THESIS

on

ULTIMATE STRENGTH OF PRESTRESSED CONCRETE BEAMS IN SHEAR

submitted by

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NOTATION

As = area of tensile reinforcement.

a = shear span, i.e. horizontal distance between a load point and the corresponding support point.

a_1 = flexural span, i.e. distance between the load points.

b = flange width.

b_1 = web thickness.

C = compressive force.

d = effective depth.

d_1 = overall depth of test specimens.

e_u = limiting strain at which concrete crushes in a beam.

e_{cc} = concrete strain on upper surface of a beam at a loading point, corresponding to "inclined tension cracking".

e_{ce} = concrete strain at level of reinforcement, due to effective prestress.

e_{cu} = strain in concrete adjacent to the steel at point of maximum moment.

e_{se} = steel strain corresponding to effective prestress.

e_{sc} = steel strain at "inclined tension cracking".

e'_{sc} = $e_{sc} - (e_{se} + e_{ce})$

e_{su} = steel strain at failure of beam.

e'_{sa} = $e_{su} - e_{sc}$ = increase in steel strain after "inclined tension cracking".

F = correction factor.

F_{se} = effective prestressing force.

f_{cu} = concrete cube crushing strength.

f_h = effective concrete prestress at the centre of area of the concrete section.

f_r = modulus of rupture.

f_{se} = effective steel prestress.

f_{su} = stress in reinforcement at failure of beam.

f_t = briquette tensile strength of concrete.

f'_c = compressive strength of concrete determined by 6 ins x 12" cylinders.

f'_t = tensile strength of concrete determined by the cylinder splitting test.

J, J_1, J_2 = strain compatibility factors.

K = factor relating to the stress distribution in the compressive zone in a shear-compression failure (see section 15).

k = friction constant.

k_2 = factor relating to the stress distribution in the compressive zone in a shear-compression failure (see section 15).

k_c = ratio of neutral axis depth at "inclined tension cracking" to effective depth.

k_d = ratio of average upper flange depth to effective depth.

k_u = ratio of depth to the inclined crack at failure to effective depth.

M_u = total ultimate bending moment.

p = $\frac{As}{bd}$ = reinforcement ratio.

P_c = total diagonal cracking load.

P_u = total ultimate load.

T = tensile force.

t_u = ultimate tensile stress of the steel reinforcement.

V = shear force.

V_1 = shear force transmitted by the tensile reinforcement.

V_c = total shear force at diagonal cracking.

V_{c_1} = total shear force acting when inclined cracking subsequent to cracking at the junction of the web and the upper flange crosses the web.

V_u = total ultimate shear force at failure following diagonal cracking.

V_{u_1} = total ultimate shear force at failure following cracking at the junction of the web and the upper flange.

σ = normal stress.

$\sigma_1, \sigma_2, \sigma_3$ = principal stresses.

$\sigma_I, \sigma_{II}, \sigma_{III}$ = principal stresses at failure.

σ_a = mean normal stress acting over the surface of an infinitesimal spherical volume.

τ = shearing stress.

τ_a = mean shearing stress acting over the surface of an infinitesimal spherical volume.

I INTRODUCTION

1. Shear Failures

Any prestressed concrete beam which fails within a zone where stresses due to shear exist or fails in some manner associated with cracking or other phenomenon within such a zone may generally be said to have failed, in some part, due to shear. The extent of the influence of the shear stresses, however, varies with the beam specifications and the type of loading.

For the purposes of this thesis, shear failures are defined as failures occurring, wholely or partly due to stresses within those parts of the beam which are subject to shear.

2. Object and Scope

The experimental studies described in this thesis were undertaken to provide more information of the behaviour of prestressed concrete beams without web reinforcement when subjected to shear. Studies of the influence of the following major variables are included.

1) Amount of longitudinal reinforcement. A wide range of percentages of tension reinforcement was used in order to co-ordinate work of other observers(1,2,3,4) who had concentrated on small unconnected ranges.

2) Length of the shear span. This, together with the shape of cross-section, was considered to be one of the major influences

affecting the mode of failure and the ultimate load of a beam. Previous work in this respect[1,2,3,4] had mostly been concentrated on rectangular beams. This variable is therefore included in the investigation to discover its influence on I-section beams.

3) Shape of cross-section. Previous work on the influence of this variable[1,2,3,4] is limited and generally lies outside ranges of other variables considered in this investigation. This variable is therefore studied with the aim of co-ordinating results of the present investigation with previous work.

4) Type of curing. It was considered that the type of curing employed and consequent shrinkage properties of the beams may influence the shear failure of prestressed concrete beams. Six otherwise identical beams were cured in different ways and the influence of the curing method is studied.

Altogether 54 simply-supported beams were tested under two concentrated loads. Some failed in flexure and a number of independent modes of shear failure were observed. Where the results of other investigators of tests on simply-supported prestressed concrete beams with no shear reinforcement or reinforcement of any kind in the web are available they are included in the analysis. Expressions are offered for cracking and failure loads.

II MATERIALS, FABRICATION AND TEST SPECIMENS

3. Materials.

(a) Cement. The cement used was Earle's Ferrocrete rapid hardening Portland cement.

(b) Aggregates. The fine aggregate used was Finningley sand. The coarse aggregate was a "pea" gravel of maximum size $\frac{3}{4}$" obtained from Bawtry, Doncaster. Before use both aggregates were dried and stored indoors in order to eliminate errors in the water-content of the concrete mix.

(c) Concrete Mixes. Two concrete mixes were used, the water/cement ratio be weight being either 0.40 or 0.50. For each mix the ratio by weight of cement : sand : coarse aggregate was 1 : $1\frac{1}{2}$: 3. The mix used for each beam is included in Table 2.

(d) Grout Mix. The grout mix used for all beams consisted of a mixture of Earle's Ferrocrete rapid hardening Portland cement and water with a water/cement ratio of 0.5. The grout attained a strength of 4000 p.s.i. at seven days as determined by 3" cubes.

(e) Steel Reinforcement. Three types of steel were used; Macalloy bars and 0.276" dia. and 0.200" dia. hard-drawn wire from 8 ft. coils. For convenience, in the tables and elsewhere, these are referred to as Types I, II and III respectively.

To improve the bond characteristics, the surfaces of all steel used were well cleaned and then rusted slightly by placing the reinforcement in the open air for a few days. This produced a slightly pitted surface which improved the bond characteristics. Before use all steel was cleaned with a wire brush to remove loose rust.

The stress-strain relationships for the steel used are shown in Figs. 7-9.

(f) Grout Channel. The grout channel was formed either by the use of flexible lead-coated steel tubing ("Unitube") or inflatable rubber tubing ("Ductube").

4. Description of Test Specimens.

All beams tested in the course of the investigation described in this report were simply supported post-tensioned prestressed concrete beams with straight reinforcement. The precompressive stress in the concrete at the centre of area of cross-section and the quality of the concrete used was kept as constant as possible except that for some beams, which were expected to fail in the shear-compression mode, (to be described in detail later), the quality of the concrete was varied in order to maintain, as far as possible, a constant ratio of concrete strength to ultimate steel strength.

Basically, three shapes of cross-section were employed, rectangular and two shapes of I-section with nominal ratios of web thickness to flange thickness of 0.50 and 0.34. Overall dimensions of the beams were nominally 4" x 12", 3" x 6" or $3\frac{1}{2}$" x 6". Full details of the nominal dimensions of all I-section specimens are shown in Figs. 1 to 5. Accurate properties of all specimens are listed in Table 1.

A wide range of amounts of longitudinal reinforcement was used, the percentage, p%, varying from 0.21% to 4.61%, eighteen different values being used. The widest range of values was employed for those beams with a ratio of web thickness to flange thickness of 0.50 but wide ranges were also used for beams of other cross-sections.

The ratio of the shear span to effective depth of each beam varied from 2.8 to 5.5, four different values being employed. At each of the four values the shape of cross-section and the amount of longitudinal reinforcement was allowed to vary as described above.

The six beams which were used to examine the influence of curing on the behaviour of prestressed concrete beams in shear also form part of the major investigation as outlined above. The beams, which were otherwise as near identical as possible, were cured in different ways and later tested in a similar way to the other beams of the investigation.

The test specimens are now described in some detail.

(a) Rectangular Beams.

(i) S1, S2. The nominal overall dimensions of these beams were 4" x 12" x 8'0". Two macalloy bars were used in each beam for reinforcement, each in separate grout channels made of Unitube, the inside diameter of which was, in each case, at least ¼" larger than the diameter of the bar.

(ii) S46 - S48. The nominal overall dimensions of these beams were 3" x 6" x 6'0". The reinforcement consisted of two 0.276" hard-drawn wires for each beam, the grout channel being formed by inflated rubber Ductube.

(b) I - Section Beams.

(i) S3 - S7. Each of these beams had overall nominal dimensions of 4" x 12" x 10'0", the web thickness for each beam being nominally 2". In order to reduce stresses at the ends of the beam due to prestressing and to allow for greater stability in support during loading the ends of the beam were widened to 6" over a short length. The beams were reinforced by one macalloy bar and the grout channel in each case was formed of Unitube.

In order to reduce the possibility of flexural failure to a minimum the central part of each beam in this group was made solid 4" x 12".

(ii) S8 - S32, S34 - S36. The nominal overall dimensions of the cross-sections of these beams are 4" x 12". The length and web thickness of the beams were varied in order that a fuller study could be made of the influence of the shear span/depth ratio and the shape of cross-section. The ends of the beam were widened to 6" as for beams described in the above group. It was felt that the use of a central solid section in the zone of the beam which was to be subjected to pure flexure was not realistic and the practice was therefore discontinued.

The reinforcement for each beam consisted of one macalloy bar in a grout channel formed of Unitube.

(iii) S44 & S45. These beams were of the same type as those described in section (ii) above except that the steel reinforcement consisted of two 0.276" hard-drawn wires for each beam, the grout channel being formed by inflated rubber Ductube.

(iv) S33 & S37 - S39. Overall nominal dimensions of these beams were 3" x 6" x 5'6", the nominal web thickness being 1.5". The ends of the beams were widened to a solid section of $4\frac{1}{2}$" x 6". The grout channel was formed in each instance by inflated rubber Ductube, the tensile reinforcement consisting of a variety of combinations of 0.20" and 0.276" hard-drawn wires.

(v) S40 - S43. The overall nominal dimensions of these beams were 3.5" x 6" x 5'6", with a web thickness of 2". The end blocks had a cross-section of 5" x 6". Reinforcement consisted of one macalloy bar, the grout channel being formed by inflated rubber Ductube.

(vi) S49 - S54. Nominally these beams were identical in all respects except for the method of curing used. Nominal overall dimensions were 4" x 12" x 7'6" with a web thickness of 1 3/8", the end-block being a solid section of 6" x 12". Each beam was reinforced by one $\frac{3}{4}$" macalloy bar, the grout channel being formed by inflated rubber Ductube. Full details of the curing of these beams are given in section 5.

5. Casting and Curing.

All concrete was mixed in a non-tilting drum-type mixer. Great care was taken to ensure that the cement and aggregates were well mixed before the addition of water. The concrete was mixed in batches of approximately one cubic foot, the number of batches used for each beam varying with the size of beams. The mixing time for each batch was about five minutes.

For all beams metal formwork was used. Basically the sides and bases of the beams were formed by the use of channel-section rolled steel joists. I-section beams were formed by the insertion of welded sheet-steel forms which were themselves welded to the channels forming the external sides of the mould. The end-blocks were made of wood and each was drilled at the appropriate location in order to allow for the formation of the grout channel.

The grout channels for the beams were formed in either of two ways. For those beams in which the flexible metallic "Unitube" was used, the Unitube was suspended at each end by the end blocks through which it passed and at a number of points along its length by wire hangers. Before casting, the sides of the mould were clamped together at the end blocks and the prestressing bar to be used in the beam was passed through the grout channel and tightened-up against the end blocks. In this way it was possible to minimise movement of the grout channel during casting. At about six inches from the ends of the beam, 3/8" diameter access holes from the top or end of the beam to the grout channel were formed with short lengths of copper tubing.

For those beams in which inflated rubber "Ductube" was used, the tube was passed through the mould at a location determined by the end-blocks and inflated to an appropriate size. Access holes were formed as described above.

Before casting, all joints of the mould were sealed with plasticine and the mould was well greased. The sides of the mould were each firmly bolted to the base and clamps were used over the top of the mould to minimise lateral movement.

The separate batches of concrete were each placed to form a layer of uniform height through the beam. Concrete for control specimens was taken from appropriate batches. For instance, for those control specimens to be used for the determination of the tensile strength of concrete, concrete was used from the same batch as that used for placement in zones of the beam where the tensile strength of concrete was thought to be a controlling factor, namely the bottom flange and the web. Concrete from the batches placed in the upper flange of the beam was used to form control specimens designed to test its compressive strength.

Throughout casting the concrete in the test beam and in the control specimens was well vibrated. The top of the test beam was trowelled smooth immediately after casting. With each beam the following control specimens were cast:

Item	No.	Dimension
Cubes	6	4" x 4" x 4"
Cylinders	2	6" x 12"
Briquettes	8	B.S.S.
Columns	2	4" x 4" x 36"

Two 6" x 6" cylinders were cast with some beams and were later used to determine a value of the tensile strength of concrete, by the cylinder splitting method to be described further on in the text. The value of the tensile strength of concrete determined by this method is given in Table 2 for each beam with which these specimens were cast.

All beams except beams S51 - S54 were covered with wet sacks immediately after the initial set and remained so covered for seven days, after which they were stored openly in the laboratory in which they were to be tested.

Beams S51 and S52 were cured by placing them under wet sacks for the whole period between the initial set and the time of testing. Beams S53 and S54 were totally immersed in water for as much of the same period as possible. It was necessary to remove the beams for prestressing and during this time the beams were covered with wet sacks.

In all cases the control specimens were stored under the same conditions as the test beams.

6. Prestressing.

Two methods of prestressing were used in the fabrication of the test beams, Lee-McCall and Gifford-Udall. All macalloy bars were stressed by use of the Lee-McCall method and the Gifford-Udall system was employed for all hard-drawn wires. All beams were stressed not less than 10 days after casting and were grouted immediately afterwards. The tensioning force in each bar or group of wires was determined directly by the use of hollow cylindrical metal dynamometers which were placed on the reinforcement between the bearing plate at the end of the beam and another plate against which the nut or wedging dolly was to bear. Figure 6 shows the anchorage details for the Lee-McCall method. The anchorage details for the Gifford-Udall are essentially the same in principle. The dynamometers are described in greater detail later in this section under the relevant heading.

(a) <u>Lee-McCall Method</u>.

The ends of the beam were trimmed smooth and the standard steel end-plates were fixed at each end with a thin layer of plaster of Paris. Gauge points for use with the De Mec mechanical strain gauge[5,6] were attached to the sides of the beam in order that the horizontal deformation due to creep and shrinkage of the concrete at the level of the reinforcement between the time of stressing and the time of test could be measured.

It was not always practicable to place the dynamometer at the same end of each beam in relation to the end used for stressing. Account of this fact is taken in section 7 in which the method used to calculate the prestressing force is described. The dynamometer was placed as centrally as possible against one end-plate and a further end-plate was placed against it. The normal procedure as recommended by the manufacturers of the equipment was then followed. The prestressing force was applied to the beam by the Lee-McCall jack which was operated by a hydraulic pump. The amount of force applied to the beam was such that on tightening up the nut against the end-plate and releasing the hydraulic pressure the resulting prestressing force as indicated by the dynamometer was as required.

Immediately after prestressing, initial readings were taken of the lengths between the gauge points on the sides of the beam. Further readings were taken at intervals up to the time of test.

The dynamometers which were used in all cases where the Lee-McCall method was employed were made of hollow noral cylinders of appropriate internal and external diameters according to the size of the bar on which they were to be placed. Electrical resistance strain gauges were attached to the external surface of the cylinder, parallel to its axis. The gauges were placed equidistant from one another and wired in series thus giving a strain which was the average of the strain in the three gauges so eliminating, as far as possible, the effects of eccentricity of load.

The dynamometers were calibrated using a 15-ton Denison testing machine and the calibrations were regularly checked during the main stressing programme.

(b) Gifford-Udall Method.

The ends of the beam were trimmed smooth and steel end-plates were fastened at each end with a thin layer of plaster of Paris. The end-plates were drilled at suitable points in order to allow for an adequate spacing between the separate wires. Gauge points were attached to the sides of the beam as described above. The dynamometer was placed as centrally as possible against one end-plate, the wires passing through it and through a further end-plate.

The prestressing force was applied to the beam by the Gifford-Udall jack which was operated by a hydraulic pump. The amount of force which was applied to each wire was such that, as far as possible, when the wedging dollies were fastened up against the end-plates and all external force was removed the resulting pre-stressing force as recorded by the dynamometer was as required.

Again, immediately after prestress, initial readings were taken of the lengths between the gauge points on the sides of the beam. Further readings were taken at intervals up to the time of test.

The dynamometers used in all cases when the Gifford-Udall method was employed were made of 10 ins. lengths of steel tube of appropriate size according to the number and spacing of the wires used. The average strain induced in these dynamometers is calculated by measuring the longitudinal deformation of two 8 ins. gauge lengths on opposite sides of each dynamometer with the De Mec mechanical strain gauge.[5,6]

The dynamometers were calibrated using a 15 ton Denison testing machine or, when greater sensitivity was required for lower values, a 15,000 lbs. hand-operated Denison testing machine. The calibrations were regularly checked during the stressing programme.

Grouting.

All beams were grouted immediately after prestressing, the mix consisting of a mixture of water and cement with a water/cement ratio of 0.5. The grout was thoroughly mixed by hand and passed through a No. 14 B.S.S. sieve into a pressure pot. The grout was forced through the grout channel under pressure and a steady flow through the channel was achieved before the flow was stopped and the access holes plugged. Examination of the beams after test showed that in all cases the grout channels were completely filled with grout.

III TEST PROCEDURE AND MEASUREMENTS

7. The Prestressing Force.

The prestressing force is known to be one of the major factors influencing the shear strength of prestressed concrete beams. Great care was therefore taken to ensure that the effective prestressing force for each beam is known as accurately as possible. Attention has been drawn to the variables influencing the prestressing force by a number of writers[7,8,9,10,11,12,13] and methods used for elimination or accurate determination of all known possible variations in the prestressing forces of the test specimens are described in this section.

The force applied to each beam was measured directly at one end of the beam at the time of stressing by the use of hollow cylindrical metal dynamometers as described in section 6. In the cases where two macalloy bars were used in a beam (S1 & S2) separate dynamometers were used to measure the force applied in each bar. In all other cases all reinforcement was passed through one dynamometer so that the total force of the separate tendons was measured. Errors and losses which might have arisen by use of the jack gauge reading for the prestressing force, namely, friction within the jack, friction within the anchorage unit and slipping of the wires (where wires were used) within the wedges, were thus eliminated. The use of dynamometers also precluded the necessity of calculating the losses due to elastic contraction of the concrete where more than one tendon was used.

The only losses of force for which allowances must be made, therefore, were those arising from friction between the reinforcement and grout channel, relaxation of the steel and beam contraction due to creep and shrinkage of the concrete.

The loss of force due to friction between the reinforcement and the grout channel arises from deviation of the grout channel from the desired straight line. It is assumed that nominally straight reinforcement actually deviates through an angle which is directly proportional to its length.

The tension at any point of the reinforcement may be expressed, therefore, in an adaptation of the conventional friction formula, as

$$T = To \ e^{-kx} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where k is a constant depending on the type of duct and stressing method and x is the distance of the point from the point at which the tension, To, is applied.

Values of k were taken from the following table which is based on experimental work by Cooley.[14]

	$k(ft^{-1} \times 10^{-4})$		
<u>Lee McCall</u>	excess of duct size over bar size		
	1/8"	$\frac{1}{4}$"	$\frac{1}{2}$"
Ductube	15	10	5
Unitube	10	5	0

Gifford Udall	(moderate vibration)
Ductube	25
Unitube	10

The maximum loss in the applied force due to friction at any point does not exceed 1% for any of the test specimens. The maximum deviation from the mid-span value of the applied force in the reinforcement is, therefore, never more than 0.5%. Since the loss of stress due to friction is the only loss dependent on the length of the reinforcement the effective prestress in the shear span at the time of test cannot differ substantially from the value of the effective prestress at mid-span. The effective prestressing force at the time of test, therefore, is taken to be the applied force at mid-span minus the losses due to relaxation of the steel and beam shortening due to creep and shrinkage of the concrete. Values of the effective prestress listed in Table 1. are calculated on this basis.

The value of the applied force at mid-span of any beam is

$$T = To \; e^{-k\, l/2} \qquad \ldots\ldots\ldots\ldots (2)$$

where To = the force recorded by the dynamometer

and l = length of the beam in feet when the dynamometer is placed at the stressing end, and,

$$T = To \; e^{+k\, l/2} \qquad \ldots\ldots\ldots\ldots (3)$$

when the dynamometer is placed at the non-stressing end.

The loss of force due to relaxation of the steel arises from the fact that when steel is maintained under tension over a constant or nearly constant length there is a relaxation of stress. Clarke and Walley[15], in tests on steel similar to types II and III used in the present investigation have shown that the stress loss is a function of the ratio of the applied stress to the ultimate stress, the size of wire and the number of hours in the stressed condition. Evans and Bennett[11] report similar reults of tests conducted at the University of Leeds.

On the basis, therefore, estimations of the percentage relaxation of the reinforcement in all test specimens were made from information made available in the above references.

Whilst friction between the tendons and the duct, and steel relaxation induce significant losses of prestress, the most substantial losses result from longitudinal shortening of the test specimens and hence the reinforcement, due to creep and shrinkage of the concrete. The shortening of the reinforcement due to these phenomena can be measured directly, thus eliminating the necessity of estimating the individual effect of each.

Al already mentioned in section 6, longitudinal deformations of the test specimen between the time of prestressing and the time of test were recorded by the use of the DeMec demountable mechanical strain gauge. The DeMec gauge and its use are adequately described in two papers by Morice and Base[illegible] and Base[6]. The precautions for accuracy outlined in the papers were adhered to.

After test, the true depth of the reinforcement was measured and the deformation due to creep and shrinkage of the concrete at the depth of the steel was calculated. The loss of stress was computed using the stress/strain relationship for the type of steel used in the test.

8. Loading Arrangements

The test specimens were loaded in one of the three following testing machines, the choice of machine depending on the size of the specimen and the expected failure load.

1. A 50t Denison electric-driven screw-type testing machine.
2. A 15t Denison electric-driven screw-type testing machine.
3. A 15,000lb. Denison hand-operated screw-type testing machine.

In all cases the load was measured by obtaining, with a movable load, a position of balance of a graduated lever arm which was connected by a series of levers with the bed of the testing machine, on which the supports of the test specimen were placed.

All beams were simply supported on two rollers which were reduced in shape to provide a three-point contact in order to eliminate possible torsional stresses. The same arrangement was employed to support the steel girder which was used to distribute the load applied from the loading ram to two points on the beam.

In all cases the loading arrangement was symmetrical about a vertical plane which contained the cross-section at mid-length of the test specimen. In this way the two shear spans of each beam were of equal length and resisted equal shears and moments.

9. Test Procedure

All beams were given a white wash with a diluted solution of plaster of Paris just before testing, in order to facilitate the detection of cracks.

Load was applied in increments and after each increment of load, deflection and strain measurements were taken and cracks marked. Photographs were taken at significant stages in some tests. The magnitude of the increments of loading depended on the development of the crack pattern.

In some cases where diagonal cracking was observed in one of the shear spans of a beam, the shear span exhibiting the cracking was reinforced externally by the use of steel bolts as shown in Plate 1. By so restraining the development of failure in the already diagonally cracked shear span the other shear span frequently exhibited diagonal cracking, so making a useful extra result available. Beams, with the relevant shear span, on which such "strapping" was employed are marked in Table 3.

All specimens were loaded to failure, each test taking about three hours.

Control specimens were tested concurrently with, or soon after the main test.

10. Measurements

Measurements undertaken during the course of the investigation may be divided into two categories, namely, measurements taken on the test specimens themselves and measurements of the properties of the materials used in them.

(a) Measurements taken on the Test Specimens

Throughout the early stages of test and, in fact, until the specimens became unstable or failure was imminent, the deflection of all test specimens was measured at mid-span with a 0.001" dial indicator. A constant check on the rate of deflection/rate of loading ratio was kept during the test so providing a sensitive guide to the behaviour of the beam.

Longitudinal strains in the zones of the flanges adjacent to the load points were measured by a DeMec gauge[5,6], except for the smaller beams for which the DeMec gauge length (8") was too long to provide pertinent information. Knowledge of deformations within these zones was useful in determining the depth of the compression area at various stages during test and indicating when failure by crushing of the concrete within the zones was imminent.

Relevant loads at significant stages in the development of the failure of each test specimen were recorded.

After completion of the test relevant measurements were taken of the test specimen. Measurements of flange width, web thickness and effective depth were taken for each shear span (left-hand and right-hand) and are recorded in Table 1. In the case of shear-compression failures, the flange width recorded is the width of the flange where crushing occurred and the effective depth is the depth to the reinforcement where it meets the crack which ultimately led to failure. In the case of diagonal cracking, the web thickness recorded is the average thickness along the length of the crack or cracks at formation.

(b) Measurements of the Properties of Materials.

(i) The Compressive Strength of Concrete.

It has been shown[16,17,18,19,20] that the compressive strength of concrete cannot be determined absolutely, the values obtained from tests being dependent upon the size and shape of test specimen, the rate of loading, whether the specimen is capped or not and other less significant variables. Values obtained from test specimens, therefore, can be said only to represent a measure of the compressive strength of the concrete under certain conditions.

Two values of compressive strength of concrete were obtained for each beam.

The 4" cubes which were cast with the main beam were tested under compressive load in accordance with B.S. 1881:1952.

The 6" dia. x 12" cylinders were tested in compression in accordance with the standard procedure adopted by the American Society for Testing Materials. The cylinders were capped with plaster of Paris in order to relieve eccentricities of load and tested about one hour after the plaster had set.

(ii) The Tensile Strength of Concrete.

No really suitable method for testing concrete in tension is available. Several methods of obtaining an acceptable value of the tensile strength of concrete have, however, been suggested by a number of writers and three of these were used in the present investigation. In none of the methods is it possible to guarantee a uniform distribution of tensile stress over an area of concrete together with zero perpendicular stress. The tests used, whilst not giving absolute values of the tensile strength of concrete, are similar to tests used by other workers and therefore provide useful comparable figures.

The eight briquettes which were cast with each specimen were tested in tension according to the method specified in B.S. 12. Tallow was used to lubricate the contact between the briquettes and the jaws of the testing machine, so minimising the eccentricity of load. Evans(21) has drawn attention to the variations which may be expected in individual results of this test and has recommended that all low values should be ignored since they arise from eccentricity of the applied load. The briquette strength therefore is taken to be the average of the top four results of the group of eight.

The modulus of rupture, used by some writers as a value of the tensile strength of concrete, was measured by testing the two small beams 36" x 4" x 4", which were cast with the test beam, by loading at third-points over a 30" span. The modulus of rupture is taken to be that value of extreme bottom fibre stress, calculated on the assumption that concrete exhibits a linear stress/strain relationship in tension up to the point of failure, which causes rupture. Loading was applied at such a rate that the maximum bottom fibre stress calculated on the above basis increases at the rate of 100 p.s.i. per 10 seconds. No allowance is made for the Seewald effect. The value given by this test can only be a very indirect measure of the tensile strength of concrete since it is well known that concrete does not obey Hooke's Law and the distribution of stress in the bottom fibres just before failure is by no means uniform. Several mechanical features of the test have also been shown[22,23,24] to influence the result.

A value of the tensile strength of concrete was also determined for some beams by the cylinder splitting test described by Azakawa[25] and Thaulow[26]. The cylindrical specimen (6"dia. x 6") is loaded to failure in the direction of its diameter. It can be shown that, under these conditions and assuming Hooke's Law applies, an almost uniform tensile stress of value $\frac{2P}{\pi DL}$, where P = force applied, D = diameter and L = length of the cylinder, occurs at the plane containing the diameter and acts perpendicularly to it.

Assuming Hooke's Law applies up to failure, a value of the tensile stress at failure can be calculated and used as a value of the tensile strength of concrete. Again, this value cannot be said to represent the pure tensile strength of concrete for it can be shown that at all points where the uniform tensile stress acts there acts a perpendicular compressive stress, the value of which is at least three times that of the tensile stress and in areas near the loading points considerably more than this.

(iii) Stress/Strain Relationship for Concrete.

Several of the beams which were used to determine the modulus of rupture of the concrete were tested beforehand in longitudinal compression in order to obtain the stress/strain relationship for the concrete. Strains were measured by a Mercer dial gauge reading to 0.0001" over a guage length of 20" with the use of a Tass frame. Stress/Strain relationships for the two concrete mixes used are shown in Fig. 10. Each curve represents the application of stress in the first cycle since it is this condition which most nearly approximates to conditions within the beam when tested.

(iv) Stress/Strain Relationship for Steel.

The Stress/Strain relationships for the different types of steel were determined from tests on samples of each of the types cut from the same coils or bars from which the reinforcement used in the test specimens was taken. The samples were tested in either a 50t Avery hydraulic testing machine or a 5t Denison electric-driven screw-type testing machine. Strains were measured over an 8" gauge length with a Ewing extensometer. The stress/strain relationships for the three types of steel used are shown in Figs. 7, 8 and 9.

IV MODES OF FAILURE

The majority of the beams tested in the course of this investigation failed by one of the shear failure modes as described in sections 11, 13 and 14. The few beams which did not fail by one of these modes failed in flexure. In all cases these beams failed by crushing of the concrete in the upper fibres of the beam in a region of maximum moment following the penetration of flexural cracks which had initiated at the bottom surface of the beam. Data for these beams are included in the tables.

The failure mode of all beams tested is specified in Table 3.

