Computer-Aided Optimal Open Pit Design With Variable Slope Angles

by

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The candidate confirms that the work submitted is his own and that appropriate credit has been given where reference has been made to the work of others

In The name of Allah,

Most Gracious,

Most Merciful

To my wife Zahra, my daughter Negin,

my parents and all my family

for their encouragement and support throughout all these years which have helped me to finish this research

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Abstract

The use of open pit mining has increased to extract large and low grade deposits with the growth in demand for raw materials, with the advances in mining technology and with the depletion of high grade and readily accessible orebodies. Development and extraction of minerals by this method is a complex operation that may extend over several decades and require very large investments. Before starting the operation, it is necessary to design the size and final shape of the pit in order to determine minable reserves and amount of waste to be removed. It is also needed to locate the waste dump, processing plant, access roads and to develop a production program. The ultimate pit limit depends upon many factors. One of the most important factors is the pit slopes which affect the stripping ratio and amounts of waste to be removed. When dealing with complex deposits in which the pit slopes may vary in different parts of the orebody due to slope stability requirements, it is necessary to take into account variable pit slopes in the designing of the pit limit.

Determination of the pit limit in open pit mining is one of the most important design factors which may be considered many times during the life of the mine as the design parameters change in the future or more information is obtained during the operation. Therefore the use of a computer is essential in order to design the pit as rapidly as possible. As a result, a number of algorithms such as the various versions of the moving cone method, Lerchs-Grossmann algorithm, network or maximal flow techniques, Korobov algorithm, dynamic programming and parameterization techniques have been developed to determine the optimum ultimate pit limit since the advent and wide spread use of computers. The main objective of these algorithms is to determine the optimum pit limit in order to maximise the overall mining profit within the designed pit limit subject to the mining constraints.

Of these, the Lerchs-Grossmann algorithm is well known for being the only method which always yields the true optimum pit limit. However, the algorithm which utilises graph theory was based on fixed slope angles that are governed by the block dimensions when it was introduced. In spite of the fact that many attempts have been made to incorporate variable slope angles, none of them provide an adequate solution where there are variable slopes controlled by complex structures and geology. This algorithm is reconsidered and modified to deal with variable slope angles. It is assumed that the orebody and the surrounding waste are divided into regions or domain sectors within which the rock characteristics are the same and each region is specified by four principal slope angles including North, South, East and West face slope angles. Consequently slope angles can vary through the deposit to follow the rock characteristics and are independent of the block dimensions. In addition, two methods were also developed to estimate the four principal slope angles from geotechnical information to use as input parameters in the optimal pit design algorithm.

A general PC software was also developed to determine the optimum pit limit with variable slope angles for an open pit mine. The software is a Windows application that can be implemented under 32-bit operating systems such as Windows 95, Windows NT and Windows 98. It is capable of taking advantage of all the computer memory and designing the optimum pit limit for complex, large and low grade deposits due to solving the memory limitation. The software includes both graphical and numerical presentation of the input data and the results of optimisation. Two case studies have been used to validate the software developed.

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CHAPTER 1

Introduction

1.1- General background

Mineral deposits are generally extracted from the earth either by underground or surface mining methods with the objective of extracting the ore at a profit. In underground mining, extraction is carried out below the surface and the mineral deposit is reached by driving openings such as vertical shafts, inclined shafts or adits from the surface to the orebody and the mineral is mined by a variety of stoping methods. In surface mining, on the other hand, all the operations are carried out from the surface.

Open pit mining is one of the most important methods of surface mining in which any waste material or overburden is stripped and transported to a waste dump prior to, and sometimes during, mining in order to uncover, and gain access to, the mineral deposit. In general, mining proceeds from the top to the bottom of the orebody. Both stripping and mining are carried out in a series of horizontal layers, usually of uniform thickness, called benches. The choice between an underground mining and a surface mining method depends on the depth, grade and tonnage of the orebody and consequently on technical and economic criteria. Surface mining methods in the form of quarries or open pits are extensively used throughout the world to extract ore at or near the surface. Mineral deposits at depth may be extracted by underground mining methods but in general, significantly higher grades are required for profitability as the depth increases.

Open pit mining operations tend to be on a larger scale (i.e. higher production tonnages) than underground operations partly because of the usually lower grades but also because they are amenable to the use of highly mechanised, high production equipment which, although capital intensive, has relatively low labour costs. The advantages of open pit mining are high productivity, low operating costs, high production rates, good recovery, low labour requirements and good health and safety conditions. The method is also suited to the use of large equipment and the extraction of low-grade ore. However, the disadvantage of the method is that it is limited by depth and progressively more waste must be mined to reach ore at greater depths. Ultimately, deeper ore must either be mined by an underground mining method or left in place.

The development and extraction of ore by open pit mining is a complex operation that may extend over several decades and require very large investments. The major exception to this is open pit mining of gold which is often on a relatively small scale with a mining life as short as five years. Before extraction, it is necessary to determine the size and final shape of the pit at the end of its life. This final shape, or ultimate pit limit, represents the volume beyond which further extraction of ore, using current or assumed economic and technical parameters, is uneconomic. The ultimate pit limit determines minable reserves and the total amount of waste to be removed. It is also used to determine locations for the waste dump and surface infrastructure (such as processing plant and access roads) and to develop a production programme. Determination of the ultimate pit limit is one of the most important design considerations in open pit mining and it may be recalculated many times during the life of the mine as prices, costs, technical considerations and geology change. There are a number of factors which affect the size and shape of the ultimate pit. These include geology, grade, topography, production rate, bench height, pit slopes, mining and processing costs, metal recovery, marketing and cut-off grade. Some of these factors are discussed below:

Cut-off grade

Cut-off grades can be defined for any number of purposes. In general, a cut-off grade is any grade that is used to distinguish two courses of action, e.g. to classify as ore or waste.

The open pit mining method is usually used for low-grade deposits in which the ore is not contained within well-defined geological boundaries. In such deposits ore and waste are defined by a cut-off grade as opposed to a geological boundary. This cut-off grade is a very important factor in mine planning as it determines the overall ore reserve and the physical location of ore as well as the amount of waste to be removed. It is a complex function of many variables such as grade, price, pit slopes, size of mining (selection) equipment, and mining and processing costs. As the cut-off grade increases, the tonnage of ore above the cut-off decreases and its average grade increases. Up to a certain point the quantity of metal contained in the ore above the cut-off grade (product of tonnage and average grade) will remain constant. Beyond this point (and certainly for cut-off grades at or above the mean grade of the orebody) the quantity of contained metal, and hence the profit, will decline.

There are several commonly used cut-off grades. The break-even cut-off grade is one of the simplest and most widely used. This is the grade at which the recoverable revenue is exactly equal to the cost of mining, processing and marketing. Cut-off grades may be classified as either planning or operating cut-off grades depending on the time scales to which they refer. Planning cut-off grades are usually used to define geological or minable reserves before the start of operations or for long periods of time. Operating cut-off grades are usually used during the operation to make short-term to medium-term decisions, e.g. to mine or not, to stockpile or process. A review of earlier work on the determination of optimum cut-off grades is given in Chapter 2.

Bench height

In open pit mining, the extraction of ore and the stripping of waste material are done in a series of layers called benches. Figure 1.1 shows an idealised cross section through a hypothetical open pit mine. The vertical distance between each horizontal level in the pit is called the bench height. This height depends on the type of metal/mineral, the manner in which it is dispersed in the host rock, the size and type of equipment used to extract the ore, the blasting method, the production rate and the geotechnical characteristics of the orebody. The height is usually set as high as possible with regard to the size and type of equipment selected for the operation.

The bench height is often reflected in the vertical dimension of the blocks that comprise the orebody block model used by computer methods to determine pit limits.

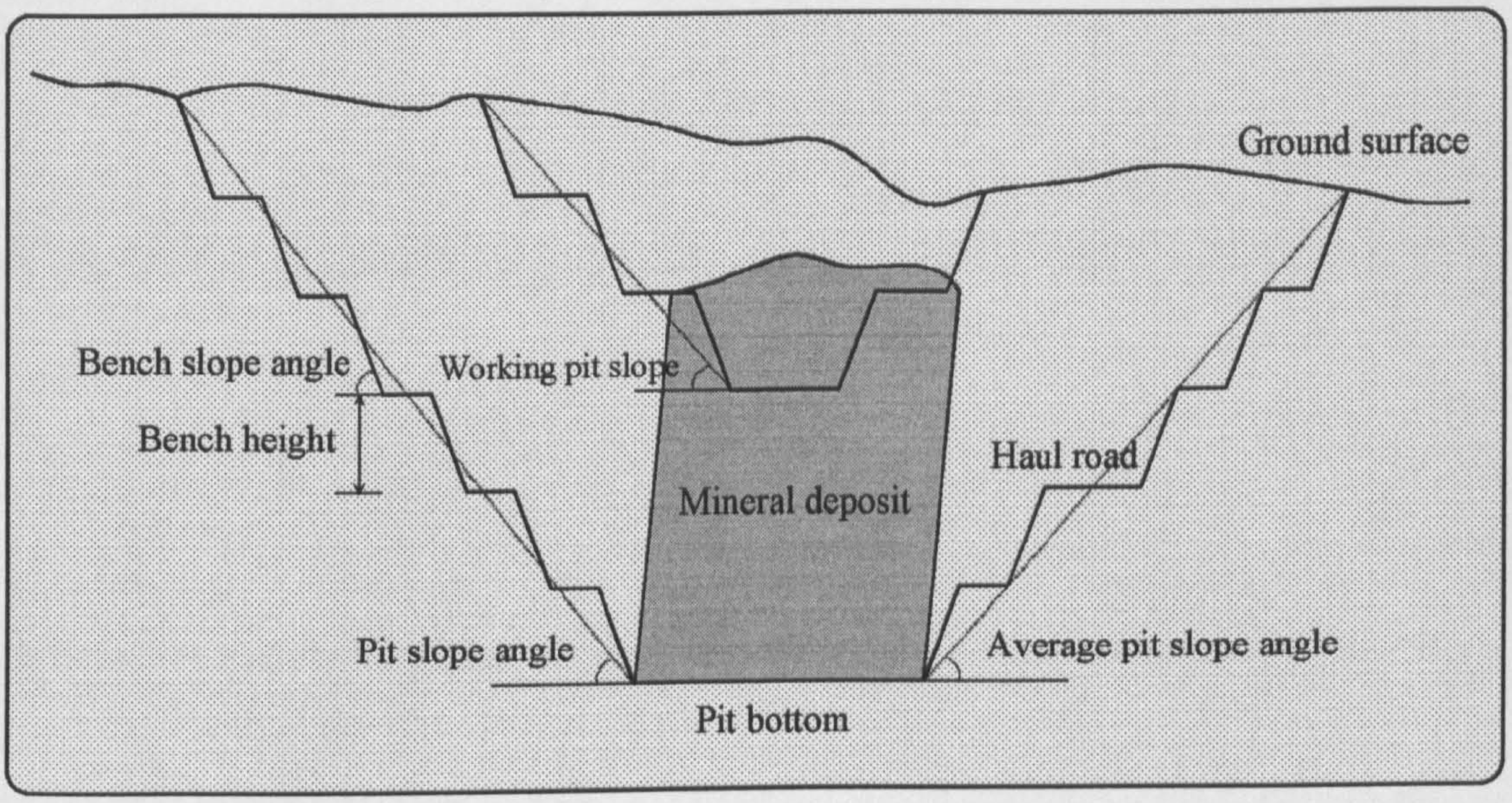


Figure 1.1- A vertical section through a hypothetical open pit mine

Stripping ratios

The stripping ratio is another important factor in open pit mining as it has a major bearing on profitability, scheduling and pit design. The stripping ratio is the ratio of the amount of waste that must be removed in order to mine a unit quantity of ore and is usually expressed as:

$$SR = \frac{Waste\ (tonne)}{Ore\ (tonne)}$$

The break-even stripping ratio (*BESR*) is calculated from the following equation:

$$BESR = \frac{R - C}{W}$$

where

R	is the revenue per tonne of ore
C	is the production cost of per tonne of ore (including all costs except stripping cost)
W	is the striping cost per tonne of waste

In some circumstance a minimum acceptable profit is included in the equation:

$$BESR = \frac{R - (C + P)}{W}$$

where

P	is the minimum acceptable profit per tonne of ore

Pit slopes

The pit slope is a major factor affecting the size and shape of an open pit. Stability is an over-riding consideration, as pit walls must remain stable during the life of the mine. The steeper the final slope can be designed, the smaller the amount of waste that has to be removed. However, as slopes become steeper, the probability of failure increases. An optimum mine plan should, therefore, have the steepest final pit limit commensurate with stability throughout the period of mining activity. As illustrated in Figure 1.1, there are four types of pit slope used in open pit mining:

1- Pit slope angle, ultimate pit slope or overall slope angle which is the angle between the horizontal and the line connecting the toe of the lowest bench to the crest of the uppermost bench. This slope makes no allowance for safety berms and haul roads.

2- Average pit slope angle or average ultimate pit slope which is the angle between the horizontal and the line connecting the toe of the lowest bench to the crest of the uppermost bench allowing for the haul road or access ramp and safety berms.

3- Bench slope angle which is the angle between the bench face and the horizontal.

4- Working pit slope angle which is the slope of the pit wall during the mining operation. This is usually smaller than the ultimate pit slope angle so as to ensure stability and provide a wider space for operation. The working pit slope increases during the mining operation until it reaches the pit slope angle at the end of mining.

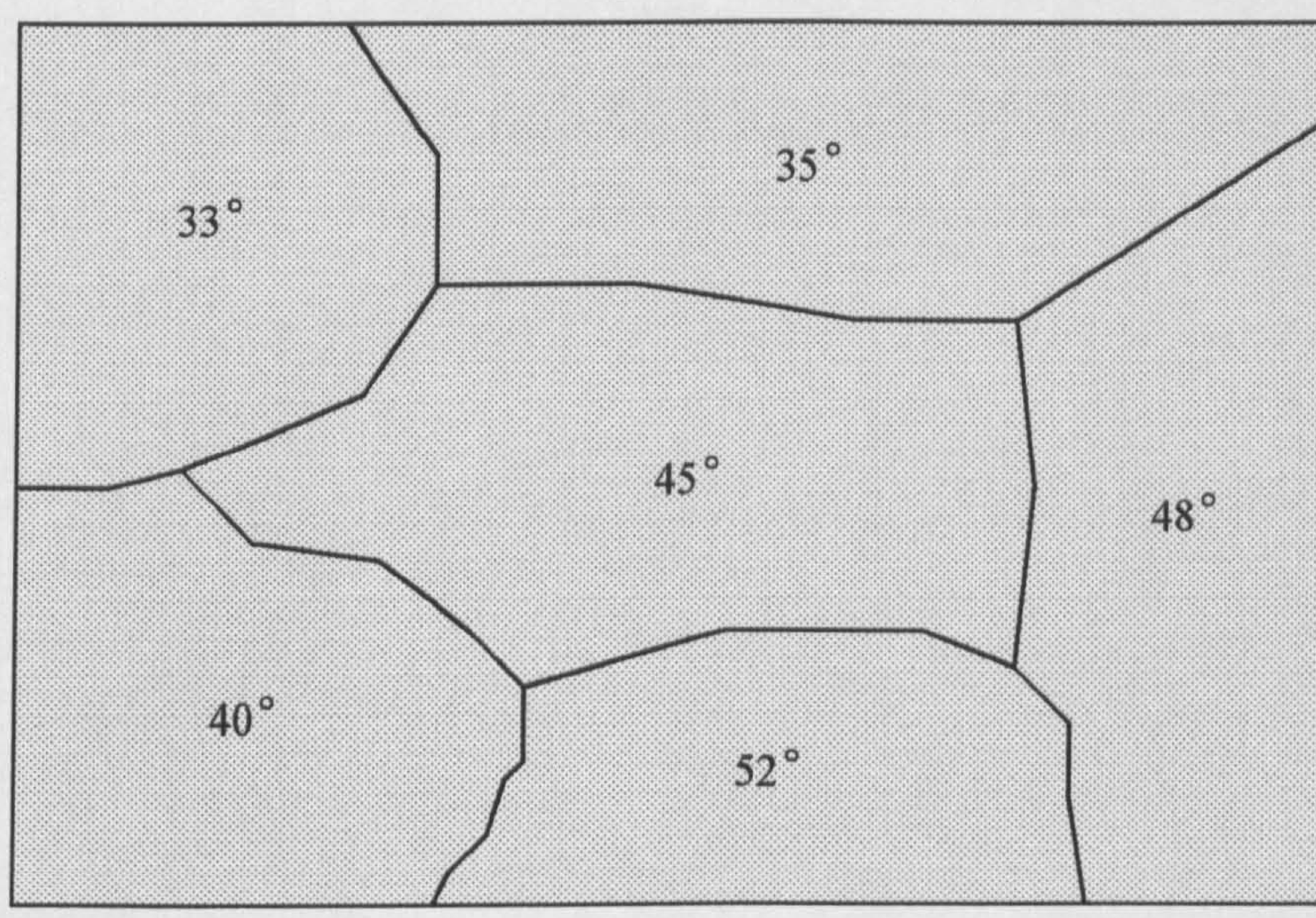


Figure 1.2- An example of pit slope varying through the deposit. (Plan view of deposit)

In general, the overall slope is designed to be as steep as possible in order to reduce the stripping ratio. Pit slope angles may vary through the orebody and may vary with direction due to changes in geological structure and stability requirements. They may also vary with elevation. Any realistic method of pit design must, therefore, take into account variable slope angles. Figure 1.2 provides an example (in plan view) of how pit slopes may vary though a deposit due to changes in lithology and geological structure.

The determination of pit slopes is essential and must be done before planning the pit limit. These are determined mainly by slope stability methods from geotechnical information gathered during the site investigation.

1.2- The objective of the research project

The present study is focused on the initial stages of open pit design and planning in which the main task is to establish the ultimate pit limit. The main objective of the research project was to develop general PC-based software for optimal open pit design with variable slope angles and incorporating geotechnical design. The stages in reaching this objective were:

1- The development of a computer program to determine the optimum pit outline based on the Lerchs-Grossmann method with variable slope angles.
2- Designing the optimal pit outline with an unlimited number of blocks in the deposit block model.
3- Developing a graphical and numerical interface for data entry and the display and interpretation of the results.
4- Incorporation of geotechnical information in the design of slope angles.

Since the advent and widespread use of computers a number of algorithms have been developed to determine the optimum ultimate pit limit. Of these, the algorithm developed by Lerchs and Grossmann (1956), based on graph theory, is the only algorithm that can be proved mathematically always to generate the true optimum pit limit. The original algorithm, however, was limited to a single slope angle defined by the block dimensions and was incapable of taking into account variable slope angles. Since pit slope angles may vary through the deposit, solving the general problem of slope constraints in the Lerchs-Grossmann algorithm makes it much more flexible, practical and reliable. This problem is dealt with in Chapter 3.

Almost all computer algorithms are based on a block model of the deposit which requires a large amount of memory to store block characteristics. Under the DOS operating system the memory limitation makes it impossible to implement optimal pit design algorithms for block models with large numbers of blocks. DOS programs can use only 640 kb of RAM memory, irrespective of the amount of total memory. To overcome this limitation and to take advantage of all available PC computer memory a general Windows-based program for optimal open pit design was developed to run under a 32-bit operating system such as Windows 95, Windows 98 or Windows NT in which dynamic memory allocation can be used to store block characteristics.

The functionality of software is enhanced by meaningful presentation of output. One of the easiest ways to present optimum pit limit results is by means of plans and sections. Several programs were developed to present the results of optimisation and input data both in numerical and graphical forms. Graphical presentation includes the optimum pit limit both in plan and sections. These are explained in Chapter 5.

Having modified the Lerchs-Grossmann algorithm to take account of variable slope angles, the next step was to determine slope angles based on geotechnical information. Chapter 4 presents two methods for estimating the safe slope angles to use as input parameters in the optimal pit design algorithm.

1.3- Organisation of the thesis

The remainder of this thesis is organised into the following chapters:

Chapter 2 **<u>Literature review and survey of previous work</u>**- This chapter presents a literature review and survey of the previous work including the various algorithms for optimal pit design together with the different methods of slope design used in open pit mining.

Chapter 3 <u>**The Lerchs-Grossmann algorithm with variable slope angles**</u>- This chapter deals with the Lerchs-Grossmann algorithm and incorporation of variable slope angles into the algorithm. It also covers the calculation of the revenue block model as input for the optimal pit design algorithm and a method of smoothing the pit bottom.

Chapter 4 <u>**Slope design procedure**</u>- This chapter provides two methods which are incorporated in the optimal pit design software to estimate a set of safe angles from geotechnical information as input parameters to determine the optimum pit limit.

Chapter 5 <u>**The software**</u>- This chapter describes the structure of the software, numerical and graphical presentation of the results, data requirements and operation of the software.

Chapter 6 <u>**Case studies**</u>- This chapter presents two case studies using real data from an operating open pit mine and from a mineral prospect currently undergoing a feasibility study. Both data sets are used to validate and evaluate the software.

Chapter 7 <u>**Conclusions and recommendations for future work**</u>- This chapter presents a final evaluation of the significance of this work, and directions for future research.

CHAPTER 2

Literature review and survey of previous work

2.1- Introduction

The ultimate limits of an open pit define its size and shape at the end of the mine's life. In addition to defining total minable reserves and determining total profitability, these limits are needed to locate the waste dump, processing plant and other facilities. They are also required for the design of overall production schedules within the planned pit shape. Since the advent and widespread use of computers a number of algorithms have been developed to determine the optimal pit outline all with a common objective: to maximise the overall mining profit within the designed pit limit. This chapter presents a literature review and survey of the previous work including an assessment of the different algorithms for optimal pit design together with the methods of slope design used in open pit mining.

2.2- Optimal open pit design

2.2.1- General introduction

One of the most important objectives in open pit mining is to design the ultimate pit outline and thereby determine the minable ore reserve and the size and shape of the pit at the end of its life. The size and shape of the ultimate open pit depends upon many factors including geology, grade, topography, production rate, bench height, pit slope angles, mining and processing costs, percentage metal recovery, marketing and cut-off grade. The most basic form of a cut-off grade is a grade that is used to classify mineralised material as ore or waste. Taylor (1972) defines cut-off grade as:

> ***"Any grade that for any specified reason is used to separate any two courses of action, e.g. to mine or to leave, to mill or to dump...."***

Many attempts have been made to devise a general theory of cut-off grades within the context of which an optimal sequence of cut-off grades can be defined and, in practice, determined, for the life of a mine. The criterion for optimality is almost universally taken to be the maximum Net Present Value of production that results from applying a sequence of cut-off grades. The most advanced approach is that of Lane (1964 and 1988) which is based on the assumption that there are three stages in the mining operation comprising mining, concentrating or processing, and refinery and/or marketing. Each stage has its own associated costs and a certain capacity. The optimum cut-off grade is determined by the manner in which these three stages limit the operation. Lane proposed three sets of cut-off grades, each set comprising three cut-off grades determined by the three stages.

The first set of cut-off grades, called limiting cut-off grades, is determined by considering the three stages of the operation, on the assumption that each stage limits only the total capacity of the operation. These cut-off grades depend indirectly on the grade distribution of the deposit and directly on the price of the final product and the cost of producing it.

The second set of cut-off grades, called balancing cut-off grades, is determined by balancing the capacities of each pair of stages.

The third set of cut-off grades comprises three optimum cut-off grades – one for each pair of stages. These optimum cut-off grades are determined by considering the balancing cut-off grades between upper and lower limits imposed by the limiting cut-off grades.

Finally, the effective optimum cut-off grade is the middle value of the three optimum cut-off grades. Lane's method is regarded as a landmark in the determination of optimum cut-off grades. His method, however, relies on the assumption that prices, costs and recovery remain constant throughout the operation. Dowd (1973), Dowd and Xu (1995) and Whittle and Wharton (1995) have coded Lane's method into a computer program.

Blackwell (1970) revised Lane's method, again with three stages, and incorporated an additional time cost by using a computer-based algorithm to determine the optimum cut-off grade. Taylor (1972) presented a general theory of cut-off grades with the inclusion of Lane's method. He also differentiated between planning and operational cut-off grades. A planning cut-off grade is normally used to distinguish between ore and waste for long-term planning prior to the start of the operation. An operational cut-off grade is used for short-term planning during production. In this study the planning cut-off grade is a grade that is used to classify ore and waste on the basis of a break-even analysis.

Roman (1973) introduced dynamic programming to determine production rates and Dowd (1976) extended this work to optimise both production rates and cut-off grades. Both Lane's method and the dynamic programming approach developed by Dowd have shown that in order to maximise present value the operation must begin with a relatively high grade which then declines over the life of the mine.

The open pit mining method is normally used to extract orebodies at or near the surface. It is usually a large-scale method and requires very large expenditure. The development and extraction of ore by this method may extend over several decades. Before extraction is started it is necessary to locate the waste dump and processing plant, and determine the size and final shape of the mine at the end of its life. There are many ways of designing an open pit. These methods can be classified broadly into two groups: manual methods and computer methods (Dowd, 1994a).

Manual methods of designing pit limits are based on stripping ratios and involve considerable time and engineering judgment. The pit limits are set on various sections by applying economic and technical criteria. The types of section used depend on the shape of the orebody and include vertical, longitudinal and radial sections. The pit limits are placed on each section independently using the stripping ratio and pit slope angle. The stripping ratio is calculated and compared to the break-even stripping ratio. If the calculated stripping ratio is less than the break-even stripping ratio the pit limit is expanded. Otherwise it is reduced. This process is repeated until the pit limit is set at a point where both stripping ratios become equal. Once the pit limits have been established on the individual sections, they are evaluated as a group to examine how they fit together and any discontinuities between sections are smoothed out (Armstrong, 1990). This method is time consuming and the result depends on the engineer's knowledge and skill. It is also difficult to use this method on a large or complex deposit. Another disadvantage of the method is that when smoothing is carried out to combine the various sections, the result may not yield the best overall pit.

Computer methods can be divided into two groups: computer-assisted hand methods and optimal open pit design algorithms. The first group of methods, which are the computerised versions of manual techniques (e.g. those in SURPAC), follow the same procedure as the manual methods. They have been developed to assist the mining engineer to make the necessary calculations in a short time. These methods also allow an evaluation of a large number of potential designs quickly and cheaply.

The objective of optimal open pit design algorithms is to determine the ultimate pit outline for an orebody together with the associated grade and tonnage that optimise some specified economic and/or technical criteria whilst satisfying practical operational constraints. The most common criteria used in optimisation are: maximum net profit, maximum net present value, maximum metal content and optimal mine life. Of these the most widely used criterion is maximum net profit.

Dowd (1994a) provides an overview and classification of the various approaches to optimal open pit design and their relationship to the types of orebodies to which they are applied. Apart from elementary methods which are used for some stratiform deposits, most computer algorithms for open pit design use block models of the orebody which are either a:

- **Block grade model** obtained by considering the deposit as a large box, covering the entire orebody, and then subdividing it into smaller blocks and assigning estimated grades to each block.

or a

- **Revenue block model** created by applying costs and prices to the grade block model of the deposit.

There are many types of block models including 3D fixed-block model, 3D variable block model, 2D irregular block model and 3D irregular block model (Kim, 1978). Among these, the three-dimensional fixed-block model is the most widely used. This model is shown in Figure 2.1 and is obtained by dividing the orebody into three-dimensional blocks of fixed size. Each block is identified within the model by its location co-ordinates comprising Easting, Northing and vertical.

The dimensions of the blocks depend on the size of mining equipment, topography and shape of the orebody and the data available for estimating the block grades. A number of papers have been written about the selection of the size of blocks namely Whittle (1989), Cai (1992), Hulse (1992) and Dowd (1994b). Larger blocks decrease the number of blocks in the block model which results in reduced computing time. On the other hand using a larger block results in a loss of definition of grade variation within the deposit. Smaller blocks increase computing time and require more memory to store the information. Furthermore, blocks with dimensions that are significantly smaller than the drilling, or sampling, grid will give rise to significant estimation errors and the variance of the estimated values will be artificially low. If the block size is too small in relation to the drilling grid the estimation errors will render the estimates meaningless. The vertical height of the blocks is usually set to the bench height planned for the mining operation. The horizontal dimensions of the blocks depend on the information gathered during exploration and evaluation and the technique used to assign grades to the blocks. As a general rule of thumb the horizontal dimensions should not be less than one-third of the average sample spacing (Dowd, 1994b).

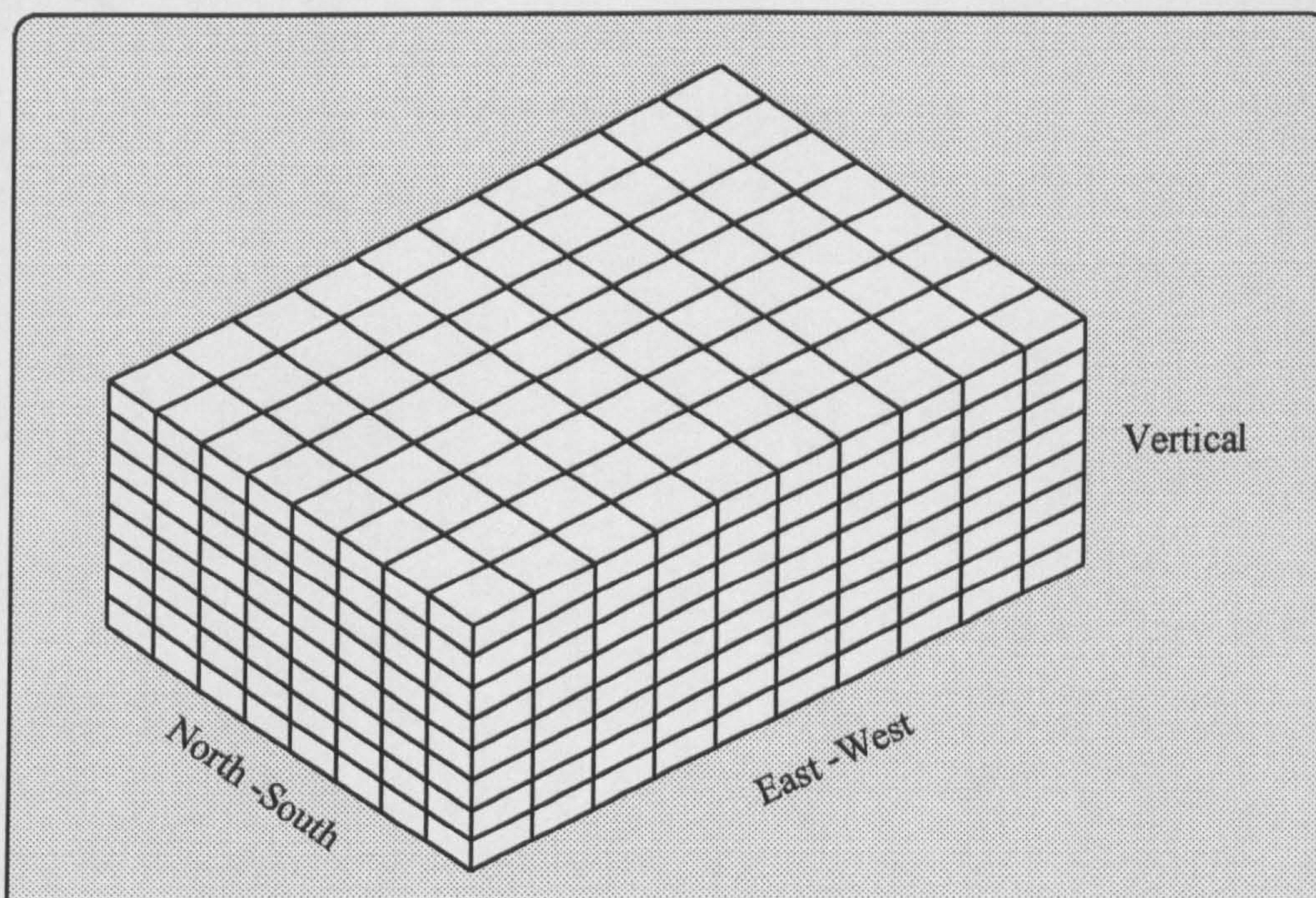


Figure 2.1- Three-dimensional fixed block model

After deciding on the block size a grade must be assigned to each block. Methods of doing so include the inverse distance squared method, the polygonal method, the triangulation method and the various geostatistical methods. Once the block grade model of the deposit is obtained, the next step is to create a revenue block model of the deposit. In other words to assign monetary value to each block. This is the value that would be obtained by mining and treating the block and selling its mineral contents. The revenue block model can be obtained by applying costs and prices to the block grade model providing a net revenue or net profit value for each block. In general, the following equation is used to determine a net value of each block:

$$NV = R - C$$

Where

NV	is the net value of a block
R	is the recoverable metal value
C	is the cost of mining and processing

In the revenue block model of the deposit air blocks have a zero value, waste blocks have a negative value representing the stripping cost and the ore block with sufficient grade have a positive value. A mine may consist of a large number of blocks each of which with an estimated grade or net profit. Since not all blocks are extracted, the objective of optimal pit design now can be defined as selecting a set of blocks which should be mined to maximise some criteria such as net profit or metal content.

Almost all computer algorithms of optimal design except the parameterization approach use a net profit as a criteria and their objective is to the find ultimate pit limit that maximise the net profit. Ideally, optimal pit limits must be determined on the basis of maximising net present value. As Whittle (1989) stated and paraphrased by Dowd and Onur (1993):

> ***"The pit outline with the highest net present value can not be determined until the block values are known; the block values are not known until a mining sequence is specified; and a mining sequence can not be specified until a pit outline is available."***

Thus the problem is intractable. One of the most common approaches to this problem is to design the pit limits on the basis of maximising net profit, and then determine the sequence of extraction of blocks, which is called the scheduling program. This is undertaken in such a way that the net present value will be a maximum. The scheduling program is excluded from this study.

Lerchs and Grossmann (1965) published the first paper describing a true optimal pit design algorithm in 1965. Since then many algorithms have been developed for determining optimal pit outlines. Some authors, namely, Kim (1978), Dowd and Onur (1993) and more recently Gill, Robey and Caelli (1996), have provided surveys and comparative studies of these methods. Kim classified the various methods of optimal design as "rigorous" and "heuristic" techniques. He used the word "rigorous" for the methods that have mathematical proofs such as graph theory and dynamic programming. Kim defined "heuristic" as an algorithm that works in nearly all cases but lacks a rigorous mathematical proof; examples include the various versions of the floating or moving cone method. The most common optimal design methods may be listed as follows:

1- Graph theory
2- Network or maximal flow techniques
3- Various versions of floating or moving cone
4- Korobov algorithm
5- Dynamic programming
6- Parameterization techniques
7- Other methods

2.2.2- Graph theory

Although many algorithms have been developed since 1965, only the Lerchs-Grossmann method (1965), based on graph theory, can be proved, rigorously, always to yield the optimal solution. However, disadvantages of this approach are complexity of the method, computing time and difficulty in incorporating variable pit slopes. This method converts the revenue block model of the deposit into a directed graph which is a simple diagram consisting of a set of small circles, called nodes or vertices, and a set of connecting arcs (lines with direction) used to indicate the relationship between the vertices. A vertex represents each block. Each vertex is assigned a mass that is equal to the net value of the corresponding block. Vertices are connected by arcs in such way as to represent the mining constraints. These arcs indicate which blocks should be removed before a particular block can be mined. Figure 2.2 shows a directed graph for a simple two-dimensional example in which the pit slope angle is 45° and the blocks are squares. In this example, to mine block 10 it is first necessary to remove blocks 2,3 and 4.

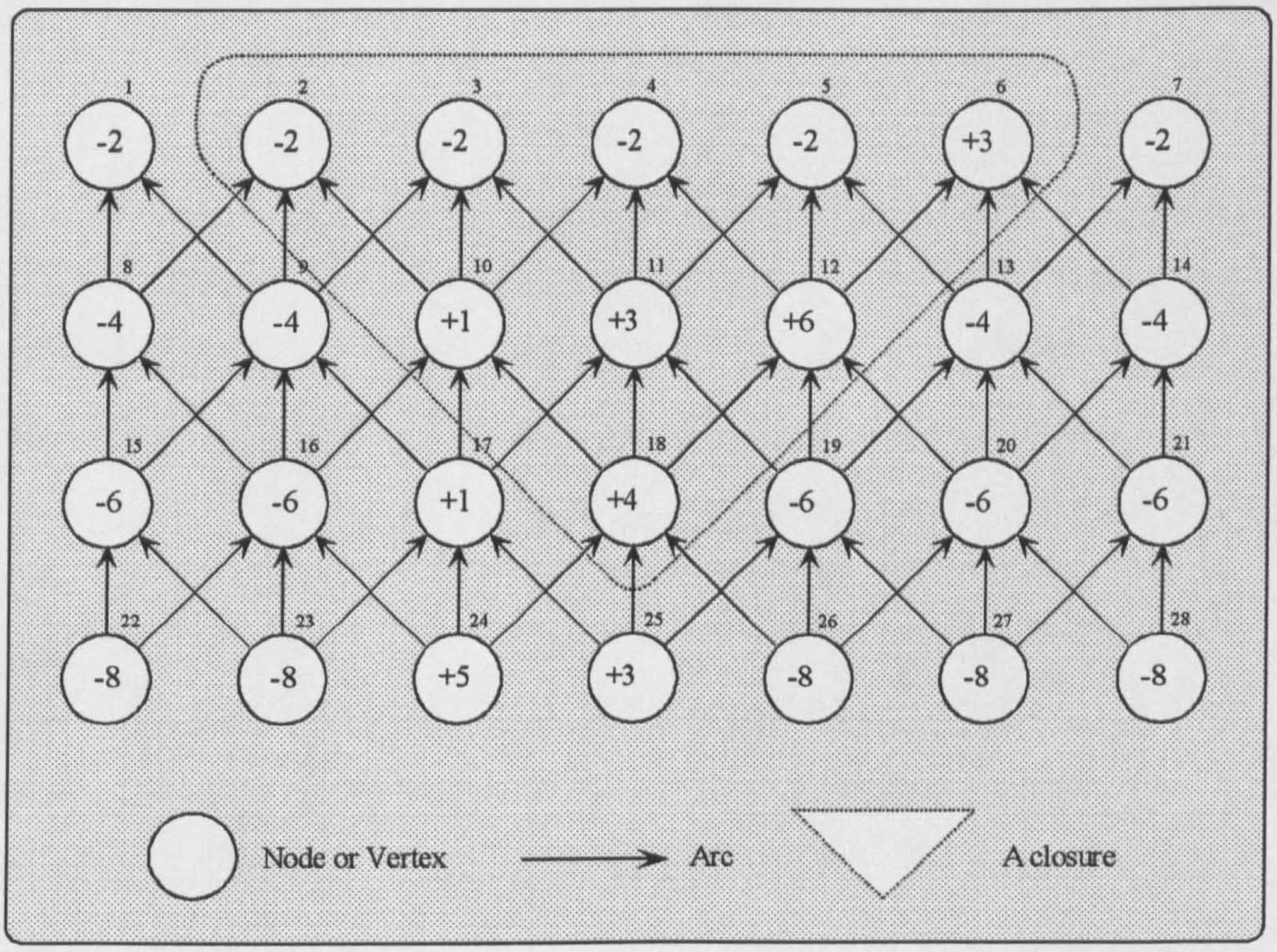


Figure 2.2- Directed graph representing a vertical section

A set of vertices is defined as closure if they represented as a feasible pit. Thus the vertices 2, 3, 4 and 10 is a closure. The value of a closure is the sum of the masses of the vertices within it. A maximum closure of the graph is the solution. Therefore this algorithm involves the finding of the maximum closure of the graph, in other words, the set of vertices or blocks yielding the maximum total value of the deposit. This is found by a set of rules. The algorithm of this method will be discussed in more details in the following chapter.

This algorithm is the only method that can be proved, rigorously, always to yield the true optimum pit. However, due to the disadvantages, such as complexity of the method, computing time and difficulty to incorporate variable pit slopes, many workers have continued to seek alternative methods. Many of the disadvantages are perceived, rather than real, and one of the objectives in this thesis is to demonstrate that the Lerchs-Grossmann algorithm can be adapted to overcome them.

The most recent algorithm based on graph theory is that of Zhao and Kim (1992). They reported that their algorithm is more efficient in terms of computer representation of arcs and, therefore, tree construction compared to the Lerchs-Grossmann algorithm. Zhao and Kim also claimed that their algorithm requires less memory and computing time to reach a solution than the Lerchs-Grossmann algorithm. The claim remains to be independently verified.

Computing time

There is no doubt that the Lerchs-Grossmann algorithm requires more computing time than other methods to reach a solution. Because of this problem many alternative algorithms have been developed to reduce computing time. The longer computing time is the price to be paid for a guaranteed optimum pit design. Computing time is fast becoming irrelevant with the advent of faster PC computers using 32-bit operating systems provided that the software is adapted to the improved hardware. Modern PCs can now run the Lerchs-Grossmann algorithm for block models that only a few years ago would have required large mainframe computers.

Complexity

Another perceived disadvantage of the method is the complexity of the mathematics and programming required. This argument implicitly holds that both are beyond the capability of the average mining engineer. This is not, however, a valid reason for not using the method. It is not necessary for the engineer to have a detailed knowledge of the algorithm once it has been coded into a validated software package. Few, if any, engineers would understand the algorithms employed in CAD or contouring packages but this does not impede their widespread use. It is, however, essential for the user to be aware of any limitations on software implementations of this (or any other) algorithm.

Difficulties in incorporating variable slope angles

There may be many types of rocks, joints and discontinuities within a deposit and, because of stability requirements, pit slopes may vary throughout the pit. Any truly optimal pit design algorithm must, therefore, take into account variable slope angles. The original formulation of the Lerchs-Grossmann algorithm was limited to one vertex to the left, one vertex to the right and one vertex above to define mining constraints (and, implicitly, pit slopes) which makes it difficult to incorporate variable slope angles. This procedure implicitly assumes that the dimensions of the blocks determine the pit slope. Different pit slopes, therefore, require different block dimensions that may not correspond to the required bench height. Furthermore, it is not possible to use different pit slopes for different parts of the deposit. Many attempts have been made to overcome this difficulty, e.g., Chen (1976), Lipkewich and Borgman (1969), Zhao and Kim (1992), Dowd and Onur (1993). Alford and Whittle (1986) have also reported incorporation of variable pit slopes into the algorithm but they give no details. Lipkewich and Borgman (1969) proposed a 'knight's move' pattern to approximate a conical expansion to the surface. Zhao and Kim (1992) defined a method based on cone templates.

Dowd and Onur (1993) used the idea of cone templates to derive a general technique to deal with the problem. This method involves the construction of a cone, or extraction volume, from the block on a given level to the surface by joining rings or envelopes of blocks corresponding to the pit slope angles. If the midpoint of any block lies within the extraction cone it is assumed that it must be removed before removing the base block. The program developed by Onur (1992) using this technique does not give the correct solution in all cases.

It is apparent that all these previous works rely on using one slope angle in the design. In practice, it may be necessary to use different slope angles for each direction and for each domain sector. In the work presented here the method of Dowd and Onur (1993) has been modified to derive a general technique for variable slope angles which will be discussed in the following chapter.

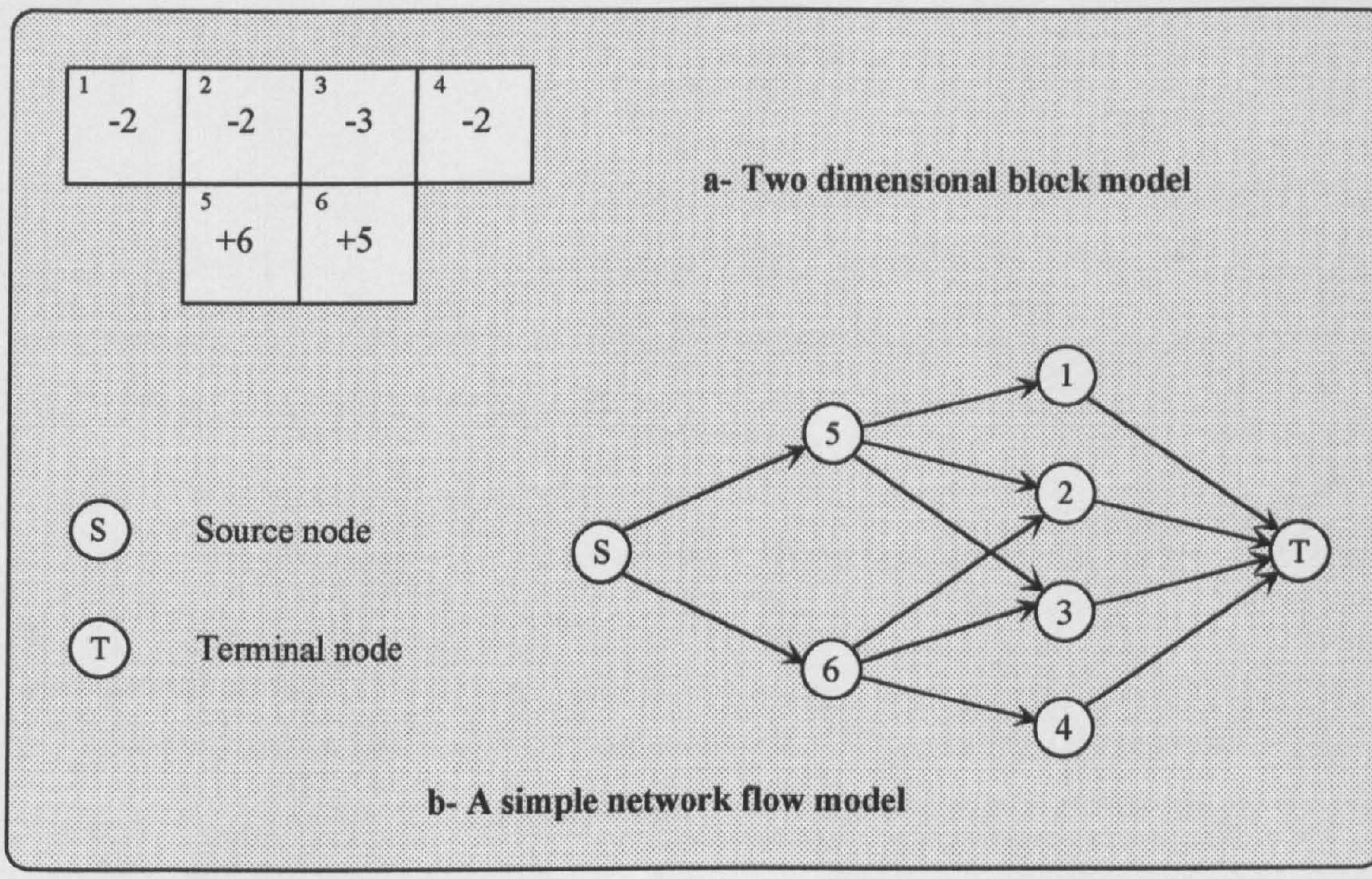


Figure 2.3- Simple network representing a vertical section

2.2.3- Network or maximal flow techniques

The maximum, or network, flow problem is a classical problem in the field of operational research. It is possible to formulate the ultimate optimum pit limit problem as an equivalent network flow problem. Johnson (1968) was the first to apply this technique to the ultimate pit outline problem. To apply this method to the optimum open pit problem, a network is constructed with a source and terminal node. A node represents each block.

Nodes representing ore blocks are connected by arcs to the source node and all nodes representing waste blocks are connected to the terminal node. In addition, each ore block node is connected to any overlying waste block nodes in such a way that the arcs represent pit slope constraints. Figure 2.3 shows an example of a simple network representing a cross-section of an orebody for which the pit slope is 45° and the blocks are squares. The arc capacity from the source node to an ore block node is assigned the block value. The arc capacity from a waste block node to the terminal node is assigned the absolute block value. Infinity capacity is assigned to the arcs connecting ore block nodes to overlying waste block nodes. Once the network is created the problem is to find the maximum flow through the terminal node. A number of algorithms have been developed to solve the maximum flow problem (Ahuja and Orlin, 1989). Johnson and Barnes (1988) applied the labeling algorithm developed by Ford and Fulkerson (1956) to solve this problem. Yegulalp and Arias (1992) used an excess scaling algorithm that was presented by Ahuja and Orlin (1989).

Although this technique has been used with some success to determine optimal pits it is apparent from the literature that it has not been adopted to any great extent largely because it has the same perceived disadvantages as the Lerchs-Grossmann algorithm.

2.2.4- Floating or moving cone method

The floating, or moving, cone approach is the simplest method for determining the optimal pit outline and it is perhaps the most popular and widely used of heuristic algorithms. This method, which is the most common alternative to the Lerchs-Grossmann algorithm, works on a revenue block model of the deposit and was first described by Carlson, Erickson, O'Brain and Pana (1966). The floating, or moving, cone method involves, for each positive (ore) block, constructing a cone, with sides oriented parallel to the pit slope angles, and then determining the value of the cone by summing the values of blocks enclosed within it. If the value of the cone is positive, all blocks within the cone are mined. This process starts from the uppermost level and moves downward searching for positive blocks. The process continues until no positive cones remain in the block model. A flow-chart of the algorithm for this method is shown in Figure 2.4. The method is very simple, is easy to program and reaches a solution in a shorter time than any other method and is an order of magnitude faster than the Lerchs-Grossmann algorithm. These methods do not, however, always yield a true optimum.

This method can best be explained by a simple example applied to a vertical section as shown in Figure 2.5. For the sake of simplicity slope angles are assumed to be 45° and the blocks are squares. There are five positive blocks in this section and there are, therefore, five cones. The optimal pit outline on this section can be determined from the following steps:

1- The procedure starts from the first level. There is only one ore block on this level, block (1, 6). As there are no overlying blocks and the value of this cone is +1, it is removed (Figure 2.6a).

2- There are two ore blocks on the second level, blocks (2, 2) and (2, 5). The extraction cone of block (2, 2) comprises blocks (1, 1), (1, 2) and (1, 3) and its value is:

   $$-1-1-1+2 = -1$$

   As the value is negative, this cone is not removed.

3- The value of the cone corresponding to block (2, 5) is:

   $$-1-1+3 = +1$$

   The cone is positive and all blocks within it are mined (Figure 2.6b).

4- There are two ore blocks on the third level, blocks (3, 3) and (3, 5). The value of the cone corresponding to block (3, 3) is:

   $$-1-1-1+2-2-2+7 = +2$$

   Thus the cone is positive and all blocks within it are mined (Figure 2.6c).

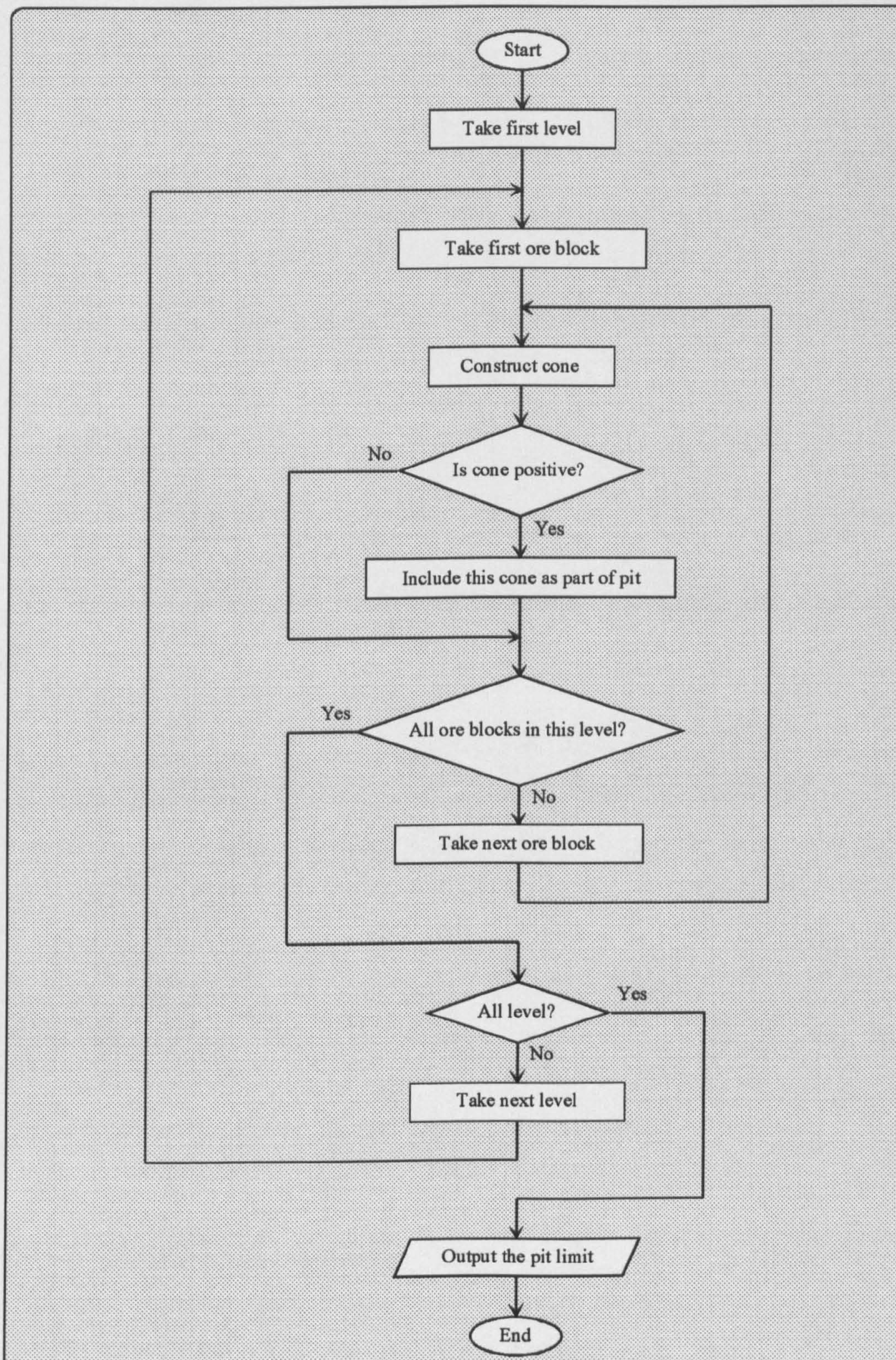


Figure 2.4- Flow chart of the floating cone method

	1	2	3	4	5	6	7
1	-1	-1	-1	-1	-1	+1	-1
2	-2	+2	-2	-2	+3	-2	-2
3	-3	-3	+7	-3	+1	-3	-3

Figure 2.5- Block values on a vertical section

5- Finally, the value of the cone corresponding to block (3, 5) is negative, and thus it is not mined.

Hence, the total value of this pit is +4 and the overall stripping ratio is 7/4 (Figure 2.6d).

The use of the floating cone method for three-dimensional cases is the same as the for the two-dimensional case presented above, except that a true three-dimensional cone must be constructed from ore blocks to the surface with the side angles corresponding to the pit slope angles.

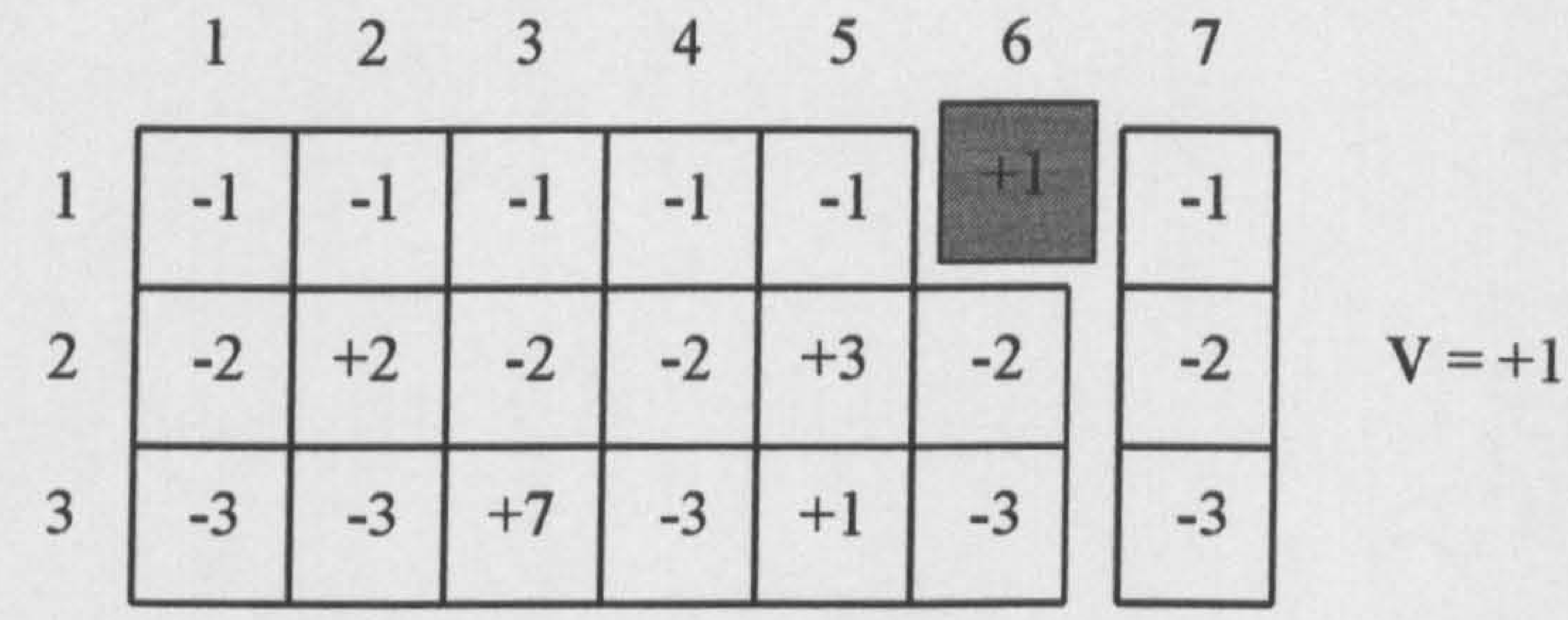
An example demonstrating a case in which the floating cone method does not yield a true optimum pit is shown in Figure 2.7a

The cone for the block in row 3 and column 4 has a value of:

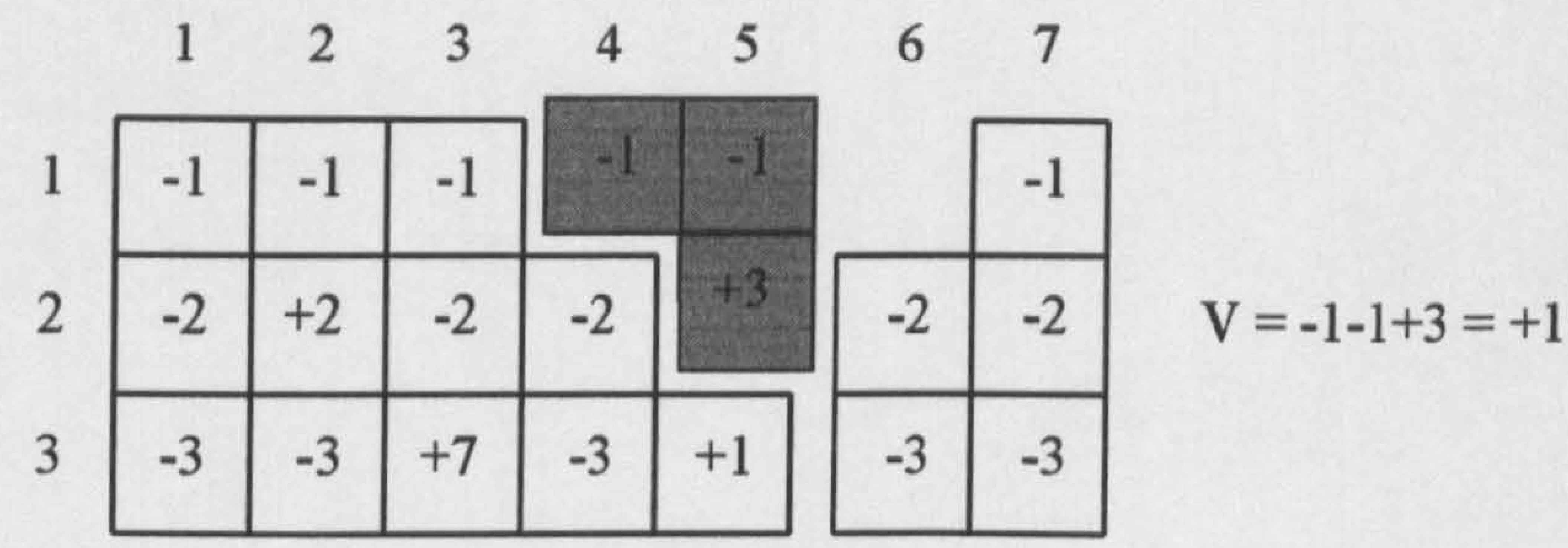
$$V = -1-1-1-1-1-2-2-2+8 = -3$$

As the value of the cone is negative (Figure 2.7b), the cone is not mined. Similarly, the cone for the block in row 3 and column 5, has a value of (Figure 2.7c):

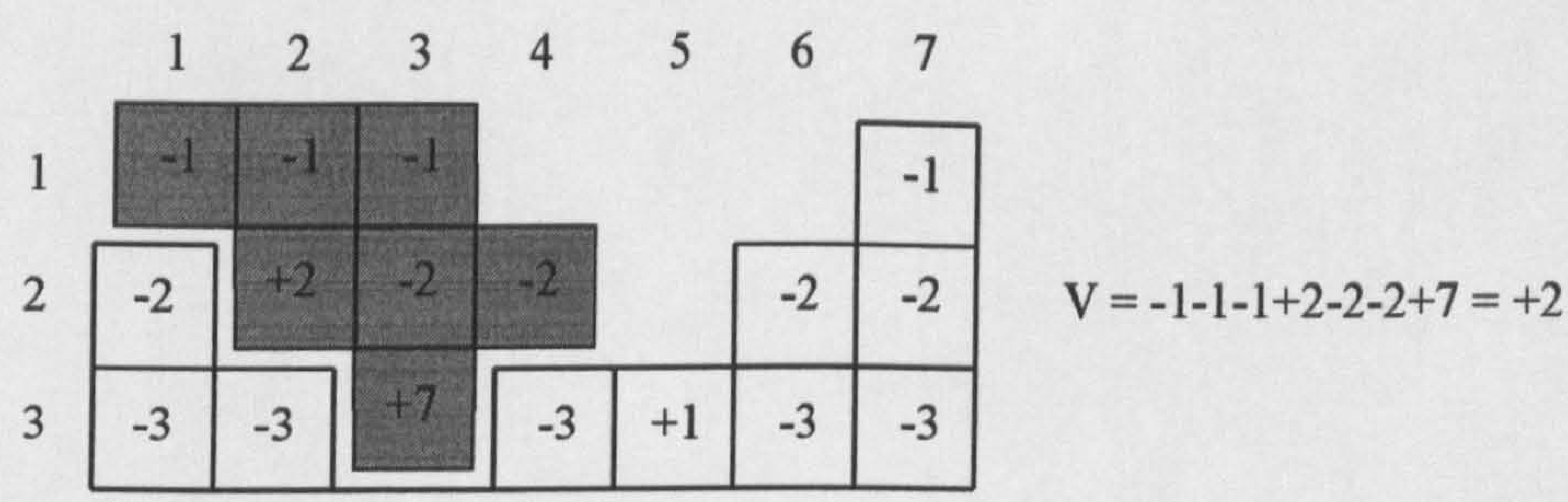
$$V = -1-1-1-1-1-2-2-2+9 = -2$$



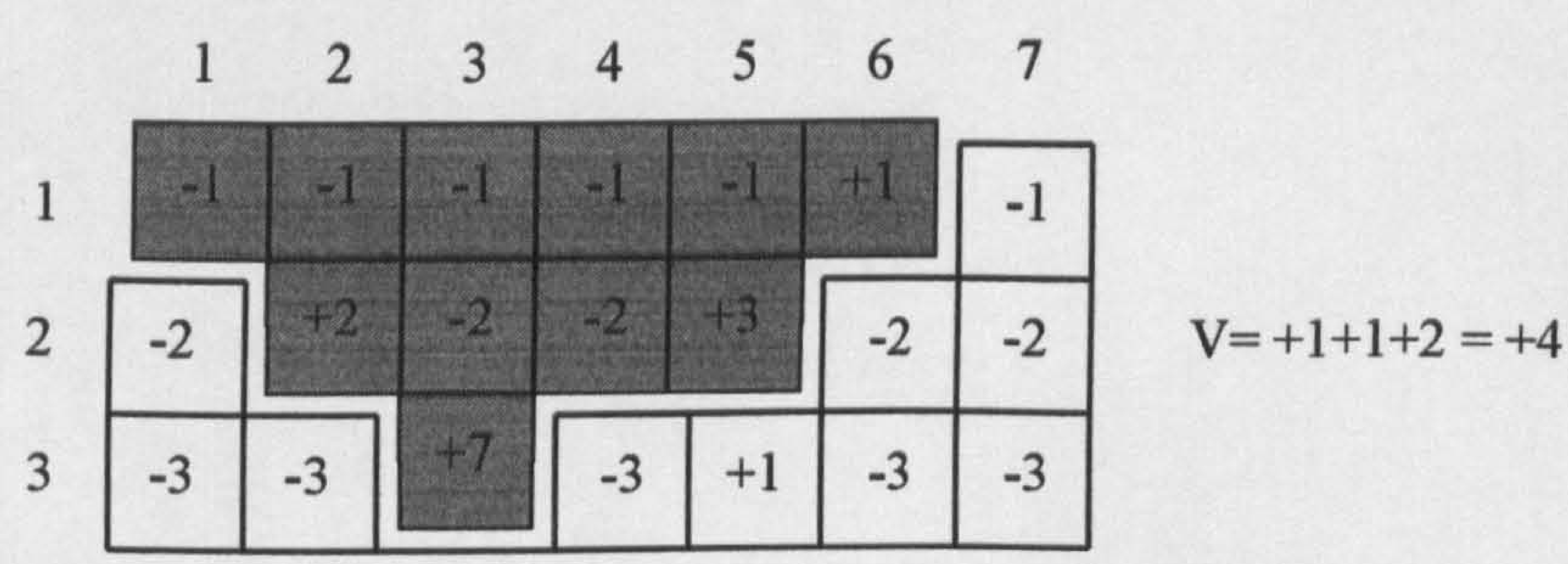
a- First positive cone



b- Second positive cone

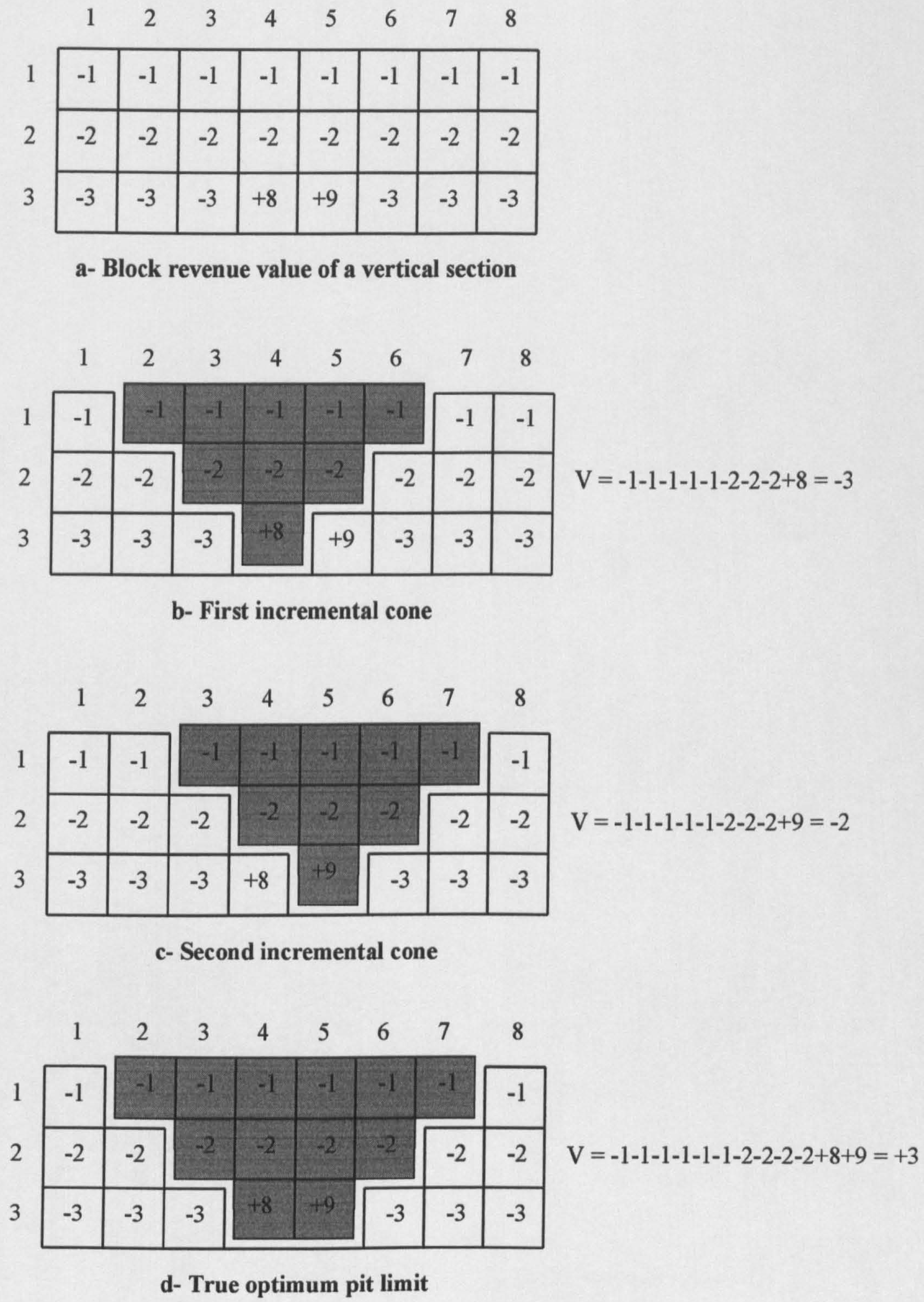


c- Third positive cone



d- The optimum pit limit

Figure 2.6- Floating cone method applied to a vertical section



a- Block revenue value of a vertical section

b- First incremental cone

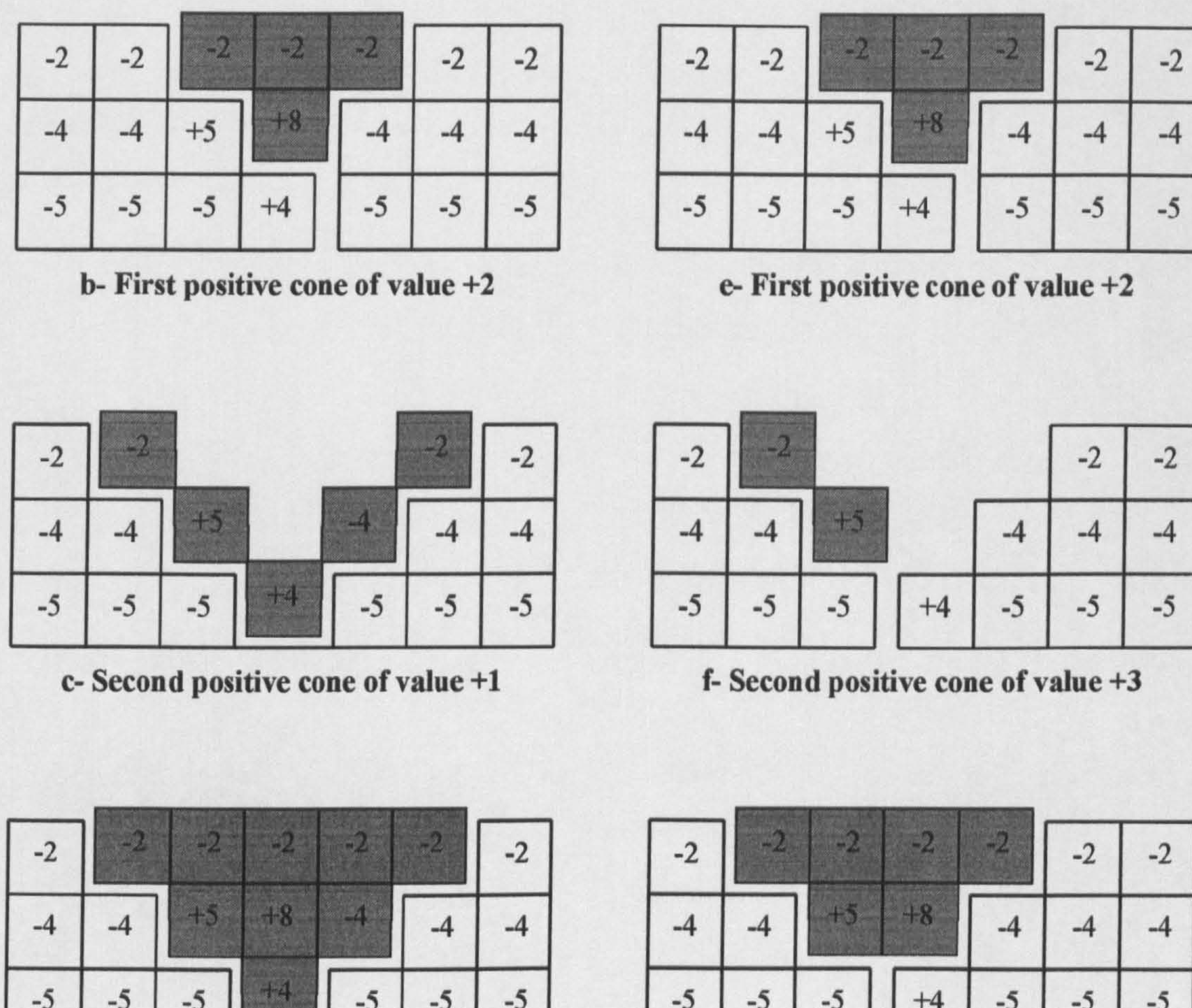
c- Second incremental cone

d- True optimum pit limit

Figure 2.7- An example in which the floating cone method does not yield a true optimum

	1	2	3	4	5	6	7
1	-2	-2	-2	-2	-2	-2	-2
2	-4	-4	+5	+8	-4	-4	-4
3	-5	-5	-5	+4	-5	-5	-5

a- Block revenue values of a section



b- First positive cone of value +2

e- First positive cone of value +2

c- Second positive cone of value +1

f- Second positive cone of value +3

d- Pit limit of value +3

g- Pit limit of value +5

Direction of search for ore block:

Figure b to d	**top down from left to right**
Figure e to g	**top down from right to left**

Figure 2.8- An example where the floating cone method yields different results

Again this cone is not be mined. Using this method, therefore, no blocks would be mined. However, if two blocks are considered together, as shown in Figure 2.7d, the removal cone for these blocks have value of:

$$V = -1-1-1-1-1-1-2-2-2-2+8+9 = +3$$

and the removal cone for the combined blocks is positive indicating that they can be mined profitably. In a similar way, it would be necessary to consider all combinations of any number of blocks. This situation often occurs in real deposits and the floating cone method is incapable of yielding the true optimum condition.

Another problem with this method is that it produces different results depending upon the direction of search for the ore blocks. For example, consider the simple example shown in Figure 2.8. If the search is carried out top down from left to right, this method will yield a pit of value +3 (Figure 2.8b, 2.8c and 2.8d). However, if the search for the ore blocks were carried out top down from right to left, a pit of value +5 would be obtained (Figure 2.8e, 2.8f and 2.8g). This method may, therefore, produce a different ultimate pit limit for the same deposit depending on the selected starting point.

Lemieux (1979) proposed a number of techniques to overcome these problems, but the various forms of the floating cone method remain heuristic due to the lack of any mathematical proof. In spite of these problems, the method is the most widely used because of the following advantages:

1- The method is straightforward, easy to understand and mining engineers feel comfortable with it.
2- It is easy to use different pit slopes in different parts of the orebody.
3- Writing a computer program based on this algorithm is easy.
4- The method requires significantly less computing time than any other method to reach a solution.

	1	2	3	4	5	6	7
1	-1	-1	-1	-1	-1	+1	-1
2	-2	+2	-2	-2	+3	-2	-2
3	-3	-3	+7	-3	+1	-3	-3

a- Block revenue values of a section

	1	2	3	4	5	6	7
1	-1	-1	-1	-1	-1		-1
2	-2	+2	-2	-2	+3	-2	-2
3	-3	-3	+7	-3	+1	-3	-3

V = +1

(b)

	1	2	3	4	5	6	7
1	0	0	-1	0	0		-1
2	-2	0	-2	-2	+1	-2	-2
3	-3	-3	+7	-3	+1	-3	-3

(c)

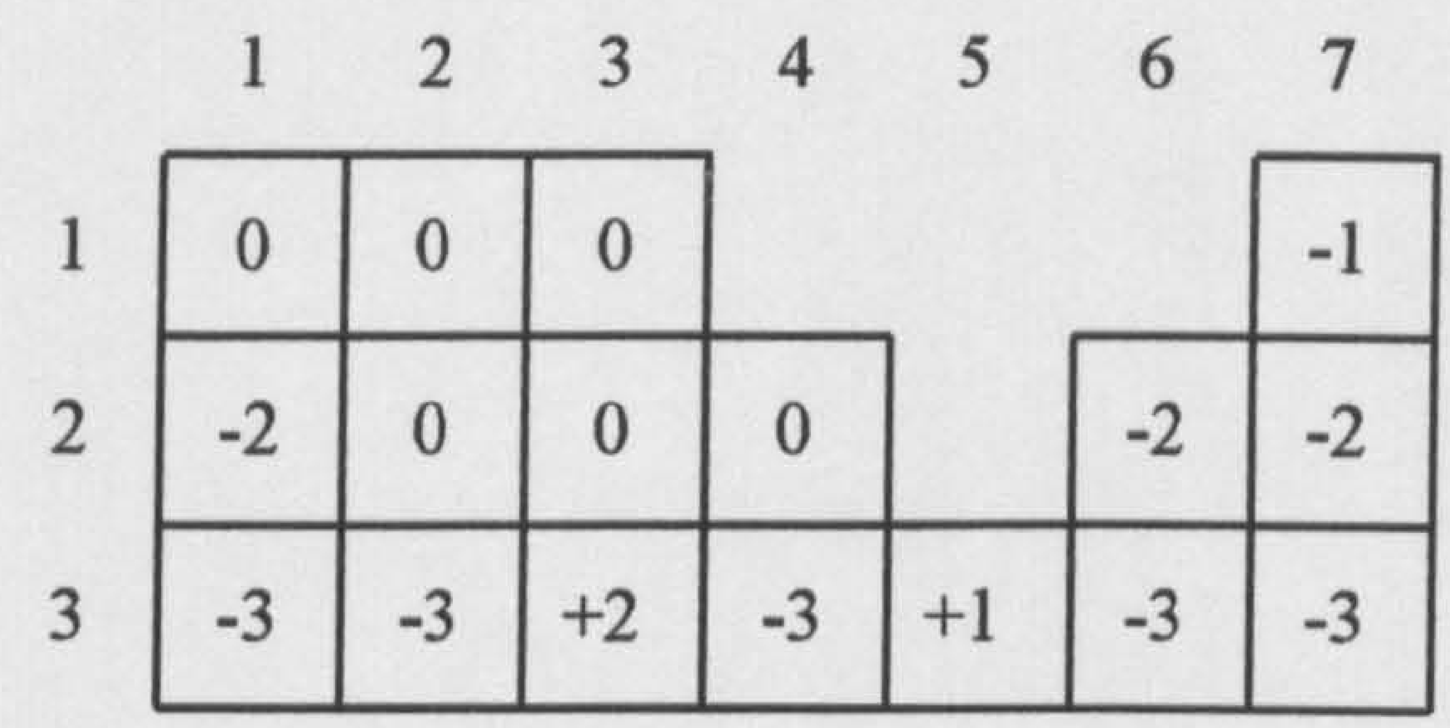
	1	2	3	4	5	6	7
1	-1	-1	-1				-1
2	-2	+2	-2	-2		-2	-2
3	-3	-3	+7	-3	+1	-3	-3

V = +1+1 = +2

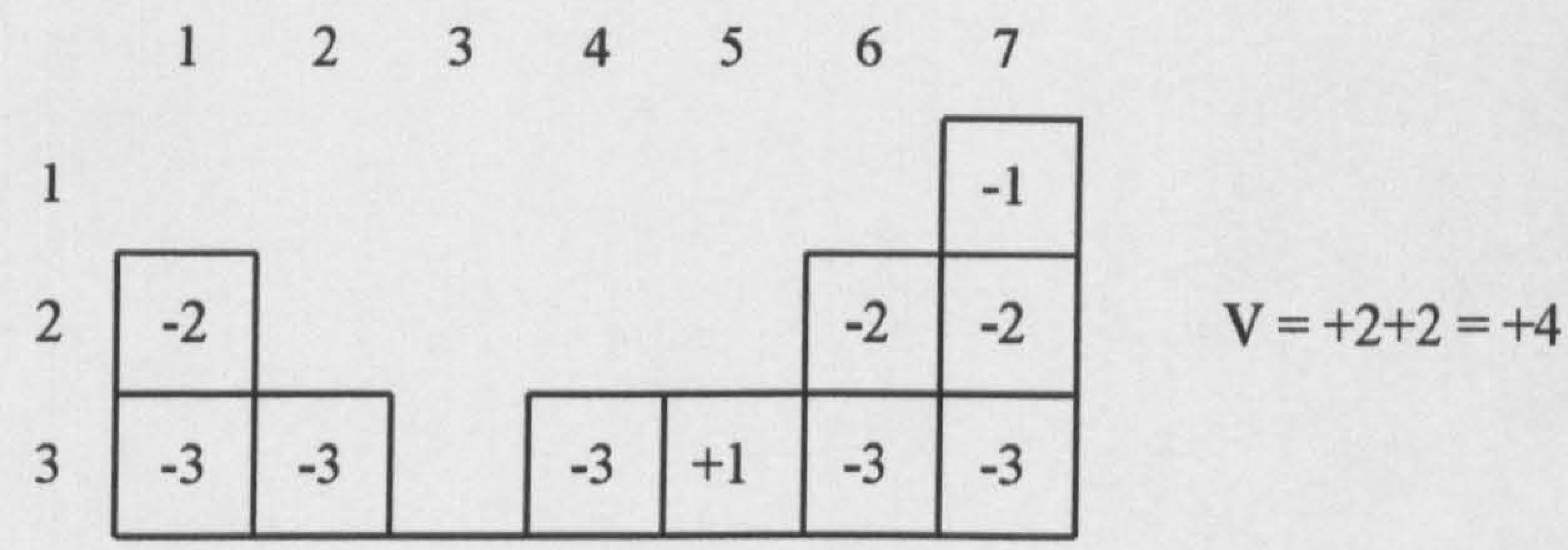
(d)

Figure 2.9- Korobov algorithm applied to a simple example

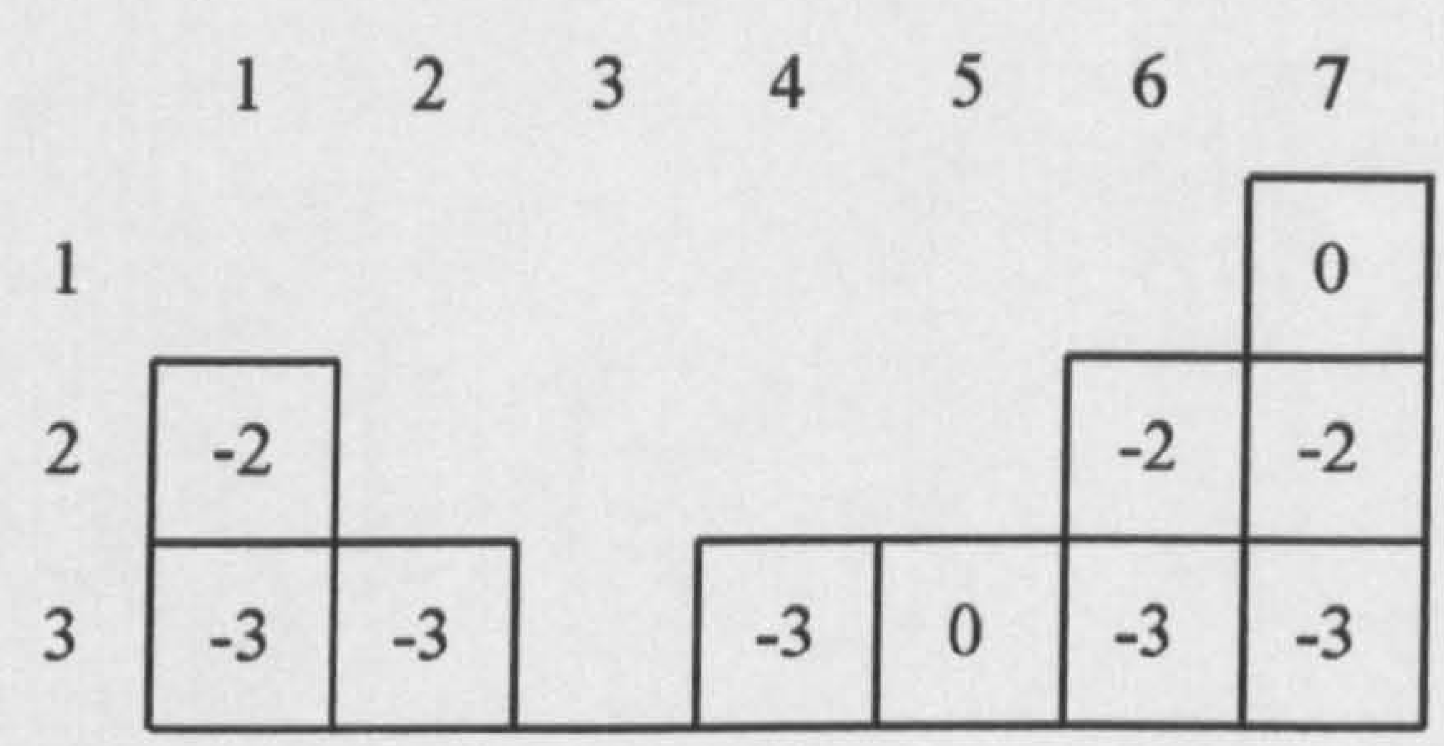
(continued ....)



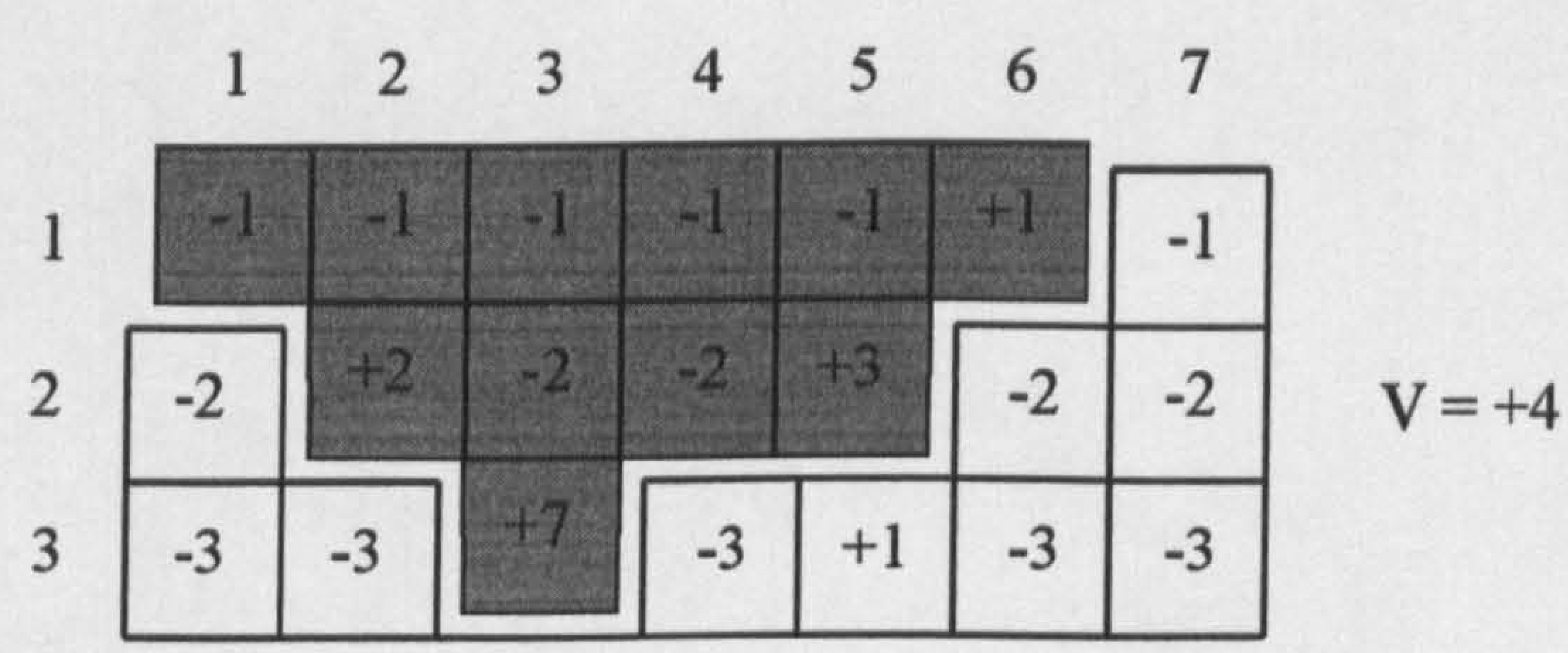
(e)



(f)



(g)



h- The optimum pit limit

Figure 2.9- (........ continued ....)

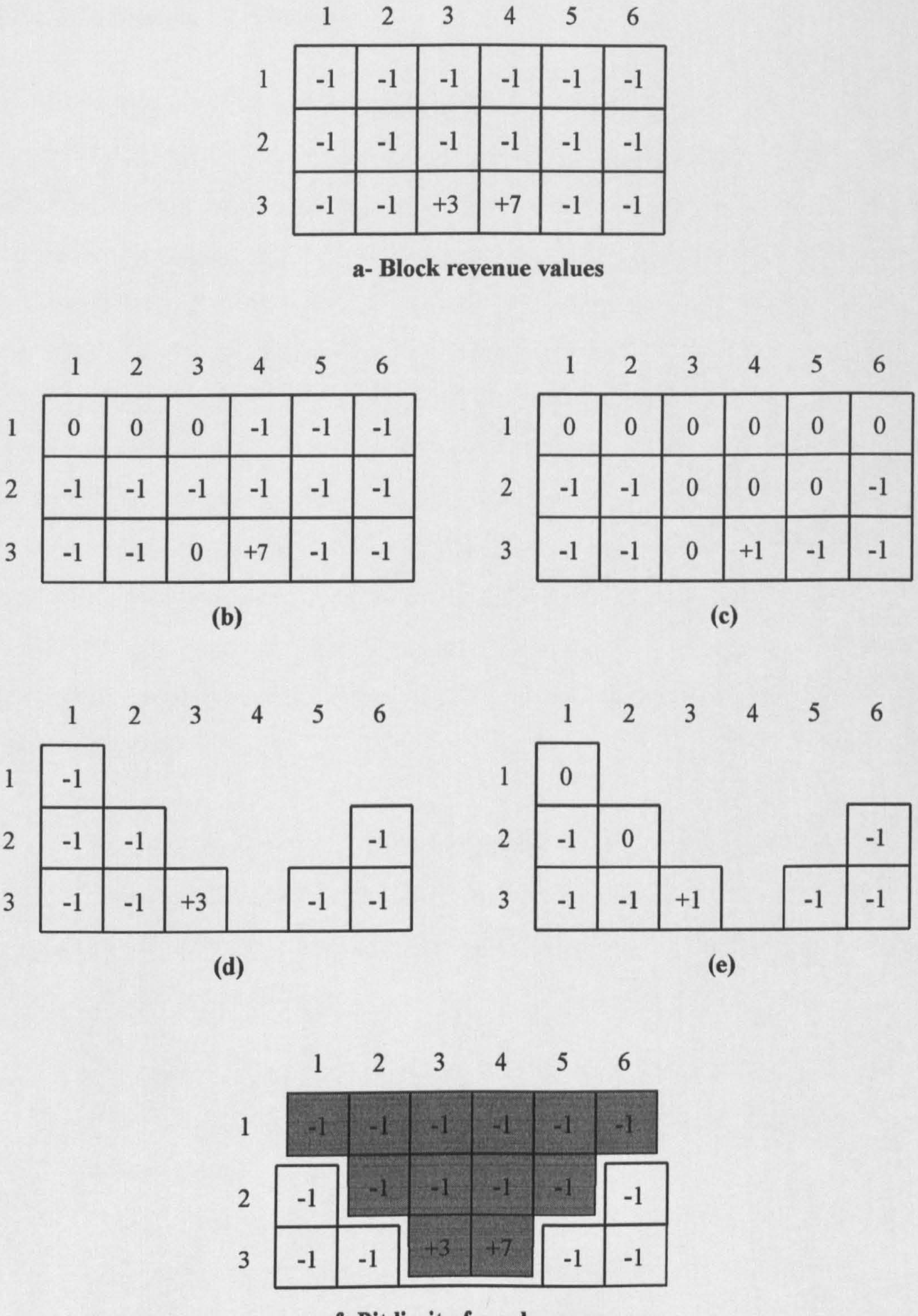


Figure 2.10- An example of the Korobov algorithm, which does not yield an optimal solution

2.2.5- The Korobov algorithm

This approach is a cone-based algorithm originally developed by Korobov (1974) and was reported by David, Dowd and Korobov (1974), and Dowd and Onur (1992 and 1993). The first step of the algorithm is to construct, for every ore block in the revenue block model, an extraction cone with sides parallel to the pit slope angles. The second step is to allocate positive values within each extraction cone against negative values within the cone until no negative block remains or until the total value of the positive blocks have been allocated. If the value of the block on which an extraction cone is constructed remains positive after the allocation has been completed, then this extraction cone is included as a member of the optimum solution set. When a non-empty extraction cone is added to the solution set, the algorithm starts again from the beginning with the original block values restored to the blocks not yet extracted from the block model. If an extraction cone is empty, the positive block is added to the solution and the algorithm continues for the next ore block. This process is repeated until no positive block remains in the block model.

