

Design optimisation of serpentine mini-channel heat sink

By

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Declaration

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Abstract

Novel designs of serpentine mini-channel heat sink with both single inlet-outlet and double inlet-outlet configurations with the application of secondary channels using fins are created to explore their thermal-hydraulic performance based on total thermal resistance, average nusselt number and pressure drop parameters which are being solved by COMSOL Multiphysics CFD software in three dimensional space. The single inlet-outlet configuration thermal-hydraulic performance will be tested by embedding the different shapes of fins of oblique, trapezium and diamond with different configurations each. The use of secondary channels are very effective in serpentine channels in improving heat transfer and reducing pressure drop by allowing fluid passing through it reducing fluid velocity as well as increasing the surface area for heat transfer. Similarly, for double inlet-outlet configuration, diamond fins were used and similar trends obtained where both total thermal resistance and pressure drop are reduced simultaneously.

For optimisation process, a set of Design of Experiment (DOE) points (input design variables) are created to be fed into COMSOL Multiphysics calculating the output objective functions. Radial Basis Function method was used to train the surrogate model by learning the relationship of input design variables with its corresponding output objective functions. Surrogate models enable the prediction of output objective functions within specified design space of input design variables without having to run further simulations using CFD software which is time consuming. The surrogate models created will be used for optimisation searching for optimal solutions for both deterministic and probabilistic (Robust Design Optimisation (RDO)) frameworks. Multi-objective optimisation process was conducted to create Pareto optimal solutions between conflicting objective functions. Monte Carlo sampling (MCS) and Polynomial Chaos Expansion (PCE) were utilised to create probabilistic distribution within every single DOE points by specified means and standard deviations to calculate the distribution of corresponding output objective functions based on output means and standard deviations using previously developed surrogate models. The output means and standard deviations give the information of the distribution of output objective functions to search for the solution which has small standard deviation indicating flatten output distribution so that moving around the curve does not change objective function significantly. The solutions of RDO has less performance compared to deterministic solutions but more flexible to change.

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Nomenclature

A_{base}	Mini-channel base area [m^2]	$T_{f,in}$	Temperature of fluid inlet [K]
A_{ch}	Mini-channel cross-sectional area [m^2]	$T_{f,out}$	Temperature of fluid outlet [K]
A_{eff}	Effective area (available area for heat transfer) [m^2]	$T_{w,avg}$	Average mini-channel wall temperature [K]
Cp_f	Fluid specific heat [$J/kg.K$]	W	Width of heat sink [m]
D_h	Hydraulic diameter [m]	W_{ch}	Width of mini-channel [m]
h	Coefficient of convective heat transfer [$W/m^2.K$]	W_{sc}	Width of secondary channel [m]
H_b	Thickness of substrate [m]	W_w	Width of fin [m]
H_{ch}	Height of mini-channel cross section [m]	J	Physical parameter
k	Turbulent kinetic energy [m^2/s^2]	TSD	Temperature Standard Deviation
k_f	Fluid thermal conductivity [$W/m.K$]	<i>Abbreviations</i>	
k_s	Copper substrate thermal conductivity [$W/m.K$]	1-D	One Dimensional
k_T	Turbulent thermal conductivity [$W/m.K$]	3-D	Three Dimensional
L	Length of heat sink [m]	CFD	Computational Fluid Dynamics
L_{ch}	Length of mini-channel [m]	GA	Genetic Algorithm
l_f	Fin length [m]	OLHC	Optimal Latin Hypercube Criterion
l_{ff}	Second fin length [m]	MCHS	Micro-channel Heat Sink
l_{sc}	Length of secondary channel [m]	NSGA-II	Non-Dominated Sorting Genetic Algorithm II
n	Number of mini-channel	DNN	Deep Neural Network
N	Number of fins	<i>Greek symbols</i>	
ΔP	Total pressure drop [Pa]	ρ_f	Density of fluid [kg/m^3]
p_f	Fin pitch ($= l_f + l_{sc}$) [m]	μ_f	Fluid dynamic viscosity [$kg/m.s$]
P_w	Wetted parameter [m]	μ_T	Turbulent viscosity [$kg/m.s$]
U_{in}	Inlet fluid velocity [m/s]	θ_{fin}	Fin angle [$^\circ$]
q	Rate of heat transfer [W]	ω	Specific dissipation rate [$1/sec$]
Re	Reynolds number	<i>Subscripts</i>	
R_{th}	Average thermal resistance [K/W]	avg	average
$T_{f,avg}$	Average temperature of fluid bulk	max	maximum
$T_{surf,max}$	Maximum surface temperature	f	Fluid
Nu_{avg}	Average Nusselt Number	in	Inlet
		out	Outlet
		s	Solid
		Γ	Fluid and solid interface

Chapter 1 : Project background and motivation

The transistor usage from the Integrated Circuit (IC) invention system in 1960s has increase significantly [1]. Gordon Moore, Intel founder, deduced the Moore's Law which functions as an empirical relation with an expression for the density of components that can be integrated within microchips that increase by twice [2] usually in every two years where more superior performance for electronics devices achieved and until now the law has been very accurate which can be seen in Figure 1-1.

For the past twenty years of technological advancement, processing speeds have been improving significantly. From this rapid development, very high heat dissipation from devices especially microwave weapon systems, high performance electronics for electrical vehicle, energy laser, avionics, nuclear fission and fusion reactor and high performance computers. The localised rate of heat flow of energy per unit area which is also called as heat flux from microprocessors have been greater than $100\text{W}/\text{cm}^2$, more than $1000\text{W}/\text{cm}^2$ for defence system application (e.g. lasers, radars and microwave devices) and nuclear industries already exceeds $10000\text{W}/\text{cm}^2$ [3]. Miniaturisation of devices and rapid improvement in technological advancement have also contributed to the continuous increase of heat dissipation rate. High performance cooling systems are essential to remove high heat fluxes to ensure that quick development of technological systems can maintain its performance without having being damaged or degraded from very high temperature due to high heat dissipation. The use of liquid cooling has been very important for an effective heat removal method compared to air cooling due to its high heat capacity value carrying larger amount of heat away from electronic devices [4].

The requirements of data storage is significantly affected from the growth of digital systems usage, leading to the data centres size and energy consumption enlargement that house an enormous infrastructure of digital systems [5, 6]. For the last ten years, there has been an average annual growth of 11% from the summation of worldwide energy consumption of 238 billion kWh electronics cooling of data centres in 2010 [7]. Estimated amount of 1.8% of electricity consumption of data centres in the United States (U.S.) giving 82 billion kWh of energy at a cost of \$6.1 billion [8-10]. In the U.S., the availability of electrical energy will diminish when yearly energy consumption increases by 1% by year of 2030 [11]. Therefore, the management of efficiency and power consumption of data centres is very crucial. By 2015, International Technology Roadmap of Semiconductors [12] predicted that power dissipation of

microprocessor chip will top 270W and expected to exceed 800W by 2026. The thermal management of electronic devices must be improve as future electronic systems such as radio frequency (RF) power amplifier, Satellite, radar systems and central processing units (CPUs) are relying on the effectiveness on the cooling systems.

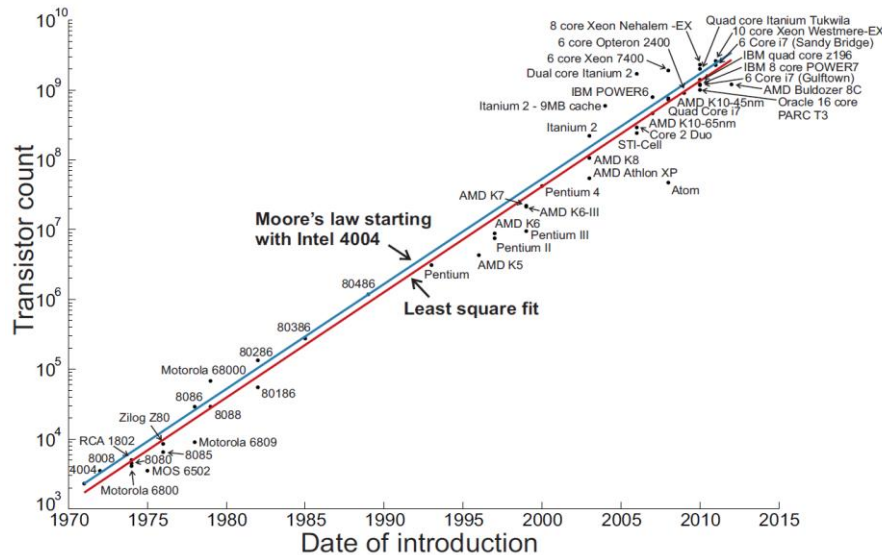


Figure 1-1: Shows the yearly increase of transistor number. The blue line representing the Moore's Law and red line representing the least square fit for the whole data set [13].

The conventional Micro-channel Heat Sink (MCHS) using air as working fluid is not suitable for cooling of high heat flux application due to its very low heat capacity [14]. High heat flux exceeding $100\text{W}/\text{cm}^2$ requires large area and high speed air flow for an effective cooling. For cooling of high heat flux devices, heat sink of 2000 times bigger than area of device required. The cooling of electronics using method of two-phase flow such as vapour chamber and heat pipes provide promising solution due to fluid rotation from pump is not needed and the coolant is confined within closed chamber. In two-phase cooling system, there are three main components of evaporator, compressor and condenser. The evaporator will be placed in contact with the heat source allowing heat to be absorbed turning the fluid into vapour state. Then, the hot vapour will be compressed by the compressor to be pushed through the condenser turning back into liquid state to be sent to the evaporator again. However, these types of devices which are capillary-driven is not suitable for cooling of high performance chips due to wick thickness and pipe cross-sectional area limitations [15]. The utilisation of pulsating and loop heat pipes are also unsuitable for high heat flux cooling [16].

The use of liquid jet impingement for high heat flux dissipation application has been studied by many researchers. Mudawar et al. [17] used dielectric fluid HFE-7100 to remove 150W/cm² to 200W/cm² of heat fluxes while Zhao et al. [18] used porous structured substrates with utilisation

of water impingement for $674\text{W}/\text{cm}^2$ of heat flux removal. But, requirement of high pumping power leading to the risk of surface erosion caused by spray cooling integration with closed loop system [16, 19].

Cooling method of flow boiling (two-phase flow) and single-phase liquid are suitable for high heat fluxes system (exceeding $100\text{W}/\text{cm}^2$) [20] due to flow boiling uses large latent heat of vaporisation but it can be unstable. The initial concept of MCHS from [21] with advantages of compactness, small mass, and large heat transfer surface area where Leonard and Phillips [22] deduced that 60% of energy consumption can be reduced effectively. However, the increase of high pressure drop and temperature gradients are required. The latter have been explored by researchers because of their capability for high heat flux dissipation using very low pumping power [23] and exploitation of liquid latent heat of vaporization [16]. But, for high heat flux, the use flow boiling MCHS suffers from flow instabilities and fluctuation of pressure inducing large reduction of heat transfer due to liquid dry out [20, 24].

Figure 1-2 shows the variation of convective thermal resistances for heat source area of 10cm^2 with different types of coolant using velocity range of 2m/s to 8m/s . Reduction from $100\text{K}/\text{W}$ to $33\text{K}/\text{W}$ of convective thermal resistance, R_{conv} , when using air as working fluid for both natural and forced convection. The use of fluorocarbon liquids as working fluid under the condition of boiling heat transfer, R_{conv} reduced to $0.5\text{K}/\text{W}$.

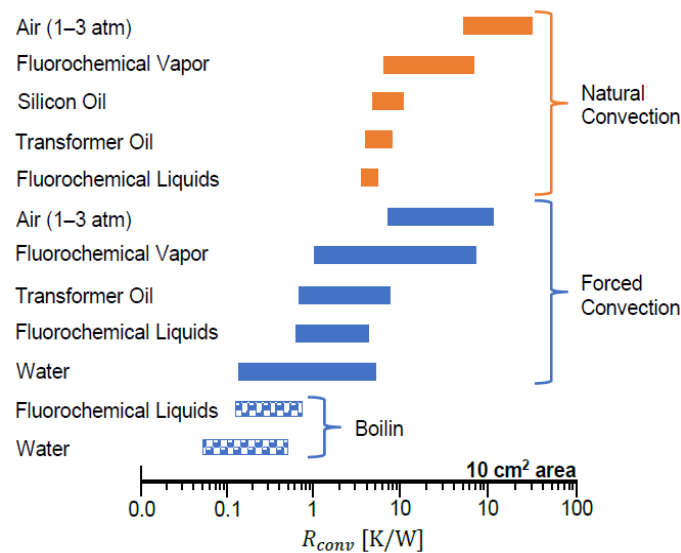


Figure 1-2: Shows the convective thermal resistances under 10cm^2 area of heat source [25].

1.1 Micro-channel heat sink with liquid as working fluid

Recently, the challenge of which is faced by thermal designer is to solve the miniaturization of heat sink device due to limited space such as in laptops where smaller heat sink can be fit in, the

utilisation of restricted space suitable for smaller heat sink and also to reduce the generation of noise from fans installed together with heat sink. An effective solution is on demand to remove excessive heat from electronics so that it will not heat up to reach the junction temperature (maximum operating temperature). Based on this issue, different methods of liquid cooling have been introduced splitting between direct or indirect contact styles. Almaneea considered direct contact method where the electronics is being submerged into dielectric liquid for cooling [26]. For this work, the indirect contact liquid method is used where the cold plate heat sink is a separated device attached to electronics with liquid flows within removing heat (open-loop single-phase system). The use of cold plate heat sink provides an effective solution to solving the key problem of over-heating electronics [27]. The use of liquids for cooling offer higher heat transfer [28] since they have larger specific heat capacities and thermal conductivities compared when using air. Different liquids used for cooling include dielectric, nano-fluids and pure water respectively. Micro-channel heat sink is an effective device for removing heat from chip effectively [29]. To begin, Tuckerman and Pease have developed a miniaturised system called micro-channel heat sink (MCHS) which has the ability of removing high heat fluxes without having a significant temperature change due large surface area enabling heat dissipation effectively [21] (refer Figure 1-3). The substrate materials for MCHS are usually using copper or silicon due to their high thermal conductivity and normally they are made by micro-fabrication technology which has high precision and quality surface finish. The characteristic dimension of MCHS is between $10\mu\text{m}$ and $1000\mu\text{m}$ [30] allowing fluids to pass through. MCHS has the ability to remove high heat fluxes due to its physical characteristics of high surface area to volume ratio. Furthermore, small mass and volume of MCHS with large heat transfer convective coefficient as well as small space of coolant makes it very suitable to be a highly efficient cooling system for applications such as laser diode arrays, radars, laser mirrors (high energy) and microprocessors [31]. The micro-channel hydraulic diameter, D_h , has an inverse variation with the average coefficient of heat transfer, h_{avg} , while pressure drop is inversely proportional to the D_h^{-4} [32]. The decrease of hydraulic diameter will definitely increase the rate of heat transfer but with very much larger pressure drop.

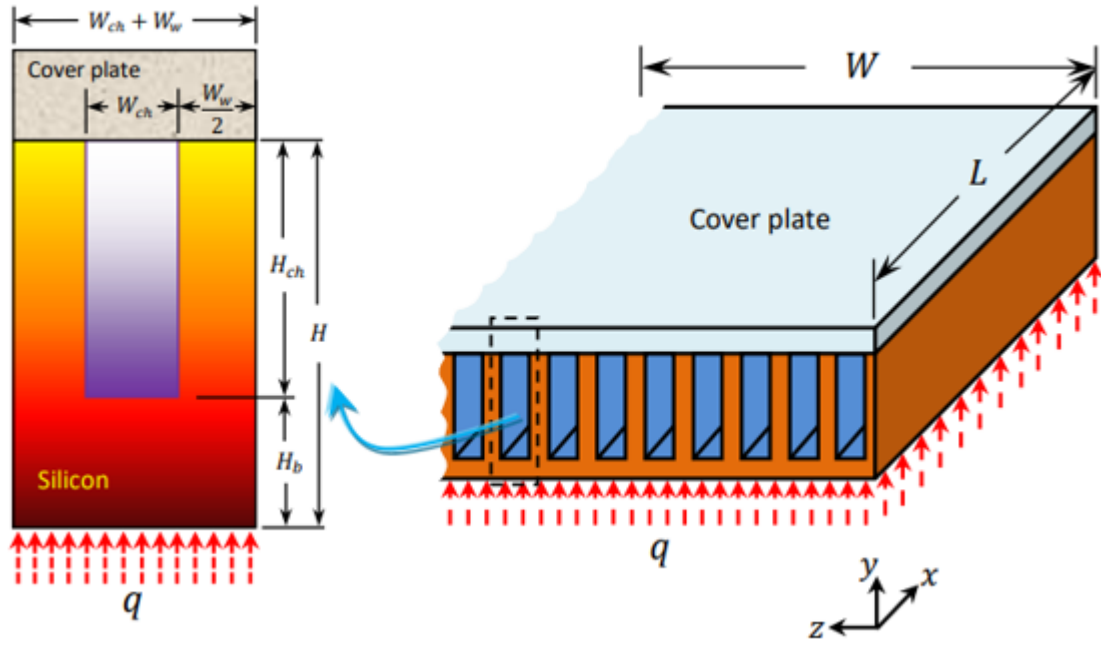


Figure 1-3: Shows the micro-channel heat sink illustration [33].

1.2 Problem statement

It is essential to grow an effective method for electronics cooling application. The ability to dissipate heat from electronics using MCHS has been the interest by many researchers. The need of miniaturised high-density heat flux remover is crucial as rapid advancement of microelectronics technology leading to smaller components, which uses more number of transistors emitting large heat energy inducing higher thermal-mechanical stress towards electrical components [34]. The heating problem affects the micro-electronics performance, reliability, design and cost significantly [1]. The application of MCHS for electronics cooling has been widely used for its capability of removing high heat flux. The key challenge is to search for an effective cooling method using liquid as coolant in MCHS which is still greatly on demand. The use of conventional straight micro-channel heat sink conveys a significant problem where temperature experience an increase along the micro-channel length [35]. To improve the performance of MCHS as a cooling system, geometrical design improvement as well as boundary conditions configurations are required. Different design of MCHS will affect fluid flow structure which is one of the main reasons for higher performance cooling system due to formation of vortex and recirculation region that breaks the boundary layer which has slow heat diffusion, thus, by breaking the boundary layers the flow velocity near the boundary is higher inducing increased rate of heat transfer as well as leads to more viscous losses.

For this project, an extension of research on the cooling ability under high heat flux of more than 100 W/cm^2 , using single inlet-outlet serpentine mini-channel with fins is further investigated continuing from successful previous work [36] on utilising the thin secondary flow reducing both total thermal resistance (the ability of a system or material to resist heat flow) and pressure drop (the decrease of pressure as the fluid flowing through a device or system) simultaneously due to continuous breaking of boundary layers and fluid velocity is greatly reduced. In this project, similar base serpentine design by [[36] will be adopted here but instead of chevron fins, three different shape of oblique (SMOF), trapezium (SMTF) and diamond (SMDF) fins are used. Each shape of fin will have four different configurations respectively (see Figure 1-4, Figure 1-5 and Figure 1-6). It can be seen that using fins creating secondary channels technically changed the serpentine heat sink into pin-fin heat sink (see sub-chapter 2.8 for comparison). The performance of all serpentine mini-channel heat sink design with fins will be compared with the conventional serpentine heat sink with plate fin (SMPF). Parameters used to compare the performance of each design are total thermal resistance, R_{th} , pressure drop, ΔP , and Average Nusselt number, Nu_{avg} , respectively.

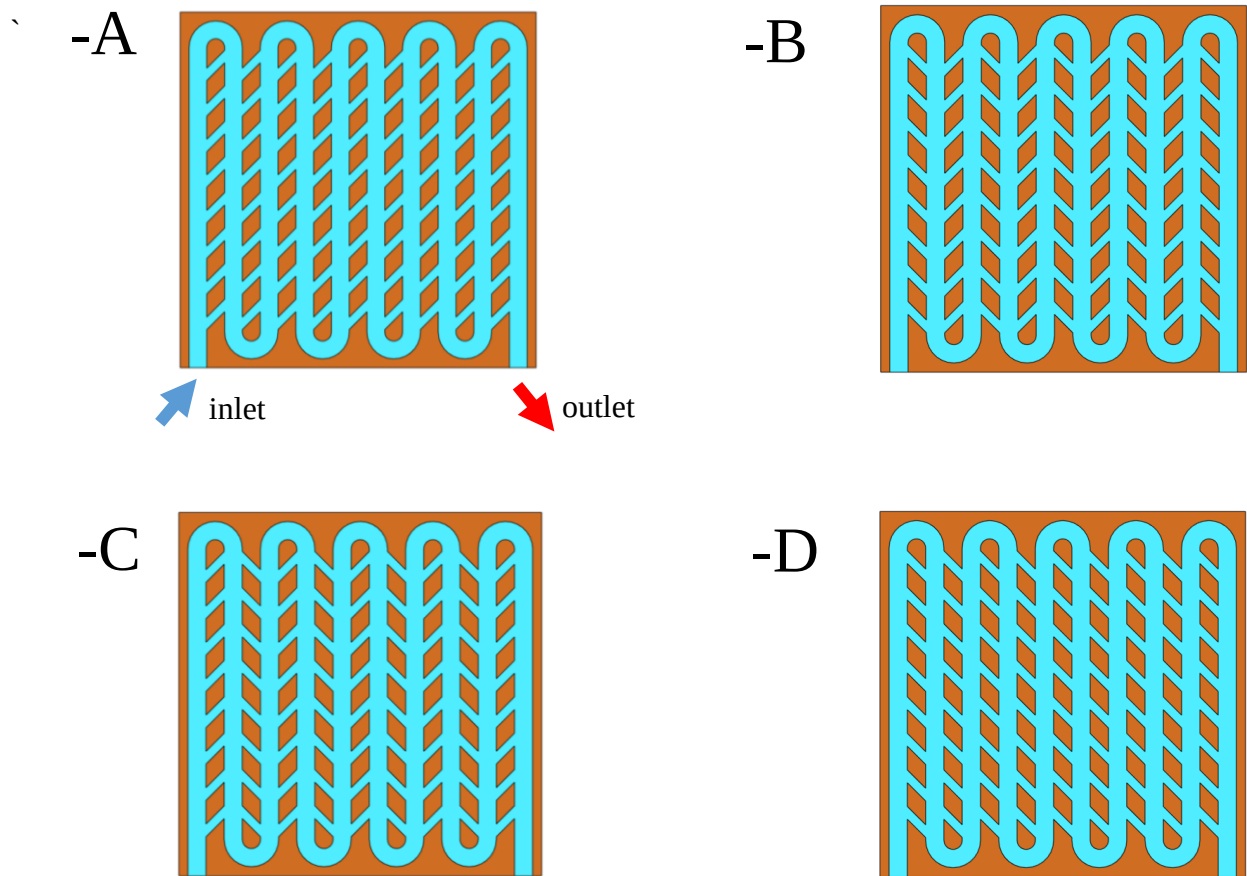


Figure 1-4: Shows the design configurations of SMOF design heat sinks.

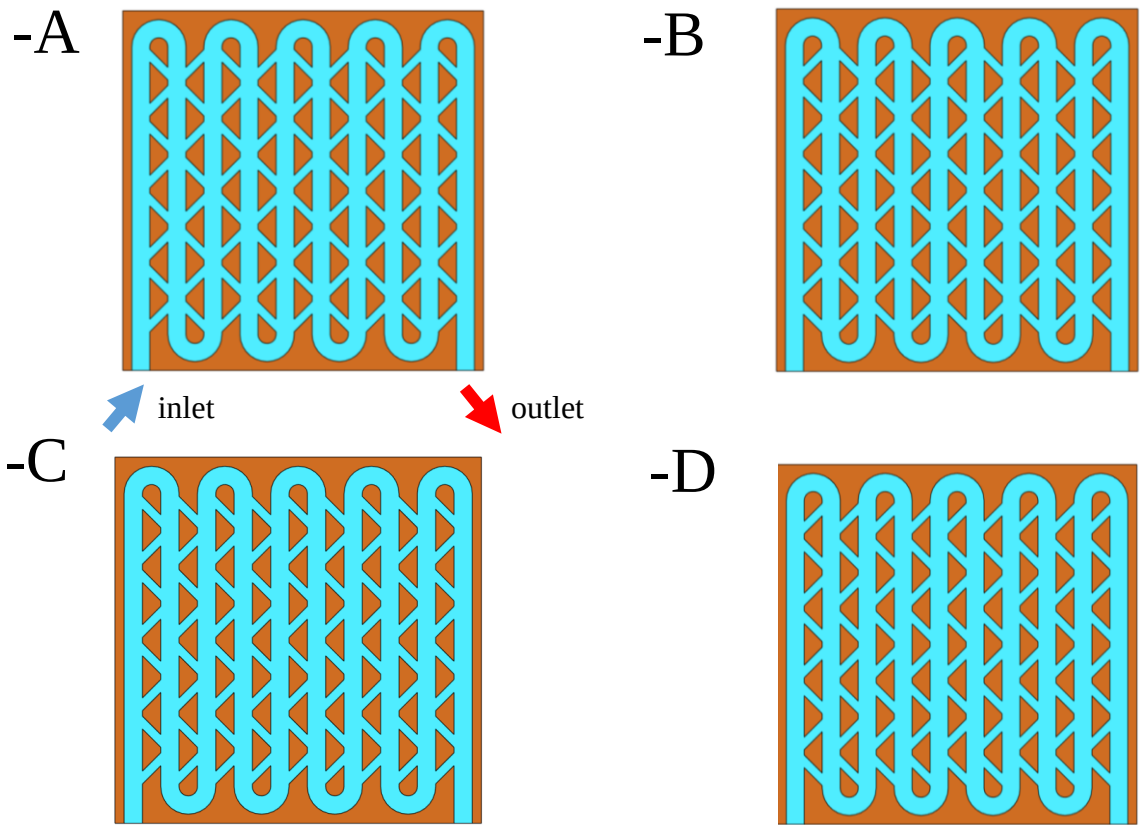


Figure 1-5: Shows the design configurations of SMTF heat sinks.

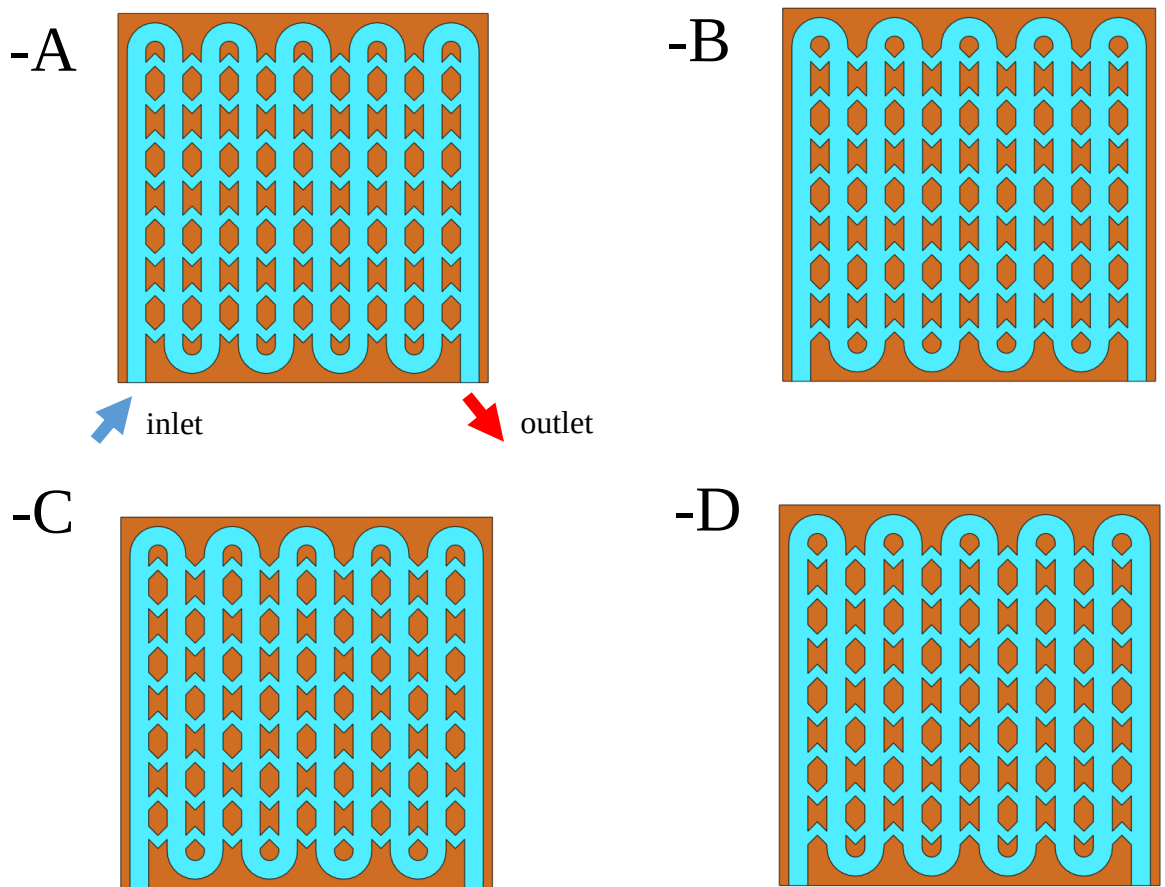


Figure 1-6: Shows the design configurations of SMDF heat sinks.

The main reason of choosing the serpentine heat sink design for electronics cooling is due the fact that the thermal and hydrodynamics boundary layers are periodically disturbed to enhance heat transfer rate. But a significant increase of pressure drop occurs with the augmentation of heat transfer due to drastic direction change of velocity of fluid around bends creating vortices. In addition, since the serpentine-channel heat sink possesses sinusoidal like channel with bends, the effective surface area, A_{eff} , which is the area available for heat to transfer from solid to fluid domain is larger compared to the conventional straight-channel heat sink for higher heat transfer [36].

Furthermore, a published work on double inlet-outlet serpentine mini-channel heat sink (D-SMPF) has shown great superiority in terms of thermal performance (refer Figure 1-7 (A)) [37]. They also considered through-holes combining two separated mini-channels which reduces the ΔP drastically and increasing the thermal-hydraulic performance. Based on the knowledge from literature reviews, the use of thin secondary channel on D-SMPF heat sink has not been researched yet. Thus, to further the research on the performance of D-SMPF heat sink, diamond fins are embedded within the plate fin allowing the control of boundary layers for improved thermal performance as well as reducing the ΔP . Diamond fin is chosen since it offer the most superior performance during the analysis on single inlet-outlet serpentine mini-channel heat sink. The application of diamond fins with D-SMPF heat sink is split into two major case studies. First investigation will cover only on the effect of diamond fins (D-SMDF) towards the D-SMPF heat sink while the second one will be the same as the first but with existence of through-holes (D-SMDF-TH) connecting the two separate serpentine mini-channels (refer Figure 1-7 (B) and (C)).

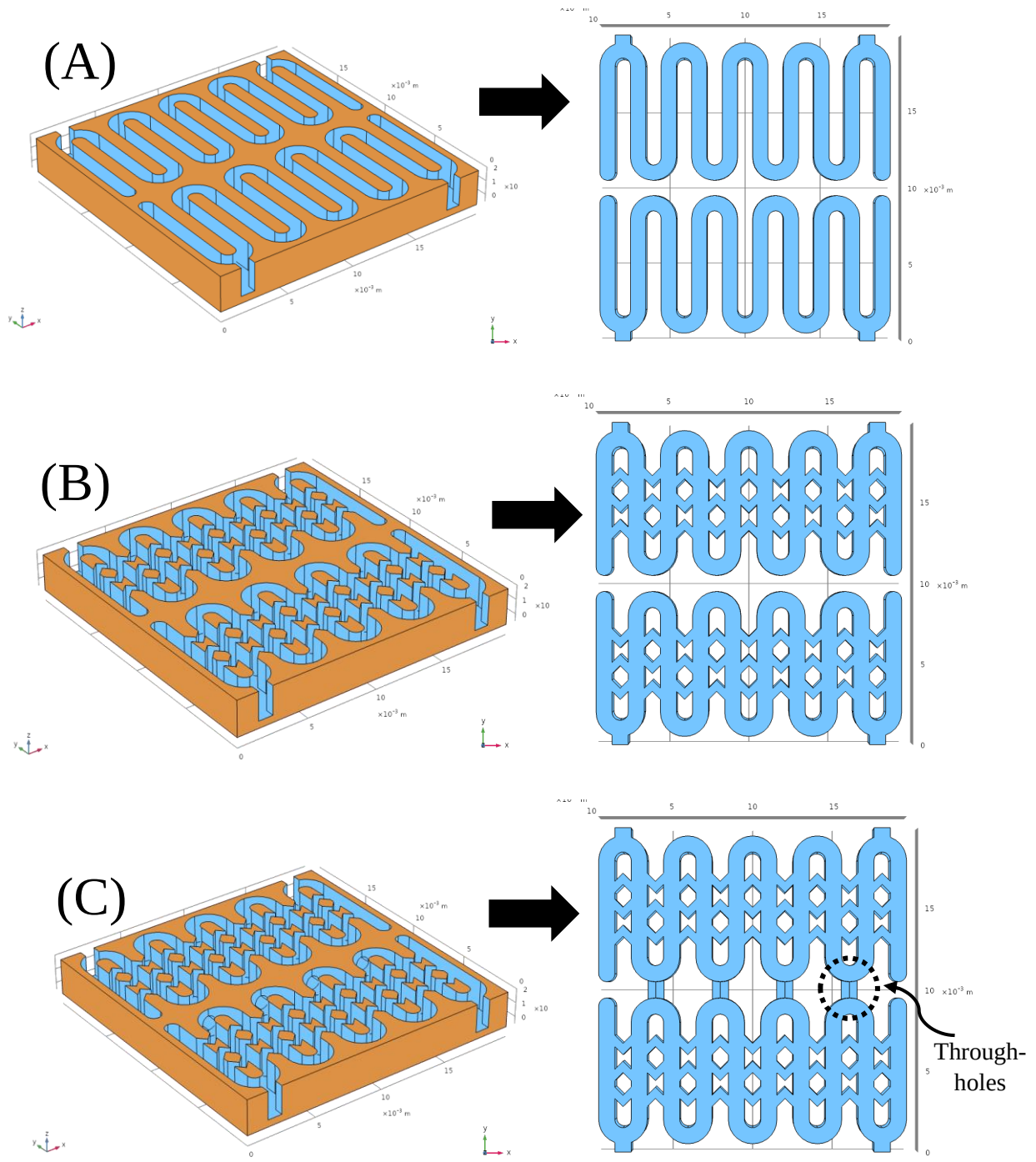


Figure 1-7: Shows the (A) D-SMPF (B) D-SMDF (C) D-SMDF-TH heat sinks.

1.3 Contributions of research

To mention all contributions for this work, it can be summarised as follows:

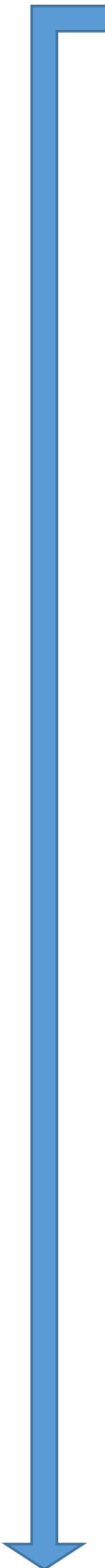
- A) Completed the design of novel designs for single and double inlet-outlet serpentine mini-channel heat sinks with different shapes of secondary channels and through-holes.
- B) Compared the thermal-hydraulic performance of single inlet-outlet serpentine mini-channel heat sink novel designs with new shape of secondary channels (oblique, trapezium and diamond) with published work (chevron shape) [38] based on hydraulic and heat transfer characteristics.
- C) Compared the thermal-hydraulic performance of double inlet-outlet serpentine mini-channel heat sink novel design with new diamond secondary channels with published work by [37] based on hydraulic and heat transfer characteristics.
- D) Compared the thermal-hydraulic performance of double inlet-outlet serpentine mini-channel heat sink novel design with new diamond secondary channels and through-holes with published work by [37] based on hydraulic and heat transfer characteristics.
- E) Used the parameters of heat transfer coefficient, h_{avg} , Average Nusselt number, Nu_{avg} , total thermal resistance, R_{th} , and pressure drop, ΔP , to compare the performance between heat sinks.
- F) Created the surrogate models for optimisation studies using Artificial Neural Network (ANN) method.
- G) Used the deterministic optimisation process for both single and multi-objective optimisation by using the surrogate models created within Genetic Algorithm (GA) framework.
- H) Used the Robust Design Optimisation process for both single and multi-objective optimisation by implementing uncertainty within the input design variables and measured the distribution output using the created surrogate models by measuring the statistical moments of means and standard deviations.

1.4 Summary of chapter

This chapter has presented that the importance of having powerful thermal management is crucial for electronics cooling to ensure component of electronics can perform its function well without having damaged by the excessive heat dissipation. The use of serpentine mini-channel heat sink has been proven for its superiority to remove heat effectively due to periodic breaking of boundary layers for greater heat transfer. Some published work has considered using single and double inlet-outlet serpentine mini-channel heat sink. In this work, both single and double

inlet-outlet serpentine mini-channel heat sink will be utilised with the application of using secondary channel and through-holes to reduce the total thermal resistance and pressure drop simultaneously.

1.5 Work-scope



0: Introduces the research motivation and background information. The objectives and work-scope for the thesis are outlined

Chapter 2: Perform the literature review related to the current work. The reviews comprises the numerical methods and experimental works for mini-micro scale heat sinks under the condition of single or two-phase flow

Chapter 3: The fundamentals of heat and fluid flow are presented such as convection heat transfer physical mechanism, types of fluid flow, concept of boundary layers, heat transfer and hydraulic characteristics and Performance evaluation criteria.

Chapter 4: Describes the numerical analysis such as meshing techniques, computational model, governing equations, boundary conditions and assumptions as well as post-processing.

Chapter 5: Explains the design optimisation procedure, formulation, Design of Experiments, sampling methods, surrogate modelling, and optimisation processes.

Chapter 6: Analysis of the single inlet-outlet serpentine-mini channel heat sink with different shapes of fins of oblique, trapezium and diamond. Parametric analysis conducted for the most superior heat sink in thermal performance. Optimisation is conducted using both deterministic and robust design optimisations.

Chapter 7: Analysis of the double inlet-outlet serpentine-mini channel heat sink with diamond fins. Parametric analysis conducted for the most superior heat sink in thermal performance. Optimisation is conducted using both deterministic and robust design optimisations.

Chapter 8: Analysis of the double inlet-outlet serpentine-mini channel heat sink with diamond fins and through-holes. Parametric analysis conducted for the most superior heat sink in thermal performance. Optimisation is conducted using both deterministic and robust design optimisations.

Chapter 9: Discussions and conclusions are explained as well as the future works recommendations.

Chapter 2 : Literature Review

An enormous number of investigations have been made by researchers using the method of numerical and experimental analysis on MCHS for cooling systems efficiency improvement. In 1981, researchers from Stanford University, Tuckerman and Pease, pioneered the first idea of miniaturised cooling system using MCHS which possessed the capability of cooling high heat flux systems without significant temperature change and being referred by many other researchers [6]. The MCHS device was firstly proposed with the application of very large scale integrated (VLSI) electronics components utilising rectangular straight micro-channel. Further study consisting experimental and theoretical analysis on heat transfer and flow structure contributed to massive amount of information for a better understanding. The flow of coolant within heat sink induced thermal resistance as well as pressure drop. Many cooling techniques utilised for microprocessor chips and IC cooling consisting jet impingements and heat pipes for application of high heat flux dissipation systems. The MCHS device has been capable and proven effective for high heat flux removal and maintaining the critical temperature which is the junction temperature of 85°C to 100°C [12]. In this chapter, a literature review regarding different cooling devices and micro-channel systems is presented.

2.1 Introduction to heat sink

The function of heat sink is as a heat exchanger where heat flows from heat source (e.g. electronic devices) to heat sink to fluid within the heat sink (e.g. air or water) to dissipate the heat. Heat sinks utilisation usually applied to high heat flux system such as power transistors and optoelectronics [39]. A simple heat sink usually consisting of fins series and thermal performance improvement from fins surface area, fins thermal conductivity and coefficient of heat transfer [40]. The presentation of classification of Heat sink presented on Table 2-1 [41].

Table 2-1: Shows a heat sink classification [41].

Type of Heat Sink	Advantages	Disadvantages	Applications	Examples
Passive	Economical, readily accessible and consumer friendly	Limited power dissolution capacity	Air flow non-dependent systems/Natural convection Used for simple power intensity system	Metal plate
Semi Active	Comparing to Passive type, for same are the thermal resistance is low	Power dissolution ability is limited	Used for moderate power intensity system	Finned heat sink
Active	Integration of heat sink and fan together giving higher heat dissolution capacity compared to previous types	Reliability cost in long term	Used for high power intensity system	Fan-fins heat sink
Liquid Cooling	Even higher capacity of heat dissipation compared to previous types	Expensive, complicated	Used for high power intensity system Constant heat required	Liquid cold plate
Phase Change Cooling	Heat is dispersed equally	Expensive, complex and large area needed	Used for high power intensity system	Vapour firmness phase change cooler

Referring to Table 2-1, there are five types of heat sink which are commonly used. Passive cooling refers to cooling techniques which use very minimal energy consumption for energy efficiency enhancement [42, 43]. Active heat sink considers force cooling with the help of machinery such fan or pump [44]. Semi active heat sinks are in between active and passive heat sinks where the amount of energy used is not as much as active cooling but not as low as passive cooling due to there are still presence of a pump or fan but these mechanical devices only operates when the temperature sensor achieved allowed maximum temperature pushing the fluid through. Liquid cooling heat sink uses liquid as coolant which is very popular now due its capability to remove high heat fluxes since liquid has thermal properties of large specific heat capacity. Phase change cooling involves the state change of a material medium which is thermally conductive from gas to liquid and vice versa [45].

2.2 Heat sink channel classification

Many authors have addressed the issue of channel size classification. Mehendale et al. [46] and Kandlikar and Grande [47] used hydraulic diameter, D_h , parameter to classify tube or smallest dimensions of other shapes of cross section. The D_h is an important parameter because it affects the flow of fluid inducing different flow resistances and rates of heat transfer. Channel size classification can be referred to Table 2-2.

Hydraulic diameter (D_h) [m], of a rectangular cross section is as follows:

$$D_h = \frac{4 \cdot A_{ch}}{P_w} = \frac{2(W_{ch} \cdot H_{ch})}{W_{ch} + H_{ch}} \quad (2.1)$$

Where A_{ch} is the cross sectional area of channel [m^2]; P_w is the wetted perimeter of channel [m]; W_{ch} is the width of channel [m]; H_{ch} is the depth of channel [m].

Table 2-2: Shows channel size classifications.

Mehendale et al. (2000)		Kandlikar and Grande (2003)	
Conventional Channels	$D_h > 6mm$	Conventional Channels	$D_h > 3mm$
Compact Passages	$1mm < D_h \leq 6mm$	Mini-channels	$200\mu m < D_h \leq 3mm$
Meso-channels	$100\mu m < D_h \leq 1mm$	Micro-channels	$10\mu m < D_h \leq 200\mu m$
Micro-channels	$1\mu m < D_h \leq 100\mu m$	Transitional channels	$0.1\mu m < D_h \leq 10\mu m$
		Transitional Micro-channels	$1\mu m < D_h \leq 10\mu m$
		Transitional nano-channels	$0.1\mu m < D_h \leq 1\mu m$
		Molecular nano-channels	$D_h \leq 0.1\mu m$

Obot [48] regarded micro-channels to have $D_h < 1mm$ where is it also used by Bayraktar and Pidugu [49] and Bahrami et al. [50].

As a consequence, by only using channel size alone the differentiation between macrochannel and micro-channel flows cannot be distinguished, thus, Kew and Cornwell [51] suggested the use of confinement number (Co) as a transition criterion between micro and macroscale flow boiling referring to Eq. (2.2). They proposed that at Co between 0.1 to 10, the flow is within the

transitional regime where characteristic length is in the same order as mean free path. In this regime, the continuum and rarefied (gas molecules density is so small where there are less collisions among them and gas behaviour acts like an individual molecules collection rather as a continuous fluid) effects are important. The use of Co is very important because it gives the information on the flow confinement degree where at low Co the flow of fluid is unconfined and has the potential to exhibit the mixing and turbulent flow behaviour. On the other hand, at high Co the fluid particles are more restricted with less mixing and turbulent flow behaviour becoming more like a laminar flow.

$$Co = \left[\frac{\sigma}{g(\rho_l - \rho_v)D_h^2} \right]^{\frac{1}{2}} \quad (2.2)$$

where σ is the surface tension (N/m); g is the acceleration due to gravity (m/s^2). ρ_l and ρ_v are densities for liquid and vapour case (kg/m^3). For Co value, $Co > 0.5$ (microscale flow) and $Co < 0.5$ (macroscale flow) where the transition occurred at $Co = 0.5$.

2.3 Natural and forced convection of MCHS

Many investigations have been made in the literature regarding MCHS using either natural or forced convection of fluid flow. Here a detailed explanation regarding the two conditions used for cooling of electronics is given.

2.3.1 Natural convection method

Natural convection condition arises when there is existence of fluid convection current due to body forces induced from density gradients without forced fluid velocity. Net body forces create the free convection currents and buoyancy force. Usually, the body forces are affected by gravitational field and density gradient is affected by temperature gradients [52]. The advantages of using natural convection are minimised cost, very simple, low of noise and manageable size [53]. Natural convection together with radiation are considered as passive cooling where it is a weak heat transfer mechanism [54, 55] based on its dependability only on fluid buoyancy effect due to difference in temperature in different parts of fluid. Natural convection is often used for cooling electronics due to its benefits of silent, reliable and low cost. Air cooling is considered as a crucial thermal design technique due to its cost effectiveness and reliability with safety when utilised in antagonistic environments [56, 57].

2.3.2 Forced convection method

Forced convection condition arises when the flow of fluid is exerted using external force such as pump, fan or atmospheric winds [52]. Generally, forced convection method has the highest number of implementations from literature due to its significant benefits and advantages as part of a cooling system [58] where very high heat fluxes can be removed away from the system. The main reason of utilisation of forced convection is due to its simplicity and low cost as well as providing the traits of high quality and non-complex maintenance [59]. The most famous system of cooling electronics is using forced convection liquid cooled MCHS [60]. In general, forced convection is more favourable over natural convection in cooling system as its ability to remove higher heat fluxes with larger rate of heat transfer [61].

2.4 Conventional MCHS

In the years of 1981 and 1982, Tuckerman and Pease [21] considered single phase heat transfer system with the fabrication of miniaturised MCHS using water as coolant. From the first test experiment, they proved that using forced convective flow through small volume silicon MCHS has the capability as a cooling device for VLSI circuit. Heat sink fabricated with series of rectangular straight micro-channels etched in 1cm^2 by 1cm^2 of silicon wafer. The channels are separated with $50\mu\text{m}$ thick wall and the dimensions of channels were $50\mu\text{m}$ wide and $302\mu\text{m}$ respectively. From thermal performance, the value of R_{th} has a very low magnitude (0.09K/W) per centimetre squared area using 1.84W of pumping power. The temperature of chip was maintained to be below than 110°C with the ability of dissipating $790\text{W}/\text{cm}^2$ of heat fluxes and there was no phase change occurrence. High pressure drop (220kPa) measured as large mass flow rate required ($5700\text{kg}/\text{m}^2$) to remove $790\text{W}/\text{cm}^2$ of heat flux. The increase of the substrate temperature can reach 71°C from inlet water temperature of 23°C resulting outlet temperature to be 94°C . Since the first invention of micro-scale cooling devices, many studies and investigations have been made further searching for room of improvement of MCHS performance.

Another significant accomplishment when Phillips [58] continued further on the work in [21] considering both regions of fully developed laminar and turbulent flow for straight rectangular MCHS using experimental study. The material of heat sink was indium phosphide while water flows through the channel. Dimensions of channel were $W_{ch} = 0.22\text{mm}$, $H_{ch} = 0.165\text{mm}$ and $L_{ch} = 9.7\text{mm}$ respectively. Predictions of overall R_{th} and characteristics of fluid flow obtained using computer model developed (MICROHEX). The R_{th} measured was in the order of $0.072\text{K}/(\text{W}/\text{cm}^2)$ but higher pressure drop penalty with 2.5bar of excess in drop of pressure.

From the investigation of Kawano et al. [62], they have considered the three-dimensional (3-D) numerical simulation and experimental analysis to study the pressure drop and heat transfer of MCHS with straight rectangular design. From the setup of experimental analysis, an area of $15 \times 15 \text{ mm}^2$ made up by 110 micro-channels which are etched with silicon material substrate. A fixed dimension of $57 \text{ }\mu\text{m}$ was set for the width of micro-channel. The micro-channel height is set to be at $180 \text{ }\mu\text{m}$ or $370 \text{ }\mu\text{m}$. From the experimental setup, the coolant used was water and an assumption is made where the numerical simulations are set for fully developed laminar flow. Measuring the loss of pressure, the chip is subjected to a condition where no heat is applied. At a range between 0 to 200 of Reynolds number, a good correlation is achieved between the experimental and numerical simulations for pressure drop results but not for thermal resistance when measured at the entrance of micro-channel heat sink which differs significantly. The discrepancies between thermal resistance results are due to the heat loss to surrounding from the experimental setup. To add, within these conditions of low Reynolds number, they are considered unreliable due to the large temperature-induced viscosity-gradients from the portion at inlet location. At Reynolds number larger than 300, the pressure drop for simulations are lower compared to experimental results. Thermal resistance experiences changes of about $0.1\text{K}/(\text{W}/\text{cm}^2)$ along the micro-channel span between inlet and outlet when $100\text{W}/\text{cm}^2$ of heat flux is applied.

Fedorov and Viskanta [63]; Qu and Mudawar [64]; and Li et al. [65] investigated heat transfer and fluid flow in three-dimensions (3-D) using numerical analysis on silicon MCHS due to its high melting point and good thermal conductor with straight rectangular micro-channels, continuing from experiments by Kawano et al. [62]. From the results in [63], almost uniform trend of average wall temperature of channel along the direction of flow while temperature gradient is very large at region close to the inlet of micro-channel. On the other hand, results from [64] showed that linear temperature rise in the region of fluid bulk and wall along in the direction of flow. In [65], an explanation provided on why there is a difference in the trend of wall temperature between [63] and [64]. From [64] the thermophysical properties such as viscosity and density of water were assumed to be temperature-dependent while in [63] ignored the assumption.

Wei and Joshi [66] used stacked MCHS and numerically investigated its thermal performance (refer Figure 2-1). Optimisation study was conducted searching for the minimum R_{th} using the developed model of a simple network of thermal resistance. The pumping power and layers number were maintained to be fixed. The overall R_{th} for two layered micro-channel stack was

less than single layered micro-channel by 30% due to its doubled area of heat transfer. Furthermore, channel length and number of layers effect on R_{th} were also investigated. From results, the optimal number of layers for stacked MCHS was 3 under constant pumping power (0.01W) where lowest total thermal resistance achieved. Thermal resistance of 0.213K/W was found to be the minimal value for two-layered MCHS with optimal dimensions of fin and channel width of 41 μ m and 107 μ m respectively. The optimal inverse aspect ratio ($\alpha^* = H_{ch}/W_{ch}$ where $H_{ch} > W_{ch}$) is 3.8.

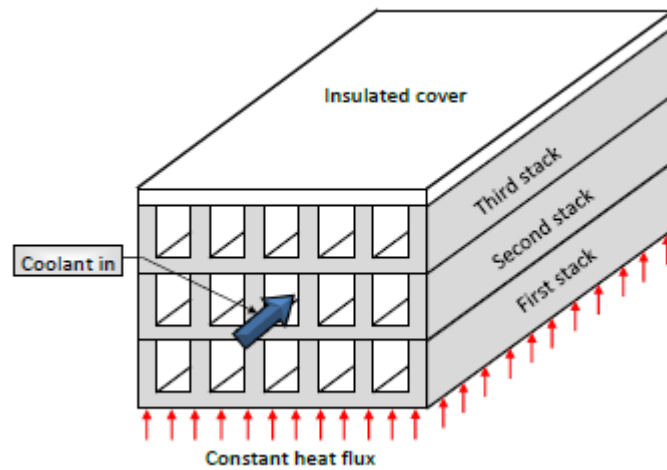


Figure 2-1: Shows the stacked MCHS in 3-D view [66].

Wei and Joshi [67] continued by using numerical study to analyse the conjugate heat transfer of multilayer MCHS. The heat sink base area was set to be 10mm x 10mm along with a range in height between 1.8mm and 4.5mm (2 to 5 layers). Variation of flow rate between 50ml/min and 400ml/min was considered. Investigation of layers number, aspect ratio, pumping area per unit area and length of channel effects on optimal R_{th} were conducted with water as the working fluid. The overall R_{th} was measured and computational analysis results were compared with single-layered MCHS.

2.5 Effect of surface roughness on MCHS performance

Using both numerical and experimental analysis, the effect of surface roughness on single-phase liquid flow in micro-channels has been studied. Continuous miniaturisation of channel, consideration of surface roughness has to be taken into account especially in the region of laminar flow. A significant role for surface roughness is to predict the fluid flow structure and it has a major effect on the laminar to turbulent transition of the flow.

Wu and Little [68] used micro-miniature Joule-Thompson refrigerators with cross-sections of rectangular and trapezoidal to measure the friction factor for different gases flow of nitrogen

(N₂), hydrogen (H₂) and Argon (Ar). The experiment covers both regimes of laminar and turbulent flow. Micro-channels were etched either in silicon or glass and sealed by using anodic bonding with Pyrex material. The D_h values were ranged between 45.5 μm and 83.1 μm . A unique feature of micro-channels is very relatively rough heat transfer surfaces.

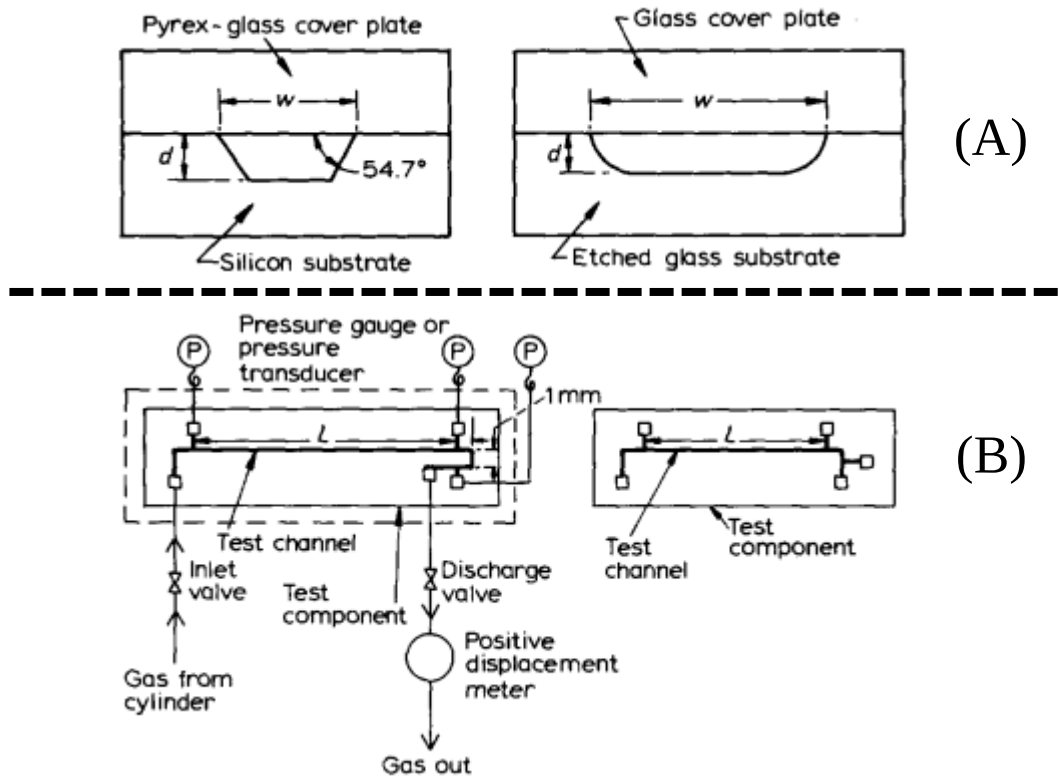


Figure 2-2: Shows the (A) Test channels cross-section (B) Test system and test channels pattern [68].

Using glass micro-channels, laminar to turbulent transition occurred at $Re = 350$ with the absent of heat treatment and at $Re = 900$ with heat treatment. These transitions occur at an early Re compared with traditional transitional which occurred at $Re = 2300$ [69]. The reason for these difference in transitional occurrence is due to the effect of relatively high surface roughness caused by the fabrication technique and channel dimensions uncertainty. To compare the results of friction factors data from experimental analysis with established correlation for macroscale pipes, experimental results show a higher value. However, an inconsistent trend observed from results by Choi et al. [70] and Wu and little [68] who utilised N₂ in both laminar and turbulent regime flowing through micro-tubes. The friction factors measurement for laminar and turbulent region has lower value compared with the conventional correlations prediction. The equation for correlations are as follows:

The Darcy friction factor, f_D , is dependent with Re for fully developed flow:

$$f_D = \left(\frac{64}{Re}\right) \text{ for } Re < 2300 \quad (2.3)$$

Colebrook [71] did an experimental analysis for smooth and rough pipes giving new equation for f_D under turbulent regime:

$$\frac{1}{\sqrt{f_D}} = -2 \log_{10} \left(\frac{\varepsilon/D_h}{3.7} + \frac{2.51}{Re\sqrt{f_D}} \right) \text{ for } 3000 \leq Re \leq 10^5 \quad (2.4)$$

From Eq. (6), $\sqrt{f_D}$ is dependent on relative roughness (ε / D_h), where ε is the surface roughness (m) and Re . (ε / D_h) = 0 for smooth pipe giving Eq. (7):

$$\frac{1}{\sqrt{f_D}} = 2 \log_{10} Re\sqrt{f_D} - 0.8 \quad (2.5)$$

Eq. (7) is representing Prandtl equation. Von Karman's expression used to obtain the equation for rough friction factor shown in Eq. (8):

$$\frac{1}{\sqrt{f_D}} = 2 \log_{10} \left(\frac{3.7}{\varepsilon/D_h} \right) \quad (2.6)$$

Wu and Little [72] further investigated the experiment on micro heat exchangers under both laminar and turbulent regime using N_2 as working fluid to study the characteristics of heat transfer. Counterflow heat exchangers micro-channels fabrication utilised the same photolithography method used in previous work with D_h ranges between 134 μ m and 164 μ m. This type of channel possessed unique features such as asymmetric roughness (very rough surface on part of channel walls while smooth on the other), large relative roughness and large heat flux and temperature variation over channel walls. For one sided heated channel, heat source positioning has an effect on the heat transfer at laminar regime but not turbulent. Transitional zone occurred at range of $1000 < Re < 3000$. Two-sided heated channel has larger Nu_{avg} compared to one-sided heating. Larger rate of heat transfer for roughened channel due to surface irregularities increasing the contact area between solid and fluid, thus, heat transfer is improved but friction factor is increased. Both laminar and turbulent flow regimes, the measured coefficient of heat transfer has higher value compared with correlations prediction which represented by Eqs (2.7 – 2.9) [73] due to higher roughness of channels surface.

Sieder-Tate's correlation for simultaneously developing laminar flow:

$$Nu = 1.86 \left(Re \cdot Pr \frac{D_h}{L} \right)^{\frac{1}{3}} \left(\frac{\mu_f}{\mu_w} \right)^{0.14} \text{ for } Re \leq 2300 \quad (2.7)$$

Hausen's correlation for transitional regime:

$$Nu = 0.116 \left(Re^{\frac{2}{3}} - 125 \right) Pr^{\frac{1}{3}} \left(\frac{\mu_f}{\mu_w} \right)^{0.14} \left(1 + \left(\frac{D_h}{L} \right)^{\frac{2}{3}} \right) \text{ for } 2300 \leq Re \leq 10000 \quad (2.8)$$

Dittus-Boelter's correlation for fully developed flow under turbulent regime:

$$Nu = 0.023 Re^{0.8} Pr^{\frac{1}{3}} \text{ for } Re \geq 10000 \quad (2.9)$$

Where $\frac{\mu_f}{\mu_w}$ is the viscosity correction; μ_w as wall temperature; μ_f as dynamic viscosity of fluid (kg/m.s) measured from bulk fluid.

Wu and Cheng [74] experimentally investigated heat transfer and friction factor under laminar regime for silicon micro-channel with deionised water as working fluid using 13 different surface conditions. The micro-channels have trapezoidal and triangular cross-sections. Different geometric has a significant effect on the Nusselt number, Nu , and apparent friction coefficient (C). Furthermore, Nu and C increase with increasing surface roughness and surface hydrophilicity, and more significant increase at larger Re . Experimentally, linear increase between Nu and Re at very low value of Re ($10 < Re < 100$), at $Re > 100$ ($100 < Re < 1500$) the increase rate of Nu decreases. From experiments, Nu dimensionless correlation and C are represented in Eqs. (2.10 – 2.12) as follows:

At $10 < Re < 100$

$$Nu = C_1 Re^{0.946} Pr^{0.488} \left(1 - \frac{W_b}{W_t} \right)^{3.547} \left(\frac{W_t}{H_{ch}} \right)^{3.577} \left(\frac{\varepsilon}{D_h} \right)^{0.041} \left(\frac{D_h}{L} \right)^{1.369} \quad (2.10)$$

At $100 < Re < 1500$

$$Nu = C_2 Re^{0.148} Pr^{0.163} \left(1 - \frac{W_b}{W_t} \right)^{0.908} \left(\frac{W_t}{H_{ch}} \right)^{1.001} \left(\frac{\varepsilon}{D_h} \right)^{0.033} \left(\frac{D_h}{L} \right)^{0.798} \quad (2.11)$$

At $10 < Re < 1500$

$$C = f_{app} \cdot Re = C_3 Re^{0.089} Pr^{0.163} \left(1 - \frac{W_b}{W_t} \right)^{4.359} \left(\frac{W_t}{H_{ch}} \right)^{4.444} \left(\frac{\varepsilon}{D_h} \right)^{0.028} \left(\frac{D_h}{L} \right)^{1.023} \quad (2.12)$$

For silicon surfaces: $C_1 = 6.7$, $C_1 = 47.8$, $C_1 = 508.7$.

For thermal oxide surfaces: $C_1 = 6.6$, $C_1 = 54.4$, $C_1 = 540.5$.

W_t , W_b and H_{ch} are the top width, bottom width and depth of trapezoidal micro-channel respectively.

Kandlikar et al. [75] studied the effect of surface roughness on characteristics of heat transfer and pressure drop of stainless steel tubes with inner diameters of 0.62mm and 1.067mm using distilled water as working fluid. Acid solutions used to etch the tubes to achieve three different roughness for both tube diameters. For 0.62mm diameter the surface roughness varies between 0.00161 and 0.00355 while for 1.067mm diameter the surface roughness varies between 0.00178 and 0.00281. The value of Re ranges between 900 to 3000 for 0.62mm diameter while Re ranges between 500 to 2600 for 1.067mm diameter. Surface roughness effect on heat transfer and pressure drop was negligible for larger tube but not for the smaller one. At 0.00355 relative surface roughness, the highest pressure drop and heat transfer was recorded.

From the literature review on surface roughness effect on MCHS performance, channel with structured surface roughness enhance the rate of heat transfer due to flow modification and increase in heat transfer area but with higher pressure drop penalty which can affect the fluid effective viscosity in turbulent flow to increase due to additional frictional forces but negligible change in laminar flow.

2.6 Effect of geometric parameter on MCHS performance

Different studies have been made by researchers to study the effect of geometric modification on the thermal-hydraulic performance of MCHS as a cooling device.

2.6.1 Cross-section channel shape effect on MCHS performance

A remarkable effect of size and shape of channel cross section on the thermal and hydrodynamic performance of MCHS were reported over the years. For this subchapter, different channel shapes of rectangular, circular and triangular will be discussed to understand its effect on thermal performance using both experimental and numerical analysis.

Jiang et al. [76] used experimental method to investigate the behaviour of microfluid flow using different shapes of micro-channel cross section (circular, triangular, rectangular and trapezoidal). Micro-channels were etched using silicon material and glass wafer utilised for

bonding purpose. All shapes except circular bear the same range of height between 13.4 μm and 46 μm , width between 25 μm and 100 μm and length between 2.5mm and 10mm. For circular shape, the diameter is varied between the range between 0.008mm and 0.042mm with length varied between 1cm and 10cm. Friction factor for experimental data showed good agreement with theoretical equation.

Jiang et al. [77] performed another study on MCHS with trapezoidal and circular cross-section shapes using experimental method under the regime of laminar flow and water as coolant. The D_h is varied to be between 35 μm and 120 μm with Re ranges between $1 < Re < 30$. The coefficient of friction shows good agreement between experimental measurement and theoretical prediction. The Poiseuille number ($Po = f \cdot Re$) is affected by shape and sizes of the cross section as well as the length for trapezoidal case. The measurements of friction factor for experimental data has lower value compared with conventional theory due to micro-channel surface roughness.

Qu et al. [78] experimentally studied the characteristics of water flow through trapezoidal silicon micro-channels with heating source on three walls. The D_h ranges between 51 μm and 169 μm and a sidewall angle at 54.7°. The measured friction factor and pressure drop are compared to the conventional laminar flow theory predictions at steady state condition. The pressure gradient and flow friction in micro-channels from experimental data have larger value compared with numerical calculations due to the surface roughness effect of micro-channels. Qu et al. [79] continue the experiments on the study of heat transfer characteristics for the same test sections using trapezoidal micro-channels and water as coolant. The data collected from the experiments were compared with the numerical calculations from the solution obtained using conjugate heat transfer analysis. From the comparison, the Nusselt number, Nu , from experimental data was significantly lower than the numerical data due to the surface roughness effects on the walls of micro-channel. To interpret the experimental data a roughness-viscosity model was proposed for both experimental and numerical studies. From the results, experimental data and predictions showed good correlation for both Nu and friction factor when an adjusted water viscosity introduced taking account for micro-channels surface roughness.

Zhuo et al. [80] used the finite volume method (FVM) to analyse the 3-D laminar fluid flow and heat transfer characteristics of symmetric silicon micro-channels using trapezoidal and triangular cross-sections. Good agreement achieved from numerical results obtained with data from experimental analysis by Wu and Cheng [74]. An assumption made for constant thermal properties of the solid and water except the viscosity of water which is temperature dependent. Numerically, the results show that highest intensity of heat transfer between fluid and solid

recorded at the inlet of both triangular and trapezoidal cross-sections but decreases along the direction of flow. At fully developed region, the heat transfer coefficient for micro-channel with trapezoidal cross-section is higher than the triangular one which indicate the cross-sectional effect on thermal performance of micro-channel system.

Alfaryjat et al. [81] used three different cross-sectional profile shapes of hexagonal, circular and rhombus MCHS (refer Figure 2-3). With Reynolds number ranging between 100 and 1000 while having constant heat flux of 500kW/m^2 . From their findings, the rhombus profile shape MCHS has the lowest average temperature distribution and top wall temperature due to the larger rhombus cross-sectional area for higher heat transfer between solid and fluid domains. For the smallest channel, the hexagonal profile shape MCHS possesses the highest heat transfer coefficient and pressure drop compared to the other two profile shapes. Even though all of the profile shapes has the same hydraulic diameter, looking as hexagonal profile shape MCHS, the vertical distance between the top and bottom walls as well as horizontal distance between two vertexes are significantly smaller compared to the other two profile shapes, hence, the hexagonal profile shape lead to high velocity of fluid inducing higher convective heat transfer and friction factor. The rhombus profile shape MCHS has the lowest both thermal resistance and friction factor followed by hexagonal and circular profile shapes MCHS. To conclude, the hexagonal profile shape MCHS, has the highest heat transfer coefficient while having highest pressure drop while rhombus profile shape MCHS has the lowest thermal resistance, pressure drop, friction factor and temperature distribution.

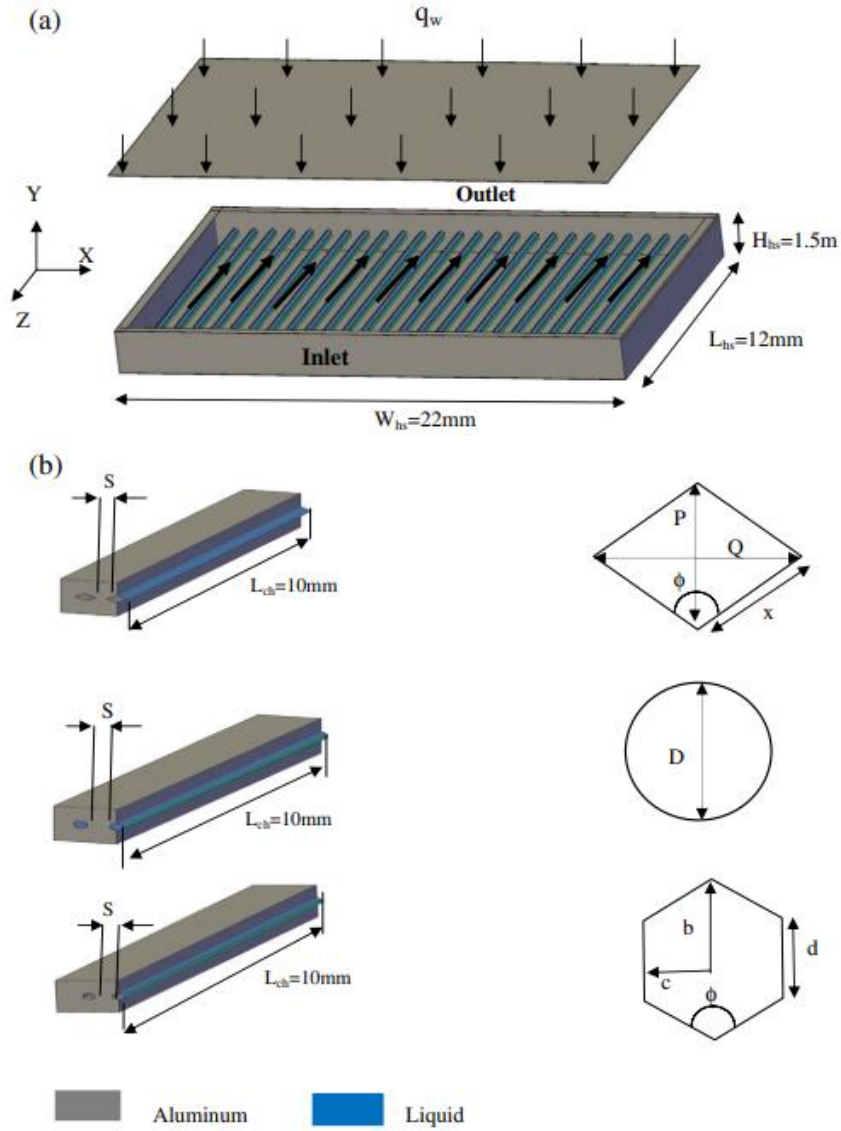


Figure 2-3: Shows the rhombus, circular and hexagonal cross-section profile shapes MCHS [81].

Gunnasegaran et al. [82] investigated the effect of changing the cross-sectional profile shapes of micro-channels on the thermal-hydraulic performance of heat sink. They have considered the profile shapes of rectangular, triangular and trapezoidal respectively (refer Figure 2-4). Different geometrical parameters of height of channel, H_c , width of channel, W_c , and D_h were tested. From their findings, at the smallest size of D_h the MCHS with rectangular shape profile possesses the highest coefficient of heat transfer value which is followed by trapezoidal and triangular shapes. The dimensionless parameters of width to height ratio (W_c/H_c) for rectangular profile, affects the Poiseuille number (multiplication of friction factor and Reynolds number) significantly where at $W_c/H_c = 0.974$, Poiseuille number is the highest and at $W_c/H_c = 0.391$, the Poiseuille number is the lowest. This is due the at higher W_c/H_c , the flow area decreases inducing higher

vortex effect, thus, led to higher friction factor. For the ratios of height-to-top (H/a), bottom-to-top (b/a) and length-to-hydraulic diameter (L/D_h), the Poiseuille number decreases when H/a and L/D_h increases, and b/a decreases. For the triangular shape, the increase of tip angle, β , will lead to the increase of Poiseuille number. They have also concluded that overall heat transfer augmentation can be achieved by using rectangular shape with trapezoidal and triangular shapes come second and third.

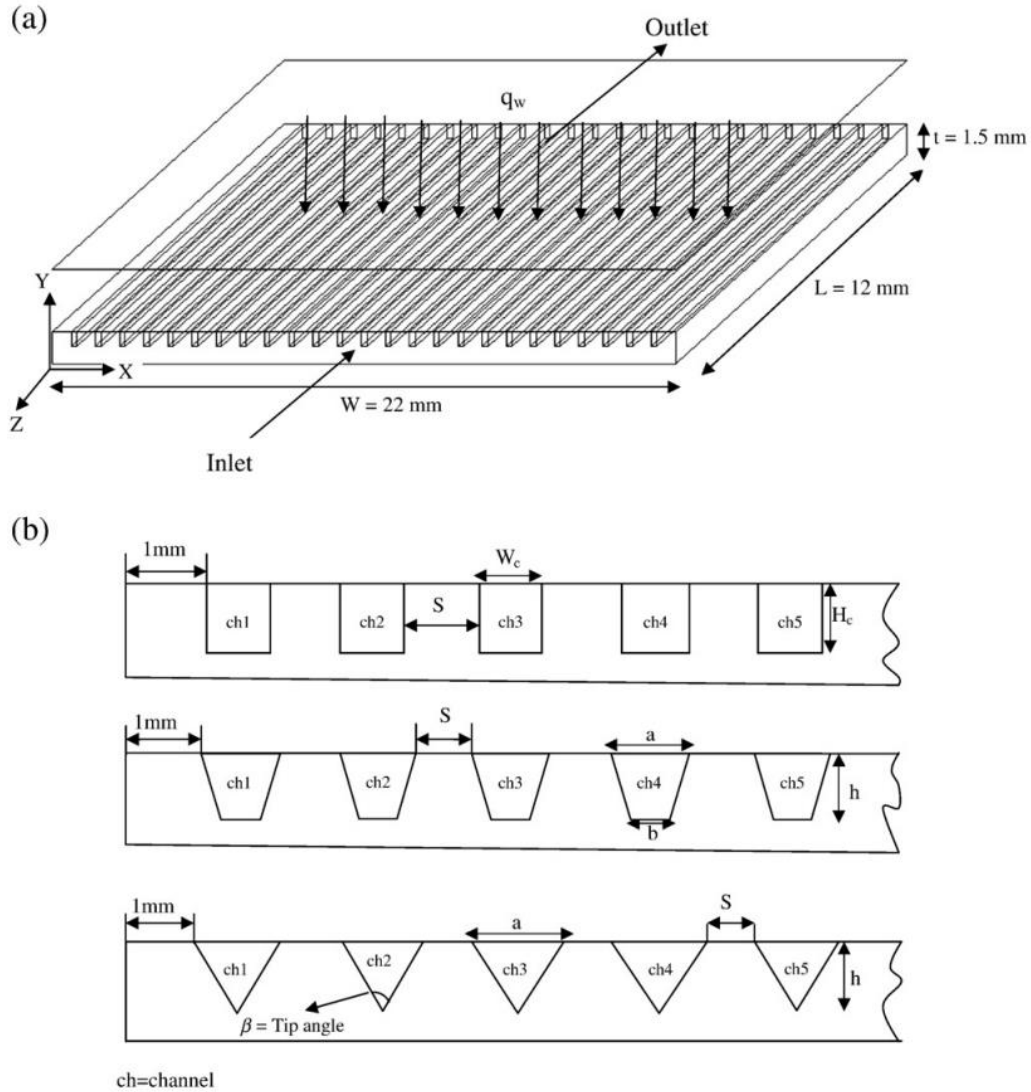


Figure 2-4: Shows the different micro-channel profile shapes of rectangular, trapezoidal and triangular tested [82].

Wang et al. [83] also investigated the rectangular, trapezoidal and triangular shaped micro-channel profiles. They have concluded that micro-channel cross-sectional profile shape have great effect onto the thermal-hydraulic performance of MCHS. From their findings, all profile shapes with high aspect ratio, long wetted perimeter and small D_h , will lead to low total thermal

resistance and large pressure drop. The rectangular profile has the lowest total thermal resistance while triangular is the highest and trapezoidal in between when they have the same porosity, channels number, area of cross-section and D_h . Even though, rectangular profile has the lowest thermal resistance but with expense of high pressure drop while the triangular profile has the lowest pressure drop.

From this sub-chapter, different cross-sectional profile shapes of channel have been tested by researchers to study its effect on the performance of MCHS while comparing the numerical with experimental results. Some of the parameters such as Nusselt number and friction factor have different values between experimental and numerical results due to surface roughness effect. In conclusion, the triangular cross-sectional profile has lowest pressure drop but highest thermal resistance. The cross-sectional profile shapes of rhombus and hexagonal shows promising in improving the heat transfer performance of heat sink and rectangular cross-sectional profile has the lowest thermal resistance value with high pressure drop, thus, for simplicity of this work the rectangular profile shape is used.

2.6.2 Aspect ratios effects on MCHS performance

Wang and Peng [84] performed experiments to study the heat transfer characteristics on six single-phase micro-channel heat sink made of plate of stainless steel under forced convective condition. The plate dimensions are 125mm long and 18mm wide. Each micro-channel has the same size of height (0.7mm) but varied widths. The length and thickness were 45mm and 2mm respectively. Different combinations of micro-channel characteristics were tested (Height (mm), Width (mm) and number of channels) (0.7, 0.8, 4), (0.7, 0.6, 4), (0.7, 0.4, 4), (0.7, 0.4, 6), (0.7, 0.2, 4) and (0.7, 0.2, 6). There were four or six micro-channels possessing the same geometry which are even distributed onto the plate. Deionised water and methanol were used as working fluid with velocity of 0.2 to 2.1 m/s and 0.2 to 1.5 m/s respectively. Working liquid temperature for deionised water ranges between 10°C and 35°C while methanol ranges between 14°C and 19°C. From the results, a considerable insight and significant data obtained gives an understanding on the behaviour of forced convective flow. The variation of heat transfer resulted in three different trends observed when using the micro-channels. For the first trend using combination of (0.7, 0.8, 4), coefficient of heat transfer has a smooth increase with temperature of wall. For the second trend of (0.7, 0.6, 4), (0.7, 0.4, 4) and (0.7, 0.4, 6), the coefficient of heat transfer has a steep increase at low temperature of wall while having moderate increase at high temperature of wall. For the third trend of (0.7, 0.2, 4) and (0.7, 0.2, 6), a decrease of coefficient of heat transfer observed first and then increase moderately when temperature of wall increased.

A conclusion made where the characteristics of heat transfer for laminar and transitional region are significantly more complex in comparison with conventional channel. The conclusion was supported with fluid thermo-physical properties change due to the large variation of the temperature of liquid along micro-channel length. The start of turbulent flow occurred at Re between 1000 and 1500 which is so much earlier for the flow of liquid in micro-channels which is commonly at Reynolds number of 4000 and beyond.

Peng et al. [85, 86] studied the heat transfer characteristics and friction flow of rectangular stainless steel micro-channels with water as working fluid using experimental analysis. The D_h is varied between 0.133mm and 0.367mm and aspect ratios (α) ranges between 0.33 and 1.0. As the flow reached fully developed and steady-state, measurement were taken for temperature of liquid, flowrates of liquid, pressures of inlet and outlet, surface temperatures of wall and substrate heat input. For the calculation of heat transfer coefficient (h) at micro-channel downstream, two thermocouples used to measure the temperature of fluid at micro-channel inlet ($T_{f,in}$) and three thermocouples used to measure the local temperature of wall (T_w) at downstream end. From experimental analysis, deviation of value observed for heat transfer and flow friction from experimental results with the classical correlations (refer Eqs. (5 – 11)). The laminar heat transfer occurred at $200 < Re < 700$, and fully turbulent convective heat transfer occurred at $400 < Re < 1500$. The size of D_h and α affect significantly on the fluid flow structure. At laminar flow regime, experimental Nusselt number, Nu , values has smaller magnitudes compared with the predicted values and dependent on Re , Nu is being to $Re^{0.62}$. At turbulent flow, the experimental Nu has higher value compared with the predicted one even though the trend captured by the correlation is correct. The values of h are affected significantly when the values of D_h and α are varied. The optimal value of α for laminar and turbulent region were 0.75 and 0.5 to 0.75 respectively. For laminar and turbulent flow, the friction factor is proportional to $Re^{-1.98}$ and $Re^{-1.72}$ respectively.

2.6.3 Inlet/Outlet arrangement effect on MCHS performance

The maldistribution of flow caused by headers design and improper channels has a major effect on performance of thermal and hydraulic parameters of MCHS. Flow of fluid entering on certain direction can exit on another, depending on the locations of inlet and outlet port. For example, fluid can enter from a direction of vertical inlet and exit from a lateral outlet or vice versa.

Lu and Wang [87] considered five inlet configurations of I-, Z-, J-, L-, and Γ - types (refer Figure 2-5) to study the effect of locations of inlet on the performance of hydrothermal of cold-plates

with multichannel. Dimensions of cold plates for all inlet configurations were the same with length, width and depth to be 66mm, 60mm, and 7.8mm respectively. The diameters for both inlet and out port were 5.8mm. Using J- type inlet, uniform flow distribution was achieved with smallest pressure drop and performance of heat transfer was moderate. On the other hand, I- and Γ - type inlets gives the largest magnitude of pressure drop and high performance of rate of heat transfer caused by impingement configuration. Maldistribution flow recorded using Z- and L- type inlets configurations inducing low performance of heat transfer. These observations featured by the recirculation of flow which induced the significant temperature difference along cold plate surface and also maldistribution.

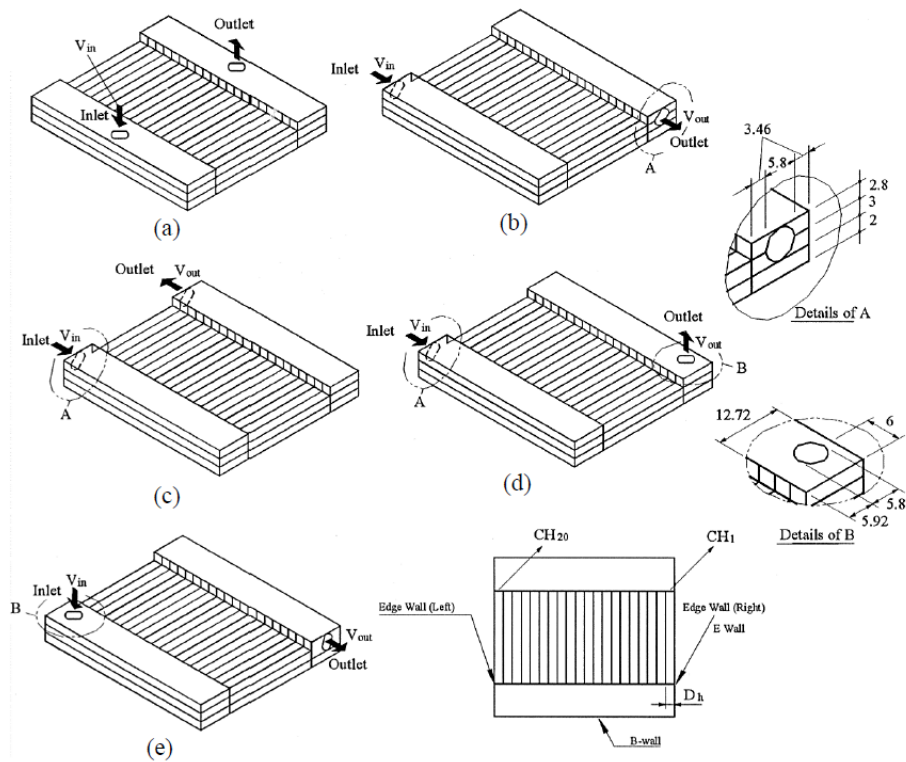


Figure 2-5: Shows the inlet and outlet configurations (a) I-type; (b) Z-type (c) J-type; (d) L-type; and (e) Γ -type [87].

Chein and Chen [88] performed 3-D numerical analysis on MCHS with different arrangements of inlet/outlet. Six different arrangements are considered to investigate the heat transfer and flow of fluid in MCHS using D, I, N, S, U and V-types (refer Figure 2-6). Fixed size of all six arrangements of MCHS with depth, length and width to be 0.5mm, 18mm and 6.2mm respectively. The channel dimensions were $W_{ch} = 200\mu m$, $H_{ch} = 400\mu m$, $L_{ch} = 10mm$. The horizontal arrangements of inlet/outlet (D, I, N and S-type), significant maldistribution observed leading to non-uniform temperature. This observation was caused by the recirculation at corners of combining and dividing manifolds. On the other hand, the V-type and U-type (vertical flow

inlet) configurations provides smaller maldistribution of flow caused by the jet impingement effect at surface of manifold suppressing flow recirculation appearance. To sum up, the heat sink with V-type configuration provides best performance of hydrothermal compared with other type of configurations. All heat sinks investigated, highest temperature located at the heat sink edge due to no heat dissipation by fluid convection.

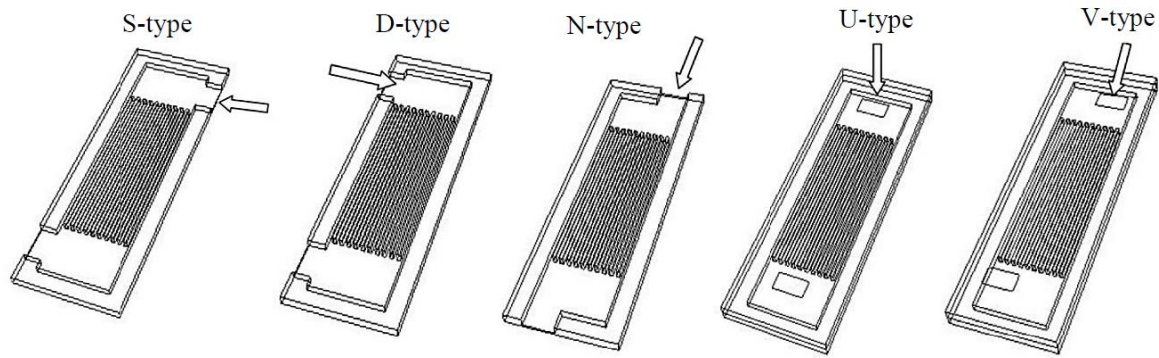


Figure 2-6: Shows the S, D, N, U and V-type configurations for straight rectangular MCHS [88].

Sehgal et al. [89] studied the different flow arrangements effect on the performance of hydrothermal of straight rectangular MCHS using experimental method. The flow arrangements utilised were S-type, U-type, and P-type (refer Figure 2-7). The arrangement of these inlets is similar to the ones in Figure 2-5 but with different dimension of inlet size, number of micro-channels used, height and width of single micro-channel, heating power leading to different fluid flow mechanisms as well as heat transfer and hydraulic characteristics. The magnitude of heat sources used were 125W, 225W and 375W applied at heat sink bottom with $224 < Re < 1121$. The U-type arrangement induces the highest rate of heat transfer compared with the other two configurations. This is contributed by the fact that U-type arrangement provides longest path of fluid flow giving the highest absorption of heat by fluid. Highest pressure drop recorded when using the S-type due to the direction of fluid changes in two bends. Using P-type arrangement provides the lowest pressure drop with acceptable performance of heat transfer rate, becoming the optimal condition for applications.

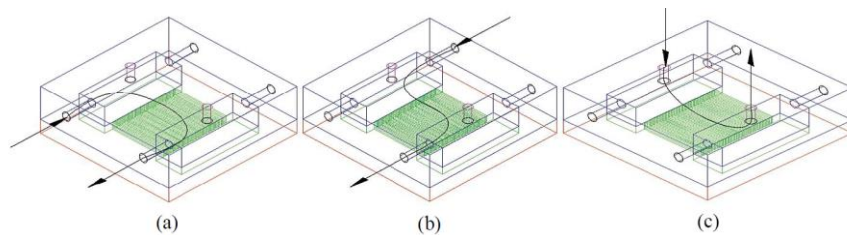


Figure 2-7: Shows the flow arrangements of straight rectangular MCHS with (a) U-type (b) S-type (c) P-type [89].

Vinodhan and Rajan [90] carried out numerical analysis on newly constructed MCHS with four different configurations comprising of four compartments where each compartment has distinct liquid inlet and outlet plenums (refer Figure 2-8) and its performance is compared to conventional MCHS. Under applied heat flux of 100 W/cm^2 with constant pumping power, the thermal resistances of the new design configurations recorded are lower compared with conventional MCHS. The substrate temperature gradient also appeared to be lower relative to conventional MCHS.

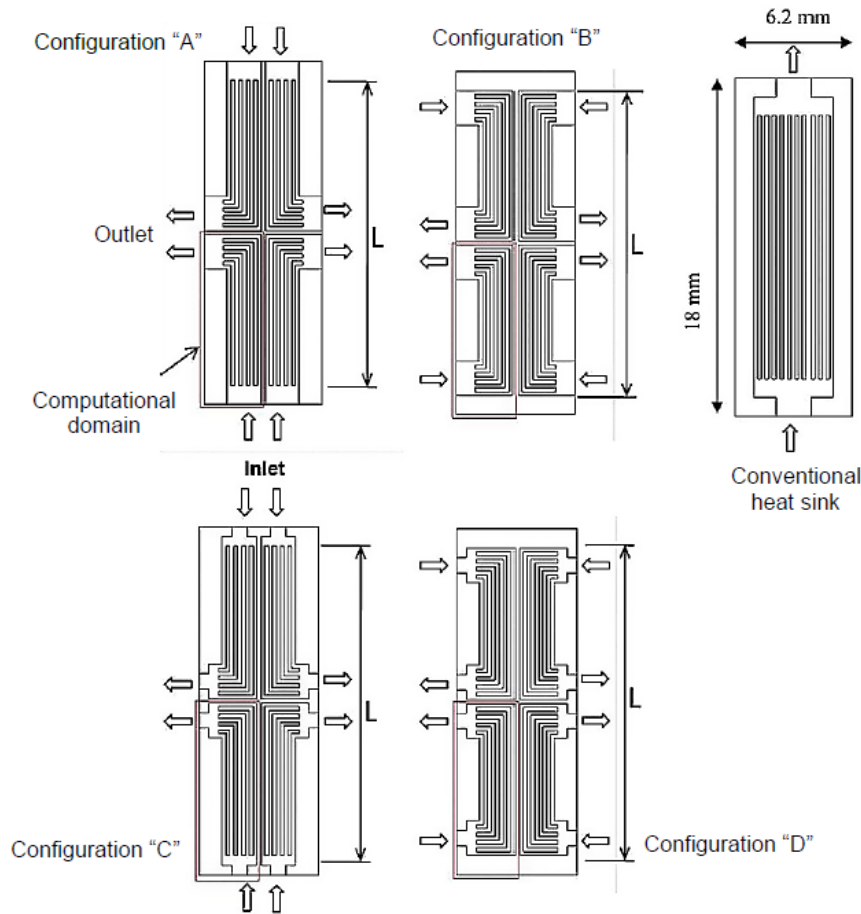


Figure 2-8: Shows the schematic diagram for different configurations of heat sink.

Anbumeenakshi et al. performed experiments to study the flow distribution over multiple micro-channels with rectangular and trapezoidal as header geometries under different flow rates at different inlet flow arrangements. From the experimental analysis, they have used 25 rectangular channels with dimensions of 100mm in length, 4.2mm in depth of channel and 0.42mm width of channel. From their CFD study, the results is in agreement with experimental study. They have found from experiments carried out where MCHS with rectangular header induce more flow maldistribution compared with trapezoidal header

From this sub-chapter, the flow maldistribution affects the MCHS performance greatly to deteriorate when inappropriate manifold designs are used. The U-type inlet arrangement from [89], V-type inlet arrangement from [88] and I-type inlet arrangement [87] have the highest thermal performance. The U-type arrangement is used in this work for both single and double inlet-outlet serpentine mini-channel heat sink for its high thermal performance in [89] and smallest pressure drop in [87].

2.7 Nanofluid effects on MCHS performance

Adding additives to working fluid has been one of the method to improve the heat transfer rate in microfluidic cooling devices. Thus, one possible solution is by improving the working fluids thermal conductivity. Nanofluids, i.e., nanoparticles are suspended into the liquid such as water ethylene glycol, oil and water, have been effective caused by improved thermophysical properties, enhancing the coefficient of convective heat transfer since has larger viscosities and thermal conductivities compared to conventional fluids. Nanofluid comprises of base fluid and nano-size particles (metallic or non-metallic). The standard nanoparticles used among by many researchers consisting of metals - Zn, Ag, Au, Fe, Al, and Cu; metal oxides – CuO, Al₂O₃, Fe₃O₄, Fe₂O₃, SiO₂, TiO₂, and ZrO₂; nitride ceramics – SiN and AlN; carbide ceramics – TiC and SiC; carbon based materials – graphite, carbon nanotubes and diamond [91]. The use of suspended nanoparticles in liquids has been a tremendously researched due to the proven stability even when nanoparticles with small volume fraction utilised [92].

Lee and Mudawar [93] performed an investigation on straight rectangular MCHS using two different Al₂O₃ nanoparticles concentration (1% and 2%) with water as base fluid on the effect to coefficient of heat transfer. The micro-channels (twenty-one channels) were etched on copper plate with dimensions of length and width to be 4.48cm and 1cm respectively. Micro-channels cross-sectional area was 821µm deep and 215µm wide. The *Re* were set to be between 140 and 941 with total power ranges between 100W and 300W. The inlet temperature was set to be 30°C with inlet and outlet pressure as 1.17-1.36bar and 1.12bar respectively. The results consisting of analysis on both single- and two-phase convective heat transfer.

From results, at the entrance region the heat transfer coefficient was enhanced due to the high thermal conductivity of nanoparticles, while at downstream only small effect of nanoparticles on to the development of thermal boundary layer. Using 2% concentration of Al₂O₃ in water improve the heat transfer by 13%. For the same *Re*, the increase in concentration of nanoparticles would induced the increase in single-phase pressure drop compared to only using base fluid (water). No clear influence recorded on the effect of increasing nanoparticles concentration to

the friction factor. It was noticed that heat flux increment has no significant effect on to the coefficient of heat transfer for pure water but significant effect observed when using nanofluid and higher concentration of Al_2O_3 would give a greater effect. For single-phase turbulent flow regime, weak heat transfer improvement when using nanofluids compared with laminar flow. The coefficient of heat transfer has weaker dependence on thermal conductivity at high fluid velocity and specific heat is decreased. Pressure drop is increase when using nano-fluid due to higher pumping power to push heavier liquid.

For two-phase cooling, the use of nanoparticles in MCHS is not recommended. Even though these particles are very small, it can cause a catastrophic failure for the cooling system when boiling reached when nanoparticles are deposited in large clusters near exit of channel because of the localized evaporation. Nanoparticles will prevent coolant from entering the heat sink as they will fill the entire space of micro-channels.

Jung et al. [92] investigated the effect of Al_2O_3 nanoparticles (170nm diameter) suspension on the friction factor and coefficient of heat transfer in straight rectangular micro-channels. Different volume of fraction of particles were tested to study the effect of nanoparticles volume fraction (ϕ) on the convective heat transfer and fluid flow structure. At 1.8% of Al_2O_3 volume fraction within the base fluid (water), under laminar flow regime, the measured convective coefficient of heat transfer improved by 32% compared to only using distilled water, with tolerable rise of friction loss. Considering nanoparticles volume fraction, the Nusselt number, Nu , measurements for different nanofluids under the regime of laminar flow was correlated:

$$Nu = 0.014\phi^{0.095}Re^{0.4}Pr^{0.6} \quad (2.13)$$

Mohammed et al. [94] numerically investigated the laminar nanofluids flow and characteristics of heat transfer using MCHS with triangular shape cross-section and aluminium as substrate. Water is used as base fluid with suspension of different nanofluids particles such as Ag, Al_2O_3 , CuO, diamond, TiO and SiO_2 at 2% of volume fraction. From results, it is preferred to have dispersion of diamond nanoparticles attaining the overall enhancement of heat transfer. Furthermore, Ag nanoparticles dispersion in water provides low pressure drop and low wall shear stress in comparison with pure water.

Even though, the use of nanofluid particles are very efficient in improving the performance of heat transfer by increasing the thermal conductivity but the pressure drop induced is very high compared to the use conventional liquid such as water. There are still challenges where particle sedimentation, erosion, fouling and channel clogging occurs.

2.8 Micro Pin-Fins and Offset-Strip Fins effect on MCHS performance

The application of micro-scale protruded out of surface pin-fin has been significant due to its effectiveness in removing heat. Different cross-section shapes of fins such as circles, squares, rhombus, elliptical, eyeshaped, and sine-shaped are utilised. High thermal conductivity materials such as aluminium, silicon and copper used as substrate. This type of heat sinks provide larger surface area for heat transfer and flow interruption for better thermal performance due to fluid mixing. For heat transfer improvement, micro pin-fins can be represented by different sizes and shapes as well as different patterns of fins (staggered and aligned).

Peles et al. [95] considered pin-fin micro heat sink studying its pressure drop and convective heat transfer using experimental analysis and compare the thermal resistance with MCHS (refer Figure 2-9). From results, the thermo-hydraulic performance for pin-fin (cylindrical shape) heat sink is greater than MCHS due to high heat fluxes removed without having large rise in wall temperature across heat sink. For fin diameters larger than $50\mu\text{m}$, less sensitive thermal resistance change towards fin diameter variation and short pins need to be used for increased efficiency.

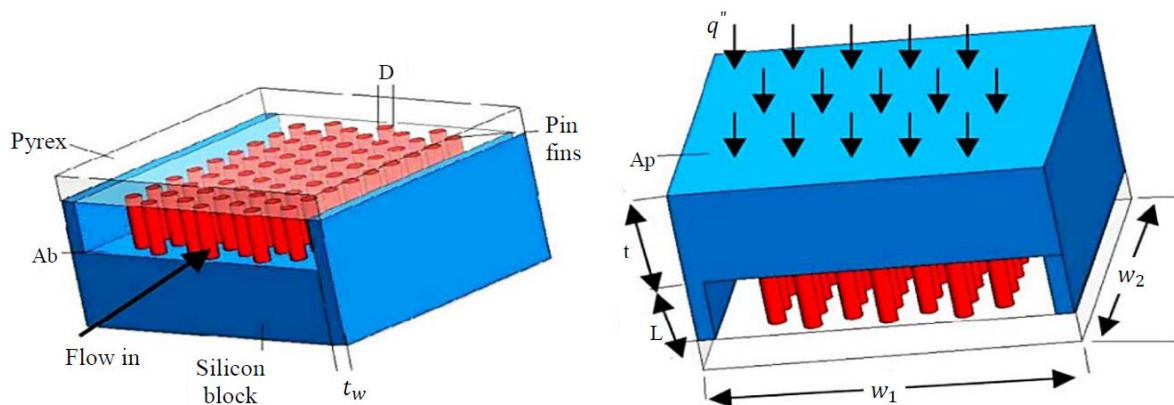


Figure 2-9: Shows the 3-D schematic view of micro pin-fin heat sink [95].

Colgan et al. [96] bonded single-phase flow offset strip fin silicon micro-channel cooler on top of high power density chip with heat flux value of $300\text{W}/\text{cm}^2$ (refer Figure 2-10). From their analysis, offset strip fins provides several time higher heat transfer enhancement compared to plain micro-channel, depending on the length of fin, but at higher pressure drop penalty. To maintain reasonable pressure drop, the length of flow was kept short using several entry and exit ports.

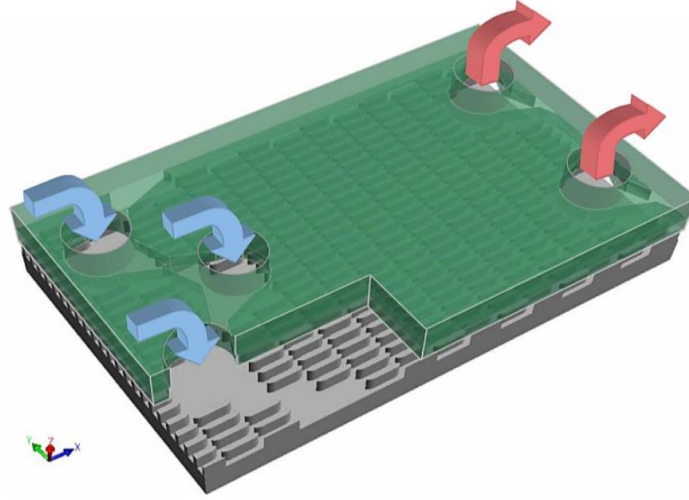


Figure 2-10: Shows the rendered 3-D micro-channel cooler assembly [96].

Siu-Ho et al. [97] and Qu and Siu-Ho [98, 99] experimentally investigated the heat transfer and pressure drop characteristics of single-phase flow micro pin-fin heat sinks. Their work comprises of an array of 1950 staggered square micro pin-fins, which has individual cross-sectional area of $0.2 \times 0.2 \text{ mm}^2$ and height of 0.67 mm . The work in [97] considered the range of Re to be between 93 and 634 while in [98, 99] considered Re range between 46 and 180 at two temperatures of 30°C and 60°C . From the results of the experiment, Qu and Siu-Ho come out with more accurate heat transfer correlations at low Re ($Re < 1000$) as follows:

$$Nu = 0.0285 Re_{avg}^{0.932} Pr_{f,avg}^{1/3} \quad (2.14)$$

$$Nu = 0.0241 Re_{avg}^{0.953} Pr_{f,avg}^{0.36} \left(\frac{Pr_{f,avg}}{Pr_{w,avg}} \right)^{0.25} \quad (2.15)$$

Where Re_{avg} is average Re . Water Pr at average water bulk temperature and average micro pin-fin temperature are $Pr_{f,avg}$ and $Pr_{w,avg}$ respectively. $\frac{Pr_{f,avg}}{Pr_{w,avg}}$ term is called property correction factor taking account varied property of fluid effect on heat transfer. The local Nusselt number, Nu , prediction from the two correlation has mean absolute error (MAE) of 13.7% and 13.0% respectively. The newly proposed correlation for friction factor of micro pin-fin array (f_{fin}) provides good agreement with 6.9% of MAE which is as follows:

$$f_{fin} = 20.09 Re^{-0.547} \quad (18)$$

Nguyen et al. [100] studied the behaviour and heat transfer improvement of nanofluid using Al_2O_3 nanoparticles in distilled water on copper pin-fin heat sink using experimental analysis. Two different particle diameters (36nm and 47nm) and varied concentration of nanoparticles volume were considered in the experiment under single-phase turbulent flow regime. From

experiments, the increase of particle volume concentration resulted in decrease of the heated block average temperature. At mass flow rates of 0.06kg/s, average temperature of heated block has values of 40.9°C, 39.3°C, 38.7°C and 37.3°C respectively for nanofluids with water at particle concentration of 1%, 3.1% and 6.8% respectively. Coefficient of heat transfer increased by 40% using 6.8% of particle volume concentration compared when using distilled water. Particle size influence on the heat transfer behaviour also investigated only by using 6.8% of particle volume concentration. More rate of heat transfer achieved when smaller particle size is utilised. Using 36nm particle diameter gives higher convective heat transfer coefficients compared with 47nm diameter.

Rubio Jimenez et al. [101] considered online and offset micro pin-fin heat sinks in variable configurations of fin density using numerical analysis for cooling IC with area of 10 x 10mm². For individual configuration, it is split into three zones of SI, SII, and SIII with varied densities (refer Figure 2-11). Water was used as working fluid with single-phase under laminar flow for flow condition which is non-developed. Fins number of 4748 was used with radius of 25µm on silicon substrate with thickness of 200 and divided into three different sections along the direction of flow. The sizes of 20, 100 and 200µm were used for width, length and height respectively. Formation of fins on the MCHS with 33 rectangular channels, 1.0 aspect ratio and 100µm for space between channels.

Thermal resistance as low as 0.1K/W achieved at 0.45W pumping power when offset micro pin-fin is utilised. More uniform temperature profile accomplished using offset micro pin-fin configuration compared with the online fin arrangement. This is due to the fact that large part of fluid passing through straight gaps between fins which has less fluid disturbance breaking the boundary layer. Fluid interaction between water and fins walls is reduced greatly, affecting the diffusion of heat. However, it is not recommended to use offset micro pin-fin heat sink as even larger pressure drop is induced to cool down system to get thermal resistance lower than 0.1K/W. This will lead to inefficiency of the overall thermal-hydraulic performance.

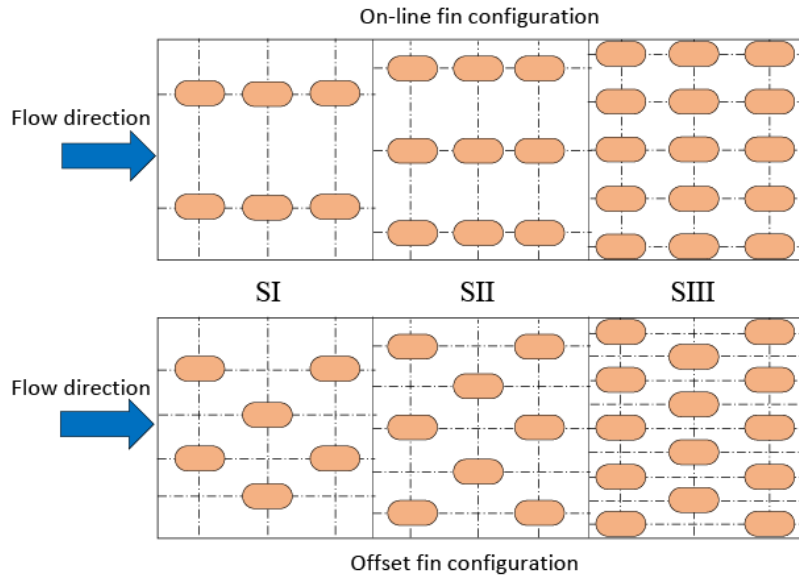


Figure 2-11: Shows the configurations of online and offset micro pin-fin heat sink [101].

Yu et al. [102] investigated the use of Piranha Pin Fins (PPFs) within micro-channel under single-phase fluid flow (refer Figure 2-12). The Reynolds number ranges between 508 to 2114 depending on the hydraulic diameter of channel. Using CFD analysis the conjugate heat transfer model was developed where the correlation between experimental results is achieved. The Nusselt number and friction factor calculated from PPF heat sink were compared conventional straight pin-fin channels available correlations. From their findings, PPFs heat sink possesses higher friction factor compared to plain channel. However, the friction factor obtained is still so much lower when compared with friction factor obtained by Kosar et al. [103], Chilito and Generaux [104], Metzger et al. [105], and Jacob [106]. The of PPFs improves heat transfer due to the heat spreads on a larger area mixing with cold region of fluid. To add, the PPFs extrusion breaks the velocity field inducing the separations and mixing for better rate of heat transfer.

Yang et al. investigated five different staggered pin-fin configurations using both experimental and numerical methods. The configurations include circle, hexagon, pentagon, square and triangle geometries respectively. On the top surface of MCHS, uniform heat flux was applied while the coolant used was deionised water. From the results, the staggered hexagonal pin-fins heat sink shows the lowest thermal resistance while having the most temperature uniformity on the top surface of the chip. On the other hand, the staggered circular pin-fins heat sink shows the lowest pressure drop and while having flow that is continuous and stable. From the staggered triangular pin-fins heat sink, the highest blocking effect occurs when coolant flows due to the

existence of adjacent pin-fins which contributes to the narrowest channel and back side area that is very large inducing pressure drop increase.

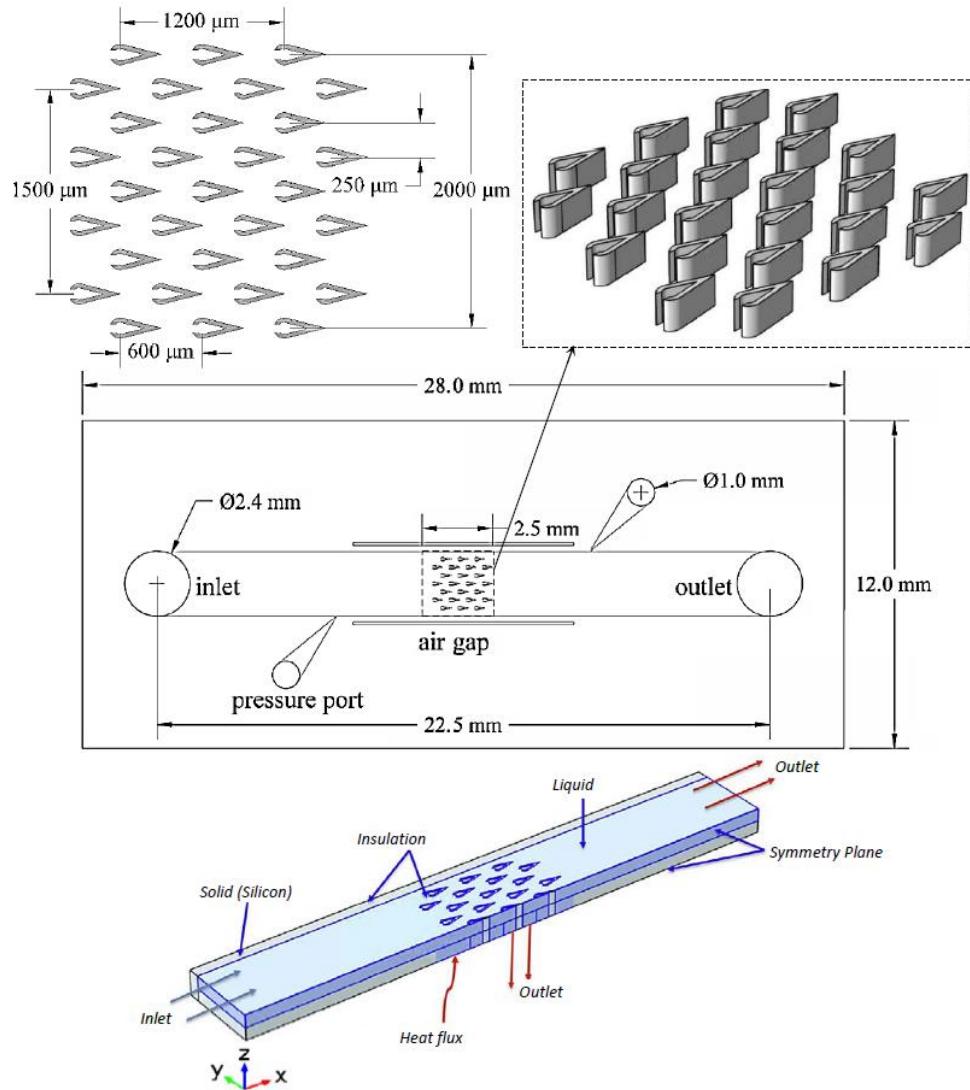


Figure 2-12: Shows the Piranha Pin Fins (PPFs) heat sink design [102].

From this sub-chapter, it can be seen that pin-fins heat sink has the ability to transfer heat to coolant effectively due to the large surface area and continuous breaking boundary layers improving heat transfer. Within this work, the secondary channels are used within serpentine mini-channel heat sink which technically converts the serpentine design into finned designs which has been shown briefly in Chapter 1 which can be referred from Figure 1-4 to Figure 1-7 respectively. The secondary channels help in breaking boundary layers continuously and increase the area for heat transfer to occur.

2.9 Method of heat augmentation to improve MCHS performance

The use of MCHS with conventional straight rectangular has reached its limit for electronics cooling due to the increase in surface temperature along direction of flow, where efficiency of MCHS is limited. For convective heat transfer enhancement, flow disruption offers a good solution to improve convective heat transfer with more homogeneous distribution of temperature along direction of flow due to hydrodynamic and thermal boundary layers are reinitialised.

The augmentation techniques are used to achieve the following objectives of: (1) prime surface area reduction (2) approach temperature reduction for process streams (3) pumping power consumption reduction (4) heat transfer capacity amplification. In general, the mechanisms of heat transfer augmentation can be categorised into three major division: (i) active method (ii) passive method and (iii) hybrid method [107].

Active method requires external power input to maintain enhancement of heat transfer mechanisms. Passive method does not utilise external power sustaining the characteristics of thermal enhancement and this method usually performed by increasing the heat transfer surface area. Hybrid method utilises both active and passive methods. In several applications, external power supply is not easy to implement, thus not much potential owing to design complexity. The applications of active method comprises fluid stirring, surface vibration, seeded light particles disturbance by magnetic field, electrostatic fields, jet impingement, cam induced pulsation and reciprocating plungers, etc [108]. The applications of passive method include rough surfaces, smooth surfaces, extended surfaces, additives for fluids, displaced enhancement devices, coiled tubes, etc [109]. For hybrid method, complex design is required which reduces the number of applications.

Passive methods, inserts are utilised within flow passage increasing the rate of heat transfer which is beneficial and received more attention compared to active methods due to the absence of power requirement aiding the enhancement. Passive method system is also very simple to be manufactured [110] and easily integrated with existing MCHS. Thus, passive augmentation techniques review are presented for this section.

2.9.1 Corrugated channel effect on MCHS performance

One of the method to enhance the performance of heat transfer is using corrugated channels such as zig-zag, wavy, serpentine and convergent-divergent. Utilising this method, flow mixing between cold fluid at core region and hot fluid near wall of channel is improved [23]. The operational principles are explained in this subchapter.

Tsuzuki et al. [111] optimised the wavy, zigzag and S-shaped fins geometry using CFD method. From the study, angle of fin, guiding wing, length, thickness and roundness were varied. The performance of thermal and hydraulic are measure using heat transfer and pressure drop calculation. From results, the angle of fin parameter showed the most influence on the micro-channel heat exchanger thermal performance.

Sui et al. [112] numerically studied the laminar water flow and heat transfer characteristics of 3-D wavy micro-channels with cross-section of rectangular shape under constant wall temperature and conjugate conditions and constant wall heat flux (Refer Figure 2-13). From the figure, the conjugate domain is described as different sub-domains with different thermal properties which coupled together suing heat transfer equations to describe the thermal energy transfer among them. From the study, general-purpose finite-volume based CFD was used to solve the Navier-Stokes equations. Dynamical system technique (Poincaré section - using plane or surface where fluid trajectory intersects periodically) was utilised to analyse the fluid mixing. From results, during the flow of liquid coolant through the wavy channel the secondary flow may change considerably along the direction of flow channel. This will lead to chaotic advection based on Poincaré sections technique, which will significantly improve the convective fluid mixing, therefore, higher performance of heat transfer with greatly lowered pressure drop penalty compared with convectional straight micro-channels achieved.

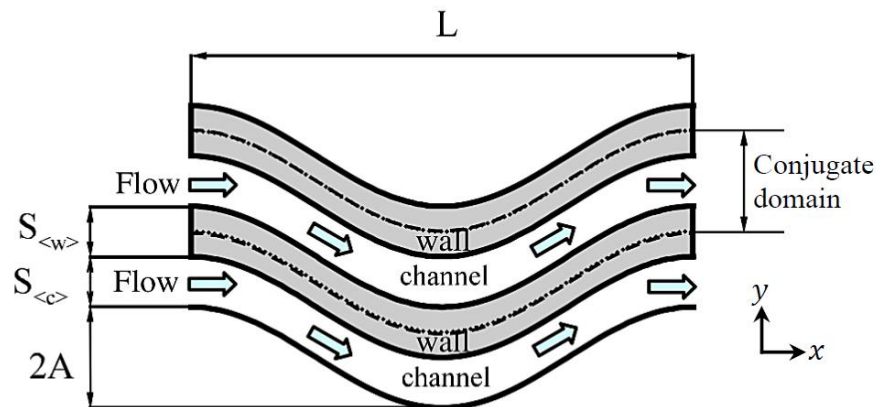


Figure 2-13: Shows the wavy MCHS cross-sectional view [112].

Sui et al. [113] continued their investigation using experimental method to study the friction flow and heat transfer of three wavy micro-channels with rectangular cross-section using deionised water as working fluid. The test pieces has dimensions of width and depth of $205\mu\text{m}$ and $404\mu\text{m}$ respectively and varied wavelengths of $0\mu\text{m}$, $138\mu\text{m}$ and $259\mu\text{m}$. The test pieces are made from copper with 60 – 62 wavy micro-channels arranged in parallel. The Re used ranges between 300 and 800. Using wavy micro-channels had superior heat transfer performance

compared to straight micro-channels. Experimental analysis were also carried out for validation purpose and good agreement achieved between experimental data and 3-D conjugate simulations results.

Mohammed et al. [114] used FVM method to solve the numerical analysis of 3-D steady-state and conjugate heat transfer governing equations. Different channel shapes used (zigzag, curvy and step micro-channels) to compare the performance of such channel with straight and wavy channels (Refer Figure 2-14). To analyse the performance of MCHS, the parameters of temperature profile, pressure drop, heat transfer coefficient, friction factor and wall shear stress were used. From results, the pressure drop penalty of tested channel compared to conventional straight MCHS is higher.

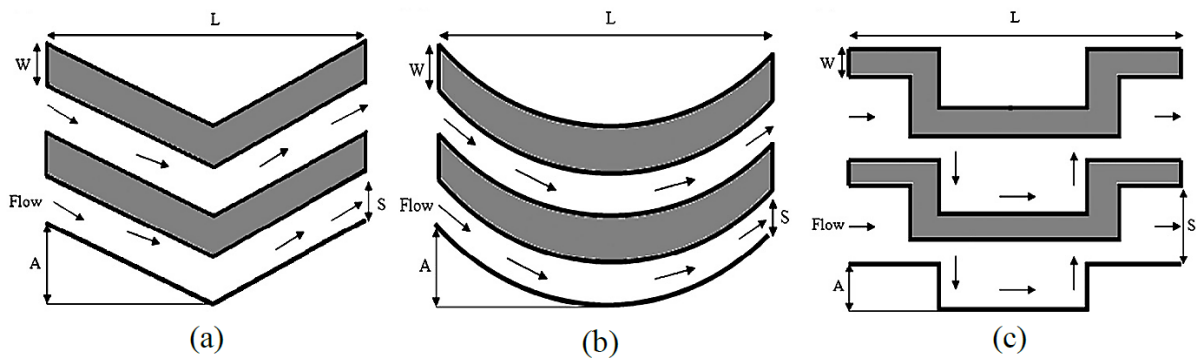


Figure 2-14: Shows the heat sink schematic view (a) zigzag channels (b) curvy channels (c) step channels [114].

Yong and Teo [115] used numerical (CFD), theoretical and experimental study on heat transfer of converging-diverging channels with rectangular cross-sections under fully developed flow. They used 15 to 20 converging-diverging passages, with 15 to 20 converging-diverging repeating units of each passage. Constant wavelength used at 1.5mm (refer Figure 2-15). The simulations of heat transfer performed under constant wall temperature condition with water as coolant. Different channel aspect ratios used to study its effect on recirculating vortices transformation and flow behaviour. From results, optimal aspect ratio of channel is between 0.5 to 1.0.

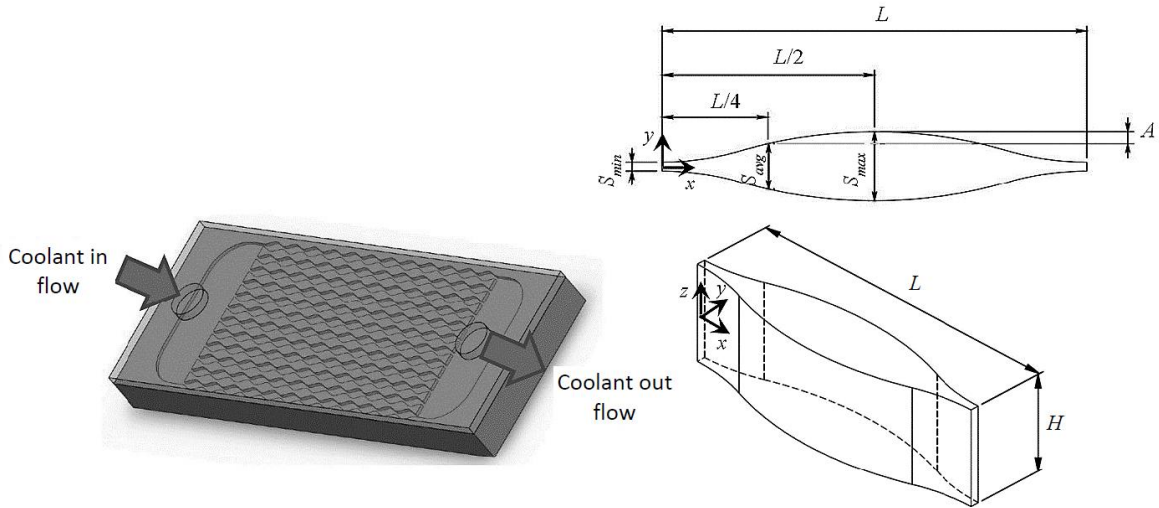


Figure 2-15: Shows the cross-section and 3-D view of MCHS with flow passages of converging-diverging type [115].

Also, they investigated the enhancement of heat transfer and pressure penalty of converging-diverging channels with varied amplitude ratios under constant sinusoidal profiles and curvature. They showed that the improvement of thermal-hydraulic performance by 60% compared to conventional straight micro-channels. Formation of a pair of symmetrical recirculating vortices at both lower and upper furrows of channel (refer Figure 2-16). These recirculating vortices enhances the mixing of fluid, thus, improves the rate of heat transfer to reduce the temperature further which is lower than conventional straight channel heat sink.

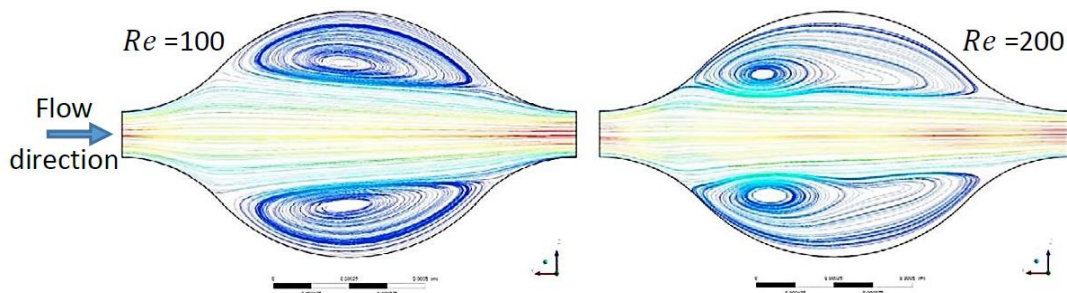


Figure 2-16: Shows the streamline plots at Re of 100 and 200 [115].

