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FRAME BEHAVIOUR WITH SEMI-RIGID CONNECTIONS

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A Thesis Submitted to the University of Sheffield for the Degree
of Doctor of Philosophy

December, 1989

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Acknowledgements

The author wishes to express his sincere gratitude to his supervisor Dr. P.A.Kirby, Senior Lecturer in the Department of Civil and Structural Engineering at the University of Sheffield, for his excellent guidance, consistent encouragement and interest in the work throughout the author's study.

The assistance provided by Professor T.H.Hanna, Head of the Department and all of the staff in the Department is appreciated.

Thanks are extended to Professor D.A.Nethercot for his assistance to the author in conducting this study.

The financial support from the British Council and from Aleppo University (Syria) made it possible for the author to start, let alone finish the study.

The assistance provided by all of the staff in the University Computing Center is acknowledged.

Declaration

This is to certify that, except when specific reference to other investigation is made, the work described in this Thesis is the result of the investigation of the candidate.

Candidate

Supervisor.

Summary

In this thesis both analytical and design studies have been conducted on the behaviour of fully and semi-rigidly connected multistorey steel frames. Many topics have been addressed, current design approaches examined and new design methods proposed to allow for such effects based upon a first order linear elastic analysis, which is the most commonly used in normal design. These topics are as follows:

1) The well known amplified sway method (see BS5950:1985:Part1) which can be used to incorporate the effect of the presence of axial loads on the behaviour of sway frames was studied. A modification to this method has been proposed and the validity of the proposed method was checked using an existing computer program [1] to verify the proposed method against a second order analysis.

2) The influence of the action of semi-rigid connections on frame behaviour was examined and the need to conduct a systematic investigation into the problem verified. A simple hand calculation method to incorporate this influence to any of the conventional design methods has been proposed. A second computer program [2] was slightly modified to suit the university of Sheffield IBM3083 mainframe machine and this program was later used in this study.

3) The stiffening effect due to partial sway bracing resulting from the presence of block or brickwork walls in a practical multistorey frame accompanied by the weakening effect due to the finite stiffness of semi-rigid joints on frame serviceability was investigated and a suitable design method is recommended.

4) The behaviour of columns in sway frames with and without par-

tial sway bracing, resulting from the presence of infill panels in practical frames, has been examined. Design charts which can be used to predict a reasonably conservative estimation of the inelastic ultimate load of a framed column in a sway structure are given. These charts are particularly helpful in assisting a designer to make a reasonably good initial selection for the column section sizes in a flexibly connected frame. In addition an empirical formula has been proposed to incorporate the beneficial effect on the column behaviour resulting from the presence of infill panels in real steel frames. In order to conduct this study a computer program developed by Rifai [3] has been modified to simulate the behaviour of a flexibly connected sway subassemblages.

5) Finally general conclusions and recommendations for future work are given.

Notation

a , b	distances from centroid of area A^* to ends i and j of the conjugate beam respectively.
a_o	initial deflection at the centre of a beam element.
A	bracing element area of partially braced frame.
Af_i	simple amplification factor of the ith storey of a frame.
Af_w	simple amplification factor of the weakest storey of a frame.
$Af_{i,b}$	simple amplification factor of the beams of the ith storey of a flexibly connected frame.
$Af_{i,c}$	simple amplification factor of the columns of the ith storey of a flexibly connected frame
A_f	area of both flanges in eq.8.15 and area of one flange in eq.8.16
A_w	area of the web of the section .
A^*	area under free bending moment diagram of conjugate beam due to external loads.
b	the width of braced bay of a frame.
C_{j1}	semi-rigid joint stiffness for end 1 of a beam element used in the formulations employed in writing the finite element program mentioned in chapter 8
C_{j2}	semi-rigid joint stiffness for end 2 of a beam element used in the formulations employed in writing the finite element program mentioned in chapter 8
C_s	reduction factor applied to flexibly-connected sway frames subjected to horizontal loads only.
E	Young's modulus of steel.

E_p	modulus of elasticity of panel material.
E_t	column tangent modulus.
E_r	column reduced modulus.
EA	axial flexural rigidity.
EI	bending flexural rigidity.
F	spring resistance force.
h	storey height.
h_i	the height of ith storey.
H	horizontal load.
H_i	storey fictitious horizontal load in P-Δ method.
I	element second moment of area.
I_c	second moment of area of a column.
I_g	second moment of area of a beam.
$\sum \frac{I_c}{h}$	sum of column flexural rigidities.
$\sum \frac{I_g}{L_g}$	sum of beam flexural rigidities.
k_e	column effective length factor.
k^2	equal to $\frac{P}{EI}$
K	semi-rigid joint stiffness.
K_a, K_b	joint stiffnesses at end A and B of a beam element respectively.
K_i	estimated semi-rigid joint stiffness.
K_3	a stiffness coefficient (the ratio of panel stiffness to column stiffness).
L	element length.
L_c	column height.
L_c/r	column slenderness.
L_g	beam length.
Lc_i	column length of the ith storey of a frame.
M	joint's bending moment.

M-Φ	moment-rotation relationship of a semi-rigid joint.
M_{AB}	bending moment at end A of a element.
M_{BA}	bending moment at end B of a element.
$M_{f_{AB}}$,$M_{f_{BA}}$	fixed end moments at end A and B of a beam element AB.
M_{hf}	bending moment of a element due to horizontal load assuming full rigid joint behaviour.
M_{hs}	bending moment of a element due to horizontal load assuming semi rigid joint behaviour.
M_{pc}	column plastic moment.
M_{vf}	bending moment of a element due to vertical load assuming full rigid joint behaviour.
M_{vs}	bending moment of a element due to vertical load assuming semi rigid joint behaviour.
$M_{v,h}$	semi-rigid joint moment due to both vertical and horizontal loads
$M_{bav,o}$	average joint bending moments used in chapter 6.
Ms_i , Ms_j	moments at ends i and j of conjugate element for a semi-rigidly connected element.
M^F	fixed end moment.
N_1 *to* N_6	shape functions in standard finite element analysis.
P	vertical load.
P	column axial load.
P_{cr}	elastic critical load.
P_e	Euler load.
$\sum P_i$	the sum of column axial loads at ith storey.
Q	horizontal test load used in Bolton's model.
R	relative angle between conjugate beam ends due to vertical displacement.

$\overline{R}_k$	column rotational end restraints including semi-rigid joint behaviour.
S	lateral spring stiffness.
$\overline{S}$	ratio of a spring stiffness to a column stiffness.
S_b	spring stiffness in Bolton's model.
$\sum S_p$	sum of spring stiffnesses of panels.
t	thickness of a panel .
u , v	the deformations at a general point of a element.
Um_f	unit rotation moment of rigidly-connected element.
Um_s	unit rotation moment of flexibly-connected element.
v	column lateral deflection.
v_o	initial imperfection at a general point of a element.
v" and v""	the second and the fourth derivatives of v respectively.
V_i , V_j	shears at ends i and j of conjugate beam due to the free bending moment diagram A^*.
V_i'	the additional shear calculated at $P - \Delta$ method due to sway forces.
$W_{i,t}$	total gravity load in the ith storey.
Z_x	elastic modulus of a steel section.

Greek letters

α	equal to $\frac{2EI}{K_a L}$,
α_j , β_j	coefficients used in B-spline fitting technique determined by least square method.
α_n	nodal semi-rigid reduction factor.

α_s	semi-rigid joint reduction factor applied to flexibly-connected frame subjected to vertical loads.
$\overline{\alpha}$	nondimensionalised beam end restraint.
β	equal to $\frac{2EI}{K_b L}$
β_e	the ratio between the column applied load and the elastic critical load.
δ	deformation vector at any point of a beam element.
δ^e	nodal deformation vector of a beam element.
Δ_i , Δ_{i+1}	the lateral deflections at level i and i+1 respectively of a multistorey frame.
Δ^*	lateral movement.
$\Delta P/P$	ratio of increase of a column's ultimate load due to the presence of cladding.
ϵ	average strains at a cross section of a beam-element.
θ_i	rotation at node i assuming semi-rigid joints.
$\overline{\theta_i}$	rotation at node i assuming fully-rigid joints.
λ	load factor.
λ_{cr}	load factor at elastic instability.
$\lambda_{cr,f}$	elastic critical load factor of a frame.
λ_u	load factor at failure.
σ_y	yield stress.
$\sigma - \epsilon$	material stress-strain relationship.
ϕ_i	sway index of the ith storey .
ϕ'_i	enhanced sway index of the ith storey.
ϕ_{max}	largest sway index of any storey in a frame.
ϕ_i , ϕ_j	change of angles between beam and column at end i and j.
Φ	joint's rotation.

Φ_{bio}	the rotation of the ith flexibly connected beam element calculated assuming that the beam is simply supported at its both ends.
η	semi-rigid reduction factor described by eq.5.17.
ω	ratio of distributed load intensities of beams (b1,b2) shown in fig.5.5.

Matrices and vectors

A	shape function matrix.
D	element elasticity matrix.
K_E	elastic stiffness matrix based on small deflection theory.
K_G	geometrical stiffness matrix which depends on the axial load of the element.
K_L	displacement stiffness matrix which depends on the deflected shape of the element.
P	structural load vector.
P_o	imaginary set of lateral forces and moments resulting from initial deflections.
U	structural displacement vector.
ΔP	incremental load vector.
$\Delta P_{unbalanced}$	incremental unbalanced load vector.
ΔU	incremental displacement vector.

Chapter 1

Introduction

During the last fifty years much research has been undertaken to investigate the true behaviour of steel framed structures. As a result the well known first order linearly elastic analysis, where the material is assumed to be perfectly linear and the displacements are proportional to the loads at any load stage, has been made increasingly complex due to the inclusion of many factors. Among these are second order effects, e.g. the geometric deformation due to the presence of the axial loads, the inelastic behaviour of the steel material, semi-rigid joint behaviour and out of plane behaviour. Nevertheless, since utilising the resultant analysis would enforce designers to utilise very sophisticated methods and to use time-consuming computer programs, which are not in general available to them, many practical codes allow the use of the traditional linear elastic analysis with recommendations to account for some of the previously mentioned factors in an approximate manner. Hence the gap between the research capabilities and design methods is great and the requirement for more investigation to utilize and incorporate the results of the progressive research is obvious.

1.1 Secondary Effects (geometrical deformation effects)

In a first order linearly elastic analysis the element stiffnesses of a frame are calculated according to their initial undeformed shapes. Nevertheless, in reality, with increasing applied loads the elements sustain deformations which will inevitably alter the geometry and hence the resistance capability of the elements. In order to more closely represent the behaviour of a steel frame where nonlinear effects are significant the analysis should take this into account. One way is by dividing the applied loads into small load steps and determining the element stiffnesses for each load step according to the current element deformed shapes rather than the initial shapes.

However since employing this procedure provides undesirable complications making it unsuitable for everyday use only approximate methods such as the $P - \Delta$ method, the effective length method and the amplified sway method have been adopted as design tools.

1.2 The Influence of Joint Flexibility on Steel Frame Behaviour

It is customary when designing practical frames to assume that the frame joints act either as if fully rigid or as frictionless pins. Nevertheless most steel joints have been proved to behave in a fashion which is intermediate between these two extreme cases (see figure 1.1). Figure 1.2 shows the bending moment diagram of an isolated beam element applied by a uniformly distributed load with fully fixed, semi-rigid and pinned joints. It can be easily seen from this figure that utilising semi-rigid jointshas shifted the beam bending moment diagram,

when compared to the corresponding fully rigid joint response.
In a multistorey frame the effect of joint flexibility is much more complex and depends on the frame type, i.e. sway or non-sway, the applied loads, the joint characteristics and on the beam and column sections. Most practical steel codes [4] and [5] allow the use of semi-rigid joints in practical frames but without providing any clear guidance about how it should be done.
The only known design method for flexibly connected frames is the 'wind connection method' in which the semi-rigid joints are assumed to behave like perfect hinges when vertical loads are considered and to behave like fully rigid joints when the combination of vertical and horizontal loads are considered.

1.3 The Effect of The Presence of Infill Panels In Practical Frames

In practical frames the presence of infill panels are normally neglected in design. However the presence of these panels will significantly increase the frame resistance against lateral movements and possibly slightly improve the ultimate load. Therefore it is desirable to develop a method to include this beneficial effect in design in order that a more economical design can be obtained.

1.4 Objectives of the Present Study

Most advanced research which has been carried out into the behaviour of steel frameworks has highlighted the differences between the analysis used in engineering practice and the true behaviour of the structure and there is a need to implement the finding of this research into simple design methods. These design methods have to provide satisfactory accuracy without causing unwarranted complications. The present study is aimed at implementing the most recently available theory particularly concerning semi-rigid action of joints to modify the currently used design method leading to more rational and safer structures. The following aspects of the in-plane behaviour of frame structures are investigated:

1) Calculation of design moments in sway frames

A doubt was raised about the accuracy of the well known the 'amplified sway method' which has been used in BS5950:1985:Part1 to account for the secondary effect resulting from the presence of axial loads and lateral deflections of sway frames in continuous construction, i.e. assuming rigid joints. Therefore this method was checked for a large number of multistorey frames against a finite element program [1] which is available at the University of Sheffield. The method was found to be unsatisfactory for some multistorey frames where irregular lateral deflections of the frame stories occurred. It was noted that BS5950 did not require any condition to be checked before utilising this method and in most cases the error obtained from the current method was observed to be non conservative. A modified form was proposed to overcome this limitation and to ensure that more accuracy is obtained with only minimal extra effort.

2) Joint flexibility in frame response

The influence of joint flexibility on frame behaviour was examined and found to be an influential factor. Little research has been conducted to provide a simple and sufficient design method. Therefore in chapters 4,5 and 6 the influence of the presence of semi-rigid joints on steel frames is highlighted and a design procedure to account for such influence is proposed. The method can be incorporated into any of the conventional currently used design methods.

3) Semi rigid joints in sway frames with partial sway bracing.

Utilising semi-rigid joints in practical frames causes the corresponding bare frames to undergo horizontal deflections which are often larger than the maximum permitted deflections. Therefore some discrete bracing system may need to be provided to these frames to assist in reducing lateral flexibility. Alternatively including the stiffening effect resulting from the presence of infill panels might provide the required stiffness against the frame's lateral movements avoiding the necessity of providing discrete bracing. In chapter 7 the effect of partial sway bracing on the lateral deflections of a flexibly connected multistorey frame has been studied and design recommendations have been proposed.

4) The first step in designing a practical frame is making a primary selection for the beam and the column sections of the frame. This selection can be accomplished with ease in simple construction (where the joints are assumed to be perfect hinges). In continuous structures however this selection is more complex due to the continuity between the beams and the columns. Therefore a parametric study has been conducted aimed at investigating the behaviour of framed columns in sway or partially braced structures. Conducting this study required the use of an analytical tool therefore an already existing computer program, written by Rifai [3] which could be used to analyse non-sway flexibly connected column subassemblages, has been modified by the author to simulate the behaviour of flexibly connected sway subassemblages. This parametric study has resulted

in the production of design charts which are particularly helpful to make a reasonably good selection for the sections of framed columns in sway structures. In addition an empirical equation has been proposed to represent the improvement of framed column behaviour resulting from the presence of infill panels in practical sway frames.

1.5 Limitation in the Present Study

The present work is bounded by the following limitations:

1) Only the in-plane behaviour of steel frames has been considered.

2) Semi-rigid joints are assumed to have the same behaviour for loading and unloading paths. This assumption has been adopted in the method proposed in chapters 5 and 6 to provide an estimation of the bending moment diagram of a flexibly connected sway frame including the nonlinear behaviour of the utilised semi-rigid joints. This assumption is likely to lead to a conservative design method since taking the unloading path of the semi-rigid joints in a flexibly connected frame as parallel lines to the initial tangent of their $M-\Phi$ relationship would lead to a reduction of the frame lateral deflection and, in many cases, to an improvement in the frame load carrying capacity.

3) Including the beneficial effect resulting from the presence of infill panels in a multistorey frame has been accomplished, in this study, according to the recommendations of Appendix E of BS5950. These recommendations are compared to the only known and limited experimental results available concerning the actual behaviour of such panels.

4) The modified version of Rifai's subassemblage program mentioned in chapter 8 accepts only nodal loads. The program simulates the behaviour of only limited subassemblages which consist of four beams and a single column, i.e. I shape subassemblages.

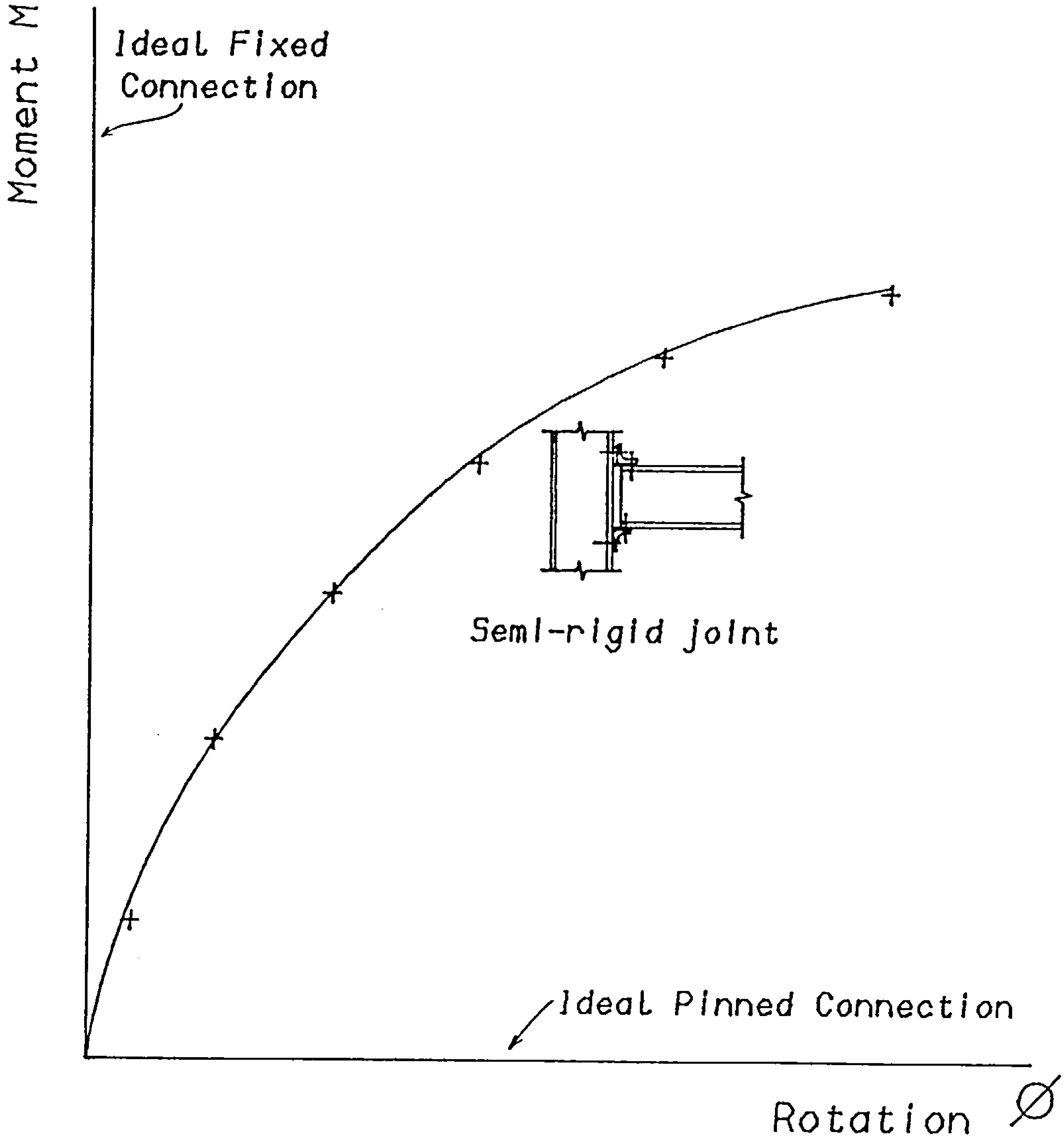

Figure 1.1: Typical moment-rotation relationship

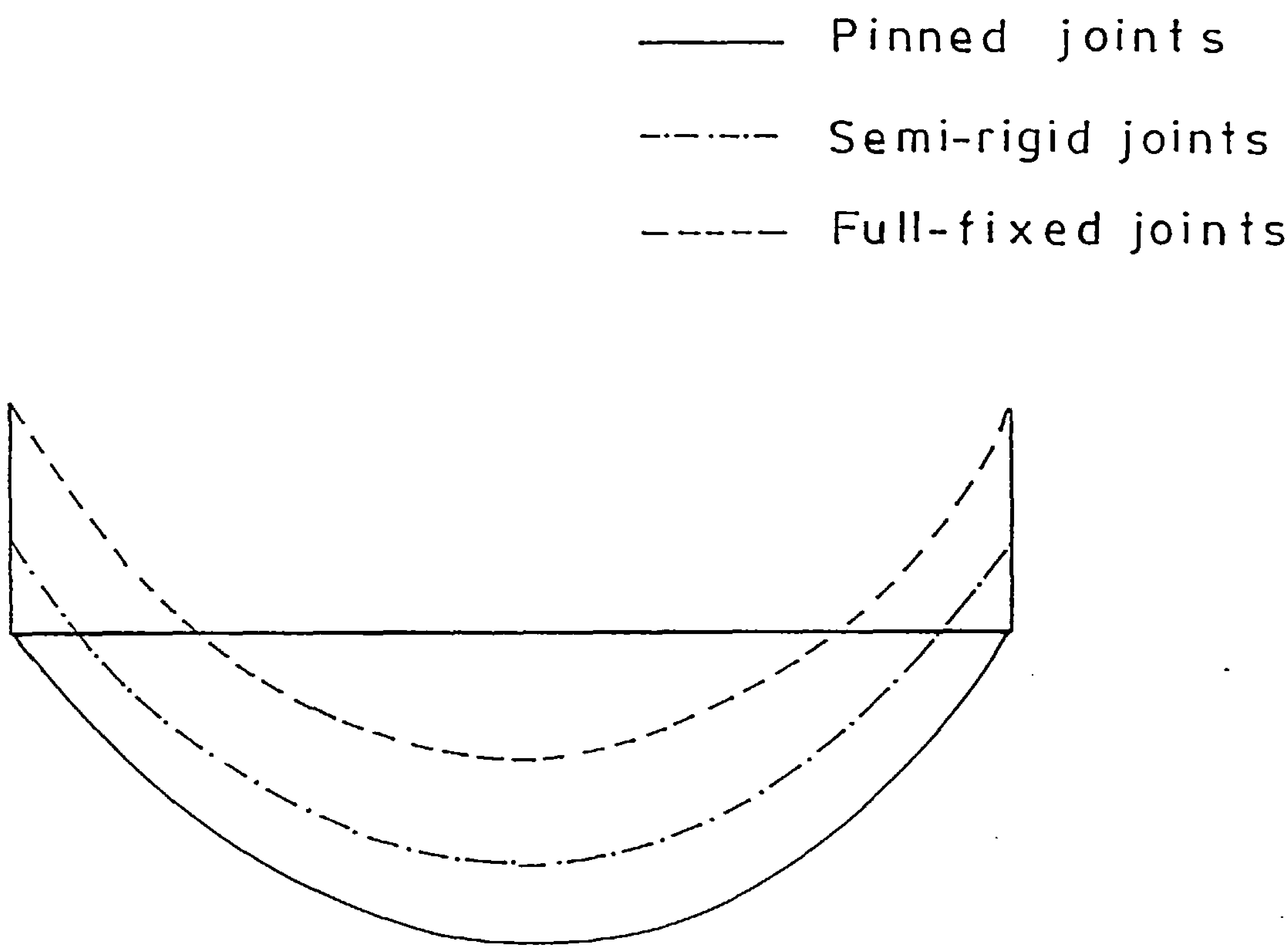

Figure 1.2: Typical bending moment diagram for an isolated beam-element with pinned, fully fixed and semi-rigid joints

Chapter 2

Review of Literature of Steel Framework Behaviour

In recent years, there has been a considerable amount of theoretical work carried out on steel framework behaviour. Some of the topics considered in this research concern geometric deformations due to axial loads, in and out-of-plane semi-rigid joint behaviour and its influence on frame behaviour, plastic behaviour of the material and out-of-plane response of a steel structure, i.e lateral torsional buckling and twisting. In this chapter a brief review of literature about steelwork behaviour will be discussed with a special reference to design implications for in-plane response.

2.1 Frame Stability

With the recent use of high strength steel and limit state design methods, many practical multistorey frames are more likely to fail by instability before developing a plastic collapse mechanism than was the case when lower grade steels were used. Therefore the stability problem has become increasingly important and much work has been done to provide analytical and design tools to ensure against structural instability.

2.1.1 Effective Length Factor

Due to its simplicity the effective length factor has been used widely in the design of structural steelwork. In this method a framed column can be designed independently by calculating its elastic critical load using one of the many available charts which are to be found in steelwork codes [4] and [5]. Only the column end restraints are required to calculate the effective length factor. The first attempt to solve a stability problem of a structure might be related to Euler [6] 1759. He suggested the well known Euler formula to calculate P_{cr} the elastic critical load for a column which is simply supported at its both ends.

$$P_{cr} = \frac{\pi^2 EI}{{L_c}^2}$$

where L_c is the column height and EI is the column's flexural rigidity. In 1788 Lagrange extended this theory to cover higher modes with an analysis procedure which is still being used today. Over the years this theory has been advanced by incorporating the influence of residual stresses, initial crookedness and load eccentricity. However, columns exist as parts of a frame structure in which these columns are connected to other members by different joint types. Thus some end restraints which are provided to these columns lead to an increase in their ultimate loads. In 1959 Julian and Laurance [7] provided a subassemblage stability solution by means of nomographs for the effective length factors in frame. Although the solution included many assumptions, it provided a simple and rational design procedure. The Column Research Council's alignment charts [8] provided a very convenient method of calculating the effective length factor of a column with rigid connections which takes into account the relative stiffness of adjoining members. Modifying these alignment charts to suit flexibility connected frames was first proposed by De Falco and Marino [9] in

1966. They suggested a formula to update the effect of joint flexibility by reducing the girder's second moments of area. The initial tangent was used to represent semi-rigid joint behaviour. Wood 1974 [10] provided a very comprehensive study on frame effective length factors. He used a stiffness distribution method to provide charts where the effective length factor k_e, for non-sway, partially braced and sway frames, is expressed in terms of column upper and lower end Hardy Cross distribution coefficients. Provided that the structure remains perfectly elastic the principle of effective length factor gives a reasonable prediction of the critical load of the column. This case can be met for a very slender column. However, no column can sustain more than its squash load which is simply equal to the column cross-section area multiplied by its yield stress σ_y. Thus the squash load forms an upper limit of the column strength curve. Fig.2.1 shows a typical column strength curve from which it can be seen that for a very stocky column, i.e. column with a very low slenderness factor, the effect of elastic stability can be ignored and the ultimate load of the column can be approximately taken as the column squash load. However, for a very slender column the elastic critical load obtained from effective length factor can be taken as the column's ultimate load. In most practical multistorey frames, where columns have intermediate slendernesses, the column failure is usually caused by inelastic behaviour of the columns, i.e. the interaction of plasticity and elastic instability. Yura [11] modified the effective length factor procedure to allow for the previously mentioned inelastic behaviour. Lui and Chen [12] studied the effect of small end restraint on the strength of H columns. Only the initial tangent stiffness of the semi-rigid joints was considered. Lui and Chen [13] criticized the use of both the Column Research Council Curve (CRC) and the Structural Stability Research Council multiple curves (SSRC) in a limit state design. In the CRC design procedure the column's initial imperfection is accounted for by means of the tangent modulus concept. This implicit way of including column geometrical imperfections is not desirable in the limit state design approach. However, although in the SSRC curves a stability method is used to derive these curves, only an initial imperfection equal to 1/1000 of the column height is considered. A more generalised procedure has been suggested to remedy these limitations. The proposed method can be used to handle any practical column with various values of initial imperfection, provided that a constant factor attributable to section plasticity is defined by experiment or by calibration against existing curves. In the currently used effective length factor procedure only positive end restraint is expected. However Bridge and Frasen [14] demonstrated the existence of negative end restraint in some practical structures. They used a continuous three lift column with different storey heights subjected to a unique axial load over the entire range of the column. Modified charts to account for such a situation have been proposed. In the recent AISC effective length factor alignment charts, the far end of the columns above and below the column being considered are assumed to be rigidly fixed. Such an assumption might not always be satisfied. A non-conservative result from the current charts was recorded by Duan and Chen [15] for unbraced frames. Hence a modified method has been proposed. This method considers various far end restraints, e.g both ends are fixed, both ends are pinned or one far end is fixed while the other is pinned.

2.1.2 P- Δ Method

An approximate method to allow for stability in multistorey frames has been described by Adams [1] in 1972. In this method the effect of lateral movements of the frame stories on the overall frame behaviour is included by means of applying fictitious equivalent horizontal loads (see fig.2.2).

The method proceeds as follows:

1) The frame lateral displacements are calculated utilising a first order analysis.

2) The shear at each storey level is calculated as

$$V_i' = \frac{\sum P_i}{h_i}(\Delta_{i+1} - \Delta_i)$$

in which

V_i'	is the additional shear at level i due to sway forces;
$\sum P_i$	is the sum of column axial loads in that storey ;
h_i	is the storey height and
Δ_i , Δ_{i+1}	are the lateral deflections at level i and i+1 respectively.

3) The fictitious horizontal load is calculated at each level as

$$H_i = V_{i-1}' - V_i'$$

This load should be used to update the external horizontal loads to $H + H_i$ at each storey level and a new first order analysis should be performed. The method is considered to have converged when the difference of the lateral displacements obtained from one iteration to the next one are negligible.

The use of this method in practical design offices was limited since this method requires undesirable iteration to converge to the solution.

2.1.3 Amplified Sway Method

This is an indirect method to allow for stability effect in a sway structure. Only a linear elastic analysis is required to analyse the structure and the influence of axial loads on member end moments is then incorporated by enhancing the member bending moments. This method has been proposed by LeMessurier [16] 1977 and converted into design method by Kanchanalai and Lu [17] 1979 and Lu [18] 1983. A similar procedure has been adopted by the current British Standard BS5950 [4].

2.1.4 Stability Functions

Stability functions were first developed during 1935 by extending the Hardy Cross [19] method of moment distribution. James [20] demonstrated the validity of the method when the effects of member axial loads are included. In this method the stiffness and carry over factor of each member is calculated in term of the member axial load. Livesley and Chandler [21] tabulated these functions in terms of the ratio of the axial load to the Euler load of the member.

2.1.5 Stability Analysis of Frames

It is evident that, the recent wide use of digital computers has encouraged the use of matrix analysis and the stability problem of frames using matrix analyses has been examined by many authors notably Halldorsson and Wang [22] and Przemieniecki [23]. The stability of a structure can be determined by monitoring the determinant of its global stiffness matrix. In small deflection theory, i.e. ignoring the nonlinear term of the strain-displacement relationship, the overall stiffness matrix of a structure, i.e. the tangent stiffness matrix, is composed of two separate matrices which are:

1) The normal stiffness matrix K_E which depends on the physical properties of the elements of the structure and;

2) The geometrical stiffness matrix K_G which depends on axial loads and element lengths of the structure alone. The geometrical stiffness

matrix accounts for the destabilising effect due to the deformations caused by the presence of axial loads. Thus the tangent stiffness matrix of a structure K_T is given by

$$K_T = K_E + K_G$$

and frame instability occurs when the determinant of K_T equals zero, i.e.:

$$Det\ K_T = 0$$

2.2 Behaviour of Frames with Semi-Rigid Connections

The influence of semi-rigid joints on frame response was realised over seventy years ago when Wilson and Moore(1917) [24] reported the flexibility of riveted structural connections. In the 1930's several researchersin Britain [25], [26] and [27], Canada [28] and the United State [29] measured the relationships between moment transmitted by the connections and relative angle changes at beam-to-column connections. Although the nature of semi-rigid joints has long been recognised and after significant work in the 1930's, only recently has a real interest in semi-rigid connection behaviour and its influence on steel framework been shown. This is because the cost of labour has been increased more rapidly than material cost. Thus, if designers are willing to produce economical design, they have to consider utilising inexpensive connections which have significant flexibility and also be willing to take advantage of some elements which has been assumed as non-structural elements present in real structure such as cladding and brick-walls.

2.2.1 Flexural Behaviour of Connections

According to experimental tests, steel frame joints show various degree of flexibility. Even a web-cleat connection, which is normally treated as a pure shear joint, is found to provide a small end restraint while an extended end plate joint, which is frequently assumed in design to be a fully rigid joint, allows small relative angle to develop between the column and the beam as moment is applied. Therefore, it is more correct to consider all structural frame joints as semi-rigid joints.

2.2.2 Descriptions of Joint Flexural Behaviour

A moment-rotation characteristic is the best representation of the flexural behaviour of a joint. This is the relationship between the moment transmitted by the connection (between the beam to the column) and the the relative angular movement between the beam and the column. Fig.2.3 shows typical semi-rigid moment-rotation curves for three semi-rigid joints.

The vertical axis of this figure represents fully rigid joint behaviour where the joint stiffness is infinite for any bending moment value, while the horizontal axis represents pinned joint behaviour where the joint stiffness is zero for any joint bending moment. These axes which represent two extreme cases have been used in practical design methods, hence the semi-rigid behaviour is either overestimated or underestimated. This figure shows clearly that none of these joints can be represented accurately as fully rigid or perfectly pinned. Neither is the joint behaviour linear over most loading range. In general, as the joint bending moment increases the joint stiffness which is the tangent of the moment-rotation curve, decreases.

2.2.3 Frame Analyses with Semi-Rigid Connections

Several conventional frame analysis methods have been modified to allow for joint flexibility where the extra deformations due to semi-rigid joints are added to those due to bending of members. Baker [26], [27] and Rathbun [29] independently proposed methods to incorporate semi-rigid end restraint into classical methods assuming a linear moment-rotation relationship. More details about other modifications made to some basic analysis methods such as the method of three moments can be found in ref. [29]

2.2.4 Beam Line Method

The beam line method is a graphical method which can be used to approximately predict the end restraint of a beam element utilising the actual moment-rotation curve. This method was first proposed by Batho and Rowan [26] and later developed by Batho [27] alone. The principles of the moment-area approach were used to derive the relation of end moment M as a linear function of the angle of connection rotation, as indicated in eq.2.1

$$M = M^F - \frac{2EI\phi}{L} \tag{2.1}$$

where L is the element length which is assumed to have uniform flexural rigidity EI. M^F is the calculated fixed end moment due to the applied loads. The beam line presented by eq.2.1 intersects the actual moment-rotation curve (see fig.2.4) defining the end bending moment and the angle of the element.

2.2.5 Modified Slope Deflection Method

The generalised slope-deflection equations for a beam of length L and flexural rigidity EI and connected at its ends by two semi-rigid joints with stiffness equal to K_a and K_b respectively are:

$$
\begin{aligned}
M_{AB} = A'[\frac{2EI}{L}(C_{AA}\theta_A + C_{AB}\theta_B &- C_{AC}\frac{\Delta^*}{L}) \\
&- F_{AA}M_{f_{AB}} - F_{AB}M_{f_{BA}}]
\end{aligned}
\tag{2.2}
$$

$$
\begin{aligned}
M_{BA} = A'[\frac{2EI}{L}(C_{BB}\theta_B + C_{BA}\theta_A &- C_{BC}\frac{\Delta^*}{L}) \\
&+ F_{BB}M_{f_{BA}} + F_{BA}M_{f_{AB}}]
\end{aligned}
\tag{2.3}
$$

where

A' , C_{AA} , C_{AB} , C_{AC} , F_{AA} , F_{AB} , C_{BB} , C_{BA} , C_{BC} , F_{BB} and F_{BA} are constants which depend on joint stiffness. Their values are tabulated in table 2.1. Δ^* is the lateral movement of end B relative to end A taken as positive if tends to rotate the element clock-wise and moments and rotations are assumed to be positive if clock-wise. $M_{f_{AB}}$ and $M_{f_{BA}}$ are the element fixed end moments at ends A and B respectively.

2.2.6 Modified Moment Distribution Method

The original moment distribution method was developed from the slope deflection method. Since this method does not require the solution of simultaneous equations, it was widely accepted after it was first proposed by Hardy Cross [19]. The procedure of this method starts with the calculation of the fixed end moments and then the joints of the structure are released in succession. When releasing any joint the moments at the member ends meeting at the joint are not, in general, in equilibrium. Thus a balancing moment is distributed between the members according to their distribution factors which depend on the relative member stiffnesses. Also a moment at the far end of every member meeting at the considered joint is induced as a result of releasing the near joint.

Constants	Semi-rigid joint with zero length	Conventional method
A'	$\frac{1}{1+2\alpha+2\beta+3\alpha\beta}$	1
C_{AA}	$2+3\beta$	2
$C_{AB} = C_{BA}$	1	1
C_{BB}	$2+3\alpha$	2
C_{AC}	$3(1+\beta)$	3
C_{BC}	$3(1+\alpha)$	3
F_{AA}	$1+2\beta$	1
F_{AB}	β	0
F_{BB}	$1+2\alpha$	1
F_{BA}	α	0
$\alpha = \frac{2EI}{K_a L}$ $\beta = \frac{2EI}{K_b L}$		

Table 2.1: Modified slope deflection constants

Factor	Semi-rigid connection with zero length	Conventional method
End Moments	$\frac{[(1+2\beta)M_{f_{AB}}+\beta M_{f_{BA}}}{1+2\alpha+2\beta+3\alpha\beta}$	M_{fAB}
Carry-over Factor	$\frac{1}{2+3\beta}$	$\frac{1}{2}$
Rotation-stiffness Factor	$\frac{2EI(2+3\beta)}{L(1+2\alpha+2\beta+3\alpha\beta)}$	$\frac{4EI}{L}$
α *and* β are defined in table 2.1		

Table 2.2: The modified moment-distribution factors

N.B This table represents the end moment and carry over factor and stiffness rotation factor considering end AB of a beam element. For the end BA same factors can be used after replacing the value β by α

Including the effect of joint flexibility can be achieved by calculating the distribution and carry over factors (see table 2.2) in terms of joint flexibility and the method can be then carried out as usual.

2.2.7 Finite Element Method

A very flexible and advanced method, which has been widely used in recent years, is the finite element method. This method is well explained in many specialized text books [30] and [31]. The procedure of the method is very similar to that of stiffness matrix analysis for beam-column analysis. In this method the deformations at any point of an element are uniquely defined from the nodal deformations by employing a set of shape functions. Hence the element stiffness matrix can be obtained by applying the approaches based on the principles of virtual work or energy conservation in conjunction with the assumed strain-displacements relationship. These element stiffness matrices are then assembled to form the structural stiffness matrix. The structural load vector is assembled and the structural displacement vector is obtained from:

$$P = K_T \, . \, U$$

where K_T is the structural tangent stiffness matrix and P and U are the structural load and displacement vectors respectively.

Next the strains, stresses and the resultant forces at any point along any element can be calculated.

El-Zanaty [1] developed a computer program for frame analysis, which utilizes the finite element method. Residual stresses, material strain hardening and the effects of the presence of axial loads on element geometry were all taken into account but, only rigid connections were considered. Corradi and Poggi [32] also developed a computer program for frame analysis based on the finite element method. Nonlinear behaviour was assumed and the spread of plasticity was allowed for over the section and the length of a element. This was done by introducing a set of shape functions which are independent from those used to present the element displacements in term of its nodal displacements. However, material strain hardening, residual stresses, and the non-linear term in the strain-displacement relationship are not included in this program. Poggi and Zandonini [2] extended this program to include the influence of semi-rigid joints. Cosenza et al [33] used the stiffness matrix method of analysis to develop a computer program in which semi-rigid joints were modelled as extra sub-elements (see fig.2.5) and a static condensation procedure was used to reduce the degrees of freedom of any beam-element to six. Chen and Lui [34] used a similar procedure to that used by Cosenza et al to represent semi-rigid joints (see fig.2.6)

2.3 Theoretical and Experimental Study on in-Plane Flexibly Connected Frames

The influence of semi-rigid joints on the response of steel frameworks has only quite recently started to receive further attention from the research community. This sudden reawakening is related to observing that, a considerable waste of time, effort and money has occured due to designers trying to provide connections which can be truly presented as either rigid or pinned. Because of this a huge body of experimental and theoretical work on flexibly connected frames have been carried out to shed light into the implications of the existence, in practical structures, of semi-rigid joints and to highlight the economical benefits which might be gained from utilising these joints in practical frames. A significant amount of this research has been undertaken in the University of Sheffield. Both in-plane and out-of-plane behaviour of isolated joints and full-scale structural frames have been explored experimentally.

In this section only a brief review of the research into in-plane behaviour of steel frames with semi-rigid connection will be included, starting with the Sheffield study and then moving to work conducted elsewhere. More details can be found in refs. [3], [35] and [36].

Numerous investigations on flexibly jointed structures has been undertaken in Sheffield University. Starting in 1977 the research program on semi-rigid action began with Jones [35] who was one of the first to interpret semi-rigid joint nonlinearity into structural analysis by adopting the B-spline technique. This modelling was found to be capable of closely representing the actual joint behaviour obtained from experimental tests. Jones developed a finite element analysis to investigate the effect of semi rigid connections on column strength allowing for joint nonlinear behaviour. The initial formulation assumed that the joints were attached to fully rigid supports. Both material and geometrical nonlinearity was included. He concluded that semi-rigid joints can have a great influence on the column buckling load. For example he found that web-cleat connections, which are normally regarded as shear joints, provide a significant increase on the buckling loads for columns with geometric slenderness greater than 80.

The computer program written by Jones was extended by Rifai [3] to handle non-sway column sub-assemblages. Initial imperfections, residual stress, material and geometrical nonlinearities were all accounted for in the analysis. One of the most important features of the modification done by Rifai was the inclusion of semi-rigid joint action by modifying the conventional shape functions which are normally used in finite element analysis and updating the energy stored in semi-rigid joints to the strain energy of the element. This work was subsequently extended by the author to represent unbraced column sub-assemblages. Following this work Davison [36] conducted several tests on non-sway column sub-assemblages and used the computer program developed by Rifai [3] to simulate the experimentally obtained result. A good agreement was reported. Davison concluded that semi-rigid joints have a substantial influence on steel frames regardless the existence of beam loads. Two full size 3 storey 2 bay flexibly connected frames was also tested by Davison et al [36] and [37] to investigate the real behaviour of a more realistic frame with realistic semi-rigid joints and these tests were used to validate the computer program SERVAR which has been developed by Poggi and Zandonini [2]. A good agreement has been reported [36].

Lindsay et al [38] investigated the possibility of using semi-rigid joints on practical multistorey frames. Special recommendation were given concerning stability and serviceability problems when semi-rigid joints are used. It was concluded that the main obstacle to adopting semi-rigid joints in practical frames is probably due to the fact that flexibly connected bare frames undergo substantial lateral drifts which cannot be accepted in practical frames for service reasons. Poggi and Zandonini [2] investigated the main parameters which governs the behaviour and the strength of semi-rigidly jointed frames by considering simple frames. The standardised moment-rotation relationship suggested by Fry and Morris [39] was used to define the law of the extended end plate joints which were incorporated. Initial imperfections of the frames were allowed for as initial lack of verticality. Various load patterns were considered and the required element sections were defined using the 'Wind-Connection Design Method'. These frames were then rigorously analysed using a computer program [40]. Many models for the employed semi-rigid joint were used for comparison. They concluded that:

1) A design method which makes use of an elastic linear analysis seems to lead to satisfactory accuracy;

2) The wind-connection method was found to provide a safe design. The second conclusion however, contrasts with Gerstle's conclusion [41] in which he stated that using this method for a practical frame causes the beams of the frame to be overdesigned and the columns to be underdesigned.

Stelmack et al [42] conducted a series of tests for a frame which was flexibly connected. Cyclic loading for both horizontal and vertical loads were used. In these tests permanent damage to the utilised members were avoided for economic reasons.

A linearly elastic displacement matrix analysis was used to simulate the experimentally obtained result. Both nonlinear behaviour and unloading of semi-rigid joints were allowed for in the analyses. The finding of this study agreed with Gerstle [41] and Poggi and Zandonini [2] in the sense that a linearly elastic analysis is found to be sufficient to predict practical frame behaviour at service conditions. Gerstle [41] investigated the influence of semi-rigid joints in unbraced frames. He found that the decrease of joint stiffnesses causes, in general, decreases of the frame's ultimate load. However, this conclusion is violated for a frame with long span beams and only a few stories high. The problem of frame behaviour with semi-rigid connections was examined by Lui and Chen [43] using the finite element method. In this study elastic-perfectly-plastic behaviour was considered, i.e. an element remains elastic until the plastic moment capacity of the section is reached. Loading and unloading behaviour of the employed semi-rigid joints was included. The influence of semi-rigid joints on the behaviour and ultimate strengths of steel frames was highlighted.

2.4 Design Methods for Steel Frames with Flexible Joints

Most modern steelwork building codes [4] and [5] recognise semi-rigidly connected frames. BS5950 1985:Part1, for instance, has identified three categories of design approaches.

1) Simple design: where the connections between members are assumed to be unable to develop any significant bending moments. Beams of this structural category can be designed as being simply

supported while columns are assumed to resist only axial loads plus moments due to eccentricities.

2) Rigid design (Continuous Structure): the connections are assumed to provide full continuity between members. Elastic or plastic design procedures can be used as appropriate.

3) Semi-rigid design: the connections are assumed to transmit some bending, but these joints are insufficient to provide full continuity between members. The standard does not give a clear procedure to handle frames fall in this category.

It is believed that the lack of specific guidance to handle flexibly connected frames can be related to the fact that most of the research which has been done so far has directed at investigating the influence of semi-rigid connections on steel frame behaviour rather than providing a simple design procedure. There appears to have been only a little work concerning design application of the problem. It is worth mentioning that designers have shown clear resistance to and disinterest in capitalising on the potential of semi-rigid joints in practical frames. Some of the reasons for this are :

1) Complexities of finding suitable and previously tested moment-rotation data for most known semi-rigid joints.

2) The non-linear behaviour of semi-rigid joints which prevents the use of some simplified design methods such as the modified slope deflection method, or the modified moment-distribution method.

3) Lack of training in dealing with these type of joints and the obvious desire to resist any dramatic change in the currently used design analysis

Until these difficulties are overcome, semi-rigid analysis is likely to remain a purely research concept.

2.4.1 Moment Rotation Curves of Semi-rigid Joints

Experimental tests have proved that the behaviour of a real semi-rigid joint is nonlinear over the most range of its moment-rotation $M - \Phi$ curve. This nonlinearity can be related to the fact that some of the joint components might have yielded in certain local positions causing a change in the stiffness of the compression side of the connection relative to the tension side.

Incorporating semi-rigid behaviour into structural analysis requires moment-rotation curves for the utilised semi-rigid joints. In an attempt to avoid conducting extensive and expensive testing on semi-rigid joints, some researchers [39] and [44] have proposed empirical equations to provide moment-rotation relationships for a large variety of semi-rigid joints. It was claimed that these equations present, within satisfactory accuracy, the behaviour of the semi-rigid joints being considered. However, it was clearly shown by Jones [35] that for some cases these equations might overestimate or underestimate the experimentally tested joint behaviour by up to 43%. Therefore, it is believed that at the present time only experimentally tested moment-rotation relationships can be trusted to represent semi-rigid joints response. Therefore the need for conducting more tests on semi-rigid joints and storing the available data about the previously tested joints is justified. Nethercot [45] provided a comprehensive study on the previously tested semi-rigid joints collected from over 70 separate tests on steel column to beam connections. In addition, a project has just started in the University of Sheffield [45] aimed at collecting these scattered data and storing them into a computer databank which should provide a rapid and convenient access to the available data of M-Φ relationship of semi-rigid joints.

2.5 Brick-Wall and Cladding in Practical Frames

whilst it is generally recognised that brick-walls and cladding do provide resistance against lateral drifts it is rarely included in the design of practical frames. Ignoring these beneficial effects can be related to the mystery concerning the actual behaviour of these elements in a real structures. Only few experimental tests and and little theoretical work about this aspect could be traced by the author. Wood [46] has proposed a design procedure to include the stiffening effect due to the presence of brick-walls in real structures. In this procedure equivalent steel bracing elements can be determined which depend upon the material property and the dimensions of the infill panel being considered. To allow for the possible damage during loading or lack of intimate contact between frame and panel a large safety factor was proposed [46], i.e. 20. A remarkable improvement in the structural response from including this effect in design analysis regardless of the large assumed safety factor, was recorded by Wood. Appendix E of BS5950 proposed a similar procedure but with larger safety factor given as 80.

Smith and Riddington [47] have proposed a design procedure to handle masonry infill steel frames. The method accounts for all possible failure modes of such frames, i.e. by diagonal tension, horizontal shear or corner compression. Also a method for estimating the lateral drift of these type of frames is included. Smith and Riddington have concluded that the compressive failure of the infill is highly dependent upon the relative stiffness of the column and the infill panel. However, the shear and diagonal tensile failure are governed by the length to height ratio of the infill.

Riddington [48] tested two rigidly connected box steel frames with and without the presence of a blockwork. This study was mainly aimed at studying the influence of the presence of initial gaps on the behaviour of the infill frames. In these tests the frames were subjected to purely horizontal loads until failure. Riddington has concluded that:

1) Initial gaps between the steel frames and the panel reduce the effectiveness of the panel against lateral drifts.

2) The presence of the panel enormously improved the frame stiffnesses against lateral drifts when compared to the equivalent bare frame.

2.6 Conclusion

It is now easy to reach the following conclusions:

1) Stability problems in practical steel frames has become more important with the increased use of higher strength steels than formerly and a limit state design which increases both stability aspects and deformations at serviceability loading. Therefore any improvement in the currently used design analysis is significant.

2) A simple design procedure to incorporate the effect of employing semi-rigid connections in practical frames is needed together with a computer databank program to supply researchers and designers with moment-rotation curves for large variety of most commonly used semi-rigid joints.

3) The limited experimental tests available on the influence of infill panels of practical steel frame behaviour have shown that great beneficial effects can be gained from including such panels in practical design methods. However more experimental and theoretical works

are needed in order that more knowledge on the actual behaviour of these panels can be obtained.

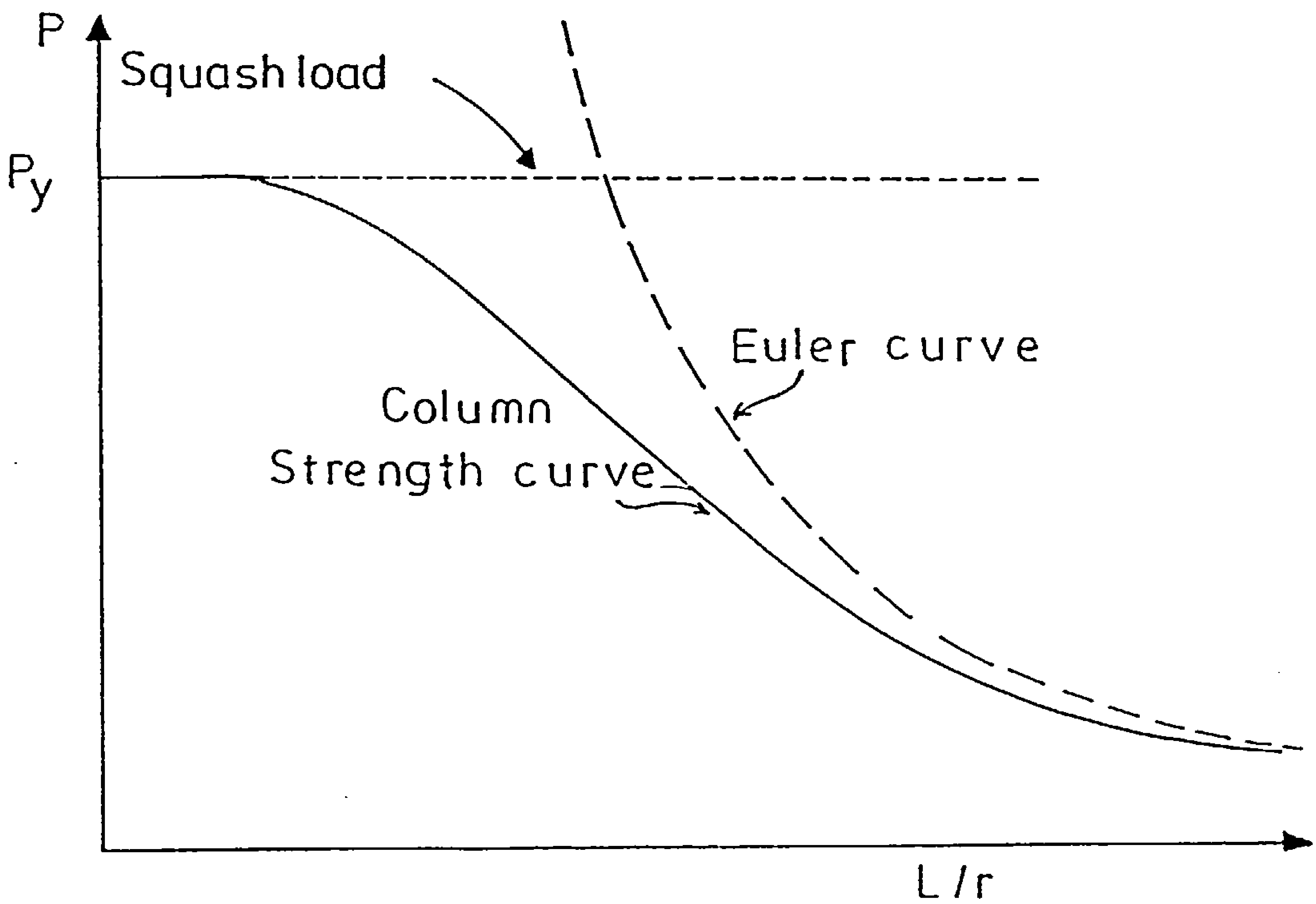

Figure 2.1: Typical column strength curve

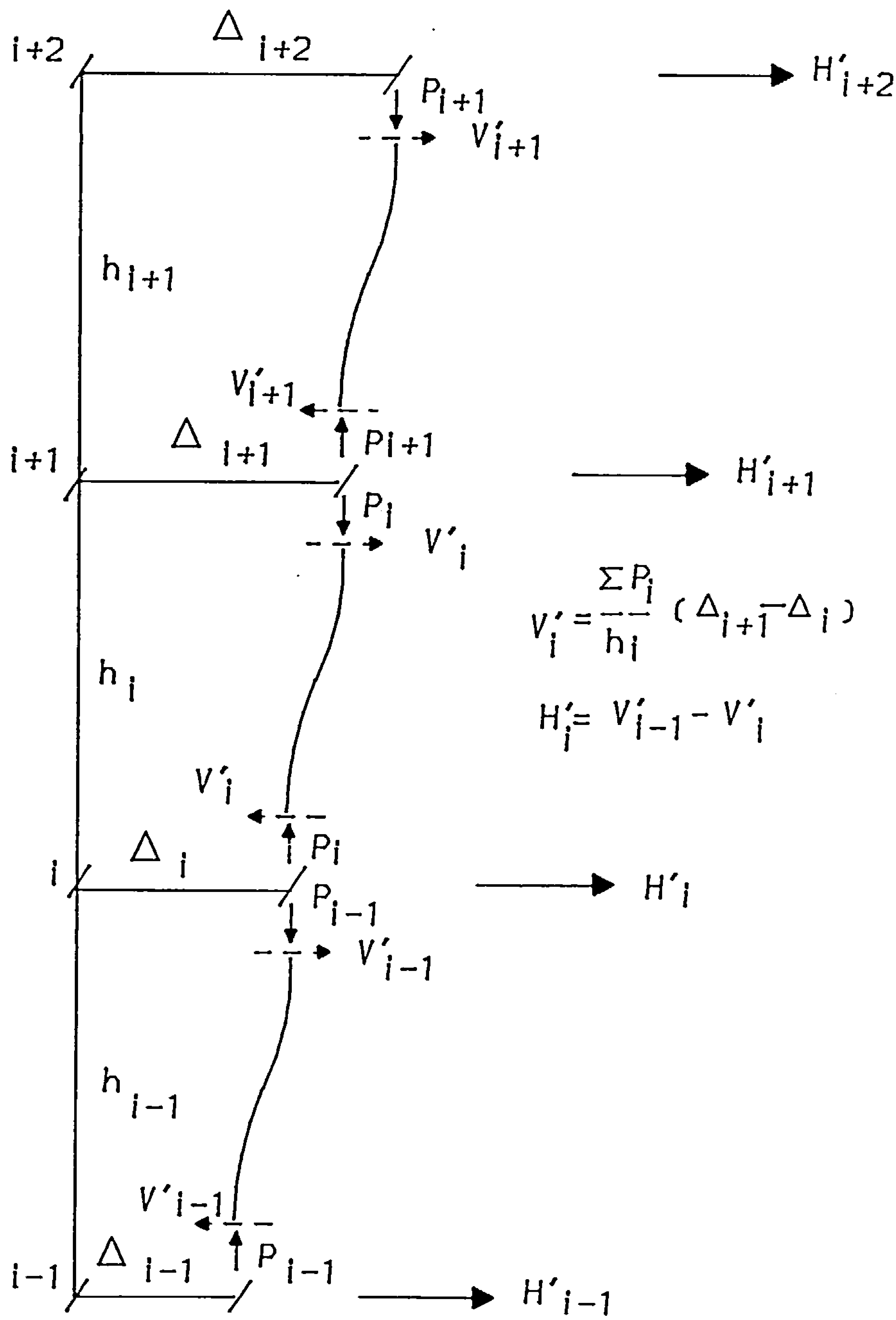

Figure 2.2: Sway forces due to vertical loads

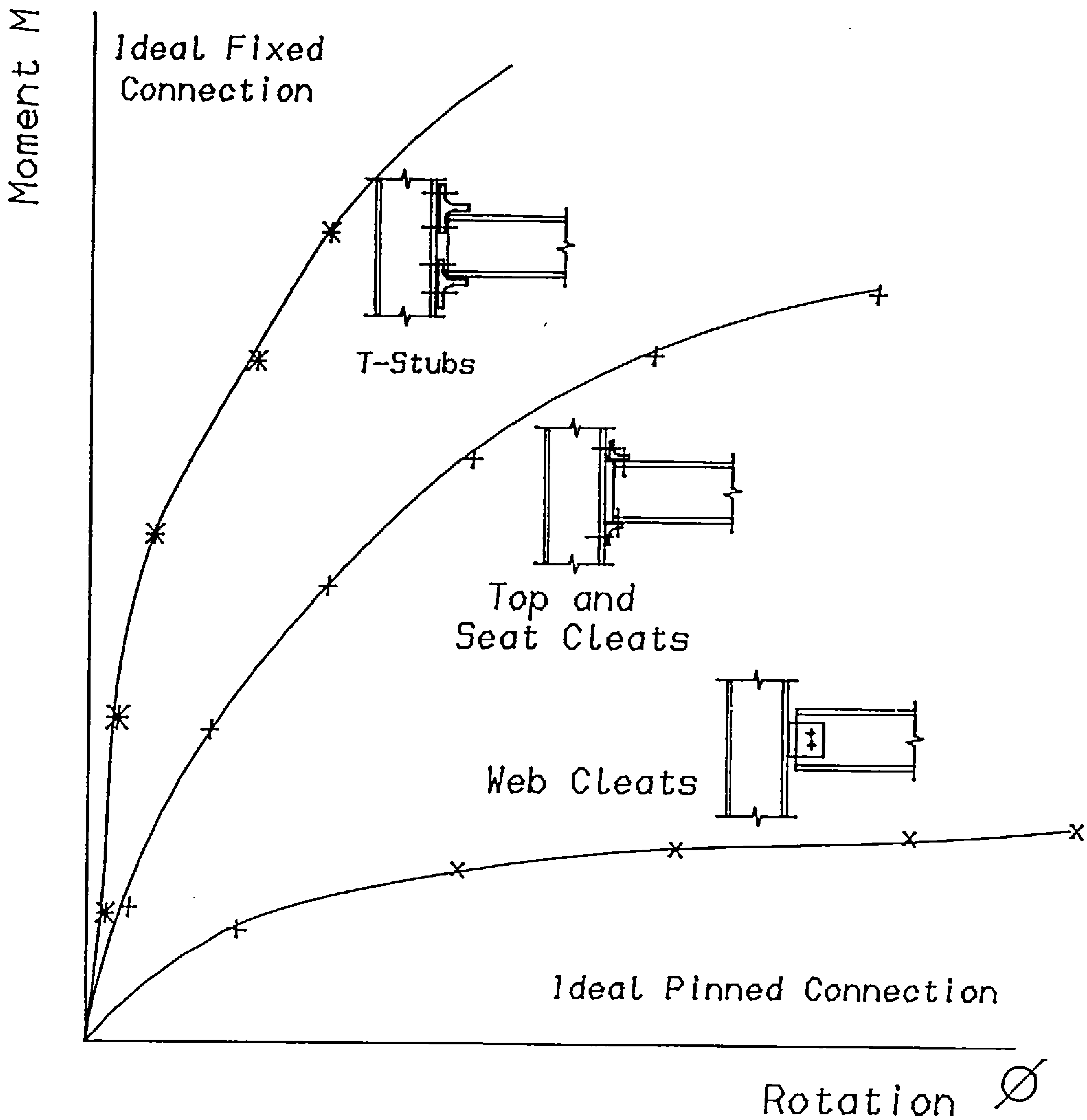

Figure 2.3: Typical moment-rotation curves for semi-rigid joints

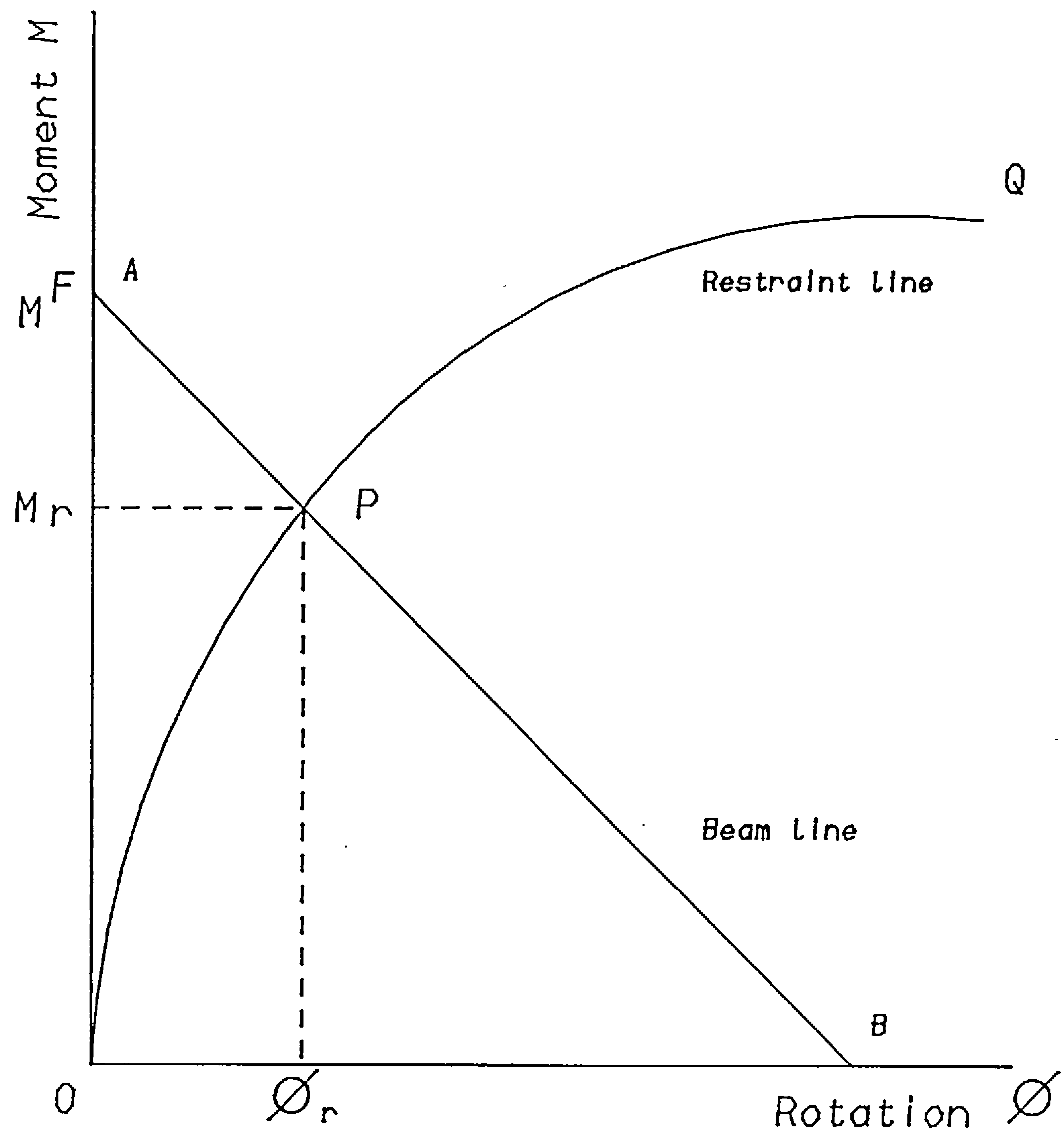

Figure 2.4: Beam line method

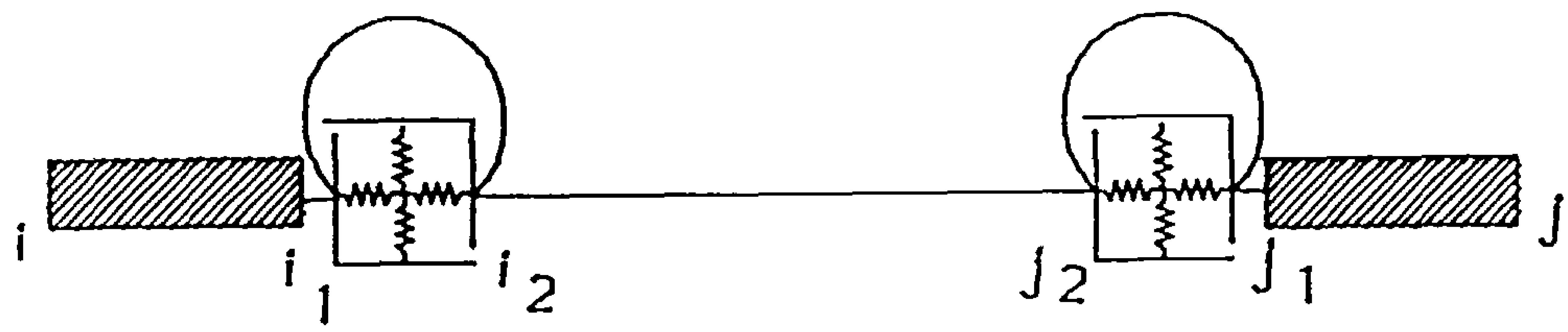

Figure 2.5: Model assumed by Cosenza et al for beam-column element with semi-rigid joints

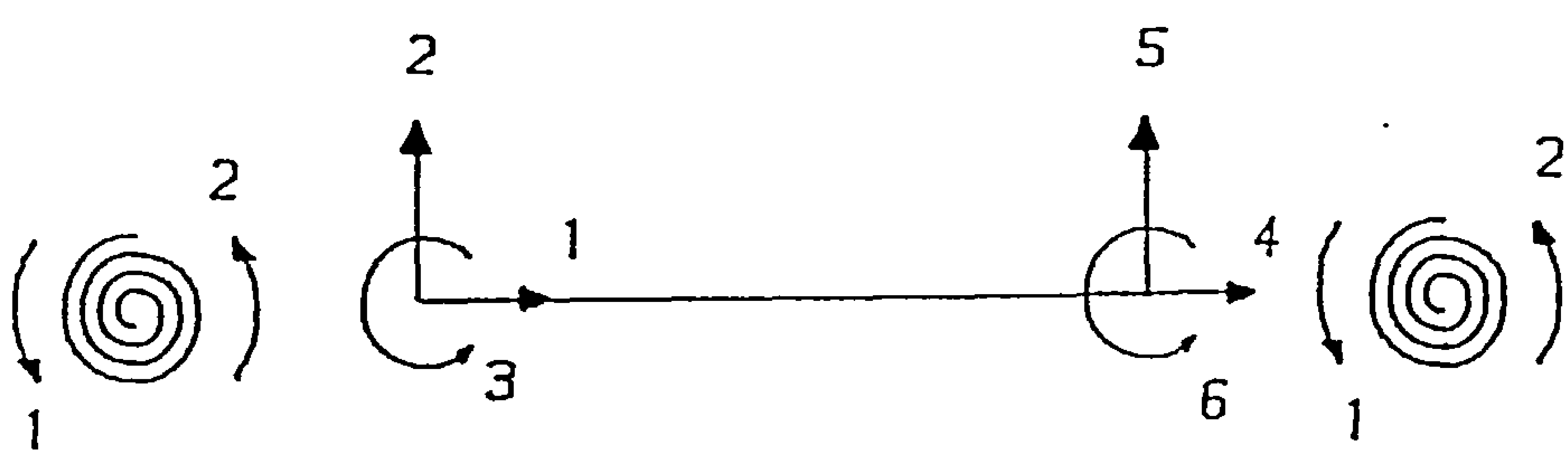

Figure 2.6: Model assumed by Chen and Lui for beam-column element with semi-rigid joints

Chapter 3

Instability of Steel Frame Structures

3.1 Introduction

If a stiff structure is subjected to relatively small loads the relationship between the loads and displacements can be assumed to be approximately linear. However, in most practical steel structures the applied loads will significantly alter the geometry of the structures and thus this proportionality between loads and displacements no longer exists (fig.3.1).

Fig.3.2 shows a beam-column carrying both axial and lateral loads. The lateral loads alone will produce lateral deflections and the latter is increased by the presence of an axial load. Thus, the lateral deflections is related to both the axial and the lateral loads in a nonlinear manner and the deflections of one loading case cannot be superimposed on those of the other. Initial imperfections or load eccentricities have the same effects on element behaviour.

3.2 Instability of Columns in Steel Frames

Two different approaches have been used in columns analysis. These approaches are:

1) Eigenvalue approach (Bifurcation): In this approach the ultimate load of a column can be defined in a direct manner by noting that when an ideal column is loaded by a concentrated axial load, it will remain in a perfectly straight position as long as the load is lower than its buckling load. However, when the applied load reaches the elastic critical load the column lateral deflection increases without limit. This method is wholly elastic and a modification to this approach is usually incorporated to include the inelastic behaviour of practical steel columns.

2) Load-deflection (Stability) approach: This approach attempts to solve the column problem by tracing its load-deflection curve and defining the peak point of this curve as the column's ultimate load. This approach is known to represent real behaviour more accurately than the previously mentioned one since column initial imperfections and residual stresses can be easily incorporated. However, a more involved numerical procedure is required to calculate the deflection in the elastic-plastic range.

3.2.1 Eigenvalue Approach

In the eigenvalue approach, a column is assumed to be in a perfectly straight position until the bifurcation point is reached (fig.3.3). At this point equilibrium is possible in both straight and slightly bent positions. This defines element buckling and the load at this point denotes the elastic critical load.

3.2.1.1 A Simply Supported Column (Euler Load)

Considering a straight simply supported column shown in fig.3.3.a Under the assumption of small deflection theory and a constant second moment of area of the element, I, the equilibrium equation for this element is

$$v'' + k_2 \, v = 0$$

where

$$k^2 = \frac{P}{EI} \tag{3.1}$$

and where v" denotes the second derivatives of v with respect to z (see fig.3.3.a). For constant k, the solution for this differential equation is

$$v = A \sin kz + B \cos kz \tag{3.2}$$

A and B are integration constants which can be defined from the element boundary conditions. These are in this case:

$$v = 0 \quad at \quad z = 0 \qquad v = 0 \quad at \quad z = l$$

Substituting in eq.3.2 leads to:

$$B = 0$$

and

$$A \sin kl = 0 \tag{3.3}$$

Eq.3.3 can be satisfied by either A=0 which means that the lateral deflection is always zero and the member remains straight or by

$$k_n L = n \pi \tag{3.4}$$

Substituting eq.3.1 into eq.3.4 gives:

$$P_n = \frac{n^2 \pi^2 EI}{L^2} \tag{3.5}$$

from eq.3.5 several value of critical load can be obtained by substituting n as (1,2,3,4 ..etc). However, Only the smallest values of P_n has significance in practice. This value is known as 'Euler load' and can be obtained by setting n=1.

$$P_e = \frac{\pi^2 EI}{L^2} \tag{3.6}$$

The load deflection relationship for this case is shown in fig.3.3.b where it can be shown the lateral deflection is uniquely defined for each load value, i.e. as zero, until the bifurcation point where several deformed shapes are possible for one load value, i.e. the elastic critical load.
It has been assumed that the column is perfectly straight, but in practical cases columns usually have unavoidable small initial geometric imperfections.

3.2.1.2 Initially Crooked Column

Bifurcation is possible only for a column which remains perfectly straight during loading. However, small initial geometrical imperfections are inevitable in practical columns. Assuming now the column has initial imperfection v_o at zero applied axial load as shown in fig.3.4.a and that under the application of a load P further lateral deflection v occurs then the moment at any section along the column is:

$$M = P\,(v + v_o)$$

Noting eq.3.1 the equilibrium equation for this column becomes:

$$v'' + k^2\, v = -\, k^2\, v_o \tag{3.7}$$

This is a linear inhomogeneous differential equation and the solution consists of two parts. The first one is for homogeneous solution, i.e. identical to eq.3.2, and the second part is any particular solution which satisfies eq.3.7.

If the initial shape of the column is assumed to be

$$v_o\,(z) \;=\; a\;\sin\;\frac{\pi\;z}{L} \tag{3.8}$$

Substituting into eq.3.7 leads to

$$v" \;+\; k^2\;v \;=\; -k^2\;a\;\sin\;\frac{\pi\;z}{L}$$

The general solution for this equation (see ref. [6])is given as:

$$v \;=\; A\;\sin\;kz \;+\; B\;\cos\;kz \;+\; \frac{1}{(\frac{\pi^2}{k^2L^2})\;-\;1}\;a\;\sin\;\frac{\pi\;z}{L} \tag{3.9}$$

Satisfying the boundary conditions (v=0 at z=0 and z=l) requires that A = B = 0 and thus eq.3.9 becomes:

$$v \;=\; \frac{1}{(\frac{\pi^2}{k^2L^2})\;-\;1}\;a\;\sin\;\frac{\pi\;z}{L} \tag{3.10}$$

Defining now the ratio between the applied load and the elastic critical load as β_e gives:

$$\beta_e \;=\; \frac{P}{P_{cr}} \;=\; \frac{P}{\frac{\pi^2\;EI}{L^2}} \;=\; \frac{k^2L^2}{\pi^2}$$

then eq.3.10 becomes:

$$v = \frac{\beta_e}{1-\beta_e}\;a\;\sin\;\frac{\pi\;z}{L}$$

The column updated shape can be obtained by adding eq.3.10 to eq.3.8 which yields:

$$v = \frac{a}{1\;-\;\beta_e}\;\sin\;\frac{\pi\;z}{L} \tag{3.11}$$

From eq.3.11 it can realised that the initial deflection is multiplied by the ratio $1/(1-\beta_e)$ as a result of the presence of the axial load and as P approaches the elastic critical load P_{cr} the deflection increases without limit (fig.3.4.b).

Columns with end eccentricities or with lateral loads can be treated in same way since only the right side of eq.3.7 will be changed according to the case under investigation.

3.2.1.3 Inelastic Behaviour of Real Columns

In the eigenvalue problem, it is assumed that a column fails by elastic instability. This presents the behaviour of a slender column as long as the column stress remains below the material yield stress. When the column axial stress reaches the yield stress its failure will be accelerated due to the plastic behaviour of the material. Thus, the column behaviour at failure is inelastic where the combination of elastic instability and material plasticity together influence the behaviour. Because of this it was realised that practical columns usually fail at loads lower than the Euler load.

3.2.1.4 Tangent Modulus Theory

Engesser (1889) proposed that, to account for inelastic behaviour of practical columns, the tangent modulus E_t should be used instead of Young's modulus E in conjunction with the 'Euler Formula' to calculate ultimate loads. E_t is the slope of the strain-stress relationship, $\sigma - \epsilon$, of the material as shown in fig.3.5. For a very slender column the axial stresses at elastic instability should fall in the linear region of the $\sigma - \epsilon$ relationship, thus Euler load can be used to represent column ultimate load. However, for more stocky columns the stresses at failure will reach the nonlinear region of the $\sigma - \epsilon$ relationship, hence E in Euler formula should be replaced by E_t.

This modification can be justified by assuming that the columns possess no lateral deflection until the yield stress is reached and the strain-stress characteristics are uniform throughout the column sections.

E_t depends on the material strain-stress relationship and on the section residual stresses. The Column Research Council (CRC) [13] recommended the use of tangent modulus theory to estimate column ultimate loads in the inelastic range.

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3.2.1.5 Reduced Modulus Theory

The assumption adopted in the tangent modulus theory which suggests that strain-stress characteristics to be uniform throughout a column section until failure is rather idealised. As soon as the column starts bending the stresses on the convex side will be decreased while the stresses at the concave side are increased (fig.3.5). Hence the column section is governed by two moduli, E for convex side and E_t on the concave side. For this reason this type of behaviour is called double modulus theory. Based on the above reasoning Engesser(1895) realised the contradictions inherent in the tangent modulus theory and recommended the use of the 'Reduced modulus theory ' where the reduced modulus E_r is used instead of E_t to replace E in the Euler formula. The magnitude of E_r lies between E_t and E and depends on the material strain-stress relationship and on the section properties.
Van Karman (1910) conducted a series of experiments on columns to verify the concept of the reduced modulus theory. He found that the actual buckling loads obtained from his specimens were closer to the tangent modulus values than the reduced modulus theory. Shanley (1947) explained this contradiction between the reduced modulus theory and experimental tests. He concluded that bending of columns starts as soon as the tangent modulus value is reached (fig.3.6), thus the real critical loads of columns are higher than tangent modulus load and lower than the reduced modulus load.

3.2.2 Load-Deflection Approach (Stability)

The load-deflection approach takes into account the fact that columns have initial geometric imperfections therefore they start bending as soon as the loads are applied. Unlike the bifurcation approach where the critical load can be obtained by means of solving the governing differential equations, a column strength curve (fig.3.7) can be only obtained by calculating load-deflection curves for a large number of columns with different slenderness ratios(L_c/r). The maximum load is obtained from each curve and a curve fitting technique is required to plot the relationship of column loads against column slenderness ratios (L_c/r).

3.3 Elastic Instability of Steel Frames

Defining frame instability requires the determination of the buckling condition of interconnected members as well as the determination of the maximum capacity of the whole structure.

3.3.1 Eigenvalue Approach

Columns rarely occur in isolation but are usually connected to other members of a structure hence eq.3.5 which has been used for a pin-ended column cannot be used directly. If a column is fixed at both ends and loaded as shown in fig.3.8 the corresponding differential equation can be expressed as:

$$EI\, v'' \;=\; -\, M(z) \tag{3.12}$$

where M(z) is the moment at any point of the column. Due to the boundary conditions, the transverse loads influence M(z) in an indeterminate manner. However, it was shown by Timoshenko and Gere (1961) that a single fourth order equation is applicable to any prismatic column regardless of the boundary conditions.

Differentiating eq.3.12 twice with respect to z yields

$$EI\, v'''' \;+\; P\, v'' \;=\; q(z)$$

where v"" is the fourth derivatives of v with respect to z. Dividing by EI and using eq.3.1 produces

$$v'''' \; + \; k^2 \; v'' \; = \; \frac{1}{EI} \; q\,(z) \qquad (3.13)$$

The general solution [6] of eq.3.13 is

$$v \; = \; C_1 \; + \; C_2 \; z \; + \; C_3 \; \sin \; kz \; + \; C_4 \; \cos \; kz \; + \; v_p(z) \qquad (3.14)$$

C_1 , C_2 , C_3 , C_4 are integration constants can be defined from the boundary condition of the case being considered.
Eq.3.14 is the base of both stiffness and flexibility analyses. A more detailed discussion of the derivatives of both analyses can be found in ref. [1]

3.3.2 Finite Element Analysis

In the finite element analysis the 'tangential' stiffness matrix, K_T , is calculated for each element. This matrix is given by:

$$K_T \; = \; K_E \; + \; K_G \; + \; K_L$$

where

K_E is the elastic stiffness matrix based on small deflection theory.

K_G is the geometric stiffness matrix which depends on the axial force of the element. This matrix updates the element deformation due to the presence of the axial load.

K_L is the displacement matrix which depends on the deflected shape of the element.

For a frame structure, the element tangential stiffness matrices are then assembled to form the global tangent matrix of the structure. Instability of the structure is usually considered to occur when the determinant of the global tangent matrix is equal to zero.

An incremental procedure should be used to calculate the deformations for the applied loads and another iterative procedure such as the Newton-Raphson process is required to converge the solution inside each load step. Details on the procedure for a beam element will be given in chapter 8. Many computer programs [1], [40], [3] have been developed to accurately solve instability problem of steel frames. A computer program, which was originally developed at The University of Alberta [1] has been used by the author to conduct this study. Brief details about this program follows

3.3.2.1 Alberta Program (Instaf)

A computer program called 'Instaf' was available at The University of Sheffield at starting time of this project. This program uses variational principles of the finite element analysis and large deformation theory to predict the response of rigidly connected frames assuming in plane behaviour. Both material and geometrical nonlinearity were included. To converge on the solution the well known Newton-Raphson iterative technique was used. Residual stresses and strain hardening are included. More details about this program can be found in ref. [1]

3.4 Design Analysis of Frame Instability

Despite the fact that many methods have been proposed to rigorously solve instability problems in steel structures, most design codes [4] and [5] still allow the use of more simple but less accurate approximate methods. This contradiction can be related to the following reasons:

1) Utilising of an exact method to solve instability problems of a frame requires the use of a numerical procedure. Therefore a powerful computer with a large memory is needed. This however is not (as yet) available in most design offices.

2) When designing a steel frame many influential factors have to considered. Some of these factors are out of plane response, plasticity, instability, semi-rigid joint behaviour and local buckling. Therefore using an exact analysis which account for all of these factors simultaneously is impractical and almost impossible.

3) Defining in advance the most influential of the previously mentioned factors is rather difficult since this will depend on the structure type, section properties, external loads..etc.

Because of the above reasons, only simple procedures are justified as design tools to include or to check against these factors. For instability two methods are currently available in modern design codes for use in design offices. These methods are the 'Effective length factor method' and 'Amplified sway method'. In the first and most popular method columns of a practical frame are designed independently considering the applied vertical loads and the end restraint provided by the existence of beams connected to the columns being considered. However, in the second method instability is included by enhancing the member bending moments due to horizontal loads by a factor which depends on the frame load factor at elastic instability.

3.4.1 The Effective Length Approach

In both the Euler formula and the column strength curves mentioned in section 3.2.2 columns are assumed to be pinned at their both ends. Nevertheless, in practical structures columns are usually connected to other members hence end restraint is provided. These end restraints can be incorporated by employing 'the effective length method '. In this method the length of an equivalent simply supported column, which has an elastic critical load equal to that of the actual column, is calculated. Utilising this method allows the conversion of a column with any end conditions to the simply supported column case.

Due to its simplicity the effective length approach has been widely used in design via the provision of simple idealised nominal effective length factors.

In order to improve on the rather crude but generally conservative estimates, alignment charts AISC are provided in specification [5] and effective length contour charts appear in BS5950. The concept of the effective length has significance only for members subjected to axial loads and, as such, it has a specific physical interpretation as shown in fig.3.9 It is the length between theoretical points of inflection in the buckled mode shape which depends on the rigidity of the connected boundary elements. If the effective length ratio k_e is known, the critical load for the framed column can be then determined by reference to column strength tables, or alternatively, the allowable stress may be determined directly from the specification of allowable stress or reduced strength formulae or tables.

3.4.2 The Simple Amplification Factor in Timoshenko's Method

An approximate method of predicting the behaviour of a beam-column element is presented by Timoshenko (fig.3.10) in which the stresses and deformations which would occur in the absence of the axial loads are multiplied by the simple amplification factor $\lambda_{cr}/(\lambda_{cr} - 1)$ where , λ_{cr} is the elastic critical load factor of the element.

It should be noted here that, the simple amplification factor is identical to the factor appearing in eq.3.11 which clearly shows that this factor might be used with good accuracy to predict deformations of a beam-column element. However this factor is less accurate when stresses are considered. A comparison which has been presented in ref. [49] confirms this conclusion. In this reference the simple amplification factor was used to estimate the deformations and stresses of a beam-column element and those were compared with the corresponding exact values.

Consider now a multistorey sway frame, which is a more complex structure, subjected to both vertical and horizontal loads. The buckling effects due to the gravity loads acting on the columns and due to the horizontal loads acting on the beams will influence the response of the structure. In practical multistorey frames however, beams are usually stiffer than columns and the applied horizontal loads are small compared with the gravity loads, therefore the beams have negligible buckling deformations compared to those in the columns. Therefore, the approximate methods (such as the P- Δ method, which has been mentioned in chapter 2, the effective length method and the amplified sway method) consider only the buckling effects on the columns.

3.4.3 The Amplified Sway Method of BS5950

This approach includes the buckling effects for sway frames by evaluating enhanced values of bending moment whilst using effective length factors of unity. Initially a linear elastic analysis is used to calculate the moments which are caused by horizontal and vertical loads acting separately. The load factor at elastic instability $\lambda_{cr,f}$, is calculated for the whole frame and it is suggested in BS5950 that the deflection method be used which is turn utilises the analysis which has probably been carried out in a sway /non-sway categorisation assessment. The moments which are caused by horizontal loads are increased by the simple amplification factor of the frame $\lambda_{cr,f}$ / ($\lambda_{cr,f}$ - 1) before being added to the moments which are caused by vertical loads. It should be noted that:

1) The method assumes that the deformation effects are identical in all members of the whole frame but in real structures this assumption is rarely satisfied.

2) The method ignores the effect of the axial loads on the individual member rotational stiffnesses.

3) The method ignores about the possibility of variation of storey stiffnesses against lateral drifts of real frames.

3.4.4 Development of the Method

In a practical multistorey frame, such as the one shown in fig.3.11, the sum of the gravity loads increases from upper to lower stories whilst sections tend to get larger. Thus the buckling effect, which depends upon the axial loads, the cross-sections and length of the columns, will in general vary from storey to storey. Therefore, applying the current amplified sway method might cause significant undesirable errors generally overestimating the buckling effects on upper stories of a multistorey frame. Although the buckling effects are likely to be overestimated it does not follow that the design moments will be conservatively assessed as will be seen later.

Consider now the ith storey of the frame of fig.3.11 and adopt the model described by Bolton [50] in which the buckling effect of the storey is simplified to that of a single element (fig.3.12) subjected to vertical load, $W_{i,t}$. A horizontal test load, Q, is used to check the stability of this element which is freely pivoted at the bottom, B, and restrained by a spring, which represents the storey stiffness at its upper end A. Taking moments about B the equilibrium equation for this element is :

$$W_{i,t} \times \Delta + Q \times L_c - F \times L_c = 0$$

The force F, in the spring is obtained by multiplying its stiffness, S_b, by its extension, Δ.

Therefore

$$W_{i,t} \times \Delta + Q \times L_c - (S_b \times \Delta) \times L_c = 0$$

or

$$\frac{Q \times L_c}{\Delta} = (S_b \times L_c) - W_{i,t}$$

The structural stiffness, Q / Δ, is zero when $W_{i,t}$ reaches its critical value, $W_{i,cr}$ thus

$$W_{i,cr} = S_b \times L_c \tag{3.15}$$

Assuming now that the test load, Q, equals $W_{i,t}$ and that the resultant horizontal displacement is U, the spring stiffness of the simple structure is $W_{i,t}/ U$. Substituting in eq.3.15 gives

$$\frac{W_{i,cr}}{W_{i,t}} = \frac{L_c}{U}$$

thus $\lambda_{cr} = \frac{W_{i,cr}}{W_{i,t}}$ is given by

$$\lambda_{cr} = \frac{1}{\phi} \tag{3.16}$$

where ϕ, the sway index is $\frac{U}{L_c}$ (see fig.3.12).

Ignoring the interaction between the stories allows the ith storey to resist a vertical load equal to $W_{i,cr}$ before instability. Hence, the simple amplification factor for this storey can be obtained from the calculated approximate load factor at elastic instability of the storey $\lambda_{cr,i}$.

Applying this argument to the other stories of the frame indicates that the resultant sway index ϕ_i can be obtained at ith storey level by subjecting the frame to a horizontal shear equal to the sum of the axial columns loads in that storey.

Whilst not strictly true, the behaviour of any storey of a multistorey frame is taken as being independent from the others. With this assumption good estimates of the approximate load factor at elastic instability $\lambda_{cr,i}$ and the simple amplification factor Af_i for every storey can be obtained readily. Thus, for the multistorey frame illustrated in fig.3.11, if the applied vertical loads are increased, the weakest storey, probably the second one here, would be most sensitive to instability while the stiffer stories would be less sensitive.

This can be conceived as one storey becoming unstable at a certain load level and by increasing the vertical loads another storey would become unstable and so on. However, this behaviour is idealised and not realistic because there is only one value of the load factor at elastic instability for this frame, which is equal to the lowest value of $\lambda_{cr,i}$. If the vertical loads reach this critical value the whole frame would collapse and not just the weakest storey.

In the proposed method this sensitivity factor is incorporated as follows. The simple amplification factor for the weakest storey (the storey which has the largest sway index) is obtained in the usual manner from $\lambda_{cr,min}$. In order to incorporate the variable influence of instability between stories, the sway index of the ith storey of the frame (except the weakest one) is multiplied by the simple amplification factor of the weakest storey, Af_w , with the restriction that it must not exceed the sway index obtained from the weakest storey, i.e.

$$\phi_i' = \phi_i \times Af_w \quad but \leq \phi_{max} \tag{3.17}$$

This modification is to enforce an interaction between the stories. When the applied loads reach their elastic critical value the sway index of the weakest storey, ϕ_{max}, equals unity and the simple amplification factor of the weakest storey, Af_w, then becomes equal to infinity. Hence by substituting into eq.3.17 the enhanced sway index at each storey, ϕ_i', reaches unity at the same time and the whole frame collapses simultaneously; thus satisfying the collapse condition.

$$\phi_i' = \phi_i \times \infty \; but \leq 1.0 \quad \rightarrow \quad \phi_i' = 1$$

Having obtained the enhanced value of the sway index at the ith storey ϕ_i' (except the weakest storey) and the load factor at elastic instability $\lambda_{cr,i}$, the simple amplification factor Af_i can be obtained in the usual manner but from ϕ_i' instead of ϕ_i.

3.4.5 Implementation of the Modified Amplified Sway Method

To apply the modified method, instead of using a constant multiplier which acts as a uniform deformation coefficient for the whole frame, special deformation coefficients for each individual storey are obtained. These more closely reflect the buckling mode shape and hence represent an improved assessment of the real influence of instability. They may be determined using a first order linear elastic analysis. It should noted that each storey is defined as all columns in the lift plus the beams which are connected at the top of these columns. The structure may thus be imagined as a series of single storey portal frames stacked vertically above each other. The modified method proceeds as follows:

1) Calculate the member moments due to vertical loads and due to horizontal loads acting individually by linear elastic analyses.