11. The Shear-Compression Failure.

A beam is said to fail in shear-compression when the concrete crushes under compressive stress above an inclined crack which has formed in the shear span and which itself extends to or from the level of the horizontal tensile reinforcement. In almost all cases of failure of this type the inclined crack is initiated by a vertical tension crack occurring at the bottom fibre of the beam in a zone where the shear force and the bending moment are high.

A few failures in which the concrete has crushed above a diagonal crack which forms as described under section 12 of this thesis (i.e. for present purposes, not extending to the bottom surface of the beam) have been observed by the author and Thornton[27]. Although these failures may technically be described as shear-compression failures the author prefers to deal with them as failures resulting from diagonal cracking. These failures are therefore described in greater detail in section 13 (a).

For the purpose of this thesis, therefore, unless otherwise stated, shear-compression failures are taken to mean failures in compression above an inclined crack which itself was initiated by a vertical tension crack at the bottom fibre of the beam. Shear-compression failures of simply supported prestressed concrete beams have been reported by Zwoyer, Sozen and Hosny[1,4,3] and have also been observed by the author.

The initial stage in the development of a shear-compression failure is the formation of the vertical tension crack in the region of maximum moment and shear. This occurs after the formation of vertical cracks in the zone of pure flexure. The load at which the crack first forms is not precise since at this stage of loading the beam is constantly losing rigidity due to the penetration of other flexural cracks and a deflection/load or a strain/load graph is of little assistance.

The next stage in the development of the failure crack is one in which the direction of the crack becomes inclined. As load is increased the once vertical crack begins to extend both upwards and inwards towards the loading point. The inclination of the crack is quite steep and is usually about or greater than 45° to the horizontal at mid-depth of the beam. The significance of the inclined crack is that it penetrates more rapidly than the vertical purely flexural cracks. Again, no precise load can be given for this stage of development since, although the crack is clearly visible to the naked eye, the 'bend-over' and penetration is gradual.

The next significant development of the failure crack is that it becomes horizontal or nearly horizontal at its upper end and invades the region of pure flexure. Failure then follows by crushing of the concrete above the upper extremity of the failure crack and within the zone of pure flexure. Two such failures are shown in Plates 2 and 3.

The above described mechanism of failure in shear-compression has been observed by a number of other observers. Zwoyer[1] states that a beam failing in shear behaves with the same degree of elasticity in the early stages of loading as a corresponding beam failing in flexure. "Diagonal tension" cracks then occur and are diverted towards the point of application of load causing a "concentration" of rotation near this point. The concrete strain increases rapidly at the top surface of the beam at a location over the "diagonal tension" crack and when the limiting value is reached the concrete begins to crush. In the majority of beams tested, the diagonal tension crack became horizontal at its upper end and invaded the region of pure flexure before crushing occured. This mode of failure appears to be essentially the same as the shear-compression mode described above and all beams reported by Zwoyer known to have failed in this manner are therefore included in the analysis of shear-compression failures.

Sozen,[4] reporting tests conducted on beams of a variety of cross-sections indicates that one of the modes of failure observed is the shear-compression mode as described above. In the majority of beams tested in the course of the investigation the cracks in the shear span were initiated by horizontal tension, and thus originated vertically. However, these cracks "bent over" in a very short distance. Except for a few extreme cases, the "inclined tension" cracks formed near the point of application of load and developed towards it. Failure of some beams occurred when the beam failed by crushing of the concrete at or near the top of an inclined tension crack which had, in most instances, penetrated into the flexure span. Again it seems clear that the failure of these beams is essentially similar to failure in shear-compression as described above and therefore data given for beams known to have failed in this manner are also used in the establishment of expressions used to determine the shear-compression failure moments of prestressed concrete beams.

Both Zwoyer[1] and Sozen[4] also report tests on beams which failed by crushing of the concrete above an inclined crack initiated at the bottom fibre of the beam when the beam has been subjected to single point loading. It is not made clear, however, how these beams behaved in the final stages of loading. It is thought probable that the inclined crack became not quite horizontal and that failure occurred by crushing adjacent to the load point. For analytical purposes the failure of these beams is treated as in no way significantly different from the shear-compression failure of beams under four point loading.

A report of tests carried out by Hosny[3] on a number of prestressed concrete beams gives information of three rectangular beams which, it was shown, failed in the shear-compression mode. These beams are therefore also included in the analysis of this mode of failure.

The essential development of a shear-compression failure has been described above. It should be noted, however, that other phenomena related to the development of the failure often occur during loading.

During the stages of the development of the failure crack between its inclined state and its horizontal state further inclined cracking may occur as a continuation downwards of the inclined part of the failure crack. The addition of further load may induce small inclined cracks at the level of the tensile reinforcement and subsequently all these cracks may become joined to form one long crack (see Plate 4). The loads at which each of these phenomena occurs are not precise, the whole formation of the long crack being a rather gentle process. Failure in shear-compression may occur at any point in relation to the development of the latter mentioned crack. Frequently, failure in shear-compression is accompanied by an extension of the crack at the level of the horizontal reinforcement as far as the end of the beam. (see Plate 5). In tests conducted by the author any such "splitting" or "tearing" occurred instantaneously as the beam failed in shear-compression or immediately afterwards. Never has the splitting been observed to occur before failure by crushing over the inclined crack.

Whether the "splitting" or "tearing" in itself is the cause of failure is a matter of some dispute. Zwoyer(1) and Sozen(4) reporting on separate series of tests conducted at the University of Illinois claim that such splitting can be the cause of failure. Sozen(4) reports that the inclined crack was sometimes "followed by a single horizontal crack or by a series of short almost horizontal inclined cracks slightly above or at the level of the reinforcement.... These cracks when fully developed transformed a bonded beam into essentially an unbonded beam..... In general, if it were not for special conditions of the tests such as the transversely prestressed end-blocks and the end-anchorages, these cracks would themselves have been the direct cause of failure". Zwoyer(1) states, similarly, that cracking at the level of the steel may extend nearly to the end of the beam. A bond failure is highly probable under these conditions. Such a failure, it is stated, may occur at any stage of loading.

Hosny(3) describes cracking at or near the level of the steel reinforcement similar to that experienced by the author and described by Zwoyer(1) and Sozen(4). Failure, however, "was caused by the rapid increase of the diagonal tension cracks resulting in a reduced compressed area which was followed by crushing of the concrete over the inclined cracks and adjacent to the load point."

In the author's tests, (all on beams with unreinforced end-blocks) in view of this phenomenon, careful observation was made of the sequence of events. As reported above, in no case was splitting or bond failure noticed to precede failure in shear-compression. On this basis, therefore, all beams which exhibited splitting with, or subsequent to, shear-compression failure are considered to be shear-compression failures for analytical purposes.

12. Diagonal Cracking

Frequently, during the testing of I-section beams a diagonal crack, or (less often) cracks, occurs within the shear span independently of the vertical cracks initiated at the bottom surface of the beam. Such cracking occurs entirely within the web or extends only a little into the flanges and usually the crack or cracks lie midway between the support point and the load point. The inclination of the crack or cracks is never more than, and usually considerably less than, 45° to the horizontal. Plates 6,7 show typical examples of such cracking.

In general the load at which such cracking occurs is precise since the cracking is usually accompanied by readily audible noise and there is also an immediate increase in deflection, resulting in loss of load with the testing machines used. It should also be noted, however, that in some instances the diagonal crack forms a short while after loading is stopped. It is possible that if loading is not interrupted the beam may carry a greater load before diagonal cracking occurs. Conversely, if loading is carried out very slowly lower diagonal cracking loads may be recorded.

Diagonal cracking may be observed at any stage in the development of a shear-compression or flexural failure. In cases where the shear span is short and the amount of prestress is high, diagonal cracking may occur before flexural cracks are observed.

The essential difference between the diagonal cracking described above and the inclined cracking which may lead ultimately to a shear-compression failure is that diagonal cracking always occurs quite independently of cracks initiated by horizontal tension at the bottom fibre of the beam, whereas this is not the case with inclined cracking leading to a shear-compression failure.

Diagonal cracking of simply supported prestressed concrete beams as described above has also been reported by a number of other observers. Wilson[28], reporting tests on prestressed I-section beams failing in shear, states that all the diagonal cracks occurred in the web before any vertical cracks became visible; that their detection was achieved quite easily as the cracking occurred at quite a definite load and the angle of inclination of the cracks was about 25° to the horizontal. Zwoyer[1] states that "in beams with an I-shaped cross-section, diagonal tension cracks may occur suddenly over the entire depth of the web before the formation of the first crack in the region of pure flexure". Similar phenomena have also been reported by Hosny, Sozen, Thornton, Hicks, Warner and Hall, Bernhardt, Abeles and Sethunarayanan [3,4,27,29,30,31,32,33]. Where it is clear that beams referred to in the reports fall within the defined limits of the present study and where sufficient information is made available the results of these workers are used, together with the author's results to establish the expressions developed in section 16.

It should be noted that diagonal cracking may be preceded by the formation of short 'minor' cracks. Attention has been drawn to this possibility by Hosny[3] and Thornton[27]. Hosny[3] reports that in tests on prestressed concrete I-section beams without web reinforcement the first sign of cracking in the web is a group of small inclined cracks appearing in the bottom half of the web and very near the bottom flange. In one instance the first cracks appeared just beneath the top flange. The inclination of the cracks varies from 20°-30°. As load is applied the cracks extend steadily in both directions and in many cases the cracks penetrate the zone of pure flexure. Diagonal cracking occurs suddenly and without relation to the already existing cracks.

Thornton[27] reports a similar phenomenon mainly with highly prestressed beams. Diagonal cracking, it is stated, occurs some time after the 'minor' cracks and apparently with little relation to them.

The author has observed only a few isolated instances of similar phenomena; in each case the 'minor' cracks are short, inclined and occur along the junction of the web and the lower flange. Plate 8 shows an example of such cracking when it has developed a little and just before diagonal cracking occurred. The existence of the 'minor' cracks has very little influence on the behaviour, load bearing capacity and rigidity of the beam. Such cracks, therefore, are considered to be of secondary significance.

13. Failure Following Diagonal Cracking.

The formation of a diagonal crack in the shear span transforms a prestressed concrete beam into a substantially new structure. In general, the behaviour of the beam after diagonal cracking is quite different from that which may have been expected had not diagonal cracking occurred. In almost all cases observed by the author where the diagonal cracking was not restrained by strapping the shear-span in which the cracking occurred, the diagonal crack or cracks were the dominant feature in the failure mechanism.

As already stated the formation of a diagonal crack in a shear span is accompanied by a drop in the test load by virtue of the construction of the testing machine. In some cases, particularly those beams with a high ratio of shear span to effective depth, beams collapse entirely at the formation of the diagonal crack. Examination of the fragments of the beam after collapse indicates that failure in such cases has followed, very rapidly, one of the possible failure modes, to be described later in this section, which may occur after diagonal cracking.

As in the above instance some beams are unable to resist even the residual load after diagonal cracking; others may resist loads higher than the residual load but less than the load required originally to cause diagonal cracking, and others may sustain loads higher than the diagonal cracking load before failure occurs. The maximum load sustained by a beam which cracks diagonally therefore, is frequently the load causing such cracking and even in cases where higher loads than the diagonal cracking load are sustained the ultimate load is usually only slightly higher than the diagonal cracking load. A full examination of the cracking and ultimate loads is made in Chapter V.

Failure following diagonal cracking may occur in either of two ways. The compression zone above the upper end of a diagonal crack may fail or the web may disintegrate.

(a) Failure of the Compression Zone above the Diagonal Crack.

Two beams, S29 and S30, tested by the author developed diagonal cracking in both shear spans. In one instance (S30) the shear span which first developed diagonal cracking was strapped as described in Section 9. The addition of load to the residual load after diagonal cracking had occurred in both shear spans of each beam, resulted in a lengthening in both directions of all unrestrained diagonal cracks. Ultimately in each beam the flexural zone was invaded by one of the diagonal cracks and failure occurred by crushing of the concrete above the diagonal crack and within the zone of pure flexure. Plate 9 shows the failure of beam S29.

The only other known failures of this kind are reported by Thornton(27). In reporting the failure mechanism of a number of beams, Thornton(27) says that "as more load was applied the major crack spread into the flanges until failure occurred.... The development near the load point was governed by the location of the end of the crack relative to the load point. If it reached the top flange near the load point it just extended along its original line, but if it reached the top flange some distance from the load point, it spread along the web/flange junction before penetrating the flange". Failure occurred by shearing of the compression zone above the inclined crack.

In each of the two instances of this type of failure observed by the author the inclined crack reached the upper flange near to the load point. The behaviour of cracks reaching the upper flange some distance from the load point cannot therefore be confirmed.

(b) Web Distress

After or at the formation of the diagonal crack or cracks, vertical or nearly vertical cracks form in either or both of the flanges of the beam, within the shear span, extending from the upper and lower surfaces of the beam. Plates 10, 12, show the formation of such cracks.

In beams with a low ratio of shear span to effective depth the vertical cracks form gently and it is difficult to give a precise value to the load at which they occur. In beams with high ratios of shear span to effective depth the formation of such cracks may be accompanied by immediate failure and in this case the load at which they occur is quite precise.

With the addition of load the vertical cracks extend towards the diagonal cracks which themselves lengthen in both directions. More cracks in the flanges may form. Failure occurs by crushing of the concrete within the web usually at the toe of one of the upper vertical cracks. A complete sequence of a failure by web-distress is shown in Plates 11, 12, 13. Plates 14, 15 show two other instances of failure by this mode.

Web distress failures are often violent and very destructive. They have been reported by Thornton[27], Hosny[3], Sozen[4] and Hicks[9]. All report similar phenomena except that Hicks[29] reporting "diagonal compression" failures says that crushing may occur in the web before cracks occur in the top and bottom flanges and Thornton refers to the possibility of crushing occurring within the web without the formation of the vertical cracks in the flanges.

The failures reported earlier which occur instantaneously with the formation of the diagonal cracks are believed to belong to this category. Inspection of the fragments of the beams after failure indicates that failure was in all cases caused by crushing of the concrete within the web and there is also considerable evidence of the existence of vertical cracks. It is impossible in such cases, however, to determine whether web crushing followed or preceded the formation of the vertical cracks.

14. Other Types of Shear Failure.

Shear failures of simply-supported prestressed concrete beams without web reinforcement reported by a number of workers and shear failures observed by the author show that the large majority of such failures follow one of the failure mechanisms described in sections 11-13. A number of shear failures observed by the author and a few failures reported by Sozen[(4)], however, do not readily fit into the categories already discussed. Such failures, therefore, are now discussed in some detail.

1. The Failure of Beam S25. During the initial stages of loading the beam behaved in a manner similar to the initial behaviour of other beams tested in the course of the investigation. Flexural cracks were observed in the zone between the load points and as more load was applied these lengthened and became more numerous. Later, vertical cracks were observed outside the purely flexural zone and these subsequently became inclined towards the load points.

Failure occurred suddenly and violently by a complete disintegration of the right-hand shear span. Fragments of the beam were scattered around the base of the testing machine. An examination of these fragments shows that, in all probability, the web became separated from the upper and lower flanges at failure and crushing also took place within the web. None of the fragments showed any evidence of diagonal cracking within the shear span.

It was impossible to determine the sequence of events at failure and it was also impossible to establish reliably which cracks within the shear span occurred at failure and which occurred when the fragments hit the ground.

2. A number of highly prestressed beams with narrow webs tested by the author and a number of similar beams tested by Sozen(4) exhibited cracking described by Sozen as "secondary inclined tension cracking". The first signs of cracking within the shear span are minute inclined cracks at the junction of the upper flange and web of the beam. The formation of these cracks is followed by a widening and lengthening of the cracks until that nearest the load point may extend well under it and that furthest from the load point may cross the web and enter the end blocks. Plates 16 and 17 show this phenomenon in one of the author's beams and one of Sozen's beams respectively.

The significant differences between this type of cracking and the 'minor' cracks described in section 12 is, firstly, that the present cracks always occur at the junction of the web and top flange whereas the 'minor' cracks do not necessarily do so, occurring more frequently, in fact, at the junction of the web and the bottom flange and, secondly, that the present cracks always play a substantial part in the ultimate failure mechanism.

The crack which may cross the web and enter the end block could be described as a diagonal crack within the web but is significantly different in its formation from the diagonal cracking described in Section 12 in that it is initiated by some other form of cracking not occurring in the web and lies much nearer the end blocks than the normal diagonal crack.

With the type of testing machine used sustained deformation or slow rate of deformation may induce a reduction in the load applied. The ultimate load of the beam therefore may be less than the load which originally caused the cracking at the junction of the web and the flange. Failure occurs by crushing within the web and may be accompanied by diagonal cracking of the more usual form, and by separation of the web from the flanges.

V ANALYSIS OF TEST RESULTS

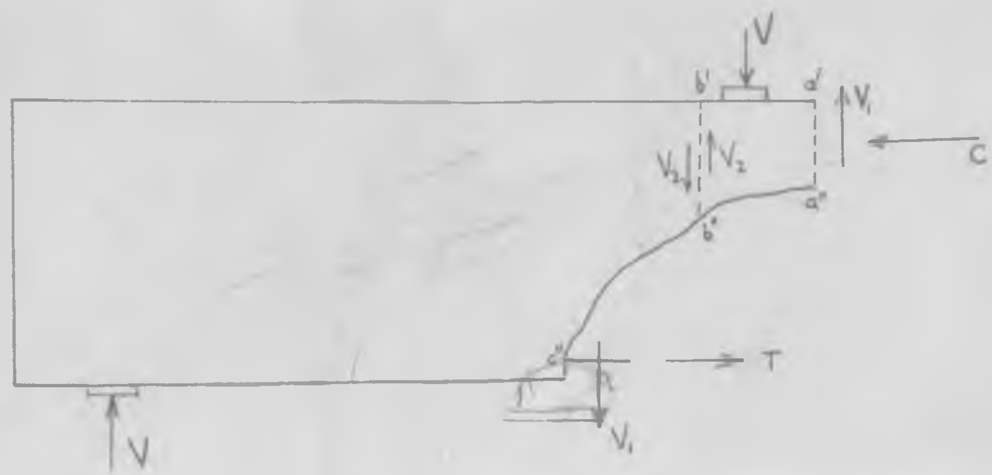
A simply-supported prestressed concrete beam without web reinforcement may fail in shear in a number of ways as described in Chapter IV. Basically, the modes of failure fall into three categories, namely, the shear compression mode, failure following diagonal cracking and, thirdly, a number of isolated and rarely observed types of failure which do not readily fit into the two previous categories.

Experimental results obtained by the author and other workers are discussed analytically in this chapter and expressions are developed for the shear-compression failure load and the diagonal cracking load of prestressed concrete beams. Failure following diagonal cracking and other modes of failure are also discussed analytically.

15. Analysis of Shear-Compression Failures and Related Phenomena

The behaviour of beams failing in shear-compression has been described in section 11. The observed phenomena are now discussed analytically and a semi-rational theory capable of predicting the shear-compression failure load of prestressed concrete beams is presented. 55 simply supported prestressed concrete beams without web reinforcement, 43 rectangular and 12 I-section, tested by a number of workers including the author are known to have failed in shear compression. Data from these beams are used to establish relationships necessary for the analysis.

Let us consider that portion of a beam under symmetrical two-point loading which lies above the crack leading to a shear-compression failure at a time just before failure occurs. The forces acting on such a portion are as shown below.



The shear is resisted jointly by the concrete above the crack and the steel reinforcement. If V is the total shear to be resisted, V_1 = the shear resisted by the steel, and V_2 = the shear resisted by the concrete above the diagonal crack and to the left of the load point in the above diagram, then

$$V = V_1 + V_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

By resolving the vertical forces, the shear resisted at the section a'- a" to the right of the load point is shown to be equal and opposite to the shear resisted by the reinforcement.

The shear resistance V_1 is provided by the reinforcement and the concrete below it acting as a beam supported on an elastic foundation. This resistance is frequently referred to as dowel action.

Under the condition described above, failure may occur in any one of a number of ways as described in chapter IV. For the existence of a crack such as the crack a" b" c" in the above diagram does not necessarily indicate that failure will be in the shear-compression mode. Three types of failure are only possible, however, if a crack such as the crack a" b" c" exists, for their mechanism depends upon its existence. The remainder of this section, therefore, is confined to a discussion of these three possibilities. Other possible failures are considered analytically in sections 16, 17 and 18.

Firstly, the magnitude of the shear transferred by the reinforcement, V_1, may be such that the horizontal section at the level of the steel is unable to resist it. In these circumstances cracking of the concrete may be expected at the level of the steel for it is at this level that the concrete cross-section is at a minimum. The occurence of such cracking should result in a decrease of the force V_1 and only when the shear force V is increased is V_1 expected to increase to a magnitude such as to cause further cracking. Such cracking has been observed by Zwoyer(1), Hosny(3), Sozen(4) and the author and is described in detail in section 11. It is possible that cracking of this kind will extend to the end of the beam so completely separating the reinforcement from the upper portion of the beam and causing complete collapse.

Alternatively, failure may occur in the zones surrounding sections a' - a" or b' - b" , these being the zones where there are the greatest concentrations of forces. Failure will occur if the system of stresses and/or deformation at one of the sections satisfies the failure criteria for the concrete at that section.

It is immediately clear that which of these types of failure occurs and the value of the shear force V at failure are largely dependent upon the shear V_1 and the crack pattern. In analytical work on ordinary reinforced concrete beams, Jones[34] and Krefeld[35] have shown that, assuming there to be no cracking at the level of the steel, the shear V_1 is dependent upon the bar size, the number and spacing of the bars, the breadth of the section, the 'cover', the strength of the concrete and the distance from the end of the beam. But since the force V_1 derives from the resistance offerred by the steel and the concrete surrounding it, any cracking at the level of the steel must affect its value. V_1, then, is largely dependent upon the crack pattern in that both the position of the major crack and the extent of subsequent cracking influence its value. Since in our present state of knowledge crack patterns are largely unpredictable, the shear force, V_1, is virtually indeterminable at any given stage of loading. Some valuable qualitative information on the force V_1 has, however, been reported by Watstein and Mathey[36] following tests on ordinary reinforced concrete beams. It was reported that as the ultimate load was approached the magnitude of vertical shear transferred by the longitudinal reinforcement across the crack decreased and neared zero. There is little to suggest that same situation should not obtain in prestressed concrete beams.

As stated above, the crack pattern is also of some considerable significance in determining where failure occurs and at what load. For the stresses in the zones surrounding sections a' - a" and b' - b" will be very largely determined by the shape of the cross-section above the diagonal crack.

Failures by splitting of the concrete at the level of the steel are a very real possibility for they have been reported by Zwoyer[1] and Sozen[4], who prefer to refer to them as bond failures rather than shear failures. Unfortunately no data are available for such failures and a quantitative analysis cannot, therefore, be undertaken. It is likely however that these failures can be avoided by the provision of a small amount of vertical reinforcement at the level of the steel.

In regard to failure above the inclined crack, the observations of Zwoyer[1], Hosny[3], Sozen[4] and the author indicate that failure may be expected at section a' - a" rather than at section b' - b" . Paez[37] and Ghali[38], however, in analytical work on prestressed concrete beams assume that the critical section is b' - b" . In order to assess the possibility of failure at either of the two zones and the loads at which failure in these zones may occur, we need to know the shape of the section above the inclined crack, the failure criteria for concrete and the principal stresses acting in the zones.

The shapes of the sections above the inclined crack are entirely dependent upon the line of the crack, which, as a whole, is virtually impossible to predict before test. There are grounds for believing, however, that the depth to the inclined crack from the upper surface of the beam at the section of failure can be predicted by an empirical expression. This is developed later in this section.

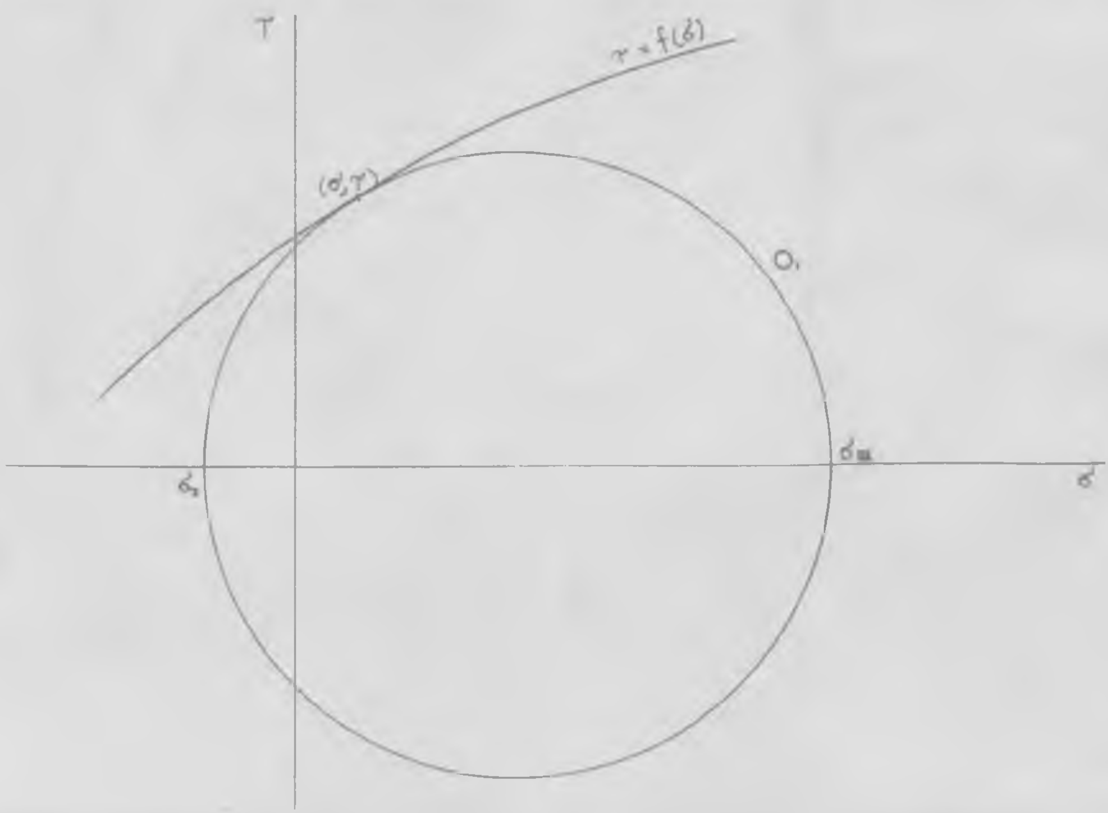
The failure of concrete under combined stresses has been the object of a considerable amount of research in recent years. The most commonly proposed theories are those of maximum stress and internal friction. Generally it may be said that these theories of failure are not adequately substantiated for plain concrete. A third theory suggesting a correlation between mean normal and shearing stresses at failure proposed by Bresler and Pister[39,40] has shown wide agreement with experimental observations.

The maximum principal stress theory which states that concrete fails when one of the principal stresses acting in the concrete reaches a limiting value has been commonly used in theoretical work. The theory is quite unsatisfactory for our purposes for it has been shown by Richart, Brandtzaeg and Brown[41] and McHenry and Karni[42] that it fails to agree with experimental evidence of non-uni-axial states of stress.

A second theory which has been frequently used is Mohr's generalisation of the internal friction theory. Basically the theory states that failure occurs by 'slipping' on a definite plane and that the normal and shearing stresses acting at the plane, σ and τ, are connected by a relationship

$$\tau = f(\sigma) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

If the maximum and minimum principle stresses at failure are σ_{I} and σ_{III}, the relationship $\tau = f(\sigma)$ can be represented by a curve which touches the circle drawn on a diameter, the co-ordinates of the ends of which are (σ_{I}, o) and (σ_{III}, o) as shown below[43]



The curve $\tau = f(\sigma)$ is an envelope of circles representing states of failure such as the circle O_1 on the above diagram.

The essential assumption of Mohr's theory is that the value of the intermediate principal stress is of no significance. But the validity of this assumption is seriously open to doubt.[44] No experimental evidence, however, is available so far which would question the theory's applicability to the case of a two-dimensional stress system, the two stresses being of opposite sense. Since the stresses in the zones of the beam which we are attempting to examine are two-dimensional and of opposite sense we may suppose that the Mohr theory does apply for our purposes.

Unfortunately, although much research has been undertaken to determine the shape of the envelope $\tau = f(\sigma)$, its form has not been satisfactorily established.

The theory proposed by Bresler and Pister[39,40] suggests that at failure there exists a unique relationship between the mean normal stress acting at a point and the mean shear stress acting at the point. If we consider an infinitesimal spherical volume of concrete the mean shearing stress over the surface can be expressed as

$$\tau_a = \lim_{S \to 0} \left[\frac{1}{S} \int_S \tau_s^2 \, ds \right] \qquad \text{. . . . . (6)}$$

where τ_s is the shear stress at the a point on the surface, S, of the spherical element.

The mean normal stress can be represented as

$$\sigma_a = \lim_{S \to 0} \left[\frac{1}{S} \int_S \sigma_s \, ds \right] \qquad \text{. . . . . (7)}$$

where σ_s is the normal stress at a point on the surface of the spherical element.