2.9.2 Flow disruption effect on MCHS performance

The disruption of development of hydrodynamic and thermal boundary layers achieved by introducing flow disturbance techniques. The disrupted flow creates instabilities inducing the augmentation of flow mixing and rate of heat transfer. Many appropriate geometrical modifications performed to side walls of conventional micro-channels to promote mixing of fluid and flow instabilities. Periodic disturbance achieved by using interrupted-wall channel, offset strip fins, re-entrant cavities, ribs, grooves and hybrid of both ribs and grooves along the

direction of flow which are proven to be effective to disrupt the flow. From the techniques listed, creation of secondary flow is possible and boundary layer growth disruption, improved leading edge flow mixing, thus, an increased rate of heat transfer is achieved.

Wirtz et al. [116] and Greiner et al. [117] investigated the effect of V-grooved micro-channels on the pressure drop and heat transfer. From results, the Nusselt number, Nu , and pressure drop of V-grooved channel were higher compared to flat channels. Steinke and Kandlikar [118, 119], Kandlikar and Grande [120] suggested different techniques promoting heat transfer in MCHS comprising heat transfer area increment and mixing flow improvement utilising staggered and interrupted strip fin design. From among the methods, work in [118] covers two techniques for secondary flow generation in MCHS. From Figure 2-17 (a), the design proposed includes cross mixing from neighbouring liquid streams. From their design, secondary channels were introduced which promote the secondary flow between main channels and this application can be used in different arrangements of plates and fins. From Figure 2-17 (b), Venturi effect is utilised where fluid pressure is reduced when flowing through constricted section which in this case is the secondary flow passage. From these two methods, increasing fluid mixing improves the convective heat transfer without having large pressure drop.

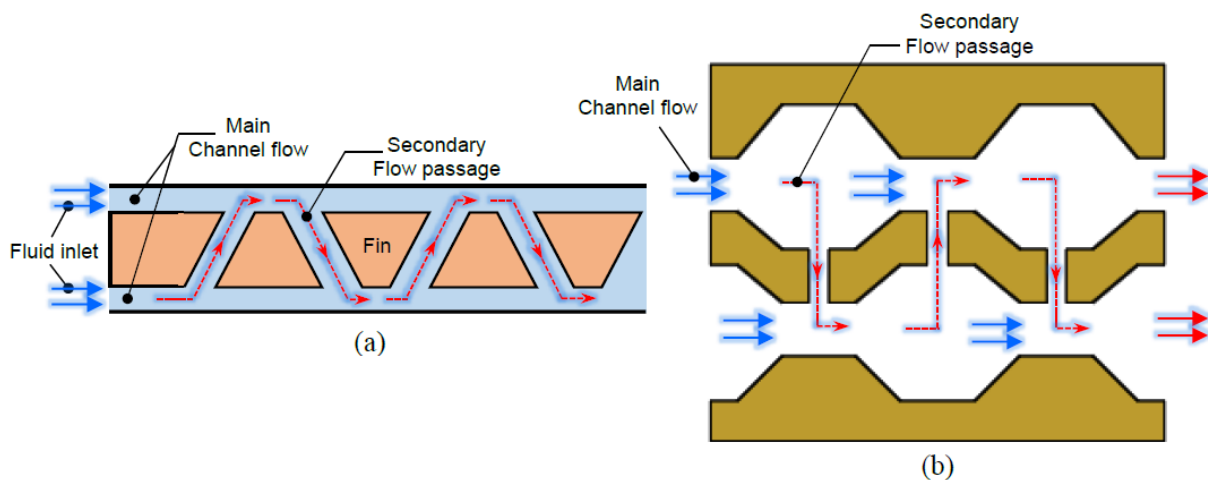


Figure 2-17: Shows the MCHS schematic view of (a) secondary flow channels (b) Venturi based secondary flow [118].

The advancement of micro-channel structures include double-layered micro-channels [121], stacked micro-channels [122], tree-shaped micro-channel networks [123, 124], offset strip-fin micro-channels [125], Y-shaped bifurcation plate with micro-channel [126] and micro pin-fins [95, 97, 127, 128] have been proposed enhancing the uniformity of temperature ensuring temperature is well distributed over the entire domain to avoid local over-heating and pressure drop reduction.

Within this sub-Chapter 2.91 and Chapter 2.92, the effect of using corrugated channel and application of secondary channels have been discussed. These kind of designs will contribute to the separating and reattachment of fluid breaking the boundary layers for higher heat transfer. The use of secondary channel is very useful in diverting the flow from the main channel through the secondary channels which not only improving heat transfer but also reducing pressure drop by lowering down the fluid velocity within channel.

2.9.2.1 Interrupted-wall channel

Xu et al. [129] used new silicon MCHS and compared its performance with conventional heat sink based on the distribution of temperature and characteristics of heat transfer using experimental analysis. The novel work utilised straight MCHS with 10 triangular micro-channels along the direction of flow and 5 transverse trapezoid microchambers uses to divide the straight micro-channels into six different zones (refer Figure 2-18). Transverse micro-channel is used to disrupt the layer of thermal boundary inducing the coefficient of heat transfer enhancement. The pressure drop reduced by 27% when using interrupted channel compared to conventional MCHS design. Low pressure drop was induced by shortened flow length and very low water mass flow rate. Different parameters of inlet temperature, inlet pressure, mass flow rate, heat flux and pressure drop were varied which were 30 – 70°C, 1 – 2bar, 534.8 – 4132.85kg/m², 10 – 100W/cm² and 10 – 100kPa respectively.

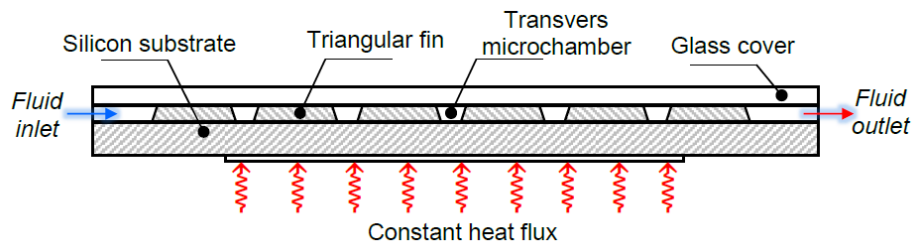


Figure 2-18: Shows the MCHS cross section view [129].

Xu et al. [130] numerically analysed the conventional and interrupted channel MCHS studying the conjugate heat transfer continuing their work in [129]. From analysis, mass flux and received heat predictions were larger by 0.2% and 2% respectively in central channels compared to edge channels. Redeveloping of hydraulic and thermal boundary layers at each separated zone entrance, hence, rate of heat transfer increased in MCHS. Smaller pressure drop in interrupted MCHS compared to conventional heat sink. Good correlation achieved between numerical analysis results and experimental data.

Lee et al. [131, 132] used the same technique as in [118], but the conventional straight fin micro-channels are split into oblique sections (refer Figure 2-19), with one direction secondary flow. The results for new arrangement of MCHS are compared with straight conventional micro-channel and there is an increase by 1.6 of the enhancement factor at $Re = 300$ with slight pressure drop penalty. The enhancement factor is define as the ratio of Nu_{avg} of oblique fins to Nu_{avg} of conventional straight channels. From results, the boundary layer in the main channels was reduced by oblique sections, triggering the re-development of hydrodynamic and thermal boundary layers at each oblique fin's leading edge. Further mixing occurred when small fraction of flow is diverted by oblique channels to adjacent main channels, thus, heat transfer was improved.

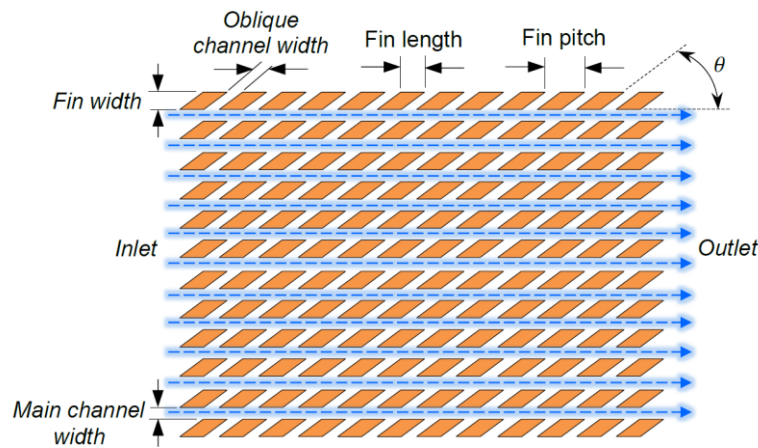


Figure 2-19: Shows the oblique fins cross-section [131, 132].

Lee et al. [133] used sectional oblique fins within MCHS to improve heat transfer. Two variables were utilised for design optimisation which are the fin pitch and oblique angle. From the results, smaller of both fin pitch and oblique angle will induce enhanced rate of heat transfer.

Khan et al. [134, 135] investigated the plate heat exchangers for asymmetric and symmetric chevron angle plates which mixed together with Reynolds number between 500 and 2500. From the results of experimental analysis, the chevron angle and Reynolds number has significant effects on the friction factor and coefficient of heat transfer. They have also used these results to estimate correlation of friction factor and Nusselt number.

From this sub-chapter, the use of fins are very useful to induce secondary flow and breaking both hydrodynamic and thermal boundary layer for improved heat transfer which is used in this work.

2.9.2.2 Cavities in MCHS

Chai et al. [136] performed numerical and experimental analysis on the characteristics of heat transfer and pressure drop of MCHS with fan-shaped and triangular re-entrant cavities. The cavities are placed periodically with expansion-constriction cross-sections (refer Figure 2-20). Ten parallel micro-channels used on MCHS with individual dimensions of depth and width of 0.2mm and 0.1mm respectively. The Nu_{avg} of periodic expansion-constriction cross-sections has 1.8 times higher compared to straight rectangular micro-channel and acceptable pressure drop achieved. This was due to the boundary layer formation interruption and larger heat transfer surface area.

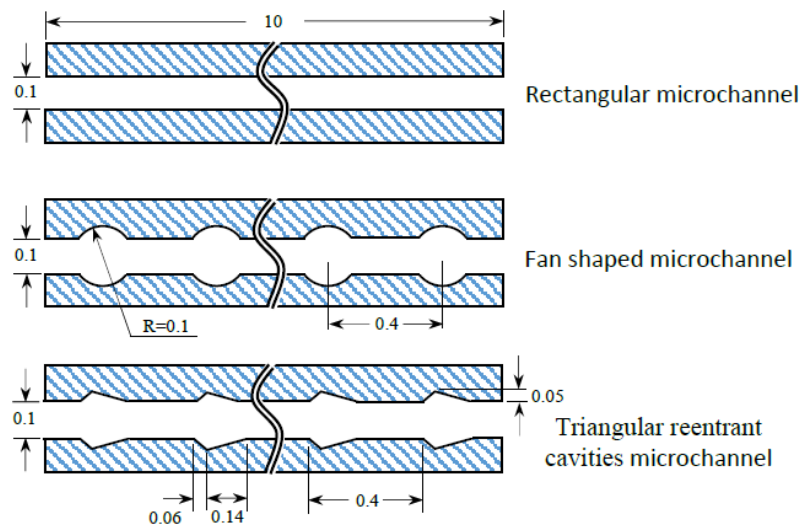


Figure 2-20: Shows the rectangular, fan shaped and triangular reentrant cavities micro-channels [136]. The dimensions are in mm.

For the periodic expansion-constriction MCHS, low pressured drop recorded at Re less than 300, but surged rapidly at Re more than 300 ($300 < Re < 750$). For component of thermal resistances, convective and capacitive, showed descending trend when volumetric flow rate (Q_{in}) is increased for all heat sinks. On the other hand, fixed conductive thermal resistance recorded at all value of Q_{in} (refer Figure 2-21). To add, new design of heat sinks possessed lower convective thermal resistance compared with the conventional design due to the coefficient of heat transfer enhancement when Q_{in} is increased.

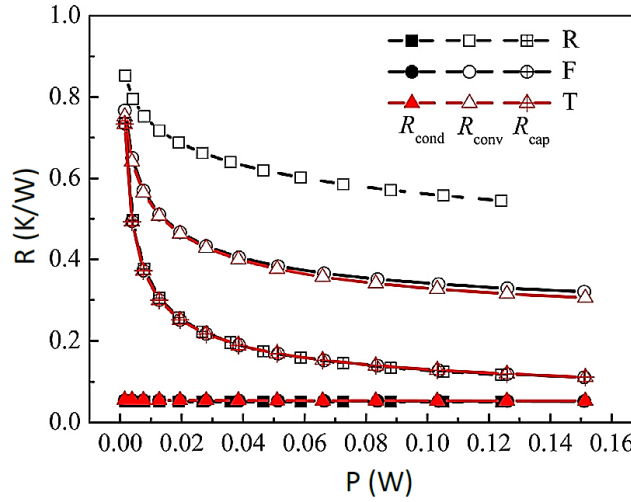


Figure 2-21: Shows the graph of thermal resistance against pumping power [136].

Deng et al. [137] investigated the single-phase and performance of heat transfer of Ω -shaped re-entrant copper MCHS using numerical and experimental analysis (refer Figure 2-22). The heat sink's thickness, length and width were 2mm, 45mm and 20mm respectively. Heat sink is made up of 14 parallel Ω -shaped reentrant micro-channels with $D_h = 781\mu\text{m}$. Two inlet deionised water temperatures used were 30°C and 60°C at Re ranges between 150 – 1100. Three different heat fluxes applied at MCHS's base (133, 200 and 267kW/m^2). For the new proposed micro-channel, heat transfer is enhanced by 10 – 30% with reduction of thermal resistance by 22%. The frictional factor ratios increase by 10%. The enhancement of heat transfer is due to the separation of flow and fluid mixing caused by the throttling effect (fluid flow is restricted by the narrow slot while sudden protrusion between circular cavity's bottom and upper vertical narrow slot disrupted the layers of hydrodynamic and thermal). To add up, for both micro-channels, increase in heat flux would result in the decrease of pressure drop. Cases of higher inlet temperature showed smaller thermal resistance.

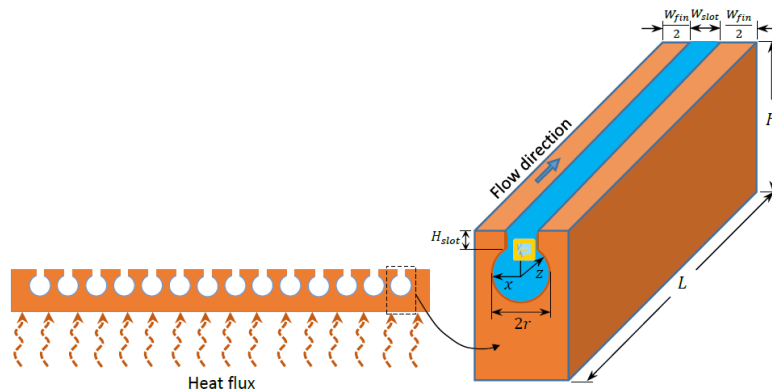


Figure 2-22: Shows the Computational domain schematic diagram of reentrant micro-channels [137].

Within sub-chapter, the use of cavities within micro-channel are very useful to interrupt boundary layer formation and increase heat transfer area which are useful to improve heat transfer rate but with the expense of higher pressure drop.

2.9.2.3 Ribs and grooves in MCHS

Liu et al. conducted an experimental investigation on the heat transfer and flow of fluid of rectangular micro-channels with longitudinal vortex generators (LVGs). The Re is varied between 170 and 1200. The LVGs are installed in five of the micro-channels in the experiment while the sixth one as a smooth micro-channel. The micro-channels were etched with silicon wafer with diameter of 101.6mm and thickness of 0.55mm. The length, height and width were 20mm, 0.1mm and 1.5mm respectively while the D_h is 0.1875mm. LVGs's length and width were 0.4mm and 0.5mm respectively. Number of LVG pair and angle of attack effect were also studied.

The transition of laminar to turbulent region using micro-channel with LVG occurred at Re of 600 – 720 while smooth micro-channel at Re of 2300. The transition of laminar to turbulent region occurrence can be delayed when changing the angle of attack of one pair of LVG. The rate of heat transfer were increased using micro-channel with LVG but at higher drop penalty. From the transition of laminar to turbulent region, the performance of heat transfer was enhanced by 9 – 90% with very large pressure drop increase by 34 – 169%.

Xie et al. [138] numerically investigated the thermal and fluid flow characteristics of silicon straight rectangular MCHS with vertical Y-shaped bifurcation plates under the regime of laminar flow. The bifurcation installed so that it will not in contact with the walls of micro-channels (refer Figure 2-23), thus, this will increase the mixing of fluid. Size optimisation study was performed using design variables of plate lengths and angles between Y-shapes bifurcation two arms aiming to achieve the highest heat transfer while maintaining the pressure drop to be minimal. The plate lengths were varied to be 10mm, 15mm and 25mm respectively while the angles were varied to be 60°, 90°, 120°, 150° and 180°. The silicon MCHS has 100 micro-channels with 35 x 35mm² surface area and 35mm length for removing heat fluxes of 24.49W/cm² and water as base fluid. The fin, height and width of channel were 0.035mm, 0.4mm and 0.315mm respectively while the cover plate and heat sink substrate's thickness were 0.5mm. The arm of bifurcation was set to be 0.15mm (fixed).

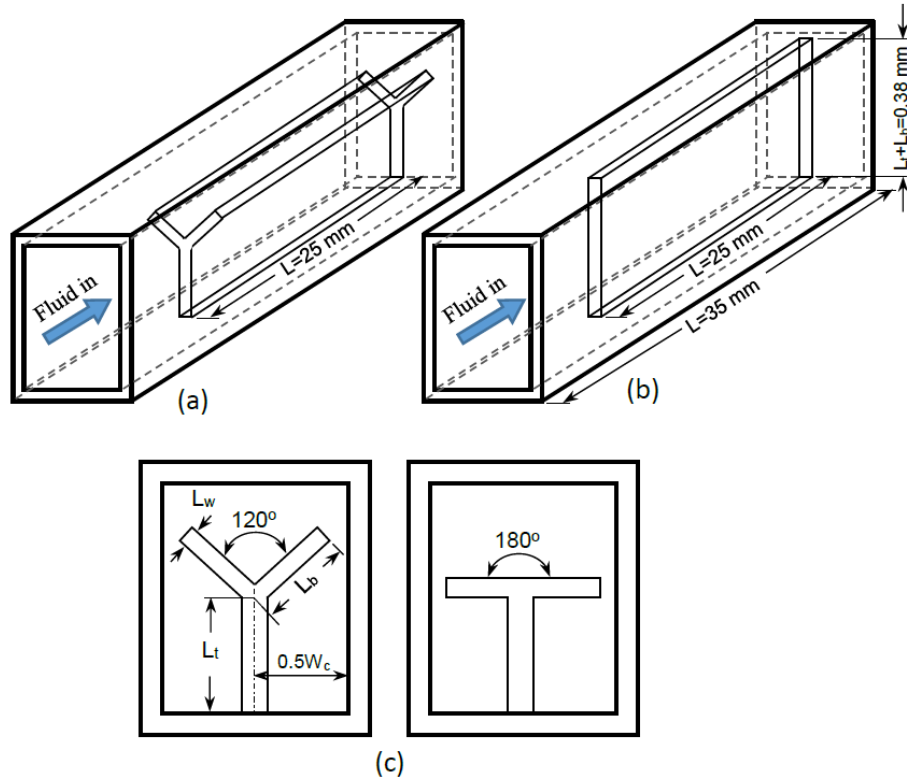


Figure 2-23: Shows the schematic diagram of (a) Y-shaped bifurcation micro-channel (b) vertical fin (c) Y-shaped bifurcation profile at 120° and 180° angles [138].

The predictions from the numerical analysis showed that 25mm Y-shaped bifurcation plates within micro-channel possessed better performance compared with the other two other bifurcations plates as well as traditional rectangular micro-channel. There was a reduction of thermal resistance by 41% achieved compared to straight rectangular micro-channel when using Y-shaped bifurcation. Y-shaped with 180° of angle induced the highest thermal performance and but with the expense of increased pressure drop.

Chai et al. [139] studied the effect of having offset ribs on sidewalls of MCHS on laminar flow and heat transfer using numerical analysis. They considered five different offset ribs shapes including semicircular, forward triangular, backward triangular, isosceles triangular and rectangular (refer Figure 2-24). From results, MCHS with forward triangular offset ribs possessed the largest performance evaluation criteria (PEC) at Re less than 350, but at Re more than 400 the micro-channels with semicircular offset ribs having the highest PEC.

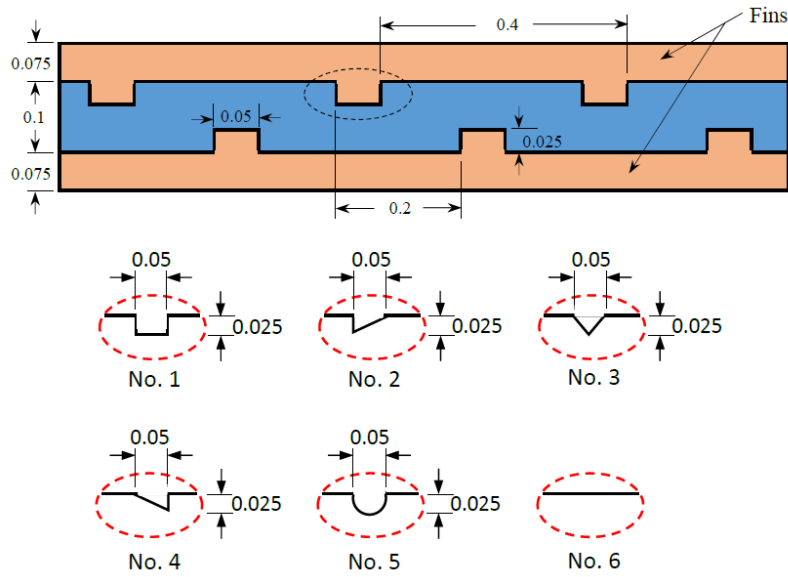


Figure 2-24: Shows the dimensions of all of offset ribs [139].

Ghani et al. [140] investigated the heat transfer characteristics and fluid flow of MCHS with sinusoidal cavities and rectangular ribs (MC-SCRR) using 3-D numerical analysis (refer Figure 2-25). New design's performance factor was compared with micro-channel with sinusoidal cavities, with rectangular ribs and conventional rectangular micro-channel. Using new design, the mixing of flow was heightened and larger area of heat transfer and flow compared with other channels. This will help to significantly improve the rate of heat transfer and pressure drop reduction. They also investigated three geometrical parameters of relative rib length, relative rib width and relative cavity amplitude to study the effect on pressure drop and heat transfer. From results, at an increase in relative cavity, the Nusselt number, Nu , increases and friction factor decreases. Moreover, when there is an increase in relative rib length and width rib length, the friction factor and Nu also increases.

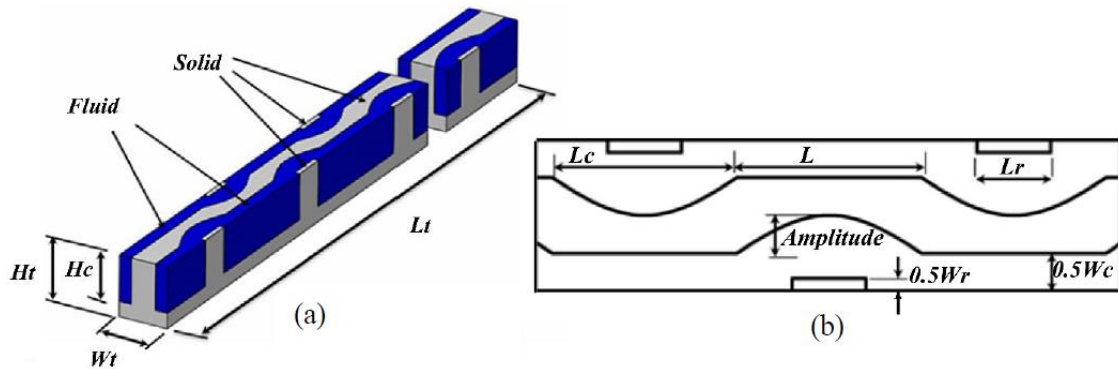


Figure 2-25: Shows the MC-SCRR (a) schematic diagrams (b) geometric parameters [140].

From the sub-chapter, using ribs and grooves are very beneficial to improve heat transfer since the flow is disturbed breaking boundary layers but with the expense of higher pressure drop due the channel is constricted.

2.10 Serpentine-based channel on MCHS performance

Serpentine-based channel has been investigated by many researchers as its capability to remove heat fluxes effectively compared with conventional straight channel. The key idea behind utilising the serpentine-based channel MCHS for electronics cooling because of the periodic interruption on the thermal and hydrodynamic boundary layers at corner or bends. Thus, the restart of both thermal and hydrodynamic layers periodically, disrupts the formation boundary layer which constrain the rate of heat transfer in designs for MCHS. Using serpentine channel, improves the heat transfer performance but at a large rise in pressure drop. To add up, serpentine channel MCHS possesses large ratio of surface area for heat transfer to volume due to bends compared with the conventional heat sink, leading higher absorption of heat from heat source (e.g. electronics chip) [141].

Hao et al. [142] considered serpentine channel heat sink with 180° bend to study the thermal resistance and pressure drop using thermal resistance network analytical model and CFD analysis (refer Figure 2-26). Thermal resistance network model was used to obtain the total thermal resistance by using equivalent thermal circuit method. The derivation of pressure drop was based on the straight channel and bend loss which periodically disrupt the hydrodynamics boundary layer. To consider the laminar flow development and redevelopment effect, the coefficient of bend loss is obtained as function of aspect ratios, Re , fins widths and turn clearances using three-regime correlation. Experimental validation for the model conducted by measuring the heat sink pressure and temperature characteristics at different Re and parameters geometry. Lastly, the pressure distribution and temperature rise at Re of 500, 1000 and 1500 are studied using the model.

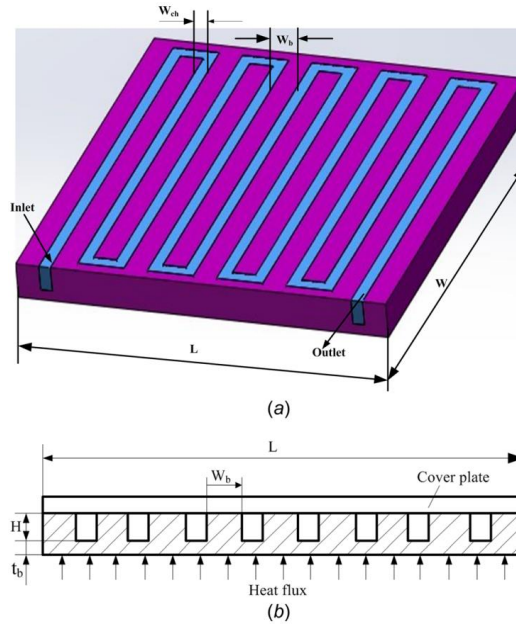


Figure 2-26: Shows the schematic diagram of 180 bend serpentine channel (a) 3-D view model (b) cross-sectional view [142].

The ratio of H/w_{ch} was varied between 1 and 6 while w_b/D_h was varied between 0.25 and 2, the Re is varied between 1000 and 2200. From the experiments conducted, the eight heat sinks with different values of H/w_{ch} , w_b/D_h and Re were verified indicating that the model proposed was accurate to predict the characteristics of heat transfer. The coefficient of bend loss was independent of Re . The pressure drop and thermal resistance were significantly dependent of Re while less dependent on parameters of geometry. For the same heat sink, as Re increase from 500 to 1500, the pressure drop also increased from 13kPa to 60kPa. Further optimisation process can be implemented using the results obtained, noting that the model proposed works for $1 < H/w_{ch} < 6$, $0.25 < w_b/D_h < 2$ and $1000 < Re < 2200$.

Imran et al. [143] considered 3-D serpentine mini-channel heat sink (SMCHS) optimisation. Four configurations of SMCHS (A, B, C and D) utilised (refer Figure 2-27). Different configurations were analysed using both experimental and numerical analysis. All SMCHSs have the same dimensions of channel with total channel length of 900mm and cross-section of $W_{ch} = 4\text{mm}$ and $H_{ch} = 5\text{mm}$. The heat sink has dimension of $L \times W \times H_b$ as 150mm x 100mm x 10mm. The SMCHS heat sinks have plates for cover and base with insulation at top surface and uniform heating at bottom surface. Water is used as working fluid. FVM CFD technique used to perform modelling on single-phase forced convection flow with water as working fluid. The experiments conducted cover the analysis of mass flow rate of water and heat load on the performance of SMCHS thermal and hydraulic parameters. From results, the serpentine with double inlet and outlet improved the performance of SMCHS significantly compared with single

inlet and outlet. Configuration A has the worst distribution of temperature with the highest pressure drop. Heat transfer enhancement percentage using configuration B, C and D compared to A were 136%, 104% and 91% respectively. Pressure drop enhancement using configuration B, C and D compared to A were 28%, 50% and 47% respectively. The least thermal resistance achieved when using configuration B. Good agreement between numerical and experimental analysis with percentage difference of 11% and 8% for baseplate temperature and pressure drop respectively.

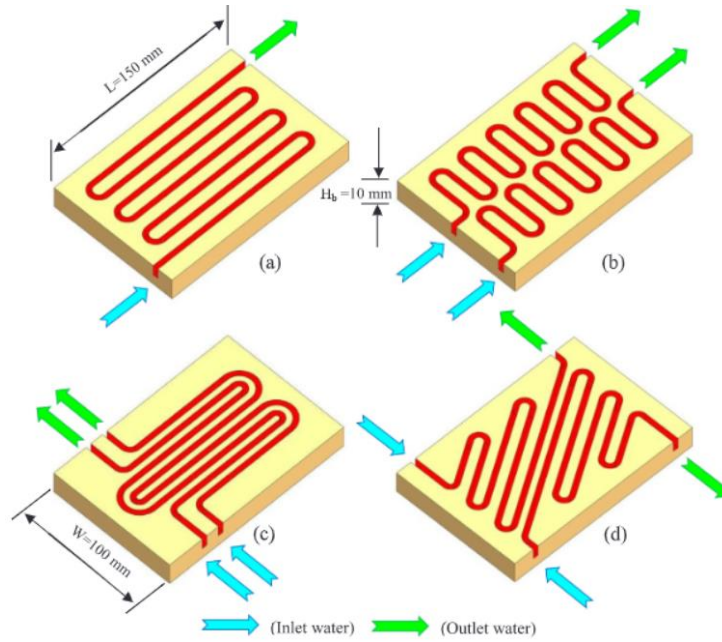


Figure 2-27: Shows the configurations of (a) A (b) B (c) C and (d) D [143].

Al – Neama et al. [144] investigated the single-phase flow and heat transfer in serpentine micro-channel embedded in copper block with heat applied at the base using combined experimental and numerical analysis. Four different configurations of serpentine channel utilised of straight rectangular micro-channels (SRMs), single path (SPSMs), double path (DPSMs) and triple path (TPSMs) multi-serpentine rectangular micro-channels (refer Figure 2-28 and Figure 2-29). For all serpentine channels configurations, a 3-D conjugate heat transfer models were developed under both single-phase laminar and turbulent flows. The governing energy and flow equations were solved by using finite elements method. All heat sinks has surface area dimensions of 41 mm x 45 mm with height (H) of 5.5 mm for straight and 6.5 mm for serpentine designs. The straight channels have width (W_{ch}), depth (H_{ch}) and length (L_{ch}) of 1 mm, 2 mm and 21 mm respectively while all of the serpentine designs have width (W_{ch}), depth (H_{ch}) of 1.5mm and 2mm respectively. The Nu_{avg} and pressure drop predicted numerically showed good agreement between experimental data. Using SPSM provides 35% enhancement in Nu_{avg} when volumetric

flow rate is at 0.5 l/min. The total thermal resistance reduced by 19% when using SPSM compared with the SRM heat sink. However, the SPSM heat sink induced very large pressure drop compared to SRM, thus, suitable compromise between the rate of heat transfer and pressure drop must be achieved. The DPSM and TPSM heat sinks has much lower pressure drop compared to SPSM heat sink due to liquid pressure is being distributed among mini-channels, hence, reducing the liquid velocity for lower friction factor.

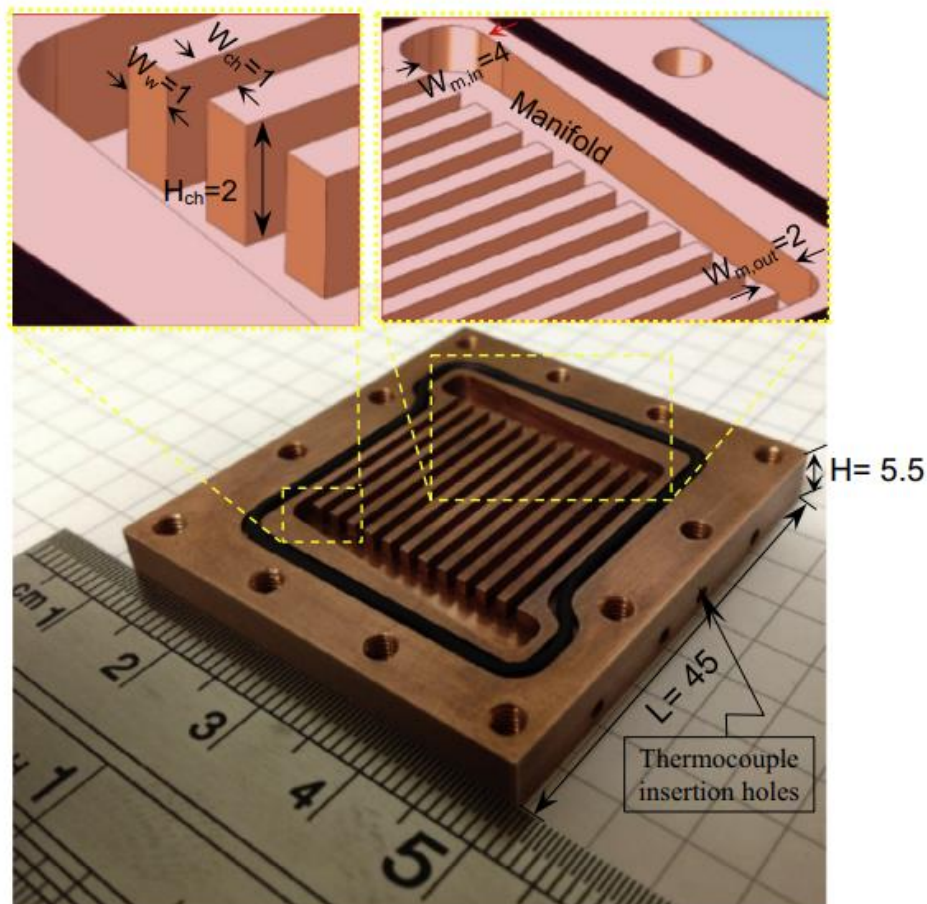


Figure 2-28: Shows the SRM design with dimensions in mm [144].

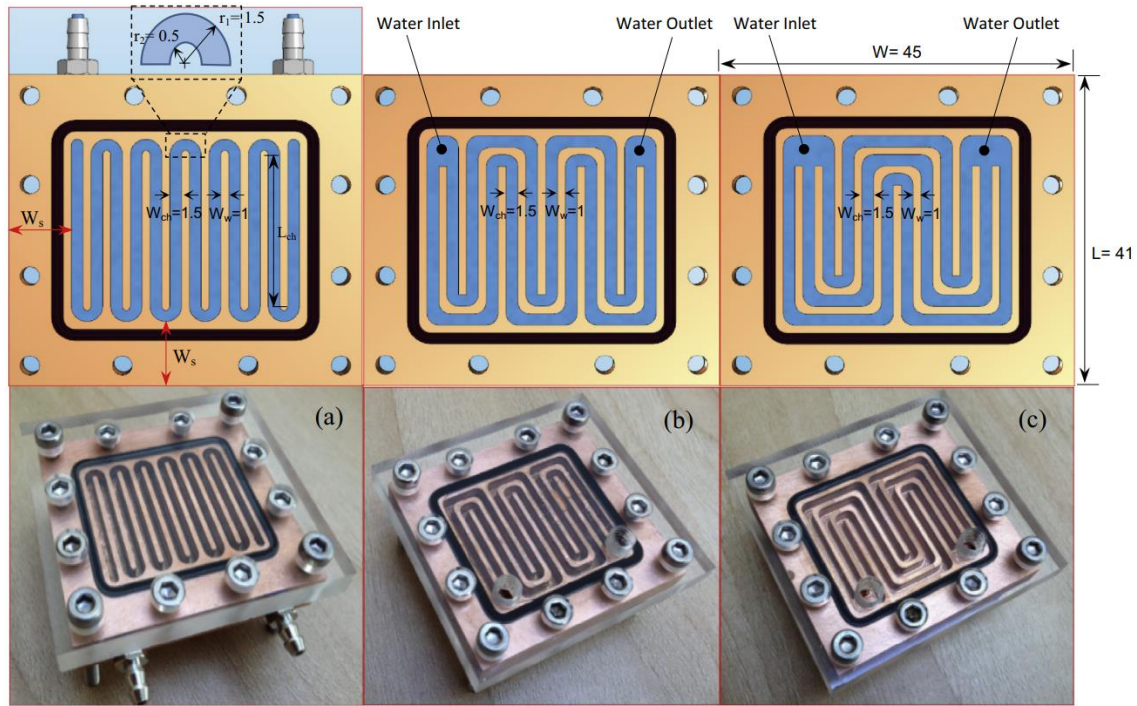


Figure 2-29: Shows the SPSM, DPSM and TPSM designs with dimensions in mm [144].

Further work by Al-Neama et al. [145] who considered the use of chevron fins within single-phase flow serpentine mini-channel heat sink with copper as substrate (SMCF) which can be referred in Figure 2-30 to solve the high pressure drop issue in [144]. The three dimensional conjugate heat transfer models were developed for both serpentine mini-channel heat sink base design and the one with chevron fins. Both laminar and turbulent flow regime were tested. Finite element method is used to solve the flow and energy equations respectively. The copper block has dimension for width x length x height of 38mm x 38mm x 4 mm respectively. The width of channel, W_{ch} , height of channel, H_{ch} , and substrate thickness, H_b are 1.5 mm, 2 mm and 2 mm respectively. Using SMCF heat sink offers reduction in both total thermal resistance and pressure drop by 10% and 60% differences respectively. The Nu_{avg} improved by 15% difference. The reduction of total thermal resistance and increase of Nu_{avg} is due to the continuous breaking of boundary layers while having larger heat transfer area for higher rate of heat transfer. The reduction of pressure drop is due to the diversion of fluid through the secondary channels reducing the fluid velocity within main channel.

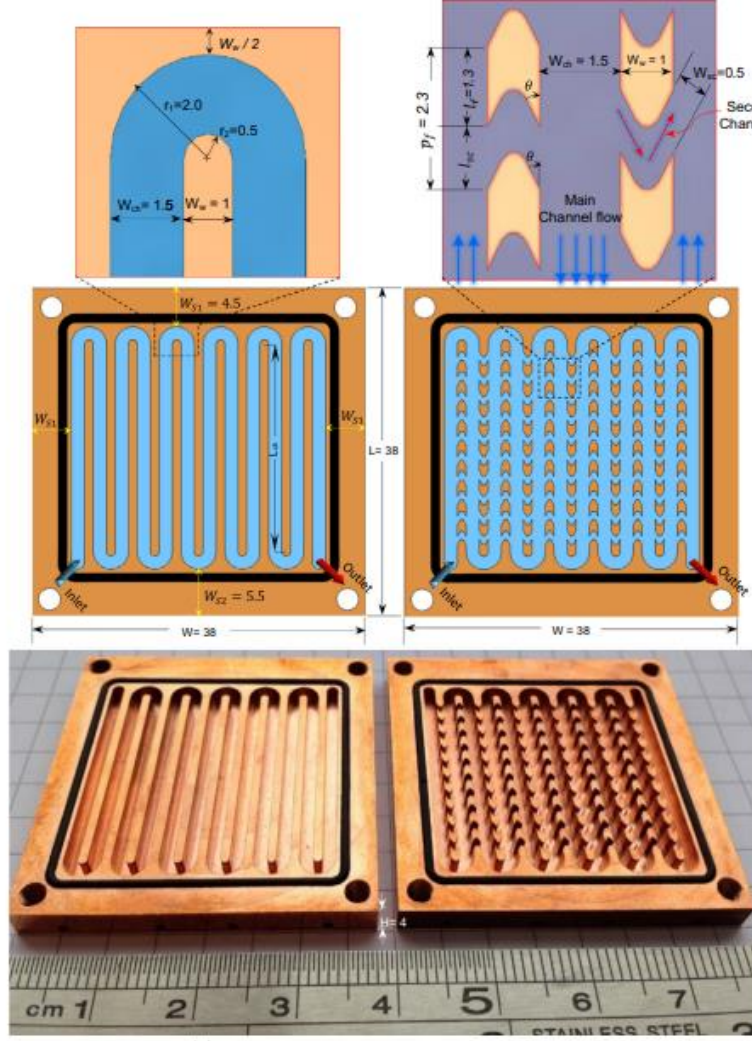
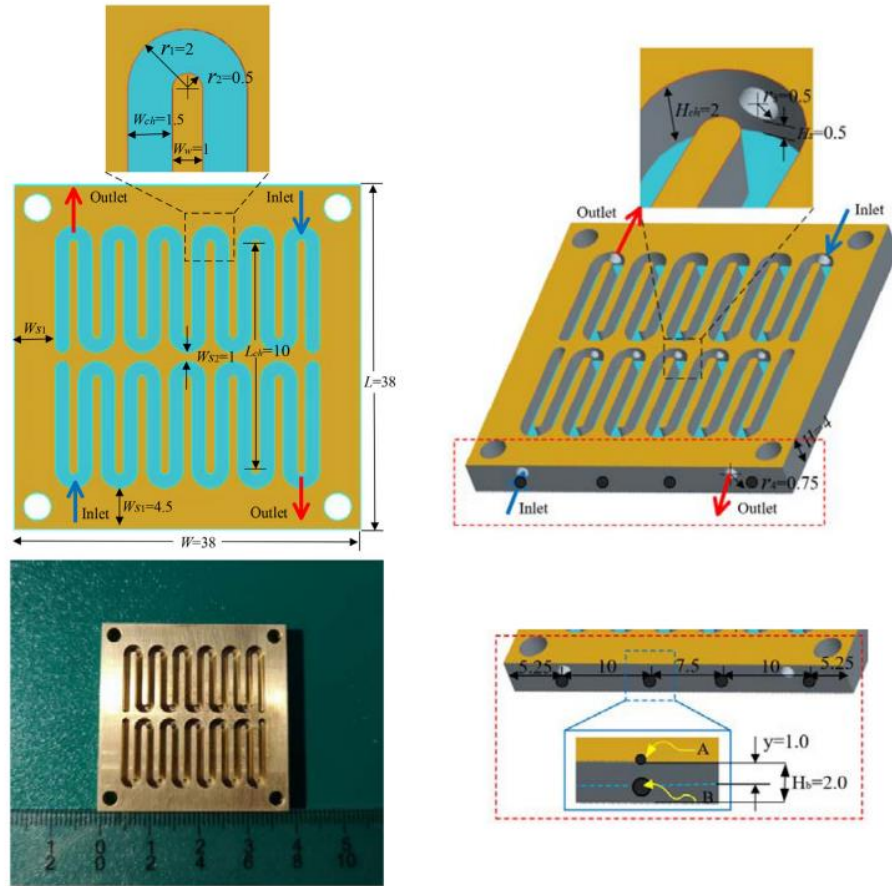


Figure 2-30: Shows the SMCF heat sink design and dimensions.

Cao et al. [146] utilised the double inlet-outlet serpentine mini-channel heat sink with (D-SMTH) and without (D-SMPF) through-holes for thermal-hydraulic performance study (refer Figure 2-31 (A)). Three different inlet boundary conditions with the same configuration using D-SMPF with through-holes of D-SMTH ‘A’, D-SMTH ‘B’ and D-SMTH ‘C’ are tested respectively (refer Figure 2-31 (B)). For both laminar and turbulent regimes, three dimensional conjugate heat transfer models were developed. All governing equations were solved using finite element method. The heat sink substrate material is copper with dimension for width x length x height of 38mm x 38mm x 4 mm respectively. The mini-channel dimensions are made equal to the work by Al-Neama et al. [145]. Based on results, D-SMTH ‘A’ heat sink design offers improvement of Nu_{avg} by 13.7% when compared with the SMCF heat sink by [145]. D-SMTH ‘A’ heat sink design has the lowest R_{th} compared to D-SMTH ‘B’ and D-SMTH ‘C’ heat sink designs respectively. D-SMTH ‘C’ heat sink design has significantly lower ΔP compared to D-SMTH ‘B’ heat sink.

(A)



(B)

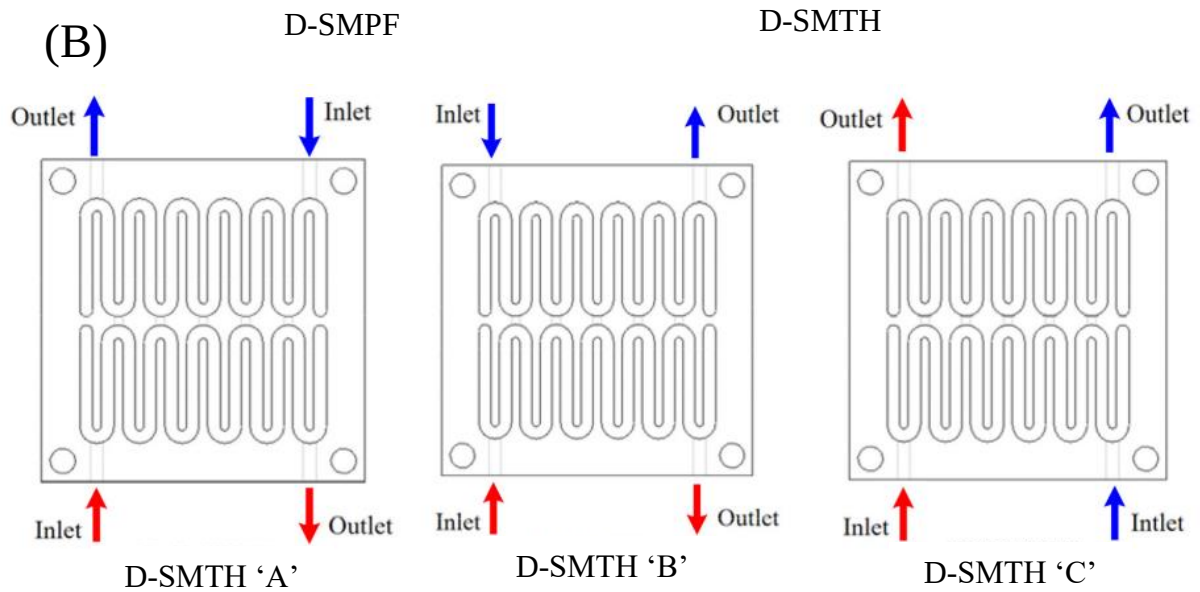


Figure 2-31: Shows the design for (A) D-SMPF and D-SMTH (b) D-SMTH configurations.

From this sub-chapter, the serpentine mini-channel heat sink has the ability to reduce thermal resistance and temperature uniformity due to large surface area from serpentine channel design and continuous breaking of boundary layers periodically. Many different designs of serpentine

channel heat sink has been explored with variation thermal-hydraulic performance. Furthermore, the use of secondary channels within serpentine channel heat sink has been proven to improve both thermal and hydraulic performance simultaneously due to larger heat transfer area and continuous breaking of boundary layers while diverting a portion of liquid through the secondary channels reducing fluid velocity leading to the reduction of friction factor.

2.11 Multi-objective optimisation and surrogate modelling

From the reviewed works, it can be concluded that heat sink systems can be classified as a multi-objective optimisation problem where the desired objectives can be achieved based on the related constraints and design variables respectively. The common objective functions used in the heat sink application are often total thermal resistance, R_{th} , pressure drop, ΔP , mass, M , and temperature distribution, TSD, of heat sink where all of them will be undergo minimisation. Heat sink miniaturisation will often induce large ΔP since smaller channel is used, thus this is also important to the structural integrity of heat sink so that it will not burst. The objective functions are affected by the design variables used such as height of channel, width of channel, inlet fluid velocity, fin angle, heating power and substrate thickness respectively. The utilisation of design optimisation most often occurs within the automotive and aerospace industries since in many occasions complex problems are always needed to be dealt with. Design optimisation offers accurate optimisation techniques which is useful in reducing the time period for optimisation of very sophisticated computational fluid dynamic (CFD) problem [147]. The computational time for flow simulation can be reduced drastically for example utilising the multi-scale wavelet method providing highly efficient transferring dynamic (interactions on micro-scale level to simulations of microscopic flow) surrogate modelling [148] where wavelets are utilised to decompose objective function into multiple scales and at each scale the solution is modified iteratively. Furthermore, multi-fidelity modelling can also be used to enable cheaper computational power [149]. The multi-fidelity modelling combines both low-fidelity and high-fidelity models evaluating the objective functions (output variables) to reduce time taken for computation and resources as well as achieving accurate solutions. Multi-fidelity modelling includes adaptive sampling, model fusion and surrogate models techniques. Surrogate models used both low-fidelity and high-fidelity models capturing the relationship between objective function input design variables. Adaptive sampling utilises low-fidelity model to study the design space and recognising the regions where high-fidelity model can used to perform evaluation. For model fusion techniques, the multiples models predictions are used for obtaining high accuracy objective function estimation. For achieving statistical surrogates, Gaussian

Process (GP) Emulators is very useful. GP emulator uses the original simulation training data for learning the probabilistic model of relationship between input design variables and objective function. The advancement of exploration of design space methods proving the feasible timescales for large-scale optimisation. According to the work by Haftka et al. [150], the use of adjoint methods are effective to solve design problems with large number of design variables (more than 1000). For design optimisation problem with design variables of less than 100, the use of surrogate modelling methods is more effective. Surrogate models such as Moving Least Squares (MLS), Radial Basis Functions (RBFs) and Support Vector Machines (SVMs) are very efficient for numerical noise minimisation that has a deleterious effects [151]. The confidence around intervals can be determined while the mixed categorical or continuous design variables is solved efficiently [152]. The use of Supervised Machine Learning (ML) methods are very efficient for complex fluid flow systems providing very effective surrogate modelling method. Neural Networks (NNs) and RBFs are the example of supervised ML [153]. The surrogate models developed with acceptable accuracy will be used within optimisation algorithm (e.g. Non-Dominated Sorting Algorithm-II (NSGA-II)) [154] searching for optimal solution(s) either for single-objective or multi-objective optimisations where for multi-objective optimisation the compromise between conflicting outputs must be achieved which is also called Pareto optimal solutions (see Chapter 5 for further explanation of optimisation algorithm and surrogate modelling method).

To conclude, using the methods of surrogate modelling such as NN, RBF, MLS and SVM is beneficial to estimate the output variables accurately from the variation of input variables within the design space while reducing computational time significantly and surrogate modelling will be used in this work due to the number of input design variables used are less than 100.

2.11.1 Deterministic optimisation of micro-channel heat sink

Optimisation process are very important for achieving the superior performance of heat sink. Husain and Kim [155] considered the micro-channel heat sink with silicon as substrate for optimisation. To ensure high accuracy and less time consumption, they utilised the surrogates for immediate approximation. From the surrogate model developed, fin width (W_w), channel's depth (H_{ch}) and width (W_{ch}) are chosen as design variables while having objectives to be pressure drop and thermal resistance respectively. The design variables were first normalized by dividing both W_w and W_{ch} with H_{ch} to be ϕ and θ respectively. Working fluid chosen was water where the properties are calculated as function of temperature which was suggested by Li et al. [156] initially. They have exploited the use of Design of Experiment (DOE) points to create

surrogates based on the polynomial method. Using the non-dominated sorting genetic algorithm-II (NSGA-II) algorithm (evolutionary algorithm-based on biological evolution concept), the surrogate models are used for optimisation and creating the Pareto optimal solutions. From their findings, the pressure drop (ΔP) and thermal resistance (R_{th}) are greatly affected by the change of ϕ and θ respectively. From the plotted Pareto optimal solutions, it gives a flexibility for more choices of different constraints of manufacturing and design.

Chen et al. [157] use the same design as Li et al. [158] who uses serpentine channel heat sink for optimisation purposes. The design variables chosen were a mixture of boundary condition (inlet fluid velocity) and physical dimension change (fin width (W_w), height (H_{ch}) and width of channel (W_{ch})). The corresponding objectives for the optimisation framework are ΔP and R_{th} respectively. Using artificial swarm fish algorithm, the dataset for the Pareto optimal solutions are created and being compared experimentally. Similarly in [156], Pareto optimal solutions show a conflicting relationship between ΔP and R_{th} .

Reddy et al. [159] investigated the thermal-hydraulic performance of pin-fin heat sink using 3-D conjugate heat transfer model for integrated circuit cooling. The pin-fins are varied with three different shapes of symmetric airfoils and convex lenses and circles respectively (refer Figure 2-32). The integrated circuit is having dimensions of 4mm by 3mm with uniform heat flux of 500 W/cm² applied underneath. The surrogate modelling is based on the Hardy's Multiquadrics (includes multiple quadratic terms) Radial Basis Function method while having Genetic algorithm to create the Pareto optimal solutions. Design variables chosen are the cross section pin fin, chip height and fluid inlet velocity while the pumping power and maximum surface temperature as objectives. The use of symmetry convex lens shape pin-fin offer the most minimum pressure drop.

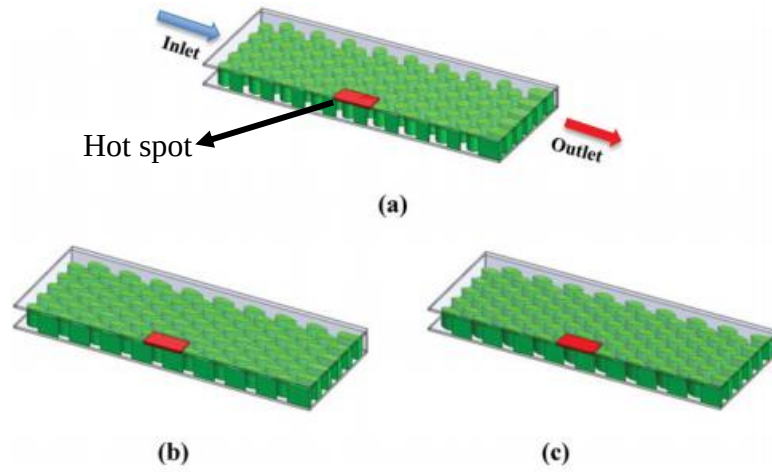


Figure 2-32: Shows the pin-fin heat sinks with different shapes of fin (a) Circular (b) Symmetric airfoil (c) Symmetric convex with only half of the computational domain [159].

Al-Neama et al. [144] optimised serpentine mini-channel heat sink. They have used two design variables of channel number (N_{ch}) and width of channel (W_{ch}) for the minimisation of both ΔP and R_{th} simultaneously. The generation of DOE points is based on the Optimal Latin Hypercube (OLHC) sampling method which will be used for the creation of surrogates. Moving Least Square (MLS) estimation method was used to perform surrogate modelling while having polynomial of second order base. Multi-objective genetic algorithm (MOGA) is utilised based on the surrogates created searching for Pareto optimal points. Al-Neama et al. [38] also investigated the effect of using chevron fins within serpentine mini-channel heat sink with an aim to reduce both total thermal resistance and pressure drop simultaneously. Multi-objective optimisation takes place with oblique angle of fin (θ), channel number (N_{ch}) and width of channel (W_{ch}) as design variables respectively. Same methods as [144] to create DOE points and surrogate modelling are being used here. MOGA framework will be used to perform optimisation for both ΔP and R_{th} . Conflicting objectives gives room for designers to choose the appropriate values for specific applications.

Shi et al. [160] utilises secondary channel to reduce both thermal resistance and pressure drop (refer Figure 2-33). They have used the design variables as an angle of secondary channel (γ), secondary channel half pitch and width of secondary channel which were normalised with respect to the width of micro-channel as β and α respectively. The use of α shows the most significant effect towards the thermal-hydraulic performance of heat sink. From the optimisation process, the reduction of both pumping power and thermal resistance can be achieved with 22.9% and 28.7% differences respectively when compared with conventional smooth channel.

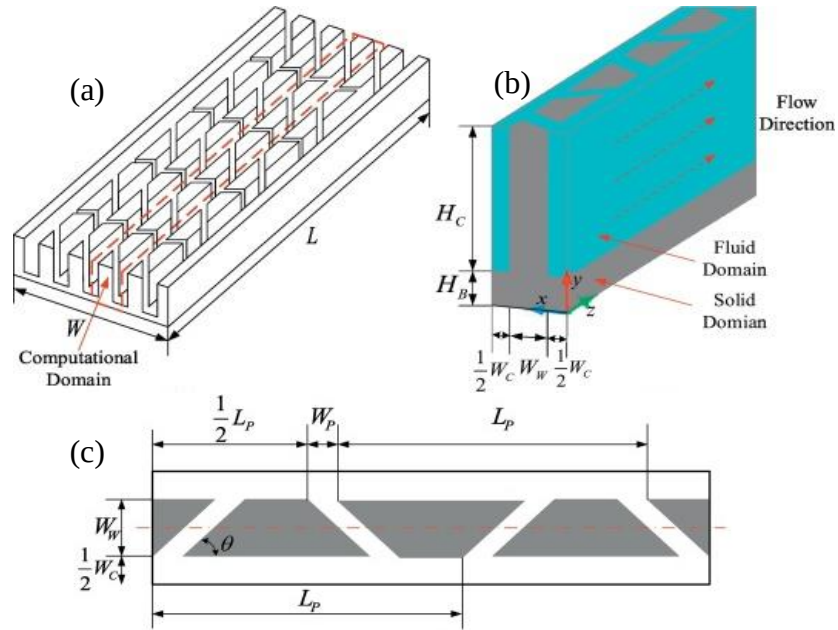


Figure 2-33: Shows the micro-channel heat sink with trapezoidal fin shape [160].

So many other studies have been conducted involving optimisation of micro-channel heat sink [81, 161, 162]. To improve the hydrothermal performance of micro-channel heat sink, Ahmed and Ahmed [163] and Fan et al. [164] modifies the dimensions of geometry to perform optimisation. From the recent review by Ahmad et al., they have not included the works related to the optimisation with uncertainty for the desired design variables. This methodology is very crucial when dealing with manufacturing errors due to the variation of geometry or changing in boundary conditions which can definitely effects the performance hydrothermally. The use of uncertainty within design variables is so important for industry like aerospace and automotive which practice critical safety.

2.11.2 Probabilistic optimisation of micro-channel heat sink

The two major classifications of uncertainties are called aleatoric and epistemic. For aleatoric uncertainty, the word originates from Latin called 'alea' which gives the meaning of dice rolling. Different names of aleatory uncertainty includes type (A) (the name comes from classification system by International Organization for Standardization (ISO)), irreducible or stochastic uncertainty. This type of uncertainty gives the representation of process or system randomness for example parameters of geometry which are varied caused by the imperfectness of manufacturing, property constants of material which are uncertain and unpredictable conditions of weather. Probabilistic theory can easily be used with this kind of uncertainty which is physical in nature. For the other kind of uncertainty called epistemic which originates from Greek word giving the meaning of knowledge. Other names for epistemic uncertainty are type (B) and uncertainty which is reducible. Epistemic uncertainty occurs when knowledge or data regarding

a field is limited. The lack of understanding of a physical phenomenon or not enough experimental data for distribution of probability are classified as epistemic uncertainty [165-167]. The application of uncertainty within optimisation framework is very useful for to create designs which are reliable or robust. The use of uncertainty in optimisation can also called probabilistic optimisation, non-deterministic optimisation or optimisation under uncertainty (OUU).

For the last twenty years, the use of probabilistic optimisation method has been increasing. Two major different formulations of probabilistic design optimisation are Reliability-Based Design Optimisation (RBDO) and Robust Design Optimisation (RDO) respectively (refer Figure 2-34). RBDO method uses the probabilistic analysis and optimisation method to ensure the products design are within the reliability requirements while RDO aims to ensure the minimisation of the effect of uncertainty on system performance benchmarks [168, 169]. During early 1920s, in order to increase the production of crops within the agricultural sector, Fisher developed the method to create DOE with statistical feature. The father of robust design, Dr. Genichi Taguchi, who found the robust design fundamentals [170].

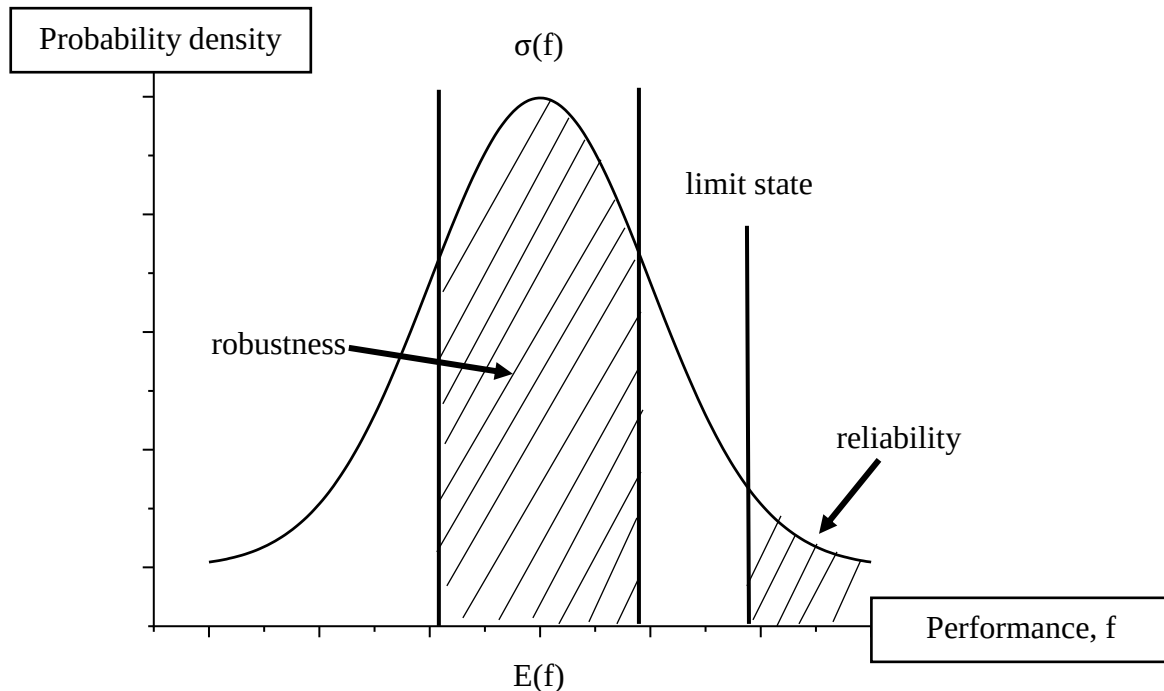


Figure 2-34: Shows the comparison between robustness and reliability based on probability density function [168].

Previously, structural analysis and optimisation field use probabilistic framework study very extensively. Doltsinis and Kang [168], introduced stochastic parameters for the optimisation of structures using robust design technique. From the structural response (objective function)

sensitivity which is measured by means of mean and variance, it is affected by the perturbed equations under the analysis of stochastic study. They have structured the multi-objective optimisation using probabilistic framework to perform minimisation of the mean and standard deviation of the objective functions. The scalar optimisation problem is derived from the multi-criteria optimisation framework and based on optimisation algorithm of gradient based method where the solution is acquired. Marano et al. [171] considered optimisation of structure using both deterministic and probabilistic framework for a comparison study within the application of single tunes mass damper (TMD) device. Under the probabilistic optimisation, that studied both system performance as well as the uncertainty parameters that affect the sensitivity. From the probabilistic optimisation influenced design, different solutions are acquired compared to the deterministic solution, but more robust.

2.11.2.1 Probabilistic optimisation for other field

From the reviewed literature, some researchers have utilised the probabilistic design optimisation of the aerodynamic field both the statistical moments (mean and standard deviation) of objective functions minimisation from the pre-set design variables under uncertainty. Zhao et al. [172] used numerical method performing probabilistic optimisation for supercritical airfoil under the conditions of transonic and laminar flow in high region Reynolds number. In order to predict the boundary layer from the transition of laminar to turbulent flow, they have used the $\gamma - \theta$ transition and shear stress transport (SST) $k - w$ turbulence models for analysis methodology. Kriging model was used to create surrogates and after that it was used within the multi-objective optimisation framework of NSGA-II to search for Pareto optimality solutions. NASA0412 airfoil type was utilised with the Mach number as an uncertain variable. From the comparison of airfoil which has been optimised using probabilistic framework, the achievement of natural laminar flow is possible while having the drag coefficient to be reduced. Using probabilistic optimisation framework also enables the airfoil drag and its divergence characteristics to achieve a compromised relationship. The values of mean and standard deviation can be balanced using the multi-objective optimisation framework.

Shahbaz et al. [170] considered the Rae 2822 airfoil for probabilistic optimisation study where probabilistic density function was used to create large number of random samples around each point of input design variables. The objective function used were the mean and variance of the coefficient of drag (Cd) while the angle of attack and Mach number as design variables respectively. Using the MC sampling method on the design variables, the mean and variance were calculated for the corresponding objective functions based on the surrogates formed using

a Kriging model. Results from probabilistic optimisation were compared with the actual CFD data, and good correlation obtained. The C_d experience reduction in range between 37.48% and 30.19% respectively when different values of Mach number and angle of attack used.

Chakraborty et al. proposed low-fidelity (LF) and high fidelity (HF) polynomial correlated function expansion (PCFE) under the framework of differential evolution algorithm (DEA). The LF method is used to calculate the properties of performance functions statistical moments while HF method is used to find the solution optimisation study. To quantify the performance of the two newly suggested methods, it was used under the three critical RDO studies, measuring its performance based on the efficiency and accuracy which in return give more superiority compared to conventional methods such as tensor product quadrature, Taylor's series and Kriging. Using LF method, the estimation of the initial state for RDO problems can defined first which is very efficient. Next, large scale problem such as hydroelectric dam can be solved using LF and HF methods.

2.11.2.2 Probabilistic optimisation of conjugate heat transfer system

Bodla et al. [173] and Sarangi et al. [174] used the probabilistic optimisation to optimise the micro-channel heat in terms of its thermal-hydraulic performance. Bodla et al. [173] used heater block together with pin-fin heat sink under the condition of impinging flow. The numerical method used to analyse the heat sink was based on CFD analysis and optimisation solutions are calculated using gradient-based optimiser called Fletcher-Reeves conjugate gradient method (negative gradient is combined with previous search direction so that new search direction can be calculated). Surrogate models are created based on the generalised polynomial chaos (gPC) method which can reduce the computational time of CFD software. The gPC works by using polynomial expansion to approximate stochastic process which involves the choosing of orthogonal polynomials as basis functions. The combination of polynomials expands the stochastic process which can be utilised to statistical quantities of means and standard deviations. The probabilistic optimisation study using Dakota optimisation platform is based on the nested approach. Uncertain input parameters can be assessed effectively using probabilistic optimisation framework within Dakota. The use of robust probabilistic optimisation framework is to reduce the system fluctuations when input variables are varied while the reliable framework deals failure probability when design variables are uncertain.

Sarangi et al. [174] considers the parameters of geometry of manifold micro-channel heats sink for probabilistic optimisation. From their investigation, they used porous-medium model and simple unit-cell model for numerical study. Both deterministic and probabilistic optimisation

framework have been used to search heat sink optimal geometries while having reduction for both pumping power and thermal resistance respectively. The same probabilistic study as in [173] is used here. From the comparison between deterministic and probabilistic optimisation, heat transfer will be the highest when inlet to outlet ratio is three.

2.12 Chapter summary

The literature review has shown a substantial amount of extensive analysis on micro-channel heat sinks for electronics cooling for the past thirty years using single-phase flow. Analytical, experimental and numerical investigations were utilised to assess the thermal and hydraulic performance of mini/micro-channel heat sink with different shapes and geometries, working fluids and materials for substrate based on their behaviours of heat transfer and fluid flow. The predictions from numerical and experimental analysis has successfully discovered the essentiality of using single-phase forced convection with liquid as base fluid in micro-channel heat sinks compared to conventional heat sink (air cooled heat sink and heat pipes) due to their ability to remove higher heat fluxes. However, micro-channels having large hydraulic diameter has low heat transfer coefficients (heat transfer coefficients and hydraulic diameter are inversely proportional to each other). Thus, having higher heat transfer coefficients would mean higher pressure drop induced due to more constricted channel resulting in higher pumping power.

Two-phase flow has not been mentioned extensively in the review since our focus is really on single-phase flow. To sum up, single-phase flow has been the popular choice of researchers due to its effectiveness but there is still plenty room for improvement.

From the literature review, analysis made from the year of 1981 to 1999 uses mainly more on the analytical and experimental side. Starting from the year 2000, massive amount of works exploit the use of numerical simulations study and evolutionary algorithms analysis.

A wide examination on the behaviours of heat transfer and fluid flow using the conventional micro-channel heat sinks with micro-channel cross-section shapes of trapezoidal, triangular and rectangular, the use of micro-pin-fin heat sinks and many other structures of channel between the inlet and outlet boundaries such as zigzag, wavy and curved are investigated. Very few studies on serpentine channel heat sink have been found in the literature covering multi-serpentine channel with both round and 180-degree sharp bend channel. Minimal research on different new configurations of serpentine channel heat sink are shown in the literature and only one study that covered serpentine channel heat sink with chevron fins [145] which has been proven effective in enhancing the heat transfer while reducing the pressure drop at the same time.

The use of fins are essentially converting the serpentine mini-channel heat sink into almost like a pin-fin heat sink design which is very effective in removing heat from system. Most of the research on serpentine channel heat sinks only covers the single inlet-outlet boundary condition but very little number of papers covering the analysis on double inlet-outlet boundary condition serpentine channel heat sink [37, 143] and only one paper considers using through-holes connecting two separated serpentine channels which is effective in reducing total thermal resistance and pressure drop simultaneously [37].

The use of serpentine channel heat sink is still in its infancy stage which has to be explored further understanding its behaviour in terms of fluid flow and heat transfer characteristics as well as pressure drop. Thus, the current studies focus on discovering further of using different shaped fins (oblique (SMOF), trapezoidal (SMTF) and diamond (SMDF) shaped fins) in single inlet-outlet serpentine channel heat sink on thermal-hydraulic performance which are used by researchers for only straight channel heat sinks [131, 132, 175-178].

Furthermore, there is only a single study on double inlet and outlet serpentine mini-channel heat sink with through-holes [37] but no application of secondary flow through secondary channel utilised together with double inlet-outlet serpentine mini-channel heat sink which will be analysed in this investigation work (D-SMDF heat sink). Next, the use of through-holes utilised together with the same double inlet-outlet with secondary channel (D-SMDF-TH). All results from the novel design will be compared with the conventional base serpentine mini-channel heat sink design. All serpentine heat sink design will have rectangular cross-section profile since it is the most efficient in transferring heat and has a simple design.

The serpentine mini-channel heat sink designs with the most superior thermal-hydraulic performance will be used for deterministic and probabilistic optimisation framework study. All results are extracted from the numerical analysis of 3-D computational simulations of conjugate heat transfer model available in COMSOL Multiphysics v5.4.

Chapter 3 : Heat and fluid flow fundamentals

3.1 Introduction

The basic concepts comprise of the convection heat transfer physical mechanism, fluid flows types and boundary layers of velocity and thermal. The heat sink characteristics of hydraulic and heat transfer are presented below.

3.2 Convection heat transfer

The heat transfer phenomenon can occur through fluids by means of conduction or convection based on the existence of any bulk fluid motion. If there is motion of bulk fluid, convection heat transfer occurs (refer Figure 3-1 (a) and (b)) while conduction will occur if there is no motion of bulk fluid (refer Figure 3-1 (c)). In reality, the heat transfer by convection includes conduction and fluid motion. The movement of heat starts from the hot surface of solid to the lower temperature fluid layer at surface by means of conduction and further transfer of heat to the next layer of lower temperature fluid compared to fluid layer by the surface by means of fluid motion. Therefore, it can be concluded that the heat transfer by convection has significantly higher rate compared to conduction [179].

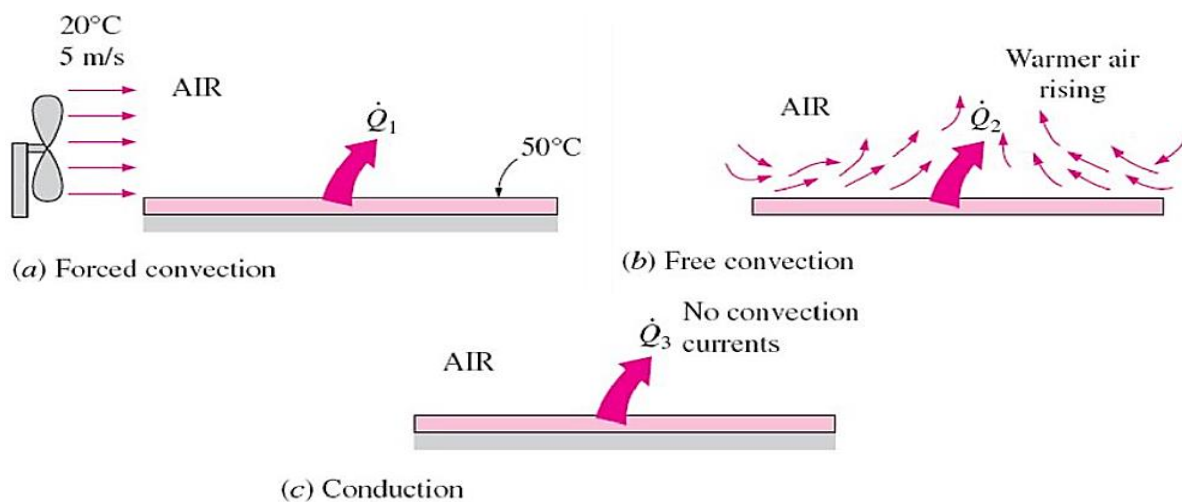


Figure 3-1: Shows the heat transfer mechanism from hot surface solid to the cool air of surrounding through conduction and convection [179].

Convection heat transfer is dependent on the properties of fluid such as fluid velocity (U), density (ρ), thermal capacity (C_p), thermal conductivity (k) and dynamic viscosity (μ). In addition, geometrical shape, surface roughness and fluid flow type also affects the rate of convection heat transfer. Thus, convection heat transfer has a complex mechanism [179]. As the temperature difference between hot surface and cold fluid increases, the rate of convection heat transfer increases and can be defined by Newton's law of cooling:

$$\dot{Q}_{conv} = hA_s(T_s - T_{\infty}) \quad (3.1)$$

Where h is the average convective heat transfer; A_s is the heat transfer surface area; T_s is the surface temperature; T_{∞} is the ambient fluid temperature.

3.3 Fluid flow types basic concept

Fluid mechanics is the major component of convection heat transfer, fluid flow types are further explained to achieve better understanding on fluid flow behaviour that affects the convection heat transfer enhancement [179].

3.3.1 Viscous, stokes and inviscid flow

If fluid viscosity is taken into account, it has a significant effect on the fluid flow behaviour due to the internal resistance, it is called viscous flow. The Reynolds number is commonly smaller than 1 which makes the viscous forces is much larger than inertial forces, but inertial forces is still significant and important. For stokes flow, the inertial forces are considered insignificant compared to viscous forces where particles of fluid move very smoothly and slowly with no eddies or turbulence are induced. On the other hand, inviscid flow holds the assumption of no viscosity due to its effect is unimportant for certain flows and it is negligible.

3.3.2 External and internal flow

External flow can be described as fluid moving around or over an object with no constraints from the adjacent surfaces. For internal flow, the fluid is moving through a channel, pipe or duct where it is surrounded by confined surfaces.

3.3.3 Compressible and incompressible flow

Flow of gas that vary in fluid density is considered compressible flows such as jet engines, rocket motors and high-speed craft due to changes in velocity, temperature and pressure causing significant change on properties of fluid. The density of fluids that are constant or very small change is considered an incompressible flow which is true for fluids at low speeds and pressure

change has negligible effect on properties of fluid. For sufficiently low of air velocity, it is also modelled as incompressible flow.

3.3.4 Laminar and turbulent flow

Laminar flow occurs when the fluid particles streams in parallel layers with disturbance between them. Fluid particle layers are moving without mixing among them inducing predictable flow pattern and highly ordered. The Reynolds number of laminar flow for internal flow is commonly at less than 2300 and external flow at less than 200000. On the other hand, turbulent flows comprises of chaotic local fluid velocity fluctuations, inducing fluid layers disruption and non-orderly flow causing fluid mixing in many directions and formation of eddies as well as vortices. Turbulent flow occurs at high speed where the inertial forces dominates the viscous forces with high Reynolds number (more than 4000 for internal flow or more than 500000 for external flow).

3.3.5 Natural and forced flow

Forced convection or forced fluid flow occurs when the fluid is forced to flow using external mechanical device such as pump or fan (refer Figure 3-1 (a)). Natural convection (thermosiphon effect) depends on the buoyancy effect of fluid where warm fluid goes up (less mass due to low density) and cold fluid goes down (higher density) (refer Figure 3-1 (b)).

3.3.6 Steady and unsteady flow

Steady flow occurs when the properties of fluid (velocity, pressure, etc) do not change with time. For unsteady flow, the flow of fluid having characteristics of fluid (velocity, pressure, etc) which is at any point change with time.

3.4 Hydrodynamic and Thermal Entrance Region

For straight rectangular mini- or micro-channel, in order to calculate the coefficient of heat transfer (h) and pressure drop ΔP , it is very crucial to identify the three major developing flow structure of thermally developing flow or/and hydrodynamic fully developed flow or both occurring at the same time. The type of flow can be identified by calculating the length of hydrodynamic entry (L_{hy}) first. The L_{hy} is defined as the distance between entrance of channel and to the location where merging of hydrodynamic boundary layers occur (refer Figure 3-2).

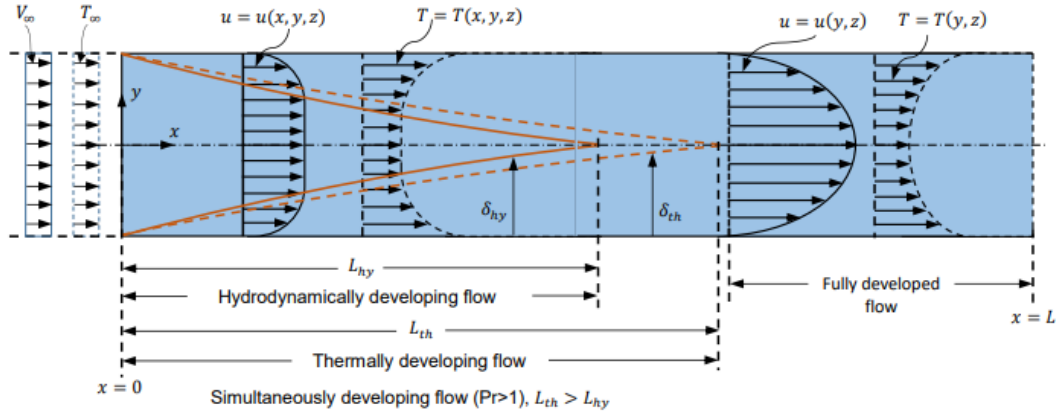


Figure 3-2: Shows the developing flow (hydrodynamic and thermal) followed by fully developed flow (hydrodynamic and thermal) within channel under isothermal condition [180].

At the entrance region, the profile of velocity develops (hydrodynamic developing flow). Velocity profile has the shape of parabolic curve at the region of hydrodynamic entrance. This profile will stay the same until the flow has reached the fully developed hydrodynamic flow. Within the regime of laminar flow, according to Shah and London [181], they have proposed an accepted equation to calculate the L_{hy} which is as follows:

$$L_{hy} = L_{hy}^+ \cdot Re \cdot D_h \quad (3.2)$$

Where dimensionless hydrodynamic entry length is denoted by L_{hy}^+ which is dependent to the channel geometry aspect ratio (α) (where $\alpha = \frac{W_{ch}}{H_{ch}}$) and it is set to be 0.05 before reaching the conditions of fully developed flow [182]. For rectangular channel, under the regime of laminar flow the value of L_{hy} can be calculated using equation proposed by Shah and London [181] which is as follows:

$$L_{hy} = (0.06 + 0.07 \alpha - 0.04 \alpha^2) Re \cdot D_h \quad (3.3)$$

The entry length of thermal (L_{th}) is also need to be calculated. L_{th} has the definition of distance of entrance of channel to the location of merging of thermal boundary layer around centreline (refer Figure 3-2). Thermal developing flow is located at the entrance region where the temperature profile develops. When the flow is thermally fully developed, thermal profile stays the same. Under the flow of laminar regime, L_{th} is calculated also using the equation proposed by [181] which is as follows:

$$L_{th} = L_{th}^* \cdot Re \cdot D_h \cdot Pr \quad (3.4)$$

Where the dimensionless length of thermal entry is denoted by L_{th}^* where the value is set to be 0.05 based on the proposal of Lee et al. [183] under the conditions of fully developed flow.

Looking at the region of fully developed flow both thermally and hydrodynamically, the values of coefficient of heat transfer (h) and friction factor (f) stay equal along the channel length and are the lowest respectively [184]. The temperature and velocity profile developed at the same time is called simultaneously developing flow (refer Figure 3-2). The value of friction factor and coefficient of heat transfer vary along the direction of flow when simultaneously developing flow occurs. From Table 3-1 by Rohsenow et al. [185] shows the relationships between velocity and temperature distributions, boundary layers, flow types, friction factor and Nusselt number (Nu) respectively.

Table 3-1: Shows the types of flow.

Type of Flow	Hydrodynamic Boundary layer	Flow direction velocity distribution	Friction factor	Thermal boundary layer	Dimensionless temperature distribution in the flow	Nusselt number
Fully developed flow	Developed	Invariant	Constant	Developed	Invariant	Constant
Hydrodynamically developing flow	Developing	Variant	Variant	----	----	----
Thermally developing flow	Developed	Invariant	Constant	Developing	Variant	Variant
Simultaneously developing flow	Developing	Variant	Variant	Developing	Variant	Variant

3.5 Boundary layers

The boundary layer concept is crucial in order to understand the heat transfer by convection between a surface and flow of fluid over it. In this section, the velocity and thermal boundary layer are described.

3.5.1 Velocity boundary layer (δ_u)

From Figure 3-3, it shows the formation of velocity boundary layer when fluid is flowing over a flat surface. The x-axis is representing the direction of fluid flow over flat surface while y-axis is representing the perpendicular flow of fluid with respect to the flat surface. At the plate's leading edge, fluid velocity is uniform in x-direction.

Fluid particles velocity (V) at first layer (adjacent to flat surface) has zero velocity due to no-slip condition. The first layer will affect the next adjacent fluid layer motion due to the frictional forces presence from the difference in velocity between fluid layers and this process continues onto the next layers. At a certain distance from the flat surface, the velocity reaches the free

stream velocity (U_0). This specific distance is the thickness of boundary layer (δ_u) when $u = 0.99U_0$. Velocity boundary layer (δ_u) comprises four different flow regions which are laminar sublayer, buffer layer and turbulent layer and free stream region. The laminar sublayer is a very thin layer located just above the flat surface and it is also called viscous sublayer where the fluid velocity increases linearly. The laminar sublayer is the closest layer to the solid boundary where on this layer fluid viscosity dominates other properties of fluid. Laminar sublayer thickness depends on the fluid viscosity, velocity and solid boundary surface roughness. The buffer layer is the layer adjacent to the viscous sublayer and turbulent flow is beginning to develop. For buffer sublayer, the fluid velocity increases quickly where fluid flow becomes more turbulent and has thickness several times thicker than laminar sublayer. The overlap layer is where the significant turbulent fluctuations occurs where fluids velocity fluctuates more randomly and fully turbulent flow. The free stream region has a large distance to the flat surface where $u = U_0$. The fluid viscous region is at the flat surface where the fluid velocity is zero [179].

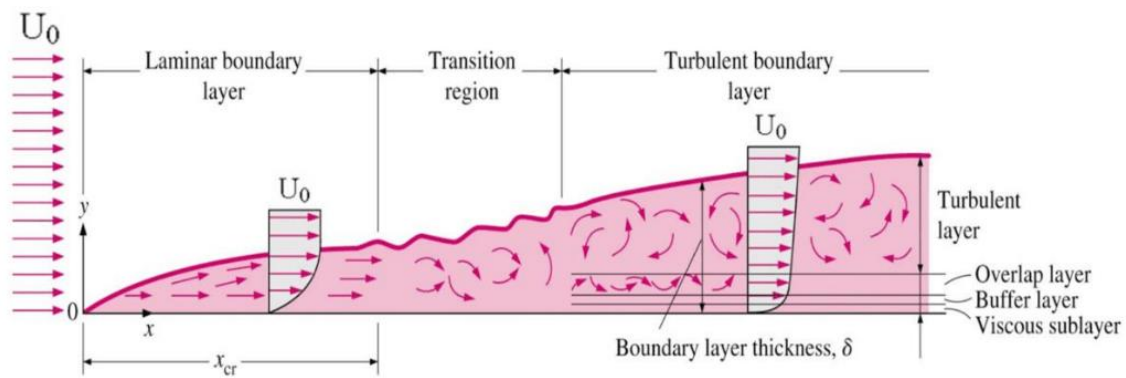


Figure 3-3: Shows the development of velocity boundary layer on flat surface [179].

3.5.1.1 Effect of Transition on Local Heat Transfer Coefficient

Based on Figure 3-4, it is showing the local coefficient of heat transfer (h) and thickness of velocity boundary layer (δ_u) for a flow over a flat plate which has a constant temperature (isothermal). The δ_u is the thin fluid layer formed at solid boundary which has fluid velocity going from zero to freestream velocity. The formation of δ_u is based on the no-slip boundary condition. The δ_u is consisting of laminar sublayer, buffer layer and overlap layer respectively (see sub-section 3.5.1). To explain simply, boundary layers can be recognised as layer of insulation. Rate of heat transfer is increase when the boundary layer thickness decreases and vice versa. In the region of laminar flow, the coefficient of heat transfer has a very high value around the region of entrance since the thickness of boundary layer is zero. When the value of δ_u increase there should be a decrease in the coefficient of heat transfer. Reaching the turbulent

regime, mixing of fluid occurs (cold and warm fluid mixes) inducing an increase of coefficient of heat transfer. When the δ_u increase again, the coefficient of heat transfer reduces. Looking at the region of transition, local coefficient of heat transfer increases drastically even though thickness of boundary layer increases. Explaining this phenomena, the turbulent mixing starts at the transitional region mixing. The mixing of fluid is the greatest during the first stages due to the thin boundary layer. After that, moving through the plate thickening of boundary layers continue to rise while fluid mixing still happening but rate of heat transfer decreases gradually.

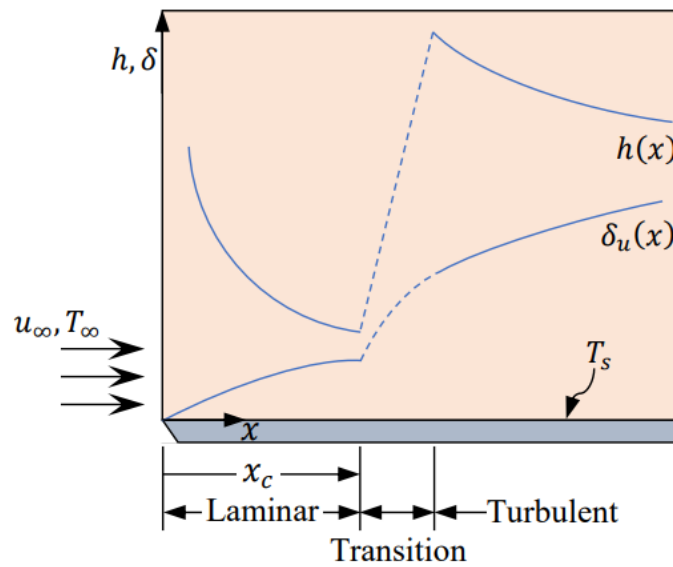


Figure 3-4: Shows the thickness of velocity boundary layer variation and coefficient of local heat transfer flow over constant temperature flat plate [32].

3.5.2 Thermal boundary layer (δ_{th})

From Figure 3-5, cold fluid is flowing with thermal boundary layer formation at certain temperature uniformity (T_∞) over hot flat surface (T_s). Thermal boundary layer possesses similar formation analogy of that velocity boundary layer but with temperature difference between flow fluid and flat surface. The flat surface with heat energy will reach thermal equilibrium with the flow of fluid layer next to it. The energy exchange will occur with the next fluid layers and so on. The thermal boundary layer temperature gradients can be expressed as follows:

$$\frac{T_s - T}{(T_s - T_\infty)} = 0.99 \quad (3.5)$$

As the distance from leading edge in the direction of flow increases, the thermal boundary layer thickness increases, since the heat transfer effects penetrate further into the flow of fluid. The conditions and convection heat transfer at any distance of x from leading edge are related through

the application of Fourier's law at the flat surface and fluid layer interface when $y = 0$ as follows:

$$k_f \left. \frac{dT_f}{dy} \right|_{y=0} = k_s \left. \frac{dT_s}{dy} \right|_{y=0} \quad (3.6)$$

Where k_s and k_f are the solid flat surface and fluid thermal conductivity respectively [186].

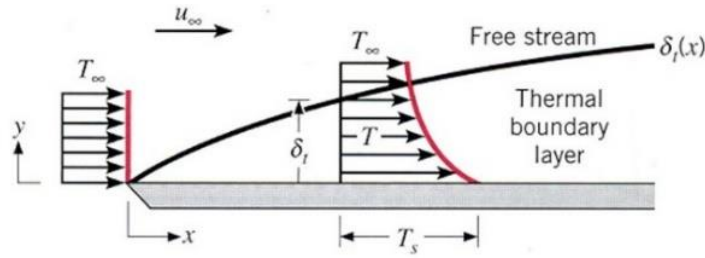


Figure 3-5: Shows the development thermal boundary layer on an isothermal flat surface [186].

3.5.3 Boundary layer separation

To understand the separation of boundary layer phenomenon, the flow of fluid around cylinder is considered due to its importance in many engineering applications such as circular pinned heat sinks, shell-and-tube heat exchangers, bundle circular tubes heat exchangers, etc.

The free stream velocity (u_∞) and upstream velocity (V) around cylinder that dependent on the x distance from the point of stagnation are shown (refer Figure 3-6). At stagnation point ($\theta = 0$), the fluid velocity is equal to zero. During the acceleration of fluid flow due to the favourable pressure gradient ($du/dx > 0$, $dp/dx < 0$), flow of fluid will the maximum velocity at $dp/dx = 0$, then deceleration of fluid flow occurs from adverse pressure gradient ($du/dx < 0$, $dp/dx > 0$) (refer Figure 3-7). As the velocity gradient decreases due to deceleration, at certain point it will reach the value of zero ($du/dy|_{y=0} = 0$) at surface which known as the separation point.

At separation point, there is no sufficiency of momentum overcoming the pressure gradient and fluid movement continuation downstream is not possible. Furthermore, the fluid flow that keeps on coming is preventing the flow back upstream, hence the separation of boundary layer must occur. At downstream region, the boundary layer separates from surface and behind the cylinder, there are formation of vortices and reversed flow [186].

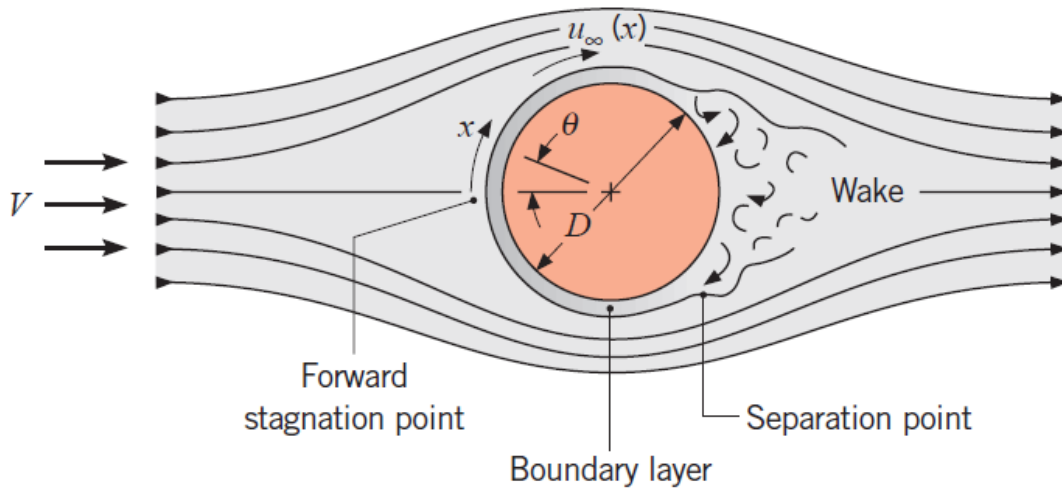


Figure 3-6: Shows the separation of boundary layer and eddies formation at region of downstream [186].

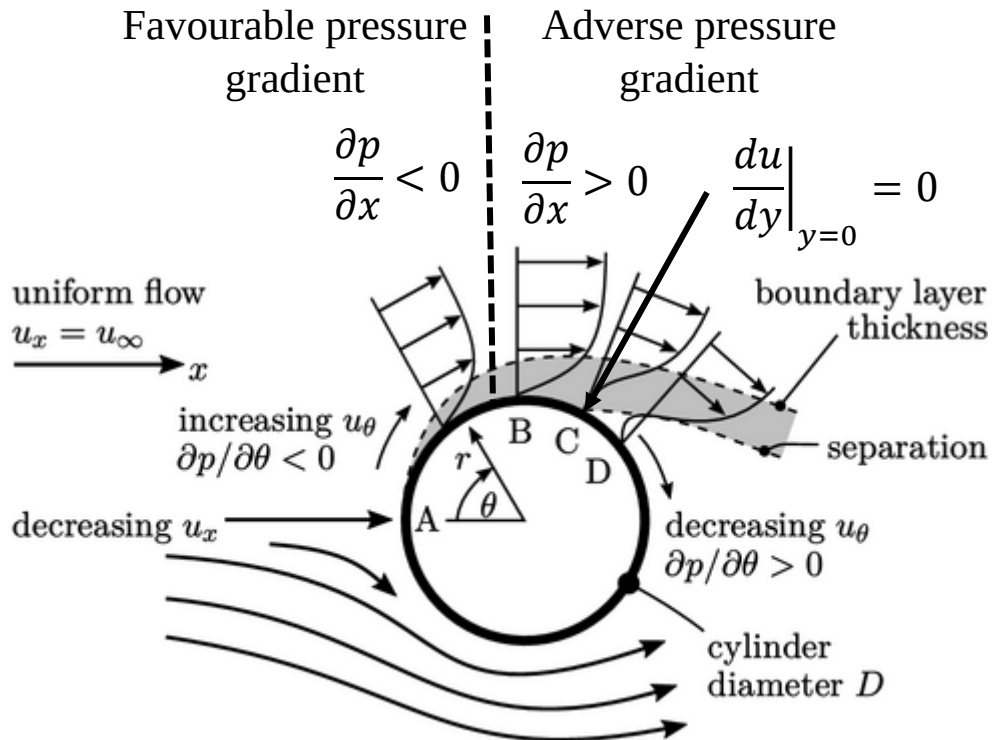


Figure 3-7: Shows the velocity profile with pressure gradient and separation point on cylinder [186].

3.6 Surface thermal Conditions for heat transfer by convection

Using constant wall temperature boundary condition and constant heat flux boundary condition, these two conditions can approximate the thermal conditions on surface which is explained below. Referring Figure 3-8, along the tube length, the temperature variations based on these two conditions is explained. Let say, a tube having diameter (D) and length (L) with fluid flowing through while having heat convection through the wall of tube, the steady flow energy balance

equation is applied for a controlled volume of fluid (refer Figure 3-8) which has been proposed by Bergman et al. [32] as follows:

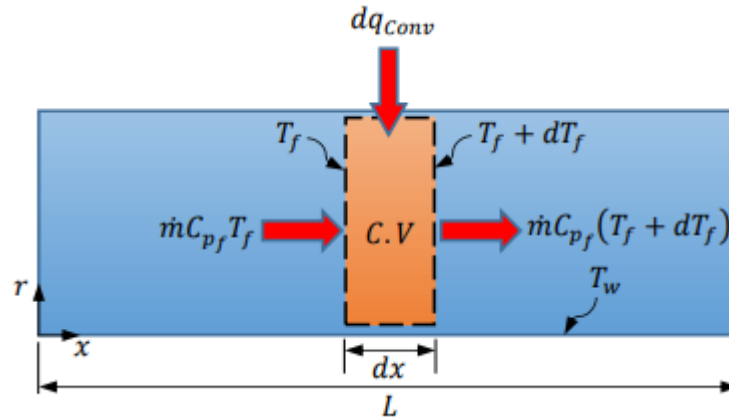


Figure 3-8: Shows the tube with fluid flowing through in a controlled volume [33].

Heat flow rate through wall + Energy inflow rate = Energy outflow rate

$$dq_{conv} + \dot{m} \cdot C_{p_f} \cdot T_f = \dot{m} \cdot C_{p_f} (T_f + dT_f) \quad (3.7)$$

Through convection heat transfer, the heat applied (dq_{conv}) to the controlled volume of fluid is as follows:

$$dq_{conv} = q_s'' \cdot dA \quad (3.8)$$

Where the term $dA = Pdx$. The perimeter of tube is denoted by P [m]. Based on Newton's law of cooling, heat transfers through convection moving from wall of tube to the stream of fluid can expressed as follows:

$$q_s'' = h(T_w - T_f) \quad (3.9)$$

Using substitution method, Eq. 3.9 and Eq. 3.8 are substituted into Eq. 3.7 resulting as follows:

$$\frac{dT_f}{dx} = \frac{P}{\dot{m} \cdot C_{p_f}} h(T_w - T_f) \quad (3.10)$$

3.6.1 Constant Wall Heat Flux (CWHF) Boundary Condition

The wall temperature (T_w) has a linear relationship with the mean temperature of fluid ($T_{f,x}$) in the region fully developed flow. Thus, the difference between these two temperature is fixed (refer Figure 3-9 (a)). On the other hand, the T_w varied non-linearly in the fluid flow developing region.

Under the applied constant wall heat flux (CWHF) boundary condition, the coefficient of average (h_{avg}) and local (h_x) heat transfer remained the same within the region of fully

developed flow. This is caused by the constant fluid properties when fluid flows. The derivative of Eq. 3.9 with respect to x is derived as follows:

$$\frac{dT_f}{dx} = \frac{dT_w}{dx} \quad (3.11)$$

Based on the differentiation of Eq. 3.9 to become Eq. 3.11, it can be conclude that the gradient $(T_w - T_f)$ does not change and the profile of temperature along the tube remained fixed.

3.6.2 Constant Wall Temperature Boundary Condition

From Figure 3-9, the behaviour of the temperature plot for constant wall heat flux (CWHF) boundary condition is different compared with constant wall temperature (CWT) boundary condition.

Applying integration of Eq. 3.10, two locations of $x = 0$ and $x = L$ will give the temperature of $T_f = T_{f,in}$ and $T_f = T_{f,out}$ respectively:

$$\int_{T_{f,in}}^{T_{f,out}} \frac{d(T_w - T_f)}{(T_w - T_f)} = -\frac{h \cdot P}{\dot{m} \cdot C_{P_f}} \int_0^L dx \quad (3.12)$$

$$\ln \left(\frac{T_w - T_{f,out}}{T_w - T_{f,in}} \right) = -\frac{h \cdot A}{\dot{m} \cdot C_{P_f}} \quad (3.13)$$

Where T_w , T_f , h , A , $\dot{m}$ and C_{P_f} are wall temperature, fluid temperature, coefficient of heat transfer, surface area, mass flow rate and specific heat capacity of fluid.

Heat transfer rate can be expressed as follows:

$$q = h \cdot A (T_w - T_f) = \dot{m} \cdot C_{P_f} (T_{f,out} - T_{f,in}) \quad (3.14)$$

Since the fluid has constant wall temperature boundary condition, the coefficient of heat transfer is not changed and can be factored outside integral (see Eq. 3.15).

Substituting the term of $\dot{m} \cdot C_{P_f}$ from Eq. 3.13 into Eq. 3.14 will result as follows:

$$q = h \cdot A \Delta T_{ln} \quad (3.15)$$

The logarithmic mean temperature difference is denoted by ΔT_{ln} which is expressed as follows:

$$\Delta T_{ln} = \frac{T_{f,in} - T_{f,out}}{\ln \left(\frac{T_w - T_{f,out}}{T_w - T_{f,in}} \right)} = \frac{\Delta T_{out} - \Delta T_{in}}{\ln \left(\frac{\Delta T_{out}}{\Delta T_{in}} \right)} \quad (3.16)$$

Where $\Delta T_{out} = T_w - T_{f,out}$ and $\Delta T_{in} = T_w - T_{f,in}$ respectively.

To get Eq. 3.16 from Eq. 3.12, the derivation is assumed where constant heat transfer rate along the length of wall that affect the fluid temperature distribution.

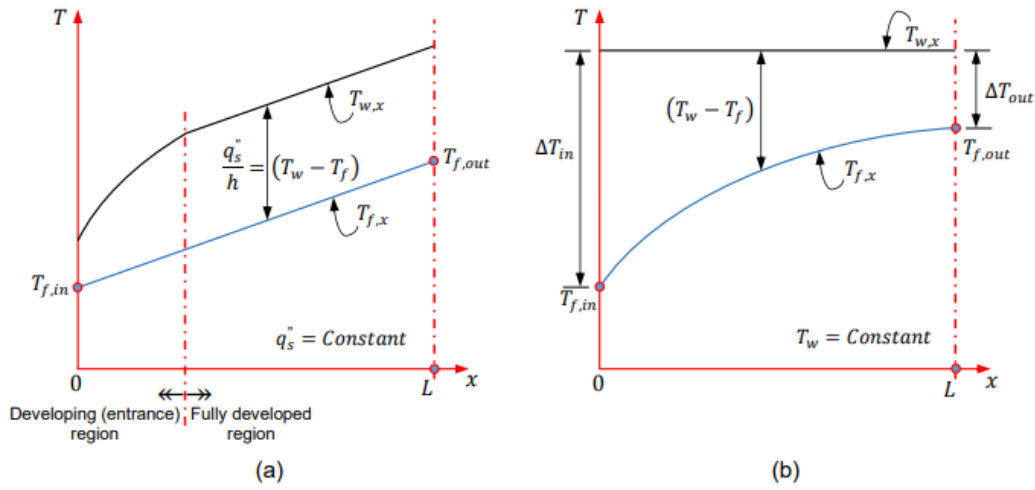


Figure 3-9: Shows the variation of axial temperature within a channel; (a) Fixed surface heat flux (b) Fixed temperature of surface [33].

3.7 Different types of Thermal Boundary conditions

The thermal boundary conditions are categorised into three different types. First type, the boundary condition of wall temperature is constant, T , where the wall temperature is uniform circumferentially and axially. Heat flux boundary conditions are the other two types of thermal boundary conditions. The first wall heat flux boundary condition, H1, is when the wall heat transfer per unit area is unchanged leading to wall temperature varies with the rate of heat transfer. The second wall heat flux boundary condition, H2, is when the wall heat transfer is controlled by coefficient of convective heat transfer depending on the geometry and properties of fluid flow. From Shah and London, they have proposed the Nusselt number (Nu) correlation for T , H1 and H2 boundary condition respectively which are as follows [181]:

$$Nu_T = 7.541(1 - 2.61 \alpha + 4.97 \alpha^2 - 5.119 \alpha^3 + 2.702 \alpha^4 - 0.548 \alpha^5) \quad (3.17)$$

$$Nu_{H1} = 8.235(1 - 2.0421 \alpha + 3.0853 \alpha^2 - 2.4765 \alpha^3 + 1.0578 \alpha^4 - 0.1861 \alpha^5) \quad (3.18)$$

$$Nu_{H2} = 8.235(1 - 10.6044 \alpha + 61.1755 \alpha^2 - 155.1803 \alpha^3 + 176.9203 \alpha^4 - 72.923 \alpha^5) \quad (3.19)$$

3.8 Heat transfer and hydraulic characteristics

The characteristics of airflow and heat transfer comprises Reynolds number, total thermal resistance, pressure drop and performance evaluation criteria (PEC), maximum base temperature of heat sink (temperature of electronics), temperature and fluid velocity distribution along the serpentine channel heat sink. These parameters are crucial to perform the heat sink design evaluation which can be obtained from flow simulations The factors are as follows:

3.8.1 Reynolds number (Re)

The Reynolds number (Re) is the ratio of inertial forces to viscous forces which can be expressed as follows:

$$Re = \frac{\rho_f U_{in} D_h}{\mu} \quad (3.20)$$

Where ρ_f is the density of fluid (water) [kg/m^3]; U_{in} is the inlet water velocity (m/s^{-1}); D_h is the hydraulic diameter of channel [m] (refer Eq. 2.1); μ is the water viscosity [kg/m.s].

3.8.2 Prandtl number (Pr)

The Prandtl number (Pr) is a parameter within convection heat transfer analysis which is dimensionless. Pr is the measure of fluid relative ability for energy and momentum transportation. The definition of Pr is the measure of ratio of diffusivity of momentum to diffusivity of heat which can be defined as follows:

$$Pr = \frac{\mu_f \cdot C_{p_f}}{k_f} \quad (3.21)$$

Where the specific heat capacity and thermal conductivity are denoted by C_{p_f} [J/kg.K] and k_f [W/m.K] respectively. The Prandtl number can also be defined as the ratio of velocity boundary layer (δ_u) to the thermal boundary layer (δ_{th}) to the power of $1/n$ which is as follows:

$$Pr = \left(\frac{\delta_u}{\delta_{th}} \right)^{\frac{1}{n}} \quad (3.22)$$

Where n must always be a positive number. Referring Figure 3-10, the plots of Pr against temperature at atmospheric pressure for different fluids are shown. At low $Pr \ll 1$ and within laminar flow regime, velocity boundary layer is thinner compared to thermal boundary layer, meaning fluid has a low viscosity (e.g., mercury) and high conductivity (thermal energy can be transported easily compared to momentum energy transport). For high $Pr \gg 1$, the velocity boundary layer is thicker compared to thermal boundary layer where fluid is viscous (e.g., engine

oils) and non-conductive (momentum energy can be transported easily compared to thermal energy).

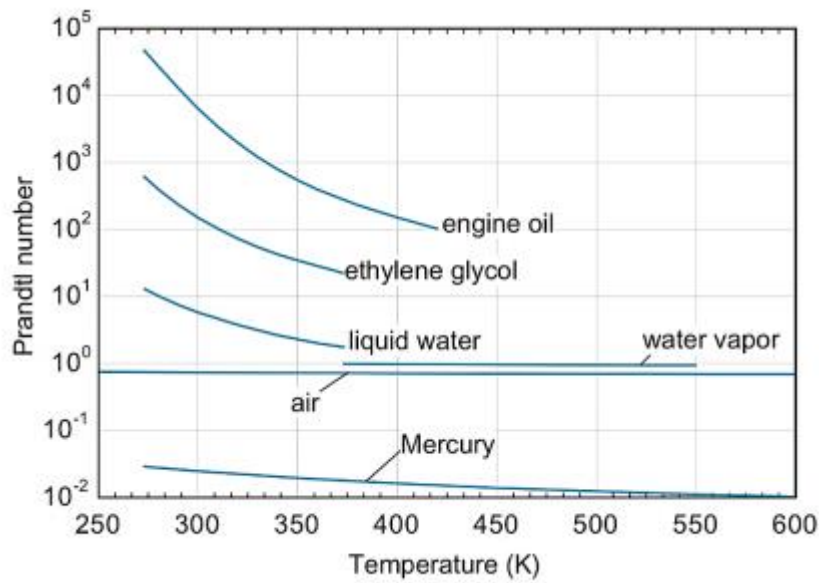


Figure 3-10: Shows the Prandtl number (Pr) against temperature for different fluids [187].

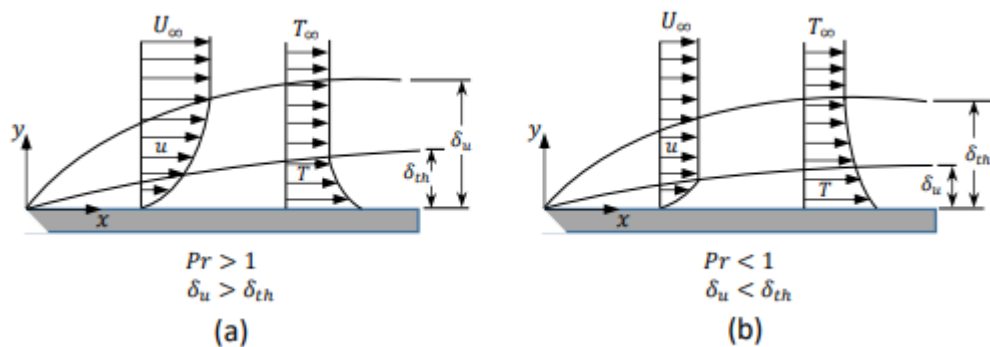


Figure 3-11: Shows the thickness off hydrodynamic and thermal boundary layers (a) $Pr > 1$ (b) $Pr < 1$ [33].

Under the regime of turbulent flow, fluid random fluctuations affects the development of boundary layer significantly but molecule diffusion does not. The thicknesses of both boundary layers are equal where $\delta_u = \delta_{th}$.

3.8.3 Average heat transfer coefficient (h_{avg})

The average heat transfer coefficient (h_{avg}) [$W/m^2.K$] can be calculated as follows:

$$h_{avg} = \frac{q}{A_{eff}(T_{w,avg} - T_{f,avg})} \quad (3.23)$$

Where q is the total heat flow supplied through the MCHS ($q = q'' A_s$ where q'' is the heat flux applied at the bottom surface of heat sink [W/m^2] and A_s is the heated surface area of heat sink's bottom surface) [m^2], A_{eff} is the effective area (A_{eff} formula is based on design of heat sink used) [m^2], $T_{w,avg}$ is the average wall temperature of mini-channel [K] and $T_{f,avg}$ is the fluid bulk average temperature ($T_{f,avg} = (T_{f,in} + T_{f,out})/2$) [K].

3.8.4 Average Nusselt Number (Nu_{avg})

The Average Nusselt number (Nu_{avg}) can be expressed as follows:

$$Nu_{avg} = \frac{h_{avg} \cdot D_h}{k_f} \quad (3.24)$$

Where D_h is the hydraulic diameter [m], h_{avg} is the average heat transfer coefficient [$\text{W/m}^2\cdot\text{K}$] and k_f is the thermal conductivity of water measured at $T_{f,avg}$ [$\text{W/m}\cdot\text{K}$].

Using water as coolant and under the condition of four-sided heated channel while having fully developed laminar flow, Shah and London [181] proposed the correlation of Nusselt number equation as follows:

$$\begin{aligned} Nu_{H1} = & 8.235(1 - 2.0421 \\ & \alpha + 3.0853 \alpha^2 - 2.4765 \alpha^3 + 1.0578 \alpha^4 \\ & - 0.1861 \alpha^5) \end{aligned} \quad (3.25)$$

Based from Eq. 3.25, it useful to calculate the Nusselt number for the geometries that are regularly used under the condition of temperature of circumferential wall is fixed and axial wall heat flux (H1 boundary condition) that is uniform with the heat load of heating on four surface sides [182]. Under the thermally fully developed flow condition within the laminar regime, the correlation of Nusselt number under the boundary condition of H1 is as follows:

$$\begin{aligned} Nu_{H1} = & 8.235(1 - 1.883 \\ & \alpha + 3.767 \alpha^2 - 5.814 \alpha^3 + 5.361 \alpha^4 - 2 \alpha^5) \end{aligned} \quad (3.26)$$

Using the case of heating for three sides of surfaces, which is the real boundary condition, under the thermally developing flow where the Nusselt number can be calculated based on correlation proposed by Phillips [188, 189] which is as follows:

$$Nu_{x,3}(x^*, \alpha) \approx Nu_{x,4}(x^*, \alpha) \frac{Nu_{fd,3,lam}(\alpha)}{Nu_{fd,4,lam}(\alpha)} \quad (3.27)$$

Where under the condition of fully developed region, the Nusselt numbers for four-sided and three-sided heating are denoted by $Nu_{fd,4,lam}$ and $Nu_{fd,3,lam}$ respectively. All of these values

can be easily determined using Eq. 3.25 and Eq. 3.26 respectively and using data proposed by Phillips [190] which can be referred to Table 3-2. The aspect ratio varies between 0.1 and 10 respectively

Table 3-2: Nusselt number under laminar regime with fully developed flow condition. The three and four walls experience heat transfer under the H1 boundary condition along with uniform heat flux axially [190].

$\alpha = \frac{W_{ch}}{H_{ch}}$	$Nu_{fd\_4,lam}$	$Nu_{fd\_3,lam}$	$\alpha = \frac{W_{ch}}{H_{ch}}$	$Nu_{fd\_4,lam}$	$Nu_{fd\_3,lam}$
0	8.235	8.235	0.9	3.62	----
0.1	6.785	6.939	1	3.608	3.556
0.2	5.738	6.072	1.43	3.75	3.195
0.3	4.99	5.393	2	4.123	3.146
0.4	4.472	4.885	2.5	4.472	3.169
0.5	4.123	4.505	3.33	4.99	3.306
0.6	3.895	----	5	5.738	3.636
0.7	3.75	3.991	10	6.785	4.252
0.8	3.664	----	∞	8.235	5.385

At a distance x , for the four-sided heating under the condition of thermally developing region, the Nusselt number is denoted by $Nu_{x,4}$. Based on the data in Table 3-3, the $Nu_{x,4}$ values can be obtained which is suggested by Phillips [189, 190]. The $Nu_{x,4}$ is calculated based on different of distance of dimensionless thermal axial (x^*) and inverse aspect ratio (α^*) respectively.

Table 3-3: Nusselt number of developing flow in laminar regime [189, 190].

$x^* = \frac{x}{Re.D_h.Pr}$	$Nu_{x,4}$					
	$\alpha^* \leq 0.1$	$\alpha^* = 1$	$\alpha^* = 2$	$\alpha^* = 3$	$\alpha^* = 4$	$\alpha^* \geq 10$
0.0001	31.6	25.2	23.7	27	26.7	31.4
0.0025	11.2	6.9	9.2	9.9	10.4	11.9
0.005	9	7.1	7.46	8.02	8.44	10
0.00556	8.8	6.86	7.23	7.76	8.18	9.8
0.00625	8.5	6.6	6.96	7.5	7.92	9.5
0.00714	8.2	6.32	6.68	7.22	7.63	9.3
0.00833	7.9	6.02	6.37	6.92	7.32	9.1
0.01	7.49	5.69	6.05	6.57	7	8.8
0.0125	7.2	5.33	5.7	6.21	6.63	8.6
0.0167	6.7	4.91	5.28	5.82	6.26	8.5
0.025	6.2	4.45	4.84	5.39	5.87	8.4
0.033	5.9	4.18	4.61	5.17	5.77	8.3
0.05	5.55	3.91	4.38	5	5.62	8.25
0.1	5.4	3.71	4.22	4.85	5.45	8.24
>1	5.38	3.6	4.11	4.77	5.35	8.23

The average Nusselt number under the condition of developing laminar flow can be calculated when the effect of thermal entrance is considered which is as follows:

$$Nu_{avg} = \sqrt[3]{\left(\frac{2.22}{(x^*)^{1/3}}\right)^3 + (Nu_{fd,lam})^3} \quad (3.28)$$

Copeland [191] proposed the correlation of Eq. 3.28, who used the data from [181]. The expression of distance of dimensionless thermal axial (x^*) is as follows:

$$x^* = \frac{x}{Re.Pr.D_h} \quad (3.29)$$

Under the flow of fully developed laminar region, the Nusselt number can be calculated using equation proposed by Copeland [191] specifically for rectangular mini- and micro-channel which is as follows:

$$Nu_{fd,lam} = 8.31 \times G - 0.02 \quad (3.30)$$

Under the H1 boundary conditions, based on the heating of three-sided and four-sided wall of a rectangular micro-channels where Hwu and Qu [192] found a new correlation for. The correlation has the ability to perform prediction of heat transfer for laminar region with single-phase flow and undergoes thermally developing region. The correlation equations for the four- and three-sided heated wall under the conditions of thermally developing region are set as function of inverse aspect ratio and dimensionless thermal axial distance respectively which are as follows:

$$Nu_{x,4} = Nu_{fd,4,lam} + 8.68(x^* \times 10^3)^{-0.506} \cdot e^{(-8.779\alpha^* - 23.157)\alpha^*} \quad (3.31)$$

$$Nu_{x,3} = Nu_{fd,3,lam} + 8.68(x^* \times 10^3)^{-0.506} \cdot e^{(-9.427\alpha^* - 23.472)\alpha^*} \quad (3.32)$$

From the correlations proposed by [192], it is being compared with two other proposed correlations from other researchers. Substituting the Eq. 3.31 with the one proposed by Phillips [189, 190], a good agreement was obtained. Another correlation comparison with Lee and Garimella [193] was made. They have considered the usage of CFD software to find the solution of conjugate heat transfer under the laminar flow regime for rectangular micro-channel. They have set the H1 boundary condition for the heat sink with the influence of thermally developing region for four-sided heating in order to calculate the average Nusselt number where α^* is varied to between 1 and 10 respectively. Under the thermally developing region, the average Nusselt number can be calculated if the distance of dimensionless thermal axial (x^*) is smaller than length of dimensionless thermal entrance (x_{th}^*). The equation of average Nusselt number is as follows:

$$Nu_{x,4} = \frac{1}{C_1(x^*)^{C_2+C_3}} + C_4 \text{ where } 1 \leq \alpha^* \leq 10 \text{ and } x^* \leq x_{th}^* \quad (3.33)$$

The formula of x_{th}^* and C_1 to C_4 can be calculated as follows:

$$x_{th}^* = \frac{-1.275 \times 10^{-6}}{\alpha^6} + \frac{4.709 \times 10^{-5}}{\alpha^5} + \frac{6.902 \times 10^{-4}}{\alpha^4} + \frac{5.014 \times 10^{-3}}{\alpha^3} - \frac{1.769 \times 10^{-2}}{\alpha^2} + \frac{1.845 \times 10^{-2}}{\alpha} + 0.05691 \quad (3.34)$$

$$C_1 = \frac{-2.757 \times 10^{-3}}{\alpha^3} + \frac{3.274 \times 10^{-2}}{\alpha^2} - \frac{7.464 \times 10^{-5}}{\alpha} + 4.476 \quad (3.35)$$

$$C_2 = 0.6391 \quad (3.36)$$

$$C_3 = \frac{1.604 \times 10^{-4}}{\alpha^2} - \frac{2.622 \times 10^{-3}}{\alpha} + 2.568 \times 10^{-2} \quad (3.37)$$

$$C_4 = 7.301 - \frac{13.11}{\alpha} + \frac{15.19}{\alpha^2} - \frac{6.094}{\alpha^3} \quad (3.38)$$

To obtain the three-sided heating wall Nusselt number, $Nu_{x,4}$ is substituted into Eq. 3.27. When $x^* \geq x_{th}^*$, assumption of fully developed is assumed. The Eq. 3.26 is used for the calculation of Nusselt number under the condition of micro-channels with three-sided heating walls.