2) Calculate the lateral deflections at each floor level after subjecting the frame to 0.5% of the sum of the applied vertical loads at each storey level acting horizontally by a linear elastic analysis. This factor of 0.5% is used for convenience as it is used elsewhere in BS5950:Part:1 thus eq.3.16 should be modified to $\lambda_{cr,f} = \frac{1}{200\,\phi}$.

3) Determine the sway index at each storey by using eq.3.18 and from the storey which has largest sway index $\lambda_{cr,f}$ for the whole frame is obtained using eq.3.19.

$$\phi_i = 200 \frac{(\Delta_{i+1} - \Delta_i)}{Lc_i} \tag{3.18}$$

$$\lambda_{cr,min} = \frac{1}{(\phi_{max})} = \lambda_{cr,f} \tag{3.19}$$

Here Δ_{i+1} and Δ_i are the lateral deflections at the top and the bottom of the ith storey respectively, Lc_i, is the column length for this storey and ϕ_{max} is the largest sway index of the frame. The simple amplification factor for the weakest storey Af_w can be obtained now from:

$$Af_w = \frac{\lambda_{cr,f}}{\lambda_{cr,f} - 1} = \frac{\frac{1}{\phi_{max}}}{\frac{1}{\phi_{max}} - 1} = \frac{1}{1 - \phi_{max}} \tag{3.20}$$

4) The sway indices obtained in step 3) must now be increased at each storey (except for the weakest storey, i.e. the storey which has the largest sway index) by the amplification factor Af_w.

This modification can be accomplished by using the following equation to amplify the sway index at each storey determined from step 3)

$$\phi_i' = 200 \times Af_w \frac{(\Delta_{i+1} - \Delta_i)}{Lc_i} = Af_w \times \phi_i \quad but \leq \phi_{max} \tag{3.21}$$

5) The simple amplification factor Af_i for all stories of the frame other than the weakest can be obtained from

$$Af_i = \frac{\lambda_{cr,i}}{\lambda_{cr,i} - 1} = \frac{\frac{1}{\phi_i'}}{\frac{1}{\phi_i'} - 1} = \frac{1}{1 - \phi_i'} \tag{3.22}$$

where ϕ_i' is the sway index which is calculated from eq.3.21.

It should be noted that:

1) Using a special simple amplification factor Af_i for each storey of a multistorey frame leads to the amplification of the end moments of two columns, connected to same node but belonging to different stories, by different factors. In fig.3.13 the bending moment due to horizontal load at end A of the column of the ith storey is amplified by Af_i whilst the bending moment at end B of the column of the ith+1 storey is amplified by Af_{i+1}. Thus small unbalanced nodal moments may arise. These unbalanced moments are small and normally negligible because the simple amplification factors of frames generally vary gradually from one storey to another. In addition only the member end moments due to horizontal loads are modified.

2) The modified method leads to predicted bending moments which are identical to those obtained using the current amplified sway method for a single storey frame.

3) It has been assumed that vertical loads do not produce sidesway of the structure, i.e. symmetrical frames applied by symmetrical vertical loads. However, for non regular cases, a slight modification has been suggested by the draft of EC3 in which the sway index at ith storey should be calculated from the following equation:

$$\phi_i = \frac{P_i}{H_i} \frac{(\Delta_{i+1} - \Delta_i)}{Lc_i}$$

where

P_i is the sum of the applied vertical loads at the ith storey;

Δ_{i+1}, Δ_i are the lateral deflections at the top and the bottom of the ith storey respectively calculated after subjecting the frame to the actual horizontal and vertical loads;

H_i is the applied horizontal load at the ith storey.

Having determined the sway index at each storey, the proposed method should be carried out as usual.

In the method of calculating a simple amplification factor Af_i for each storey of a multistorey frame summarized above, the first three steps are required by the current amplified sway method. The remaining additional steps require only minimal extra effort.

3.4.6 Application of the Method

The proposed method has been applied to many examples and comparisons have shown that the modified amplified sway method represents the effect of stability on a multistorey frame significantly more accurate than the current amplified sway method.

3.4.6.1 Worked Example 3.1

The use of the method is demonstrated by the example of the frame shown in fig.3.11 which has the following properties:

h = 3.75 m , L_b = 6.00 m , W = 200 kN , H = 20 kN.

The beam sections of the frame are IPE300.

The column sections of the frame are HE200B.

Young's modulus E is taken as 21000 kN/cm^2.

The example is fully worked below, while the accuracy of the modified method for practical multistorey frames is then discussed.

The Solution

1) A first order linear elastic method of analysis is utilized to calculate the member end moments of the frame for actual horizontal loads and for vertical loads separately.

2) In table 3.1 the lateral deflections are calculated by a linear elastic analysis after subjecting the frame to the notional loads which are 0.5% of the sum of the vertical loads applied at each floor level acting in a horizontal direction. The simple amplification factor for each storey is then calculated.

In this table SN is the storey number and Δ_{i+1}, Δ_i are the upper and the lower end deflections of each storey. The weakest storey is identified as the second one. Here, $\phi_2 = \phi_{max}$ and $\lambda_{cr,min}$ are obtained from the eqs.3.18 and 3.19 respectively as indicated

$$\phi_{max} = 200 \times \frac{(0.693 - 0.288)}{375} = 0.216$$

$$\lambda_{cr,min} = \frac{1}{0.216} = 4.63 = \lambda_{cr,f}$$

The simple amplification factor for this storey Af_w, the weakest one, is obtained from eq.3.20 as

$$Af_2 = \frac{1.00}{(1.00 - 0.216)} = 1.28 = Af_w$$

It should be noted here that, Af_2 is used by the current method of BS5950 as the simple amplification factor of the entire frame.

Now computing the sway index ϕ_i' and the simple amplification factor Af_i for the first storey, for example, using eqs.3.18, 3.21 and 3.22 gives:

$$\phi_1 = 200 \left[\frac{0.288 - 0.00}{375}\right] = 0.154$$

$$\phi_1' = 1.28 \times 0.154 = 0.20 \ but \leq 0.216 \rightarrow \phi_1' = 0.20$$

$$Af_1 = \frac{1.0}{(1.0 - 0.20)} = 1.25$$

S.N	$\Delta_{i+1}(cm)$	$\Delta_i(cm)$	ϕ_i	$\lambda_{cr,f}$	Af_w	ϕ'_i	Af_i
1	0.288	0.00	0.154	-	1.28	0.20	1.25
2	0.693	0.288	0.216	4.63	-	-	1.28
3	1.066	0.693	0.199	-	1.28	0.22	1.28
4	1.384	1.066	0.170	-	1.28	0.22	1.28
5	1.644	1.384	0.137	-	1.28	0.18	1.22
6	1.845	1.644	0.107	-	1.28	0.14	1.16
7	1.985	1.845	0.075	-	1.28	0.10	1.11
8	2.069	1.985	0.045	-	1.28	0.06	1.06

Table 3.1: Simple amplification factors of the frame indicated in fig.3.1

Repeating the same steps for the third storey gives:

$$\phi_3 = 200\left[\frac{1.066 - 0.693}{375}\right] = 0.199$$

$$\phi'_3 = 1.28 \times 0.199 = 0.250 \ but \leq 0.216 \rightarrow \phi'_3 = 0.216$$

$$Af_3 = \frac{1.0}{(1.0 - 0.216)} = 1.28$$

Finally for the top storey floor, number 8,

$$\phi_8 = 200\left[\frac{2.069 - 1.985}{375}\right] = 0.045$$

$$\phi'_8 = 1.28 \times 0.045 = 0.06 \ but \leq 0.216 \rightarrow \phi'_8 = 0.06$$

$$Af_8 = \frac{1.0}{(1.0 - 0.057)} = 1.06$$

3) Finally the member end moments due to the horizontal loads only, obtained from step 1 for the ith storey, are multiplied by the corresponding Af_i before being added to the member end moments which are caused by the applied vertical loads.

The member end moments, due to the vertical and the horizontal loads, obtained from utilizing the current amplified sway method and the proposed method are compared with those obtained from utilizing a rigorous nonlinear analysis. This comparison is shown in fig.3.14 where the member end moments predicted by the current amplified sway method are represented by a broken line, the member end moments predicted by the proposed method by a chain dotted line and the member end moments obtained from using an exact second order analysis by a continuous line.

Since, the prediction of the current and the modified methods are identical for the second, the third and the fourth stories, the member end moments predicted by the two methods are represented by a single line (broken line) for these stories.

However fig.3.14 indicates that more accuracy is achieved by predicting the member end moments by computing special amplification factors for stories 1,5,6,7,8 directly from eqs. 3.21 and 3.22 and where differences occur substantial improvements in accuracy accrue.

Two realistic practical multistorey frames, which are selected from reference [51] are utilised to demonstrate the use of the modified method for practical frames. However, since the horizontal loads are not considered in the previously mentioned reference, values are assumed.

3.4.6.2 Example 3.2

The frame shown in fig.3.15 is examined below. Since the values of the member bending moments due to the applied vertical loads are relatively large compared to those obtained due to the horizontal loads, only the member bending moments due to the horizontal loads accounting for the geometrical deformations caused by the presence of the vertical loads will be considered. Hence, in the modified and the current amplified sway methods the member end moments are due to the horizontal loads only and are obtained from utilizing a first order elastic analysis and amplified by the corresponding simple amplification factors. In the exact nonlinear analysis the member end moments due to the vertical loads which are obtained from using a first order elastic analysis are subtracted from the member end moments due to the vertical and horizontal

Storey	Current Method		Modified Method	
	Af_i (1)	Average Error % (2)	Af_i (3)	Average Error % (4)
1	1.29	3.68	1.29	3.68
2	1.29	21.85	1.08	6.13
3	1.29	29.56	1.032	15.93

Table 3.2: Summary of the result of the frame indicated in fig.3.15

Storey	Current Method		Modified Method	
	Af_i (1)	Average Error % (2)	Af_i (3)	Average Error % (4)
1	1.124	1.12	1.124	1.12
2	1.124	1.06	1.124	1.06
3	1.124	1.15	1.124	1.15
4	1.124	2.74	1.10	1.15
5	1.124	6.21	1.06	1.83

Table 3.3: Summary of the result for the frame indicated in fig.3.16

loads obtained from an exact nonlinear analysis. This permits clarification of the accuracy which is obtained from utilizing the modified method.

The result is summarized in table 3.2 where the simple amplification factor for each storey Af_i obtained from employing the current and the modified method are tabulated in columns 1 and 3. The average value of the absolute error per cent of the member end moments which are calculated from the current BS5950 and the modified method are compared with those obtained from an exact nonlinear analysis and tabulated in columns 2 and 4.

3.4.6.3 Example 3.3

Applying same process to the frame shown in fig.3.16 gives table 3.3.

An efficient improvement of the accuracy can be observed by applying the modified method to estimate the member end moments for the second and the third stories of the frame used in example 3.2. However, since in the frame of example 3.3 the storey stiffnesses are almost uniform throughout the entire frame, i.e. the horizontal deflections due to the notional loads subjected horizontally are approximately constant, the improvement which was gained from employing the modified method is, as expected, small.

It can be concluded that the proposed method predicts the spread of the instability effect in a multistorey frames more accurately than the current amplified method but, if the storey stiffnesses are uniform throughout the frame the prediction of the current and the proposed method are identical. Nevertheless, it is always beneficial to adopt the modified method for practical frames, which do not in general have uniform storey stiffnesses.

3.5 Conclusion

The limitation of the current BS5950:Part1 method to allow for the buckling effect in a multistorey sway frame has been discussed. A modified form of the current amplified sway method has been developed. The proposed method is based upon estimating the buckling effect at each storey of a multistorey frame depending on the applied vertical loads.

Comparing with the current method the proposed method features as follows:

1) It is realistic in nature since the simple amplification factor, calculated for each storey of a frame, depends on both storey stiffness against lateral drift and the elastic critical load of the frame. This does not apply in the current approach.

2) It is more accurate which has been shown by many worked examples conducted to investigate the accuracy of the modified method. Comparison with the current method has shown that significantly increased accuracy is obtained.

3) It is still simple to use since the proposed method requires only minimal additional calculation.

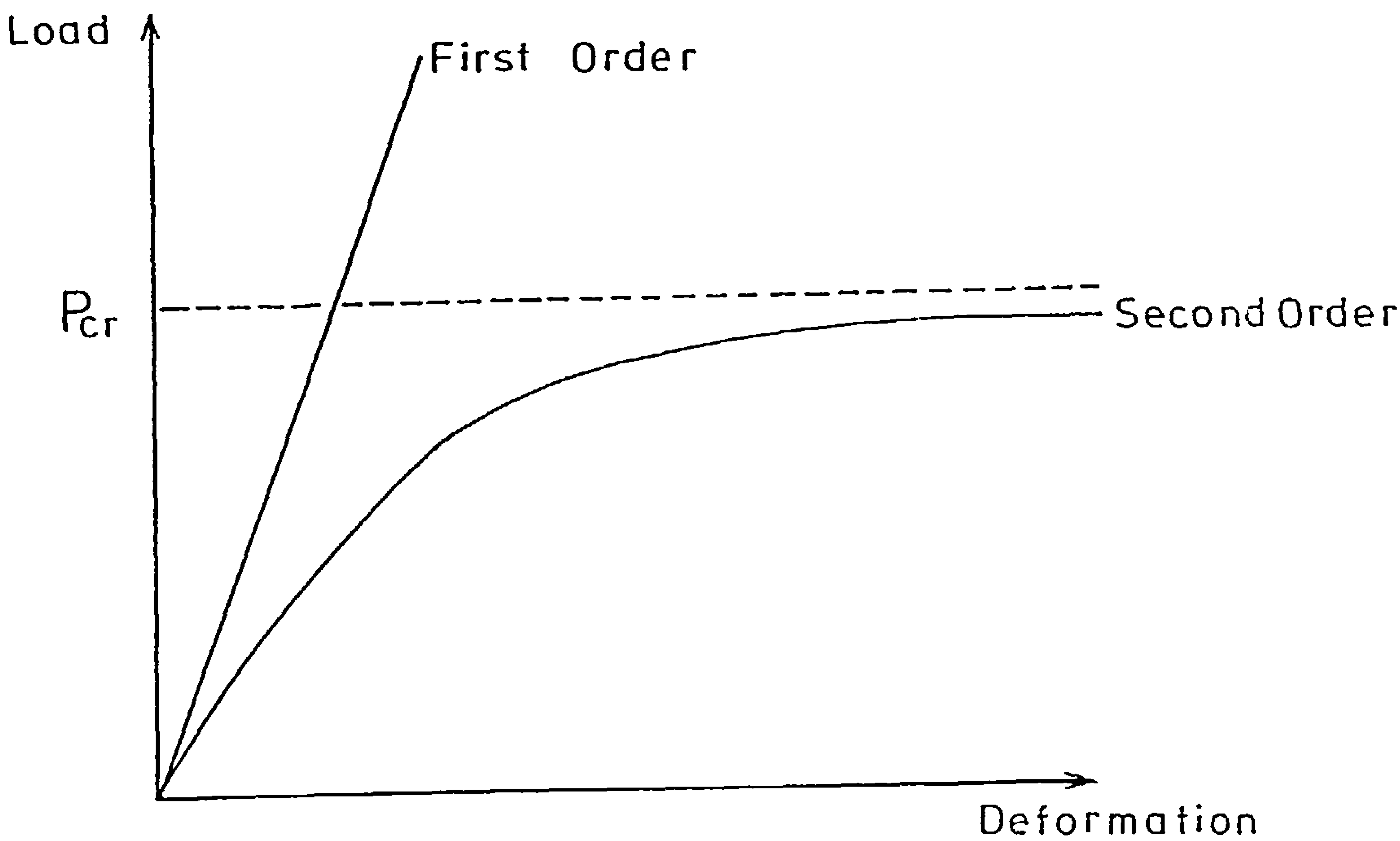


Figure 3.1: First and second order analyses.

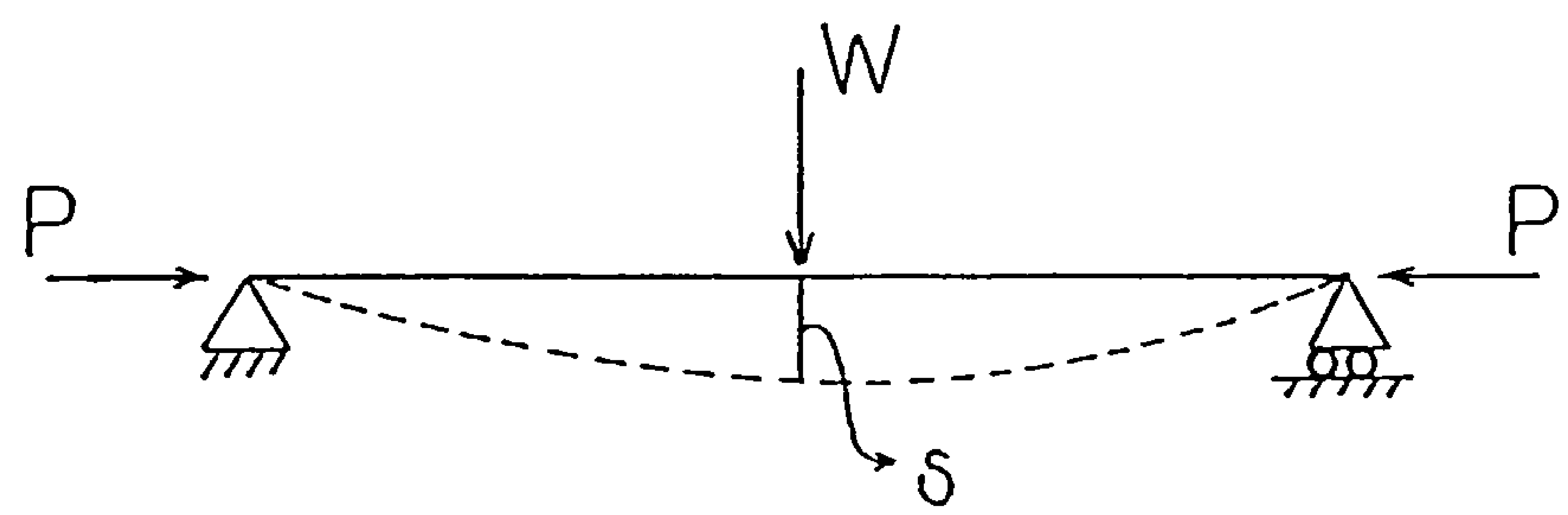


Figure 3.2: Beam-column element.

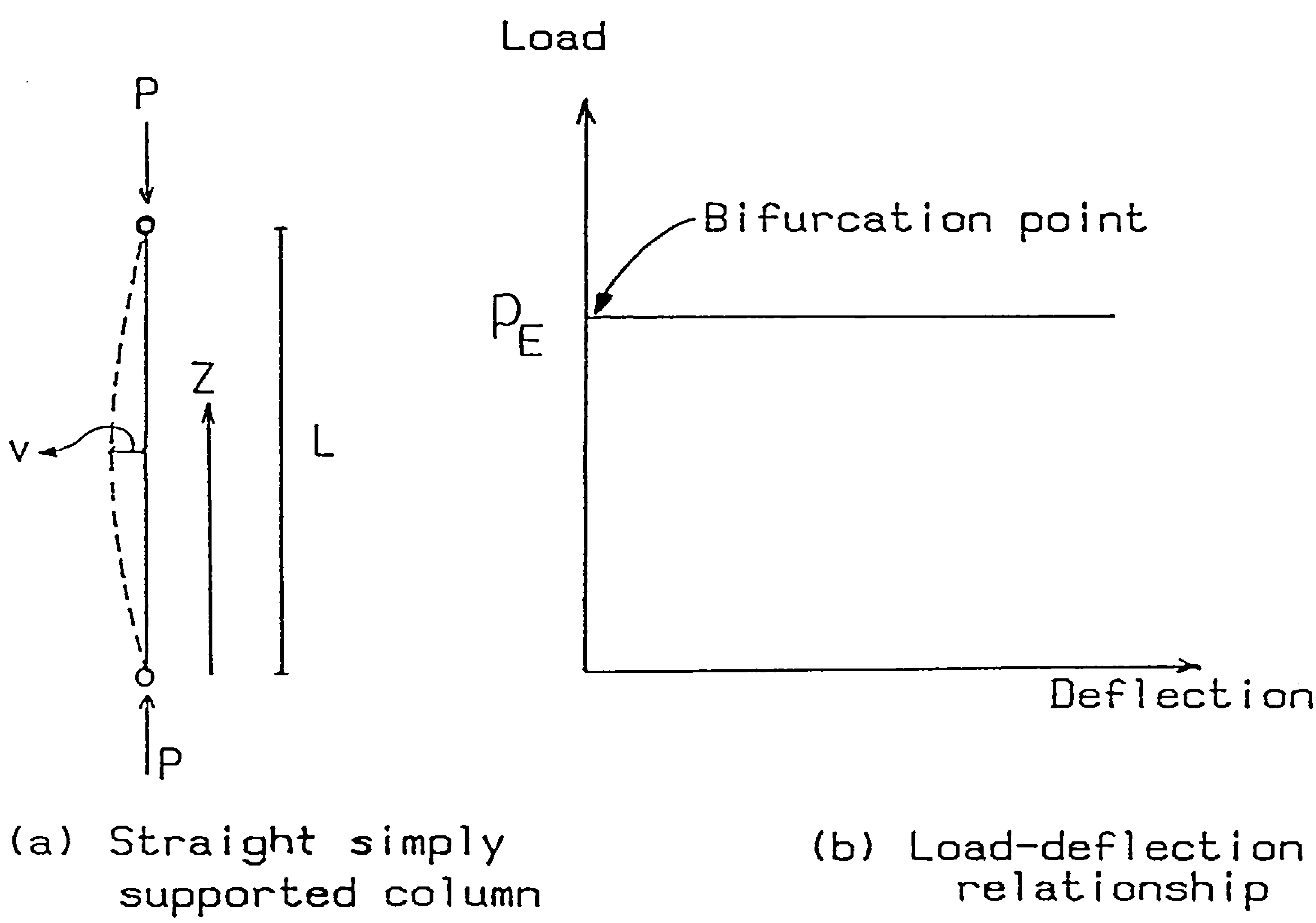


Figure 3.3: Eigenvalue problem for a perfect pinned column.

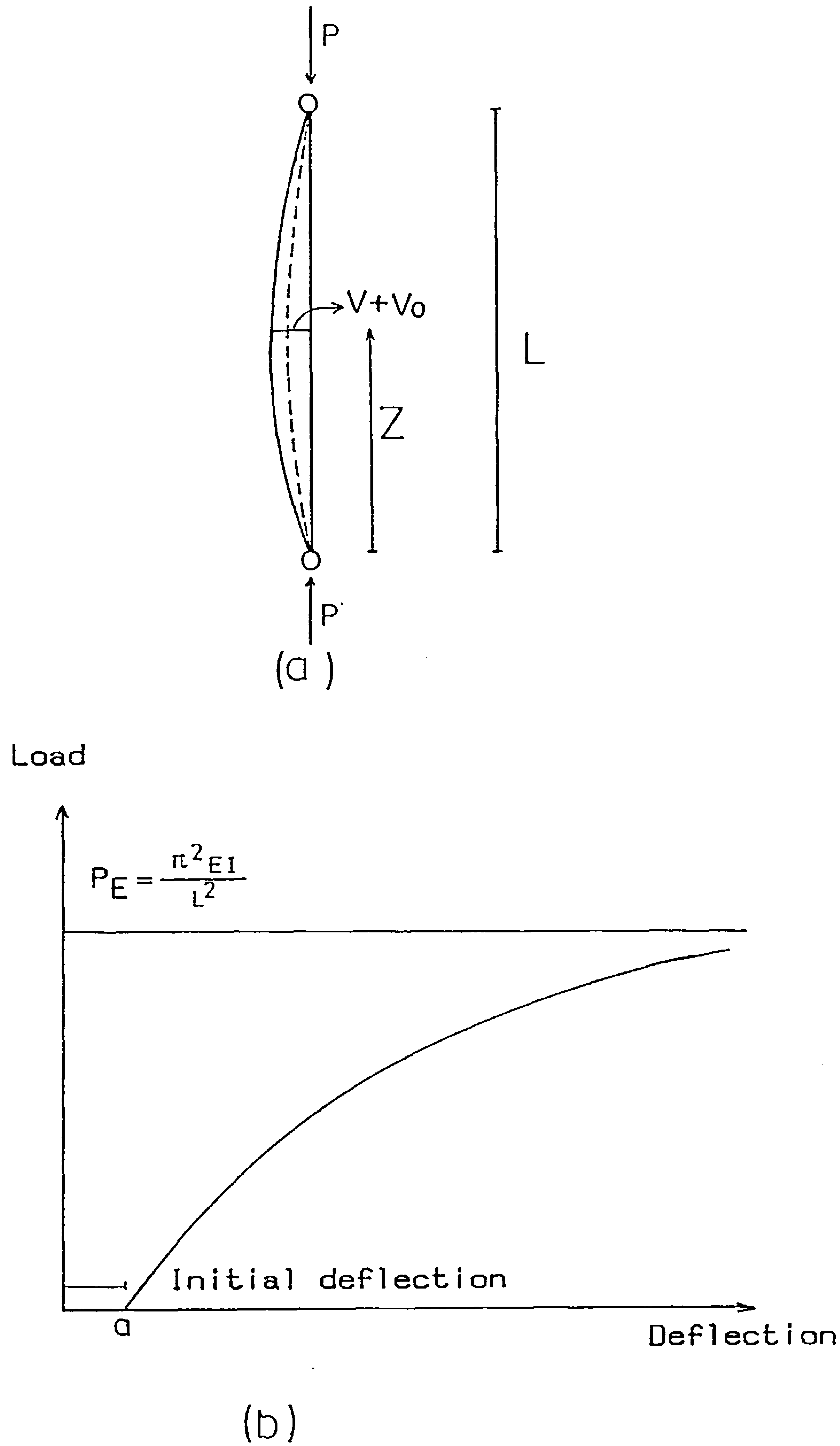


Figure 3.4: Load-deflection relationship for initially crooked column.

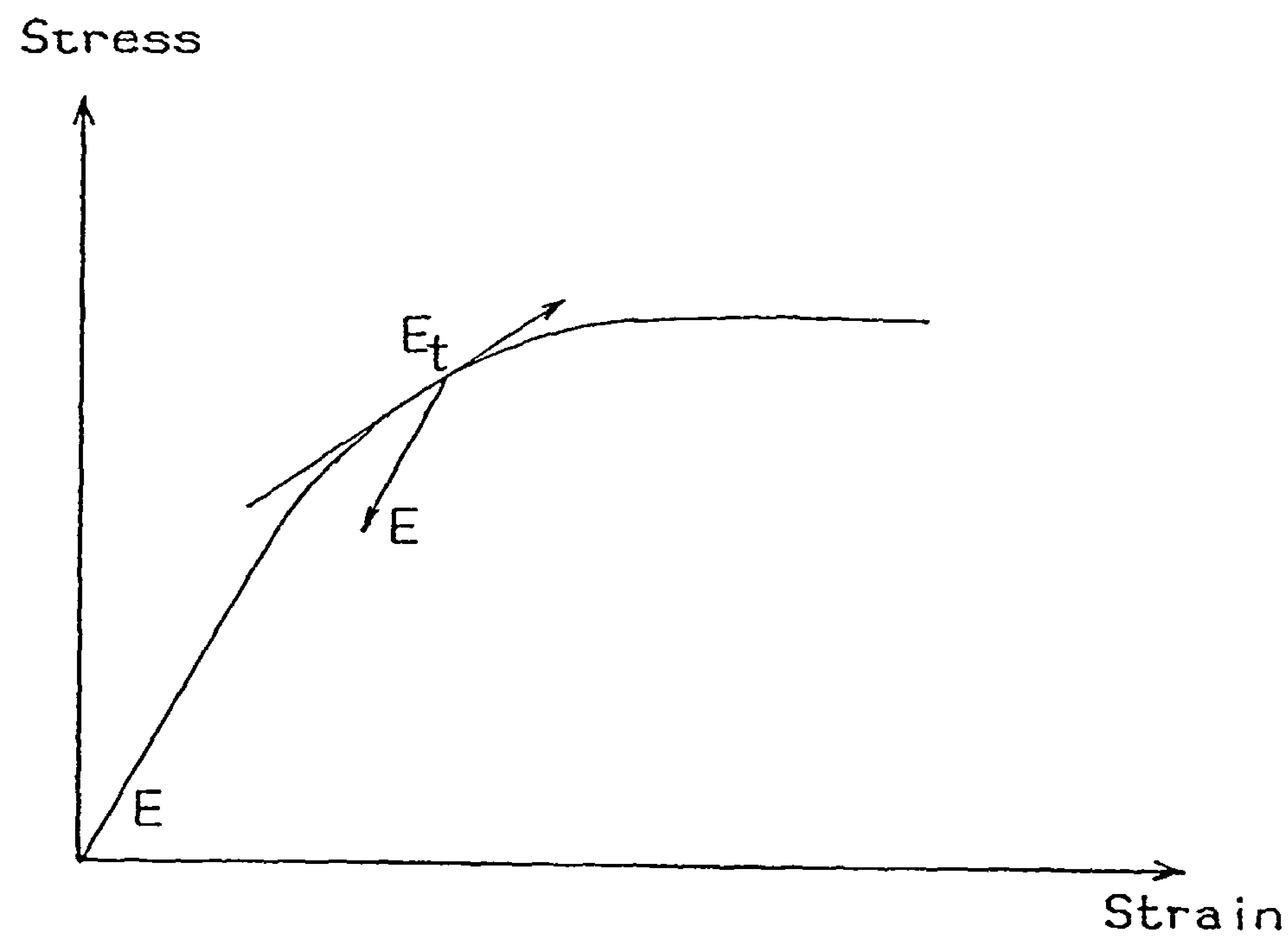


Figure 3.5: Generalised strain-stress relationship.

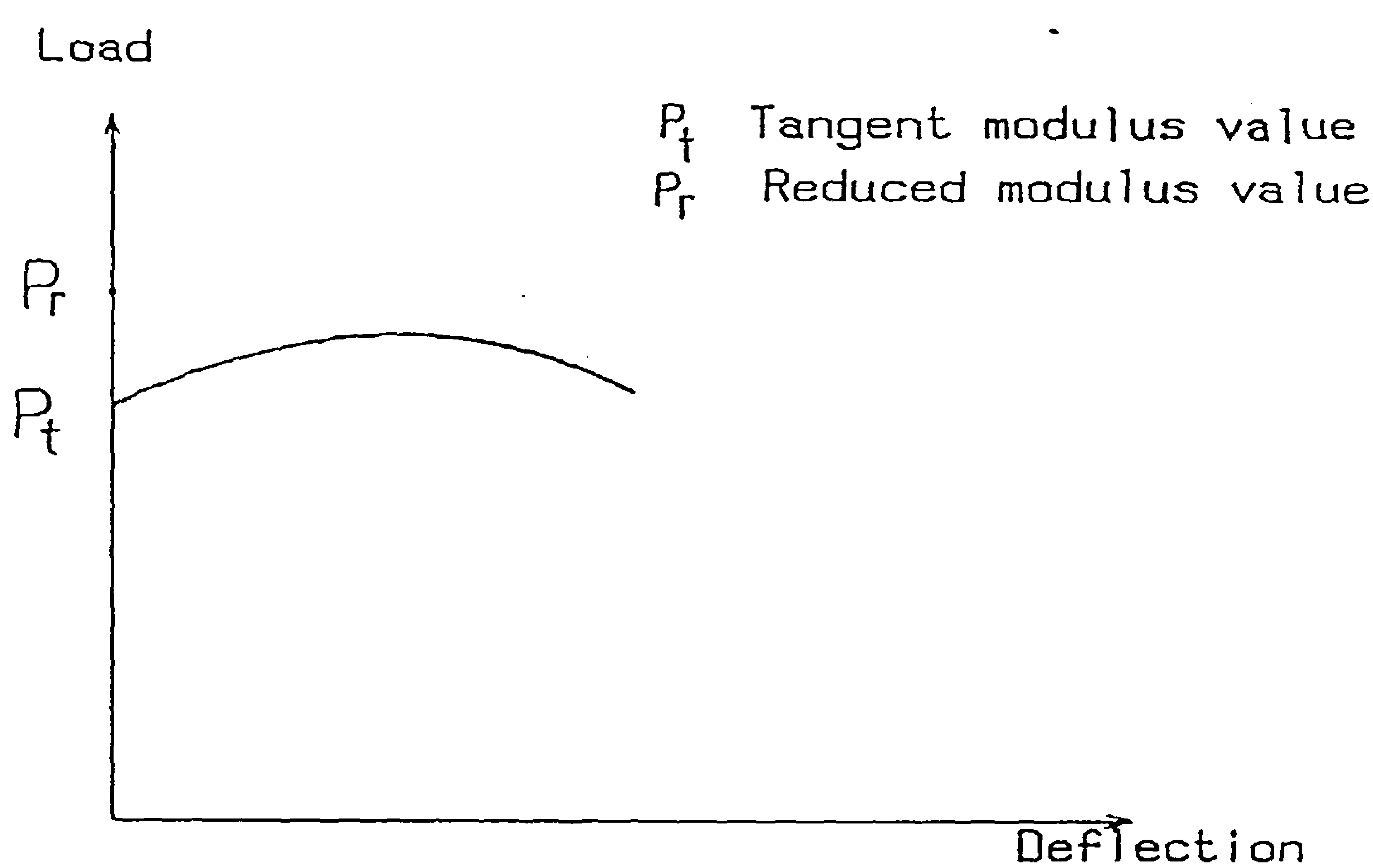


Figure 3.6: Load-deflection relationship for perfect column with generalised strain-stress relationship.

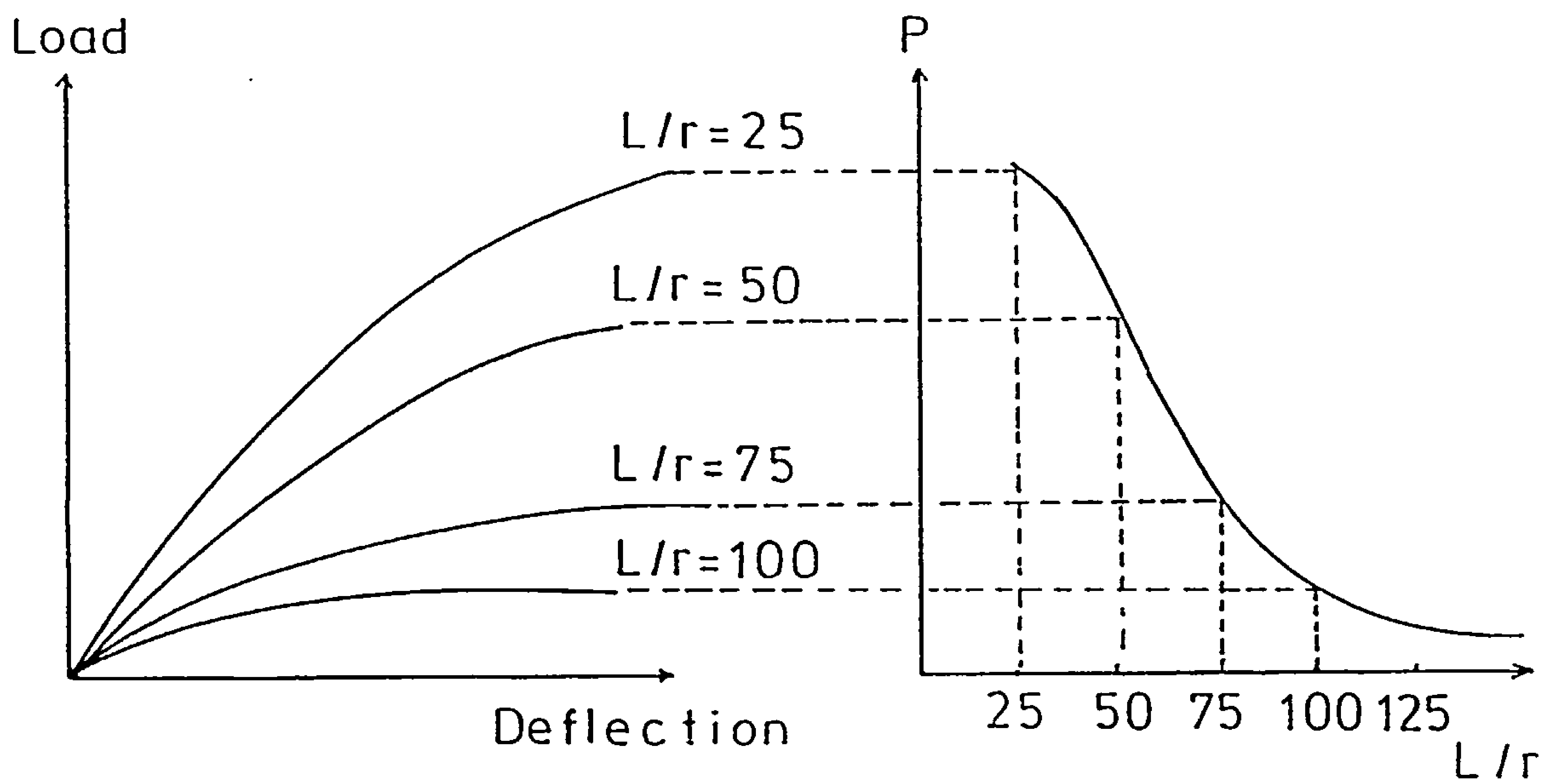


Figure 3.7: Construction of a column strength curve.

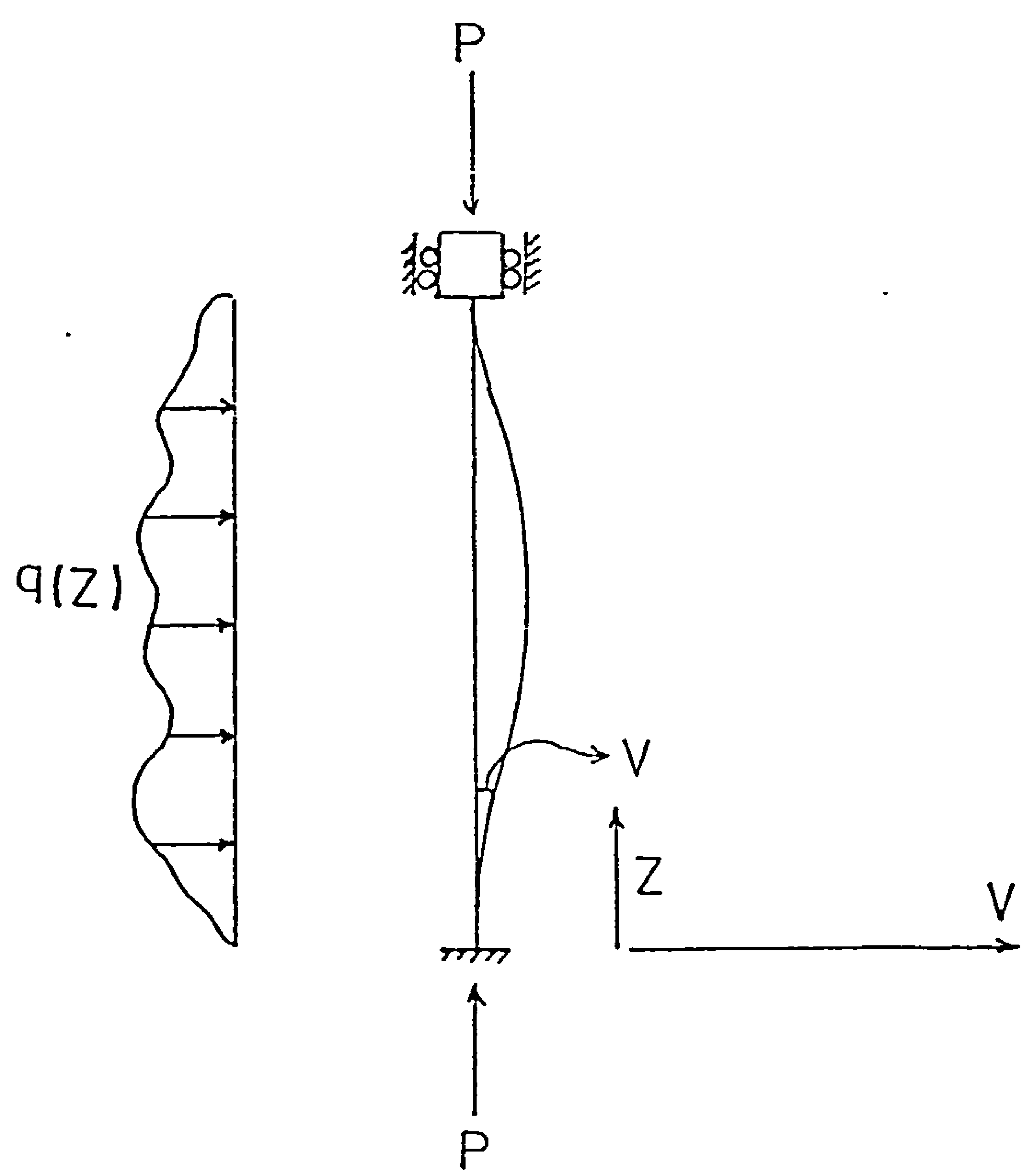


Figure 3.8: Fix ended column under generalised loading.

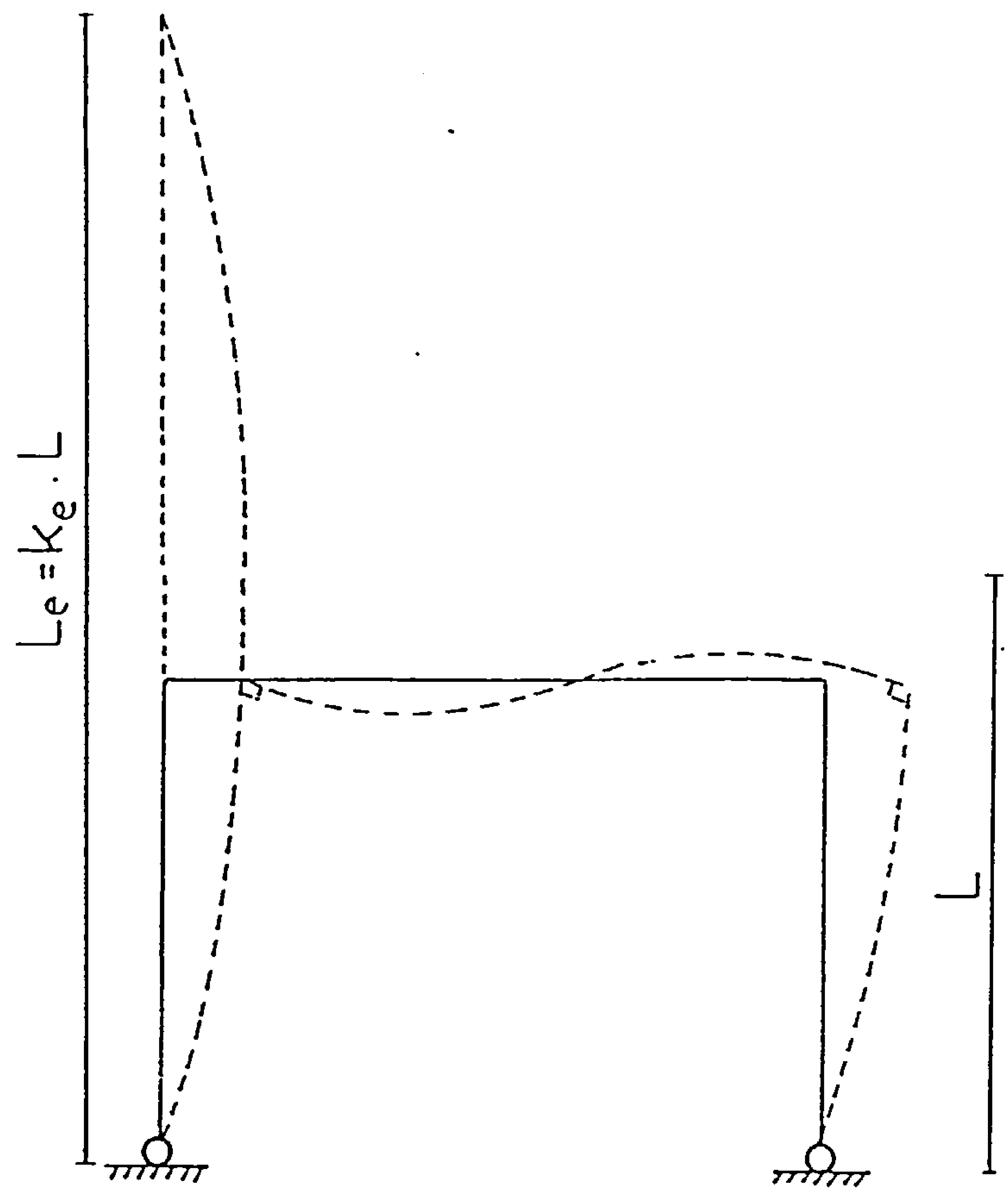


Figure 3.9: Effective length factor (k_e) of a column in a sway frame.

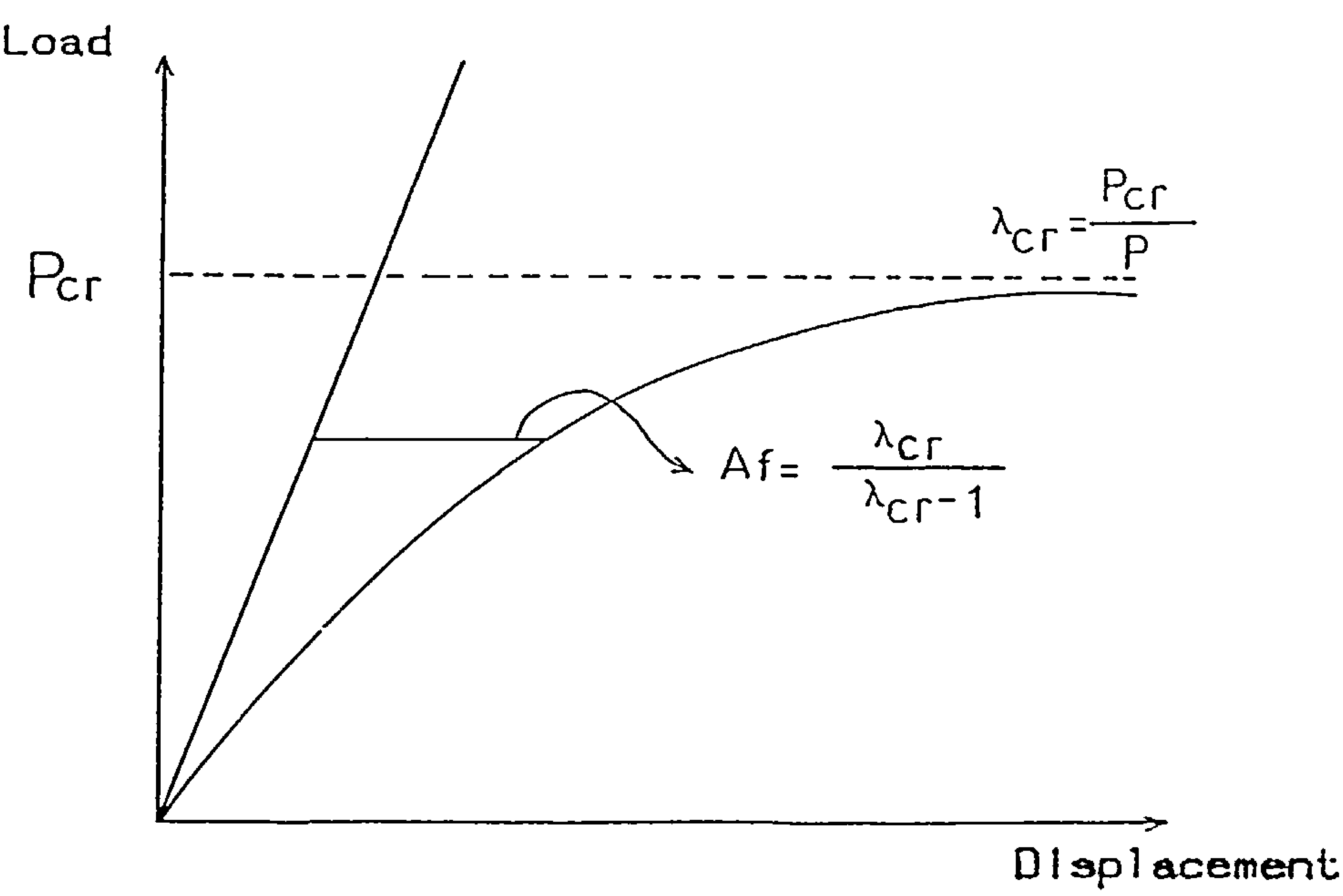


Figure 3.10: The simple amplification factor in Timoshenko's method

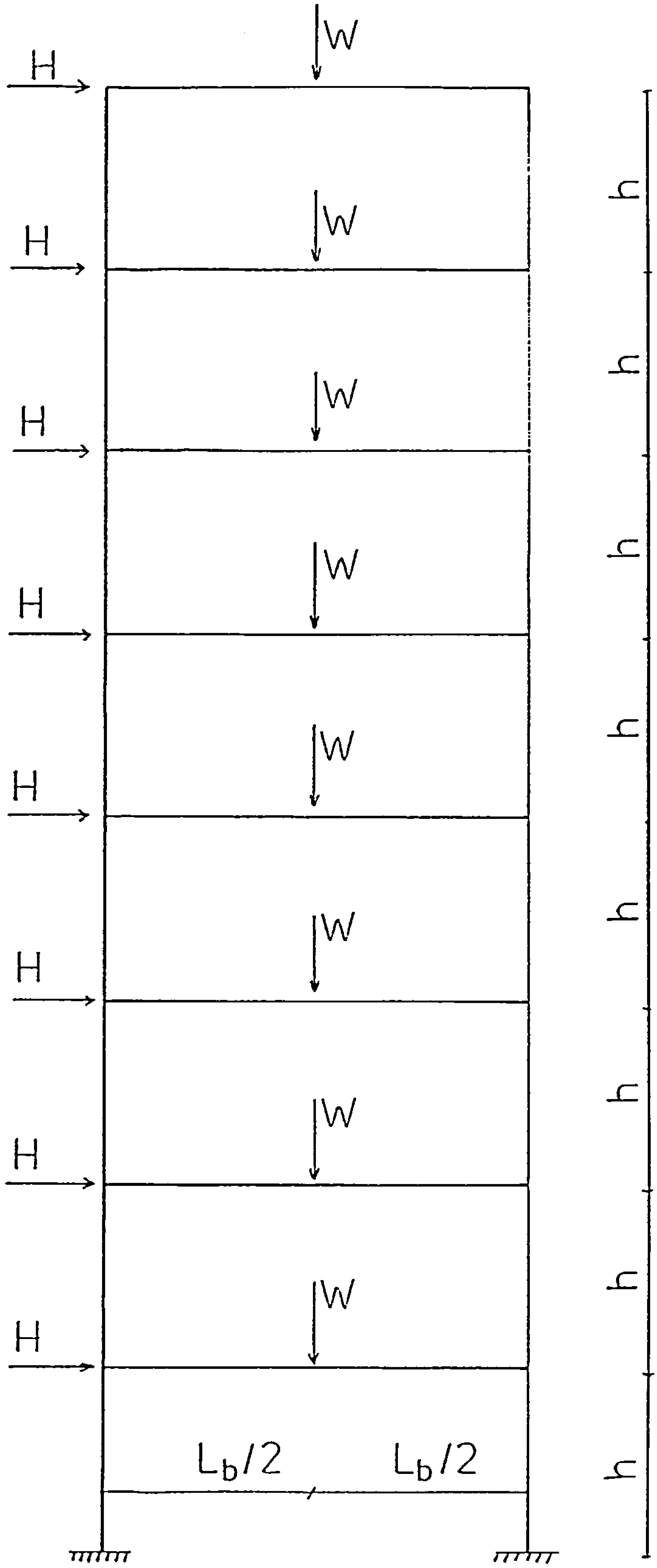


Figure 3.11: The frame of example 3.1.

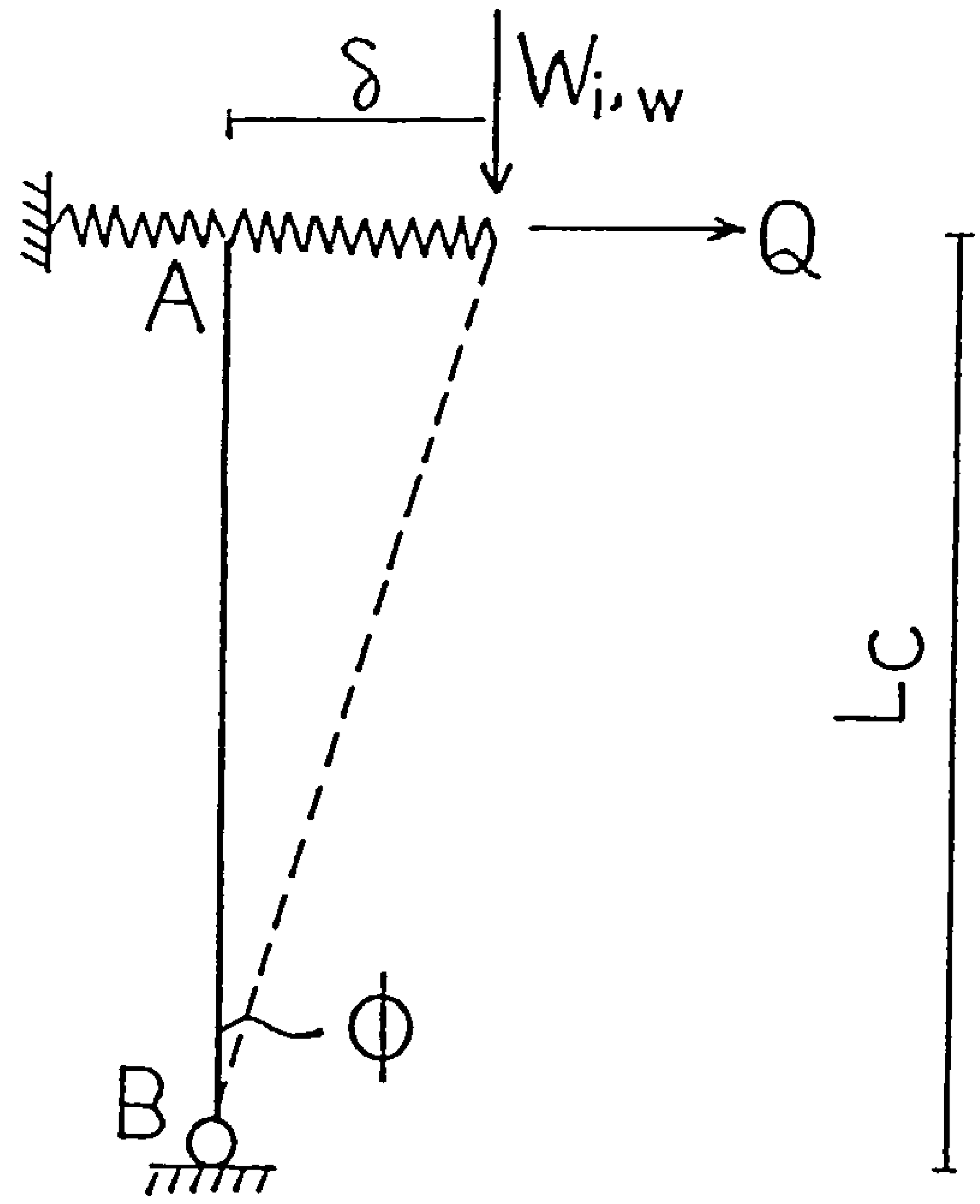


Figure 3.12: Bolton's model.

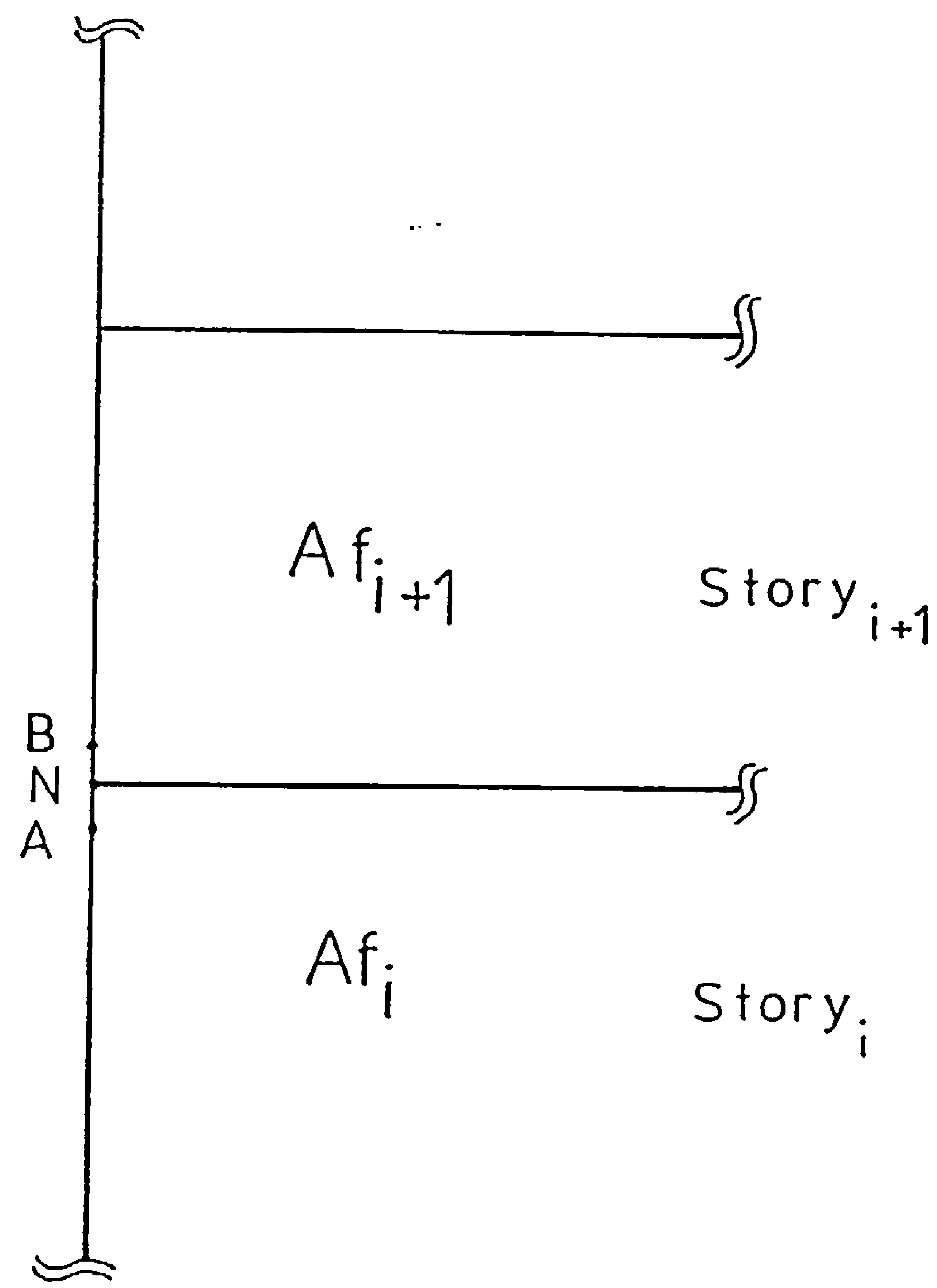


Figure 3.13: Simple amplification factor in the modified method.

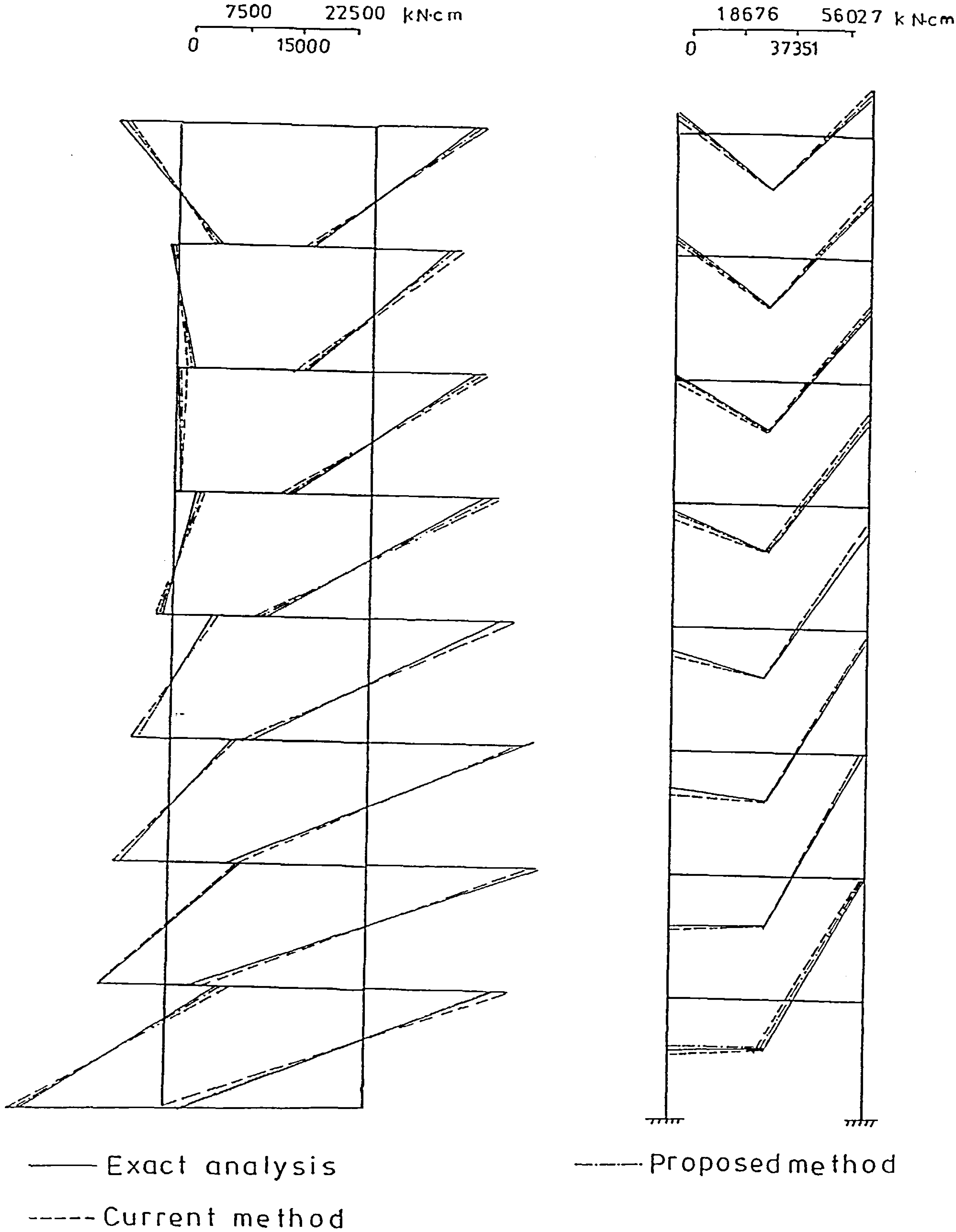
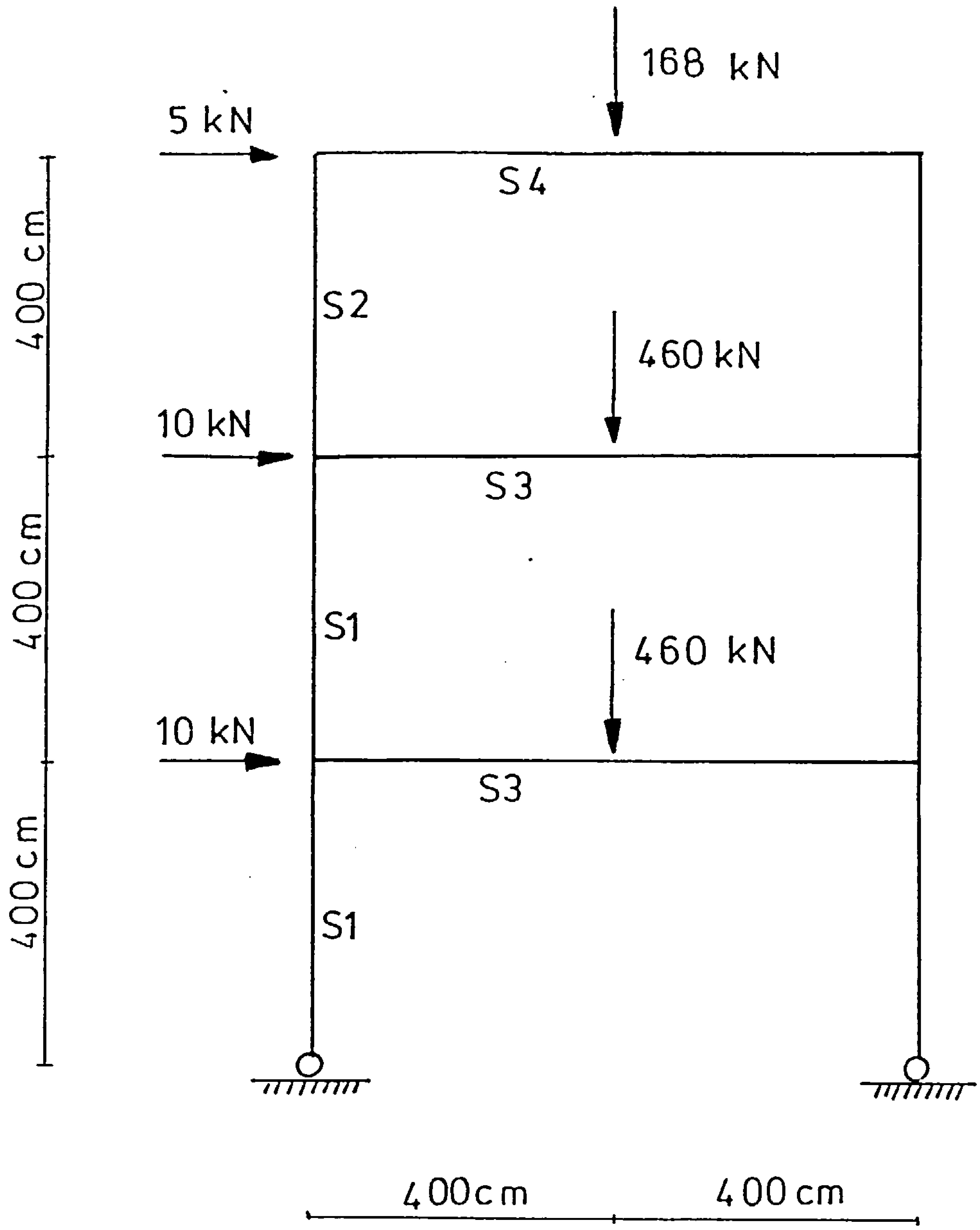


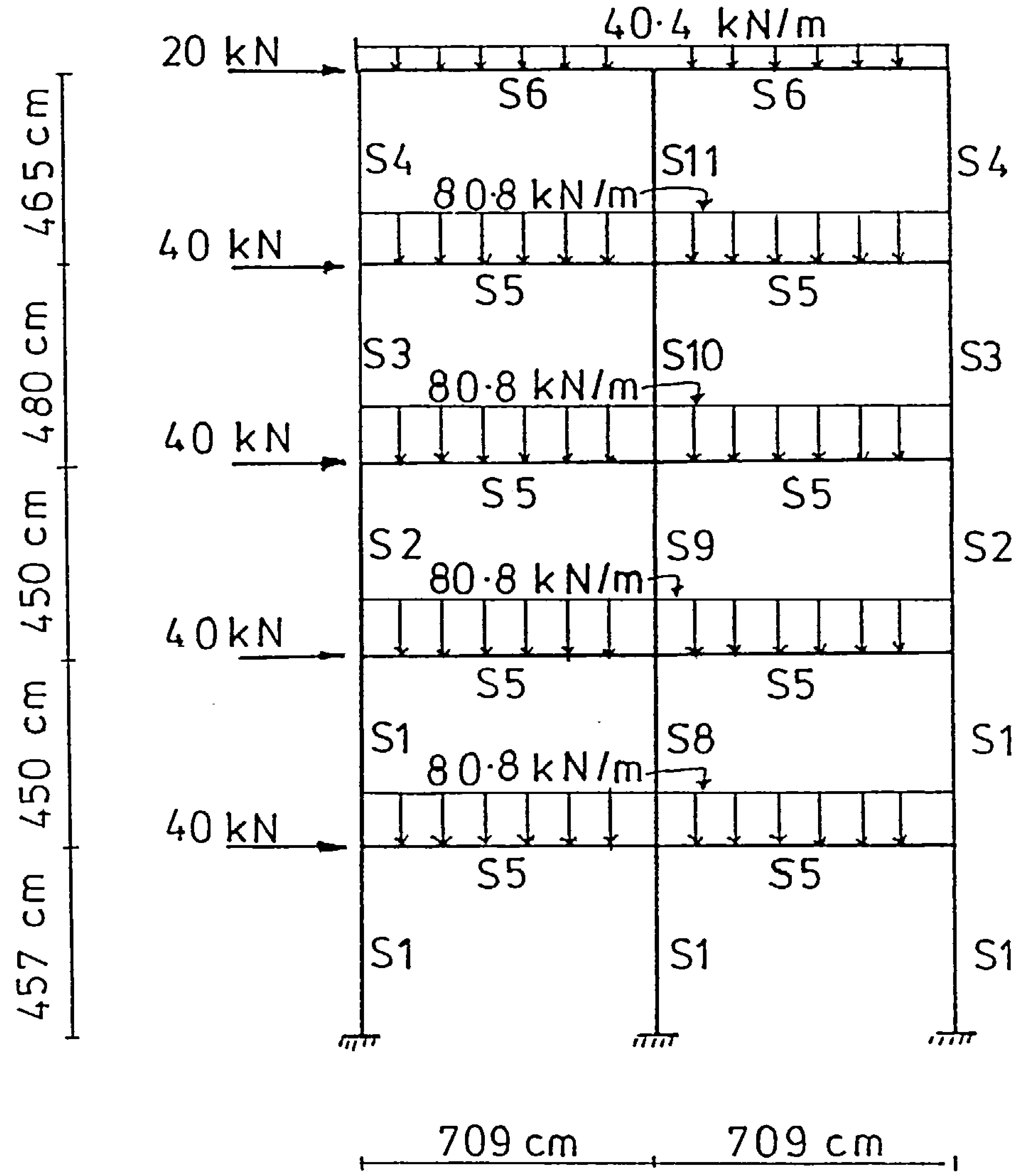
Figure 3.14: Comparisons between the prediction of the current BS5950 method, the proposed method and an accurate second order elastic analysis.



SECTIONS

S1	203 × 203 × 86 UC
S2	203 × 203 × 52 UC
S3	457 × 152 × 82 UB
S4	356 × 171 × 45 UB

Figure 3.15: The frame of example 3.2.



SECTIONS

S1	254×254×89UC	S6	406×178×60 UB
S2	254×254×73UC	S7	305×305×118 UC
S3	203×203×86UC	S8	254×254×137 UC
S4	203×203×46UC	S9	254×254×89 UC
S5	533×210×92UB	S10	203×203×86UC
		S11	203×203×52UC

Figure 3.16: The frame of example 3.3.

Chapter 4

Influence of Joint Flexibility on a Multistorey Frame Behaviour

4.1 Introduction

During the last fifty years the influence of semi-rigid connections on the design of framed steel structure has been under investigation. Although the connections constitute a small percentage of the weight of a structure, they have a great influence on its strength and stiffness. According to the British research workers [25], an average saving in the weight of beams of as much as 20% could be expected by taking advantage of the partial end restraint provided by semi-rigid connections when compared with simple construction.

In early stages [25] and [52] semi-rigid connections were represented into structural analysis as elastic springs with constant stiffness throughout the entire loading range. However, recently the wide use of digital computers has made it possible to incorporate the true behaviour of semi-rigid joints with the joint bending moment being related to the relative beam-column rotation in non-linear manner normally with a decreasing slope as the joint moment increases. Since semi-rigid connections are cheaper and easier to erect than fully rigid joints (accepting that very stiff semi-rigid joints behave in a way similar to fully-rigid joints), there have been several attempts to provide simple design procedures which allow for joint flexibility. An example of these procedures is the well-known 'simple wind-connection method'. In order to explain the philosophy behind the wind connection method, a brief historical review about this method follows.

4.2 The Simple Wind-Connection Method

It has long been recognised that neither the so-called 'simple construction' nor 'continuous construction' assumptions represent the true behaviour of flexibly connected frames, which lies between these extreme idealisations. An attempt has been proposed to use both idealisations simultaneously to represent such intermediate behaviour. In the wind-connection method joint flexibility is included in an indirect way by assuming that semi-rigid joints behave like perfect hinges when gravity loads are considered whilst it is assumed that the same joints behave as fully-rigid when considering wind loads. Although fig.4.1 shows that none of these assumptions represents the real behaviour of semi-rigidly connected frames, this method has been used to design some practical frames. McGuire [53] provided an extensive historical review about the development of this method. He concluded that the wind-connection method has been met with a mixed reaction from designers. In the United State, for example, engineers to the east of the Hudson River have used this method while those west of this border of New York did not. He also indicated that a first attempt to justify the wind-connection method was made by Sourochnikoff [54] who used the simple portal frame shown in fig.4.2 to explain the semi-rigid joints behaviour under repeated wind loads. He argued that under the effect of repeated wind loads semi-rigid joints will behave elastically. This conclusion

Type of connection	Rigid	Flexible semi-rigid	Stiff semi-rigid
Lateral sway Tolerable sway 9.2mm	9.7mm	34.9mm	26.00
Column stress Status	Substantially overstressed	overstressed	overstressed

Table 4.1: The result of the simple frame shown in fig.4.4 as presented in ref.[53]

can be explained as follows:

Assuming that the bending moment of the semi-rigid joint at end A of the frame shown in fig.4.2, due to gravity load, is represented by the point a1 in fig.4.3 If the wind load H is assumed to blow from the left side of the frame, the joint bending moment will move to a2 through the unloading path which is assumed to be parallel to the joint initial stiffness. By removing the wind load the moment moves from a2 to a3. If the wind load H is applied from the right side, this would have caused the joint moment to leave a3 to a4. Finally removing H leaves this moment at a5. From fig.4.3 it can be seen that the final bending moment of semi-rigid joints are significantly less than their original values. It was claimed that [53] this behaviour can be used to verify the the philosophy of the wind-connection method. However, as can be seen from fig.4.3 maintaining this behaviour requires that the ratio of the applied horizontal to vertical loads to be relatively large which is rarely satisfied in most practical steel structures. McGuire [53] has used the wind-connection method to design the simple portal frame shown in fig.4.4.a. For the sake of comparison he then checked the frame stresses considering three different types of joints, i.e. rigid joints, stiff semi-rigid joints (fig.4.4.b,curve b) and flexible semi-rigid joints (fig.4.b,curve a). His finding is summarised in table 4.1 which clearly shows that the wind-connection method grossly underestimate both column stresses and lateral drifts. This conclusion agrees with Moncarz and Gerstle [41]. Therefore, McGuire [53] and Nethercot [55] recommend designers to use this method cautiously and only for frames where the structural influence of cladding is sufficient to compensate the loss of frame stiffnesses resulting from utilising semi-rigid joints. Considering this brief review about the wind connection method, the following questions are still unanswered.

1) How the method caters for a frame with joints ranging from extended-end plate joints to web-cleat joints.

2) Is it always guaranteed that the presence of cladding, in practical frames, will compensate the loss of frame stiffnesses caused by semi-rigid joint deformations and if not which criterion should be used to check that.

In the following section two simple structures are analysed using a procedure incorporating semi rigid joint action and the results compared with the corresponding values for rigidly connected frames. This limited study has been conducted to shed light into the effect of semi-rigid joints characteristics on frame deflections, bending moments and load carrying capacity. The influence of partial bracing combined with the presence of semi-rigid joints on steel frames will be examined in chapter 7.

4.3 The Behaviour of Flexibly Connected Frames

To explore the effect of joint flexibility on the behaviour of a multistorey frame, many frames have been analysed and compared with the corresponding values for rigidly connected frames.

4.3.1 Example 4.1

The simple frame which is shown in fig.4.5 is analysed by the author using a sophisticated computer program which was developed at the Politecnico di Milano [40]. It incorporates an elasto-plastic second order analysis including semi-rigid joint action assuming joint behaviour to be multi-linear.

The frame consists of two columns European HE200B sections supporting a beam of IPE300 section. The Young's modulus E and the yield stress σ_y have been taken as 21000 kN/cm^2 and 25 kN/cm^2 respectively. Joints to be investigated are extended end plates and flush end plates being the most common moment resisting connections.

Fig.4.6 shows the moment-rotation characteristics which have been used to incorporate the behaviour of extended end plate and flush end plate connections. These relationships are obtained from references [56] and [57] respectively and though not corresponding to the exact sections of the frame, were taken as typical of the connection types.

The horizontal displacements versus the load factor are plotted in fig.4.7 where it can be seen that utilizing the extended end plate connection only slightly influences the frame behaviour when compared with rigid joint response. However, employing flush end plate connections causes the frame to undergo much larger horizontal displacements and consequently also reduces the ultimate load factor.

Fig.4.8 shows the relationship between the bending moment at the left hand end of the beam (end i) against load factor. It can be seen that throughout the loading, and in particular at the failure condition, the flush end plate joints behave rather like hinges. Noting that the ratio between the joint plastic moment to the beam plastic moment is approximately 0.6 the beam behaviour may be approximated to that of a simply supported beam and failure occurs with the formation of one hinge at the centre of the span (see fig.4.9.c)

Also it may be concluded from figs 4.7 and 4.8 that extended end plate joint response might be approximated by fully rigid connection behaviour as shown by load-displacement and load-bending moment history. Also the mechanisms at collapse are identical though occuring at slightly different load factors figs 4.9(a) and (b). However it is important, when making such assumption, to ensure that the joint stiffnesses are comparable to the element stiffnesses of the frame.

It can seen from this example that semi-rigid joints characteristics are highly influential factors in the response of a steel structure. Generally speaking a decrease of joint stiffnesses cause a decrease in the frame's ultimate load accompanied by larger horizontal deflections with even the possibility of altering the collapse mode of the frame (see fig.4.9). Nevertheless, this behaviour cannot be predicted using the wind connection method since the behaviour of all flexibly connected frames are assumed to be independent from the utilised joint behaviour.

4.3.2 Example 4.2

As a second example consider the simple multistorey frame designed by Kahl [58] which is shown in fig.4.10. It consists of beams attached to the columns by top and seat angle connections. The joints possess significant flexibility and the moment-rotation curves which were obtained experimentally are illustrated in fig.4.11 which also shows the approximations used in the analysis. In fig.4.12 the lateral deflections of the first and the second stories of the frame are plotted against the load factor and compared with those obtained assuming fully-rigid joints. It can be concluded that the inclusion of semi-rigid behaviour of the analysis causes the frame to undergoes much larger horizontal displacements typically 2-3 times that assessed from using rigid joints.

Plotting the joint bending moment at end, i, of the upper and the lower beams of the frame versus the load factor with different joint characteristics produces fig.4.13 where it can be seen that the joint influence on the element bending moment of the frame is dependent upon the load factor and that this influence is greatest below failure and occur when the load factor λ is approximately equal to 1.50 However, as the applied loads increase this difference is sharply reduced. This aspect of the frame behaviour can be explained as follows:

1) When rigid joints are utilised, plastic hinges would occur at the ends of both columns 3 and 4 (see fig.4.14) at load factor $\lambda > 1.50$, i.e. before reaching the failure load of the frame. This leads to a significant reduction of the frame stiffness against the lateral loading (see fig.4.12) which causes considerable reduction of the bending moment at the end i for both beams. This reduction occurred due to the loss of the frame resistance against lateral deflections allowing the applied horizontal load to dominate the element bending-moment diagrams. Noting that on the windward side of the frame the bending moments due to the horizontal loads oppose those due to the vertical loads, a reduction of the bending moments due to the combination of the vertical and the horizontal loads is expected as the load factor is increased.

2) If semi-rigid connections are used smaller bending moments, than would occur if rigid connections are employed, are transfered to the columns from the beams. Therefore neither column developed any hinges for the load factor from 1.5 to 1.75 and two plastic hinges occured at the mid-spans of the beams instead (see fig.4.14). This behaviour allowed the rate of change of the beam-moment versus

load factor diagrams to be positive throughout the whole loading path. Hence by increasing the load factor beyond $\lambda = 1.5$ the beam bending moments at ends i of both beams will be increased for semi-rigid analysis whilst they will be decreased for fullyrigid analysis causing these values to be very close to each other at failure.

The semi-rigid joints used are strong compared with the frame elements, i.e. throughout the loading path the semi-rigid joints did not develop plastic hinges(see fig.4.14). Frame failure was initiated by the formation of plastic hinges located on the beams and the columns of the frame coupled with stability effects in the columns, the effect of the joint flexibility on the frame at ultimate load factor was negligible and the load factors at failure for the semi-rigid and rigid joint analyses are within 1% of each other. It can concluded from this example that using stiff semi-rigid joints might in a way, particularly at the failure condition, agree with the wind connection method in the sense that the frame response is similar to that corresponding to fully rigid frames. However, it should be kept in mind that this is not necessarily true if the frame working loads are considered which are, in ultimate load design, smaller than the ultimate loads by the value of the adopted safety factors. This argument is supported by consideration of fig.4.13 where it can be noticed that the element stresses of the frame with top and seats angles connections, at a load factor 1.5, are significantly different from those obtained from the full rigidly connected frame.

4.4 Conclusion

Throughout the analysis of the multistorey frames, it has been found that the effect of the presence of semi-rigid joints is dependent upon

1) the joint moment-rotation characteristic.

2) the element stiffnesses of the frame.

3) the frame type, i.e. sway or non-sway, and the applied horizontal and the vertical loads.

Thus the inclusion of semi-rigid effects in the analysis leads to a more realistic representation of the structure response which must inevitably lead to safer and more economical designs.

The limitation of the wind connection method in representing flexibly connected steel frames has been briefly explained and the need for a simple design method which account for semi-rigid joint behaviour is therefore justified.

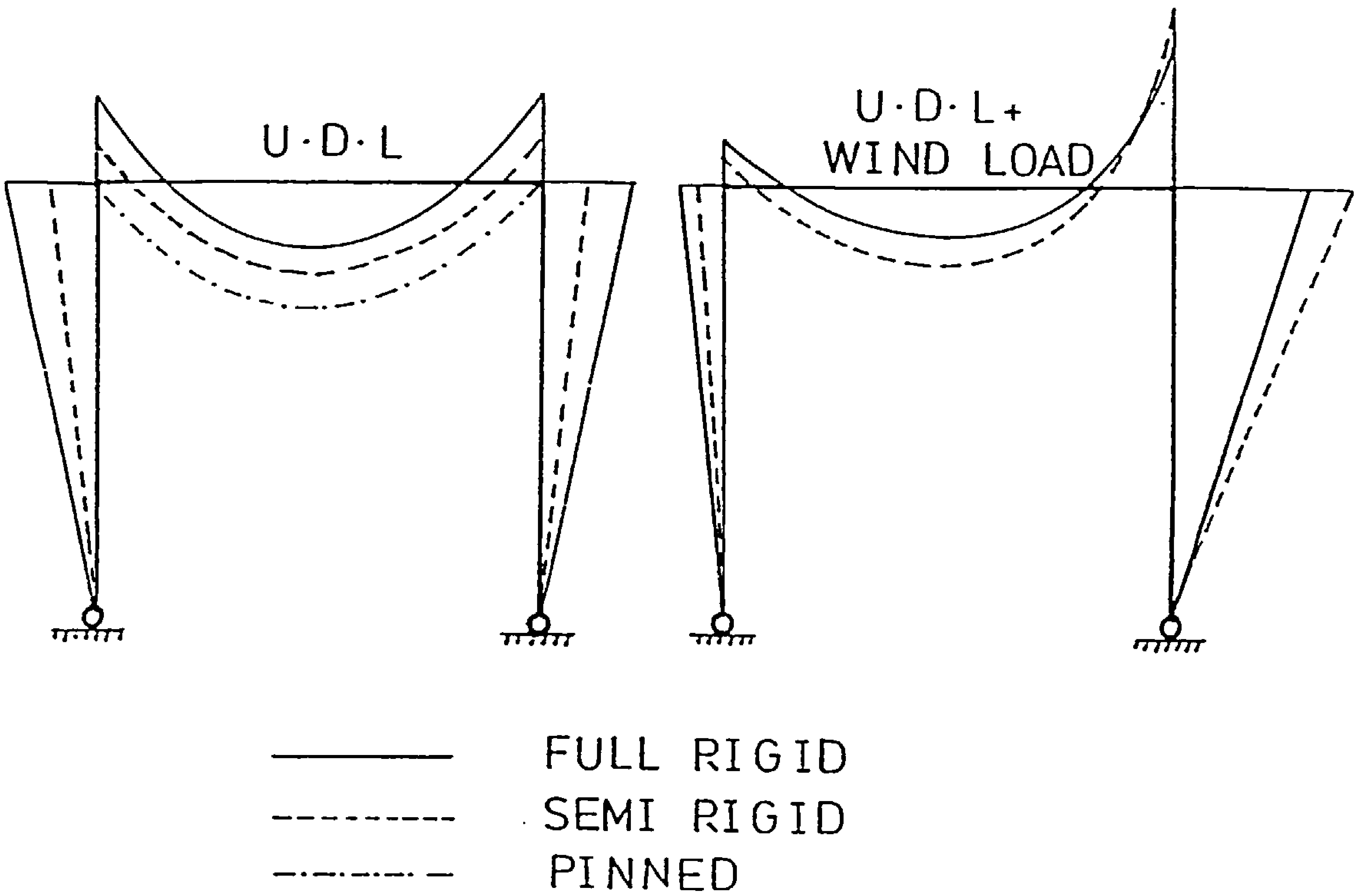


Figure 4.1: Typical bending moment diagrams for a simple frame.

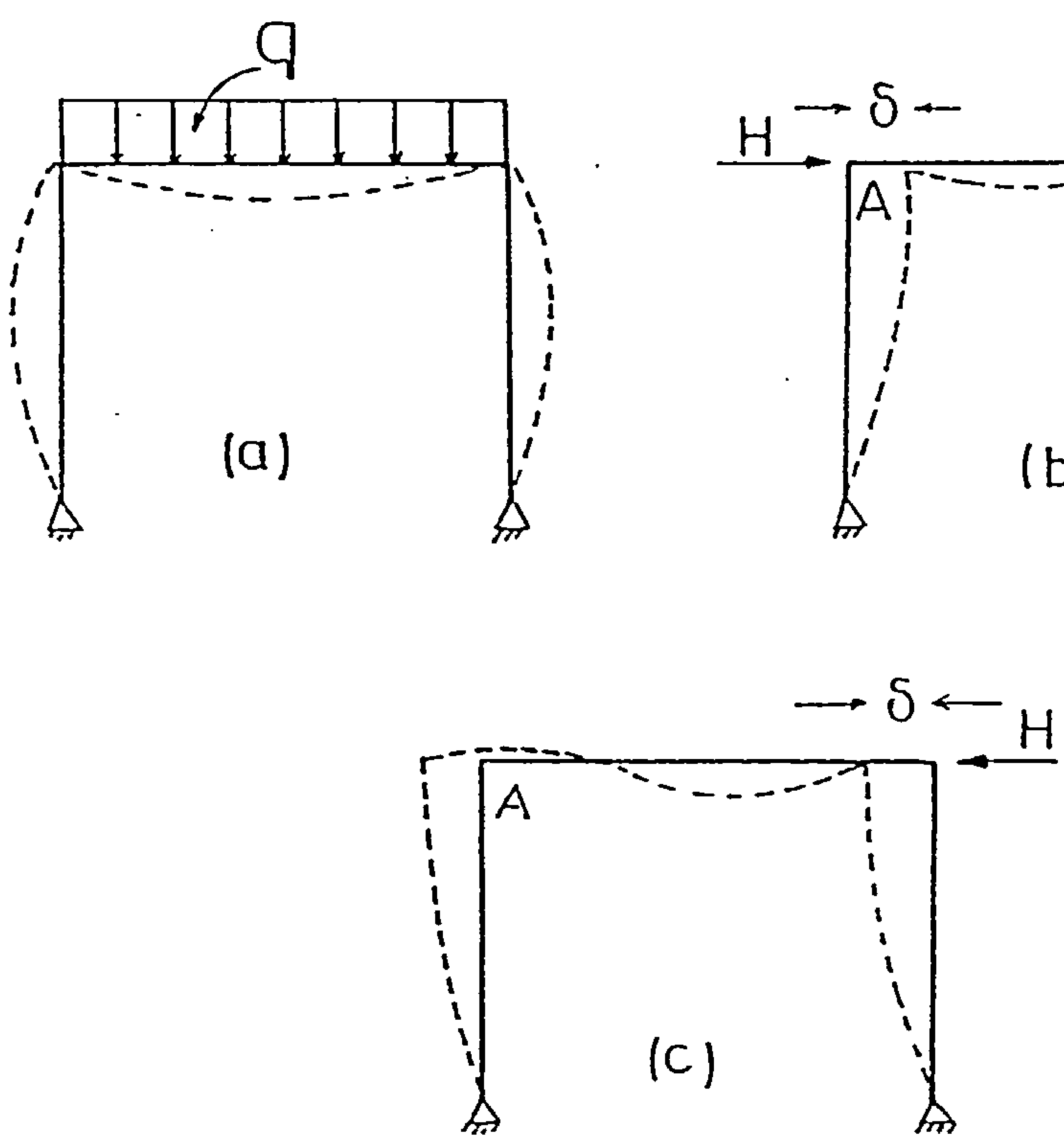


Figure 4.2: The behaviour of a simple frame subject to vertical and horizontal loads.

