no entry cost, the exit condition requires:

$$\pi_a(z_a^*/\bar{z}_a) = e_a(z_a^*/\bar{z}_a)(1-\theta_a) - \lambda_d = 0 \tag{2.16}$$

The productivity cutoff in the domestic case is z_a^*, such that if $z \geqslant z_a^*$ all firms with productivity z stay in the market, and otherwise, they all leave the market. This productivity cutoff is therefore described by the following condition, rearranging Eq. (2.16):

$$e_a\left(1-\theta_a\right)\left(z_a^*/\bar{z}_a\right) = \lambda_d \tag{2.17}$$

Note, in equilibrium I obtain that:

$$\bar{z}_a(z_a^*) = \int_{z_a^*}^{\infty} z\mu(z)dz \tag{2.18}$$

where:

$$\mu(z) = \begin{cases} \frac{g(z)}{1-G(z_a^*)} & \text{if } z \geqslant z_a^*, \\ 0 & \text{otherwise,} \end{cases} \tag{2.19}$$

It follows that I can express $\bar{z}_a$ in terms of z_a^*:[3]

$$\bar{z}_a = \frac{k}{(k-1)} z_a^* \tag{2.20}$$

[2] See Appendix A.1.
[3] See Appendix A.2.

Using Eq. (2.20) in Eq. (2.17) I have the following expression for the Exit Condition under autarky EC_a:

$$e_a = \frac{\lambda_d}{(1-\theta_a)z_a^*/\bar{z}_a} = \frac{k\lambda_d}{(k-1)(1-\theta_a)} \quad \boldsymbol{(EC_a)} \tag{2.21}$$

Note that under the Pareto distribution, the EC_a condition is independent of the productivity cutoff. If we draw the graph with a horizontal axis representing z and a vertical axis showing e, Eq. (2.21) is expressed as a horizontal line in this graph. That is, Eq. (2.21) determines the expenditure per differentiated firm e as a function of the tail parameter of the entry distribution k, the fixed production cost λ_d and the equilibrium markup θ_a, where θ_a depends on the elasticity of substitution across varieties and the number of firms n per product line. Eq. (2.21) can also be explained as $z_a^*/\bar{z}_a$ is a constant under the Pareto distribution and only relates to the shape parameter of the Pareto productivity distribution, k.

I now derive the Labour Market Clearing condition under autarky, MC_a, which will yield a relationship between e_a and z_a. In the steady state, I assume that the total number of non-exporters in the market remains constant over time. For this to happen, the following condition must hold:

$$(1-M)(1-G(z_a^*)) = \delta M. \tag{2.22}$$

This condition states that the exit flow, δM, equals the entry flow defined by the number of potential new varieties, $(1-M)$, times the probability of surviving, $1-G(z_a^*)$. Consequently:

$$M(z_a^*) \equiv \frac{1-G(z_a^*)}{1+\delta-G(z_a^*)} = \frac{z_a^{*-k}}{z_a^{*-k}+\delta} \tag{2.23}$$

Note, Eq. (2.23) describes a decreasing relationship between M and the productivity cutoff z_a^*, given $M \in (0, 1/(1+\delta))$. Since this relationship between M and z is not dependent on a specific market or trading conditions in this model, it gives rise directly to the following general result:

Lemma 2.1. *The equilibrium mass of produced varieties, M, is strictly decreasing in the productivity cutoff, z^*, for $M \in \left(0, \frac{1}{(1+\delta)}\right)$.*

Lemma 2.1 is in line with Impullitti and Licandro (2018), which considers the condition for endogenous variety and results in the decreasing relationship between the mass of operative variety and the productivity threshold, while the number of entrants responds endogenously to the changes in variable transportation costs as in Melitz (2003) and Atkeson and Burstein (2010).

I now set out the labour market clearing condition, which can be written:

$$nM\left[\int_{z_a^*}^{\infty}\left(e\theta_a\left(z_a/\bar{z}_a\right)+\lambda_d\right)\mu(z)dz\right]+(1-\gamma)E=1 \tag{2.24}$$

The first element on the left-hand side of Eq. (2.24) indicates the total amount of labour devoted to the differentiated sector. In contrast, the second element is the total amount of labour devoted to the homogeneous sector. Since:

$$\int_{z_a^*}^{\infty}\mu(z)dz=\int_{z_a^*}^{\infty}\left(z_a/\bar{z}_a\right)\mu(z)dz=1 \tag{2.25}$$

after integrating over all varieties and using Eqs. (2.13b) and (2.25) in Eq. (2.24), I have:

$$nM(z_a^*)(e_a\theta_a+\lambda_d+\frac{e_a}{\gamma}(1-\gamma))=1$$

Rearranging I obtain the following condition:

$$e_a(z_a^*)=\frac{\frac{1}{nM(z_a^*)}-\lambda_d}{\theta_a+\frac{1-\gamma}{\gamma}}\quad \boldsymbol{(MC_a)} \tag{2.26}$$

Eq. (2.23) established that $M(z_a^*)$ is a decreasing function of z_a^*. Hence, Eq. (2.26), which is the Market Clearing condition under autarky, establishes an increasing relationship between e and z_a^*. That is to say, the MC_a condition describes an increasing line in the graph of (e, z_a^*), with the decreasing number of $M(z_a^*)$.

Note, in order to obtain the equilibrium productivity threshold, the following is assumed $\lambda_d \geqslant \bar{\lambda_a}$[4], where:

$$\bar{\lambda_a}=\frac{1+\delta}{n\left(1+\frac{k}{k-1}\frac{\theta_a+\frac{1-\gamma}{\gamma}}{1-\theta_a}\right)}$$

This assumption can be derived from the consideration of (EC$_a$)$\geqslant (MC_a)$ at the $z_{\min}=1$. It is because there are two lines in the graph of (e, z_a^*), a horizontal line of (EC_a) and an increasing line of (MC$_a$), by imposing the condition of (EC$_a$)$\geqslant (MC_a)$ at the $z_{\min}=1$, I can solve the unique intersection of two lines. At the extreme case, I can derive $z_{\min}=z_a^*=1$ and $(EC_a)=(MC_a)$.

The productivity cutoff under autarky is obtained by equating EC_a and MC_a. Setting (EC_a) equal to (MC_a), using Eqs. (2.21) and (2.26) and substituting $M(z_a^*)$

[4] $(EC_a)\geqslant(MC_a)$ at the minimum $z_a^*=1$,

$$\frac{k\lambda_d}{(1-\theta_a)(k-1)}\geqslant\frac{\frac{z_a^{*-k}+\delta}{nz_a^{*-k}}-\lambda_d}{\theta_a+\frac{1-\gamma}{\gamma}}$$

using Eq. (2.23) I get:

$$\frac{k\lambda_d}{(1-\theta_a)(k-1)} = \frac{\frac{z_a^{*-k}+\delta}{nz_a^{*-k}} - \lambda_d}{\theta_a + \frac{1-\gamma}{\gamma}}$$

Rearranging gives the following expression for the autarky productivity threshold:

$$z_a^* = \left(\frac{n\lambda_d(1 + \frac{k}{k-1}\frac{\theta_a + \frac{1-\gamma}{\gamma}}{1-\theta_a}) - 1}{\delta} \right)^{1/k} \tag{2.27}$$

Lemma 2.2. *Under the assumption $\lambda_d \geqslant \bar{\lambda}_a$, there exists a unique interior solution $(e_a,\ z_a^*)$ at the intersection of MC_a and EC_a, with $M(z_a^*)$ determined by Eq.* (2.23).

Proof. See Appendix A.3.

The technique of seeking the equilibrium productivity cutoff threshold is similar to Impullitti and Licandro (2018), which emphasizes the case where all are exporters. In my chapter, I incorporate the scenario where all are non-exporters to extend the discussion. Specifically, I pin down the exit condition (EC) and the labour market condition (MC) to derive equilibrium variables in the steady state, the productivity cutoff and average expenditure per heterogeneous firm in a case in which all are exporters in two-symmetric countries. However, here I investigate the scenario where all are non-exporters with similar methods and find a unique interior solution of the equilibrium productivity cutoff and average expenditure per firm for sector h, under a certain constraint on the fixed production costs, λ_d.

2.3.4 Trade Openness

In this section, I analyze the scenario in which all the domestic firms can serve the foreign market via exports. It implies that the cost of opening a plant in a foreign market is so large that no firm would like to do this, compared to the scenario where all are multinational firms via FDI in the following section. This will allow us to compare the welfare implications of opening up to trade and multinational production in the welfare analysis. When the firm serves the foreign market via exports, it must bear a transportation cost, τ, $(\tau \geq 1)$, of the 'iceberg' type, which means τ units of a good must be shipped to obtain one unit at the destination. For exporting, I consider two subcases: the scenario where there is no fixed exporting cost and then one where exporting firms bear a fixed cost of $\lambda_x \leqslant \lambda_d$.

Under the Cournot assumption, firms behave non-cooperatively and maximize profits subject to the inverse demand function in Eq. (2.4), taking competitor output in their variety as given. As a result, exporters compete simultaneously in domestic and foreign markets, which are referred to by subindices d_x and f_x, respectively, and they treat each market as segmented.

Let q_{d_x} and q_{f_x} denote the quantities sold by a domestic firm in the domestic and foreign market, respectively, and p_{d_x} and p_{f_x} denote the associated prices. Hence, total quantity produced by a firm is given by $q_x = q_{d_x} + \tau q_{f_x}$. Note, this includes $(\tau - 1)q_{f_x}$ which is the amount of output that the firm needs to produce to bear the 'iceberg' transportation costs. Let $\hat{x}_{d_x}$ and $\hat{x}_{f_x}$ denote the quantities sold by a firm's competitors in the domestic and foreign markets, respectively. Note, in the domestic market, under exporting, a domestic firm's rivals include domestic competitors and foreign firm exporters of the same variety. Let x_{d_x} and x_{f_x} denote the total quantities sold in the separate domestic and foreign markets for a traded variety, v. If all firms export, there will be $2n$ firms serving any traded variety in the domestic and foreign markets. Firms, in the 'all exporting' scenario, solve the problem:

$$\max_{q_{d_x}, q_{f_x}} \pi_x = \underbrace{\frac{\gamma E}{X^\alpha}(\hat{x}_{d_x} + q_{d_x})^{\alpha-1}}_{p_{d_x}} q_{d_x} + \underbrace{\frac{\gamma E}{X^\alpha}(\hat{x}_{f_x} + q_{f_x})^{\alpha-1}}_{p_{f_x}} q_{f_x} - z^{\frac{\alpha-1}{\alpha}}(q_{d_x} + \tau q_{f_x}) - \lambda_d \quad (2.28)$$

which yields the following first-order conditions for the domestic and foreign markets, respectively:

$$\frac{\gamma E}{X^\alpha}((\alpha - 1)(\hat{x}_{d_x} + q_{d_x})^{\alpha-2} q_{d_x} + (\hat{x}_{d_x} + q_{d_x})^{\alpha-1}) = z^{\frac{\alpha-1}{\alpha}} \quad (2.29)$$

$$\frac{\gamma E}{X^\alpha}((\alpha - 1)(\hat{x}_{f_x} + q_{f_x})^{\alpha-2} q_{f_x} + (\hat{x}_{f_x} + q_{f_x})^{\alpha-1}) = \tau z^{\frac{\alpha-1}{\alpha}} \quad (2.30)$$

Given competition is of the Cournot type; each firm takes as given the output of its competitors in variety v of sector h, $\hat{x_x}$, but also aggregate expenditure, E, and aggregate consumption of heterogeneous varieties X in each sector. Country symmetry implies: $E_{d_x} = E_{f_x} = E$, $X_{d_x} = X_{f_x} = X$. Further, since all firms producing the same variety are identical, denote the aggregate output of firms in given a variety under 'all exporting', by $x_x = x_{d_x} = x_{f_x} = n(q_{d_x} + q_{f_x})$, and associated price $p_x = p_{d_x} = p_{f_x}$.

Applying symmetry, $x_{d_x} = x_{f_x} = x_x = n(q_{d_x} + q_{f_x})$, to the ratio between Eqs. (2.29) and (2.30):

$$\frac{\frac{\gamma E}{X^\alpha}((\alpha - 1)(\hat{x}_{d_x} + q_{d_x})^{\alpha-2} q_{d_x} + (\hat{x}_{d_x} + q_{d_x})^{\alpha-1})}{\frac{\gamma E}{X^\alpha}((\alpha - 1)(\hat{x}_{f_x} + q_{f_x})^{\alpha-2} q_{f_x} + (\hat{x}_{f_x} + q_{f_x})^{\alpha-1})} = \frac{z^{\frac{\alpha-1}{\alpha}}}{\tau z^{\frac{\alpha-1}{\alpha}}}$$

from which it follows:

$$q_{d_x} = \frac{(\alpha - 1) + (1 - \tau)n}{\tau(\alpha - 1) + (\tau - 1)n} q_{f_x} \quad (2.31)$$

It is useful to refer, henceforth, to the following:

$$\beta \equiv \frac{q_{f_x}}{q_{d_x}} = \frac{\tau(n+\alpha-1)-n}{n+\alpha-1-n\tau} \tag{2.32}$$

where:

$$\frac{\partial\beta}{\partial\tau} = \frac{(2n+\alpha-1)(\alpha-1)}{(n+\alpha-1-n\tau)^2} < 0$$

Analysis of β reveals several notable properties. First, $\beta \in [0,1]$, hence, the quantity supplied by a domestic firm to the foreign market is smaller or equal to that it supplies in the domestic market. This comes from the fact that the marginal cost of a domestic firm serving the foreign market is, under an 'iceberg' cost, weakly greater than one of its foreign competitor firms in the overseas market with equality when $\tau = 1$, so transportation costs are zero and the firm supplies the same output in both markets.

Second, from Eq. (2.32) it follows that the prohibitive level of trade costs, associated with $\beta = 0$, is given by:

$$\bar{\tau} = \frac{n}{n+\alpha-1} \tag{2.33}$$

Hence, for $\tau \in (1, \bar{\tau})$ then $\beta > 0$. At the extreme case, variable trade costs are at the prohibitive level $\tau = \bar{\tau}$, then $q_{f_x} = 0$ and $\beta = 0$. For $\tau > \bar{\tau}$, the markup in foreign markets turns negative, and firms will not export.

Applying symmetry to the first order conditions of the firm in Eqs. (2.29) and (2.30), I obtain the equilibrium (symmetric) price in the domestic and foreign markets:

$$p_x(z,n) = \frac{z^{\frac{\alpha-1}{\alpha}}}{\theta_{d_x}} = \frac{\tau z^{\frac{\alpha-1}{\alpha}}}{\theta_{f_x}} \tag{2.34}$$

where:

$$\theta_{d_x} = \frac{2n+\alpha-1}{n(1+\tau)}, \quad \theta_{f_x} = \tau\theta_{d_x}, \tag{2.35}$$

and θ_{d_x} and θ_{f_x} represent the inverse of the markups of a domestic firm in the domestic and foreign markets, respectively. Note also that firms in this model will, accordingly, charge a higher markup on their domestic sales for $\tau > 1$. This is the result of the fact that the marginal cost of serving the foreign market under $\tau > 1$ is higher while the equilibrium prices in the domestic and foreign markets are the same.

The derivatives of θ_{d_x}, θ_{f_x} with respect to $\tau \in (1, \bar{\tau})$ are:

$$\frac{\partial\theta_{d_x}}{\partial\tau} = \frac{\partial\frac{2n+\alpha-1}{n(1+\tau)}}{\partial\tau} = -\frac{(2n+\alpha-1)}{n(1+\tau)^2} = -\frac{\theta_{d_x}}{(1+\tau)} < 0$$

$$\frac{\partial \theta_{f_x}}{\partial \tau} = \frac{\partial(\tau\theta_{d_x})}{\partial \tau} = \tau\frac{\partial \theta_{d_x}}{\partial \tau} + \theta_{d_x} = \frac{-\tau\theta_{d_x} + \theta_{d_x}(1+\tau)}{(1+\tau)} = \frac{\theta_{d_x}}{(1+\tau)} > 0$$

Hence, lowering the trade cost, τ, leads to an increase in θ_{d_x} because the domestic market becomes more competitive due to the penetration of foreign firms, as in Impullitti and Licandro (2018). This pro-competitive effect, consistent with the empirical evidence, will have significant implications for our welfare analysis. Besides, a reduction in trade costs, τ, induces higher markups on a firm's foreign sales, $1/\theta_{f_x}$, since exporters enjoy the benefit of cost reduction in their shipment while domestic firms do not. The claim can be found in Impullitti et al. (2018) as well.

Based on Eq. (2.32), I can express the ratio of total production to total consumption of a domestic firm within an exported variety (in line with Brander and Krugman, 1983), as Φ:

$$\Phi \equiv \frac{q_{d_x} + \tau q_{f_x}}{q_{d_x} + q_{f_x}} = \frac{(1-n-\alpha)(1+\tau^2) + 2n\tau}{(1-\alpha)(1+\tau)} = \frac{1+\tau\beta}{1+\beta} \geq 1 \tag{2.36}$$

Φ captures the losses related to trade due to the iceberg cost, which means the cost of importing goods that could be otherwise produced locally. It is straightforward to see that Φ is hump-shaped in τ and strictly greater than one, for $\tau \in (1, \bar{\tau})$, and equals one in the extreme cases of free trade, $\tau = 1$, and the prohibitive trade cost, $\bar{\tau}$. Intuitively, when the iceberg trade costs are at a prohibitive level, export sales, q_{f_x}, are zero, a special case of autarky: no share of production is wasted in transportation, implying $\Phi = 1$. A reduction in the iceberg cost results in firms having incentives to export and reduce domestic sales. Consequently, the losses associated with trade costs become positive and Φ increases above one. At the other extreme of free trade, there is no waste in transportation, that is, $\Phi = 1$, and any increase in trade cost increases Φ above one.

I define the inverse of an exporting firm's average markup and derive it as an expression in terms of Φ using Eqs. (2.34) and (2.36):

$$\theta_x \equiv \frac{q_{d_x}\theta_{d_x} + q_{f_x}\theta_{f_x}}{q_{d_x} + q_{f_x}} = \Phi\theta_{d_x} \tag{2.37}$$

Note that θ_x is a weighted average of the respective inverse markups for a firm in the domestic and foreign markets. The firm's average markup captures the ratio of total revenue to variable production costs. When iceberg trade costs are prohibitive, $\bar{\tau} = n/(n+\alpha-1)$, $q_{f_x} = 0$ and this average collapses to $\theta_x = \theta_{d_x} = (n+\alpha-1)/n$. When trade is costless ($\tau = 1$), I find, $\theta_x = \theta_{d_x} = (2n+\alpha-1)/2n$, since $\theta_{f_x} = \theta_{d_x}$. I also identify that θ_x is decreasing in τ for $\tau \in (1, \bar{\tau})$:

$$\frac{\partial \theta_x}{\partial \tau} = -\frac{2(\tau-1)(2n-1+\alpha)^2}{n(1+\tau)^3(1-\alpha)} = -\frac{2n(\tau-1)\theta_{d_x}^2}{(1-\alpha)(1+\tau)} < 0 \tag{2.38}$$

This means trade liberalisation decreases a firm's average markup on total sales. Intuitively, with regard to decreasing variable transportation costs τ, I can explain that the decrease in the markup of an exporter in the domestic market is sufficiently substantial to offset the increase of the markup in the foreign market. This leads to an overall pro-competitive effect of trade liberalisation.

Using Eqs. (2.4) and (2.34), I find that equilibrium output in a country for a given variety under 'all exporting' and zero exporting fixed cost is:

$$x_x(z,n) = \left(\frac{\gamma E}{X^\alpha}\right)^{\frac{1}{1-\alpha}} \theta_{d_x}^{\frac{1}{1-\alpha}} z^{\frac{1}{\alpha}} \tag{2.39}$$

The following expression is an exporting firm's variable production cost:[5]

$$l_x(z,n) - \lambda_d = z^{\frac{\alpha-1}{\alpha}} \left(q_{d_x}(z,n) + \tau q_{f_x}(z,n)\right) = e\theta_x(z/\bar{z}) \tag{2.40}$$

where l_x is the labour of an exporting firm devoted to the production of goods for both domestic and foreign markets.

Entry and Exit

Analogous to the case under autarky, the cutoff productivity z, with all firms exporting and zero exporting fixed cost, is determined by the exit condition. Using Eqs. (2.4), (2.39) and (2.40), firm profit in the all exporting with no exporting fixed cost case is:

$$\begin{aligned} \pi_x\left(z/\bar{z}\right) &= p_x(q_{d_x} + q_{f_x}) - z^{\frac{\alpha-1}{\alpha}}(q_{d_x} + \tau q_{f_x}) - \lambda_d \\ &= e\left(1-\theta_x\right)\left(z/\bar{z}\right) - \lambda_d \end{aligned} \tag{2.41}$$

Eq. (2.41) defines the operating profits of the firm as a function of two endogenous variables, $\bar{z}$ and e. Consider the case of the variety whose firms just break even in the market and denote their productivity with z_x^*. The condition defining this productivity threshold is then given by:

$$\pi_x(z_x^*/\bar{z}_x) = e_x\left(1-\theta_x\right)\left(z_x^*/\bar{z}_x\right) - \lambda_d = 0 \tag{2.42}$$

Note that in equilibrium I obtain that:

$$\bar{z}_x = \int_{z_x^*}^{\infty} z\mu(z)dz \tag{2.43}$$

[5] This follows directly from the derivation in Appendix A.1 under autarky with θ_x replacing θ_a.

where:

$$\mu(z) = \begin{cases} \frac{g(z)}{1-G(z_x^*)} & \text{if } z \geqslant z_x^*, \\ 0 & \text{otherwise,} \end{cases} \tag{2.44}$$

Similar reasoning that produced Eq. (2.20) under autarky results in the following holding under all exporting with no exporting costs:

$$\bar{z}_x = \frac{k}{(k-1)} z_x^* \tag{2.45}$$

Using Eq. (2.45) in Eq. (2.42) gives the following expression for the Exit Condition under all exporting with no exporting fixed cost:

$$e_x = \frac{\lambda_d}{(1-\theta_x) z_x^* / \bar{z}_x} = \frac{k\lambda_d}{(k-1)(1-\theta_x)} \quad \boldsymbol{(EC_x)} \tag{2.46}$$

Note that under the Pareto distribution, the EC_x condition is independent of the productivity cutoff. I now use the condition for entrants together with the labour market condition to find another relationship between e_x and z_x^*.

Analogous to the case of autarky, preserving a steady state number of firms in the context of entry and exit of varieties, the following condition must hold $(1-M)(1-G(z_x^*)) = \delta M$ and hence:

$$M(z_x^*) \equiv \frac{1-G(z_x^*)}{1+\delta - G(z_x^*)} \tag{2.47}$$

Note, as per Lemma 2.1, Eq. (2.47) describes a decreasing relationship between M and the productivity cutoff z_x^* for $M \in (0, \frac{1}{(1+\delta)})$.

In line with the case of autarky, the labour market clearing condition can be written as:

$$e_x(z_x^*) = \frac{\frac{1}{nM(z_x^*)} - \lambda_d}{\theta_x + \frac{1-\gamma}{\gamma}} \quad \boldsymbol{(MC_x)} \tag{2.48}$$

Above, I established that $M(z_x^*)$ is a decreasing function of z_x^*. Eq. (2.48) implies an increasing relationship between e_x and z_x^*.

Analogous to the case of autarky, in order to obtain an equilibrium productivity cutoff, the following is assumed $\lambda_d \geqslant \bar{\lambda}_x$ where:

$$\bar{\lambda}_x = \frac{1+\delta}{n\left(1 + \frac{k}{k-1}\frac{\theta_x + \frac{1-\gamma}{\gamma}}{1-\theta_x}\right)} \tag{2.49}$$

Eq. (2.49) can be derived from the consideration that at the $\min z = 1$, $(EC_x) \geqslant (MC_x)$. Equating EC_x and MC_x, and manipulating in the same way as with autarky, the productivity cutoff is given by:

$$z_x^* = \left(\frac{n\lambda_d (1 + \frac{k}{k-1} \frac{\theta_x + \frac{1-\gamma}{\gamma}}{1-\theta_x}) - 1}{\delta} \right)^{1/k} \tag{2.50}$$

Similar to Lemma 2.2 in the case of autarky, here, in the scenario in which all are exporters with zero export fixed cost, I can claim that, under the assumption of the fixed production costs, $\lambda_d \geqslant \bar{\lambda_x}$, for $\tau \in (1, \bar{\tau})$, there exists a unique interior solution of average expenditure per differentiated exporting firm e_x and equilibrium productivity cutoff z_x^* for sector h.

It is useful to note the following result:

Lemma 2.3. $\frac{\theta + \frac{1-\gamma}{\gamma}}{1-\theta}$ *is strictly increasing in* θ *for* $\theta \in (0, 1)$*, and hence,* z^* *is strictly increasing with respect to its associated* θ *in that interval.*

In lemma 2.3, the component related to the variable markups in the productivity threshold at equilibrium can be used to identify how the change in markups will affect the productivity threshold and furthermore the welfare implications for firms.

The following Proposition 2.1 sets out some comparable properties of the model across the different scenarios considered thus far.

Proposition 2.1. *(i)* $\alpha \leq \theta_a \leq \theta_{d_x} \leq \theta_x \leq \theta_{f_x} \leq 1$ *and hence firm mark-ups are weakly highest under autarky and weakly lowest for exported goods, and (ii) productivity cutoffs satisfy* $z_x^* \geq z_a^*$ *and hence selection is greater under all exporting.*

Proof. See Appendix A.4.

Inspection of Eqs. (2.37), (2.47) and (2.50) gives rise to the following Proposition 2.2, which establishes some properties of the all exporting equilibrium with respect to trade liberalisation.

Proposition 2.2. *Under all firms exporting with zero fixed exporting cost, a reduction in* τ *(trade liberalisation) (i) reduces the domestic markup for a domestic firm within a variety,* $1/\theta_{d_x}$*, (ii) increases its markup in the foreign market,* $1/\theta_{f_x}$*, (iii) triggers a reduction in the average markup* $1/\theta_x$*, (iv) raises the equilibrium productivity cutoff* z_x^**, and (v) reduces the number of operative varieties* $M(z_x^*)$*.*

For an exogenous number of firms, n, within a variety for each country in the basic setup, a reduction in τ decreases exporters' markups on domestic sales, increases markups on foreign sales and decreases the average markup on total sales. In other words, although the export markup increases, it is not sufficiently strong enough to offset the pro-competitive effect on the domestic market since the average markup decreases when considering trade liberalization. Notice that the pro-competitive effect of trade operating through the increased θ_{d_x}, decreases the markup on domestic sales. This results in increased θ_x, which decreases the average markup of exporters due to a reduction in trade cost, τ. In conclusion, when the number of firms is exogenous, there is an overall pro-competitive effect of trade in an economy with the oligopolistic competition. The statement is in line with Impullitti et al. (2018), although they consider an endogenous number n within each variety and pick up the different entry strategy compared to my model. Specifically, in their paper, a more sophisticated entry structure, where entry strategy is focused on a particular product line, is applied to pin down the number of firms in each variety. Therefore, different varieties could produce different markups. There exist pro-competitive effects and selection effects of trade liberalization in the global economy, which lead to an increase in the equilibrium productivity cutoff z_x^* and a decrease in the mass of operative varieties, as shown in Eq. (2.47). Here, different from the Melitz model, the number of entrants can respond endogenously to changes in trade costs, which tame or even offset the loss of varieties due to selection. While our model does not allow this, selection can result in the decrease of operative varieties, therefore producing a negative welfare effect as stated in Impullitti and Licandro (2018).

2.3.5 Costly Trade

Augmenting the all exporting model to include a fixed exporting cost, λ_x, under the assumption of $\lambda_d + \lambda_x \geqslant \bar{\lambda}_x$, where $\bar{\lambda}_x$ is given by Eq. (2.49), the relative productivity cutoff threshold, Eq. (2.50) becomes:

$$z_x^{*\prime} = \left(\frac{n(\lambda_d + \lambda_x)(1 + \frac{k}{k-1}\frac{\theta_x + \frac{1-\gamma}{\gamma}}{1-\theta_x}) - 1}{\delta} \right)^{1/k} \tag{2.51}$$

The following Proposition 2.3 follows straightforwardly, given Eq. (2.51) is strictly increasing in λ_x.

Proposition 2.3. *$z_x^{*\prime} > z_x^*$ for $\lambda_x > 0$, and hence the higher productivity cutoff threshold and associated selection effect under all exporting with zero fixed costs relative to autarky, which occurs through the 'pro-competitive effect' of trade liberalization is further enhanced and increased in the fixed cost of exporting.*

The derived equilibrium productivity cutoff, the case where all operating firms are exporters with non-zero fixed costs of exporting, increases with the 'pro-competitive' effect of trade liberalization and fixed export costs under the constraints of total fixed costs. As stated in Proposition 2.2, trade liberalization generates a selection effect through the 'pro-competitive effect' if there is no fixed cost of exporting. When I consider the fixed export costs, there would be a higher equilibrium productivity cutoff; that is, there are two channels inducing a firm's selection effect, the 'pro-competitive effect' and 'fixed export cost'. Compared to Melitz (2003), there are constant markups, but the fixed cost of exporting can explain the self-selection of producers into the export market. Impullitti and Licandro (2018) introduce the exporters only with the assumption of firms without fixed exporting costs. However, my model complements them by suggesting that fixed export costs generate a larger selection effect than their study.

2.3.6 Multinational Production

This part will address the case when the firm serves the foreign market through horizontal FDI, as in Helpman et al. (2004). It means that a multinational firm based in the domestic market bears an additional fixed cost of λ_m in the foreign market, which includes the costs of planting a subsidiary in the foreign country and duplicating some of the overhead production costs involved in λ_d. I assume $\lambda_m \leqslant \lambda_d$. Similar to the case in which all firms export, domestic multinational firms producing the same variety with productivity level z compete in two separate Cournot games simultaneously in both domestic and foreign markets via FDI, which are referred to by subindices d_m and f_m, respectively, in each sector h. Where q_{d_m} denotes domestic consumption and production of the domestically produced good, q_{f_m} denotes foreign consumption of the domestically produced good via FDI. Therefore, a domestic multinational firm will produce $q_m = q_{d_m} + q_{f_m}$ and consumers will consume $x_m = n(q_{d_m} + q_{f_m})$ for a particular domestic variety, since n is the number of firms within a variety domestically from each country playing a symmetric Cournot game in each market, domestic and foreign. $\hat{x}_{d_m}$ and $\hat{x}_{f_m}$ denote the production of competitors in the domestic and foreign markets via FDI, where firms take their competitors' strategies as given. So, in the case in which all are multinational firms, the firm's problem is as follows:

$$\max_{q_{d_m}, q_{f_m}} \pi_m = \underbrace{\frac{\gamma E}{X^\alpha}(\hat{x}_{d_m} + q_{d_m})^{\alpha-1}}_{p_{d_m}} q_{d_m} + \underbrace{\frac{\gamma E}{X^\alpha}(\hat{x}_{f_m} + q_{f_m})^{\alpha-1}}_{p_{f_m}} q_{f_m} - z^{\frac{\alpha-1}{\alpha}}(q_{d_m} + q_{f_m}) - \lambda_d - \lambda_m \tag{2.52}$$

Eq. (2.52) shows the profit function of a domestic multinational firm manufacturing a variety with productivity level z, which means the profits of multinational firms domestically that serve the foreign market via FDI, consist of both domestic and foreign parts, according to the cost function Eq. (2.5) and the inverse demand functions Eq. (2.4). The first order conditions for domestic sales, q_{d_m}, and foreign sales, q_{f_m} , of multinational firms are, respectively:

$$\frac{\gamma E}{X^\alpha}((\alpha - 1)(\hat{x}_{d_m} + q_{d_m})^{\alpha-2} q_{d_m} + (\hat{x}_{d_m} + q_{d_m})^{\alpha-1}) = z^{\frac{\alpha-1}{\alpha}} \tag{2.53}$$

$$\frac{\gamma E}{X^{\alpha}}((\alpha-1)(\hat{x}_{f_m}+q_{f_m})^{\alpha-2}q_{f_m}+(\hat{x}_{f_m}+q_{f_m})^{\alpha-1})=z^{\frac{\alpha-1}{\alpha}} \tag{2.54}$$

Since I focus on the symmetric equilibrium, total consumption of the domestic multinational firms in a variety via FDI is $x_m = n(q_{d_m} + q_{f_m})$. Given symmetry across countries, consumption of multinational firms' domestically within a variety are the same and equal to x_m, that is, $x_{d_m} = x_{f_m} = x_m$. All multinational firms producing the same variety are identical and 'all via FDI'.

Using the symmetry implied above and Eqs. (2.53) and (2.54), I can derive:

$$q_{d_m} = q_{f_m} \tag{2.55}$$

It indicates that domestic and foreign consumption of domestically produced goods via FDI are equal. That is, multinational firms within a variety domestically will create the same domestic and foreign sales.

Applying Eq. (2.55) and symmetry to Eqs. (2.53) and (2.54), I have:

$$\underbrace{\frac{\gamma E}{X^{\alpha}}x_m^{\alpha-1}}_{p_m}\left(\frac{(\alpha-1)q_{d_m}}{n(q_{d_m}+q_{f_m})}+1\right)=z^{\frac{\alpha-1}{\alpha}} \tag{2.56}$$

$$\underbrace{\frac{\gamma E}{X^{\alpha}}x_m^{\alpha-1}}_{p_m}\left(\frac{(\alpha-1)q_{f_m}}{n(q_{d_m}+q_{f_m})}+1\right)=z^{\frac{\alpha-1}{\alpha}} \tag{2.57}$$

The equilibrium price of domestic multinational firms within a variety of both domestic and foreign markets then follows straightforwardly:

$$p_m(z,n)=\frac{z^{\frac{\alpha-1}{\alpha}}}{\theta_{d_m}}=\frac{z^{\frac{\alpha-1}{\alpha}}}{\theta_{f_m}} \tag{2.58}$$

where:

$$\theta_m=\theta_{d_m}=\theta_{f_m}=\frac{2n+\alpha-1}{2n} \tag{2.59}$$

θ_{d_m} and θ_{f_m}represent the inverse of markups of multinational firms within a variety charged in the domestic and foreign markets via FDI. Obviously, since there are no transport/trade costs under FDI, τ does not appear in θ_{d_m} and θ_{f_m}. In other words, variable trade costs have no relationship with markups of domestic multinational firms within a variety, since multinational firms domestically within a variety do not suffer iceberg transportation costs but have to pay fixed cost λ_m in order to establish a plant in the foreign market. Hence, in terms of variable costs, the FDI case is analogous to trade where $\tau = 1$, which captures the markup of the exporter and multinational firm on domestic sales.

According to Eqs. (2.4) and (2.58), I have that equilibrium consumption of domestic multinational firms for a given variety under 'all are multinational firms' is given by:

$$x_m(z,n) = \left(\frac{\gamma E}{X^{\alpha}}\right)^{\frac{1}{1-\alpha}} \theta_{d_m}^{\frac{1}{1-\alpha}} z^{\frac{1}{\alpha}} \tag{2.60}$$

The following expression is a multinational firm's variable production cost:[6]

$$l_m(z,n) - \lambda_d - \lambda_m = z^{\frac{\alpha-1}{\alpha}}(q_{d_m}(z,n) + q_{f_m}(z,n)) = e\theta_{x_m} z/\bar{z} \tag{2.61}$$

where l_m is the labour cost of production of goods produced by a multinational firm domestically for both domestic and foreign markets.