To demonstrate how the Korobov algorithm works, it is applied to the simple two-dimensional revenue block value shown in Figure 2.5 in which the pit slope is 45° and the blocks are squares. The following procedure is carried out to determine optimum pit limits based on this algorithm:

1- The algorithm starts from the uppermost level. There is only one positive block on this level and this is removed and included in the solution set (Figure 2.9b).

2- There are two positive blocks on the second level, blocks (2, 2) and (2, 5). The extraction cone of block (2, 2) includes blocks (1, 1), (1, 2) and (1, 3). A value of +1 from block (2, 2) is allocated to block (1, 1), leaving block (1, 1) with a value of zero and block (2, 2) with a value of +1. The remaining +1 value of block (2, 2) is then allocated to block (1, 2), leaving both blocks with a value of zero. In the same way, the extraction cone of block (2, 5) comprises blocks (1, 4), (1, 5) and (1, 6) of which block (1, 6) has already been removed. The value of block (2, 5) is allocated against blocks (1, 4) and (1, 5), leaving both blocks with values of zero and block (2, 5) with a residual value of +1 (Figure 2.9c). As this block remains positive and its extraction cone is non-empty, it is added to the optimal solution set which now has a value of +2 and the algorithm starts from the beginning with the original block values restored to all non-removed blocks (Figure 2.9d).

3- Two units of the value of block (2, 2) are allocated against blocks (1, 1) and (1, 2), leaving all three blocks with a value of zero. Now consider block (3, 3). Its extraction contains only blocks (1, 3), (2, 3) and (2, 4) with negative values. One unit is allocated for block (1, 3) and four units are allocated against blocks (2, 3) and (2, 4). After the allocation (Figure 2.9e) block (3, 3) remains positive with a value of +2. This block and all blocks within its extraction cone are added to the solution set. The net value of this extraction cone is +2, and the net pit value becomes +4. As the extraction cone for block (3, 3) is non-empty, the algorithm starts from the beginning with the original block values restored to all non-removed blocks (Figure 2.9f).

4- Finally, only one positive block remains, block (3, 5). Its extraction cone contains blocks (1, 7) and (2, 6) both with negative values. The value of block (3, 5) is allocated against block (1, 7), leaving both blocks with a value of zero. As the extraction cone for this block contains a block with a negative value, block (2, 6), and the total value of the positive block been allocated, the cone cannot be added to the solution set (Figure 2.9g). This algorithm gives the final pit shape shown in Figure 2.9h with a value of +4.

This method is simple and the computing time required is shorter than that of the Lerchs-Grossmann method. Although the Korobov algorithm overcomes some weakness of the floating cone method, in some circumstances the method does not yield a true optimum solution. For example, consider the simple, two-dimensional example shown in Figure 2.10a, taken from Dowd and Onur (1993). There are two positive blocks in the revenue block model. The value of the first positive block, block (3, 3), is allocated against blocks (1, 1), (1, 2) and (1, 3), leaving all four blocks with a value of zero (Figure 2.10b). Now consider the second positive block, block (3, 4). The extraction cone for this block contains six blocks with negative values, each of which becomes zero after the allocation of the value of block (3, 4). When allocation has been completed, this block remains positive with a value of +1 (Figure 2.10c), and therefore this block and its extraction cone are added to the solution set. As the net value of this cone is -1, the net value of the pit is also -1 at this stage. As the extraction cone for block (3, 4) is non-empty, the algorithm starts from the beginning with the original block values restored to all non-removed blocks (Figure 2.10d).

There are now two blocks with negative values (-1) within the extraction cone of block (3, 3), each of which becomes zero when the value is allocated from block (3, 3). After allocation, this block remains positive with a value of +1 (Figure 2.10e). Thus this block and its extraction are added to the solution set which yields a pit with a net value of zero (Figure 2.10f). This algorithm therefore yields a pit with a net profit of zero.

In the above example, blocks that are common to both extraction cones cause the error. Dowd and Onur (1992 and 1993) identified the fault in the Korobov algorithm and modified the algorithm to yield a true optimum. They refer to the process of allocating positive block values to negative block values as the latter "paying" for the former; a negative block is "paid for" when the allocation process increases its value to zero. Their correction to the Korobov algorithm is based on the following logic:

> ***"If two or more cones have blocks in common, then blocks not in common must be paid for first; common blocks are only paid for after all blocks not in common have been paid for."***

For example, consider again the two-dimensional example shown in Figure 2.10a. Blocks (1, 2), (1, 3), (1, 4), (1, 5), (2, 3) and (2, 4) are common to the extraction cones of blocks (3, 3) and (3, 4). Blocks (1, 1) and (2, 2) belong to the extraction cone of block (3, 3) but are not members of the extraction cone of block (3, 4). The same is true for blocks (1, 6) and (2, 5) which are members of the extraction cone of block (3, 4).

	1	2	3	4	5	6
1	0	-1	-1	-1	-1	0
2	-1	0	-1	-1	0	-1
3	-1	-1	+1	+5	-1	-1

(a)

	1	2	3	4	5	6
1	0	0	0	0	0	0
2	-1	0	0	0	0	-1
3	-1	-1	0	0	-1	-1

(b)

Figure 2.11- An example of the corrected Korobov algorithm applied to a simple example

The non-common blocks must be paid for first. Starting with the extraction cone of block (3, 3), one unit is allocated to each of blocks (1, 1) and (2, 2), leaving both these blocks with a value of zero and block (3, 3) with a residual value of +1. In the same way, a total of two units from block (3, 4) are allocated against blocks (1, 6) and (2, 5) leaving both these blocks with a value of zero and block (3, 4) with a value of +5 (Figure 2.11a).

When the allocation is completed for the non-common blocks, the algorithm starts again by paying for the blocks that are common to the extraction cones of both blocks. One unit from block (3, 3) is allocated to block (1, 2), leaving both blocks with a value of zero. Five units from block (3, 4) are allocated to (1, 3), (1, 4), (1, 5), (2, 3) and (2, 4), leaving all the blocks with a value of zero (Figure 2.11b).

After allocation, none of the positive blocks, on which the extraction cones are constructed, remain positive, so no blocks are added to the optimum solution set. As there are no other positive blocks, the algorithm stops and the solution yielded is not to mine any blocks. Using the original version of the algorithm, both extraction cones were part of the solution set; in the revised version neither extraction cone belongs to the solution set.

It is apparent that the corrected form of the Korobov algorithm produces good results and the method is faster and simpler than the Lerchs-Grossmann algorithm. However, due to the lack of any rigorous mathematical proof, the method has not been accepted as a general method for the determination of the optimum pit.

2.2.6- Dynamic programming

Dynamic programming is a reasonably well used method in the field of operational research and can be used to find an optimum for sequential decision problems. The problem is divided into small sub-problems (stages) for each of which an optimal solution is found. The technique was developed by Bellman (1957) and it has been applied to a number of mining problems such as optimal cut-off grades and production rates (Dowd, 1976 and 1980), optimal open pit design (Lerchs and Grossmann, 1965, Johnson and Sharp, 1971, Koenigsberg, 1982) production scheduling (Onur and Dowd, 1993) and grade control in sub-level open stoping (Dowd and Elvan, 1987).

The first two-dimensional, dynamic programming algorithm for determining the optimal pit was presented by Lerchs and Grossmann (1965) in the same paper in which they introduced the general graph theory method of pit optimisation. Like the manual method, this technique designs the pit on vertical sections and some smoothing will normally be required between sections. Although the pit is optimal on each section, the ultimate pit resulting from the smoothing is probably not optimal.

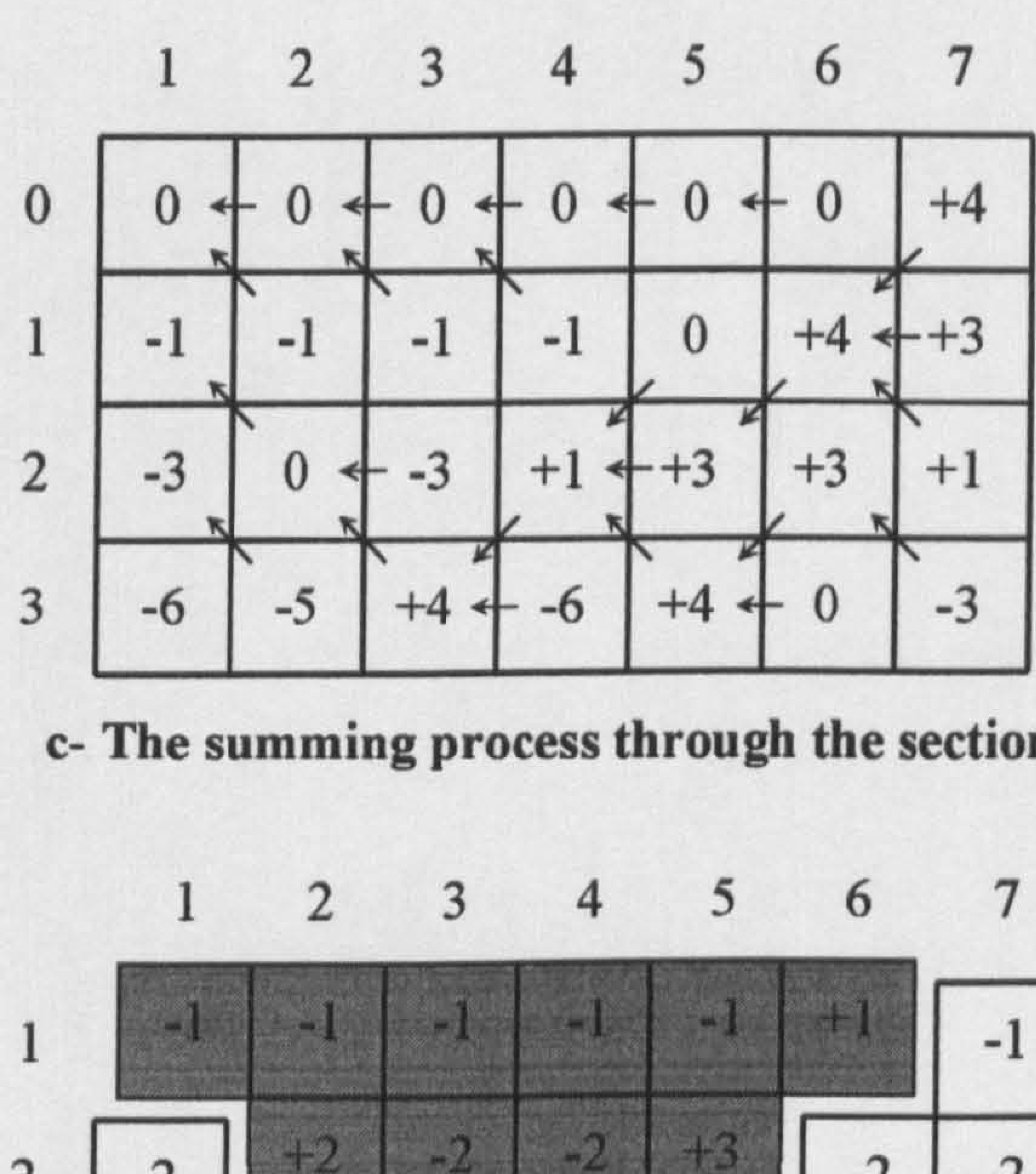
The two-dimensional dynamic programming algorithm is simple and easy to program. The algorithm can best be expressed in the following steps and by reference to vertical section shown in Figure 2.5:

	1	2	3	4	5	6	7
1	-1	-1	-1	-1	-1	+1	-1
2	-2	+2	-2	-2	+3	-2	-2
3	-3	-3	+7	-3	+1	-3	-3

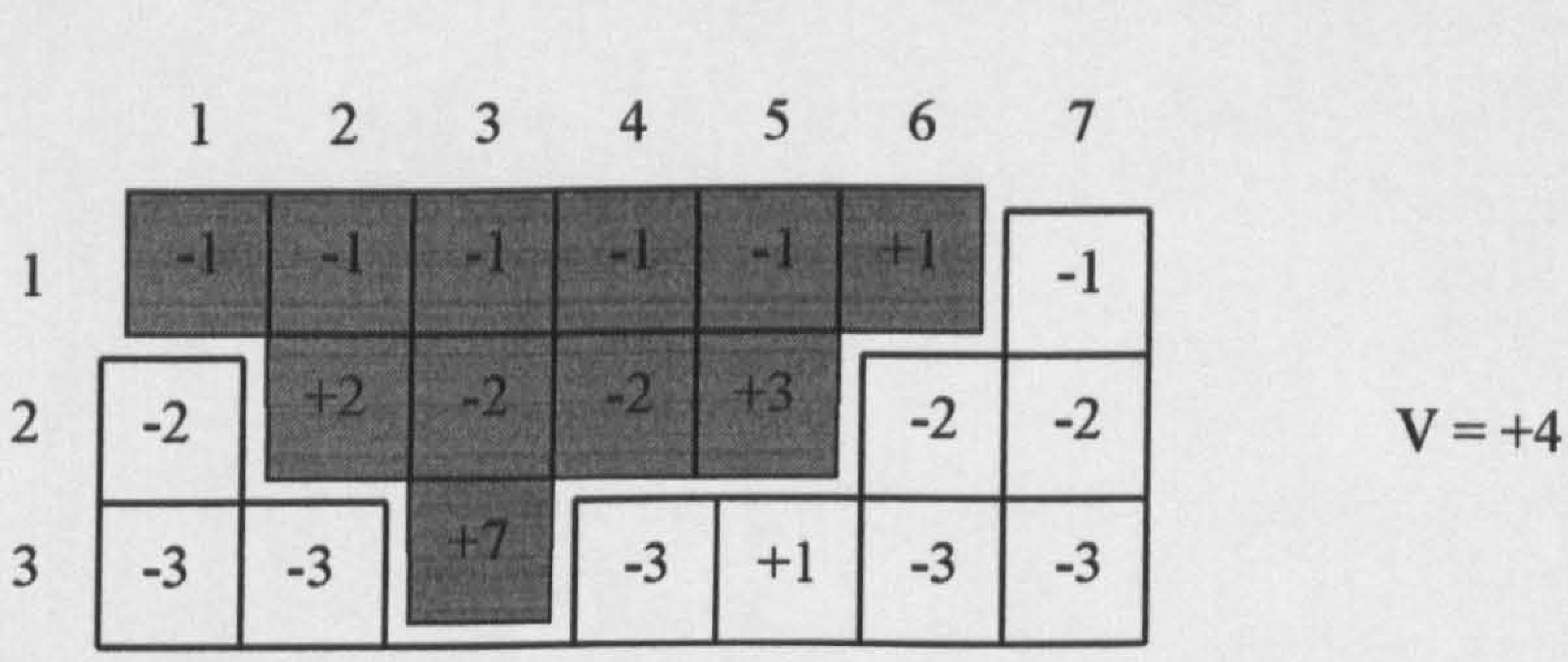
a- Block revenue values of a vertical section

	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0
1	-1	-1	-1	-1	-1	+1	-1
2	-3	+1	-3	-3	+2	-1	-3
3	-6	-2	+4	-6	+3	-4	-6

b- Completed cumulative sums



c- The summing process through the section



d- The optimum pit limit

Figure 2.12- Dynamic programming method applied to a vertical section

Step 1- Add row zero containing profit values of zero and calculate cumulative the column value for each block starting from the top and working down (Figure 2.12b), that is:

$$M_{ij} = \sum_{k=1}^{i} m_{kj}$$

Where

m_{kj} is the value of the block in row k and column j

M_{ij} is the cumulative column value for the block in row i and column j

Step 2- Start from the first column at the left end of the section and work downwards. Calculate the overall cumulative value (P_{ij}) which is the sum of the cumulative value of the block (M_{ij}) and the highest value in:

a- the block directly above and to the left

b- the block on the left

c- the block directly below and to the left

$$P_{ij} = M_{ij} + Max\{P_{i-1,j-1}, P_{i,j-1}, P_{i+1,j-1}\}$$

After calculation of P_{ij} draw an arrow pointing from the original block to the block with the highest value among a), b) and c). This process should be repeated for all blocks and the new value P_{ij} should be replaced with the cumulative value M_{ij} (Figure 2.12c).

Step 3- If the maximum value of P_{ij} in the first row is positive the optimum pit is obtained by following the arrows from, and to the left of, the block that has the maximum value. If all values of P_{ij} in the first row are negative then there is no positive profitable pit (Figure 2.12d).

The two-dimensional dynamic programming algorithm requires additional effort to fit and adjust sections together and to smooth out the pit bottom and end sections. This smoothing process almost always results in a sub-optimal solution. Johnson and Sharp (1971) proposed the simplest extension of the algorithm to the three-dimensional case. Their method, which is referred to as $2\frac{1}{2}$-dimensional dynamic programming, is based on adjusting the sections row by row so that they fit together and satisfy three-dimensional slope constraints. Although their algorithm often gives a good, or near optimum, result it does not guarantee optimality. In addition, short of checking the result against the output from the Lerchs-Grossmann algorithm, there is no way of knowing how close the pit is to the optimum. Koenigsberg (1982) was the first to apply dynamic programming to the three-dimensional case using the same principle as Lerchs and Grossmann. Wilke and Wright (1984) proposed a modified version of the Koenigsberg algorithm. A recently published three-dimensional dynamic programming method (Yamatomi, Mogi, Akaike and Yamaguchi 1995) yields optimum or near-optimum results. However, due to the difficulties of establishing the true optimum in three dimensions, dynamic programming has not gained widespread acceptance or use as a method of optimal open pit design.

2.2.7- Parameterisation

All the methods of optimal pit design described above are based on a revenue block model obtained by calculating the net profit value of each block by applying costs, price and technical parameters to a block grade model of a deposit. The algorithm then searches among all possible technically feasible pits to find the optimum pit with the highest net profit value. It is assumed that the values of the variables, such as grade, costs, prices and recovery factor, used to create a revenue block model are constant. The values of these variables are usually estimated and are, therefore subject to error.

In reality the values of these variables will vary in a stochastic manner over time or, in the case of grade, over space. Often small changes in variables such as grade and price have significant effects on the optimum pit. Any realistic pit design should at least address the effects of using fixed values of variables that are known to be uncertain and/or to vary. One possible way to deal with the uncertainty associated with the variables is to conduct sensitivity or risk analyses (see for example Dowd, 1994c and 1997). In sensitivity analysis the value of a variable is changed systematically and its effect on the result is assessed while the values of other variables are fixed. For the open pit problem sensitivity analysis involves changing the value of one variable (say, metal price) by a fixed amount, calculating the revenue block model values and then generating the optimum pit. The procedure is repeated by changing the value of the variable again and determining the new optimum pit. A typical application might begin by determining the optimum pit for a starting metal price (sometimes called the base value) and then varying the base price by ±5%, ±10% and ±20% and determining the optimum pit corresponding to each of these changed values. The objective would be to measure the corresponding change in each pit – e.g. total profit, tonnage, average grade, pit shape and depth.

Risk analysis assesses the effects on the result of simultaneous variation in the values of all variables. For the open pit problem risk analysis essentially involves repeated simulation of the values of the variables. Each simulation consists of selecting a value of each variable from a specified probability distribution. The pit is then designed on the basis of the set of simulated values. The whole procedure is then repeated a number of times, each time with a newly simulated set of values. This generates a set of pits – one for each set of simulated values – from which risk can quantified. For example, the proportion of pits that did not generate a specified minimum profit would quantify the risk, or probability, that the mining operation would fail to achieve the specified level of profits. These methods are described in Dowd (1997).

Both sensitivity and risk analysis involve multiple generations of optimum pits and this can consume enormous amounts of computing time. The cost and time involved in doing this must, of course, be set against the cost of the mining operation and the cost of failure. Even so, it is often critical to be able to generate results very quickly. Quite apart from sensitivity and risk analysis it is often necessary to design pits on the basis of different assumptions about certain variables (e.g. bench heights, prices via sales contracts). It would certainly be feasible to generate multiple pits for small block models in very short periods of time but this rapidly becomes impossible as the size of the block model increases. These observations are especially true of the Lerchs-Grossmann algorithm.

A radically different approach to this problem is the parameterisation method that involves expressing the solution (i.e. the optimum pit) as a function of an input parameter (e.g. price). In this approach all optimum pits are generated simultaneously for all possible values of the input parameter. This is called single parameterisation. The process of generating solutions as functions of two input parameters is called double parameterisation and in theory, at least, this could be extended to multiple parameterisation. To date, however, only single parameterisation has been completely developed. This method which was pioneered by Matheron (1975) has been discussed in a number of papers (François-Bongarçon and Maréchal, 1976, François-Bongarçon and Guibal, 1982, Dagdelen and François-Bongarçon,1982 and Coléou,1989).

Parameterisation uses the block grade model rather than a revenue block model and the parameterisation is normally done as a function of cut-off grade. The problem is divided into two separate parts: technical and economic. The objective of the technical part is to find a series of technically optimum pits that maximise the quantity of metal for a given total tonnage and selected tonnage without assuming any values for the economic parameters. The result of the algorithm is a series of maximum metal pits nested within one another, each corresponding to a cut-off grade determined by the specified block grades. These pits can then be analysed by applying economic parameters to determine the economically optimum pit. The method is not rigorously optimal in the sense used in the discussion of the previous methods. The main criticism levelled against it is the simple (some critics say simplistic) manner in which the economic optimisation is handled. It does, however, retain significant unrealised potential for solving many problems including pit optimisation by maximising net present value.

Whittle (1988) used the concepts of the parameterisation technique and introduced a method to deal with the uncertainty in economic parameters used in the calculation of the economic block model. He reduced to two the number of variables used to create the revenue block model: one major variable (metal cost of mining) and one minor variable (ratio between processing and mining cost). Whittle also suggested that a number of pit designs can be generated for risk assessment for a range of values of the major variable.

2.2.8- Other methods

The methods of optimal pit design discussed thus far are the most commonly used. In addition to these a number of other methods have been devised but have found little use. Among these is an adaptation of the transportation algorithm (Huttagosol and Cameron, 1992), which involves the formulation of the ultimate pit problem as a transportation model which is then solved by the standard simplex method. Another is the use of genetic algorithms (Denby and Schofield, 1994). It is, however, apparent from the literature that these methods have not been adopted to any great extent

Genetic algorithms are search procedures based on the mechanics of genetics and natural selection. They can be applied to complex optimisation problems in which optimum or near optimum solutions are required. The technique has been applied to the optimal open pit problem and to the open pit scheduling problem by Denby and Schofield (1994), and Denby, Schofield and Hunter (1996). The method involves the following fundamental steps in pit optimisation:

1- An initial population of pits is generated at random.
2- For each pit, a fitness value is calculated by using an objective function, e.g. net present value.

3- On the basis of their fitness values, pairs of pits are selected for combination to produce a new pit. This process is termed reproduction. Pits with higher fitness values are most likely to produce a new, better pit during reproduction.
4- Each pit is altered at random to produce a new pit.
5- The new pits are replaced with the old pits and the procedure is repeated from step 2 until the system reaches an optimum.

The main advantage of this technique is that both pit limit and production schedule can be optimised simultaneously and the optimisation criterion is the maximisation of net present value. However, at present the technique can only handle deposits defined by a small number of blocks and are impractical for large deposits. It is also heuristic and there is no way of guaranteeing that an acceptable solution could be reached.

2.3- Slope design in open pit mining

2.3.1- Introduction

The ultimate objective of a mining operation whether by underground or surface mining (open pit mining) methods is to extract ore at a profit. The profitability of an operation depends on adequate technical and economic planning on the basis of all relevant parameters. One of these parameters is the pit slope angle. The determination of pit slopes is of vital importance in pit planning as it affects the size and shape of the final pit. A steeper slope decreases the stripping ratio and thereby increases the profitability of a mine. On the other hand, the pit wall must remain stable and the use of a steeper slope increases the probability of failure. An optimum mine plane should, therefore, have the steepest final pit slope that will remain stable throughout the life of the mine.

An increase or decrease in slope angle in a medium to large open pit will significantly alter the stripping ratio, i.e. the amount of waste that must be removed to mine a unit quantity of ore. A total of 23 million extra tonnes of waste would have to be removed as a result of an average slope being reduced by 5° in an open pit mine with the following dimensions and specific gravity: 1900m length from crest to crest, the width of the bottom floor 75m from toe to toe, vertical depth 150m and the specific gravity of material 2.65 (Seegmiller, 1979).

On the other hand, improper design of slopes may place the entire mining activity in jeopardy. Disruption of mining operations caused by slope instability can be severe. Some consequences are loss of ore, extra cost due to cleaning up of failures, production delays, and damage to equipment and probability of injuries. Many slope failures have been recorded as a result of improper slope design (Seegmiller, 1979). For instance, a large-scale failure that happened in Afton mine in 1986 (Reid, and Stewart, 1986).

In general, pit slopes are designed to be as steep as possible in order to reduce the stripping ratio. Singh (1986) showed the importance of pit slope angles on the design and economics of open pits and found that the steeper slope not only yields higher net present value but also a shorter pay back period. The determination of pit slopes is a critical step during the pit planning process. The steeper slope required to minimise the stripping ratio must be balanced against the flatter slopes that are needed to ensure stability. The slope designer must, thus, reconcile these two conflicting requirements. There is also a circular problem involved in the determination of pit slopes, clearly stated by Call and Savely (1990):

> ***"On one hand the slope designer needs the position, orientation and the height of the pit walls to design the slopes. On the other hand, the mine planner needs the slope angles in order to design pit limits."***

Due to the complexity and unknowns associated with the rock mass, any decision in rock engineering requires a considerable amount of experience. It is very important to recognise factors affecting slope stability during the design stages. These factors include geological conditions, structural features, groundwater conditions, shear strength of discontinuities, geometry of slopes and the excavation technique used in creating the slope. Slope stability planning involves the following activities:

1- Collecting relevant data.
2- Identification of critical geological features.
3- Determination of shear strength parameters of discontinuities.
4- Determination of groundwater conditions and pore pressure acting on critical discontinuities.
5- Identification of potential failure modes.
6- Analysing potential failure modes by using an appropriate method.

To design slope angles in open pit mining the first step is to detect a potential failure mode and then to use one of the slope stability methods to assess the stability. This requires judgment as to whether the slope is stable or not together with the decisions to be made as a consequence. There are several methods that can be used to evaluate stability or instability of a given slope. These are: stereographic projection, limit equilibrium methods, probabilistic methods and numerical methods.

Pit slopes in the optimal pit design algorithm are considered as constraints and they are defined in term of blocks which must be removed in order to provide access to a particular block in the block model. There is no significant evidence of the incorporation of geotechnical information into the design of pit slopes in optimal pit design algorithms. The main reason is that they either use a pattern, or a set of blocks, to define the mining slopes or they use predefined pit slopes which are assumed to have already been designed. In this section a brief review of the available methods of slope design, types of failure and shear strength is given.

2.3.2- Types of failure

The first step in the design of the slope in open pit mining, is to identify the potential failure mode. The types of failure that may occur within a slope depend mainly on the presence or absence of structural features such as faults, bedding planes and joints. They depend also on their geometrical relationship with the slope face that will determine which parts of the rock mass are free to slide or fall. Many authors including Ross-Brown (1979), Hoek and Bray (1981), Walton (1988) and Giani (1992) discuss various types of failure. Instabilities in open pit mines may be classified using the criterion of whether or not they are controlled by well-defined structural discontinuities. They fall into two categories: structural instability and non-structural instability.

The first comprises slopes in which the failure is controlled by geological features such as bedding planes, faults and joints. These failures depend on the orientations of the discontinuities and their relationship to the orientation of the cut slope. Some of these instabilities are:

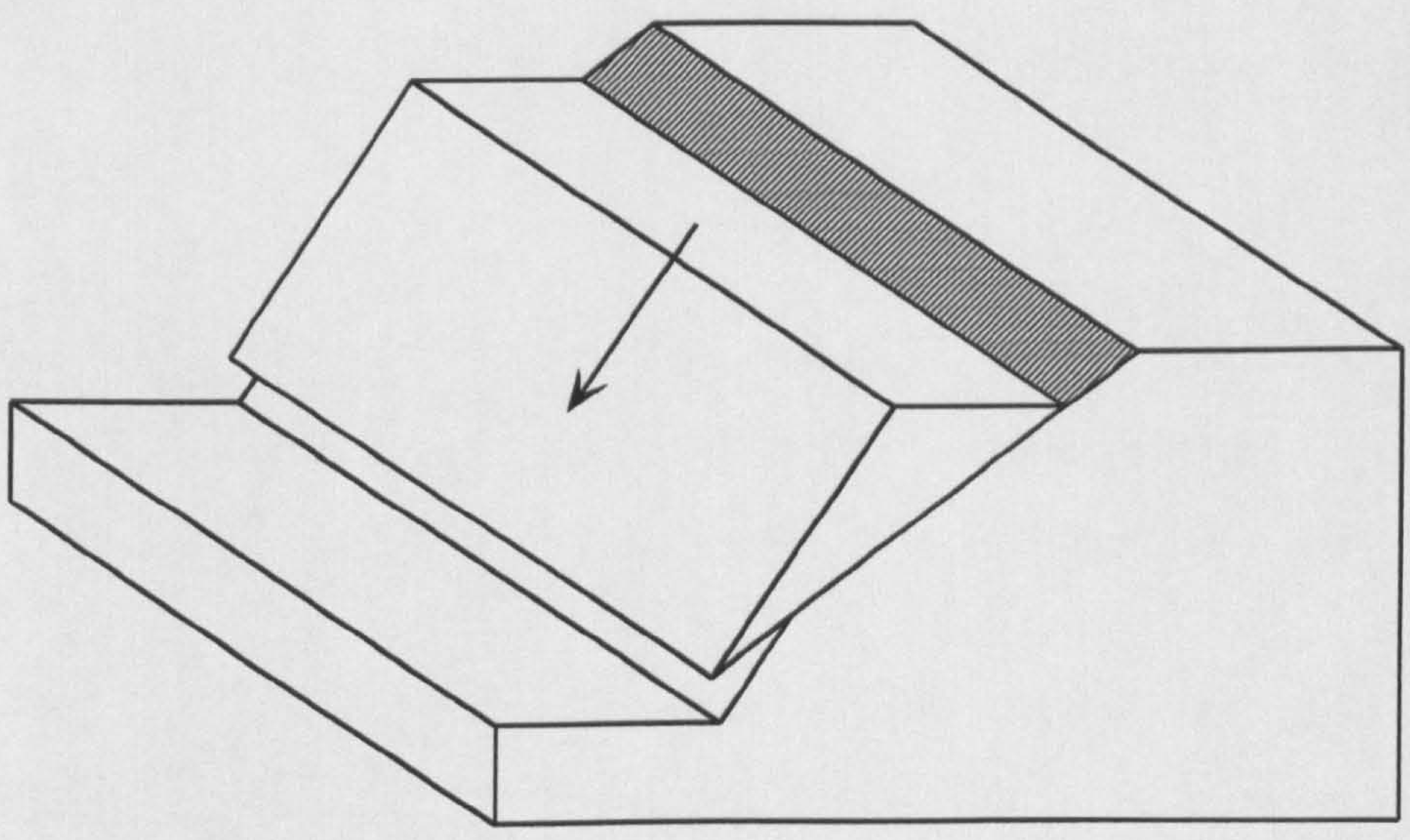


Figure 2.13- Planar failure

Plane failure- Planar instability involves the sliding of a block of rock mass along a simple planar surface inclined towards the slope face. This failure is likely to occur when a geological discontinuity, such as a bedding plane, has a strike parallel, or nearly parallel (within about 20°), to the strike of the slope face, a dip lower than the slope angle and intersects the slope face (Figure 2.13). Variations on this type of failure mode can occur when a tension crack forms either in the upper surface or in the slope face.

Wedge failure- This failure occurs when two planar, or nearly planar, geological structures intersect to form a wedge of material in such way that the line of intersection daylights into the slope face (Figure 2.14).

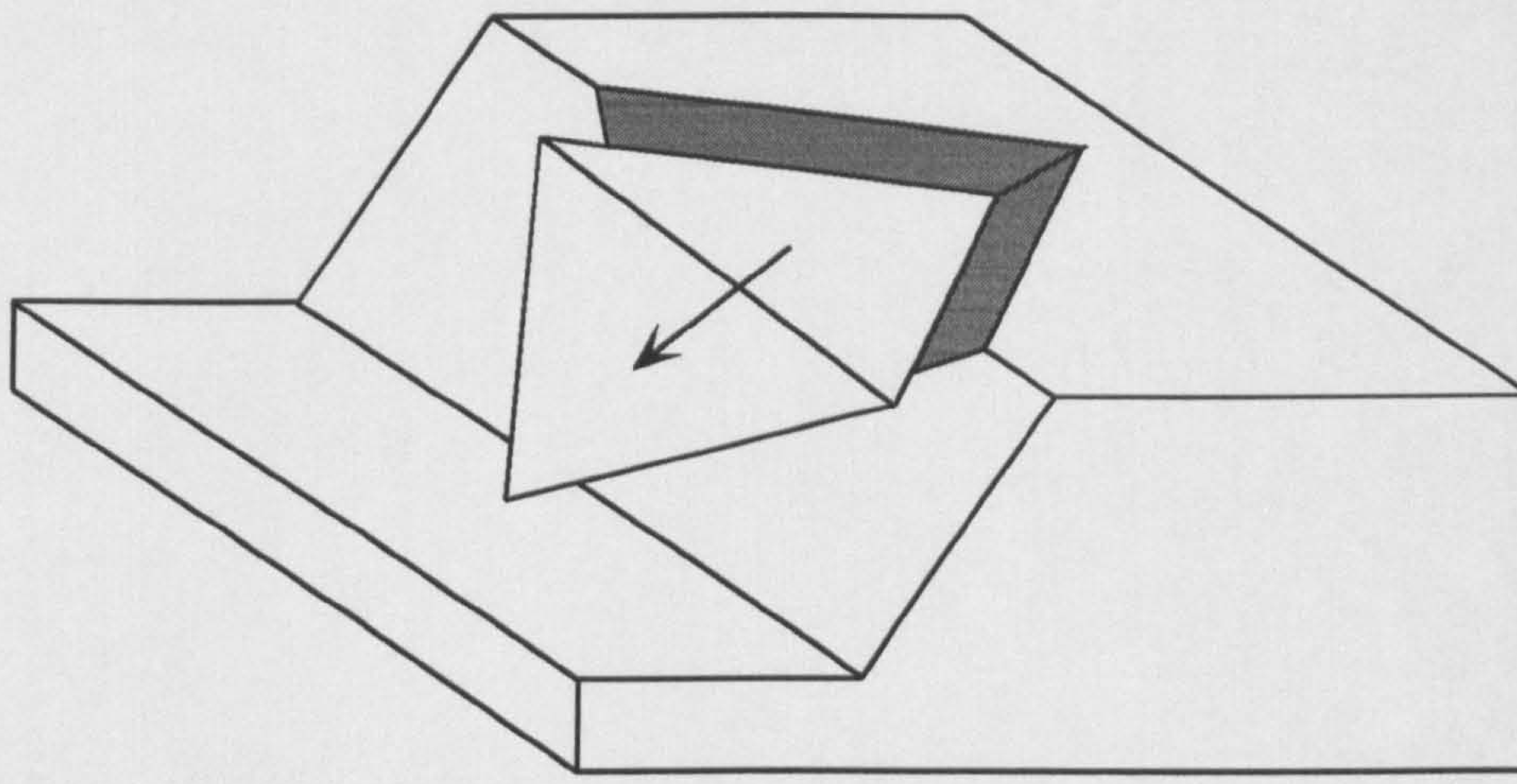


Figure 2.14- Wedge failure

Toppling failure- This type of failure involves rotation of one or more blocks. This may occur in slopes having near vertical joints. Toppling failure and rock falls are excluded from this study since there is no general way of dealing with these failures.

The second category comprises slopes in which well-defined discontinuities do not exist and the failure surface is free to occur along the line of least resistance through the slope. These are failures, which can occur in soil slopes or in rock slopes, when one or more of the following conditions exist:

a- The rock mass is highly fractured throughout or exhibits random jointing.

b- The rock mass is very weak so that its properties approach those of a soil.

c- Very high angle slopes exist in closely jointed rock.

d- A bench or haul road cuts into soil.

These failures are classed as **circular** or **non-circular failures**.

Circular failure- This type of failure is common in soil masses where it is generally assumed that the cross-sectional form of the failure surface is a circular arc. This is usually referred to as a slip circle as illustrated in Figure 2.15. The position of the critical slip (i.e. where it is most likely to occur) is determined by analytical methods, which are a function of slope geometry, material strength, unit weights, and pore pressure.

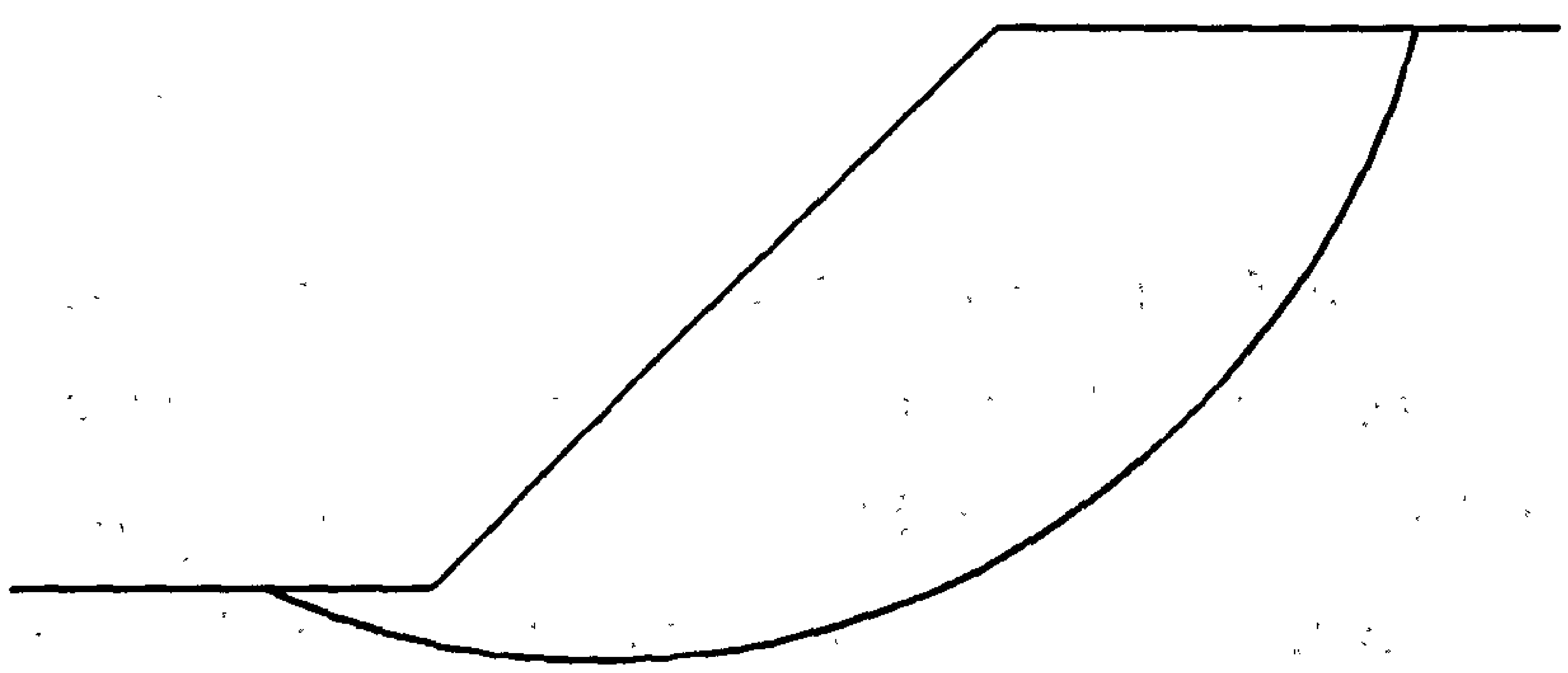


Figure 2.15- Circular failure

Non circular failure- This type of failure occurs under similar circumstances to the circular failure. The failure surface, which tends to occur parallel to a set of weakness planes, consists of a mixture of curved and linear segments.

2.3.3- Shear strength

One of the most important factors in analysing the stability of a rock slope is the shear strength. A small change in shear strength causes significant change in the stability of a slope. Determination of the shear strength is, therefore, a critical part of any stability analysis. It is clear that in order to obtain shear strength parameters for engineering design some form of testing is necessary. Depending on the nature of the problem being investigated, testing may take the form of a very sophisticated laboratory test or a simple *in situ* test. The classical analysis of shear strength involves both cohesion and angle of friction and can be stated by the Mohr-Coulomb criterion. This criterion can be expressed by the following equation:

$$\tau = c + \sigma.Tan\phi$$

Where

τ	is the shear strength
c	is the cohesion
ϕ	is the angle of friction
σ	is the normal stress

Both cohesion and angle of friction can be found from laboratory or field tests. The above equation for shear strength indicates a linear failure criterion as illustrated in Figure 2.16. However, it has been recognised that in reality the failure criterion is non-linear. A variety of non-linear criteria have been proposed in the past and a summary of these can be found in Hoek and Bray (1981).

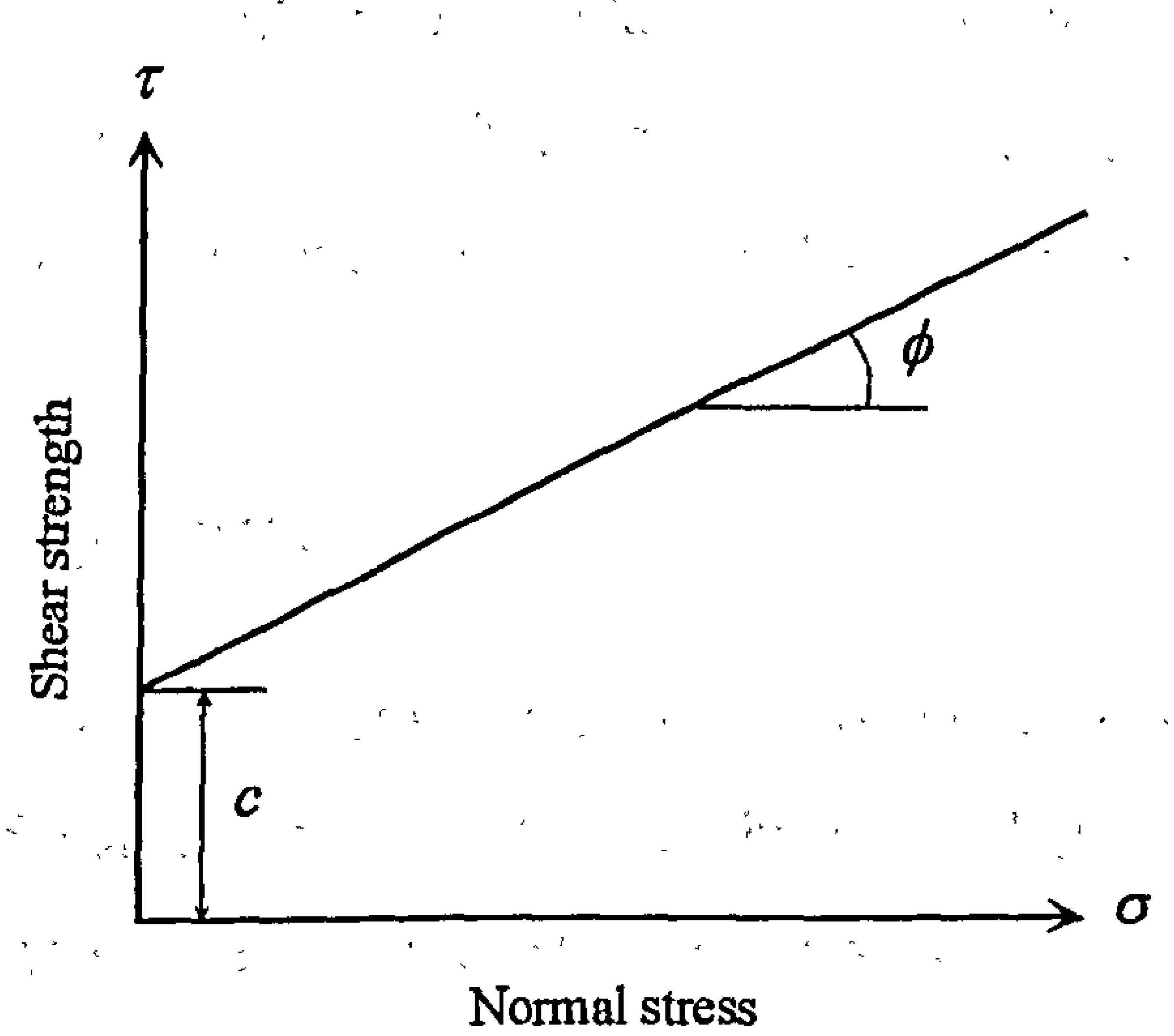


Figure 2.16- Mohr-Coulomb linear failure criterion

It is difficult to measure rock strength by the direct shear test if the failure is likely to occur through the rock mass and the rock mass is heavily fractured. For this situation Hoek and Brown (1980) developed a criterion, based on their wide empirical and theoretical experience, for estimating the rock mass strength. The general criterion is given by the following equation (Hoek, Kaiser, and Bawden, 1995):

$$\sigma_1' = \sigma_3' + \sigma_c\left(m_b \frac{\sigma_3'}{\sigma_c} + s\right)^a$$

Where

σ_1'	is the axial effective principal stress
σ_3'	is the confining effective principal stress
σ_c	is the uniaxial compressive strength of intact rock pieces
m_b	is the value of constant m for the rock mass
s and a	are constants that depend on the characteristics of the rock mass

These authors introduced a procedure for estimating the material constant from a rock mass classification system, for example *RMR* (Rock Mass Rating system). Many of the analyses used to calculate the safety factor are based on the Mohr-Coulomb criterion. They also presented a method for determining the equivalent values of the Mohr-Coulomb criterion, the friction angle (ϕ) and the cohesive strength (c) from the tangent to the envelope to the principal stress defined by the Hoek-Brown failure criterion.

2.3.4- Stereographic projection

Stereographic projection is a powerful graphical method used to present and analyse the three-dimensional orientation of planes and linear geological structures in two dimensions. These methods are widely used in rock mechanics and geological studies for analysing planar discontinuities such as fractures, faults, joints and bedding planes that occur at various orientations within rock masses. Two types of stereographic projection are in common use: equal angle and equal area. Although there are some differences between these two methods, the representation of features within both methods is the same. The principle of the techniques and their use in structural geology and geotechnical studies have been described in more detail by Phillips (1971), Priest (1985), Goodman (1980) and Hoek and Bray (1981).

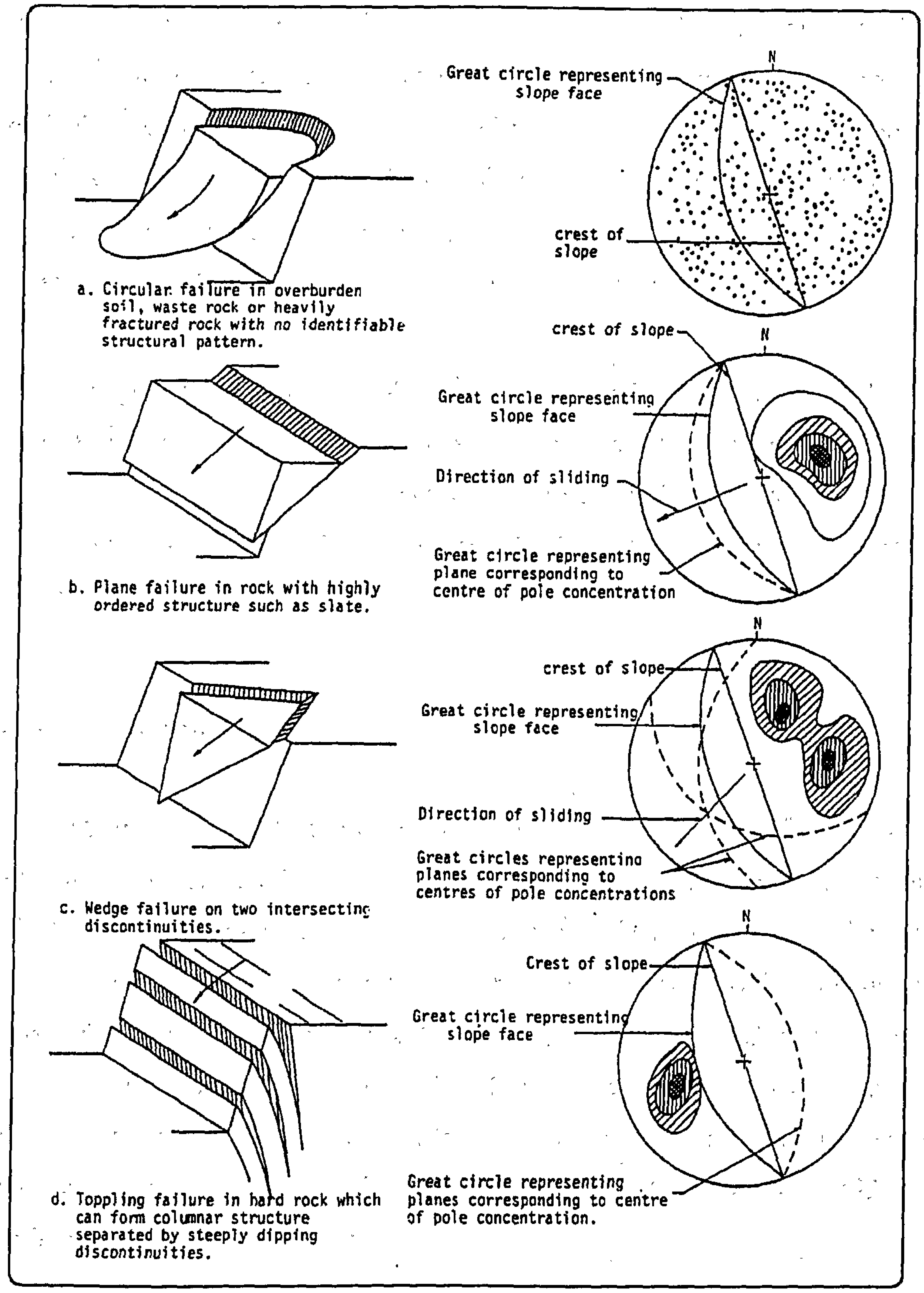


Figure 2.17- Stereographic projection of geological features and corresponding likely mode of failure (After Hoek and Bray, 1981)

Stereographic projection is a useful method to use in the early stages of slope design to identify likely modes of failure and critical areas that require more rigorous analysis. As an example consider Figure 2.17 which shows the data from four discontinuity sets together with the mean plane of the slope face on an equal area projection with the corresponding likely mode of failure on the left. These techniques ignore several important parameters such as the effect of ground water, slope height and cohesion, all of which are closely related to the stability of a slope.

Stereographic projection is also a useful method for carrying out kinematic analyses. These analyses refer to the motion of bodies without taking into account the forces causing them to move. They are used either to assess the stability of a given slope when its orientation is known (Hoek and Bray, 1981), or to determine the steepest safe angle (Goodman 1980).

2.3.5- Limit equilibrium

Limit equilibrium is the most common method for evaluating slope stability. The method is based on the calculation of a factor of safety defined as the ratio of total forces tending to resist failure, to the total forces tending to cause instability. For a given failure mode, a factor of safety in excess of 1 indicates that the slope is unlikely to fail. By contrast, a factor of safety less than 1 indicates that failure is likely to occur. Hoek and Bray (1981) pointed out that a factor of safety of 1.3, is the minimum acceptable value for a temporary slope in open pit mines, while for a permanent slope, such as those carrying the haul road, a factor of safety value should exceed 1.5. Walton (1991) divided limit equilibrium methods into two groups:

1- Those that consider the material above the slip surface. These methods of analysis, developed in rock engineering, calculate a factor of safety for structural instability such as plane and wedge failures.
2- Those that divide the sliding mass into a number of vertical slices. These methods, developed in soil mechanics, are known collectively as the **method**

of slices, and can be used to analyse non-structural instability such as circular and non-circular failures.

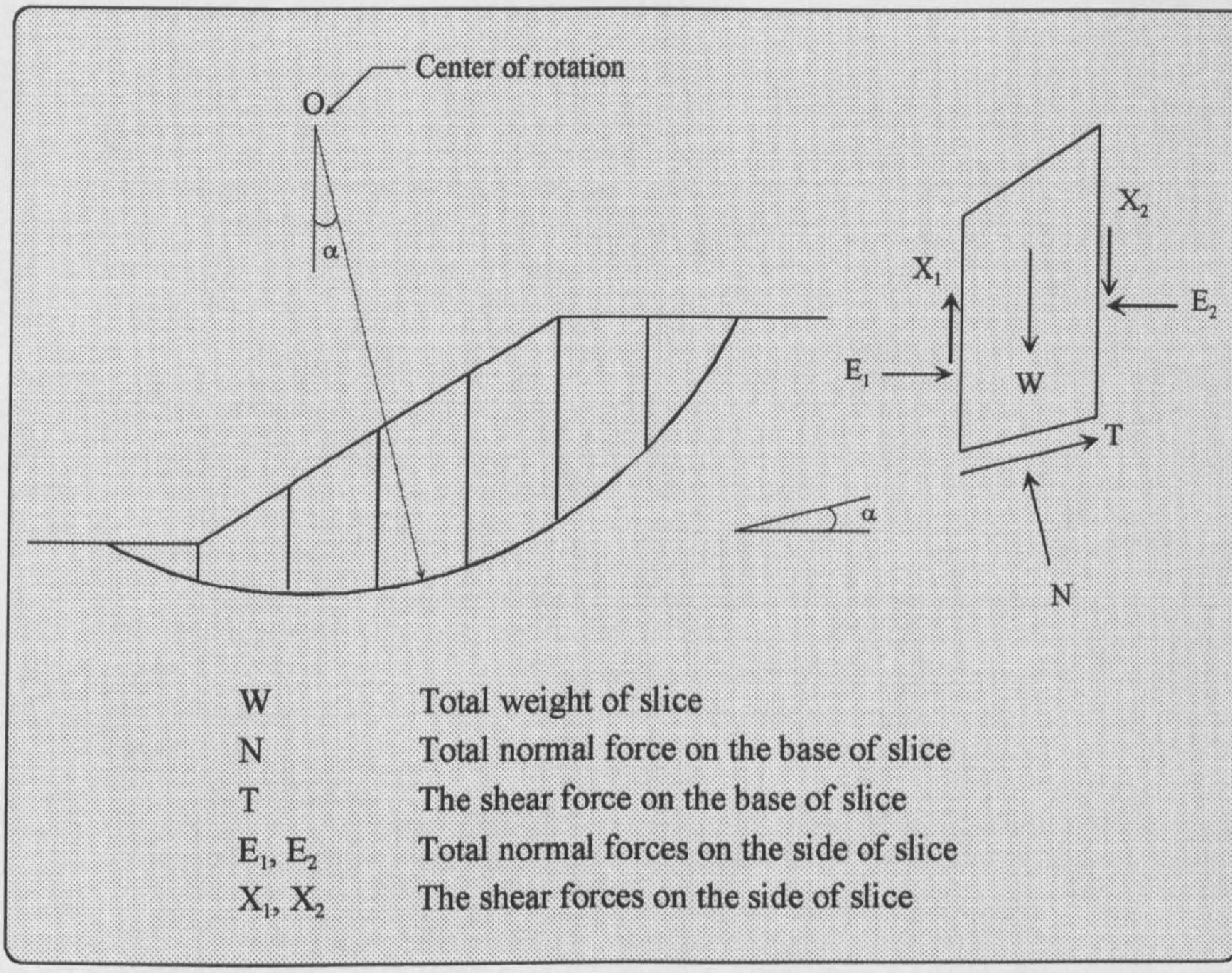


Figure 2.18- Circular failure and forces on a typical slice

In the first method, the slope is usually considered in two dimensions on a cross-section of unit thickness. The forces acting on the sliding mass are found and resolved to obtain the factor of safety. Several reference books, "Rock Slope Engineering" (Hoek and Bray, 1981) and "Pit Slope Manual" (Major, Kim, and Ross-Brown, 1977), provide reviews and detailed descriptions of the various analytical techniques used to calculate the factor of safety. A variety of design charts are also given by Hoek and Bray to make a rapid assessment of the stability of a slope.

In the second method, the slope is considered on a cross-section of unit thickness and the mass above the failure surface is divided by vertical planes into a series of slices as shown in Figure 2.18. The factor of safety is determined as the ratio of the available shear strength to the shear strength required for stability.

There are a number of different methods that have been developed over the years in soil mechanics using the method of slices. Nash (1987) gives a comparative review of these methods. The most common methods of slices are those developed by Fellenius (1936), known as the Ordinary or Swedish method, Bishop (1955), Janbu (1973), Morgenstern and Price (1965), Spencer (1967) and Sarma (1973). The first two methods of analysis, Ordinary or Swedish method and Bishop's simplified method are used for circular failure and the others are used to analyse non-circular failures. The general problem with these methods is that there are more unknowns than equations. For a slope divided into n slices there are, in general, 5n-2 unknowns while there are only 3n equations for the static variables. The problem, therefore, is statically indeterminate and in order to obtain a solution assumptions must be made regarding the interslice forces E and X (Figure 2.18).

Different methods of analysis make different assumptions about the position and inclination of the interslice forces. Since all methods of analysis involve assumptions, none will yield the correct value for the factor of safety. Apart from the Ordinary or Swedish method, which ignores the interslice forces and assumes that for each slice the resultant of the interslice forces is zero, most of the methods yield an acceptable result. Bishop's simplified method is the most widely used and accepted procedure in the analysis of circular failure. In this method, it is assumed that the side forces are horizontal and the forces are resolved vertically to determine the stress acting on the base of each slice. A number of trial and error attempts are required to determine the least factor of safety and a computer is therefore necessary to obtain a solution.

2.3.6- Probabilistic methods

In the conventional deterministic methods, the values of variables such as rock or joint properties, density, groundwater condition and orientation of discontinuities are assumed to be unique and fixed. These parameters are estimated either from field sampling and laboratory results or from *in situ* tests. There are significant uncertainties in the estimation and measurement of the values of these variables. These uncertainties arise from natural variability and/or lack of data used to estimate or to measure the input parameters. The uncertainties associated with the input variables used in deterministic methods may have a significant effect on the stability of a slope. Probabilistic approaches have been developed to assess the effects of the uncertainties on the stability of excavated slopes, see, for example, Major, Kim, and Ross-Brown (1977), Priest and Brown (1983) and Pine (1992). The advantage of these methods over conventional deterministic methods is that they examine the complete range of relevant parameters rather than unique values.

The classical approach of safety measurement in engineering design involves the determination of the factor of safety, *F*, which is defined as the ratio between capacity, *C*, or the resisting force, and the demand, *D*, or disturbing forces, ($F = C/D$). In the deterministic method the mean value of *C* and *D* are used to calculate the mean value of the factor of safety; failure is assumed to occur when *F* is less than 1. An alternative to the factor of safety is the safety margin, *SM*, which is defined as capacity minus demand ($SM = C - D$) and failure is indicated when *SM* is less than zero.

In general, both capacity and demand result from many uncertain variables. They are, therefore, random variables, and hence their ratio (factor of safety) and their difference (safety margin) are themselves random variables and the probability of failure can then be described either:

- by the probability of the factor of safety, *F*, being less than one $[p(F < 1)]$ as illustrated in Figure 2.19 (shaded area), on the assumption that the factor of safety follows a normal distribution.

or

- by the probability of the safety margin, SM, being less than zero

$[p(SM < 0)]$ on the assumption that the safety margin follows a normal distribution. This is the shaded area shown in Figure 2.20

The complement of the probability of failure is called the reliability, *R*, which is defined as the probability of success. Hence: $R = 1 - p$

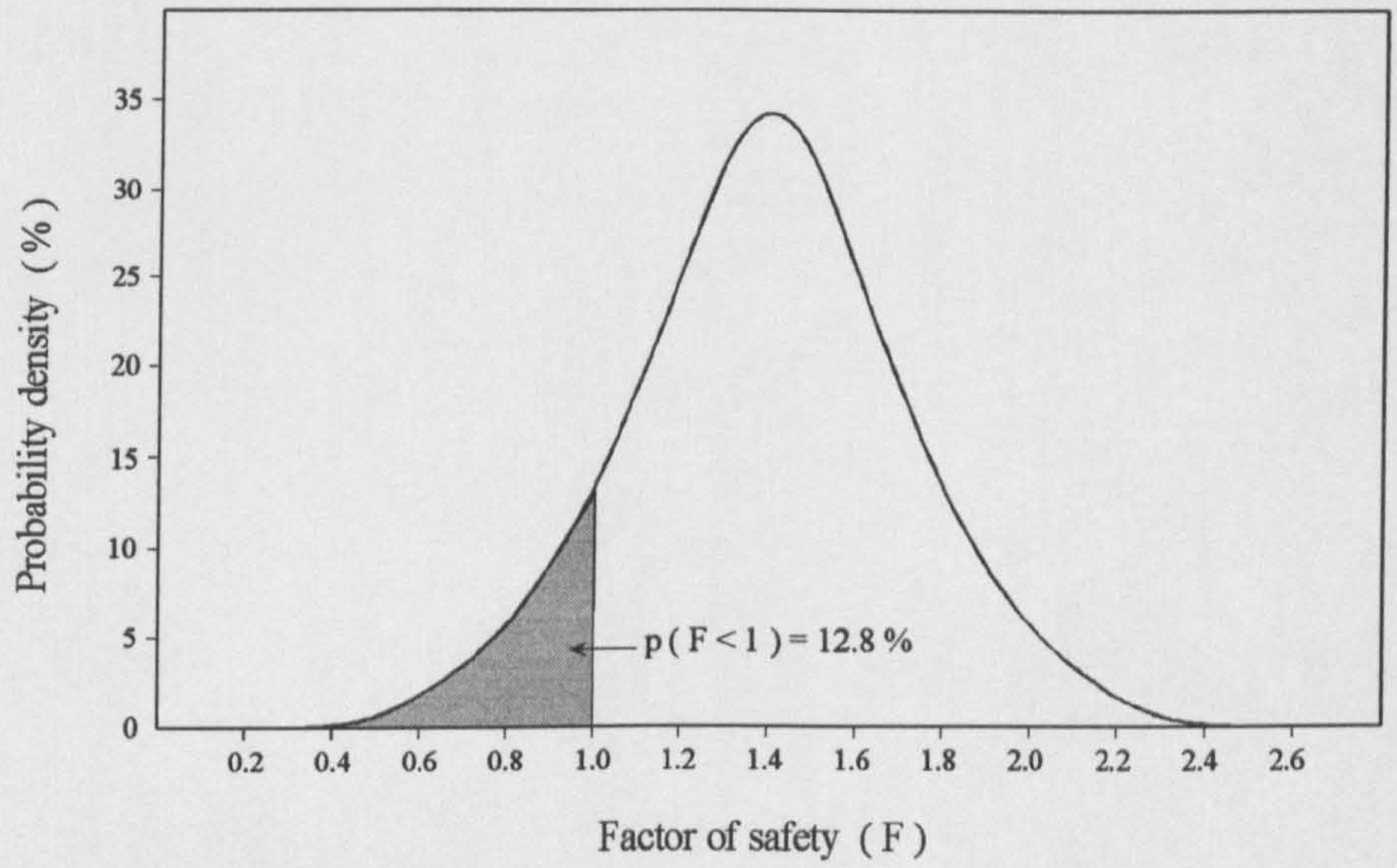


Figure 2.19- Factor of safety probability distribution

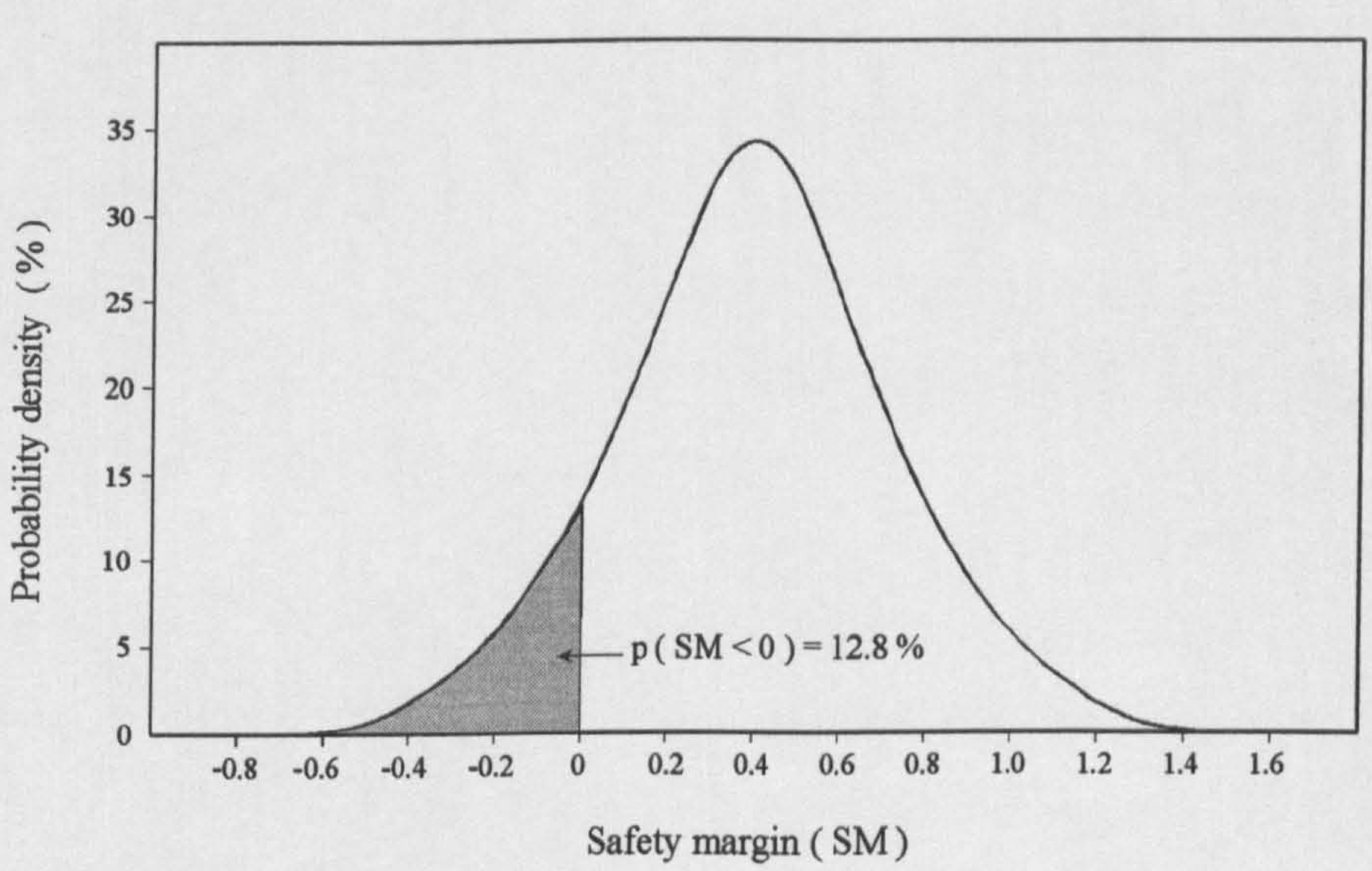


Figure 2.20- Safety of margin probability distribution

Due to the inherent uncertainties associated with the input variables, it is necessary to incorporate the use of probability in the design of slopes. To determine the probability of failure, it is first necessary to define the probability distribution of the input parameters, and then to choose an appropriate method of probabilistic analysis.

There are several methods for conducting probabilistic analyses of slope stability problems, including:

- First order, second moment method
- General point estimate method
- Latin hypercube sampling technique
- Monte Carlo simulation

Harr (1987) has described all these methods, except the Latin hypercube sampling technique.

The **first order, second moment** method is based on truncating the Taylor series expansion of the output which must be expressed as a function of input variables. Both output and input variables are characterised by their mean and standard deviation. The advantage of the method is that only the mean and standard deviation of input variables are required. On the other hand it gives the output as mean and standard deviation rather than a complete description of the probability distribution (unless of course a normal distribution is assumed). The method also requires the evaluation of derivatives that may range from being simple to cumbersome or impossible.

The **general point estimate method** was developed by Rosenblueth (1981) and was described in detail by Harr (1987). Pine (1992) applied this method to the analysis of slope stability and other mining problems. The general point estimate method involves the use of only two sample values at one standard deviation on either side of the mean for each input variable. The factor of safety or safety margin is calculated for every possible combination of these two selected values. There are 2^n combinations where n is the number of input variables. Hence the output comprises 2^n factors of safety or safety margins from which the mean and standard deviation can be calculated. Although the method is very simple, it does not provide a full distribution of the output variable.

The **Latin hypercube sampling technique** is a relatively recent development in mathematics and numerical analysis. It is based on a form of stratified sampling that gives comparable results to those of Monte Carlo simulation, but with fewer samples. The method was developed by Startzman and Watterbarger (1985) and an example of its use in both surface and underground excavation was given by Pine (1992).

Monte Carlo simulation is a completely general technique which can be used for any input and output distribution. The technique uses random or pseudo-random numbers to sample from the distribution. It has been applied to a wide variety of problems including slope stability analysis (Major, Kim, and Ross-Brown, 1977 and Priest and Brown, 1983) and mining finance (Dowd and Xu 1995, Dowd 1997). The Monte Carlo method for probabilistic risk analysis for slopes, as illustrated in Figure 2.21, is implemented via the following steps:

Step 1- Define a probability distribution function for the input variables from the histogram of experimental data.

Step 2- Select a value at random from the probability distribution of each input variable by using a random number sampling technique.

Step 3- Use the selected values as input parameters and determine the safety factor or safety margin.

Step 4- Repeat steps 2 and 3 until a stable histogram of the factor of safety or safety margin is obtained. This histogram is interpreted as the estimated probability distribution of the factor of safety or safety margin.

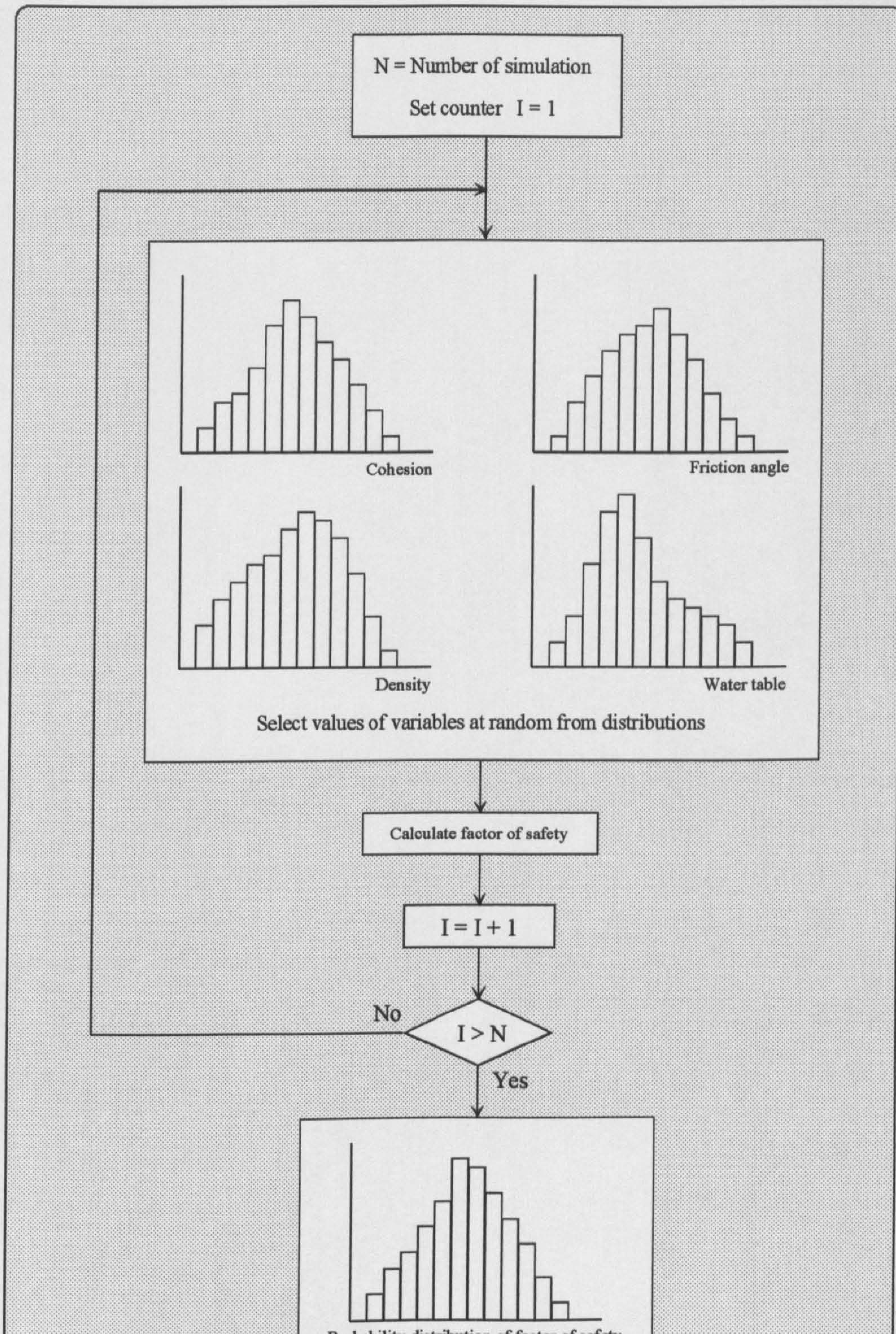


Figure 2.21- Flow chart of Monte Carlo simulation

Step 5- Calculate the probability of failure, mean and standard deviation from the estimated probability distribution of the factor of safety or the safety margin.

A large number of iterations should be performed in the Monte Carlo approach and a computer is, therefore, essential.

Category	Consequence of Failure	Examples	Acceptable criteria		
			Maximum values (%)		Minimum value
			$p(F<1.0)$	$p(F<1.5)$	Mean F
1	Not serious	Non-critical	10	20	1.3
		benches	**10**	**80/75**	**1.3**
2	Moderately serious	Semi-permanent	1.0	10	1.6
		slopes	**2.0**	**36/39**	**1.6**
3	Very serious	High/permanent	0.3	5.0	2.0
		slopes	**0.3**	**8/12**	**2.0**

Table 2.1- Acceptable criteria to define the level of risk as proposed by Priest and Brown and modified by Pine

When risk analysis or probabilistic approaches are applied to the design of slope angles, the acceptable risk of failure must be considered in the design. Priest and Brown (1983) suggested some criteria for this based on their experience and that of others. The modified versions of these criteria given by Pine (1992) are shown in Table 2.1. The values in this Table should only be used as guides. The first value in each category is based on the original criteria and the second value shown in bold is based on the modified criteria. For the modified criteria there are two values for $p(F<1.5)$ corresponding to the normal and the lognormal distributions, respectively, of the factor of safety.

There is no doubt that probabilistic approaches have advantages over the deterministic methods since they consider the complete range of input variables rather than a unique value. However these methods have not been accepted to any great extent largely due to the difficulty of determining the appropriate probability distribution for the input variables and the mathematics involved.

2.3.7- Numerical methods

Where the geology is complex and potential failure modes may involve a combination of several mechanisms, the analysis of rock slope stability is more complicated and perhaps impossible by limit equilibrium methods. Examples of this complexity include cases in which failure involves sliding along discontinuities, rotation of blocks (toppling) and failure through intact rock. In these cases numerical methods may provide a useful tool, particularly to study rock mass behaviour or to investigate progressive failure. With high-speed computers these methods are capable of effective analysis of complex geometrical problems.

Numerical methods have been used to study stress-strain behaviour of rock masses to determine the stability of both underground excavations and slopes, cf. Coggan and Pine (1996) Hencher, Liao and Monaghan (1996). There are several different types of numerical methods that have been coded into computer programs. Two of the most widely used programs in rock mechanics are FLAC and UDEC (Universal Distinct-Element Code). The first code is a continuum program in which the rock mass is considered as a continuous material. The distinct-element method (UDEC) is perhaps the most widely used of the discontinuum methods for the modelling of rock masses. Cundall (1971) originally described this method. In this method the rock mass is treated as a collection of individual blocks and the method is capable of dealing with complex masses.

Numerical methods have not been used in this work, as they are more suitable for studying the stability of complex slopes using information that is only be available once production starts.

CHAPTER 3

The Lerchs-Grossmann algorithm with variable slope angles

3.1- Introduction

The development of an open pit mine involves the determination of the size and final shape of the pit. Determination of this ultimate pit limit is one of the most important design requirements in open pit mining. Different methods may be employed to do this but all, implicitly or explicitly, use some form of optimising criterion. The most widely accepted criteria are the maximisation of total profit and the maximisation of net present value. Although the latter is the better measure of profitability there is an inherent difficulty in using it for optimal open pit design. The net present value of a block can not be determined until it is known when the block is to be mined. The time at which the block is to be mined, however, cannot be known until the optimal pit is determined. This circularity prevents the use of standard approaches to optimal pit design when maximisation of net present value is the criterion and for this reason maximisation of total profit is the most widely used criterion.

Many algorithms for optimal open pit design have been developed over the past 30 years. Of these only the Lerchs-Grossmann algorithm (1965) can be shown mathematically always to yield the pit that generates the maximum total profit. The original algorithm, which uses graph theory, was based on a single, fixed slope angle and many attempts have been made to incorporate variable slope angles. In this chapter a general method is introduced to accommodate variable slope angles. First, relevant aspects of graph theory are outlined and details of the original algorithm are given. Then the general method for dealing with variable slope angles is discussed. This chapter also introduces programming techniques to overcome memory limitations so that the algorithm can be implemented on PC computers. There is also an account of the calculation of the revenue block model as the input for the optimal pit design algorithm together with a method of smoothing the pit bottom layout.

3.2- Graph theory

Graph and directed-graph methods are used as models in many fields of study such as electrical and civil engineering, communication networks, industrial management, operational research and computer science. In their simplest form they comprise a diagram consisting of a set of nodes and connecting lines used to indicate the relationships between the nodes. The following definitions of terms and concepts are required for the application of graph theory to the optimal open pit problem:

A **graph** G = (X, E) consists of a set of elements called vertices or nodes, X, and a set of connecting lines, E, called edges such that each edge connects two vertices.

A **directed graph** is a graph in which the edges have directions. Directed edges are called arcs. The directed graph represented by G = (X, A) where X is the set of vertices and A is the set of arcs. As an example, the directed graph illustrated in Figure 3.1 may be described by the following sets:

Vertices: $\{x_1, x_2, x_3, x_4, x_5, x_6\}$

Arcs: $\{ (x_2, x_1), (x_2, x_3), (x_3, x_4), (x_4, x_5), (x_5, x_6), (x_6, x_4)), (x_6, x_3) \}$

In a directed graph, each arc is used to connect two vertices. For an arc,

$a_i = (x, y)$, the vertex x is called its **initial point** and the vertex y its **terminal point**. The vertex y is said to be the **successor** of vertex x if there exists an arc between them which is pointed towards the vertex y. The set of all successors of x is denoted by Γx. For example, in Figure 3.1 vertices x_1 and x_3 are the successors of vertex x_2 and $\Gamma x_2 = \{x_1, x_3\}$.

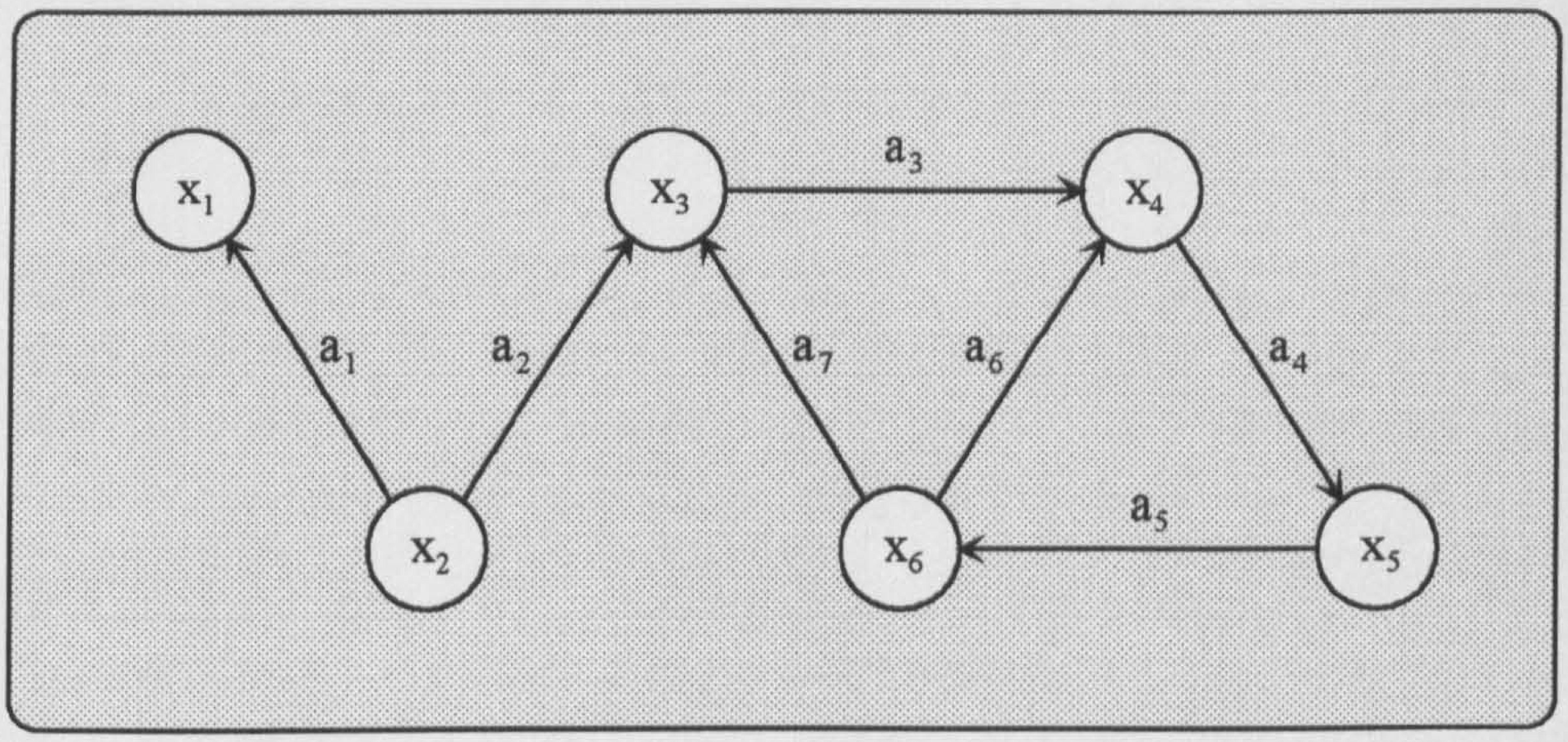


Figure 3.1- An example of a directed graph

A sequence of arcs is called a **path** such that the terminal vertex of each arc corresponds to the initial vertex of the succeeding arc. The arcs a_2, a_3, a_4 and a_5 in Figure 3.1 constitute a path.

A **circuit** is a closed path. In other words it is a path in which the initial vertex coincides with the terminal vertex. For example, the arcs a_4, a_5 and a_6 in Figure 3.1 constitute a circuit. Note that the arcs a_3, a_6 and a_7 do not constitute a circuit.

A **chain** is a sequence of edges in which each edge has a common vertex with an adjacent edge and no vertex in common with more than two edges.

A **cycle** is a chain in which the first and the last edge having a common vertex. The edges (x_3, x_4), (x_4, x_6) and (x_6, x_3) in Figure 3.1 constitute a cycle.

A directed **subgraph** G(Y) is a subset of a directed graph G(X, A). It comprises a set of vertices of Y belonging to X together with the arcs that connect the vertices of G(Y).

A **closure** of a directed graph G(X, A) is a subgraph G(Y) such that if $x \in Y$ then $\Gamma x \in Y$. Closure from the view-point of pit design is a subgraph of a directed graph, representing the block model of a deposit, which indicates a feasible pit. For example, in the directed graph illustrated in Figure 3.2, representing a simple two-dimensional block model, the vertices 4, 5, 6 and 12 form a closure.

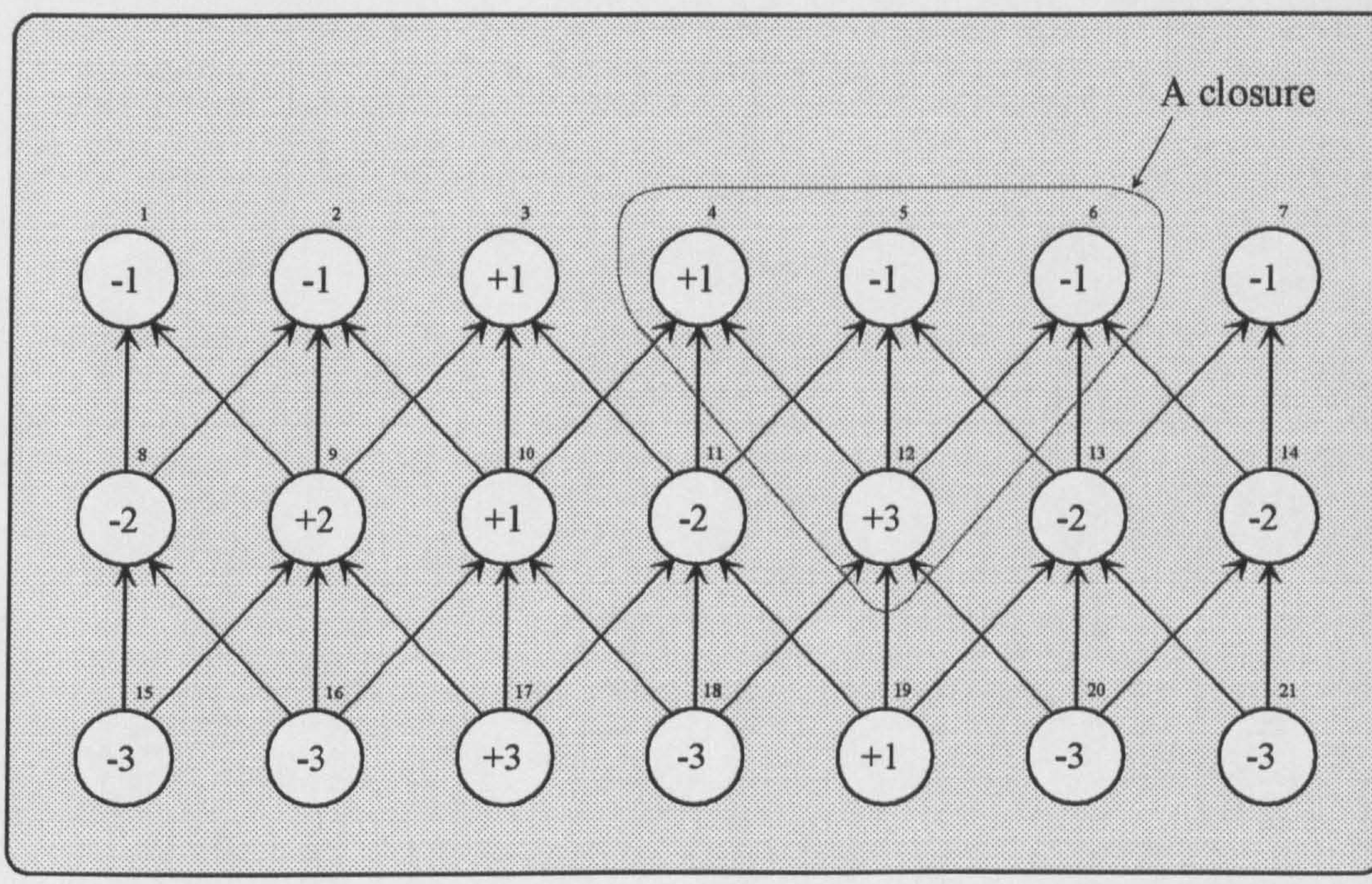


Figure 3.2- A two-dimensional directed graph

In this graph, each vertex is assigned the value of the block that it represents. This value is called a **mass.** The value of a closure is the sum of the masses of all vertices that comprise the closure. Each vertex is also assigned an identification or sequence number which indicates its location within the block model. The vertices are connected by arcs that define access or mining constraints; in the simple example shown in Figure

3.2 it is assumed that the blocks are squares and the pit slopes are 45° in all directions. For example, to mine vertex 12 requires the prior removal of vertices 4, 5 and 6 and these four vertices define a possible pit (closure). This closure has a cumulative value of +2 that is obtained by summing the values of the vertices within it.

The closure G(Y), of the directed graph G(X, A), with the maximum value is called a **maximum closure** of G(X, A) and represents the optimum pit limit.

A **tree** is a connected and directed graph that contains no cycles. It is represented as T = (X, A). For example in Figure 3.1 X = {x_1, x_2, x_3}, A = {a_1, a_2} is a tree.

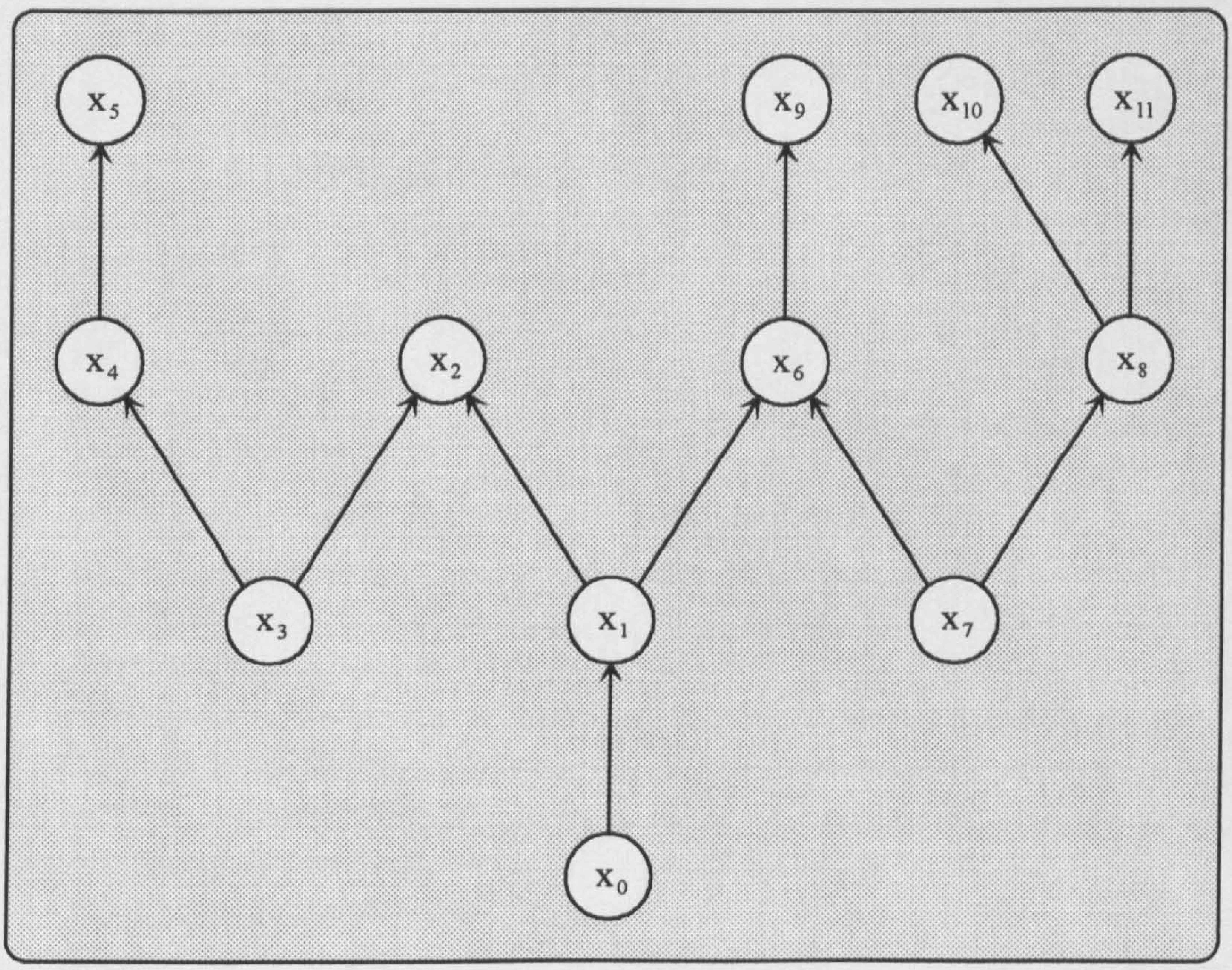


Figure 3.3- An example of a rooted tree

A **rooted tree** is a tree that contains one unique vertex called a root. Any vertex can be designated as a root for any specified purpose. Figure 3.3 shows an example of a rooted tree which contains a vertex x_0 as its root.