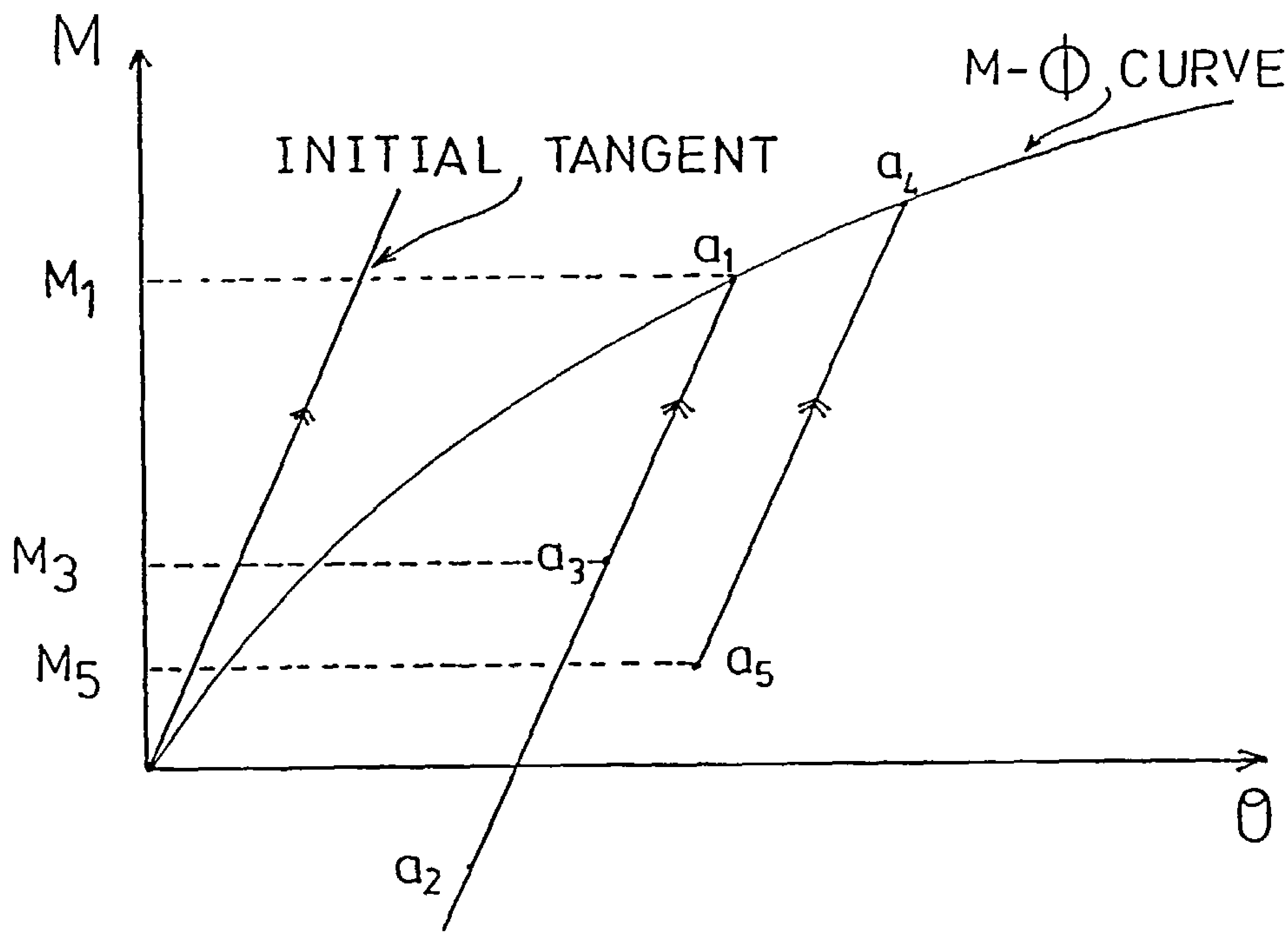


Figure 4.3: Loading and unloading behaviour of a semi-rigid joint.

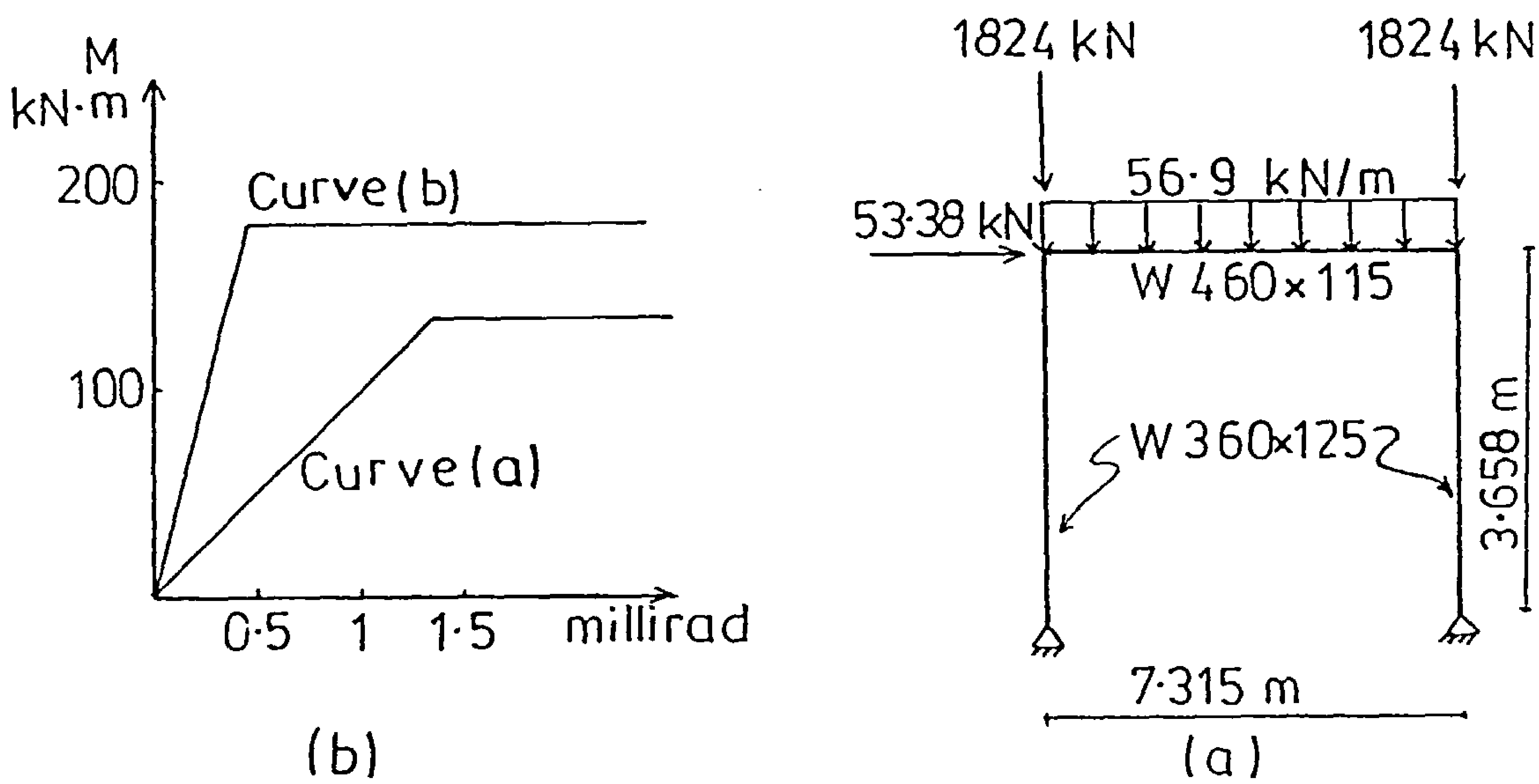


Figure 4.4: Simple frame designed by McGuire

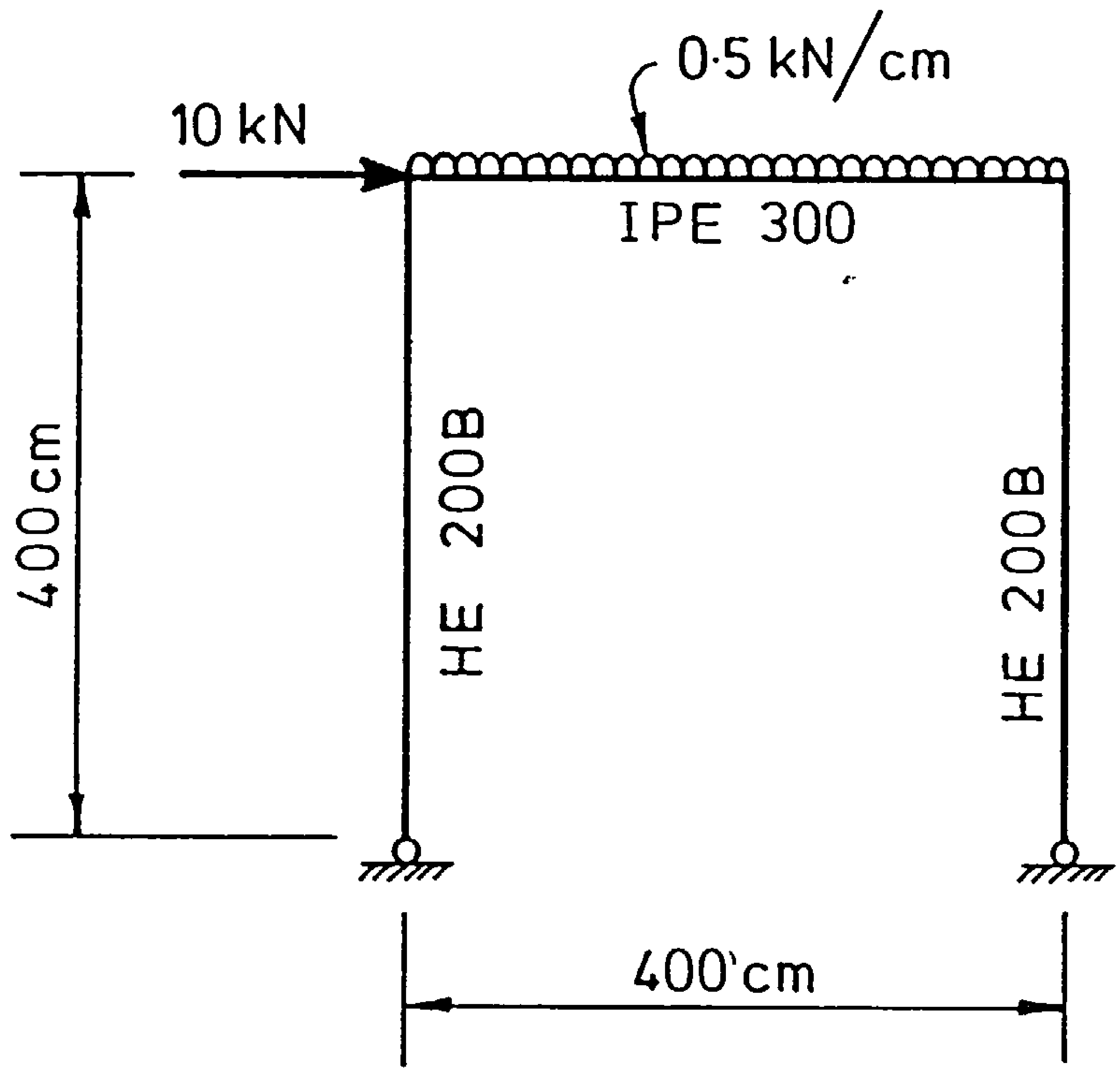


Figure 4.5: The frame of example 4.1

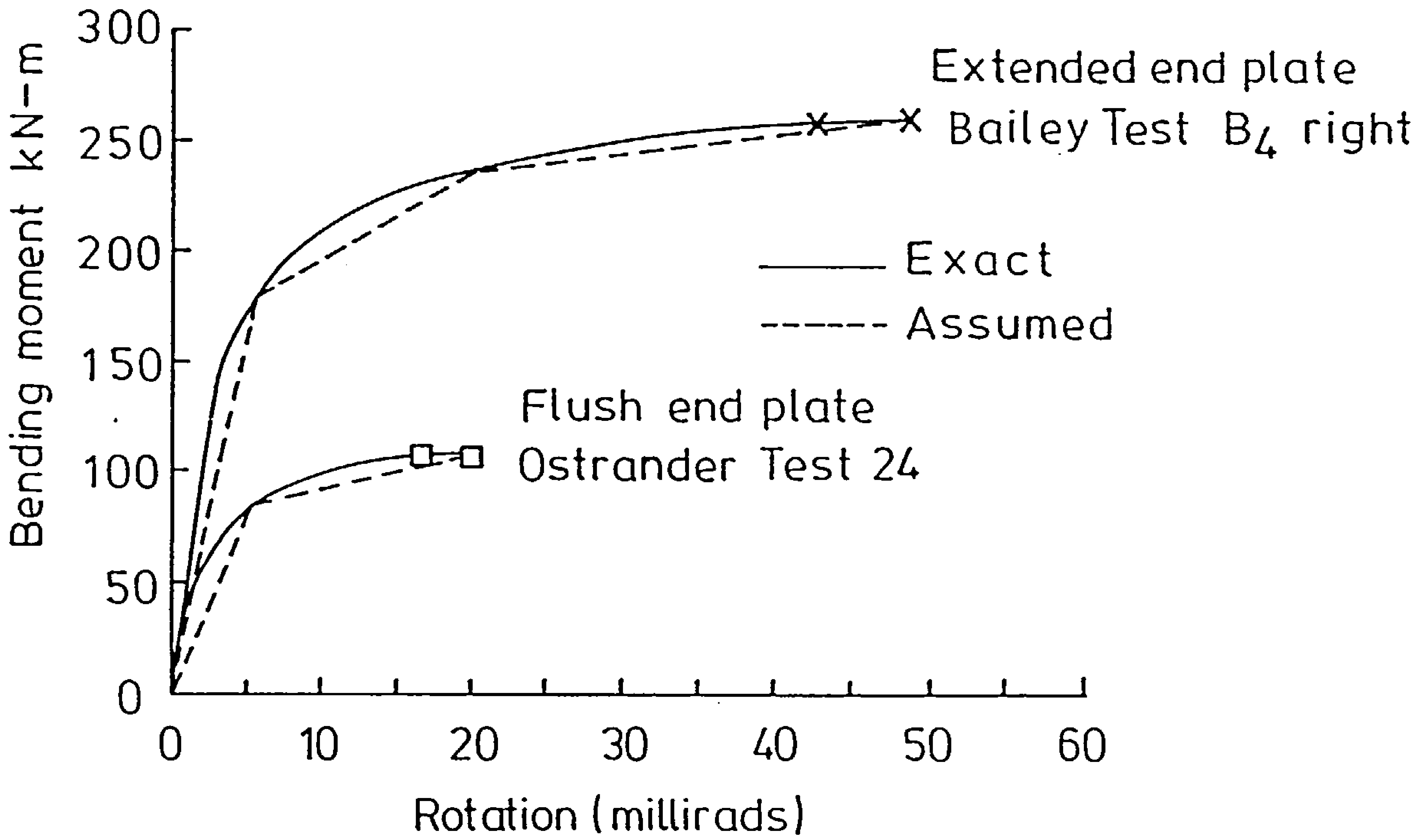


Figure 4.6: Moment-rotation relationship for semi-rigid joints utilised in the frame of figure 4.5.

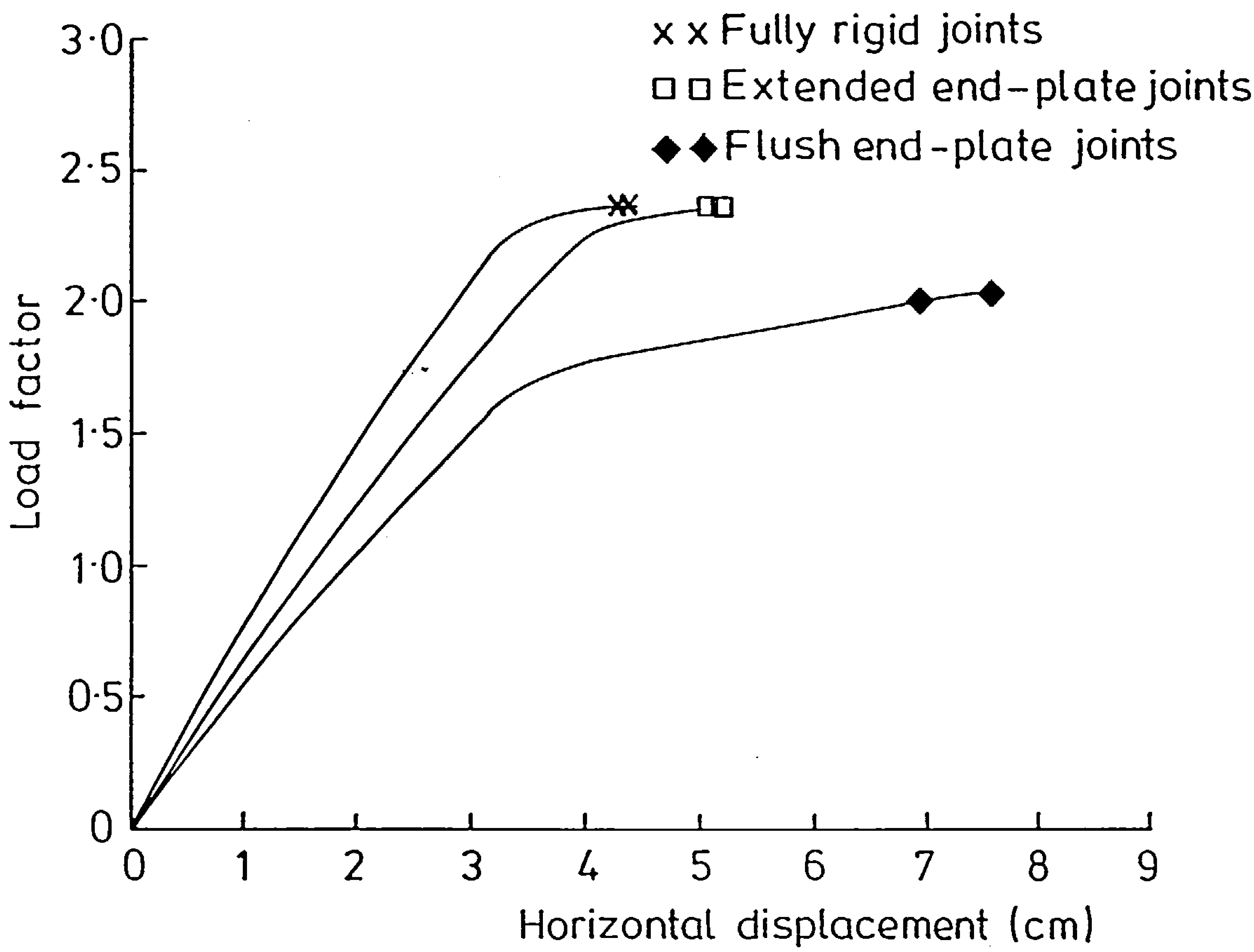


Figure 4.7: Horizontal displacement versus load factor for the frame of figure 4.5 with different joint types.

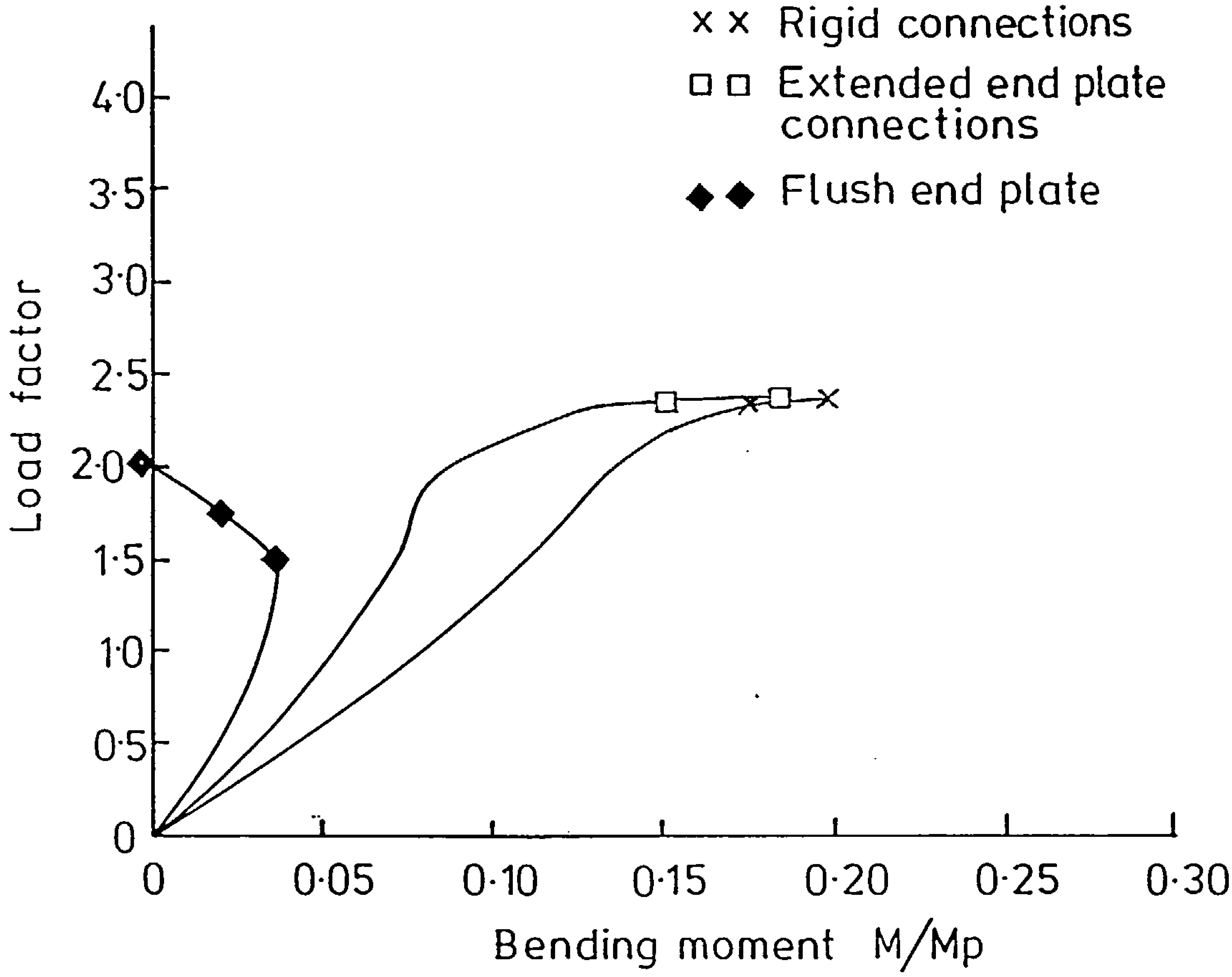


Figure 4.8: The bending moment at end i of the beam versus load factor for different joint types.

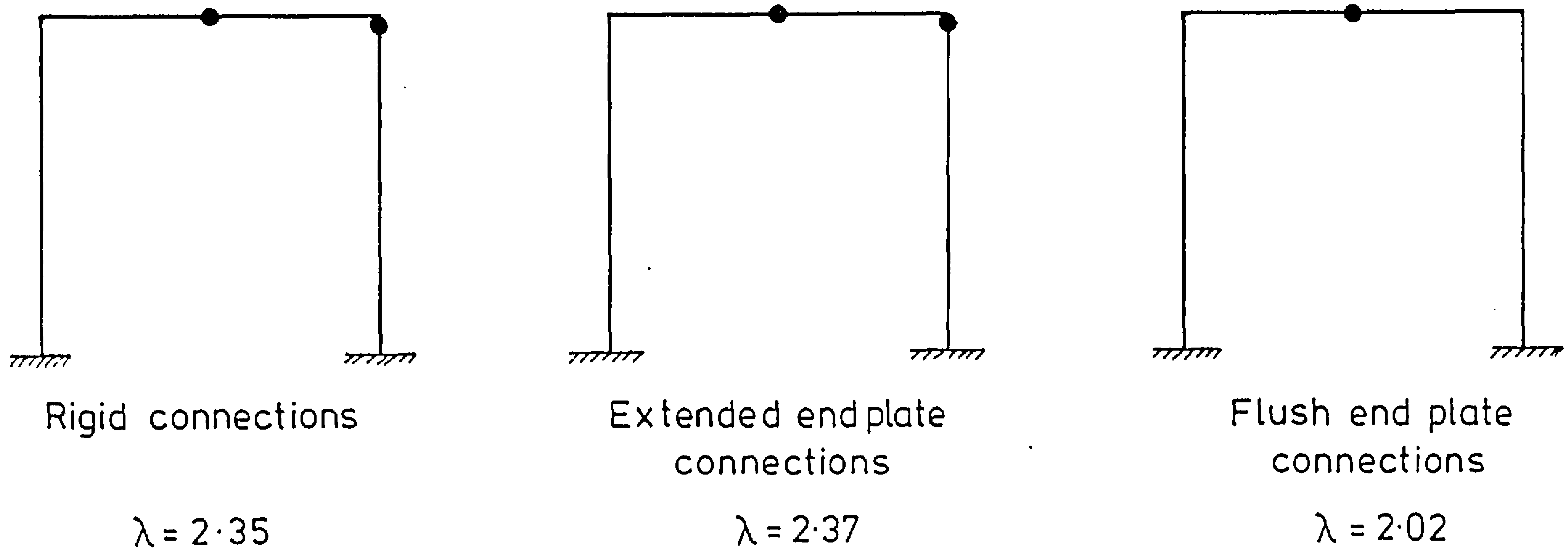
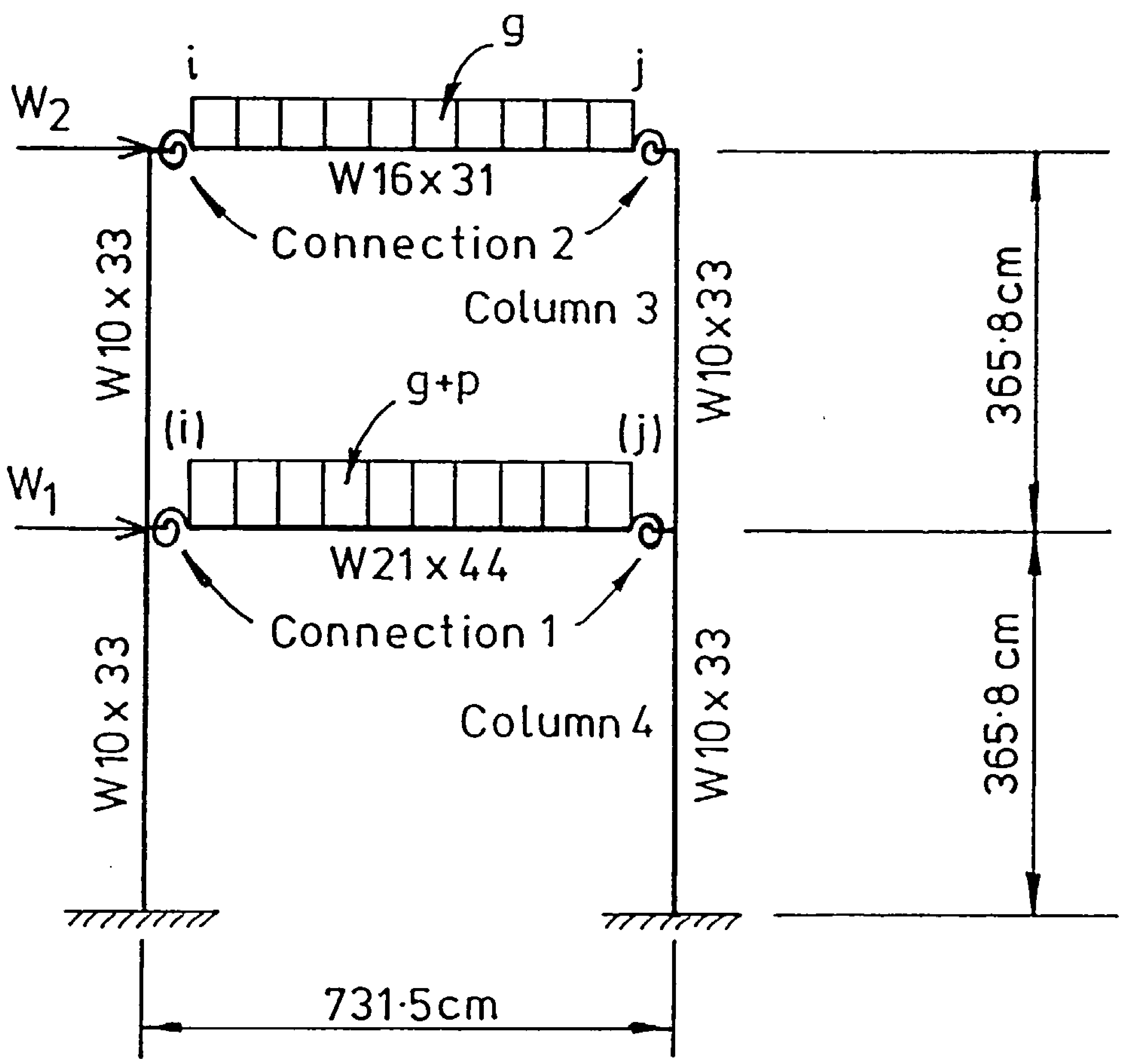


Figure 4.9: Plastic hinges formed at collapse for the simple frame of figure 4.5 with different joint types.



$g = 27{\cdot}14$ kN/m
$p = 17{\cdot}51$ kN/m
$W_1 = 25{\cdot}62$ kN
$W_2 = 12{\cdot}81$ kN

Figure 4.10: The frame of example 4.2.

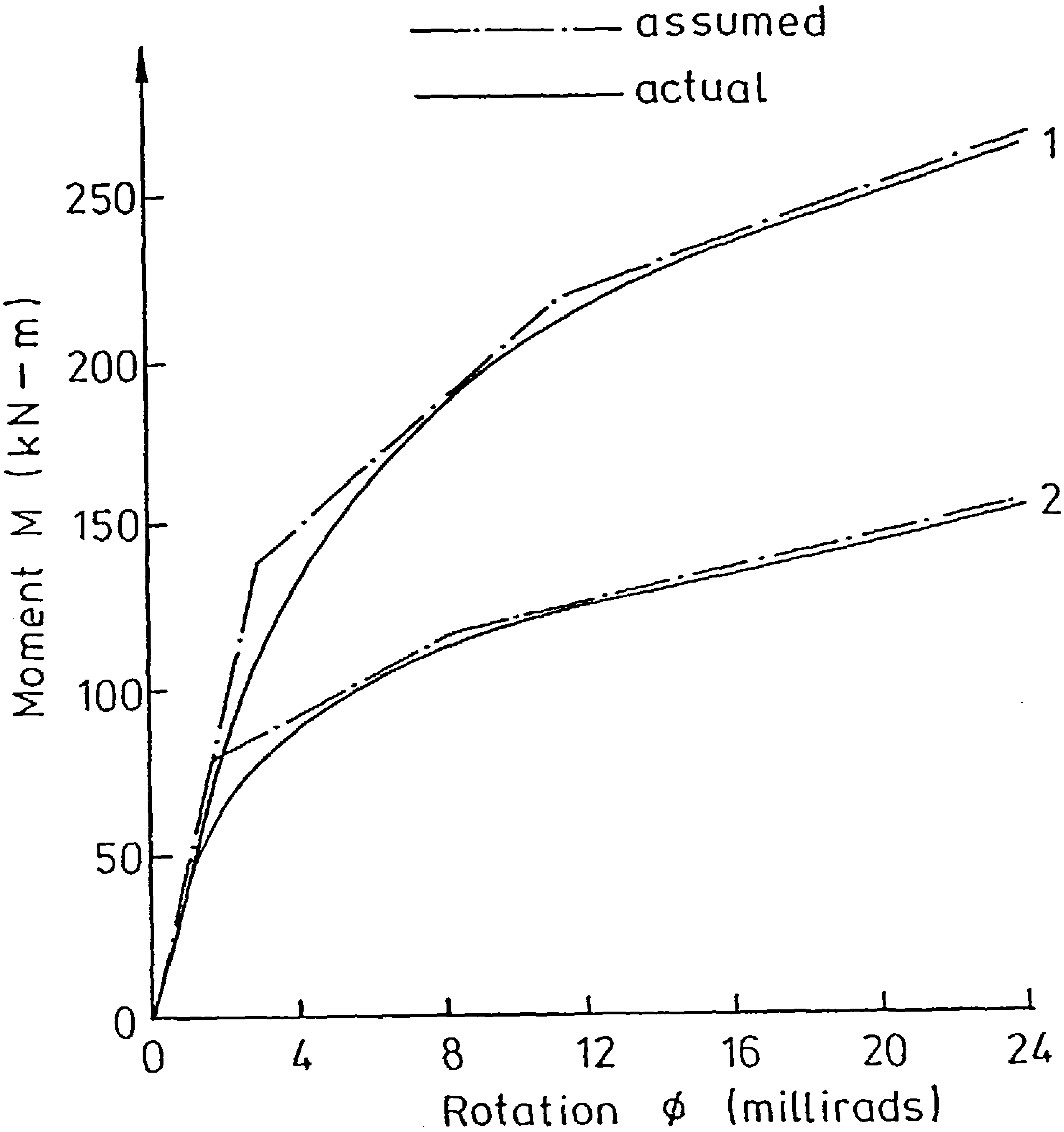


Figure 4.11: Moment-rotation relationship for joints utilised in the frame of figure 4.10.

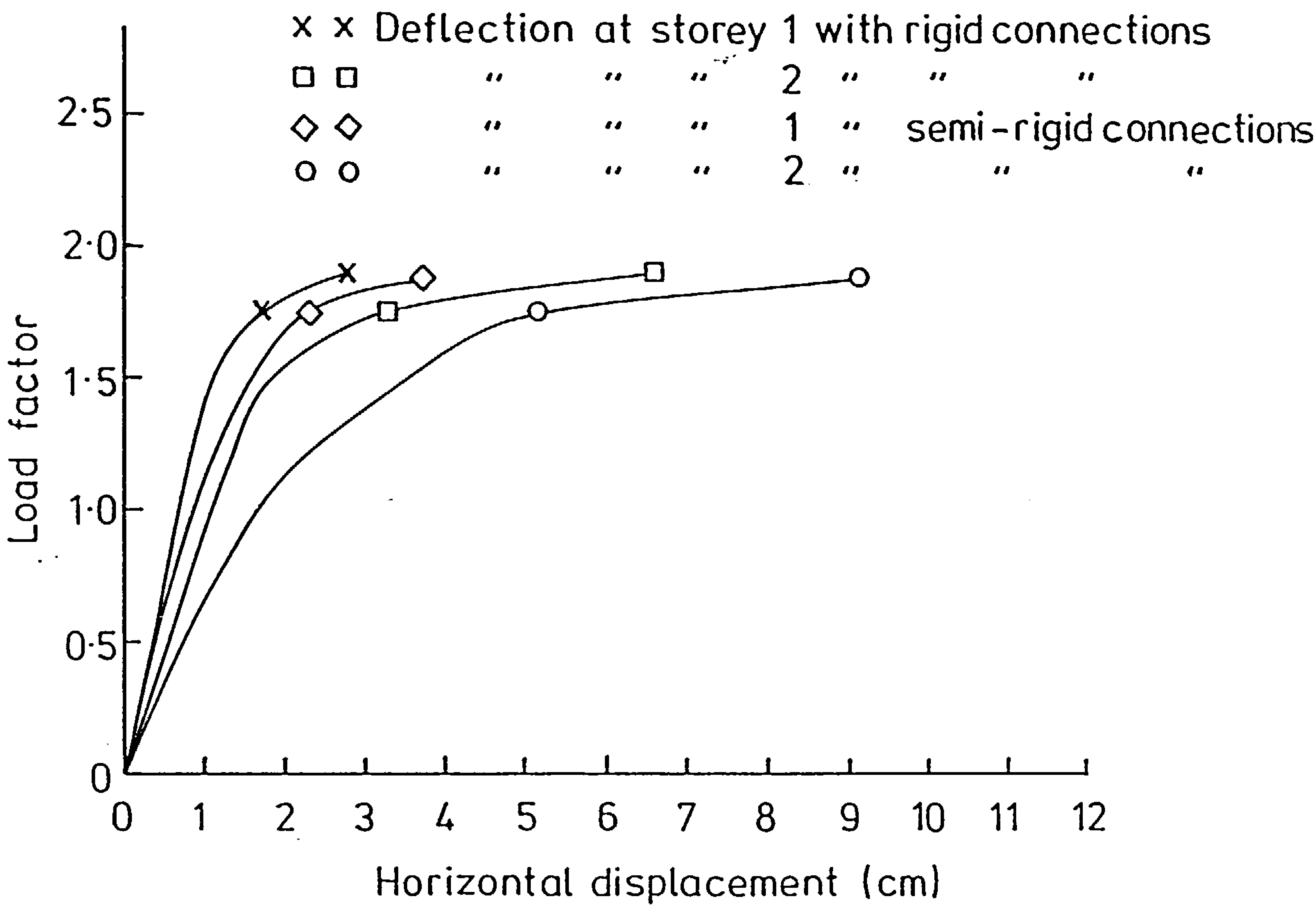


Figure 4.12: Lateral deflections for the frame of figure 4.10 with different joint characteristics.

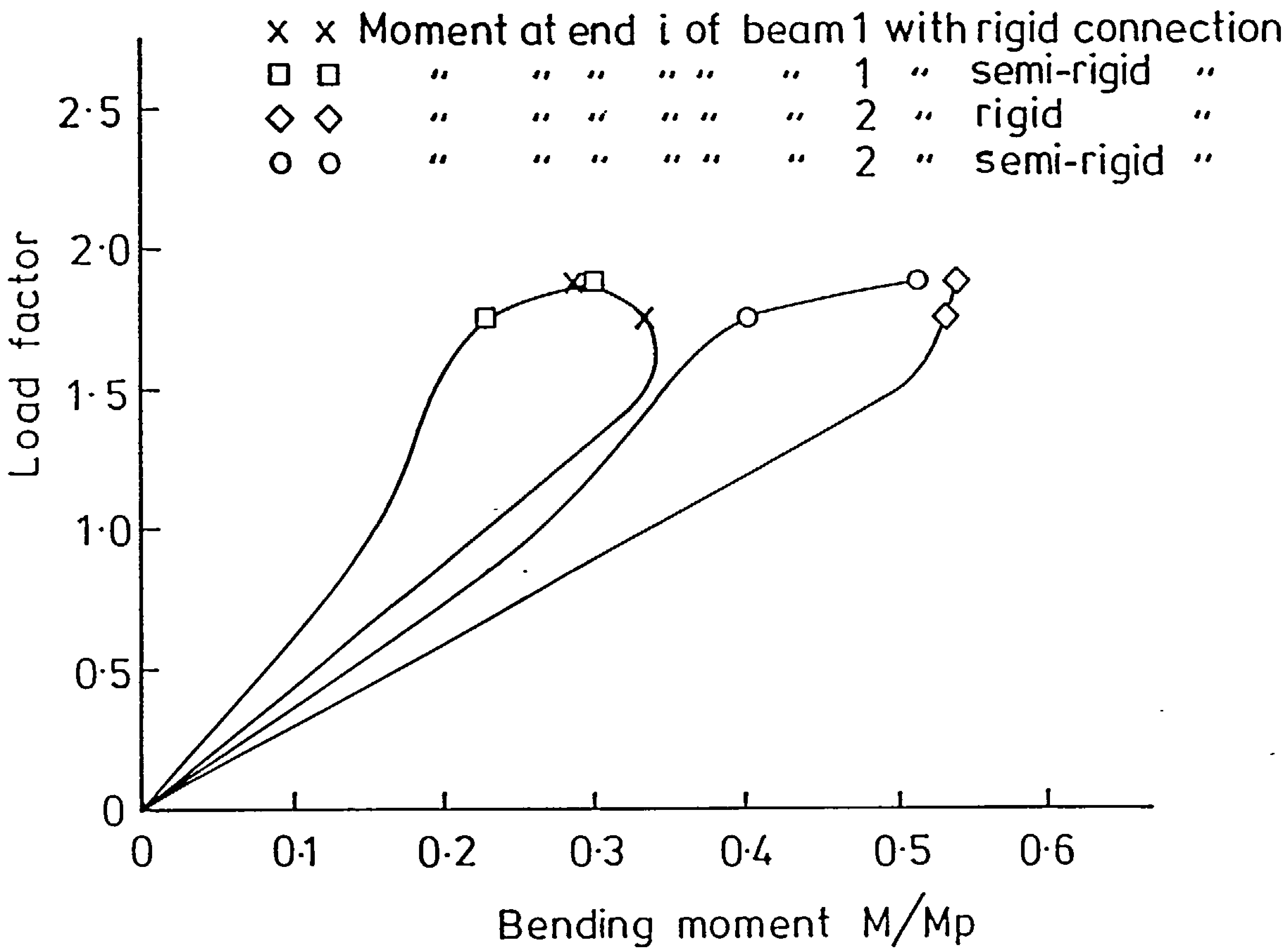


Figure 4.13: Beam end moment versus load factor for the frame of figure 4.10.

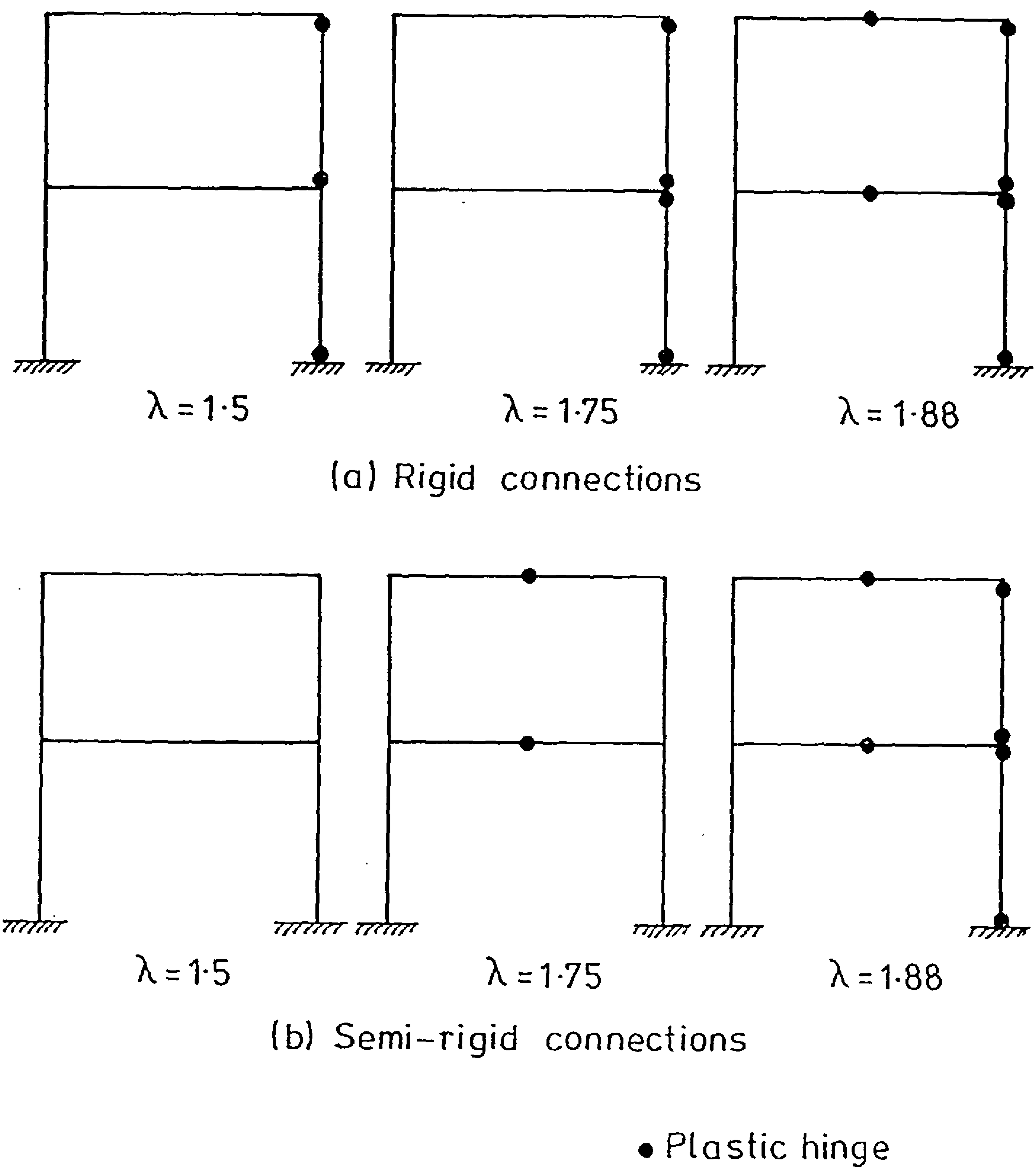


Figure 4.14: Collapse mechanism for the frame of figure 4.10.

Chapter 5

An Approximate Method for Estimation of Bending Moments in Continuous and Semi-Rigid Construction

5.1 Introduction

To allow for semi-rigid behaviour of a framed steel structure, modifications to the conventional methods of indeterminate frame analysis have been suggested by different researchers. Examples of these methods are the modified slope deflection method, the modified moment-distribution method, the beam line method and the modified three moment method. Nevertheless, these approaches proved to be too cumbersome and not practical for normal design use. This is because they require undesirable complicated hand calculation and therefore only a few designers have adopted them.

Experimental tests have proved that all steel joints exhibit different degrees of flexibility. Only a very stiff joint can be approximated to be fully rigid and a very flexible joint assumed to act like a perfect hinge. The majority of steel joints have moderate stiffnesses and thus neither assumptions can properly represent these joints and special treatments are needed. However, using a rigorous analysis, which can be only achieved by utilising complex computer programs, is likely to prove inconvenient for designers due to the requirement of a dramatic change in the currently used design analysis. Also the wind connection method outlined in chapter 4 suffers from a lack of logic in representing flexibly connected frames. It is thought therefore that a simple and logical design procedure which can be used with any of the currently used fully rigid analyses with only minor modifications is justified as a design tool.

In this chapter an approximate method which includes semi-rigid effects is presented and uses only a simple modification to the fully rigid method of analysis. As the process involves only small additional labour, it is believed that the method can be used efficiently for design purposes.

5.2 Assumptions

The following assumptions are made:

1) The material is assumed to be linearly elastic.

This assumption is justified for many practical frames up to working load level.

2) The moment-rotation relationship of the joint is taken as linear over the whole loading range.

Although, a typical moment-rotation relationship of a semi-rigid joint is, in general, nonlinear for the sake of simplicity a linear representation of a moment-rotation curve, i.e. the initial tangent or a secant of the joint $M - \Phi$, has been adopted. This assumption might, in some cases, produces satisfactory results for design purposes. However, if more accuracy is needed a better representation of the actual semi-rigid behaviour has to be included in the analysis and a simple hand calculation method will be presented in chapter 6 to allow for joint nonlinearity in multistorey sway frames subjected to both horizontal and vertical loads.

3) The principle of superposition is valid.
This can be simply justified by recognizing that assumptions 1 & 2 lead to a first order linearly elastic analysis including the behaviour of the semi-rigid connections.

4) Frames are assumed to have identical connections at both ends of every beam. However in multi-bay frames, the beams in the external bays might have different characteristics for their two end joints, therefore the average value of the both joint stiffnesses of each beam should be used to represent the joint stiffness K for both joints (at either end of the beam).

5.3 Equilibrium of a General Semi-Rigidly Connected Member

The conjugate beam method [59] is used to establish the relation between the forces and displacements for a member under loading as indicated in fig.5.1

$$\begin{aligned} V_i &= \theta_i - \phi_i - R = \frac{Ms_iL_g}{(3EI_g)} - \frac{Ms_jL_g}{(6EI_g)} + \frac{A^*\,b}{(EI_gL_g)} \\ V_j &= \theta_j - \phi_j - R = \frac{Ms_jL_g}{(3EI_g)} - \frac{Ms_iL_g}{(6EI_g)} - \frac{A^*\,a}{(EI_gL_g)} \end{aligned} \qquad (5.1)$$

where

V_i , V_j	are the shears at the ends of the conjugate beam and
L_g	is the element length.
ϕ_i , ϕ_j	are the changes of angle between the beam and the column at ends i and j.
I_g	is the second moment of area for the element.
E	is Young's modulus
R	is the relative angle between the both element ends due to relative end vertical displacements.
A^*	is the area under the free bending moment diagram for the element due to external loads.
θ	is the rotation of the node which is connected to the element.
Ms_i, Ms_j	are the semi-rigid member end moments of the element and
a , b	are the distances from the centroid of area A^* to the ends, i and j, of the beam respectively.

For a non-sway frame structure the joint flexibility inherent in semi-rigid construction might produce, when compared with rigid jointed structure, some beneficial effects by distributing some bending moments from the beam ends to its mid-span, therefore these two moments are likely to be more nearly balanced. However, for a sway frame, if semi-rigid joints are assumed, the structure will undergo larger deformations causing a reduction of the structure strength relative to rigid jointed frames.

For the sake of simplicity, semi-rigid joint behaviour in steel framed structures will be considered in two separate stages:

1) Frames subject to horizontal loads only.

2) Frames subject to symmetrical vertical loads only.

Noting that since the proposed method is linearly elastic, braced frames can be treated according to case 2) alone. However sway frames requires the superimposing of both 1) and 2).

5.3.1 Sway Frames Subjected to Horizontal Loads Only

When a sway frame is loaded by horizontal loads alone, the beams respond by double curvature behaviour assisting the columns in resisting lateral displacements and causing bending moments at their ends (fig.5.2). If fully rigid connections are assumed, the relative rotations between the beams and the columns vanish and the beams ends carry a certain value of bending moment. On the other hand, if semi-rigid connections are assumed, relative rotations between the beams and the columns occur and this will cause a reduction of the bending moments at the beams ends and allow the frame to undergo larger horizontal displacements (fig.5.2). Hence introducing semi-rigid connections in the behaviour of a sway frame loaded by horizontal loads only causes a reduction in contribution of the bending resistance of the beams in limiting horizontal displacements, which can in turn be visualised as a reduction in the effectiveness of the second moment of area of the beams. Therefore, in a flexibly connected frame, including the influence of semi-rigid connection behaviour on the flexural stiffness of the beams allows the use of a fully rigid method of analysis to predict the frame behaviour.

Considering an isolated unloaded semi-rigidly connected beam with equal end restraint (fig.5.1) and note that in this case:

As the beam is unloaded	(i) $A^*=0$
Due to symmetry	(ii) $\theta_i = \theta_j$
	(iv) $\phi_i = \phi_j$
	(v) $Ms_i = Ms_j = M_{hs}$

As the relative vertical displacement is zero   (ii) R = 0

where M_{hs} is the semi-rigid moment due to horizontal load.

Substituting into either of eqs.5.1 yields

$$V_i = \theta_i - \phi_i = \frac{M_{hs}L_g}{(3EI_g)} - \frac{M_{hs}L}{(6EI_g)} = \frac{M_{hs}L}{(6EI_g)}$$

Assuming a linear relationship between moment and rotation which may be taken as the initial tangent, or a secant stiffness of the utilized semi-rigid joints $M - \Phi$ relationships as appropriate and defining the joint stiffness K as M/ϕ, solving for Mh_s/θ_i yields

$$Mh_s/\theta_i = \left[\frac{6EI_g}{\frac{6EI_g}{K} + L_g}\right] = Um_s \qquad (5.2)$$

where Um_s is the unit rotation moment of an element allowing for two identical semi-rigid connections located at each end of the element.

For fully rigid connections $K = \infty$ and the unit rotation-moment Um_f is

$$M_{hf}/\overline{\theta_i} = \left[\frac{6EI_g}{L_g}\right] = Um_f \qquad (5.3)$$

where

$\overline{\theta_i}$ is the node rotation assuming fully-rigid connections.

Introducing the ratio $C_s = Um_s \,/\, Um_f$ from eqs. 5.2 and 5.3 gives

$$C_s = \frac{1}{1 + \frac{6EI_g}{L_g K}} \qquad (5.4)$$

This indicates that introducing semi-rigid connections into element behaviour would cause a reduction in the moment resisting effectiveness by the coefficient C_s.

5.3.2 Frames Acted on by Symmetrical Vertical Loads

Initially if a beam element is assumed to be rigidly connected to columns the bending moments at the end of the beam, which are created as a result of applying vertical loads, will reach their maximal values. However, if semi-rigid connection behaviour is considered, smaller bending end moments are resisted by the beam ends leading to a reduction in the bending end moments of the columns (fig.5.3). Thus the effects of semi-rigid connection behaviour in a frame, loaded by symmetrical vertical loads alone, can be allowed for by reducing the beam end moments in the whole frame and reducing the magnitude of the column end moments by appropriate factors. This modification is dependent upon connection types and the beam and the column stiffnesses.
Considering this case the following relationships can be identified:

Due to symmetrical response	(i) $\theta_i \;=\; -\theta_j$
As the relative vertical displacement zero	(ii) R = 0
Due to asymmetry	(iii) loads a = b = $L_g/2$
	(iv) $Ms_i = -Ms_j = M_{vs}$
For equal end restraints	(v) $K_i = K_j = K$

Substituting these into one of eqs. 5.1 yields:

$$\theta_i - \frac{M_{vs}}{K} = \frac{M_{vs}\,L_g}{2EI_g} + \frac{A^*}{2EI_g}$$

Rearranging and solving for M_{vs} gives:

$$M_{vs} = \frac{-\,A^* + 2EI_g\,\theta_i}{\frac{2EI_g}{K} + L_g} \tag{5.5}$$

For fully rigid connections $K = \infty$ and the beam end moment M_{vf} is

$$M_{vf} = \frac{-\,A^* + 2EI_g\,\overline{\theta_i}}{L_g} \tag{5.6}$$

Introducing the ratio $\alpha_s = M_{vs}/M_{vf}$ from eqs. 5.5 and 5.6 and assuming now that the beam is connected to very stiff columns therefore θ_i and $\overline{\theta_i}$ are very small hence $\theta_i \simeq \overline{\theta_i}$ gives:

$$\alpha_s = \frac{1}{1 + \frac{2EI_g}{K\ L_g}} \tag{5.7}$$

eq.5.7 has been suggested by Lui and Chen [12] to approximate the end restraint for columns in a box-frame with H sections assuming a single curvature beam distortion.

5.3.3 Practical Frames Acted on by Uniformly Distributed Vertical Loads

In practical multi-storey frames the columns have limited stiffnesses, thus eq.5.7 cannot be used without a modification to account for the effect of joint stiffness K on the node rotations, i.e. θ_i must be calculated in term of the joint stiffness K (see eq.5.5).

The model indicated in fig.5.4 is utilized where the beams and the columns are assumed to bent in a perfect single curvature. Considering the semi-rigid joint located on the right-side of the node (i) and making use of the slope-deflection equations one can write the following relationship between the bending moments and rotations for the columns c1 and c2

$$(M_A)_{c1} = \frac{2EI_c\,\theta_i}{h} = (M_A)_{c2} \tag{5.8}$$

in which

I_c is the second moment of area of the columns and

h is the storey height.

Using the modified slope-deflection equations (see reference [35]) for the beams (b1) and (b2) and noting that the positive direction for the beam fixed end moment is clock-wise gives:

$$(M_A)_{b1} = \frac{2EI_g}{L_g} A' [B'\theta_i - C'\theta_i] - D' A' \frac{w L_g^2}{12} \tag{5.9}$$

$$(M_A)_{b2} = \frac{2EI_g}{L_g} A' [B'\theta_i - C'\theta_i] + D' A' \frac{\omega w L_g^2}{12} \tag{5.10}$$

where

$$A' = \frac{1}{(3\alpha\beta + 2\alpha + 2\beta + 1)} ;$$

$$B' = (3\beta + 2) ;$$

$$C' = 1 \quad , \quad D' = (3\beta + 1);$$

$$\alpha = \frac{2EI_g}{K_A L_g} \quad , \quad \beta = \frac{2EI_g}{K_B L_g} \tag{5.11}$$

and where ω is the ratio of the distributed load intensities for beams (b2 and b1) indicated in fig.5.4.

Equilibrium at node A requires

$$(M_A)_{c1} + (M_A)_{c2} + (M_A)_{b1} + (M_A)_{b2} = 0 \tag{5.12}$$

Introducing eqs.5.8, 5.9 and 5.10 into eq.5.12 and solving for θ_i gives:

$$\theta_i = \frac{A' D' \frac{(1 - \omega) w L_g^2}{12}}{\frac{4EI_c}{h} + \frac{4EI_g}{L_g} A' (B' - C')} \tag{5.13}$$

Now substituting eq.5.13 into eq.5.5 gives:

$$M_{vs} = \frac{-A^* + \frac{2EI_g A' D' \frac{(1 - \omega) w L_g^2}{12}}{\frac{4EI_c}{h} + \frac{4EI_g}{L_g} A' (B' - C')}}{\frac{2EI_g}{K} + L_g} \tag{5.14}$$

For uniformly distributed loads $A^* = \frac{w L_g^3}{12}$ and substituting this into eq.5.14 gives:

$$M_{vs} = \frac{\frac{w L_g^2}{12}\left[-1 + \frac{(1 - \omega)\frac{2EI_g}{L_g} D' A'}{\frac{4EI_c}{h} + \frac{4EI_g}{L_g} A' (B' - C')}\right]}{1 + \frac{2EI_g}{K L_g}} \tag{5.15}$$

If fully rigid connections are assumed eq.5.15 becomes:

$$M_{vf} = \frac{w\ L_g^2}{12}\left[-1 + \frac{(1 - \omega)\frac{2EI_g}{L_g}}{\frac{4EI_c}{h} + \frac{4EI_g}{L_g}}\right] \tag{5.16}$$

Since the beams are assumed to have identical connections at both ends:

$$K_A = K_B = K \quad and \quad \alpha = \beta$$

Assuming now that

$$D'\ A' = A'\ (B' - C') = \frac{3\alpha + 1}{3\alpha^2 + 4\alpha + 1} = \eta \tag{5.17}$$

$$\psi_s = \frac{\frac{2EI_g}{L_g}\eta\ (1 - \omega)}{2E(\sum\frac{I_c}{h} + \sum\frac{I_g}{L_g}\eta_s)} \tag{5.18}$$

and

$$\psi_f = \frac{\frac{2EI_g}{L_g}(1 - \omega)}{2E(\sum\frac{I_c}{h} + \sum\frac{I_g}{L_g})} \tag{5.19}$$

Using eqs. 5.15-5.19 and introducing the ratio $\alpha_s = M_{vs}/M_{vf}$ gives :

$$\alpha_s = \frac{1}{1 + \alpha}\ \frac{\psi_s - 1}{\psi_f - 1} \tag{5.20}$$

Eq.5.20 provides an attractive way of including the behaviour of semi rigid joint behaviour into a fully rigid analysis for a steel frame subjected to only symmetrical vertical loads. That is because including the effect of the presence of semi-rigid joints in a practical frame can be accomplished by simply reducing the beam end moments, obtained from fully rigid analysis, by the coefficient α_s. Consequently an appropriate action should be taken to reduce the column end moments.

5.4 Comment

Against the background of the theory of the model indicated in fig.5.4, eq.5.20 can be used for general practical frames, i.e. it can be utilized for unsymmetrical multi-storey multi-bay frames subjected to unsymmetrical vertical loads, providing that ω is taken as the ratio of the fixed end moments due to the external vertical loads of the beams(b2 and b1) respectively. Moreover, if any element indicated in the model is not present its flexural rigidity should be taken as zero.

5.5 Design Implications

In a design office a 'trial and error procedure' is usually employed for designing practical frames. In this procedure a primarily selection of beam and column sections is made using simple and sometimes empirical design rules. This is normally followed by the checking of strength and stiffness.

The checking is usually carried out to ensure that the assumptions used in selecting the frame sections did not violate safety. The method which is to be proposed is ideal for checking stresses in practical frames once the sections have been selected. In the proposed method inclusion of semi-rigid joint behaviour can be accomplished explicitly, hence it can be used without altering the currently used fully rigid analyses.

The proposed method proceeds as follows:

1.a) Calculate the bending moment due to the actual vertical loads for each element of the frame from utilizing a fully rigid method of analysis.

1.b) Reduce the bending moment of each semi-rigid joint of the entire frame by a corresponding coefficient α_s, where α_s should be calculated from eq.5.20 for each semi-rigid joint independently

1.c) Reduce the absolute values of the column end moments at each node by the coefficient α_n, where α_n is the ratio between the modified and non-modified beam end moment which is connected to this node.

1.d) The absolute column end moment at the base of each column should be reduced by the same factor, i.e. α_n, which is used for top end of that column.

It should recognized from eqs.5.13, 5.18 and 5.19 that when $\omega = 1$, i.e. the distributed loads on the beams (b2 and b1) are identical, the node rotation θ equals to zero and the second part of eq.5.20 is equal to unity. This case, which is equivalent to a frame in which the columns are very stiff compared with the beams, might occur in a multi-bay frame when considering intermediate joints. However, for external joints ω equals zero.

2) To include the effect of semi-rigid connections for a frame subjected to horizontal loads only, the second moments of area of all beams in the entire frame should be reduced by the corresponding coefficient C_s (eq.5.4), and then a fully rigid method of analysis can be used.

3) Superimposing the element bending moments obtained from steps 1 and 2.

Comparing this method with a precise method using a computer program which includes true semi-rigid connection behaviour gives good correlation for member moments and deflections.
It should be noted that in the proposed method the only modification suggested for frames subjected to vertical loads (see section 5.3.3) is required to allow for the behaviour of semi-rigid connections of a non-sway frame, since in a non-sway frame the horizontal loads are resisted by the bracing elements alone.

5.6 Example 5.1

Many examples have been evaluated and compared with the exact method using a sophisticated computer program which properly represents semi-rigid connection behaviour. This has demonstrated that the approximate method gives member end moments which are satisfactory for design purposes. A three-storey one-bay frame indicated in fig.5.5, in which the section properties, the loading pattern and the frame geometry are arbitrarily chosen by the author, is given as an example.

For generality an unsymmetrical frame with flexible connections, K = 125000.0 kN-cm/rad. which is typical of web cleat joints for the size of members used, and subjected to unsymmetrical concentrated loads is considered. This example is chosen to demonstrate the ability of the proposed method to tackle practical frames. For other cases, such as symmetrical frames with reasonably stiff connections and subjected to symmetrical vertical loads, more accuracy should be expected from the method.

The solution proceeds according to the following steps:

1-a) The actual frame is subjected to actual vertical loads alone and a fully rigid method of analysis is used to calculate the member end moments for each element in the whole frame. These values are encircled in fig.5.6

1-b) Since the beam sections are uniform throughout the frame, α & η the beam semi-rigid factors can be calculated from eqs. 5.11 and 5.17 for all semi-rigid joints of the whole frame.

$$\alpha = \frac{2 \times 21000 \times 8356}{125000 \times 500} = 5.62$$

According to Wood's classification [10] these semi-rigid joints are very flexible.

$$\eta = \frac{3 \times 5.62 + 1}{3\,(5.62)^2 + 4 \times 5.62 + 1} = 0.15$$

Since the boundary elements of the semi-rigid joints are different, they should be divided into four groups (S_1 , S_2 , S_3 *and* S_4) as indicated in fig.5.5

Now computing ψ_{s1} *and* ψ_{f1} for the first joint group (S_1) from eqs.5.18 and 5.19 respectively and noting that, in this case, $\omega = 0$ gives:

$$\psi_{s1} = \frac{\frac{8356}{500} \times 0.15}{\frac{5696 \times 2}{400} + \frac{8356}{500} \times 0.15} = 0.079$$

$$\psi_{f1} = \frac{\frac{8356}{500}}{\frac{5696 \times 2}{400} + \frac{8356}{500}} = 0.369$$

Thus α_{s1} can be obtained directly from eq.5.20

$$\alpha_{s1} = \frac{1}{1 + 5.62}\,\frac{0.079 - 1}{0.369 - 1} = 0.22$$

Repeating same argument for S_2 , S_3 *and* S_4 gives:

$$\alpha_{s2} = 0.202 \quad , \alpha_{s3} = 0.28 \quad and \quad \alpha_{s4} = 0.246$$

Reduce the absolute value of the bending moment at each semi-rigid joint of the frame, which is obtained from a fully rigid analysis , by the corresponding coefficient α_s (fig.5.6)

1.c) Reduce the absolute values of the column end moments at each node by the factor α_n, which is the ratio of the modified to non-modified beam end moment which is connected to this node. The resultant member end moments are indicated in fig.5.6 as uncircled values.

1.d) The absolute value of the column end moment at the base is reduced by the same factor α_n which is used for top end of the column (see fig.5.6).

2) Substituting K into eq.5.4 yields values for the beam semi-rigid stiffness reduction factors C_s for horizontal loading

$$C_1 = C_2 = C_3 = \frac{1}{1 + \frac{6 \times 21000 \times 8356}{500 \times 125000.0}} = 0.056$$

The equivalent beam second moment of area for the frame stories are therefore

$$I'_{g1} = I'_{g2} = I'_{g3} = I'_g = 0.056 \times 8356 = 467 \ cm^4$$

To obtain the substitute frame for horizontal loads only the beam second moment of area of each storey is replaced in the actual frame by I'_g and then a fully rigid method of analysis is used to calculate the member end moments of the frame.

3) Now take the sum of the member end moments due to horizontal and vertical loads obtained from steps 1 and 2. These are compared with values obtained from an exact method which gives the values indicated in fig.5.7 from which it can be seen that close correspondence occurs throughout the frame.

5.7 $\lambda_{cr,f}$ for a Semi-Rigidly Connected Frame

In addition to determining member end moments for frames with semi-rigid connections, the proposed method can also be used to compute an approximation of the elastic critical load factor $\lambda_{cr,f}$ for a semi-rigidly connected frame using Horne's method. In this method the modification adopted for a frame subject to horizontal loads only is required. The method proceeds as follows:

	Semi-rigid analysis			Rigid analysis
	Exact analysis	Proposed method	Error %	
$\lambda_{cr,f}$	3.70	3.48	6%	17.50

Table 5.1: $\lambda_{cr,f}$ for the frame indicated in fig.5.5

1) The second moment of area for beams in the entire frame should be reduced by the appropriate factors C_s using eq.5.4

2) The sway index is calculated at each storey level after applying 1/2% of the factored vertical loads in a horizontal direction at each floor level of the frame.

3) $\lambda_{cr,f}$ is taken as equal to the inverse of two hundred times the maximum sway index of the frame stories.

5.7.1 Example 5.2

To demonstrate the use of the proposed method for estimating the elastic critical load factor of a semi-rigidly connected frame $\lambda_{cr,f}$, the frame indicated in fig.5.5 is examined and the result is compared, in table 5.1, with an exact value obtained from utilizing a sophisticated computer program. It can be seen from this table that, the accuracy of the proposed method is satisfactory for design purposes.

5.8 Conclusion

An approximate method which accounts for the behaviour of flexible connections of a multistorey frame has presented in this chapter. Since the proposed method requires only any of the conventional fully rigid first order elastic method of analysis, it is straight-forward and ideally suited for practical design office use.

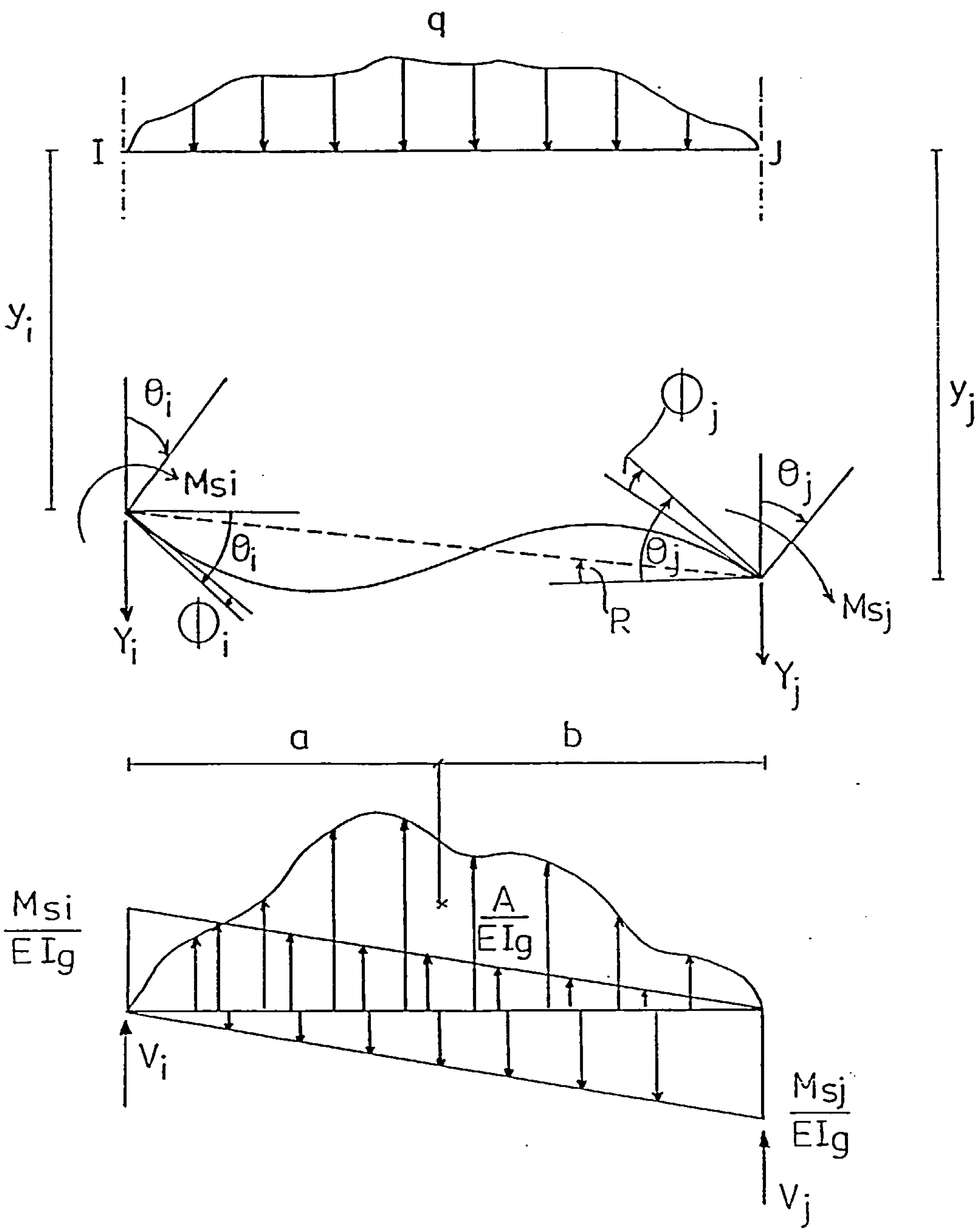


Figure 5.1: Conjugate beam for a semi-rigidly connected element.

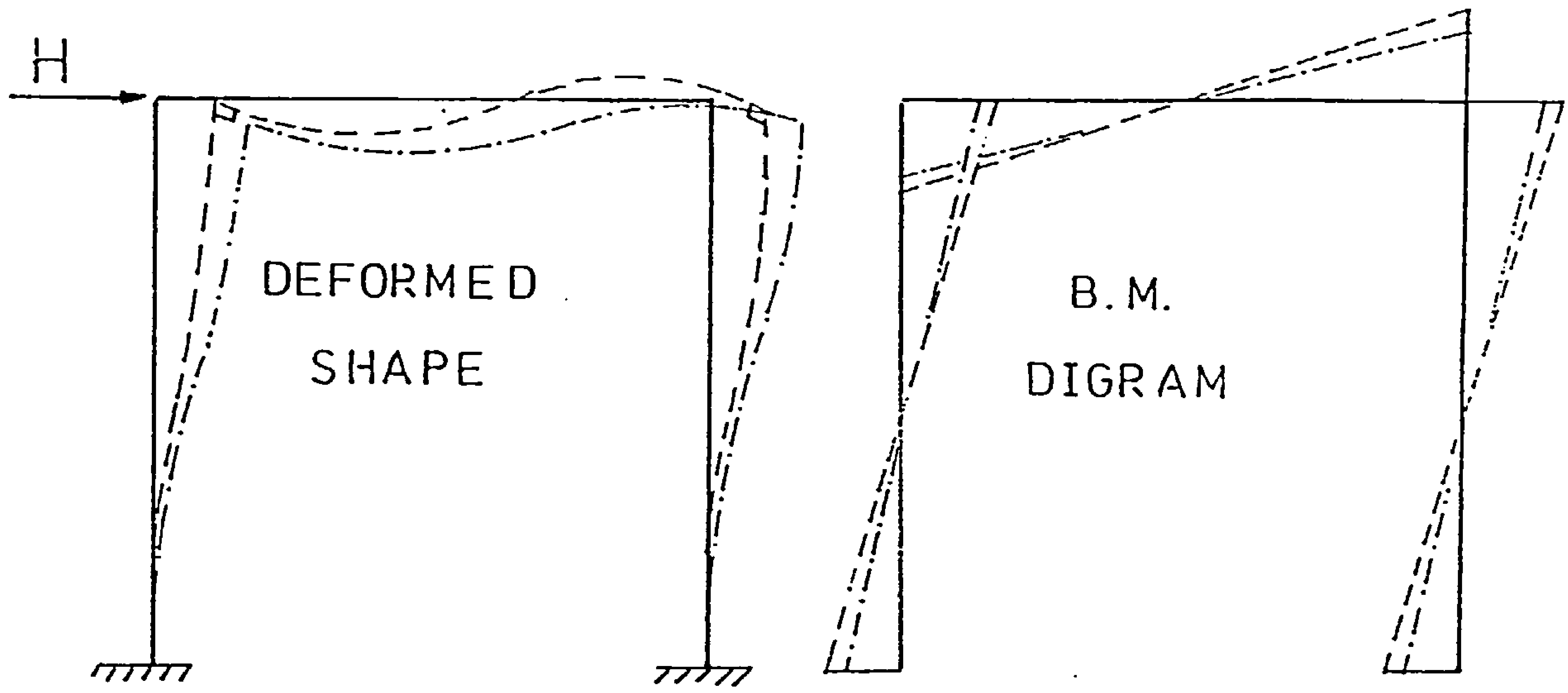


Figure 5.2: Typical behaviour of a semi and fully rigidly connected simple frame subject to horizontal load.

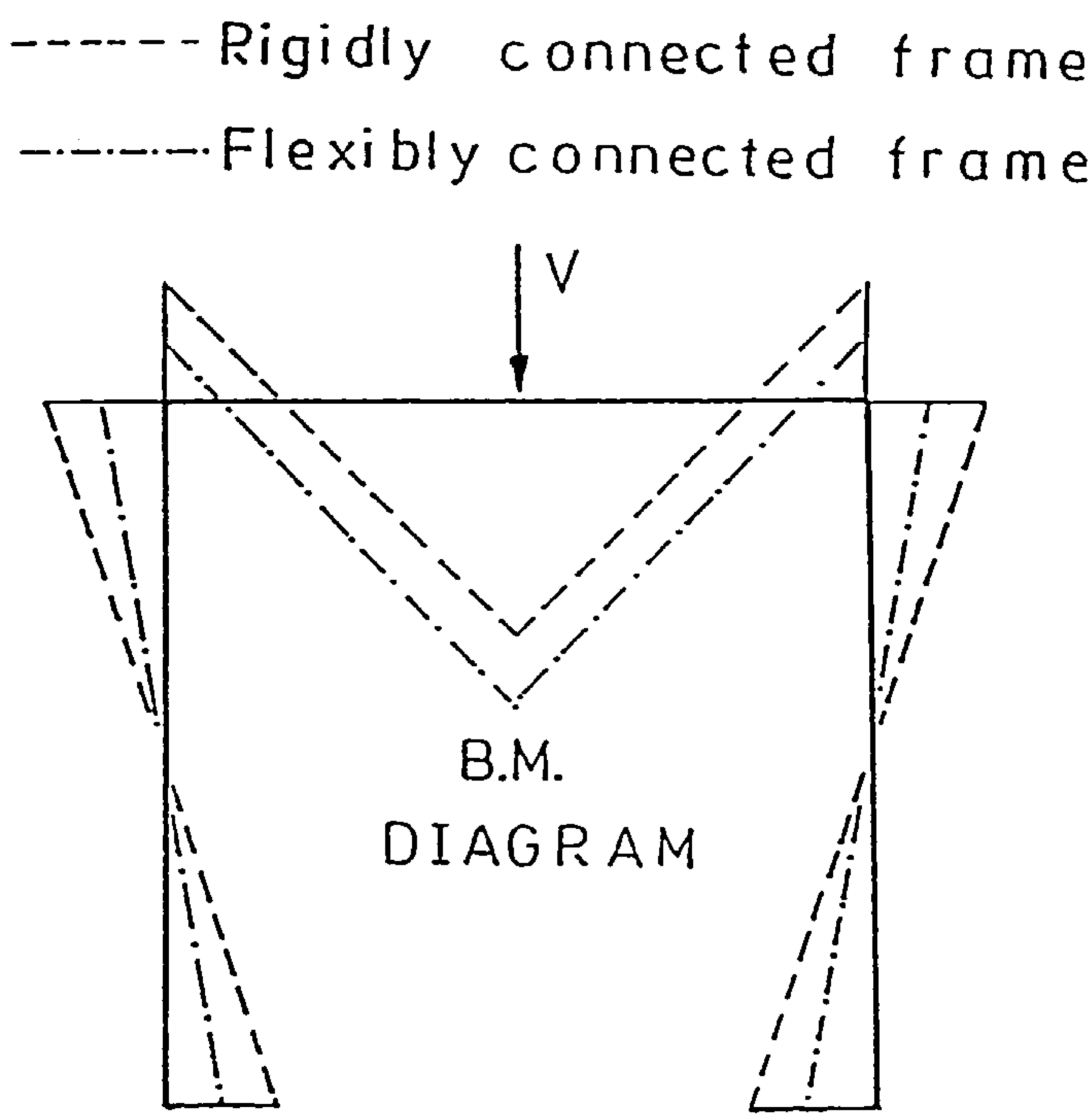
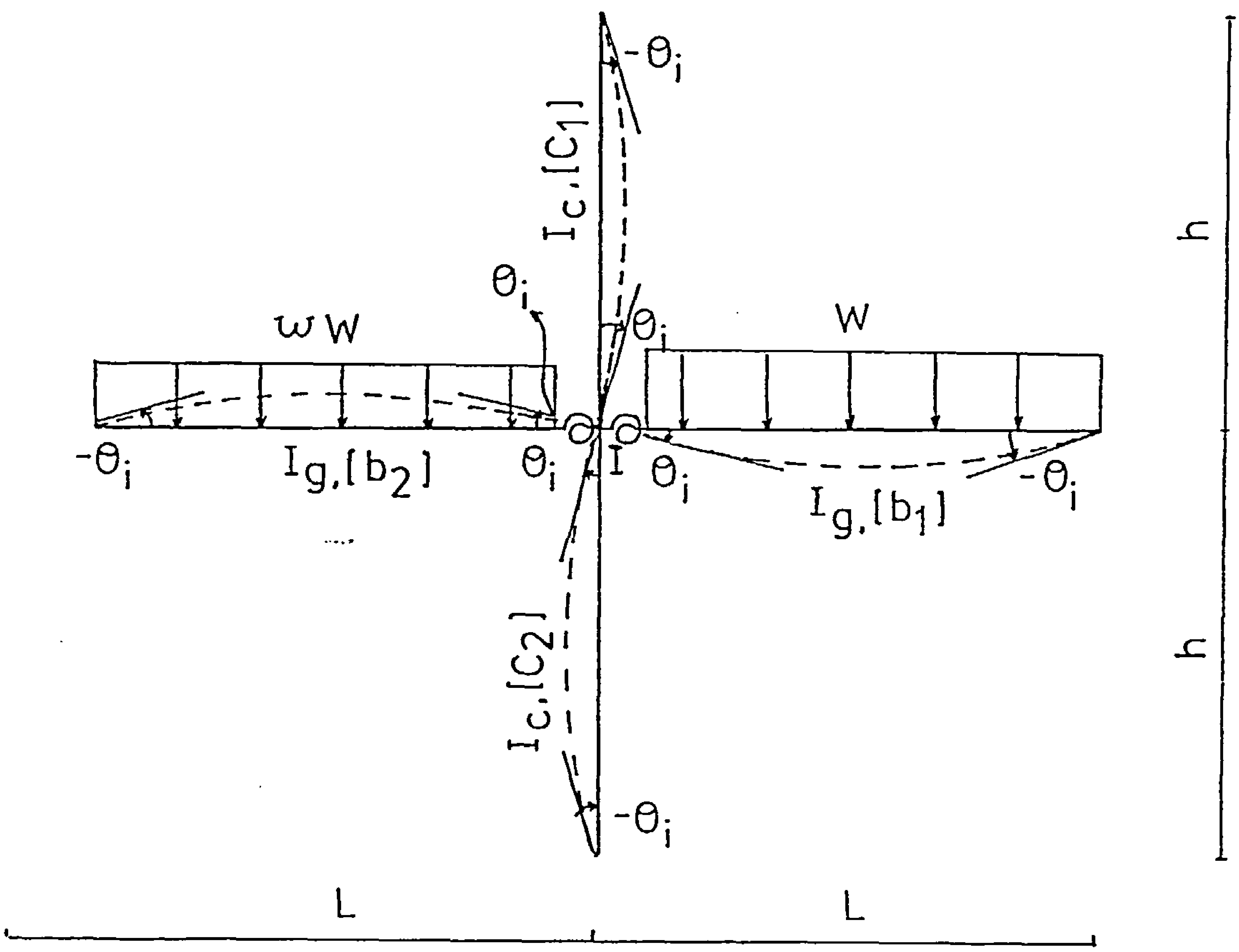


Figure 5.3: Typical bending moment diagrams for a semi and fully rigidly connected simple frame subject to symmetrical vertical load.



[cn] Column number

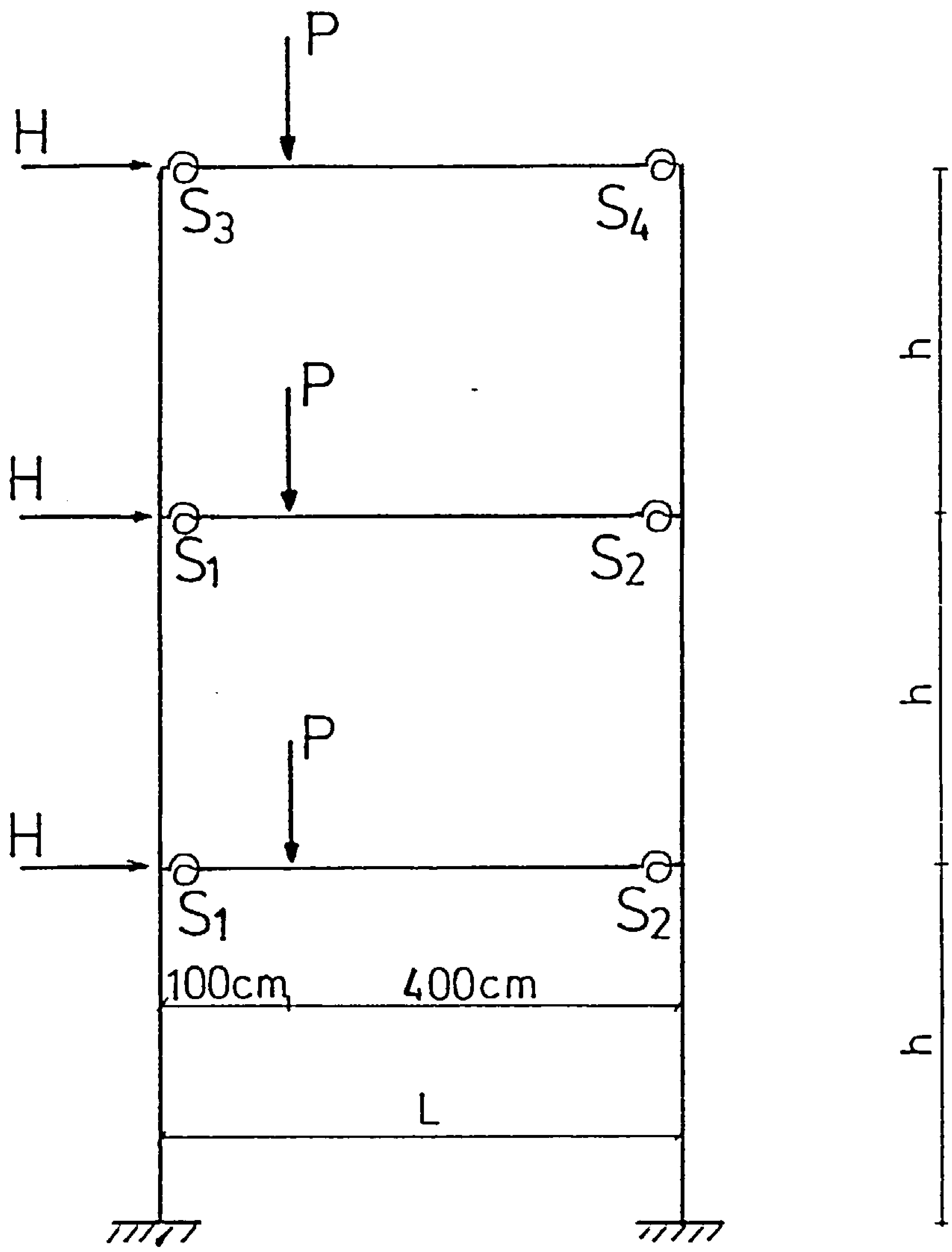
[bn] Beam number

I_c Column second moment of area

I_g Beam second moment of area

W U.D.L Load

Figure 5.4: Subassemblage used in the derivation of equation 5.20.



All beams are
IPE 300

All columns are
HE 200B

L = 500 cm

h = 400 cm

H = 20 kN

P = 200 kN

Figure 5.5: Three storey frame used in example 5.1.

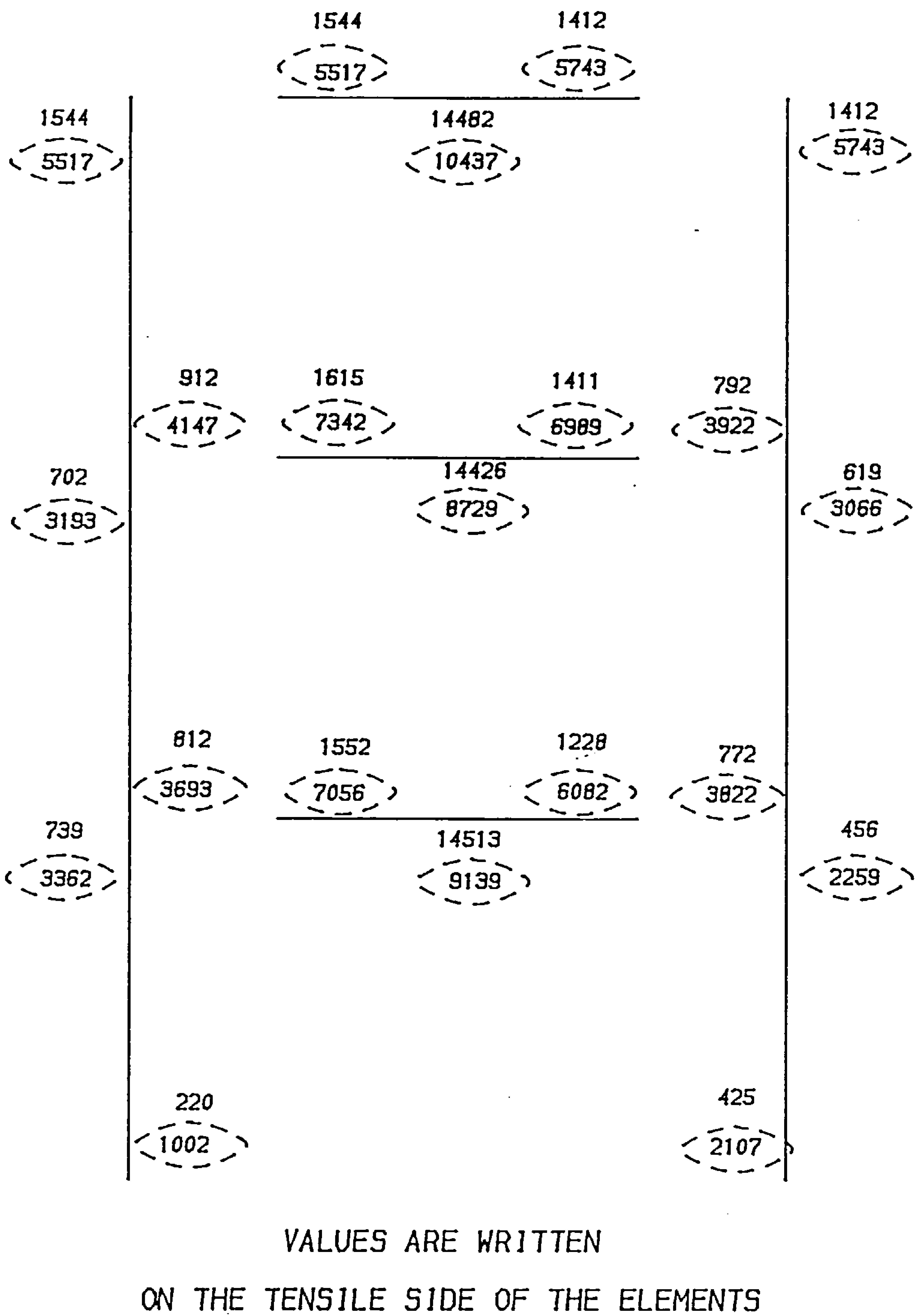


Figure 5.6: Comparison between bending moments obtained from the proposed method and an exact analysis for the frame of figure 5.5 considering vertical loads only.

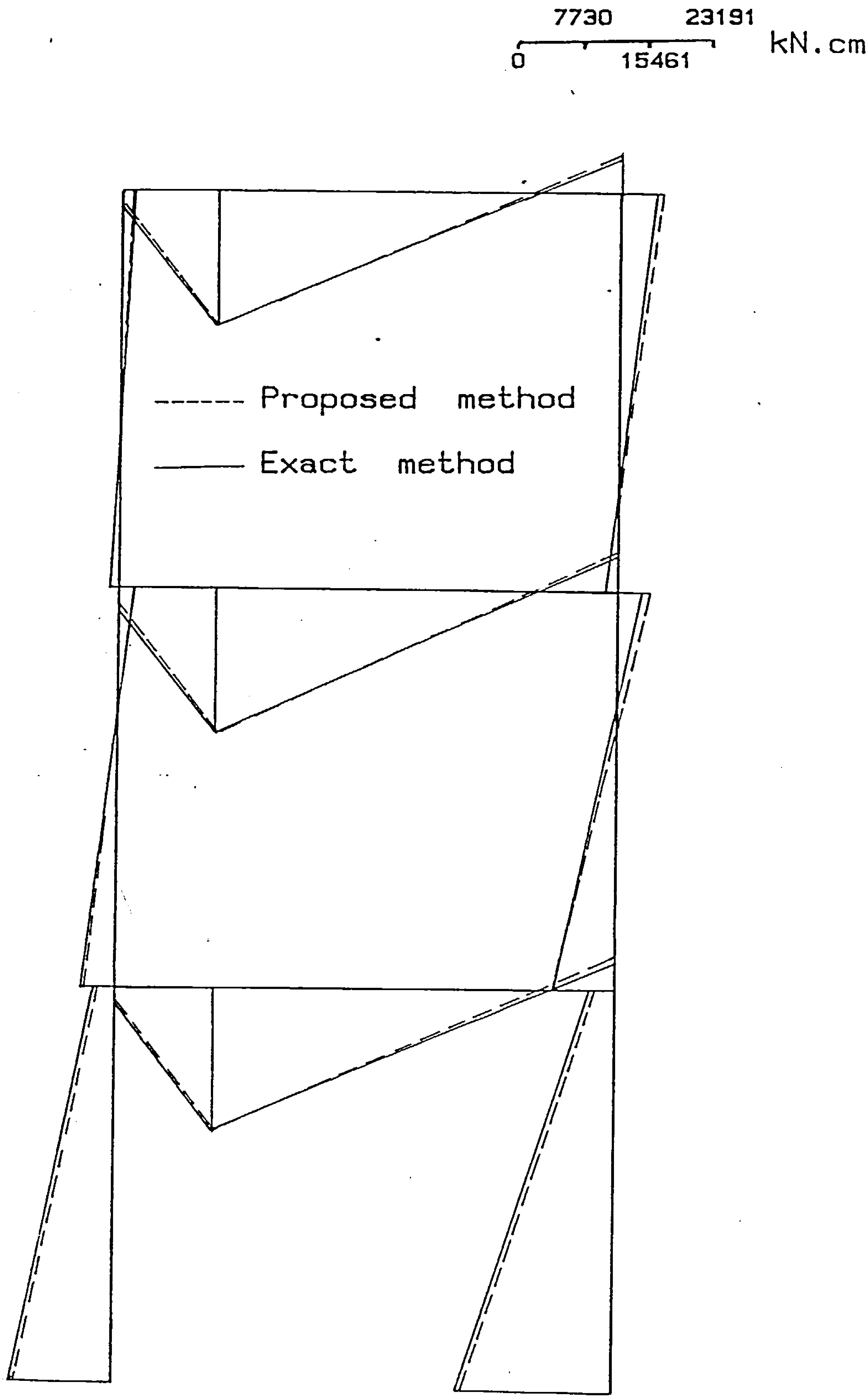


Figure 5.7: Comparison between bending moments obtained from the proposed method and an exact analysis for the frame of figure 5.5 considering both vertical and horizontal loads.