Entry and Exit

As before, the cutoff productivity, z, with all firms undertaking FDI, is determined by the exit condition. Using Eqs. (2.58), (2.60) and (2.61), firm profit of 'all are multinational firms' case with an extra plant fixed cost in the foreign market is:

$$\begin{aligned} \pi_m\left(z/\bar{z}\right) &= p_m(q_{d_m} + q_{f_m}) - z^{\frac{\alpha-1}{\alpha}}(q_{d_m} + q_{f_m}) - \lambda_d - \lambda_m \\ &= e\left(1 - \theta_{d_m}\right)\left(z/\bar{z}\right) - \lambda_d - \lambda_m \end{aligned} \tag{2.62}$$

Eq. (2.62) defines the operating profits of the multinational firm as a function of two endogenous variables, $\bar{z}$ and e. Consider the case of the variety whose multinational firms just break even in the market and denote their productivity with z_m^*. The condition defining this productivity cutoff is then given by:

$$\pi_m(z_m^*/\bar{z}_m) = e_m\left(1 - \theta_{d_m}\right)\left(z_m^*/\bar{z}_m\right) - \lambda_d - \lambda_m = 0 \tag{2.63}$$

Note that in equilibrium I obtain that $\bar{z}_m = \int_{z_m^*}^{\infty} z\mu(z)dz$, where:

$$\mu(z) = \begin{cases} \frac{g(z)}{1-G(z_m^*)} & \text{if } z \geqslant z_m^*, \\ 0 & \text{otherwise,} \end{cases}$$

Similar reasoning that produced Eq. (2.20) under autarky results in the following holding under all firms engaging with FDI:

$$\bar{z}_m = \frac{k}{(k-1)} z_m^* \tag{2.64}$$

[6] This follows directly from the derivation in Appendix A.1 under autarky with θ_m replacing θ_a.

Using Eq. (2.64) in Eq. (2.63) gives the following expression for the Exit Condition under all undertaking FDI:

$$e_m = \frac{\lambda_d + \lambda_m}{(1-\theta_{d_m})z_m^*/\bar{z_m}} = \frac{k(\lambda_d + \lambda_m)}{(k-1)(1-\theta_{d_m})} \quad \boldsymbol{(EC_m)} \tag{2.65}$$

Note that under the Pareto distribution, the EC_m condition is independent of the productivity cutoff.

I now use the condition for entrants and labour market conditions to find another relationship between e_m and z_m^*. As before, deriving a steady state number of firms in the context of restricted entry and exit of varieties requires $(1-M)(1-G(z_m^*)) = \delta M$ to hold, and hence:

$$M(z_m^*) \equiv \frac{1 - G(z_m^*)}{1 + \delta - G(z_m^*)} \tag{2.66}$$

Note that Eq. (2.66) describes a decreasing relationship between M and the productivity cutoff z_m^* for $M \in (0, 1/(1+\delta))$.

Following the same manipulations as the case under autarky, the labour market clearing condition under all firms engaging with FDI can be written as:

$$e_m(z_m^*) = \frac{\frac{1}{nM(z_m^*)} - \lambda_d - \lambda_m}{\theta_{d_m} + \frac{1-\gamma}{\gamma}} \quad \boldsymbol{(MC_m)} \tag{2.67}$$

Since I have established $M(z_m^*)$ is a decreasing function of z_m^*, then Eq. (2.67) implies an increasing relationship between e_m and z_m^*.

The equilibrium productivity threshold is obtained by equating EC_m and MC_m. Note that the EC_m condition is independent of the productivity cutoff under the Pareto distribution.

In line with the case of 'all exporting', in order to obtain an equilibrium productivity threshold for multinational firms, the following is assumed $\lambda_d + \lambda_m \geqslant \bar{\lambda}_m$ where:

$$\bar{\lambda}_m = \frac{1+\delta}{n\left(1 + \frac{k}{k-1}\frac{\theta_m + \frac{1-\gamma}{\gamma}}{1-\theta_m}\right)} \tag{2.68}$$

Eq. (2.68) can be derived from the consideration that at the min $z = 1$, $(EC_m) \geqslant (MC_m)$. In the case of the multinational firm, the domestic firm sets up production in the foreign country at a fixed cost $\lambda_m \leqslant \lambda_d$, and enjoys production at the same marginal cost as its foreign rivals in the same variety (there is no iceberg cost for serving the foreign market). The relevant productivity cutoff threshold can then be derived straightforwardly in the same way as under autarky:

$$z_m^* = \left(\frac{n(\lambda_d + \lambda_m)(1 + \frac{k}{k-1} \frac{\theta_m + \frac{1-\gamma}{\gamma}}{1-\theta_m}) - 1}{\delta} \right)^{1/k} \tag{2.69}$$

Similar to the case of 'all exporting' and proposition 2.1 in autarky, in the case in which all are multinational firms, I can state that, under the assumption of the fixed production costs, $\lambda_d + \lambda_m \geqslant \bar{\lambda_m}$, there exists a unique interior solution of average expenditure per differentiated multinational firm e_m and equilibrium productivity cutoff z_m^* for sector h. The solution is the intersection of MC_m and EC_m with $M(z_m^*)$ determined by Eq. (2.66).

Now, I collect together (from Propositions 2.1 and 2.3) and complete comparisons across all scenarios regarding firms' inverse of markups and equilibrium productivity cutoffs.

Proposition 2.4. *(i) $\alpha \leq \theta_a \leq \theta_{d_x} \leq \theta_x \leq \theta_m \leq \theta_{f_x} \leq 1$ and hence firm markups are weakly highest under autarky and weakly lowest for exported goods, but the average markup is the lowest for multinational firms, and (ii) productivity cutoffs satisfy $z_m^* \geq z_x^{*'} \geq z_x^* \geq z_a^*$ and hence the selection is greatest under all multinational firms. If $\lambda_m > 0$ and/or $\tau > 1$ then $z_m^* > z_x^*$ and if $\lambda_m > \lambda_x$ then $z_m^* > z_x^{*'}$.*

Proof. See Appendix A.5.

$\theta_{d_m} = (2n+\alpha-1)/2n$, is the largest one between θ_a, θ_x, θ_{d_m}, and leads to the case 'all via FDI' having the highest equilibrium productivity threshold. This can also be explained with the calculation in the case 'all exporting', since the derivative of z_x^* with respect to θ_x is positive, which means $z_a^* < z_x^* < z_m^*$ because of $\theta_a < \theta_x < \theta_{d_m}$, a larger selection effect happens in the economy with all multinational firms. That is to say, in an economy with all multinational firms, the productivity cutoff threshold would be larger than it would be with the economy under all exporters, and these properties are employed in the welfare analysis in the following section.

I compare three scenarios as follows, for a given exogenous number of firms within each variety with productivity z, exporters charge lower markups on their domestic sales compared to non-exporters. Moreover, multinational firms possess lower markups on their domestic sales than exporters, as shown above. The reason is that multinational firms have the most competitive pressure forcing them to reduce their markup on domestic sales, which indicates a stronger pro-competitive effect than the case where all operating firms are exporters. The pro-competitive effect of trade operates through decreased average markups of firms in the case where all potential firms are exporters compared to the markup of non-exporters. Proposition 2.4 is similar to Impullitti et al. (2018) for non-exporters and exporters in the same economic environment, but this study does not include FDI. This statement indicates that multinational firms and trade both exert pro-competitive effects but that it is stronger for multinational firms. This could have important implications for policy-making (e.g., Chor, 2009; De Santis and Stähler, 2004; Egger et al., 2005; Fumagalli, 2003) regarding incentivising firms to be multinational firms in order to increase welfare gains. For example, Chor (2009) explores the welfare analysis of subsidies to attract multinational firms with firm heterogeneity and finds that the policy of small subsidies will increase welfare in the host country of multinational firms through a selection effect, which indicates the subsidies will lead the most productive exporters to switch to serve the foreign market via multinational production, with the consumption gains from inducing more multinational firms overtaking the costs of the subsidy program.

2.4 Welfare Analysis

This section will identify and decompose the welfare benefits from autarky, trade openness and multinational firms via FDI and compare the overall gains. In the following analysis, I generalise the notation related to the three scenarios.

2.4.1 Decomposition of Welfare Effects

In line with Impullitti and Licandro (2018), I decompose the welfare effects of trade into three different channels. Notice that love-for-variety exists in the model, which means the positive welfare effects through greater selectivity may be offset by the reduction in the mass of potential entrants, M, produced by the same process. Indeed, since Lemma 2.1 applies across all scenarios, selection always results in a reduction in the equilibrium mass of varieties. $(1 - M)$ is the mass of potential entrants and is bounded above by one. Here, I focus on the equilibrium aggregate welfare effect derived from the given utility function rather than a specific sector h as calculated in the above section. First, let us denote:

$$\bar{\gamma} = \sum_{h=1}^{H} \gamma_h \tag{2.70}$$

and the average expenditure per heterogeneous firm among all sectors:

$$\bar{e} \equiv \frac{\bar{\gamma} E}{nM} \tag{2.71}$$

I now decompose aggregate steady state welfare gains arising through three channels, as follows:[7]

$$U = \underbrace{\bar{\gamma}\frac{1-\alpha}{\alpha}\ln M\bar{z}}_{\text{Productivity/LFV}} + \underbrace{\bar{\gamma}\ln\theta\bar{e}nM}_{\text{Consumption}} + \underbrace{(1-\bar{\gamma})\ln\frac{(1-\bar{\gamma})}{\bar{\gamma}}\bar{e}nM}_{\text{Homogeneous good}} \tag{2.72}$$

There are three different channels: the first two are related to composite goods consumption, while the third is associated with the homogeneous sector. The first term indicates the net effect of the productivity gains from selection effects, which are increased with higher average productivity $\bar{z_x}$, and the welfare losses through Love-For-Value (LFV) caused by fewer varieties, M. The second term is associated with the consumption of the composite goods $\bar{\gamma}E$ and the oligopolistic distortions in these sectors. I derived $\theta_d = \theta_x/\Phi$, which represents the pro-competitive effect of trade with the cross-hauling effect, as measured by Φ. The third channel measures utility from homogeneous goods consumption.

In line with Impullitti and Licandro (2018), welfare gains from selection operate through $\bar{z}$, M and $\bar{e}$, all of which depend on z^*. Differentiating Eq. (2.72) with respect to z^*, the selection gains can be collected into two sources:[8]

$$\begin{aligned}\text{Selection gains} &= \underbrace{\bar{\gamma}\frac{1-\alpha}{\alpha}\left[\frac{1}{\bar{z}}\frac{\partial\bar{z}}{\partial z^*} + \frac{1}{M}\frac{\partial M}{\partial z^*}\right]}_{\text{Productivity/LFV}} + \underbrace{\left\{\frac{1}{\bar{e}}\frac{\partial\bar{e}}{\partial M} + \frac{1}{M}\right\}\frac{\partial M}{\partial z^*}}_{\text{Fixed Cost}} \\ &= \underbrace{\frac{\bar{\gamma}}{z^*}\frac{1-\alpha}{\alpha}\left[1 - \frac{k\delta}{z^{*-k}+\delta}\right]}_{\text{Productivity/LFV}} + \underbrace{\left\{\frac{n\delta k\lambda_d z^{*k-1}}{(1+\delta z^{*k})^2(1-nM\lambda_d)}\right\}}_{\text{Fixed Cost}}\end{aligned} \tag{2.73}$$

Proposition 2.5. *Selection produces (i) unambiguous welfare gains through the fixed cost channel, and (ii) for sufficiently small values of the exogenous death rate δ, the productivity/LFV trade-off results in positive welfare gains as well:*

$$\delta < \frac{z^{*-k}}{k-1} \tag{2.74}$$

Proof. see Appendix A.8.

It means that when there is a sufficiently small value of the exogenous death rate δ, the productivity/LFV trade-off leads to positive welfare gains. The second component represents the change in labour allocated to the production of the composite good (excluding the fixed costs). Selection forces some firms to exit the market, reducing

[7]See Appendix A.6 for a proof.
[8]See Appendix A.7 for a derivation.

the resources needed to cover fixed production costs. These resources are allocated to surviving firms, leading to more production and consumption.

As the statement shown above, I can illustrate that the productivity effect is always positive and independent of δ, but the welfare effect for the reduced mass of variety is negative and increases in δ. However, I can restrict the condition about the value of δ for which the total effect regarding the trade-off between productivity and LFV is positive. This parameter constraint can also be explained with reference to literature assessing the welfare gains from selection, where the probability of firm death could be very small, like Arkolakis et al. (2012). In addition, they suggest a classical method to measure aggregate welfare gains from trade liberalisation, which examines the welfare effect when two statistics are pinned down: the share of expenditure on domestic goods and the elasticity of imports with respect to variable trade costs. However, unlike their work, I calculate the welfare effect by considering the utility function with both homogeneous sector and heterogeneous firms, which allows wages to be equal across two countries and furthermore assumes that two symmetric countries comprise a global economy. This is significant because it will enable us to explore the influences of oligopolistic competition and firm heterogeneity on the welfare effect in the two-way reciprocal trade and multinational firms via FDI.

2.4.2 Welfare Comparison

As represented by the welfare analysis regarding the scenario where all operating firms are exporters, it is similar to the case where all are non-exporters and all potential producers are multinational firms. Combining the previous equilibrium productivity cutoff threshold calculated in the above section and comparing those three scenarios, in the condition of $\delta < \frac{z_m^{*-k}}{k-1}$, I have that

Proposition 2.6. *Under the specific condition for exogenous death rate δ, welfare gains from the case in which are all multinational firms are larger than the scenario where all are exporters and all are non-exporters in the economy.*

According to $z_a^* < z_x^* < z_m^*$, which is shown in proposition 2.4, I derived that multinational firms generate the largest selection gains, where the selection gains always increase in z^* through the 'Fixed cost' Channel and will rise in z^* through the Productivity/LFV channel under the specific condition $\delta < z_m^{*-k}/(k-1)$, as presented by proposition 2.6, like Impullitti and Licandro (2018). That is to say, I extend Impullitti and Licandro (2018) to multinational firms and find that under the specific condition for an exogenous death rate δ, welfare gains from all multinational firms are the largest, compared to the scenario where all are exporters and the environment in which all are non-exporters. The fundamental reason for the result is the largest 'pro-competitive effect' via lower markups and the toughest selection if all potential firms are multinational producers.

2.5 Conclusion

In this chapter, I have explored welfare gains from trade and FDI following Ethier (1986) in an economy with firm heterogeneity and variable markups under oligopolistic competition. I have shown that trade liberalisation increases product market competition by reducing markups like Impullitti and Licandro (2018). The pro-competitive effect operates through the decreased average markup of firms in a scenario where all operating firms are exporters compared to the markups of non-exporters in the environment where all are non-exporters. Specifically, trade liberalisation decreases an exporter's markup in domestic sales while increasing the markup in a foreign market, but it decreases the average markup on total sales. Therefore, although the export markup increases, that is not sufficient to offset the pro-competitive effect on the domestic market since the average markup decreases when there is a reduction in iceberg transportation costs. It is in line with the findings of Impullitti and Licandro (2018).

The most important contribution of my chapter is that I incorporate multinational production via FDI following Helpman et al. (2004) as an alternative way for firms to serve a foreign market. In order to explore their effect on welfare, my model features Cournot competition in each heterogeneous variety. I consider horizontal FDI and find that multinational production generates the highest competitive pressure forcing a reduction in the markup, indicating a more substantial pro-competitive effect than trade openness. The pro-competitive channel of FDI can also be found in Ahn (2014), which built a theoretical model with firm heterogeneity and without oligopolistic competition. The economy in the case of 'all multinational production' would collapse to an economy of 'all exporters' when I consider there are no variable trade costs and the same fixed exporting costs as multinational firms' fixed costs for setting up a plant in a foreign market.

In my model, with firm heterogeneity and variable markups stemming from oligopolistic competition, I conclude that exporters and multinational firms both generate a pro-competitive effect through variable markups from trade openness. Multinational firms produce the most significant welfare due to the lowest markups and largest selection effect by comparing all three scenarios with all non-exporters, all exporters and all multinational firms. I assume that the number of firms in each variety is exogenous, and there is no interaction between non-exporters, exporters and multinational firms in the economic environment for the simplicity of the model. However, there is a limitation of the chapter that generates a challenge for further research, which would consider the endogeneity of the total number of firms in the model (e.g., Impullitti et al., 2018, 2022). In the fourth chapter, I complement it by incorporating an endogenous mass of product lines and consider that non-exporters, exporters, and multinational firms can coexist in the same industry. In addition, in the third chapter of the thesis, I also extend to introduce R&D into the model framework to investigate how innovation affects the welfare implications of trade and FDI.

Appendix A

A.1 Equation (2.14)

I seek to obtain an expression for X in the case in which all are non-exporters. Using Eqs. (2.2) and (2.11) I have:

$$X = \left[\int_0^M \left(z^{\frac{1}{\alpha}} \left[\frac{\gamma E}{X^\alpha} \theta_a \right]^{\frac{1}{1-\alpha}} \right)^\alpha dv \right]^{\frac{1}{\alpha}}$$

Rearranging and using Eq. (2.13a):

$$X^\alpha = \left(\frac{\gamma E}{X^\alpha} \theta_a \right)^{\frac{\alpha}{1-\alpha}} \int_0^M z_v dv = \left(\frac{\gamma E}{X^\alpha} \theta_a \right)^{\frac{\alpha}{1-\alpha}} M \bar{z}$$

Rearranging the above terms, I focus on the case where all are non-exporters and identify the aggregate composite goods X for each sector h:

$$X^{\frac{\alpha}{1-\alpha}} = (\gamma E \theta_a)^{\frac{\alpha}{1-\alpha}} M \bar{z} \tag{A.1}$$

Then, I derive a non-exporter's variable production cost, based on Eq. (2.5) and using Eqs. (2.11), (2.13b), (A.1), and symmetry $x_a = q_a + \hat{x}_a = n q_a$:

$$\begin{aligned}
l_a(z,n) - \lambda_d &= z^{\frac{\alpha-1}{\alpha}} q_a(z,n) \\
&= z^{\frac{\alpha-1}{\alpha}} \frac{q_a}{x_a} x_a = z^{\frac{\alpha-1}{\alpha}} \frac{q_a}{x_a} \left(\frac{\gamma E}{X^\alpha} \right)^{\frac{1}{1-\alpha}} \theta_a^{\frac{1}{1-\alpha}} z^{\frac{1}{\alpha}} \\
&= \frac{q_a}{x_a} \frac{(\gamma E)^{\frac{1}{1-\alpha}}}{(\gamma E \theta_a)^{\frac{\alpha}{1-\alpha}}} \theta_a^{\frac{1}{1-\alpha}} \frac{z}{M \bar{z}} \\
&= \frac{q_a}{n q_a} \gamma E \theta_a \frac{z}{M \bar{z}} \\
&= \frac{\gamma E}{n M} \theta_a \frac{z}{\bar{z}} \\
&= e \theta_a \frac{z}{\bar{z}}
\end{aligned}$$

A.2 Equation (2.20)

From the Pareto distribution $G(z)$, I have $1-G(z)=\left(\frac{1}{z}\right)^k$, $g(z)=kz^{-k-1}$,$zg(z)=kz^{-k}$. Since I have defined $\bar{z_a}(z_a^*)=\int_{z_a^*}^{\infty} z\mu(z)dz$, combined with the definition of $\mu(z)$ and $\bar{z_a}$, then

$$\begin{aligned}
\bar{z_a}\left(z_a^*\right) &= \int_{z_a{}^*}^{\infty} z\mu(z)dz = \int_{z_a{}^*}^{\infty} \frac{g(z)}{1-G(z_a{}^*)} zdz \\
&= \int_{z_a{}^*}^{\infty} \frac{kz^{-k-1}}{z_a{}^{*-k}} zdz = \frac{k}{z_a{}^{*-k}} \int_{z_a{}^*}^{\infty} z^{-k}dz \\
&= \frac{k}{z_a{}^{*-k}} \frac{1}{1-k} z^{1-k}|_{z_a{}^*}^{\infty} \\
&= \frac{k}{k-1} z_a{}^*
\end{aligned}$$

A.3 Lemma 2.2

With $e'_a(z_a^*)=0$ from the EC_a condition, Eq. (2.21), $e'_a(z_a^*)>0$ under MC_a in Eq. (2.26), and the condition on $\bar{\lambda}_a$ ensuring $EC_a \geq MC_a$ at the minimum $z=1$, completes the proof.

A.4 Proposition 2.1

(i) From Eq. (2.7), $\theta_a \geq \alpha$ follows directly from inspection, with equality at the limiting case of $n=1$. Let:

$$\Omega^{dxa} = \frac{\theta_{d_x}}{\theta_a} = \frac{2n+\alpha-1}{(n+\alpha-1)(1+\tau)}$$

where $\Omega^{dxa}_{\tau} = \frac{\alpha-1}{[n+\alpha-1]^2(1+\tau)} < 0$. Given the relevant range of τ, with $\Omega^{dxa}(\tau=1) = \frac{2n+\alpha-1}{2(n+\alpha-1)} > 1$ and $\Omega^{dxa}(\tau=\bar{\tau})=1$, establishing $\theta_a \leq \theta_{d_x}$. Given $\theta_{f_x}=\tau\theta_{d_x} \geq \theta_{d_x}$, by definition of the average $\theta_{d_x} \leq \theta_x \leq \theta_{f_x}$. Finally, given $\frac{\partial\theta_{f_x}}{\partial\tau} = \frac{2n+\alpha-1}{n(1+\tau)^2} > 0$ and $\theta_{f_x}(\tau=\bar{\tau})=1$, completes the proof.
(ii) This follows directly from Lemma 2.3 and Proposition 2.1(i).

A.5 Proposition 2.4

(i) Let $\Omega^{mx} = \frac{\theta_m}{\theta_x} = \frac{(1+\beta)(1+\tau)}{2(1+\tau\beta)}$. Noting, $\Omega^{mx}(\tau=1)=1$ and $\Omega^{mx}(\tau=\bar{\tau}) = \frac{(n+\alpha-1+2n+n)-\beta(1-\alpha)}{(n+\alpha-1+2n+n)-(1-\alpha)} \geq 1$ given $\beta\in[0,1]$ establishes $\theta_m \geq \theta_x$. Let $\Omega^{mdx} = \frac{\theta_m}{\theta_{d_x}} = \frac{(1+\tau)}{2}$. Noting, $\Omega^{mdx}(\tau=1)=1$ and $\Omega^{mdx}(\tau=\bar{\tau})>1$ establishes $\theta_m \geq \theta_{d_x}$. Finally,

$\Omega^{mfx} = \frac{\theta_m}{\theta_{fx}} = \frac{(1+\tau)}{2\tau}$. Noting, $\Omega^{mfx}(\tau = 1) = 1$ and $\Omega^{mfx}(\tau = \bar{\tau}) < 0$ establishes $\theta_m \leq \theta_{fx}$.

(ii) This follows from Lemma 2.3 and the recognition that the fixed cost under multinational firms further enhances z_m^* such that under $\lambda_m > 0$ (in addition to $\tau > 1$) guarantees $z_m^* > z_x^*$ and $\lambda_m > \lambda_x$ guarantees $z_m^* > z^{*\prime}_x$.

A.6 Equation (2.72)

Applying Eq. (A.1) to the case where the markup is that in the domestic market under any given regime, and aggregating across all sectors, I have:

$$X = (M\bar{z})^{\frac{1-\alpha}{\alpha}} \bar{\gamma} E \theta_d$$

where $\bar{\gamma} = \sum_{h=1}^{H} \gamma_h$. Substituting into the aggregate utility function, Eq. (2.1), for a given country (domestic), and using Eq. (2.13b), (2.70), (2.71) and $Y = (1-\bar{\gamma})E$, I have:

$$\begin{aligned} U &= \bar{\gamma} \ln X + (1-\bar{\gamma}) \ln Y \\ &= \bar{\gamma} \frac{1-\alpha}{\alpha} \ln M\bar{z} + \bar{\gamma} \ln \theta_d \bar{\gamma} E + (1-\bar{\gamma}) \ln (1-\bar{\gamma}) E \\ &= \underbrace{\bar{\gamma} \frac{1-\alpha}{\alpha} \ln M\bar{z}}_{\text{Productivity/LFV}} + \underbrace{\bar{\gamma} \ln \theta_d \bar{e} n M}_{\text{Consumption}} + \underbrace{(1-\bar{\gamma}) \ln \frac{(1-\bar{\gamma})}{\bar{\gamma}} \bar{e} n M}_{\text{Homogeneous good}} \end{aligned} \tag{A.2}$$

A.7 Equation (2.73)

This is derived using $\bar{z}$ from Eq. (2.20), M from Eq. (2.23) and e from Eq. (2.26). Starting from $M = (1+\delta z^{*k})^{-1}$ and $e = \frac{1+\delta z^{*}k - n\lambda_d}{nT}$ where $T = \theta_d + \frac{1-\gamma}{\gamma}$:

$\frac{\partial e}{\partial z^*} \frac{1}{e} = \frac{k\delta z^{*k-1}}{nT} \frac{nT}{1+\delta z^{*k} - n\lambda} = \frac{k\delta z^{*k-1}}{1+\delta z^{*k} - n\lambda}$ and $\frac{\partial M}{\partial z^*} \frac{1}{M} = \frac{-\delta k z^{*k-1}(1+\delta z^{*k})^{-2}}{(1+\delta z^{*k})^{-1}} = \frac{-\delta k z^{*k-1}}{(1+\delta z^{*k})}$

Hence, $\frac{\partial e}{\partial z^*} \frac{1}{e} + \frac{\partial M}{\partial z^*} \frac{1}{M} = \frac{k\delta z^{*k-1}}{(1+\delta z^{*k})(1+\delta z^{*k} - n\lambda)} \left[1 + \delta z^{*k} - 1 - \delta z^{*k} + n\lambda\right] = \frac{kn\lambda\delta z^{*k-1}}{(1+\delta z^{*k})(1+\delta z^{k*} - n\lambda)}$

Finally, noting that given $M = (1+\delta z^{*k})^{-1}$, the denominator can be written $(1+\delta z^{*k})^2(1 - nM\lambda_d)$, completing the proof.

A.8 Proposition 2.5

(i) This follows from inspection of Eq. (2.26), where, given the numerator is positive, for $e > 0$, requires $1 - \lambda_d n M > 0$, which ensures $\{.\} > 0$, completing the proof.

(ii) The inequality in Eq. (2.74) follows directly from setting the term $[.] > 0$ in Eq. (2.73) and rearranging for δ, completing the proof.

Chapter 3

Firms with Innovation in Autarky, Costly Trade and Multinational Production

Abstract

This chapter extends the first main chapter by incorporating innovation and analysing how innovation affects the welfare gains of international trade and horizontal FDI with firm heterogeneity and variable markups from oligopolistic competition. The economic environment is similar to the first chapter building a multi-sector general equilibrium framework in a global world with two symmetric countries and three scenarios where all firms in a variety are domestic firms, exporters or multinational firms. However, I allow firms to invest in R&D technology to increase productivity after they learn about their initial productivity. Then, general equilibrium is reached by combining firms' zero cutoff condition and a labour market clearing condition. This model finds that innovation, as a new mechanism, complements the channels of the pro-competitive effect by decreasing markups and a selection effect by increasing aggregate productivity, which increases welfare gains from trade and horizontal FDI. The most significant welfare enhancement is under the case of 'all multinational firms via horizontal FDI' through the above three channels compared to the scenarios 'all exporters' and 'all non-exporters'.

Keywords: heterogeneous firms; horizontal FDI; Innovation; oligopolistic competition; welfare gains
JEL Classification: F12, F13, O31, D60

3.1 Introduction

Innovation, as the critical element of productivity growth for firms, industries, and countries, has attracted the attention of many researchers (e.g., Doraszelski and Jaumandreu, 2013; Impullitti and Licandro, 2018; Klette and Kortum, 2004; Melitz and Redding, 2021; Navas, 2018; Petit and Sanna-Randaccio, 2000). The main reason for the rapid transformation of the global economy is the acceleration of a firm's international expansion, primarily through Foreign Direct Investment (FDI) (e.g., Carril-Caccia et al., 2018; Helpman et al., 2004; Lin, 2010; Petit and Sanna-Randaccio, 2000). In the last few decades, the growth in multinational production has been an increasingly important component of economic globalisation. There was a massive surge in worldwide Foreign Direct Investment (FDI) from 1986 to 1990. Specifically, FDI flows increased by 23% while exports grew by only 9%, with the world income rising over the same period by about 6% per annum (e.g., Irarrazabal et al., 2013; Petit and Sanna-Randaccio, 2000). From 2000 to 2016, the stock of FDI divided by world GDP increased approximately by 60%, from 22% to 35% (e.g., Carril-Caccia et al., 2018; Goerke, 2020). According to firms' different modes of serving foreign markets through export and FDI, this chapter investigates how exporters and multinational firms allocate different amounts of labour to undertake R&D activities and generate various welfare gains. In my chapter, I construct a theoretical model with heterogeneous firms undertaking innovation and variable markups under oligopolistic competition to examine the corresponding welfare effects of international trade and FDI.

The availability of rich firm-level datasets from the mid-1990s onwards reveals a significant level of firm heterogeneity (e.g., Bernard et al., 2007; Pavcnik, 2002; Roberts and Tybout, 1997; Trefler, 2004; Tybout and Westbrook, 1995). We observe several stylised facts associated with firm heterogeneity and variation in exporting behaviour across firms within an industry. For example, few firms export, demonstrating that exporting is a rare event. Second, their export share remains low even for the comparative advantage industries, Bernard and Jensen (1999). Third, there is self-selection, whereby more productive firms are more likely to export (e.g., see Clerides et al., 1998), and trade liberalisation favours market share reallocations towards more productive firms and therefore increases industry aggregate productivity (e.g., see Pavcnik, 2002). As the empirical work revealed these stylised facts, Melitz (2003), a seminal milestone built a theoretical model to explain them with firm heterogeneity, CES utility and constant markup under autarky and exporting.

How does firm heterogeneity relate to multinational production? Helpman et al. (2004) include horizontal FDI into the theoretical framework of Melitz (2003). They assume that there is a trade-off for firms to decide to be exporters or multinational producers as exporting involves lower fixed costs while FDI involves lower variable costs. They find that the most productive firms choose to serve the foreign market via FDI, while more productive firms will export, the less productive firms stay in the domestic market, and the least productive firms exit the market. Like Helpman et al. (2004), my model introduces horizontal FDI with CES preferences and firm heterogeneity. However, I introduce variable markups from oligopolistic competition within each variety rather than constant markups of monopolistic competition in Melitz (2003) and Helpman et al. (2004). Another component which distinguishes my work from Melitz (2003) and Helpman et al. (2004) is that I introduce process innovation into the framework to examine how firms' different modes of foreign expansion, through exports or FDI, would affect their incentives for R&D activities.

Why do I consider oligopolistic competition? Hansen and Hoenen (2016) undertook a comprehensive literature review and found that strategic interaction among multinational corporations (MNCs) in oligopolistic industries often drives FDI. In addition, some literature focuses on the international trade model of oligopoly in general equilibrium and examines the effects on the industrial structure and the gains from trade due to trade liberalization (e.g., Impullitti and Licandro, 2018; Neary, 2003, 2007, 2016). I consider oligopolistic competition in my model but the difference between my model and theirs (e.g., Impullitti et al., 2022; Neary, 2003, 2007) is that I introduce two modes of serving the foreign market, exporting or horizontal FDI, for firms to investigate their welfare effect. Moreover, R&D activities are taken into account to identify the firms' potential investment differences according to their different international strategies for serving the foreign market. Some empirical evidence shows that the degree of product market competition and trade openness is related to innovation in the heterogeneous sector (e.g., see Navas, 2015). In my model, I incorporate firms' decisions to innovate with trade and horizontal FDI to investigate by which mechanism trade and FDI impact innovation and productivity. Last but not least, my model allows an assessment of the welfare gains for each scenario where firms choose to stay in the domestic market, firms can export, and firms are capable of undertaking FDI as well as a comparison among these three scenarios. Therefore, my work aims to fill a gap in the literature by presenting welfare analysis of a model of trade and FDI with innovation and variable markups under oligopolistic competition with heterogeneity across varieties.

My model's economy is characterised by a continuum of imperfectly substitutable product lines, or varieties, comprised of firms with different productivities. It differs from Melitz (2003), where there is only one firm in each variety; in my model, each product line is manufactured by a small number of identical firms, which is similar to Impullitti and Licandro (2018). In other words, firms in a particular product line interact strategically with their direct competitors who are identical to them in the product line but compete with the indirect competitors from other product lines. Conditional on paying fixed sunk costs, a small number of firms within a particular variety draw their productivity from the distribution of Pareto. If firms' entry is successful, firms play Cournot games with their direct rivals within a unique variety and undertake innovation to improve their productivity, and this follows Impullitti et al. (2018). However, the difference between my work and theirs is that my model considers horizontal FDI, another mode of serving the foreign market. The global economy comprises two symmetric countries with costly, two-way trade, as in Brander and Krugman (1983), and strategic interaction between identical firms producing perfect substitute goods within each variety. Similarly, considering the oligopolistic competition, I analyse three separate scenarios: horizontal FDI with no variable transportation costs but higher fixed costs in a global economy; exporters and domestic firms. The number of identical oligopolistic firms per variety is exogenously fixed. The three specific cases are that all firms are domestic, all firms choose to export, and all firms become multinational producers. Through analysing the model, I find that trade liberalisation produces welfare gains through selection, the pro-competitive effect and cost-reducing innovation under reasonable parameter restrictions. Comparing the welfare gains of the three different scenarios, with innovation, the economy where all are multinational firms generates the most extensive welfare gains, followed by the economy where all are exporters, and the least welfare gains are related to the case of autarky. In other words, there are three channels for the most extensive welfare gains of a multinational firm at work: highest investments in innovation, toughest selection and lowest markups.

The rest of the chapter is structured as follows. Section 2 introduces the model and develops the simple model in the previous chapter to include innovation. Section 3 describes the benchmark scenario of autarky to investigate the domestic firms' productivity threshold. Section 4 presents the model with innovation in the case of 'all exporters' with fixed costs of exporting. Section 5 provides the scenario of 'all multinational firms' with the associated higher fixed costs and innovation and compares different investment effects on innovation due to different modes of serving the foreign market. In section 6, I conduct the welfare analysis, which compares the above three scenarios. Finally, section 7 presents the conclusion.

3.2 The Model

This section extends the simple and tractable theoretical benchmark model in the previous chapter by incorporating innovation to present the basic properties of the theory. I consider a world economy consisting of two identical countries with symmetric technologies, preferences and endowments. In each country, any product line or variety v is manufactured by n identical firms producing perfectly substitutable goods with oligopolistic competition, which is different from Melitz (2003) with only one firm in each variety. At entry, each firm within a particular variety with the other $(n-1)$ identical oligopolistic firms choose to jointly draw their productivity from a given Pareto distribution, where productivity differs across varieties. After the firm receives its initial draw, there is an R&D technology that firms may use to increase their productivity. Three scenarios are considered: a domestic case where all firms are domestic firms, exporting case where all are exporters and a multinational case where all are multinational firms.

3.2.1 Preferences and Demand

There are two countries identical in all features constituting a global economy. Each country is populated by a continuum of identical individuals, which I normalize to 1 for simplicity.