If a rooted tree T is separated into two parts by the elimination of one arc a_i, the part of tree which does not contain the root is called a **branch**, denoted by $T_i = (X_i, A_i)$. The root of the branch is the vertex of the branch adjacent to the arc a_i. For example, if the arc (x_1, x_6) is eliminated from the rooted tree illustrated in Figure 3.3, then the branch shown in Figure 3.4 is obtained. The root of this branch is vertex x_6.

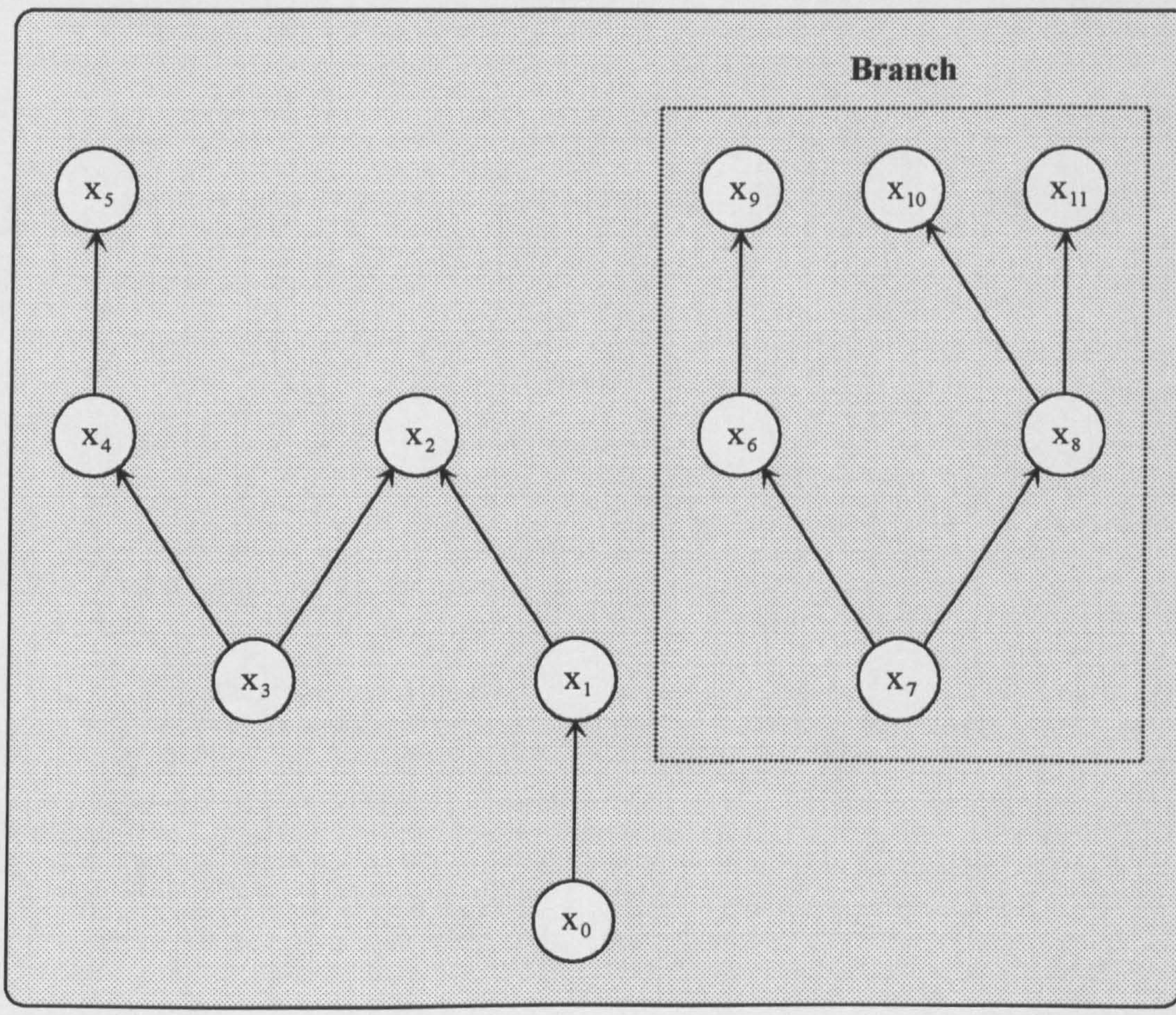


Figure 3.4- Rooted tree and branch obtained by the elimination of arc (x_1, x_6)

Each arc a_i of a tree T defines a branch, denoted by $T_i = (X_i, A_i)$. If numerical values are assigned to the vertices, mass M_i of branch T_i can be obtained by summing the masses of all of vertices of T_i. The arc a_i is said to **support** the tree T_i and the mass M_i.

For example, the mass of the branch obtained by the elimination of the arc (x_1, x_6) in Figure 3.5 is +3+5-2-1-1-1 = +3.

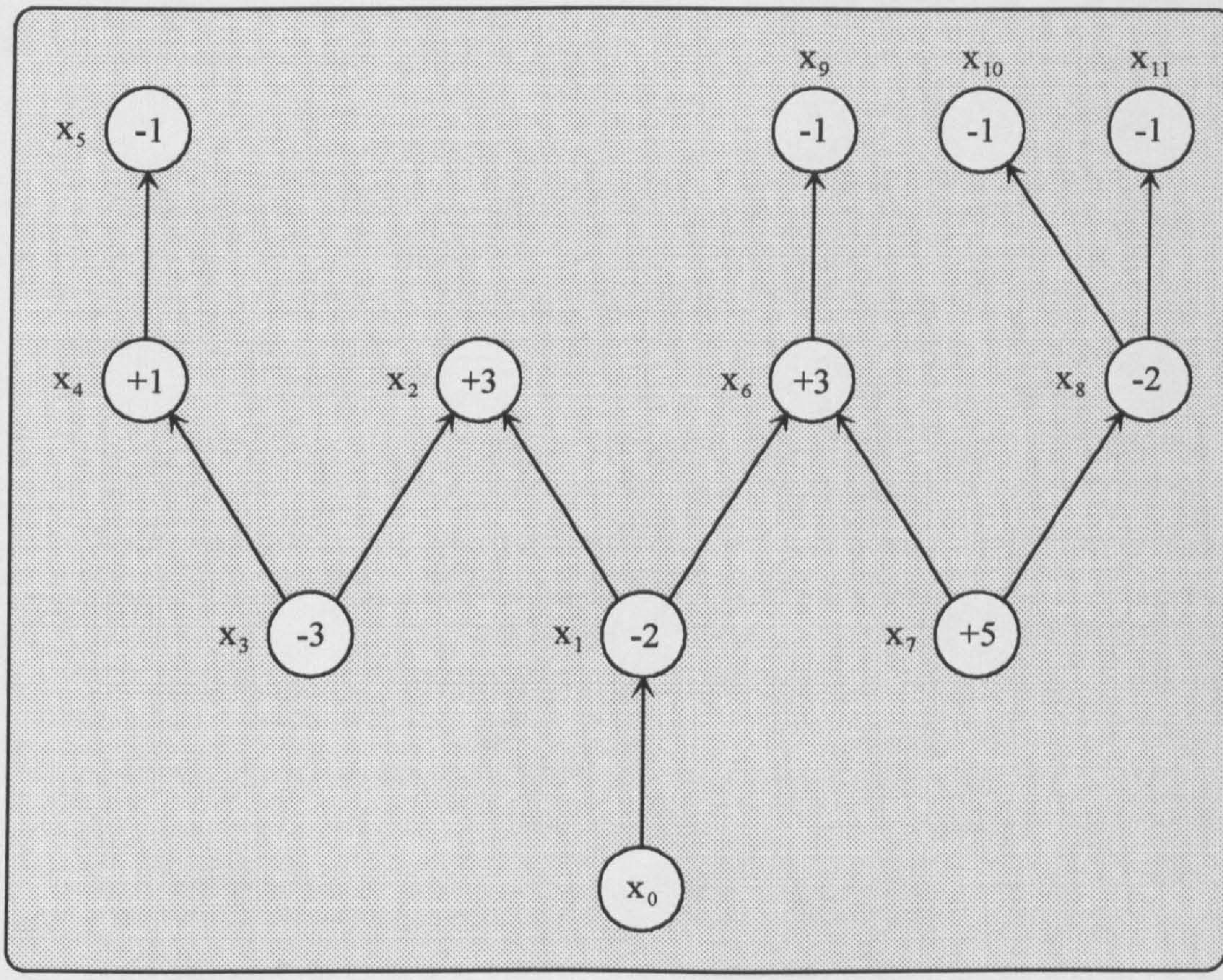


Figure 3.5- A rooted tree in which numerical values are assigned to the vertices

In a rooted tree any arc or branch can be characterised by its orientation with respect to the root. If an arc a_i points toward the branch, it is called a **p-edge** (plus-edge) - in other words a p-edge is one for which the terminal vertex of its associated arc, a_i, is part of the branch T_i. T_i is then called a **p-branch**. As an example, consider the arc (x_1, x_6) in Figure 3.3. If this arc is eliminated the branch shown in Figure 3.4 is obtained. The terminal vertex of the arc (x_1, x_6) is part of the branch. Therefore this arc is a p-edge and the branch obtained by the elimination of this arc is a p-branch.

If an arc a_i points away from the branch, then a_i is called an **m-edge** (minus-edge) and T_i is an **m-branch**. For example, consider the arc (x_3, x_2) in Figure 3.5. If this arc is eliminated the branch shown in Figure 3.6 is obtained. The terminal vertex of the arc (x_3, x_2) is not part of the branch. Therefore this arc is an m-edge and the branch obtained by the elimination of this arc is an m-branch.

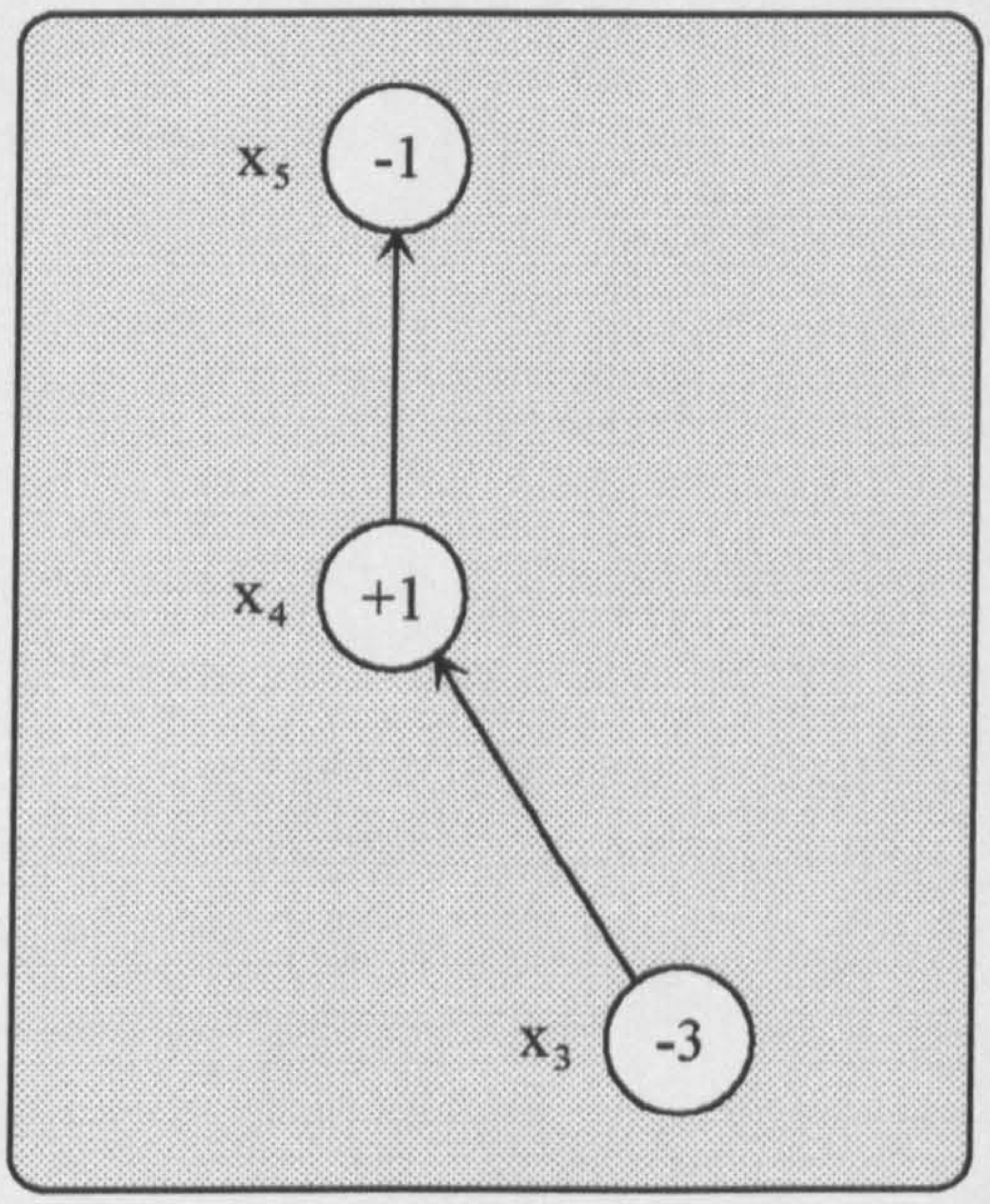


Figure 3.6- Branch obtained by the elimination of arc (x_3, x_2) in Figure 3.5

Any arc or branch can also be classified as **strong** or **weak**. A p-edge or p-branch is strong if it supports a mass that is strictly positive. An m-edge or m-branch is strong if it supports a mass that is negative or null. Arcs or branches that are not strong are said to be weak. This is summarised in Table 3.1.

The branch obtained by the elimination of arc (x_1, x_6) in Figure 3.5, is a p-branch and it supports a mass of +3. Therefore both branch and arc (x_1, x_6) are strong.

The branch shown in Figure 3.6, is an m-branch and it supports a mass of -3. Thus both branch and arc (x_3, x_2) are strong.

The result of classifying the arcs of the tree in Figure 3.5 as strong or weak is illustrated in Figure 3.7.

Case	Direction	Support	Label
1	Plus	Positive	Strong
2	Plus	Null or negative	Weak
3	Minus	Positive	Weak
4	Minus	Null or negative	Strong

Table 3.1- Labelling guide for arcs and branches

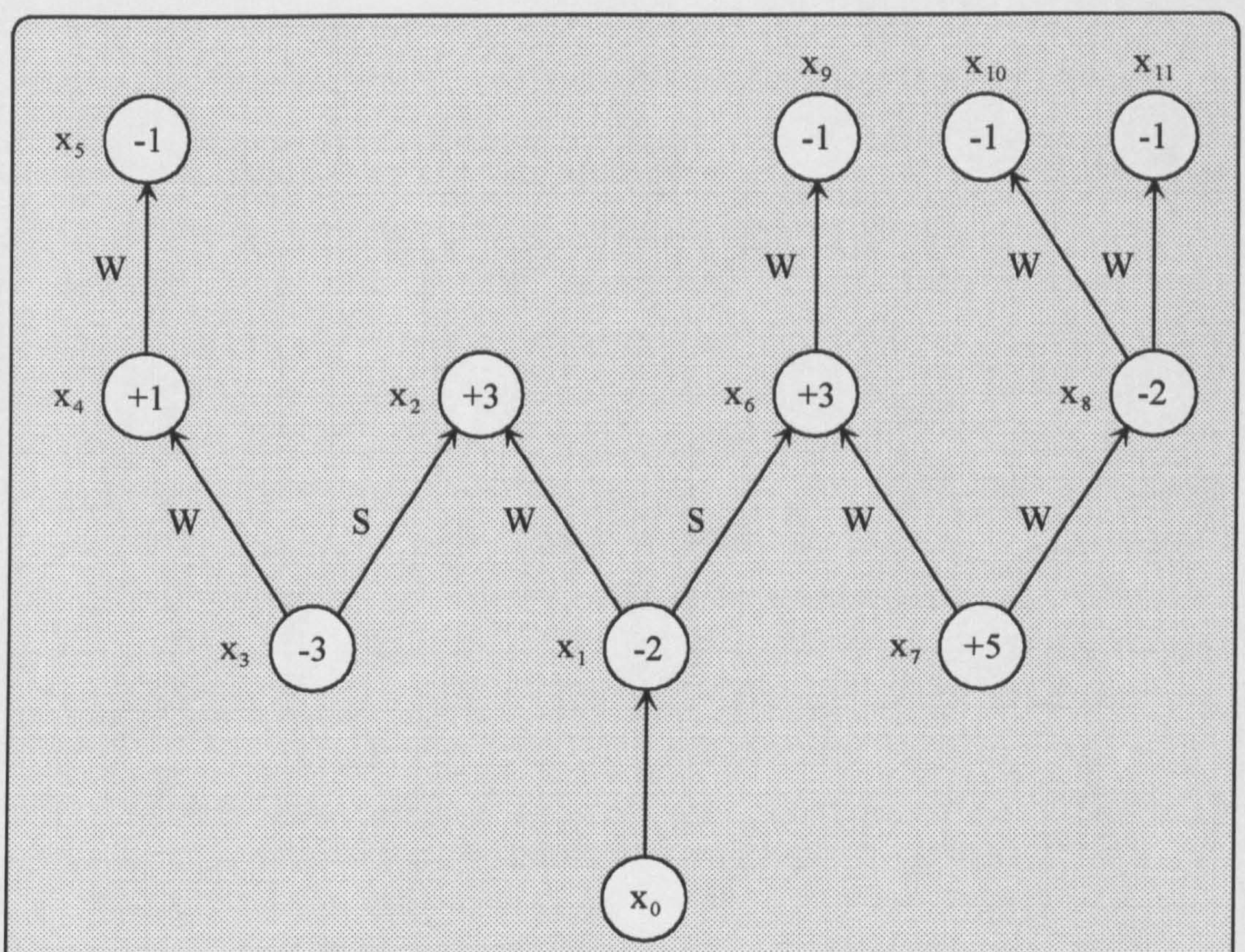


Figure 3.7- Arcs are classified as strong (S) or weak (W)

A vertex x_i is labelled strong if there exists at least one strong edge on the chain of T joining it to the root x_0. Vertices that are not strong are labelled as weak. For example, in Figure 3.7 vertex x_6 is a strong vertex and vertex x_2 is a weak vertex.

A tree is **normalised** if the root is common to all strong edges. Any tree T of a graph G can be normalised by changing it such that all strong edges have the root as one of the vertices. This is achieved by selecting a strong arc (x_i, x_j) not connected to the root, and replacing it with a new arc from the root x_0 to either vertex x_j if the arc (x_i, x_j) is a p-edge or to the vertex x_i if the arc (x_i, x_j) is an m-edge.

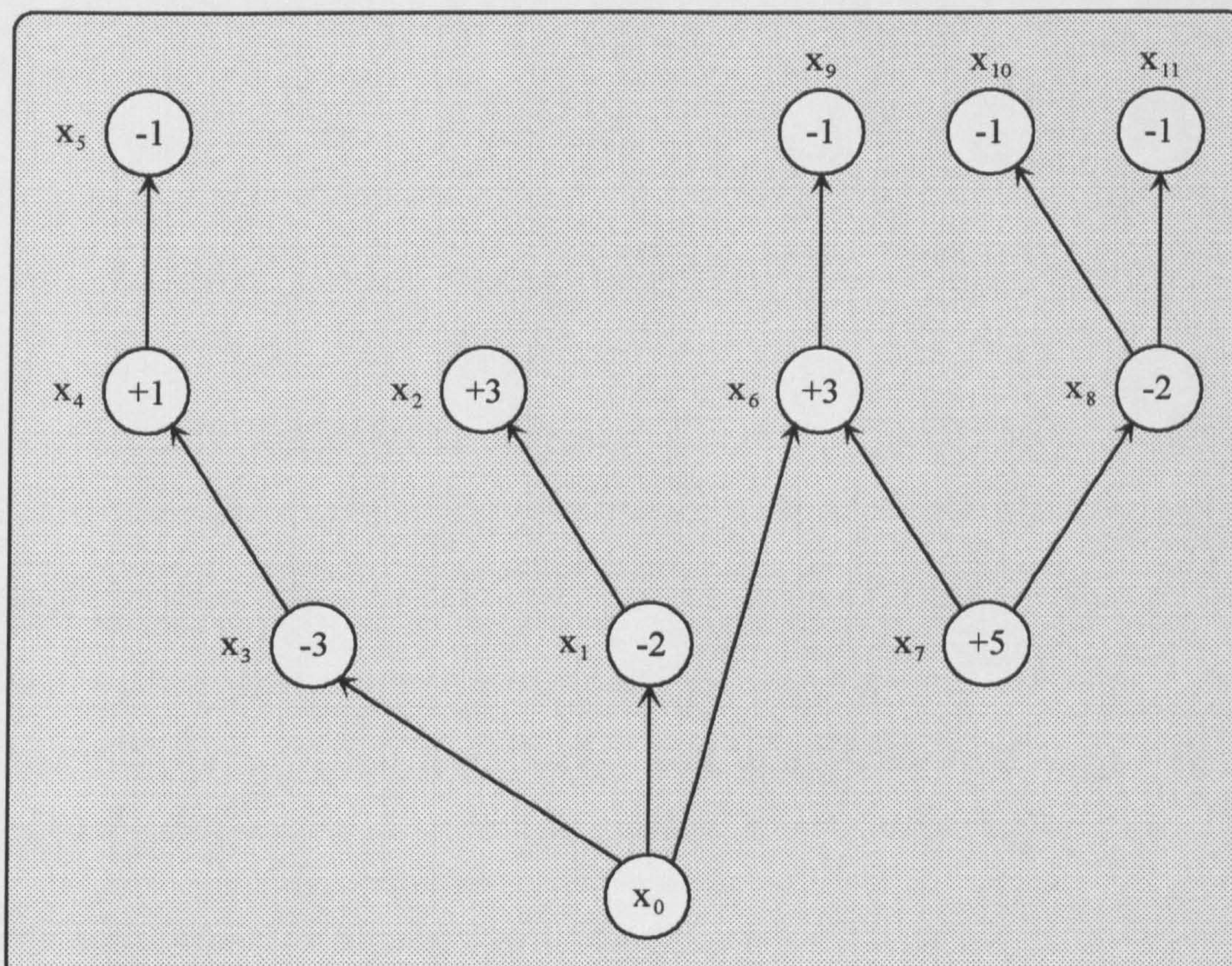


Figure 3.8- The result of normalisation of the tree in Figure 3.7

For example, the tree shown in Figure 3.7 is not normalised because the strong arcs (x_1, x_6) and (x_3, x_2) do not have the root as one of their vertices. To normalise the tree these two arcs must be changed. The arc (x_1, x_6) of the strong p-edge is replaced by the dummy arc (x_0, x_6) and the arc (x_3, x_2) of the strong m-edge is replaced by the dummy arc (x_0, x_3) to give the normalised tree shown in Figure 3.8.

In graph theory terms the optimal pit problem becomes that of finding a maximal closure, Y, of the directed graph G = (X, A) representing the orebody block model in which vertex x_i is assigned a mass m_i equal to the corresponding block value. In other words, the problem is to find a set of vertices Y belonging to X such that if $x_i \in Y$ then $\Gamma x_i \in Y$ and $\sum_{x_i \in Y} m_i$ is maximum. The optimum pit (maximum closure Y of a graph) comprises all vertices in the branches of a normalised tree connected to the root by a strong edge provided that this normalised tree satisfies all pit slope constraints. The mathematical proof of this conclusion is due to Lerchs and Grossmann (1965).

3.3- The Lerchs-Grossmann algorithm

The Lerchs-Grossmann algorithm (1965) takes the orebody revenue block model as the input and determines which blocks should be mined to obtain maximum net profit. The algorithm involves the construction of a normalised tree for each successive level of the pit. It starts with the construction of a normalised tree T_0 in a directed graph G representing the revenue block model of the deposit. This tree is then iteratively transformed into successive trees T_1, T_2, . . . , T_n, until no further transformation is possible. Iteration i+1 transforms a normalised tree T_i into a new normalised tree T_{i+1}. Each tree $T_i = (X, A_i)$ is characterised by its set of arcs A_i and its set of strong vertices Y_i. The maximum closure Y is found by summing the vertices of the branches connected to the root by strong edges. The algorithm formulated by Lerchs and Grossmann (1965) and described by Lipkewich and Borgman (1969) may be summarised in the following steps:

Step 1 Consider level i. Add arcs from the root, vertex x_0, to all vertices on level i.

Step 2 Classify arcs as weak or strong.

Step 3 Check for negative mass (i.e. the sum of all blocks connected beyond the terminal point of an arc) overlying a strong vertex. If none go to step 5. If a weak vertex j overlies a strong vertex k add the arc (k, j) to the tree T_i. Replace the arc (x_0, T_i), i.e. the arc connecting the tree T_i to the root (vertex x_0) by the arc (x_0, j).

Step 4 If the tree T_i contains any strong arcs not connected to the root, vertex x_0, normalise the tree. Go to step 3.

Step 5 Remove all strong vertices; these form part of the maximum closure. Go to the next level i+1 and start again from step 1.

3.3.1- A two-dimensional example

The algorithm can best be explained by a simple two-dimensional example illustrated in Figure 3.9. There are two numbers in each block: the upper is the block number and the lower is the value of the block. For the sake of simplicity slope angles are assumed to be 45° and the blocks are squares. The corresponding directed graph representing this example is shown in Figure 3.2 in which each block has three successors (three arcs pointing away from the block). The successor vertices must be removed before a particular block is mined.

1 -1	2 -1	3 +1	4 +1	5 -1	6 -1	7 -1
8 -2	9 +2	10 +1	11 -2	12 +3	13 -2	14 -2
15 -3	16 -3	17 +3	18 -3	19 +1	20 -3	21 -3

Figure 3.9- Block revenue value

The procedure starts from the first level. The initial tree is constructed by adding arcs from the dummy root, vertex x_0, to the all the vertices representing the blocks on this level (step 1). These arcs are classified as weak (W) or strong (S) as illustrated in Figure 3.10 (step 2).

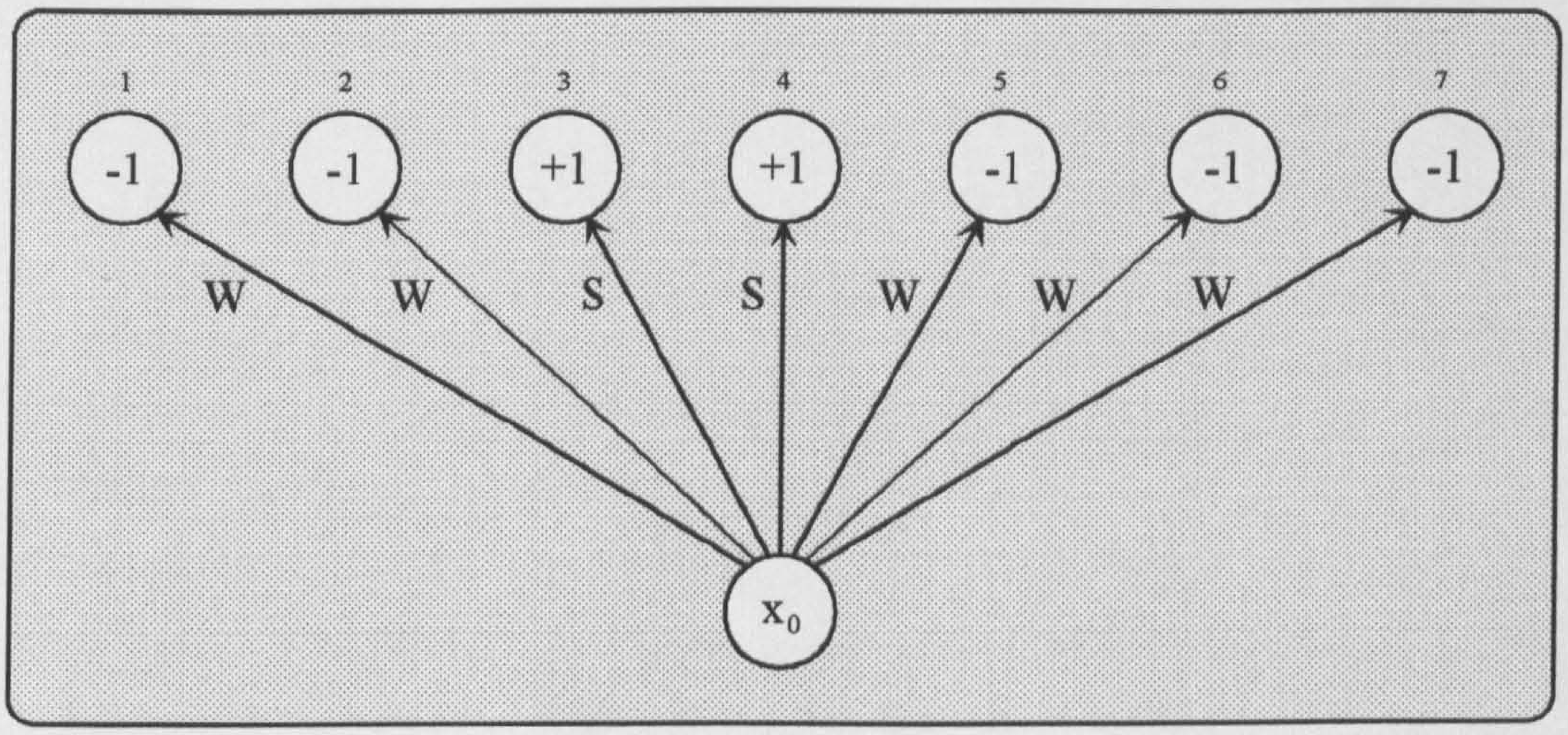


Figure 3.10- Data for the first level

In the tree, there is no weak vertex overlying a strong arc or vertex since only the first level is involved (step 3). This tree is consolidated by deleting strong vertices 3 and 4 as shown in Figure 3.11. The blocks on the second level are then added (step 5).

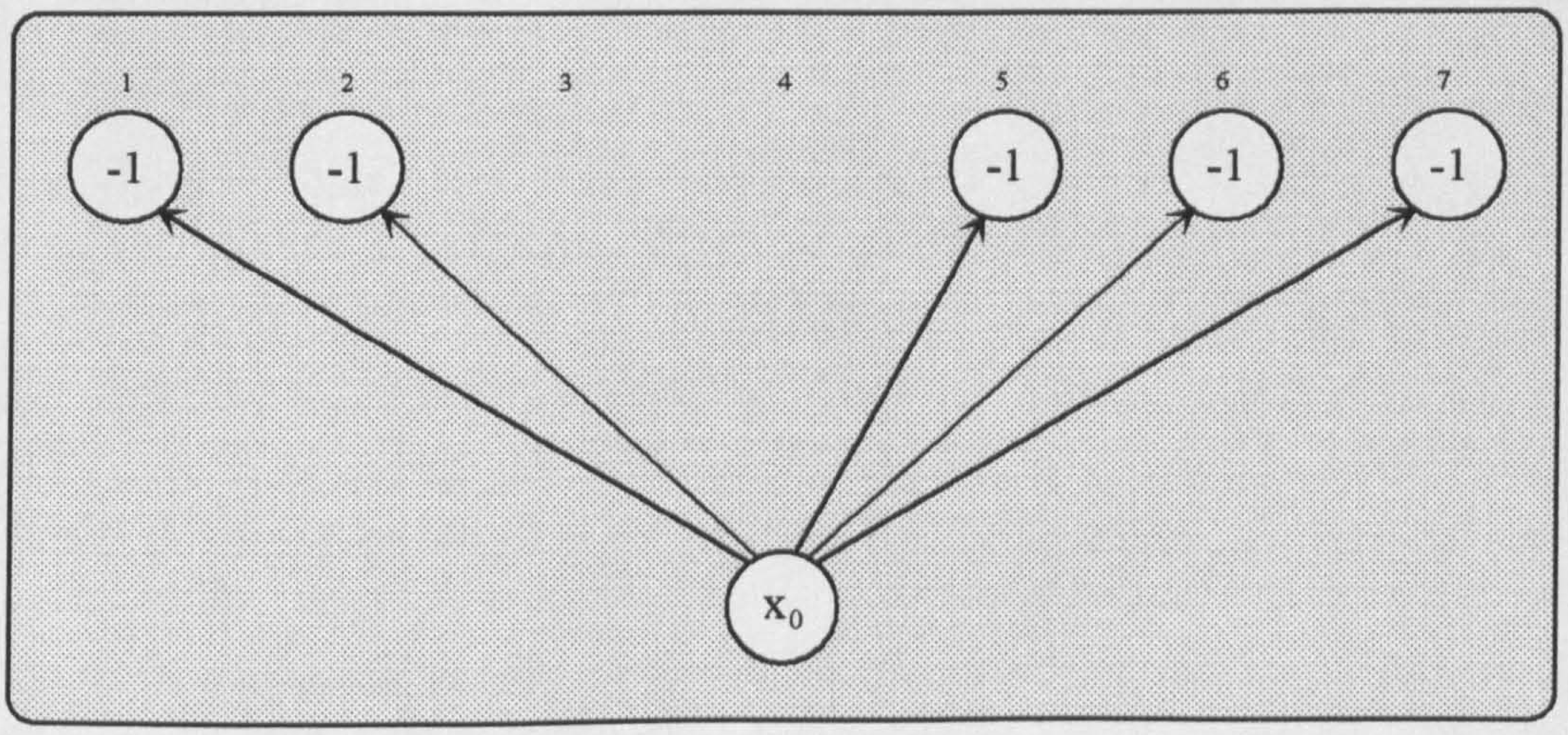


Figure 3.11

Again, arcs are added from the root, vertex x_0, to the all vertices on the second level (step 1). They are also classified as strong or weak as shown in Figure 3.12 (step 2). There are three strong vertices and two weak vertices on this level.

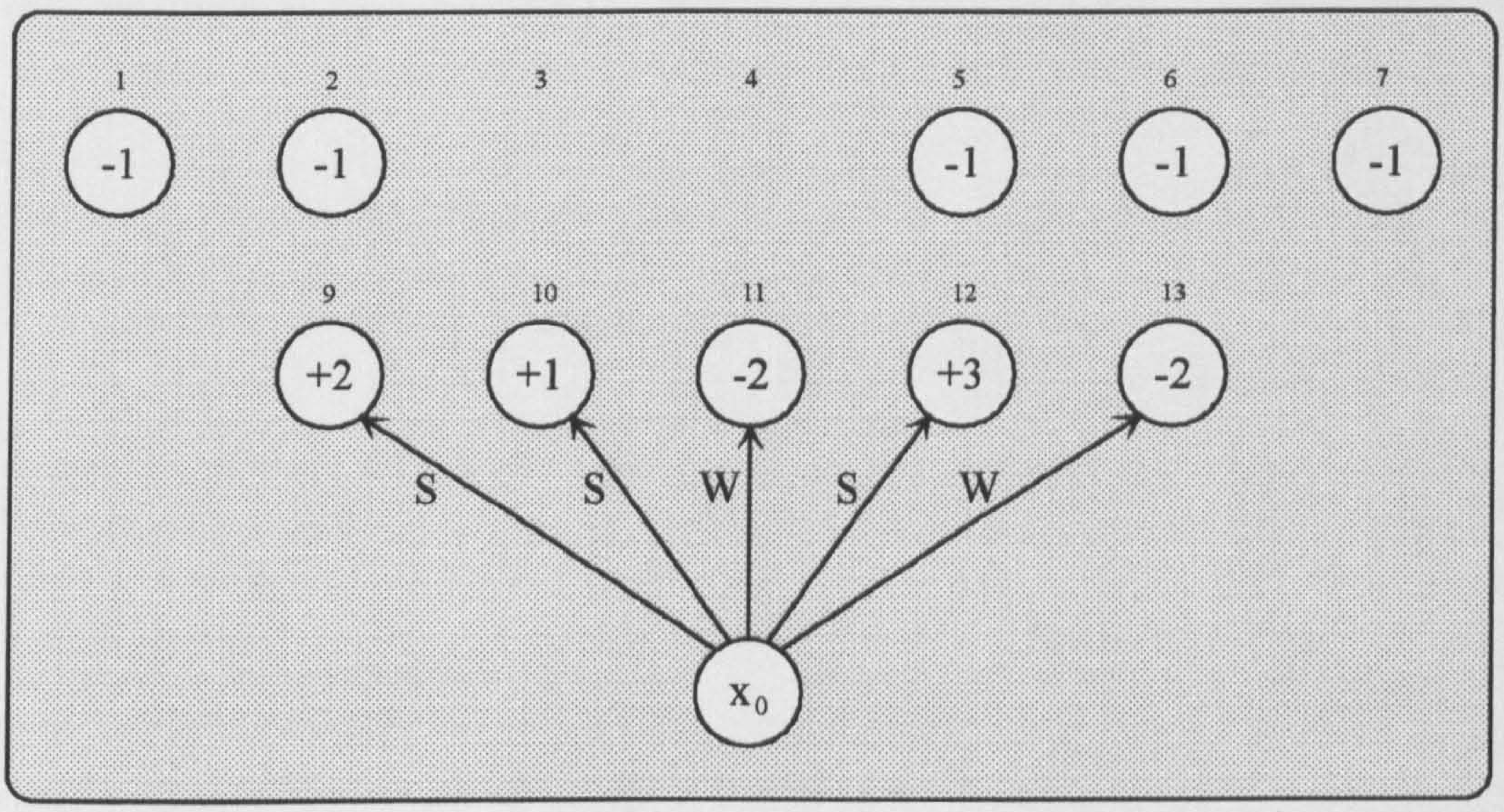


Figure 3.12- Data for the second level

Now consider step 3 of the algorithm to check for negative masses overlying a strong vertex. The weak vertex 1 overlays strong vertex 9 so one arc is added from vertex 9 to vertex 1. The arc from the root to vertex 9 is replaced by the arc from the root to vertex 1. The result is illustrated in Figure 3.13.

Now consider the tree which is connected to the root by the arc from vertex x_0 to vertex 1. If this arc is eliminated from the tree, its terminal vertex will be part of the branch. So it is a p-edge and it supports a total mass of $2 + (-1) = +1$. Therefore this arc is classified as a strong arc.

The weak vertex 2 overlays the strong vertex 9 so it is connected to vertex 9 by adding an arc from vertex 9 to vertex 2. The tree is connected to the root by the arc from vertex x_0 to vertex 1. This arc is removed and replaced by the arc from vertex x_0 to vertex 2 as shown in Figure 3.14.

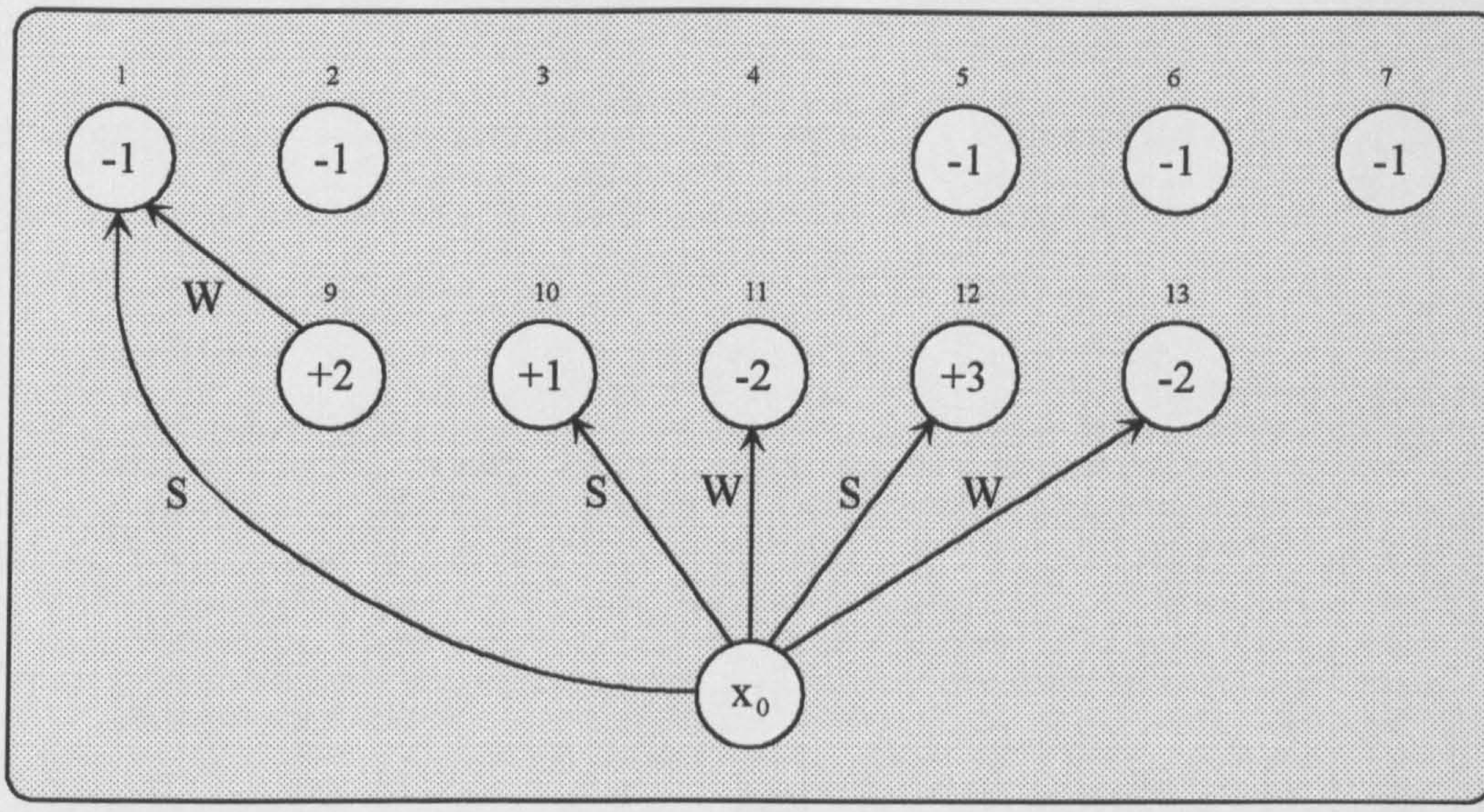


Figure 3.13

The arc from vertex x_0 to vertex 2 has its terminal vertex in the tree and is thus a p-edge. It supports a total mass of (-1) +2 + (-1) = 0 and thus this arc is classified as a weak arc (Figure 3.14).

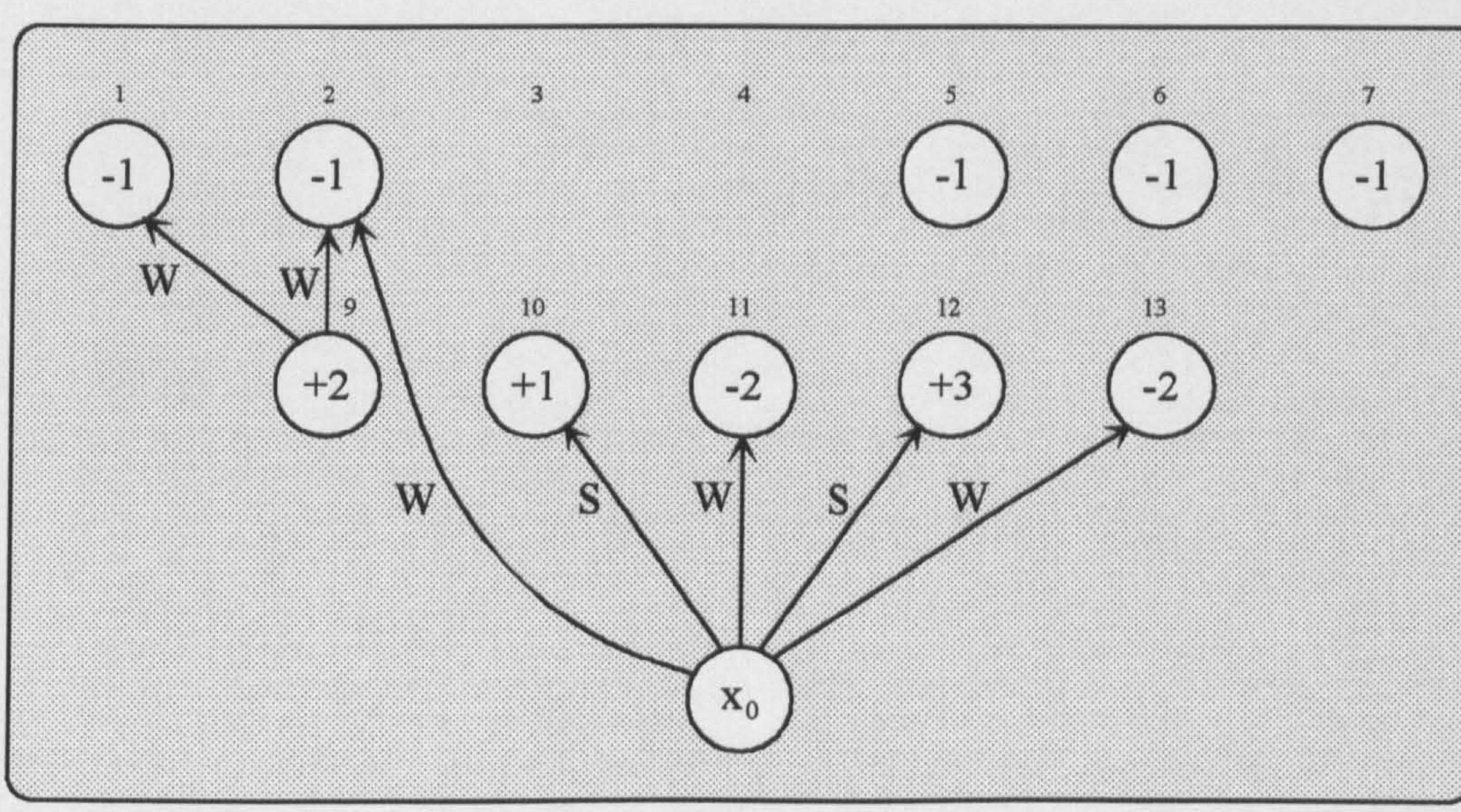


Figure 3.14

The weak vertex 2 overlays strong vertex 10. Therefore the arc from vertex 10 to vertex 2 is added to the tree and the arc from x_0 to vertex 10 is removed and replaced by the arc from x_0 to vertex 2 as shown in Figure 3.15. This arc (from x_0 to vertex 2) has its terminal vertex in the tree and supports a total mass of (-1) +2 + (-1) +1 = +1, thus it is classified as a strong arc.

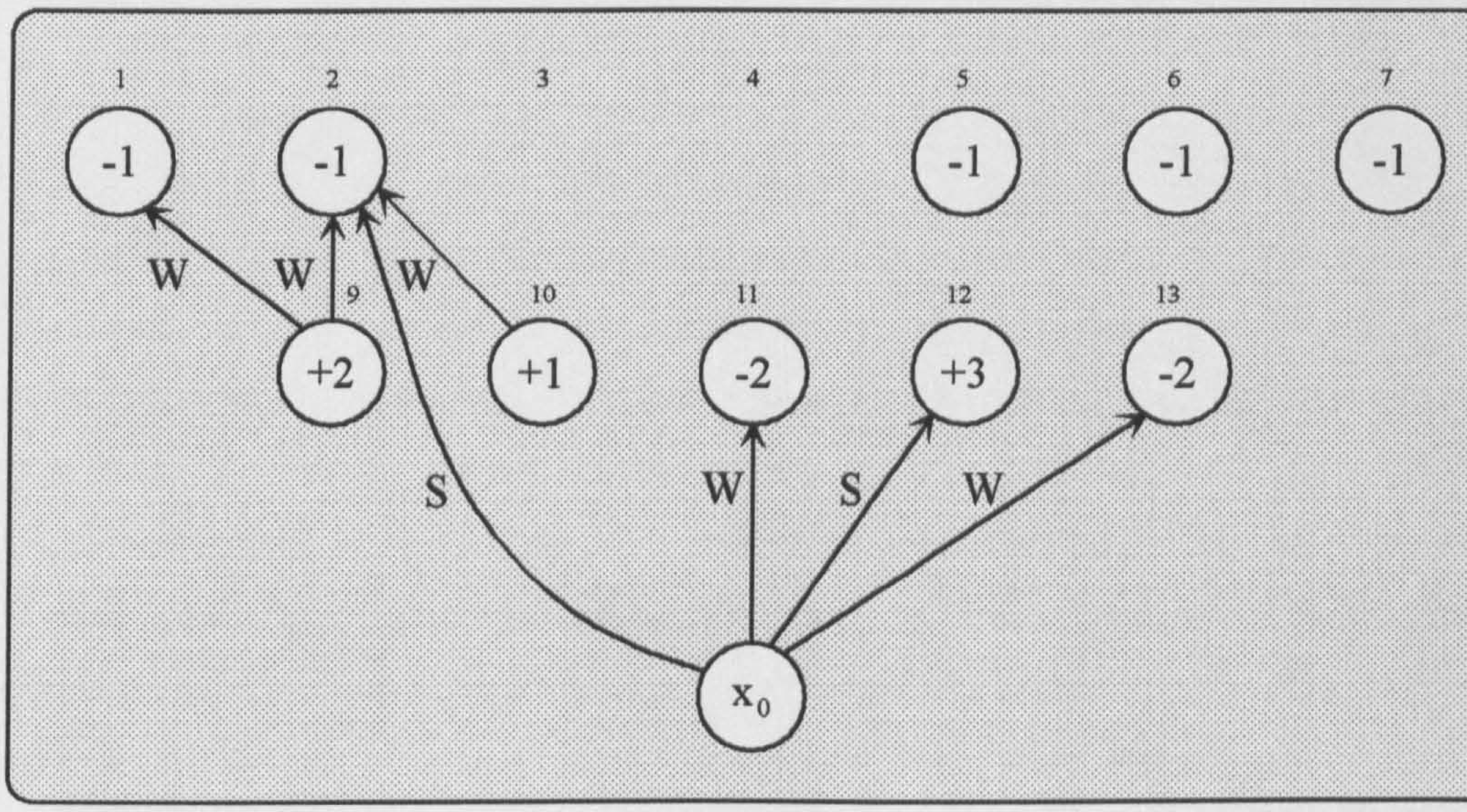


Figure 3.15

The weak vertex 5 overlays strong vertex 12. The arc from vertex 12 to vertex 5 is added to the tree and the arc from x_0 to vertex 12 is replaced by the arc from x_0 to vertex 5 as shown in Figure 3.16. This arc (from x_0 to vertex 5) is a p-edge and supports a total mass of (-1) +3 = +2, thus it is strong.

Similarly, weak vertex 6 overlays strong vertex 12. Therefore vertex 6 is connected to the tree by drawing an arc from vertex 12 to vertex 6 and the arc from x_0 to vertex 5 is replaced by the arc from x_0 to the vertex 6 as shown in Figure 3.17. This arc (from x_0 to vertex 6) has its terminal vertex in the tree and supports a total mass of (-1) +3 + (-1) = +1, thus it is classified as a strong arc.

There are now no negative masses overlying strong arcs or vertices so step 3 of the algorithm is completed. The tree is normalised since all strong arcs are connected to the root (step 4). There are two strong arcs connected to the root. These strong nodes are removed and included as part of the ultimate optimum pit as illustrated in Figure 3.18 (step 5).

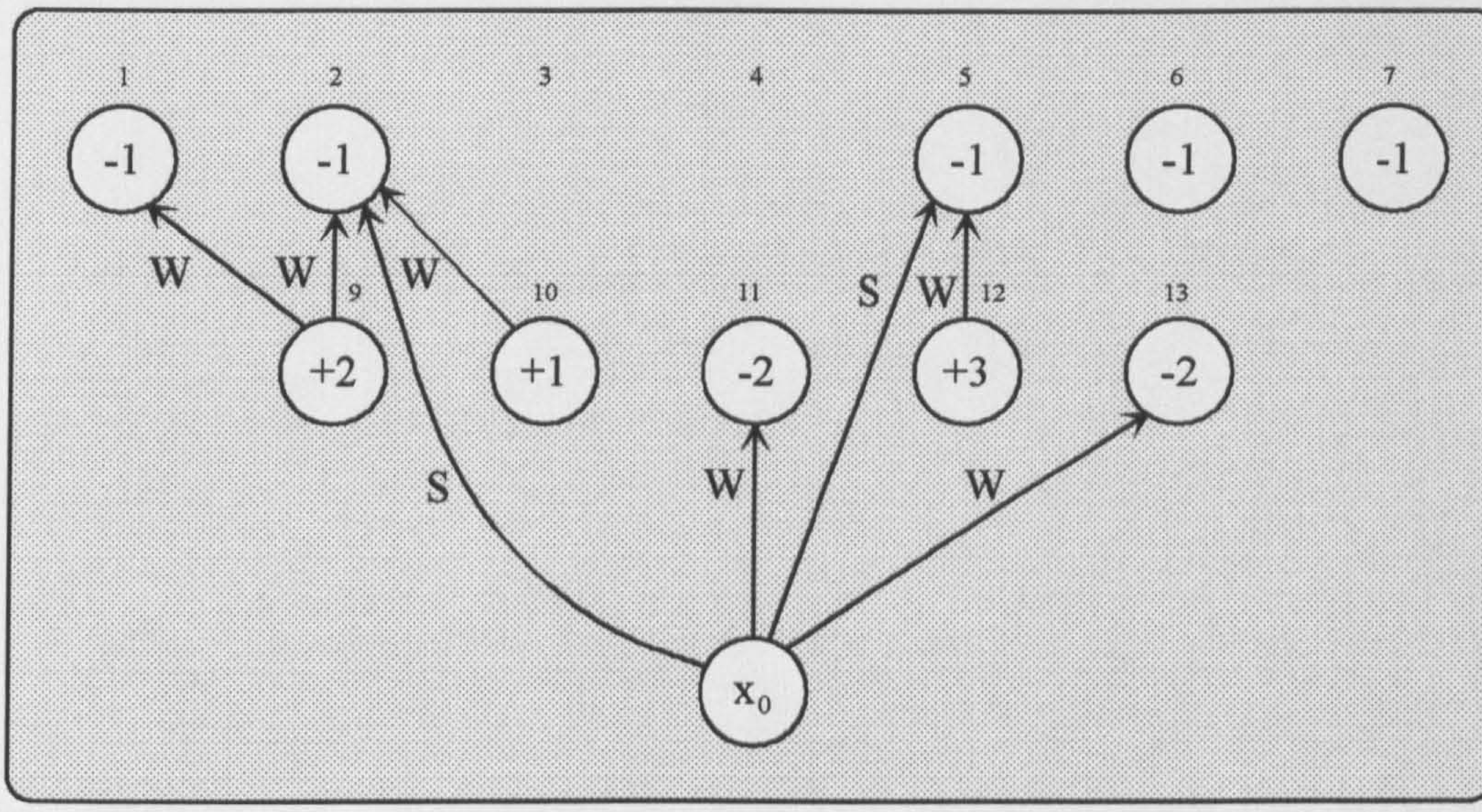


Figure 3.16

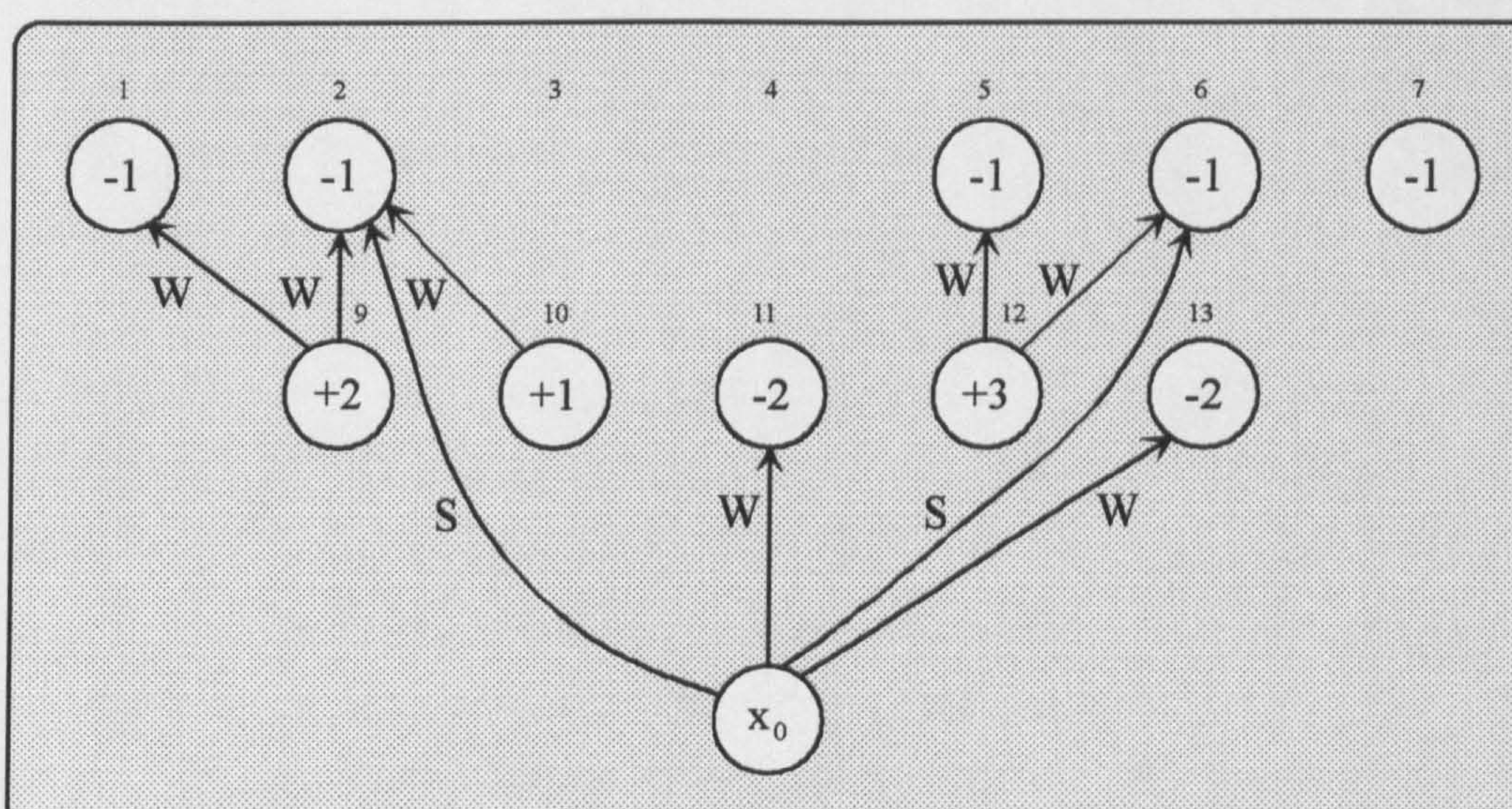


Figure 3.17

Finally, the blocks from the third level are added and joined to the root which gives the graph shown in Figure 3.19 (step 1). The arcs from the root to these vertices of the third level are classified as indicated in Figure 3.19 (step 2). There are two strong arcs and one weak vertex on the third level.

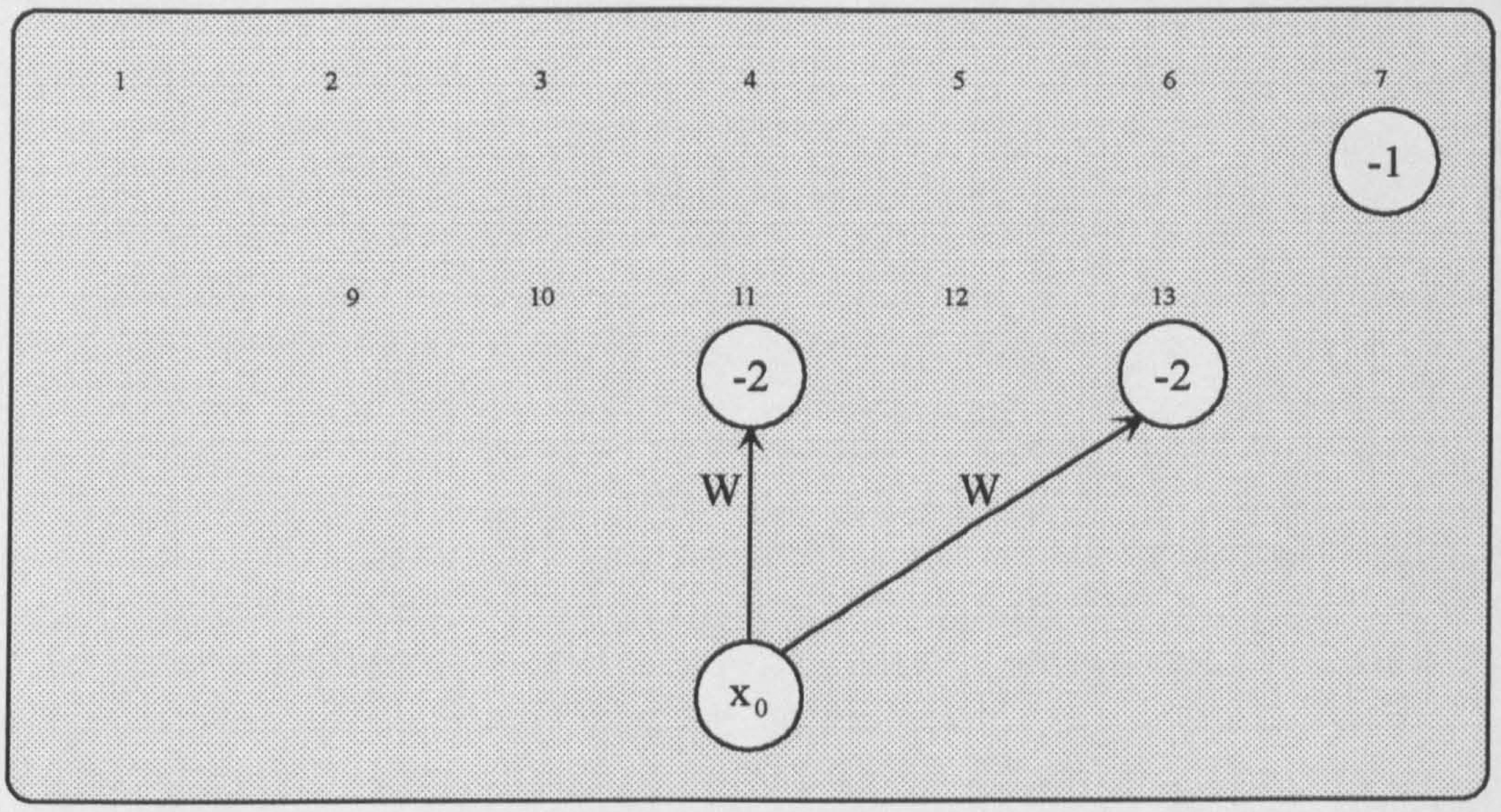


Figure 3.18

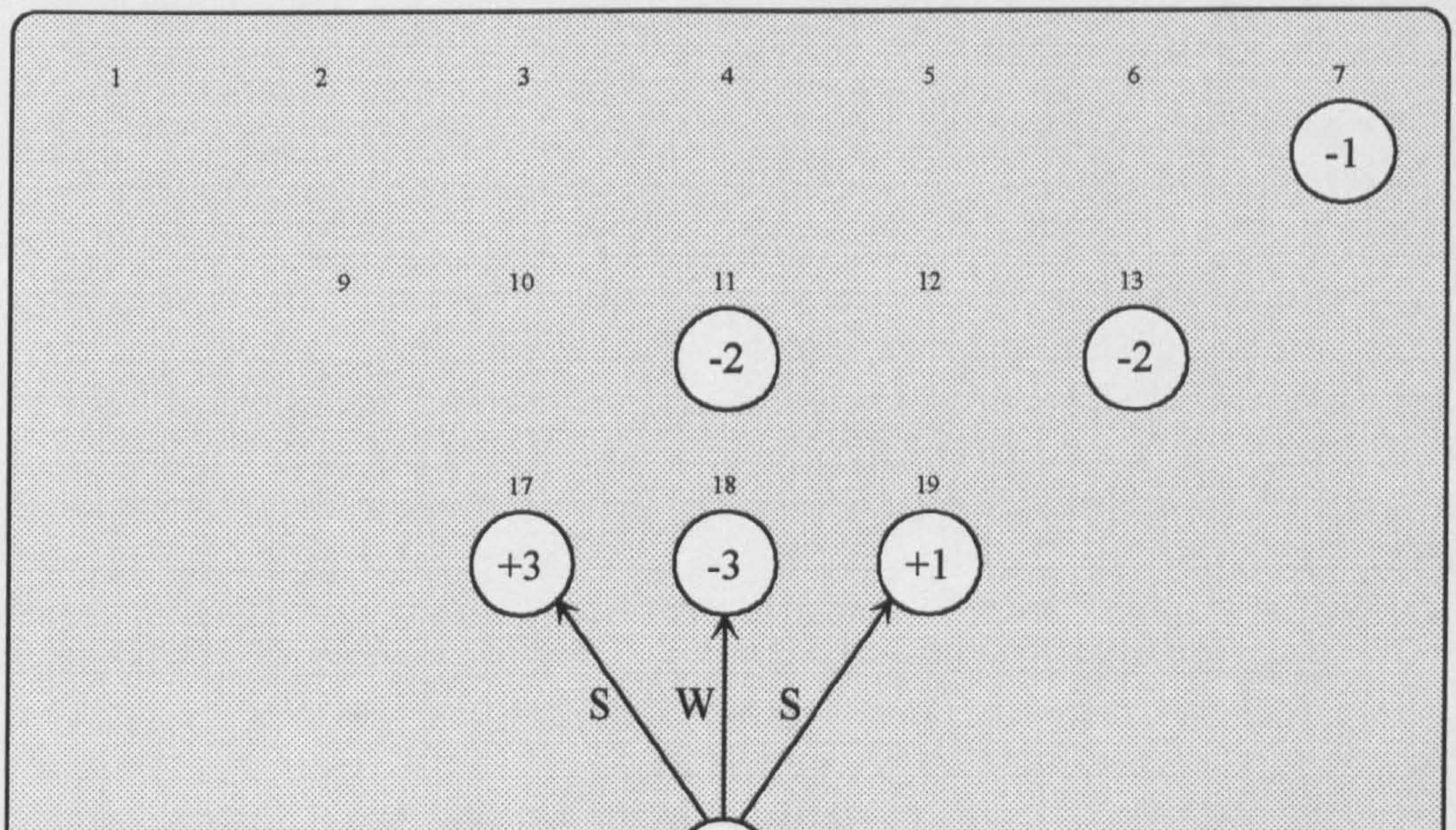


Figure 3.19- Data for the third level

There is a weak vertex (vertex 11) overlaying strong vertex 17. Thus the arc from vertex 17 to vertex 11 is added to the tree and the arc from x_0 to vertex 17 is replaced by the arc from x_0 to vertex 11 as shown in Figure 3.20. The arc from x_0 to vertex 11 is a p-edge and supports a total mass of (-2) +3 = +1, hence it is classified as strong.

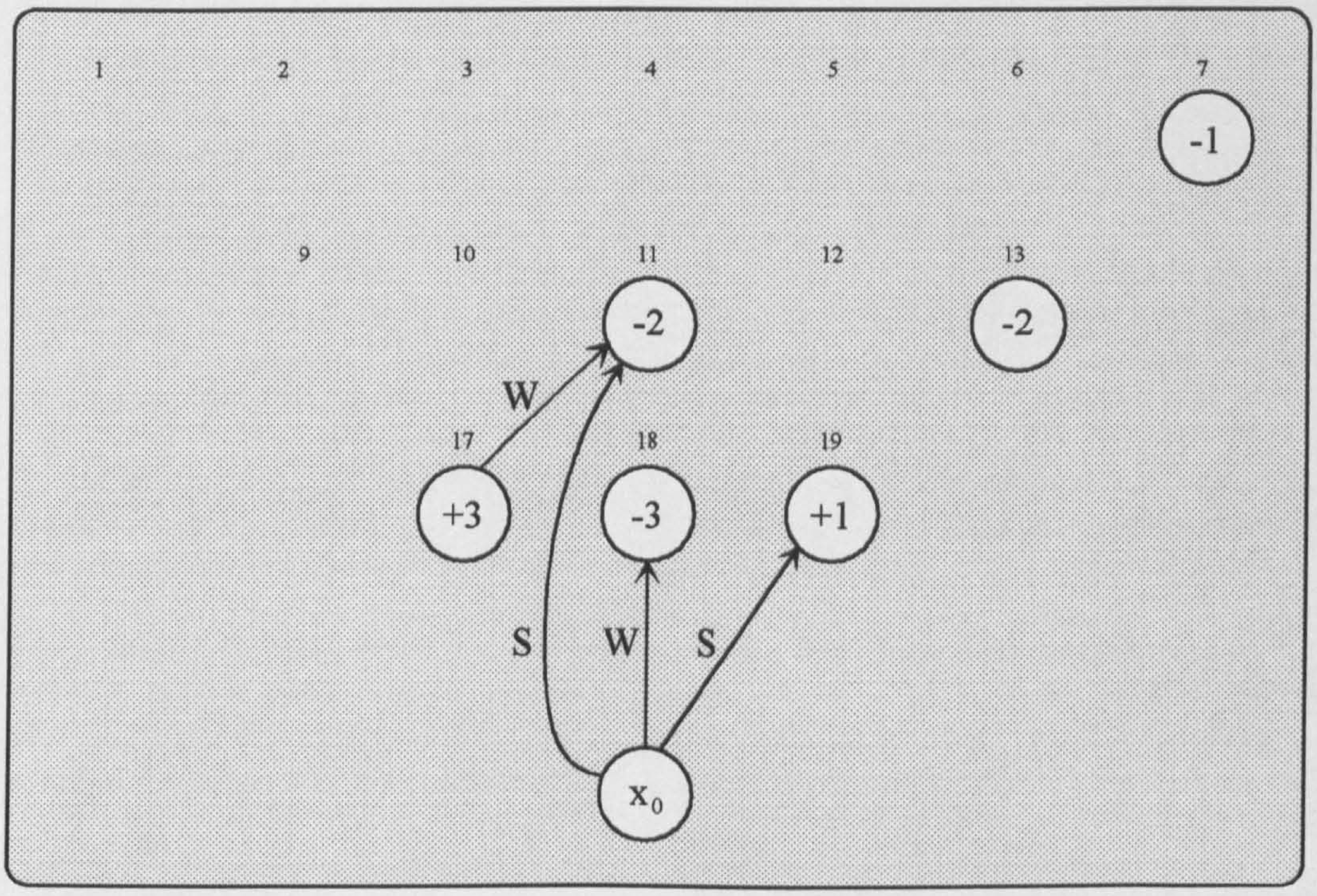


Figure 3.20

The weak vertex 13 overlays strong vertex 19. The arc from vertex 19 to vertex 13 is added to the tree and the arc from x_0 to vertex 19 is replaced by the arc from x_0 to vertex 13 to give the graph shown in Figure 3.21. The arc from x_0 to vertex 13 is a p-edge and supports a total mass of (-2) +1 = -1, hence it is classified as weak.

There is now no negative mass overlying a strong vertex and the tree is normalised so step 3 and step 4 of the algorithm are completed.

There is only one strong arc in the tree – that which joins vertices 11 and 17.

These vertices are removed and included in the ultimate optimum pit. The algorithm is completed and the optimal pit with a value of +5 is obtained (Figure 3.22).

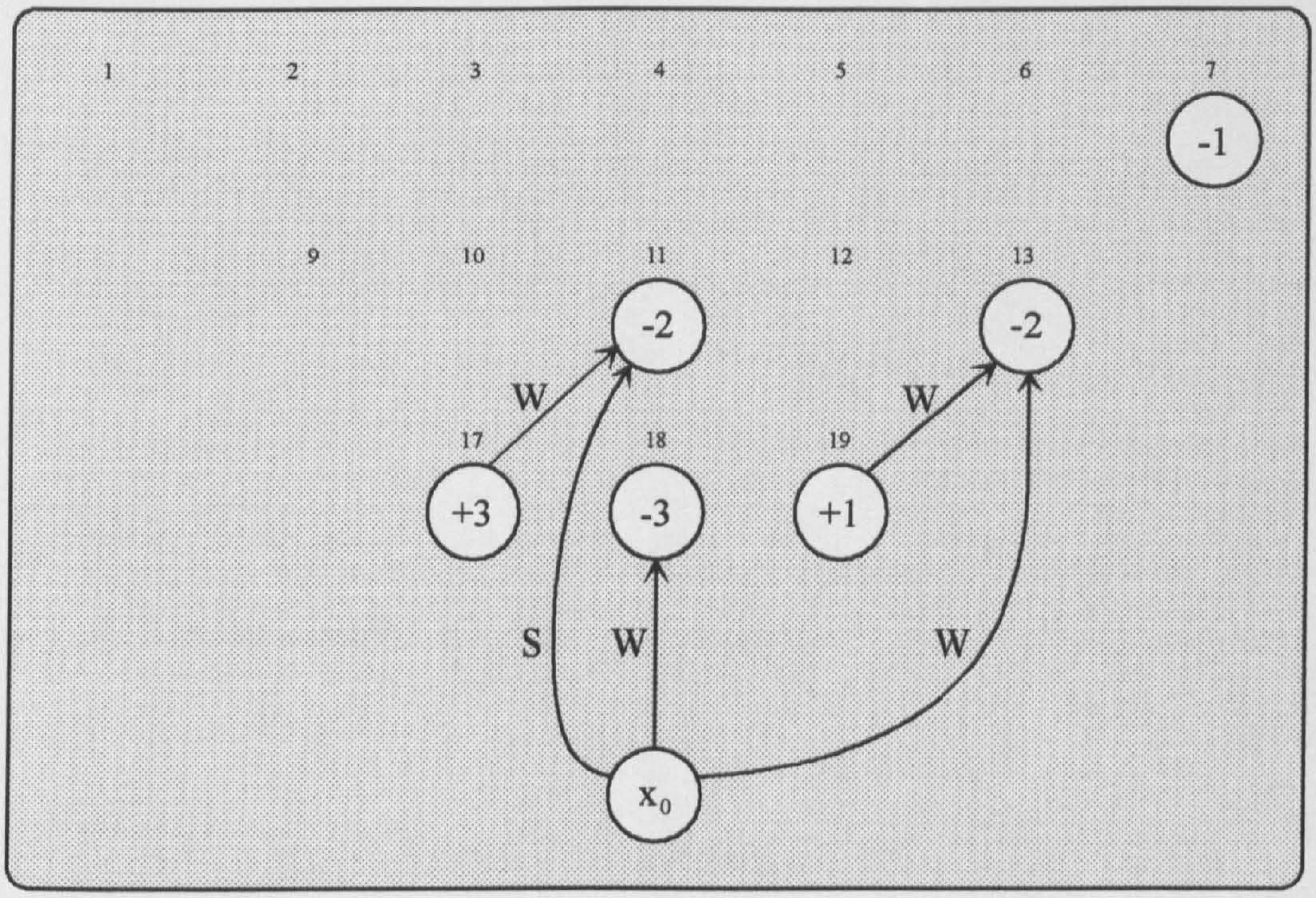


Figure 3.21

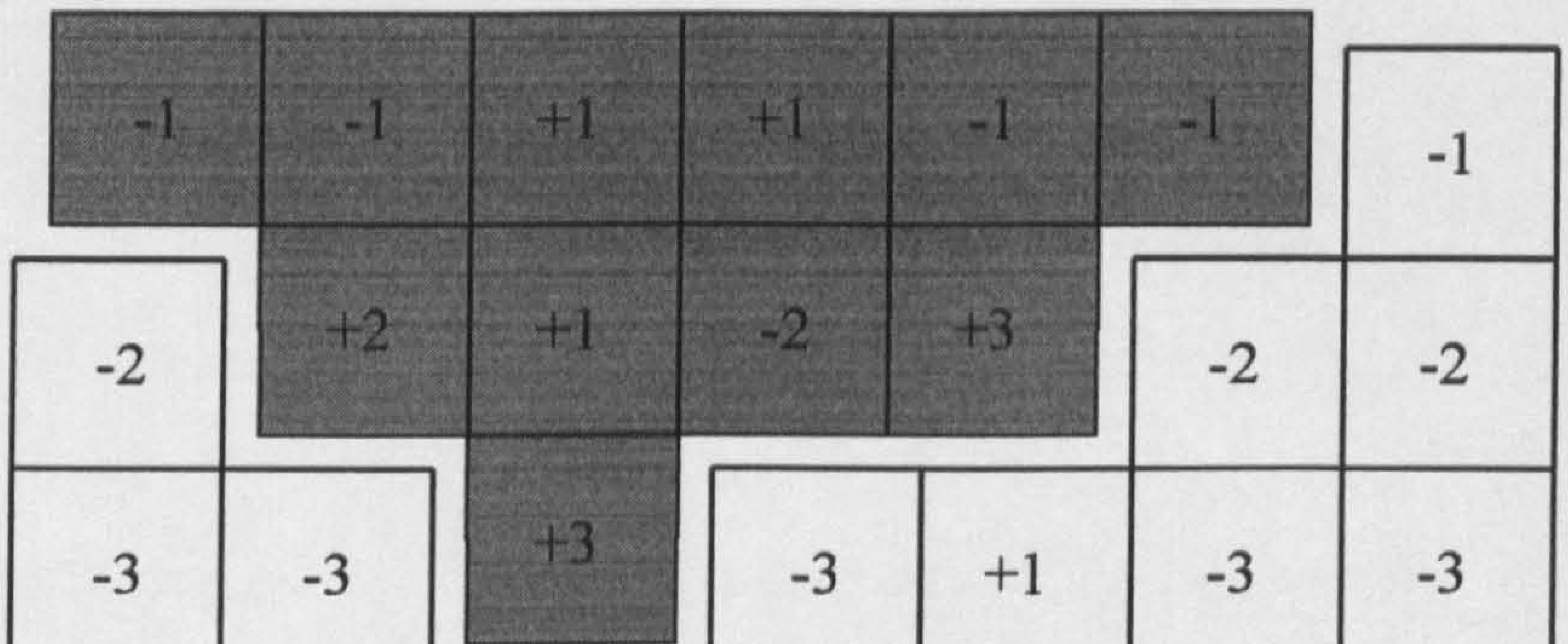


Figure 3.22- The optimum pit limit

3.4- Mining and access constraints

For a deposit represented as a grade or revenue block model, pit slopes are specified in terms of blocks which must be removed in order to provide access to each block within the block model. In the Lerchs-Grossmann algorithm, these restrictions are imposed by directed arcs. They indicate which blocks should be removed before a particular block can be mined. For example, consider Figure 3.2 in which each block has three successors (the arcs pointing away from the block). The successor blocks (vertices) of any specified block (vertex) must be removed before that block can be mined. Various procedures are used to specify mining and access constraints for block models. They may be classified into two categories:

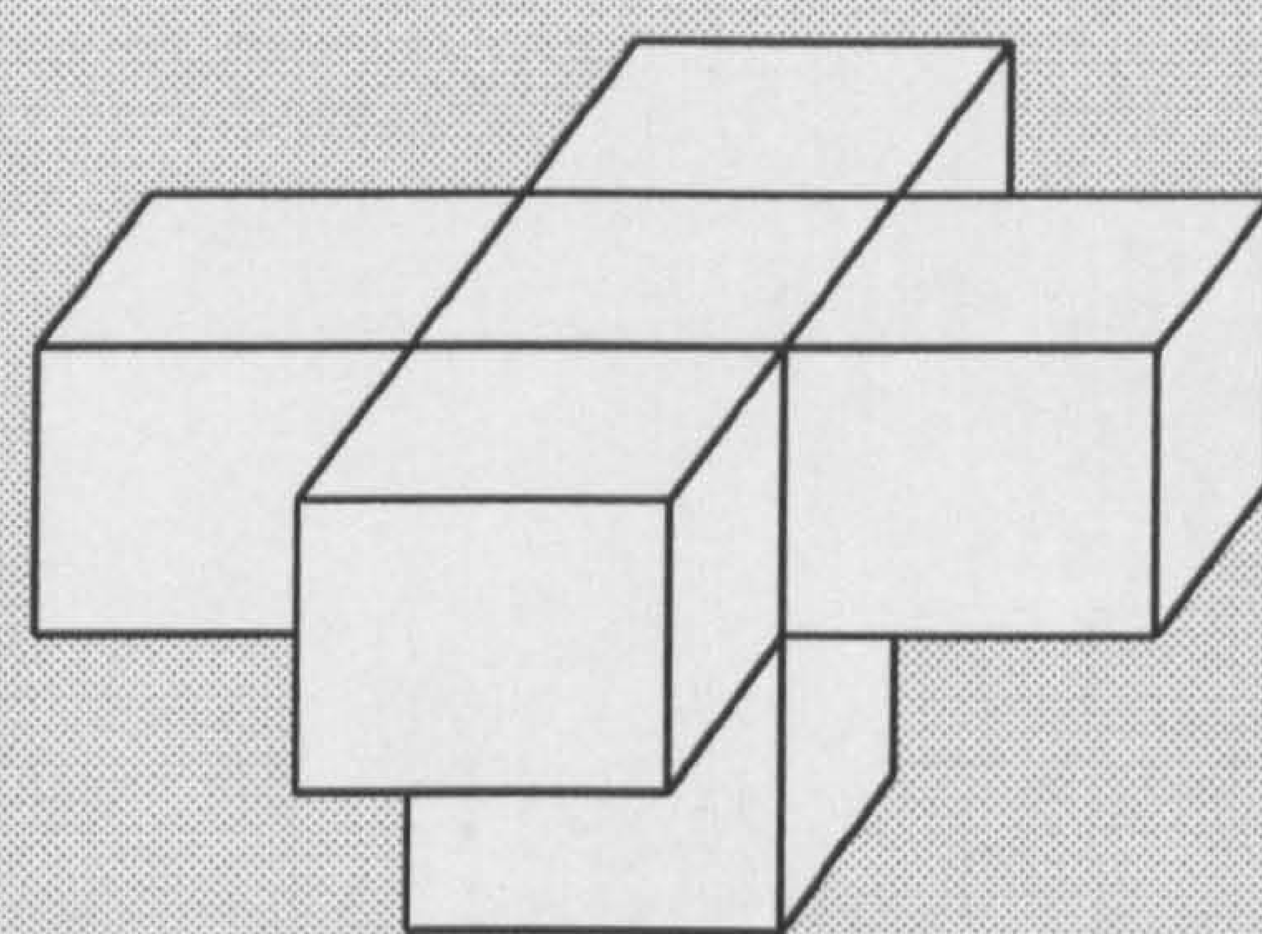
a- Non-cone-based methods

b- Cone-based methods

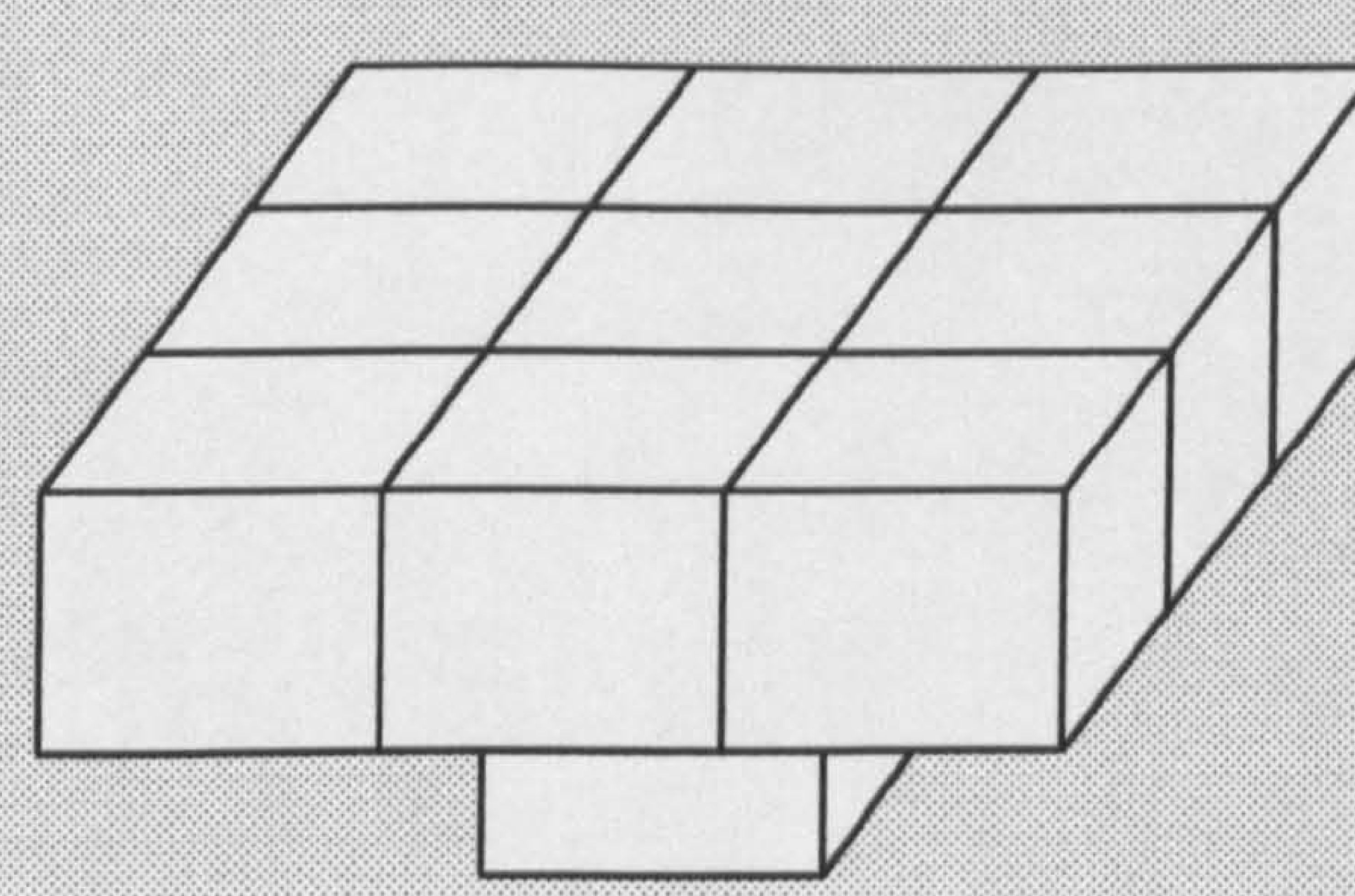
The first category, non-cone-based methods, involves the use of a pattern, or a set of blocks, to define mining slopes. For example:

- 1:5 block configuration
- 1:9 block configuration
- Combination of both or 1:5:9 pattern

In the original formulation of the Lerchs-Grossmann algorithm, the 1:5 block pattern is used to specify mining slopes. In this pattern, in order to gain access to one block, five over-lying blocks, one-up and one-over, as illustrated in Figure 3.23a, must first be removed. This pattern requires the use of five arcs pointing away from each vertex (block) to satisfy the mining constraints. If this support is carried up over several levels, as indicated by Lipkewich and Borgman (1969), an undesirable wall slope would be obtained. For example, consider a 45° slope in a cubic block model as shown in Figure 3.24 which illustrates the pit shape for mining a block on the fifth level in row 5 and column 5. The numbers indicate the levels from the surface down to and including this level that must be mined. As can be seen from Figure 3.24, only on cross-sections A-A and B-B is the average slope angle 45°. On cross-sections C-C and D-D the average slope angle would approximately 55°.



a- 1:5 block pattern in which five overlying blocks must be removed to mine one block

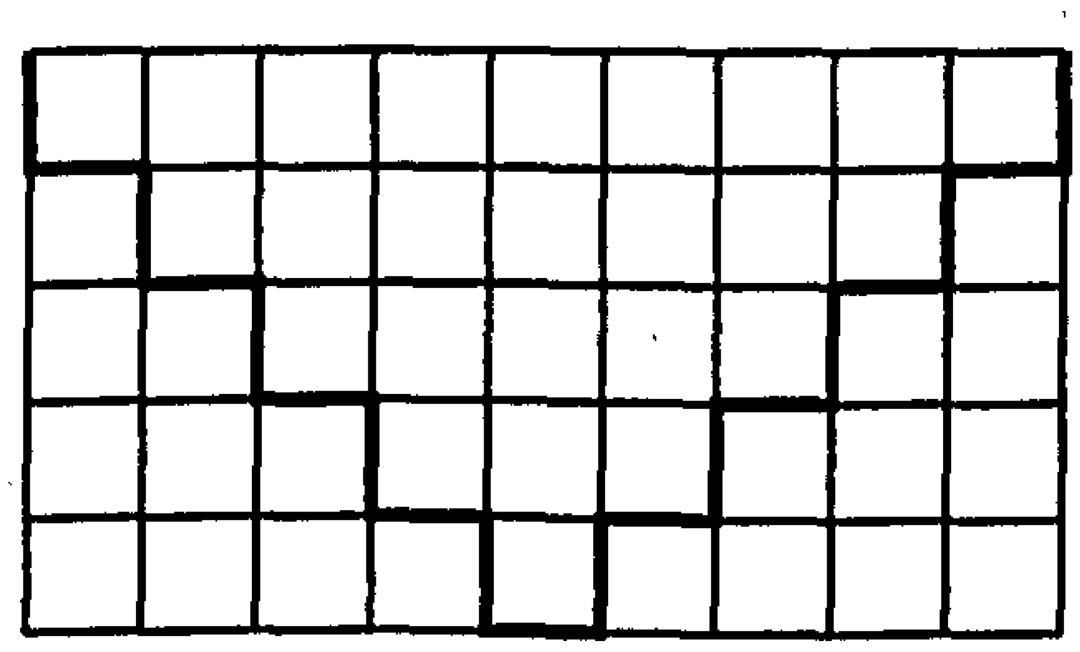


b- 1:9 block pattern in which nine overlying blocks must be removed to mine one block