For smooth circular tube, Dittus and Boelter [194] proposed the calculation of Nusselt number under the condition of turbulent flow for the case of micro-channel with rectangular cross-section. The Dittus-Boelter equation is as follows:

$$Nu_{D-Bfd} = 0.023. Re^{0.8}. Pr^n \quad (3.39)$$

The value of n is equals to 0.3 and 0.4 when the wall is cooling and heating the fluid respectively. Petukhov [195] also considered smooth circular tube under the condition of turbulent flow which is thermally fully developed where the correlation equation is as follows:

$$Nu_{P-tfd} = \frac{\left(\frac{f_D}{8}\right) Re. Pr}{C + 12.7 \sqrt{\frac{f_D}{8}} \left(Pr^{\frac{2}{3}} - 1\right)} \quad (3.40)$$

Where the value of C can be calculated using as follows:

$$C = 1.07 + \frac{900}{Re} - \frac{0.63}{1 + 10Pr} \quad (3.41)$$

Based on Eq. 3.40, the Darcy friction factor of f_D can be calculated as follows:

$$f_D = (0.79 \ln(Re) - 1.64)^{-2} \text{ where } 3000 < Re < 5 \times 10^6 \quad (3.42)$$

To consider the effects of region of thermal entrance towards micro-channel, Al-Arabi [196] used the circular tube with both constant heat flux and wall temperature where Rohsenow et al. [185] proposed the correlation equation of Nusselt number as follows:

$$Nu_{P\_tdf} = Nu_{P\_tfd} \left(1 + \frac{B}{\frac{L_{ch}}{D_h}} \right) \quad (3.43)$$

For the region of thermally developing turbulent flow, The Nusselt number is denoted as Nu_{P_tdf} where the value of B has the formula as follows:

$$B = \frac{\left(\frac{L_{ch}}{D_h}\right)^{0.1} \left(0.68 + \frac{3000}{Re}\right)}{Pr^{\frac{1}{6}}} \quad (3.44)$$

The complete equation for the Petukhov's correlation under the condition of turbulent flow which is thermally developing is given as follows:

$$Nu_{P\_tdf} = \left(\frac{\left(\frac{f_D}{8}\right) Re \cdot Pr}{1.07 + \frac{900}{Re} - \frac{0.63}{1 + 10Pr} + 12.7 \sqrt{\frac{f_D}{8}} (Pr^{\frac{2}{3}} - 1)} \right) \left(1 + \frac{\left(\frac{L_{ch}}{D_h}\right)^{0.1} \left(0.68 + \frac{3000}{Re}\right)}{Pr^{\frac{1}{6}} \left(\frac{L_{ch}}{D_h}\right)} \right) \quad (3.45)$$

The friction factor within the Eq. 3.45 must be used based on the equation set by Haaland's of Eq. 3.46 [197] to consider the surface roughness effect which is as follows:

$$f_{Haaland} = \left(-1.8 \log_{10} \left(\frac{6.9}{Re} + \left(\frac{\frac{\varepsilon}{D_h}}{3.7} \right)^{1.11} \right) \right)^{-2} \quad (3.46)$$

To add, an adjustment can be made towards the Dittus-Boelter equation based on Al-Arabi's proposed correlation equation including the effects of flow which is thermally developing. The use of Dittus-Boelter equation is originally for smooth surface which can be modified to include surface roughness based what has been proposed by Norris [198] which is as follows:

$$\frac{Nu_{rough}}{Nu_{smooth}} = \left(\frac{f_{rough}}{f_{smooth}} \right)^n \quad (3.47)$$

The function of $n = 0.68Pr^{0.215}$. From the findings by Norris, the increase of surface roughness will not led to the increase of Nusselt number when $\left(\frac{f_{rough}}{f_{smooth}}\right) > 4$. The $\left(\frac{f_{rough}}{f_{smooth}}\right)$ is calculated based on the Haaland's equation for micro-channel's with both smooth and rough walls. The correlation by Dittus-Boelter under the flow of turbulent with thermally developing condition can be modified to be as follows:

$$Nu_{D-B\_tdf} = (0.023.Re^{0.8}.Pr^{0.4})\left(\frac{f_{rough}}{f_{smooth}}\right)^{0.68Pr^{0.215}}\left(1 + \frac{\left(\frac{L_{ch}}{D_h}\right)^{0.1}\left(0.68 + \frac{3000}{Re}\right)}{Pr^{\frac{1}{6}}\left(\frac{L_{ch}}{D_h}\right)}\right) \quad (3.48)$$

Gnielinski, [199] proposed the empirical correlation equation based on the Petukhov equation (Eq. 3.40) which is as follows:

$$Nu_{Gn-tdf} = \frac{\left(\frac{f_D}{2}\right)(Re - 10^3).Pr}{1 + 12.7\sqrt{\frac{f_F}{2}}\left(Pr^{\frac{2}{3}} - 1\right)} \quad (3.49)$$

The Gnielinski correlation equation is only true when Re has minimum and maximum limit of 3×10^3 and of 5×10^6 while Pr has to be lower than 2000 and higher than 0.5. Under the flow of fully developed turbulent, the smooth circular tubes will have Fanning friction factor (f_F) based on what is proposed by Filonenko [200] which is as follows:

$$f_F = (1.58 \ln(Re) - 3.28)^{-2} \text{ where } 4000 < Re < 5 \times 10^6 \text{ and } 0.5 < Pr < 10^6 \quad (3.50)$$

The region of thermal entry effect consideration will be added where this will change the Eq. 3.49 to what has been proposed by Kakac et al. [201] which is as follows:

$$Nu_{Gn-tdf} = \frac{\left(\frac{f_F}{2}\right)(Re - 10^3).Pr}{1 + 12.7\sqrt{\frac{f_F}{2}}\left(Pr^{\frac{2}{3}} - 1\right)}\left(1 + \left(\frac{D_h}{L_{ch}}\right)^{2/3}\right) \quad (3.51)$$

The equation of Gnielinski [199] is being utilised by Phillips [190]for the Nusselt number prediction based on the region of turbulent developing flow in circular duct.

$$Nu = 0.012 \left(1 + \left(\frac{D_h}{L_{ch}} \right)^{\frac{2}{3}} \right) (Re^{0.87} - 280) Pr^{0.4} \quad (3.52)$$

3.8.5 Total thermal resistance analysis (R_{th})

Heat is transferred by means of conduction and convection from the chip to heat sink and heat sink to coolant respectively [202]. The capability of micro-channel heat sink for cooling electronics is measured by using parameter of total thermal resistance, R_{th} [K/W]. R_{th} is defined as ratio of temperature difference from electronics die along module to coolant fluid per watt of energy transferred [203] which can be referred to Eq. (3.53).

$$\begin{aligned} R_{th} &= \frac{T_{surf,max} - T_{f,in}}{A_s q} = R_{cond} + R_{conv} + R_{bulk} \\ &= \frac{H_b}{k_s \cdot A_s} + \frac{1}{h_{avg} \cdot A_{eff}} + \frac{1}{\dot{m} \cdot C_{Pt}} \end{aligned} \quad (3.53)$$

Where,

$$R_{cond} = R_{cond,base} + R_{cond,fin}$$

$$R_{conv} = R_{conv,base} + R_{conv,fin}$$

Where $T_{surf,max}$ is the heat sink maximum base surface temperature [K]; $T_{f,in}$ is inlet temperature of fluid [K]; A_s is the heat sink base surface area [m²]; q'' is the heat flux applied at base [W/m²]; H_b is the thickness of substrate [m]; k_s is the thermal conductivity of heat sink material (copper) [W/(m·K)]; h_{avg} is the average heat transfer coefficient [W/(m²·K)] ($h_{avg} = \frac{q}{A_{eff}(T_{w,avg} - T_{f,avg})}$) with A_{eff} as the effective area available for heat transfer [m²]; $T_{w,avg}$ is the average is average surface base temperature [K] and $T_{f,avg}$ is the average is average fluid temperature [K] ($T_{f,avg} = (T_{f,in} + T_{f,out})/2$); $\dot{m}$ is the total mass flow rate through the channel [kg/s] ($\dot{m} = \rho_f U_{in} A_{ch}$) with A_{ch} as cross-sectional area of channel; C_{Pt} is the fluid specific heat capacity [J/(kg·K)] measured at $T_{f,avg}$.

At low thermal resistance, the electronics temperature rise is minimised. For a system, the total thermal resistance, R_{total} , can be defined as the addition of individual layer's thermal resistance dependent on thermal conductivity and material thickness referring to Eq. (3.53). From Figure 3-12, conventional cooling method using MCHS under forced convection with water as base fluid. Symmetry boundary condition used to simplify the MCHS, and thermal resistance analysis is illustrated. Shao et al. [204] investigated the main thermal resistances on MCHS from Figure 3-12. Conduction thermal resistance for bottom base of heat sink and fins are $R_{cond,base}$ and

$R_{cond,fin}$ respectively. Convection thermal resistance for bottom base of heat sink and fins wall are $R_{conv,base}$ and $R_{conv,fin}$ respectively. R_{bulk} is the thermal resistance dependent to bulk temperature rise. From Eq. (3.53), the individual component of conduction and convection thermal resistances are presented.

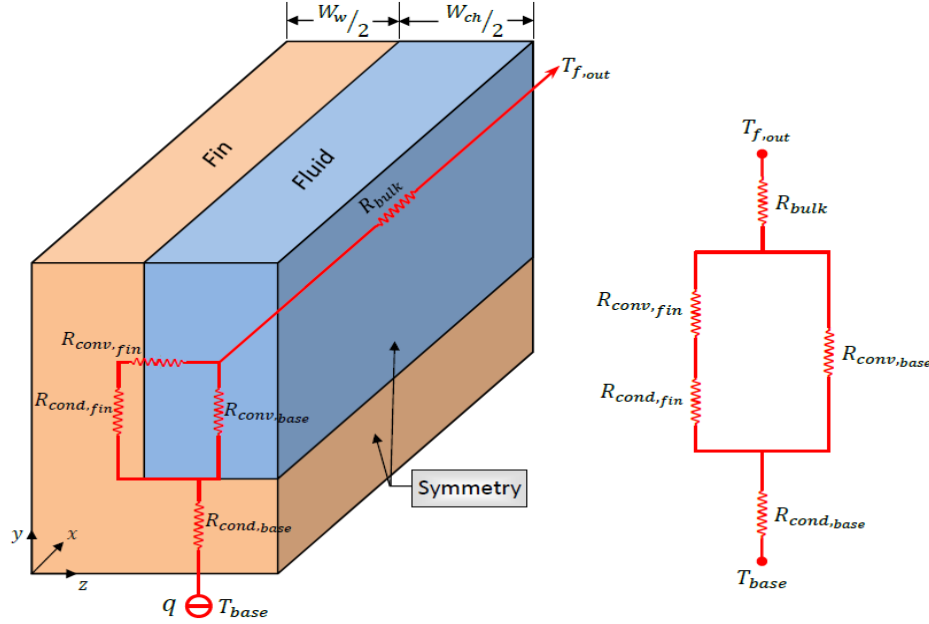


Figure 3-12: Shows the networks of thermal resistances [33].

3.8.6 Pressure drop (ΔP)

Pressure drop (ΔP) is defined as the pressure difference between inlet and outlet of heat sink as follows:

$$\Delta P = P_{inlet} - P_{outlet} \quad (3.54)$$

Where ΔP is the difference in pressure between inlet and outlet (pressure drop) [Pa] and P_{inlet} and P_{outlet} is the inlet and outlet pressure of fluid of heat sink [Pa]. The Eq. 3.54 will be used in Chapter 6 since single inlet-outlet channel is used.

For Chapter 7, double inlet-outlet mini-channels are used with separated serpentine, therefore, the newly constructed ΔP formula is as follows:

$$\Delta P = P_{inlet,1} + P_{inlet,2} - P_{outlet,1} - P_{outlet,2} \quad (3.55)$$

For Chapter 8, almost similar heat sink design executed compared with the design in Chapter 7 with a slight difference of adding through-holes connecting two separated serpentine, thus, the suitable ΔP formula can be defined as follows:

$$\Delta P = \frac{P_{inlet,1} + P_{inlet,2} - P_{outlet,1} - P_{outlet,2}}{2} \quad (3.56)$$

3.8.7 Temperature standard deviation (TSD)

Temperature distribution standard deviation (TSD) can be calculated as follows:

$$T_{TSD} = \sqrt{\frac{\sum (T_i - T_{ave})^2}{N}} \quad (3.57)$$

Where T_i is the local point temperature at each node of the mesh grid, T_{ave} is the average temperature of the whole heat sink and N is the number of nodes within the mesh grid. TSD is calculated to quantify the temperature uniformity of the heat sink. Lower magnitude of TSD gives an indication of higher uniformity of temperature distribution along the domain of heat sink.

3.9 Performance evaluation criteria (PEC) index

A thermal-hydraulic performance evaluation criteria (PEC) index [205, 206] is utilised to perform the enhancement of heat transfer effectiveness using serpentine channel heat sink with fins and taking account the penalty paid from the calculated pressure drop. The initial formula of PEC was firstly introduced using the terms of friction factors and Nusselt numbers as contrasted measures of hydraulic and thermal characteristics. In this work, the PEC index is presented using the term of direct computed quantities which are equivalent in [205, 206] to the average Nusselt number (Nu_{avg}) and pressure drop (ΔP) proposed in [145]. Thus, PEC index is presented as follows:

$$PEC = \frac{R_{th,conventional}/R_{th,new}}{(\Delta P_{new}/\Delta P_{conventional})^{1/3}} = \frac{E_{R_{th}}}{\sqrt[3]{E_{\Delta P}}} \quad (3.56)$$

Where $R_{th,conventional}$ and $R_{th,new}$ are total thermal resistances of the conventional serpentine heat sink and novel design serpentine heat sink respectively. $\Delta P_{conventional}$ and ΔP_{new} are the pressure drop of the conventional serpentine hat sink and novel design serpentine heat sink respectively. $E_{R_{th}}$ and $E_{\Delta P}$ are the ratio of thermal resistance and pressure drop of the novel design to the conventional design.

3.10 Chapter summary

Within this chapter, the fundamentals of heat and fluid flow for this work are presented. Fundamentals from basic concept of fluid flow, boundary layers and characteristics of heat transfer and hydraulic are explained in details. For this work, fully developed (to ensure constant velocity profile across channel and symmetric flow) forced convection internal incompressible (to reduce computational time) flow is considered. Unsteady flow is assumed to ensure

simplicity of CFD models. Turbulent flow model is used for validation and optimisation analysis while laminar flow is used for the detailed analysis to compare the thermal-hydraulic performance between different heat sinks and parametric analysis in order to reduce computational time. Breaking of boundary layers analysis are important to use for studying its effect on improving heat transfer around secondary channels. The Reynolds number parameter is used to vary the fluid velocity within the CFD software. The parameters used to compare the thermal performance are Average Nusselt number and total thermal resistance while for hydraulic performance, pressure drop parameter is used. The Performance Evaluation Criteria parameter is used to calculate the overall thermal-hydraulic performance of heat sink by including both thermal and hydraulic parameters in one equation.

Chapter 4 : Numerical Analysis

4.1 Introduction to Numerical analysis

Computational Fluid Dynamics (CFD) numerical analysis method has been widely used to analyse the complex multi-physics systems comprising heat transfer, fluid flow and mass transport study using computer-based simulation. To give an example, CFD is exploited in electronics industry for energy systems and heat transfer optimisation for electronics cooling. Discretised partial differential equations (PDEs) are formed using CFD to search for approximate solutions of governing equations for fluid flow and heat transfer from the predetermined elements grid giving a certain corresponding number of points within the boundary of geometry [207, 208].

For the current problem, all CFD models are solved using the finite-element method (FEM) utilised by COMSOL Multiphysics version 5.4 which is very suitable for problems relating multiphysics and it is stable compared to other numerical methods. The Galerkin method used by COMSOL Multiphysics converts the PDEs into integral scheme of FEM [209]. The FEM is a numerical method, utilised governing equations integral form as its starting point [210]. The algebraic form of governing equations using FEM are being discretized over domain entirely. An element is the smallest sub-geometry consisting nodes (refer Figure 4-1). From this elemental analysis, the interaction between adjacent elements induced continuous change in various properties [211].

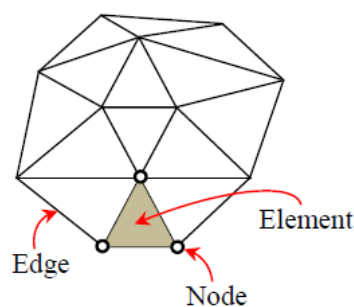


Figure 4-1: Shows the typical mesh of finite element [211].

Three major stages using CFD in carrying out numerical analysis modelling of heat transfer and fluid flow [208] can be referred schematically in Figure 4-2.

- 1) Pre-processing: In this initial step, the geometry is defined which are the computational domain, sub-domains from the entire domain (generation of

grid), the physical and chemical phenomena selection, the properties of fluid and solid are defined and boundary conditions specifications.

- 2) Simulation phase (processing): The governing equations of heat transfer and fluid flow are solved simultaneously within the domain of computational analysis.
- 3) Post-processing: The analysis of results obtained from numerical solutions are carried out. This comprises the analysis results, plots, graphics, animations and writing full reports presentation.

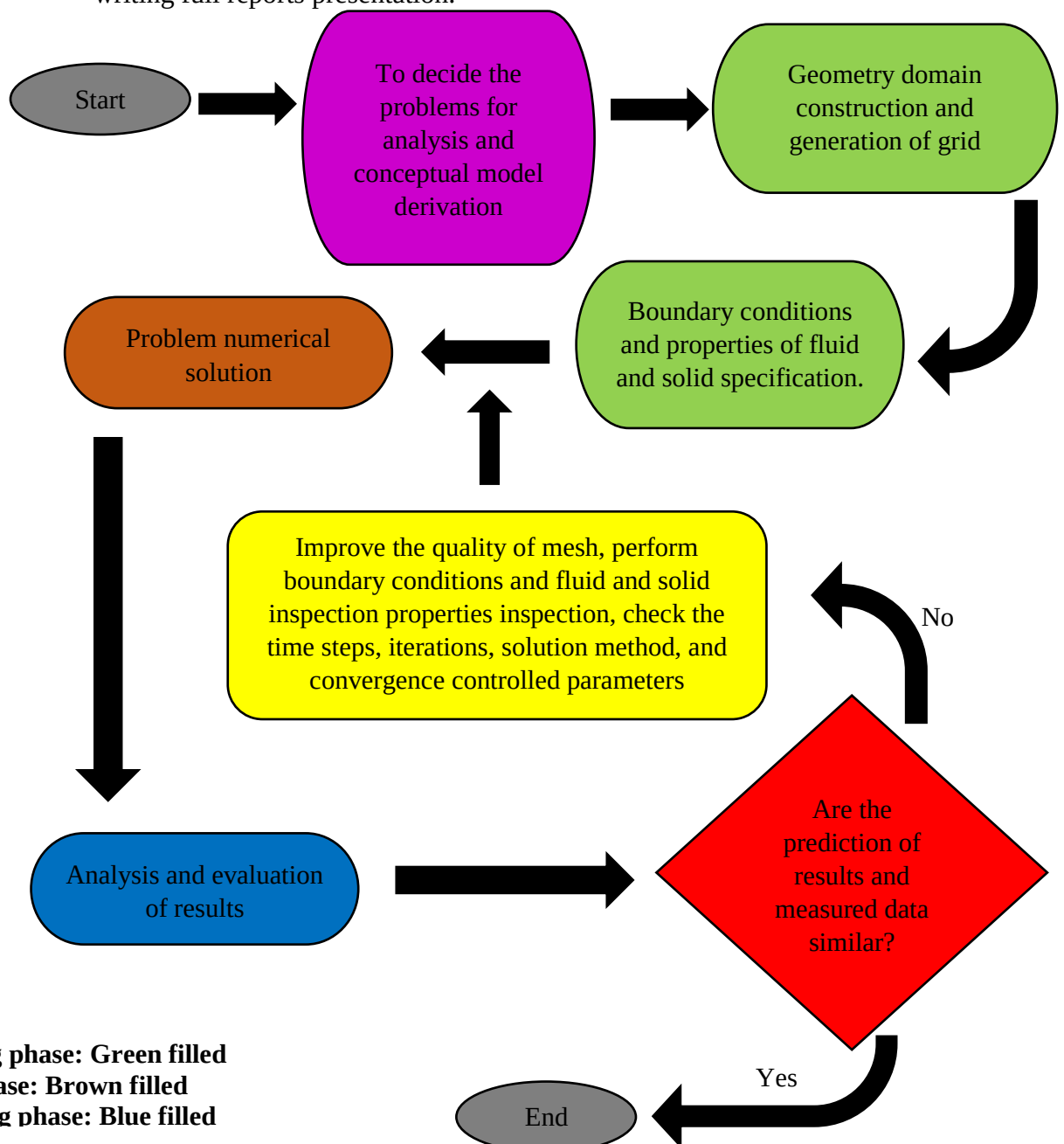


Figure 4-2: The stages of CFD simulation.

4.2 Pre-processing (Generation of grid)

COMSOL Multiphysics version 5.4 utilises FEM solving PDE equations for both regimes of laminar and turbulent flow. Mesh generation of domain for computational analysis consisting of two different types of 2-D elements (which are called quadrilateral and triangular respectively). To solve 3D models, mesh is generated initially to discretise the entire domain using four main element types of tetrahedral (tets), hexahedral (bricks), triangular prisms (prisms) and pyramids which can be utilised in different combinations. Element types can be referred in Figure 4-3.

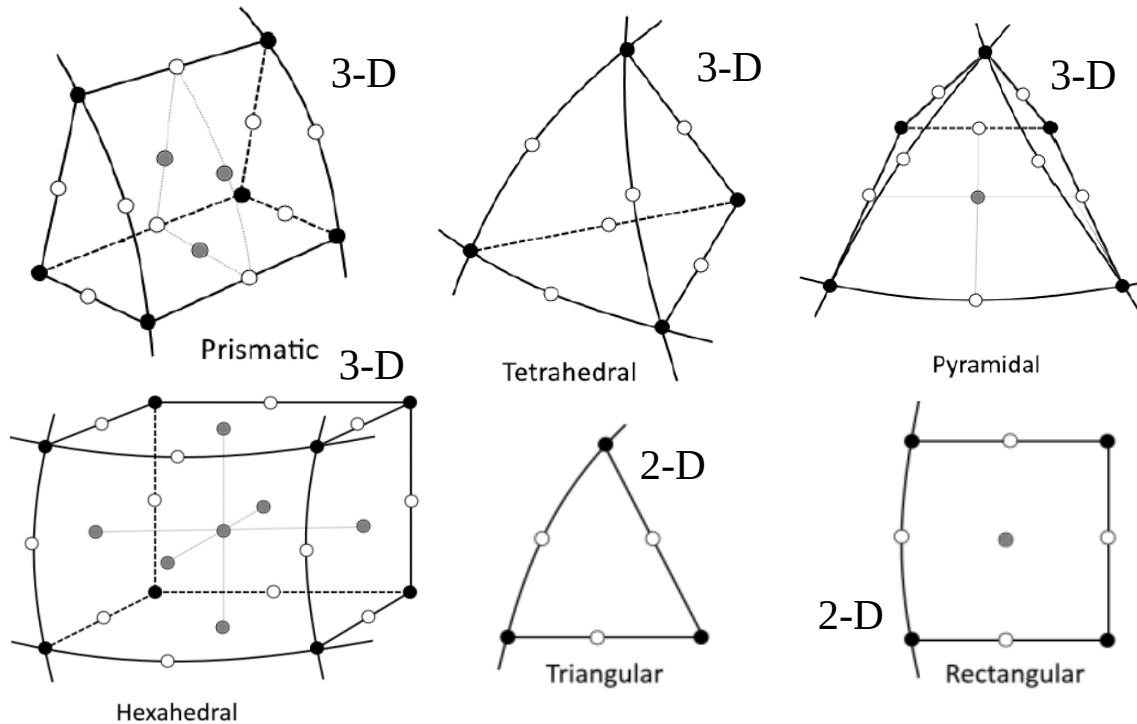


Figure 4-3: Shows the different element types in 3D space [212].

From Figure 4-3, the use of pyramids element type only in creating mesh transition between hexahedral and tetrahedral element types. For complex domain geometry, higher difficulties faced in creating the mesh using bricks due to very low mesh quality and difficult pressure solver convergence. For the present work, the tetrahedral mesh is applied to the entire domain which also the default element type provided in COMSOL Multiphysics. For higher accuracy, second order tetrahedral mesh is used due to the existence of mid nodes (white filled nodes) between nodes (black filled nodes).

4.2.1 Mesh of structured or unstructured grids

The generation of mesh can be divided into three major methods of structured, unstructured mesh and combination of both which is sole affected by the shape of geometry. Commonly, the geometries with simple design would use the structured mesh. Unstructured mesh uses 3-D

elements in Figure 4-3 and it is more favourable than structured mesh due to its flexibility to assemble the elements freely for highly complex geometries and domain decomposition is not required [213]. The advantage of structured mesh is less numerical diffusion since the grid has well defined geometric shape which can be aligned with direction of flow while having so much lower computational power to run it. Oppositely, the unstructured mesh has the advantage of no domain decomposition and easily applied for very complex geometries. Referring Figure 4-4, the difference between structured and unstructured are shown in 2-D.

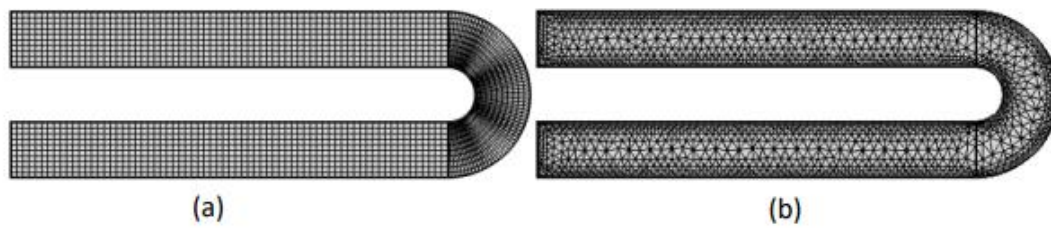


Figure 4-4: Shows the (a) structured (b) unstructured mesh in 2-D space [33].

4.2.2 Quality of mesh

Mesh quality (Q_{mesh}) affects solution accuracy and COMSOL is capable to automatically check the quality of mesh element under both regimes of laminar and turbulent flow. Since the current work involves complex geometry, the whole domain is meshed using free tetrahedral elements. COMSOL uses the combination of Delaunay and front algorithms for free tetrahedral element filling around domains. At contact surface between solid and fluid boundaries, boundary element solver is used to create boundary element mesh. Grid refinement is applied on small geometry and curved edges to capture the solution at high accuracy.

For the regime of laminar flow, Q_{mesh} for tetrahedral element is calculated by COMSOL using the following formula:

$$Q_{mesh} = \frac{72\sqrt{3}V}{(h_1^2 + h_2^2 + h_3^2 + h_4^2 + h_5^2 + h_6^2)^{1.5}} \quad (4.1)$$

Where V is the volume and h_1, h_2, h_3, h_4, h_5 and h_6 are the element lengths of edge [214]. To have high element quality, the Q_{mesh} must be more than 0.1 to achieve reasonable edge ratio, volume ratio and aspect ratio avoiding stretched or distorted elements.

For the regime of turbulent flow, the wall lift-off value in viscous units (y^+) can be calculated using the following formula [215]:

$$y^+ = \frac{\delta_w \cdot u_\tau}{\nu} \quad (4.2)$$

The wall lift-off value (δ_w) is the distance between the wall and the closest boundary layer mesh of the fluid (refer Figure 4-5). The ν is the fluid kinematic viscosity and u_τ is the friction velocity defined as the following formula:

$$u_\tau = C_\mu^{\frac{1}{4}} \sqrt{k} \quad (4.3)$$

Where k is the Von Karman constant ($k = 0.418$ for wall lift-off); C_μ is the empirical constant ($C_\mu = 0.09$) [215].

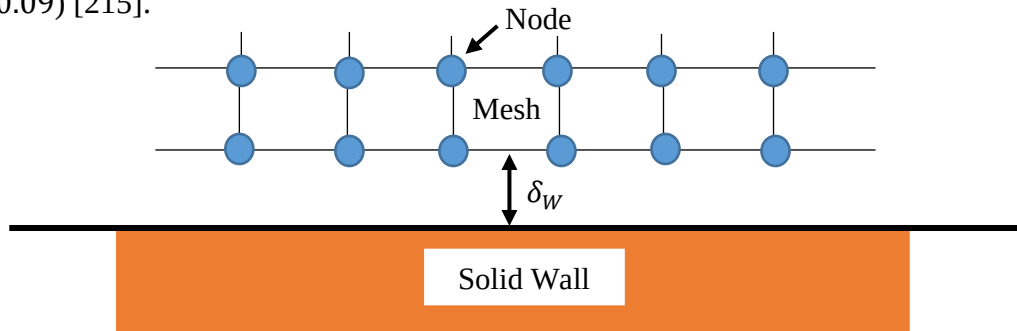


Figure 4-5: Shows the distance δ_w from wall to the nearest boundary layer mesh.

The y^+ value has a great impact on the solution accuracy to resolve the boundary layers effectively. To achieve the highest accuracy the value of y^+ must be equal to 11.06 according to [215] based on the point where viscous sublayer meets the logarithmic layer (region where fluid flow is observed near surface or wall boundary where the fluid velocity profile is characterised using logarithmic relationship between distance from boundary and fluid velocity). The recommended range of y^+ value is between 11.06 and 300 suggested by [215] because from their findings beyond 300, the results of simulations becomes less reliable and accuracy since near-wall region resolution is lacking.

4.3 Processing (Computational model)

The modelling of 3D flow and heat transfer of mini-channel heat sink (Mini-CHS) was established based on the assumptions in subchapter 4.4.1. The fluid flow is modelled using the governing equations (continuity and Navier-Stokes momentum equations) for both regimes of laminar and turbulent flow. The derivation of governing equations starts from compressible flow going to incompressible flow by applying assumptions for simplicity which is discussed in detailed under sub-chapter 4.3.1.

4.3.1 Navier-Stokes Equations

The heat transfer and fluid flow of Mini-CHS are solved by the governing equations of continuity, momentum and energy for both fluid and solid domain which described as follows [208, 216].

4.3.1.1 Continuity equation

In 3D space, the continuity equation is as follows:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0 \quad (4.4)$$

The vector form of Eq. 4.4 can be written as follows:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \quad (4.5)$$

Where ρ as the density of fluid (kg/m³), ∇ is the gradient operator and $\mathbf{u}$ is the vector of fluid velocity which are as follows:

$$\nabla = \frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}k, \quad \mathbf{u} = ui + vj + wk$$

Where u , v and w are the component of velocities in 3D space for directions of x –, y – and z –. Referring to Eq. 4.4, it applies for the case of compressible and unsteady flow in 3D space. Thus, to have an incompressible flow, the term of $\frac{\partial \rho}{\partial t}$ is equal to zero as ρ is constant regardless if the flow are either steady or unsteady [217]. Therefore, Eq. 4.5 can be reduced as follows:

$$\nabla \cdot \mathbf{u} = 0 \quad (4.6)$$

4.3.1.2 Momentum equation

The momentum equation under laminar flow (single-phase) is as follows:

$$\frac{\partial(\rho \mathbf{u})}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = -\nabla p + \nabla \cdot \tau_{ij} + \mathbf{F}_b \quad (4.7)$$

Where $\mathbf{F}_b$ is the vector of body force per unit volume (N/m³) due to gravitational forces; p is the pressure of fluid (Pa); τ_{ij} is the viscous stress tensor (Pa) which can be expressed as follows:

$$\tau_{ij} = \begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{bmatrix} \quad (4.8)$$

For compressible Newtonian fluid, Eq. 4.7 is expressed as follows:

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \nabla \cdot \mathbf{u} = -\nabla p + \nabla \cdot \left[\mu (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) - \frac{2}{3} \mu (\nabla \cdot \mathbf{u}) \mathbf{I} \right] + \rho \mathbf{g} \quad (4.9)$$

Where μ as coefficient of molecular viscosity; $\mathbf{I}$ as unit diagonal matrix; $\mathbf{g}$ as vector of gravity (m²/s) ($\mathbf{g} = g_x i + g_y j + g_z k$). The formula of strain-rate tensor, $\dot{\gamma}$, is as follows:

$$\dot{\gamma} = \mu (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) \quad (4.10)$$

To get viscous stress tensor, τ , the $\dot{\gamma}$ is multiplied by μ which yields, $\tau = \mu(\dot{\gamma} - \frac{2}{3}\mu(\nabla \cdot \mathbf{u})\mathbf{I})$. To achieve incompressible flow, the velocity vector divergence is equal to zero ($\nabla \cdot \mathbf{u} = 0$), thus Eq. 4.9 can be simplified to be as follows:

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \nabla \cdot \mathbf{u} = -\nabla p + \nabla \cdot [\mu(\nabla \mathbf{u} + (\nabla \mathbf{u})^T)] + \rho \mathbf{g} \quad (4.11)$$

Eq. 4.11 can be further reduced into component form when μ is constant which is as follows:

x-direction:

$$\rho \frac{\partial u}{\partial t} + \rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + \rho g_x \quad (4.11a)$$

y-direction:

$$\rho \frac{\partial v}{\partial t} + \rho \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) + \rho g_y \quad (4.11b)$$

z-direction:

$$\begin{aligned} \rho \frac{\partial w}{\partial t} + \rho \left(u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) \\ = -\frac{\partial p}{\partial z} + \mu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) + \rho g_z \end{aligned} \quad (4.11c)$$

The equations of 4.11a, 4.11b and 4.11c are the momentum equations in 3D space for incompressible and unsteady flow. The vector form of these equations is as follows:

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho(\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla P + \mu(\nabla^2 \mathbf{u}) + \rho \mathbf{g} \quad (4.12)$$

When the term of $\frac{\partial \mathbf{u}}{\partial t} = 0$, the flow of fluid is assumed steady.

The compressible and unsteady Newtonian fluid momentum equation under turbulent flow (single-phase) is as follows:

$$\begin{aligned} \rho \frac{\partial \mathbf{u}}{\partial t} + \rho(\mathbf{u} \cdot \nabla) \mathbf{u} \\ = \nabla \cdot \left[-p\mathbf{I} + (\mu + \mu_T)(\nabla \mathbf{u} + (\nabla \mathbf{u})^T) - \frac{2}{3}(\mu + \mu_T)(\nabla \cdot \mathbf{u})\mathbf{I} \right. \\ \left. - \frac{2}{3}\rho k\mathbf{I} \right] + \mathbf{F}_b \end{aligned} \quad (4.13)$$

To capture the physics well for conjugate heat transfer model, $k - \omega$ turbulence model is used to solve the governing equations for single-phase turbulent flow [218, 219] since it predicts flow-

separation better and works well with flow near wall region where the development of adverse pressure gradient. The $k - \omega$ turbulence model has two additional variables: specific dissipation rate, ω (1/s) and turbulent kinetic energy, k (m^2/s^2) with respective transport equations which is as follows [220]:

$$\rho \frac{\partial k}{\partial t} + \rho(\mathbf{u} \cdot \nabla)k = P_k - \beta^* \rho \omega k + \nabla \cdot [(\mu + \mu_T \sigma_k^*) \nabla k] \quad (4.14)$$

$$\rho \frac{\partial \omega}{\partial t} + \rho(\mathbf{u} \cdot \nabla)\omega = \alpha \frac{\omega}{k} P_k - \rho \beta \omega^2 + \nabla \cdot [(\mu + \mu_T \sigma_\omega) \nabla \omega] \quad (4.15)$$

The turbulent viscosity and production term are as follows:

$$\mu_T = \rho \frac{k}{\omega} \quad (4.16)$$

$$P_k = \mu_T \left[\nabla \mathbf{u} : (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) - \frac{2}{3} (\nabla \cdot \mathbf{u})^2 \right] - \frac{2}{3} \rho k \nabla \cdot \mathbf{u} \quad (4.17)$$

The parameters of empirical turbulent model constants are as follows [214]:

$$\alpha = \frac{13}{25}, \quad \sigma_\omega = \frac{1}{2}, \quad \sigma_k^* = \frac{1}{2}, \quad \beta = \frac{9}{125}, \quad \beta^* = \frac{9}{100} \quad (4.18)$$

4.3.1.3 Energy equation

The energy equation conservation under laminar flow (single-phase) can be expressed in Cartesian coordinates which is as follows:

$$\rho C_p \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right) = k_f \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \mu \Phi \quad (4.19)$$

Where T [K], k_f $\left[\frac{W}{m.K} \right]$, C_p $\left[\frac{J}{kg.K} \right]$ and Φ are the fluid absolute temperature, thermal conductivity, specific heat and viscous dissipation function as follows:

$$\begin{aligned} \Phi = & 2 \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial z} \right)^2 \right] \\ & + \left[\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right)^2 \right] \\ & - \frac{2}{3} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right)^2 = \tau : \nabla \mathbf{u} \end{aligned} \quad (4.20)$$

From Eq. 4.19, it can be expressed in vector form as follows:

$$\rho C_p \left(\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T \right) = \nabla \cdot (k \nabla T) + \mu (\tau : \nabla \mathbf{u}) \quad (4.21)$$

For solid domain, conduction heat transfer occurs which can be expressed as follows:

$$\frac{\partial}{\partial x} \left(k_s \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k_s \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k_s \frac{\partial T}{\partial z} \right) = 0 \quad (4.22)$$

From Eq. 4.22, it can be expressed in vector form as follows:

$$\nabla \cdot (k_s \nabla T) = 0 \quad (4.23)$$

The energy equation conservation under turbulent flow (single-phase) can be expressed as follows:

$$\rho C_p \left(\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T \right) = \nabla \cdot ((k_f + k_T) \nabla T) \quad (4.24)$$

Where k_s and k_f are the thermal conductivity of solid and fluid respectively. k_T is the thermal conductivity for turbulent flow $\left(k_T = \frac{\mu_T \cdot C_p}{Pr_T} \right)$ where Pr_T as turbulent Prandtl number [221]. At $\mu_T = 0$, the flow is considered laminar.

4.3.2 Conjugate heat transfer

The heat transfer through conduction and convection in solid and fluid domain are coupled by the temperature and heat flux continuity at fluid and solid walls interface surface. The temperature distribution within the domain of fluids and solids caused by solid and fluids thermal surface interaction can be expressed using the model of conjugate heat transfer. The fluid and solid conjugate heat flow can be referred in Figure 4-6 with interface wall where solid wall and fluid have equal temperature ($T_{s,\Gamma} = T_{f,\Gamma}$). Fourier's law is used to describe the heat flux at interface wall which is as follows:

$$-k_s \frac{\partial T_s}{\partial n} \Big|_{\Gamma} = -k_f \frac{\partial T_f}{\partial n} \Big|_{\Gamma}, \quad u = v = w = 0 \quad (4.25)$$

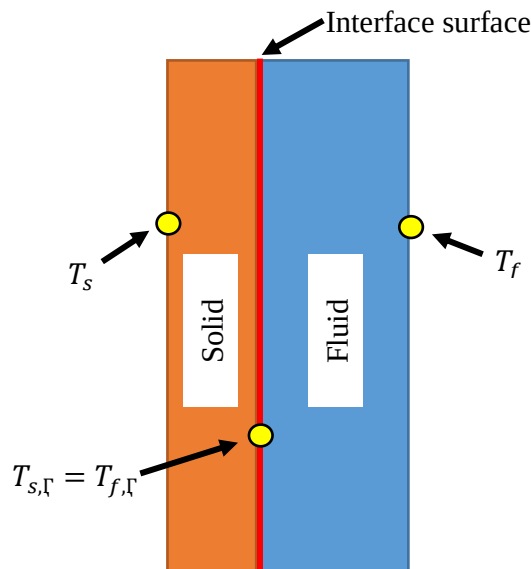


Figure 4-6: Shows the conjugate heat transfer between fluid and solid domains.

4.4 Boundary conditions and assumptions

4.4.1 Assumptions

In order to simplify the numerical solution and reduce computation time effectively, several simplifications have been applied using published studies [31, 63, 64, 141, 222, 223]. The assumptions are as follows:

- 1) The heat transfer and flow of fluid are assumed steady and three-dimensional.
- 2) The fluid flow is incompressible and single-phase fluid is considered for the simplicity of CFD models.
- 3) All surfaces of Mini-CHS are assumed to be adiabatic (no heat flow) except the bottom wall where the heat flux is applied.
- 4) The natural convective heat transfer and radiation effect are neglected.
- 5) Viscous dissipation effect is neglected since there is small conversion of mechanical energy to thermal energy caused by fluid layers frictional forces.
- 6) The channel having smooth surface due to only CFD analysis conducted to reduce model complexity.
- 7) “No-slip” velocity boundary condition applied ($\mathbf{u}_s = 0$) with temperature of solid wall equals to temperature of fluid at solid boundary ($T_s = T_f \text{ at wall}$).

Summarisation of boundary conditions for creation of model of conjugate heat transfer are shown in Table 4-1.

4.4.2 Boundary conditions

The simulations of the Mini-CHS are based specified boundary conditions as follows:

- 1) The inlet flow boundary condition is fluid inlet velocity, U_{in} (m/s) specified based on Reynolds number. The inlet temperature (T_o) is set to be 293.15 K (20°C). Under the regime of turbulent flow, the $k - \omega$ turbulence model is used. Thus, the turbulent intensity (I_T) and turbulent length scale (L_T) at inlet boundary has to be set. For all simulations, the value of I_T and L_T are specified to be 5% and $0.07D_h$ respectively [214].
- 2) The pressure outlet boundary condition (fluid is assumed coming out to open atmosphere):

$$p = p_o = 0$$

- 3) To simplify the models, the all surfaces on the heat sink except the bottom of Mini-CHS are considered adiabatic (no heat flow):

$$-\mathbf{n} \cdot (-k\nabla T) = 0$$

Where $\mathbf{n}$ as the outward normal vector with respect to the domain boundary.

- 4) Uniform heat flux is applied at the bottom wall of the Mini-CHS as follows:

$$-\mathbf{n} \cdot (-k_s \nabla T) = q''$$

Where q_o is the heat flux applied to solid boundary.

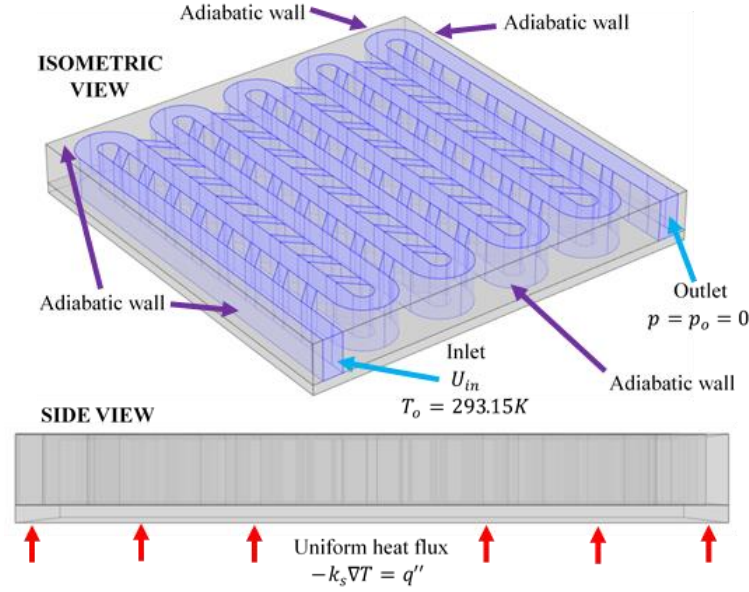


Figure 4-7: Boundary conditions of 3D domain serpentine channel with fins with copper (grey domain) and water (blue domain).

Table 4-1: Boundary conditions for the model of conjugate heat transfer.

Boundary	Location	Condition
Thermal	At inlet	$T_f = T_{in} = 293.15K$ (for water) $-k_s \left(\frac{\partial T_s}{\partial x} \right) = 0$ (for solid)
	At outlet	$-k_f \left(\frac{\partial T_f}{\partial x} \right) = 0$ (for water) $-k_s \left(\frac{\partial T_s}{\partial x} \right) = 0$ (for solid)
	At all walls except bottom	$u = v = w = 0 \text{ ms}^{-1}$ $-k_s \left(\frac{\partial T_s}{\partial z} \right) = 0$
	At bottom wall	$-k_s \left(\frac{\partial T_s}{\partial z} \right) = q''$
Hydrodynamic	At solid-fluid interface	No penetration and no-slip $u = v = w = 0 \text{ ms}^{-1}$ $-k_s \left(\frac{\partial T_s}{\partial n} \right) = -k_f \left(\frac{\partial T_f}{\partial n} \right)$, where n is perpendicular to the wall
	At inlet	$u_f = u_{in}$ $v = w = 0 \text{ ms}^{-1}$
	At outlet	$P_f = P_{out} = 0Pa$

4.5 Discretisation schemes

COMSOL Multiphysics uses the approach of fully coupled or segregated solver for solving the governing equations. For non-linear problems, the segregated solve required iterations. The segregated solver also can utilised the direct or iterative linear solvers. The Jacobian matrix is being split by segregated solver into sub-problems which is based on the type of degree of freedom (DoF). The number of variables assigned for solving either linear or non-linear algebraic system that describes the operational unit is the degrees of freedom. The computational time and memory required to compute a solution is significantly depending on the degrees of freedom number. For coupled solver, governing equations are solved simultaneously while segregated solver solves them sequentially. For this work, the segregated solver is used to balance the accuracy and memory requirements growth.

Within the COMSOL Multiphysics software, three discretisation elements types can be choose from which are P1+P1, P2+P1 and P3+P2 which is discussed as follows in Table 4-2:

Table 4-2: Discretisation element types.

Element types	Velocity field	Pressure field
P1+P1	First-order	First-order
P2+P1	Second-order	First-order
P3+P2	Third-order	Second-order

For this work, the P1+P1 element type will be used together with quadratic elements for temperature field. Many number of CFD studies only used relative error tolerance between the order of 10^{-3} to 10^{-6} . Typically, using smaller relative error tolerance will results in the increase of accuracy of solutions but with higher computational time. Thus, for this work the convergence occurs between the order of 10^{-4} to 10^{-5} with 0.5 relaxation factor.

4.6 Post-processing

After the solutions of governing equations using continuity, Navier-Stokes, energy and transport equations obtained using the boundary conditions and assumptions specified, the analysis of results are carried out. However, to ensure the reliability of the results, it is crucial that the mesh verification and validation studies are accomplished. The mesh study will be explained in the results section in Chapter 6 to 8.

4.7 Chapter summary

In this chapter, numerical analysis process are introduced. The numerical method process used in this project can be summarised as follows:

- A) Initially the design of the micro-channel heat sink solid and fluid domains can be constructed using design software such as Computer Aided Design (CAD) software or built in design tools within CFD software.
- B) The solid and fluid domains are assigned according to the materials used which in this work will be copper and water.
- C) The heat transfer module will be utilised for both solid and fluid domains while the fluid flow module will be assigned for fluid domain only to solve the conjugate heat transfer analysis.
- D) The governing equations like continuity, Navier-Stokes, energy and transport equations will be used for solving the heat sink physics.
- E) All outer surfaces except bottom surface of heat sink will have adiabatic condition where no heat transfer occurs to exclude heat loss to surrounding. Heat flux is applied only at the bottom surface of heat sink for flow of heat.
- F) At inlet boundary, a perpendicular to boundary of fully developed flow condition with specified velocity magnitude is applied to ensure steady state flow where properties of flow does not changing along the direction of flow. At outlet boundary, pressure outlet is set to zero to indicating boundary pressure is equal to the atmospheric pressure to simplify the mathematical equations for faster and stable solution.
- G) The meshing technique will include using unstructured mesh for domain discretisation based on the tetrahedral elements. The meshing is must have conformal mesh between solid and fluid boundaries to ensure accuracy of solutions. Finer mesh will used for fluid domain and mesh refinement applied around bends and secondary channels. This is to ensure the simulations can capture complex flow phenomena, resolving boundary layers, fluid separation and reattachment as well as vortices.
- H) Segregated solver with P1+P1 discretisation element types is used to balance between the growth of memory requirements and computational time and order of relative error tolerance between the order of 10^{-4} to 10^{-5} .

Chapter 5 : Design optimisation

5.1 Design optimisation formulation

The process of optimisation aims to achieve the optimal solutions for a certain design problem while obeying specified constraints [224]. For many application of engineering such as construction and design would require the designers to ensure that maximum performance of results can be achieved [225, 226]. A complete optimisation problem is comprised of objective(s), design variables and the design space constraints. The objective(s) are the output parameters of a system which should be minimised or maximised. One or more than one objectives to be optimised is called single-objective optimisation and multi-objective optimisation respectively. For multi-objective optimisation, it often occur that the objectives conflicts each other where minimisation of one objective would result in the maximisation of the other and vice versa. For the design variables, they are used to obtain the corresponding objectives. Design variables are chosen based on the application problem specifications. Next, design variables can be of either two forms whether it is discrete or continuous. The design variables must be within design space which has an upper and lower limits called constraints where the feasibility of the design totally depend on it [225-227].

The goal is to search the design variables, $\mathbf{x}$, which minimises or maximises the single objective function, $f(\mathbf{x})$, for single-objective optimisation or multi-objective optimisation, $F(\mathbf{x})$. The objective functions are the output results from CFD analysis which are total thermal resistance (see sub-section 3.8.5), pressure drop (see sub-section 3.8.6) and mass of copper heat sink (see appendix section A 4 for volume formula). The formulation of the optimisation problem can be expressed as follows:

(Minimisation or maximisation) $\mathbf{y} = F(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_m(\mathbf{x}))$

$$h_i(\mathbf{x}) \leq 0, \text{ where } i = 1, 2, \dots, n$$

$$g_i(\mathbf{x}) = 0, \text{ where } i = 1, 2, \dots, p$$

$$\mathbf{x} = (x_1, x_2, \dots, x_q), \in \mathbf{X} \subset R^q \text{ where } x_i^l \leq x_i \leq x_i^u$$

$$\mathbf{y} = (y_1, y_2, \dots, y_m), \in \mathbf{Y} \subset R^m \quad (5.1)$$

Where $h_i(\mathbf{x})$ and $g_i(\mathbf{x})$ are the inequality and equality constraints which has to be satisfied, while $\mathbf{X}$ and $\mathbf{Y}$ are the design and search space respectively from the corresponding design vectors of $\mathbf{x}$ and $\mathbf{y}$. x_i is the design variable at a specific point i which has lower and upper limits of x_i^l and x_i^u respectively. In this section, the process of optimising the heat sinks exploiting the

CFD enabled optimisation is explained in detail, from design variables selection stage to the optimum design of heat sink obtained.

5.2 Optimisation problems classification

The categorisation of optimisation problem can be divided into two major division which are deterministic and probabilistic optimisation [228]. For single-objective function, the deterministic optimisation can be split into three categories depending on the design variables data structural type either discrete or continuous or both combined together. The method of deterministic optimisation is based on the assumption where the results of Pareto optimality is known with the real precision. In many occasions, the deterministic results are possible to be implemented within physical world for manufacturing but sometimes it is not possible due to manufacturing constraints such as tolerance and measurement errors. Thus, to solve this issue, probabilistic optimisation framework offers the opportunity for more flexible solutions which is more feasible in real life engineering problems. Furthermore, probabilistic optimisation can be split into two major method of robust and reliability-based optimisations respectively. The presence of constraints on design variables will define the optimisation framework whether it is a constrained or unconstrained problem [225, 229-231] for both deterministic and probabilistic optimisation framework. Based on the optimisation problem nature, it can either be local or global optimisation.

5.3 Design of Experiments (DOEs)

To build the response surface (meta-model) over a certain range number of design variables, the use of Design of Experiments (DOE) points is exploited. Meta-model is known as surrogate model where the relation between the input and output is estimated. Different methods of creating the points of DOE can be utilised such as Full Factorial, Random, Latin Hypercube (LHC) and Optimal Latin Hypercube (OLHC) [232]. For this study, the OLHC is used based on several advantages compared to other methods. The reason on the choice of OLHC is due to its effectiveness giving uniform sampling within the whole design space with the least design points count. It is crucial to fill the whole design space obtaining the highest accuracy of response surface within the range of design variables as well as to achieve the reduction in performing expense of each experiment [233]. In addition, the lacking in DOE points number will induce low quality response representation and reduces the accuracy of meta-model. Creating the DOE points requires appropriate design parameters to build the response surface.

5.4 Sampling and Probability Distributions

The motivation of using the sampling and probability distributions are to ensure cost-effectiveness when collecting data. Furthermore, accuracy and precision can be achieved estimating the means, variances and proportions as well as making predictions based on past data or for future events. When certain values are drawn probabilistically (probability distribution), it is called a sampling process [234]. The distribution of probability can also be converted in the form of cumulative curve (refer Figure 5-1). Distribution of probability explains by where random variables are assign to its own probability values using pre-set mathematical functions. Model historical and experimental data are some of the applications where probability distributions can be used for generating the estimation or prediction and large number outcome analysis (e.g., Monte Carlo simulations). Normal or Gaussian and Weibull distributions are the examples of parametric distributions where parameters data fitting is established.

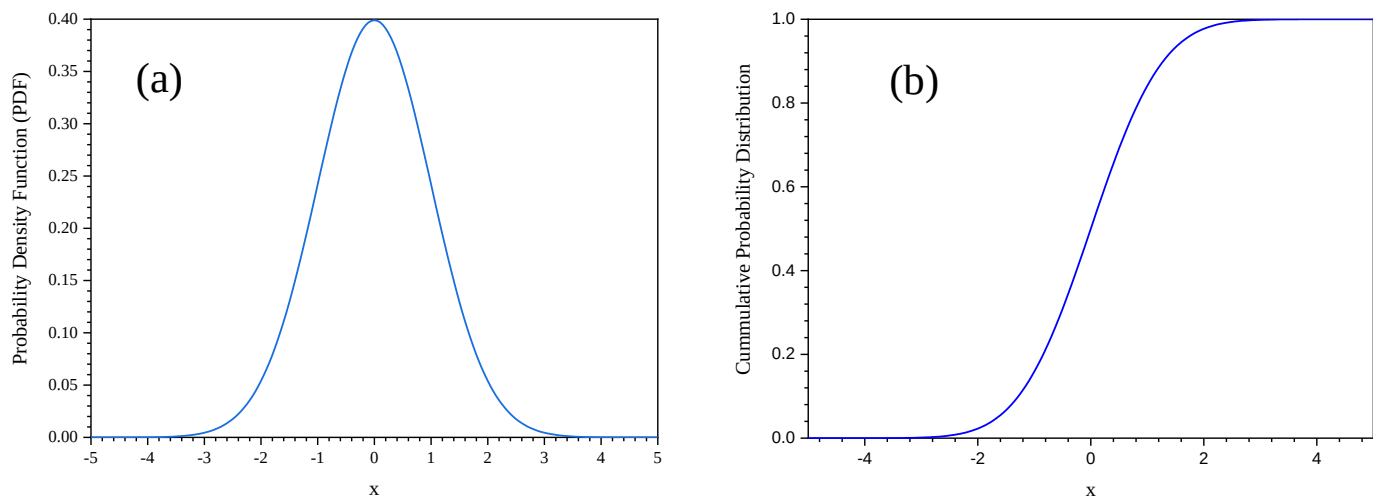


Figure 5-1: Shows the (a) probability density function (PDF) (b) cumulative form of PDF [235].

Referring Figure 5-1, it shows the plots of Probability Distribution Function (PDF) and its corresponding cumulative form. To give a brief explanation, let say we have $x = 0$, looking at Figure 5-1 (b) it can be seen that cumulative probability distribution gives a corresponding value of 0.5 meaning 50% of data starting from the most negative value have been added (refer Figure 5-1 (a) for area under the graph at $x = 0$). Similarly when $x = -0.7$, cumulative distribution shows 0.26 value meaning 26% of the data are below at $x = -0.7$ while the other 74% of data above at $x = -0.7$.

To measure the sampling method efficiency, it can be based on three criteria of running time for simulations, iteration number and accuracy. Therefore, it can be concluded as inefficient if the

sampling method requires longer time and larger iteration number. Common methods of sampling are Monte Carlo and Latin Hypercube samplings.

Based on Zacks [236], he described the uncertainty fundamental theory. Let the random variables vector $X = x_1, x_2, \dots, x_n$ under the variability of probability density function (PDF) and cumulative distribution function (CDF) (refer Figure 5-1 for plots). The PDF and CDF are denoted by $pX(x)$ and $PX(x)$ respectively.

To compute the probability of X , the area under the curve is calculated based on the limits of x_1 and x_2 respectively. The calculation of PDF is as follows:

$$Pr[x_1 < X < x_2] = \int_{x_1}^{x_2} p(x) = P_x(x_2) - P_x(x_1) \quad (5.2)$$

Using Gaussian distributions, it will be used in this work with a description of the value of mean $\mu(x)$ and standard deviation $\sigma(x)$ to perform the estimation of random variable of $E(X)$ and $Var(x)$ values. The equations of $E(X)$ and $Var(x)$ are as follows:

$$E(X) = \mu(x) = \int_{-\infty}^{\infty} xp(x)dx \quad (5.3)$$

$$Var(X) = \sigma^2 x = \int_{-\infty}^{\infty} (x - \mu x)^2 p(x)dx \quad (5.4)$$

5.4.1 Monte Carlo Sampling Method (MCSM)

From a probability distribution, the generation of random samples often exploits Monte Carlo sampling method. Common complex problems which exhibits probabilistic behaviour uses Monte Carlo sampling method [237]. Many generating random samples algorithms are accessible using probability distribution of different types. High randomness when using Monte Carlo sampling methods, where samples that have been generated are distributed randomly within the distribution of input range. Within the distributions region where the chance of occurrence is higher, this is where samples are more probable to be drawn. From Figure 5-2, a new random number where individual Monte Carlo sample use which is in between 0 and 1. The input distributions can be recreated when sampling has enough iterations. Referring Fig., in the middle of the distribution where all five samples are distributed, led to no samples at the outer ranges meaning not simulations occur in these regions. The sampling within these regions is important since it influences the outcome. But having very low probability, small iterations of Monte Carlo sampling might not be enough to sample these outcomes with sufficient quantities in order to have probability representation which is accurate enough. Therefore, the use of Monte Carlo sampling method (MCSM) requires a very large number of samples to approximate the

probability distribution accurately and small number of samples has shown some issues before. Often, the approximation of the Monte Carlo sampling is not sufficient for the actual probability distribution. Since MCSM uses the memory-less system, samples points that are newly generated is not based on the previous sample points that has been generated. From this issue, different distribution methods have been developed with less sampling needed compared to the MCSM such as the Latin Hypercube sampling (LHS).

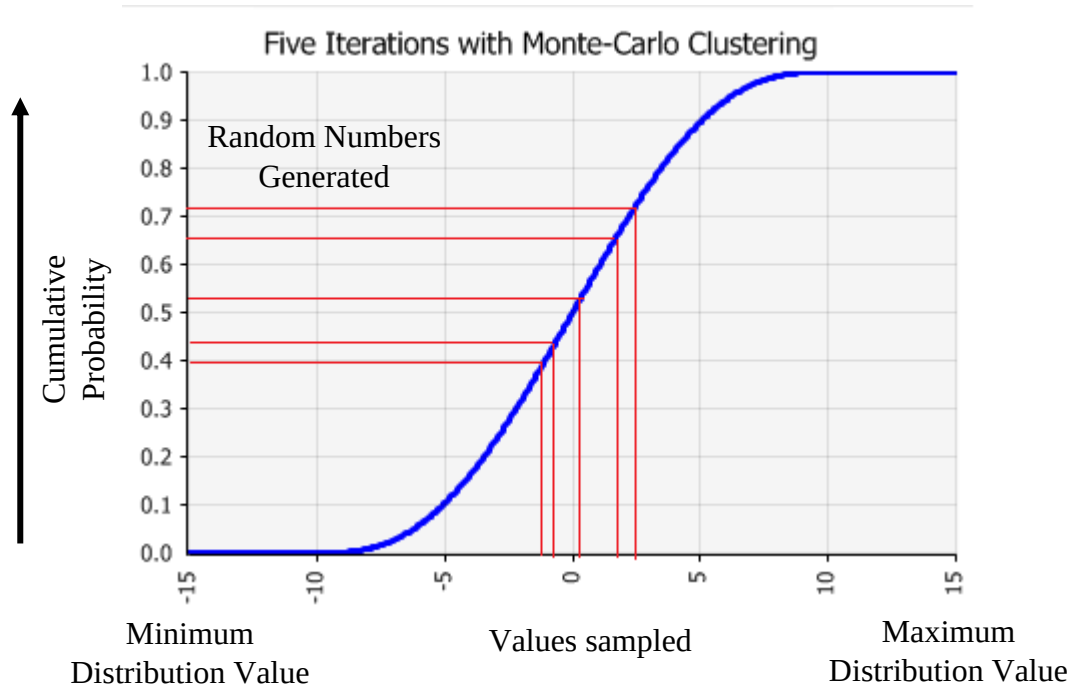


Figure 5-2: Monte Carlo sampling for 5 iterations [238].

5.4.2 Latin Hypercube Sampling (LHS) method

The application of Latin Hypercube sampling (LHS) was introduced to replace the MCSM for a more efficient, random and simple sampling method with better sampling variables [239, 240]. The LHS method is then further studied by Iman Conover [241]. Based on the conclusion made by Swidzinski et al. [242], the effectiveness of LHS method is five times more powerful compared to traditional methods in terms of yield estimation since LHS only need 200 points compared to MCSM which requires 1000 points. The method of LHS utilised the stratified sampling technique to achieve the decrease in simulations number computing the uncertainty of response. The method of stratification is the equal division of random variables distribution function (Refer vertical axis in Figure 5-3) to become non-overlapping segments where the computer running number is based on the intervals number with an aim to improve the design space input variables distribution. In contrast with MCSM, the LHS method is to ensure the

creation of input distribution with high accuracy without having to iterate with large number of samples. LHS uses the method of taking the sample from an interval at a time while having the new sample taken from a new segment instead of the same segment. To compare with sampling based on MCSM (refer Figure 5-2), the use of LHS method for sampling has a very higher accuracy since the whole input probability is approximated with less number of iterations (refer Figure 5-3) [243].

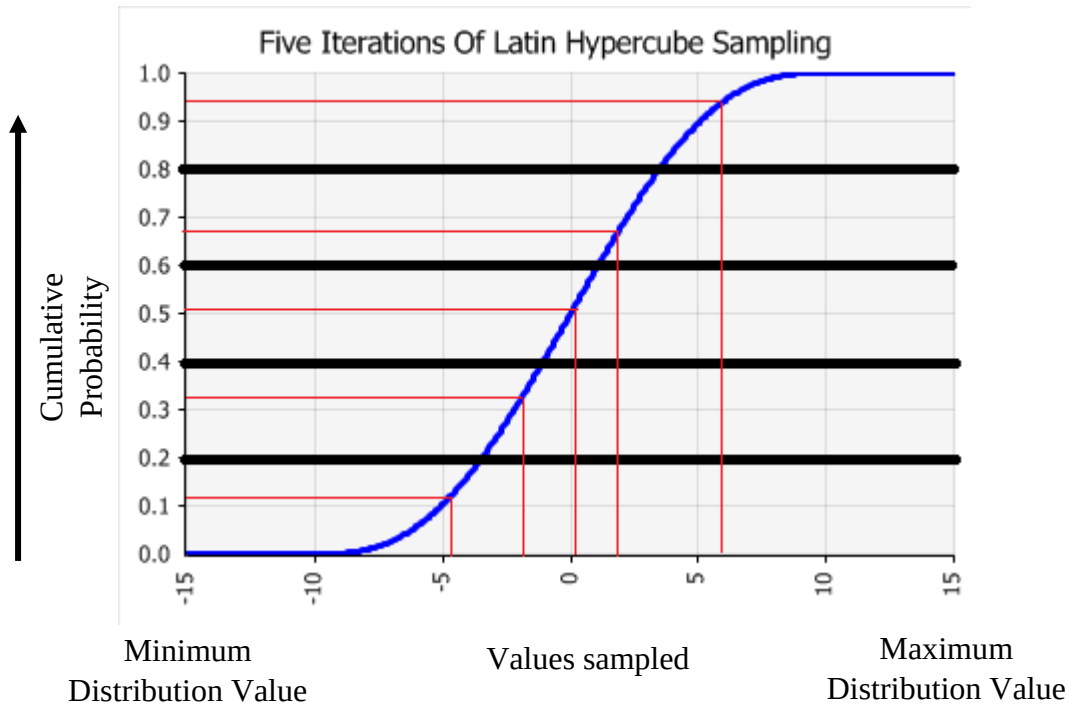


Figure 5-3: Shows the Latin Hypercube Sampling (LHS) for five iterations [238].

5.4.3 Latin Hypercube Sampling (LHS) method with multi-dimension variables

LHS generate the independent variable random samples (x_1 , x_2) using grids with only one sample generated for each column and row. For each variable of x_1 and x_2 , individually will be created into one-dimensional space and then it will be pooled together into two-dimensional space (refer Figure 5-4 (a)). The domain of design for LHS will be split into orthogonal grids with equal length of elements of N number only if the variables are distributed uniformly. The use of LHS method is more advanced as well as highly efficient compared to MCSM. The points generated by LHS are randomly placed throughout the design space within the square grids with a condition where there are no equal values for any two points of the same parameter. Therefore, for each row and column of a point will be uniquely placed without having any other duplications. With the same accuracy as MCSM, LHS method only need 20% to 40% less number of simulation iterations [238]. Still, the exploitation of LHS method is entirely dependent to the application problem. Under the multi-dimensional (more than two) grid with N number,

the sub-volumes can be specified similarly when only with two-dimensional space by specifying the range of each of the N dimensions. Based on the grid with N number of dimensions, the sub-volumes placed in each row and column only exist once without repetition. Based on Figure 5-4 (b), under the multi-dimensional grids of N number, the sub-volumes (represented by the black cubes) can be compared to the crossword-like illustration. The sample will be chosen randomly to be placed to a specific sub-volume. It is very crucial specific sub-volumes are chosen to avoid inter-dimensional spurious correlations or sample spreading throughout the whole design space which can have the risk of generated samples from grid do not represent the entire design space.

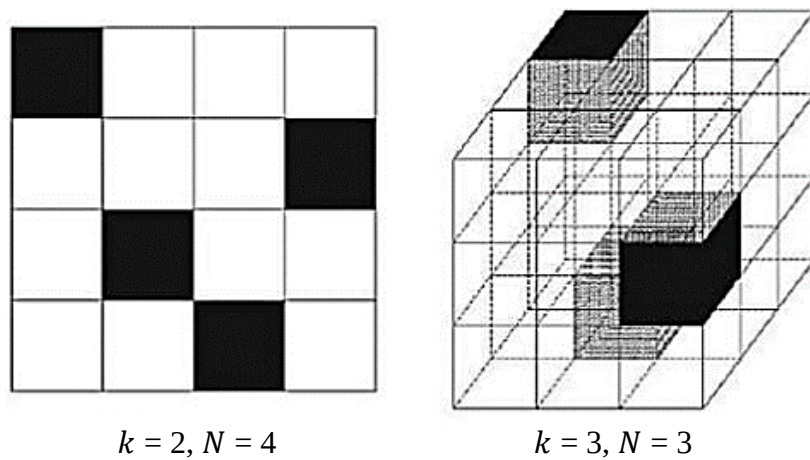


Figure 5-4: Latin Hypercube Sampling (LHS) [238].

Filling up the design space with a diagonal manner distribution would definitely satisfy the LHS criteria, but the other empty spaces would be left out. Different techniques can be used to ensure the correlations reduction within the designs of LHS. From Figure 5-5, the correlation reduction is shown using LHS method with $k = 2$ and $N = 10$ respectively.

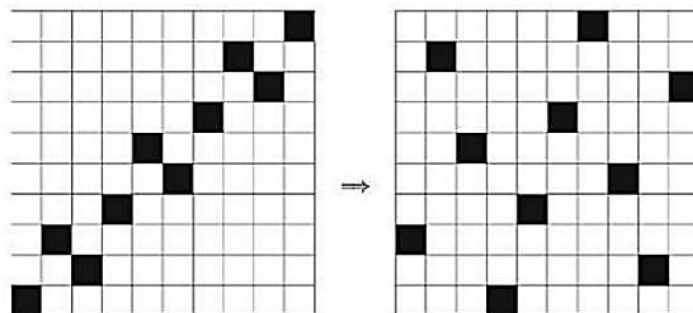


Figure 5-5: Shows the LHS correlation reduction [238].

5.4.4 Optimal Latin Hypercube Sampling (OLHS) method

From the explanation in previous sub-chapter 5.4.3, LHS method functions by dividing the parameter of design with N intervals of equally sized and only once of occurrence of each parameter value. Different types of Latin Hypercube sampling can be divided into two major methods of Random Latin Hypercube (RLH) and Optimised Latin Hypercube (OLH). It is required for uniform sampling of DOE points distribution to ensure the surrogate model quality. The use of RLH or conventional LH sampling design is only based on the number of DOE points and factors, N, of the input data. Referring to Figure 5-6 (a), using the conventional LHS method the DOE points are arranged randomly causes some unfilled areas within the design space [244]. Samples that are clumped together when using conventional LHS can be solved by increasing the distance between design points using Audze-Eglais criterion as follows:

$$\sum_{p=1}^P \sum_{q=p+1}^P \frac{1}{(r_{pq})^2} \quad (5.5)$$

The two sample points distance is denoted by $(r_{pq})^2$. Using permutation genetic algorithm, Eq. 5.2 can be minimised to achieve Optimised Latin Hypercube sampling.

The aims of OLHS method are to ensure that factors level number is equal to the DOE points number while having only one experiment point on each level. Furthermore, the distribution of DOE points are ensured to be very uniform within the design space. From Figure 5-6, it can be observed that using OLHS method provides more uniform distribution of design points for the two design variables of x_1 and x_2 .

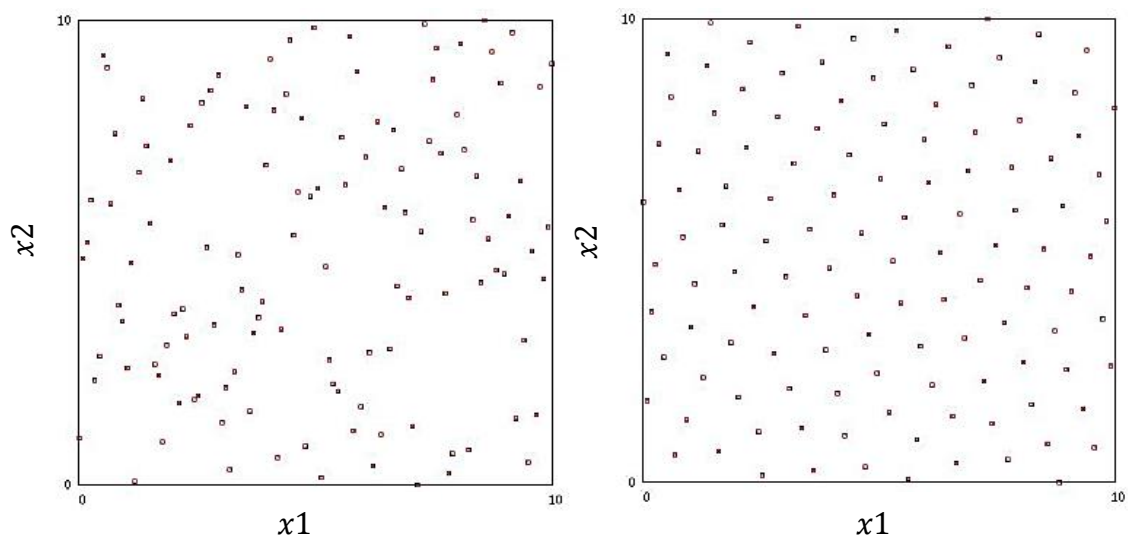


Figure 5-6: (a) LHS (b) OLHS for 100 design points for 2 design variables [245].

Under optimisation process, the accuracy of the results are significantly dependent to the distribution of DOE points. The DOE points also will be used to create surrogates or meta-models for computational cost reduction. The surrogates then utilised for optimisation purpose creating Pareto Front which will be discussed further under sub-chapter 5.5

5.5 Surrogate modelling

Using the DOE points extracted from using the OLHS method, the output results of system will then be calculated using CFD models created. The CFD results with its associated DOE points will be exploited for the creation of surrogates. The use of surrogate models (Response Surface Model (RSM) or meta-models [246]) can be utilised to map the relationship of the change in inputs to the outputs of a system without having further CFD simulations which is very time consuming [247]. By using surrogates, the output at new untested input can be predicted easily as long as within the limits of the design space. The numerical noise effect and missing data can be reduced by using surrogate models [245, 248]. Surrogates will be used within optimisation framework to search for global minimum of objective function from associated design variables. A good distribution of DOE points will be very useful to have accurate surrogate models [248].

The creation of surrogates are usually based on two major different schemes of interpolation-based or approximation-based. Different methods under the approximation based-scheme are least square, moving least square and polynomial response surface. The use of approximation-based models are very easy to work with as well as cheap resources and parameter sensitivity is straight forward, but, lower accuracy of results especially when dealing with non-linear system. Approximation-based models does not do interpolation of sample points and type of function provides limitation to the approximation [249]. Methods that used interpolation technique are Kriging function, Radial Basis Function (RBF) and artificial neural network (ANN) [250] where the surrogates formed will pass through all DOE points. For this work, the ANN method will be exploited for surrogate modelling.

5.5.1 Radial Basis Functions (RBFs)

To reduce the computational simulation resources, surrogate models can be used based on Radial Basis Functions (RBFs) method [251]. Since RBFs have the capability of non-linear systems and arbitrary behaviour functions prediction as well as handling multi-dimensional design space with scattered points. Surrogate models can be easily constructed using RBFs interpolation scheme from the DOE points [252]. Based on the design variables (ndv) and n number of DOE points, the estimation of y can be defined as follows:

The design variables and DOE points are given by $X_i = [x_{(i,1)}, x_{(i,2)}, \dots, x_{(i,ndv)}]$ corresponds to the $Y_i = [y_{(i,1)}, y_{(i,2)}, \dots, y_{(i,ndv)}]$ where $i = 1, \dots, n$. The prediction of y can be for every points of X_i can be defined as follows:

$$\hat{y}(x) = \sum_{i=1}^n \lambda_i \phi(r(x, x_i)) \quad (5.6)$$

Where the prediction of true response function is denoted by $\hat{y}(x)$, at i -th number of basis function the weighted coefficient is denoted by λ_i , samples number is denoted by n , the $r(x, x_i)$ is the Euclidean distance between testing point x and i_{th} sample x_i given as $r(x, x_i) = \|\mathbf{x} - \mathbf{x}_i\|$, thus, Eq. 5.3 can be modified to be as follows:

$$\hat{y}(x) = \sum_{i=1}^n \lambda_i \phi(\|\mathbf{x} - \mathbf{x}_i\|) \quad (5.7)$$

The matrix form of the whole RBFs can written as follows:

$$\begin{bmatrix} \phi\|\mathbf{x}_1 - \mathbf{x}_1\| & \phi\|\mathbf{x}_2 - \mathbf{x}_1\| & \dots & \phi\|\mathbf{x}_n - \mathbf{x}_1\| \\ \phi\|\mathbf{x}_1 - \mathbf{x}_2\| & \phi\|\mathbf{x}_2 - \mathbf{x}_2\| & \dots & \phi\|\mathbf{x}_n - \mathbf{x}_2\| \\ \vdots & \vdots & \ddots & \vdots \\ \phi\|\mathbf{x}_1 - \mathbf{x}_n\| & \phi\|\mathbf{x}_2 - \mathbf{x}_n\| & \dots & \phi\|\mathbf{x}_n - \mathbf{x}_n\| \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_m \end{bmatrix} = \begin{bmatrix} \hat{y}(x)_1 \\ \hat{y}(x)_2 \\ \vdots \\ \hat{y}(x)_n \end{bmatrix} \quad (5.8)$$

The short form of matrix 5.8 above can be reduce to be as follows:

$$\phi(r). \lambda = \hat{y}(x) \quad (5.9)$$

The working principle of RBF is based on interpolation where to get exact RBF approximation for each point of DOE, the weights, λ_i , are chosen accurately. For example, $yrbf(x_i) = \hat{y}(\mathbf{x}_i)$ where $i = 1, \dots, n$. The values of λ_i can be calculated using the equation of linear matrix as follows:

$$\lambda = \phi(r)^{-1} \hat{y}(x) \quad (5.10)$$

Where the Gram matrix is denoted by $\phi(r)$ with formula of $\phi(r) = \phi\|\mathbf{x} - \mathbf{x}_i\|$. The function of RBF has different forms which listed as follows:

- 1) Cubic, $\phi(r, c) = (r + c)^3$
- 2) Thin plate spline, $\phi(r, c) = r^2 \ln(cr)$
- 3) Gaussian, $\phi(r, c) = \exp(-cr^2)$
- 4) Inverse multi-quadric, $\phi(r, c) = \frac{1}{\sqrt{r^2 + c^2}}$
- 5) Multi-quadric, $\phi(r, c) = \sqrt{r^2 + c^2}$

For this work, the cubic function of RBF is used where $\phi\|\mathbf{x} - \mathbf{x}_i\| = \beta\|\mathbf{x} - \mathbf{x}_i\|^3$. The cubic RBG parameter of calibration is denoted by β . Based on the relatively simple Leave-One-Out (LOO) calibration, the value of β is determined for individual surrogate model. The left out sample of each point of RBF function will be exploited for value prediction at the point of DOE which has been omitted. The predicted and actual DOE points discrepancies will be calculated using Root Mean Square Error (RMSE) will then be minimised with respect to the parameter of β for intervals which is suitable [253]. The RMSE formula is defined as follows:

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (y_i - \hat{y}_i)^2} \quad (5.11)$$

Where the DOE points is denoted by N , and the predicted and actual response values are denoted by $\hat{y}_i$ and y_i respectively.

5.5.2 Moving Least Squares Method (MLSM)

Surrogate model can be formed using Moving Least Square Method (MLSM) which can reduce the computational time and resources. Even though, limited number of CFD solution formed from small number DOE points, MLSM can capture relationship of inputs and outputs effectively. Numerical noise issue from results obtained by CFD software also address when using MLSM surrogate modelling [254].

The design variables and DOE points are given by $X_i = [x_{(i,1)}, x_{(i,2)}, \dots, x_{(i,ndv)}]$ corresponds to the $Y_i = [y_{(i,1)}, y_{(i,2)}, \dots, y_{(i,ndv)}]$ where $i = 1, \dots, n$. The prediction of y can be for every points of X_i can be defined as follows:

$$\hat{y}(x) = \sum_{i=1}^n \omega_i y_i \quad (5.12)$$

Where the true response function is denoted by $\hat{y}(x)$, Weight decay function is denoted by ω_i which is used to determine the basis function at i -th position and samples number is denoted by n . The formula of ω is as follows:

$$\omega = \exp(-\theta r_i^2) \quad (5.13)$$

Where Euclidean distance between testing point x and i_{th} sample x_i is denoted by $r(x, x_i) = \|\mathbf{x} - \mathbf{x}_i\|$ and fit parameter closeness is denoted by θ .

Significant influence of θ on surrogates which is very important to choose from. Manual estimation of θ will induce numerical noise due to inefficient fit which will be required for tuning

resulting in reduced numerical noise effect. The determination of θ will be based on the LOO calibration method mentioned in sub-chapter 5.5.1. The same parameter of RMSE (refer Eq. 5.8) will be used here.

5.5.3 Artificial Neural Network (ANN)

Based on biological nervous systems, the Artificial Neural Network (ANN) for creating surrogates has been a very important technique. The structure for ANN model is the combination of the input layer, hidden layers and output layer [255]. The number of hidden layers can be one or more respectively. Hidden layers has the function of learning the relationship between the input and output layers respectively which has the ability of mapping the highly non-linear and sophisticated relationship between input and output parameters. ANN has the hidden layers which has its own number of neurons specified individually, each neuron will hold a certain specific weight. The neurons has a network connectivity between them and activation function is used to guide the neurons for mapping the relationship between input and output parameters accurately.

5.5.3.1 Understanding the concept of neuron

To easily understand the concept of training an ANN model, a simple ANN model with only one neuron is constructed (refer Figure 5-7). The model comprises of input values, weights, bias, transfer function and output values. Any predicted values from ANN model will be compared with the actual values extracted from CFD software. Percentage difference between predicted and actual values that are less than 2% are considered very accurate, but higher than that will need an adjustment between weights and bias (refer Figure 5-8) using backpropagation method. Explaining backpropagation method, firstly, during forward propagation the input design variables are fed into the network where using the current weights the output is calculated. Then, the relative error is calculated between predicted output and actual data. After that, using backward propagation, the relative error calculated is being propagated back into the network where error derivative for each weight is calculated using differentiation chain rule. The update of weights is conducted based on the error derivative with respect to each weight. On the opposite direction of gradient, the weights are adjusted by multiplying with the learning rate for weight size update. The processes are repeated until accurate solution achieved.

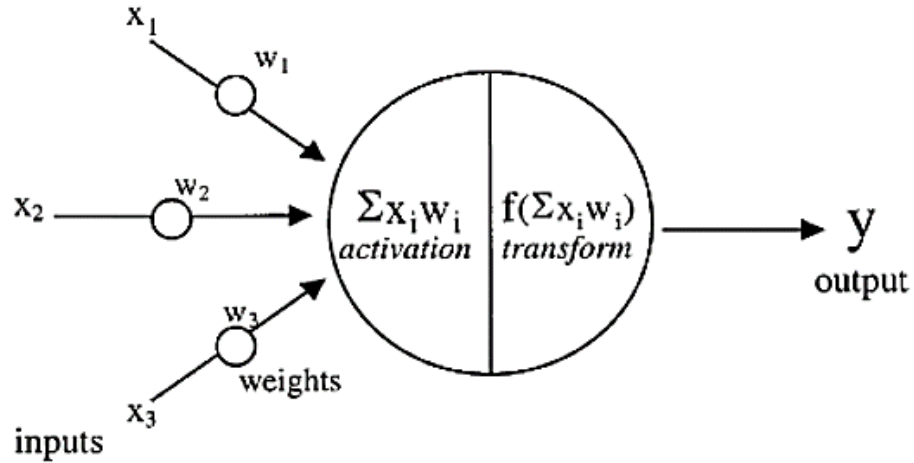


Figure 5-7: A neuron schematic diagram [256].

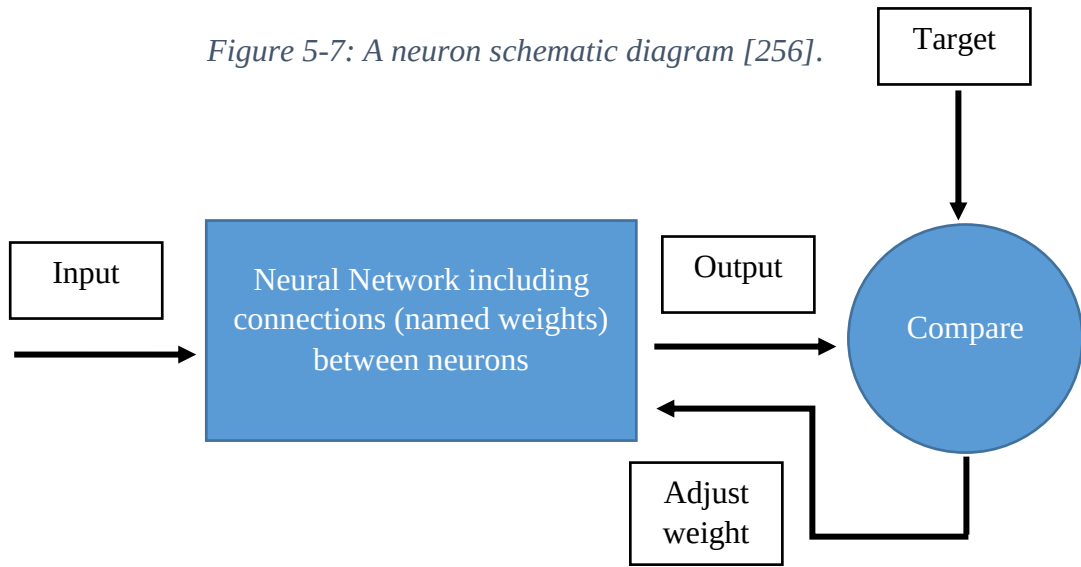


Figure 5-8: The Artificial Neural Network (ANN) diagram of control [256].

Using mathematical operations, network training of neuron calculating the output results which will satisfy the target solution based on the input variables. The transfer function converts vector to scalar function by adjusting the input values weights to be as follows:

$$y = \sum_{i=1}^n [x_i \cdot \omega_i] = [x_1 \cdot \omega_1 + x_2 \cdot \omega_2 + x_3 \cdot \omega_3 \dots x_n \cdot \omega_n] \quad (5.14)$$

The structure of ANN model will have hidden layers with at least one layer. From Figure 5-9, the input layer, hidden layers (contain neurons) and output layer are shown schematically. The process principle of using ANN are listed as follows:

- 1) The vectors of both input (DOE points of design variables) and output (objective functions) data are prepared.
- 2) The initial dataset (DOE points) of design variables are split into training and validation sets.

- 3) The architecture of ANN model is structured by specifying the number of hidden layers and number of neurons per hidden layer.
- 4) Training of ANN model.

Different methods available including supervised, unsupervised and reinforcement learning can be used to train the ANN model by adjusting the inter-nodes connection weights. Using the method of supervised learning, adjustment for the connecting weights are made from evaluation of both data from the training and predicted data from the ANN model results [257]. Back-propagation introduced by Rumelhart et al. [258] uses optimisation algorithm based on the gradient descent method which also exploits the algorithm of supervised learning for weights connection adjustment achieving error minimisation [259, 260]. The algorithm of Stochastic Gradient Descent (SGD) reduces the error of back-propagation between estimated and actual values by adjusting the connection weights [261]. SGD only uses small random subset of training data to minimise the lost function efficiently while avoiding being stuck in local minima which faster than gradient descent optimiser. The minimisation of cost function is performed based on SGD method which is as follows:

$$cost = \sum_{i=1}^{N_{train}} \|y_i^{predict} - \bar{y}_i^{train}\| / N_{train} \quad (5.15)$$

The accuracy of the ANN model is measured by comparing the predicted data with test data which is as follows:

$$err = \sum_{i=1}^{N_{train}} \|y_i^{predict} - \bar{y}_i^{test}\| / N_{test} \quad (5.16)$$

Where the tested functions number is denoted by N .

Another parameter used to measure the accuracy or discrepancy of a model is the R-squared value which ranges in between 0 to 1 where closer to unity indicates close relationship and vice versa [262]. The determination coefficient (R^2) is defined as follows:

$$R^2 = 1 - \frac{\sum_{i=1}^{N_{train}} (\|y_i^{predict} - y_i^{true}\|)^2}{\sum_{i=1}^{N_{train}} (\|y_i^{true} - \bar{y}_i\|)^2} \quad (5.17)$$

The $\bar{y}_i$ is defined as $\bar{y}_i = \frac{\sum_{i=1}^{N_{train}} y_i^{true}}{N_{train}}$.

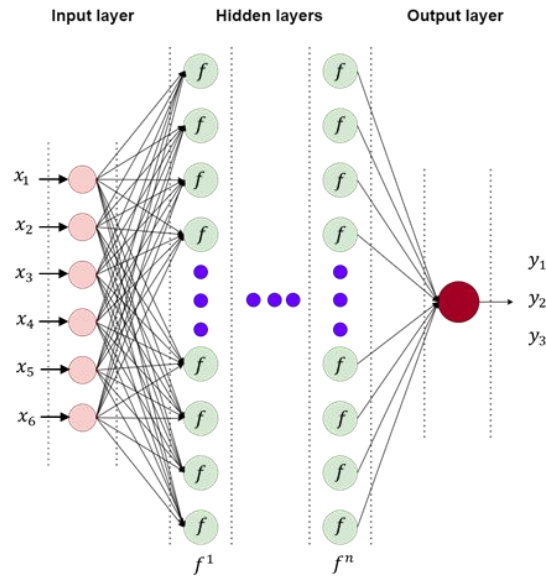


Figure 5-9: Shows the Structure of Artificial Neural Network (ANN) model.

5.6 Optimisation

To search for global minimum point the surrogates developed will be used within the optimisation framework. From the inspiration of biological evolution, natural selection is used within the Genetic Algorithms (GAs) for optimisation process. The use of GAs are mainly for the search of global optimum based on the problems which has both constrained and unconstrained condition [263]. Optimisation based on the random population is the method of GAs determining the optimal solution for current solutions for new generation of solutions by applying the random changes [264]. Many different kind of optimisation problems can be solved using genetic algorithm methods, sometimes an optimisation problem is not able to be solved by standard optimisation algorithms such as stochastic objective function, non-differentiable discrete dataset and highly non-linear system [265]. Global minimum(s) can be easily searched within entire design space using GAs methods. Even though, using GAs is very efficient in searching the global minimum, sometimes it is more realistic to find the local minimum instead due to manufacturing constraints because sometimes at global minimum the use of equipment and materials can be very costly and hard to be available. To search for local minima, another method can be used such as robust optimisation which will be explained further under sub-chapter. In this work, Non-dominated sorting Genetic Algorithm-II (NSGA-II) will be used since it offers non-dominated sorting, an elitist strategy, it uses the approach for parameterless problem and very efficient for optimisation problem with constraints to handled.

5.6.1 Understanding Genetic Algorithms (GAs)

Repetitive modification of individual solutions population which have been randomly generated is the first principle step of genetic algorithm [264]. Selecting the individuals from current population to be parents for producing the next generation of offspring is second step. Then, the evolution of populations will occur for the following generations in order to search for the solutions which is optimal. The process of Genetic Algorithms is explained schematically in Figure 5-10, involving different stages of creating initial population, producing next generation of offspring by means of selection, crossover and mutation and termination.

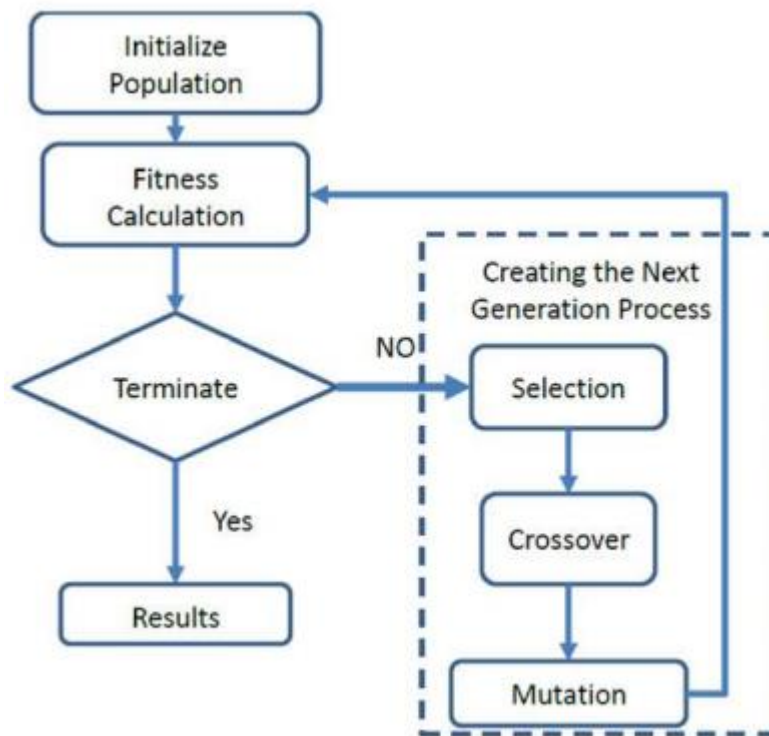


Figure 5-10: Shows the flow chart for the working process of genetic algorithm method [266].

From the first step of Genetic Algorithm where potential solutions, P , are generated. Individual solution are being generated randomly to be the first population of $P = (p_1, p_2, \dots, p_{pop\_size})$ and $P_i = (p_{i,1}, p_{i,2}, \dots, p_{i,j}, p_{no\_var})$ where $i = 1, 2, \dots$, population size and $j = 1, 2, \dots$, number of design variables. The $p_{i,j}$ has the range where $para_{min}^j \leq p_{i,j} \leq para_{max}^j$. Population size is denoted by pop_size , number of design variables are denoted by no_var and the design variables lower and upper limits are represented by $para_{min}^j$ and $para_{max}^j$ respectively.

To measure the performance of chromosome created by GAs individually, fitness function is utilised to perform the assessment where it need to be optimised. Higher values of fitness

function obtained if chromosome quality is high. The formulation of fitness function can be structured as follows:

$$\text{Fitness function} = f(x_1), \dots, f(x_n) \quad (5.18)$$

Where n is the number design variables.

To explain in a detailed manner, GAs will generate the population randomly, which will force the fitness function to evaluate the whole first population. This is to ensure the determination suitable parents to produce children by passing the genes. Higher solution quality will have higher value of fitness function. Commonly, individuals will be chosen based on the parents that has the best value of fitness function. An improved generation will be newly produced from the initial population based on the chromosomes. To perform identification of the best quality parents giving the best chromosomes, the calculation of fitness will be conducted. Next generation classification will be based on the three-different sub-steps of following:

- a) Selection stage: The chromosomes with high fitness values will be chosen to produce offspring.
- b) Crossover stage: Using the parents' chromosomes, the best pair will be combined creating children.
- c) Mutation stage: For an individual gene, random mutation is introduced.

From step 2, where the determination of individual fitness is completed, the step 3 will cover the criterion satisfied function assessing the entire population fitness. For the last step of step 4, the search for best population is terminated when criteria is met. The criteria can be either generations has reached the maximum number, time limitation or solutions that are acceptable obtained (refer Figure 5-10 for algorithm). The generation of new population is made up of two populations of last generation which has high fitness quality and new children from the crossover and mutation process. Looping through the algorithm from Figure 5-10, after the best individuals obtained the algorithm will converge and the problem will have their optimal solutions [267].

5.6.2 Single-objective Optimisation

After the surrogate models have been developed for each objective function, the single-objective optimisation will only cover single objective function at one without optimising the other. The weakness of optimising only single objective is that, while optimising one objective will lead the deterioration of performance of the other [268]. One of the way to solve this issue is by combining the objectives together using weighted sum approach where weight factor is

associated to each objective depending on their importance but it not covered in this work since Pareto optimisation is chosen. This condition single-objective optimisation is not feasible for certain applications if only one objective optimisation is considered. Thus, the study of multi-objective optimisation is more applicable to search for design variables giving balanced performance of multi-objectives which is the Pareto optimal solutions.

5.6.3 Multi-objective Optimisation

The concept of multi-objective optimisation is to search the most optimal solutions compromising between conflicting objectives. For this work, the surrogates developed using RBF and ANN for each objective function studied will be used within the multi-objective optimisation framework. The multi-objective optimisation will provide the Pareto optimal solutions for different objectives tested. Both RBF and ANN surrogates will be used within Platypus optimisation package in Python with optimisation algorithm of NSGA-II. The function of Pareto optimal solutions is to provide flexibility to the designers choosing between competing objectives for at least to minimise all of them. The solutions from deterministic optimisation study often lead to only absolute results without taking into account that there might be other solutions that perform as good but at different design points. The formulation of multi-objective optimisation can be constructed as follows:

$$\begin{aligned}
 & \text{(Minimisation or maximisation)} \mathbf{y} = F(\mathbf{x}) = \\
 & \quad (f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_m(\mathbf{x})) \\
 & \quad h_i(\mathbf{x}) \leq 0, \text{ where } i = 1, 2, \dots, n \\
 & \quad g_i(\mathbf{x}) = 0, \text{ where } i = 1, 2, \dots, p \\
 & \quad \mathbf{x} = (x_1, x_2, \dots, x_q), \in \mathbf{X} \subset R^q \\
 & \quad \mathbf{y} = (y_1, y_2, \dots, y_m), \in \mathbf{Y} \subset R^m
 \end{aligned} \tag{5.19}$$

The variable of q and m are the input design variables and objective functions number. The number set of equality and inequality constraints are denoted by p and n respectively. The design and goal space are denoted by $\mathbf{X}$ and $\mathbf{Y}$ respectively where the $\mathbf{x}$ and $\mathbf{y}$ are design vectors. The function of equality and inequality are represented by $g_i(\mathbf{x})$ and $h_i(\mathbf{x})$ respectively.

Deterministic solutions often result in a design where it is not feasible for manufacturing due to restrictions from tolerances to manufacture. Therefore, solving this issue, uncertainty quantification is added within design variables to search different design points that give almost equally superior performance. For the next sub-chapter, the optimisation with uncertainty quantification is explained in details.

5.6.4 Optimisation Under Uncertainty (OUU)

Optimisation Under Uncertainty (OUU) includes the process of adding uncertainty within the design points and measure the performance of it, and after that optimisation is performed [269, 270]. OUU is split into two major methods of robust design and reliability-based design respectively. Robust design aims to search for the design that is insignificantly affected by the small change of the geometry under the condition of uncertainty while reliability-based design takes into account of failure probability when obtaining the optimal solutions of the design.

5.6.4.1 Robust Design Optimisation (RDO)

The method of Robust Design Optimisation (RDO) performs minimisation of performance of system sensitivity towards the application of uncertainty onto the design input variables contributed by the factor of environment or manufacturing conditions [271]. For this work, aleatoric uncertainty is used which contributed by the manufacturing tolerances causing uncertainty. Using probabilistic framework, uncertainties are included by imposing the normal distribution around a design point (refer Figure 5-11 showing the optimisation robustness working principle) contributing to distribution of output, f . It can be observed even though the normal distribution of both inputs are similar but each of them results into different features of normal distribution of the outputs where one of them has more steep curve compared to the other indicating high gradient region. The application of uncertainty will definitely affects the response f to be varied and has to go for quantification and minimisation. The robust design from uncertainty application which has been optimised must have a balance between the system mean and variance performance (Pareto optimality) [272]. The formulation of RDO is as follows:

$$\begin{aligned} & \text{find } x \text{ that satisfies} \\ & \text{minimising } \{E(f(x, y)), \sigma(f(x, y))\} \\ & \text{subject to } \sigma(f, x) \leq \sigma_1 \end{aligned} \tag{5.20}$$

Where performance function mean value and standard deviation are denoted by $E(f)$ and $\sigma(f)$ respectively. The maximum limit of standard deviation allowed is denoted by σ_1 .

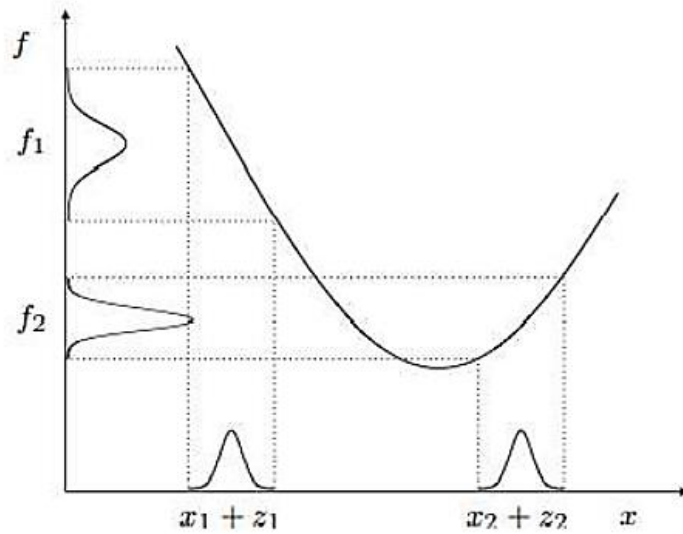


Figure 5-11: Shows the Robust Design Optimisation (RDO) principle [273].

The RDO is mainly used to study the response variation when input design variables are subjected to uncertainty while identifying what parameters are causing them. To perform uncertainty study, a few assumptions are to be followed as follows:

- A) Individual design variables will be subjected with Gaussian distribution.
- B) The magnitude of standard deviation will be between 5% to 10% away from the value of mean [274] because the range is considered appropriate to be large enough having meaningful variability degree but not too large causing unrealistic input values.
- C) The normal distribution is set to follow the 3-sigma rule where one, two and three sigma away mean are set to have the percentage of 68%, 95% and 99.7% respectively. The illustration of normal distribution can be referred to Figure 5-12.

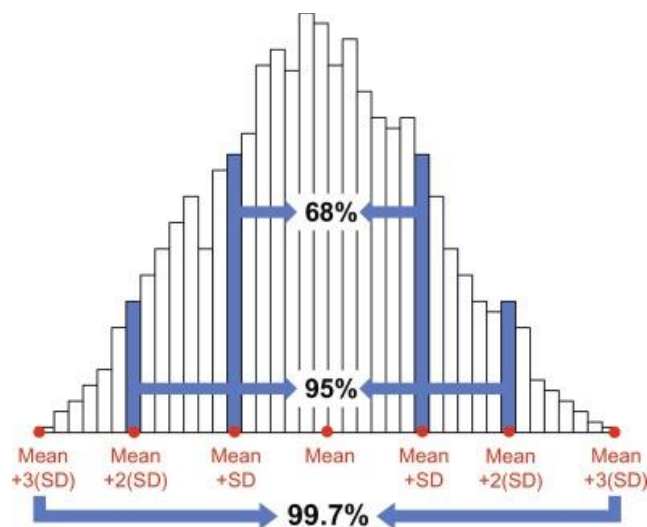


Figure 5-12: Shows the normal distribution applied onto the input design variables [275].

Initially, let $\sigma^2 > 0$, $\mu \in \mathbb{R}$ and $X \sim \mathcal{N}(\mu, \sigma^2)$

Then, the X has a finite value of variance, $\sigma^2 = v(x)$ and $E(X)$ as follows:

$$\begin{aligned}
 E[X] &= \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} x e^{-(x-\mu)^2/(2\sigma^2)} dx \\
 &= \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} (x + \mu) e^{-(x)^2/(2\sigma^2)} dx \\
 &= \mu + \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} x e^{-(x)^2/(2\sigma^2)} dx = \mu
 \end{aligned} \tag{5.21}$$

The simplification of variance similarly can be defined as follows:

$$\mathbf{Var}[X] = \mathbf{E}[X^2] - \mu^2 = \dots = \sigma^2 \tag{5.22}$$

The standard deviation, σ , of X can be calculated by $\sigma = \sqrt{\mathbf{Var}[X]}$.

The optimisation using RDO method will often target to minimise the objective function(s) based on the design domain pre-set. To search for robust design, the optimisation is performed to compromise between mean and standard deviation (noise factor) of output. The RDO method utilises the surrogates searching for objective function that is least affected by the input parameters uncertain variation [276]. The input parameters that are uncertain has standard deviation values which are adjustable [277]. Performing robust optimisation, surrogates are used based on published work of Jin et al. [278], Nejadseyfi et al. [279], Shin et al. [280]. The key steps of performing the robust design optimisation are as follows:

- A) The robust design optimisation problem is formulated based on pre-defined design space, design variables, objective functions and constraints.
- B) From the pre-set DOE points, each DOE points will be subjected by uncertainty forming a normal distribution of points. The new sets of points will be within the initial design space for deterministic solution.
- C) Each DOE point will be fed into the CFD model to calculate the objective functions or use the pre-constructed surrogates from the deterministic solution.
- D) Utilising the distribution output objective function calculated, the mean and standard deviation can be computed for each DOE point [268]. The calculation of mean and standard deviation can be performed using either Monte Carlo Sampling (MCS) or Polynomial Chaos Expansion (PCE) methods (refer subchapters 5.6.5.3 and 5.6.5.4)

- E) Surrogates are created for the newly calculated mean and standard deviation to map the relationship between the input and output data.
- F) The surrogates will then be fed within the optimisation algorithm for optimisation process to minimise the conflicting mean and standard deviation results which is called the Pareto Front.

The flow chart of robust design optimisation can be referred to Figure 5-13.

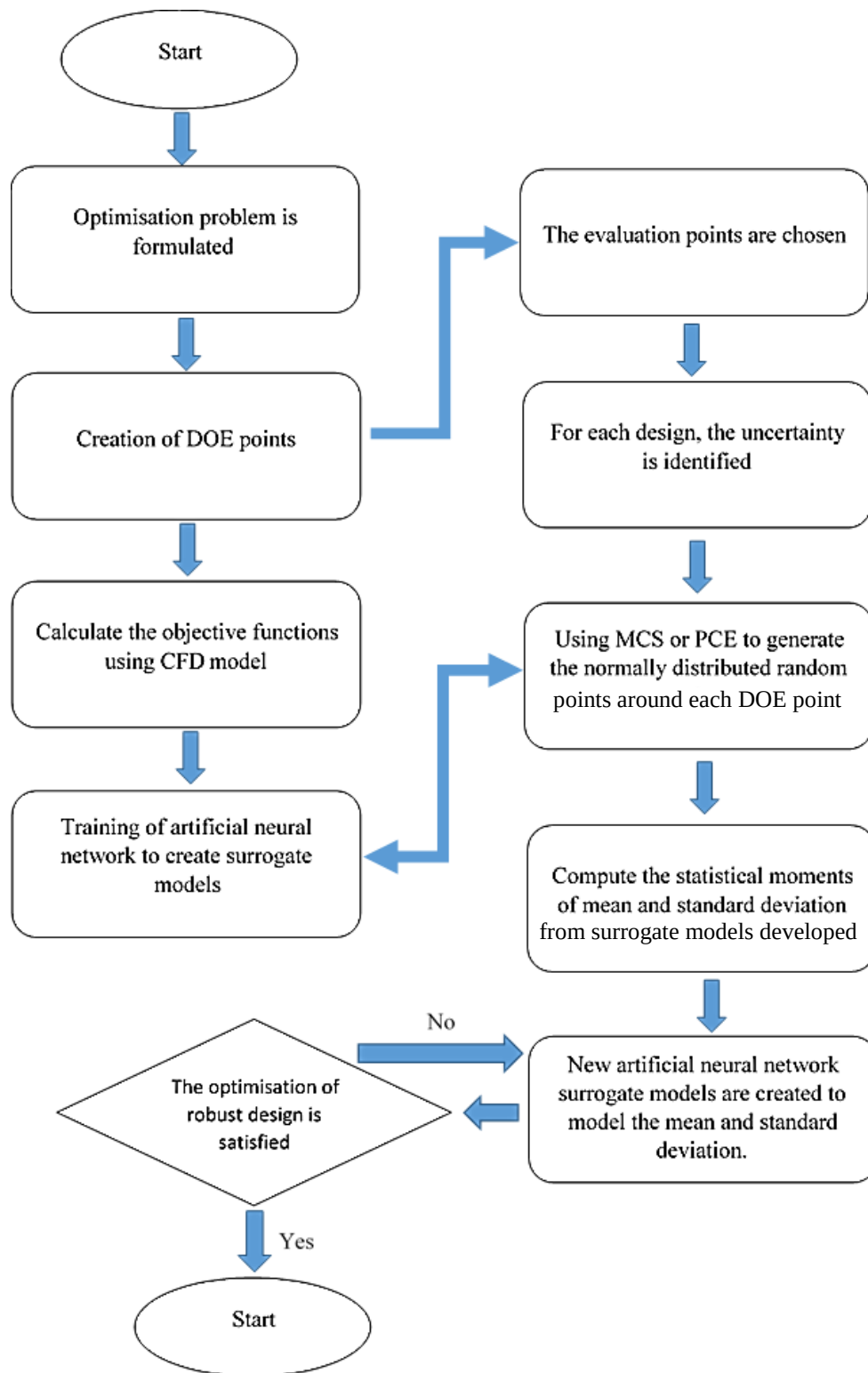


Figure 5-13: Flow chart for the steps of robust optimisation process.

5.6.5 Description of Uncertainty

To measure the product quality (Measurand-measured quantity [281]), uncertainty measurement is used as a fundamental process. Using the measurement uncertainty expression guide (GUM) for assessing and measurement uncertainty reporting, it can be used to perform uncertainty evaluation providing complete guidance, framework and reference point in an uncertainty estimation [282].

GUM methodology can listed as follows:

- 1) The input sources and measurand are defined respectively. The input sources variables identification which affects the measurement estimation either directly or indirectly is conducted.
- 2) Input sources results prediction is carried out by model modelling. Model modelling help to establish the relationship of the input and output sources and capturing the influence of inputs towards the measurements, thus, uncertainty can be predicted very well if strong model is developed.
- 3) Input sources uncertainties estimation. Estimation for all input sources are needed where there are two type methods to quantify the uncertainty evaluation [283, 284]. The methods are as follows:
 - A) **Type A:** From the observation repetitions where the uncertainty is calculated. The parameter used for this is usually standard deviation of measurements that have been repeated [285]. Let say, the dataset is represented by x where repeated data is given by $x_1, x_2, \dots, x_n$ and number of readings is denoted by n . Probability distribution standard deviation can be calculated using formula as follows:

$$s(x) = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2} \quad (5.23)$$

The uncertainty $u(x)$ is represented as follows:

$$u(x) = \frac{s(x)}{\sqrt{n}} \quad (5.24)$$

- B) **Type B:** measurement of uncertainty is based on non-statistical procedures. Very careful uncertainties estimation for input sources of Type B will be based on observational analysis or scientific judgement with high accuracy, utilising the procedure of measurement from the collections of data.

5.6.5.1 Uncertainties Propagation

The uncertainties propagation law (LPU) is the law which the GUM framework is based on where calculations are simplified based on the set of approximations and it can be used for different models range. The measured quantity of y as function of design variables with N number and using first-order Taylor series to express the y uncertainty are as follows:

$$y = f(x_1, x_2, \dots, x_n) \quad (5.25)$$

$$u_y^2 = \sum_{i=1}^N \left(\frac{\partial f}{\partial x_i} \right)^2 u_{x_i}^2 + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^N \left(\frac{\partial f}{\partial x_i} \right) \left(\frac{\partial f}{\partial x_j} \right) cov(x_i, x_j) \quad (5.26)$$

Based on Eq. 5.26, since there is no input variables correlation, the second term is neglected due to the cross-terms of partial derivatives products can be assumed to be zero which reduces the equation of be as follows:

$$u_y^2 = \sum_{i=1}^N \left(\frac{\partial f}{\partial x_i} \right)^2 u_{x_i}^2 \quad (5.27)$$

If the relationship between the input sources and output is non-linear where short version of the Taylor's series expansion until the terms of first-order does not has the ability to perform propagation error correction. There are numerous restrictions when using the GUM method [286-288] which are as follows:

- 1) Model must be linear for valid use of GUM methods, highly non-linear model is not solvable using Taylor's series expansion since it has only up to first-order terms where propagation of error cannot be corrected. The GUM method is not suitable for current work as non-linear datasets are being used
- 2) The same manner of input sources is required for application of GUM methods. If one or more input sources have uncertainty or sensitivity are larger compared to the others, GUM method is not valid to use due to overall uncertainty domination and hard to get accurate results. In addition, when there is no symmetric properties of input quantities distribution, GUM method is also not valid to be used.

From the limitations mentioned above, different solutions can be used for input quantities probability distribution propagation such as numerical and analytical integrations and numerical simulations (i.e., Monte Carlo methods).

5.6.5.2 Monte Carlo simulations

The use of Monte Carlo simulation (MCS) is a substitute approach instead of using GUM method when it does not work. With different model measurements, MCS has the upper hand compared to the GUM method providing more fast, robust and reliable results due to its ability to deal with non-linear complex model. When dealing with non-linear models, MCS has the ability to perform an estimation of it. Based on the information available in terms of quality and reliability, the uncertainty study can also be achieved using different methods. Using MCSM, random sampling is applied on the design variables (input data) to obtained the desired objectives (output data). If the number of samples for probability distribution is sufficient, then accurate desired output can be calculated. The normal distribution is used onto the input data where the values are quantified in terms of pre-set mean and standard deviation respectively. Based on respective PDF, the generation of random drawn numeric value is conducted by MCS for individual input [289]. The Gaussian distribution of PDFs are plotted in Figure 5-14 with fixed value of $\mu = 0$ and varied values of σ to be 0.5, 1.0 and 1.5 respectively.

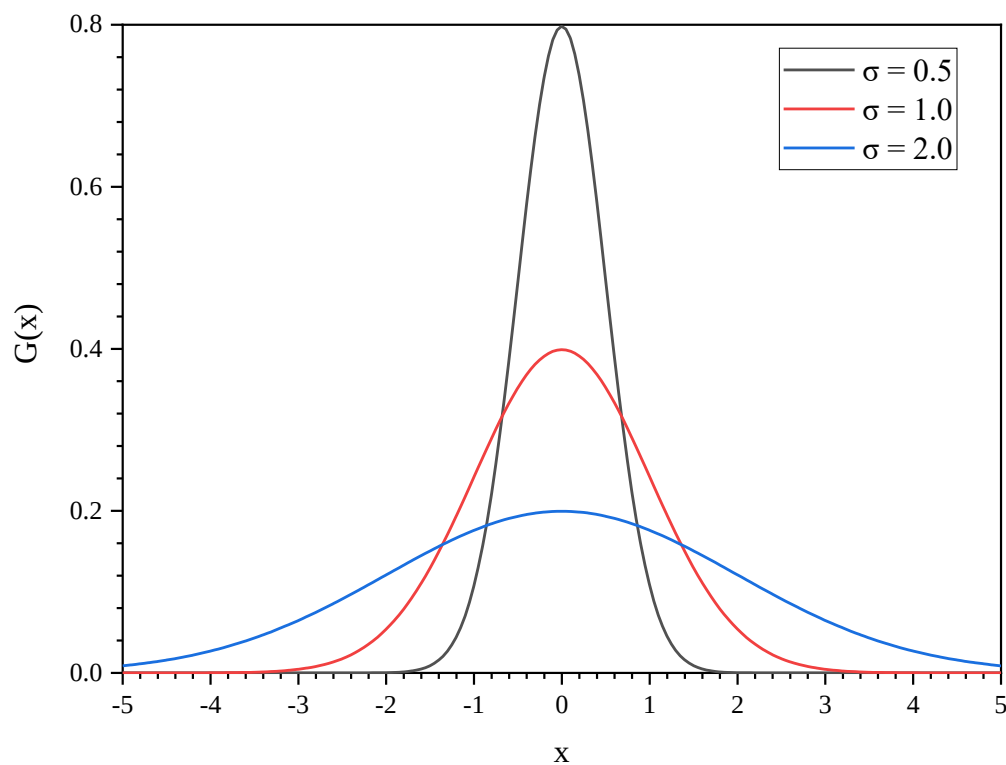


Figure 5-14: Shows the PDF with different values of standard deviation [290].

5.6.5.3 Polynomial Chaos Expansion (PCE) method

The Polynomial Chaos Expansion (PCE) method is initially proposed by Wiener [291]. PCE method utilises the Hermite polynomials and random variables obeying the normal distribution to solve stochastic processes. Cameron and Martin [292] validate the exploitation of Hermite polynomials to be used as orthogonal polynomials. From the introduction by Wiener [291], stochastic processes are solved by using Hermite polynomials chaos approach where it is implemented to be orthogonal polynomials. This method has been very effective for many engineering analyses [293-296]. Although the method by Wiener's works very well, some problems are present due to non-Gaussian based method and convergence issue [297, 298]. To solve this concern, new polynomial chaos method based the collaboration of Wiener-Askey is presented. Using the suitable orthogonal polynomials basis where the problems are solved instead of when using initial proposal of Wiener Hermite Polynomials method [299, 300].

The input design variables of X and output objectives of Y for a stochastic system is considered. Referring Table 5-1, the scheme proposed by Askey and its linkage to the orthogonal polynomial groups with its corresponding unique input distribution are shown. From Eq. 5.28, the surrogates of the objectives are developed using the orthogonal polynomials method [301].

$$y(x) \approx \hat{y}(x) = \sum_{i=0}^p c_i \psi_i(x) \quad (5.28)$$

Where the truncation order is denoted by p , coefficient is denoted by c_i and at the i -th order of orthogonal polynomial is denoted by ψ_i .

PCE model can be approached from two sides of different methods either using regression or pseudo-spectral projection. By using generated evaluations and samples, regression method functions by fitting the PCE model into it [302]. On the other hand, Pseudo-spectral projection method achieved the correspondent coefficient approximation by applying rules of quadrature.

Table 5-1: Shows the Orthogonal polynomial basis scheme by Askey and specified density function linkage with stochastic input variables [303].

Type of Distribution	Density Function	Orthogonal Polynomial basis	Weight function	Domain
Normal	$\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)$	Hermite $He_n(x)$	$\exp\left(-\frac{x^2}{2}\right)$	$[-\infty, +\infty]$
Beta	$\frac{(1+x)^\beta (1-x)^\alpha}{2^{\alpha+\beta+1} B(\alpha+1, \beta+1)}$	Jacobi $P_n^{(\alpha, \beta)}(x)$	$(1+x)^\beta (1-x)^\alpha$	$[-1, 1]$
Uniform	0.5	Legendre $P_n(x)$	1	$[-1, 1]$
Exponential	e^{-x}	Laguerre $L_n(x)$	e^{-x}	$[0, +\infty]$
Gamma	$\frac{x^\alpha e^{-x}}{\Gamma(\alpha+1)}$	Generalised Laguerre $L_n^\alpha(x)$	$x^\alpha e^{-x}$	$[0, +\infty]$

5.6.5.4 Uncertainty Quantification (UQ)

The study of Uncertainty Quantification (UQ) is based on the application of uncertainty onto the input variables achieving a set of output values. The probability distribution applied around input data will lead to the probability distribution of output data. Initially different parameters representing the design variables are introduced and then each design point within each design variable will have a normal distribution. For every new design point in the distribution data will then be fed into the CFD software for solutions but this will consume very high computational power and time. Thus, surrogates which have been developed earlier will be utilised for predicting the output results from the new input values (design points).

The Monte Carlo method is applied to obtain the propagated distribution where for every input data will have a normal distributed data which will be calculated through the CFD model or surrogates for thousands of iterations. After that, the output PDF distribution will be calculated based on the standard deviation σ and mean μ using the following formulas:

$$\mu_{fi} = \frac{1}{N} \sum_{j=1}^N f_i^j \quad (5.29)$$

$$\sigma_{fi}^2 = \frac{1}{N} \sum_{j=1}^N (f_i^j - \mu_{fi})^2 \quad (5.30)$$

5.6.5.5 Method of Robust Design Optimisation (RDO)

For Robust Design Optimisation (RDO), the mean of outputs (refer Eq. 5.29) undergoes minimisation. The solving of robust optimisation problem requires uncertainty with surrogates and confidence intervals. RDO formulation is as follows:

$$\begin{aligned} \text{minimisation } \mu_{\omega} &= \omega\mu_{f_1(x)} + (1 - \omega)\mu_{f_2(x)} \\ x &\in \Omega_0 \\ \text{subject to } \sigma_{f_1(x)} &< \sigma_1 \text{ and } \sigma_{f_2(x)} < \sigma_2 \end{aligned} \quad (5.31)$$

Where the range $\omega \in [0,1]$.

5.7 Chapter summary

Within this chapter, possible design formulation is explained in details. Different sampling methods are reviewed. The concept of surrogate modelling is also introduced where computational time and resources can be reduced drastically without having to simulate further CFD models. Furthermore, optimisation techniques covering deterministic and probabilistic framework were also presented. The methods that will be used to solve optimisation problem in this work can be summarised as follows:

- A) The DOE points of input design variables will be based on the chosen physical parameters defining the heat sink. The DOE points are created using Optimal Latin Hypercube Sampling method (Audze-Eglais optimality criterion) ensuring even distribution of points within the design space specified for each design variable. The number of points should be at least 10 to 15 times the number of design variables for acceptable representation of points distribution within the design space [304].
- B) Using the DOE points created, these points will be used in CFD software to calculate the corresponding of chosen output (objective functions) which in this work will total thermal resistance (R_{th}), pressure drop (ΔP). For mass of copper heat sink (M_{Cu}), the calculated is based on the analytical method proposed in sub-chapters of 6.5, 7.5 and 8.5 respectively.
- C) Based on the DOE points of input design variables and calculated objective functions, the Radial Basis Function (RBF) and Artificial Neural Network (ANN) methods will be used to map the relationship between input and output variables creating surrogate models.
- D) The surrogate models will be utilised within optimisation algorithm (NSGA-II) to search for optimal solution for both single- and multi-objective deterministic optimisations. Only single solution will be obtained for single-objective optimisation while for multi-

objective optimisation a Pareto optimal solutions are obtained compromising between conflicting objectives.

- E) To include uncertainty study based on Robust Design Optimisation (RDO) method, Monte Carlo simulation method and Polynomial Chaos Expansion (PCE) methods are used to quantify the normal distribution with specified statistical moments of standard deviations and means (the DOE point itself) applied on every single DOE points. The distributions will be used on the pre-developed surrogate models to calculate the corresponding distribution of outputs objective functions where also the statistical moments calculated.
- F) Surrogate models are further developed between the original input design variables with newly calculated output statistical moments. These surrogates will be used to find the Pareto optimal solutions between means and standard deviations searching for optimal output of means which has corresponding small standard deviations indicating the output curve at that particular position does not have very steep gradient.

Chapter 6 : Design optimisation of single inlet-outlet serpentine mini-channel heat sinks

6.1 Introduction

From the previous work on the analysis of serpentine mini-channel heat sink performance both thermally and hydraulically, it was found that using serpentine channel heat sink offers good heat-removal ability compared to conventional straight channel heat sink and offers better temperature distribution. Using single path serpentine mini-channel (SPSM) provides better thermal performance compared to double path (DPSM) and triple path (TPSM) serpentine mini-channel [144]. But using SPSM design induces very large pressure drop, ΔP , up to ten times larger compared to straight rectangular mini-channel (SRM) design. From the findings, thermal enhancement comes at the expense of very high pressure drop. In order to reduce the magnitude of pressure drop while improving the rate of heat transfer, the SPSM design has been modified by researcher with addition of integrated small chevron fins which was proven to be effective for pressure drop reduction and simultaneously enhancing the coefficient of heat transfer [38]. The addition of small fins within serpentine channel heat sink is still at its infancy level which needs to be explored further since adding fins generate secondary flow improve heat transfer and reducing pressure drop significantly at the same time. Therefore, this work focuses on the use of novel design of oblique, trapezium and diamond shapes fins within serpentine mini-channel heat sink where the fin shapes have only been used by researchers in straight channel.

To explore further on the performance of serpentine mini-channel heat sink with fins, different shapes of fins are tested including oblique, trapezium and diamond fins to study the effect on thermal and hydraulic performance of heat sink. The tests on secondary channel within straight channel using fins have been explored in the literature covering oblique fins [131, 132] and trapezoidal fins [175, 176] which has been proven effective to improve heat transfer and reducing pressure drop simultaneously. The use of diamond shape has not been utilised enough for research in heat sink study where diamond shape vortex generator used to induce flow disturbance [177] and diamond shape perforation for pin fin heat sink [178], thus, to explore further using diamond shape, diamond fins creating secondary flow in serpentine mini-channel heat sink is investigated for the present work. In this chapter, the descriptions of novel designs of heat sinks are presented. The CFD solutions will be presented using post processing tools by COMSOL Multiphysics v5.4. Four major parameters utilised to test the performance of heat sink

are pressure drop, ΔP , total thermal resistance, R_{th} , Average Nusselt Number, Nu_{avg} , and Performance Evaluation Criteria (PEC) respectively. Design optimisation process is performed on serpentine mini-channel heat sink with fins that gives the most superior thermal performance while having very high performance hydraulically.

6.2 Design descriptions of single inlet-outlet of serpentine mini-channel heat sink with fins

For current work, single inlet-outlet serpentine mini-channel heat sink is investigated numerically for plate (SMPF), oblique (SMOF), trapezium (SMTF) and diamond (SMDF) fins. Each type of serpentine mini-channel with fins (excluding SMPF) has four different configurations (-A, -B, C and -C) which are completely designed using COMSOL Multiphysics v5.4 geometry tools. In order to make a fair performance comparison between serpentine mini-channel heat sink with different shapes of fins, all heat sinks share the same height of channel (H_{ch}), width of fin (W_w), width of channel (W_{ch}), area of footprint ($W \times L$) and thickness of substrate (H_b) respectively. The heat sinks are made from copper and uses water as cooling fluid. The serpentine mini-channel heat sink with fins (excluding SMPF) has continuous plate fin with 7 smaller fins embedded per row. The dimensioning of SMPF is shown in Figure 6-1 while for four different configurations of SMOF, SMTF and SMDF heat sinks can be referred in Figure 6-2, Figure 6-3 and Figure 6-4 respectively. The detailed dimensions for all heat sinks are listed in Table 6-1. To have a fair comparison between the performances of heat sinks, all configurations of SMOFs, SMTFs and SMDFs are having their fins spread out equally within the plate fin. To provide good balance between pressure drop and heat transfer, Suga and Aoki [305] recommended the range of louver angles between 20° to 45° . All heat sinks with oblique, trapezium and diamond fins will have the same fin angle, θ_{fin} , of 45° which is within range proposed in [305] and secondary channel length, l_{sc} , of $0.7mm$. For Figure 6-1 to Figure 6-4, the blue domain represents the fluid (water) and grey domain represents the solid part of heat sink (copper).