Utility Function

There are two-tier preferences, with Cobb-Douglas in the upper tier and CES utility in the lower tier. In each country, there are $S+1$ sectors producing final goods. Each sector s in S produces a continuum of differentiated goods characterised by variety v while sector 1 produces a homogeneous good. Consumers have preferences over a set of varieties V as follows:

$$U = (1 - \sum_{s=1}^{S} \gamma_s) \ln Y + \sum_{s=1}^{S} \frac{\gamma_s}{\alpha_s} \ln \left(\int_{v \in V_s} x_s(v)^{\alpha_s} dv \right)$$

where Y is aggregate consumption of the homogeneous good, $x_s(v)$ is consumption of a heterogeneous variety v from sector s and V_s is the set of all potential varieties in sector s. A consumer will spend a fraction γ_s of income on the differentiated goods and $(1-\gamma_s)$ on homogeneous goods in the sector s. Let σ_s be the elasticity of substitution between any two varieties in sector s and $\alpha_s \in (0,1)$ defined according to: $\sigma_s = 1/(1-\alpha_s) > 1$. [1]

Demand Function

Consumers then maximize their utility function subject to the following budget constraint:

[1]Note that the utility function presented here is a monotonic transformation of the following utility function:

$U = (Y)^{(1-\sum_{s=1}^{S} \gamma_s)} \prod_{s=1}^{S} (X_s)^{\gamma s}$ where $X_s = \left(\int_{v \in V_s} x_s(v)^{\frac{\sigma_s - 1}{\sigma_s}} dv \right)^{\frac{\sigma_s}{\sigma_s - 1}}$ with $\alpha_s = \frac{\sigma_s - 1}{\sigma_s}$.

$$Y + \sum_{s=1}^{H} \int_{s \in V_s} p_s(v) x_s(v) dv = E \tag{3.1}$$

The homogeneous good Y is considered the numéraire, and consumers are endowed with one unit of labour supplied inelastically. E is the aggregate level of expenditure, including homogeneous goods and composite goods. Consumers spend $\gamma_s E$ on composite goods and $(1-\gamma_s E)$ on homogeneous goods in a sector s. Solving the utility maximization problem yields the following inverse demand function for each variety v from a particular sector s:

$$p_s(v) = \frac{\gamma_s E}{X_s^{\alpha}} x_s(v)^{\alpha-1} \tag{3.2}$$

where

$$X_s = \left(\int_0^M x_s(v)^{\alpha_s} dv \right)^{\frac{1}{\alpha_s}} \tag{3.3}$$

I have $Y = (1-\bar{\gamma})\, E$ where $\bar{\gamma} = \sum_{s=1}^{S} \gamma_s$ is the share of heterogeneous goods on the aggregate level. A continuum of varieties of endogenous mass $M \in (0,1)$ in sector s illustrates the aggregate differentiated good X from sector s as Eq.(3.3). Notice that I omit the subscript s for a particular sector in the following sections for simplicity.

3.2.2 Production and Firm Behaviour

There are a fixed number of n identical firms within a specific product line, which means exogenous n identical firms compete *à la* Cournot for market share in each specific variety in each country.

Initial Productivity and R&D

At entry, n firms within a specific variety v jointly draw their productivity z from a continuous Pareto distribution, that is to say, firms within the same product line are identically productive, but productivity differs across product lines. The entry process for firms is quite similar to Impullitti and Licandro (2018). However, innovation in my model is different from knowledge spillovers which generate sustained growth in their model. R&D activity in my model is quite similar to Impullitti et al. (2018), which applies R&D technology to the initial productivity distribution and derives the actual productivity by allocating labour resources. In other words, after entry, process innovation could be improved by firms through allocating resources to increase their productivity after they receive the initial draw of productivity from the given continuous Pareto distribution:

$$G(z) = 1 - \left(\frac{\text{z}}{z} \right)^k = 1 - z^{-k}, z \geq 1, k \geq 1 \tag{3.4}$$

The productivity z denotes the draw of initial productivity at the entry for a firm within a specific product line. Eq.(3.4) is the entry distribution of productivity across product lines, where $\underline{z}= 1$, indicates the lower productivity bound, and k represents a distribution parameter.

In order to invest in R&D, firms use labour as a production factor. More precisely, the R&D technology is described by the following functional form:

$$\tilde{z} = Ah^{\eta}z \tag{3.5}$$

Firms can invest in R&D activity, decreasing marginal production costs. h represents labour resources allocated to R&D activity. Post innovation, productivity is denoted $\tilde{z}$. $\eta \in (0, 1)$ measures the degree of decreasing marginal returns to labour associated with innovation. A is the technology shift parameter representing differences in technological opportunities across varieties, and $A > 0$ is constant. Notice that here I consider firm-specific innovation without technological spillovers among firms.

Technology and Market Structure

There is only one production factor, labour. The homogeneous good Y is produced in perfect competition with a linear technology transforming one unit of labour into one unit of output. Combining that Y is numéraire, I have a unit wage $w = 1$. Given the above assumption, a firm manufactures a variety with actual productivity $\tilde{z}$ using the following cost function:

$$C(\tilde{z}) = l(\tilde{z}) = \tilde{z}^{\frac{\alpha-1}{\alpha}}q(\tilde{z}) + \lambda \tag{3.6}$$

where, $q(\tilde{z})$ is the units of a firm's output, $C(\tilde{z})$ is the total cost of the firm and $l(\tilde{z})$ is the total amount of labour for the production after innovation. $\lambda > 0$ is the fixed costs. Notice that there are different types of fixed costs, λ_d, λ_x, and λ_m, as I consider three scenarios, each of them is in a global economy with two symmetric countries and contains all domestic firms, all exporters and all multinational firms, separately. To be more specific, λ_d indicates a firm's fixed operating cost in the domestic market, λ_x and λ_m represent an exporter's fixed exporting cost and a multinational firm's fixed cost of creating a new plant in the foreign market, respectively. Eq.(3.6) expresses a firm with the actual productivity $\tilde{z}$, which undertakes innovation and will pay production costs $\tilde{z}^{\frac{\alpha-1}{\alpha}}q(\tilde{z})$ to produce $q(\tilde{z})$ units of output and its fixed costs λ.

3.3 Autarky

I begin the analysis with the benchmark case of autarky with the subscript a. Here, firms will be able to serve only the domestic market, indicating that only domestic firms in a global economy exist. A firm producing a variety with the actual productivity $\tilde{z}$ after process innovation will maximize its profits subject to the inverse demand function of Eq.(3.2), taking the production of its competitors as given. Solving the firm's problem:

$$\pi_a = \max_{q_a h_a} \underbrace{\frac{\gamma E}{X^\alpha}(\hat{x}_a + q_a)^{\alpha-1}}_{p_a} q_a - \underbrace{(Ah^\eta z)}_{\tilde{z}}^{\frac{\alpha-1}{\alpha}} q_a - \lambda_d - h_a \tag{3.7}$$

Eq.(3.7) expresses the profit function of a domestic firm under autarky by which the firm can augment productivity by employing labour. Here, q_a is the firm's output and $\hat{x}_a$ is the output of the firm's direct competitors within the same product line v, also with the initial productivity z. Hence, total production of a domestic variety in the domestic country is $x_a = q_a + \hat{x}_a$. The first order conditions for sales, q_a, and labour allocated to innovation activities, h_a, are, respectively,

$$\frac{\gamma E}{X^\alpha}\Big((\alpha-1)(\hat{x}_a + q_a)^{\alpha-2} q_a + (\hat{x}_a + q_a)^{\alpha-1}\Big) = \tilde{z}^{\frac{\alpha-1}{\alpha}} \tag{3.8}$$

$$\hat{\eta}\tilde{z}^{\frac{\alpha-1}{\alpha}} q_a / h_a = 1 \tag{3.9}$$

where $\hat{\eta} = \frac{1-\alpha}{\alpha}\eta > 0$ is a parameter related to the degree of marginal returns of labour devoted to innovation. Given symmetry and the relationship of production for variety and their firms, $x_a = nq_a$, Eq.(3.8) can be written:

$$\underbrace{\frac{\gamma E}{X^\alpha} x_a^{\alpha-1}}_{p_a} \left(\frac{(\alpha-1)q_a}{nq_a} + 1\right) = \tilde{z}^{\frac{\alpha-1}{\alpha}} \tag{3.10}$$

Simplifying Eq.(3.10), I have:

$$p_a = \frac{\tilde{z}^{\frac{\alpha-1}{\alpha}}}{\theta_a} \tag{3.11}$$

and rearranging Eq.(3.10), total quantity of a domestic variety in the domestic market:

$$x_a(\tilde{z}) = (\frac{\gamma E}{X^\alpha})^{\frac{1}{1-\alpha}} \theta_a^{\frac{1}{1-\alpha}} \tilde{z}^{\frac{1}{\alpha}} \tag{3.12}$$

combining Eqs.(3.10) and (3.11), θ_a indicates the inverse of the markup of a firm within a domestic variety:

$$\theta_a = \frac{n-1+\alpha}{n} \tag{3.13}$$

Here, I define

$$\tilde{Z} = (1/M)\int_0^M \tilde{z}_v dv$$

$$e = \gamma E/(nM)$$

where $\tilde{Z}$ is the average actual productivity $\tilde{z}$ after firms' innovation with initial productivity z, it measures the aggregate level of actual productivity for sector s and can be used to find productivity cutoff points. e is the consumer's expenditure received by each differentiated firm in sector s.

Then, building on Eqs.(3.3), (3.4) and (3.12), the variable production cost of a domestic firm undertaking R&D is shown as follows[2]:

$$l_a(\tilde{z}) - \lambda_d = \tilde{z}^{\frac{\alpha-1}{\alpha}} q_a(\tilde{z}) = e\theta_a \frac{\tilde{z}}{\tilde{Z}} \tag{3.14}$$

where l_a is the labour of a domestic firm after innovation devoted to the output of goods in autarky. The expression indicates that labour demand is positively associated with the productivity level. More productive firms with a higher actual productivity level $\tilde{z}$ after innovation acquire more inputs and produce more. Rearranging the first order condition for h_a, in Eq.(3.9), and using Eq.(3.14) for labour demand, R&D effort is shown as:

$$h_a = \hat{\eta}(l_a(\tilde{z}) - \lambda_d) \tag{3.15}$$

Eq.(3.15) indicates that the innovation effort is positively associated with the firm's variable labour demand and hence with the firm size. Firms benefit more from R&D if they can apply the reduction in costs to a larger quantity because cost-reducing innovation is assumed. Therefore, more productive firms produce more, demand more labour, and make a greater R&D effort. Substituting optimal h_a of Eq.(3.15) into the R&D technology of Eq.(3.5), the productivity of this variety is given by:

$$z = \frac{\tilde{Z}^{\eta}}{A(\hat{\eta}e\theta_a)^{\eta}} \tilde{z}^{1-\eta} \tag{3.16}$$

Simplifying Eq.(3.16), I can also represent the equilibrium distribution of $\tilde{z}$ as follows:

$$z = \mathscr{A} \tilde{z}^{1-\eta}$$

since z is distributed Pareto with tail parameter k, then $\tilde{z}$ is distributed Pareto with tail parameter $(1-\eta)k$, [3] where $\mathscr{A}$ depends on parameters and some aggregates as shown in Eq.(3.16).

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[2]See Appendix B.1

[3]I can derive it through

$$G(z) = 1 - \left(\frac{z^*}{z}\right)^k = 1 - \left(\frac{z^*}{\mathscr{A}\tilde{z}^{1-\eta}}\right)^k = 1 - \left(\frac{\tilde{z}^*}{\tilde{z}}\right)^{(1-\eta)k}$$

3.3.1 Entry and Exit

There is a unit endogenous mass of potential varieties of which $M \in (0,1)$ are operative, as in the first chapter. In other words, $(1-M)$ mass of non-operative new varieties, each produced by n identical firms with oligopolistic competition, are trying to enter the economy at zero cost. As before, initial productivity z is jointly drawn for each of the varieties from a Pareto distribution $G(z)$ as in Eq.(3.5). In addition, an exogenous death shock δ can result in n identical firms within a variety exiting the market simultaneously.

In order to establish the productivity cut-off point, I need to find the domestic firm's break-even point. Starting with the equilibrium profit for a domestic firm with the actual productivity $\tilde{z}$ after innovation given average actual productivity $\tilde{Z}$, the non-exporting firm's profit can be expressed as:

$$\pi_a \left(\tilde{z}/\tilde{Z}\right) = p_a q_a - l_a - h_a = \left(1-(1+\hat{\eta})\theta_a\right) e \frac{\tilde{z}}{\tilde{Z}} - \lambda_d \tag{3.17}$$

With restricted entry and no entry cost, the break even point of a domestic firm or its exit condition requires:

$$\pi_a \left(\tilde{z}_a^*/\tilde{Z}_a\right) = (1-(1+\hat{\eta})\theta_a) e_a \frac{\tilde{z}_a^*}{\tilde{Z}_a} - \lambda_d = 0 \tag{3.18}$$

The actual productivity cut-off after innovation in the domestic case is $\tilde{z}_a^*$ with initial productivity z_a^*, such that if initial productivity $z > z_a^*$, which means actual productivity $\tilde{z} > \tilde{z}_a^*$, all firms with actual productivity $\tilde{z}$ stay in the market, and otherwise, they all leave the market. Hence, the productivity cut-off is demonstrated by the following condition, rearranging Eq.(3.18), the expression of the cutoff condition:

$$e_a \left(1-(1+\hat{\eta})\theta_a\right) \left(\tilde{z}_a^*/\tilde{Z}_a\right) = \lambda_d \tag{3.19}$$

where

$$\tilde{Z}_a(z_a^*) = \int_{z_a^*}^{\infty} \tilde{z}\mu(z)dz$$

represents the average actual productivity observed at equilibrium and the equilibrium density is identified by

$$\mu(z) = \begin{cases} \frac{g(z)}{1-G(z_a^*)} & \text{if } z \geqslant z_a^*, \\ 0 & \text{otherwise,} \end{cases}$$

Therefore, I can express $\tilde{Z}_a$ in terms of $\tilde{z}_a^*$: [4]

[4] See Appendix B.2

$$\tilde{Z}_a = \frac{k}{k - \frac{1}{1-\eta}} \tilde{z}_a^* \tag{3.20}$$

Eq.(3.20) can be compared with Eq.(2.20), it is explained through Eq.(3.16) and the simplifying item $z = \mathscr{A} \tilde{z}^{1-\eta}$. This indicates actual productivity $\tilde{z}$ is distributed Pareto with tail parameter $(1-\eta)k$ while potential productivity z is distributed Pareto with tail parameter k. Substituting Eq.(3.20) into Eq.(3.19), the expression EC_a is derived for the Exit Condition under autarky:

$$e_a = \frac{\lambda_d}{(1-(1+\hat{\eta})\theta_a)\left(\tilde{z}_a^*/\tilde{Z}_a\right)} = \frac{k\lambda_d}{(1-(1+\hat{\eta})\theta_a)(k - \frac{1}{1-\eta})} \quad \textbf{(}\boldsymbol{EC_a}\textbf{)}$$

Note that under the Pareto distribution given in Eq.(3.4), the EC_a condition illustrates that the consumer's expenditure received by each differentiated domestic firm e_a is independent of its productivity cut-off z_a^*. If I consider a graph with horizontal axis z and vertical axis e, it shows a horizontal line in the graph of (e, z) because e_a does not rely on z_a^*. Eq.(3.4) can be explained as $\tilde{z}_a^*/\tilde{Z}_a$ is a constant as in Eq.(3.20) and only associated with the shape parameter of the Pareto productivity distribution, k and the degree of marginal returns of labour devoted to innovation η, where $\eta \in (0, 1)$. The logic here is that I pin down the equilibrium value of (e, z) through a system with two equations from the Exit Condition and Labour Market Clearing condition of a domestic firm.

I now turn to derive the Labour Market Clearing Condition under autarky, MC_a, which will yield another relationship between e_a and z_a. I focus on a condition for entrants and assume that the total number of non-exporting firms in the market remains constant over time in the steady-state. For this to happen, the following condition must hold:

$$(1 - M)(1 - G(z_a^*)) = \delta M \tag{3.21}$$

where M is the level of the mass of operative varieties. This expression demonstrates that the exit flow of mass, δM, is equivalent to the entry flow of mass, which is identified by the number of potential new varieties, $(1 - M)$, times the probability of surviving, $1 - G(z_a^*)$. Then, I derive the equilibrium mass of the non-exporting varieties by rearranging Eq.(3.21) and substituting Eq.(3.4), which depends on the productivity cutoff z_a^*:

$$M(z_a^*) = \frac{1 - G(z_a^*)}{1 + \delta - G(z_a^*)} = \frac{z_a^{*-k}}{z_a^{*-k} + \delta} \tag{3.22}$$

Eq.(3.22) describes a decreasing relationship between M and the productivity cutoff z_a^*, given $M \in (0, 1/(1+\delta))$. It means the operative mass of variety will decrease when the productivity cutoff z_a^* increases. In addition, I can identify $M(z_a^*) = 0$ when z_a^* approaches infinity and $M(z_a^*) = 1/(1+\delta)$ when $z_{a\ \min}^* = 1$.

I now focus on the labour market and combine Eq.(3.17) to derive the labour market clearing condition, which can be shown:

$$nM\left[\int_{z_a^*}^{\infty} \underbrace{\left((1+\hat{\eta})e\theta_a\frac{\tilde{z}_a}{\tilde{Z}_a} + \lambda_d\right)\mu(z)dz}_{l(\tilde{z})+h(\tilde{z})}\right] + (1-\gamma)E = 1 \tag{3.23}$$

The first element on the left side hand side of Eq.(3.23) explains the total amount of workers devoted to the differentiated goods, including labour involved with innovation activities for a sector s, which is the total number of firms in all operative varieties, times total labour cost per heterogeneous firm, while the second element is the total amount of labour devoted to the homogeneous goods. Since:

$$\int_{z_a^*}^{\infty} \mu(z)dz = \int_{z_a^*}^{\infty} (\tilde{z}_a/\tilde{Z}_a)\mu(z)dz = 1 \tag{3.24}$$

Eq.(3.24) shows the aggregate equilibrium density; by definition, it equals 1. It is used when I simplify Eq.(3.23). After integrating over all varieties and inserting the definition of the consumer's expenditure received by each differentiated firm $e = \gamma E/(nM)$, Eq.(3.24) into Eq.(3.23), I have

$$e_a(z_a^*) = \frac{\frac{1}{nM(z_a^*)} - \lambda_d}{(1+\hat{\eta})\theta_a + \frac{1-\gamma}{\gamma}} \quad \boldsymbol{(MC_a)}$$

Similar to the model without innovation in the previous chapter, the labour market clearing condition for domestic firms (MC_a) with consideration of innovation also describes an increasing relationship between e_a and z_a as the mass of operative varieties M established a negative relationship with z_a^*, shown from Eq.(3.22). In other words, including innovation, (MC_a), the condition shows an increasing relationship with e and z, with the decreasing number of $M(z_a^*)$. However, compared with the model excluding innovation, the model with innovation will shift both the (EC_a) and (MC_a) conditions to the right. That is to say, a horizontal line in (e, z) for (EC_a) would move upwards, and an increasing line of (MC_a) would shift to the right, thus, it leads to a higher cutoff z_a^*.

3.3.2 Equilibrium

Then, equaling the Exit Condition (EC_a) and the Labour Market Clearing Condition (MC_a) yields the following productivity threshold for a non-exporting firm to create the incentive to produce its non-exporting variety, with substitution of $M(z_a^*)$ in Eq.(3.21), I have:

$$\frac{k\lambda_d}{(1-(1+\hat{\eta})\theta_a)(k-\frac{1}{1-\eta})} = \frac{\frac{z_a^{*-k}+\delta}{nz_a^{*-k}} - \lambda_d}{(1+\hat{\eta})\theta_a + \frac{1-\gamma}{\gamma}}$$

Rearranging yields the following equation for the productivity threshold of autarky with the consideration of innovation technology that could be chosen by domestic firms:

$$z_a^* = \left(\frac{n\lambda_d(1+\frac{k}{k-\frac{1}{1-\eta}}\frac{(1+\hat{\eta})\theta_a+\frac{1-\gamma}{\gamma}}{1-(1+\hat{\eta})\theta_a}) - 1}{\delta} \right)^{1/k} \tag{3.25}$$

Notice that I have the assumption of $\lambda_d \geq \bar{\lambda}_a$ where $\bar{\lambda}_a$ is the lowest bound for the fixed operating cost λ_d in the domestic market including innovation, and it is lower than the bound of the model without innovation[5]. The assumption can be derived from the setting of $(EC_a) \geq (MC_a)$ at the minimum productivity $z_{\min} = 1$ because I derive the equilibrium value of (e, z) through the intersection between two lines in (e, z), a horizontal line of EC_a and an increasing line MC_a, separately. Only by restricting the condition of $(EC_a) \geq (MC_a)$ at the $z_{\min}$, I can solve the unique intersection of the two lines. At the extreme case, I have $z_{nin} = z_a^* = 1$ and $(EC_a) = (MC_a)$. Where

$$\bar{\lambda}_a = \frac{\delta+1}{n\left(1+\frac{k}{k-\frac{1}{1-\eta}}\frac{(1+\hat{\eta})\theta_a+\frac{1-\gamma}{\gamma}}{1-(1+\hat{\eta})\theta_a}\right)} \tag{3.26}$$

Proposition 3.1. *Compared to a model without innovation, under autarky, introducing innovation for a domestic firm: i) the survival productivity threshold is higher, ii) the price of the composite goods is lower, iii) the per firm volume of output is higher, iv) mark-ups are the same.*

Proof. See Appendix B.3

This statement is in line with several papers. From a theoretical perspective, like Impullitti and Licandro (2018), they explain that the innovation channel, as the engine of economic growth, produces larger welfare by lowering the equilibrium price, increasing a firm's output and productivity threshold. In addition, R&D

[5]Here, comparing Eq.(3.25) with the solution of the model without innovation Eq. (2.21) in the first main chapter, by the different components in the denominator, I get: $\frac{k}{k-\frac{1}{1-\eta}} > \frac{k}{k-1}$ as $\eta \in (0,1)$; $\frac{(1+\hat{\eta})\theta_a+\frac{1-\gamma}{\gamma}}{1-(1+\hat{\eta})\theta_a} > \frac{\theta_a+\frac{1-\gamma}{\gamma}}{1-\theta_a}$ as $\frac{\partial \frac{\theta+\frac{1-\gamma}{\gamma}}{1-\theta}}{\partial\theta} > 0$ which means $\frac{\theta+\frac{1-\gamma}{\gamma}}{1-\theta}$ is increasing in θ and $(1+\hat{\eta}) > 1$, where $\hat{\eta} = \frac{1-\alpha}{\alpha}\eta > 0$.

activity also varies across firms within the same industry or firms across industries, leading differentiated firms to be more productive and produce more output with a lower price (e.g., Klette and Kortum, 2004; Navas, 2018; Petit and Sanna-Randaccio, 2000). These all support proposition 3.1 to explain that R&D activity as a facilitator enhances productivity, increases output, and decreases prices for domestic firms. Notice that the model with innovation can reduce to the model without innovation by setting the degree of decreasing marginal returns $\eta = 0$ and technology shift parameter $A = 1$.

3.4 Costly Trade

Consider now that firms can export, and it entails both a variable transportation cost of the iceberg type τ, $\tau \geqslant 1$, which indicates the delivery cost of a unit product to go abroad and a fixed cost of exporting $\lambda_x > 0$. A firm producing a traded variety with an initial draw of potential productivity level z competes simultaneously in both domestic and foreign markets, which are referred to by subindices d_x and f_x, respectively. q_{d_x} denotes domestic consumption and production of domestically produced goods. q_{f_x} denotes foreign consumption of the domestically produced good, and its associated products are τq_{f_x}. The firm will produce $q_x = q_{d_x} + \tau q_{f_x}$ but consumers will consume $x_x = (q_{d_x} + q_{f_x})n$ for a particular exporting variety, which is smaller than total production nq_x.

Firms manufacturing the same exported variety compete in two separate Cournot games in domestic and foreign markets. $\hat{x}_{d_x}$ and $\hat{x}_{f_x}$ denote the production of competitors in the domestic and foreign markets, where firms take the production of their competitor as given. Solving the firm's problem:

$$\begin{aligned} \pi_x \quad = \quad & \max_{q_{d_x}, q_{f_x}, , h_x} \underbrace{\frac{\gamma E}{X^\alpha}(\hat{x}_{d_x} + q_{d_x})^{\alpha-1}}_{p_{d_x}} q_{d_x} + \underbrace{\frac{\gamma E}{X^\alpha}(\hat{x}_{f_x} + q_{f_x})^{\alpha-1}}_{p_{f_x}} q_{f_x} \\ & - \underbrace{(Ah^\eta z)}_{\tilde{z}}^{\frac{\alpha-1}{\alpha}} (q_{d_x} + \tau q_{f_x}) - h_x - \lambda_d - \lambda_x \end{aligned} \tag{3.27}$$

Eq.(3.27) expresses the profit function of a firm manufacturing an exporting variety with potential productivity level z relating to actual productivity $\tilde{z}$ after innovation. That is to say, an exporting firm's profits consist of domestic and foreign parts, according to the cost Eq.(3.6) and the inverse demand functions Eq.(3.5). Exporters maximize profits subject to the corresponding domestic and foreign inverse demand functions. Differentiating Eq.(3.27) with regard to its three arguments, domestic sales q_{d_x}, export q_{f_x}, and R&D labour h_x, I have the following expressions from the first order conditions:

$$\frac{\gamma E}{X^{\alpha}}((\alpha-1)(\hat{x}_{d_x}+q_{d_x})^{\alpha-2}q_{d_x}+(\hat{x}_{d_x}+q_{d_x})^{\alpha-1})=\tilde{z}^{\frac{\alpha-1}{\alpha}} \tag{3.28}$$

$$\frac{\gamma E}{X^{\alpha}}((\alpha-1)(\hat{x}_{f_x}+q_{f_x})^{\alpha-2}q_{f_x}+(\hat{x}_{f_x}+q_{f_x})^{\alpha-1})=\tau\tilde{z}^{\frac{\alpha-1}{\alpha}} \tag{3.29}$$

$$\hat{\eta}\tilde{z}^{\frac{\alpha-1}{\alpha}}(q_{d_x}+\tau q_{f_x})/h_x=1 \tag{3.30}$$

where $\hat{\eta}=\frac{1-\alpha}{\alpha}\eta>0$ is the same as it is in autarky. Since I focus on two symmetric countries, the total consumption in both the domestic and foreign markets of a domestically traded variety is defined as $x_x=n(q_{d_x}+q_{f_x})$, which are the same as the total quantities sold in the separate domestic and foreign markets for a traded variety, v, that is, $x_{d_x}=x_{f_x}=x_x$. In other words, there is the same amount of total consumption of a traded variety in the separate domestic and foreign markets, represented by

$$x_x=(q_{d_x}+q_{f_x})n=x_{d_x}=x_{f_x} \tag{3.31}$$

Applying the symmetric relationship implied in Eq.(3.31) to the ratio between Eqs.(3.28) and (3.29), I can derive the ratio between foreign sales, q_{f_x}, and domestic sales, q_{d_x}, of a firm within a traded variety, denoted by β

$$\beta=q_{f_x}/q_{d_x}=\frac{\tau(n+\alpha-1)-n}{n+\alpha-1-n\tau} \tag{3.32}$$

where

$$\frac{\partial\beta}{\partial\tau}=\frac{(2n+\alpha-1)(\alpha-1)}{(n+\alpha-1-n\tau)^2}<0$$

Here, β is the same as the model excluding innovation activity and the characteristics of β are shown as follows: $\beta\in(0,1)$, when $\tau\in(1,\bar{\tau})$.

$$\bar{\tau}=\frac{n}{n+\alpha-1}$$

$\bar{\tau}$ denotes prohibitive trade costs, a limit above which the export markup becomes negative and firms will not export. It is the same as the markups for domestic firms under autarky. Specifically, when $\bar{\tau}=\frac{n}{n+\alpha-1},\beta=0$; while $\tau=1,\beta=1$. When τ is equal to the prohibitive level, $\bar{\tau}$, foreign sales would disappear, $q_{f_x}=0$, thus, $\beta=0$. While for $\tau=1$, free trade happens so that the output for exporters in the foreign market will be the same as in the domestic market; hence, $\beta=1$.

Then, applying symmetry, inserting the Eq.(3.31) to Eqs.(3.28) and (3.29), the optimal price of an exporter in the domestic and foreign markets is given by:

$$p_x=\frac{\tilde{z}^{\frac{\alpha-1}{\alpha}}}{\theta_{d_x}}=\frac{\tau\tilde{z}^{\frac{\alpha-1}{\alpha}}}{\theta_{f_x}} \tag{3.33}$$

where

$$\theta_{d_x} = (2n + \alpha - 1)/n(1 + \tau),\ \theta_{f_x} = \tau\theta_{d_x} \tag{3.34}$$

θ_{d_x} and θ_{f_x} represent the inverse of the markups of exporters charged in the domestic and foreign markets, respectively. These are identical to the case without innovation, as outlined in the first chapter. It shows that exporters charge a higher markup on their domestic sales, $1/\theta_{d_x}$, than on their export sales, $1/\theta_{f_x}$, due to the relationship between θ_{d_x} and θ_{d_x}, $\theta_{f_x} = \tau\theta_{d_x}$. In other words, variable iceberg trade costs result in a lower markup charged in the foreign market than the domestic market for exporters. Notice that lowering the trade cost τ leads to a rise in θ_{d_x}, because the domestic market becomes more competitive due to the penetration of foreign firms, like Impullitti et al. (2018). The pro-competitive effect of trade operates through this mechanism. Besides, a reduction in trade costs τ induces higher markups on export sales, $1/\theta_{f_x}$, since exporters enjoy a cost reduction in their shipment while domestic firms do not benefit from it.[6] When considering trade liberalization, exporters can charge a higher markup on their export sales, not passing the whole cost reduction onto foreign consumers. It is called 'pricing to market', which is a characteristic of oligopoly trade models, like Atkeson and Burstein (2008).

According to Eq.(3.32), the ratio of total production to total consumption of the same traded variety, in line with Brander and Krugman (1983),

$$\frac{q_{d_x} + \tau q_{f_x}}{q_{d_x} + q_{f_x}} = \frac{(1 - n - \alpha)(1 + \tau^2) + 2n\tau}{(1 - \alpha)(1 + \tau)} = \frac{1 + \tau\beta}{1 + \beta} \equiv \Phi > 1 \tag{3.35}$$

Φ measures losses related to international trade due to iceberg costs, and it is the same as the model without the consideration of innovation, which means the cost of importing goods that could be otherwise produced locally. The property of Φ is hump-shaped in τ, for $\tau \in (1, n/(n + \alpha - 1))$, it equals one in the extreme cases of free trade, $\tau = 1$, and the prohibitive trade cost level, $\bar{\tau} = n/(n + \alpha - 1)$, and above one for values between the range. Intuitively, when iceberg trade transportation costs are at the prohibitive level, export sales, q_{f_x}, are zero. In a special case, in autarky, there is no share of production wasted in transportation, implying $\Phi = 1$. A reduction in iceberg cost results in firms having an incentive to export and reduce domestic sales, consequently, the losses associated with trade costs become positive, and Φ increase above one. At the other extreme, which is free trade, there is no waste in transportation costs, that is, $\Phi = 1$, then, any increase in trade cost increases Φ above one, like Impullitti et al. (2018).

[6] See Appendix B.4

In addition, Φ could be applied to define an exporting firm's average markup:

$$\theta_x = \frac{q_{d_x}\theta_{d_x} + q_{f_x}\theta_{f_x}}{q_{d_x} + q_{f_x}} = \Phi\theta_{d_x} \tag{3.36}$$

where θ_x is a weighted average of the respective inverse markups for a firm in the domestic and the foreign markets and the same as the model without innovation. The gap between a firm's average and domestic markup equals the cross-hauling ratio, Φ. According to the definition of Φ, $\theta_{f_x} = \tau\theta_{d_x}$. When iceberg trade costs are at the prohibitive level, $\bar{\tau} = n/(n+\alpha-1)$, I find that $\theta_x = \theta_{d_x} = \theta_a = (n+\alpha-1)/n$, and $\theta_{f_x} = 1$. This is the case which collapses to autarky and there are no foreign sales, $q_{f_x} = 0$. Under free entry, $\tau = 1$, I can arrive at $\theta_x = \theta_{d_x} = (2n+\alpha-1)/2n > \theta_a$, since $\theta_{f_x} = \theta_{d_x}$ [7]. The expression indicates that trade liberalisation decreases a firm's average markup on total sales from the derivative of θ_x with respect to τ. Intuitively, with regard to decreasing variable transportation cost τ, the decrease of the markup of an exporting firm in the domestic market is sufficiently substantial to offset the increase of the markup in the foreign market. It leads to an overall pro-competitive effect of trade liberalisation. Considering Eqs.(3.2) and (3.33), the equilibrium quantities in a country for a given variety denoted by actual productivity $\tilde{z}$ relating to potential productivity level z before innovation under 'all exporters' and exporting fixed cost is:

$$x_x(\tilde{z}) = (\frac{\gamma E}{X^{\alpha}})^{\frac{1}{1-\alpha}}(\theta_{d_x})^{\frac{1}{1-\alpha}}\tilde{z}^{\frac{1}{\alpha}} \tag{3.37}$$

Combining Eqs.(3.3) and (3.37) leads to the following expression for exporters' variable production costs [8]

$$l_x(\tilde{z}) - \lambda_d - \lambda_x = \tilde{z}^{\frac{\alpha-1}{\alpha}}(q_{d_x} + \tau q_{f_x}) = e\theta_x\tilde{z}/\tilde{Z} \tag{3.38}$$

where l_x is the labour cost of production of goods for both domestic and foreign markets.

Rearranging the first order condition for h_x, i.e. Eq.(3.30), and using the expression for labour demand Eq.(3.38), R&D effort is given by:

$$h_x = \hat{\eta}(l_x(\tilde{z}) - \lambda_d - \lambda_x) \tag{3.39}$$

Substituting optimal h_x into the R&D technology, the productivity of the variety is given by:

[7] Notice that θ_x is decreasing in variable trade costs τ: $\frac{\partial\theta_x}{\partial\tau} = -\frac{2(\tau-1)(2n+\alpha-1)^2}{n(1+\tau)^3(1-\alpha)} \leq 0$ reaching its maximum $(2n+\alpha-1)/2n$ when $\tau = 1$.

[8] This follows directly from the derivation in Appendix B.1 under autarky with θ_x replacing θ_a.

$$z = \frac{\tilde{Z}^{\eta}}{A(\hat{\eta}e\theta_x)^{\eta}}\tilde{z}^{1-\eta} \tag{3.40}$$

Similar to the case of autarky, simplifying Eq.(3.40), I show that $\tilde{z}$ is the Pareto distribution with tail parameter $(1-\eta)k$, and z is distributed Pareto with tail parameter k.