Figure 3.23- Non-cone-based pattern

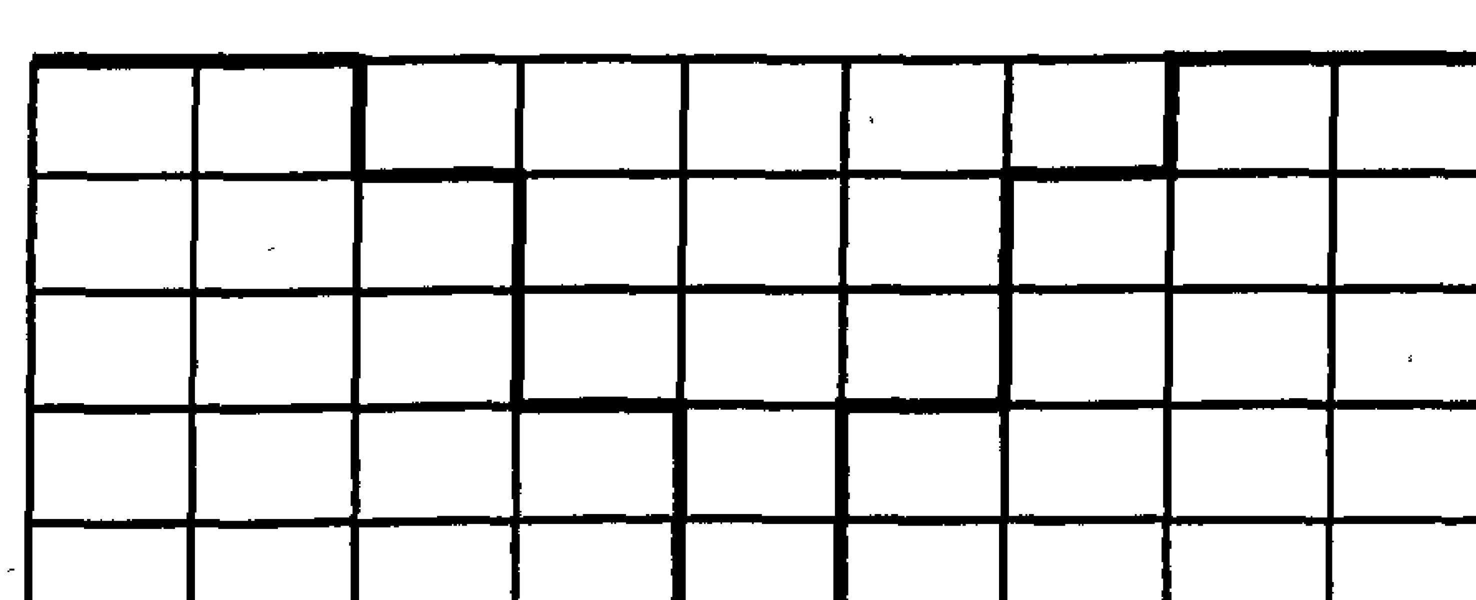
	1	2	3	4	5	6	7	8	9
1	0	0	0	0	1	0	0	0	0
2	0	0	0	1	2	1	0	0	0
3	0	0	1	2	3	2	1	0	0
4	0	1	2	3	4	3	2	1	0
5	1	2	3	4	5	4	3	2	1
6	0	1	2	3	4	3	2	1	0
7	0	0	1	2	3	2	1	0	0
8	0	0	0	1	2	1	0	0	0
9	0	0	0	0	1	0	0	0	0

a- Plan



$$Slope = ArcTan\left(\frac{4a}{4a}\right) = 45°$$

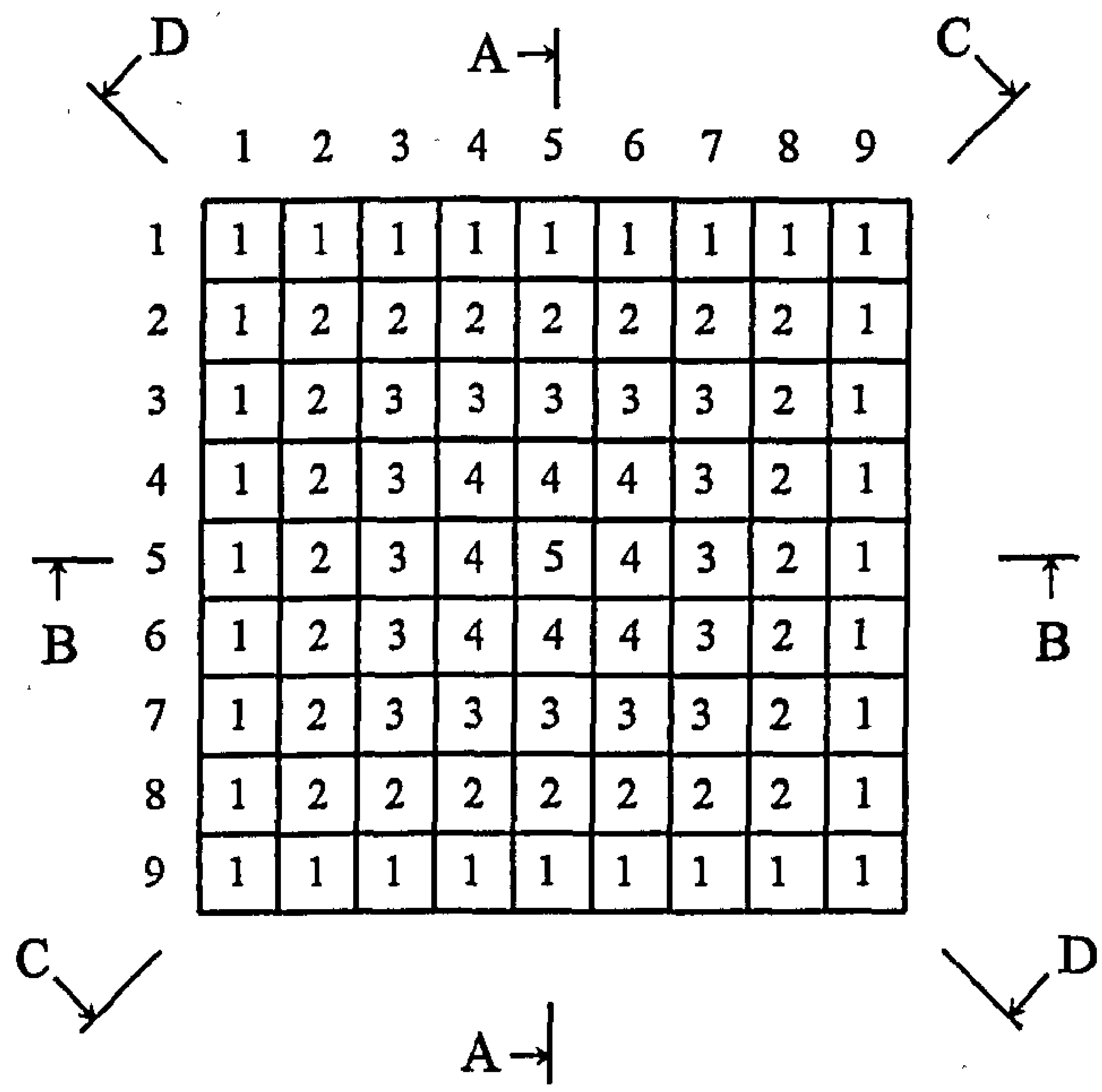
b- Section A-A or B-B



$$Slope = ArcTan\left(\frac{4a}{2\sqrt{2}a}\right) = 55°$$

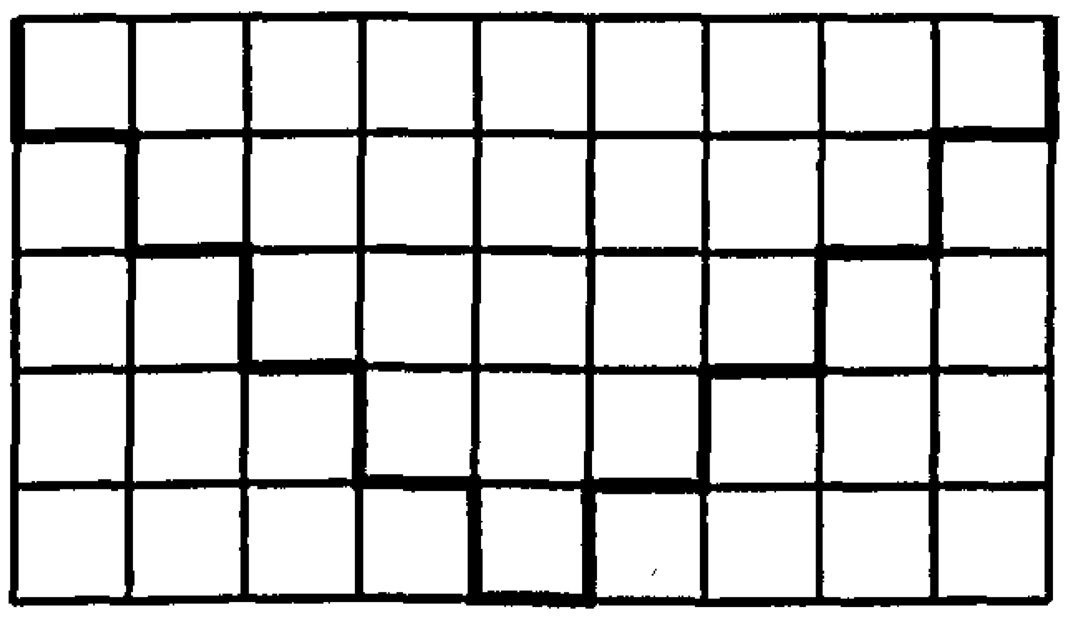
c- Section C-C or D-D

Figure 3.24- 1:5 block pattern applied to a cubic revenue block model



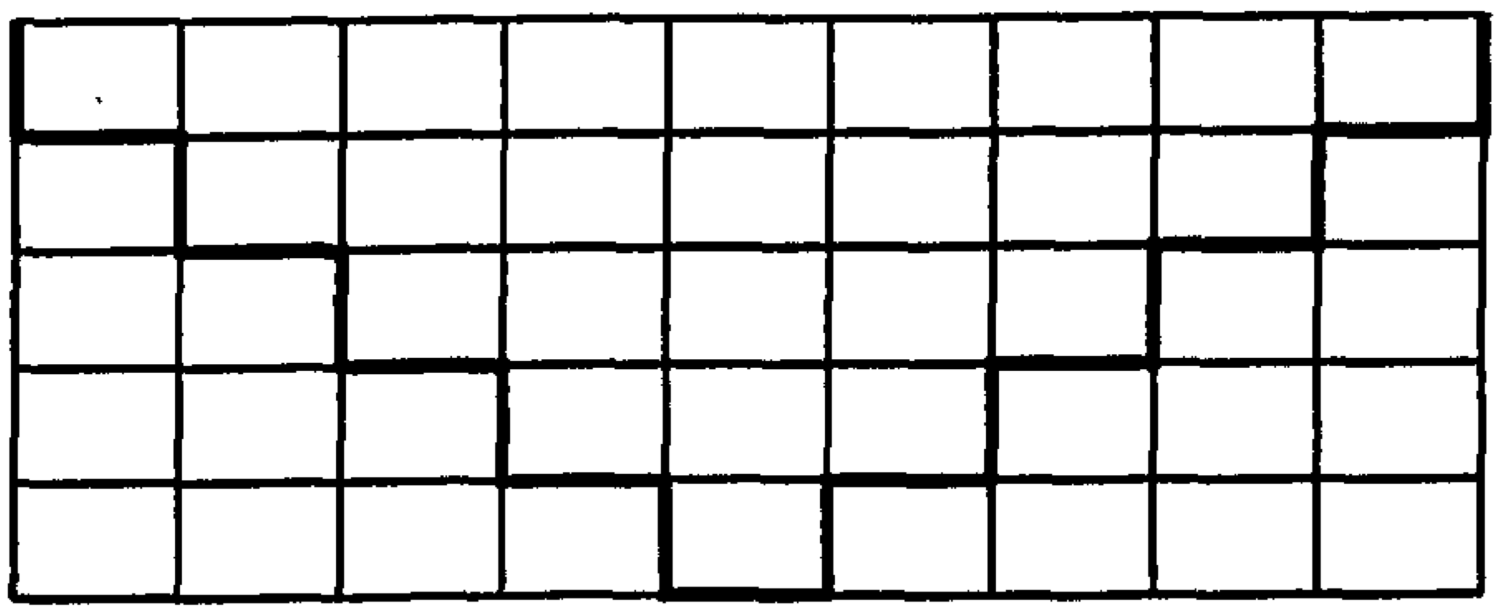
	1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	1	1	1	1
2	1	2	2	2	2	2	2	2	1
3	1	2	3	3	3	3	3	2	1
4	1	2	3	4	4	4	3	2	1
5	1	2	3	4	5	4	3	2	1
6	1	2	3	4	4	4	3	2	1
7	1	2	3	3	3	3	3	2	1
8	1	2	2	2	2	2	2	2	1
9	1	1	1	1	1	1	1	1	1

a- Plan



$$Slope = ArcTan\left(\frac{4a}{4a}\right) = 45°$$

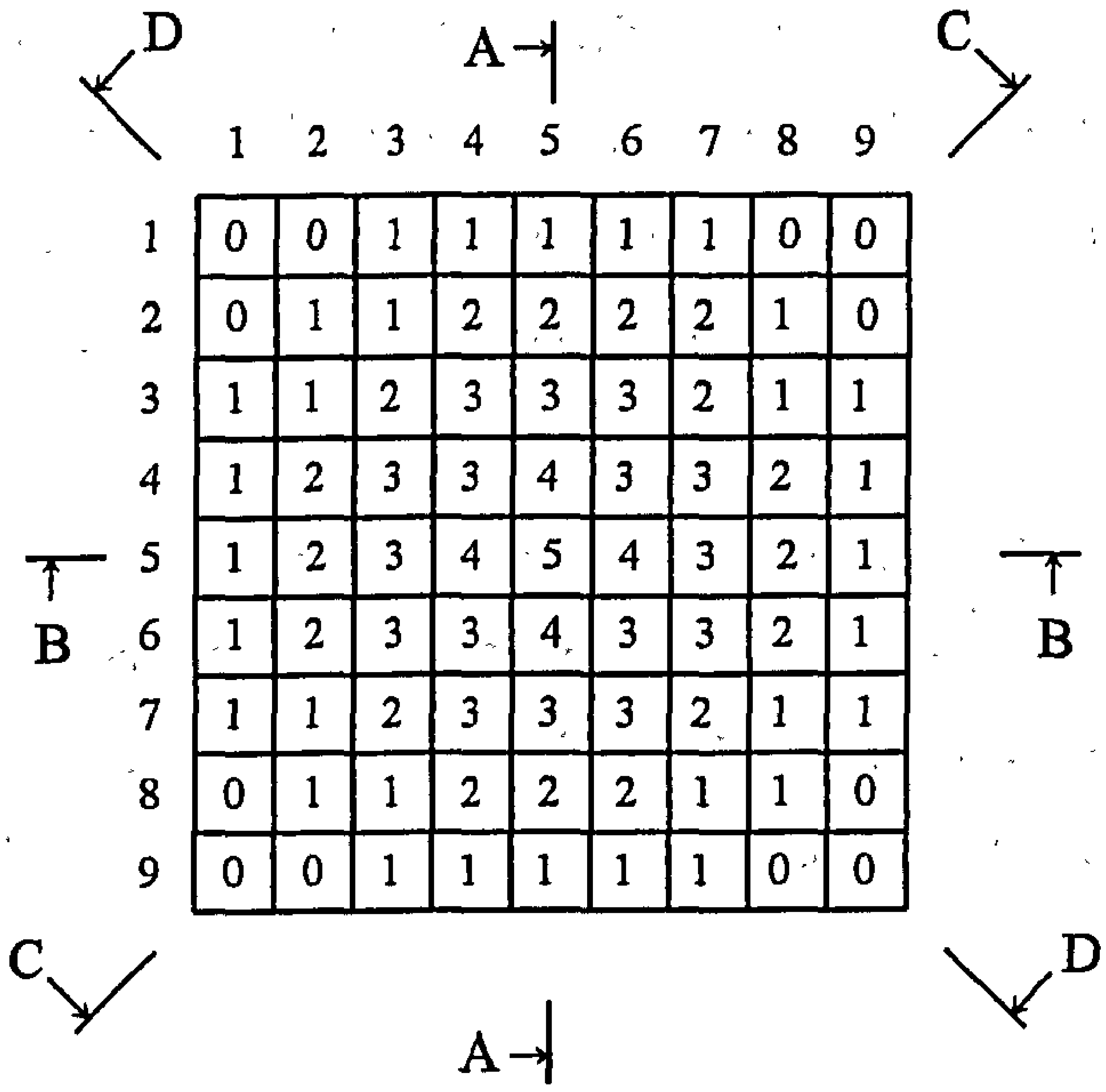
b- Section A-A or B-B



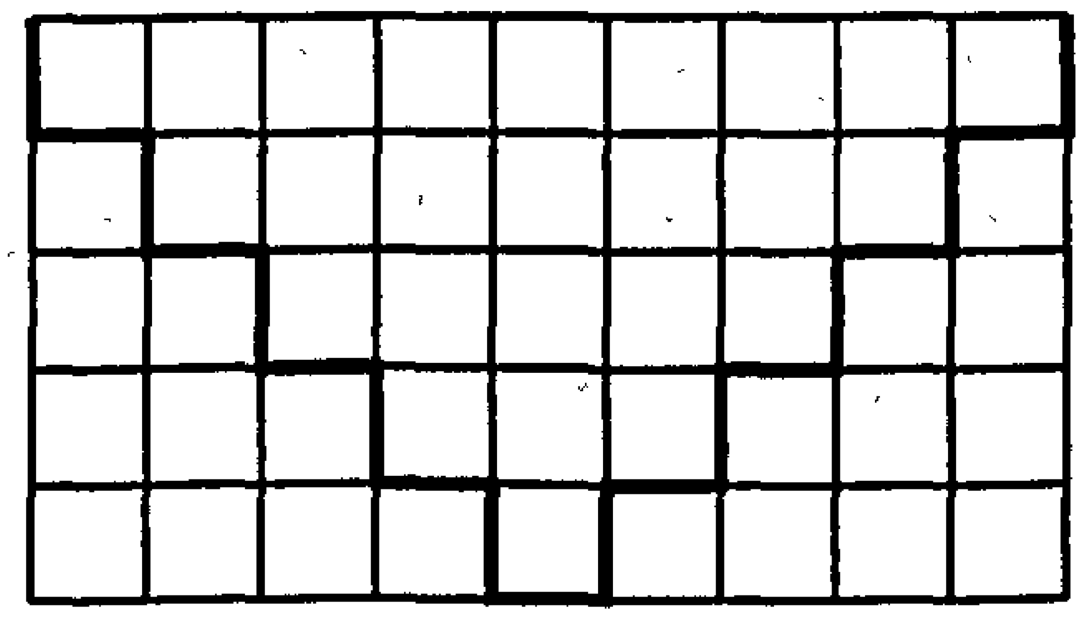
$$Slope = ArcTan\left(\frac{4a}{4\sqrt{2}a}\right) = 35°$$

c- Section C-C or D-D

Figure 3.25- 1:9 block pattern applied to a cubic revenue block model

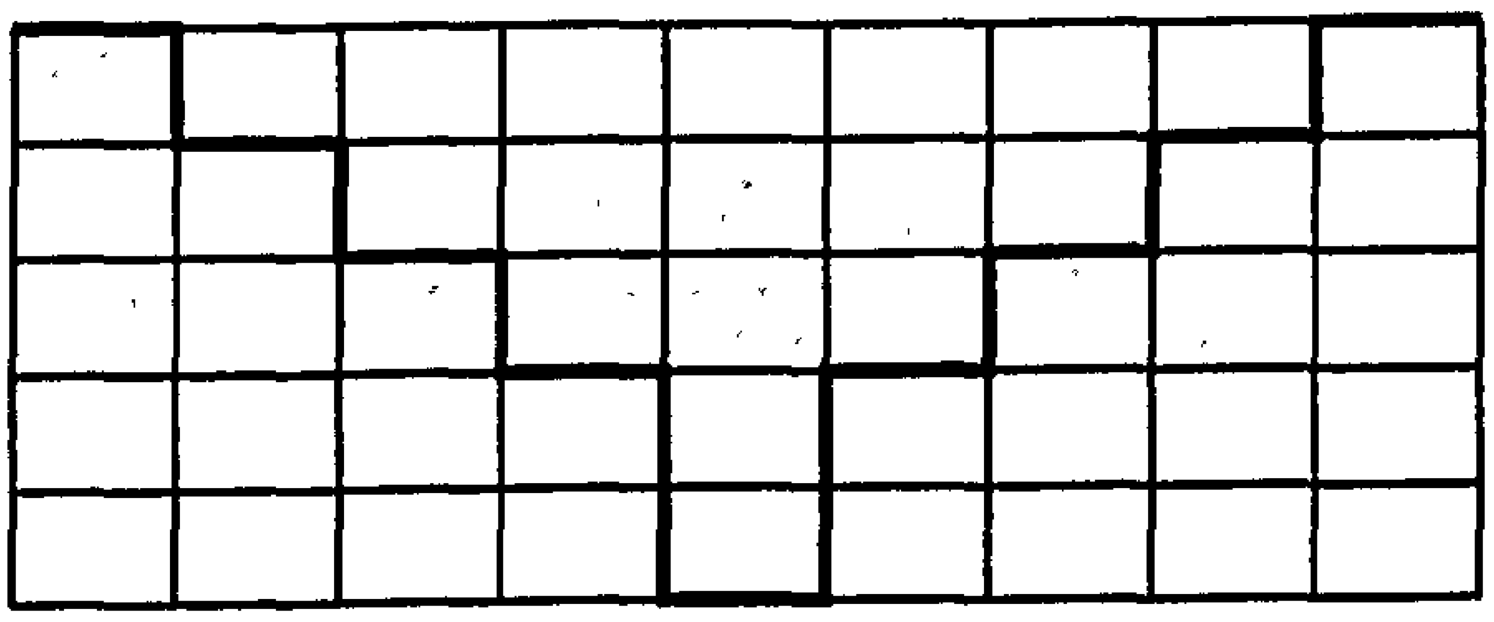


a- Plan



$$Slope = ArcTan\left(\frac{4a}{4a}\right) = 45°$$

b- Section A-A or B-B



$$Slope = ArcTan\left(\frac{4a}{3\sqrt{2}a}\right) = 43°$$

c- Section C-C or D-D

Figure 3.26- 1:5:9 block pattern applied to a cubic revenue block model

The second configuration is a 1:9 block pattern, in which nine overlying blocks must be removed to mine one block (Figure 3.23b). As illustrated in Figure 3.25, this approximation to slopes produces a cone with slopes ranging from 35° (cross-sections C-C and D-D) to 45° (cross-sections A-A and B-B).

A close approximation to a 45° slope in the cubic block model is obtained by combining a 1:5 block pattern for the first level above the base block with a 1:9 pattern for the second level above the base block (Figure 3.26). An example of the use of this pattern in the Lerchs-Grossmann algorithm can be found in the program published by Dowd (1994a).

One of the main disadvantages of using the first category is the difficulty of establishing optimum pit outlines with variable slope angles. The slope angles are assumed to be defined by the dimensions of the blocks. For example, if a 1:5:9 pattern is used in the general rectangular revenue block model of an orebody, then for 10m x 10m x 5m blocks, the slope angles in different cross-sections would be:

Section A-A or BB:

$$Slope = ArcTan\left(\frac{4*5}{4*10}\right) = 26^{\circ}$$

Section C-C or DD:

$$Slope = ArcTan\left(\frac{4*5}{3\sqrt{2}*10}\right) = 25^{\circ}$$

Therefore, using this procedure, different slope angles would require different sizes for the blocks in the orebody block model and these may not correspond to required bench heights. The grades of different block sizes estimated from a given configuration of data would have different estimation errors thus creating difficulties in assessing reliability and confidence levels associated with the final pit values (the optimal pit is commonly used to define minable reserves with stated levels of confidence). In addition, different parts of the orebody may require different slope angles. It is impossible in this method to have different angles for different parts of the mine.

The second category, cone-based methods, involves the use of a cone to define the mining slope. For example, Lipkewich and Borgman (1969), Zhao and Kim (1992), Dowd and Onur (1993) have all used cones to define variable slopes. Lipkewich and Borgman (1969) proposed a knights move pattern to approximate a conical expansion to the surface. Zhao and Kim (1992) defined a method based on cone templates. Dowd and Onur (1993) used the idea of cone templates to derive a technique to establish the optimum pit with variable slope angles. These works rely on the use of only one slope angle in the design whereas a complex deposit may require the use of different slope angles for each direction and for different parts of the orebody. The method presented by Dowd and Onur (1993) has been adopted in the current work and has been modified to derive a general method for variable slope angles. This procedure is incorporated in the Lerchs-Grossmann algorithm given later in this chapter.

3.5- The Lerchs-Grossmann algorithm with variable slope angles

One of the main disadvantages of the original Lerchs-Grossmann algorithm is that it was based on a fixed slope angle. Although solutions were reported to overcome this limitation, none of them were adequate for variable slopes controlled by complex structures and geology. When dealing with a deposit represented as a block model, a method must be found to define mining slopes and to take into account variable slope angles. The method adopted here is based on the cone method to approximate the conical shape of the pit. The cone is defined with four slope angles in four principal directions. It is assumed that the orebody has been divided into regions or domains based on geotechnical information. It is further assumed that within each region or domain the rock characteristics are the same and can be characterised by a set of slopes. Depending on the number of regions, the problem is treated in two different ways:

- **Variable slope angles** in which only one region or domain sector is specified to define the pit slopes.

- **Multiple variable slope angles** in which more than one region or domain sector is specified to define the pit slopes.

For each region or domain sector, pit slopes are assumed to be defined by four principal slope angles in four principal directions which in the current study are termed as:

North face slope or Northing slope:	Slope face to the North.
East face slope or Easting slope:	Slope face to the East.
South face slope or Southing slope:	Slope face to the South.
West face slope or Westing slope:	Slope face to the West.

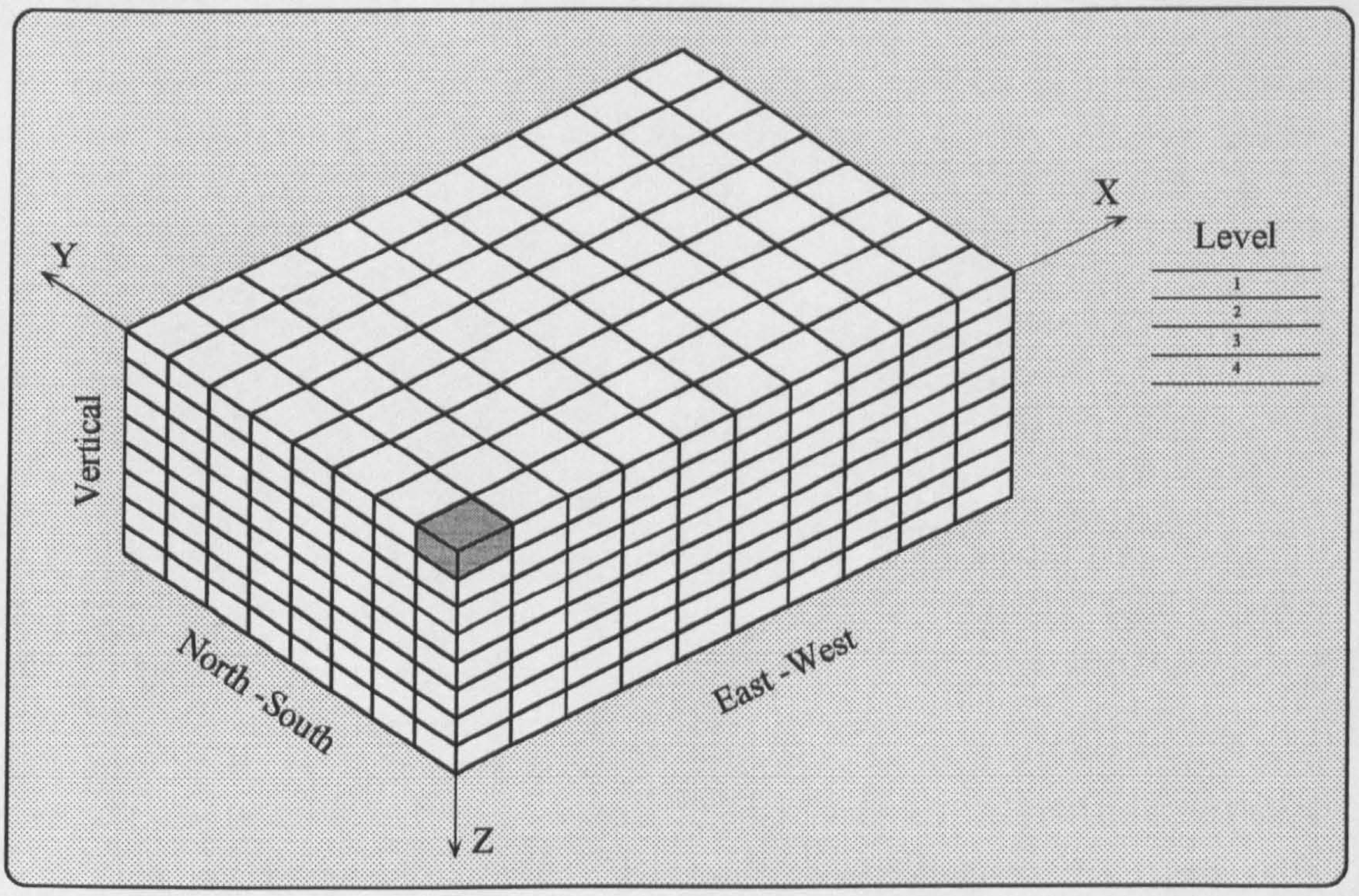


Figure 3.27- Block model of deposit and co-ordinate systems

Two types of co-ordinate system, as illustrated in Figure 3.27, are used throughout this study for pit optimisation for orebodies represented by block models. The first is the X, Y, Z system in which the X axis runs East-West, the Y axis runs South-North and the Z axis vertically. The origin of the system is located in the Southwest of the upper most level, the shaded block shown in Figure 3.27. The second system, an i, j, k co-ordinate index system, is also used. The i, j, k co-ordinate increases along the line of increasing X, Y, Z co-ordinate respectively. In addition, the following parameters are used to define the block model for the deposit:

xdim	Dimension of block in x direction (West-East)
ydim	Dimension of block in y direction (South-North)
zdim	Dimension of block in z direction (Elevation)
numx	Number of blocks in x direction (West-East)
numy	Number of blocks in y direction (South-North)
numz	Number of blocks in z direction or number of levels

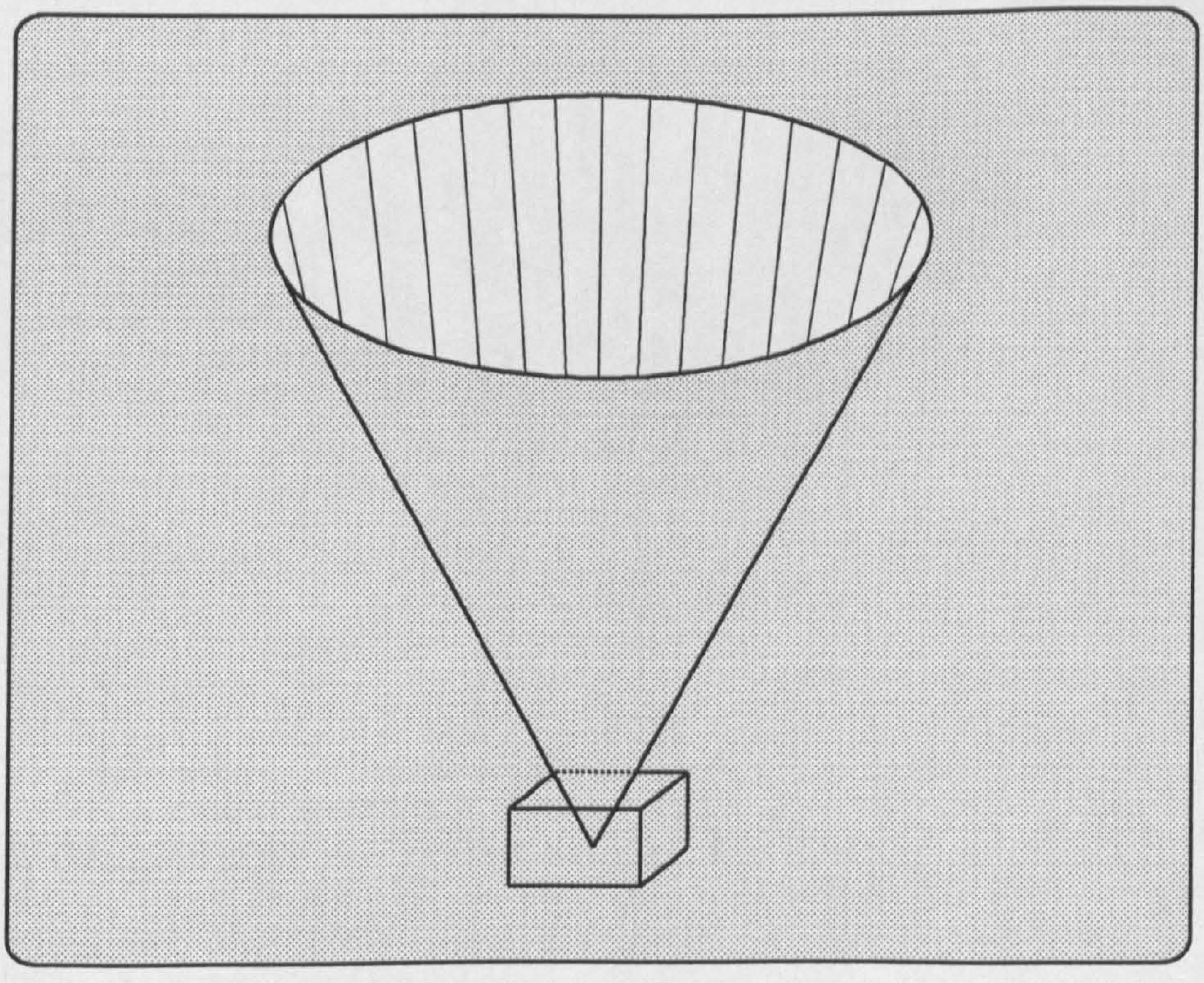


Figure 3.28- Constructing a cone from the base block

3.5.1- Variable slope angles

Pit slopes can be approximated by constructing a cone representing an extraction volume. This can be done by creating rings, or envelopes, from the mid-point of the base block and extending them to the surface (Figure 3.28) in such way that the side angles of the cone are equal to the four principal slope angles: North face angle, East face angle, South face angle and West face angle.

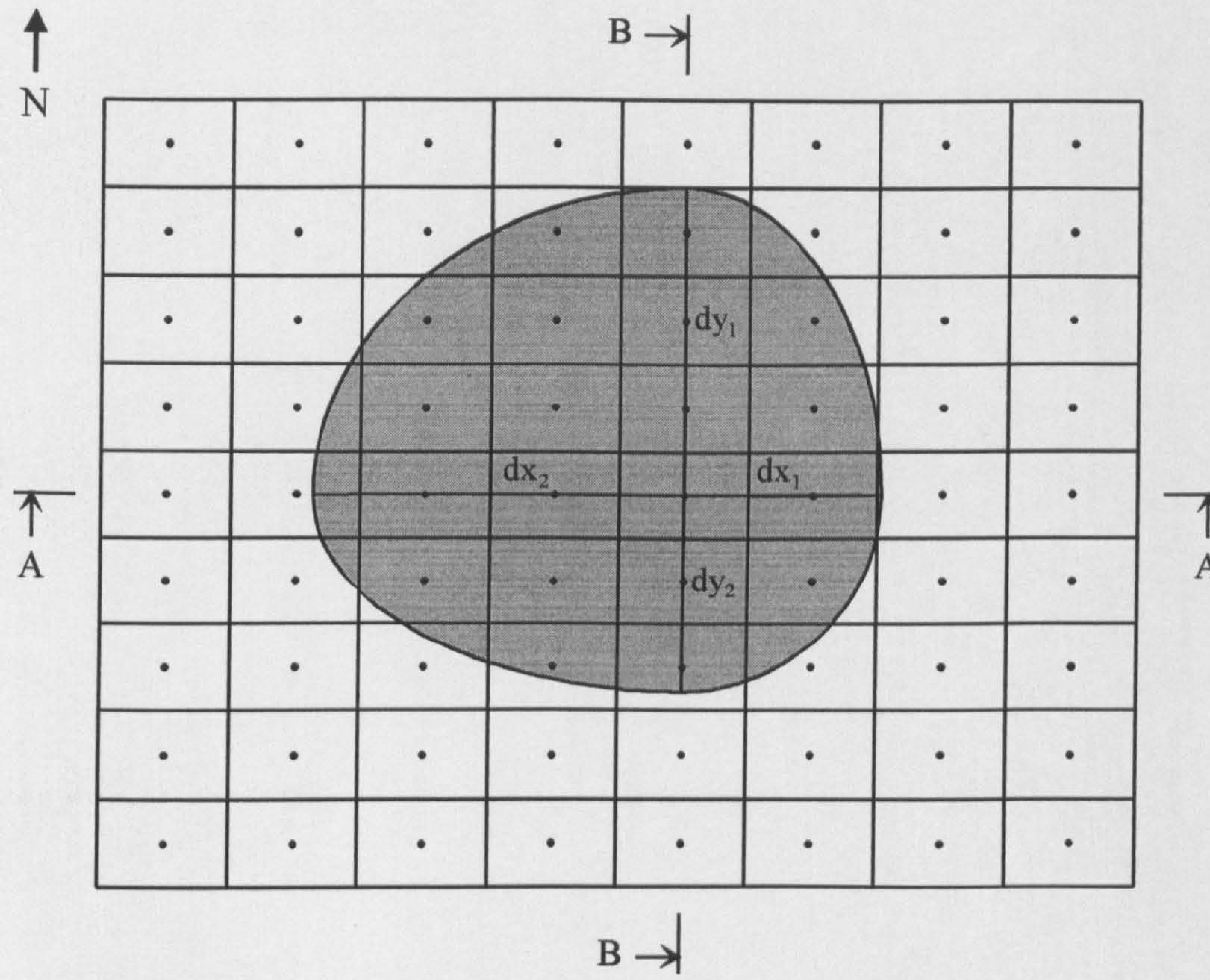
If the pit wall slopes in the four principal directions are not the same the upper area of the cone on each level (intersection of the cone with level) will consist of four quadrants of different ellipses. If the pit wall angles are the same the upper area of the cone will be a circle. Figure 3.29 shows the extraction cone and the blocks within it on the first level and on the two cross-sections. On each level the value of two semi-major axes and two semi-minor axes depend on the four principal slope angles and the vertical distance of the mid-point of the base block from the overlying blocks. These parameters can be found by using trigonometric functions. The number of blocks in the principal directions on any level above the base block can be calculated by dividing these parameters by the corresponding block dimensions. Consider a block $X_{i,j,k}$ on level k, these parameters and the number of blocks in the principal directions, as illustrated in Figure 3.29, can be calculated from the following equations:

$$dx_1 = \frac{(k-t) * z\,dim}{Tan(West\ face\ angle)} \tag{3.1}$$

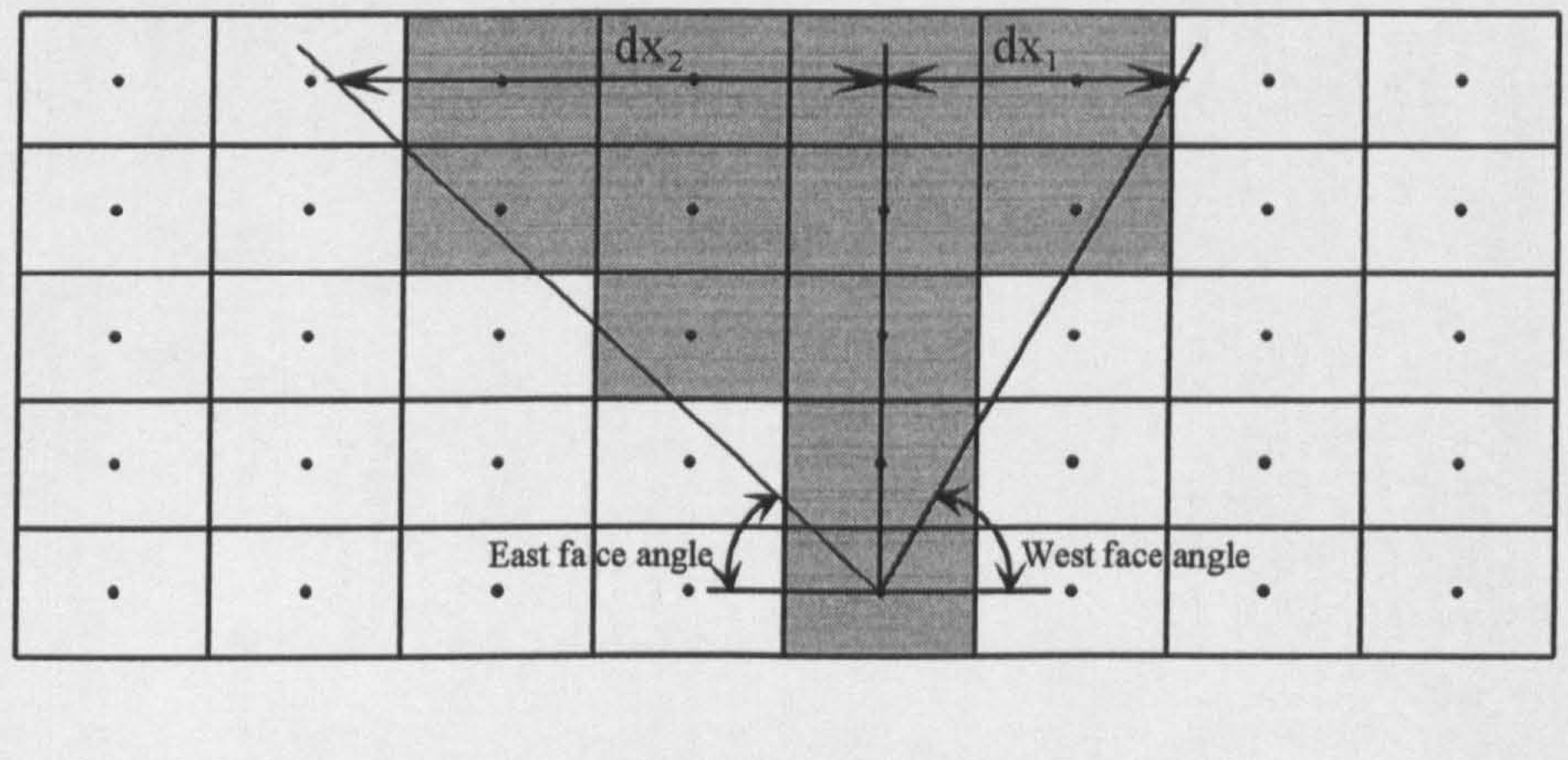
$$dy_1 = \frac{(k-t) * z\,dim}{Tan(South\ face\ angle)} \tag{3.2}$$

$$dx_2 = \frac{(k-t) * z\,dim}{Tan(East\ face\ angle)} \tag{3.3}$$

$$dy_2 = \frac{(k-t) * z\,dim}{Tan(North\ face\ angle)} \tag{3.4}$$



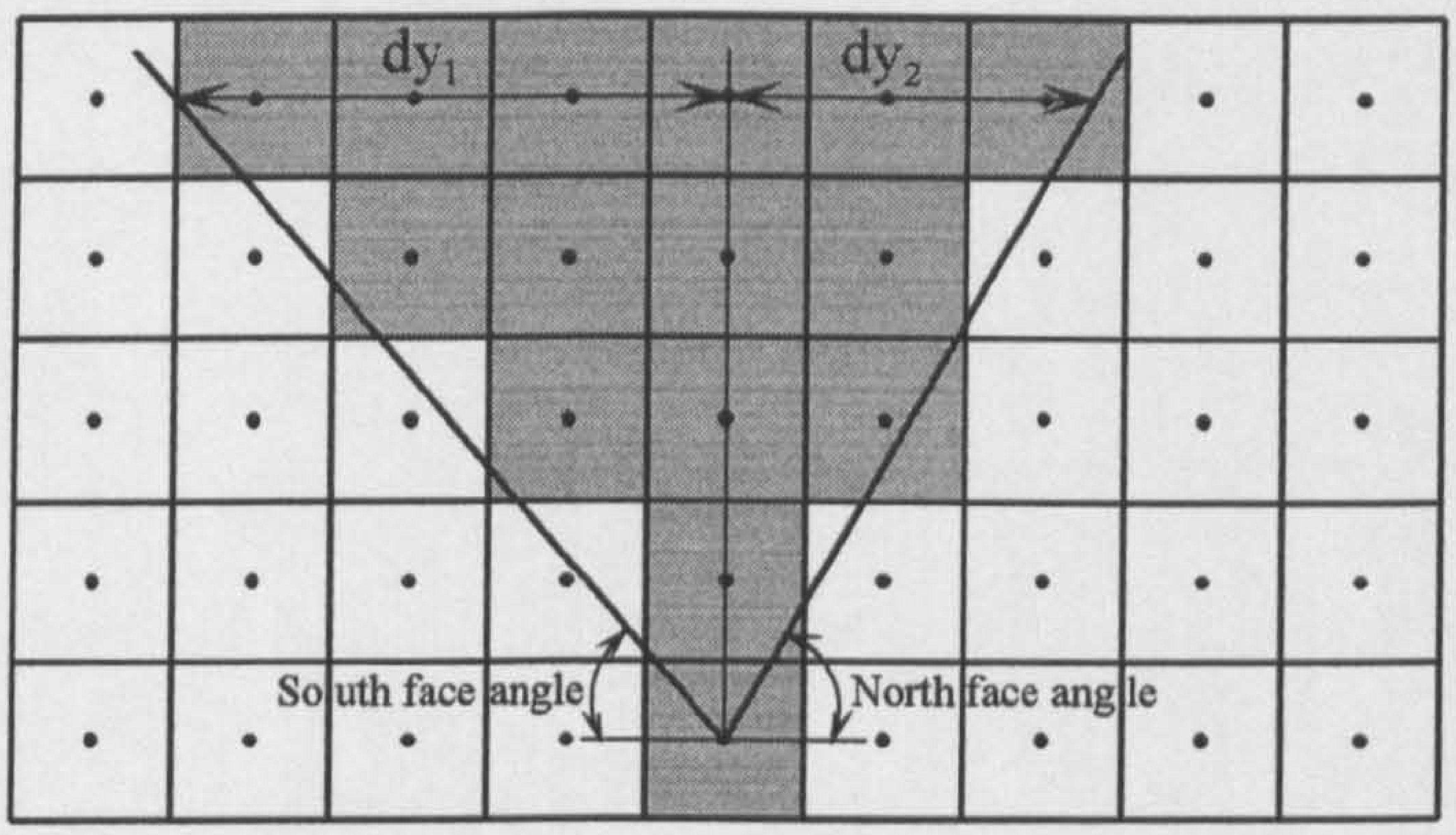
a- The upper area of the cone on the first level



b- Northing section A-A

Figure 3.29- Extraction cone and blocks within it of a base block

(continued ....)



c- Easting section B-B

Figure 3.29- (........ continued)

$$m_1 = \frac{dx_1}{x\,dim} \tag{3.5}$$

$$n_1 = \frac{dy_1}{y\,dim} \tag{3.6}$$

$$m_2 = \frac{dx_2}{x\,dim} \tag{3.7}$$

$$n_2 = \frac{dy_2}{y\,dim} \tag{3.8}$$

where

t	is the level above the base block and varies from 1 to k-1
m_1	is the number of blocks from the base block to the East
n_1	is the number of blocks from the base block to the North
m_2	is the number of blocks from the base block to the West
n_2	is the number of blocks from the base block to the South

When the numbers of blocks within the upper area of the cone on any level, say the tth level above the base block, are calculated in the four principal directions, all the blocks $X_{m,\, n,\, k\text{-}t}$, where m = i-m_2, i+m_1 and n = j-n_2, j+n_1, must be examined to determine whether they are within the extraction volume. This can be done by using the ellipse equation as follows:

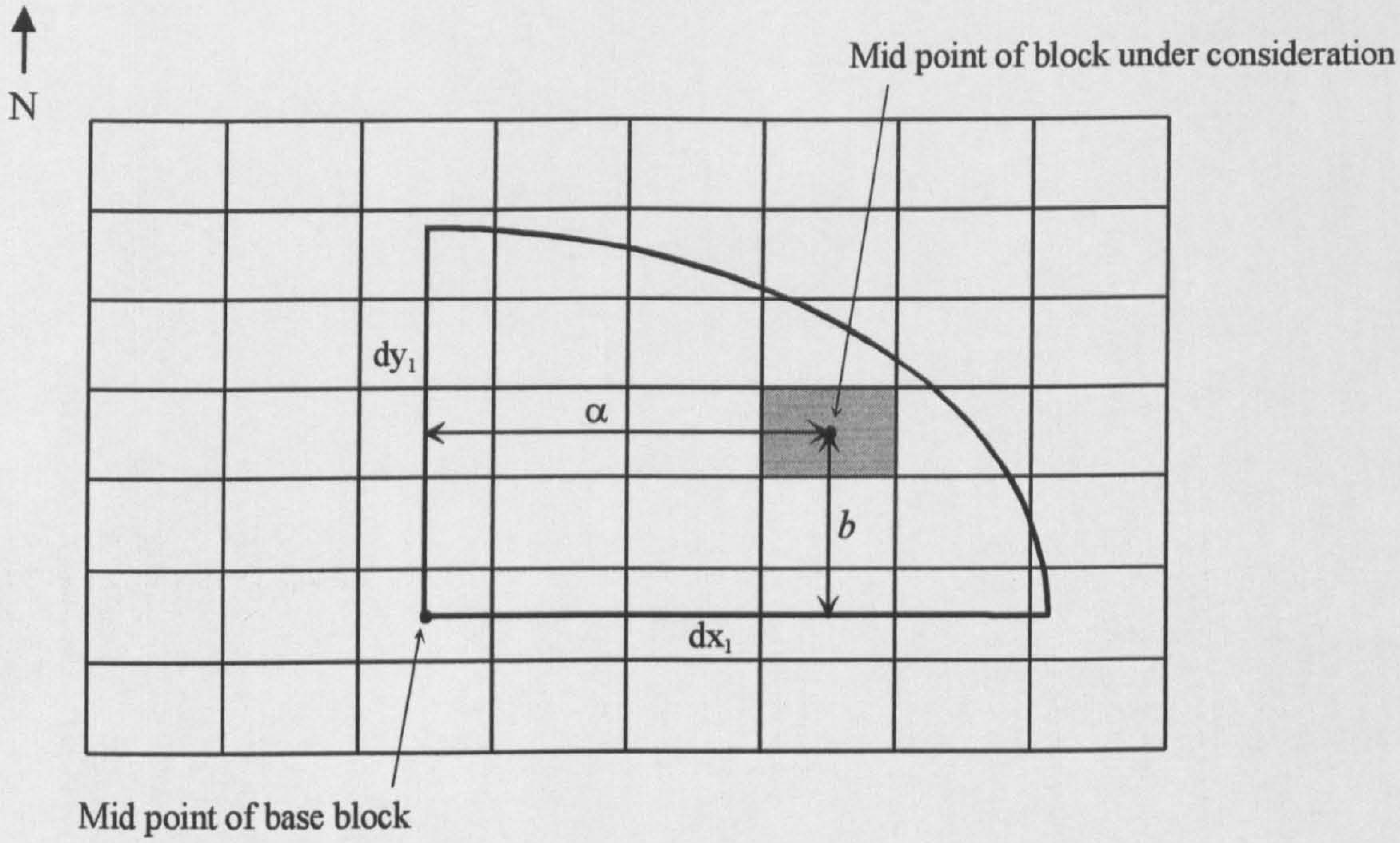


Figure 3.30- The value of parameters α and *b*

$$a = x\dim^* (i - m) \tag{3.9}$$

$$b = y\dim^* (j - n) \tag{3.10}$$

If m ≥ i and n ≥ j then $Value = \frac{a^2}{(dx_1)^2} + \frac{b^2}{(dy_1)^2}$ (3.11)

If m ≥ i and n ≤ j then $Value = \frac{a^2}{(dx_1)^2} + \frac{b^2}{(dy_2)^2}$ (3.12)

If m ≤ i and n ≤ j then $Value = \frac{a^2}{(dx_2)^2} + \frac{b^2}{(dy_2)^2}$ (3.13)

If $m \leq i$ and $n \geq j$ then $$Value = \frac{a^2}{(dx_2)^2} + \frac{b^2}{(dy_1)^2} \quad (3.14)$$

Where α and *b* are the horizontal distances from the mid-point of the block under consideration to the base block measured in the East-West and South-North directions respectively, as illustrated in Figure 3.30. If the 'Value' is less than or equal to 1, it is assumed that the block is within the extraction cone and it must be removed before the base block. Otherwise it is assumed that the block is outside the extraction cone. Blocks that lie within the extraction cone are submitted to the graph algorithm. The program was written in such a way that extraction cones are established only for ore blocks. This prevents unnecessary increases in computing time and prevents waste blocks being considered many times.

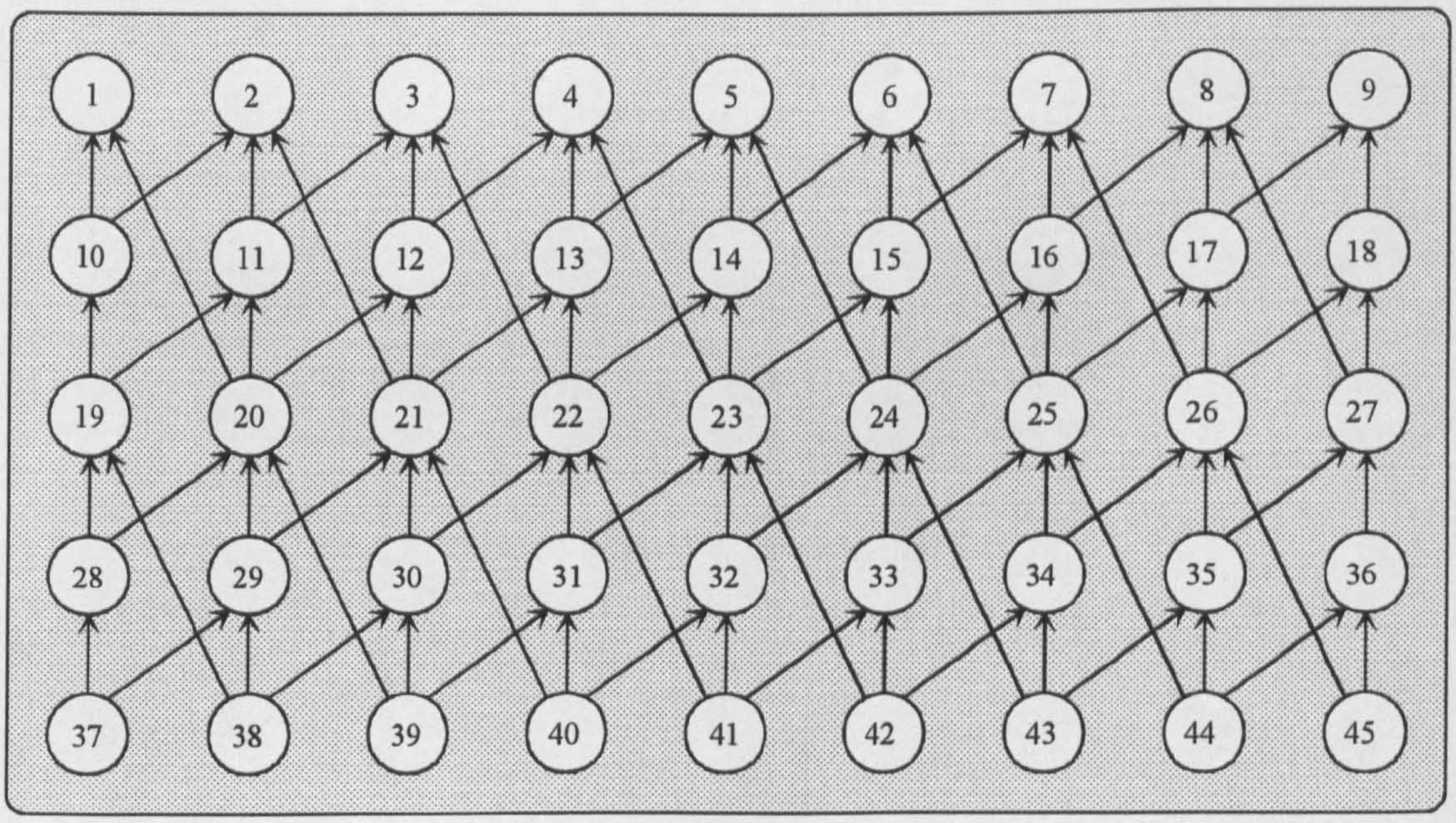


Figure 3.31- Directed graph representing a north-south cross-section in a cubic block model with East face angle of 60° and West face angle of 45°

Using this procedure, pit slopes are no longer fixed and are not limited to one-up and one-over patterns. They can vary in principal directions and are independent of block dimensions. Figure 3.31 illustrates a directed graph representing a Northing section in cubic block model in which East face angle and West face angle are assumed to be 60° and 45° respectively. In this graph, Vertices 4, 5, 6, 7, 14 and 15 are in the extraction cone of block 23.

3.5.1.1- An example

This procedure has been incorporated into the Lerchs-Grossmann algorithm to determine optimum pit limits with variable slope angles. To show how the program works, a simple example is used in which the block dimensions and wall slopes are as follows:

Block dimensions:

East - West	15 m
North - South	10 m
Vertical	10 m

Pit slope angles:

North face angle	60°
East face angle	50°
South face angle	30°
West face angle	45°

The graphical results of the optimum pit limit are illustrated in Figures 3.32, 3.33 and 3.34. Figure 3.32 shows the optimum pit in plan in which the location of the pit is shown with coloured rectangles representing the number of levels from the surface that must be mined. Figures 3.33 and 3.34 illustrate the pit limits on Easting and Northing sections respectively in which the pit limit is shown with a thick line indicating that the blocks above this line are part of the optimum pit. It should be noted that the program also works with rectangular blocks. Slope angles are independent of the block dimensions and can vary in all four principal directions.

Figure 3.32 - Pit limit without pit bottom smoothing - variable slope angles

Figure 3.33 - Pit limit without pit bottom smoothing - variable slope angles

Figure 3.34 - Pit limit without pit bottom smoothing - variable slope angles

3.5.2- Multiple variable slope angles

In complex cases in which the pit slopes vary in different parts of the orebody due to slope stability requirements, it is necessary to divide the orebody into regions or domain sectors, within which the rock characteristics are the same, and to use different slope angles for each region. In these cases, slope angles are assigned to each block in four principal directions within each region – this is discussed later in this chapter.

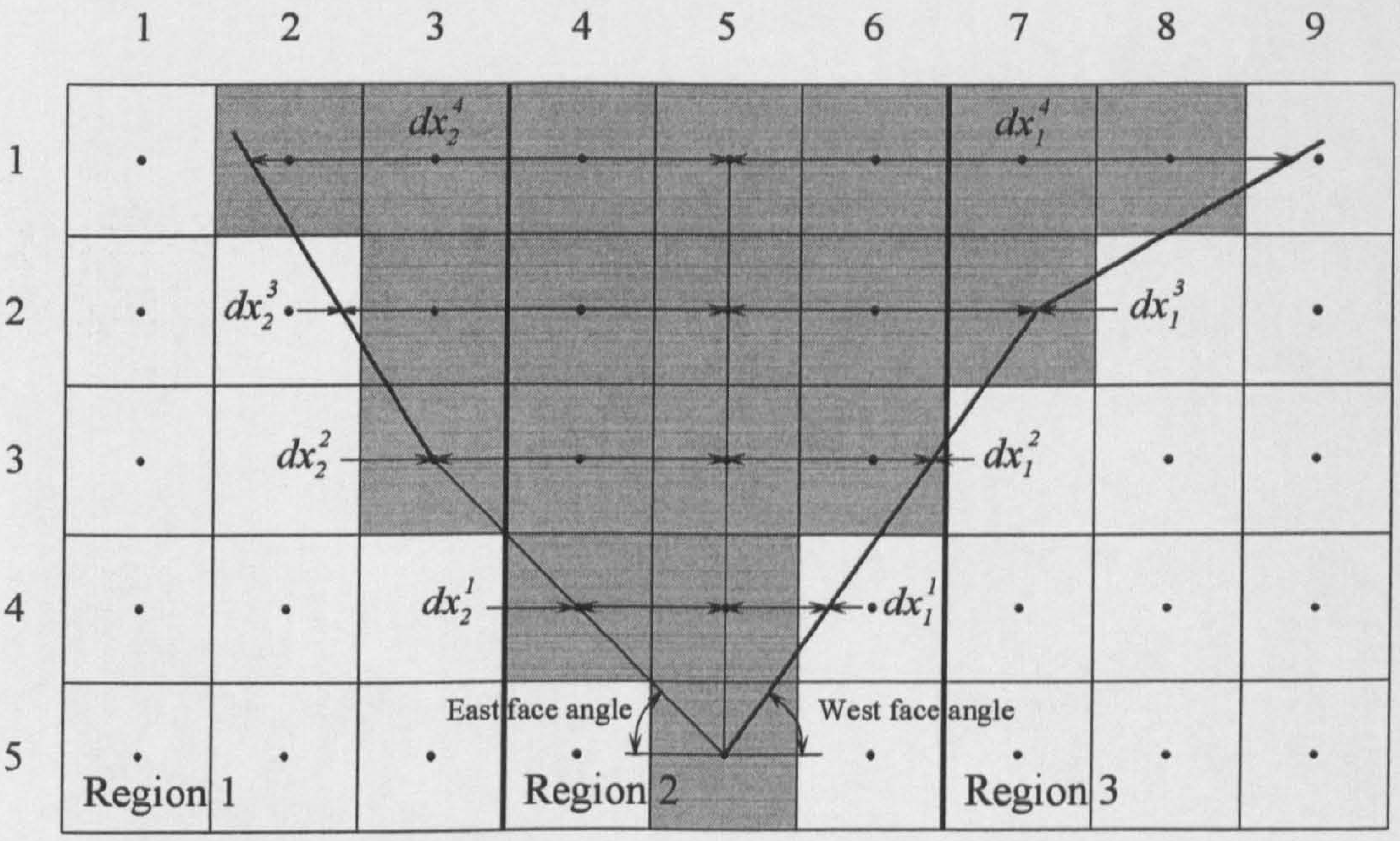


Figure 3.35- Extraction cone of a block for a two-dimensional example with three different regions

In the case of multiple variable slopes, an extraction volume is constructed level by level by creating rings or envelopes from the base block and extending them to the surface having regard for pit slopes which have already been assigned to blocks. The extraction volume is constructed from the base block to the next overlying block and then is constructed from the point of intersection of the cone with this level to the second level above the base block. This procedure is continued to the surface.

This procedure can best be explained by means of the two-dimensional example shown in Figure 3.35 in which the orebody is divided into three different parts. Let us assume that this is the Northing section and two slope angles – the West face angle and the East face angle - have already been assigned to each block. Consider a block in level k (row) and column j, for instance block (5, 5) on which the cone will be constructed. To determine the blocks within the extraction cone of this block the following procedure is used.

With reference to Figure 3.35, an extraction cone is constructed from the base block to the next overlying level using the slope angle of this block. In the two-dimensional case, two lines are drawn from the mid-point of the base block to the left and right with regard to the East and West face angle of the base block respectively and they are extended to the level above. The values of parameters dx_1^1, dx_2^1 and the number of blocks to the East, m_1^1, and to the West, m_2^1, on the first level above the base block are determined by the following equations:

$$dx_1^1 = \frac{z\,dim}{Tan\left[West\,face\,angle\,of\,block(k,j)\right]} \quad (3.15)$$

$$dx_2^1 = \frac{z\,dim}{Tan\left[East\,face\,angle\,of\,block\,(k,j)\right]} \quad (3.16)$$

$$m_1^1 = \frac{dx_1^1}{x\,dim} \quad (3.17)$$

$$m_2^1 = \frac{dx_2^1}{x\,dim} \quad (3.18)$$

Where zdim and xdim are the block dimensions in the vertical and horizontal directions respectively. On the first level above, the blocks $X_{k-1,\,m}$, where m = j-m_2^1, j+m_1^1, are considered as part of the extraction cone.

There are two intersection points of the extraction cone with the level above the base block (lines drawn from the mid-point of the base block to the next overlying level). The extraction cone is extended from these two points to the next overlying block (second level above the base block) by using the slope angles of the blocks containing the points of intersection. In order to determine the slope of the block to be used (in other words, to find the block in which the intersection lies) the values of parameters dx_1^1 and dx_2^1 are divided by the block dimension and the result is rounded up. This means that a value of 0.5 is added to the result of division and then the integer part is taken. That is:

$$ml_1^1 = \frac{dx_1^1}{x\,dim} + 0.5 \qquad (3.19)$$

$$ml_2^1 = \frac{dx_2^1}{x\,dim} + 0.5 \qquad (3.20)$$

Then the values of parameters dx_1^2, dx_2^2 and the number of blocks in both directions m_1^2 and m_2^2 on the second level above the base block are determined as:

$$dx_1^2 = dx_1^1 + \frac{z\,dim}{Tan\left[West\,face\,angle\,of\,block\left(k-1, j+ml_1^1\right)\right]} \qquad (3.21)$$

$$dx_2^2 = dx_2^1 + \frac{z\,dim}{Tan\left[East\,face\,angle\,of\,block\left(k-1, j-ml_2^1\right)\right]} \qquad (3.22)$$

$$m_1^2 = \frac{dx_1^2}{x\,dim} \qquad (3.23)$$

$$m_2^2 = \frac{dx_2^2}{x\,dim} \qquad (3.24)$$

Again, the blocks $X_{k-2,\,m}$, where m = j-m_2^2, j+m_1^2, are considered as part of the extraction cone on the second level above the base block. This procedure is continued to the surface.

In general, for the n[th] level above the base block, The values of parameters dx_1^n, dx_2^n and the number of blocks to the East, m_1^n, and to the West, m_2^n, are determined by the following equations:

$$ml_1^{n-1} = \frac{dx_1^{n-1}}{x\,dim} + 0.5 = \frac{\sum_{i=1}^{n-1} dx_1^i}{x\,dim} + 0.5 \tag{3.25}$$

$$ml_2^{n-1} = \frac{dx_2^{n-1}}{x\,dim} + 0.5 = \frac{\sum_{i=1}^{n-1} dx_2^i}{x\,dim} + 0.5 \tag{3.26}$$

$$dx_1^n = \sum_{i=1}^{n-1} dx_1^i + \frac{z\dim}{Tan\left[\text{West face angle of block}\left(k-n+1,\ j+ml_1^{n-1}\right)\right]} \tag{3.27}$$

$$dx_2^n = \sum_{i=1}^{n-1} dx_2^i + \frac{z\dim}{Tan\left[\text{East face angle of block}\left(k-n+1,\ j-ml_2^{n-1}\right)\right]} \tag{3.28}$$

$$m_1^n = \frac{dx_1^n}{x\,dim} \tag{3.29}$$

$$m_2^n = \frac{dx_2^n}{x\,dim} \tag{3.30}$$

On the n[th] level above the base block, the blocks $X_{k-n,\,m}$, where m = j-m_2^n, j+m_1^n, are considered as part of the extraction cone.

As an example, the extraction cone of the block in level 5 (row) and column 5 in Figure 3.35 with the slope angles shown in Table 3.2 is determined as follows:

Region	East face angle	West face angle
1	58°	40°
2	45°	55°
3	42°	30°

Table 3.2- Slope angles for two-dimensional example

The first level above the base block (level 4):

$$dx_1^1 = \frac{z\,dim}{Tan[West\,face\,angle\,of\,block(5,5)]} = \frac{10.0}{Tan55} = 7.0 \qquad (3.31)$$

$$dx_2^1 = \frac{z\,dim}{Tan[East\,face\,angle\,of\,block\,(5,5)]} = \frac{10.0}{Tan45} = 10.0 \qquad (3.32)$$

$$m_1^1 = \frac{7.0}{10.0} = 0 \qquad (3.33)$$

$$m_2^1 = \frac{10.0}{10.0} = 1 \qquad (3.34)$$

Thus, the blocks (4, 4) and (4, 5) are in the extraction of block (5, 5).

The second level above the base block (level 3):

$$ml_1^1 = \frac{dx_1^1}{x\,dim} + 0.5 = \frac{7.0}{10.0} + 0.5 = 1 \qquad (3.35)$$

$$ml_2^1 = \frac{dx_2^1}{x\,dim} + 0.5 = \frac{10.0}{10.0} + 0.5 = 1 \qquad (3.36)$$

$$dx_1^2 = dx_1^1 + \frac{z\,dim}{Tan[West\,face\,angle\,of\,block(4,\ 6)]} = 7.0 + \frac{10.0}{Tan55} = 14.0 \qquad (3.37)$$

$$dx_2^2 = dx_2^1 + \frac{z\,dim}{Tan\left[East\ face\ angle\ of\ block\ (4,4)\right]} = 10.0 + \frac{10.0}{Tan45} = 20.0 \quad (3.38)$$

$$m_1^2 = \frac{dx_1^2}{x\,dim} = \frac{14.0}{10.0} = 1 \quad (3.39)$$

$$m_2^2 = \frac{dx_2^2}{x\,dim} = \frac{20.0}{10.0} = 2 \quad (3.40)$$

On level 3 blocks (3, 3), (3, 4), (3, 5) and (3, 6) are in the extraction cone of block (5, 5).

The third level above the base block (level 2):

$$ml_1^2 = \frac{dx_1^2}{x\,dim} + 0.5 = \frac{14.0}{10.0} + 0.5 = 1 \quad (3.41)$$

$$ml_2^2 = \frac{dx_2^2}{x\,dim} + 0.5 = \frac{20.0}{10.0} + 0.5 = 2 \quad (3.42)$$

$$dx_1^3 = dx_1^2 + \frac{z\,dim}{Tan\left[West\ face\ angle\ of\ block(3,\ 6)\right]} = 14.0 + \frac{10.0}{Tan55} = 21.0 \quad (3.43)$$

$$dx_2^3 = dx_2^2 + \frac{z\,dim}{Tan\left[East\ face\ angle\ of\ block\ (3,3)\right]} = 20.0 + \frac{10.0}{Tan58} = 26.2 \quad (3.44)$$

$$m_1^3 = \frac{dx_1^3}{x\,dim} = \frac{21.0}{10.0} = 2 \quad (3.45)$$

$$m_2^3 = \frac{dx_2^3}{x\,dim} = \frac{26.2}{10.0} = 2 \quad (3.46)$$

On level 2, the blocks (2, 3), (2, 4), (2, 5), (2, 6) and (2,7), as shown in Figure 3.35, are in the extraction cone of block (5, 5).

The fourth level above the base block (level 1):

$$ml_1^3 = \frac{dx_1^3}{x\,dim} + 0.5 = \frac{21.0}{10.0} + 0.5 = 2 \tag{3.47}$$

$$ml_2^3 = \frac{dx_2^3}{x\,dim} + 0.5 = \frac{26.2}{10.0} + 0.5 = 3 \tag{3.48}$$

$$dx_1^4 = dx_1^3 + \frac{z\,dim}{Tan\left[West\,face\ angle\ of\ block(2,\ 7)\right]} = 21.0 + \frac{10.0}{Tan30} = 38.3 \tag{3.49}$$

$$dx_2^4 = dx_2^3 + \frac{z\,dim}{Tan\left[East\,face\ angle\ of\ block\,(2,2)\right]} = 26.2 + \frac{10.0}{Tan58} = 32.4 \tag{3.50}$$

$$m_1^4 = \frac{dx_1^4}{x\,dim} = \frac{38.3}{10.0} = 3 \tag{3.51}$$

$$m_2^4 = \frac{dx_2^4}{x\,dim} = \frac{32.4}{10.0} = 3 \tag{3.52}$$

In the first level, the blocks (1,2), (1, 3), (1, 4), (1, 5), (1, 6), (1,7) and (1, 8) are in the extraction of block (5, 5) (Figure 3.35).

The procedure presented for multiple variable slopes in two dimensions can be applied to the three-dimensional case. In the same way as the procedure used for variable slope angles, the pit shape is assumed to be defined by an irregular, elliptical outline on each level. The outline on each level consists of four quadrants of different ellipses defined by the pit slope angles in the four directions. The value of two semi-major axes, two semi-minor axes and the number of blocks in the principal directions on any level above the base block should be calculated in both sections in the same way as presented above for one section. When these parameters are determined, again by using the ellipse formula, any block whose mid-point lies inside the ellipse is considered to be part of the cone. The value of four axes and number of blocks in four directions for t^{th} level above the base block (block $X_{i,j,k}$) can be found from the following equations:

$$ml_1^{t-1} = \frac{\sum_{i=1}^{t-1} dx_1^i}{x\,dim} + 0.5 \quad (3.53)$$

$$nl_1^{t-1} = \frac{\sum_{i=1}^{t-1} dy_1^i}{y\,dim} + 0.5 \quad (3.54)$$

$$ml_2^{t-1} = \frac{\sum_{i=1}^{t-1} dx_2^i}{x\,dim} + 0.5 \quad (3.55)$$

$$nl_2^{t-1} = \frac{\sum_{i=1}^{t-1} dy_2^i}{y\,dim} + 0.5 \quad (3.56)$$

$$dx_1^t = \sum_{i=1}^{t-1} dx_1^i + \frac{z\dim}{Tan\left[\text{West face angle of block}\left(i + ml_1^{t-1},\ j,\ k - t + 1\right)\right]} \quad (3.57)$$

$$dy_1^t = \sum_{i=1}^{t-1} dy_1^i + \frac{z\dim}{Tan\left[\text{South face angle of block}\left(i,\ j + nl_1^{t-1},\ k - t + 1\right)\right]} \quad (3.58)$$

$$dx_2^t = \sum_{i=1}^{t-1} dx_2^i + \frac{z\dim}{Tan\left[\text{East face angle of block}\left(i - ml_2^{t-1},\ j,\ k - t + 1\right)\right]} \quad (3.59)$$

$$dy_2^t = \sum_{i=1}^{t-1} dy_2^i + \frac{z\dim}{Tan\left[\text{North face angle of block}\left(i,\ j - nl_2^{t-1},\ k - t + 1\right)\right]} \quad (3.60)$$

$$m_1^t = \frac{dx_1^t}{x\,dim} \quad (3.61)$$

$$n_1^t = \frac{dy_1^t}{y\,dim} \quad (3.62)$$

$$m_2^t = \frac{dx_2^t}{x\,dim} \quad (3.63)$$

$$n_2^t = \frac{dy_2^t}{ydim} \tag{3.64}$$

In general, for the three-dimensional case the following steps can be used to construct an extraction cone for block $X_{i,j,k}$ on level k.

1- Start from the base block $X_{i,j,k}$
2- Assign zero to dx_1, dy_1, dx_2 and dy_2
3- Assign 1 to t
4- Calculate:

$$ml_1 = dx_1 / x\dim + 0.5$$
$$nl_1 = dy_1 / ydim + 0.5$$
$$ml_2 = dx_2 / x\dim + 0.5$$
$$nl_2 = dy_2 / xdim + 0.5$$

5- Read slope angle from the block model of the deposit:

$dip0$ = North face angle of block ($i, j - nl_2, k - t + 1$)
$dip90$ = *East* face angle of block ($i - ml_2, j, k - t + 1$)
$dip180$ = *South* face angle of block ($i, j + nl_1, k - t + 1$)
$dip270$ = *West* face angle of block ($i + ml_1, j, k - t + 1$)

6- Calculate dx_1, dy_1, dx_2 and dy_2 from:

$$dx_1 = dx_1 + zdim / \tan dip270$$
$$dy_1 = dy_1 + zdim / \tan dip180$$
$$dx_2 = dx_2 + zdim / \tan dip90$$
$$dy_2 = dy_2 + zdim / \tan dip0$$

7- Calculate the number of blocks for the four principal directions:

$$m_1 = dx_1 / xdim$$
$$n_1 = dy_1 / ydim$$
$$m_2 = dx_2 / xdim$$
$$n_2 = dy_2 / ydim$$

8- Examine all the blocks $X_{m,n,k-t}$, where $m = i-m_2, i+m_1$ and $n = j-n_2, j+n_1$, to determine whether they are within the cone:

$$a = x\dim * (i-m)$$

$$b = y\dim * (j-n)$$

If $m \geq i$ and $n \geq j$ then $Value = \frac{a^2}{(dx_1)^2} + \frac{b^2}{(dy_1)^2}$

If $m \geq i$ and $n \leq j$ then $Value = \frac{a^2}{(dx_1)^2} + \frac{b^2}{(dy_2)^2}$

If $m \leq i$ and $n \leq j$ then $Value = \frac{a^2}{(dx_2)^2} + \frac{b^2}{(dy_2)^2}$

If $m \leq i$ and $n \geq j$ then $Value = \frac{a^2}{(dx_2)^2} + \frac{b^2}{(dy_1)^2}$

If the 'Value' is less than or equal to 1 the block $X_{m,n,k-t}$ is part of the extraction cone of block $X_{i,j,k}$ and is submitted to the graph algorithm otherwise the next block is examined.

9- Add 1 to t

10- If t is less than k go to step 4

Again, when the shape of the cone has been established all blocks within the cone are submitted to the graph algorithm.

3.5.2.1- An example

This procedure has also been incorporated into the Lerchs-Grossmann algorithm to determine optimum pit limits for complex deposits in which the pit slopes vary throughout the orebody. In order to show how the algorithm works for multiple regions, refer to the simple example illustrated in Figure 3.36 in which the deposit is divided into four different regions with the slope angles shown in Table 3.3.

Region	North face angle	East face angle	South face angle	West face angle
1	30°	40°	42°	38°
2	41°	37°	50°	46°
3	35°	35°	35°	35°
4	39°	39°	46°	46°

Table 3.3- Slope angles applied to example shown in Figure 3.36

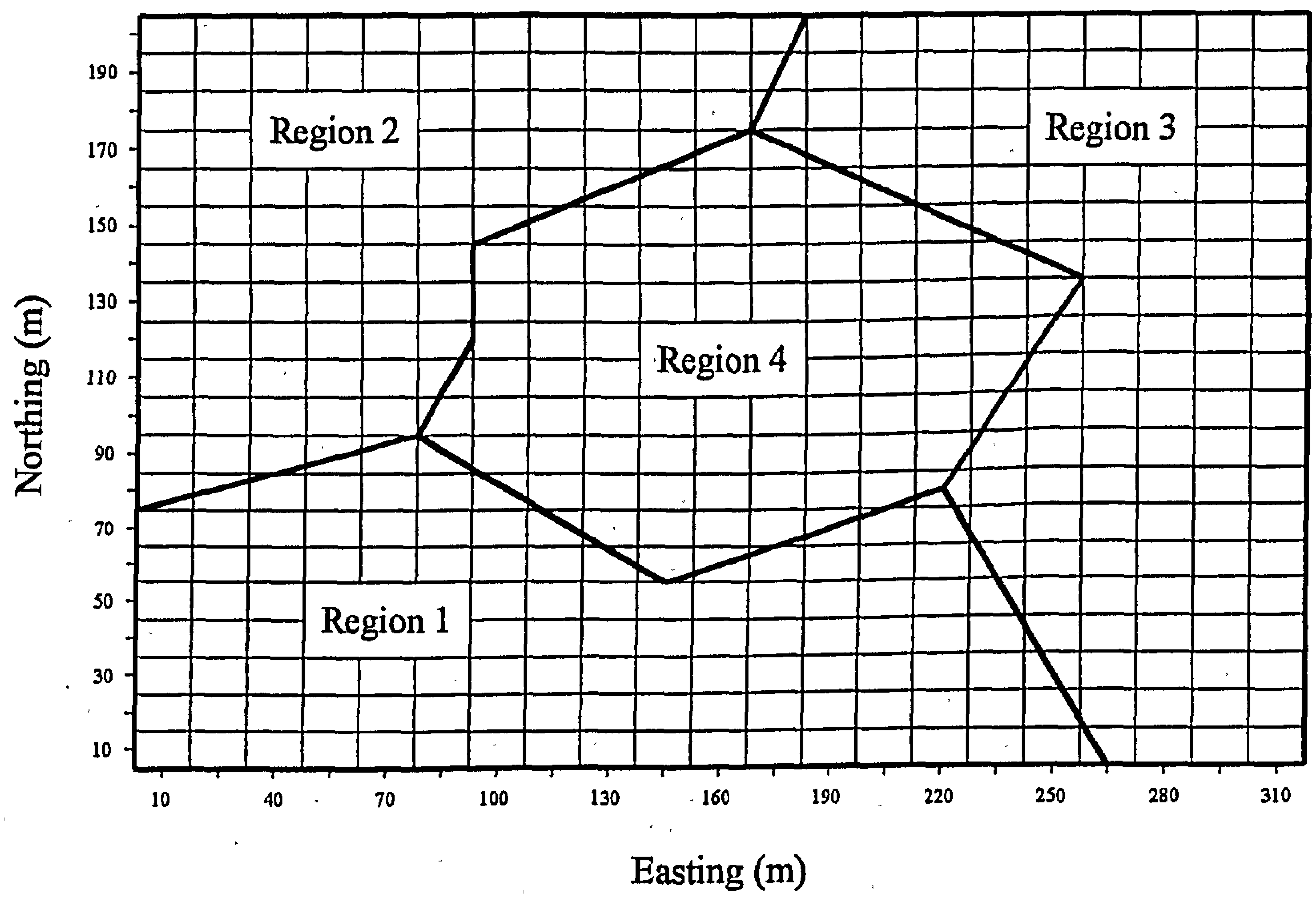


Figure 3.36- Dividing a deposit into four different regions

The block dimensions are 15m x 10m x 10m in the East, North and vertical directions respectively. The graphical results of the optimisation are shown in Figures 3.37, 3.38 and 3.39. As can be seen from the graphical presentation the algorithm works well and slope angles can vary throughout the deposit without changing the block dimensions.

Figure 3.37 - Pit limit without pit bottom smoothing - multiple variable slope angles

Figure 3.38 - Pit limit without pit bottom smoothing - multiple variable slope angles

Figure 3.39 - Pit limit without pit bottom smoothing - multiple variable slope angles

3.5.3- Programming

Optimal pit design algorithms require an amount of memory to be allocated in the computer. This is required to store the block model either in the form of block revenue values or in the form of block grades. For instance the Lerchs-Grossmann algorithm programmed in FORTRAN 77 by Dowd (1994a) and based on the 1:5:9 pattern to define mining slopes, uses two three-dimensional arrays and six two-dimensional working arrays (matrices). The three-dimensional arrays include the revenue block values and a logical variable that indicates whether each block is inside or outside the pit. The numbers of elements in these arrays are equal to the number of blocks used to define the orebody block model. The numbers of elements in the working matrices depend on the complexity of the deposit and the number of blocks in the model. This is usually assumed to be one-twentieth of the number of blocks in the orebody block model. To implement the algorithm requires an amount of computer memory to store these arrays and, thus, imposes a storage limitation on the program, especially for personal computer (PC) implementations due to the limitations of DOS memory. This limitation has been overcome by the development of a Windows program and the use of dynamic memory allocation to take full advantage of hardware capability.

To date, almost all the PC software for the mining industry has been written in FORTRAN running under the DOS operating system. There are some limitations which are imposed by both the language and the operating system. For instance, DOS is unable to use all the computer's memory. It uses conventional memory that is usually limited to 640 kb. Not all this memory is available for DOS programs since DOS uses some memory to store memory resident programs. FORTRAN programs also require definition of the dimensions or size of the arrays with constant values and these have to be large enough to store all potential elements. In other words, it does not use dynamic memory and cannot allocate space required during program implementation. On the other hand the Lerchs-Grossmann algorithm requires an amount of memory which depends on the number of blocks in the orebody model. Using PC computers running under DOS this algorithm can only be implemented for relatively small deposits. As an example, the FORTRAN 77 code developed by Dowd (1994a) can only deal with deposits containing less than 50 x 50 x 10 blocks. Dowd and Onur (1993) overcame this limitation by using random access storage and only retrieving blocks when they are required for processing. Whilst this approach can handle any number of blocks computing time increases almost exponentially as the number of blocks increases primarily because of storage and retrieval times. In practice, this approach is limited to block models with up to 10^6 blocks depending on the complexity of the distribution of grade in the deposit; the maximum number will decrease as the complexity increases. One way to overcome the memory limitation and deal with larger deposits is to implement the algorithm on a work station. The primary disadvantage here is that workstations have limited availability especially on mine-sites.

At first it was decided to modify Dowd's FORTRAN 77 code of the Lerchs-Grossmann algorithm in order to take into account variable slope angles. After considering the limitations of DOS programs it was then decided to develop a Windows program to take full advantage of hardware capability. Windows applications can access all the computer memory and have many advantages over DOS programs. Some of these are better graphics, device independence and the use of 32-bit numbers rather than 16-bit. In addition, Windows is rapidly becoming the standard environment for PC users. On the downside Windows requires a different style of programming which is slightly more difficult than that for DOS programs.

Windows applications can be divided into two categories: 16-bit Windows such as Windows 3.1, or standard Windows, and 32-bit Windows such as Windows 95 and Windows NT. These two forms of Windows manage memory in two different ways. The first divides memory into segments each of which up to 64 kb long. With the use of 16-bit Windows, it is possible to allocate 16,000 elements for the working matrix which is sufficient to deal with a deposit having around 320,000 blocks. In 32-bit Windows, memory is like an empty large box and it is not segmented. There is no limitation and any matrix with any dimension can be allocated.

A new computer program was developed in C++ code to determine optimum open pits. This program runs under the 32-bit Windows operating system and uses dynamic memory allocation for the storage of block characteristics and working matrices. For example, the following C++ code is used to allocate space required to store the block revenue value.

```
float ***val;

try{
        val = new float ** [numx];

        for (i = 0; i < numx; i++)
                val[i] = new float * [numy];

        for (i = 0; i < numx; i++)
                for (j = 0; j < numy; j++)
                        val[i][j] = new float [numz];
}
catch ( xalloc ){
        cout << "Couldn't allocate memory.";
}
```

This program has been successfully tested for a real data set having more than 400,000 blocks in the model on a PC computer with 16 Mb of RAM memory. Due to the low price of computer memory and also with this new program, it is now possible to implement the Lerchs-Grossmann algorithm on a PC for an unlimited number blocks in the orebody block model.

3.5.3.1- Memory requirements

In order to develop a computer program using dynamic memory allocation, the

FORTRAN 77 code developed by Dowd (1994a) was adopted and converted to C++ code. It was then validated by comparing the results of both programs from a number of trial data sets. After validation of the C++ code of the Lerchs-Grossmann algorithm, it was modified to deal with variable slope angles using the procedure described above. Two programs were developed to determine the optimum pit outline. The first program determines the optimum pit limit with variable slope angles in which only one region is used to define the pit slopes. The second program determines the optimum pit limit for multiple regions in each of which the mining and access slopes are defined by four principal slope angles. This program can also be used for one slope angle for each region. Both programs require two input data files which are previously created by the revenue calculation program explained later in this chapter. The first file is a text file that contains the block model characteristics - block dimensions, number of blocks in the model - and the four principal slope angles if a single region is used to define the mining slopes. The second file is a random access file and contains one record for each block including either the block value and logical value for the variable slope or, for a multiple slope pit, the block value, logical value and four principal slope angles. The programs read the first file and allocate the memory required to store the information from the second file; they then determine the optimum pit limit and write the results to an output file for further use. The memory required to implement the algorithm on a PC computer is described in the next section.

3.5.3.1.1- Memory required for the variable slope angles algorithm

The program uses two three-dimensional arrays and six two-dimensional working matrices. The three-dimensional arrays are used to store block values and logical values that take a value of 0, 1 or 2. A value of 0 indicates that block is ready to be considered, a value of 1 indicates that a block has been added to the optimum solution and a value of 2 indicates that a block is still being considered. The numbers of elements in these two arrays are equal to the number of blocks in the deposit. The number of elements in the working matrices are set to one-twentieth of the number of blocks in the model. Four bytes are required to store the value of one block and for each block one byte is required for storage of the logical value. A total of 40 bytes is also required to store each of the elements of the working matrices. The memory required is therefore:

$n = numx * numy * numz$	number of blocks in the model
$m_1 = 5 * n$	memory required for three-dimensional arrays
$m_2 = 40 * n / 20$	memory required for working matrices
$m = m_1 + m_2 = 7 * n$	total memory required

For example, for a deposit having 400,000 blocks in the model, almost 2.8 Mb memory is required for PC implementation of the Lerchs-Grossmann algorithm with variable slope angles. It should be noted that this program does not use any array for storage of slope angles as they are constant through the deposit and only four variables are required to store them.

3.5.3.1.2- Memory required for the multiple variable slope angles algorithm

In addition to the arrays used in the program for variable slope angles, this program uses an extra four three-dimensional arrays for the storage of slope angles which have already been assigned to the blocks. Four bytes are also required to store the four principal slope angles for each block. Therefore the total memory required for storage of a block's characteristics and working matrices are:

$m = 7 * n + 4 * n = 11 * n$ total memory required

For example, almost 4.4 Mb memory is required for PC implementation of the Lerchs-Grossmann algorithm with multiple variable slope angles for a deposit having 400,000 blocks in the model.

3.5.3.2- Computing time

Computing time to reach a solution depends on many factors including the number of blocks in the orebody model, the complexity of grade (and, hence, revenue) variation in the orebody and the pit slope angles. It is obvious that a deposit having a large number of blocks in the model requires longer computing time to reach a solution than a deposit containing less blocks. As the complexity of the deposit increases, the computing time required also increases. Flat slopes require more blocks to be removed to mine a particular block than steeper slopes which results in increased computing time.

As mentioned before using random access files in the optimal pit design algorithm significantly increases the computing time to reach a solution. For instance, under the 32-bit Windows operating system, a real data set, with 59 x 96 x 15 blocks, takes 45 minutes with the use of random access files. It takes less than two minutes for the same data on the same computer if using computer memory for the storage of required arrays. In addition, the new program is faster than the similar code running under DOS, since it takes advantage of the 32-bit operating system rather than 16 bit.

3.6- Revenue block model

The Lerchs-Grossmann algorithm requires a revenue block model of the orebody as input and determines which blocks should be mined to achieve maximum net profit. To create a revenue block model and determine the optimum pit limit the following information is required:

1- Block grade model of deposit
2- Physical and economic parameters
3- Pit slopes

The orebody block grade model can be obtained by subdividing the orebody into a regular, rectangular array of blocks and assigning grade values to each block. Such a model can be achieved by using geostatistical or other estimation methods. In the current study, it is assumed that the block grade model of a deposit has already been determined and represented in a text file. This file must contain one record for each block and each record can be one of the two following types:

Type I- Each block has single grade only

- Co-ordinate of the mid-point of each block
- Estimated grade (mean grade of the block)

Type II- Each block has grade and recoverable tonnage

- Co-ordinate of the mid-point of each block
- Grade above cut-off grade
- Recoverable tonnage

The three-dimensional co-ordinates are the Easting, Northing and vertical of the mid-point of the blocks. The records can be in any order and the information can be recorded in free format. It is assumed that blocks whose co-ordinates are missing from the file (i.e. gaps in the rectangular array) define the surface topography. Any negative values for grade and tonnage denote missing values and are regarded as waste blocks when the revenue value of a block is calculated. These are blocks whose grades or tonnage have not been estimated because of lack of data.

The physical and economic information includes block dimensions, specific gravity of ore and waste, cut-off grade if the block grade is of type I (each block contains only a grade) is used, cost of mining of ore and waste, processing cost, recovery factor and price of metal. The cost of mining of ore and waste can be fixed or can vary as a function of depth. Recovery factor can also be fixed or can vary as a function of grade.

The third type of information required for optimum pit design is the mining slopes which are considered as constraints on the pit design. Pit slope angles, which depend on many factors such as rock strength, presence or absence of discontinuities, ground water conditions and other geotechnical factors, may vary through the deposit. To define mining slopes, the orebody must be sub-divided into the regions or domain sectors in which rock characteristics are the same and can be characterised by a single set of slopes. This can be done by the evaluation of geological and geotechnical information which is obtained during site investigation or at the exploration stage. It is assumed that the upper and lower area of the region is approximated by a polygon. Figure 3.36 illustrates a simple example in which the deposit is divided into four different regions.

The information required to define mining slopes depends on the number of regions. If only one region is specified only the four principal slope angles are required. Otherwise, for each region the minimum and maximum depth, co-ordinates of the points defining the region and four principal slope angles are required. A maximum of 20 different regions can be specified and there is no limit on the number of co-ordinates defining each region.

A revenue block model is constructed by applying economic and physical factors to the block grade model of the deposit. Air blocks (blocks whose co-ordinates are missing from the grade block file) are assigned a value of zero, waste blocks are assigned negative values and the ore blocks have positive values. The profit value for each block is the net amount of income obtained by extracting the block and processing and selling its contents. For type I block grade models, in which each the entire block is assigned an average grade, the cut-off grade distinguishes ore blocks from waste blocks. All blocks with a grade below the cut-off grade are treated as waste. The value of each block is determined from the following formulas for the two types of block model:

$volume = xdim * ydim * zdim$ Volume of block

Type I- Each block has an average grade for the entire block:

If the grade of the block is less than the cut-off grade, or it is negative, then the block is treated as waste and its value is determined by:

*Value = -volume * specific gravity of waste * cost of mining of waste*

If the grade of the block is greater than the cut-off grade it is considered to be ore and its value is calculated as:

*Value = volume * specific gravity of ore * grade * price * recovery factor*
*- volume * specific gravity of ore * (cost of mining ore + processing cost)*

Type II- Each block is assigned a recoverable tonnage above a cut-off grade and the average grade of the proportion of the block above the cut-off grade:

If the grade of the block is negative or its recoverable tonnage is negative, the entire block is regarded as waste and its value is determined as:

*Value = -volume * specific gravity of waste * cost of mining of waste*

If both grade and recoverable tonnage of the block are positive its value is determined as:

*Value = tonnage of ore in the block * grade * price * recovery factor*
*- tonnage of waste in the block * cost of mining of waste*
*- tonnage of ore in the block * (cost of mining ore + processing cost)*

When the value of each block is calculated, the results are written to a random access file that is used as input for the optimal pit design programs. One record is used for each block. The number of fields in each record differs according to the number of regions specified. If only one region is specified each record will contain the block value and one logical variable. If multiple regions are specified each record will contain the block value, one logical variable and four principal slope angles.