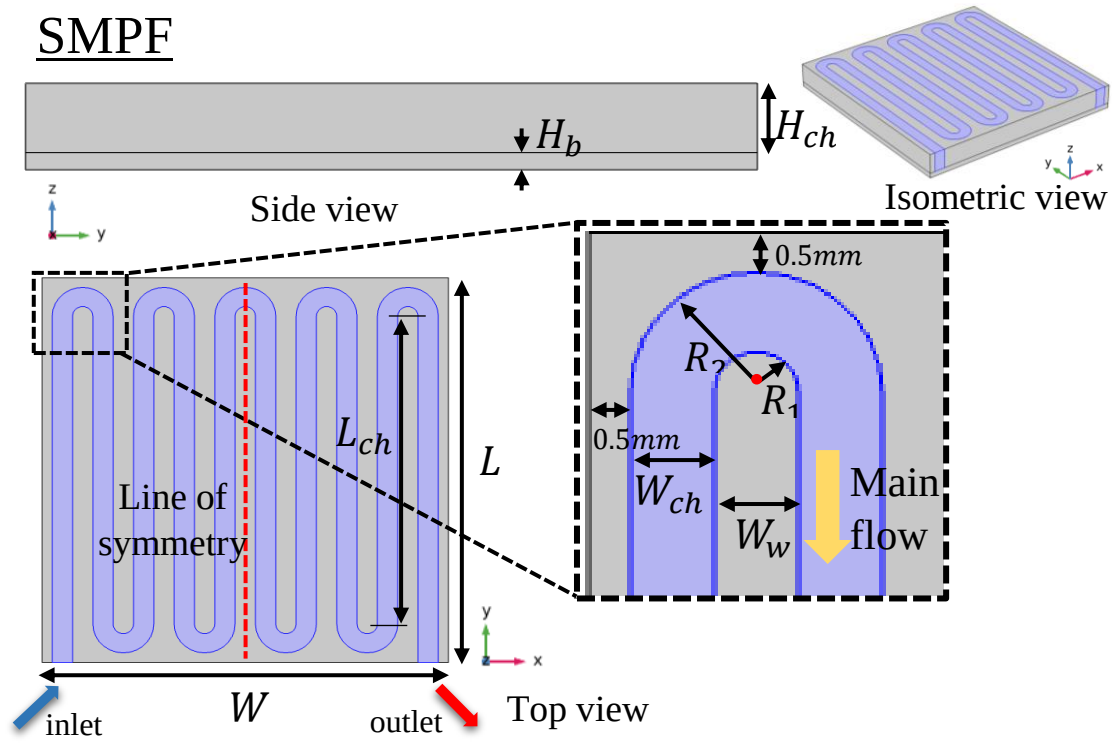


Figure 6-1: Shows the detailed dimensioning for SMPF heat sink.

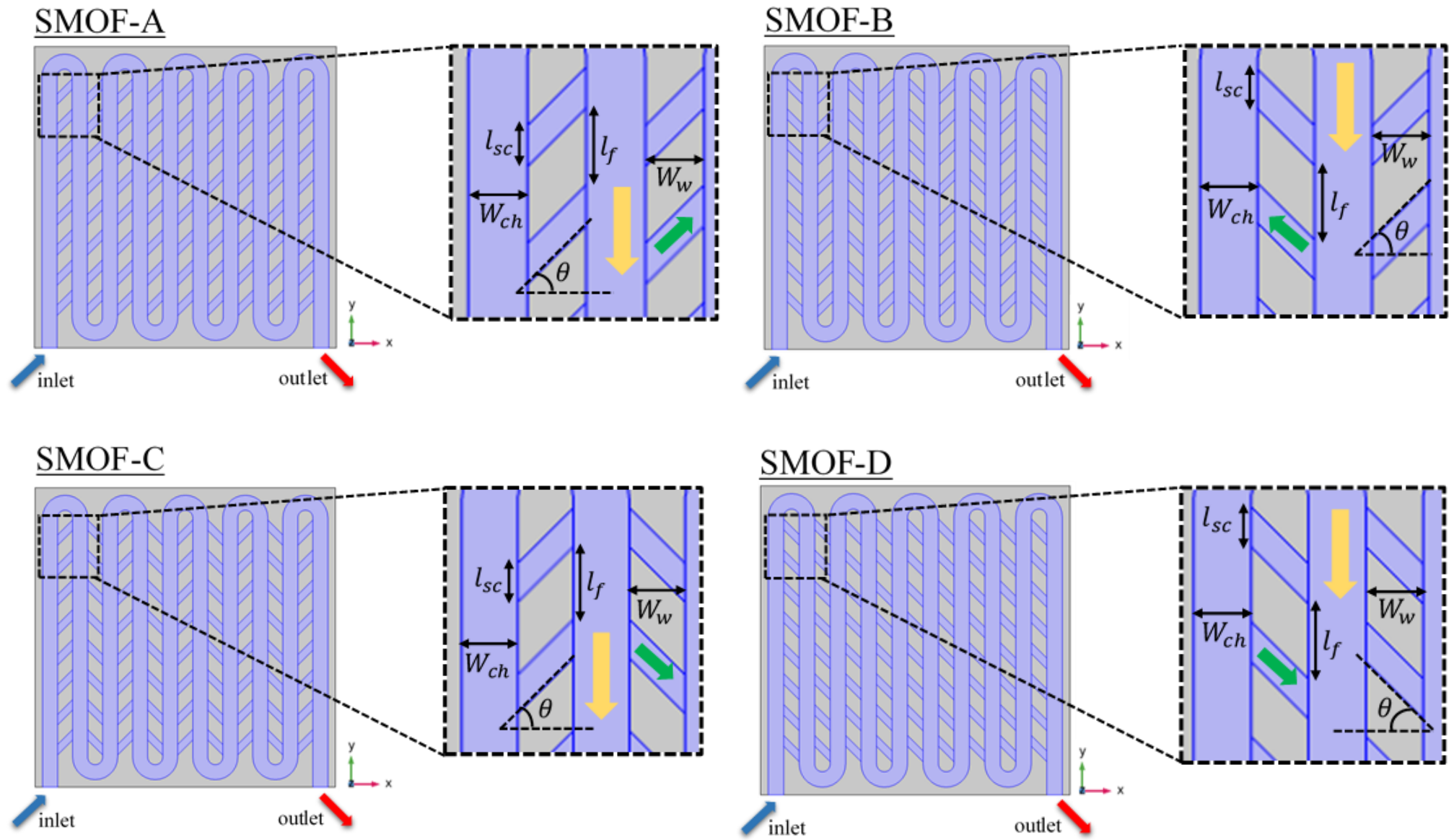
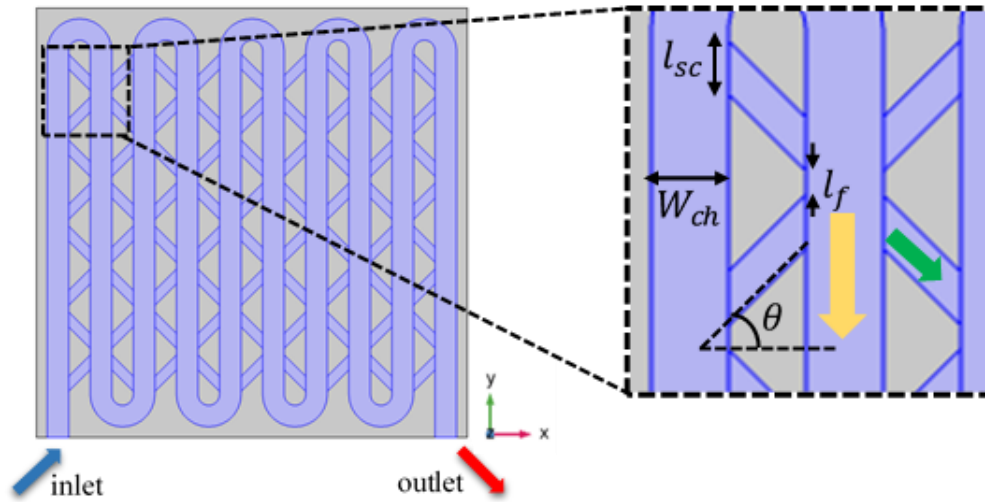
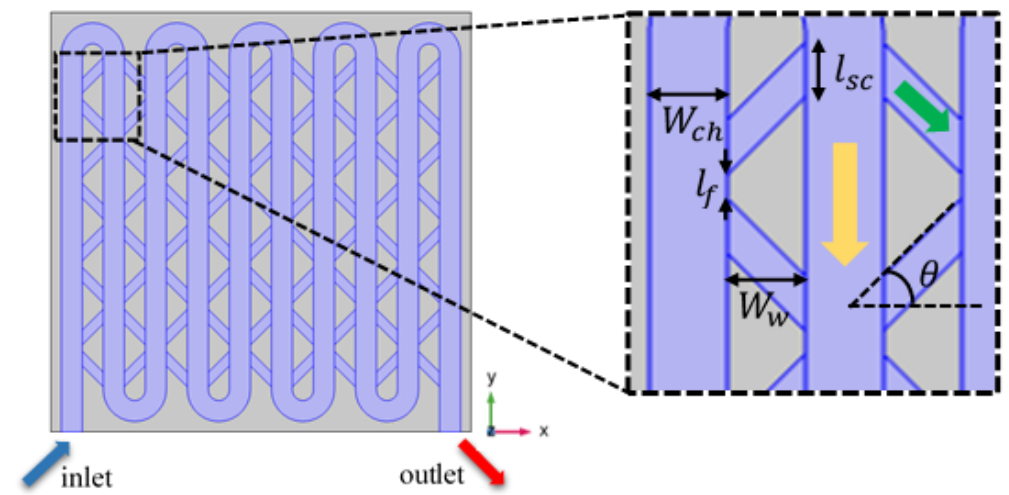


Figure 6-2: Shows the different configurations of SMOF heat sink design.

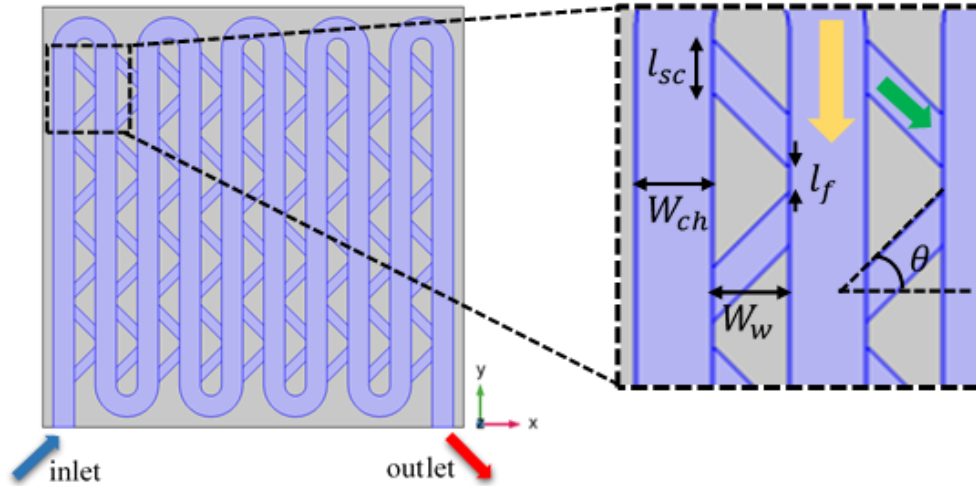
SMTF-A



SMTF-B



SMTF-C



SMTF-D

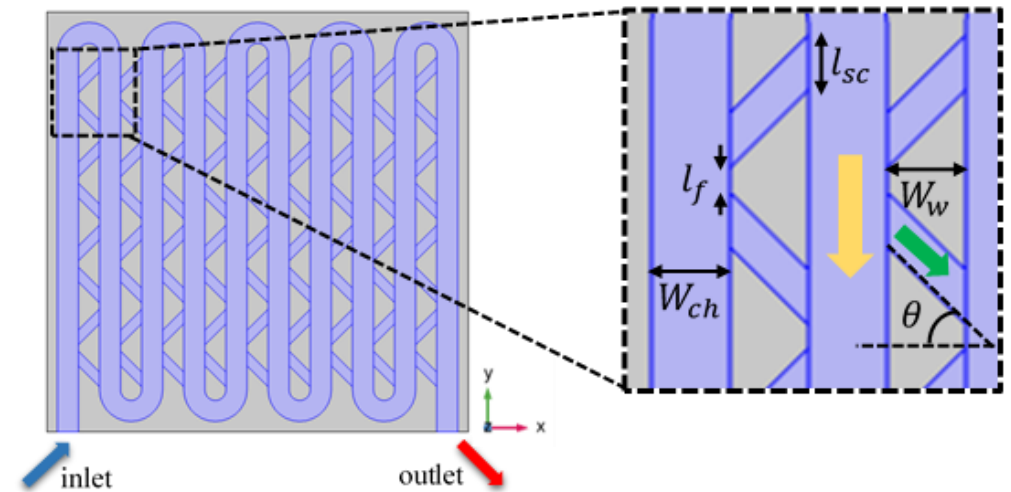


Figure 6-3: Shows the different configurations of SMTF heat sink design.

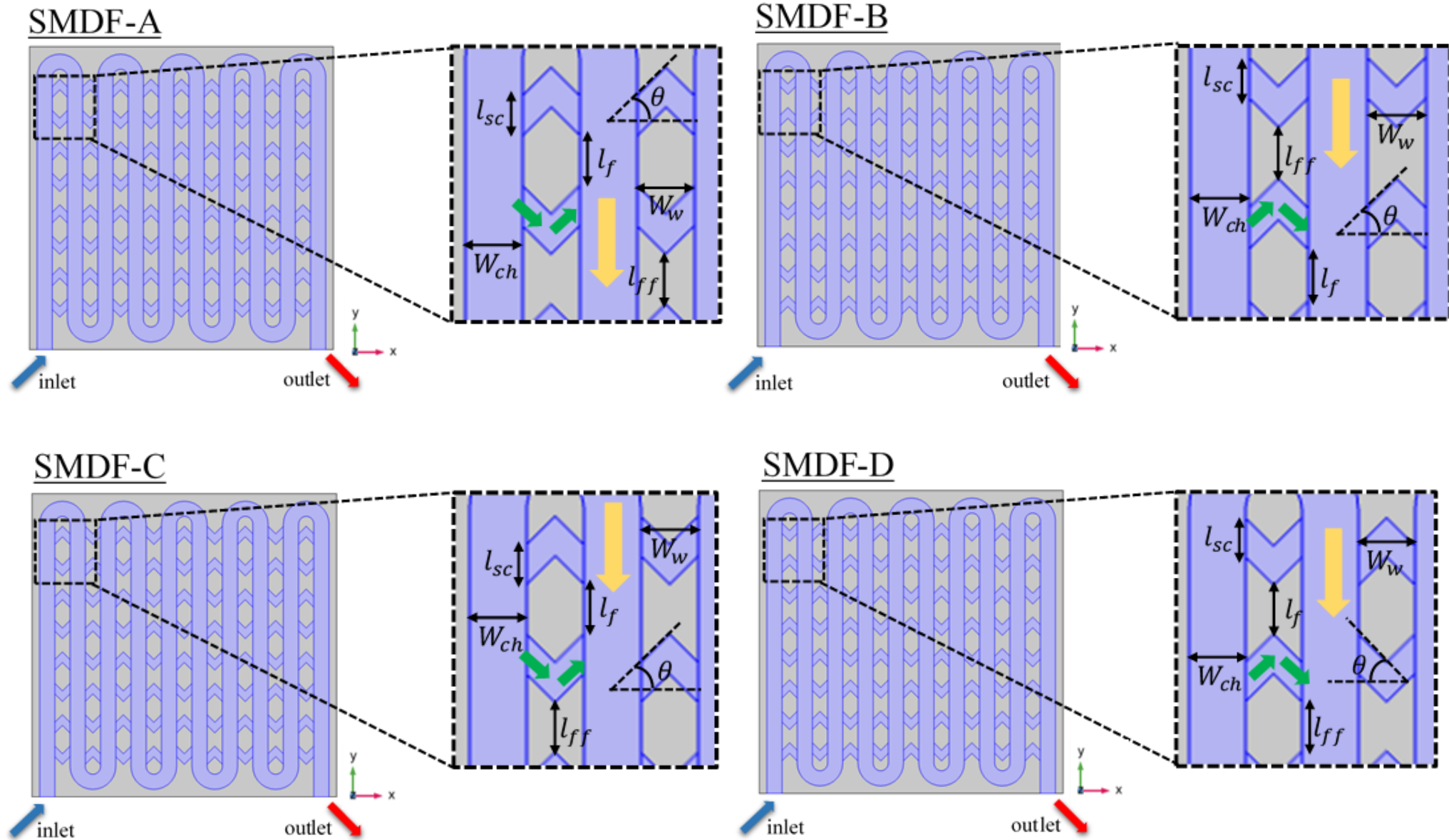


Figure 6-4: Shows the different configurations of SMDF heat sink design.

Table 6-1: Shows the dimensions for all serpentine mini-channel heat sink designs.

Characteristics	SMPF	SMOF-A	SMOF-B	SMOF-C	SMOF-D	SMTF-A	SMTF-B	SMTF-C	SMTF-D	SMDF-A	SMDF-B	SMDF-C	SMDF-D
Material							Copper						
Heat sink dimensions, width x length x height, $W \times L \times H(mm)$							20 x 20 x 2.5						
Length of channel, $L_{ch} (mm)$							16						
Main channel width, $W_{ch}(mm)$							1						
Fin Width, $W_w (mm)$							1						
Inner radius, $R_1 = W_w / 2 (mm)$							0.5						
Outer radius, $R_2 = W_w / 2 + W_{ch} (mm)$							1.5						
Channel depth, $H_{ch} (mm)$							2						
Substrate thickness, $H_b (mm)$							0.5						
Number of mini-channels, n							10						
Hydraulic diameters, $D_h (mm)$							1.333						
Number of fins per row, N_{cf}	-						7						
Secondary channel width, $W_{sc} = l_{sc} \cos \theta_{fin} (mm)$	-						0.4950						
Secondary channel length, $l_{sc} (mm)$	-						0.7						
fin length, $l_f (mm)$	-	1.3	1.3	1.3	1.3	0.3	0.3	0.3	0.3	0.8714	0.8714	0.8714	0.8714
fin pitch, $P_f = l_f + l_{sc} (mm)$	-	2	2	2	2	1	1	1	1	1.571	1.571	1.571	1.571
second fin length, $l_{ff} (mm)$	-	-	-	-	-	-	-	-	-	0.8714	0.8714	0.8714	0.8714
Fin angle, $\theta_{fin} (^\circ)$	-						45						

For current work, the transferring of heat from solid to moving fluid (convection heat transfer) is through the area available for heat transfer which is called the effective area (A_{eff}). The A_{eff} only covers all mini-channel walls except the top wall which is considered adiabatic. The effective area are comprises of area of mini-channel base (A_{base}) and wall (A_{wall}). Thus, the available area can be calculated using the following formula:

$$A_{eff} = A_{base} + A_{wall} \quad (6.1)$$

For SMPF design the total effective area ($A_{eff,SMPF}$) can be calculated as follows:

$$\begin{aligned} A_{eff,SMPF} &= [A_{base,SMPF}] + [A_{wall,SMPF}] \\ A_{eff,SMPF} &= \left[nW_{ch}L_{ch} + 2W_{ch} \left(\frac{L - L_{ch}}{2} \right) \right. \\ &\quad \left. + \pi(n-1) \left(\frac{W_w W_{ch} + W_{ch}^2}{2} \right) \right] \\ &\quad + \left[2nH_{ch}L_{ch} + 4H_{ch} \left(\frac{L - L_{ch}}{2} \right) \right. \\ &\quad \left. + \pi H_{ch}(n-1) \left(\frac{W_w}{2} + 2W_{ch} \right) \right] \end{aligned} \quad (6.2)$$

Where n is the number of mini-channels ($n = 10$).

The total effective area (A_{eff}) for all other designs of SMOFs, SMTFs and SMDFs can be calculated using the same formula:

$$\begin{aligned} A_{eff,fins} &= [A_{base\ with\ fins}] + [A_{wall\ with\ fins}] \\ A_{eff} &= \left[nW_{ch}L_{ch} + 2W_{ch} \left(\frac{L - L_{ch}}{2} \right) + \pi(n-1) \left(\frac{W_w W_{ch} + W_{ch}^2}{2} \right) \right. \\ &\quad \left. + (m+1)(n-1)W_w l_{sc} \right] \\ &\quad + \left[2nH_{ch}L_{ch} + 4H_{ch} \left(\frac{L - L_{ch}}{2} \right) \right. \\ &\quad + \pi H_{ch}(n-1) \left(\frac{W_w}{2} + 2W_{ch} \right) \\ &\quad \left. + 2(m+1)(n-1)(H_{ch}W_w^2 \tan\theta - H_{ch}l_{sc}) \right] \end{aligned} \quad (6.3)$$

Where m is the number of fins integrated within one plate fin ($m = 7$).

The parameter of A_{eff} will be used extensively in this chapter to compare the performance of conventional serpentine mini-channel heat sink with the new heat sink designs with fins. Firstly, because the A_{eff} effects the performance of heat transfer significantly. Secondly, for the comparison between different shapes of secondary channels used, the A_{eff} is made to be constant.

6.3 The approach for CFD simulations

For all CFD simulations, all heat sinks are designed in COMSOL Multiphysics v5.4 for more effective parametric analysis. The governing equations, boundary conditions and assumptions, validation and mesh independent studies of the numerical results will be presented in this sub-chapter.

6.3.1 Governing Equations

To understand the phenomena of heat transfer and fluid flow of the single inlet-outlet serpentine mini-channel heat sinks, Finite Element Analysis (FEA) method is exploited within COMSOL Multiphysics v5.4 to find the solution of 3-D CFD simulations. The water velocity (U_{in}) ranges between 0.3348 ms^{-1} to 1.540 ms^{-1} with corresponding Re of 500 to 2300 when the hydraulic diameter, $D_h = 1.333 \text{ mm}$. COMSOL Multiphysics has the capability to solve the governing equations which are the continuity, momentum and energy equations for the serpentine mini-channel heat sinks. For thermal-hydraulic performance of heat sinks with different shapes secondary channels comparison and parametric study the Reynolds number only ranges within the laminar region to reduce computational duration as the turbulence region has been modelled within the validation code in sub-section 6.3.3 used in optimisation study, thus, this shows that turbulence modelling technique has been shown correctly without having to use it again since turbulence modelling is computationally expensive. For single-phase and steady laminar regime, an incompressible fluid Navier-Stokes momentum equations together with continuity equation are used (refer Eq. 4.6 and Eq. 4.12). The energy equations (heat transfer) used for solid and liquid (water) are describes in Eq. 4.23 and Eq. 4.24 respectively.

6.3.2 The assumptions and boundary conditions

For numerical solution simplification, the thermal and flow characteristics analysis on all designs of heat sinks are simplified which are also used in other studies [113, 136, 141, 223, 306]. The assumptions used for the development of 3-D flow and heat transfer numerical model are as follows: (A) steady fluid flow and heat transfer; (B) single-phase and incompressible laminar

flow; (C) neglecting the effect of radiation and buoyancy; (D) viscous dissipation is negligible; (E) channel surfaces are smooth.

The boundary conditions as well as computational domain can be referred in Figure 4-7. No-slip velocity condition ($\mathbf{u}_s = 0$) applied to the solid walls and temperatures of wall can be defined as $T_s = T_f \text{ at wall}$. At the boundaries of solid-liquid, heat transfer through convection and conduction to fluid are coupled using heat flux continuity at fluid and solid walls interface (refer Figure 4-6). Inlet flow boundary conditions are inlet velocity, U_{in} (m/s) based on corresponding Re and temperature ($T_{f,in} = 20^\circ C$). Outlet flow boundary condition is set to be $p = p_0 = 0 \text{ Pa}$ (fluid is considered flowing out to open atmosphere where gauge pressure is zero). All outer surfaces of a heat sink except bottom side are considered adiabatic. Uniform heat flux is applied at the bottom of heat sink using $-\mathbf{n} \cdot (-k_s \nabla T) = q''$.

6.3.3 Validation code with published work

The validation process is conducted based on the published work by Al-Neama et al. [38] who investigated the effect of using chevron fins within serpentine mini-channel heat sink on the thermal-hydraulic performance. They used water as coolant with Reynolds number, Re , set between 747 and 4482 respectively. A total of 62 W/cm^2 of heat flux applied underneath of heat sink from two heaters used. The oblique angle (θ), length of fin (l_f) and pitch of fin (p_f) are set to be 30° , 1.3 mm and 2.3 mm respectively (refer [38] for further reference of diagram). Under both regimes of laminar and turbulence flow the results of R_{th} are compared for current and published work. From Figure 6-5, the correlation of the current and published work is in good agreement. Based on the validation, it can be decided that further simulations can be confidently conducted with high accuracy.

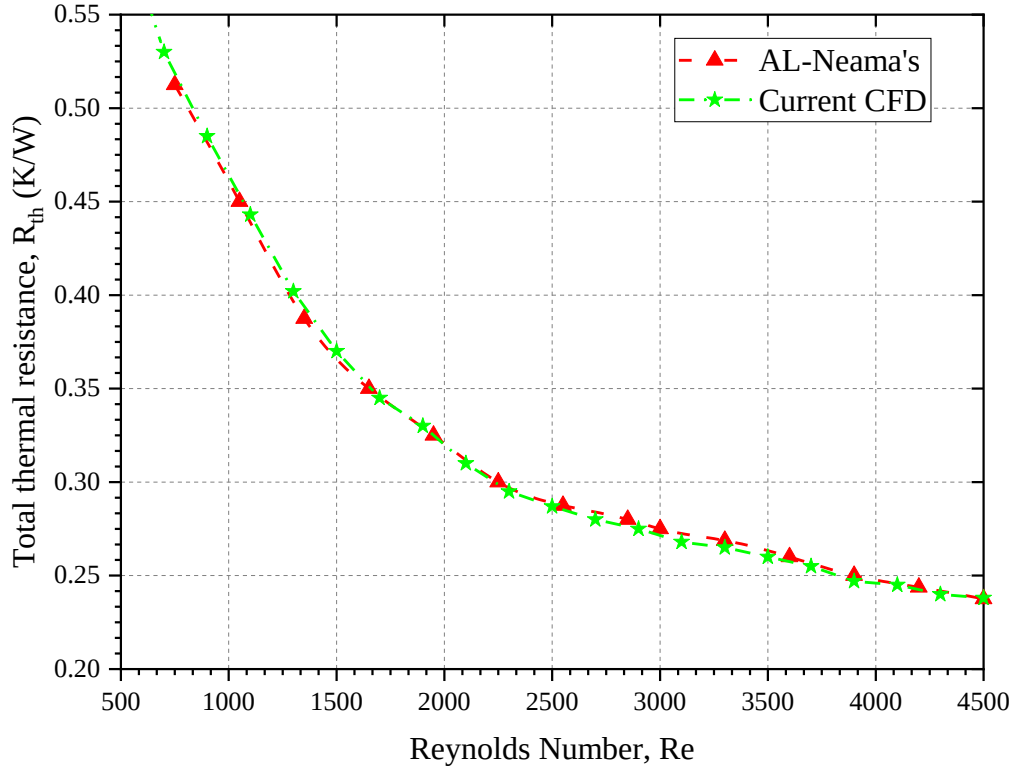


Figure 6-5: Shows the plots of R_{th} against Re for both published and current work.

6.3.4 Grid independence test

Grid independence study is performed on the numerical solutions for all heat sinks designs. To obtain accurate results from CFD solution, a mesh independence study has to be carried out searching for the minimum number of cells of entire domain (solid and fluid) ensuring accuracy and simulation running time is minimised. The corresponding results of physical parameters are based on different number of elements used listed in Table 6-2. Grid 1 has the least number of elements while grid 4 has the most for each design of heat sink. The meshes within the domain of fluid are refined around bends and fins. Fine mesh is used within the remaining geometry for all heat sink designs. The percentage error (E) of each grid is calculated with respect to Grid 4 using parameters of R_{th} and ΔP . Relative error is employed to find the lowest number of mesh with reference to the highest number of mesh which is as follows [209]:

$$e\% = \left| \frac{J_2 - J_1}{J_1} \right| \times 100 \quad (6.4)$$

Where the physical parameter is denoted by J . The physical parameter is parameter that be calculated straight from the CFD model such as R_{th} , ΔP , h_{avg} and Nu_{avg} respectively. For this chapter, the mesh independent study is conducted with both R_{th} and ΔP parameters respectively. The Grid 3 has been chosen to run CFD simulations due to its high accuracy and efficiency with

small percentage relative error ($< 2\%$). Refer Appendix A.2.1 to A.2.3 for the mesh structures of Grid 3 for all cases. The CFD simulations for mesh independence study for all heat sink designs are conducted using the same boundary conditions with Re of 2300, temperature of water inlet at 293.15 K and uniform heat flux of 100 W/cm^2 applied at the base surface of heat sinks (refer Figure 4-7 for the diagram of boundary conditions).

Table 6-2: Shows the parameters of R_{th} and ΔP for grid independence study.

Heat sink model	Grid No.	Elements number	Rth (K/W)	E%	ΔP (Pa)	E%
SMPF	1	469012	0.11521	5.688	17404	8.099
	2	510535	0.11314	3.789	16721	3.857
	3	554182	0.11014	1.037	16319	1.360
	4	645865	0.10901	-	16100	-
SMOF-A	1	3068626	0.08936	6.002	5522.2	8.270
	2	3383214	0.08725	3.499	5313.1	4.170
	3	3797051	0.08525	1.127	5173.5	1.433
	4	6845817	0.08430	-	5100.4	-
SMOF-B	1	3079747	0.09164	6.126	3975.3	8.817
	2	3397060	0.08941	3.544	3810.0	4.292
	3	3800329	0.08724	1.031	3719.3	1.809
	4	6873522	0.08635	-	3653.2	-
SMOF-C	1	3075087	0.09628	5.674	5815.8	9.707
	2	3386066	0.09421	3.402	5611.8	5.859
	3	3791398	0.09214	1.131	5396.8	1.803
	4	6703388	0.09111	-	5301.2	-
SMOF-D	1	3090549	0.09599	5.891	4311.2	8.245
	2	3410001	0.09391	3.596	4202.3	5.511
	3	3806055	0.09182	1.291	4049.1	1.665
	4	6781542	0.09065	-	3982.8	-
SMTF-A	1	3054644	0.10095	5.873	4185.7	8.029
	2	3365750	0.09855	3.356	4079.1	5.278
	3	3713430	0.09643	1.133	3958.4	2.163
	4	6001880	0.09535	-	3874.6	-
SMTF-B	1	3064009	0.09693	2.193	3589.1	8.429
	2	3374694	0.09897	4.344	3463.2	4.625
	3	3922965	0.09693	2.193	3383.5	2.217
	4	6356570	0.09485	-	3310.1	-
SMTF-C	1	3054599	0.08997	6.840	5784.2	8.926
	2	3367213	0.08764	4.073	5589.1	5.252
	3	3719842	0.08564	1.698	5404.0	1.766
	4	6369335	0.08421	-	5310.2	-
SMTF-D	1	3064818	0.0911	7.303	4795.4	7.929
	2	3380790	0.0889	4.711	4680.9	5.352
	3	3773917	0.0868	2.238	4533.8	2.041
	4	6374563	0.0849	-	4443.1	-
SMDF-A	1	3317415	0.08977	6.325	4987.4	7.501
	2	3639538	0.08787	4.074	4899.3	5.602
	3	4547602	0.08598	1.836	4734.0	2.039
	4	6181908	0.08443	-	4639.4	-
SMDF-B	1	3589579	0.08978	6.085	4190.1	8.126
	2	3928250	0.08799	3.963	4085.3	5.422
	3	4530888	0.08589	1.488	3937.5	1.608
	4	7123438	0.08463	-	3875.2	-
SMDF-C	1	3470557	0.08628	7.729	5500.3	8.08
	2	3800125	0.08431	5.269	5310.2	4.345
	3	4399489	0.08212	2.535	5192.1	2.024
	4	6626969	0.08009	-	5089.1	-
SMDF-D	1	34485567	0.08728	6.465	4905.1	7.326
	2	3784656	0.08533	4.086	4776.9	4.52
	3	4459289	0.08375	2.159	4666.2	2.098
	4	6768542	0.08198	-	4570.3	-

6.4 Characteristics of fluid flow and heat transfer using different design of single inlet-outlet serpentine mini-channel heat sink

6.4.1 The effect of changing the Re on R_{th} , Nu and ΔP

The effect of changing the values of Re were investigated on all SMCHS designs proposed in terms of different parameters of thermal resistance (R_{th}), average Nusselt number (Nu) and pressure drop (ΔP). For this work, single-phase laminar flow is considered with Re ranges between 500 and 2300 based on the hydraulic diameter (D_h) of 1.333mm for all designs. Uniform heat flux of 100W/cm^2 is applied underneath the heat sink. For all SMCHS designs, the geometrical parameters can be referred in Table 1 where all of them shares the same fin angle (θ_{fin}) and secondary channel length (l_{sc}) of 45° and 0.7mm respectively. The smaller fins are distributed evenly throughout the plate fin to have a fair comparison between different types of fins used. Within this sub-chapter, the results obtained are based on the pre-set dimensioning of serpentine mini-channel heat sink with boundary conditions applied where it might be not suitable for smaller (e.g. micro-channel or nano-channel) or bigger (e.g. automotive industry) applications where the heat flux produced is significantly higher.

From Figure 6-6, it is observed that as Re increases R_{th} (refer Eq. 3.53) decreases monotonically for all SMCHS designs since the highest surface temperature ($T_{surf,max}$) of heat sink decreases. At higher Re , higher liquid velocity induces the increase rate of convective heat transfer thus reducing the surface temperature of heat sink [307]. Referring Figure 6-7, it shows the percentage difference in R_{th} against the SMPF design. As Re increases, generally the percentage difference increases indicating that the rate of R_{th} decrease for the SMCHS with smaller fins is higher than SMPF showing that rapid increase in heat transfer due enhanced fluid flow mixing due to flow re-initialisation but there are existence of positive and negative percentage differences indicating decrement and increment of R_{th} demonstrating that having smaller fins do not always lead to improved R_{th} compared to SMPF heat sink which will be discussed further in subchapter 3.2.2.1.

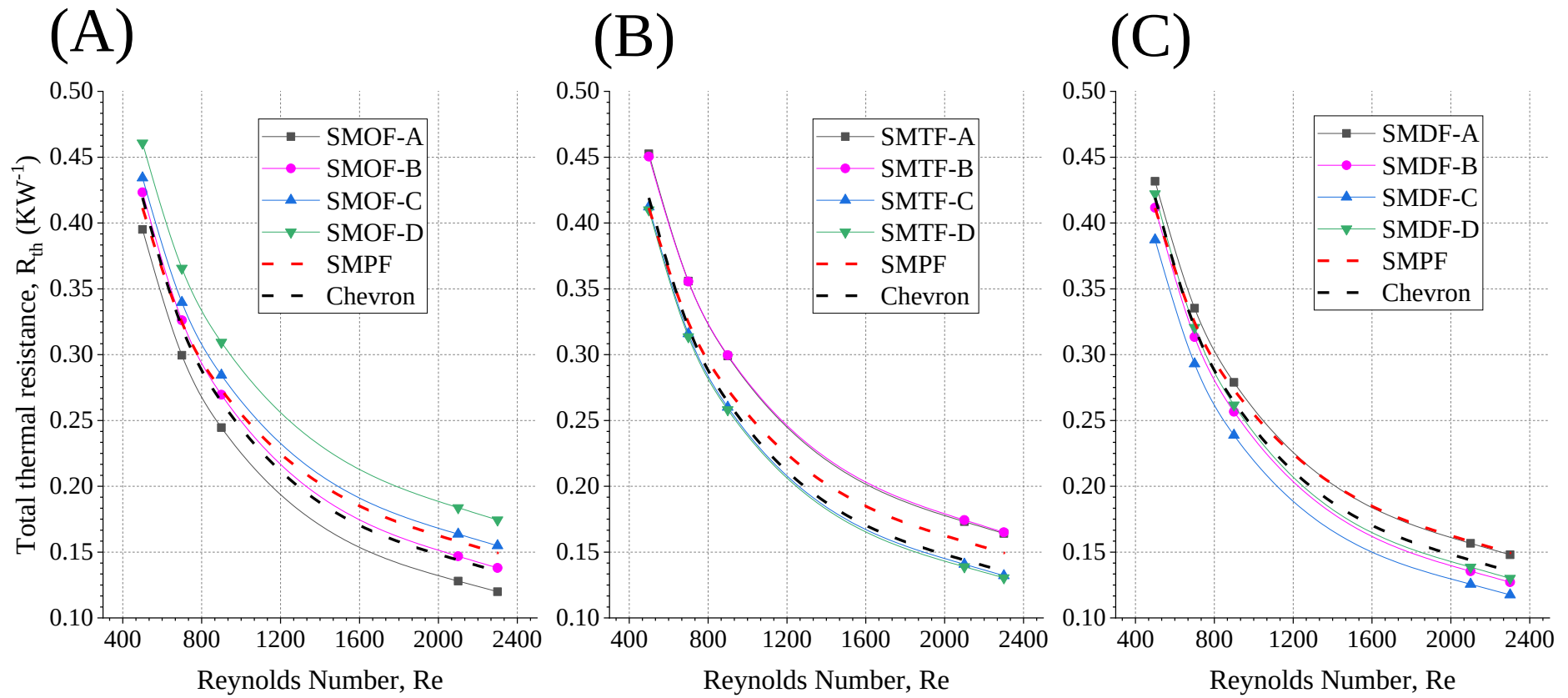


Figure 6-6: Shows the total thermal resistance, R_{th} , plots against Re for all heat sink designs.

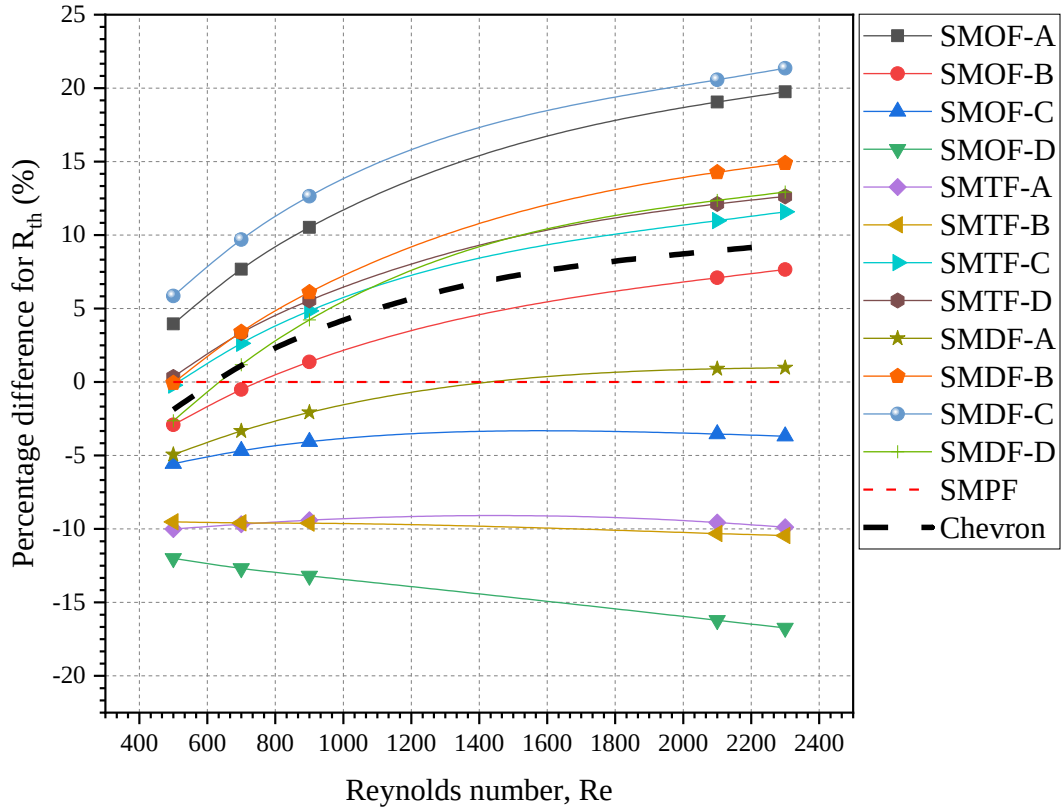


Figure 6-7: Shows the percentage difference of total thermal resistance, R_{th} , for all serpentine mini-channel heat sink with fins against the SMPF heat sink at increasing Re .

Referring to Figure 6-8, it shows the Nu_{avg} (refer Eq. 3.24) plots for all serpentine mini-channel heat sink design designs with increasing Re . The Nu_{avg} increases as Re increases. From Eq. 3.24, Nu is directly proportional to heat transfer coefficient (h_{avg}) which is greatly affected by change in Re since at higher liquid velocity the convective heat transfer improves leading to increase in h_{avg} value.

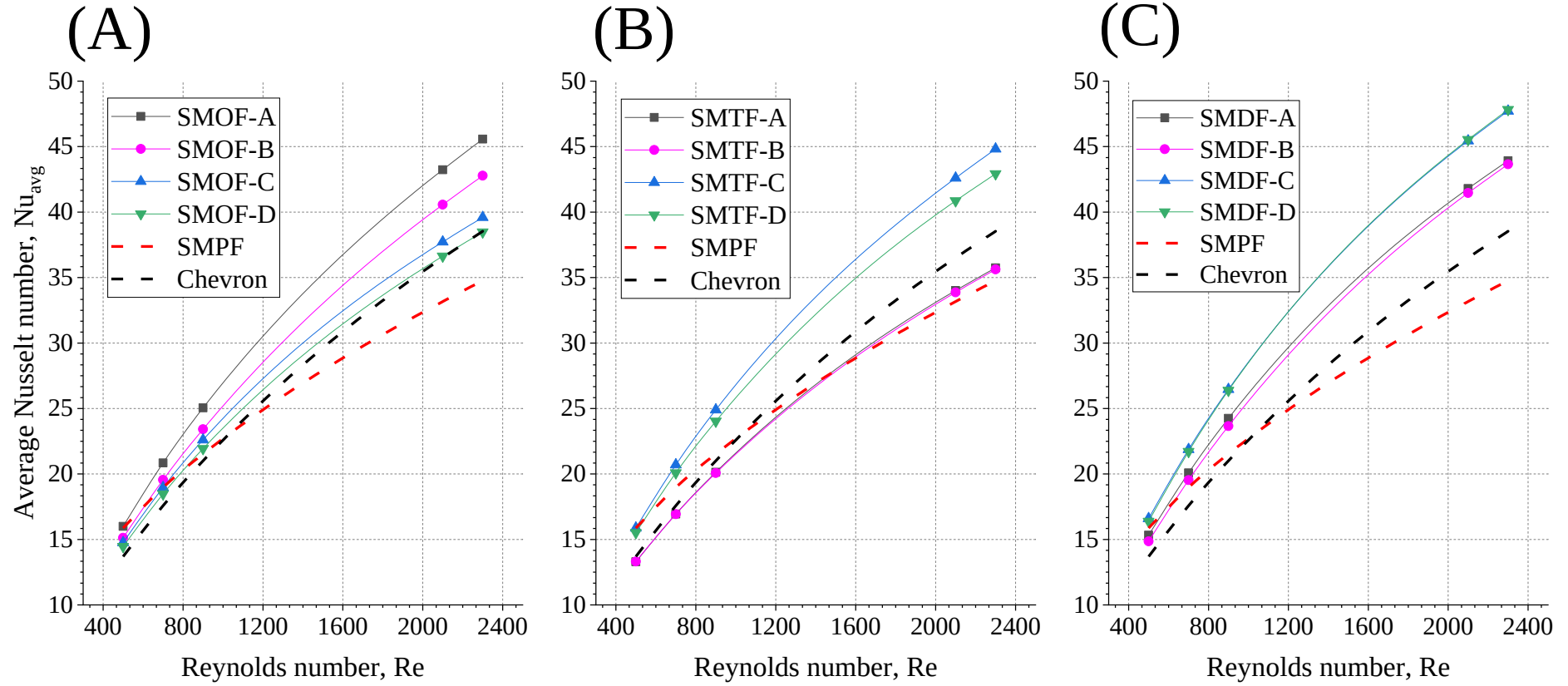


Figure 6-8: Shows the average Nusselt number, Nu_{avg} , plots against Re for all serpentine mini-channel heat sink designs.

Figure 6-10 shows the ΔP (refer Eq. 3.54) plots for all SMCHS designs studied. The increase in Re results in increase in ΔP exponentially for all SMCHS heat sink designs. This is due to fact that at higher Re , the fluid dynamic pressure increases with increasing liquid velocity and when liquid moves through narrow channel and bends creating higher frictional and head losses leading to larger ΔP [308]. From Figure 6-9, the percentage difference of ΔP decreases gradually as Re increases for all SMCHS with fins designs against SMPF since ΔP has an exponential increase with Re .

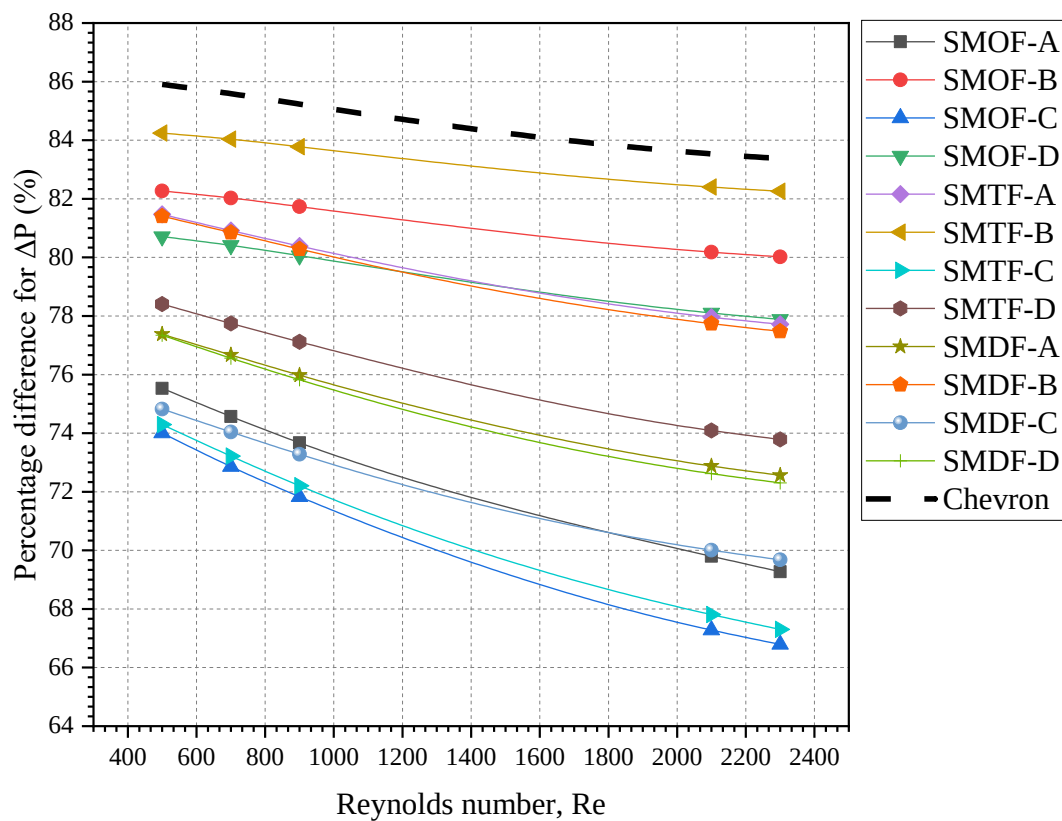
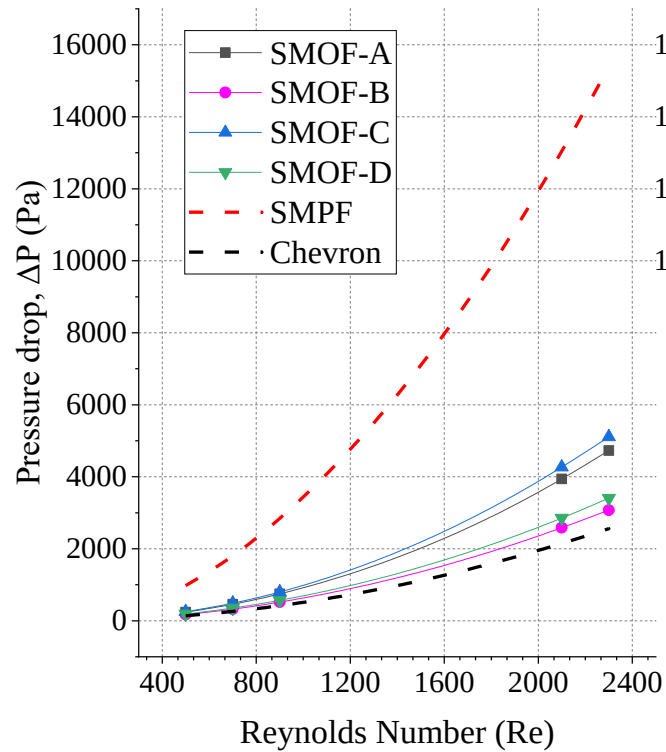
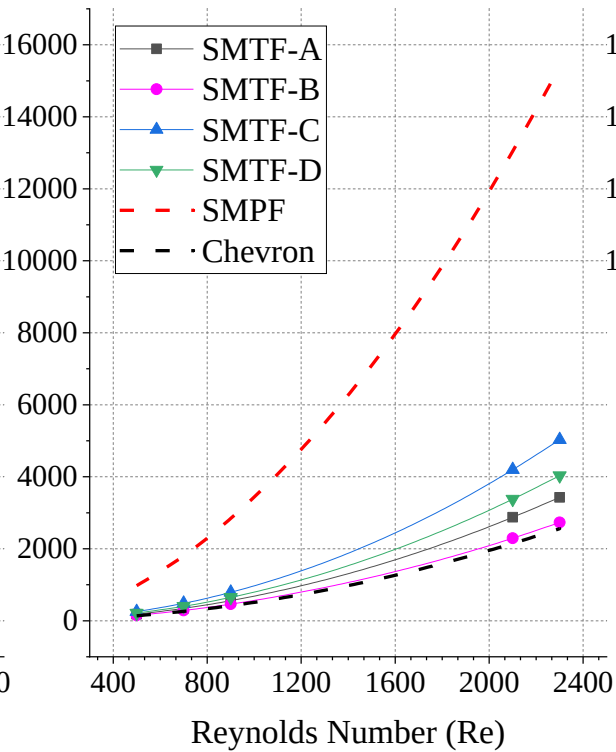


Figure 6-9: Shows the percentage difference of ΔP for all serpentine mini-channel heat sink design with fins against SMPF at increasing Re .

(A)



(B)



(C)

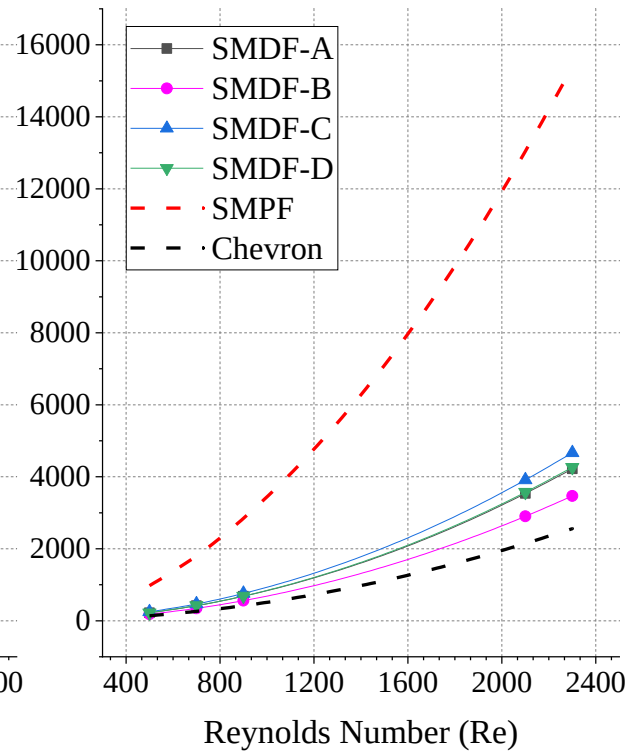


Figure 6-10: Shows the pressure drop (ΔP) plots against Re for all heat sink designs with reference to SMPF heat sink.

6.4.2 The effect of different serpentine mini-channel heat sink designs and configurations on R_{th} , Nu_{avg} and ΔP

Different designs of SMCHS with smaller fins have been explored thoroughly to improve the conventional design of SMPF utilising the secondary channels. The shape of fins studied were oblique (SMOF), trapezium (SMTF) and diamond (SMDF) which hold four different configurations of A, B C and D each referring to Figure 6-2, Figure 6-3 and Figure 6-4 respectively where the dimension parameters are stated in Table 6-1. The SMCHS designs considered will be assessed individually to compare its thermal-hydraulic performance using R_{th} , Nu and ΔP with heat flux applied underneath of heat sink at 100 W/cm². To have fair comparison between SMCHS designs, the fin angle, θ_{fin} , and secondary channel length, l_{sc} , are set to be 45° and 0.7mm respectively. The thermal-hydraulic performance of SMCHS designs with new shape of fins will be compared to the serpentine-minichannel heat sink with chevron fins proposed by Al-Neama et al. [38] and the SMPF heat sink design.

6.4.2.1 Effect on the temperature distribution, R_{th} and Nu_{avg}

Referring Figure 6-6, three different graphs have been constructed according to the shape of fins considered with R_{th} of SMPF heat sink and serpentine mini-channel heat sink with chevron fins plotted in each one of them. Serpentine mini-channel heat sink with chevron fins has lower R_{th} compared to the SMPF heat sink with average percentage reduction of 4.17%. Looking at the SMOF heat sink design plot (Figure 6-6 (A)), SMOF-A design has the lowest calculated R_{th} at all values of Re ranges between 500 to 2300 with average percentage decrease of 12.2% with reference to the SMPF heat sink (refer Figure 6-6) and the temperature distribution can be observed in Figure 6-11. For SMOF-B design, at Re < 700 the values of R_{th} are higher than SMPF heat sink while at Re > 700, R_{th} values are lower. The SMOF-C and D design configurations are having R_{th} values higher than SMPF heat sink at all values of Re with SOMF-D configuration is the highest with average percentage increase between 4.3% and 14.2% respectively. These observations can be explained by referring to Figure 6-12 showing the temperature plots with streamline plots embedded for all SMOF design configurations. Lowest R_{th} for SMOF-A design configuration in Figure 6-12 induced by flow separation and re-attachment around oblique fins breaking both thermal and hydrodynamic boundary layers thus improving heat transfer [309]. The flow re-initialisation is heightened due to the existence of side-to-side recirculation region which are having opposing rotation. Flow from the trailing edge transferred effectively to leading edge of oblique fin improving fluid mixing, hence also lead to enhanced rate of heat transfer. The SMOF-B design configuration also gives lower R_{th} compared

to SMPF heat sink (Figure 6-6) since there are small vortices formation at the side of oblique fin and within the secondary channel breaking the boundary layers. The liquid is travelling freely through the main and secondary channel for SMOF-B configuration enabling higher heat transfer along the effective area (A_{eff}). From Figure 6-6, higher values of R_{th} captured compared to SMPF heat sink at increasing Re for both SMOF-C and D configurations are caused by very low localised velocity region which represent by low density streamline plot mainly around bends (see Figure 6-12) reducing the heat transfer rate. Even though at the middle of the heat sink, the liquid flow for both SMOF-C and D has about similar flow structure compared to SMOF-A and B configurations but these localised low velocity region deteriorates the rate of heat transfer leading to very high localised temperature region, thus reducing the overall thermal performance of heat sink since the R_{th} is depending on maximum surface temperature ($T_{surf,max}$).

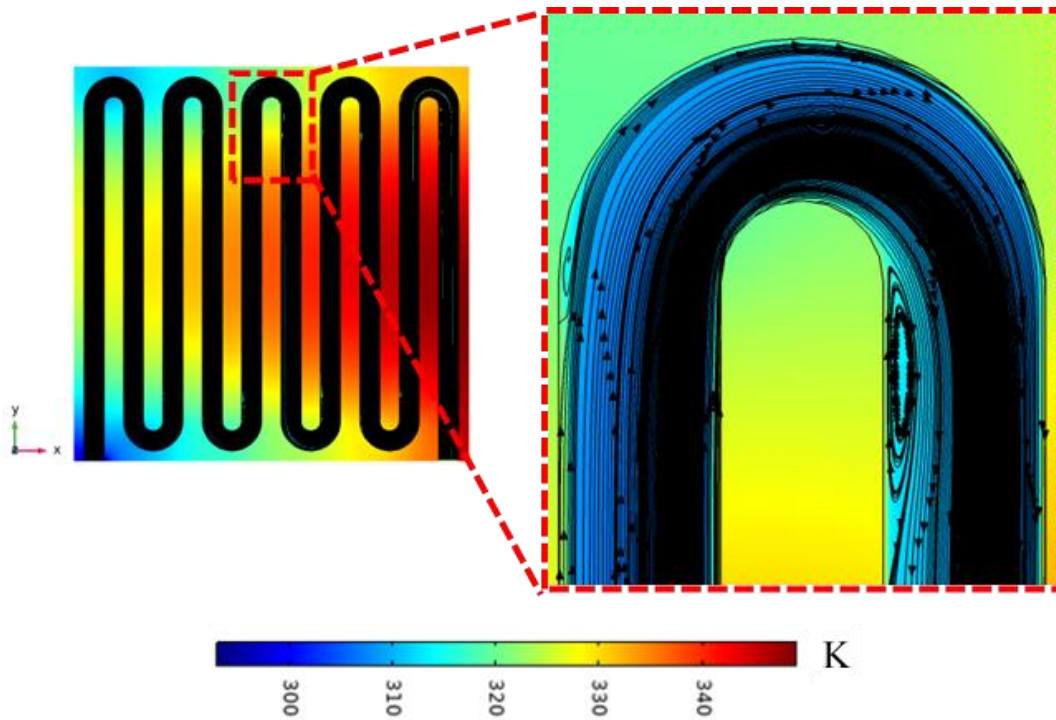


Figure 6-11: Shows the plane of temperature plot integrated with streamline plot at mid-depth of the channel ($H_{ch}=1mm$) for SMPF heat sink design.

From Figure 6-6 (B), both SMTF-C and D design configurations are having R_{th} lower than SMPF heat sink at all values of Re with average percentage decrease of 6.0% and 6.8% respectively (refer Figure 6-7). On the other hand, both SMTF-A and B have higher R_{th} value compared to the conventional SMPF heat sink with average percentage increase of 9.7% and 9.9% respectively at increasing Re . Referring Figure 6-13 of SMTF-C and D configurations, it is observed that the streamline plots show down-wash formation at the longest side of fins which is very beneficial in thinning the boundary layers thickness and transferring the cold fluid at the core of main channel to the hot wall of fins, thus improved rate of heat transfer is achieved [310]. Higher water velocity around bends also aid in transferring heat from heat sink to liquid. For SMTF-A and B design configurations (refer Figure 6-13), similar observations to SMOF-C and D where very low liquid velocity region captured around bends and also between two longest sides of adjacent fins even though there are breaking in boundary layers and flow re-initialisation for heat transfer enhancement but localised high temperature region affecting the $T_{surf,max}$ to increase inducing R_{th} increment.

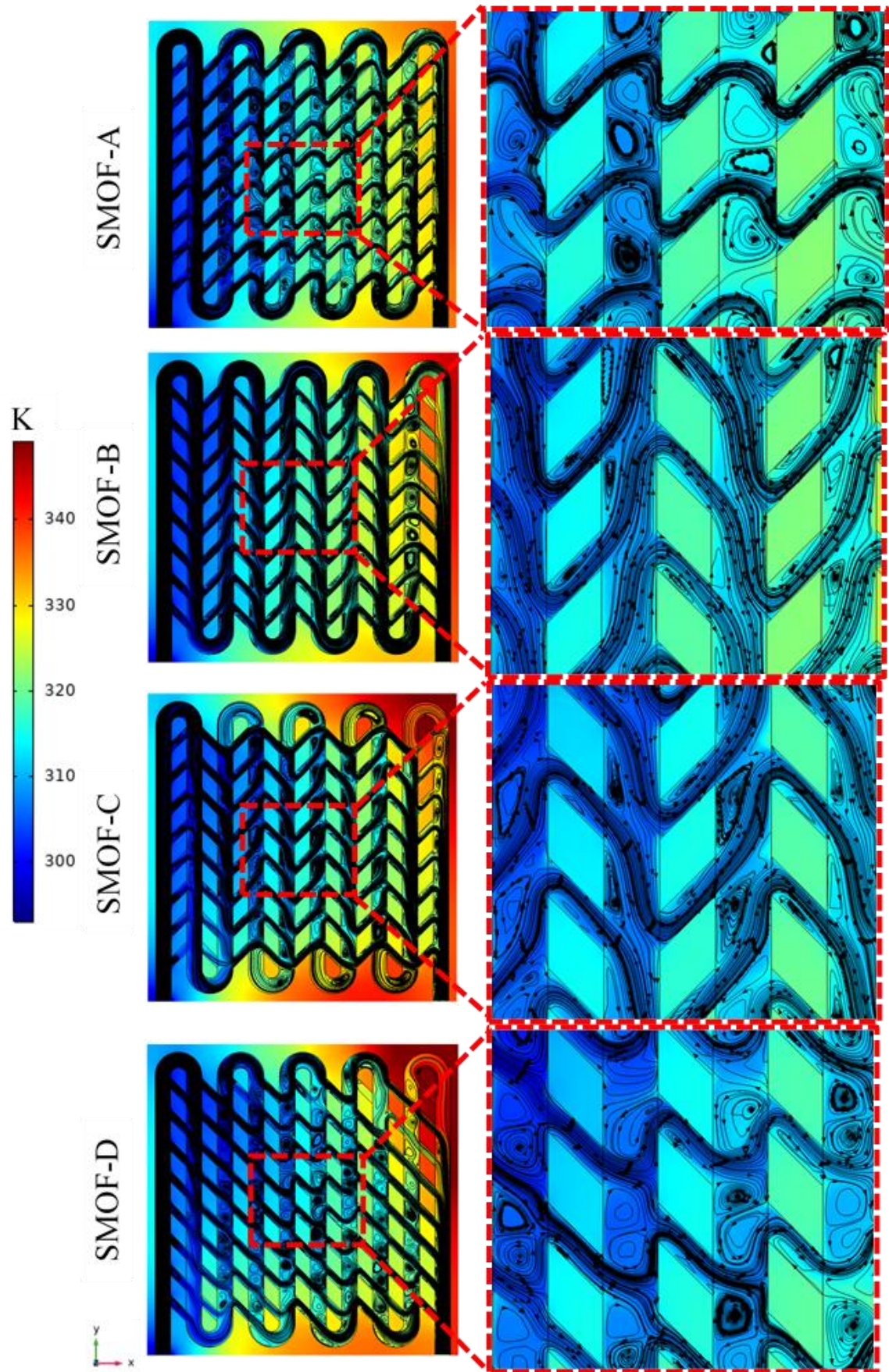


Figure 6-12: Shows the plane of temperature plot integrated with streamline plot at mid-depth of the channel ($H_{ch}=1\text{mm}$) for all SMOF design configurations.

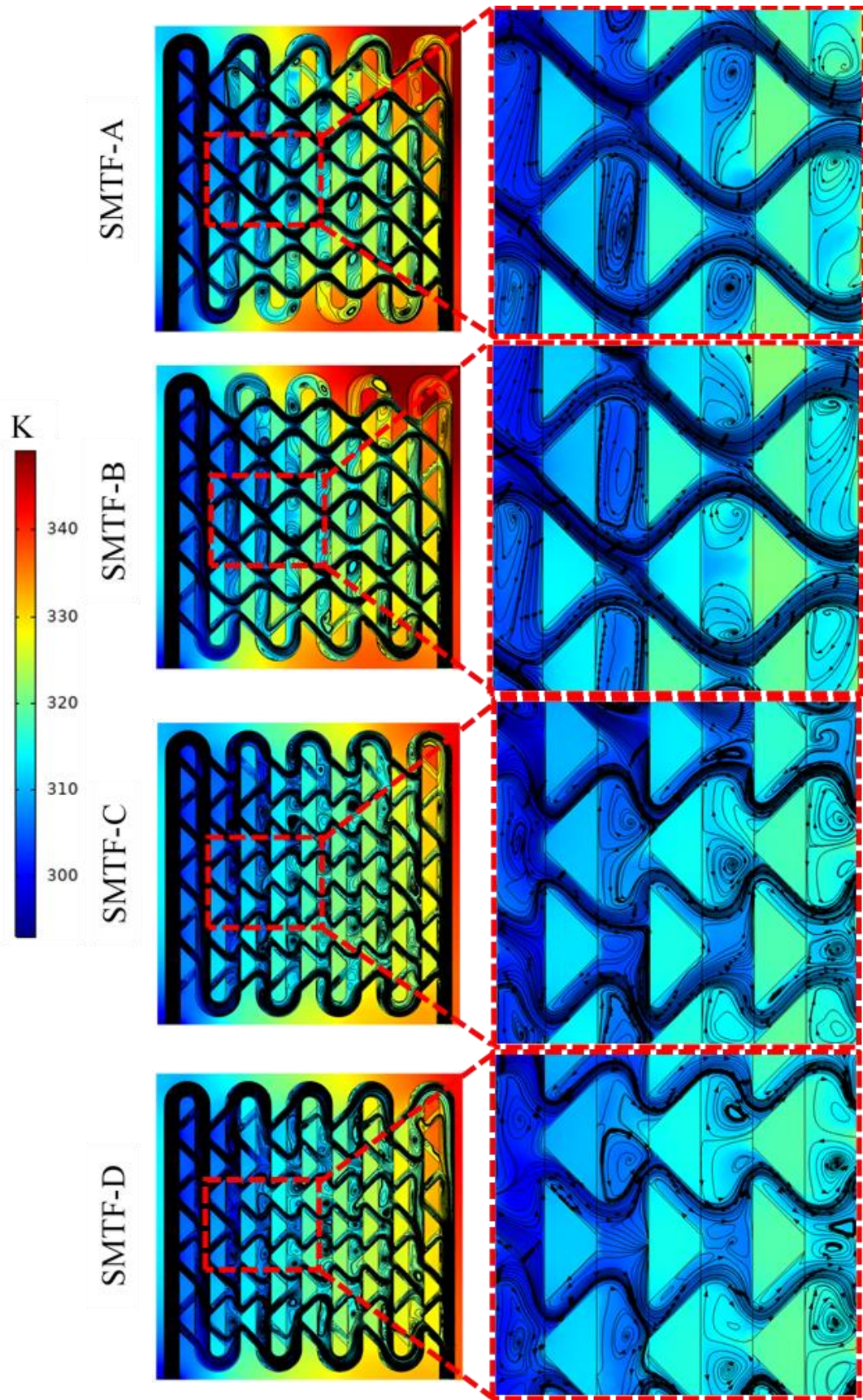


Figure 6-13: Shows the plane of temperature plot integrated with streamline plot at mid-depth of the channel ($H_{ch}=1\text{mm}$) for all SMTF design configurations.

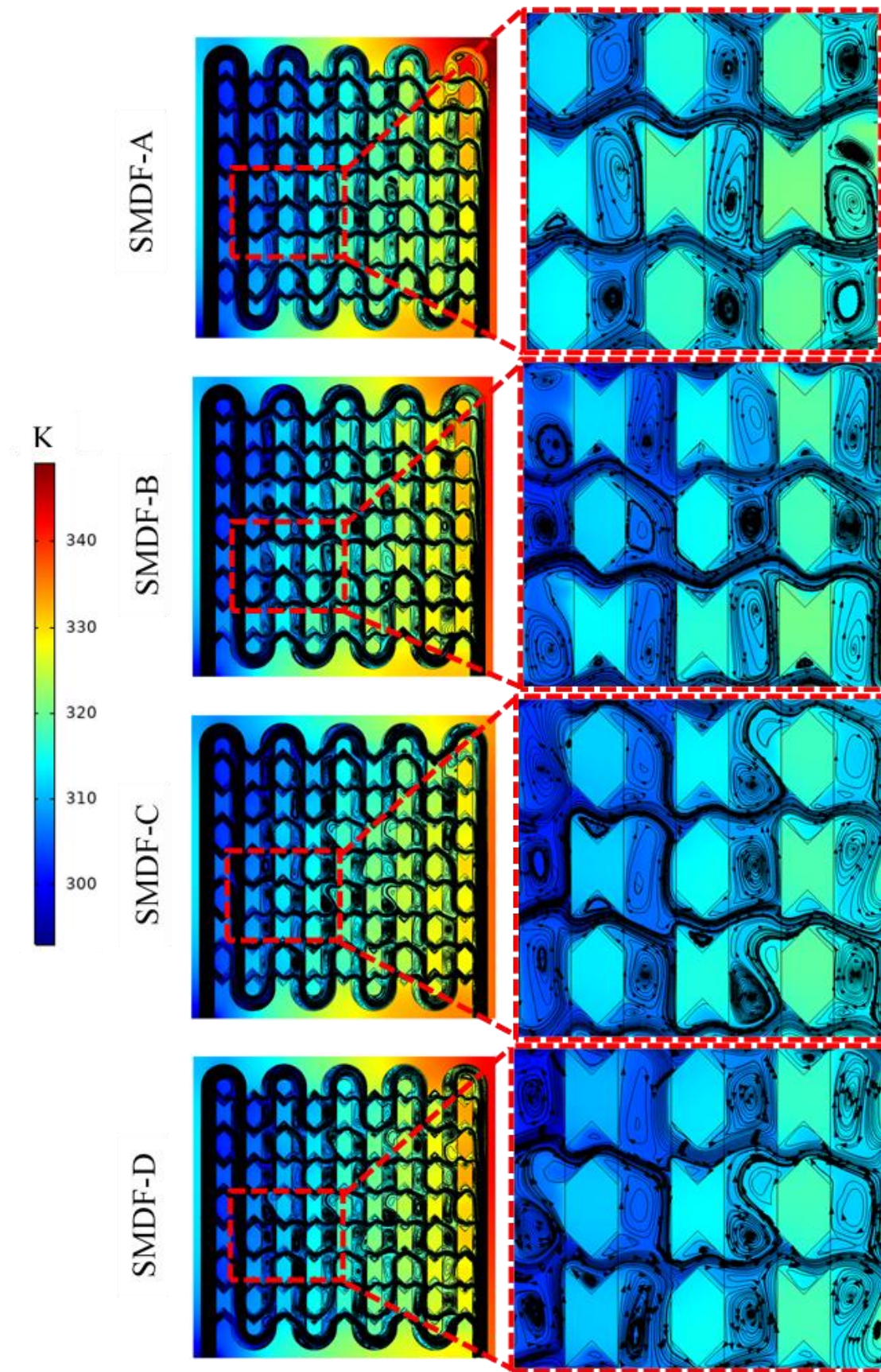


Figure 6-14: Shows the plane of temperature plot integrated with streamline plot at mid-depth of the channel ($H_{ch}=1\text{mm}$) for all SMDF design configurations.

Looking at Figure 6-6, SMDF-C configuration has the lowest R_{th} compared to other SMDF configurations at increasing Re where the average percentage reduction of R_{th} from SMPF heat sink is 14.0%. Lowest R_{th} by SMDF-C configuration is due to the higher streamline uniformity and density indicating higher liquid velocity enhancing the rate of heat transfer (refer Figure 6-14) along with continuous separation and re-attachment of flow due to formation of vortices. The SMDF-B configuration also experience reduction in R_{th} compared to SMPF heat sink with average percentage reduction of 7.7% (refer Figure 6-6). For SMDF-D configuration, at $Re < 700$ the R_{th} is slightly higher than SMPF heat sink while at $Re > 700$ much lower R_{th} is observed. From Figure 6-14, the SMDF-B and D design configurations has less uniformity of streamlines plot inducing less rate of heat transfer. The highest R_{th} plot by SMDF-A showing at $Re < 1400$, R_{th} is slightly higher than SMPF heat sink while at $Re > 1400$ the R_{th} is slightly lower. High R_{th} for SMDF-A configuration is due to the same reason stated earlier for SMOF-C,D and SMTF-A, B configurations where localised low velocity regions exist at bends reducing local heat transfer leading to increase in R_{th} (refer Figure 6-14). Generally the SMDFs configuration has the lowest R_{th} compared to the SMOFs and SMTFs configurations due to more formation of vortices breaking the boundary layers effectively to improve the heat transfer.

Based on Figure 6-8, all configurations of SMOFs, SMTFs and SMDFs (refer Figure 6-2 to Figure 6-4) except SMTF-A and B show significant increase in Nu_{avg} at higher Re compared to the SMPF heat sink (refer Fig. 14). This is due to the fact that, having secondary channel promotes heat transfer enhancement caused by flow re-initialisation and continuous breaking of boundary layers which means large heat transfer coefficient (h_{avg}) value attained since the Nu_{avg} is solely affected by h_{avg} (refer Eq. 3.24). Even though some of the designs considering secondary channel are having higher R_{th} compared to conventional SMPF heat sink due to localised deterioration of heat transfer caused by localised low velocity region but in terms of the whole heat sink, significant improvement of heat transfer is achieved with high value of h_{avg} . Based on Figure 6-8, comparing between heat sink with smaller fins the highest Nu_{avg} achieved for serpentine mini-channel heat sink with oblique, trapezium and diamond fins are SMOF-A, SMTF-C and SMDF-C while the lowest Nu achieved are SMOF-D, SMTF-B and SMDF-B respectively with average percentage increase of 17.7%, 16.4%, 23.4%, 2.3%, -5.8% and 11.4% respectively. SMTF-B heat sink design shows negative percentage which means Nu is lower compared to SMPF heat sink since higher number of local low velocity regions reducing the rate of heat transfer. In general, for all temperature plots of SMPF heat sink and serpentine mini-channel heat sink with oblique, trapezium and diamond fins in Figure 6-12 to Figure 6-14,

it can be concluded that having smaller fins improves heat transfer effectively where very low temperature region captured in the middle of heat sink. Major area of high temperature region captured when using the SMPF heat sink from Figure 6-11.

6.4.2.2 Effect on the pressure distribution, velocity distribution and ΔP

From Figure 6-10, it is observed that all configurations of serpentine mini-channel heat sink with oblique, trapezium and diamond fins as well as chevron fins show great reduction in ΔP compared to SMPF heat sink at all range of Re with minimum average percentage reduction of 70.6% change which is represented by SMOF-C design while the highest average percentage reduction hold by chevron fins with 84.7% change with SMTF-B design following closely at 83.3% change (refer Figure 6-9 for the percentage difference). The reduction in ΔP of serpentine mini-channel heat sink with oblique, trapezium and diamond fins with reference to the SMPF heat sink can be observed in Figure 6-15 where all the ΔP contours plotted. From Figure 6-15, the ΔP contours experienced great change when using serpentine mini-channel heat sink with smaller fins compared to SMPF heat sink. All serpentine mini-channel heat sink with smaller fins have uniform bluish contour throughout the liquid domain resembling great reduction in ΔP while SMPF heat sink showing sudden change in ΔP contour. The reason behind this is explained by Figure 6-16 where velocity contours are plotted for all heat sinks. Looking at Figure 6-16, the velocity distributions show that the liquid velocity is greatly reduced when using SMCHS with oblique, trapezium and diamond fins compared to SMPF heat sink. This is due the existence of secondary channels when using smaller fins which divert a portion of liquid from the main channel to the secondary channel reducing the velocity of liquid within the main channel, hence, pressure and frictional losses are reduced greatly.

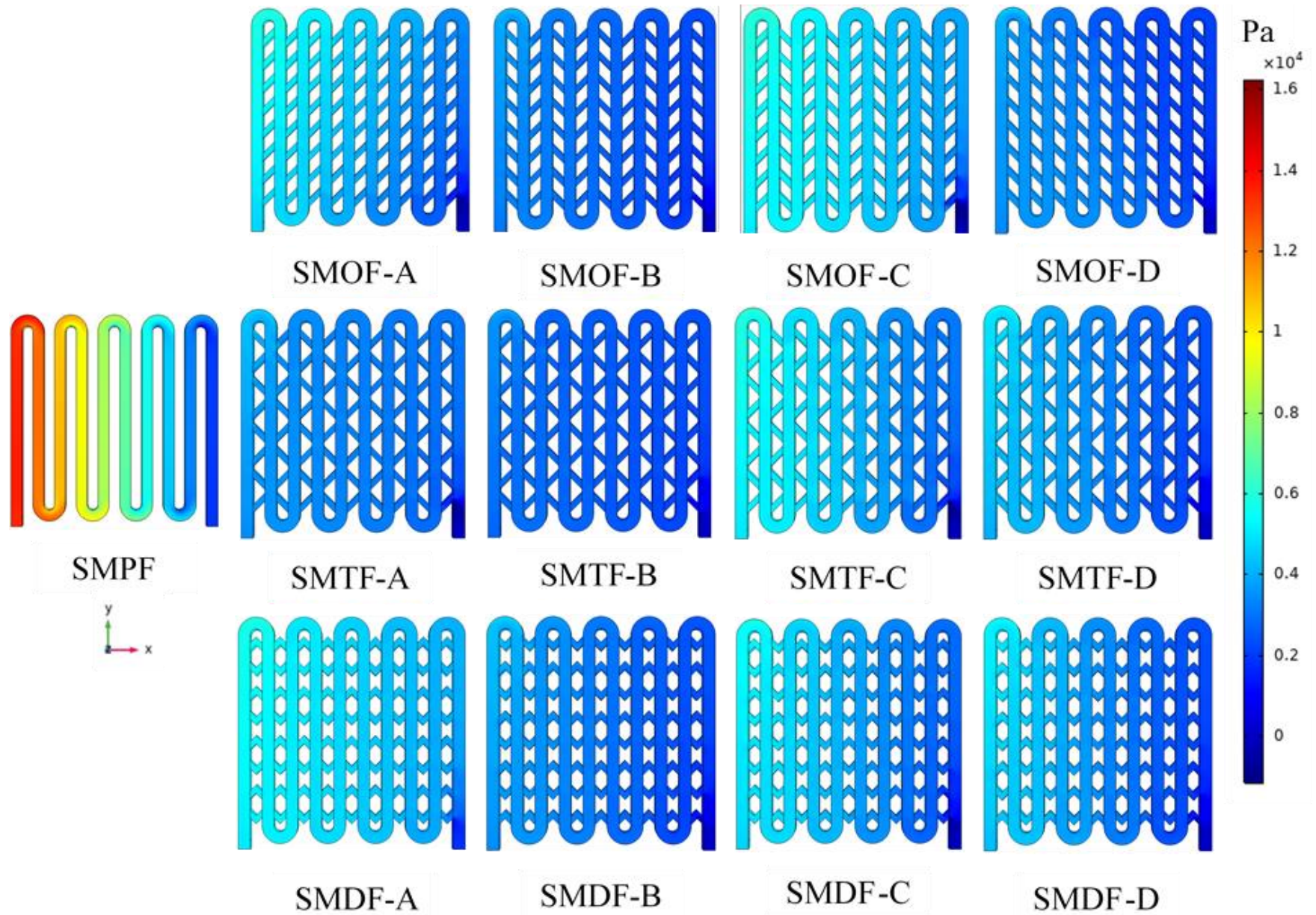


Figure 6-15: Shows all pressure drop (ΔP) contour plots of SMPF heat sink and serpentine mini-channel heat sink with oblique (SMOF), trapezium (SMTF) and diamond (SMDF) fins for all configurations.

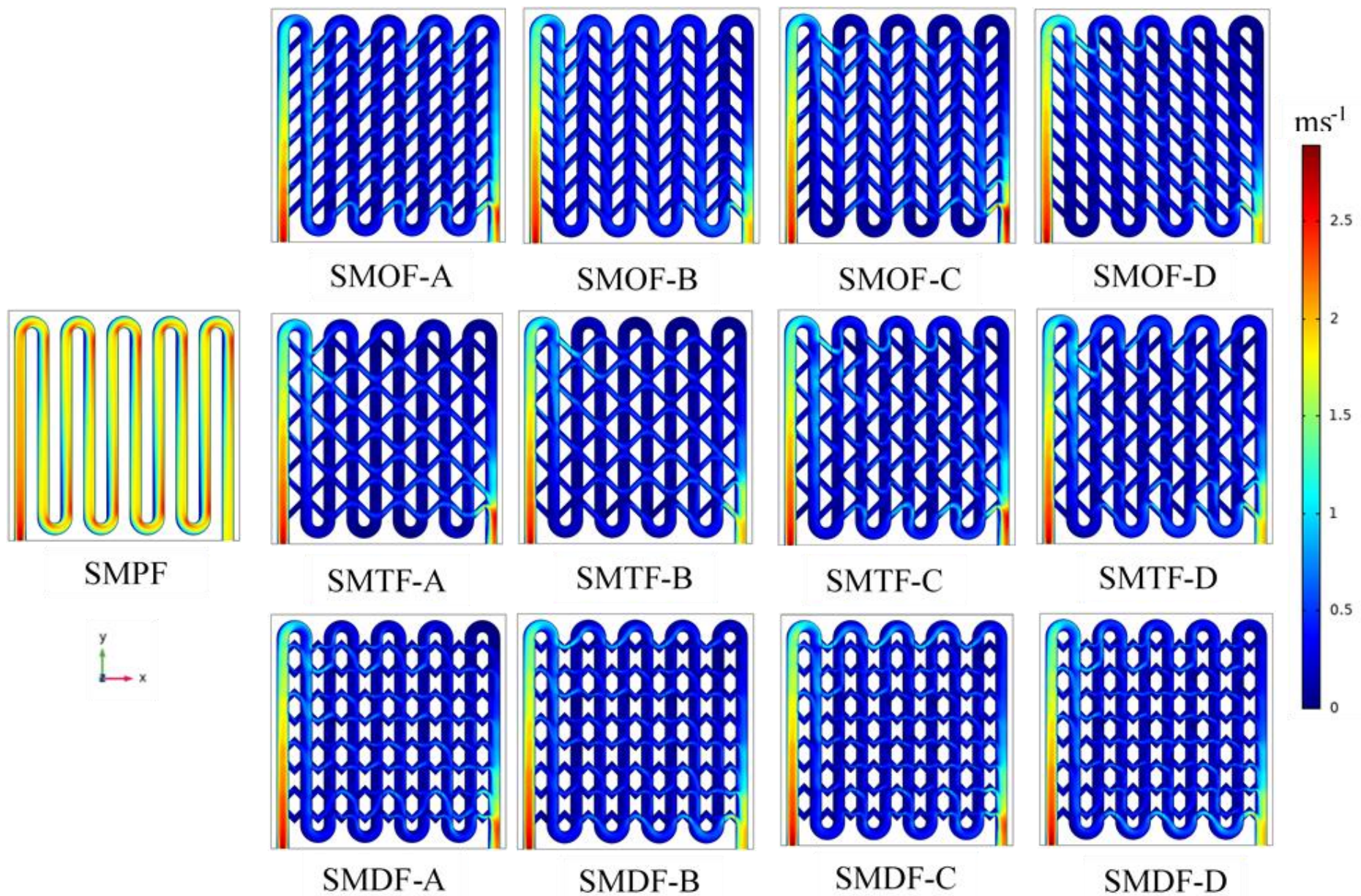


Figure 6-16: Shows all velocity distribution contour plots of SMPF heat sink and SMCHS with oblique (SMOF), trapezium (SMTF) and diamond (SMDF) fins for all configurations.

6.4.2.3 Performance Evaluation Criteria (PEC) index study

To understand the effect of having smaller fins within SMCHS on both R_{th} and ΔP , performance evaluation criteria (PEC) index is utilised assessing the thermal-hydraulic performance of all SMCHS designs [205, 206]. The PEC index is comprising of both of Nu_{avg} and ΔP terms ensuring the compromise between the two terms effecting the overall performance of heat sink. Instead of using the Nusselt number (Nu) and friction factors (f) from the initial suggestion by [205, 206], for this work the PEC index is defined using computed quantities of Nu_{avg} and ΔP which is equivalent to Nu and f which is suggested in [311]. Thus, the PEC index can be re-written as follows:

$$PEC = \frac{Nu_{avg,new} / Nu_{avg,SMPF}}{(\Delta P_{new} / \Delta P_{SMPF})^{1/3}} = \frac{E_{Nu}}{\sqrt[3]{E_{\Delta P}}} \quad (6.5)$$

Where $Nu_{avg,SMPF}$ and ΔP_{SMPF} are the average Nusselt number and pressure drop for SMPF heat sink while $Nu_{avg,new}$ and ΔP_{new} are the average Nusselt number and pressure drop for serpentine mini-channel heat sink with oblique, trapezium and diamond fins. The E_{Nu} and $E_{\Delta P}$ are the normalised parameters for Nu_{avg} and ΔP of serpentine mini-channel heat sink with fins with respect to the SMPF heat sink.

From Figure 6-17, there is a great surge of PEC index of all SMCHS with oblique, trapezium and diamond fins as well as chevron fins from unity which is the SMPF heat sink. The significant increase of PEC index of serpentine mini-channel heat sink with smaller fins is contributed by the massive reduction of ΔP achieved when using secondary channel compared to conventional SMPF heat sink. Looking at Figure 6-17, the highest PEC index is achieved by SMDF-D design while the lowest by SMOF-C design. The highest PEC index by SMDF-D design is due to high ΔP reduction of 74.9% average change (refer Figure 6-9) while having Nu_{avg} higher than SMPF heat sink with 23.0% average increment (refer Figure 6-8) at increasing Re . Lowest PEC index by SMOF-C design since it has the lowest ΔP reduction of 70.6% average change from Figure 6-9 and Nu_{avg} slightly higher than SMPF heat sink of 5.1% average increase when looking at Figure 6-8 at increasing Re . Referring Figure 6-17, all SMCHS designs, the PEC index increases with Re but with decreasing rate due to ΔP increases exponentially from Figure 6-10 while Nu_{avg} increases with decreasing rate, thus PEC index gradient must reduce to pay for higher ΔP penalty.

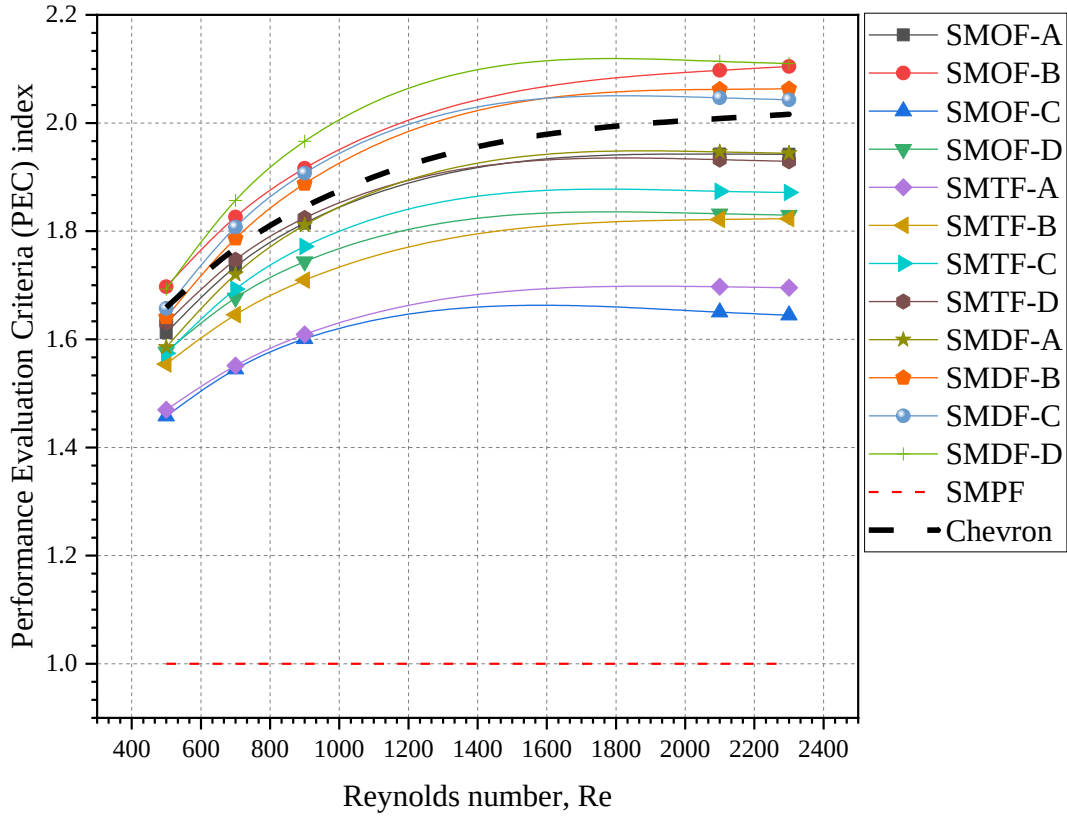


Figure 6-17: Shows the Performance evaluation criteria (PEC) index plots for all serpentine mini-channel heat sink with oblique, trapezium and diamond fins against the SMPF heat sink.

The outcome of PEC index is depending on both Nu_{avg} and ΔP as well as the dimensions of heat sink. The Nu_{avg} formula include the difference between the terms of average wall temperature of mini-channel, $T_{w,avg}$ and temperature difference between inlet and outlet boundaries, $T_{f,avg}$. This indicates that PEC index is directly influenced by the temperature distribution of the solid and fluid domains and the temperature distribution is also directly influenced by dimensioning of heat sinks affecting rate of heat transfer and fluid velocity distribution.

6.4.3 Parametric analysis of single inlet-outlet serpentine mini-channel heat sink with fins

From the PEC index study under subchapter 6.4.2.3, it can be seen that the highest PEC index is contributed by SMDF-D design. Even though SMDF-D possesses the highest PEC index but for parametric study will utilise the SMDF-C design since it has the lowest R_{th} and highest Nu_{avg} compared to SMPF heat sink since thermal performance is more crucial for heat sink as a heat removal device and also the ΔP reduction for SMDF-C design is also very large which is not far from ΔP reduction of other heat sink cases.

6.4.3.1 Effect of changing the secondary channel length (l_{sc}) of SMDF-C design on R_{th} and ΔP

To study the effect of variation of secondary channel length (l_{sc}) on the values of R_{th} and ΔP where four values of l_{sc} used were 0.7mm, 0.8mm, 0.9mm and 1.0mm respectively. The Re used ranges between 500 and 2300 with uniform heat flux of 100 W/cm² applied underneath heat sink. Other constant values apart from l_{sc} were based from parameter values in Table 6-1. Figure 6-18 shows the effect of changing the values of l_{sc} on R_{th} and ΔP at increasing Re . From Figure 6-18, the R_{th} decreases and ΔP increases as l_{sc} decreases.

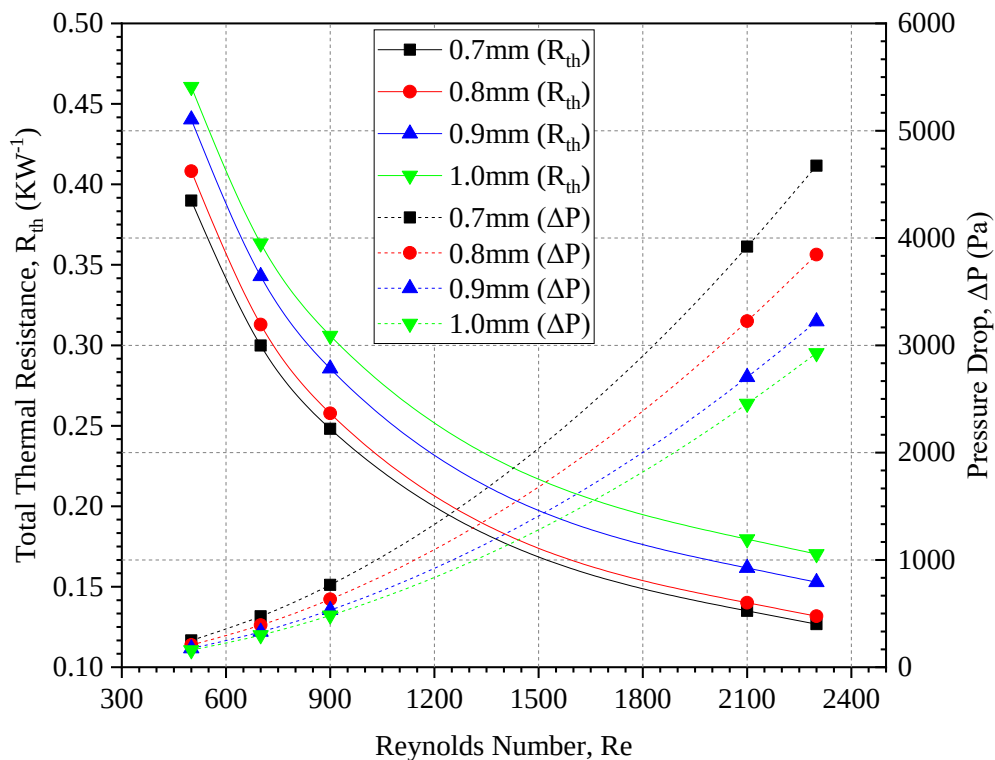


Figure 6-18: The R_{th} and ΔP plots at different values of l_{sc} .

The average percentage decrease in R_{th} when l_{sc} changes from 1.0mm to 0.7mm along increasing Re is at 26%. The decrease in R_{th} as l_{sc} decreases is due to the increase in effective

area (A_{eff}) which leads to the enhancement of convective heat transfer between mini-channel walls and liquid (water). The associated A_{eff} with l_{sc} at 1.0mm and 0.7mm are 1152.5mm² and 1217.3mm² respectively which gives 5.6% increment. Furthermore, having an increase in A_{eff} induce temperature reduction of the maximum surface temperature ($T_{surf,max}$) since it is directly proportional to R_{th} . Referring to Figure 6-19, reducing the size of l_{sc} would result in the increase of velocity within the main channel and bends since less fluid is diverted through the secondary channel. In addition, having smaller l_{sc} promotes thermal boundary layer disruption effectively increasing the convective heat transfer, hence reducing the R_{th} .

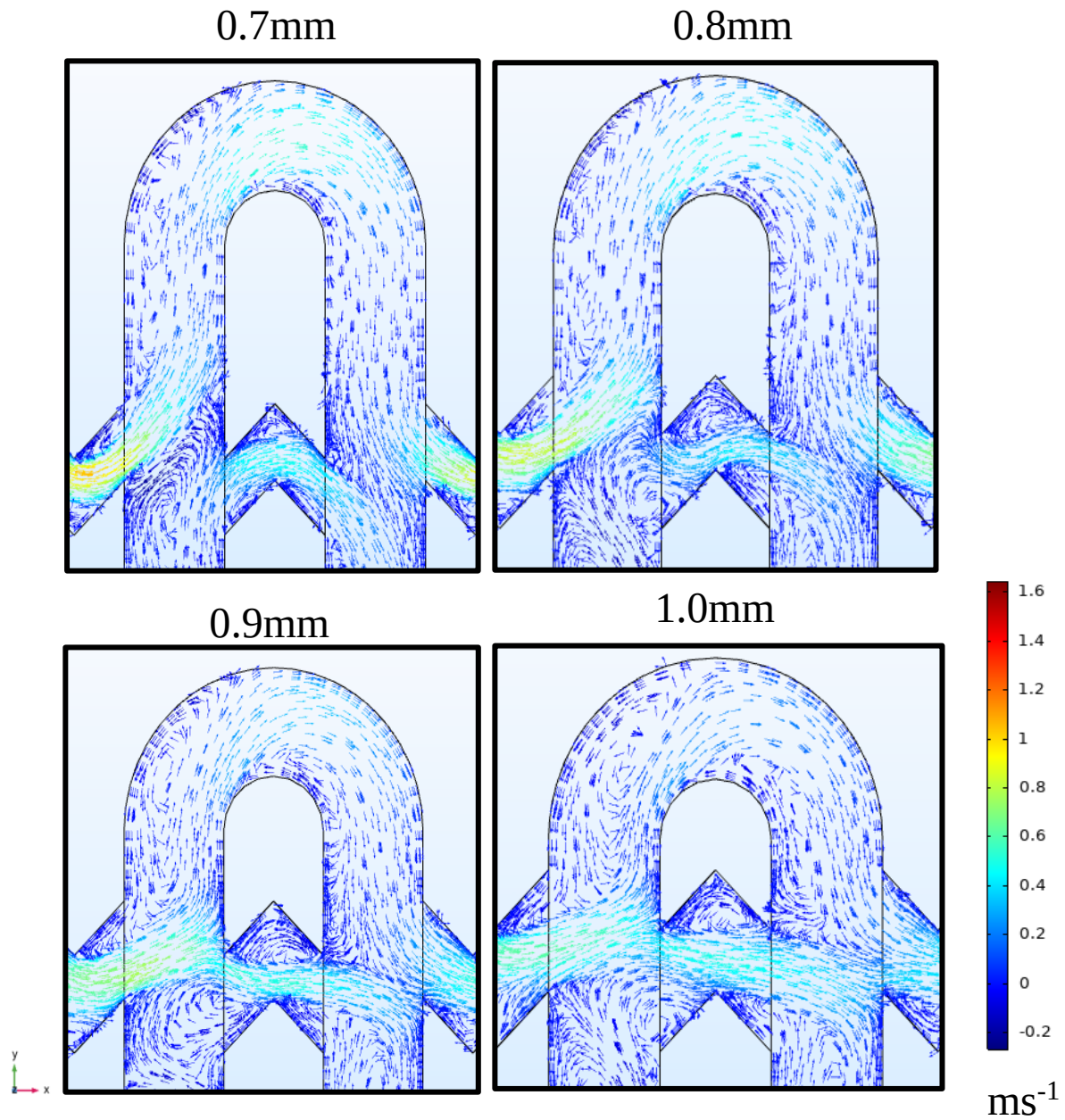


Figure 6-19: Shows the arrow surface plotted at $H_{ch}=1\text{mm}$ on the xy -plane with velocity contour distribution for different values of l_{sc} .

The increase in average percentage of ΔP when l_{sc} varied between 1.0 mm and 0.7mm is 37% at increasing Re . The reasoning applies where at smaller l_{sc} , the fluid velocity go through the small secondary channel inducing higher velocity within the secondary channel giving higher pressure and frictional losses. The limit size of l_{sc} depends on how well the diamond fins can fit in along the plate-fin.

6.4.3.1.1 PEC index study on different values of secondary channel length (l_{sc})

From Figure 6-20, the plots of PEC index, $E_{Nu_{avg}}$ and $E_{\Delta P}$ are displayed for different values of l_{sc} . The values of PEC index are calculated using $E_{Nu_{avg}}$ and $E_{\Delta P}$ based on Eq. 6.5. To achieve the highest PEC, $E_{Nu_{avg}}$ must be the highest while $E_{\Delta P}$ the lowest. The values of $E_{Nu_{avg}}$ for $l_{sc} = 0.7\text{mm}$ and 0.8mm at increasing Re are more than 1 showing superior performance thermally compared to SMPF heat sink while at $l_{sc} = 0.9\text{mm}$ and 1.0mm is the other way around with $E_{Nu_{avg}} < 1$ at $Re < 800$ and 1200 respectively. This is due the fact that at higher size of l_{sc} , the recirculation region gets larger which reduces the rate of convective heat transfer [312]. Additionally, the A_{eff} gets smaller at larger l_{sc} reducing the performance of heat transfer. Furthermore, more fluid diversion through the secondary channel when l_{sc} increases deteriorating the heat transfer since lower velocity within main channel affecting the convective heat transfer.

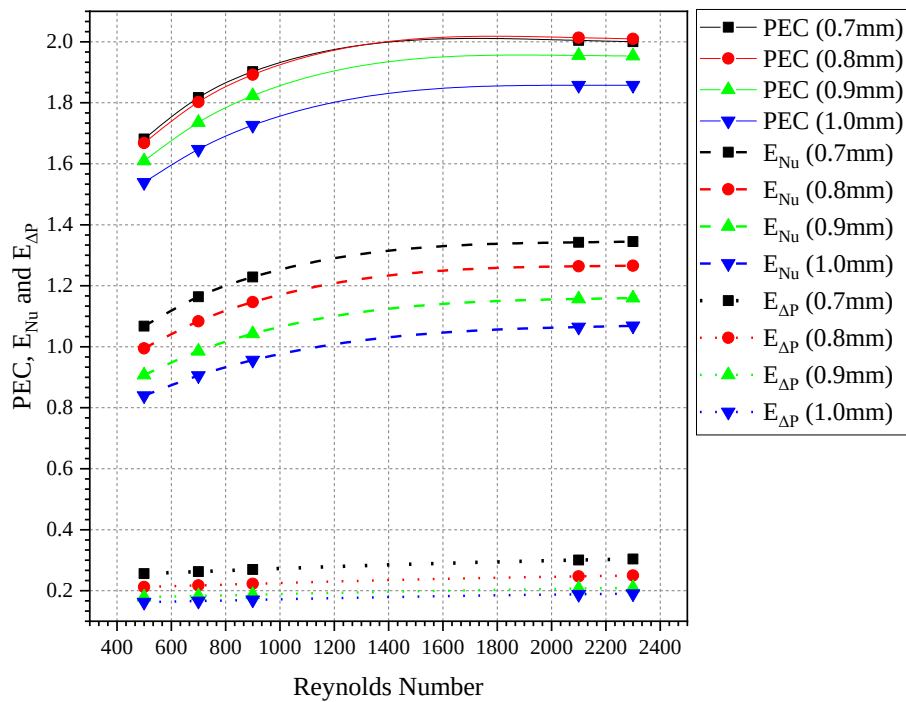


Figure 6-20: Shows the PEC, $E_{Nu_{avg}}$ and $E_{\Delta P}$ plots for different values of l_{sc} at increasing Re .

At $l_{sc} = 1.0\text{mm}$, lowest value of $E_{\Delta P}$ achieved at increasing Re while at $l_{sc} = 0.7\text{mm}$, $E_{\Delta P}$ is the highest. Larger $E_{\Delta P}$ at smaller size of l_{sc} induced by the smaller portion of fluid diversion through the secondary channel increasing the average velocity within the main channel and bends, thus, giving higher pressure loss.

The highest PEC index achieved when $l_{sc} = 0.8\text{mm}$ while the lowest at $l_{sc} = 1.0\text{mm}$. The PEC index increases gradually with increasing Re but with decreasing rate. From Figure 6-20, the PEC index values are majorly affected by E_{Nu} since they have similar trend. The increase of PEC index is caused by the increment of Nu when Re increases but as it goes up, ΔP experiences an exponential increase giving higher ΔP penalty reducing the gradient of PEC index against Re .

6.4.3.2 Effect of changing the fin angle (θ_{fin}) of SMDF-C design on R_{th} and ΔP

To investigate the effect of variation of fin angle (θ_{fin}) on the values of R_{th} and ΔP , the values of θ used are 25° , 35° , 45° and 55° respectively. The values of Re were set to be between 500 and 2300 with $100\text{W}/\text{cm}^2$ heat flux applied. The value of l_{sc} is set to be 0.8mm for all cases of θ_{fin} tested. Figure 6-21 shows the effect of varying of θ_{fin} values on R_{th} and ΔP at increasing Re . From Figure 6-21, the R_{th} decreases and ΔP increases as θ_{fin} increases.

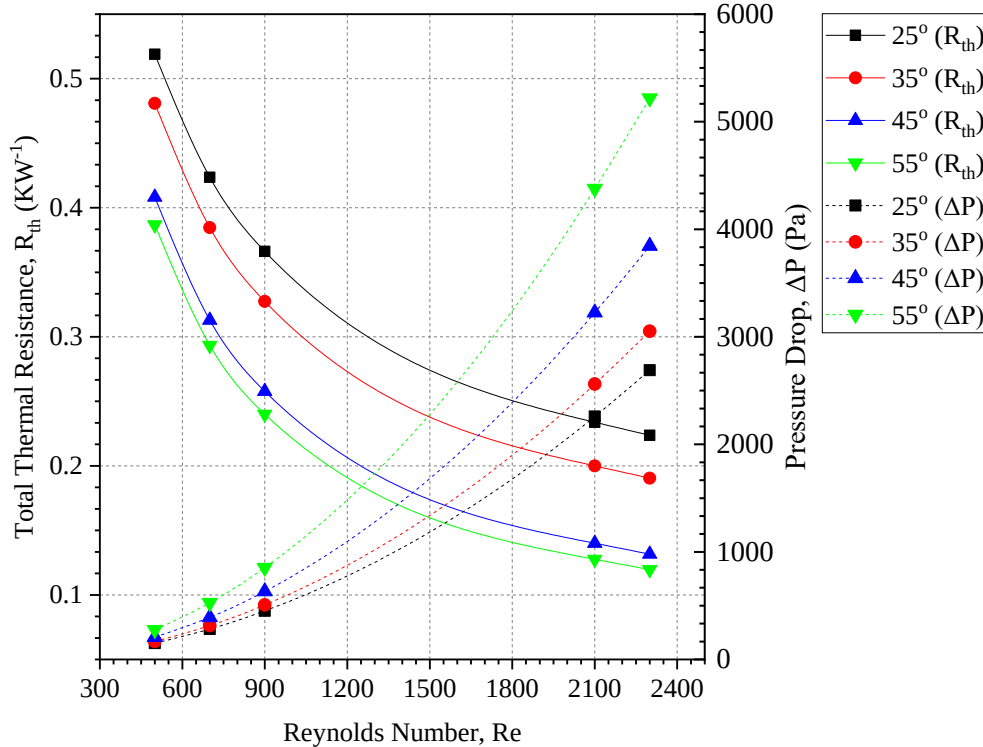


Figure 6-21: Shows the R_{th} and ΔP plots at different values of θ_{fin} .

Changing of θ_{fin} from 25° to 55° gives an average percentage decrease of R_{th} at 36.5% along increasing Re (refer Figure 6-21). R_{th} reduction is due to the increase of A_{eff} when θ increases leading to enhanced convective heat transfer between contact of liquid (water) and mini-channel walls. At θ_{fin} of 25° and 55° , the A_{eff} calculated were 1106.2mm^2 and 1290.5mm^2 respectively which gives 16.7% surface area increment. As mentioned in subchapter 6.3.6.1, having higher A_{eff} induces reduction in $T_{surf,max}$. From , the increase of θ_{fin} contributes to the increase of velocity in the main channel since less portion of water gets diverted through the secondary channel due to secondary channel width (W_{sc}) is getting smaller. To add up, more obvious flow disruption to break the thermal boundary layer at higher θ_{fin} increasing the rate of convective heat transfer lading to smaller value of R_{th} .

Varying of θ_{fin} from 25° to 55° induces an increase of ΔP with average percentage increase of 88.9% along increasing Re . Similar reason applies where at smaller θ_{fin} , the average velocity in the main channel is very high since less fluid moving through the secondary channel giving higher pressure and frictional losses.

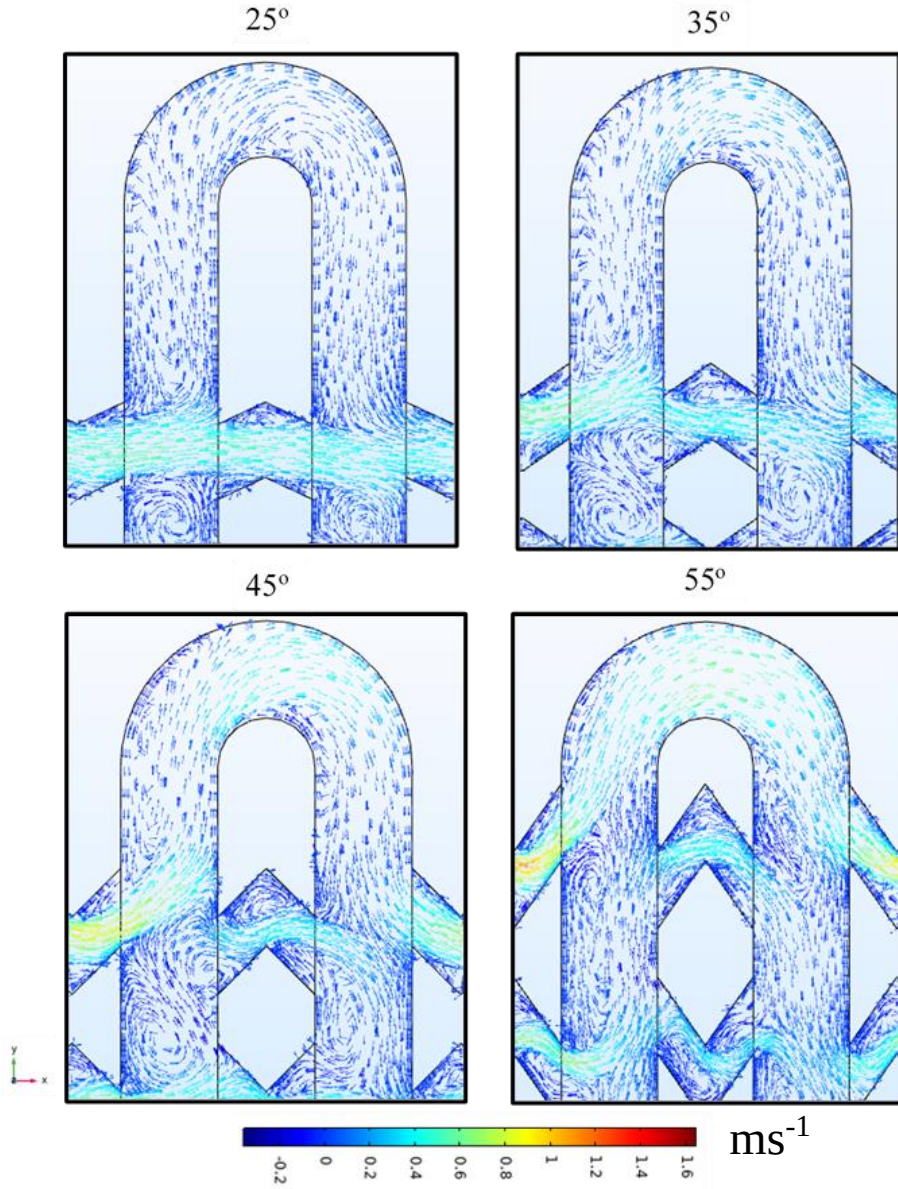


Figure 6-22: Shows the arrow surface plotted at $H_{ch}=1\text{mm}$ on the xy -plane with velocity contour distribution for different values of θ_{fin} .

6.4.3.2.1 PEC index study on different values of fin angle (θ_{fin})

Referring to Figure 6-23, the PEC index, $E_{Nu_{avg}}$ and $E_{\Delta P}$ are plotted at different values of θ_{fin} . High PEC index can be obtained by having lowest $E_{\Delta P}$ and highest $E_{Nu_{avg}}$. At θ equals to 45° and 55° , the values of $E_{Nu_{avg}} > 1$ at increasing Re while at θ_{fin} equals to 25° and 35° the values of $E_{Nu_{avg}} < 1$ at Re less than 700 and 900 respectively. Having E_{Nu} values more than 1 indicating superior thermal performance of SMCHS with fins compared to SMPF heat sink. Having larger θ_{fin} values promotes vortices formation to break thermal boundary layer for improving the rate of heat transfer (refer Figure 6-22). In addition, higher value of θ_{fin} having larger A_{eff} for more heat transfer. Also, larger value of θ_{fin} gives more flow through the main channel increasing the average velocity, hence convective heat transfer is enhanced.

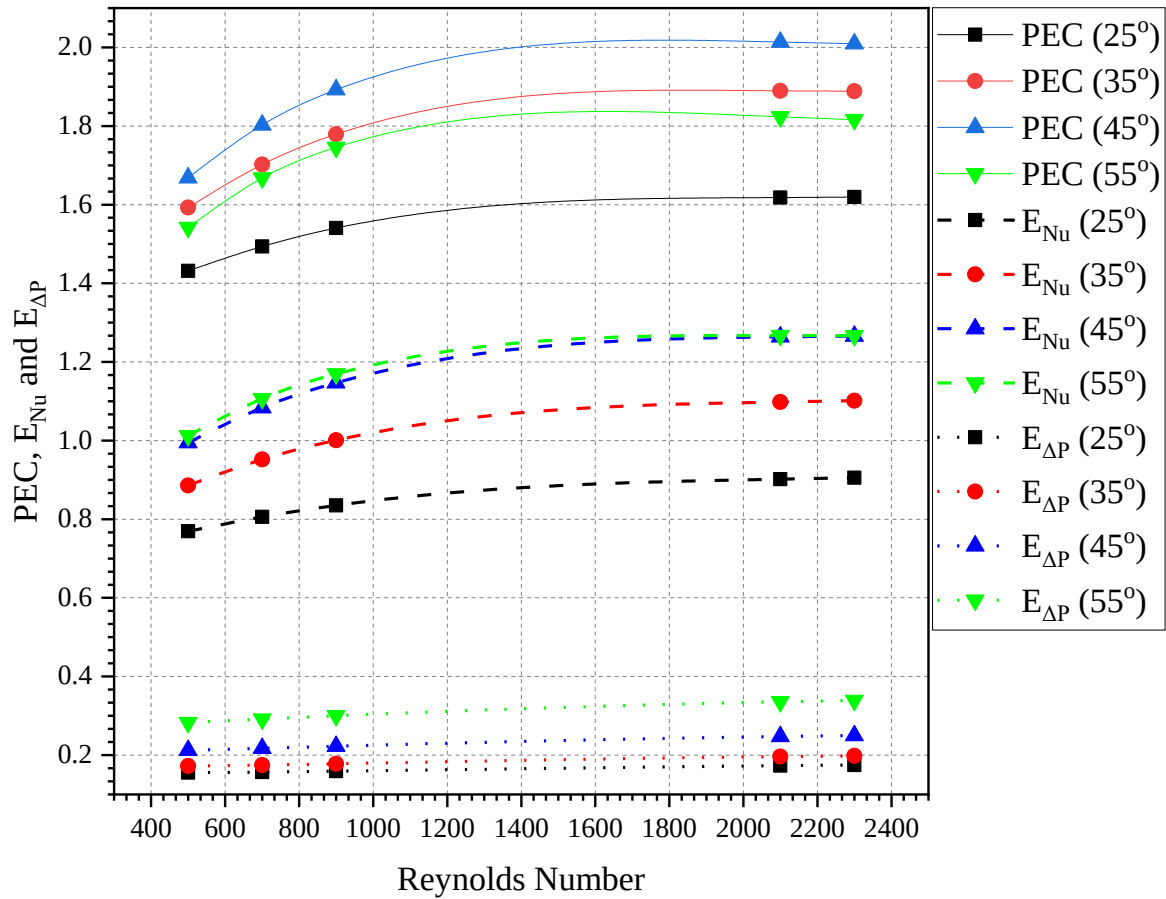


Figure 6-23: Shows the PEC , $E_{Nu_{avg}}$ and $E_{\Delta P}$ plots for different values of θ_{fin} at increasing Re .

The lowest and highest values of $E_{\Delta P}$ at increasing Re contributed by θ_{fin} at 25° and 55° respectively. Smaller values of $E_{\Delta P}$ occurs at $\theta_{fin} = 25^\circ$ since larger portion of fluid is diverted through the secondary channel reducing the average velocity within main channel leading to lower ΔP .

At $\theta_{fin} = 45^\circ$ and 25° , the PEC index is the highest and lowest respectively. PEC index increases gradually with increasing Re but with reducing gradient. From Figure 6-23, the PEC index and E_{Nu} has similar trend and shape along increasing Re . The increase in E_{Nu} causes PEC index to increase along increasing Re but at higher Re the ΔP increase suppresses the increment of E_{Nu} leading to curve flattening of PEC index.

6.4.3.3 Effect of changing the mini-channel height (H_{ch}) of SMDF-C design on both R_{th} and ΔP

The magnitude of height of channel (H_{ch}) is varied at 0.75mm, 1.00mm, 1.25mm and 1.50mm respectively to study its effect on both R_{th} and ΔP values. The Re is ranges between 500 and

2300 with $100\text{W}/\text{cm}^2$ heat flux applied underneath. All other dimensions are set to be as in Table 6-1. Referring Figure 6-24, the effects of changing the H_{ch} on both R_{th} and ΔP are plotted as a function Re . From Figure 6-24, as H_{ch} increases, the R_{th} decreases and ΔP increases respectively.

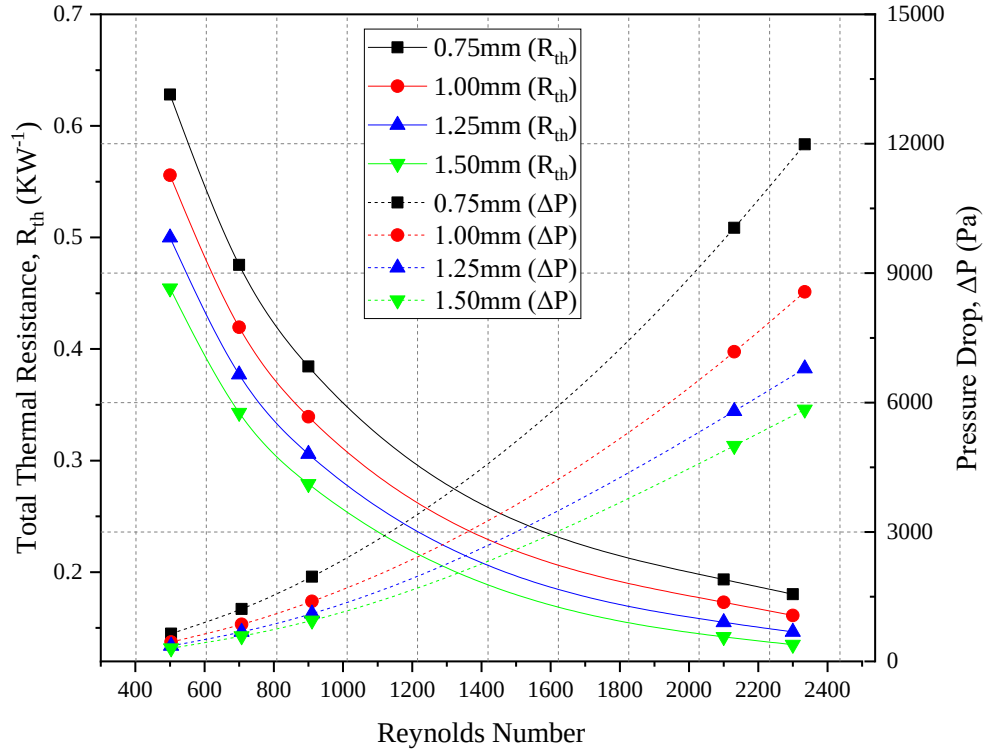


Figure 6-24: Shows the R_{th} and ΔP plots at different values of H_{ch} .

Increasing the values of H_{ch} from 0.75mm to 1.50mm induce reduction of R_{th} at an average percentage of 36.8% at increasing Re (refer Figure 6-24). The reason of the decrease in R_{th} is due to the increase of A_{eff} as the value of H_{ch} increases improving the convective heat transfer between liquid and mini-channel walls. As H_{ch} is varied between 0.75mm and 1.50mm, the corresponding A_{eff} would result in 608.1mm^2 and 973.6mm^2 respectively giving surface area percentage increase of 60.1%. Higher A_{eff} would definitely lead to the reduction in $T_{surf,max}$.

Changing the H_{ch} from 0.75mm to 1.50mm leads to the decrease of ΔP with average percentage decrease of 51.4% at increasing Re . Referring to Figure 6-25, increasing H_{ch} would result in the decrease of liquid velocity within the main channel at the same number of Re since the hydraulic diameter, D_h , is proportional to H_{ch} . Thus, lower pressure is recorded and less frictional losses.

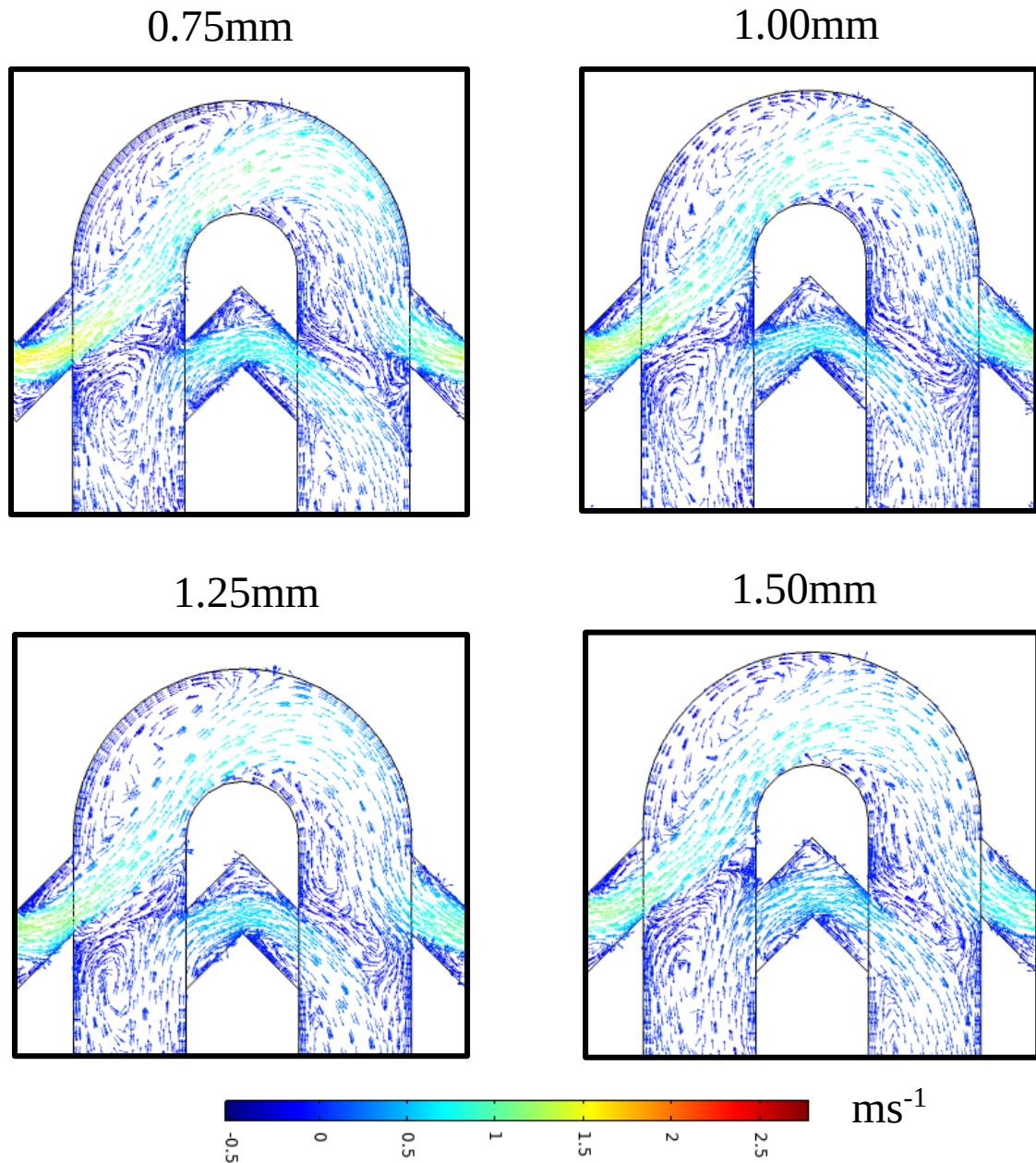


Figure 6-25: Shows the arrow surface plotted at $H_{ch}/2$ mm on the xy -plane with velocity contour distribution for different values of H_{ch} .