It can be shown that

$$\tau_a = \frac{1}{\sqrt{15}}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2} \quad . \quad (8)$$

where $\sigma_1, \sigma_2, \sigma_3$ are the principal stresses.

$$\& \quad \sigma_a = \tfrac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) \quad . \; . \; . \; . \; . \; . \quad (9)$$

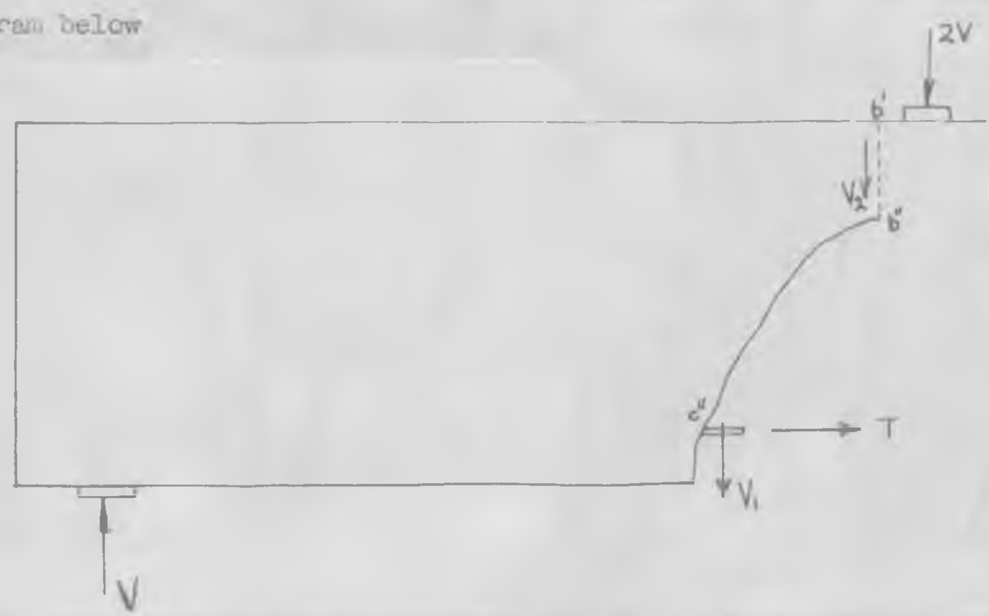
The relationship between τ_a and σ_a can be established with experimental data and it has been shown that the relationship

$$\frac{\tau_a}{f'_c} = 0.050 + 1.224\left(\frac{\sigma_a}{f'_c}\right) + 0.826\left(\frac{\sigma_a}{f'_c}\right)^2 \quad . \; . \; . \; . \; . \quad (10)$$

shows a wide measure of agreement with all known test results of plain concrete under combined stresses.

If it is possible, therefore, to estimate the shape of the cross-sections of the potential failure zone and the Bresler and Pister theory of concrete failure is adopted, a knowledge of the principal stress distributions in the zones should make possible a generally applicable solution. However, the principal stresses acting in the possible failure zones cannot be determined with any certainty. No value can be given to the shear V_4 with any accuracy and the distribution of compressive stress in the zones is by no means clear. Even an estimation of the principal stresses within the zones would involve several assumptions about the behaviour of the beam which may never be sufficiently accurate to justify such elaborations. Were it possible to establish accurately the distribution of principal stresses in the possible failure zones it remains doubtful whether a solution in general terms would be justified. For an assumption would have to be made as to whether σ_a and τ_a would be required to satisfy the failure criteria at one point only, or at a number of points or whether some mean values of σ_a and τ_a in the failure zone would be required. The problem of shear-compression failure, therefore, has been approached on a semi-rational basis.

Hitherto, this section has been concerned only with beams under symmetrical two-point loading. The general principles which were applied to the foregoing analysis are expected, however, to be equally applicable to beams under single point loading. Considering the diagram below



failure dependent on the existence of the crack b" c", which is the only type of failure considered in this section, may occur in one of two ways. Firstly the concrete may split at the level of the reinforcement in a manner similar to that explained for beams under two-point loading, and secondly, the concrete may fail at section b'-b".

Failures by splitting at the level of the reinforcement have been observed by Zwoyer[1] and Sozen[4] but no data have been made available. As for beams under two-point loading it is expected that such failures can be eliminated by the provision of a small amount of vertical reinforcement at the level of the steel.

Since shear-compression failures have been observed to be essentially similar to flexural failures the analysis has been carried out on a similar basis to that normally used for beams failing in flexure. The proposed theory is semi-rational and is not dependent on a knowledge of the shear forces acting in the zones above the inclined crack. Therefore, although these forces are substantially different for beams under single point loading as opposed to beams loaded at two points, no distinction is made between these cases. It will be demonstrated that the proposed theory shows a wide measure of agreement with experimental results regardless of the method of loading.

Initially, the analysis is confined to rectangular beams. Modifications are subsequently proposed to cater for beams of a different cross-section.

It is assumed that the forces acting at failure can be represented by the diagram below

The terms K and k_2 are determined by the stress distribution in the zone above the inclined crack.

By taking moments about the line of action of the tensile force, T, the failing moment

$$M_u = Kbd^2 f'_c k_u (1 - k_2k_u) \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (11)$$

The term k_2 is entirely dependent upon the shape of the stress block, which, unfortunately, is impossible to determine with precision. For even if the complete distribution of strain in the zone above the inclined crack were determined accurately the relationship between stresses and strains at failure under the combined stresses which are acting is not sufficiently well substantiated for an estimation to be made of the horizontal stress distribution. It is unlikely, however, that k_2 would be less than 0.33 which would be its value if the stress distribution were triangular or more than 0.50 which would be its value were the distribution rectangular. For convenience the average value $k_2 = 0.42$ is assumed to apply for all conditions. The effect of assuming some other value for k_2 is to slightly alter the derived expression for K and k_u. The selection of a semi-arbitrary value for k_2, therefore, does not seriously affect the analysis.

Assuming that the properties of the beams are also known, therefore, if K and k_u can be determined Equation (11) can be used to determine the ultimate load of a prestressed concrete beam in shear-compression. Zwoyer[1] has proposed that an expression for K can be determined by measuring the depth of the compressive zone at failure, $k_u d$, and the failing moment M_u. Values of K can be determined from equation (11) for each failure and an attempt made to relate K to relevant variables. An expression for k_u is developed by assuming that there exists a unique relationship, modified by the position of the neutral axis, between the concrete and steel strains at failure.

Zwoyer suggests that, at prestress, the compressive strain in the concrete at the level of the steel is e_{ce}, and the steel strain is e_{se}. As load is applied to the beam, the compressive strain in the concrete at the steel level decreases and at some stage reaches zero. At this stage the steel strain is equal to $e_{se} + e_{ce}$. As loading continues beyond this stage the elongation at the steel level is designated e_{cu}, whilst the compressive strain in the concrete at the top fibres of the beam increases until the limiting value e_u is reached. At ultimate load

$$e_{su} = e_{se} + e_{ce} + e_{cu} \quad \ldots\ldots\ldots\ldots \quad (12)$$

It is proposed that e_u and e_{cu} can be related by a simple equation

$$e_{cu} = Je_u \left(\frac{1 - k_u}{k_u} \right) \quad \ldots\ldots\ldots\ldots \quad (13)$$

Balancing the horizontal forces acting on the beam at the section of failure

$$Kbdk_u f'_c = pbdf_{su} \quad \ldots\ldots\ldots\ldots \quad (14)$$

$$\therefore \quad k_u = \frac{pf_{su}}{Kf'_c} \quad \ldots\ldots\ldots\ldots \quad (15)$$

substituting this value in equation (13)

$$e_{cu} = J\, e_u \left(\frac{Kf'c}{pf_{su}} - 1 \right) \quad \ldots\ldots\ldots\ldots \quad (16)$$

substituting this value in equation (12)

$$f_{su} = \frac{Je_u \dfrac{Kf'c}{p}}{e_{su} - e_{se} - e_{ce} + Je_u} \quad \ldots\ldots\ldots\ldots \quad (17)$$

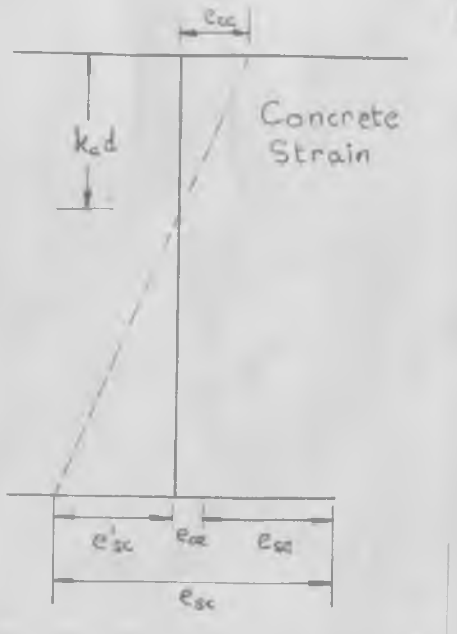
Equation (17) represents one relationship between the steel stress and strain at failure for any given beam and the steel stress/strain curve is another relationship. Only one combination of stress will satisfy these relationships simultaneously.

Assuming that a value of K can be determined for each beam using the empirical expression derived as indicated above, values of the term Je_u can be found using equation (17) and the stress/strain relationship for steel. An attempt can then be made to find an expression relating the term Je_u to the relevant variables.

An estimate can now be made of the shear-compression failing moment of any beam whose properties fall within the ranges of variables used in the test specimens. Values of the term Je_u and K are determinable from the derived empirical expressions and thus values of f_{su} and e_{su} are found by the simultaneous solution of equation (17) with the stress/strain relationship for steel. Equations (11) and (15) are used to obtain a value of the failing moment.

Sozen[4] has suggested that the term, K, can be assumed to be $\frac{1}{2}\,\dfrac{f'_c + 6{,}000}{f'_c + 1{,}500}$, which expression was derived by Billet and Appleton[45] from tests of prestressed concrete beams failing in flexure. An expression for k_u is developed by assuming that failure occurs when the concrete above the inclined crack reaches a limiting strain and that there exists a unique relationship, modified by the position of the neutral axis, between the concrete and steel strains at failure.

It is proposed that the behaviour of the beam may be divided into two significant stages terminated by "inclined tension cracking" and ultimate failure. At "inclined tension cracking" the strains at the critical section may be represented by the diagram below

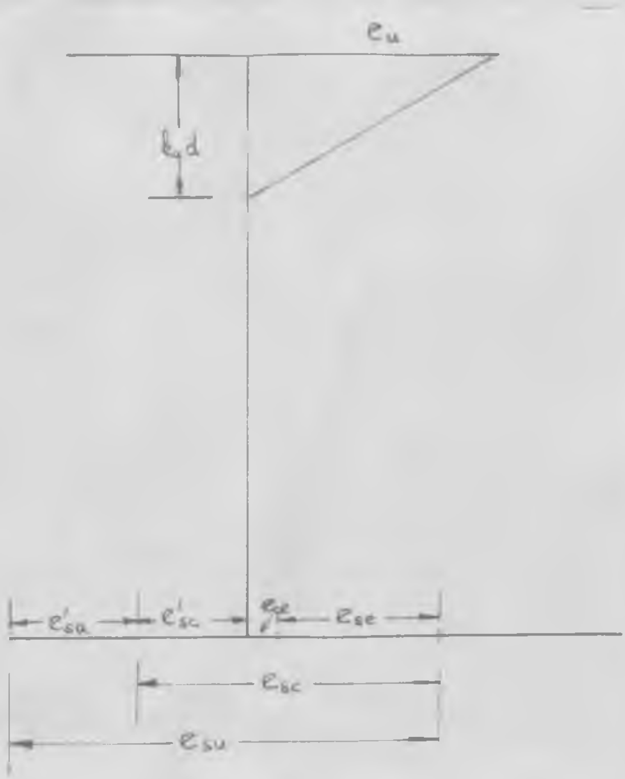


The steel strain, e_{sc}, = steel strain at prestress, e_{se} + the concrete strain at prestress, e_{ce} + the steel elongation after zero stress in the concrete at the level of the steel is reached, e'_{sc}. The depth of the neutral axis at this stage of loading is $k_c d$ and the concrete strain at the extreme fibre is e_{cc}. The relationship

$$e_{sc} = J_1 \, e_{cc} \left[\frac{1 - k_c}{k_c}\right] + e_{ce} + e_{se} \quad \ldots \ldots \quad (18)$$

is suggested as a relationship between the steel and concrete strains at this stage of loading.

At ultimate load the strains at the critical section may be represented by this diagram



It is assumed that the additional steel strain, e'_{sa}, over the steel strain at "inclined tension cracking" may be related to the additional extreme fibre concrete strain, $e_u - e_{cc}$, as below

$$e'_{sa} = J_2 \left(e_u - e_{cc}\right) \frac{(1 - k_u)}{k_u} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (19)$$

The ultimate steel stress, e_{su}, then becomes

$$e_{su} = J_1 \, e_{cc} \left[\frac{1 - k_c}{k_c}\right] + J_2 \left(e_u - e_{cc}\right) \left[\frac{1 - k_u}{k_u}\right] + e_{ce} + e_{se} \; . \; . (20)$$

Balancing the horizontal forces acting at the critical section at failure we get

$$K\,bdk_u\,f'_c = pbd\,f_{su} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (14)$$

$$\therefore \quad k_u = \frac{pf_{su}}{Kf'_c} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (15)$$

The equations (15) and (20) can be solved to find a steel stress and consequently the strength of the beam if J_1, J_2, k_c, k_u, K, e_{cc} and e_u are known or assumed. By making pertinent assumptions and using test results Sozen derives empirical expressions which enable us to determine the value of f_{su} for any given beam. Equations (11) and (15) can then be used to find a value of the failing moment in shear compression.

Zwoyer's theory proposes, therefore, that a value of k_u can be found for any given beam by the use of an empirical expression and the simultaneous solution of two relationships between steel stress and strain at failure, one of which is the stress/strain relationship for the steel. Sozen's theory proposes that with the use of empirical expressions one of which is for the moment causing "inclined cracking", a term which is not clearly defined, and the solution of a relationship between the steel stress and strain at failure with the stress/strain relationship for the steel, a value of k_u can be determined for any given beam.

The assumptionsmade in these theories, however, may never be sufficiently justified to warrant such elaborate analyses and the resulting methods for the determination of the ultimate shear-compression load are so complex as to be of little practical value. It is believed that the terms K and k_u can be determined directly as functions of the controlling variables and that values of K and k_u found by the use of these expressions are no less valid than values obtained by the methods of Zwoyer and Sozen described above.

A value of K for any given beam failing in shear-compression can be found from the following adaptation of equation (11)

$$K = \frac{M_u}{bd^2 f'_c k_u (1 - 0.42 k_u)} \quad \ldots\ldots\ldots\ldots (21)$$

provided M_u, b, d, f'_c and k_u are known. Values of K for all rectangular beams known to have failed in shear compression and for which values of M_u, b, d, f'_c and k_u are known were obtained from equation (21) and are listed in Table 4. It should be noted that for the author's beams, the failing moment used, M_u, corresponds to the total of the applied load and the appropriate proportion of the dead load. For other workers' beams, M_u is as listed. In some cases it is not clear whether the failing load recorded is the total load or the applied load. At failure, however, the self load of a beam is small in comparison with the applied load and any discrepancy is, therefore, considered to be of little significance.

Values of K are shown graphically as a function of the cylinder strength of concrete in Fig. 11. Since, as has already been explained, it is thought that the amount of shear transferred by the tensile reinforcement is a factor influencing the stress pattern in the failure zone at the time of failure it is possible that the scatter shown is due to this shear force. The beam properties influencing this force were, however, sensibly constant for all the specimens, and details of cracking at the level of the reinforcement are not available. It is therefore not known whether the scatter can be related to the amount of shear transferred by the longitudinal reinforcement. Other possible causes of scatter are the shear span to effective depth ratio, the amount of prestress and the amount of tensile reinforcement. Each of these variables were examined but no correlation with the scatter was apparent.

Although it is expected that K will approach some limiting value at higher values of f'_c, sufficient extra information is not available to suggest a modification of Zwoyer's expression

$$K = 1.500 - 0.715 \times 10^{-4} f'_c \quad \ldots\ldots\ldots\ldots \quad (22)$$

Zwoyer[1] has noted that the results of the tests show that the average compressive stress in the concrete of the beam just before failure may be higher than the cylinder crushing strength; and, of course, if the distribution of the compressive stress is non-uniform, as is expected, the stress in some parts of the zone may be higher than that. It is thought likely that this phenomenon is due to the fact that the high compressive stresses exist only over a short length in relation to the cross-sectional area of the zone where the stresses act. For it has been shown[20] that, in general, a decrease in the ratio of the length to diameter of concrete test specimens may be expected to result in an increase in the measured compressive strength.

It was found that the measured value of the depth of the compression zone at failure, $k_u d$, could be determined by the use of the empirical relationship

$$k_u = \frac{pt_u}{f'_c}(0.427 + 0.349 \times 10^{-3} pf_{se}) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; (23)$$

which is shown graphically on Figs. 12 and 13. In an attempt to assess the cause of the scatter of the results values of

$$\frac{k_u}{\frac{pt_u}{f'_c}(0.427 + 0.349 \times 10^{-3} pf_{se})}$$

were plotted against a/d, the shear span to depth ratio, this being the only remaining variable capable of measurement. No correlation was detected. Although, as explained earlier, the shear transferred by the reinforcement is expected to have some influence on the shear-compression failure load of prestressed concrete beams, it is not possible to make any quantitative assessment of its value at failure. Whilst it is likely, therefore, that this shear is influencing failure it is not possible to attempt to relate scatter to its value.

Equation (23) is not valid for values of $\frac{pt_u}{f'_c}$ and pf_{se} higher than those used in the tests. It is expected that k_u should approach some upper limiting value but the scatter of data for beams with $\frac{pt_u}{f'_c}$ less than 0.7 and pf_{se} less than 1000 p.s.i. and the lack of data for beams with values of $\frac{pt_u}{f'_c}$ and pf_{se} above these values does not warrant a more refined expression. The measured depth to the top of the inclined crack from the upper surface of the beam at the point of failure was never less than 0.091 and never more than 0.444 of the measured effective depth.

Equations (22) and (23) were established by the use of data from the tests of Zwoyer[1] and the author on rectangular beams for which the measured depth to the inclined crack at the point of failure was known. These equations are now used to examine the failing moments of all beams known to have failed in shear compression. Values of the failing moments of 55 beams tested in four separate investigations were compared with those computed with the use of equation (22) and (23). 43 of the beams were rectangular. The remaining 12 beams, all tested by the author, had I-shaped cross-sections. A detailed study of the influence of the shape of cross-section on the shear-compression failing load cannot be made, however, for all the I-beams which failed in shear-compression were basically of one type. That is, the ratio of web thickness to flan e width was of the order of 0.50 and the average flange depths were each a quarter of the overall depth of the beam. Data for the additional rectangular beams are listed in Table 5 and data for the I-beams are listed in Table 6. For the I-beams the computed value of k_u was taken as the lesser of the value calculated by equation (23) and the ratio of the mean flange depth to effective depth.

It is apparent that the extra information provided demands a revision of the expression for the calculated failing moment. Divergence from the calculated values depends on $\frac{pt_u}{f'_c}$ and the shape of the cross-section. Figures 14 and 15 show graphically the relationship between the moments calculated by equation (21) (i.e. $M_u$calc) and the test failure moments ($M_u$test) for the rectangular and the I-beams respectively. It is suggested that a correction factor, F, can be applied to the right-hand side of equation (11) such that now,

$$M_u = F\, bd^2 f'_c K k_u (1 - k_2 k_u) \quad \ldots \ldots \ldots \ldots (24)$$

where

$$F = 1.40 - 1.11 \frac{pt_u}{f'_c} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (25)$$

for rectangular beams, and

$$F = (1.40 - 1.11 \frac{pt_u}{f'_c})(2.76 \frac{pt_u}{f'_c} - 0.19) \quad \ldots \ldots (26)$$

for the I-beams.

For rectangular beams, the expression

$$F = 1.40 - 1.11 \frac{pt_u}{f'_c} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (25)$$

should not be regarded as valid for values of $\frac{pt_u}{f'_c}$ higher than 0.9.

It is expected that at higher values of $\frac{pt_u}{f'_c}$, F should tend to some lower limiting value. The scatter of the results shown in Fig. 14 and the absence of data for beams with values of $\frac{pt_u}{f'_c}$ greater than 1.0 does not justify a more refined expression. For the I-beams, the expression

$$F = \left(1.40 - 1.11\frac{pt_u}{f'_c}\right)\left(2.76\frac{pt_u}{f'_c} - 0.19\right) \quad \ldots\ldots \quad (26)$$

has been so arranged as to allow easy comparison of the failing moments of the I-beams with the rectangular beams. The expression should not be regarded as valid for values of $\frac{pt_u}{f'_c}$ outside the range $0.3 < \frac{pt_u}{f'_c} < 1.0$. The scatter of results and the lack of information of beams with values of $\frac{pt_u}{f'_c}$ outside this range does not warrant a more refined expression than that given. An attempt was made to relate the scatter to other known variables of the tests but no consistent relationship could be determined.

It is clear that the term, F, in part, represents the influence of the shape of cross-section of the beam. It is unfortunate, though, that this influence cannot be fully investigated for reasons given above. The expression

$$F = 1.40 - 1.11\frac{pt_u}{f'_c} \quad \ldots\ldots\ldots\ldots\ldots \quad (25)$$

for rectangular beams, however, represents substantial deviation of the measured failing moments from those moments calculated by the use of empirical expressions established with experimental evidence of tests on beams which were in no way substantially different from those which were not included in the analysis. It must be concluded, therefore, for rectangular beams, that F represents a refinement of the expressions for K and k_u; and for the I-beams, F represents partly the influence of the shape of the cross-section and partly refinements in the expressions for K and k_u. To what extent each of these terms should be modified, however, is indeterminable from the information which is available. Further research is necessary before more accurate expressions for K and k_u can be established.

One of the main reasons for undertaking the tests described in this report was to determine the influence of the shape of cross-section of the beam on the modes of failure in shear and the failing loads. However, no beams with narrow-webbed I-sections failed in shear-compression and, therefore, no widely valid assessment can be made of the influence of the shape of cross-section on the shear-compression failure of prestressed concrete beams. Fig. 15 indicates, however, that, at least for beams with cross-sections not significantly dissimilar from those discussed in this section, there exists a critical value of the term $\frac{pt_u}{f'_c}$ above which I-beams fail at moments higher than those predicted for otherwise similar rectangular beams and below which the beams fail at lower moments. Further research is necessary before the influence of the shape of cross-section can be established in quantitative terms.

16. Analysis of the Phenomenon of Diagonal Cracking.

The behaviour of beams exhibiting diagonal cracking has been described in section 12. The observed phenomena are now discussed quantitatively. Data from 40 beams, tested by Hosny[3] and the author, which are known to have shown diagonal cracking as defined in section 12 are used to establish an expression capable of predicting the diagonal cracking load of prestressed concrete beams.

It is suggested that diagonal cracking occurs when the stress pattern in the shear span satisfies the failure criteria for the concrete in that zone. A complete knowledge of the stresses within the shear span under all possible conditions would therefore make possible a generally applicable solution provided that the actual criteria of failure of the concrete in the shear span are known. Unfortunately, although it may be stated that the general theory of failure of concrete proposed by Bresler and Pister[39,40], which was briefly described in the previous section, is reasonably well substantiated, it is not clear in practice whether failure occurs when the stresses at one point only or at a number of points satisfy the failure criteria, or whether failure would occur if the average stresses in a zone met the criteria. In any case the distribution of stress just before diagonal cracking occurs cannot be estimated with accuracy. For at such a stage the concrete behaves plastically and follows no well-defined natural laws which would enable us to assess accurately the value of stresses acting. Furthermore, the penetration of flexural cracks presents an additional factor the influence of which is by no means definite. It is clear therefore that a number of assumptions about the behaviour of the beam would be required if it were particularly necessary to make available a general solution. It is doubtful, however, in our present state of knowledge, whether such assumptions would be sufficiently accurate to justify such a 'correct' analysis.

The problem of diagonal cracking must, therefore, be approached at an empirical or semi-empirical level.

A possible semi-empirical method of correlating results would be to select a parameter the value of which was felt to be of major significance in the formation of diagonal cracks and attempt to relate this parameter to pertinent variables. Such a parameter would most usefully be some stress (e.g. principal tensile stress) or some function of stresses existing at a known point in the web just before diagonal cracking occurs or possibly the maximum value of such a function throughout the web. This method is, however, open to serious objections.

As already stated the actual stresses existing in the web just before diagonal cracking occurs cannot be calculated with a great degree of accuracy. Values of stresses acting just before cracking, calculated using some form of idealised concrete stress/strain relationship and assuming ideal beam conditions, represent, therefore, only a measure of the true stresses existing. An expression involving a parameter calculated in this manner would therefore be quite empirical and no more representative of the true diagonal cracking criteria than an empirical expression relating the applied load to significant variables. The solution,would, furthermore, lead to a cumbersome method of assessing the possibility of the occurrence of diagonal cracks. It is, in addition, potentially misleading in that it conveys the impression that one particular factor (i.e. the parameter) is of overriding significance in the formation of diagonal cracking whereas it is clear that, unless the parameter in some way represents the complete stress pattern in the shear span, this cannot be so.

The problem has therefore been approached on a completely empirical level and an expression relating shear across the section when cracking occurs to the pertinent variables is offerred. Data from the beams used to establish the expression are listed in Table 7.

In cases where one of the shear spans of a beam has been strapped following diagonal cracking and where diagonal cracking has subsequently occurred in the other shear span, data relating to each of the shear spans are used. It is considered that since the beam remains statically determinate a result obtained from the shear span exhibiting later diagonal cracking is as equally valid as that obtained from the shear span showing earlier diagonal cracks.

For beams tested by the author the shears causing diagonal cracks as listed in Table 7 are those considered to be acting at mid-length of the shear span. That is they correspond to a total of the applied load and the appropriate proportion of the dead load. It is not clear whether this also applies to the beams tested by Hosny but since the dead load forms only a very small part of the total load at diagonal cracking it is not felt that any discrepancy will be of major significance.

The most simple and consistent expression relating the shear causing diagonal cracking to the influencing variables was found to be

$$V_c = \left(7.2 + 10.0\,\frac{f_h}{f_t}\right)\left(\frac{44}{a/d} + 16\right)\frac{f_t bd}{480}\left(\frac{b_1}{b}\right) \quad \ldots\ldots \quad (27)$$

where V_c = shear force at diagonal cracking,

f_h = average effective prestress in the concrete, i.e. the effective prestress at the centre of area of the concrete section.

f_t = briquette strength of concrete,

a/d = shear span/effective depth ratio,

$\frac{b_1}{b}$ = web thickness/flange width ratio.

It should be noted that the above expression is entirely derived from experimental evidence and is, therefore, not known to be valid for beams other than those similar to those which have shown reasonable correlation with it. Equation (27) therefore is presented as being valid over the following ranges.

$$2.3 < a/d < 5.5$$

$$0.28 < \frac{b'}{b} < 0.50$$

$$400 < f_t < 700 \text{ p.s.i.}$$

$$300 < f_h < 900 \text{ p.s.i.}$$

$$0.3\% < p < 3.0\%$$

$$0.8 < \frac{f_h}{f_t} < 1.6$$

d of the order of 10"

b of the order of 4"

Equation (27) may be expected to produce conservative values of V_c for beams with $\frac{f_h}{f_t} < 0.8$. This expression is represented graphically in relation to test results in Figs. 16 to 20. The figures show that major influencing factors affecting diagonal cracking in prestressed concrete beams are the ratio of the shear span to effective depth, the shape of cross-section, the strength of the concrete and the amount of prestress. It is realised, however, that the actual size of the beam will also be a controlling factor of some significance but, unfortunately, the result of only one beam (S4.2) which was substantially different in size from others was available.

The dependence of the diagonal cracking load on the size of the beam could not be ignored in the development of equation (27) and it was therefore assumed, in the absence of significant contrary evidence, that the diagonal cracking load was directly proportional to the product of the overall breadth of the beam and the effective depth. This assumption is, of course, open to correction when sufficient evidence is made available. In the meantime, however, it must be regarded as a limitation on the validity of the expression.

Figs. 16 and 17 show the influence of the strength of the concrete and the level of prestress on the diagonal cracking load. The value used to show the influence of the concrete strength is the briquette strength of concrete, values of which were obtained as described in section 10 for the author's beams and are as listed for Hosny's beams. The briquette strength was selected as a measure of concrete strength since it was considered that the conditions of the briquette test were more similar to the conditions acting within the shear span than those of any other test except possibly the cylinder splitting test, also described in section 10, values from which are not available for all test specimens.

Since the stresses in the shear span due to prestress act in superposition to those existing due to the application of load it was felt that diagonal cracking would occur when the stresses due to the applied load plus the stresses due to prestress satisfy some failure criteria dependent on the strength of the concrete. This may be expressed mathematically as

$$F_1(V_c) + F_2(f_h) = F_3(f_t) \quad \ldots\ldots\ldots\ldots\ldots \quad (28)$$

where $F_1(V_c)$ represents the influence of the stresses due to applied load on failure

$F_2(f_h)$ represents the influence of the stresses due to the prestressing force on failure

and $F_3(f_t)$ represents the failure criteria of the concrete.

This equation may be rewritten as

$$V_c = F_4(f_t) \left[1 + F_5 \left(\frac{f_h}{f_t}\right) \right] \quad \ldots\ldots\ldots\ldots \quad (29)$$

The terms f_t and $\left(\frac{f_h}{f_t}\right)$ were therefore selected as parameters against which experimental evidence could be examined.

As may be seen from Fig. 17 equation (27) may be expected to predict conservative values of the shear causing diagonal cracking for beams with values of $\frac{f_h}{f_t}$ less than 0.8. It is possible that a more refined expression than that offered would represent the data available more accurately but the lack of data for values of $\left(\frac{f_h}{f_t}\right)$ lower than 0.5 and the scatter of results at values above this do not justify any refinement at present.

The effect of variation of the ratio of the shear span to effective depth on the shear causing diagonal cracking is shown in Fig. 18. The parameter of the ratio of shear span to effective depth was chosen since it was felt that it would present a reliable basis for the examination of the effect of the applied moment whilst eliminating any consideration of the size of test specimens. It is immediately noticed that, all other factors being equal, an increase in this ratio leads to a decrease in the amount of shear which can be carried before diagonal cracking occurs. For otherwise similar beams resisting the same applied loads the effect of increasing the length of the shear span is to produce a higher applied moment. It is therefore considered that the dependence of the diagonal cracking load of the beam on the shear span is indicative of the influence of the flexural stresses acting in the shear span. Precisely how the flexural stresses influence the stress pattern within the shear span is not clear. For whilst it is certain that such stresses play a significant part in the structure of the stress pattern in the shear span the distribution of these stresses is dependent upon the extent of flexural cracking, a factor about which little, in quantitative terms, is known at present.