Two programs were written to create the revenue block model. The first is used to calculate the revenue block model for variable slope angles in which only one slope region is specified. For a single region the slope angles are not assigned to each block so as not to allocate unnecessary memory in the pit design program. The second program calculates the revenue block model values for multiple regions in which pit slopes are assigned to each block. Both programs create three output files: the block file, the revenue block model and the grade block file. The first file is a text file that contains block model characteristics - block dimensions, number of blocks in the model and, for a single region of mining slopes, the four principal slope angles. The next two files are random access files that contain one record for each block. The revenue block model file is used in the optimal pit design program and the block grade file is used to determine the mean grade of the blocks within the pit. The basic steps involved in creating the revenue block model are:

- Read data: block grade file, physical and economic factors and pit slopes.
- Calculate the number of blocks in the east-west, north-south and vertical directions.
- Construct a random access working file and assign zero to each block.
- If multiple slope regions are used assign slope angles to each block.
- Calculate the profit value of each block and output to the working file.
- Calculate the number of blocks to add at the borders.
- Construct a random access revenue file.
- Output the values in the working file to the revenue file.
- Add waste block around the borders.

Note that, in the revenue block model, an air block indicates surface topography and has a value of zero; ore and waste blocks have positive and negative values respectively. An air block takes a logical variable of 1, indicating that it is not added to the graph when determining the optimum pit. Waste and ore blocks take a logical variable of 0, denoting that they are considered during the optimisation.

3.6.1- Adding additional waste blocks to the model

The orebody block grade model does not usually have sufficient peripheral blocks to allow the removal of the bottom-most edge blocks. Additional waste blocks must be added around the boundaries of the block grade model to ensure that all blocks are technically minable and to ensure the stability of the pit. For example, consider Figure 3.40 in which additional waste blocks are added to the model in order to make minable the ore blocks at the bottom edge of the deposit.

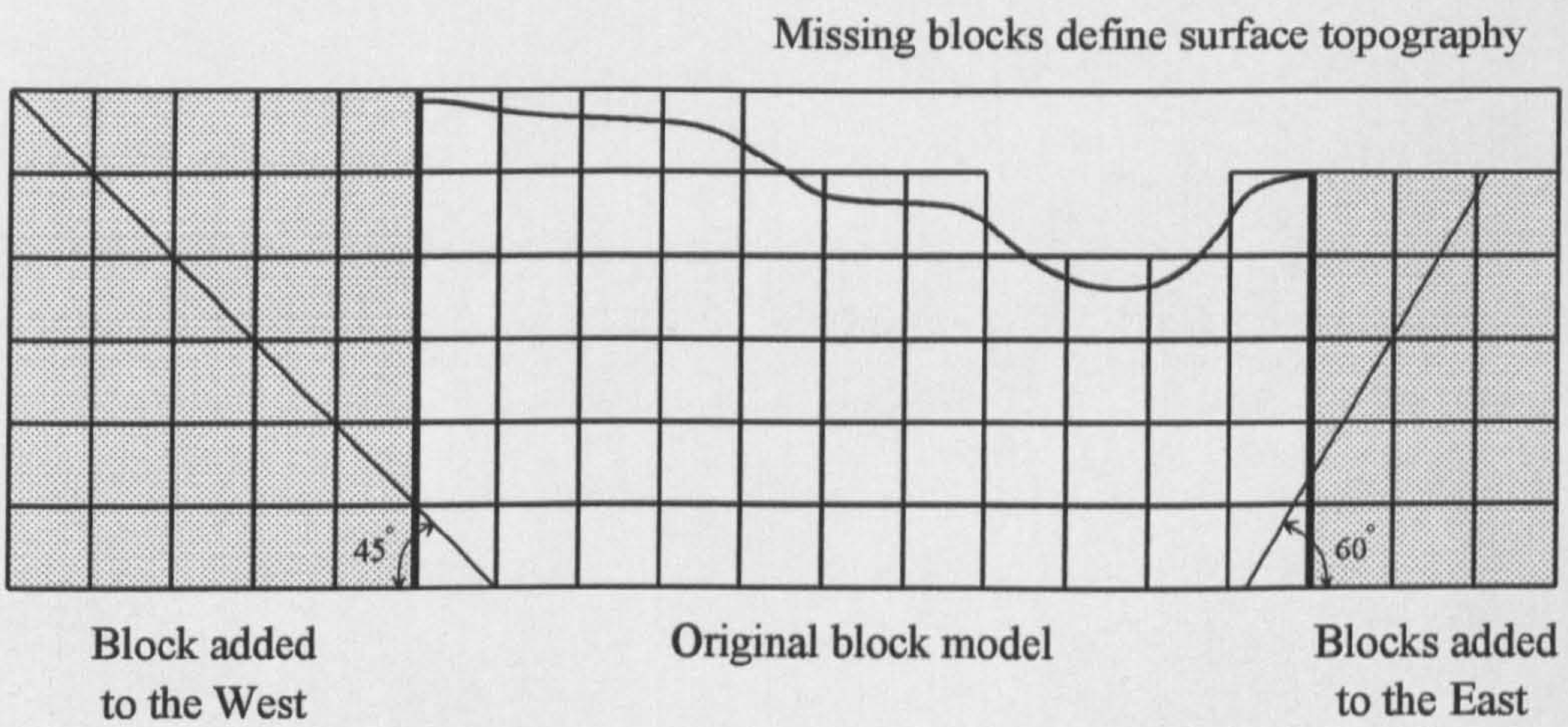


Figure 3.40- Blocks added to the West and East of Northing section

The number of blocks that are added to the orebody model on the four horizontal, rectangular boundaries must satisfy the lowest slope angle of any area of the deposit. To determine the number of blocks to be added to the model, it is necessary to find the lowest slope on the boundaries of the model. The lowest slope angles for variable slopes in which only one slope region is specified are the four principal slope angles. For multiple regions the lowest slope angles must be found among the slopes of the blocks on the boundaries of the deposit, that is:

For variable slope angles:

d_1 = *North face angle*

d_2 = *East face angle*

d_3 = *South face angle*

d_4 = *West face angle*

For multiple variable slope angles:

d_1 = *The lowest angle of the North face angle of blocks (i,0,k)*

d_2 = *The lowest angle of the East face angle of blocks (0,j,k)*

d_3 = *The lowest angle of the South face angle of blocks (i,numy-1,k)*

d_4 = *The lowest angle of the West face angle of blocks (numx-1,j,k)*

When the lowest angles are found, the number of blocks to add to the deposit in the four principal directions can be determined from the following equations:

$$num\_N = \frac{zdim*(numz-1)}{ydim*tan(d_3)} + 0.5 \tag{3.65}$$

$$num\_E = \frac{zdim*(numz-1)}{xdim*tan(d_4)} + 0.5 \tag{3.66}$$

$$num\_S = \frac{zdim*(numz-1)}{ydim*tan(d_1)} + 0.5 \tag{3.67}$$

$$num\_W = \frac{zdim*(numz-1)}{xdim*tan(d_2)} + 0.5 \tag{3.68}$$

Where

num-N	is the number of blocks added to the northern boundary
num-E	is the number of blocks added to the eastern boundary
num-S	is the number of blocks added to the southern boundary
num-W	is the number of blocks added to the western boundary

It should be noted that the optimal open pit program is written in a such way that, when a search is made to find ore blocks in the revenue block model, the waste blocks that are added at the borders are excluded from the search. These blocks are only added to the graph when it is necessary to consider ore blocks at the bottom edge of the orebody.

3.6.2- Assigning slope angles to the blocks

If more than one region, or domain sectors, are specified to define mining slopes, it is necessary to assign slope angles to each block. To assign slope angles to the blocks, the first step is to determine which blocks are inside the particular region. A block is deemed to be inside a region if its mid-point lies within that region. Blocks deemed to be within a given region have the slopes of that region assigned to them. Two different methods have been used to determine whether a point is inside, outside or on the boundary of a polygon. They are the ray-crossing and angle sum methods.

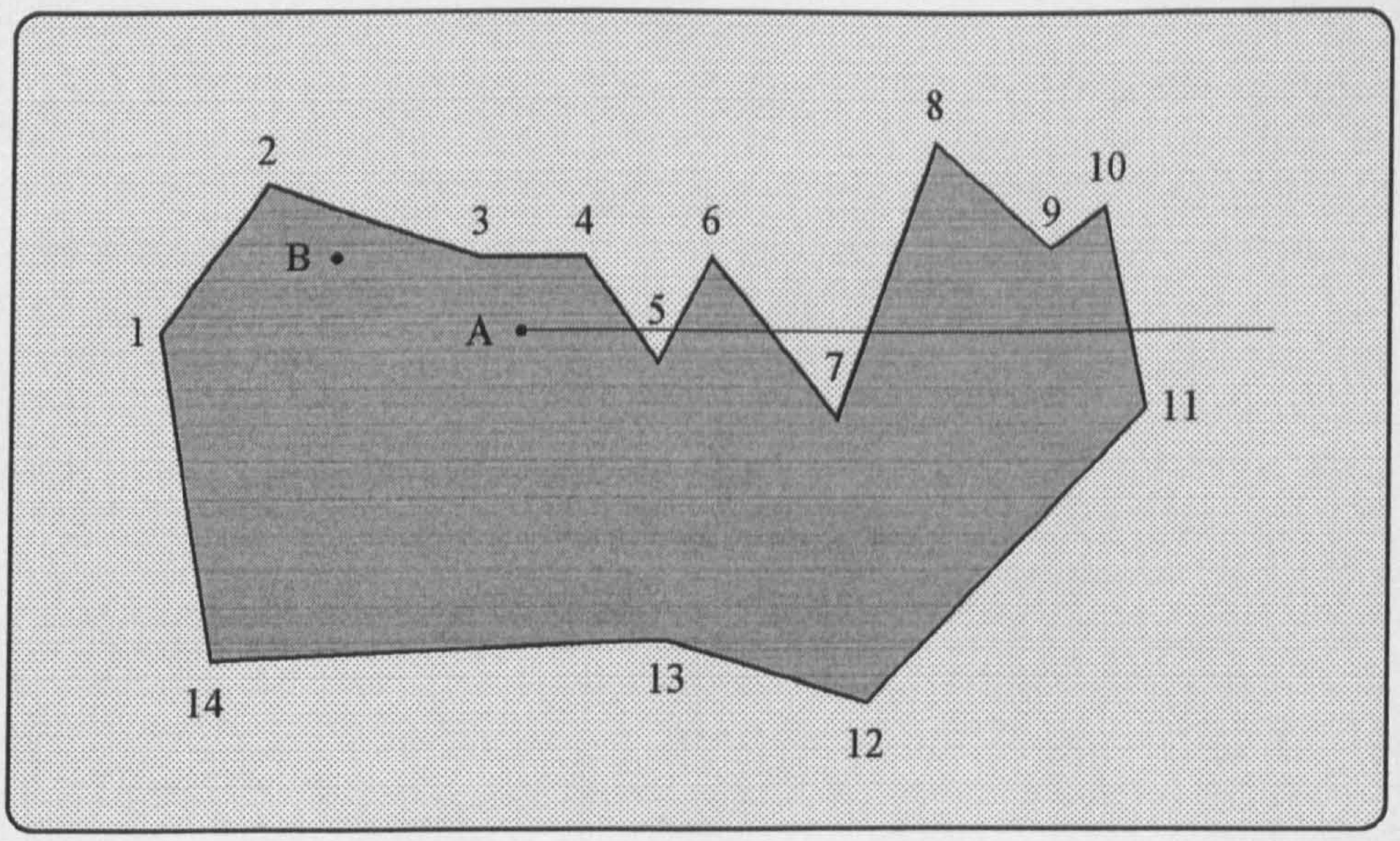
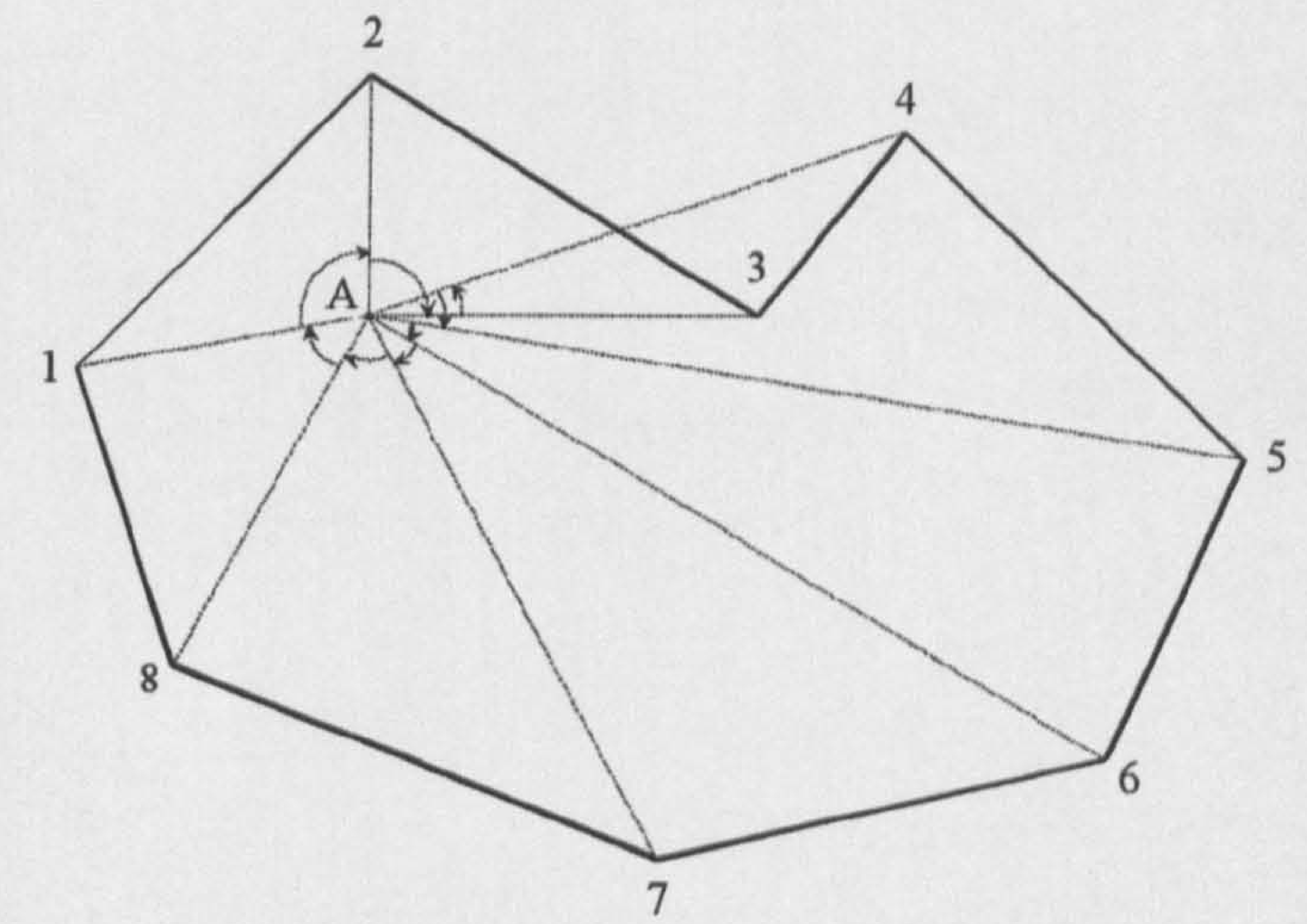
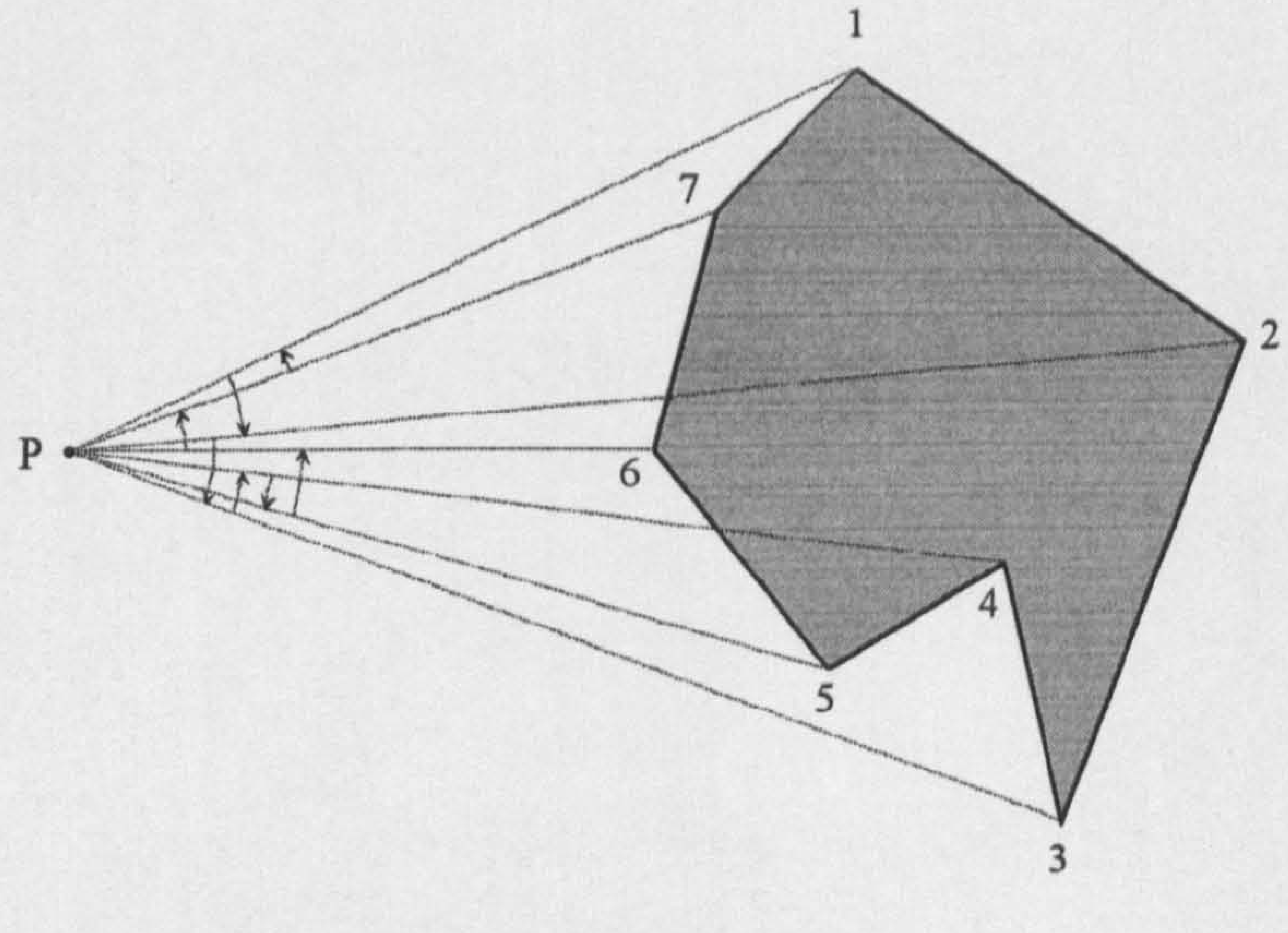


Figure 3.41- Ray-crossing method

Ray-crossing method- Consider the polygon and point A shown in Figure 3.41. Choose any point outside the polygon and draw a line, or ray, from this point to point A. Count the intersections of this line with the edges of the polygon. If the number of intersections is odd then the point is inside the polygon, otherwise it is outside the polygon. This procedure is simple but care must be taken to account for lines that pass through one of the vertices and lines that are collinear with an edge. For an example see point B in Figure 3.41, if a horizontal line is drawn from this point it will be collinear with the edge 3-4 and pass through vertex 6.



a- Point A inside the polygon ($\sum \theta_i = 360°$)



b- Point P outside the polygon ($\sum \theta_i = 0°$)

Figure 3.42- Angle sum method

Angle sum method- Consider any polygon defined by lines joining a set of vertices and any point A that may be inside, outside or on the boundary of the polygon (e.g. Figure 3.42a). Draw lines from A to each of the vertices defining the boundary of the polygon. Measure the angle between each successive pair of lines defining clockwise as positive and anticlockwise as negative. The point is inside the polygon if the sum of the angles is 2π radians and is outside if the sum is zero as illustrated in Figure 3.42.

The basic requirement for this method is the signed angle between pairs of lines from the point to successive pairs of vertices defining the boundary of the polygon. These angles can be determined either by using the dot product of two vectors or using the equation of a triangle. The sign of angles can also be determined from the cross-product of two vectors as follows:

$a^2 = b^2 + c^2 - 2bcCos\theta$	triangle equation
$v_1.v_2 = \|v_1\|\|v_2\|Cos\theta$	dot product of two vectors
$v_1 x v_2 = \|v_1\|\|v_2\|Sin\theta$	cross-product of two vectors

The angle sum method has been employed in this work and is based on coding originally written by Dowd (1973).

A block whose mid-point lies within a slope region is assigned the slopes of that region. The method is implemented by first imposing a bounding box around the region – this is the smallest rectangle that contains the region or polygon (Figure 3.43). Then the mid-points of all the blocks in the first level of the region that are inside the bounding box are examined to see whether or not they are inside the region. If the mid-point of any block lies inside the region, slope angles are assigned to all blocks at this location from the minimum to the maximum depth of the region.

Other methods, such as the scan line method, could be used to assign slope angles to the blocks. This is also used in computer graphics to fill polygons.

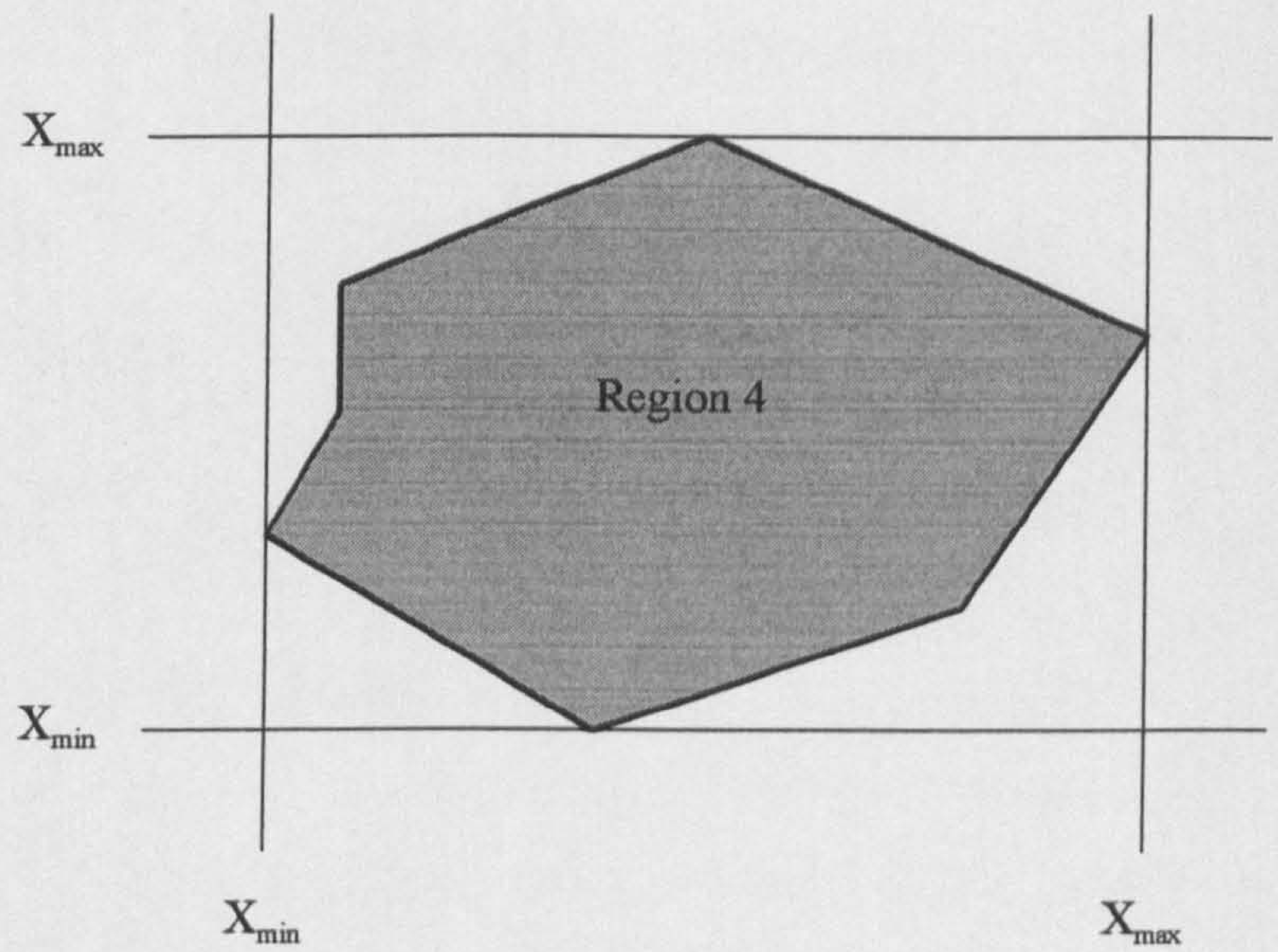
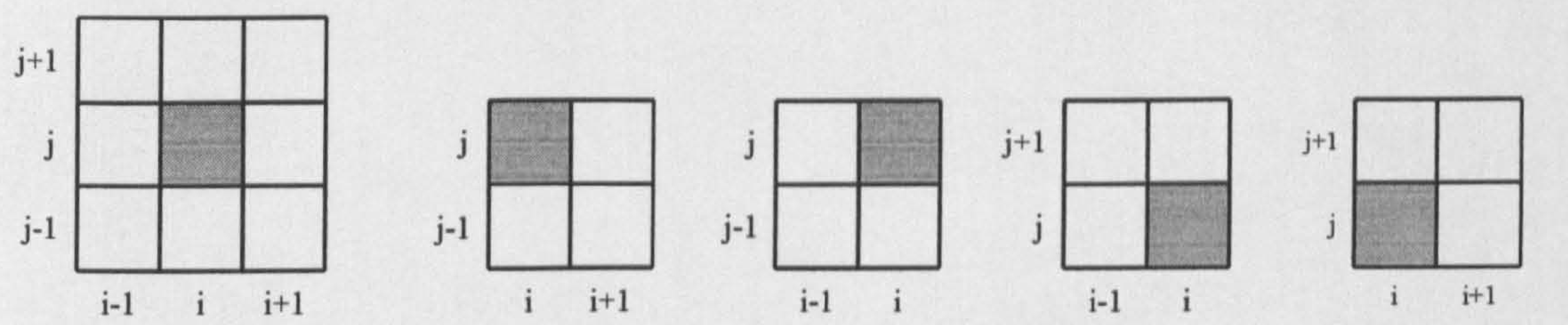


Figure 3.43- Bounding box of region 4 in Figure 3.36

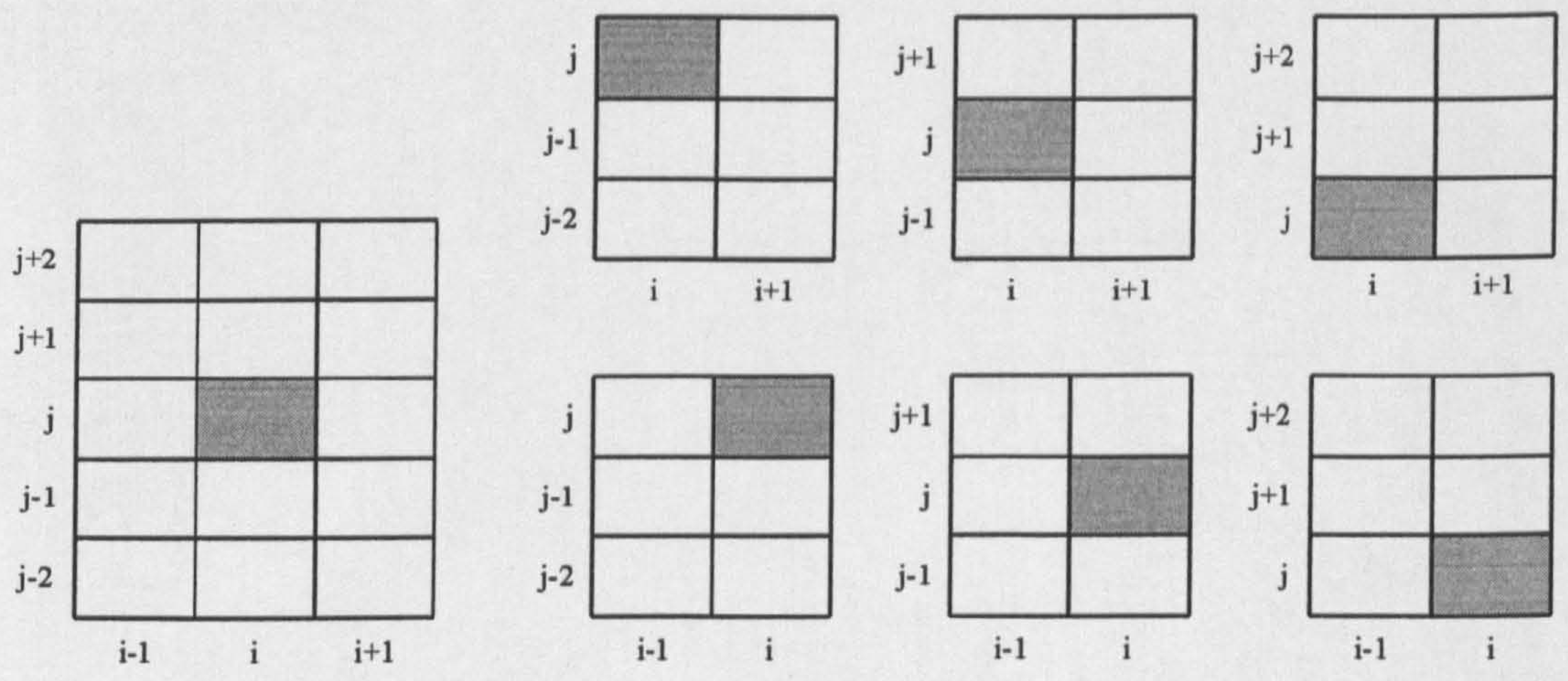
3.7- Pit bottom smoothing

The Lerchs-Grossmann algorithm takes the revenue orebody block model and determines those blocks that should be mined to obtain the maximum profit value from the pit. This, or any other, optimal pit design algorithm is likely to produce a very irregular outline on the floor of the pit which may not be practical due to the presence of isolated blocks or the lack of adequate space for mining equipment. As an example, consider the pits shown in Figures 3.32-3.34 and 3.37-3.39. Although these pits generated by the Lerchs-Grossmann algorithm are economic optima, they may not be practically feasible because of minimum space requirements for mining equipment to operate freely. It is also possible to incorporate minimum space requirements in the algorithm but this significantly increases the computing time to reach a solution. Another approach is to design the pit without any minimum access constraints and then to smooth the pit bottom in order to allow sufficient space for mining equipment.

A modified version of the pit smoothing approach presented by Onur and Dowd (1993) has been adopted in this work. The user is asked to specify the required minimum space in which the mining equipment can operate freely. The minimum space can be defined in terms of the number of blocks to be mined together at any one time which is a function of the horizontal block dimensions and the minimum space required. For example, for an orebody model comprising 10m x 10m x 10m blocks with a minimum space requirement of 20m, four blocks must be mined together to gain the required space. Six blocks must be mined together for the minimum space of 30m for a block model comprising 15m x 10m x 10m blocks. There are usually several combinations of blocks that will provide the required space. As illustrated in Figure 3.44, there are four possible alternatives in a 10m x 10m x 10m block model, to gain a minimum space of 20m x 20m (Figure 3.44a) and six possible combinations of blocks to provide a 30m x 30m space in a 15m x 10m x 10m block model (Figure 3.44b) .



a- Cubic block (10m) and minimum space 20m



b- Rectangular block (15m x 10m) and minimum space 30m

Figure 3.44- Possible combinations of blocks around a specific block (shaded)

Figure 3.45 - Pit limit with pit bottom smoothing - multiple variable slope angles

Figure 3.46 - Pit limit with pit bottom smoothing - multiple variable slope angles

Figure 3.47 - Pit limit with pit bottom smoothing - multiple variable slope angles

The pit-bottom smoothing algorithm starts with a search for all blocks within the pit. Each of these blocks is then examined in turn to determine whether minimum space requirements are met. For each block all combinations of surrounding blocks are examined. If any combination satisfies the minimum space requirement no smoothing is done. Otherwise the net value of each of the combinations that would be obtained by mining the additional block and corresponding overlying blocks is calculated. Among these, the combination chosen is the one with the highest value and the least effect on the overall profit. This value is added to the value of the block under consideration. If the result is still positive then all blocks in this combination are included as part of the pit. Otherwise the block under consideration is removed from the pit.

Two programs were written to implement pit bottom smoothing. The first involved the smoothing of the pit bottom for variable slope angles in which only one region is specified. The second program is for pit bottom smoothing for the case of multiple slope angles in which more than one region is specified. These programs use files containing records that have different fields and use different procedures to construct an extraction cone. As an example of the result of pit bottom smoothing, consider Figures 3.45, 3.46 and 3.47, in which the algorithm has been applied in the case of multiple regions with a minimum space of 30m x 30m, introduced in section 3.5.2.1.

3.8- Conclusion

The Lerchs-Grossmann algorithm is well-known for being the only method that can be proved, rigorously, always to yield the true optimum pit. However, when the algorithm was first introduced it was based on a fixed slope angle governed by the block dimensions. The methods presented in this chapter have been incorporated into the algorithm to overcome this limitation and to take account of variable slope angles. As demonstrated by simple examples, the algorithm is able to generate an optimal ultimate limit with variable slopes. It works well and can be used for both cubic and rectangular block models. Slope angles can vary in different parts of the orebody without changing the block dimensions and the block dimensions are independent of the slope angles.

The Lerchs-Grossmann algorithm uses a three-dimensional fixed block model to determine the optimum pit limit. Implementation of the algorithm on a PC requires an amount of memory for storage of the block characteristics. Due to the limitations of DOS memory it is not possible to use the algorithm for large block models. This limitation has been overcome by developing a new computer program in C++ code that can be implemented under the 32-bit Windows operating system. This program is able to use all the computer memory and can be used for large deposits on PC computers providing that the computer used has sufficient memory.

The first step for determination of the optimal pit outline is to create a revenue block model of the deposit. This has been done by applying physical and economic factors to the orebody block grade model previously created by geostatistical or other methods. Furthermore, slope angles have been assigned to the blocks with the use of the angle sum method if multiple regions are specified to define mining slopes.

The optimum pit limit generated by the Lerchs-Grossmann algorithm may have a very irregular bottom that may not be feasible in practice because of the minimum space required for mining equipment to operate freely. The method presented in this chapter smoothes the pit bottom to obtain the minimum space required.

CHAPTER 4

Slope design procedure

4.1- Introduction

The objective of a mining operation by open pit mining is to extract ore at a profit. The profitability of this operation depends on the ability of the pit slopes to be as steep as possible without resulting in a large scale failure. Pit slopes of an open pit, which are considered as constraints in the determination of the optimum pit limit, affect the size and shape of the final pit. The effect of steeper slopes at the pit limit is to decrease the stripping ratio and therefore reduce the amount of waste to be removed. On the other hand, pit wall needs to remain stable during the life of the mine. The determination of this slope angle is very important. The steeper the final slope that can be designed safely, the lower the amount of waste that has to be removed. However, as the slope becomes steeper, the probability of slope failure increases. Therefore an optimum mine plane should have the steepest final pit limit remaining stable as long as mining activity is continued in that area.

To determine the optimum pit limit with variable slope angles, an estimate of a set of average and safe slope angles is required. Various procedures may be used to determine pit slope angles which may vary throughout the deposit to follow the rock characteristics. If little geotechnical information is available they may be found from other mines of similar size and geological conditions. If the mine is in operation, predefined or existing pit slopes can be used to obtain the optimum pit limit. This chapter also presents two approaches and associated computer programs for estimation of a set of safe and average pit slopes to use in the optimal pit design program. The first is to determine the steepest safe angle with kinematic analysis. The second includes methods to design slopes with limit equilibrium analysis in terms of the calculation of the factor of safety or the probability of failure using the Monte Carlo simulation technique.

4.2- General information

4.2.1- Orientation of a plane

The orientation and inclination of any structural plane can be described as either dip and dip direction or strike and dip. Dip and dip direction are more useful for engineering purposes, while strike and dip are usually used in geological studies. The definitions of these terms follow and are illustrated in Figure 4.1.

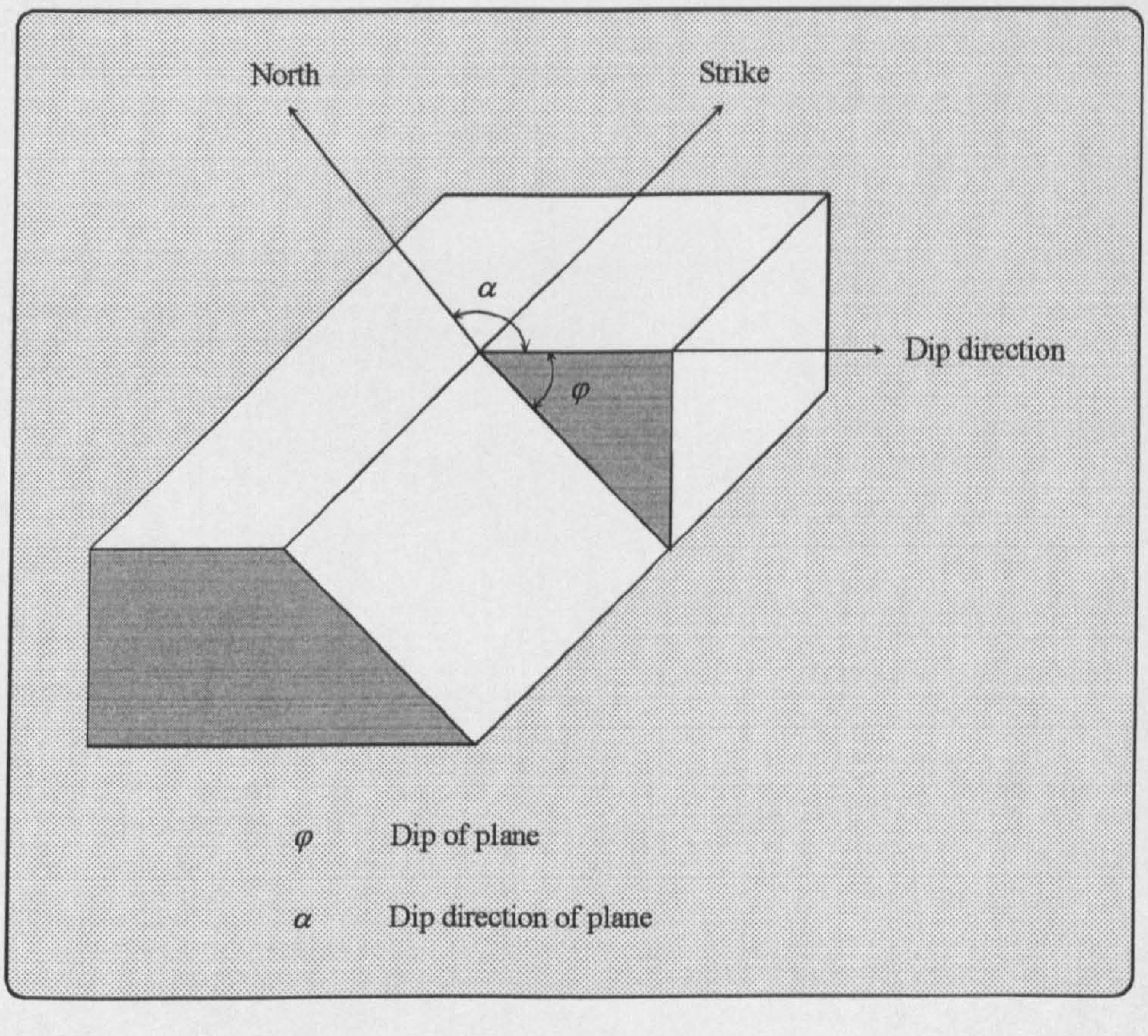


Figure 4.1- Definition of geometrical terms

Strike- The strike of a plane is the direction of a horizontal line contained within the plane.

Dip- Dip is the maximum inclination of the structural plane to the horizontal which is measured in a vertical plane perpendicular to its strike.

Dip direction- Dip direction is the direction of the horizontal projection of the line of maximum inclination measuring clockwise from the North. It varies between 0° and 360°.

The relation between strike and dip direction is: Strike = Dip direction ± 90°

4.2.2- Orientation of a line

The orientation of any line in space can be described in terms of its trend and plunge, which are defined as:

Trend- The trend of a line is the direction of the horizontal projection of the line which is measured clockwise from the North.

Plunge- Plunge, which is the dip of line, is the angle made between the line and the horizontal which is measured in a vertical plane.

4.2.3- Four principal slope angles

The dip direction of the four principal slope angles used in the current study to develop the optimal pit design with variable slope angles are:

North face slope or Northing slope: the slope with dip direction of 0°.

East face slope or Easting slope: the slope with dip direction of 90°.

South face slope or Southing slope: the slope with dip direction of 180°.

West face slope or Westing slope: the slope with dip direction of 270°.

4.3- Design procedure

The optimal pit design algorithm with variable slope angles presented in the previous chapter requires an estimate of a set of average and safe slope angles to determine the optimum pit limit. For this purpose, it is necessary to divide the orebody into domain sectors or regions based on the geotechnical data gathered during the exploration stage and the results of the field investigation in which the rock characteristics are the same and can be characterised by a single set of slopes. These domains require separate stability analyses and design procedures.

The first step of slope design is to identify the likely mode of failure. The second step is to use one of the slope stability methods to determine whether the slope is stable or not. To ensure stability, it may be required to reduce the slope angle. A review of the available methods and associated software for slope stability are given by Coggan, Stead and Eyre (1998). Traditionally the stability of the slope has been determined in vertical section in which the location of the slope and its geometrical relationship with discontinuities within it are known or assumed. Whereas in the optimal pit design, the location and position of the pit wall is not known until the pit limit is determined. On the other hand, to determine optimum pit limit an estimate of pit slope angles is required.

To overcome this limitation, two different approaches and associated computer programs were developed to determine the slope angles: the steepest safe angle from kinematic analysis and limit equilibrium analysis in terms of factor of safety or the probability of failure. Both methods determine slope angles for the slopes with dip directions from 0° to 360° in 15° increments. Then these angles are modified to find the four principal slope angles for each region. In the design of pit slopes, the analysis applies to major discontinuities occurring in the pit wall and kinematic analysis is used to detect potential failure modes. Toppling and other types of failure are excluded from this study. The following sections describe these methods which have been developed to determine the four principal slope angles for use as input in the optimal pit algorithm.

4.3.1- Steepest safe angle with kinematic analysis

Kinematic analysis, which refers to the study of movement without taking into account the forces causing them to move, can be used to identify structural instabilities (Hoek and Bray ,1981) or to determine steepest safe angle (Goodman ,1980). This method, which is a quite simple and useful technique as the first stage of slope stability planning, is usually carried out with the use of stereographical projection techniques. The approach is used in this study to determine the steepest safe angle is based on the analytical solution. This analysis is carried out with regard to the structural instabilities such as plane and wedge failure for each region and for various slopes with dip directions from 0° to 360° in 15° steps. For each slope, the program investigates all the discontinuities to find the steepest safe angle due to plane failure for each discontinuity and also determines the line of intersection for all pairs of discontinuities to find the steepest safe angle due to wedge failure. Then among these angles the minimum angle is chosen as the steepest safe angle for that direction. Figure 4.2 shows the flow chart to determine the steepest safe angle together with the potential failure mode in which it is assumed that input data for the algorithm includes dip and dip direction of all the discontinuities and also the dip direction of the slope. When the steepest safe angle is determined for all directions they are modified to find the four principal slope angles which will be explained later in this chapter.

4.3.1.1- Steepest safe angle with regard to plane failure

This failure is likely to occur when a geological discontinuity such as bedding plane has a strike parallel to or nearly parallel to the strike of the slope face and daylights into the slope face. To consider the kinematic analysis of plane failure, the following three criterion are examined:

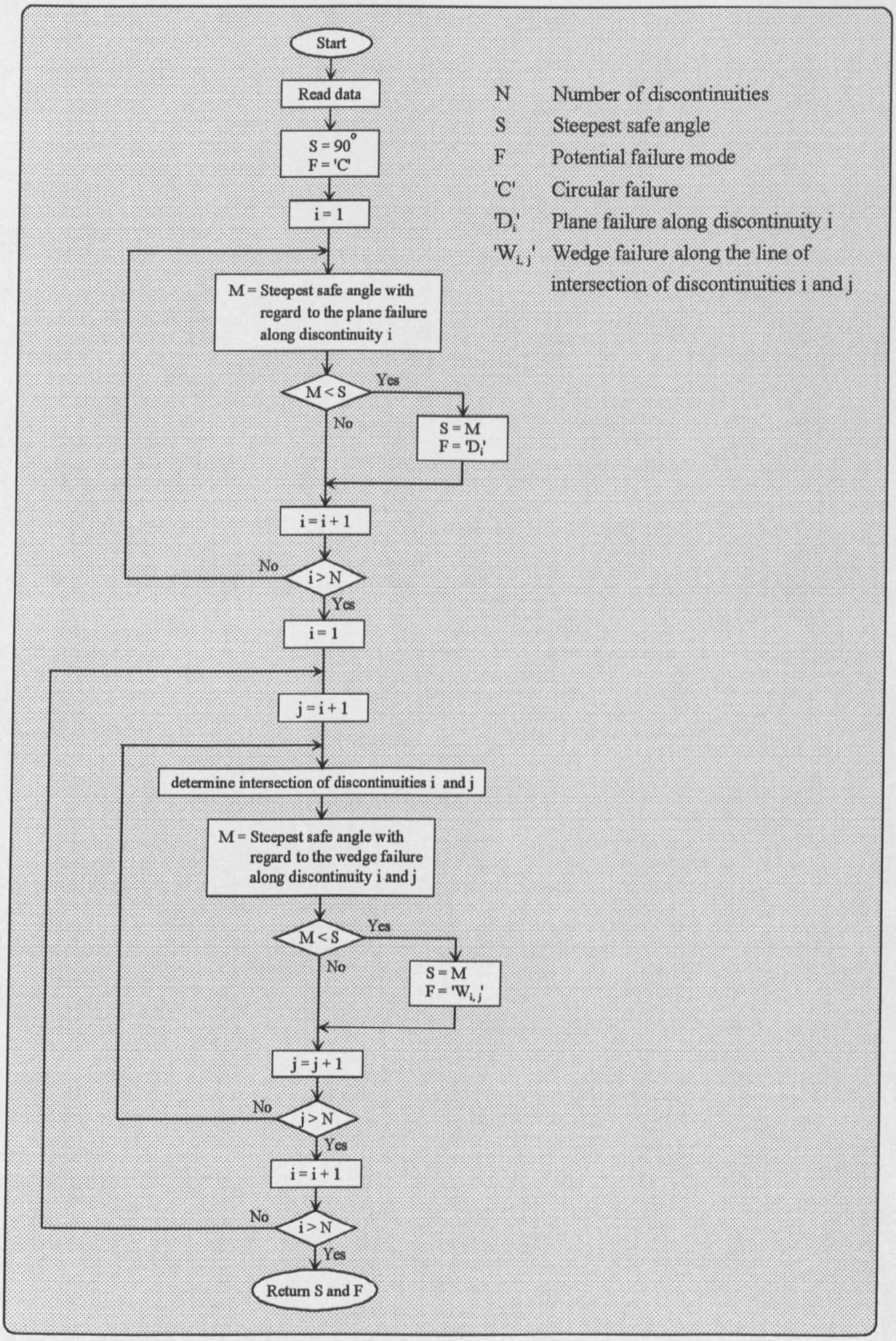


Figure 4.2- Flow chart to determine steepest safe angle and failure mode

(a) The dip direction of the sliding plane should lie within approximately ±20° of the dip direction of the slope.

(b) The sliding plane must daylight in the slope face. This means that its dip must be less than the angle of slope face.

(c) The dip of the sliding plane must be greater than its angle of friction.

The program investigates all structures within the region. Then the above criteria are considered for each discontinuity. If these conditions are satisfied the steepest safe angle is determined from the following equation. Otherwise the steepest safe angle is considered as 90 degrees for this discontinuity.

$$\theta = ArcTan\left(\frac{Tan\varphi}{Cos\alpha}\right) \qquad (4.1)$$

where

θ is the steepest safe angle

φ is the dip of the sliding plane

α is the difference between the dip direction of the sliding plane and the slope

4.3.1.2- Steepest safe angle with regard to wedge failure

This failure can be considered as a variation of plane instability. It is likely to occur when the line of intersection of two discontinuities daylights into the slope face. To consider the kinematic analysis of wedge failure, the following two criterion are examined:

(a) The line of intersection of two planes must daylight in the slope face. This means that its plunge must be less than the angle of slope face which is measured in the direction of the line of intersection.

(b) The plunge of the line of intersection of two planes must be greater than the angle of friction of the planes.

To determine the steepest safe angle with regard to the wedge instability, the trend and plunge of the line of intersection of all pairs of discontinuities within the region are determined and the above criteria are considered if these conditions are satisfied the steepest safe angle is determined from equation 4.1 except that plunge and trend of the line of intersection is used instead of the dip and dip direction of the discontinuity respectively. Otherwise the steepest safe angle is set to 90°.

Trend and plunge of the line of intersection of two plane A and B when their dip and dip direction are known can be determined from the equations 4.2 and 4.3 respectively. These equations are given by Hoek , Bray and Boyd (1973).

$$\alpha_i = ArcTan\left(\frac{Tan\varphi_A.Cos\alpha_A - Tan\varphi_B.Cos\alpha_B}{Tan\varphi_B.Sin\alpha_B - Tan\varphi_A.Sin\alpha_A}\right) \tag{4.2}$$

$$\varphi_i = ArcTan\left[Tan\varphi_A.Cos\left(\alpha_A - \alpha_i\right)\right] \tag{4.3}$$

where

α_i	is the trend of the line of intersection
φ_i	is the plunge of the line of intersection
α_A	is the dip direction of plane A
α_B	is the dip direction of plane B
φ_A	is the dip of plane A
φ_B	is the dip of plane B

4.3.2- Design with the limit equilibrium method

The steepest safe angle with kinematic analysis determines the slope angles with regard to the structural instability without taking into account the non structural instability and groundwater conditions. This method can be used when little information is available.

For example during the feasibility study and preliminary design stages of a project when very detailed information is not available and an estimate of the safe angle is required to determine the pit outline and stripping ratio, the steepest safe angle method can be a useful approach.

The limit equilibrium method is the most widely accepted and commonly preferred design tools in slope design. The first step of analysis is to detect potential failure mode and then determine the factor of safety. Then a decision must be made whether the slope is stable or not. In the case of instability, the slope angle must be reduced or the slope height must be decreased to obtain the minimum acceptable factor of safety.

In general after establishing boundaries for the design sectors or regions, the following steps are carried out for each region to determine the four principal slope angles as input parameters for use in the optimal pit design program. For each region, the analysis is carried out for the slopes with dip directions from 0° to 360° in 15° increments and for each direction depending on the input data the factor of safety or probability of failure is determined versus slope angles from 20° to 80° in 5° steps. If the input data are fixed values the factor of safety is calculated, otherwise the probability of failure is obtained. The outputs are put in a text file from which the four principal slope angles are determined with regard to the minimum acceptable of factor of safety or maximum acceptable of risk of failure. The steps are as follow:

(a) Performing kinematic analysis to identify potential failure mode and critical discontinuities.

(b) Analysing all the failure modes with the limit equilibrium method in terms of the calculation of the factor of safety if the input data are deterministic values or in terms of probability of failure if the input data are random variables.

(c) Drawing the factor of safety or the probability of failure versus slope angle.

(d) Determination of slope angles with regard to the minimum acceptable factor of safety or the maximum acceptable risk of failure.

(e) Modifying slope angles to the four principal angles in the four principal directions to use as input in the optimal pit design program.

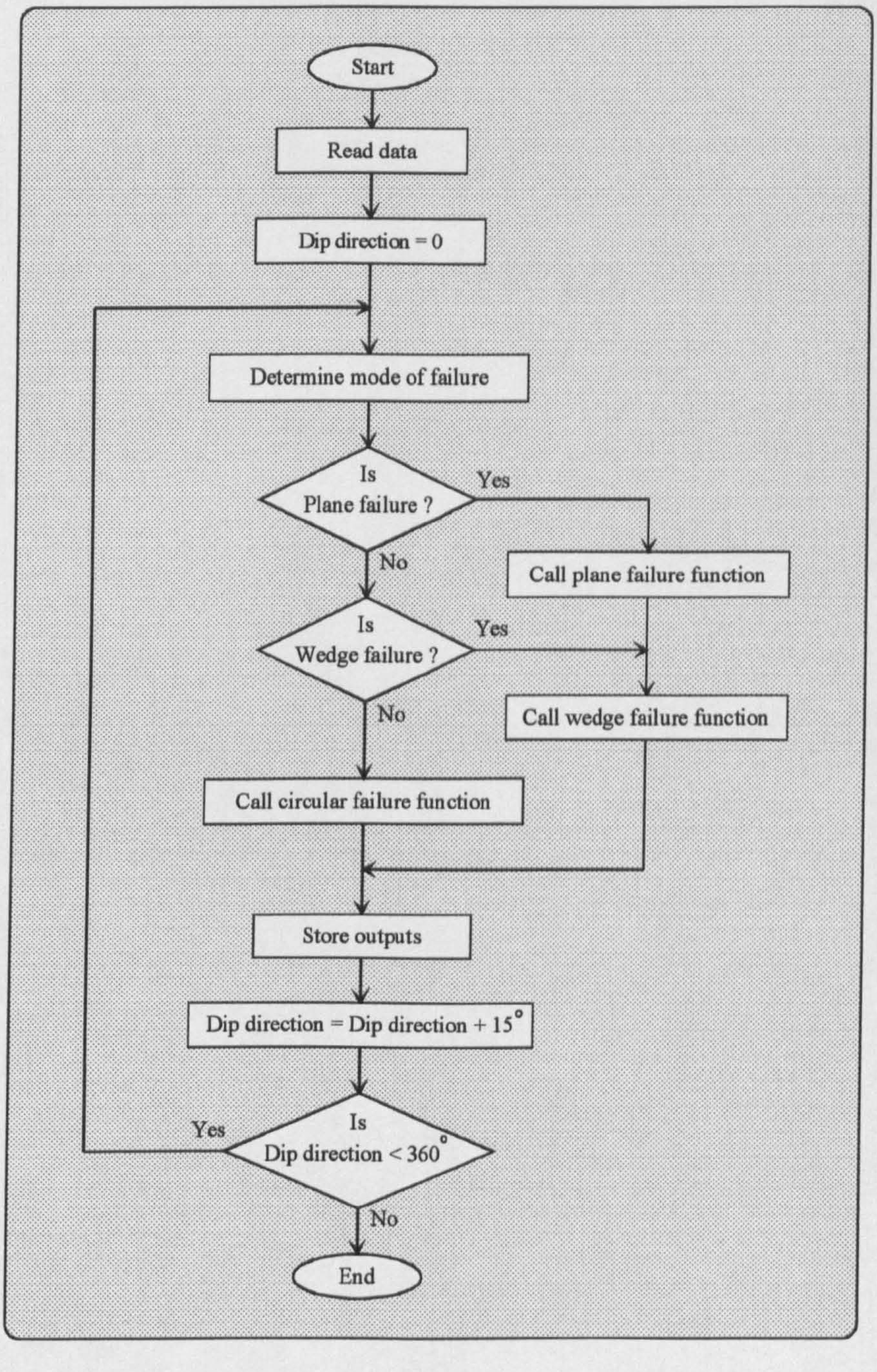


Figure 4.3- Flow chart to determine potential failure mode

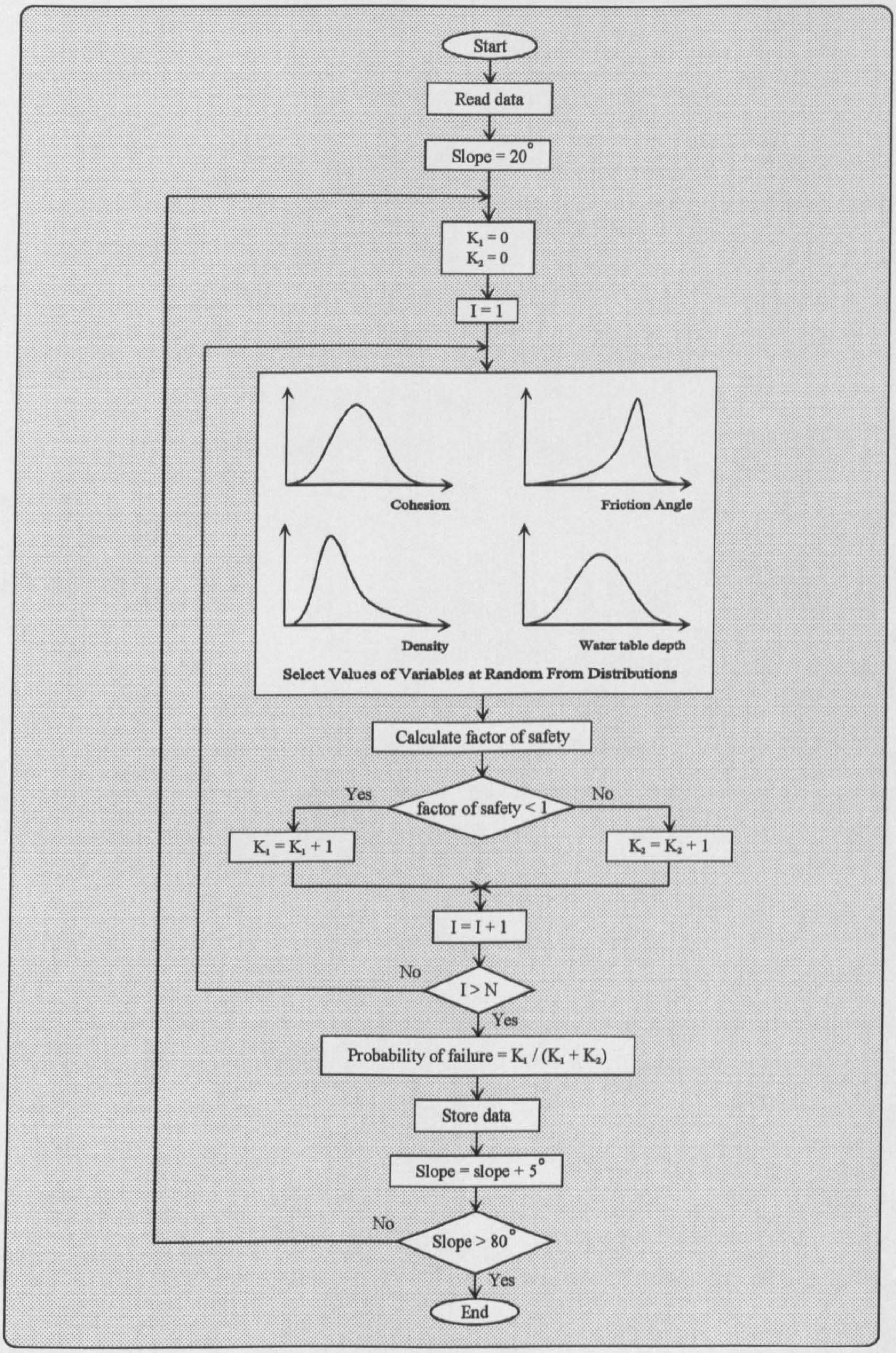


Figure 4.4- Flow chart to determine probability of failure

4.3.2.1- Identification of potential failure mode

The first step in slope stability analysis with the limit equilibrium method is to detect which type of failure is likely to occur. To do this, kinematic analysis is used to find the possible failure mode. By assuming a circular excavation for each region, as can be seen from the flow charts illustrated in Figure 4.2 and 4.3 the program investigates the likely mode of failure for all the slopes of dip direction from 0° to 360° in 15° steps. When the type of failure is detected the corresponding program is used to calculate the factor of safety or probability of failure versus slope angles from 20° to 80° in 5° increments. Figure 4.4 shows the algorithm which was developed to determine the probability of failure versus slope angle for circular failure.

The program is also capable of displaying the results of kinematic analysis graphically, for which some examples will be given later in this chapter. It should be noted that the kinematic analysis can identify only structural instability. In the case of non-detection of structural instability, circular failure is used to design the slope. To identify potential failure mode the mean dip and dip direction of discontinuities are used when the input data are specified as variable values.

4.3.2.2- Calculation of factor of safety

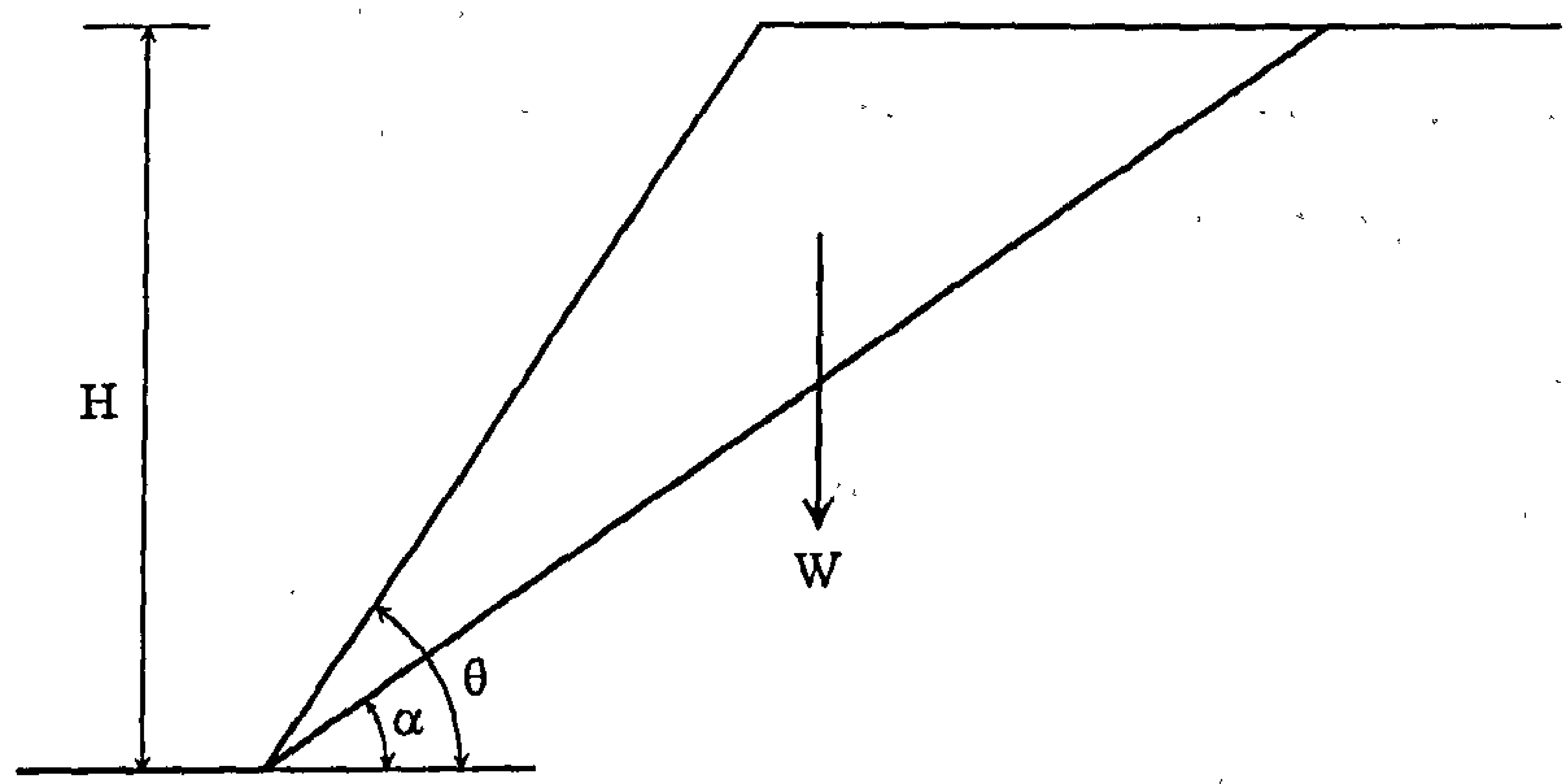
In order to design safe slope angles with the limit equilibrium method, this involves the determination of the factor of safety for each type of failure mode. The factor of safety is defined as the ratio of total forces available to resist failure to the total forces tending to cause instability. The following gives a brief description of the required formulas used to calculate the factor of safety for each type of failure mode. To calculate the factor of safety the influence of any external forces due to bolts, cables or seismic acceleration are not taken into account, since it would only be necessary to consider these influences when the critical slopes are being considered.

4.3.2.2.1- Factor of safety for plane failure

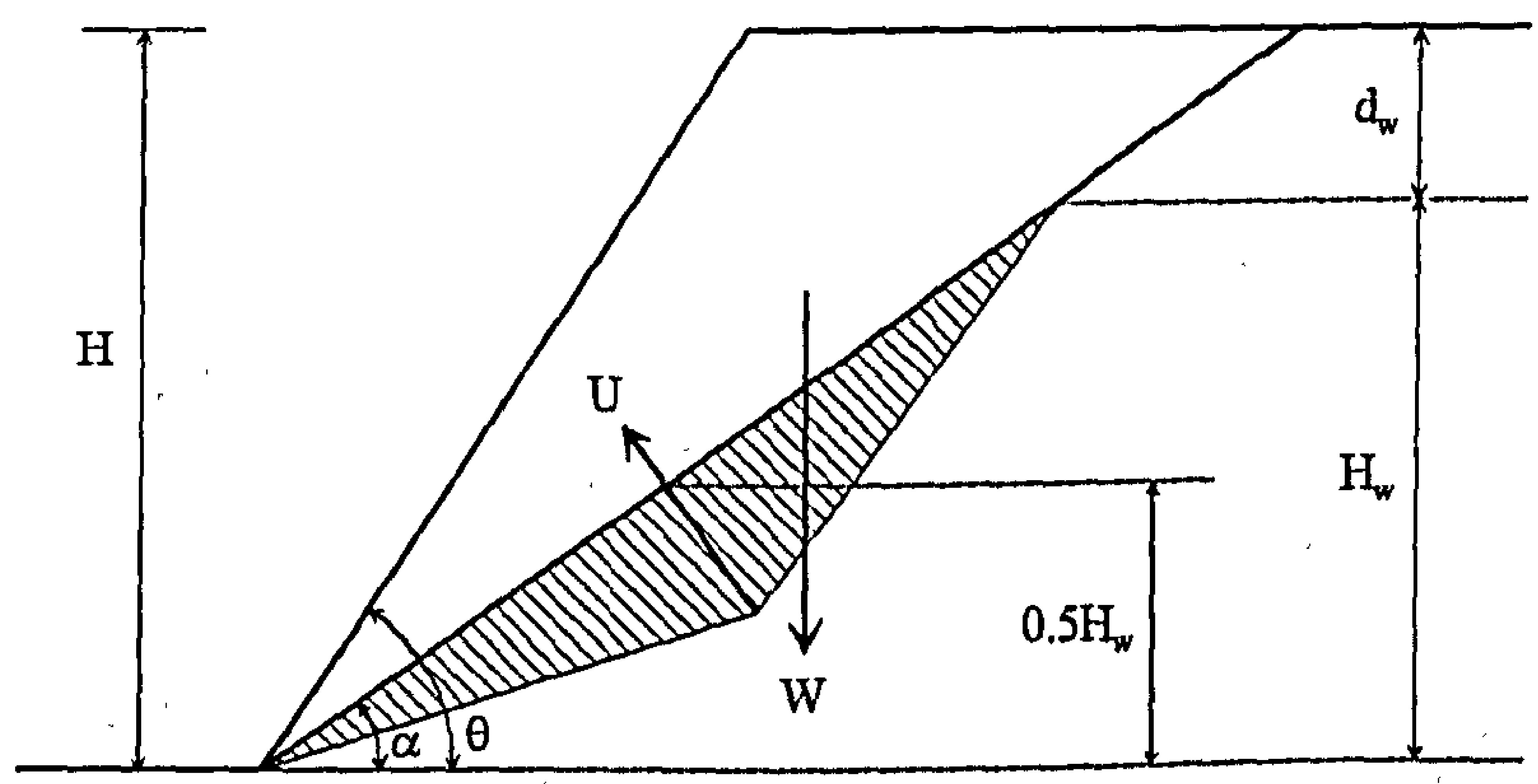
As mentioned before this type of failure is likely to occur when a geological discontinuity daylights in the slope face. Three types of plane failure are considered and for each a factor of safety is determined from the following equations. These are slopes without tension cracks under both dry and wet conditions and slopes with a critical tension crack under wet conditions (Figure 4.5). The factor of safety of plane failure for the general case of model III in Figure 4.5c, can be determined by the following equation given by Hoek and Bray (1981) in which the shear strength of the sliding surface is assumed to be defined by the Mohr-Coulomb criterion. The program also includes a kinematic check to test whether the plane failure is possible or not. This is needed in probabilistic analysis where the distribution of discontinuity orientation may result in some cases where the plane of weakness does not daylight into the slope face. This case is regarded as a factor of safety being greater than 1.

Notation

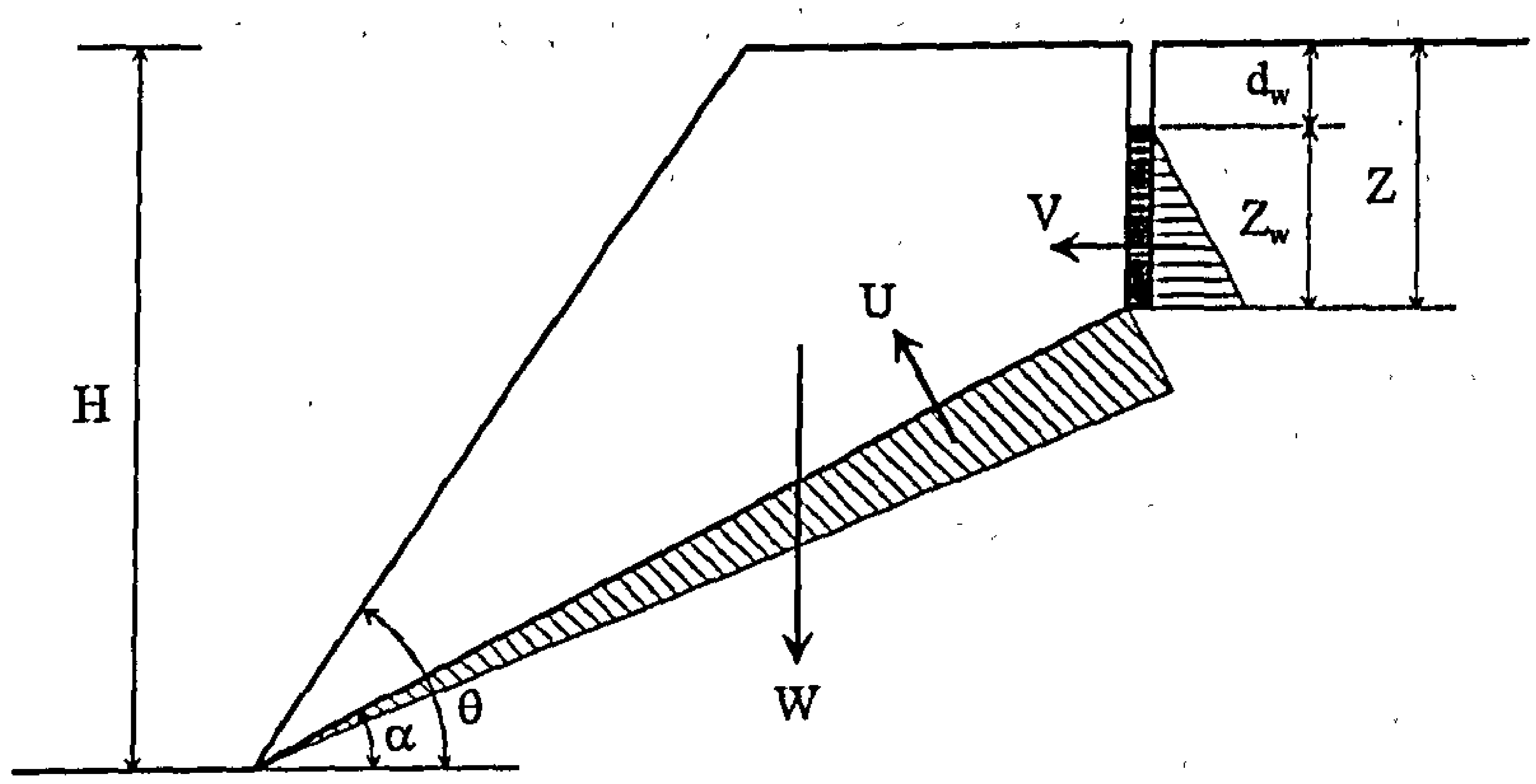
F	Factor of safety
c	Cohesion
ϕ	Angle of friction
α	Angle of failure plane
θ	Slope angle
γ	Rock density (unit weight)
γ_w	Water density (unit weight)
d_w	Depth of water table
H	Slope height
Z	Height of tension crack
Z_w	Height of water in tension crack
H_w	Height of water table
A	Surface area of sliding block



a- Model I- Dry slope without tension crack



b- Model II- Wet slope without tension crack



c- Model III- Wet slope with tension crack

Figure 4.5- Types of plane failure

W	Weight of sliding mass
U	Uplift force due to water pressure on the sliding surface
V	Horizontal force due to water pressure in the tension crack

$$F = \frac{c.A + (W.Cos\alpha - U - V.Sin\alpha).Tan\phi}{W.Sin\alpha + V.Cos\alpha} \tag{4.4}$$

where

$$A = (H - Z).\frac{1}{Sin\alpha} \tag{4.5}$$

$$U = \frac{1}{2}\gamma_w.Z_w.\frac{H - Z}{Sin\alpha} \tag{4.6}$$

$$V = \frac{1}{2}\gamma_w.Z_w^2 \tag{4.7}$$

$$W = \frac{1}{2}\gamma.H^2.\left\{\left[1 - \left(\frac{Z}{H}\right)^2\right].Cot\alpha - Cot\theta\right\} \tag{4.8}$$

Model I- Slope without tension crack under dry conditions- This model as shown in Figure 4.5a is the simplest case in which the slope is assumed to be dry and no tension crack to be formed. The factor of safety for this type can be obtained from the general case in which the uplift water force, horizontal water force and height of tension crack are zero. Thus:

$$F = \frac{c.A}{W.Sin\alpha} + Cot\alpha.Tan\phi \tag{4.9}$$

where

$$A = H.\frac{1}{Sin\alpha} \tag{4.10}$$

$$W = \frac{1}{2}\gamma.H^2.(Cot\alpha - Cot\theta) \tag{4.11}$$

Model II- Slope without tension crack under wet conditions- This model is shown in Figure 4.5b and is the same as the previous model except that it is assumed that water is present along the failure surface. The factor of safety for this type can be obtained from the general case except that the value of V and Z are zero. Thus:

$$F = \frac{c.A + (W.Cos\alpha - U).Tan\phi}{W.Sin\alpha} \tag{4.12}$$

where

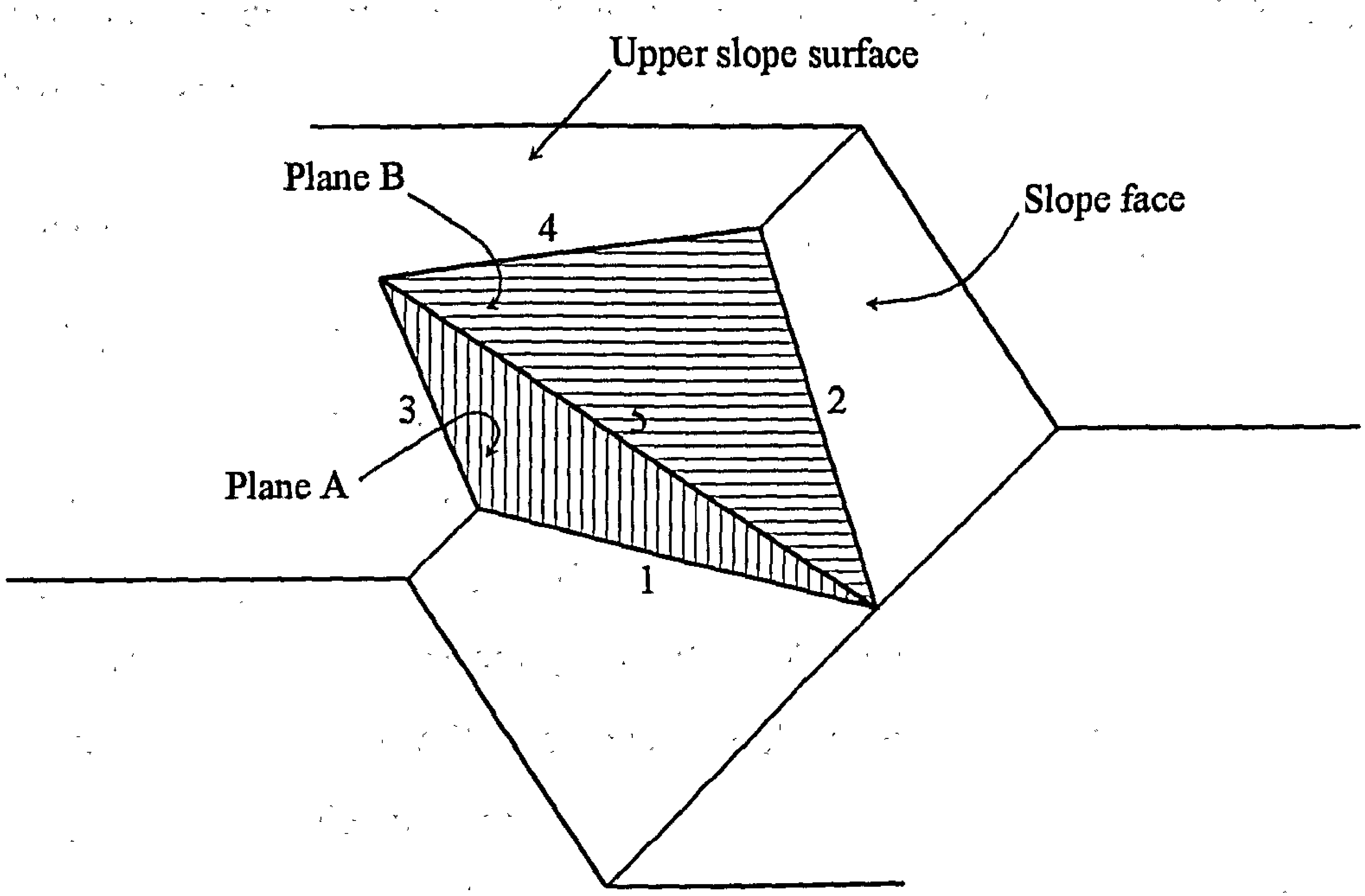
$$U = \frac{1}{4}\gamma_w.H_w^2.\frac{1}{Sin\alpha} \tag{4.13}$$

Model III- Slope with tension crack under wet conditions- This is the general case in which it is assumed that tension crack is formed at the rear of the slope and water presents both on the sliding surface and in the tension crack. The height of the tension crack is determined from the following equation given by Hoek and Bray (1981):

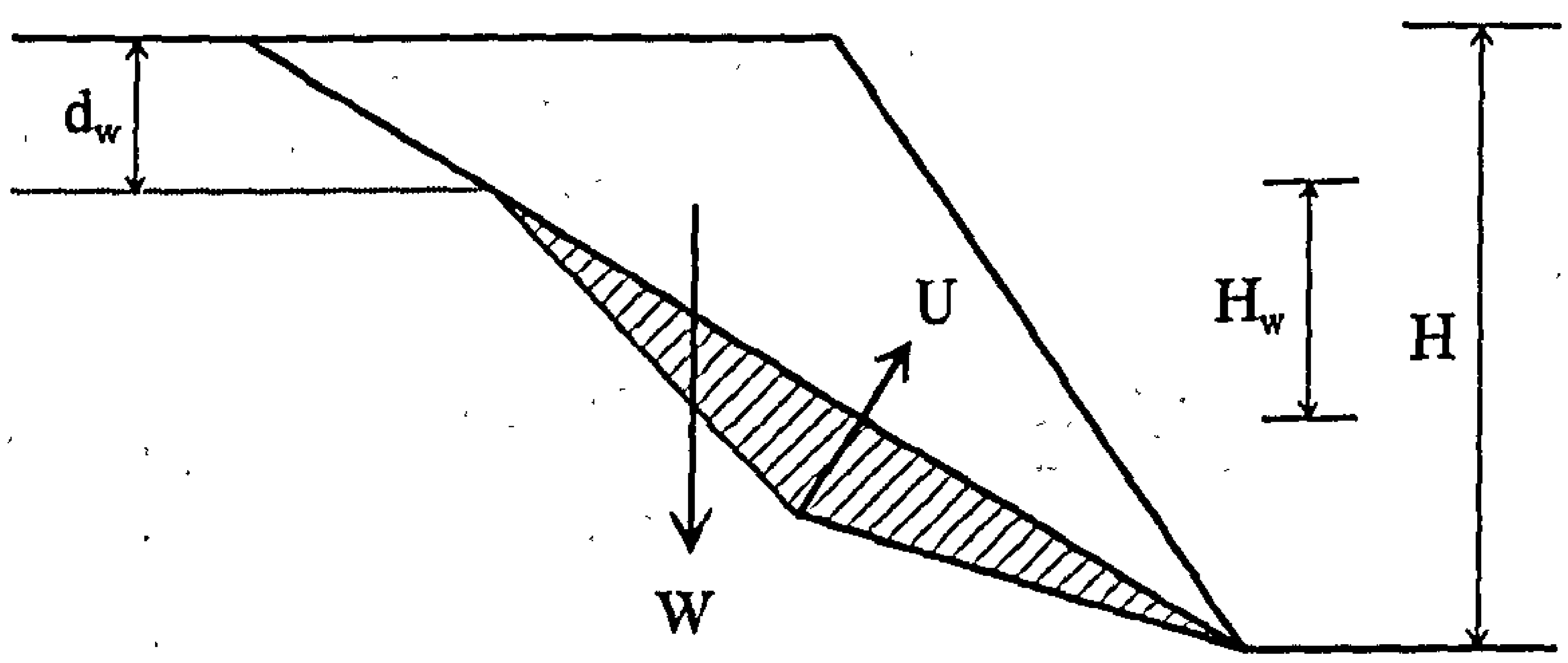
$$Z = H.\left(1 - \sqrt{Cot\theta.Tan\alpha}\right) \tag{4.14}$$

4.3.2.2.2- Factor of safety for wedge failure

Wedge failure is likely to occur when the line of intersection of two discontinuities daylights to the slope face and can not be directly approximated in two dimensions. This failure is rarely involved at large scale unless steeply dipping discontinuities are present in the slope. Various methods have been developed to deal with wedge failure such as the use of stereographical projection, utilising engineering graphics, design chart and analytical approach (Hoek and Bray, 1981). Disturbing and resisting forces involved in wedge failure are more complex than other failure modes. The method adopted here is based on the analytical method using vector analysis developed by Hoek, Bray and Boyd (1973). This method calculates the factor of safety and assumed that sliding of the wedge is kinematically possible along the line of intersection of two planes which daylights on the slope face. This solution deals with wedge failure in terms of dip and dip direction of the planes and slope face and involves the determination of the geometry of the wedge, area of the potential sliding planes and the forces due to water pressure.



a- Geometry of wedge



b- Assumed water pressure in wedge failure

Figure 4.6- Geometry of the wedge and assumed water pressure distribution used in the analysis

Figure 4.6 shows the geometry of the wedge and also the numbering of the lines of intersection of the various planes involved in this problem together with the assumed water pressure used in the analysis. The following gives the factor of safety for this failure which is derived from the analytical method develop by Hoek, Bray and Boyd (1973) on the assumption that the shear strength of the sliding surfaces is defined by the Mohr-Coulomb criterion.

Input data:

φ_A	dip of plane A
α_A	dip direction of plane A
φ_B	dip of plane B
α_B	dip direction of plane B
c_A and c_B	cohesive strength of planes A and B
ϕ_A and ϕ_B	angle of friction on planes A and B
φ_f	dip of slope face
α_f	dip direction of slope face
H	height of wedge
γ	rock density (unit weight)
γ_w	water density (unit weight)
d_w	water table depth

$$F = \frac{c_A.A_A + c_B.B_B + (q.W - U_A).Tan\phi_A + (x.W - U_B).Tan\phi_B}{W.Sin\varphi_s} \quad (4.15)$$

Where

U_A and U_B	are the uplift force due to the water pressure on planes A and B
W	is the weight of the wedge
φ_s	is the plunge or dip of the line of intersection of planes A and B
q and x	are the dimension-less factors which depend on the geometry of the wedge and can be determined from the equations 4.16-4.20

$$m_{na.nb} = Sin\varphi_A.Sin\varphi_B.Cos(\alpha_A - \alpha_B) + Cos\varphi_A.Cos\varphi_B \quad (4.16)$$

$$m_{W.na} = -Cos\varphi_A \tag{4.17}$$

$$m_{W.nb} = -Cos\varphi_B \tag{4.18}$$

$$q = \frac{m_{na.nb}.m_{W.nb} - m_{W.na}}{1 - m_{na.nb}^2} \tag{4.19}$$

$$x = \frac{m_{na.nb}.m_{W.na} - m_{W.nb}}{1 - m_{na.nb}^2} \tag{4.20}$$

In order to use equation 4.15 to find the factor of safety for wedge failure, it is necessary to calculate weight of the wedge and uplift forces acting on both planes A and B when groundwater exists in the slope. The weight of the wedge and the area of the sliding planes are determined from the following equations:

$$AC = H.\frac{Sin\theta_5}{Sin\varphi_1.Sin\theta_{35}} \tag{4.21}$$

$$A_A = \frac{1}{2}.AC^2.\frac{Sin\theta_{13}.Sin\theta_{35}}{Sin\theta_{15}} \tag{4.22}$$

$$A_B = \frac{1}{2}.AC^2.\frac{Sin^2\theta_{13}.Sin\theta_{25}.Sin\theta_{45}}{Sin^2\theta_{15}.Sin\theta_{24}} \tag{4.23}$$

$$K = \sqrt{1 - Cos^2\theta_{34} - Cos^2\theta_{35} - Cos^2\theta_{45} + 2Cos\theta_{34}.Cos\theta_{35}.Cos\theta_{45}} \tag{4.24}$$

$$W = \frac{1}{6}\gamma.K.AC^3.\frac{Sin^2\theta_{13}.Sin^2\theta_{25}}{Sin^2\theta_{15}.Sin^2\theta_{24}} \tag{4.25}$$

Where A_A and A_B are the areas of the sliding planes A and B respectively. θ_{ij} is the angle made by the line of intersection i with j (line 1-5 in Figure 4.6a). It can be determined from the equation 4.26 in which α_i and φ_i are the trend and plunge of line i respectively.

$$\theta_{ij} = ArcCos\left[Cos\varphi_i.Cos\varphi_j.Cos(\alpha_i - \alpha_j) + Sin\varphi_i.Sin\varphi_j\right] \tag{4.26}$$

The trend and plunge of the line of intersection of two planes can be found from equations 4.2 and 4.3 respectively.

The water pressure distribution is shown in the Figure 4.6b in which it is assumed that the wedge is impermeable and water in the slope will be transmitted along both discontinuities A and B. The forces due to water pressure are calculated assuming of a tetrahedral distribution of pressure in which it is assumed that the maximum pressure occurs along the line of intersection 5 and the pressure being zero along lines 1, 2, 3 and 4. The uplift forces U_A and U_B are determined as:

$$H_w = \frac{1}{2}.(H - d_w) \tag{4.27}$$

$$P = \gamma_w.H_w \tag{4.28}$$

$$U_A = \frac{1}{3}.P.S_A \tag{4.29}$$

$$U_B = \frac{1}{3}.P.S_B \tag{4.30}$$

Where S_A and S_B are the area over which the water pressure acts. For the case of a saturated slope S_A and S_B are equal to the area of sliding planes A and B and H_w is equal to the half of the wedge height. S_A and S_B can be determined form the following equations:

$$t = 1 - \frac{d_w.Sin\theta_{15}.Sin\theta_{13}}{AC.Sin^2\theta_{35}} \tag{4.31}$$

$$S_A = t.A_A \tag{4.32}$$

$$S_B = t.A_B \tag{4.33}$$

The program also contains a kinematic check to examine whether wedge failure is possible or not. This test is necessary in the probabilistic approach where the distribution of joint orientations may result in some cases where the line of intersection of two discontinuities does not daylight into the slope face. It is considered in this case that the factor of safety is greater than 1. Two models of wedge failure including the slope under both dry and wet condition are considered in the program.

