A parametric design process of shell structures. Proposal for optimised strategies linking design and fabrication

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The candidate confirms that the work submitted is her own, except where work which has formed part of jointly authored publications has been included. The contribution of the candidate and the other authors to this work has been explicitly indicated below. The candidate confirms that appropriate credit has been given within the thesis where reference has been made to the work of others.

In the first two publications the candidate carried out all the works and prepared the articles, the co-authors provided guidance and support. As regards the last article, the work has been carried out together with a multi-disciplinary team including the candidate and one of the co-author and supervised by Dr Iuorio.

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Abstract

The design of shell structures has been the object of investigations for decades characterised by the necessity to optimize the form for efficient and cost-effective structures, either by using analytical methods or graphical methods. However, the importance of optimizing the design process has been acknowledged only partially in the architecture practice especially regarding shell structures. The use of parametric modelling is paving the way for new approaches that can improve the design process in terms of structural performance, geometrical rationalization and realization; the definition of design workflows by parametric approaches contributes to deliver structurally and geometrically optimized outcomes.

This thesis addresses the development of design workflows related to shell structures, covering geometrical, structural and realization aspects. Such workflows are implemented in a parametric environment and they are described in a step-by step process in order to foster their application amongst students and professionals. The scope is to provide practical guidelines in the design of shell structure by means of parametric tools in order to promote its use within architecture practice. The work covers different aspects of the processes, starting from geometrical considerations, form-finding, planarization, structural analysis and optimisation by Genetic Algorithms. The primary scope of the design process is to allow the realization of the structures, developing strategies that can simplify this phase. This is particularly crucial for shells since they present many challenges in the realization process given their geometrical features in terms of curvature and supports as well as the use of massive formworks to erect them. In the second part of this dissertation, two designs of shell structures have been carried out with the common purpose to minimize formwork for the construction. Digital fabrication techniques have been implemented in order to realize scaled models for the first project and a full-scale model for the second project. In both cases, it was possible to optimize the realization phase and minimize supporting structures by developing ad-hoc connection systems. Digital fabrication reveals to be an efficient tool to evaluate, analyse and test structures with different scales, being able to provide

immediate feedbacks of the structures and representing a powerful medium thanks to its high interoperability with parametric tools.

Keywords: digital fabrication, shell-optimisation, parametric design, workflows

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1. Introduction

The first chapter provides a general overview of the topics discussed in the next chapters, focusing on core principles of the design of shell structures and highlighting the approach taken in this thesis. In addition, aims and objectives linked to research methodologies are discussed with the aim to contextualise the study and describe its limitations. Finally, the structure of the thesis is described.

1.1 Historical background

Shell structures can be defined and categorized according to their geometry, structural efficiency, and their aesthetics. They can be defined as curved surfaces in which one dimension is smaller than the other two. Shell structures have been largely used in the architecture scenario since antiquity for their aesthetical qualities combined with an efficient structural form. The earliest examples of shell structure date back to the third millennium B.C., and they were discovered in Egypt and Mesopotamia (Dahmen and Ochsendorf, 2012). Curved structures allow covering large spans in a harmonious way by exploiting structural peculiarities. By definition, thanks to the curvature they carry membrane loads providing a state of equilibrium, but their structural definition is instead delicate and directly connected to several parameters: a small irregularity in the design may cause buckling and compromise the stability. An appropriate design and definition of the supports allow shells to work by membrane actions. History is the proof of how a correct design process is crucial for the durability of these shapes. In the last 5000 years or so, many civilisations contributed to the further development of vaulted constructions to such an extent that it became one of the most widely structural systems. This structural system has undergone a long process of evolution since its origin.

Trilithon made up of overlapping blocks, laid with a slight protrusion and converging at the top was used in Egypt and Mesopotamia, Romans used arches and vaults extensively, developing the pointed arch and then increasing stiffness in their structure by using the opus caementicium technique. In the Middle Ages, the vault system was less resistant and it relied mainly on the supports. Indeed, while Medieval architects' static system relied on the supporting structure of the aisles to ensure the stability of the main nave, in the Renaissance there was a re-discovery of Roman forms and domes became the most used curved structure, to the extent that S. Maria del Fiore dome became the symbol of this historical period (Tomasoni, 2007). It was realised by a technique, which allowed the structure to be free-standing during the construction, consisting of two overlapping domes connected by ribs and masonry chains, which reproduce exactly the meridians and parallels evolution. The masonry technique adopted "herringbone" pattern courses, whose arrangement interlocks the bricks, and as a result, the dome was built without using framework.

Similarities with these key-principles can be found in the "Catalan" vault used by R. Guastavino and A. Gaudì at the beginning of the 20th century. In this historical period, the importance of the early design phases to optimize the shape and therefore efficient structures with a minimum amount of material started to be acknowledged. Until this point, one of the main components in the design was provided by the realization of buildings whose stability relied on the size and mass of their elements.

An "optimisation" of the shape intended to be harmonious in the proportions, without compromising structural efficiency, was considered crucial in the design process of shell structures. Therefore, techniques such as form-finding were used in the earliest design. The first examples of form-finding dates back to the 17th century with the catenary arch by Robert Hooke, who expressed it with a straightforward concept in a form of anagram, successively interpreted as:

"Ut pendet continuum flexile, sic stabit contiguum rigidum inversum", or, "As hangs a flexible cable so, inverted, stand the touching pieces of an arch." (Hooke, 1675).

The catenary arch as intended by Robert Hooke exploits the great advantages provided by its shape, which under self-weight represents an efficient structural form. The *thrust line*, representing the path of the resulting forces, need to be within the thickness in order to guarantee stability. This has a very close connection, with the basic principles of graphic statics. Indeed, Hooke's hanging chain represents the thrust line (Heyman, 1997), which allows to detect the equilibrium state of the arch.

This principle found its main formulation in graphical methods such as graphic statics, which was developed in its first complete version by Karl Culmann at ETH in Zurich in 1866 (Culmann, 1866).

This powerful methodology paired with innovative building techniques like Catalan vaulting allowed a wide diffusion of shell architectures in this historical period: Guastavino was able to build, based on graphic statics principles, impressive vaults that are resistant with the minimum thickness and minimising bending stresses throughout the United States. His construction method was based on setting the tiles into three herringbone-pattern courses with a sandwich of thin layers of Portland cement (Dahmen et al., 2012). He built approximately 1000 buildings in North America between 1881 and 1962. Moreover, physical methods represented by the use of scaled models involving hanging chains also presented efficient solutions in the design phase, as Gaudì proved. A hanging chain in the shape of catenary represents a stable configuration since its inverted position is in equilibrium with compression forces. The catenary arch was one of Gaudi's most emblematic geometrical forms, which he realized by exploiting the strengths of such geometry.

Masonry has played a key role until the first half of the 20^{th} century providing a large contribution in shell architecture and taking advantage of its great compression resistance. This was due to the great ability to exploit the advantages of this material by realizing compression structures. However, in the second half of the 20^{th} century, the modus operandi changed approach in favour of materials that are strong in both compression and in tension, such as reinforced concrete and/or reinforced masonry, and graphical methods were replaced by analytical methods. As a result, a new era of thin shells emerged, with authors such as Candela, Isler, Dieste and Otto being the pioneers of this era.

Different reasons have contributed to the increased adoption of shell structures during the 20^{th} century. The use of reinforcement, first allowed building lighter shell covering the same large spans. Moreover, *economy* of the realization intended as optimisation of the costs and material was crucial in this development. Most of the designed shells relied on analytical methods, which provided good tools to predict stresses. In the 20^{th} century, the approach to the design of shell structures relied mainly on mathematical methods to predict buckling and displacements. Moreover, mathematical methods based on geometric rules were used to obtain well-known geometrical shapes, such as the Candela's hyperbolic paraboloids (Billington, 2010), and were easy to realize since generated by straight lines. Form-finding techniques developed by Isler (Chilton, 2012) and Otto (Liddell, 2015) have proved the importance to use physical models in order to define double curvature surfaces. Another main factor regards the great knowledge and expertise in materials like concrete or bricks with steel reinforcement, which were used generously in the second half of the 20th century. For instance, Dieste was a master in the use of reinforced bricks and pre-stressing methods to withstand tensions in an efficient way (Ochsendorf, 1999).

This historical analysis in the shell design can help to understand how shell designs have been processed according to their historical and architectural contextualization. In the contemporary age, the critical evaluation of the past techniques can provide the great privilege to absorb the right knowledge to design shell structures, understanding the advantages that can be exploited during the design process.

But unfortunately, as a general rule, the design of shell structures has been for a long time driven by the desire to pursue specific objectives in terms of formal aspects rather than by seeking optimal structural solutions based on the understanding of their behaviour and the intrinsic advantages. This led to a misbehaviour that in technical terms it translates into expensive projects with structural inefficiency. The knowledge of the “form” and the importance of the boundary conditions represent essential requirements for a correct design. Nowadays architects and designers have become aware of their importance in the design process and the development of computational modelling has allowed to translate the traditional techniques into powerful digital tools. This led to a reborn interest in shell structure, which goes beyond the limitation dictated by the past.

In the contemporary scenario, the design of shell structures by means of new digital tools guarantees a high level of effectiveness and interoperability delivering successful outcomes. All these important contributions are implemented within a parametric modelling background, which has radically changed the way of designing, improving the efficiency of the process. Projects like the Armadillo Vault at the Venice Architecture Biennale 2016 (Block et al., 2016), the Mapungubwe Interpretation Center located in Limpopo, South Africa (López et al., 2016) or the Landesgartenschau Exhibition Hall in Stuttgart, Germany (Menges et al., 2015) represent the cutting-edge of the innovation where the traditional techniques have been brought to life combined with the latest developments in digital computation and digital fabrication. Over the last few decades, the design process in architecture has experienced outstanding advancements: complex structural systems can be realized without comprising structural efficiency and exploiting innovations in fabrication techniques. Operations in the design process, such as form-finding, structural optimisation, rationalization of free-form surfaces (whose definitions will be provided in the literature review chapter) allow to reach a high level of quality in the projects that delivers a most enduring architecture with minimal intervention of special maintenance.

It is fundamental for designers to find appropriate forms according to their boundary conditions, otherwise there is a likely risk to create astonishing architecture but with serious structural disadvantages.

The interest in shell structures has been accelerated by the developments carried out by several research groups that in the last decades have made available digital tools able to investigate and optimize the design for shell structures. These important innovations have contributed to a "digital revolution" where the search of optimisation in the shape taking into account structural requirements and fabrication requirements has become a priority. The Armadillo Vault provides a clear insight of a design approach based on the geometry whereas the Landesgartenschau Exhibition Hall exploits digital computation to produce an efficient panelization of the structure with planar elements and connections realized through digital fabrication techniques. Further investigation and study about the latest innovations in design and fabrication are carried out in the next sections in order to give a clear explanation of the potentialities in computational design that have to be used in the shell design to such an extent as to replace traditional modelling.

1.2 Problem definition and thesis contribution

Nowadays, the use of shell structures is a small portion of contemporary architecture compared to other structural forms. This is extremely true in the case of shells working in compression. In the contemporary architecture scenario, a design process implemented by considering structural and realization requirements does not represent common practice for the main reason that there is a lack of knowledge, a lack of design guidance in the use of optimisation strategies in the professional practice. In the research and academic activity, many promising projects like the aforementioned ones have been carried out but they still represent a challenge since they require a considerable amount of economic resources and high level of expertise. Many innovations in the investigation of geometrical aspects of the shell structures have been achieved, however their practical implementation with software largely used by architects and engineers has not been covered yet, with the consequence to have a disconnected flow of information between academia and practice.

In order to design efficient shell structures it is fundamental that a renewed awareness, dictated by the lessons of the past and supported by new and powerful digital tools, is raised through the knowledge and its implementation in the digital computation, leading to simplified and informed processes that aim to optimize the outcomes in terms of structural efficiency and realization.

Although this approach is still non-dominant, it is necessary for architects and designers to change their approach where the formal aspects are functions of the structural behaviour of the shell. In the next chapters, design guidance has been provided by the use of optimisation strategies in the design of shell structures in order to overcome a wide range of problems.

This thesis addresses the definition of workflows within the design process of shell structures aiming to support their use in architecture practice by straightforward and simplified approaches. In particular, the investigation of the main issues related to shell design such as geometrical rationalization, planarization, structural efficiency and realization, has been carried out by making use of parametric tools largely used among professionals. In particular, their realization through basic digital fabrication techniques represents one of the main priorities in the design process.

In the first part, an in-depth survey regarding the influence of geometrical parameters within the design process has been carried out. Moreover, the role of the constraints in the design process has been defined to formulate important observations in the research of improved design solutions, with a particular focus on the rationalization and planarization. The parameters involved in the design process represent the key elements to fulfil requirements in terms of structural efficiency and fabrication.

Successively, such investigation of the design parameters has been implemented by a comparative structural analysis of the several design outcomes to formulate final considerations of their influence in terms of structural efficiency. The investigation has progressed with a focus on structural optimisation with the aim to improve the design of asymmetrical shells by means of heuristic search method such as Genetic Algorithms; geometrical subdivision and height of the structures have been parametrized according to structural goals. Hence, the study of a range of different cases has allowed to analyse a significant amount of boundary conditions and design parameters, providing an exhaustive formulation of the guidelines in the shell design from a digital perspective. In the last part, the realization of two cases studies by using basic digital fabrication techniques have provided feedback regarding their structural behaviour and proved to be an efficient tool both for the evaluation and for the assembling process. In particular, by developing two different connection systems, two different approaches have been taken: on the one side, a scaled model approach has been used as mean of analysis in order to understand strengths and weaknesses of the shell, on the other side a full-scale structure has been realized by adopting minimum formwork in the assembling process.

1.3 Research questions

As stated in the problem definition, the design process of shell structures still presents many challenges especially when contextualised in the professional practice. At this stage, the following research questions can be formulated:

- How structural informed and geometrical-aware processes can be integrated in the architectural design?

The importance of a structural informed and geometrical aware processes in the early design phases will be demonstrated and justified in order to increase efficiency of shells. In detail, the implementation of structural form-finding techniques and geometrical operations of discretization and planarization in the architecture practice can provide remarkable contribution in the quality of the projects from a structural and geometrical perspectives, but they still represents a challenge in delivering successful outcomes. Therefore, the integration of such an approach within the design process needs to be promoted by acknowledging its potentialities in the practice and making use of intuitive and powerful resources.

- Can parametric strategies related to design and fabrication for curvilinear structures be implemented within the professional architectural practice?

The main condition to promote parametric strategies is by providing design guidance that link the academic findings with the professional activity. Such indications if implemented in open source software can contribute effectively to raise awareness in the contemporary scenario and shorten the distance between academia and practice. Parametric strategies include also digital fabrication techniques to be used in the realization phase, leading to a further control of the process and a stronger connection between design and production. Digital fabrication techniques that do not require highly customized setups represent an efficient tool in the architecture scenario for scaled and real scale prototypes.

1.4 Research aims and objectives

The principal aim of the research is to develop straightforward guidance for the design of digitally fabricated shell structures, creating a bridge between theoretical principle and practical applications to promote parametric workflows for architecture practice. The principal research objectives are:

1. To identify weaknesses in the past and current procedures used to design shell structures;
2. To gain understanding of the current state of the art regarding:

i. the evolution of parametric modelling in the architecture practice;

ii. the role of the form-finding in the design process;

iii. the definition of structural optimisation process with its different classifications;

iv. the potential of the digital fabrication techniques highlighting how they might be used for the evaluation and realization of shell structures.

3. To develop digital workflows for shell structures, intended to be digitally fabricated, within Grasshopper® environment that allows us to:

i. identify the main parameters that govern the design process of shell structures, in order to fulfil structural and fabrication targets;

ii. establish the influence of constraints applied in the early phase of the design process in order to optimize the solutions;

iii. explore different subdivisions and define their potential on asymmetrical shells;

iv. structurally optimize shell structures with a minimum intervention on the shape by supporting the use of Genetic Algorithms;

v. assess the feasibility of adopting subdivisions and understanding their relationship with the structural performance.

4. To explore the potentialities of basic digital fabrication techniques which do not require customized setups and to develop systems with minimum formwork.

1.5 Scope and limitations

This work considers the design of shell structures for the development of a full workflow that includes design and fabrication phases from conceptual design, to geometrical and structural performance optimisation, to digital fabrication. It takes into account:

1. an investigation of mesh subdivisions techniques, constraints for form-finding and planarization involved in the digital workflow;
2. a range of geometrical requirements (e.g. clear spans, heights, the number, size and location of opening);
3. a range of different support conditions;

4. a range of static vertical and lateral load combinations;
5. a range of geometrical subdivision to carry out a rationalization of the shell.

At this stage of the research, some limitations were necessary and they regard some structural considerations, since the structural analysis and computational structural modelling do not form the focus of the research described in this thesis:

1. materials working in compression without the use of reinforcement have been considered;
2. load conditions have been estimated in a simplified approach given the focus of such a research, however the definition of their values and directions lie within the admissibility bounds with the goal to generate tensile stresses in the material;
3. seismic conditions have not been object of investigation in the structural analysis.

1.6 Thesis structure

This thesis is divided into 5 chapters and one appendix.

The first chapter concerns the introduction, aimed to provide a general contextualisation and recall important remarks regarding the historical evolution of shell structure and the identification of aims, objectives and research methodology.

The second chapter is a review of the literature, which is fundamental to provide an introduction of the subjects that are discussed next. In detail, an evaluation of the design approaches has been carried out, highlighting the innovations of parametric modelling and the translation of past methodologies into new digital tools; the literature review includes a critical analysis of the form-finding techniques, representing a key part in the design phase. The optimisation processes, aimed to improve structural efficiency, are integrated in the architecture practice and they have been analysed with a focus on Genetic Algorithms. Moreover, geometrical aspects have been discussed to outline the strict connection with improved design outcomes. In the last part of the literature review, the digital fabrication techniques have been examined related to contemporary architecture cases to comprehend the enormous potentialities that digital process is giving to architects and designers.

The third chapter involves the core investigation of this work: firstly, a study regarding the design parameters has been carried out to provide useful insights for the process; being aware of the influence of the parameters involved in the design process makes it possible to generate more efficient processes. Secondly, a parametric workflow is defined in the case of the symmetrical shell, addressing issues such as planarization and rationalization. This part is fundamental for the next steps regarding the generation of a physical prototype by using 3D printing technique. The digital process has been elaborated also by considering other aspects: starting from an investigation of the polygonal subdivision, a comparative analysis of hexagonal and Voronoi subdivision has been carried out, through Genetic Algorithms (GA). In this way, the importance of a heuristic approach is identified and such results are evaluated with Finite Element Analysis. A further validation of the potentialities of GA has been done through the application on one of Heinz Isler's unrealized projects taken as starting reference, with the aim to redesign it without the use of reinforcement and improve the structural behaviour.

Finally, the proposed processes have been analysed and validated through the digital fabrication of a series of 3D printed prototypes in the fourth chapter. The fabrication of such models facilitates investigation of the requirements that are essential for an efficient realization, highlighting potential issues in terms of stability. Furthermore, the development of a connection system minimized the use of formwork during assembly in the full-scale model. The prototypes have been realized through 3D printing and successively assembled.

The fifth chapter presents the conclusions providing the contributions of this research and the recommendation for future work.

An appendix at the end of the thesis addresses the design workflow from a technical perspective, aiming to provide guidance in the digital computation. For such a purpose, a step-by-step approach has been taken according to the main topics covered in this work in order to guide the reader from the beginning to the end of the processes.

2 Literature review

2.1 Design process fundamentals

2.1.1 Introduction

The necessary condition at the base of architecture is its visual representation. Its representation is fundamental to allow its realization, based on a transfer of information, details and knowledge, which allows architecture to exist.

Drawing is the link between ideas and their materialisation. The first definition dates back to 350 BC with the mathematician Euclid of Alexandria, who defined a system to describe geometry by the use of axioms and theorems. Indeed, Euclidean geometry represents the basis of any representations and even today digital tools are based on these assumptions.

Any 3D object can be described with orthogonal projections by 2D operations according to a system of coordinates *x, y, z* (Fig. 2.1): orthogonal projections technique was developed by Gaspard Monge at the École Polytechnique in Paris (Carreiro and Pinto, 2013): since then, Descriptive Geometry has represented any 2D and 3D object providing a sense of depth by the combined use of axonometric projections and shadows on the drawings.

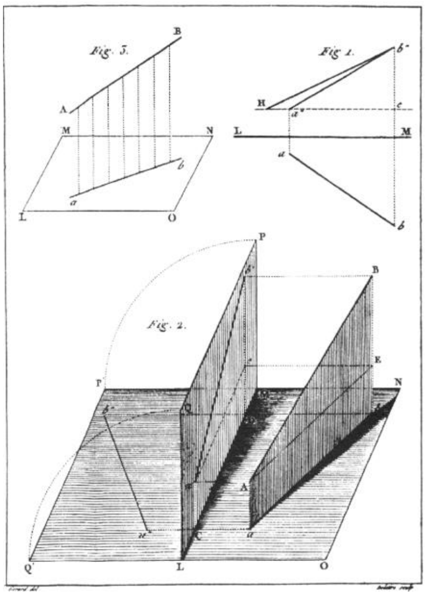


Figure 2.1 Orthographic projection example taken from Géometrie Déscriptive (Monge, 1799)

In architecture, the orthogonal projections represent a fundamental part in the design process.

The design process allows the materialisation of architecture through different stages: it is a process that begins with the concept and arrives at the realization of a structure. It is possible only by dividing the whole process into phases, each of which presents proposals and solutions with different grades of complexity and details. In a conventional approach, the design process employs 2D drawings such as plan, sections and elevation views to define structurally the building and axonometric and perspective views to articulate the space in three dimensions. The design process can be considered as an additive process, where the architect/designer acts directly on the geometry, "adding" information on different levels.

Until the second half of the 20th century, the sketching of accurately scale drawings by hand was the only practice to design in architecture. It is in 1963 that this physical mean starts to be digitalised. The first appearance of a

Computer Aided Design (CAD) changes the way of understanding the design process.

The development of intuitive, fast and powerful software as described in the next sections determined a radical change in the perception of the design process. From then, the diffusion of CAD software was unprecedented to such an extent that nowadays, they represent the foundation for architecture training.

2.1.2 Parametric modelling

Over a considerable length of time, architects and engineers have dealt with a huge variety of classical forms defined by geometrical rules in orthogonal systems. This has led to an extensive production of architecture but at the same time with no remarkable variations in terms of formal aspects. Shell structures are generally double curvature surfaces. Approaching complex and free-form shapes by using conventional approaches would prove to be a time consuming and intricate process. Where shapes are not determined by geometrical and mathematical principles, their representation might become an issue.

Parametric modelling may be a powerful approach for the design of shell structures: a process regulated by the use of parameters and their relationships represents a good approach to manage complex architecture not attributable to forms already established but characterized by a significant amount of artistic freedom. A parametric approach, when applied in architecture, affects the shapes of structures by variating parameters and defining relationships between them.

The term *parametric* has its primordial origin in mathematics. In a general context, a parametric equation is based on parameters and variables that can be modified in order to get results that can satisfy specific goals. A catenary arch, for instance, is defined on a parametric equation:

$$y = a \cosh\left(\frac{x}{a}\right) \qquad (1)$$

Where, $cosh$ is the hyperbolic cosine function, x and y are the Cartesian coordinates and a is the parameter, and as a changes its numeric value, than the scale of the arch is affected (Weisstein, MathWorld).

A parametric design is based indeed on parameters; and they can be of any kind according to the requirements imposed in the process.

A parametric system has a very clear structure in order of being defined as such, and it is composed of input variables, an equation, a rule and a solution, so that a wide range of solutions can be created through the manipulations of the input. In architecture, the manipulation of parameters and variables can provide multiple versions of the same starting geometry by simply interacting with few parameters in the system. The application of parametric thinking in architecture is not a contemporary innovation, but it existed before the digitalization of such approaches (Peteinarelis and Yiannoudes, 2018). Gaudì is considered one of the pioneers of parametric approach, especially for arches and double curvature geometries (Fig. 2.2). For the development of catenary arches, he used strings and weights to simulate the shape of the catenary, which is stable under its self-weight. Its inverted position provides a shape working in compression (Mazanek, 2016).

Figure 2.2 Parametric model for Colonia Guell from Antoni Gaudì (available from arch2o.com)

Gaudì applied this "parametric" approach to develop many of his projects (Fig. 2.3), and among them, the Sagrada Familia is probably the most widely known.

Figure 2.3 Hanging model of Sagrada Familia

Simulating parametric modelling in a physical way is a laborious approach since every change and modification requires a re-computation of the whole system and its correctness in building the model requires a great ability and knowledge of its functionality.

However, it finds its first definition in a digital context with the expression "Architettura Parametrica" by Luigi Moretti (1971), who defined the fundamental correlation between Parametric architecture as the relations between parameters. His research findings were shown at the 12th Milan Triennial in 1960 with a series of models for stadiums for soccer, tennis and swimming (Fig. 2.4). In that exhibition, he focused on the importance of the relationship between the parameters and how it is fundamental to process this paradigm with computers that enables designing the connections. By using an IBM 610 computer, he is considered to be the first architect able to control parametric equations through digital computation (Davis, 2013)

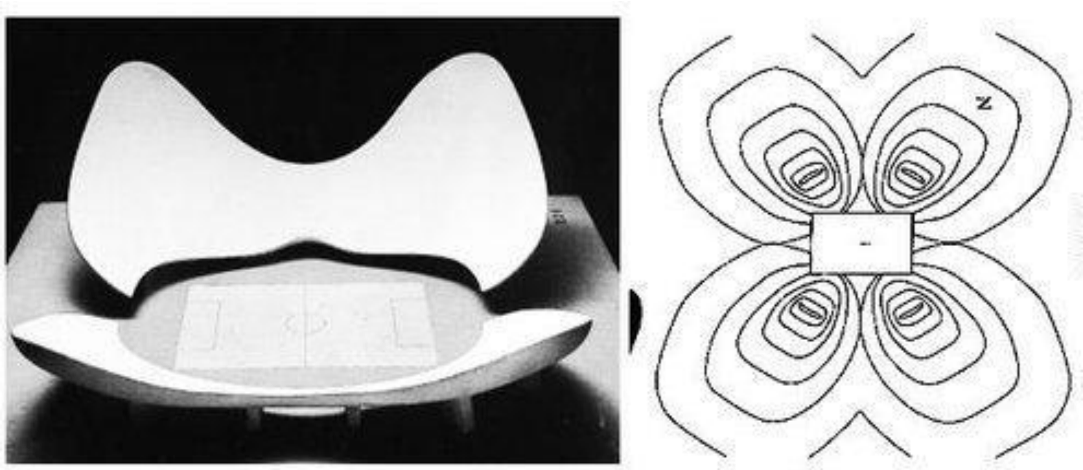


Figure 2.4 A model of the stadium composed of 19 parameters by Luigi Moretti during the exhibition of Milan Triennial in 1960 (Bucci and Mulazzani, 2002)

It was a logical consequence that, with the advent of computers, parametric modelling increased in popularity. As a result, the digitalisation of architecture brought to a new way of intending architecture with many positive implications but also some negative effects.

Its impact has been so strong to such an extent that some architects consider it as a movement, or an architectural style. Patrick Schumacher in (2008) used the term "Parametricism" to define the style typical of the contemporary age (Fig. 2.5). In detail, his vision of "Parametricism" is founded on concepts like fluidity, adaptability, complexity, variability and it relies on the use of algorithms and equations. What is considered rigid and repetitive is abandoned, while dynamicity in the shapes, smoothness and variations is encouraged. Schumacher's vision of architecture reflects many works realized in the last decades where the project is seen as a challenging mission to obtain bold structures.

However, parametric architecture is intended as architectonic style, therefore with its own features, may turn into self-referencing architecture, disconnecting itself from the context and its functions. Parametric should be the methodology rather than the style that sometimes may give forced directions.

This revolutionary approach is providing new solutions in the architecture scenario but at the same time it may become too ambitious towards architecture. The idea to associate to parametric architecture a style comprised of formal expressions may give forced directions: it is not always fundamental to reach a high complexity in the formal aspects of the projects neglecting what is considered "standard" or traditional. Architecture includes a wide range of aspects in its materialisation and in some cases it does not need to be intricate. It is necessary to consider parametricism as an innovative and powerful style that needs to be contextualised and used in a wise way. Although the results are quite fascinating and attractive to architects and designers, parametric is an important tool that is able to optimize projects from various perspectives, not necessarily implying the design of bold structures and complex geometries. The importance of this approach lies also in a more optimized workflow that allows analysis of structural behaviour, environmental impacts, and management of the time and costs of the production phases at the same time, so that any adjustments in the design phase do not compromise the results.

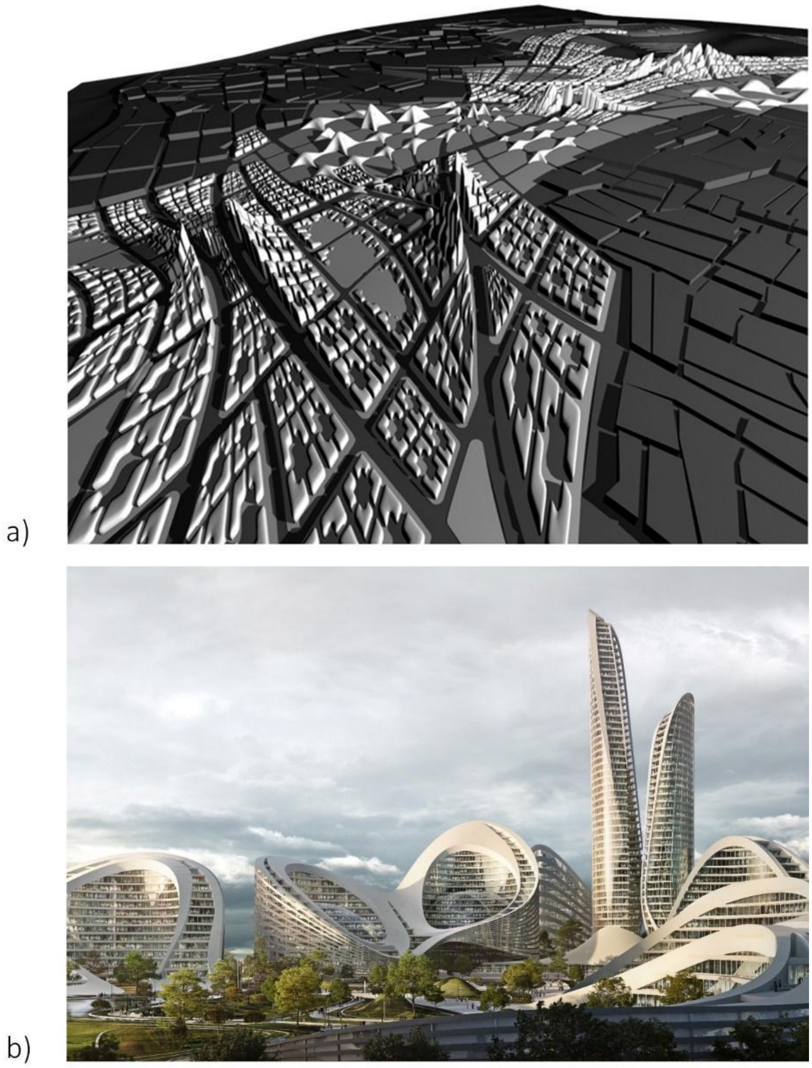


Figure 2.5 Examples of parametric architecture: a) Zaha Hadid Architects, Kartal-Pendik Masterplan, Istanbul, Turkey, 2006 (available from patrikschumacher.com); b) project of a new neighbourhood named Rublyovo-Arkhangelskoye in the west of Moscow (available from fyingarchitecture.com)

2.1.3 Computational tools

The evolution of digital tools in the last decades has facilitated a more sophisticated approach to parametric design in architecture. With the development of parametric computational tools, the process is accelerated and physical models are replaced by digital simulations that allow the parameters to be varied according to the results. The first parametric tool was Sketchpad (Sutherland, 1963) whose system is based on constraints called atomic constraints. As such, it can be considered as a precursor of the modern parametric software.

The user could draw lines and arches and relationships between these elements could be defined. Its closest heir regarding the evolution of digital tools was Pro/ENGINEER, which makes use of constraints in 3D, underpinning connection between several elements in the design. At the base of the development of such tools, there was remarkable research activity regarding programming languages based on algorithms. Endless possibilities can be explored thanks to the investigation of visual programming and their algorithms; and it is thanks to their development that nowadays we can use sophisticated software, such as CATIA, Revit and Grasshopper®.