3.4.1 Entry and Exit

Equivalent to the case under autarky, the actual productivity threshold $\tilde{z}$ is determined by the exit condition and labour market clearing condition together. According to Eqs.(3.2), (3.37) and (3.38), firm profit with innovation in the 'all exporters' with exporting fixed cost case is:

$$\begin{aligned} \pi_x(\tilde{z}/\tilde{Z}) &= p_x(q_{d_x} + q_{f_x}) - \tilde{z}^{\frac{\alpha-1}{\alpha}}(q_{d_x} + \tau q_{f_x}) - h_x - \lambda_d - \lambda_x \\ &= e(1-(1+\hat{\eta})\theta_x)\tilde{z}/\tilde{Z} - \lambda_d - \lambda_x \end{aligned} \tag{3.41}$$

Then consider the case of the variety with productivity level $\tilde{z}_x^*$ undertaking innovation whose firms break even in the market. The condition defining the actual productivity cut-off is then given by:

$$\pi_x(\tilde{z}_x^*/\tilde{Z}_x) = e_x(1-(1+\hat{\eta})\theta_x)(\tilde{z}_x^*/\tilde{Z}_x) - \lambda_d - \lambda_x \tag{3.42}$$

Notice that I obtain the average actual productivity observed at equilibrium

$$\tilde{Z}_x(z_x^*) = \int_{z_x^*}^{\infty} \tilde{z}\mu(z)dz$$

where equilibrium density is given by:

$$\mu(z) = \begin{cases} \frac{g(z)}{1-G(z_x^*)} & \text{if } z \geqslant z_x^*, \\ 0 & \text{otherwise,} \end{cases}$$

By similar logic of Eq.(3.20) under autarky, I derive the result of the relationship between $\tilde{Z}_x$ and $\tilde{z}_x^*$, under all exporters with exporting costs:

$$\tilde{Z}_x = \frac{k}{k - \frac{1}{1-\eta}}\tilde{z}_x^* \tag{3.43}$$

with Eqs.(3.42) and (3.43), I derive the expression for the Exit Condition under 'all exporters' with exporting fixed cost

$$e_x = \frac{\lambda_d + \lambda_x}{(1-(1+\hat{\eta})\theta_x)\left(\tilde{z}_x^*/\tilde{Z}_x\right)} \quad = \frac{k(\lambda_d + \lambda_x)}{(1-(1+\hat{\eta})\theta_x)(k - \frac{1}{1-\eta})} \quad \boldsymbol{(EC_x)}$$

Notice that the above equation is derived under the Pareto distribution, the (EC_x) condition is independent of the potential productivity cut-off z_x^*. I now use the condition for entrants together with the labour market condition to find another relationship between e_x and z_x.

In line with the case of autarky, the condition for the mass of product lines M requires $(1-M)(1-G(z_x^*)) = \delta M$. This condition states that the exit flow, δM, equals the entry flow defined by the number of the entrants, $(1-M)$, times the probability of surviving, $1-G(z_x^*)$. Consequently,

$$M = M(z_x^*) \equiv \frac{1-G(z_x^*)}{1+\delta-G(z_x^*)} \tag{3.44}$$

a decreasing function of the productivity cutoff z_x^*, $M \in (0, 1/(1+\delta))$.

Analogous to the case of autarky, the labour market clearing condition can be written as:

$$e_x(z_x^*) = \frac{\frac{1}{nM(z_x^*)} - \lambda_d - \lambda_x}{(1+\hat{\eta})\theta_x + \frac{1-\gamma}{\gamma}} \quad \boldsymbol{(MC_x)}$$

the above equation illustrates an increasing relationship between e_x and z_x as $M(z_x^*)$ is a decreasing function of z_x^*.

3.4.2 Equilibrium

Then, the combination of exit condition (EC_x) and labour market clearing condition (MC_x) yields the following productivity threshold for an exporter within an exporting variety to generate an incentive to produce:

$$z_x^* = \left(\frac{n(\lambda_d + \lambda_x)(1 + \frac{k}{k-\frac{1}{1-\eta}} \frac{(1+\hat{\eta})\theta_x + \frac{1-\gamma}{\gamma}}{1-(1+\hat{\eta})\theta_x}) - 1}{\delta} \right)^{1/k} \tag{3.45}$$

under the assumption of $\lambda_d + \lambda_x \geq \bar{\lambda}_x$, where

$$\bar{\lambda}_x = \frac{\delta + 1}{n\left(1 + \frac{k}{k-\frac{1}{1-\eta}} \frac{(1+\hat{\eta})\theta_x + \frac{1-\gamma}{\gamma}}{1-(1+\hat{\eta})\theta_x}\right)}$$

It would be derived from $(EC_x) \geq (MC_x)$ at the minimum productivity $z_{\min} = 1$. Here, under 'all exporters', the lowest bound of the fixed operative cost with the chance for exporters to innovate is lower than the model without innovation[9].

3.4.3 Trade Liberalisation

How would trade liberalisation affect an exporter's productivity threshold? According to Eq.(3.45), I can identify that an exporter's productivity threshold will increase at equilibrium because of trade liberalisation as the derivative of z_x^* with respect to θ_x is positive [10], and θ_x is decreasing in τ. That is to say; exporters have a selection effect when the scenario of all exporters of the economy is considered. Compared to the model without innovation, as in the case of autarky in the above section, the productivity threshold of an exporter here is higher because an exporter performs innovation.

Proposition 3.2. *Introducing innovation, a movement from autarky to free trade, i) increases investment in R&D activity, $h_x > h_a$, ii) induces a pro-competitive effect in the domestic market, iii) results in a higher survival productivity cutoff compared to the model without innovation.*

Proof. See Appendix B.5.

Proposition 3.2 shows that the R&D effort for exporters is greater than domestic firms through the pro-competitive effect. Several pieces of literature support the above statement (e.g., Atkeson and Burstein, 2010; Navas and Licandro, 2011; Peretto, 2003). For example, Navas and Licandro (2011) illustrate that openness to trade enhances innovation through a pro-competitive effect. Navas (2015) extends the framework of Navas and Licandro (2011) and documents that the movement from autarky to free trade induces innovation in those less competitive sectors when he considers sectoral heterogeneity with different markups and the degree of trade openness. Peretto (2003) built a world economy model with oligopolistic manufacturing firms and suggests that firms in a larger and more competitive market will increase their investment in R&D. Atkeson and Burstein (2010) indicate that trade liberalisation results in more investment in process innovation, leading to a larger increase in the volume of trade. In my model, innovation, as a new channel, interacting with the pro-competitive effect and selection effect, leads to openness to trade, produces a higher productivity threshold, more output and a lower price than the model without innovation in chapter two.

[9] This is similar to the 'Autarky'.

[10] In equilibrium, the component of $\frac{(1+\hat{\eta})\theta + \frac{1-\gamma}{\gamma}}{1-(1+\hat{\eta})\theta}$ in the productivity threshold of Eq.(3.45) is strictly increasing in θ for $\theta \in (0,1)$ and hence, z^* is strictly increasing with respect to its associated θ in that interval.

3.5 Multinational Production

Let us now consider the scenario in which all firms serve the foreign market via horizontal FDI rather than exporting. In order to serve the foreign markets, firms are going to bear a higher fixed cost λ_m than the fixed exporting cost λ_x, and with no iceberg transportation costs, where λ_m is associated with building a new plant in the foreign market, like Helpman et al. (2004). λ_m is lower than the overhead production cost involved in the domestic market λ_d, as the firm chooses to duplicate some of the overhead production costs. That is, I assume $\lambda_m < \lambda_d$ since only by doing it does a firm have the incentive to undertake FDI because of the reduced fixed costs in the foreign market. Similar to the case of 'all exporters', domestically multinational firms producing the same variety with actual productivity level $\tilde{z}$ compete Cournot games simultaneously in both domestic and foreign markets via FDI, which are referred to by subindices d_m and f_m, respectively. q_{d_m} denotes domestic consumption and production of the domestically produced good, q_{f_m} denotes foreign consumption of the domestically produced good through FDI. Therefore, a multinational firm will produce $q_m = q_{d_m} + q_{f_m}$ and consumers will consume $x_m = n(q_{d_m} + q_{f_m})$ for a particular variety with actual productivity $\tilde{z}$, as n identical multinational firms within a variety play a symmetric Cournot game. $\hat{x}_{d_m}$ and $\hat{x}_{f_m}$ are the production of direct competitors within a particular variety doing FDI in the corresponding domestic and foreign markets through FDI, where firms take their competitors' strategy as given. So, the firm's problem is given by:

$$\begin{aligned}\pi_m \quad = \quad & \max_{q_{d_m}, q_{f_m},, h_m} \underbrace{\frac{\gamma E}{X^\alpha}(\hat{x}_{d_m} + q_{d_m})^{\alpha-1}}_{p_{d_m}} q_{d_m} + \underbrace{\frac{\gamma E}{X^\alpha}(\hat{x}_{f_m} + q_{f_m})^{\alpha-1}}_{p_{f_m}} q_{f_m} \\ & -\underbrace{(Ah^\eta z)}_{\tilde{z}}^{\frac{\alpha-1}{\alpha}}(q_{d_m} + q_{f_m}) - h_m - \lambda_d - \lambda_m\end{aligned} \tag{3.46}$$

Eq.(3.46) shows the profit function of a multinational firm manufacturing a special product line, v, with actual productivity level $\tilde{z}$ which means the profits of a multinational firm that is capable of doing FDI, consist of both domestic and foreign parts, according to the cost function Eq.(3.6) and the inverse demand Eq.(3.2). Multinational firms maximize profits subject to the corresponding domestic and foreign inverse demand functions of Eq.(3.2). The first order conditions for domestic sales, q_{d_m}, and foreign sales, q_{f_m}, for a multinational firm within a specific variety doing FDI are, respectively,

$$\frac{\gamma E}{X^\alpha}((\alpha - 1)(\hat{x}_{d_m} + q_{d_m})^{\alpha-2} q_{d_m} + (\hat{x}_{d_m} + q_{d_m})^{\alpha-1}) = \tilde{z}^{\frac{\alpha-1}{\alpha}} \tag{3.47}$$

$$\frac{\gamma E}{X^{\alpha}}((\alpha-1)(\hat{x}_{f_m}+q_{f_m})^{\alpha-2}q_{f_m}+(\hat{x}_{f_m}+q_{f_m})^{\alpha-1})=\tilde{z}^{\frac{\alpha-1}{\alpha}} \tag{3.48}$$

The first order condition for R&D labour for a multinational firm within a variety doing FDI with actual productivity $\tilde{z}$ is

$$\hat{\eta}\tilde{z}^{\frac{\alpha-1}{\alpha}}(q_{d_m}+q_{f_m})/h_m=1 \tag{3.49}$$

where $\hat{\eta}=\frac{1-\alpha}{\alpha}\eta>0$ is the same as shown in the above scenarios of autarky and 'all exporters'. Total consumption in both domestic and foreign markets of a domestic specific variety doing FDI is defined as $x_m=(q_{d_m}+q_{f_m})n$. Since I assume perfect symmetry across two countries, the total quantities sold in the separate domestic and foreign markets for a specific variety v doing FDI, denoted by x_{d_m} and x_{f_m}, are equal and equal to x_m, that is,

$$x_{d_m}=x_{f_m}=x_m=(q_{d_m}+q_{f_m})n \tag{3.50}$$

Using the symmetric relationship implied above and Eqs.(3.47) and (3.48), I can derive

$$q_{d_m}=q_{f_m} \tag{3.51}$$

This indicates that, with the innovation process, domestic and foreign consumption of domestically produced goods via FDI are equal. That is, a multinational firm within a specific variety doing FDI has the same production for domestic and foreign sales.

Then, inserting Eqs.(3.50) and (3.51) into Eqs.(3.47) and (3.48), I have

$$\underbrace{\frac{\gamma E}{X^{\alpha}}x_m^{\alpha-1}}_{p_m}(\frac{(\alpha-1)q_{d_m}}{n(q_{d_m}+q_{f_m})}+1)=\tilde{z}^{\frac{\alpha-1}{\alpha}}$$

$$\underbrace{\frac{\gamma E}{X^{\alpha}}x_m^{\alpha-1}}_{p_m}(\frac{(\alpha-1)q_{f_m}}{n(q_{d_m}+q_{f_m})}+1)=\tilde{z}^{\frac{\alpha-1}{\alpha}}$$

The optimal price of a domestic multinational firm within a variety doing FDI for both domestic and foreign markets with actual productivity $\tilde{z}$ after the innovation process is as follows:

$$p_m=\frac{\tilde{z}^{\frac{\alpha-1}{\alpha}}}{\theta_{d_m}}=\frac{\tilde{z}^{\frac{\alpha-1}{\alpha}}}{\theta_{f_m}} \tag{3.52}$$

where

$$\theta_{d_m}=\theta_{f_m}=\theta_m=(2n+\alpha-1)/2n \tag{3.53}$$

θ_{d_m} and θ_{f_m} represent the inverse of markups of a domestic multinational firm within a variety doing FDI charged in the domestic and foreign markets, respectively, even when I consider there is an opportunity for firms to consider the process of innovation. The iceberg transportation costs τ don't appear in the equation of θ_{d_m} and θ_{f_m}. In other words, variable trade costs have no relationship with markups of multinational firms within a variety via FDI, since multinational firms within a variety do not suffer iceberg transportation costs but have to pay fixed cost λ_m to establish a plant in the foreign market. By observing that θ_{d_m} equals θ_{d_x}, when $\tau = 1$, which means the markup of the exporter and multinational firm on domestic sales are the same when there is no iceberg trade cost. According to Eqs.(3.2) and (3.52), I have that equilibrium consumption of a domestic firm in a country for a given variety is given by:

$$x_m(\tilde{z}) = (\frac{\gamma E}{X^\alpha})^{\frac{1}{1-\alpha}}(\theta_{d_m})^{\frac{1}{1-\alpha}}\tilde{z}^{\frac{1}{\alpha}} \tag{3.54}$$

The following expression is a domestic multinational firm's variable production costs [11]

$$l_m(\tilde{z}) - \lambda_d - \lambda_m = \tilde{z}^{\frac{\alpha-1}{\alpha}}(q_{d_m} + q_{f_m}) = e\theta_m\tilde{z}/\tilde{Z} \tag{3.55}$$

where l_m is the labour cost of production of goods produced by a multinational firm domestically for domestic and foreign markets within a variety doing FDI with innovation.

Rearranging the first order condition for h_m, i.e. Eq.(3.49), and using the expression for labour demand above, R&D effort is given by:

$$h_m = \hat{\eta}(l_m(\tilde{z}) - \lambda_d - \lambda_m) \tag{3.56}$$

Substituting optimal h_m into the R&D technology, the productivity of this variety is given by:

$$z = \frac{\tilde{Z}^\eta}{A(\hat{\eta}e\theta_m)^\eta}\tilde{z}^{1-\eta} \tag{3.57}$$

Similar to the above case of autarky and 'all exporters', simplifying Eq.(3.57), $\tilde{z}$ is Pareto distribution with tail parameter $(1-\eta)k$ and z is distributed Pareto with tail parameter k.

[11] This follows directly from the derivation in Appendix B.1 under autarky with θ_m replacing θ_a.

3.5.1 Entry and Exit

Analogous to the scenarios under autarky and ‘all exporters’, the actual cutoff productivity, $\tilde{z}$, is derived by the exit condition. Applying Eqs.(3.52), (3.54) and (3.55), a firm’s profit is:

$$\begin{aligned}\pi_m(\tilde{z}/\tilde{Z}) &= p_m(q_{d_m} + q_{f_m}) - \tilde{z}^{\frac{\alpha-1}{\alpha}}(q_{d_m} + q_{f_m}) - h_m - \lambda_d - \lambda_m \\ &= e(1-(1+\hat{\eta})\theta_m)\tilde{z}/\tilde{Z} - \lambda_d - \lambda_m\end{aligned} \tag{3.58}$$

The above equation indicates the operating profits of the multinational firm with the innovation process as a function of two endogenous variables, $\tilde{z}$ and e. Then consider the case of the variety with actual productivity level $\tilde{z}^*_m$ whose multinational firms break even in the market. The condition defining the actual productivity cut-off is then given by:

$$\pi_m(\tilde{z}^*_m/\tilde{Z}_m) = e_m(1-(1+\hat{\eta})\theta_m)(\tilde{z}^*_m/\tilde{Z}_m) - \lambda_d - \lambda_m \tag{3.59}$$

Notice that in equilibrium, I obtain that average actual productivity is:

$$\mu(z) = \begin{cases} \frac{g(z)}{1-G(z^*_m)} & \text{if } z \geqslant z^*_m, \\ 0 & \text{otherwise,} \end{cases}$$

where the equilibrium density is given by:

$$\tilde{Z}_m(z^*_m) = \int_{z^*_m}^{\infty} \tilde{z}\mu(z)dz$$

Similar reasoning of Eq.(3.20) under autarky, I derive the result of the relationship between $\tilde{Z}_m$ and $\tilde{z}^*_m$, under all undertaking FDI with an extra fixed plant cost in the foreign market:

$$\tilde{Z}_m = \frac{k}{k - \frac{1}{1-\eta}}\tilde{z}^*_m \tag{3.60}$$

Using Eqs.(3.59) and (3.60), I derive the expression for the exit condition with a domestic multinational firm:

$$e_m = \frac{\lambda_d + \lambda_m}{(1-(1+\hat{\eta})\theta_m)\left(\tilde{z}^*_m/\tilde{Z}_m\right)} = \frac{k(\lambda_d + \lambda_m)}{(1-(1+\hat{\eta})\theta_m)(k - \frac{1}{1-\eta})} \quad \boldsymbol{(EC_m)}$$

Notice that under the Pareto distribution, the above exit condition (EC_m) for a multinational firm within a particular variety undertaking FDI with the consideration of the innovation process for firms to choose is independent of the potential productivity cut-off z_m^*. Then I use the condition for entrants and labour market conditions to find another relationship between $e_m$and z_m. In line with the cases of autarky and 'all exporters', determining a steady state mass of product lines, M requires $(1-M)(1-G(z_m^*)) = \delta M$ to hold. Consequently,

$$M = M(z_m^*) \equiv \frac{1-G(z_m^*)}{1+\delta-G(z_m^*)} \tag{3.61}$$

it indicates a decreasing function of M containing the productivity cutoff z_m^*, for $M \in (0, 1/(1+\delta))$

Analogous to the logic of the scenarios with autarky and 'all exporters', the labour market clearing condition under all firms undertaking FDI can be written as follows:

$$e_m(z_m^*) = \frac{\frac{1}{nM(z_m^*)} - \lambda_d - \lambda_m}{(1+\hat{\eta})\theta_m + \frac{1-\gamma}{\gamma}} \quad \boldsymbol{(MC_m)}$$

it implies an increasing relationship between e_m and z_m as I have established $M(z_m^*)$ is a decreasing function of z_m^*.

3.5.2 Equilibrium

I can equate the exit condition (EC_m) and labour market clearing condition (MC_m) to yield the following equilibrium productivity threshold for a multinational firm within a particular variety undertaking FDI, with the incentive to produce under the innovation procedure:

$$z_m^* = \left(\frac{n(\lambda_d + \lambda_m)(1 + \frac{k}{k-\frac{1}{1-\eta}} \frac{(1+\hat{\eta})\theta_m + \frac{1-\gamma}{\gamma}}{1-(1+\hat{\eta})\theta_m}) - 1}{\delta} \right)^{1/k} \tag{3.62}$$

under the assumption of $\lambda_d + \lambda_m \geq \bar{\lambda}_m$, where:

$$\bar{\lambda}_m = \frac{\delta+1}{n\left(1 + \frac{k}{k-\frac{1}{1-\eta}} \frac{(1+\hat{\eta})\theta_m + \frac{1-\gamma}{\gamma}}{1-(1+\hat{\eta})\theta_m}\right)}$$

With similar reasoning to 'autarky', $\bar{\lambda}_m$ is derived from $(EC_m) \geq (MC_m)$ at the minimum productivity $z_m^* = 1$. Compared to the model without innovation, $\bar{\lambda}_m$ here with innovation expresses a lower bound for total fixed costs, including λ_d and λ_m. It guarantees that there exists a unique interior solution of the average consumer's expenditure received by each multinational firm e_m and equilibrium productivity threshold z_m^* for sector s. The solution of $M(z_m^*)$ determined by the intersection of MC_m and EC_m is shown in Eq.(3.61).

Proposition 3.3. *Introducing innovation, a movement from autarky to multinational production, i) increases investment in R&D activity, $h_m > h_x > h_a$, ii) increases the survival productivity cutoff $z_m^* > z_x^* > z_a^*$ compared to the model without innovation, iii) induces increasingly lower markups in the domestic market, $\theta_m > \theta_x > \theta_a$, which is the same as the model with no innovation.*

Incorporating horizontal FDI and innovation, my model shows how a multinational firm produces the highest productivity threshold, the largest output and the lowest price than exporters and non-exporters through three channels: the lowest markup from the oligopolistic competition, the toughest selection effect and the largest R&D investment. Notice that free trade without variable transportation costs is the extreme case of 'all multinational firms' but with lower fixed exporting costs in the foreign market. Because of that, the higher R&D investment and productivity threshold hold for multinational firms compared to exporters with free trade, while there are the same markups for both scenarios. The finding is related to Petit and Sanna-Randaccio (2000), they find a positive relationship between the multinational firm and R&D investment, which indicates that multinational production invests more in research. This investment increases the expansion of multinational firms simultaneously, with a model of two countries with imperfect competition allowing firms to face different decisions, which endogenize the market structure.

3.6 Welfare Analysis

This section will compare the welfare gains of three separate scenarios: all are domestic firms, exporters and multinational firms, by identifying and decomposing welfare.

3.6.1 Decomposition of Welfare Effects

Like Impullitti and Licandro (2018), the welfare effect of trade can be decomposed into three different channels considering the firm's willingness to innovate. Here, love for variety exists in the model, which indicates the positive welfare gains stemming from the selection effect may be offset by the reduction in the mass of varieties $M \in (0, 1)$, shown by Eqs.(3.22), (3.44) and (3.61) for each scenario. In contrast to the above section for a specific sector s, here, I investigate the welfare gains of the whole economy at equilibrium in the aggregate level, deriving from the given utility function:

$$\bar{\gamma} = \sum_{s=1}^{S} \gamma_s$$

and the average consumer's expenditure received by each differentiated firm among all sectors:

$$\bar{e} = \frac{\bar{\gamma} E}{nM}$$

I generalise the notation in the welfare analysis for the scenarios of 'autarky', 'all exporters', and 'all multinational firms' separately and decompose the welfare effects of trade into their different channels. Notice that as long as love-for-variety matters, selection always results in a reduction in a mass of varieties. Aggregate equilibrium welfare can be decomposed into three terms as follows[12]:

$$U = \underbrace{\bar{\gamma}\frac{1-\alpha}{\alpha} \ln M\, \tilde{Z}}_{\text{Productivity/LFV}} + \underbrace{\bar{\gamma} \ln \bar{e} n M \theta}_{\text{Consumption}} + \underbrace{(1-\bar{\gamma}) \ln \frac{(1-\bar{\gamma})\bar{e} n M}{\bar{\gamma}}}_{\text{Homogeneous good}} \tag{3.63}$$

There are three different channels: the first two are associated with the consumption of differentiated goods, while the third is related to the homogeneous sector. The first reflects the productivity gains due to selection, which increases average actual productivity $\tilde{Z}$ after the process innovation and the welfare losses due to fewer varieties M. The second is associated with the consumption of composite goods and the oligopolistic distortions in these sectors. Remember that $\theta_{d_x} = \theta_x / \Phi$, combining the pro-competitive effect of trade with the cross-hauling effect, as measured by Φ. The third component measures the utility of homogeneous goods.

In line with Impullitti and Licandro (2018), welfare gains from selection operate through $\tilde{Z}$, M and $\bar{e}$, all depending on z^*. I can decompose Eq.(3.63) by differentiating it with respect to z^* [13]:

$$\begin{aligned}
\text{Selection gains} &= \underbrace{\bar{\gamma}\frac{1-\alpha}{\alpha}\left(\frac{1}{\tilde{Z}}\frac{\partial \tilde{Z}}{\partial z^*} + \frac{1}{M}\frac{\partial M}{\partial z^*}\right)}_{\text{Productivity/LFV}} + \underbrace{\left(\frac{1}{\bar{e}}\frac{\partial \bar{e}}{\partial M} + \frac{1}{M}\right)\frac{\partial M}{\partial z^*}}_{\text{Fixed Cost}} \\
&= \underbrace{\frac{\bar{\gamma}}{z^*}\frac{1-\alpha}{\alpha}\left[\frac{1}{1-\eta} - \frac{k\delta}{z^{*-k}+\delta}\right]}_{\text{Productivity/LFV}} + \underbrace{\left\{\frac{n\delta k \lambda_d z^{*k-1}}{(1+\delta z^{*k})^2 (1-nM\lambda_d)}\right\}}_{\text{Fixed Cost}}
\end{aligned} \tag{3.64}$$

The above Eq.(3.64) can be derived from $\bar{e} \equiv \bar{\gamma} E/(nM)$, where $\bar{e}$ is the aggregate level of the consumers' expenditure received by each differentiated firm. $\partial \bar{e}/\partial M$ is

[12] See Appendix B.6

[13] See Appendix B.7

obtained by differentiating (MC), $\partial M/z^*$ by differentiating Eqs.(3.22), (3.44) and (3.61), $\partial\tilde{Z}/\partial z^*$by differentiating the definition of $\tilde{Z}$, the actual average productivity. Beginning with the first component, I identify that the selection effect induces an increase in average productivity as we know $\frac{1}{\tilde{Z}}\frac{\partial\tilde{Z}}{\partial z^*} > 0$. This component also includes love-for-variety (LFV) losses, leading to the reductions in the mass of available varieties as I have $\frac{1}{M}\frac{\partial M}{\partial z^*} < 0$ (See Appendix B.7). The two factors mean that the productivity/LFV trade-off is a positive welfare effect only through a sufficiently small enough value of the exogenous death rate δ. The second component illustrates the change in labour applied to producing the composite good (excluding the fixed costs). Selection expels some firms from the market, reducing the resources needed to cover fixed production costs. These resources are used by surviving firms, producing more production and consumption. Therefore, I denote the second part of Eq.(3.64) as 'fixed costs' since the mechanism is conducted through the fixed cost channel.

Proposition 3.4. *Introducing innovation, the selection provides (i) welfare gains through the fixed cost channel, and (ii) the productivity/LFV trade-off channel generates positive welfare effects for sufficiently small values of the exogenous death rate δ, where the boundary of δ is larger than it is without innovation:*

$$\delta < \frac{z^{*-k}}{(1-\eta)k-1} \tag{3.65}$$

Proof. See Appendix B.8

This is similar to the selection effect in the stationary equilibrium of Impullitti and Licandro (2018). It means the productivity/LFV trade-off would gain welfare when there is a sufficiently small value of the exogenous death shock δ. The restriction of the upper boundary of δ is larger than it is without innovation, which indicates that, including innovation, there are more options for value δ compared to the model without innovation. In other words, incorporating R&D releases some restrictions on δ, to some extent. Another component explains the change in labour allocated to the heterogeneous goods through the fixed costs channel, which always generates welfare gains.

3.6.2 Welfare Comparison

This part aims to compare the welfare gains from three scenarios I have considered 'autarky', 'all exporters', and 'all multinational firms'. Combining the productivity threshold and the welfare gains at equilibrium in the above section for three scenarios, in the condition of $\delta < \frac{z_m^{*-k}}{(1-\eta)k-1}$, I have that

Proposition 3.5. *Introducing innovation, a movement from autarky to multinational production, welfare gains: i) increase through the pro-competitive effect, selection* *channel and innovation channel, ii) generate more significant gains compared to the model without innovation.*

The innovation effect is in line with Impullitti and Licandro (2018). Including innovation, welfare gains from 'all multinational firms' are the largest, as they have the lowest markups from the oligopolistic competition, the highest productivity threshold at equilibrium for the selection effect and the highest R&D investment from the innovation channel, compared to the scenarios of 'all exporters' and 'autarky'. That is to say, innovation, as the engine of firms' development, complements the first chapter as the third mechanism, which would interact with firms' selection, generates higher productivity by undertaking R&D activities compared to the model without innovation. The welfare effect from multinational firms is the largest compared to exporters and domestic firms through the mechanism of the highest level of the pro-competitive effect, selection effect and innovation. Let us consider the extreme case of 'all exporters', which is free trade, with the case of 'all multinational firms'. I can find the largest welfare gains for multinational firms as they bear higher fixed costs of introducing plants in the foreign market compared to the fixed exporting cost of exporters, stemming from higher consumer expenditure received by each multinational firm, which is $\bar{e_m} > \bar{e_x}$.

3.7 Conclusion

This chapter investigates the welfare gains of trade and multinational production via horizontal FDI, providing the opportunity for firms to undertake R&D activities. I consider process innovation, which represents the investment to an existing firm's specific factor (e.g., Impullitti et al., 2018; Navas, 2015) rather than including technological spillovers among firms or product innovation (e.g., Atkeson and Burstein, 2010; Impullitti and Licandro, 2018; Klette and Kortum, 2004).

The analysis presented in this chapter introduces cost-reducing innovation into the simple version of the model of the first main chapter to investigate how innovation affects the welfare effect of trade and FDI with firm heterogeneity and variable markups from oligopolistic competition. I find that innovation increases firms' productivity thresholds, profits, and welfare gains of domestic firms, exporters and multinational firms via horizontal FDI by allocating reduced labour costs. In addition, trade liberalisation increases R&D efforts when there is a movement from autarky to free trade, which is in line with the less competitive sector in Navas (2015). Moreover, innovation would interact with the selection effect, which enlarges the welfare gains of international trade and multinational firms, compared to the model without innovation. Multinational firms, a better strategy for serving the foreign market, would generate the highest welfare gains in the global economy through the strongest competition from the oligopolistic competition, the highest productivity thresholds of the selection effect and the largest investment of R&D activities, compared to exporters and domestic firms.

This chapter could be extended in two dimensions. Firstly, a global economy including domestic firms, exporters and multinational firms via horizontal FDI coexisting in the same sector could be considered to explore the welfare effect of trade liberalisation for trade and FDI with firm heterogeneity, variable markups from the oligopolistic competition and process innovation. It would make the model more interesting by adding innovation and the coexistence of domestic firms, exporters and multinational firms. However, it would be complex and potentially intractable with the possibility of the loss of the closed solution of the economic system. Secondly, this chapter assumes a world economy with two identical countries, explaining the welfare effect of regional integration among similar countries. Another extension would be the study of economies with different initial conditions, for example, different factor endowments or technological levels, which generates possible study of the interaction between developed and developing countries, like Navas and Licandro (2011). However, incorporating strategic interaction among firms through oligopolistic competition with firm heterogeneity and multinational firms into different initial conditions for two countries would be extremely challenging and beyond the scope of this thesis.

Policy implications for the government are based on my finding that investment in innovation and being a multinational firm to serve the foreign market would increase the welfare gains in the economy. Therefore, it is potentially significant for policymakers to consider improving firms' R&D investment to reduce their marginal costs and increase productivity. For example, the government could guarantee intellectual property rights and provide incentives to encourage cooperative research development between universities and industries. In addition, given that the findings suggest that being a multinational firm to serve the foreign could be a better strategy to increase the economy's welfare, the government could issue tax subsidies, such as tax holidays and job-creation subsidies, to encourage firms to be multinational corporations, as suggested by Chor (2009). These arguments are also supported by several studies (e.g., Atkeson and Burstein, 2010; Impullitti et al., 2018; Klette and Kortum, 2004; Navas, 2015; Petit and Sanna-Randaccio, 2000). For example, Petit and Sanna-Randaccio (2000) construct a theoretical model to explain the relationship between foreign strategies and innovation by considering a three-stage game, which includes 1) the strategic expansion to a foreign market, 2) the investment in R&D and 3) the amount they will serve in each market. They found that the positive association between multinational firms and innovation, especially R&D investment, motivates firms to be multinationals, and multinationals spend more on innovation;

therefore, consumer welfare will increase for multinationals rather than exporters. Accordingly, they suggest policy implications for the government that multinational firms undertaking R&D could be a better strategy for firms to serve the foreign market to avoid the variable transportation costs than exporters, increasing the welfare gains of the economy.

Appendix B

B.1 Equation (3.14)

Using Eqs.(3.3) and (3.12) so that I can obtain an expression for X in the case in which all are domestic firms:

$$X = [\int_0^M (\tilde{z}^{\frac{1}{\alpha}}[\frac{\gamma E}{X^\alpha}\theta_a]^{\frac{1}{1-\alpha}})^\alpha dv]^{\frac{1}{\alpha}}$$

Rearranging and using the definition of the average productivity of the overall economy, I have: $\tilde{Z} = (1/M)\int_0^M \tilde{z}_v d_v$

$$X^\alpha = (\frac{\gamma E}{X^\alpha}\theta_a)^{\frac{\alpha}{1-\alpha}} \int_0^M \tilde{z}_v d_v = (\frac{\gamma E}{X^\alpha}\theta_a)^{\frac{\alpha}{1-\alpha}} M\tilde{Z}$$

Rearranging the above terms, I focus on the case that all are domestic firms and identify the aggregate composite goods for each sector s:

$$X^{\frac{\alpha}{1-\alpha}} = (\gamma E\theta_a)^{\frac{\alpha}{1-\alpha}} M\tilde{Z} \tag{B.1}$$

Then, I derive a domestic firm's variable production cost, based on Eqs.(3.4) and (3.12), $e = \gamma E/(nM)$ and the above equation, and symmetry $x_a = q_a + \hat{x}_a = nq_a$:

$$\begin{aligned}
l_a(\tilde{z}) - \lambda_d &= \tilde{z}^{\frac{\alpha-1}{\alpha}} q_a(\tilde{z}) \\
&= \tilde{z}^{\frac{\alpha-1}{\alpha}} \frac{q_a}{x_a} x_a = \tilde{z}^{\frac{\alpha-1}{\alpha}} \frac{q_a}{x_a} (\frac{\gamma E}{X^\alpha})^{\frac{1}{1-\alpha}} \theta_a^{\frac{1}{1-\alpha}} \tilde{z}^{\frac{1}{\alpha}} \\
&= \frac{q_a}{x_a} \frac{(\gamma E)^{\frac{1}{1-\alpha}}}{(\gamma E\theta_a)^{\frac{\alpha}{1-\alpha}}} \theta_a^{\frac{1}{1-\alpha}} \frac{\tilde{z}}{M\tilde{Z}} \\
&= \frac{q_a}{nq_a} \gamma E\theta_a \frac{\tilde{z}}{M\tilde{Z}} \\
&= e\theta_a \frac{\tilde{z}}{\tilde{Z}}
\end{aligned}$$

B.2 Equation (3.20)

From the Pareto distribution $G(z)$, I have $1 - G(z) = (\frac{1}{z})^k$, $g(z) = kz^{-k-1}$, $zg(z) = kz^{-k}$. Since I have defined $\tilde{Z}_a(z_a^*) = \int_{z_a^*}^{\infty} \tilde{z}\mu(z)dz$, combining with the definition of $\mu(z)$, $\tilde{Z}_a$, and rearranging Eq.(3.16), I have $\tilde{z} = [A(\hat{\eta}e\theta_a)^\eta \tilde{Z}^{-\eta}]^{\frac{1}{1-\eta}} z^{\frac{1}{1-\eta}}$, then substituting $\tilde{Z}_a(z_a^*) = \int_{z_a^*}^{\infty} \tilde{z}\mu(z)dz$, I have:

$$
\begin{aligned}
\tilde{Z}_a(z_a^*) &= \int_{z_a^*}^{\infty} \tilde{z}\mu(z)dz = \frac{[A(\hat{\eta}e\theta_a)^\eta]^{\frac{1}{1-\eta}}}{\tilde{Z}_a^{\frac{\eta}{1-\eta}}} \int_{z_a^*}^{\infty} z^{\frac{1}{1-\eta}}\mu(z)dz \\
\tilde{Z}_a^{\frac{1}{1-\eta}} &= [A(\hat{\eta}e\theta_a)^\eta]^{\frac{1}{1-\eta}} \int_{z_a^*}^{\infty} z^{\frac{1}{1-\eta}}\mu(z)dz \Rightarrow \\
[\frac{\tilde{Z}_a}{A(\hat{\eta}e\theta_a)^\eta}]^{\frac{1}{1-\eta}} &= \int_{z_a^*}^{\infty} z^{\frac{1}{1-\eta}}\mu(z)dz = \int_{z_a^*}^{\infty} z^{\frac{1}{1-\eta}} \frac{g(z)}{1 - G(z_a^*)}dz \\
&= \int_{z_a^*}^{\infty} z^{\frac{1}{1-\eta}} \frac{kz^{-k-1}}{z_a^{*-k}}dz = kz_a^{*k} \int_{z_a^*}^{\infty} z^{-k-1+\frac{1}{1-\eta}}dz \\
&= \frac{kz_a^{*k}}{-k + \frac{1}{1-\eta}} z^{-k+\frac{1}{1-\eta}}|_{z_a^*}^{\infty} \\
&= \frac{k}{k - \frac{1}{1-\eta}} z_a^{*\frac{1}{1-\eta}} \Rightarrow \\
\tilde{Z}_a^{\frac{1}{1-\eta}} &= \frac{k}{k - \frac{1}{1-\eta}} z_a^{*\frac{1}{1-\eta}} [A(\hat{\eta}e\theta_a)^\eta]^{\frac{1}{1-\eta}} \Rightarrow \\
\tilde{Z}_a &= \frac{k}{k - \frac{1}{1-\eta}} [A(\hat{\eta}e\theta_a)^\eta \tilde{Z}_a^{-\eta}]^{\frac{1}{1-\eta}} z_a^{*\frac{1}{1-\eta}} \Rightarrow \\
\tilde{Z}_a &= \frac{k}{k - \frac{1}{1-\eta}} \tilde{z}_a^* \Rightarrow \\
\frac{\tilde{Z}_a}{\tilde{z}_a^*} &= \frac{k}{k - \frac{1}{1-\eta}}
\end{aligned}
$$

B.3 Proposition 3.1

There are two ingredients at force to represent the innovation effect on the productivity threshold. The first component $\frac{k}{k-\frac{1}{1-\eta}}$ in the numerator of the equilibrium threshold is strictly increasing in $\eta \in (0, 1)$), hence $\frac{k}{k-\frac{1}{1-\eta}}$ becomes larger compared to the extreme case with no innovation $\eta = 0$. The second component is $\frac{(1+\hat{\eta})\theta+\frac{1-\gamma}{\gamma}}{1-(1+\hat{\eta})\theta}$, where $\frac{\theta+\frac{1-\gamma}{\gamma}}{1-\theta}$ is strictly increasing in θ for $\theta \in (0, 1)$ and I know $\hat{\eta} > 0$ as $1+\hat{\eta} > 0$. Therefore,

$\frac{(1+\hat{\eta})\theta_a+\frac{1-\gamma}{\gamma}}{1-(1+\hat{\eta})\theta_a} > \frac{\theta_a+\frac{1-\gamma}{\gamma}}{1-\theta_a}$ which indicates the second component operates to enhance the innovation effect in the productivity threshold. Combining Eqs. (3.11),(3.12) and (3.16), under autarky, I get the lower equilibrium price, and that the equilibrium output of a domestic firm is higher when I consider the chance for firms to undertake innovation compared to the model without innovation.