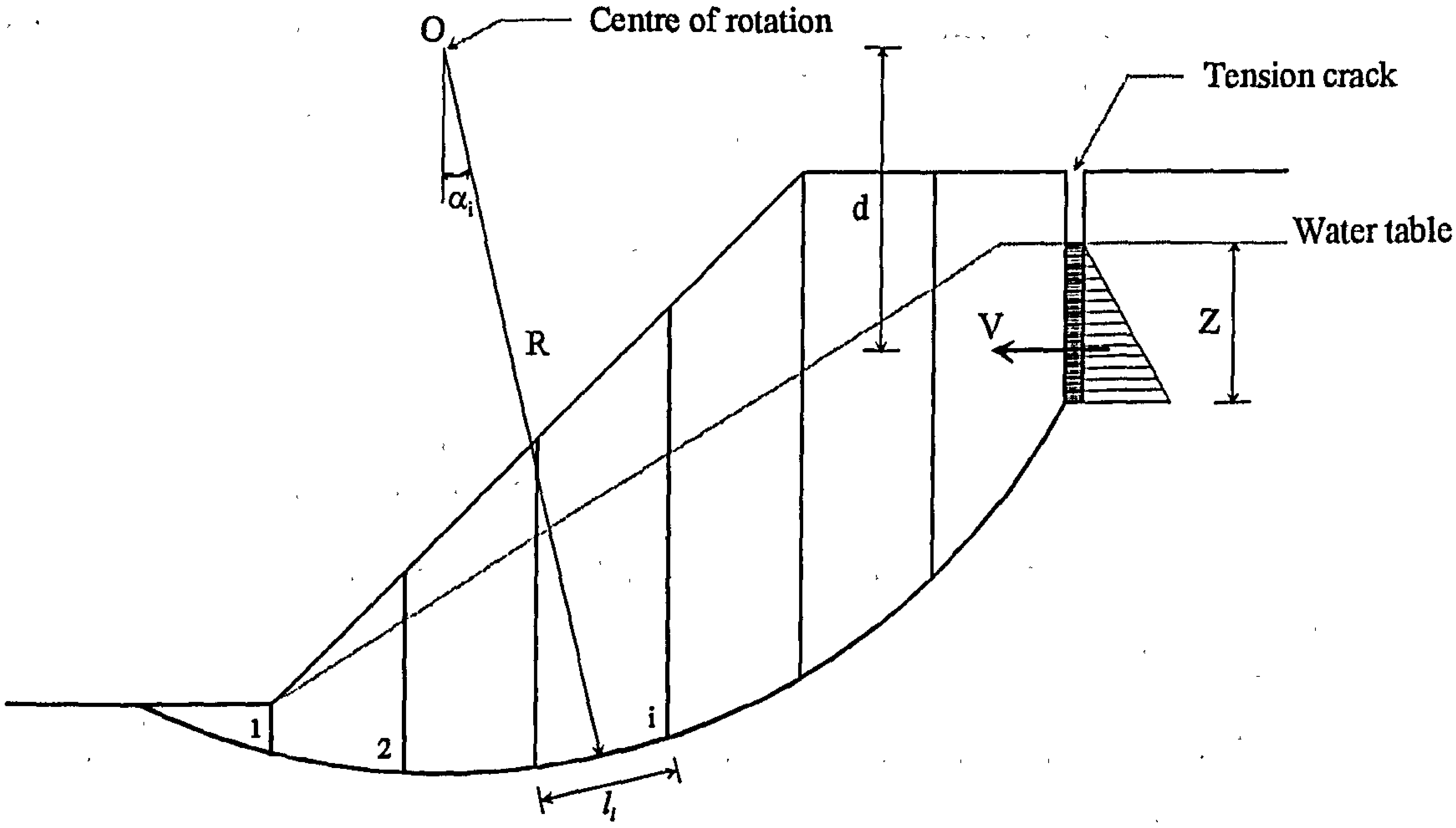
Model I- Slope without tension crack under dry conditions- In this model, it is assumed that the slope is dry and no tension crack is formed. The factor of safety for this type can be obtained from the equation 4.15 in which the uplift water forces U_A and U_B are zero. Thus:

$$F = \frac{c_A.A_A + c_B.B_B + q.W.Tan\phi_A + x.W.Tan\phi_B}{W.Sin\varphi_s} \qquad (4.34)$$

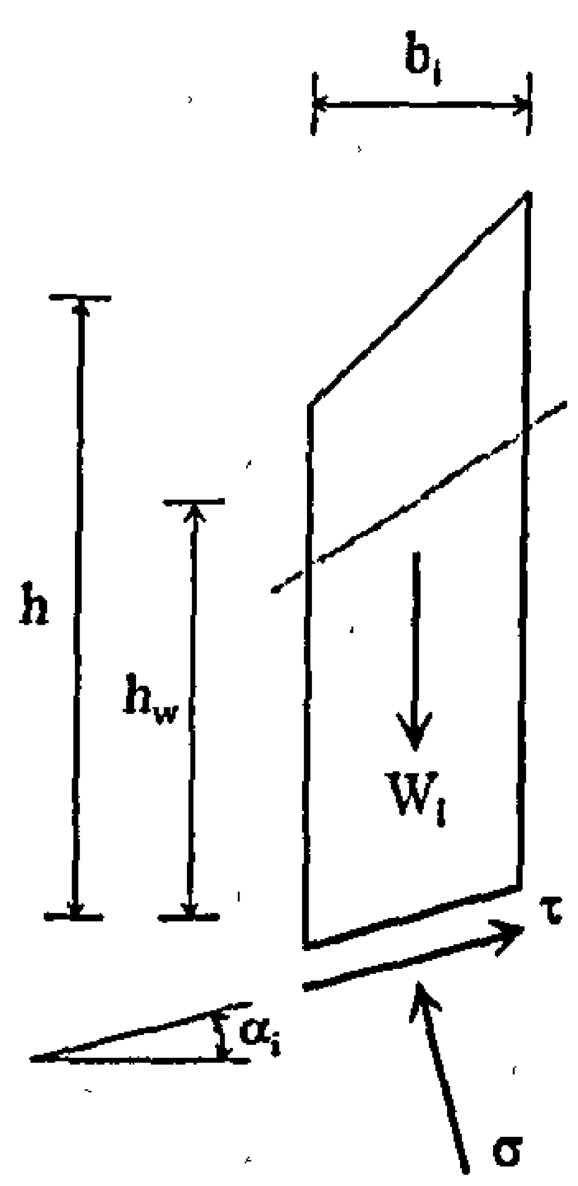
Model II- Slope without tension crack under wet conditions- In this model, it is assumed that water is present along the sliding surfaces. The factor of safety for this type can be obtained from the equation 4.15.

4.3.2.2.3- Factor of safety for circular failure

In the absence of structural instability and where the rock is very weak or heavily fractured, the failure is assumed to occur by sliding of a block of ground on a circular slip surface. A number of methods have been devised to assess this type of failure. Almost all of these techniques use the method of slices in which the mass above the failure surface is divided into a number of vertical slices and the factor of safety is determined by resolving forces acting on each slice and examining the overall moment of equilibrium. The method used for circular failure in which failure can occur through intact rocks or rock masses, corresponds to the Bishop simplified method (Bishop, 1955) as illustrated in Figure 4.7. As can be seen from Figure 4.7 the factor of safety, F, appears on both sides of the equation, therefore it requires iterative analysis. The trial and error approach is used to calculate the factor of safety which involves assuming F on the right side of the equation, then solving for F on the left. If the difference between the assumed F and the computed F is significant, the computed F is used on the right side and the procedure repeated until a satisfactory F is determined. The ordinary method of slices (Fellenius, 1936) is used to estimate an initial value of F (equation 4.35). The rock mass above the failure surface is divided into between 5 and 20 slices.



$$F = \frac{1}{\sum_{i=1}^{n} W_i . Sin\alpha_i + Q} . \sum_{i=1}^{n} \frac{c.b_i + (W_i - U_i.b_i).Tan\phi}{\left(1 + \frac{Tan\alpha_i . Tan\phi}{F}\right).Cos\alpha_i}$$



Where

$W_i = \gamma . h$

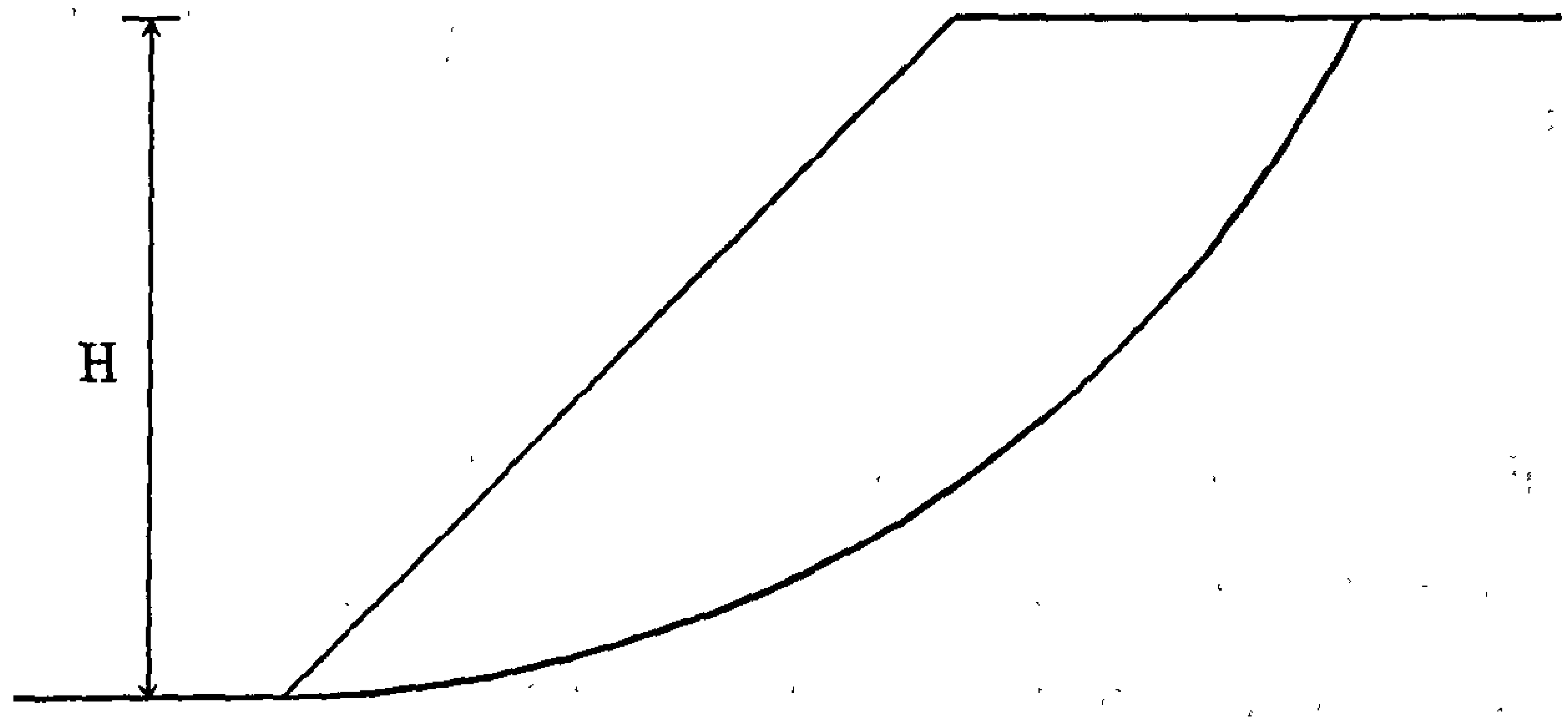
$V = \frac{1}{2} . \gamma_w . Z^2$

$Q = V . \frac{d}{R}$

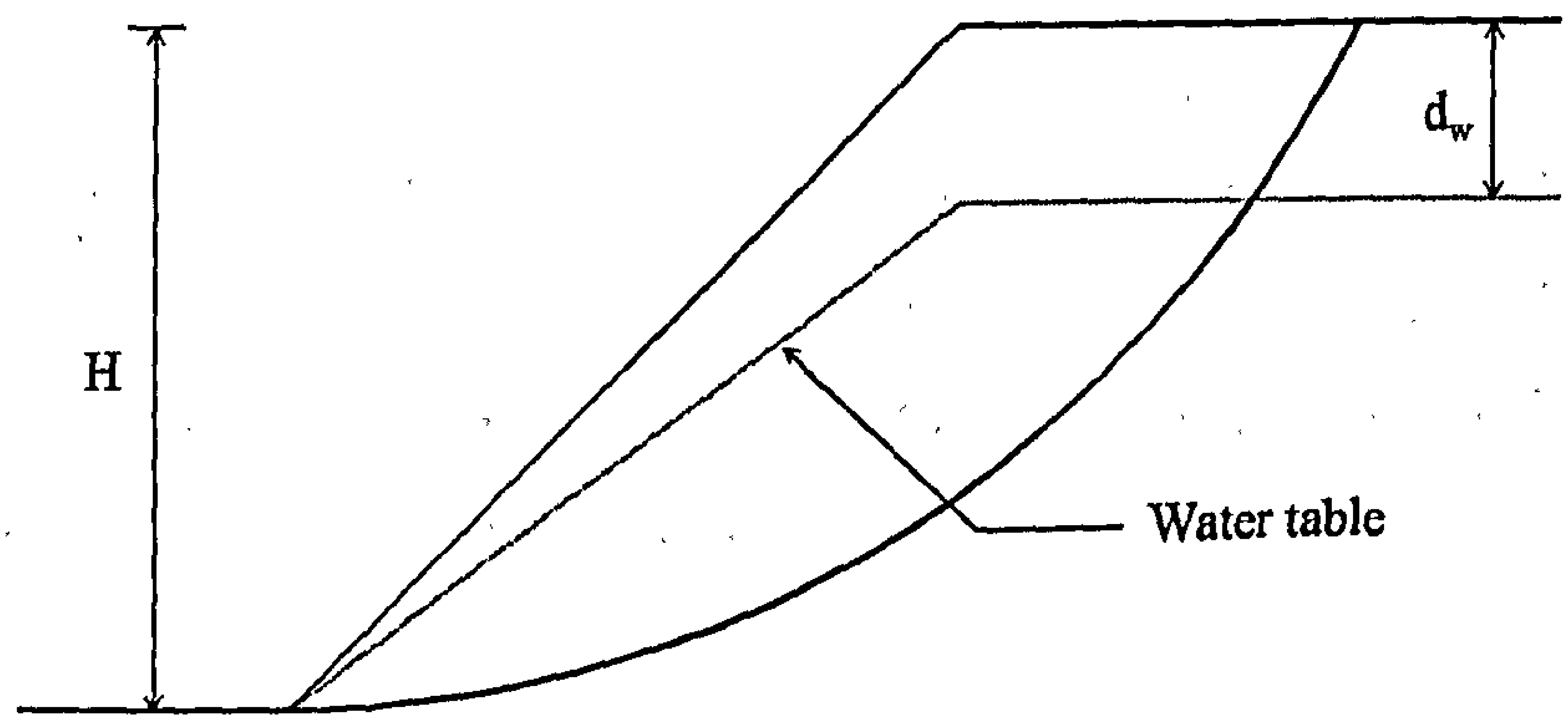
$U_i = \gamma_w . h_w$

n	is the total number of slices
c and ϕ	are the cohesion and friction angle of material
γ and γ_w	are the rock and water density

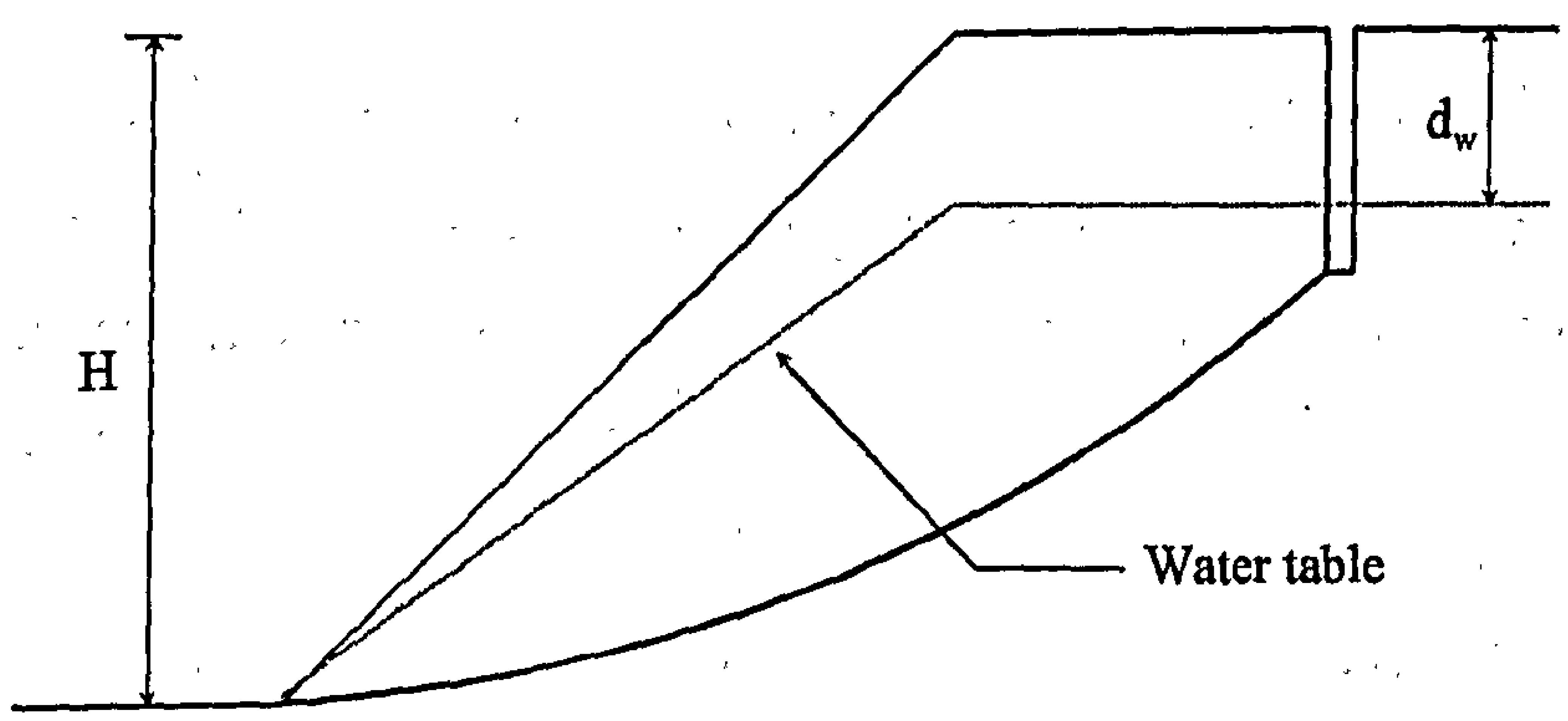
Figure 4.7- Bishop's simplified method of slices for circular failure (After Hoek and Bray, 1981)



a- Model I- Dry slope without tension crack



b- Model II- Wet slope without tension crack



c- Model III- Wet slope with tension crack

Figure 4.8- Types of circular failure

$$F = \frac{\sum_{i=1}^{n}\left[c.l_i + \left(W_i.Cos\alpha_i - U_i.l_i\right).Tan\phi\right]}{\sum_{i=1}^{n} W_i.Sin\alpha_i + Q} \tag{4.35}$$

Three models are used in the analysis. As can be seen from the Figure 4.8 these include the dry slope without tension crack, wet slope without tension crack and wet slope with tension crack. In all these models, it is assumed that the failure surface is occurring through the toe of the slope. This is a valid assumption since failure is likely to occur under the toe of the slope if the angle of friction becomes less than 5°. Such a condition is unlikely to occur in mining slopes (Hoek and Bray, 1981).

Model I- Slope without tension crack under dry conditions- This model as shown in Figure 4.8a is the simplest case in which the slope is assumed to be dry and no tension crack is formed. The factor of safety for this type can be obtained from the general case by substituting zero instead of Q and U.

$$F = \frac{1}{\sum_{i=1}^{n} W_i.Sin\alpha_i}.\sum_{i=1}^{n}\frac{c.b_i + W_i.Tan\phi}{\left(1 + \frac{Tan\alpha_i.Tan\phi}{F}\right).Cos\alpha_i} \tag{4.36}$$

Model II- Slope without tension crack under wet conditions- This model as shown in Figure 4.8b is the same as the previous model in which the effect of ground water is taken into account. The factor of safety for this type can be obtained from the general case except that the value of Q is zero.

Model III- Slope with tension crack under wet conditions- This is the general case as shown in Figure 4.8c in which it is assumed the tension crack is formed to the rear of the crest of the slope and is filled with water below the ground water table. The height of the tension crack is determined by the following formula:

$$h_c = 2.\frac{c}{\gamma}.Tan\left(45+\frac{\phi}{2}\right) \tag{4.37}$$

Where h_c is the critical depth of the tension crack.

The location of the most critical slip surface usually can be found through a rigorous search by a variation of the centre of rotation. For this purpose, the factor of safety is calculated for several points and contours of F value are drawn. These contours are roughly elliptical so that their centre indicates where the centre of the critical circle will be located. In this study a systematic search is used to find the location of the critical circle.

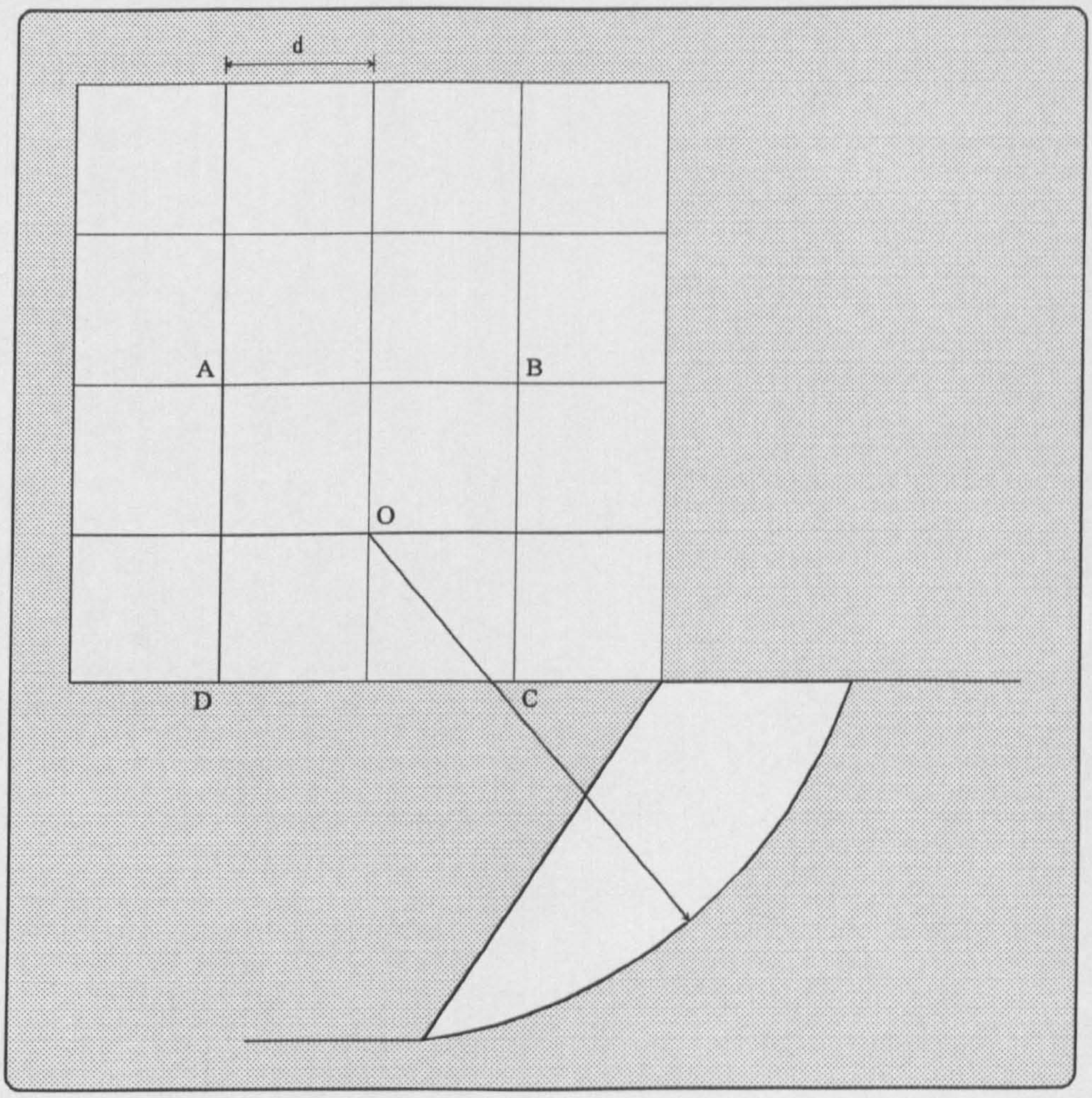


Figure 4.9- Procedure to find the location of the critical circle

In order to find rapidly the location of the most critical circle, as illustrated in Figure 4.9, the area over which the centre of rotation is likely to be found is divided into the 4*4 grid and the factor of safety is determined for each grid point. The distance between each point of the grid is d units. Then, among these 25 F values, the point containing the minimum factor of safety is selected. Let us assume that point O in Figure 4.9 is the centre of the circle which has the minimum factor of safety. Next, the four squares around this point (square ABCD) is divided into the 4*4 grid for which the factor of safety is calculated for each point (Figure 4.10). The distance between each point of this grid is d/2 units. Again the point containing the minimum factor of safety is chosen, let suppose point O' in Figure 4.10. The method is applied to square A'B'C'D' and this process is continued until the distance between points in the grid become a satisfactory value. After a few trials the location of the most critical circle is obtained.

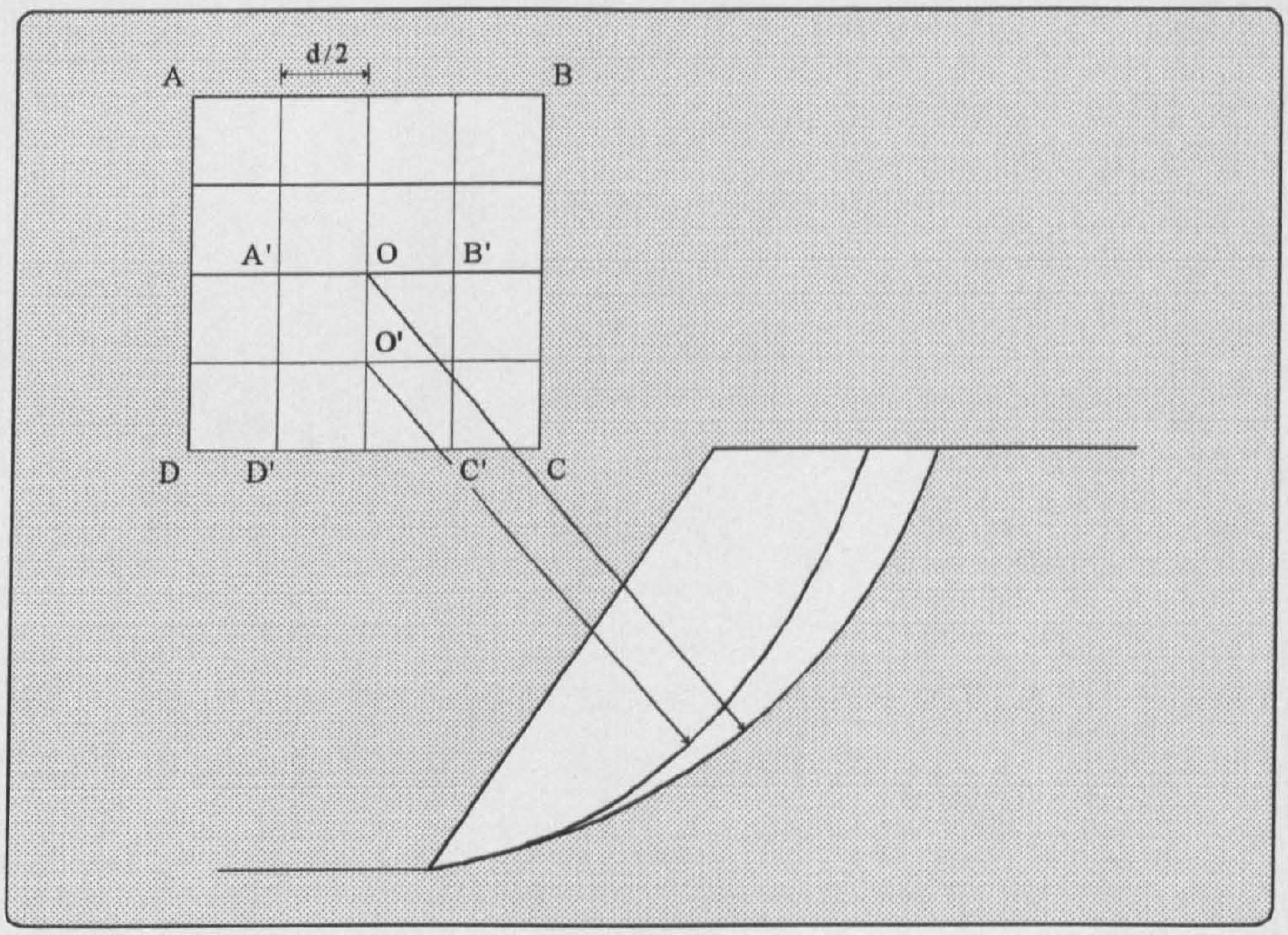


Figure 4.10- Procedure to find the location of the critical circle

Both failure criteria including the Mohr-Coulomb and Hoek-Brown criterion are used to analyse circular failure. If the first criterion is used only the cohesion and friction angle of the material is required. If the Hoek-Brown criterion is used the material constant of intact rock, the geological strength index and uniaxial compressive strength of intact rock are required. In the case of using the Hoek-Brown criterion, the program finds the equivalent values of the Mohr-Coulomb criterion, the friction angle (ϕ) and the cohesive strength (c) from the tangent to the envelope to the principal stress defined by the Hoek-Brown failure criterion (Hoek, Kaiser, and Bawden, 1995).

4.3.3- Probabilistic approach

Some parameters such as cohesion, friction angle and orientation of discontinuities within the rock mass are subject to variation and do not have a single value, but may assume any number of values. These variables are known as random variables. Conventional deterministic methods of limit equilibrium use fixed values to calculate the factor of safety. This approach does not take account of the uncertainty associated with the parameters used in the calculation. Since the input data used to design the slope angle are subject to variation, it is necessary to take account of these uncertainties in the design by using a probabilistic approach. The Monte Carlo simulation is adopted to determine the probability of failure, p_f, which can be defined as the probability of the factor of safety being less than one.

$$p_f = P(F < 1) \tag{4.38}$$

To carry out this analysis, when the potential failure mode is determined for the slopes with dip directions from 0° to 360° in 15° steps, for each failure mode the probability of failure is determined versus slope angles from 20° to 80° in 5° increments. Values are selected at random from the predefined probability distribution of the input data and used to calculate the factor of safety. This process is repeated a number of times and the probability of failure is calculated as:

$$p_f = \frac{\text{number of factors of safety being less than 1}}{\text{total number used in simulation}} \tag{4.39}$$

Figure 4.4 shows the flow chart for calculation of the probability of failure versus slope angles for circular failure in which it is assumed that the shear strength is determined by the use of the Mohr-Coulomb criterion. The probability of failure is determined from a direct count. This method has been adopted as in some cases when analysing wedge failure where the factor of safety obtained has no physical significance. This is the case when values are sampled for which sliding is kinematically impossible. This case is regarded as having a factor of safety greater than one and is theoretically stable.

4.3.3.1- Random number generators

The simulation involved the selection of variables from probability distribution functions of input parameters at random. For this purpose a random number, *r*, generated in the range (0, 1) and then from the cumulative probability distribution, $F(x)$, variable x is selected by the inverse transformation such that $r = F(x)$ as illustrated in Figure 4.11.

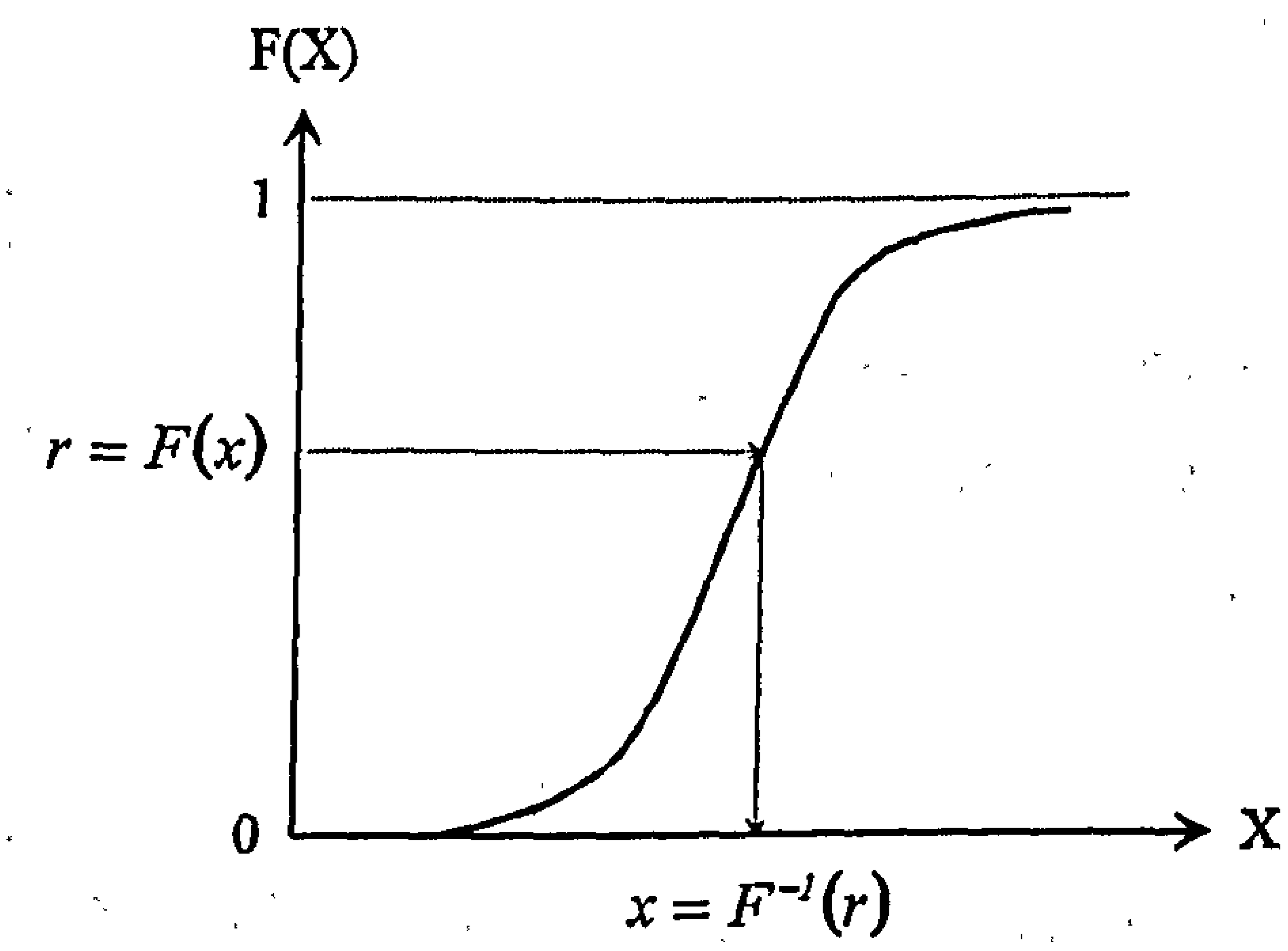


Figure 4.11- To sample a value from the cumulative probability distribution

The most common way to generate a random number is by using the following equation:

$$R_n = \left(a.R_{n-1} + c\right) \bmod m \quad (4.40)$$

Where

R, a and c >= 0

$m > R_0$ $m > a$ $m > c$

This means that the value of $a.R_{n-1} + c$ is divided by m and the remainder is taken as the value of R_n. The initial value of R is called the *seed*, R_0, and *a*, *c* and *m* are integer and constant values. The value of *m* should be chosen to be a large prime number that can be fitted to the computer word size to generate a large sequence of random numbers.

Equation 4.40 generates a sequence of integer numbers in the range *0* to *m-1* and the quantity of R_n/m which is in the range of (0, 1) is taken to sample from the probability distribution function. This equation with the value of $R_0 = 1000 * seed$, $a = 125$, $c = 0$ and $m = 2796203$ is used to generate random numbers in which it is assumed that the seed value is given by the user.

4.3.3.2- Probability distribution

The program uses the five most common probability distributions to define random variables. These are: normal, uniform, triangle, lognormal, reverse lognormal distributions. The following describes the types of probability functions and their inverse transformation functions.

The normal distribution- The normal or Gaussian distribution is one of the most commonly used probability distributions in engineering applications (Figure 4.12). Many random variables conform to this distribution. It can be defined by its *mean* and *standard deviation.* The probability density function for the normal distribution with a *mean*, μ, and *standard deviation*, σ, is given by:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \quad \text{for } -\infty \le x \le \infty \qquad (4.41)$$

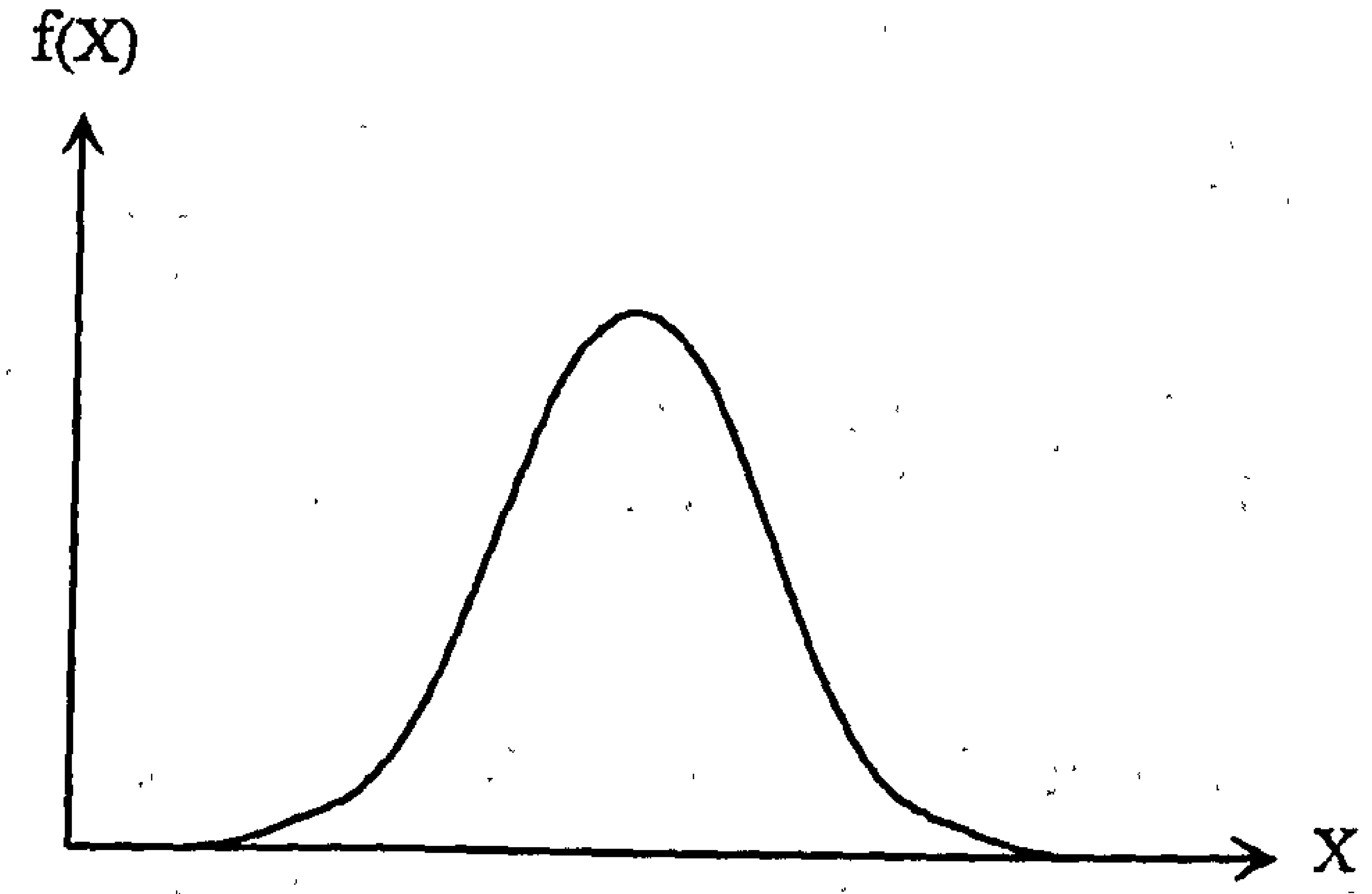


Figure 4.12- A normal distribution

and its cumulative distribution function is:

$$F(x) = \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(t-\mu)^2}{2\sigma^2}} dt \tag{4.42}$$

Equation 4.42 can not be integrated but it can be found by numerical methods. In the Monte Carlo simulation, which involves the inverse transformation, the problem is to find the value of variable x for the random number r, such that:

$$r = F(x) = \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(t-\mu)^2}{2\sigma^2}} dt \tag{4.43}$$

This can be found from the following equations (Schmidt and Taylor, 1970):

if $r \leq 0.5$

$$v = \sqrt{-2\ln(r)} \tag{4.44}$$

$$x = m - \sigma\left(v - \frac{2.515517 + 0.802853v + 0.010328v^2}{1 + 1.432788v + 0.189269v^2 + 0.001308v^3}\right) \tag{4.45}$$

if $r \geq 0.5$

$$v = \sqrt{-2\ln(1-r)} \tag{4.46}$$

$$x = m + \sigma\left(v - \frac{2.515517 + 0.802853v + 0.010328v^2}{1 + 1.432788v + 0.189269v^2 + 0.001308v^3}\right) \tag{4.47}$$

The uniform distribution- This distribution describes a random variable which can take any value between a specified minimum and maximum value with equal probability. The distribution function of this type (Figure 4.13) by the minimum and maximum a and b is:

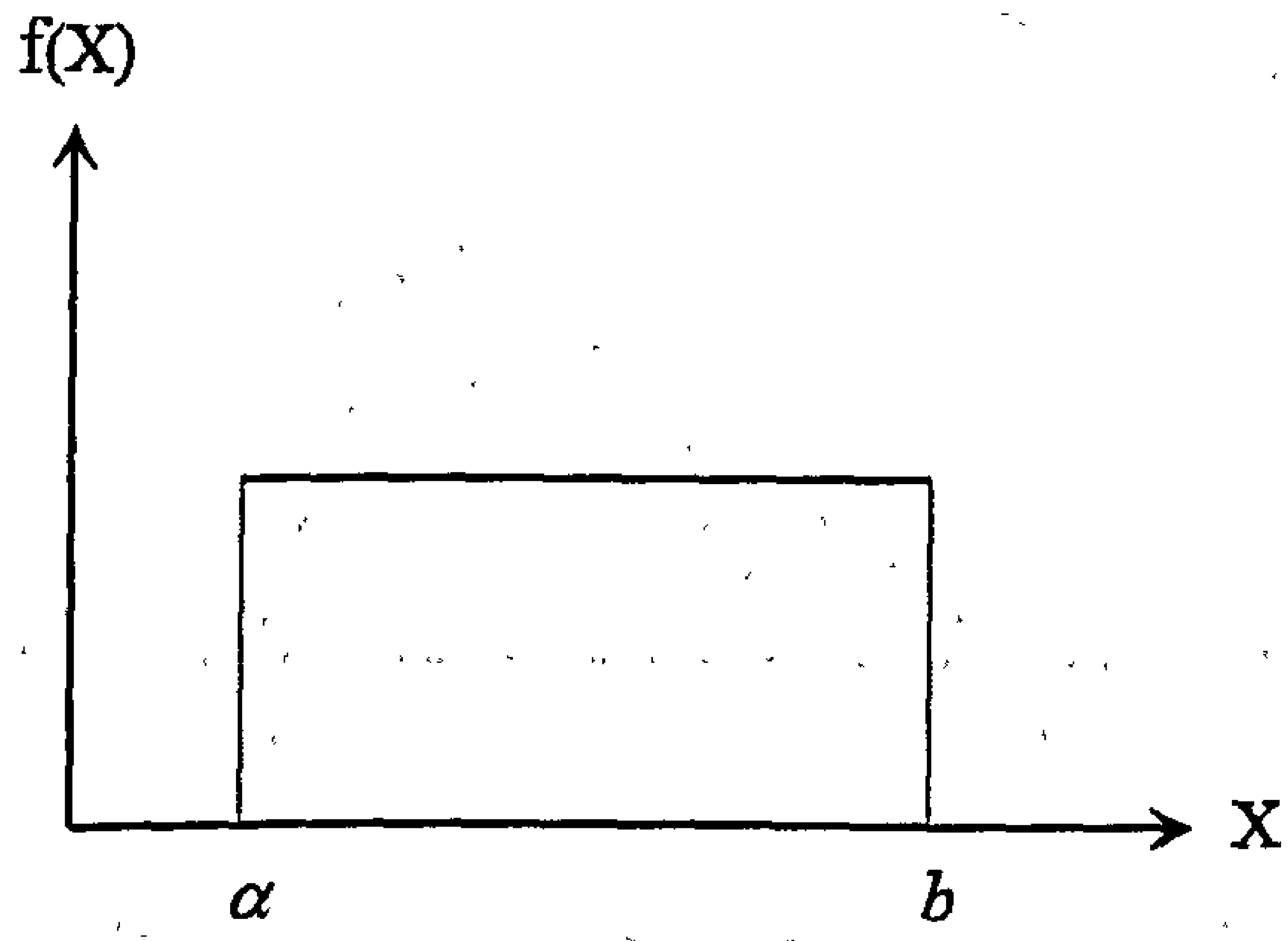


Figure 4.13- A uniform distribution

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & otherwise \end{cases} \tag{4.48}$$

the *mean*, μ, and *standard deviation*, σ, of this distribution are:

$$\mu = \frac{b+a}{2} \qquad \sigma = \sqrt{\frac{(b-a)^2}{12}} \tag{4.49}$$

The inverse transformation for the uniform distribution can be obtained as:

$$r = F(x) = \int_a^x \frac{1}{b-a}dt \quad \Rightarrow \quad x = a + r(b-a) \tag{4.50}$$

The triangular distribution- This distribution is illustrated in Figure 4.14 and requires three parameters to be defined. These are: lower bound (a), upper bound (b) and most likely value (c). Its probability density function can be defined as:

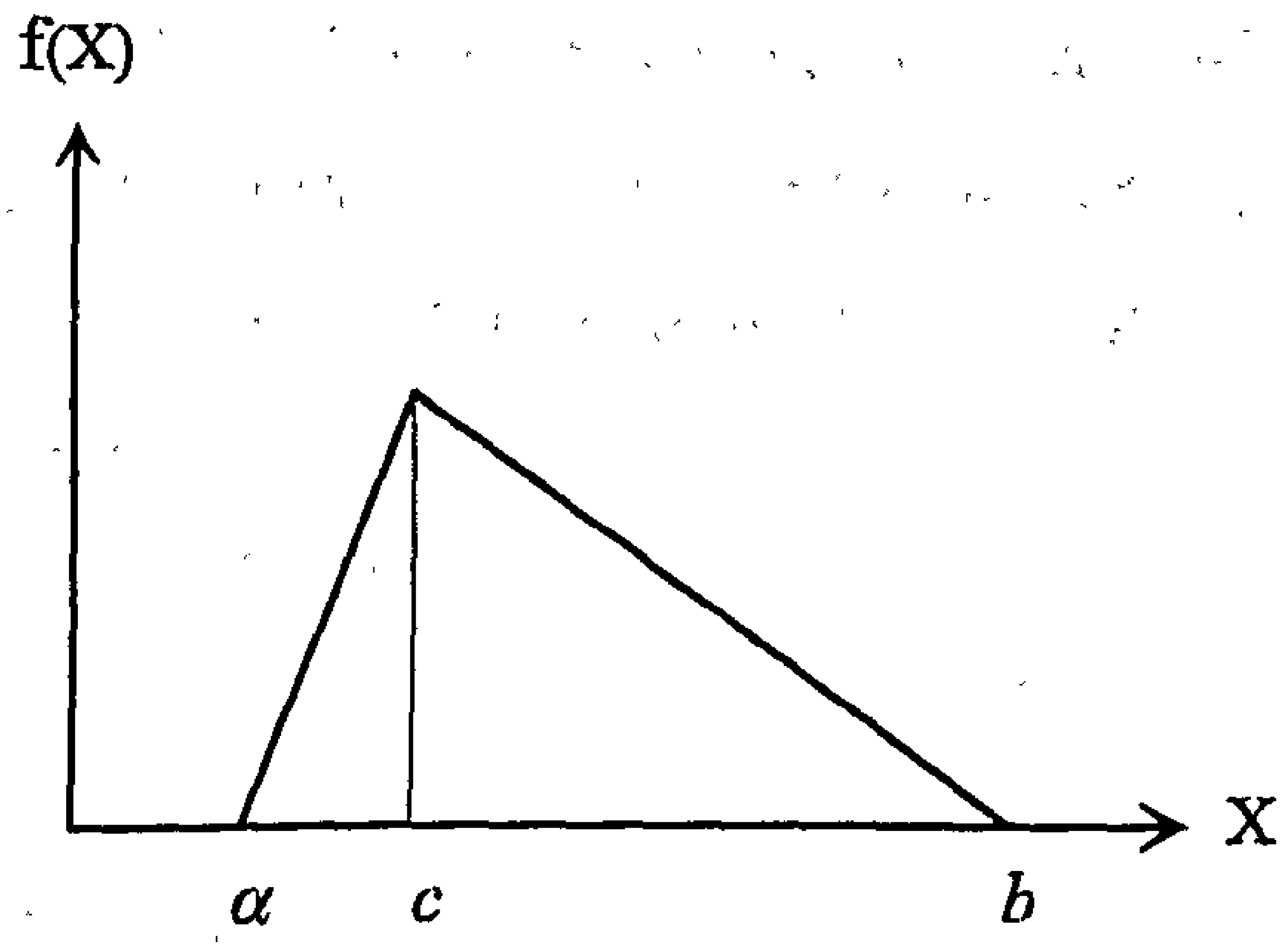


Figure 4.14- A triangular distribution

$$f(x) = \begin{cases} \frac{2}{b-a}\left(\frac{x-a}{c-a}\right) & a \le x \le c \\ \frac{2}{b-a}\left(\frac{b-x}{b-c}\right) & c \le x \le b \end{cases} \tag{4.51}$$

and its *mean*, μ, and *standard deviation*, σ, are:

$$\mu = \frac{1}{3}(a+b+c)$$
$$\sigma = \sqrt{\frac{1}{18}\left(a^2+b^2+c^2-ab-ac-bc\right)} \tag{4.52}$$

The inverse transformation for the triangle distribution can be obtained from the following equation:

$$x = \begin{cases} a + \sqrt{r(b-a)(c-a)} & 0 \le r \le \dfrac{c-a}{b-a} \\ b - \sqrt{(1-r)(b-a)(b-c)} & \dfrac{c-a}{b-a} \le r \le 1 \end{cases} \tag{4.53}$$

The lognormal distribution- A lognormal distribution (Figure 4.15) is one for which the logarithm of the variable is normally distributed. This distribution which is positively skewed can be defined by its mean and standard deviation. Sampling from a lognormal distribution with the *mean*, μ, and *standard deviation*, σ, is the logarithm of a corresponding sample from a normal distribution N(m, s) whose parameters are related by the following equation:

$$\mu = e^{m+\frac{1}{2}s^2}$$
$$\sigma^2 = \mu^2(e^{s^2} - 1) \tag{4.54}$$

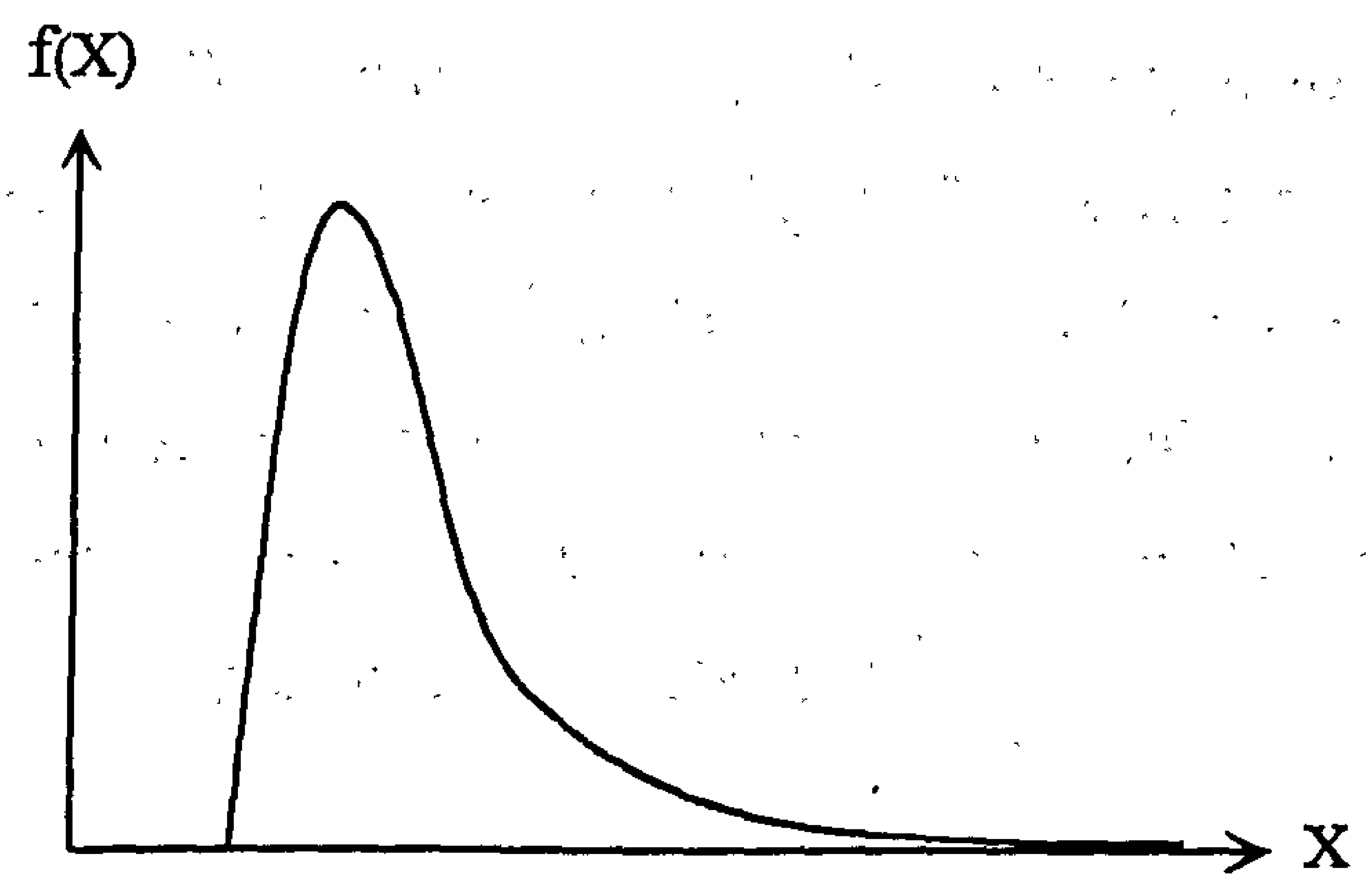


Figure 4.15- A lognormal distribution

The reverse lognormal distribution- This distribution as illustrated in Figure 4.16 is negatively skewed and it can be converted to a lognormal distribution by subtracting variables from a large number. Sampling from this distribution is carried out in the same way as for the lognormal distribution.

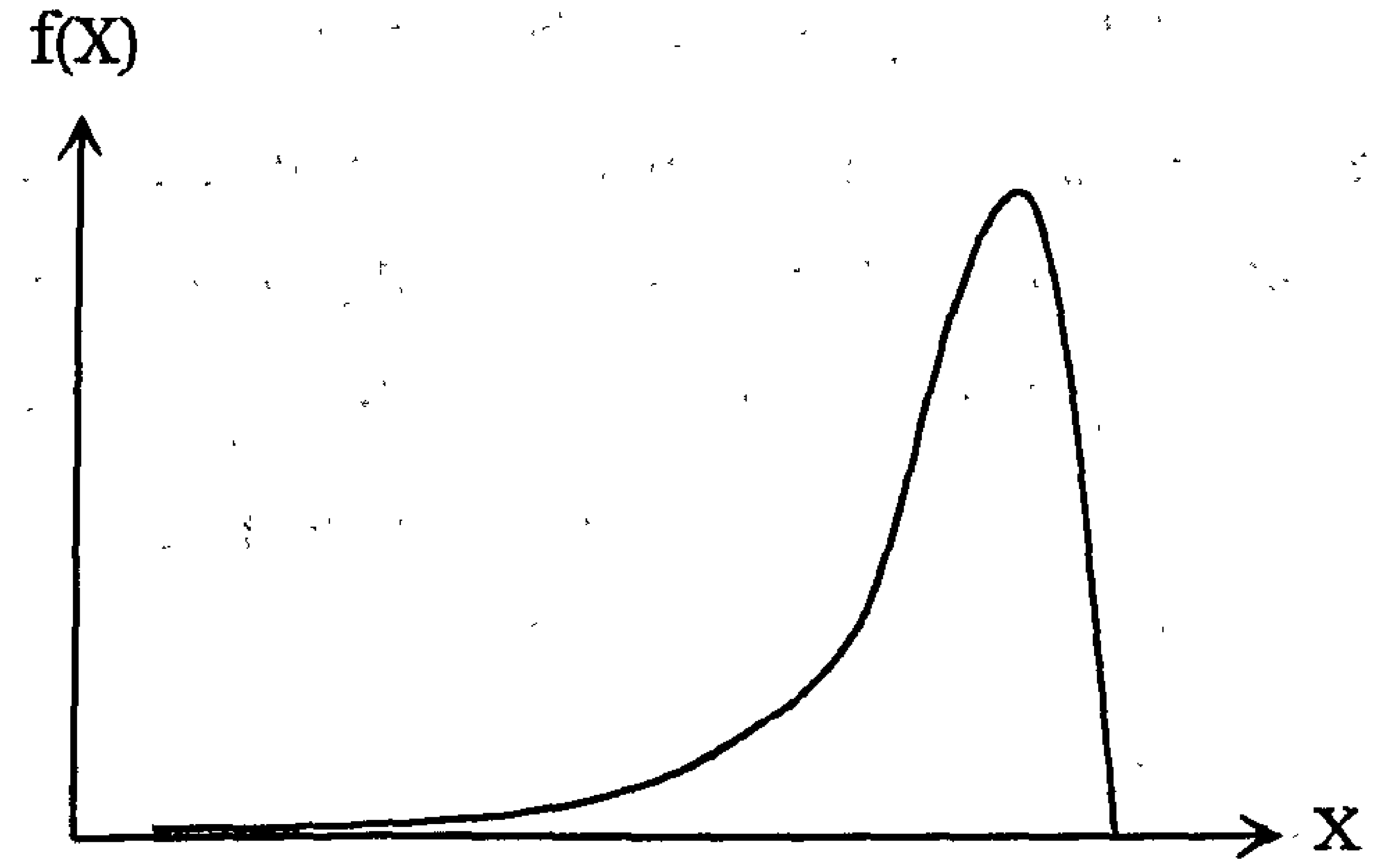


Figure 4.16- A reverse lognormal distribution

4.3.4- Factor of safety or probability of failure versus slope angle

Depending on the input data, the program calculates the factor of safety or the probability of failure versus slope angles from 20° to 80° in 5° steps using the procedures mentioned before for the slope of dip direction from 0° to 360° in 15° increments and puts the result in a text file. A program also has been developed to provide a graphical display of the factor of safety or the probability of failure versus slope angle for each region and for all directions from which slope angles can be obtained by specifying a minimum value for the factor of safety or maximum acceptable of risk of failure. Some examples of the graphical display for the factor of safety or the probability of failure versus slope angle will be given later in this chapter.

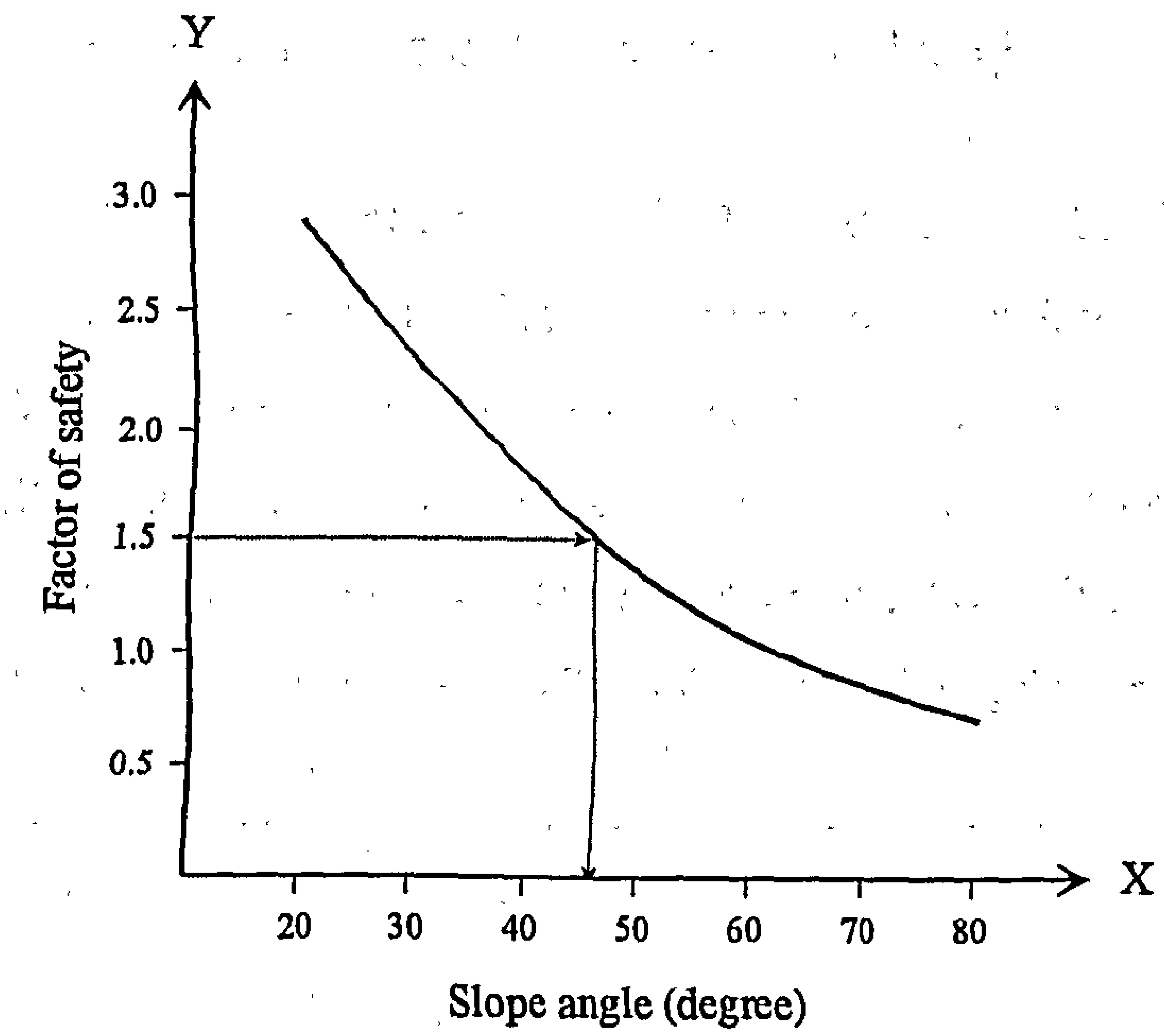
4.3.5- Determination of slope angles

Once the factor of safety or the probability of failure have been determined versus slope angle, a decision must be made to select the slope angles for each direction. For this purpose, depending on the method applied to design slopes, a minimum value for the factor of safety or maximum acceptable risk of failure is required. Hoek and Bray (1981) pointed out that a factor of safety of 1.3, is the minimum acceptable value for a temporary slope in open pit mines, while for a permanent slope such as those carrying the haul road, the factor of safety value should exceed 1.5. In the case of using the deterministic limit equilibrium method, the minimum factor of safety of 1.5 should be used. When probabilistic methods are used to design slopes, Table 2.1 in chapter 2 and Table 4.1 which was suggested by McCracken (1983) can be used as guidelines to choose the maximum acceptable risk of failure.

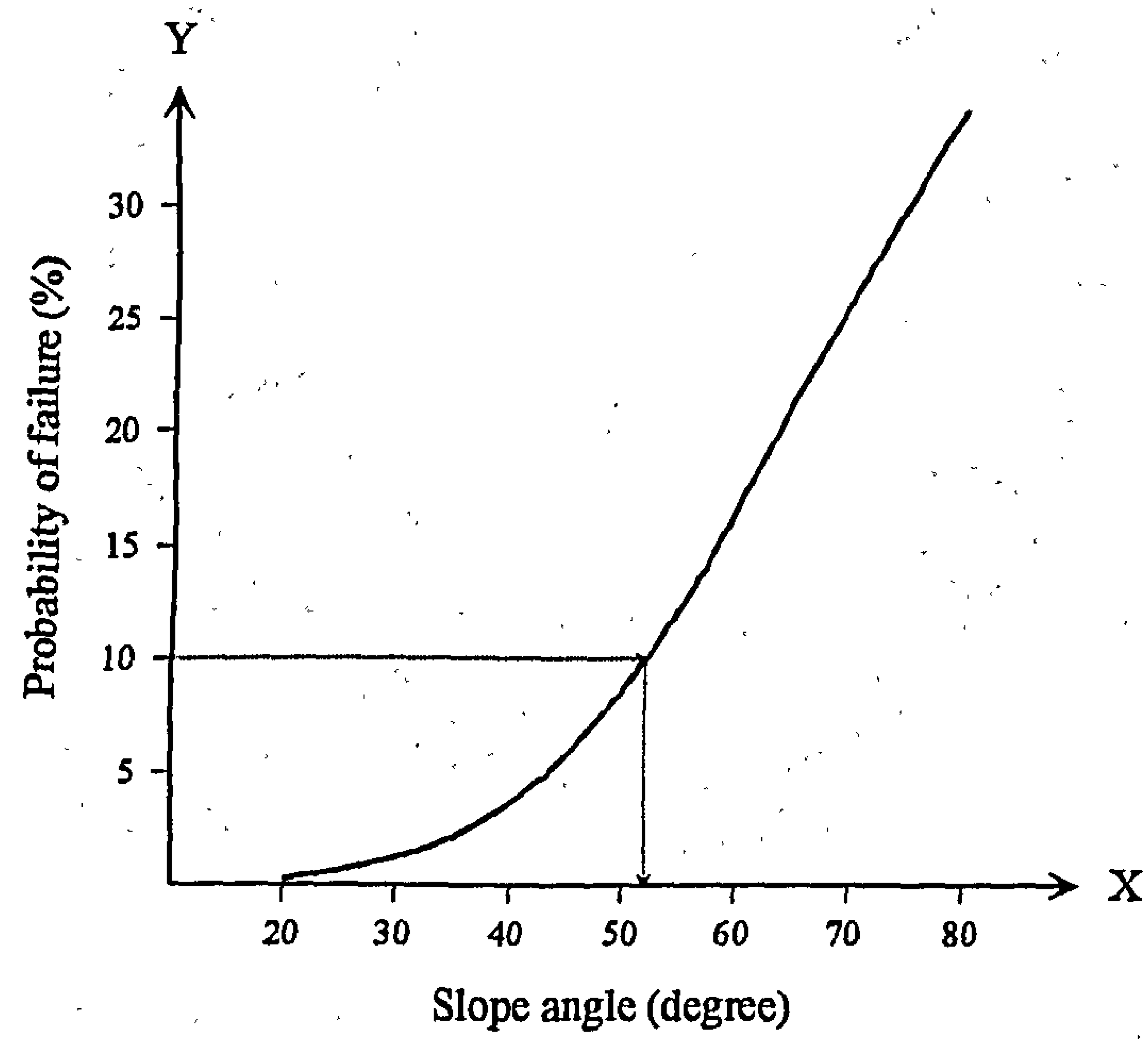
Operation	Stand-up time	Probability of failure (%)
Permanent civil works	Very long	< 0.5
civil works	Long	0.5 - 1.5
Small open pit / quarry final slopes	Medium long	1.5 - 5
Large open pit final slopes	Medium	5 - 15
Small open pit / quarry final slopes	Medium	5 - 15
Large open pit final slopes	Short	15 -30
Temporary benches	Very Short	30 - 50

Table 4.1- Maximum acceptable risk of failure for use in mining and civil works (McCracken, 1983)

When the minimum acceptable value for the factor of safety or maximum acceptable value for the risk of failure and type of slopes have been specified, the program determines the slope angles for all directions by using linear interpolation as illustrated in Figure 4.17. Therefore for each direction one slope angle is obtained. These slopes must be smoothed and four principal slopes must be determined for use in the optimal pit design program.



a- Factor of safety versus slope angle



b- Probability of failure versus slope angle

Figure 4.17- To determine slope angles by specifying the minimum acceptable factor of safety or maximum acceptable risk of failure

4.3.6- Modification of slope angles to four principal angles

When the slope angles with dip directions of 0°, 15°, 30°, . . . and 345° are determined for each region either by kinematic analysis or by the limit equilibrium method , the next step is to modify them to four principal slope angles for the use in the optimal pit design program. Consider Figure 4.18 which shows a plan of how the crest of a pit with a circular floor would appear. Is this acceptable or is modification of the slope required? It is obvious that these slopes have to be modified for use in the optimal pit algorithm.

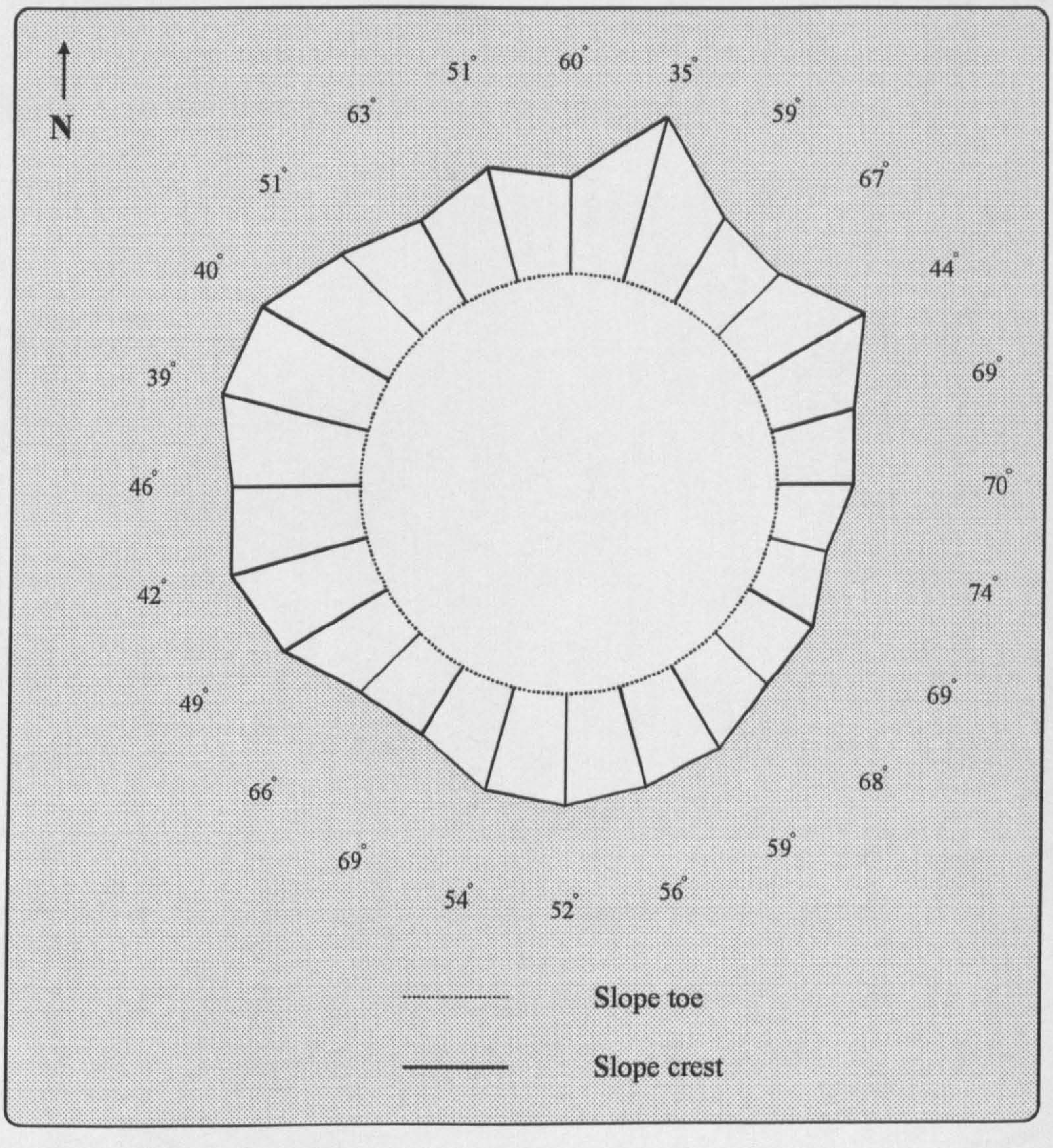


Figure 4.18- An example of a circular floored excavation

One possible way is to select the minimum slope and use it for all directions in order to ensure stability. As an example, for the case shown in Figure 4.18, it is possible to use a 35° slope for all directions. It is obvious that this solution increases the stripping ratio, in other words, it increases the amount of waste to be removed. The other alternative is to use an elliptical excavation. For this purpose, the excavation is divided into four different quadrants each of which contains six slope angles. Each quadrant is considered separately and two alternatives are examined. In the first alternative the minimum slope angle is chosen for all directions in the quadrant as illustrated in Figure 4.19a for which the 35° slope is used. In the second alternative, an elliptical excavation is assumed for each quadrant and the value of the two semi-major and semi-minor axes are found by simple geometry from which two principal slopes are determined.

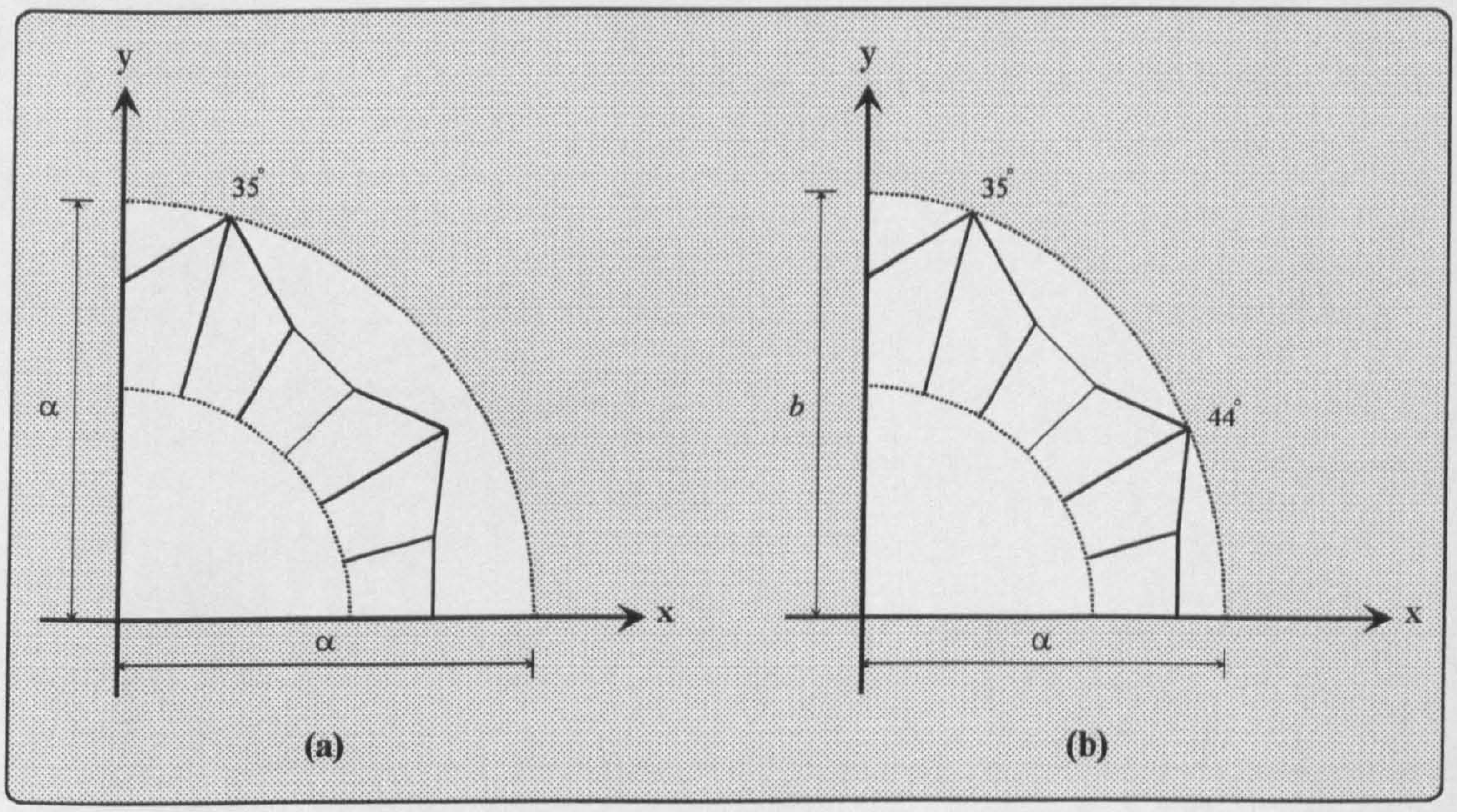


Figure 4.19- To modify slope angles in the first quadrant

To find slopes for the second alternative, the first two minimum slope angles are selected as illustrated in Figure 4.19b. Then by using the ellipse formula passing through the co-ordinates of these slopes for any assumed slope height, two of the four principal slopes are obtained. Consider the first quadrant shown in Figure 4.19b, the co-ordinates of the two points corresponding to the least slopes can be calculated from the following equations:

$$d = \frac{h}{Tan(\alpha)}$$
$$x = d.Sin(\theta) \tag{4.55}$$
$$y = d.Cos(\theta)$$

where

h	is the slope height
x and y	are the co-ordinates
θ	is the dip direction

If it is assumed that the co-ordinates of the two mentioned points are x_1, y_1 and x_2, y_2, by using the ellipse formula we have:

$$\begin{aligned} Ax_1^2 + By_1^2 &= 1 \\ Ax_2^2 + By_2^2 &= 1 \end{aligned} \tag{4.56}$$

where

$$\begin{aligned} a &= \frac{1}{\sqrt{A}} \\ b &= \frac{1}{\sqrt{B}} \end{aligned} \tag{4.57}$$

a and b	are the semi-major and semi-minor axes respectively

By solving the above equation A and B can be obtained as:

$$\begin{aligned} A &= \frac{y_2^2 - y_1^2}{x_1^2 y_2^2 - x_2^2 y_1^2} \\ B &= \frac{x_2^2 - x_1^2}{x_2^2 y_1^2 - x_1^2 y_2^2} \end{aligned} \tag{4.58}$$

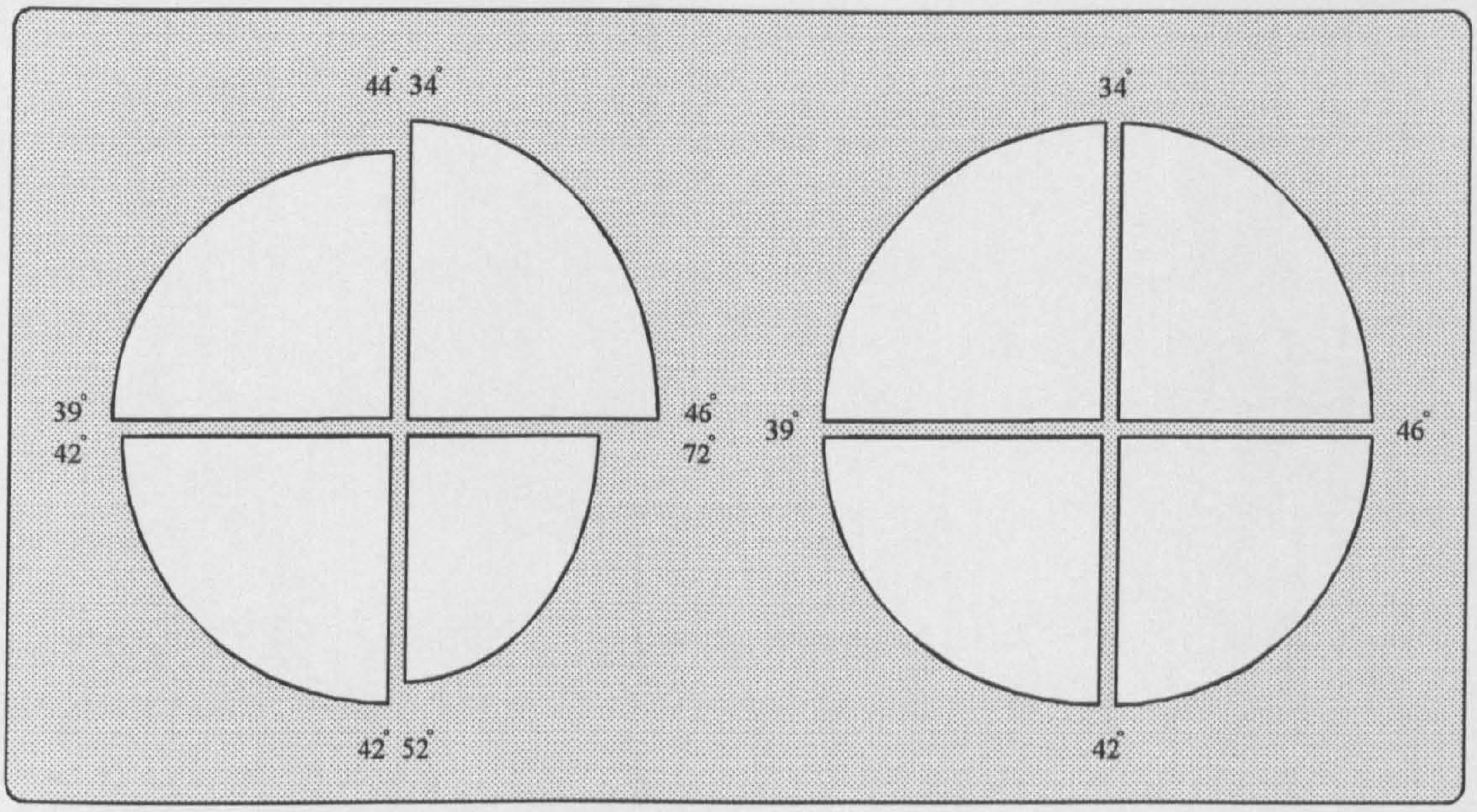


Figure 4.20- To modify slope angles to the four principal slopes

Then the two principal slopes for the first quadrant are:

$$\begin{aligned} dip1 &= ArcTan\left(d.\sqrt{B}\right) \\ dip2 &= ArcTan\left(d.\sqrt{A}\right) \end{aligned} \qquad (4.59)$$

Between these two alternatives the one with least area is selected which decreases the stripping ratio. The above procedure is carried out for all the quadrants. Consequently, two slope angles are obtained for each principal direction from which the lower one is selected as the principal slope. Figure 4.20 shows the result of modification of the slopes for the example illustrated in Figure 4.18.

4.4- Data required

The required input data depends on the method applied to design slopes. If the steepest safe angle is used, only the orientation and properties of the major discontinuities within each region or domain sector including dip, dip direction and angle of friction are required. In the case of using the limit equilibrium method either deterministic or probabilistic, for each region the orientation and properties of major discontinuities (dip, dip direction, cohesion and friction angle), rock mass properties including either cohesion and friction angle or geological strength index, uniaxial compressive strength and material constant of intact rock, rock mass density and water table depth are required. All these values can be specified in terms of either fixed value or as a random variable with corresponding density function.

One of the most important variables in slope design is the slope height. This is calculated by the program and is set to the maximum depth of the region or the maximum depth of the pit in that region. If the optimum pit outline has already been determined the maximum depth of the pit is selected, otherwise the maximum depth of the region is used. When the optimum pit outline is obtained, the program shows the maximum depth of the pit in each region together with the slope height used in the calculation. If the difference between these is significant, the user can change the slope height and design slopes until it becomes a satisfactory value.

4.5- Example 1

In order to show how the algorithm works, two examples are examined. The first example is applied to the example of variable slope angles introduced in section 3.5.1.1 (chapter 3) in which it is assumed that only one region is specified to define the mining slopes. It is also assumed that there are three major discontinuity sets in the region. The geotechnical information required for the design of slopes is given in Table 4.2. Both methods are used to determine four principal slopes for this example. Figure 4.21 shows the result of kinematic analysis which indicates that the mode of failures for different directions includes circular failure and plane failure along joint sets 1, 2 and 3. Also the results of the limit equilibrium method in terms of calculation of the probability of failure versus slope angle are illustrated in Figure 4.22, 4.23, 4.24 and 4.25 for the slopes with dip directions of 0° to 360° in 15° steps under dry conditions and saturated slopes with and without a tension crack. Slope angles can be found from these graphs with regard to the maximum acceptable value for the probability of failure. Table 4.3 shows the four principal slopes obtained by the steepest safe angle method together with the limit equilibrium approach for which the maximum of 10% probability of failure or 90% reliability is used to select slope angles.

The four principal slope angles obtained by the probabilistic method under dry slope conditions is used to determine the optimum pit outline which is illustrated in Figure 4.26. After determination of this, the program shows the maximum depth of the pit in the region which is the same as the slope height used in the calculation. Therefore it is not necessary to design the slopes again.

		Probability distribution	Mean	Standard deviation	Lower bound	Upper bound
Joint set 1	Dip (degree)	Normal	27	1.5	-	-
	Dip direction (degree)	Normal	80	2.5	-	-
	Cohesion (kPa)	Normal	75	2.4	-	-
	Friction angle (degree)	Normal	23	1.2	-	-
Joint set 2	Dip (degree)	Normal	32	2.0	-	-
	Dip direction (degree)	Normal	165	4.0	-	-
	Cohesion (kPa)	Normal	87	2.7	-	-
	Friction angle (degree)	Normal	25	1.5	-	-
Joint set 3	Dip (degree)	Normal	31	3.0	-	-
	Dip direction (degree)	Normal	296	2.8	-	-
	Cohesion (kPa)	Normal	72	3.5	-	-
	Friction angle (degree)	Normal	22	1.0	-	-
Rock mass	Density (t/cubic m)	Normal	2.7	0.5	-	-
	Geological index ,GSI	Normal	30	3.2	-	-
	Compressive strength (MPa)	Normal	32	4.1		
	Material constant, m_i	Normal	9	2.5	-	-
	Water table depth (m)	Uniform	-	-	0	20

Table 4.2- The geotechnical information for example 1

Figure 4.21 - Results from kinematic analysis - Example1

Figure 4.22 - Results of slope angle design - Example1

Figure 4.23 - Results of slope angle design - Example1

Figure 4.24 - Results of slope angle design - Example1

Figure 4.25 - Results of slope angle design - Example1

Figure 4.26 - Pit limit without pit bottom smoothing - Example1

Principal slope angles	Steepest safe angle	Limit equilibrium		
		Dry slope	Wet slope with tension crack	Wet slope without tension crack
North face angle	27°	41°	23°	39°
East face angle	27°	49°	23°	47°
South face angle	32°	49°	27°	45°
West face angle	31°	41°	25°	39°

Table 4.3- The four principal slope angles for example 1

4.6- Example 2

The second example is applied to the example of multiple variable slope angles introduced in section 3.5.2.1 (chapter 3) in which the deposit is divided into four domain sectors or regions as illustrated in Figure 3.36. It is assumed that region 1, 2 and 3 contain 2, 1 and 3 set of discontinuities respectively. Region 4 does not have any through going discontinuities and it is assumed that the slopes are determined by use of circular failure analysis. The geotechnical information for each region is given in tables 4.4 to 4.7. Again the results of kinematic analysis and some of the output from the program for the probability of failure or factor of safety versus slope angle are illustrated in Figures 4.27 to 4.31. The four principal slope angles obtained for the minimum factor of safety 1.5 and maximum probability of failure 10% are given in Table 4.8.

Again the four principal slope angles obtained by the program under dry conditions are used to determine the optimum pit outline which is illustrated in Figure 4.32. After determining this, the program shows the maximum depth of the pit in each region which is the same as the slope height used in the calculation for that region. Therefore it is not necessary to design the slopes again.

		Probability distribution	Mean	Standard deviation	Lower bound	Upper bound
Joint set 1	Dip (degree)	Normal	35	-	-	-
	Dip direction (degree)	Normal	302	-	-	-
	Cohesion (kPa)	Normal	108	-	-	-
	Friction angle (degree)	Normal	20	-	-	-
Joint set 2	Dip (degree)	Normal	29	-	-	-
	Dip direction (degree)	Normal	90	-	-	-
	Cohesion (kPa)	Normal	108	-	-	-
	Friction angle (degree)	Normal	20	-	-	-
Rock mass	Density (t/cubic m)	Normal	2.6	-	-	-
	Cohesion (kPa)	Normal	180	6.5	-	-
	Friction angle (degree)	Normal	30	3.2	-	-
	Water table depth (m)	Normal	20	-	-	-

Table 4.4- The geotechnical information for example 2 - region 1

		Probability distribution	Mean	Standard deviation	Lower bound	Upper bound
Joint set 1	Dip (degree)	Normal	27	2.6	-	-
	Dip direction (degree)	Normal	35	3.4	-	-
	Cohesion (kPa)	Normal	95	4.5	-	-
	Friction angle (degree)	Normal	18	1.5	-	-
Rock mass	Density (t/cubic m)	Normal	2.7	-	-	-
	Cohesion (kPa)	Normal	180	-	-	-
	Friction angle (degree)	Normal	35	-	-	-
	Water table depth (m)	Normal	20	-	-	-

Table 4.5- The geotechnical information for example 2 - region 2

		Probability distribution	Mean	Standard deviation	Lower bound	Upper bound
Joint set 1	Dip (degree)	Normal	34	-	-	-
	Dip direction (degree)	Normal	60	-	-	-
	Cohesion (kPa)	Normal	78	-	-	-
	Friction angle (degree)	Normal	25	-	-	-
Joint set 2	Dip (degree)	Normal	50	-	-	-
	Dip direction (degree)	Normal	130	-	-	-
	Cohesion (kPa)	Normal	78	-	-	-
	Friction angle (degree)	Normal	25	-	-	-
Joint set 3	Dip (degree)	Normal	47	-	-	-
	Dip direction (degree)	Normal	250	-	-	-
	Cohesion (kPa)	Normal	78	-	-	-
	Friction angle (degree)	Normal	25	-	-	-
Rock mass	Density (t/cubic m)	Normal	2.9	-	-	-
	Cohesion (kPa)	Normal	210	-	-	-
	Friction angle (degree)	Normal	33	-	-	-
	Water table depth (m)	Normal	20	-	-	-

Table 4.6- The geotechnical information for example 2 - region 3

		Probability distribution	Mean	Standard deviation	Lower bound	Upper bound
Rock mass	Density (t/cubic m)	Normal	2.8	0.7	-	-
	Cohesion (kPa)	Normal	182	6.2	-	-
	Friction angle (degree)	Normal	35	3.5	-	-
	Water table depth (m)	Normal	20	-	-	-

Table 4.7- The geotechnical information for example 2 - region 4

Figure 4.27 - Results from kinematic analysis - Example 2

Figure 4.28 - Results of slope angle design (region 1) - Example 2

Figure 4.29 - Results of slope angle design (region 2) - Example 2

Figure 4.30 - Results of slope angle design (region 3) - Example 2

Figure 4.31 - Results of slope angle design (region 4) - Example 2

Figure 4.32 - Pit limit without pit bottom smoothing - Example 2

Region	Principal slope angles	Limit equilibrium		
		Dry slope	Wet slope with tension crack	Wet slope without tension crack
1	North face angle	36°	32°	35°
	East face angle	36°	32°	35°
	South face angle	36°	32°	35°
	West face angle	40°	36°	39°
2	North face angle	41°	31°	35°
	East face angle	41°	31°	35°
	South face angle	46°	35°	35°
	West face angle	46°	35°	35°
3	North face angle	39°	35°	35°
	East face angle	39°	38°	38°
	South face angle	55°	44°	44°
	West face angle	45°	35°	35°
4	North face angle	58°	46°	46°
	East face angle	58°	46°	46°
	South face angle	58°	46°	46°
	West face angle	58°	46°	46°

Table 4.8- The four principal slope angles for example 2

4.7- Conclusion

The optimal pit design program with variable slope angles requires an estimate of the safe slope angles to determine the optimum pit outline. Two methods, the steepest safe angle and the limit equilibrium method in terms of the calculation of either the factor of safety or the probability of failure were incorporated into the optimal pit design software to determine the slope angles. To perform both methods, first the orebody must be divided into domain sectors or regions based on the results of the field investigation each of which is considered separately. The first approach determines the steepest safe angle with regard to structural instability including plane and wedge failures without taking into account cohesion, density, ground water conditions and non structural failure. This approach can be used when only a little information is available during design such as at the feasibility stage. If enough information is available the second approach can be used. The program calculates the factor of safety or the probability of failure depending on the input data. If they are defined as fixed values the factor of safety is calculated otherwise the probability of failure is determined. The probabilistic approach has the advantage over the deterministic method as it takes into account the uncertainty inherent in the input data. Both approaches require engineering judgment to select the slope angles from the graph provided by the program. As demonstrated by the two examples with corresponding outputs, the program works well and is able to generate true optimum ultimate pit limits with variable slopes.

It should be noted that the program analyses potential failure modes which can be determined kinematically. If the geology is complex such that the potential failure modes may involve a combination of several mechanisms and when the geometry of the slopes are well known, slope angles must be determined by other sophisticated method such as numerical modelling. This is considered to be an area for further study.

CHAPTER 5

The Software

5.1- Introduction

All the methods described in the previous chapters - the algorithm for optimal open pit design with variable slope angles, the procedure for creating an orebody revenue block model, pit bottom smoothing and the method of designing four principal slope angles - have been coded into an interactive Windows software package "**PITWIN32**". This can be implemented under a 32-bit Windows operating system such as Windows 95, Windows NT or Windows 98. The software was written entirely in C++ code using Borland Object Window Library (OWL) which consists of a number of classes for use in Windows programming. The software has been tested on trial data and real data sets. This chapter describes the structure and operation of the software. There is also a brief description of the programs for numerical and graphical displays of the input data and the results of optimisation.

5.2- System requirements

The software is a 32-bit Windows program that requires the following minimum configuration for implementation:

- An IBM compatible personal computer with an 80486, or higher, processor (Pentium is recommend).

- 16 megabytes of random access memory (32 megabytes is recommend for a deposit with a large number of blocks).
- A hard disk with sufficient free space.
- SVGA, or compatible, colour display.
- A Microsoft Mouse or other compatible pointing device.
- Microsoft Windows 95, Windows NT or Windows 98 operating system.