CATIA appeared for the first time in 1977 and it still is one of the most used parametric software. CATIA is a multi-platform initially developed for aerospace design and then its use was extended to other fields, such as architecture and engineering. It is able to manage the life cycle of a 3D object, from the design to the production. It was used by Frank Gehry to design curvilinear sections of his iconic project the Guggenheim Museum Bilbao (Fig. 2.6-2.7). CATIA principles were elaborated in a new software called Digital Project that the Gehry studio developed in its own computational tool for architectural projects.

The concept of controlling and managing the full cycle of a product allows to maximise efficiency in the production and checking potential errors with the result to save money and time in the process (Architizer).

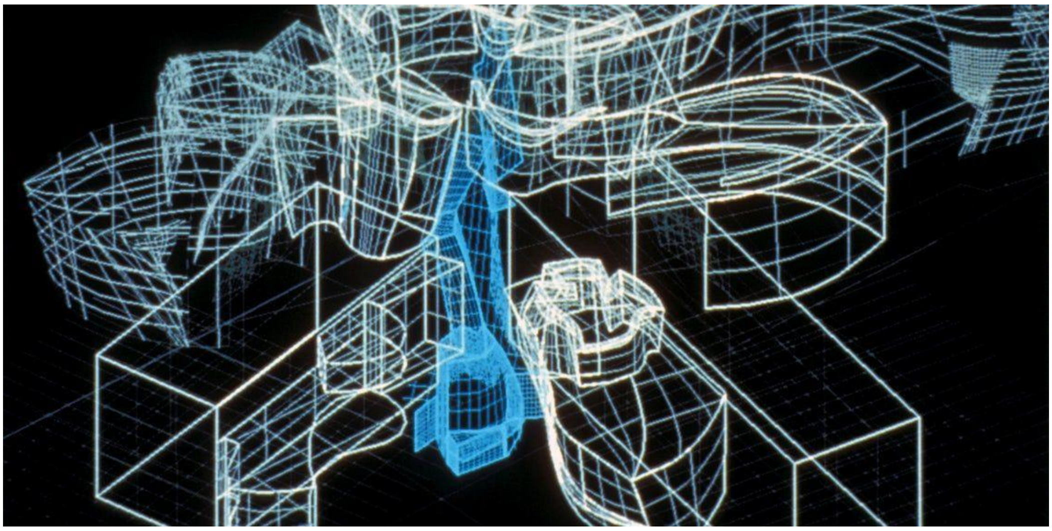


Figure 2.6 Digital model of the Guggheineim Museum (Gehry Technologies and Gehry Partners LLP)

Figure 2.7 View of the Guggenheim Museum in Bilbao (available from archdaily.com)

The idea of managing a project in architecture within its life cycle from the concept to the manufacturing is the approach behind Revit (2001), developed by Autodesk, based on the concept of Building Information Modelling (BIM).

BIM is a system of coordination and model management that optimizes the whole process through the creation of intelligent projects. Different aspects of the process are included such as design, maintenance, building performance.

BIM's creation is rather recent but its quick diffusion is enabling more intelligent design where the project is not considered anymore as a single object but as part of the whole that communicates with the context. Although the first release was in 2000, it has experienced a strong growth in architecture and engineering fields. Revit potentialities have been extended thanks to the use of a visual algorithm editor, that allows creating own workflows and establishing relations with the parameters in a dynamic way. The use of an editor allows elaborate scripts which require specific functions not included in the software. It interacts directly with Revit providing high flexibility in the process.

"Traditional solid modeling defines objects as regular point sets. (…) that they can be constructed according to a representation based on some decomposition of space. In this sense, the representation scheme is a language in which words (sequence of symbols) are the descriptions of geometric solids and sentences (sequence of words) are descriptions of three-dimensional scenes. Algorithmic modeling incorporates the symbol generation mechanisms into the representation. Consequently, instead of the static structure used in geometric modeling, we are dealing with a dynamic structure in which the rules for generation of symbols are encapsulated into the model itself, producing a geometric description whenever that is required. Note that such scheme allows for adapted realizations of a shape, sensitive to external environment conditions. The procedural representation is actually a meta-representation that introduces another step into the model creation." (Velho, 1993, pag.14)

Parametric modelling in its typical digital context relies on algorithms, on its functioning and the relations between them. Algorithms are used through visual components to create endless possibilities in design, acting on important aspects of the process such as acoustic, environmental analysis, structural analysis without the need of programming experience.

Grasshopper® represents one of the most popular visual algorithm editors. It is a plugin for Rhinoceros, and in the last version Rhino 6 it is completely integrated in Rhinoceros. Grasshopper® was developed in 2007 by David Rutter and Robert McNeel & Associates, as a computational tool for architects and engineers. It relies on algorithmic modelling by the use of visual components. The procedure of algorithmic modelling is organized according to a set of instructions in the following way: a set of inputs is given that have to follow the instructions dictated by the algorithms; based on that an output is generated. The output is a digital system able to communicate with other processes within the same platform (Tedeschi, 2014).

Such a process relies on programming languages that provide instructions in order to perform a task. For instance, Grasshopper includes VB.net and C# components. Therefore, the use of programming languages embedded in design tools can extend their potentialities gaining more control in the process as well. The architect learns new skills, approaching the design in a mathematical way with an in-depth understanding how an algorithmic approach can be beneficial for an improved process. This is particularly valid in the case of dealing with free-form structures that can be properly represented by custom scripts implemented in parametric tools rather than by direct modelling: the latter presents many problems and limitations in the generation of free-form forms, since they are not connected to any rigorous mathematical formulas.

For instance, a line is a very simple geometrical figure that connects 2 points. By drawing we connect two points picking the inclination and the length. To change any feature of the line we should repeat the drawing by re-making the line. There are several algorithms to define a line such as Digital Differential Analyzer (graphics algorithm), Bresenham's line algorithm and Xiaolin Wu's line algorithm (Shirley et al., 2002). Such algorithms provide instructions in order to generate lines by taking into account points coordinates and mathematical operations.

2.1.4 Form-finding in the past

In the evolution process regarding parametric modelling, special focus was given to the computational tools that enable the development of a new vision of architecture. However, beyond the digital tools it is fundamental to draw attention to the techniques whose principles underlie the generation of models.

There are different classes of models:

- models based on parametrization in order to define shapes in the process;
- models based on explicit functions where the geometry is generated by using functions of space, namely by using primitives and transformations;
- models based on topology grammars, such as fractals;
- models based on a physics system where the shape is generated through mathematical physics.

In this last class, the geometry is obtained through a simulation which considers a physical law and a set of boundary conditions. This approach is the so-called form-finding technique (Velho, 1993).

Form-finding and form-making have the same final outcome but with two opposite approaches. Form-making is at the base of the creativity of the designer (Fig. 2.8), it is a process of translating perceptions in geometrical forms and transforming them into structures by a language universally recognized. However, the desire to pursue formal objectives has always been the first approach of generating architecture, conceived by an initial concept following a clear geometrical definition that takes into account the relationship of existing buildings, the clients' requirements, the material to use, the time and budget for the realization among others. The architect's sketch is the first step in the elaboration of a first interaction between the final project, the physical object and the designer's intentions. This is the starting point of a form-making process that will be re-elaborated and developed in a computational program.

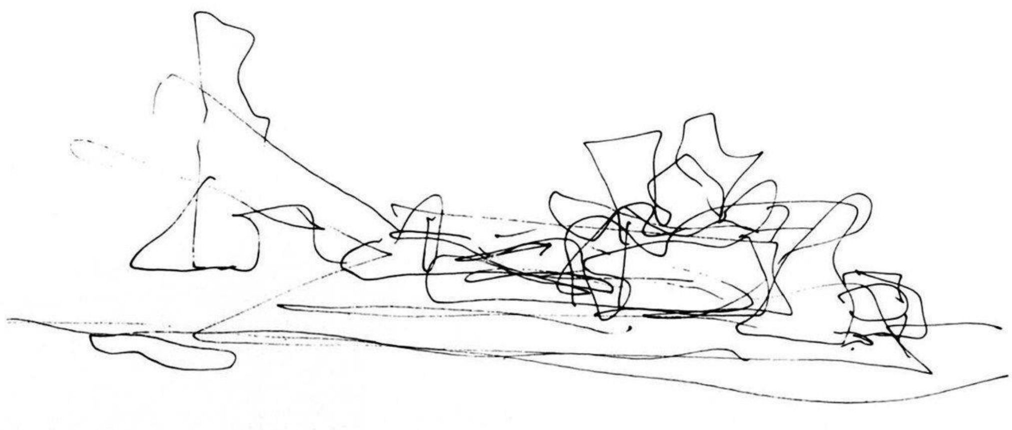


Figure 2.8 Sketch of Guggenheim project in Bilbao by Frank Gehry (available from guggenheim-bilbao.eus)

For some aspects, it represents a sort of personal vision where the team or the person engaged with this phase try to translate their inner ideas about how architecture should be in a precise context. This intimate process is the foundation of architecture practice; in the formation process as architects, as creators of the forms, we have been dealing with this approach for centuries.

On the other side, there is form-finding, which contrasts this conventional way of designing on many points. The main concept behind form-finding is undoubtedly "form from the force": the generation of a shape not depending on personal creative expression. It is a result of a simulation based on a physical law that takes into account a set of constraints, parameters and boundary conditions. The form-finding technique's results are particularly effective in the design of curved geometries such as shells.

It aims to find the best configuration, considering design constraints and a given specific load. Different ways to control the shape during the design process have been developed, especially regarding the use of real models: indeed, the use of physical models has contributed to a total understanding of the structural behaviour of the shell structures. As aforementioned, hanging chains were used by Gaudì, providing one of the first examples of parametric approach. His procedure consisted of using an array of space hanging chains, constrained at the supports to retrieve sections of his elegant structures, such as the sections of the vaults of Colonia Guell (Burry, 2007). Another important technique was graphic statics (Fig. 2.9), it did not rely on physical simulations, but on drawings and calculations.

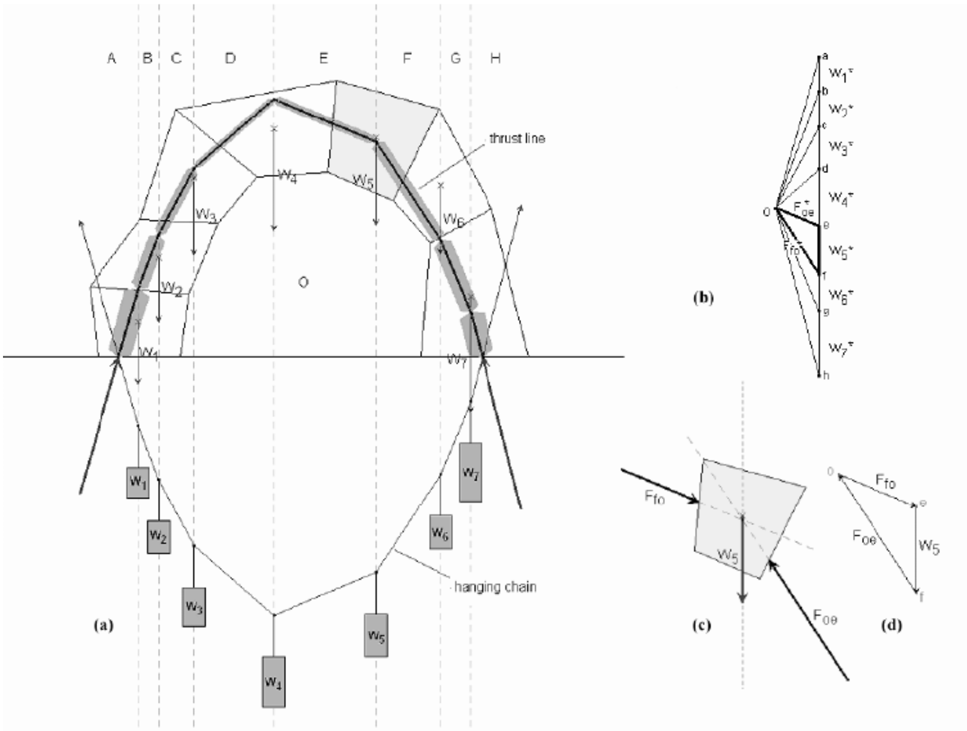


Figure 2.9 Graphic statics for a cross section (Allen and Zalewski, 2010)

It was developed in its first complete version by Karl Culmann at ETH in Zurich in 1866 (Rippmann, 2016) representing an important methodology to find the best solution for vaulted structures. Based on the force polygon, it is possible to find an equilibrium state, represented by a closed polygon; the force involved in the process, the gravity force and the two inclined compressed forces must form a closed triangle. In this way, a funicular shape is found which allows to have the right spatial configuration for a given load acting in pure compression and using the minimum amount of material. Graphic statics was used widely by different architects and in different contexts. For instance, Rafael Guastavino Jr. (1842-1908) was familiar with this technique in his numerous works (Fig. 2.10). He used graphic statics to design thin shell masonry domes throughout America in the 20th century (Ochsendorf, 2010).

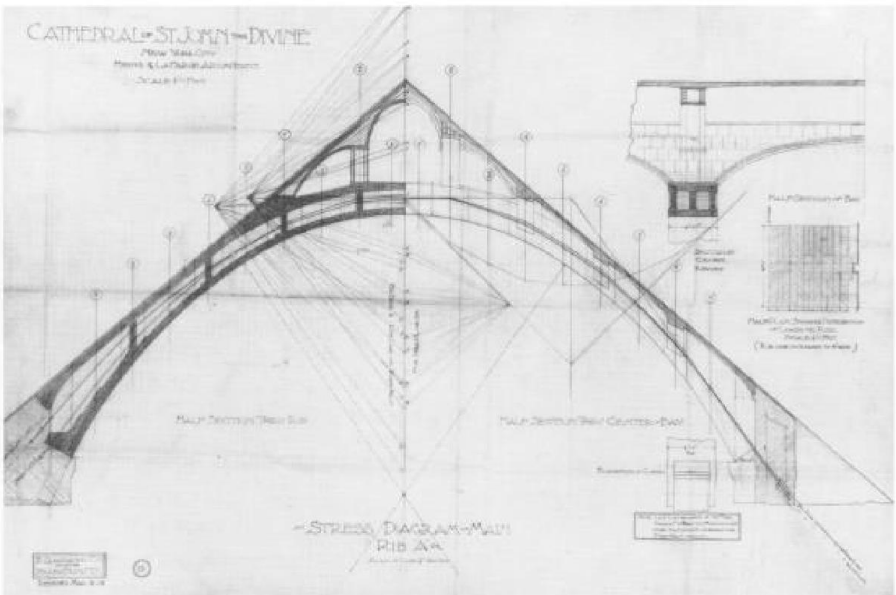


Figure 2.10 Guastavino's drawings. (Guastavino, 1910)

In the second half of the 20th century the process of form-finding for shell structures was expertly used by various architects that have contributed in a better definition of the great advantages of this technique.

Although the materials used throughout history have been changed and with it the building techniques, the aim has always been to realize long spans with the minimum use of material by generating membrane stresses and no bending in the structures with little or no tensile strength.

Frei Otto and Heinz Isler were the main exponents for the form-finding process in the second half of the 20th century. For both the main aim was efficiency but they developed different techniques: Frei Otto's investigation was based mainly on the study of tensile stresses and the search of correct shapes where main elements are in tension assuring rigidity. After years of experimenting, he developed his famous theory of minimal surfaces where he used soap film in order to generate shells with the smallest area (Fig. 2.11). By exploring this kind of form-finding Otto was able to exploit the structural features of anticlastic curvatures and therefore tensile structures, with their specific formal characteristics (Glaeser, 1972).

Frei Otto was considered the father of tent structures thanks to his recognisable system, Heinz Isler was well known for his variety in the technique and hence in the resulting shapes. His structural elegance found expression in different ways according to his paper titled "New shapes for shells" for the first International Association for Shell and Spatial Structures conference (IASS) in 1959, describing three form-finding methods (Fig. 2.12): the free-form hill, the inflated membrane, the inverted hanging cloth (Isler, 1961). The latter was one of the techniques that we can find in most of his projects, where he was able to express the elegance and lightness of his forms.

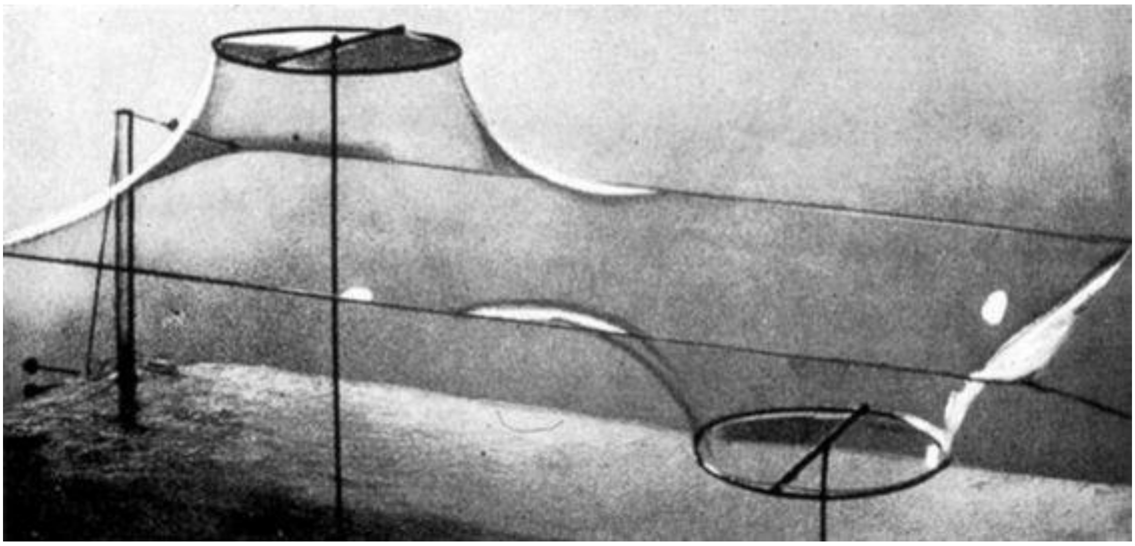


Figure 2.11 Soap bubble experiments by Frei Otto (Zexin and Mei, 2017)

"The process consists of pouring a plastic material onto a cloth resting on a solid surface. Once the material is evenly spaced on the cloth, the solid surface is lowered and the plastic-covered cloth, now in pure tension, is freely suspended from its corners. In that position, the plastic and the solid shell model is turned upside down, giving a shell form in pure compression" (Isler, 1980).

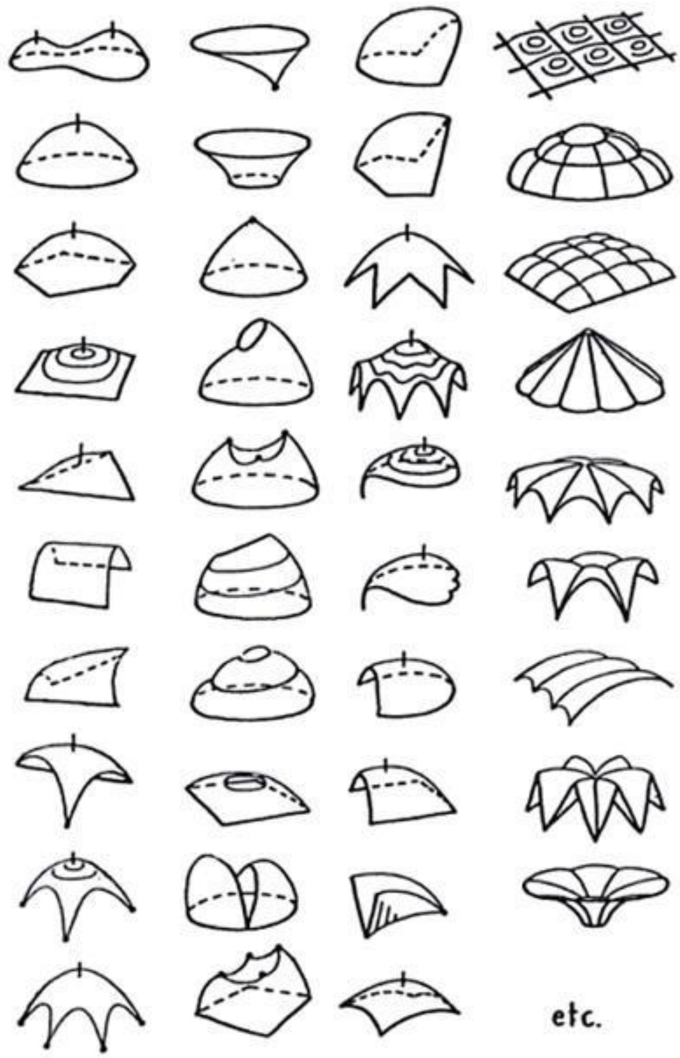


Figure 2.12 'New Shapes for Shells' taken from Isler's paper (IASS Archive)

2.1.5 Form-finding in the contemporary scenario

The use of physical models is a time consuming process, it is evident that this system is not flexible, the change of some parameters within the simulation involves a total re-computing of the simulation. The translation of this system in a digital environment has enabled designers to improve the efficiency of the process, understanding the great advantages of form-finding and investigating the variables involved in the process in a quick and intuitive way. Powerful methods whose application in computational tools is straightforward have been elaborated to overcome limitations due to use of physical models.

Thrust Network Analysis (TNA) was developed starting from the principles of graphic statics but applied to find spatial configurations. Like graphic statics, TNA is based on reciprocal diagrams, the form and force diagram, and they are the corresponding funicular and force polygon, where *"each node with a valence higher than one in a diagram corresponds to a space formed by a polygon of edges in the other, and vice versa"*, recalling the features of graphic statics (Rippmann, 2016, p.34). The form diagram represents *"the layout of the forces in plan"*, and it is the first step in the working process. In the case of RhinoVAULT, the form diagram is represented by a polygon of forces based on support conditions, unsupported edges and the target edge lengths. The force diagram is based on the form diagram and it represents the "force polygon" of the form given previously, a sort of first draft before reaching the solution. The solving process of this method is based on solving the horizontal and vertical equilibrium (Fig. 2.13). The main condition in order to have the equilibrium is that the corresponding edges of the form and force diagram must be parallel within a certain tolerance, given by an angle of deviation.

So the process is repeated until the corresponding edges are parallel and have the same direction in order to have structure working mainly in compression.

After the horizontal equilibrium is reached, the second step concerns the vertical equilibrium where the thrust network shape is finally obtained. In this phase the equilibrium is obtained when the displacement of all vertices in the shape is below a defined value, it is important to highlight that the vertices involved in this process are non-supportive vertices. Moreover, it is possible to have more control, by setting particular bounds during the design process, on the edge length or on vertex displacement.

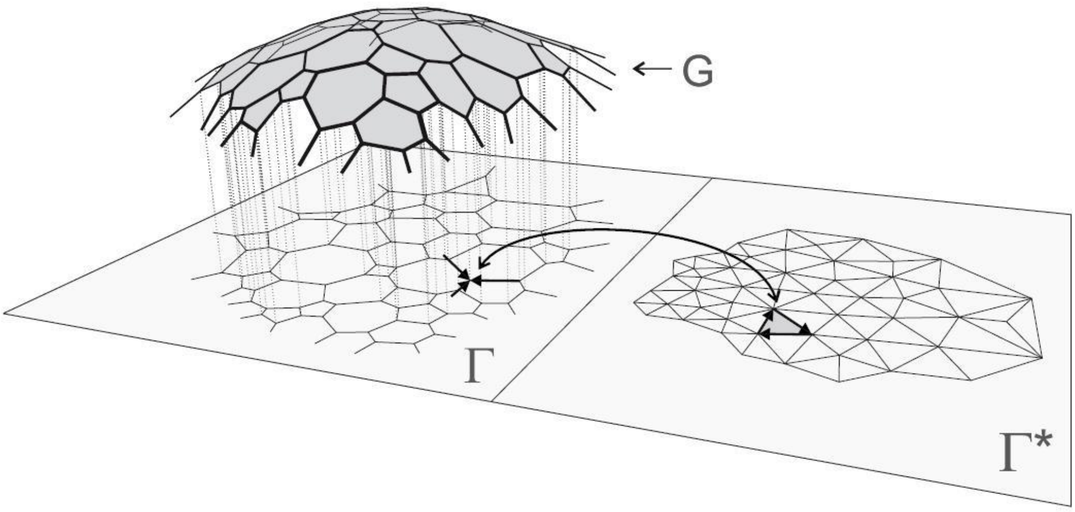


Figure 2.13 TNA workflow for a compression membrane (Block, 2009)

Edge length control consists of setting bounds in the edges of form and force diagram and it can be used to control and handle the force distribution in the thrust network.

Vertex movement control can be used when the user wants to control the displacement of some vertices: if the factor is 0 the position is fixed, if it is 1 the vertices are free to move. Moreover, it is possible to control vertices by taking account specific curves. Surely, all these operations are used for specific purposes by the user and they require a good experience with this tool. With constraints the control is more precise and the constraint can be applied on the lengths of the edges in the force diagram. Different actions can be taken: such as the creations of openings and unsupported edges, the change of boundary conditions in order to modify the shape, alter loading conditions, modify the flow of forces on the geometry to attract forces in specific areas amongst them.

In the Force Density Method the form-finding is computed for tensile membranes and pre-stressed cable nets. This method has been used for many important projects like the roofs of Mannheim Mutihalle (Fig. 2.14) of which a 1:100 model was built and then it was processed by the engineers in the Institut Fur Anwendungen der Goedesie im Bauwesen at the University of Stuttgart by using this innovative method (Liddell, 2015).

Figure 2.14 Hanging model of the Mannheim Multihalle structure designed by Frei Otto and Ted Happold (Liddell, 2015)

The relationship between pre-stressed forces and their elongation are controlled through Hookes's law of elasticity, defining material behaviour in the form-finding process. Referring to the equilibrium of a single node, the force density is defined as the ratio between the axial force of a branch and its stresses length.

$$q_i = \frac{F_i}{l_i} \tag{2}$$

The Force Density Method is really efficient, non-linear equations are reformulated as linear equations that can solve equilibrium equations for cable nets and membranes because the initial coordinates of the nodes do not necessitate to be defined.

The following expressions regard the equilibrium in nodes (Gidak and Fresl, 2012):

$$\frac{S_a}{l_a}(x_i - x_m) + \frac{S_b}{l_b}(x_i - x_g) + \frac{S_c}{l_c}(x_i - x_k) + \frac{S_d}{l_d}(x_i - x_l) = 0,$$

$$\frac{S_a}{l_a}(y_i - x_m) + \frac{S_b}{l_b}(y_i - y_g) + \frac{S_c}{l_c}(y_i - y_k) + \frac{S_d}{l_d}(y_i - y_l) = 0, \tag{3}$$

$$\frac{S_a}{l_a}(z_i - z_m) + \frac{S_b}{l_b}(z_i - z_g) + \frac{S_c}{l_c}(z_i - z_k) + \frac{S_d}{l_d}(z_i - z_l) = 0.$$

where:

S_a, S_b, S_c, S_d are prestressed forces for a given set of cables a, b, c, d

l_a, l_b, l_c, l_d … are nonlinear functions given by:

$$l_j = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2} \tag{4}$$

Hooke's law is at the base of Particle Spring Systems as well (PSS). PSS allows to compute equilibrium by exploiting properties that are present in the physical model previously described like hanging chains (Fig. 2.15).

Killian and Ochsendorf (2005) have explained thoroughly this methodology, highlighting a potential application in architecture. This method was used originally for animation based on dynamic simulations. In the case of axial forces, it may be a good tool to find funicular shapes. It consists of a discretization where the surface is transformed into a system composed of particles and spring: the particles have a position and a velocity and forces represented by vectors are applied on them, while the springs are the connection between the points and, during the process, they "behaves according Hooke's law" (Tedeschi, 2014, p.362).

$$f = k \times e = k \times (l - l_0) \tag{5}$$

Where f represents the forces, k the stiffness constant and e is given by the difference between the original length l_0 and the current length. In order to find equilibrium the second law from Newton is applied and the particles of the system has an unknown acceleration which is function of the particles mass m_i and a net force.

$$a = M^{-1}r(x, v) \tag{6}$$

Where M is the diagonal mass matrix and r the residual force vector.

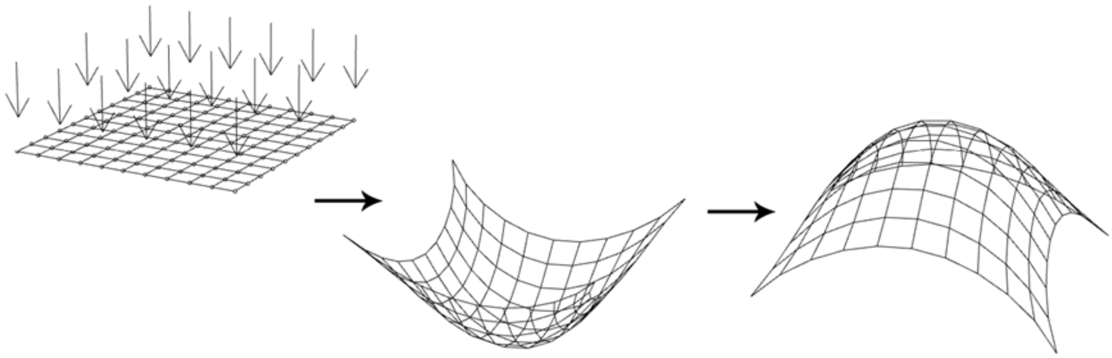


Figure 2.15 PSS workflow (discretization in particles and springs, simulation with forces applied on particles and reversed model)

This system presents analogies with the Force Density Method, in fact we can consider the spring forces also as force density of each spring that connects particles (Adriaenssen et al., 2014). Indeed, by manipulating some parameters during the computation a behaviour comparable to force density is obtained.

2.1.6 Computational tools for form-finding techniques

All the methods previously described are well established for their application by using effective digital tools within parametric modelling. These specific methods have been selected for various reasons: the aim is to consider digital tools that are at disposal of designers and architects alike, whose design path is clearly familiar to architecture practice, by relying on graphical simulations with manipulation of parameters in a straightforward way; moreover, the use of the unique platform Rhinoceros® is considered crucial at this stage. Rhinoceros® is a popular modelling software tool that embeds Grasshopper® environment and TNA plugin with a high degree of interoperability. In detail, the main peculiarities of two plugins are given below: RhinoVAULT implements TNA in the digital environment and Kangaroo, a plugin of Grasshopper® follows dynamic relaxation methods (such as Particle-Spring Systems and Force Density Method).

One of the main peculiarities of the Thrust Network Analysis is its application through a plugin operating within Rhinoceros background: RhinoVAULT. All the principles have been translated in this open source tool developed by the Block Research Group in 2012 (Rippmann et al., 2012). Its interface is easy to use and familiar for designers and architects.
Starting from the drawing of NURBS surface, discretisation of the surface is carried out by generating a form diagram and defining supports and unsupported edges. With simple commands, a force diagram is generated where the horizontal equilibrium is computed and the edges need to be parallel, moreover the user may manipulate either geometry of force diagram, or form diagram or both. In the last part of the simulation the vertical equilibrium is calculated and the 3D shape is generated (Fig. 2.16). This plugin relies mainly on the immediate use of commands and graphical operations to run form-finding.