6.4.3.3.1 PEC index study on different values of height of mini-channel (H_{ch})

Looking at Figure 6-26, the PEC index, $E_{Nu_{avg}}$ and $E_{\Delta P}$ are plotted at different values of H_{ch} . Large value of PEC index resulted from highest E_{Nu} and lowest $E_{\Delta P}$. At H_{ch} equals to 1.25mm and 1.50mm, $E_{Nu_{avg}} > 1$ at increasing Re while at H_{ch} equals to 0.75mm and 1.00mm, the $E_{Nu_{avg}} < 1$ at $Re < 900$ and 700 respectively. At $E_{Nu_{avg}} > 1$ indicates that greater thermal performance of SMCHS with fins compared to SMPF heat sink. At higher H_{ch} , A_{eff} increases leading to enhanced convective heat transfer. Increasing the values of H_{ch} would definitely improve the

heat transfer since $E_{Nu_{avg}}$ increases but the liquid velocity decreases greatly as well inducing the decreases in change between $E_{Nu_{avg}}$ values at different H_{ch} (refer Figure 6-26).

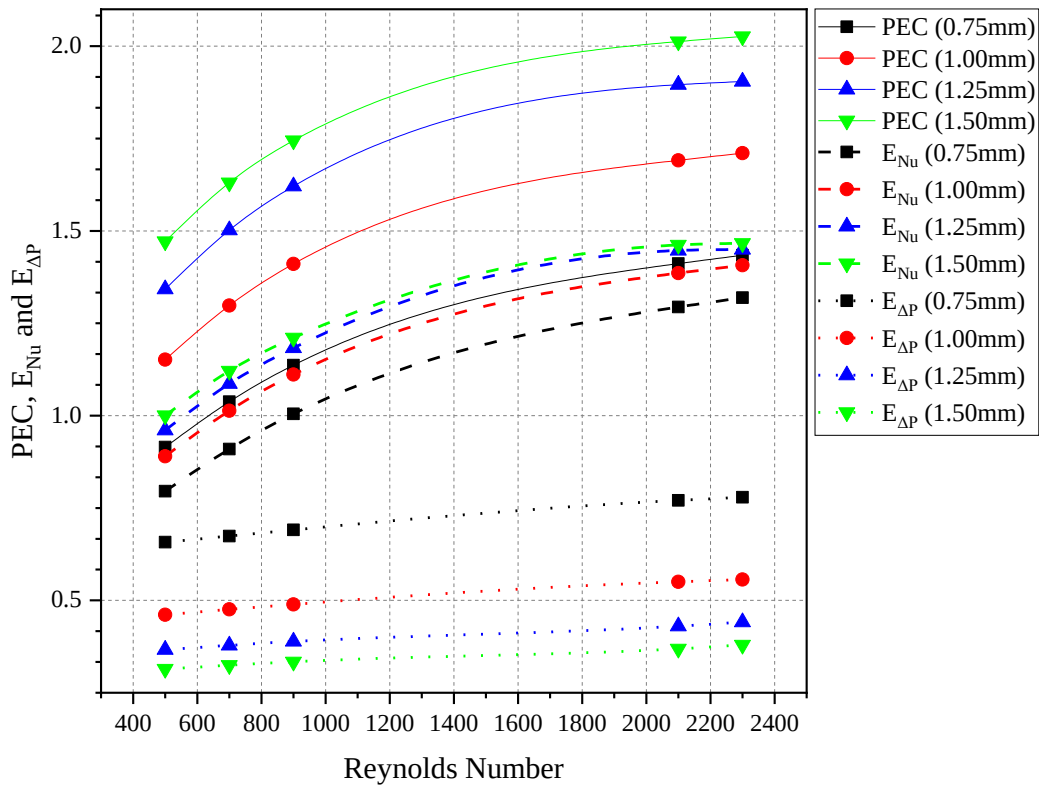


Figure 6-26: Shows the PEC, $E_{Nu_{avg}}$ and $E_{\Delta P}$ plots for different values of H_{ch} at increasing Re .

The highest and lowest $E_{\Delta P}$ values along increasing Re are hold by H_{ch} equals to 0.75mm and 1.50mm respectively. Lower magnitude of $E_{\Delta P}$ at $H_{ch}=1.5$ mm is due to larger D_h at same Re would result in lower liquid velocity and vice versa.

At $H_{ch}=0.75$ mm and 1.50mm, the PEC index recorded are the lowest and highest respectively along increasing Re . The PEC index increases gradually with decreasing rate along increasing Re . Looking at Figure 6-26, similar trend is observed between PEC index and E_{Nu} plots. E_{Nu} increment will result in the increase of PEC index at increasing Re but since the ΔP increase overthrows the increment of E_{Nu} , the PEC index gradient diminishes.

6.4.3.4 Effect of changing the mini-channel width (W_{ch}) of SMDF-C design on both R_{th} and ΔP

To study the effect of changing the size of width of mini-channel (W_{ch}) on both R_{th} and ΔP values, it is varied at 1.0mm, 1.1mm, 1.2mm and 1.3mm respectively. The Re is varied between 500 and 2300 respectively with $100\text{W}/\text{cm}^2$ heat flux applied at the base of heat sink. For this work, the fin width is set to be 0.7mm and all other dimensions can be referred in Table 6-1. In Figure 6-27, the effect of changing the W_{ch} on both R_{th} and ΔP are plotted as a function of Re . From Figure 6-27, the R_{th} decreases and ΔP increases as W_{ch} increases.

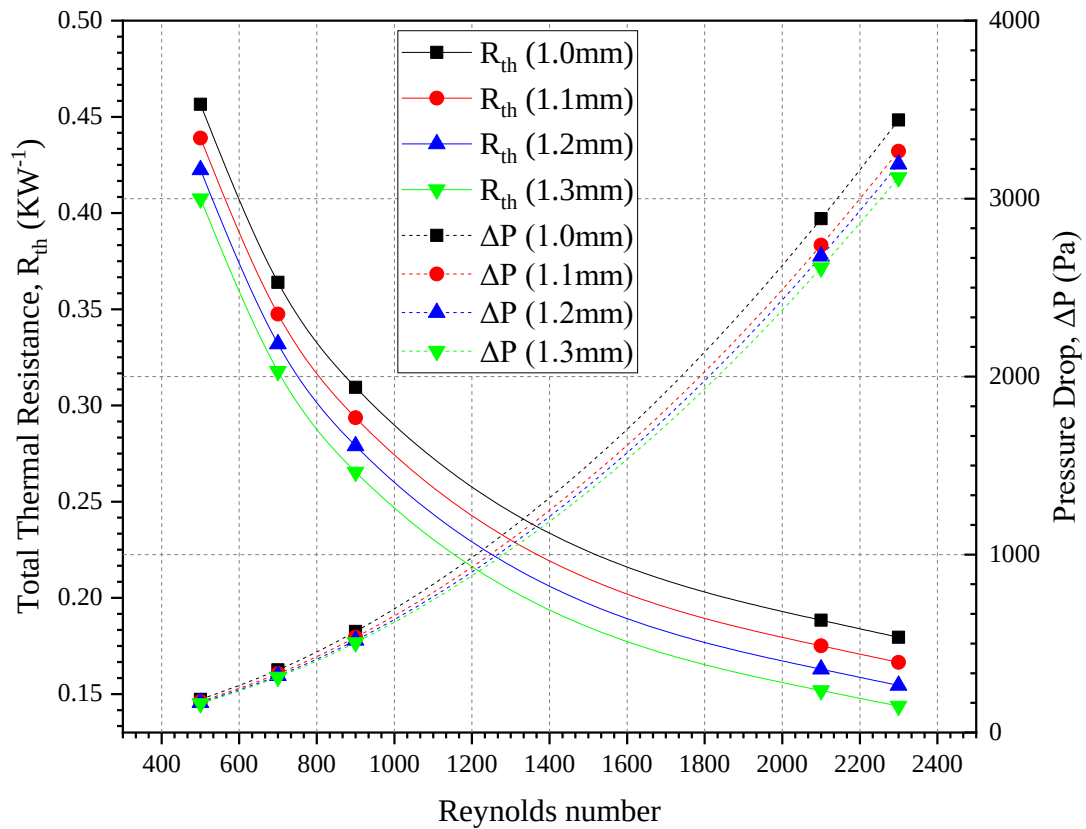


Figure 6-27: Shows the R_{th} and ΔP plots at different values of W_{ch} .

Changing the value of W_{ch} from 1.0mm to 1.3mm, R_{th} reduces with an average reduction of 18% difference along increasing Re . Reduction of R_{th} is mainly caused the increase in A_{eff} where more convective heat transfer occurs. The A_{eff} at W_{ch} equals to 1.0mm and 1.3mm are 1058.8mm^2 and 1137.6mm^2 respectively which gives a percentage difference of 7.4%. Since $T_{surf,max}$ is proportional to R_{th} , therefore, at higher A_{eff} the $T_{surf,max}$ is lower.

Varying the W_{ch} from 1.0mm to 1.3mm results in reduction of ΔP with average percentage of 10.8% change along increasing Re . Based on Figure 6-27 and the calculated percentage, the ΔP

does not change much when W_{ch} is varied. This is due to the very small change in liquid velocity within the channel when W_{ch} is varied (refer Figure 6-28).

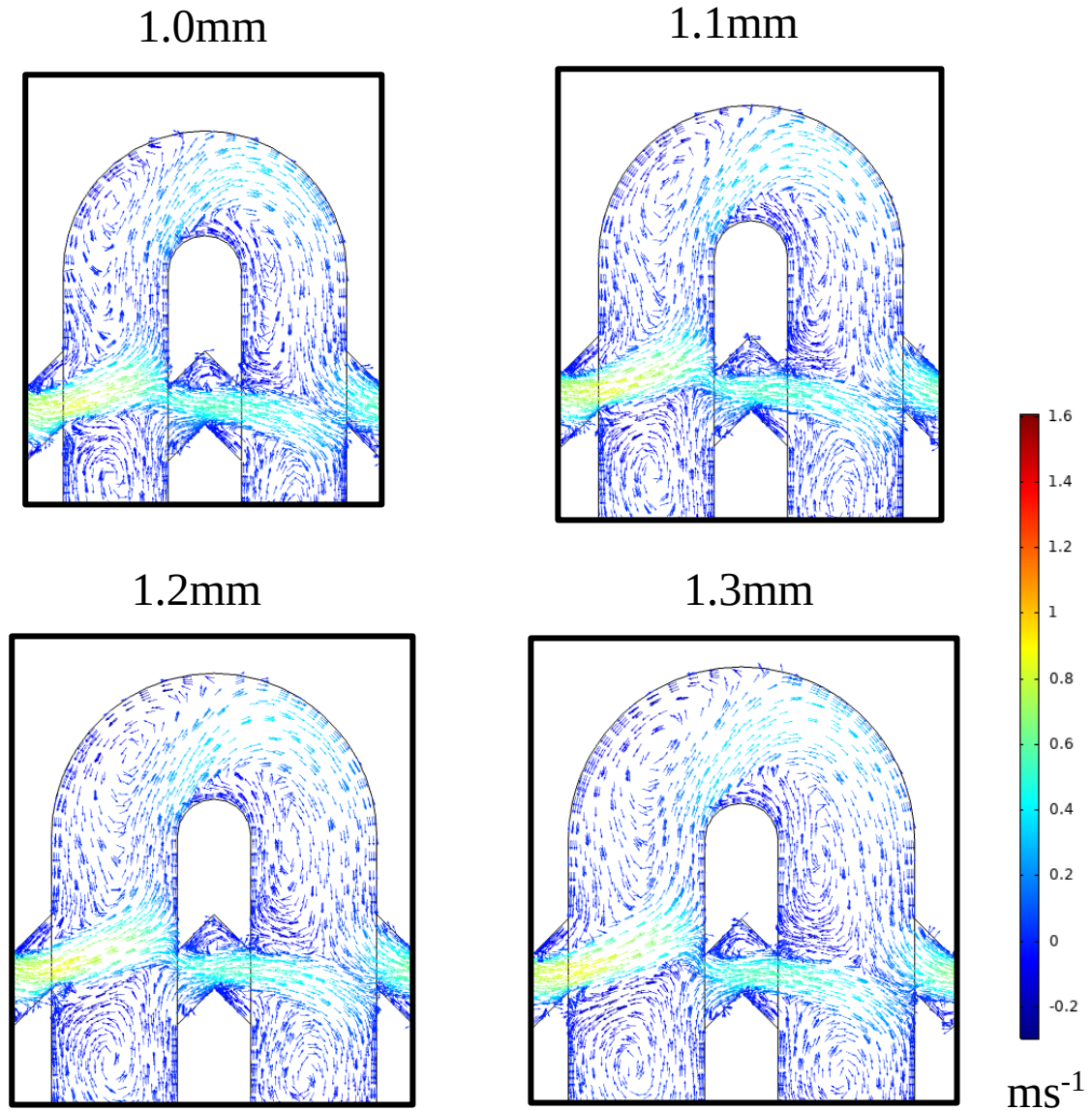


Figure 6-28: Shows the arrow surface plotted at $H_{ch} = 1\text{mm}$ on the xy -plane with velocity contour distribution for different values of W_{ch} .

6.4.3.4.1 PEC index study on different values of width of mini-channel (W_{ch})

Referring to Figure 6-29, the PEC index, $E_{Nu_{avg}}$ and $E_{\Delta P}$ are plotted at different values of W_{ch} . For all plots of E_{Nu} at all W_{ch} values, the $E_{Nu_{avg}} > 1$ along increasing Re . $E_{Nu_{avg}}$ more than unity showing superior performance against SMPF heat sink. At higher W_{ch} , the convective heat transfer increases since the A_{eff} is larger.

All $E_{\Delta P}$ plots are very close with each other since ΔP does not change much when W_{ch} is varied. In all W_{ch} cases tested, the change in D_h does not affects the velocity within the main channel and secondary channel.

At $W_{ch} = 1.0\text{mm}$ and 1.3mm , lowest and highest PEC index recorded along increasing Re . The trend of PEC is main contributed by $E_{Nu_{avg}}$ plots. The increase of PEC index with decreasing gradient when Re increases is due to reduction of R_{th} suppressed by the increment of ΔP .

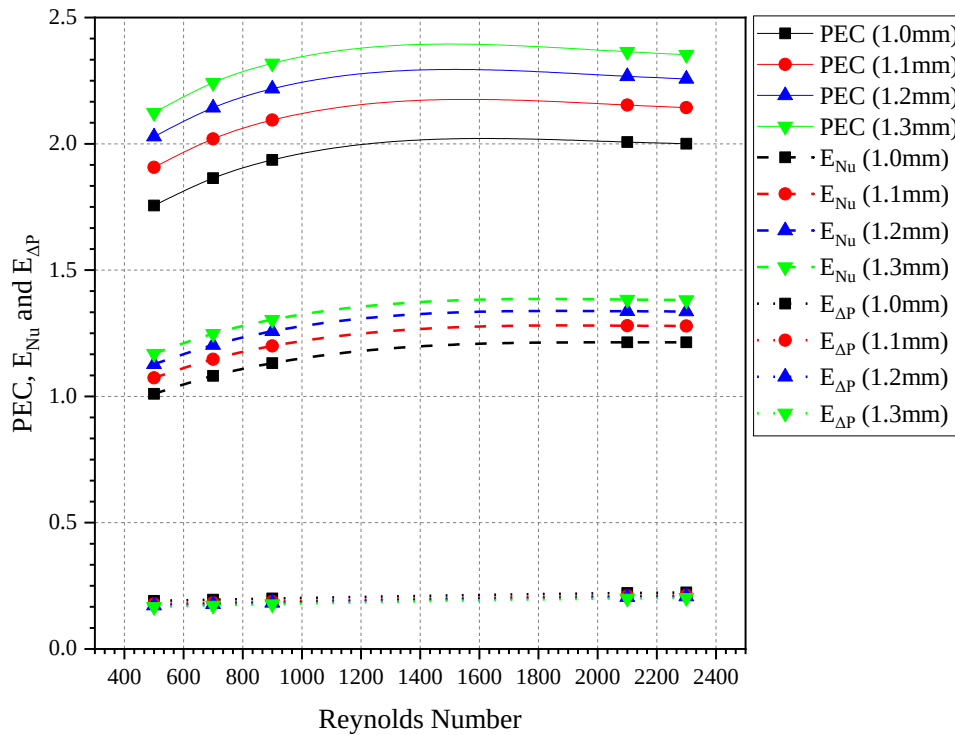


Figure 6-29: Shows the PEC, $E_{Nu_{avg}}$ and $E_{\Delta P}$ plots for different values of W_{ch} at increasing Re .

6.4.3.5 Effect of changing heat flux of SMDF-C design on both $T_{surf,max}$ and ΔP

To study the effect changing heat flux applied underneath of heat sink on both $T_{surf,max}$ and ΔP , three different values were used which are 50W/cm^2 , 75W/cm^2 and 100W/cm^2 respectively. All heat flux magnitudes were tested under range of Re between 500 and 2300. All dimensions of SMDF-C design can be referred in Table 6-1. Looking at Figure 6-30, ΔP decreases slightly as heat flux increases with 3.8% average percentage difference between the highest and lowest ΔP . This is due to the high temperature liquid affects its density and viscosity to be reduced.

From Figure 6-30, the $T_{surf,max}$ experiences an increase in magnitude when heat flux increases due to more heat supplied. The $T_{surf,max}$ reduces at increasing Re since larger convective heat

transfer when liquid velocity is higher. The average percentage increase in $T_{surf,max}$ when heat flux increase from 50W/cm^2 to 100W/cm^2 is at 11.4% change.

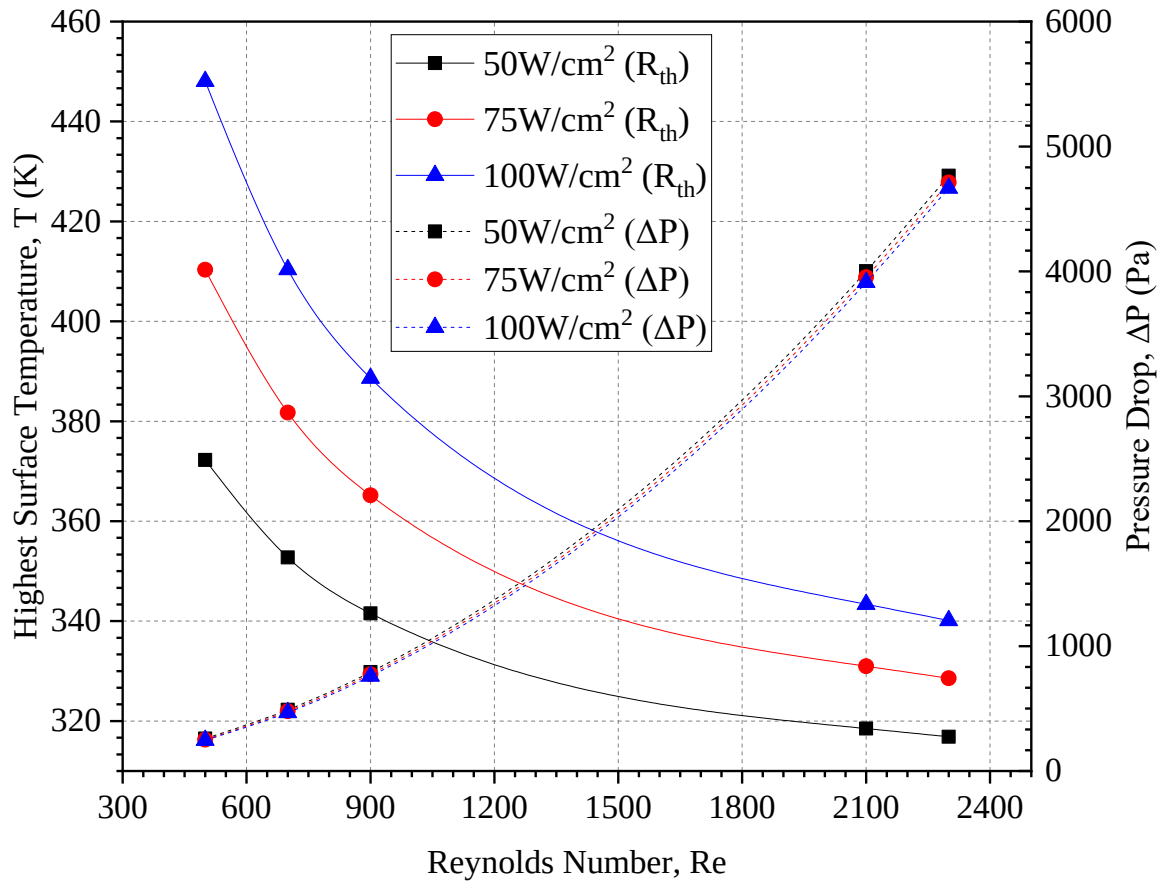


Figure 6-30: Shows the R_{th} and ΔP plots at different values of heat flux applied.

6.5 Design optimisation of SMDF-C heat sink

For electronics cooling, the use of heat sinks is mainly for the removal of heat fluxes dissipation from electronic devices maintaining the temperature to be below than the junction temperature. In this subchapter, the optimisation the SMDF-C heat sink will be conducted covering single-objective optimisation, multi-objective optimisation for both deterministic and robust design optimisation (RDO) framework since it possesses the lowest R_{th} and highest Nu_{avg} . For this work, three objectives are used to be optimised simultaneously which are the total thermal resistance (R_{th}) pressure drop (ΔP) and mass of heat sink (M_{Cu}) respectively.

Choosing the input design variables are very important to search the most optimal design points giving the highest thermal-hydraulic performance. For the optimisation study, the design variables chosen are channel width (W_{ch}), angle of fin (θ_{fin}), channel height (H_{ch}) and thickness of substrate (H_b) respectively since they induce significant effect to the thermal-hydraulic performance. The description with the corresponding limits of the design variables are stated in Table 6-3 below.

Table 6-3: Shows the description of design variables.

Design variables	Lower limit	Upper limit
Width of channel, W_{ch} (mm)	1.0	1.5
Angle of fin, θ_{fin} ($^{\circ}$)	45	60
Height of channel, H_{ch} (mm)	0.5	2.0
Thickness of substrate, H_b (mm)	0.5	1.0

An important note to be mention, the limits of design variables are made to ensure that the design space is as large as possible for optimisation can be conducted over a very wide span of range of design variables. The Design of Experiment (DOE) points are first created based on the limits set for the design variables in Table 6-3. A set of 310 points were created using the Optimal Latin Hypercube (OLHC) sampling method (refer subchapter 5.4) for uniform distribution throughout the design space (refer Figure 6-31 for the distribution of DOE points). The DOE points then will be utilised for the creation of surrogates using Artificial Neural Network (ANN) within TensorFlow framework and Radial Basis Functions (RBFs) methods mapping the relationship of the design variables and the objectives chosen. For optimisation analysis, Platypus package is utilised. All of data analysis are conducted using Python open-sourced software.

Carrying out the optimisation process, the secondary channel length, l_{sc} , is set to be constant at $0.7mm$. Water volumetric flow of $2.67 \times 10^{-6} m^3s^{-1}$ is set at the inlet boundary. A $100 Wm^{-2}$ of heat flux applied underneath of heat sink. The heat sink area, $W \times L$, is set at $20 \times 20 mm^2$. The aim of optimisation is to search for the optimal points of design variables that gives the lowest objectives which will be discussed further on the next subchapters.

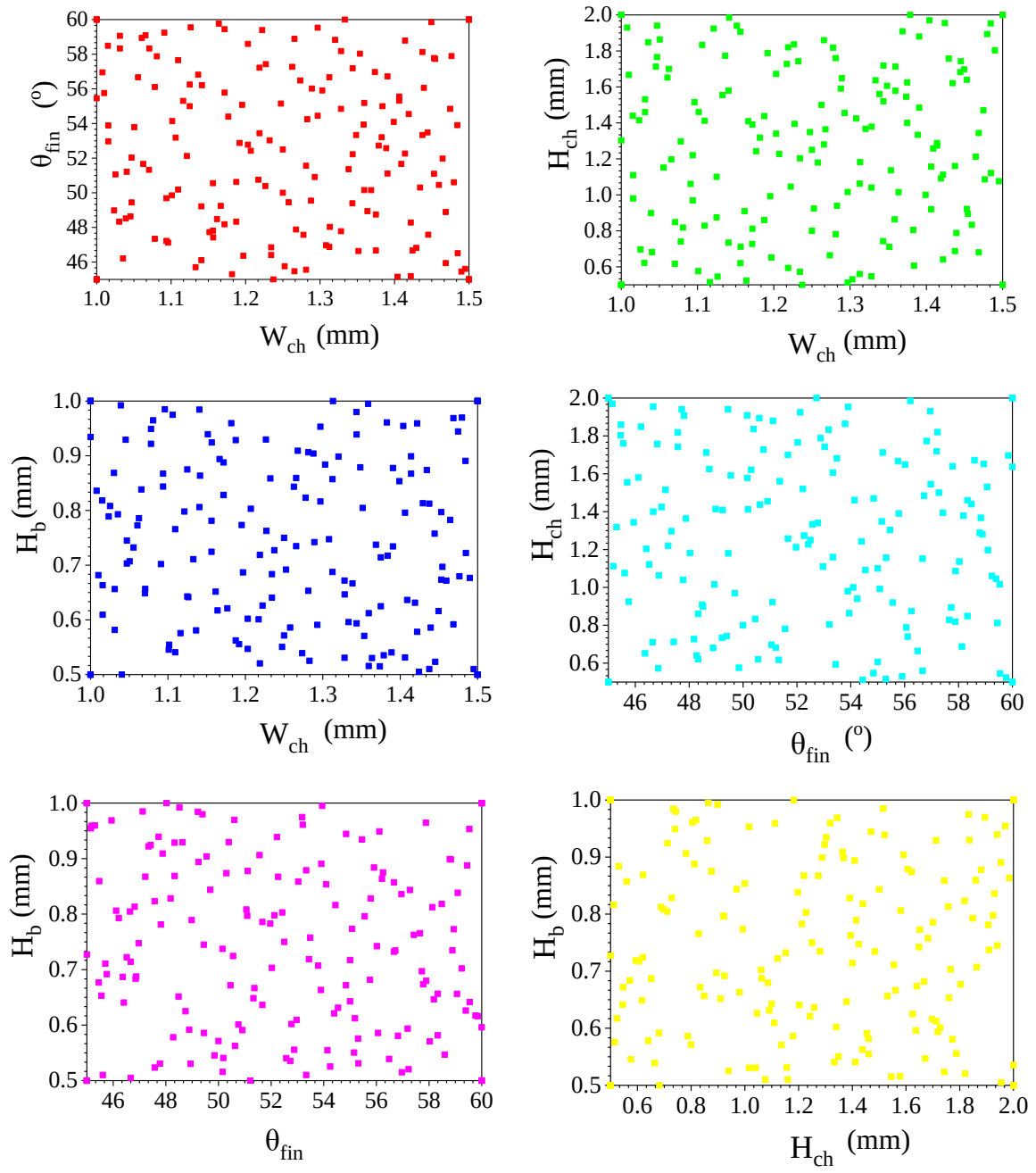


Figure 6-31: Shows the distribution of DOE points for different design variables.

6.5.1 Surrogate modelling

Before the creation of surrogate models, each of the DOE point will be fed into the model developed within the commercial CFD software COMSOL Multiphysics to obtain the output results which are the objectives mentioned in subchapter 6.5. Then, the input design variables will be normalised based on the lower and upper limit stated in Table 6-3 and objective functions are normalised using the largest and smallest calculated outputs. This step is very important to minimise the variation of the input design variables for better convergence of the surrogate modelling later. The normalisation of input design variables can be performed using the following formula:

$$\bar{y}_k = \frac{y_k - y_{k,min}}{y_{k,max} - y_{k,min}} \quad (6.6)$$

where y_k is the k^{th} number for the component y , $y_{k,min}$ and $y_{k,max}$ are the minimum and maximum magnitude of the entire dataset.

The dataset of normalised input design variables and objectives will be trained using the feedforward type of network where the propagation of information is passed from the input layer (design variables) through the hidden layers specified and lastly the output layer (objective functions). Complete explanation of training a model using ANN method can be found under subchapter 5.5.3. Three objective functions have been chosen are total thermal resistance (R_{th}), pressure drop (ΔP) and mass of copper heat sink (M_{Cu}) respectively. The R_{th} and ΔP can be calculated using Eq. 3.53 and Eq. 3.54 respectively under subchapter 3.8 and needs to be trained with the input design variables. The M_{Cu} can be calculated directly using analytical method where no surrogate model needed based on the dimensioning of serpentine mini-channel heat sink by multiplying the total volume of the solid copper to its density as follows:

$$M_{Cu} = V_{Cu} \times \rho_{Cu} \quad (6.7)$$

Where V_{Cu} and ρ_{Cu} are the volume and density of copper respectively. The density of copper is $8.96 \times 10^{-3} \text{ g/mm}^3$. The total volume of solid copper can be calculated as follows:

$$\begin{aligned} V_{Cu} = & WL(H_{ch} + H_b) \\ & - \left[\left[\frac{\pi}{2} (W_{ch}W_w + W_{ch}^2)m \right] - l_{sc}W_wmn \right. \\ & \left. - L_{ch}W_{ch}p - [(L - L_{ch})W_{ch}] \right] H_{ch} \end{aligned} \quad (6.8)$$

Where m , n and p are the number of channel bends, number of secondary channels per column and number of main channels respectively. Where $m = 9$, $n = 8$ and $p = 10$.

During the training of the surrogate models, the surrogates are trained individually for faster convergence rate. If the objective functions are combined and trained together, the neural network optimisation to find convergence is very slow since it has to optimise many objective functions simultaneously which makes the optimisation problem very complex and also sophisticated network architecture is required. To separate the training and validation set, a ratio of 80% to 20% is applied to the whole DOE dataset.

6.5.1.1 Artificial Neural Network (ANN) and Radial Basis Functions (RBFs) methods

To ensure the accuracy of the ANN models, convergence studies are carried out based on the hyperparameters tuning which are the optimizer, number of neurons, type of activation function and learning rate respectively. The Adam optimizer is used in this work since it offers high validation accuracy, low memory requirements and straight-forward implementation [313, 314]. As for learning rate, 0.00001 magnitude is utilised for all three surrogates for faster convergence rate and high models accuracy while avoiding over-fitting of the training model if smaller learning rate used. Larger learning rate experiences slow convergence rate and less accuracy. Training of the design variables with both R_{th} and M_{Cu} objective functions used Rectified Linear Unit (ReLU) as activation function within every hidden layer while with ΔP objective function, Scaled Exponential Linear Unit (SELU) activation function was utilised since faster convergence resulted and also increase ANN model performance. Both ReLU and SELU activation functions can be defined as follows:

$$\text{ReLU}(x) = f(x) = \begin{cases} 0 & := x < 0 \\ x & := x \geq 0 \end{cases} \quad (6.9)$$

Where, if the input $x < 0$, the input is set to be 0. Meanwhile, if the input $x \geq 0$, the input remains the same.

$$\begin{aligned} \text{SELU}(x) = f(x) &= \lambda \begin{cases} x & := x > 0 \\ \alpha e^x - \alpha & := x \leq 0 \end{cases} \\ \alpha &\approx 1.6732632423543772848170429916717 \\ \lambda &\approx 1.0507009873554804934193349852946 \end{aligned} \quad (6.10)$$

The values of α and λ are initially predetermined using extensive calculation shown by Klambauer et al. [315].

After the surrogate models have been trained the loss functions, Mean Absolute Error (MAE) and Root Mean Square Error (RMSE) between the training and validation set are plotted on the same axis for two objective functions of R_{th} and ΔP respectively (refer Figure 6-34 to Figure 6-35).

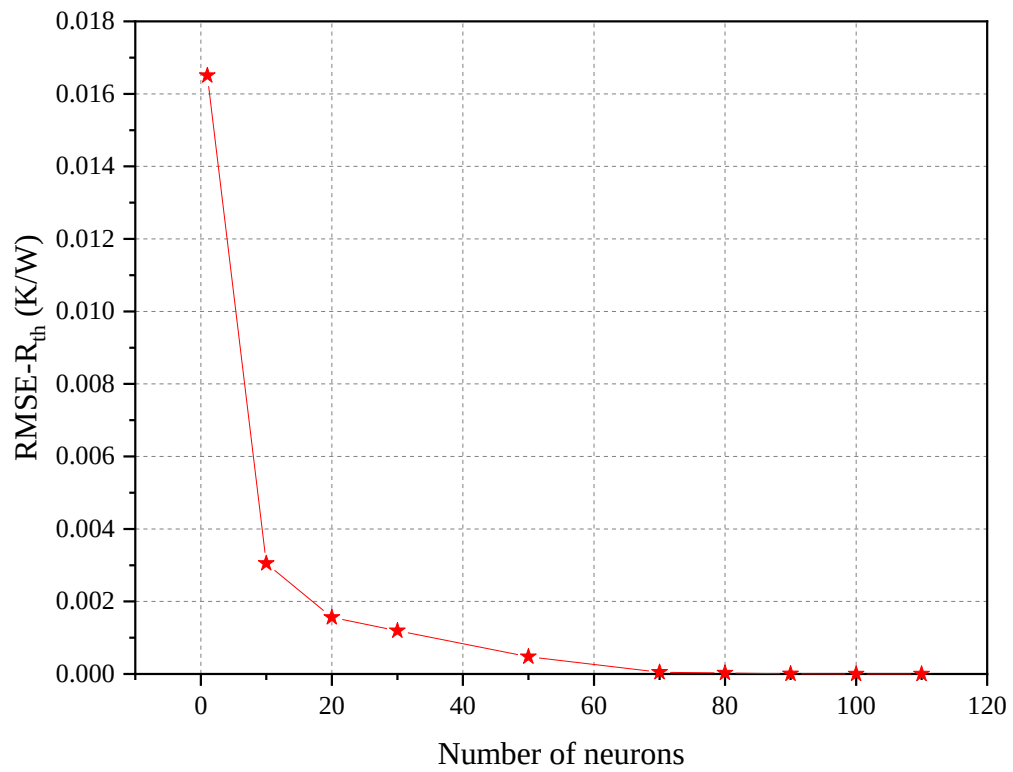


Figure 6-32: Shows the RMSE convergence plot for R_{th} objective function.

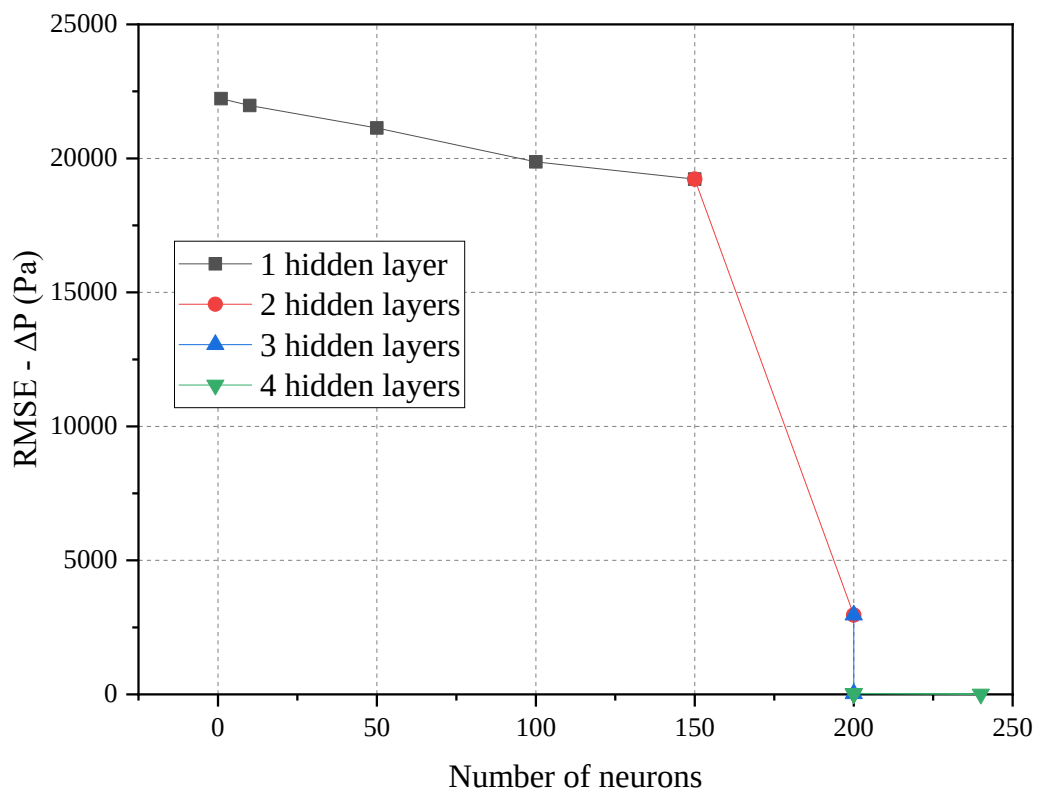


Figure 6-33: Shows the RMSE convergence plot for ΔP objective function.

Based on Figure 6-32 to Figure 6-33, the convergence plots based on the RMSE parameter are shown for the three objective functions. For both RMSE convergence plots of R_{th} objective function, only a single layer of hidden layer is required while for ΔP , four hidden layers are needed for convergence to arise. The RMSE for R_{th} decreases drastically when number of neurons changed from 1 to 5 and after that it reduces gradually. As for ΔP , the RMSE decreases gradually at 1 hidden layer and increasing the hidden layers to 2 and 3 layers, the RMSE decreases drastically until very close to zero. For R_{th} objective function, the convergence of RMSE parameter occur at 80 and 60 neurons respectively while for ΔP , convergence happen at 200 neurons with three hidden layers required. Surrogate models that have reached the convergence will be used for optimisation process since high accuracies of surrogates achieved.

Referring Figure 6-34 to Figure 6-35, it can be seen that the training and validation plots separate after certain epoch number which are 12000 and 27000 for R_{th} and ΔP respectively. The separation of the plots indicate that the ANN models training start to over-fit. Over-fitting essentially specify that the ANN models are being trained too well which can affect the accuracy of results of the output objective functions when new set of design variables are being added for prediction. Thus, always stop the model training when overfitting occurs. Other ways to prevent over-fitting is to introduce dropout within the network where some randomly chosen neurons are ignored during training. Apart from that is to increase the number of datasets.

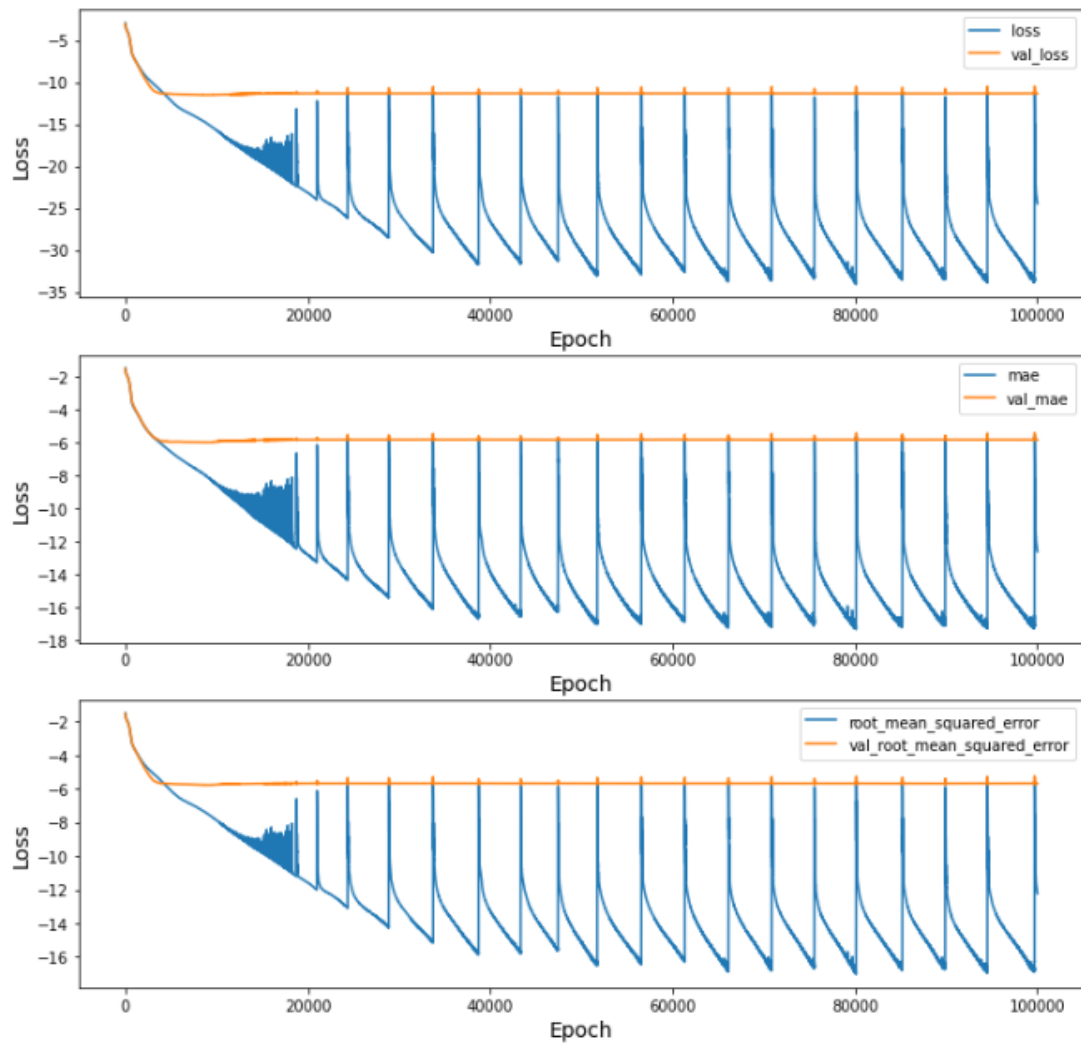


Figure 6-34: The log plot of loss functions, MAE and RMSE between the training and validation dataset for R_{th} objective function.

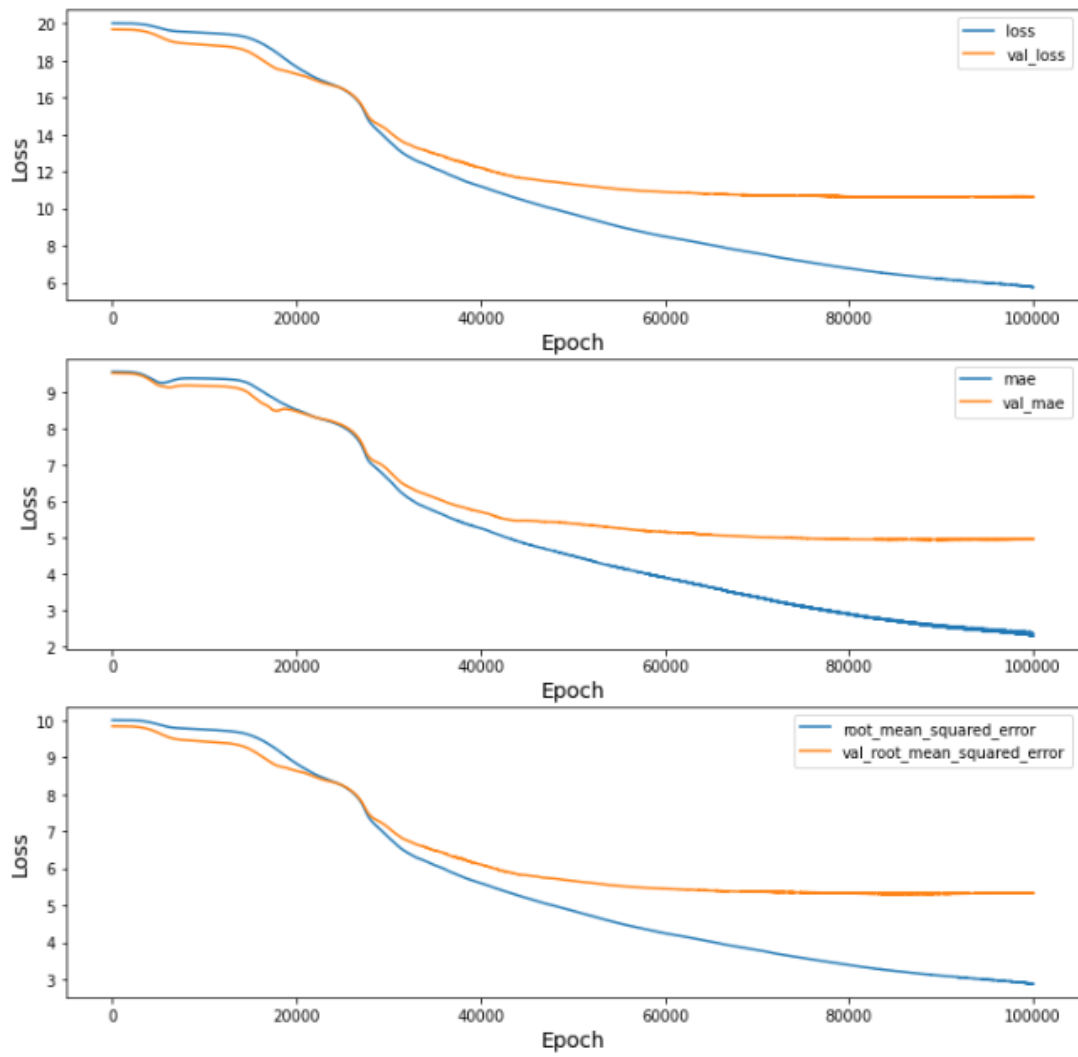


Figure 6-35: Shows the log plot of loss functions, MAE and RMSE between the training and validation dataset for ΔP objective function.

Table 6-4: The accuracy parameters for training, validation and testing sets of surrogate models using ANN.

R_{th}			
Accuracy parameter	Training set	Validation set	Testing set
RMSE (K/W)	2.820116×10^{-7}	5.382246×10^{-7}	3.590711×10^{-7}
R^2	0.999999	0.999842	0.999936
Maximum Residual Error (%)	0.00111305	0.00263114	0.00184323
ΔP			
Accuracy parameter	Training set	Validation set	Testing set
RMSE (Pa)	13.627698	20.443897	16.357453
R^2	0.999999	0.999221	0.999875
Maximum Residual Error (%)	1.119512	1.647251	1.347862

To demonstrate the accuracy of the trained two surrogate models for R_{th} and ΔP objective functions, the comparison between the predicted output and numerical results output are conducted by calculating the RMSE, R^2 and maximum residual error (refer Table 6-4). From the table, it can be seen that predicted outputs are in correlation with the original CFD results for training, validation and testing sets because the R^2 values are very close the unity indicating high accuracy in surrogate models created. Furthermore, looking at maximum residual errors all of them are less 2% also indicating high accuracy of predicted results compared to the original CFD data. The difference between RMSE values for R_{th} and ΔP are due to the nature of data which either can be considered as very high or low. The RMSE results also show low values compared to respective objective functions (refer Table 6-6 for typical magnitudes) signifying very low discrepancies between predicted data and original numerical data.

Table 6-5: The accuracy parameters for training, validation and testing sets of surrogate models using RBF.

R_{th}			
Accuracy parameter	Training set	Validation set	Testing set
RMSE (K/W)	9.561642×10^{-5}	1.632908×10^{-4}	1.024188×10^{-4}
R^2	0.999991	0.993351	0.999781
Maximum Residual Error (%)	0.390062	0.744512	0.531193
ΔP			
Accuracy parameter	Training set	Validation set	Testing set
RMSE (Pa)	22.751682	31.668441	25.412112
R^2	0.999998	0.995481	0.999112
Maximum Residual Error (%)	1.196326	1.792756	1.471123

From Table 6-5, the accuracy parameters of RMSE, R^2 and maximum residual error are recorded. It can be seen that using RBF method to create surrogate models is very effective as well compared to ANN method due to the accuracy parameter exhibiting accurate results but with very small discrepancies. The training of surrogate models using ANN method took about 10 minutes while using RBF almost instant results are obtained. Therefore, to save computational time and resources, the RBF method will be used for further surrogate models training in further chapters.

6.5.2 Optimisation process

Under the optimisation procedure, two different approaches are taken here which the deterministic and robust design optimisation framework. Single- and multi-objective optimisations will be covered to explore the different solutions generated and compare the thermal-hydraulic performance between different methods. The optimisation will cover the serpentine mini-channel heat sink of SMDF-C configuration due to the reason mentioned under subchapter 6.3.6.

6.5.2.1 Deterministic optimisation

Within this subchapter, the use of deterministic optimisation framework will be utilised under the condition of single- and multi-objective optimisations respectively.

6.5.2.1.1 Single-objective optimisation

To optimise a single objective function at a time is a direct process where the minimum or maximum value of objective can be obtained easily with the associated input design variables using NSGA-II method. The associated input design variables that corresponds to the minimisation of individual objective of R_{th} , M_{Cu} and ΔP are recorded in Table 6-6.

Table 6-6: Shows the input design variables associated with single-objective functions optimisation..

W_{ch} (mm)	θ_{fin} (°)	H_{ch} (mm)	H_b (mm)	R_{th} (K/W)	ΔP (Pa)	M_{Cu} (g)
1.0	60	0.5	1.0	0.122915	92790.85	4.288819
1.5	45	2.0	0.5	0.264280	1277.900	3.340082
1.5	45	0.5	0.5	0.285003	22264.55	2.179020

From Table 6-6, the minimisation of single objective at a time would result in high magnitudes of other objectives. The green fill is the minimisation of that particular objective. At very low R_{th} of 0.123 K/W, a very high ΔP of 92970.9 Pa is recorded as well as larger M_{Cu} of 4.289 g. The same thing happened when ΔP and M_{Cu} are minimised individually, it would results in a drastic increase of the other two objectives respectively. To ensure the compromise minimisation between the three objectives, the multi-objective minimisation framework is applied in the next sub-chapter.

6.5.2.1.2 Multi-objective optimisation

Using the pre-trained surrogate models using RBF method (refer subchapter 6.5.1) where it will be exploited under the framework of multi-objective optimisation for competing objectives to achieve compromise minimisation between objectives of R_{th} , ΔP and M_{Cu} respectively. Using the input design variables and output objective function defined under subchapter 6.5.1, the formulation of multi-objective optimisation is as follows:

$$\begin{aligned}
 &\text{Objective functions: } \min(R_{th}), \min(\Delta P) \text{ and } \min(M_{Cu}) \\
 &\text{Constraints: } W_w + W_{ch} = 2mm \\
 &\text{Design space: } 1.0mm \leq W_{ch} \leq 1.5mm \\
 &\quad 45^\circ \leq \theta_{fin} \leq 60^\circ \\
 &\quad 0.5mm \leq H_{ch} \leq 2.0mm \\
 &\quad 0.5mm \leq H_b \leq 1.0mm
 \end{aligned} \tag{6.11}$$

During the optimisation process, the other parameters and boundary conditions apart from the design variables are kept constant which can be referred in Table 6-1. The aim of multi-objective optimisation is to create the Pareto Front curve consisting of non-dominated solutions of the three objective functions defined which allow the designers flexibility in choosing the appropriate objective functions.

As mentioned earlier in subchapter 6.5, Platypus optimisation package in Python software will be utilised to perform multi-objective optimisation using Non-dominated Sorting Genetic Algorithm-II (NSGA-II) since it is fast, reliable and initial guess is not required [316].

Referring Figure 6-36 to Figure 6-36 , the Pareto Front plots and DOE points between two objective functions of ΔP vs R_{th} , M_{Cu} vs R_{th} and M_{Cu} vs ΔP are shown respectively. The Pareto Front plots shows the relationship between two competing objectives which means an increase in one objective will decrease in another. To understand better of the Pareto Front plots, the numerical data of the competing objectives with its associated input design variables are tabulated in Table 6-7 and the Pareto Front generated from the NSGA-II algorithm are compared with CFD results for accuracy testing.

From the tabulated data of ΔP vs R_{th} ignoring the variation of M_{Cu} , going down the table where increasing the values of R_{th} from 0.128 K/W to 0.152 K/W (18.8% increment) will result a decrease in ΔP from 5396.792 Pa to 2724.1 Pa (49.5% decrement) and vice versa. Similarly for M_{Cu} vs R_{th} ignoring the variation of ΔP , going down the table R_{th} values increases from 0.125 K/W to 0.146 K/W (17% increment) gives M_{Cu} decrease from 3.042 g to 2.359 g (22.4%

decrement) respectively. Lastly, for M_{Cu} vs ΔP ignoring the variation of R_{th} , increasing the ΔP from 2272.751 Pa to 13683.6 Pa (502.1% increment) will result in the decrease of M_{Cu} from 2.957 g to 2.401 g (18.8% decrement) respectively. From any Pareto Front relationship between two objective functions, the other untouched objective will have a very large magnitude which is undesirable. Thus, all three objectives must be considered for multi-objective optimisation.

To consider all three objective functions, the Pareto Front distribution is plotted on Figure 6-39 which is the 2-D contour plot. It can be observed that moving towards the zero location of the Cartesian plane, ΔP and R_{th} experience reduction while M_{Cu} increases. Thus, a compromise between three objectives can be conducted to search for the best combination of the magnitudes of objective functions to better suit the desired applications.

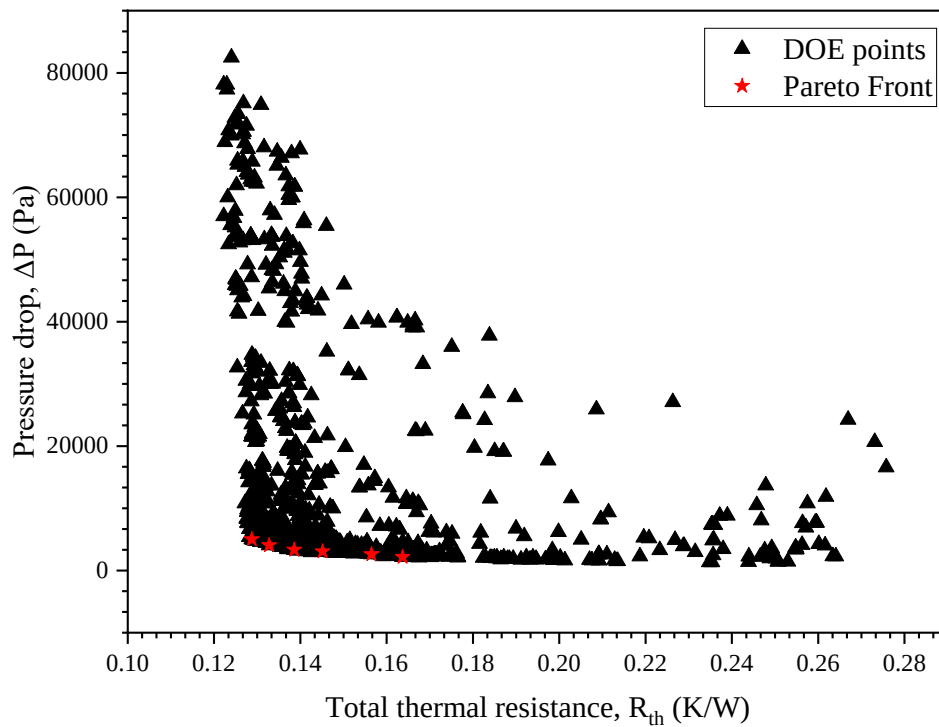


Figure 6-36: Shows the DOE points and Pareto front between competing objectives of ΔP and R_{th} .

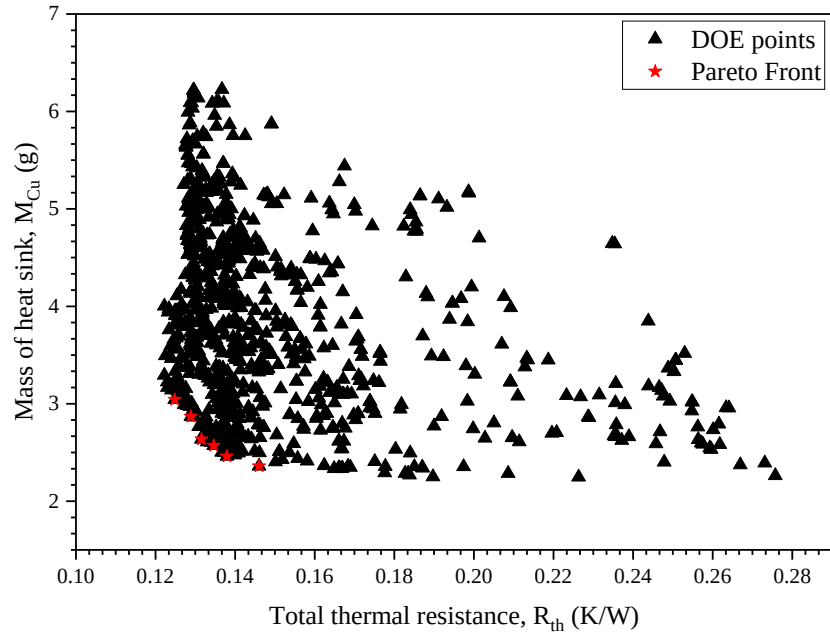


Figure 6-37: Shows the DOE points and Pareto front between competing objectives of M_{Cu} and R_{th} .

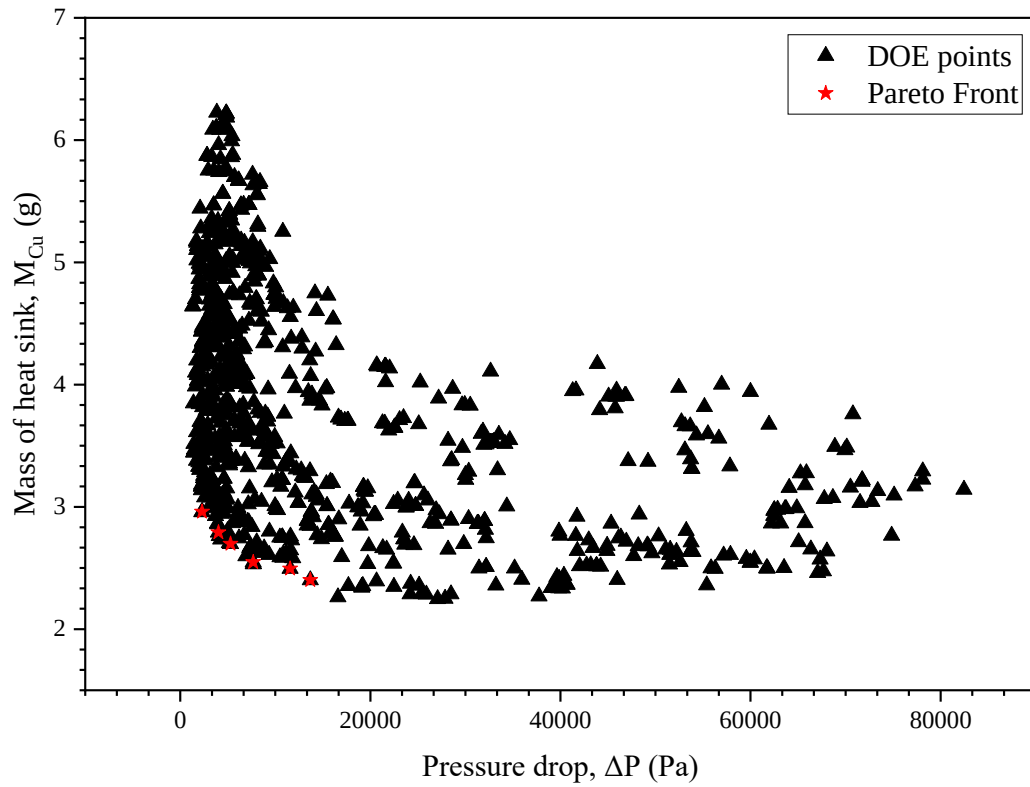


Figure 6-38: Shows the DOE points and Pareto front between competing objectives of M_{Cu} and ΔP .

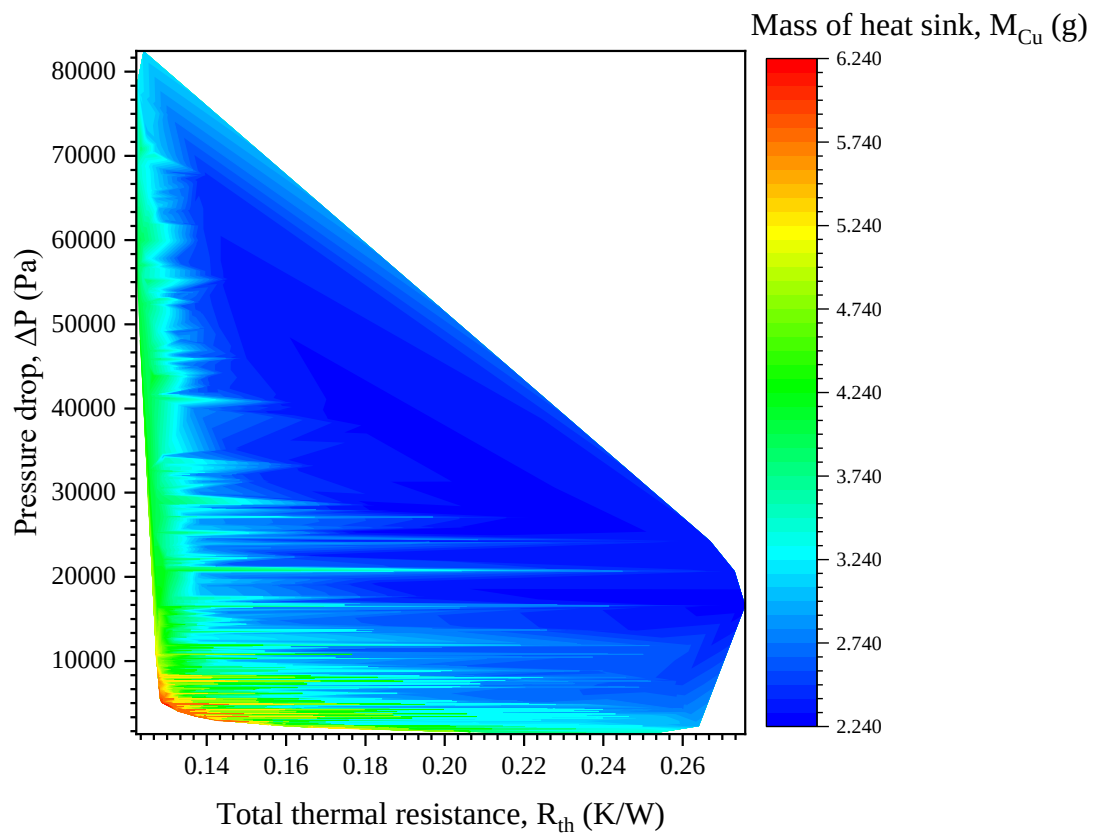


Figure 6-39: Shows the 2-D contour plot of the Pareto front for three objective functions.

Table 6-7: Shows the Pareto front points plotted in Figure 6-36 to Figure 6-38.

ΔP vs R_{th}												
Design variables				R_{th} (K/W)			ΔP (Pa)			M_{Cu} (g)		
W_{ch} (mm)	θ_{fin} (°)	H_{ch} (mm)	H_b (mm)	Pareto	CFD	Error (%)	Pareto	CFD	Error (%)	Pareto	Analytical	Error (%)
1.011014	56.9039	1.912791	0.929134	0.12820	0.12769	0.399405	5396.792	5473.26	1.397120	5.992162	5.993010	0.014150
1.022409	45.88616	1.908912	0.964776	0.135371	0.13682	1.059056	3815.183	3865.33	1.297354	6.098745	6.098732	0.000213
1.143833	50.1295	1.966486	0.962299	0.138587	0.13793	0.476329	3316.632	3370.56	1.599972	5.863458	5.862532	0.015795
1.239061	51.62061	1.972629	0.851011	0.141368	0.14211	0.522131	3087.136	3103.48	0.526635	5.242014	5.242113	0.001889
1.227322	50.26061	1.99929	0.733369	0.144579	0.14563	0.721692	2966.404	3004.63	1.272237	4.883610	4.883752	0.002908
1.227839	47.68663	1.926336	0.831576	0.152314	0.15319	0.571839	2724.128	2770.31	1.667034	5.144217	5.144236	0.000369

M_{Cu} vs R_{th}												
Design variables				R_{th} (K/W)			ΔP (Pa)			M_{Cu} (g)		
W_{ch} (mm)	θ_{fin} (°)	H_{ch} (mm)	H_b (mm)	Pareto	CFD	Error (%)	Pareto	CFD	Error (%)	Pareto	Analytical	Error (%)
1.0192	55.55904	0.527614	0.639522	0.124831	0.125234	0.321798	72781.29	73657.33	1.189345	3.041843	3.041833	0.000329
1.022763	51.69331	0.518033	0.601072	0.128895	0.128903	0.006206	65735.30	66241.66	0.764413	2.868686	2.868731	0.001569
1.033478	52.71614	0.512011	0.542693	0.131573	0.131493	0.060840	68039.91	68113.45	0.107967	2.636626	2.636541	0.003216
1.068676	59.03755	0.554866	0.520975	0.134646	0.134651	0.003713	67331.25	68455.76	1.642681	2.570318	2.570413	0.003696
1.049513	53.73738	0.531873	0.500388	0.136638	0.136642	0.002927	63525.77	64013.97	0.762646	2.501919	2.501201	0.028706
1.209968	52.88823	0.507435	0.500273	0.146012	0.146095	0.056812	55397.06	56323.68	1.645169	2.359043	2.359001	0.001780

M_{Cu} vs ΔP

Design variables				$R_{th} (K/W)$			$\Delta P (Pa)$			$M_{Cu} (g)$		
W_{ch} (mm)	θ_{fin} (°)	H_{ch} (mm)	H_b (mm)	Pareto	CFD	Error (%)	Pareto	CFD	Error (%)	Pareto	Analytical	Error (%)
1.495466	45.11062	1.514963	0.502796	0.264103	0.263912	0.072373	2272.751	2265.332	0.327502	2.957083	2.957101	0.000615
1.488588	48.04717	1.164665	0.507441	0.25628	0.25699	0.276275	4110.219	4123.564	0.323628	2.762224	2.762324	0.00362
1.490249	52.73603	1.095272	0.50118	0.219605	0.219713	0.049155	5283.77	5379.681	1.782838	2.697343	2.697442	0.00367
1.466612	47.87961	0.924783	0.511447	0.257159	0.257555	0.153754	6906.585	7015.67	1.554876	2.597452	2.597762	0.011933
1.466612	47.87961	0.882017	0.503836	0.259594	0.259667	0.028113	7686.173	7734.232	0.621380	2.53327	2.533298	0.001105
1.444812	49.20806	0.702723	0.501835	0.247845	0.247881	0.014523	13683.61	13793.41	0.796032	2.400829	2.400934	0.004373

6.5.2.2 Robust design optimisation (RDO)

Based on the deterministic optimisation study conducted under subchapter 6.5.2.1, it does not have the ability to capture the challenges of real life manufacturing process issues such as tolerances and operating conditions variation. Thus, the existing of environment or input parameters fluctuations cause the design to be unreliable. To take account the uncertainties effect on the performance of the design, Robust Design optimisation (RDO) method is utilised. The use of RDO method is very effective when dealing with the application of uncertainties onto the input design variables, which corresponds to the distribution of output objective functions enabling the calculation of statistical moments of mean (μ) and standard deviation (σ) of that particular point. Within this subchapter, the probabilistic optimisation will be utilised to achieve the robust design of serpentine mini-channel heat sink. The calculation of statistical moments of the output objective functions are based on two different methods of Monte Carlo sampling (MCS) and Polynomial Chaos Expansion (PCE).

6.5.2.2.1 Model description

Based on the detailed explanation under subchapter 6.5, the same working processes will be used here where the same four input design variables are chosen which are W_{ch} , θ_{fin} , H_{ch} and H_b respectively. But, under this subchapter, each point of the input design variables are subjected to an uncertainty quantity enabling the distribution of the input design variables which corresponds to distribution of the output objective functions. The same conjugate heat transfer model and boundary conditions (refer subchapter 6.3) will be used here to calculate the output objective functions from the pre-set input design variables with uncertainty application through the CFD model.

6.5.2.2.2 Validation of RDO method with published work

To validate the RDO framework using MCS and PCE methods, a case study by Couto et al. [288] is used for comparison which is the Brinell hardness test (refer Figure 6-40). The MCS and PCE methods are written within Python open-sourced programming software to calculate the statistical moments. A sphere is used to indent the sample test as it unloads. The sample material hardness is inversely proportional to the mark diameter. Initially, the input design variables are identified which are the Load (F), Diameter of indenter (D) and Mark diameter (d). Then, a model to map the relationship between the input design variables and output objective function is prepared which is as follows:

$$HB = \frac{0.204F}{\pi D(D - \sqrt{D^2 - d^2})} \quad (6.12)$$

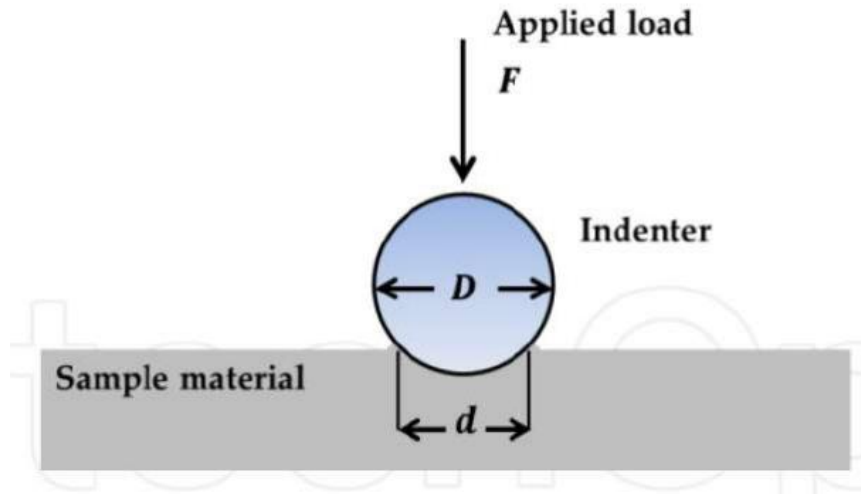


Figure 6-40: Shows the Brinell hardness test schematic representation [288].

After that, the uncertainty information is obtained. The value of F is set to be constant at 29400 N with 2% uncertainty, 95% probability coverage while having $k = 2$ where the results can be represented by Gaussian distribution. The mean, μ , is set at 29400 N while the standard deviation, σ , is calculated using $U/k = \frac{0.02 \times 29400 \text{ N}}{2} = 294 \text{ N}$. The statistical moments for both D and d are calculated using the same procedure. The information for all statistical moments are recorded in Table 6-8.

Table 6-8: Shows the uncertainty associated with the respective PDF parameters.

Input source	Type	PDF	PDF parameters
Load (F)	B	Gaussian	$\mu = 29400 \text{ N}, \sigma = 294 \text{ N}$
Diameter of indenter (D)	B	Gaussian	$\mu = 10 \text{ mm}, \sigma = 0.005 \text{ mm}$
Mark diameter (d)	A	Gaussian	$\mu = 3 \text{ mm}, \sigma = 0.035 \text{ mm}$

To compare the accuracy between results of using the MCS and PCE methods with the results in [288], the results obtained are tabulated in Table 6-9. Number of samples and polynomial order used within the MCS and PCE are 200000 and 3 respectively. It can be observed that the results of MCS and PCE show a very good correlation with the original result improving the confidence of using these methods.

Table 6-9: Shows the statistical moments calculated using MCS and PCE compared with the original result.

Statistical moments	Couto et al. [288]	MCS	PCE
Mean (μ)	415 HB	414.68 HB	414.62 HB
Standard deviation (σ)	11 HB	10.77 HB	10.74 HB

6.5.2.2.3 Uncertainty quantification (UQ)

The use of Uncertainty Quantification (UQ) can be utilised to perform an assessment of uncertainty applied onto the input design variables corresponding to the output objective functions. The Gaussian distributions applied on the input will result in distributions of the output. The framework of using uncertainty analysis involves some key parameters to be followed:

- A) Using the same input design variables stated under subchapter 6.5, each of the DOE point will be subjected to a normal Gaussian distribution. The uncertainties subjected to the input design variables are recorded in Table 6-10.

Table 6-10: Shows the uncertainty associated for each design variable.

Input design variables	Distribution	μ_{in}	σ_{in}
W_{ch} , mm	normal	W_{ch_i} , mm	0.0025mm
θ_{fin} , °	normal	θ_{fin_i} , °	0.025°
H_{ch} , mm	normal	H_{ch_i} , mm	0.0025mm
H_b , mm	normal	H_{b_i} , mm	0.0025mm

- B) Each DOE point with normal distribution will be fed into CFD model constructed to calculate the output objective function, but, this method is considered to be unfeasible since very high time consumption required. Therefore, the use of surrogate models created under subchapter 6.5.1 will be exploited to save time and automate the process instantly.
- C) Using either MCS or PCE methods, the solutions are propagated from the input to the output. From the analysis, PCE method (30 minutes) performs so much faster compared to MCS method (7 hours). Thus, the distribution of output objective functions can be determined [173].

D) The mean (μ) and standard deviation (σ) of the output will be calculated using Eq. 5.29 and Eq. 5.30 respectively [317].

6.5.2.2.4 Convergence study for MCS and PCE methods

Under the optimisation under uncertainty, during the calculation of the output statistical moments of means and standard deviations, a convergence study has to be carried out to ensure that the calculated statistical moments reached the acceptable accuracies without having to increase the number of variable parameters. The convergence study for both MCS and PCE methods are entirely dependent on the number of samples and polynomial order respectively. The convergence plots for statistical moments using two different methods of MCS and PCE are plotted in Figure 6-41 to Figure 6-42 respectively. Figure 6 41, using MCS method, it can be seen that for all statistical parameters of $\mu_{R_{th}}$, $\sigma_{R_{th}}$, $\mu_{\Delta P}$, $\sigma_{\Delta P}$, $\mu_{M_{Cu}}$ and $\sigma_{M_{Cu}}$ converge when number of samples reach the order of $\times 10^5$. Ensuring accurate values to be obtained, number of samples are set to be at 200000 for all statistical parameters. Similarly, using PCE method (refer Figure 6-42), the increase of polynomial order from 0 to 5 gives almost similar results for all statistical parameters convergence study. It also can be observed that increasing the polynomial order for more than 5, the magnitude of statistical parameters increases drastically which turns to be unreliable. Using the convergence study, the mentioned number of samples and polynomial order will be utilised onto the input design variables.

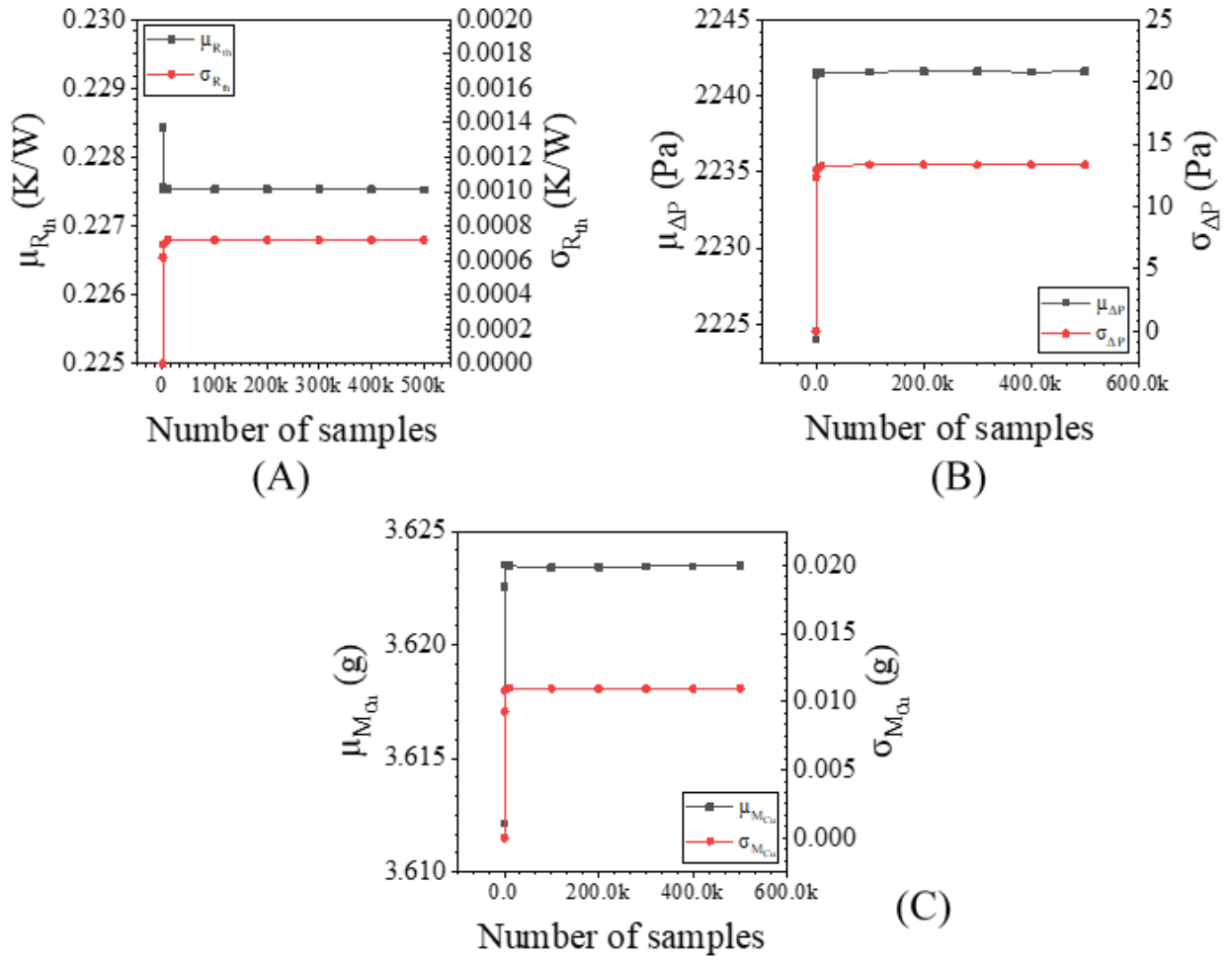


Figure 6-41: Shows the convergence plots using MCS method.

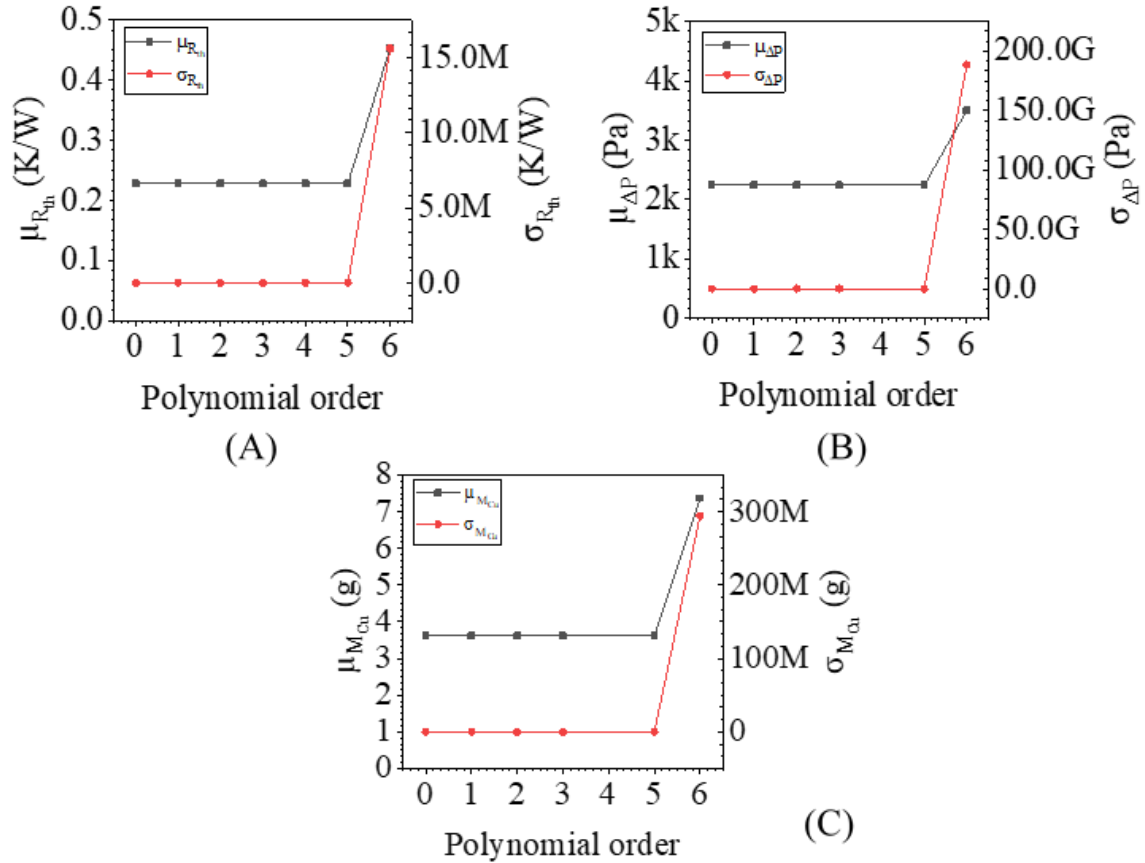


Figure 6-42: Shows the convergence plots using PCE method.

6.5.2.2.5 Comparison of results between MCS and PCE methods

To check the correlation between two different methods using MCS and PCE calculating the statistical moments of the output objective functions, the determination coefficient parameter (R^2) is used here (refer Eq. 5.18). The R^2 values calculated between the results of MCS and PCE methods are tabulated in Table 6-11 for all means and standard deviations data of the three objective functions pre-defined (refer subchapter 6.5.1). All R^2 values appear to be very close to unity indicating very high accuracy.

Table 6-11: Shows the R^2 values of each statistical moment objective function comparing MCS and PCE methods.

Data type	Statistical moments	R^2
Mean	$\mu_{R_{th}}$ (K/W)	0.999999997
	$\mu_{\Delta P}$ (Pa)	0.999990240
	$\mu_{M_{Cu}}$ (g)	0.999999999
Standard deviation	$\sigma_{R_{th}}$ (K/W)	0.999995182
	$\sigma_{\Delta P}$ (Pa)	0.999999999
	$\sigma_{M_{Cu}}$ (g)	0.999344196

6.5.2.2.6 Surrogate modelling for RDO

To map the relationship between the newly constructed DOE points having the same input design variables under subchapter 6.5 with the output objective functions which has been divided into statistical moments of mean and standard deviation, new RBF models are trained individually. Similarly, based on subchapter 6.5.1, the input design variables and output objective functions are normalised to improve the convergence rate of the RBF model training using Eq. 6.6. As mentioned earlier (refer subchapter 6.5.1), the three objective functions of total thermal resistance (R_{th}), pressure drop (ΔP) and mass of copper heat sink (M_{Cu}) are computed as means and standard deviations which are tabulated in Table 6-12. For the other training of RBF models of objective functions of statistical moments, the same procedures are applied. To check the accuracy of trained RBF models for statistical moments, it can be based on the pre-trained RBF models under subchapter 6.5.1 where the same surrogate models are used to calculate the statistical moments of means and standard deviations which are already been checked for accuracy under subchapters 6.5.1 and 6.5.1.1. From Table 6-13, the accuracy parameters of RMSE, R^2 and maximum residual error are recorded. The R^2 value is very close to unity while having very low magnitude of RMSE parameter indicating the trained RBF model is accurate. Similarly, other trained RBF models are checked to be accurate but the results are not included here.

Table 6-12: Shows the means and standard deviations of three objective functions.

Objective functions	mean	standard deviation
R_{th} (K/W)	$\mu_{R_{th}}$ (K/W)	$\sigma_{R_{th}}$ (K/W)
ΔP (Pa)	$\mu_{\Delta P}$ (Pa)	$\sigma_{\Delta P}$ (Pa)
M_{Cu} (g)	$\mu_{M_{Cu}}$ (g)	$\sigma_{M_{Cu}}$ (g)

Table 6-13: The accuracy parameters for training, validation and testing sets of surrogate model of $\sigma_{R_{th}}$ objective function using RBF.

$\sigma_{R_{th}}$			
Accuracy parameter	Training set	Validation set	Testing set
RMSE (K/W)	7.454198×10^{-5}	1.360912×10^{-4}	1.014006×10^{-4}
R^2	0.999998	0.992212	0.999882
Maximum Residual Error (%)	0.590453	1.266542	0.845111

6.5.2.2.7 Multi-objective robust design optimisation on single objective function

To perform robust design optimisation, the same procedure used like under subchapter 6.5.2.1.2 where the surrogate models that have been trained will be fed into optimisation algorithm of NSGA-II for optimisation process. Based on the mean and standard deviation surrogate models with the same unit will be optimised simultaneously to search for competing objectives and finding the robust design points which has the least sensitivity of mean towards its corresponding standard deviation. The constraint of the normalised standard deviation of output parameter can also give different range of objective functions values introducing robust design. The optimisation under robust framework can be formulated as follows [317]:

$$\begin{aligned} &\textbf{Minimise } \mu_{R_{th}} \text{ and } \sigma_{R_{th}} \\ &\textbf{such that } \sigma_{R_{th},norm} < \epsilon_1 \end{aligned} \quad (6.13)$$

$$\begin{aligned} &\textbf{Minimise } \mu_{\Delta P} \text{ and } \sigma_{\Delta P} \\ &\textbf{such that } \sigma_{\Delta P,norm} < \epsilon_2 \end{aligned} \quad (6.14)$$

$$\begin{aligned} &\textbf{Minimise } \mu_{M_{Cu}} \text{ and } \sigma_{M_{Cu}} \\ &\textbf{such that } \sigma_{M_{Cu},norm} < \epsilon_3 \end{aligned} \quad (6.15)$$

To analyse the single objective multi-objective optimisation of robust design framework, the results of optimisation are shown where the corresponding Pareto fronts for the means and standard deviations of the same units are plotted in Figure 6-43 to Figure 6-45. From Figure 6-43, the Pareto front plot and DOE points are plotted together based on the $\mu_{R_{th}}$ and $\sigma_{R_{th}}$ statistical moments. From the figure, it can be seen that competing objectives on the Pareto front shows that having an increase of $\sigma_{R_{th}}$ will results in the decrease of $\mu_{R_{th}}$ and vice versa. The numerical data from the Pareto front of $\sigma_{R_{th}}$ vs $\mu_{R_{th}}$ are tabulated in Table 6-14 where it can be seen that going down the table $\mu_{R_{th}}$ increases from 0.126 K/W to 0.130 K/W (3.3% change) while $\sigma_{R_{th}}$ decreases from 0.000100 K/W to 0.0000145 K/W (85.5% change). Small change in $\mu_{R_{th}}$ leads to very large change in $\sigma_{R_{th}}$. Another observation to be made, on the Pareto front where large $\sigma_{R_{th}}$ will have very small $\mu_{R_{th}}$ indicating sharp curve distribution with steep gradient on the output result and vice versa.

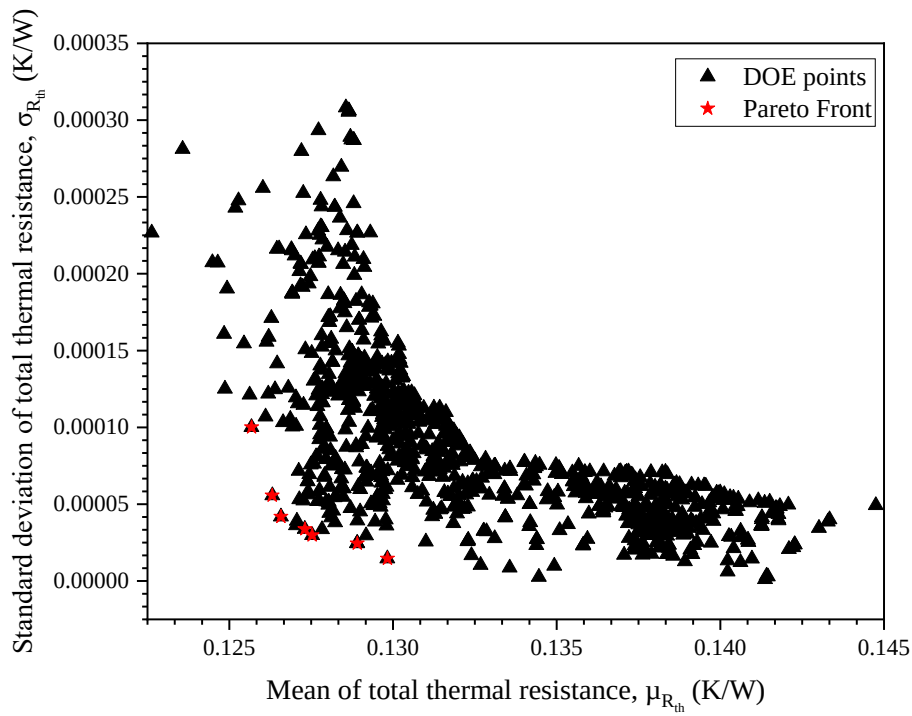


Figure 6-43: Shows the Pareto front and DOE points plots for $\sigma_{R_{th}}$ vs $\mu_{R_{th}}$.

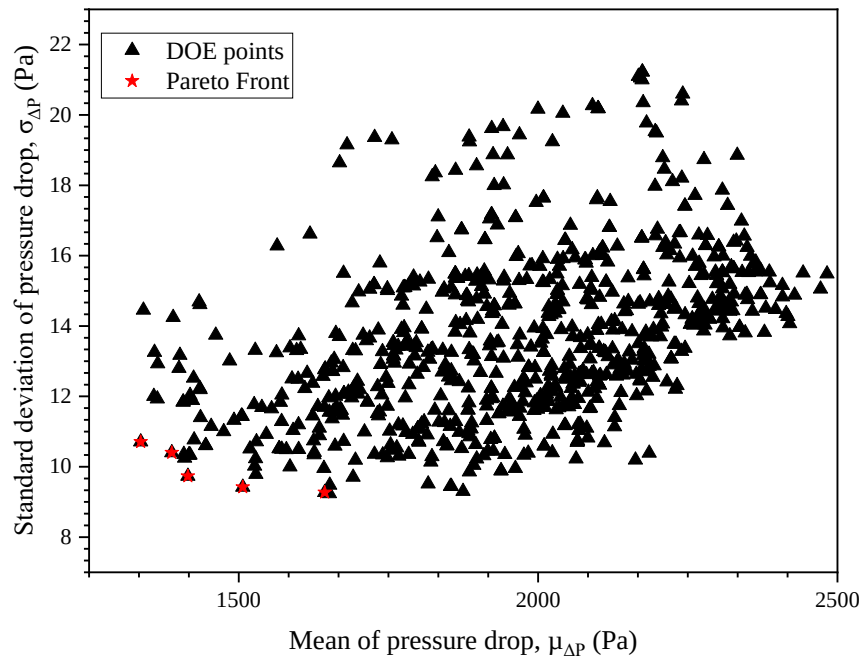


Figure 6-44: Shows the Pareto front and DOE points plots for $\sigma_{\Delta P}$ vs $\mu_{\Delta P}$.

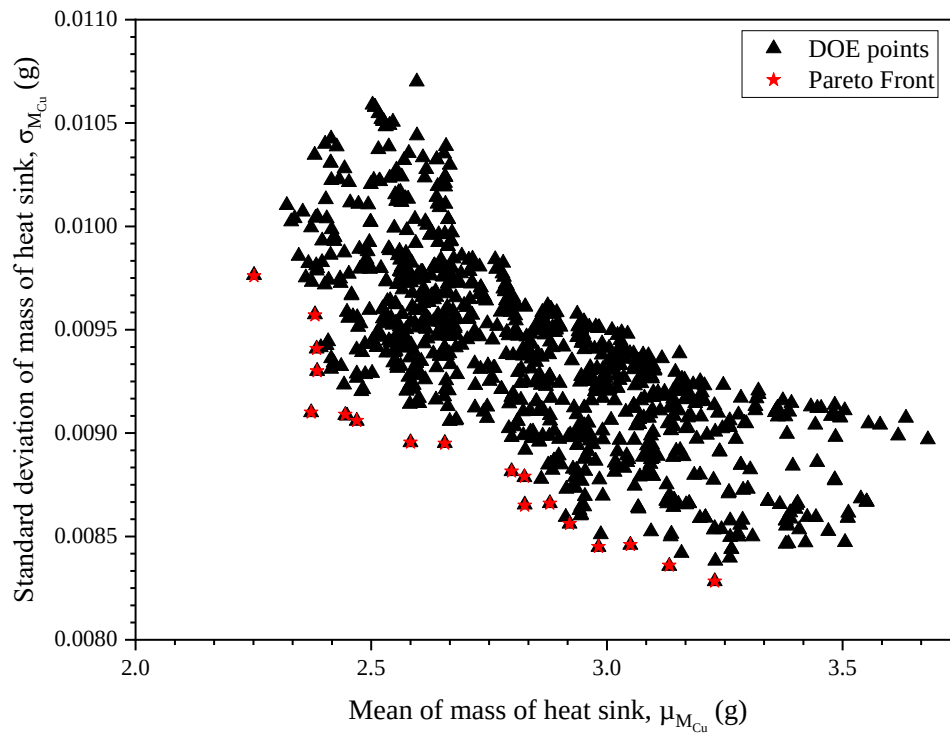


Figure 6-45: Shows the Pareto front and DOE points plots for $\sigma_{M_{Cu}}$ vs $\mu_{M_{Cu}}$.

Table 6-14: Shows the Pareto Front data for $\sigma_{R_{th}}$ vs $\mu_{R_{th}}$.

Design variables				$\mu_{R_{th}}$, K/W	$\sigma_{R_{th}}$, K/W
W_{ch} , mm	θ_{fin} , mm	H_{ch} , mm	H_b , mm		
0.07252	0.84171	0.10229	0.82410	0.12568	1.00048E-4
0.07252	0.83049	0.13887	0.87605	0.12631	5.56150E-5
0.02266	0.96409	0.95206	0.93058	0.12658	4.18339E-5
0.04317	0.95811	0.97997	0.89278	0.12731	3.37790E-5
0.03820	0.98836	0.98907	0.87872	0.12753	2.97069E-5
0.02322	0.73773	0.98949	0.77563	0.12891	2.43941E-5
0.04158	0.70381	0.97057	0.89278	0.12983	1.45447E-5

Table 6-15: Shows the Pareto Front data for $\sigma_{\Delta P}$ vs $\mu_{\Delta P}$.

Design variables				$\mu_{\Delta P}$, Pa	$\sigma_{\Delta P}$, Pa
W_{ch} , mm	θ_{fin} , mm	H_{ch} , mm	H_b , mm		
0.96116	0.09587	0.99251	0.92517	1336.3173	10.70815
0.97568	0.09861	0.96384	0.92517	1388.2130	10.40050
0.91374	0.04071	0.99062	0.92517	1415.0201	9.731830
0.88841	0.01604	0.97904	0.79502	1506.7360	9.418960
0.93462	0.06020	0.90058	0.92598	1643.6283	9.270790

Table 6-16: Shows the Pareto Front data for $\sigma_{M_{Cu}}$ vs $\mu_{M_{Cu}}$.

Design variables				$\mu_{M_{Cu}}$, g	$\sigma_{M_{Cu}}$, g
W_{ch} , mm	θ_{fin} , mm	H_{ch} , mm	H_b , mm		
0.95012	0.91122	0.02434	0.02846	2.25124	0.00976
0.94780	0.64204	0.06266	0.06759	2.38077	0.00957
0.65558	0.81376	0.04269	0.02727	2.38428	0.00941
0.86096	0.19400	0.08755	0.02463	2.38517	0.00930
0.90241	0.02920	0.10981	0.07350	2.44505	0.00909
0.97610	0.25270	0.09616	0.08194	2.46928	0.00906
0.96032	0.22944	0.12087	0.12837	2.58345	0.00895
0.96091	0.07703	0.20311	0.13377	2.65607	0.00895
0.94218	0.02426	0.26148	0.17258	2.79784	0.00882
0.96145	0.37331	0.11050	0.26737	2.82509	0.00879
0.92172	0.14863	0.21724	0.19690	2.82550	0.00865
0.96447	0.25270	0.20224	0.24056	2.87888	0.00866
0.96495	0.25270	0.23534	0.24056	2.92074	0.00856
0.86648	0.43926	0.23068	0.24977	2.98218	0.00845
0.95024	0.39994	0.19315	0.34044	3.04961	0.00846
0.96145	0.44288	0.35891	0.26737	3.13216	0.00836
0.95055	0.92733	0.11217	0.48625	3.22884	0.00828

Similarly, the Pareto fronts for $\sigma_{\Delta P}$ vs $\mu_{\Delta P}$ and $\sigma_{M_{Cu}}$ vs $\mu_{M_{Cu}}$ show the same trend where an increase in one objective function would lead to a decrease in another and vice versa. For $\sigma_{\Delta P}$ vs $\mu_{\Delta P}$, decreasing Table 6-15 the $\mu_{\Delta P}$ experience an increase of 23% changing from 1336.300 Pa to 1643.6 Pa while the $\sigma_{\Delta P}$ decrease by 13% going from 10.7 Pa to 9.3 Pa. For $\sigma_{M_{Cu}}$ vs $\mu_{M_{Cu}}$, going down Table 6-16 the $\mu_{M_{Cu}}$ increases from 2.250 g to 3.230 g with 43% change while $\sigma_{M_{Cu}}$ changes from 0.00976 g to 0.00828 g with 15.2% decrease.

To compare the solutions of multi-objective optimisation of single objective function with the single-objective deterministic optimisation, comparison can be made with tabulated deterministic results in Table 6-6. From the results related to R_{th} , deterministically the lowest value of R_{th} occurs at 0.123 K/W. Based on the robust optimisation results of $\sigma_{R_{th}}$ vs $\mu_{R_{th}}$, the Pareto front values of $\mu_{R_{th}}$ (refer Table 6-14) shows an increase in magnitudes compared with the deterministic result ranging from 2.25% to 5.63% change. The robust results have slightly higher magnitude compared with deterministic results but enabling designers to choose different sets of input design variables meeting the condition of manufacturing tolerances and also having output objectives which insensitive towards the change in input data. Similarly, for results related to ΔP , the deterministic result lowest magnitude of 1277.9 Pa. Using the results of robust optimisation of $\sigma_{\Delta P}$ vs $\mu_{\Delta P}$, the values of $\mu_{\Delta P}$ on Pareto front recorded on Table 6-15, an increase in value between 4.57% and 28.62% of the robust optimisation results allowing variation of different of input design variables to be chosen with insensitive output results. For the M_{Cu} results, deterministically lowest value of M_{Cu} is at 2.179 g. The values of $\mu_{M_{Cu}}$ based on Table 6-16 shows an increase in mass compared to the deterministic solution with range of 3.31% and 48.17% change giving flexibility to make choices of input design variables with insensitive output results.

6.5.2.2.7.1 Multi-objective robust design optimisation on single objective optimisation with constrained output standard deviations

To further analyse the robust optimisation study, from pre-defined formulations of Eq. 6.13, 6.14 and 6.15, the constraints are varied to observe the effect on the performance of multi-objective optimisation of single objective function using Pareto Front plots. The plots are shown in Figure 6-46 to Figure 6-48 respectively. For R_{th} parameter, referring Figure 6-46, having constraint of $0.0 < \sigma_{R_{th},norm} < 1.0$ shows the closest Pareto front to the origin having the lowest values of both $\sigma_{R_{th}}$ and $\mu_{R_{th}}$ parameters respectively. Constraining the $0.0 < \sigma_{R_{th},norm} < 0.7$, shows shifting of the Pareto front plot to the right inducing higher $\mu_{R_{th}}$. Similarly, for $0.0 <$

$\sigma_{R_{th},norm} < 0.2$, higher $\mu_{R_{th}}$ values are recorded. For constraint $0.4 < \sigma_{R_{th},norm} < 0.7$, the rate of increase of $\mu_{R_{th}}$ is slower compared to constraint $0.5 < \sigma_{R_{th},norm} < 1.0$ across the Pareto fronts. The shifting of Pareto fronts when the constraints are introduced is simply because of new sets of limits are applied to the input design variables thus constraining the surrogate models to perform less effective.

For ΔP parameter, referring Figure 6-48, $0.0 < \sigma_{\Delta P,norm} < 1.0$ constraint has the lowest $\sigma_{\Delta P}$ and $\mu_{\Delta P}$ values. Constraining at $0.0 < \sigma_{\Delta P,norm} < 0.7$ and $0.0 < \sigma_{\Delta P,norm} < 0.2$ show slight shifting of Pareto front plot to the right with almost equal magnitudes. At constraints of $0.4 < \sigma_{\Delta P,norm} < 0.7$ and $0.5 < \sigma_{\Delta P,norm} < 1.0$, the Pareto fronts are recorded to be very far from the origin with a large increase in both $\sigma_{\Delta P}$ and $\mu_{\Delta P}$ respectively showing massive deterioration in ΔP performance. This is due to the new limits of input design variables corresponds to very large ΔP .

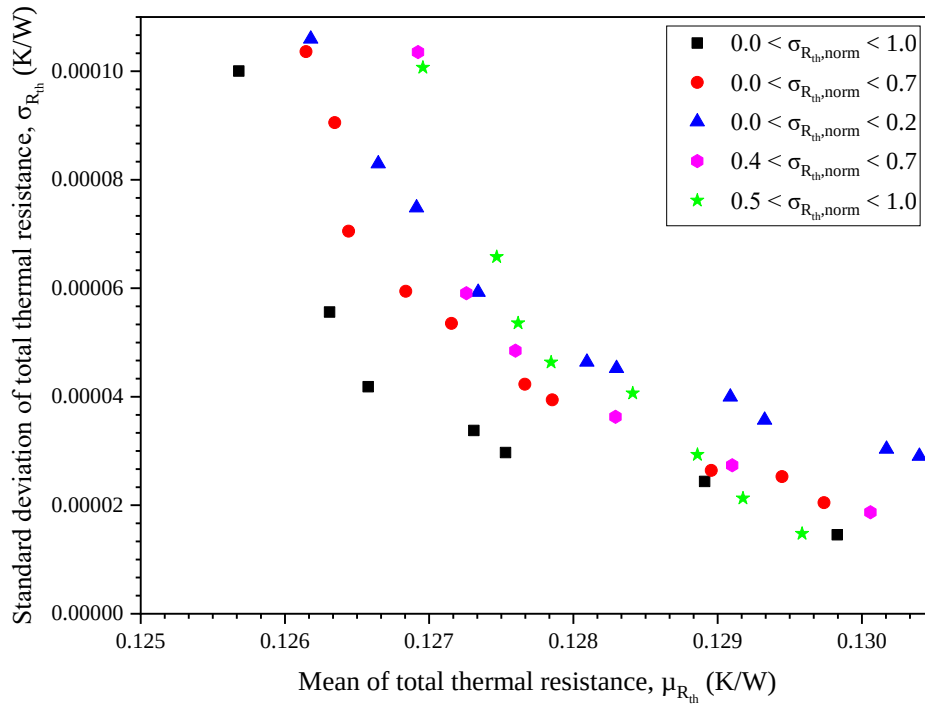


Figure 6-46: Shows the Pareto front of $\sigma_{R_{th}}$ vs $\mu_{R_{th}}$ for different constraints of $\sigma_{R_{th},norm}$

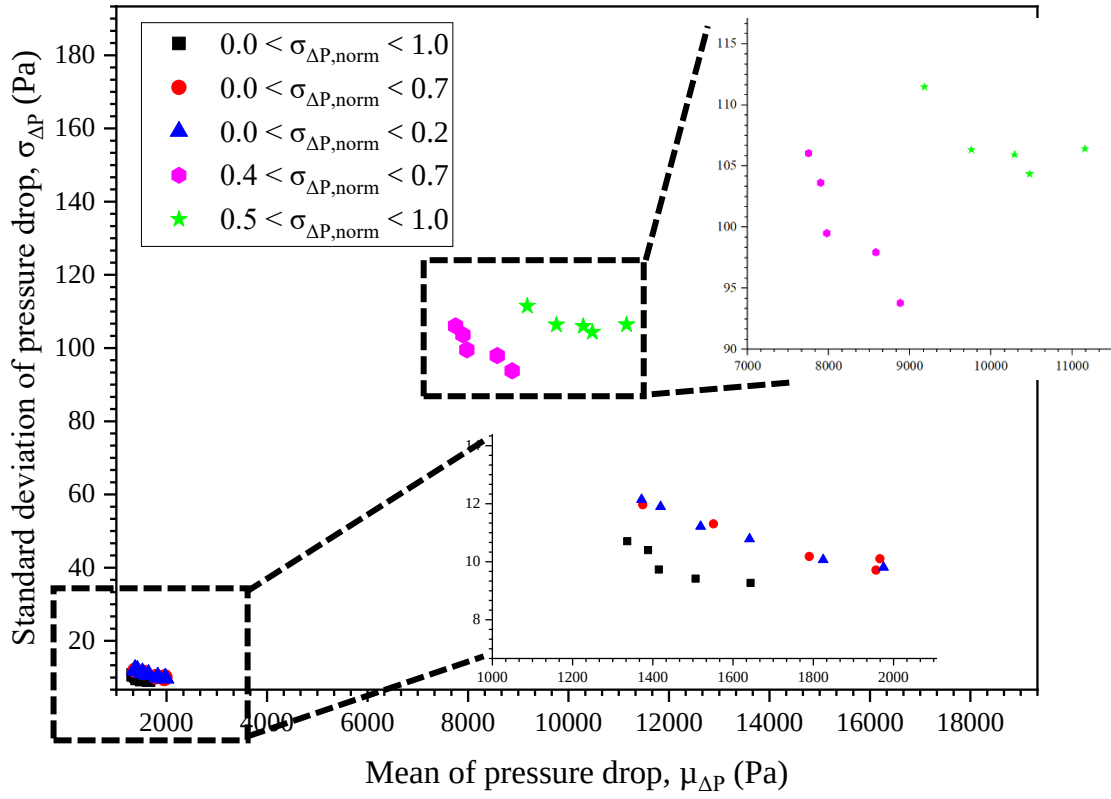


Figure 6-47: Shows the Pareto front of $\sigma_{\Delta P}$ vs $\mu_{\Delta P}$ for different constraints of $\sigma_{\Delta P, norm}$.

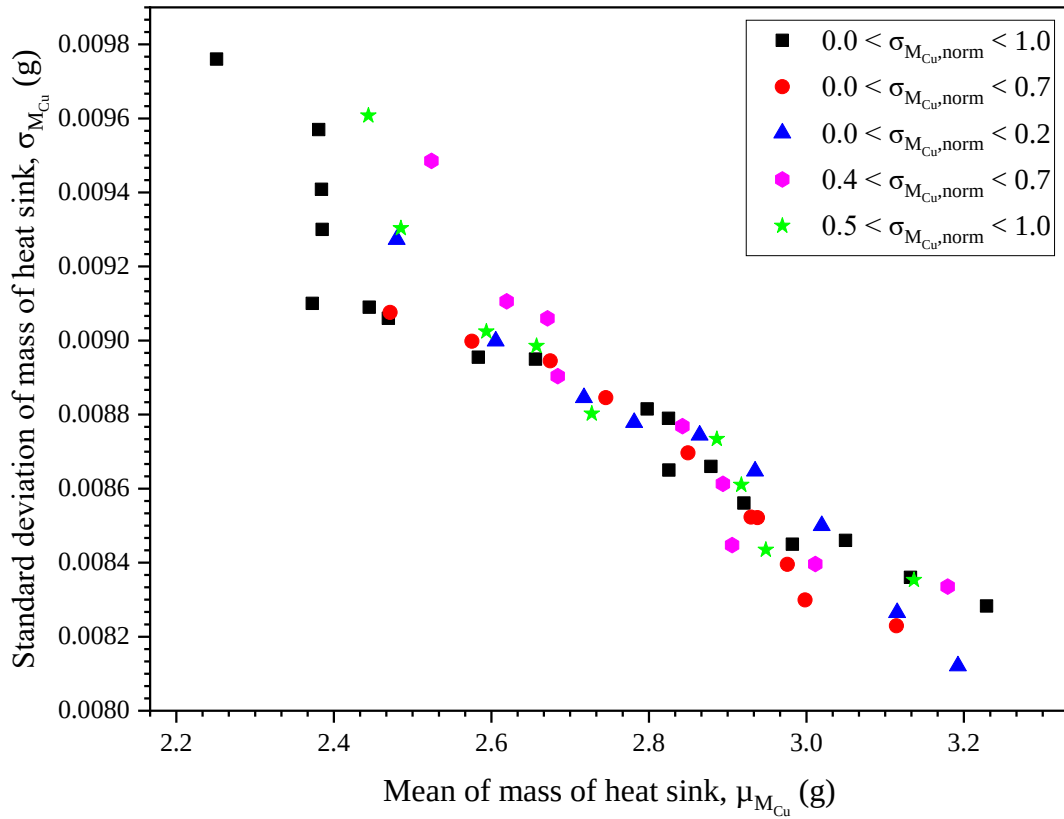


Figure 6-48: Shows the Pareto front of $\sigma_{M_{Cu}}$ vs $\mu_{M_{Cu}}$ for different constraints of $\sigma_{M_{Cu}, norm}$.

For M_{Cu} parameter, referring Figure 6-48, constraining at $0.0 < \sigma_{M_{Cu},norm} < 1.0$, $0.0 < \sigma_{M_{Cu},norm} < 0.7$, $0.0 < \sigma_{M_{Cu},norm} < 0.2$, $0.4 < \sigma_{M_{Cu},norm} < 0.7$ and $0.5 < \sigma_{M_{Cu},norm} < 1.0$ respectively do not differ in terms of Pareto Front plots. This is due to the fact that the new limits of input design variables for constrained $\sigma_{M_{Cu},norm}$ are very close to each other inducing very much similar Pareto front magnitudes and trends.

6.5.2.2.8 Multi-objective robust design optimisation on multi-objective functions with constrained output standard deviation.

Based on the analysis conducted under subchapter 6.5.2.2.7, only single objective function was considered based on a multi-objective optimisation on the statistical moments. Thus, for this subchapter the multi-objective optimisation of multiple objective functions are considered with constraints of $\sigma_{R_{th},norm}$, $\sigma_{\Delta P,norm}$ and $\sigma_{M_{Cu},norm}$ included to see its behaviour on to the optimisation results based on the Pareto front plots.

From Figure 6-49, the Pareto fronts at different constraints for ΔP vs R_{th} are plotted. It can be observed that the Pareto fronts at deterministic solution and constraint combination of (A) the same Pareto front magnitudes and trends overlapping with each other since the combinations of input design variables change insignificantly. For constraints of (B) and (C), the Pareto fronts shifted upwards into the region of ΔP between 15000 Pa to 27500 Pa due to the constriction of H_{ch} and W_{ch} . Similarly, at constraint (D), the increase in ΔP is the highest showing the largest shifting of Pareto front upwards because of even smaller H_{ch} and W_{ch} . The R_{th} does not shift so much due to the θ_{fin} compensation maintaining the heat transfer.

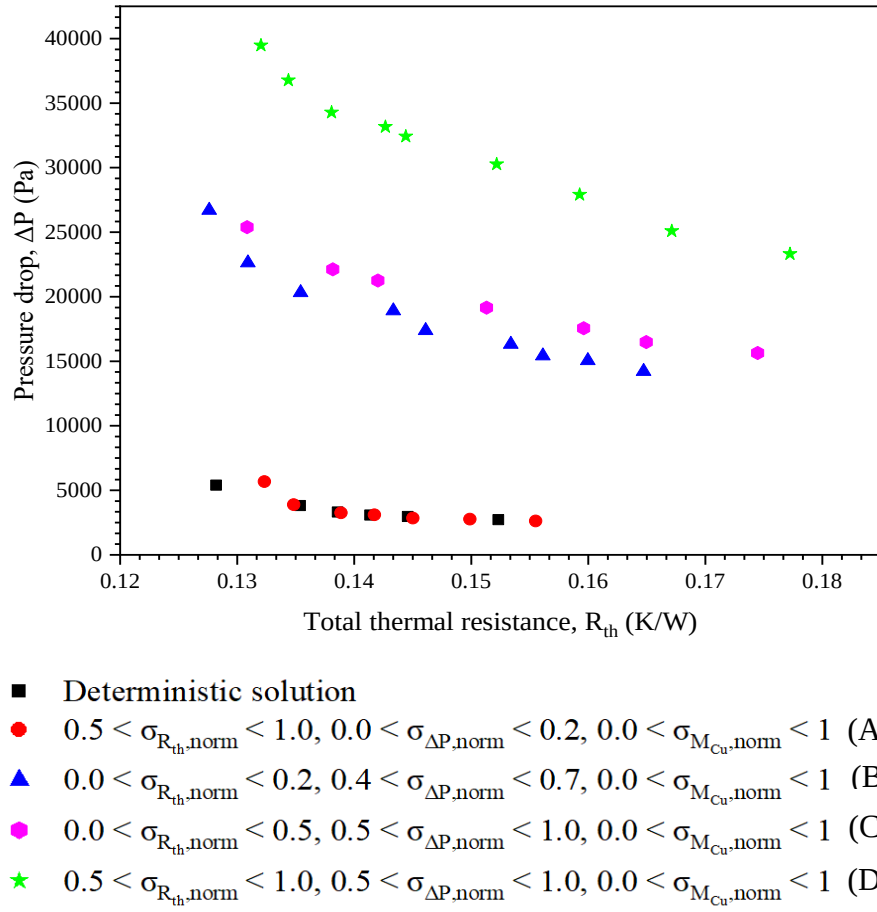


Figure 6-49: Shows the Pareto fronts plot of ΔP vs R_{th} for different values of constraints of $\sigma_{R_{th,norm}}$, $\sigma_{\Delta P,norm}$ and $\sigma_{M_{Cu,norm}}$ parameters respectively.

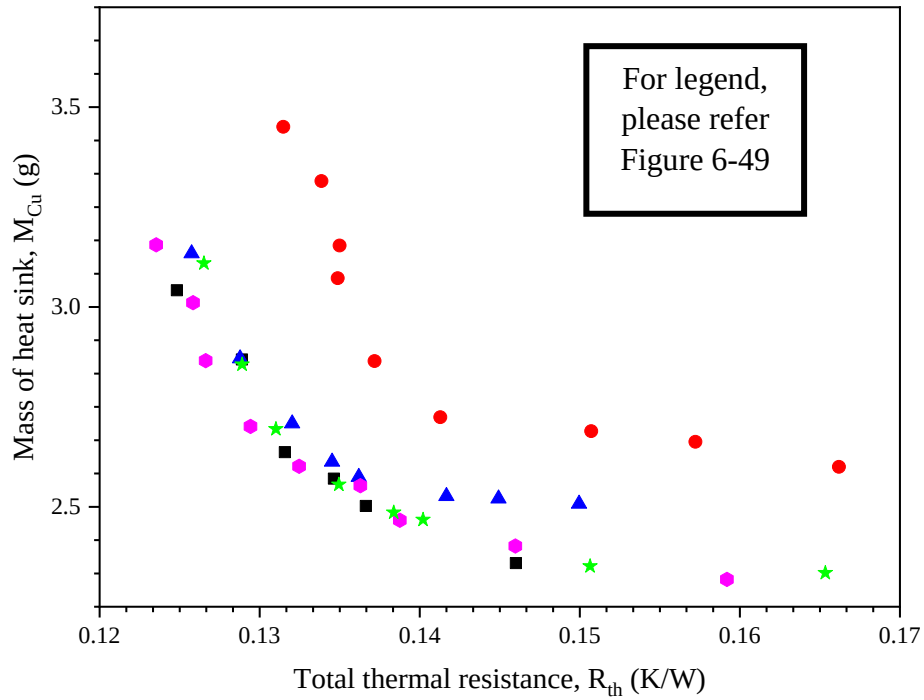


Figure 6-50: Shows the Pareto fronts plot of M_{Cu} vs R_{th} for different values of constraints of $\sigma_{R_{th,norm}}$, $\sigma_{\Delta P,norm}$ and $\sigma_{M_{Cu,norm}}$ parameters respectively.

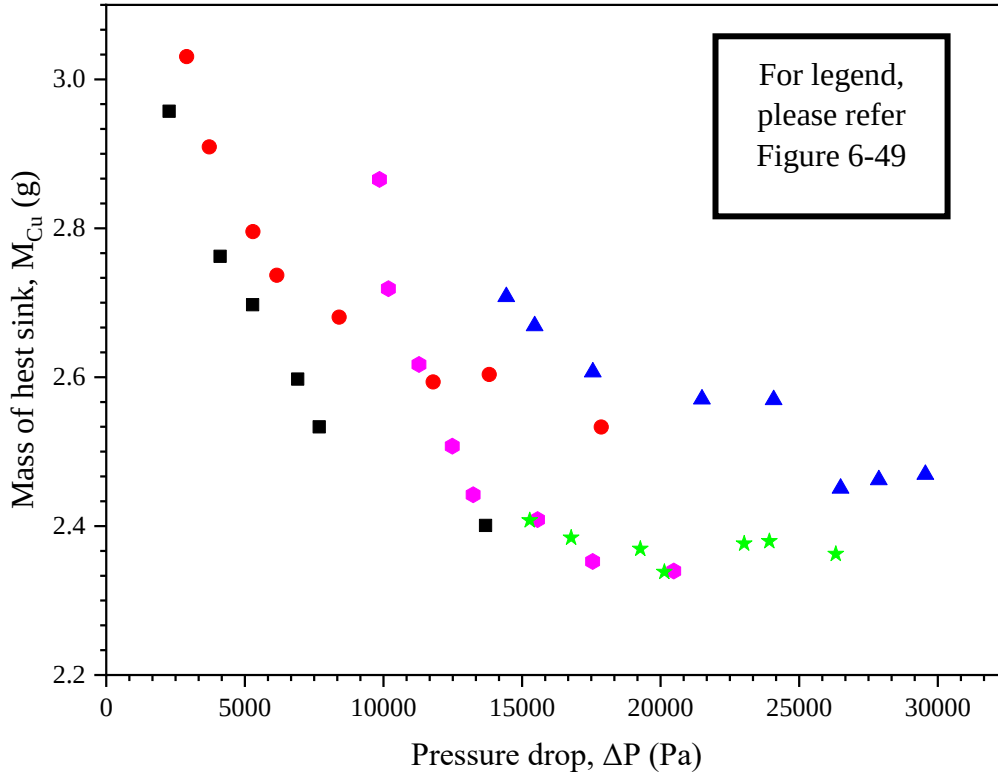


Figure 6-51: Shows the Pareto fronts plot of M_{Cu} vs ΔP for different values of constraints of $\sigma_{R_{th,norm}}$, $\sigma_{\Delta P,norm}$ and $\sigma_{M_{Cu,norm}}$ parameters respectively.

From Figure 6-50, the deterministic solution shows the same Pareto front plot when compared with Pareto fronts with constraints of (C) and (D) respectively. At constraint of (B), the Pareto front almost has the same trend compared to the constraints of (C) and (D) but with higher increase rate of R_{th} . For constraint of (A), the Pareto front plots shifted upwards showing increase of M_{Cu} , this is due to the A_{eff} does not change so much on this Pareto front but the range H_{ch} and W_{ch} are larger to increase the M_{Cu} .

Looking at Figure 6-51, to compare with the Pareto front of the deterministic solution, at constraint of (A) the Pareto front plot shifted upwards due to the increase in H_{ch} and W_{ch} sizes but θ_{fin} aids to maintain the ΔP . On the other hand, at constraint of (C), the Pareto front shifted to the right, while at constraint (B) is even further caused by the enlargement of θ_{fin} but does not affect the M_{Cu} insignificantly. At constraint of (D), the Pareto front located at the most bottom and right among the plots mainly because very small H_{ch} and W_{ch} sizes effecting the M_{Cu} to be lower.

6.6 Summary

The use of liquid cooling for removing high heat flux for electronics cooling application has been an interest by many researchers. Within this chapter, the use of secondary channels from fins embedded within single inlet-outlet serpentine mini-channel heat sink have been employed for disrupting both thermal and hydrodynamic boundary layers by displacing a portion of fluid through it from the main channel reducing both total thermal resistance and pressure drop simultaneously as well as increasing coefficient of heat transfer. Three major shapes of fin have been used which are oblique, trapezium and diamond and each shape holds four different configurations for testing.

From the results obtained, total thermal resistance (R_{th}) experiences monotonic decrease when liquid velocity increases since heat transfer through convection enhances for all fins shapes and configurations. Generally, serpentine mini-channel heat sinks with diamond shaped fins have the lowest magnitudes of R_{th} since more frequency of breaking of boundary layers occurring and flow re-initialisations. The serpentine mini-channel heat sink with diamond fins of configuration SMDF-C has the lowest R_{th} with 14% reduction compared to the SMPF heat sink. In terms of Average Nusselt Number (Nu_{avg}), SMDF-C configuration has the highest increase with 23.4% change when compared to the SMPF heat sink. In terms of pressure drop (ΔP), trapezoidal shaped fin with configuration of SMTF-B has the highest reduction with 83.3% change with reference to the SMPF heat sink. Generally, the serpentine mini-channel heat sink with fins experience great reduction of ΔP compared to the SMPF heat sink since portions of liquid are displaced through the secondary channel reducing the velocity of liquid within the main channel greatly. Taking into account of heat transfer and hydraulic components of heat sink, Performance Evaluation Criteria (PEC) study was conducted. From the PEC analysis, configuration of SMDF-D holds the highest value followed closely by SMDF-C configuration, but to further for parametric analysis configuration of SMDF-C is chosen since its holds the lowest R_{th} which is more essential towards the functionality of heat sink.