5.3- Structure of the software

The software is written entirely in C++ using the Borland Object Window Library (OWL) that consists of a group of classes used to represent the Windows structure. It is a 32-bit Windows application and comprises the following programs for each of which a brief description is given below:

5.3.1- Program PITWIN32.CPP

This is the main program and it contains two main classes: a class to manage the window and a class to manage the application. The second class provides both graphical and numerical presentations from which other programs can be executed. This class also contains the two following functions for determination of the optimum pit limit.

5.3.1.1- Function variable _ pit

This function finds the optimum open pit using the Lerchs-Grossmann algorithm with variable slope angles when the definition of mining slopes is limited to one domain sector or region.

5.3.1.2- Function multiple _ pit

This function determines the optimum open pit using the Lerchs-Grossmann algorithm with multiple variable slope angles when mining slopes are defined for more than one domain sector or region.

Both functions use block and revenue files created by the revenue programs as input and allocate space required for storage of the block model and working matrices. They both then determine the optimum pit and store the results in two files for further use.

5.3.2- Program PITWIN32.RC

This is a resource script file used by the main program to construct the program's menu, dialogue boxes and related functions.

5.3.3- Program PITWIN32.RH

This is a resource header file that contains command constants for the menu items together with various controls such as edit and list boxes of the dialogue boxes.

5.3.4- Program PITWIN32.DEF

This is the module definition file that specifies various attributes of the executable program running under the Windows operating system such as the program's heap and stack size.

5.3.5- Program PITCLASS.CPP

This program includes a number of classes designed to construct various dialogue boxes for data entry and user interaction.

5.3.6- Program PITCLASS.H

This is the main header file that contains function prototypes and the declaration of constants, data types and classes used to construct dialogue boxes.

5.3.7- Program PITTOOL.CPP

This program contains a number of functions, each of which performs a specific task. These functions are called from other programs and include the conversion of random access files to text files, provision of summaries of the orebody grade block model or revenue block model and changing the direction of the vertical co-ordinates in the orebody grade model.

5.3.8- Program PITSLOPE.CPP

This program contains a number of functions used in the calculation of the four principal slope angles. These functions include determination of potential failure mode, random sampling from a probability distribution, determination of the safety factor or the probability of failure versus slope angles and calculation of the four principal slope angles.

5.3.9- Program PITFUNCT.CPP

This program includes the following functions for creating an orebody revenue block model and for smoothing of the pit bottom for both variable and multiple slope angles.

5.3.9.1- Function variable _ revenue

This function creates a revenue block model from a block grade model of a deposit by applying costs, prices and slope angles when slopes are defined for a single domain sector or region.

5.3.9.2- Function multiple _ revenue

This function creates a revenue block model from a block grade model of the deposit by applying costs, prices and multiple-domain variable slope angles. The program also assigns slope angles to the blocks using the angle sum method described in Chapter 3.

Both functions, **variable_revenue** and **multiple_revenue** use two text files as input and create a revenue block model, in the form of three files, as output. The input data comprise a block grade file (previously created by means external to the software) and a file containing the physical and economic factors and pit slopes. The latter file can either be created by the software or supplied from an external source. The output files comprise one text file containing the block model characteristics (block dimensions, number of blocks in the model and four principal slope angles if a single region is used), a random access revenue model file and a random access grade block file. The random access files contain one record for each block. The revenue block model file is used in the optimal pit design program and the block grade file is used to determine the mean grade of the blocks within the optimal pit.

5.3.9.3- Function variable _ pitbot

This function smoothes the bottom of an economically optimum open pit with variable slope angles. It is assumed that the minimum space required for mining equipment to operate freely is given by the user.

5.3.9.4- Function multiple_pitbot

This function performs pit bottom smoothing on an economically optimum pit for the case in which slopes are specified for more than one domain sector or region. It is assumed that the user gives the required minimum space.

Both smoothing functions use as input the revenue block model file and the pit generated by the optimum pit programs. Both functions generate two output files for further use.

5.4- An overview of the program

The program is initiated by double clicking the icon in Windows Program Manager. As shown in Figure 5.1, the program comprises the following components:

- The frame window with the title, menu, minimise and maximise icons. The system menu contains numerous commands, each of which performs a specific task.
- The client area used to display dialogue boxes, error messages and graphical and numerical outputs.
- The status line, located at the bottom of the program window, displays a brief explanation of the various commands in the menus.

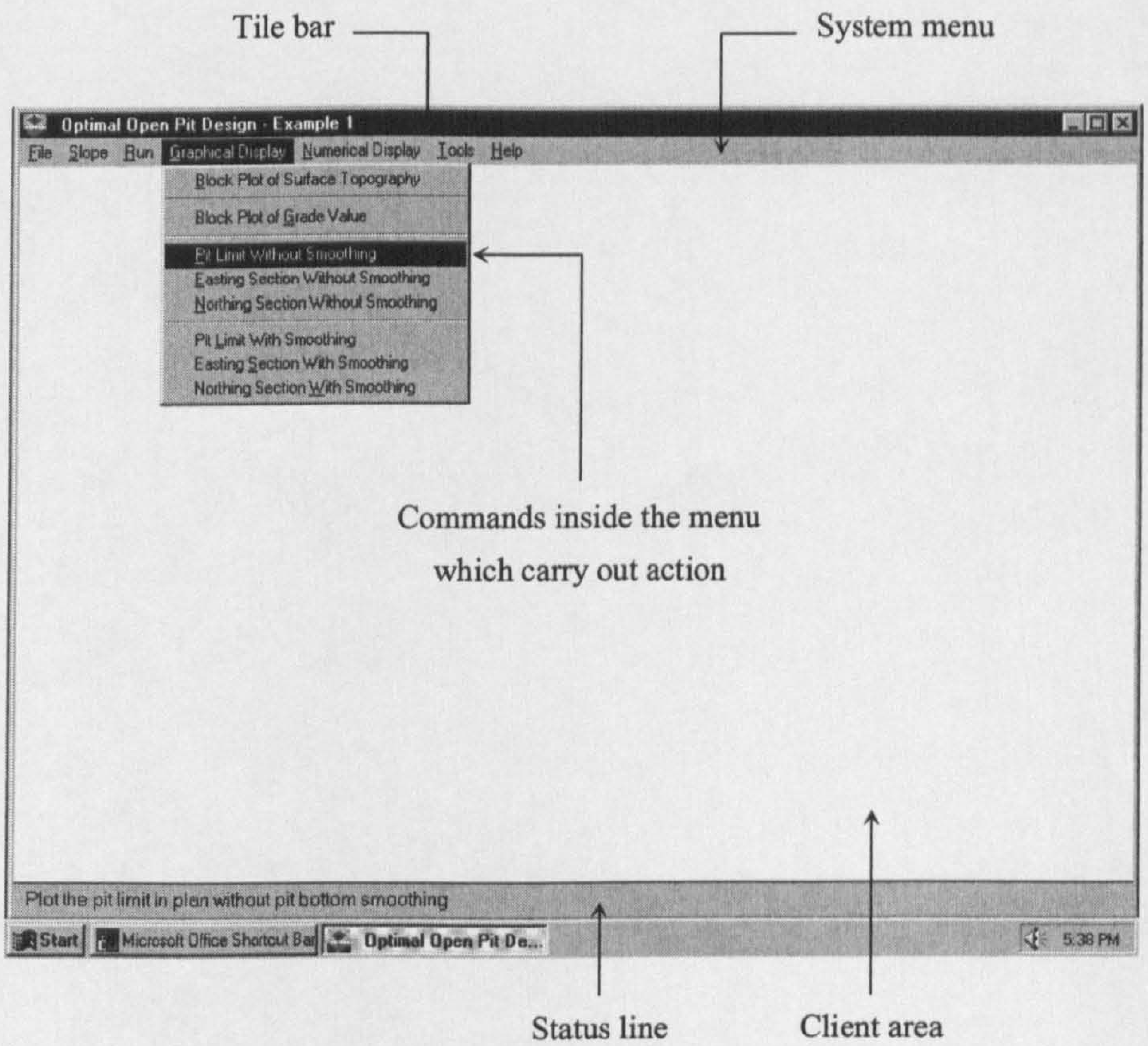


Figure 5.1- The components of the software

In addition to the main components of the Windows program it is useful to define the following key terms used in Windows programming.

Dialogue box- Dialogue boxes are special pop-up windows that contain controls such as radio buttons, edit boxes and list boxes serving either to display or to input data. Windows applications usually use a dialogue box to exchange information with the user.

Radio button- A radio button control is a circular button with a title that is usually used to select an option from two or more options. When a radio button is selected a tiny, filled circle appears inside the circular button.

Edit box- An edit control box is an editing tool in the dialogue box that can be used to enter and edit information.

List box- List box controls are input tools in a dialogue box and display a list of items. The user can browse the list and choose one or more items.

Scroll bar- Scroll bars are visual components of a window that assist in scrolling through the window's contents. A window can have a vertical scroll bar, a horizontal scroll bar or both. A scroll bar has an arrow box at each end and a scroll thumb. The arrow boxes are used to scroll the window's contents to either end or to either side.

All menus are accessed using a mouse or by pressing the <Alt> key plus the underlined letter. Commands from the menu are activated either by pressing the underlined letter or by using the mouse or the cursor keys to select from the menu. The main menu is displayed in Figure 5.1 and shows the available options which can be used to activate operations. These options are **F**ile, **S**lope, **R**un, **G**raphical Display, **N**umerical Display, **T**ools and **H**elp. A brief summary of the menus and the commands in each menu is given below.

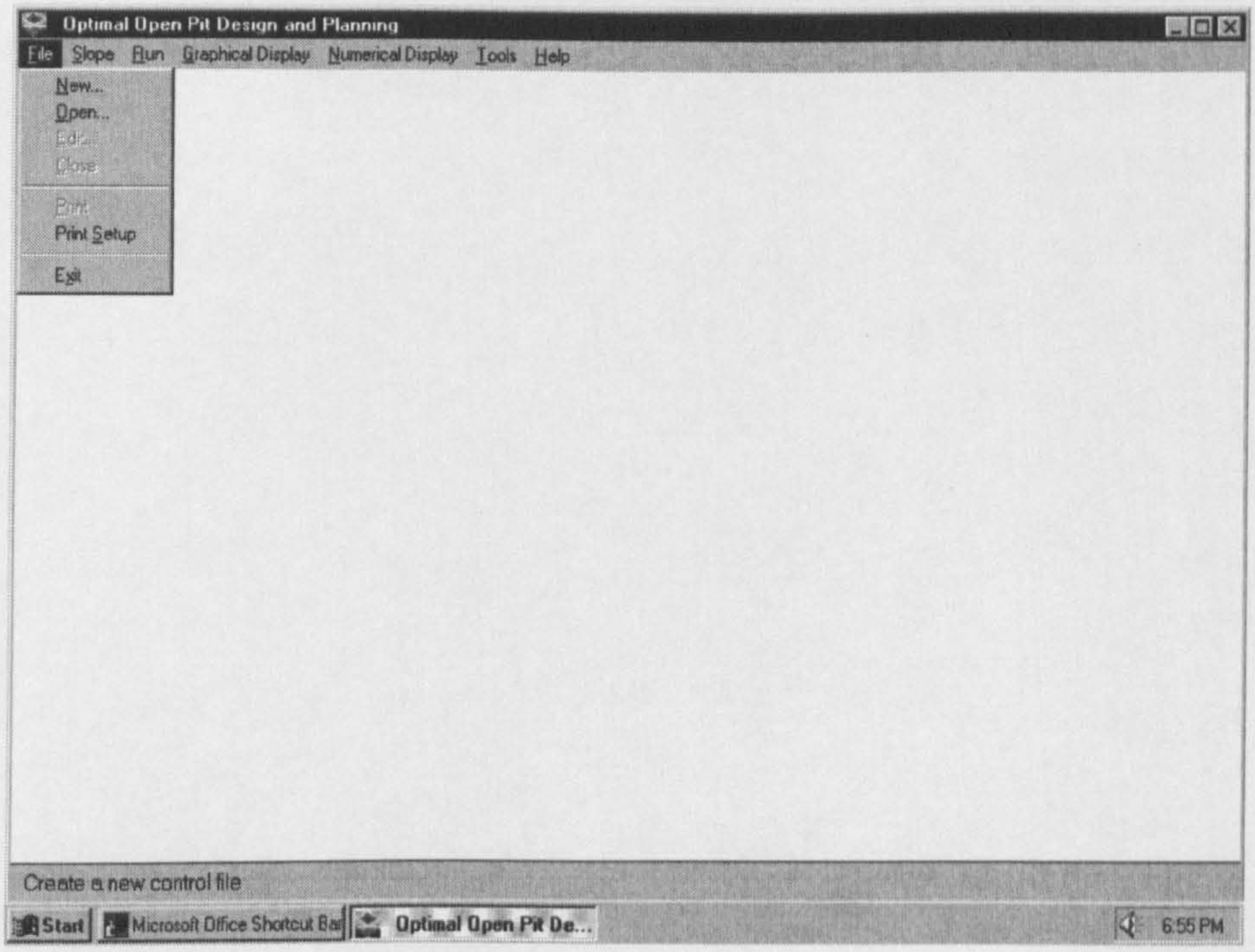


Figure 5.2- The File menu

5.4.1- The File Menu

The **File** menu, shown in Figure 5.2, contains commands to create, load, edit and close the control file which contains information required to create a revenue block model and to determine the optimum pit. This menu also includes commands to select and set up a printer and to generate hard copy of the numerical or graphical presentation displayed in the client area of the program. Table 5.1 summarises the available commands in this menu.

5.4.1.1- The New command

The first step of the optimum open pit design is to create or load a file called the control file that contains information required for the design of the optimum pit. This information includes the name of the mine, grade file name, block dimensions, specific gravity of the ore and waste, cut-off grade, costs of mining ore and waste, the processing cost, recovery factor, mineral/metal price and pit slopes. The New command can be used to enter these data and store them in a text file for further use. The default extension of the control file is "CFL". It is, however, also possible to use a different extension. This command invokes different dialogue boxes for entering information and storing it in a file. When a control file is successfully created, the name of the mine is added to the title bar.

Command	Function
New	Create a new control file
Open	Load an existing control file
Edit	Open an existing control file for editing
Close	Close the control file and update the client area
Print	Print the contents of the client area
Print Setup	Select and set up the printer
Exit	Exit the program

Table 5.1- Summary of the commands in the File menu

5.4.1.2- The Open command

The Open command can be used to load an existing control file. This option invokes the **Open** dialogue box that contains several list boxes that can be used to locate a control file and then select it. When a control file is successfully loaded, the name of the mine is added to the title bar.

5.4.1.3- The Edit command

The Edit command can be used to edit or to view the contents of an existing control file. This option invokes various dialogue boxes to edit the contents of a control file. The Edit command is only available if a control file has already been loaded. Otherwise this command is not available.

5.4.1.4- The Close command

The Close command can be used to remove the active control file. This option also updates the client area of the program and removes the name of the mine from title bar.

5.4.1.5- The Print command

The Print command can be used to generate a hard copy of any numerical or graphical presentation displayed in the client area of the program. This command is only available when any result has already been displayed in the client area. Before using the Print command, it is necessary to select and setup the printer.

5.4.1.6- The Print Setup command

Before generating any hard copy of the output displayed in the client area, it is necessary to select and set up the printer as well as to chose the appropriate paper size and its orientation. The Print Setup command can be used to select and setup the printer.

5.4.1.7- The Exit command

The Exit command can be used to quit the program.

5.4.2- The Slope Menu

The Slope menu, shown in Figure 5.3, contains commands for entering geotechnical information and for designing slope angles by two methods: steepest safe angle and the limit equilibrium method in terms of the calculation of the factor of safety or the probability of failure. It also includes options for determining the four principal slope angles together with the graphical display of the factor of safety or the probability of failure versus slope angles. Table 5.2 summarises the available commands in this menu.

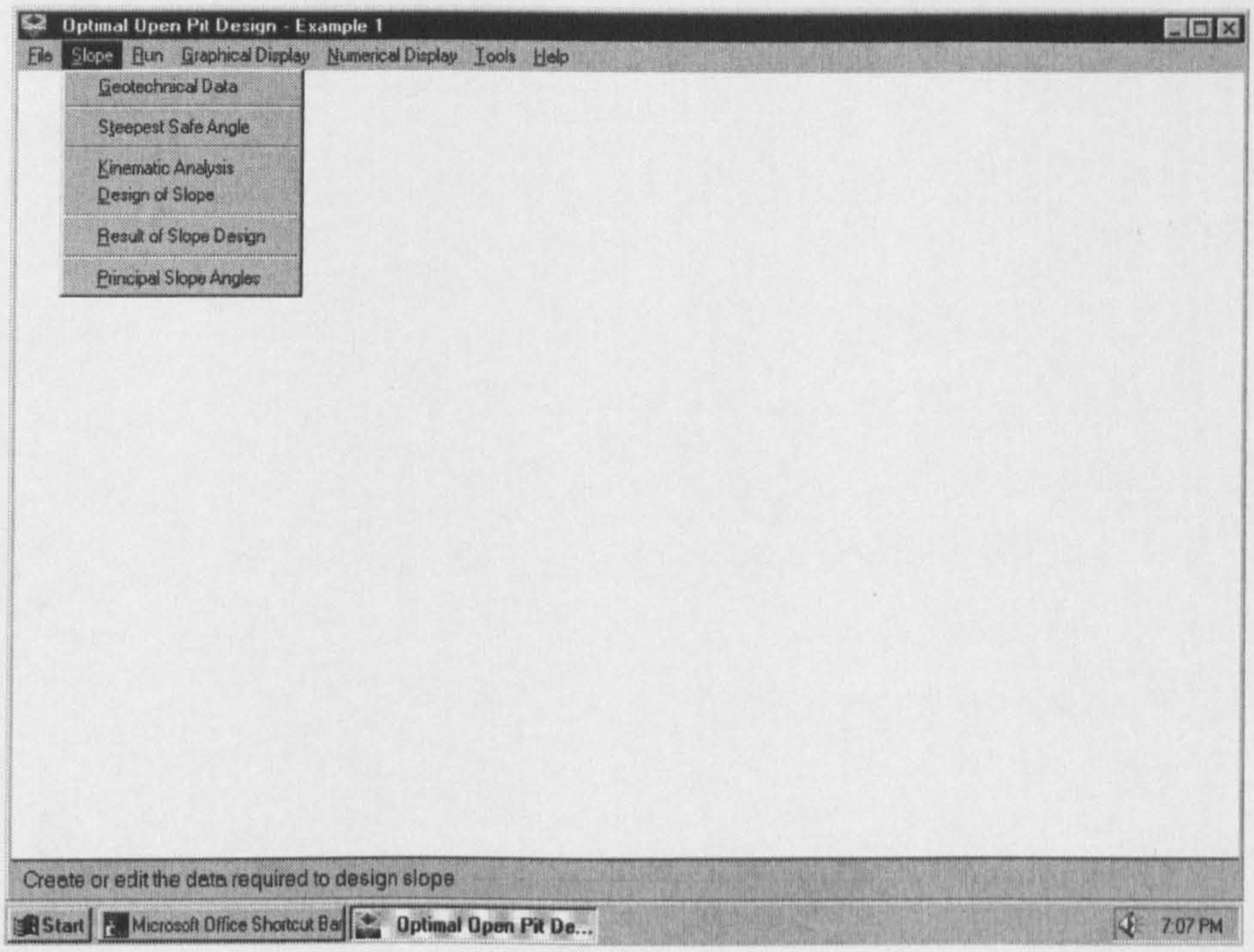


Figure 5.3- The Slope menu

Command	Function
Geotechnical Data	Create or edit data required to design slope angles
Steepest Safe Angle	Determine steepest safe angles for each region
Kinematic Analysis	Display the result of kinematic analysis graphically
Design of slope	Determine the factor of safety or the probability of failure versus slope angle
Result of slope design	Display the factor of safety or the probability of failure versus slope angle
Principal slope angles	Determine four principal slope angles

Table 5.2- Summary of the commands in the Slope menu

5.4.2.1- The Geotechnical Data command

The **Geotechnical Data** command is only available if a control file has already been created or loaded from the **File** menu and can be used to enter or edit the information required for calculating the slope angles. This includes the orientation and strength of discontinuities, strength of rock mass and ground water conditions. It invokes various dialogue boxes for entering data and storing them in a text file for further use. The name of this file is the same as that of the control file but with the extension **"SLD"**.

5.4.2.2- The Steepest Safe Angle command

The Steepest Safe Angle command is only available if the geotechnical data have already been entered and stored in a text file. This command can be used to determine the steepest safe angle with regard to plane and wedge failure. This is a simple method and is useful when only a little information is available. If this command is selected four principal slope angles for each region are determined.

5.4.2.3- The Kinematic Analysis command

The Kinematic Analysis command is only available if the geotechnical data have already been entered and stored in a text file. This command provides a graphical display of the kinematic analysis results superimposed on a plan view of the orebody.

5.4.2.4- The Design of Slope command

The Design of Slope command is only available if the geotechnical data have already been entered and stored in a text file. This command can be used to determine the factor of safety or the probability of failure versus slope angles. If all the input data are defined as deterministic values, the factor of safety is obtained otherwise the probability of failure is calculated for various slope angles from 20° to 80° in 5° steps with different dip directions from 0° to 360° in 15° increments. The results are stored in a text file with the same name as control file name but with the extension of **"PRO"**.

5.4.2.5- The Result of Slope Design Option

The Result of Slope Design command is only available if the factor of safety or the probability of failure has already been determined and stored in a text file by the **D**esign of Slope command. This command displays the factor of safety, or the probability of failure versus slope angles, for slopes with dip direction from 0° to 360° in 15° increments. There are four figures for each domain sector or region for slopes with a dip direction from 0° to 90° (quadrant I), 90° to 180° (quadrant II), 180° to 270° (quadrant III) and 270° to 360° (quadrant IV); each figure contains six graphs. The number of the quadrant and the number of the region must be specified by the user.

5.4.2.6- The Principal Slope Angles command

The **P**rincipal Slope Angles command is only available if the factor of safety or probability of failure has already been determined and stored in a text file by the **D**esign of Slope command. This option can be used to determine the four principal slope angles provided that the required data - minimum acceptable factor of safety and maximum acceptable risk of failure together with the type of slope including slope under dry conditions, wet slope without tension crack and wet slope with tension crack - are given by the user. The user can edit, abandon or use these principal slopes to determine the optimum pit. If they are accepted these slopes replace those defined in the control file.

5.4.3- The Run Menu

The **R**un menu, illustrated in Figure 5.4, includes commands to create an orebody revenue block model, to determine the optimum pit and to execute pit bottom smoothing. Table 5.3 summarises the available commands in this menu.

5.4.3.1- The Revenue Block Model option

The optimal pit design algorithm requires a revenue block model of the deposit in which a profit or revenue value is assigned to each block. This option can be used to create an orebody revenue block model by applying the information stored in the control file. It is only available when a control file has already been created or loaded from the **F**ile menu. If a revenue block model has already been created an indicative message will be displayed. Depending on the number of regions stored in the control file, this option uses either the **variable_revenue** function or the **multiple_revenue** function to create the orebody revenue block model. If a single region is specified **variable_revenue** is used otherwise the **multiple_revenue** function is executed.

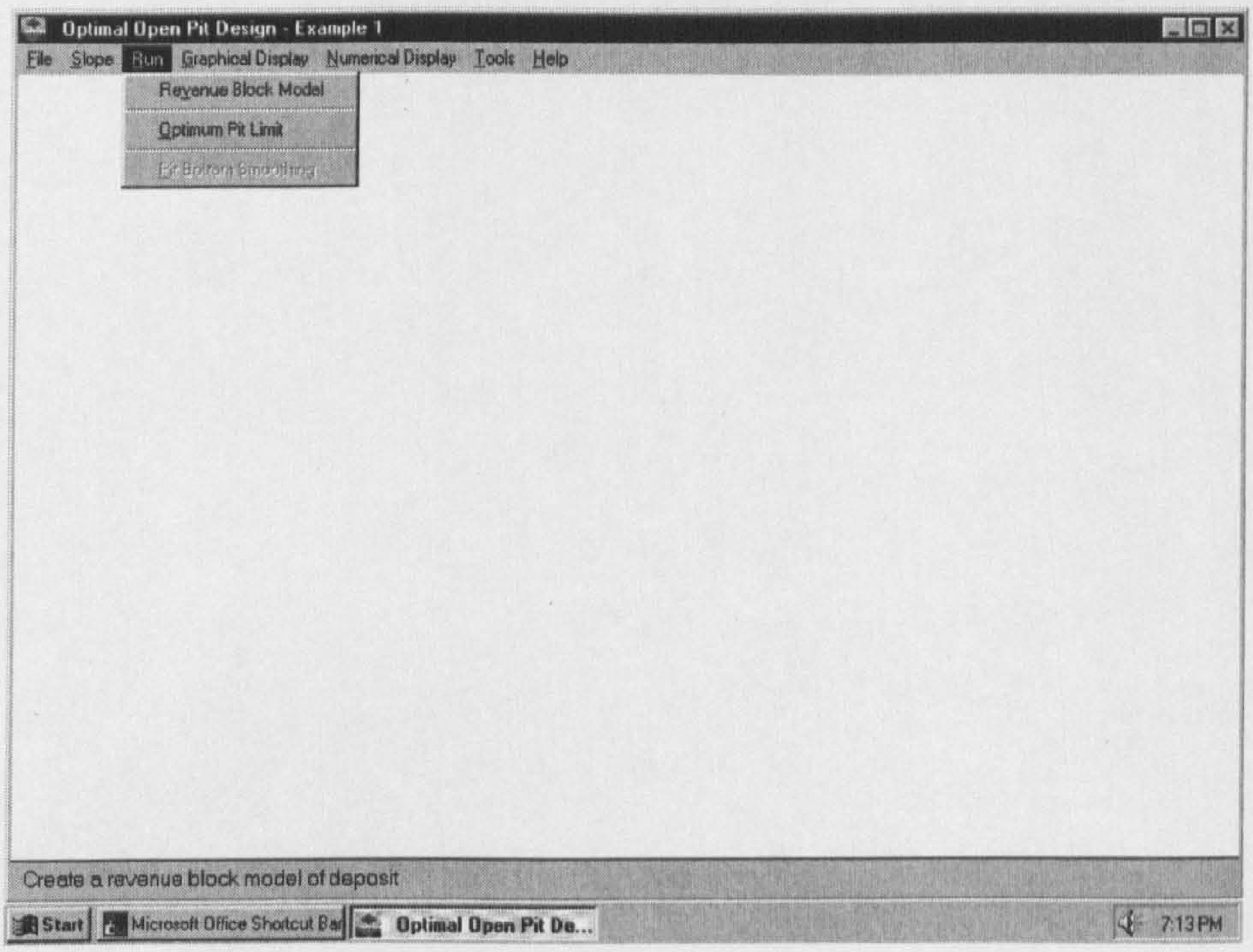


Figure 5.4- The Run menu

5.4.3.2- The Optimum Pit Limit option

The **O**ptimum Pit Limit option can be used to determine the optimum pit from a revenue block model of the deposit. This option is only available if an orebody revenue block model has already been created. Depending on the number of regions stored in the control file, this option uses either the **variable_pit** or the **multiple_pit** function to determine the optimum pit limit with variable slope angles. If a single region is specified **variable_pit** is used otherwise the **multiple_pit** function is executed.

Command	Function
Revenue Block Model	Create a revenue block model of deposit
Optimum Pit Limit	Determine the optimum pit limit
Pit Bottom Smoothing	Carry out pit bottom smoothing

Table 5.3- Summary of the available options in the Run menu

5.4.3.3- Pit Bottom Smoothing option

The optimum pit limit generated by the algorithm may have a very irregular bottom that may not be feasible in practice because of the minimum space required for mining equipment to operate freely. The Pit Bottom Smoothing option can be used to smooth the pit bottom by specifying a required minimum space in order to obtain a technically optimum pit. This option is only available if the optimum pit limit has already been obtained. Depending on the number of regions stored in the control file, this command uses either the **variable_pitbot** function or the **multiple_pitbot** function to smooth the pit bottom and determine the technically optimum pit. If a single region is specified **variable_pitbot** is used otherwise the **multiple_pitbot** function is executed.

5.4.4- The Graphical Display Menu

The Graphical Display menu, shown in Figure 5.1, provides options for displaying graphical presentations of the optimum pit in plan and sections for pits with and without pit bottom smoothing. It also includes options for the display of block plots of the surface topography and block plots of grade values for any level and interval. Table 5.4 summarises the options available in this menu.

5.4.4.1- The Block Plot of Surface Topography command

The Block Plot of Surface Topography command is only available if a revenue block model of the deposit has already been created. This option can be used to display a block plot of the surface topography.

Command	Function
Block Plot of Surface Topography	Plot the surface topography
Block Plot of Grade Value	Plot the grade value for any level
Pit Limit With Smoothing	Plot the pit limit in plan without pit bottom smoothing
Easting Section Without Smoothing	Plot the pit limit in Easting section without pit bottom
Northing Section Without Smoothing	Plot the pit limit in Northing section without pit bottom
Pit Limit With Smoothing	Plot the pit limit in plan with pit bottom smoothing
Easting Section With Smoothing	Plot the pit limit in Easting section with pit bottom
Northing Section With Smoothing	Plot the pit limit in Northing section with pit bottom

Table 5.4- Summary of the commands in the Graphical Display menu

5.4.4.2- The Block Plot of Grade Value command

This command is only available if an orebody revenue block model of the deposit has already been created. It can be used to display a block plot of the grade values provided that the user gives the level number and required interval.

5.4.4.3- The Pit Limit Without Smoothing command

The Pit Limit Without Smoothing command is only available if the optimum pit limit has already been determined. This option generates a graphical display of the pit limit without pit bottom smoothing.

5.4.4.4- The Easting Section Without Smoothing command

The Easting Section Without Smoothing command is only available if the optimum pit limit has already been determined. This option generates a graphical display of an east-west cross-section of the pit without pit bottom smoothing provided that the user first specifies the Easting of the cross-section.

5.4.4.5- The Northing Section Without Smoothing command

The Northing Section Without Smoothing command is only available if the optimum pit limit has already been determined. This option generates a graphical display of a north-south cross-section of the pit without pit bottom smoothing. To do so the user must first specify the Northing of the cross-section.

5.4.4.6- The Pit Limit With Smoothing command

This command is only available if the technical optimum pit has already been obtained. It generates a graphical display of the pit with pit bottom smoothing.

5.4.4.7- The Easting Section With Smoothing command

The Easting Section With Smoothing command is only available if the optimum pit limit has already been smoothed by the Pit Bottom Smoothing command. This option generates a graphical display of an east-west cross-section of the pit with pit bottom smoothing provided that the user first specifies the Easting of the required cross-section.

5.4.4.8- The Northing Section With Smoothing command

This command is only available if the technical optimum pit limit has already been obtained. It generates a graphical display of a north-south cross-section of the pit with pit bottom smoothing. To do so, the user must first specify the Northing of the required cross-section.

5.4.5- The Numerical Display Menu

The Numerical Display, shown in Figure 5.5, provides options for displaying numerical presentations of output including optimisation results with and without pit bottom smoothing. Table 5.5 summarises the options available in this menu.

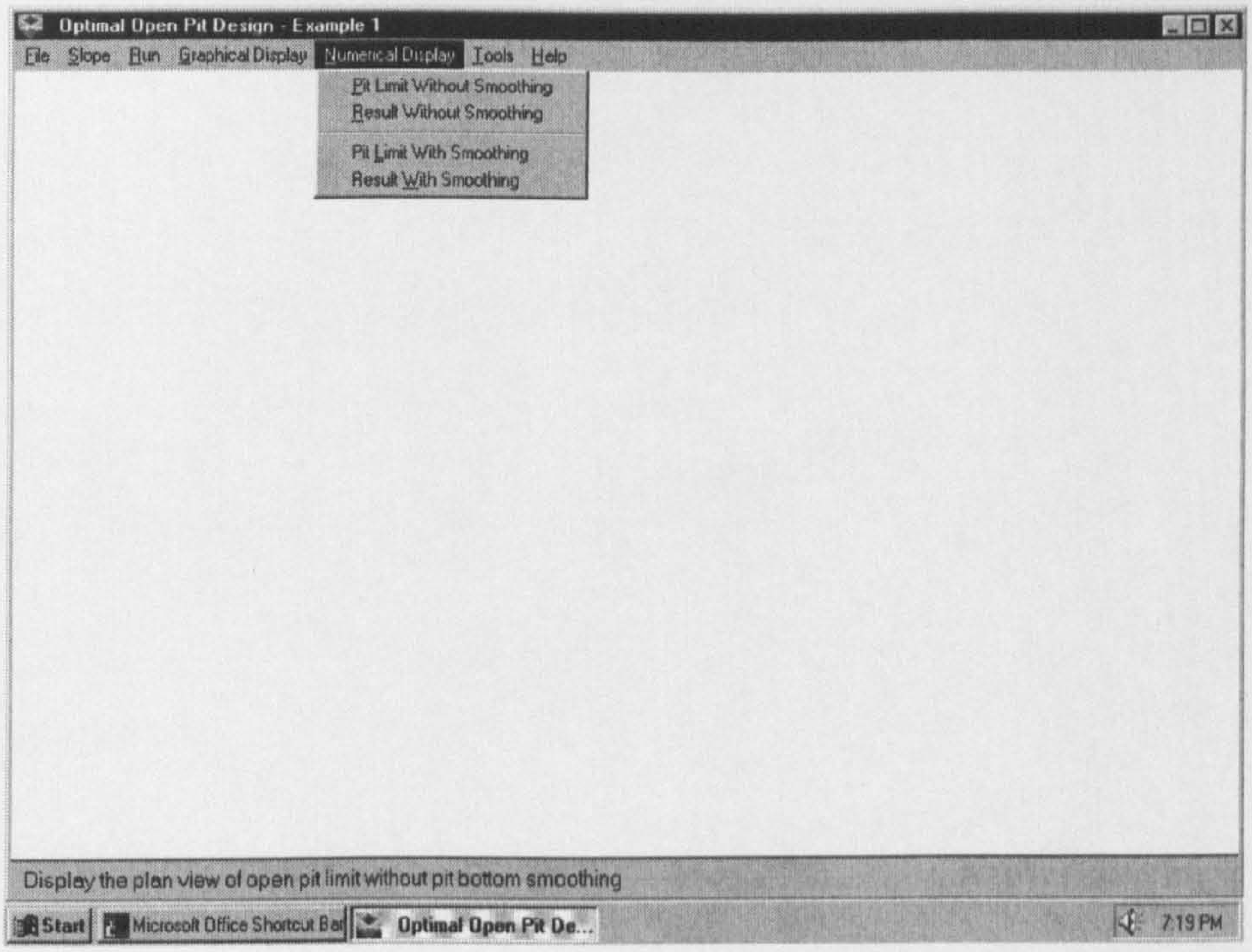


Figure 5.5- The Numerical Display menu

5.4.5.1- The Pit Limit Without Smoothing command

The **P**it Limit Without Smoothing command is only available if the optimum pit has already been determined. This option generates numerical displays of the pit without pit bottom smoothing.

5.4.5.2- The Result Without Smoothing command

The **R**esult Without Smoothing command is only available if the optimum pit has already been determined. This option generates numerical displays of the results of pit optimisation without pit bottom smoothing. These results include level-by-level summaries of the number of blocks in the pit, the tonnage of ore and waste, the monetary values of ore and waste and the mean grade of ore.

5.4.5.3- The Pit Limit With Smoothing command

The Pit Limit With Smoothing command is only available if the economically optimum pit has already been smoothed by using the Pit Bottom Smoothing command. This option generates a numerical display of the smoothed pit.

Command	Function
Pit Limit Without Smoothing	Display the plan view of the pit limit without pit bottom smoothing
Result Without Smoothing	Display the results of optimisation without pit bottom smoothing
Pit Limit With Smoothing	Display the plan view of the pit limit with pit bottom smoothing
Result With Smoothing	Display the results of optimisation with pit bottom smoothing

Table 5.5- Summary of the commands in the Numerical Display menu

5.4.5.4- The Result With Smoothing command

The Result With Smoothing command is only available if the economically optimum pit has already been smoothed by using the Pit Bottom Smoothing command. This option generates summaries of the smoothed pit optimisation results. These results include level-by-level summaries of the number of blocks in the pit, the tonnage of ore and waste, the monetary value of ore and waste and the mean grade of ore.

5.4.6- The Tools Menu

The Tools menu, illustrated in Figure 5.6, contains commands for:

- displaying summaries of the orebody grade and revenue block models
- changing the graphical display options
- converting a random access file to a text file
- creating a file containing the co-ordinates, grade and monetary value of all blocks

- creating a file of ore and waste blocks inside either the economical or technical optimum pit limit
- creating an equivalent orebody block model for a multi-mineral deposit
- changing the direction of the vertical co-ordinates of the orebody block model.

Table 5.6 summarises the commands available in this menu.

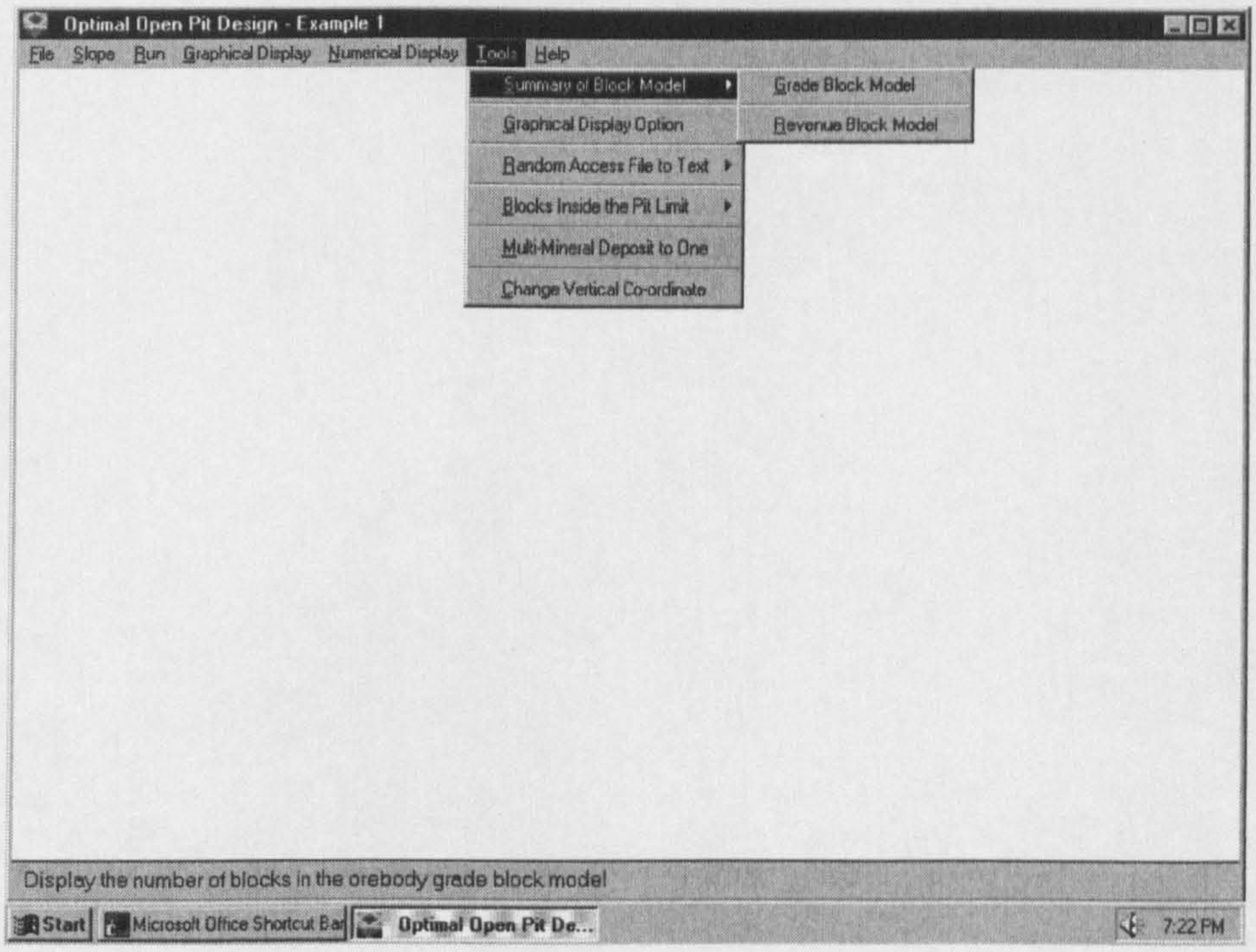


Figure 5.6- The Tools menu

5.4.6.1- The Summary of Block Model command

This command contains two options: **G**rade Block Model and **R**evenue Block Model. The former is only available if the control file has already been loaded. It displays the number of blocks in the East-West, North-South and vertical directions together with the minimum and maximum co-ordinates of blocks in the orebody block grade model. The latter is accessible when the orebody revenue block model is created. As illustrated in

Figure 5.7, this option displays the number of blocks in East-West, South-North and vertical directions, the number of ore, waste and air blocks, the number of blocks added to the borders together with the minimum and maximum co-ordinates of the blocks in the orebody revenue block model.

Command	Function
Summary of Block Model	
Grade Block Model	Display the number of blocks in the orebody grade block model
Revenue Block Model	Display the number of blocks in the orebody revenue block model
Graphical Display Option	Change scale and the X axis in the graphical display
Random Access File to Text	
Revenue Block Model	Convert the grade and revenue orebody block model to a text file
Pit Limit Without Smoothing	Convert the optimum pit limit without smoothing to a text file
Pit Limit With Smoothing	Convert the optimum pit limit with smoothing to a text file
Blocks Inside the Pit Limit	
All Blocks in Optimum Pit	Generate the 3 dimensional co-ordinates of the blocks inside the optimum pit limit
Ore Blocks in Optimum Pit	Generate the 3 dimensional co-ordinates of the ore blocks inside the optimum pit limit
Waste Blocks in Optimum Pit	Generate the 3 dimensional co-ordinates of the waste blocks inside the optimum pit limit
All Blocks in Technical Pit	Generate the 3 dimensional co-ordinates of the blocks inside the technical pit limit
Ore Blocks in Technical Pit	Generate the 3 dimensional co-ordinates of the ore blocks inside the technical pit limit
Waste Blocks in Technical Pit	Generate the 3 dimensional co-ordinates of the waste blocks inside the technical pit limit
Multi-Mineral Deposit to One	Convert the multi-mineral orebody block model to one mineral
Change Vertical Co-ordinate	Change the vertical co-ordinate of the block model

Table 5.6- Summary of the commands in the Tools menu

5.4.6.2- The Graphical Display Option command

This command displays the Graphical Display Option dialogue box which contains radio buttons for choosing whether the graphical outputs are displayed with or without scale. It also contains options for changing the orientation of the X-axis between east-west and north-south for plan view graphical displays.

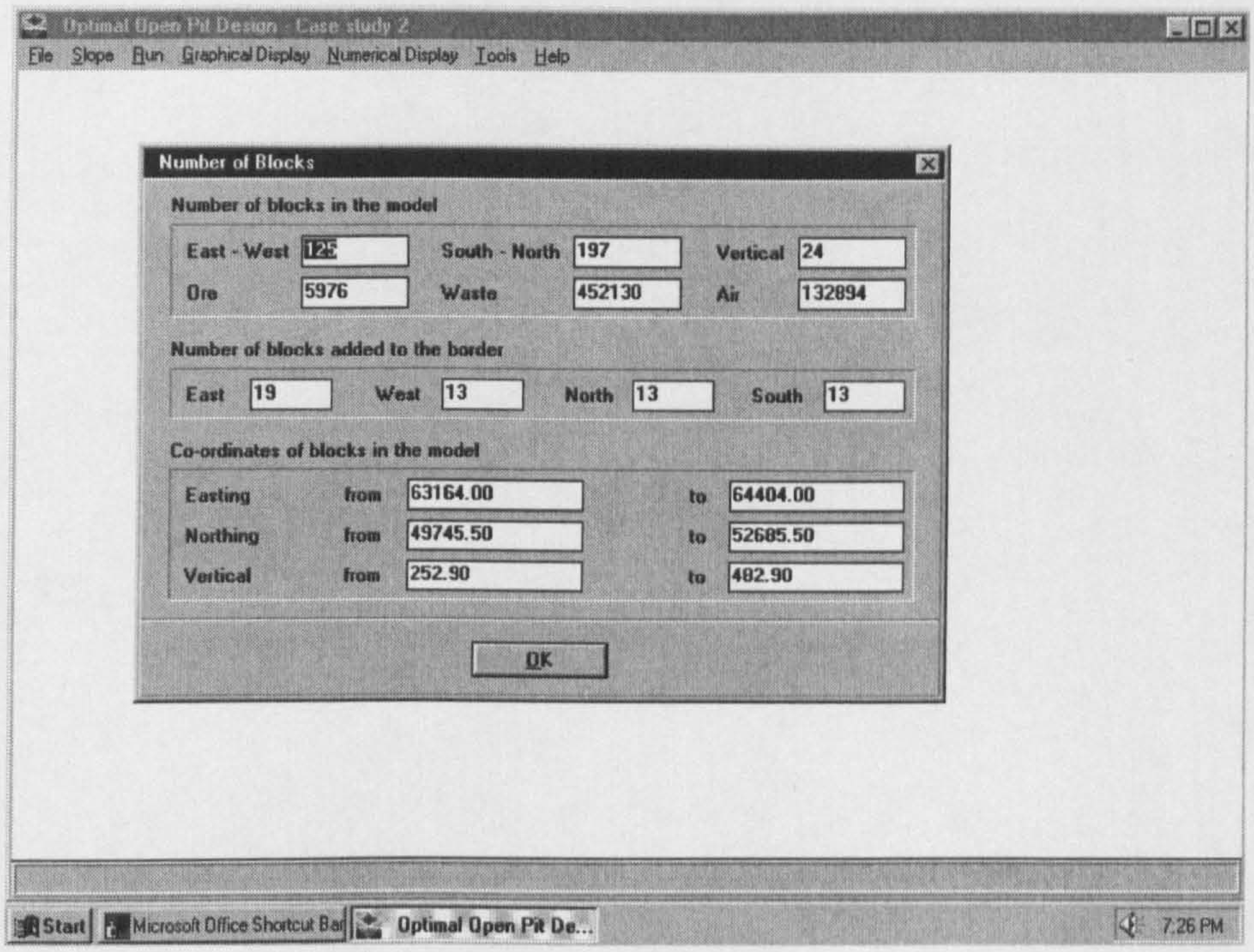


Figure 5.7- Summary of orebody revenue block model

5.4.6.3- The Random Access File to Text command

The **R**andom Access File to Text command is used to convert the random access files generated from the **R**un menu to text files. It contains three options: Revenue **B**lock Model, **P**it Limit Without Smoothing and Pit Limit **W**ith Smoothing. These options are only available after the corresponding files have been created. The first option, Revenue **B**lock Model, converts the orebody revenue and block grade model generated by the Revenue Block Model command to a text file. The next two options, **P**it Limit Without Smoothing and Pit Limit **W**ith Smoothing, convert, respectively, the economical and technical optimum pit, stored in the random access files, to text files. The user must specify the names of the text files before these options can be implemented.

5.4.6.4- The Blocks Inside the Pit Limit command

As shown in Table 5.6, this command contains six options that can be used to create text files:

- containing the three dimensional co-ordinates of the blocks, grade, tonnage (only for grade model type II) and the monetary value of all blocks
- containing ore and waste blocks inside either economical or technical optimum pit provided that the name of output file is given by the user.

It should be noted that these options are available only after the related commands have been executed.

5.4.6.5- The Multi-Mineral Deposit to One command

This command is used to create an equivalent orebody grade block model for a multi-mineral deposit. It displays a dialogue box for entering the multi-mineral file name, output file name and the required coefficients for each mineral. It is assumed that the multi-mineral file contains two lines as a title and one record for each block. Each record must contain the Easting, Northing and vertical co-ordinates of the block together with the grade of each mineral.

5.4.6.6- The Change Vertical Co-ordinate command

For the orebody block grade model, illustrated in Figure 3.27, it is assumed that the Z-axis (representing the vertical direction) increases from top to bottom. Block models for which the Z-axis decreases from top to bottom must be modified so that the vertical co-ordinate increases from top down. The Change Vertical Co-ordinate command can be used to do this by specifying the file names and block dimensions in a dialogue box.

5.4.7- The Help Menu

This option displays the version number of the program.

5.5- Input requirements

5.5.1- Data required to determine the optimum pit

The input data required to design the optimum open pit are the block grade model of the deposit, physical and economic factors and pit slopes. The only predetermined input is the orebody block grade model; all other information can be entered during program execution. Two types of block model can be used for open pit design. The orebody block grade model must be represented in a text file. This file must contain one record for each block and each record can be one of the two following types:

Type I- Each block has a single grade only

- Easting (X)
- Northing (Y)
- Vertical (Z) co-ordinate of mid-point of the block
- Estimated grade (mean grade of the entire block)

Type II- Each block has grade and recoverable tonnage of that portion of the block that is above a specified cut-off grade

- Easting (X)
- Northing (Y)
- Vertical (Z) co-ordinate of mid-point of the block
- Grade above cut-off grade
- Recoverable tonnage above cut-off grade

The records of the file can be in any order and information can be recorded in free format. Blocks without co-ordinates describe surface topography. Any negative values of grade or tonnage denote missing values and the corresponding block is considered to be waste. Missing values generally denote blocks for which there are too few data to provide an estimate. The following data are specified by the user and are used to create a control file in text format:

- Block dimensions (metres)
- Specific gravity of ore and waste (tonnes/cubic metre)
- Cut-off grade (grams/tonne)
- Cost of mining of ore and waste (monetary units per tonne)

(This can be a fixed value or variable value depending on the depth)

- Processing cost (monetary units per tonne)
- Recovery factor (%)

(This can be a fixed value or variable value depending on the grade)

- Price of metal (monetary units per gram)
- Pit slopes characteristic

The information required to define the mining slope depends on the number of regions. If one region is specified only four principal slope angles are required. Otherwise, for each region the user must specify minimum and maximum depth, co-ordinates of the points defining the region and the four principal slope angles. It should be noted that the co-ordinates of the points defining the region must be in sequence either in a clockwise or anticlockwise direction. Up to a maximum of 20 different regions can be specified and there is no limitation on the number of co-ordinates defining each region.

5.5.2- Data required to design slope angles

If the steepest safe angle is used, only the orientation and properties of the discontinuities (dip, dip direction and angle of friction) within each domain sector are required to determine slope angles. When the deterministic or probabilistic limit equilibrium method is used the following data are required for each region to determine the four principal slope angles as input parameters for the optimal pit design:

1- Orientation and properties of the discontinuities including dip, dip direction, cohesion and friction angle either as a fixed value or as a random variable with corresponding density function.

2- Rock mass properties including either cohesion and friction angle or geological strength index (*GSI*), uniaxial compressive strength (σ_c) and material constant of intact rock (m_i) either as a fixed value or as a random variable with corresponding density function.
3- Rock mass density either as a fixed value or as a random variable with corresponding density function.
4- Water table depth either as a fixed value or as a random variable with corresponding density function.

5.6- Outputs

A number of programs were written to produce graphical and numerical presentations of the results of the optimisation. These programs generate various forms of output all designed to fit A4 paper size. Outputs are displayed in the client area of the program window and hard copy can be generated as well. Vertical and horizontal scroll bars are provided to scroll the window content to view information not in the current viewing portion of the window. Examples of all outputs are given in Chapter 6. In general, program outputs can be categorised as graphical and numerical displays.

5.6.1- Graphical display

Various forms of graphical presentation were developed to display the input data and the output generated by the software. These are displayed in the client area with an appropriate scale for which the X-axis is oriented east-west in plan. It is also possible to display these outputs without scale and to change the orientation of the X-axis to north-south. The graphical presentation includes the following output:

1- Block plot of surface topography.
2- Block plot of grade value

3- Optimum pit limit in plan

4- Optimum pit limit in cross-section

5- Results of kinematic analysis

6- Factor of safety or probability of failure versus slope angle

5.6.1.1- Block plot of surface topography

In the orebody revenue block model, air blocks, indicating the surface topography have a value of zero. When an orebody revenue block model has been created, the software is able to represent the surface topography in two dimensions. An example of a surface topography block plot of real data is given in Chapter 6. In the surface topography block plot, each block is identified by its easting and northing and is shown by a colour-filled rectangle corresponding to a level number indicating that all blocks from the first level down to and including this level are air blocks. For example, the number 2 indicates that there are two air blocks in this location. These are blocks whose co-ordinates are missing from the orebody grade model and they are treated as air blocks when the orebody revenue block model is created. A value of zero indicates that blocks reach the surface at this location.

5.6.1.2- Block plot of grade value

Once an orebody revenue block model has been created and the block grade file is obtained, it is possible to plot the grade value for each level in two dimensions. For this purpose, the level number and the number of class intervals for displaying colour-coded grade values are required. For display purposes the block grades are colour-coded automatically by colours assigned to each class interval.

Some examples of grade block plots are given in Chapter 6. It should be noted that air blocks and blocks with zero values are not displayed on block plots of grade values.

5.6.1.3- Optimum pit limit in plan

When an optimum pit has been obtained, with or without pit bottom smoothing, the software can provide a graphical display of a two-dimensional plan view of the optimal pit. In this plan view the pit is shown as colour-filled rectangles corresponding to the level number down to which blocks must be mined. Some examples of graphical displays of the optimum pit are provided in Chapter 3 (Figure 3.32, 3.37 and 3.45). For example, the number 6 in Figure 3.32 indicates that, at this horizontal location, all blocks from the surface down to and including level 6 are part of the optimum pit. The number 0 indicates that no blocks are mined at this horizontal location.

5.6.1.4- Optimum pit limit in cross-section

The program can create cross-sections in both directions (East-West, South-North) for both the economical and technical optimum pits. The Easting cross-section specifies a vertical cross-section as seen looking from south to north with the Easting of the section constant. The Northing section specifies a vertical cross-section as seen looking from west to east with the Northing of the section constant. This is achieved by specifying the Easting or Northing of the section required. The program finds the group of blocks on the cross-section and shows the optimum pit limit with a thick red line indicating that all blocks above this line are part of the optimum pit.

5.6.1.5- Results of kinematic analysis

When the geotechnical data are entered, it is possible to display the results of kinematic analysis. The program determines the likely mode of failure and displays the results of kinematic analysis graphically and superimposed on the orebody. Some examples of this plot are shown in Chapter 4 (Figures 4.21 and 4.27).

5.6.1.6- Factor of safety or probability of failure versus slope angle

Once the factor of safety or probability of failure has been determined the software can generate graphical displays of the slope design results. There are four figures for each domain sector or region for slopes with a dip direction from 0° to 90° (quadrant I), 90° to 180° (quadrant II), 180° to 270° (quadrant III) and 270° to 360° (quadrant IV) each of which contains six graphs. When the number of the quadrant and the number of the region are specified, the program displays the factor of safety or the probability of failure versus slope angle using the information stored in the file with the same name as the control file name but with the extension **"PRO"**. Some examples of this plot are shown in Chapter 4 (Figures 4.22-2.25 and 4.28-4.31).

5.6.2- Numerical presentation

Once the economically or technically optimum open pit has been determined the software can generate numerical representations of the optimum pit with and without pit bottom smoothing. The numerical representations include:

1- Plan view of the pit
2- Results of the optimisation

5.6.2.1- Plan view of the pit limit

In the plan view of the pit each block is associated with a level number which indicates the number of blocks from the surface down that must be mined. For example, the number 5 indicates that all blocks down to and including level 5 must be mined or are above the topographical surface. The number 0 indicates that no blocks are mined at this horizontal location. Figure 5.8 shows the plan view of the pit for the multiple regions example given in Chapter 3. In this example, there are ten levels (10m, 20m, ...,100m). The number 4 in Figure 5.8 indicates that at this location all blocks down to and including level 4 (10m, 20m, 30m and 40m) are part of the optimum pit or are above the topographical surface.

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
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0	0	1	2	3	5	6	7	9	9	9	10	10	9	10	9	9	10	9	10	9	9	8	10	10	10	9	9	9	7	6	5	4	3	2	1	0	0
0	0	1	2	3	5	6	7	8	10	10	9	10	8	9	10	9	10	10	9	10	9	8	10	10	9	10	9	8	7	6	5	4	3	2	1	0	0
0	0	1	2	3	5	6	7	8	9	10	8	10	9	9	10	10	10	10	10	9	10	9	9	9	10	9	8	8	7	6	5	4	3	2	1	0	0
0	0	1	2	3	4	6	7	8	9	9	8	9	9	8	9	9	9	9	9	8	10	8	8	8	9	9	8	7	6	6	5	4	3	2	1	0	0
0	0	1	2	3	4	5	6	7	8	8	8	8	9	8	8	8	9	8	8	9	9	9	8	8	8	8	7	7	6	5	4	3	3	2	1	0	0
0	0	0	2	3	4	5	6	7	7	8	7	8	8	7	8	8	8	8	8	8	8	8	7	7	7	7	7	6	5	5	4	3	2	1	0	0	0
0	0	0	1	3	4	5	6	6	7	7	7	7	7	7	7	7	7	7	7	7	8	7	7	6	7	7	6	6	5	4	3	3	2	1	0	0	0
0	0	0	1	2	3	4	5	6	6	7	6	7	7	6	7	7	7	7	7	7	7	7	6	6	6	6	6	5	4	4	3	2	1	1	0	0	0
0	0	0	1	2	3	4	5	5	6	6	6	6	6	6	6	6	6	6	6	6	7	6	6	5	5	5	5	4	4	3	2	2	1	0	0	0	0
0	0	0	1	2	3	4	4	5	5	5	5	5	6	5	5	5	6	5	5	6	6	6	5	5	5	4	4	4	3	3	2	1	0	0	0	0	0
0	0	0	0	1	2	3	4	4	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	4	4	4	4	3	3	2	1	1	0	0	0	0	0
0	0	0	0	1	2	3	3	4	4	4	4	4	4	4	4	4	4	4	4	5	5	5	4	4	3	3	3	3	2	1	1	0	0	0	0	0	0
0	0	0	0	1	1	2	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	3	3	2	2	2	1	1	0	0	0	0	0	0	0
0	0	0	0	0	1	2	2	3	3	3	3	3	3	3	3	3	3	3	3	4	4	4	3	3	2	2	2	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	2	2	2	3	2	3	3	3	3	3	3	3	3	3	3	3	3	2	2	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	3	2	2	2	1	1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	1	1	1	1	1	2	1	1	1	2	1	2	2	2	2	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

East - West

Figure 5.8 - Plan view of the open pit without pit bottom smoothing - multiple variable slope angles

5.6.2.2- Results of the optimisation

When the optimum pit is obtained, with or without pit bottom smoothing, the software can generate level-by-level summaries of the optimisation results including the number of blocks in the pit, the tonnage of ore and waste, the monetary value of ore and waste and the mean grade of ore. An example of this is given in Chapter 6.

5.6.3- Output written to files

Apart from the control file with the default extension **"CFL"**, a number of data files are created during program implementation to record the results of each process. These files have the same name as the control file but have different extensions. Table 5.7 shows the types of data files and their extensions created by the program.

Extension	Type	Contents
.REV	Binary	Revenue block model
.GRD	Binary	Grade block model
.BLK	Text	Deposit characteristic
.SLD	Text	Geotechnical data
.PRO	Text	Result of design of slope angles
.PIT	Binary	Output of optimisation
.RST	Text	Result of optimisation
.BOT	Binary	Output of pit bottom smoothing
.RPB	Text	Result of pit bottom smoothing

Table 5.7- Output files generated by the software

The output files are created by revenue calculation programs (variable_revenue or multiple_revenue) have the extensions **"BLK"**, **"REV"** and **"GRD"**. They contain, respectively, block model characteristics, revenue block model and grade values. The programs for determining the optimum pit (variable_pit or multiple_pit) produce two output files with extensions of "**PIT**" and "**RST**" that contain, respectively, the optimum pit limit and the overall results. Similarly, two output files, with extensions of "**BOT**" and "**RPB**", are created by the pit bottom smoothing programs (variable_pitbot or multiple_pitbot) and contain, respectively, the technical optimum pit and the overall results. The geotechnical data and the results of the slope design are stored in files with extensions of "**SLD**" and "**PRO**" respectively.

It should be noted that it is possible to convert random access files to text files or to create files containing three-dimensional co-ordinates of all blocks, ore and waste blocks in the pit together with their grades, tonnages (only for grade model type II) and monetary values.

5.7- Program operations

The operation of the program is straightforward. Any user-induced error during operation generates an error message that will be displayed in the dialogue message box. The software can be loaded by simply clicking the **PITWIN32** icon or by double clicking the **PITWIN32.EXE** from the file manager. When the program is initiated, some commands are not immediately available. These are commands related to other options and are only available after the corresponding related commands have been executed. For example, the **O**ptimum pit limit command is operational only after the orebody revenue block has been created. Although the commands are related to each other, it is also possible to execute each of them at different times.

Appendix A provides a detailed set of instructions for executing all functions of the software.

5.8- Conclusion

The objective of the present study was to develop general, PC-based software for optimal open pit design with variable slope angles. The **PITWIN32** software presented in this chapter is able to generate the true optimum pit with variable slope angles. The program has been validated on trial data sets and two case studies. The software can be used for both cubic and rectangular block models and slope angles can vary in different parts of the orebody without changing the block dimensions. It is also able to generate output in the form of graphical and numerical displays or to write the results in text files. The different style of programming makes for much faster execution times than the similar FORTRAN code.

The main features of the software are:

- It is easy to use.
- The user can specify fixed or variable mining costs. The latter can vary with depth over an unlimited number of intervals.
- The user can specify fixed or variable recovery. The latter can vary with grade over an unlimited number of intervals.
- Slope angles can vary in both vertical and horizontal directions.
- Up to 20 different regions can be specified for the definition of mining slopes.
- An unlimited number of vertices can be used to define approximate regions.
- Graphical and numerical of outputs are provided.
- Hard copy can be generated.
- The program can handle an unlimited number of blocks in the deposit.

CHAPTER 6

Case studies

6.1- Introduction

Two case studies were employed to illustrate and test the application of the software for determination of optimum open pit limits. The data for both cases come from real deposits in Europe. The first is a low grade gold deposit and the second is a zinc-silver-gold-lead deposit. The orebody block grade models for both cases were generated by geostatistical methods. This chapter presents the results obtained from applying the optimal open pit design software for both cases.

6.2- Case study 1

The data for the first case study come from the Björkdal gold mine which is located approximately 35 km northwest of Skellefteå in the north of Sweden. The mine, which started production in 1988, is operated by Terra Mining AB.

Gold mineralisation in the Björkdal area occurs within a network of steeply dipping quartz veins in the contact between granodiorite and limestone (chalk)/acid volcanic rocks. The gold is erratically distributed but is mainly concentrated in and around high grade quartz veins. It occurs as both fine and coarse grains and is free-milling (Dowd, 1995).

The block grade model is type II in which each block is assigned the estimated (kriged) recoverable tonnage of ore above a cut-off grade and the estimated (kriged) average grade of this tonnage. The method of estimation is described in Dowd (1995). The deposit is divided into 15m (east-west) x 10m (north-south) x 5m (vertical) blocks and the recoverable tonnage is based on a selective mining unit of 5m (east-west) x 4m (north-south) x 5m (vertical). The numbers of blocks in the east-west, north-south and vertical directions are 101, 82 and 36 respectively. Some blocks have not been estimated because of lack of data and their co-ordinates are omitted from the block grade model. In addition, some blocks in the surrounding waste have been omitted but these do not affect the determination of the optimum pit. Other input data are:

a- Economic and technical factors

Specific gravity of ore and waste	2.71 $t.m^{-3}$
Cost of mining of ore and waste	(See Table 6.1)
Processing cost	SEK52 t^{-1} of ore
Price of gold	SEK90 g^{-1}
Recovery	91%

Level (m)		Cost of Mining (SEK / tonne)	
From	To	Waste	Ore
0	120	11.0	11.0
120	130	11.30	11.30
130	140	11.60	11.60
140	150	11.90	11.90
150	160	12.20	12.20
160	170	12.50	12.50
170	180	12.80	12.80
180	200	13.20	13.20

Table 6.1- Cost of mining of ore and waste

b- Geotechnical information

The main rock types in the Björkdal mining area are granodiorite, chalk and vulcanite (greywacke). Generally within the mine area these rocks dip 25-30° to the north. Although there are no major discontinuities in the mine area there are numerous small scale discontinuities. The strength of the rock masses and their classification with the 1989 version of the Rock Mass Rating (RMR) system have been estimated by Terra Mining AB from drill core samples and the results are shown in Table 6.2.

Rock type	RMR	Strength (MPa)	Density ($t.m^{-3}$)	Cohesion (KPa)	Friction angle (degree)
Granodiorite	78	221.2	2.73	5200	59.2
Chalk	68	83.7	2.70	1700	35.5
Vulcanite	72	111.6	2.75	2400	49.1

Table 6.2- Geotechnical information for case study 1

With regard to the RMR rating these rocks can be classified as good. Since there are no major discontinuities in the mine area, no structural instability is likely to occur and the slope angle is determined by the use of circular failure analysis. In order to ensure stability, chalk, which is the weakest among the rock types in the mine zone, is chosen to determine slope angles. Figure 6.1 shows the safety factor versus slope angles for this rock for which the slope is assumed to be saturated. Using acceptable safety factors of 2 and 2.5, slope angles of 67° and 54°, respectively, were obtained for saturated slopes in all directions. The mine is currently operating with an overall slope angle of 58° in all directions. A slope angle of 58° was used to design the optimum pit and it is assumed that this angle is the same for the entire area .

The orebody revenue block model created by the software contains 115, 104 and 36 blocks in the east-west, north-south and vertical directions respectively, yielding a total of 438,048 blocks. Figure 6.2 shows a block plot of the surface topography for this deposit. The block plot of grade value (distribution of the gold grades) on level 27 is illustrated in Figure 6.3. The optimum pit limits in plan and sections are shown in Figures 6.4 to 6.6. Table 6.3 shows the overall results of this case study.

Figure 6.1 - Results of slope angle design - Case study 1

Figure 6.2 - Block plot of surface topography - Case study 1

Figure 6.3 - Block plot of grade values (Vertical 152.5) - Case study 1

Figure 6.4 - Pit limit without pit bottom smoothing - Case study 1

Figure 6.5 - Pit limit without pit bottom smoothing - Case study 1

Figure 6.6 - Pit limit without pit bottom smoothing - Case study 1

Level No.	Number of blocks			Tonnage (tonnes)		Value (*10000)		Mean grade (g / t)
	Pit	Ore	Waste	Ore	Waste	Ore	Waste	
1	0	0	0	0.0	0.0	0.0	0.0	0.000
2	7	0	7	0.0	14227.5	0.0	-15.7	0.000
3	20	0	20	1261.2	39388.8	0.0	-44.2	0.689
4	47	6	41	8321.0	87206.5	28.2	-72.1	1.534
5	89	19	70	18943.7	161948.8	71.4	-133.7	1.516
6	98	24	74	23070.9	176114.1	120.4	-149.2	1.642
7	116	23	93	26635.0	209135.0	172.8	-180.7	1.787
8	115	22	93	30349.5	203388.0	223.7	-183.4	1.831
9	132	31	101	38982.5	229307.5	313.7	-208.1	1.890
10	188	70	118	88491.2	293618.8	986.6	-243.1	2.241
11	304	109	195	149660.5	468219.5	1638.9	-406.3	2.195
12	381	160	221	229524.3	544858.2	2661.4	-462.7	2.258
13	652	186	466	257278.2	1067911.8	2804.6	-1009.1	2.179
14	833	192	641	274901.7	1418170.9	2903.7	-1398.5	2.131
15	912	199	713	294251.1	1559389.0	3259.6	-1543.8	2.193
16	884	213	671	290347.6	1506382.4	2632.3	-1443.5	1.966
17	899	183	716	230409.9	1596807.6	2006.2	-1523.7	1.956
18	853	181	672	216524.7	1517197.8	2190.4	-1428.1	2.140
19	793	181	612	224820.8	1386951.8	2453.2	-1295.3	2.227
20	730	182	548	225121.9	1258603.1	2529.7	-1171.2	2.257
21	734	212	522	270892.7	1220962.2	3666.3	-1133.8	2.536
22	723	228	495	286762.7	1182734.8	3542.2	-1045.6	2.405
23	663	209	454	285203.3	1062344.2	3118.6	-986.9	2.217
24	638	233	405	312822.6	983912.4	3298.2	-862.0	2.173
25	650	281	369	389399.2	931725.8	4327.1	-766.2	2.244
26	652	344	308	464991.1	860198.9	5262.1	-619.0	2.268
27	625	324	301	441324.8	828987.8	5652.3	-606.6	2.460
28	579	342	237	478614.2	698203.3	6984.8	-456.9	2.667
29	479	296	183	415863.8	557703.7	5937.7	-381.9	2.623
30	384	241	143	338962.9	441517.1	4642.5	-290.7	2.554
31	300	223	77	295231.1	314518.9	3920.7	-153.8	2.516
32	220	181	39	249775.2	197374.8	4036.9	-66.2	2.856
33	150	137	13	195474.3	109400.7	3117.2	-28.0	2.816
34	100	94	6	140512.9	62737.1	2686.1	-12.1	3.192
35	57	56	1	83122.6	32729.9	1964.2	-2.7	3.741
36	23	23	0	35842.0	10905.5	700.1	0.0	3.230
Total	15030	5405	9625	7313691.1	23234784.1	89853.7	-20324.9	2.380

Table 6.3 - Pit result without pit bottom smoothing - Case study 1

Figure 6.7 - Pit limit with pit bottom smoothing - Case study 1

Figure 6.8 - Pit limit with pit bottom smoothing - Case study 1

Figure 6.9 - Pit limit with pit bottom smoothing - Case study 1

Level No.	Number of blocks			Tonnage (tonnes)		Value (*10000)		Mean grade (g / t)
	Pit	Ore	Waste	Ore	Waste	Ore	Waste	
1	0	0	0	0.0	0.0	0.0	0.0	0.000
2	7	0	7	0.0	14227.5	0.0	-15.7	0.000
3	23	0	23	1569.4	45178.1	0.0	-49.5	0.787
4	57	6	51	10601.1	105251.4	28.2	-89.1	1.402
5	107	19	88	19553.7	197923.8	71.4	-171.4	1.504
6	116	24	92	23970.6	211799.4	120.4	-186.3	1.621
7	142	23	119	27388.1	261226.9	172.8	-235.5	1.771
8	136	23	113	30461.6	245958.4	223.0	-229.3	1.828
9	168	30	138	41592.6	299867.3	313.4	-280.5	1.834
10	243	72	171	94148.6	399748.9	990.2	-344.6	2.177
11	359	111	248	157585.0	572082.5	1640.5	-503.7	2.138
12	451	159	292	235239.9	681417.6	2657.8	-592.5	2.230
13	753	186	567	264089.0	1266383.5	2782.7	-1205.0	2.143
14	974	192	782	286783.1	1692872.0	2910.8	-1674.4	2.088
15	1041	202	839	307245.3	1808587.2	3275.1	-1784.5	2.152
16	1009	214	795	300355.6	1750436.9	2626.6	-1689.8	1.933
17	1030	184	846	241852.8	1851622.2	1993.2	-1771.5	1.909
18	986	185	801	229267.3	1774777.8	2203.5	-1685.4	2.085
19	902	186	716	233148.4	1600166.6	2463.2	-1511.8	2.189
20	819	186	633	228408.5	1436209.0	2523.0	-1346.6	2.243
21	841	215	626	277264.3	1432068.1	3638.4	-1338.9	2.498
22	824	232	592	294185.4	1380594.6	3523.2	-1241.2	2.368
23	760	211	549	295028.9	1249671.1	3114.3	-1173.7	2.180
24	720	233	487	319485.8	1143914.2	3276.7	-1033.1	2.141
25	766	280	486	400590.3	1156304.8	4326.5	-997.8	2.214
26	753	348	405	476634.7	1053837.8	5267.2	-819.4	2.241
27	739	327	412	457686.2	1044331.3	5657.7	-825.1	2.413
28	708	349	359	497410.0	941600.0	6982.5	-703.4	2.607
29	596	294	302	425643.9	785726.1	5911.9	-634.5	2.583
30	495	239	256	350543.2	655544.3	4623.9	-526.8	2.500
31	387	223	164	309017.7	477559.8	3885.9	-332.9	2.437
32	311	187	124	261608.8	370498.7	4042.4	-245.8	2.785
33	237	142	95	210002.6	271699.9	3125.1	-202.5	2.704
34	169	101	68	153993.9	189498.6	2696.1	-150.0	3.013
35	125	65	60	97969.4	156093.1	1978.2	-132.0	3.354
36	48	24	24	38001.9	59558.1	676.1	-49.1	3.063
Total	17802	5472	12330	7598327.6	28584237.6	89722.3	-25773.3	2.330

Table 6.4 - Pit result with pit bottom smoothing - Case study 1

This case study took 42 minutes of CPU time to determine the optimum pit limits with the use of a Pentium 200 PC computer. A minimum space of 30m x 30m is also used to carry out pit bottom smoothing for which the technical optimum pit limits in plan and sections are illustrated in Figures 6.7 to 6.9. Table 6.4 shows the overall result for the technical optimum pit limit for case study 1.

6.3- Case study 2

The second case study is a low grade zinc-silver-gold-lead deposit. The block grade model file contains the co-ordinates of the mid-points of the blocks together with the grade of each metal in grams per tonne. Block dimensions are 10m (east-west) x 15m (north-south) x 10m (vertical). The block model has also 93, 171 and 24 blocks in the east-west, north-south and vertical directions respectively, giving a total of 381,672 blocks. Other input data are listed below:

Specific gravity of ore and waste	2.8 $t.m^{-3}$
Processing cost	SEK40 t^{-1} of ore
Cost of mining of ore and waste	SEK8.10 t^{-1} (for level 1 and increasing at SEK0.15 t^{-1} every 10m)
Base prices:	
Gold	SEK78.01 g^{-1}
Silver	SEK1.59 g^{-1}
Zinc	SEK9.13 kg^{-1}
Lead	SEK4.26 kg^{-1}
Number of regions	1
Pit slope angles:	
North face angle	50°
South face angle	50°
East face angle	60°
West face angle	50°

Figures 6.10 to 6.13 show the distribution of gold, silver, zinc and lead grades on levels 12, 16, 14 and 15 respectively. A block plot of the surface topography for this case is shown in Figure 6.14.

Level	Number of blocks			Tonnage (tonne)		Value (*10000)		Mean grade (g/t)			
No.	Pit	Ore	Waste	Ore	Waste	Ore	Waste	Au	Ag	Zn	Pb
1	0	0	0	0	0	0	0	0	0	0	0
2	48	0	48	0	201,600	0	-166.3	0	0	0	0
3	197	15	182	105,000	722,400	430,3	-669.8	0.549	3.504	7,348	1,612
4	294	43	251	205,800	1,029,000	1,495.8	-925.9	0.835	3.535	5,427	1,607
5	846	61	785	315,000	3,238,200	1,473.6	-2,902.5	0.622	3.205	4,845	1,737
6	1,139	131	1,008	659,400	4,124,400	2,718	-3,812	0.232	4.959	7,718	3,953
7	1,075	217	858	1,071,000	3,444,000	4,596.9	-3,297.1	0.181	4.843	8,632	3,907
8	972	218	754	1,121,400	2,961,000	5,343.0	-2,927.8	0.248	4.697	9,048	3,578
9	912	219	693	1,150,800	2,679,600	5,501.3	-2,771.6	0.221	4.807	9,648	3,523
10	829	243	586	1,188,600	2,293,200	6,390.1	-2,382.4	0.300	4.632	9,146	3,230
11	684	248	436	1,205,400	1,667,400	6,883.5	-1,803.7	0.294	4.771	9.653	3.329
12	496	249	247	1,129,800	953,400	7,258.8	-1,036.4	0.309	5.001	9,816	3,474
13	329	215	114	970,200	411,600	6,566.6	-510.8	0.293	5.194	10,291	3.755
14	188	134	54	609,000	180,600	4,600.8	-227.4	0.339	5.626	10,738	4,172
15	96	84	12	357,000	46,200	2,325.7	-51.2	0.191	6.041	10,222	4,705
16	35	33	2	147,000	0	1,208.7	-12.7	0.062	9.288	12,773	7,990
17	6	6	0	25,200	0	385.8	0	0.052	18.026	15,562	18,040
18	2	2	0	8,400	0	143.6	0	0.050	10.305	23,656	10,166
19	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0
total	8,148	2,118	6,030	10,269,000	23,952,600	57,322.5	-23,497.9	0.285	4.979	9,321	3,657

Table 6.5- Pit result without pit bottom smoothing - Case study 2

To evaluate the effect of the price and recovery factor on the optimum pit design, two alternatives were examined. For the first alternative, the recovery factor is taken as 100%. For the second alternative it is taken as 90%, 75%, 55% and 55% for gold, silver, zinc and lead respectively. A metal-equivalent orebody block model is created for each alternative by using the Multi-Mineral Deposit to One command from the Tools menu for which the coefficients of each mineral are the product of its price and recovery factor. For each alternative five different metal-equivalent prices, SEK0.70, 0.85, 1.0, 1.2 and 1.4, are used to create the orebody revenue block model.

The orebody revenue block model contains 591,000 (125 x 197 x 24) blocks to which 13, 19, 13 and 13 blocks are added, respectively, to the eastern, western, southern and northern boundaries of the deposit. The optimum pit limit in plan and the results for the second alternative with a price of SEK1.4 are shown in Figure 6.15 and Table 6.5 respectively. The overall results for both alternatives are summarized in Table 6.6.

Alternative	Price	Tonnage (tonne)		Net Value	Stripping	Mean grade (g/t)				Optimum
		Ore	Waste	(*10000)	ratio	Au	Ag	Zn	Pb	depth (m)
1	0.70	5,808,600	11,050,200	16,407.5	1.90	0.375	5.094	10,379	3,720	150
	0.85	10,676,400	25,061,400	31,785.6	2.35	0.260	4.979	9,512	3,669	180
	1.0	13,288,800	34,679,400	52,533.5	2.61	0.228	4.757	8,904	3,429	190
	1.2	18,261,600	59,442,600	87,282.7	3.26	0.184	4.526	8,497	3,232	220
	1.4	23,482,200	86,751,000	130,761.5	3.69	0.158	4.426	7,932	3,105	240
2	0.70	1,591,800	1,885,800	2,870.1	1.18	0.555	6.049	12,995	4,805	130
	0.85	3,032,400	4,548,600	6,147.4	1.50	0.515	5.610	11,700	4,216	130
	1.0	4,662,000	8,681,400	11,663.6	1.86	0.494	5.264	10,731	3,878	140
	1.2	6,270,600	12,999,000	21,086.8	2.07	0.403	4.974	10,000	3,594	150
	1.4	10,269,000	23,952,600	33,824.6	2.33	0.285	4.979	9,321	3,657	180

Alternative 1:	Recovery = 100%	for all minerals
Alternative 2:	Recoveries	Au: 90%
		Ag: 75%
		Zn: 55%
		Pb: 55%

Table 6.6- Comparison of results obtained with different prices and recovery factors for case study 2

Figure 6.10 - Block plot of grade values (Vertical 362.9) - Case study 2 (Gold)

Figure 6.11 - Block plot of grade values (Vertical 402.9) - Case study 2 (Silver)

Figure 6.12 - Block plot of grade values (Vertical 382.9) - Case study 2 (Zinc)

Figure 6.13 - Block plot of grade values (Vertical 392.9) - Case study 2 (Lead)

Figure 6.14 - Block plot of surface topography - Case study 2

Figure 6.15 - Pit limit without pit bottom smoothing - Case study 2

As can be seen from Table 6.6, an increase in the price leads to higher net profit and an increase in the optimum pit depth. It also generates a higher tonnage of ore with a lower mean grade and a higher stripping ratio. The consequence of a decrease in the recovery factor are lower values of total tonnage of ore and waste, net profit, stripping ratio and optimum pit depth and an increase in the mean grade of all metals.

Price	Number of blocks			Computing time
	Ore	Waste	Air	Minute
0.70	1,631	456,475	132,894	1
0.85	2,653	455,453	132,894	8
1.0	3,483	454,623	132,894	38
1.2	4,545	453,561	132,894	237
1.4	5,302	452,804	132,894	338

Table 6.7- Computing time for the second alternative

Table 6.7 shows the computing times to determine the optimum pit for the second alternative with the use of a Pentium 400 PC. All the revenue block models for this case study contain 591,000 blocks but each has a different number of ore and waste blocks as illustrated in Table 6.7. The results indicate that the computing time increases significantly as the price (and, consequently, the number of ore blocks) increases. Thus computing depends not only on the number of blocks in the model but also on the complexity of the deposit as defined by the spatial distribution of ore blocks. As the complexity increases, the computing also increases. As it is difficult to measure the degree of the complexity of a deposit, it is also difficult to predict the computing time.

6.4- Conclusion

The application of the software to two case studies demonstrates that it works and is able to generate an optimum pit limit with variable slope angles, in particular for complex and low grade orebodies. Moreover, the software is capable of generating solutions on a

PC within reasonable times. It also generates numerical and graphical displays of the optimization providing a means of visualizing and interpreting the results of open pit design.

The result shows that the computing time required to reach a solution depends primarily on the complexity of the deposit. As the complexity of the deposit increases computing time increases. However, with the use of a high-speed computer, it is now possible to determine the optimum pit limits with the variable slope version of the Lerchs-Grossmann algorithm for complex and large orebodies.

CHAPTER 7

Conclusions and recommendations for future work

Mineral deposits are generally extracted from the earth either by underground mining or by surface mining methods (open pit mining) and the objective is normally to extract ore at a profit. The growth in demand for the raw material together with advances in mining technology and the depletion of high-grade, readily accessible sources of ore has led to an increase in the use of open pit mining to extract large, low grade deposits. This method permits the use of highly mechanised, large-scale production equipment, which is capital intensive but low in labour costs. Large-scale open pit mining is a complex operation that may extend over several decades and require very large investments.

Smaller scale operations, usually for precious metals, are often more complex because of the erratic nature of the mineralisation. These operations generally have a much shorter life (current developed world average of 5-7 years for gold) but proportionately they require an equally high capital investment. The erratic nature of the mineralisation means that these types of orebodies must be mined in a highly selective manner. In addition, pit designs and mining schedules are very sensitive to changes in prices, costs and grades.

Many studies have shown that the maximisation of net present value requires an operation to start with a relatively high-grade ore, which declines over the life of the mine.

The size, location and final shape of an open pit are important in planning the location of waste dumps, stock piles, processing plant, access roads and other surface facilities and to develop a production programme. The pit design also defines minable reserves and the associated amount of waste to be removed during the life of the operation. This must be determined before the operation starts. The pit design, which is a function of numerous variables, may be re-evaluated many times during the life of the mine as design parameters change or more information is obtained during the operation. The use of computer methods is necessary in order to re-design the pit as rapidly as possible.

The use of computers in designing open pits begins with the development of a block model of the orebody. For this purpose, a large rectangular block is defined, sufficient to cover all the mineralisation. This large block is then sub-divided into a finite number of smaller blocks. Each block in the model is assigned an estimated ore grade obtained by the use of geostatistical or other block grade estimating techniques. Although there are many types of block models, the three-dimensional, fixed block model, obtained by sub-dividing the deposit into three-dimensional fixed-size blocks, is the most widely used and the only one generally applicable to optimum pit design. The vertical height of the blocks is usually set to the planned bench height that will be used in the mining operation. The horizontal dimensions of the blocks depend upon the density of drilling and sampling available to estimate the attributes (e.g. grade) of the blocks.

With the advent and wide spread use of computers a number of algorithms have been developed to determine the optimum ultimate pit limits. The main objective of these algorithms, all of which use the block model, is to find groups of blocks that should be removed to yield the maximum overall mining profit under specified economic conditions and technological constraints. The most common methods are: graph theory, network or maximal flow techniques, various versions of the floating or moving cone, the Korobov algorithm, dynamic programming and parameterization techniques. Of these, the algorithm developed by Lerchs and Grossmann, based on graph theory, is the only algorithm that can be proved, rigorously, always to generate the true optimum pit limit. However, the original algorithm was limited to only one slope angle defined by the block dimensions and was incapable of taking into account variable slope angles.

Many factors govern the size and shape of an open pit. The pit slope, which may vary throughout the deposit, is one of the key factors governing the amount of waste to be removed in order to gain access to the mineral deposit. Small changes in slope angle can change the amount of waste to be removed and significantly affect the degree of selectivity in mining operations. For this reason, it is very important to change slope angles through the deposit in order to follow different structures and rock types and keep the total amount of waste as small as possible. Any truly optimal pit design algorithm must, therefore, take into account variable slope angles. Incorporating variable slope constraints into the Lerchs-Grossmann algorithm will make it much more flexible, practical and reliable. Although many attempts have been made to incorporate variable slope angles into the algorithm, none of them provide an adequate solution where there are variable slopes controlled by complex structures and geology. In chapter 3, the Lerchs-Grossmann algorithm was reconsidered and modified to deal with variable slope angles. It is assumed that the orebody and surrounding waste are divided into regions or domain sectors within which the rock characteristics are the same and each region is specified by four principal slope angles: north, south, east and west faces. Pit slopes were then approximated by constructing a cone representing an extraction volume from the base block and extending it to the surface in such way that the side angles of the cone are equal to the four principal slope angles. This method was incorporated into the algorithm and its operation demonstrated by examples and two case studies in chapter 6. One of the most important outcomes of the work described here is that slope angles can vary throughout the deposit to follow rock characteristics without changing the block dimensions.

The first step for determination of the optimal pit shape is to create a revenue block model of the deposit. This model is the basis on which the pit is constructed. To do this, an orebody block grade model is required. In the work described here it is assumed that this model has already been created and that the block attributes have been estimated by geostatistical or other block grade estimation methods. The method presented in Chapter 3 creates an orebody revenue block model in which for each block a net value is calculated by applying physical and economic factors to the block grade model. In addition, when multiple mining slope regions are specified, four principal slope angles are assigned to the blocks. Finally, additional waste blocks are added around the borders of the block model in order to render minable the ore blocks at the bottom-most edge of the deposit. The optimum pit generated by the Lerchs-Grossmann algorithm may have a very irregular bottom, which may not be feasible in practice because of the minimum space required for mining equipment to operate freely. It is possible to smooth the pit floor by incorporating minimum access space requirements into the algorithm but this significantly increases the computing time required to reach a solution. The method presented in Chapter 3 determines the technical optimum pit limit and smooths the pit by removing non-practical blocks on the pit bottom and/or adds blocks to the optimum solution.

The Lerchs-Grossmann algorithm with variable slope angles requires an estimation of a set of average and safe slope angles to determine the optimum pit limits. Various procedures may be used to determine pit slope angles. If little geotechnical information is available they may be found from other mines of similar size and geological conditions. If the mine is in operation, predefined or existing pit slopes can be used to find the optimum pit limits. In addition, in Chapter 4 two approaches and associated programs - steepest safe angle and the limit equilibrium method - were presented to estimate a set of safe slope angles from the geotechnical information. The first approach determines the steepest safe angle with regard to structural instability including plane and wedge failures without taking into account cohesion, density, ground water conditions and non-structural failure. This approach can be used at design stages when little information is available (e.g. the pre-feasibility stage). The second approach, which includes methods for the design of slopes using limit equilibrium analysis, can be used when the input data are defined as fixed values (factor of safety) or as random variables (probability of failure). In the latter case the Monte Carlo simulation technique can be used to generate values of the variables. The advantage of a probabilistic approach over deterministic methods is that it takes into account the uncertainty inherent in the input data. Both approaches require engineering judgment to select slope angles from the graph provided by the program. It should be noted that structural instabilities that can be determined kinematically are used to design slope angles. If the geology is complex potential failure modes may involve a combination of several mechanisms and when the geometry of the slopes is well-known, slope angles must be determined by more sophisticated methods such as numerical modelling.

Almost all computer algorithms are based on a block model of the deposit, which requires a large memory to store block characteristics. Due to the limitation of memory under the DOS operating system, it is not possible to implement the Lerchs-Grossmann algorithm for block models that contain large numbers of blocks. DOS programs can use only 640 kb RAM memory, no matter how large the computer memory. Only relatively small deposits with up to approximately 50 x 50 x 10 blocks can be implemented on PC computers operating under DOS. One possible solution to overcome the memory limitation is to implement the algorithm on a workstation. However, workstations are not as widely used and available as PCs. In addition, PCs are rapidly approaching workstations in the storage and computational capabilities. Another solution is to use random access files to store block characteristics but this significantly increases the computing time required to determine the optimum pit. The recent developments in both PC hardware and software have been so rapid that many applications, including those in the minerals industries, have still to take advantage of the enormous potential now available. The **PITWIN32** software, written in C++ and presented in Chapter 5, is a Windows application running under 32-bit operating systems such as Windows 95, Windows NT and Windows 98. It overcomes all memory limitations by using dynamic memory allocation to store block characteristics. The application of the software to optimal open pit design is very simple and non-computer professionals can use it.

Because of a different style of programming, the software is much faster than similar programs written in FORTRAN code.

Any software without a good output presentation loses much of its functionality and purpose. Optimal pit design programs must be able to display the results of optimisation in such way that the user can readily understand all aspects of the design. As shown in Chapter 6, the **PITWIN32** software provides graphical and numerical displays of input data and results as well as creating output files that can be subjected to further processing. These outputs display both the economic and technical optimum pits in plan and cross-sectional views and give results level by level including a summary of blocks in the pit, tonnage of ore and waste, value of ore and waste and mean grade of ore. The software provides displays of the surface topography and block plots of grade values.

The main objectives of this work were to solve the variable slopes problem, incorporate slope design in optimum open pit design and implement the methods in general PC-based software. It is now possible to use the Lerchs-Grossmann algorithm with variable slope angles in planning the optimum pit limit for large, complex orebodies in which the pit slopes can vary throughout the deposit at every stage of the operation. However, many improvements can still be made and the following is a summary of recommendations for future work.

- Production scheduling was excluded from this study. It is essential to develop and investigate an efficient method for production scheduling within the planned pit with, for example, the use of artificial intelligence methods or other operation research techniques, and incorporate it into the software.
- Haul roads are another important feature of open pit design that was not considered in this study. It is recommended that a method for incorporating haul roads to be investigated and added into the software. For example, the method of Onur and Dowd (1993) could be readily adapted and incorporated into the software described here.

- To develop and incorporate a three-dimensional graphical display for visualisation of the optimum pit and the topography.
- To develop and incorporate a graphical display for the optimum pit limits on cross-sections for any direction.
- To add and incorporate geotechnical information for designing bench face angles.

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Appendix A

Instructions for executing software functions

A.1- To create control file

The New command in the File menu invokes various dialogue boxes for entering the data required for pit optimisation and writing them into a text file (control file) for further use. These data include the name of the file containing the block grade model of the deposit, cost of mining, recovery factor and pit slopes. The following steps are required to create a control file:

1- If necessary, start the **PITWIN32** software.

2- From the File menu, select the New command.

   The program displays the **Control File Dialog (Basic Information)** dialogue box illustrated in Figure A.1.

3- Enter the required information and use radio buttons to specify the type of the grade file, cost of mining and recovery factor.

4- Choose **OK**.

   Depending on the specification of the cost of mining, different dialogue boxes are used for data entry. If the user defines cost of mining as a fixed value the dialogue box shown in Figure A.2, otherwise the dialogue box shown in Figure A.3 is displayed.

5- Enter the cost of mining.

6- Choose **OK**.

   Depending on the specification of the recovery factor, different dialogue boxes are displayed for data entry. If the user defines the recovery factor as a fixed value the dialogue box shown in Figure A.4 is displayed, otherwise the dialogue box illustrated in A.5 is displayed.