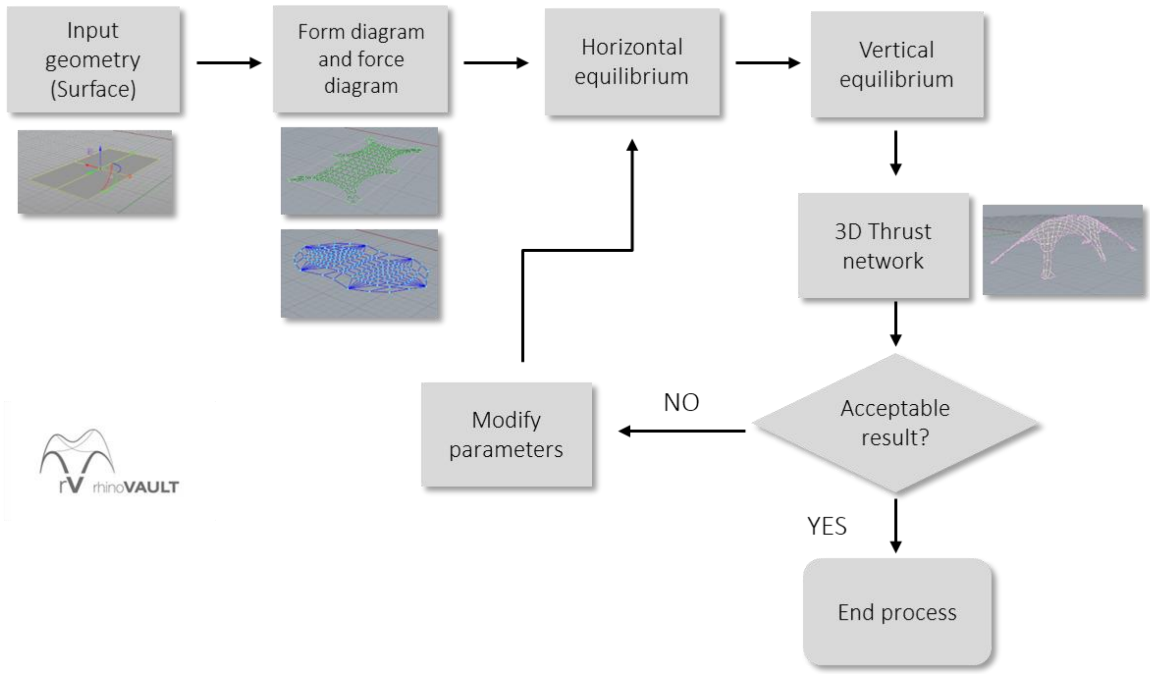


Figure 2.16 Workflow of the TNA implemented by RhinoVault

Kangaroo is a solver based on dynamic relaxation, which is a numerical method that processes non-linear equations. The method is implemented in Grasshopper® environment (Piker, 2013) and it works in parallel with the typical Rhinoceros® modelling window; in this context, the design of the structure is generated and controlled by specific parameters, set by the user. In this way, architects and engineers can handle all the process with a consequent major control over the design process, governed indeed by the parameters and their relations. In detail, within the parametric modelling, this plug-in was analysed since it reveals a good tool for the form finding approach.

The architecture of this solver is depicted in Figure 2.17, referring to Particle-Spring Systems workflow, which is an efficient method for the design of shell structures.

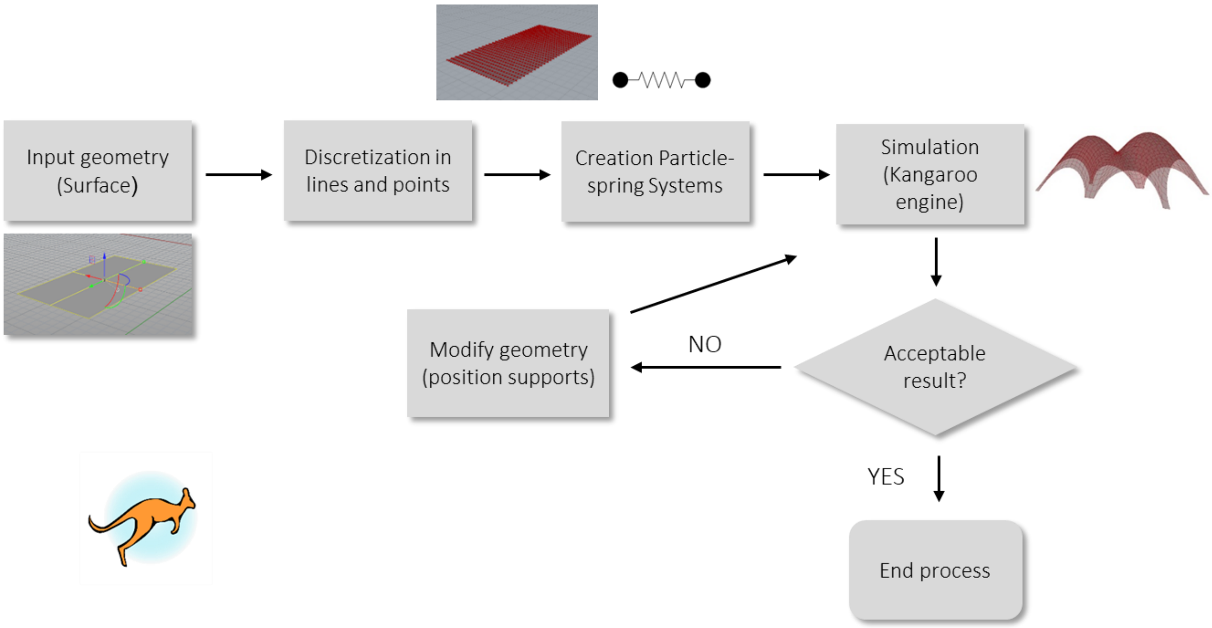


Figure 2.17 Workflow of PSS implemented by Kangaroo

It starts with a discretization essential for assigning mass and velocity to the particles and to apply loads on them. The nature of simulation is influenced on the set of goal objects defined, but generally parameters involved are load conditions, the support definition and the setting of rest length.

According to the manipulation of some parameters like goals, elements, forces, Kangaroo may be used for different scopes, following specific principles. For instance, it is possible to simulate constant Force Density Method in Grasshopper® by setting rest length as zero for all edges. With this value set to zero and gravity with a low value, the simulation reproduces the force density system and anticlastic geometries are generated. Moreover, tension values are proportional to extension, and in this case, the current length is the extension.

For simulations regarding hanging cloth like the emblematic examples carried out by architects like Gaudì and Isler, Kangaroo has proved to be an efficient tool as explained in the following chapter. In this case, important factors are represented by gravity loads condition, whose rule is crucial for the generation of synclastic geometries, moreover rest length should be set equal to the original length. With these assumptions, a particle spring system is performed creating shell structures that satisfy these conditions.

2.1.7 Summary

In this section, several aspects concerning shell architecture have been investigated, in order to provide a general understanding of the legacy of the past and of the potentialities that can be exploited in the present context. The process of generating architecture starts with a simple hand that transfers architects thoughts on a sheet of paper. This creative process has undergone a substantial evolution over the centuries until reaching a radical change in its paradigm, where algorithms rather than geometrical functions regulate the design. The advent of the parametric approach has consequently led to a total overturning in architecture to such an extent that a new style was codified. The practice to develop more bold shapes is now common thanks to intuitive computational tools paired with techniques that allow designers to generate shapes according to the force instead of the form. It was highlighted how these techniques have roots in the past in reality but it is in the contemporary scenario that they are becoming easier to access thanks to digital translation. Among these, different techniques have been analysed with the aim to familiarise with their features and in particular with their digital adoption. Therefore, it is crucial to gain understanding of the current innovations in order to produce an improved design process, but as it has been pointed out, an informed approach is the key-factor for successful projects without exacerbating aspects for reasons driven more by complexity than efficiency of the shapes.

2.2 Geometric control of surfaces

2.2.1 Introduction

In this chapter, shell structures are evaluated from a purely geometrical perspective, considered as surfaces controlled by a set of variables, with double curvature and that can be approximated in a collection of panels. In detail, such surfaces can be considered free-form due to their degree of irregularities in terms of curvature and not attributable to defined geometries such as conic surfaces, domes, etc. Starting from very basic definitions, a general contextualization is provided in order to cover a wide range of geometrical aspects, such as geometrical representation of curve and surfaces, curvature definition, rationalization and planarization. By carrying out a study that takes into account geometrical considerations it is possible to deal with more control in the design process and to gain understanding of limitations that can arise depending on the geometry and how they can be overcome.

2.2.2 NURBS and mesh

A fundamental part in the modelling process, especially in the case of free-form surfaces is to understand geometries we are dealing with in architecture. With the advent of computational modelling, we can generate an infinity of shapes by using a broad vocabulary of elements with specific features and purposes. In every CAD software at the base of the drawing process there are two macro-categories: NURBS and meshes.

NURBS (Non-Uniform Rational Splines) is one of the principal families of geometries that enables the generation of elements ranging from a simple line to freeform surfaces. They are characterized by a set of specific features such as control points, degree, nodes, evaluation rule that determines the typology of geometrical entity (Altman).

NURBS geometry is affected by its control points and by their position, by the degree whose number establish the kind of geometry 1 for lines and polylines, 2 for circles and 3 or 5 for free-form curve, the knots are the number of the control points therefore accountable for the smoothness and finally a rule regulated by mathematical functions establishes the positions of the points.
For a mathematical definition of NURBS geometry it is sufficient to provide a sequence of n 3D control point $P_1, P_2, \dots, P_n$ and their respective weights $w_1,\ w_2, \dots, w_n$. The degree m of the curve is defined by $n + k$ knots $t_1, \dots, t_{n+k}$.

$N_i(t)$ is the i-th b-spline basis function of degree m derived from the knots $t_1, \dots, t_{n+k}$:

$$C(t) = \frac{\sum_{i=1}^{n} N_i(t) w_i P_i}{\sum_{i=1}^{n} N_i(t) w_i} \tag{7}$$

NURBS geometries include surfaces (Fig. 2.18) as well, which are regulated by the same method: they can be controlled through control points that manipulate the geometry. NURBS surfaces have the characteristic of smoothness because the surface presents itself as a single element.

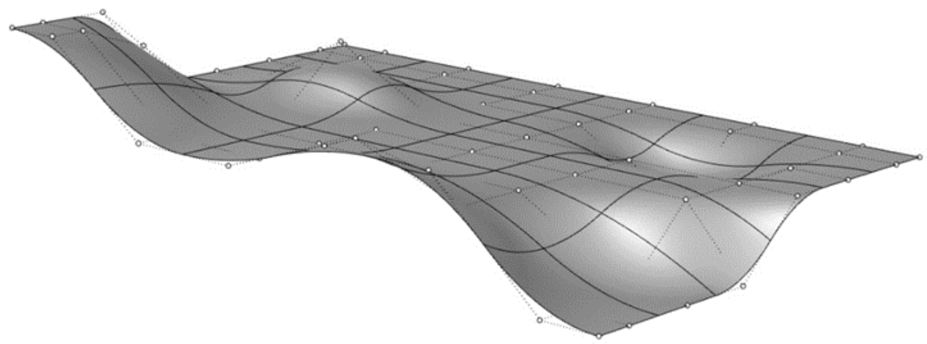


Figure 2.18 Freeform NURBS surface created in Rhino

Another way of representing 3D objects is polygonal modelling through meshes. In such a way, the object is approximated by a set of polygons that compose the mesh, with a substantial difference: the smoothness of mesh is given by the number of the polygons; the greater the number of the polygons, the smoother the mesh. Each polygon is defined by a set of vertices, edges and faces. The use of meshes in freeform architecture represents an important topic dealt with by researchers and architects, in detail the discretization of freeform surfaces is mainly based on the use of meshes of different nature. Meshes can be triangular, quadrilateral, hexagonal according to specific requirements as described afterwards (Fig. 2.19).

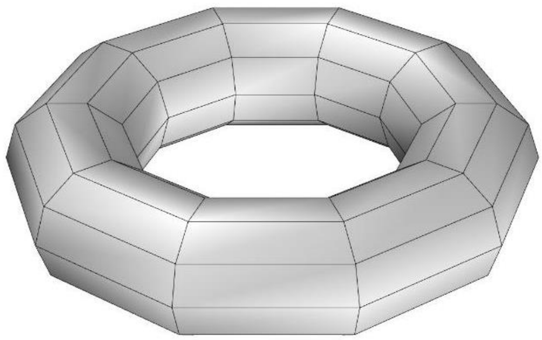


Figure 2.19 Polygon mesh (torus) made of quads

2.2.3 Surface curvature

Curvature is a geometric property that represents an important parameter in the design process. Geometrically, it is defined by the deviation from the tangent plane in a given point P on the surface (Tedeschi, 2014).
The definition of curvature can be explicated through the Gaussian curvature. Principal curvatures k_1 and k_2 for a given point P on the surface.

Gaussian curvature is, by definition, the product of the two principal curvatures, calculated as tangent vectors at a given point P of the surface (Fig. 2.20):

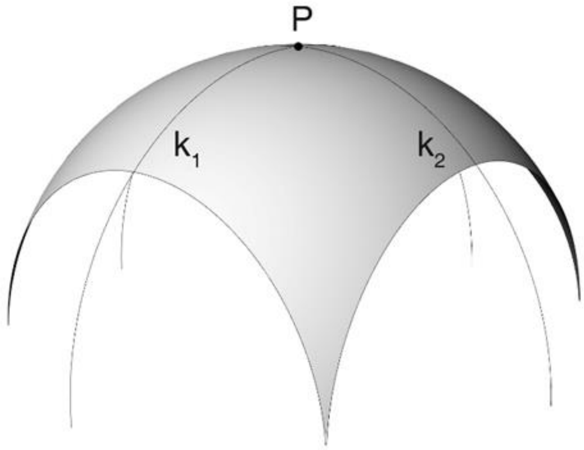


Figure 2.20 Principal curvatures on a surface

$$K = k_1 \cdot k_2 \qquad (8)$$

Gaussian curvature allows to categorise surfaces in three groups as shown in Figure 2.21:

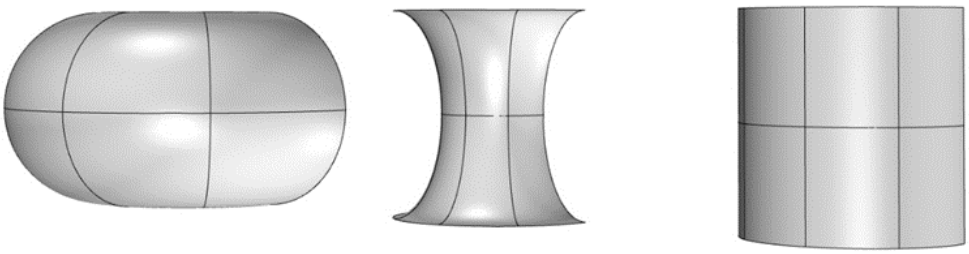


Figure 2.21 Gaussian curvature: a) positive curvature; b) negative curvature; c) null curvature

Positive curvature surfaces are where K > 0, negative curvature surfaces are K < 0 and surfaces with K are null. In the case where Gaussian curvature is null one deals with the so called developable surface, meaning that the surface can be flattened on a plane without deformation. Gaussian curvature sign is an important parameter that affects the process of discretization of surfaces. In the design of a shell, generally the surface is by definition with a positive Gaussian curvature, especially in the form-finding simulation where the shape is affected by gravity load.

2.2.4 Rationalization of free-form architecture

Architecture is materialised through a series of operations that can be quite challenging in the context of freeform contemporary projects. Different aspects need to be taken into account such as the goal to have on one hand very complex shapes but on the other hand the necessity of subdividing components must be easy to realize. Therefore, discretization is an imperative in order to realize architecture.

Pottman used the expression Architectural Geometry (2010) to define a new challenging research area. With the advent of CNC machines, parametric modelling and powerful computational tools have been paired with the desire to push boundaries with more and more complex shapes in architecture. A detailed knowledge needs to be developed to pave the way for efficient workflow, construction aware processes that meet requirements dictated from design aims and good solutions for realization. There is an obvious trend to design architecture free from traditional formal schemes and this has led to a necessity to re-formulate geometry. This involved other disciplines too such as mathematics, differential geometry, engineering as well as computational graphics. Pottmann highlights the importance of Architectural Geometry to such an extent it can be considered a field of design.

Discretization is the process where a surface is an object of re-elaboration and it is translated into a collection of panels that they need to be assembled together

(Dunn, 2012). Part of the discretization process is also the choice of the technique that provides an appropriate pattern for the panelization. At this stage, several parameters affect this decision, mostly depending on structural and realization requirements as well as manufacturing cost. In detail, the surface is parametrized and therefore subdivided into u and v coordinates as a first step for the discretization; it is carried out by considering the 2D pattern to use, which is projected on the surface, with a high likelihood that it will be subject to geometrical deformations (Stavric et al. 2011).

The geometrical discretization of free-form geometries is a fundamental part of the design process that may result in time-consuming and costly operations (Fig. 2.22). The typology of discretization can affect the success of such operations.

Figure 2.22 One of the four Zaha Hadid projects for the "Innsbrucker Nordkettenbahn" (Verner Huthmacher, available from bollinger-grohmann.com)

The main reason behind that is the attempt to approximate as much as possible free-form surfaces and this translates into panels highly customized and different from each other. For instance, Stavric and Wiltsche (2011) have outlined the current situation of materialization in architecture through two main solutions: the typical solution regarding concrete and a more innovative combination of steel and glass, which allows to realize double curved structures. However, in both cases it was demonstrated that they require a high level of complexity, in particular with the construction of moulds in the case of glass (to follow double curvature) and laborious formwork and reinforcement in the case of concrete (Sauter, 2008) with the main consequence of making these structures not economical from different point of views.

It is quite clear that the materialization for freeform architecture is not a cost effective process but it is possible now to find improved solutions by selecting the right pattern, standardising the panelization through optimisation processes and to find middle ground between design goals and production in order to get more feasible ways of making architecture.

Subdivision from polygonal discretization

According to the number of edges, panels can be of different nature: triangular panels are a common technique for discretization (Fig. 2.23). Thanks to their shape the approximation of the surface works quite well and planar panels are obtained (a triangle is always contained in one plane). Moreover, in terms of computational design plenty of algorithms for the subdivision are available and easy to use. On the other side, high node valency and torsion of the nodes result in quite complex structures to design together with the need to have structural elements as supports. The combination of all these things implies an increase of the manufacturing costs (Frolli and Tonelli, 2014).

Figure 2.23 British museum roof made of triangular glass panels realized by Foster + Partners (available from fosterandpartners.com)

Quadrangular panels present advantages in terms of geometric complexity thanks to a lower node valence that it is 4 in this case, they allow more flexibility in terms of design of the panels providing more natural light in the case on transparent surfaces; however, quadrangular elements are not necessarily planar and their planarization can require a more significant amount of work.

In the context of quadrangular panels, quad meshing represents an efficient tool that has found many applications in computational design. Quad meshing consists of generating mesh composed of quadrilaterals. There can be distinctions according to the "regularity" of such meshes (Fig. 2.24) (Bommes et al., 2012):

- regular mesh when it can be subdivided in rectangular grid obtained from square;
- semi-regular mesh is the product of combination of rectangular patches whose number is smaller than the number of faces composing the mesh;
- valence-regular mesh has valence 4 for most of its faces;
- unstructured mesh when the majority of the vertices are irregular.

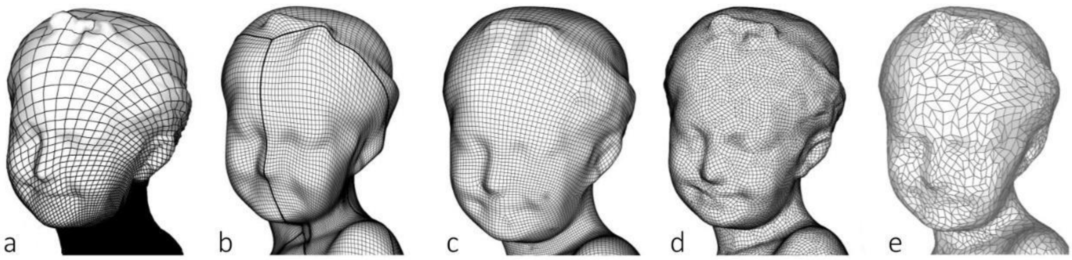


Figure 2.24 Quad meshes: a) regular mesh; b) semi-regular; c) valence regular; d and e) unstructured (Bommes et al., 2012)

These different categories find different applications but semi-regular meshes allow many applications because of more flexibility.

Another class of discretization is characterised by hexagonal subdivision (Fig. 2.25). Undoubtedly, they represent a new way for panelization but at the same time it results in a challenging technique: although hexagonal panels have a very low valence node (3) and an efficient structural system, operations such as planarization can result more elaborate than quadrangular meshing.

Hexagonal subdivision is paving the way in the architecture scenario thanks to recent developments in the computational design too that provide innovative subdivision algorithms and geometrical procedures that allow better control of the hexagonal cells. Moreover, it is an organic pattern for the pattern largely present in nature, becoming more attractive for its aesthetic qualities.

Figure 2.25 Landesgartenschau Exhibition Hall (© ICD/ITKE/IIGS University of Stuttgart)

Different approaches have been investigated to carry out hexagonal subdivision and duality represents a key operator in this process: it is well established that triangles and hexagons are duals (Fig. 2.26) thus given a mesh defined by a set of vertices and edges, a new mesh is generated by a new set of vertices that corresponds to the original faces and edges defined through the connection of the faces.

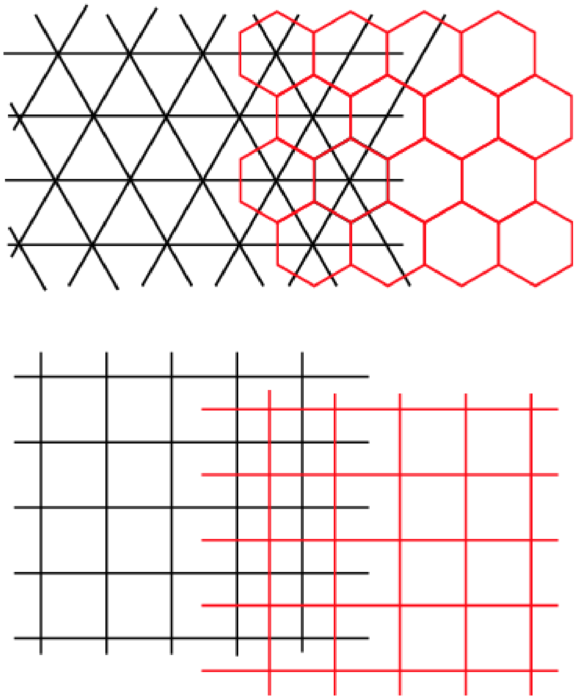


Figure 2.26 Duality between triangles and hexagons and squares (available from wolfram.com)

Honeycomb discretization refers to a hexagonal pattern with an organic configuration: it resembles the cells built from the honeybees. Besides this particular aspect, such subdivision results of particular interest in the topic of subdivision surfaces. Moreover, in this case we talk about duality properties starting from triangular schemes where after interactions vertices become three-valent.

Akleman et al. (2003) have addressed this topic by elaborating an algorithm that consists of creating new faces from vertices computed as linear combinations of old vertices and from then edges of the final honeycomb mesh are generated through connecting the points obtained from the faces (Fig. 2.27).

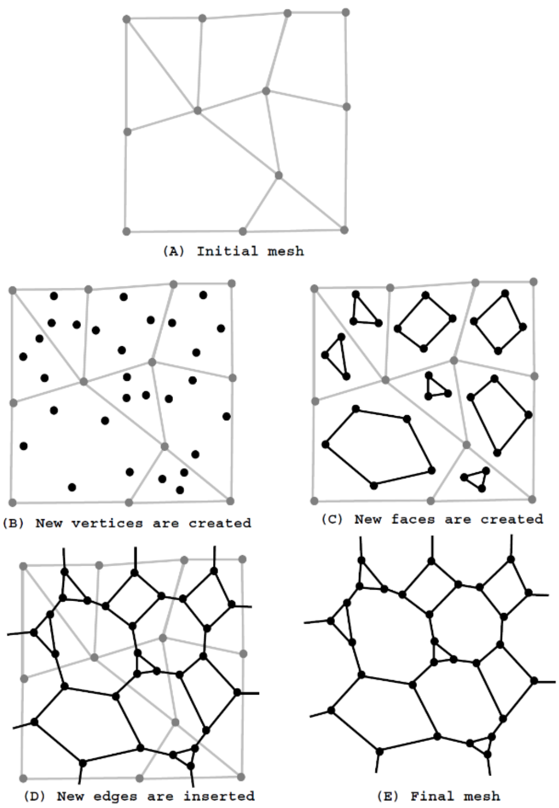


Figure 2.27 Algorithm procedure for honeycomb elaborated (Akleman et al., 2003)

Together with honeycomb subdivision, Voronoi is also a form of polygonal subdivision. Its properties are making it a valid subdivision scheme in many applications, with a new rising interest in its application in architecture.

Georgy Fedoseevich Voronoy provided important advancements in this methodology and his name to this diagram is proof of that. Voronoi diagram is based on the decomposition of a given space into portions. Every portion of such space has its own "generator" where the distances of the points included in the region is less than the distances to the generator of other regions. This tessellation can be better controlled by applying a centroidal Voronoi tessellation where the generator of each region is also the barycentre of each partition. Therefore, it is necessary to compute the centroid of each region and associate it to the point generator of such region. This process is iterative in order to reach the convergence between these two points (Lloyd algorithm). Taking into account the same set of points *S*, the dual of Voronoi subdivision is Delanuay triangulation, whose algorithms allows to compute a more regular triangulation avoiding thin triangles. The triangulation operates in such a way: given a set of points P, the choice of three points is justified by the generation of circumcircle that must not contain any other points of P. Given a set of points $\{z_i\}_{i=1}^{k}$ the mathematical definition of Voronoi tessellation is (Burns, 2009):

$$\widehat{V_i} = \{x \in \Omega | |x - z_i| < |x - z_j| for\ j = 1, \dots, k, j \neq \qquad (9)$$

Subdivision from single curved panels: ruled and developable surfaces

So far, subdivision has been addressed focusing on the typology of polygons applied. However, subdivision can be carried out by taking into account features that imply a different approach. Rationalization is part of this informed process aimed to exploit digital technologies by using computational tools that can decompose free-form shapes. In a more general context, the approach categorizes in double curved panels, single curved panels and planar panels. Double curved panels require a high customization of the projects, with a perfect approximation of the original surface, but on the other hand, the production for such panels is not considered optimized in terms of production time and costs. Between double curved panels and planar panes that represent the best solution for the optimisation of the design process, single curved panels are undoubtedly a good alternative without compromising the artistic freedom in architecture.

A ruled surface (Fig. 2.28) has the peculiarity to have a clear parametric definition, consisting of a movement of a straight line along a directrix (Patrikalakis et al., 2009):

$$r(u, v) = \alpha(u) + v\,\beta(u) \tag{10}$$

where $\alpha(u)$ is a directrix and $\beta(u)$ is a unit vector which provides the direction of the ruling.

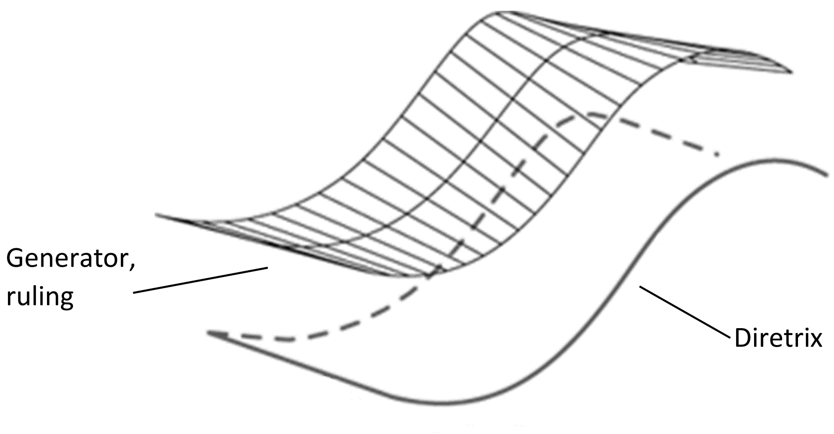


Figure 2.28 Ruled surface with its directrix and generatrix (available from iam.tugraz.at)

Rationalization for non-positive surfaces is suited for ruled surfaces. They have the remarkable property of being generated through a straight line called generatrix that translates into advantages in the realization, namely mould productions or support structures among others. Double curved panels typical of freeform architecture can be decomposed in a series of ruled surfaces by using traditional CAD software. This method was perfectioned with the aid of a workflow involving basic algorithms. Firstly, a surface is approximated with strips of ruled surfaces when the curvature analysis provides suitable results and successively it is optimized in order to minimize the distance from the target surface S (Flory et al., 2010). The authors of such a research have proved this approach by applying this workflow to Cagliari Contemporary Arts Center where the façade is divided into patches of single ruled surface.

A particular class of ruled surface is represented by developable surfaces. They have been used extensively by Frank Gehry in his architecture expressed through single curved panels (Fig. 2.29). These surfaces have one of the principal curvature null (k=0) meaning that they can be unfolded on a plane without undergoing any deformation. Single panels represent a good solution in architecture thanks to aesthetic properties that make them more attractive than polyhedral subdivision and less expensive than double curved panels. Their investigation through semi-discrete geometry allows the formulation of several algorithms starting from a conjugate curve network, or from a PQ mesh (Pottmann et al., 2008).

Figure 2.29 Disney Concert Hall by Frank Gehry (available from archinect.com)

2.2.5 Planarization and geometric remarks

In the computation of polygonal meshes, planarization plays a fundamental role either dealing with quad-meshing or hex-meshing. Planarization has been investigated with different approaches and its application in panelization in freeform architecture is having a growing impact in the realization. Particularly with the use of new materials like glass planarization is necessary for carrying out panelization. However, regardless of the material, it represents a process that optimizes realization, lowering manufacturing costs and increasing the standardisation of the process.

The investigation of such topics has been made from different perspectives relying on differential geometry principles. The derivation of a network of conjugate curves was addressed by Liu et al. (2011) proving to be an efficient method: two families of curves are generated in order to cover the surface whose tangent vectors define a cross field throughout the surface called conjugate direction field (CDF). It was demonstrated that the generation of PQ (planar quad) meshes is controlled by CFD. Other approaches take into consideration the geometrical properties of principal meshes such as circular and conical meshes. They present many advantages in architecture and therefore in the fabrication of panels. Meshes can be defined circular (Fig. 2.30) if the faces can be inscribed in a circumcircle; conical meshes (Fig. 2.31) have faces that meet in a vertex tangent to a circular cone. This topic was thoroughly addressed by Pottmann et al (2008) analysing the relationship between conical and circular meshes as well as offset mesh at constant face distance and constant vertex distance. These advancements provide important applications in freeform architecture. Especially for conical meshes, that has been studied by Liu et al. (2006), the properties concerning their geometry are quite remarkable: the offset mesh at constant face distance and the parallelism between corresponding faces and edges affect positively the realization process for structure that require constant thickness.

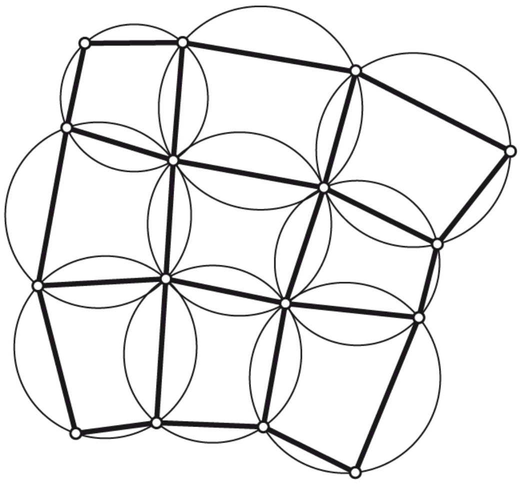


Figure 2.30 Circular mesh (Liu, 2007)

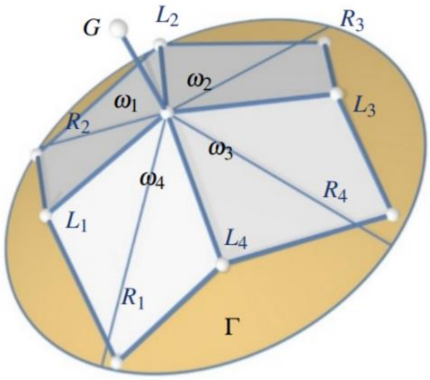


Figure 2.31 Conical mesh with valence of four vertices (Wang et al., 2006)

Planar hexagonal (P-hex) meshes are contextualised in a similar context of PQ meshes but with few dissimilarities. Starting from duality property the relationship between triangular and P-hex meshes has been analysed by duality which represents an efficient tool to compute planarization. Wang et al. (2008) developed Dupin duality that consists of a perturbation of vertices that exploits the tangent duality between a triangle mesh and its corresponding hexagonal mesh. P-hex meshes can be computed through quad meshes too via a method that includes conjugate curves and non-linear optimisation. The method Wang et al. have elaborated is based on a step-by-step process starting from the approximation of surface S with the derivation of its conjugate curves. Successively a quad mesh is obtained from the conjugate curves and by shifting every row of the quad panel a staggered pattern is obtained. By a second-step optimisation the planar hexagons are obtained taking into account constraints such as a minimization function that constrains vertices to the target surfaces and functions that define as null the volumes of tetrahedral of the 4 point subset of the vertices of the hexagon.