Changing the parameter of secondary channel length (l_{sc}) from 1.0 mm to 0.7 mm will lead to the reduction of R_{th} by 26 % (due to increased effective surface area, A_{eff}) and increase of ΔP by 37% since l_{sc} is smaller inducing higher velocity of liquid. When the fin angle (θ_{fin}) increases from 25° to 55° , the R_{th} decreases with 36.5% change due to A_{eff} increases and ΔP increases as secondary channel width (W_{sc}) decreases reducing liquid displacement through secondary channel. Next, increasing the value of height of channel (H_{ch}) from 0.75 mm to 1.5 mm lead to the decrease in both R_{th} by 36.8% due to the increase of A_{eff} and ΔP by 51.4% since the channel

cross sectional area is getting larger reducing fluid velocity. For the width of channel (W_{ch}) parameter, there is an 18% reduction in R_{th} changing from 1.0 mm to 1.3 mm because A_{eff} increases while ΔP also reduces with 10.8% change since liquid velocity decreases going through larger channel. Lastly, increasing the heat flux (q'') from 50 W/cm² to 100 W/cm² would results in the increase of maximum surface temperature ($T_{surf,max}$) by 11.4% change.

Two different frameworks of deterministic and robust design optimisation are considered. For the single-objective deterministic optimisation very low objective functions of R_{th} , ΔP and M_{Cu} can be achieved individually but with the expense of very high of other objectives. This makes the heat sink to be inefficient and heavier at certain design points. Thus, multi-objective deterministic optimisation is carried out to solve this issue by creating Pareto fronts which compromise between conflicting objectives searching for low values of all objectives at the same time. Multi-objective optimisation for two objectives at a time are tested first. Looking at Pareto front of ΔP vs R_{th} , increasing the R_{th} by 18.8% lead to the decrease of ΔP by 49.5%. Similarly, for M_{Cu} vs R_{th} Pareto front, the increase of R_{th} by 17% would lead to the decrease of M_{Cu} by 22.4%. Lastly, for Pareto front of M_{Cu} vs ΔP , the decrease of M_{Cu} by 18.8%, the ΔP increases by five folds. For the multi-objective deterministic optimisation for three objective functions altogether, moving towards the zero value of Cartesian plane, both ΔP and R_{th} reduces while the M_{Cu} increases.

Next, multi-objective robust design optimisation studies are carried out. The input design variables are subjected to a specified uncertainties corresponding to distributions of output enabling for the statistical moments of means and standard deviations can be calculated. Similarly, Pareto front is used to visualise the conflicting objectives to choose from. The robust design optimisations are carried based on multi-objective optimisation of single objective function and multi-objective function. Furthermore, the effect of constraining the output standard deviations towards the performance of Pareto fronts are carried out enabling the designers to choose the design points which are not as good as deterministic solution but has just slightly lower performance and insensitive towards the change of input design variables. The constraints are chosen based on the location either towards to the smallest, intermediate or largest magnitude so that throughout the dataset, the trends of Pareto fronts can be captured.

Chapter 7 : Design optimisation of double inlet-outlet serpentine mini-channel heat sink with diamond fins (D-SMDF)

7.1 Introduction

The use of single inlet-outlet serpentine mini-channel heat sink has been widely researched in the literature since it offers better thermal performance and temperature distribution compared to conventional straight channel heat sink. But, with only single inlet-outlet configuration, the thermal improvement of serpentine mini-channel heat sink can only reach a certain best performance. Thus, under this chapter, the use of double inlet-outlet serpentine mini-channel heat sink is utilised for better thermal performance. From the literature, the use of double inlet-outlet serpentine mini-channel heat sink has been utilised on a few papers [37, 143] who considered different configurations to test the thermal-hydraulic performance and very superior thermal performance achieved compared to the single inlet-outlet serpentine mini-channel heat sink. In this study, the work by Cao et al. [37] is adapted with modifications from their original design improving the thermal-hydraulic performance of heat sink. They have used double inlet-outlet serpentine mini-channel heat sink with D-SMPF configuration and application of through-holes. To modify the original design from [37] for this work, D-SMPF heat sink design is being used with the application of secondary channels from embedded diamond fins (D-SMDF) as novel design since the fin shape holds greater thermal performance of serpentine mini-channel heat sink (refer Chapter 6) but without the through-holes application. Within this chapter, the novel heat sink design detailing is presented. The COMSOL Multiphysics v5.4 CFD software is used to solve the numerical simulations of the heat sink models. The parameters used to measure the performance of tested heat sinks are total thermal resistance (R_{th}), pressure drop (ΔP), Average Nusselt Number (Nu_{avg}) and Performance evaluation Criteria (PEC) respectively. Next, design optimisation process will be conducted to optimise the novel design heat sink.

7.2 Description of design of double inlet-outlet serpentine mini-channel heat sink with diamond fins

In this work, the double inlet-outlet serpentine mini-channel heat sink with D-SMPF configuration with the application of diamond fins and no through-holes will be utilised. The design of heat sink for numerical analysis will be created using geometry tools available in COMSOL Multiphysics v5.4. To compare the performance of novel design with the original design in [37] fairly, the same base design will be used with equal channel width (W_{ch}), channel height (H_{ch}), fin width (W_w), area of base of heat sink ($W \times L$) and substrate thickness (H_b) respectively. The material for heat sink will be copper and water is used as working fluid. For the D-SMDF heat sink, only two diamond fins will be placed evenly on individual plate fin and symmetrically mapped. Based on Suga and Aoki [305] recommendation, louver angles are set to between 20° to 45° in order to have the balance between both of heat transfer and pressure drop where it is adapted in this work to make a fair comparison between heat sink with and without fins. Therefore, the fin angle (θ_{fin}) of the diamond fins embedded will have 45° magnitude which is within range of proposed values. The secondary channel length (l_{sc}) of fins is set to be 0.7 mm. The diagram representation of D-DMPF and D-SMDF heat sinks are shown in Figure 7-1 and Figure 7-2 respectively and dimensions in Table 7-1.

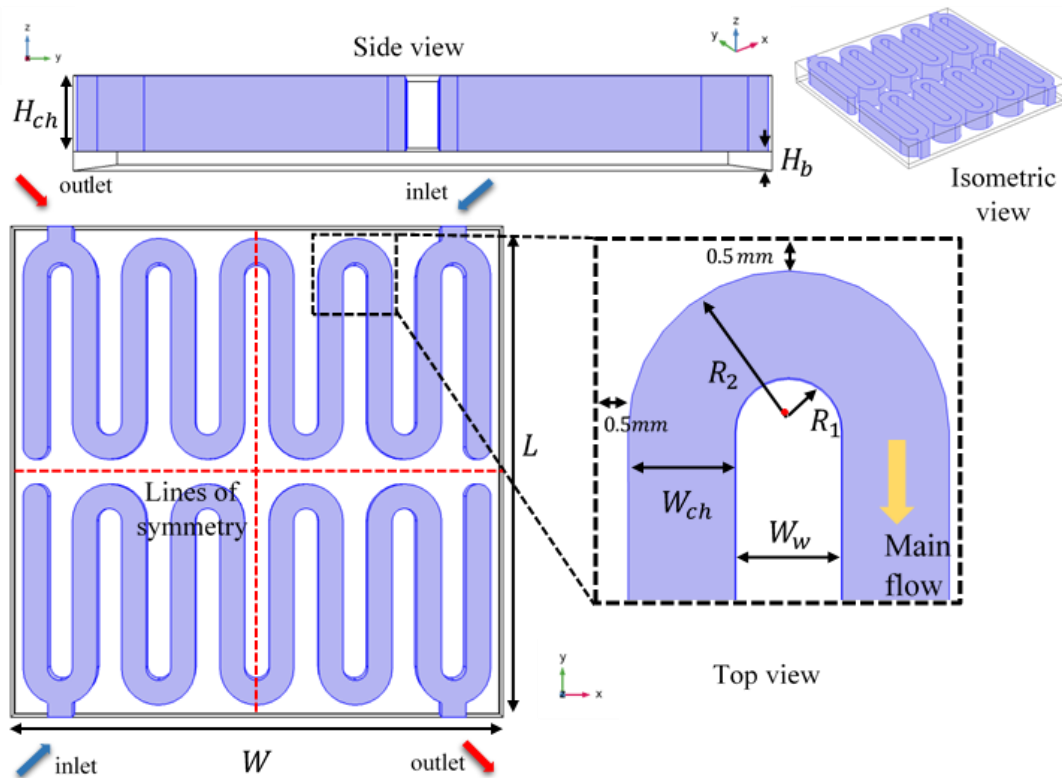


Figure 7-1: Shows the detailed dimensioning of the D-SMPF heat sink design from [37].

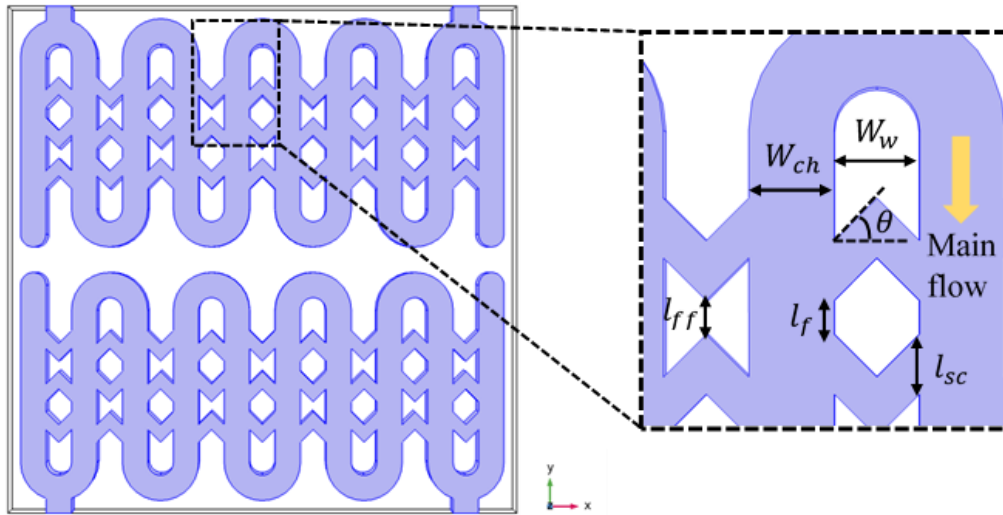


Figure 7-2: Shows the detailed dimensioning of the D-SMDF heat sink.

Table 7-1: Shows the tabulated data for dimensions of D-SMPF and D-SMDF heat sinks.

Characteristics	D-SMPF	D-SMDF
Material	Copper	
Heat sink dimensions, width x length x height, $W \times L \times H$ (mm)	20 x 20 x 2.5	
Main channel width, W_{ch} (mm)	1	
Fin Width, W_w (mm)	1	
Channel depth, H_{ch} (mm)	2	
Substrate thickness, H_b (mm)	0.5	
Number of mini-channels, n	10	
Hydraulic diameters, H_b (mm)	1.333	
Outer radius, R_1 (mm)	1.5	
Inner radius, R_2 (mm)	0.5	
Number of fins per column, N_{cf}	-	2 (each side)
Secondary channel width, W_{sc} (mm)	-	0.495
Secondary channel length, l_{sc} (mm)	-	0.7
fin length, l_f (mm)	-	0.4
fin pitch, p_f (mm)	-	1.6
second fin length, l_f (mm)	-	0.4
Fins oblique angle, θ_{fin} (deg)	-	45

Effective area (A_{eff}) is the contact area where the transfer of heat moving from the solid to the liquid domain. The A_{eff} comprises the area of the walls (A_{wall}) and bases (A_{base}) within the main and secondary mini-channels and the top wall is considered adiabatic (no heat transfer). Similarly, the available area can be calculated based on the following formula:

$$A_{eff} = A_{base} + A_{wall} \quad (7.1)$$

To calculate the A_{eff} for D-SMPF and D-SMDF heat sink, refer appendix A 3.1 for full equations.

7.3 CFD simulation approach

The simulations of all CFD models are conducted through the COMSOL Multiphysics v5.4 software. Under this chapter, the boundary conditions, assumptions and governing equations are presented as well as code validation and grid independence studies.

7.3.1 Governing Equations

Refer sub-chapter 6.3.1 and 6.3.2 for the process flow of solving the governing equations with the assumptions and boundary conditions.

7.3.3 Validation code with literature

To gain confidence in running CFD simulations, validation code study is carried out with published literature of Cao et al. [37]. They have investigated the performance of double inlet-outlet serpentine mini-channel heat sink with through-holes. The volumetric flow-rate is set to be between 0.053 l/min to 0.318 l/min corresponding from Re of 747 and 4482 respectively. A 62 W/cm^2 of heat flux is applied from underneath heat sink using two heating elements. The footprint of heat sink has dimension of $38 \text{ mm} \times 38 \text{ mm}$ with thickness of 2.5 mm . The comparison with the literature uses the plot of Average Nusselt Number (Nu_{avg}) against increasing pumping power (P_p) from the paper. Good correlation can be observed between published work and current simulation from Figure 7-3, thus, running further simulations can be conducted with high confidence.

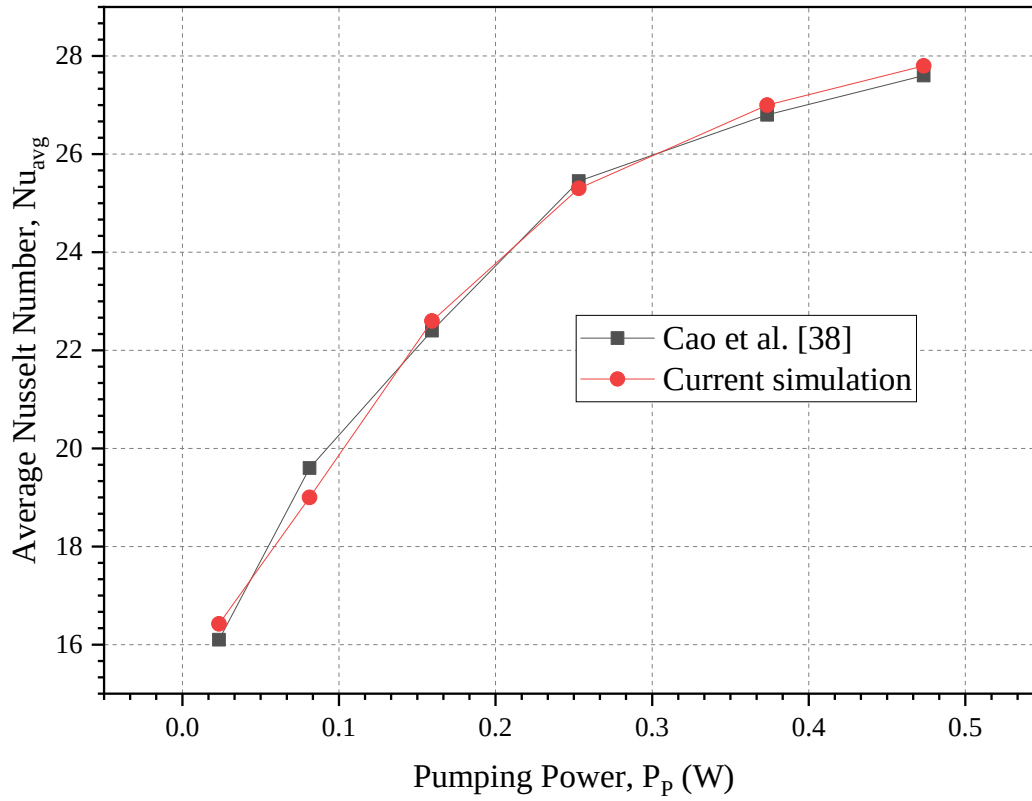


Figure 7-3: Shows the plot of Average Nusselt Number (Nu_{avg}) vs Pumping power (P_p).

7.3.4 Grid independence study

To run the simulations with the least time consumption while having high accuracy, the mesh independence grid number study is conducted. Four different grids are used with Grid 1 to Grid 4 has the lowest to the highest number of meshes. To choose which mesh number possesses acceptable accuracy, the lower number of meshes will be compared with the highest number of meshes using relative error formula which is as follows [209]:

$$e\% = \left| \frac{J_2 - J_1}{J_1} \right| \times 100 \quad (7.2)$$

Where physical parameters is denoted by the term J which can be physical quantities obtained from the heat sink such as total thermal resistance (R_{th}) and pressure drop (ΔP). All simulations are conducted at Re of 2300 and uniform heat flux of 100 W/cm^2 . Based on Table 7-2, the values of R_{th} and ΔP at different number of meshes are recorded. Referring to the data, both R_{th} and ΔP have Grid 3 with relative errors to be less than 2% compared to Grid 4 indicating the highest accuracy. Grid 3 will be used to further the simulations ensuring high accuracy while having the least computational time (refer appendix A 2.4 for mesh structure).

Table 7-2: Shows the R_{th} and ΔP at different mesh number for grid independence test.

Heat sink model	Grid No.	Elements number	R_{th} (K/W)	R_{th} -E%	ΔP (Pa)	ΔP -E%
D-SMPF	1	704074	0.10843	3.56	30132	8.1
	2	1034678	0.110599	1.63	29385	5.4
	3	1536266	0.111683	0.66	28050	0.6
	4	2086576	0.11243	-	27879	-
D-SMDF	1	3455049	0.079235	3.18	19167	7.5
	2	4450827	0.079933	2.33	18736	5.1
	3	7338971	0.080743	1.34	18067	1.3
	4	8548526	0.081837	-	17828	-

7.4 Characteristics of fluid flow and heat transfer using double inlet-outlet serpentine mini-channel heat sink with diamond fins

7.4.1 Effect of Reynolds number (Re) on R_{th} , Nu_{avg} and ΔP

To compare the performance of D-SMDF (novel design) with the previous design studied by Cao et al. [37], three different parameters are utilised which are the Average Nusselt Number (Nu_{avg}), total thermal resistance (R_{th}) and pressure drop (ΔP) respectively with Reynolds number (Re) ranges between 500 and 2300. The hydraulic diameter (D_h) is calculated from channel height (H_{ch}) and channel width (W_{ch}) of 2 mm and 1 mm respectively which corresponds to $D_h = 1.333$ mm (refer Eq. 2.1). A heat flux is uniformly applied underneath of heat sink with 100 W/cm^2 in magnitude. The diamond fins are evenly distributed throughout the plate fin with two fins per plate. Based on the design proposed by [37], the D-SMTH 'A' design will have through-holes diameter of 1 mm.

Referring to Figure 7-4, for all plots, as the Re increases the R_{th} is monotonically decreasing. Since the values of R_{th} is calculated using $T_{surf,max}$ parameter (refer Eq. 3.53) which also decreases at increasing Re . When the Re increases, indicating the liquid velocity also increase which inducing the convective heat transfer to be greater [307].

Referring to Figure 7-5, the increase in Re leads to the increase of Nu_{avg} for all plots. Heat transfer by convection increases with Re as mentioned earlier, therefore, the heat transfer coefficient (h_{avg}) also increases leading to the increase of Nu_{avg} (refer Eq. 3.24). As can be observed in the figure, the D-SMDF and D-SMPF plots moves further away from D-SMTH 'A' plot at increasing Re due to the reason of both $T_{w,avg}$ and $T_{f,avg}$ are monotonically decreasing.

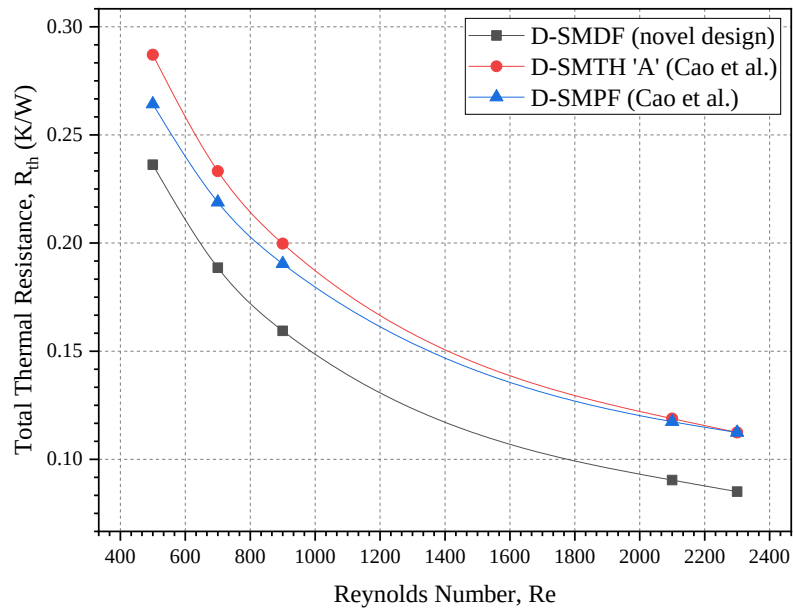


Figure 7-4: Total thermal resistance (R_{th}) against Re for different designs of double inlet-outlet serpentine mini-channel heat sink.

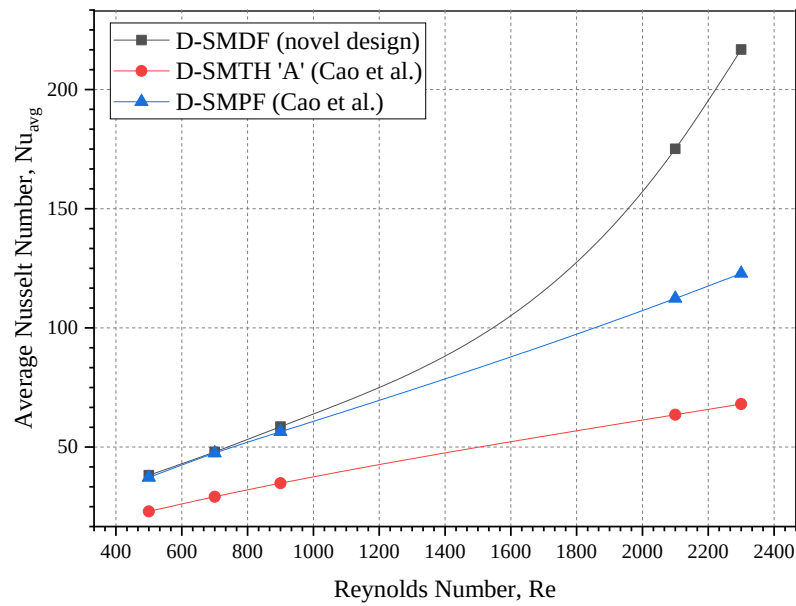


Figure 7-5: Average Nusselt number (Nu_{avg}) against Re for different designs of double inlet-outlet serpentine mini-channel heat sink.

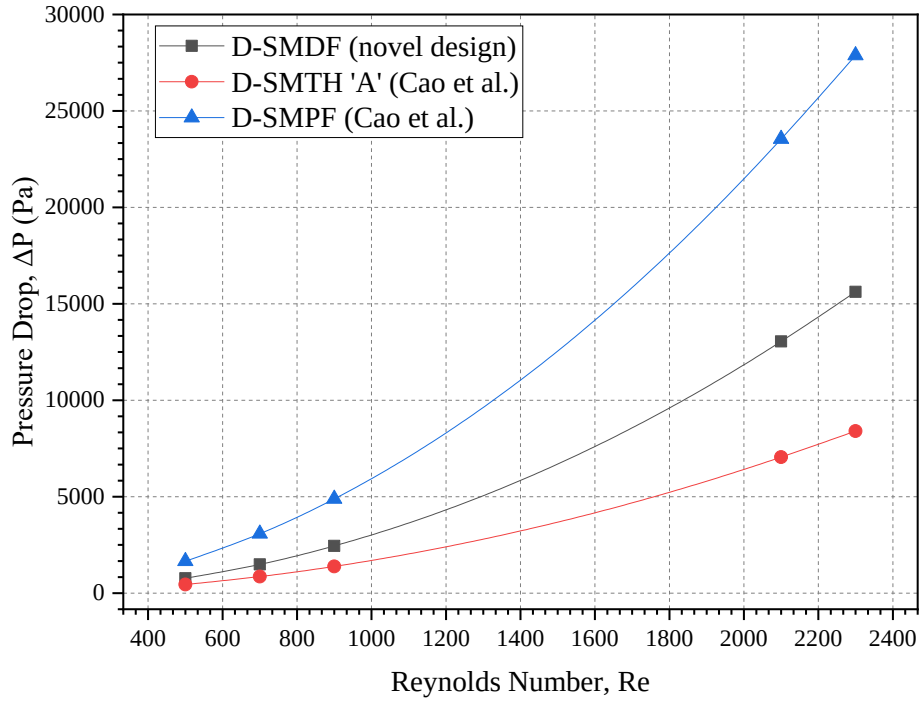


Figure 7-6: Pressure drop (ΔP) against Re for different designs of double inlet-outlet serpentine mini-channel heat sink.

Looking at Figure 7-6, the ΔP (refer Eq. 3.55) increases exponentially as Re increases for all plots. When Re increases (increasing liquid velocity), fluid dynamic pressure increases due higher head losses and frictions when liquid flows through the bends and channels [308]. The ΔP is proportional to the squared of water velocity [318].

7.4.2 The effect of the D-SMDF heat sink design on its thermal-hydraulic performance

The D-SMDF heat sink uses the diamond shaped fins introducing the secondary channels to provide higher thermal-hydraulic performance and will be compared with the design suggested in [37]. All dimensioning of D-SMDF heat sink can be referred in Table 7-1.

7.4.2.1 Effect of D-SMDF heat sink on R_{th} and Nu_{avg}

From Figure 7-4, along the increase of Re , the D-SMDF heat sink design possesses the lowest R_{th} compared with the D-SMPF and D-SMTH 'A' heat sink designs. Highest R_{th} performance for D-SMDF heat sink can be explained using Figure 7-7 where the temperature contours are plotted at $Re = 2300$. Referring Figure 7-7, temperature distribution of D-SMDF heat sink shows the lowest temperature magnitude compared to D-SMPF and D-SMTH 'A' heat sinks. The reason behind this is based on the continuous breaking of thermal and hydrodynamic boundary layers along the serpentine mini-channels due to liquid flows through secondary channel. From the formation of vortices, the liquid also separates and re-initialised continuously improving heat

transfer [309]. The R_{th} of D-SMDF heat sink improves by 17.6% change when compared with the D-SMPF heat sink due to D-SMPF heat sink design has poor design where the channel has dead ends on both sides of heat sink allowing liquid to be trapped reducing heat transfer. Looking at D-SMPF heat sink, higher temperature recorded on the sides due to the water is trapped within the channel on the sides and no mixing occurs, hence, inducing localised of very low velocity region (low streamline density) affecting the convective heat transfer to deteriorates. On the other hand, D-SMTH 'A' heat sink studied by [37] shows higher temperature on both middle and side part of heat sink with similar reason of localised low velocity region reducing convective heat transfer (refer Figure 7-7).

Referring Figure 7-5, D-SMDF heat sink shows the most high magnitudes of Nu_{avg} as Re increases followed by D-SMPF and D-SMTH 'A' respectively. Average increase in percentage of Nu_{avg} by 28% recorded going from D-SMPF heat sink to D-SMDF heat sink. Based on Figure 7-7, due to the continuous breakage of thermal and hydrodynamic boundary layers from the diversion of liquid through the secondary channel, average mini-channel wall temperature drops since more heat is removed from solid domain as convection improves. The effective area (A_{eff}) of D-SMDF heat sink is 1111.9 mm^2 which is larger than A_{eff} of D-SMPF at 919.83 mm^2 , thus, aids in transferring more heat from solid to liquid domain. From Figure 7-5, the D-SMTH 'A' heat sink has lower Nu_{avg} compared to D-SMPF heat sink along increasing Re with an average of 40.6% reduction. The existence of hot region spots are due to the localised low velocity region affecting heat transfer performance (refer Figure 7-7). Even though, D-SMTH 'A' has larger A_{eff} compared to D-SMDF heat sink by 0.8%, it not capable to improve the transfer of heat effectively.

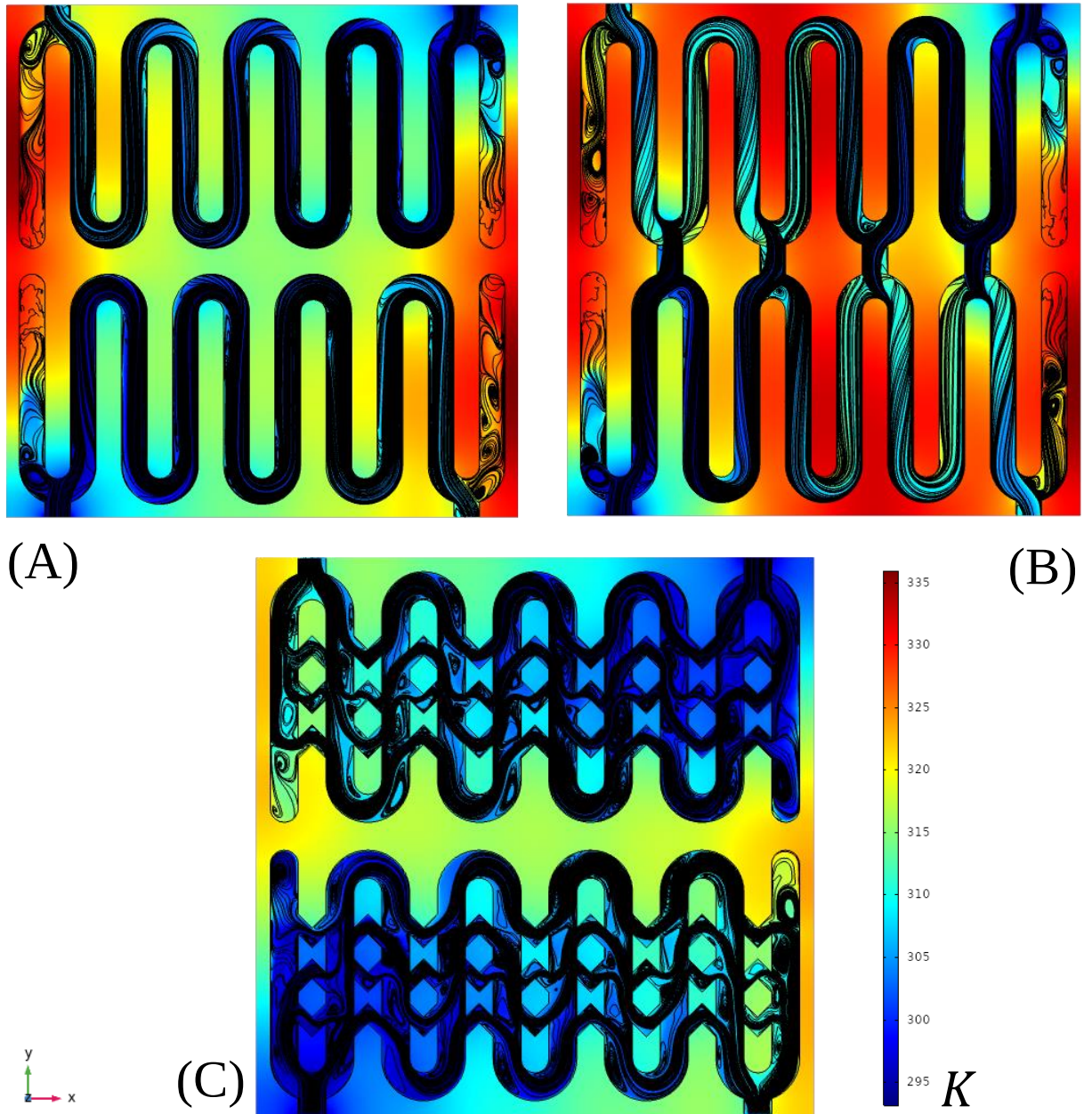


Figure 7-7: Shows the temperature contours of (A) D-SMPF [37] (B) D-SMTH 'A' [37] (C) D-SMDF (novel design) at $Re = 2300$ at $H_{ch} = 1$ mm.

7.4.2.2 Effect of D-SMDF heat sink on velocity and pressure distribution, and ΔP

From Figure 7-6, the use of D-SMPF heat sink holds the highest pressure drop (ΔP) followed by D-SMDF heat sink and D-SMTH 'A' heat sink has the lowest ΔP along increasing Re . When comparing with D-SMPF heat sink, D-SMDF heat sink experiences reduction by 48.7% while D-SMTH 'A' heat sink reduces by 71.3% relatively. By looking at Figure 7-8, the D-SMPF heat sink design holds a very large change in pressure distribution due to very large water velocity within the primary channel (refer Figure 7-9 (A)) increasing the head loss and friction factor, thus, the ΔP increases. On the other hand, D-SMDF and D-SMTH 'A' heat sinks have more uniform pressure distribution with much lower ΔP within the main channels, secondary channels and through-holes. This reason is liquid being diverted through the secondary channels and through-holes reducing the velocity of liquid within the main channel greatly (refer Figure 7-9 (B) and (C)). Comparing the ΔP between D-SMDF and D-SMTH 'A' heat sinks, lower ΔP of D-SMTH 'A' heat sink is due to the fact that through-holes connecting between serpentine channels helps to achieve pressure equilibrium reducing the ΔP about half.

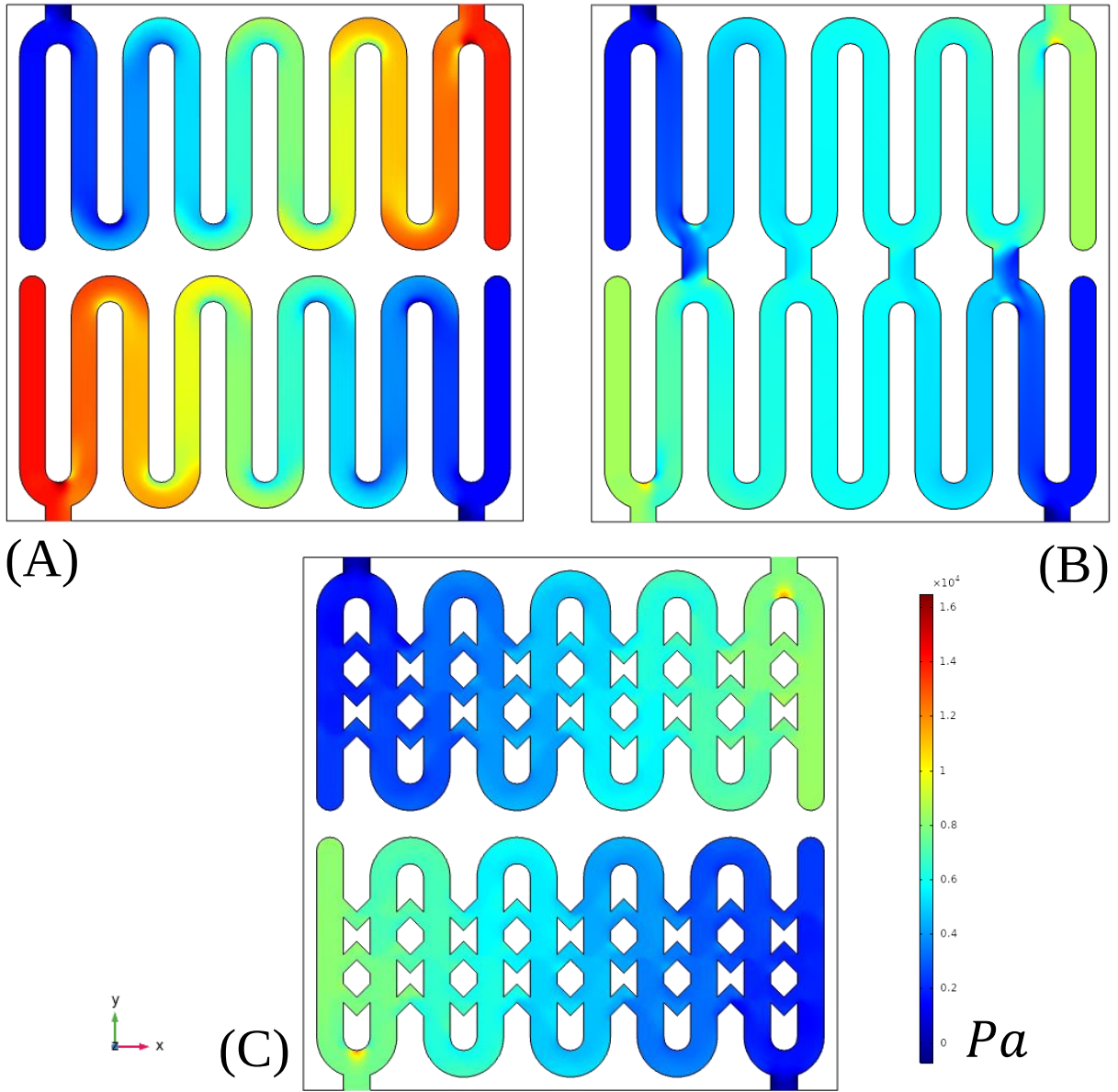


Figure 7-8: Shows the pressure contours of (A) D-SMPF [37] (B) D-SMTH 'A' [37] (C) D-SMDF (novel design) at $Re = 2300$ at $H_{ch} = 1$ mm.

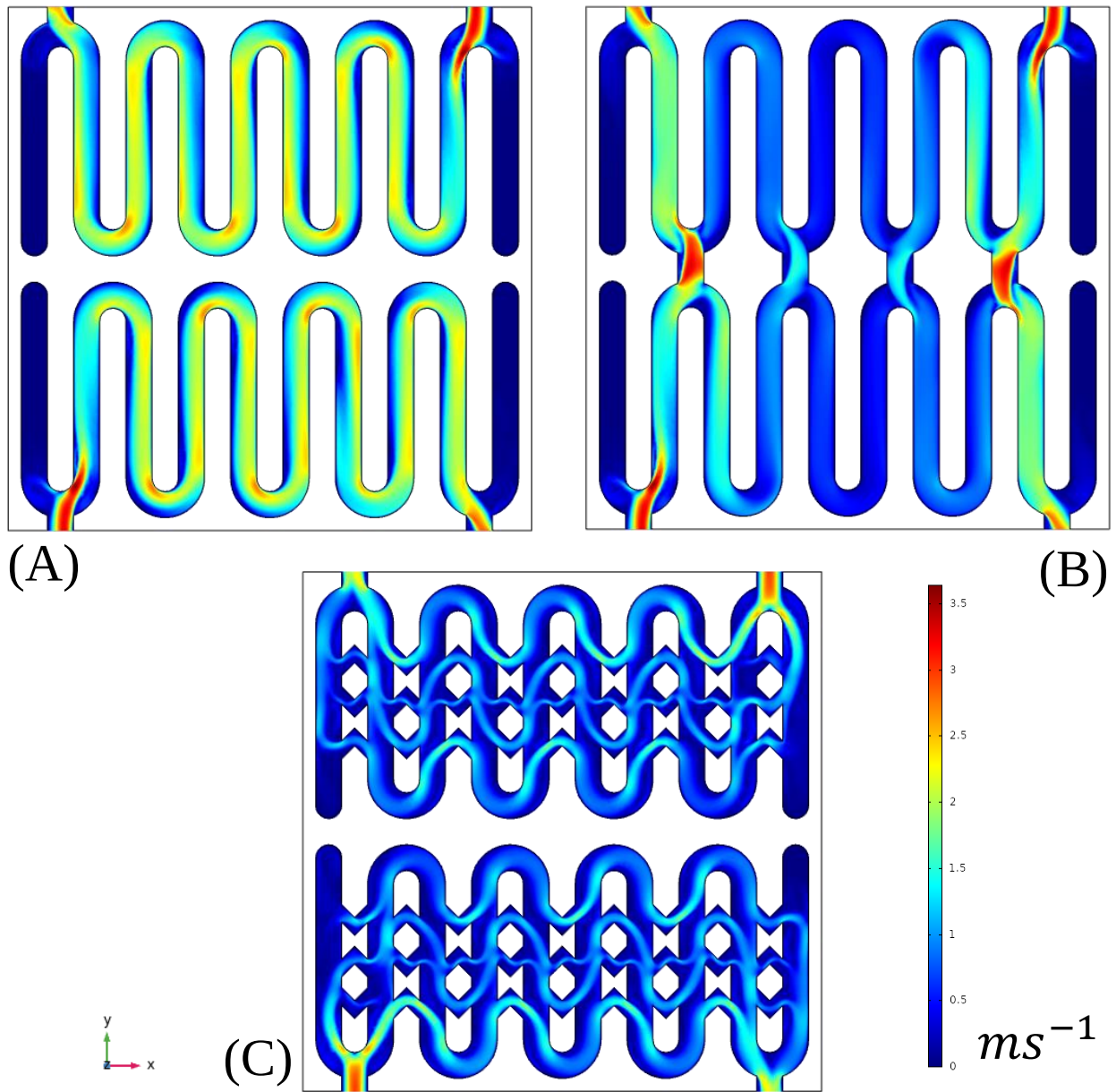


Figure 7-9: Shows the velocity contours of (A) D-SMPF [37] (B) D-SMTH 'A' [37] (C) D-SMDF (novel design) at $Re = 2300$ at $H_{ch} = 1$ mm.

7.4.2.3 Performance Evaluation Criteria (PEC) index study

The concept of Webb's index has been use widely to calculate the performance evaluation criterion that utilised the Nusselt number and friction factor [319]. But the Webb's index does not include the study of occupied space within the equation which is very crucial for small scale cooling systems. To include all parameters of thermal-hydraulic and occupied space components together within one equation, the performance evaluation criterion formula used is based on Sajadi et al. [320] but with a slight modification inspired from the formula proposed by Al-Asadi et al. [311]. The newly constructed PEC index formula is as follows:

$$PEC = \frac{\frac{R_{th,base}/R_{th,new}}{\sqrt[3]{\Delta P_{new}/\Delta P_{base}}}}{\frac{OS_{new}/OS_{base}}{\sqrt[3]{E_{\Delta P}}}} = \frac{E_{R_{th}}}{E_{OS}} \quad (7.3)$$

Where R_{th} , ΔP and OS is the total thermal resistance, pressure drop and occupied space respectively. From the PEC equation proposed by [320], they have used the effectiveness (ε) and pumping power (P_p) which have been replaced by R_{th} and ΔP parameters respectively based on [311].

Under subchapter 7.4.2.1, the D-SMDF heat sink has shown high superiority in terms of R_{th} and Nu_{avg} compared to the D-SMPF and D-SMTH 'A' heat sinks. From Figure 7-10, the PEC index for D-SMDF heat sink shows the highest compared to the D-SMTH 'A' heat sink. D-SMDF heat sink has 1.73 times higher while SMTH 'A' heat sink has 1.46 times when compared to D-SMPF heat sink. Very high PEC index of D-SMDF heat sink is mainly contributed to the very high performance of its R_{th} performance.

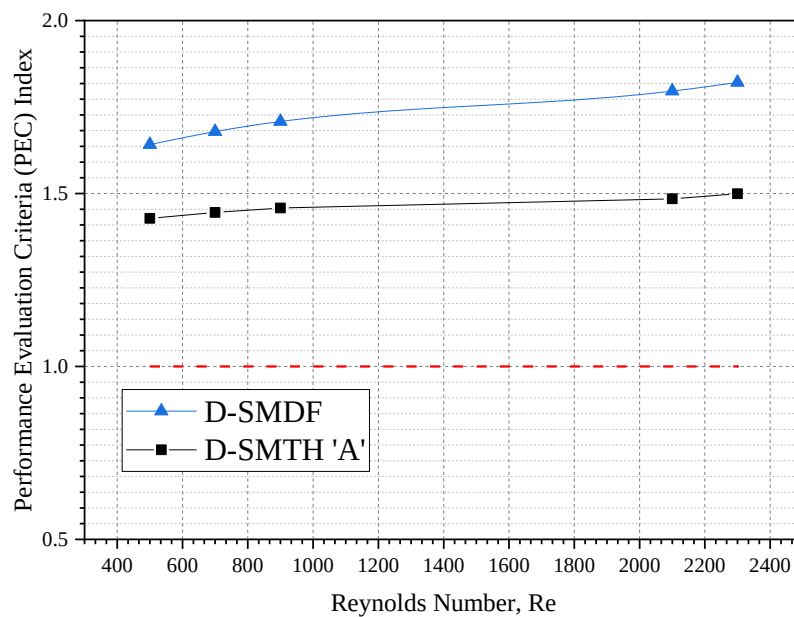


Figure 7-10: Shows the PEC index plots of different heat sinks.

7.4.3 Parametric analysis of double inlet-outlet serpentine mini-channel heat sink

The study of parametric analysis covers parameters changes affecting the performance of thermal-hydraulic components of D-SMDF heat sink. Under subchapter 7.4.2, the D-SMDF heat sink has been proven to have very high performance in terms of thermal and hydraulic components compared to the D-SMPF heat sink. The parameters chosen for parametric analysis will be mini-channel height (H_{ch}), mini-channel width (W_{ch}), fin angle (θ_{fin}) and heat flux applied (q'').

7.4.3.1 Effect of varying the height of mini-channel (H_{ch}) of D-SMDF heat sink on parameters of R_{th} , ΔP and PEC index

The variation of H_{ch} values affecting the thermal-hydraulic performance of D-SMDF heat sink are tested at 1.5 mm to 2.0 mm respectively. The heat flux applied underneath heat sink is at 100 W/cm^2 with Re ranges between 500 and 2300. All other dimensions are set to be constant which can be referred in Table 7-1.

Referring Figure 7-11, it can be observed that increasing the H_{ch} leads to the decrease of both R_{th} and ΔP respectively. The reduction of R_{th} and ΔP goes by 21.6% and 18.4% respectively when H_{ch} increases from 1.5 mm to 2.0 mm along increase Re . The R_{th} is reduced is due to the fact that A_{eff} enlarges from 895.4 mm^2 to 1119.9 mm^2 (25.1% increase) increasing the heat transfer. The reduction of ΔP caused by enlargement of the mini-channel reducing the liquid velocity, thus, the head losses and friction factor decreases.

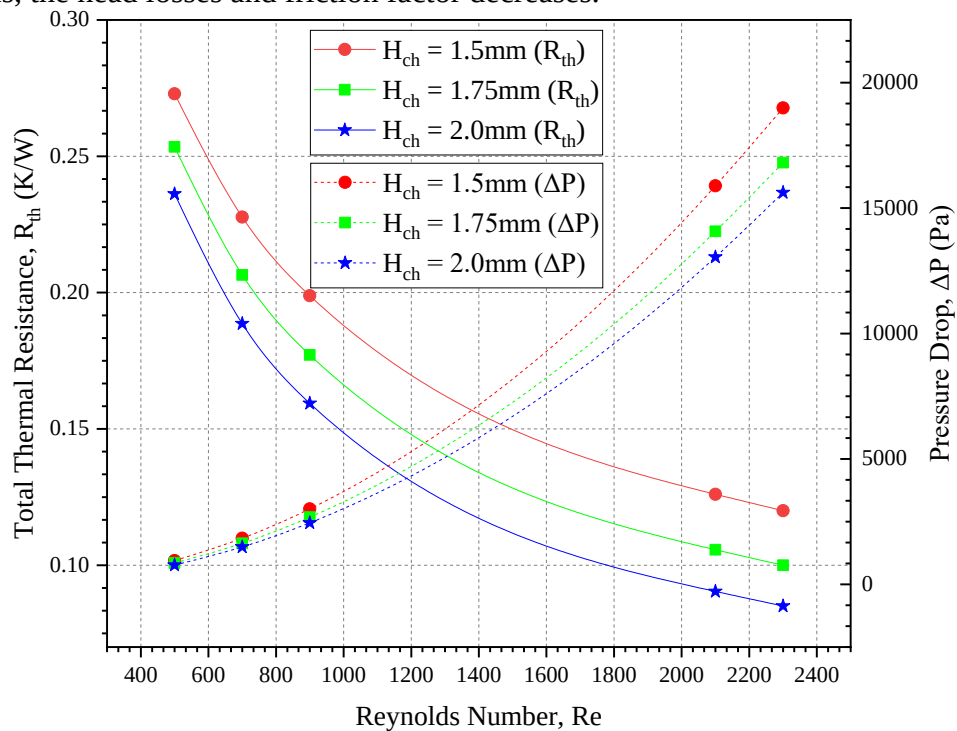


Figure 7-11: Shows the plots of R_{th} and ΔP at different values of H_{ch} .

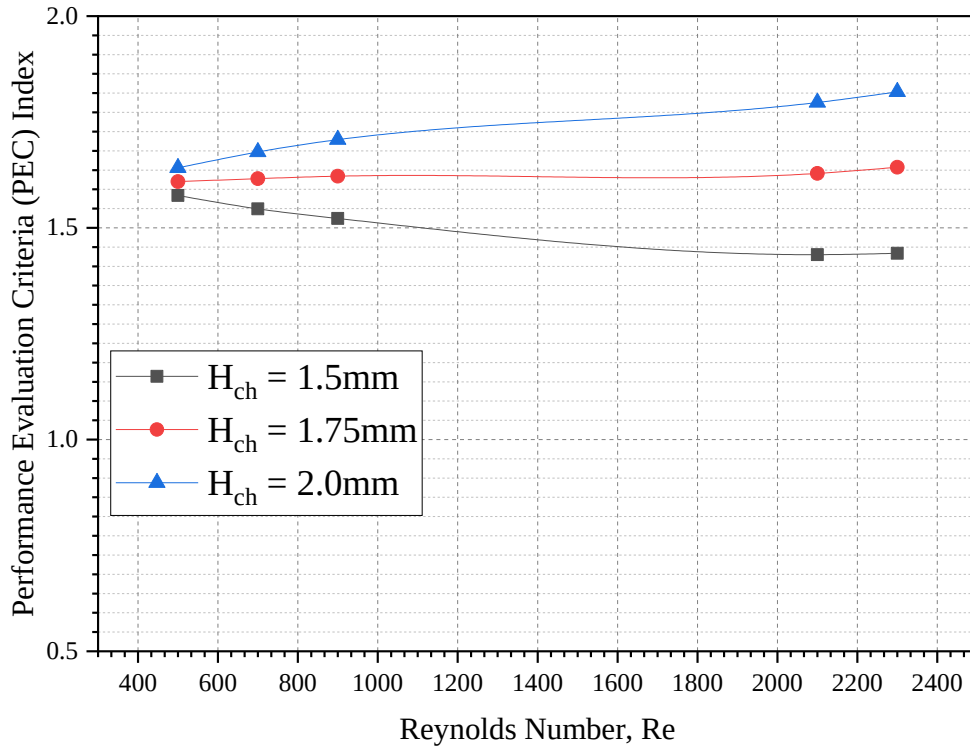


Figure 7-12: Shows the plots of PEC at different values of H_{ch} .

From Figure 7-12, at different H_{ch} values, the PEC index are plotted along increasing Re . From Eq. 7.5, increasing of PEC values can be obtained by the highest values of $E_{R_{th}}$ while having the lowest values of $E_{\Delta P}$ and E_{OS} respectively. From the figure, at $H_{ch} = 2.0 \text{ mm}$, the PEC index is the highest while at $H_{ch} = 1.5$, PEC is the lowest.

7.4.3.2 Effect of varying the W_{ch} of D-SMDF heat sink on parameters of R_{th} , ΔP and PEC index

The changing of values of W_{ch} from 0.8 mm to 1.0 mm will be tested for its effect on the thermal-hydraulic performance of D-SMDF heat sink. The Re is set to be between 500 and 2300 while having heat flux applied at the bottom of heat sink with 100 W/cm^2 in magnitude. The dimensioning apart from W_{ch} will be kept constant (refer Table 7-1).

From Figure 7-13, when W_{ch} changes from 0.8 mm to 1.0 mm , the R_{th} and ΔP decreases by 9.8% and 9.4% along increasing Re . As W_{ch} increases, the A_{eff} increases from 1040.1 mm^2 to 1117.1 mm^2 (7.4% increase) respectively helping the heat transfer to improve. Similarly, the mini-channel size increases as W_{ch} increases, thus, water velocity is reduced while reducing the head losses and friction factor.

From Figure 7-14, the PEC index at $W_{ch} = 1.0 \text{ mm}$ has the highest value while at $W_{ch} = 0.8 \text{ mm}$ has the lowest. At $W_{ch} = 1.0 \text{ mm}$, the $E_{R_{th}}$ and $E_{\Delta P}$ are the lowest and highest contributing the highest PEC index.

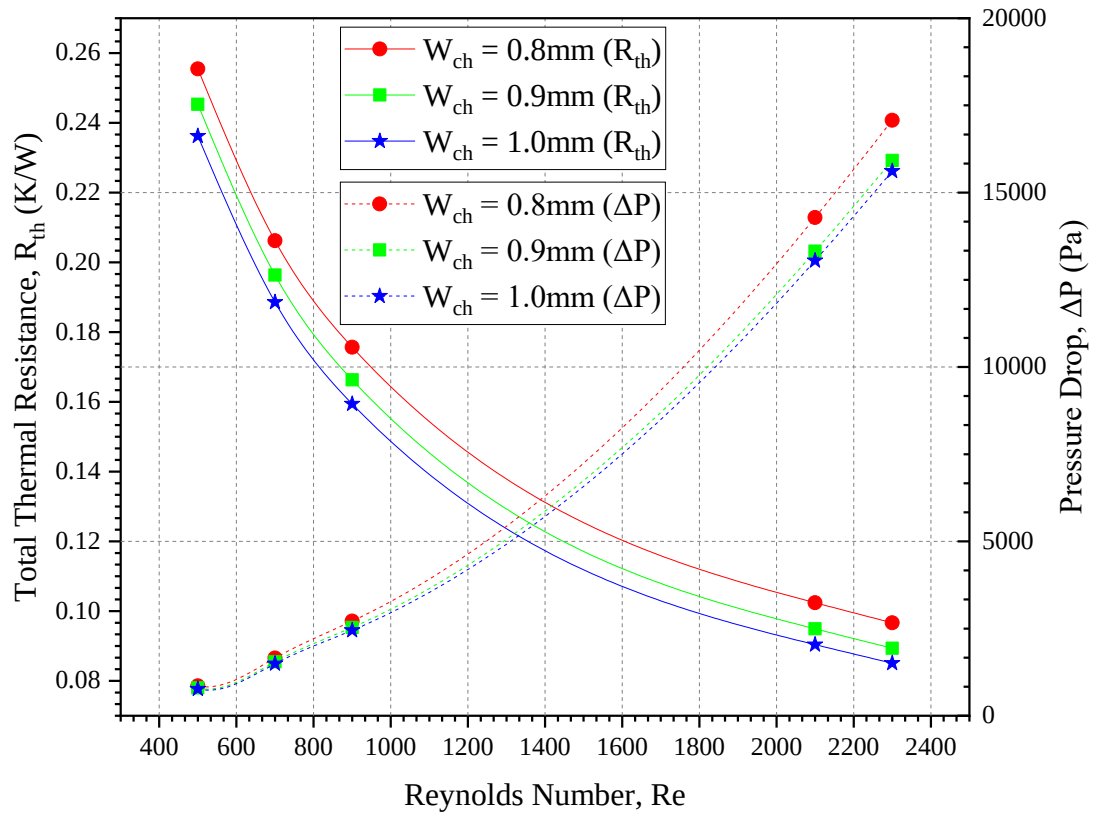


Figure 7-13: Shows the plots of R_{th} and ΔP at different values of W_{ch} .

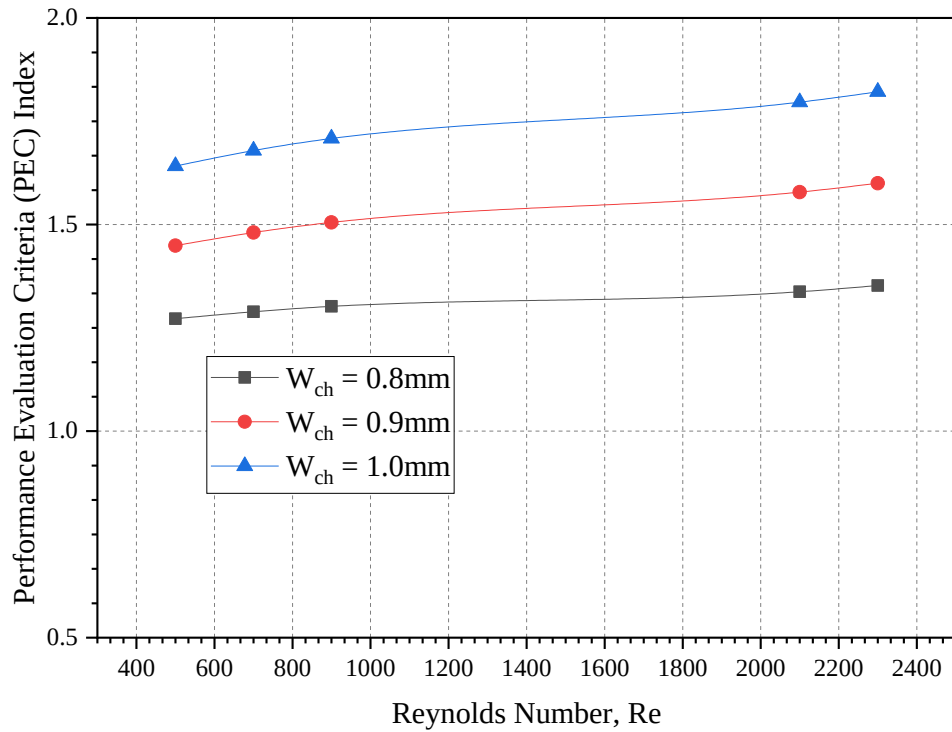


Figure 7-14: Shows the plots of PEC at different values of W_{ch} .

7.4.3.3 Effect of varying the θ_{fin} of D-SMDF heat sink on parameters of R_{th} , ΔP and PEC index

The θ_{fin} is varied to be between 45° to 60° to study its effect on the thermal-hydraulic performance of D-SMDF heat sink. Same conditions are applied where 100 W/cm^2 of heat flux applied underneath and Re is varied between 500 and 2300 respectively.

From Figure 7-15, changing the θ_{fin} from 60° to 45° leads to the increase of R_{th} by 3.7% and decrease of ΔP by 18.5% respectively. The increase of R_{th} is due to the fact that reduction of θ_{fin} will lead to the increase of secondary channel width (W_{sc}), thus, reducing the liquid velocity passing through secondary channel, hence, reducing the convective heat transfer. The A_{eff} also decreases from 1246.4 mm^2 to 1117.1 mm^2 (10.4% reduction) reducing the heat transfer area, hence, less heat flow. As the liquid velocity is reduced, ΔP also reduces due to reductions of head losses and friction factor.

Looking at Figure 7-16, at $\theta_{fin} = 50^\circ$ the PEC is the highest while at $\theta_{fin} = 60^\circ$ the lowest. The increase of R_{th} while having the decrease of ΔP has caused the PEC to behave this way when θ_{fin} experiences reduction. The decrease of θ_{fin} causes the drop of ΔP is more than the rise of R_{th} .

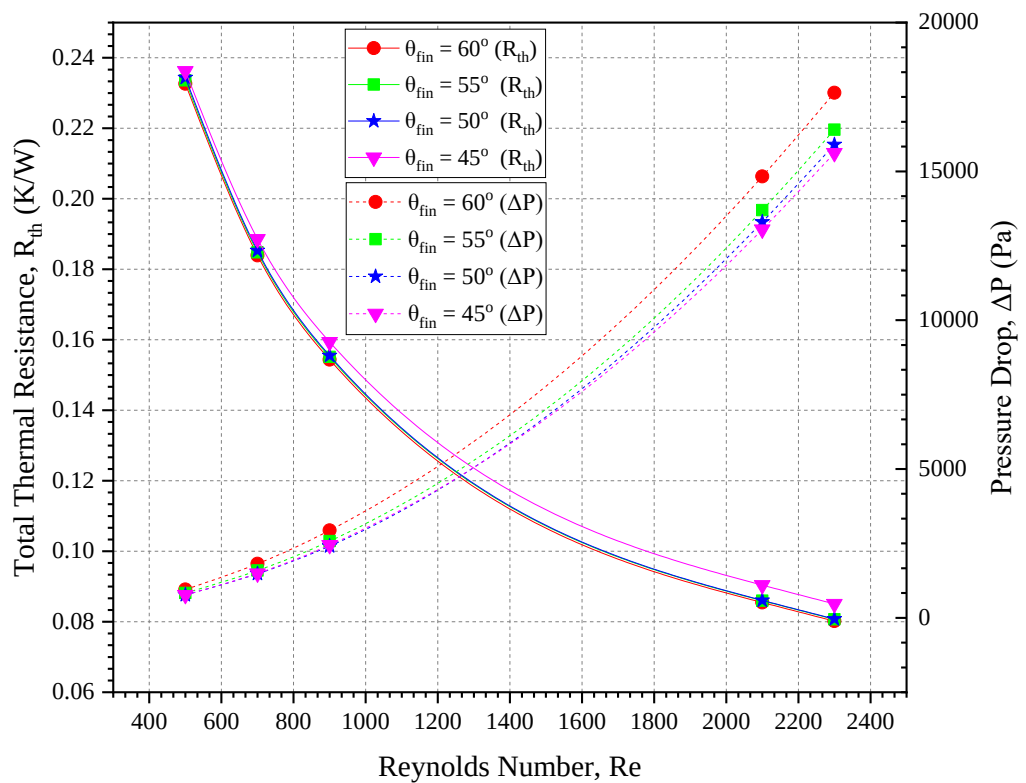


Figure 7-15: Shows the plots of R_{th} and ΔP at different values of θ_{fin} .

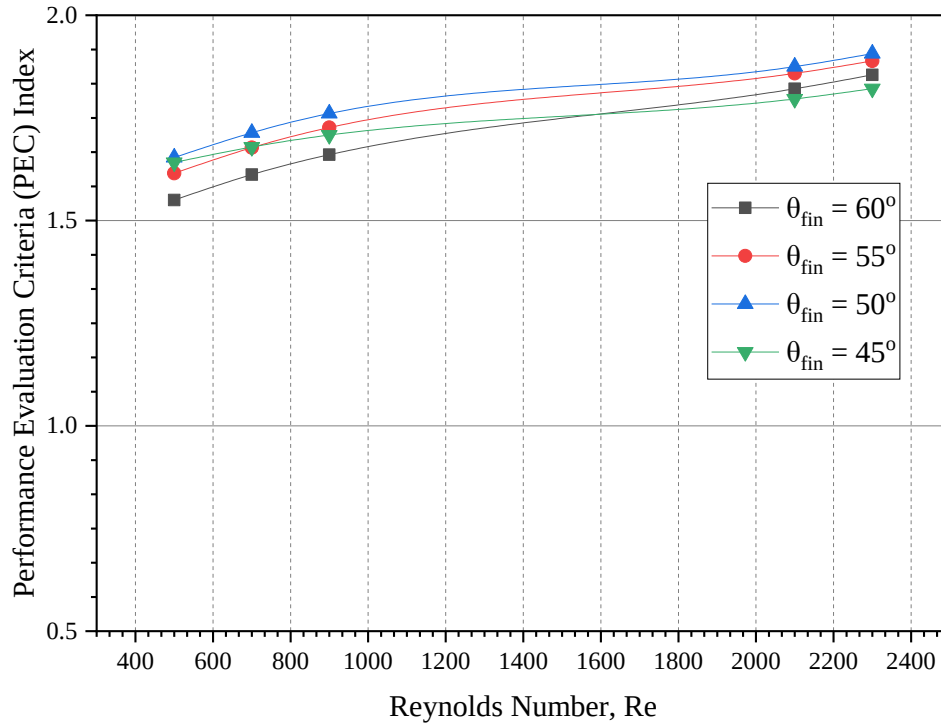


Figure 7-16: Shows the plots of PEC at different values of θ_{fin} .

7.4.3.4 Effect of varying the q'' of D-SMDF heat sink on parameters of $T_{surf,max}$ and ΔP

Three different heat fluxes (q'') applied underneath of heat sink have been tested of 100 W/cm^2 , 150 W/cm^2 and 200 W/cm^2 respectively to see its effect on the thermal-hydraulic performance of heat sink. Similarly, the Re is varied between 500 and 2300. Dimensions of heat sink are the same in Table 7-1.

From Figure 7-17, there is a slight decrease in ΔP when q'' increases since liquid density and viscosity reduces along increasing temperature. When the q'' increases, the maximum surface temperature ($T_{surf,max}$) increases enormously since more heat is transferred. The increase of $T_{surf,max}$ change by 18.8% when q'' increases from 100 W/cm^2 to 200 W/cm^2 .

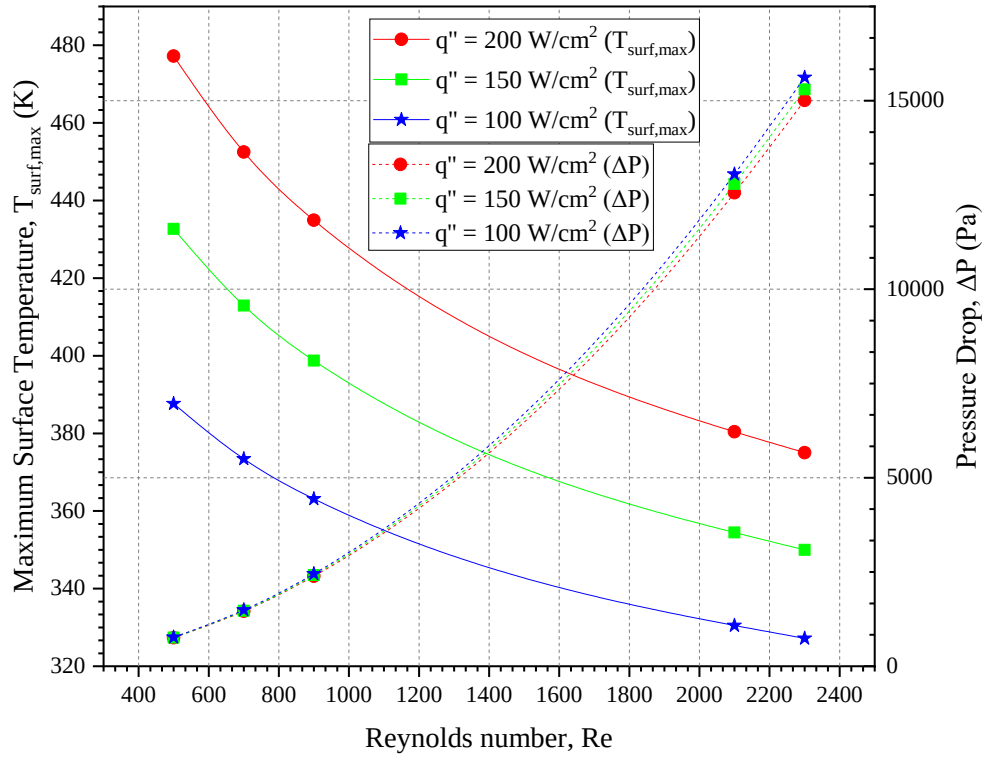


Figure 7-17: Shows the maximum surface temperature, $T_{surf,max}$, and ΔP plots when Re is varied.

7.5 Design optimisation of D-SMDF heat sink

Optimisation of heat sink is very crucial to maintain the temperature of heat sink under junction temperature. Similarly like under subchapter 6.5, the optimisation of D-SMDF heat sink will be covering optimisation of single-objective and multi-objective within the frameworks of deterministic and robust design study. Three same objectives are used here which are total thermal resistance (R_{th}) pressure drop (ΔP) and mass of copper heat sink (M_{Cu}) respectively.

In this optimisation study, six input design variables are chosen to study its effect on the thermal-hydraulic performance of heat sink. The input design variables are channel height (H_{ch}), channel width (W_{ch}), angle of fin (θ_{fin}), thickness of substrate (H_b), heat flux (q'') and Reynolds number (Re) respectively due to its huge effect on the thermal-hydraulic performance. The limits on each input design variables are tabulated in Table 7-3.

Table 7-3: Shows the description of design variables for D-SMDF heat sink.

Design variables	Lower limit	Upper limit
Width of channel, W_{ch} (mm)	1.0	1.4
Angle of fin, θ_{fin} ($^{\circ}$)	45	60
Height of channel, H_{ch} (mm)	0.5	2.0
Thickness of substrate, H_b (mm)	0.5	1.0
Heat flux, q'' (W/cm^2)	100	200
Reynolds number, Re	2000	4000

Limits of the individual input design variables are made to be as wide as possible to ensuring the optimisation process can cover large range of design variables. Based on the limits set in Table 7-3, the Design of Experiment (DOE) points are created. A number of 350 DOE points are constructed using Optimal Latin Hypercube (OLHC) sampling since the most uniform distribution can be achieved (refer Figure 7-18 to see the distribution). The created DOE points will be used for creating surrogate models based on the Artificial Neural Network (ANN) method, in order to capture the link between the input design variables and objective functions. The analysis of data, creation of surrogates (Tensorflow) and optimisation (Platypus) are processed utilising the Python open-sourced software.

When performing the optimisation process, the secondary channel length (l_{sc}) is kept fixed at 0.7 mm. The foot print area ($W \times L$) of heat sink is set at $20 \times 20 \text{ mm}^2$. Optimisation process aims to achieve the lowest magnitude of the objective functions chosen and will be discussed in detail within the next subchapters.

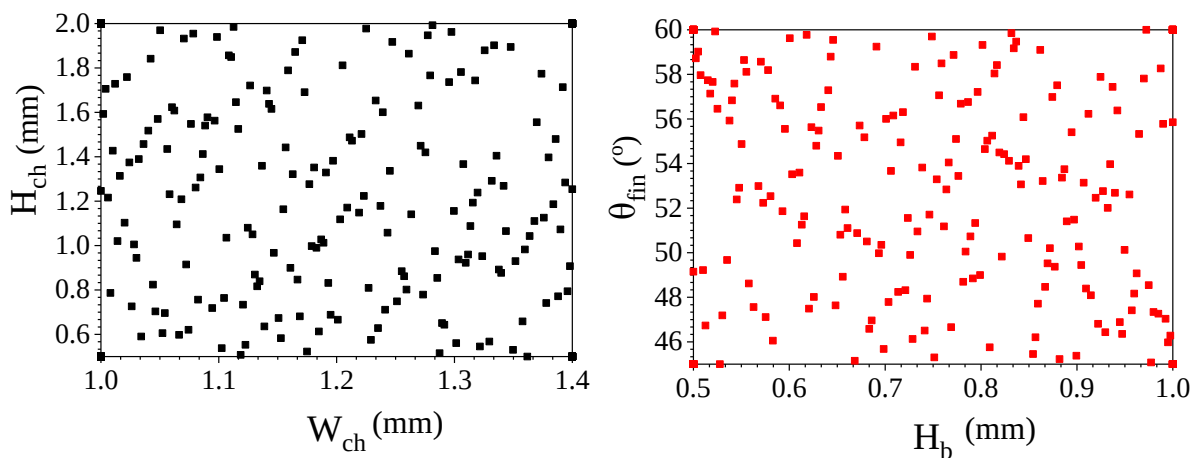


Figure 7-18: Shows the typical DOE points of different input design variables.

7.5.1 Surrogate modelling

To create surrogate models, the DOE points will be fed into COMSOL Multiphysics to calculate the individual objective function (refer subchapter 7.5). Then, the input design variables will be normalised using Eq. 6.6 based on the limits stated in Table 7-3. The normalisation of design variables helps in improving the convergence rate of surrogate modelling.

Based on the surrogate modelling analysis in sub-chapter 6.5.1, RBF method will be used here to create surrogate models. The objectives of R_{th} and ΔP are calculated using Eq. 3.53 and Eq. 3.54 respectively from the CFD models. The M_{Cu} is calculated analytically using Eq. 6.7 but the volume of copper heat sink (V_{Cu}) is based on equation in appendix A 4.1.

Training of RBF surrogate models requires an individual training since a faster convergence rate can be achieved. To train the RBF surrogate models, the DOE dataset is split into 80 to 20 ratio representing as training and validation set of ΔP and R_{th} are shown respectively. The accuracy parameters of RMSE, R^2 and maximum residual error are tabulated in Table 7-4. It can be seen that the R^2 shows magnitude close to unity for all training, validation and testing sets for both R_{th} and ΔP objective functions indicating RBF surrogate models predictions are in correlation with the CFD results. The RMSE parameters show very small magnitudes when compared with their respective objective functions magnitudes (refer for typical values of objective functions). It can also be seen that the maximum residual error shows less than 2% for all training, validation and testing sets indicating the RBF model approximates the relationship between input design variables and objective functions effectively.

Table 7-4: The accuracy parameters for training, validation and testing sets of surrogate models using RBF.

R_{th}			
Accuracy parameter	Training set	Validation set	Testing set
RMSE (K/W)	3.678958×10^{-5}	9.295782×10^{-5}	7.336201×10^{-5}
R^2	0.999997	0.997162	0.999736
Maximum Residual Error (%)	0.265691	1.187324	0.698261
ΔP			
Accuracy parameter	Training set	Validation set	Testing set
RMSE (Pa)	62.142933	114.445981	90.849163
R^2	0.999999	0.999301	0.999955
Maximum Residual Error (%)	1.396070	2.284662	1.856382

7.5.2 Optimisation process

To perform optimisation process, the framework of deterministic and robust design optimisation are considered. The single- and multi-objective optimisations process will be utilised for generation of different solutions and comparison between them in terms of thermal-hydraulic performance. The D-SMDF heat sink will be optimised since it has the most superior thermal performance.

7.5.2.1 Deterministic optimisation

Under this subchapter, single- and multi-objective optimisation process using the framework of deterministic study will be conducted.

7.5.2.1.1 Single-objective optimisation

Optimising the objective function individually is very straight-forward procedure where the minimum or maximum value of objective can be easily pinpointed. From single-objective optimisation, it is very hard to locate the set of input design variables which corresponds to the minimisation of all three objective functions. Table 7-5 shows the minimisation of each individual objective function (represented by the green fill). From Table 7-5, the M_{Cu} objective has the lowest value at seven different sets of input design variables while R_{th} and ΔP on have one lowest point each.

Table 7-5: Shows the input design variables at individual minimisation of objective functions.

W_{ch} (mm)	H_{ch} (mm)	H_b (mm)	θ_{fin} (°)	q'' (W/cm ²)	Re	M_{Cu} (g)	R_{th} (K/W)	ΔP (Pa)
1.4	0.5	0.5	60	200	4000	2.316475	0.074222	310851.0
1.4	0.5	0.5	60	200	4000	2.316475	0.074222	310851.0
1.4	0.5	0.5	60	100	4000	2.316475	0.076423	313083.4
1.4	0.5	0.5	45	100	4000	2.316475	0.086834	221376.3
1.4	0.5	0.5	45	100	4000	2.316475	0.086834	221376.3
1.4	0.5	0.5	45	200	4000	2.316475	0.084996	218362.8
1.4	0.5	0.5	60	100	4000	2.316475	0.076423	313083.4
1.3	2.0	1.0	60	100	4000	4.196749	0.046636	49175.56
1.4	2.0	1.0	45	200	2000	5.699301	0.092756	10018.61

From Table 7-5, at minimisation of M_{Cu} at 2.316 g using analytical method (refer sub-chapter 7.5.1), it corresponds to seven different values of R_{th} and ΔP respectively. The seven values of R_{th} shows quite low magnitudes but the with seven different of very high ΔP which is not feasible. For very low R_{th} at 0.0466 K/W, moderately high ΔP recorded of 49175.6 Pa but the M_{Cu} of heat sink is moderately high of 4.197 g. At very low ΔP of 10018.6 Pa, large M_{Cu}

recorded of 5.699 g with moderately low R_{th} of 0.0928 K/W. Therefore, to make sure that all objective functions experience minimisation, multi-objective optimisation process is carried within the next subchapter.

7.5.2.1.2 Multi-objective optimisation

Using the created surrogate models trained earlier based on the RBF method (refer subchapter 7.5.1), the models will be employed under the multi-objective optimisation to achieve the compromise between competing objectives of M_{Cu} , R_{th} and ΔP respectively. Using the pre-defined input design variables and output objective functions within subchapter 7.5, the multi-objective optimisation formulation can be defined as follows:

$$\begin{aligned}
 &\text{Objective functions: } \min(M_{Cu}), \min(R_{th}) \text{ and } \min(\Delta P) \\
 &\text{Constraints: } W_w + W_{ch} = 2mm \\
 &\text{Design space: } 1.0mm \leq W_{ch} \leq 1.4mm \\
 &\quad 45^\circ \leq \theta_{fin} \leq 60^\circ \\
 &\quad 0.5mm \leq H_{ch} \leq 2.0mm \\
 &\quad 0.5mm \leq H_b \leq 1.0mm \\
 &\quad 1 \times 10^6 \leq q'' \leq 2 \times 10^6 \\
 &\quad 2000 \leq Re \leq 4000
 \end{aligned} \tag{7.4}$$

Performing optimisation process, apart from the design space mentioned in Eq. 7.7, all other boundary conditions and dimensions are kept to be fixed (refer Table 7-1). The importance of multi-objective optimisation is to create the Pareto front for competing objectives to enable the designer to have multiple options depending on the application. The use of Platypus optimisation package within Python software will be employed here. Non-dominated Sorting Genetic Algorithm-II (NSGA-II) algorithm will be utilised due to its reliability, quickness and the initial guess is not a requirement [316].

Referring Figure 7-19 to Figure 7-21, Pareto fronts and DOE points are plotted between pairs of objective functions which are R_{th} vs M_{Cu} , ΔP vs R_{th} and ΔP vs M_{Cu} respectively. The use of Pareto Front plots are to provide the results for competing objectives which means a decrease of one objective will results in an increase of the other. To further analyse the Pareto fronts, the numerical data between conflicting objectives are extracted and tabulated in Table 7-6. The results of Pareto fronts are compared with the CFD results for validation purpose. From Table 7-6, looking at R_{th} vs M_{Cu} while ignoring the variation of ΔP , moving down the table, the value of R_{th} reduces by 32.5% going from 0.0733 K/W to 0.0494 K/W respectively. The M_{Cu} increases from 2.643 g to 4.667 g respectively corresponding to 76.6% decrease. For the ΔP vs R_{th} ,

ignoring M_{Cu} changes, going down the table R_{th} increases by 56.4% going from 0.0543 K/W to 0.0849 K/W respectively. For ΔP , it reduces from 30352.5 Pa to 12412.0 Pa corresponding to 59.1% reduction.

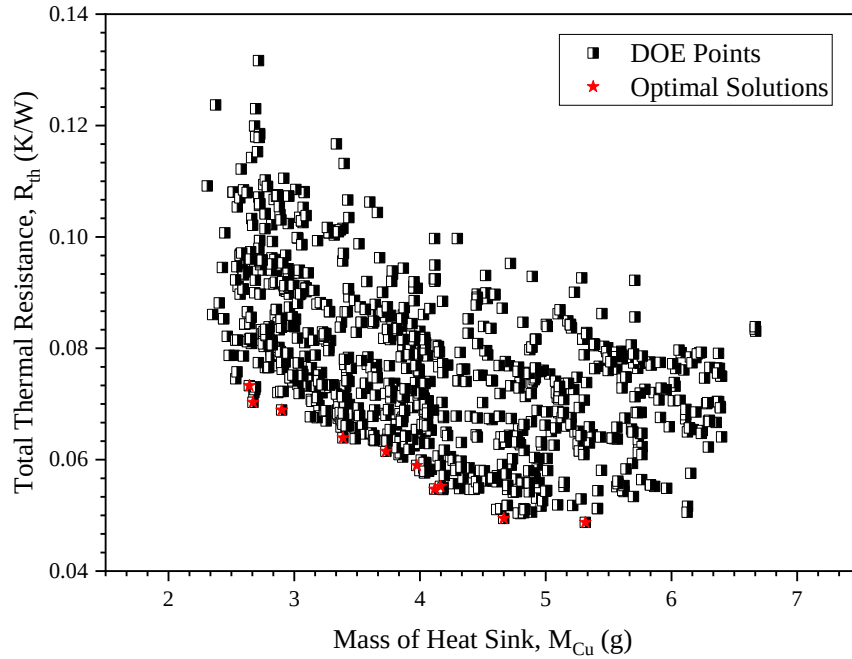


Figure 7-19: Shows the DOE points and Pareto front between competing objectives of M_{Cu} and R_{th} .

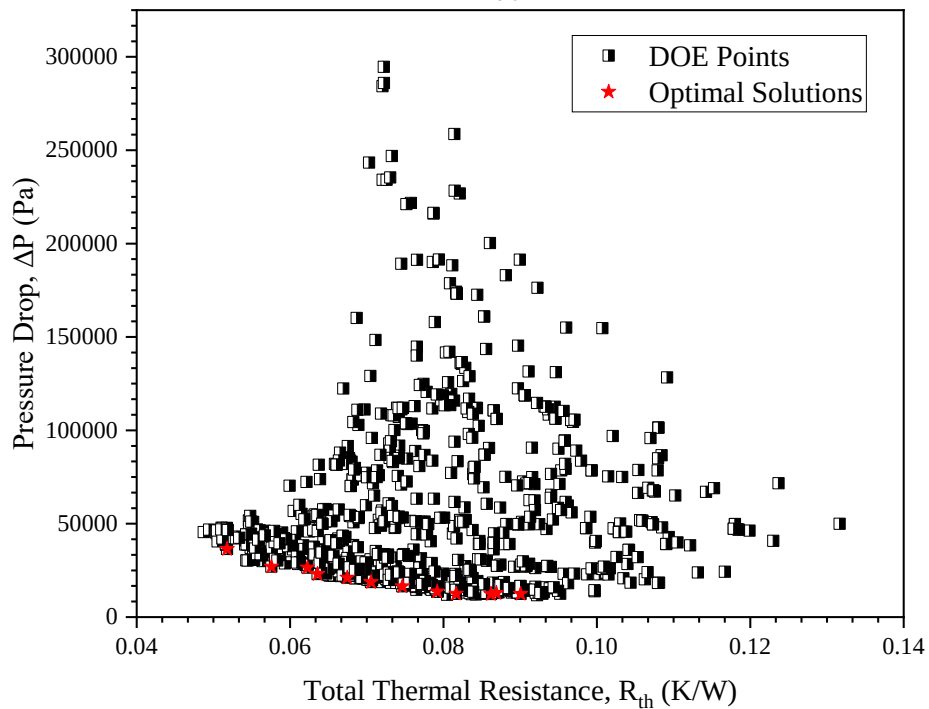


Figure 7-20: Shows the DOE points and Pareto front between competing objectives of R_{th} and ΔP .

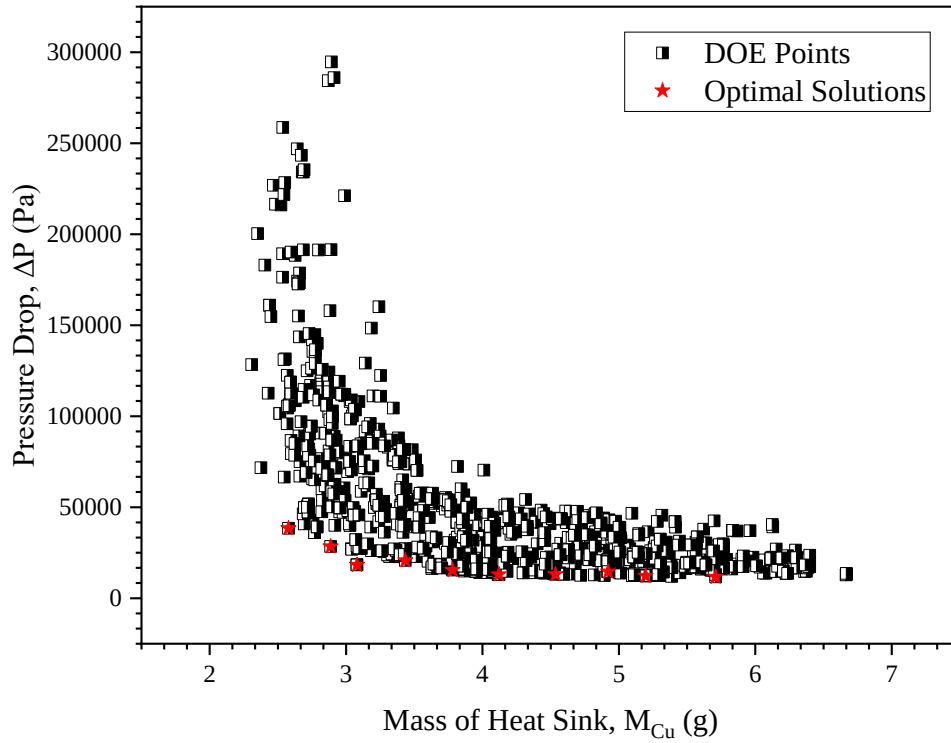


Figure 7-21: Shows the DOE points and Pareto front between competing objectives of M_{Cu} and ΔP .

Looking at ΔP vs M_{Cu} , ignoring R_{th} variation, the M_{Cu} increases from 2.769 g to 4.720 g giving 70.5% change. On the other hand, the ΔP decreases by 65.3% going from 36003.8 Pa to 12483.9 Pa respectively. From any Pareto front relationship between two objective functions, the other objective seems to be not desirable since very large magnitude recorded.

Considering the three objective functions, the Pareto surface distribution has been plotted in Figure 7-22 using 2-D contour plot. From the figure, going towards zero position of Cartesian plane, both R_{th} and M_{Cu} experience reduction but ΔP increases. Therefore, finding the right solution is crucial to compromise between objectives for certain application related.

Table 7-6: Shows the Pareto front points plotted in Figure 7-19 to Figure 7-21.

R_{th} vs M_{Cu}

Design variables						M_{Cu} (g)			R_{th} (K/W)			ΔP (Pa)		
W_{ch} (mm)	H_{ch} (mm)	H_b (mm)	θ_{fin} ($^{\circ}$)	q'' (W/m ^{2})	Re	Pareto	Analytical	Error (%)	Pareto	CFD	Error (%)	Pareto	CFD	Error (%)
1.329037	0.502090	0.581579	51.91579	1751853	3957.221	2.64295	2.642883	0.002535	0.073275	0.073356	0.110420	246910.6	246771.3	0.056441
1.297425	0.502090	0.581579	51.83242	1511315	3946.198	2.67297	2.672886	0.003143	0.070295	0.070452	0.222847	243331.8	243867.6	0.219709
1.147761	1.154398	0.516766	59.25425	1598677	3994.643	3.485219	3.485232	0.000373	0.063742	0.063701	0.064363	81576.16	81669.12	0.113825
1.311223	1.776083	0.553395	59.70418	1968520	3689.052	4.039682	4.039773	0.002253	0.057926	0.057902	0.041449	46138.46	46256.43	0.255035
1.121219	1.869887	0.559093	47.56627	1270546	3953.651	4.666905	4.666813	0.001971	0.049447	0.049533	0.173622	46828.07	46934.68	0.227145

ΔP vs R_{th}

Design variables						M_{Cu} (g)			R_{th} (K/W)			ΔP (Pa)		
W_{ch} (mm)	H_{ch} (mm)	H_b (mm)	θ_{fin} ($^{\circ}$)	q'' (W/m ^{2})	Re	Pareto	Analytical	Error (%)	Pareto	CFD	Error (%)	Pareto	CFD	Error (%)
1.330397	1.974713	0.927840	55.18509	1067505	3134.733	5.622689	5.622737	0.000854	0.054287	0.054331	0.080985	30352.52	30400.41	0.157531
1.005497	1.977023	0.880548	59.45194	1299710	2800.789	6.294716	6.294332	0.006101	0.062259	0.062267	0.012848	26438.41	26444.12	0.021593
1.136667	1.925423	0.912202	57.39836	1057313	2755.589	5.983089	5.983101	0.000201	0.065916	0.065993	0.116679	22518.97	22793.34	1.203729
1.087399	1.857707	0.912108	53.36570	1188116	2486.249	6.015557	6.015669	0.001862	0.070558	0.070645	0.123151	18767.53	18993.21	1.188214
1.030551	1.830072	0.813160	47.53327	1310013	2271.230	5.75117	5.751123	0.000817	0.077286	0.077313	0.034923	16008.38	16015.88	0.046829
1.028797	1.868030	0.638785	50.89107	1677355	2042.069	5.176924	5.177032	0.002086	0.084913	0.084892	0.024737	12411.93	12432.21	0.163125

ΔP vs M_{Cu}

Design variables						M_{Cu} (g)			R_{th} (K/W)			ΔP (Pa)		
W_{ch} (mm)	H_{ch} (mm)	H_b (mm)	θ_{fin} (°)	q'' (W/m ²)	Re	Pareto	Analytical	Error (%)	Pareto	CFD	Error (%)	Pareto	CFD	Error (%)
1.294192	0.841780	0.500447	47.62626	1298760	2228.893	2.769396	2.769444	0.001733	0.104127	0.104236	0.104570	36003.77	36112.25	0.300397
1.312679	1.080434	0.509917	45.41216	1626040	2007.233	3.08021	3.080350	0.004545	0.107962	0.107873	0.082504	18285.42	18376.65	0.496445
1.361902	1.742473	0.508240	47.50966	1434511	2310.306	3.70189	3.701930	0.001081	0.090058	0.090174	0.128640	16635.31	16623.12	0.073332
1.305564	1.822267	0.558380	45.47868	1762810	2107.298	4.114227	4.114591	0.008847	0.092436	0.092381	0.059536	13034.46	13025.91	0.065638
1.386497	1.981935	0.720662	45.05115	1020513	2112.811	4.720642	4.720525	0.002479	0.095232	0.095398	0.174008	12483.86	12521.44	0.300125

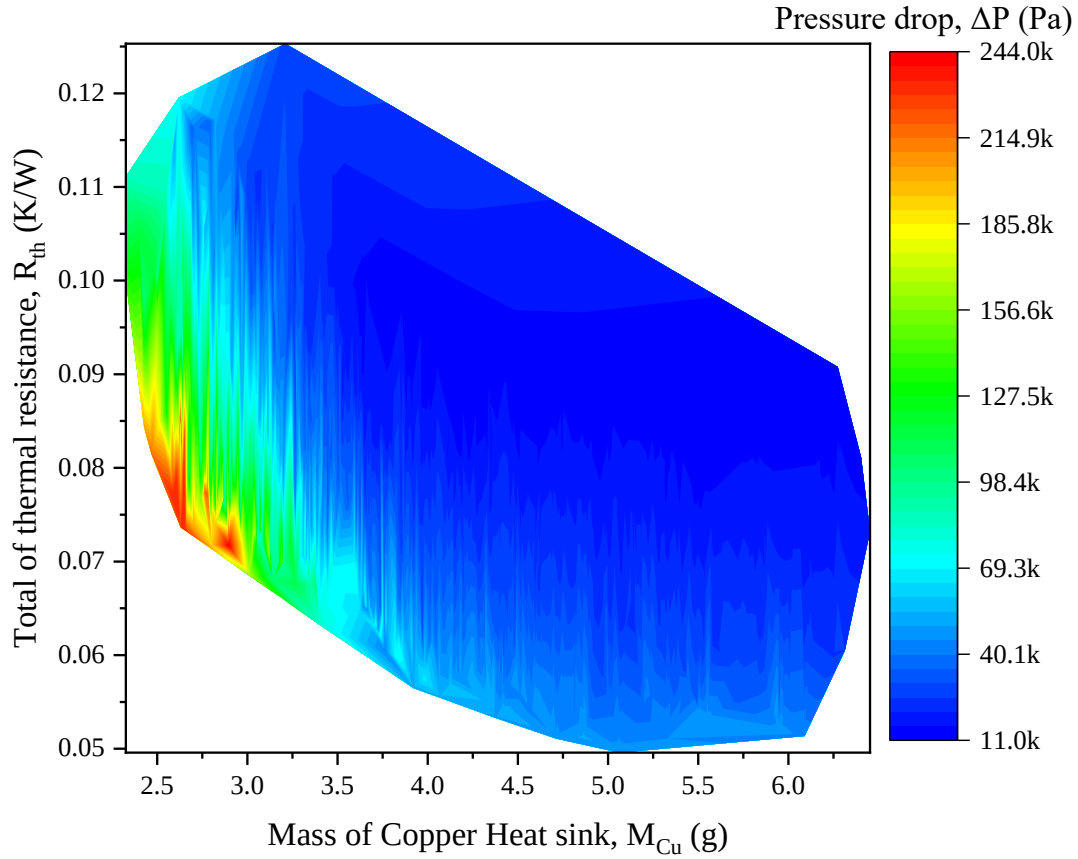


Figure 7-22: Shows the Pareto surface for all three objective functions.

7.5.2.2 Robust design optimisation (RDO)

From the previous study of deterministic optimisation under subchapter 7.5.2.1, the method does not cover the real life manufacturing processes issue which involves the operating conditions and tolerances. Therefore, the fluctuations of parameters within the environment can cause the design of unreliability. Using Robust Design optimisation (RDO) method, uncertainties are applied onto the input design variables to check its effect on the design performance. Input design variables with uncertainty application will correspond to the distribution of the output objective function, so that the statistical moments of mean (μ) and standard deviation (σ) can be calculated on each point of output. Under this subchapter, the double inlet-outlet serpentine mini-channel heat sink optimisation will be conducted using robust design method. From the previous subchapter 6.5.2.2, the Monte Carlo sampling (MCS) method has been proven not effective in terms of time consumption for simulation, only Polynomial Chaos Expansion (PCE) method will be considered here since it is accurate and low computational time consumption needed.

7.5.2.2.1 Model description

From the explanation under subchapter 7.5 for optimisation process, the same six input design variables of H_{ch} , W_{ch} , θ_{fin} , H_b , q'' and Re will be used respectively. To differentiate the optimisation methods, the input design variables will be subjected to uncertainties which will correspond to the distribution of output objective functions where statistical moments can be calculated. Similarly like under subchapter 7.3, the conjugate heat transfer model and boundary conditions used are the same. This model will be employed to calculate the objective functions from the pre-set input design variables.

7.5.2.2.2 Uncertainty quantification (UQ)

Under this subchapter, the same Uncertainty quantification (UQ) procedure will be used (refer subchapter 6.5.2.2.3 for details). The only difference will be for this subchapter is the number of input design variables and the parameters used which is tabulated in Table 7-7.

Table 7-7: Shows the uncertainty associated for each design variable.

Input design variables	Distribution	μ_{in}	σ_{in}
W_{ch} , mm	normal	W_{ch_i} , mm	0.0025mm
H_{ch} , mm	normal	H_{ch_i} , mm	0.0025mm
H_b , mm	normal	H_{b_i} , mm	0.0025mm
θ_{fin} , °	normal	θ_{fin_i} , °	0.025°
q'' , W/cm^2	normal	q''_i , W/cm^2	1 W/cm^2
Re	normal	Re_i	20

7.5.2.2.3 Convergence study for PCE method

To optimise the heat sink under uncertainty, a convergence study has to be carried out when using the PCE method ensuring the calculation of means and standard deviations of output objective functions are accurate enough without having to increase the variable number parameters which is in this case is polynomial order. The convergence plot study is shown in Figure 7-23. From the figure, for all $\mu_{R_{th}}$, $\sigma_{R_{th}}$, $\mu_{\Delta P}$, $\sigma_{\Delta P}$, $\mu_{M_{Cu}}$ and $\sigma_{M_{Cu}}$ statistical parameters, the polynomial order going from 0 to 4 shows almost equal results while at value 5 the statistical parameters skyrocketed drastically which turns out to be inaccurate and unreliable. Thus, at polynomial order of 2 will utilised for all DOE points of input design variables.

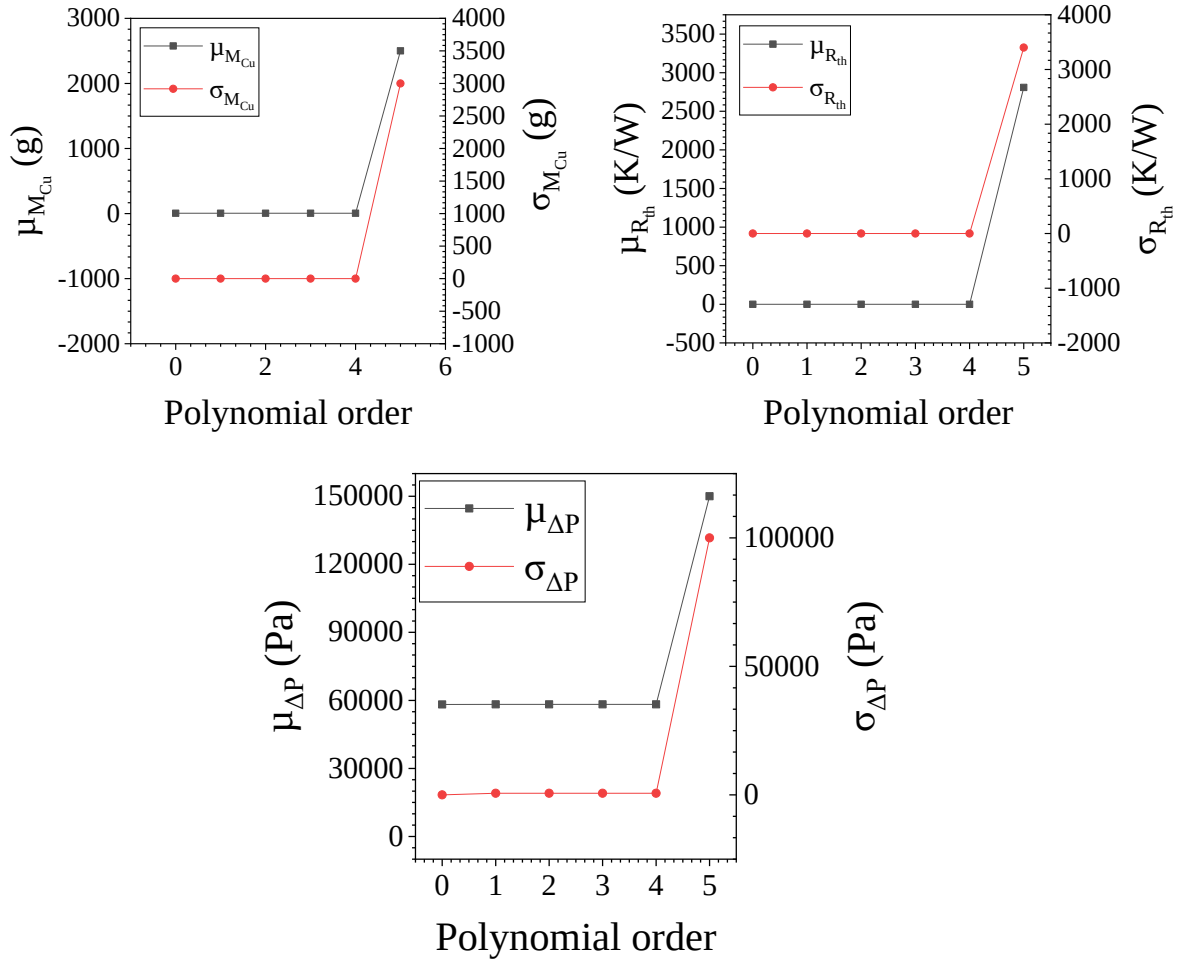


Figure 7-23: Shows the convergence plots using PCE method.

7.5.2.2.4 Surrogate modelling for RDO

From statistical moments of means and standard deviations of output objective functions calculated, new RBF surrogate models are formed. Normalisation of input design variables are conducted using Eq. 6.6 for RBF models coverage rate improvement. From subchapter 7.5.1, the objective functions of mass of copper heat sink (M_{Cu}), total thermal resistance (R_{th}) and pressure drop (ΔP) will be divided into means and standard deviations (refer Table 6-12). The accuracy of RBF models for statistical moments can be assured since the surrogate models are used from the same pre-trained RBF models in subchapter 7.5.1. The example of training the $\sigma_{\Delta P}$ statistical moment is shown in Table 7-8 where the accuracy parameters of RMSE, R^2 and maximum residual error tabulated. The parameters indicates accurate RBF model compared to numerical data. Other RBF surrogate models are not shown here.

Table 7-8: The accuracy parameters for training, validation and testing sets of surrogate model of $\sigma_{\Delta P}$ objective function using RBF.

$\sigma_{\Delta P}$			
Accuracy parameter	Training set	Validation set	Testing set
RMSE (Pa)	3.854736	6.334764	4.084721
R^2	0.999901	0.996172	0.999662
Maximum Residual Error (%)	0.0385368	0.136382	0.079372

7.5.2.2.5 Multi-objective robust design optimisation on single objective function

Using the procedure in subchapter 7.5.2.1.2, the ANN models of statistical moments will be utilised within NSGA-II algorithm for optimisation process. Surrogate models with the same units will be optimise simultaneously based on their means and standard deviations. The simultaneous optimisation will aim to achieve the Pareto front of competing objectives in order to search for mean which is insensitive towards the standard deviation. The standard deviations of output parameter which have constraints will lead to robust design giving range objective functions values to choose from. The robust design optimisation formulations can be referred in Eq. 6.13 to Eq. 6.15 respectively.

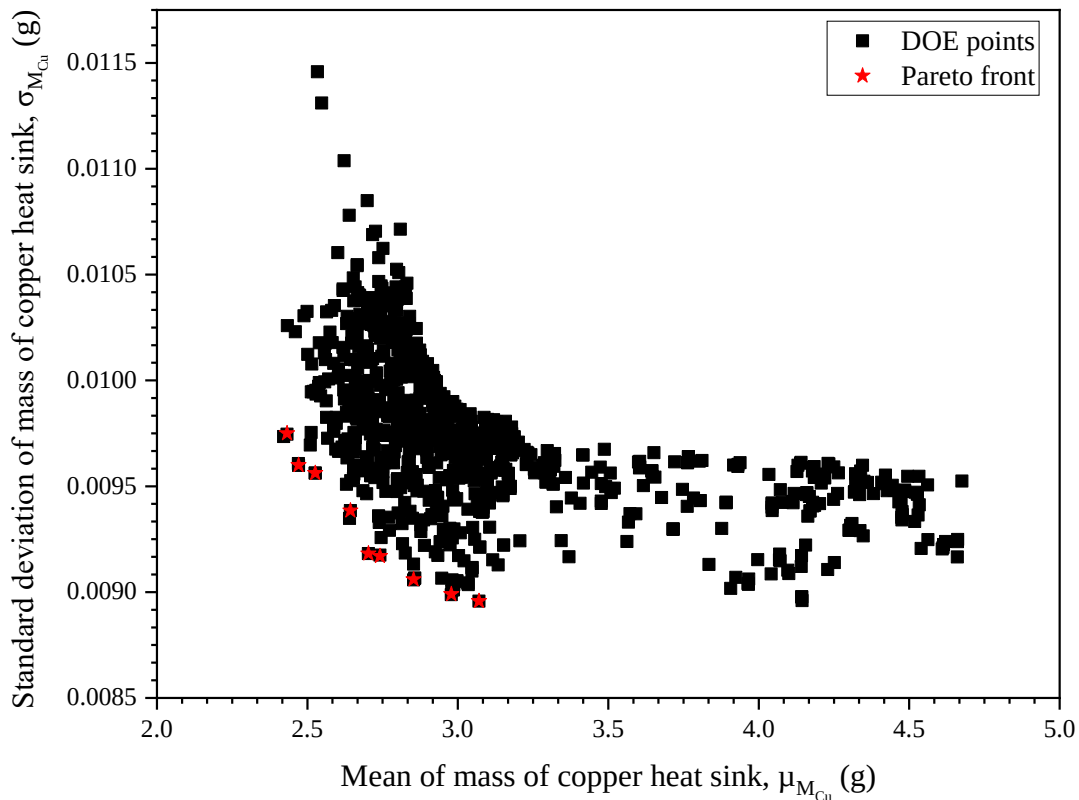


Figure 7-24: Shows the Pareto front and DOE points plots for $\sigma_{M_{Cu}}$ vs $\mu_{M_{Cu}}$.

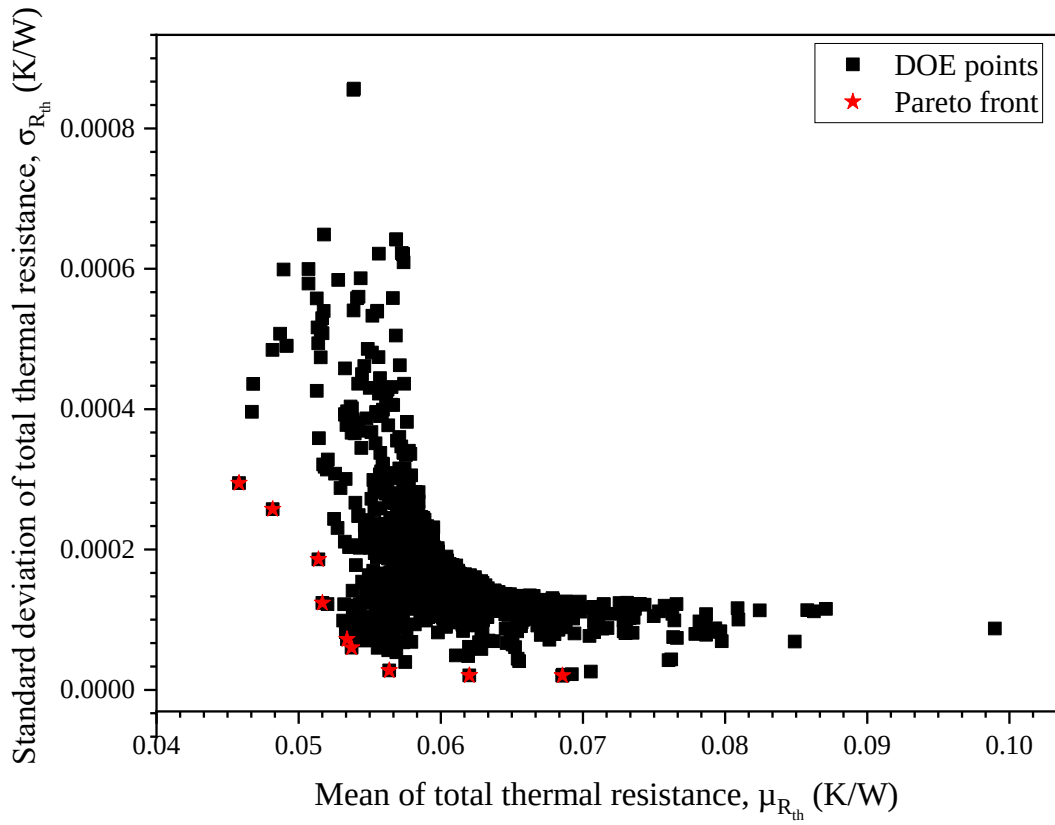


Figure 7-25: Shows the Pareto front and DOE points plots for $\sigma_{R_{th}}$ vs $\mu_{R_{th}}$.

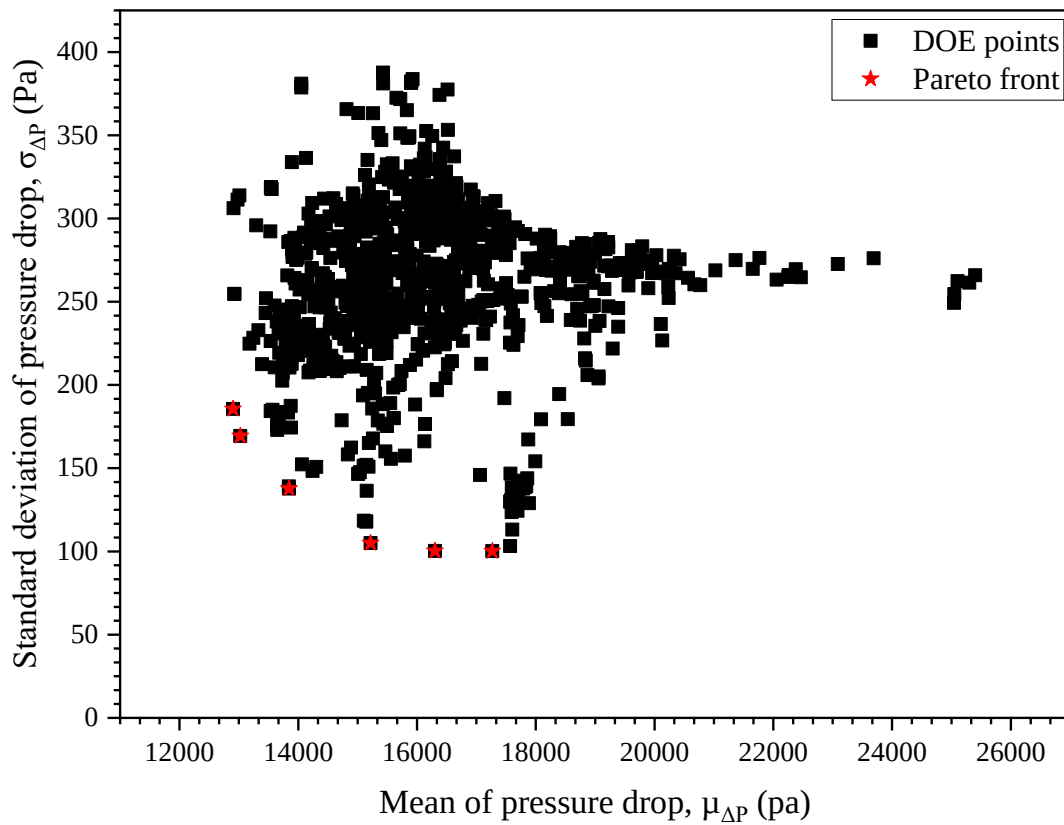


Figure 7-26: Shows the Pareto front and DOE points plots for $\sigma_{\Delta P}$ vs $\mu_{\Delta P}$.

Table 7-9: Shows the Pareto Front data for $\sigma_{M_{Cu}}$ vs $\mu_{M_{Cu}}$.

Design variables						$\mu_{M_{Cu}}, \text{ g}$	$\sigma_{M_{Cu}}, \text{ g}$
$W_{ch}, \text{ mm}$	$H_{ch}, \text{ mm}$	$H_b, \text{ mm}$	$\theta_{fin}, \text{ mm}$	$q'', \text{ W/m}^2$	Re		
1.378812	0.524012	0.517883	59.39460	1892448	2275.650	2.432398	0.009746
1.312698	0.535853	0.524006	50.58695	1372901	2795.904	2.470626	0.009598
1.229435	0.536517	0.515009	56.95853	1172998	3014.467	2.526401	0.009562
1.207093	0.593736	0.517021	55.77318	1051625	3313.763	2.640309	0.009348
1.265769	0.565657	0.569358	57.97334	1757774	3456.589	2.702733	0.009182
1.302527	0.836036	0.523499	54.85939	1118507	3071.952	2.853275	0.009058
1.036049	0.526826	0.580805	45.29765	1094727	2146.253	2.978614	0.008988
1.171744	0.764212	0.562740	59.89202	1214502	3530.480	3.070272	0.008957

Table 7-10: Shows the Pareto Front data for $\sigma_{R_{th}}$ vs $\mu_{R_{th}}$.

Design variables						$\mu_{R_{th}}, \text{ K/W}$	$\sigma_{R_{th}}, \text{ K/W}$
$W_{ch}, \text{ mm}$	$H_{ch}, \text{ mm}$	$H_b, \text{ mm}$	$\theta_{fin}, \text{ mm}$	$q'', \text{ W/m}^2$	Re		
1.08680	1.821677	0.672248	59.59142	1898882	3828.851	0.048176	0.000257
1.055689	1.973280	0.539250	47.27230	1390632	3802.012	0.051392	0.000186
1.048929	1.930310	0.622214	46.13180	1241126	3626.969	0.051670	0.000124
1.329507	1.940447	0.854072	53.35436	1321724	3753.168	0.053401	7.21E-05
1.347476	1.940447	0.854072	53.01570	1321724	3753.168	0.053724	6.00E-05
1.360150	1.881433	0.988370	59.59006	1845522	3828.731	0.056371	2.75E-05
1.267706	1.605307	0.942933	57.66984	1842218	3473.692	0.062009	2.06E-05
1.042199	1.962527	0.571381	49.33732	1205421	2803.705	0.068562	2.04E-05

Table 7-11: Shows the Pareto Front data for $\sigma_{\Delta P}$ vs $\mu_{\Delta P}$.

Design variables						$\mu_{\Delta P}, \text{ Pa}$	$\sigma_{\Delta P}, \text{ Pa}$
$W_{ch}, \text{ mm}$	$H_{ch}, \text{ mm}$	$H_b, \text{ mm}$	$\theta_{fin}, \text{ mm}$	$q'', \text{ W/m}^2$	Re		
1.128796	1.900957	0.572092	56.13824	1488231	2083.358	12903.20	185.5887
1.044511	1.858448	0.966258	46.28208	1782439	2080.588	13024.86	169.3285
1.159337	1.948759	0.600741	47.89835	1086451	2118.947	13842.51	139.0150
1.013603	1.719916	0.765540	46.34814	1137479	2195.703	15218.18	105.0308
1.036354	1.779472	0.887856	45.57986	1166344	2273.623	16302.50	100.2488
1.036181	1.970532	0.821216	45.29806	1166344	2372.551	17268.03	100.2060

To study the multi-objective robust design optimisation of single objective function, the Pareto fronts plotted are in terms of means and standard deviations of output objectives. Figure 7-24 to Figure 7-26 show the Pareto front of three objective functions based on their statistical moments. Looking at Figure 7-24, the DOE points and Pareto front plots are based on the $\sigma_{M_{Cu}}$ vs $\mu_{M_{Cu}}$ statistical moments. From the figure Pareto front, it can be seen that an increase in $\sigma_{M_{Cu}}$ will lead to the decrease of $\mu_{M_{Cu}}$ and vice versa. The numbers of the Pareto front plot are tabulated in Table 7-9. Going down the table it can be seen that the $\mu_{M_{Cu}}$ increases from 2.4324 g to 3.0703 g inducing 26.2% increase while $\sigma_{M_{Cu}}$ reduces from 0.00975 g to 0.00896 g giving 8.1% decrease. Small change in $\sigma_{M_{Cu}}$ leads to large change in $\mu_{M_{Cu}}$. At high magnitude of $\sigma_{M_{Cu}}$, small value of $\mu_{M_{Cu}}$ recorded meaning the output curve is steep.

Similarly, for the Pareto fronts of both $\sigma_{R_{th}}$ vs $\mu_{R_{th}}$ and $\sigma_{\Delta P}$ vs $\mu_{\Delta P}$ respectively, the same trend of Pareto front trends are recorded where an increase in one objective will result in the decrease of the other. Going down the Table 7-10 for $\sigma_{R_{th}}$ vs $\mu_{R_{th}}$, the $\mu_{R_{th}}$ increases by 48.7% changing from 0.0451 K/W to 0.0686 K/W respectively. On the other hand, the $\sigma_{R_{th}}$ changes from 0.000295 K/W to 0.0000204 K/W with 93.1% reduction. For $\sigma_{\Delta P}$ vs $\mu_{\Delta P}$, going down Table 7-11, the $\mu_{\Delta P}$ increases from 12903.2 Pa to 16302.5 Pa producing 26.3% change while $\sigma_{\Delta P}$ reduces from 185.6 Pa to 100.2 Pa with 46% change.

Comparing the robust design optimisation solutions with the deterministic solutions under subchapter 7.5.2.1.1 in Table 7-5, the M_{Cu} has the lowest deterministic solution of 2.316 g while based on robust design optimisation the $\mu_{M_{Cu}}$ in ranges between 2.432 g to 3.070 g giving 5% to 32.5% increase from the deterministic solution. Similarly, for R_{th} , the deterministic solution shows at 0.0466 K/W. From the robust design solution, the $\mu_{R_{th}}$ ranges between 0.0482 K/W and 0.0686 K/W contributing to the percentage increase of 6.7% to 32.7% respectively. Next, for ΔP , the deterministic solution holds the magnitude at 10018.61 Pa. Based on the robust design solution, the $\mu_{\Delta P}$ ranges between 12903.2 Pa to 16302.5 Pa giving percentage increase of 28.8% and 62.7% respectively.

The robust design solutions have slightly higher value compared to deterministic solution which is useful when dealing with manufacturing tolerances while still having acceptable superior performance. The output results would have insensitive relationship towards the input design variables variation.

7.5.2.2.5.1 Multi-objective robust design optimisation on single objective optimisation with constrained output standard deviations

To study further on the robust optimisation, based on the formulations that have been defined in subchapter 6.5.2.2.7, the normalised standard deviation constraints will be treated as variable to observe the effect on the change onto the Pareto front plots of multi-objective optimisation of single objective function. The Pareto front plots are shown in Figure 7-27 to Figure 7-29 respectively. For the M_{Cu} objective, referring Figure 7-27, from the constraint of $0.0 < \sigma_{M_{Cu},norm} < 1.0$ it has the closest Pareto front to the origin indicating the lowest magnitudes of $\sigma_{M_{Cu}}$ and $\mu_{M_{Cu}}$ parameters.

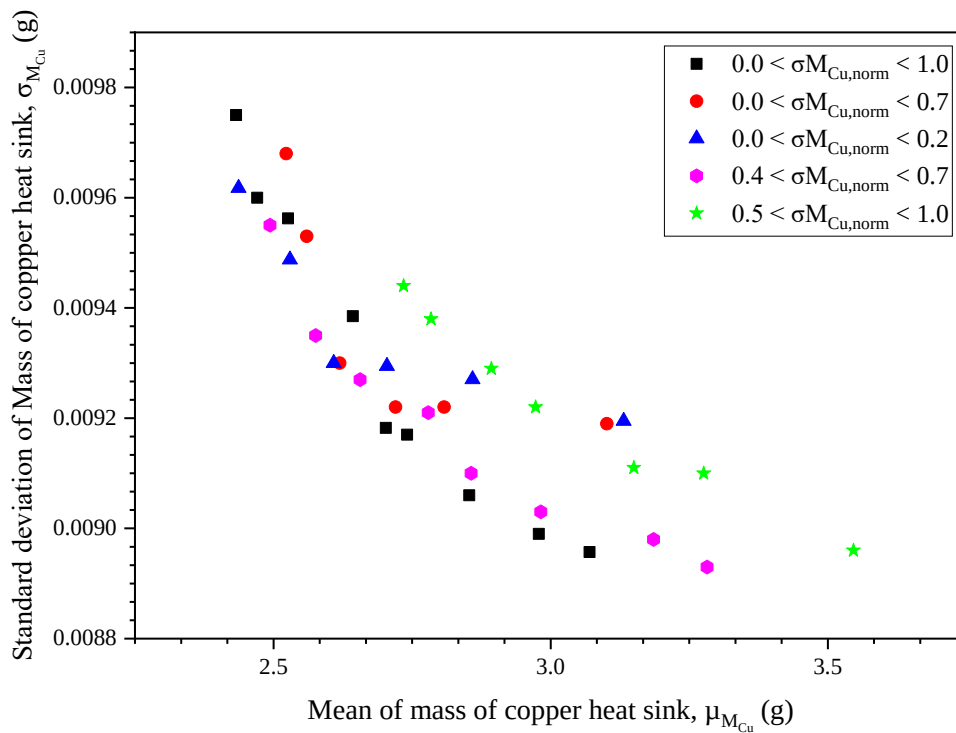


Figure 7-27: Shows the Pareto front of $\sigma_{M_{Cu}}$ vs $\mu_{M_{Cu}}$ for different constraints of $\sigma_{M_{Cu},norm}$.

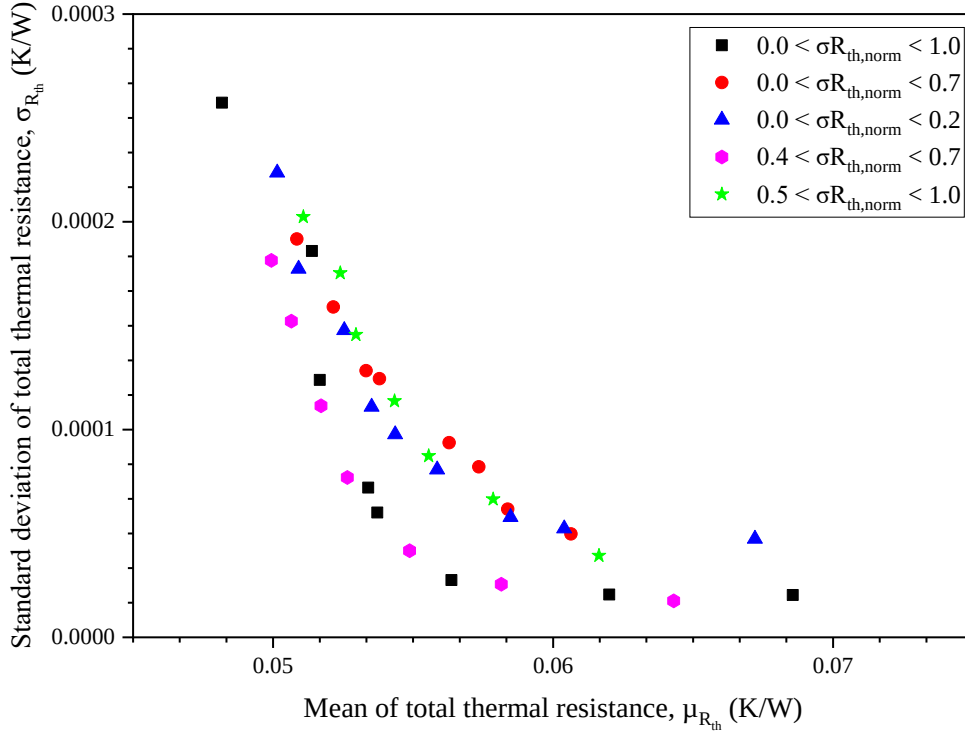


Figure 7-28: Shows the Pareto front of $\sigma_{R_{th}}$ vs $\mu_{R_{th}}$ for different constraints of $\sigma_{R_{th},norm}$.