The term $(\frac{b_1}{b})$ in equation (27) represents the influence of the shape of cross-section of the beam on the diagonal cracking load. The effect of this factor is shown graphically in Fig. 19. The ranges of the variation of the term $(\frac{b_1}{b})$ are unfortunately limited and although it is thus possible that a more refined expression than that indicated by equation (27) would more accurately represent the influence of the shape of the cross-section of the beam, it was felt that sufficient data were not available to justify any adjustment to the expression offered at present.

In the group of beams which was specially designed to examine the influence of the method of curing on the shear failure of the test specimens, four instances of diagonal cracking were noted. Two of these instances occurred in different shear spans of the same beam. Other beams of the group failed following cracking at the junction of the web and the upper flange as described in section 14.2. and are discussed separately in section 18.

The results of the beams belonging to this group showing diagonal cracking are plotted in Figs. 16 to 20. It can be seen that diagonal cracking in the shear spans of beam S50, which was cured in the normal manner used for the rest of the tests described in this report, occurred at lower loads than in beams S52 and S53 which were cured more rigorously. It should be noted, however, that many other beams with comparable properties cured in the same manner as that used for beam S50 showed diagonal cracking loads in excess of those recorded for beams S52 and S53. It is concluded, therefore, that these results do not necessarily indicate that improved curing has any marked effect on the diagonal cracking load of the beam. Whilst it is considered that internal stresses due to shrinkage during the drying out of a beam are likely to influence the diagonal cracking load of prestressed concrete beams it is thought that any shrinkage effects in these beams are minimal, since the cross-section of the beams tested were such that no part of the section was really remote from the surface in relation to other parts. It is therefore likely that drying out was reasonably uniform; in which case stresses due to differential shrinkage would have been at a minimum. Further research will be necessary before the influence of the method of curing on the diagonal cracking load of prestressed concrete beams can be established.

The result of beam S42 marked on Figs. 16 to 20 shows substantial deviation from other test results. Such deviation may be due to a number of factors, for this beam is in many ways different from others showing diagonal cracking. The most important differences are that it is markedly smaller than other beams whose results are plotted, its ratio of web thickness to flange width is higher, the amount of longitudinal reinforcement is higher and the ratio of its effective prestress at the centre of area to the tensile strength of the concrete is lower. The deviation of this result may be due to any or a combination of any of these factors. Further research will be necessary before any comparative assessment of these factors can be made. In the meantime the result of beam S42 emphasises the limitations of the validity of equation (27).

Since the amount of longitudinal reinforcement used in the test beams was one of the major variables of this study a careful examination of the test results was made in order to determine whether the amount of reinforcement influenced the diagonal cracking load of prestressed concrete beams. No consistent trends were observed. Fig. 20 shows values of

$$\frac{480\ V_c}{\left(\frac{44}{a/d}+16\right)\left(7.2 + 10.0\ \frac{f_h}{f_t}\right) bdf_t\left(\frac{b_1}{b}\right)},$$

that is the fractional deviation of test results from values given by equation (27), plotted against percentage of reinforcement. It is clear from this figure that the amount of longitudinal reinforcement of a prestressed concrete beam is unlikely to influence the diagonal cracking load of that beam except possibly at very high values of p%. The scatter of data at such values, however, shows that any possible dependence of the diagonal cracking load on the amount of longitudinal reinforcement cannot be confirmed without further evidence.

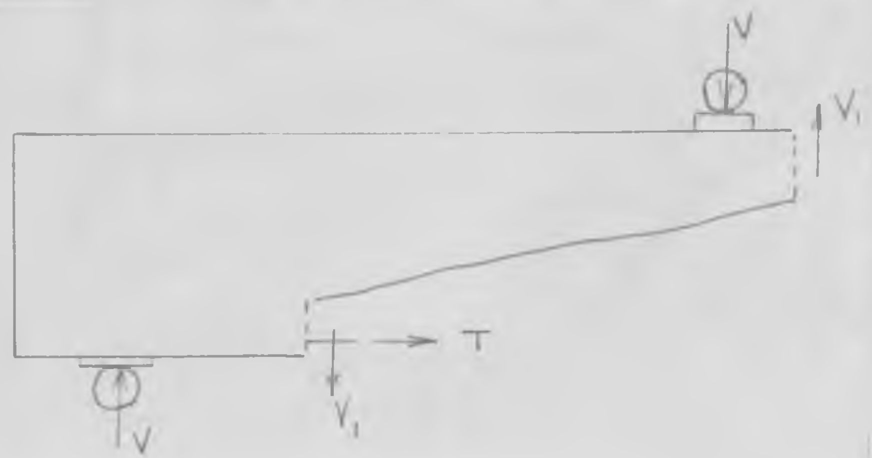
17. Discussion of Failures following Diagonal Cracking.

The failure of prestressed concrete test beams following the occurrence of diagonal cracking has been described in section 13. It is clear from the observation of the test phenomena that, generally, failure may occur either by a failure of the concrete zone above the upper extremity of the diagonal crack or by a failure of the concrete within the web.

(a) Failure of the Compression Zone above the Diagonal Crack.

The details of the behaviour of two beams, tested by the author, S29 and S30, which failed by crushing of the concrete above the upper extremity of the diagonal crack have been given in section 13 (a).

Let us consider that portion of one of the beams which lies above the diagonal cracks at a time just before failure occurs. The forces acting on such a section are as shown



It will be noted that the section and the forces acting on it are essentially similar to those of a like section of a beam which has developed an inclined crack leading to a shear-compression failure. It is expected, therefore, that an analysis of this type of failure could be developed in much the same way as that presented for shear-compression failures in section 15. It is clear that the empirical expressions presented for the terms K and k_u would need adjustment for it is expected that they are dependent to some extent on the crack pattern of the beam. Too few failures of this kind have been reported, however, to allow the development of any expressions for K and k_u applicable to this type of failure.

The results of beams S29 and S30 have been checked against the estimated shear compression failure moments as given by equations (24) and (26) and are compared below

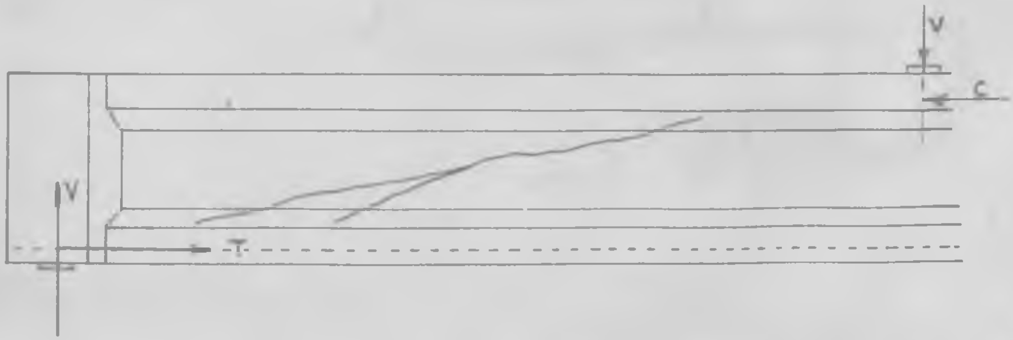
Beam	Test $M_u \times 10^{-3}$ lbs ins	Computed $M_u \times 10^{-3}$ lbs ins	Test Value / Computed Value
S29	406	464	0.88
S30	467	301	1.55

Whilst it is clear that there is poor correlation between the test and computed values of M_u it is to be noted that in neither case is the test result substantially lower than the computed value.

(b) Failure of the Web.

The failure mechanism of those beams which fail in the web following diagonal cracking has been described in section 13 (b). The observed behaviour of these beams is now discussed analytically. Data from 14 beams, all tested by the author, are used to establish an empirical expression relating the failure load to pertinent variables. Although it is known that other failures of the kind now discussed have been reported by other workers, either sufficient information is not available for their inclusion in this analysis or the specimens exhibiting such failures do not fall within the defined limits of this investigation.

Let us consider the forces acting on a shear span which has cracked diagonally. The diagram below shows such a shear span.



The applied shear is transferred across the shear span by a system of stresses and forces dependent upon the geometry and elastic properties of the component parts of the structure, the resultant of which is an inclined thrust. The stresses acting at given point in the shear span are therefore dependent on the applied load, the properties of the beam section, the position of the diagonal crack or cracks and the extent of localised failure (e.g. tensile cracks at the upper or lower surfaces of the beam) in other parts of the shear span. The stresses acting at a given point are expected to be to some extent dependent on the amount of shear transferred by the tensile reinforcement but since this factor is in turn dependent on the factors already noted it is not listed separately.

The observation of test specimens shows that a beam may be expected to fail when the concrete in the web crushes, usually at the toe of one of the vertical tensile cracks extending from the upper surface. A complete analysis of the stresses acting within the shear span would therefore make possible a generally applicable solution assuming that the criteria of failure of the concrete are known or can be satisfactorily approximated. It is clear, however, that no accurate assessment can be made of the stresses acting within the web just before failure occurs. For as already stated the stresses are largely dependent on the crack pattern of the shear span which cannot be predicted with accuracy and is in fact very variable. The problem of the failure of the web following diagonal cracking has therefore been pursued at an empirical level and an expression relating to the shear across the section when failure occurs to the relevant variables is offered. Data for the beams used to establish the expression are listed in Table 8.

Data from 14 beams, all tested by the author, known to have failed by web crushing following diagonal cracking are plotted in Fig. 21. Results from those beams which failed at lower loads than the loads causing diagonal cracking are included. For it is considered that such results are valid as being representative of the load carrying capacity of the new, diagonally cracked, structure. The following simple equation was fitted to the data over the range shown

$$V_u = bb_1 f_{cu} (0.376 - 0.044\ a/d) \quad . \ . \ . \ . \ . \ . \ (30)$$

where V_u = shear force at ultimate load

b = flange width

b_1 = web thickness

f_{cu} = cube strength of concrete

a/d = shear span/effective depth ratio

The above expression should not be assumed to be valid other than for beams similar to those whose results are plotted in Fig. 21. The ranges of its validity therefore are

$$2.7 < a/d < 4.5$$

$$6000 < f_{cu} < 8000 \text{ p.s.i.}$$

$$1.3'' < b_1 < 2''$$

b of the order of 4"

It is expected that at higher values of a/d the term $\frac{V_u}{bb_1 f_{cu}}$ should tend to some limiting value but the complete lack of data for beams with values of a/d over 4.5 and the scatter of results at values lower than this do not justify a more refined expression.

That the ultimate failure load appears to be dependent on the terms b, b_1 and a/d in addition to the concrete strength is attributed to the fact that it is these factors which basically determine the geometry of the cross-section of concrete above and below the diagonal crack and the direction and value of the applied thrust through the sections.

It is expected that the flange depth will also be an influencing factor since the geometrical properties of the cross-sections of the concrete resisting the thrust are also dependent upon its value. This factor was, however, sensibly constant for all beams whose results are plotted and therefore its influence cannot be established.

That the ultimate failure load decreases with an increase of the ratio of the shear span to the effective depth of the beam is to be expected. For the higher the value of a/d the less inclined is the direction of the thrust through the shear span. Since the vertical component of the thrust must always equal the applied shear the less inclined the direction of the thrust the greater will be its value. Thus the stresses acting in the shear span are higher and failure more likely.

Of the two instances of diagonal cracking in the beams which were more rigorously cured than others of the test specimens, in neither case was diagonal cracking followed by failure of the web in the same shear span as that in which the diagonal cracking occurred. Both failed by a failure of the other shear span following the formation of cracks at the junction of the web and the upper flange as described in section 14.2. No assessment can therefore be made of the influence of the method of curing on the failure of beams by crushing in the web following diagonal cracking.

It has already been noted that beams failing in the web following diagonal cracking can be expected to sustain little more load, if any, than the load required to cause the diagonal cracking. A comparison of the diagonal cracking loads and the failure loads of the 14 beams discussed in this section is made in Fig. 22. Two observations can immediately be made from this figure. Firstly, those beams which do sustain more load than that required for diagonal cracking to occur, show that the extra load which can be sustained is small in relation to the diagonal cracking load. Secondly, the higher the ratio of the shear span to the effective depth, the more likely is it that the ultimate load will be lower than the load causing diagonal cracking. In addition to the above observations, which are based on the results of 14 beams which failed subsequently to diagonal cracking it should be noted that in the case of many beams the residual load after diagonal cracking is too high to be sustained by the new structure, failure occurring almost instantaneously with diagonal cracking. It has been observed that this is more frequently so in the case of beams with a high ratio of shear span to effective depth, so confirming to some extent the conclusion drawn in Fig. 22.

Since failures of the web following diagonal cracking are frequently violent and since beams can be expected to sustain only a little, if any, more load than that causing diagonal cracking, it is suggested that unless some form of shear reinforcement is provided with the aim of preventing web failures the diagonal cracking load should be regarded as the limit of usefulness of the beam.

18. Discussion of other Types of Shear Failure.

The failure of a number of beams following the formation of small cracks at the junction of the web and the upper flange has been described in section 14. This type of failure occurred only in those beams which were specially designed to examine the influence of the method of curing on the failure of test specimens. It is not believed, however, that the method of curing is a major influencing factor in this type of failure for, with other factors sensibly constant, improvement in the method of curing produced no consistent trend in test results. Other than the method of curing used there were no significant variations between the beams. It should be noted, though, that all the beams failing in this manner were relatively highly prestressed, but since there was little variation of this factor between the beams the influence of the level of prestress on this type of failure could not be observed.

The various localised failures which constitute the mechanism of this type of failure may be expected to occur when the system of stresses and/or deformations in the appropriate parts of the shear span satisfy the failure criteria for the concrete. As has already been indicated in section 16, however, an accurate stress analysis in the shear span just before failure occurs is virtually impossible. An empirical analysis is also unwarranted because of the very limited amount of data available. These failures are therefore discussed in comparison with the proposed expressions for diagonal cracking and web failure following diagonal cracking, which phenomena show some similarities to those at present under discussion. Data for those of the author's beams showing this type of failure are given in Table 9. Data for those of Sozen's beams showing similar phenomena are not available.

Values of $\dfrac{480\ V_{c'}}{(\frac{44}{a/d} + 16)\ f_t\ bd\ (\frac{b'}{b})}$, where $V_{c'}$ is the shear acting when the inclined cracking subsequent to cracking at the junction of the web and the upper flange crosses the web, are plotted against $\dfrac{f_h}{f_t}$ in Fig. 23 and are compared with equation (27). It will be seen that equation (27) only slightly overestimates values of $V_{c'}$. It will also be noted, as stated earlier, that the method of curing has little influence on the test results.

Values of $\dfrac{V_{u'}}{bb'f_{cu}}$, where $V_{u'}$ is the shear acting at ultimate load, are plotted against a/d in Fig. 24 and are compared with equation (30). It will be noted that equation (30) predicts $V_{u'}$ reasonably well in these instances and that, again, the method of curing has little effect on the result.

Although failures of beams following the formation of small inclined cracks at the junction of the web and the flange are substantially different from failures of the web following diagonal cracking in their mechanisms, some limited conclusions may be drawn from Figs. 23 and 24. Fig. 23 shows that when the inclined cracking in the former instance crosses the web the load acting may be expected to be not substantially different from that load expected to cause diagonal cracking. Fig. 24 shows that failure following either type of cracking may be expected at similar loads. It is therefore suggested that equation (27) should be used to predict loads causing any substantial form of inclined or diagonal cracking occurring in the web except that originating at the bottom fibre of the beam, and equation (30) should be used to predict loads at failure subsequent to this type of cracking.

The Failure of Beam S25

The failure of beam S25, which occurred by a sudden and complete disintegration of one of its shear spans, has already been described in some detail in section 14. An examination of the fragments of the failed beam gave no indication of the mechanism of failure. It has therefore not been possible to use data from this beam in establishing any of the expressions presented. The failing load of this beam was, however, compared against the computed shear compression failing load and the computed diagonal cracking loads in order to determine whether the use of the expression offered for these computed loads would have satisfactorily safeguarded against such a failure as that which occurred. The comparison is made below

Test Failing Load R.H. Shear Span lbs	Computed Shear-Compression Failing Load lbs	Computed Diagonal Cracking Loads L.H. Shear Span lbs	Computed Diagonal Cracking Loads R.H. Shear Span lbs
23,800	24,800	20,400	21,400

It will be noted that the expression for the diagonal cracking loads produced slightly conservative values for the failing load. Failure in shear-compression may have been expected at a load slightly higher than that causing failure. Whilst no generalisations may be made from the evidence provided by one beam it is considered likely that in this one instance failure would not have occurred in the manner in which it did had the beam been designed against the occurrence of diagonal cracking and a shear-compression failure.

VI SUMMARY AND CONCLUSIONS

The object of this investigation was to provide more information of the behaviour of prestressed concrete beams without web reinforcement when subjected to shear. Studies of the following major variables were included

1) Amount of longitudinal reinforcement
2) Length of the shear span
3) Shape of cross-section of the beam
4) Type of curing

A total of 54 beams was tested. Five of the beams were rectangular in cross-section and the remainder were I-section beams of basically two types. All the beams were simply supported and were tested under symmetrical two-point loading. The tests have been described and the results have been presented graphically and in tables.

The observation of test specimens shows that shear failure of prestressed concrete beams may occur in a number of ways. The most frequently observed modes of failure were shear-compression failures and failures following diagonal cracking. A shear-compression failure occurs when the concrete in the compression zone above an inclined crack, which was initiated by a vertical tension crack at the bottom fibre of the beam within the shear span, crushes. It should be noted that this type of failure is often followed, almost instantaneously, by 'splitting' of the concrete at the level of the tensile reinforcement.

Frequently, during test, I-section beams exhibit diagonal cracking within the shear span initiated independently of the vertical cracks which may have formed at the bottom surface of the beam. In almost all such cases the cracking becomes the dominant feature of the failure mechanism. Failures following the formation of diagonal cracks are often violent and any additional load which the beam can carry before failure occurs is small. A number of less frequently occurring modes of failure are fully described in the text.

On the basis of 55 results of the author and a number of other workers an analysis for the strength in shear-compression of prestressed concrete beams is presented and is similar to the analysis normally used for strength in flexure. It is proposed that shear-compression failing moment of prestressed concrete beams can be represented by the following equation

$$M_u = Fbd^2 f'_c K k_u (1 - k_2 k_u) \quad \ldots\ldots\ldots\ldots (24)$$

where

$$K = 1.500 - 0.715 \times 10^{-4} f'_c \quad \ldots\ldots\ldots\ldots (22)$$

$$k_u = \frac{pt_u}{f'_c} (0.427 + 0.349 \times 10^{-3} pf_{se}) \quad \ldots\ldots\ldots (23)$$

$$F = 1.40 - 1.11 \frac{pt_u}{f'_c} \quad \ldots\ldots\ldots\ldots\ldots (25)$$

for rectangular beams

$$\text{and} \quad F = \left(1.40 - 1.11 \frac{pt_u}{f'_c}\right) \left(2.76 \frac{pt_u}{f'_c} - 0.19\right) \quad \ldots\ldots (26)$$

for I-beams similar to those whose results are used in the analysis.

Data from 40 beams, tested by Hosny[3] and the author were used to establish an expression capable of predicting the diagonal cracking load of prestressed concrete beams. The shear causing diagonal cracking in a shear span can be related to the pertinent variables by the following expression

$$V_c = \left(7.2 + 10.0\,\frac{f_h}{f_t}\right)\left(\frac{14}{a/d} + 16\right)\frac{f_t bd}{480}\left(\frac{b'}{b}\right) \quad . \; . \; . \; . \; . \; . \; . \; . \quad (27)$$

An expression for the ultimate failure load by crushing of the web following diagonal cracking is presented in the text and cracking and failure loads of other modes of failure are compared with the expressions already presented. It is concluded, however, that, in general, for prestressed concrete beams without shear reinforcement the lesser of the shear-compression failing load and the diagonal cracking load as given by equations (24) and (27) should mark the limit of usefulness of a beam.

It should be noted that the expressions given above are largely derived from observation of test specimens and therefore should not be regarded as valid for beams which are not comparable with those whose results are used in their establishment. In particular, before more widely valid expressions can be presented for cracking and failure loads of prestressed concrete beams, further research will have to be undertaken to determine more accurately the influence of the shape of cross-section of the beam.

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TABLE 1

PROPERTIES OF BEAMS

Mark	Flange Width L.H.	Flange Width R.H.	Web Thickness L.H.	Web Thickness R.H.	Overall Depth L.H.	Overall Depth R.H.	Effective Depth L.H.	Effective Depth R.H.	Type of Steel	Longit. Reinf.	Effective Prestressing Force
	b ins		b_1 ins		d_1 ins		d ins			p %	$F_{se} \times 10^{-3}$ lbs
S1	4.0	4.0	-	-	12.0	12.0	10.1	10.1	I	2.96	10.2
S2	4.0	4.0	-	-	12.0	12.0	9.9	9.9	I	2.01	13.1
S3	4.0	4.0	2.0	2.0	12.0	12.0	10.5	10.5	I	1.05	12.3
S4	4.0	4.0	2.0	2.0	12.0	12.0	10.5	10.5	I	2.36	12.3
S5	4.0	4.0	2.0	2.0	12.0	12.0	10.5	10.5	I	1.43	16.6
S6	4.0	4.0	2.0	2.0	12.0	12.0	10.5	10.5	I	0.47	12.2
S7	4.0	4.0	2.0	2.0	12.0	12.0	10.5	10.5	I	1.87	18.8
S8	4.0	4.0	2.0	2.0	12.0	12.0	10.1	10.1	I	1.49	29.2
S9	4.0	4.0	1.9	1.9	12.0	12.0	10.1	10.1	I	1.95	28.9
S10	4.1	4.1	1.3	1.4	12.0	12.0	10.0	10.0	I	1.92	27.6
S11	4.0	4.0	1.9	2.1	12.0	12.0	10.1	10.1	I	1.09	22.4
S12	4.1	4.1	1.4	1.4	12.0	12.0	10.1	10.1	I	1.45	22.5
S13	4.1	4.1	1.3	1.4	12.0	12.0	10.1	10.1	I	1.07	21.3
S14	4.0	4.0	2.0	2.0	12.0	12.0	10.1	10.1	I	1.95	21.8
S15	4.0	4.0	2.0	2.0	12.0	12.0	10.0	10.0	I	1.11	21.9

TABLE 1 (cont.)

Mark	Flange Width L.H.	Flange Width R.H.	Web Thickness L.H.	Web Thickness R.H.	Overall Depth L.H.	Overall Depth R.H.	Effective Depth L.H.	Effective Depth R.H.	Type of Steel	Longit. Reinf.	Effective Prestressing Force
	b ins		b_1 ins		d_1 ins		d ins			p %	$P_{se} \times 10^{-3}$ lbs
S16	4.0	4.0	2.0	2.0	12.0	12.0	10.0	10.0	I	1.50	22.1
S17	4.1	4.1	1.3	1.4	12.1	12.1	10.3	10.3	I	1.86	16.2
S18	4.1	4.1	1.3	1.4	12.0	12.0	10.1	10.1	I	1.45	20.7
S19	4.0	4.0	2.0	2.0	12.0	12.0	10.1	10.1	I	1.94	20.6
S20	4.1	4.1	1.3	1.4	12.0	12.0	10.0	10.0	I	1.08	23.3
S21	4.0	4.0	2.0	2.0	12.0	12.0	10.1	10.1	I	1.09	24.6
S22	4.1	4.1	1.3	1.4	12.0	12.0	10.3	10.3	I	1.86	19.5
S23	4.0	4.0	2.0	2.0	12.0	12.0	10.0	10.0	I	0.49	18.5
S24	4.1	4.1	1.3	1.4	12.0	12.0	10.1	10.1	I	1.07	19.9
S25	4.0	4.0	2.0	2.1	12.0	12.0	10.1	10.1	I	1.49	20.8
S26	4.1	4.1	1.3	1.4	11.9	12.0	9.8	10.0	I	1.49/1.46	20.0
S27	4.0	4.0	2.0	2.0	12.0	12.0	10.1	10.1	I	1.95	19.4

TABLE 1 (cont.)

Mark	Flange Width L.H.	Flange Width R.H.	Web Thickness L.H.	Web Thickness R.H.	Overall Depth L.H.	Overall Depth R.H.	Effective Depth L.H.	Effective Depth R.H.	Type of Steel	Longit. Reinf.	Effective Prestressing Force
	b ins		b_1 ins		d_1 ins		d ins			p %	$F_{se} \times 10^{-3}$ lbs
S28	4.2	4.2	1.4	1.3	12.0	12.0	10.0	10.0	I	1.47	13.8
S29	4.0	4.0	2.0	1.9	12.0	12.0	10.1	10.1	I	1.49	18.4
S30	4.0	4.0	2.0	2.0	12.0	12.0	10.1	10.1	I	1.09	23.7
S31	4.0	4.0	2.0	2.0	12.0	12.0	10.1	10.1	I	0.48	13.9
S32	4.1	4.1	1.5	1.4	12.0	12.0	10.0	10.0	I	1.08	18.3
S33	3.0	3.0	1.5	1.5	6.0	6.0	5.0	5.0	III	0.21	2.96
S34	4.0	4.0	1.2	1.1	12.0	12.0	10.1	10.1	I	1.49	18.3
S35	4.0	4.0	1.3	1.3	12.1	12.1	10.3	10.3	I	1.91	17.6
S36	4.0	4.0	1.2	1.2	12.1	12.1	10.3	10.3	I	0.47	14.5
S37	3.0	3.0	1.5	1.5	6.0	6.0	5.0	5.0	II	0.40	4.88
S38	3.0	3.0	1.5	1.5	6.0	6.0	5.0	5.0	III	0.61	7.45
S39	3.0	3.0	1.5	1.5	6.0	6.0	5.0	5.0	II	0.80	7.40
S40	3.5	3.5	2.0	2.0	6.0	6.0	5.0	5.0	I	1.12	10.4
S41	3.6	3.6	2.1	2.1	6.0	6.0	5.4	5.4	I	2.27	8.75
S42	3.6	3.6	2.1	2.2	6.0	6.0	5.0	5.0	I	3.33	12.4

TABLE 1 (cont.)

Mark	Flange Width L.H.	Flange Width R.H.	Web Thickness L.H.	Web Thickness R.H.	Overall Depth L.H.	Overall Depth R.H.	Effective Depth L.H.	Effective Depth R.H.	Type of Steel	Longit. Reinf.	Effective Prestressing Force
	b ins		b_1 ins		d_1 ins		d ins			p %	$F_{se} \times 10^{-3}$ lbs
S43	3.6	3.5	2.2	2.0	6.0	6.0	4.8	4.8	I	4.54/4.77	11.7
S44	4.3	4.2	1.2	1.2	12.2	12.2	10.4	10.4	II	0.27	11.3
S45	4.0	3.9	2.0	1.9	12.0	12.0	10.0	10.0	II	0.30/0.31	10.3
S46	3.1	3.1	-	-	6.0	6.0	4.3	4.3	II	0.90	11.8
S47	3.0	3.0	-	-	6.0	6.0	3.9	3.9	II	1.02	13.6
S48	3.1	3.1	-	-	6.0	6.0	4.0	4.0	II	0.96	14.1
S49	4.1	4.1	1.3	1.4	12.0	12.0	10.0	10.0	I	1.08	24.1
S50	4.3	4.3	1.4	1.3	11.8	11.7	9.4	9.2	I	1.09/1.12	24.2
S51	4.1	4.1	1.3	1.3	12.1	12.0	10.2	10.0	I	1.06/1.08	24.6
S52	4.1	4.0	1.2	1.1	12.2	12.2	10.3	10.4	I	1.05/1.06	22.7
S53	4.1	4.0	1.2	1.2	12.0	12.0	10.0	10.0	I	1.08/1.10	22.7
S54	4.1	4.1	1.2	1.2	12.0	12.0	10.0	10.0	I	1.08	24.0

TABLE 2

PROPERTIES OF CONCRETE MIXES

Mark	Cement:Sand:Gravel by weight	Water/Cement by weight	f_{cu} p.s.i.	f'_c p.s.i.	f_t p.s.i.	f_r p.s.i.	f'_t p.s.i.
S1	1.0:1.5:3.0	0.4	8150	7300	746	810	-
S2	1.0:1.5:3.0	0.5	7070	5630	714	648	-
S3	1.0:1.5:3.0	0.5	7280	5380	535	722	-
S4	1.0:1.5:3.0	0.5	7750	5060	654	778	-
S5	1.0:1.5:3.0	0.5	6550	5100	390	655	-
S6	1.0:1.5:3.0	0.5	7560	6530	665	740	-
S7	1.0:1.5:3.0	0.5	7350	4900	480	732	-
S8	1.0:1.5:3.0	0.5	5990	4150	613	545	-
S9	1.0:1.5:3.0	0.5	5840	4430	592	568	-
S10	1.0:1.5:3.0	0.5	5840	4430	592	568	-
S11	1.0:1.5:3.0	0.5	6620	4730	651	738	-
S12	1.0:1.5:3.0	0.5	6620	4730	651	738	-
S13	1.0:1.5:3.0	0.5	7040	5220	616	654	-
S14	1.0:1.5:3.0	0.5	7270	4390	668	623	-
S15	1.0:1.5:3.0	0.5	7700	4040	549	585	-
S16	1.0:1.5:3.0	0.5	6990	4930	585	665	-
S17	1.0:1.5:3.0	0.5	8050	5420	607	562	-
S18	1.0:1.5:3.0	0.5	6720	4350	409	570	-

TABLE 2 (cont.)