Moreover, it was proved that planar hexagons generate special surfaces such as minimal surface and surface with constant mean curvature (Bobenko et al., 2006).

As shown, many theories have been elaborated in the field regarding discrete geometry with remarkable results, especially if one deals with planarization issues. Nevertheless, if we want to address this problem from a practical point of view that involves the use of computational modelling, some limitations can occur: the translations of these principles in a digital environment is still hard to define with a few exceptions and in several cases geometry represents a big challenge to implement planarization, as well as self-intersections can occur without the possibility to control them.

An approach that results to be quite efficient includes the principle of "conformal". By establishing a Christoffel dual construction minimal surfaces are generated, the main principle is that the "dual of an isothermic parametrisation of a sphere is an isothermic parametrisation of a minimal surface and vice versa" (Müller, 2011).

Muller explored this property for hexagonal meshes highlighting the relationship between the Christoffel dual of a discrete isothermic surface covering a sphere and discrete minimal surface formed of planar hexagons. A discrete isothermic surface is a mesh which has vertices in a sphere where the faces are conformal hexagons. To define conformal hexagons the concept of circular mesh results quite useful for this aim. A mesh is considered to be circular when their faces are circular polygon and the cross ration is real.

$$cr(z_0, z_1, z_2, z_3) = \frac{(z_0 - z_1)(z_2 - z_3)}{(z_1 - z_2)(z_3 - z_0)} \tag{11}$$

A hexagon which is defined by a set of vertices $(z_0,\ldots,z_5)$ is called *conformal* (Fig. 2.32) if the cross ratios (cr) are equal to cr$(z_0; z_1; z_2; z_3)$ = -1/2 and cr$(z_0; z_5; z_4; z_3)$ = -1/2.

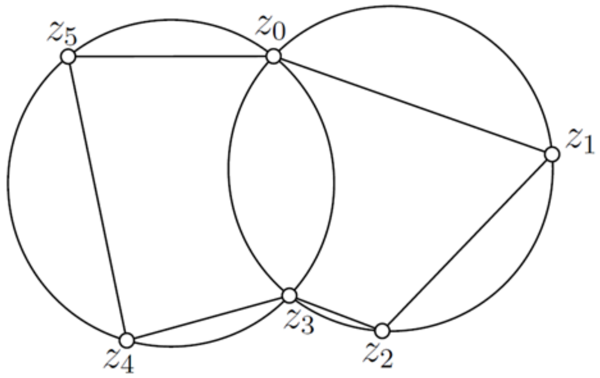


Figure 2.32 Conformal hexagon with two circles (Muller, 2011)

2.2.6 Volumetric subdivision

So far, the tessellation of free-form surfaces has been investigated from a geometrical point of view intended as subdivision techniques by means of algorithms definitions. The new technologies allow to deal with very complex situations but at the same time holding a good control of the process. This particularly regards the subdivision in 2D of surfaces and meshes. What has not been considered yet is the tradition aspect in architecture of the tessellation intended as the art of cutting solids and assembling them to form structures in a 3D volumetric way. Generally, this refers to “stereotonomy”, whose meaning is from the greek, "stereo"- solid and "tomia" – section and its first use can be found in many theoretical works of the past. The importance of having a correct tessellation guided by specific rules dates back to Philibert Delorme in Le Premier Tome de l’Architecture in the 16th century (de l'Orme, 1567) where the importance of controlling construction requirements is considered fundamental for a correct design. Its materialisation in its most prolific period still remains in the 17th century culminating with the Descriptive Geometry of Gasparde Monge (de Azambuja Varela et al., 2016) where the use of stone in architecture was quite predominant (Fig. 2.33)

Figure 2.33 Vault of the Hotel de Viell, Arles (available from pinterest.com)

Although this practice has known a strong decline with the appearance of new materials starting from the Industrial Revolution, in these last decades the tessellation of unreinforced structures has become an important topic within the design process, further supported by digital technologies that aim to translate historical theoretic principles into applicative processes. Ongoing research is providing significant contributions, such as that of Giuseppe Fallacara from Politecnico di Bari, who is undoubtedly one of the main investigators of such topics. His works deal with the investigation of the stereotomy principles in historical stone buildings and their reinterpretation in contemporary projects through traditional techniques by means of digital technologies (Andrusko, 2014). The Ponte Truchet is a remarkable example of the evolution of the design presented by Jean Truchet in the 18th century where interlocking elements were able to cover a space of a flat ceiling. The interlocked system was assured by translation and rotation of a single ashlar type, representing a further improvement of the Abeille's flat vault (Frézier, 1738). Fallacara used these features to realize a skewed barrel vault by using digital fabrication techniques (Fig. 2.34-2.35).

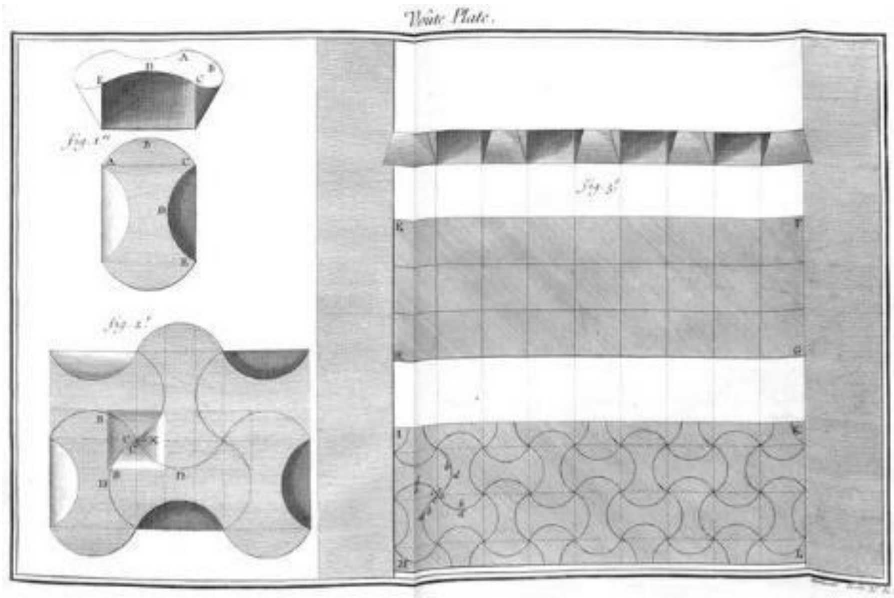


Figure 2.34 Truchet's flat vault patent (de Azambuja Varela and Sousa, 2016)

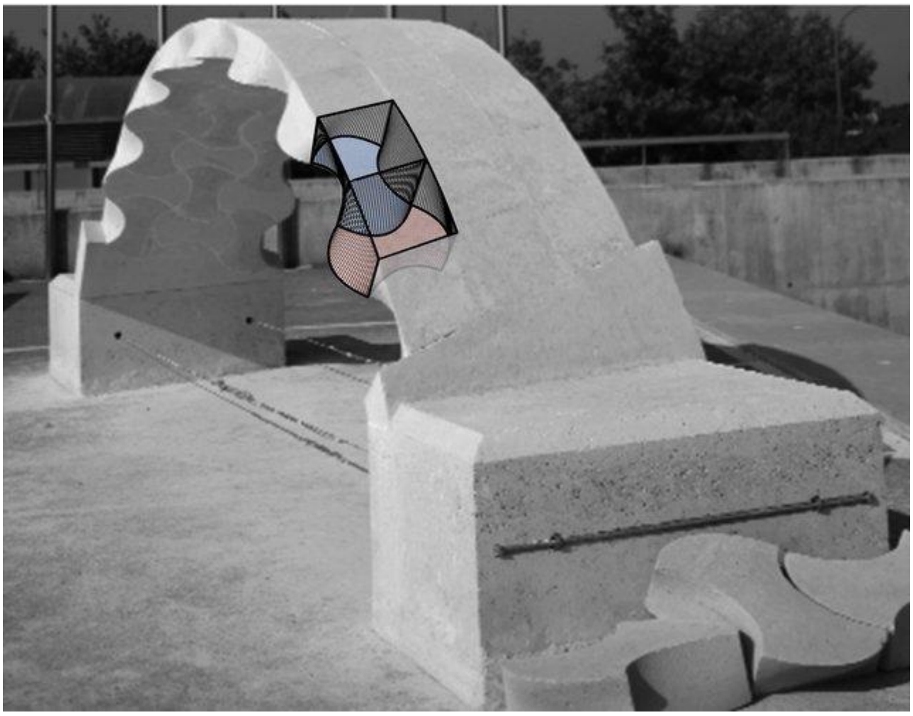


Figure 2.35 Fallacara's vault based on Truchet system (de Azambuja Varela and Sousa, 2016)

Digital stereotomy is addressed by several research groups from various perspectives: exploring potentialities of vault system in compression with sustainable materials (de Azambuja Varela and Sousa, 2006) or re-defining joint systems applicable to shells (Li, 2017) or aiming to include stone stereotomy in contemporary projects (Aau Anastas, 2017).

Moreover, the Block Research Group (Rippmann and Block, 2013) defined a workflow for the tessellation of free-form masonry shells. Thus, it represents one of the main references for the work carried out in the digital fabrication chapter (chapter 4). Although the volumetric approach has been taken only for specific cases described in the digital fabrication chapter, at this stage it was considered necessary to mention the main principles underpinning the digital tessellation of such case studies.

In detail:

- staggered tessellation;
- suitable thickness to assure safety of the structure;
- voussoirs (wedge-shaped elements in stone architecture) have to be perpendicular to the thrust surface.

Staggered tessellation prevents the structure from the alignment of the blocks that can cause instability and it is connected with the mesh subdivision used in the process of form-finding. Moreover, the voussoirs need to follow normal vectors direction for their extrusion in order to prevent sliding and an adequate thickness evaluated by structural analysis provides an efficient behaviour (Fig. 2.36). Therefore, an informed process involving every phase of the design is fundamental for a correct implementation (Rippmann et al. 2016).

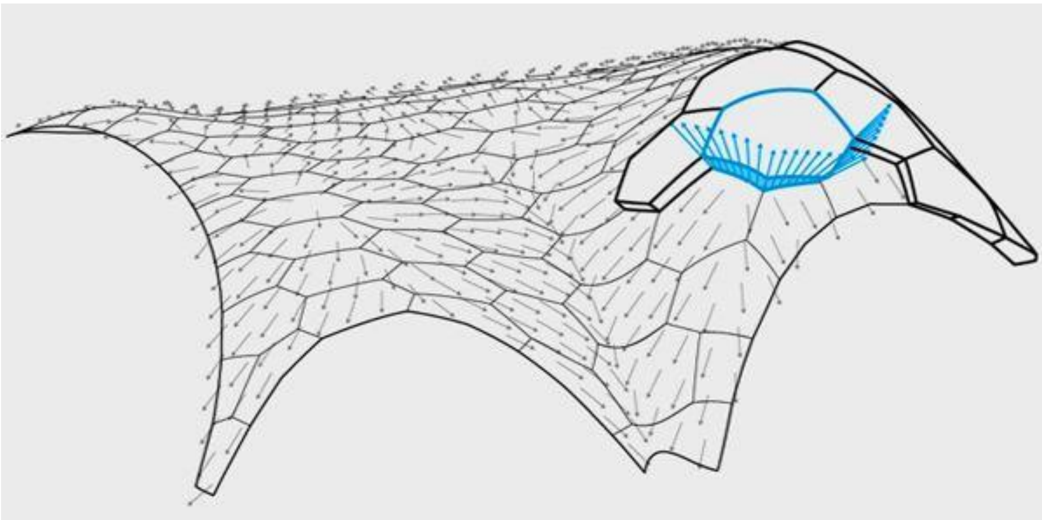


Figure 2.36 Force of flow and tessellation of a free-form shell (Rippmann and Block, 2013)

These main principles regard the structural efficiency of the shell structure, meaning that in order to guarantee stability, a compression structure without the use of reinforcement needs to fulfil these requirements. In the contemporary scenario, very few structures are designed and realized following this method, since it still represents a challenging approach that involves elaborated design workflows and customised fabrication, especially when free-form shapes are involved. This implies operations that are cost and time intensive. However, it is possible to optimize the process from the fabrication perspective by means of digital operations within parametric modelling that indeed allows to standardize some parts of the process.

2.2.7 Summary

This chapter has focused on the geometric aspects related to architecture, especially the importance of rationalization for free-form architecture by several techniques. The contemporary vocabulary of architecture requires a more accurate degree of attention for rationalization process due mainly to the complexity of the projects together with the priority of optimisation of the production and realization. Different ways to approximate surfaces have been presented, each of them characterized by strengths and weaknesses. Polygonal subdivision and planarization have been taken into account as well as the discretization according to the curvature of the panels. It is not possible to define a "winner" among the different techniques described because every choice is dictated by precise requirements related to specific projects that justify the use of a subdivision over the other. However, important properties concerning specific typologies of meshes such as circular and conical meshes need to be considered because of the issues arising in the realization to be overcome. What is important is the need to gain knowledge of the geometric features during the process of creation in order to obtain the best solutions. Rationalization must be an upstream phase of the design process, embedded in the preliminary formulation of the design. In such a way, many issues that successively need to be overcome can be solved in a more straightforward way at the beginning of the process from the designers responsible for the creation of the shape.

2.3 Optimisation as necessary design phase

2.3.1 Introduction

The following chapter addresses optimisation processes. Starting with a general overview in order to understand the main principles behind optimisation and its main categorisation, the work focuses on the potential use in architecture and how it can be integrated in the design process. The main advancements obtained in the design of shell structures by means of optimisation are discussed and with it the advantages and disadvantages associated with the different methods. In such a way, by gaining more awareness of such a practice, optimisation can become a powerful tool for architects who deal with free-form structures.

2.3.2 Optimisation: general overview

In a general overview, optimisation means a process where an objective function has to be minimized or maximized according to a set of variables, regardless of the principles and the theoretical process used for reaching that result. Conceptually it regards the selection of best solutions in a range of possible solutions for a specific problem. Its peculiarities have been explored first in mathematics where it reveals an important tool for the solution of problems of different natures.

An optimisation problem can be generally expressed as follows:

$$\text{minimize } f_0(x) \tag{12}$$

$$\text{subject to } f_{i1}(x) \leq b_i,\ i = 1, \dots, m \tag{13}$$

(12) is the objective function, $f_i(x)$ is the constraints functions, the vector $x = (x_0, \dots x_n)$ is the variable and $b1, \dots bm$ are the bounds of the constraints. A vector x^* is a solution of the problem if it has the smallest objective value taking into account the constraints (Boyd and Vandenberghe, 2004).

While single-objective optimisation reaches the solution in a straightforward way based on their performance, multi-objective optimisation defines not a single solution but a set of solutions that are considered “non-dominant”.

Multi-objective optimisation is defined as follows:

$$min[\, f_1(x), f_2 2(x), \dots, f(x)] \quad (14)$$
$$x \in S$$

S represents the set of constraints and $n > 1$.

As already mentioned, the optimisation acts in a different way in a multi-objective context depending on Pareto principle: “a vector $x^* \in S$ is said to be Pareto optimal for a multi-objective problem if all other vectors $x \in S$ have a higher value for at least one of the objective functions f_i, with $i = 1, \dots, n$, or have the same value for all the objective functions” (Caramia and Dell’Olmo, 2008, pag.12). This implies the non-dominance of the solutions with the possibility to reach a result that takes into account various scenarios.

Figure 2.37 shows a typical example in a two dimensional case where the Pareto principle is represented by a curve considering two objectives.

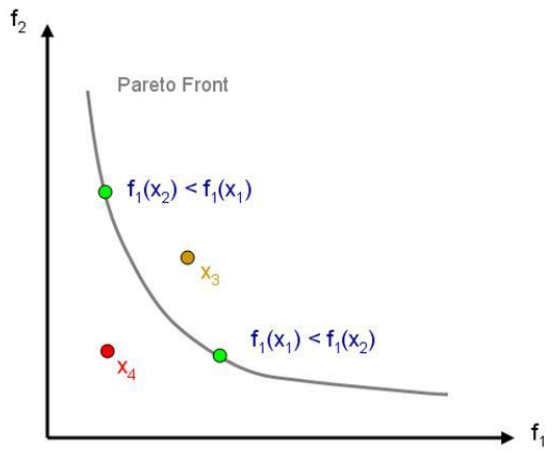


Figure 2.37 Pareto front defined by the curve between two objective functions (available from insider.altairhyperworks.com)

This mathematical principle can be applied in various fields and it found one of its maximum expressions in structural problems, proving to be the most reliable tool to address efficient structures in architecture. In this context one deals with structural optimisation where objective functions may include the minimization of material, the minimization of the mass, the minimization of displacements or the maximization of stiffness in a structure among others (Adriaenssen et al., 2014).

Structural optimisation is beneficial for the design process since the search of solutions for given objectives to satisfy translates into improved economy and efficiency. The use of solvers and strategies for optimisation embedded in the design may give enormous advantages from different perspectives.

The first examples of optimisation go back to the investigations carried out regarding form-finding; indeed these processes aimed to find an optimum shape for a set of boundary conditions.

Although the concept of optimisation has always been recurrent in architectural practice, its theorization in architecture application has quite recent origin: the first validation happened in 1993 through Evolutionary Structural

Optimisation (ESO) and its successive variants (Xie and Steven, 1993), but it is worth mentioning the first investigation carried out 100 years before by Michell who developed an optimal layout for a truss with a force applied in a point (Fig. 2.38). In such a way, if alignment of the truss with the principal stress direction occurs, it is possible to have an optimal configuration (Michell, 1904).

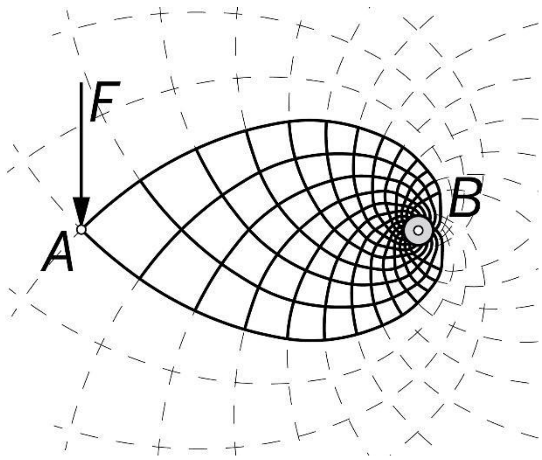


Figure 2.38 Michell truss layout (Michell, 1904)

2.3.3 Classification of optimisation processes

Nowadays, a wide range of strategies can be adopted according to the requirement to fulfil and the nature of the objectives. Three macro categories are defined.

Topology optimisation. It is quite certain that it represents one of the branches in structural optimisation that has been object of research and investigation for decades. The term topology indicates the study of properties of geometric elements when they undergo deformation analysing their relationship. At the base of topology, there is homeomorphism, which is considered the main topological equivalence. Homeomorphism relies on the idea of a continuous deformation of a body without cuts or overlapping, leading to a transformation of the object into another object (Weisstein, n.d.).

Such principles have been investigated in topological optimisation (TO). TO acts on the topology of the structures, which is defined as relationships and connectivity of elements in the structures, such as nodes. Different methods can be used to carry out topological optimisation such as density based methods, evolutionary procedures, bubble method, topological derivative (Xia et al., 2016). Topological optimisation relies on the topological features of the geometries, namely the study of the geometric properties when an entity undergoes deformations. Evolutionary Structural Optimisation is surely noteworthy regarding its application as an architecture tool; it has represented one of the main topics that have undergone impressive developments since its formulation. Based on the evolutionary process of nature that the topology and shape structures tend to find an optimum solution adapting in the surrounding environment (Fig. 2.39), it addresses the removal of material whose position within the structure is considered inefficient.

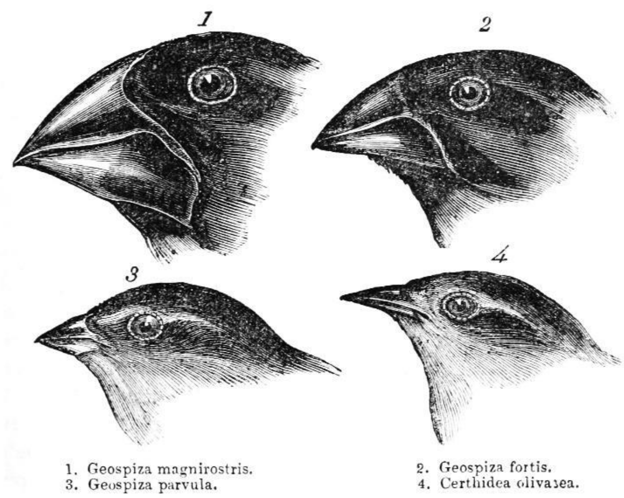


Figure 2.39 Darwin's finches or Galapagos finches (Darwin, 1845)

This is defined by the stress level of each elements, which takes into account its von Mises stress σ_{e}^{vm} of the element over a prescribed critical or maximum von Mises stress of the whole structure σ_{max}^{vm}. In such way, a rejection ratio (c_{rr}) is obtained and values below this threshold are considered as material to remove during the simulation (Fig. 2.40).

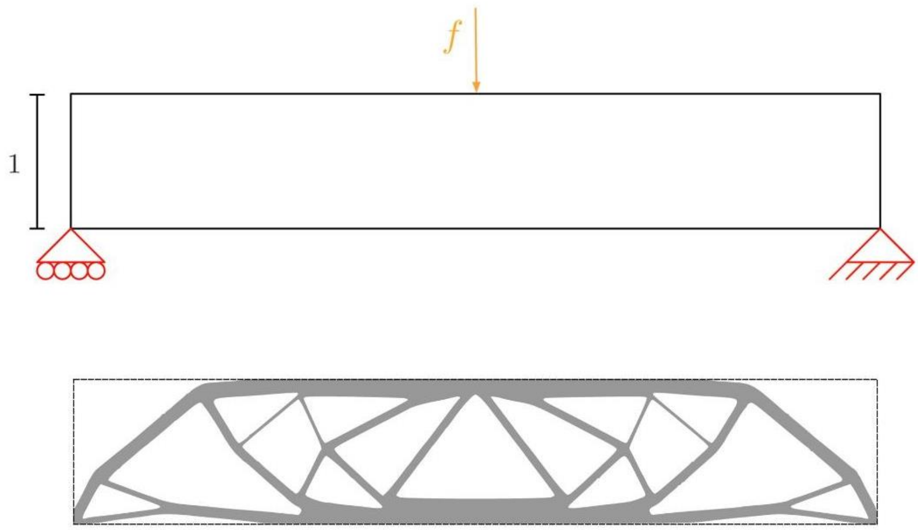


Figure 2.40 TO simulation applied on a beam structure with symmetrical load condition

This is an iterative process that aims to give a topological configuration where only the material active in the structural performance is preserved. While with ESO only material removal can be operated and therefore some issues can occur on the generation of an appropriate shape, especially due to its irreversibility, a process that operates in an opposite way has been developed by Querin et al. (2000) called additive ESO, where the material is added in presence of high stress values. A further development of ESO approach has overcome its main limitation regarding the irreversible aspect, once material is added or removed, it is not possible to retrieve precedent actions of optimisation, providing a bi-directional ESO (BESO) always formulated by Querin et al. (1998). BESO allows to compute an optimal solution by combining principle of ESO and

AESO, meaning that initial setup does not influence in a significant way the process that may include both material removal and addition according to the conditions satisfied:

$$\sigma_e^{vm} < c_{rr} \cdot \sigma_{max}^{vm} \rightarrow element\ removal, \quad (15)$$

$$\tilde{\sigma}_e^{vm} > cir \cdot \sigma_{max}^{vm} \rightarrow element\ addition. \quad (16)$$

Shape optimisation can be implemented in different ways such as using non-linear force density methods or thrust network analysis but one of the most efficient approaches concerns the use of Genetic Algorithms (GA).

Size optimisation regards mainly a later phase of the design process since it consists of finding optimal dimensions of building elements interacting on variables such as cross sections of the elements composing the structures or the thickness. In architecture like gridshells for instance the areas of the cross sections represent an important variable within the optimisation (Adriaenssen et al., 2014).

The implementation of optimisation processes regarding the geometry and cross-sectional dimensions may occur through the use of Genetic Algorithms (GA), that are revealing an efficient tool in the search of optimal solutions regardless of the variables and the objective to satisfy.

Mitchell's book (1996) provides a useful description regarding Genetic Algorithm. Genetic algorithms represent an efficient tool in the search of optimal solutions given a specific problem definition. Their formulation has been explored for a wide range of problems, including engineering, biology and social sciences. Their application in architecture has surely revolutionised the process of making architecture.

Developed in the 1960s by John Holland at the University of Michigan, his study focused mainly on the adaptive behaviour present in nature and how this adaptation could be translated in the computation. By analysing this mutable process and by applying these characteristics, solutions for a wide range of problems can be found.

Due to the nature of Genetic Algorithms the terminology used refers to the biology environment, therefore terms like *chromosomes, genes* will be used.

A Genetic Algorithm is based on a population of elements called chromosomes that represent a potential solution for a problem as well as the parameters of the process. In order to allow that these parameters can operate a fitness function as already mentioned, necessary for the implementation of GA. Its procedure is based on three specific "operators": starting from a population of chromosomes, which are defined as codes, a new population of candidates is generated through the evaluation of their performances. This is done by using the operators:

- the selection consists of "selecting" the best candidates for the new generation of a population;
- the crossover combines the codes of the selected elements in order to generate a new population;
- the mutation is based on a random solution modifying the codes of the new chromosomes.

All these operations are repeated until a result that satisfies the fitness function is obtained. Regardless of the fitness function to fulfil, the process results the same for all the different applications.

2.3.4 Optimisation processes for the design of shell structures

Optimisation process has been revealed to be an efficient tool in contemporary architecture. According to its typology, its use can be integrated in different phases of the design process although its potentialities can be fully exploited in the early phases where it provides a strong support in the definition of the shapes. This is particularly valid in the case of free-form surfaces where the form can be modelled in order to fulfil structural requirements and therefore improve the structural efficiency. In the contemporary scenario, the practice to integrate optimisation in the design process is still not dominant for different reasons: firstly because of the lack of background within architects and professionals due to the fact it represents a new field addressed mainly in academic activity; secondly for the difficulty to deal with shapes obtained from such simulations that can provide issue in the realization for their complexity.

One of the first typologies of optimisation to be explored in architecture was the topological optimisation. Topological optimisation has been used as an efficient architecture tool in the design starting from the end of 1990s. The main representatives were the Japanese architect Arata Isozaki, who with structural engineer Matsuro Sasaki started a very productive collaboration regarding the design of emblematic structures through topology optimisation. Noteworthy examples are undoubtedly a multifunctional centre Illa de Blaned created by using an ESO algorithm and Extended ESO algorithm able to control better the process together with a bi-directional approach. Another contribution in this direction was the project for Santa Maria Novella train station in Florence where a giant tree like structure dominated the station. Both projects were never realized due to the high cost of realization but the materialisation of such innovations took place in the realization of The Qatar National Convention Centre (QNCC) in Doha. Isozaki designed an impressive

(Białkowski, 2016) structure with his iconic tree-like supports (Fig. 2.41) realized through Extended ESO strategy considering mainly the self-load of the structure. In the shell design, research in topological optimisation applied on shell structures has been done throughout the years. Dienemann et al. (2017) have investigated a new method by considering mid surfaces of shell structures and by using manufacturing constraints to the 3D topology optimisation, Ansola et al. (2002) have combined topological and shape optimisation always by considering the mid surface. Such combination of two different typologies has been also explored simultaneously by controlling shape and topology as design variables (Hassani et al, 2013). The investigation in this field has delivered promising results but with a massive intervention on the shape and therefore with a limitation on the design freedom.

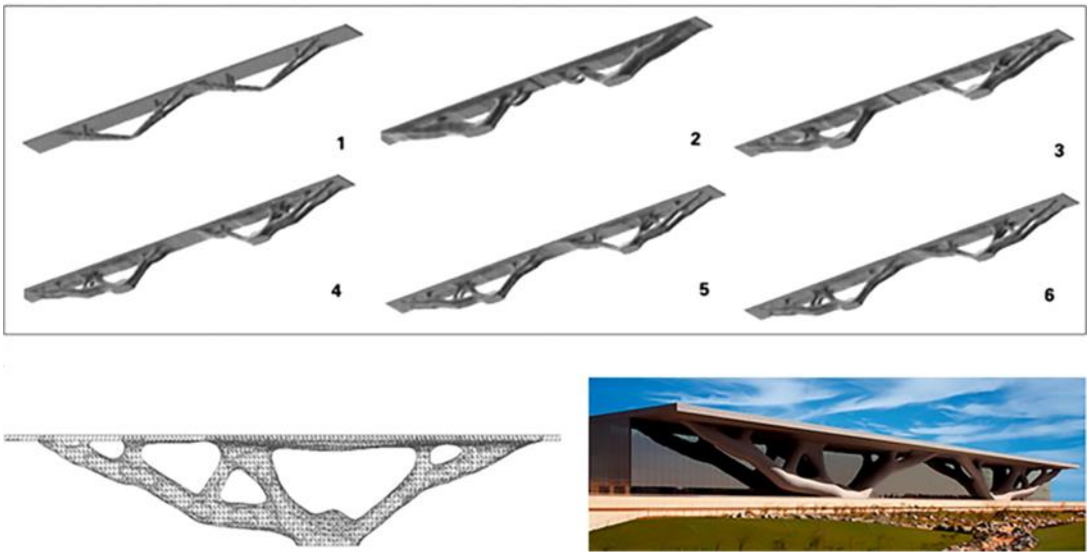


Figure 2.41 Qatar National Convention Centre (2011). On the top form-finding process with evolutionary algorithm iterations; result for the design of the supports and final realization (Sasaki, 2007)

However, in the shell design scenario, Genetic Algorithms (GA) for the implementation of shape and/or size optimisation are becoming a valid

approach: this powerful tool in the last decades has been investigated in the structural optimisation problems dealing with the definition of large span structures and successively for shell and freeform structures. In the case of structural optimisation, the implementation of GA allows to refine the design process in a remarkable way providing definition of spatial solutions that represent the best spatial configurations for a given objective to satisfy. In the case that one needs to formulate structural problems, the design variables are translated into the definition of chromosomes and the fitness functions. For instance, a problem of structural optimisation can be defined in the following way: the chromosomes may be represented by nodal positions or sizing of cross-section members and fitness functions may consider the minimization of nodal displacement or of the strain energy among others. It is important to define a correct domain so that the process can operate correctly. The definition of the domain is given by the variables used in the formulation of GA. Indeed, they represent the limit of such functions in which they can be implemented. By providing a correct space domain, it guarantees the search of a global and therefore more efficient solutions instead of finding local solutions.
By using GA in architecture a new way of considering it is therefore quite obvious: architecture is no longer considered as a standalone object not subject to mutation throughout the time, it responds to evolution laws, it is an organism that changes, that adapts itself according to objectives to satisfy.
The use of GA in architecture refers to a specific expression that best summarizes its main features: computational morphogenesis. In fact, its meaning involves the development of an organism and of its "morphogenic" characteristics by the use of computers.

In the contemporary scenario, computational morphogenesis is a powerful design tool for engineers and architects by means of evolutionary solvers operating in a parametric environment. In such a way, the design undergoes a series of iterations aimed to find the best solution for a given objective function (or a set of objective functions). The use of GA for the computation of free-form shells has been explored with remarkable results. Their use integrated in the design process has been addressed in the last years, drawing attention to concrete shells and gridshells as well (Dimcic and Knippers, 2011; Bertagnoli et al., 2014). Approaches like the one proposed by Yang et al. (2013) have considered the shell as NURBS geometry and have modified a GA in order to improve their convergence in the search of solution. Such a method was proposed on a project based in Shanghai, whose efficiency was improved by combining shape and thickness. NURBS geometries have been defined and analysed by Finite Element Analysis in Adha et al. (2020) and Pugnale and Sassone (2017). In the first case, the study has regarded the implementation of the framework via numerical approach in order to evaluate its effectiveness in order to minimize the material; the latter case regards the application of GA for the new Kakamigahara crematorium in Japan (Fig. 2.42), realized by Tokyo Ito with the structural collaboration of Mutsuro Sasaki. The free-form reinforced concrete roof has been designed taking into account an optimisation process to minimize strain energy and supported by supports positioned in a random position.