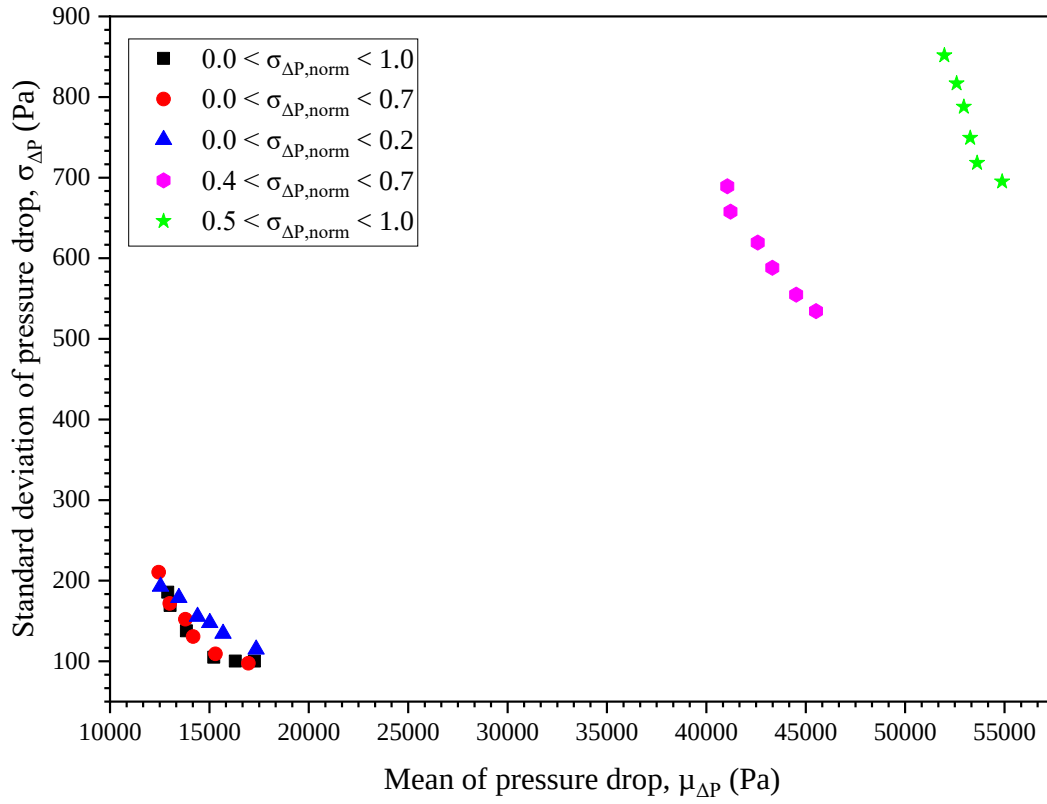


Figure 7-29: Shows the Pareto front of $\sigma_{\Delta P}$ vs $\mu_{\Delta P}$ for different constraints of $\sigma_{\Delta P,norm}$.

At $0.0 < \sigma_{M_{Cu},norm} < 0.7$ constraint, the Pareto front plot changes with higher rate of increase of $\mu_{M_{Cu}}$. Constraining at $0.0 < \sigma_{M_{Cu},norm} < 0.2$, the Pareto front is almost similar trend recorded like $0.0 < \sigma_{M_{Cu},norm} < 0.7$ constraint. For constraint of $0.4 < \sigma_{M_{Cu},norm} < 0.7$, the Pareto front trend is equal to $0.0 < \sigma_{M_{Cu},norm} < 1.0$. Finally, at constraint of $0.5 < \sigma_{M_{Cu},norm} < 1.0$, the Pareto front shifts to the right indicating higher M_{Cu} . Higher M_{Cu} is due to the fact that new limits of input design variables are enforced.

For R_{th} objective, looking at Figure 7-28, at constraint of $0.0 < \sigma_{R_{th},norm} < 1.0$ and $0.4 < \sigma_{R_{th},norm} < 0.7$, the Pareto fronts are the closest to the origin and having the same trend. The other three constraints of $0.0 < \sigma_{R_{th},norm} < 0.7$, $0.0 < \sigma_{R_{th},norm} < 0.2$ and $0.5 < \sigma_{R_{th},norm} < 1.0$ respectively, all of them show the same trend of Pareto fronts and the plots are shifted to the right showing higher R_{th} recorded. Similarly, the increase of R_{th} is caused by the input design variables are subjected to new limits.

For ΔP objective, referring Figure 7-29, the three constraints of $0.0 < \sigma_{\Delta P,norm} < 1.0$, $0.0 < \sigma_{\Delta P,norm} < 0.7$ and $0.0 < \sigma_{\Delta P,norm} < 0.2$ shows overlapping Pareto fronts and the closest to the origin. For the $0.4 < \sigma_{\Delta P,norm} < 0.7$ constraint, the Pareto front plot increases drastically in both directions of $\sigma_{\Delta P}$ and $\mu_{\Delta P}$ respectively. The same thing occur for $0.5 < \sigma_{\Delta P,norm} < 1.0$ constraint with even higher magnitude of $\sigma_{\Delta P}$ and $\mu_{\Delta P}$ compared to $0.4 < \sigma_{\Delta P,norm} < 0.7$ constraint. As discussed earlier, the limits of input design variables changes with the varied constraints inducing the increase of both $\sigma_{\Delta P}$ and $\mu_{\Delta P}$ statistical moments.

7.5.2.2.6 Multi-objective robust design optimisation on multi-objective functions with constrained output standard deviation.

From subchapter 7.5.2.2.5, the multi-objective optimisation studies on a single objective function have been carried out optimising the statistical moments simultaneously. Therefore, within this subchapter the multi-objective optimisation on multiple objective functions will be conducted using the constraints of all three $\sigma_{R_{th},norm}$, $\sigma_{\Delta P,norm}$ and $\sigma_{M_{Cu},norm}$ statistical moments to see the effect on the performance of heat sink based on the Pareto front plots. Referring Figure 7-30, the Pareto fronts of R_{th} vs M_{Cu} are plotted based on different statistical moment constraints. The deterministic solution shows the lowest value of Pareto front closer to the origin. For constraint of (C) and (B) show almost similar trend and higher magnitudes of Pareto fronts compared to deterministic solution due to the enlargement of H_{ch} and W_{ch} increasing the A_{eff} for more heat flow. For (D) constraint the Pareto front shifted even higher

with the same trend for the same reason. Looking at constraint of (A), it has the highest Pareto front in terms of R_{th} while lower rate of increase in M_{Cu} compared to other pareto fronts due to smaller θ_{fin} size.

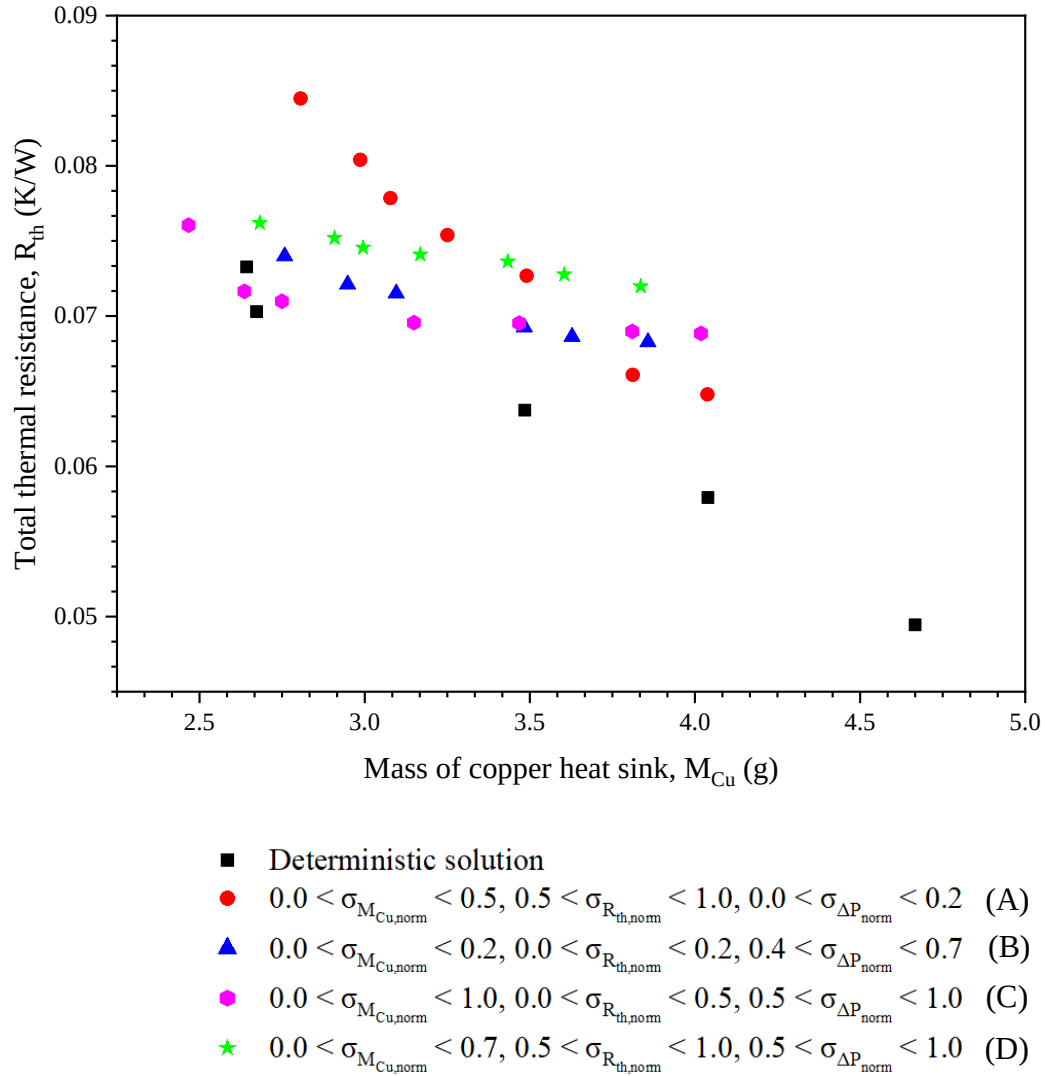


Figure 7-30: Shows the Pareto fronts plot of R_{th} vs M_{Cu} for different values of constraints of $\sigma_{R_{th,norm}}$, $\sigma_{\Delta P,norm}$ and $\sigma_{M_{Cu,norm}}$ parameters respectively.

Looking at Figure 7-31, the ΔP vs M_{Cu} Pareto fronts are plotted using the same constraints. Based on the figure, the deterministic solution and at constraint of (A), the Pareto fronts overlapped on top of each other indicating the same magnitudes and trends since the input design variables are similar. For the constraints of (C) and (D), the Pareto fronts shift way higher indicating larger values ΔP recorded and they have the same magnitudes and trends due to θ_{fin} getting smaller with almost similar sizes of H_{ch} and W_{ch} respectively. For constraint of (B), the highest Pareto front plotted making it to be unfeasible and not desirable.

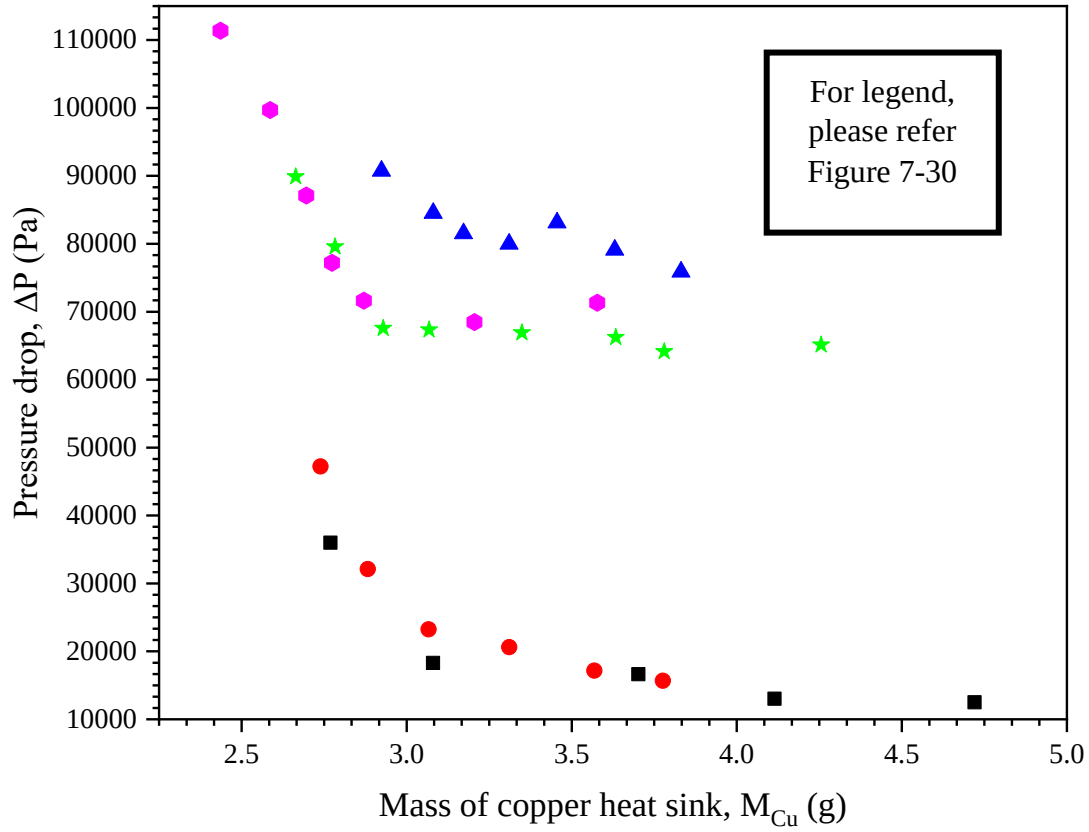


Figure 7-31: Shows the Pareto fronts plot of ΔP vs M_{Cu} for different values of constraints of $\sigma_{R_{th,norm}}$, $\sigma_{\Delta P,norm}$ and $\sigma_{M_{Cu,norm}}$ parameters respectively/.

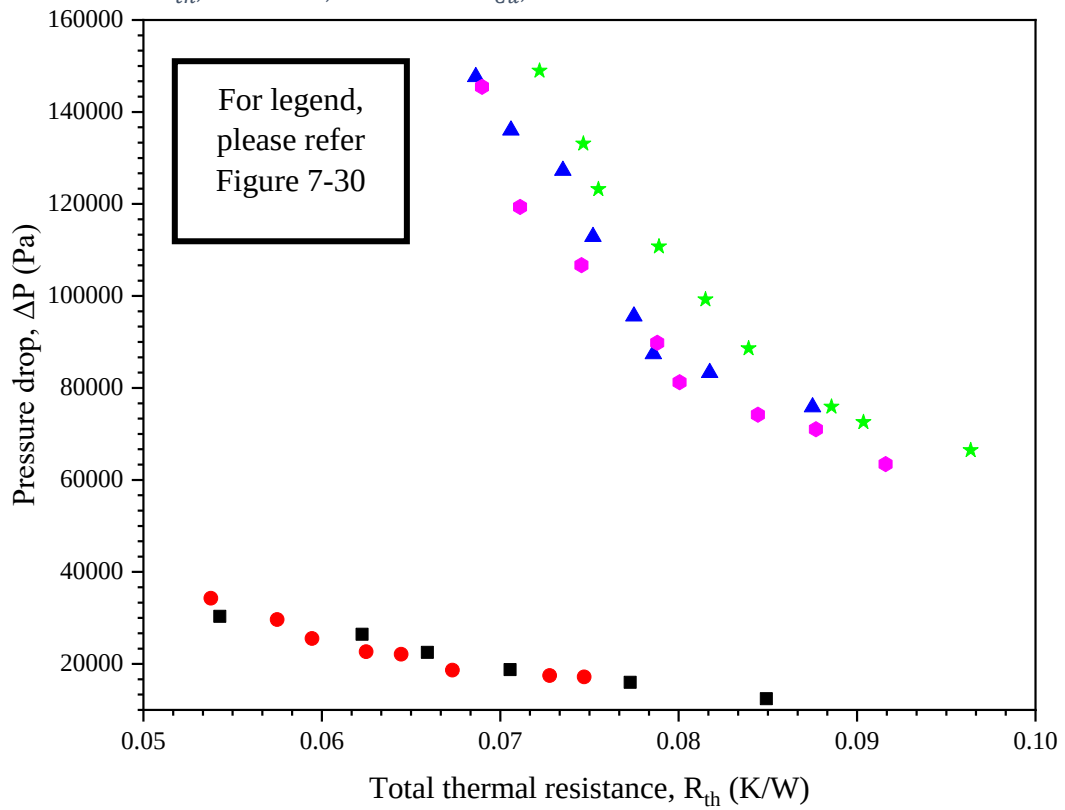


Figure 7-32: Shows the Pareto fronts plot of ΔP vs R_{th} for different values of constraints of $\sigma_{R_{th,norm}}$, $\sigma_{\Delta P,norm}$ and $\sigma_{M_{Cu,norm}}$ parameters respectively.

Based on Figure 7-32, the ΔP vs R_{th} Pareto fronts are plotted. From the figure, deterministic solution has the same magnitudes and trends when compared with (A) constraint which possess the lowest ΔP and R_{th} respectively. The other three constraints of (B), (C) and (D) respectively, the Pareto fronts increases significantly in both direction of ΔP and R_{th} making the performance non-desireable due to the significant decrease of both H_{ch} and W_{ch} leading to very small A_{eff} reducing heat transfer rate.

7.6 Summary

The employment of liquid cooling for high heat flux removal has been researched widely. Within this chapter, the utilisation of secondary channel within double inlet-outlet serpentine mini-channel heat sink has been conducted to improve thermal-hydraulic performance of heat sink by breaking the thermal and hydrodynamic boundary layers and reducing liquid velocity within channel improving the heat transfer and pressure drop respectively. Diamond shaped fin has been used since it holds the most superior thermal performance (refer Chapter 6).

From the results, the total thermal resistance (R_{th}) reduced monotonically as liquid velocity increases for D-SMDF heat sink. The use of D-SMDF heat sink improves the R_{th} value by 17.6% reduction compared to D-SMPF heat sink due to the breaking of thermal boundary layers and flow re-initialisation. For the Average Nusselt number (Nu_{avg}) parameter, D-SMDF heat sink improves by 28% change compared to D-SMPF heat sink, since more heat transfer occurred. For the ΔP , the D-SMDF heat sink improves by 48.7% change compared to D-SMPF heat sink, due the liquid velocity reduces within main channel as liquid diverted through the secondary channel. From the PEC index perspective, the D-SMDF heat sink has 1.73 times higher PEC index compared to D-SMPF heat sink. The D-SMDF heat sink undergoes parametric analysis to study the effect of changing different parameters and boundary conditions onto the thermal-hydraulic performance of heat sink.

For the parametric analysis, changing the height of channel (H_{ch}) from 1.5 mm to 2.0 mm will have reduction in both R_{th} and ΔP by 21.6% and 18.4% respectively. This is due to the increase in effective area (A_{eff}) for heat transfer and larger channel reducing the liquid velocity. For channel width (W_{ch}), changing it from 0.8 mm to 1.0 mm would also lead to the decrease of both R_{th} and ΔP by 9.8% and 9.4% due to the same reason as changing H_{ch} . For fin angle (θ_{fin}), going from 45° to 60° results in the decrease of R_{th} by 3.7% and increase of ΔP by 18.5% respectively. As the θ_{fin} increases, the liquid velocity and A_{eff} increases when going through the secondary channel, therefore, the convective heat transfer and dynamic pressure increases.

For heat flux (q'') variation, increasing the magnitude from 100 W/cm^2 to 200 W/cm^2 will increase the $T_{surf,max}$ by 18.8%. The ΔP is only slightly affected since the viscosity and density of water decreases.

Single- and multi-objective optimisation process were conducted to further optimise the D-SMDF heat sink. Two different approach of deterministic and robust design optimisation framework were used. Under the deterministic optimisation study, single-objective optimisation study is carried out individually which can be achieved easily to find the minimum value of objective functions of R_{th} , ΔP and M_{Cu} . But single-objective optimisation is not feasible since only one objective is minimised while the other two will have a very large or moderately large value. To solve this issue, multi-objective optimisation process is conducted to optimise all objective functions simultaneously obtaining the competing objectives. For optimising R_{th} vs M_{Cu} , the R_{th} drops by 35% while having M_{Cu} increase by 76.6%. Optimizing ΔP vs R_{th} , the R_{th} rose by 56.4% and ΔP reduces by 59.1% respectively. Lastly, optimising ΔP vs M_{Cu} , the M_{Cu} increased by 70.5% and ΔP drops by 65.3% respectively. Optimising M_{Cu} , R_{th} and ΔP leads to Pareto surface, therefore, going close to the zero value in the Cartesian plane, the ΔP increases while the other two objective functions decreases.

Robust design optimisation framework covers the uncertainty analysis where it is subjected to the input design variables corresponding to the distribution of output objective functions. The importance of including uncertainty is due to the manufacturing tolerances and operating conditions which are very unpredictable. The multi-objective optimisation of single objective function are carried first in terms of means and standard deviations. Then, multi-objective optimisation of multiple objective functions are processed for all objective functions of M_{Cu} , R_{th} and ΔP respectively. Then, constraining the normalised standard deviations is executed to observe the effect on the Pareto fronts of multi-objective optimisation.

Chapter 8 : Design optimisation of double inlet-outlet serpentine mini-channel heat sink with diamond fins and through-holes (D-SMDF-TH)

8.1 Introduction

To continue the analysis of double inlet-outlet serpentine mini-channel heat sink, the exact heat sink design within Chapter 7 is used where secondary channel are utilised by embedding the diamond fins within the plate fins but the novelty of new design in this chapter is by including the through-holes which recommended by Cao et al. [37]. The numerical simulations of conjugate heat transfer model of double inlet-outlet serpentine mini-channel heat sink with diamond fins and through-holes (D-SMDF-TH) are conducted using COMSOL Multiphysics v5.4 CFD software. Similarly like under subchapter 7.1, three major parameters used to compare the performance between heat sinks are total thermal resistance (R_{th}), pressure drop (ΔP), Average Nusselt Number (Nu_{avg}) respectively. The Performance evaluation Criteria (PEC) index is calculated to measure the combination of thermal-hydraulic performance while taking account the occupied space which will be discussed in detail under subchapter 8.4.2.3. After that, the optimisation of D-SMDF-TH heat sink will be executed to further optimise the thermal-hydraulic performance of heat sink.

8.2 Description of design of double inlet-outlet serpentine mini-channel heat sink with diamond fins and through-holes

To describe the design of D-SMDF-TH heat sink, the same exact design of D-SMDF heat sink within subchapter 7.2 will be utilised with a minor difference of the application of through-holes in this work. All design procedures and descriptions are explained in details within subchapter 7.2 for reference. Looking at Figure 8-1, only the dimensioning of through-holes is shown since the other dimensions have been labelled clearly in Figure 7-1. The detail dimensioning of the D-SMDF-TH heat sink is tabulated in Table 8-1.

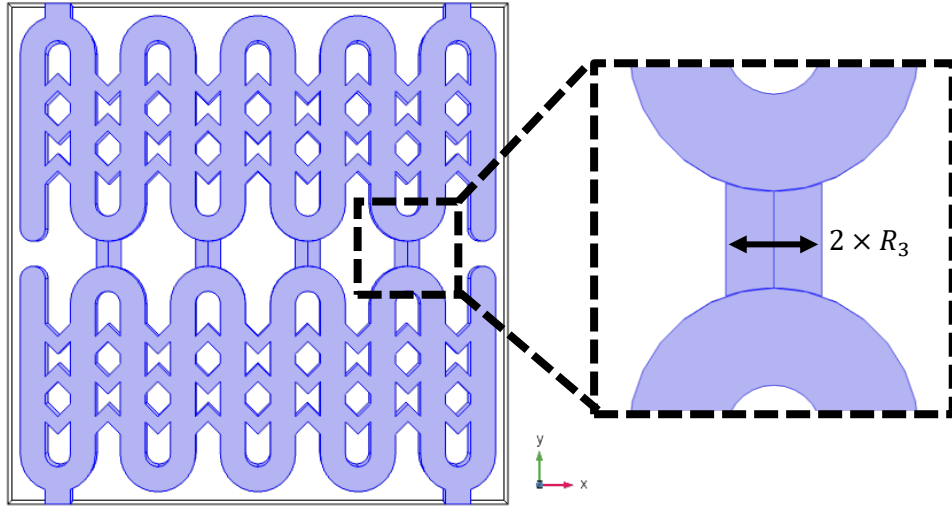


Figure 8-1: Shows the D-SMDF-TH heat sink with through-holes dimension.

Table 8-1: Shows the tabulated data for dimensions of D-SMDF-TH heat sink.

Characteristics	D-SMDF-TH
Material	Copper
Heat sink dimensions, width x length x height, $W \times L \times H$ (mm)	20 x 20 x 2.5
Main channel width, W_{ch} (mm)	1
Fin Width, W_f (mm)	1
Channel depth, H_{ch} (mm)	2
Substrate thickness, H_b (mm)	0.5
Number of mini-channels, n	10
Hydraulic diameters, H_b (mm)	1.333
Outer radius, R_1 (mm)	1.5
Inner radius, R_2 (mm)	0.5
Through-hole radius, R_3 (mm)	0.5
Number of fins per column, N_{cf}	2 (each side)
Secondary channel width, W_{sc} (mm)	0.495
Secondary channel length, l_{sc} (mm)	0.7
fin length, l_f (mm)	0.4
fin pitch, p_f (mm)	1.6
second fin length, l_f (mm)	0.4
Fins oblique angle, θ_{fin} (deg)	45

To calculate the effective heat transfer area (A_{eff}) where heat transfers from solid to liquid domain, it is based on two major components of walls area (A_{wall}) and bases area (A_{base}) of the primary and secondary mini-channels. The top walls are adiabatic where no exchange of heat is considered. The formulation of A_{eff} can be referred to Eq. 7.1 To calculate the A_{eff} for D-SMDF-TH heat sink, refer appendix A 3.2 for full equations.

8.3 CFD simulation approach

Refer sub-chapter 6.3.1 and 6.3.2 for the process flow of solving the governing equations with the assumptions and boundary conditions. The only difference there is the existence of through-holes, which gives extra volume of liquid domain.

8.3.1 Validation code with literature

To further the simulations of D-SMDF-TH heat sink, a validation code process is carried out to compare the CFD model created with results in literature conducted by Al-Hasani et al. [321] using Model F. They have investigated the performance of using serpentine mini-channel with single inlet and double outlets and the application of pin-fin configuration within the mini-channel. The heat flux applied underneath heat sink is at 26.67 kW/m^2 with Reynolds number varied between 800 and 1600 respectively. The heat sink foot print is at $75 \text{ mm} \times 50 \text{ mm}$ with thickness of 5 mm . The comparison of current CFD model with published work is conducted using Nusselt number (Nu) vs Reynolds number (Re) plot which can be referred in Figure 8-2. From the figure, the results correlate with each other indicating high accuracy. Therefore, the further simulations can be executed with confidence.

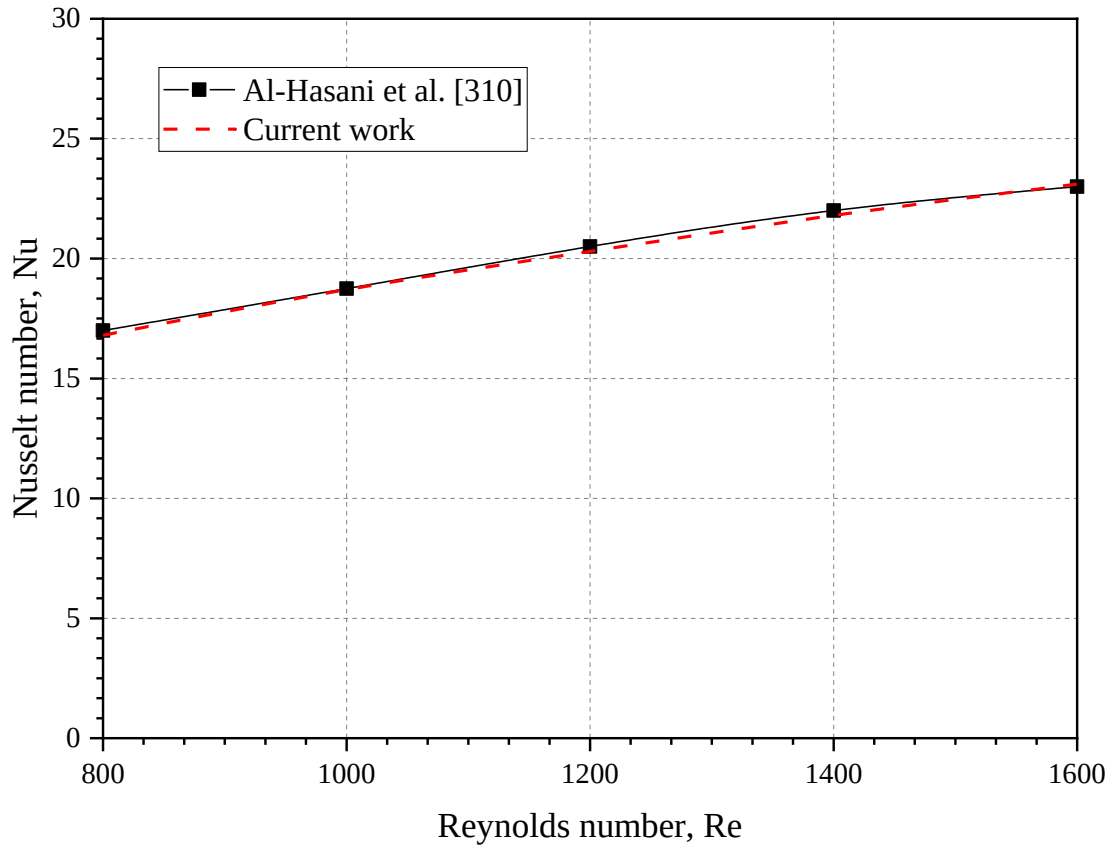


Figure 8-2: Shows the plot of Nusselt Number (Nu) vs Reynolds number (Re)

8.3.2 Grid independence study

Using the same procedure stated in subchapter 7.3.4, four different grids are chosen with different mesh number to check which mesh number possesses the highest accuracy with the lowest computational time. To make sure the mesh number gives high accuracy results, relative error formula is used (refer Eq. 7.4), lower mesh number will be compared with highest mesh number. From Table 8-2, it can be observed that at Grid 3, the relative errors for both R_{th} and ΔP parameters are less than 2% which is considered accurate. Therefore, Grid 3 will be utilised for further computational simulations (refer appendix A 2.5 for mesh structure).

Table 8-2: Shows the R_{th} and ΔP at different mesh number for grid independence test

Heat sink model	Grid No.	Elements number	R_{th} (K/W)	R_{th} -E%	ΔP (Pa)	ΔP -E%
D-SMDF-TH	1	4264738	0.076451	2.8	5675.2	7.7
	2	5167356	0.077653	1.3	5541.6	5.1
	3	7979064	0.078552	0.1	5335.1	1.2
	4	9321275	0.078633	-	5270.3	-

8.4 Fluid flow and heat transfer characteristics using double inlet-outlet serpentine mini-channel heat sink with diamond fins and through-holes

8.4.1 Effect of Reynolds number (Re) on R_{th} , Nu_{avg} and ΔP

To measure the performance of using D-SMDF-TH heat sink compared to the D-SMDF heat sink from Chapter 7 and the base design of D-SMPF heats sink, three distinct parameters are used which are total thermal resistance (R_{th}), Average Nusselt Number (Nu_{avg}) and pressure drop (ΔP) respectively. The liquid velocity is governed by the Reynolds number (Re) ranges between 500 and 2300. The hydraulic diameter is kept constant at 1.333 mm calculated from channel width (W_{ch}) and channel height (H_{ch}) at 1 mm and 2 mm respectively (refer Eq. 2.1). A heat flux of 100 W/cm^2 is applied at the bottom surface of heat sink. Two diamonds fins are embedded evenly within the single plate fin. Four through-holes are used to connect the separated serpentine with radius (R_3) of 0.5 mm .

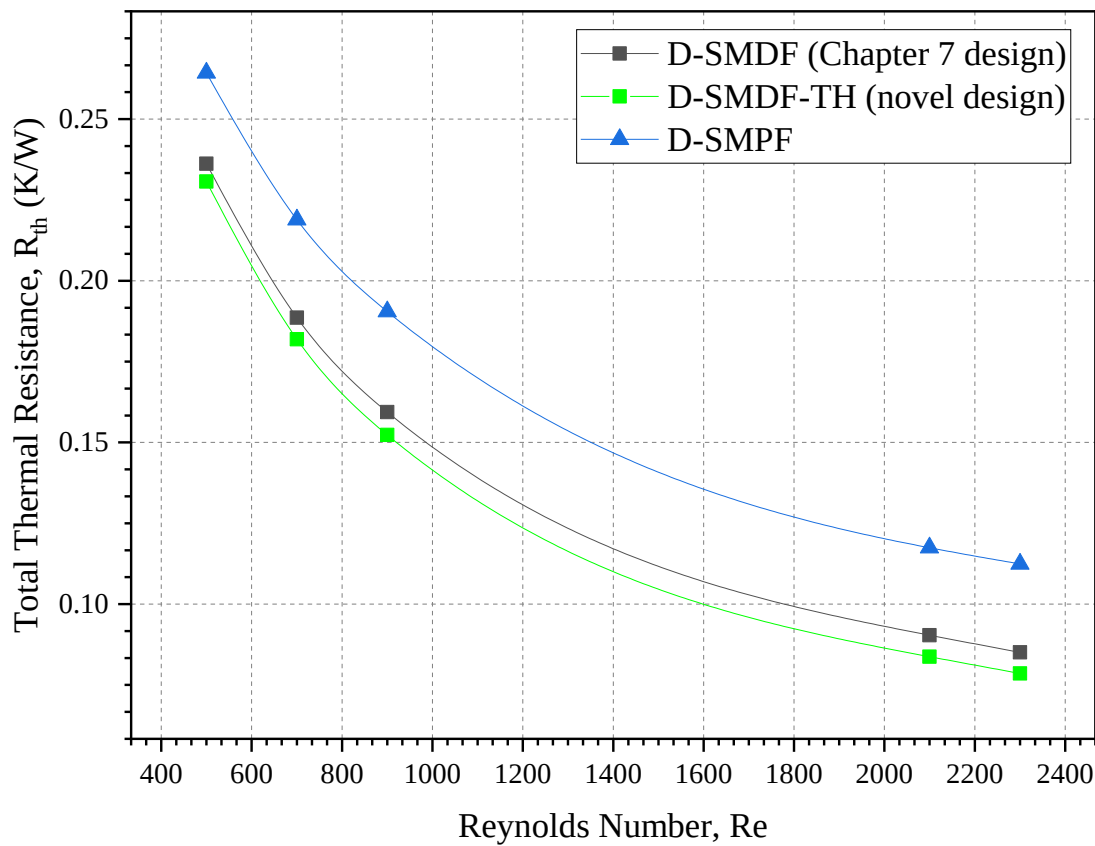


Figure 8-3: Shows the plots of total thermal resistance (R_{th}) against Re for different designs of double inlet-outlet serpentine mini-channel heat sink.

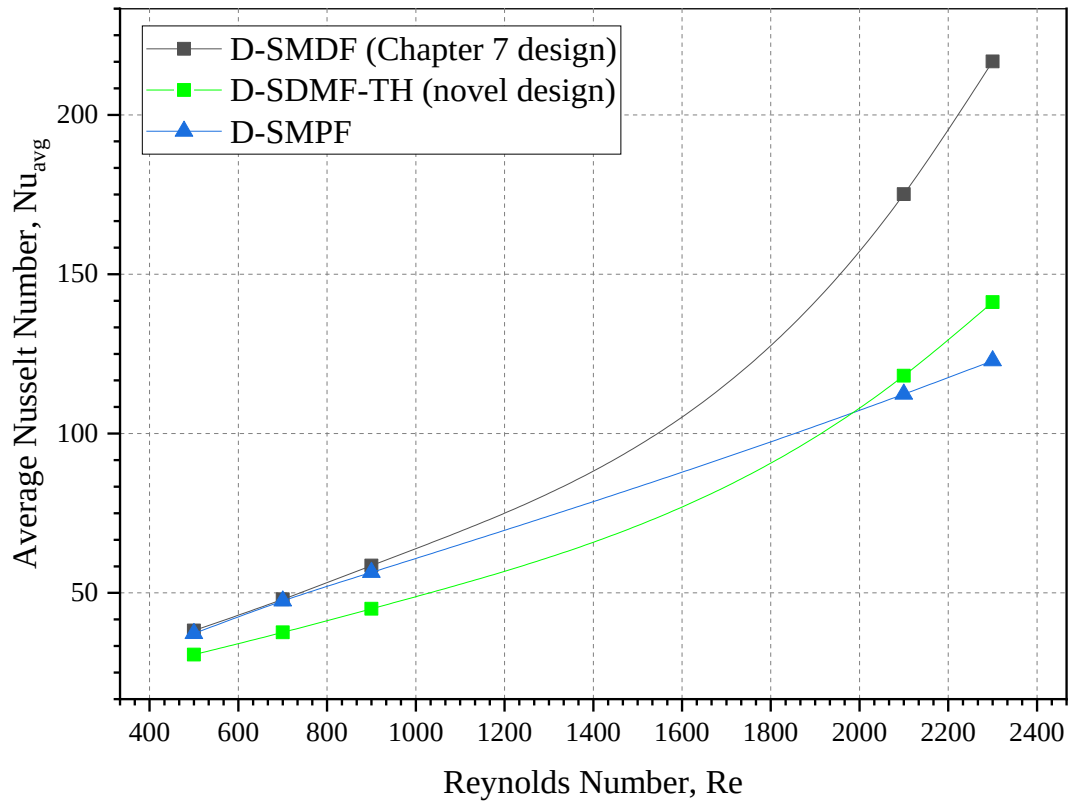


Figure 8-4: Shows the plots of Average Nusselt number (Nu_{avg}) against Re for different designs of double inlet-outlet serpentine mini-channel heat sink.

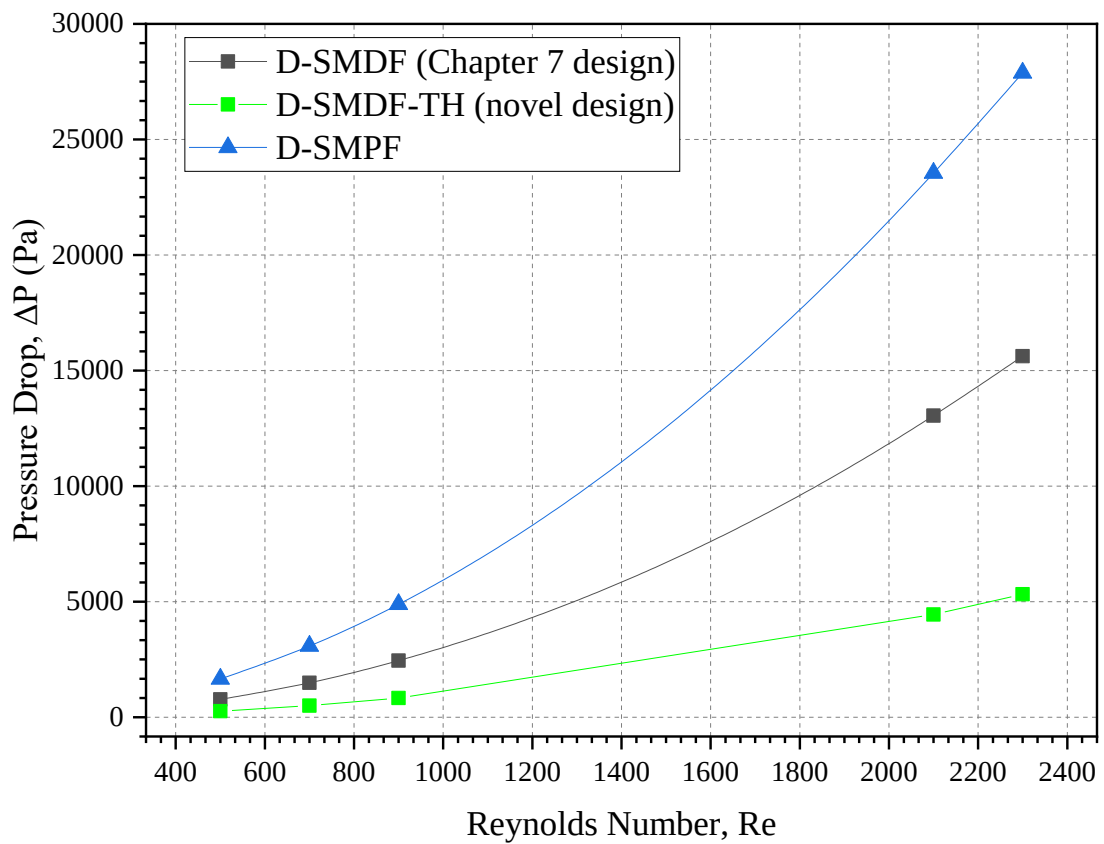


Figure 8-5: Shows the plots of pressure drop (ΔP) against Re for different designs of double inlet-outlet serpentine mini-channel heat sink.

From Figure 8-3, the R_{th} decreases monotonically as Re increases for all plots. Since the R_{th} is derived from $T_{surf,max}$ parameter (refer Eq. 3.53) where the magnitude also decreases as Re gets larger. At higher Re , the liquid velocity becomes faster increasing the rate of heat transfer by convection [307].

Looking at Figure 8-4, the Nu_{avg} increases as Re increases for all plots. As mentioned earlier, the heat transfer by convection improves when Re increases, thus, inducing to the enhancement of heat transfer coefficient (h_{avg}) leading to Nu_{avg} increase (refer Eq. 3.24).

Referring Figure 8-5, at increasing Re the ΔP (refer Eq. 3.56) also increases exponentially for all plots. This due to the fact that at large Re , the dynamic pressure of liquid increases due to the larger head losses and friction factors as it flows through channels and bends [308]. A point to be mentioned, ΔP is directly proportional to the squared of water velocity [318].

8.4.2 The effect of the D-SMDF-TH heat sink design on its thermal-hydraulic performance

The D-SMDF-TH heat sink uses both diamond fins and through-holes to improve its thermal-hydraulic performance and will be compared with both D-SMDF heat sink from Chapter 7 and D-SMPF heat sink. The dimensioning of D-SMDF-TH heat sink is tabulated in Table 8-1 and boundary conditions in sub-chapter 6.3.2 respectively.

8.4.2.1 Effect of D-SMDF-TH heat sink on R_{th} and Nu_{avg}

Referring Figure 8-3, within the range of Re tested, the D-SMDF-TH heat sink shows the lowest values of R_{th} compared to D-SMDF and D-SMPF heat sinks. By referring Figure 8-6 (C), it can be seen that base surface temperature contour of D-SMPF heat sink shows hot spots on both sides of the heat sink which is expected. From Figure 8-6 (A) and (B), the D-SMDF heat sink shows more bluish colour region compared to D-SMDF-TH heat sink, but looking at D-SMDF-TH heat sink the highest temperature on the surface is lower than the highest surface temperature of D-SMDF heat sink. Therefore, the D-SMDF-TH heat sink calculation of R_{th} will show the lowest value since it is derived from the highest surface temperature ($T_{surf,max}$) parameter. The D-SMDF heat sink has very low localised velocity region within the channels at the sides of heat sink, making heat transfer to be worse (Figure 7-7), hence, inducing localised high temperature region. The use of D-SMDF-TH heat sink will have an improvement of R_{th} relative to D-SMDF and D-SMPF heat sinks by 5.1% and 21.7% change respectively.

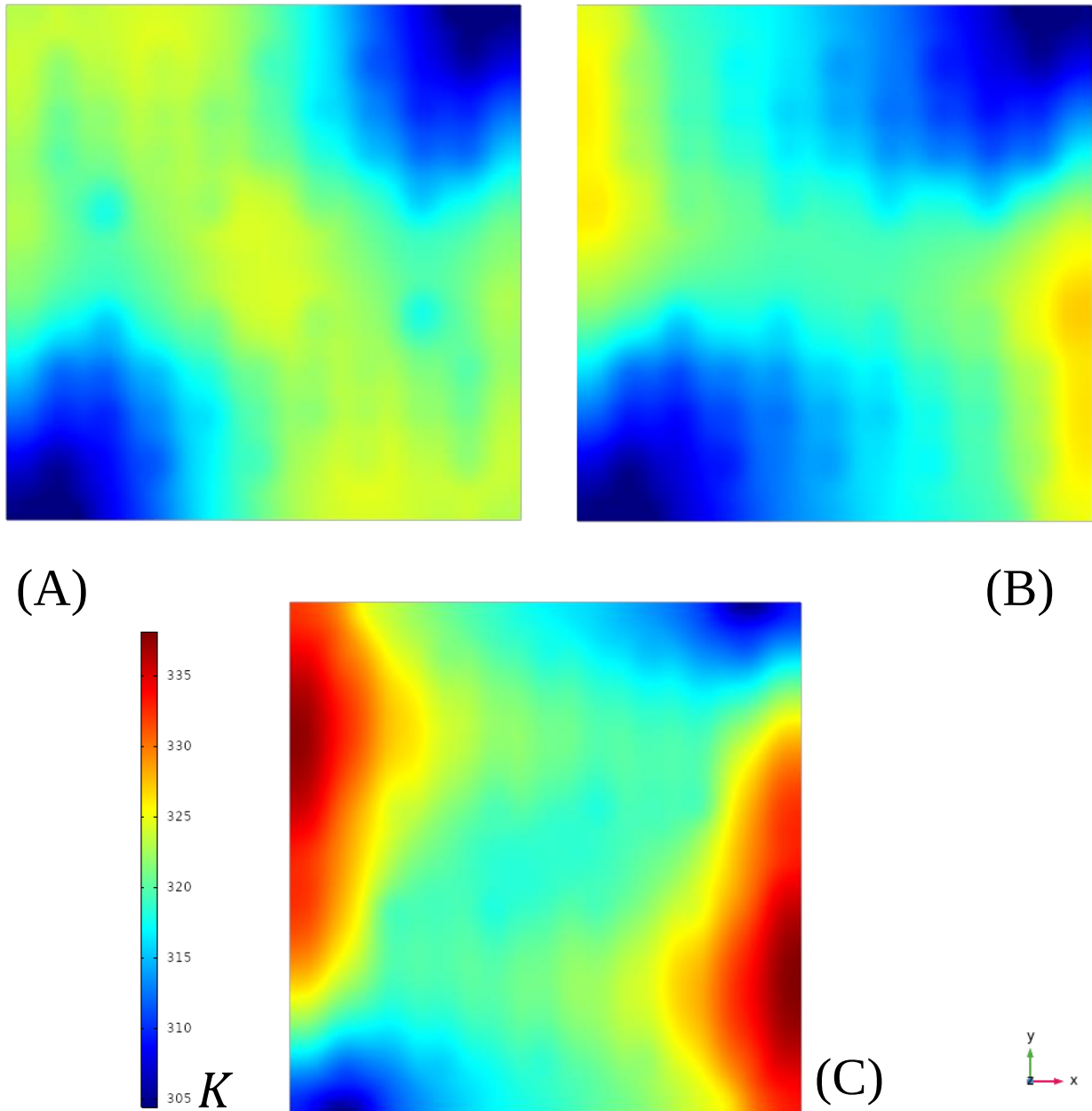


Figure 8-6: Shows the temperature contour of the bottom surface of (A) D-SMDF-TH (novel design) (B) D-SMDF (refer Chapter 7) and (C) D-SMPF heat sinks.

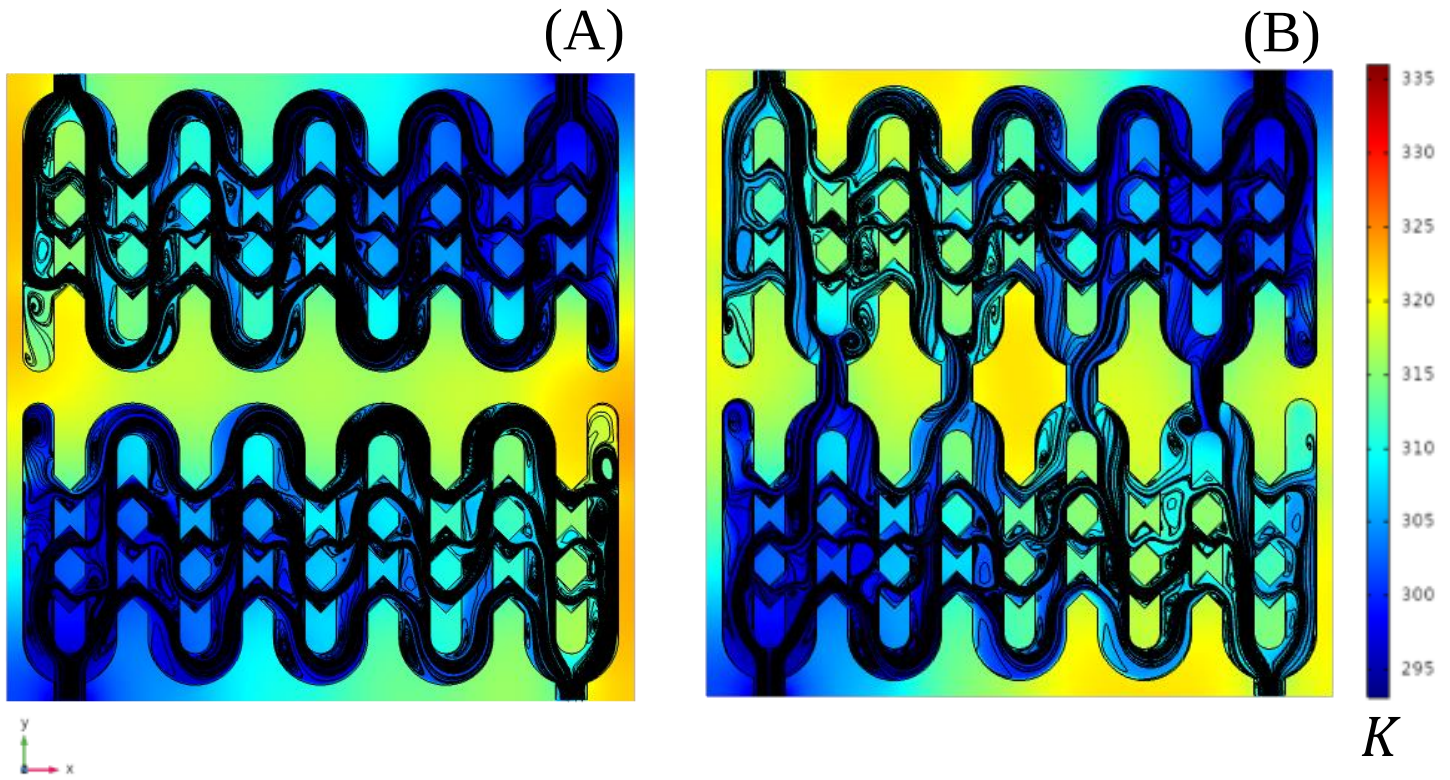


Figure 8-7: Shows the temperature contours of (A) D-SMDF (Chapter 7 design) (B) D-SMDF-TH (novel design) heat sinks at $Re = 2300$ and $H_{ch} = 1$ mm.

From Figure 8-4, it can be seen using D-SMDF-TH heat sink at lower Re , the Nu_{avg} has slightly lower magnitudes compared to D-SMDF and D-SMPF heat sinks. The reason behind this is due to the fact that at lower Re the existence of localised very low velocity region affecting the heat transfer in the middle channels of heat sink. Even for Nu_{avg} of D-SMDF heat sink, the magnitudes are almost equal to the Nu_{avg} of D-SMPF heat sink at lower Re caused by the same problem. As the Re increases, Nu_{avg} of D-SMDF-TH heat sink increases almost exponentially bypassing the Nu_{avg} of D-SMPF heat sink showing superior performance. The values of Nu_{avg} of D-SMDF-TH heat sink reduces by 26.3% along increasing Re . Even though, D-SMDF-TH heat sink possesses lower R_{th} values compared to D-SMDF heat sink, in terms of Nu_{avg} the D-SMDF heat sink is more superior due to the less formation of localised low velocity regions, thus, inducing more heat transfer within the middle section of heat sink showing more blueish region (refer Figure 8-7 (A) and (B)).

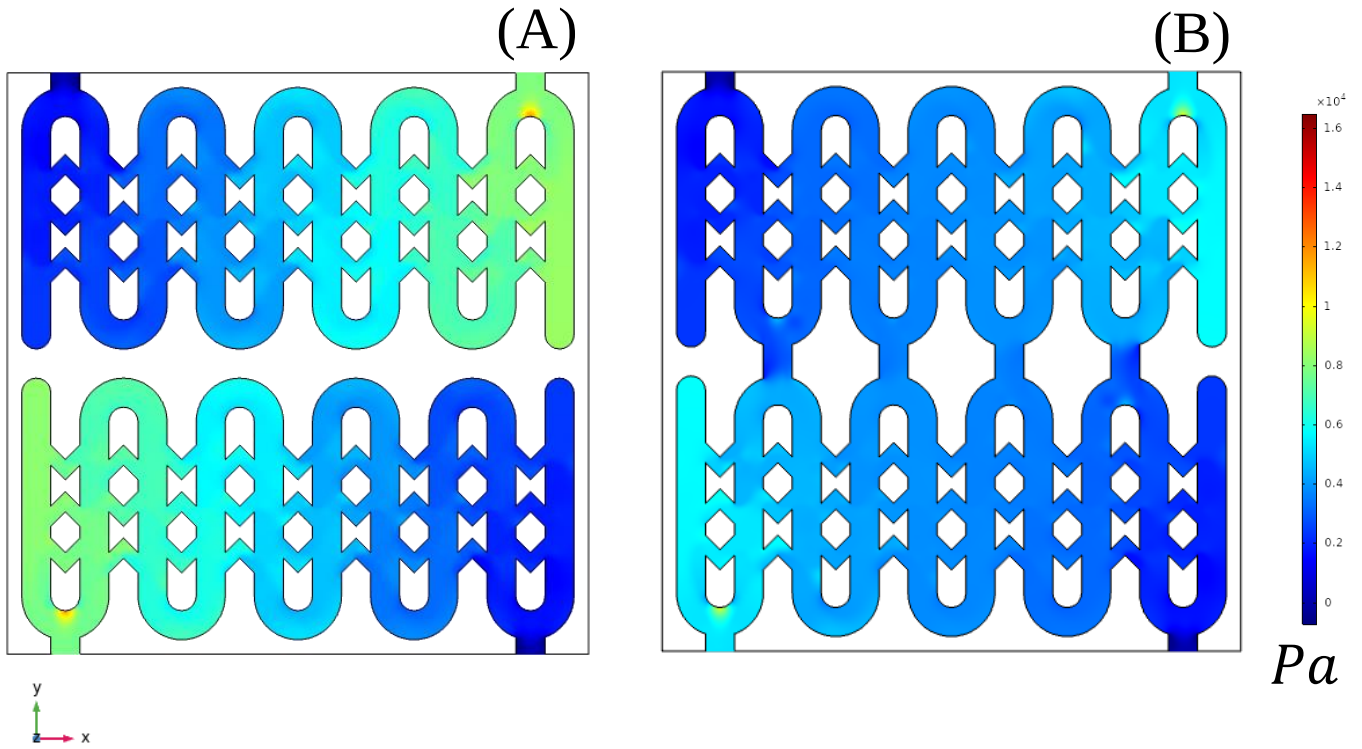
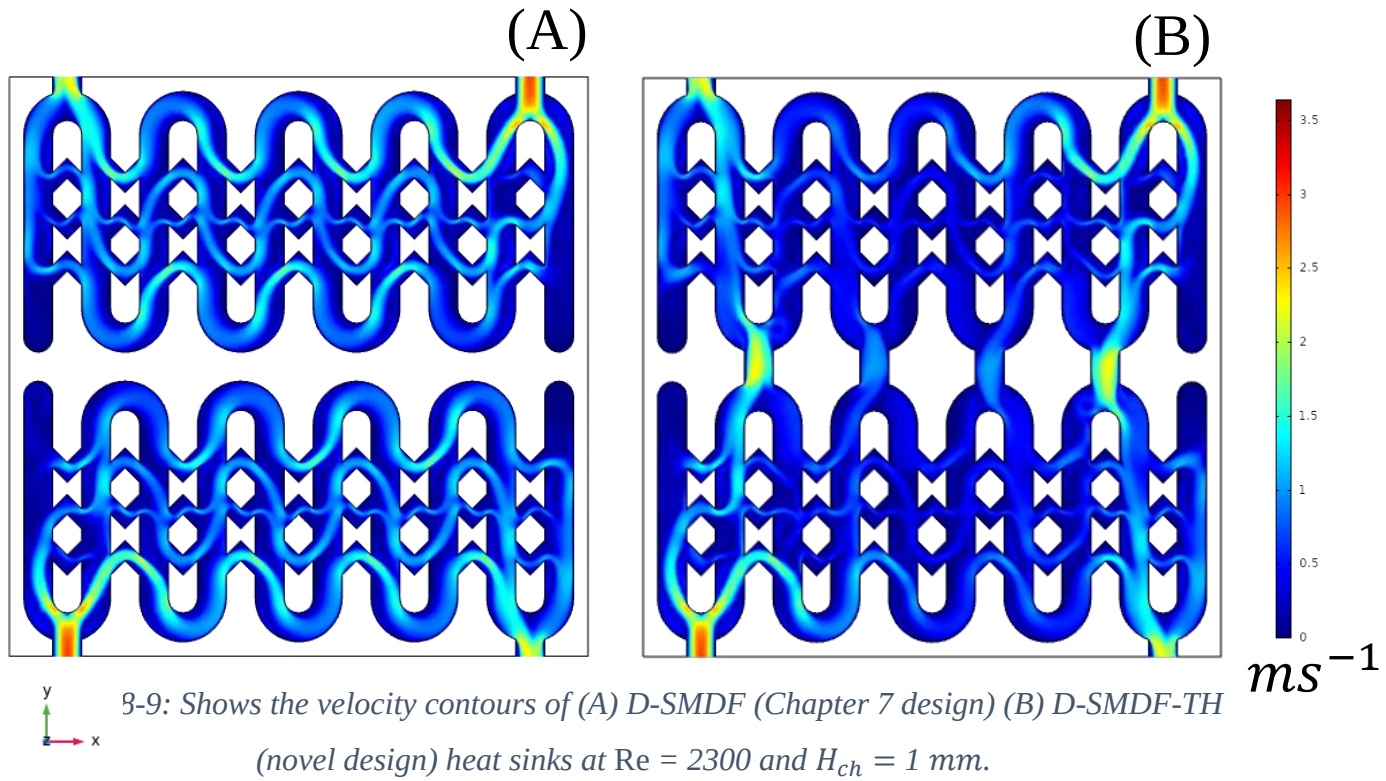


Figure 8-8: Shows the pressure contours of (A) D-SMDF (Chapter 7 design) (B) D-SMDF-TH (novel design) heat sinks at $Re = 2300$ and $H_{ch} = 1 \text{ mm}$.

8.4.2.2 Effect of D-SMDF heat sink on velocity and pressure distribution, and ΔP

Looking at Figure 8-5, the D-SMDF-TH heat sink has the lowest pressure drop (ΔP) while D-SMPF heat sink has the highest and D-SMDF is in the middle along increasing Re . The relative ΔP percentage reduction of D-SMDF-TH heat sink compared to D-SMPF and D-SMDF heat sinks are 82.6% and 66.1% respectively which are massive drops. To explain the reason behind this, referring Figure 8-8, the pressure distribution of D-SMDF-TH heat sink show very low pressure compared with D-SMDF heat sink. This is due to the fact that from Figure 8-9, the D-SMDF-TH heat sink experiences a great reduction in liquid velocity from the pressure equilibrium achieved by diverting the liquid between two serpentine through through-holes, hence, reducing both head losses and friction factors.



8.4.2.3 Performance Evaluation Criteria (PEC) index study

Using the same procedure from subchapter 7.4.2.3, the PEC index is calculated using the formula by Eq. 7.3. From Figure 8-10, the PEC index for D-SMDF-TH heat sink performed 163% and 52.1% better compared to D-SMPF and D-SMDF heat sinks. The D-SMDF-TH heat sink has highly superiority of PEC index performance compared to D-SMPF and D-SMDF heat sink is due the significant reduction in ΔP .

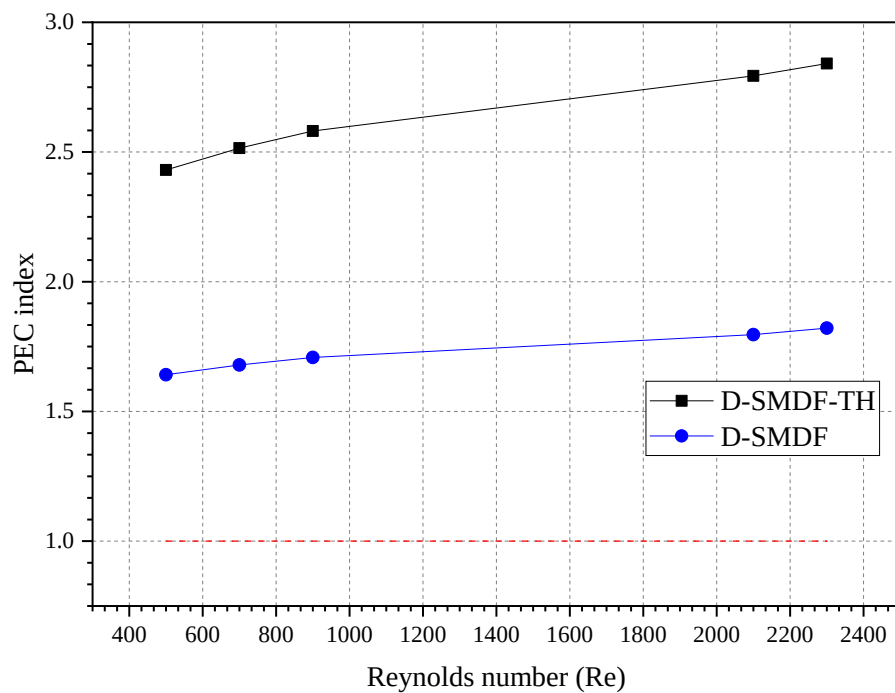


Figure 8-10: Shows the PEC index plots of different heat sinks.

8.4.3 Parametric analysis of D-SMDF-TH heat sink

The parameters analysis of D-SMDF-TH heat sink covers the variation of chosen parameters to observe its effect onto the thermal-hydraulic performance of heat sink. From subchapter 8.4.2.3, the D-SMDF-TH heat sink shows very superior performance in term thermal-hydraulic components. The parameters utilized under this subchapter are mini-channel height (H_{ch}), mini-channel width (W_{ch}), fin angle (θ_{fin}) and heat flux applied (q'') respectively.

8.4.3.1 Effect of varying the H_{ch} of D-SMDF heat sink on parameters of R_{th} , ΔP and PEC index

The changing of value of H_{ch} parameter is tested at a range of 1.5 mm to 2.0 mm to see its effect onto the thermal-hydraulic performance of D-SMDF-TH heat sink. The heat flux applied at bottom surface of heat sink is at 100 W/cm² with Re ranges from 500 to 2300. The other parameters are kept constant (refer Table 8-1).

From Figure 8-11, as H_{ch} increase both R_{th} and ΔP decreases simultaneously. The percentage reduction of both parameters are at 18.2% and 12.2% respectively. The reduction of R_{th} is based on the increase of A_{eff} from 897.0 mm² to 1119.2 mm² (24.7% increase) which allows more convective heat transfer to occur. Furthermore, the decrease of ΔP is due to less head losses and friction factors since liquid velocity is reduced.

Referring to Figure 8-12, the PEC index plots at different values of H_{ch} are plotted. From Eq. 7.5, to achieve high PEC index values, high magnitude of $E_{R_{th}}$ while having low magnitudes of both $E_{\Delta P}$ and E_{OS} must be obtained. From the figure, PEC index is the highest and lowest at H_{ch} equals to 2.0 mm and 1.5 mm respectively.

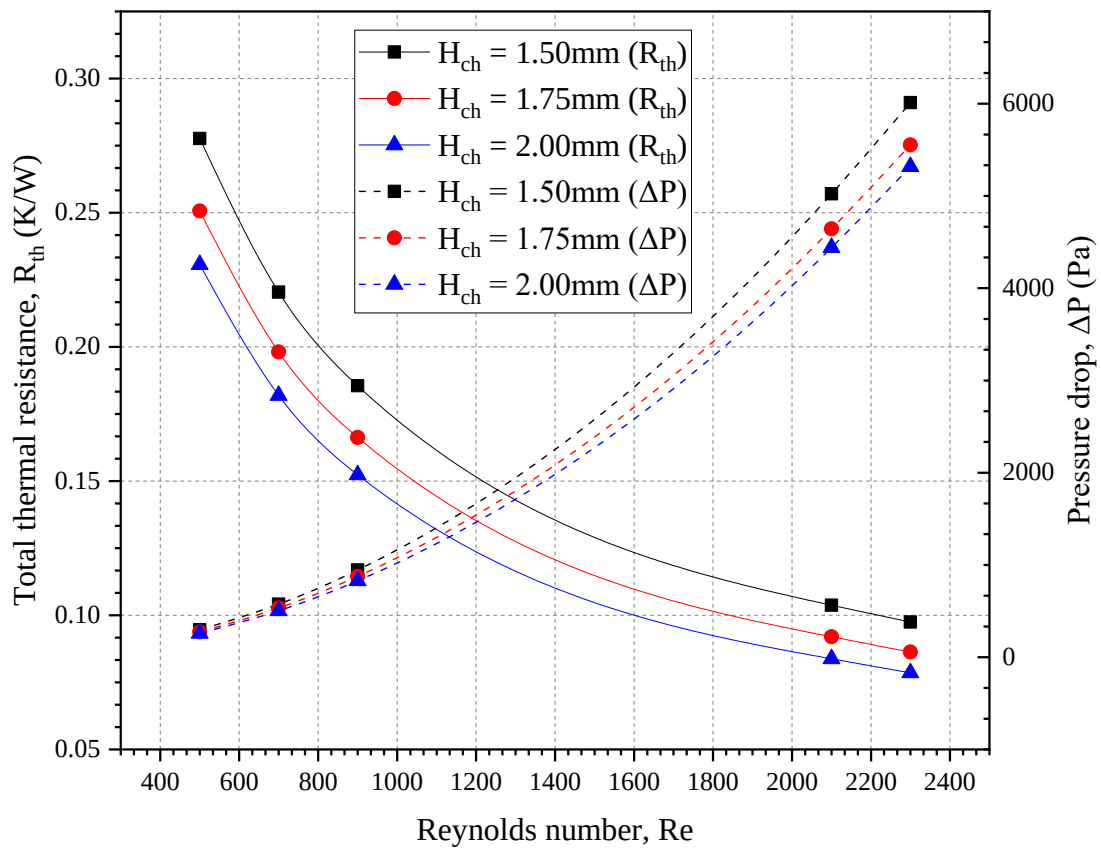


Figure 8-11: Shows the plots of R_{th} and ΔP at different values of H_{ch} .

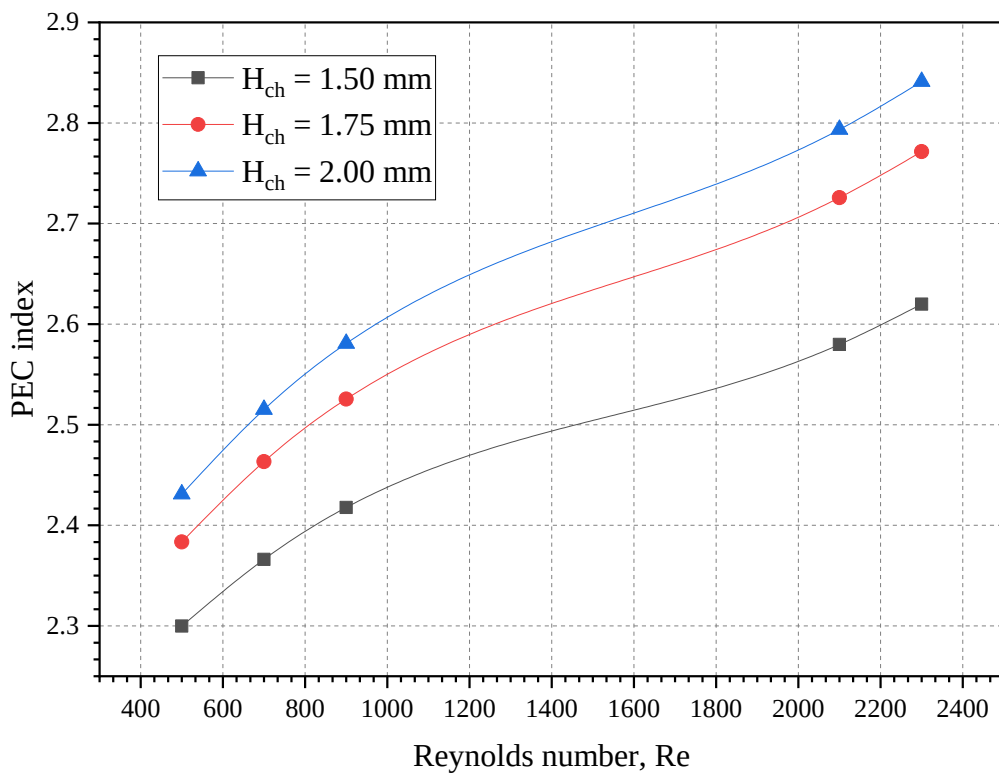


Figure 8-12: Shows the plots of PEC at different values of H_{ch} .

8.4.3.2 Effect of varying the W_{ch} of D-SMDF heat sink on parameters of R_{th} , ΔP and PEC index

The changing of W_{ch} from 0.8 mm to 1.0 mm is tested on thermal-hydraulic performance of D-SMDF-TH heat sink. The heat sink is applied of heat flux at 100 W/cm^2 in magnitude while having Re ranges between 500 to 2300. All other dimensions are kept unchanged (refer Table 8-1).

Looking at Figure 8-13, varying the W_{ch} from 0.8 mm to 1.0 mm, both R_{th} and ΔP decreases at 9.9% and 15.0% respectively. As W_{ch} gets larger, the A_{eff} becomes bigger from 1070.3 mm^2 to 1119.2 mm^2 respectively contributing to 4.6% increase. The ΔP decreases when W_{ch} increases is due to the fact that liquid velocity get reduced when channel size enlarges, thus, less head losses and friction factors.

From Figure 8-14, at $W_{ch} = 1.0 \text{ mm}$, the PEC index is the highest and at $W_{ch} = 0.8 \text{ mm}$, it is the lowest. At $W_{ch} = 1.0 \text{ mm}$, the PEC index is the highest is due to the $E_{R_{th}}$ and $E_{\Delta P}$ are the lowest and highest.

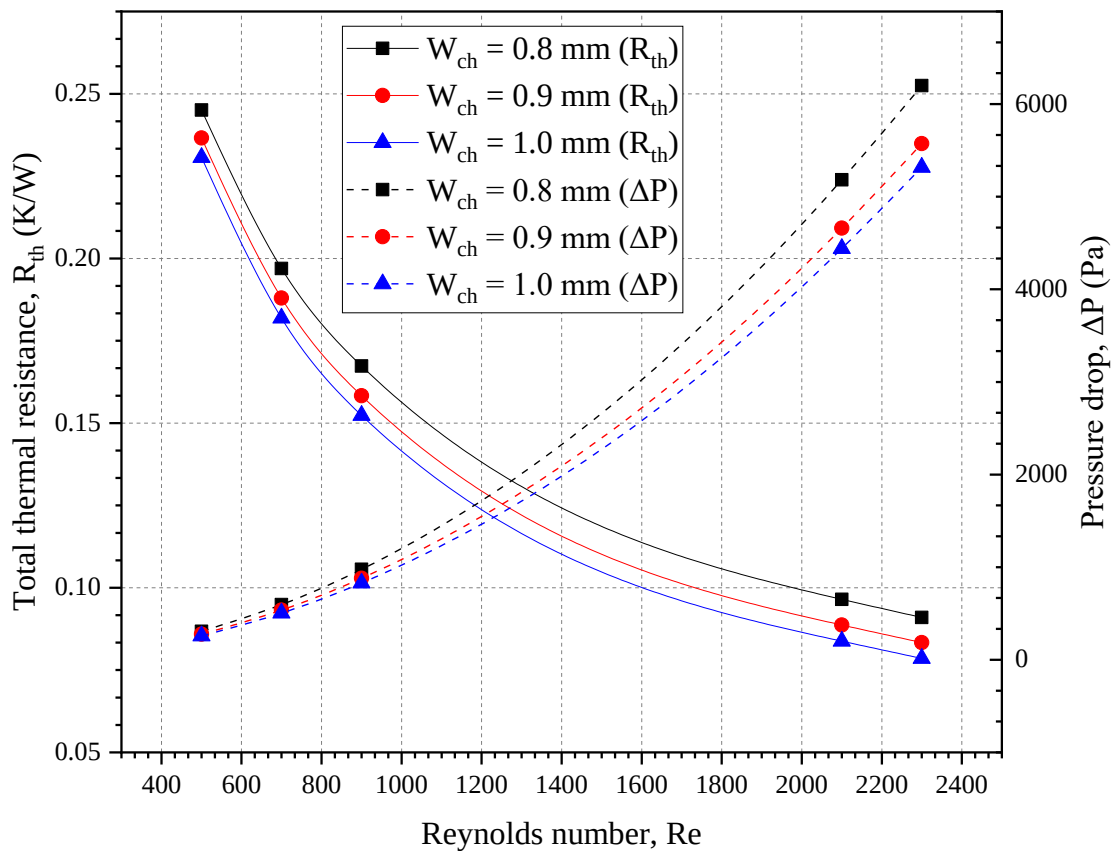


Figure 8-13: Shows the plots of R_{th} and ΔP at different values of W_{ch} .

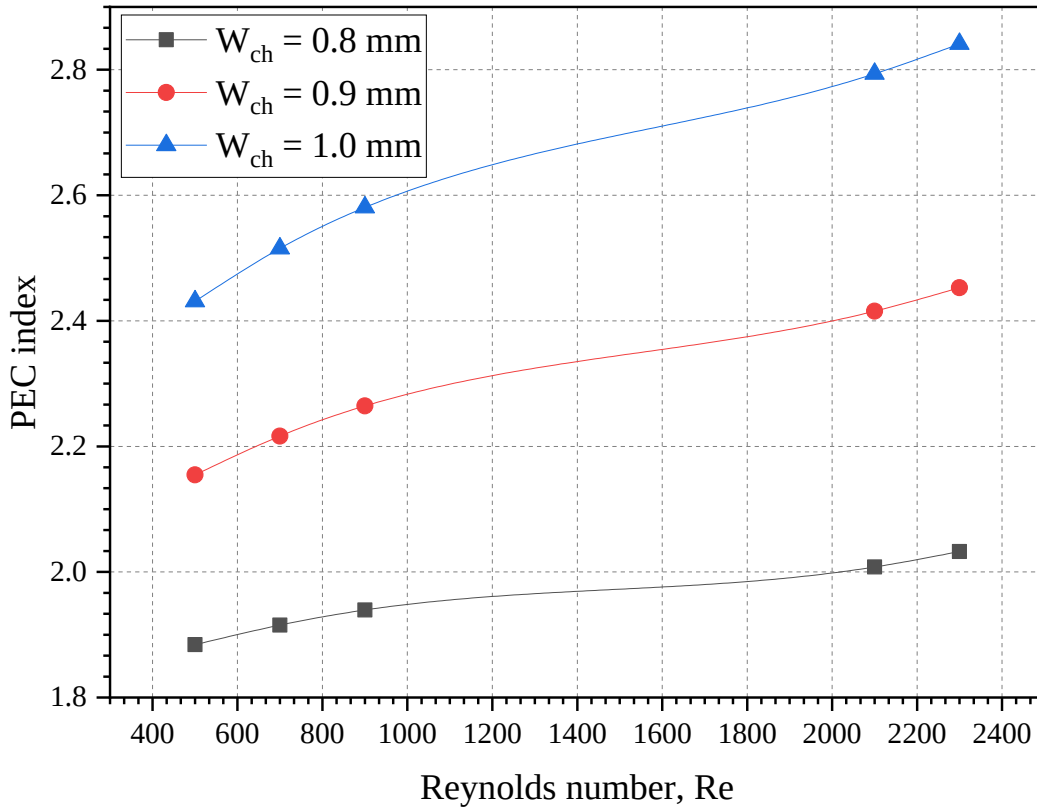


Figure 8-14: Shows the plots of PEC at different values of W_{ch} .

8.4.3.3 Effect if varying the θ_{fin} of D-SMDF heat sink on parameters of R_{th} , ΔP and PEC index

The θ_{fin} is varied to be between 45° to 60° for analysing its effect on the thermal-hydraulic performance of D-SMDF-TH heat sink. Heat flux of 100 W/cm^2 is applied underneath heat sink with Re set in between 500 and 2300.

From Figure 8-15, increasing the θ_{fin} from 45° to 60° leads to the increase of both R_{th} and ΔP by 4.9% and 21.3% respectively. For the R_{th} even though, the A_{eff} increases from 1119.2 mm^2 and 1246 mm^2 respectively, the slight increase in surface area for higher heat transfer cannot overcome the high inertia of liquid at $\theta_{fin} = 45^\circ$ due to larger portion of liquid is diverted through the secondary channel making breaking of thermal and hydrodynamic boundary layers more effective.

Referring Figure 8-16, at $\theta_{fin} = 45^\circ$ the PEC index is the highest while at $\theta_{fin} = 60^\circ$ is the lowest. As the θ_{fin} increases of both R_{th} and ΔP lead to the deterioration of PEC index.

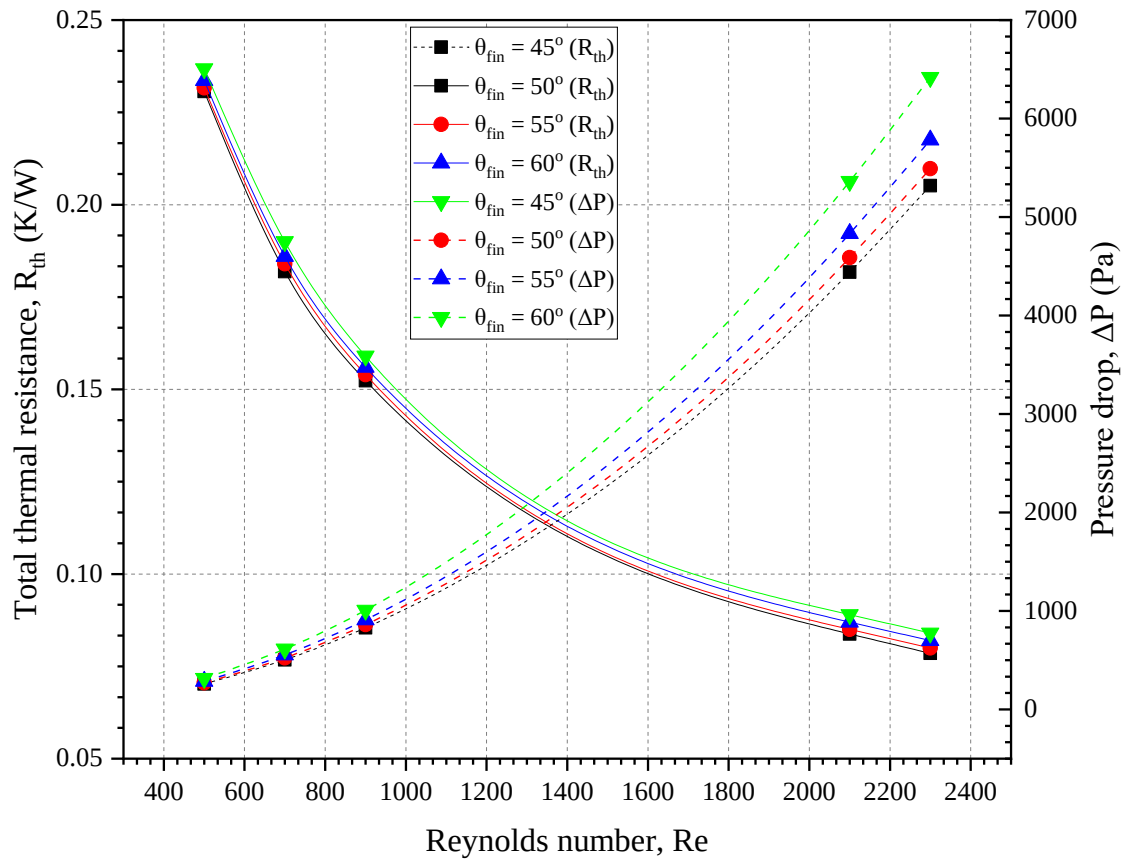


Figure 8-15: Shows the plots of R_{th} and ΔP at different values of θ_{fin} .

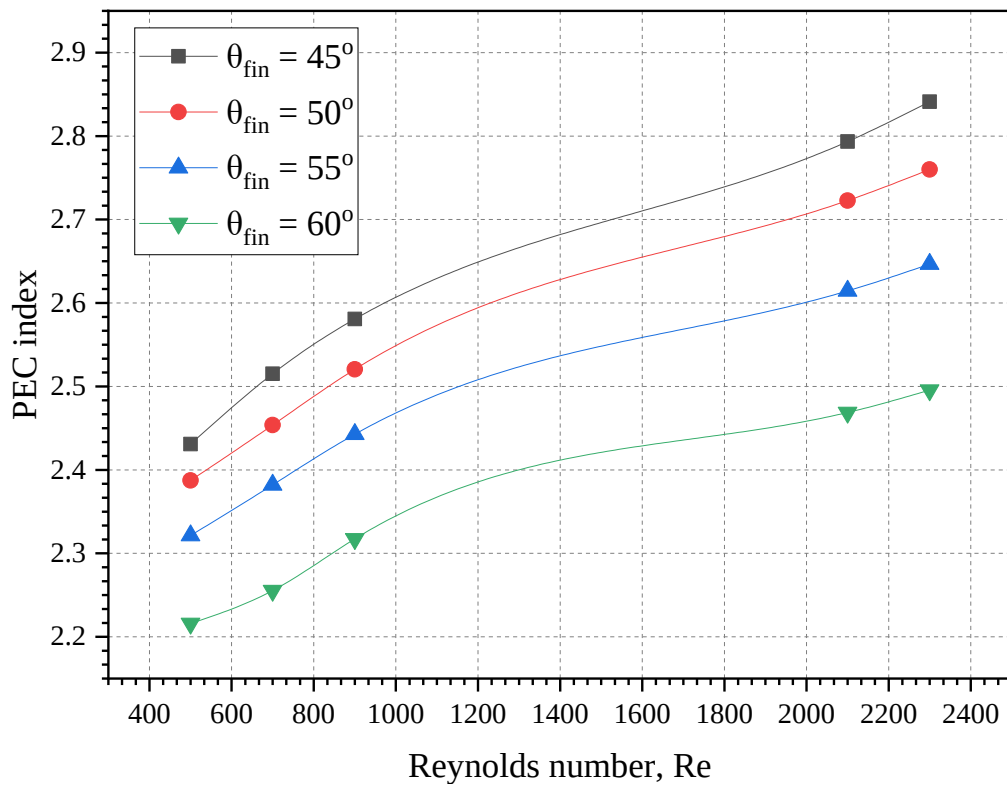


Figure 8-16: Shows the plots of PEC at different values of θ_{fin} .

8.4.3.4 Effect if varying the q'' of D-SMDF heat sink on parameters of $T_{surf,max}$ and ΔP

To test variation of heat fluxes (q''), three magnitudes chosen which are 100 W/cm^2 , 150 W/cm^2 and 200 W/cm^2 respectively analysing the effect of thermal-hydraulic performance of D-SMDF-TH heat sink. The Re is set in between 500 and 2300 respectively. All dimensions can be referred in Table 8-1.

From Figure 8-17, as q'' the $T_{surf,max}$ increases massively by 22.3% since more heat is supplied to the heat sink. The ΔP experience small reduction as q'' increases due to the liquid viscosity and density decrease at high temperature.

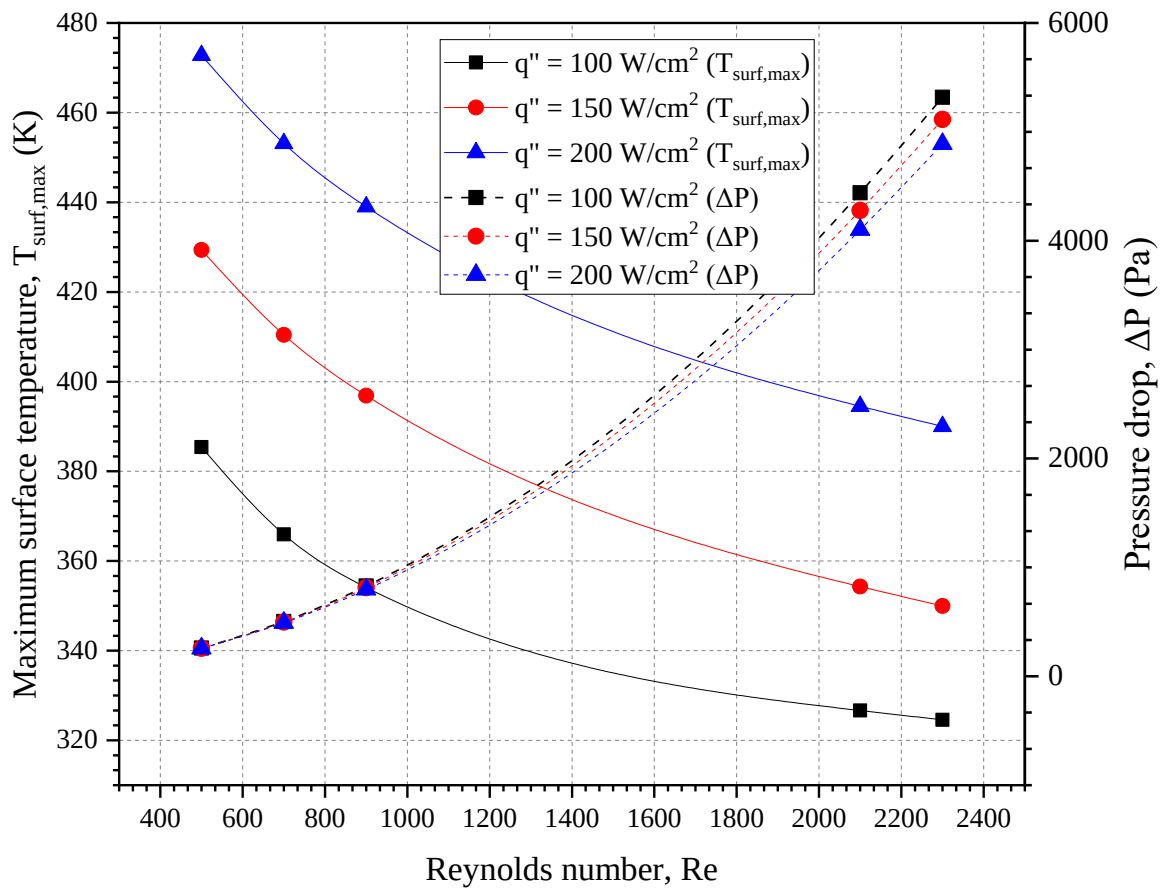


Figure 8-17: Shows the maximum surface temperature, $T_{surf,max}$, and ΔP plots when Re is varied.

8.5 Design optimisation of D-SMDF-TH heat sink

The optimisation process is essential to maintain the heat sink temperature under the junction temperature for electronics to work properly. The optimisation of single- and multi-objective functions will be carried out within the frameworks of deterministic and robust design optimisation study frameworks. The objective functions used are the same as the subchapter 7.5 which are mass of copper heat sink (M_{Cu}), total thermal resistance (R_{th}) and pressure drop (ΔP) respectively.

For this optimisation work, six input design variables are chosen to see its variation affecting the thermal hydraulic performance of D-SMDF-TH heat sink. The input design variables chosen are channel width (W_{ch}), channel height (H_{ch}), thickness of substrate (H_b), angle of fin (θ_{fin}), through-holes diameter ($2R_3$) and heat flux (q'') respectively. The limits of the input design variables are tabulated in Table 8-3.

Table 8-3: Shows the description of design variables for D-SMDF heat sink.

Design variables	Lower limit	Upper limit
Width of channel, W_{ch} (mm)	1.0	1.3
Height of channel, H_{ch} (mm)	0.75	2.0
Thickness of substrate, H_b (mm)	0.5	1.0
Angle of fin, θ_{fin} ($^\circ$)	45	60
Through-holes diameter, $2R_3$ (mm)	0.5	1.75
Heat flux, q'' (W/cm^2)	100	200

The individual input design variables limits is extended to be as wide as possible to make sure that optimisation covers design variables with the largest range. From the limits tabulated in Table 8-3, a set of 304 Design of Experiment (DOE) points are created using Optimal Latin Hypercube (OLHC) sampling method for even distribution of points (see Figure 8-18). The DOE points will then be used to calculate the output objective functions using COMSOL Multiphysics software. The input design variables and output objective functions are trained using Artificial Neural Network (ANN) to map the relationship between them. Surrogate modelling will be conducted utilising Tensorflow package and optimisation process will be executed using Platypus package in python. During optimisation process, the secondary channel length (l_{sc}) is made fixed at 0.7 mm while the footprint area ($W \times L$) is kept at at $20 \times 20 \text{ mm}^2$. At inlet boundary, fully developed flow of volumetric flow rate at $2.67 \times 10^{-6} \text{ m}^3 \text{ s}^{-1}$ is applied.

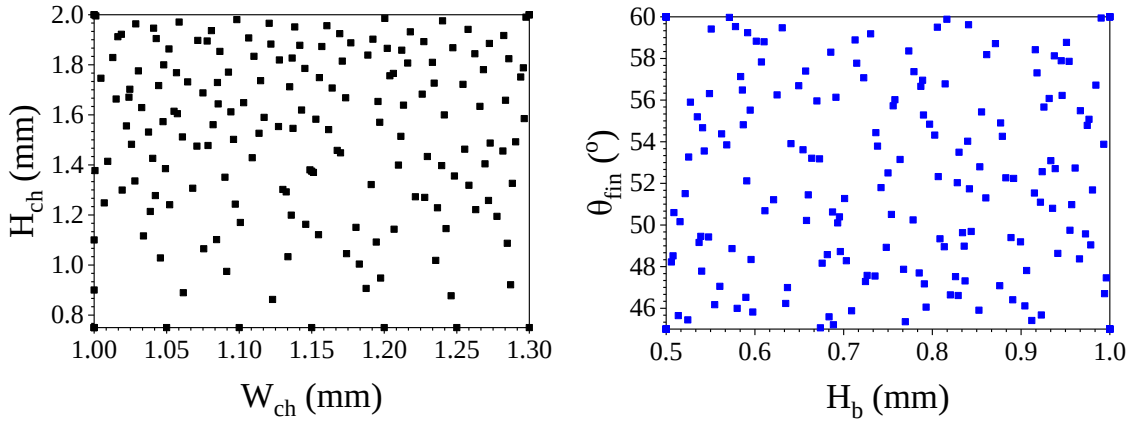


Figure 8-18: Shows the typical DOE points of different input design variables.

8.5.1 Surrogate modelling

Creating surrogate models involves the feeding of DOE points of input design variables into COMSOL Multiphysics calculating the output objective functions. After that, the normalisation of input design variables will be conducted based on Eq. 6.6 using the limits tabulated in Table 8-3. The act of normalising improves the convergence rate when training the surrogate models.

The objective functions of R_{th} and ΔP are calculated based on Eq. 3.53 and Eq. 3.56 respectively. The calculation of M_{Cu} is based on Eq. 6.7 which utilises analytical method with different formula of volume of copper heat sink (V_{Cu}) which shown in appendix A 4.2. Surrogate models are trained individually to reach faster convergence rate. The training procedure using RBF method requires the DOE dataset to be split into 80% to 20% ratio which are the training and validation set. From Table 8-4, the RBF models for R_{th} and ΔP objective functions are very accurate since RMSE is very small compared to calculated objective functions from CFD models while having R^2 close to unity and Maximum residual error less than 2%.

Table 8-4: The accuracy parameters for training, validation and testing sets of surrogate models using RBF.

R_{th}			
Accuracy parameter	Training set	Validation set	Testing set
RMSE (K/W)	1.273558×10^{-5}	2.371634×10^{-5}	1.984628×10^{-5}
R^2	0.999999	0.999252	0.999933
Maximum Residual Error (%)	0.139261	0.773628	0.467182
ΔP			
Accuracy parameter	Training set	Validation set	Testing set
RMSE (Pa)	0.734719	1.084628	0.827161
R^2	0.999999	0.999977	0.999991
Maximum Residual Error (%)	0.176231	0.593238	0.271728

8.5.2 Optimisation process

For the optimisation procedure, both deterministic and robust design optimisation frameworks are utilised. The optimisation using single- and multi-objective study are carried out and the results will be compared in terms of thermal-hydraulic components.

8.5.2.1 Deterministic optimisation

Within this subchapter, the single- and multi-objective optimisations are conducted based on deterministic study framework.

8.5.2.1.1 Single-objective optimisation

To perform single-objective optimisation, the process is straight-forward, where the maximum and minimum values can be obtained easily. The downside of single-objective optimisation, as one objective is minimised but the other objectives might not be and this issue makes it hard to search the set of input of design variables which corresponds to the minimisation of all objectives. From Table 8-5, it shows the tabulation data of the individual objective function which represented by the green fill. From Table 8-5, the R_{th} and ΔP objectives only have one lowest point each while M_{Cu} objective has the lowest value at eight different sets of input design variables.

Table 8-5: Shows the input design variables at individual minimisation of objective functions.

W_{ch} (mm)	H_{ch} (mm)	H_b (mm)	θ_{fin} (°)	D_3 (mm)	q'' (W/cm ²)	R_{th} (K/W)	ΔP (Pa)	M_{Cu} (g)
1.0	0.75	0.689	49.16	0.50	139.5	0.066083	33742.9	3.52084
1.3	2.00	1.000	45.00	1.75	200.0	0.111500	1962.55	5.93440
1.3	0.75	0.500	60.00	0.50	200.0	0.089480	24824.0	2.67807
1.3	0.75	0.500	60.00	0.50	100.0	0.092390	24921.9	2.67807
1.3	0.75	0.500	45.00	0.50	200.0	0.093650	22661.7	2.67807
1.3	0.75	0.500	45.00	0.50	100.0	0.096850	22855.8	2.67807
1.3	0.75	0.500	45.00	0.50	100.0	0.096850	22855.8	2.67807
1.3	0.75	0.500	45.00	0.50	200.0	0.093650	22661.7	2.67807
1.3	0.75	0.500	60.00	0.50	100.0	0.092390	24921.9	2.67807
1.3	0.75	0.500	60.00	0.50	200.0	0.089490	24824.0	2.67807

Referring to Table 8-5, at the lowest value of R_{th} of 0.0661 K/W, it corresponds to high values ΔP and M_{Cu} at 33742.9 Pa and 3.52084 g respectively. For the smallest value of ΔP of 1962.55 Pa, the R_{th} is quite high at 0.112 K/W while having the very high mass of heat sink of 5.934 g. For M_{Cu} , the lowest magnitude occurs at 2.678 g at seven different high values of R_{th} and ΔP

respectively. To solve this issue, multi-objective optimisation is carried out to minimise all objectives simultaneously by plotting the Pareto front of competing objectives.

8.5.2.1.2 Multi-objective optimisation

Based on the surrogate models trained earlier under subchapter 8.5.1, the models will be into multi-objective framework achieving the compromise between competing objectives of M_{Cu} , R_{th} and ΔP altogether. Utilising the input design variables and output objective functions defined under subchapter 8.5, the formulation of multi-objective optimisation is defined as follows:

$$\begin{aligned}
 &\text{Objective functions: } \min(M_{Cu}), \min(R_{th}) \text{ and } \min(\Delta P) \\
 &\text{Constraints: } W_w + W_{ch} = 2mm \\
 &\quad H_{ch} - 2R_3 \geq 0.25mm \\
 &\text{Design space: } 1.0mm \leq W_{ch} \leq 1.3mm \\
 &\quad 45^\circ \leq \theta_{fin} \leq 60^\circ \\
 &\quad 0.75mm \leq H_{ch} \leq 2.0mm \\
 &\quad 0.5mm \leq H_b \leq 1.0mm \\
 &\quad 1 \times 10^6 \leq q'' \leq 2 \times 10^6 \\
 &\quad 0.5mm \leq 2R_3 \leq 1.75mm
 \end{aligned} \tag{8.4}$$

During the optimisation process, other than the design space set in Eq. 8.4, the other boundary conditions and dimensions are constant (refer Table 8-1). The use of multi-objective function is to construct the Pareto front for conflicting objectives enabling the designer to choose between different thermal-hydraulic performance based on the specific application. Platypus optimisation package from python software is utilised. The Non-dominated Sorting Genetic Algorithm-II (NSGA-II) algorithm is used since is very reliable, fast and no initial guess is required [316].

Looking at Figure 8-19 to Figure 8-21, the Pareto fronts as well as DOE points are plotted between two objectives of ΔP vs R_{th} , M_{Cu} vs ΔP and M_{Cu} vs R_{th} respectively. The function of Pareto fronts are to show the conflicting between objectives which means an increase of one objective will results in a decrease of another and vice versa. To perform detailed analysis on the Pareto fronts, the numerical data between competing objectives are tabulated in Table 8-6. The results of Pareto front are compared with the CFD data for validation process.

Based on Table 8-6, referring to ΔP vs R_{th} , ignoring the variation of M_{Cu} , going down the table, the R_{th} increases from 0.0711 K/W to 0.105 K/W with 48.4% increase while ΔP decreases from 16544.0 Pa to 2615.4 Pa giving 84.2% change. For M_{Cu} vs ΔP , ignoring the variation of R_{th} ,

going down the table, the ΔP increases from 2181.7 Pa to 15632.0 Pa (616.5% increase) while the M_{Cu} decreases from 4.350 g to 2.733 g yielding 37.2% reduction. For M_{Cu} vs R_{th} , ignoring the variation of ΔP , going down the table, R_{th} increases from 0.071438 K/W to 0.103986 K/W (45.6% increase) while the M_{Cu} reduces from 3.865 g to 2.778 g (28% decrease).

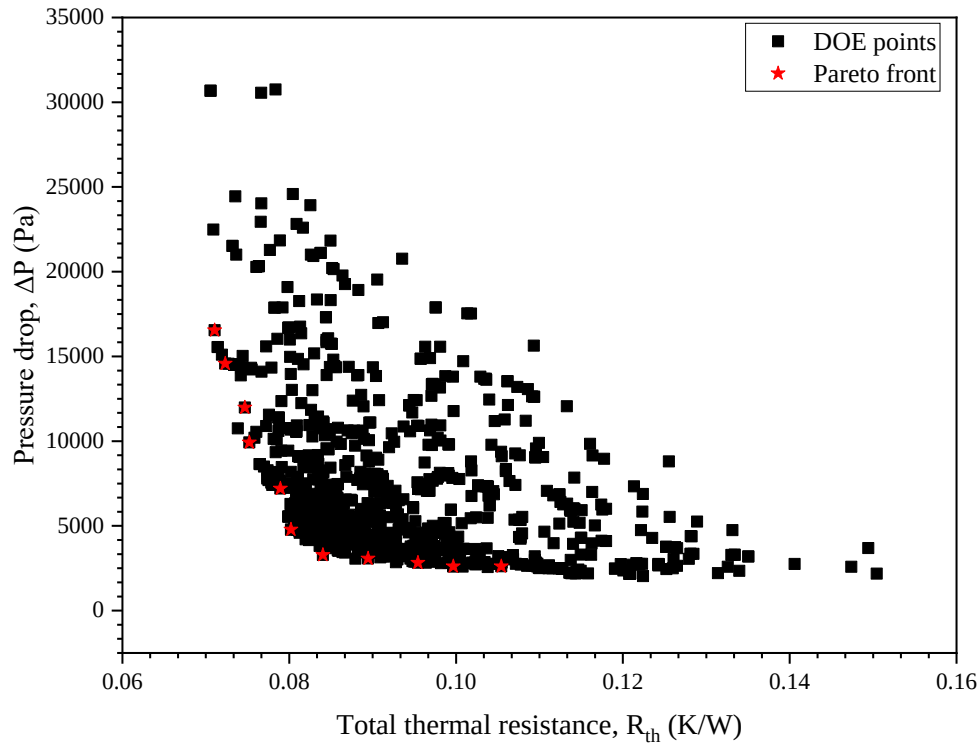


Figure 8-19: Shows the DOE points and Pareto front between competing objectives of ΔP and R_{th} .

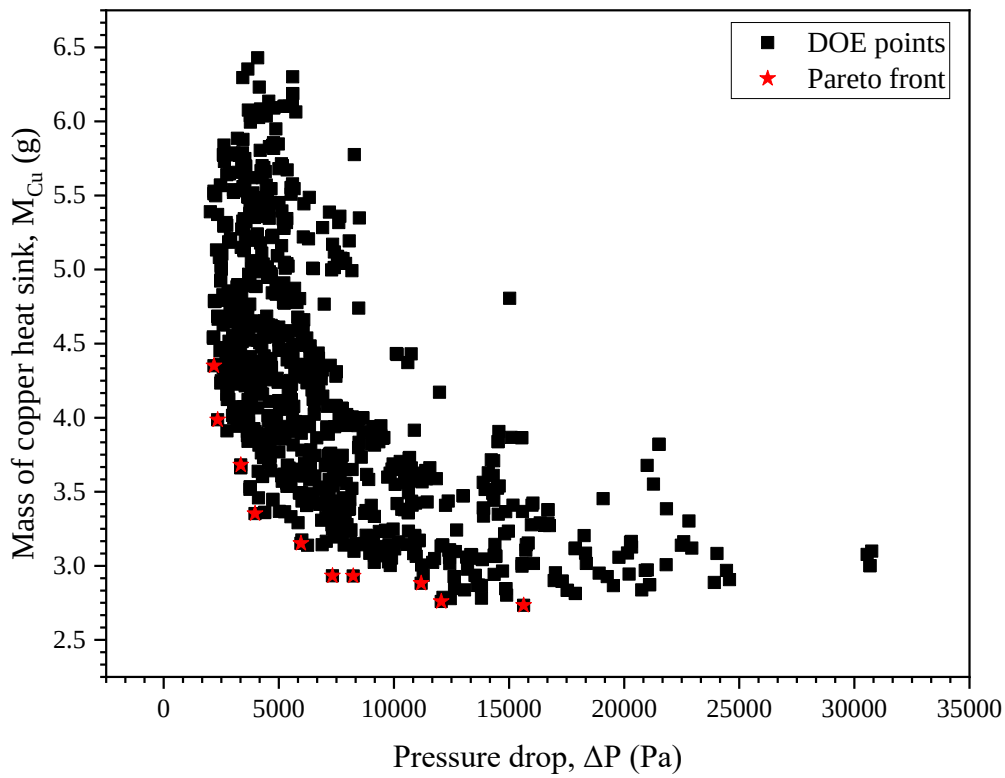


Figure 8-20: Shows the DOE points and Pareto front between competing objectives of M_{Cu} and ΔP .

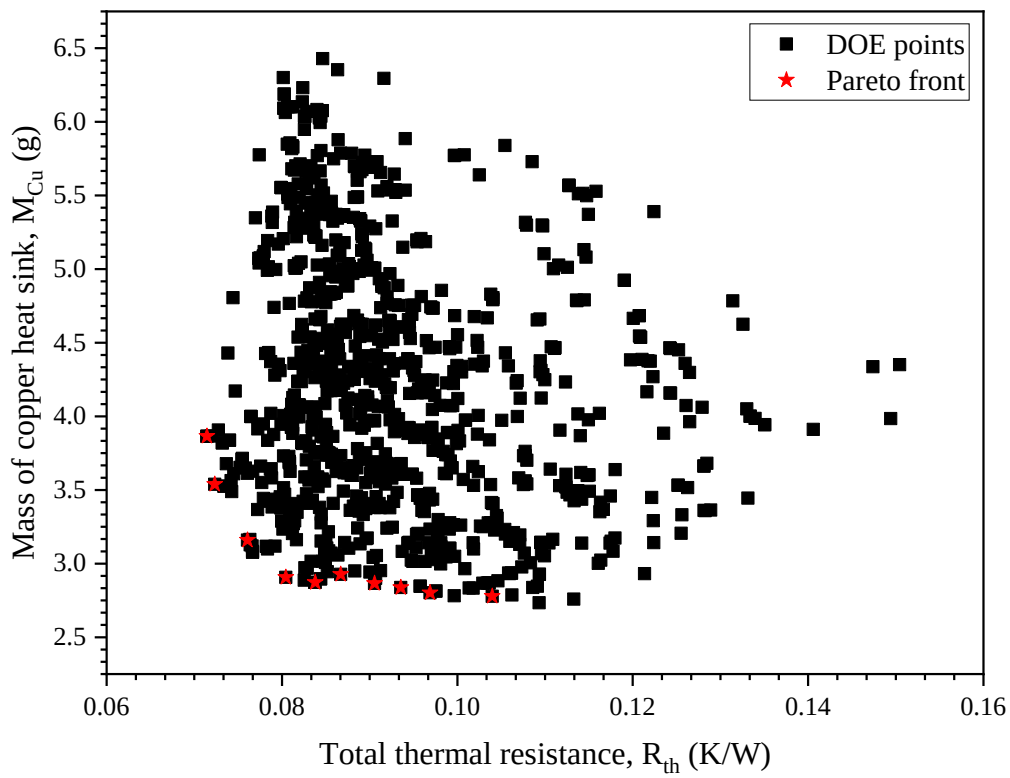


Figure 8-21: Shows the DOE points and Pareto front between competing objectives of M_{Cu} and R_{th} .

To consider all three objectives for optimisation, a Pareto front distribution in a form of 2-D contour plot is shown in Figure 8-22. From the figure, it can be seen that going towards zero value on the Cartesian plane would cause both R_{th} and ΔP to drop their values while M_{Cu} experience increment. Thus, to choose the suitable combination of objective functions values, it will be based on the application desired and its requirements.

Table 8-6: Shows the Pareto front points plotted in Figure 8-19 to Figure 8-21.

ΔP vs R_{th}														
Design variables						R_{th} (K/W)			ΔP (Pa)			M_{Cu} (g)		
W_{ch} (mm)	H_{ch} (mm)	H_b (mm)	θ_{fin} ($^{\circ}$)	D_3 (mm)	q'' (W/m ^{2})	Pareto	CFD	Error (%)	Pareto	CFD	Error (%)	Pareto	Analytical	Error (%)
1.048587	0.944984	0.525519	45.53919	0.720374	1236826	0.071063	0.071152	0.125084	16543.99	16513.21	0.186396	3.272125	3.272331	0.006295
1.072306	1.065790	0.550443	45.98197	0.570907	1143046	0.072337	0.072445	0.149079	14574.08	14612.44	0.262516	3.538274	3.539003	0.020599
1.089802	1.193944	0.67216	45.74400	0.533314	1124156	0.074672	0.074663	0.012054	11981.82	11903.12	0.661171	4.17142	4.171520	0.002397
1.089613	1.285080	0.504483	45.75832	0.610066	1049849	0.075217	0.075201	0.021276	9919.684	9925.732	0.060933	3.652927	3.653005	0.002135
1.036936	1.585541	0.819678	45.38194	0.544729	1380972	0.078932	0.078994	0.078487	7201.934	7256.343	0.749813	5.387061	5.387003	0.001077
1.012526	1.888193	0.874363	45.86367	0.798972	1317101	0.080216	0.080334	0.146887	4783.642	4785.461	0.038011	6.091775	6.091673	0.001674
1.258109	1.977441	0.92469	45.97625	0.859649	1485271	0.08403	0.084076	0.054712	3286.852	3313.436	0.802309	5.770526	5.771486	0.016633
1.251218	1.998318	0.923146	49.76795	1.018195	1586954	0.089457	0.089552	0.106084	3070.125	3115.445	1.454688	5.773212	5.773333	0.002096
1.270603	1.855532	0.812921	45.36185	1.400301	1442105	0.095423	0.095523	0.104687	2821.275	2901.777	2.774231	5.185082	5.185095	0.000251
1.270912	1.927538	0.96133	46.37482	1.474888	1096559	0.099676	0.099773	0.097221	2611.729	2653.431	1.571626	5.771403	5.771512	0.001889
1.229270	1.997025	0.925523	48.03046	1.475838	1854617	0.10543	0.10551	0.075822	2615.394	2613.243	0.082312	5.839814	5.839794	0.000342

M_{Cu} vs ΔP														
Design variables						R_{th} (K/W)			ΔP (Pa)			M_{Cu} (g)		
W_{ch} (mm)	H_{ch} (mm)	H_b (mm)	θ_{fin} ($^{\circ}$)	q'' (W/m ^{2})	Re	Pareto	CFD	Error (%)	Pareto	CFD	Error (%)	Pareto	Analytical	Error (%)
1.299211	1.981202	0.557678	59.80881	1.740652	1779548	0.150439	0.150523	0.055805	2181.679	2192.573	0.496859	4.350421	4.350398	0.000529
1.288878	1.875760	0.503861	51.91929	1.685322	1830783	0.133957	0.133888	0.051536	2346.021	2366.032	0.845762	3.986020	3.986650	0.015803
1.275331	1.574512	0.527587	50.39172	1.747489	1819445	0.128466	0.128541	0.058347	3349.585	3340.662	0.267103	3.680973	3.680811	0.004401
1.299028	1.444844	0.505910	47.76233	1.508226	1004817	0.116262	0.116323	0.052440	3969.075	3974.065	0.125564	3.352140	3.352231	0.002715
1.292410	1.137991	0.529708	48.82173	1.533787	1329393	0.117548	0.117532	0.013613	5963.968	5964.154	0.003119	3.151816	3.151903	0.002760

1.284865	0.943483	0.521324	50.07788	1.716056	1486534	0.121346	0.121451	0.086455	7333.372	7329.111	0.058138	2.932117	2.932231	0.003888
1.215191	0.969203	0.523201	48.32595	1.501997	1075407	0.105965	0.105891	0.069883	8228.934	8256.471	0.333520	2.931362	2.931402	0.001365
1.090827	0.848225	0.508394	48.24871	1.529862	1246350	0.104630	0.104512	0.112906	11184.34	11190.51	0.055136	2.882494	2.882533	0.001353
1.198260	0.774463	0.516028	56.16617	1.446463	1364651	0.113291	0.113403	0.098763	12058.78	12099.91	0.339920	2.759126	2.759130	0.000145
1.282606	0.753200	0.520048	56.38807	0.957099	1325159	0.109331	0.109421	0.082251	15632.41	15579.22	0.341416	2.733378	2.733401	0.000841

M_{Cu} vs R_{th}

Design variables						R_{th} (K/W)			ΔP (Pa)			M_{Cu} (g)		
W_{ch} (mm)	H_{ch} (mm)	H_b (mm)	θ_{fin} ($^{\circ}$)	q'' (W/m ^{2})	Re	Pareto	CFD	Error (%)	Pareto	CFD	Error (%)	Pareto	Analytical	Error (%)
1.002803	1.083398	0.620212	45.74400	0.533314	1069943	0.071438	0.071397	0.057425	15550.7	15563.83	0.084362	3.86486	3.86472	0.003623
1.072306	1.065790	0.550443	45.98197	0.570907	1143046	0.072337	0.072389	0.071834	14574.08	14541.37	0.224944	3.538274	3.538305	0.000876
1.146408	0.860890	0.553841	46.20433	0.572455	1265731	0.076061	0.076055	0.007889	20278.99	20289.18	0.050223	3.159277	3.159323	0.001456
1.087634	0.819354	0.508394	48.24871	0.512733	1730997	0.080435	0.080386	0.060956	24580.51	24493.43	0.355519	2.906425	2.906401	0.000826
1.229270	0.819919	0.52193	48.03046	0.529851	1862109	0.083799	0.083782	0.020291	21103.34	21141.63	0.181111	2.872311	2.872386	0.002611
1.120214	0.795480	0.529899	52.33927	0.813694	1001445	0.086689	0.086677	0.013845	19267.01	19323.42	0.291925	2.926843	2.926795	0.00164
1.120220	0.795480	0.504864	52.33927	0.813694	1892509	0.090563	0.090491	0.079566	19533.2	19500.69	0.166712	2.865562	2.865543	0.000663
1.213326	0.767426	0.519738	57.82455	0.719221	1758762	0.093548	0.093633	0.09078	20769.36	20780.16	0.051972	2.837139	2.837155	0.000564
1.214024	0.816264	0.509606	51.76264	0.947374	1354322	0.096891	0.096702	0.195446	14895.4	14933.22	0.253260	2.801437	2.801397	0.001428
1.222732	0.765363	0.533749	52.15253	1.332252	1438558	0.103986	0.103892	0.090479	12451.66	12503.85	0.417391	2.777793	2.777745	0.001728

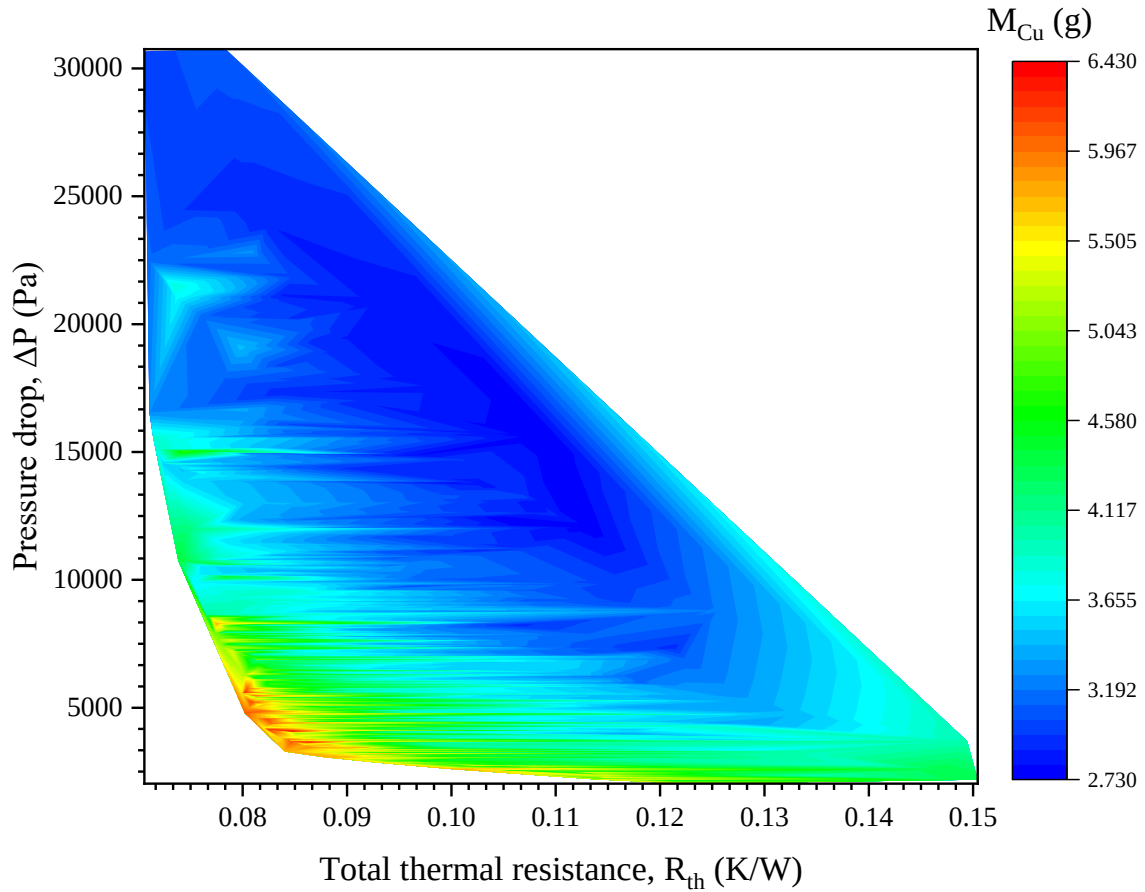


Figure 8-22: Shows the 2-D contour plot of the Pareto front for three objective functions.

8.5.2.2 Robust design optimisation (RDO)

Refer sub-chapter 7.5.2.2 for detailed explanation.

8.5.2.2.1 Model description

As it has been explained within subchapter 8.5, the same six input design variables will be employed which are H_{ch} , W_{ch} , θ_{fin} , H_b , D_3 and q'' respectively. Uncertainty will be used onto the input design variables to achieve distributions of output objective functions so that the statistical moments can be calculated. Using the same model of conjugate heat transfer and boundary conditions is exploited here. The distribution of objective functions will be calculated using the same model.

8.5.2.2.2 Uncertainty quantification (UQ)

The same procedure of subchapter 6.5.2.2.3 will be used here to perform Uncertainty quantification (UQ) process with only slight difference of the parameters used and number of input design variables. The UQ results will be tabulated in Table 8-7.

Table 8-7: Shows the uncertainty associated for each design variable.

Input design variables	Distribution	μ_{in}	σ_{in}
W_{ch} , mm	normal	W_{ch_i} , mm	0.0025mm
H_{ch} , mm	normal	H_{ch_i} , mm	0.0025mm
H_b , mm	normal	H_{b_i} , mm	0.0025mm
θ_{fin} , °	normal	θ_{fin_i} , °	0.025°
D_3 , mm	normal	D_{3_i} , mm	0.0025mm
q'' , W/cm ²	normal	q''_i , W/cm ²	1 W/cm ²

8.5.2.2.3 Convergence study for PCE method

Performing the optimisation under uncertainty, the use of PCE method to calculate the statistical moments of the output objective functions has to be tested for convergence study to see which polynomial order will give the highest accuracy while having the lowest computational time. The convergence plot study is shown in Figure 8-23. Looking at the figure, for all statistical moments parameters of $\mu_{R_{th}}$, $\sigma_{R_{th}}$, $\mu_{\Delta P}$, $\sigma_{\Delta P}$, $\mu_{M_{Cu}}$ and $\sigma_{M_{Cu}}$, the magnitudes stays constant when increasing the polynomial order from 0 to 4. At polynomial order of 5, the values increases drastically leading to such inaccuracies and not reliable. Therefore, in this work the polynomial order of 2 will be utilised for input design variables DOE points.

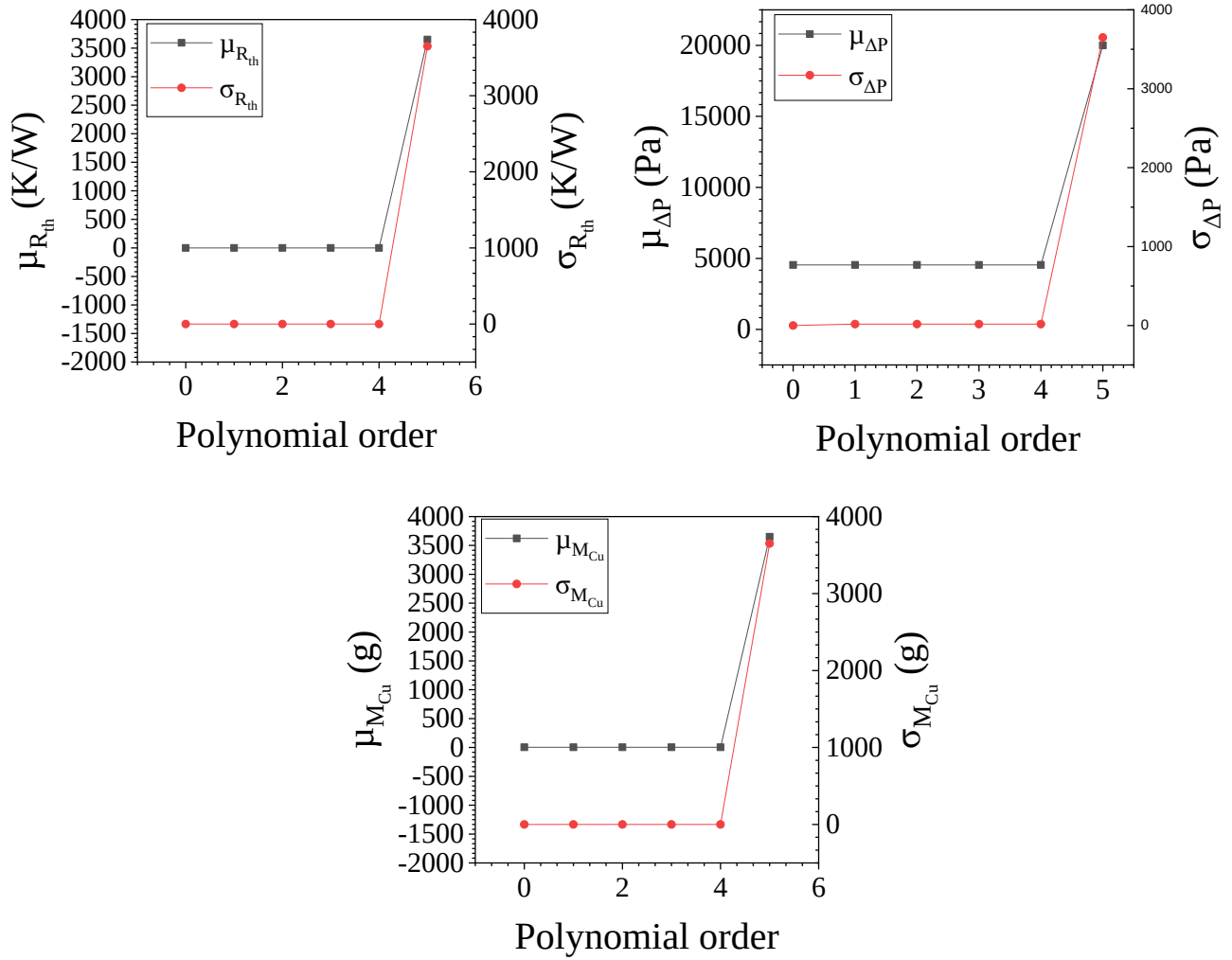


Figure 8-23: Shows the convergence plots using PCE method.

8.5.2.2.4 Surrogate modelling for RDO

After the statistical moments of means and standard deviations have been calculated, new surrogate models will be formed using the RBF method. Based on Eq. 6.6, the normalisation of the input design variables is conducted to enhance the rate of convergence. Accuracy of the RBF models for means and standard deviations are confirmed since they are derived from the surrogate models from subchapter 8.5.1.

Table 8-8: The accuracy parameters for training, validation and testing sets of surrogate model of $\sigma_{\Delta P}$ objective function using RBF.