Chapter 6

Nonlinear Behaviour of Semi-Rigid Joints

6.1 Introduction

It has been proved (see chapter 4) that connection flexibility in framed structures can be a primary determinant of both deformations and internal force distributions. Hence, it appears important for professional designers to have the capability of analysing flexibly connected frames. A simple design procedure which can be used to modify any of the conventional rigid analysis techniques has been proposed in chapter 5 in which semi rigid joint behaviour is assumed to be linear throughout the whole loading range.

The actual behaviour of a typical semi-rigid joint is in general nonlinear. Although the spring stiffness of a semi-rigid joint is usually taken as the initial slope of the joint's moment-rotation M-Φ curve, this is true only for M equal to zero (see fig.6.1) and deviates from it as moment increases. For a joint with modest stiffness this deviation might cause significant errors due to the overestimation of the joint stiffness.

Hence, to represent the joint behaviour more accurately, the joint stiffness, K, should be taken as the secant to the M-Φ curve at the relevant moment rather than the initial tangent.

In this chapter a graphical hand calculation procedure to estimate semi-rigid joint stiffnesses in a practical frame, subject to horizontal and vertical loads is proposed. The method calculates the joint stiffnesses considering the actual $M - \Phi$ relationships of the joints and taking into account the relevant moment in the joint. This procedure is then incorporated into the approximate method for estimating the element bending moments in flexibly connected frames which has been discussed earlier in chapter 5.

6.2 Nonlinear Behaviour of Semi-Rigid Connections

In order to explain the philosophy behind the proposed method, consider the semi-rigid joint used in the two element frame ABC indicated in fig.6.2. The end A is encastre' and the moment-rotation relationship of the joint at B is indicated in fig.6.3 whilst the joint at C is on a knife edge and roller system. Suppose that after subjecting the frame to a loading pattern, the bending moment of the joint reaches point M1 on the moment rotation curve passing through the nonlinear path O-M1. Using a linear analysis and taking the joint stiffness as the secant O-M1 should, for this particular frame under this loading pattern, represent accurately the nonlinear behaviour of the frame. Therefore this nonlinear problem has been converted into an equivalent linear one providing that the secant stiffness, in this case O-M1, can be assessed with satisfactory accuracy. This secant, as it can be seen from fig.6.3, depends upon the joint bending moment and the latter depends on the element sections of the frame, the loading pattern and the joint moment-rotation relationship.

In the proposed method this secant is estimated by utilising an iterative procedure. Thus a computer program which uses a first order elastic analysis including joint flexibility, or an equivalent method, is required to allow for the behaviour of two semi-rigid joints located at both ends of every beam in a multistorey frame.

The basis of the method can be explained by means of assuming an initial value of a joint stiffness K, i.e. this could be either the initial tangent of the moment-rotation relationship or any other value, and correcting this value by iteration until the joint's resisting moment M converges.

The method proceeds via the following steps:

1) Assume an initial stiffness value K_o for the utilised semi-rigid joint. In this chapter since the initial tangent of the moment-rotation relationship forms an upper bound to a joint stiffness, it is adopted as a first estimation to the joint's stiffness.

2) Locate M on the initial stiffness line of the $M - \Phi$ curve fig.6.3, and rotate this line about the origin O until M cuts the moment rotation curve defining the point M1. Take the resultant secant K_1 as the new joint stiffness.

3) Calculate the new beam end moment M' as in step 1

4) Define M' on K_1 fig.6.3 and if M' is approximately located on the secant K_1 and the moment-rotation curve simultaneously the solution has converged, otherwise rotate K_1 around O clockwise or anti-clockwise until M ' cuts the moment rotation curve defining a revised joint stiffness K_2.

5) Repeat steps 3 and 4 until the resultant beam end moment obtained from step 3 is located on, or is sufficiently close to, the secant and the moment rotation curve simultaneously.

	Exact Nonl.Method	Proposed Method(Linear)	
		1^{st} Estimation	2^{nd} Estimation
1	2	K_o =750000 kN.cm/rad. 3	K_1 =182600 kN.cm/rad. 4
Δ (cm)	2.95	4.38	2.97
M_a (kN.cm)	3062.	4388.	3087.
M_b (kN.cm)	2063.	3389.	2088.
M_m (kN.cm)	-24000.	-23300.	-24000.0

Table 6.1: Comparison between the proposed and an exact method for the member end moment and lateral displacement of the two element frame indicated in fig.6.2

The proposed method converges to the result of an exact nonlinear analysis providing that the loading and unloading paths are identical in M- Φ curve.

6.2.1 Example 6.1

To demonstrate the use of the proposed method the two element frame, indicated in fig.6.2, has been worked by hand with the following properties:

h = 5.00 m, L = 5.00 m, H = 2 kN., P = 80 kN/m.

The beam and the column sections are HE200B and IPE300 respectively.

The joint's M- Φ relationship is indicated in fig.6.4

It should be noted here that, in the 'exact nonlinear method', the only nonlinearity is considered is that due to connection response. The results are summarized in fig.6.4 and table 6.1. In this table Δ denotes the horizontal deflection calculated at point B and M_a, M_b and M_m denote the bending moments of the frame calculated at points A, B and the mid-span of the beam AB respectively. Utilising an exact nonlinear analysis produces column 2 whilst employing the proposed method and taking the stiffness of the joint B as the initial tangent of the $M - \Phi$ relationship indicated in fig.6.4 gives column 3. Defining the resulting joint moment, i.e. M_b, on the initial tangent line (point M of fig.6.4) and rotating this line clock-wise until the joint moment intersects the actual $M - \Phi$ relationship at point M'. Taking the resulting secant stiffness, i.e. K_1 = 182600 kN.cm/rad., as a revised value of the joint's stiffness and repeating the same procedure gives column 4 in which a good correlation with the exact method (column 2) can be observed.

6.3 Comments

Due to the variation of the resisting bending moment M at each semi-rigid joint of a multistorey frame which is subjected to combination of vertical and horizontal loads, the use of the proposed method would involve a lot of hand calculation.

In addition, the proposed method requires a linear elastic analysis accounting for semi-rigid joint behaviour, such an analysis is not in general available to designers.

To overcome the above difficulties, and to permit the use of the approximate method, described earlier in chapter 5, which requires only any of the conventional fully rigid method to account for joint flexibility behaviour, the following assumptions are made:

1) All semi-rigid joints of a multistorey frame have the same moment rotation characteristics.

2) The vertical loads to which the frame are subjected are symmetric.

3) The vertical loads of a multistorey frame are applied first. The joint behaviour may be nonlinear throughout the whole loading path, in which case the joint stiffnesses should be found by trial and error.

In a practical multistorey frame it is common for only one type of semi-rigid joint to be usually utilized to join beams to columns and the vertical loads applied to each storey are often almost the same. Hence, for the sake of simplicity, the response of all semi-rigid joints of the frame can be assumed to be uniform, and a unique value of joint's stiffness K can be computed to represent the behaviour of all semi-rigid connections. Nevertheless, a frame with unsymmetrical vertical loads may still be solved by the proposed method with reduced but normally satisfactory accuracy.

4) When considering the horizontal loads, the same joint stiffness defined for vertical loads should be utilized to represent the joint behaviour, thus the joint stiffnesses are either slightly underestimated or slightly overestimated (fig.6.5). In this figure M_v indicates the bending moment of a semi-rigid joint resisting the vertical loads and the joint's stiffness can be estimated as the secant K_1 of the joint moment-rotation curve. In the proposed method, when considering the horizontal loads the joint's resisting moment might reach either $M'_{v,h}$ or $M"_{v,h}$. Utilizing an exact nonlinear analysis and assuming that loading and unloading paths in the $M - \Phi$ curve are identical, the joint's resisting moment reaches either $M1_{v,h}$ or $M2_{v,h}$ (dependent on whether the joint situated in the leeward or windward side of the frame).

This overestimation of the joint stiffnesses should not produce gross errors since, in a practical multistorey frame, the horizontal loads are usually small compared to vertical loads. However, if unloading path of the $M - \Phi$ curve is assumed, as is usual for joints in structural steelwork, to be parallel to the initial tangent of the joint's $M - \Phi$ curve the joint resisting moment moves to $M3_{v,h}$ instead of $M2_{v,h}$ if the joint is located on the windward side of the frame. Thus the joint's stiffness is underestimated by the proposed method. It should be noted here that, in a practical steel structure, there are additional beneficial effects which resist sway such as cladding including partial bracing due to infill panels. Therefore, this assumption should provide a satisfactory accuracy for design purposes.

6.4 Design Implications

In order to incorporate the nonlinear behaviour of semi-rigid joints in practical steel frames, the method mentioned in chapter 5 is modified to proceed using the following steps:

1) Calculate member end moments due to the applied vertical loads utilising a fully rigid method of analysis.

2) Calculate the initial joint stiffness K_o, i.e. the initial tangent of the moment-rotation of the joints.

3) Calculate α_s for each semi-rigid joint from eq.5.20 and then reduce the joint bending moment, which is obtained from step 1, by the computed value of α_s.

4) Take the average of the modified beam end moments, $M_{bav,o}$, obtained from step 3.

5) Define $M_{bav,o}$ on the joint's stiffness line and rotate this line about the origin O until $M_{bav,o}$ intersects the joints moment rotation curve. The resultant secant should be used as a new value of the joints stiffness.

6-a) Repeat steps (3 to 5) until the resultant $M_{bav,i}$ is located on, or is sufficiently close to, the joint stiffness line and the moment rotation curve simultaneously. The resultant value K_i should be used as K for all semi-rigid joints of the frame. The beam end moments which are obtained from the last iteration of this step should be stored as the modified beam end moments due to the applied vertical loads.

6-b) Reduce the absolute value of the column end moments at each node, obtained from step 1, by the ratio α_n where α_n is the ratio between the modified to non-modified beam end moment which is connected to the node.

6-c) The absolute value of the column end moment at the base, obtained from step 1, should be reduced by the same factor, i.e. α_n, which is used for the top end of the column.

7) Use the joint stiffness obtained from last iteration of step(6-a) to calculate C_s for each storey from eq.5.4 and reduce the beam's second moment of area at that storey by C_s and utilise a fully rigid method of analysis to calculate member end moments of the frame due to the horizontal loads only.

8) Add the member end moments obtained from step 6 to those obtained from step 7 to determine the moments in the frame.

It should be noted here that:

1) For a non-sway frame only the vertical loads need to be considered, since the horizontal loads in this case are resisted by the bracing elements alone.

2) In a practical frame where there is a severe change in beam stiffnesses, in the applied vertical loads or in the joint moment-rotation characteristics throughout the frame stories, satisfactory accuracy can be still obtained from the proposed method by dividing the

actual frame into several sub-assemblages, i.e. individual storey, pairs of stories or each bay, and employing steps(2-6) to calculate an approximate value of joint stiffness K for each sub-assemblage independently. These subassemblages should be chosen in a way such that all semi-rigid joints in a subassemblage are expected to have similar behaviour under the externally applied loads.

6.4.1 Example 6.2

The frame indicated in fig.6.6, in which the sections sizes of the elements, the loading pattern and the geometry of the frame have been arbitrarily selected by the author, is fully worked below:

The moment-rotation relationship is assumed to be tri-linear as indicated in fig.6.7. This assumption has been adopted to facilitate the use of the computer program SERVAR, in which the $M - \Phi$ relationships of semi-rigid joints are incorporated as multi-linear models, to verify the validity of the proposed method. However, the principle of the proposed method is workable with any sort of $M - \Phi$ relationship; thus the actual joint $M - \Phi$ relationships can be used in designing practical steel frames.

1) The member end moment of the frame due to the vertical loads are calculated using a fully rigid method of analysis. These values are encircled on fig.6.8

2) K_o = is evaluated from fig.6.7 as the initial stiffness given by

$K_o = \frac{1500.}{0.002} = 750000.\ kN - cm/rad.$

3) Since the beam sections are uniform α and η can be obtained for the whole frame from eqs. (5.11 and 5.17) respectively.

$$\alpha = \frac{2 \times 21000 \times 8356}{750000 \times 500} = 0.93 \quad , \quad \eta = \frac{3 \times 0.93 + 1}{3(0.93)^2 + 4 \times 0.93 + 1} = 0.518$$

In this example the semi-rigid joints should be divided, according to boundary elements, into two groups (S_1 , S_2) as indicated in fig.6.6 Calculating α_{s1} for

		α	η	ψ_s	ψ_f	α_s	$M_{bav,i}$	K_{i+1}
K_o =750000	s1	0.93	0.518	0.233	0.369	0.629	3172	190476
	s2	0.93	0.518	0.378	0.539	0.699		
K_1 =190500	s1	3.68	0.214	0.112	0.369	0.30	1564	257143
	s2	3.68	0.214	0.20	0.539	0.370		
K_2 =257100	s1	2.73	0.268	0.135	0.369	0.367	1895.8 (OK. see fig.6.7)	
	s2	2.73	0.268	0.239	0.539	0.44		

Table 6.2: Summary of the result obtained from the proposed method to estimate the joint stiffness for the frame indicated in fig.6.6

the first group of the semi rigid joints S_1, for example, from eqs.(5.18, 5.19 and 5.20) and noting that in this case ω =0. gives:

$$\psi_{s1} = \frac{\frac{8356}{500} \times 0.518}{\frac{5696 \times 2}{400} + \frac{8356}{500} \times 0.518} = 0.233$$

$$\psi_{f1} = \frac{\frac{8356}{500}}{\frac{5696 \times 2}{400} + \frac{8356}{500}} = 0.369$$

$$\alpha_{s1} = \frac{1}{1+0.93} \frac{0.233-1}{0.369-1} = 0.629$$

Repeating same argument for S_2 gives:

$$\alpha_{s2} = 0.699$$

4) $M_{bav,o}$ is computed as the reduced average moment in the frame, i.e.

$$M_{bav,o} = \frac{0.629 \times (4994 + 5481) + 0.699 \times (4189)}{3} = 3172\ kN - cm.$$

5) Plotting $M_{bav,o}$ on the joints stiffness line fig.6.7 and rotating this line about the origin O until $M_{bav,o}$ cuts the joint $M - \Phi$ curve gives : $K_1 = 190500$. kN-cm/rad.

6-a) Repeating steps 3-5 for the new joints stiffness $K_1 = 190500\ kN - cm.$ and carrying out the iteration until the resultant $M_{bav,i}$ converges. The result is summarised in table 6.2 from which it can be seen that two iterations are required. From fig.6.7 the resultant $M_{bav,2}$, which is obtained when $K_2 = 257100\ kN - cm/rad.$, is approximately located on the joint stiffness line and the joint $M - \Phi$ curve. Therefore there is no need for further iteration. It should be mentioned here that, only two iteration were required to converge $M_{bav,i}$, although in this example there is a great reduction of the joint stiffness compared to the initial tangent stiffness K_o, i.e. 65%. For other cases in which the vertical loads are lighter or the semi-rigid joints are stiffer, a single iteration might suffice.

6-b) Reducing the column end moments for the entire frame, as suggested in steps 6-b) and 6-c) gives the modified member end moments. These values are indicated in fig.6.8

In fig.6.9 the member end moments obtained from step 6 are plotted as are those obtained from an exact second order analysis for vertical loads only.

7) In order to determine the moment arising due to the horizontal loads, it is necessary first to compute the reduction factor C_s for all beams of the frame using eq.5.4 thus:

$$C_{s1} = C_{s2} = C_{s3} = \frac{1}{1 + \frac{6 \times 21000 \times 8356.}{257143. \times 500.}} = 0.1088$$

The reduced values of the beam's second moment of area of the frame is

$$I_g' = 0.1088 \times 8356. = 909.4 \quad cm.^4$$

Substitute I_g' for the actual second moment of areas and carry out a fully rigid method of analysis to calculate the bending end moments of the elements of the frame due to the horizontal loads only.

8) Adding the resultant member end moments to those obtained from step 6 and comparing them to the member end moments obtained from an exact analysis due to the actual vertical and horizontal loads applied simultaneously gives fig.6.10. The member end moments predicted by the proposed method are represented by the dotted chain lines while the member end moments predicted by the exact nonlinear analysis are represented by continuous lines. This fig-

ure shows that good accuracy has been obtained from utilizing the proposed method.

6.5 Variability of Joint Response

Predicting the behaviour of a semi-rigidly connected frame accurately requires a precise description of moment-rotation relationship for the adopted semi-rigid joints. These relationships are usually obtained from testing isolated beam-column assemblages. Therefore they might be influenced by some factors such as the test apparatus, the loading system, fabrication of test specimens etc. Although a good agreement (see ref. [36]) has been reported between the moment-rotation relationships obtained from these simple subassemblages with those obtained from full-scale frame tests, still some differences will occur from inevitable variations in material properties, constructional tolerance etc. Hence a safety factor to account for this possible variations should be employed when utilizing these relationships in a design procedure. This requires the conduct of a large number of experimental tests to estimate a safety factor to cover all the previously mentioned influential factors. Since no previous work on this aspect has been found by the author, the principal way of allowing for such effects will be outlined without proposing any value to be utilized for such purpose.

Suppose that the $M - \Phi$ relationship of a semi-rigid joint is as indicated in fig.6.11. Hence it is desirable to consider an envelope of curves which can be obtained by reducing and increasing the rotation for a specific joint's bending moment obtained from the actual curve (see fig.6.11) by a factor of variation (say 10%). Therefore when considering different aspects of the frame behaviour, the most unfavourable of these curves, i.e. C_u or C_l in fig.6.11, should be employed, e.g. (i) when considering column end moments the upper curve should be used, (ii) for calculating the frame lateral drifts in sway frames lower curve should be adopted and (iii) beam stresses should be checked against both curves.

6.6 General Discussion

The proposed method provides a convenient way in which designers should be able to predict the behaviour of such frames with ease. That is because utilising this method provides the following advantages:

1) When considering vertical loads, for a practical frame, only one analysis is required assuming the joints to be fully rigid and the resultant member bending moments can be stored as 'reference values'. Knowing the joint stiffnesses the reduced value of the reduction factor α_s can be calculated for each semi-rigid joint of the frame (see eq.5.20) and then the reference values of member bending moments can be modified accordingly. If, for some reasons, a designer decides to replace the first selection of semi-rigid joints by different joints with different characteristics, different values of α_s can be calculated and the reference values of bending moment can be modified according with the new joint's characteristics.

This feature facilitates the inclusion of joint nonlinear behaviour as mentioned earlier and it can be used to allow for variability of joint stiffnesses, as recommended in the previous section, since this will simply mean employing the proposed method twice for upper and lower curves.

2) In a rigidly connected frame, due the assumption that the joints have infinite stiffnesses over the whole loading range, the beams provide end restraints to columns until failure. Nevertheless, in a flexibly connected frame, semi-rigid joints have limited bending moment capacities (joint plastic moments). After exceeding these moments, the joints behave like perfect hinges for increasing moments. Thus if the joints fail before the occurrence of frame instability, the frame columns which are located on the leeward side of the frame behave like

cantilevers extended over the whole height of the frame which might practically mean a sudden failure or at least a great loss of the frame stiffness against lateral drifts and thus the frame undergoes large horizontal deflections due to small horizontal loads. This behaviour cannot be accepted in practical frames for serviceability reasons and designers therefore are required to check flexibly connected frames against such a failure. In circumstances where this failure is predicted, the previous selection of semi-rigid joints should be replaced by stiffer onces or alternatively a bracing system should be provided to assist the frame against lateral drift.

The proposed method can be used with ease to check against such failure. Recalling the three multi-storey frame used in example 6.2 and noting from figure 6.7 that the final beam average bending moment $M_{bav,2}$ is equal to 75% of the joint plastic moment therefore no joint failure will occur when subjecting the structure to the vertical loads only. When considering the horizontal loads, the semi-rigid joints located at the windward side of the frame are normally subjected to a reduction of their bending moments through the unloading path, hence the stiffnesses of these joints are dramatically increased. Nevertheless an increase of bending moments will occur on the joints which are located in the leeward side of the frame, therefore the resultant joint moments should be checked against the plastic moments of these joints. In this example according to figure 6.10 the resultant bending moment are once again below the joint plastic moment, i.e. the joint bending moments are less than the plastic moments of the joints which are equal to 25 kN.m as shown in figure 6.7.

It should be noted here that the proposed method slightly overestimates the bending moments for these joints, as can been seen from fig.6.10. That is due to the fact that, in the proposed method, the joint stiffnesses are overestimated when considering the horizontal loads for the joints located on the leeward side of the frame. Therefore the resultant joint bending moments obtained from the proposed method can be safely compared with the plastic bending moments of the joints.

In the author'sview the failure of semi-rigid joints should checked when the frame is subjected to vertical loads only, since in real steel frames the presence of cladding such as infill panels will inevitably assists the bare frame in resisting lateral drifts. This aspect of the behaviour of flexibly connected frame will be discussed in detail in chapter 7.

6.7 Conclusion

An approximate method which accounts for the nonlinear behaviour of flexible connections in a multistorey frame has presented in this chapter. Since the proposed method requires only a conventional fully rigid first order elastic method of analysis, it is relatively straight-forward and ideally suited for practical design office use.

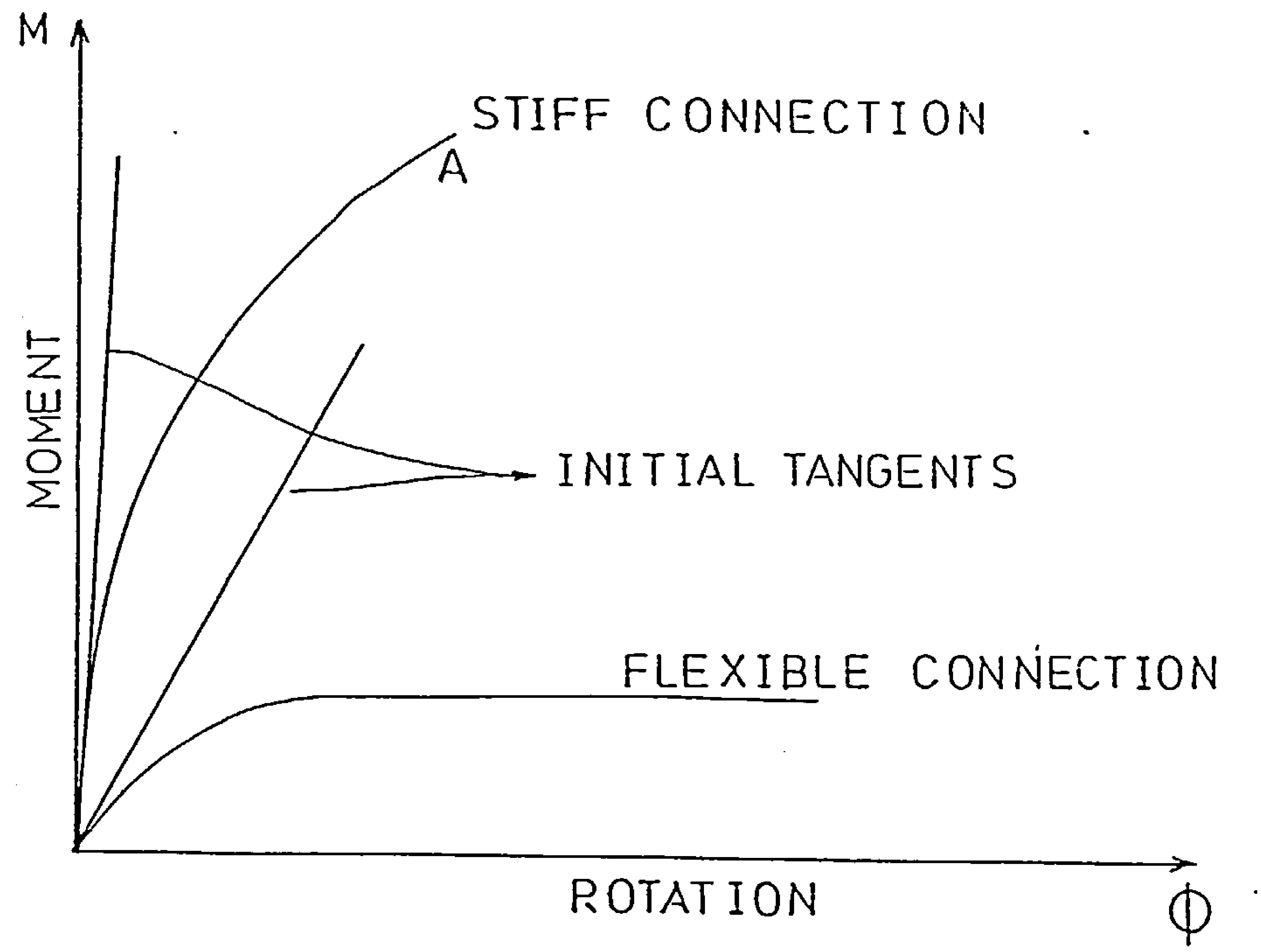


Figure 6.1: Typical moment-rotation curve for stiff and flexible semi-rigid joints.

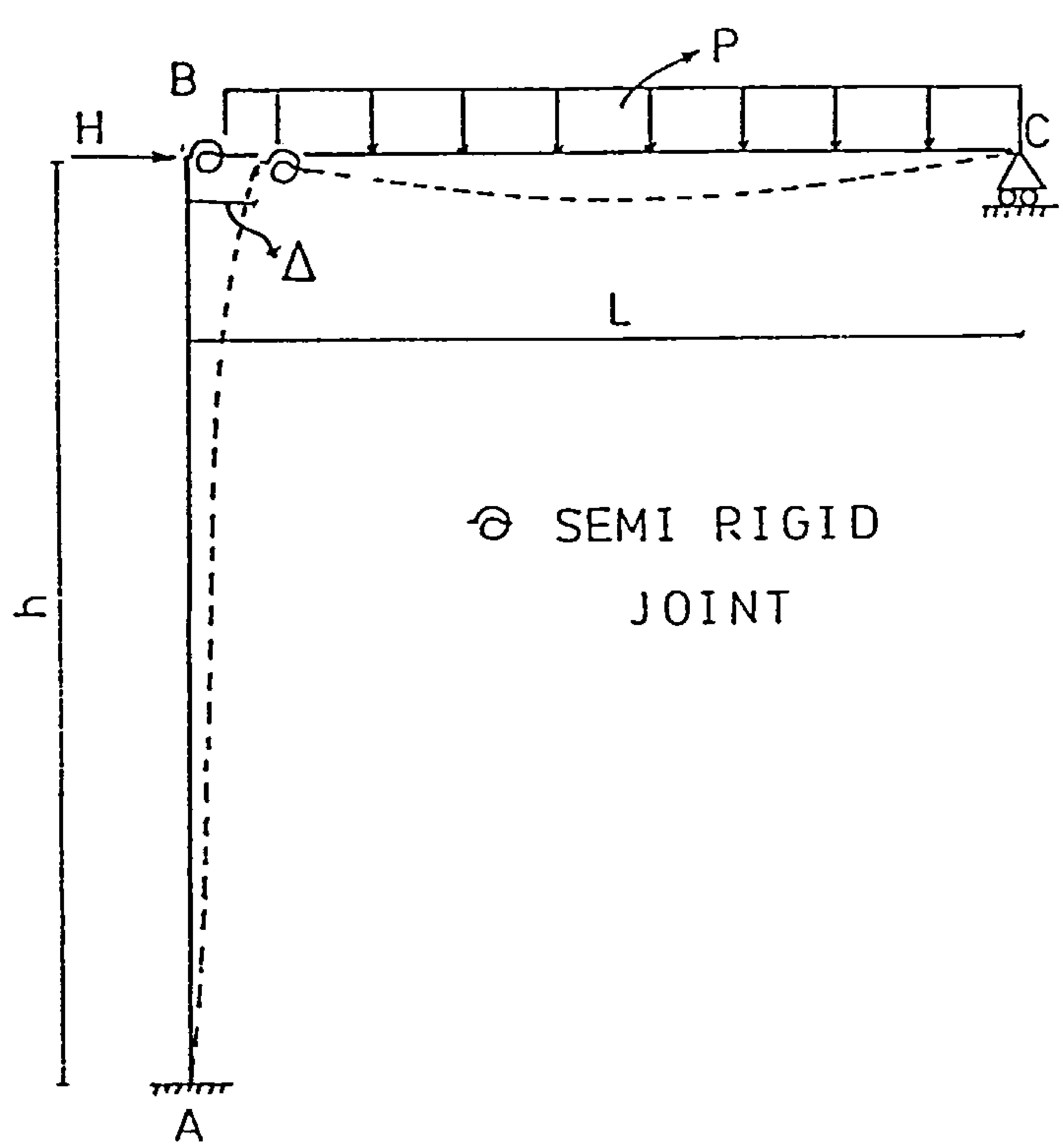


Figure 6.2: Two element frame with a semi-rigid joint.

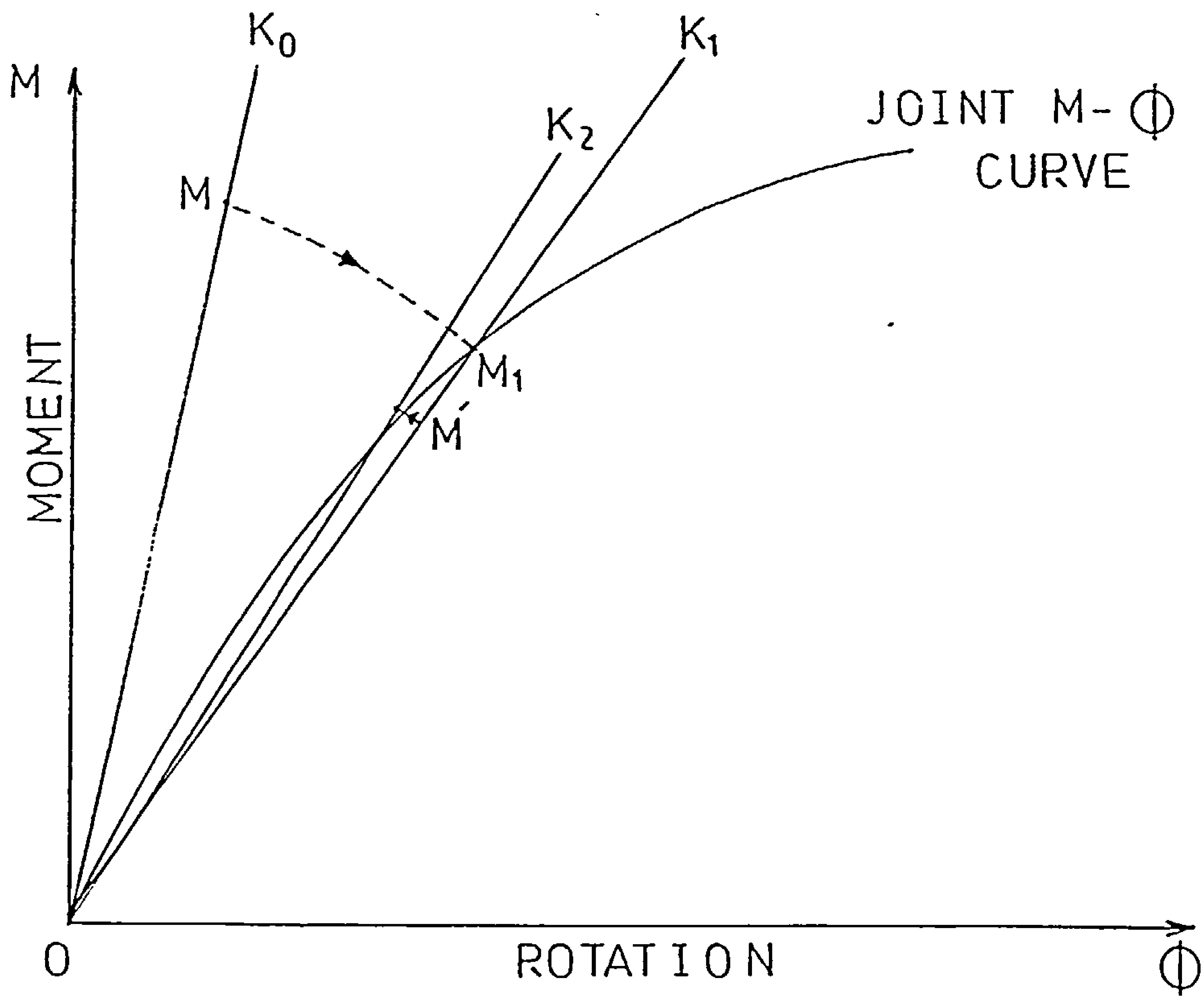


Figure 6.3: The inclusion of semi-rigid joint nonlinearity in the proposed method

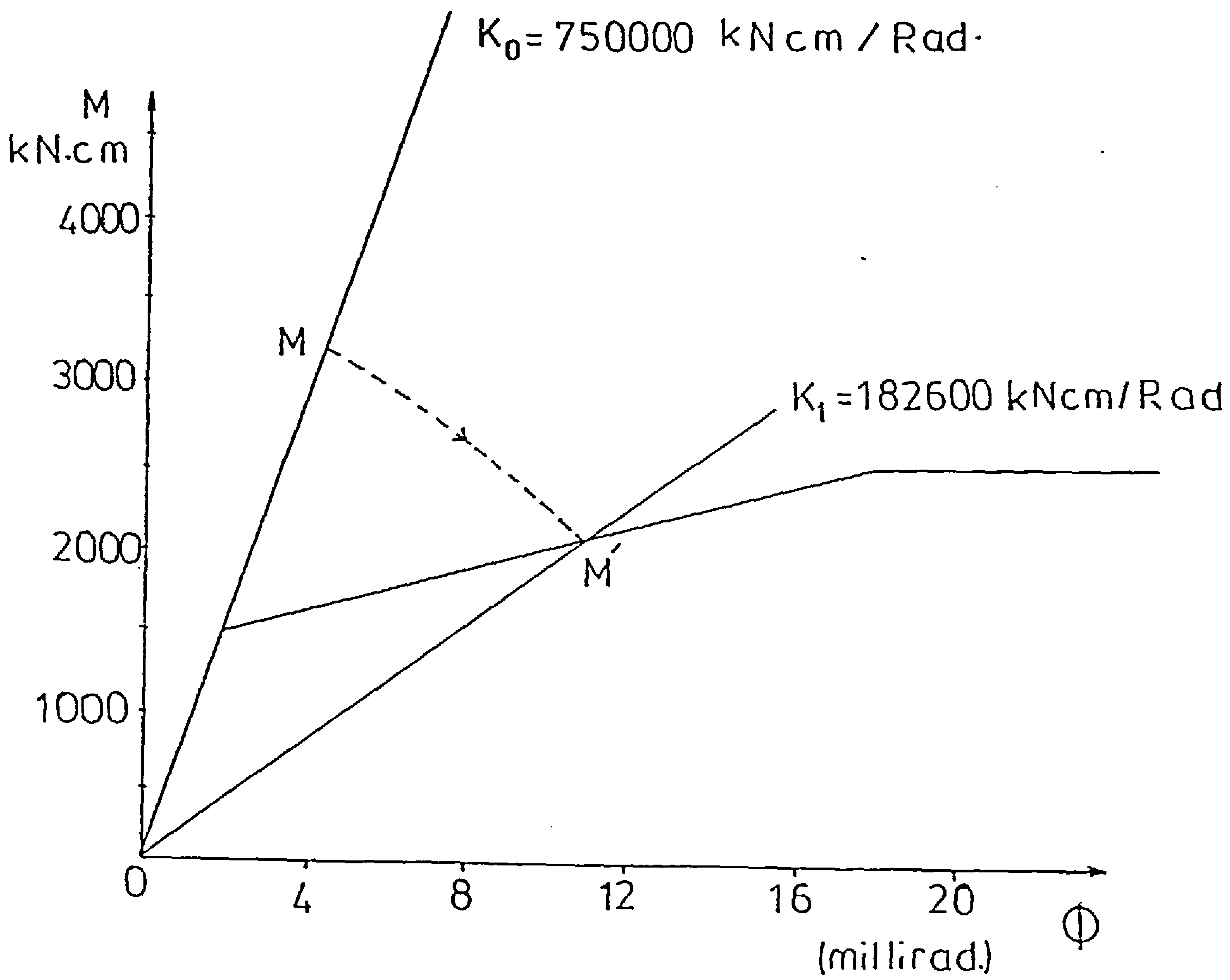


Figure 6.4: Moment rotation relationship for the semi-rigid joint used in the two element frame shown in figure 6.2.

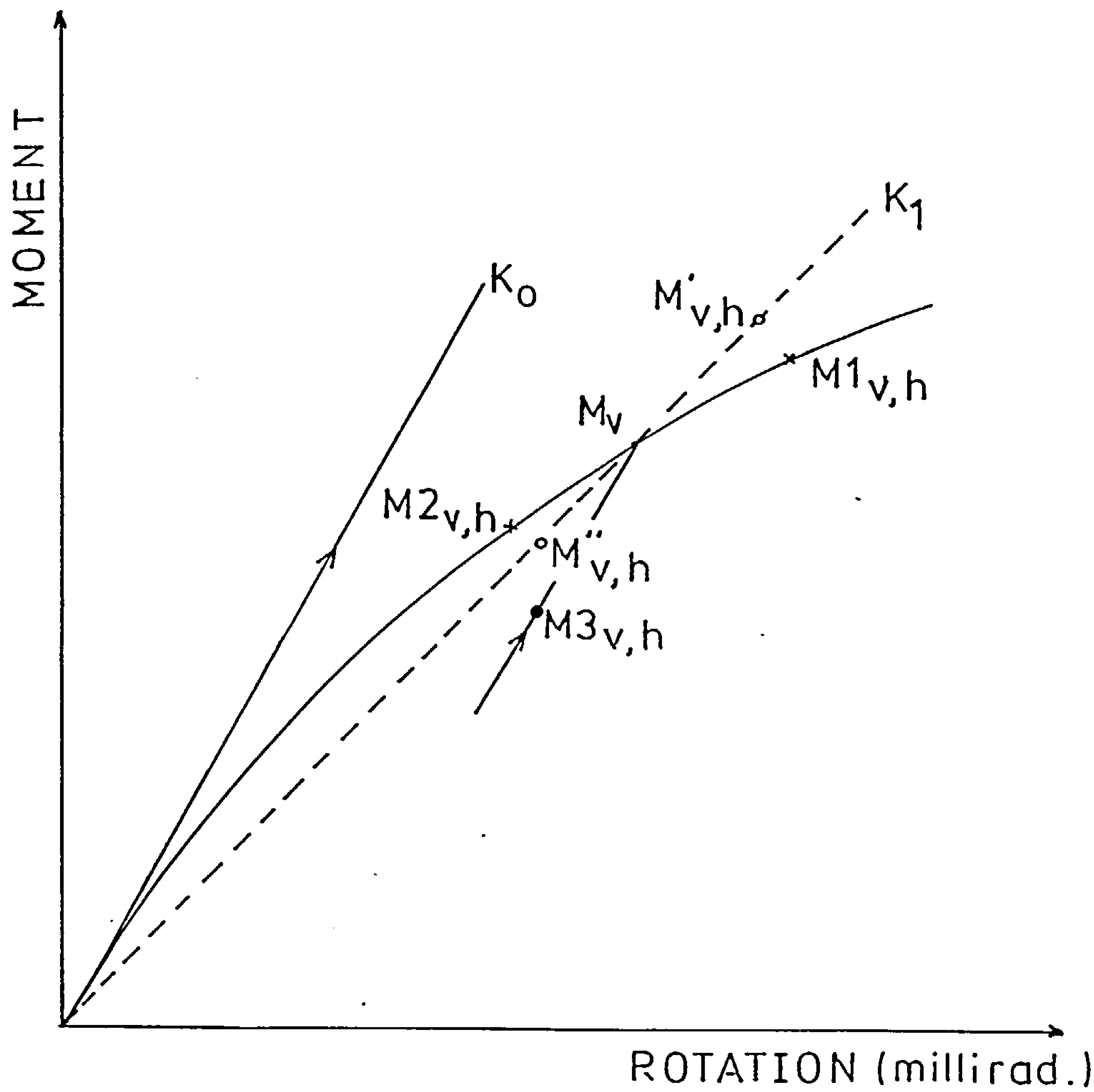
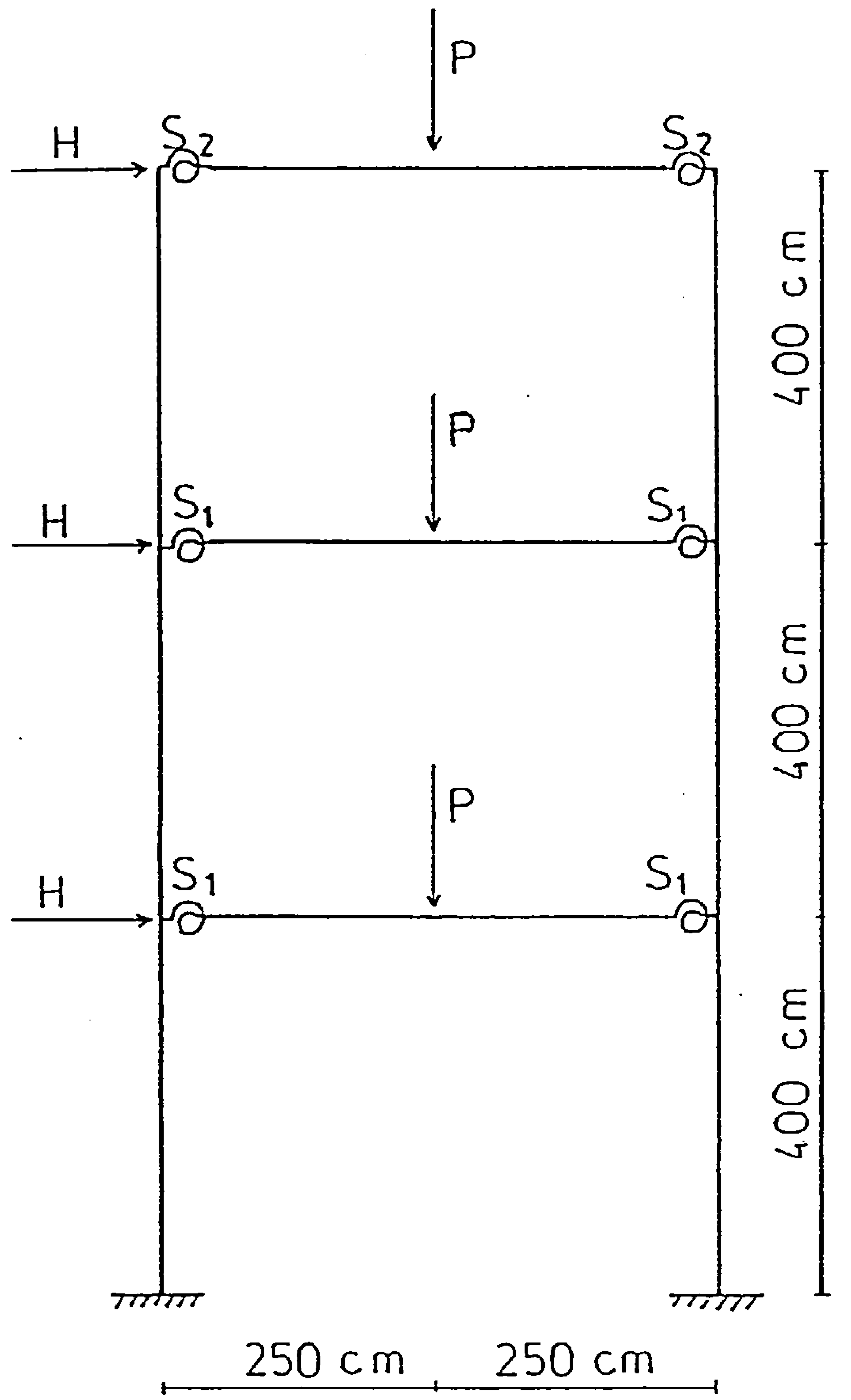


Figure 6.5: Assumption used in the proposed method to estimate a semi-rigid joint stiffness.



All columns are HE200B section

All beams are IPE300 section

H = 5 kN

P = 100 kN

Figure 6.6: The frame of example 6.2

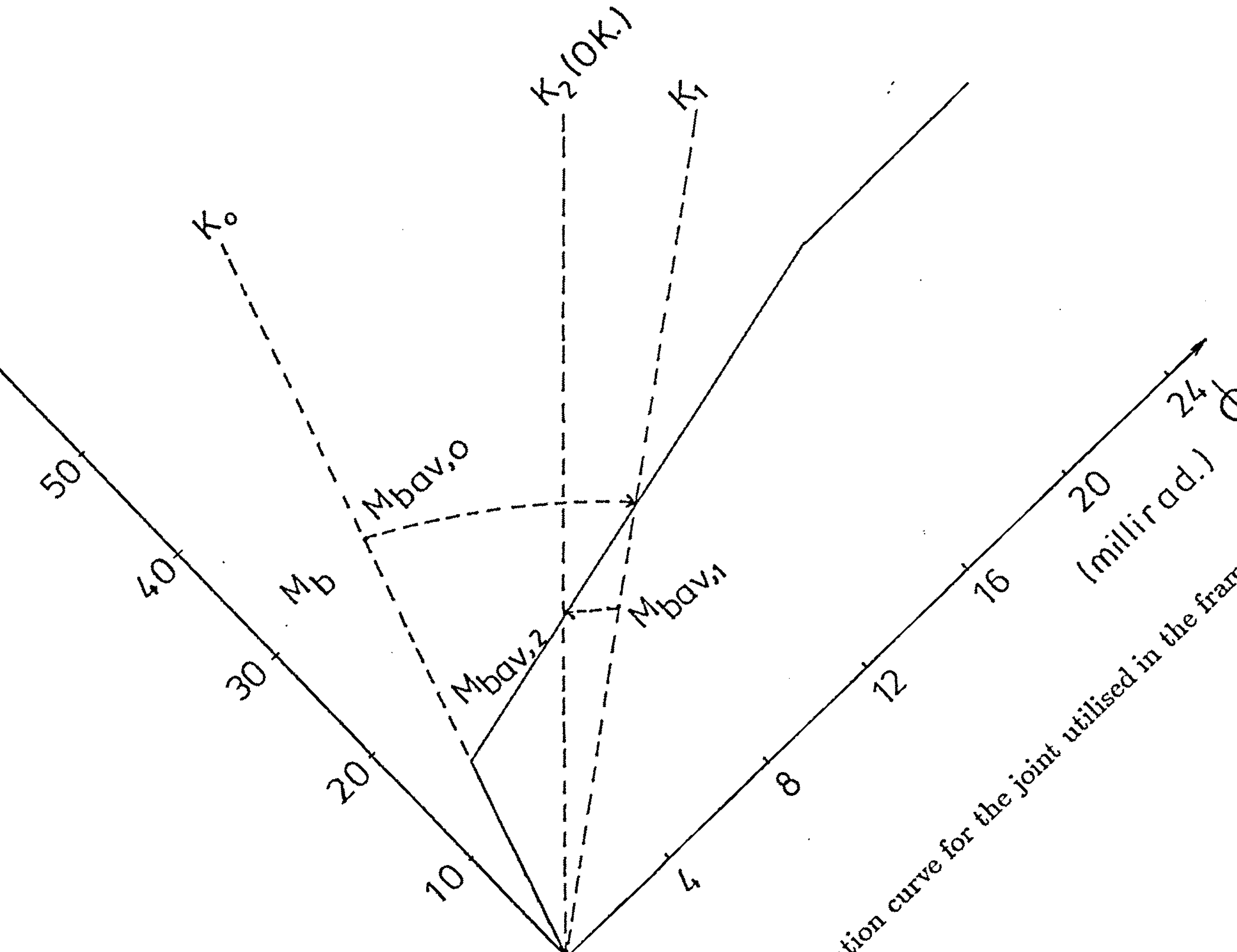
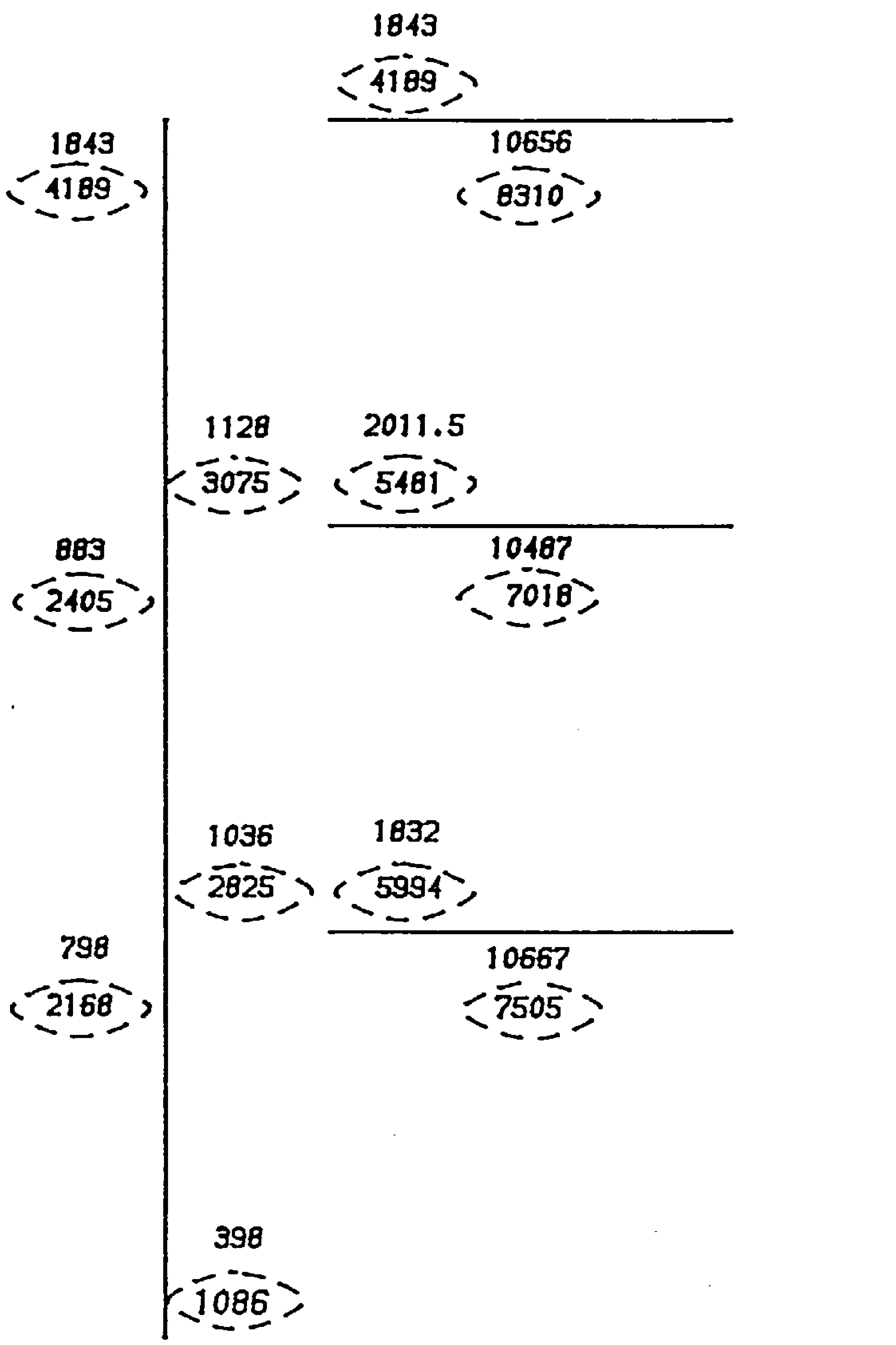


Figure 6.7: Moment rotation curve for the joint utilised in the frame
figure 6.6.



THE BENDING MOMENT
DIAGRAM IS SYMMETRICAL

VALUES ARE WRITTEN
ON THE TENSILE SIDE OF THE ELEMENTS

Figure 6.8: Comparison between bending moment values obtained from the proposed method and an exact analysis for the frame of figure 6.6 considering vertical loads only.

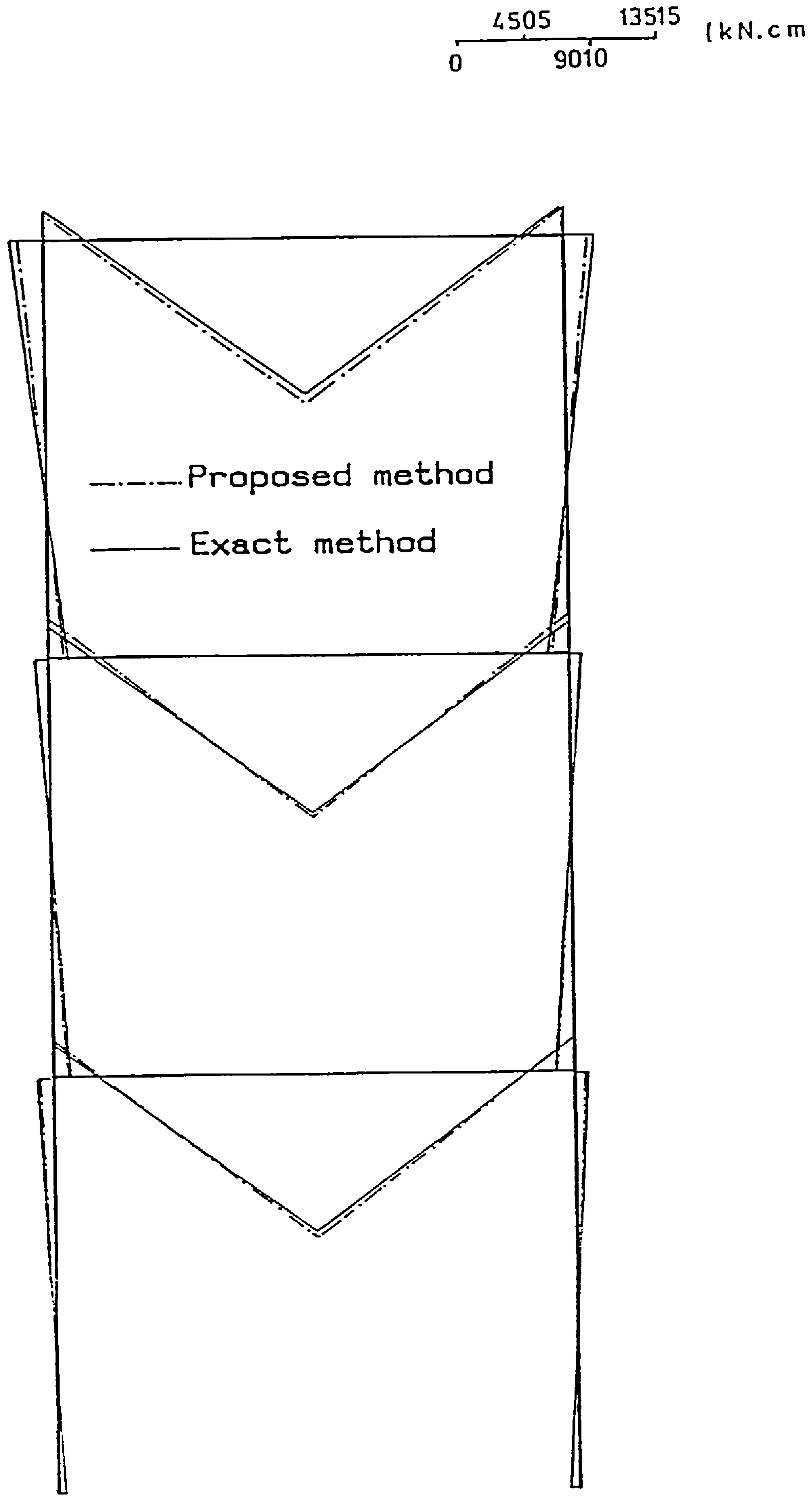


Figure 6.9: Comparison between bending moments obtained from the proposed method and an exact analysis for the frame of figure 6.6 considering vertical loads only.

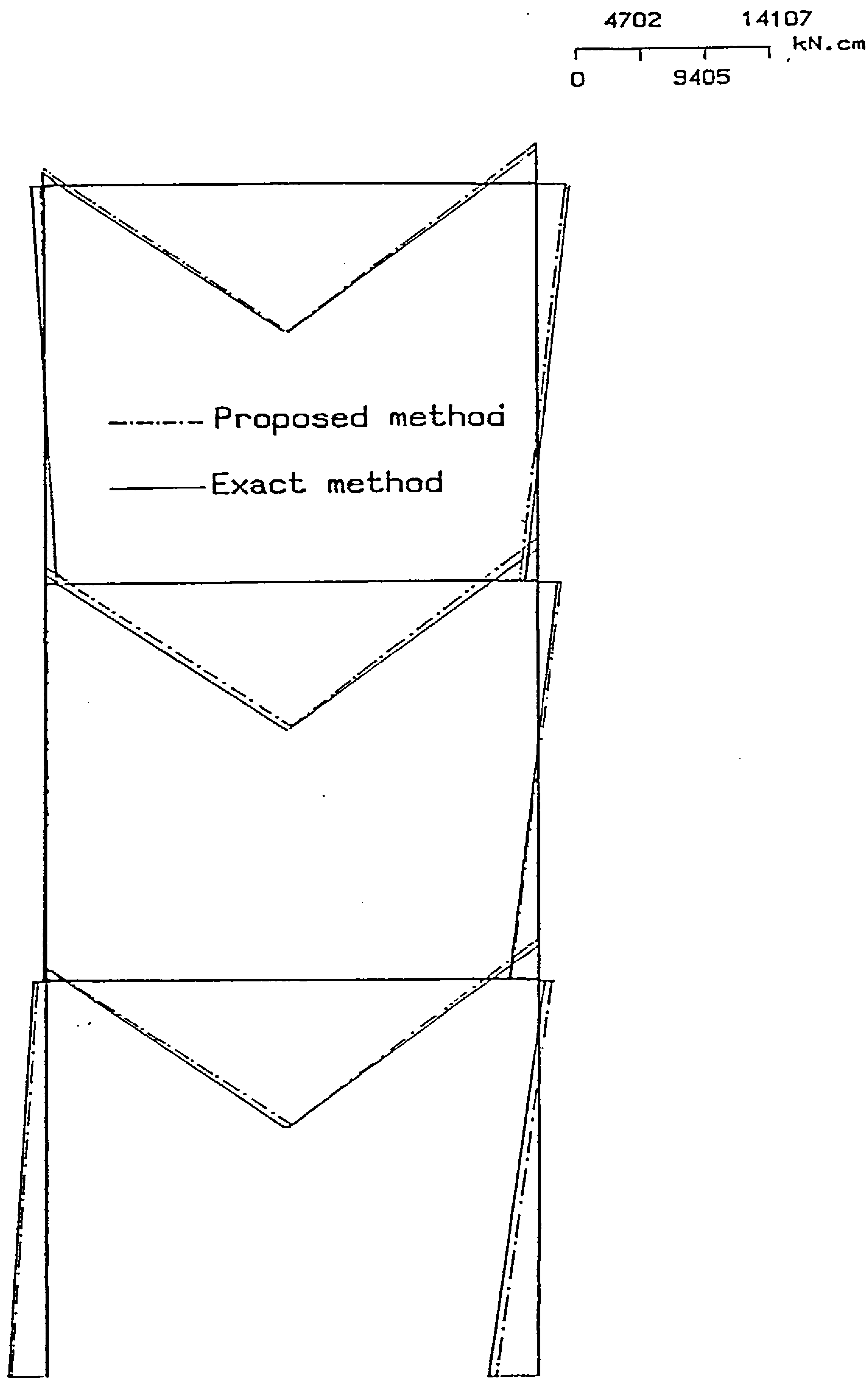


Figure 6.10: Comparison between bending moments obtained from the proposed method and an exact analysis for the frame of figure 6.6 considering both vertical and horizontal loads.

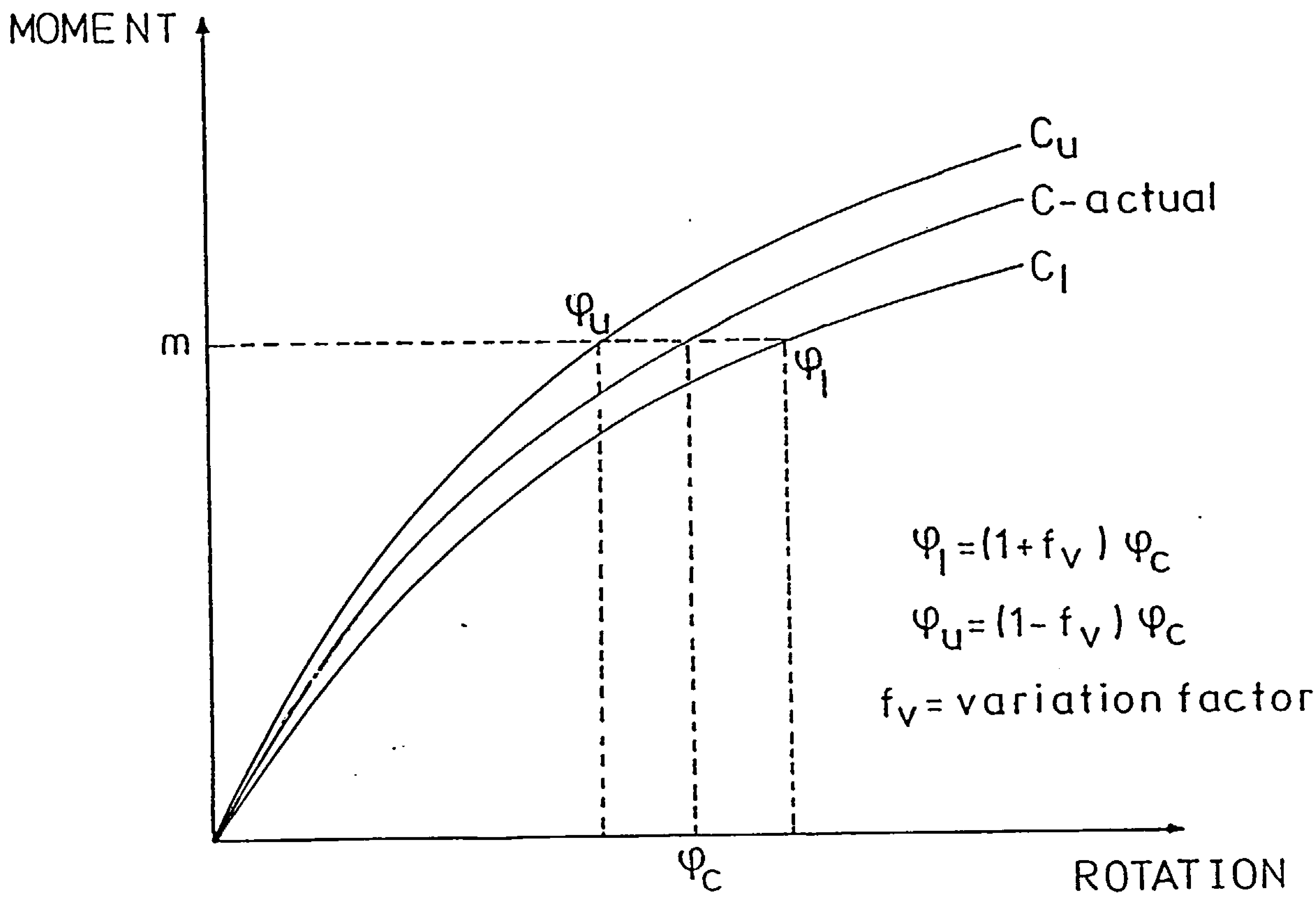


Figure 6.11: Variation of joint $M - \Phi$ relationship.

Chapter 7

Partial Sway Bracing in Semi-Rigidly Connected Steel Frames

7.1 Introduction

Much work has been conducted on stability aspects of rigid jointed steel beam and column building frames including the influence of bracing. It has been recognised that not all systems which provide resistance to racking action in building frames e.g. wall panels, are sufficiently stiff to prevent the P- Δ effects from being important, yet the presence of such features can have a considerable influence in reducing the P- Δ effect. Such systems are known as partial sway bracing. Traditionally two methods are utilised by designers to provide resistance to lateral loading in steel frames which are:

1) Bracing

2) Rigid Frame Action

These two arrangements are illustrated in fig.7.1 The use of bracing to resist sway deformation means that the structures will be stable even if the joints have no rotational stiffness and function as perfect pins. In design, it is therefore customary to adopt the concept of 'simple framing' and to neglect any beneficial effect of joint stiffness in assisting with resistance to lateral loads. On the other hand, frame action normally assumes the joints to be completely rigid, i.e. the original angle between the beams and columns remains unchanged whatever the value of moment transmitted. The practical realisation of the latter therefore entails the use of very stiff connections which, in the designers view, approximate to the fully rigid case. Almost certainly this will require heavy and therefore expensive joints employing such features as significant degree of stiffening, site welding etc.

Previous work on the effects of semi-rigid joint action on the behaviour of sway frames has suggested that once even modest degrees of joints flexibility are allowed for, the resulting structure will behave in a significantly more flexible fashion than the rigid jointed equivalent. An illustration of this is given in fig.7.2. What is not yet fully understood is the effect that a strategically positioned and comparatively modest degree of bracing would have in reducing the lateral flexibility of sway frames with less than fully rigid connections. Provisions of such bracing, either as specific structural items or by utilising the inherent stiffness of supposedly non-structural parts of the frame e.g cladding, infill panels, etc., acting in combination with reasonable levels of joint stiffness should be able to limit lateral flexibility of the structure to reasonable levels.

The computer program SERVAR [2] which represents the behaviour of flexibly connected frames utilising second order elasto-plastic analysis has been employed to conduct this study. Initial investigations were aimed at defining the range of variables to be addressed in a parametric study of the problem.

The results of some illustrative studies are presented in this chapter. The influence of partial sway bracing on flexibly connected frames is investigated. It is concluded that in some practical frames the presence of partial sway bracing compensates for the loss of frame stiffness against horizontal deflections. A simple design procedure has been developed which can be used with ease in design offices.

7.2 Ultimate Load and Sidesway of a Semi-Rigidly Connected Frame.

The main feature which occurs when using semi-rigid joints in bare steel sway structures is the undesirable increase of lateral drifts when compared to the assumption of fully rigid behaviour. Hence the serviceability condition cannot in general be satisfied and designers might find themselves compelled to use full rigid connections conflicting with the desire to keep the cost of the structure as low as possible.

However, experiments [48] have confirmed that the ability of a steel frame with infill panels to resist horizontal movements is significantly greater than the corresponding bare frame. This is due to the presence of the panels which provide additional resistance against lateral deflections.

A simple portal frame shown in fig.7.3 is used to study the effect of introducing two pinned diagonal elements (these elements represent an infill panel) into the frame behaviour. The calculations are conducted with the portal beam to column connections being

1) fully rigid joints

2) extended-end-plate joints and

3) flush-end-plate joints.

Accounting for semi-rigid connection behaviour in an analysis requires a good description of the joint moment-rotation characteristics. The moment-rotation curves for the semi-rigid connections illustrated in fig.7.4 were adopted from refs. [56] and [57] for extended end plate joints and for flush end plate joints respectively. A second order elasto-plastic procedure was utilized to analyse the bare frame without bracing and the resultant horizontal deflections are plotted against the frame load factors in fig.7.5 for the different joint types. The obvious conclusion from this figure is that, when the joint flexibility is increased the frame undergoes larger horizontal displacement but with only a slight reduction of the ultimate load factor.

In the design of framed steel structures two conditions normally govern the member selection. These are: (i) the strength of each member of the frame must not be exceeded and (ii) the sidesway at any storey level must not exceed a permissible value which typically ranges from 1/300 to 1/500 of the storey height depending on the structure type. Hence, to study the benefits which may be obtained from incorporating the presence of the infill panels of a frame, the effect of these panels on the element stresses and the horizontal displacements needs to be investigated.

However, since in a practical multistorey frame flexible joints and infill panels have only a modest effect on the frame ultimate load factor, the study will concentrate on the frame sidesway.

Work by Wood [10] has led to the only known codified formulation for partial sway bracing which is in BS5950:Part1:1985. This document contains clauses which represent a highly conservative way of allowing for the presence of panels by converting their influence into an equivalent diagonal brace before conducting a first order elastic analysis to determine frame sidesway.

To examine the effect of the presence of the bracing elements in a rigidly connected frame, two bracing elements with rectangular cross-section are included. The area of the axial elements, for this example, were increased gradually from $A = 0.0$, i.e. no bracing, to $A = 1.2\ cm^2$. Since in this type of steel structure, nonlinear geometrical deformations are small and can be neglected, BS5950 allows the use of a linear elastic analysis to calculate the lateral deflections of practical frames. A first order elasto-plastic analysis was utilized and the resultant horizontal deflections are plotted against the load factors in fig.7.6. It can be seen from these curves that incorporating very light bracing elements into the analysis produces a considerable reduction of the sidesway while the ultimate load factor is only slightly increased. However, if typical semi-rigid rather than rigid joints are assumed (fig.7.7 and fig.7.8) this effect on the frame sidesway is generally reduced, therefore stiffer bracing is required to compensate for the loss of the beam effectiveness in restricting the lateral deflection due to the presence of semi-rigid connections.

It can be concluded that the presence of infill panels in a frame causes a considerable reduction of the sidesway of the frame while the presence of semi-rigid joints tends to reduce the effectiveness of the beam in controlling lateral deflections. Thus, if semi-rigid connections are considered, the frame resistance against lateral deflection should be checked and, if the infill walls and cladding are not capable of controlling such deflection to satisfy the serviceability condition, additional bracing elements should be provided or the initially selected semi-rigid joints replaced by stiffer ones.

7.3 Sway of Flexibly-Connected Frames

It has been proved in chapter 5 that the effect of two identical spring elements with stiffness K attached to a beam in a frame subjected to horizontal loads only, can be allowed for by reducing the second moment of area of the beam by the coefficient C_s. This coefficient should be calculated from eq.5.4.

A fully rigid method of first order elastic analysis may then be employed to calculate the horizontal deflections of the frame.

7.4 Sway of Partially Braced Frames

Many practical multistorey frames contain wall panels and whilst these only slightly increase the ultimate load capacity of the frame, they will inevitably stiffen the frame against lateral displacement and it will be beneficial for designers to take advantage of this effect. Wood [10] has provided a comprehensive study of the effect of infill walls on the behaviour of rigid jointed multistorey frames and has suggested a very conservative method to incorporate such effects into conventional frame design methods. Basically, in his study, the infill panels were assumed to behave in a linearly elastic manner and a large safety factor has been employed to ensure against possible damage of the panels.

Appendices E & F of BS5950 [4] contain clauses which allow the inclusion of the stiffening effect resulting from the presence of infill panels at each storey by introducing a single diagonal axial element at that storey. The cross section of this member A is dependent upon the storey height, the storey width, the thickness of the panel and the modulus of elasticity of the panel material as indicated below.

$$A = \frac{K_3 \sum \frac{I_c}{h}}{h(h/b)} \left[1 + (h/b)^2\right]^{3/2} \tag{7.1}$$

where

$$K_3 = \frac{h^2 \sum S_p}{80E \sum \frac{I_c}{h}} \tag{7.2}$$

$$S_p = \frac{0.6(h/b)}{(1 + (h/b)^2)^2}\, t \times E_p \tag{7.3}$$

in which

h	is the storey height;
$\sum S_p$	is the sum of the spring stiffnesses (horizontal force per horizontal unit deflection)of the panels in that storey of the frame;
E	is the modulus of elasticity of steel;
$\sum \frac{I_c}{h}$	is the sum of the stiffnesses I/L of the columns in that storey of the frame;
b	is the width of the braced bay;
t	is the thickness of the panel;
E_p	is the modulus of elasticity of the panel material.
K_3	is a coefficient which effectively measures the ratio of a conservative estimate of the stiffness of the panels or bracing system to the stiffness of the columns of a storey of the frame.

BS5950 requires the following conditions to be satisfied:

- The wall panel material must be capable of resisting a diagonal compressive force.
- The wall panel must be located in the plane of the frame and extend over the full clear height of the storey.

It should be noted here that the BS5950 method uses two safety features to reduce the area of the equivalent braced element accounting for possible cracks of the infill panels during loading stages. These are

1) a safety factor of value 80 in eq.7.3.

2) restricting K_3 not to exceed 2.

7.4.1 Example 7.1

This example demonstrates the use of eq.5.4 to account for the presence of two identical semi-rigid beam to column joints idealised as linear spring elements in a practical multistorey frame. The frame shown in fig.7.9, which was designed by Kahl [58] is utilised with the following properties:

$E = 21000\ kN/cm^2$; $g = 27.14\ kN/m$; $p = 17.51\ kN/m$;

$W_1 = 25.62\ kN$; $W_2 = 12.81\ kN$

The spring stiffness for connection 1 is $K_1 = 3014.1 \times 10^3\ kN.cm/rad.$ and the spring stiffness for connection 2 is $K_2 = 2260.6 \times 10^3\ kN - cm/rad.$

7.4.1.1 The Solution

The reduced factor C_s are calculated from eq.5.4 for lower and upper beams respectively as

$$C_{sl} = \frac{1}{1 + \frac{6 \times 21000 \times 34224}{3014. \times 10^3 \times 731.}} = 0.338$$

$$C_{su} = \frac{1}{1 + \frac{6 \times 21000 \times 15282}{2260.6 \times 10^3 \times 731}} = 0.462$$

Thus the equivalent second moment of areas for the lower and upper beams are

$$I'_{gl} = 0.338 \times 34224 = 11577.70\ cm^4;$$

$$I'_{gu} = 0.462 \times 15282 = 7060.3\ cm^4.$$

Using a first order fully-rigid analysis the horizontal displacements of the frame are computed to produce column 1 in table 7.1. However, utilising a first order analysis, accounting for semi-rigid connection behaviour to calculate the horizontal displacements of the actual frame gives column 2 in table 7.1 which is virtually identical to column 1. The third column, which was obtained from utilizing a first order elastic analysis assuming fully-rigid joints, is included for comparison.

Storey	First Order Analysis		
	Modified Full-Rigid (1)	Semi-Rigid (2)	Fully-Rigid (3)
Δ Storey.1 (cm)	0.87	0.874	0.669
Δ Storey.2 (cm)	1.64	1.63	1.093

Table 7.1: Horizontal displacement of the frame indicated in fig.7.9

This demonstrates that utilizing eq.5.4 produces a very good prediction of the sidesway of semi-rigidly connected frames providing that the joint's characteristics are linearly elastic.

7.4.2 Example 7.2

In order to check the BS5950 method, recommended to include the presence of infill panels in real steel structures, the results of two tests by Riddington [48] on rigid jointed frames with infill brickwork panels were used (see fig.7.10). These experimental tests were conducted to investigate the influence of initial gaps between steel frames and infill panels on the lateral stiffness of their infill frames. Two frames were employed. Frame a) was flexible and made from light sections. Frame b) was fabricated from much heavier sections. For each frame the following four different cases were considered.

1) No infill panel.

2) Infill panel is present with a top gap equal to 3mm and both side gaps equal to 1.5mm.

3) Infill panel is present but with a top gap equal to 3mm and no side gaps.

4) Infill panel is present which is firmly attached to the steel frame (no gaps).

In all tests the frames were subjected to horizontal loads only until failure of the panel. In figures 7.11 and 7.12 the frame horizontal displacements are plotted against the load factor and it can be seen that

1) Initial gaps between the steel frames and the panel reduce the effectiveness of the panel against lateral drifts.

2) The presence of the panel enormously improved the frame stiffnesses against lateral drifts when compared to the equivalent bare frame.

7.4.2.1 The Analytical Comparison

a) In the flexible frame the members and the panel material properties are as follows:

All beam and column sections are 152 × 152 × 30 UC sections.

Panel dimension are $271 \times 271\ cm^2$, $E = 20000\ kN/cm^2$, $E_p = 1540\ kN/cm^2$, Panel thickness $t = 10\ cm$.

Four analytical predictions are undertaken.

a_0) Using a first order elasto-plastic analysis to analyse the bare frame produces curve a_0 in fig.7.11.

a_1) Eqs. 7.1-7.3, taken from BS5950, are used to calculate the equivalent braced element area A as indicated

$$S_p = \frac{0.6}{(1+1)^2} \times 10 \times 1540 = 2310\ kN/cm$$

$$K_3 = \frac{(286)^2 \times 2310}{80 \times 20000 \times 2 \times \frac{1740}{286.75}} = 9.7\ but\ \leq 2.0\ thus\ =\ 2.0$$

$$A_{K_3=2} = \frac{2.0 \times 6.06}{286}\left[1+1^2\right]^{3/2} = 0.240\ cm^2$$

Including the equivalent braced element in the analysis gives curve a_1 in fig.7.11 from which it can be seen that the BS5950 method is very conservative.

a_2) Ignoring the upper limit of 2 for K_3, i.e taking $K_3 = 9.7$, but otherwise using the advice of BS5950 and repeating the above calculation, produces curve a_2 in fig.7.11 which is once again significantly lower than the experimental curves.
a_3) Replacing the value 80 in eq.7.3 (which is an arbitrary safety factor) by 40 and repeating the computation gives curve a_3 which is now approaching the weakest of the experimental curves E_1 obtained for the infill panel with both top and side gaps.
b) The second frame (the stiff frame) consists of $406 \times 140 \times 39$ UB sections for all beam and column elements. Infill panel dimension are $248.5 \times 248.5\ cm^2$, $E = 20400\ kN/cm^2, E_p = 1540\ kN/cm^2$, the panel thickness $t = 10\ cm$.
b_0) Analysing the bare frame using a first order elasto-plastic analysis produces curve b_0 in fig.7.12.
b_1) Calculating A for this case gives:
$S_p = 2310\ kN/cm,\ K_3 = 1.35,\ A = 1.14\ cm^2$.
Updating the braced element in the analysis and repeating the calculation produces curve b_1 in fig.7.12.
Note that $K_3 \prec 2$. Therefore b_2 would be identical to b_1.
b_3) Repeating the analysis but replacing the value 80 in eq.7.3 by 40 gives curve b_2 which is now approaching the lowest of the experimental curves (E_1)
This example of the two frames shows clearly that:

1) The BS5950 method provides a safe and conservative manner for including the action of infill panels in a real rigid jointed steel frame.

2) The upper limit of 2 on K_3 appears to be unduly conservative.

3) The safety factor used in eq.7.3 (a value of 80) is large and a factor of 40 seems to represent a safe correspondence with the limited available experimental test data. However, this aspect requires more investigation.

7.4.3 Example 7.3

To study the combined effect of the presence of flexible joints and infill panels in a practical frame, the structure shown in fig.7.13 was used. This frame was designed by Anderson [61], assuming fully rigid connections, to satisfy the sway deflection limitation, i.e. the sidesway at each storey level does not exceed 1/300.

For the sake of clarity and simplicity the semi-rigid joints are assumed to be identical throughout the entire frame with a linear moment rotation behaviour. The joint's stiffness is taken identical to that of connection 2 mentioned in example 7.1, i.e. $K = 2260.6 \times 10^3\ kN - cm/rad.$

Since the frame has two bays, it is beneficial to use a Grinter substitute frame which has, in each storey, one column equal in stiffness to the total of the column stiffnesses and also one beam of stiffness equal to three times the total of the beam stiffnesses in the real frame. The effect of fixity at the base can be allowed for by introducing a ground beam with a very high stiffness (see fig.7.14). Calculating the reduction factor C_s for the beams in each storey of the frame from eq.4.1 gives:

$$C_{s1} = C_{s2} = \frac{1}{1 + \frac{6\text{x}21000\text{x}18626}{2260\text{x}10^3\text{x}600}} = 0.366;$$

$$C_{s3} = \frac{1}{1 + \frac{6\text{x}21000\text{x}12091}{2260\text{x}10^3\text{x}600}} = 0.47;$$

$$C_{s4} = C_{s5} = \frac{1}{1 + \frac{6\text{x}21000\text{x}7162}{2260\text{x}10^3\text{x}600}} = 0.60;$$

$$C_{s6} = \frac{1}{1 + \frac{6\text{x}21000\text{x}4439}{2260\text{x}10^3\text{x}600}} = 0.708$$

Reducing the second moments of area of the beams in the actual frame by the appropriate C_s factor and calculating the element stiffnesses gives the substitute frame of fig.7.14. Employing any of the conventional fully-rigid methods to

	Semi-rigid				Fully Rigid
	without infill panel		with infill panel		without infill
storey	substitute (1)	exact (2)	substitute (3)	exact (4)	exact (5)
1	$\frac{1}{302}$	$\frac{1}{296}$	$\frac{1}{422}$	$\frac{1}{422}$	$\frac{1}{441}$
2	$\frac{1}{190}$	$\frac{1}{189}$	$\frac{1}{304}$	$\frac{1}{299}$	$\frac{1}{347}$
3	$\frac{1}{185}$	$\frac{1}{184}$	$\frac{1}{313}$	$\frac{1}{310}$	$\frac{1}{318}$
4	$\frac{1}{216}$	$\frac{1}{216}$	$\frac{1}{382}$	$\frac{1}{379}$	$\frac{1}{341}$
5	$\frac{1}{276}$	$\frac{1}{272}$	$\frac{1}{463}$	$\frac{1}{452}$	$\frac{1}{382}$
6	$\frac{1}{559}$	$\frac{1}{552}$	$\frac{1}{986}$	$\frac{1}{938}$	$\frac{1}{798}$

Table 7.2: Sway indices of the frame shown in fig.7.13

calculate the sway indices gives column 1 in table 7.2.

However, using a computer program which represents a first order analysis accounting for the semi-rigid joints as elastic springs and calculating the sway indices for the full frame gives column 2 in table 7.2. It can be concluded that:

1) Using eq.5.4 in conjunction with the Grinter frame produces good predictions of sidesways of a flexibly connected frame and

2) The use of the semi-rigid joints causes, in this example, the frame to undergo sidesways which are larger than the permissible.

Consider the presence of blockwork infill panels, which are assumed to be throughout the full height but over one bay of the frame only. Taking the panel thickness, t, equal 15 cm and E_p as 700 kN/cm^2. These values are arbitrarily chosen by the author. Computing the cross-section of the equivalent bracing element for the first and the second stories, for instance, gives:

$$S_{p(1,2)} = \frac{0.6(375/600)}{(1+(375/600)^2)^2} 15.0 \times 700.0 = 2036.1 \; kN/cm$$

$$K_{3(1,2)} = \frac{(375)^2 \times 2036.1}{80 \times 21000 \times 107.3} = 1.59$$

$$A(1,2) = \frac{1.59 \times 107.3}{375(375/600)} \left[1 + (375/600)^2\right]^{3/2} = 1.193 \; cm^2$$

Repeating for the rest of the frame storeys gives:

$A_{(3,4)} \; = \; 0.99 \; cm^2$ and $A_{(5,6)} \; = \; 0.568 \; cm^2$

Including the equivalent bracing element for each storey of both the substitute (fig.7.14b) and the actual frame fig.7.13 and using a first order analysis to obtain the frame sidesways produces columns 3 and 4 respectively in table 7.2.

It can shown here that including the stiffening effect due to an infill panel extending over one bay of the frame (see columns 3, 4 and 5 of table 7.2), compensates for the loss of the beam stiffnesses due to the presence of semi-rigid connections. Hence, the sidesway limitation is satisfied without the need to incorporate discrete bracing elements.

7.5 Conclusion and Discussion

It has been shown that the inclusion of the effect of semi-rigid joints in estimating the sidesways of a multi-storey frame can be achieved provided that:

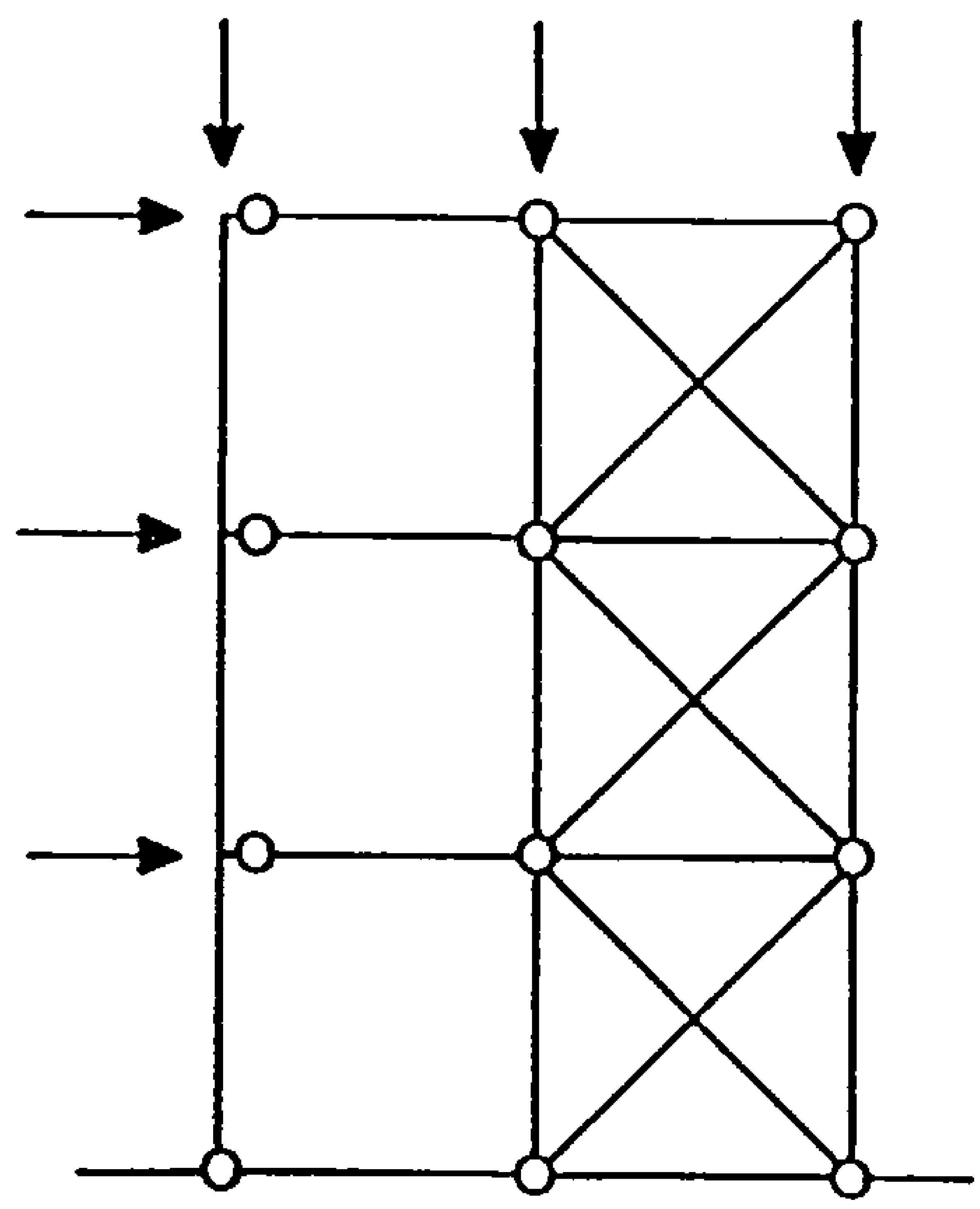
1) The semi-rigid joints behaviour can be assumed to be approximately linear. It is well known that the behaviour of a semi-rigid joint is, in general, curvilinear as indicated in fig.7.15. Therefore if vertical and horizontal loads are considered, and if the loading and unloading paths are assumed to be identical in the joint's moment-rotation relationship, then the joint stiffnesses K should be calculated with reference to the moment due to the applied vertical loads, i.e. if the bending moment is assumed to reach position M_v in fig.7.15 due to the vertical loads then when applying the horizontal loads the joint bending moment might reach either $M_{v,h}$ or $M'_{v,h}$. Hence the secant $M_{v,h} \; - \; M'_{v,h}$ which approximates the tangent stiffness at M_v, should be used to represent the semi-rigid joint

stiffness when considering the horizontal deflections due to the horizontal loads. In real steel structures however, the unloading path of $M - \Phi$ relationship of a semi-rigid joint is normally assumed to be parallel to the initial tangent of the relationship. Therefore the secant $M_{v,h} - M'v, h$ represents a safe underestimate of the joint stiffness if this joint is located on the windward side of the frame. A hand calculation method has been suggested in chapter 6 to predict the stiffnesses of semi-rigid joints in a practical multistorey frame subjected to vertical and horizontal loads including the nonlinear behaviour of the joints.

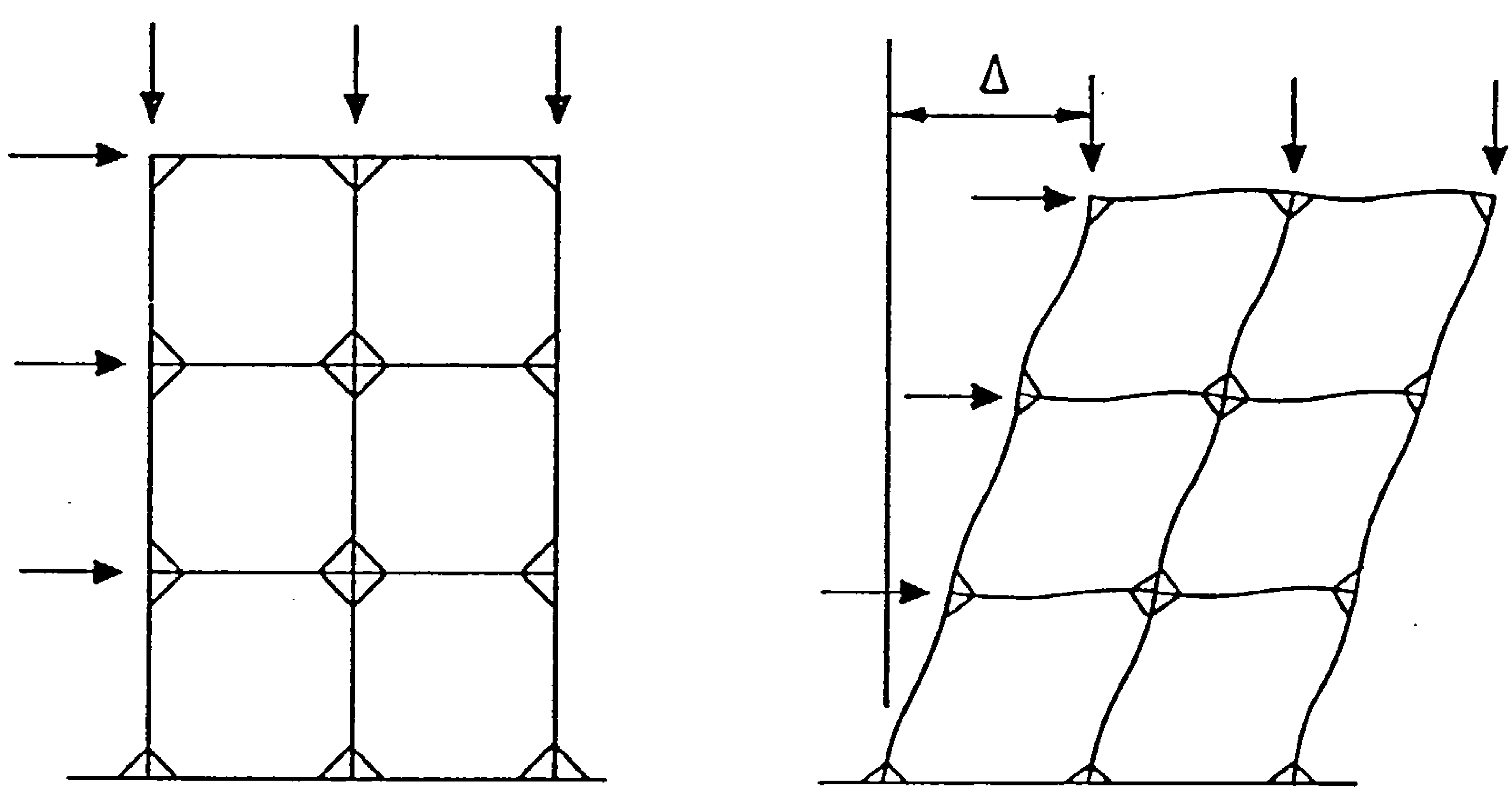
2) Each beam of the frame is assumed to have identical semi-rigid joints attached to its both ends. However in multibay frames, the beams in the external bays might have slightly different characteristic for their two end joints. Under such circumstances it is suggested that the average value of the joint stiffnesses of each beam should be used in conjunction with eq.5.4 to calculate the reduced factor C_s of that beam.