Mark	Cement:Sand:Gravel by weight	Water/Cement by weight	f_{cu} p.s.i.	f'_c p.s.i.	f_t p.s.i.	f_r p.s.i.	f'_t p.s.i.
S19	1.0:1.5:3.0	0.5	6720	4350	409	570	-
S20	1.0:1.5:3.0	0.5	6190	4080	525	512	-
S21	1.0:1.5:3.0	0.5	6190	4080	525	512	-
S22	1.0:1.5:3.0	0.5	5950	3480	392	510	-
S23	1.0:1.5:3.0	0.5	5950	3480	392	510	-
S24	1.0:1.5:3.0	0.5	6160	5350	464	691	-
S25	1.0:1.5:3.0	0.5	6160	5350	464	691	-
S26	1.0:1.5:3.0	0.5	6300	5150	454	610	-
S27	1.0:1.5:3.0	0.5	7280	5130	630	692	-
S28	1.0:1.5:3.0	0.5	6030	4140	372	576	-
S29	1.0:1.5:3.0	0.5	6030	4140	372	576	-
S30	1.0:1.5:3.0	0.5	7110	5100	634	603	-
S31	1.0:1.5:3.0	0.5	6120	4670	384	698	-
S32	1.0:1.5:3.0	0.5	6120	4670	384	698	-
S33	1.0:1.5:3.0	0.4	8800	6430	709	795	-
S34	1.0:1.5:3.0	0.5	7420	4910	680	680	-
S35	1.0:1.5:3.0	0.5	7460	5700	712	733	-
S36	1.0:1.5:3.0	0.5	6480	5180	412	628	-

TABLE 2 (cont.)

Mark	Cement:Sand:Gravel by weight	Water/Cement by weight	f_{cu} p.s.i.	f'_c p.s.i.	f_t p.s.i.	f_r p.s.i.	f'_t p.s.i.
S37	1.0:1.5:3.0	0.4	9250	6580	720	770	-
S38	1.0:1.5:3.0	0.4	8750	6420	685	725	554
S39	1.0:1.5:3.0	0.4	8400	6730	696	725	662
S40	1.0:1.5:3.0	0.4	10030	5980	616	690	515
S41	1.0:1.5:3.0	0.4	9140	6890	718	750	456
S42	1.0:1.5:3.0	0.4	9240	6330	516	610	376
S43	1.0:1.5:3.0	0.4	9560	7010	731	678	515
S44	1.0:1.5:3.0	0.5	6020	4710	494	600	583
S45	1.0:1.5:3.0	0.5	6550	4550	446	610	-
S46	1.0:1.5:3.0	0.5	7320	4790	656	656	495
S47	1.0:1.5:3.0	0.5	7320	4790	656	656	495
S48	1.0:1.5:3.0	0.5	7320	4790	656	656	495
S49	1.0:1.5:3.0	0.5	7070	5260	541	612	436
S50	1.0:1.5:3.0	0.5	7560	5740	715	690	535
S51	1.0:1.5:3.0	0.5	7280	5300	651	778	485
S52	1.0:1.5:3.0	0.5	7600	4910	611	828	515
S53	1.0:1.5:3.0	0.5	7280	5500	628	840	515
S54	1.0:1.5:3.0	0.5	6860	5260	607	723	465

TABLE 3

DETAILS OF METHOD OF LOADING, CRACKING AND FAILING LOADS

Mark	Shear Span a ins	Flexural Span a_1 ins	Mode of Failure (see end of table)	Diagonal Cracking load Shear Span L.H. $P_o \times 10^{-3}$ lbs	Diagonal Cracking load Shear Span R.H.	Ultimate loads Shear Span L.H. $P_u \times 10^{-3}$ lbs	Ultimate loads Shear Span R.H.
S1	28.0	16.5	F	-	-	42.4	
S2	24.0	24.0	S-C	-	-	47.5	
S3	42.5	24.0	S-C	-	-		21.1
S4	42.5	24.0	S-C	-	-		29.5
S5	42.5	30.0	S-C	-	-	26.0	
S6	42.5	30.0	F	-	-	13.1	
S7	36.0	36.0	S-C	-	-		33.9
S8	55.0	55.0	S-C	-	-	16.4	
S9	50.8	64.5	S-C	-	-	22.0	
S10	55.0	55.0	D.C.	15.4	-	15.4	
S11	55.0	55.0	S-C	-	-	17.2	
S12	55.0	55.0	D.C.	15.2	-	15.2	
S13	55.0	55.0	D.C.	16.9	-	16.9	
S14	45.0	45.0	D.C.	22.8	-	22.8	
S15	45.0	45.0	S-C	-	-		19.3

TABLE 3 (cont.)

Mark	Shear Span a ins	Flexural Span a_1 ins	Mode of Failure (see end of table)	Diagonal Cracking load Shear Span L.H. $P_c \times 10^{-3}$ lbs	Diagonal Cracking load Shear Span R.H.	Ultimate loads Shear Span L.H. $P_u \times 10^{-3}$ lbs	Ultimate loads Shear Span R.H.
S16	45.0	45.0	S-C				21.5
S17	45.0	45.0	D.C./W.C.+	15.0		12.4	
S18	45.0	45.0	D.C./W.C.+	14.8		13.1	
S19	37.0	37.0	D.C./W.C.+	22.2		21.4	
S20	45.0	45.0	D.C./W.C.+	15.0		13.8	
S21	37.0	37.0	S-C				24.0
S22	37.0	37.0	D.C./W.C.+	15.0*	17.0		12.5
S23	37.0	37.0	F			12.8	
S24	37.0	37.0	D.C./W.C.+	14.9*	16.3		15.9
S25	37.0	37.0	See text				23.8
S26	37.0	37.0	D.C./W.C.+	15.3*	16.7		13.9
S27	28.0	28.0	D.C./W.C.	22.7		29.8	
S28	37.0	39.0	F		12.5*	14.2	
S29	28.0	28.0	D.C./S-C	19.8	16.8	29.0	
S30	28.0	28.0	D.C./S-C+	27.7*	35.4		33.3

TABLE 3 (cont.)

Mark	Shear Span a ins	Flexural Span a_1 ins	Mode of Failure (see end of table)	Diagonal Cracking load Shear Span L.H. $P_c \times 10^{-3}$ lbs	Diagonal Cracking load Shear Span R.H. $P_c \times 10^{-3}$ lbs	Ultimate loads Shear Span L.H. $P_u \times 10^{-3}$ lbs	Ultimate loads Shear Span R.H. $P_u \times 10^{-3}$ lbs
S31	28.0	28.0	F		—	18.8	
S32	28.0	28.0	D.C./W.C.	15.8	16.3		18.2
S33	20.0	20.0	F			3.4	
S34	28.0	28.0	D.C./W.C.	15.0*	16.9		21.8
S35	28.0	28.0	D.C./W.C.	15.7*	16.3		26.6
S36	28.0	28.0	D.C./W.C.	13.4	13.4*		13.4
S37	20.0	20.0	F			5.9	
S38	20.0	20.0	F			8.3	
S39	20.0	20.0	F			8.6	
S40	20.0	20.0	F			9.5	
S41	20.0	20.0	F			14.1	
S42	20.0	20.0	S-C		13.3	14.1	
S43	20.0	20.0	S-C			13.0	
S44	28.0	28.0	D.C./W.C.	13.7	13.1		13.7
S45	20.0	20.0	F			17.6	

TABLE 3 (cont.)

Mark	Shear Span a ins	Flexural Span a_1 ins	Mode of Failure (see end of table)	Diagonal Cracking Load Shear Span L.H. $P_c \times 10^{-3}$ lbs	 R.H.	Ultimate loads Shear Span L.H. $P_u \times 10^{-3}$ lbs	 R.H.
S46	20.0	20.0	S-C			7.9	
S47	20.0	20.0	F				7.1
S48	20.0	20.0	S-C			8.4	
S49	28.0	28.0	X	15.7°		16.1	
S50	28.0	28.0	D.C./W.C.	15.8	15.7°	20.1	
S51	28.0	28.0	X		19.2°		16.8
S52	28.0	28.0	X	17.6	17.0°		19.4
S53	28.0	28.0	X	18.1°	17.0	16.8	
S54	28.0	28.0	X	16.5°		14.3	

Notes

* indicates that the shear span was 'strapped' after diagonal cracking occurred.

+ denotes that the ultimate failing load was less than the diagonal cracking load.

° the values referred to are those loads at which the cracking originating at the junction of the upper flange and the web extended across the web.

Failure Modes

F Flexural failure

S-C	Shear-Compression failure
D.C.	Failure occurring simultaneously with diagonal cracking.
D.C./W.C.	Failure by web-crushing following diagonal cracking.
D.C./S-C	Failure by crushing of the compression zone above a diagonal crack.
X	Failure of the web following the formation of cracks at the junction of the web and the upper flange.

TABLE 4

DATA RELATING TO SHEAR COMPRESSION FAILURES OF RECTANGULAR BEAMS

Mark	b ins	d ins	f'_c p.s.i.	$\frac{pt_u}{f'_c}$	pf_{se} p.s.i.	Derived from Test Results K	k_u	$M_u \times 10^{-3}$ lbs ins
Beams tested by the author								
S2	4.0	9.90	5630	0.535	331	1.03	0.283	570
S46	3.1	4.30	4790	0.413	885	0.97	0.349	79
S48	3.1	4.00	4790	0.441	1138	1.26	0.326	84
Beams tested by Zwoyer								
S1	6.0	9.16	3620	0.212	32	1.19	0.109	225
S3	6.0	9.38	4290	0.175	75	1.24	0.094	253
S5	6.0	8.30	6260	0.338	915	0.99	0.241	559
S6	6.0	8.20	7990	0.268	982	0.90	0.244	594
S7	6.0	8.44	3550	0.587	905	1.19	0.329	505
S8	6.13	8.20	6120	0.297	38	1.17	0.113	333
S9	6.0	8.20	4760	0.390	439	1.40	0.162	413
S10	6.0	8.20	5790	0.321	557	1.07	0.204	475
S11	6.0	8.80	2580	0.309	I20	1.24	0.138	193

TABLE 4 (cont.)

Mark	b ins	d ins	f'_c p.s.i.	$\frac{pt_u}{f'_c}$	pf_{se} p.s.i.	Derived from Test Results: K	k_u	$M_u \times 10^{-3}$ lbs ins
Beams tested by Zwoyer (cont.)								
S12	6.0	8.20	4760	0.390	235	1.16	0.180	376
S14	6.0	8.20	4660	0.368	940	1.10	0.276	503
S15	6.0	8.35	2890	0.291	277	1.41	0.169	265
S16	6.0	8.06	3530	0.299	326	1.31	0.160	268
S18	6.0	8.31	4150	0.272	277	1.14	0.132	248
S19	6.0	8.38	3850	0.218	210	1.04	0.158	249
S20	6.0	8.45	5350	0.156	188	1.30	0.091	260
S22*	6.0	8.12	5630	0.422	566	1.27	0.172	463
S23*	6.0	8.02	4360	0.441	966	1.02	0.342	503
S24*	6.0	8.41	2900	0.790	1071	1.15	0.444	507
S25*	6.0	8.44	2900	0.420	560	1.22	0.282	374
S26*	6.0	8.95	3130	0.321	238	1.26	0.189	293
S27*	6.0	8.45	3350	0.228	187	1.76	0.089	216
S28	6.0	8.80	3470	0.192	163	1.45	0.114	253
S29	6.0	8.53	3320	0.454	711	0.98	0.359	435

TABLE 4 (cont.)

Mark	b ins	d ins	f'_c p.s.i.	$\frac{pf_u}{f'_c}$	pf_{se} p.s.i.	Derived from Test Results: K	k_u	$M_u \times 10^{-3}$ lbs ins
Beams tested by Zwoyer (cont.)								
S30	6.0	8.50	3350	0.362	576	1.19	0.251	384
S31	6.0	8.42	2440	0.574	655	1.14	0.356	358
S32	6.0	8.35	3350	0.322	509	1.36	0.199	347
S33	6.0	8.60	3400	0.448	653	1.11	0.291	435
S34	6.0	8.64	5800	0.256	683	0.83	0.246	478

* these beams were loaded centrally.

TABLE 5

DATA RELATING TO SHEAR COMPRESSION FAILURES OF RECTANGULAR BEAMS

Mark	b ins	d ins	f'_c p.s.i.	$\frac{pt_u}{f'_c}$	pf_{se} p.s.i.	Computed K	Computed k_u	$M_u \times 10^{-3}$ lbs ins
Beams tested by Hosny								
H2	4.0	10.0	5100	0.721	263	1.14	0.374	407
H3	4.0	10.1	5010	0.880	287	1.14	0.464	393
H4	4.0	10.1	5175	0.539	359	0.92	0.298	539
Beams tested by Sozen								
A.12.23	6.1	9.33	5650	0.192	499	1.10	0.115	492
A.12.36	6.1	9.19	3440	0.294	471	1.25	0.174	396
A.12.56	6.0	8.59	3790	0.473	447	1.23	0.342	483
A.12.69	6.1	8.12	2950	0.584	801	1.29	0.413	451
A.12.81	6.0	8.66	2600	0.689	833	1.31	0.495	423
A.22.28	6.1	8.75	34[illegible]0	0.226	161	1.25	0.109	240
A.32.19	6.1	9.03	4990	0.151	0	1.14	0.064	212
A.32.27	6.0	9.16	2800	0.274	33	1.30	0.120	233

TABLE 6

DATA RELATING TO SHEAR COMPRESSION FAILURE OF I-BEAMS

Mark	b ins	d ins	k_d	$f'c$ p.s.i.	$\frac{pt_u}{f'_c}$	pf_{se} p.s.i.	Computed [illegible]	Computed k_u	$M_u \times 10^{-3}$
S3	4.0	10.5	0.286	5380	0.293	293	1.12	0.155	254
S4	4.0	10.5	0.286	5060	0.680	293	1.14	0.360	708
S5	4.0	10.5	0.286	5100	0.409	395	1.14	0.231	390
S7	4.0	10.5	0.286	4900	0.557	448	1.15	0.325	610
S8	4.0	10.1	0.297	4150	0.524	793	1.20	0.369	451
S9	4.0	10.1	0.297	4430	0.643	715	1.18	0.435	558
S11	4.0	10.1	0.297	4730	0.336	554	1.16	0.208	473
S15	4.0	10.0	0.300	4040	0.401	548	1.21	0.248	434
S16	4.0	10.0	0.300	4930	0.444	553	1.15	0.275	4[illegible]4
S21	4.0	10.1	0.297	4080	0.390	609	1.21	0.249	444
S42	3.6	5.0	0.300	6330	0.768	689	1.05	0.513	141
S43	3.6	4.8	0.313	7010	0.960	687	1.00	0.640	130

TABLE 7

DATA RELATING TO DIAGONAL CRACKS

Mark	Shear Span	$V_c \times 10^{-3}$ lbs	f_h p.s.i.	a/d	$\frac{b_1}{b}$	b ins	d ins	f'_t p.s.i.
			Beams tested by the author					
S10	L.H.	7.70	886	5.50	0.32	4.1	10.0	592
S12	L.H.	7.60	708	5.45	0.34	4.1	10.1	651
S13	L.H.	8.45	674	5.45	0.32	4.1	10.1	616
S14	L.H.	11.4	628	4.45	0.50	4.0	10.1	688
S17	L.H.	7.50	576	4.37	0.32	4.1	10.3	607
S18	L.H.	7.40	664	4.45	0.32	4.1	10.1	409
S19	L.H.	11.1	594	3.66	0.50	4.0	10.1	409
S20	L.H.	7.50	738	4.50	0.32	4.1	10.0	525
S22	L.H.	7.50	626	3.59	0.32	4.1	10.3	392
S22	R.H.	8.50	614	3.59	0.34	4.1	10.3	392
S24	L.H.	7.45	630	3.66	0.32	4.1	10.1	464
S24	R.H.	8.15	618	3.66	0.34	4.1	10.1	464
S26	L.H.	7.65	642	3.78	0.32	4.1	9.8	454
S26	R.H.	8.35	630	3.78	0.34	4.1	10.0	454
S27	L.H.	11.4	560	2.77	0.50	4.0	10.1	630

TABLE 7 (cont.)

Mark	Shear Span	$V_c \times 10^{-3}$ lbs	f_n p.s.i.	a/d	$\frac{b_1}{b}$	b ins	d ins	f_t p.s.i.
Beams tested by the author(cont.)								
S28	R.H.	6.25	424	3.70	0.33	4.2	10.0	372
S29	L.H.	9.90	530	2.77	0.50	4.0	10.1	372
S29	R.H.	8.40	540	2.77	0.48	4.0	10.1	372
S30	L.H.	13.8	676	2.77	0.50	4.0	10.1	634
S30	R.H.	17.7	676	2.77	0.50	4.0	10.1	634
S32	L.H.	7.90	580	2.80	0.32	4.1	10.0	384
S32	R.H.	8.15	570	2.80	0.34	4.1	10.0	384
S34	L.H.	7.50	610	2.77	0.30	4.0	10.1	680
S34	R.H.	8.45	624	2.77	0.28	4.0	10.1	680
S35	L.H.	7.85	568	2.72	0.33	4.0	10.3	712
S35	R.H.	8.15	568	2.72	0.33	4.0	10.3	712
S36	L.H.	6.70	466	2.72	0.30	4.0	10.3	442
S36	R.H.	6.70	466	2.72	0.30	4.0	10.3	442
S42	R.H.	6.65	756	4.00	0.58	3.6	5.0	516
S44	L.H.	6.85	338	2.69	0.28	4.3	10.4	494
S44	R.H.	6.55	346	2.69	0.29	4.2	10.4	494
S50	L.H.	7.90	726	2.98	0.33	4.3	9.4	715
S50	R.H.	7.85	738	2.98	0.30	4.3	9.2	715

TABLE 7 (cont.)

Mark	Shear Span	$V_c \times 10^{-3}$ lbs	f_b p.s.i.	a/d	$\frac{b'}{b}$	b ins	d ins	f_t p.s.i.
			Beams tested by the author (cont.)					
S52	L.H.	8.80	732	2.72	0.29	4.1	10.3	611
S53	R.H.	8.50	746	2.80	0.30	4.0	10.0	628
			Beams tested by Hosny					
H5		16.0	339	2.80	.50	4.0	10.0	635
H6		14.5	310	2.77	.48	4.0	10.1	600
H8		13.5	376	2.77	.47	3.95	10.1	605
H9		14.3	456	2.77	.47	3.95	10.1	610
H10		13.4	502	2.80	.47	3.95	10.0	605
H11		16.8	769	2.77	.47	3.95	10.1	657
H12		15.3	671	2.80	.47	3.95	10.0	590
H13		15.3	862	2.80	.47	3.95	10.0	590
H14		14.9	753	2.77	.47	3.95	10.1	565
H15		11.9	404	2.40	.47	3.95	10.0	487
H16		15.7	826	2.37	.47	3.95	10.1	585
H17		15.5	871	2.37	.47	3.95	10.1	620
H18		9.2	432	3.20	.47	3.95	10.0	541
H19		12.8	963	3.17	.47	3.95	10.1	600
H21		9.5	450	3.60	.47	3.95	10.0	550
H22		12.3	448	3.56	.47	3.95	10.1	619

TABLE 8

DATA RELATING TO WEB FAILURE FOLLOWING DIAGONAL CRACKING

Mark	Shear Span	f_{cu} p.s.i.	b ins	b_1 ins	a/d	$V_u \times 10^{-3}$ lbs	$V_c \times 10^{-3}$ lbs
S17	L.H.	8050	4.1	1.3	4.37	6.20	7.50
S18	L.H.	6720	4.1	1.3	4.45	6.55	7.40
S19	L.H.	6720	4.0	2.0	3.66	10.7	11.1
S20	L.H.	6190	4.1	1.3	4.50	6.90	7.50
S22	R.H.	5950	4.1	1.4	3.59	6.25	8.50
S24	R.H.	6160	4.1	1.4	3.66	7.95	8.15
S26	R.H.	6300	4.1	1.4	3.78	6.95	8.35
S27	L.H.	7280	4.0	2.0	2.77	14.9	11.4
S32	R.H.	6120	4.1	1.4	2.80	9.10	8.15
S34	R.H.	7420	4.0	1.1	2.77	10.9	8.45
S35	R.H.	7460	4.0	1.3	2.72	13.3	8.15
S36	R.H.	6480	4.0	1.2	2.72	6.70	6.70
S44	R.H.	6020	4.2	1.2	2.69	6.85	6.55
S50	L.H.	7560	4.3	1.4	2.98	10.1	7.90

TABLE 9

DATA RELATING TO BEAMS SHOWING CRACKING AT THE JUNCTION OF THE UPPER FLANGE AND THE WEB

Mark	Shear Span	f_{cu} p.s.i.	f_t p.s.i.	b ins	b_1 ins	d ins	a/d	f_h p.s.i.	$V_{c_1} \times 10^{-3}$ lbs	$V_{u_1} \times 10^{-3}$ lbs
S49	L.H.	7070	541	4.1	1.3	10.0	2.80	734	7.85	8.05
S51	R.H.	7280	651	4.1	1.3	10.0	2.80	772	9.60	8.40
S52	R.H.	7600	611	4.0	1.1	10.4	2.70	762	8.50	9.70
S53	L.H.	7280	628	4.1	1.2	10.0	2.80	732	9.05	8.40
S54	L.H.	6860	607	4.1	1.2	10.0	2.80	774	8.25	7.15

Plate 1 Strapped shear span

Plate 2 Shear-compression failure

Plate 3 Shear-compression failure

Plate 4 Cracking at the level of the reinforcement following the formation of the inclined crack

Plate 5 "Splitting" following a shear-compression failure

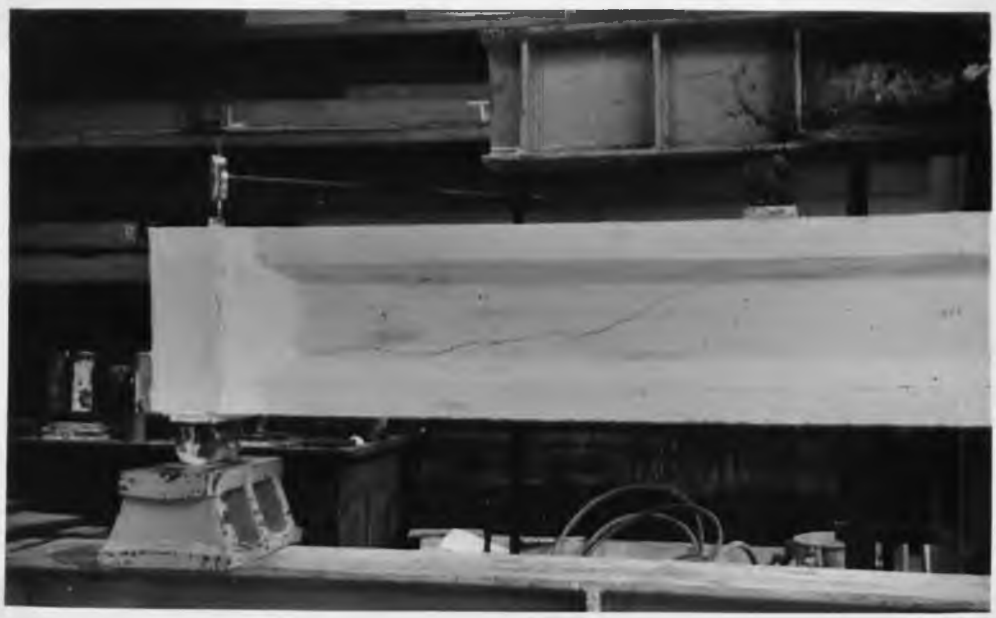


Plate 6 Diagonal cracking

Plate 7 Diagonal cracking

Plate 8 'Minor' cracking

Plate 9 Failure of the compression zone above a diagonal crack

Plate 10 Vertical cracking in the lower flange following diagonal cracking

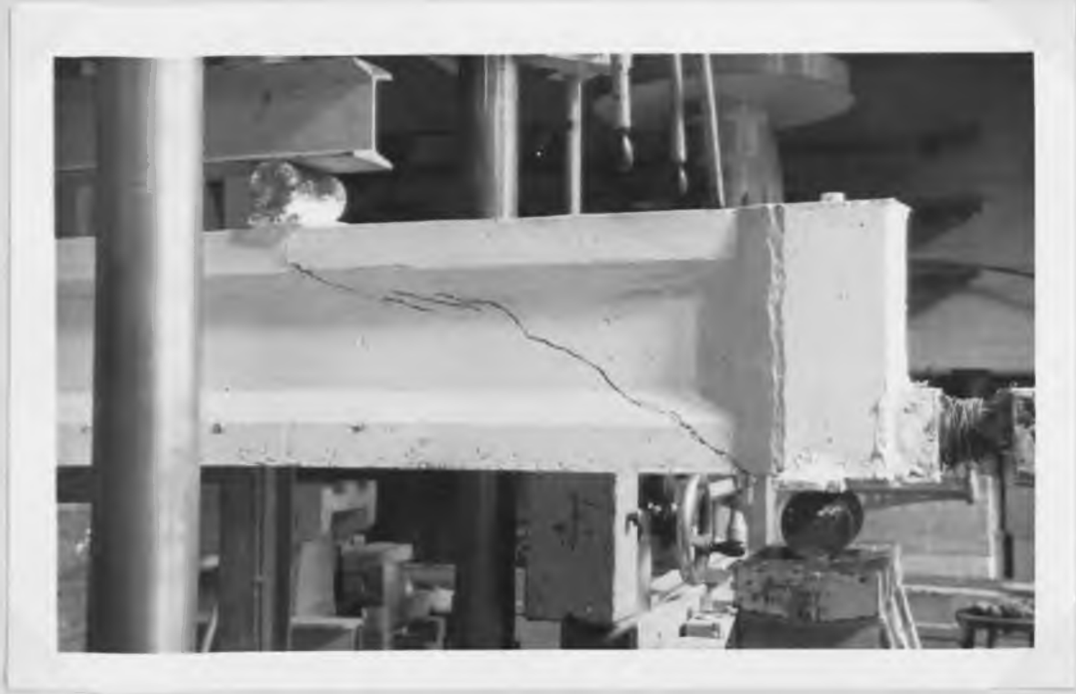


Plate 11 Extended diagonal cracking

Plate 12 Vertical cracking in the upper flange following diagonal cracking

Plate 13 Failure by crushing of concrete within the web

Plate 14 Failure by crushing of concrete within the web

Plate 15 Failure by crushing of concrete within the web

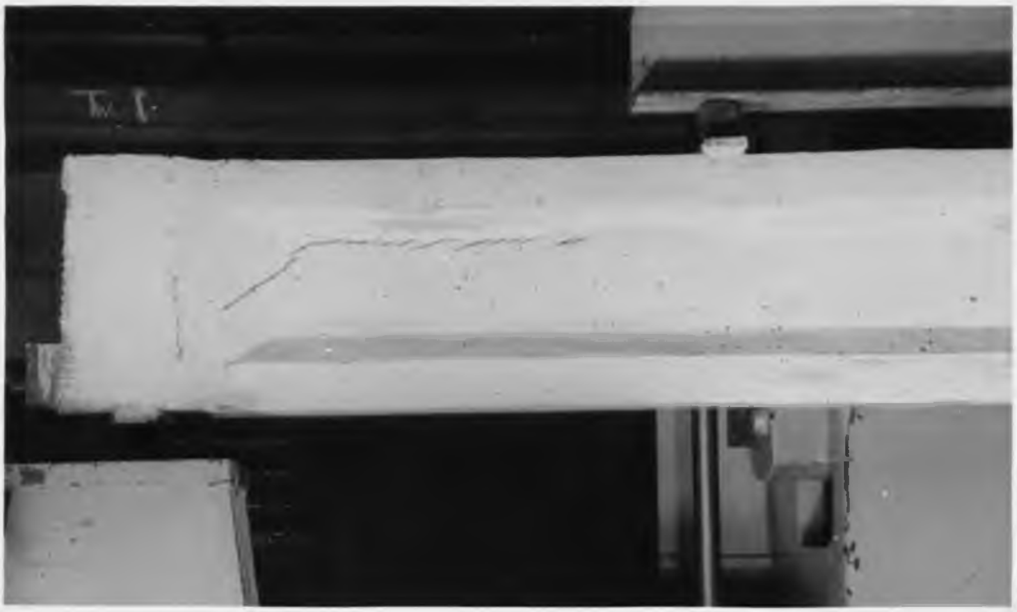


Plate 16 Cracking at the junction of the web and the upper flange

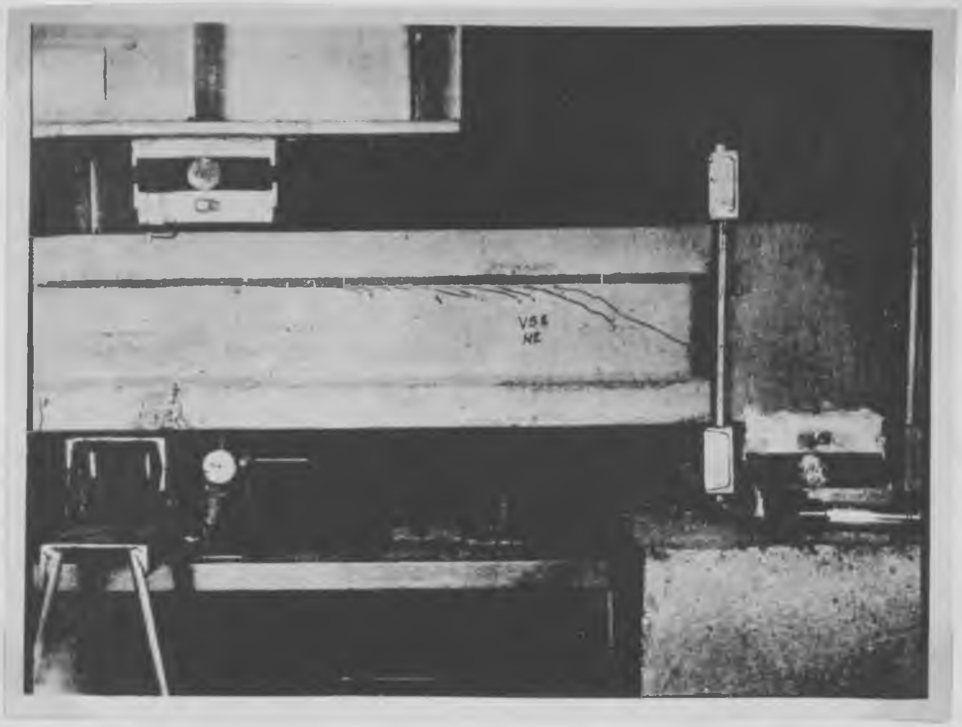


Plate 17 Cracking at the junction of the web and the upper flange in one of the beams tested by Sozen