$\sigma_{R_{th}}$			
Accuracy parameter	Training set	Validation set	Testing set
RMSE (Pa)	4.282643×10^{-7}	8.472871×10^{-7}	6.432991×10^{-7}
R^2	0.999989	0.999449	0.999821
Maximum Residual Error (%)	0.492842	1.382874	0.899154

Looking at Table 8-8, the accuracy parameters of RMSE, R^2 and maximum residual error are tabulated. The R^2 value is close to unity and RMSE is very small demonstrating very high accuracy of RBF model trained. The remaining RBF models are also tested for their accuracy which are not presented here.

8.5.2.2.5 Multi-objective robust design optimisation on single objective function

Based on the same procedure explained under subchapter 8.5.2.1.2, using NSGA-II optimisation algorithm, the ANN models of statistical moments will be employed here. The surrogates with the same unit will be optimised in terms of its means and standard deviations. Multi-objective optimisation will produce the Pareto front of conflicting objectives looking for the mean values which is insensitive towards the standard deviation. To achieve robust design the standard deviations are constrained which will affect the mean values giving flexibility to have options of objective functions for choosing. The robust design optimisation formulation can be referred to Eq. 6.13 to Eq. 6.15 respectively.

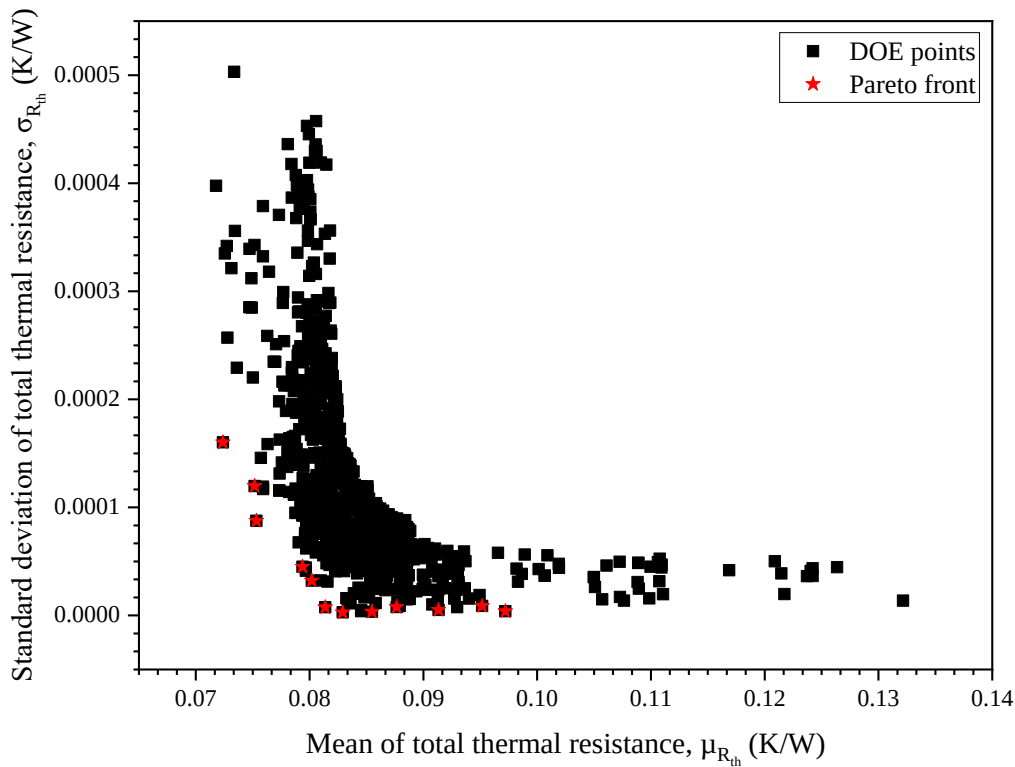


Figure 8-24: Shows the Pareto front and DOE points plots for $\sigma_{R_{th}}$ vs $\mu_{R_{th}}$.

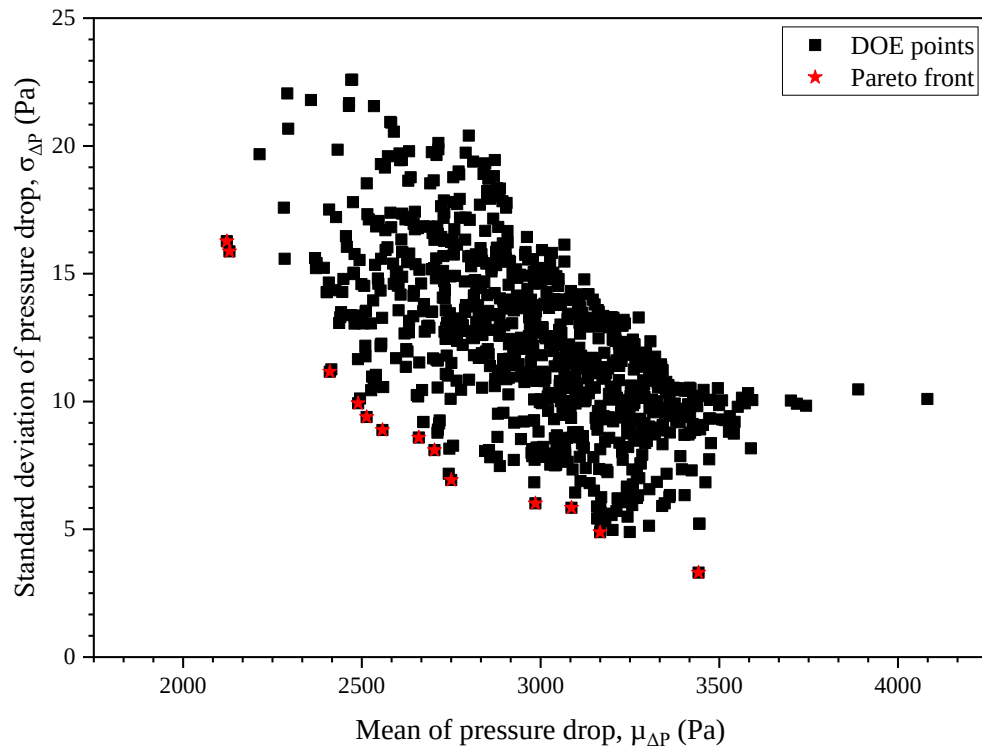


Figure 8-25: Shows the Pareto front and DOE points plots for $\sigma_{\Delta P}$ vs $\mu_{\Delta P}$.

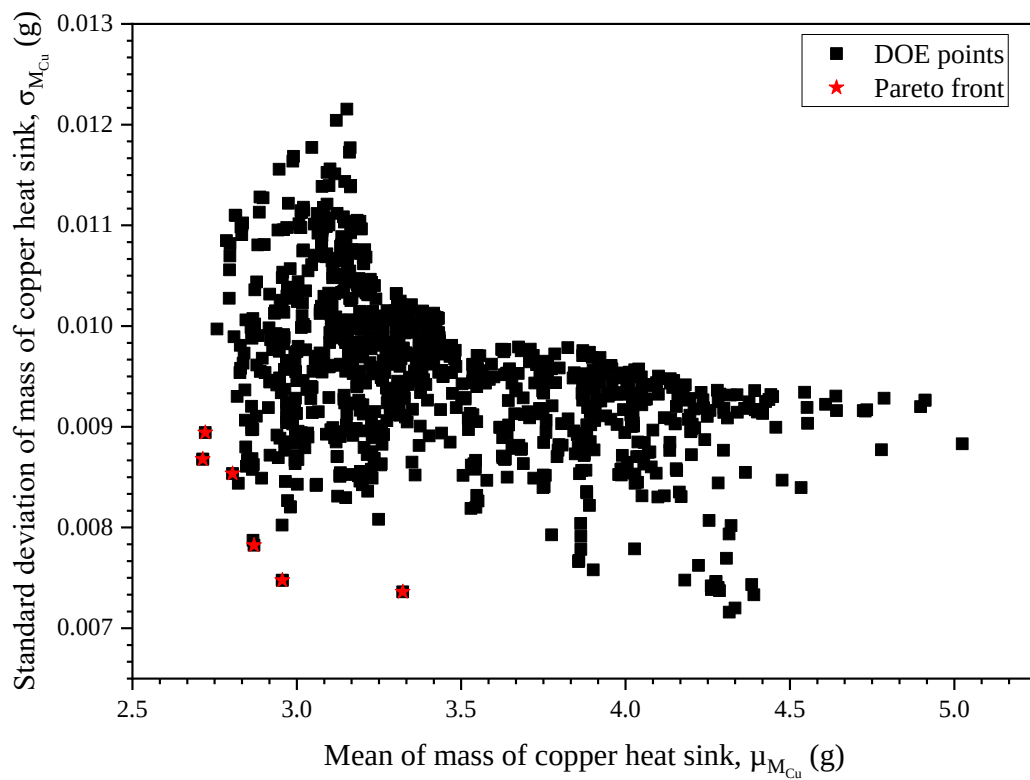


Figure 8-26: Shows the Pareto front and DOE points plots for $\sigma_{M_{Cu}}$ vs $\mu_{M_{Cu}}$.

Table 8-9: Shows the Pareto Front data for $\sigma_{R_{\Delta P}}$ vs $\mu_{R_{\Delta P}}$.

Design variables						$\mu_{\Delta P}$, Pa	$\sigma_{\Delta P}$, Pa
W_{ch} , mm	H_{ch} , mm	H_b , mm	θ_{fin} , mm	D_3 , mm	q'' , W/m^2		
1.285955	1.968322	0.546656	51.61861	1.701559	1412692	2122.120	16.27298
1.274398	1.985136	0.515098	51.75367	1.701559	1412692	2129.889	15.87017
1.266555	1.987346	0.639475	53.69389	1.425166	1784302	2410.241	11.16060
1.255713	1.912627	0.610716	47.74514	1.580595	1148520	2489.601	9.932048
1.219705	1.954180	0.617928	47.01305	1.645128	1183665	2513.679	9.390561
1.266461	1.936722	0.584697	54.04561	1.407455	1684200	2557.728	8.883336
1.284224	1.970017	0.572285	47.83113	1.297032	1336717	2659.594	8.587688
1.218333	1.913571	0.856917	50.82650	1.580355	1174173	2703.416	8.096545
1.204887	1.895282	0.813819	50.06376	1.625352	1141932	2750.281	6.927760
1.043110	1.950855	0.789111	47.17179	1.702575	1243323	2985.884	6.014546
1.110539	1.917415	0.806839	47.17273	1.693557	1228130	3086.472	5.842505
1.060814	1.959515	0.563514	45.37474	1.599593	1355531	3166.723	4.885184
1.049837	1.917807	0.963654	45.33704	1.512732	1246302	3442.421	3.303952

Table 8-10: Shows the Pareto Front data for $\sigma_{R_{th}}$ vs $\mu_{R_{th}}$.

Design variables						$\mu_{R_{th}}$, K/W	$\sigma_{R_{th}}$, K/W
W_{ch} , mm	H_{ch} , mm	H_b , mm	θ_{fin} , mm	D_3 , mm	q'' , W/m^2		
1.05688	1.193691	0.67143	47.92861	0.514786	1195354	0.072390	0.00016
1.043139	1.320812	0.83613	49.84455	0.545350	1210542	0.075167	0.00012
1.050305	1.067759	0.793305	52.75233	0.511024	1097595	0.075322	8.77E-05
1.087942	1.293648	0.869013	49.25140	0.521906	1357959	0.079378	4.51E-05
1.249550	0.814200	0.708284	45.21020	0.514325	1726226	0.080195	3.22E-05
1.016678	1.701022	0.833703	56.35693	0.715144	1424111	0.081376	7.61E-06
1.105215	1.460815	0.628650	56.57961	0.620996	1143522	0.082917	2.91E-06
1.056057	1.193691	0.688810	52.13887	0.800109	1195354	0.085484	3.44E-06
1.072059	1.350882	0.681653	55.80569	0.803216	1126863	0.087653	7.82E-06
1.164612	1.610651	0.583194	59.28486	0.789914	1219753	0.091347	5.1E-06
1.187065	1.958526	0.685285	53.44692	1.123162	1539348	0.095165	8.82E-06
1.127245	0.791962	0.914162	46.83694	1.047386	1579595	0.097209	3.9E-06

Table 8-11: Shows the Pareto Front data for $\sigma_{M_{Cu}}$ vs $\mu_{M_{Cu}}$.

Design variables						$\mu_{M_{Cu}}, \text{g}$	$\sigma_{M_{Cu}}, \text{g}$
W_{ch}, mm	H_{ch}, mm	H_p, mm	θ_{fin}, mm	D_3, mm	$q'', \text{W/m}^2$		
1.207405	0.786738	0.514798	52.98730	1.693606	1047522	2.721835	0.008942
1.222211	0.786738	0.514798	52.98730	1.693606	1047543	2.713994	0.008678
1.252584	0.825652	0.521331	49.82574	1.638057	1205127	2.804766	0.008535
1.184710	0.780718	0.533346	45.75624	1.322993	1262232	2.870160	0.007824
1.222211	0.786416	0.565400	47.05287	1.665139	1047522	2.955783	0.007477
1.026279	0.83115	0.580645	46.29061	1.621520	1114697	3.322375	0.007360

To perform multi-objective robust design optimisation on single objective function, the Pareto fronts obtained are plotted based on their respective means and standard deviations which can be referred in Figure 8-24 to Figure 8-26 respectively. Based on Figure 8-24, the Pareto front and DOE points are plotted based on statistical moments of $\sigma_{R_{th}}$ vs $\mu_{R_{th}}$. From the Pareto front plot, the increase of $\sigma_{R_{th}}$ will results to the decrease of $\mu_{R_{th}}$. The numerical data are tabulated in Table 8-10. Going down the table, the $\mu_{R_{th}}$ increases from 0.0724 K/W to 0.0972 K/W (34.3% change) while the $\sigma_{R_{th}}$ decreases from 0.00016 K/W to 0.0000039 K/W yielding to 97.6 % decrease. Small change in $\mu_{R_{th}}$ leads to big change in $\sigma_{R_{th}}$. Figure 8-25

For both of the $\sigma_{\Delta P}$ vs $\mu_{\Delta P}$ and $\sigma_{M_{Cu}}$ vs $\mu_{M_{Cu}}$ plots, similar Pareto fronts trends recorded where the increase on one objective will have a decrease in another. Going down Table 8-9, for $\sigma_{\Delta P}$ vs $\mu_{\Delta P}$, the $\mu_{\Delta P}$ increases from 2122.12 Pa to 3442.4 Pa (62.2% change) while the $\sigma_{\Delta P}$ decreases from 16.273 Pa to 3.304 Pa respectively giving 79.7% change. For $\sigma_{M_{Cu}}$ vs $\mu_{M_{Cu}}$, going down the Table 8-11, the $\mu_{M_{Cu}}$ increases for 22.1% going from 2.722 g to 3.322 g respectively while the $\sigma_{M_{Cu}}$ decreases from 0.00894 g to 0.00736 g with 17.7% change.

To compare the robust design solutions with the deterministic solutions (refer 8.5.2.1.1), under Table 8-5, the R_{th} has a lowest value of 0.0661 K/W and to compare with robust design solutions, the $\mu_{R_{th}}$ has the lowest and highest values of 0.0724 K/W and 0.0972 K/W giving 9.5% and 47.1% increase from the deterministic solution. For ΔP , deterministically the lowest value is at 1962.6 Pa and comparing with the robust design solutions, the range of 2122.1 Pa to 3442.4 Pa leads to an increase of 8.1% and 75.4% respectively. For M_{Cu} , the lowest deterministic value is 2.678 g when compared with the robust design solution with range of 2.722 g to 3.322 g leads to an increase of 1.6% and 24.1% respectively.

The solutions from robust design optimisation show higher magnitudes compared to the deterministic solutions but to solve the issue from tolerances and operating conditions during manufacturing, it is very useful while still having acceptable thermal-hydraulic performance of heat sink.

8.5.2.2.5.1 Multi-objective robust design optimisation on single objective optimisation with constrained output standard deviations

To further the study of robust design optimisation, from the pre-defined formulation under subchapter 6.5.2.2.7, within this subchapter the normalised standard deviation is varied to see its effect onto the Pareto front plots for multi-objective optimisation of single objective function. The plots of Pareto fronts are plotted on Figure 8-27 to Figure 8-29 respectively. For the R_{th} , referring Figure 8-27, the constraint of $0.0 < \sigma_{R_{th},norm} < 1.0$ has the Pareto front closest to the origin. For other constraints of $0.0 < \sigma_{R_{th},norm} < 0.7$, $0.0 < \sigma_{R_{th},norm} < 0.2$, $0.4 < \sigma_{R_{th},norm} < 0.7$ and $0.5 < \sigma_{R_{th},norm} < 1.0$ respectively, all of them show Pareto fronts slightly higher in the direction $\sigma_{R_{th}}$ with almost similar trends among them.

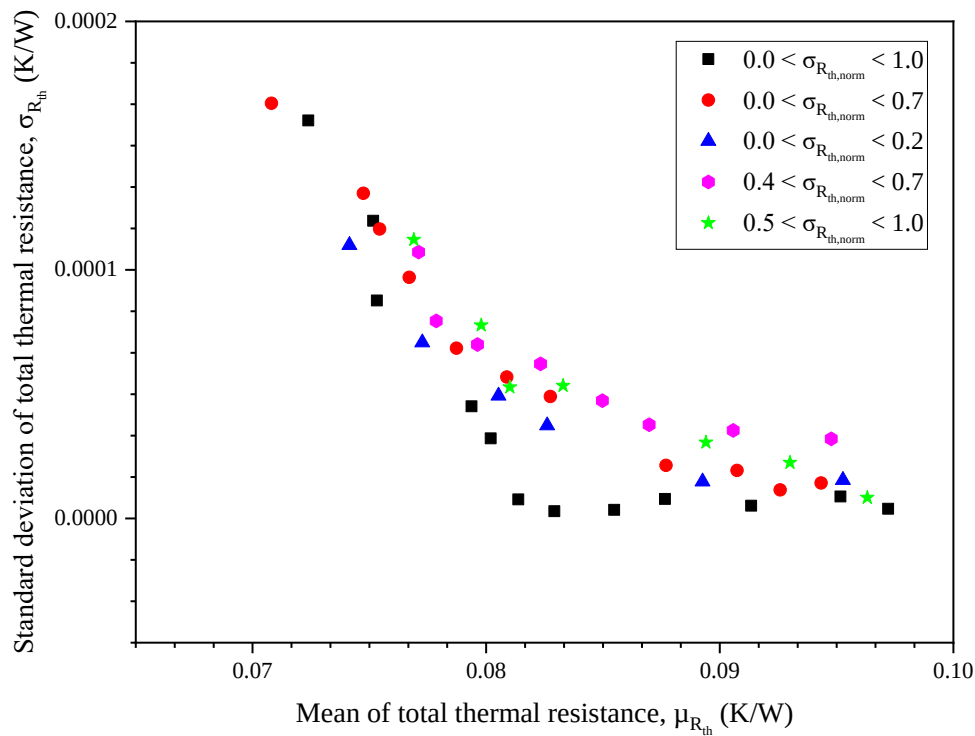


Figure 8-27: Shows the Pareto front of $\sigma_{R_{th}}$ vs $\mu_{R_{th}}$ for different constraints of $\sigma_{R_{th},norm}$.

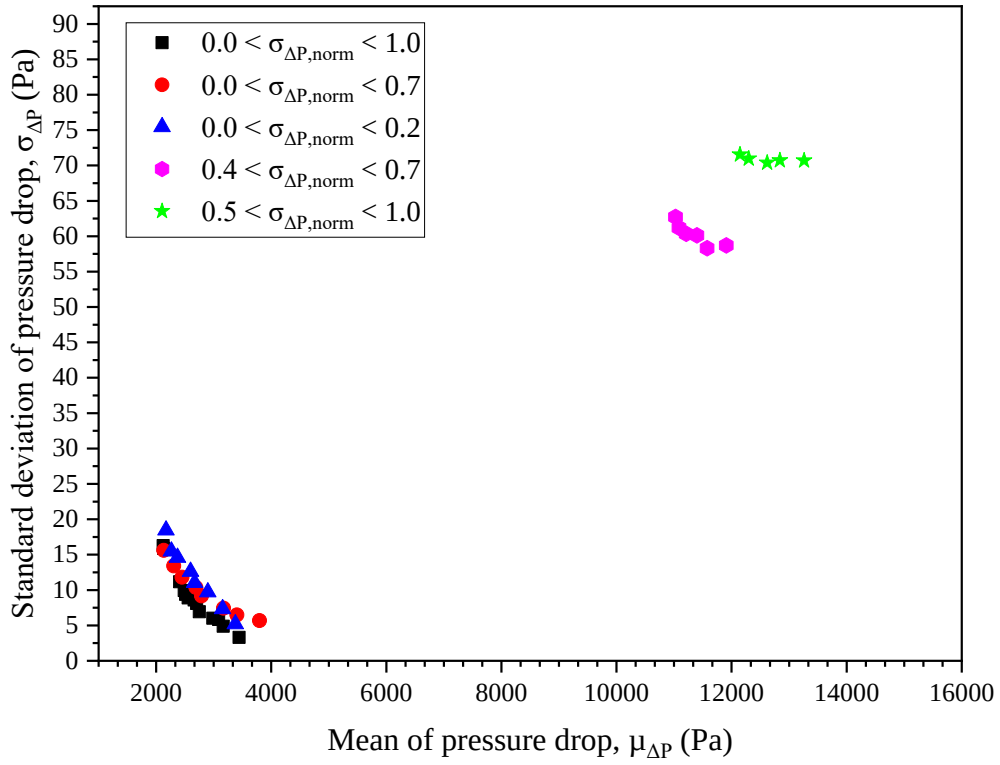


Figure 8-28: Shows the Pareto front of $\sigma_{\Delta P}$ vs $\mu_{\Delta P}$ for different constraints of $\sigma_{\Delta P, norm}$.

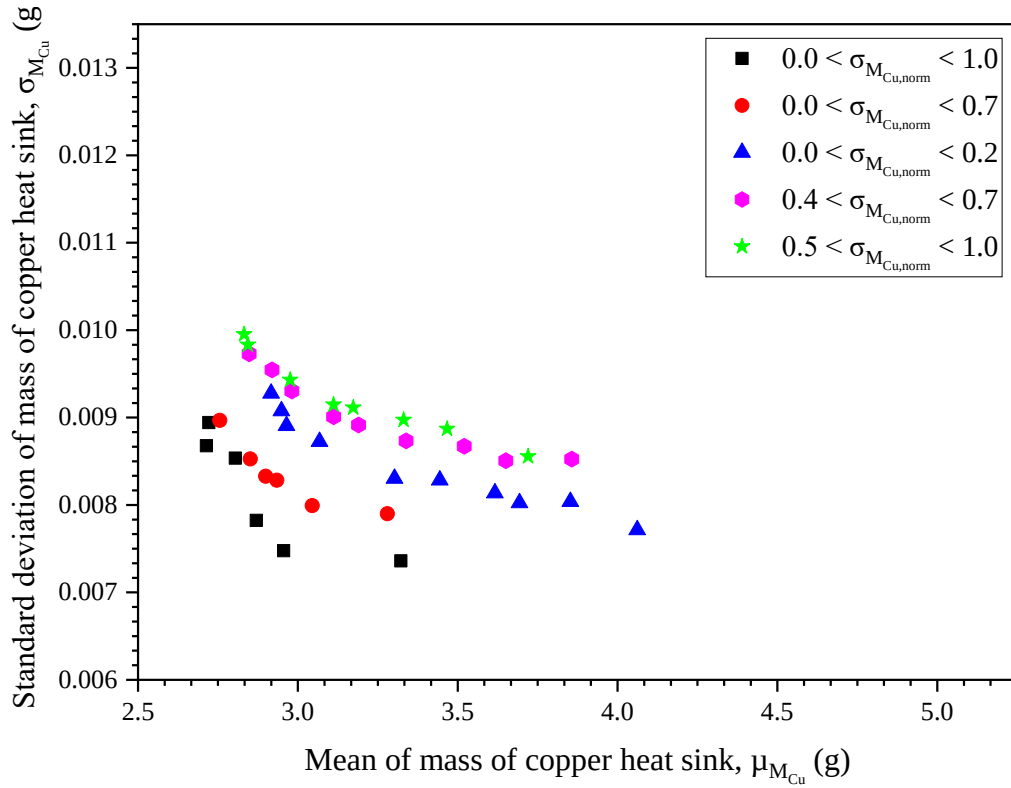


Figure 8-29: Shows the Pareto front of $\sigma_{M_{Cu}}$ vs $\mu_{M_{Cu}}$ for different constraints of $\sigma_{M_{Cu}, norm}$.

For ΔP objective, looking at Figure 8-28, all three constraints of $0.0 < \sigma_{\Delta P, norm} < 1.0$, $0.0 < \sigma_{\Delta P, norm} < 0.7$ and $0.0 < \sigma_{\Delta P, norm} < 0.2$ respectively shows the lowest magnitudes in terms of both $\sigma_{\Delta P}$ and $\mu_{\Delta P}$ with almost the exact similar trends between them. Constraining at $0.4 < \sigma_{\Delta P, norm} < 0.7$, the Pareto front increases in both directions of $\sigma_{\Delta P}$ and $\mu_{\Delta P}$ with a massive change. At constraint of $0.5 < \sigma_{\Delta P, norm} < 1.0$, the increase Pareto front shifted even higher compared to $0.4 < \sigma_{\Delta P, norm} < 0.7$ constraint showing even worse performance in ΔP .

For M_{Cu} objective, based on Figure 8-29, the constraint of $0.0 < \sigma_{M_{Cu}, norm} < 1.0$ shows the lowest Pareto front in term of both $\sigma_{M_{Cu}}$ and $\mu_{M_{Cu}}$ respectively. Going to constraint of $0.0 < \sigma_{M_{Cu}, norm} < 0.7$, the Pareto front shifted upwards slightly and at constraint $0.0 < \sigma_{M_{Cu}, norm} < 0.2$ is even higher. For both constraints of $0.4 < \sigma_{M_{Cu}, norm} < 0.7$ and $0.5 < \sigma_{M_{Cu}, norm} < 1.0$ respectively, they possess the highest Pareto fronts with the same trends. Different levels of Pareto fronts at different constraints is due to the limits of input design variables changes with the constrained normalisation standard deviations.

8.5.2.2.6 Multi-objective robust design optimisation on multi-objective functions with constrained output standard deviation.

Referring to subchapter 8.5.2.2.5, the study of multi-objective optimisation on a single objective function was considered, thus, within this subchapter the multi-objective optimisation on multiple objective functions are studied utilising all three $\sigma_{R_{th}, norm}$, $\sigma_{\Delta P, norm}$ and $\sigma_{M_{Cu}, norm}$ statistical moments in order to observe its effect on the Pareto front plots based M_{Cu} , R_{th} and ΔP objective functions.

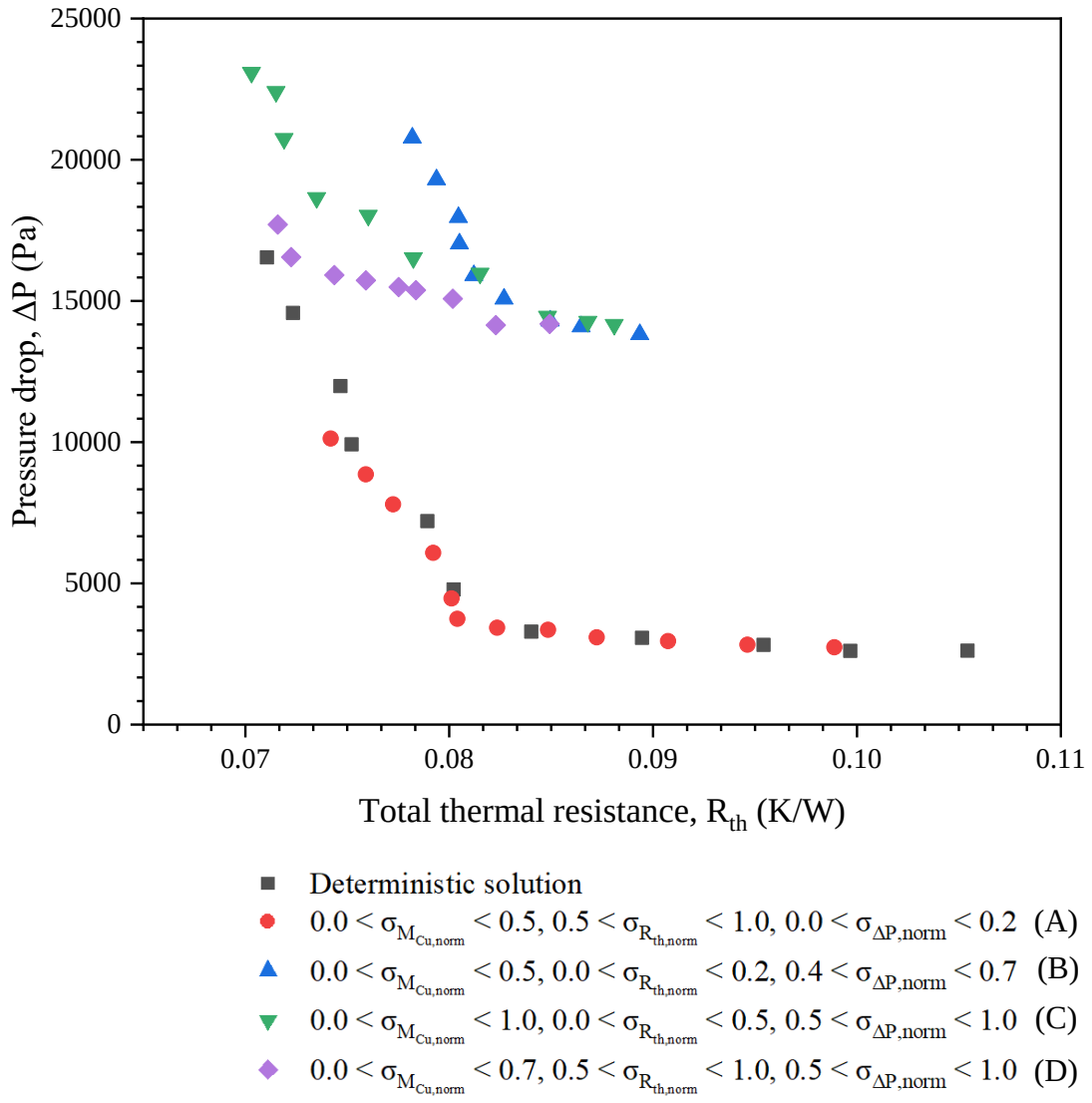


Figure 8-30: Pareto fronts plot of ΔP vs R_{th} for different values of constraints of $\sigma_{R_{th,norm}}$, $\sigma_{\Delta P,norm}$ and $\sigma_{M_{Cu,norm}}$ parameters respectively.

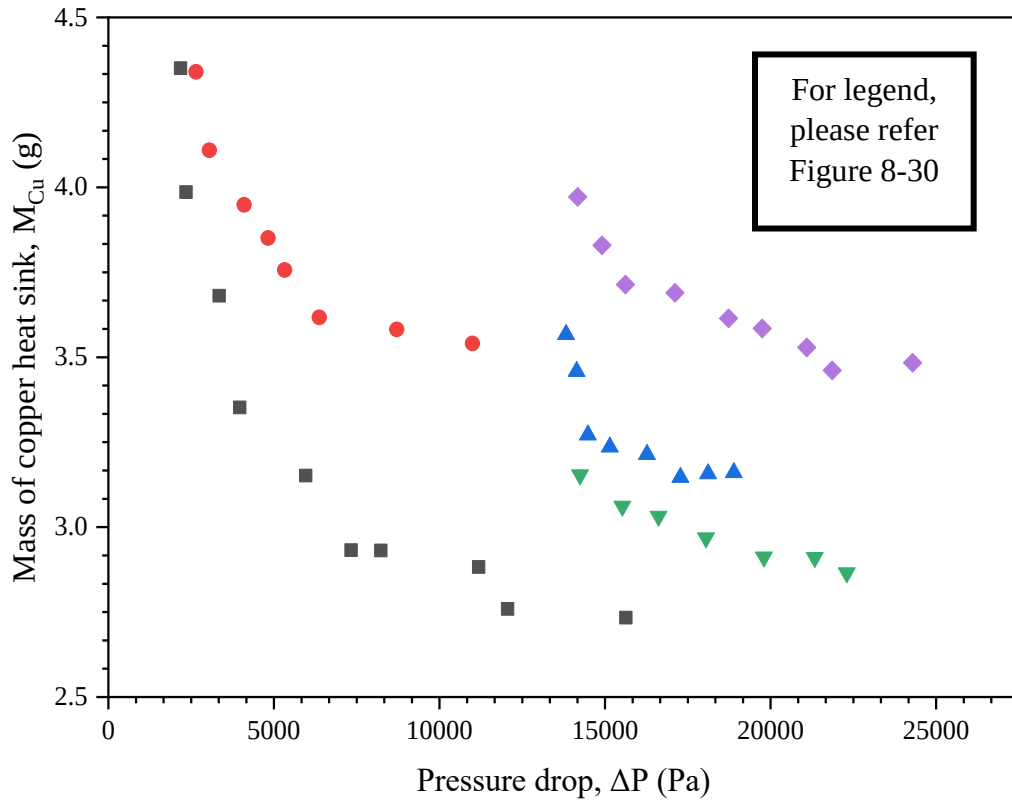


Figure 8-31: Pareto fronts plot of M_{Cu} vs ΔP for different values of constraints of $\sigma_{R_{th},norm}$, $\sigma_{\Delta P,norm}$ and $\sigma_{M_{Cu},norm}$ parameters respectively.

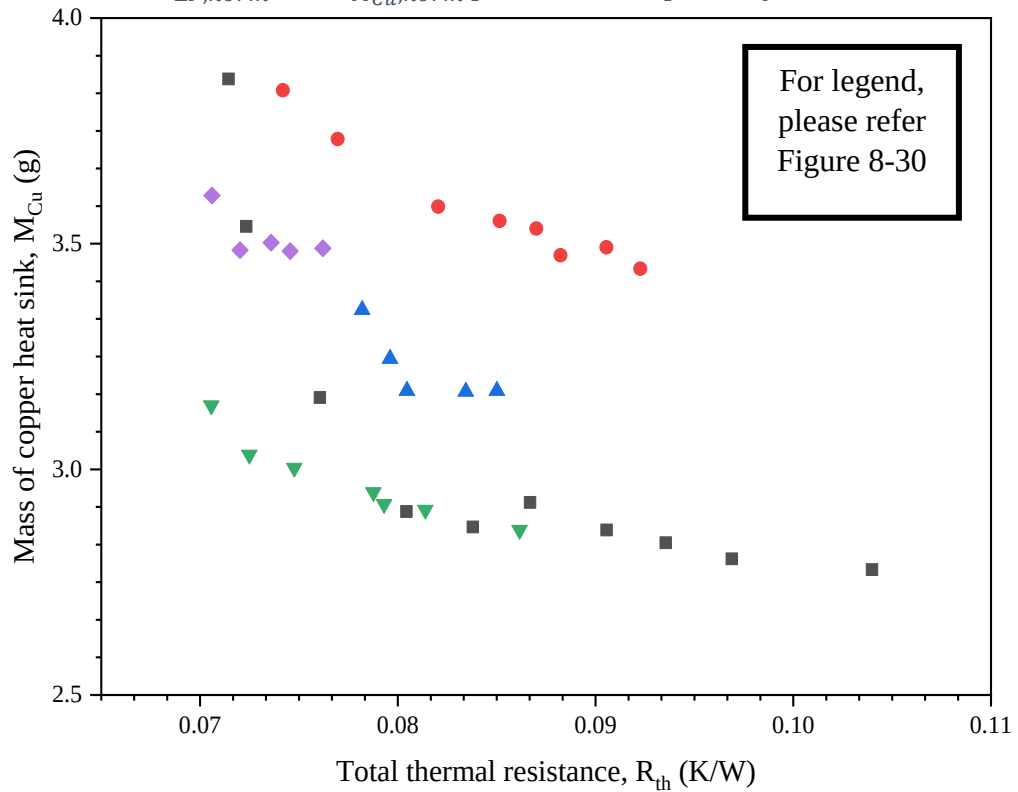


Figure 8-32: Pareto fronts plot of M_{Cu} vs R_{th} for different values of constraints of $\sigma_{R_{th},norm}$, $\sigma_{\Delta P,norm}$ and $\sigma_{M_{Cu},norm}$ parameters respectively.

Referring to Figure 8-30, for the ΔP vs R_{th} , deterministic solution shows the same Pareto front and trend when compared with constraint at (A) and the Pareto fronts are the closest to the origin. For constraint of (D) the Pareto front shifted upwards in the direction of ΔP with higher rate on the increase of R_{th} due to the decreasing size of both H_{ch} and W_{ch} parameters constricting the channel while having high value of θ_{fin} to compensate the total thermal resistance values. At constraint of (C), the Pareto front shifted even higher while at constraint of (B), the Pareto front shifted to the right with the trend.

From Figure 8-31, for M_{Cu} vs ΔP , the deterministic solution shows the lowest Pareto front in terms of both M_{Cu} and ΔP respectively. At constraint of (A), the Pareto front shifted upwards with high rate of increase of ΔP due to the constricting channel profile but θ_{fin} is small enough to maintain the ΔP . For constraint of (D), the Pareto front shifted upwards and rightwards showing the highest magnitudes of both M_{Cu} and ΔP while having the same trend as constraint of (A) since the combination of large values of H_{ch} and W_{ch} with higher θ_{fin} maintaining the M_{Cu} but increasing the ΔP . For constraint of (B), the Pareto front also shifts upwards and rightwards but lower compared to the highest Pareto front which has small H_{ch} and W_{ch} but large θ_{fin} magnitude. At constraint of (C), the Pareto front shifted rightwards with higher ΔP .

Looking at Figure 8-32, for M_{Cu} vs R_{th} , the deterministic solution and at constraint of (C) shows the lowest Pareto fronts compared with the other constrained Pareto fronts. At constraint of (B), the Pareto front shifted upwards slightly with the same trend as deterministic solution due to larger θ_{fin} . For constraint of (D), the Pareto front shifted higher but with lower magnitudes of R_{th} due to very large θ_{fin} with similar channel size but thicker H_b . Finally at constraint of (A), the Pareto front has the highest values of M_{Cu} but has the same trend with deterministic solution due to very large channel size for highest A_{eff} and very thick H_b size.

8.6 Summary

The utilisation of heat sink with liquid coolant has been studied extensively to search for an effective heat sink system. For this chapter, the employment of secondary channel and through-holes within double inlet-outlet serpentine mini-channel heat sink is used for the enhancement of thermal-hydraulic performance of heat sink. The performance augmentation is due to the continuous breaking of thermal and hydrodynamic boundary layers as well as achieving liquid velocity reduction to simultaneously improve the heat transfer and pressure distribution. The use of diamond shaped fins will be embedded within plate fins creating secondary channels.

Referring to findings, the total thermal resistance (R_{th}) monotonically reduced when Reynolds number (Re) increases. The employment of D-SMDF-TH heat sink shows an average of 5.1% and 21.7% increase in R_{th} when compared with D-SMDF and D-SMPF heat sinks since higher effective area (A_{eff}) and more frequency in thermal boundary layers breaking and re-initialisation of flow. The Average Nusselt number (Nu_{avg}) for D-SMDF-TH heat sink shows slight lower magnitudes at lower Re compared to the D-SMDF and D-SMPF heat sink, but at higher Re the Nu_{avg} of D-SMDF-TH heat sink surpassed the D-SMPF heat sink. Looking at ΔP parameter, the D-SMDF-TH heat sink improves by 82.6% and 66.1% compared to the D-SMPF and D-SMDF heat sinks since the liquid velocity within channel reduced greatly due to pressure equilibrium between serpentine through through-holes. The PEC index for D-SMDF-TH heat sink improves by 163% and 52.1% compared to the D-SMPF and D-SMDF heat sinks. Parametric analysis are performed for D-SMDF-TH heat sink to check its effect on the thermal-hydraulic performance of heat sink.

Under the parametric analysis, the variation of channel height (H_{ch}) from 1.5 mm to 2.0 mm reduces both R_{th} and ΔP parameters by 18.2% and 12.2% respectively. This is caused by the increase of A_{eff} for more heat transfer and bigger channel size reducing liquid velocity. Changing the channel width (W_{ch}) from 0.8 mm to 1.0 mm also lead to the decrease of both R_{th} and ΔP parameters by 9.9% and 15% respectively due the same reason mentioned earlier. Increasing the θ_{fin} from 45° to 60° result in the increase of both R_{th} and ΔP parameters by 4.9% and 21.3% respectively. The smaller value of R_{th} at lower θ_{fin} is due to the higher inertia of liquid passing through the secondary channel improving the breaking of boundary layers more effectively. For heat flux (q''), changing the magnitude from 100 W/cm^2 to 200 W/cm^2 increases the $T_{surf,max}$ by 22.3% and slightly affects the ΔP due to the decrease of liquid viscosity and density at high temperature.

Using the framework of single- and multi-objective optimisation method, the D-SMDF-TH heat sink is further optimised. The use of deterministic and robust design optimisations methods were also utilised. Within the deterministic optimisation framework, single-objective is easily conducted for individual objective searching the minimum magnitude of either R_{th} , ΔP or M_{Cu} respectively. The use of single-objective function is not entirely desirable since only individual objective is optimised while other objectives will be very large and not feasible. Solving this problem, multi-objective optimisation is carried out optimising all objectives simultaneously obtaining the Pareto front of conflicting objectives. From ΔP vs R_{th} , the Pareto

front shows the increase of R_{th} by 48.4% while ΔP reduces by 84.2%. For M_{Cu} vs ΔP , the ΔP increases by 616.5% while M_{Cu} reduces by 37.2% respectively. Looking at M_{Cu} vs R_{th} , the R_{th} rises by 45.6% while M_{Cu} drop by 28% respectively.

The optimisation by robust design uses uncertainty on the input design variables to obtain the distribution of output objective functions. The use of including uncertainty study is due to the fact that in real life manufacturing processes, the tolerances and unpredictable operating conditions are taken into account which require range solutions to have. The multi-objective optimisation of single objective function is conducted based on the means and standard deviations. After that, multi-objective optimisation of multiple objective functions are carried out to all objectives of M_{Cu} , R_{th} and ΔP respectively. Lastly, the constrained of normalised of standard deviations are used to see the effect on the Pareto-front plots of multi-objective optimisations.

Chapter 9 : Discussions and conclusions

9.1 Introduction

For this work, the numerical analysis of single and double inlet-outlet serpentine mini-channel heat sinks were studied to their respective thermal-hydraulic performances. Under Chapter 6, the use of single inlet-outlet serpentine mini-channel heat sink with application of secondary channel was explored due to simultaneous improvement of heat transfer coefficient and reduction of pressure drop altogether. Three different shapes of fins which are oblique (SMOF), trapezium (SMTF) and diamonds (SMDF) were tested. Each shape of fin had four different configurations of -A, -B, -C and -D respectively. The single inlet-outlet serpentine mini-channel heat sinks with fins was compared to the conventional serpentine mini-channel heat sink with plate fin (SMPF) in terms of its total thermal resistance (R_{th}), Average Nusselt number (Nu_{avg}) and pressure drop (ΔP). The configuration of SMDF-C heat sink offers the highest thermal performance. Within Chapter 7, newly configured design of heat sink (D-SMDF) of double inlet-outlet serpentine mini-channel heat sink with the application of secondary channel was tested for its capability to remove high heat flux. The diamond fin shape was used for this design since it offers the capability to enhance the heat transfer from the findings during the analysis of single inlet-outlet serpentine mini-channel heat sink. For Chapter 8, the exact base design used in Chapter 7 is used here with one modification of adding through-holes connecting two serpentine channels (D-SMDF-TH) to distribute the dynamic pressure distribution reducing the ΔP massively as well as enhancing the reduction of R_{th} at the same time. The performance of D-SMDF-TH heat sink was compared with the double inlet-outlet serpentine mini-channel heat sink with through-holes and plate fin (D-SMTH). During the comparison studies for all heat sinks, the Reynolds number (Re) were set to be from 500 to 2300. A total of 100 W/cm^2 of uniformly distributed heat flux was applied at the bottom surface of heat sink.

At the end of the chapters of Chapter 6, Chapter 7 and Chapter 8 respectively, the most superior in thermal performance of SMDF-C, D-SMDF and D-SMDF-TH heat sinks were optimised aiming to achieve reduction of R_{th} , ΔP and mass of heat sink (M_{cu}) altogether. The optimisation procedures covered two major frameworks of deterministic and Robust Design Optimisation (RDO) method. The findings from the analysis are summarised in subchapter 9.2, also recommending the future works proposal and conclusion is made.

9.2 Discussions

For the discussion, the results from Chapter 6, Chapter 7 and Chapter 8 will be discussed into three major subchapters of 9.2.1, 9.2.2 and 9.2.3 respectively followed by the findings from the optimisation processes.

9.2.1 Single inlet-outlet serpentine mini-channel heat sink with fins

From Chapter 6, numerical analysis on the single inlet-outlet serpentine mini-channel heat sink with the application of secondary channels was conducted. From the study, a few crucial conclusions were determined as follows:

- A) From the analysis, the reduction of R_{th} occurred monotonically when liquid velocity increases due to the thinning of the thermal boundary layer for heat sink designs and configurations tested.
- B) The use of diamond fins showed more superior in thermal performance due to more breaking of boundary layers and flow re-initialisation as well as less region of localised low velocity region inhibiting fluid mixing between hot and cold liquid. The lowest R_{th} recorded was by SMDF-C heat sink with 14% improvement compared to SMPF heat sink. The Nu_{avg} was also enhanced by 23.4% compared to SMPF heat sink.
- C) The highest reduction of ΔP was contributed by SMTF-B heat sink with 83.3% reduction due to the lowest liquid velocity achieved reducing the dynamic pressure distribution.
- D) For PEC index study, the SMDF-D heat sink showed the highest PEC index due to the optimal combination of heat transfer and hydraulic components.
- E) For the parametric analysis, the SMDF-C heat sink was considered due its superiority in terms of R_{th} and Nu_{avg} which is more important when it comes to cooling. Changing the secondary channel length (l_{sc}) from 1.0 mm to 0.7 mm led to reduction of R_{th} by 26% since more effective surface area (A_{eff}) available for heat transfer but higher ΔP increase due to less portion of liquid was displaced through the secondary channel increasing the velocity within main channel. Similarly, increasing the fin angle (θ_{fin}) from 25° to 55° led to the reduction of R_{th} by 36.5% and increased the ΔP since the secondary channel width (W_{sc}) was constricting the liquid flow. For the height of channel (H_{ch}) and width of channel (W_{ch}), increasing both parameters definitely led to the decrease of R_{th} since larger A_{eff} and the ΔP also reduced due to bigger channel profile area reducing the liquid velocity. The heat flux (q'') applied would definitely

affects maximum surface temperature ($T_{surf,max}$) since they are proportional to each other but slightly affects the ΔP due to the physical properties of liquid changes with temperature.

The use of pin-fin heat sink was very useful to improve the heat transfer effectively since the thermal boundary layers were stopped from forming and allows better mixing of fluid. These types of heat sink also increase the A_{eff} for more rate of heat transfer. On the other hand, the downside of using pin-fin heat sink was more pumping power required. For this work, the novel design used was very effective compared to pin-fin heat sink since both ΔP and R_{th} experience reduction.

9.2.2 Double inlet-outlet serpentine mini-channel heat sink with fins

From Chapter 7, double inlet-outlet serpentine mini-channel heat sink with the application of secondary channels was numerically analysed. The concluding points are listed as follows:

- A) Similarly, the R_{th} reduced monotonically as Re increases for the D-SMDF heat sink due to the increased convective transfer and reduction of thermal boundary layer thickness.
- B) The employment of D-SMDF heat sink provided 17.6% improvement of R_{th} compared with D-SMPF heat sink since the thermal boundary layers were continuously breaking and high frequency flow re-initialisations. The Nu_{avg} of D-SMDF heat sink was improved massively with 28% increase compared to D-SMPF heat sink caused by larger A_{eff} .
- C) For ΔP , the D-SMDF heat sink had lower magnitude compared to D-SMPF heat sink by 48.7% reduction due to the liquid velocity within the main channel reduced drastically as the liquid was displaced through the secondary channel.
- D) The PEC index for D-SMDF heat sink offered 1.73 times superiority compared to the D-SMPF heat sink based on the high performance in terms of heat transfer and hydraulic components as well as being lighter in mass.
- E) For the parametric analysis, changing the θ_{fin} from 45° to 60° improved R_{th} by 3.7 % and increased ΔP by 18.7% respectively due to higher A_{eff} and less portion of fluid diverted through the secondary channel. The variation of H_{ch} and W_{ch} from 1.5 mm to 2.0 mm and 0.8 mm to 1.0 mm respectively led to the reduction of R_{th} by 21.6% and 9.8% while the ΔP was reduced by 18.4% and 9.4% respectively since the channel size was larger reducing the liquid velocity and higher A_{eff} for more heat transfer.

Similarly, the increased of q'' affects the $T_{surf,max}$ to be higher as more heat was supplied while having slight changes to the ΔP due to the slight variation of liquid physical properties with temperature.

9.2.3 Double inlet-outlet serpentine mini-channel heat sink with fins and through-holes

From Chapter 8, the effect of using the double inlet-outlet serpentine mini-channel heat sink together with secondary channels and through-holes on thermal-hydraulic performance was numerically analysed. The conclusions obtained from analysis are as follows:

- A) The same trend here where the R_{th} was monotonically decreasing with liquid velocity for D-SMDF-TH heat sink due to the thinning of thermal boundary layer and increasing in heat transfer by convection.
- B) The D-SMDF-TH heat sink had superior performance in terms of R_{th} compared to D-SMDF and D-SMPF heat sinks by 5.1% and 21.7% enhancement. This was due to larger A_{eff} for more heat transfer to occur high frequency of flow re-initialisation and breaking of thermal boundary layers.
- C) For Nu_{avg} parameter, at low Re D-SMDF-TH heat sink had slightly lower magnitudes compared to the D-SMDF and D-SMPF heat sinks due to the formation of localised low velocity region affecting heat transfer to deteriorate, but at higher Re , the Nu_{avg} of D-SMDF-TH heat sink had surpassed the D-SMPF heat sink since localised low velocity region disappeared improving heat transfer.
- D) For ΔP , the D-SMDF-TH heat sink improved by 82.6% and 66.1% compared to the D-SMPF and D-SMDF heat sink since the dynamic pressure of liquid reach equilibrium through the through-holes reducing the liquid velocity significantly.
- E) For PEC index study, the D-SMDF-TH heat sink was more superior compared to D-SMPF and D-SMDF heat sink with 163% and 52.1% increased respectively.
- F) Under the parametric analysis, changing the θ_{fin} from 45° to 60° led to the increase both R_{th} and ΔP by 4.9% and 21.3% respectively. This was due the fact at smaller θ_{fin} the liquid inertia passed through the secondary channel was larger improving the breaking of thermal boundary layers more effectively. Higher ΔP at higher θ_{fin} was due to the less portion of liquid was diverted through the secondary channel hence increasing the liquid velocity within the main channel. The variation of H_{ch} and W_{ch} from 1.5 mm to 2.0 mm and 0.8 mm to 1.0 mm respectively led to the reduction of R_{th} by 18.2% and 9.9% while the ΔP reduced by 12.2% and 15% respectively since the

channel size was larger reducing the liquid velocity and higher A_{eff} for more heat transfer. Similarly, the increased of q'' affects the $T_{surf,max}$ to increase due to higher heat supply and only slight changes occur to the ΔP due to the liquid physical properties varied with temperature.

9.2.4 Optimisation method

Under the optimisation analysis, initially the input design variables were chosen first which are tabulated in Table 9-1. Then, the Design of Experiment (DOE) points were constructed using the input design variables in Table 9-1 using Optimal Latin hypercube (OLHC) sampling method for even distributed points throughout the design space. Each DOE point was fed into the COMSOL Multiphysics CFD software to generate the output results (objective functions) and used to create surrogate models based on Radial Basis Function (RBF) method (refer Table 9-1). After the surrogate models had been built, the models were utilised within the optimisation algorithm of non-dominated sorting genetic algorithm-II (NSGA-II) for optimisation process. The single- and multi-objective optimisations processes were conducted under both deterministic and robust design frameworks.

Table 9-1: Shows the tabulated input design variables used for different heat sinks.

Heat sink type	Input design variables	Output objective functions
SMDF-C	$W_{ch}, \theta_{fin}, H_{ch}, H_b$	$R_{th}, M_{Cu}, \Delta P$
D-SMDF	$H_{ch}, W_{ch}, \theta_{fin}, H_b, q'', Re$	$R_{th}, M_{Cu}, \Delta P$
D-SMDF-TH	$H_{ch}, W_{ch}, \theta_{fin}, H_b, 2R_3, q''$	$R_{th}, M_{Cu}, \Delta P$

For the single-objective optimisation, only a single objective function was optimised without considering the other objectives. Only one objective experienced minimisation without considering the other objectives which caused non-optimised objective to be very large for all heat sink cases. Thus, to solve this issue, the multi-objective optimisation framework was used to optimise all objectives where the Pareto front of competing objectives was created.

Under the deterministic framework, single- and multi-objective optimisations were conducted for SMDF-C, D-SMDF and D-SMDF-TH heat sinks. After that, Robust design optimisation was conducted for all SMDF-C, D-SMDF and D-SMDF-TH heat sinks respectively. Firstly, the means and standard deviations of single objective function was calculated using pre-developed surrogate models based on the Monte Carlo sampling (MCS) or Polynomial Chaos Expansion (PCE) methods. The newly constructed means and standard deviations of outputs

were trained further using RBF method to create surrogate models for robust design optimisation.

Under robust design optimisation, the multi-objective optimisation of single objective function was conducted first. Pareto front was created based on the standard deviation vs mean plot. The results were compared with the single-objective optimisation results under deterministic framework. Further analysis of multi-objective robust design optimisation of single objective function covered the constrained of normalised standard deviations to observe the effect on the performance of Pareto front. The results from robust design optimisation showed the performance of R_{th} , M_{Cu} and ΔP were worse than deterministic results but with less sensitivity towards the normalised standard deviation variation.

Lastly, the multi-objective robust design optimisation of all objective functions of R_{th} , M_{Cu} and ΔP respectively was conducted based on constrained normalised standard deviations of $\sigma_{R_{th},norm}$, $\sigma_{\Delta P,norm}$ and $\sigma_{M_{Cu},norm}$ to observe the effect on Pareto front plots and compared with the results from multi-objective optimisation under deterministic framework. Similarly, the Pareto fronts of robust design optimisation had less performance compared to the deterministic results but possesses the insensitivity towards normalised standard deviations. Robust design optimisation offers the solution when dealing with tolerances and operating conditions of real life manufacturing processes which can be unpredictable.

9.3 Recommendations for future works

From the findings and achievements from this numerical works, possible future works to be conducted are listed as follows:

- A) In this work, the serpentine mini-channel heat sink analysis has been extensively researched with water as coolant which is commonly used and also only single phase is considered. Therefore, to further the study, different fluids can be utilised such as Perfluorinated poly ketones (Galden Ht series), dielectrics (HFEs and HFOs) and Nano-fluids (Al_2O_3 , Fe_3O_4 , Fe_2O_3 etc.) as well as heat transfer by flow boiling within serpentine mini-channel heat sink with diamond fins.
- B) For this work, only copper (Cu) material used for heat sinks due to its high thermal conductivity of 398 W/m.K. The use of aluminium (Al) can be used since it is so much cheaper but the thermal performance would degrade since it has lower thermal conductivity at 200 W/m.K compared to Cu which requires more simulations to be conducted.

- C) For this work, the optimisation only up to six design variables and three objective functions were considered. Thus, adding more input design variables such as secondary channel length (l_{sc}), size of particle of nano-fluids (if nano-fluid is used) and distance between fins can be considered. Adding more objective functions such as Nu_{avg} and heat transfer coefficient (h_{avg}), temperature distribution standard deviation (T_{TSD}) are also possible.
- D) The use of vortex generator within the channels can be applied but the downside of it is the ΔP will increase significantly.
- E) The application of wavy channel within serpentine channels also useful to disrupt the flow improving heat transfer.
- F) Under the optimisation framework, the use of topology optimisation with the application Radial Basis Function (RBF) is appealing.

9.4 Conclusions

Thermal management of electronics is becoming increasingly challenging and liquid cooled heat sinks are required rather than simply air ones heat sinks have received much attention in the literature. For this work, the use of diamond fins within serpentine mini-channel heat sink had been studied extensively in both single and double inlet-outlet configurations. Using diamond fins had shown superior thermal performance compared to other shaped of fins of oblique and trapezium.

Based on double inlet-outlet configuration heat sink with diamonds fins but without through-holes, it offered massive superiority in terms of thermal performance compared to single inlet-outlet configuration with diamond fins but with the expense of higher pressure drop since two inlet liquid velocities were used.

Using double inlet-outlet configuration heat sink with both diamonds fins and through holes provided superiority in thermal performance as well as hydraulically since more heat transfer rate due to larger effective area and dynamic pressure distributed evenly to be very small.

For companies that have an interest in cooling solutions, they can actually benefitted from the analysis conducted in this optimisation works.

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Appendix

A.1 The water thermo-physical properties

For this work, the equations of water thermo-physical properties of density of fluid (ρ_f), specific heat (C_{p_f}), thermal conductivity (k_f) and dynamic viscosity (μ_f) respectively are crucial which has been built in COMSOL Multiphysics. The equations are as a function of temperature defined as follows:

$$\rho_f = 838.466 + 1.20T - 0.0030T^2 + 3.72 \times 10^{-7}T^3 \quad (\text{A1.1})$$

$$\mu_f = 1.38 - 0.0212T + 1.36 \times 10^{-4}T^2 - 4.646 \times 10^{-7}T^3 + 8.90 \times 10^{-10}T^4 - 9.08 \times 10^{-13}T^5 + 3.846 \times 10^{-16}T^6 \quad (\text{A1.2})$$

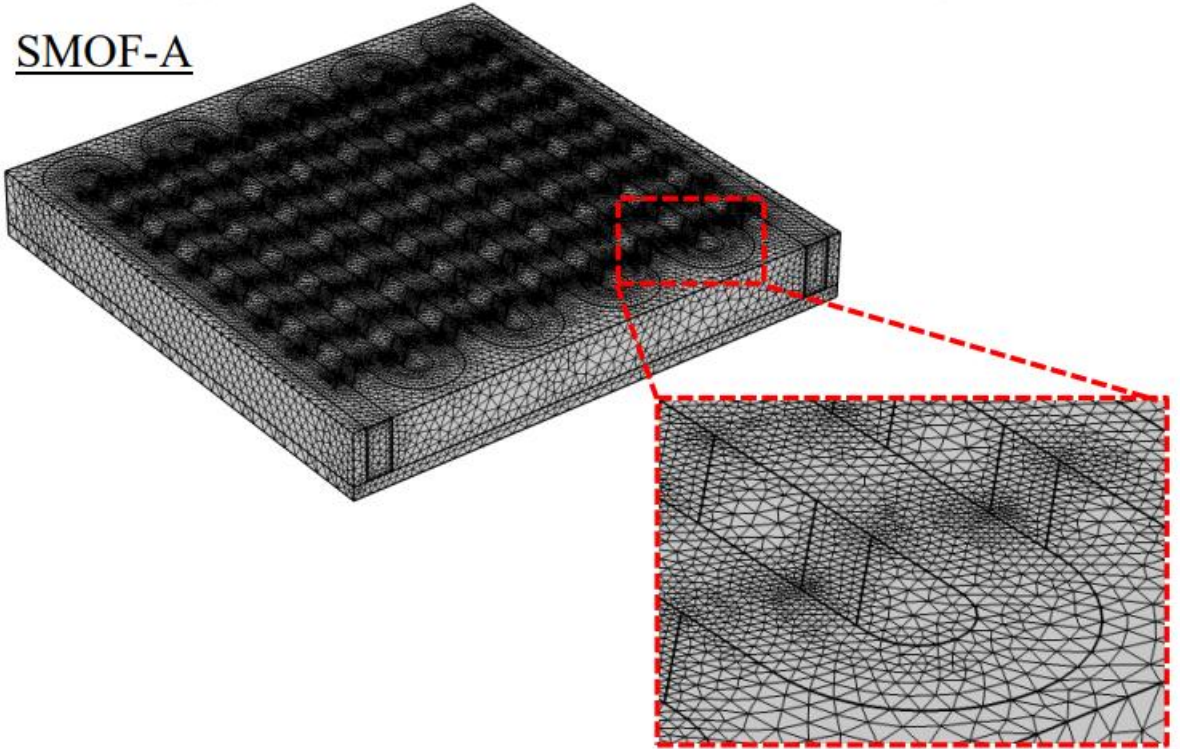
$$C_{p_f} = 12010.147 - 80.407T + 0.3099T^2 - 5.382 \times 10^{-4}T^3 + 3.625 \times 10^{-7}T^4 \quad (\text{A1.3})$$

$$k_f = -0.869 + 0.00895T - 1.584 \times 10^{-5}T^2 + 7.975 \times 10^{-9}T^3 \quad (\text{A1.4})$$

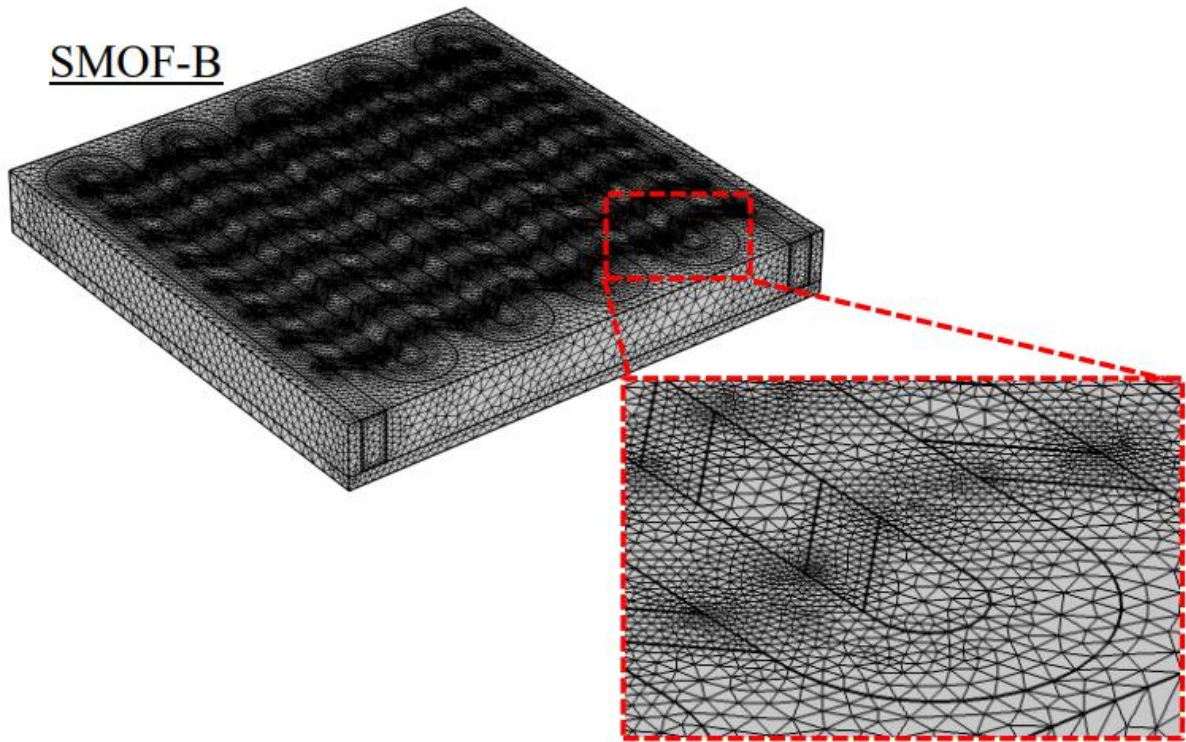
A.2 Mesh structure

A 2.1 Single inlet-outlet serpentine mini-channel heat sink with oblique secondary channels

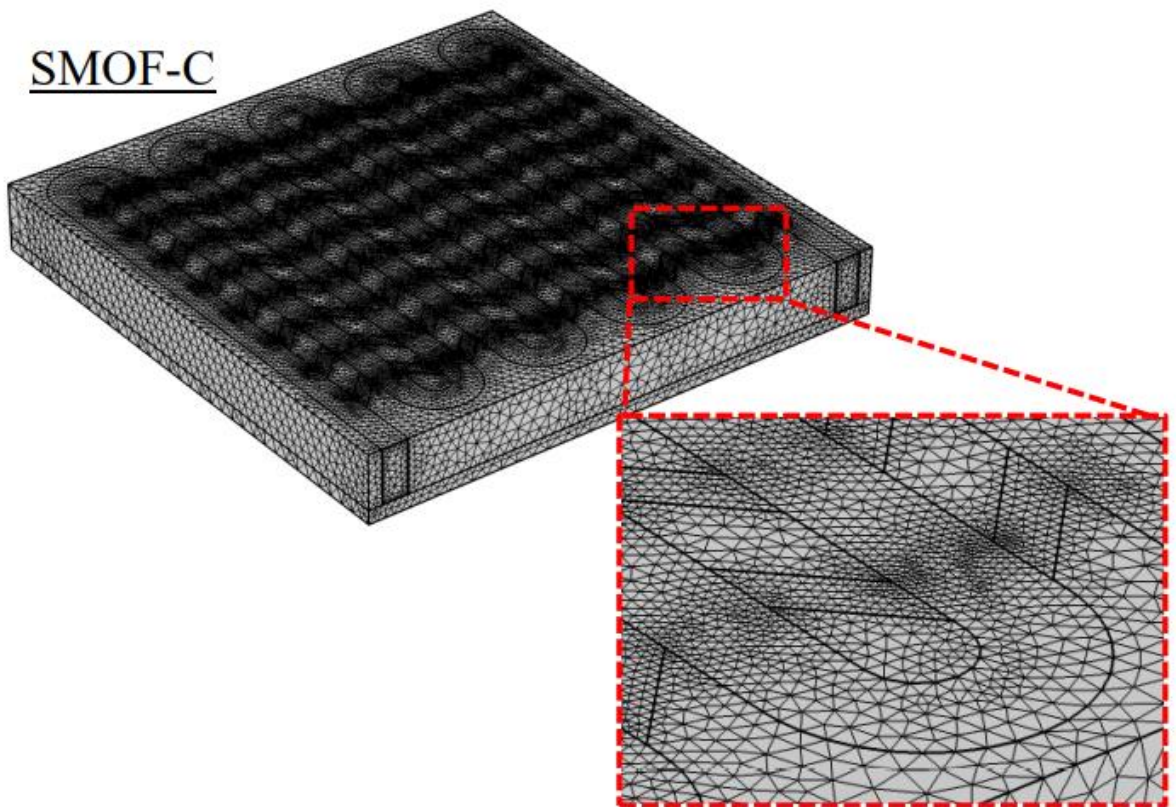
SMOF-A



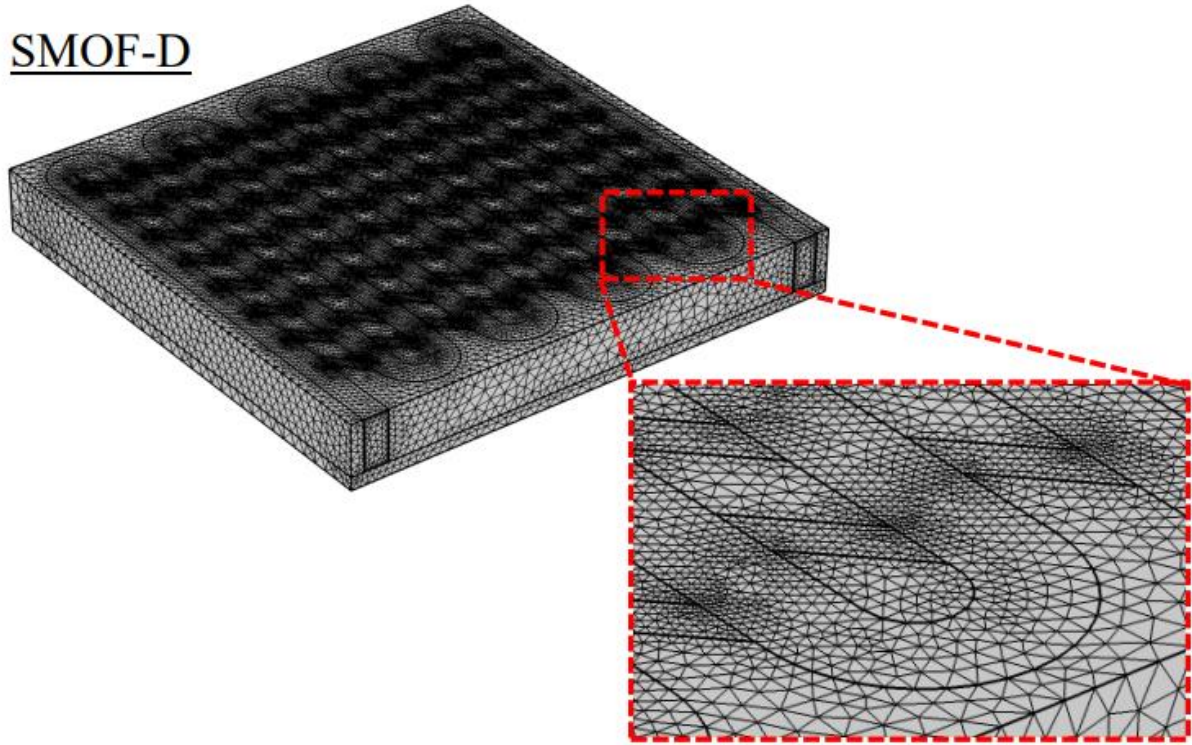
SMOF-B



SMOF-C

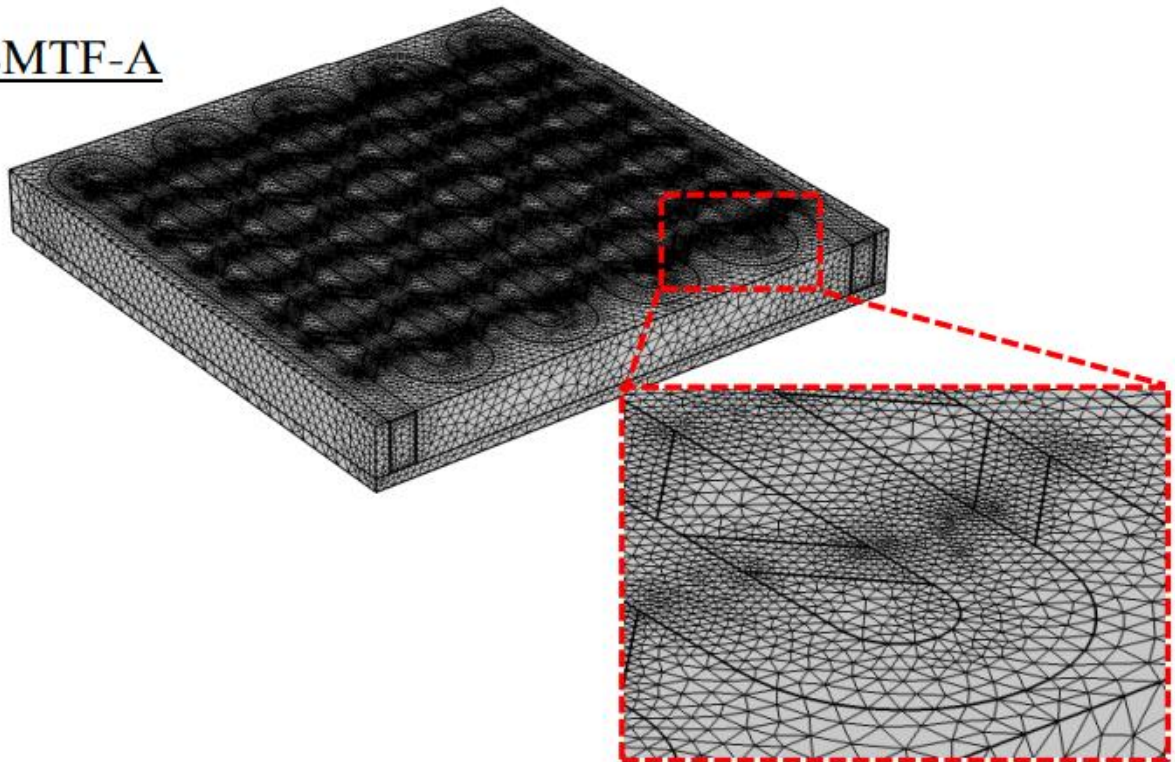


SMOF-D

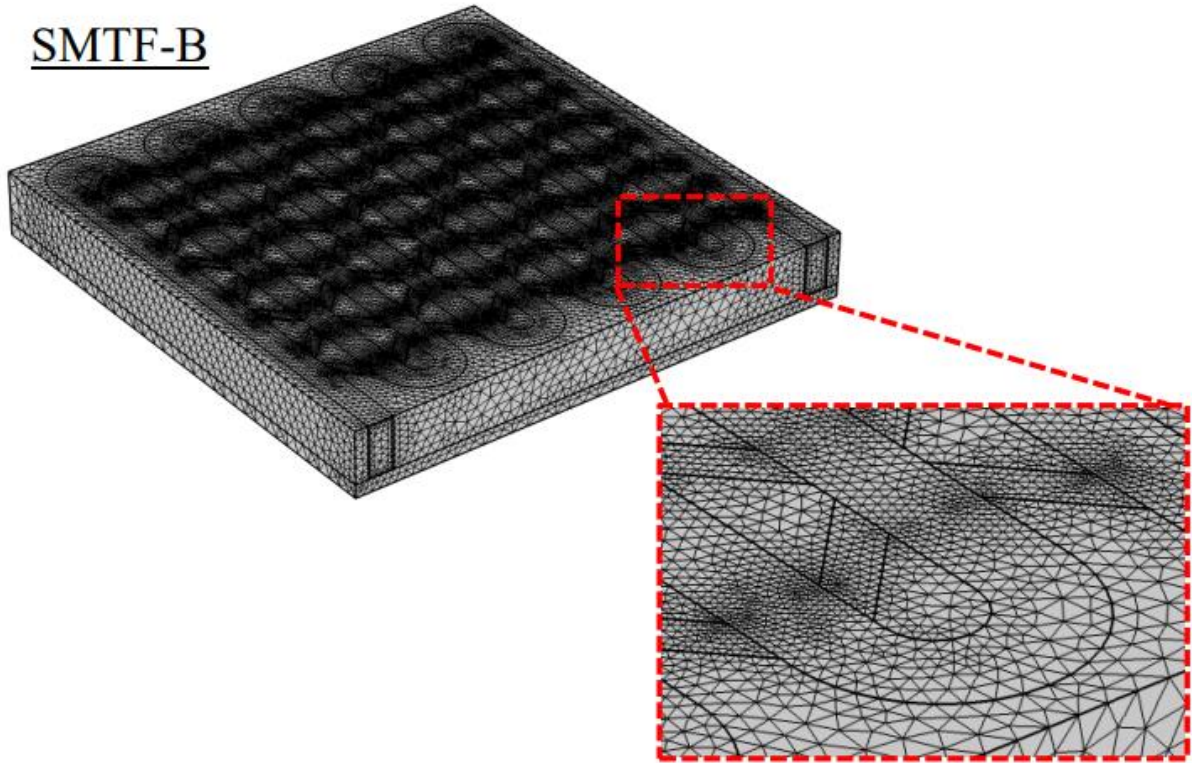


A 2.2 Single inlet-outlet serpentine mini-channel heat sink with trapezium secondary channels

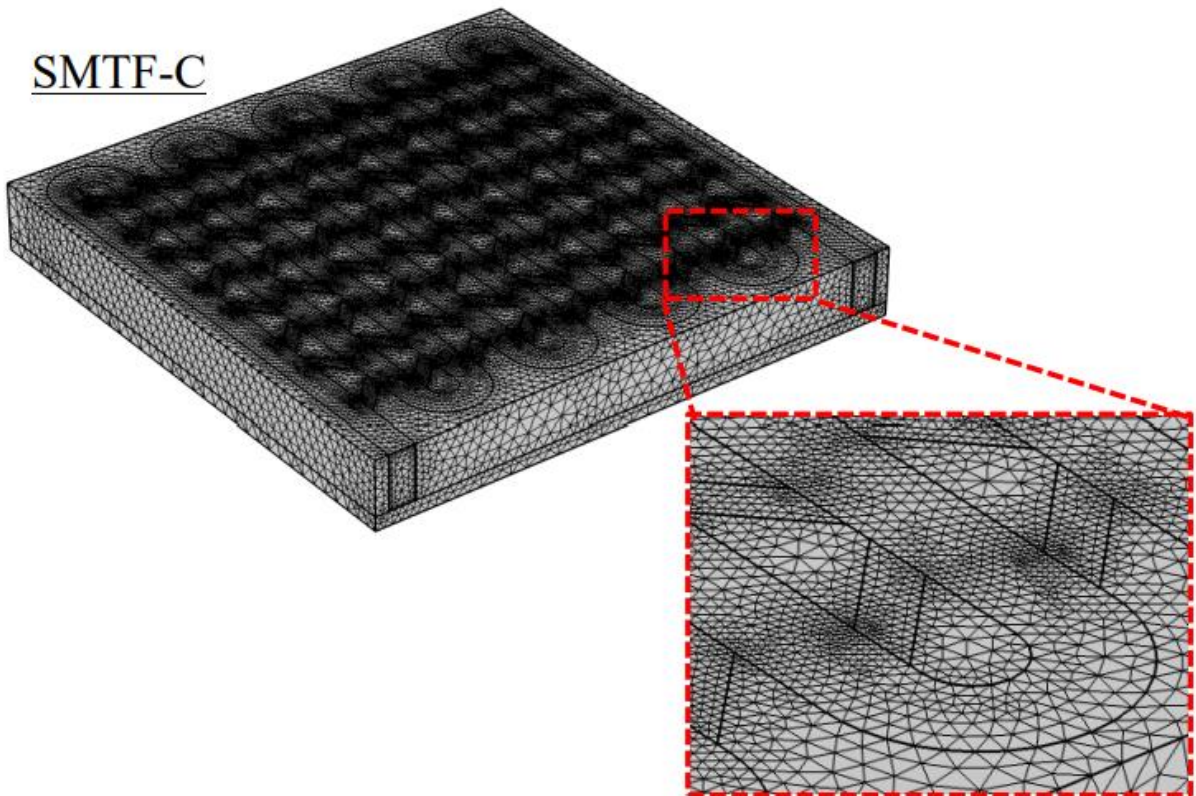
SMTF-A



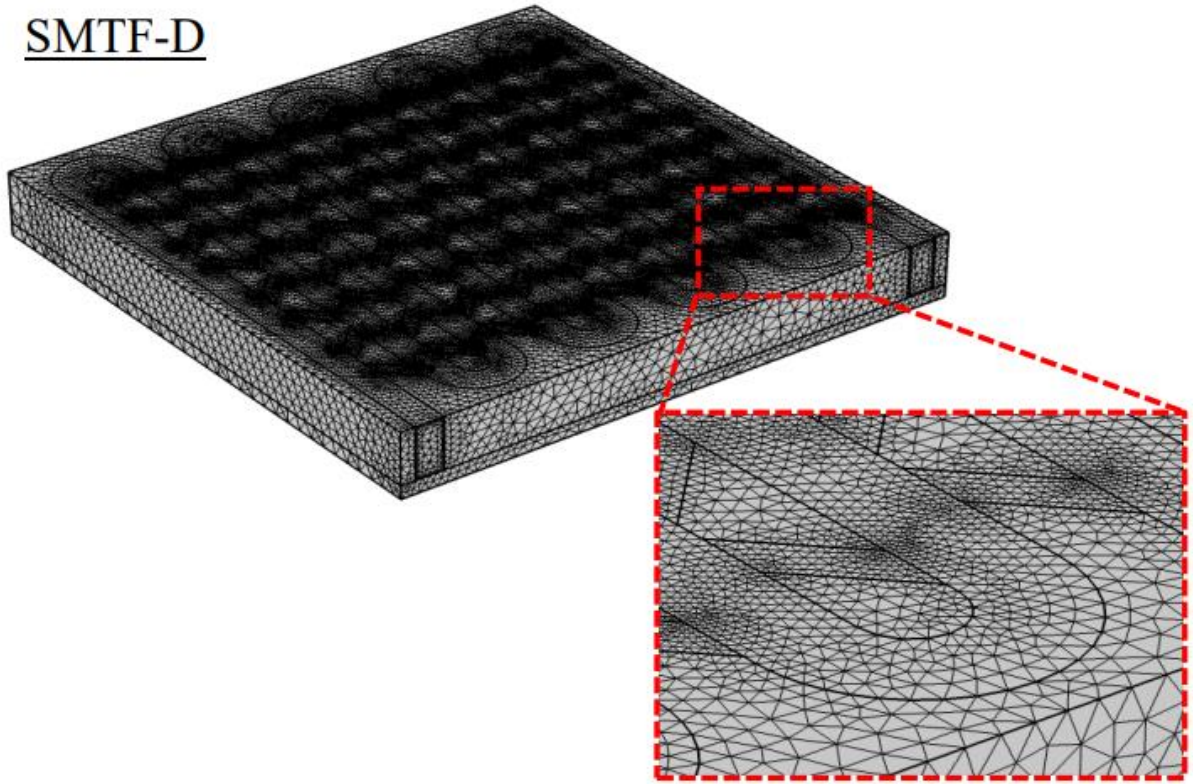
SMTF-B



SMTF-C

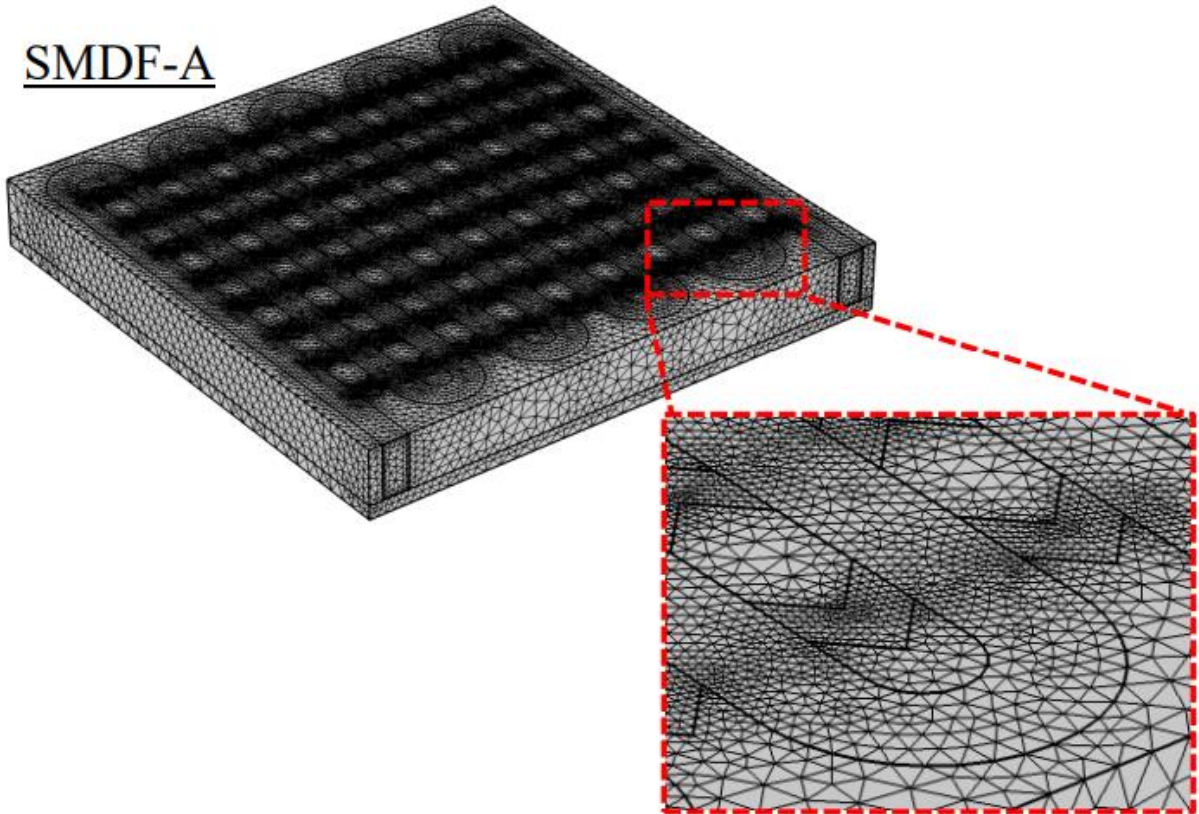


SMTF-D

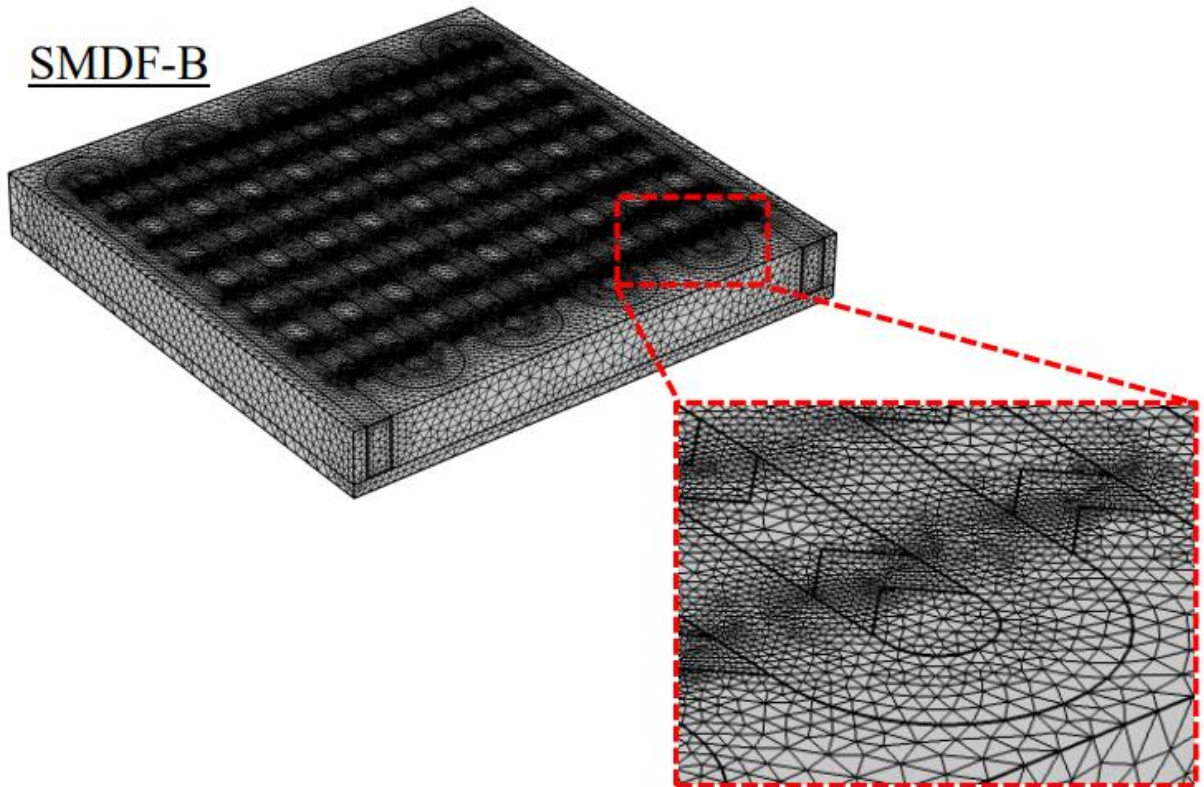


A 2.3 Single inlet-outlet serpentine mini-channel heat sink with diamond secondary channels

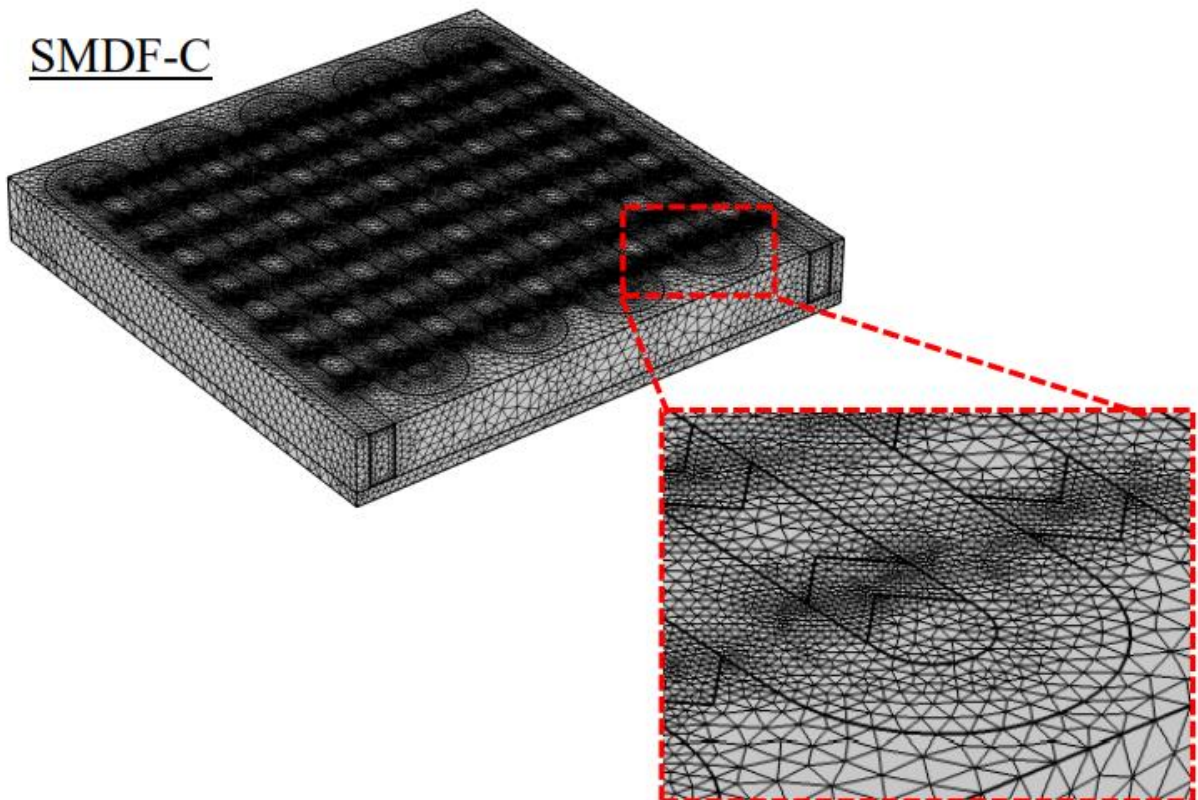
SMDF-A



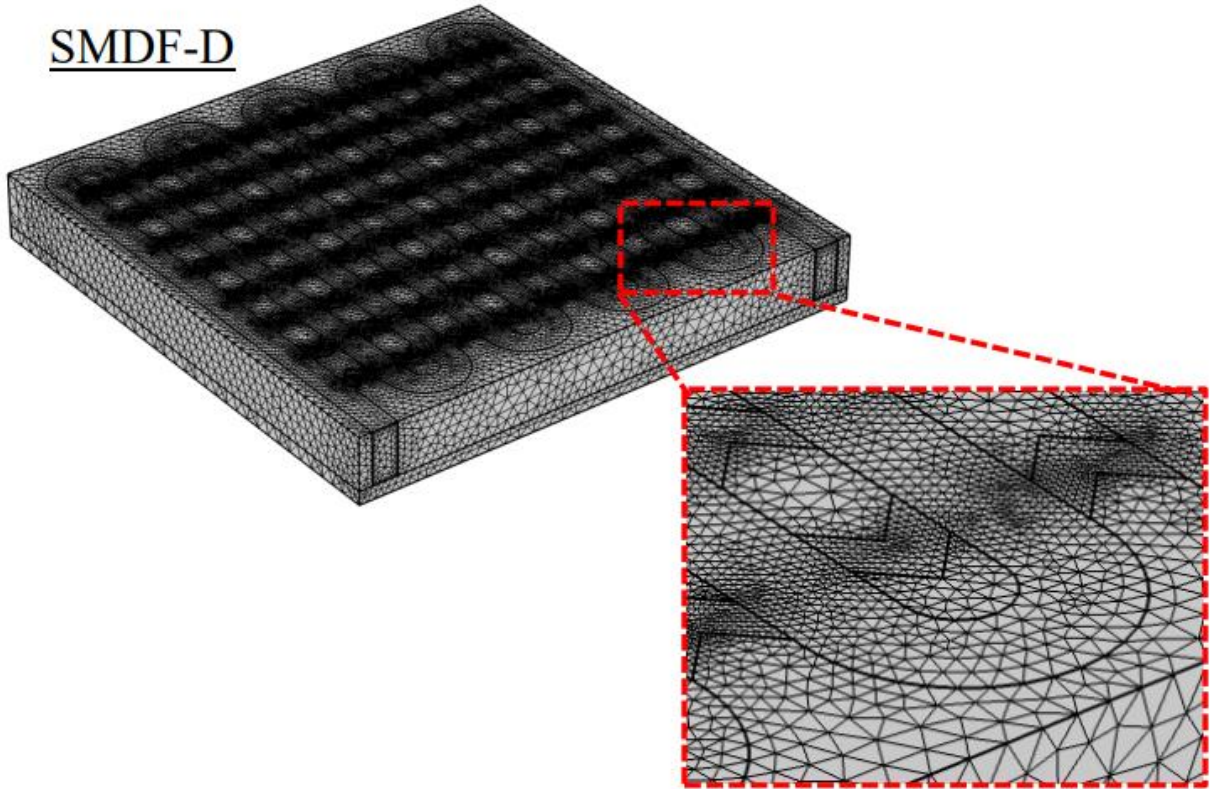
SMDF-B



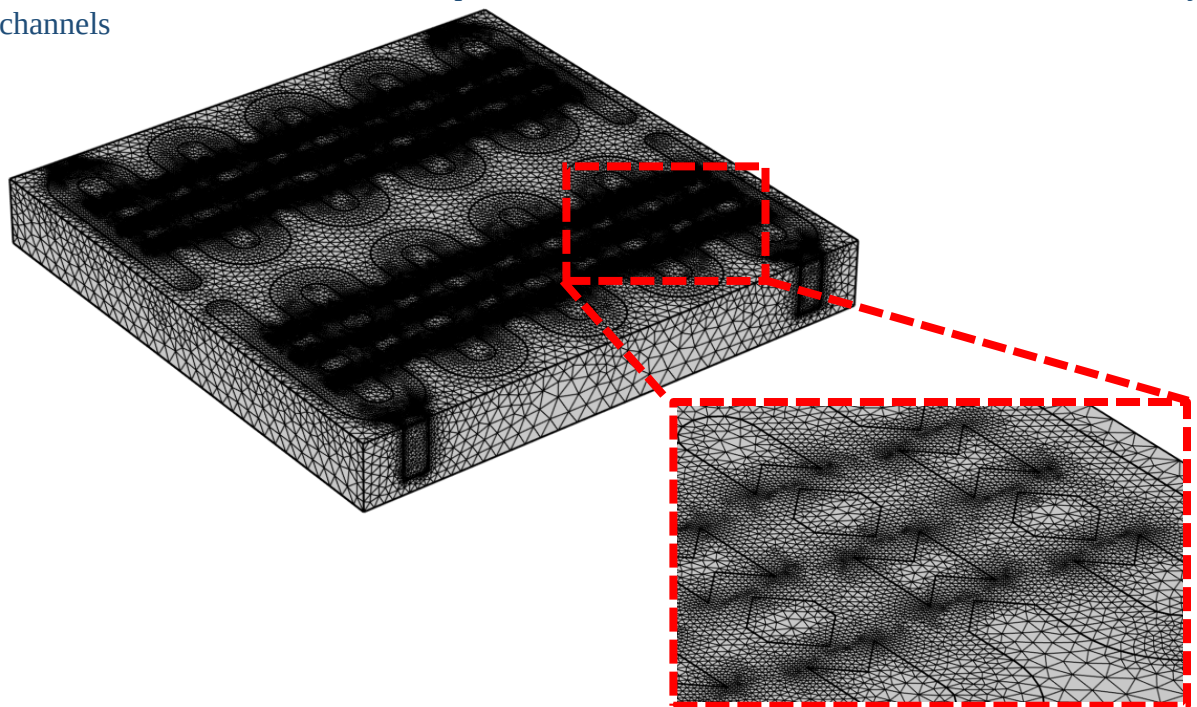
SMDF-C



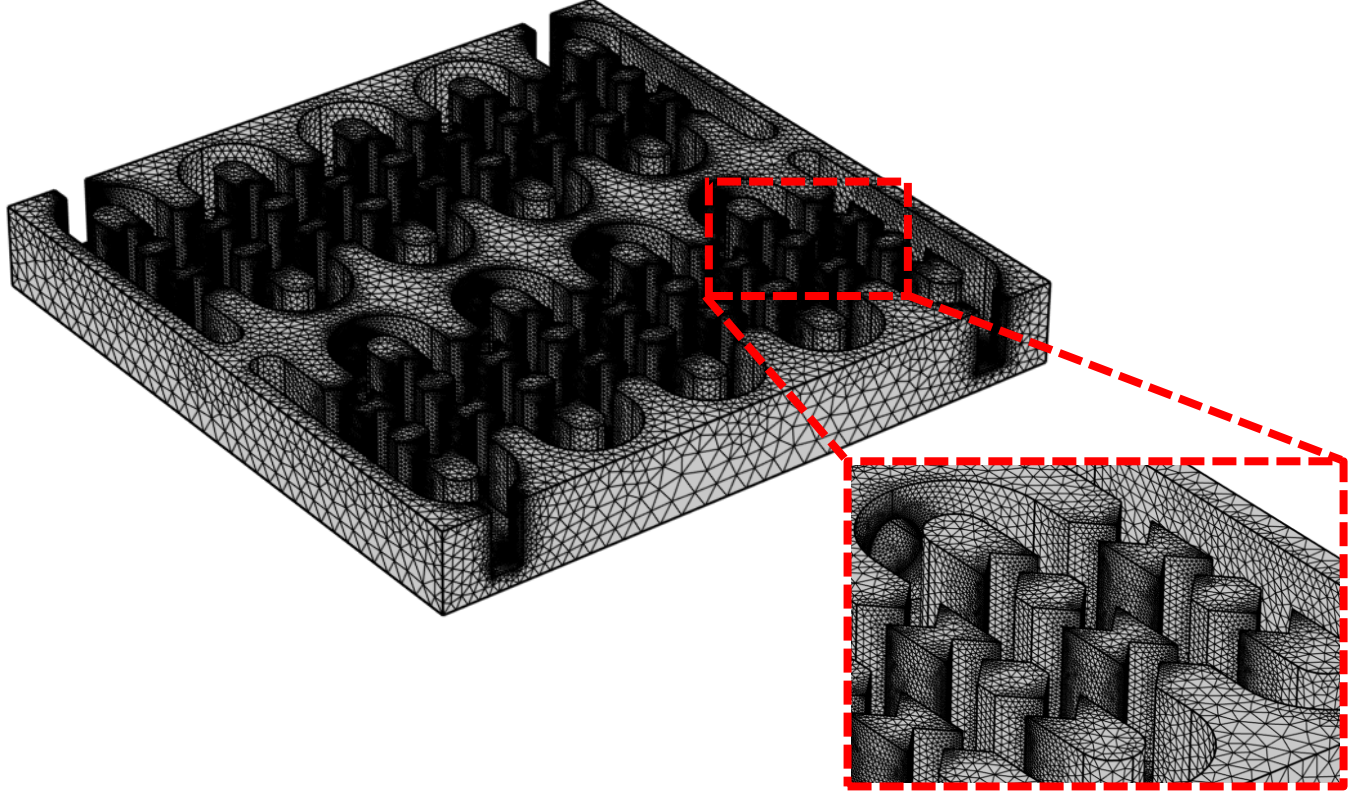
SMDF-D



A 2.4 Double inlet-outlet serpentine mini-channel heat sink with diamond secondary channels



A 2.5 Double inlet-outlet serpentine mini-channel heat sink with diamond secondary channels and through-holes



A.3 The analytical equations to calculate A_{eff} .

A 3.1 Double inlet-outlet serpentine mini-channel heat sink with diamond secondary channels

For D-SMPF, the A_{eff} can be calculated as follows:

$$\begin{aligned}
 A_{eff,D-SMPF} &= [A_{base,D-SMPF}] + [A_{wall,D-SMPF}] \\
 A_{eff,D-SMPF} &= \left[m \frac{\pi \left(\frac{W_W}{2} + W_{ch} \right)^2 - \pi \left(\frac{W_W}{2} \right)^2}{2} \right] + [n L_{ch} W_{ch}] + \\
 &\quad \left((W_{ch}) \left(W_{S,1} + \frac{W_W}{2} + W_{ch} - \sqrt{\left(\frac{W_W}{2} + W_{ch} \right)^2 - \left(\frac{W_{ch}}{2} \right)^2} \right) - \right.
 \end{aligned} \tag{A 3.1.1}$$

$$\begin{aligned}
& \left(\pi \left(\frac{W_W}{2} + W_{ch} \right)^2 \frac{2 \left(\tan^{-1} \left(\frac{W_{ch}}{\sqrt{\left(\frac{W_W}{2} + W_{ch} \right)^2 - \left(\frac{W_{ch}}{2} \right)^2}} \right) \right)}{2\pi} - \right. \\
& \left. \frac{1}{2} (W_{ch}) \left(\sqrt{\left(\frac{W_W}{2} + W_{ch} \right)^2 - \left(\frac{W_{ch}}{2} \right)^2} \right) \right) p \\
& + \left[\left(L_{ch} + \left(\frac{W_W}{2} + W_{ch} - \frac{W_{ch}}{2} \right) \right) (W_{ch}) q \right] \\
& + \left[\frac{\pi \left(\frac{W_{ch}}{2} \right)^2}{2} r \right] \\
& + \left[\frac{2\pi \left(\frac{W_W}{2} + W_{ch} \right)}{2} H_{ch} m \right] \\
& + \left[\frac{2\pi \left(\frac{W_W}{2} \right)}{2} H_{ch} m \right] \\
& + [L_{ch} H_{ch} s] \\
& + \left[\frac{2\pi \left(\frac{W_{ch}}{2} \right)}{2} H_{ch} r \right] \\
& + \left[2 \left(L_{ch} + \left(\frac{W_W}{2} + W_{ch} - \frac{W_{ch}}{2} \right) \right) H_{ch} q \right] \\
& + \left[2 \left(W_{S,1} + W_{ch} + \frac{W_W}{2} - \sqrt{\left(\frac{W_W}{2} + W_{ch} \right)^2 - \left(\frac{W_{ch}}{2} \right)^2} \right) H_{ch} p \right]
\end{aligned}$$

$$- \left[\frac{2 \tan^{-1} \left(\frac{W_{ch}}{\sqrt{\left(\frac{W_W}{2} + W_{ch}\right)^2 - \left(\frac{W_{ch}}{2}\right)^2}} \right)}{2\pi} 2\pi \left(\frac{W_W}{2} + W_{ch}\right) H_{ch} p \right]$$

For D-SMDF, the A_{eff} can be calculated as follows:

$$A_{eff,D-SMDF} = [A_{base,D-SMDF}] + [A_{wall,D-SMDF}]$$

$$A_{eff,D-SMDF} = \left[m \frac{\pi \left(\frac{W_W}{2} + W_{ch}\right)^2 - \pi \left(\frac{W_{ch}}{2}\right)^2}{2} \right] + [n L_{ch} W_{ch}] +$$

$$\left[x \frac{W_W}{2} l_{scyz} \right] + \left((W_{ch}) \left(W_{S,1} + \frac{W_W}{2} + W_{ch} - \right. \right.$$

$$\left. \left. \sqrt{\left(\frac{W_W}{2} + W_{ch}\right)^2 - \left(\frac{W_{ch}}{2}\right)^2} \right) - \pi \left(\frac{W_W}{2} + \right. \right.$$

$$\left. \left. \sqrt{\left(\frac{W_W}{2} + W_{ch}\right)^2 - \left(\frac{W_{ch}}{2}\right)^2} \right) \right] \quad (A\ 3.1.2)$$

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$$- \left[\frac{2 \tan^{-1} \left(\frac{W_{ch}}{\sqrt{\left(\frac{W_W}{2} + W_{ch}\right)^2 - \left(\frac{W_{ch}}{2}\right)^2}} \right)}{2\pi} 2\pi \left(\frac{W_W}{2} + W_{ch}\right) H_{ch} p \right]$$

Where m, n, p, q, r, s, x, y and x are the number of bends, number of channel, number of entrance, number of end channels, number of circular ends, number of sidewalls, number of parallelogram per fin, number of fin per row and number of plate fin respectively.

A 3.2 Double inlet-outlet serpentine mini-channel heat sink with diamond secondary channels and through-holes

For the D-SMDF-TH heat sink, the A_{eff} can be formulated by the following:

$$A_{eff,D-SMDF-TH} = [A_{base,D-SMDF-TH}] + [A_{wall,D-SMDF-TH}]$$

$$A_{eff,D-SMDF-TH} = \left[m \frac{\pi \left(\frac{W_W}{2} + W_{ch}\right)^2 - \pi \left(\frac{W_W}{2}\right)^2}{2} \right] + [n L_{ch} W_{ch}] +$$

$$\left[x \frac{W_W}{2} l_{scyz} \right] + \left((W_{ch}) \left(W_{S,1} + \frac{W_W}{2} + W_{ch} - \right. \right.$$

$$\left. \left. \sqrt{\left(\frac{W_W}{2} + W_{ch}\right)^2 - \left(\frac{W_{ch}}{2}\right)^2} \right) - \pi \left(\frac{W_W}{2} + \right. \right.$$

(A 3.2.1)

$$\begin{aligned}
& + \left[2t \left[\frac{W}{2} - W_{S,1} - W_{ch} - \frac{W_W}{2} - L_{ch} \right. \right. \\
& \quad \left. \left. - \sqrt{\left(\frac{W_W}{2} + W_{ch} \right)^2 - R_3^2} \right] (2\pi R_3) \right. \\
& \quad \left. - 8t \int_{\theta_2=0}^{\theta_2=\frac{\pi}{2}} R_3^2 \sin \theta_2 d\theta_2 \right] \\
& \quad - [l_{sc} H_{ch} x y z] \\
& - \left[\frac{2t \tan^{-1} \left(\frac{W_{ch}}{\sqrt{\left(\frac{W_W}{2} + W_{ch} \right)^2 - \left(\frac{W_{ch}}{2} \right)^2}} \right)}{2\pi} 2\pi \left(\frac{W_W}{2} + W_{ch} \right) H_{ch} p \right] \\
& - \left[8t \int_{\theta_2=0}^{\theta_2=\frac{\pi}{2}} R_3 \left[\frac{W_W}{2} + W_{ch} \right]^2 \left[\cos \left[\sin^{-1} \left[\frac{R_3 \sin \theta_2}{\frac{W_W}{2} + W_{ch}} \right] \right] - \cos \left[\sin^{-1} \left[\frac{R_3}{\frac{W_W}{2} + W_{ch}} \right] \right] \right] d\theta_2 \right]
\end{aligned}$$

Where $m, n, p, q, r, s, t, x, y$ and x are the number of bends, number of channel, number of entrance, number of end channels, number of circular ends, number of sidewalls, number of through-holes, number of parallelogram per fin, number of fin per row and number of plate fin respectively.

A.4 The analytical equations to calculate V_{Cu} .

A 4.1 Double inlet-outlet serpentine mini-channel heat sink with diamond secondary channels

$$\begin{aligned}
 V_{Cu} = & WL(H_{ch} + H_b) - \left[m \frac{\pi \left(\frac{W_W}{2} + W_{ch} \right)^2 - \pi \left(\frac{W_W}{2} \right)^2}{2} H_{ch} \right] - \\
 & \left([nL_{ch}W_{ch}H_{ch}] - \left[x \frac{W_W}{2} l_{sc} y z H_{ch} \right] - (W_{ch}) \left(W_{S,1} + \frac{W_W}{2} + \right. \right. \\
 & \left. \left. W_{ch} - \sqrt{\left(\frac{W_W}{2} + W_{ch} \right)^2 - \left(\frac{W_{ch}}{2} \right)^2} \right) - \pi \left(\frac{W_W}{2} + \right. \right. \\
 & \left. \left. W_{ch} \right)^2 \frac{2 \left(\tan^{-1} \left(\frac{W_{ch}}{\sqrt{\left(\frac{W_W}{2} + W_{ch} \right)^2 - \left(\frac{W_{ch}}{2} \right)^2}} \right) \right)}{2\pi} - \right. \\
 & \left. \left. \frac{1}{2} (W_{ch}) \left(\sqrt{\left(\frac{W_W}{2} + W_{ch} \right)^2 - \left(\frac{W_{ch}}{2} \right)^2} \right) \right) H_{ch} p \right. \\
 & \left. - \left[\left(L_{ch} + \left(\frac{W_W}{2} + W_{ch} - \frac{W_{ch}}{2} \right) \right) (W_{ch}) q \right] H_{ch} \right. \\
 & \left. - \left[\frac{\pi \left(\frac{W_{ch}}{2} \right)^2}{2} r \right] H_{ch} \right)
 \end{aligned} \tag{A 4.1.1}$$

Where m, n, p, q and r are the number of bends, number of channel, number of entrance, number of end channels and number of circular ends respectively.

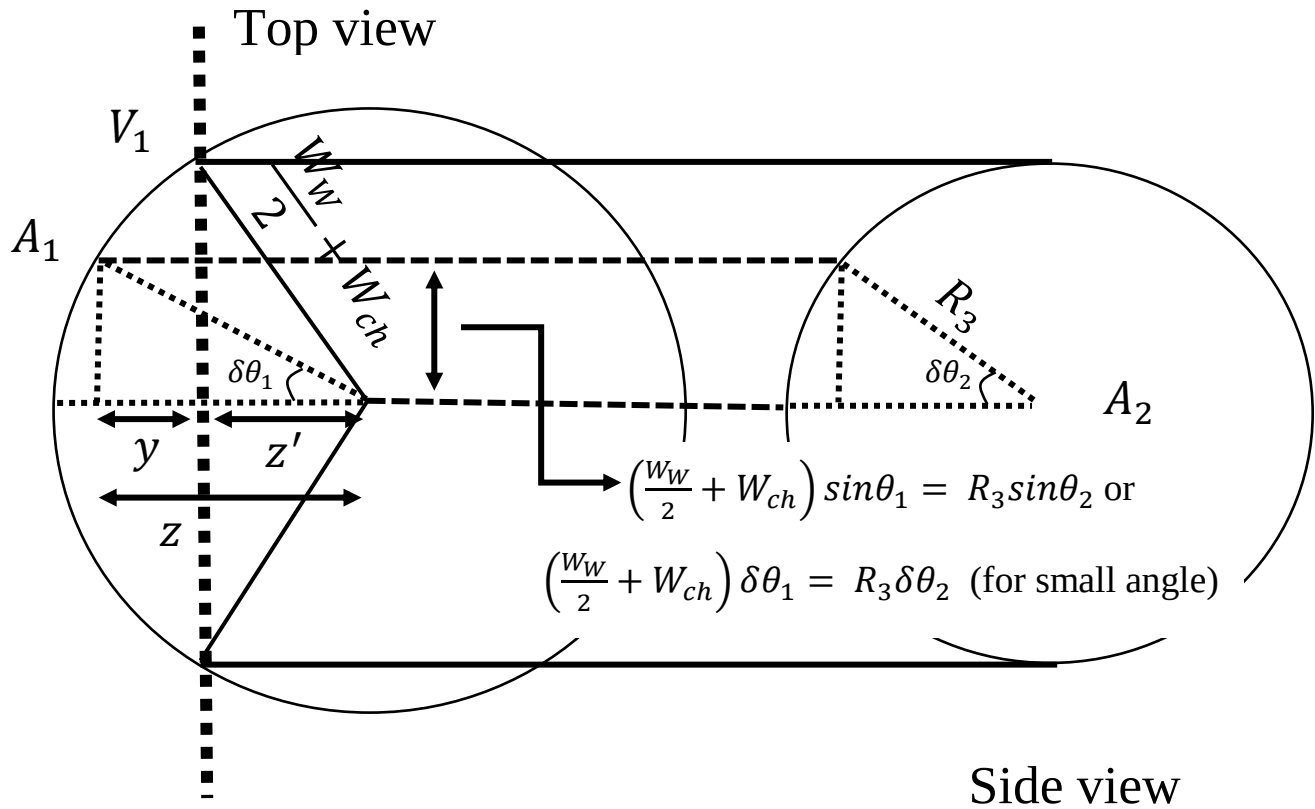
A 4.2 Double inlet-outlet serpentine mini-channel heat sink with diamond secondary channels and through-holes

$$\begin{aligned}
 V_{Cu} = & WL(H_{ch} + H_b) - \left[m \frac{\pi \left(\frac{W_W}{2} + W_{ch} \right)^2 - \pi \left(\frac{W_W}{2} \right)^2}{2} H_{ch} \right] - \\
 & \left([nL_{ch}W_{ch}H_{ch}] - \left[x \frac{W_W}{2} l_{sc} y z H_{ch} \right] - (W_{ch}) \left(W_{S,1} + \frac{W_W}{2} + \right. \right. \\
 & \left. \left. W_{ch} - \sqrt{\left(\frac{W_W}{2} + W_{ch} \right)^2 - \left(\frac{W_{ch}}{2} \right)^2} \right) - \pi \left(\frac{W_W}{2} + \right. \right. \\
 & \left. \left. W_{ch} \right)^2 \frac{2 \left(\tan^{-1} \left(\frac{W_{ch}}{\sqrt{\left(\frac{W_W}{2} + W_{ch} \right)^2 - \left(\frac{W_{ch}}{2} \right)^2}} \right) \right)}{2\pi} - \right. \\
 & \left. \left. \frac{1}{2} (W_{ch}) \left(\sqrt{\left(\frac{W_W}{2} + W_{ch} \right)^2 - \left(\frac{W_{ch}}{2} \right)^2} \right) \right) H_{ch} p \right. \\
 & \left. - \left[\left(L_{ch} + \left(\frac{W_W}{2} + W_{ch} - \frac{W_{ch}}{2} \right) \right) (W_{ch}) q \right] H_{ch} \right. \\
 & \left. - \left[\frac{\pi \left(\frac{W_{ch}}{2} \right)^2}{2} r \right] H_{ch} \right]
 \end{aligned} \tag{A 4.2.1}$$

$$\begin{aligned}
& - \left[\left[2t \left[\frac{W}{2} - W_{S,1} - W_{ch} - \frac{W_W}{2} - L_{ch} - \sqrt{\left(\frac{W_W}{2} + W_{ch} \right)^2 - R_3^2} \right] (\pi R_3^2) \right] \right] \\
& + 8t \left[R_3 \int_{\theta_2=0}^{\theta_2=\frac{\pi}{2}} \sqrt{\left[\frac{W_W}{2} + W_{ch} \right] \left[\cos \left[\sin^{-1} \left[\frac{R_3 \sin \theta_2}{\left[\frac{W_W}{2} + W_{ch} \right]} \right] \right] - \cos \left[\sin^{-1} \left[\frac{R_3}{\left[\frac{W_W}{2} + W_{ch} \right]} \right] \right]} \right] d\theta_2 \right]^2
\end{aligned}$$

Where m, n, p, q, r and t are the number of bends, number of channel, number of entrance, number of end channels, number of circular ends and number of through-holes respectively.

A.5 Derivation of integrals in Eq. A 3.2.1 and Eq. A 4.2.1 from through-holes structure



Derivation of the term $\int_{\theta_2=0}^{\theta_2=\frac{\pi}{2}} R_3^2 \sin\theta_2 d\theta_2$ in Eq. A 3.2.1 is as follows:

Delta area of segment of δA_1 :

$$\delta A_1 = R_3 \sin\theta_2 \left(\frac{W_W}{2} + W_{ch} \right) \delta\theta_1$$

We know that:

$$\delta\theta_1 = \frac{R_3 \delta\theta_2}{\frac{W_W}{2} + W_{ch}}$$

Therefore, new δA_1 is as follows:

$$\delta A_1 = R_3^2 \sin\theta_2 \delta\theta_2$$

To find the summation of the whole area of segment of A_1 is as follows:

$$A_1 = \sum_{\theta_2=0}^{\theta_2=\frac{\pi}{2}} R_3^2 \sin\theta_2 \delta\theta_2$$

By integration we get:

$$\int_{\theta_2=0}^{\theta_2=\frac{\pi}{2}} R_3^2 \sin\theta_2 d\theta_2$$

Derivation of the term $\int_{\theta_2=0}^{\theta_2=\frac{\pi}{2}} R_3 \left[\frac{W_W}{2} + W_{ch} \right]^2 \left[\cos \left[\sin^{-1} \left[\frac{R_3 \sin\theta_2}{\frac{W_W}{2} + W_{ch}} \right] \right] - \right.$

$\left. \cos \left[\sin^{-1} \left[\frac{R_3}{\frac{W_W}{2} + W_{ch}} \right] \right] \right] d\theta_2$ in Eq. A 4.2.1 is as follows:

Delta area of segment of δA_2 :

$$\delta A_2 = \left[\frac{W_W}{2} + W_{ch} \right] \left[\cos \left[\sin^{-1} \left[\frac{R_3 \sin\theta_2}{\frac{W_W}{2} + W_{ch}} \right] \right] - \cos \left[\sin^{-1} \left[\frac{R_3}{\frac{W_W}{2} + W_{ch}} \right] \right] \right] \left[\frac{W_W}{2} + W_{ch} \right] \delta\theta_1$$

We know that:

$$\delta\theta_1 = \frac{R_3\delta\theta_2}{\frac{W_W}{2} + W_{ch}}$$

Therefore, new δA_2 is as follows:

$$\begin{aligned} \delta A_2 = & \left[\frac{W_W}{2} + W_{ch} \right] \left[\cos \left[\sin^{-1} \left[\frac{R_3 \sin \theta_2}{\frac{W_W}{2} + W_{ch}} \right] \right] - \cos \left[\sin^{-1} \left[\frac{R_3}{\frac{W_W}{2} + W_{ch}} \right] \right] \right] \left[\frac{W_W}{2} \right. \\ & \left. + W_{ch} \right] \frac{R_3 \delta \theta_2}{\frac{W_W}{2} + W_{ch}} \end{aligned}$$

By simplification we get:

$$\delta A_2 = R_3 \left[\frac{W_W}{2} + W_{ch} \right] \left[\cos \left[\sin^{-1} \left[\frac{R_3 \sin \theta_2}{\frac{W_W}{2} + W_{ch}} \right] \right] - \cos \left[\sin^{-1} \left[\frac{R_3}{\frac{W_W}{2} + W_{ch}} \right] \right] \right] \delta \theta_2$$

To find the summation of the whole area of segment of A_1 is as follows:

$$\begin{aligned} A_2 = & \sum_{\theta_2=0}^{\theta_2=\frac{\pi}{2}} R_3 \left[\frac{W_W}{2} + W_{ch} \right] \left[\cos \left[\sin^{-1} \left[\frac{R_3 \sin \theta_2}{\frac{W_W}{2} + W_{ch}} \right] \right] \right. \\ & \left. - \cos \left[\sin^{-1} \left[\frac{R_3}{\frac{W_W}{2} + W_{ch}} \right] \right] \right] \delta \theta_2 \end{aligned}$$

By integration we get:

$$\begin{aligned} A_2 = & \int_{\theta_2=0}^{\theta_2=\frac{\pi}{2}} R_3 \left[\frac{W_W}{2} + W_{ch} \right]^2 \left[\cos \left[\sin^{-1} \left[\frac{R_3 \sin \theta_2}{\frac{W_W}{2} + W_{ch}} \right] \right] \right. \\ & \left. - \cos \left[\sin^{-1} \left[\frac{R_3}{\frac{W_W}{2} + W_{ch}} \right] \right] \right] d\theta_2 \end{aligned}$$

Derivation of the term $\left[\int_{\theta_2=0}^{\theta_2=\frac{\pi}{2}} R_3 \left[\frac{W_W}{2} + W_{ch} \right]^2 \left[\cos \left[\sin^{-1} \left[\frac{R_3 \sin \theta_2}{\frac{W_W}{2} + W_{ch}} \right] \right] - \cos \left[\sin^{-1} \left[\frac{R_3}{\frac{W_W}{2} + W_{ch}} \right] \right] \right] d\theta_2 \right]^2$ in Eq. 8.3 as follows:

To calculate the delta volume of δV_1 :

$$\delta V_1 = \left[\frac{W_W}{2} + W_{ch} \right] \left[\cos \left[\sin^{-1} \left[\frac{R_3 \sin \theta_2}{\frac{W_W}{2} + W_{ch}} \right] \right] - \cos \left[\sin^{-1} \left[\frac{R_3}{\frac{W_W}{2} + W_{ch}} \right] \right] \right] R_3^2 \delta \theta_2^2$$

By simplification we get:

$$\sqrt{\delta V_1} = \sqrt{\left[\frac{W_W}{2} + W_{ch} \right] \left[\cos \left[\sin^{-1} \left[\frac{R_3 \sin \theta_2}{\frac{W_W}{2} + W_{ch}} \right] \right] - \cos \left[\sin^{-1} \left[\frac{R_3}{\frac{W_W}{2} + W_{ch}} \right] \right] \right] R_3 \delta \theta_2}$$

To find the summation of the whole volume of segment of V_1 is as follows:

$$\begin{aligned} & \sqrt{V_1} \\ &= \sum_{\theta_2=0}^{\theta_2=\frac{\pi}{2}} R_3 \sqrt{\left[\frac{W_W}{2} + W_{ch} \right] \left[\cos \left[\sin^{-1} \left[\frac{R_3 \sin \theta_2}{\frac{W_W}{2} + W_{ch}} \right] \right] - \cos \left[\sin^{-1} \left[\frac{R_3}{\frac{W_W}{2} + W_{ch}} \right] \right] \right] \delta \theta_2} \end{aligned}$$

By integration we get:

$$V_1 = \left[\int_{\theta_2=0}^{\theta_2=\frac{\pi}{2}} \sqrt{\left[\frac{W_W}{2} + W_{ch} \right] \left[\cos \left[\sin^{-1} \left[\frac{R_3 \sin \theta_2}{\frac{W_W}{2} + W_{ch}} \right] \right] - \cos \left[\sin^{-1} \left[\frac{R_3}{\frac{W_W}{2} + W_{ch}} \right] \right] \right]} d\theta_2 \right]^2$$