3) Although accounting for the presence of infill panels in a practical frame is seen to be relatively simple, it has to be recognized that the implementation of the BS5950 method, which is summarized in eqs 7.1-7.3, grossly underestimates the beneficial effect which may be gained due to the presence of the infill walls. Moreover, restricting K_3 in eq.7.3 not to exceed 2 causes some identical infill panels to provide different resistances against lateral deflections, e.g. $A_{(5,6)} \prec A_{(3,4)} \prec A_{(1,2)}$ in the multistorey frame due to the variation in column stiffnesses.

It should noted that the BS5950 method is in its current form due to uncertainty regarding the precise influence of real infill panels in actual structures. With increasing knowledge this method may be revised to permit an even more advantageous allowance to be included in design.



(a) Simple connections



(b) Rigid connections

Figure 7.1: Braced and rigid jointed frames.

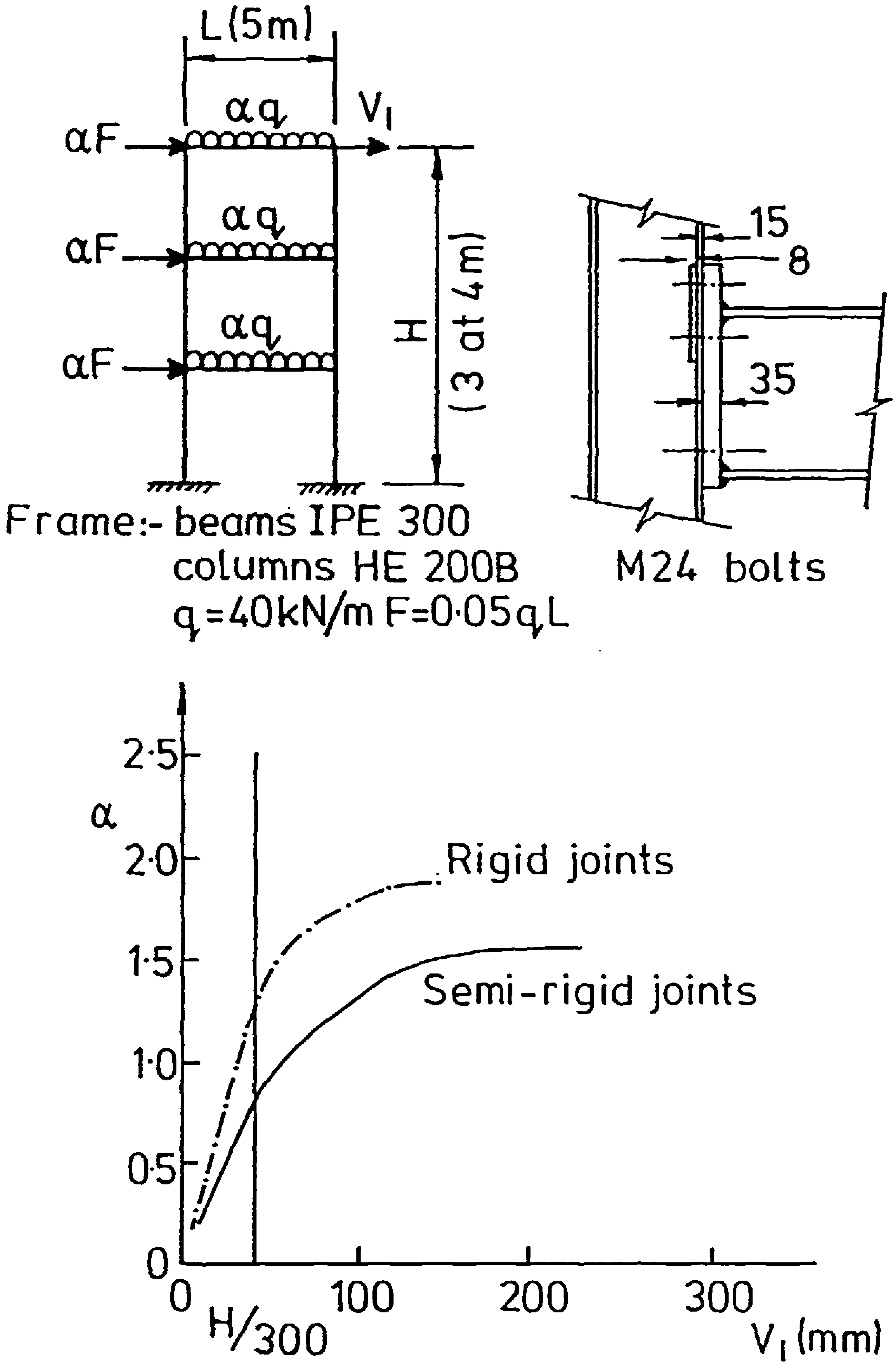


Figure 7.2: Comparison of rigid and flexibly jointed frames.

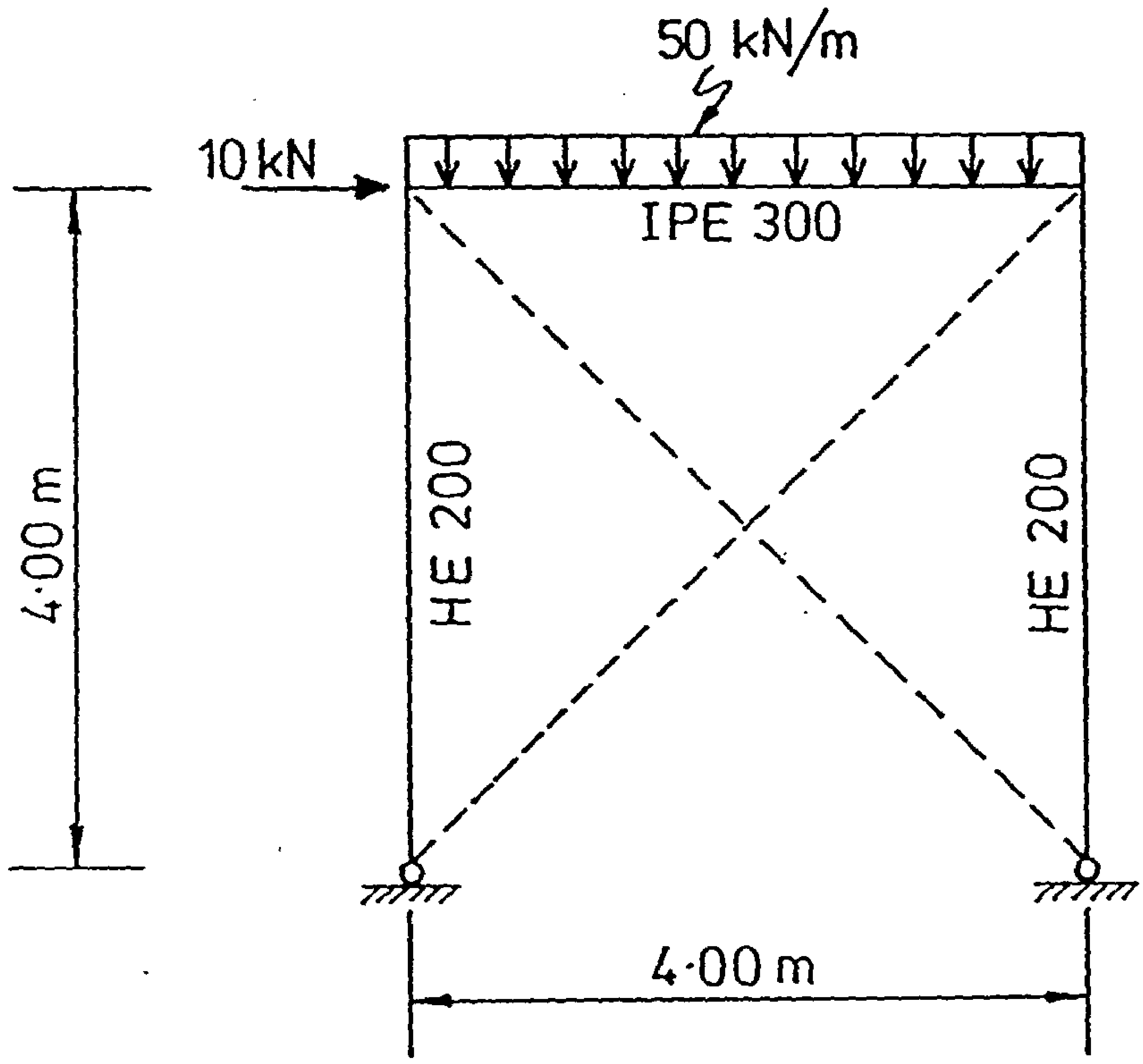


Figure 7.3: The frame of example 7.1

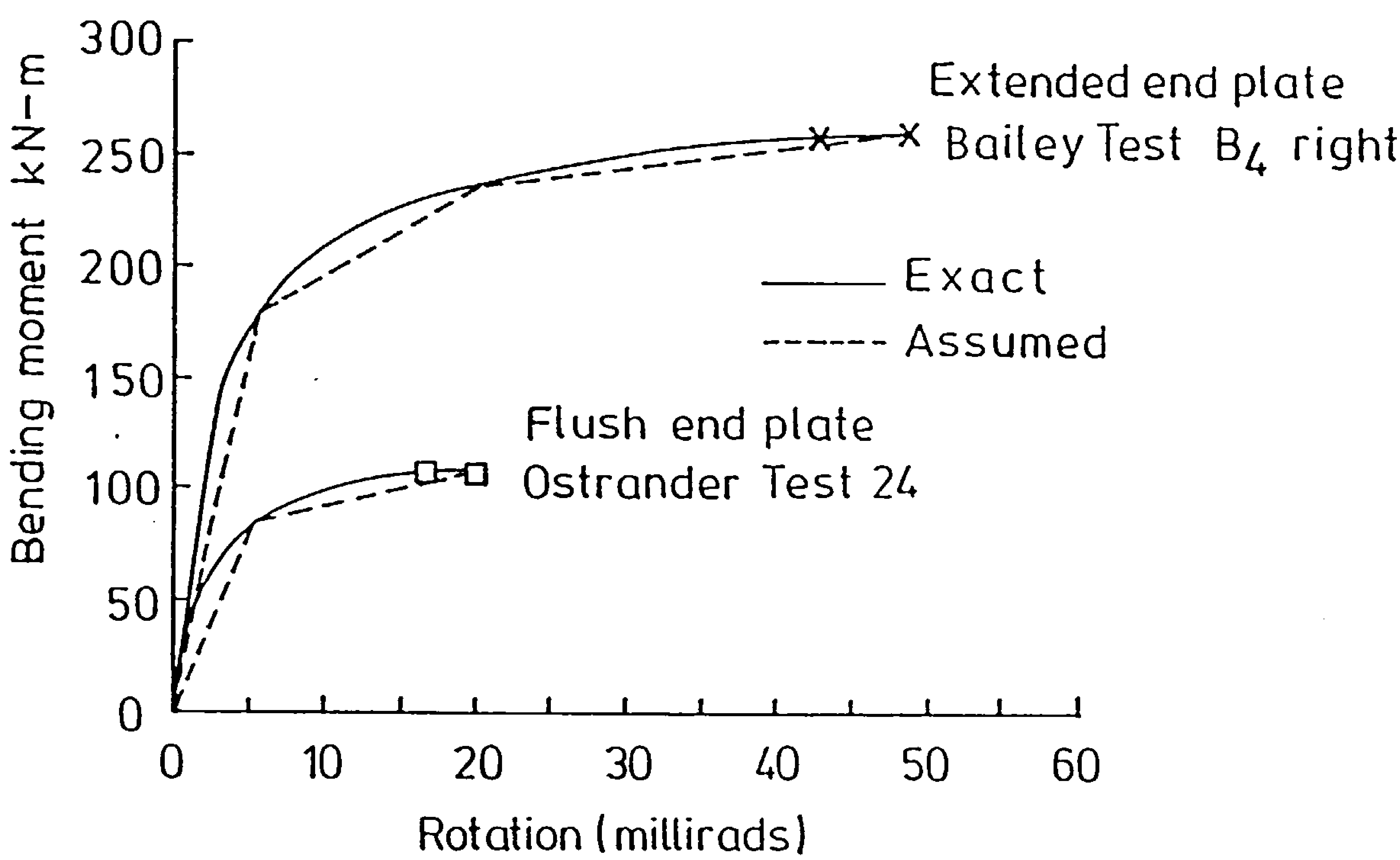


Figure 7.4: Moment rotation curves for semi-rigid joints used in the frame of figure 7.3.

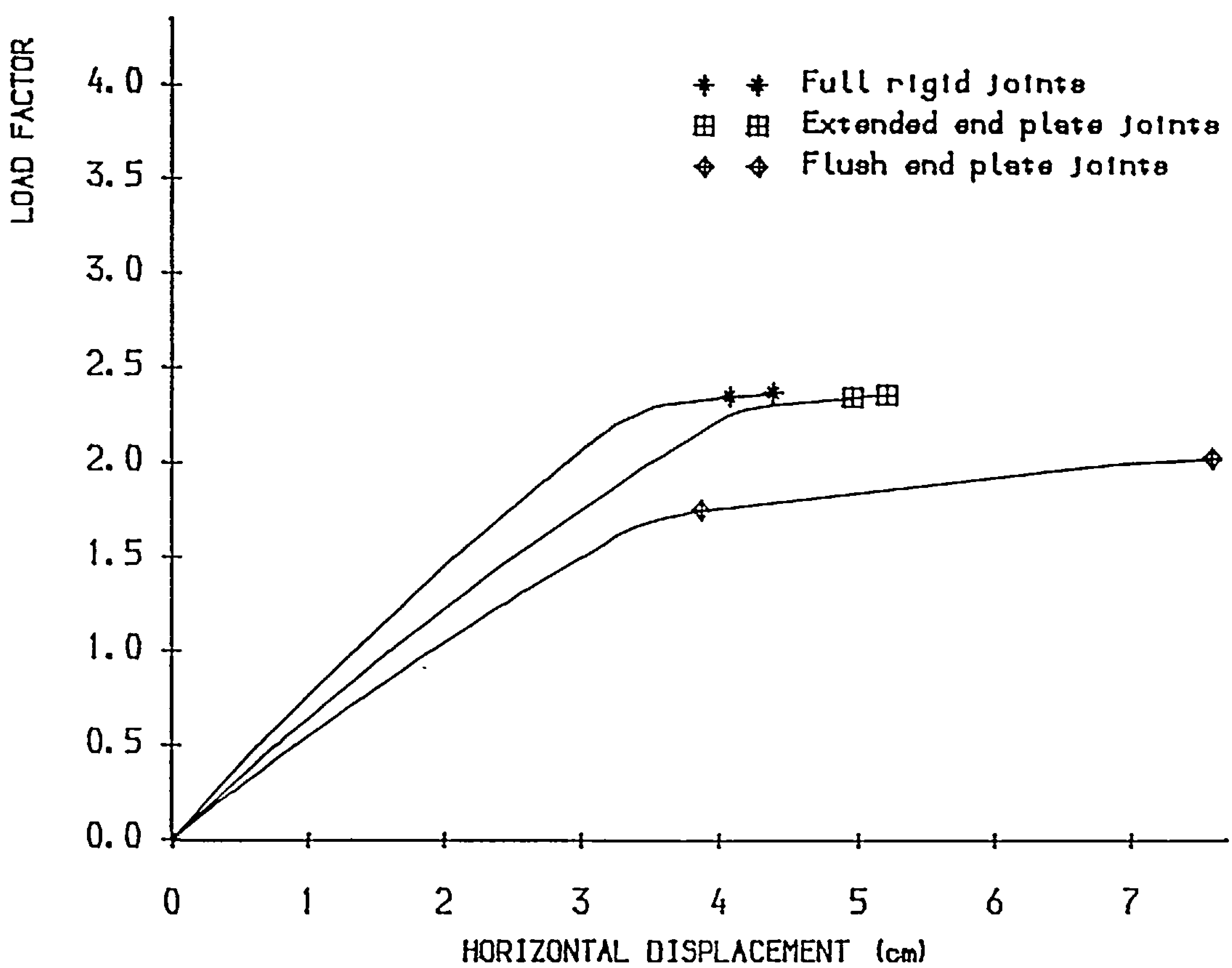


Figure 7.5: Horizontal displacements for the frame of figure 7.3 when unbraced.

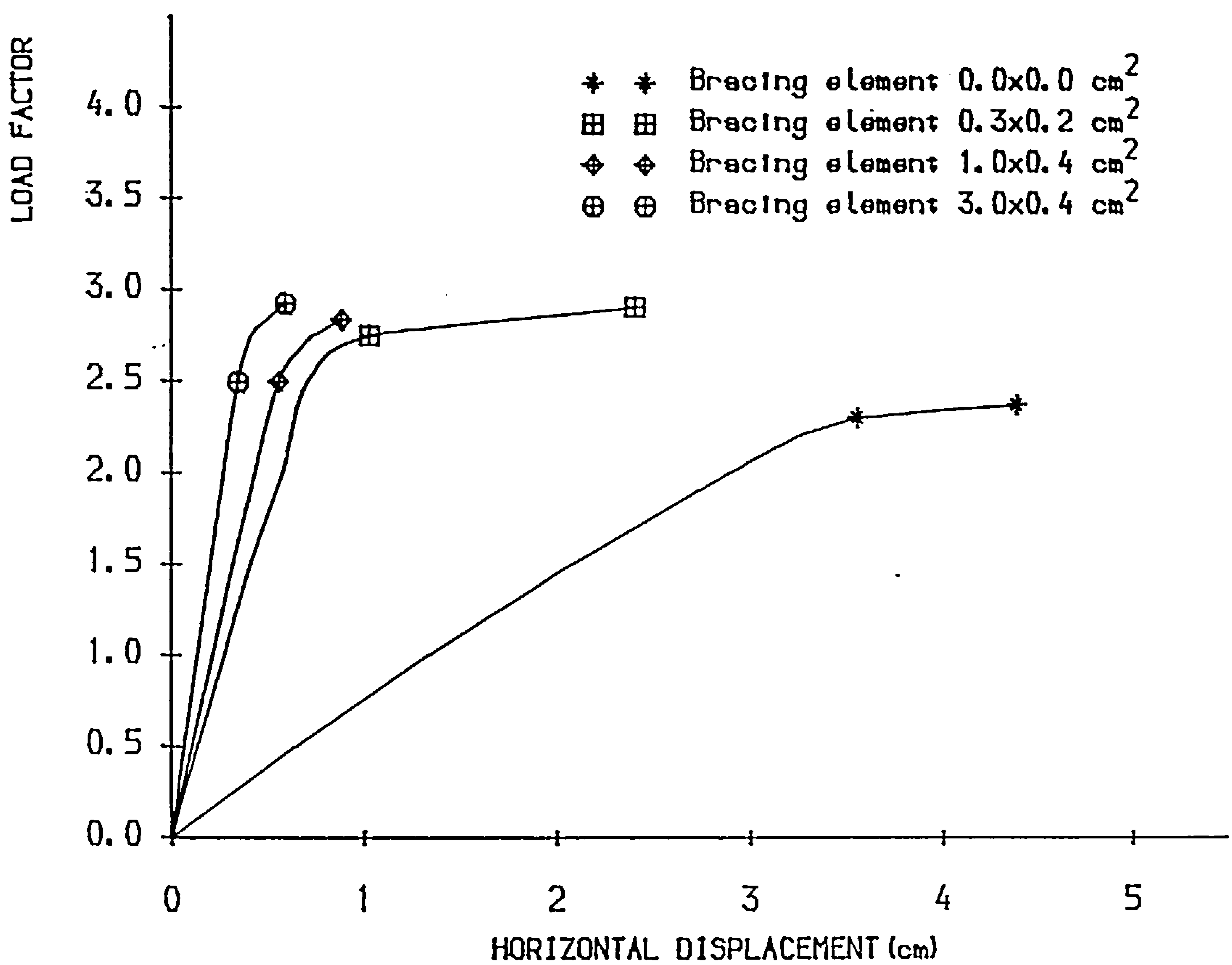


Figure 7.6: Horizontal displacement for the frame of figure 7.3 with rigid joints for varying bracing elements.

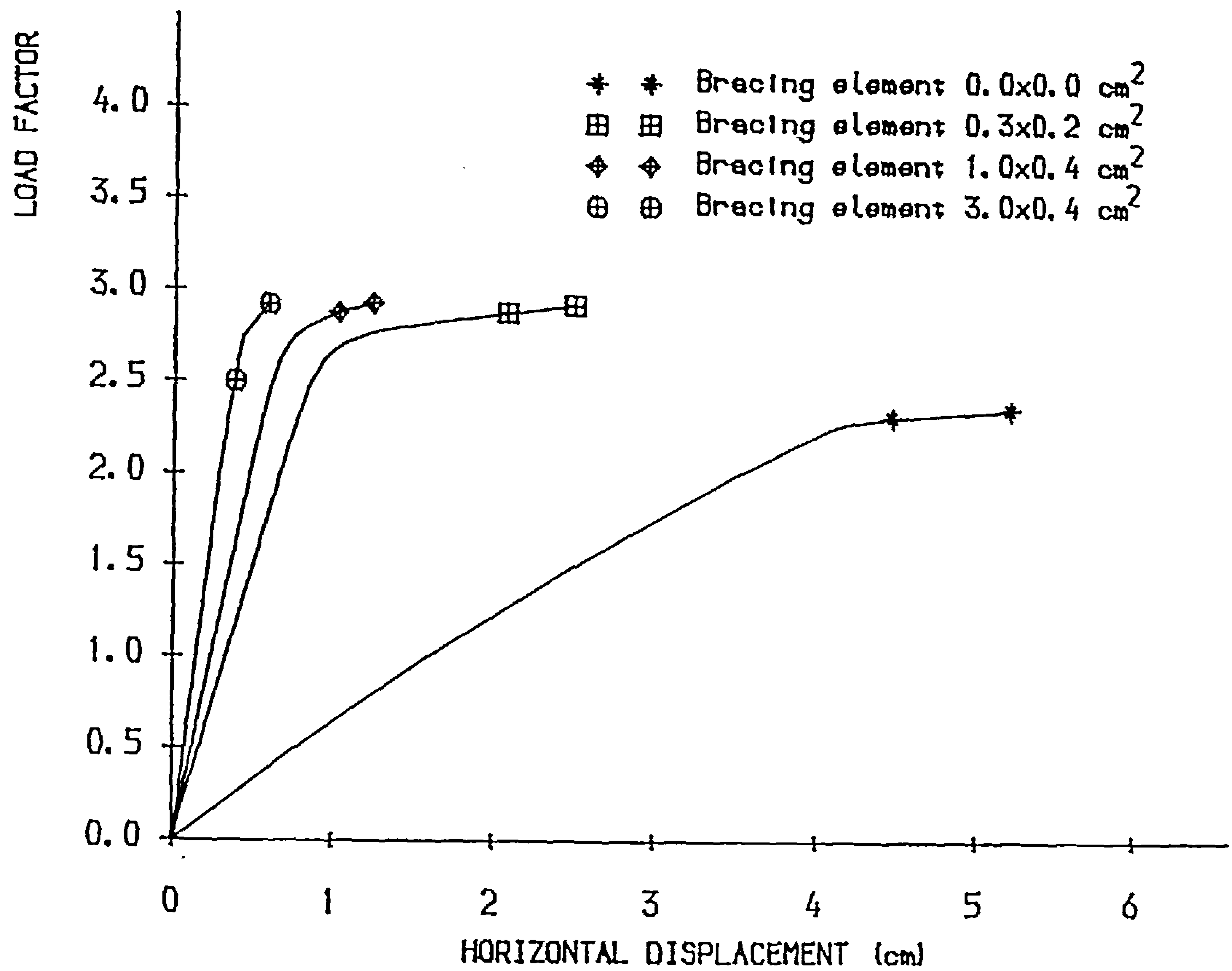


Figure 7.7: Horizontal displacement for the frame of figure 7.3 with extended end plate joints for varying bracing elements.

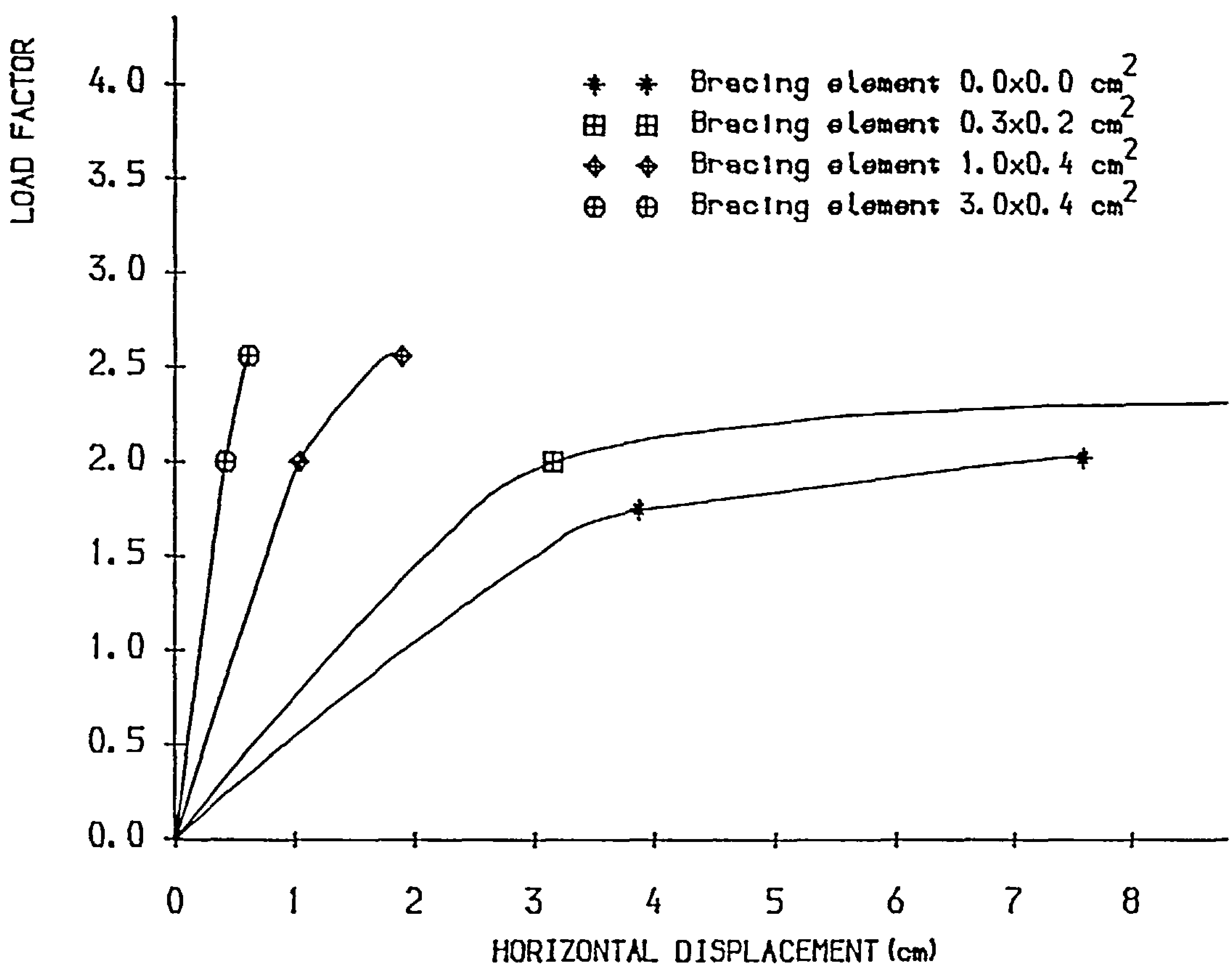


Figure 7.8: Horizontal displacements for the frame of figure 7.3 with flush end plate joints for varying bracing elements.

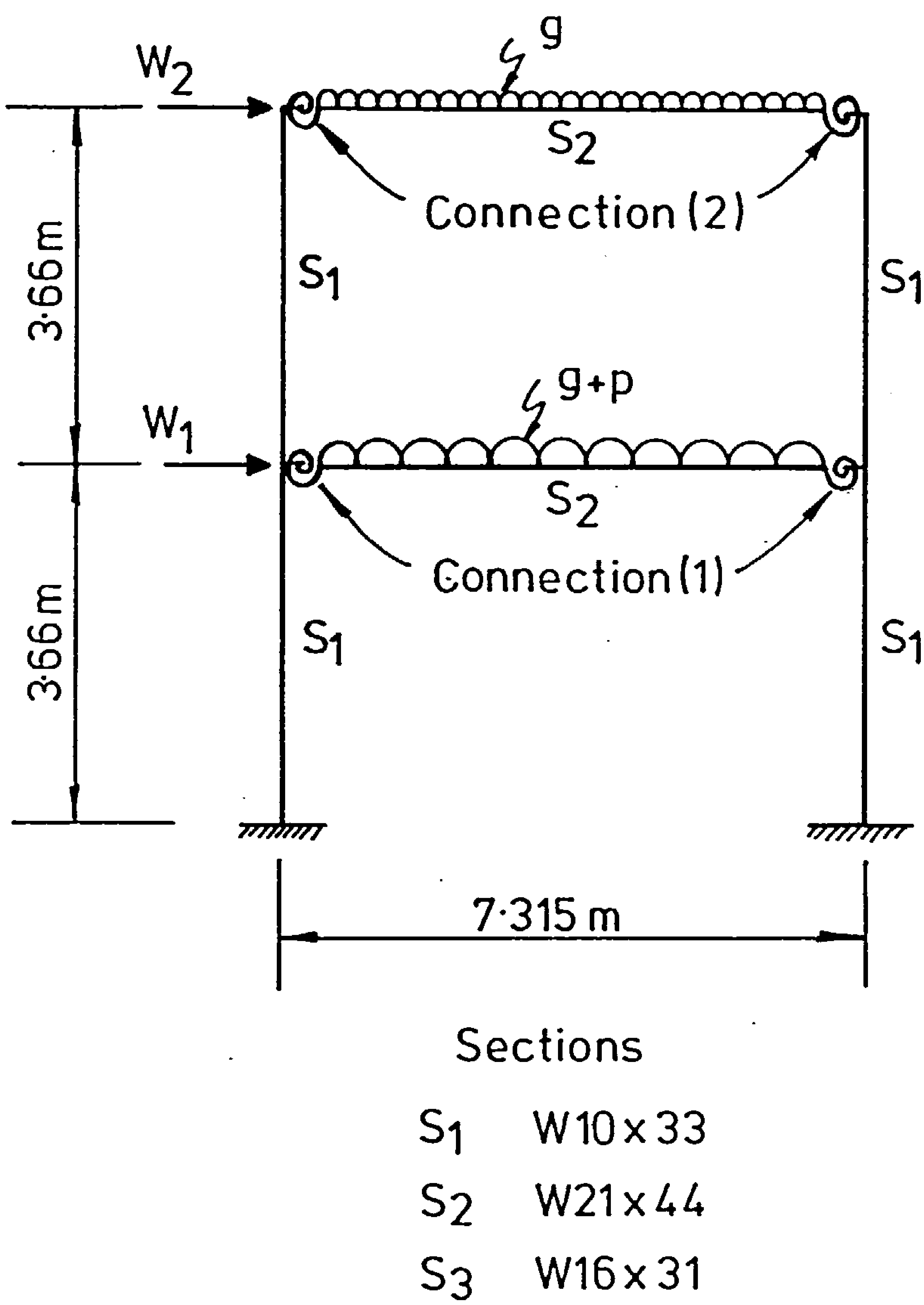


Figure 7.9: The frame of example 7.2.

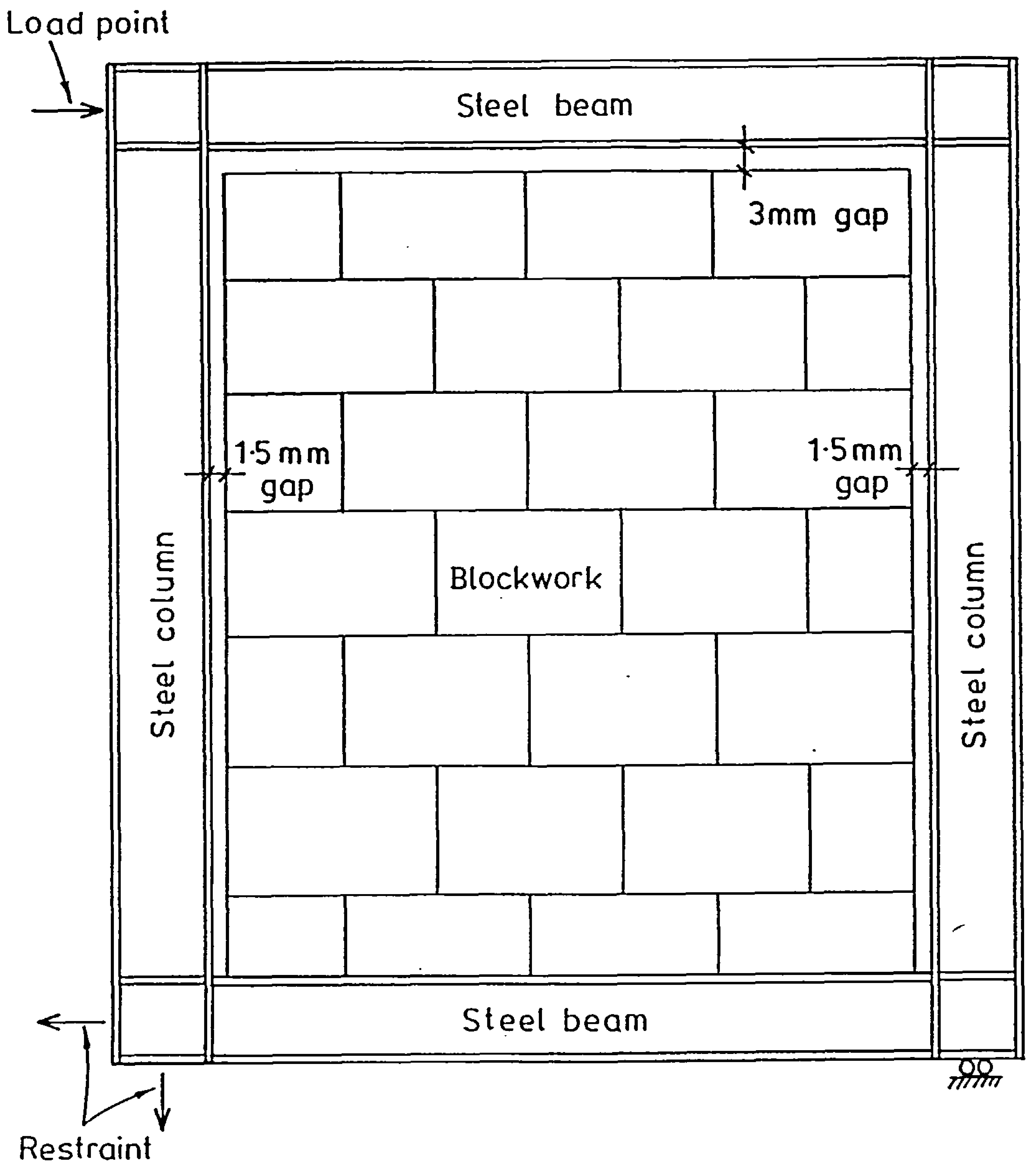


Figure 7.10: The frame of example 7.3 (tested by Riddington).

Theoretical		Experimental	
a_o	no infill	E_o	no infill
a_1	$A = 0.24\ cm^2$	E_1	infill with top and side gaps
a_2	$A = 1.16\ cm^2$	E_2	infill with top gap
a_3	$A = 2.32\ cm^2$	E_3	infill with no gaps

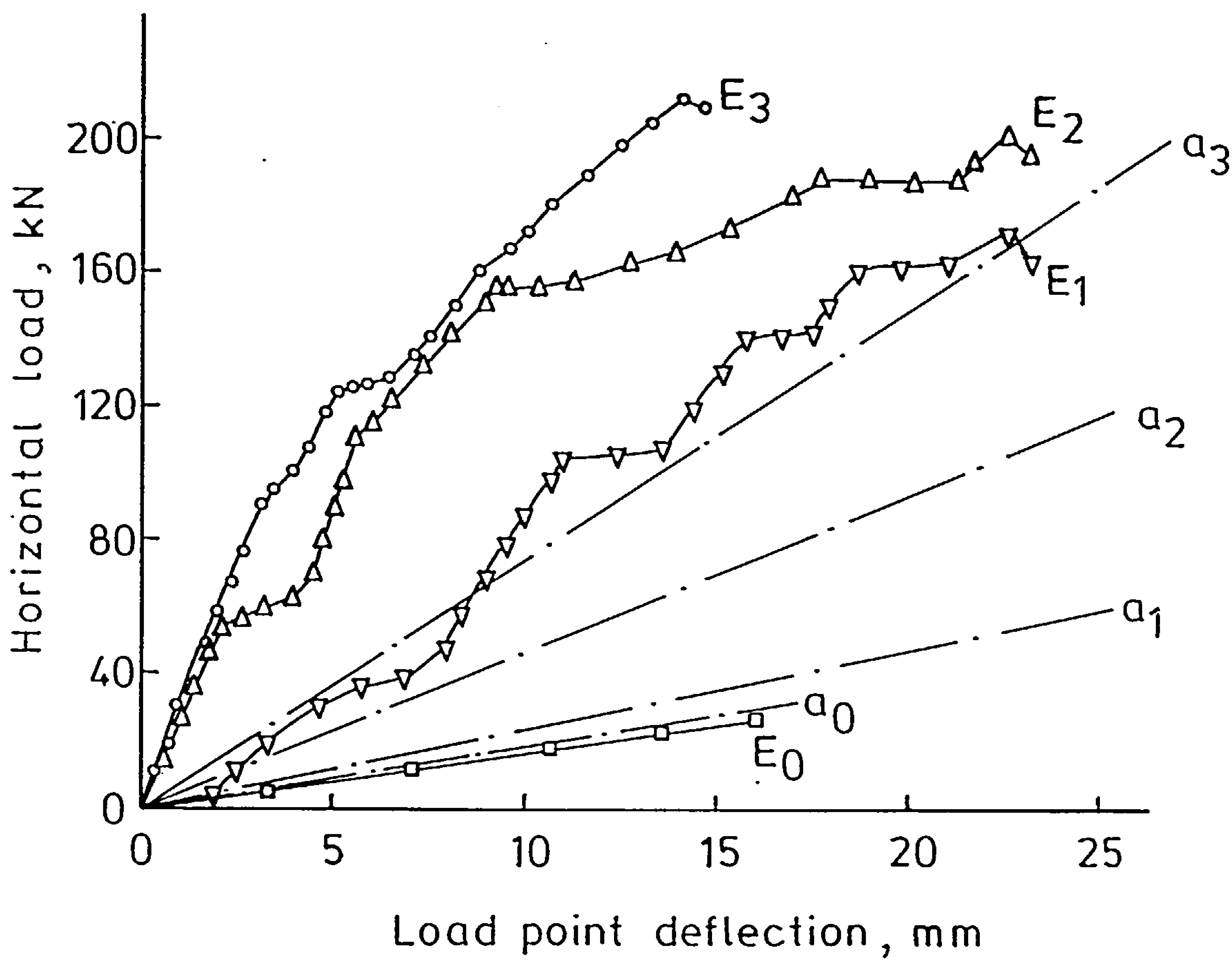


Figure 7.11: Horizontal displacements for the frame of figure 7.10 (flexible frame) with and without infill panel.

Theoretical		Experimental	
b_o	no infill	E_o	no infill
b_1	A = 1.14 cm^2	E_1	infill with top and side gaps
b_2	A = 2.28 cm^2	E_2	infill with top gap
		E_3	infill with no gaps

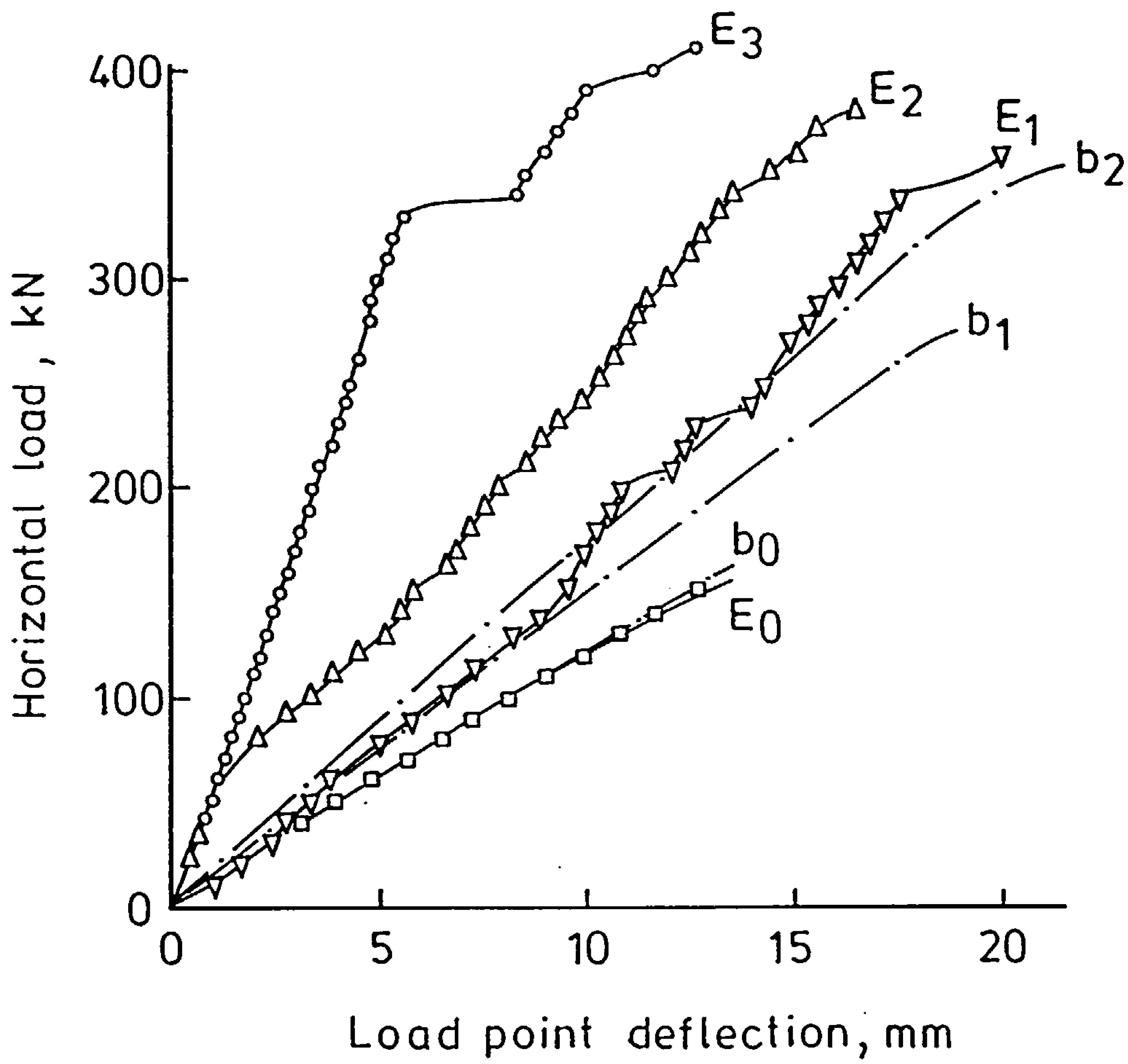
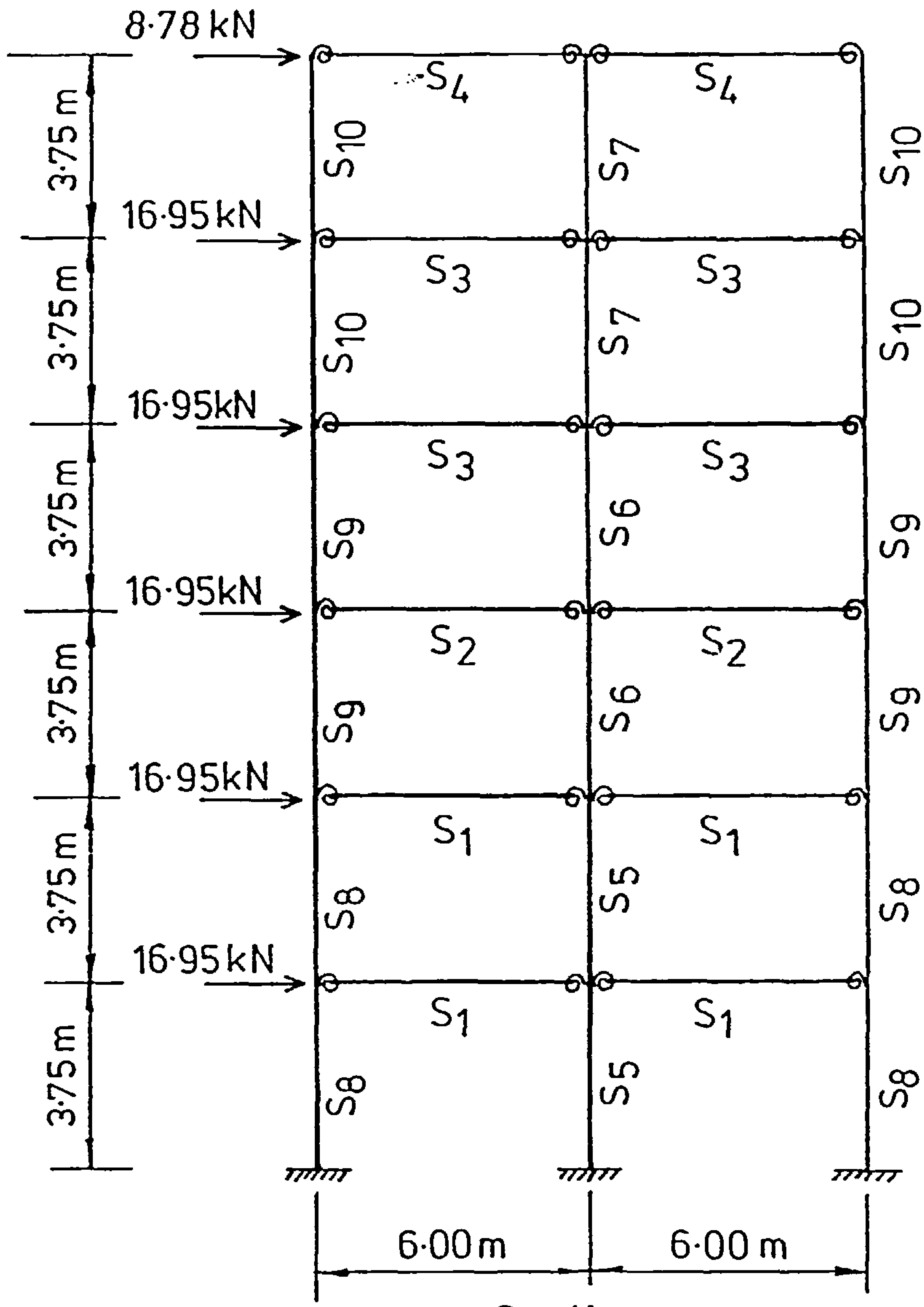


Figure 7.12: Horizontal displacements for the frame of figure 7.10 (stiff frame) with and without infill panel.



Sections

S_1	406x178x54UB	S_6	254 x 254 x 89 UC
S_2	356x171x45UB	S_7	203 x 203 x 60UC
S_3	305x127x37UB	S_8	254 x 254x 73UC
S_4	254 x 146x 31UB	S_9	203 x 203x 60UC
S_5	254 x 254x107UC	S_{10}	203 x 203x 46UC

Figure 7.13: The frame of example 7.4.

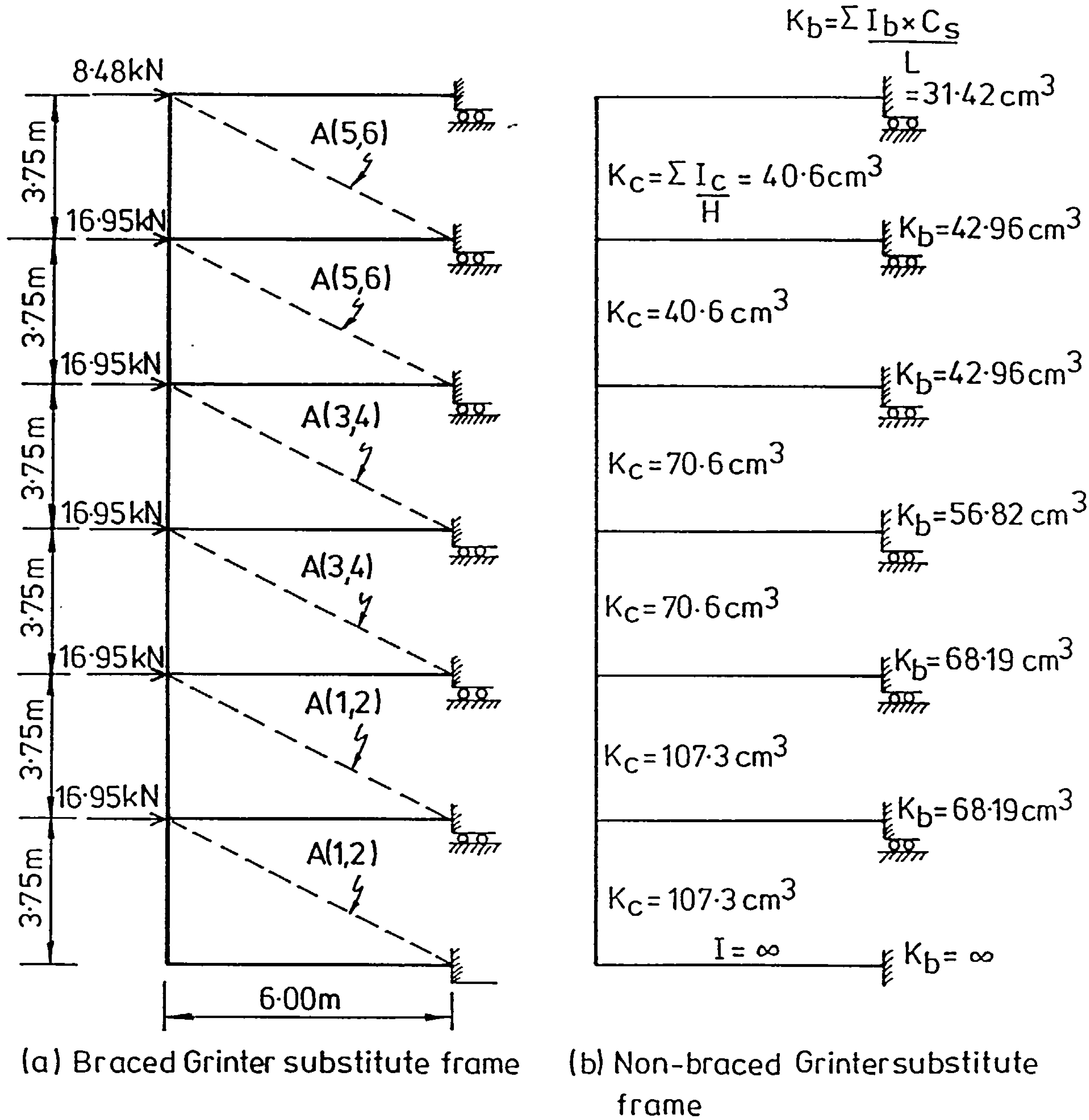


Figure 7.14: Grinter substitute arrangement for the frame of figure 7.13.

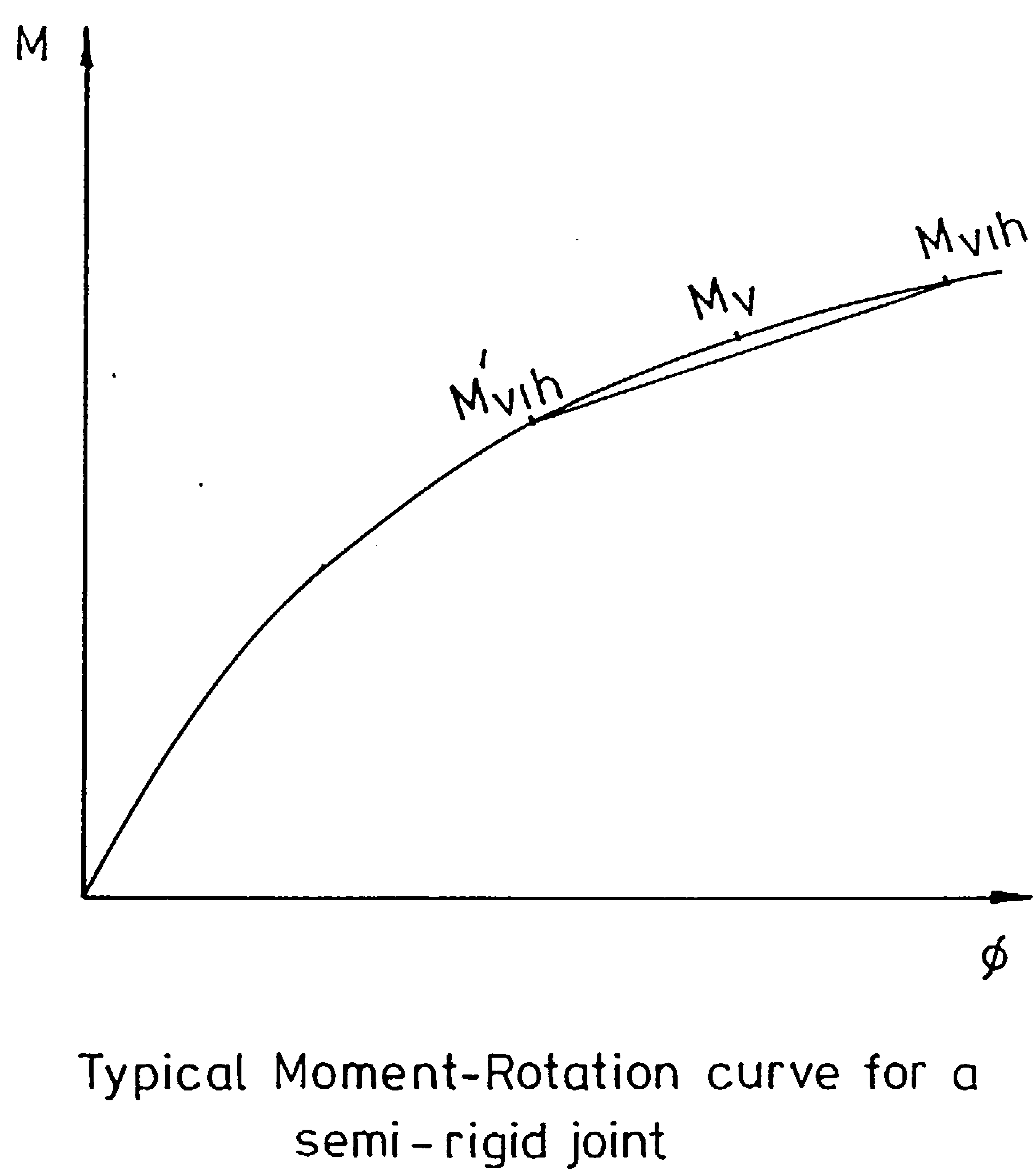


Figure 7.15: Generalised stiffness of a semi-rigid joint.

Chapter 8

Analysis of Flexibly Connected Sway Column Subassemblages

8.1 Introduction

In order to investigate the behaviour of flexibly connected sway subassemblages, an analytical tool is required. Nonlinear methods of finite element analyses have been widely used to represent the response of steel structures including several factors which have been proved to influence such response. Examples of these factors are:

1) the behaviour of semi-rigid joints which has been shown to be nonlinear over most of the loading range.

2) the nonlinear elasto-plastic behaviour of steel material.

3) residual stresses which are present in steel sections after manufacturing processes. These stresses have a vital role in the spread of plasticity over the cross section of steel members.

4) the nonlinear effect due to the presence of axial loads, i.e. the change of geometry effect particularly on columns.

5) the nonlinear term in the strain-displacement relationships arising from the consideration of large deflection theory.

In a standard finite element analysis the nonlinear aspect of the problem is usually dealt with by dividing the applied loads into reasonably small increments and conducting a linear analysis for each load step using the following equilibrium equation:

$$K_T \, . \, \Delta U \; = \; \Delta P \qquad (8.1)$$

where:

K_T is the tangential stiffness matrix ;

ΔU is the incremental displacement vector and

ΔP is the incremental load vector.

An iterative procedure is then employed, such as the Newton-Raphson iteration, to converge on to the solution in each load step.

8.2 The Existing Program

During the period of the Sheffield study into semi-rigid frame response, a computer program which uses finite element analysis has been developed. Starting in 1977 Jones [35] wrote a program to investigate the in-plane behaviour of a non-sway steel column allowing for its ends to be semi-rigidly connected to infinitely stiff supports. This program was then modified by Rifai [3] to represent the in-plane behaviour of non-sway column subassemblages consisting of a column plus up to two beams framing in at each end via semi-rigid joints.

For the sake of simplicity semi-rigid joints have been represented in most existing computer programs (see ref. [2]) as independent elements with stiffnesses dependent upon the moment-rotation relationships of the joints. However, although including semi-rigid joint behaviour in this manner reduces the complexity of the problem, it causes a considerable increase in the size of the global stiffness matrices of the structures due to an increase of the number of elements in the frame, i.e. each flexibly connected beam is treated as three independent elements. This problem is avoided in Rifai's formulation since the semi-rigid joint behaviour is included in the standard beam-element theory by means of modifying the standard shape functions, which have been used for fully rigidly connected beam-element, and including the energy stored in the semi-rigid joints in the virtual work equation.

8.3 Basic Formulations

In this section the basic formulations used in Rifai's program will be reviewed. For details about the derivations of these formulations the reader is referred to ref. [3].

8.3.1 Shape Functions

Considering the beam-element indicated in fig.8.1 in which three positions 1-G-2, 1'-G'-2', 1"-G"-2" are shown. In this figure 1-G-2 represents the undeformed shape of the element. Applying the load P^e leaves the element in the position 1'-G'-2' (which is assumed here to be an equilibrium position of the element under the applied load). Applying a small incremental set of loads ΔP^e causes the element to have additional deformations $\Delta\delta^e$ thus reaching the final position denoted by the curve 1"-A"-2". From this figure it can be seen that the semi-rigid joints, which are located at both ends of the element, locations 1 and 2, have two rotations. One is just to the left of the joint and the other is just to the right and the difference between these rotations is the joint rotation itself.

The deformation along the element, in the final position, can be expressed as:

$$\delta = A \cdot \delta^e \tag{8.2}$$

where

$$\delta = [\ u\ ,\ v\ ]^T$$

in which u and v are the deformations at a general point on the element.

$$\delta^e = [\ u_1\ v_1\ \theta_1\ u_2\ v_2\ \theta_2\ ]^T$$

and

$$A = \begin{vmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 \\ 0 & N_3 & N_4 & 0 & N_5 & N_6 \end{vmatrix} \tag{8.3}$$

in which N_1 *to* N_6 are the shape functions which define the deflected shape of the element. These can be obtained from the following set of equations:

$$\begin{aligned}
N_1 &= 1 - r \\
N_2 &= r \\
N_3 &= 1 - \overline{\theta_{j1}}^1 Lr - (3 - 2\overline{\theta_{j1}}^1 L - \overline{\theta_{j2}}^1 L)r^2 + (2 - \overline{\theta_{j1}}^1 L - \overline{\theta_{j2}}^1 L)r^3 \\
N_4 &= L[(1 - \overline{\theta_{j1}}^2)r - (2 - 2\overline{\theta_{j1}}^2 - \overline{\theta_{j2}}^2)r^2 + (1 - \overline{\theta_{j1}}^2 - \overline{\theta_{j2}}^2)r^3] \\
N_5 &= \overline{\theta_{j1}}^3 Lr + (3 - 2\overline{\theta_{j1}}^3 L - \overline{\theta_{j2}}^3 L)r^2 - (2 - \overline{\theta_{j1}}^3 L - \overline{\theta_{j2}}^3 L)r^3
\end{aligned}$$

and

$$N_6 = L[-\overline{\theta_{j1}}^4 r - (1 - 2\overline{\theta_{j1}}^4 - \overline{\theta_{j2}}^4)r^2 + (1 - \overline{\theta_{j1}}^4 - \overline{\theta_{j2}}^4)r^3] \tag{8.4}$$

in which

$$\begin{aligned}
\overline{\theta_{j1}}^1 &= \frac{A_1 B_1}{H}\ ,\quad \overline{\theta_{j2}}^1 = \frac{A_1 B_2}{H} \\
\overline{\theta_{j1}}^2 &= \frac{A_2 B_3}{H}\ ,\quad \overline{\theta_{j2}}^2 = \frac{\frac{2EI}{L} C_{j1}}{H} \\
\overline{\theta_{j1}}^3 &= \overline{\theta_{j1}}^1\ ,\quad \overline{\theta_{j2}}^3 = \overline{\theta_{j2}}^1 \\
\overline{\theta_{j1}}^4 &= \frac{\frac{2EI}{L} C_{j2}}{H}\ ,\quad \overline{\theta_{j2}}^4 = \frac{A_2 B_4}{H}
\end{aligned}$$

where

$$
\begin{aligned}
A_1 &= \frac{6EI}{L^2}\ ,\ A_2 = \frac{4EI}{L} \\
B_1 &= \frac{2EI}{L} + C_{j2}\ ,\ \ B_2 = \frac{2EI}{L} + C_{j1} \\
B_3 &= \frac{3EI}{L} + C_{j2}\ ,\ \ B_4 = \frac{3EI}{L} + C_{j1} \\
H &= \frac{1}{2}A_1 A_2 + A_2(C_{j1} + C_{j2}) + C_{j1}C_{j2}
\end{aligned}
$$

and

(8.5)

I	is the second moment of area of the element.
L	is the element length.
E	is Young's modulus.
C_{j1}	is the stiffness for the joint located at end 1 of the element.
C_{j2}	is the stiffness for the joint located at end 2 of the element.
r	is the ratio x/L.
x	is the abscissa of the considered point taken on the local axes of the element.

8.3.2 Calculating Strains and Stresses

According to the large deflection theory, the average strains at a cross section of a beam-element can be expressed in term of the element displacement at that section as:

$$\epsilon = \left[-\left(\frac{du}{dx} + \frac{1}{2}(\frac{dv}{dx})^2 \right), -\frac{d^2v}{dx^2} \right]^T \tag{8.6}$$

Differentiating this equation with respect to the element nodal displacement δ^e results in a relation between the incremental strain $d\epsilon$ and incremental nodal displacement $d\epsilon^e$.

$$d\epsilon = - \left[\begin{vmatrix} \frac{dN_1}{dx} & 0 & 0 & \frac{dN_2}{dx} & 0 & 0 \\ 0 & \frac{d^2N_3}{dx_2} & \frac{d^2N_4}{dx_2} & 0 & \frac{d^2N_5}{dx_2} & \frac{d^2N_6}{dx_2} \end{vmatrix} - \left(\frac{dv}{dx}\right) \begin{vmatrix} 0 & \frac{dN_3}{dx} & \frac{dN_4}{dx} & 0 & \frac{dN_5}{dx} & \frac{dN_6}{dx} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{vmatrix} \right] d\delta^e \tag{8.7}$$

Knowing the element strain the stresses can be calculated from

$$\sigma = D\ \epsilon \tag{8.8}$$

where D is the element elasticity matrix given by

$$D = \begin{vmatrix} EA & 0 \\ 0 & EI \end{vmatrix} \tag{8.9}$$

8.3.3 Tangent Stiffness Matrix

In order to carry out a nonlinear finite element analysis, the tangent stiffness matrix for each element should be calculated and then assembled into the global tangential stiffness matrix of the whole frame. For a flexibly connected element the tangent stiffness matrix is given by:

$$K_T = K_E + K_G + K_L \tag{8.10}$$

where

K_E is the matrix given by

$$K_E = \int_0^L \begin{vmatrix} {B_o^a}^T\ EA\ B_o^a & 0 \\ 0 & {B_o^b}^T\ EI\ B_o^b \end{vmatrix} dx + [{N_{j1}}^T\ C_{j1}\ N_{j1} + {N_{j2}}^T\ C_{j2}\ N_{j2}] \tag{8.11}$$

K_G is

$$K_G = - \begin{vmatrix} 0 & 0 \\ 0 & \int_0^L G^T\ P\ G\ dx \end{vmatrix} \tag{8.12}$$

K_L is

$$K_L = \int_0^L \begin{vmatrix} 0 & {B_o^a}^T\ EA\ B_L^b \\ {B_L^b}^T\ EA\ B_o^a & {B_L^b}^T\ EA\ B_L^b \end{vmatrix} dx \tag{8.13}$$

in which

$$
\begin{aligned}
B_o{}^a &= -\left[\left| \frac{dN_1}{dx} \;\; 0 \;\; 0 \;\; \frac{dN_2}{dx} \;\; 0 \;\; 0 \right| \right] \\
B_o{}^b &= -\left[\left| 0 \;\; \frac{d^2N_3}{dx_2} \;\; \frac{d^2N_4}{dx_2} \;\; 0 \;\; \frac{d^2N_5}{dx_2} \;\; \frac{d^2N_6}{dx_2} \right| \right] \\
G &= \left[\left| \frac{dN_3}{dx} \;\; \frac{dN_4}{dx} \;\; \frac{dN_5}{dx} \;\; \frac{dN_6}{dx} \right| \right] \\
B_L{}^b &= -\left(\frac{dv}{dx}\right) G \\
N_{j1} &= \left| 0 \;\; \frac{dN_3}{dx} \;\; -1 + \frac{dN_4}{dx} \;\; 0 \;\; \frac{dN_5}{dx} \;\; \frac{dN_6}{dx} \right| \\
N_{j2} &= \left| 0 \;\; \frac{dN_3}{dx} \;\; \frac{dN_4}{dx} \;\; 0 \;\; \frac{dN_5}{dx} \;\; -1 + \frac{dN_6}{dx} \right|
\end{aligned}
\tag{8.14}
$$

The derivatives $\frac{dN_3}{dx}$, $\frac{dN_4}{dx}$, $\frac{dN_5}{dx}$, $\frac{dN_6}{dx}$ are evaluated at x=0 and x=L for N_{j1} and N_{j2} respectively

8.4 Nonlinear Finite Element Analysis

Knowing the flexural rigidity of a beam-element together with its joints' stiffnesses, i.e. the stiffnesses of the semi-rigid joints located at the element ends, the element tangent stiffness matrix can be calculated in conjunction with eqs.8.2-8.5 and 8.10-8.14. These equations compute the tangent stiffness matrix of a beam element with reference to its local axes including the influence of

1) the current deformed shape;

2) the remaining unyielded regions at the current load stage;

3) the stiffnesses of the utilised semi-rigid joints;

4) the residual stresses when calculating the resultant stresses of the elements due to the externally applied loads using eqs 8.6-8.9.

Assembling the tangent stiffness matrices for all elements to give the global tangent stiffness matrix, adding the structure's boundary conditions and utilising eq.8.1 gives the structural incremental displacements. Next a solution technique is employed to converge on to the solution in each load step. Initially, the existing program claimed (see ref. [3]) to be capable of handling such problems. However in using this program to analyse flexibly connected sway sub-assemblages, some difficulties arose making it necessary to modify the program to overcome these obstacles. In the following sections modifications accomplished by the author are described.

8.4.1 Incorporating The Spread of Yielding

It is true to assume that the response of a steel cross section is elastically linear when the applied loads are small. By increasing the loads some fibres of the section might reach plasticity, i.e. permitting deformations without increase of stresses. This behaviour needs to be included in the analysis. A simple but approximate manner in which such effect can be included is given by Nethercot [62]. This is accomplished by:

1) Dividing the cross section into very thin layers, i.e. horizontally for flanges and vertically for webs,

2) Calculating the axial and flexural rigidities, i.e. EA and EI, for each layer independently taking into account the stress-strain relationship of steel and the resultant strains due to the externally applied loads calculated at each load step including the residual strains.

3) Taking the summation of these quantities for all layers of the section.

In order to calculate the shape functions of a beam element, the element properties EI and EA have to be determined. These values are uniform for a beam-element in the elastic stage of the behaviour. However as the applied loads increase some sections of the element may develop plasticity in certain locations which leads to an alteration in the values of EI and EA through the element length. Hence a variation of these values will occur through the beam length, which depends upon the spread of plasticity, and this must be included in the analysis when calculating the tangent stiffness matrix and the strains and the stresses in the element.

For the sake of simplicity and to avoid the complexity of determining the spread of plasticity, an alternative procedure has been widely used in structural analyses. In this procedure the presence of yielded fibres of the element sections is ignored and some assumptions are used to facilitate the calculation of the quantities EI and EA. In the original program, for example, the quantities EA and EI are calculated at each end of every element and average values are then used in the analysis. This assumption was found to be unconservative when compared with the computer program SERVAR where the spread of plasticity has been incorporated in a more accurate manner. In the modified subassemblage program however the quantities EI and EA are calculated at both ends of each element, as before, but the smallest value of these has been adopted in the analysis. This assumption has been found to provide reasonable and safe results when compared with SERVAR. It should be noted here that, in order to increase the accuracy of the analysis a structural element should be divided into several elements, the author found that the use of three or four elements gave good accuracy.

8.4.1.1 Stress-strain Relationship

Supplying the quantities EA and EI to calculate the shape functions (eqs 8.3 -8.5) requires the value of E which depends on the element stresses. In order to determine E, it is essential to adopt a strain-stress relationship which simulates the behaviour of the material. In the present study a general elastic-perfectly plastic curve with strain hardening has been used (see fig.8.2). By knowing the strain at any point of a section the relevant E value can be easily calculated by reference to the material strain-stress curve. This is achieved in the current program by

i) calculating the total strain at both nodes of a element;

ii) using the adopted strain-stress relationship to calculate E and then computing the values EI and EA and

iii) finally taking the smallest value of each of EI and EA to calculate the shape functions, the tangent matrix and the resultant stresses of the element.

8.4.2 Incorporating Joint Nonlinearity

At the present time the most reliable $M - \Phi$ relationships are those obtained by experimental tests. Utilising these relationships in frame structural analyses requires the representation of these relationships into mathematical formulae. This enables the determination of the joint stiffnesses whenever needed by the analysis.

Many approximate models have been proposed. The simplest is a linear representation (see fig.8.3) which utilises the constant initial tangent of a joint's moment-rotation relationship to represent the joint stiffness throughout the whole loading range. This modelling suffers lack of accuracy as the joint's moment increases. A better approximation is bi-linear relationship (fig.8.3) in which the initial slope is replaced by a shallower line at a certain level of rotation. More advanced representations are tri-linear, multi-linear and a polynomial fitting. Even the latter has been criticized by Jones [35] for having poor representations especially at higher level of rotation where unacceptable negative slope may occur.

In order to more closely represent the $M - \Phi$ curves Jones proposed a cubic B-spline representation which is uniquely defined by

$$\Phi(M) = \sum_{j=0}^{3} \alpha_j M^j + \sum_{j=1}^{h_n} \beta_j (< M - k >)^3$$

where:

h_n is number of "knots" in the entire range;

k are suitable knots locations which may be chosen by the users;

$$< M - k > \quad = M - k \quad for \quad (M - k) \succ 0$$
$$= 0 \quad for \quad (M - k) \prec 0$$

in which

α_j , β_j are coefficients determined by the least squares method.

A cubic B-spline technique was adopted in the existing program to represent semi-rigid joint $M - \Phi$ relationships. However cubic B-spline modelling itself has been found to suffer lack of accuracy for some semi-rigid joints in regions at either end of the range where the $M - \Phi$ relationships are very close to linear. Because of this a cubic B-spline fitting might produce for some semi-rigid joints ficticious negative stiffnesses right at the start of of their $M - \Phi$ relationships. This lack of accuracy is probably due to lack of slope control at the end of these regions. In real semi-rigid joints linear behaviour might occur at the beginning and at the end of $M - \Phi$ relationships. Therefore, this modelling has been improved by using linear parts located at the beginning and at the end of $M - \Phi$ relationships and nonlinear part located in the middle where a cubic B-spline can be used effectively to model this region. This way of representing semi-rigid joints has been proved to provide more accurate values for joint stiffnesses throughout the whole loading range.

8.4.3 Inclusion of Imperfections

The theory, which has been mentioned earlier, analyses a straight member without initial residual stresses. However, practical steel members have usually some residual stresses due to welding or due to differential cooling after rolling. Hence these factors needs to be included in the analysis.

8.4.3.1 Inclusion of Residual Stresses

Recent experimental work on steel beams and columns have shown that the distribution of the residual strains are highly irregular due to the cold straightening processes now employed. Nevertheless simple patterns are usually adopted in analysis as representing the worst conditions which might arise. These patterns have derived experimentally with some modification to facilitate their use.
The most common of these are:

1) A pattern with parabolic distributions of residual stresses across steel sections as indicated in figure 8.4. This has been proposed by Young [63] and usually used for hot rolled sections produced in the U.K. The values for the residual stresses at the flange tips σ_f, the flange to column junction σ_{fw} and the centre of the web σ_w are given by the following empirical equations:

$$\begin{aligned}
\sigma f &= 165 \left[1 - \frac{A_w}{1.2\ A_f}\right] N/mm^2 \\
\sigma f w &= -100 \left[0.7 + \frac{A_w}{A_f}\right] N/mm^2 \\
\sigma w &= 100 \left[1.5 + \frac{A_w}{1.2\ A_f}\right] N/mm^2
\end{aligned} \tag{8.15}$$

where

A_w is the area of the web of the section ;

A_f is the area of both flanges

A parabolic equation, which must satisfies eq.8.15, can be used to estimate the residual stress at each point of the section.

2) In the U.S.A, for simplicity, a linear pattern is usually assumed [35] to represent the residual stresses distribution (fig.8.4) with the following values;

$$\begin{aligned} \sigma_f &= 0.3\,\sigma_y \\ \sigma_{fw} &= -0.3 \times \sigma_y \left(\frac{A_f}{A_w + A_f}\right) \end{aligned} \tag{8.16}$$

in which

σ_y is the material yield stress;

A_w is the area of the web and

A_f is the area of one flange only of the section.

3) For a welded section, a simpler pattern is usually assumed (see fig.8.4) [35] which consists of a simple rectangular tensile pattern for residual stresses around webs of value $-0.9\ \sigma_y$ while a simple compressive residual stresses pattern with value $0.1\ \sigma_y$ occurs around the flanges.

All these patterns were claimed to be accepted in the original program [3]. However, the corresponding subroutines were rechecked and partly rewritten since some errors were found.

8.4.3.2 Inclusion of Initial Deflection

In practical columns small geometrical imperfections are inevitable due to manufacturing and erection conditions. The shapes of these initial deflections are often complicated therefore a simple representative shape has been used in steel frame analysis ([3] and [35]). This formula assumes:

1) the maximum initial deflections a_o allowed in practical columns are one thousandth of these column lengths.

2) a half sine wave is assumed to represent the initial deflection mode v_o. Therefore by knowing a_o the initial deflection can be calculated at each point of a steel column from the following equation

$$v_o = a_o \sin \pi \frac{x}{l}$$

In a finite element analysis this effect can be incorporated by computing the initial rotation and lateral deflections v_o of a steel column and converting those into an imaginary set of lateral forces and moments P_o by multiplying v_o by the geometrical stiffness matrix K_G, i.e.

$$P_o = K_G \times v_o$$

8.4.4 Boundary Conditions

Before applying the boundary conditions, the tangential stiffness matrix of a structure is singular therefore the matrix cannot be decomposed. In steel structures a variety of different boundary conditions exist such as

1) a fixed condition, i.e. the corresponding degree of freedom equals zero;

2) a flexible elastic boundary conditions in which the boundary condition is represented by its elastic spring stiffness and

3) a free boundary condition.

In the original program only fixed or free boundary conditions are accepted. This has been modified to represent more realistically frame boundary conditions by incorporating their elastic stiffnesses (three springs corresponding to the nodal degrees of freedom for each node). Hence for a fixed boundary conditions a large number should be assigned to the spring stiffness, e.g. 10^{15}, and for a flexible boundary condition any value can be given to the spring stiffness to allow for a prescribed resistance whilst an unrestrained condition may be represented by zero.

8.4.5 Solution Technique

Since eq.8.1 is an incremental equation and since the nature of the problem is nonlinear, an iteration technique is needed to converge on to the solution for each load step. The most widely used technique is the Newton-Raphson technique where the unbalanced forces, i.e. the difference between the external to internal load vectors, are calculated and reapplied on the structure until those loads are negligible. This procedure is used in each load step until satisfactory convergence is obtained. Details about the procedure can be found in references [1], [30], [31].

8.4.5.1 Unbalanced Forces

Calculating unbalanced forces in a finite element analysis can be accomplished by employing two different techniques:

1) After calculating the strains and stresses over the cross-sections of all nodes, the resulting forces at these nodes can be obtained, and by subtracting those from the external nodal loads, the unbalanced forces are obtained. This method suffers lack of convergence when the structure is partly or fully yielded due to the discontinuity

between the elements, although the structure might be in equilibrium. This method was employed in the original program which has been used extensively and successfully for non-sway subassemblages. However the program frequently failed when it was used by the author to analyse sway sub-assemblages due to the relatively large deformations when compared to those occuring in non-sway structures. Moreover, the original program checks convergence for axial loads on columns and lateral loads on beams only [3], i.e. secondary effects on beams are ignored. This assumption might be tolerated for non-sway column sub-assemblages but it cannot be used for sway sub-assemblages since the secondary effect on beams is significant due to the large lateral drifts expected in such structures.

2) In the second method the tangential stiffness matrix of the structure is used in conjunction with the current incremental displacement vector to calculate the resultant force vector, i.e. internal forces of the elements of the structure.

This second method has been used in the modified program which is thought to provide the following advantages:

1) Includes the nonlinear geometrical deformations of beams.

2) Calculates unbalanced forces in a standard manner which suits any steel structure, i.e. a single element, sub-assemblages and frames. This feature also facilitates further subsequent development of the program.

Fig.8.5 describes the solution technique which has been employed. Assume that the point E represent an equilibrium state of the structure. Now applying an incremental load vector ΔP, calculating the tangential stiffness matrix of the structure K_{T_o} and solving eq.8.1 results in the structural deformation vector ΔU_o. These incremental displacements will inevitably alter the geometry of the frame, hence the global tangential stiffness matrix of the structure K_{T1} needs to be recomputed taking into consideration the current deformed shape of the structure. This certainly involves the calculation of the tangent stiffness matrix for each element according to its deformed shape and taking into account the stiffnesses of its semi-rigid joints together with the properties EI and EA of the element sections as mentioned in section 8.4.1. Now the unbalanced force vector $\Delta\ P_{unbalanced}$ can be obtained from:

$$\Delta P_{unbalanced} \ = \ \Delta P \ - \ K_{T1}\ \Delta U_o$$

Applying this unbalanced load vector would result in further displacements vector $\Delta U_{unbalanced}$. Adding this to the initial displacement vector ΔU_o gives the total displacement vector of the structure ΔU_1 in this incremental step. For the next iteration, the global tangent matrix K_{T2} is computed and the unbalanced force vector is calculated (see fig.8.5) as:

$$\Delta P_{unbalanced} \ = \ \Delta P \ - \ K_{T2}\ \Delta U_1$$

or in general

$$\Delta P_{unbalanced} \ = \ \Delta P \ - \ K_{Ti}\ \Delta U_{i-1} \tag{8.17}$$

in which

$$\Delta U_{i-1} \ = \ \Delta U_o \ + \ \sum_{1}^{i-1} \Delta U_{unbalanced} \tag{8.18}$$

This procedure should be repeated for each load step until satisfactory convergence is obtained.

8.4.5.2 Convergence Criterion

In the present analysis convergence is checked for the incremental loads and displacements, resulting after each iteration, simultaneously. In other words, the program will proceed for next load step when both of the following conditions are satisfied:

1) The ratios of all unbalanced force vector components to the corresponding incremental load vector components are less than a prespecified small number (c) which can be defined by the user.

2) Similarly, the ratios of all incremental nodal displacement vector components, resulting from subjecting the structure to the last unbalanced load vector, to those corresponding to the total nodal displacements are less than (c).

Typically the value of (c) adopted by the author and found to be satisfactory is 0.001.

8.5 The Modified Program

The original program mentioned in section 8.2 has been modified by the author to simulate the response of flexibly connected sway or non-sway column subassemblages. The program traces the nonlinear load-displacement response and determines ultimate loads of the structures. An initial pattern of loads is usually specified by the user and this pattern will be increased automatically by the program until failure. Since many analyses suffer from numerical ill-conditioning when the structures are near collapse and in order to increase the accuracy of predicting ultimate loads of flexibly connected column subassemblages, the program proceeds using the appropriate one of the two option below.

1) If this is the first indication of failure in the structure, it steps back by the last two load increments and proceeds utilising smaller load steps equal to one fifth of the original size.

2) Otherwise it stops after printing an appropriate message telling the user about the occurrence of the structure failure.

8.6 Verification of The Modified Computer Program

Before conducting a parametric study on the behaviour of flexibly connected sway sub-assemblages, the ability of the program to handle such problems needed to be verified. This verification would be best achieved be comparing the behaviour predicted by the program to response of some previously tested sub-assemblages. No test results could be traced by the author probably due to the difficulties involved in conducting tests on sway structures. Therefore, the modified program has been verified against another existing computer program called SERVAR [2]. A sub-assemblage which consists of four beams and a single column and shown in fig.8.6 has been used. The upper beams are connected at far ends by simple rollers which allow horizontal displacements. All beams are assumed to be identical with cross-section denoted as SECTION S.1. Also shown in this figure the column cross-section, SECTION S.2. The beam and column sections are assumed to be free from initial imperfections and residual stresses. This has been assumed to make the utilised programs, SERVAR and the modified program, comparable since these effects are not included in SERVAR. A horizontal load H equal to 0.4 kN and a vertical load P equal to 40.0 kN are both applied at the node which connects the upper beams to the columns. This load pattern has been increased by a single load factor until failure.

Three type of steel joints have been utilised to connect the beams to the column. These are:

1) fully rigid joints

2) flush end plate joints

3) web cleat joints.

The $M - \Phi$ characteristics for these joints are indicated in fig.8.7. These characteristics are taken from Rifai's work [3] and originated from experimental tests conducted by Davison [36]. Fig.8.8 show the relationships between the load factors against lateral deflections of the subassemblage for the different joint types where it can be seen that, good agreement between these two programs has been obtained. The small differences between the predictions obtained from both programs can be related to the differences between the analyses used by these programs. These differences can be summarised as follows:

1) semi-rigid joint characteristics are modelled in SERVAR as multi-linear relationshipswhile a B-spline curve joining two linear regions has been used in the modified program. (see fig.8.7) 2) the spread of plasticity is modelled by means of a polynomial in SERVAR while the approximate method mentioned earlier in section 8.4.1 has been used in the modified program.

3) the nonlinear term in the strain-displacement relationship is not included in SERVAR, i.e. small deflection theory is assumed, while this term is included in the modified program.

8.7 Conclusion

An existing computer program has been modified to simulate the behaviour of steel sway sub-assemblages. The program uses nonlinear finite element analysis and includes several sources of nonlinearity, i.e. material and geometrical non-linearity, residual stresses, initial imperfections and semi-rigid joint behaviour.

Due to lack of experimental tests on steel sway structures, the validity of the modified program has been verified against another computer program. Good correlation has been obtained hence the reliability of the modified program to predict the behaviour of sway column subassemblages has been established as far as is currently practical.

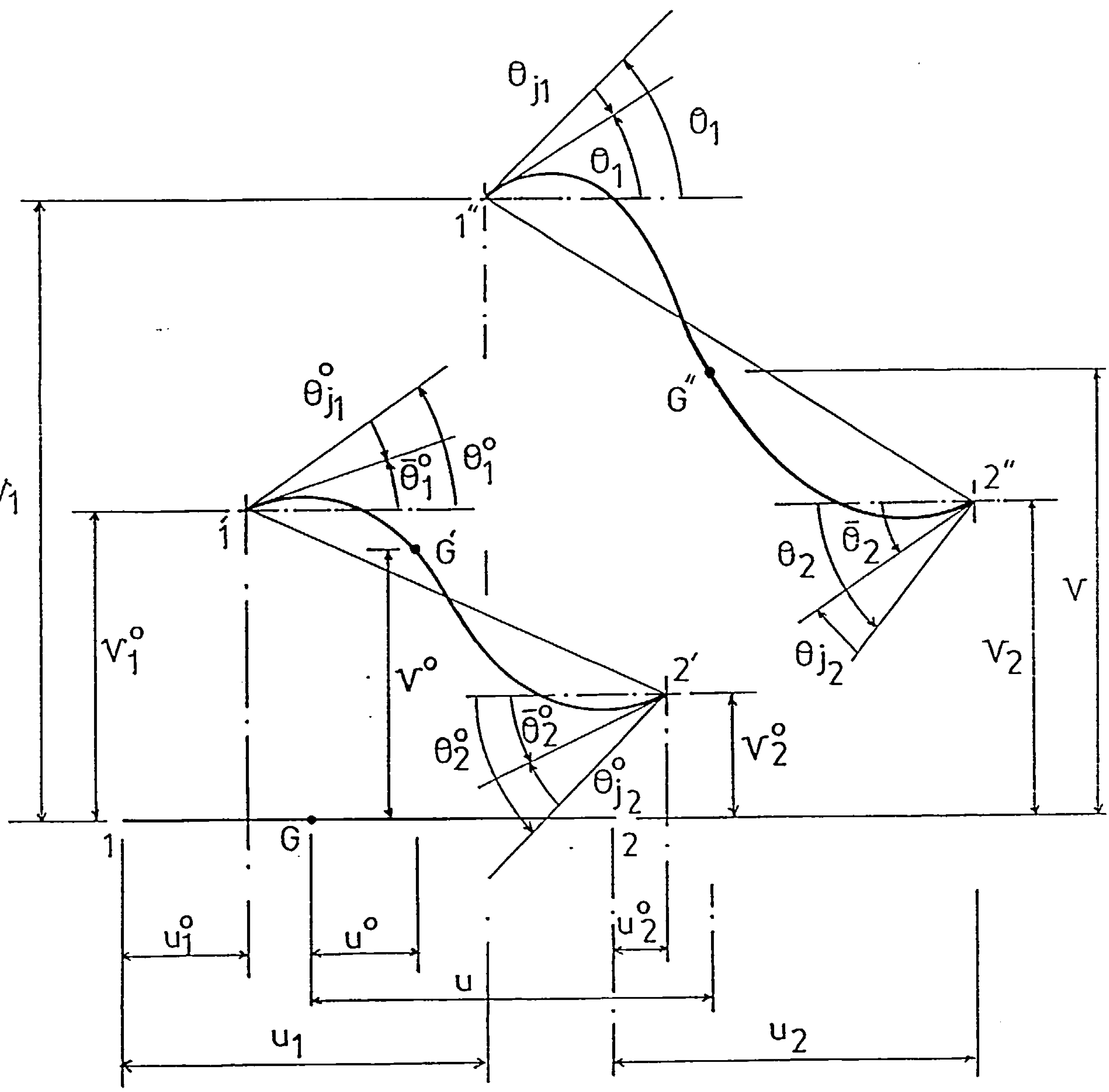


Figure 8.1: Deformed shape of a beam element with semi-rigid joints.

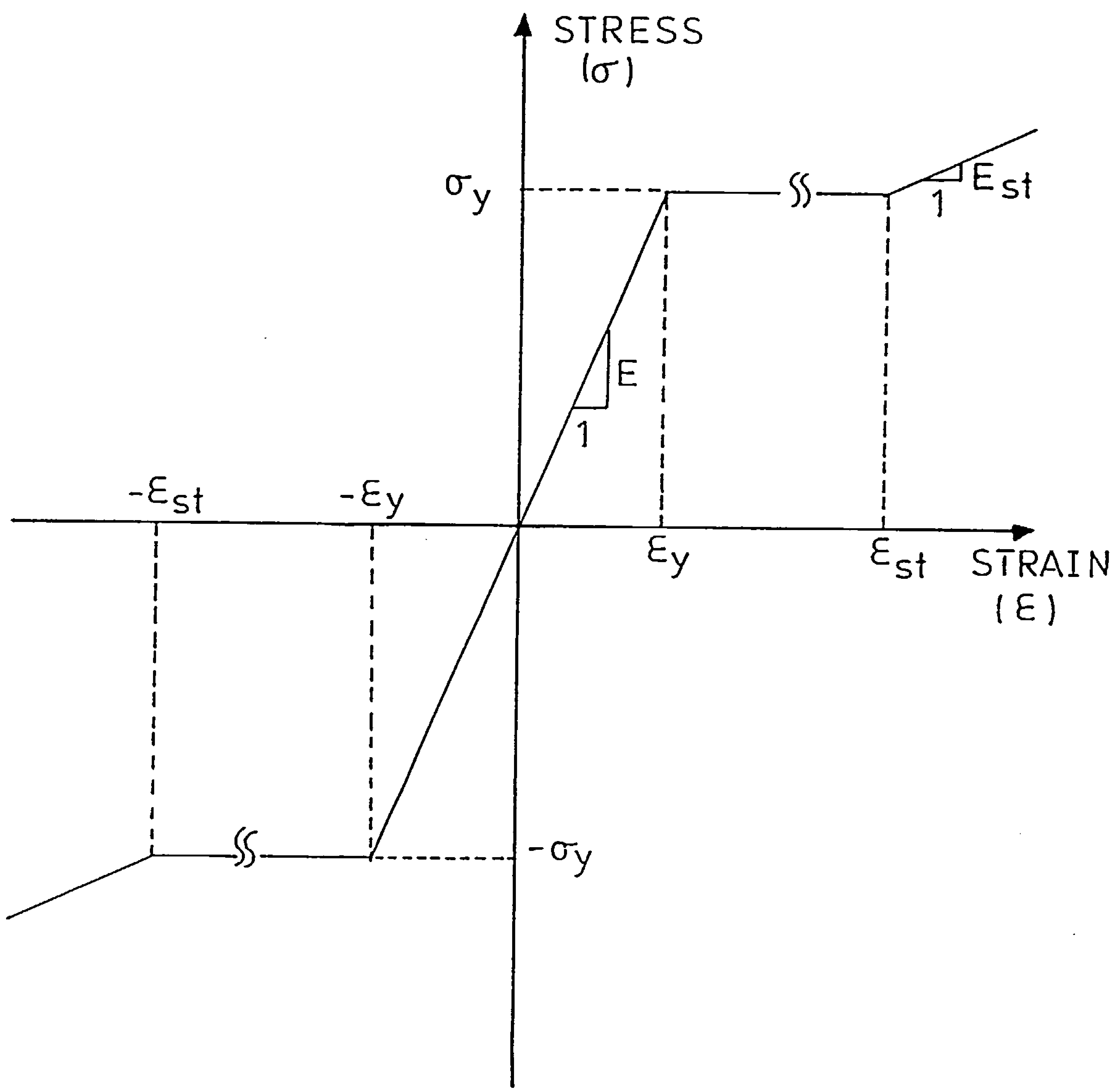


Figure 8.2: Standardised stress-strain relationship of steel.