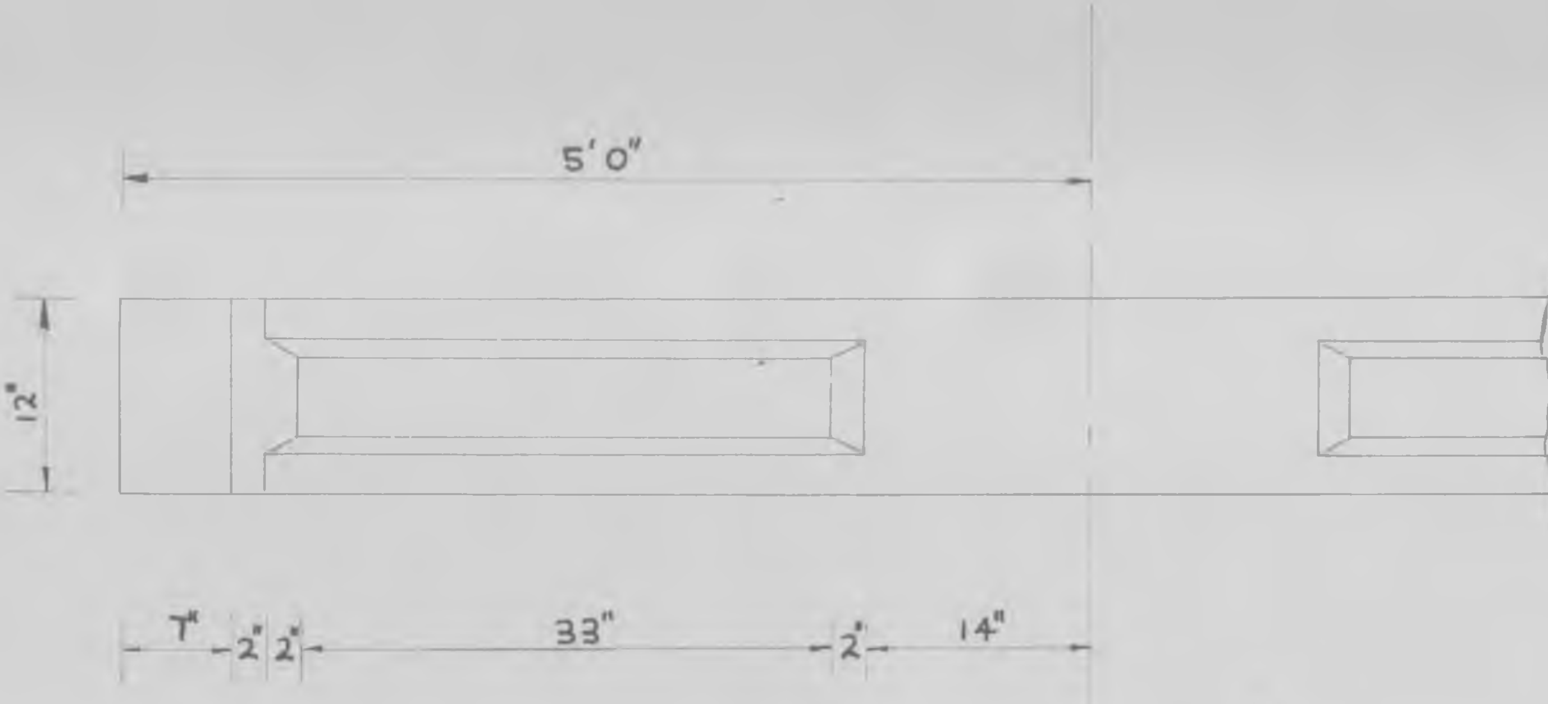


FIG. 1 DETAILS OF BEAMS S3-7

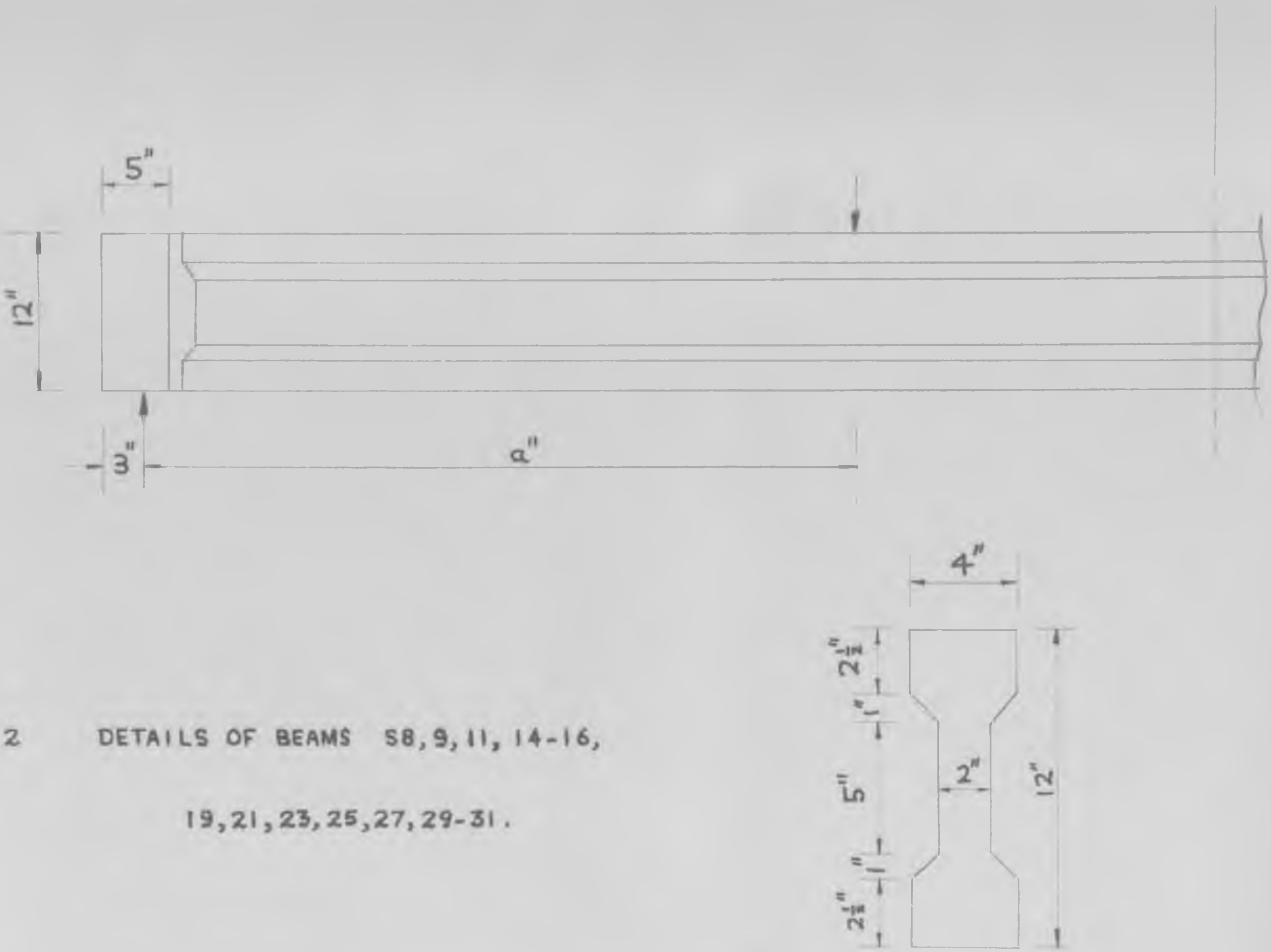


FIG. 2 DETAILS OF BEAMS S8, 9, 11, 14-16,
19, 21, 23, 25, 27, 29-31.

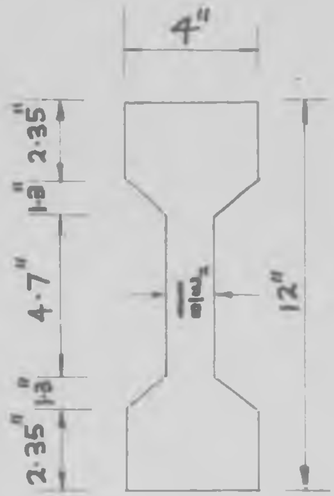
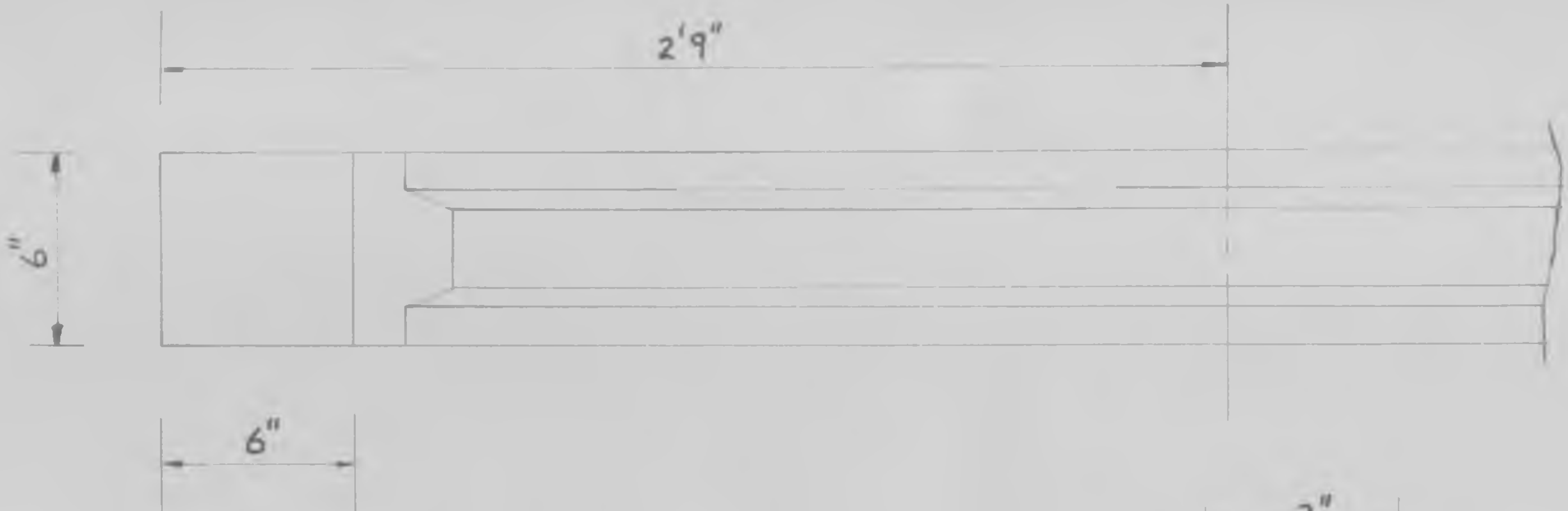


FIG. 3 DETAILS OF BEAMS S10, 12, 13, 17, 18, 20,

22, 24, 26, 28, 32, 34-36, 44, 45, 49-54.



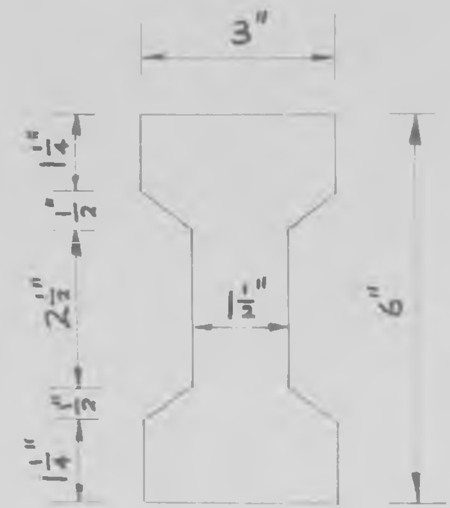
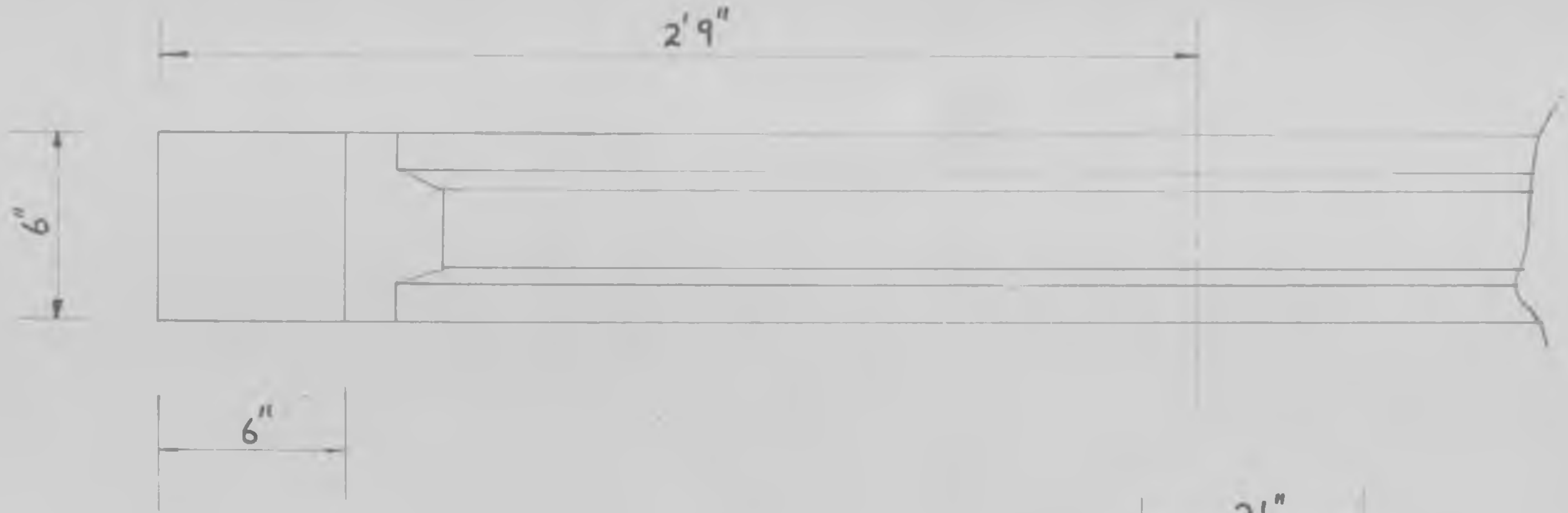


FIG. 4 DETAILS OF BEAMS S33, 37-39.



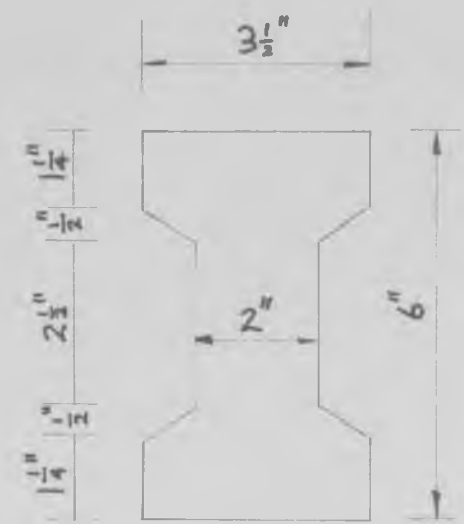


FIG. 5 DETAILS OF BEAMS S40-43.