B.4 Derivatives θ_{d_x} and θ_{f_x} with regard to τ

I can derive the derivatives of θ_{d_x}, $\theta_{f_x}$with respect to $\tau \in (1, \bar{\tau})$

$$\frac{\partial \theta_{d_x}}{\partial \tau} = \frac{\partial \frac{2n+\alpha-1}{n(1+\tau)}}{\partial \tau} = -\frac{(2n+\alpha-1)}{n(1+\tau)^2} = -\frac{\theta_{d_x}}{(1+\tau)} < 0$$

$$\frac{\partial \theta_{f_x}}{\partial \tau} = \frac{\partial(\tau\theta_{d_x})}{\partial \tau} = \tau\frac{\partial \theta_{d_x}}{\partial \tau} + \theta_{d_x} = \frac{-\tau\theta_{d_x} + \theta_{d_x}(1+\tau)}{(1+\tau)} = \frac{\theta_{d_x}}{(1+\tau)} > 0$$

B.5 Proposition 3.2

I can find that the labour devoted to innovation technology from Eqs. (3.38) and (3.39) for ‘all exporters’ is larger than its innovation cost for domestic firms from Eqs. (3.14) and (3.15). In addition, the consumer’s expenditure received by each exporter e_x is larger than the consumer’s expenditure received by each domestic firm e_a for sector s, according to the definition of the mass of varieties M of Eq. (3.21) which is decreasing by production threshold z^* from Eq. (3.22).

B.6 Equation (3.63)

Applying Eq. (3.63) to the case where the markup is that in the domestic market under any given regime, and aggregating across all sectors, I have:

$$X = (M\tilde{Z})^{\frac{1-\alpha}{\alpha}}\bar{\gamma}E\theta_d$$

where $\bar{\gamma} = \sum_{s=1}^{H}\gamma_s$. Substituting into the aggregate utility function, for a given country (domestic), and using $\bar{\gamma} = \sum_{s=1}^{S}\gamma_s$, $\bar{e} = \frac{\bar{\gamma}E}{nM}$ and $Y = (1-\bar{\gamma})E$, I have:

$$\begin{aligned} U &= \bar{\gamma} \ln X + (1-\bar{\gamma}) \ln Y \\ &= \bar{\gamma} \frac{1-\alpha}{\alpha} \ln M\tilde{Z} + \bar{\gamma} \ln \theta_d \bar{\gamma} E + (1-\bar{\gamma}) \ln (1-\bar{\gamma}) E \qquad \text{(B.2)} \\ &= \underbrace{\bar{\gamma} \frac{1-\alpha}{\alpha} \ln M\tilde{Z}}_{\text{Productivity/LFV}} + \underbrace{\bar{\gamma} \ln \theta_d \bar{e} n M}_{\text{Consumption}} + \underbrace{(1-\bar{\gamma}) \ln \frac{(1-\bar{\gamma})}{\bar{\gamma}} \bar{e} n M}_{\text{Homogeneous good}} \end{aligned}$$

B.7 Equation (3.64)

This is derived using $\bar{z}$ from Eq. (3.20), M from Eq. (3.22) and (MC_a). Starting from $M = (1+\delta z^{*k})^{-1}$ and $e = \frac{1+\delta z^* k - n\lambda_d}{nT}$ where $T = (1+\hat{\eta})\theta_d + \frac{1-\gamma}{\gamma}$:

I can derive the productivity/LFV trade-off as follows:

$$\frac{1}{\tilde{Z}} \frac{\partial \tilde{Z}}{\partial z^*} + \frac{1}{M} \frac{\partial M}{\partial z^*} = \frac{1}{1-\eta} z^{*-1} - \frac{k\delta z^{*-1}}{\delta + z^{*-k}}$$

Which is positive iff:

$$\delta < \frac{z^{*-k}}{(1-\eta)k - 1}$$

$\frac{\partial e}{\partial z^*} \frac{1}{e} = \frac{k\delta z^{*k-1}}{nT} \frac{nT}{1+\delta z^{*k} - n\lambda_d} = \frac{k\delta z^{*k-1}}{1+\delta z^{*k} - n\lambda_d}$ and $\frac{\partial M}{\partial z^*} \frac{1}{M} = \frac{-\delta k z^{*k-1}(1+\delta z^{*k})^{-2}}{(1+\delta z^{*k})^{-1}} = \frac{-\delta k z^{*k-1}}{(1+\delta z^{*k})}$
Hence, $\frac{\partial e}{\partial z^*} \frac{1}{e} + \frac{\partial M}{\partial z^*} \frac{1}{M} = \frac{k\delta z^{*k-1}}{(1+\delta z^{*k})(1+\delta z^{*k} - n\lambda_d)} \left[1 + \delta z^{*k} - 1 - \delta z^{*k} + n\lambda_d\right] = \frac{kn\lambda_d \delta z^{*k-1}}{(1+\delta z^{*k})(1+\delta z^{k*} - n\lambda_d)}$
Finally, noting that given $M = (1+\delta z^{*k})^{-1}$, the denominator can be written $(1+\delta z^{*k})^2(1 - nM\lambda_d)$, completing the proof.

B.8 Proposition 3.4

(i) This follows from inspection of (MC_a), where, given the numerator is positive, for $e > 0$, requires $1 - \lambda_d nM > 0$, which ensures $\{.\} > 0$, completing the proof.
(ii) The inequality in Eq. (3.65) follows directly from setting the term $[.] > 0$ in Eq. (3.64) and rearranging for δ, completing the proof.

Chapter 4

The Coexistence of Non-exporters, Exporters and Multinational Production

Abstract

This chapter develops the first main chapter by considering an inclusive economy where non-exporters, exporters and multinational firms via horizontal FDI can coexist and endogenise the mass of variety through a different condition compared with the first chapter. The model builds a multi-sector economy with two symmetric countries, variable markups from oligopolistic competition and firm heterogeneity. By doing this, the model allows us to explore the welfare gains from trade liberalisation in the presence of multinational firms. My model shows that multinational firms generate the largest pro-competitive effect because they increase product market competition through the lowest markups, similar to the first chapter. The most significant contribution of this chapter is that I quantify the results from the theoretical model by numerical simulation analysis for the global economy moving from autarky to free trade. The simulation results indicate that trade liberalisation has a pro-competitive effect only on exporters as the markups of exporters depend on iceberg-type variable costs. Besides, trade liberalisation also generates firm selection for non-exporters by forcing the least productive firms out of the market, increasing aggregate productivity. In contrast, when trade liberalises, firstly, productivity thresholds for exporters decrease as more firms choose to serve the foreign market. Secondly, multinational firms and the ratio of productivity thresholds for multinational firms to exporters increases, so the proximity-concentration trade-off between multinational firms and exporters holds. Finally, trade liberalisation will increase welfare by approximately 0.12% from autarky to free trade through three channels: selection effect for non-exporters, pro-competitive effect for exporters and efficiency from the switch from FDI to exporters via the proximity-concentration effect between exporters and multinational firms.

Keywords: firm heterogeneity; horizontal FDI; oligopolistic competition; trade liberalisation; welfare gains
JEL Classification: F12, F13, D60

4.1 Introduction

How significant are the welfare effects of trade and multinational production? How would trade liberalisation affect welfare gains in the presence of multinational production? Multinational production (MP) refers to the products manufactured by firms outside the original country through foreign affiliates (e.g., Irarrazabal et al., 2013; Ramondo et al., 2015). During the last three decades, a significant component of economic globalisation has been the growth of multinational production. For example, from 1990 to 2006, inward foreign direct investment (FDI, aka MP) flows increased by 9.5% annually while world exports increased by only 8%, and the value added by MP accounted for one-fourth of global GDP by 2010 (e.g., Irarrazabal et al., 2013; Sun et al., 2020). Despite this, very few papers have investigated the welfare gains of multinational production or how the presence of multinational production affects the gains from trade liberalisation. This chapter is a contribution in this direction.

In the first main chapter of this thesis, I considered a simple global economy model with two symmetric countries, examining the welfare gains from international trade and horizontal FDI under firm heterogeneity and variable markups in an oligopolistic competition framework. This setting allows us to identify a new source of gains from globalisation in welfare: the increase in competition, as explained in the first main chapter. I assume the number of firms n within each variety is exogenously fixed, and the mass of operative varieties, M, is determined by the firm's productivity threshold in equilibrium, capping the maximum number of varieties to 1. I compare the gains from trade and financial liberalisation under three different scenarios: a domestic scenario, another one in which all firms serve the foreign market via exports and another one in which all firms serve the foreign market by becoming multinational firms. By means of this analysis, I explored how trade openness and multinational production contribute to welfare via increased competition.

In this chapter, I extend the previous framework by considering an environment where the number of varieties, M, is endogenous through a different condition of entrants from the above second and third chapter, but like the way of Melitz (2003). As emphasised by Etro (2014), considering an endogenous number of competitors in a more realistic form of oligopolistic competition is critical, as firms' strategies affect entry and vice versa; furthermore, endogenous conditions for entrants on preference and technology affect the equilibrium, they have substantial implications for industrial organisation, international finance and international trade. Therefore, I explore the welfare implications of international trade and industrial organisation by incorporating the endogenous number of entrants in oligopolistic competition. In addition, I allow for the possibility that non-exporters, exporters, and multinational producers coexist in the same industry. This extension is of potential significance as gains from trade liberalisation may be overestimated without FDI. Multinational firms are not directly affected by trade liberalisation (e.g., Ethier, 1986; Sun et al., 2020) and trade liberalisation only affects a firm via FDI if it switches from a multinational firm to an exporter due to the reduction of variable transportation costs. I consider a substitution relationship between horizontal FDI with higher fixed marginal costs and exports with higher variable marginal costs, like Helpman et al. (2004), which affects the distribution of multinational firms and exporters due to trade liberalisation. My framework is similar to the general version of the model in Impullitti and Licandro (2018). However, I incorporate multinational production via horizontal FDI as Helpman et al. (2004) and consider a different condition for entrants like Melitz (2003) rather than a free entry condition as in Impullitti and Licandro (2018) to investigate how welfare gains would be due to trade liberalisation because of the tractability of my model.

This chapter relates to literature investigating welfare gains from trade liberalisation for international trade. For example, Arkolakis et al. (2012) demonstrate that the aggregate welfare gains from international trade are summarised by the statistics from a class of theoretical trade models with heterogeneous and homogeneous firms. Calibrating the statistics for the US economy, they show that trade creates very few welfare gains. Melitz and Redding (2015) compare a heterogeneous firm model to a homogeneous firm model and identify a new source of welfare gains, endogenous firm selection with firm heterogeneity. Melitz and Redding (2014) also found a new channel for welfare improvement of trade liberalisation, which is an endogenous increase in domestic productivity within a model of sequential production. My contribution is to include multinational production in the economy and investigate how trade liberalisation affects the welfare gains in such an economy. Several studies quantify the welfare gains from trade (e.g., Chang and Jin, 2017; Hsu et al., 2020, 2019), my framework contributes to this literature by introducing multinational production via horizontal FDI and examining the welfare gains of trade liberalisation. This is estimated to be around 0.12% when there is a movement from autarky to free trade incorporating multinational firms, as shown in my numerical simulation analysis. The value of welfare implications differs from Sun et al. (2020); they found that trade liberalisation brought around 8% welfare gains for the economy with exports and FDI by setting parameters identical to Melitz and Redding (2015). In contrast, my parameter settings of the numerical simulation analysis are quite similar to Impullitti and Licandro (2018). My research also features firm heterogeneity as is the case in some related literature (e.g., Bombarda and Marcassa, 2020; Chor, 2009; Eckel and Neary, 2010; Edmond et al., 2015; Kim, 2009; Lin and Saggi, 2007). Similarly, Chor (2009) investigates the welfare implications of subsidies to attract multinational corporations with heterogeneous firms, and finds welfare gains stem from a selection effect since the subsidy leads the most productive exporters to opt to serve the host's market via FDI. I also incorporate firm heterogeneity, find the selection effect for the welfare gains of trade liberalisation, and contribute by considering variable markups from the oligopolistic competition.

This chapter also relates to the literature investigating multinational production's welfare gains. For example, Ramondo and Rodríguez-Clare (2013) estimate the welfare gains from trade and FDI through the force of substitutability and complementarity between trade and MP, building a model with the interaction between trade and FDI in three ways: 'horizontal' FDI, there are two competing ways to serve the foreign market; 'vertical' FDI, foreign affiliates import inputs from the home country; and firms choose another country as an export platform to serve a particular market. They extend the model of Eaton and Kortum (2002) for perfect competition to include MP, but there is no other channel for welfare gains except comparative advantage. As in their work, my research captures the interaction between trade and MP in a way *à la* Helpman et al. (2004), some varieties are flipped from being served via exports to via multinational production, where there are two competing ways of serving the foreign market. The distribution of firms into different ways of serving the foreign country is altered by trade liberalisation since more firms choose to export as the variable transportation trade costs decrease. However, my model endogenises the mass of varieties available in the economy when trade liberalisation happens, while for Eaton and Kortum (2002), this is impossible as the mass of varieties available in the economy is fixed. In addition, I incorporate oligopolistic competition rather than perfect competition as in Ramondo and Rodríguez-Clare (2013) to add the new channel for welfare gains via a pro-competitive effect. In addition, Irarrazabal et al. (2013) examine a model of trade and MP with firm heterogeneity and no free entry. They ignore the proximity-concentration hypothesis but consider intra-firm trade and indicate MP leads to significant gains through international technology sharing. Moreover, Bombarda and Marcassa (2020) extend Irarrazabal et al. (2013) to explain that the higher welfare gains of exporting, including multinational production with intra-firm trade, are created by magnifying the response to trade openness when they assume symmetric country and free entry conditions. They also indicate that welfare gains rely on foreign supply strategies between exports and MP. In contrast to these studies, my chapter differs from theirs by including a pro-competitive effect through the variable markup from the oligopolistic competition and investigating the welfare of trade liberalisation in the presence of FDI with firm heterogeneity.

Lastly, this chapter relates to the literature featuring variable markups. Some papers consider variable markups originating from the monopolistic competition. For example, Feenstra and Weinstein (2017) examine the welfare effect of endogenous markups with firm heterogeneity by considering translog preferences and find the rise in varieties and the decline in markups contributed to an additional increase in welfare by one per cent in the period 1992-2005 for the US economy. Other papers investigated the variable markups through oligopolistic competition (e.g., Atkeson and Burstein, 2008; Edmond et al., 2015). Edmond et al. (2015) estimate the pro-competitive gains from international trade in a quantitative model with variable markups from the endogenous market structure and find that openness reduces markup distortions by up to one-half and significantly reduces productivity losses. While other papers estimate variable markups through monopolistic competition (e.g., Arkolakis et al., 2019; Bernard et al., 2003). For example, Arkolakis et al. (2019) explore the gains from trade liberalisation in a body of models with monopolistic competition, firm-level heterogeneity, and variable markups. They conclude that pro-competitive effects of trade are elusive as welfare effects from trade liberalisation for the model with variable markups are slightly lower than the model with constant markups. However, among these papers, MP is absent. Some papers assess the welfare gains with oligopolistic competition and multinational firms (e.g., De Santis and Stähler, 2004; Eckel and Neary, 2010; Kim, 2009; Lin and Saggi, 2007). Kim (2009) investigates the welfare implications of the FDI entry modes, greenfield investment or cross-border mergers and acquisitions (M&A), when a domestic firm buys a controlling stake in a foreign firm, influenced by the regional economic integration based on an oligopoly model. They find that greenfield investment is a welfare dominant FDI entry mode for the host country. Eckel and Neary (2010) establish a multi-product firms (MPFs) model with intra-firm trade in partial and general oligopolistic equilibrium. Firms play a Cournot game within each variety and choose the quantity of each good, and the mass of produced goods for scale or scope, which is not affected by their rivals and firms' productivity would be altered according to the changes in firms' scope. They document that globalisation affects MPF through competition and demand effects. There is a new source of gains from trade: productivity increases as firms become 'leaner and meaner' and concentrate on their core competence. De Santis and Stähler (2004) examined the impact of liberalising FDI on welfare by comparing the FDI-allowed regime with the FDI-prohibited regime with imperfect competition. Although some papers focus on MP with oligopolistic competition, they seldom examine the pro-competitive effect via variable markups like my chapter. My work explores the variable markup under oligopolistic competition and focuses on the welfare gains of trade liberalisation, including horizontal FDI.

The chapter proceeds as follows. First, in section 2, I establish the basic model framework. Then, section 3 derives the equilibrium of the model. In section 4, I present the numerical simulation analysis. Section 5 concludes.

4.2 Model Setup

This section presents the basics of the model setup. Like the main chapter 1, the model captures a continuum of imperfectly substitutable varieties. Within each product line, a small number of exogenous n identical firms producing perfectly substitutable goods compete *à la* cournot. Firms count on two different ways of serving the foreign market, exporting and via horizontal FDI. A firm only serving the domestic market bears a fixed sunk cost λ_d to produce in the domestic market. When a firm chooses to serve the foreign market via exports, it must bear a variable transportation cost τ, $\tau \geq 1$, of an iceberg-type cost. If the firm exports, it must bear an extra fixed cost of λ_x. Suppose a firm decides to undertake horizontal FDI, it has to pay an additional fixed cost of creating a new plant in the foreign market, λ_m, similar to λ_x but with no variable transportation cost τ for the foreign market, with a fixed production cost λ_d in the domestic market. Setting restrictions between the different types of fixed costs in the foreign market for exporters and multinational firms, $\lambda_x < \lambda_m$, so that the partitioning between exporters and FDI is considered. It will lead the most productive multinational firms to choose horizontal FDI, and the middle productive firms choose to serve the foreign market by exporting like Helpman et al. (2004). In addition, the model allows firms to charge the different markups between non-exporters, exporters and multinational firms. In contrast to the first chapter, I consider the endogenous number of varieties M, which determine the equilibrium of the economy by introducing the different condition similar to Melitz (2003). It is conducted by considering an exogenously fixed mass of new varieties M_e. In contrast, the mass of varieties M is not a determinant of the economy's equilibrium and relies on the productivity threshold of equilibrium.

4.2.1 Preferences and Demand

As in the first main chapter, there are two identical economies with a continuum of 1 unit mass of identical individuals. In each economy, there are $H + 1$ sectors producing goods. H sectors produce differentiated goods while one sector produces a homogeneous good. γ_h is an exogenous share of income spent on composite products of sector h. Notice that a particular sector from the H sectors, denoted by h, consists of a continuum of varieties v. Preferences of consumers are represented by the following utility function:

$$U = (1 - \sum_{h=1}^{H} \gamma_h) \log Y + \sum_{h=1}^{H} \frac{\gamma_h}{\alpha_h} \log(\int_{v \in V_h} x_h(v)^{\alpha_h} dv) \tag{4.1}$$

where Y is aggregate consumption of the homogeneous good, $x_h(v)$ is consumption of a heterogeneous variety v from sector h and V_h is the set of all potential varieties in sector h. $\alpha_h \in (0,1)$, is related to the elasticity of substitution between any two varieties in sector h, which is denoted by σ_h and $\sigma_h = 1/(1-\alpha_h) > 1$. [1] Consumers maximize utility subject to the following budget constraint

$$Y + \sum_{h=1}^{H} \int_{v \in V_h} p_h(v) x_h(v) dv = E \tag{4.2}$$

where the homogeneous good Y is considered to be the numéraire. E is the aggregate level of expenditure including homogeneous goods and composite goods. The utility maximization problem yields the following inverse demand function for each variety v from a particular sector h:

$$p_h(v) = \frac{\gamma_h E}{X_h^{\alpha}} x(v)^{\alpha-1} \tag{4.3}$$

where

$$X_h = \left(\int_0^M x_h(v)^{\alpha_h} dv \right)^{\frac{1}{\alpha_h}} \tag{4.4}$$

$Y = (1-\bar{\gamma}) E$, where $\bar{\gamma} = \sum_{h=1}^{H} \gamma_h$. M is the mass of operative varieties in sector h, which relates to the aggregate differentiated good X of sector h. For simplicity, I omit the subscript h for a particular sector in the following sections.

4.2.2 Production and Firm Behaviour

Similar to the first main chapter, the sole factor of production is labour. Since the homogeneous good Y is numéraire, its price equals 1. It is manufactured with perfect competition under constant returns to scale, with one unit of labour producing one unit of output. Therefore, the wage is 1. The cost function of a firm using labour, including both variable production costs and a fixed cost of the operation, λ, is given by the following expression:

$$C(z) = l(z) = z^{\frac{\alpha-1}{\alpha}} q(z) + \lambda \tag{4.5}$$

$q(z)$ is the quantity of a firm's output, $C(z)$ is the total cost of the firm and $l(z)$ is associated with different production activities of the firm. In particular, λ_d is the fixed production cost, λ_x is the cost the firm incurs if it decides to serve the foreign market via exporting and λ_m is the cost the firm incurs if it decides to serve the foreign market by establishing a subsidiary in the foreign market. I assume $\lambda_x < \lambda_m$

[1] Note that the utility function presented here is a monotonic transformation of the following utility function:

$U = (Y)^{(1-\sum_{h=1}^{H}\gamma_h)} \prod_{h=1}^{H} (X_h)^{\gamma_h}$ where $X_h = \left(\int_{v \in V_h} x_h(v)^{\frac{\sigma_h - 1}{\sigma_h}} dv \right)^{\frac{\sigma_h}{\sigma_h - 1}}$ with $\alpha_h = \frac{\sigma_h - 1}{\sigma_h}$.

like Helpman et al. (2004), to guarantee that only the most productive firms can be multinational firms which is in line with the empirical evidence.

At entry, firms within a variety jointly draw their productivity, z, from a continuous Pareto productivity distribution with a distribution function shown as follows:

$$G(z) = 1 - \left(\frac{\underline{z}}{z}\right)^k = 1 - z^{-k},\ z \geq 1,\ k \geq 1$$

where $\underline{z}= 1$, indicates the lower productivity bound and k represents the shape parameter.

Notice that, the firms that compete *à la* Cournot in a product line are equally productive with a particular productivity z, but productivity differs across product lines.

4.3 Equilibrium

This section provides a solution to the model equilibrium. Firstly, I provide the relevant equations of operating profits for domestic, exporting, and multinational firms. Then, I describe the cutoff conditions for each type of firm. Third, I explore the firms' entry and selection, including the condition for endogenous variety, which pins down the number of varieties, and a labour market-clearing condition to examine the general equilibrium of the economy. Lastly, the welfare analysis is conducted.

4.3.1 Cournot Competition

Like the first main chapter, a, x, and m denote non-exporters, exporters and multinational firms, with domestic and foreign markets referred to by subindices d and f separately. However, I depart from the first chapter by introducing an inclusive economy with two symmetric countries where non-exporters, exporters, and multinational firms coexist rather than three respective scenarios in which all are non-exporters, all are exporters, and all are multinational firms as in the first chapter.

Non-exporters

A firm producing a domestic variety serves only the domestic market with a productivity level of z. It maximizes its profit subject to the inverse demand function in Eqs. (4.3), taking the output produced by its direct competitors of $(n-1)$ firms within the same product line as given. I solve a non-exporter's problem:

$$\max_{q_a} \pi_a = \underbrace{\frac{\gamma E}{X^\alpha}(\hat{x}_a + q_a)^{\alpha-1}}_{p_a} q_a - z^{\frac{\alpha-1}{\alpha}} q_a - \lambda_d$$

q_a is a non-exporter's production, and $\hat{x}_a$ is the output of the firm's direct competitors in a particular product line domestically, p_a indicates the associated price. Using the first order condition for a non-exporter and combining symmetry $x_a = q_a + \hat{x}_a = nq_a$, where x_a represents the total consumption for a particular domestic variety, I derive the same equation as the first chapter shown as follows:

$$\underbrace{\frac{\gamma E}{X^\alpha} x_a^{\alpha-1}}_{p_a} \underbrace{\left(\frac{(\alpha-1)q_a}{nq_a} + 1\right)}_{\theta_a} = z^{\frac{\alpha-1}{\alpha}}$$

which indicates

$$p_a = \frac{z^{\frac{\alpha-1}{\alpha}}}{\theta_a}, \ \theta_a = (n + \alpha - 1)/n \tag{4.6}$$

$$x_a = [\frac{\gamma E}{X^\alpha p_a(v)}]^{\frac{1}{1-\alpha}} = \left(\frac{\gamma E}{X^\alpha}\right)^{\frac{1}{1-\alpha}} \theta_a^{\frac{1}{1-\alpha}} z^{\frac{1}{\alpha}} \tag{4.7}$$

Eqs. (4.6) and (4.7) are equivalent to the price of a non-exporter and quantities for a domestic variety in the first main chapter. θ_a is the inverse of the markup for a non-exporter within a particular domestic variety, as in the first chapter.

Exporters

Similar to the calculations for a non-exporter and with the same notation for an exporter in the first main chapter, an exporter within a specific exported variety with productivity level z competes *à la* Cournot simultaneously in both domestic and foreign markets referred to by d_x and f_x, and regards each of the markets as segmented. In other words, there would be $2n$ firms competing non-cooperatively within a particular exported product line in both the domestic and foreign market as I assume n identical firms operating in an oligopolistic market within each variety in each symmetric country. An exporter's problem is solved by:

$$\max_{q_{d_x}, q_{f_x}} \pi_x = \underbrace{\frac{\gamma E}{X^\alpha}(\hat{x}_{d_x} + q_{d_x})^{\alpha-1}}_{p_{d_x}} q_{d_x} + \underbrace{\frac{\gamma E}{X^\alpha}(\hat{x}_{f_x} + q_{f_x})^{\alpha-1}}_{p_{f_x}} q_{f_x} - z^{\frac{\alpha-1}{\alpha}}(q_{d_x} + \tau q_{f_x}) - \lambda_d$$

q_{d_x} and q_{f_x} are the output sold by an exporter domestically in the domestic and foreign market, separately; p_{d_x} and p_{f_x} denote the relevant prices. $\hat{x}_{d_x}$ and $\hat{x}_{f_x}$ are the quantities for an exporter's direct competitors of a particular exported product line domestically in the domestic and foreign markets. $q_x = (q_{d_x} + \tau q_{f_x})$ represents total quantity produced by a domestic exporter. $x_x = n(q_{d_x} + q_{f_x})$ denotes the aggregate quantities of exporters in a given exported variety in one country and p_x indicates the associated price. Combining the first order conditions for the domestic and foreign markets, respectively and symmetry conditions, like in the first chapter, I derive:

$$p_x = \frac{z^{\frac{\alpha-1}{\alpha}}}{\theta_{d_x}} = \frac{\tau z^{\frac{\alpha-1}{\alpha}}}{\theta_{f_x}}, \quad \theta_{d_x} = \frac{2n+\alpha-1}{n(1+\tau)}, \quad \theta_{f_x} = \tau\theta_{d_x}, \quad \tau \in (1, \bar{\tau} = \frac{n}{n+\alpha-1}) \tag{4.8}$$

$$\Phi \equiv \frac{q_{d_x} + \tau q_{f_x}}{q_{d_x} + q_{f_x}} = \frac{(1-n-\alpha)(1+\tau^2)+2n\tau}{(1-\alpha)(1+\tau)} \geq 1, \quad \theta_x \equiv \frac{q_{d_x}\theta_{d_x} + q_{f_x}\theta_{f_x}}{q_{d_x} + q_{f_x}} = \Phi\theta_{d_x} \tag{4.9}$$

$$x_x = [\frac{\gamma E}{X^\alpha p_x(v)}]^{\frac{1}{1-\alpha}} = \left(\frac{\gamma E}{X^\alpha}\right)^{\frac{1}{1-\alpha}} \theta_{d_x}^{\frac{1}{1-\alpha}} z^{\frac{1}{\alpha}} \tag{4.10}$$

Eqs. (4.8) and (4.10) are equivalent to the price of an exporter domestically and quantities for an exporting variety domestically as in the first main chapter. θ_{d_x} and θ_{f_x} are the inverse of the markups for a domestic exporter in the domestic and foreign markets separately, which are the same as in the first chapter. Similar to the first chapter, Φ is the ratio of an exporter's output to consumption, indicating losses because of the iceberg transportation costs with trade. It is related to θ_x, which is the weighted average of the inverse markups for an exporter in the domestic and foreign markets. They can be used to calculate the exporter's profit function and cutoff condition in the following analysis. Notice that there is a prohibitive transportation cost $\bar{\tau}$, which illustrates that no firms will export at the very high transportation costs.

Multinational Firms

Analogous to the derivation for an exporter within an exported product line, a firm within a particular multinational product via horizontal FDI with productivity z engages in strategic interactions with its direct competitors simultaneously in both domestic and foreign markets. I treat the domestic and foreign markets as segmented markets with the same notation as in the first main chapter. I focus on a domestic multinational firm to conduct the analysis, like exporters, as two symmetric countries are considered. I solve a multinational firm's problem:

$$\max_{q_{d_m}, q_{f_m}} \pi_m = \underbrace{\frac{\gamma E}{X^\alpha}(\hat{x}_{d_m} + q_{d_m})^{\alpha-1}}_{p_{d_m}} q_{d_m} + \underbrace{\frac{\gamma E}{X^\alpha}(\hat{x}_{f_m} + q_{f_m})^{\alpha-1}}_{p_{f_m}} q_{f_m} - z^{\frac{\alpha-1}{\alpha}}(q_{d_m} + q_{f_m}) - \lambda_d - \lambda_m$$

q_{d_m} and q_{f_m} denote the production of domestically produced goods for a multinational firm in the domestic and foreign markets separately and p_{d_m} and p_{f_m} are the associated prices. $\hat{x}_{d_m}$ and $\hat{x}_{f_m}$ are the quantities for a multinational firm's direct competitors in a particular product line via horizontal FDI domestically in the domestic and foreign markets. $q_m = (q_{d_m} + q_{f_m})$ represents total quantity produced by a multinational firm. $x_m = n(q_{d_m} + q_{f_m})$ denotes aggregate quantities of multinational firms in a given product line via horizontal FDI in one country and p_m indicates the associated price. Combining the first order conditions for the domestic and foreign markets, respectively and symmetric conditions as in the first main chapter, I derive:

$$p_m = \frac{z^{\frac{\alpha-1}{\alpha}}}{\theta_{d_m}} = \frac{z^{\frac{\alpha-1}{\alpha}}}{\theta_{f_m}}, \quad \theta_m = \theta_{d_m} = \theta_{f_m} = \frac{2n+\alpha-1}{2n} \tag{4.11}$$

$$x_m = [\frac{\gamma E}{X^{\alpha} p_m(v)}]^{\frac{1}{1-\alpha}} = \left(\frac{\gamma E}{X^{\alpha}}\right)^{\frac{1}{1-\alpha}} \theta_{d_m}^{\frac{1}{1-\alpha}} z^{\frac{1}{\alpha}} \tag{4.12}$$

Eqs. (4.11) and (4.12) are equivalent to the price of a multinational firm domestically and quantities for multinational firms in a particular product line via horizontal FDI domestically as in the first main chapter. $\theta_{d_m} = \theta_{f_m} = \theta_m$ represents the inverse of the markups charged in the domestic market and foreign markets and the inverse of the average markups for multinational firms within a variety via FDI, which is the same as the first chapter.

Unlike the first chapter, I incorporate an inclusive economy with non-exporters, exporters and multinational firms coexisting. Substituting Eqs.(4.7), (4.10) and (4.12) into Eq. (4.4) for aggregate differentiated goods X, yields [2],

$$X = M^{\frac{1-\alpha}{\alpha}} (\frac{\gamma E}{\bar{p}}) \tag{4.13}$$

where average price represents:

$$\bar{p}^{\frac{\alpha}{\alpha-1}} = [\int_{z_a^*}^{z_x^*} p_a(z)^{\frac{\alpha}{\alpha-1}} \mu(z) dz + \int_{z_x^*}^{z_m^*} p_x(z)^{\frac{\alpha}{\alpha-1}} \mu(z) dz + \int_{z_m^*}^{\infty} p_m(z)^{\frac{\alpha}{\alpha-1}} \mu(z) dz] \tag{4.14}$$

in which I denote $\mu(z)$ as the equilibrium density, and the expressions are shown as follows:

$$\mu(z) = \begin{cases} \frac{g(z)}{1-G(z_a^*)} & \text{if } z \geqslant z_a^*, \\ 0 & \text{otherwise}, \end{cases} \tag{4.15}$$

Here, z_a^*, z_x^* and z_m^* are cutoff conditions for non-exporters, exporters and multinational firms, and I will go into detail in the next part. The expression of corresponding

[2] See Appendix C.1

prices p_a, p_x and p_m are equilibrium prices for non-exporters, exporters and multinational firms, as Eqs.(4.6), (4.8) and (4.11) show. Like Melitz (2003), I apply the pricing index to calculate the main equilibrium variables in the general model. The notation of $\mu(z)$ is the equilibrium productivity distribution similar to the first chapter, where any entering firm drawing a productivity level $z < z_a^*$ will immediately exit and never produce. The initial productivity draw must determine the distribution, conditional on successful entry. Hence, $\mu(z)$ is the conditional distribution of $g(z)$ on $[z_a^*, \infty)$, which indicates that only productivity $z \geqslant z_a^*$ can produce in the market and $G(z)$ is the probability distribution of an entrant from a variety with productivity z.