Figure 2.42 View of the Crematorium in Gifu - Toyo Ito (Januszkiewicz and Banachowicz, 2017)

Pugnale and Sassone (2017) applied GA to optimize the shape regulated by a NURBS with 10 x 10 control points (Fig. 2.43). The fitness function is the inverse of the maximum vertical displacement. Moreover, other design data regard the thickness of 15 cm and the application of only dead load. At the first iterations, the edges and the centre of the roof present large displacements reduced by the algorithms, showing the weakness of the free edges, which have been stiffened during the simulation. GA have been performed within Grasshopper® and more precisely through the use of Galapagos. Computational morphogenesis represents a relatively new field but projects like those presented demonstrate the great potential to reach a higher standard for architecture.

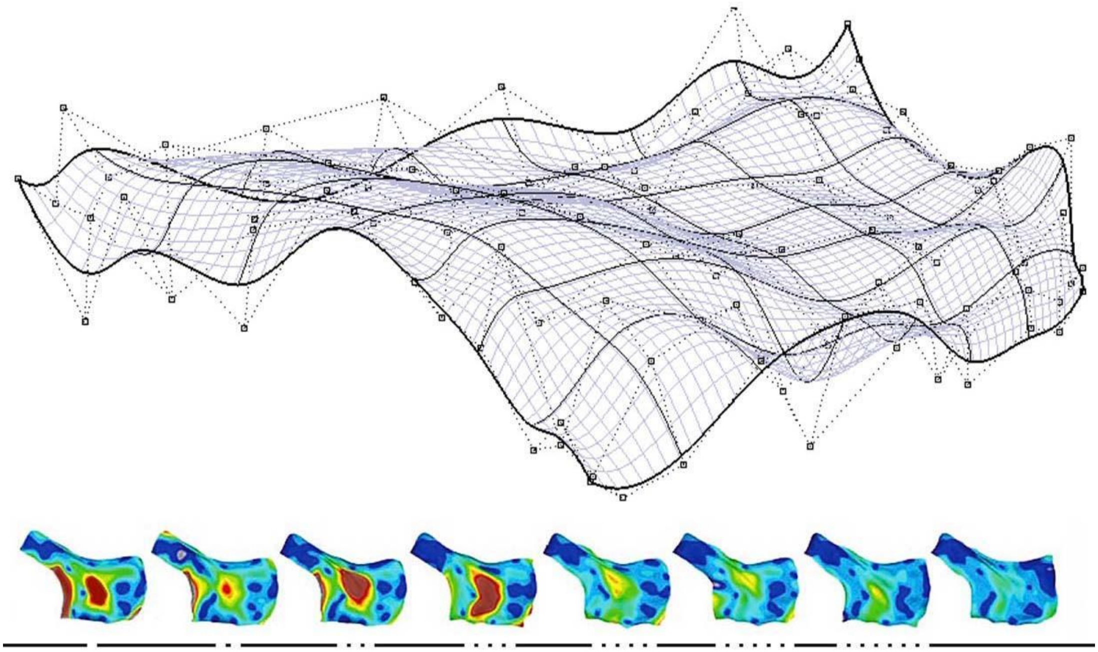


Figure 2.43 Computation of the GA applied on the roof of the crematorium (Pugnale and Sassone, 2017)

Galapagos is an evolutionary solver integrated in Grasshopper®. It is used for optimisation processes applied in architecture by defining *Genome* which are the design variables that operate in the optimisation and the *Fitness* function that needs to be satisfied. In architectural design the process is becoming more challenging through the use of multi-objectives optimisation where more than one fitness function needs to be satisfied, aiming to improve efficiency of the structure from different perspectives. In terms of digital tools, Octopus is an efficient plugin operating within Grasshopper that allows to carry out this kind of optimisation, relying on Pareto principle.

2.3.5 Summary

The concept of optimisation has a wide contextualization: its foundations lie in the biology aspects but its evolution has led to a significant application in a broad range of fields until to become a rising practice in architecture. Although its principles differ from the approach taken generally from architects and engineers, the use of solver within parametric environment is contributing to a more widespread use in the design process. However, this approach requires a thorough knowledge since outcomes derived by evolutionary simulations are not always reliable. Indeed, the reach of good solutions is not always guaranteed, this depends on the definition of objectives and a correct domain set as well. Moreover, for complicated problems it may be difficult to control the process in order to get good results. Another correlated aspect is the restricted variety in terms of formal solutions: by using topology optimisation as a design tool there would be the risk to lead to a well-known vocabulary of forms that result from the iterations of the simulation.

Nevertheless, its integration within design workflow features notable potentials as proved by significant projects and its investigation and use should start to be more and more required especially in the design of free-form structures.

2.4 Digital fabrication

2.4.1 Introduction

The development of computational tools in architectural practice has been pivotal as demonstrated in the previous sections. The evolution of formal aspects has gone hand in hand with the use of digital tools that have allowed more freedom in the design process. This artistic expression has found its implementation due to the relationship established between the design process and production. It is unimaginable to consider parametric modelling without combining its potentialities with digital fabrication. With this expression, we refer to a new era where architectural elements are completely produced through CNC (Computer numerical control machines). Therefore, the data necessary for the operation of the machines, their movements and control, are directly transferred from the digital process to the outputs. In this way, the traditional way to transmit information by scaled drawings is overcome and the building process becomes more efficient. The traditional way consists of scaled drawings and construction drawings elaborated by engineers and architects and successively provided to suppliers, responsible for the realization and assembly. A process composed of different parts involving a variety of professional figures that need to interact with each other. Digital fabrication has found many applications in engineering but its potential has only been explored in architecture in the last decade. The use of digital fabrication techniques are contributing to a new workflow for the design where there is no longer distinction between design and production but it can be considered as a continuum, like a flow of information that starts from the design concept and arrives at its materialization. It is important to highlight the latest innovations to gain understanding of the advantages digital fabrication can give in the design process. In architecture, the investigation has concerned several techniques to realize small and medium objects. However, new projects are exploring the potential of digital fabrication for full-scale projects as described in the next sections.

2.4.2 Digital fabrication techniques

According to the materials used and the typologies of the final product, there are different fabrication techniques such as 2D cutting, subtractive fabrication, additive fabrication and formative fabrication. Each technique has advantages and disadvantages regarding the geometrical requirements. At this stage, the fabrication techniques having a specific impact in architecture will be discussed.

- **Cutting techniques**

These include a variety of different approaches according to the material used. The typology of cutting is what differentiates the technologies: laser cutting, water-jet cutting and plasma cutting. Each of them has a different power, speed and temperature that makes it more appropriate for a given material. Laser cutting is undoubtedly the most common technique that allows to cut relatively thin materials. Although it presents limitations for the dimensions of the sheet materials, its great accuracy enables the reproduction of patterns and small details. Its use is mainly targeted for small and medium objects that can be assembled or used for model making (Dunn, 2012). Water-jetting as its name suggests uses a high-pressure jet of water mixed with an abrasive that cuts a wide range of materials. Plasma cutting involves the use of electric conductive materials by using the jet of plasma at a high temperature (up to 22.000° C).

- **Subtraction operations**

These consist of removing material from a volume deriving the required object. It is a well-established technique in architecture giving a good degree of freedom in terms of shape complexity. Such techniques include CNC milling and routing, which are driven by computer codes called G codes. They act as “instructions” for the machines, adjusting their movements and controlling the amount of waste material as well. Milling and routing are similar but the main difference lies in the material they generally cut: milling is more appropriate for metals, while routing refers to wood and plastic. Subtractive machines may have a different number of axes, depending on the complexity of the shape to obtain. Milling machines can have three or five axes, the latter allows rotations (one perpendicular to the other) to reach internal parts of the object, impossible to reach with three axis machines. Different tools can be used for milling processes such as hot wire cutters which can cut polystyrene foam or robotic arms whose utility may concern various operations (Bonwetsch et al., 2006). At this stage, it is important to address the potential of these techniques in the production of elements for tessellating architecture. The features of tessellating have already been described but in this section the techniques in digital fabrication to produce it are highlighted. Every cutting and milling machine presents advantages and geometrical limitation, depending on the depth of the cut or on the type of geometry they can process, therefore dealing with these characteristics becomes part of the design process, i.e. circular saw blades create planar faces or four-six axes diamond wire cutters can process ruled surfaces (Rippmann and Block, 2011). Milling techniques results are more appropriate for cutting stone providing more freedom in the geometry generation but on the other side they are time consuming and require a significant amount of energy. The choice of the machine relies on the material as well. In the case of stone material, the process results are quite delicate because it has to consider many factors concerning cut, dimensions of the block, typologies of geometry and material. In many cases, the combination of different techniques is necessary involving cutting and milling operations in the same process.

- **Additive manufacturing**

Also known as rapid prototyping, this is based on adding a liquid or powder material layer by layer, which solidifies and creates the 3D object. The first machines were released a few decades ago, bringing a new technology for the creation of prototypes. Its application has been successful in fields such as engineering, medical tooling, aeronautics among others but its potentialities have been explored in architecture. There are various deposition methods that are working on different bases. However, these processes are similar in the thermal, chemical and mechanical ways they fabricate parts. Stereolithography (SLA), Liquid Polymerization (LP), Fused Deposition Modelling (FDM), Ballistic Particle Manufacturing (BPM), Selective Laser Melting (SLM), Laser-Engineered Net-Shaping (LENS) and Binder Jet Printing (BJP) are the most common techniques. Compared to the other techniques, additive manufacturing produces less waste material with a lower final cost together with the direct link between CAD software and the machine. The projects can be easily exported through STL (Standard Tessellation Language) format by operating a triangulation of the model.

Fused deposition modelling (FDM) consists of the deposition of a filament coming out from a nozzle at a temperature similar to the melting point of the material. After the deposition a milling head adjusts the surface finish and the thickness. The filament is a polymer such as acrylonitrile butadiene styrene (ABS), medical grade ABS and E20 well known for their resistance. This process does not need post processing machines (Dehghan et al., 2018).

Stereo lithography (SLA) relies on the use of liquid resin material, involving the use of light such as laser or UV. The layers are solidified after being hit by light therefore reflectivity of material is an important property in this process. This technique allows to create accurate objects but at the same time it is expensive (Tedeschi, 2014).

When dealing with metals, Binder Jet Printing (BJP) represents the most suitable technique although some research has demonstrated its applicability for polymeric and ceramic materials (Greil, 2000). A powder based material and an adhesive material are used. A computer controlled machine deposits materials by alternating layers of the build material and the binding material. The layers need to be cured after their deposition in order to complete the process adding time to the post processing.

2.4.3 Digital fabrication in architecture

In the previous paragraph, the main techniques have been described in order to understand strengths and weaknesses of their processes. The application of digital fabrication in architecture is becoming more and more predominant and this exploration is providing remarkable outcomes in terms of production improvements. The use of digital fabrication regards various approaches according to the material used, the scale of the project and the formal characteristics.

Cutting and milling machines are suitable for tessellation. One of the most emblematic projects is surely Armadillo Vault (Rippmann et al., 2012) where after computing discretization the voussoirs were produced by using 5-axis router OMAG Blade5 (Generation 3) using a circular saw blade (ø81 cm) and customised profiling tools to cut limestone (Fig. 2.44). Before getting to the finishing parts the blocks were extracted from the quarry and worked with a 2-axis cutting machine.

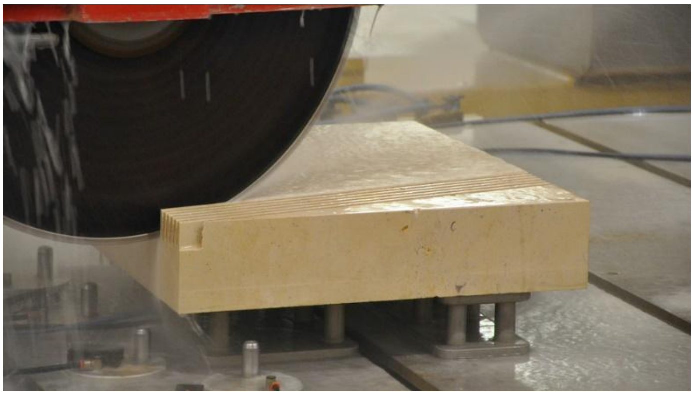


Figure 2.44 Cutting process for the intrados of a block in Armadillo Vault (David Escobedo, available from dezeen.com)

The Landesgartenschau Exhibition Hall is an example of the great efficiency of robotic fabrication, used to produce an efficient prototype composed of lightweight timber. The 243 geometrically differentiated beech plywood plates for the primary structure were generated, as well as the digital prefabrication of the insulation, waterproofing and cladding (Menges et al., 2015). Moreover, finger joints for a total of 7600 were designed and produced in order to provide stability with this interlocking system. By using a Hundegger SPM machine the plywood material is cut into plates and milled with the generation of finger joints too (Fig. 2.45). The whole process has been the object of improvements to embed cutting and milling operations.

Figure 2.45 Finger joints realization though robotic arms for the Landesgartenschau Exhibition Hall (available from archdaily.com)

Additive manufacturing can present some limitations in terms of durability of the material, depending mainly on the materials properties, but many improvements in rapid prototyping technologies as well as properties material have been accomplished thanks to a considerable synergic research activity from universities and architecture firms. This has led to explore potentialities of Additive Manufacturing in architecture either in small and full scale. The challenge mainly regards the use of additive manufacturing to create full-scale architecture. It is worth mentioning the outcomes achieved in this field: Contour Crafting, D-Shape 2006, and Freeform Construction (Buswell and De Kestelier, 2009).

Contour Crafting has been explored with materials like concrete, ceramic, polymer (Bos et al., 2016) consisting of the deposition of layers of concrete layers one on the other developed by Khoshnevis in the mid 1990s (Khoshnevis, 1998). It is used to build large-scale objects (Fig. 2.46). The extrusion is obtained by containing the filaments vertically and horizontally to the trowel and by adjusting the orientation of the side trowel. From its elaboration, many developments have occurred regarding the exploration of several materials.

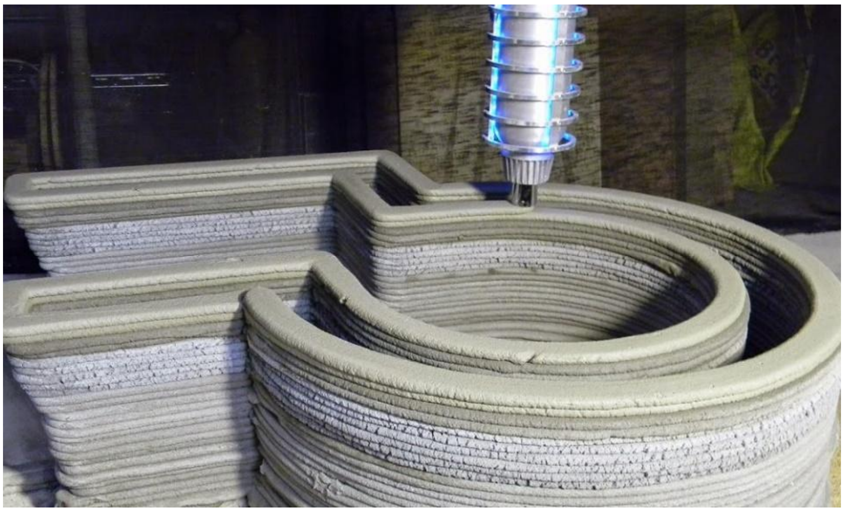


Figure 2.46 Contour crafting technique by adding filaments of concrete (available from sculpteo.com)

Loughborough University together with the UK Engineering and Physical Sciences Research Council and in collaboration with partners such as Foster+Partners have developed a similar technique to FDM that processes large components but made of concrete instead of plastic (2m x 2m x 2m). It was implemented with the realization of a wall component whose geometry is quite flexible for cavities within the wall and without the use of formwork (Xavier, 2011). Another important development comes from Enrico Dini (Dini et al., 2006) with his technology D shape (2016). It is a 3D printer that uses binder jetting and it can produce large-scale objects. Radiolaria pavilion (Fig. 2.47) was built by using this technology, in collaboration with Shiro Studio. It is a 3m x

3m monolithic structure in stone-like material without any kind of reinforcement, working in compression. The great strength of the material combined with the advanced technology of this large 3D printer provided important outcomes in the digital fabrication applied in architecture.

Figure 2.47 Radiolaria pavilion realized with D-shape technology (available from shiro-studio.com)

2.4.4 Prototyping as tool for structural evaluation

Scaled models represent one of the main evaluation tools for architects. Different reasons are connected to their importance: the possibility to have a small model of a project surely provides a more realistic perception of the design features, creating a more direct approach and a better understanding of its strengths and weaknesses. Besides the aesthetic feature, models are also known for extracting important insights regarding the structural behaviour of structures. Indeed, by testing scale models it is possible to compare results from physical tests and simulation run on software for structural analysis in order to get values that are more realistic and consequently apply correct improvements for the structure stability. In case of unreinforced masonry structure, this principle is further demonstrated by the stability that depends on the geometry than the material properties as Heyman (1996) highlighted in his book "The Stone Skeleton". This important remark provides a consistent reliability in case of physical models whose scalability does not affect the feasibility of the results obtained. Therefore, nowadays the use of physical prototypes yet represents an important tool to predict collapse of masonry shell structures.

The first examples of structural models refer to Danyzy's experiment where plaster models were used to analyse the collapse mechanisms of masonry arches and buttresses (Calvo Barentin et al., 2018). Additive manufacturing plays a fundamental role for structural models, whose techniques contribute to promote this practice in contemporary scenarios. The great advantages lie in the interoperability between the computational model and the physical model. For instance, the Block Research Group have carried out several studies regarding the use of models for testing by using Additive Manufacturing. The main cases regard the study of the collapse of masonry vaults under a set of supports displacements in three different directions, vertical diagonal and transversal by collecting data through an optical measuring system and comparing them with the results obtained from Discrete Element Modelling (Van Mele et al., 2012). This approach is also present in Calvo Barentin et al. (2012) where the behaviour of a cross vault is investigated using a force sensitive robotic arm and optical measuring system (Fig. 2.48). By using the same setup, a point load is applied in 6 different locations in order to measure the behaviour of the vault until the collapse as well as the movement of one of the supports following the direction of the horizontal thrust.

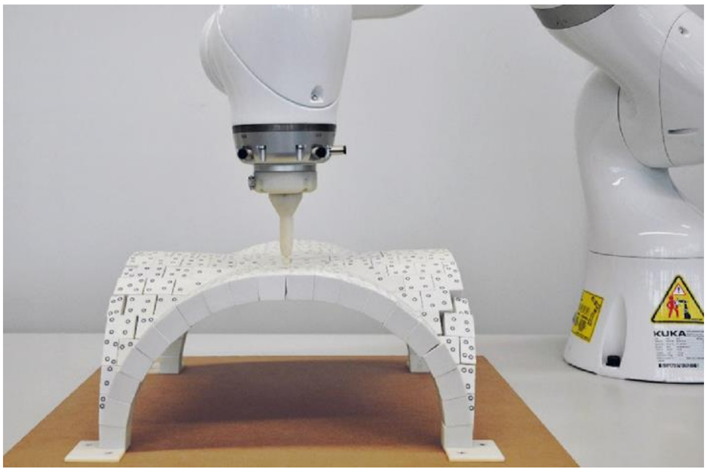


Figure 2.48 Robotic arms applying vertical load on a cross vault (Calvo Barentin et al., 2017)

A more elaborate example from an architectonic perspective is the structural model of the MLK Jr Park Vault that was realized in scale 1:33 (Rippmann and Block, 2013) as shown in Figure 2.49. The voussoirs were produced by 3D printing and successively assembled on a foam formwork. This model was used to test the shape generated by form-finding (Thrust Network Analysis) by applying concentrated loads on the shell and analysing the collapse mechanism caused by such loads. This model represented an efficient evaluation tool to refine curvature degree of the shell in order to contrast local collapses. Although some investigations include the use of robotic arms able to apply accurate vertical loads as aforementioned, tests can also be carried out by applying loads consisting of manual forces as well, resulting in a straightforward practice that at the same time able to provide very useful hints regarding the vulnerability of critical areas that need to be improved.

Figure 2.49 3D printed model of the MLK Jr Park Vault Project with a partial collapse caused by a point load (Rippmann and Block, 2013)

2.4.5 Summary

The great potential of digital fabrication for architecture is quite evident, the projects highlighted in this chapter prove the remarkable results obtained as well as the great developments that research of academics and professionals are carrying out, pushing the boundaries and extending the use of digital architecture for real-scale buildings. Although some limitations are still present due to high costs and the complexity of realizing machines able to manage a large amount of material, the latest developments are paving the way for a new materialization of architecture by digital fabrication. An integrated process where design and production are able to communicate at the same time is possible, thanks to use of parametric tools and a more synergic work between architects, engineers and manufacturers. Moreover, the digital fabrication for different scales can be used for a wide range of scopes such as building components, design objects and architecture models as described in the last section. This allows to produce in a quicker way models that are fundamental in architecture practice for the evaluation of several aspects of the project

2.5 Conclusions

The literature review has provided a general overview of the design process from historical, geometrical, technical and structural perspectives. Focusing in particular on shell structures considered as free-form geometries, it was fundamental to highlight the importance of a structural informed process especially in the early design phases. The awareness and knowledge of methods that exploit structural efficiency of curvilinear shapes is pivotal in order to define successful outcomes. In the current practice, it is still a predominantly design process that does not transmit enough information regarding the structural behaviour, that translates into post-operations of rationalization and optimisation. This kind of strategy, that tends to fix the errors done in the design, does not provide a real knowledge of the complex forms, especially valid for shell structures where the relationship between form and force is of the utmost importance. In fact, the masters of the past have taught how to design efficient structures and how to realize them without any need of reinforcement. Unfortunately, such concepts did not become common practice in contemporary architecture and the realization of complex shapes still represents a challenging achievement. In the research activity, architectural geometry and structural optimisation processes have brought many innovations in the design of free-form shapes, including planarization that has been investigated from a wide range of perspectives. However, these remarkable achievements have not found yet an extensive application in the architecture practice. Although the evolution of the design process has brought the advent of the parametric modelling and powerful digital tools are at disposal of architects and engineers, providing infinite possibilities to contemporary architecture, the practical implementation of the digital form-finding, rationalization, planarization are still little investigated. Moreover, structural optimisation, that should be considered at this stage as an integrated structural tool for architects, is still limited to very few problem definitions. For instance, Genetic Algorithms in particular are applied in a limited way to shell design.

Another crucial phase is the construction. Complex structures like shells require a challenging process, they are time-consuming and costly to realize, the formwork and the assembly process are laborious aspects of the realization. In the last decades, digital fabrication has delivered promising projects, proving to be an efficient strategy. Therefore, to define a connection between the design process and digital fabrication by using simplified approaches might be an interesting path to take in order to minimize costly operations such as the use of formwork.

Informed design processes that deal with geometrical and structural requirements in the early design phases and make use of efficient tools to optimize shell structures can provide a strong contribution for different reasons. Firstly, the definition of design workflows by using software largely used among professional can deliver useful design guides; secondly, the use of structural and in the same time intuitive tools integrated in the design can increase the awareness and the knowledge of such efficient forms; thirdly, the optimisation processes can overcome many limitations in the fabrication phase.

3. Methodology

3.1 Research overview

The academic world is providing cutting-edge technologies that strongly influences the architectural design process. In particular, optimization is becoming a key-process covering all the aspects of the design, as described in the literature review. However, these innovative techniques translate into complex and highly specific workflows that are not always applicable in the practice and that require a consistent use of resources. Architecture practice, mostly relies on standard operating processes, based on experience and the adoption of building codes and regulations that ensure consistency in terms of quality.

Overall, architecture practice still preserves a traditional approach when it is related to the design and realization: the architect defines the overall shape of a building and its structural analysis and sizing are considered only afterwards. This conventional way can still be valid when standard shapes are realized. This has been done for centuries in a successful way. However, the matter becomes more intricate when curvilinear and freeform shapes are conceived, since they require a higher degree of integration of structural design. Structural informed processes can be deployed to allow the form to be defined according to both its aesthetic and its structural performance. The integration of structural informed process in the architectural practice is becoming increasingly crucial in order to obtain good outcomes with the minimum use of resources. In the current scenario, this is a viable step that can be taken thanks to digitalization of the design processes and the enormous availability of software and powerful tools that enable the integration of innovative structural aspects in the daily life of architecture.

3.2 Research approach

The approach taken for this research consists of articulating a system of design guides that allow architects to deal with complex geometries in an efficient way. The methodology entails the definition of multiple case studies that implement practical applications related to form-finding, geometrical rationalization, planarization and structural optimization. Structural and geometrical requirements are taken into account, with the scope to provide knowledge to architects and students related to topics that typically are limited to structural engineers. However, structural considerations are investigated in a preliminary way given the target of this work.

The literature review has critically evaluated past and contemporary projects and their related design techniques, with particular focus on the latest computational tools used in the development of contemporary projects.

The case studies define a diversified set of approaches with the aim to optimize symmetrical and asymmetrical shell structures from structural and geometrical perspectives. In detail, two main strategies have been developed for symmetrical shell, which are pre-rationalization and post-rationalization, in order to compare the results geometrically and structurally. Such approaches are evaluated by Finite Element Analysis. For the asymmetrical shells two different subdivisions have been investigated, i.e. hexagonal and Voronoi subdivisions, to evaluate the feasibility of the planarization on more complex geometries. Other operations of optimization have been implemented to improve the structural efficiency of the shell by using Genetic Algorithms.

The last part of the methodology has covered the fabrication of shell structures, in particular a set of prototypes has been realized, ranging from scaled models to full- scale models, including the development of connection systems, to validate the digital process outcomes, and to evaluate the structural performance.

3.3 Research contextualization

Such a research has been developed with the intent to provide guidance to architects and students that aim to apply parametric processes for the definition of curvilinear structures. In a general context, architects and engineers represent two separate worlds with distinct and defined tasks, that complement each other. In the contemporary scenario, the role of the architect is evolving and a greater awareness of the structural aspects is fundamental to carry out successful outcomes. This does not imply the overthrow of the existing paradigm but rather the importance of a structural aware background for architects that have to deal with complex geometries. As literature review stated, form-finding approaches have demonstrated the potentiality to deliver efficient structures and nowadays geometric rationalization and optimization strategies are contributing to a new way to design. At this point, in order to fully exploit these innovations, design knowledge need to be put in place and be at disposal of architects. The presented methods rely on a combination of quantitative and qualitative data, with special attention to the qualitative aspects, which were considered more appropriate for this research. Moreover, given the target of this work, architectural design has taken priority over the structural design.

3.4 Research limitations

As aforementioned, this work has given limited focus to engineering aspects due to the nature of the methodologies applied and to the identified audience.

Therefore, the choice of a preliminary structural analysis that makes use of a macromodel approach has been considered appropriate for the typology of research conducted, providing useful insights and promoting its use within architects. The values of loading conditions used throughout this work are based on practical experience of the design of building structures in the UK, and refer to typical loading used for the design of single storey buildings. Thus, the results obtained from the simplified structural analysis are a good indication of the behaviour of the analysed shell structures, and they will need to be complemented by full structural verification prior any full scale fabrication to guarantee the compliance with stability and safety requirements.

4. Design workflows

In the last few decades the design of freeform structures has experienced a radical change from different perspectives: surface rationalization, investigation of form-finding techniques, Genetic Algorithm applied in architecture, and digital fabrication have affected the design process in such a way that the traditional design approach results are obsolete and counterproductive. The main purpose of a digital workflow is to identify its key-phases as tasks to implement an elaborate and systematic approach. Design workflows for shell structures need further investigation, especially related to their implementation in the digital computation in order to create a bridge between theoretical principles and their practical application. The previous studies regarding the latest innovations in this field represent a fundamental reference in this work. However, the aim at this stage is to focus on the formulation of straightforward workflows from a digital perspective that take into account a wide range of conditions. In detail, their implementation occurs in a parametric environment by the use of tools developed in Grasshopper®.

4.1 Design parameters

4.1.1 Introduction

As the first step of the elaboration of design workflows, the investigation of the parameters affecting the design process is carried out, strictly related to the Particle Spring Systems (PSS) approach used at this stage.

The parameters, and therefore constraints, considered for such an approach are mainly geometric and their use can affect positively (or negatively) the outcomes. The framework (Fig. 4.1) discussed aims to highlight the flexibility and interoperability of the workflow.

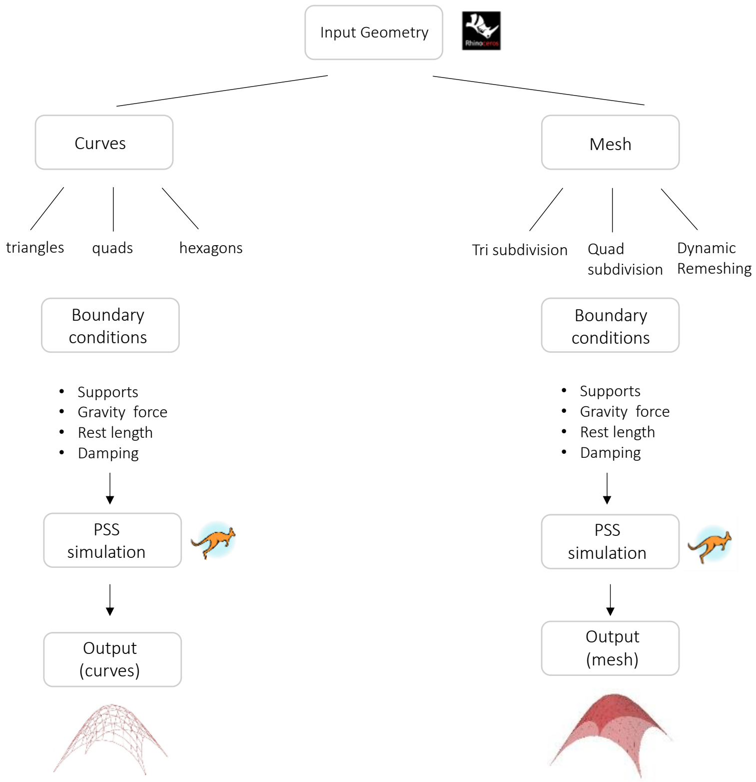


Figure 4.1 Workflow for the form-finding process

4.1.2 Form-finding

The form-finding represents the initial step of the workflow. This process is implemented in the Grasshopper® environment by the use of a Live Physics Engine in two different versions: Kangaroo and Kangaroo2. The basic approach underpinning these tools is Dynamic Relaxation that allows the computation of equilibrium for structures through dynamic iterations; in this work the approach used refers to a specific Dynamic Relaxation technique which is Particle Spring Systems. Indeed, the great advantage of these digital tools is that they can use different principles according to the goals achieved in the simulation. The PSS relies on the translation of the system in a series of components subject to forces, velocity and therefore displacement. Kangaroo and Kangaroo2 aim to compute equilibrium that translates into a final spatial configuration. In detail, the features of the approach used for this work are described as well as the influence of the different parameters involved in the simulation.

The theoretical definition has already been provided in Section 2.1.5, therefore at this stage a more pragmatic investigation regarding the digital computation is carried out to evaluate its application on practical models.

4.1.3 Discretization

The discretization process consisting of subdividing the input geometry can be implemented by means of different approaches (Fig. 4.2):

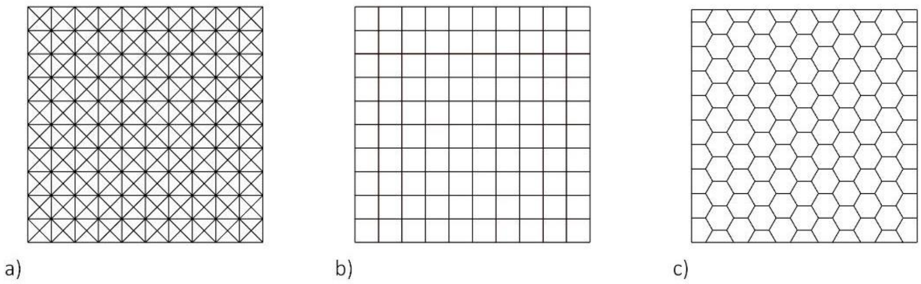


Figure 4.2 Discretization examples: a) discretization in triangles, b) discretization in squares and c) discretization in hexagons

Discretization from points and lines enables starting with an arbitrary polygonal subdivision according to the designer's decisions. In such a way, a pre-rationalization can be carried out meeting requirements relative to the future realization. The system of lines and points is essentially the PSS but in this case, the topology is given from the user/designer in the topology. By the use of components that translate such lines in springs and the points connecting lines in particles, the PSS is created and the simulation can be run. The following example shows a discretization composed of triangles, quads and hexagons.