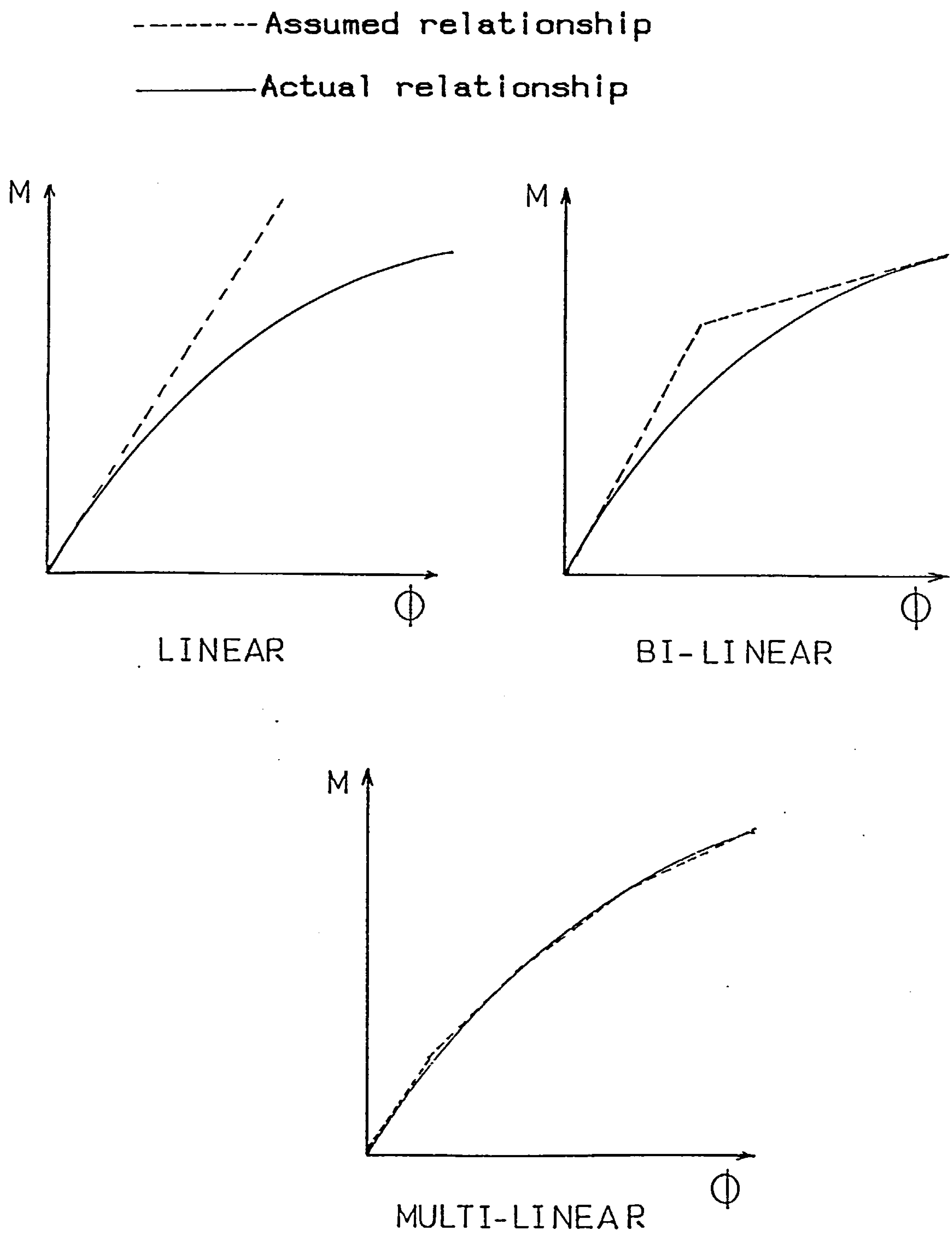


Figure 8.3: Modelling of a semi-rigid joint.

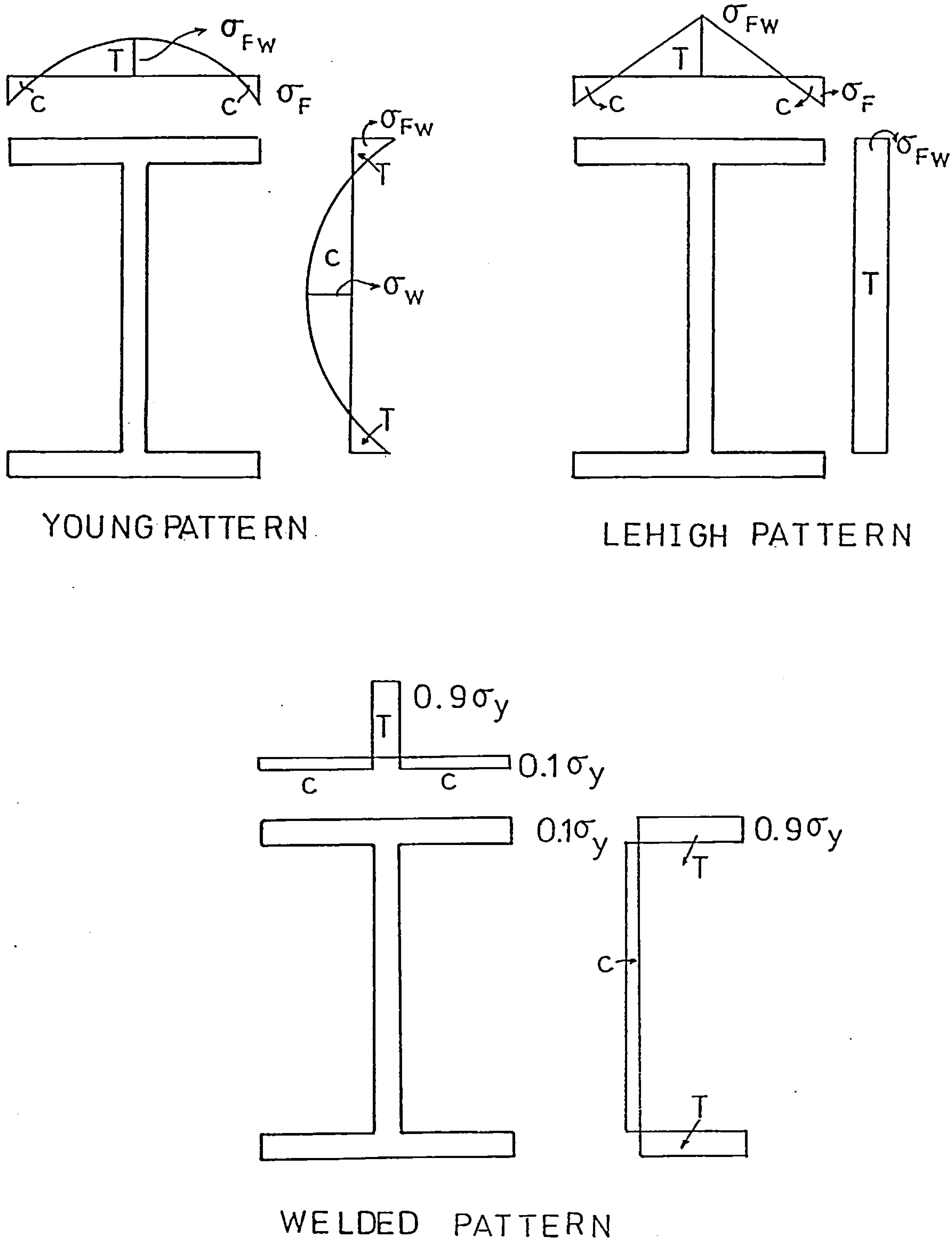


Figure 8.4: Residual stress patterns.

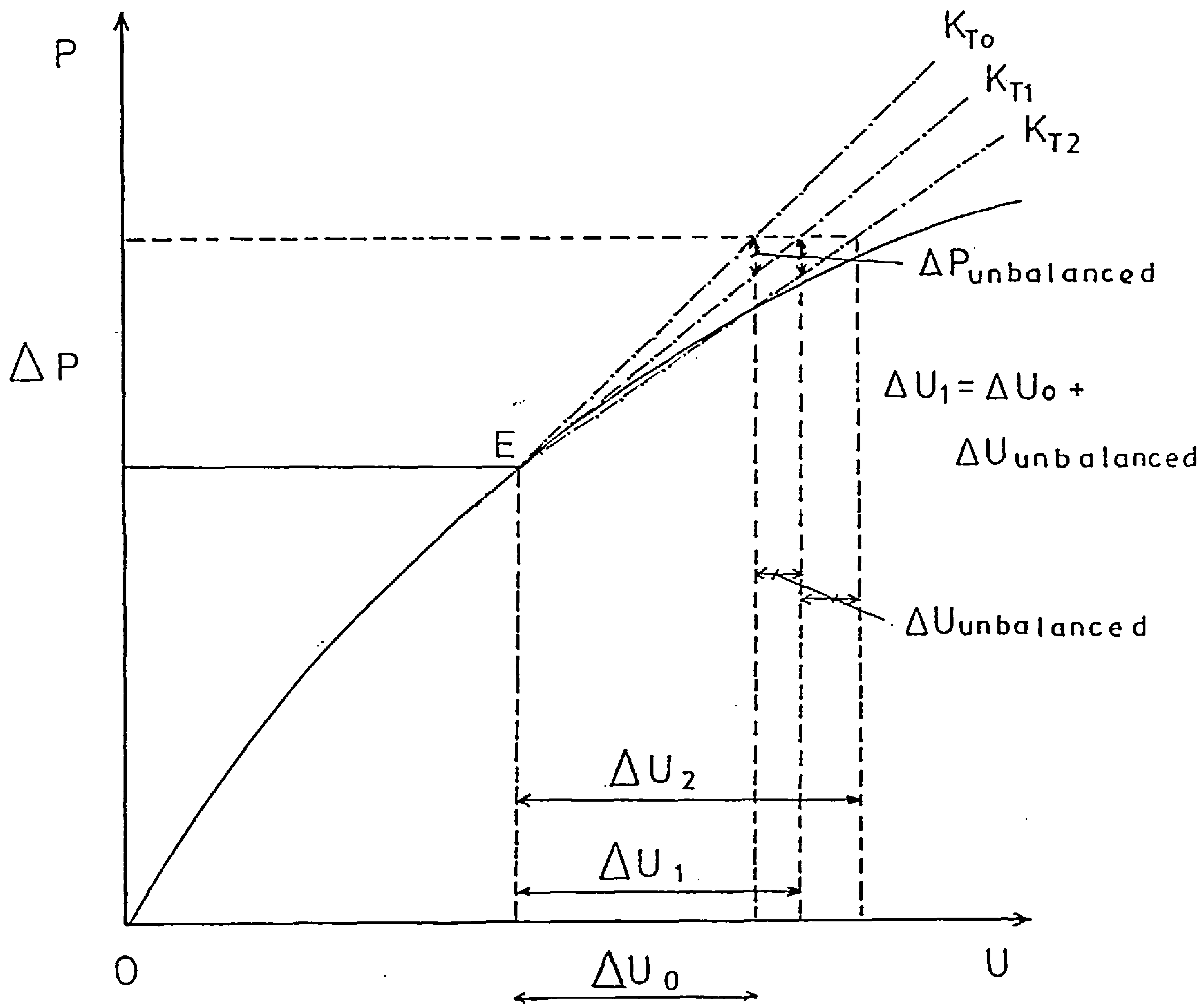


Figure 8.5: Solution technique.

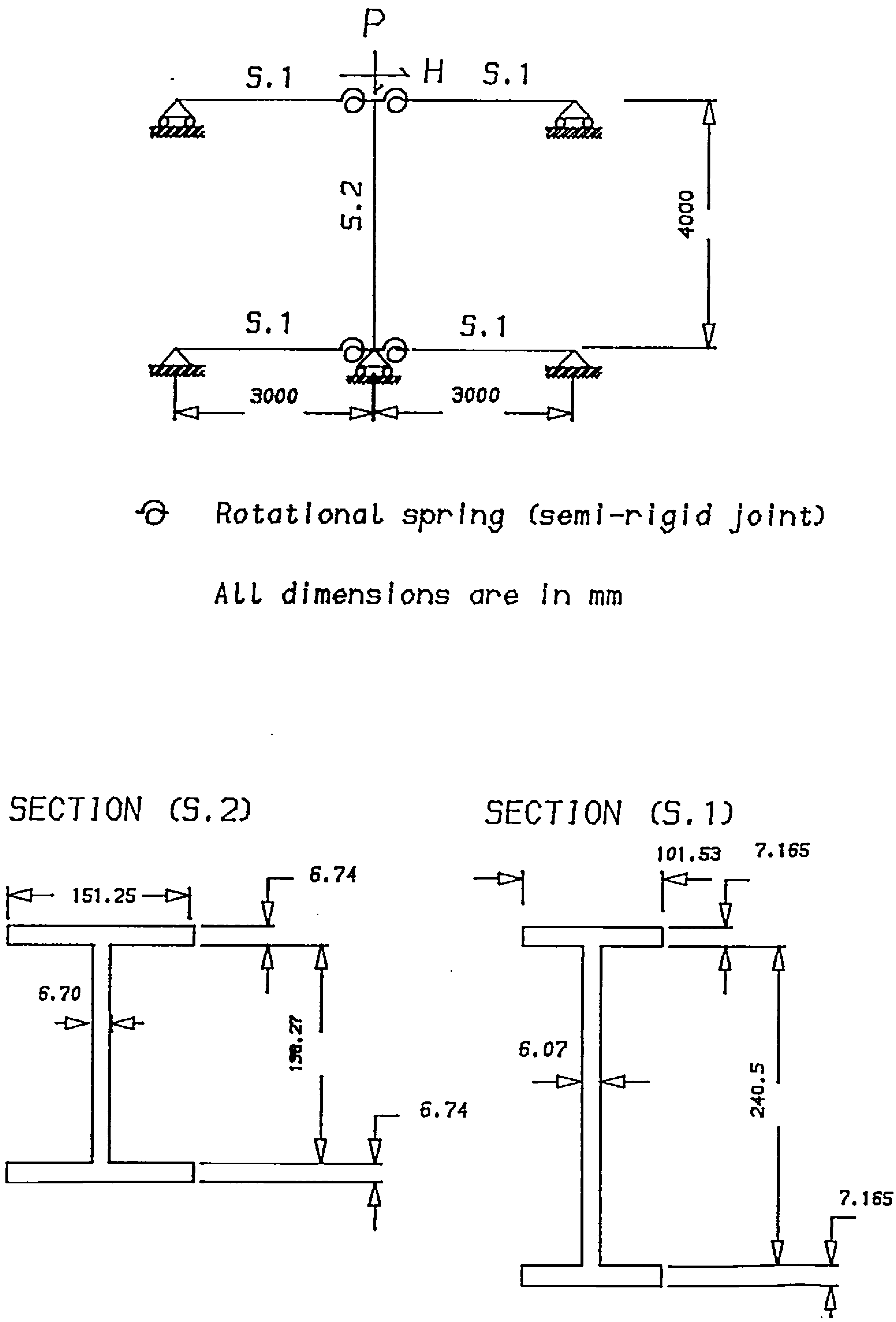


Figure 8.6: Subassemblage used in verifying the modified program against SERVAR.

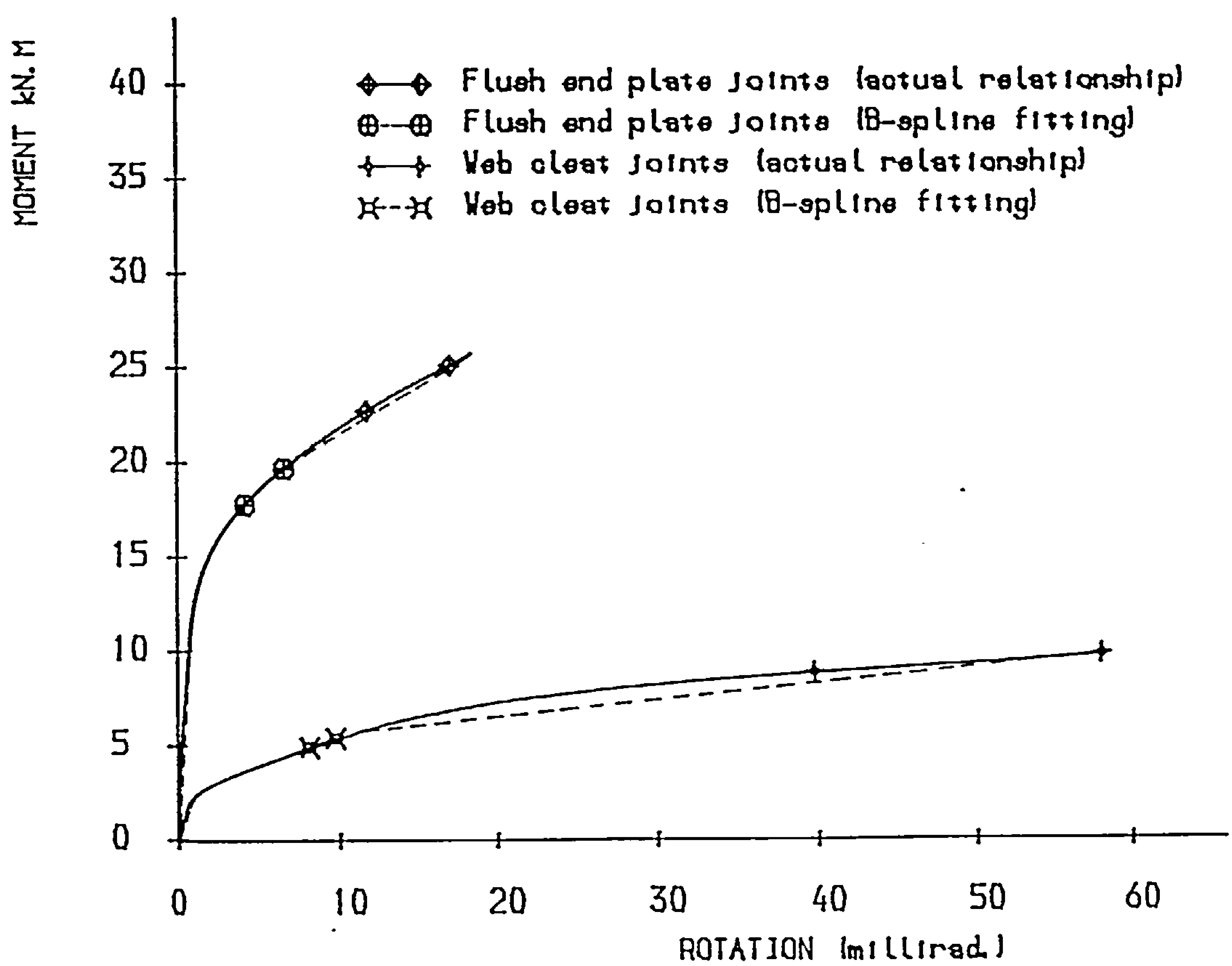


Figure 8.7: $M-\Phi$ characteristics for flush end plate and web cleat joints employed in the subassemblage shown in figure 8.6.

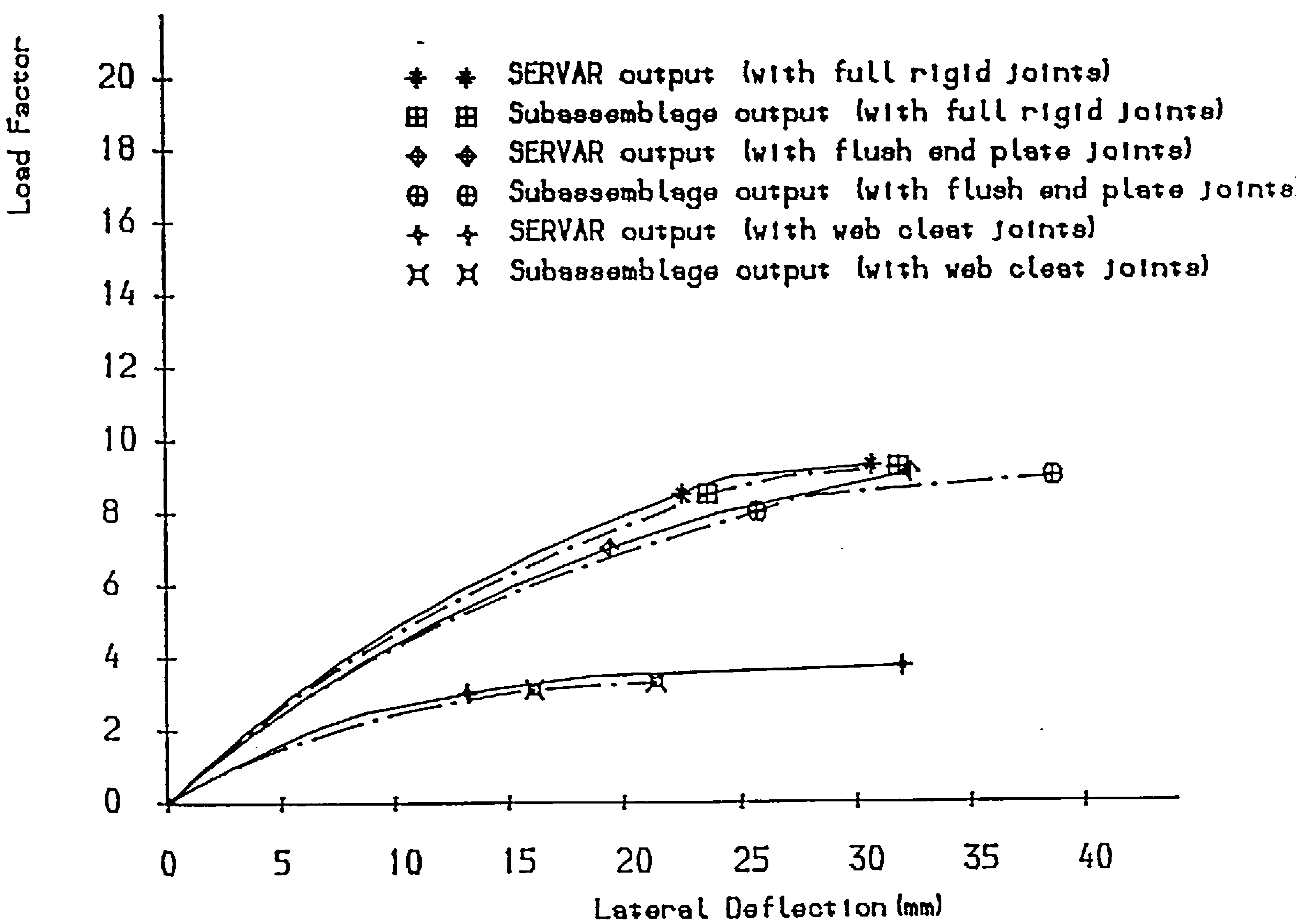


Figure 8.8: Comparison of the predicted behaviour of the subassemblage shown in figure 8.6 using SERVAR and the subassemblage program.

Chapter 9

The Influence of Partial Sway Bracing on Flexibly Connected Column Sub-Assemblages

9.1 Introduction

It has been seen in chapter 7 that any partial sway bracing system resulting for example from the presence of cladding in practical multistorey frames, has a considerable influence in limiting the lateral drift of a frame. This influence therefore provides desirable effects which tends to improve the frame performance and at least partly offsets the weakening effects caused by the presence of semi-rigid joints in real steel structures. Investigating the influence of the presence of partial sway bracing on the stability of flexibly connected steel frames requires a proper understanding of the behaviour of the columns in these frames when such bracing is present because the instability of these members is highly dependent upon the lateral drift. The modified computer program mentioned in chapter 8 has been utilised to conduct a limited parametric study. In this study some of the influential factors on column behaviour will be considered. These are:

1) Column end restraints.

2) Presence of lateral loads.

3) Geometrical imperfections.

4) Residual stresses.

Since item 1) includes the effect of the presence of semi-rigid joints and partial sway bracing, the study will discuss this factor in some detail.

9.2 Description of the Parametric Study

The subassemblage considered shown in fig.9.1. It consists of four identical beams connected to the column by four identical semi-rigid joints. The partial sway bracing is modelled as an elastic spring (with a linear lateral stiffness) attached to the far end of the top right hand beam. Grade 43 steel with yield stress and Young's modulus equal to 275 N/mm^2 and 210 kN/mm^2 respectively has been adopted in this study. The material behaviour has been assumed to be elastic-perfectly plastic, i.e. strain hardening is not considered. The load pattern adopted consists of loads applied both vertically and horizontally at the top end of the column. In each analysis this pattern of load was increased until failure.

9.3 End Restraints

In this study two categorises of end restraints are recognised which are

1) Rotational end restraints resulting from the presence of the beam flexural rigidities via the joint stiffnesses.

2) Lateral end restraints resulting from the presence of infill panels in such a structure.

9.3.1 The Influence of Rotational End Restraints on Steel Columns

In sway structures, lateral loads are resisted by frame action with rotational end restraints which are provided to the columns resulting from the presence of the beams. The magnitudes of these end restraints are dependent upon:

1) the flexural rigidity, EI/L, of the beams

2) the connections which control the continuity between the beams and the columns of the structure. Fully rigid joints provide full continuity whilst semi-rigid connections exhibit some flexibility and therefore provide only partial continuity. This continuity vanishes for ideal pinned joints.

In order to investigate the influence of rotational end restraints on the behaviour of columns in sway sub-assemblage, the arrangement shown in fig.9.1 has been used with the following:

1) a horizontal load equal to 1% of the vertical load.

2) four types of joints have been used which are:

i- rigid joints

ii- extended end plate joints

iii- flush end plate joints

iv- web cleat joints

3) no infill panel is present.This is achieved by assigning a zero value to the stiffness of the lateral elastic spring located at the far end of the top right beam

The $M - \Phi$ characteristics adopted for extended end plates, flush end plates and web cleats are shown in fig.9.2. These have been obtained from reference [3]. The column strength curves for the sub-assemblage have been presented in fig.9.3 for the four joint types assuming that the beam spans are equal to 3.0 m. In producing this figure a second order elasto-plastic analysis via the modified program has been utilised to analyse several subassemblages with different column heights. The failure criterion is taken as the subassemblage ultimate load which clearly represents the column's ultimate load since the beams are unloaded and therefore they are unlikely to cause failure. From this figure it can be seen that the use of extended end plate and flush end plate joints has, in this example, only a slight influence on the column strength curves when compared to that obtained assuming fully rigid joints. Nevertheless the use of web-cleat joints considerably effects the column behaviour causing a sharp reduction in column strength. It should be noted here that this conclusion is valid only within the limits of this study since the beams are unloaded and the bending end moments resisted by these beams are therefore relatively small. Hence these bending moments, for extended end plates and flush end plates, are located in the primary regions of their $M - \Phi$ relationships where the stiffnesses of these joints are very high. However, for web-cleats their flexibility, given by the the slope of the $M - \Phi$ curve, is significantly less and therefore an early column failure due to excessive lateral drift has occurred. This figure also shows that the beneficial influence of rotational end restraints is largest for columns with intermediate slendernesses.

In figs.9.4 and 9.5 the same study was repeated but for beam spans equals 6.0

and 12.0 m respectively. In addition to the above conclusion these figures show that increasing the beam spans (or reducing the flexural rigidity of the beams) caused a reduction of the influence of joint-flexibility on the column strength curve.

Next the column strength curves were drawn for each joint type independently taking the beam spans as 3.0 m, 6.0 m, 12.0 m which produced figs.9.6-9.9 respectively. These figures show that the beam flexural rigidity has only a modest influence on column behaviour and that this influence is largest for columns with intermediate slenderness ratios.

It should be noted that semi-rigid joints, in reality, are expected to exhibit more flexibility than are inferred in this study. This is because the beams in practical frames are usually loaded which means that larger bending moments are transmitted to the corresponding joints leading to a reduction of their stiffnesses. Hence the conclusion that extended end plate and flush end plate joints have comparable behaviour with fully rigid ones cannot be guaranteed.

Summarising, the following may be observed:

1) End restraints have vital roles in stabilising the columns in bare sway subassemblages. These restraints result from the combined action of beam flexural rigidity and connection stiffness. In real steel structures which are designed elastically most beams are usually subjected to loads which are below those which cause large plastic deformations, i.e. the beam behaviour is linearly elastic, therefore the variation of beam flexural rigidities is limited. Nevertheless, the possible variation of joint stiffnesses is relatively large and theoretically ranges from zero for pinned joints to infinite for rigid joints. Utilising relatively light connections, i.e. joints with modest plastic moments and low stiffnesses throughout their $M - \Phi$ curve, pre-

vents the columns, in sway steel structures, from developing their maximum load capacities causing a premature failure due to lack of restraints at their ends.

2) The influence of end restraints in framed columns is largest for columns with intermediate slendernesses.

3) The effect of joint flexibility is reduced when the flexural rigidities of the corresponding beams are low.

9.3.2 The Influence of Partial Sway Bracing in a Flexibly Connected Column Subassemblage

In real structures the existence of infill panels will inevitably increase the resistance of these frames against lateral drift leading to an improvement in the structural performances. BS5950:Part1:1985 allows the inclusion of this effect by means of a ficticious steel diagonal bracing element with a specified cross sectional area. This area, A, depends upon the frame geometry together with the material properties of panels. Calculating A can be accomplished utilising equations 7.1-7.3 and two examples have been given in chapter 7 to demonstrate their use.

In order to investigate the influence of the presence of lateral bracing in the behaviour of framed columns in sway structures, the subassemblage shown in fig.9.1 has been analysed. In this study the presence of an infill panel is simulated by an elastic spring connected to the top right beam with a constant stiffness S throughout the whole loading range which is equivalent to the use of a diagonal member. In this section flush end plates are adopted, as they are representative of mid range semi-rigid joints. The beam spans are taken as 3.0 m and the column height as 6.0 m. The lateral deflections of the frame are computed for increasing loads. In figure 9.10 the ordinate is the frame vertical load, which is at the same time equal to 100 times the frame horizontal load, nondimensionalised by dividing by the column squash load whilst the abscissa is lateral drift. The computation is repeated for several values of the lateral spring stiffness S multiplied by ${L_c}^3/EI_c$ to produce the nondimensional panel stiffness $\overline{S}$, i.e.$\overline{S} = S\ {L_c}^3/EI_c$ in which:

L_c is the column height;

S is the stiffness of the lateral spring and

I_c is the second moment of area of the column.

It should be noted that a conservative estimate of $\overline{S}$ for practical frames is given in appendix E of BS5950 [4] as K_3.

Fig.9.10 shows that, as expected, for any value of horizontal load the lateral deflection of the subassemblage is significantly reduced with increasing stiffness of infill panel. This improved control of lateral deflections permits the attainment of higher ultimate loads with increasing panel stiffness. Also this figure shows that, although the stiffness of the lateral elastic spring is assumed to be constant throughout the whole loading range, the horizontal load-displacement relationships are still nonlinear. That is due to the secondary effects of the sub-assembleges since the lateral elastic spring is, in this case, subjected to the axial load of the top right beam and not to the applied horizontal load.

It has been shown that the improvement of the column performance obtained from the presence of an infill panel of sway structures is mainly dependent upon $\overline{S}$, the relative stiffness of the lateral bracing provided by the panel to the column stiffness. In practical sway structures different arrangements for infill panels can be adopted and hence the corresponding columns are provided with different degrees of lateral restraint. In order to study the variation of the column ultimate load in the presence of varying degrees of lateral bracing, the lateral spring stiffness $\overline{S}$ is increased from zero (no lateral bracing) to a relatively large value, taken here as 700. Fig.9.11 has the relative stiffness of the spring $\overline{S}$ presented on the horizontal axis while the ultimate column load nondimensionalised by dividing by the column squash load, is plotted on the vertical axis. Seven values of the column height have been considered ranging from 4.0 m to 15.0 m, i.e. the column slenderness varies from a stocky 62.3 to a very slender 233.9. Each of the seven curves produced starts from a point where the panel has no resistance to sway, i.e. the stiffness of the lateral elastic spring is taken as zero. By increasing the stiffness of the spring, the ultimate load of the column is increased until it reaches a stage where independency between the spring stiffness and the column ultimate load has almost been reached. At this point failure is by column failing as non-sway member. This stage is indicated as a flat portion located at the end of each of the curves.

It is worth mentioning here that it has been assumed that the presence of infill panels controls only the lateral drift of the subassemblage. In reality the presence of such panels might also influence the forces and the bending moments of the elements of the corresponding frames. However this feature needs to be investigated experimentally before the inclusion of such influence.

It can be concluded from fig.9.11 that partial sway bracing has great influence on framed column behaviour and this influence is dependent upon the stiffness of the infill panels which depends on the dimensions and the material properties of the panels. For most practical frames the stiffnesses of infill panels normally provides modest resistance against lateral drifts, therefore the behaviour of the corresponding framed columns are expected to fall in the curved part of the characteristics.

This figure also indicates that:

1) A relatively large spring stiffness is required to reach the flat part of the curves indicated in fig.9.11 for columns with high slen-

dernesses.

2) For a column with a low slenderness, a modest value of the elastic spring stiffness $\overline{S}$ produces a significant increase in the column ultimate load and, in some cases, allows the column to develop its full squash load.

In fig.9.12 the column strength curves are drawn for a constant beam span of 3.0 m and taking the horizontal load as 1% of the vertical load, for five values of $\overline{S}$ which are 0, 1, 2, 5 and 10. This figure shows that the increase of the panel stiffness, i.e. the increase of $\overline{S}$, produces the following effects:

1) It enlarges the flat plateau in which the column resists its full squash load.

2) It improves the column performance and this improvement is most pronounced for columns with intermediate slenderness ratios.

It should be noted here that BS5950:Part1:1985 restricts the value of $\overline{S}$ not to exceed 2. However even obeying this restriction, a significant improvement can still be obtained.

It has been concluded in the previous section that the beam flexural rigidity, I/L, has modest influence on the column behaviour (see figs. 9.6-9.9). In order to study this influence where an infill panel is present, two column strength curves are plotted in fig.9.13 assuming that the relative infill stiffness $\overline{S}$ is equal to 10 and taking the beam spans as 3.0 and 12.0 m. This figure shows that the beam flexural rigidities have only a very small effect on the column ultimate loads. This conclusion can be explained by noting that the presence of infill panel (i.e. the lateral elastic spring) reduces the lateral drift of the column which reduces the nonlinear geometrical deformations of the beams. Hence the effect of the I/L values of the beams is less than that corresponding to the case where the partial sway bracing is absent. This conclusion is valid only when beams are unloaded since the nonlinear deformations of these beams are caused, in these circumstances, by the lateral drift only.

It has been demonstrated that both rotational and lateral end restraints provide similar effects in the sense that they tend to stabilise the column leading to higher ultimate column loads.

In order to assess the relationship between panel stiffness, the beam end restraints and the ultimate column load, figures 9.14-9.19 have been produced. In these figures the horizontal axis represents the ratio of the relative panel stiffness, $\overline{S}$, to the nondimensionalised beam end restraints $\overline{\alpha}$ defined as

$$\overline{\alpha} = \frac{\overline{R_k}}{M_{pc}} \tag{9.1}$$

where

$$\overline{R_k} = \frac{3EI_g}{L_g}\left[\frac{1}{1 + \frac{3EI_g}{L_g\,K}}\right] \tag{9.2}$$

in which

L_g is the beam span.

I_g is the beams second moment of area.

K is the joint stiffness which has been assumed to be constant and equal to 13.60×10^5 $kN.cm/rad.$.

M_{pc} is the column plastic moment.

$\overline{R_k}$ is the column rotational end restraints including semi-rigid joint behaviour.

Details of the derivation of eq.9.2 can be found in appendix A.

It should be noted here that $\overline{\alpha}$ has been obtained from a similar study by Lui and Chen [12] on the effect of end restraint on non-sway framed columns.

The vertical axis of these figures is the percentage increase of the ultimate column loads obtained by including the effect of the presence of partial sway bracing $P + \Delta P$ relative to those obtained when such bracing is not considered P (i.e. as 100 ΔP / P).

In figs.9.14-9.17 the ratio of horizontal to vertical loads is taken as 1% and four different values of $\overline{S}$, i.e. 1,2,5,10 respectively, have been considered. In each figure $\overline{S}$ has been kept constant while $\overline{\alpha}$ has been varied by considering different beam spans. It can be seen that over the range considered approximately linear relationships between the ratio of $\overline{S}$ to $\overline{\alpha}$ and the percentage increase of the ultimate column loads seem to hold and these relationships are not heavily dependent on the column slendernesses. This provides an attractive manner in which designers can include the effect of the presence of infill panels in practical multistorey frames by simply enhancing the ultimate loads of the framed columns obtained from neglecting the presence of infill panels. This can be achieved using the ratio of $\overline{S}$ to $\overline{\alpha}$ which can be easily calculated once the first selection of beam and column sections are determined together with the geometrical dimensions of the frames and the material properties of the infill panels. An approximate lower bound solution for the points of figures 9.14-9.17 can be represented by the following simple equation:

$$100\,\frac{\Delta P}{P} = 7 \times \overline{S} + 200\left(\frac{\overline{S}}{\overline{\alpha}}\right) \quad but \quad P + \Delta P \leq P_y \qquad (9.3)$$

In fig.9.17 several points, which belong to the column with height 4.0 m, i.e. $L_c/r = 62.36$, appear below the approximate lower bound line. This overestimation of the percentage increase in the ultimate column load resulting from eq.9.3 is related to the inelastic behaviour of the column. The low slenderness ratio (L_c/r) caused the domination of the inelastic behaviour on the ultimate column load rather than the lateral bracing, i.e. the large stiffness assigned to the lateral spring $\overline{S} = 10$ provided lateral restraint which is sufficient to increase the ultimate load of the column beyond its squash load. This is physically impossible and therefore the column fails at its squash load. This conclusion can be verified in conjunction with fig.9.13 in which it can be seen that columns which are supported laterally by an elastic spring with relative stiffness $\overline{S}$ equal to 10 and have slenderness ratios L_c/r lower than 70 sustain their full squash loads. The restriction imposed on eq.9.3 that the enhanced value of the ultimate column load ($P + \Delta P$) does not exceed its squash load P_y will insure against such overestimation.

In order to study the influence of the variation of lateral loads on the percentage increase of the column ultimate load with respect to the ratio ($\overline{S}$ *to* $\overline{\alpha}$), fig.9.14 has been repeated but for horizontal loads equal to 0.5% and 5.0% of the vertical load respectively. This has produced figs.9.18 and 9.19 where it can be seen that with an increasing value for the ratio of the horizontal to the vertical loads an even larger benefit in terms of enhanced column capacity can be obtained. However eq.9.3 provides a simple and generally conservative manner for including such an effect.

9.4 Compensation of Effects of Semi-rigid Joints and Infill Panels

It has been shown that end restraints have a great influence on framed columns in sway-permitted steel structures. Utilising semi-rigid rather than rigid joints reduces the continuity between the columns and the beams leading to larger lateral drifts therefore reducing the ultimate loads of the corresponding columns. Nevertheless, the presence of a lateral bracing system provides beneficial effects by resisting the lateral drift leading to an improvement of the column ultimate loads. In order to investigate the influence of the simultaneous presence of both semi-rigid joints and infill panels, two sets of column strength curves have been indicated in figs.9.20 and 9.21. In fig.9.20 the utilised joints are flush end plates

,the beam spans are taken as 3.0 m and the horizontal load is 1% of the vertical load. Three column strength curves are represented in this figure corresponding to three different arrangements which are:

1) a full rigidly connected subassemblage.

2) a semi-rigidly connected subassemblage without an infill panel.

3) a semi-rigidly connected subassemblage including the presence of an infill panel with a relative stiffness $\overline{S}$ equal to 1.

This figure shows how the presence of an infill panel with a small stiffness, i.e. $\overline{S} = 1$, compensated for the presence of semi-rigid joints leading to a column strength curve almost identical to that corresponding to the full rigidly connected subassemblage. In fig.9.21 this comparison has been repeated but for web-cleat joints instead, where it can be clearly seen that since web cleats are relatively weak joints, a stiffer infill panel with relative stiffness $\overline{S}$ equal to 5 is required to match the weakening effects resulting from utilising web cleat joints.

9.5 The Influence of the Presence of Lateral Loads

Sway structures are, in reality, subjected to both horizontal and vertical loads. Whilst the presence of horizontal loads might not affect the elastic behaviour of the corresponding columns, it is believed that such loads play vital roles in the inelastic behaviour of these columns. This is because these loads tend to increase the column end moments which might cause the performance of some plastic hinges hence accelerating the column failure. The isolated column (see fig.9.22.a) which is fully fixed at the base and which has perfect rotational restraint at the top, i.e. only vertical and horizontal movements are allowed at the top end, has been used to investigate the influence of the the presence of horizontal loads on the inelastic behaviour of steel columns. The adopted load pattern is also shown in figure 9.22.a which consists of horizontal and vertical loads applied to the column head. In this section no reference is made to the presence of lateral bracing or semi-rigid joints since the influence of the presence of such factors has been investigated and it can be easily concluded now that:

1) Utilising semi-rigid joints in a structure causes a reduction of the ultimate load of the corresponding columns due to a reduction of their rotational end restraints.

2) The presence of lateral bracing resulting from the presence of lateral bracing, improves the column performances as a result of reducing the lateral drift of the columns.

This section is devoted to investigate the influence of the presence of lateral loading in the absence of lateral bracing with the assumption that the beams have infinite stiffnesses. The same influence is expected to be obtained if lateral bracing or flexible joints with finite beam stiffnesses are considered.
The column strength curves are shown in figure 9.22.b for four sets of horizontal loads represented by their ratios to the applied vertical loads which are 0.5%,1%,5% and 10% .
It is evident from this figure that the column ultimate loads are very sensitive to the presence of the horizontal loads and by increasing the ratio of the horizontal to the vertical loads the column ultimate loads are sharply reduced. Therefore in order to obtain a satisfactory estimation of the carrying capacity of such a column in sway structure, this influence has to be included. A design procedure which account for such influence is included in section 9.8.1

9.6 The Effect of the Presence of Geometrical Imperfections

In sway structures the effect of lack of verticality of the columns can be simulated by subjecting these structures to fictitious horizontal loads (usually taken as 0.5% of the vertical loads). The effect of the presence of horizontal loads has been investigated in the previous section and thus the presence of lack of verticality has a corresponding effect.

9.7 The Effect of Residual Stresses on Framed Steel Columns

Two commonly used patterns for residual stresses, the British and Lehigh patterns (see figure 8.4), have been used in conjunction with the modelled column shown in figure 9.22.a to study the effect of the presence of residual stresses on sway column sub-assemblages. The horizontal load is taken as 0.5% of the vertical load. Three column strength curves are indicated in figure 9.23. These represent three different arrangements which are:

1) the column is assumed to be free from residual stresses.

2) the residual stresses distribution over the column section is assumed to be parabolic,the British pattern [63].

3) the residual stress distribution known as the Lehigh pattern widely used in U.S.A [63] is adopted.

This figure shows clearly that residual stresses have little effect on the behaviour of framed steel columnsand that this effect is not always undesirable since for columns with low or intermediate slendernesses this effect might even lead to a slightly increase of the ultimate column loads. This conclusion can be explained by noting that when residual stresses were included the column sections over the whole column length were found to yield at a lower load than the case where residual stresses were not considered. This leads to a considerable reduction of the column rigidity which therefore reduces the column resistance against lateral drifts. Thus smaller bending moments are transmitted to the column from the beams which are assumed to have infinite stiffnesses here, due to the lateral movement of the subassemblage. This causes, in some cases, a slight increase of the column ultimate load. Nevertheless if residual stresses are not considered, only the column ends were found to yield which causes the development of a relatively large bending moments at the column ends and rapidly forming a failure mechanism with most intermediate column sections still below the yield stress.

This behaviour is for rigid jointed subassemblages where the beams have infinite stiffness. To investigate the circumstances where the column is connected to beams with finite stiffnesses, the process used to develop figure 9.23 has been repeated to produce figure 9.24 but assuming that the column is connected at its ends to two rotational springs with stiffnesses $\overline{R_k}$ equal to 10 times the column rotational stiffness, i.e. $\overline{R_k}\ L_c/\ E\ I_c\ =\ 10$. This figure shows that the beneficial improvement resulting from the presence of residual stresses has now disappeared since the beam flexural rigidities are severely reduced. In general figures 9.23-9.24 suggest that residual stresses have a negligible effect on framed columns in sway frames, therefore this effect can be neglected. It should be noted that utilising semi-rigid joints in practical frames will result in reducing the column end restraints and this case has been considered in figure 9.24.

9.8 Design Application for Practical Framed Columns

9.8.1 Without Infill Panels

It can be concluded from the limited parametric study that the behaviour of columns in sway frames is primarily dependent upon the lateral sway of these elements, i.e. on the bending moments induced at their ends. In order to simulate the behaviour of practical columns in sway frame structures, the column indicated in figure 9.25.a has been adopted. It can be seen that the rotational end restraints provided by the flexural rigidities of the beams via the semi-rigid joints which are modelled as two equivalent rotational springs located one at each end of the column.

Theoretically the stiffness of the end restraints of the columns in a sway structure can range from zero upwards. In simple construction, the stiffness of both rotational springs are assumed to equal zero and the horizontal loads must be resisted by a bracing system. In such circumstances the subassemblage of fig.9.25(a) is a mechanism.

In general in a practical structure however, non-zero rotational restraints are provided to the framed column and these end restraints are dependent upon the beam flexural rigidities and on the joints used. When employing rigid framing designers are usually at pains to create very stiff joints involving typically extended end plate connections and the use of stiffeners. A variety of end restraints can be met in practical framed columns due to stiff joints and varying beam sections and it may be possible to widen this with the use of somewhat more flexible connections. To investigate the influence of rotational spring stiffness $\overline{R}_k$ on the load carrying capacity P, figure 9.25.b has been produced. In this figure the nondimensionalised column ultimate loads P/P_y is plotted against the nondimensionalised rotational stiffness $\overline{R}_k.L_c/EI_c$. Five different column heights equal to 4.0,6.0,8.0,10.0 and 12.0 m have been considered which produced the five curves indicated. The ratio of the applied horizontal to vertical loads is taken here as 1%. This figure shows that:

1) the effect of end restraints are dependent upon the slenderness ratio L_c/r of the column and

2) the rate of the increase in the column ultimate load is higher for a column with a low or intermediate slenderness than for a column with a high slenderness.

In order to facilitate the use of these curves in estimating the ultimate load of a column in sway structures two obstacles have to be overcome which are:

1) The dependency between the column slenderness ratio L_c/r and the effect of the presence of rotational end restraint on the column ultimate loads. Hence a large number of curves to cover the whole practical range of the slenderness ratio in real framed columns are required in order to provide design charts which can be used in design offices.

2) The assumption that the column has equal rotational restraints at both ends. In real steel structures however, this not always satisfied.

In order to overcome the first obstacle a search for the best formula to represent the previously mentioned curves has been achieved utilising a computer program and the result is indicated in fig.9.26. In this figure the nondimensionalised column ultimate load has been replaced by the formula $(P/P_y) \times (L_c/r)^{1.6}$ which reduced the spread of the five curves to a condition where they can be approximately represented by a single lower bound curve. Thus only one curve is required to estimate the ultimate load of a column in a sway structure regardless of its slenderness ratio providing that the ratio of the horizontal load to the vertical loads acting on the column is 1%.

To overcome the second obstacle it is suggested that the average value of the end restraints in practical steel framed columns be utilised in conjunction with fig.9.26.

To study the effect of this approximation, the column indicated in fig.9.27.a has been used. This has, at its base, a rotational spring with a stiffness equal to three times the value of the one located on the top end of the column. The average value of both spring stiffnesses is utilised to obtain two curves representing the relationship between P / P_y and $R_k L_c / E I_c$ and for column heights equal to 4.0 and 12.0 m. These curves are compared to those obtained by assuming average end restraint values and it can be seen that a good correlation has been obtained. This indicates that using the average value of the rotational end restraints of steel framed columns in sway structures in conjunction with fig.9.26 provides satisfactory accuracy for design purposes.

Finally it should be kept in mind that the curve indicated in figure 9.26 is derived for a column subjected to a horizontal load equal to 1% of its vertical load. In reality different values for the ratio of horizontal to vertical loads can be met (which are normally higher than 1%), therefore three additional curves indicated in figs.9.28-9.30 has been produced. They are similar to fig.9.26 but the ratio of the horizontal to vertical load has been taken as 2% , 5% and 10% respectively. For other different ratios an interpolation between these curves is possible. Figs.9.26 and 9.28-9.30 show that with increasing the ratio of the applied horizontal to vertical loads, the column load-carrying capacity is significantly reduced.

9.8.2 With Infill Panels

Incorporating the effect of the presence of infill panels in real semi-rigidly connected sway structures can be accomplished as follows:

1) Calculate the average value of a column rotational end restraints and use one of the curves indicated in figs.9.26,9.28-9.30, according to the ratio of the applied horizontal to vertical loads of the column, to calculate its ultimate load, neglecting the presence of infill panels.

2) Knowing the dimensions and the material properties of the infill panels, $\overline{S}$ can be calculated and an enhanced factor for the column ultimate load can be then obtained from eq.9.3 which may then be applied to the ultimate load found in 1) above.

9.9 Comment and Discussion

It should be kept in mind that the design recommendation included in this chapter is in general limited to the following condition:

1) Grade 43 steel as specified in BS5950 has been assumed. However for high yield steels different curves similar to the ones shown in fig.9.26,9.28-9.30 can be drawn to represent the relationship between the column end restraints and the column ultimate load.

In the design of framed columns, two classes of steel structures are usually recognised and these are:

1) 'Simple construction' where the columns are usually designed as axially loaded elements with a nominal moments transferred from beams to columns to account for the eccentricities between the centre line of the column and the assumedcentre of bearing at the con-

nection. BS5950 [4], for example, recommends including a nominal moment equal to the applied axial load multiplied by the greatest of either (i) the half column section depth or (ii) 100 mm. Interaction formulae are then used to check the capacity of the section and the overall stability of the column.

2) 'Continuous construction' in which a similar procedure to designing columns in 'simple construction' is normally used except of that the bending moments transmitted to to the corresponding column ends are defined by a rigid frame analysis (linear or nonlinear analysis as appropriate).

The former class are usually non-sway structures since the rotational discontinuity between the columns and the beams requires the existence of a bracing system to resist lateral loading. The second are usually sway structures since lateral loads can be resisted by the framed elements, i.e. the beams and the columns.

In reality most practical steel frames fall in between these two idealised extreme cases since all commonly used steel joints have been experimentally proved to be stiffer than pinned joints and more flexible than rigid joints. In addition the presence of cladding in real steel structures provides partial resistance against lateral drifts therefore few structures are correctly simulated as fully sway structures the exception being unclad frames such as may exist in plant structures with very heavy connections.

The design of a framed column in sway structures is normally accomplished by adopting a trial selection for the column section and then determining the column axial load and the bending moment pattern to employ the interaction formulae to check the capacity and the overall stability of the column. This, of course, involves the calculation of the effective length factor for the column considered which should include the inelastic behaviour of the column. In the proposed method all of these steps are incorporated simultaneously and only the column end restraints of the column are required to obtain a safe estimate for the column's ultimate load.

Moreover the proposed method has the advantage of including the influence of the presence of column end moments implicitly in the curves indicated in figures 9.26 and 9.28-9.30. This is because such moments result from the following sources:

1) The presence of lateral loads in sway structures.

2) The presence of rotational end restraints.

3) The presence of vertical loads applied to the beams and resulting in moments transferred to the columns through the existing full or partial continuity between the beams and the column under consideration.

Both items 1) and 2) are included in the proposed method and only item 3) has been ignored since, in the development of the design charts, the column is assumed to be axially loaded by a vertical load with no loads on the beams. In a practical frame however three categories of columns can be met which are:

1) Columns located on the windward side of the frame in which the bending moments due to vertical loads are subtracted from those arising due to horizontal loads therefore the total column bending moments are likely to be reduced to a small value. The proposed method provides conservative estimations for the inelastic column ultimate loads.

2) Interior columns where the bending moments due to the applied vertical loads are approximately balanced. The proposed method is

ideally suited this category of column.

3) Columns located on the leeward side of the frame in which the bending moments due to vertical loads are additive to those due to horizontal loads. Therefore the proposed method is unconservative here. Nevertheless the method can still be used to design these columns since the bending moment due to horizontal load is expected to dominate the ultimate column load at failure.

It is believed that the proposed method can be used by designers to make a reasonable selection for the column section and the selected section obtained can be checked against the interaction formulae available in most steel codes for designing beam-column elements. A check is required to ensure against additional possible failure modes such as out-of-plane behaviour, which is not considered in this study which is limited to in-plane behaviour.

9.9.1 The End Restraints of Framed Steel Columns

It has been shown that defining the ultimate column load requires the determination of the column rotational end restraints. In defining these restraints the flexural rigidities of all elements connected to the column ends together with the stiffnesses of the utilised joints needs to be considered. In most practical steel frames the beams are usually designed elastically therefore their full flexural rigidities I/L can be taken when calculating such restraints. However when some beams are heavily loaded, these elements are expected to develop plasticity under the externally applied loads and their flexural stiffness should be reduced or neglected. Column end restraint is also dependent upon the stiffnesses of the steel joints used to attach the column to its neighbouring elements. Assuming that the rotational stiffnesses of these joints can be considered to be constant throughout the whole loading range, an expression for the rotational

Mode	Far end condition		n
No sway	Flexible connection	single curvature bending	2
Sway	Flexible connection	double curvature bending	6
Any	Simply supported		3
Any	Fully fixed		4

Table 9.1: The constant factor (n) used in equation 9.4 to calculate column end restraints for beam with differing far end conditions

end restraints, resulting from the presence of an elastic beam element connected to the column by a semi-rigid joints with differing far end conditions, can readily be obtained as:

$$\overline{R}_k = \frac{n\,E\,I_g}{L_g}\left[\frac{1}{1+\frac{n\,E\,I_g}{K\,L_g}}\right] \tag{9.4}$$

in which

$\overline{R}_k$ is the rotational end restraint resulting from the presence of an elastic beam element with a second moment of area equal to I_g and length L_g connected to the column by an elastic spring with stiffness K.

E is Young's modulus

n is a constant value of which is dependent upon the far end condition of the beam.

The constant (n) is tabulated in table 9.1 for the most known practical cases. The first two rows of table 9.1 has been derived by Lui and Chen [60], the third case is derived in Appendix A and the final case, i.e. the far end is assumed to by fully fixed can be easily obtained by following the procedure outlined in Appendix A. It should be noted that for first two end conditions the beam is assumed to be connected at the far end by a semi-rigid joint with stiffness K, i.e. the beam has equivalent rotational springs at both ends.

In many previous studies the rotational spring stiffness has usually been taken as constant corresponding to the initial tangent of the joint $M-\Phi$ relationship.

This involves an overestimation of the joint stiffness since such a relationship has been shown to be nonlinear. This leads to an overestimation of the column end restraints and therefore the ultimate column load is also overestimated. Defining a more appropriate value to take into account the joint stiffness at the relevant joint bending moment, as has been proposed in chapter 6, causes a great increase in complexity for the proposed method. This is undesirable since the method is suggested as a rapid way to make a reasonable selection of the section for the column. To remedy this problem it is recommended that the stiffness of a semi-rigid joint be taken as a fraction of the initial tangent of its $M - \Phi$ relationship. Defining this value can be left to the designers judgement. However it should recognised here that the upper bound for a semi-rigid joint stiffness is the initial tangent while the lower bound is the secant corresponding to the simply supported beam rotation, i.e. taking the semi-rigid stiffness as the slope of the line joining the origin of the $M - \Phi$ relationship and a point of the relationship corresponding to the beam end rotations calculated assuming that the beam is simply supported at both ends. In most cases, especially where no previous knowledge of the behaviour of semi-rigid joint is available, the secant corresponding to simply supported beam rotations can be adopted safely.

9.10 The Verification of the Proposed Method

In order to check the accuracy of the proposed method in predicting the ultimate loads for framed steel columns in partially braced sway structures the subassemblage shown in fig.9.1 has been used with different values for the beams spans, column heights and the utilised joint characteristics. This has led to the production of tables 9.2-9.5 in which the ratio of the horizontal load to the vertical load is taken as 1% in tables 9.2 and 9.3 whilst in tables 9.4 and 9.5 this ratio is taken as 10%. The following procedure has been adopted

Beam Spans 3.0 m								
column height (m) (1)	L_c/r (2)	$\frac{R_k.L_c}{EI_c}$ (3)	$\frac{P}{P_y}\times (L_c/r)^{1.6}$ (4)	$P_{prop.}$ kN (5)	P_{exact} kN (6)	$\frac{P_{prop.}}{P_{exact}}-1$ kN (7)	col.B.M. kN.cm (8)	stability BS5950 (9)
2.0	31.2	9.58	399	774*	812	-0.05	-	-
4.0	62.4	19.15	451	492	549	-0.10	1596	0.78
6.0	93.6	28.74	470	267	327	-0.18	1407	0.37
8.0	124.7	38.31	470	169	214	-0.21	1275	0.28
10.0	155.9	47.89	470	118	149	-0.20	1181	0.30
12.0	187.1	57.47	470	88	111	-0.20	1129	0.31
15.0	233.8	71.84	470	62	75	-0.17	1066	0.32

Table 9.2: Ultimate load for the subassemblage shown in fig. 9.1 assuming fully rigid joints and without lateral bracing
(*) This value has been obtained from fig.9.22.b as recommended in section 9.10.1 since the column is very stocky. Utilising the design chart indicated in fig.9.26 would produce $P_{prop.}$ equal to 1318 kN. Moreover the stability of the column could not be checked according to BS5950: Part1 since the inelastic behaviour ,which dominates this column, is not included in the standard when calculating the effective length factor k_e.

1) Designing the subassemblage according to the proposed method.

2) Calculating the exact ultimate load using the modified program.

3) Calculating the column bending moment, according the predicted value for the subassemblage ultimate load obtained from step 1), using a second-order elastic analysis and then checking the section capacity and the overall stability of the column according to the current design method suggested in BS5950 [4].

9.10.1 Rigid Joints

In table 9.2 the joints are assumed to be fully rigid and the beam spans are taken to be equal to 3.0 m. Seven values for the column heights have been considered ranging from 2.0 m to 15.0 m. Computing the values shown in table 9.2 for the column with a height equal 4.0 m, for example, involves the following:

1) According to the column section which is shown in figure 9.1 the column slenderness $L_c/r = 62.4$.

2) The column rotational end restraints can be calculated for upper and lower ends of the column from eq.9.2 after taking $K = \infty$. and by noting that, in this example,

The beam second moment of area $I_g = 2921\ cm^4$;

The column second moment of area $I_c = 1220\ cm^4$;

The cross-sectional area of the column A is equal to 29.6 cm^2 ;

The column height L_c and the beam span L_g are equal to 4.0 m and 3.0 m respectively;

The yield stress σ_y is taken as $27.5\ kN/cm^2$ thus the column squash load $P_y = 814\ kN$.

Hence

$$\overline{R}_k = \frac{3 \times 21000 \times 2921.0}{300} \times 2 = 1227000 kN.cm$$

The value of $\overline{R}_k$ has been multiplied by 2 since there are two beams at each end of the column.

$$\frac{\overline{R}_k \times L_c}{E \times I_c} = \frac{1227000 \times 400}{21000 \times 1220} = 19.15 \quad \text{(see column 3)}$$

3) Employing the curve indicated in fig.9.26 gives

$$\frac{P}{P_y} \times \left(\frac{L_c}{r}\right)^{1.6} = 451$$

Thus:

$$P = P_{prop.} = 492\ kN. \quad \text{(see column 5)}$$

4) An accurate value for the ultimate column load is obtained utilising the modified computer program mentioned in chapter 8 using

a second order elasto-plastic analysis given P_{exact} as 549 kN. (column 6).

5) Checking both the column section capacity and overall stability (see clause 4.8.3) according to the beam-column design method recommended in BS5950 [4]. In this method a steel column of a sway structure can be designed (clause 5.6.3) either by calculating its bending moments using a first order elastic analysis and then incorporating the effective length factor obtained from the charts included in appendix E of the standard to obtain the column compressive strength from Table 27 . Alternatively the column bending moments obtained from a first order elastic analysis should be enhanced by the amount of the frame simple amplification factor $\lambda_{cr}/(\lambda_{cr} - 1)$ to account for secondary effects and then the actual column height should be used as the column's effective length. Nevertheless it is well known that the amplified sway method suffers from a lack of accuracy for frames with elastic critical load factors approaching unity. Therefore the second method has been used here, except that the column bending moments have been defined utilising a second order elastic analysis. Knowing the column axial load and bending moment the local strength and overall stability checks are required (see clause 4.8.3). For the column being considered, the overall stability check has been found to be more critical than local capacity check. The overall stability equation has been evaluated and the resulting values are tabulated in column 9. These values are required to be equal or less than 1 to guarantee the overall stability of the column.

It can clearly seen, from this table, that the proposed method provided desirable conservative estimations for the considered columns except the case where the column height is taken as 2.0 m, i.e. $L_c/r = 31.2$. This can be explained by noting that, due to the column low slenderness ratio, the column failure has been dominated by the inelastic behaviour limiting the benefit from the presence of relatively large rotational end restraints offered by the beam and the rigid connections, i.e. in this type of columns only small end restraint is required for the column to sustain its maximum load. Therefore they can be simulated as having infinite rotational end restraints at both ends without a significant overestimation of their load carrying capacities. Hence this type of column can be related to figure 9.22.b and only the ratio of the applied horizontal to vertical loads is required together with the column slenderness (L_c/r) to obtain an estimation for its ultimate load. Figure 9.22.b shows that a very stocky column can sustain its full squash load only when the ratio of the horizontal to vertical loads subjected to the column is small and that with higher value for this ratio less value for the ultimate column load is obtained.

Using figure 9.22.b to calculate the ultimate column load for this column and noting that the column slenderness ratio L_c/r is equal to 31.2 and the ratio of the applied horizontal to vertical loads is equal 1% gives $P \ / \ P_y$ equal to 0.95. Hence $P_{prop.} = 774$ kN.

It is well known that this type of column is rarely met in practice, nevertheless the proposed method can still be used for such instances with the condition that its prediction for the ultimate load of a practical framed column does not exceed the value of the ultimate column load obtained from figure 9.22.b.

It is thought that it is desirable when making a first selection for the section of practical steel columns to slightly underestimate their ultimate loads. This increases the possibility for these elements to satisfy more exact and sophisti-

Beam Spans 3.0 cm								
column height (m) (1)	L_c/r (2)	$\frac{R_k.L_c}{EI_c}$ (3)	$\frac{P}{P_y}\times (L_c/r)^{1.6}$ (4)	$P_{prop.}$ kN (5)	P_{exact} kN (6)	$\frac{P_{prop.}}{P_{exact}} - 1$ kN (7)	col.B.M. kN.cm (8)	stability BS5950 (9)
2.0	31.2	9.58	-	-	815	-	-	-
4.0	62.4	19.15	472	516	549	-0.06	1831	1.32
6.0	93.6	28.74	490	280	327	-0.14	1983	0.61
8.0	124.7	38.31	490	177	214	-0.17	1998	0.51
10.0	155.9	47.89	480	121	149	-0.18	1964	0.55
12.0	187.1	57.47	480	91	111	-0.18	1955	0.60
15.0	233.8	71.84	480	63	75	-o.16	1858	0.66

Table 9.3: Ultimate load for the subassemblage shown in fig. 9.1 assuming fully rigid joints and without lateral bracing. Utilising the polynomial shown in figure 9.31

cated conditions recommended by most design codes. That is why the curve indicated in fig.9.26 has been taken as lower bound of the analytical results. Nevertheless in an attempt to improve the accuracy of the proposed method, the analytical results shown in fig.9.26 have been fitted to a fifth order polynomial which has been chosen for its accuracy when compared with polynomials with lower orders. The least squares method has been used in deriving the polynomial. The result is shown in fig.9.31 and the relationship obtained for the column rotational end restraints and the nondimensionalised column ultimate loads can be represented as follows:

$$For\ X \leq 40 \quad : \quad Y = 107 \times X - 9.64 \times X^2 + 0.422 \times X^3 - 8.82 \times 10^{-3} \times X^4 + 7.025 \times 10^{-5} \times X^5 \tag{9.5}$$

$$For\ X \succ 40 \quad : \quad Y = 480$$

in which

X is equal to $\overline{R_k} L_c/EI_c$ and Y is equal to $P\ /P_y \times (L_c/r)^{1.6}$.

Utilising the polynomial and repeating the calculations, as for table 9.2, produced table 9.3 from which it can be seen that greater accuracy has been obtained. It can also be seen that for the subassemblage with column height equal to 4.0 m, the overall stability check, with respect to BS5950, is not satisfied and therefore the column section should be increased in size. Because of this the polynomial equation has been ignored and the curve indicated in fig.9.26 has been adopted instead. It should noted here that, although the overall stability checks were found to be satisfied with values which are surprisingly small for most subassemblages (see table 9.2), i.e. ranging from 0.78 to 0.28, it does not follow that the estimations for the column ultimate loads are unduly conservative since the equation governing such checks is very sensitive to the column bending moments and it is well known that these moments are nonlinearly dependent of the externally applied loads. As an example consider the subassemblage with column height 4.0 m. For an axial load equal to 492 kN. the column bending moments (see second row of table 9.2) equal 1596 kN.cm. and the column overall stability is satisfied. By increasing this load to 516 kN., i.e. by approximately 5%, the column bending moment reached 1831 kN.cm., i.e. an increase of almost 15%, and the overall stability is not satisfied.

This raises an important question on the validity of the first method recommended in BS5950 (clause 5.6.3) which allows the calculations of the bending moments of a steel column using a first order elastic analysis combined with the condition that the column effective length should be used instead of the column actual length in defining the column's compressive strength. It is well known that the column bending moments of a framed column vary nonlinearly with the externally applied loads. Using a first order analysis to calculate the column bending moments this nonlinear increase of the column bending moments cannot be predicted by the BS5950 recommended method. Moreover incorporating the column effective length instead of the column actual length cannot be claimed to allow for this effect since the effective length of a steel column is dependent only on the column and the surrounding beams flexural

Beam Spans 12.0 m						
column height (m) (1)	L_c/r (2)	$\frac{R_k.L_c}{EI_c}$ (3)	$\frac{P}{P_y} \times (L_c/r)^{1.6}$ (4)	$P_{prop.}$ kN (5)	P_{exact} kN (6)	$\frac{P_{prop.}}{P_{exact}} - 1$ kN (7)
2.0	31.2	2.15	81	268	290	-0.08
4.0	62.4	4.30	114	124	136	-0.08
6.0	93.6	6.46	138	79	83	-0.05
8.0	124.7	8.61	152	55	59	-0.07
10.0	155.9	10.76	159	40	43	-0.07
12.0	187.1	12.91	164	31	35	-0.09
15.0	233.8	16.14	171	23	24	-0.04

Table 9.4: Ultimate load for the subassemblage shown in fig. 9.1 assuming flush end plate joints and without lateral bracing

rigidities making no reference to the externally applied loads.

9.10.2 Flush End Plate Joints

It is believed that, in practice the ratio of the horizontal to vertical loads may be greater than 1%. Therefore in tables 9.4 and 9.5 the ratio of the horizontal load to vertical load has been increased to 10%. In table 9.4 the beam spans are taken as 12.0 m and the utilised joints are assumed to be flush end plates (see fig.9.2) and as for table 9.2 and 9.3 seven values for the column heights are considered repeating same procedure to obtain the ultimate column load using the proposed method and comparing those obtained from using an accurate second order elasto-plastic analysis via the modified program. However in BS5950, although the existence of semi-rigid action is recognised, no clear guidance for handling such structures is presented. The column section capacity and the overall stability of the designed columns are not checked according to the code in tables 9.4 and 9.5.

The semi-rigid joints are represented by their initial tangents with a constant stiffness equal to 13.6×10^5 $kN.cm$. This assumption can be justified here by noting that the beams are unloaded and therefore the influence of the nonlin-

Beam Spans 12.0 m					
column height (m) (1)	L_c/r (2)	$P_{prop.}$ kN (3)	$(P+\Delta P)_{prop.}$ kN (4)	$(P+\Delta P)_{exact}$ kN (5)	$\frac{(P+\Delta P)_{prop.}}{(P+\Delta P)_{exact}} - 1$ kN (6)
2.0	31.2	268	326	525	-0.38
4.0	62.4	124	151	212	-0.29
6.0	93.6	79	96	118	-0.19
8.0	124.7	55	67	77	-0.13
10.0	155.9	40	49	56	-0.13
12.0	187.1	31	38	42	-0.10
15.0	233.8	23	28	29	-0.03

Table 9.5: Ultimate load for the subassemblage shown in fig. 9.1 assuming flush end plate joints and with the presence of lateral bracing with relative stiffness $\overline{S}$ equal 2.

ear behaviour of the joints can be ignored. For other circumstances a different value for the joint stiffness should be calculated according to section 9.9.1. The column end restraints are calculated using eq.9.2, whilst the actual $M - \Phi$ relationship including the nonlinear behaviour is used in determining the exact value for the ultimate column load shown in column (6). Once again this table shows that the proposed method provides reasonable but conservative estimations for the ultimate column loads for all cases considered. Finally in table 9.5 the previously used column subassemblages, used in table 9.4, are assumed here to be connected at the top right end beam by lateral elastic springs with a relative stiffness $\overline{S}$ equal to 2. The estimations for the column ultimate load corresponding to the case when these lateral springs are not present are obtained from table 9.4 column (5) and tabulated here as column (3). These are enhanced according to eq.9.3 after calculating the column end restraints $\overline{R_k}$ from eq.9.2 and by noting that for the column section being used $M_{pc} = 5320\ kN.cm$.

$$\overline{R_k} = \frac{3 \times 21000 \times 2921}{1200}\left[\frac{1}{1 + \frac{3 \times 21000 \times 2920}{1200 \times 13.6 \times 10^5}}\right] \times 2 = 275600\ kN.cm$$

Hence

$$\overline{\alpha} = \frac{275600}{5320} = 51.8$$

and using eq.9.3 gives

$$100\,\frac{\Delta P}{P} \;=\; 7\times 2 \;+\; 200\times\frac{2}{51.8} \;=\; 21.7$$

The percentage increase of the column ultimate loads for the considered cases are uniform since both $\overline{\alpha}$ and $\overline{S}$ are uniform for the seven column subassemblages being considered. Enhancing the column ultimate loads tabulated in column (3) by 21.7% produces column (4). Then the modified computer program is used to accurately predict the ultimate loads of these subassemblages column (5) and a comparison between those both prediction are tabulated in column (6) in which it can be seen that the prediction of the proposed method are even more conservative now after incorporating the effect of partial sway bracing. This confirms the conclusion which has been reached in section 9.3.2 which states that the empirical eq.9.3 is even more conservative when the ratio of horizontal to vertical load subjected to the column is greater than 1%.

9.11 Conclusion

It has been shown that end restraints have an important influence on framed columns of sway structures and these restraints can be either rotational resulting from the presence of beams and connection stiffnesses or lateral restraints resulting from the presence of cladding in practical structures. Residual stresses have been found to have little influence on such columns and therefore this effect can be neglected. Nevertheless the presence of lateral loading or geometrical imperfections has been found to highly influence the behaviour of framed columns in sway structures since these tend to increase the column bending moments.

In this chapter a simple design method has been developed which provides a rapid and safe estimation of ultimate load for a steel framed column including

1) the effect of end restraints;

2) the effect of the presence of lateral loading;

3) including the inelastic behaviour of the column.

The presence of partial sway bracing resulting from the presence of cladding in a practical steel structure has been proved to provide a beneficial effect. An empirical equation to include such influence has been included. It is simple to use and has been found to provide safe results, within the limits of the parameters considered, therefore it is considered to be ideally suited to design office use.

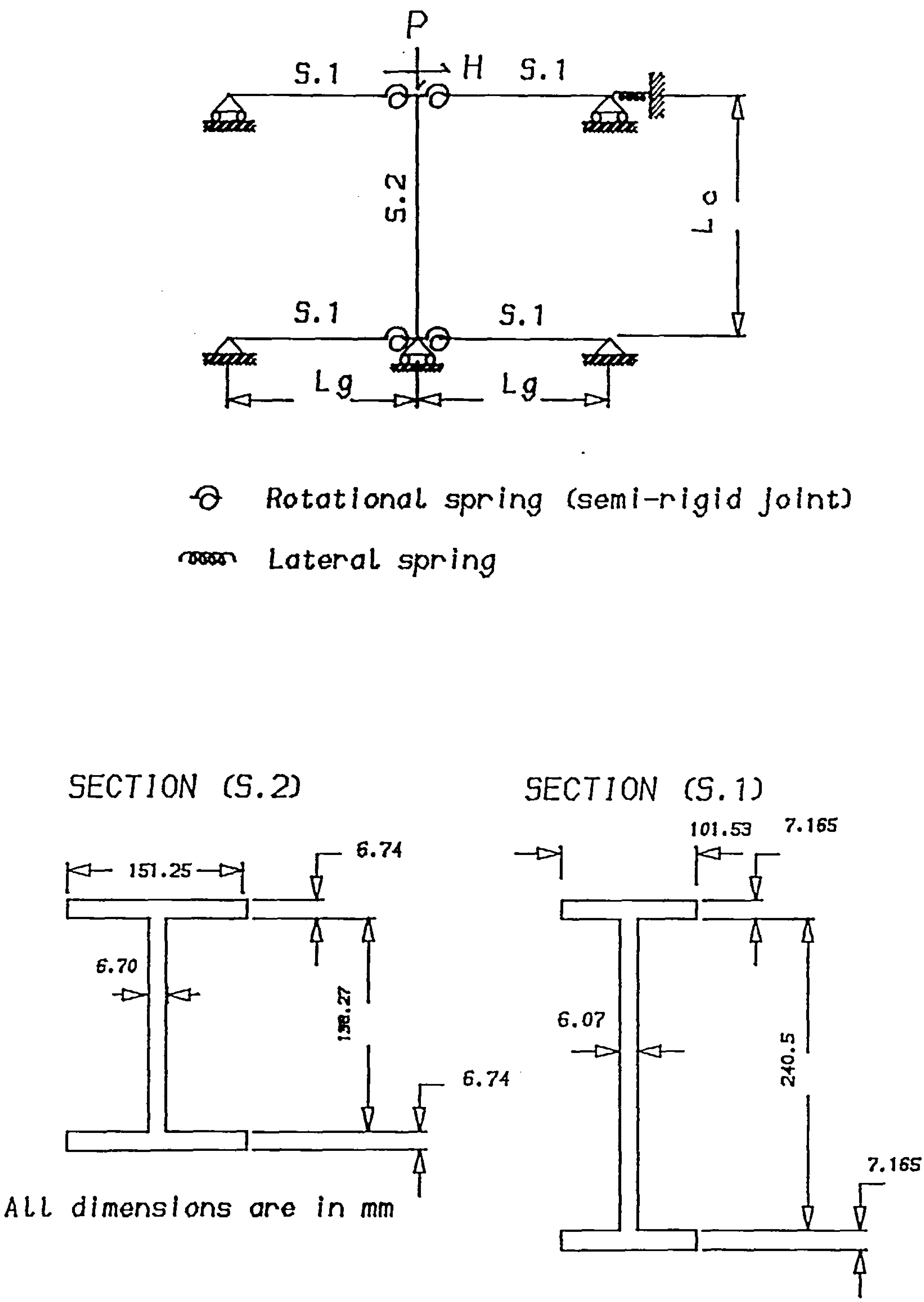


Figure 9.1: Subassemblage employed in conducting the parametric study

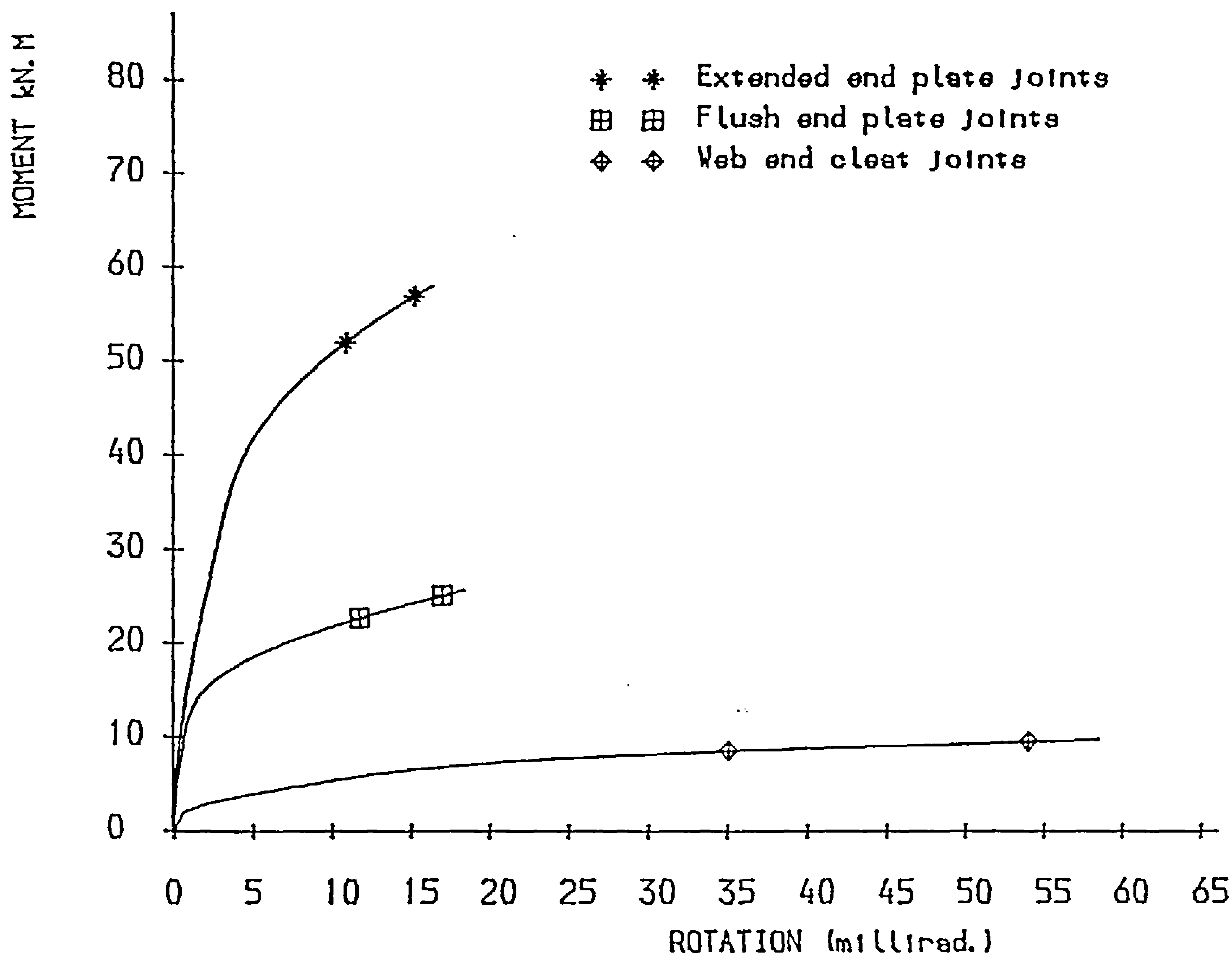


Figure 9.2: M-Φ relationship for extended end plate, flush end plate and web cleat joints.

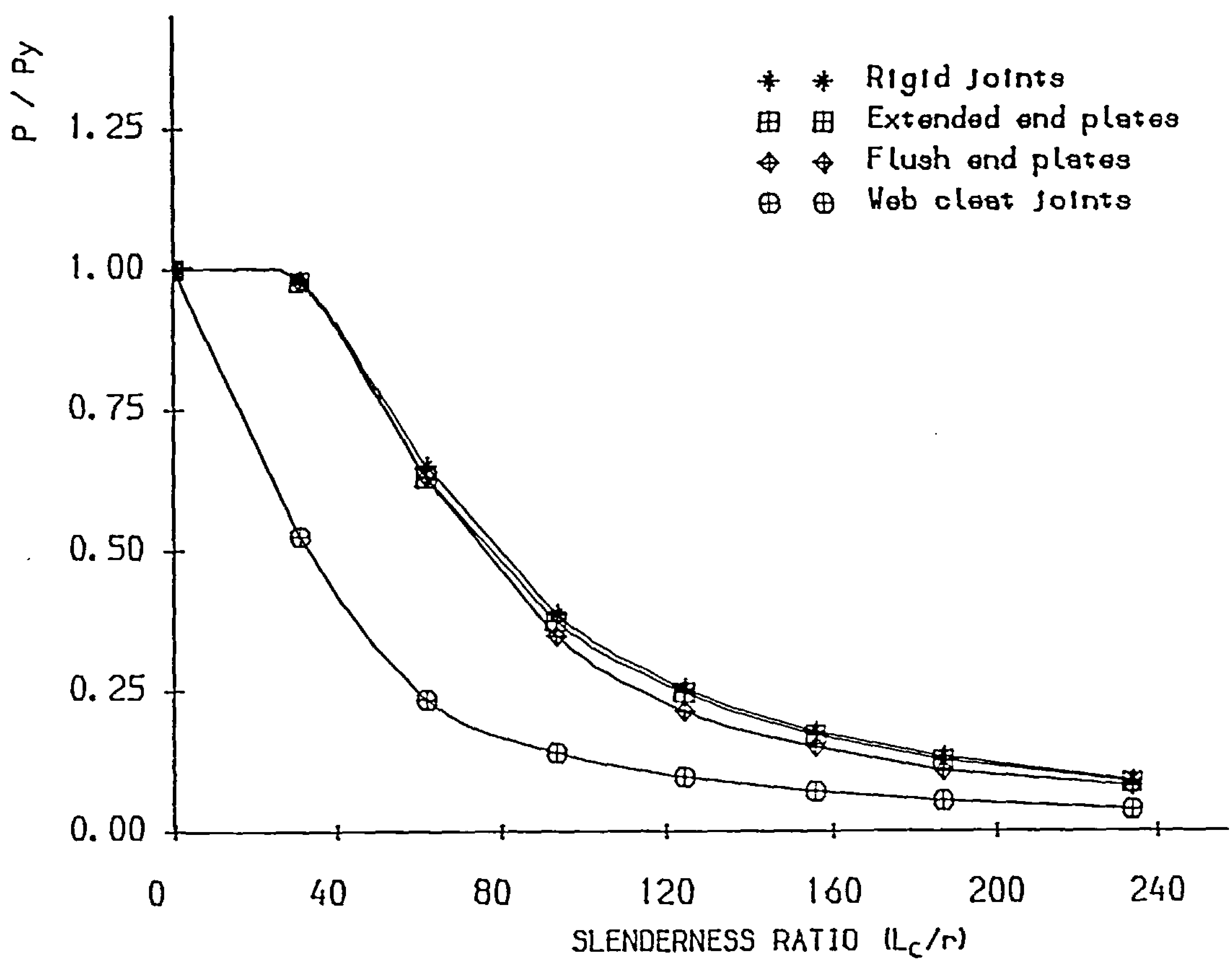


Figure 9.3: Column strength curves obtained for beam spans of 3.0 m and for different joint types without partial sway bracing.

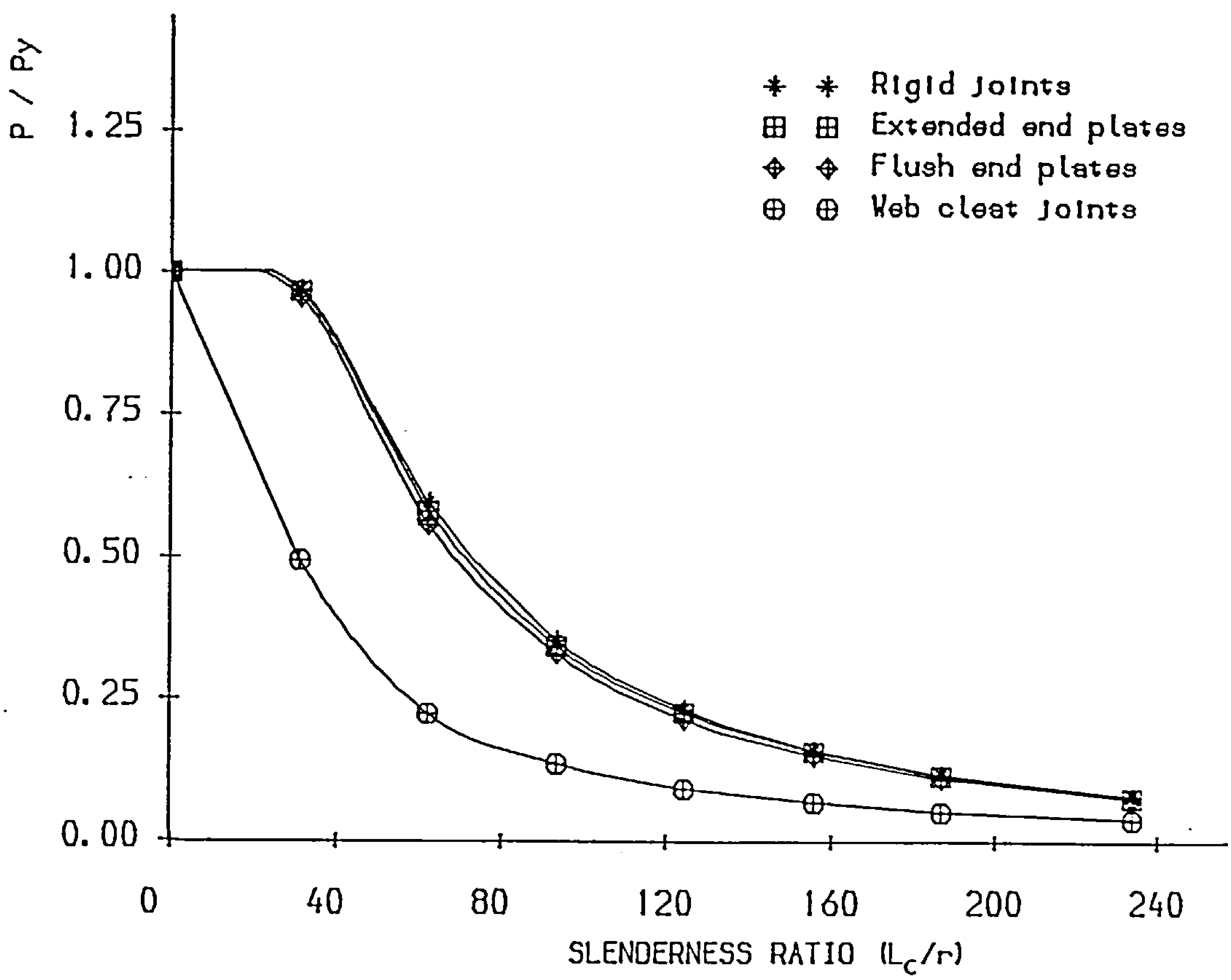


Figure 9.4: Column strength curves obtained for beam spans of 6.0 m and for different joint types without partial sway bracing.

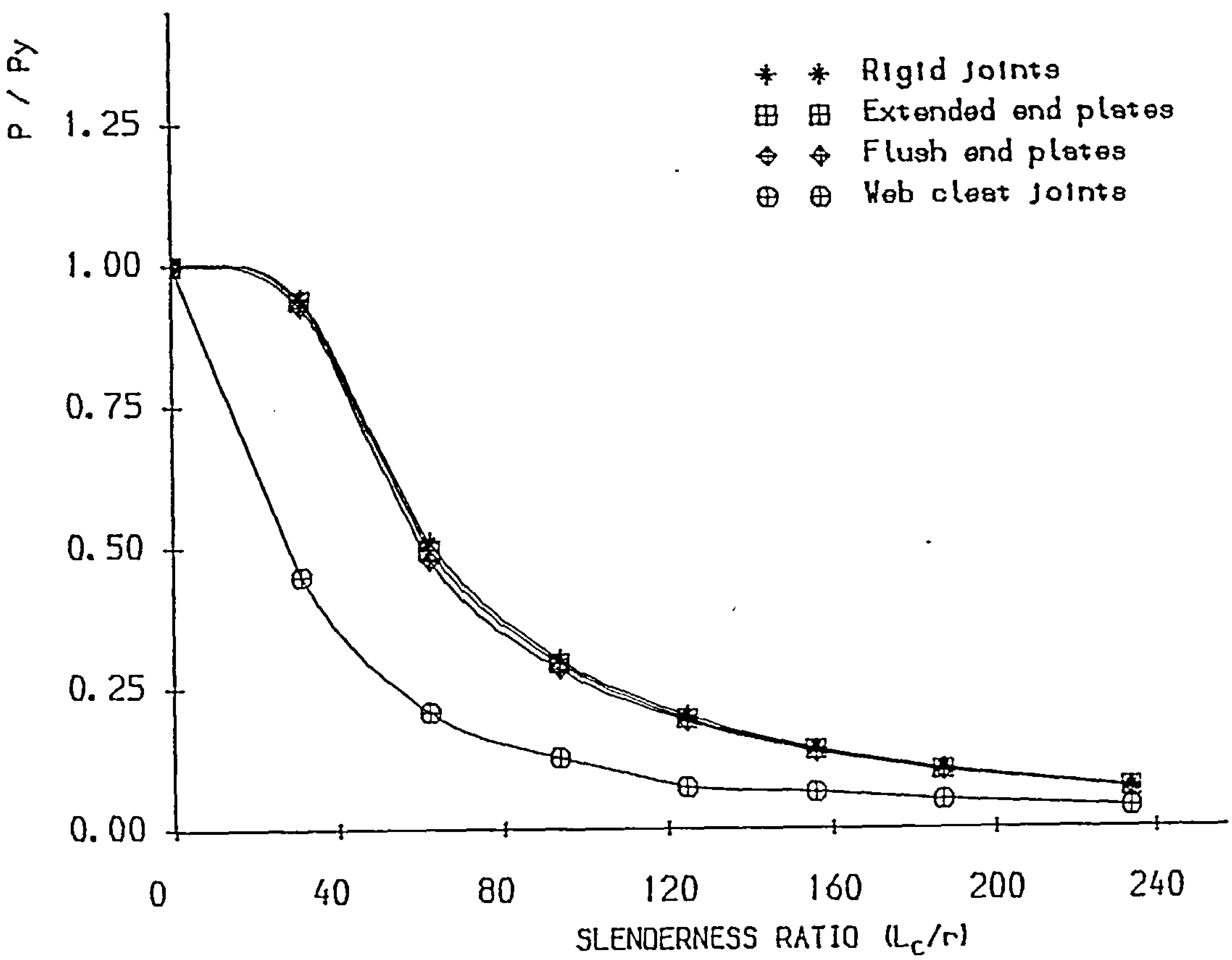


Figure 9.5: Column strength curves obtained for beam spans of 12.0 m and for different joint types without partial sway bracing.