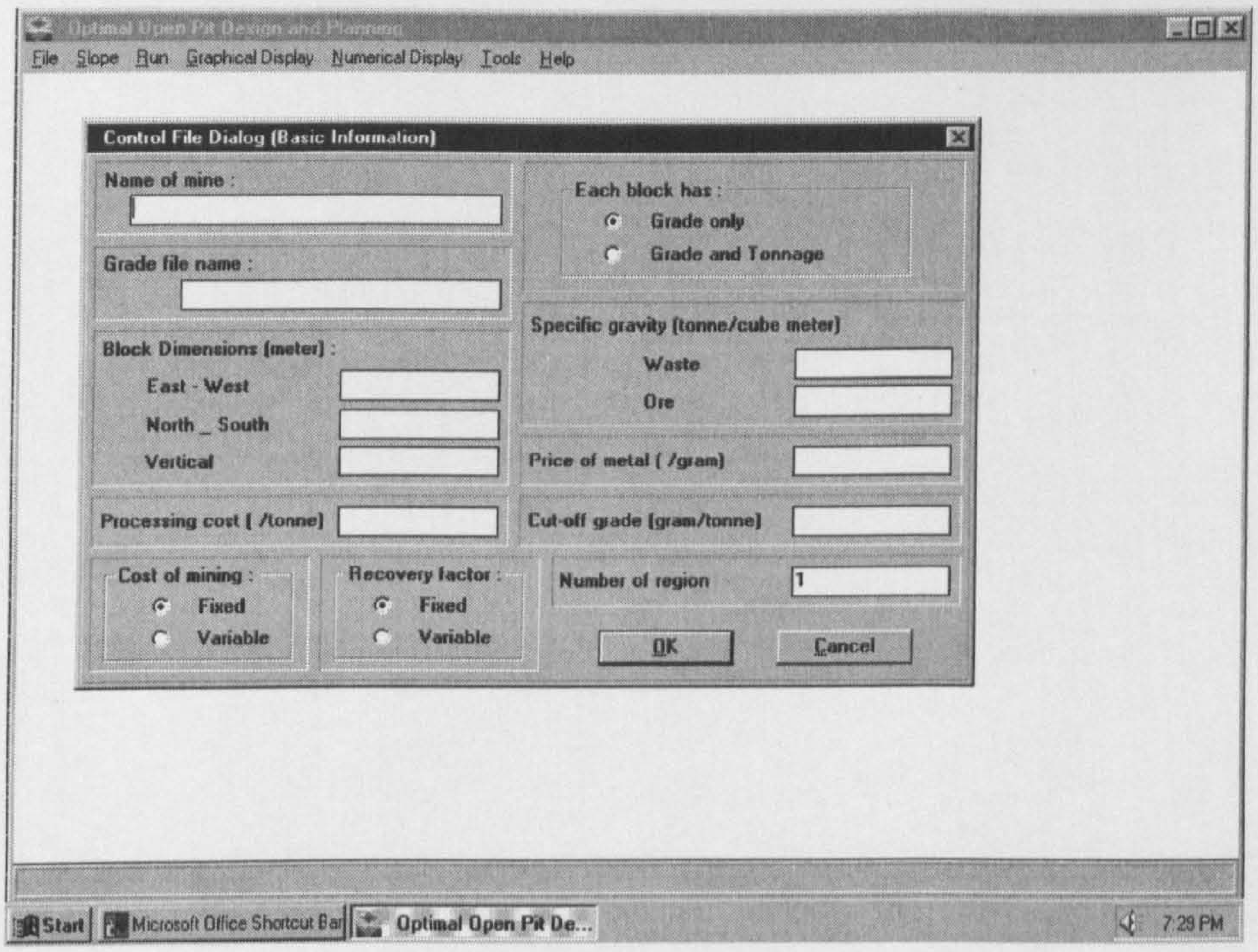

Figure A.1- Dialogue box for entering basic information

7- Enter the recovery factor.

8- Choose **OK**.

If one region is specified, only four principal slope angles are required to define pit slopes (Figure A.6). If multiple regions are specified additional data including minimum and maximum depth and co-ordinates of each region are requested (Figure A.7).

9- Enter the required information on the pit slopes.

10- Choose **OK**.

The program displays the **Save As** dialogue box shown in Figure A.8.

11- In the File name List box, enter the control file name.

12- To store the control file in a different directory or in a different drive, select the directory from the **Folders** list box and use **Drives** list box to choose an appropriate drive.

13- Choose **OK**.

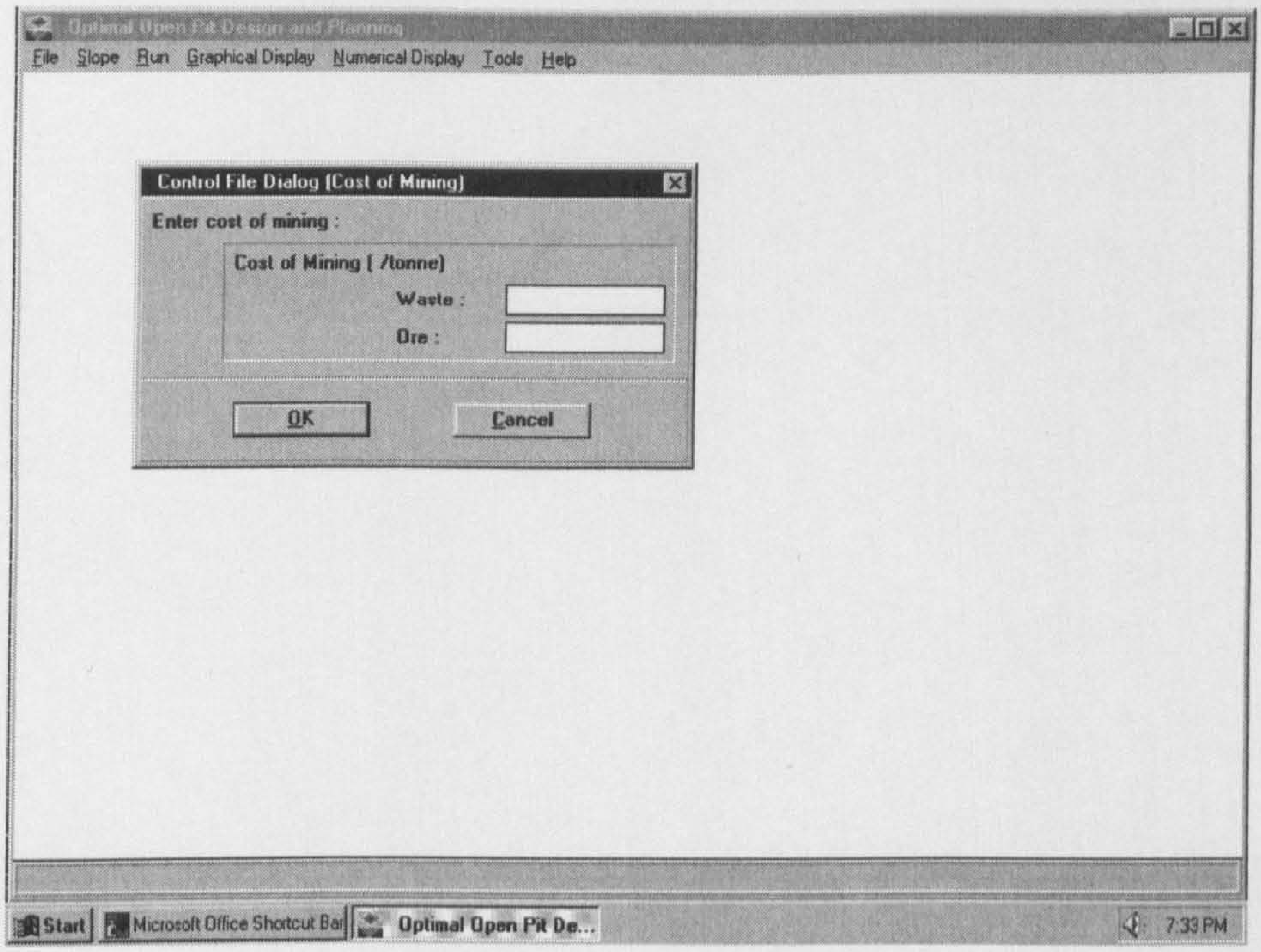

Figure A.2- Dialogue box for entering fixed cost of mining

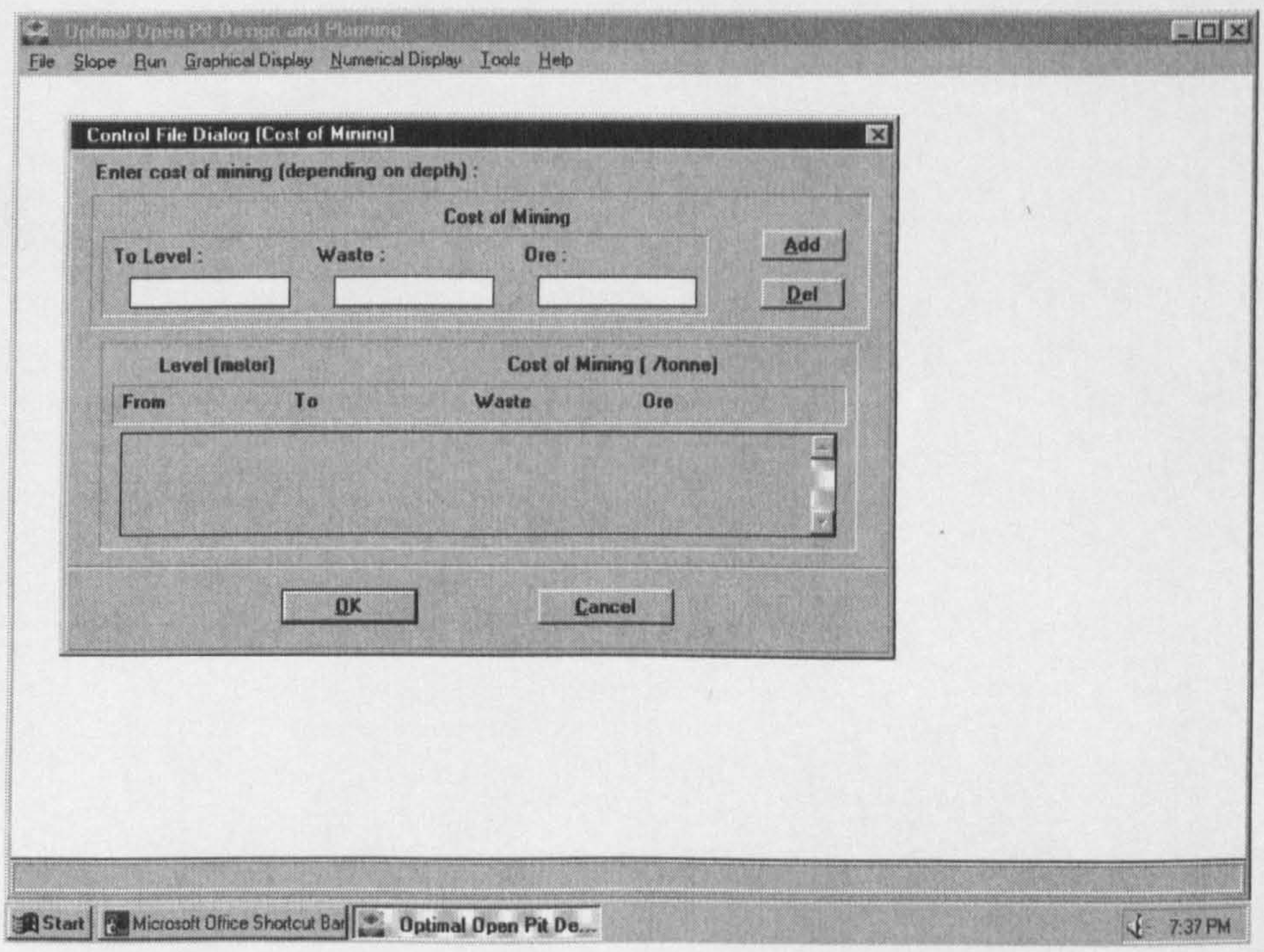

Figure A.3- Dialogue box for entering variable cost of mining

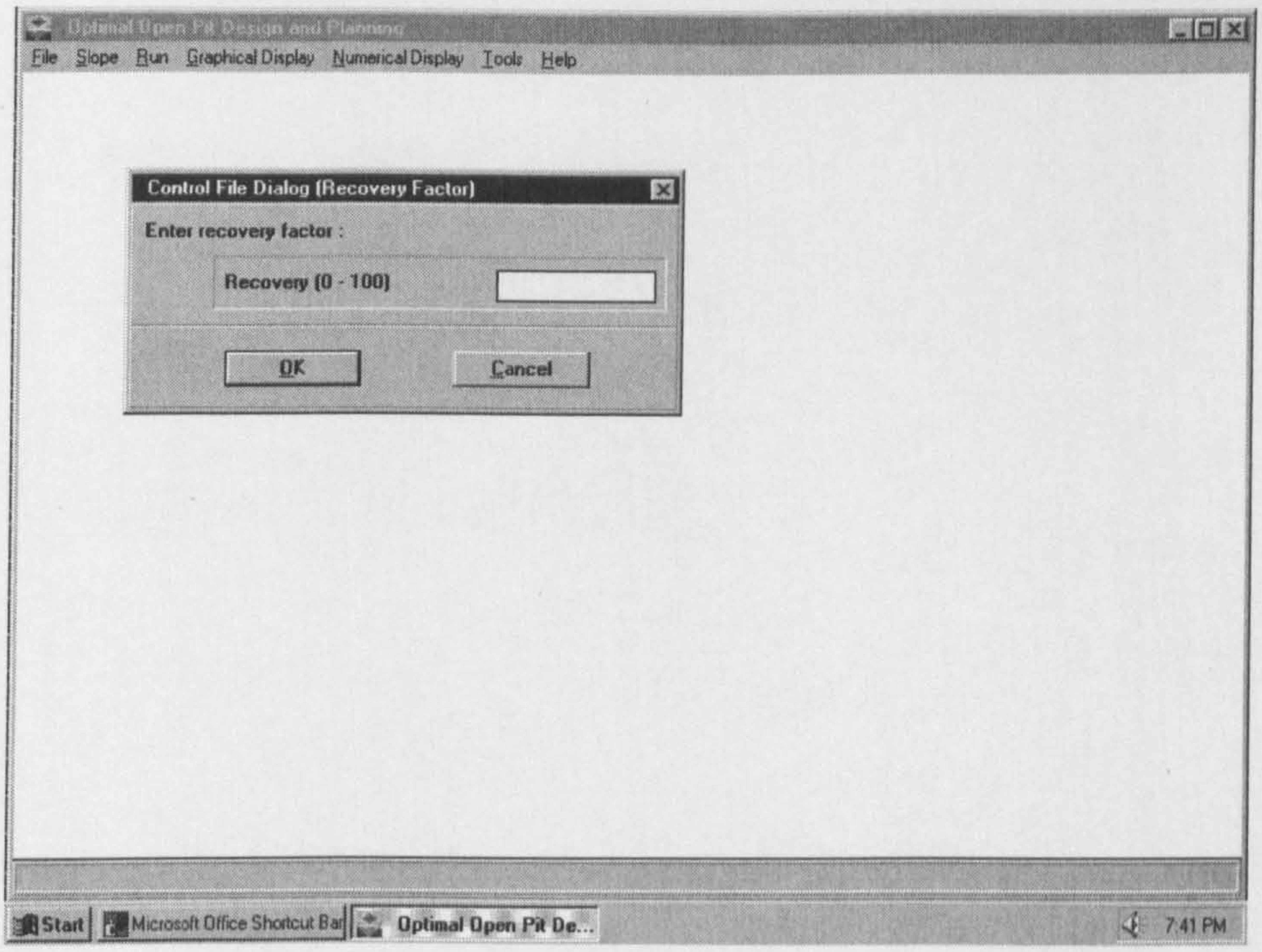

Figure A.4- Dialogue box for entering fixed recovery factor

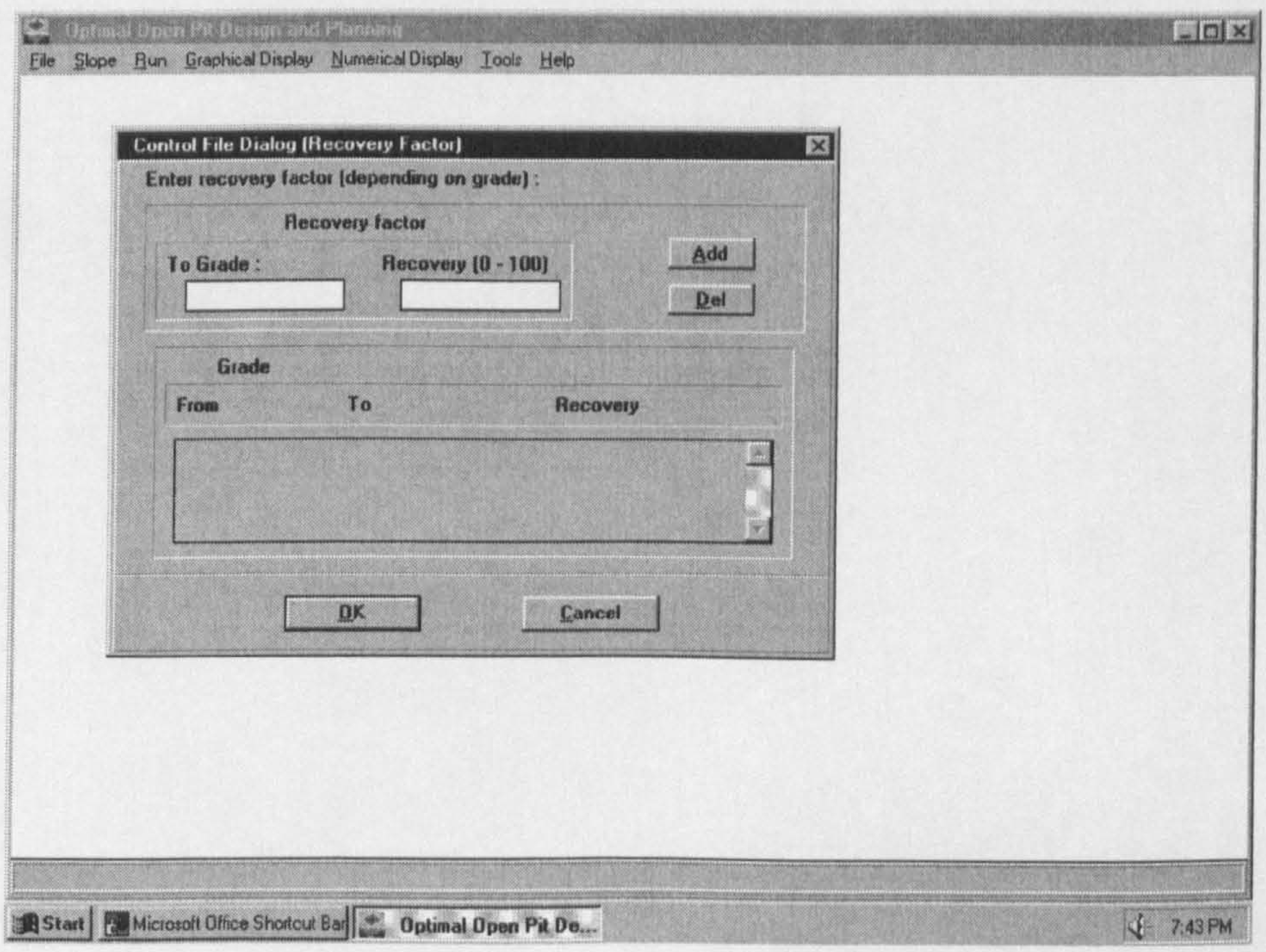

Figure A.5- Dialogue box for entering variable recovery factor

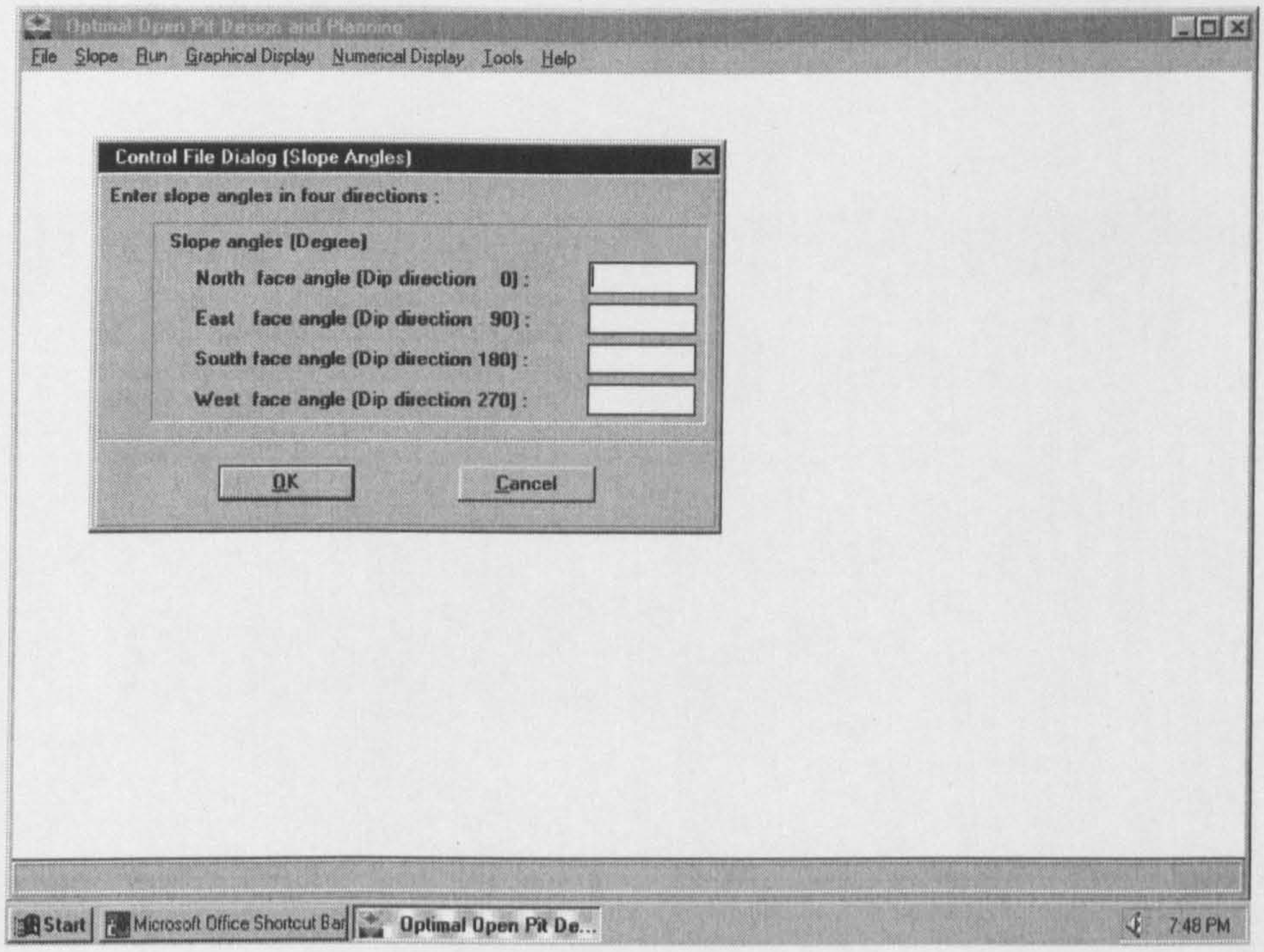

Figure A.6- Dialogue box for entering slope angles

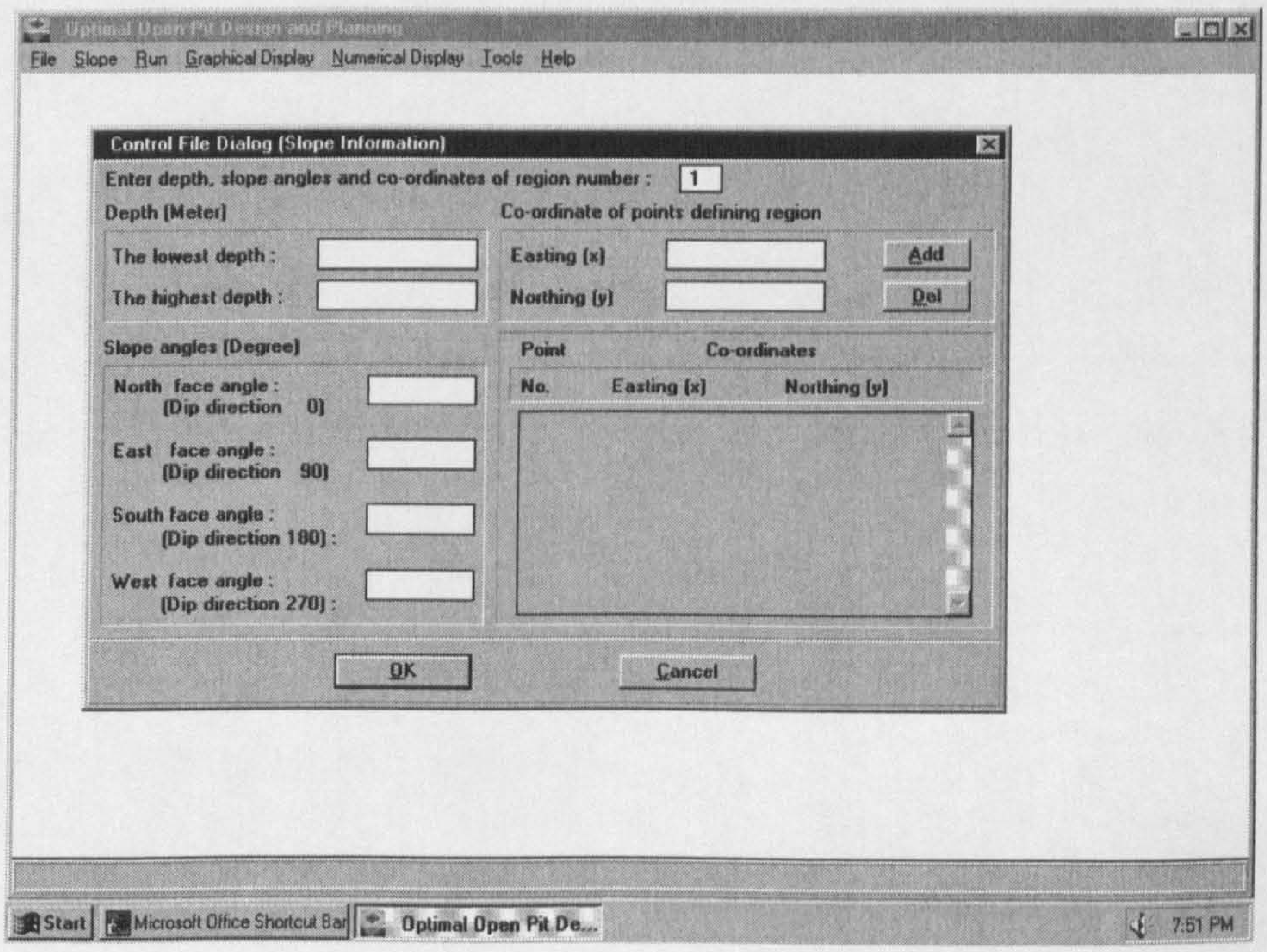

Figure A.7- Dialogue box for entering multiple slope angles

If the control file is successfully created, the name of the mine will be added to the title bar.

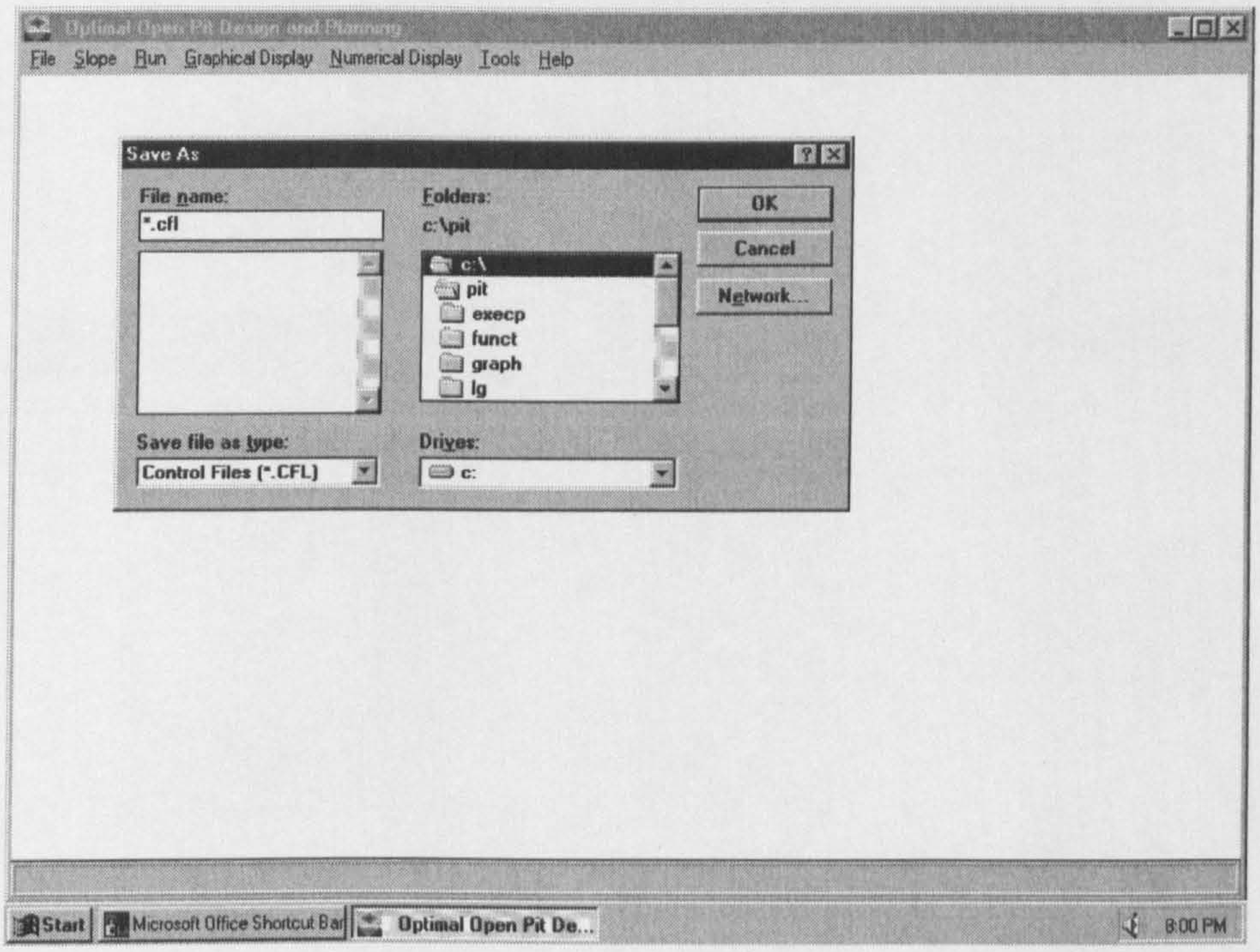

Figure A.8- Save as dialogue box for storing a control file

It should be noted that some dialogue boxes for data entry (e.g., Figures A.3 and A.5) contain two extra buttons: **A**dd and **D**el. The former can be used to transfer data from edit boxes to the list box below. The latter can be used to delete data from the list box provided that they have been transferred to the edit boxes. Data can also be transferred from the list box to edit boxes by clicking the left mouse button. It is possible to make changes to data with these two buttons.

A.2- To load an existing control file

The following steps will load a control file:

1- If necessary, start the **PITWIN32** software.

2- From the **F**ile menu, select the **O**pen command.

The software displays the **Open** dialogue box shown in Figure A.9.

3- Locate drive, directory and then select the file to load.

4- Choose **O**K.

If the control file is successfully loaded, the name of the mine will be added to the title bar.

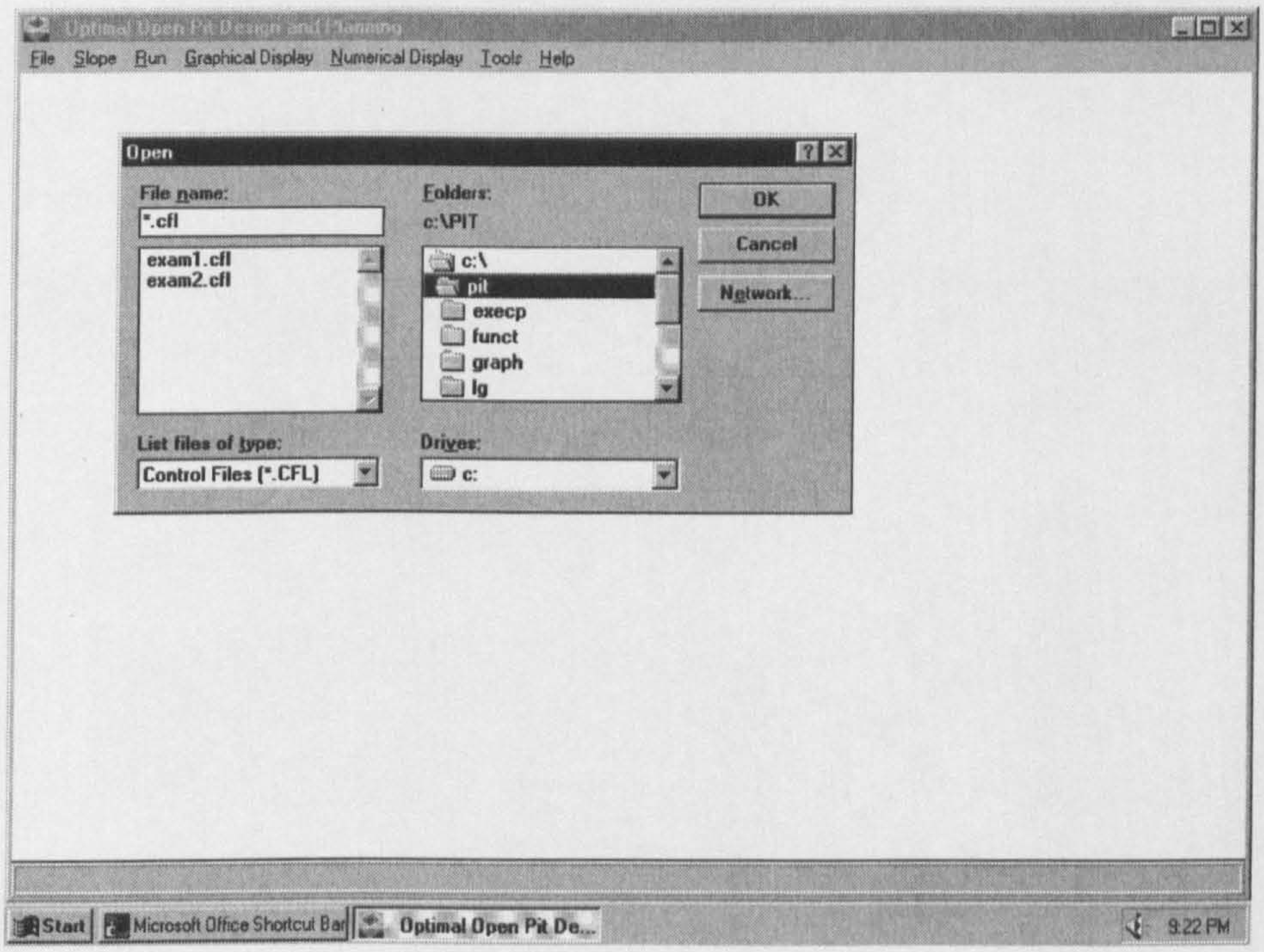

Figure A.9- Open dialogue box for loading a control file

A.3- To edit an existing control file

The following steps are used to edit an existing control file:

1- If necessary, start the **PITWIN32** software.

2- Choose the **O**pen command from the **F**ile menu to load a control file, if it has not already been loaded.

3- From the **F**ile menu, choose the **E**dit command.

The software invokes the various dialogue boxes in the same way as the

New command does.

4- Edit the information displayed in the dialogue boxes.

5- Follow the steps shown in the New command.

It should be noted that when an existing control file is stored with the previous name, the software displays a message before saving the file, pointing out that other output files have been created before are not accessible and will be overwritten. This is due to possible changes in the control file data that may invalidate the output files. For example, if the cost of mining is changed, any previously created orebody revenue block model is invalid. These files must, therefore be created again.

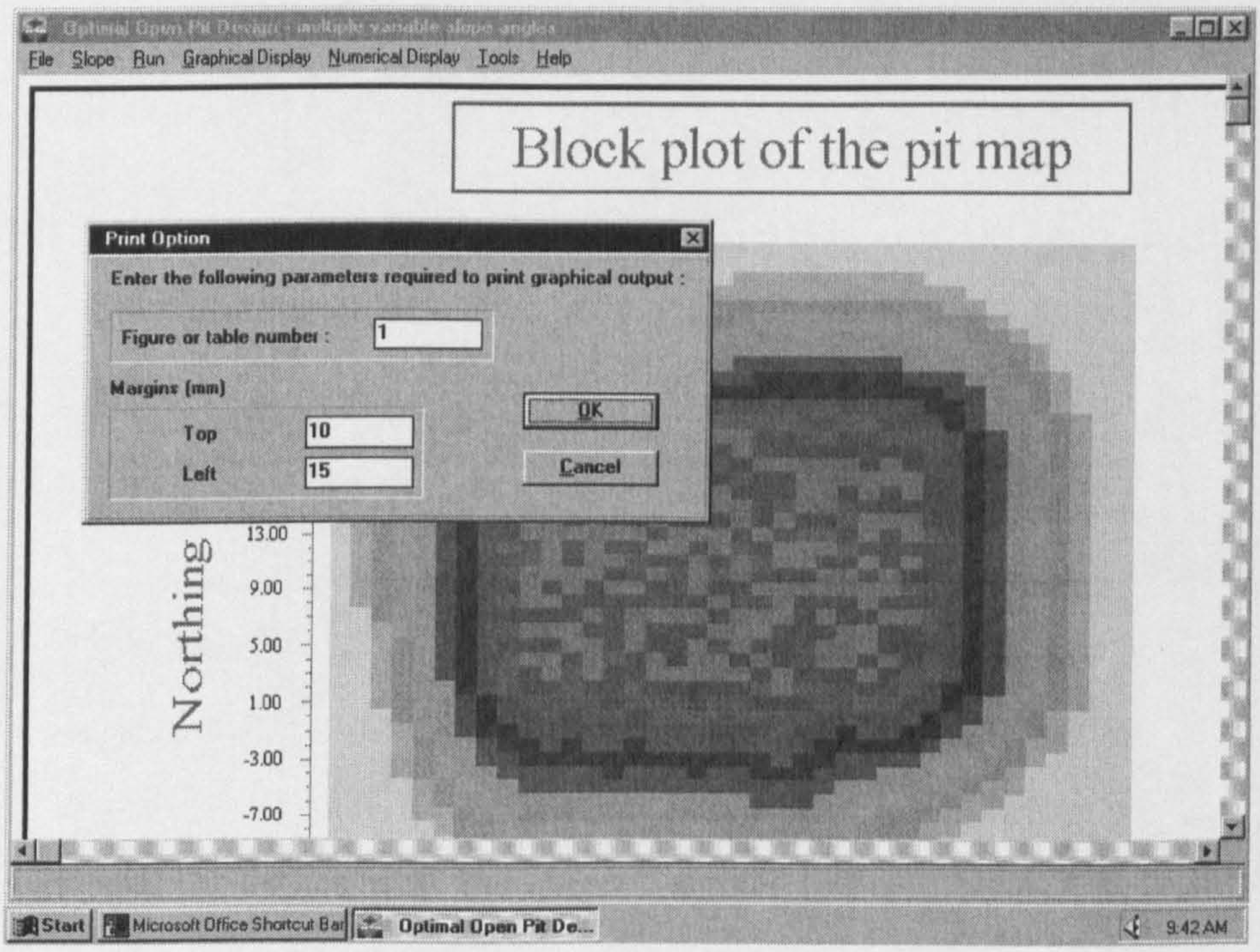

Figure A.10- Print Option dialogue box

A.4- To generate hard copy

The following steps generate a hard copy of the results displayed in the client area of the software:

1- If necessary, start the **PITWIN32** software.

2- If necessary, from the **File** menu, select **Open** to load a control file.

3- If necessary, use an appropriate command to display the required graphical or numerical results.

4- If necessary, from the **Tools** menu, select the **Graphical Display Option** command to choose whether the graphical outputs are displayed with or without scale and to choose whether the X-axis is oriented east-west or north-south.

5- If necessary, from the **File** menu, select Print **Setup** command.

The software displays the **Print Set up** dialogue box.

6- Select and set up the printer. Use portrait orientation for the pit results and landscape orientation for other outputs. Set the paper size to A4.

7- Click the right button of the mouse to enter the figure number and adjust the margins.

The program displays the **Print Option** dialogue box shown in Figure A.10.

8- Enter the figure or table number, top and left margin.

9- From the **File** menu, choose the **Print** command.

The software displays the **Print** dialogue box for entering the number of copies.

10- Enter the required number of copies.

11- Choose **OK**.

A.5- To enter or edit geotechnical data

The Geotechnical Data command in the Slope menu invokes various dialogue boxes for entering or editing the information required to design slope angles and to store these in a text file with the same name as the control file but with the extension of "**SLD**". If this file has already been created this command invokes information for editing. The information includes the simulation number, seed value for generating random numbers, type of failure criteria used in analysing circular failure, strength and orientation of discontinuities, rock mass strength, density and ground water conditions. The following steps are used to enter or edit the geotechnical data:

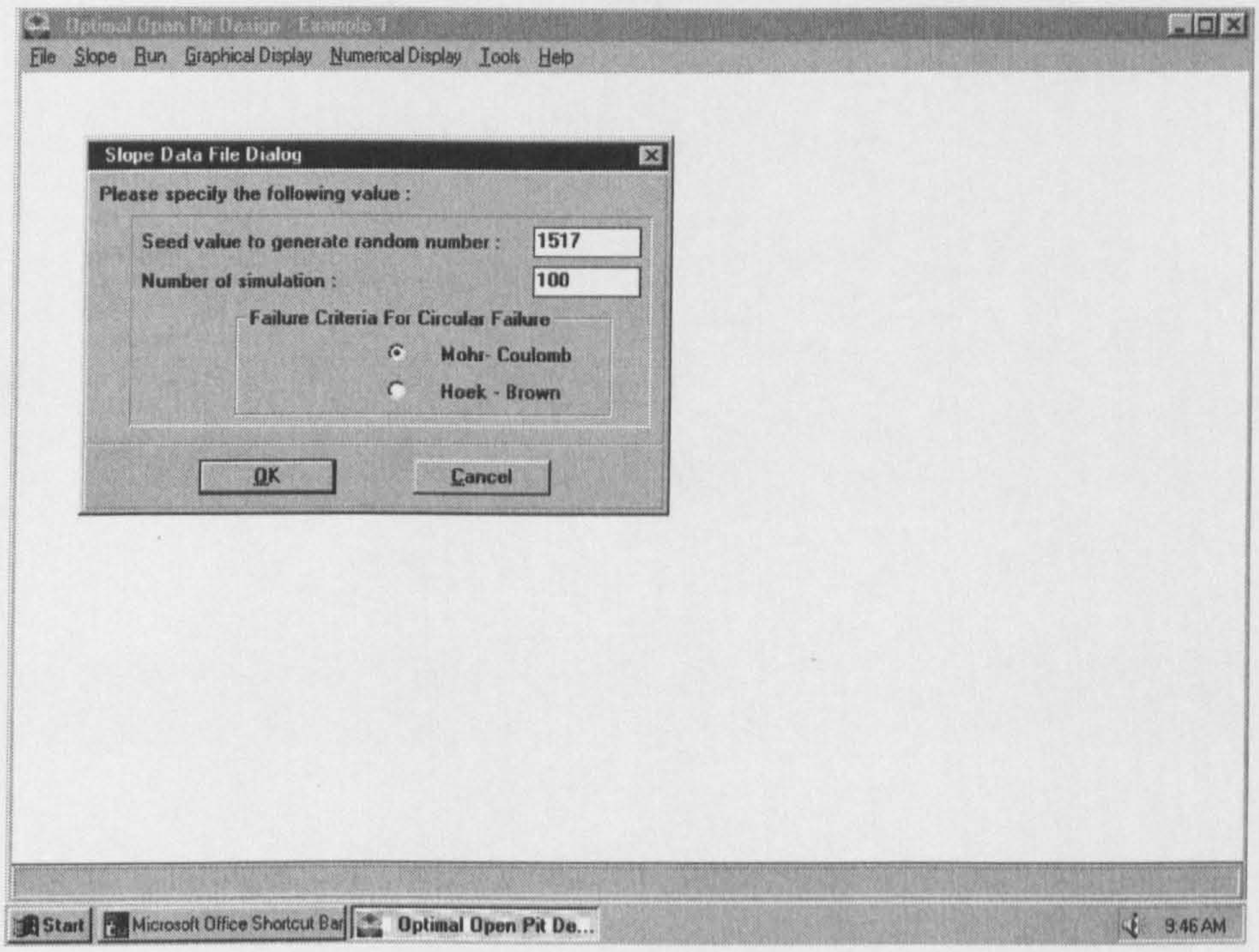

Figure A.11- Dialogue box for entering data

1- If necessary, start the **PITWIN32** program.
2- If necessary, from the **F**ile menu, select **O**pen to load a control file.
3- From the **S**lope menu, select **G**eotechnical Data option.

    The software displays the **Slope Data File Dialog** dialogue box as illustrated in Figure A.11.

4- Edit the seed value and simulation number and use the radio buttons to specify the type of failure criteria.
5- Choose **O**K.

    The program displays the **Slope Data File Dialog (Orientation of Discontinuities)** dialogue box as shown in Figure A.12.

6- Edit or enter the orientation of discontinuities including dip and dip direction for each region.

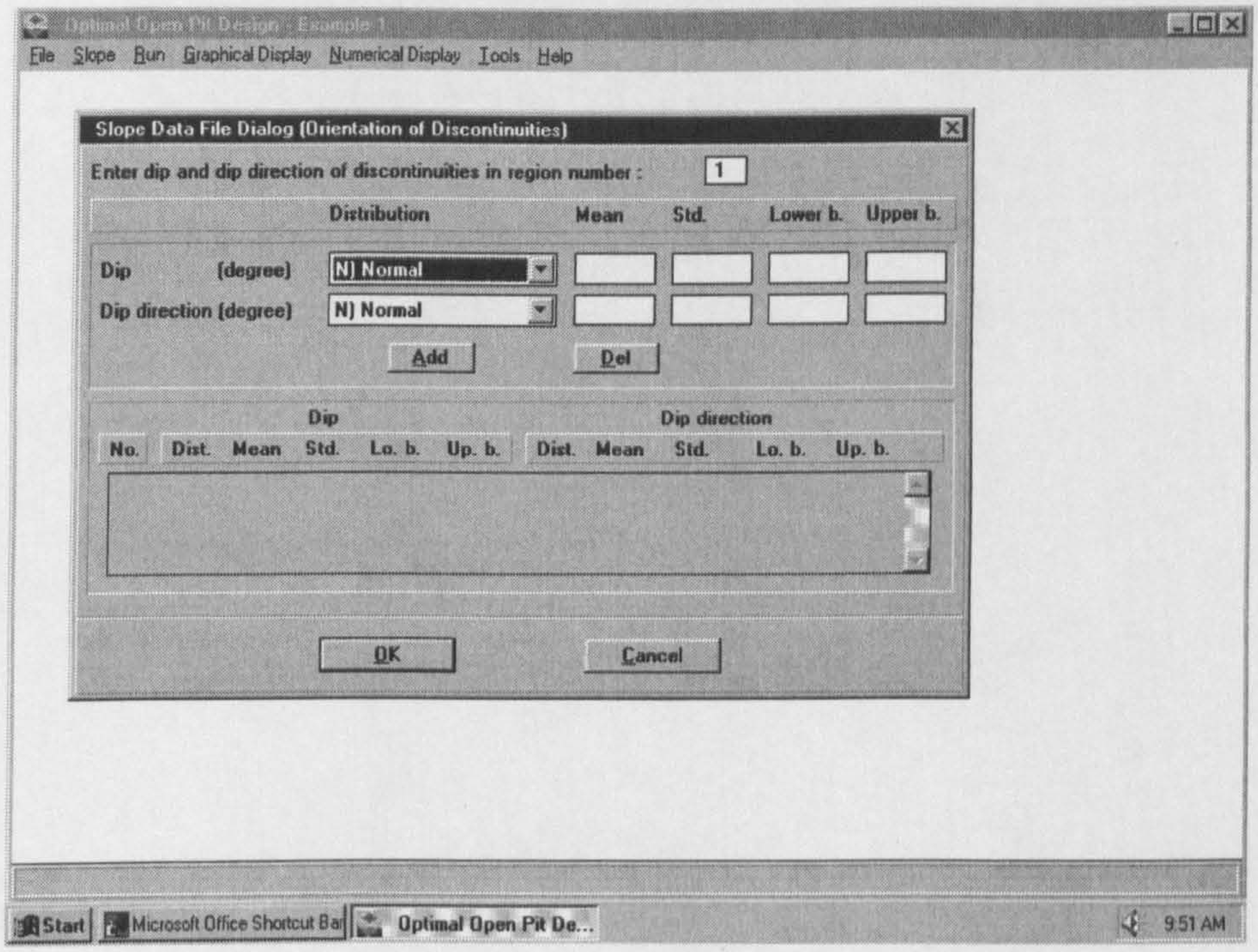

Figure A.12- Dialogue box for entering orientation of discontinuities

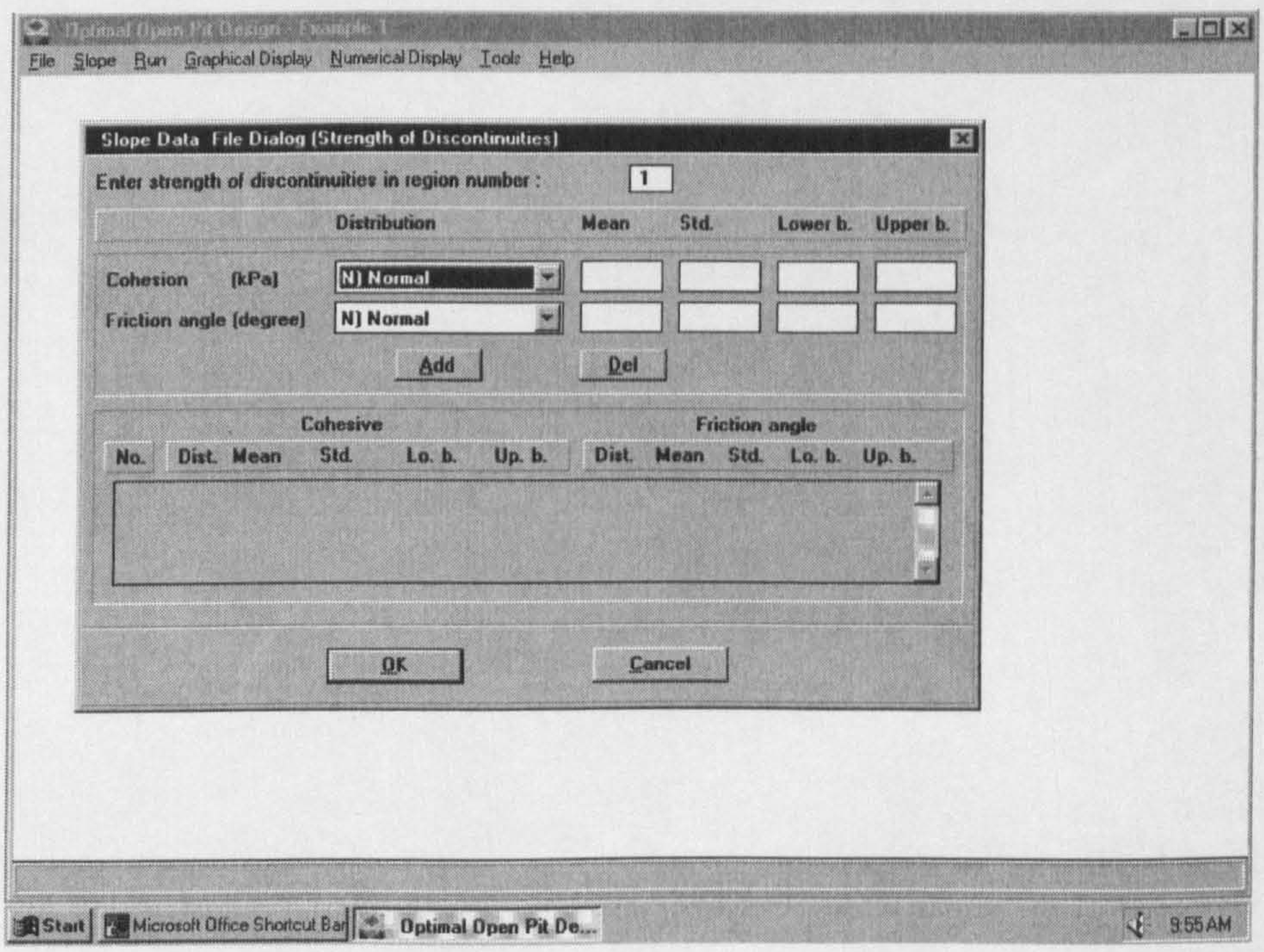

Figure A.13- Dialogue box for entering strength of discontinuities

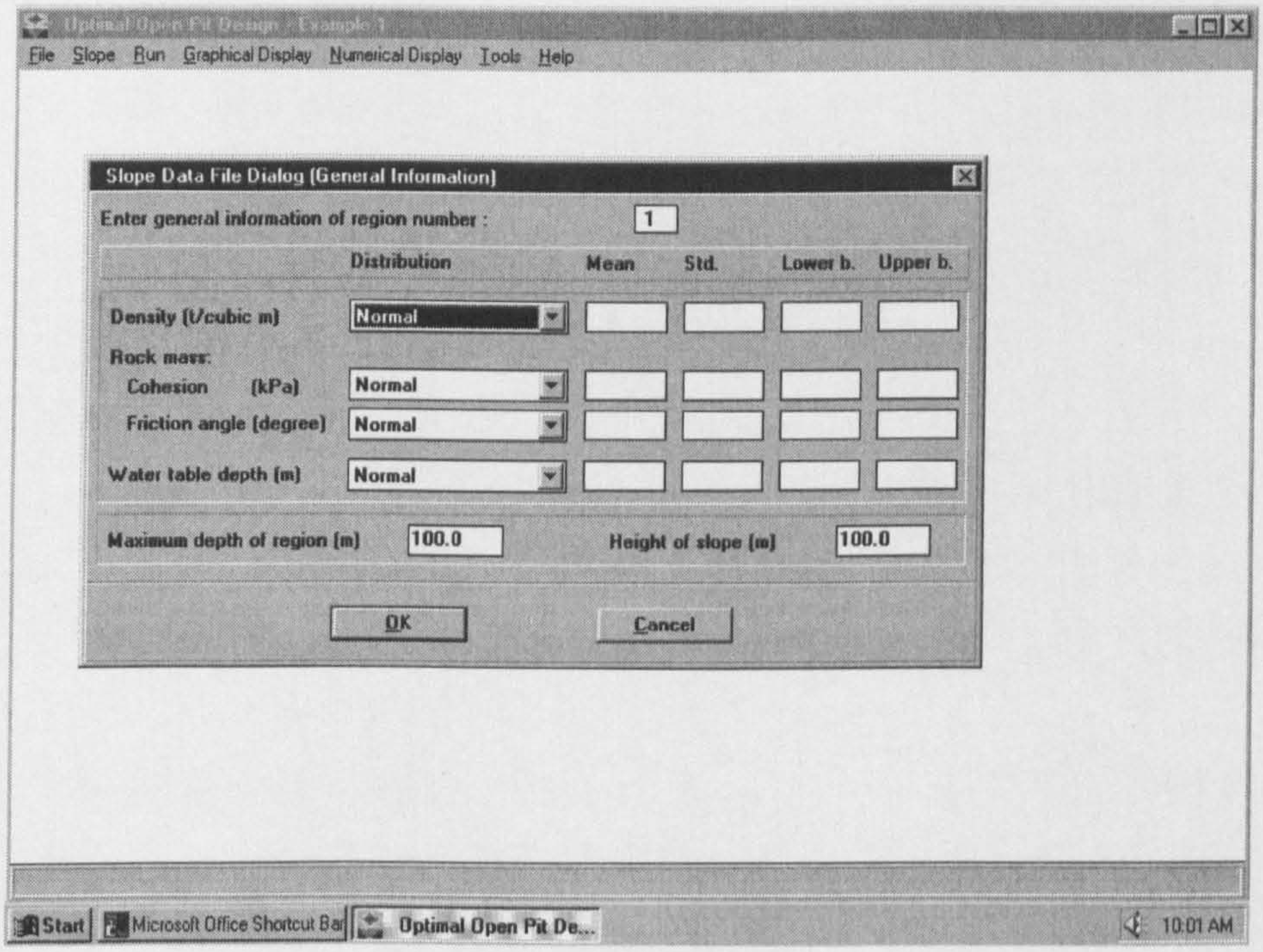

Figure A.14- Dialogue box for entering strength of rock mass

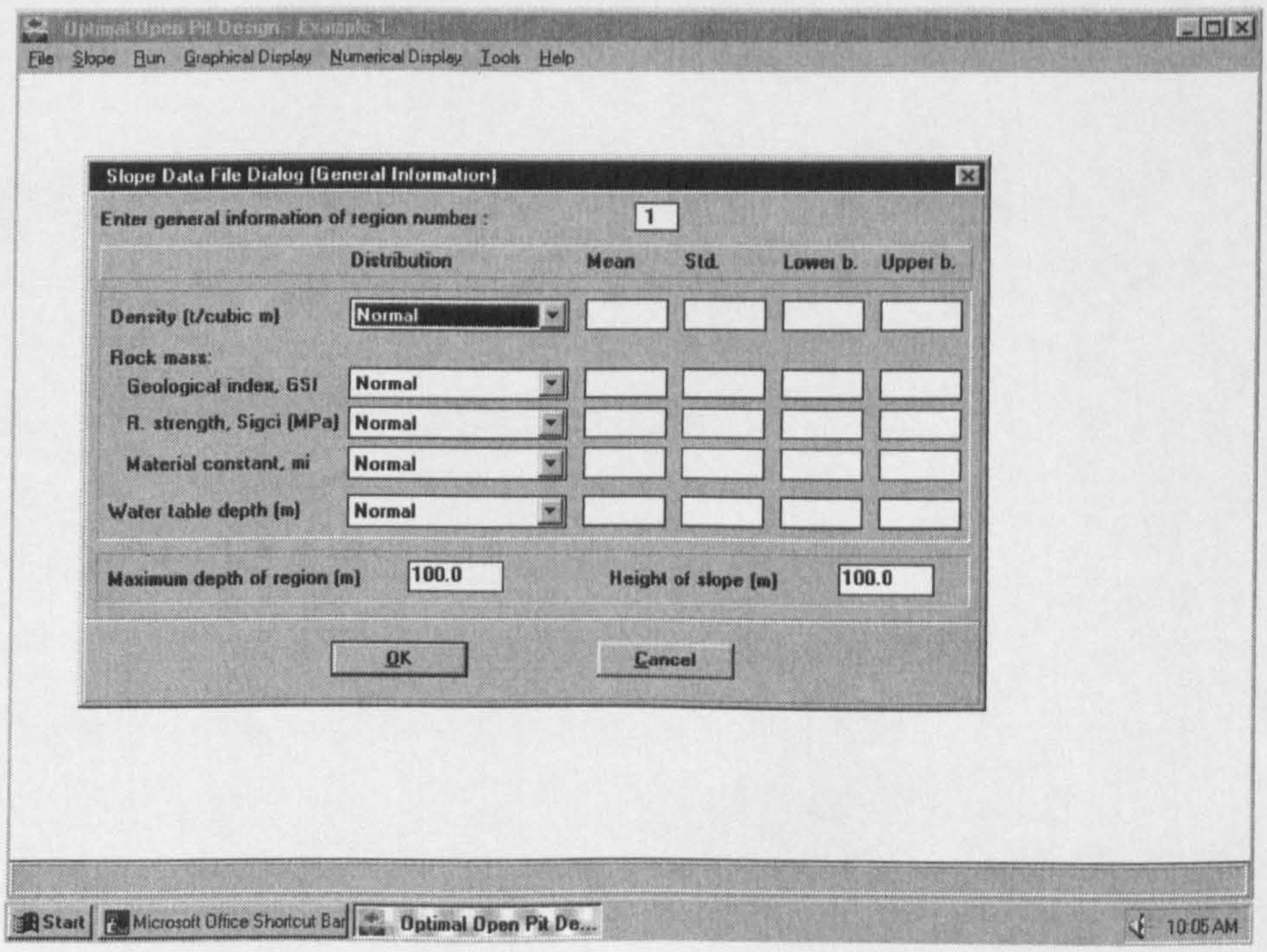

Figure A.15- Dialogue box for entering strength of rock mass

7- Choose **OK**.

The software displays the **Slope Data File Dialog (Strength of Discontinuities)** dialogue box as shown in Figure A.13.

8- Edit or enter the strength of discontinuities including the cohesion and the friction angle for each region.

9- Choose **OK**.

Depending on the type of failure criteria, different dialogue boxes are displayed for data entry. For the Mohr-Coulomb criteria the dialogue box is shown in Figure A.14, and for the Hoek-Brown criteria the dialogue box is shown in Figure A.15.

10- Edit or enter the remaining information including rock mass strength, rock density, water table depth and slope height.

11- Choose **OK**.

12- Repeat steps 6 to 11 for all regions.

It should be noted that any variable with a specified standard deviation of zero is considered to be a constant.

The dialogue boxes illustrated in Figures A.14 and A.15 contain edit boxes for the maximum depth of region and for the slope height. If the optimum pit has not yet been determined then the program sets these two values as the maximum depth of the region. Once the optimum pit has been determined the maximum depth of the region is changed to the maximum depth of the pit and the slope height remains fixed. If there is a significant difference between these two values the user may change the slope height and repeat the whole process - including designing slopes, creating an orebody revenue model and determining the optimal pit - until satisfactory results are obtained.

A.6- To determine the steepest safe angles

The following steps determine the steepest safe angle:

1- If necessary, start the **PITWIN32** software.

2- If necessary, from the **F**ile menu, select **O**pen to load a control file.

3- If necessary, from the **S**lope menu, select **G**eotechnical Data to enter the geotechnical information.

4- From the **S**lope menu, select the option of S**t**eepest Safe angle.

The software calculates and displays the steepest safe angle for all the regions in the dialogue box illustrated in Figure A.16. A slope value of -1 indicates that the angle cannot be determined by this method due to lack of discontinuities. The user can accept, change or abandon these slope angles.

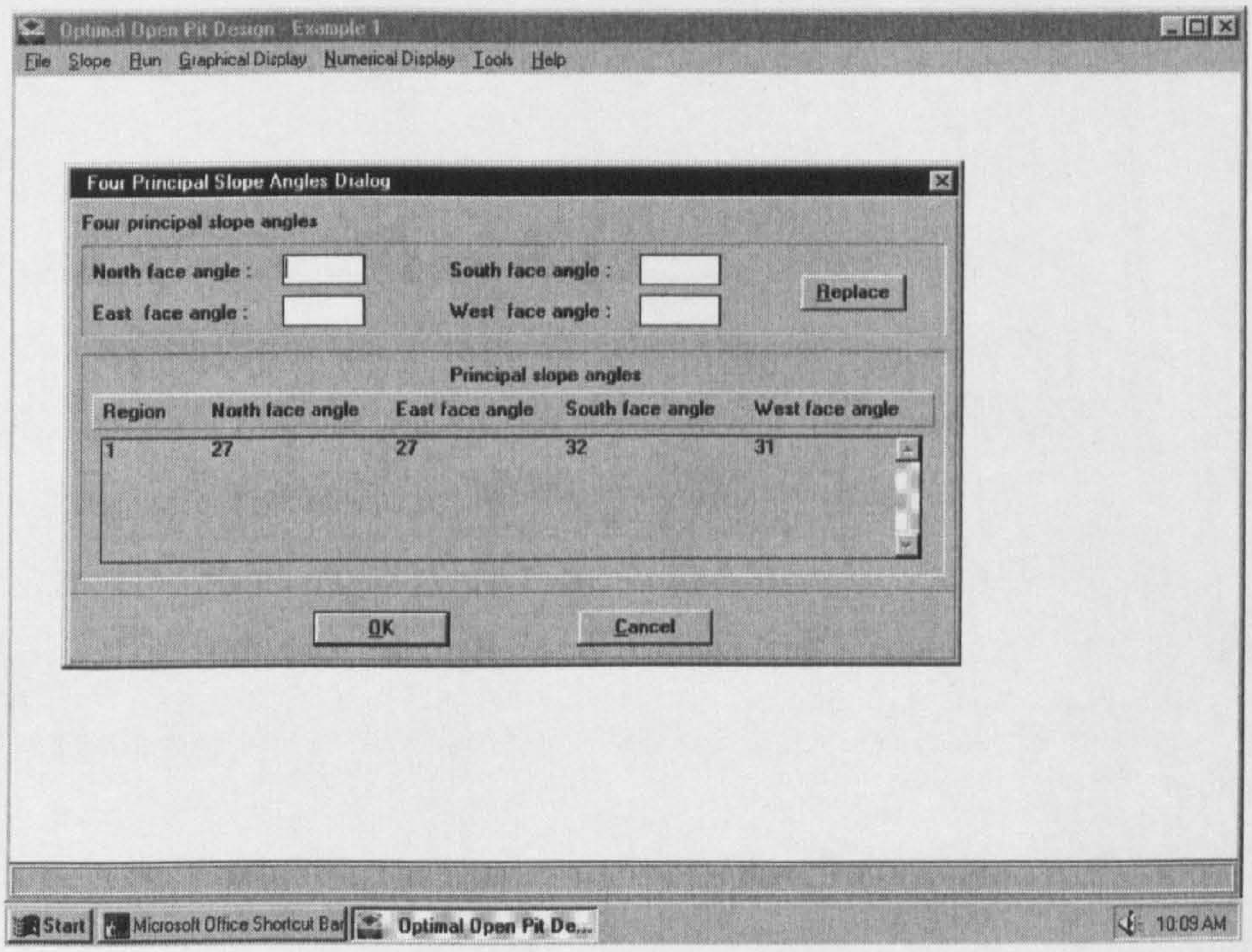

Figure A.16- Four principal slope angles

5- Choose **O**K.

The software displays a message asking the user whether or not the pit is to be designed with the slopes obtained by this method.

6- Choose **Y**es or **N**o to select whether the slope angles obtained by the steepest safe angle method are to be used to determine the optimum pit.

If the response is **Yes** these slopes are replaced by the slope angles in the control file.

The software can also take into account the effects of variation in dip and dip direction of discontinuities on the determination of the slope angles by the steepest safe angle method provided that the lower (minimum) and upper (maximum) values of the orientation of the discontinuities are given.

A.7- To display the results of kinematic analysis

The following steps will display a graphical representation of the results of kinematic analysis:

1- If necessary, start the **PITWIN32** software.
2- If necessary, from the **File** menu, select **Open** to load a control file.
3- If necessary, from the **Slope** menu, select **Geotechnical Data** to enter the geotechnical information.
4- From the **Slope** menu, select the option of **Kinematic Analysis**.

A.8- To determine the slope angles with the limit equilibrium method

The following steps will determine the factor of safety or the probability of failure versus slope angle:

1- If necessary, start the **PITWIN32** software.
2- If necessary, from the **File** menu, select **Open** to load a control file.
3- If necessary, from the **Slope** menu, select **Geotechnical Data** to enter the geotechnical information.
4- From the **Slope** menu, select the command of **Design of Slope**.
5- Wait until the program displays a message, indicating that the slopes are to be designed by the limit equilibrium method.

6- Choose **O**K.

A.9- To display the results of the slope design

The following steps will display the factor of safety or the probability of failure versus slope angle:

1- If necessary, start the **PITWIN32** software.
2- If necessary, from the **F**ile menu, select **O**pen to load a control file.
3- If necessary, from the **S**lope menu, select **G**eotechnical Data to enter the geotechnical information.
4- If necessary, from the **S**lope menu, select **D**esign of Slope to determine the factor of safety or the probability of failure versus slope angle.
5- From the **S**lope menu, select the option of **R**esult of Slope Design.

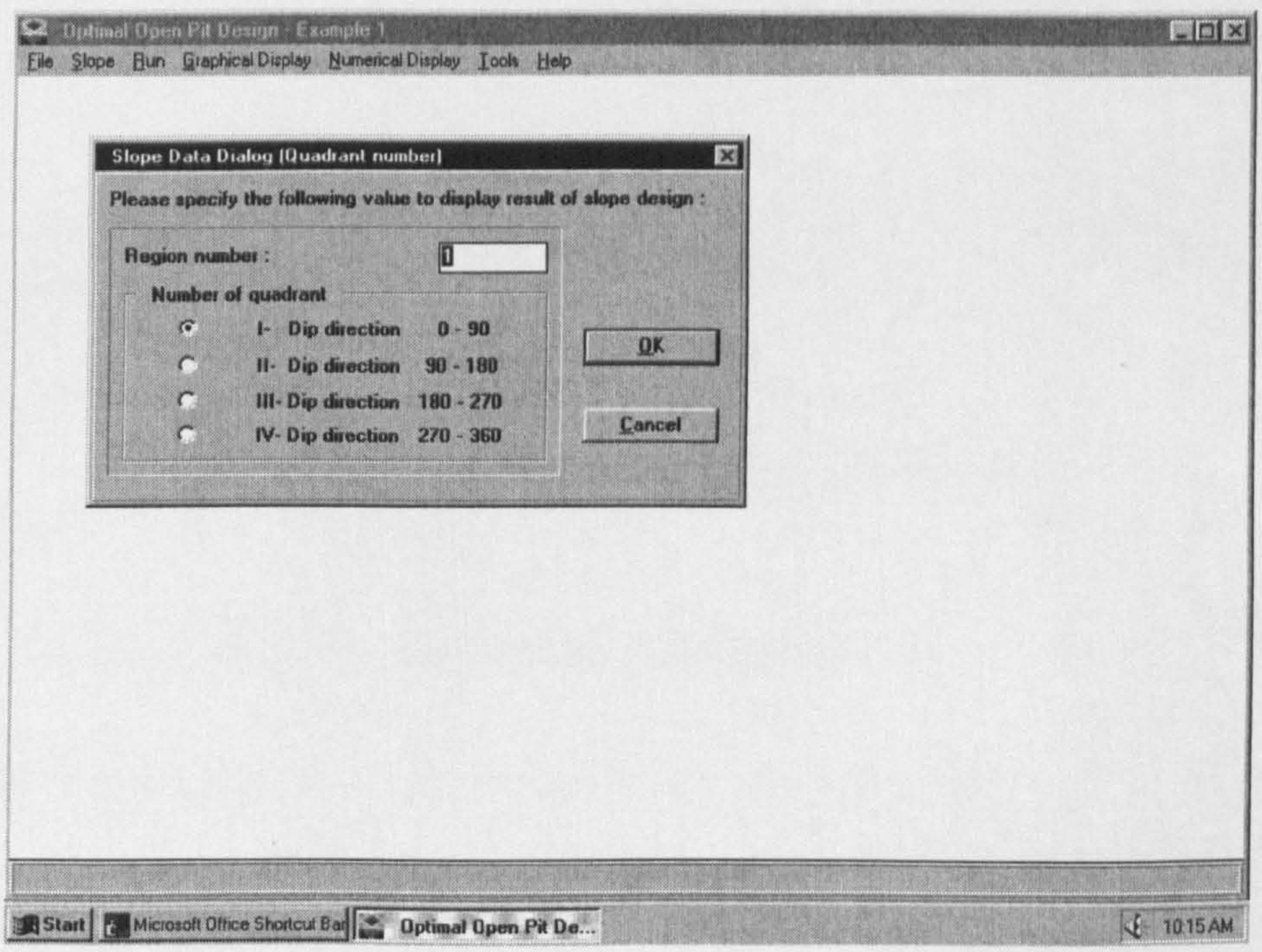

Figure A.17- Dialogue box for entering the region number and specifying the quadrant number

The software displays the **Slope Data Dialog (Quadrant number)** dialogue box illustrated in Figure A.17.

6- Enter the region number and use the radio button to specify the required quadrant.

7- Choose **OK**.

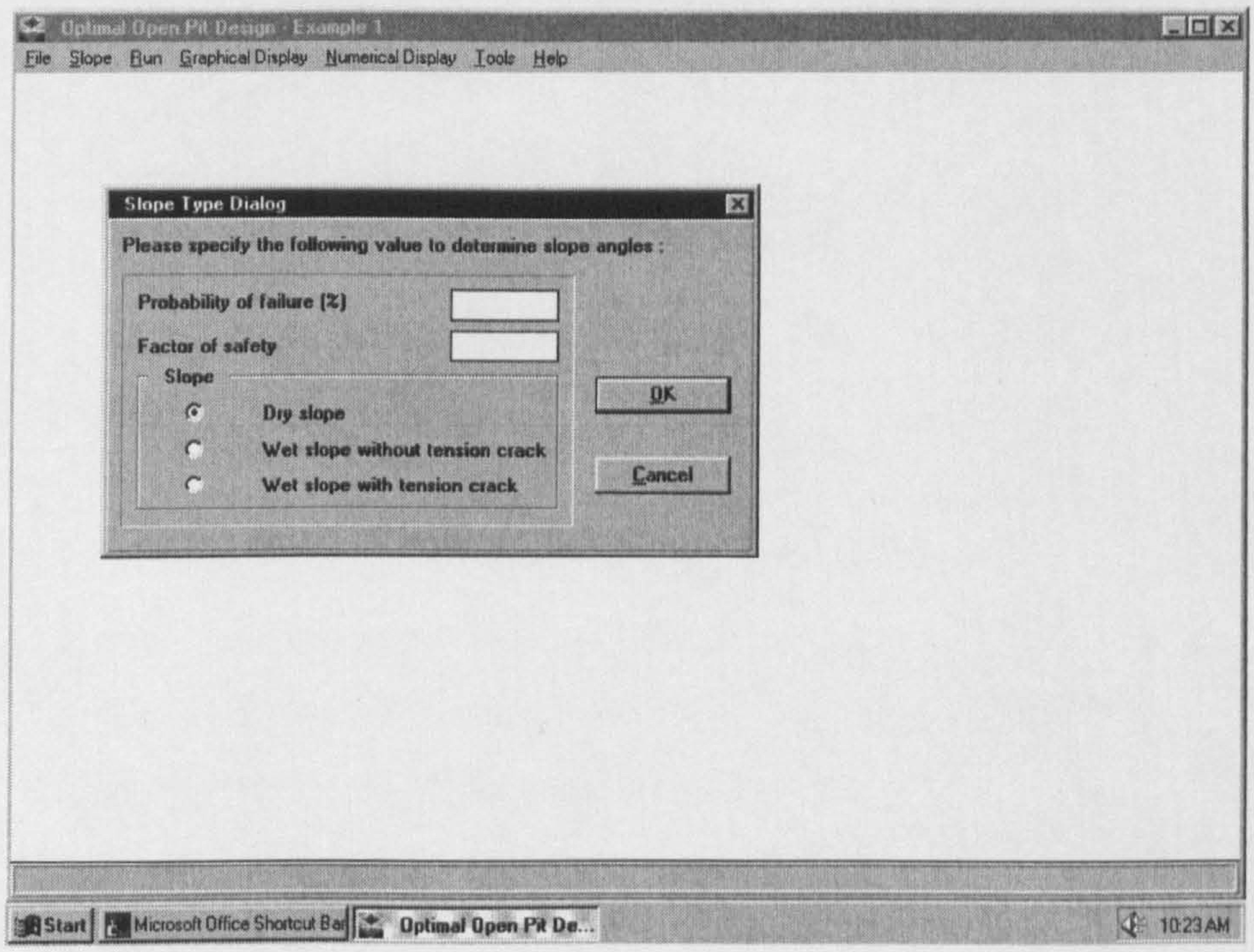

Figure A.18- Dialogue box for entering the information required to determine the four principal slope angles

A.10- To determine the four principal slope angles

The following these steps will determine four principal slope angles:

1- If necessary, start the **PITWIN32** software.

2- If necessary, from the **F**ile menu, select **O**pen to load a control file.

3- If necessary, from the **S**lope menu, select **G**eotechnical Data to enter the geotechnical information.

4- If necessary, from the Slope menu, select **Design** of Slope to determine the factor of safety or the probability of failure versus slope angle.

5- From the Slope menu, select the command of **P**rincipal Slope Angles.

   The software displays the dialogue box shown in Figure A.18.

6- Enter the minimum acceptable value for the factor of safety and maximum acceptable risk of failure and use the radio buttons to select the conditions for the slopes.

7- Choose **OK**.

   The software calculates and displays the four principal slope angles for all regions in the dialogue box illustrated in Figure A.16. The user can accept, change or abandon these slope angles.

8- Choose **OK**.

   The software displays a message asking the user whether or not the pit is to be designed with the slopes obtained by this method.

9- Choose **Y**es or **N**o to specify whether or not to use the slope angles obtained by the limit equilibrium method to determine the optimum pit.

   If **Y**es is chosen, these slopes are replaced with the slope angles in the control file.

A.11- To create an orebody revenue block model

The following steps will create a revenue block model:

1- If necessary, start the **PITWIN32** software.

2- If necessary, from the **F**ile menu, select **O**pen to load a control file. If a control file has not already been created, select the **N**ew command to create it.

3- From the **R**un menu, select the command of Revenue Block Model.

4- Wait until the program displays a message indicating that the orebody revenue block model has been created.

5- Choose **OK**.

A-12- To determine the optimum pit

The following steps will determine the optimum pit:

1- If necessary, start the **PITWIN32** software.
2- If necessary, from the **File** menu, select **Open** to load a control file.
3- Choose Revenue Block Model from the **Run** menu to create an orebody revenue block model if it has not already been created.
4- From the **Run** menu, select the **Optimum** Pit Limit command.
5- Wait until the program displays a message indicating that the optimum open pit limit has been determined.
6- Choose **OK**.

A.13- To carry out pit bottom smoothing

The following steps will smooth the pit bottom:

1- If necessary, start the **PITWIN32** software.
2- If necessary, from the **File** menu, select **Open** to load a control file.
3- Choose Revenue Block Model from the **Run** menu to create an orebody revenue block model if it has not already been created.
4- Choose **Optimum** Pit Limit from the **Run** menu to determine the optimum pit limit if it has not already been determined.
5- From the **Run** menu, select the command of **Pit** Bottom Smoothing.

   The software displays the **Smoothing width** dialogue box for entering the minimum space required to smooth the pit bottom.
6- Enter the minimum space required to carry out smoothing.
7- Choose **OK**.
8- Wait until the program displays a message indicating that the pit bottom has been smoothed.
9- Choose **OK**.

A.14- To display a block plot of the surface topography

Use the following steps to display a block plot of the surface topography:

1- If necessary, start the **PITWIN32** software.
2- If necessary, from the **F**ile menu, select **O**pen to load a control file.
3- Choose Revenue Block Model from the **R**un menu to create an orebody revenue block model if one has not already been created.
4- From the **G**raphical Display menu, select the **B**lock Plot of Surface Topography command.

A.15- To display a block plot of grade value

Use the following steps to display a block plot of the grade value for any level required:

1- If necessary, start the **PITWIN32** software.
2- If necessary, from the **F**ile menu, select **O**pen to load a control file.
3- Choose Revenue Block Model from the **R**un menu to create an orebody revenue block model if one has not already been created.
4- From the **G**raphical Display menu, select the Block Plot of **G**rade Value option.
   The program displays the dialogue box shown in Figure A.19 for entering the level number and the required interval.
5- Enter the required information.
6- Choose **OK**.

A.16- To display the pit limit in plan without pit bottom smoothing

Use the following steps to display the optimum pit without pit bottom smoothing:

1- If necessary, start the **PITWIN32** software.
2- If necessary, from the **F**ile menu, select **O**pen to load a control file.
3- Choose Revenue Block Model from the **R**un menu to create an orebody

revenue block model if one has not already been created.

4- Choose **O**ptimum Pit Limit from the **R**un menu to determine the optimum pit outline if it has not already been determined.

5- From the **G**raphical Display menu, select the **P**it Limit Without Smoothing option.

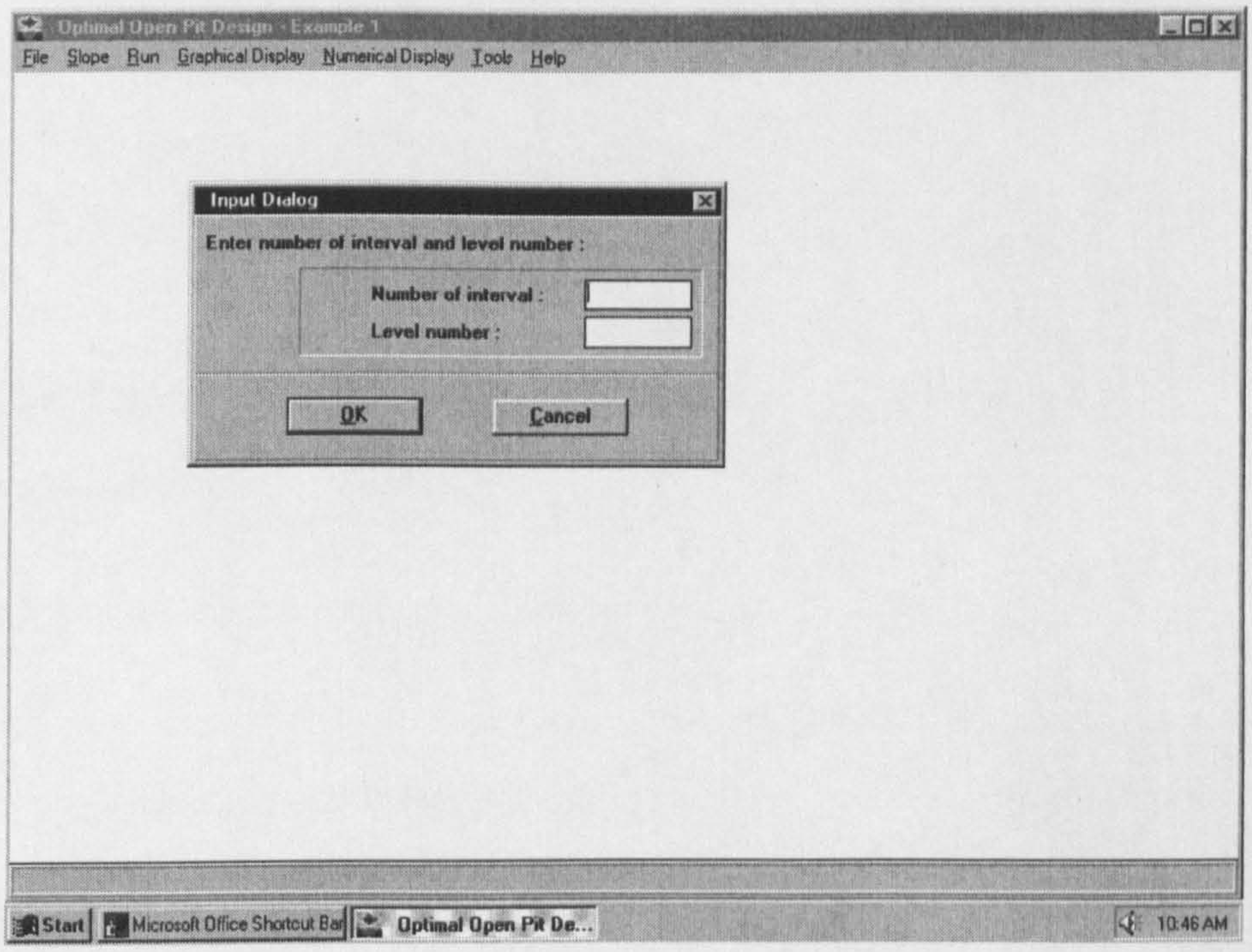

Figure A.19- Dialogue box for entering the level number and the required interval

A.17- To display an east-west cross-section of a pit without pit bottom smoothing

Use the following steps to display an east-west cross-section of a pit without pit bottom smoothing:

1- If necessary, start the **PITWIN32** software.

2- If necessary, from the **F**ile menu, select **O**pen to load a control file.

3- Choose Revenue Block Model from the **Run** menu to create an orebody revenue block model if one has not already been created.
4- Choose **O**ptimum Pit Limit from the **Run** menu to determine the optimum pit outline if it has not already been determined.
5- From the **G**raphical Display menu, select the **E**asting Section Without Smoothing option.
The program displays the **Easting Input** dialogue box.
6- Enter the required Easting of the cross-section.
7- Choose **OK**.

A.18- To display a north-south cross-section of a pit without pit bottom smoothing

Use the following steps to display a north-south cross-section of a pit without pit bottom smoothing:

1- If necessary, start the **PITWIN32** software.
2- If necessary, from the **F**ile menu, select **O**pen to load a control file.
3- Choose Revenue Block Model from the **R**un menu to create an orebody revenue block model if one has not already been created.
4- Choose **O**ptimum Pit Limit from the **R**un menu to determine the optimum pit outline if it has not already been determined.
5- From the **G**raphical Display menu, select **N**orthing Section Without Smoothing
The program displays the **Northing Input** dialogue box.
6- Enter the required Northing of the cross-section.
7- Choose **OK**.

A.19- To display a pit limit in plan with pit bottom smoothing

Use the following steps to display the optimum pit with pit bottom smoothing:

1- If necessary, start the **PITWIN32** software.
2- If necessary, from the File menu, select Open to load a control file.
3- Choose Revenue Block Model from the **Run** menu to create an orebody revenue block model if one has not already been created.
4- Choose Optimum Pit Limit from the **Run** menu to determine the optimum pit outline if one has not already been determined.
5- Choose Pit Bottom Smoothing from the **Run** menu to carry out pit bottom smoothing if the pit has not already been smoothed.
6- From the Graphical Display menu, select the Pit Limit With Smoothing option.

A.20- To display an east-west cross-section of a pit with pit bottom smoothing

Use the following steps to display an east-west cross-section of a pit with pit bottom smoothing:

1- If necessary, start the **PITWIN32** software.
2- If necessary, from the File menu, select Open to load a control file.
3- Choose Revenue Block Model from the **Run** menu to create an orebody revenue block model if one has not already been created.
4- Choose Optimum Pit Limit from the **Run** menu to determine the optimum pit if it has not already been determined.
5- Choose Pit Bottom Smoothing from the **Run** menu to carry out pit bottom smoothing if it has not already been smoothed.
6- From the Graphical Display menu, select Easting Section With Smoothing.
   The software displays the **Easting Input** dialogue box.
7- Enter the Easting of the required cross-section.
8- Choose **OK**.

A.21- To display a north-south cross-section of a pit with pit bottom smoothing

Use the following steps to display a north-south cross-section of a pit with pit bottom smoothing:

1- If necessary, start the **PITWIN32** software.
2- If necessary, from the **File** menu, select **Open** to load a control file.
3- Choose Revenue Block Model from the **Run** menu to create an orebody revenue block model if one has not already been created.
4- Choose **Optimum** Pit Limit from the **Run** menu to determine the optimum pit outline if it has not already been determined.
5- Choose **Pit** Bottom Smoothing from the **Run** menu to carry out pit bottom smoothing if the pit has not already been smoothed.
6- From the Graphical Display menu, select Northing Section With Smoothing.
   The software displays the **Northing Input** dialogue box.
7- Enter the Northing of the required cross-section.
8- Choose **OK**.

A.22- To display a numerical representation of the pit limit without pit bottom smoothing

Use the following steps to generate a numerical display of a pit without pit bottom smoothing:

1- If necessary, start the **PITWIN32** software.
2- If necessary, from the **File** menu, select **Open** to load a control file.
3- Choose Revenue Block Model from the **Run** menu to create an orebody revenue block model if one has not already been created.
4- Choose **Optimum** Pit Limit from the **Run** menu to determine the optimum pit if it has not already been determined.

5- From the Numerical Display menu, select the **Pit Limit Without Smoothing** option.

A.23- To display the results of optimisation without pit bottom smoothing

Use the following steps to display the results of optimisation without pit bottom smoothing:

1- If necessary, start the **PITWIN32** software.
2- If necessary, from the **File** menu, select **Open** to load a control file.
3- Choose Revenue Block Model from the **Run** menu to create an orebody revenue block model if one has not already been created.
4- Choose **Optimum** Pit Limit from the **Run** menu to determine the optimum pit if it has not already been determined.
5- From the Numerical Display menu, select the **Result** Without Smoothing option.

A.24- To generate a numerical display of a pit with pit bottom smoothing

Use the following steps to generate a numerical display of a pit with pit bottom smoothing:

1- If necessary, start the **PITWIN32** software.
2- If necessary, from the **File** menu, select **Open** to load a control file.
3- Choose Revenue Block Model from the **Run** menu to create an orebody revenue block model if one has not already been created.
4- Choose **Optimum** Pit Limit from the **Run** menu to determine the optimum pit if it has not already been determined.
5- Choose **Pit** Bottom Smoothing from the **Run** menu to carry out pit bottom

smoothing if the pit has not already been smoothed.

6- From the Numerical Display menu, select the Pit Limit With Smoothing option.

A.25- To display the results of optimisation with pit bottom smoothing

Use the following steps to display the results of optimisation with pit bottom smoothing:

1- If necessary, start the **PITWIN32** software.
2- If necessary, from the **F**ile menu, select **O**pen to load a control file.
3- Choose Revenue Block Model from the **R**un menu to create an orebody revenue block model if one has not already been created.
4- Choose **O**ptimum Pit Limit from the **R**un menu to determine the optimum pit if it has not already been determined.
5- Choose **P**it Bottom Smoothing from the **R**un menu to carry out pit bottom smoothing if the pit has not already been smoothed.
6- From the **N**umerical Display menu, select the Result With Smoothing option.

A.26- To display a summary of the orebody block model

Use the following steps to display a summary of the orebody block model:

1- If necessary, start the **PITWIN32** software.
2- If necessary, from the **F**ile menu, select **O**pen to load a control file.
3- Choose Revenue Block Model from the **R**un menu to create an orebody revenue block model if one has not already been created. This option is only necessary for displaying a summary of the orebody revenue block model.
4- From the **T**ools menu, select the **S**ummary of Block Model option and then choose one of the two options: **G**rade Block Model or **R**evenue Block Model.

   The software displays a summary of the orebody block model in a dialogue box.
5- Choose **OK**.

A.27- To convert random access files to text files

Use the follow these steps to convert random access files created by the program to text files:

1- If necessary, start the **PITWIN32** software.
2- If necessary, from the **File** menu, select **Open** to load a control file.
3- Choose Revenue Block Model from the **Run** menu to create an orebody revenue block model if one has not already been created.
4- Choose **Optimum** Pit Limit from the **Run** menu to determine the optimum pit if it has not already been determined. This option is only required to convert a random access file of an optimum pit to a text file.
5- Choose **Pit** Bottom Smoothing from the **Run** menu to carry out pit bottom smoothing if the pit has not already been smoothed. This option is only required to convert the technical optimum pit limit stored in a random access file to a text file.
6- From the **Tools** menu, select the **Random** Access File to Text option and then choose one of the three options: Revenue **Block** Model, **Pit** Limit Without Smoothing and Pit Limit **With** Smoothing.

   The software displays a dialogue box for entering output file name.
7- Enter the output file name.
8- Choose **OK**.
9- Wait until the software displays a message indicating that the output file has been created.

A.28- To create text files for blocks inside the optimum pit limit

Use the following steps to create text files for blocks inside either the economical or technical optimum pit:

1- If necessary, start the **PITWIN32** software.

2- If necessary, from the **File** menu, select **Open** to load a control file.

3- Choose Revenue Block Model from the **Run** menu to create an orebody revenue block model if one has not already been created.

4- Choose **Optimum** Pit Limit from the **Run** menu to determine the optimum pit if it has not already been determined.

5- Choose **Pit** Bottom Smoothing from the **Run** menu to carry out pit bottom smoothing if the pit has not already been smoothed. This option is only required to create a text file of blocks inside the technical optimum pit.

6- From the **Tools** menu, select the **Blocks** Inside the Pit Limit option and then choose one of the six options which are available in this command.

The software displays a dialogue box for entering the output file name.

7- Enter the output file name.

8- Choose **OK**.

9- Wait until the software displays a message indicating that the output file has been created.

A.29- To create an equivalent orebody block model for a multi-mineral deposit

Use the following steps to create an equivalent orebody grade block model for a multi-mineral deposit:

1- If necessary, start the **PITWIN32** software.

2- From the Tools menu, select the option of Multi-Mineral Deposit to One.

The software displays a dialogue box for entering input and output file names and the required coefficient for each mineral.

3- Enter the required information.

4- Choose **OK**.

5- Wait until the software displays a message indicating that the output file has been created.

A.30- To change the direction of the Z axis in the orebody grade block model

Use the following steps to change the direction of the vertical co-ordinate of the orebody grade block model:

1- If necessary, start the **PITWIN32** software.

2- From the Tools menu, select the option of Change Vertical Co-ordinate.

   The software displays a dialogue box for entering input and output file names, block dimensions and the type of grade file.

3- Enter the required information.

4- Choose **OK**.

5- Wait until the software displays a message indicating that the output file has been created.

A.31 - To quit the program

Follow these steps to quit the software:

1- From the **File** menu, select **Exit**.

   Program displays the exit dialogue box.

2- Choose **Yes**.