In this approach, the system is considered from the beginning as a set of lines and points (2D) arranged according to different topologies. There is no use of mesh at this stage. It is obvious that the more regular the grid used, the more uniform the spatial configuration will be. By taking into account four corner supports, the process of form finding is described by taking into account three different types of discretization as shown in Figure 4.3 with the corresponding outcomes:

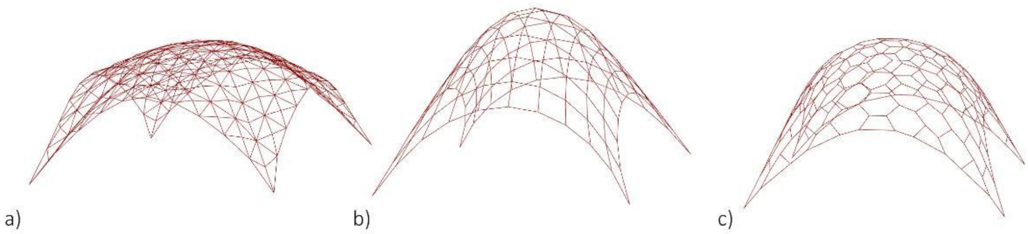


Figure 4.3 Discretized form-found shapes: a) 3D shape made of triangles; b) 3D shapes in quads and c) 3D shape with hexagons

Another approach concerns the use of a continuous *mesh*. In this case, since the rationalization intended as operation for the realization is considered successively, thus the discretization at this stage regards the translation of the mesh into a system suitable for the digital simulation finalised to the form-finding process. It is possible to start either with a surface or with a mesh, but the translation of the surface into a mesh is a mandatory phase. As described in the literature review there are different ways to generate a mesh but regarding form-finding process, it is useful to mention the operations that are more appropriate for the creation of a system subject to form-finding.

The discretization of the mesh implies a good degree of accuracy of its discretization. The following cases show a mesh generation carried out by triangulation and quad subdivision. Triangulation allows a further refinement of the mesh and in Grasshopper® different tools relying on algorithms implement this type of subdivision. It can be implemented taking into account objectives relative to edge length, density or vertices-valence. Mesh subdivisions can occur in multiple ways, but the more appropriate techniques for this scope are presented in the next paragraphs.

4.1.4 Boundary conditions

Boundary conditions represent fundamental geometric constraints provided as input for the simulation. They define the typology of geometry and they consist of:

- support conditions. They can be point supports or linear supports, and identify supported and unsupported edges;
- a potential pre-subdivision of the geometry;
- a set of variables actively involved in the form-finding simulation.

For form-finding simulation based on Particle Spring System (PSS), the main variables governing the solutions are: rest-length (*l*), stiffness (*k*) and damping (*d*). Different values of such parameters reproduce a variety of conditions.

Since the PSS approach refers to Hooke's law, each of these variables can be defined according to Hooke's law.

The rest-length is intended to be as the length of a spring after it is subjected to a force, and it is proportional to the displacement. According to the positivity or negativity of the rest length, the spring behaves as a compression or tension element. More in detail, if the rest length set is multiplied by a value below 1 this implies that the elements will behave as stretched components, while rest length equal or greater than the original length provides hanging elements (Adriaenssens et al., 2014).

The stiffness, as defined by the well-known Robert Hooke's formula, indicates the amount of the displacement with a force applied on a given spring. Stiffness value can be obtained by the following expression (12):

$$K = \frac{EA}{L} \tag{17}$$

where E is the Young's modulus, A is the cross-section area, and L is the length.

The damping force has an opposite direction of the spring force, meaning that it contributes to stabilize the simulation. It depends on two factors: the strength (damping coefficient) and the velocity (13):

$$f = d \times v \tag{18}$$

Where d is the damping coefficient and v is the velocity of the deformation.

4.1.5 Mesh subdivision

Mesh triangulation is one of the most common techniques for subdivision, as described in the literature review. In Grasshopper® there is a wide range of algorithm subdivisions to carry out mesh triangulation.

For the form-finding process, three different techniques have been used in order to understand the strengths and weaknesses of each subdivision. To this end, starting from the same design parameters (i.e. original surface and support conditions), the three techniques have been applied and the results have been compared.

Triangulation #1

Given an initial quad mesh, diagonals are added to obtain a triangulation. Generally speaking, this approach helps to improve the stability of a structure, while increasing its stiffness. The input geometry is generated thanks to the creation of triangular panels that are then discretised in PSS (Fig. 4.4a). This approach leads to a spatial configuration that presents a number of creases in several areas, due to the rigidity of such a structure (Fig. 4.4b).

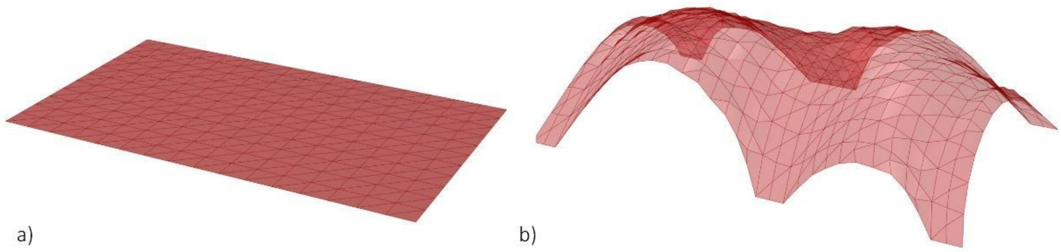


Figure 4.4 Triangulation 1 form finding process: a) input geometry; b) 3D shape after simulation

Even with a higher subdivision of the surface (by increasing *u* and *v* divisions) such irregularities cannot be completely overcome since the type of triangulation does not guarantee smooth geometries.

Triangulation #2: Dynamic Remeshing

Dynamic Remeshing is a refinement technique. It allows refining a mesh in order to get a more even distribution. By running a pre-simulation, the triangulation acts on a given mesh by setting edge lengths and boundary conditions. Dynamic Remeshing (Piker, 2012) is based on force based methods and relaxation and aims to optimize any mesh subdivision. It allows improvement of a given mesh through physics simulation. Different targets can be set in order to have regularity in the geometry or regularity in the connectivity. This depends also on the kind of geometry one is dealing with. In this case, equal edge length was provided and the resulting shape is rather smooth (Fig. 3.5a) and shows the great potentialities that Dynamic Remeshing can have when applied for form-finding purposes (Fig. 4.5b).

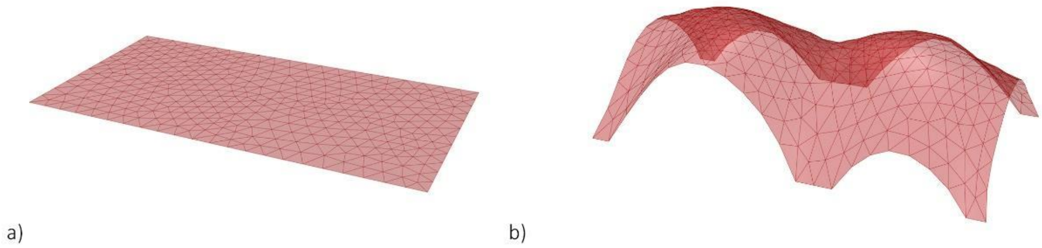


Figure 4.5 Triangulation 2 form-finding process: a) input geometry; b) 3D shape after simulation

Triangulation #3: Delaunay

Delaunay triangulation is based on the mathematical principle so that given a set of points *P*, the triangulation is operated in such a way that the circumcircle of each triangle does not contain any other point (Delaunay, 1934). Delaunay triangulation can easily be implemented in Grasshopper®. Such triangulation starts with a population of points within the input geometry, and such population is regulated by "seed" and "number" parameters defined by the user. The triangulation depends on the population of the points and a more uniform population delivers an improved triangulation. For instance, the triangulation can be optimized by reducing the numbers of the points close to the boundary edges and therefore not generating skinny triangles that can cause irregularities in the form-finding process (Fig. 4.6a). The form-finding process (Fig. 4.6b) delivers a 3D geometry with concave edges that potentially provides stability realized as edge beams in the construction process.

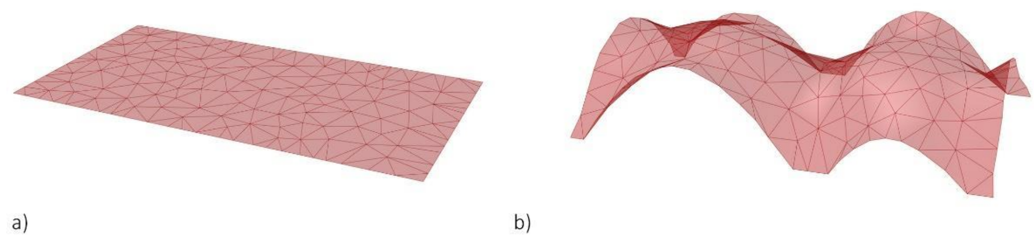


Figure 4.6 Triangulation 3 form-finding process: a) input geometry; b) 3D shape after simulation

Quad subdivision #1

For quad subdivisions different strategies can be implemented within Grasshopper®.

In case of quad subdivision connected to the PSS, the refinement depends on the number of *u* and *v* subdivisions for a given mesh. Tools like Weaverbird can be used for quad subdivision. Indeed, Weaverbird is a topological mesh editor that extracts vertices and lines from meshes, and uses them as input geometries. For regular geometries, the workflow is quite immediate because the operation of subdivision does not present any critical issue in terms of geometry regularity and it is able to approximate in an accurate way the input geometry. The next example shows the workflow by using the same design parameters of the previous cases.

This is a basic quad meshing technique where the surface is converted into a mesh and then subdivided in quads according to *u* and *v* subdivision (Fig. 4.7a). This approach works mainly with untrimmed surfaces. Indeed when applied on an untrimmed surface, a smooth surface is obtained in the form-finding process (Fig. 4.7b).

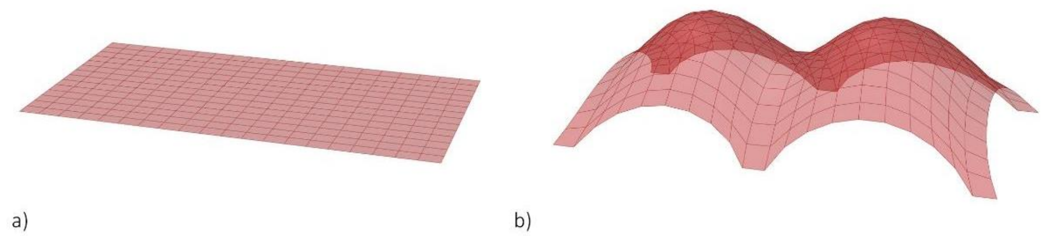


Figure 4.7 Quad subdivision 1 form-finding process: a) input geometry; b) 3D shape after simulation

For trimmed and irregular geometries, instead, other approaches would be more appropriate. However, in order to investigate this limitation, an irregular shape is taken into account to carry out quad subdivision and show potential solutions.

Quad subdivision #2

An irregular shape cannot rely on the previous strategy because the subdivision will not result equally distributed and it does not reproduce the shape in an accurate way (Fig. 4.8):

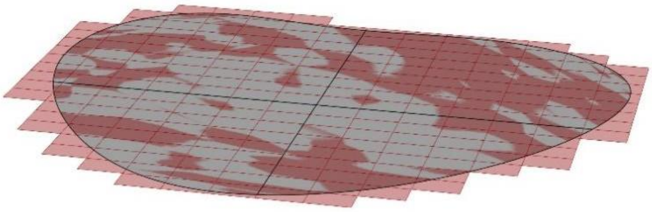


Figure 4.8 Attempt of subdivision for a trimmed surface

Different techniques can be used to overcome this limitation, depending on the kind of outcomes required. One of the common ways is represented by a manual process where a coarse mesh is generated by hand. Starting from the original input, an approximation of such geometry is carried out by the generation of a mesh with few quads as shown in Fig. 4.9a. Successively, the resulting mesh is subdivided in quads thanks to algorithms subdivisions (different algorithms can provide subdivisions, but at this stage a specific algorithm has resulted appropriately because it is based on adding a face for any edge of the original surface). Since the mesh does not approximate the input; a physics simulation is implemented through the use of forces applied on the mesh that allows to push the points of the mesh on the boundary curves and finally reproduce the initial geometry, accurately (Fig. 4.9b).

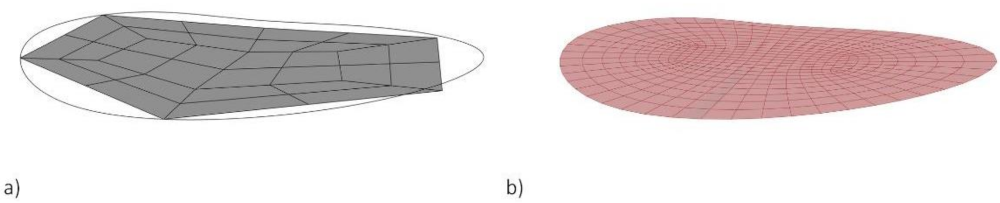


Figure 4.9 Subdivision process: a) definition of a coarse mesh according to the original input geometry; b) resulting quad mesh

With the latest versions of computational modelling, powerful tools are able to generate accurate quad mesh. One of these is the Quadmesh available on Rhino

7 where a quad mesh can be created according to different parameters such as edge length, curvature directions or potential singularities.

In case one wants to deal with different typologies of subdivision that can include pentagons, hexagons, Voronoi grid, a good way to proceed is by starting with triangulated mesh and then projecting the grid pattern on the resulting mesh.

The use of different techniques depends mainly on the user's choice. In some cases, the need to have a very refined geometry leads to a specific technique over another. Experience and trial and error process provide more knowledge on how to manage the input geometry for a Kangaroo simulation.

4.1.6 Reciprocal diagrams

In form-finding processes based on graphical methods, the visualization of the equilibrium state is a fundamental part. Reciprocal diagrams provide immediate feedback of the results of a simulation.

In such a way, the behaviour of funicular structures can be evaluated straightforwardly. It is well-established that graphic statics makes use of reciprocal diagrams that are the form and the force diagrams. This approach was translated in 3D structures by Philippe Block through Thrust Network Analysis (Block and Ochsendorf, 2007) where the use of reciprocal diagrams have been embedded in the digital computation (Fig. 4.10)

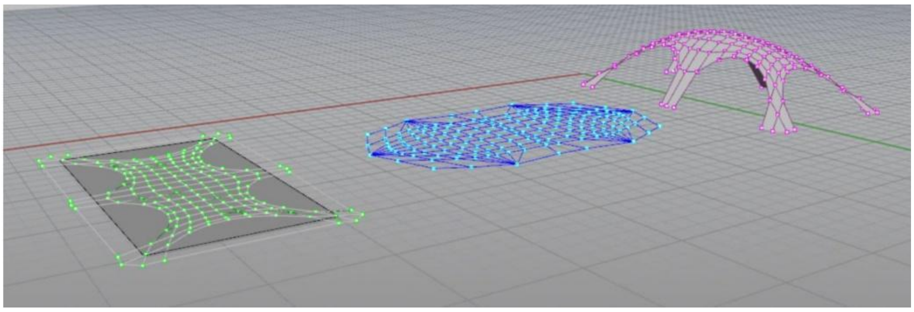


Figure 4.10 TNA form-finding: From the left, the form diagram, the force diagram and the 3-D thrust network

In a physics live simulation the form diagram is the final output of such simulation while the force diagram is not part of the design computation. However, it is possible to visualize the equilibrium state of the structure as a second step of the simulation (Piker, 2014). Kangaroo refers to dynamic relaxation and in this context case to Particle-Springs System. By setting parameters in a specific way, funicular shapes based on graphic statics principles are generated. With the generation of compression-only structures through Kangaroo, it is possible to get more insights regarding the equilibrium state of such structures. When the simulation reaches a solution, the structure has taken a configuration that in static equilibrium and the reciprocal diagram can be generated. It is important to indicate that in order to visualize reciprocal diagrams in equilibrium (it must form closed polygons) the structure has to be only in compression.

By taking into account the previous geometry (a rectangular plan used in Fig. 3.4-3.5-3.6-3.7, a funicular shape was found (Fig. 4.11a) and successively its reciprocal diagram (Fig. 4.11b) was computed:

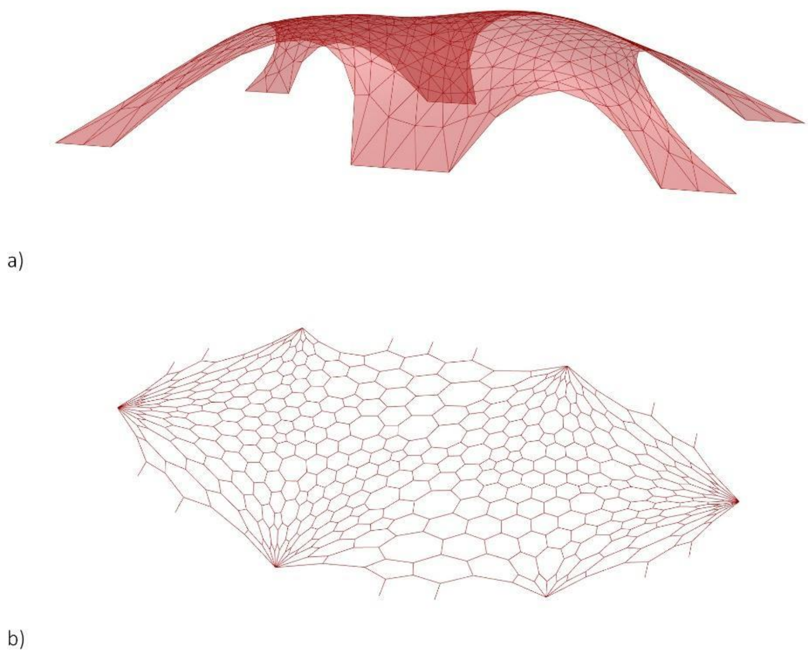


Figure 4.11 a) Funicular shape; b) Reciprocal diagram

When the polygons are not closed, then the structure is not working only in compression. In order to make the structure only in compression, rest length needs to be set to 0 and gravity loads on the vertices are applied. The force diagram is obtained by following the principle that the edges are scaled and then rotated so they correspond to the force magnitude. "The length of edge *e** in the force diagram is, up to a scale factor, equal to the magnitude of axial force in edge *e* in the form diagram" (Rippmann et al., 2012 pag.221). The simulation makes use of attractor forces to form closed polygons. By running the simulation through attraction forces, the polygons are closed and the diagram is generated. However, it can represent a verification only at the end because the approach is unilateral, it is not possible to change the force diagram in order to make adjustments in the shape as, instead, it occurs in the TNA framework described by Rippmann (2016).

4.1.7 Planarization

Planarization process has a key role in the design process. It is an optimisation of the tessellation that processes more efficient geometries for the fabrication. Many works regarding planarization have been investigated in the literature review highlighting the importance of the geometry and the type of subdivision. Such works show a significant potential in the rationalisation of free-form structures, however planarization is the result of a combination of several aspects that does not always converge to a satisfactory solution.

According to the geometry, a certain amount of constraints needs to be used in order to achieve a good solution. Besides constraints applied in the process other factors also affect the simulation, such as curvature of the surface, the size of the subdivision, and the typology of supports, among others. The common aim is to control the geometry during simulation so that deformations do not occur and compromise the process. This was done on different levels of complexity in order to investigate the parameters involved in the simulation.

For instance, forces can be applied to the edges of the panels to planarize them. Fig. 4.12 shows an example where the geometry is divided into hexagonal panels and then the panels are subjected to edge forces to compute the planarization. In this case the surface was easily planarized because the initial geometry is regular (having a double symmetry) and the supports are continuous. The parameters involved in this planarization process were:

- Constraints on the edge lengths, which provide a bound for the edge length during the simulation and help to prevent deformations;
- Planarization force applied on the points of the hexagonal cells.

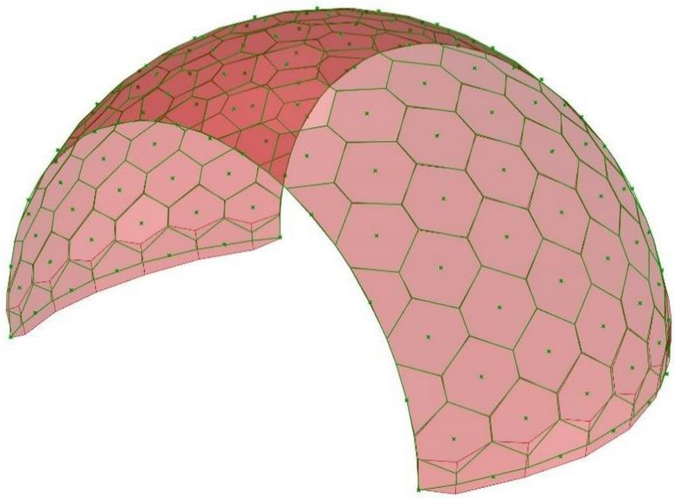


Figure 4.12 Original geometry (green polylines) and planarization of the hex cell (red panels)

When dealing with a regular geometry the result is straightforward but in case of more complex shapes, there is the need to add more constraints in the process. Providing more constraints it is possible to have control of the simulation although the outcomes may not always be optimal.

In the next case shown in Figure 4.13, the planarization process is applied on an asymmetrical shell structure with corner supports. Also in this case the hexagonal subdivision is carried out after the generation of the form-finding, and it is projected on the resulting mesh.

To achieve a fully planarized surface, it has been necessary to also include other constraints in order to avoid self-intersections. In detail:

- Constrain edge vertices of the hexagons on the boundary curve;
- Fix supports as anchor points;
- Constrain vertices of the hexagons on the mesh in order to approximate the original shape;

- Constraint the length of the lines, in order to be as equal as possible to current lengths.

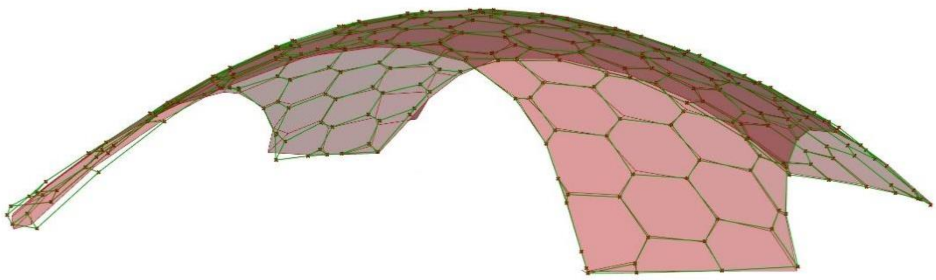


Figure 4.13 Original form-found shape (green polylines) and planarization of the hexagons (red panels)

Although this case shows a successful outcome, in other geometries, severe deformations and self-intersections can arise and other approaches may be necessary.

By considering Muller's work (2011), the planarization depends on the use of conformal hexagons: starting with a very simple definition important insights can be applied in the computation. A conformal hexagon relies on the principle of circular polygons, implying that it can be contained in circumcircle therefore in a plane. This principle can be translated into an additional constraint in Grasshopper, making polygons lie in circumcircle and in such a way controlling through them when planarization occurs. Together with planarization force, a force that constrains hexagons in circles is applied, minimizing deformations especially in areas that may be potentially critical.

Therefore, generally, the parameters to be set for the planarization of hexagons are:

- Planarization force;
- Circular force;

- Constraints on the length of the edges.

The difference between non-planar and planar cells can be minimum as shown in Figure 4.14 (green polylines are non-planar and the red surfaces are planar). Form-finding and planarization are implemented by using the same solver but this workflow will be described in detail in the next section.

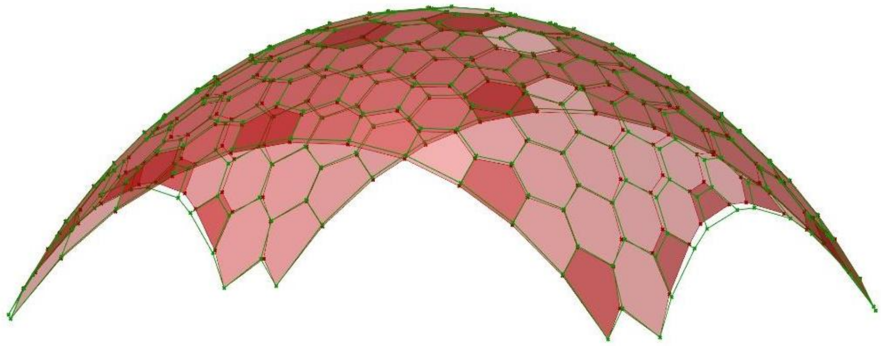


Figure 4.14 Original geometry (green polylines) and planarization process (red faces)

So far, the planarization process has been presented for hexagonal panels, but similar processes can be applied to other geometries, such as quad panels. When dealing with planar quads it is possible to take into account constraints that concern the use of diagonals on the polygons as well: by definition when a quad is planar the two diagonals have equal length, therefore such constraint can be used in the planarization by imposing equal length on the diagonals.

Figure 3.15 shows an application of planarization of an asymmetrical shell with quad panels. The starting surface is a circular plan, whose subdivision in quads is obtained by defining a coarse mesh, first manually, and then by using a subdivision algorithm as already previously described (Fig. 4.15a). After obtaining a good topology, the form-finding process (Fig. 4.15b) is carried out and the planarization (Fig. 4.15c) provides a successful outcome.

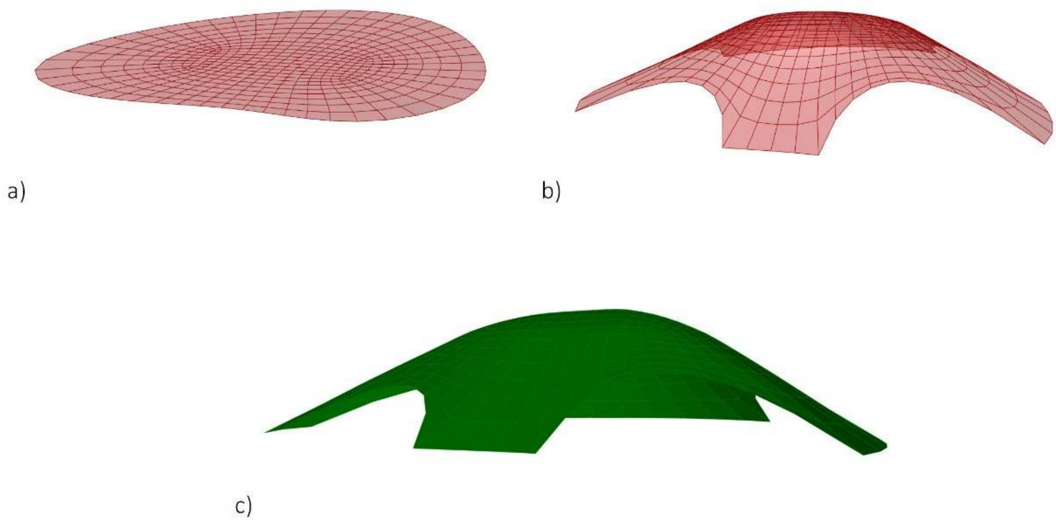


Figure 4.15 Form-finding and planarization for a quad shell: a) Quad mesh obtained from the previous workflow; b) form-finding and c) planarization of the quads

The parameters used for this simulation are the following:

- Planarization force;
- Diagonals equivalence that impose same length to the diagonals of each quad;
- Anchor boundaries to constraint edge vertices to boundary curves;
- Constraints on the edge lengths, to provide limitation to the edge dimensions during the simulation;
- Constraints applied on the mesh edges and plasticity to the vertices to prevent drift;
- Smooth function (Laplacian algorithm) to improve smoothness of the shell.

Although this last process requires a larger number of parameters to be set at the start, it provides good results for geometries composed of quad panels.

4.1.8 Summary

The aim of this section was to identify the parameters and their influence in the design process from a geometrical perspective. Such investigations have shown different approaches that can be taken when dealing with discretization, form-finding and planarization. In detail, the first step towards the definition of design workflow was the study of surface subdivision in polylines and mesh and the evaluation of their outcomes. Both frameworks result to be efficient in the generation of shell structures although some expedients may be necessary to improve quality of the results. The subdivision of the geometry, whether finalised to the fabrication or not, is a fundamental requirement for the form-finding process and it needs to be evaluated according to the geometry investigated. However, an incorrect subdivision can compromise the results; a trial and error process sometimes is necessary as well as experience of the user to deal with subdivision techniques especially in case of mesh subdivision. There are plenty of approaches that can be used but in the interest of this research, this investigation has addressed the ones appropriate for the planned objectives that take into account a rationalization of the structure for potential fabrication.

The same path was taken for the planarization phase. Planarization was analysed by considering techniques that cover different levels of complexity and different sets of parameters, depending on the typologies of geometries to planarize. Such operation is a result of a successful combination of forces and constraints. The outcomes have demonstrated that limitations can occur during the simulation and that can be overcome by a skilful use of constraints in the critical area of the structures (supports area might be critical as shown previously). However, planarization does not deliver successful outcomes in every circumstance, therefore in such a case a backward process that acts on the initial geometry may be beneficial. In light of these conclusions, the next chapter addresses specific planarization issues related to symmetrical shell

4.2 Design case #1 symmetrical shell and planarization issues: results

4.2.1 Introduction

The design process of shell structures may differ as described in the previous chapter according to the typology of shapes and goals to achieve. In the case of symmetrical shells, two main workflows have been developed, including planarization process and structural analysis.

Figure 4.16 presents these two workflows, highlighting the main features and description of each workflow is included in the following sub sections.

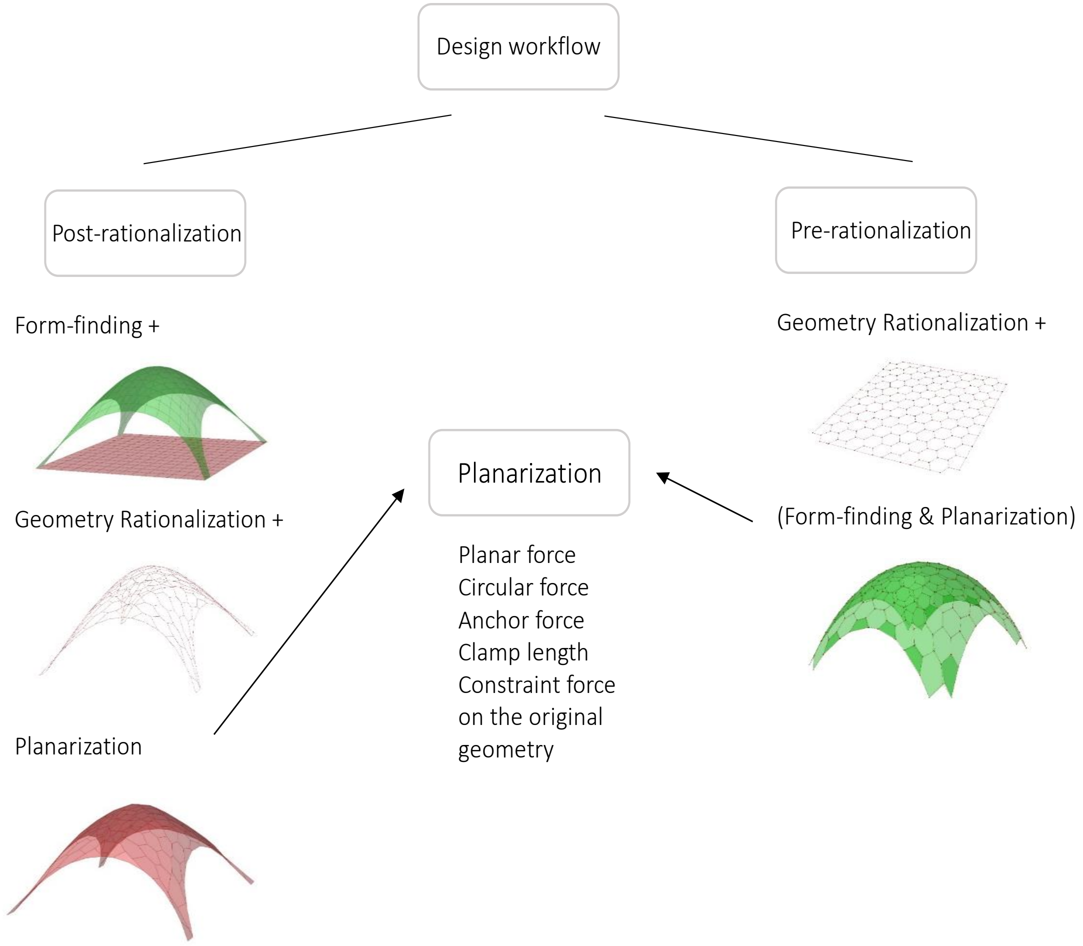


Figure 4.16 The two main approaches for the design process of a symmetrical shell

4.2.2 Form-finding, Rationalization and Planarization

The first approach is based on:

- **Form-finding + Rationalization + Planarization**

It represents an established flow for a design process: an input geometry is used for finding 3D spatial configuration, and successively operations concerning the rationalization as well as potential realization and fabrication are considered. It is a successful framework where the shape is optimized without any issues and the objectives are satisfied, but in some cases, a different solution could be necessary. In the following case, a symmetrical geometry is presented and the framework is applied in order to evaluate the outcome.