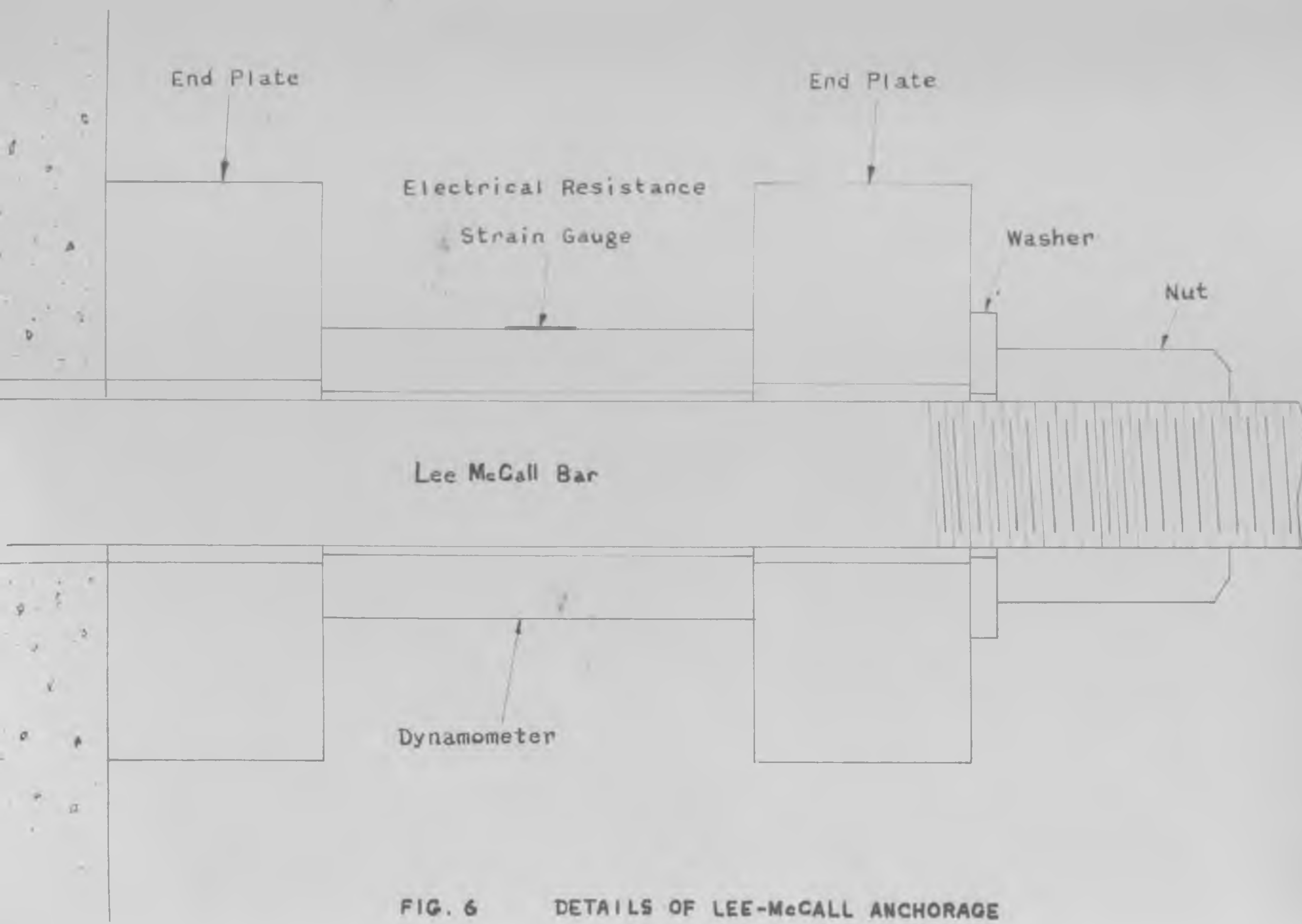


FIG. 6 DETAILS OF LEE-McCALL ANCHORAGE

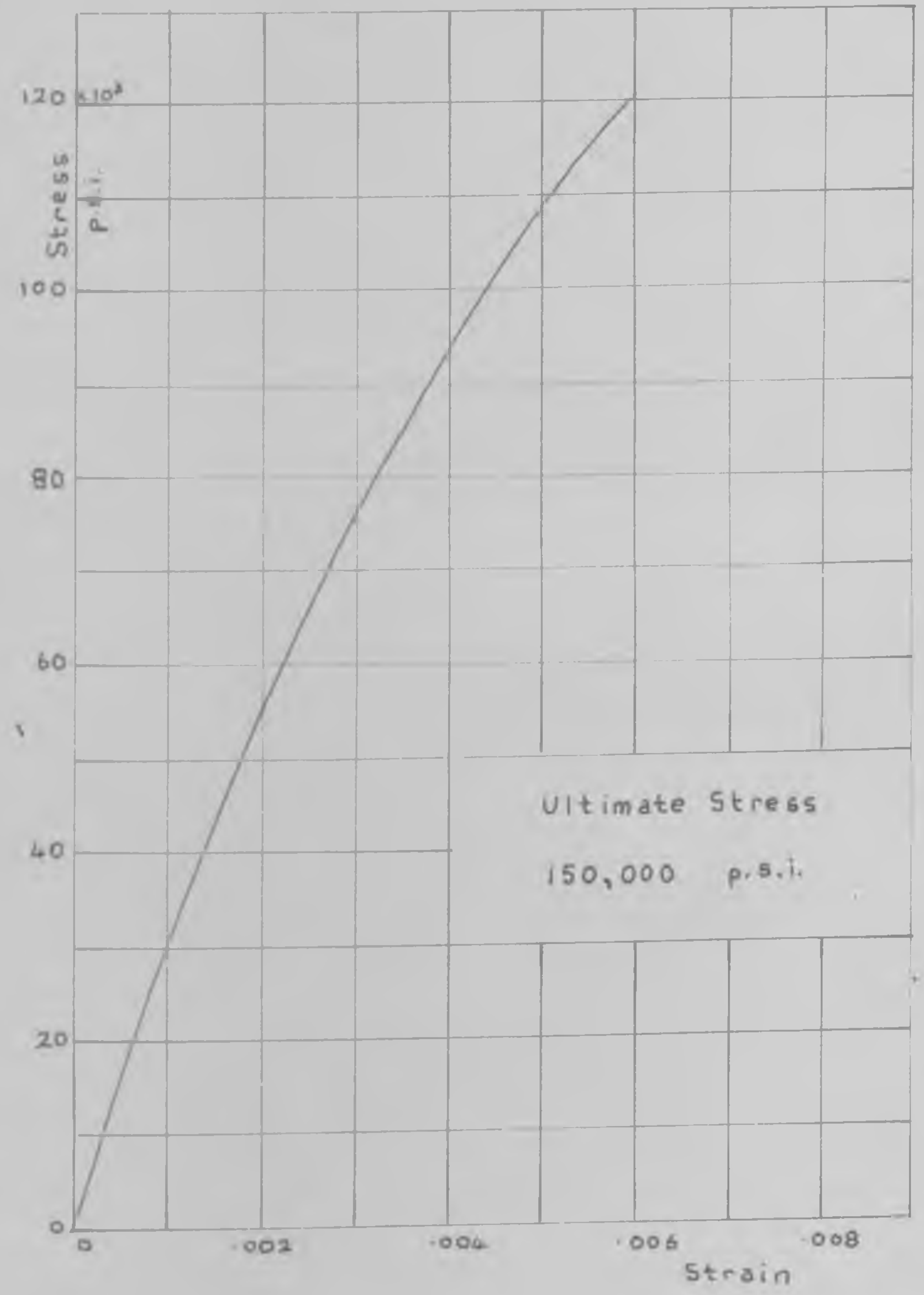


FIG. 7 STRESS/STRAIN RELATIONSHIP FOR TYPE I STEEL

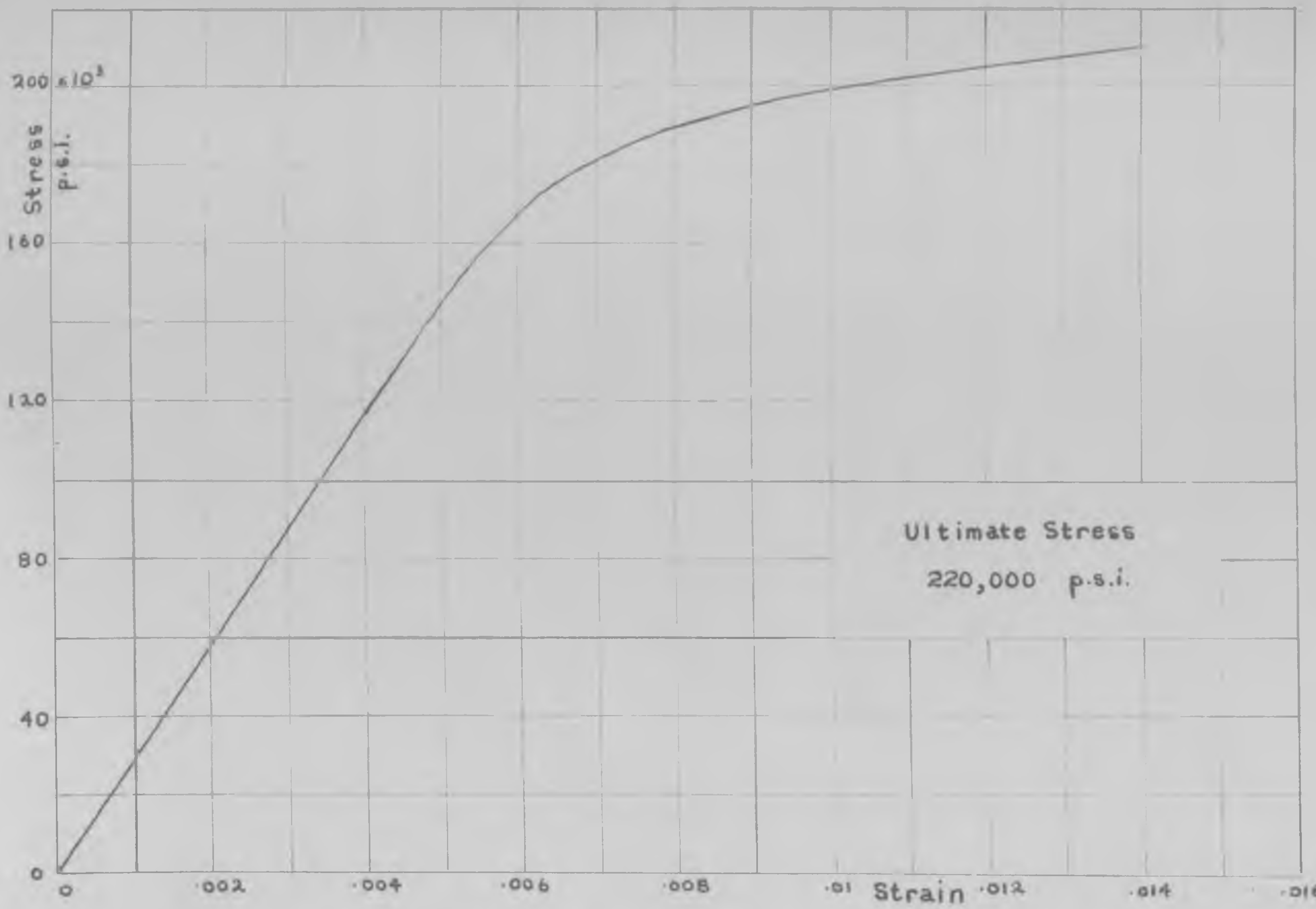


FIG. 8 STRESS/STRAIN RELATIONSHIP FOR TYPE II STEEL

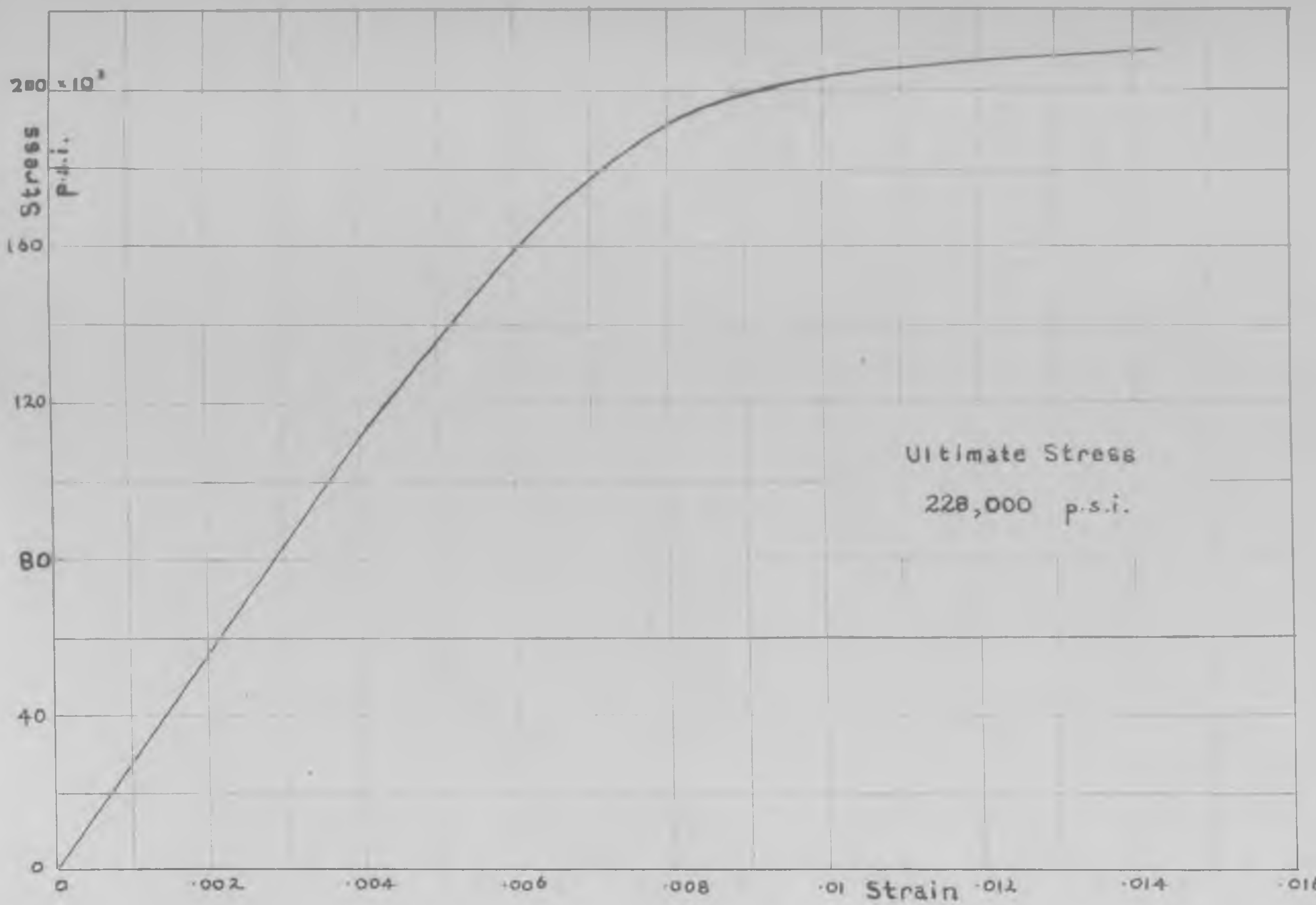


FIG. 9 STRESS/STRAIN RELATIONSHIP FOR TYPE III STEEL

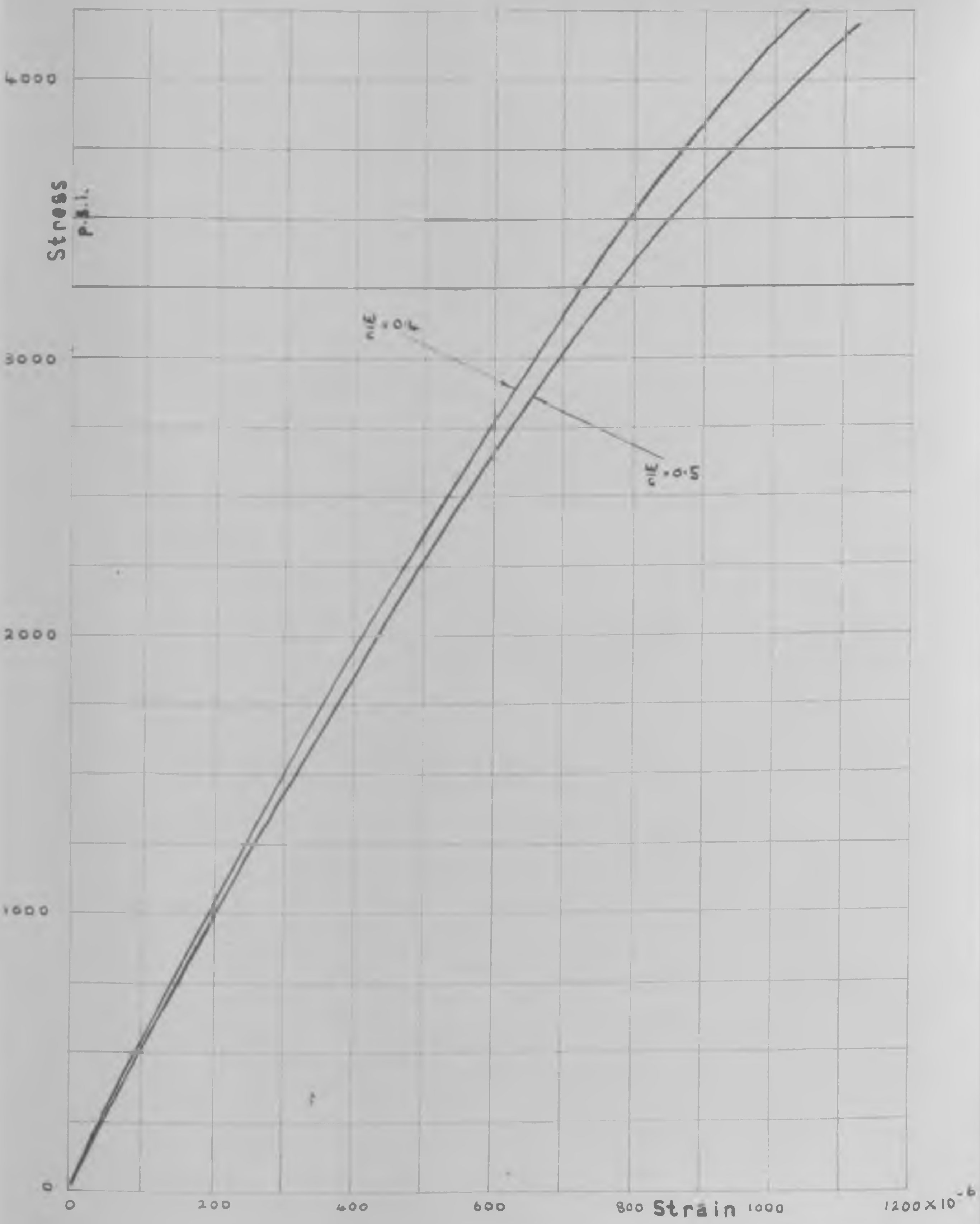


FIG. 10 STRESS/STRAIN RELATIONSHIP FOR THE CONCRETE

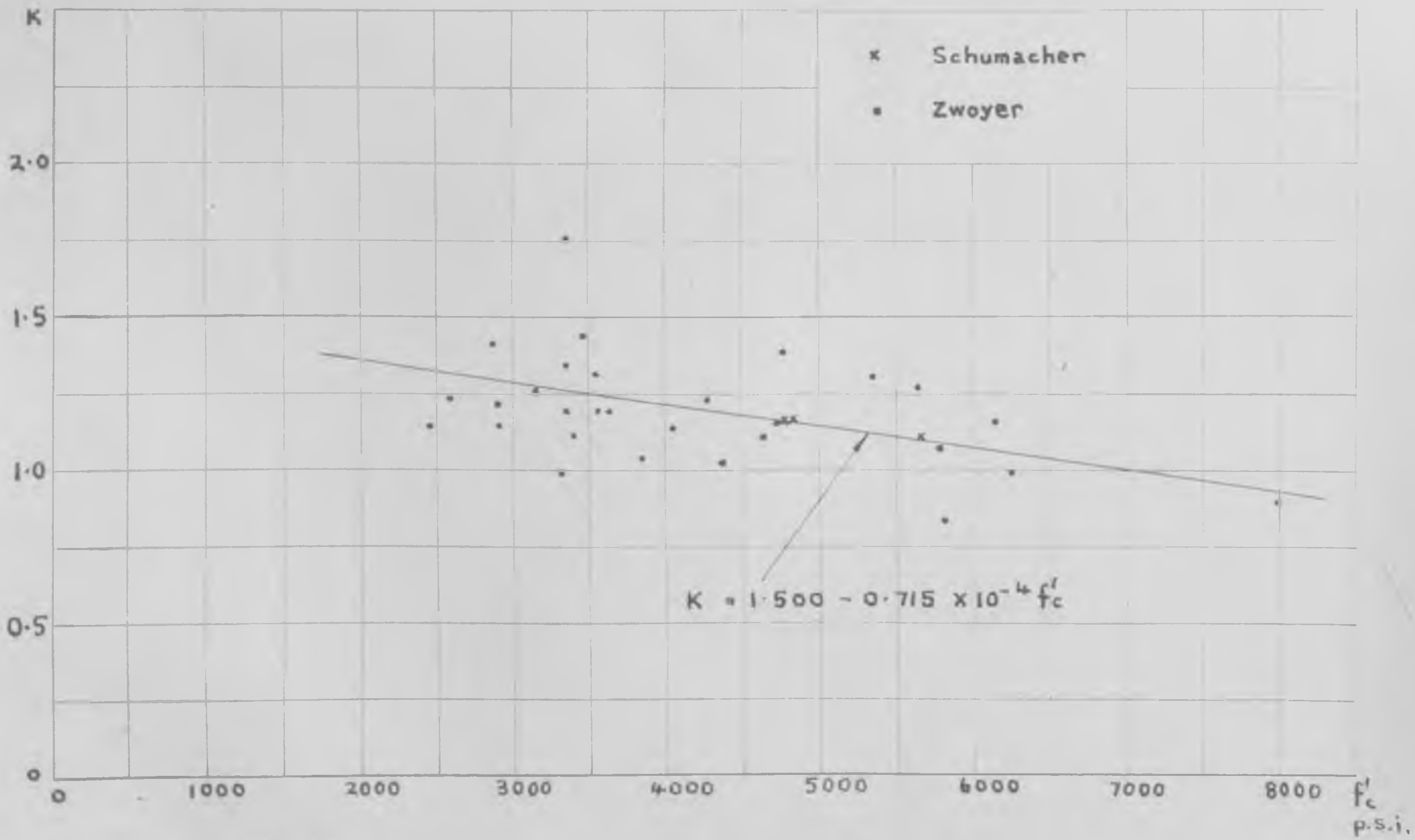


FIG. 11 COMPARISON OF K WITH CONCRETE STRENGTH

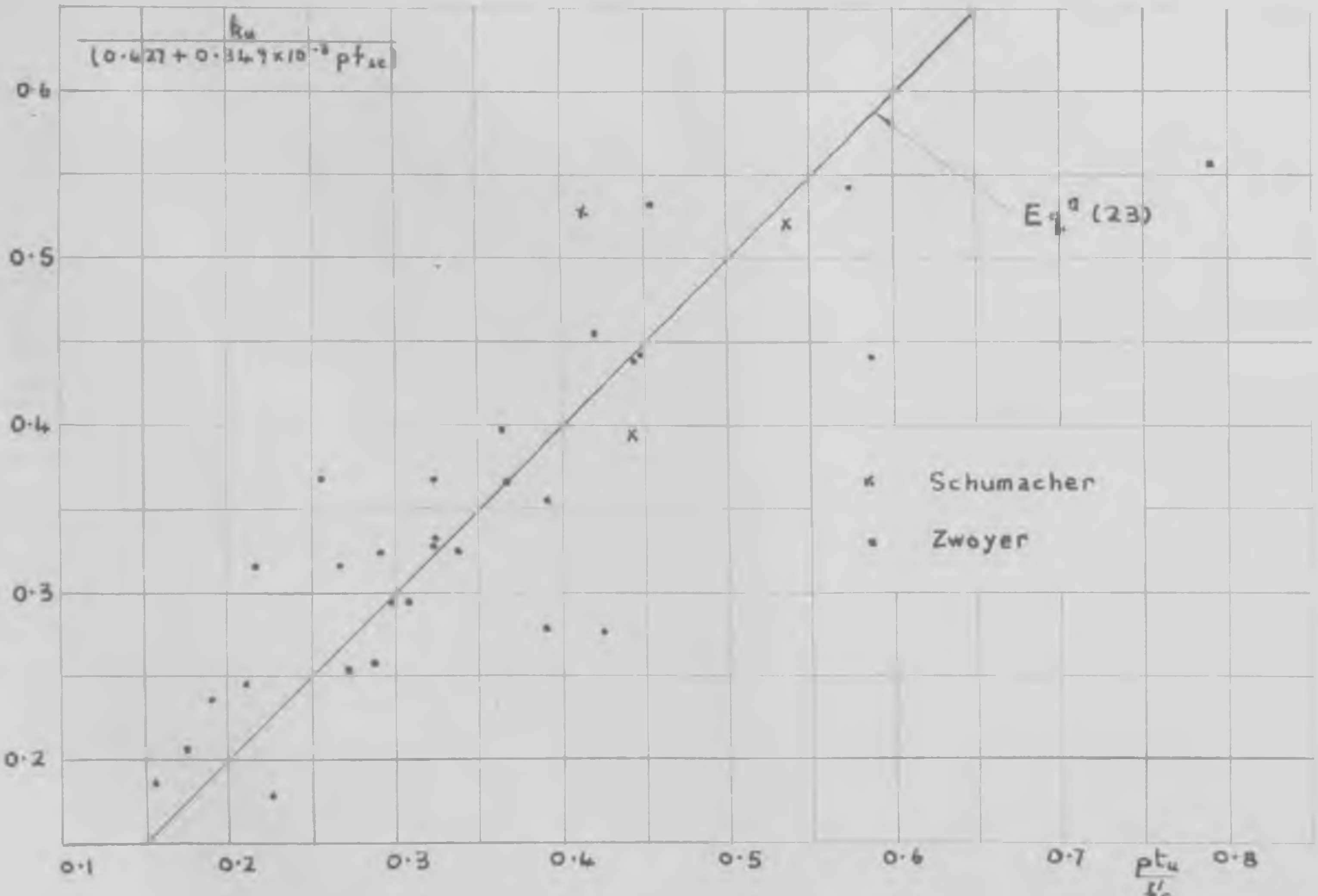


FIG. 12 INFLUENCE OF $\frac{pt_u}{f'_c}$ ON k_u

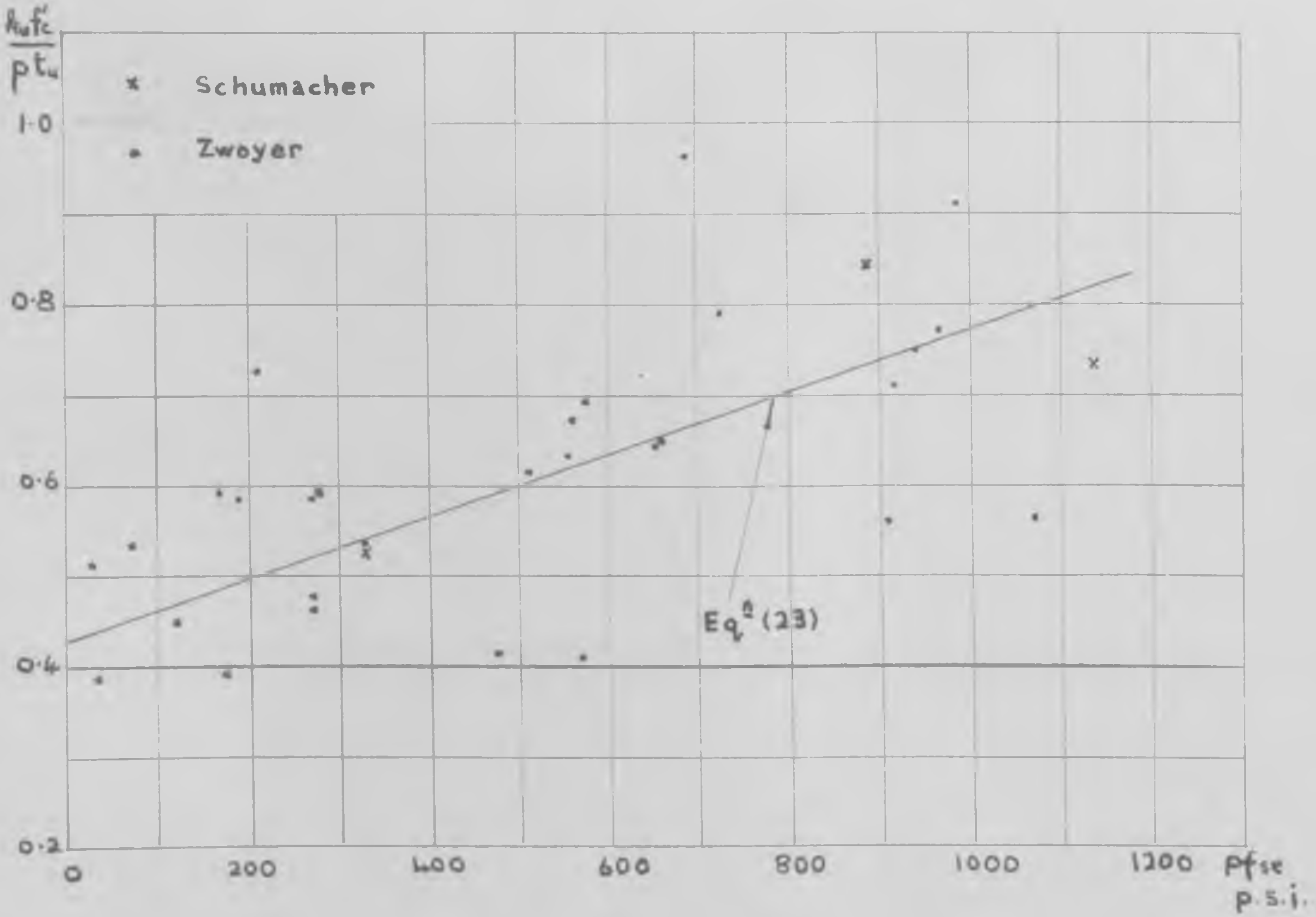


FIG. 13 INFLUENCE OF $p f_{se}$ ON k_u

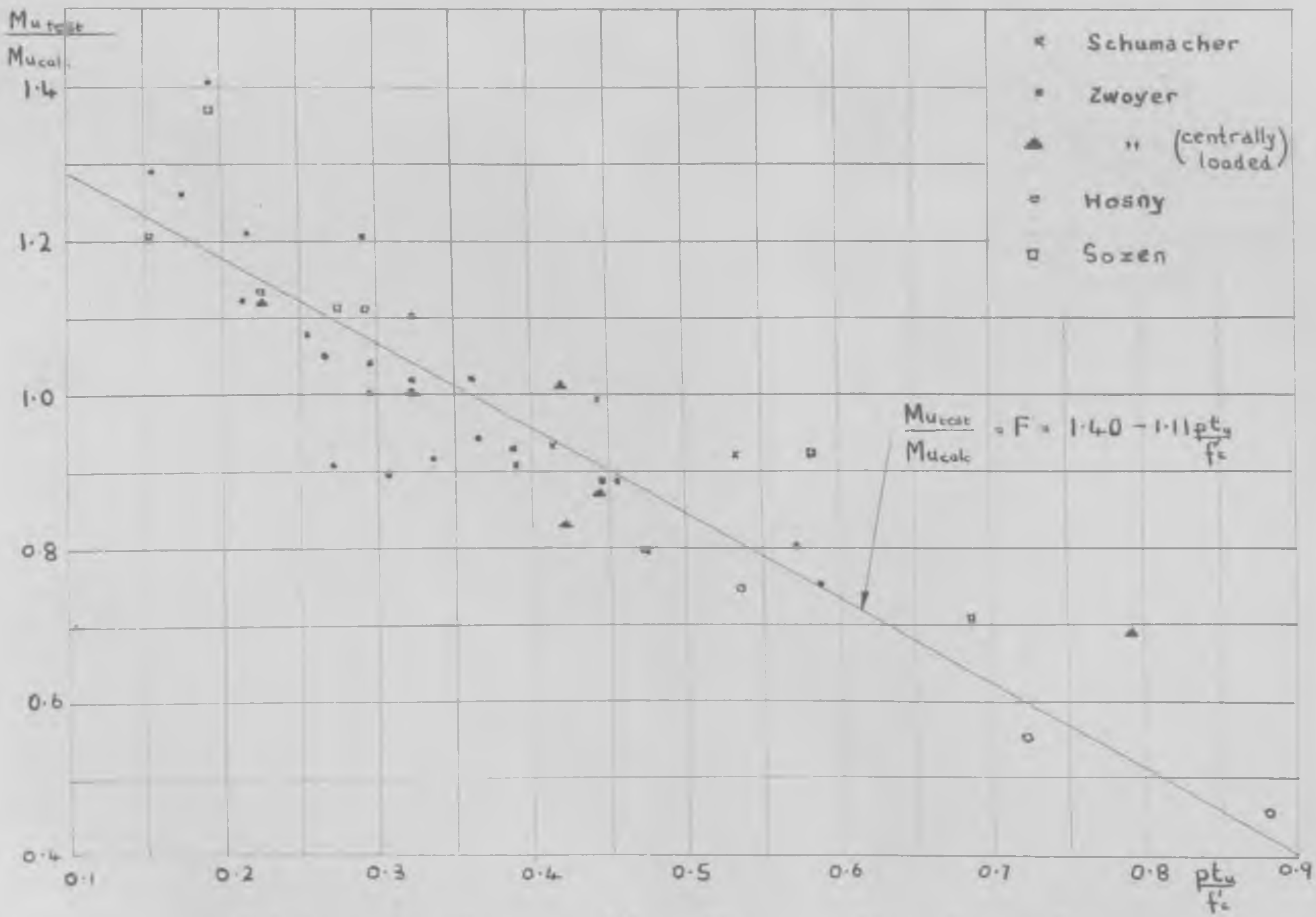


FIG. 14 INFLUENCE OF $\frac{pt_y}{f'_c}$ ON THE CORRECTION FACTOR, F FOR RECTANGULAR BEAMS

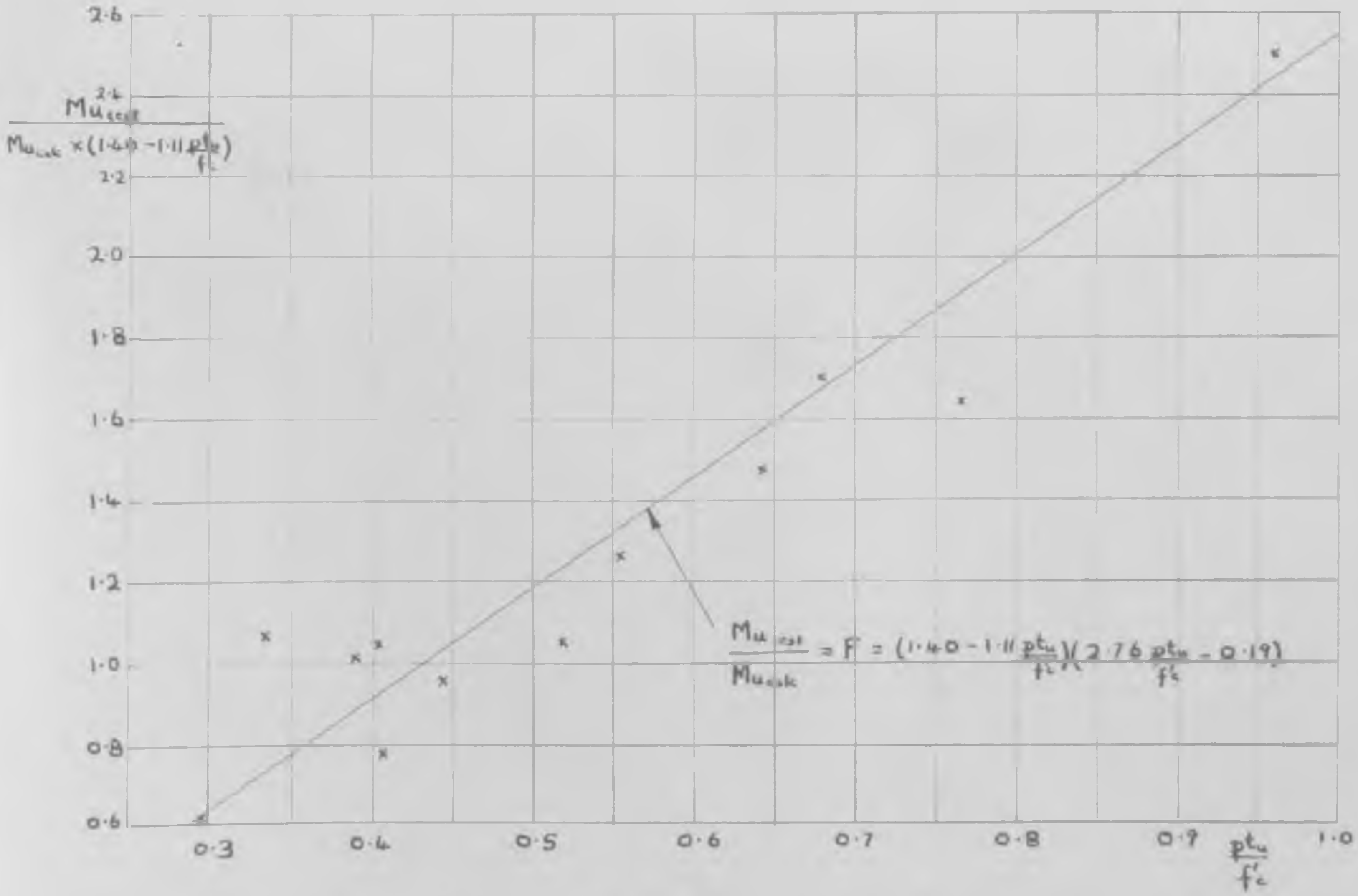


FIG. 15 INFLUENCE OF $\frac{pt_u}{f'_c}$ ON THE CORRECTION FACTOR, F FOR I-BEAMS

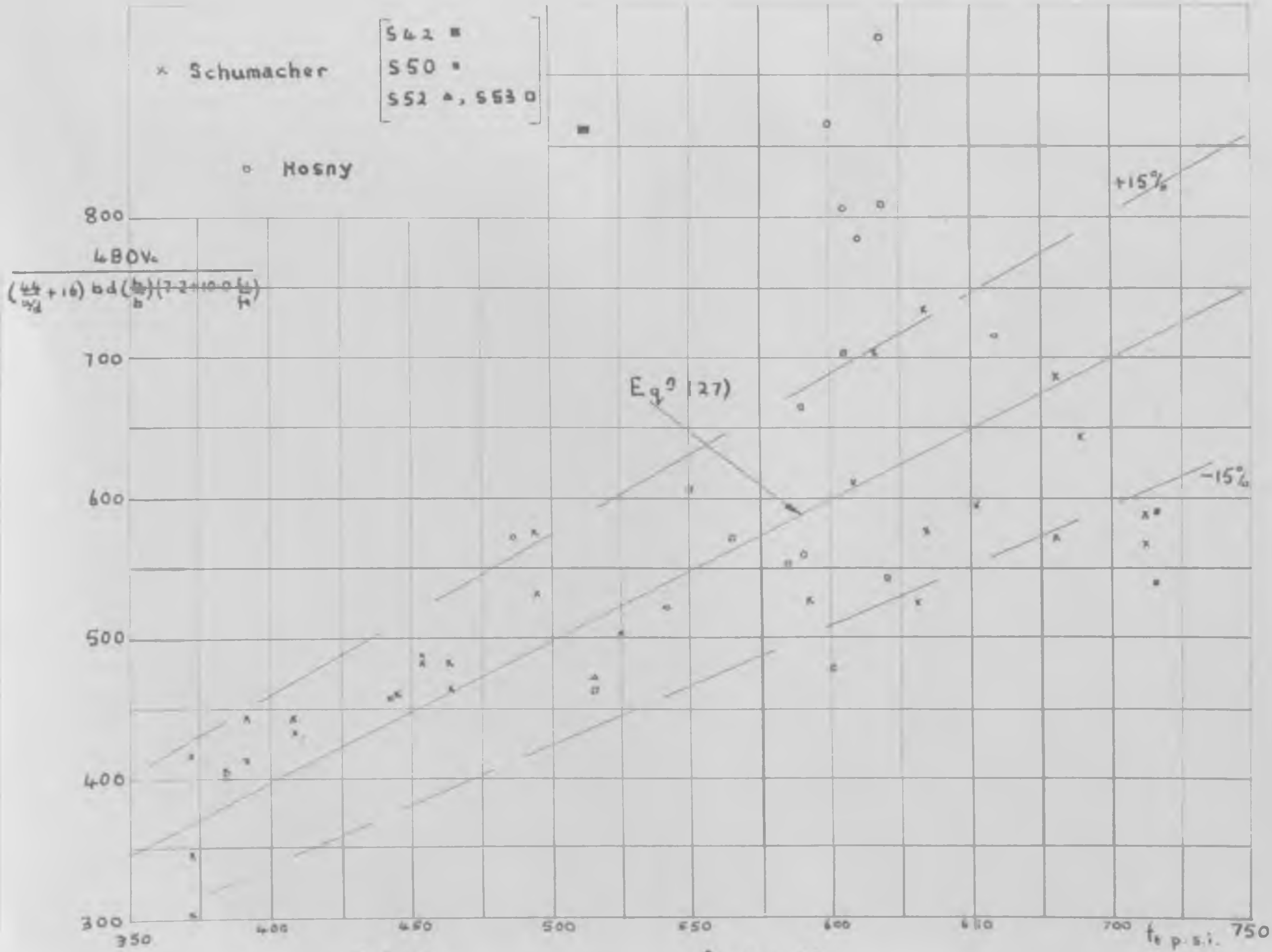


FIG. 16 INFLUENCE OF f_t ON V_c

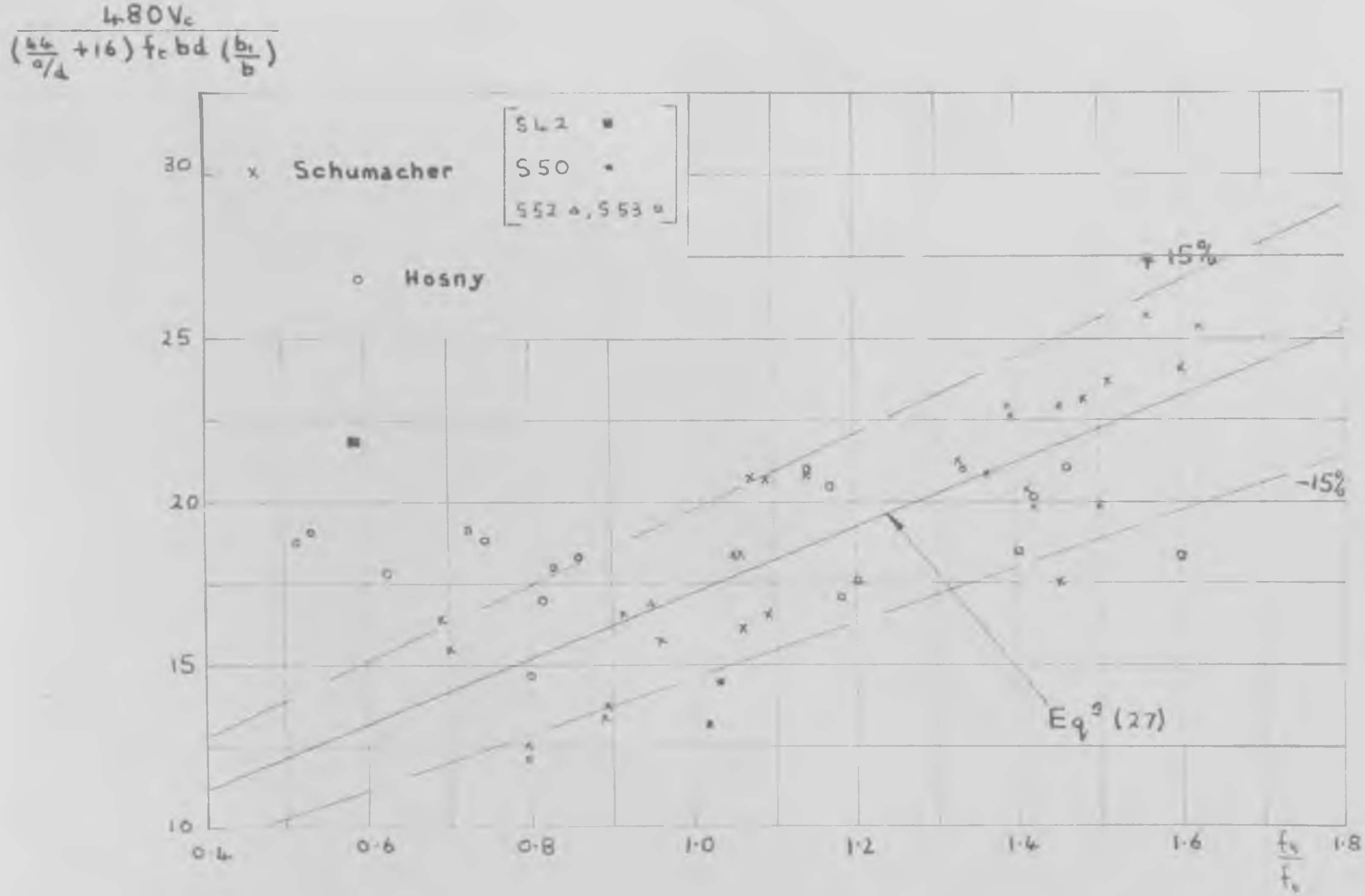


FIG. 17 INFLUENCE OF $\frac{f_s}{f_c}$ ON V_c

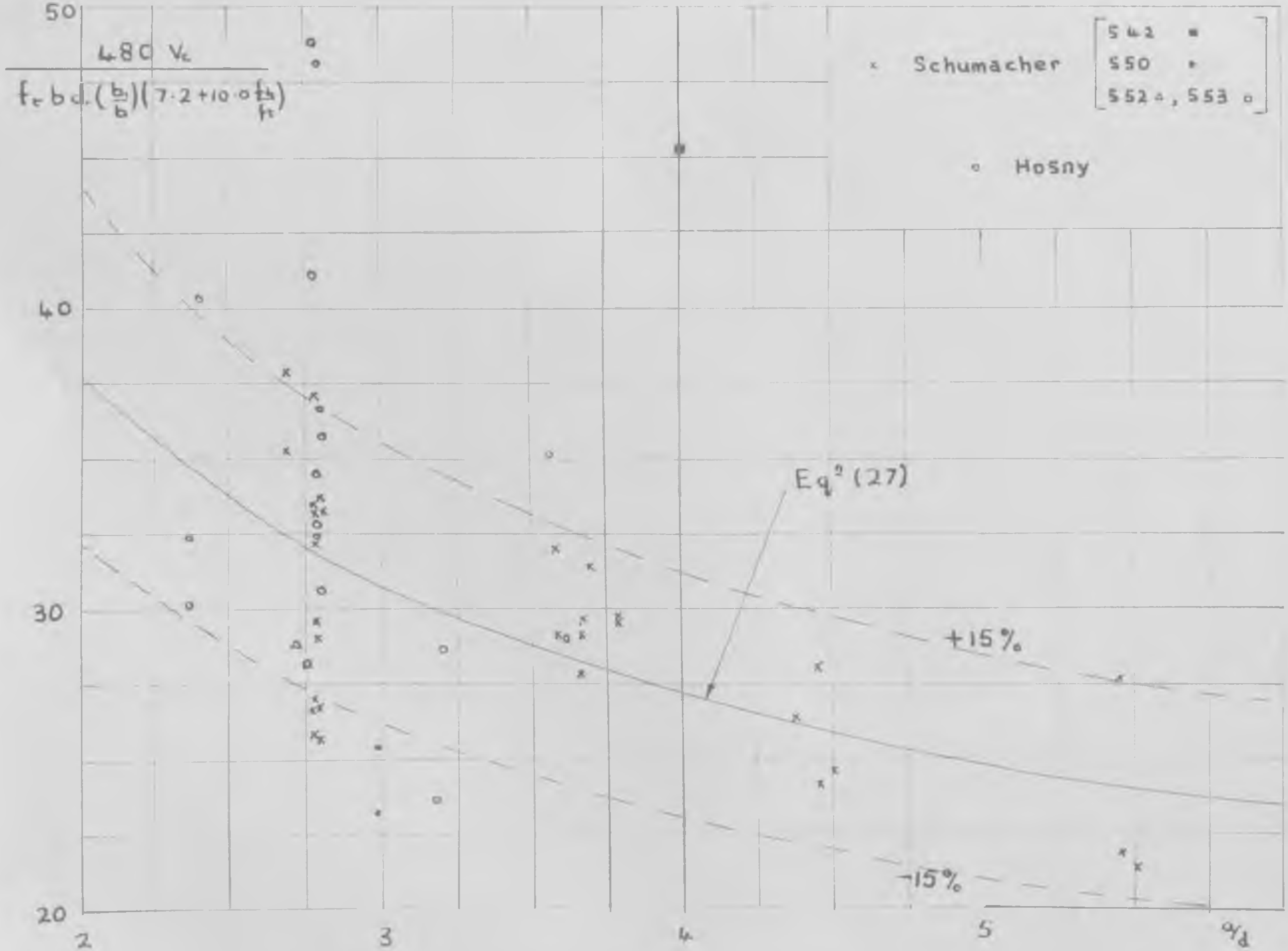


FIG. 18 INFLUENCE OF a/d ON V_c

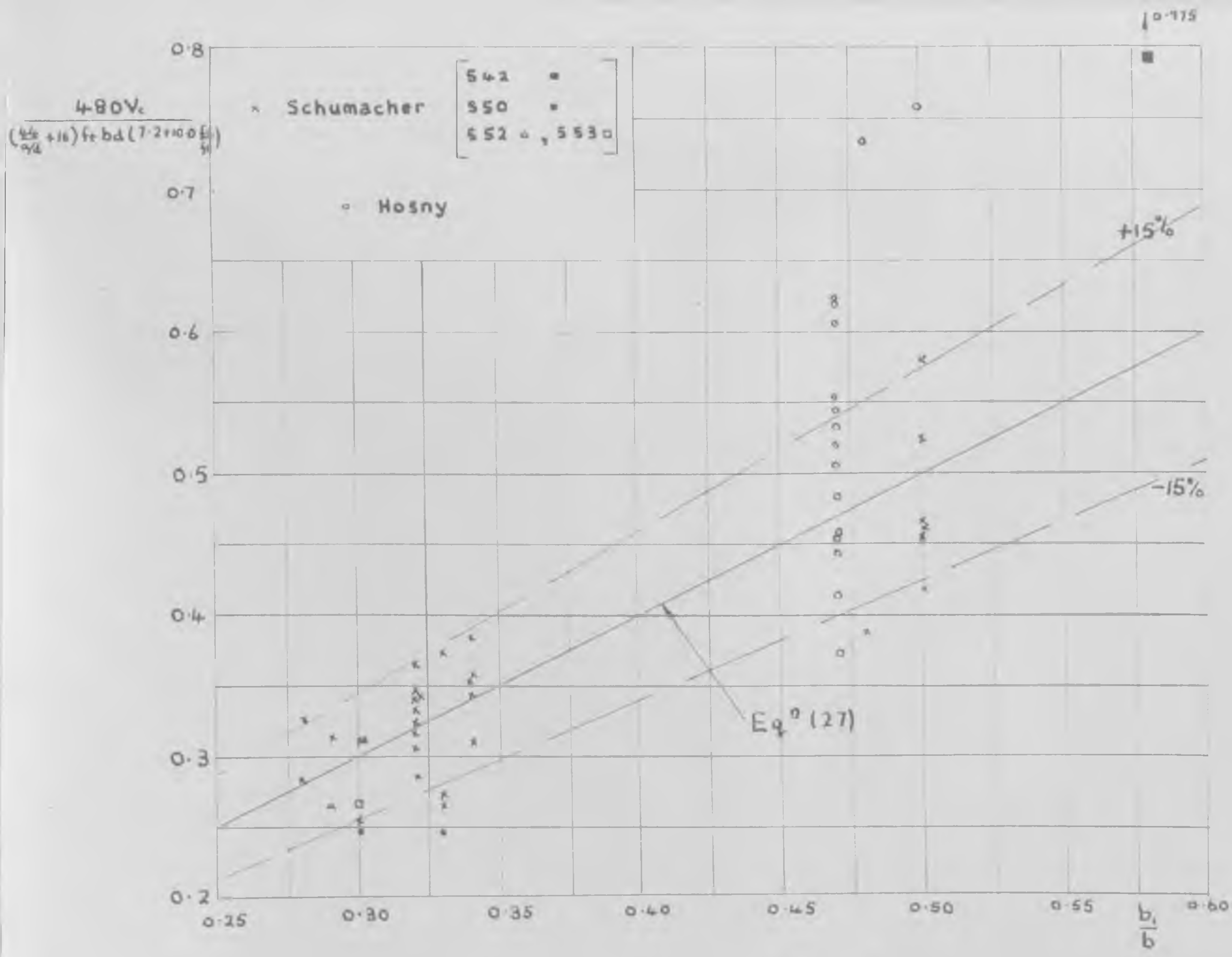


FIG. 19 INFLUENCE OF $\frac{b_1}{b}$ ON V_c

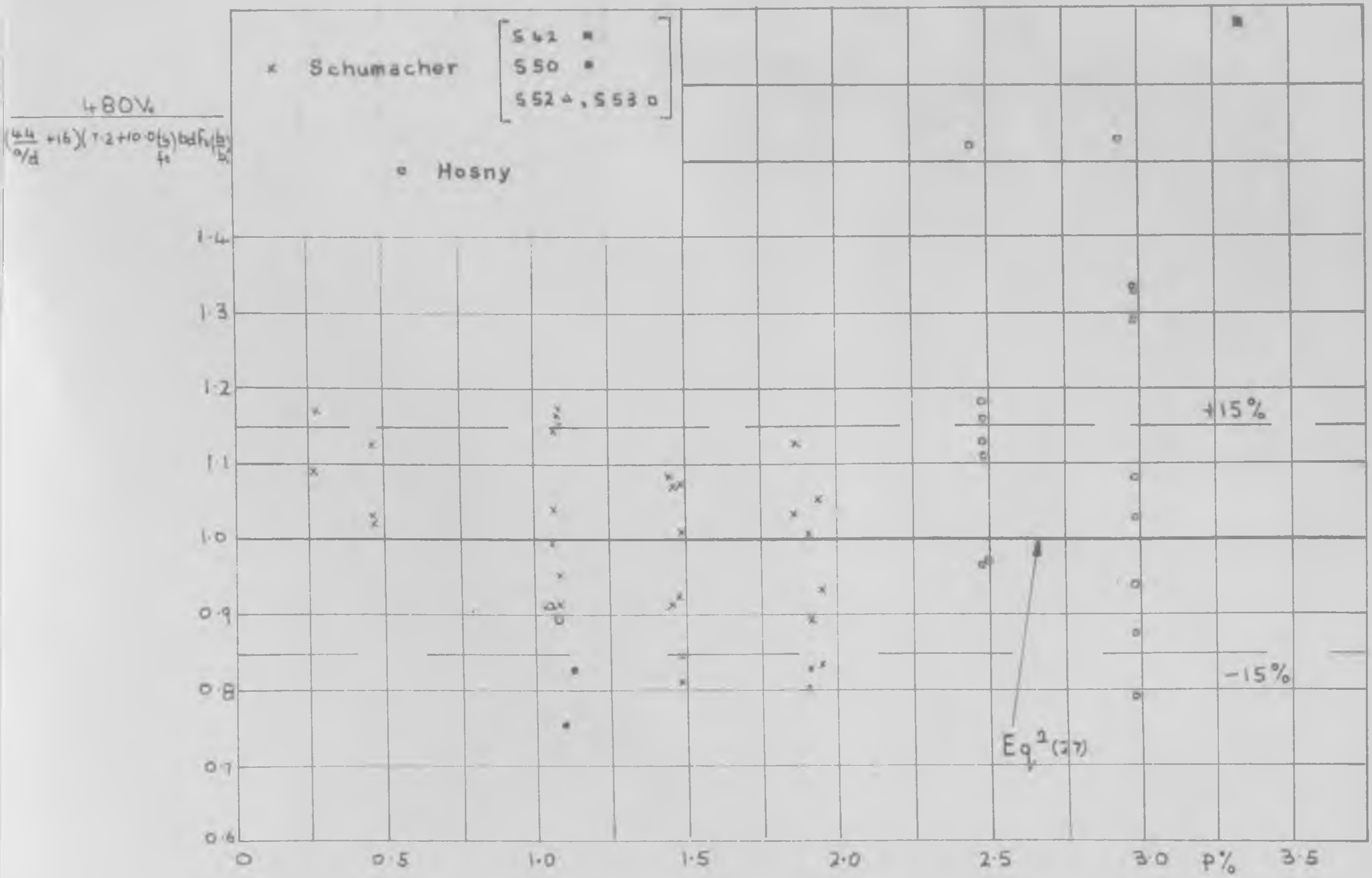


FIG. 20 COMPARISON OF $\frac{480 V_c}{\left(\frac{44}{a/d}+16\right)\left(7.2+10.0\frac{t}{f_t}\right) b d f_t \left(\frac{b}{b}\right)}$ WITH p%