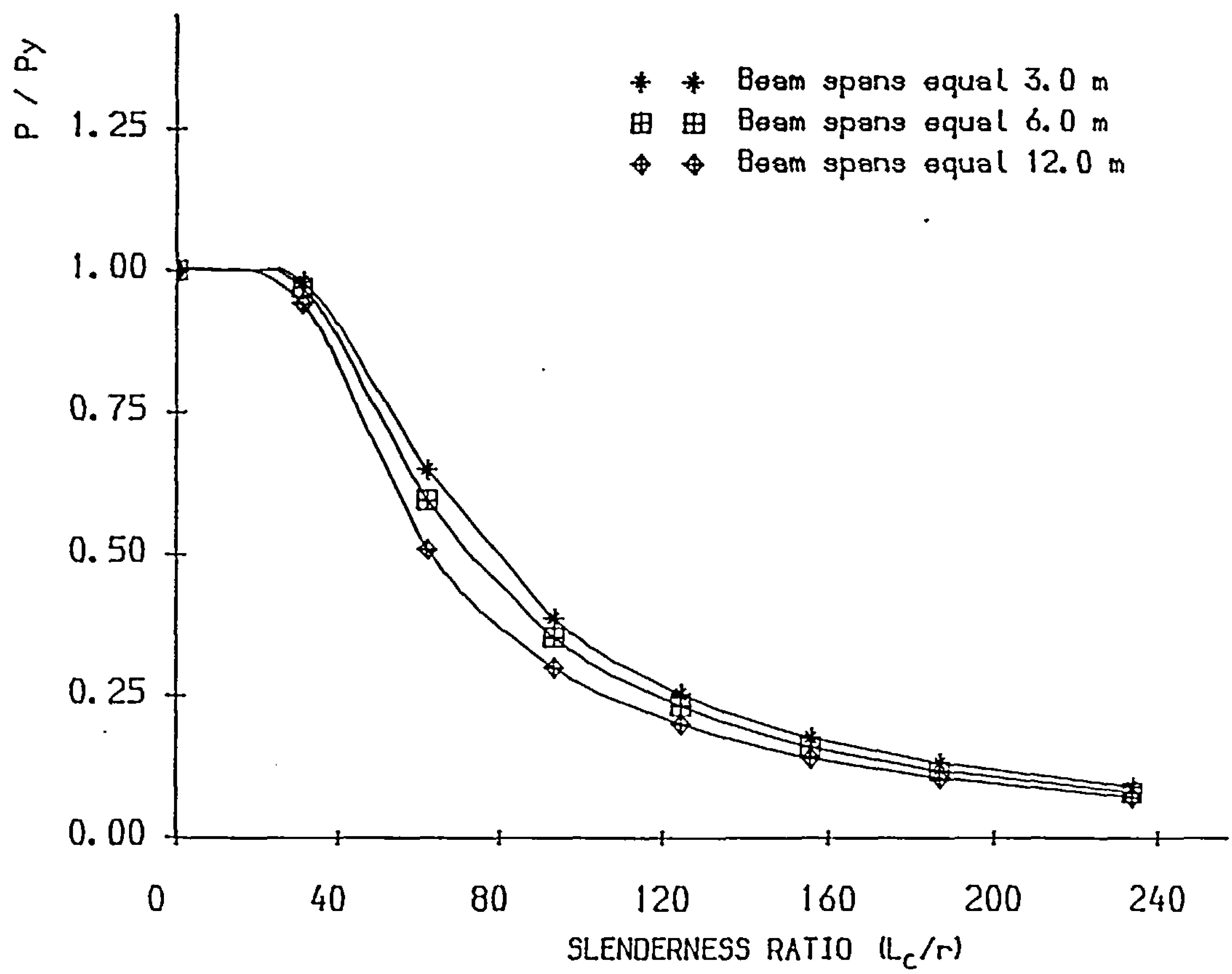


Figure 9.6: Column strength curves obtained for beam spans of 3.0, 6.0 and 12.0 m assuming rigid joints and without partial sway bracing.

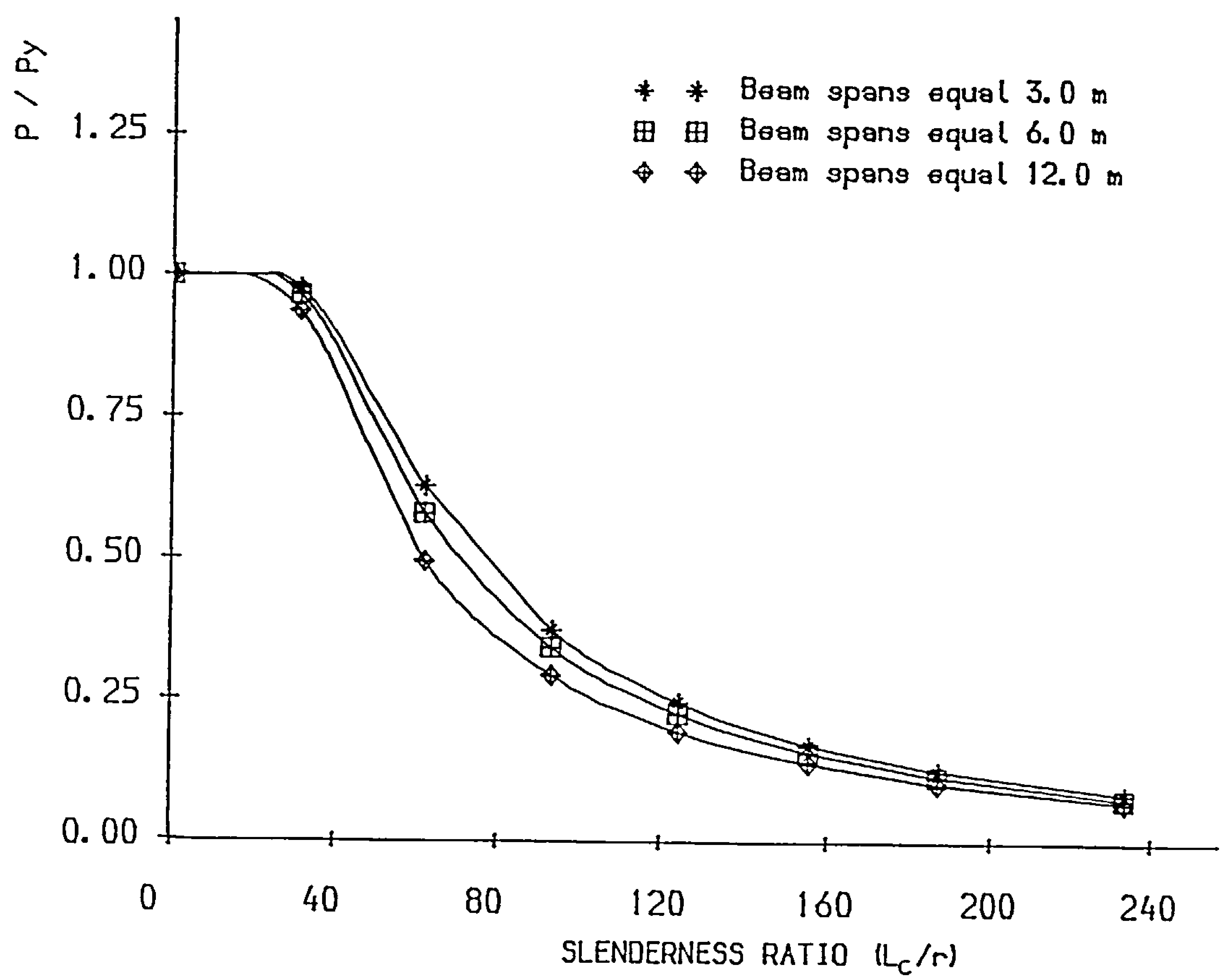


Figure 9.7: Column strength curves obtained for beam spans of 3.0, 6.0 and 12.0 m assuming extended end plate joints and without partial sway bracing.

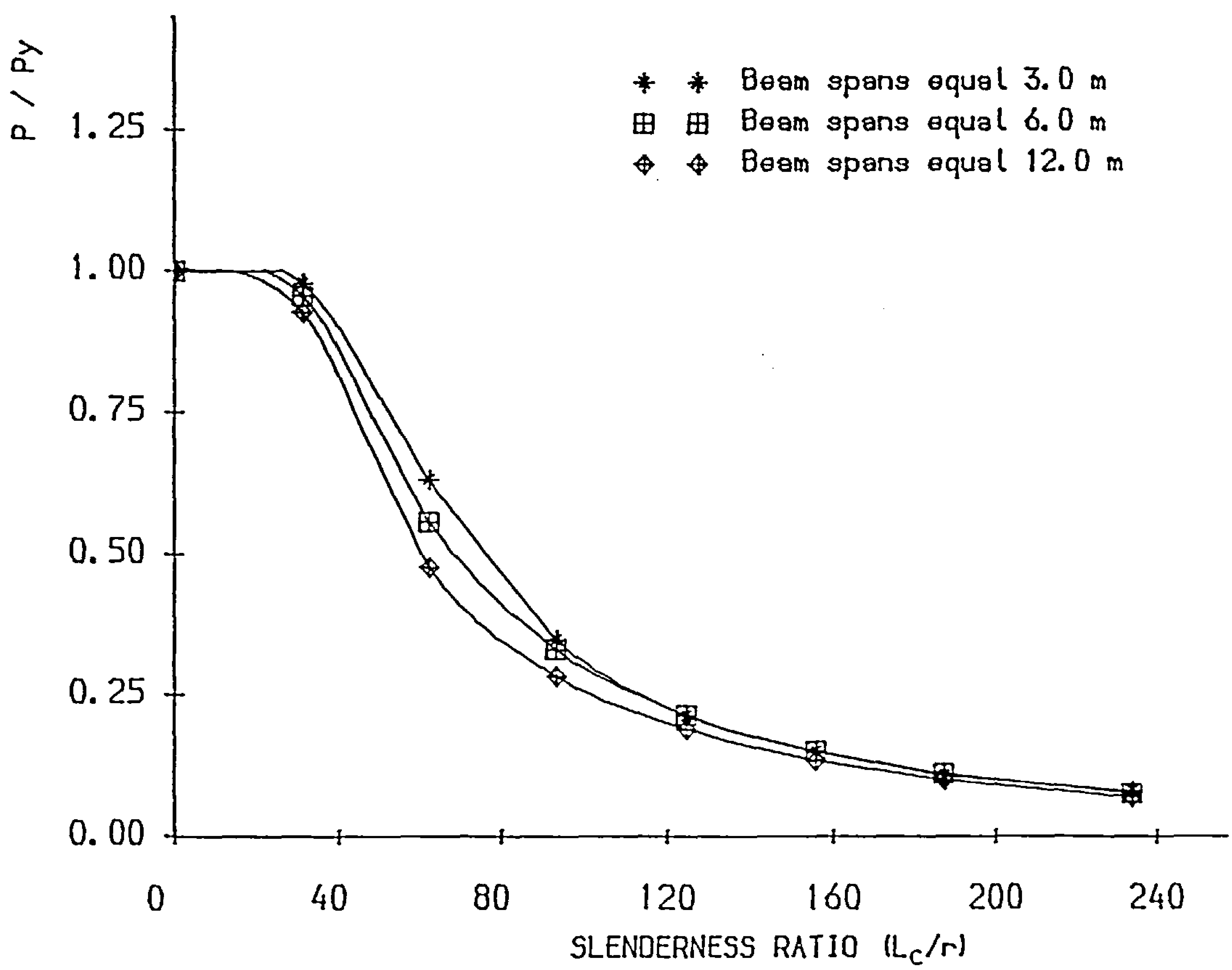


Figure 9.8: Column strength curves obtained for beam spans of 3.0, 6.0 and 12.0 m assuming flush end plate joints and without partial sway bracing.

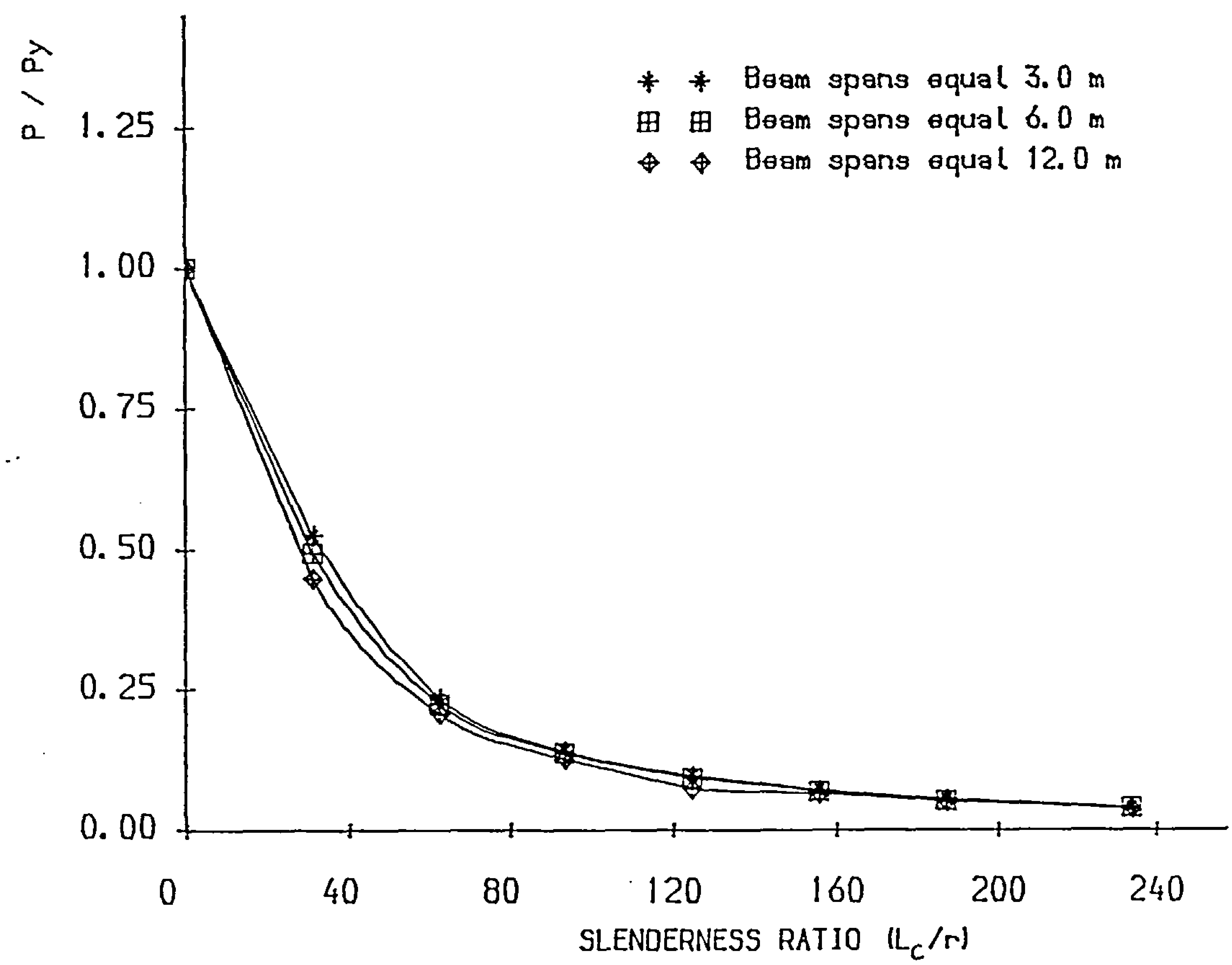


Figure 9.9: Column strength curves obtained for beam spans of 3.0, 6.0 and 12.0 m assuming web cleat joints and without partial sway bracing.

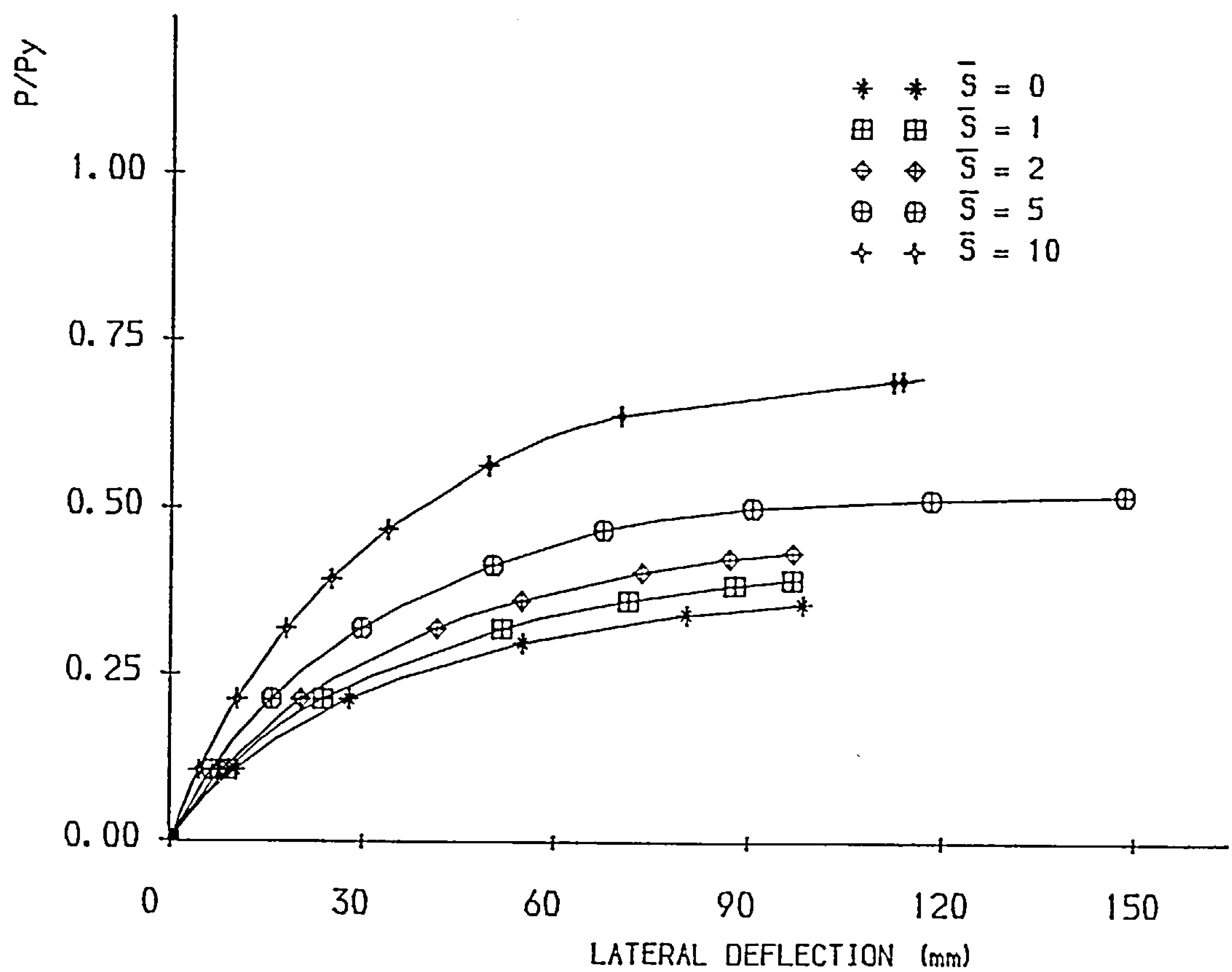


Figure 9.10: Lateral deflections obtained with the beam spans of 3.0 m, column height of 6.0 m ($L_c/r = 93.36$) and flush end plate joints. The ratio of horizontal to vertical loads is 1%.

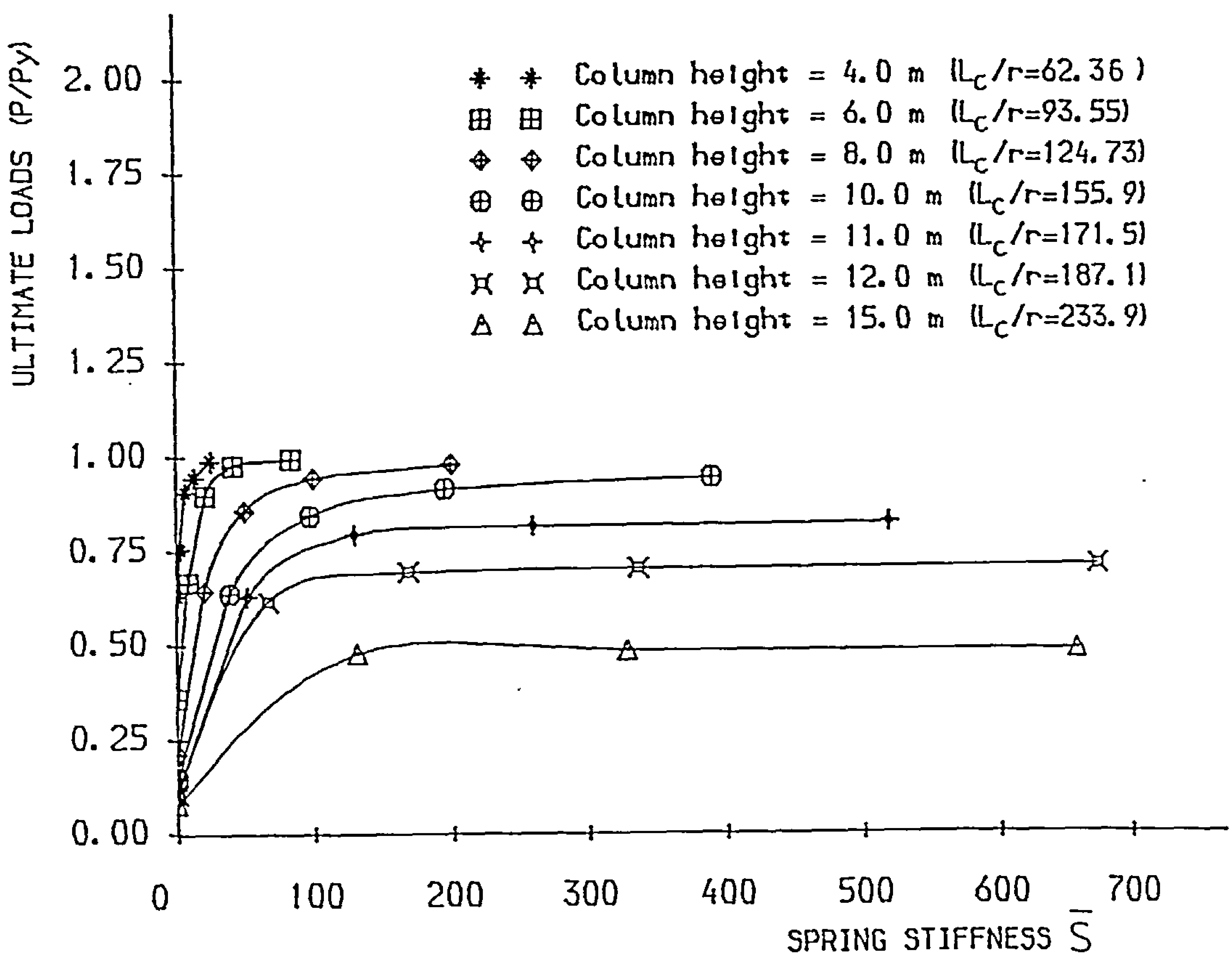


Figure 9.11: Ultimate load versus relative spring stiffness for the subassemblage shown in figure 9.1. The beam spans are 3.0 m and the ratio of horizontal to vertical loads is 1%.

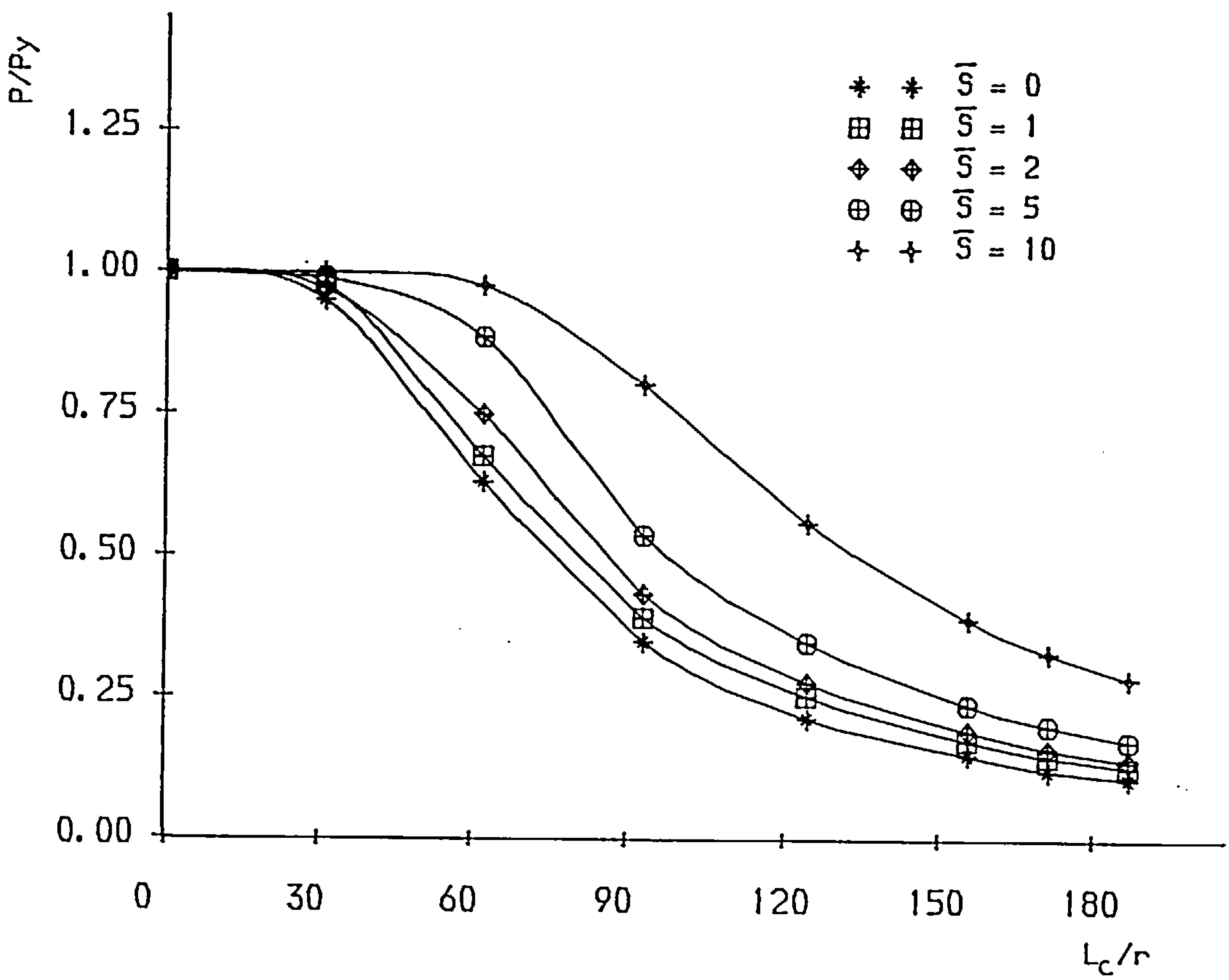


Figure 9.12: Column strength curves for the subassemblage shown in 9.1. The beam spans are 3.0 m, the joints are flush end plates and the ratio of horizontal to vertical loads is 1%.

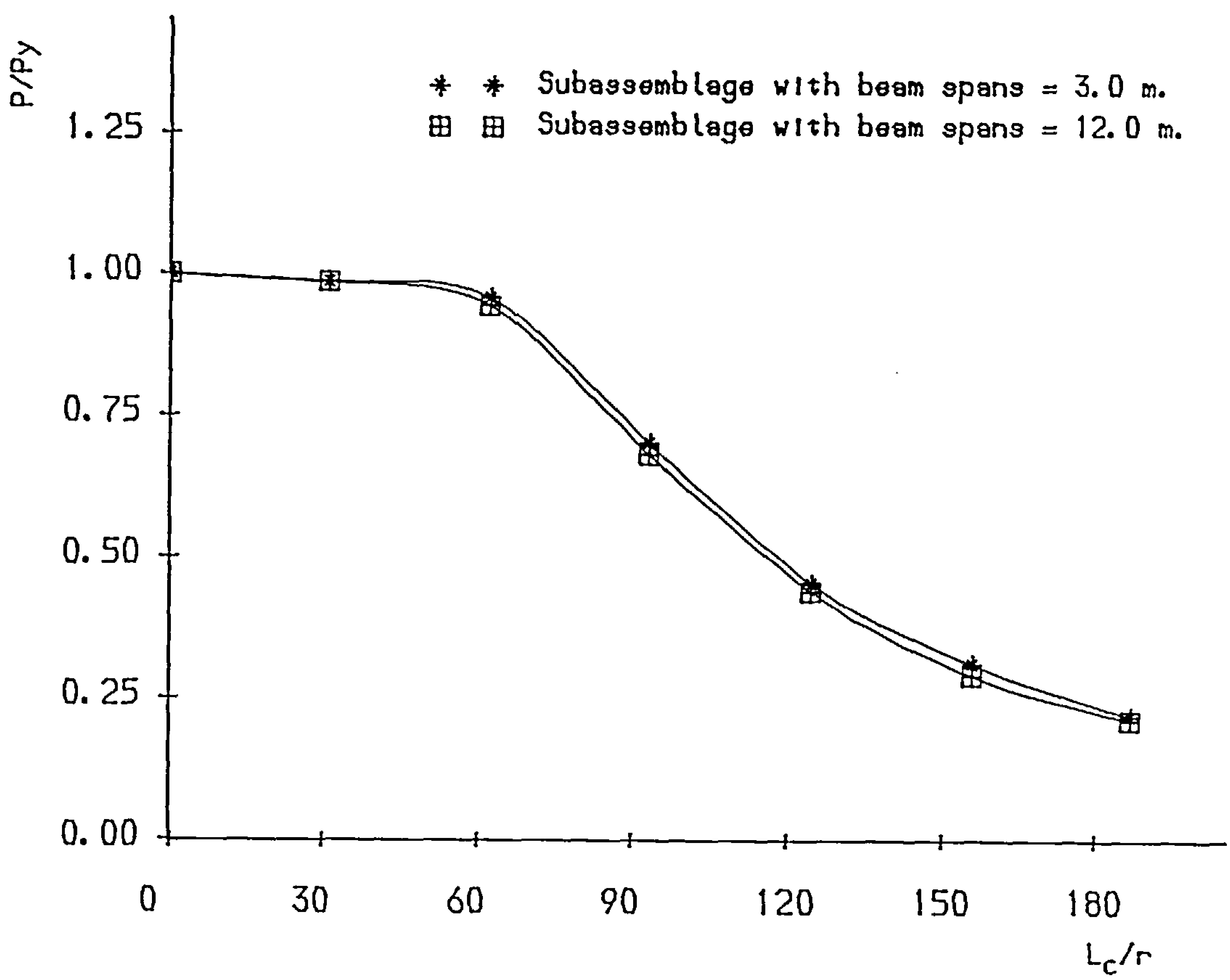


Figure 9.13: Column strength curves for the subassemblage shown in 9.1 with lateral sway bracing ($\overline{S} = 10$). The joints are flush end plates and the ratio of horizontal to vertical loads is 1%.

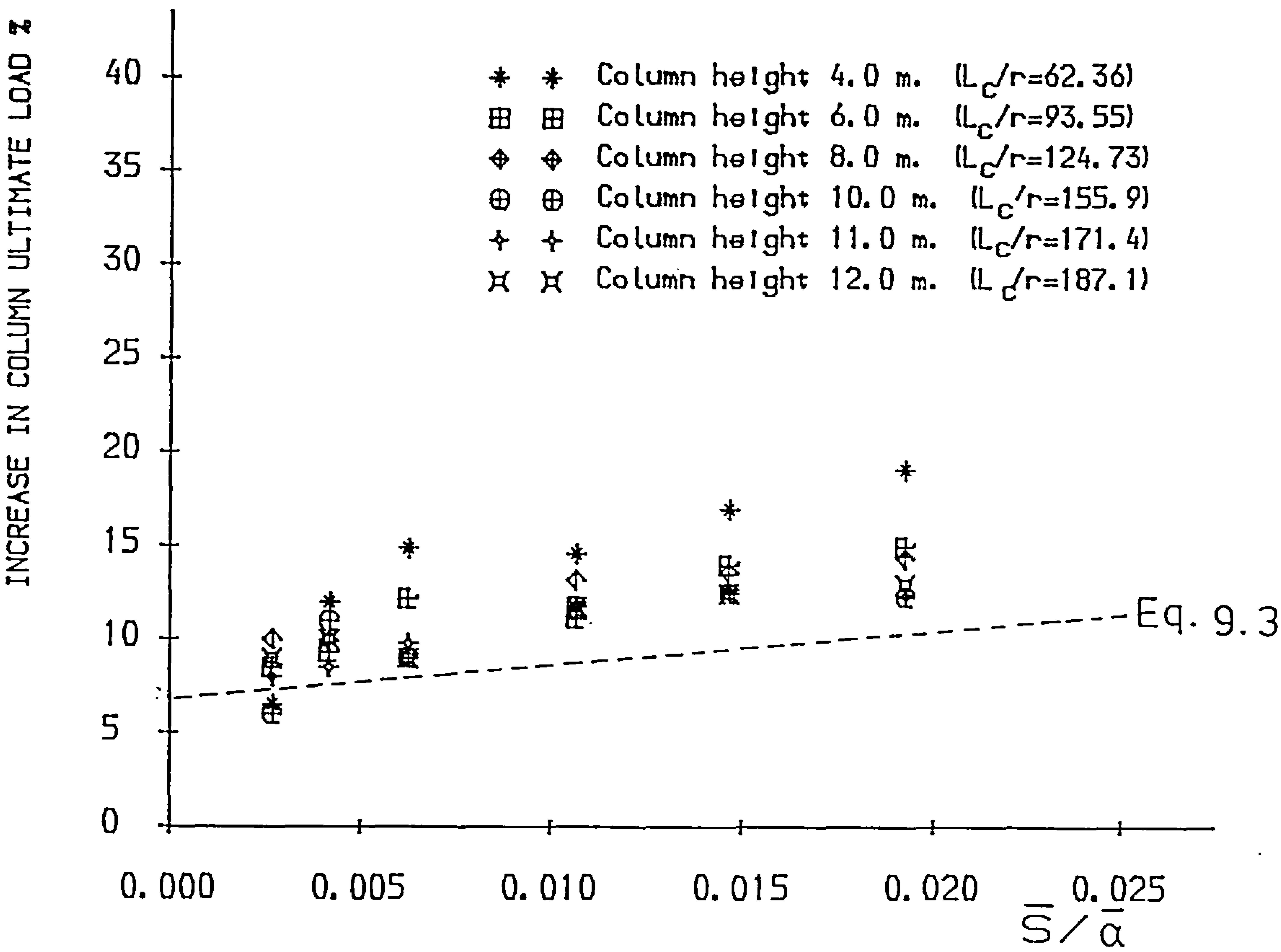


Figure 9.14: The increase of ultimate column load against the ratio $\overline{S}/\overline{\alpha}$ with lateral sway bracing ($\overline{S} = 1$). The joints are flush end plates and the ratio of horizontal to vertical loads is 1%.

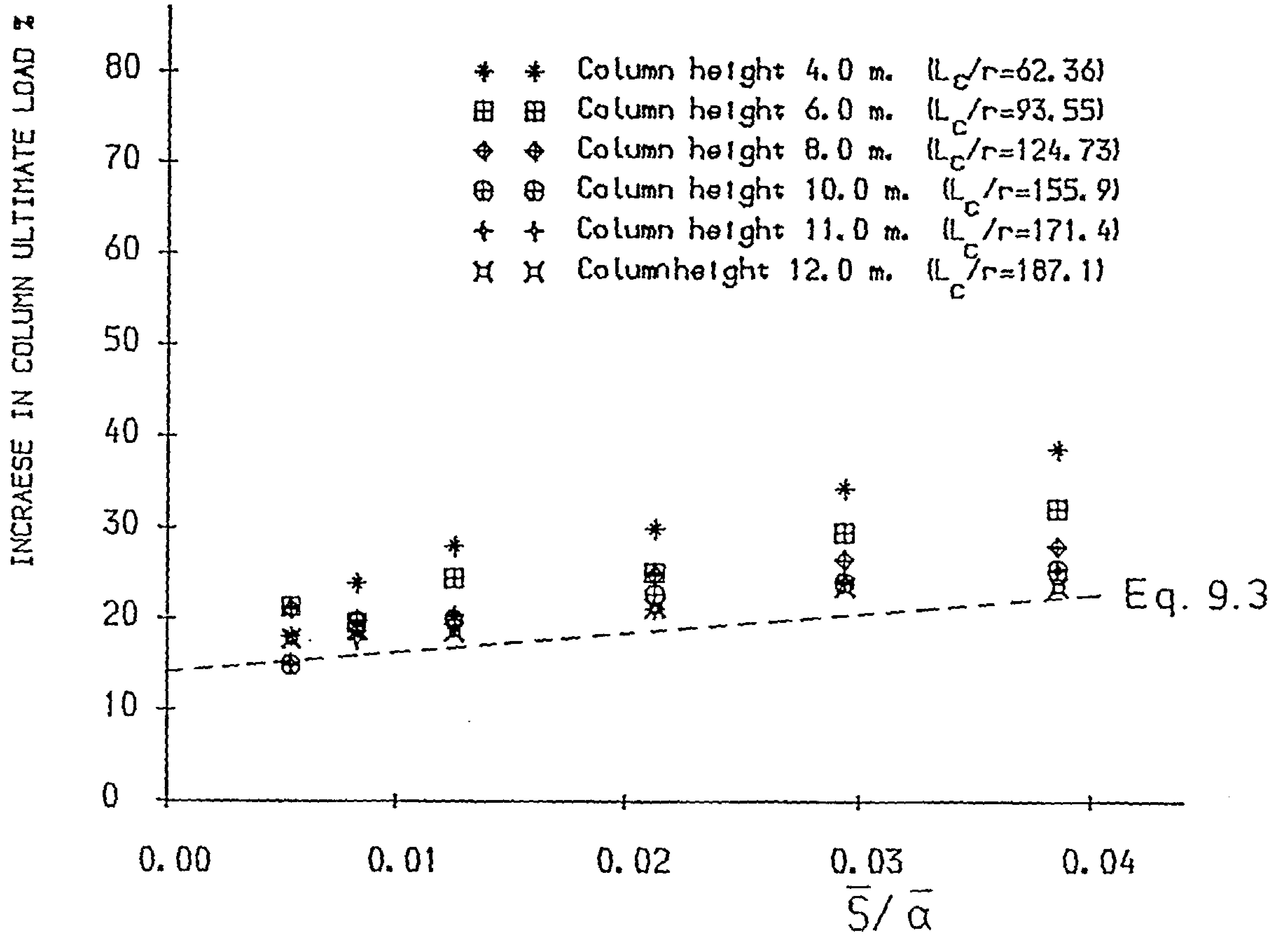


Figure 9.15: The increase of ultimate column load against the ratio $\overline{S}/\overline{\alpha}$ with lateral sway bracing ($\overline{S} = 2$). The joints are flush end plates and the ratio of horizontal to vertical loads is 1%.

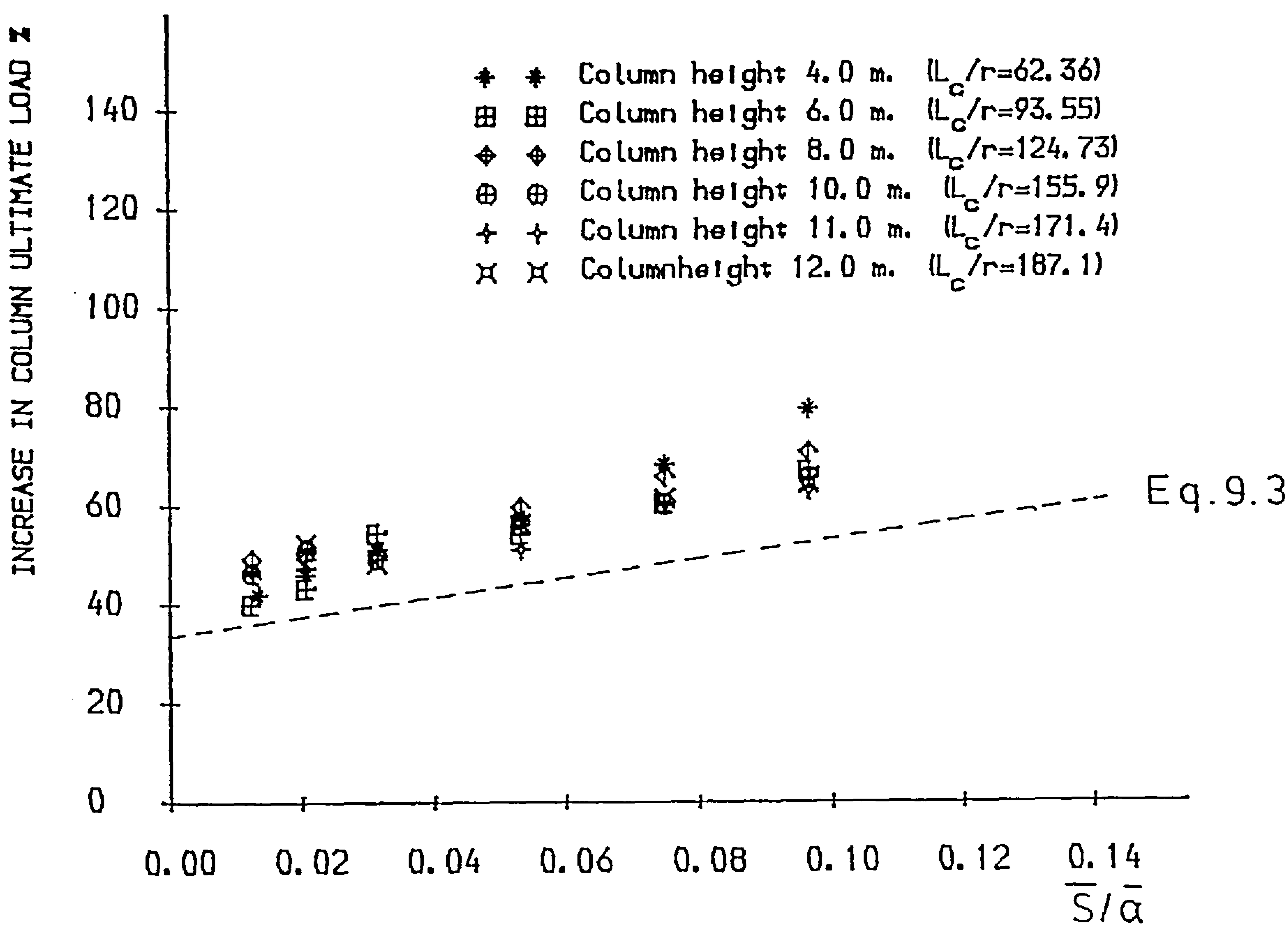


Figure 9.16: The increase of ultimate column load against the ratio $\overline{S}/\overline{\alpha}$ with lateral sway bracing ($\overline{S} = 5$). The joints are flush end plates and the ratio of horizontal to vertical loads is 1%.

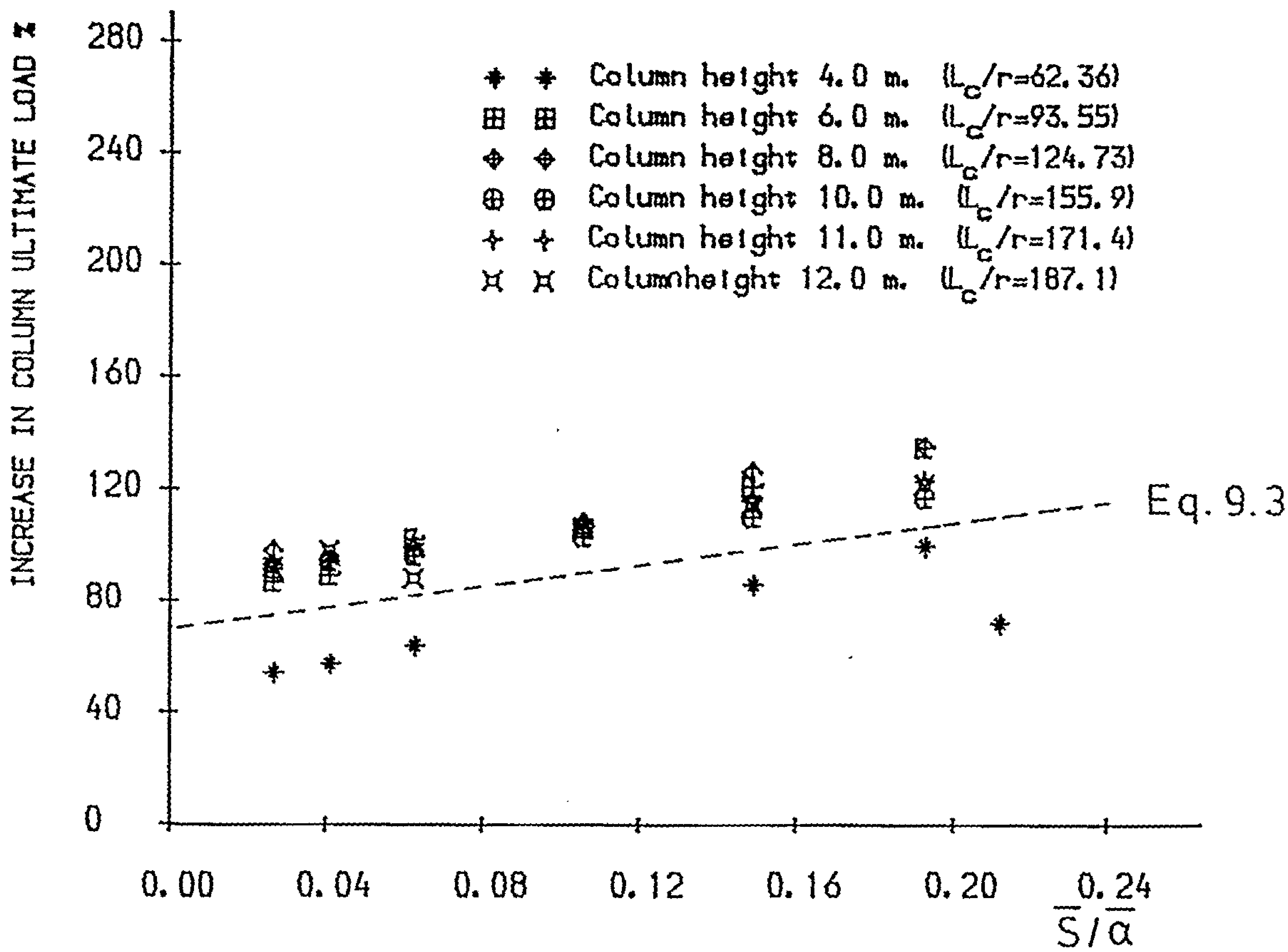


Figure 9.17: The increase of ultimate column load against the ratio $\overline{S}/\overline{\alpha}$ with lateral sway bracing ($\overline{S} = 10$). The joints are flush end plates and the ratio of horizontal to vertical loads is 1%.

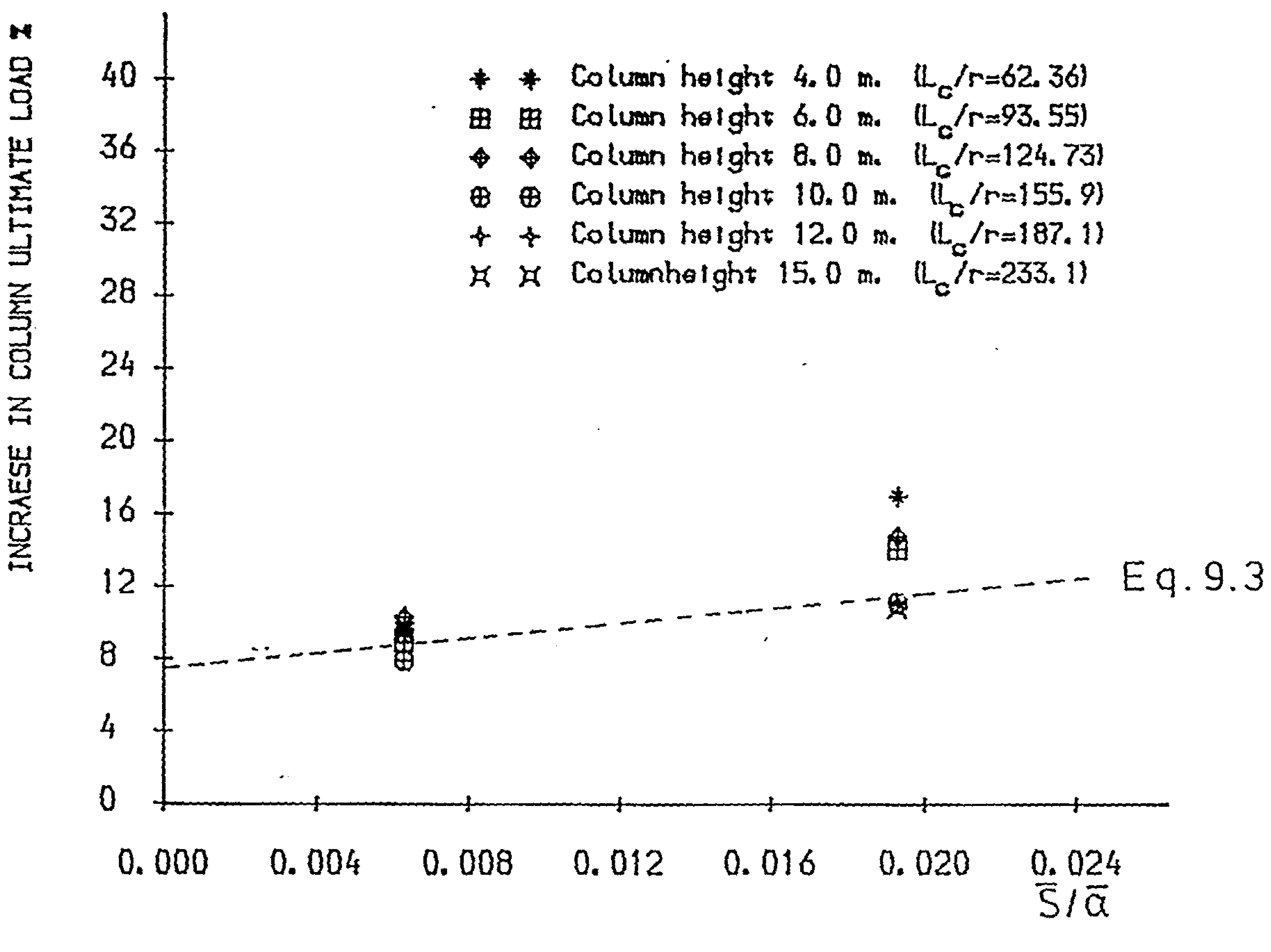


Figure 9.18: The increase of ultimate column load against the ratio $\overline{S}/\overline{\alpha}$ with lateral sway bracing ($\overline{S} = 1$). The joints are flush end plates and the ratio of horizontal to vertical loads is 0.5%.

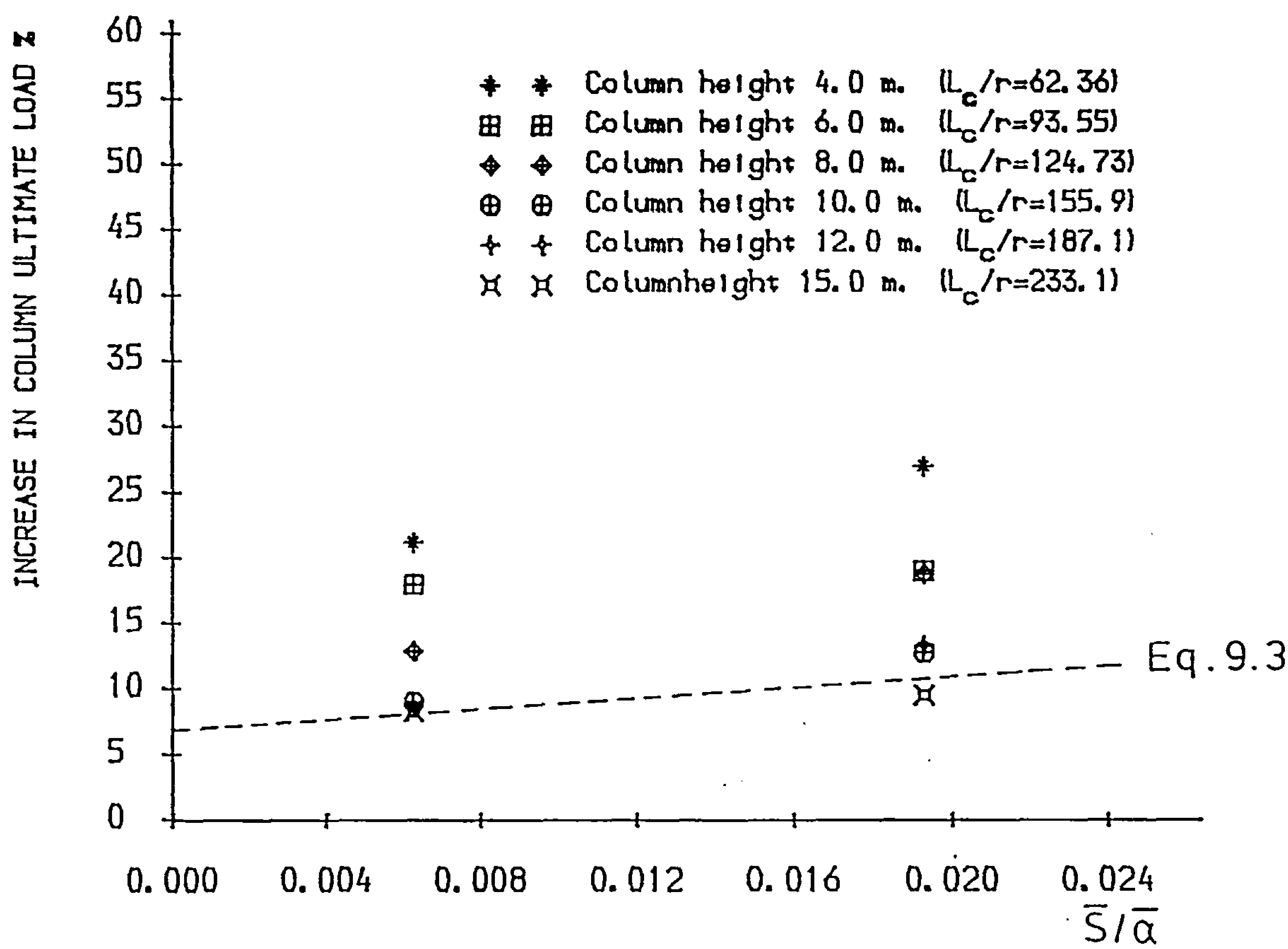


Figure 9.19: The increase of ultimate column load against the ratio $\overline{S}/\overline{\alpha}$ with lateral sway bracing ($\overline{S} = 1$). The joints are flush end plates and the ratio of horizontal to vertical loads is 5%.

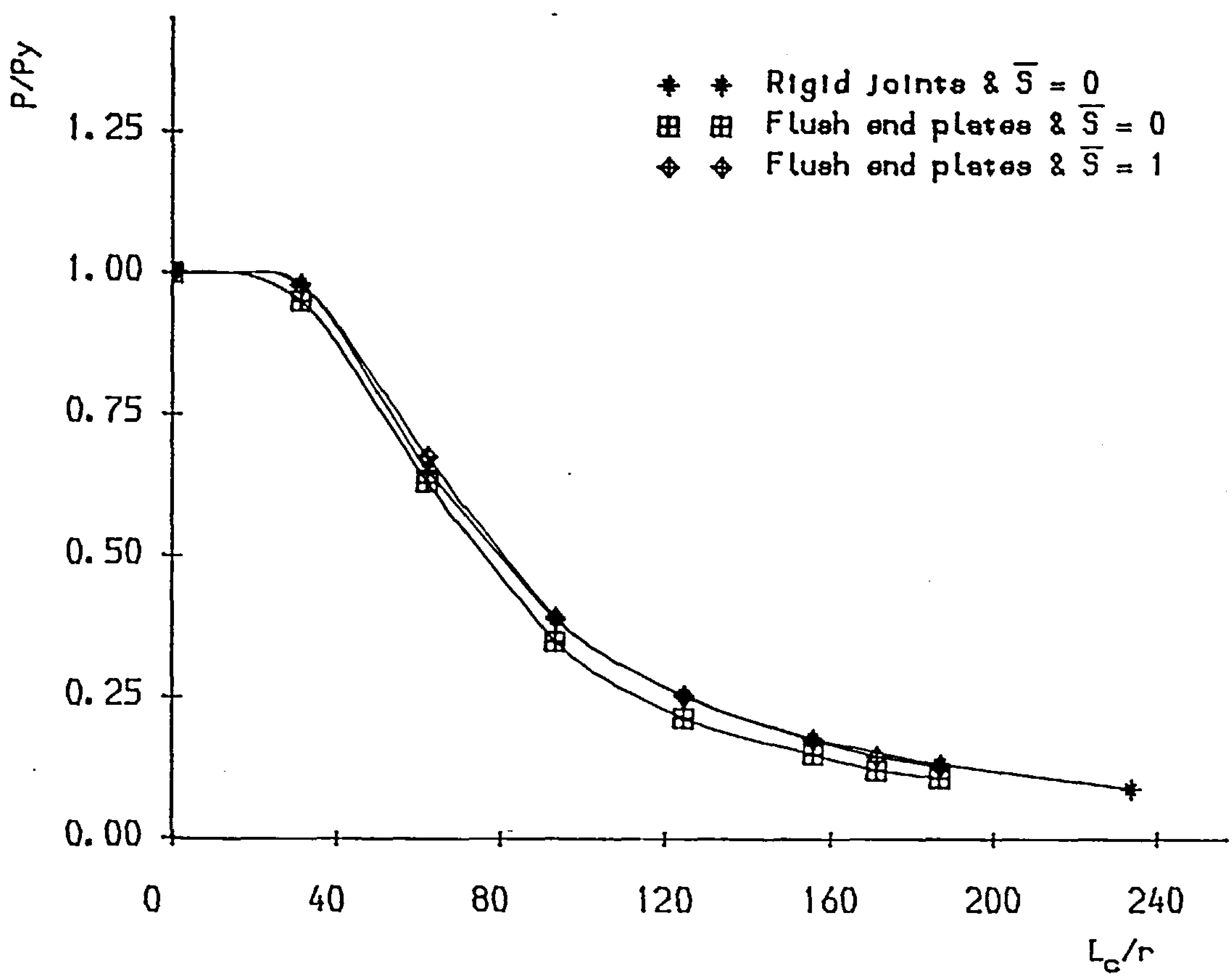


Figure 9.20: Column strength curve for rigid and flush end plate joints, assuming the ratio of horizontal to vertical loads as 1%, taking the beam spans L_g as 3.0 m with and without the presence of lateral bracing

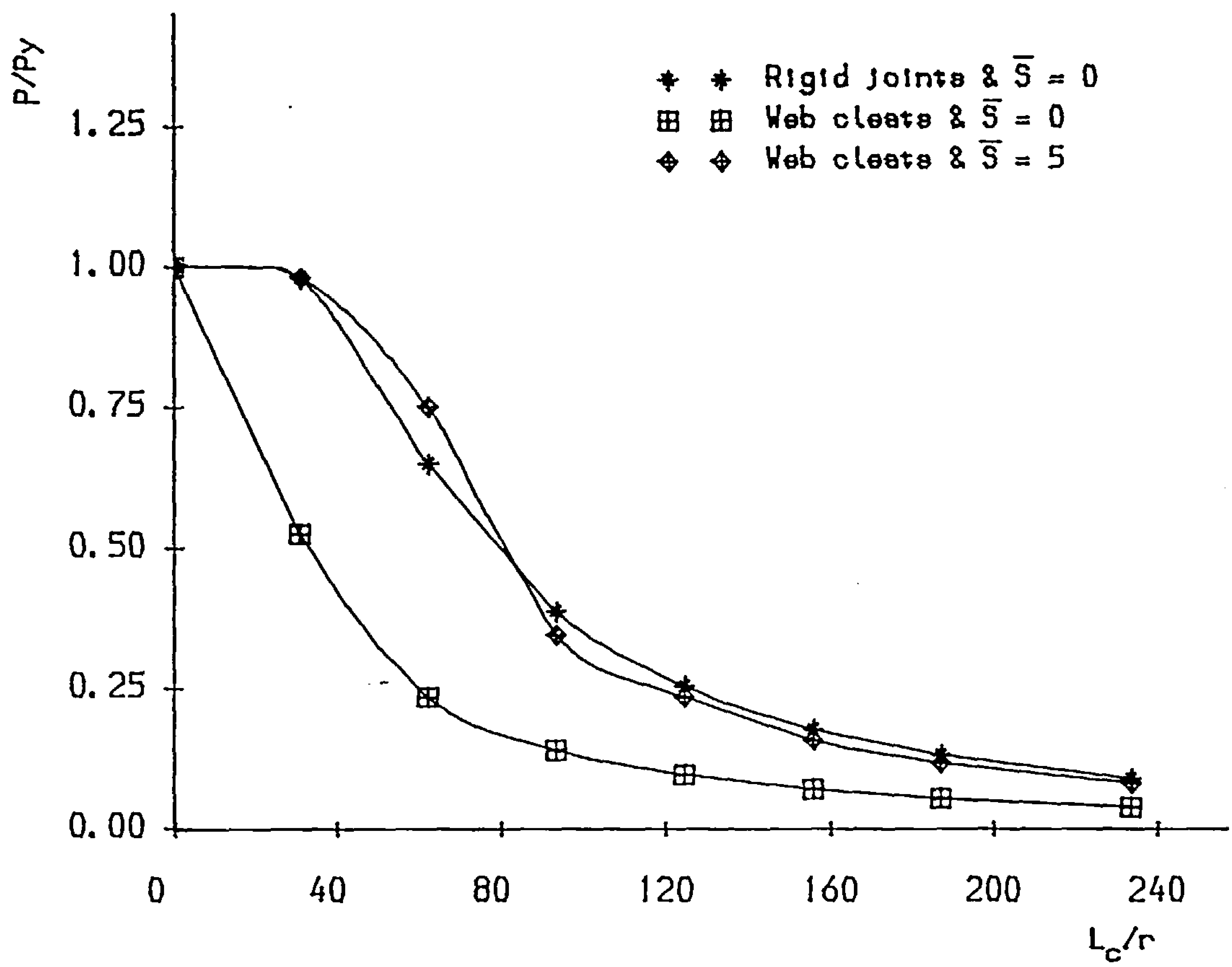


Figure 9.21: Column strength curve for rigid and web cleat joints, assuming the ratio of horizontal to vertical loads as 1%, taking the beam spans L_g as 3.0 m with and without the presence of lateral bracing

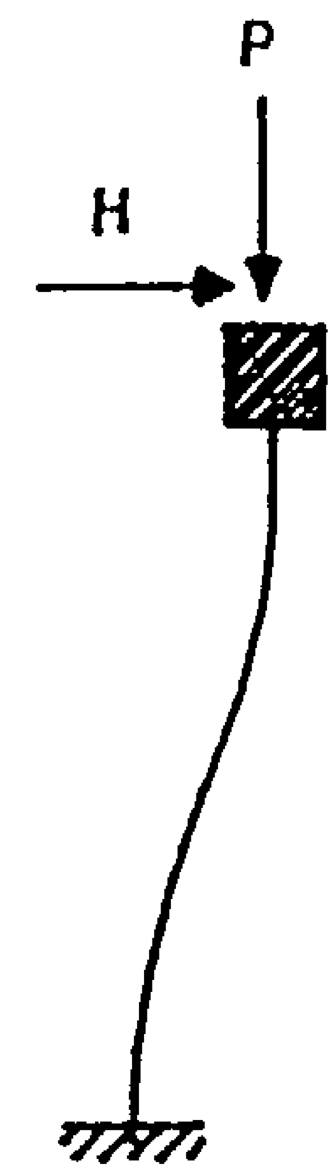


Figure 9.22.a: Column with perfect rotational end restraints

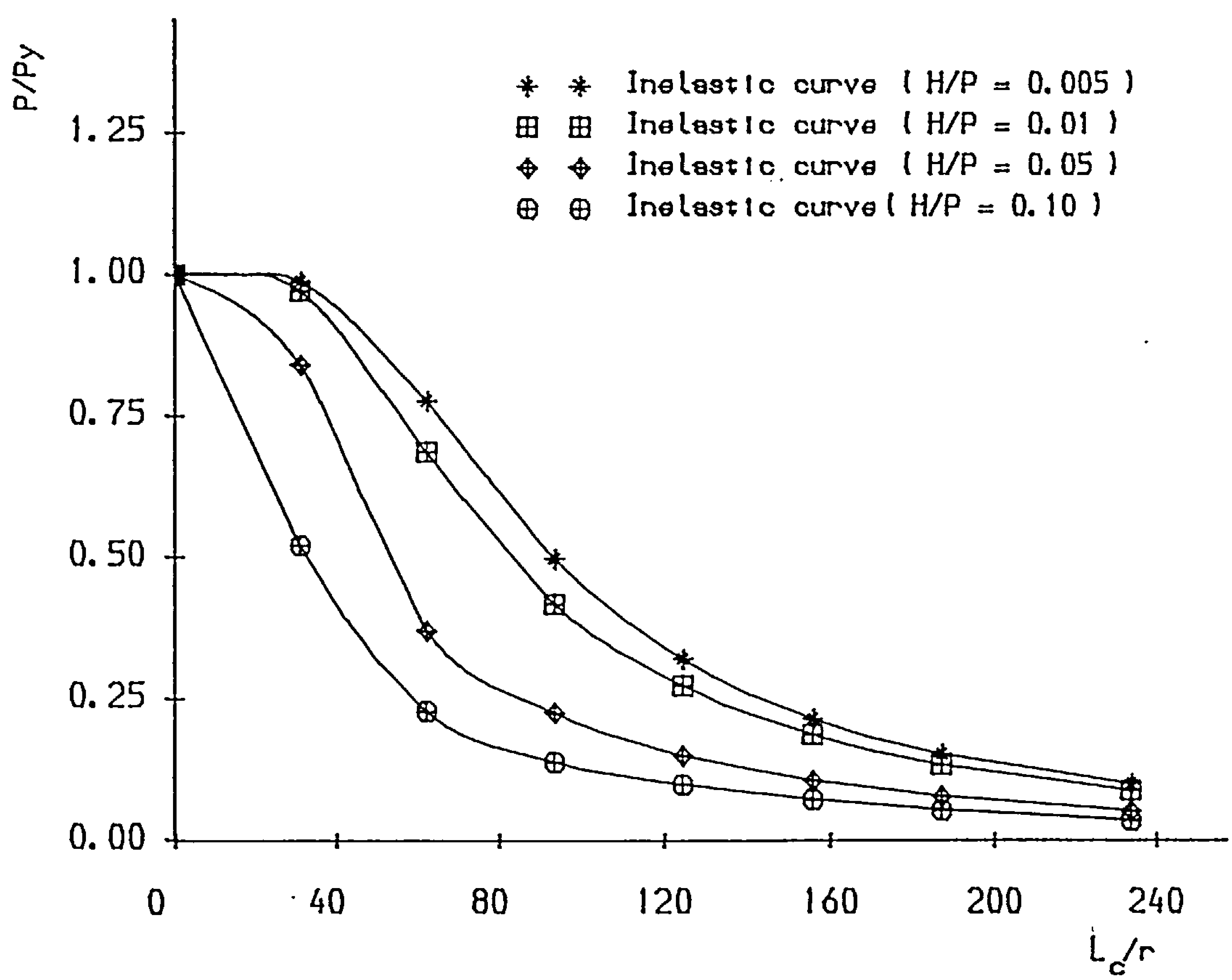


Figure 9.22b:Column strength curves for the reference column shown in fig.9.22.a considering different ratios of the applied horizontal to vertical loads.

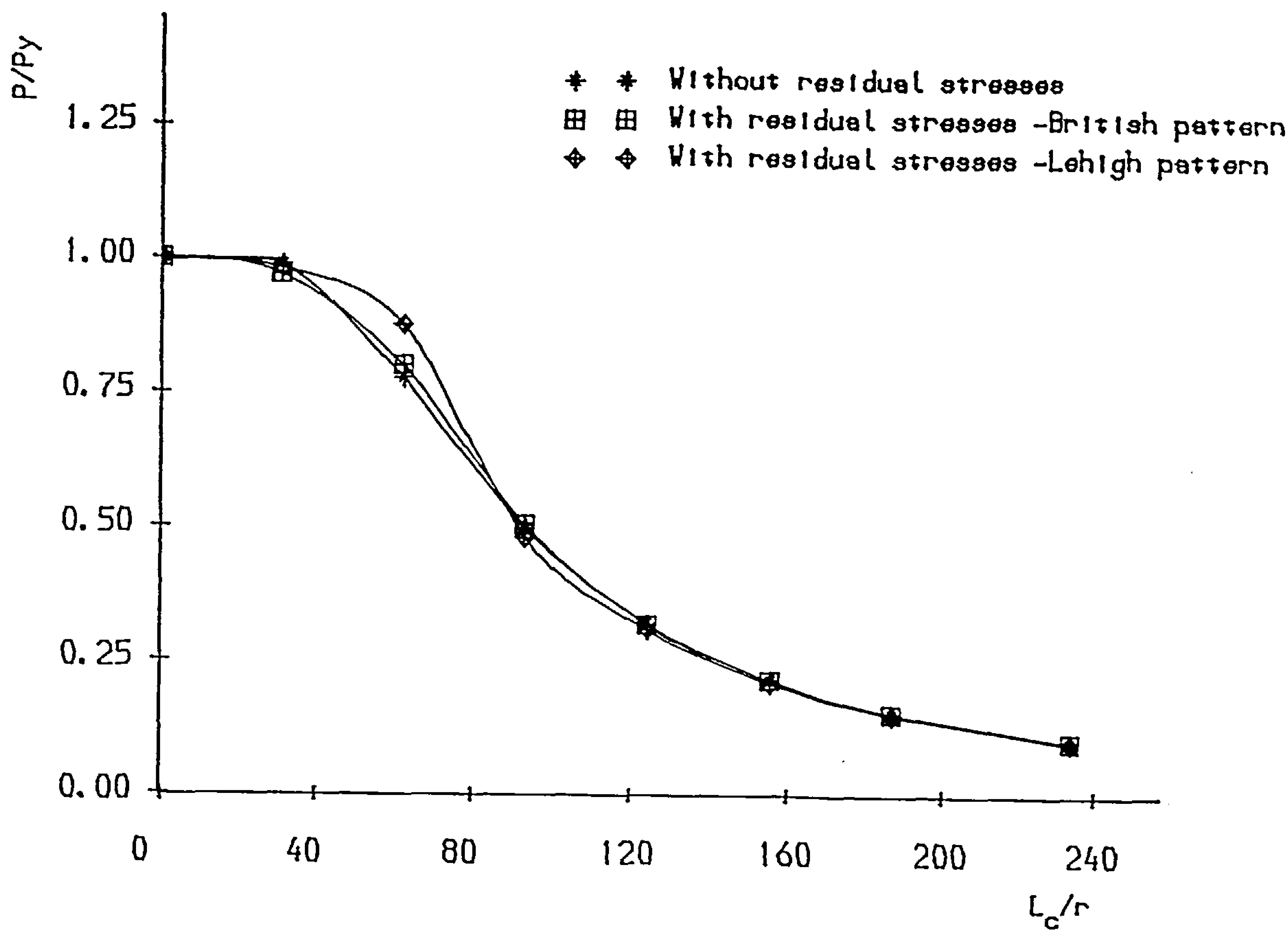


Figure 9.23: Column strength curves for the reference column shown in fig.9.22.a taking the ratio of horizontal to vertical loads as 0.5% with and without the presence of residual stresses.

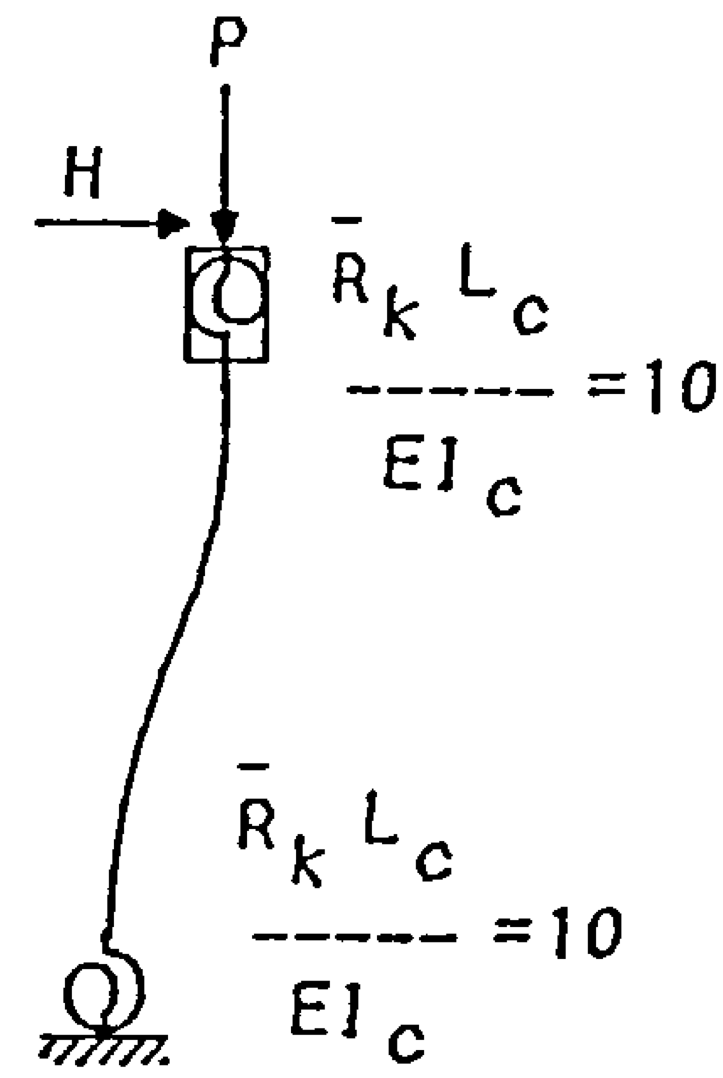


Figure 9.24.a: Column with relative rotational end restraints equal to 10.

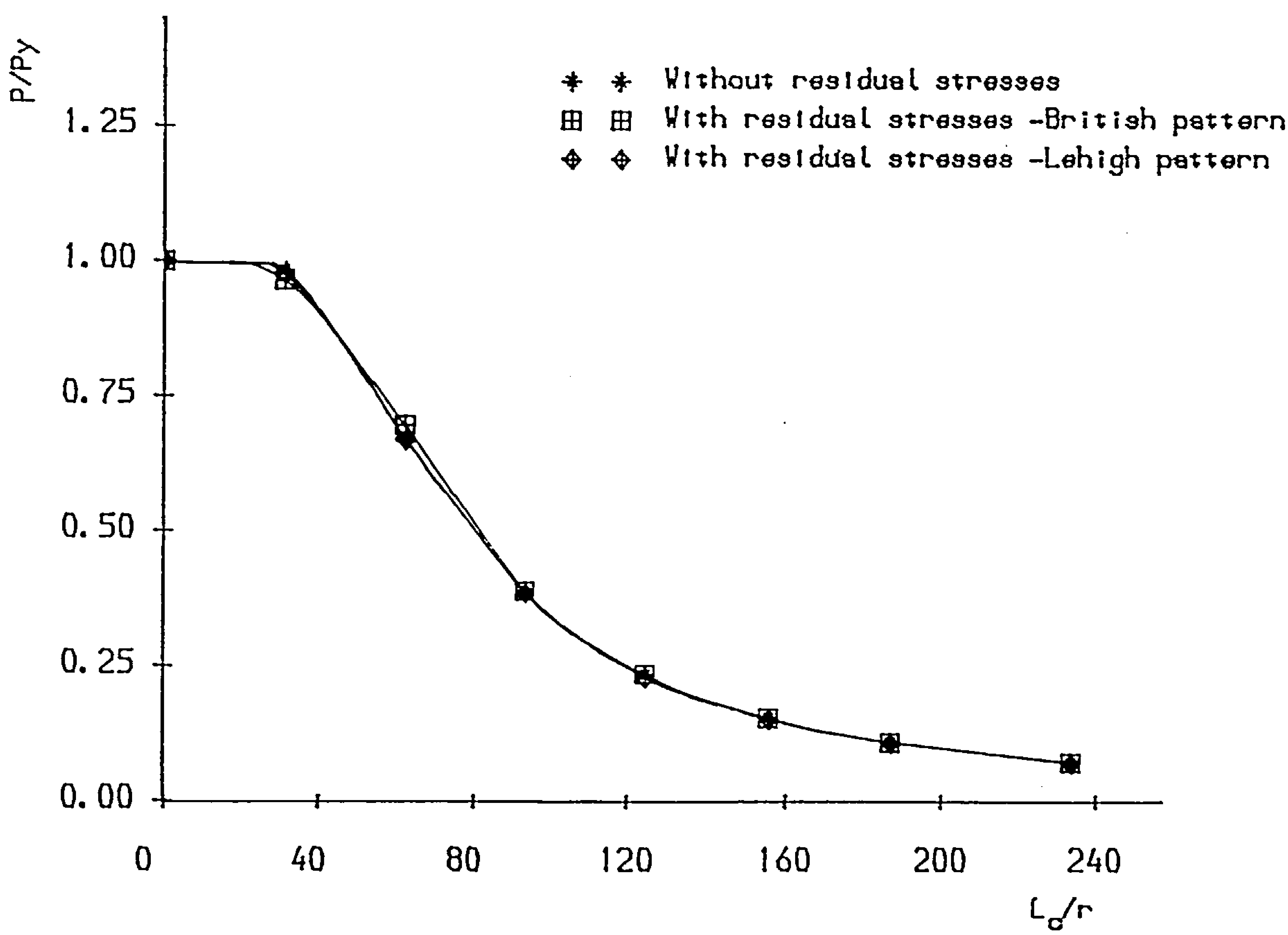


Figure 9.24b:Column strength curves for the reference column shown in fig.9.24.a taking the ratio of horizontal to vertical loads as 0.5% with and without the presence of residual stresses.

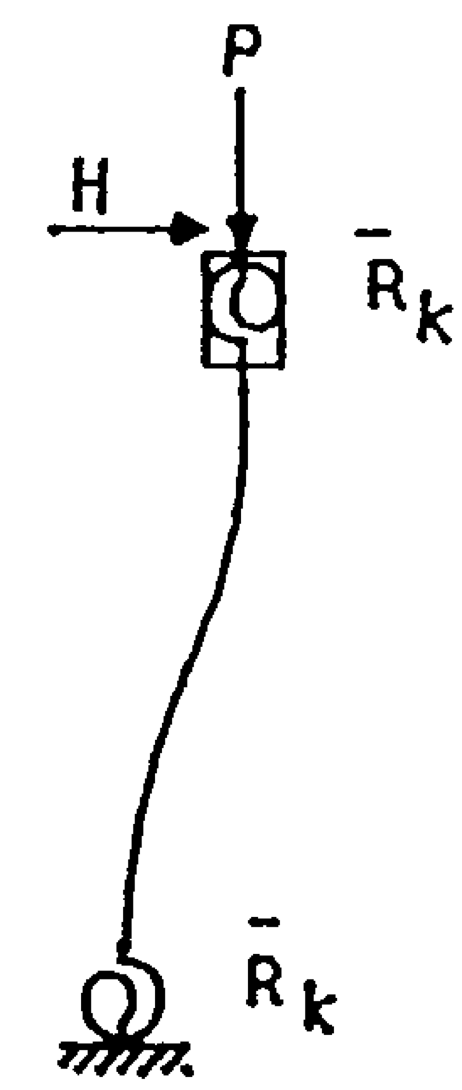


Figure 9.25.a: Column with equal rotational end restraints

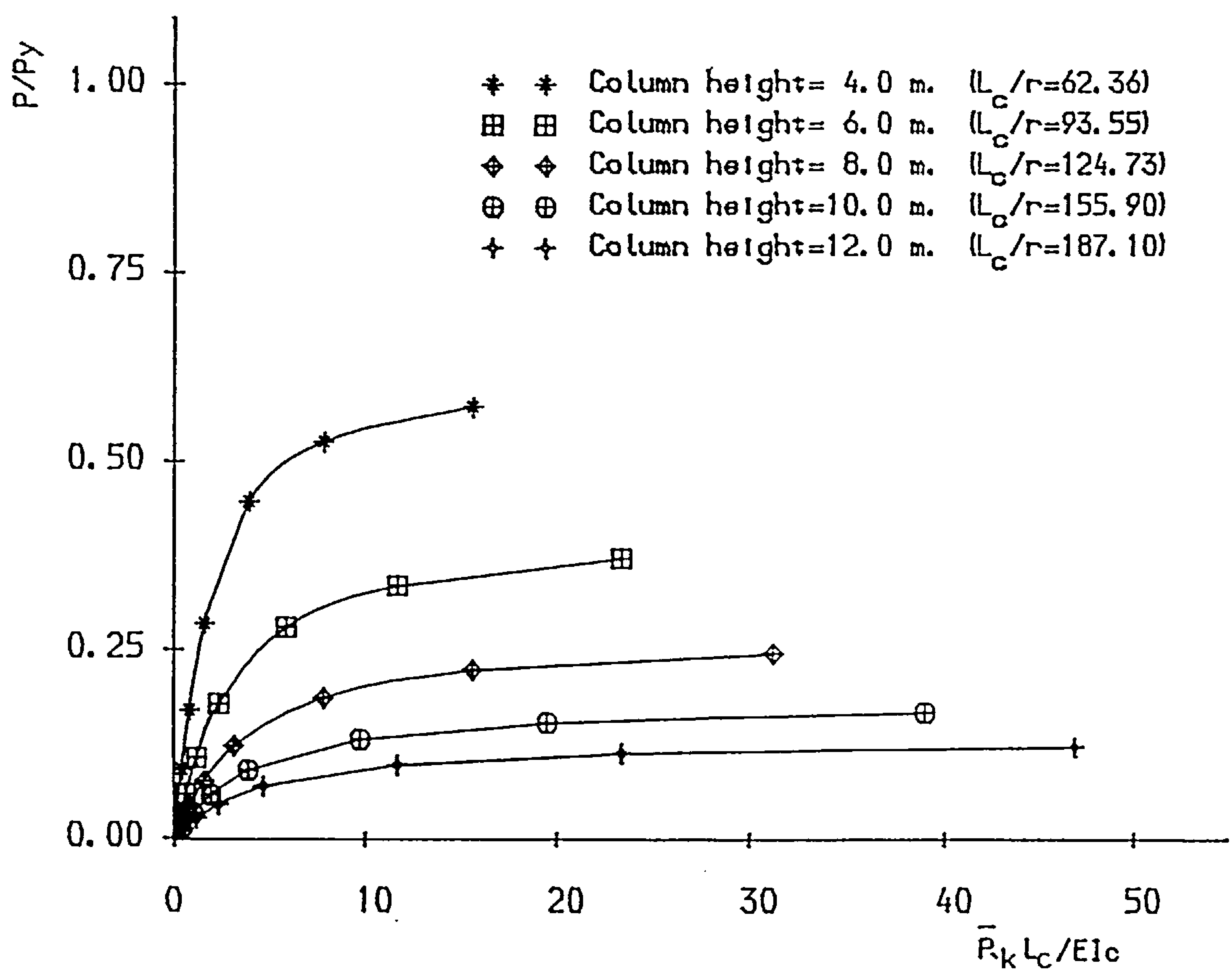


Figure 9.25.b:Relationship of ultimate column load against column end restraints for the column shown in fig.9.25.a obtained taking the ratio of horizontal to vertical loads as 1%.

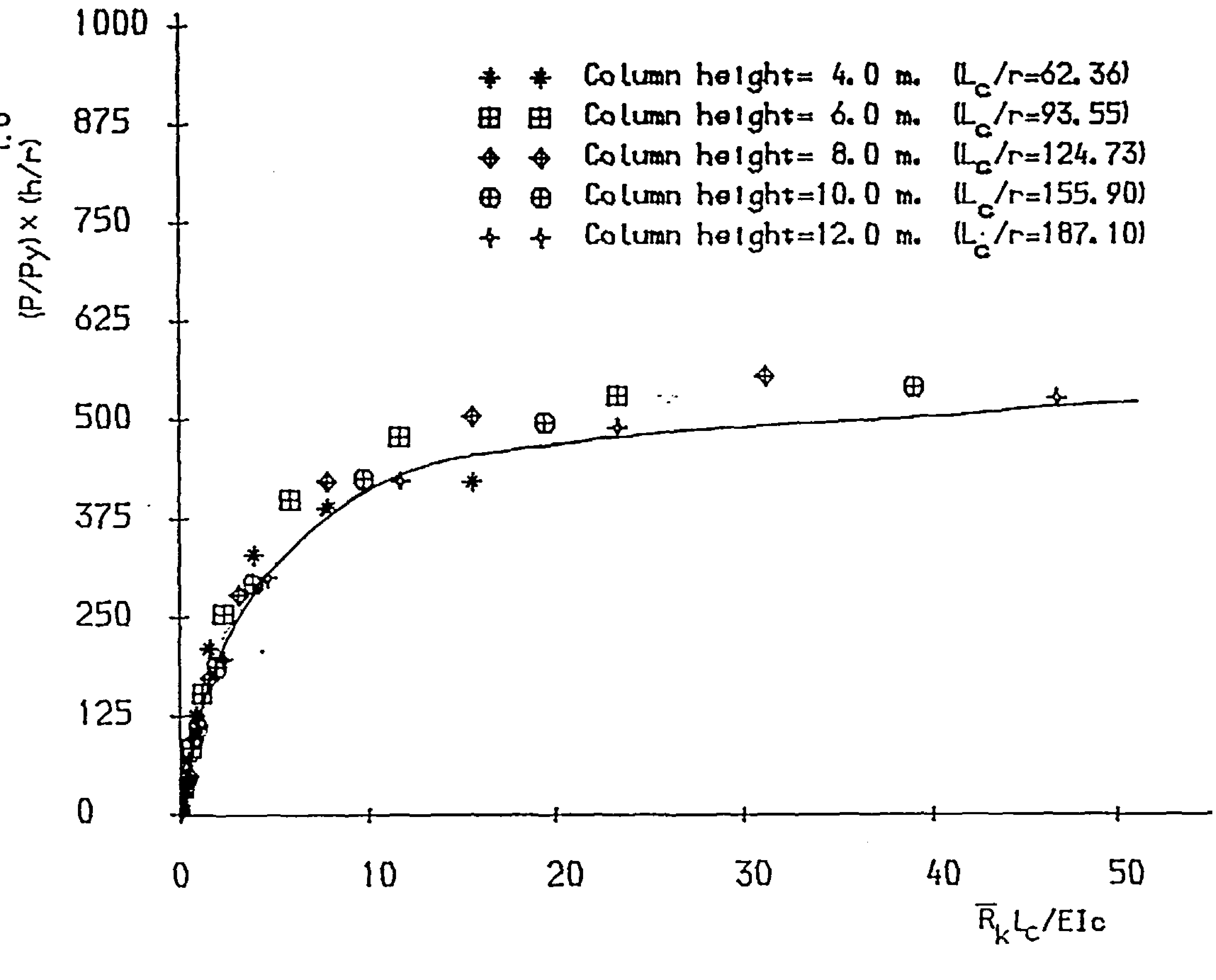


Figure 9.26: Column design chart obtained taking the ratio of horizontal to vertical loads as 1%.

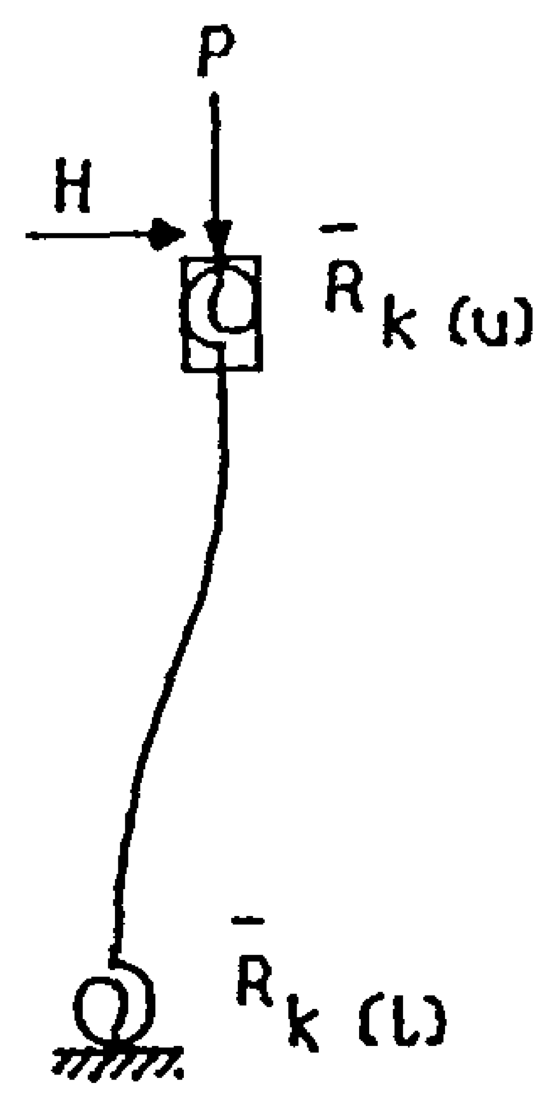


Figure 9.27.a: Column with different rotational end restraints

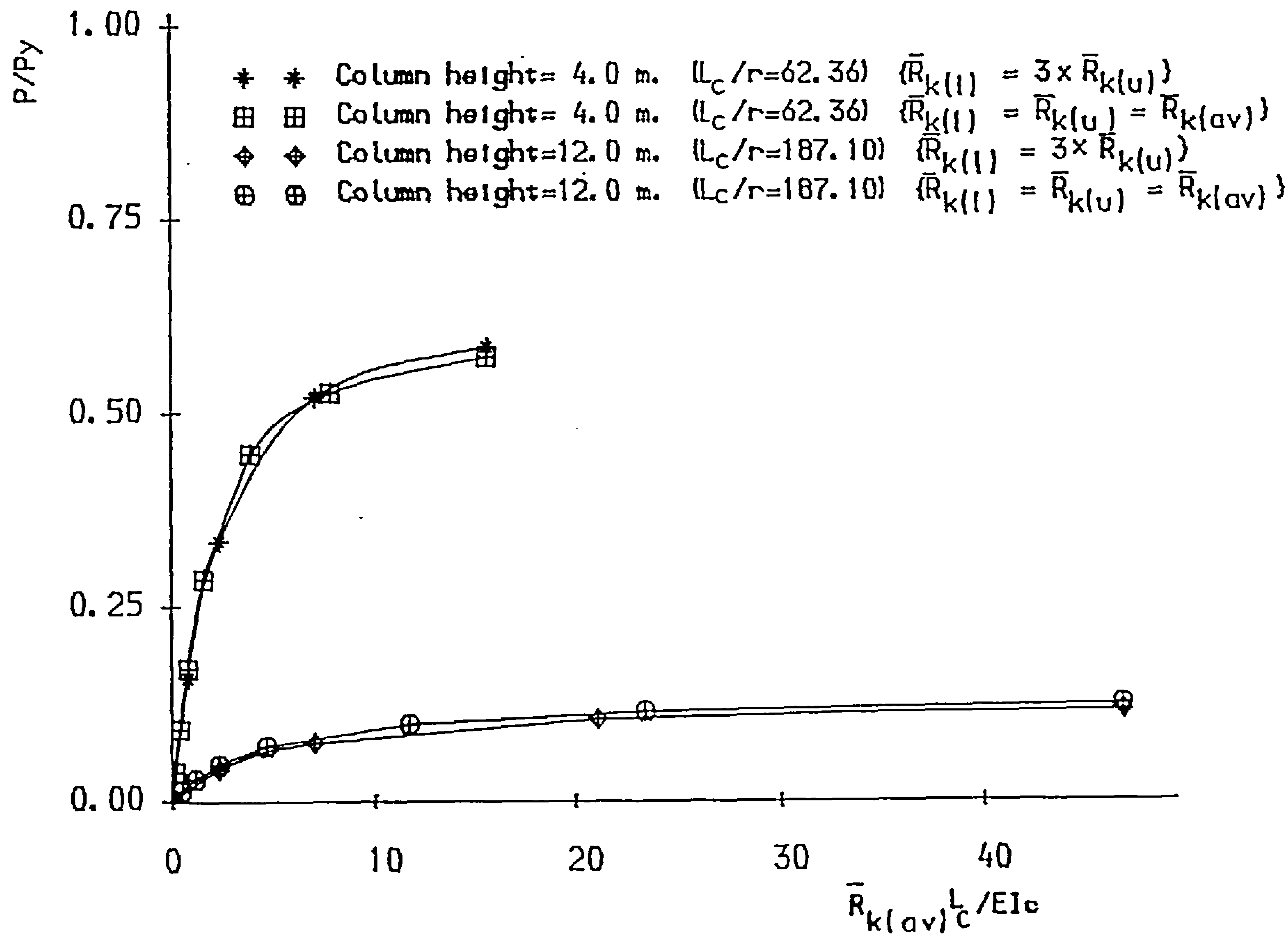


Figure 9.27b: Relationship of ultimate column load against column end restraints for the column shown in fig.9.27.a. The ratio of horizontal to vertical loads is 1%.