However, different methods for subdivision and planarization can rely on this workflow presenting clear differences in the implementation. The results for each of them are presented, starting from the same input geometry and the support conditions. The scope is to gain a thorough understanding of the different solutions proposed and the relationship between form-finding, rationalization and planarization.

The first case concerns the use of a quadrangular mesh, which is the input surface for the form-finding process (Fig. 4.17). By following the post-rationalization workflow, the hexagonal grid is applied after the form-finding (Fig. 4.18a) and in the end the planarization can be implemented (Fig. 4.18b)

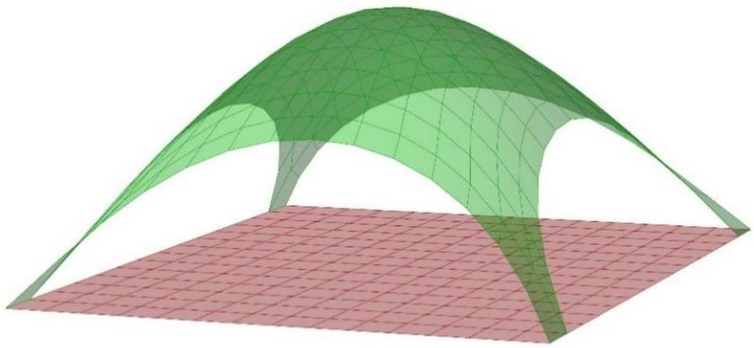
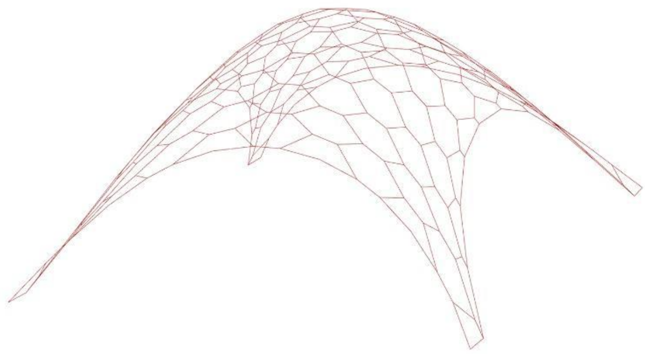
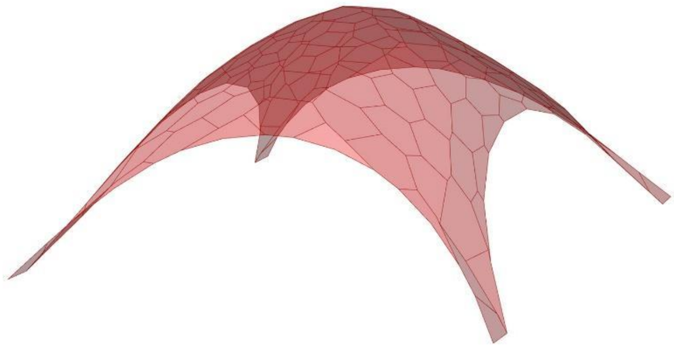


Figure 4.17 Quad mesh and its form-finding implementation



a)



b)

Figure 4.18 Post-rationalization approach: a) hexagonal subdivision applied on the shell and b) planarization

Some problems may arise when operations like planarization need to be done. In this case, planarization is affected by the degree of freedom of the surface and not always satisfactory results can be achieved, irregularity of the geometry and self-intersection as well as some creases can appear not providing smoothness to the shell. Critical areas for this process are undoubtedly the supports area where the polygons take irregular shapes.

The use of a triangulated mesh instead of a quad mesh represents another approach that can be taken. The process starts with the definition of a 2D hexagonal grid and, by exploiting the dual property between hexagons and triangles, a triangulated mesh is generated in Figure 4.19a. The mesh is the input for the form-finding process and the hexagonal grid is projected in the end of the simulation in order to implement tessellation of the shell (Fig. 4.19b).

The planarization of the hexagonal cells takes place by applying planarization forces, constraints on the edge lengths and on the mesh as well as defining anchor points. The planarized shape shows a good outcome in the central part of the shell (Fig. 4.19c), although the hexagons have changed in terms of dimensions and in the support areas some deformations have occurred.

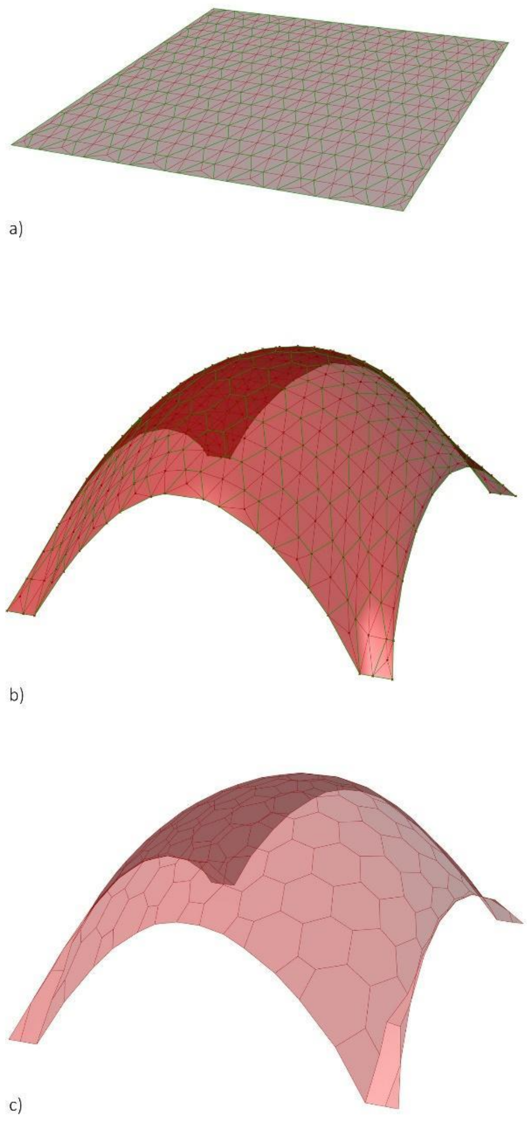
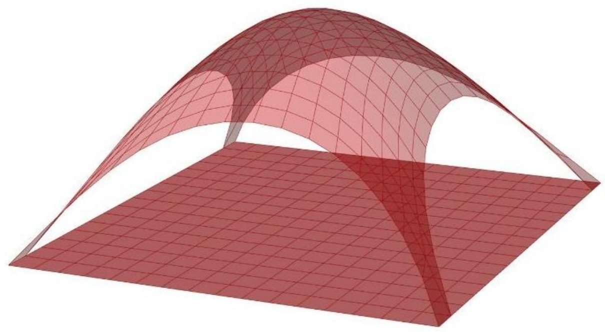


Figure 4.19 Post-rationalization approach: a) input triangular mesh obtained from the hexagonal grind (green), b) form-found shape and projection of the hexagons (green), c) planarization

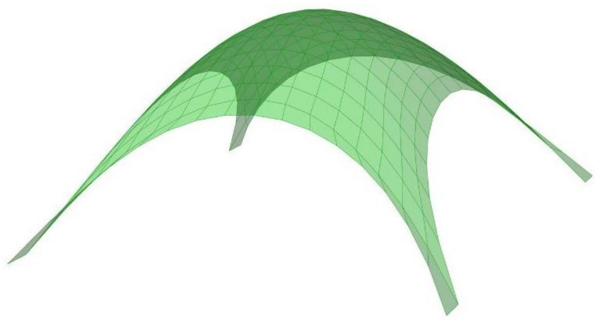
The last case that refers to the post-rationalization workflow regards the tessellation and planarization of quad panels instead of hexagonal panels.

The initial surface is a quad mesh (Fig. 4.20a) that undergoes form-finding. Successively, the planarization of quad panels is achieved, delivering satisfying outcomes since the difference between the form-found shape and the planarized shell is minimum (Fig. 4.20b).

Such a case refers to aligned quad panels that in case of compression structures does not represent the optimal solution for tessellation, since aligned panels does not guarantee a stable and interlocked configuration.



a)



b)

Figure 4.20 Post rationalization for quad panels: a) quad mesh and its form-finding implementation, b) planarization

4.2.3 Rationalization, form finding and planarization

The pre-rationalization workflow is presented and applied to hexagonal shell consisting of:

- **Rationalization + (Form-finding & Planarization)**

Rationalization is given as the first requisite of the whole process and the same rationalization is the starting point of the PSS. It is *a priori* step and it provides more control on the surface, with consequently less freedom in the form-finding simulation.

The degree of freedom in a process depends on the amount of constraints that operate in a simulation, more constraints are provided, the more limited will be the found final shape. However, in some cases a larger number of constraints can help to reduce deformations.

This approach aims to avoid deformations and self-intersections that can occur in some critical areas during the planarization process of hexagonal cells. It consists of providing more control in the form-finding process, delivering a more regular double-curved geometry (Fig. 4.21).

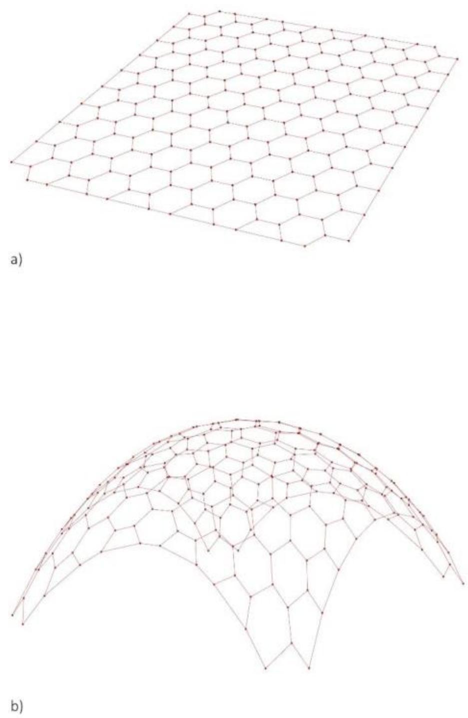
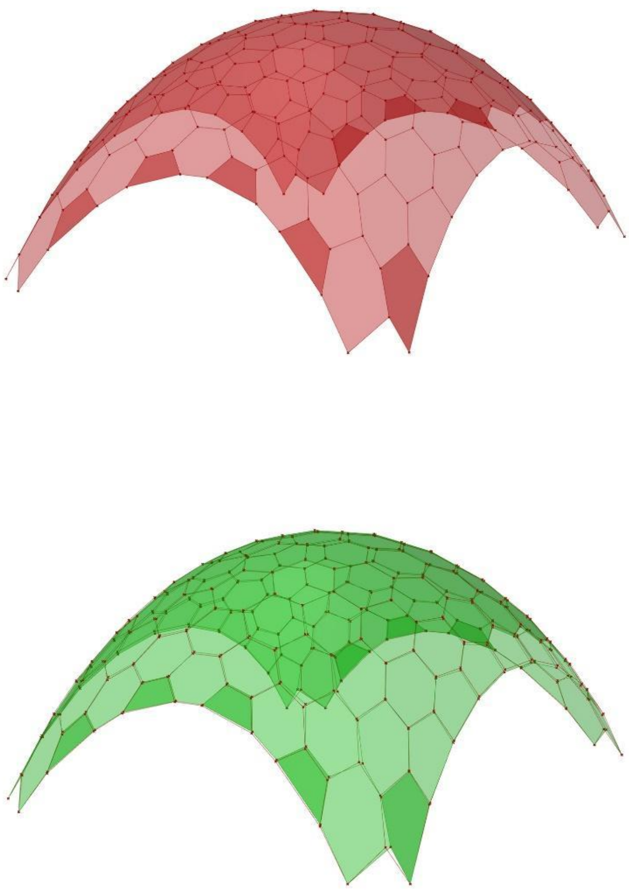


Figure 4.21 Pre-rationalization process: a) pre-rationalized input geometry and b) its form-finding result

A pre-rationalized geometry in hexagonal cells is given as input geometry. The hexagons are made of 2D polylines. The form-finding and planarization are embedded in one simulation with the use of one solver in Kangaroo2. The simulation is composed of: translation of lines into springs, a force is applied on the vertices to initiate the form-finding simulation, circular forces are applied to contain polygons in a circle, a planarization force is activated after the form-finding is complete. This process allows the attainment of two different goals (3D shape and planarization) in the same workflow.

The results are remarkable, the planarized hexagons have undergone very small deformations and the control of the geometry is considerable, as displayed in Figure 4.22.



a)

b)

Figure 4.22 Planarization process: a) form-found shape before planarization and b) after planarization

4.2.4 Structural analysis

A reliable benchmark to evaluate the outcomes obtained from the different workflows is to analyse them from a structural perspective. In such a way, it is possible to provide an objective investigation based on the same set of conditions. Three form-found shapes have been analysed according to the macro-model approach.

The structural analysis has been performed using the finite element method within the Grasshopper® platform (Preisinger 2013, Preisinger 2016). A macro-model approach has been used in accordance with López López et al. (2014). The homogenous membranes have been discretized using 2D "shell" elements, which do not take into account the tessellation of the structure. The macro-models were analysed by taking into account the following conditions:

- Four point supports;
- Material properties related to a material such as masonry or pre-cast concrete (possible materials used in the construction of the shell) which have a relatively high compressive strength and a much lower tensile strength; see Table 4.1. The properties, based on those for concrete class C30/37, are taken from Eurocode 2 (EN 1992-1-1 2005);
- Loading – see Table 4.2. This consists of dead loading acting with a horizontal wind load applied on the windward face of the structure with a vertical internal uplift wind load to create the maximum tensile stresses in the structure.

Material Parameters	**Value**

Density	1900 kg/m3
Elastic Modulus	3300 N/mm^2
Compressive Strength	30 N/mm^2
Tensile strength	2.9 N/mm^2
Shear Modulus (G)	1375 N/mm^2
Poison's ratio (V)	0.2

Table 4.1 Material properties

Load combination	**Load (kN/m^2)**
Dead load + wind load	5.7 + 0.8 (lateral wind) + 0.5 (up-lift wind)

Table 4.2 Loading condition

The study has investigated the structural performance of the shell structures for a thickness of 15 cm.

The first case is the form-found shell obtained from the post-rationalization approach (displayed in Fig. 4.17) that has been translated into a structural model. The stresses throughout the shell are displayed in Figure 4.23 that shows the generation of the tensile stresses, due mainly to the presence of wind load.

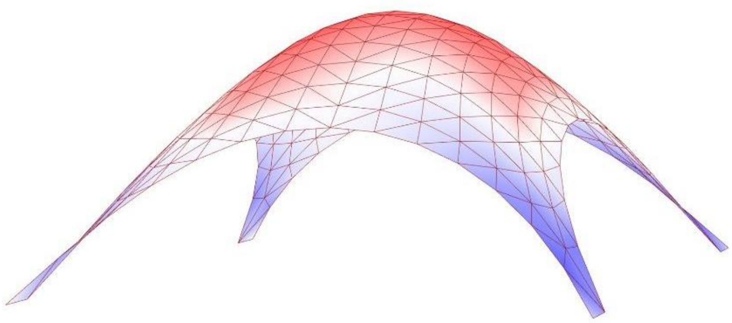


Figure 4.23 Structural analysis results in Karamba3D for the first case: stress visualization, tension (blue) and compression (red)

The second case (Fig. 4.24) refers to the form-found shape obtained in the post-rationalization approach (displayed in Fig. 4.19).

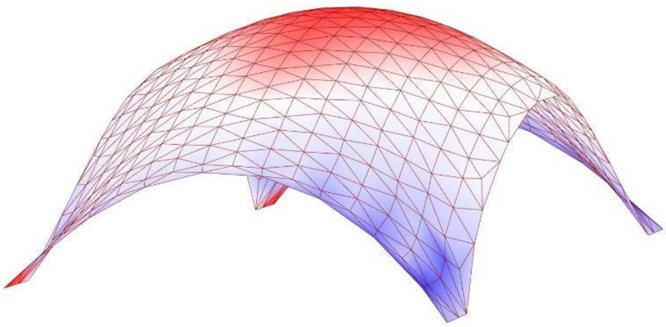


Figure 4.24 Structural analysis results in Karamba3D for the second case: stress visualization, tension (blue) and compression (red)

Tensile stresses arise around the support area affected by the lateral component of the wind load.

The last case regards the pre-rationalization approach (displayed in Fig. 4.22). In detail, since the FEA needs a mesh as input, a mesh was re-built from hexagons and successively joined and welded (Weaverbird plugin). Figure 4.25 shows the final structural model and the stresses generated on the structure.

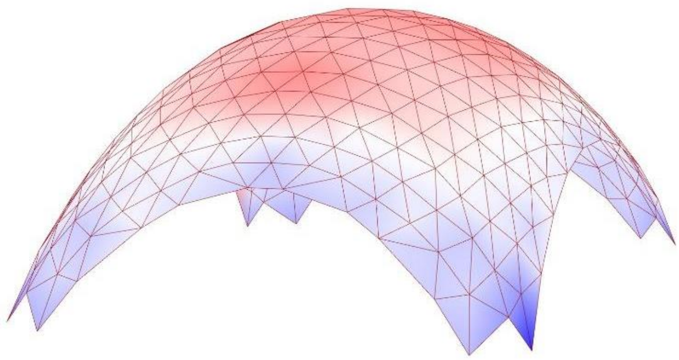


Figure 4.25 Structural analysis results in Karamba3D for the third case: stress visualization, tension (blue) and compression (red)

The results of the analysis are summarised and compared in Table 4.3 in order to provide an overall evaluation and highlight the relationship between geometrical shape and stresses. Maximum compressive stress, tension stress and displacement give a good feedback of shells' efficiency, showing its behaviour with a set of boundary conditions and a load combination applied.

	Case 1	**Case 2**	**Case 3**
Max comp. stress N/mm^2	0.13	0.12	0.40
Max tensile stress N/mm^2	0.14	0.24	0.37
Max Displacement (mm)	0.08	0.03	0.4

Table 4.3 Results extracted from Karamba3D

The results extracted form Karamba3D shows the relationship between constraints, form, and mechanical behaviour. The results have demonstrated that the use of more constraints during the form-finding process affects negatively the structural performance. This means that when very few constraints are applied, the form-finding simulation delivers funicular shapes that result more efficiently as compression structures. Indeed, the use of constraints during form-finding, like in the pre-rationalization workflow, does not generate funicular shapes, that translates in more stresses presented throughout the shell.

However, in case of high tensile stresses, specific measures might be taken in a final design. Either by locally reinforcing the highly stressed zones or by thickening the shell can help to reduce the tensile stress to an acceptably low level.

4.2.5 Summary

This chapter aimed to describe two main different frameworks for the design of symmetrical shell structures: the first one has followed a more conventional workflow while the second one is based on the use of more constraints in order to overcome potential problems arising in the first workflow. Indeed, by using constraint-based simulations it is possible to have significant control in the design process overcoming issues that can be problematic to solve in the realization phase. Rationalization has demonstrated its capacity to play an important role and its "location" within the workflow can provide variable results as shown by the previous examples. Starting with the same set of design parameters, different shapes have been generated depending mainly on the amount and types of constraints used in the design process. Therefore, the evaluation of such shapes from a structural perspective was considered a reliable way to carry out a comparison and gain understanding of the relations between form, constraints and mechanical behaviour. The results from Karamba3D have shown some dissimilarities according to the number of constraints used. Indeed, the case with more constraints (pre-rationalization approach) applied during form-finding have shown less efficiency to withstand load conditions since its form is not a pure result of a physical simulation. Although the pre-rationalization approach delivers shapes that are less efficient from a structural point of view with the appearance of higher stresses, the strength of such an approach lies mainly in the realization process. Indeed, from a geometrical aspect, the pre-rationalization workflow has been demonstrated to improve the effectiveness of the panelization and planarization operations to such an extent that there is no need to refine them at a later stage. Thus, the selection of the most appropriate approach in the case of symmetrical shells must be considered on a case-to-case basis, since each of which has strengths and weaknesses.

4.3 Design case #2 asymmetrical shells

4.3.1 Introduction

The design of symmetrical shells has been implemented by the definition of flexible workflows, depending on the typologies of parameters used in the simulation. However, in contemporary architecture the use of asymmetrical forms is becoming more and more predominant. If from one side architects and engineers want to deal with complex geometries, from another side the scope to simplify and optimize is of high interest. In this context, asymmetrical shells are obtained through form-finding. Therefore, this section investigates the adoption of geometrical subdivisions for irregular shapes.

Following a workflow similar to that already described, two different shells are developed starting with the same set of design parameters. Then hexagonal and Voronoi subdivisions are implemented to investigate the possibility to tessellate asymmetrical shells.

Hexagonal and Voronoi subdivisions are widely spread in nature, as shown in Figure 4.26. Taking inspiration from natural patterns can be very useful not only to achieve esthetical features but also for the definition of structural models.

a

b

Figure 4.26 Patterns present in nature: a) honeycomb pattern largely present in nature; b) Voronoi pattern of a leaf

The potentialities of these special discretizations have been investigated in a variety of projects from urban planning to façade design, until more recently their use has been explored in shell structures. Noteworthy examples are the Alibaba Headquarters facade in Hangzhou (Hassell Studio, 2009), the BUGA wood Pavilion (Menges, 2019) and the ICD/ITKE Research Pavilion (2013-14) (Menges, 2014).

4.3.2 Form finding

The definition of the geometrical inputs regarding the starting surface and support positions represent the requirements for defining the form-finding process.

For the form finding process of an asymmetrical shell, the input geometry can be a circular-like shape and points to define the position of the supports. The approach used in this work starts with the identification of four points that define four circles, which interacts with the original surface (Fig. 4.27). Through a region difference between the starting surface and the circles, the final input geometry is obtained and with it the remaining areas that represent the supports area (Fig. 4.28).

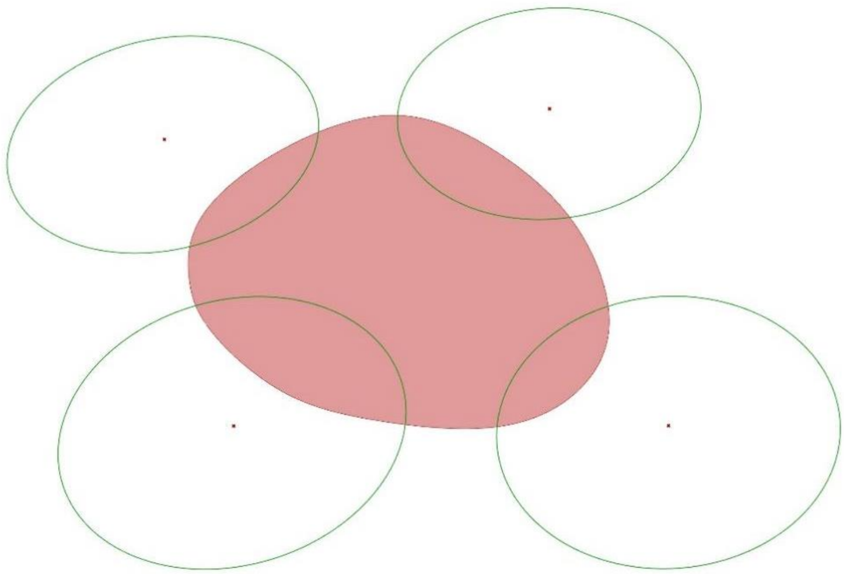


Figure 4.27 Initial input geometries to define surface and support positions

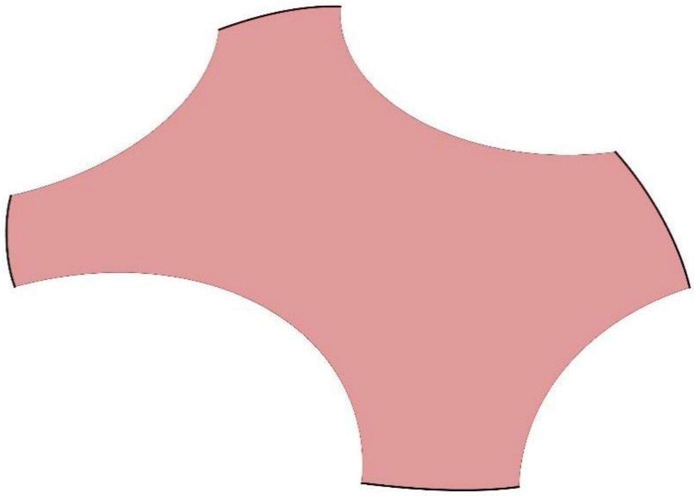


Figure 4.28 Surface and support positions highlighted in black

Such a system controls the position of the support area in a parametric way, by using the radius value of the circles.

The process of form-finding relies on the same workflow explained in the previous chapter, starting with a post-rationalization approach that makes use of projection operation to subdivide the geometry (see section 4.2.2).

4.3.3 Hexagonal shell

Initial hexagonal subdivision (Fig. 4.29) is applied on the surface, according to the logic of untrimmed surfaces. Therefore, the right approach consists of obtaining a dual mesh by starting from the geometrical discontinuities represented by the vertices of the hexagons. The resulting mesh is necessary for the form finding simulation (Fig. 4.30).

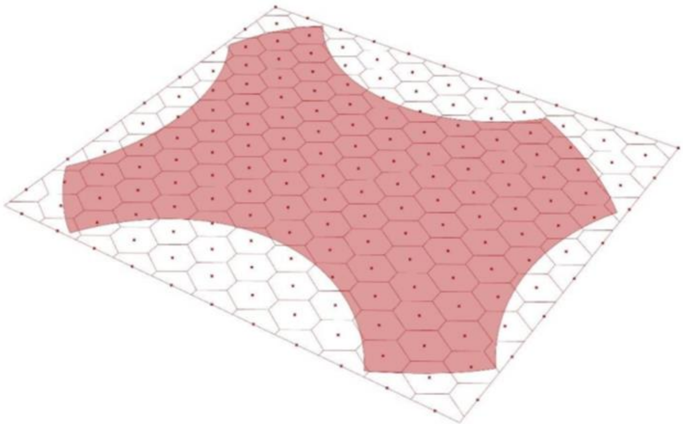


Figure 4.29 Hexagonal grid on the initial surface

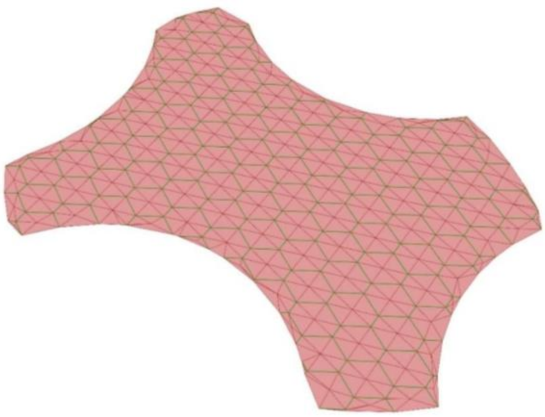
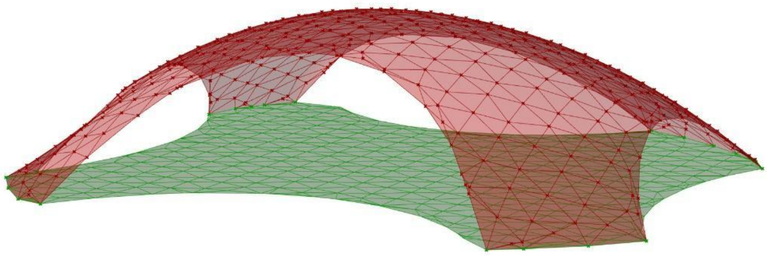


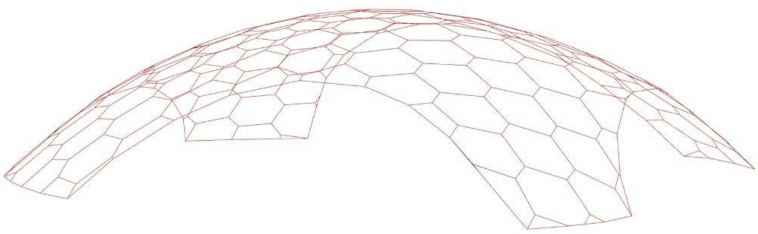
Figure 4.30 Mesh obtained from the hexagonal subdivision

Such a workflow represents a procedure to obtain a good mesh from a hexagonal subdivision, exploiting the dual properties between triangles and hexagons. This procedure allows the final operation of projection as shown successively.

The 3D spatial configuration is obtained (Fig. 4.31a). However, at this stage a continuous mesh represents the outcome of the simulation; therefore, the subdivision in hexagonal panels has been carried out (Fig. 4.31b).



a)



b)

Figure 4.31 Form-finding simulation: a) form-finding outcome; b) mapping of the hexagonal cells

The hexagonal subdivision is projected on the final mesh in reliance on the discontinuities from the hexagons, enabling process rationalization of an asymmetrical shell with an optimal approximation of the mesh.

Planarization is implemented taking into account a consistent number of constraints applied in the simulation. Due to the complexity of the shape, it needs to be carried out by considering an important parameter represented by the constraint force applied on the points on the mesh. This approach has already been described in the section regarding planarization (4.1.6) and here the outcome of this approach is validated (Fig. 4.32) showing the possibility to carry out planarization in a successful way in case of asymmetrical shells.

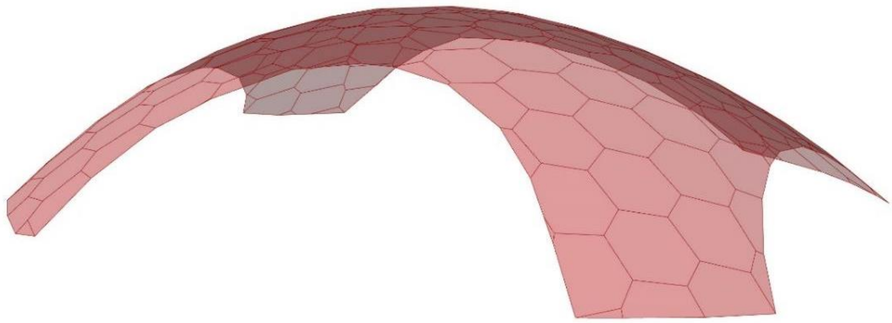


Figure 4.32 Planarization of the hexagonal panels

4.3.4 Voronoi shell

Voronoi definition in parametric modelling such as Grasshopper® results in a straightforward process by the use of different algorithms that allow to control such subdivision. In this instance, the same workflow is applied in order to obtain a 3D geometry with a Voronoi subdivision. The same design parameters have been used, therefore the principles behind the conversion in mesh and the successive mapping of the Voronoi diagram are alike.

Starting from the original surface, 2D Voronoi subdivision is obtained depending on parameters such as number and random seed (these are the parameters present in the algorithmic subdivision component) for the generation of population points (Fig. 4.33). The dual mesh of the Voronoi is defined taking into account geometrical discontinuities like in the hexagonal case (Fig. 4.34).