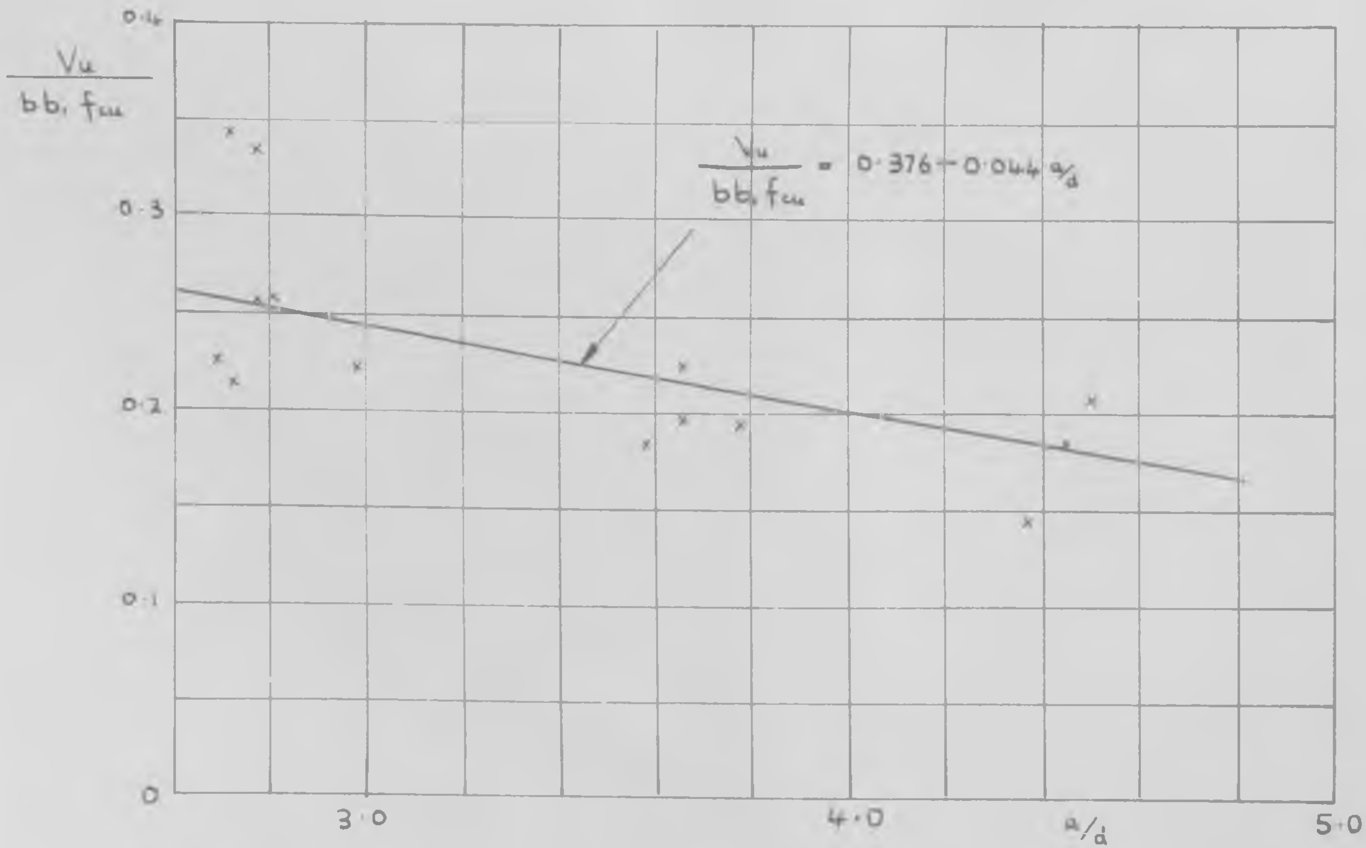


FIG. 21 INFLUENCE OF a/d ON $\frac{V_u}{bb_1 f_{cu}}$

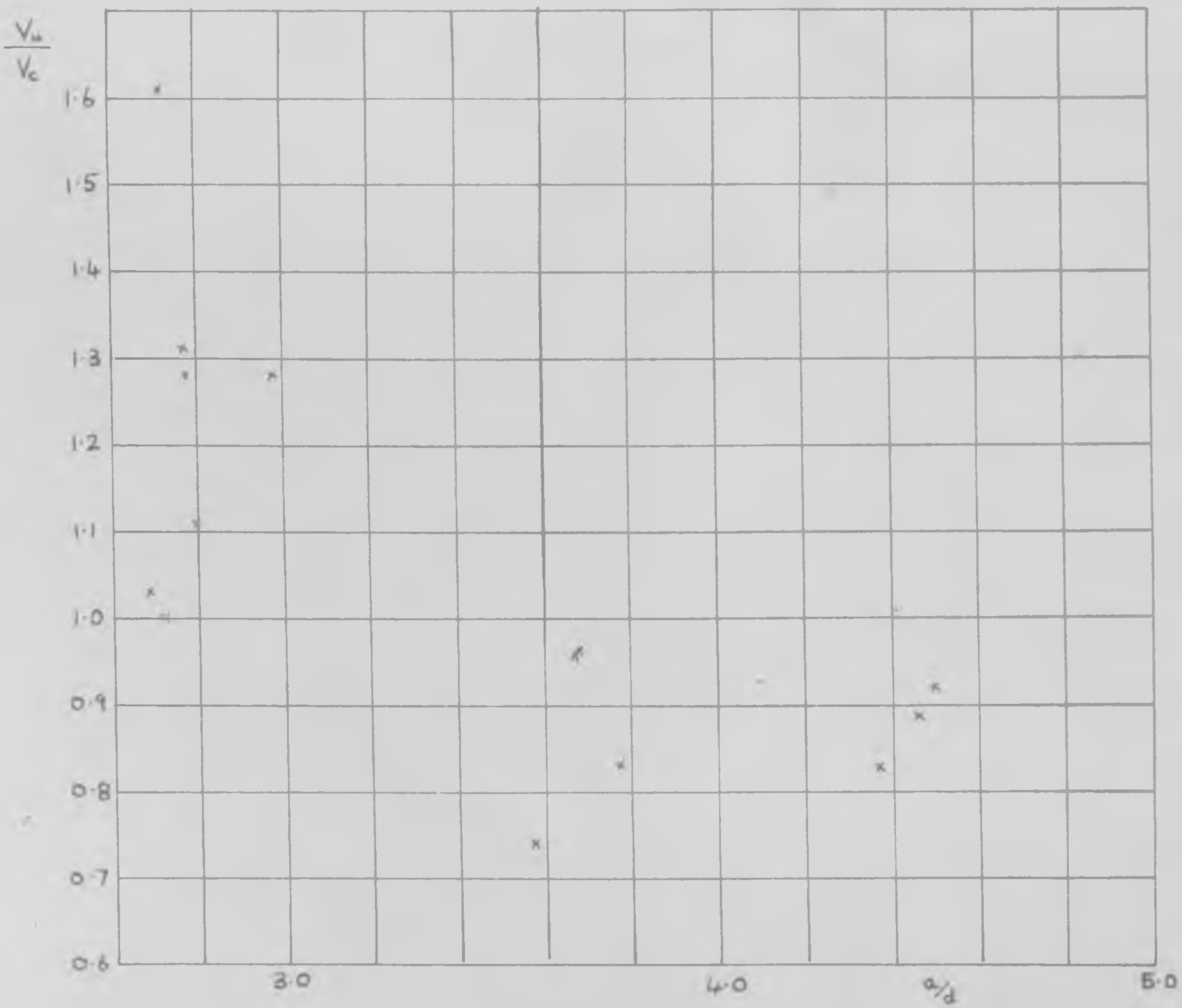


FIG. 22 COMPARISON OF DIAGONAL CRACKING AND FAILING LOADS

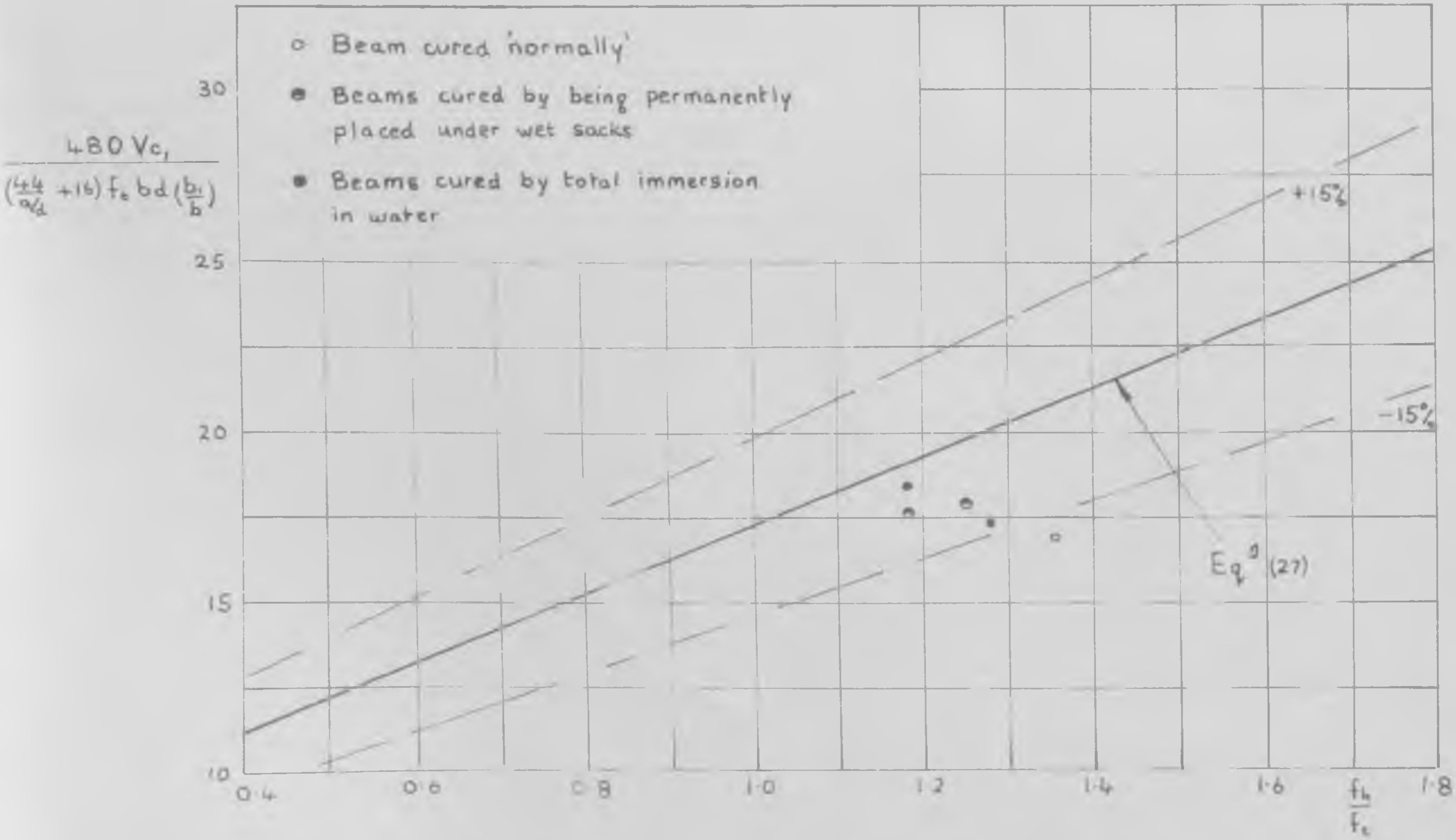


FIG. 23 COMPARISON OF LOADS CAUSING CRACKING IN THE WEB FOLLOWING CRACKING AT THE JUNCTION OF THE WEB AND THE UPPER FLANGE WITH EQUATION (27)

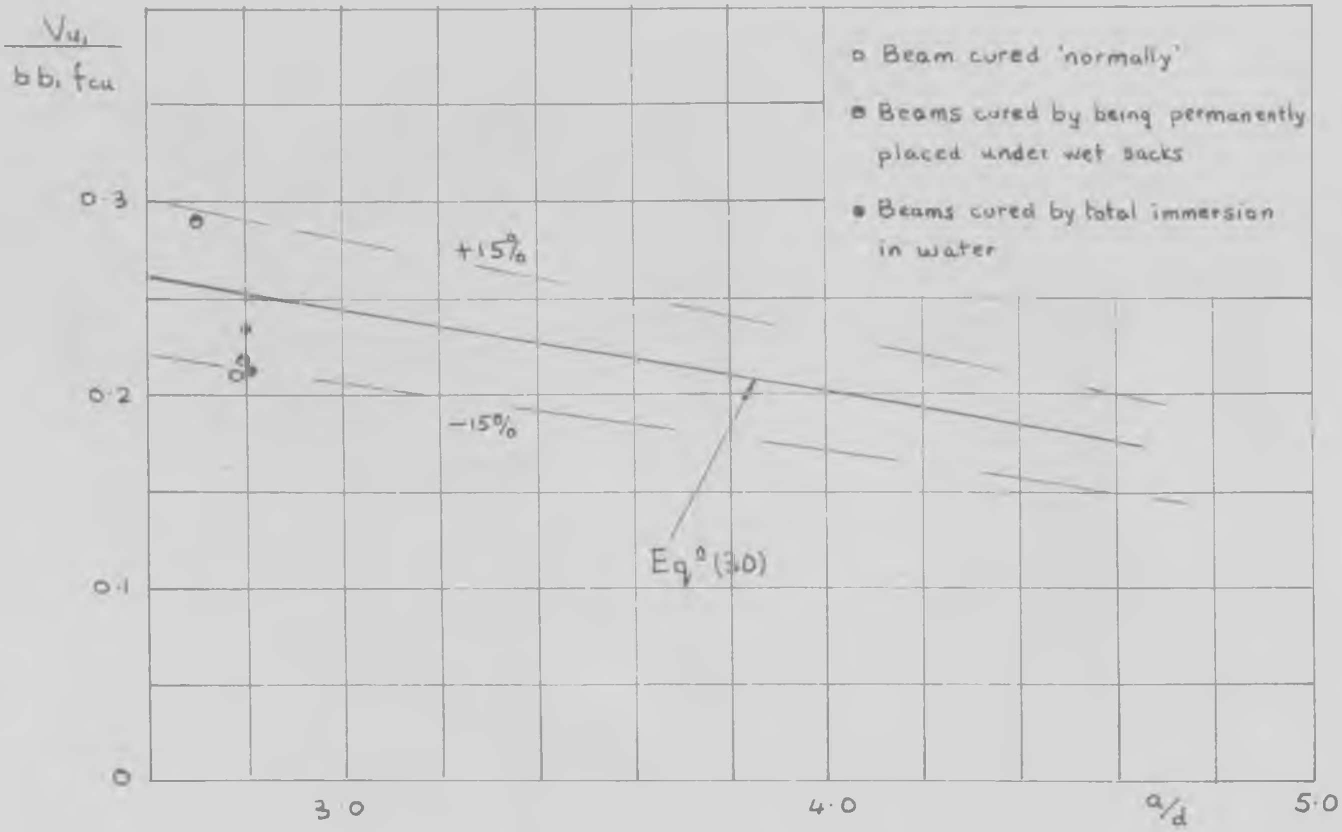


FIG. 24 COMPARISON OF LOADS CAUSING FAILURE FOLLOWING CRACKING AT THE JUNCTION OF THE UPPER FLANGE AND THE WEB WITH EQUATION (30)

What mechanism

Sudden failure – [illegible]?

Page [illegible] explain – evidence of [illegible] [illegible]

Long reinforcement as a [illegible] tie

Does shear compression always occur at [illegible] diagonal?

How [illegible] determined in design

How compare [illegible] [illegible]

Shear [illegible]?

Comments on [illegible] [illegible]

Page 65. [illegible] $\geq$ [illegible] is there no linear dist shear [illegible].

at section when M/strength [illegible] [illegible] shear crack occurs?

Page 67. – [illegible] [illegible] dependence on [illegible]

What increase of [illegible] after cracking without shear [illegible]
or load

Page 89 (17) how does compression [illegible] [illegible] formula apply to [illegible] [illegible]

[illegible] [illegible] values [illegible]

or [illegible] it shear – high shear

& low [illegible] would be critical.

Explain diagrams – p 65.