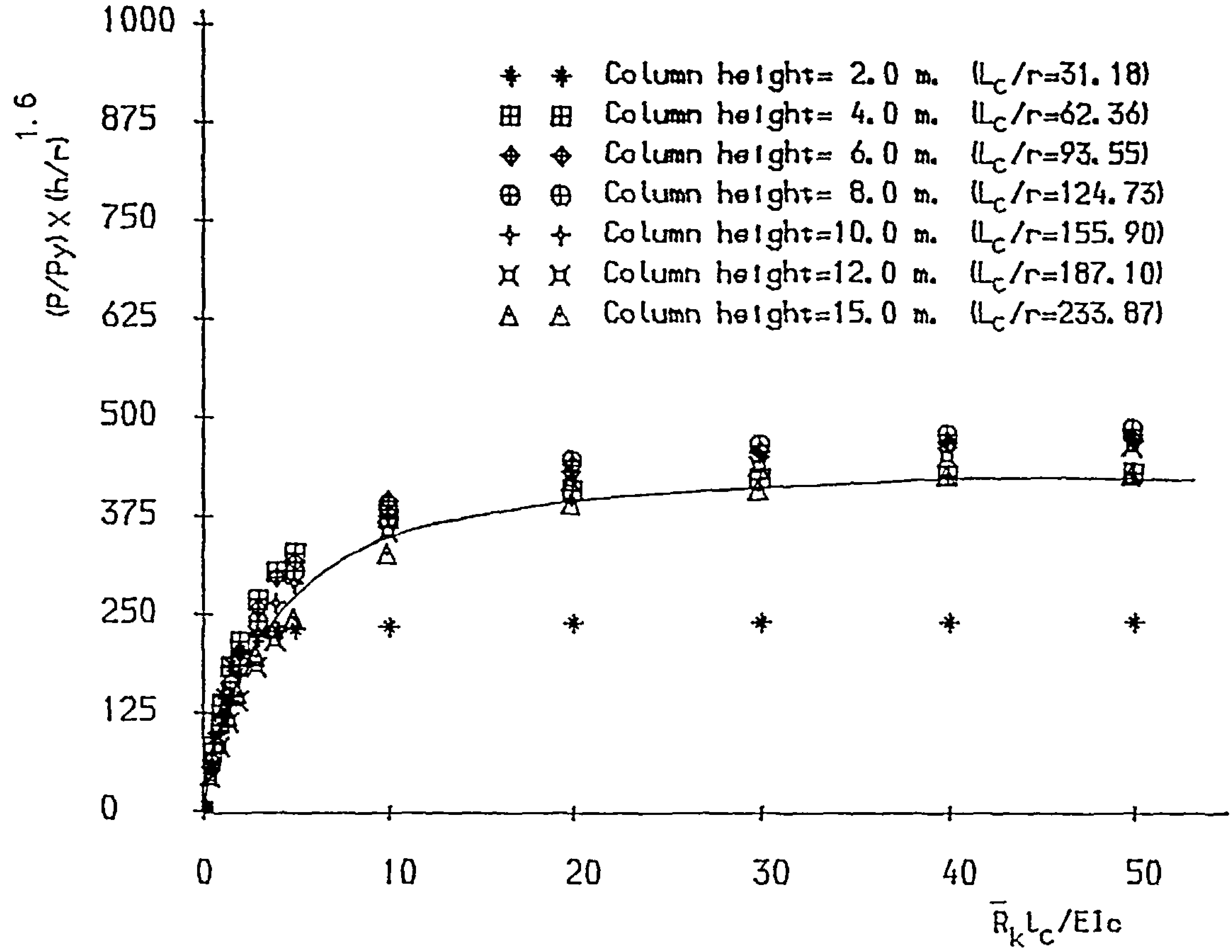


Figure 9.28: Column design chart obtained taking the ratio of horizontal to vertical loads as 2%.

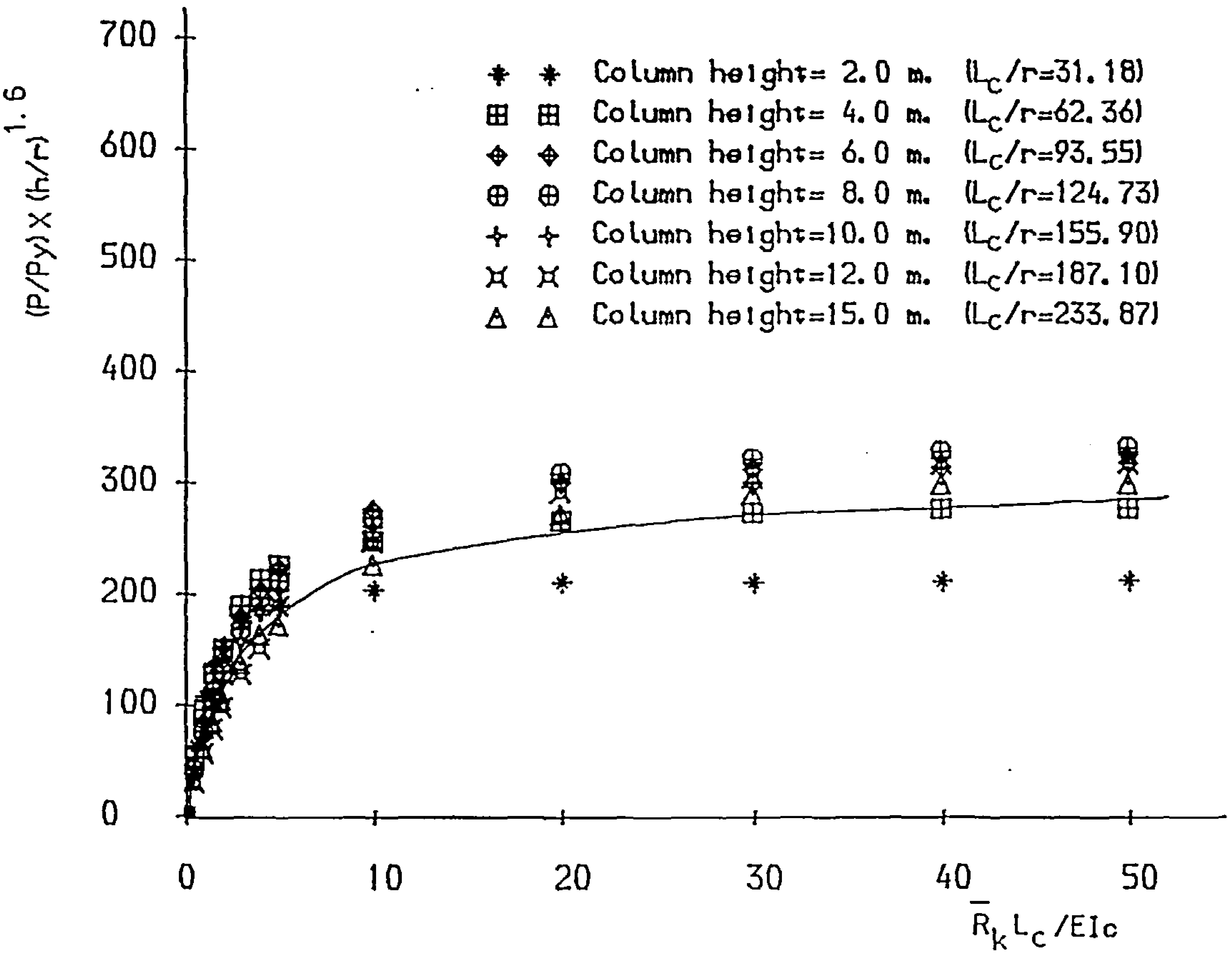


Figure 9.29: Column design chart obtained taking the ratio of horizontal to vertical loads as 5%.

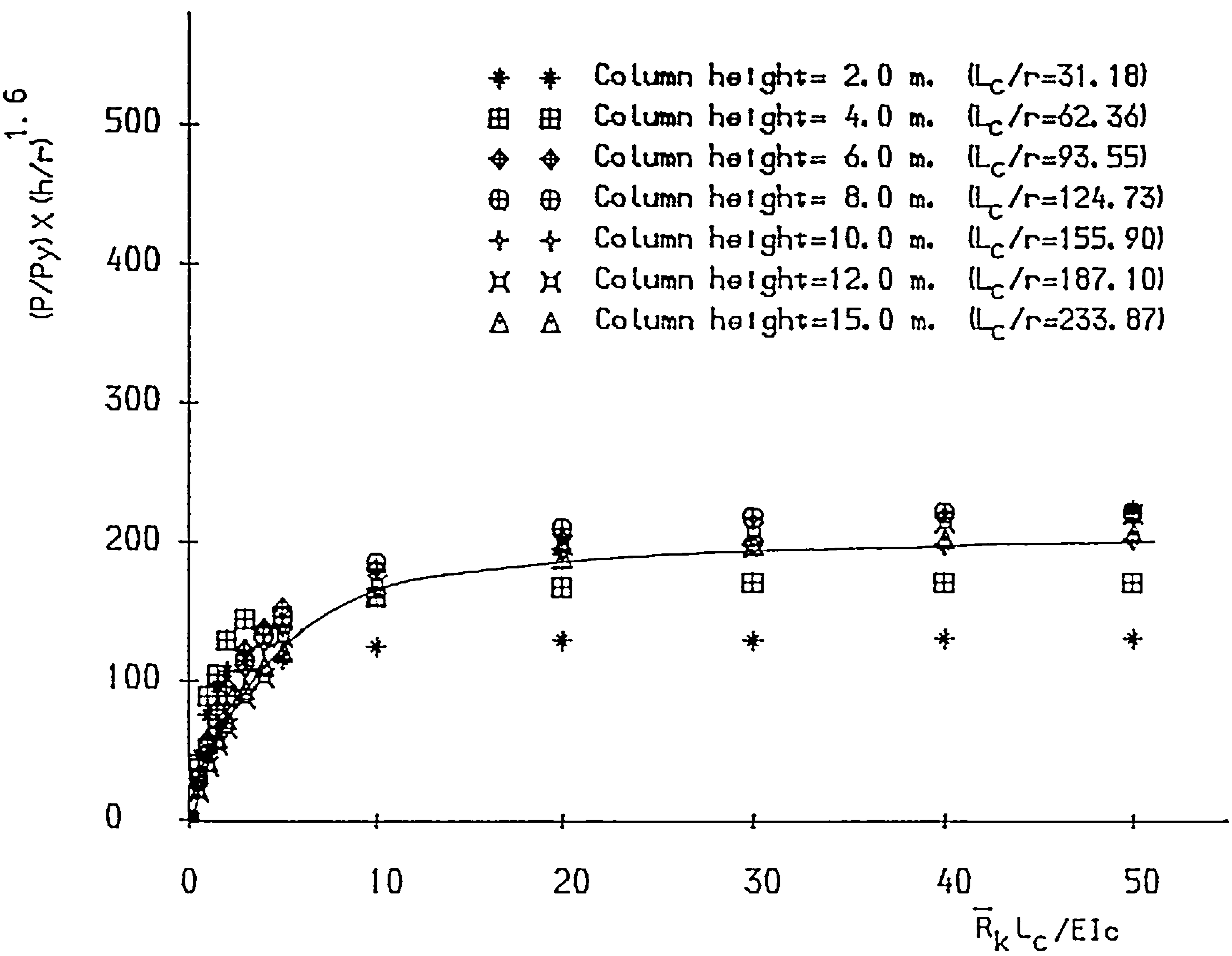


Figure 9.30: Column design chart obtained taking the ratio of horizontal to vertical loads as 10%.

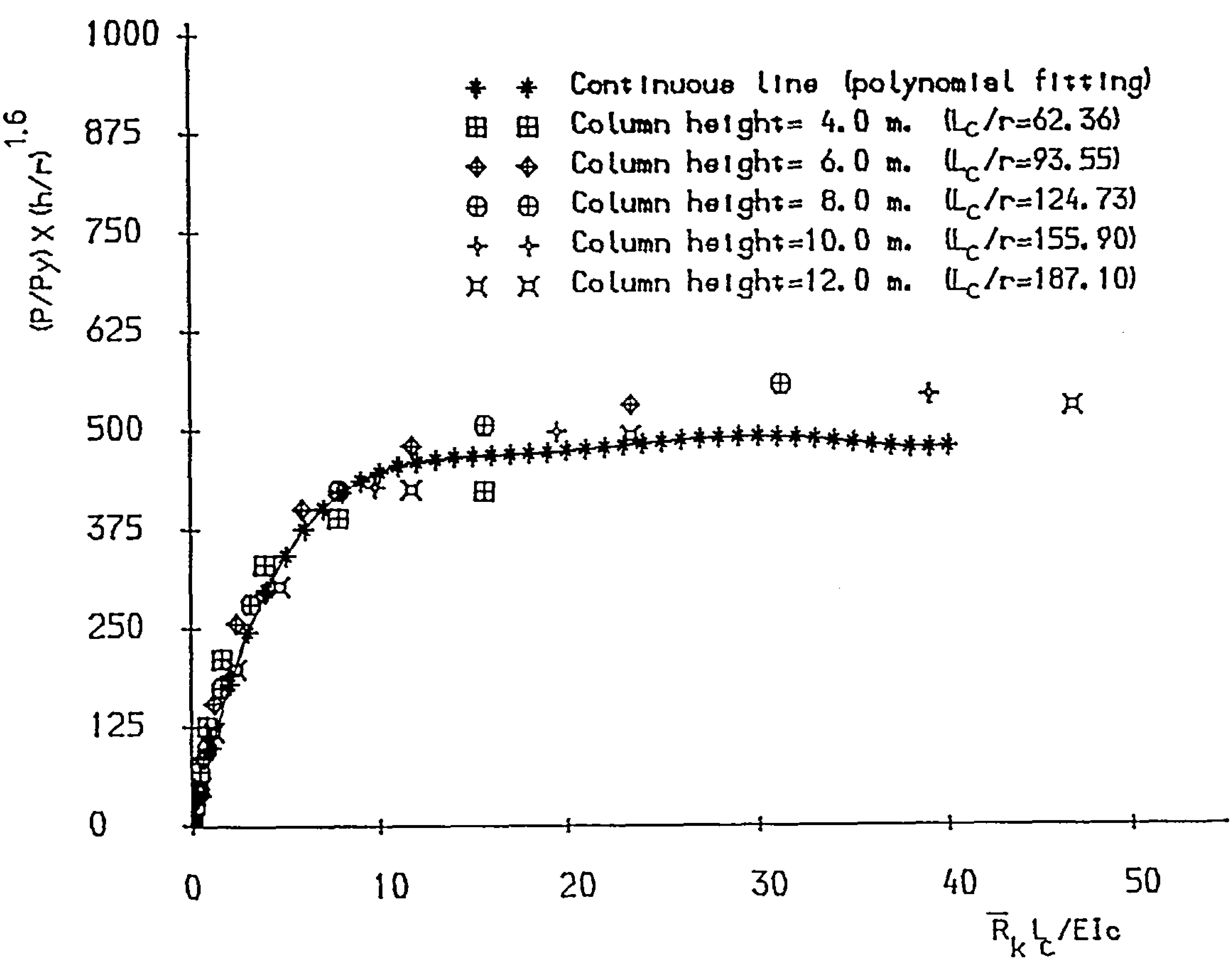


Figure 9.31: Polynomial representation of the design chart shown in 9.26.

Chapter 10

Design Example of a Flexibly Connected Steel Sway Frame

10.1 Introduction

This chapter describes the findings of an investigation of several aspects concerning the in-plane behaviour of both rigidly and semi-rigidly connected steel frames and some design methods which have been developed by the author. In order to demonstrate the use of these methods in designing practical frames, a two storey two bay frame subject to both horizontal and vertical loads, has been fully worked.

10.2 The Problem

When designing a practical frame the designer usually knows only the applied loading together with the frame geometry. For the frame shown in fig.10.1 the vertical loads are taken as typical of office loading as 56 kN/m on the floor beams and as 28 kN/m on the roof beams. The horizontal loads are arbitrarily chosen by the author as 4% of the vertical loads. In designing this frame the following steps have been undertaken:

1) Primary selections for the elements of the frame and for the beam-column joints which are assumed here to be flush end plate connections are made using the method described in section 9.8.1

2) The accuracy of this selection is checked against a computer program (SERVAR) utilising a second-order elasto-plastic analysis.

3) The approximate method mentioned in chapter 6 has been employed to calculate an estimate for the bending moments in the elements of the frame including the nonlinear behaviour of the selected semi-rigid joints. These moments have been checked against those obtained utilising an exact analysis.

4) The improved version of the amplified sway method mentioned in chapter 3 has been used to account for the secondary effects (geometrical deformations) of the frame and the resulting bending moments of the frame elements checked against exact values obtained from utilising the computer program (SERVAR).

5) The lateral deflections of the frame have been checked to ensure that they satisfy the serviceability condition and steps 3 and 4 repeated if required.

10.2.1 Primary Selection for the Frame Element Sections

In designing a steel frame, the first step required is the primary selection for the frame elements taking in account the geometry of the frame and the applied loads. Clearly a good initial choice of the beam and the column sections will reduce the complexity of the problem and lessen the requirement for repetition in the procedure. The column elements and beam elements of the frame are chosen independently as follows:

1) Beams are proportioned assuming that they are simply supported.

2) Columns are designed according to the design charts developed in chapter 9 taking into consideration their rotational end restraints and the ratio of the horizontal to the vertical loads.

10.2.1.1 The Solution

Beams

The bending moment at the beam mid-span can be calculated for the lower storey of the frame as:

$$M_{b1} = \frac{0.56 \times (600)^2}{8} = 25200 \; kN.cm$$

Therefore the elastic modulus for the required beam section is:

$$Z_x = \frac{25200}{27.50} = 916.3 \; cm^3$$

Select section 356 × 171 × 67 UB with $Z_x = 1070 \; cm^3$; $I_{b1} = 19500 \; cm^4$

Repeating same procedure for beam b_2 gives the required section for this beam as 305 × 127 × 37 UB with $Z_x = 472 \; cm^3$; $I_{b2} = 7160 \; cm^4$.

Joints

The utilised joints are assumed here to be flush end plates. However no available test data for the selected beam sections could be traced by the author therefore the $M - \Phi$ relationship shown in fig. 10.2 has been assumed as reasonable by interpolating existing data.

Columns

The first selection for the column sections of the frame can be accomplished by calculating the rotational end restraints, the applied vertical load and the ratio of the vertical to the horizontal loads of the column.

Column C1

Axially applied load $= (28 + 56)\ 3 = 252 \quad kN$

Try section $152 \times 152 \times 37\ UC$ with $I_{c1} = 2220\ cm^4$, $\frac{L_c}{r} = 87.71$ and $P_y = 1303.5 \quad kN$

In calculating the rotational end restraints of the column the stiffness of the joint S_1, indicated in fig.10.1, is taken as the secant K_{b1_o} (see the dotted line in fig.10.2). This value has been calculated according to the recommendation given in section 9.9.1, i.e. by the following steps:

1) Calculate the beam end rotation Φ_{bi_o} assuming that it is simply supported at both ends.

$$\Phi_{bi_o} = \frac{W \times (L_g)^3}{24 \times E \times I_g} \tag{10.1}$$

where:

W	is the beam U.D.L load intensity;
L_g	is the beam length ;
E	is Young's modulus and
I_g	is the beam second moment of area.

Hence

$$\Phi_{b1_o} = \frac{0.56 \times (600)^3}{24 \times 21000 \times 19500} = 0.0123 \quad rad.$$

2) Take the connection stiffness as the slope of the line which joins the origin and the point on the $M - \Phi$ relationship corresponding to the rotation Φ_{b1_o}. Hence K_{b1_o} is equal to 780000 kN.cm/rad.

Knowing the joint stiffness K_{b1_o}, the rotational restraints of the column can be computed in conjunction with eq.9.4 by noting that n in this case is equal to 6 (double curvature). However for the upper end of the column only the beam (b1) has been considered hence the presence of the upper column is ignored. This can be justified by recognising that the columns are chosen according to their ultimate loads therefore at failure only the beams of the frame are expected to provide rotational restraints. This arrangement has been assumed by the author to account for a possible occurrence of yielding in the upper column. Different arrangements can be used provided that they are justified. Therefore the rotational end restraint for upper end of the column is equal to:

$$\overline{R}_k = \frac{6 \times 21000 \times 19500}{600}\left[\frac{1}{1 + \frac{6\times 21000\times 19500}{600\times 780000}}\right] = 655200 \ kN.cm/rad.$$

For lower end and since it is fully fixed at the base, the corresponding rotational end restraint can be taken conservatively as the upper end value. Therefore

$$\frac{\overline{R}_k.L_c}{E\ I_c} = \frac{655200 \ \times 600}{21000 \ \times 2220} = 8.43$$

Taking linear interpolations between figures 9.28 and 9.29 produces:

$$\frac{P}{P_y}\left(\frac{L_c}{r}\right)^{1.6} = 265.5$$

Hence

$$P = \frac{265.5 \times \ 1303.5}{(87.7)^{1.6}} = 269.0 \ kN. \ \succ \ 252.0 \ kN \ (OK.)$$

Here 252 kN is the externally applied load.

The selected column sections are given in table 10.1.

Column	$P_{applied}$ kN	$\overline{R}_k$ kN.cm	Trial section	$\frac{\overline{R}_k L_c}{EI_c}$	$\frac{P}{P_y}\left(\frac{L_c}{r}\right)^{1.6}$	$P_{proposed}$ kN
C1	252	655200	152x152x37UC	8.43	265.5	269
C2	504	1310400	203x203x46UC	8.21	260.0	491
C3	84	482300	152x152x23UC	10.93	290	170

Table 10.1: Selected column sections for the frame indicated in figure 10.1

10.2.2 Analytical Comparison

Utilising the computer program (SERVAR) to analyse the frame employing a second-order elasto-plastic analysis to calculate the column ultimate load factor λ_u produces a value of $\lambda_u = 1.16$. Thus indicating that the frame has been designed conservatively. The semi-rigid joints are represented here by their actual $M - \Phi$ relationships. Fig.10.3 shows the location of plastic hinges at failure. This analysis shows that utilising the design charts figures 9.26,9.28-9.30, included in chapter 9 to estimate the inelastic ultimate loads for the columns, provided a safe and conveniently rapid design procedure.

Nevertheless the selected beam sections did not develop any plastic hinges at failure (see figure 10.3) and may be heavier than necessary. Therefore the beam section sizes may be made smaller and are reduced by approximately 25% according to their second moment of areas. This leads to the selection of $356 \times 171 \times 51\,UB$ and $305 \times 102 \times 28\,UB$ for beams b1 and b2 respectively. The procedure to select appropriate sections for the columns elements of the frame is repeated to produce table 10.2. Utilising SERVAR to calculate the frame ultimate load λ_u after reducing the beam section sizes gives $\lambda_u = 1.175$. The frame mechanism at failure is shown in fig.10.4.

It should be noted here that by reducing the beam section sizes, the frame ultimate load factor surprisingly increased, i.e. from 1.16 to 1.175. This conclusion can be explained by noting that reducing the beam sections causes a reduction

Column	$P_{applied}$ kN	$\overline{R}_k$ kN.cm	Trial section	$\frac{\overline{R}_k\, L_c}{EI_c}$	$\frac{P}{P_y}\left(\frac{L_c}{r}\right)^{1.6}$	$P_{proposed}$ kN
C1	252	565400	152x152x37UC	7.27	242.0	245
C2	504	1130800	203x203x52UC	6.14	227.0	491
C3	84	389500	152x152x23UC	8.83	266	156

Table 10.2: Selected column sections for the frame indicated in fig.10.1 after reducing the beam section sizes

of the end bending moments of the columns allowing for a slight increase of the frame ultimate load.

10.2.3 Estimating the Bending Moments in the Frame Elements

In order to check the adequacy of the frame elements, good prediction for these elements bending moments are required. A simple and rational design procedure to calculate these bending moments is included in chapter 6. This procedure is to be applied here to estimate the element bending moments of the frame proceeding as follows:

1) The frame is subjected to vertical loads only and a fully rigid first order linearly elastic analysis has been carried out to calculate the frame bending moment diagram. These values are encircled in fig.10.5.

2) Use eqs. 5.11 and 5.17-5.20 to calculate the reduction factor α_s for each semi-rigid joint independently. Thus for joint S_1 taking the joint stiffness as the initial tangent of the $M - \Phi$ relationship, i.e. $K_o = 1050000\ kN.cm$ (see figs.10.1 and 10.2) gives:

$$\alpha = \frac{2 \times 21000 \times 13920}{1050000 \times 600} = 0.92$$

$$\eta = \frac{3 \times 0.92 + 1}{3(0.92)^2 + 4 \times 0.92 + 1} = 0.521$$

$$\psi_{s1} = \frac{\frac{2\times21000\times13920}{600} \times 0.521}{2 \times 21000(\frac{2190}{600} + \frac{1235}{600} + 0.521 \times \frac{13920}{600})} = 0.68$$

$$\psi_{f1} = \frac{\frac{2\times21000\times13920}{600}}{2 \times 21000(\frac{2190}{600} + \frac{1235}{600} + \frac{13920}{600})} = 0.8$$

$$\alpha_{s1} = \frac{1}{1 + 0.521}\,\frac{0.68 - 1}{0.80 - 1} = 1.05$$

For this joint utilising a flush end plate joint surprisingly increases the beam end moment even in the absence of horizontal loads. This behaviour has been confirmed by SERVAR and can be explained by realising that the reduction factor α_s (see eq.5.20) has two terms. The first one is always less than unity and represents the reduction of the bending moment at the beam end resulting from the presence of the semi-rigid joint. The second term is always greater than unity and represents the reduction of the flexural rigidity of the node in which the semi-rigid joint is connected. This reduction will certainly increase the node rotation which might increase the beam end moment. In this particular case and since the corresponding columns are relatively slender, when compared with the beams and the joints, the increase of the beam end moment as a result of the reduction of the node flexural rigidity matches the reduction of the beam end moment resulting from using the semi-rigid joints. Repeating same procedure to compute the values α_{s2} and α_{s3} gives:

$$\alpha_{s2} = 0.518 \quad and \quad \alpha_{s3} = 0.94$$

Reducing the beam end moments obtained from a fully rigid analysis by the appropriate factor α_s for each semi-rigid joint independently produces the values indicated in fig.10.5. From this figure and according to the semi-rigid joints $M-\Phi$ curve (which is indicated in fig.10.2) only the semi-rigid joint labelled as S_2 needs an iteration procedure, as recommended in chapter 6, to incorporate the joint nonlinearity. This is because the corresponding bending moment obtained considering the initial tangent of the joint $M - \Phi$ relationship is equal to

11810 kN.cm which exceeds the linear stage of the relationship hence the joint exhibits nonlinear behaviour. The joint bending moment, shown in fig.10.0 as $M_{s2,1}$, is plotted on the initial line range of the $M - \Phi$ curve which is rotated clockwise (see fig.10.2) until $M_{s2,1}$ intersects the actual $M - \Phi$ curve defining a revised value for this joint stiffness $K_{s2,1} = 747000\ kN.cm/rad.$
Calculating the new value of α_{s2} corresponding to $K_{s2,1}$ gives:

$$\alpha_{s2} = 0.433$$

Reducing the joint bending moment by α_{s2} gives the revised value, $M_{s2,2}$, for the joint bending moment as 9872 kN.cm. Plotting the value $M_{s2,2}$ on the secant line $K_{s2,1}$ indicates that no further iteration is needed since $M_{s2,2}$ is sufficiently close to both the assumed joint stiffness $K_{s2,1}$ and the actual $M - \Phi$ curve.
The column bending moment is reduced as recommended in section 5.5 to obtain the modified column end moments. These values are indicated in fig.10.5. In fig.10.6 the frame bending moment diagram predicted by the proposed method is compared to the exact diagram predicted utilising the computer program SERVAR from which it can be seen that reasonably good correlation between the two diagrams is obtained. It should noted here that in both analyses only the nonlinear behaviour of the utilised semi-rigid joints is included.
3) To calculate a prediction of the frame bending moments due to the applied horizontal loads, the second moment of areas of the beams are reduced by the factor C_s (see eq.5.4) and then a fully rigid method of analysis is used to calculate the frame bending moments. Thus for upper beams (b2) the factor C_{su} is given by:

$$C_{su} = \frac{1}{1 + \frac{6 \times 21000 \times 5336}{1050000 \times 600}} = 0.484$$

and the reduced beam second moment of area I'_{gu} is

$$I'_{gu} = 0.484 \times 5336 = 2581\ cm^4$$

The initial tangent stiffness (K = 1050000 kN.cm) has been employed here in conjunction with eq.5.4 since the beam end joints have linear behaviour under the externally applied loads.

Repeating the same procedure for the beams b1 and noting that the internal joint S_2 of the beam has shown nonlinear behaviour with stiffness K= 747000 kN.cm/rad while the external joint S_1 behaved linearly elastic with stiffness K= 1050000 kN.cm/rad. Hence the corresponding beam has two semi-rigid joints with different characteristics therefore the average value of both joint stiffnesses should used in conjunction with eq.5.4 to calculate the reduced factor C_{sl}.

$$K = \frac{1050000 + 747000}{2} = 898500 \ kN.cm/rad.$$

and therefore

$$C_{sl} = \frac{1}{1 + \frac{6 \times 21000 \times 13920}{898500 \times 600}} = 0.235$$

and the reduced beam second moment of area I'_{gu} is

$$I'_{gl} = 0.235 \times 13920 = 3272 \ cm^4$$

A fully rigid linearly elastic analysis is used to analyse the frame considering the applied horizontal loads only, after replacing the actual beam sections by ficticious sections with second moment of areas equal to 2581 and 3272 cm^4 for upper and lower beams respectively.

4) Combining the frame bending moments obtained from step 3) and those obtained from step 2) and plotting the bending moment diagram of the frame produces fig.10.7. Also indicated in this figure are the frame bending moments obtained from the computer program SERVAR. Comparison shows that a reasonable accuracy of the prediction of the frame bending moment diagram has been obtained by employing a very simple hand calculation procedure.

10.2.4 Secondary Effect

It has been assumed that the geometrical deformations of the frame are small and can be neglected therefore only linearly elastic analysis including the non-linear behaviour of the semi-rigid joints has been adopted. In flexibly connected sway structures however these geometrical deformations may well be significant due to the relatively large lateral movements. Including this effect can be accomplished approximately by means of employing the improved amplified sway method mentioned in chapter 3.

The frame lateral deflections resulting from step 3) can be used to calculate the required sway index of each storey noting that, since in this analysis the frame is subjected to applied horizontal loading which is equal to 4% of the vertical load, hence the lateral deflections obtained in step 3) should be divided by 8 to correspond to the standard 1/2 % rule. The proposed method can then be carried out as usual. Using the proposed method gives the amplified sway factors A_{f1} and A_{f2} for the lower and the upper storey respectively as:

$$A_{f1} = 1.27 \qquad and \qquad A_{f2} = 1.26$$

In the development of this method the beams are assumed to be fully rigidly connected to the columns thus same amplified sway factor can be used to amplify the bending moment digram of any storey of the frame resulting from the applied horizontal loads. For flexibly connected sway frames however, only partial continuity exists between the beams and the columns and hence the geometrical deformations effect in beams is expected to be less significant than its values for the columns. Therefore the amplified sway factor calculated for a storey of the frame should be used to amplify the column bending moments and only a fraction of it should be used to amplify the bending moment of the beams of the frame. This can be accomplished by utilising the factor η (see eq.5.17) which effectively measures the continuity between the beam and the column of the corresponding semi- rigid joints, i.e. the value of η ranges between (0 and 1) and takes the values 0 and 1 for a pinned joint and a fully rigid joint respectively. The reduced amplified sway factor for the beams of a storey can be obtained from:

$$Af_{i,b} = (Af_i - 1) \times \eta + 1$$

while the amplified sway factor for the columns $Af_{i,c}$ of the storey should be taken as Af_i which is calculated from the improved amplified sway method assuming fully rigid joints.

Applying this to the frame results leaves the column factors unaltered as

$$Af_{f1,c} = 1.27 \qquad and \qquad Af_{f2,c} = 1.26$$

and reduces the beam factors to

$$Af_{f1,b} = 1.13 \qquad and \qquad Af_{f2,c} = 1.20$$

Amplifying the element bending moments resulting from step 3) by the corresponding factor and adding them to those resulting from step 2) provides the frame bending moment diagram including both the nonlinear behaviour of the semi-rigid joints and the geometrical deformations of the frame. These values are compared in fig.10.8 with the exact value obtained from SERVAR. This comparison shows that, once again a reasonably good correlation has been obtained.

10.2.5 Serviceability Condition

Most steel design codes [4], [5] requires that the sway index of any storey of a practical structures does not exceed, for a serviceability reason, a specific limit which is generally taken as 1/300. To check the serviceability condition of the designed frame, the accuracy of the predicted lateral deflections obtained

	Proposed method	Exact
Lower storey	5.00 cm	5.22 cm
Upper storey	10.21 cm	10.66 cm

Table 10.3: Comparison between the proposed method and an exact method for the prediction of the lateral deflections of the frame indicated in fig.10.1

from the proposed method (step 3) is first checked against the corresponding values obtained from utilising a first order linearly elastic analysis including the nonlinear behaviour of the utilised semi-rigid joints. In the exact analysis both horizontal and vertical loads are considered. The results are tabulated in table 10.3. Noting that the storey height is 6.00 m, table 10.3 shows that:

1) The proposed method provided a good estimation of the frame lateral deflection

2) The sway indices of the frame stories are greater than the allowable.

Hence the sections should be increased or some bracing system should be provided to assist the frame elements in resisting the lateral loads. Consider first the presence of one infill panel extending over the full height of the frame but only over one bay of the frame. Taking the panel dimension as 6.00x6.00 m^2, its elastic modulus E_p as 1540 kN/cm^2, the panel thickness t as 10 cm and employing equations 7.1-7.3 of BS5950:Part1 to calculate the equivalent diagonal bracing elements for the lower storey A_{s1} and the upper storey A_{s2} gives:

$$S_p = 2310\, kN/cm \quad and \quad K_{3,s1} = 31.1\, but \leq 2 \quad thus \quad K_{3,s1} = 2$$

and therefore

$$A_{s1} = 0.15 cm^2$$

Repeating same procedure for the upper storey of the frame gives:

$$S_p = 2310\, kN/cm \quad and \quad K_{3,s2} = 80.0\, but \leq 2 \quad thus \quad K_{3,s1} = 2$$

	Proposed method	Exact
Lower storey	4.03 cm	3.91 cm
Upper storey	8.080 cm	7.870 cm

Table 10.4: Comparison between the proposed method and an exact method for the prediction of the frame lateral deflections with an infill panel. The panel behaviour is defined according to BS5950 (see eqs.7.1-7.3)

	Proposed method	Exact
Lower storey	1.93 cm	1.83 cm
Upper storey	3.49 cm	3.39 cm

Table 10.5: Comparison between the proposed method and an exact method for the prediction of the frame lateral deflections including two diagonal bracing elements

and therefore

$$A_{s1} = 0.06cm^2$$

Incorporating the equivalent bracing elements in the substitute frame used in step 3) and computing the frame lateral deflections produces table 10.4 It can be seen that the frame sway indices are still greater than the allowable and therefore a real bracing system should be included in the frame if the section sizes are not to be increased. Two diagonal bracing elements are incorporated into the structure with areas 1.0 cm^2 and 0.5 cm^2 for the lower and the upper storey respectively. These values are chosen arbitrarily by the author. Calculating the frame lateral deflection using the substitute frame used in step 3) gives table 10.5 where the exact values obtained from utilising SERVAR are included for comparison. The table shows that the serviceability condition is now satisfied.

Incorporating the two diagonal bracing elements in the frame will inevitably influence the frame's bending moment diagram thus steps 3) and 4) are repeated. Repeating step 3) after including the diagonal bracing elements and adding the resultant element bending moments to those obtained from step 2) produces fig.10.9 in which these values are compared to exact values obtained from SERVAR.

Since the bracing elements sharply reduce the lateral deflection of the frame, the frame secondary effect (the frame geometrical deformation) can be neglected here.

It should be noted that by incorporating the bracing elements the column sections could be reduced according to the empirical eq.9.3. Nevertheless since the serviceability condition dominates the frame design and since any reduction of the column section sizes might leave the frame sway indices beyond the serviceability limit, it is desirable to retain the column sections derived.

10.3 Conclusion

A flexibly connected steel frame has been fully designed for in plane response according to design methods which have been developed in this project. At the start of the problem only the applied loads together with the geometry of the frame are known. A simple and hand calculation procedure has been used to accomplish a first selection of the frame elements which has been demonstrated to be reasonably conservative selection for strength consideration. To check the capacity of the frame members element forces and moments need to be calculated. Since the presence of semi-rigid joints is expected to influence only the bending moment diagram of the frame, an estimation for such a diagram has been achieved by means of utilising a simple hand calculating procedure including the influence of the nonlinear behaviour of the utilised semi-rigid joints together with the secondary effect (geometrical deformation effect).

A comparison has been made between the proposed method and an exact method utilising a computer program called SERVAR. This comparison has shown that a reasonably good prediction of the frame behaviour has been obtained by employing the proposed methods.

Since the whole procedure is simple and requires only few calculations, it is ideally suited for a design office use.

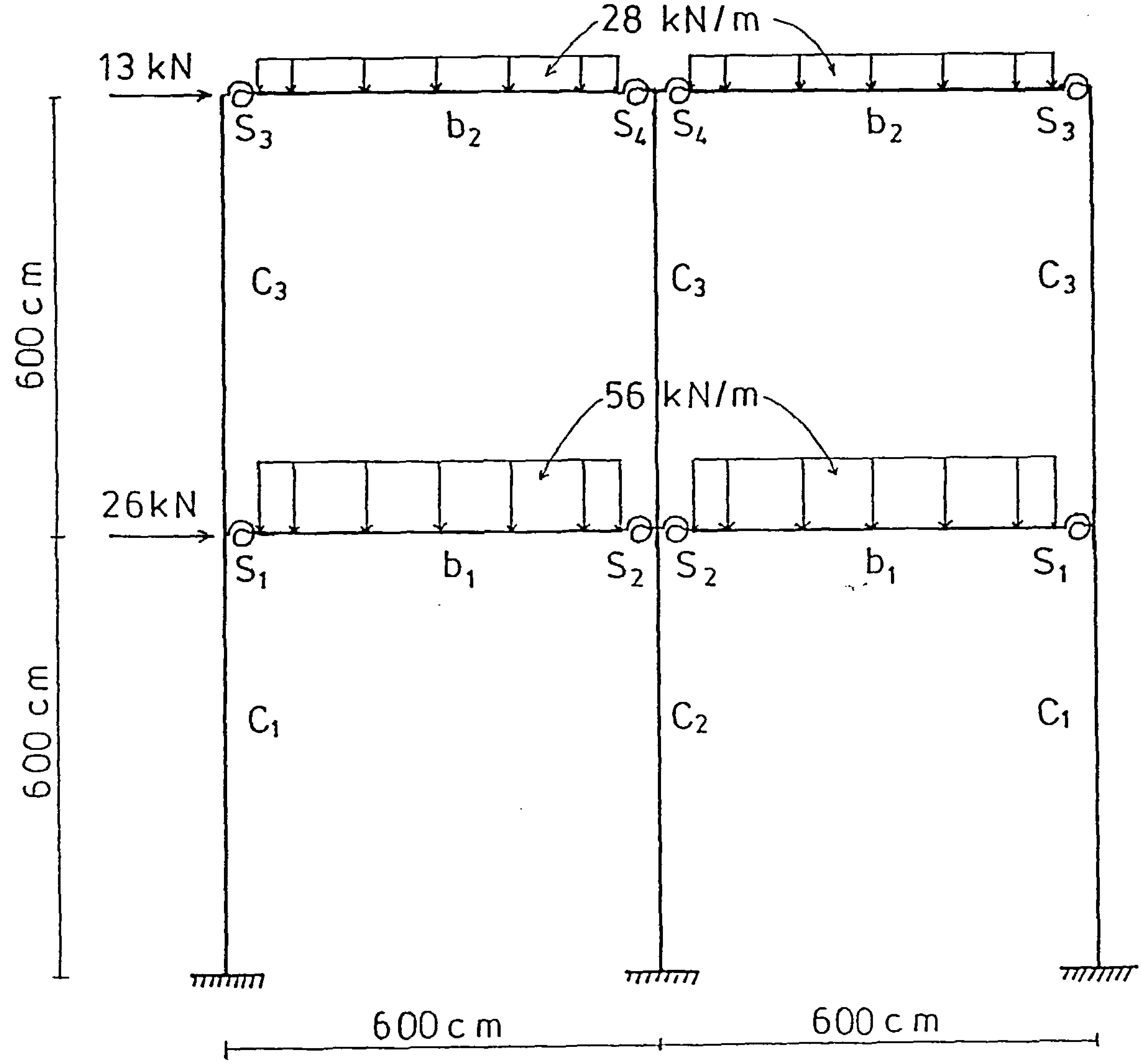


Figure 10.1: Two storey two bay flexibly connected frame.

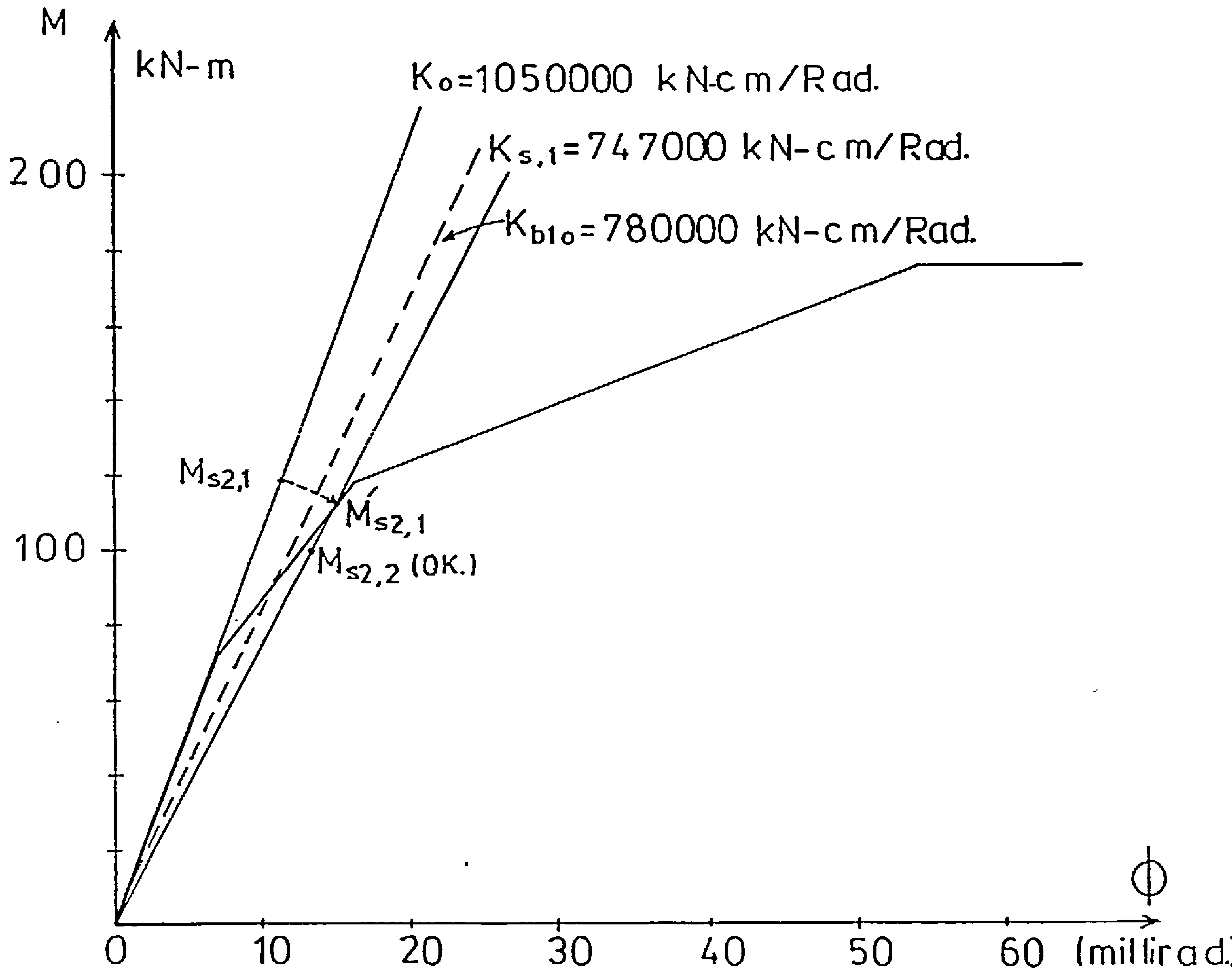


Figure 10.2: $M - \Phi$ relationship for flush end plate joints used in the frame of figure 10.1.

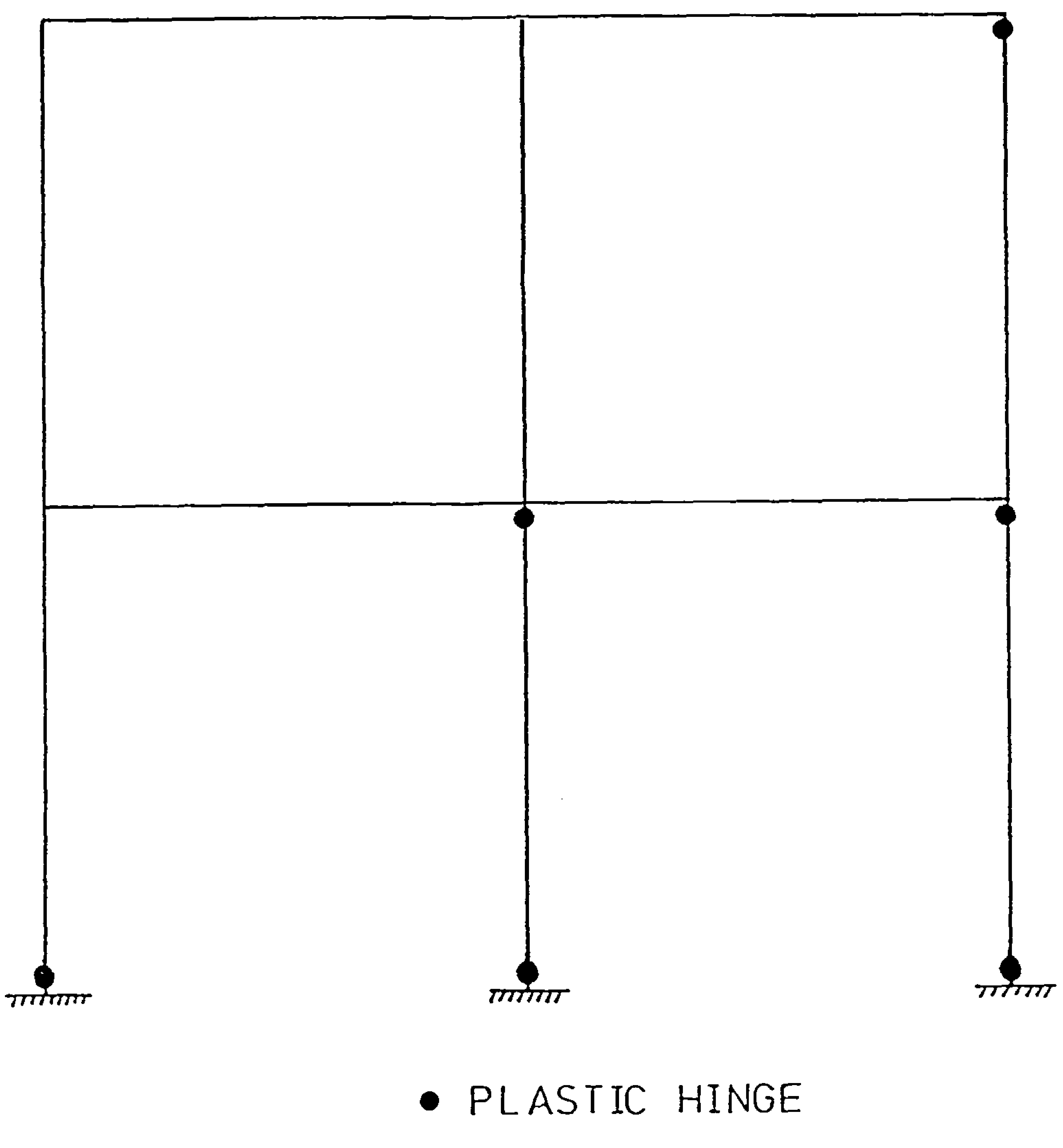


Figure 10.3: Collapse mechanism at load factor $\lambda_u = 1.16$.

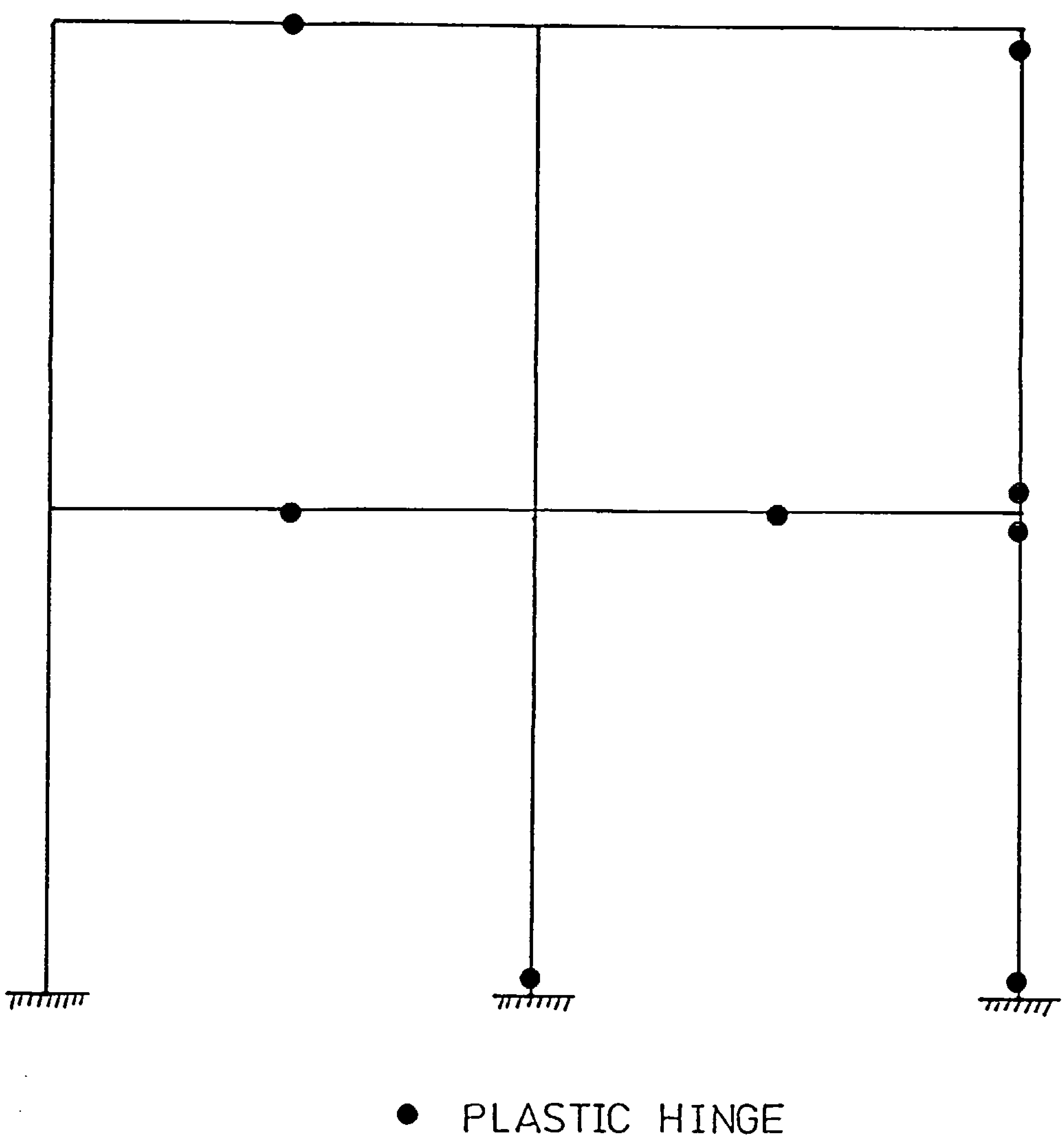


Figure 10.4: Collapse mechanism after reducing the beam sectional sizes by 25%. $\lambda_u = 1.175$.

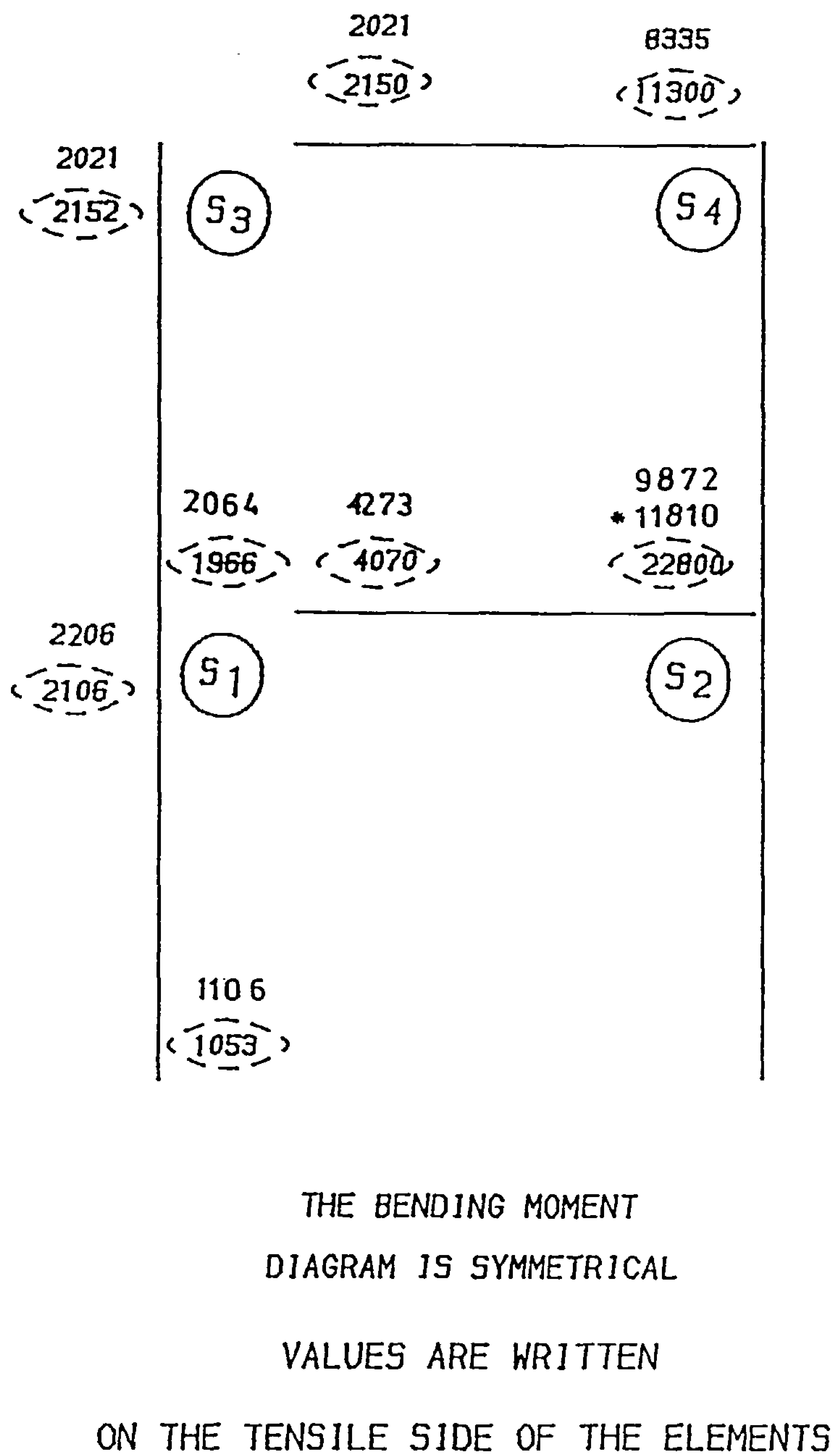


Figure 10.5: Comparison of the frame bending moment values obtained using the proposed method and an exact analysis considering vertical loads only.

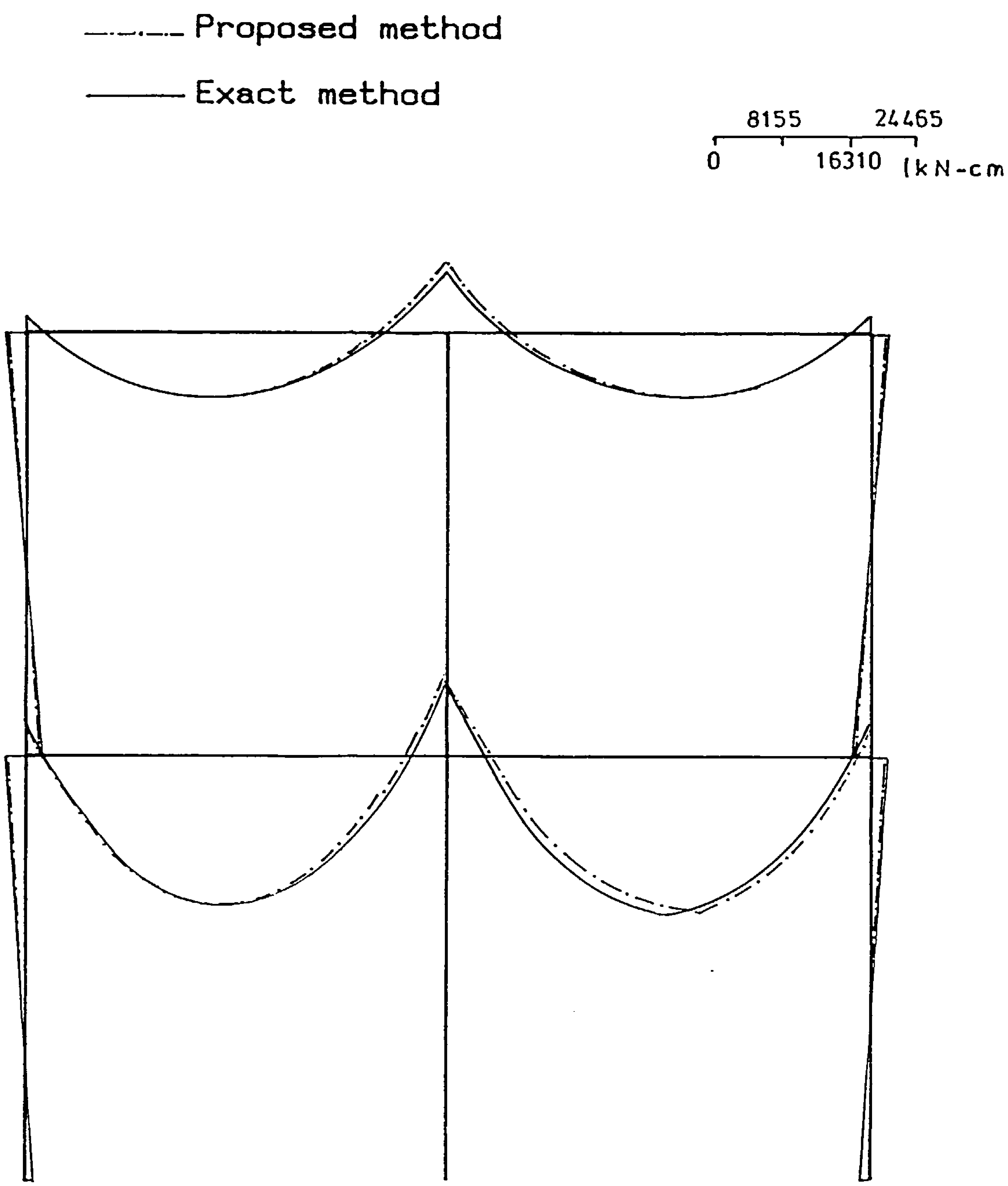


Figure 10.6: Comparison of the frame bending moments obtained using the proposed method and an exact analysis considering vertical loads only.

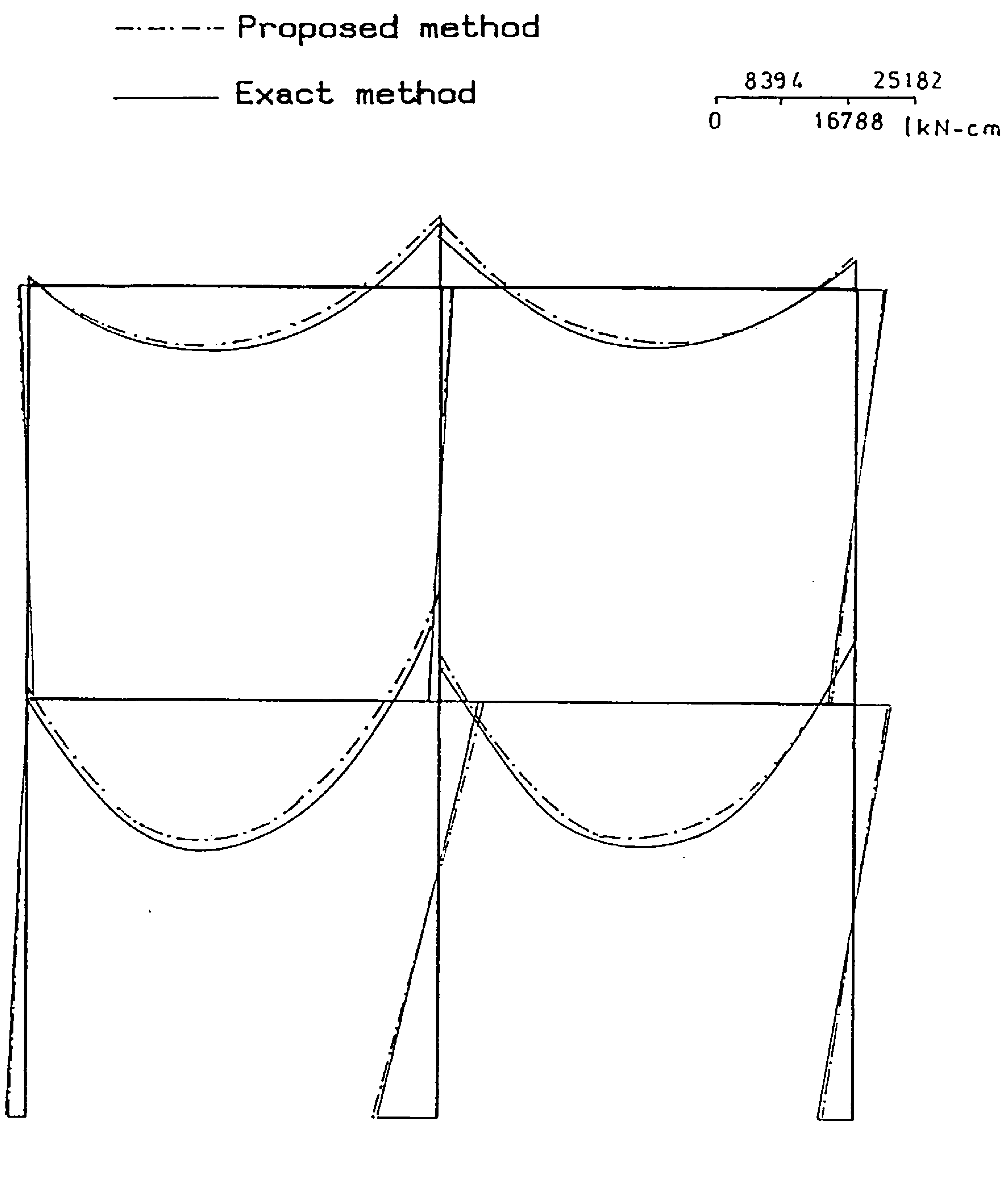


Figure 10.7: Comparison of the frame bending moments obtained using the proposed method and an exact analysis considering both horizontal and vertical loads neglecting P- Δ effects.

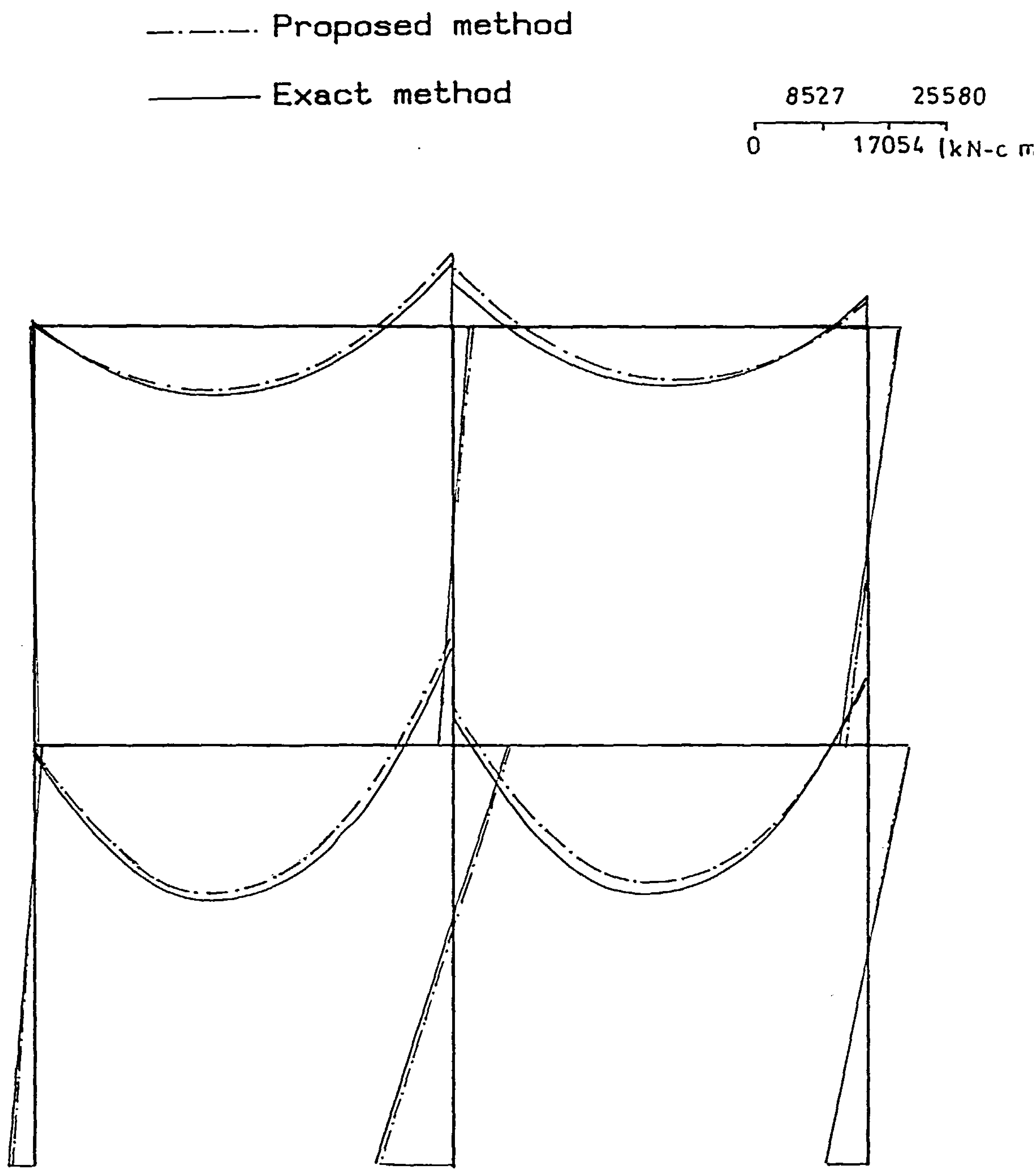


Figure 10.8: Comparison of the frame bending moments obtained using the proposed method and an exact analysis considering both horizontal and vertical loads including P- Δ effects.

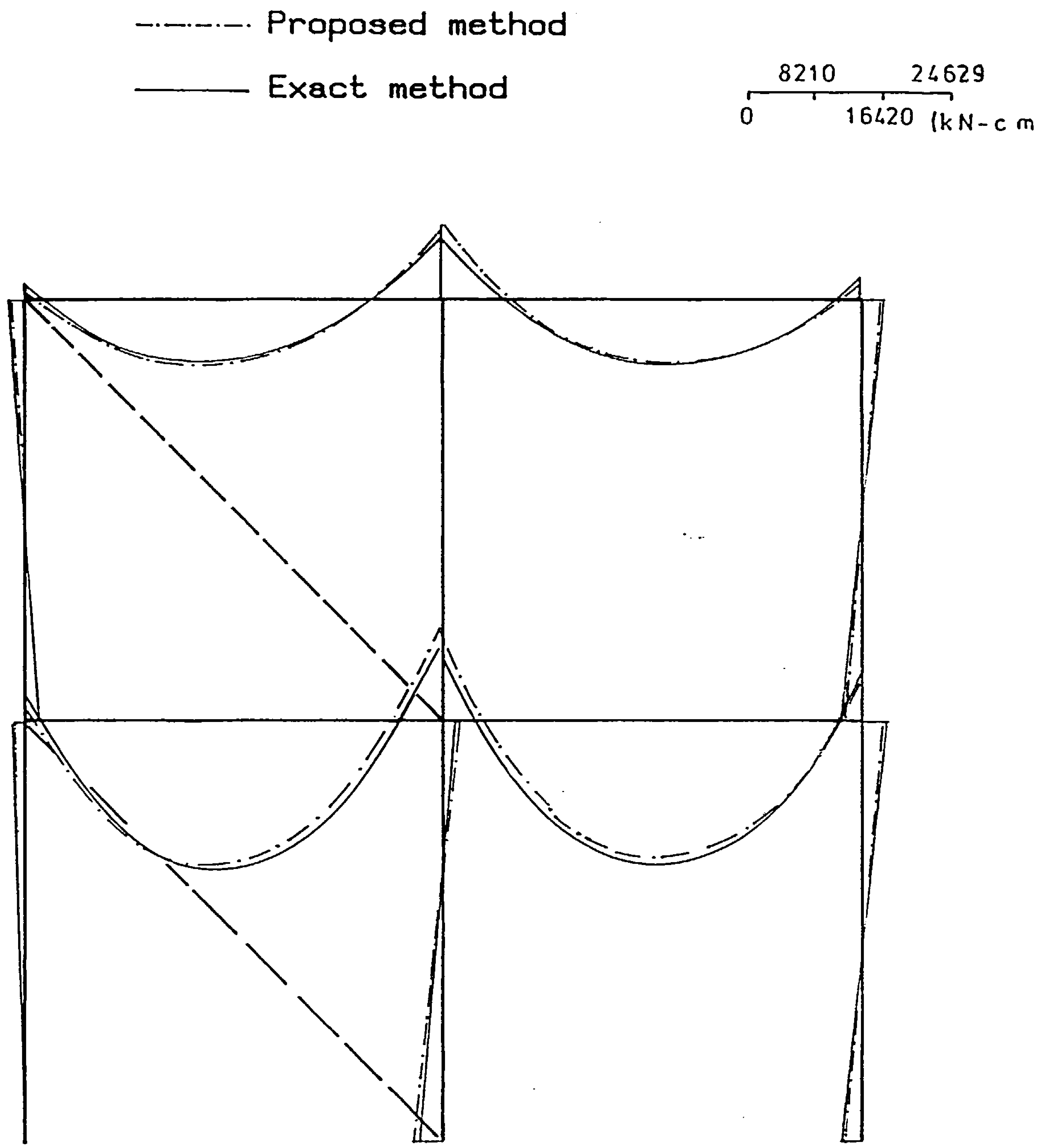


Figure 10.9: Comparison of the frame bending moments obtained using the proposed method and an exact analysis considering both horizontal and vertical loads with lateral bracing.

Chapter 11

General Conclusions and Recommendations for Future Work

11.1 Summary

Modern digital computers have made it possible to predict the behaviour of steel frame structures with remarkable accuracy. Hence several computer programs have been written [1], [32], [3] to simulate the behaviour of such frames. Nevertheless relatively little research work has been conducted with the aim of applying the new ideas to improve the currently used design methods. This project is an attempt to push ahead these methods reducing the gap between the dramatically developed research capability and the relatively simple and often outdated currently used design methods.
The most influential factor on the behaviour of sway frames is well known to be the secondary effect (geometrical deformation effect). BS5950 recommends the use of the amplified sway method in which the bending moments of a prac-

tical frame elements due to horizontal loads are amplified by a constant factor before being added to those due to vertical loads. In a steel frame, although secondary effects are mainly dependent upon the sections and the axial loads of the columns together with the frame lateral deflections, the latter governs the nonlinear behaviour of the frame. For a multistorey frame and since the lateral deflections of the frame are generally non-uniform, the current amplified sway method has been shown to provide unsatisfactory accuracy since a constant amplification factor is used in this method to represent the frame secondary effects which might vary from one storey to another. An improved version of the amplified sway method has been proposed which is almost as simple as the current method but has the superiority of reflecting the frames deformed shape hence providing a more accurate prediction of the behaviour of multistorey frames. This conclusion has been verified by means of comparing the prediction of the modified method and the currently used method for many multistorey frames and checking these against an exact method utilising a computer program called 'Instaf'. This program was available at the start of this project and represents accurately a second order elastic analysis. For all the considered cases the proposed method provided more accurate predictions of the frame behaviour.
A second influential factor which affects both the stability and the bending moment distribution of a steel frame, is the flexibility of steel joints. Experimental tests have shown that all steel joints show finite rotational stiffnesses and behave rather differently than the two extreme cases fully rigid and perfectly pinned joints. Most design codes [4] recognise the existence of semi-rigid joints and permits in a general way the inclusion of their influence in the design analysis yet without providing any clear guide as how to accomplish this. A very simple method called the 'wind connection method' has been used over many decades to handle flexibly connected structures. However with the recent dramatic in-

crease in the knowledge about the behaviour of such structures, it is believed that a more accurate method is possible. This method should account for joint flexibility in a more realistic manner including the nonlinear characteristics of semi-rigid joints with only a minimum disturbance to the currently used design methods.

A simple design procedure which can be used with any of the conventional fully rigid methods of analysis has been proposed. The inclusion of joint nonlinearity is accomplished in this method by means of a simple iterative graphical procedure, which in addition to providing a satisfactory prediction of the frame bending moment diagram, gives the designer a clear idea of the stability of the frame.

In steel sway structures the sway index of any storey should be held below a specific limit (generally taken as 1/300) for serviceability reasons. The proposed method can be used with remarkable accuracy to predict the sway indices of frames including, if required, the beneficial effects resulting from the presence of infill panels. However only an unduly conservative method recommended by BS5950 has been used in this study to account for the presence of such panels. That is due to the lack of detailed knowledge concerning the actual behaviour of these panels on real sway frames.

In order to employ the above mentioned methods to predict the bending moments of the elements of a practical frame, it is first necessary to make a preliminary selection for the frame elements. Such a selection is very complex and is usually achieved by means of trial and error. Therefore any guidance on initial selection, is significant.

Conducting a parametric study aimed at defining the major influential factors on the behaviour of the columns in practical sway frames, requires an analytical tool. The computer program, which was first developed by Jones [35] to simulate the behaviour of flexibly connected columns and which was modified later by Rifai [3] to represent the behaviour of non-sway subassemblages with semi-rigid joints, was found to be the ideal computer program to conduct the above mentioned study. That is due to the fact that this program, unlike SERVAR, accounts for the presence of both initial imperfections and residual stresses. Nevertheless this program required some modifications to suit partially-braced flexibly-connected sway subassemblages. After accomplishing these modifications, a limited parametric study has been conducted to investigate the influence of many factors on flexibly connected column subassemblages with the presence of partial bracing resulting from the existence of infill panels of practical frames. These factors are:

1) Rotational end restraints.

2) Partial sway bracing.

3) Horizontal loads.

4) Residual stresses.

5) Initial imperfections.

The most important factors on the inelastic behaviour of framed columns in sway structures have been identified as i) rotational end restraints, ii) horizontal loads and iii) partial sway bracing.

As a result of this parametric study, design charts have been produced. In these charts, a reasonably conservative estimation of a framed column, in a bare sway frame, can be obtained after estimating the column end restraints taking into account the horizontal load applied on the column storey. Hence in practical frames the beam section sizes can be assessed first assuming that these beams are simply supported and then the type of the joints used can be specified and as a result an estimation of the column end restraints can be calculated. After selecting a section for the considered column, an approximation for the column's ultimate load can be obtained from the design charts and this load should be compared with the externally applied column load. This process should be repeated until the calculated column ultimate load is sufficiently close to but still above the externally applied load.

Inclusion of the beneficial effect resulting from the presence of infill panels can be achieved by enhancing the ultimate column load obtained from the design charts by a factor which can be calculated from an empirical formula which has been developed in this project. This factor is dependent upon the column rotational stiffness, the panel stiffnesses and the rotational restraints provided to the column ends.

A two storey two bay frame has been fully designed. At the start of the problem only the geometry of the frame and the applied horizontal and vertical loads are known. The frame sections have been selected according with the design procedure mentioned above. This selection has been proved to be rapid and adequate. An estimation of the frame bending moment diagram including the nonlinear behaviour of the utilised joints (which have been assumed to be flush end plates) is accomplished by utilising the method proposed in chapter 6. Later the improved amplified sway method has been used with a minor modification to incorporate the secondary effect (geometrical deformation effect) on the frame bending moment diagram. The resulting diagram is verified against an exact one utilising the computer program SERVAR. Finally the frame sway indices have been calculated by both the proposed method and an exact method; good correlation can be seen. Since the frame sway indices were found to exceed the allowable (assumed to be 1/300), the partial bracing due to the presence of an infill panel extended over the whole storey height but one bay only has been considered, according to the design procedure recommended by BS5950, and

found to be inadequate to significantly reduce the frame's lateral deflections. Therefore a real bracing system has been incorporated to remedy this problem and the frame bending moment diagram has then been reassessed.
Only fully-rigid first order linearly elastic analysis (which can achieved by means of a simple computer program or by any of the conventional method such as the slope deflection equations etc.) has been required to design the frame and predict its bending moment diagram including the nonlinear behaviour of the utilised semi-rigid joints together with the secondary effect (geometrical deformation) resulting from the frame lateral deflections. Hence the design methods which have been developed can be conveniently used in a design office.

11.2 Recommendations for Future Work

The following recommendations are thought to be of primarily important to extend this subject.

1) In order to employ the design method, which has been proposed to obtain an estimation of the element bending moments in a flexibly connected frame, a good description of the joint moment-rotation characteristics are needed. It is believed that at this time such characteristics can be only obtained reliably by means of conducting experimental tests. Therefore moment-rotation relationship for the most commonly used semi-rigid joint types and covering the most common arrangements of practical beam and column sections are required. A data bank program is being developed at the University of Sheffield to store all the known test information on semi-rigid joints but most tests have been concentrated at the smallest end of the beam and column size spectrum.

2) As mentioned earlier including the presence of semi-rigid joints requires the presence of $M - \Phi$ curves for the utilised joints. These curves are usually obtained from limited beam to column joint sub-assemblages. Therefore these relationships might be influenced by many factors such as the test apparatus, the loading system and different arrangements adopted to conduct these tests. Hence an investigation should be conducted to highlight the variation factor on a semi-rigid joint $M - \Phi$ curve resulting from test conditions to allow including this factor in design analyses.

3) Since the presence of infill panels in a practical frame has been proved to be highly influential factor in reducing the frame lateral deflections and possibly increasing the frame ultimate load, more experimental tests to investigate the real behaviour of these panels in practical frames with the presence of a range of semi-rigid joints and including the possibility vertical loads in the panels are required. In addition the influence of these panels on the force distribution on the frame, i.e. bending moment and axial load, needs to be examined.

4) The design charts included in chapter 9 to provide a safe estimation of the inelastic ultimate load of framed columns in sway structures can be utilised in parallel study to generate similar design charts for framed columns in non-sway structures.

5) In this study it has been assumed that the loading and unloading path of a semi-rigid joint are identical. However an unloading path of a semi-rigid joint has been proved to be approximately parallel to the initial tangent of the joint $M - \Phi$ curve. This behaviour will inevitably reduce the lateral deflections of a flexibly connected frame

and reduce the bending moment of the joints located on the windward side of the frame. This beneficial effect has been neglected to provide the conservative design procedures described. Nevertheless, a parametric study should be conducted to determine the influence of this behaviour of practical flexibly connected frames.

6) The modified program mentioned in chapter 8 accepts in ,the current situation, only nodal loading, i.e. loads subjected to the nodes of a subassemblage. A modification can be easily added to this program to account for semi-rigid joints effects on member end moments hence different load patterns, i.e. uniformly distributed loads, will be accepted by the program. In this study and since only concentrated loads have been considered, this modification has not been accomplished.

7) The modified computer program represents only flexibly connected sway or non-sway subassemblages which consist of a single column and possibly four beams attached to the column ends. A further modification to this program to represent multistorey multibay frames is relatively simple and can accomplished by reordering some of fortran statements in the program.

8) This study has been limited to the in-plane behaviour of steel frames. Further extension to this work to cover the in-space behaviour is possible with the availability of the understanding of the the in-space response of such structures together with the influence of semi-rigid joints which in turn requires a knowledge of the out of plane response of semi-rigid joints.

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Appendix A

Derivation of End Restraints for the Column-subassemblage Utilised in Chapter 9

Assume that the right beam of the subassemblage of fig.9.1 deforms in a fashion similar to that shown in fig.A.1 as a result of applying a vertical and a horizontal loads at the column top end. Utilising the well known slope deflection equation method and assuming that the axial load of the beam AB can be neglected results:

$$M_{AB} = \frac{2EI_g}{L_g}(2\,\theta_{AB} + \theta_{BA}) \tag{A.1}$$

$$M_{BA} = \frac{2EI_g}{L_g}(2\,\theta_{BA} + \theta_{AB}) = 0 \;\; thus \;\; \theta_{BA} = -\frac{1}{2}\,\theta_{AB} \tag{A.2}$$

Substituting eq.A.2 into A.1 gives:

$$M_{AB} = \frac{3EI_g}{L_g}(\,\theta_{AB}\,) \tag{A.3}$$

but the joint bending moment equal in magnitude and opposite in sign to the beam end moment, i.e. opposite in direction, and this joint bending is linearly related to the joint rotation θ_j thus:

$$M_{AB} = -M_j \quad and \quad M_j = -K_j\,\theta_j \tag{A.4}$$

whereK_j is the semi-rigid joint stiffness which is assumed here to be constant. Substituting the two term of eq.A.4 into eq.A.3 produces:

$$\theta_{AB} = \frac{K_j\,\theta_j}{\frac{3EI_g}{L_g}} \tag{A.5}$$

According to fig.A.1 the joint rotation θ_j can be written as:

$$\theta_j = -(\,\theta_c - \theta_{AB}\,) \tag{A.6}$$

Substituting eq.A.5 into eq.A.6 gives:

$$\theta_j + \frac{C_j\,\theta_j}{\frac{3EI_g}{L_g}} = -\,\theta_c \tag{A.7}$$

rearranging eq.A.7 gives:

$$\theta_j = \frac{-\theta_c}{1 + \frac{K_j}{\frac{3EI_g}{L_g}}} \tag{A.8}$$

Substituting the second term of eq.A.4 into eq.A.8 and rearranging produces:

$$M_j = \frac{3EI_g}{L_g}\left[\frac{1}{1 + \frac{3EI_g}{L_g\,K_j}}\right]\theta_c \tag{A.9}$$

but

$$M_j = \overline{R}_k\,\theta_c \tag{A.10}$$

therefore

$$\overline{R}_k = \frac{3EI_g}{L_g}\left[\frac{1}{1 + \frac{3EI_g}{L_g\,K_j}}\right] \tag{A.11}$$

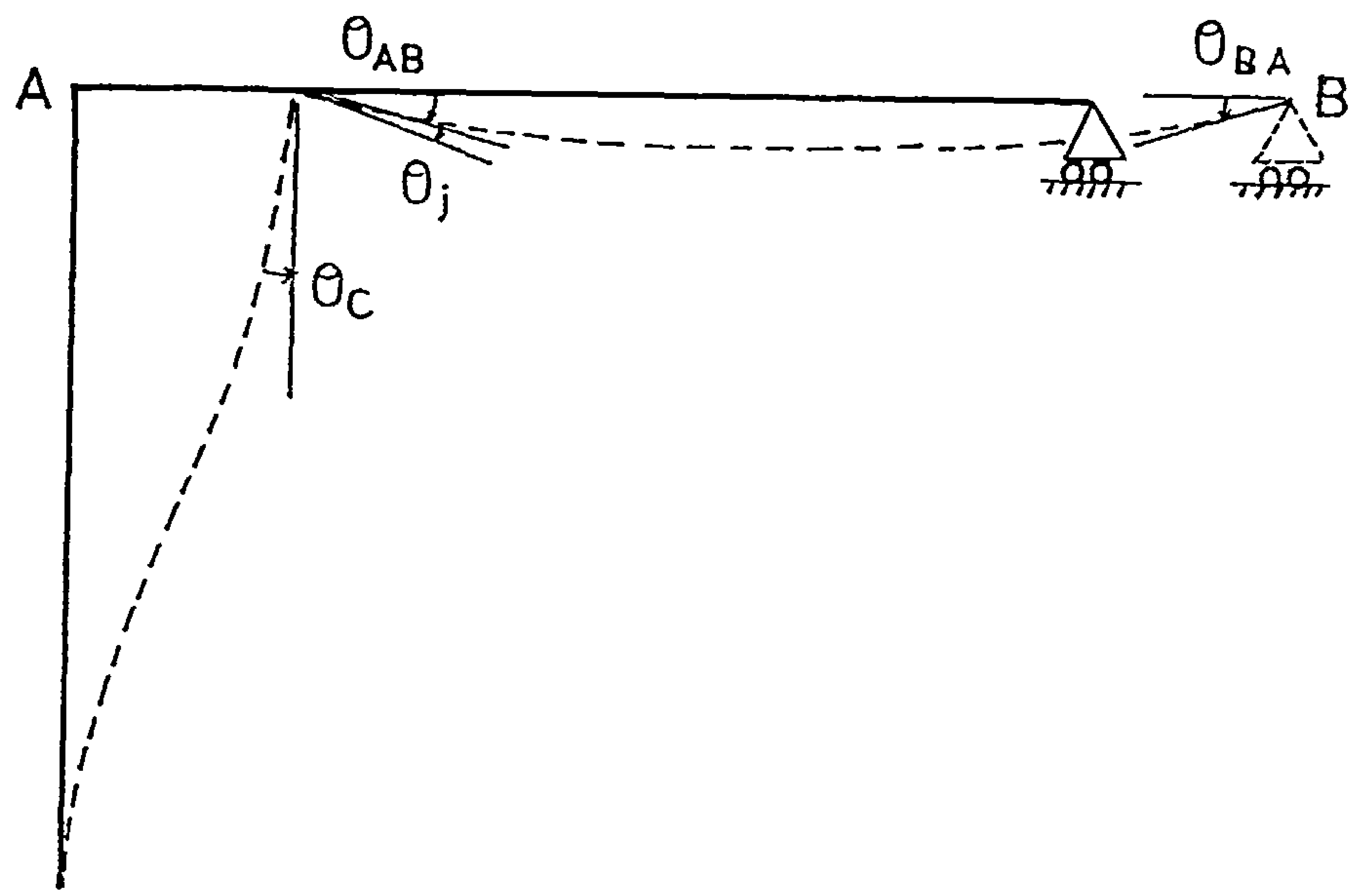

Figure A.1: The deformed shape of the beam in the flexibly connected subassemblage of figure 9.1.

Appendix B

List of publications

Paper 1 CHIKHO, A.H. and KIRBY, P.A., "Influence of Joints Flexibility on Multistorey Frame Behaviour", International Colloquium on Bolted and special Structural Connections, Moscow, Vol. 3, May 1989, pp. 53-57.

Paper 2 CHIKHO, A.H. and KIRBY, P.A., "Partial Sway Bracing in Semi-Rigidly Connected Steel Frames", Third ASCE/ASME Mechanics Conference, San Diego, July 1989, 24 pp.

Paper 3 KIRBY, P.A. and CHIKHO, A.H., "An Improved Amplified Sway Method for Rigidly Jointed Sway Frames", submitted to a committee revising BS5950 and to be submitted for publication.

Paper 4 CHIKHO, A.H. and KIRBY, P.A., "An Approximate Method for Estimating Bending Moments in Continuous and Semi-Rigid Construction", (in preparation).