Notice that $\bar{p}M^{\frac{\alpha-1}{x}}$ is the average price of the differentiated goods for a particular sector h, which is derived from Eq. (4.13) represented by $(\frac{\gamma E}{X})$. Given the definition of average price $\bar{p}$ in the above Eq. (4.14), I combine Eqs.(4.6), (4.8) and (4.11) which are the relative equilibrium prices of non-exporters, exporters and multinational firms respectively, the Pareto productivity distribution which is below Eq. (4.5) and Eq. (4.15) for the definition of equilibrium density $\mu(z)$ to simplify the average price as follows (See Appendix C.2):

$$\bar{p}^{\frac{\alpha}{\alpha-1}} = \frac{k}{k-1} z_a^* [\theta_a^{\frac{\alpha}{1-\alpha}} + (\theta_{d_x}^{\frac{\alpha}{1-\alpha}} - \theta_a^{\frac{\alpha}{1-\alpha}}) Z_1^{k-1} + (\theta_{d_m}^{\frac{\alpha}{1-\alpha}} - \theta_{d_x}^{\frac{\alpha}{1-\alpha}}) Z_2^{k-1}] \tag{4.16}$$

where I have the ratio of the cutoff condition for non-exporters to exporters $Z_1 = z_a^*/z_x^*$, and the ratio of the cutoff condition for non-exporters to multinational firms $Z_2 = z_a^*/z_m^*$. Z_1 and Z_2 simplify the equations and are used to solve the equilibrium with numerical analysis.

Then I derive a firm's revenue within a domestic variety, exported variety and a variety of multinational production via horizontal FDI, respectively. The revenue is derived by substituting X into the demand function Eqs.(4.7), (4.10) and (4.12), with Eq. (4.13) of equilibrium aggregate composite goods X and multiplying both sides by the corresponding equilibrium price denoted by p_a, p_x, p_m: [3]

$$\begin{aligned} r_a &= p_a x_a / n = \frac{\gamma E}{nM} (\frac{\bar{p}}{p_a})^{\frac{\alpha}{1-\alpha}} = e(\frac{\bar{p}}{p_a})^{\frac{\alpha}{1-\alpha}} \\ r_x &= p_x x_x / n = \frac{\gamma E}{nM} (\frac{\bar{p}}{p_x})^{\frac{\alpha}{1-\alpha}} = e(\frac{\bar{p}}{p_x})^{\frac{\alpha}{1-\alpha}} \\ r_m &= p_m x_m / n = \frac{\gamma E}{nM} (\frac{\bar{p}}{p_m})^{\frac{\alpha}{1-\alpha}} = e(\frac{\bar{p}}{p_m})^{\frac{\alpha}{1-\alpha}} \end{aligned} \tag{4.17}$$

x_a, x_x and x_m represent the total consumption of firms from a particular domestic variety, exported variety and multinational product line via horizontal FDI in both

[3] See Appendix C.3

domestic and foreign markets. I can derive the expressions of a firm's revenue in the product lines of autarky, export and the multinational production via horizontal FDI as there are n firms within each variety. Here,

$$e = \gamma E/(nM) \tag{4.18}$$

where Eq. (4.18) shows the firm's average expenditure for a specific sector h.

I have the assumption that firms in any market compete *à la* Cournot in a global economy with two symmetric economies and can opt to be non-exporters, exporters and multinational firms via horizontal FDI, leading to the following proposition 4.1.

Proposition 4.1. *Under the above assumption, the equilibrium prices and revenues among firms, not trading, exporting, and via horizontal FDI, respectively:* (i) $p_m < p_x < p_a$, *and* (ii) $r_a < r_x < r_m$.

Proposition 4.1 can be proved through the relationship of the inverse of related markups among non-exporters, exporters, and multinational firms via horizontal FDI, $\alpha < \theta_a < \theta_{d_x} < \theta_x < \theta_{f_x} < \theta_m < 1$ shown as proposition 2.4 in the first chapter and their equations for equilibrium prices Eqs.(2.10), (2.34) and (2.58). Unlike the first chapter, in which I consider three scenarios of the economy, all domestic varieties, all exported varieties, and all multinational production via horizontal FDI, respectively. In this chapter, I incorporate all non-exporters, exporters and multinational production that coexist in each industry. I can still identify their equilibrium prices from Eqs.(4.6), (4.8) and (4.11), and the inverse of the markups for non-exporters, exporters and multinational firms are the same as in the first chapter. For a given productivity level z, firms' price is negatively associated with the inverse of their markups, which means lower markups are associated with a lower price, shown in proposition 4.1 (i). It could be explained by pricing-to-market, which means firms change the relative price at which they sell their output at home and abroad in response to a change in the relative costs of production, and it depends on the presence of international trade costs (e.g., Atkeson and Burstein, 2007, 2008). In addition, given the fixed markup, notice that the relative equilibrium prices have a negative relationship with productivity, which would also explain the most productive firm charging the lowest price with a constant markup like Melitz (2003) and Helpman et al. (2004). The most productive varieties can serve the foreign market via FDI with the lowest price. I can illustrate that a firm's revenue within different product lines is negatively related to its equilibrium prices as the same average price $\bar{p}$ exists, shown in proposition 4.1 1(ii), as Eq. (4.17) shows. It indicates that a multinational firm will achieve the highest revenue because of its lowest price and highest quantity than exporters and non-exporters.

4.3.2 Cutoff Condition

This section examines the different cutoff conditions for non-exporters, exporters and multinational firms via horizontal FDI. It is distinguished from the first chapter because I include an inclusive environment with coexisting non-exporters, exporters and multinational firms; therefore, the deviation cases for exporters and multinational firms need to be considered.

Non-exporters

According to the derivation from the section on oligopolistic competition, the variable costs for a non-exporter are shown as follows, which are used to find a non-exporter's profit

$$z^{\frac{\alpha-1}{\alpha}}q_a = p_a\theta_a\frac{x_a}{n} = \frac{\gamma E}{M}(\frac{\bar{p}}{p_a})^{\frac{\alpha}{1-\alpha}}\frac{\theta_a}{n} = \theta_a e(\frac{\bar{p}}{p_a})^{\frac{\alpha}{1-\alpha}} \tag{4.19}$$

Then, the profit of a firm within a domestic product line, combining Eqs. (4.6) and (4.19), the equilibrium profit is given by

$$\begin{aligned}\pi_a &= p_a q_a - z^{\frac{\alpha-1}{\alpha}}q_a - \lambda_d \\ &= (1-\theta_a)e(\frac{\bar{p}}{p_a})^{\frac{\alpha}{1-\alpha}} - \lambda_d \\ &= e\bar{p}^{\frac{\alpha}{1-\alpha}}(1-\theta_a)\theta_a^{\frac{\alpha}{1-\alpha}}z - \lambda_d\end{aligned}$$

Therefore, I can get the productivity cutoff for a non-exporter within a domestic product line, which means a zero-profit condition for a non-exporter $\pi_a(z_a^*) = 0$, as an exit condition for non-exporters (EC):

$$e = \frac{\lambda_d}{(1-\theta_a)(\frac{\bar{p}}{p_a(z_a^*)})^{\frac{\alpha}{1-\alpha}}} \tag{4.20}$$

According to the average price of Eq. (4.16) and the equilibrium price of non-exporters Eq.(4.6), I can simplify $(\frac{p_a}{\bar{p}})^{\frac{\alpha}{1-\alpha}}$ as shown below (See Appendix C.4):

$$(\frac{p_a(z)}{\bar{p}})^{\frac{\alpha}{1-\alpha}} = \frac{kz_a^*}{(k-1)z}[1+((\frac{\theta_{d_x}}{\theta_a})^{\frac{\alpha}{1-\alpha}}-1)Z_1^{k-1}+((\frac{\theta_{d_m}}{\theta_a})^{\frac{\alpha}{1-\alpha}}-(\frac{\theta_{d_x}}{\theta_a})^{\frac{\alpha}{1-\alpha}})Z_2^{k-1}] \tag{4.21}$$

Where A_1 is defined as follows:

$$A_1 = [1+((\frac{\theta_{d_x}}{\theta_a})^{\frac{\alpha}{1-\alpha}}-1)Z_1^{k-1}+((\frac{\theta_{d_m}}{\theta_a})^{\frac{\alpha}{1-\alpha}}-(\frac{\theta_{d_x}}{\theta_a})^{\frac{\alpha}{1-\alpha}})Z_2^{k-1}]$$

Using the above expression for $(\frac{p_a}{\bar{p}})^{\frac{\alpha}{1-\alpha}}$ and substituting zero cutoff productivity z_a^* for general z, I can simplify the exit condition for non-exporters (EC) with notation A_1:

$$e = \frac{\lambda_d(\frac{p_a(z_a^*)}{\bar{p}})^{\frac{\alpha}{1-\alpha}}}{1-\theta_a} = \frac{\lambda_d k A_1}{(k-1)(1-\theta_a)}$$

This simplified cutoff condition for a non-exporter, will be applied to the numerical simulation analysis.

Exporters

The term Φ is the ratio of total production to the total consumption of a domestic exporter, and θ_x represents the inverse of a domestic exporter's average markup. Therefore, I can apply Eq.(4.6), the equilibrium price of a domestic exporter, the definition of Φ and the production for a domestic exporter $q_x = (q_{d_x} + \tau q_{f_x})$ to derive the ratio of variable costs to revenue as shown, which is equivalent to θ_x:

$$\frac{z^{\frac{\alpha-1}{\alpha}} q_x}{p_x(q_{dx} + q_{f_x})} = \theta_x \tag{4.22}$$

since $x_x = n(q_{d_x} + q_{f_x})$ represents the total consumption of a domestic exported variety in both domestic and foreign markets. According to Eq. (4.22), the total variable cost is as follows, which is applied to calculate the relative equilibrium profit:

$$z^{\frac{\alpha-1}{\alpha}} q_x = \theta_x \frac{x_x}{n} p_x = \frac{\gamma E}{nM}(\frac{\bar{p}}{p_x})^{\frac{\alpha}{1-\alpha}} \theta_x = \theta_x e(\frac{\bar{p}}{p_x})^{\frac{\alpha}{1-\alpha}} \tag{4.23}$$

Where e is shown in Eq.(4.18) as the firm's average expenditure for a specific sector h.

I calculate the equilibrium profit of a firm within domestic exported varieties using Eqs. (4.22) and (4.23):

$$\begin{aligned} \pi_x &= p_x(q_{d_x} + q_{f_x}) - z^{\frac{\alpha-1}{\alpha}} q_x - \lambda_d - \lambda_x \\ &= (1-\theta_x)e(\frac{\bar{p}}{p_x})^{\frac{\alpha}{1-\alpha}} - \lambda_d - \lambda_x \\ &= e\bar{p}^{\frac{\alpha}{1-\alpha}}(1-\theta_x)\theta_x^{\frac{\alpha}{1-\alpha}} z - \lambda_d - \lambda_x \end{aligned}$$

In order to investigate the zero cutoff condition for exporters, I split the profits of the exporter in both domestic and foreign markets, separately. From the definition of $\Phi = (q_{d_x} + \tau q_{f_x})/(q_{d_x} + q_{f_x})$, I can get $q_{f_x} = \frac{\Phi-1}{\tau-\Phi} q_{d_x}$. Then, substituting it into Eq. (4.23) and combining with the definition of total output for an exporter in both the domestic and foreign market $q_x = (q_{d_x} + \tau q_{f_x})$, I have:

$$z^{\frac{\alpha-1}{\alpha}} q_{d_x} = \frac{\tau - \Phi}{\tau - 1} \theta_{d_x} e(\frac{\bar{p}}{p_x})^{\frac{\alpha}{1-\alpha}}$$

In other words, according to the equilibrium price of an exporter of Eq. (4.8) and substituting the expression of $z^{\frac{\alpha-1}{\alpha}} q_{d_x}$ into the profit function, the profit of a firm within an exported variety in the domestic market is shown as follows:

$$\begin{aligned}
\pi_{d_x} &= p_x q_{d_x} - z^{\frac{\alpha-1}{\alpha}} q_{d_x} - \lambda_d \\
&= \frac{1-\theta_{d_x}}{\theta_{d_x}} z^{\frac{\alpha-1}{\alpha}} q_{d_x} - \lambda_d \\
&= e(\frac{\bar{p}}{p_x})^{\frac{\alpha}{1-\alpha}} \frac{(1-\theta_{d_x})(\tau-\Phi)}{\tau-1} - \lambda_d
\end{aligned}$$

In addition, I can derive the profit of an exporter in the foreign market:

$$\begin{aligned}
\pi_{f_x} &= \pi_x - \pi_{d_x} = p_x q_{f_x} - z^{\frac{\alpha-1}{\alpha}} \tau q_{f_x} - \lambda_x \qquad (4.24) \\
&= e(\frac{\bar{p}}{p_x})^{\frac{\alpha}{1-\alpha}} \frac{1-\tau\theta_{d_x}}{1-\tau}(1-\Phi) - \lambda_x \\
&= e(\frac{\bar{p}}{p_x})^{\frac{\alpha}{1-\alpha}} [(1-\theta_x) - \frac{(1-\theta_{d_x})(\tau-\Phi)}{\tau-1}] - \lambda_x
\end{aligned}$$

Here, I consider defining the zero cutoff condition for exporters as $\pi_{f_x}(z_x^*) = 0$ so that there are no incentives for firms within an exported variety to deviate by only serving the domestic market and saving the fixed export costs. In this sense, in any case of $\pi_{f_x}(z_x^*) \geq 0$, there would be no firms within a variety that would like to deviate. The deviation case can be illustrated by the alternative oligopoly model with one domestic exported variety for $(n-1)$ exporters and one deviating non-exporter. In other words, the cutoff condition for exporters (XC) is $\pi_{f_x}(z_x^*) = 0$, where $\pi_{f_x}(z)$ is the profit of a firm producing an exported variety z in the foreign market. The export condition indicates that no firm prefers deviating by not exporting and not spending on fixed export costs at equilibrium prices.[4]

Therefore, the productivity cutoff for a firm within an exported variety domestically, also known as a domestic exporter's zero cutoff profit condition, is denoted by (XC) and determined by $\pi_{fx}(z_x^*) = 0$:

$$e = \frac{\lambda_x}{(\frac{\bar{p}}{p_x})^{\frac{\alpha}{1-\alpha}}[(1-\theta_x) - \frac{(1-\theta_{d_x})(\tau-\Phi)}{\tau-1}]} \qquad (4.25)$$

Notice that I consider the exporter's zero cutoff condition (XC) as $\pi_{f_x}(z_x^*) = 0$ rather than $\pi_x(z_x^*) = 0$ as the condition of $\pi_x(z_x^*) = 0$ is necessary but not sufficient. Because

[4] See Appendix C.5 for the deviation analysis.

$\pi_x(z_x^*) = 0$ cannot guarantee that firms have incentives to export for generating non-negative profits in the foreign market.

Analogous to non-exporters, according to Eq. (4.16) and Eq. (4.8), the equilibrium price of exporters, I can simplify $(\frac{p_x}{\bar{p}})^{\frac{\alpha}{1-\alpha}}$ as shown below (See Appendix C.6):

$$(\frac{p_x(z)}{\bar{p}})^{\frac{\alpha}{1-\alpha}} = \frac{kz_a^*}{(k-1)z}[(\frac{\theta_a}{\theta_{d_x}})^{\frac{\alpha}{1-\alpha}}+((1-(\frac{\theta_a}{\theta_{d_x}})^{\frac{\alpha}{1-\alpha}})Z_1^{k-1}+((\frac{\theta_{d_m}}{\theta_{d_x}})^{\frac{\alpha}{1-\alpha}}-1)Z_2^{k-1}] \quad (4.26)$$

Where A_2 is defined as follows:

$$A_2 \;=\; [(\frac{\theta_a}{\theta_{d_x}})^{\frac{\alpha}{1-\alpha}} + ((1-(\frac{\theta_a}{\theta_{d_x}})^{\frac{\alpha}{1-\alpha}})Z_1^{k-1} + ((\frac{\theta_{d_m}}{\theta_{d_x}})^{\frac{\alpha}{1-\alpha}} - 1)Z_2^{k-1}]$$

Using the above expression for $(\frac{p_x}{\bar{p}})^{\frac{\alpha}{1-\alpha}}$ and substituting zero cutoff productivity z_x^* for general z, I can simplify the exit condition for exporters (XC) with notation A_2 shown above:

$$e = \frac{k\lambda_x A_2 Z_1}{(k-1)[(1-\theta_x) - \frac{(1-\theta_{d_x})(\tau-\Phi)}{\tau-1}]}$$

Like a non-exporter, the simplified equation is applied for numerical simulation analysis.

Multinational Firms

Here, notice that the production and consumption of a multinational firm are the same as there is no variable iceberg-type transportation cost compared to exporters. However, I assume an additional fixed cost of creating a new plant in the foreign market, λ_m, rather than λ_x of fixed export costs for the exporter in the foreign market. Then, I apply Eq. (4.11) of the equilibrium price to derive the total variable costs of a multinational firm within a product line via horizontal FDI. Then, it can be used to calculate the equilibrium profit of a multinational firm, as shown:

$$z^{\frac{\alpha-1}{\alpha}} q_m = p_m\theta_m\frac{x_m}{n} = \frac{\gamma E}{M}(\frac{\bar{p}}{p_m})^{\frac{\alpha}{1-\alpha}}\frac{\theta_m}{n} = \theta_m e(\frac{\bar{p}}{p_m})^{\frac{\alpha}{1-\alpha}} \quad (4.27)$$

Notice that θ_{d_m} and θ_{f_m} are equal, which implies the inverse of markups of a multinational firm charged the same in the domestic and foreign markets, separately, are both equal to θ_m. It is shown as Eq. (4.11) because of no variable transport costs τ under FDI. e is shown as Eq. (4.18) .

Next, I derive the equilibrium profit of a multinational firm in a product line via horizontal FDI with Eq. (4.27) and the equilibrium price of a multinational firm from Eq. (4.11):

$$\begin{aligned}\pi_m &= p_m(q_{d_m}+q_{f_m}) - z^{\frac{\alpha-1}{\alpha}}q_m - \lambda_d - \lambda_m \\ &= (1-\theta_m)e(\frac{\bar{p}}{p_m})^{\frac{\alpha}{1-\alpha}} - \lambda_d - \lambda_m \\ &= e\bar{p}^{\frac{\alpha}{1-\alpha}}(1-\theta_m)\theta_m^{\frac{\alpha}{1-\alpha}}z - \lambda_d - \lambda_m\end{aligned}$$

Analogous to the definition of the exporters' cutoff profit condition, I separate the profits of a multinational firm within a product line via horizontal FDI in both domestic and foreign markets, respectively. Using Eq. (4.11) of the equilibrium price of a multinational firm, I have:

$$\begin{aligned}\pi_{d_m} &= p_m q_{d_m} - z^{\frac{\alpha-1}{\alpha}}q_{d_m} - \lambda_d \\ &= (\frac{1}{\theta_{d_m}} - 1)z^{\frac{\alpha-1}{\alpha}}q_{d_m} - \lambda_d\end{aligned}$$

Since there is no variable transportation cost τ, I get $q_{d_m} = q_{f_m}$ and substitute it into Eq. (4.27),

$$z^{\frac{\alpha-1}{\alpha}}q_{d_m} = \frac{e}{2}\theta_{d_m}(\frac{\bar{p}}{p_m})^{\frac{\alpha}{1-\alpha}}$$

where $\theta_{d_m} = \theta_{f_m} = \theta_m$ in Eq. (4.11), I derive:

$$\pi_{d_m} = \frac{e}{2}(1-\theta_m)(\frac{\bar{p}}{p_m})^{\frac{\alpha}{1-\alpha}} - \lambda_d$$

Moreover, the profits of a multinational firm within a product line via horizontal FDI in the foreign market are:

$$\begin{aligned}\pi_{f_m} &= \pi_m - \pi_{d_m} \\ &= \frac{e}{2}\bar{p}^{\frac{\alpha}{1-\alpha}}(1-\theta_m)\theta_m^{\frac{\alpha}{1-\alpha}}z - \lambda_m \\ &= \frac{e}{2}(1-\theta_m)(\frac{\bar{p}}{p_m})^{\frac{\alpha}{1-\alpha}} - \lambda_m\end{aligned} \tag{4.28}$$

Eq. (4.28) denotes the profits in the foreign market for multinational firms via horizontal FDI, and it should be non-negative. Like the cutoff condition for exporters, $\pi_{f_m}(z_m^*) = 0$ is necessary but not sufficient. Therefore, I need to find a condition where no multinational firms would like to deviate from multinational firms to the exporter and save the higher fixed cost of locating a firm in the foreign market. In this case, I need to consider the deviation case for multinational firms. Analogous to exporters, alternative oligopoly models have only one specific multinational variety with $(n-1)$ multinational firms and one deviating exporter. That is to say, I emphasize the ways of serving the foreign market for firms rather than the domestic market when I investigate the cutoff condition for multinational firms.

In other words, the cutoff condition for multinational firms via horizontal FDI is $(\pi_{fm} - \pi_{fx(n-1,\underline{1})})(z_m^*) = 0$. Where π_{fm} is the profit in the foreign market of a firm producing in the multinational product line and $\pi_{fx(n-1,\underline{1})}$ is the profit in the foreign market of one deviating exporter with $(n-1)$ multinational firms in a deviating multinational product line. The expression implies that no firm prefers deviating from multinational products to exporting and saving the higher fixed cost of locating a subsidiary in a foreign market.

Therefore, in the extreme case, I derive the multinational firms' zero cutoff profit condition in a product line via FDI using $(\pi_{fm} - \pi_{fx(n-1,\underline{1})})(z_m^*) = 0$, referred to by (FC),

$$e = \frac{\lambda_m - \lambda_x}{[\frac{(1-\theta_{dm})}{2}\theta_{dm}^{\frac{\alpha}{1-\alpha}} - \frac{n}{D}\tau^{\frac{\alpha}{\alpha-1}}(1-\theta_{fx(n-1,\underline{1})})\theta_{fx(n-1,\underline{1})}^{\frac{\alpha}{1-\alpha}}]\bar{p}^{\frac{\alpha}{1-\alpha}}z} \tag{4.29}$$

Where $D = \frac{(2n-1+\tau)(1-\alpha)}{(1-\tau)(2n-1)+\tau(1-\alpha)}$.[5] Analogous to non-exporters, according to the average price Eq. (4.16) and the equilibrium price Eq. (4.11) of multinational firms, I can simplify $(\frac{p_m}{\bar{p}})^{\frac{\alpha}{1-\alpha}}$ as shown below (See Appendix C.8):

$$(\frac{p_m(z)}{\bar{p}})^{\frac{\alpha}{1-\alpha}} = \frac{kz_a^*}{(k-1)z}[(\frac{\theta_a}{\theta_{d_m}})^{\frac{\alpha}{1-\alpha}} + ((\frac{\theta_{d_x}}{\theta_{d_m}})^{\frac{\alpha}{1-\alpha}} - (\frac{\theta_a}{\theta_{d_m}})^{\frac{\alpha}{1-\alpha}})Z_1^{k-1} + (1-(\frac{\theta_{d_x}}{\theta_{d_m}})^{\frac{\alpha}{1-\alpha}})Z_2^{k-1}] \tag{4.30}$$

Where A_3 is defined as follows:

$$A_3 = [(\frac{\theta_a}{\theta_{d_m}})^{\frac{\alpha}{1-\alpha}} + ((\frac{\theta_{d_x}}{\theta_{d_m}})^{\frac{\alpha}{1-\alpha}} - (\frac{\theta_a}{\theta_{d_m}})^{\frac{\alpha}{1-\alpha}})Z_1^{k-1} + (1-(\frac{\theta_{d_x}}{\theta_{d_m}})^{\frac{\alpha}{1-\alpha}})Z_2^{k-1}]$$

The terms $(\frac{p_m}{\bar{p}})^{\frac{\alpha}{1-\alpha}}$ and A_3 can be used as intermediate notation to calculate the inverse of average markups of the global economy $\bar{\theta}$ when I move to the labour market clearing condition.

4.3.3 Entry and Selection

In this general set-up, I introduce a different condition for endogenous variety from the one in the first main chapter, applying the exogenous mass of potential variety M_e to determine the number of operative variety M endogenously. The restricted entry strategy performs as in the first chapter with exogenously fixed n, and I assume a mass of potential varieties M_e and consider the condition for endogenous variety like Melitz (2003). It is different from my model because each variety is produced by

[5] See Appendix C.7 for a proof

n identical oligopolistic firms in my model; only one firm is in each variety in Melitz (2003). Once firms observe their productivity draw within a product line, they decide whether they are productive enough to earn non-negative profits; otherwise, all n firms exit collectively. The condition for endogenous variety here is distinguished from the first main chapter as the mass of operative variety M in the first chapter is bounded below 1, which leads to its value relying on the firm's productivity threshold of equilibrium because of its boundary and the condition for endogenous variety. However, this chapter assumes an exogenous number of entrants M_e to derive the mass of M endogenously as a determinant of the equilibrium, which means the mass of M, through the static condition, determines the equilibrium. Finally, I assume that firms within a product line face a constant risk of death collectively in every period, as in the first chapter and Melitz (2003). In other words, firms within a variety face a bad shock that leads them to exit together with probability δ, and the shock is across all varieties with different productivity levels. Here, probability δ is the same across firms and varieties, and it is assumed to be independent across varieties of firms with different productivity. I solve the zero cutoff productivity for non-exporters, exporters and multinational firms in the above section with the notation $(EC)-(XC)-(FC)$. In the global economy, I need to pin down the system with two more conditions, a condition for endogenous variety (SC) and a labour market clearing condition (MC) to solve the general equilibrium of the parameters as a vector $\{z_a^*, z_x^*, z_m^*, e, M\}$. There is a mass of operative variety M, and the number of entrants M_e, where each of them is produced by n identical firms. The mass of varieties M_e entering the economy at zero cost with associated productivity z is jointly drawn from a Pareto distribution $G(z)$ with lower productivity bound $z = 1$ and shape parameter k.

Condition for Endogenous Variety

A condition for endogenous variety is imposed by introducing an exogenous mass M_e, like Melitz (2003). It is a different condition than in the second and third chapters, while they have the same assumption with the exogenous number of firms n within each variety. In contrast, the second and third chapters assume that M did not affect the equilibrium but is determined by the productivity threshold of equilibrium. For example, in the simple model in the second chapter, the equilibrium is derived through the Exit Condition (EC) and the Labour Market Clearing Condition (MC), determining the productivity threshold and average expenditure per firm for a particular sector h in three separate scenarios for non-exporters, exporters and multinational firms. However, in the general model, the equilibrium is considered by the condition for endogenous variety (SC), which endogenously determines the operative mass of M, combining the labour market clearing condition (MC) and three exit conditions for non-exporters, exporters and multinational firms. The solution of the equilibrium analysis of the economy can be solved by pinning down these cutoff conditions with the condition for endogenous variety and the Labour market-clearing condition, given by $(EC)-(XC)-(FC)-(SC)-(MC)$. It derives the equilibrium variables of the productivity threshold for non-exporters, exporters and multinational firms, z_a^*, z_x^*, z_m^*, the endogenous number of an operative variety M and average expenditure per firm, e. In other words, the equilibrium of the system $\{z_a^*, z_x^*, z_m^*, M, e\}$ is calculated by $(EC)-(XC)-(FC)-(SC)-(MC)$ simultaneously. Notice that I consider a particular sector h for simplification of each condition but move to welfare analysis with all sectors considered.

A specific variety is generated by a firm's entry into the economy at zero cost and n oligopolistic firms comprise each variety to produce goods with particular productivity z. I introduce a condition for endogenous variety like Melitz (2003), to endogenously determine the number of operative varieties M:

$$(1-G(z_a^*))M_e = \delta M \tag{4.31}$$

Where M_e is the mass of new varieties and given exogenously. The equation expresses that the quantities of successful varieties, $(1-G(z_a^*))M_e$, equal the exit flow δM of varieties which are affected by the bad shock. Eq. (4.31) is applied to determine the mass of operative variety M endogenously in the economy. It is distinct from the first and second main chapters, where the condition for endogenous variety is defined due to the exogenous mass of varieties.

Regarding the condition for endogenous variety (SC) of Eq. (4.31), I simplify it with the Pareto productivity distribution as above Eq. (4.5) and derive:

$$M_e = \delta M z_a^{*k}$$

Labour Market Clearing Condition

This part considers a labour market clearing condition in the inclusive economy with non-exporters, exporters and multinational firms:

$$\begin{aligned}\frac{1}{nM(z_a^*)} &= \int_{z_a^*}^{z_x^*}[z^{\frac{\alpha-1}{\alpha}}q_a+\lambda_d]\mu(z)dz + \int_{z_x^*}^{z_m^*}[z^{\frac{\alpha-1}{\alpha}}q_x+\lambda_d+\lambda_x]\mu(z)dz \\ &\quad + \int_{z_m^*}^{\infty}[z^{\frac{\alpha-1}{\alpha}}q_m+\lambda_d+\lambda_m]\mu(z)dz + \frac{(1-\gamma)}{\gamma}e\end{aligned}$$

It means the labour resources used by non-exporters, exporters and multinational firms via FDI plus labour used for homogeneous goods equals the labour endowment of the economy. Simplifying the market clearing condition with Eqs. (4.19), (4.23) and (4.27), it can be written as (MC):

$$\begin{aligned}\frac{1}{nM(z_a^*)} &= e(\frac{(1-\gamma)}{\gamma}+\bar{\theta})+\lambda_d+\frac{G(z_m^*)-G(z_x^*)}{1-G(z_a^*)}\lambda_x \\ &+\frac{1-G(z_m^*)}{1-G(z_a^*)}\lambda_m\end{aligned} \tag{4.32}$$

where

$$\begin{aligned}\bar{\theta} &= \theta_a\int_{z_a^*}^{z_x^*}(\frac{\bar{p}}{p_a})^{\frac{\alpha}{1-\alpha}}\mu(z)dz+\theta_x\int_{z_x^*}^{z_m^*}(\frac{\bar{p}}{p_x})^{\frac{\alpha}{1-\alpha}}\mu(z)dz \\ &+\theta_m\int_{z_m^*}^{\infty}(\frac{\bar{p}}{p_m})^{\frac{\alpha}{1-\alpha}}\mu(z)dz\end{aligned} \tag{4.33}$$

Therefore, I simplify the inverse of average equilibrium markups in the global economy, $\bar{\theta}$, as follows[6]:

$$\bar{\theta}=\frac{\theta_a^{\frac{1}{1-\alpha}}+(\theta_{d_x}{}^{\frac{1}{1-\alpha}}\Phi-\theta_a^{\frac{1}{1-\alpha}})Z_1^{k-1}+Z_2^{k-1}((\theta_{d_m})^{\frac{1}{1-\alpha}}-\theta_{d_x}{}^{\frac{1}{1-\alpha}}\Phi)}{\theta_a^{\frac{\alpha}{1-\alpha}}+(\theta_{d_x}{}^{\frac{\alpha}{1-\alpha}}-\theta_a^{\frac{\alpha}{1-\alpha}})Z_1^{k-1}+((\theta_{d_m})^{\frac{\alpha}{1-\alpha}}-(\theta_{d_x})^{\frac{\alpha}{1-\alpha}})Z_2^{k-1}} \tag{4.34}$$

Where $\Phi=\frac{\theta_x}{\theta_{d_x}}$, it is the ratio of total production to the total consumption of an exporter, which also relates the losses related to export to the variable transportation cost, τ.

Finally, I simplify the Market Clearing Condition (MC) condition of the model; according to Eq. (4.32), I have:

$$e(\frac{1-\gamma}{\gamma}+\bar{\theta})+\lambda_d+(Z_1^k-Z_2^k)\lambda_x+Z_2^k\lambda_m=\frac{1}{nM}$$

where the expression of $\bar{\theta}$ is shown as Eq. (4.34).

4.3.4 Welfare

In this part, I address the total welfare effect of the global economy across all sectors rather than focusing on a specific sector h as in the above section. Analogous to the main chapter 1, I firstly focus on a specific sector as shown above, then, I extend to all sectors. I have:

$$\bar{\gamma}=\sum_{h=1}^{H}\gamma_h$$

[6]See Appendix C.9 for a proof

$$\bar{e} = \frac{\bar{\gamma}E}{nM}$$

$\bar{e}$ represents the average expenditure per firm among all sectors, M is the number of operative varieties and is endogenous. Therefore, according to the utility function Eq. (4.1) and Eq. (4.13), I have the welfare gains in the whole economy:

$$U = \bar{\gamma}\ln(n\bar{e}\frac{M^{1/\alpha}}{\bar{p}}) + (1-\bar{\gamma})\ln(\frac{1-\bar{\gamma}}{\bar{\gamma}}n\bar{e}M) \tag{4.35}$$

4.4 Numerical Simulation Analysis

Since the system is not linear, I will investigate the model's main properties through numerical simulations. My main aim is to explore how the welfare gains perform as trade liberalisation happens under the presence of multinational production. In addition, I also examine the response of product market competition and the selection effect as iceberg-type trade costs are reduced from the prohibitive level to the lowest theoretical value 1 with free entry. Notice that since the theoretical model focuses on a steady state, I compare welfare effects through two global economies similar in all characteristics except for the iceberg-type cost.

Through my theoretical model, I have ten parameters $\{\alpha, \tau, \lambda_d, \lambda_x, \lambda_m, k, \bar{\gamma}, \delta, n, M_e\}$ to match with empirical research, and these parameters have been taken from the literature. The value for shape parameter k of entry distribution which follows Pareto is pinned down as 1.14, which is in the range of 1.06 estimated by Luttmer (2007) for US firms and 1.39 estimated by Head et al. (2014) with French data exporting to Belgium. I set $\alpha = 0.32$ from Impullitti and Licandro (2018) which implies an elasticity of substitution of 1.48. They indicate that the elasticity of substitution sits in the median micro elasticity of 3.1, see Feenstra et al. (2018) and macro elasticity close to one. The setting of the death rate δ is 0.09 to match the average enterprise annual death rate for manufacturers in 1998-2004 using Census 2004 data. I set $\bar{\gamma} = 2/3$ to represent the aggregate share of composite goods to be in line with Rauch (1999), which finds that the differentiated goods account for 64.6% and 67.1% of total US in manufacturing with the chosen aggregation scheme. I pin down the value of fixed exporting costs as $\lambda_x = 0.0022$ like Impullitti and Licandro (2018) and fixed production cost $\lambda_d = 0.0013$. The $\lambda_m = 0.010$ is set according to my assumption $\lambda_m > \lambda_x$ in the theory. The mass of new entrants M_e is set to 6, which is similar to the number of firms in Impullitti and Licandro (2018) considering the free entry condition, to pin down the endogenous number of operative varieties. The number of firms $n = 1.1$, which is close to the setting of the numerical analysis of Impullitti et al. (2018) and Impullitti et al. (2022). They consider a similar economic environment with variable markups stemming from the oligopolistic competition, firm heterogeneity and different free entry conditions. Finally, I set the variable transportation cost $\tau \in (1, 1.2)$ to represent the lack of barriers, and the prohibitive trade cost like Impullitti and Licandro (2018). In addition, Anderson and Van Wincoop (2004) estimate that 170 percent of the tax equivalent of the trade cost is accounted for by 21 percent of transportation costs, which contains both directly measured freight costs and tax equivalent of the time value of goods in transit with US data.

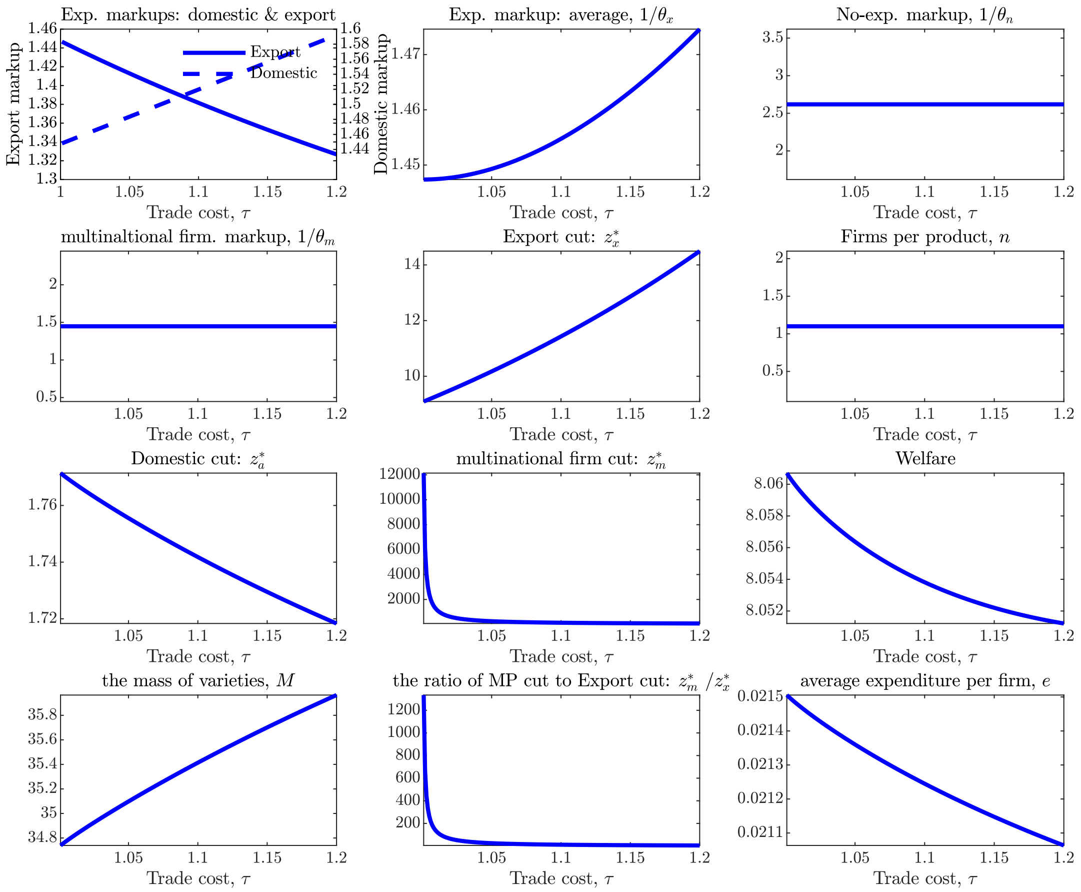

Figure 4.1. Trade Liberalisation

The figure shows that trade liberalisation has a pro-competitive effect on exporters only, while the markups of non-exporters and multinational firms are unaffected by trade liberalisation. It can be identified through Eqs. (4.6), (4.8) and (4.11), where the inverse of the markups for non-exporters θ_a and multinational firms θ_m only relate to n, the number of firms per variety and α associated with elasticity of substitution. In contrast, the inverse of the markups for exporters θ_{d_x} and θ_{f_x} in the domestic and foreign markets are also relative to variable iceberg-type costs τ, according to Eq. (4.8) and its properties as shown in the first main chapter 2.2, see panel 1. Panel 6 represents the number of firms n per product line, which is exogenously fixed with $n = 1.1$ and irrelevant to τ. We know that $1/\theta_m < 1/\theta_x < 1/\theta_a$ from proposition 2.4 of the first main chapter, which implies that multinational firms via horizontal FDI have the lowest markups compared to exporters and non-exporters, see panels 2, 3 and 4. Trade liberalisation also induces fewer operative varieties to survive in the market because of the selection effect for non-exporters, as expected in the literature. The selection effect happens in the presence of exporters because of the firm heterogeneity included. Eaton et al. (2011) document the efficiency of firm heterogeneity when they examine French manufacturing firms' trade data. In addition, Yeaple (2009) also claims the significance of heterogeneous multinational firms using US firm-level data and finds that country characteristics will affect the structure of multinational firms' activity. The export cutoff z_x^* decreases with trade liberalisation due to the increase in markups on foreign sales. It occurs since trade liberalisation induces exporters to enjoy a cost reduction in trade while domestic firms do not benefit, which encourages more firms to start exporting, and finally, z_x^* decreases when τ declines. Notice that although exporting firms increase their markups on foreign sales, it is not enough to offset the decreased makeups in their domestic sales. Therefore, average markups of exporters decrease when iceberg-type trade costs decline, like Impullitti and Licandro (2018). Panel 2 in the figure indicates that trade liberalisation produces the pro-competitive effect on exporters from a qualitative perspective, consistent with proposition 2.1 from the simple model of the first main chapter.

The pro-competitive effect of exporters increases as trade liberalises, forcing the least productive firms to exit the market. It leads to more firms choosing to serve the foreign market and fewer domestic firms as τ decreases. The selection effect of non-exporters displays an increasing cutoff threshold z_a^*, in which the minimum productivity is larger than 1 as in the assumption in the theory of the Pareto distribution, as shown in panel 7. Panel 10 displays a lower number of varieties M as trade liberalises. It can be explained with the condition for endogenous variety, as shown in Eq.(4.31), in which we have a negative association between the mass of variety and the productivity threshold for non-exporters. The productivity thresholds for multinational firms z_m^*, as shown in panel 8, increase as trade liberalises, implying that a lower number of multinational firms enter the foreign market via FDI. Because of the proximity-concentration effect between exporters and multinational firms, trade liberalisation induces more firms to serve the foreign market via exports. This is also represented as the increases in z_m^*/z_x^* see panel 11: the proportion of firms being engaged in multinational production to relative exporters decreases due to trade liberalisation because firms prefer to be exporters rather than multinational firms as the iceberg-type cost τ decreases. In addition, panels 5, 7 and 8 also represent that $z_m^* > z_x^* > z_a^*$, which implies that only the most productive firms would choose to serve the foreign market via horizontal FDI, while the medium productive firms choose to serve the foreign market via export, and the lower productivity firms can only serve the domestic market. It is in line with proposition 2.4 in the first main chapter. The consumer's expenditure received by each differentiated firm $\bar{e}$, as shown in Eq.(4.18) and panel 12, increases with trade liberalisation as nM decreases due to the fixed n and decreasing M.

In the global economy, as shown in panel 9, welfare increases arise from trade liberalisation through 1) the selection effect for non-exporters, increasing the aggregate productivity of the economy; 2) the pro-competitive effect for exporters, lowering markups of the economy; 3) efficiency gains from the switch from multinational firms to exporters with the engagement of multinational firms, decreasing the mass of multinational firm and proportion of multinational firms to exporters in the economy. Specifically, according to Eq.(4.35) with numerical simulation analysis, I found that the welfare gains of trade liberalisation increase by around 0.12% when the economy moves from autarky 8.0512 to free trade 8.0607 in the presence of horizontal FDI, where the prohibitive level of τ I set $\bar{\tau} = 1.2$ as in Impullitti and Licandro (2018) and free trade $\tau = 1$, respectively.

4.5 Conclusion

This chapter examines the welfare effects of international trade and multinational production via horizontal FDI due to trade liberalisation in an economy with oligopolistic competition and firm heterogeneity. The contribution highlights three mechanisms of enhancement of welfare effects: the pro-competitive effect, selection effect and the efficiency from the engagement of multinational firms via horizontal FDI, which is a competing (substitutable) way of serving the foreign market with exporting. This framework extends Impullitti and Licandro (2018) to examine the welfare gains of trade liberalisation by adding horizontal FDI like Helpman et al. (2004). Compared to Impullitti and Licandro (2018), I incorporate multinational production, which produces the highest pro-competitive effect through the fiercest competition of openness, the highest productivity threshold, forcing the least productive domestic firms to exit the market and the efficiency of multinational firms as an alternative method to serve the foreign market.

My work is similar to Sun et al. (2020), which develops Melitz and Redding (2015) to discuss the welfare gains from trade liberalisation, allowing the presence of horizontal FDI. However, I complement their paper by including variable markups stemming from the oligopolistic competition, which generates the pro-competitive effect of welfare gains. Specifically, I illustrate that multinational firms generate the highest pro-competitive effect from the lowest markups under oligopolistic competition.

This chapter also introduces firm heterogeneity and finds that welfare increases as trade liberalises because of firm selection, leading the least productive firms to exit the market and inducing higher aggregate productivity (e.g., Chor, 2009; Eckel and Neary, 2010; Edmond et al., 2015). Furthermore, I find that multinational firms produce the largest productivity threshold compared to exporters and non-exporters, as shown in numerical analysis, which means that only the most productive firms can be multinational firms to serve the foreign market, like Helpman et al. (2004). The selection effect for non-exporters is one source of welfare gains from trade liberalisation.

Lastly, my work complements the first main chapter with non-exporters, exporters and multinational firms simultaneously existing in the world economy. This allows us to study the interaction between international trade and horizontal FDI, where horizontal FDI is a substitutable way of serving the foreign market. I consider the mass of operative varieties as a determinant of the general equilibrium by introducing the condition for endogenous variety like Melitz (2003). I find that there is a proximity-concentration effect between exporters and multinational firms, trade liberalisation induces more firms to serve the foreign market via exports, and there is decreasing variety due to the selection effect for non-exporters and the enhancement of welfare effects with the efficiency of the engagement of multinational production.

Appendix C

C.1 Equation (4.13)

According to the inverse demand function in Eqs. (4.3), I can derive that:

$$x_i(v) = [\frac{\gamma E}{X^{\alpha} p_i(v)}]^{\frac{1}{1-\alpha}}$$

Here, I put general notation $i \in \{a, x, m\}$ in variables x_i and p_i, which indicates relative quantities and prices of domestic, exported and multinational products via horizontal FDI. Insert them into aggregate composite goods X of Eqs. (4.4)

$$X^{\alpha} = \int_0^M x(v)^{\alpha} dv = M[\frac{\gamma E}{X^{\alpha} p_i(v)}]^{\frac{\alpha}{1-\alpha}}$$

$$X^{\frac{\alpha}{1-\alpha}} = M[\frac{\gamma E}{p_i(v)}]^{\frac{\alpha}{1-\alpha}}$$

$$X = M^{\frac{1-\alpha}{\alpha}}[\frac{\gamma E}{p_i(v)}]$$

$$X = M^{\frac{1-\alpha}{\alpha}}[\frac{\gamma E}{\bar{p}}]$$

where i represents all productive varieties of domestic, export and multinational products via horizontal FDI; similar to Melitz (2003) and Helpman et al. (2004) introducing aggregate price, I take average price as follows:

$$\bar{p}^{\frac{\alpha}{\alpha-1}} = [\int_{z_a^*}^{z_x^*} p_a(z)^{\frac{\alpha}{\alpha-1}} \mu(z) dz + \int_{z_x^*}^{z_m^*} p_x(z)^{\frac{\alpha}{\alpha-1}} \mu(z) dz + \int_{z_m^*}^{\infty} p_m(z)^{\frac{\alpha}{\alpha-1}} \mu(z) dz]$$

Here, $\mu(z)$ is the equilibrium distribution of productivity.

C.2 Equation (4.16)

According to the definition of average price $\bar{p}$ and applying to the relative equilibrium price for non-exporters, exporters and multinational firms, Eqs.(4.6), (4.8) and (4.11) separately, the Pareto productivity distribution $G(z) = 1 - \left(\frac{1}{z}\right)^k$, $z \geq 1$, $k \geq 1$, which means $1 - G(z) = \left(\frac{1}{z}\right)^k$, $g(z) = kz^{-k-1}$, and $zg(z) = kz^{-k}$, and Eq. (4.15) of the equilibrium density $\mu(z)$, I get:

$$
\begin{aligned}
\bar{p}^{\frac{\alpha}{\alpha-1}} &= [\int_{z_a^*}^{z_x^*} p_a(z)^{\frac{\alpha}{\alpha-1}}\mu(z)dz + \int_{z_x^*}^{z_m^*} p_x(z)^{\frac{\alpha}{\alpha-1}}\mu(z)dz + \int_{z_m^*}^{\infty} p_m(z)^{\frac{\alpha}{\alpha-1}}\mu(z)dz] \\
&= [\int_{z_a^*}^{z_x^*} (\frac{z^{\frac{\alpha-1}{\alpha}}}{\theta_a})^{\frac{\alpha}{\alpha-1}}\mu(z)dz + \int_{z_x^*}^{z_m^*} (\frac{z^{\frac{\alpha-1}{\alpha}}}{\theta_{d_x}})^{\frac{\alpha}{\alpha-1}}\mu(z)dz + \int_{z_m^*}^{\infty} (\frac{z^{\frac{\alpha-1}{\alpha}}}{\theta_{d_m}})^{\frac{\alpha}{\alpha-1}}\mu(z)dz] \\
&= [\theta_a^{\frac{\alpha}{1-\alpha}} z_a^{*k} \int_{z_a^*}^{z_x^*} zg(z)dz + \theta_{d_x}^{\frac{\alpha}{1-\alpha}} z_a^{*k} \int_{z_x^*}^{z_m^*} zg(z)dz + \theta_{d_m}^{\frac{\alpha}{1-\alpha}} z_a^{*k} \int_{z_m^*}^{\infty} zg(z)dz] \\
&= [\theta_a^{\frac{\alpha}{1-\alpha}} z_a^{*k} \frac{k}{1-k}(z_x^{*1-k} - z_a^{*1-k}) + \theta_{d_x}^{\frac{\alpha}{1-\alpha}} z_a^{*k} \frac{k}{1-k}(z_m^{*1-k} - z_x^{*1-k}) - \theta_{d_m}^{\frac{\alpha}{1-\alpha}} z_a^{*k} \frac{k}{1-k} z_m^{*1-k}] \\
&= \frac{k}{k-1}[\theta_a^{\frac{\alpha}{1-\alpha}} z_a^* - \theta_a^{\frac{\alpha}{1-\alpha}} z_a^{*k} z_x^{*1-k} + \theta_{d_x}^{\frac{\alpha}{1-\alpha}} z_a^{*k} z_x^{*1-k} - \theta_{d_x}^{\frac{\alpha}{1-\alpha}} z_a^{*k} z_m^{*1-k} + \theta_{d_m}^{\frac{\alpha}{1-\alpha}} z_a^{*k} z_m^{*1-k}] \\
&= \frac{k}{k-1} z_a^* [\theta_a^{\frac{\alpha}{1-\alpha}} - \theta_a^{\frac{\alpha}{1-\alpha}} z_a^{*k-1} z_x^{*1-k} + \theta_{d_x}^{\frac{\alpha}{1-\alpha}} z_a^{*k-1} z_x^{*1-k} - \theta_{d_x}^{\frac{\alpha}{1-\alpha}} z_a^{*k-1} z_m^{*1-k} + \theta_{d_m}^{\frac{\alpha}{1-\alpha}} z_a^{*k-1} z_m^{*1-k}] \\
&= \frac{k}{k-1} z_a^* [\theta_a^{\frac{\alpha}{1-\alpha}} + (\theta_{d_x}^{\frac{\alpha}{1-\alpha}} - \theta_a^{\frac{\alpha}{1-\alpha}}) Z_1^{k-1} + (\theta_{d_m}^{\frac{\alpha}{1-\alpha}} - \theta_{d_x}^{\frac{\alpha}{1-\alpha}}) Z_2^{k-1}]
\end{aligned}
$$

C.3 Equation (4.17)

I use general notation i to derive the revenue of a firm within three types of varieties with the equilibrium price p_i of a firm within a non-exported variety, an exported variety and the multinational production via horizontal FDI, combined with Eqs.(4.7), (4.10) and (4.12), as follows

$$
\begin{aligned}
p_i x_i &= p_i(\frac{\gamma E}{X^\alpha p_i})^{\frac{1}{1-\alpha}} = (\frac{\gamma E}{p_i})^{\frac{1}{1-\alpha}} X^{\frac{\alpha}{\alpha-1}} p_i \\
&= (\frac{\gamma E}{p_i})^{\frac{1}{1-\alpha}} [M^{\frac{1-\alpha}{\alpha}} (\frac{\gamma E}{\bar{p}})]^{\frac{\alpha}{\alpha-1}} p_i \\
&= \frac{\gamma E}{M} (\frac{\bar{p}}{p_i})^{\frac{\alpha}{1-\alpha}}
\end{aligned}
$$

C.4 Equation (4.21)

Substituting $\bar{p}$ into each component of the definition $\bar{\theta}$ and using Eq. (4.6), (4.8) and (4.11) which are equilibrium prices for non-exporters, exporters and multinational firms, I get:

$$
\begin{aligned}
(\frac{p_a}{\bar{p}})^{\frac{\alpha}{1-\alpha}} &= (\frac{\frac{z^{\frac{\alpha-1}{\alpha}}}{\theta_a}}{\{\frac{k}{k-1}z_a^*[\theta_a^{\frac{\alpha}{1-\alpha}}+(\theta_{d_x}^{\frac{\alpha}{1-\alpha}}-\theta_a^{\frac{\alpha}{1-\alpha}})Z_1^{k-1}+(\theta_{d_m}^{\frac{\alpha}{1-\alpha}}-\theta_{d_x}^{\frac{\alpha}{1-\alpha}})Z_2^{k-1}]\}^{\frac{\alpha-1}{\alpha}}})^{\frac{\alpha}{1-\alpha}} \\
&= \frac{\{\frac{k}{k-1}z_a^*[\theta_a^{\frac{\alpha}{1-\alpha}}+(\theta_{d_x}^{\frac{\alpha}{1-\alpha}}-\theta_a^{\frac{\alpha}{1-\alpha}})Z_1^{k-1}+(\theta_{d_m}^{\frac{\alpha}{1-\alpha}}-\theta_{d_x}^{\frac{\alpha}{1-\alpha}})Z_2^{k-1}]\}}{(\theta_a)^{\frac{\alpha}{1-\alpha}}z} \\
&= \frac{kz_a^*}{(k-1)z}[1+((\frac{\theta_{d_x}}{\theta_a})^{\frac{\alpha}{1-\alpha}}-1)Z_1^{k-1}+((\frac{\theta_{d_m}}{\theta_a})^{\frac{\alpha}{1-\alpha}}-(\frac{\theta_{d_x}}{\theta_a})^{\frac{\alpha}{1-\alpha}})Z_2^{k-1}]
\end{aligned}
$$

C.5 Equation (4.24)

This is the deviation analysis for the exported variety with $(n-1)$ exporters and one deviating non-exporter. In this case, I consider the alternative oligopoly model of $(n-1)$ exporters and one deviating non-exporter within only one specific exported product line domestically, which means the aggregate level in the global economy will not be changed under this situation. The purpose is to make sure no firm would decide to deviate by only serving domestically in a particular exported variety. In other words, I investigate the case of one particular deviating exported variety v domestically with $(n-1)$ exporters and one non-exporter in the global economy. According to the inverse demand function Eq. (4.3), I have notation as follow:

$q_{dx(\underline{n-1},1)}$: Exporter quantity offered in the domestic market;
$q_{fx(\underline{n-1},1)}$: Exporter quantity offered in the foreign market;
$q^*_{dx(\underline{n-1},1)}$: Foreign exporter's quantity offered in the foreign market;
$q^*_{fx(\underline{n-1},1)}$: Foreign exporter's quantity offered in the domestic market;
$q_{a(n-1,\underline{1})}$: non-exporter's quantity offered in the domestic market;
$p_{x(n-1,1)}$: the price of the variety in the domestic market;
$p^*_{x(n-1,1)}$: the price of the variety in the foreign market;
$x_{dx(n-1,1)} = n(q_{a(n-1,\underline{1})} + q^*_{fx(\underline{n-1},1)})$: quantities sold in the domestic market;
$x_{fx(n-1,1)} = nq^*_{dx(\underline{n-1},1)} + (n-1)q_{fx(\underline{n-1},1)}$: quantities sold in the foreign market;

then, I can get the relative price of the variety in both domestic and foreign market:

$$p_{x(n-1,1)} = \frac{\gamma E}{X^\alpha}x_{dx(n-1,1)}{}^{\alpha-1}$$

$$p^*_{x(n-1,1)} = \frac{\gamma E}{X^\alpha} x_{fx(n-1,1)}{}^{\alpha-1}$$

$(n-1)$ **exporters:**

Here, I use the above notation and calculate the profit of an exporter in a particular domestic variety with $(n-1)$ exporters and one deviating non-exporter, $x_{dx(n-1,1)}$ and $x_{fx(n-1,1)}$ are the quantities sold in the domestic market and the foreign market. Similar to the main chapter 1, an exporter in this situation solves the following problem:

$$\pi_{x(\underline{n-1},1)} = \max_{q_{dx}, q_{fx}} \underbrace{\frac{\gamma E}{X^\alpha} x^{\alpha-1}_{dx(n-1,1)}}_{p_{x(n-1,1)}} q_{dx} + \underbrace{\frac{\gamma E}{X^\alpha} x^{\alpha-1}_{fx(n-1,1)}}_{p^*_{x(n-1,1)}} q_{fx} - z^{\frac{\alpha-1}{\alpha}} (q_{dx} + \tau q_{fx}) - \lambda_d - \lambda_x \quad \text{(C.1)}$$

Notice that in the above expression, I omit the subscript $(\underline{n-1}, 1)$ of q_{dx} and q_{fx} for simplicity, which means I always focus on the quantities of an exporter in an alternative oligopoly of one deviating variety with $(n-1)$ exporters and one non-exporter. As a result, Eq. (C.1) yields the following first-order conditions for the quantity sold in both domestic market and the foreign market:

$$\frac{\gamma E}{X^\alpha}((\alpha-1) x_{dx(n-1,1)}{}^{\alpha-2} q_{dx} + x^{\alpha-1}_{dx(n-1,1)}) = z^{\frac{\alpha-1}{\alpha}} \quad \text{(C.2)}$$

$$\frac{\gamma E}{X^\alpha}((\alpha-1) x^{\alpha-2}_{fx(n-1,1)} q_{fx} + x^{\alpha-1}_{fx(n-1,1)}) = \tau z^{\frac{\alpha-1}{\alpha}} \quad \text{(C.3)}$$

Then I turn to the foreign exporters.

$$\pi^*_{x(\underline{n-1},1)} = \max_{q^*_{dx}, q^*_{fx}} \underbrace{\frac{\gamma E}{X^\alpha} x^{\alpha-1}_{dx(n-1,1)}}_{p_{x(n-1,1)}} q^*_{fx} + \underbrace{\frac{\gamma E}{X^\alpha} x_{fx(n-1,1)}{}^{\alpha-1}}_{p^*_{x(n-1,1)}} q^*_{dx} - z^{\frac{\alpha-1}{\alpha}} (q^*_{dx} + \tau q^*_{fx}) - \lambda_d - \lambda_x \quad \text{(C.4)}$$

Then, the first-order conditions for both the quantity sold in the domestic and foreign market, separately:

$$\frac{\gamma E}{X^\alpha}((\alpha-1) x_{fx(n-1,1)}{}^{\alpha-2} q^*_{dx} + x^{\alpha-1}_{fx(n-1,1)}) = z^{\frac{\alpha-1}{\alpha}} \quad \text{(C.5)}$$

$$\frac{\gamma E}{X^\alpha}((\alpha-1) x^{\alpha-2}_{dx(n-1,1)} q^*_{fx} + x^{\alpha-1}_{dx(n-1,1)}) = \tau z^{\frac{\alpha-1}{\alpha}} \quad \text{(C.6)}$$

One non-exporter:

Here, I use the above notation and calculate the profit of the deviating non-exporter. The non-exporter solves the following problem:

$$\pi_{a(n-1,\underline{1})} = \max_{q_a} \underbrace{\frac{\gamma E}{X^{\alpha}} x_{dx(n-1,1)}^{\alpha-1}}_{p_{x(n-1,1)}} q_a - z^{\frac{\alpha-1}{\alpha}} q_a - \lambda_d$$

Similarly, I omit the subscript $(n-1,\underline{1})$ of q_a for simplicity, which indicates my focus on the quantities of one non-exporter in an alternative oligopoly of one deviating variety with $(n-1)$ exporters and one non-exporter, which yields the following first-order conditions:

$$\frac{\gamma E}{X^{\alpha}}((\alpha-1)x_{dx(n-1,1)}{}^{\alpha-2}q_a + x_{dx(n-1,1)}^{\alpha-1}) = z^{\frac{\alpha-1}{\alpha}} \tag{C.7}$$

According to Eqs. (C.2) and (C.7), I have $q_{dx} = q_a$. It means there are the same quantities for each exporter and one deviating non-exporter in the domestic market under the case of a particular variety with $(n-1)$ exporters and one non-exporter.

Then I combine Eqs. (C.2) and (C.6), I can get the relationship between $x_{dx(n-1,1)}$ with q^*_{fx} and q_{dx},

$$\begin{aligned}\tau(\alpha-1)x_{dx(n-1,1)}{}^{\alpha-2}q_{dx} + \tau x_{dx(n-1,1)}^{\alpha-1} &= (\alpha-1)x_{dx(n-1,1)}^{\alpha-2}q^*_{fx} + x_{dx(n-1,1)}^{\alpha-1} \\ \Rightarrow \quad x_{dx(n-1,1)} &= \frac{(\alpha-1)(q^*_{fx} - \tau q_{dx})}{(\tau-1)}\end{aligned}$$

This relates to the definition of $x_{dx(n-1,1)} = n(q_{dx} + q^*_{fx})$, I can get the relationship between q^*_{fx} and q_{dx}

$$q_{dx}\frac{\tau(\alpha-1)+n(\tau-1)}{\alpha-1+n(1-\tau)} = q^*_{fx} \tag{C.8}$$

where I denote $\frac{\tau(\alpha-1)+n(\tau-1)}{\alpha-1+n(1-\tau)} = A$, namely, $A = q^*_{fx}/q_{dx}$, where A indicates the ratio of quantities for foreign exporter to exporter domestically in the domestic market.

Similarly, comparing Eqs. (C.3) and (C.5), I can derive the relationship between x_{fx} with q^*_{dx} and q_{fx},

$$\begin{aligned}(\alpha-1)x_{fx(n-1,1)}^{\alpha-2}q_{fx} + x_{fx(n-1,1)}^{\alpha-1} &= \tau((\alpha-1)x_{fx(n-1,1)}{}^{\alpha-2}q^*_{dx} + x_{fx(n-1,1)}^{\alpha-1}) \\ \Rightarrow \quad x_{fx(n-1,1)} &= \frac{(\alpha-1)(q_{fx} - \tau q^*_{dx})}{(\tau-1)}\end{aligned}$$

The output in the foreign market $x_{fx(n-1,1)} = (n-1)q_{fx} + nq^*_{dx}$, then I derive the relationship between q_{fx} and q^*_{dx}

$$q^*_{dx} = \frac{(\alpha-1)-(n-1)(\tau-1)}{n(\tau-1)+(\alpha-1)\tau} q_{fx}$$

where I denote $B = \frac{(\alpha-1)-(n-1)(\tau-1)}{n(\tau-1)+(\alpha-1)\tau}$, namely, $B = q^*_{dx}/q_{fx}$. B shows the ratio of output for foreign exporter to exporter in the foreign market.

Then, I rewrite Eq. (C.2), which is the same as Eq (C.7) with $q_{dx} = q_a$,

$$\underbrace{\frac{\gamma E}{X^\alpha} x^{\alpha-1}_{d(n-1,1)}}_{p_{x(n-1,1)}} [\frac{(\alpha-1)}{x_{dx(n-1,1)}} q_{dx} + 1] = z^{\frac{\alpha-1}{\alpha}} \tag{C.9}$$

combining with the definition of $x_{dx(n-1,1)} = n(q_{dx} + q^*_{fx})$ and indicator A in which $Aq_{dx} = q^*_{fx}$, I derive:

$$\theta_{dx} = [\frac{(\alpha-1)}{x_{dx(n-1,1)}} q_{dx} + 1] = \frac{(\alpha-1)}{n(A+1)q_{dx}} q_{dx} + 1 = \frac{2n+\alpha-1}{n(1+\tau)}$$

θ_{dx} represents the inverse of the markup of a domestic exporter or non-exporter in the domestic market within a particular variety with $(n-1)$ exporters and one deviating non-exporter. It is the same as the inverse markup of a domestic firm in the domestic market under 'all are exporters' in the first main chapter. It can be explained as there is no deviation in the domestic market, so it should be the same situation as n exporters within a product line in the domestic market.

Similarly, I rearrange Eq. (C.3) for one of $(n-1)$ domestic exporters within a particular deviating exported variety as follows:

$$\underbrace{\frac{\gamma E}{X^\alpha} x^{\alpha-1}_{f}}_{p^*_{x(n-1,1)}} [\frac{(\alpha-1)}{x_f} q_{fx} + 1] = \tau z^{\frac{\alpha-1}{\alpha}} \tag{C.10}$$

According to the output in the foreign market $x_{fx(n-1,1)} = (n-1)q_{fx} + nq^*_{dx}$ and indicator $B = q^*_{dx}/q_{fx} = \frac{(\alpha-1)-(n-1)(\tau-1)}{n(\tau-1)+(\alpha-1)\tau}$, I derive:

$$\begin{aligned} \theta_{fx(\underline{n-1},1)} &= [\frac{(\alpha-1)}{x_f} q_{fx} + 1] = \frac{(\alpha-1)}{((n-1)+nB)q_{fx}} q_{fx} + 1 \\ &= \frac{\tau(2n-2+\alpha)}{\tau(n-1)+n} \end{aligned}$$

$\theta_{fx(\underline{n-1},1)}$ represents the inverse of the markup of a domestic exporter in the foreign market for the alternative oligopoly model of $(n-1)$ exporters and one non-exporter within a particular variety. I can show that $\theta_{fx(\underline{n-1},1)} < \theta_{fx}$ [1], where $\theta_{fx} = \tau\theta_{dx}$ is the inverse markup of a domestic firm in the foreign market with n exporters in every variety with 'all are exporters' in the economy of the first main chapter. There will be less competition in the foreign market if one firm decides to deviate by serving only domestically than in the situation no one would like to deviate with n exporters within a variety.

According to the respective prices of an exporter in both domestic and foreign markets, $p_{x(n-1,1)}$ and $p^*_{x(n-1,1)}$, within a unique variety clustered by $(n-1)$ exporters and one deviating non-exporter, I have the total output in the different domestic and foreign market:

$$x_{dx(n-1,1)} = [\frac{\gamma E}{X^\alpha p_{x(n-1,1)}}]^{\frac{1}{1-\alpha}}$$

$$x_{fx(n-1,1)} = [\frac{\gamma E}{X^\alpha p^*_{x(n-1,1)}}]^{\frac{1}{1-\alpha}}$$

where $p_{x(n-1,1)} = \frac{z^{\frac{\alpha-1}{\alpha}}}{\theta_{dx}}$ and $p^*_{x(n-1,1)} = \frac{\tau z^{\frac{\alpha-1}{\alpha}}}{\theta'_{fx}}$ as I can identify them from the Eqs. (C.9) and (C.10).

The aggregate composite goods are shown as above Eq. (4.13). Notice that aggregate and average economy levels will not be altered because of only one firm's deviation. Then, substituting Eq. (4.13) with the relative price $p_{x(n-1,1)}$ and $p^*_{x(n-1,1)}$ into the $x_{dx(n-1,1)}$ and $x_{fx(n-1,1)}$, I derive $x_{dx(n-1,1)}$ and $x_{fx(n-1,1)}$ represented by the aggregate parameters:

$$x_{dx(n-1,1)} = [\frac{\gamma E}{X^\alpha p_{x(n-1,1)}}]^{\frac{1}{1-\alpha}} = \frac{\gamma E}{M}\theta_{dx}^{\frac{1}{1-\alpha}}\bar{p}^{\frac{\alpha}{1-\alpha}} z^{\frac{1}{\alpha}}$$

$$x_{fx(n-1,1)} = [\frac{\gamma E}{X^\alpha p^*_{x(n-1,1)}}]^{\frac{1}{1-\alpha}} = \frac{\gamma E}{M}(\frac{\tau}{\theta_{fx(\underline{n-1},1)}})^{\frac{1}{\alpha-1}}\bar{p}^{\frac{\alpha}{1-\alpha}} z^{\frac{1}{\alpha}}$$

Then, combining the definition of the output in the domestic and foreign market, $x_{dx(n-1,1)} = n(q_{dx} + q^*_{fx})$, $x_{fx(n-1,1)} = (n-1)q_{fx} + nq^*_{dx}$ with indicators A and B and substituting them into the above $x_{dx(n-1,1)}$ and $x_{fx(n-1,1)}$. I have the quantities of an exporter within a particular variety of alternative oligopoly containing $(n-1)$ exporters and one deviating non-exporter in both domestic and foreign markets, $q_{dx(\underline{n-1},1)}$ and $q_{fx(\underline{n-1},1)}$, represented by the aggregate level:

$$q_{dx(\underline{n-1},1)} = \frac{x_{dx}}{n(1+A)} = \frac{\gamma E}{n(1+A)M}\theta_{dx}^{\frac{1}{1-\alpha}}\bar{p}^{\frac{\alpha}{1-\alpha}} z^{\frac{1}{\alpha}}$$

[1] I can simply prove that $\frac{\theta_{fx(\underline{n-1},1)}}{\theta_{fx}} < 1$ since $\frac{\theta_{fx(\underline{n-1},1)}}{\theta_{fx}} = \frac{(2n+\alpha-2)n(1+\tau)}{(2n+\alpha-1)(n(\tau+1)-\tau)} = \frac{1-\frac{1}{2n+\alpha-1}}{1-\frac{\tau}{n(1+\tau)}}$ in which $\frac{1}{2n+\alpha-1} > \frac{\tau}{n(1+\tau)}$ as $\tau < \bar{\tau} = \frac{n}{n+\alpha-1}$ where $\bar{\tau}$ is prohibitive iceberg-type transportation costs.

$$q_{fx(\underline{n-1},1)} = \frac{x_{fx}}{(n-1+nB)} = \frac{\gamma E}{((n-1)+nB)M}(\frac{\tau}{\theta_{fx}})^{\frac{1}{\alpha-1}}\bar{p}^{\frac{\alpha}{1-\alpha}}z^{\frac{1}{\alpha}}$$

The above equations with aggregate parameters are applied to calculate the profit of an exporter in the alternative oligopolistic competition I assumed. I need to guarantee that, at least, there will be no difference for firms to deviate or not. In other words, firms have no incentive to deviate from any varieties. According to the profit function of the deviating non-exporter and substituting $p_{x(n-1,1)} = \frac{z^{\frac{\alpha-1}{\alpha}}}{\theta_{dx}}$ identified from Eq. (C.9), q_{fx} with aggregate levels and $e = \frac{\gamma E}{nM}$, the firm's average expenditure, I have the profit of one deviating non-exporter:

$$\begin{aligned}
\pi_{a(n-1,\underline{1})} &= (\frac{z^{\frac{\alpha-1}{\alpha}}}{\theta_{dx}} - z^{\frac{\alpha-1}{\alpha}})\frac{\gamma E}{n(1+A)M}\theta_{dx}^{\frac{1}{1-\alpha}}\bar{p}^{\frac{\alpha}{1-\alpha}}z^{\frac{1}{\alpha}} - \lambda_d \\
&= \frac{1}{(1+A)}e\bar{p}^{\frac{\alpha}{1-\alpha}}z(\theta_{dx}^{\frac{\alpha}{1-\alpha}} - \theta_{dx}^{\frac{1}{1-\alpha}}) - \lambda_d
\end{aligned}$$

Where $\theta_{dx(n-1,\underline{1})} = \frac{2n+\alpha-1}{n(1+\tau)}$ is the inverse of the markup of a non-exporter in the domestic market within a particular variety with alternative oligopoly. It is the same as the inverse of the markup of domestic exporters within a product line with n exporters since there is no firm deviation in the domestic market.

In the extreme case, there will be no difference between the profit of the deviating non-exporter $\pi_{a(n-1,\underline{1})}$ in an alternative oligopoly and the profit of the exporter π_x who is not willing to deviate within a variety containing n exporters, which equalise π_x and $\pi_{a(n-1,\underline{1})}$. In other cases, only $\pi_x > \pi_{a(n-1,\underline{1})}$, no firms would have an incentive to deviate from the variety, which means I need to find a cutoff condition for the exported variety to make sure $\pi_x \geq \pi_{a(n-1,\underline{1})}$ in any case. The following calculation involves the difference between π_x and $\pi_{a(n-1,\underline{1})}$:

$$\begin{aligned}
\pi_x - \pi_{a(n-1,\underline{1})} &= (1-\theta_x)e(\frac{\bar{p}}{p_x})^{\frac{\alpha}{1-\alpha}} - \lambda_d - \lambda_x \\
&\quad -[\frac{1}{(1+A)}e\bar{p}^{\frac{\alpha}{1-\alpha}}z(\theta_{dx}^{\frac{\alpha}{1-\alpha}} - \theta_{dx}^{\frac{1}{1-\alpha}}) - \lambda_d] \\
&= e\bar{p}^{\frac{\alpha}{1-\alpha}}\theta_{d_x}^{\frac{\alpha}{1-\alpha}}[1-\theta_x - \frac{(1-\theta_{dx})}{(1+A)}]z - \lambda_x
\end{aligned} \tag{C.11}$$

where $p_x = \frac{z^{\frac{\alpha-1}{\alpha}}}{\theta_{dx}}$ from Eq. (4.8) is the symmetric equilibrium price for an exporter within a specific exported variety, including n exporters. The aim is to prove that there are always $\pi_x \geq \pi_{a(n-1,\underline{1})}$ under the zero cutoff productivity $\pi_{f_x}(z_x^*) = 0$ for exporters (XC). Notice that $\pi_{f_x}(z_x^*) = 0$ is the lowest threshold for firms to not deviate from the non-exported variety with n exporters. However, any higher profits $\pi_{f_x}(z_x^*) > 0$ provide more incentive for firms to not deviate from the product line since they can earn profit from the foreign market. Then, I compare Eqs. (4.24) and (C.11), which can be identified as the same expression since $\frac{1}{1+A} = \frac{\tau-\Phi}{\tau-1}$ from indicator $A = \beta$ and $\Phi = \frac{1+\tau\beta}{1+\beta}$. β is the ratio of the quantity supplied by a domestic exporter in the foreign market to its quantity in the domestic market and the relationship between Φ and β from Eq. (C.11) in the first main chapter, $\Phi = \frac{1+\tau\beta}{1+\beta}$.

C.6 Equation (4.26)

Substituting $\bar{p}$ into each component of the definition $\bar{\theta}$ and using Eq. (4.8), which are equilibrium prices for non-exporters, exporters and multinational firms, I get:

$$
\begin{aligned}
(\frac{p_x}{\bar{p}})^{\frac{\alpha}{1-\alpha}} &= (\frac{\frac{z^{\frac{\alpha-1}{\alpha}}}{\theta_{dx}}}{\{\frac{k}{k-1}z_a^*[\theta_a^{\frac{\alpha}{1-\alpha}} + (\theta_{d_x}^{\frac{\alpha}{1-\alpha}} - \theta_a^{\frac{\alpha}{1-\alpha}})Z_1^{k-1} + (\theta_{d_m}^{\frac{\alpha}{1-\alpha}} - \theta_{d_x}^{\frac{\alpha}{1-\alpha}})Z_2^{k-1}]\}^{\frac{\alpha-1}{\alpha}}})^{\frac{\alpha}{1-\alpha}} \\
&= \frac{\{\frac{k}{k-1}z_a^*[\theta_a^{\frac{\alpha}{1-\alpha}} + (\theta_{d_x}^{\frac{\alpha}{1-\alpha}} - \theta_a^{\frac{\alpha}{1-\alpha}})Z_1^{k-1} + (\theta_{d_m}^{\frac{\alpha}{1-\alpha}} - \theta_{d_x}^{\frac{\alpha}{1-\alpha}})Z_2^{k-1}]\}}{(\theta_{d_x})^{\frac{\alpha}{1-\alpha}}z} \\
&= \frac{kz_a^*}{(k-1)z}[(\frac{\theta_a}{\theta_{d_x}})^{\frac{\alpha}{1-\alpha}} + ((1-(\frac{\theta_a}{\theta_{d_x}})^{\frac{\alpha}{1-\alpha}})Z_1^{k-1} + ((\frac{\theta_{d_m}}{\theta_{d_x}})^{\frac{\alpha}{1-\alpha}} - 1)Z_2^{k-1}]
\end{aligned}
$$

C.7 Equation (4.29)

This is deviation analysis for a deviating multinational product line with $(n-1)$ multinational firms and one deviating exporter. This section examines the alternative oligopoly model of $(n-1)$ multinational firms and one deviating exporter in only one domestic particular multinational variety v. In contrast; there are n multinational firms in the multinational product line in the previous analysis for all multinational varieties. The alternative oligopoly in just one domestic unique variety means the aggregate levels of the global economy do not alter because of the deviation. The aim is to guarantee that no multinational firms in the variety are incentivised to deviate as an exporter to save the highest fixed costs of locating a firm in the foreign market. According to the inverse demand function Eq. (4.3), I have notation as follows:

$q_{dx(n-1,\underline{1})}$: Exporter quantity offered in the domestic market;
$q_{fx(n-1,\underline{1})}$: Exporter quantity offered in the foreign market;
$q_{fm(\underline{n-1},1)}$: Multinational quantity offered in the foreign market;
$q_{dm(\underline{n-1},1)}$: Multinational quantity offered in the domestic market;
$q^*_{dm(\underline{n-1},1)}$: Foreign multinational quantity offered in the foreign market;
$q^*_{fm(\underline{n-1},1)}$: Foreign multinational quantity offered in the domestic market;
$p_{m(n-1,1)}$: the price of the variety in the domestic market;
$p^*_{m(n-1,1)}$: the price of the variety in the foreign market;

$x_{dm(n-1,1)} = n(q_{dx(n-1,\underline{1})} + q^*_{fm(\underline{n-1},1)})$: quantities sold in the domestic market;

$x_{fm(n-1,1)} = nq^*_{dm(\underline{n-1},1)} + (n-1)q_{fm(\underline{n-1},1)} + q_{fx(n-1,\underline{1})}$: quantities sold in the foreign market;

then, I can get the relative price of the variety in both domestic and foreign markets:

$$p_{m(n-1,1)} = \frac{\gamma E}{X^\alpha} x_{dm(n-1,1)}{}^{\alpha-1}$$

$$p^*_{m(n-1,1)} = \frac{\gamma E}{X^\alpha} x_{fm(n-1,1)}{}^{\alpha-1}$$

One exporter:

Here, I use the above notation and calculate the profit of one deviating domestic exporter, $x_{dm(n-1,1)}$, and $x_{fm(n-1,1)}$ are the quantities sold in the domestic and foreign markets. One deviating domestic exporter solves the following problem:

$$\pi_{x(n-1,\underline{1})} = \max_{q_{dx},q_{fx}} \underbrace{\frac{\gamma E}{X^\alpha} x^{\alpha-1}_{dm(n-1,1)}}_{p_{m(n-1,1)}} q_{dx} + \underbrace{\frac{\gamma E}{X^\alpha} x^{\alpha-1}_{fm(n-1,1)}}_{p^*_{m(n-1,1)}} q_{fx} - z^{\frac{\alpha-1}{\alpha}}(q_{dx} + \tau q_{fx}) - \lambda_d - \lambda_x \tag{C.12}$$

Notice that in the above expression, I omit the subscript $(n-1,\underline{1})$ of q_{dx} and q_{fx} for simplicity, which means I always focus on the quantities of a deviating exporter in an alternative oligopoly of one deviating variety with $(n-1)$ multinational firms and one exporter. Consequently, Eq. (C.12) yields the following first-order conditions for both the quantity sold in the domestic market and the foreign market:

$$\frac{\gamma E}{X^\alpha}((\alpha-1)x_{dm(n-1,1)}{}^{\alpha-2} q_{dx} + x^{\alpha-1}_{dm(n-1,1)}) = z^{\frac{\alpha-1}{\alpha}} \tag{C.13}$$

$$\frac{\gamma E}{X^\alpha}((\alpha-1)x^{\alpha-2}_{fm(n-1,1)} q_{fx} + x^{\alpha-1}_{fm(n-1,1)}) = \tau z^{\frac{\alpha-1}{\alpha}} \tag{C.14}$$

$(n-1)$ **multinational firms:**

Here, I focus on the profit of a multinational firm in a deviating domestic multinational variety with $(n-1)$ multinational firms and one exporter. As the notation shown above, I solve the multinational firm's problem:

$$\pi_{m(\underline{n-1},1)} = \max_{q_{dm},q_{fm}} \underbrace{\frac{\gamma E}{X^\alpha} x_{dm(n-1,1)}{}^{\alpha-1}}_{p_{m(n-1,1)}} q_{dm} + \underbrace{\frac{\gamma E}{X^\alpha} x_{fm(n-1,1)}{}^{\alpha-1}}_{p^*_{m(n-1,1)}} q_{fm} - z^{\frac{\alpha-1}{\alpha}}(q_{dm}+q_{fm}) - \lambda_d - \lambda_m \tag{C.15}$$

Similarly, I omit the subscript $(\underline{n-1},1)$ of q_{dm} and q_{fm} for simplicity, which indicates that I focus on the quantities of one multinational in an alternative oligopoly of one deviating variety with $(n-1)$ multinational firms and one exporter. Then, the first-order conditions for the quantity of the multinational sold in both domestic and foreign markets, separately:

$$\frac{\gamma E}{X^{\alpha}}((\alpha-1){x_{dm(n-1,1)}}^{\alpha-2}q_{dm}+{x_{dm(n-1,1)}}^{\alpha-1})=z^{\frac{\alpha-1}{\alpha}} \tag{C.16}$$

$$\frac{\gamma E}{X^{\alpha}}((\alpha-1){x_{fm(n-1,1)}}^{\alpha-2}q_{fm}+{x_{fm(n-1,1)}}^{\alpha-1})=z^{\frac{\alpha-1}{\alpha}} \tag{C.17}$$

Then I turn to foreign multinational firms:

$$\pi^*_{m(\underline{n-1},1)}=\max_{q^*_{dm},q^*_{fm}}\underbrace{\frac{\gamma E}{X^{\alpha}}x^{\alpha-1}_{dm(n-1,1)}}_{p_{m(n-1,1)}}q^*_{fm}+\underbrace{\frac{\gamma E}{X^{\alpha}}{x_{fm(n-1,1)}}^{\alpha-1}}_{p^*_{m(n-1,1)}}q^*_{dm}-z^{\frac{\alpha-1}{\alpha}}(q^*_{dm}+q^*_{fm})-\lambda_d-\lambda_m \tag{C.18}$$

Then, the first-order conditions for the quantity sold in both domestic and foreign markets of a foreign multinational firm, separately:

$$\frac{\gamma E}{X^{\alpha}}((\alpha-1){x_{fm(n-1,1)}}^{\alpha-2}q^*_{dm}+x^{\alpha-1}_{fm(n-1,1)})=z^{\frac{\alpha-1}{\alpha}} \tag{C.19}$$

$$\frac{\gamma E}{X^{\alpha}}((\alpha-1)x^{\alpha-2}_{dm(n-1,1)}q^*_{fm}+x^{\alpha-1}_{dm(n-1,1)})=z^{\frac{\alpha-1}{\alpha}} \tag{C.20}$$

Like the above, the subscript $(\underline{n-1},1)$ of q^*_{dm} and q^*_{fm} is omitted for simplicity, which implies I focus on the quantities of one foreign multinational in an alternative oligopoly of one deviating foreign variety with $(n-1)$ foreign multinational firms and one foreign exporter. According to Eqs. (C.13), (C.16) and (C.20), I have $q_{dx}=q_{dm}=q^*_{fm}$. Similarly, combining Eqs. (C.17) and (C.19), I get $q_{fm}=q^*_{dm}$.

Then I combine Eqs. (C.14) and (C.17), I can get the relationship between q_{fx} and q_{fm}, where $x_{fm(n-1,1)}=nq^*_{dm(\underline{n-1},1)}+(n-1)q_{fm(\underline{n-1},1)}+q_{fx(n-1,\underline{1})}=(2n-1)q_{fm(\underline{n-1},1)}+q_{fx(n-1,\underline{1})}$

$$\begin{aligned}\frac{\frac{\gamma E}{X^{\alpha}}((\alpha-1)x_f^{\alpha-2}q_{fx}+x_f^{\alpha-1})}{\frac{\gamma E}{X^{\alpha}}((\alpha-1){x_f}^{\alpha-2}q_{fm}+{x_f}^{\alpha-1})} &= \frac{\tau z^{\frac{\alpha-1}{\alpha}}}{z^{\frac{\alpha-1}{\alpha}}}\\ &\Rightarrow\ q_{fm}=\frac{\tau-\alpha}{(1-\tau)(2n-1)+\tau(1-\alpha)}q_{fx}\end{aligned}$$

where I denote $\frac{\tau-\alpha}{(1-\tau)(2n-1)+\tau(1-\alpha)}=C$, which is $C=q_{fm}/q_{fx}$, where C is the ratio of the quantity for a multinational firm from $(n-1)$ multinational firms to the quantity for one deviating exporter within a domestic deviating multinational variety. Since the definition of $x_{dm(n-1,1)}=n(q_{dx(n-1,\underline{1})}+q^*_{fm(\underline{n-1},1)})$ and $q_{dx}=q_{dm}=q^*_{fm}$, I have $x_{dm(n-1,1)} = 2nq_{dx}$. From the definition of x_f and C, I have $x_{fm(n-1,1)} = (2n-1)q_{fm} + q_{fx} = (2n-1+1/C)q_{fm}$.

Substituting the above relationships into Eqs. (C.13) and (C.16),

$$\underbrace{\frac{\gamma E}{X^\alpha} x_d^{\alpha-1}}_{p_{m(n-1,1)}} [\frac{(\alpha-1)}{x_{dm(n-1,1)}} q_{dm} + 1] = z^{\frac{\alpha-1}{\alpha}}$$

where $\theta_{dm(\underline{n-1},1)} = \theta_{dx(n-1,\underline{1})} = \theta_{dm} = [\frac{(\alpha-1)}{x_d} q_{dx} + 1] = \frac{(\alpha-1)}{2nq_{dx}} q_{dx} + 1 = \frac{2n+\alpha-1}{2n}$, is the inverse of the markup of a domestic multinational firm or exporter in the domestic market within a deviating multinational variety with $(n-1)$ multinational firms and one deviating exporter, similar to the deviation for a particular exported variety with $(n-1)$ exporters and one deviating non-exporter. $\theta_{dm(\underline{n-1},1)}$ is the same as the inverse markup of a domestic multinational firm for the domestic market under the circumstance of 'all are multinational firms' in the first main chapter. It can be explained as there is no deviation in the domestic market, so it should be the same situation as n multinational firms within a multinational product line in the domestic market.

Similarly, by rearranging Eq. (C.14), I have:

$$\underbrace{\frac{\gamma E}{X^\alpha} x_f^{\alpha-1}}_{p^*_{m(n-1,1)}} [\frac{(\alpha-1)}{x_{fm(n-1,1)}} q_{fx} + 1] = \tau z^{\frac{\alpha-1}{\alpha}}$$

where $\theta_{fx(n-1,\underline{1})} = [\frac{(\alpha-1)}{x_f} q_{fx} + 1] = \frac{(\alpha-1)}{((2n-1)C+1)q_{fx}} q_{fx} + 1 = \frac{(\alpha-1)}{(2n-1)C+1} + 1$ is the inverse of the markup for a deviating domestic exporter in the foreign market within a deviating multinational variety with $(n-1)$ multinational firms and one exporter. Simplifying it, I get:

$$\theta_{fx(n-1,\underline{1})} = \frac{\tau(2n+\alpha-1)}{2n+\tau-1}$$

where I denote $D = (2n-1)C + 1$, and it can be calculated according to the definition of C, where $D = \frac{(2n-1+\tau)(1-\alpha)}{(1-\tau)(2n-1)+\tau(1-\alpha)}$. I apply it to derive firms' quantity at the aggregate level.

Analogous to the previous deviation of an exported variety with $(n-1)$ exporters and one non-exporter, according to the respective prices of a multinational firm in both domestic and foreign markets, $p_{m(n-1,1)}$ and $p^*_{m(n-1,1)}$, within a deviating multinational variety with $(n-1)$ multinational firms and one exporter, we have the total output in the domestic and foreign markets, respectively:

$$x_{dm(n-1,1)} = [\frac{\gamma E}{X^\alpha p_{m(n-1,1)}}]^{\frac{1}{1-\alpha}}$$

$$x_{fm(n-1,1)} = [\frac{\gamma E}{X^{\alpha} p^{*}_{m(n-1,1)}}]^{\frac{1}{1-\alpha}}$$

where $p_{m(n-1,1)} = \frac{z^{\frac{\alpha-1}{\alpha}}}{\theta_{dm(\underline{n-1},1)}}$ and $p^{*}_{m(n-1,1)} = \frac{\tau z^{\frac{\alpha-1}{\alpha}}}{\theta_{fx(n-1,\underline{1})}}$ can be identified with Eqs. (C.13) and (C.14).

Then, substituting Eq (4.13) with the related price $p_{m(n-1,1)}$ and $p^{*}_{m(n-1,1)}$ into $x_{dm(n-1,1)}$ and $x_{fm(n-1,1)}$, I derive $x_{dx(n-1,\underline{1})}$ and $x_{fx(n-1,\underline{1})}$ represented with the aggregate levels:

$$x_{dx(n-1,\underline{1})} = [\frac{\gamma E}{X^{\alpha} p_{m(n-1,1)}}]^{\frac{1}{1-\alpha}} = \frac{\gamma E}{M} \theta_{dx(n-1,\underline{1})}^{\frac{1}{1-\alpha}} \bar{p}^{\frac{\alpha}{1-\alpha}} z^{\frac{1}{\alpha}}$$

$$x_{fx(n-1,\underline{1})} = [\frac{\gamma E}{X^{\alpha} p^{*}_{m(n-1,1)}}]^{\frac{1}{1-\alpha}} = \frac{\gamma E}{M} (\frac{\tau}{\theta_{fx(n-1,\underline{1})}})^{\frac{1}{\alpha-1}} \bar{p}^{\frac{\alpha}{1-\alpha}} z^{\frac{1}{\alpha}}$$

Since $q_{dx(n-1,\underline{1})} = \frac{x_{dm(n-1,1)}}{2n}$ and $q_{fx(n-1,\underline{1})} = \frac{x_{fm(n-1,1)}}{(2n-1)C+1}$ as shown above, I need to derive the profit of the deviating exporter in a deviating multinational variety with $(n-1)$ multinational firms and one exporter, I denote it as $\pi_{fx(n-1,\underline{1})}$ to differentiate it with the variety in which all n firms are exporters in the foreign market. According to $x_{dm(n-1,1)}$ and $x_{fm(n-1,1)}$ of aggregate level and the definition of $q_{dx(n-1,\underline{1})} = \frac{x_{dm(n-1,1)}}{2n}$ and $q_{fx(n-1,\underline{1})} = \frac{x_{fm(n-1,1)}}{(2n-1)C+1}$, I have

$$\begin{aligned} \pi_{fx(n-1,\underline{1})} &= \frac{\tau z^{\frac{\alpha-1}{\alpha}}}{\theta_{fx(n-1,\underline{1})}} \frac{\gamma E}{((2n-1)C+1)M} (\frac{\tau}{\theta_{fx(n-1,\underline{1})}})^{\frac{1}{\alpha-1}} \bar{p}^{\frac{\alpha}{1-\alpha}} z^{\frac{1}{\alpha}} \\ &\quad - z^{\frac{\alpha-1}{\alpha}} \tau \frac{\gamma E}{((2n-1)C+1)M} (\frac{\tau}{\theta_{fx(n-1,\underline{1})}})^{\frac{1}{\alpha-1}} \bar{p}^{\frac{\alpha}{1-\alpha}} z^{\frac{1}{\alpha}}) - \lambda_x \\ &= \frac{ne}{D} \bar{p}^{\frac{\alpha}{1-\alpha}} \tau^{\frac{\alpha}{\alpha-1}} (1 - \theta_{fx(n-1,\underline{1})}) \theta_{fx(n-1,\underline{1})}^{\frac{\alpha}{1-\alpha}} z - \lambda_x \end{aligned}$$

where $\theta_{fx(n-1,\underline{1})} = \frac{\tau(1-2n-\alpha)}{1-2n-\tau}$ and $D = (2n-1)C + 1$.

Compared to the profits of the deviating exporter of alternative oligopoly in the foreign market with the profits of a multinational firm in the foreign market within a particular variety with n multinational firms, the deviating exporter needs to earn a lower profit than the multinational in the foreign market, so that no deviation would happen. Hence, I focus on the profit in the foreign market only. I equalise the profits of a multinational in the foreign market within a multinational product line with n multinational firms as Eq. (4.28) with the profit in the foreign market of the deviating exporter within a deviating multinational product line with $(n-1)$ multinational firms and one deviating exporter as the expression of $\pi_{fx(n-1,\underline{1})}$.

As I would like to compare the profit in the foreign market with a deviating exporter and no deviating n multinational firms as $\pi_{fm} - \pi_{fx(n-1,\underline{1})} \geq 0$, no multinational firms would like to deviate. Then, I have the difference between π_{fm} and $\pi_{fx(n-1,\underline{1})}$:

$$\pi_{fm} - \pi_{fx(n-1,\underline{1})} = e\bar{p}^{\frac{\alpha}{1-\alpha}} z[\frac{(1-\theta_{dm})}{2}\theta_{dm}^{\frac{\alpha}{1-\alpha}} - \frac{n}{D}\tau^{\frac{\alpha}{\alpha-1}}(1-\theta_{fx(n-1,\underline{1})})\theta_{fx(n-1,\underline{1})}^{\frac{\alpha}{1-\alpha}}] - \lambda_m + \lambda_x$$

C.8 Equation (4.30)

Substituting $\bar{p}$ into each component of the definition $\bar{\theta}$ and using Eq. (4.11), which are equilibrium prices for multinational firms, I get:

$$\begin{aligned}
(\frac{p_m}{\bar{p}})^{\frac{\alpha}{1-\alpha}} &= (\frac{\frac{z^{\frac{\alpha-1}{\alpha}}}{\theta_{d_m}}}{\{\frac{k}{k-1}z_a^*[\theta_a^{\frac{\alpha}{1-\alpha}} + (\theta_{d_x}^{\frac{\alpha}{1-\alpha}} - \theta_a^{\frac{\alpha}{1-\alpha}})Z_1^{k-1} + (\theta_{d_m}^{\frac{\alpha}{1-\alpha}} - \theta_{d_x}^{\frac{\alpha}{1-\alpha}})Z_2^{k-1}]\}^{\frac{\alpha-1}{\alpha}}})^{\frac{\alpha}{1-\alpha}} \\
&= \frac{\{\frac{k}{k-1}z_a^*[\theta_a^{\frac{\alpha}{1-\alpha}} + (\theta_{d_x}^{\frac{\alpha}{1-\alpha}} - \theta_a^{\frac{\alpha}{1-\alpha}})Z_1^{k-1} + (\theta_{d_m}^{\frac{\alpha}{1-\alpha}} - \theta_{d_x}^{\frac{\alpha}{1-\alpha}})Z_2^{k-1}]\}}{(\theta_{d_m})^{\frac{\alpha}{1-\alpha}} z} \\
&= \frac{kz_a^*}{(k-1)z}[(\frac{\theta_a}{\theta_{d_m}})^{\frac{\alpha}{1-\alpha}} + ((\frac{\theta_{d_x}}{\theta_{d_m}})^{\frac{\alpha}{1-\alpha}} - (\frac{\theta_a}{\theta_{d_m}})^{\frac{\alpha}{1-\alpha}})Z_1^{k-1} + (1 - (\frac{\theta_{d_x}}{\theta_{d_m}})^{\frac{\alpha}{1-\alpha}})Z_2^{k-1}]
\end{aligned}$$

C.9 Equation (4.33)

As in the main text, I substitute the complex expressions of the derivation of each component in $\bar{\theta}$ with A_1, A_2 and A_3 and identify the relationship among A_1, A_2, and A_3: $A_1 = (\frac{\theta_{d_x}}{\theta_a})^{\frac{\alpha}{1-\alpha}} A_2 = (\frac{\theta_{d_m}}{\theta_a})^{\frac{\alpha}{1-\alpha}} A_3$, then combining the above results and the definition of $\bar{\theta}$, I have:

$$
\begin{aligned}
\bar{\theta} &= \theta_a \int_{z_a^*}^{z_x^*} (\frac{\bar{p}}{p_a})^{\frac{\alpha}{1-\alpha}} \mu(z)dz + \theta_x \int_{z_x^*}^{z_m^*} (\frac{\bar{p}}{p_x})^{\frac{\alpha}{1-\alpha}} \mu(z)dz + \theta_{d_m} \int_{z_m^*}^{\infty} (\frac{\bar{p}}{p_m})^{\frac{\alpha}{1-\alpha}} \mu(z)dz \\
&= \frac{(k-1)\theta_a}{kz_a^* A_1} \int_{z_a^*}^{z_x^*} z\mu(z)dz + \frac{(k-1)\theta_x}{kz_a^* A_2} \int_{z_x^*}^{z_m^*} z\mu(z)dz + \frac{(k-1)\theta_{d_m}}{kz_a^* A_3} \int_{z_m^*}^{\infty} z\mu(z)dz \\
&= \frac{(k-1)\theta_a z_a^{*k}}{kz_a^* A_1} \int_{z_a^*}^{z_x^*} zg(z)dz + \frac{(k-1)\theta_x z_a^{*k}}{kz_a^* A_2} \int_{z_x^*}^{z_x^*} zg(z)dz + \frac{(k-1)\theta_{d_m} z_a^{*k}}{kz_a^* A_3} \int_{z_m^*}^{\infty} zg(z)dz \\
&= \frac{(k-1)\theta_a z_a^{*k}}{kz_a^* A_1} \frac{k}{1-k} (z_x^{*1-k} - z_a^{*1-k}) + \frac{(k-1)\theta_x z_a^{*k}}{kz_a^* A_2} \frac{k}{1-k} (z_m^{*1-k} - z_x^{*1-k}) \\
&\quad - \frac{(k-1)\theta_{d_m} z_a^{*k}}{kz_a^* A_3} \frac{k}{1-k} z_m^{*1-k} \\
&= \frac{\theta_a(1-Z_1^{k-1})}{A_1} + \frac{(\frac{\theta_{dx}}{\theta_a})^{\frac{\alpha}{1-\alpha}} \theta_x (Z_1^{k-1} - Z_2^{k-1})}{(\frac{\theta_{d_x}}{\theta_a})^{\frac{\alpha}{1-\alpha}} A_2} + \frac{(\frac{\theta_{d_m}}{\theta_a})^{\frac{\alpha}{1-\alpha}} \theta_{d_m} Z_2^{k-1}}{(\frac{\theta_{d_m}}{\theta_a})^{\frac{\alpha}{1-\alpha}} A_3} \\
&= \frac{\theta_a + Z_1^{k-1}((\frac{\theta_{d_x}}{\theta_a})^{\frac{\alpha}{1-\alpha}} \theta_x - \theta_a) + Z_2^{k-1}((\frac{\theta_{d_m}}{\theta_a})^{\frac{\alpha}{1-\alpha}} \theta_{d_m} - (\frac{\theta_{dx}}{\theta_a})^{\frac{\alpha}{1-\alpha}} \theta_x)}{A_1} \\
&= \frac{(\theta_a)^{\frac{\alpha}{1-\alpha}} [\theta_a + Z_1^{k-1}((\frac{\theta_{d_x}}{\theta_a})^{\frac{\alpha}{1-\alpha}} \theta_x - \theta_a) + Z_2^{k-1}((\frac{\theta_{d_m}}{\theta_a})^{\frac{\alpha}{1-\alpha}} \theta_{d_m} - (\frac{\theta_{d_x}}{\theta_a})^{\frac{\alpha}{1-\alpha}} \theta_x)]}{(\theta_a)^{\frac{\alpha}{1-\alpha}} A_1} \\
&= \frac{\theta_a^{\frac{1}{1-\alpha}} + (\theta_{d_x}{}^{\frac{1}{1-\alpha}} \Phi - \theta_a^{\frac{1}{1-\alpha}}) Z_1^{k-1} + Z_2^{k-1}((\theta_{d_m})^{\frac{1}{1-\alpha}} - \theta_{d_x}{}^{\frac{1}{1-\alpha}} \Phi)}{\theta_a^{\frac{\alpha}{1-\alpha}} + (\theta_{d_x}{}^{\frac{\alpha}{1-\alpha}} - \theta_a^{\frac{\alpha}{1-\alpha}}) Z_1^{k-1} + ((\theta_{d_m})^{\frac{\alpha}{1-\alpha}} - (\theta_{d_x})^{\frac{\alpha}{1-\alpha}}) Z_2^{k-1}}
\end{aligned}
$$

Chapter 5

Conclusions, Limitations and future work

5.1 Conclusions

This thesis explored the research question of the welfare effects of international trade and horizontal FDI under conditions of oligopolistic competition and firm heterogeneity. By constructing theoretical models, I explore how the welfare effect of trade and FDI will respond to different economic environments in the three main chapters. Our first main chapter offers the fundamental framework used in all later chapters. This allows us to further develop the models by including more components, like innovation in the second chapter and the potential for the coexistence of domestic firms, exporters and multinational production in the third.

The first main chapter considers three scenarios: autarky, where all firms are domestic firms; trade openness, where all firms are exporters and multinational production, where all firms are multinational firms. This complements Impullitti and Licandro (2018) by incorporating multinational firms via horizontal FDI like Helpman et al. (2004). I find that multinational firms generate the highest welfare gains compared to exporters and domestic firms. It is conducted through: 1) the pro-competitive effect of the lowest markup from oligopolistic competition, creating the lowest price, and 2) the highest productivity threshold from the most extensive selection effect, increasing the aggregate productivity of the economy. In addition, the expenditure received by each firm for multinational production is the highest compared with exporters and domestic firms. In contrast, multinational production creates the least mass of operative variety in the market as there is a negative association between the productivity threshold and the number of varieties through the stationarity condition.

The second main chapter extends the first by introducing process innovation captured by a cost-reducing R&D technology. The comparisons come through two dimensions: 1) firms in each scenario with innovation or not, which is the model in the first main chapter and 2) the models' properties for three scenarios, separately. I find that multinational production generates the largest investment in R&D activities with the highest volume of output, produces the lowest price due to the largest pro-competitive effect, and creates the highest survival productivity threshold because of the selection effect, generating the largest welfare gains compared to exporters and domestic firms. Moreover, compared with a firm that does not undertake R&D, a firm undertaking process innovation will generate higher output, lower prices and a higher survival productivity threshold. Notice that the first chapter is the extreme case of the second when I restrict the parameters of R&D technology in the model of the second chapter. Specifically, this is when the degree of decreasing marginal returns equals zero and the technology shift parameter equals 1.

The third main chapter develops the first by including the potential coexistence of domestic firms, exporters and multinational firms and adding a stationary condition like Melitz (2003), which is different from the first and second chapters. It allows us to explore the substitutability between exporters and multinational firms via horizontal FDI with trade liberalisation. I also examine the welfare effect of trade liberalisation in the presence of horizontal FDI. The chapter finds that trade liberalisation has a pro-competitive effect only on exporters because their average markups decrease. In addition, the trade liberalisation triggers the selection effect only for domestic firms due to firm heterogeneity, forcing the least productive firms to exit the market and reallocating productive resources to more productive firms. It differs from the first and second chapters as the selection effect holds for the scenarios of 'all exporters' and 'all multinational firms' in the first and second. Thirdly, the welfare gains are always higher in the case with FDI than those without FDI, like Sun et al. (2020). In comparison, welfare gains from trade liberalisation are smaller with the engagement of horizontal FDI than without FDI. Specifically, in my simulation, welfare gains of trade liberalisation increase by approximately 0.12% when the economy moves from autarky to free trade with FDI. Finally, this chapter also shows that only the most productive firms choose to be multinational firms. In contrast, the medium productive firms choose to be exporters, the less productive firms stay in the domestic market, and the least productive firms exit the market. Note that among different economic environments in the three chapters, multinational firms and exporters all generate the pro-competitive effect via variable markups from oligopolistic competition, where the equations of their markups are the same in three different set-ups.

5.2 Limitations and Future Work

Although the propositions and related numerical analysis of the thesis are in line with the empirical evidence, there is still potential for some extensions as follows: 1) introduce the free entry condition, 2) incorporate sector heterogeneity, 3) include endogenous growth, 4) capture 'vertical' FDI, 5) apply different demand functions, 6) add the study of the economies with different initial conditions or different factor endowments, which I discuss more fully below. However, in this thesis, I am not including these because they will complicate the model to a large extent and lie beyond the scope of this thesis.

Impullitti et al. (2018), and Impullitti et al. (2022), which are quite close to my research, both consider the sophisticated entry strategies in each product line. The difference is that they assume the number of firms is real or discrete within each variety. Therefore, it is possible to study a more complicated model environment to include FDI and the free entry condition for future research. Navas (2015) extends the framework of Navas and Licandro (2011) with sector heterogeneity in the level of product market competition and builds a multi-sector endogenous growth model to explore how trade liberalisation affects innovation, sector and aggregate productivity growth. He illustrates that a movement from autarky to free trade facilitates innovation and productivity growth for the less competitive sectors. Accordingly, it enables us to consider sector heterogeneity like this in future research. In addition, as Navas (2015), endogenous growth is also a potential dimension to extend for future research, like Impullitti and Licandro (2018) as well, which finds that endogenous productivity growth leads to substantial welfare gains through the increase in the selection gains from trade. Ramondo and Rodríguez-Clare (2013), which built a model with the interaction between trade and FDI in three ways: 'horizontal' FDI, there are two competitive ways to serve the foreign market; 'vertical' FDI, foreign affiliates import inputs from the home country; and firms choose another country as an export platform to serve a particular market. I have already considered some of the substitutable ways of trade and FDI. It is apparent that 'vertical FDI' is an interesting area for future research, focusing on symmetry.

Some recent literature suggests different utility (demand) functions to apply to the construction of the theoretical models (Mrázová and Neary, 2014, 2017; Mrázová et al., 2021) aiming to explore the central questions in trade areas like the welfare effects of trade and FDI. For example, Mrázová and Neary (2014) relax the CES preference function and consider the alternatives to the CES with trade costs and separable preferences combined in a simple model to examine the implications for the gains from trade. Therefore, there is potential to build the model using alternative utility functions rather than the CES utility function. Finally, this thesis restricts the analysis to identical countries to understand the welfare effect of trade and FDI among similar countries. Future research could consider the economy with asymmetric countries, such as studying the interaction between developing and developed countries.

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