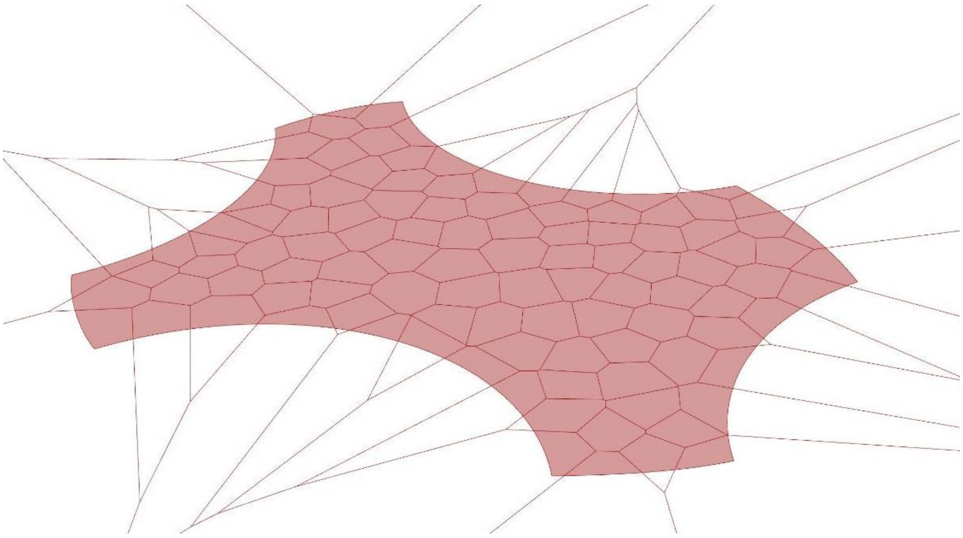


Figure 4.33 Initial Voronoi grid applied on the surface

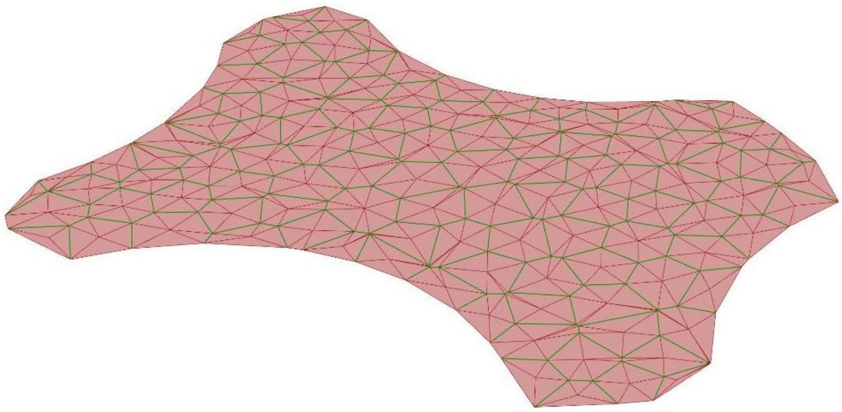
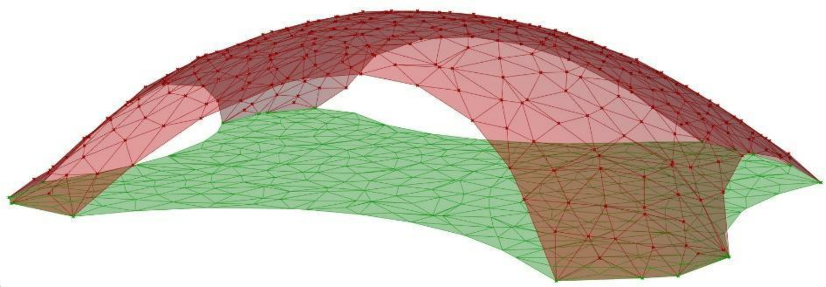
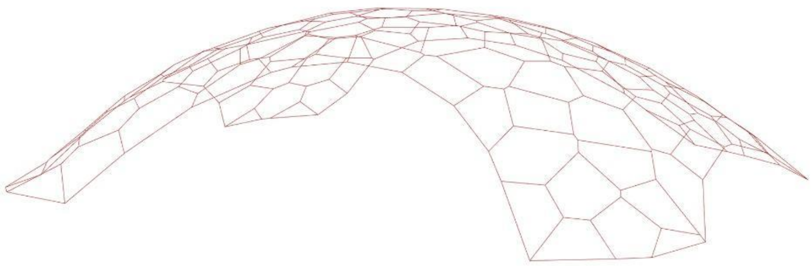


Figure 4.34 Mesh obtained from Voronoi diagram

The form-finding simulation provides a successful outcome, further validated by the final projection of the Voronoi points on the resulting mesh. Figure 4.35a and Figure 4.35b show the 3D shape.



a)



b)

Figure 4.35 Form-finding simulation: a) final mesh from form-finding process; b) projection of the Voronoi tessellation on the mesh

Also for this case, the planarization process is carried out by taking into account an adequate amount of constraints. Among them, in this simulation, constraints acting on the length of the edges and on the vertices of the Voronoi, as well as the mesh, play an important role.

The planarized cells are displayed in Figure 4.36 where planar surfaces have been generated throughout the shell.

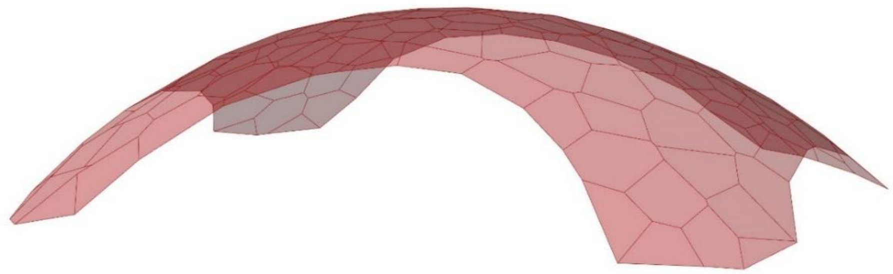


Figure 4.36 Planarization of the Voronoi cells

4.3.5 Summary

In conclusion, a design process based on post-rationalization approach has been presented, focusing mainly on the tessellation. Two different tessellations have been implemented, starting from the same set of design parameters with the aim to evaluate the outcomes obtained from the planarization process. The results have shown that when dealing with asymmetrical shells, such an approach has proved to be successful. In both cases, the process of conversion in mesh has provided good results, by exploiting the dual properties of hexagonal and Voronoi subdivision. A good quality mesh is fundamental for a correct implementation of the simulation. The projection of the initial subdivision on the 3D form found shape has demonstrated the effectiveness of such a technique and afterwards the planarization has delivered planarized shells with very few dissimilarities from the non-planarized shape. At this stage, a further optimisation of the asymmetrical shells might contribute to a thorough understanding of their behaviour, maximising their potentialities.

4.4 Structural optimisation for asymmetrical shells

4.4.1 Introduction

It is well established the potentialities of Genetic Algorithms (GA) in architecture practice (Pugnale et al. 2007) (Wortmann et al. 2017). The use of Genetic Algorithm is paving the way to a new approach in the design that is allowing architects and engineers to control the efficiency of the design process. GA have been introduced in the literature review, however at this stage, it is important to investigate their implementation in the parametric modelling by developing a straightforward workflow in order to affect minimally the overall shape. Galapagos is a solver used in Grasshopper® environment that carries out an optimisation process by GA. By defining a fitness function and design variables, the simulation aims to find an optimal solution. The obtained results are presented in the forms of *Genomes* that are generated by combinations of variables or *Genes*. The combinations depend on the fitness function that can be maximized, minimized or set to a specific value. The following images (Fig. 4.37) refer to a simple case with two genes in a fitness landscape where according to the variation of Genes good or bad combinations can be produced. The fitness function is the peak in this landscape and the aim of the solver is to find it by starting with a random population of solutions. The first population (population 0) is generated and it is possible to evaluate the closest solutions; starting from them a new population is generated that takes into account such solutions and new offsprings will explore the landscape. The iterations are repeated until the "highest peak" is found.

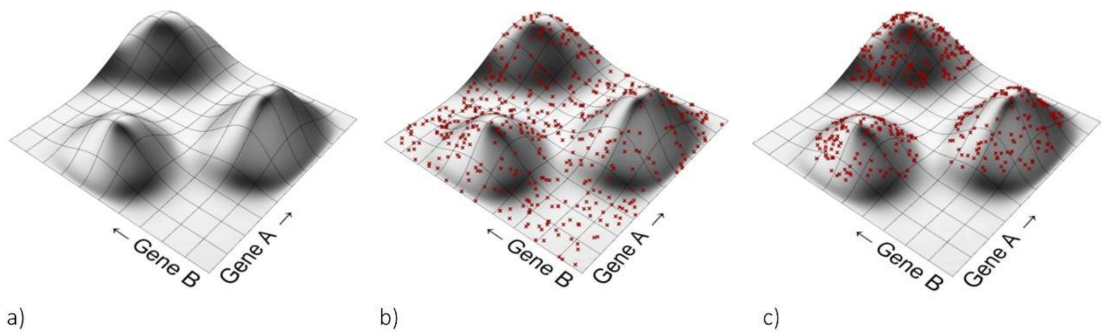


Figure 4.37 Simulation in a fitness landscape of three dimensions of two Genes: a) setting of the fitness function, b) first population, c) final solutions (Rutten, 2011)

The relation between the functions incorporated in Galapagos and their practical application is investigated in order to gain understanding of the implementation of Evolutionary Algorithms in architecture. For instance, specific functions are available in order to control the evolutionary iterations through populations, matings, and offsprings variations. Table 4.4 describes the principles behind the settings in Galapagos.

Evolutionary Solver settings in Galapagos	**Definition**
Runtime limit (Y/N)	Time restriction for simulation
Max. stagnant	Number of maximum fittest genomes with same fitness
Population (x)	Number of genomes after first population
Initial Boost (x)	Multiplication factor for the first simulation

Maintain (%)	Percentage of solutions to be maintained in each step
Inbreeding (%)	Percentage of solutions to be inbreeding for the next steps

Table 4.4 Main parameters used in Galapagos and their definition

4.4.2 Implementation in Grasshopper®

Structural optimisation by GA has been carried out for asymmetrical shells. The problem is defined by implementing the optimisation on the asymmetrical shells obtained in Chapter 4.3. Such digital workflow is described in Figure 4.38.

In a general context, structural optimisation finds its best application in the definition of the search of optimal design parameters that thus acts on the definition of a structural form. In this case, the search of a spatial configuration took place through form-finding that can be considered part of the optimisation process. Indeed, the final goal is the definition of a space through optimal design parameters and it is possible to define this approach as a form-finding process where the aim is exactly to find a spatial configuration. However, further optimisation of an asymmetrical shell can be developed by using a heuristic approach. The next cases aim to optimize the density of tessellation applied on the shell according to the displacement values.

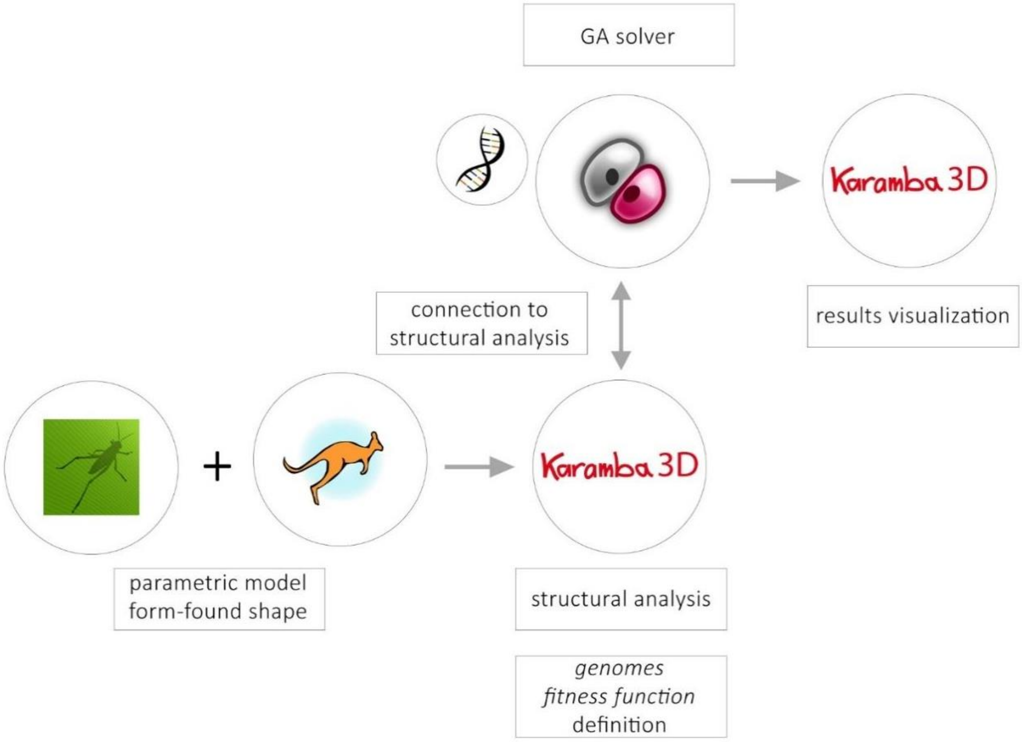


Figure 4.38 Simulation definition by digital tools

Problem definition and boundary conditions

The problem is formulated by taking into account the density of the subdivisions of the form-found shells. However, such density has been controlled with two different approaches.

The fitness function is intended to be in the general form of minimization of the displacement generated by the load conditions.

The geometrical inputs used for the generation of the starting surface are the dimensions of the circular-like curve and the location of points to define the supports and unsupported edge (Fig. 4.39):

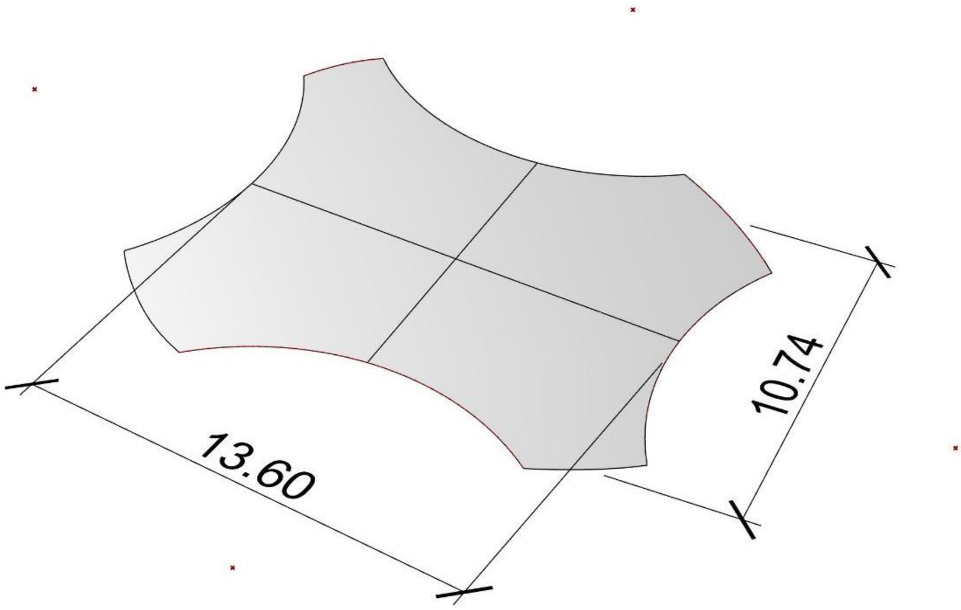


Figure 4.39 Geometrical inputs (dimensions are taken from middle point to middle point and are in meters)

The boundary conditions are the loading conditions, consisting of gravity load, self-weight load and live load, and the fixed supports. The material used is the same material adopted in the previous chapter and the shell thickness is 15 cm. Table 4.5 summarises the values.

Support conditions	Point supports
Loading conditions (kN/m^2)	Gravity load + Dead load (5.70) + Live load (0.75)
Material	Concrete C30/37
Shell thickness	15 cm

Table 4.5 Supports, loading, material and thickness properties of the structural model

4.4.3 Case 1: Hexagonal grid by Dynamic Remeshing

The process presented here is slightly different for the one carried out in the design workflow of the hexagonal symmetrical shell. This is justified by the need to control the parameter of the density grid connected to the optimisation simulation in a meaningful way in order to obtain satisfactory results.

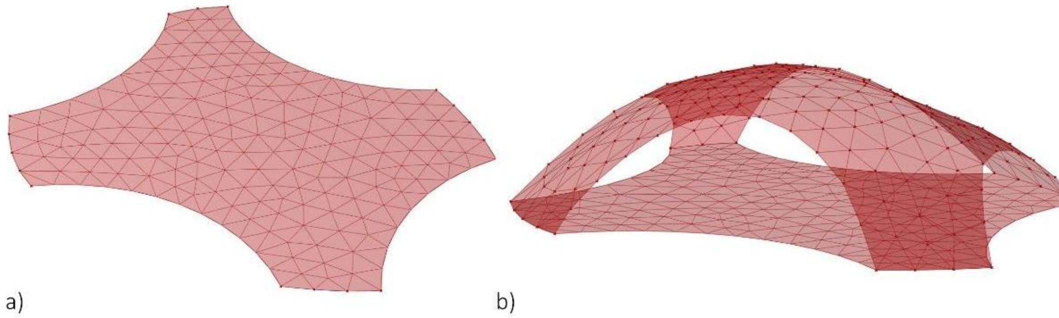
The input surface is subjected to "remeshing" operations. This is fundamental for several reasons: firstly, the remeshing allows better control of the mesh subdivision that consequently provides the hexagonal tessellation, meaning that it can be parametrised in the optimisation process; secondly the triangulation at the base of the remeshing is distributed evenly for a better result in the form finding as well; thirdly there is a high degree of freedom for the input geometry.

Dynamic Remeshing is carried out through iterations that are fed by a set of parameters:

- target edge length that allows to provide bounds for the length of the edges;
- constraints to the boundary curves of the surface and constraints to the vertices of the surface;
- pull force to the target geometry;
- a scale factor to reduce edge length at the boundary.

Figure 4.40 shows the mesh subdivision used as input geometry in the form-finding and the found spatial configuration.

The aim of this framework is to control the hexagonal grid by its dual triangles. The detailed process, which consists of obtaining the incircles of the triangles and connecting their centers to generate hexagons, is shown in Figure 4.41.



a) b)

Figure 4.40 Form-finding simulation: a) initial mesh obtained through Dynamic Remeshing, b) form-finding outcome

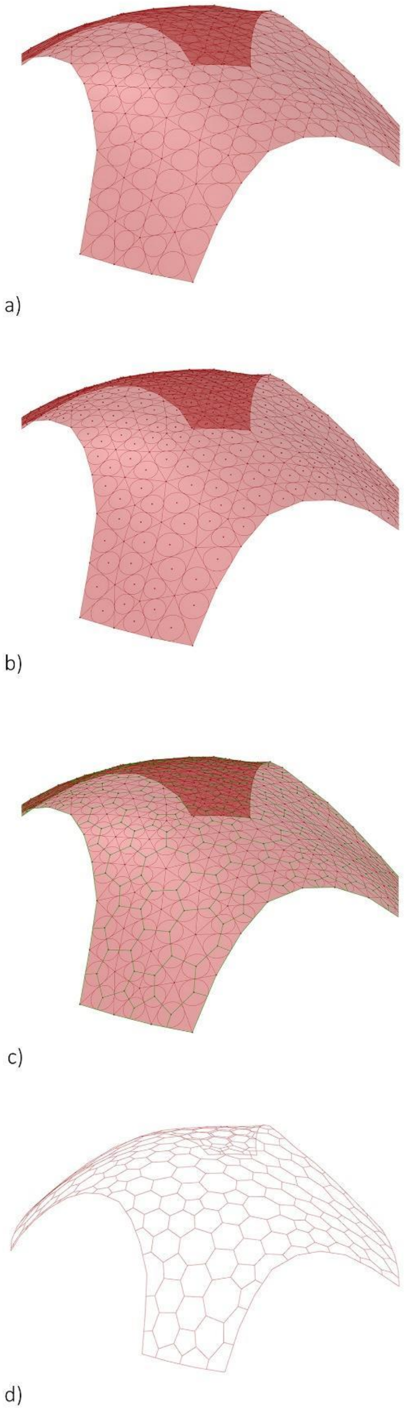


Figure 4.41 Process for the generation of the hex-dominant pattern: a) incircles obtained from the triangular mesh; b) centres of the circles; c) connection of the centres through edges; d) result

The grid is hexagonal dominant but other polygons are included in the subdivision.

At this stage the optimisation can be carried out by setting the variables within the simulations as well as by building the FE model though Karamba3D. Since the shape is already defined, the goal is to control the density of the subdivision. This can be done by the parameters that operate in the generation of the triangles of the mesh and in particular, on the dimension of the edge length: three different factors, 0.8-1.2-1.6, are shown in Figure 4.42.

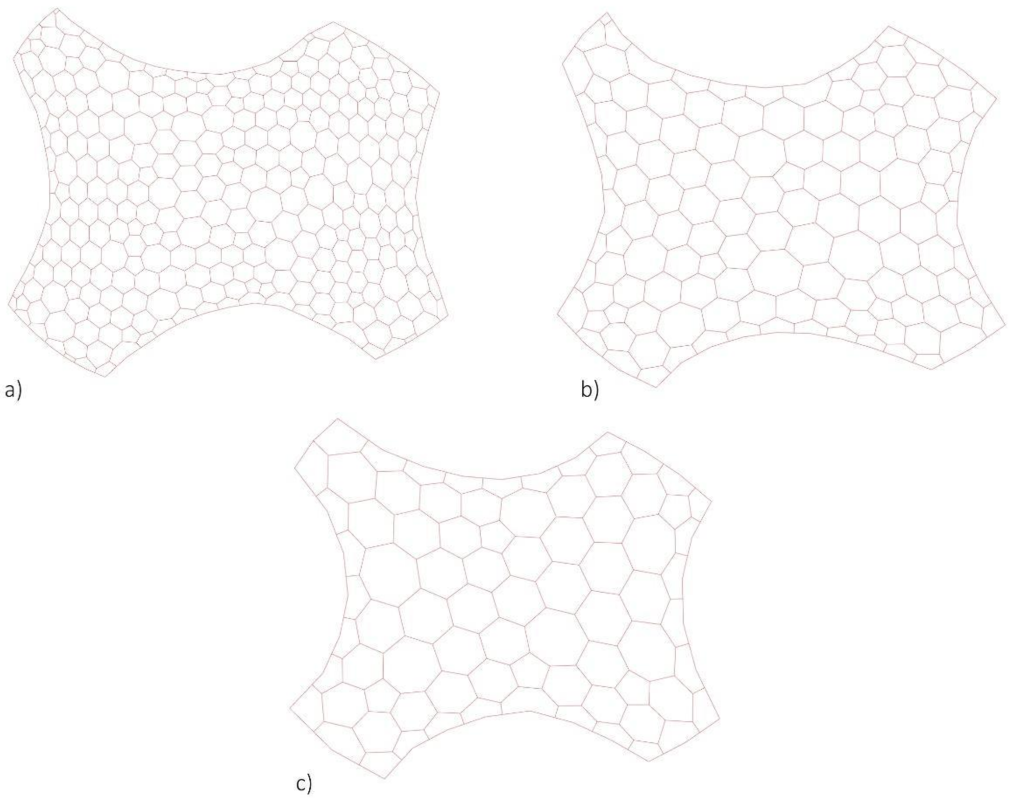


Figure 4.42 Target length values: a) subdivision with factor 0.8; b) subdivision with factor 1.2; c) subdivision with factor 1.6

The choice of a correct domain and an appropriate fitness function is fundamental for a good quality of the results since Genetic Algorithms can produce results not always reliable. For such cases, the aim is to minimize displacement and thus improve stiffness of the structure. The variables or *Genomes* used in this simulation are the target lengths of the generation of the triangular mesh and z-load value applied on the mesh points that influences the vertical movement of such points and thus the total height of the shell. However, the range needs to be carefully selected in order to have a successful form-finding result (Table 4.6). It is important to mention that this optimisation does not involve the shape generation, but it only acts to refine the form-found shape. The purpose of this investigation is to improve structural efficiency by acting on a few variables without modifying the geometrical configuration. Table 4.7 shows the settings used for the simulation before to discuss the results obtained.

Genes	**Domain**
Target edge length	Lower limit: 0.6 Upper limit: 1.8
Vertical load factor	Lower limit: 0.01 Upper limit: 0.1

Table 4.6 Definition of the Genomes

Variables for Evolutionary Algorithm	**Value**
Max Stagnant	100
Population	50
Initial Boost	3x
Mantain	5%
Imbreeding	50%

Table 4.7 Settings used for the simulation in Galapagos

Results

The simulation runs for a total of 101 generations converging towards a solution. Fig. 4.43 and Table 4.8 displays the results based on generations and fitness value.

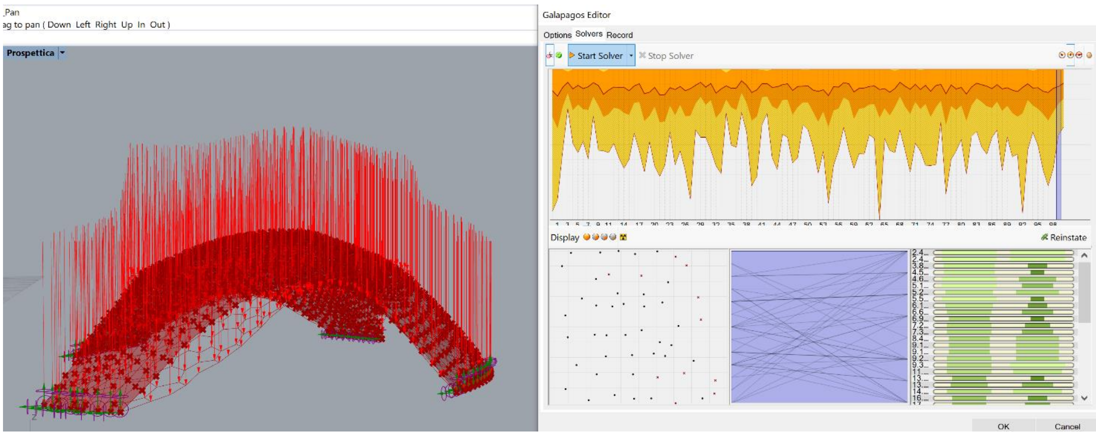


Figure 4.43 Iterations of the solver in Galapagos

Results	
Number of generations	101
Vertical load factor	0.03
Target edge length	0.6
Max displacement (cm)	2.49

Table 4.8 Results extracted from Galapagos

The displacement value ranges from a minimum of 2.4942 to 343.1529 cm based on the combinations of the two variables (Fig. 4.44). According to the tessellation, the solver provides a target length of 0.6 that translates into a subdivision (Fig. 4.45).

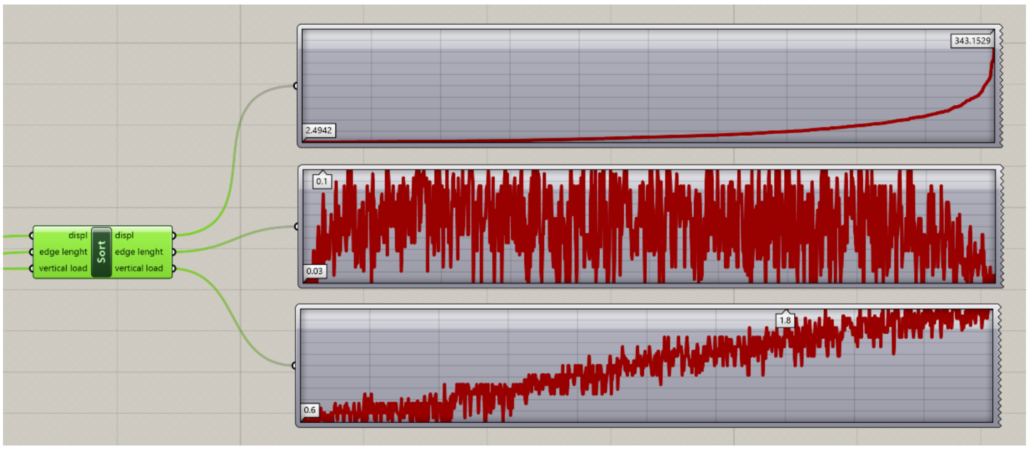


Figure 4.44 Graph showing the displacement evolution though iterations depending on the two variables

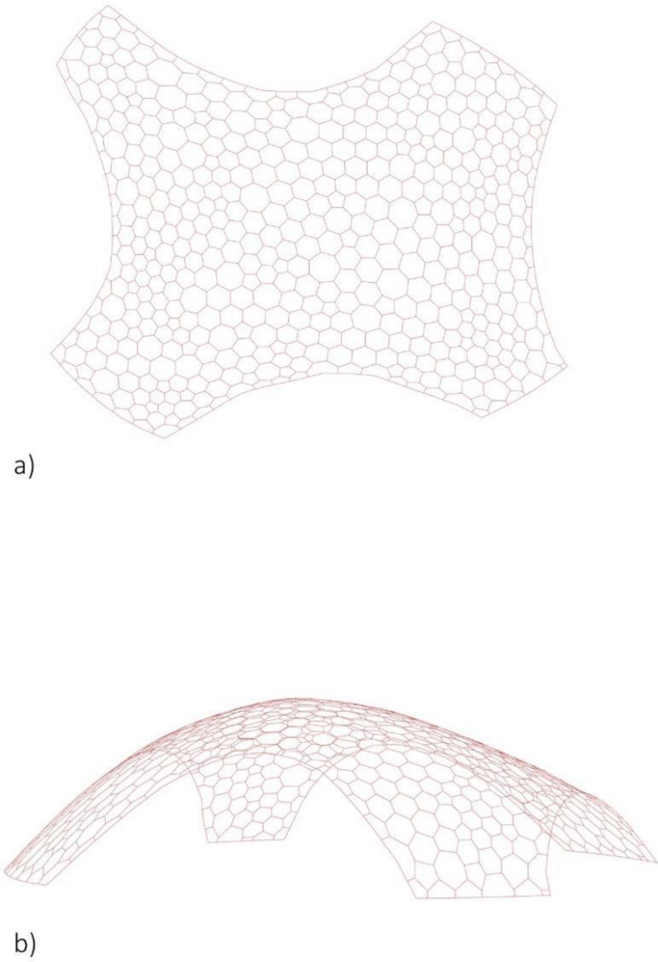


Figure 4.45 Outcomes from optimisation process: a) plan view and b) 3D view of the solution

As already stated, the tessellation is the result of a combination of different polygons and Table 4.9 shows the hex dominance of the subdivision, while showing the presence of pentagons, heptagons, etc.

Tessellation	**Total**
Number of polygons	495
Number of faces vertices n.4	5
Number of faces vertices n.5	46
Number of faces vertices n.6	331
Number of faces vertices n.7	98
Number of faces vertices n.8	15

Table 4.9 Tessellation results for the final solution

4.4.4 Case 2: Voronoi grid controlled by an attractor point

Voronoi subdivision has already been investigated in the form-finding, subdivision and planarization processes showing potentialities in architecture for the design of shell structures. At this stage, the density of the Voronoi grid is optimized by means of an attractor point. An attractor point generates a field of magnetism through attraction or repulsive forces. This can be particularly useful in the variation of certain parameters. By parametrizing the position of such control points this variation is obtained and it can be linked to a simulation of optimisation with the goal to improve structural efficiency, thus, changing density according to structural results. The principle of using attractor points can be implemented in different ways according to the kind of objectives to satisfy. In this definition, one of the requirements is to preserve a homogeneous distribution of points that translates into an even subdivision for Voronoi grid. It can be achieved through the use of a reduction factor that controls the density of the points. This range of values is parametrized and connected to the evolutionary solver to carry out optimisation. In detail, the variables are the following: a range of values that represents the factor that controls the density of the population of the points for Voronoi and the vertical load applied on the mesh points and the minimization of the displacement value set as fitness. The domain is defined according to the feasibility of the results. The generation of the mesh is implemented with the same process already illustrated in the previous Voronoi shell, although some distinctions need to be made due to the requirement to change density for evolutionary algorithms.

The definition of the shell relies on a population of points that considers the number and the seed that generates a Voronoi diagram (Fig. 4.46). This list of points is connected with a reduction factor that regulates the number of points randomly and therefore controls the density of points within the input surface (Fig. 4.47). This factor if linked to the Galapagos component can change not randomly but according to a fitness value that is the minimization of the total displacement. The seed and the original count of points is chosen thoroughly in order to have an even distribution of points considering the geometry.

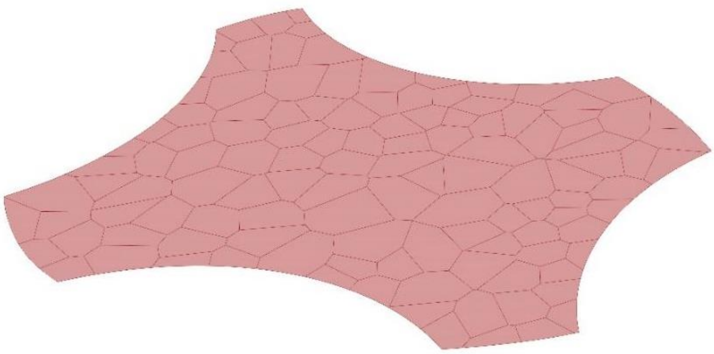


Figure 4.46 Initial subdivision of Voronoi grid applied on the surface

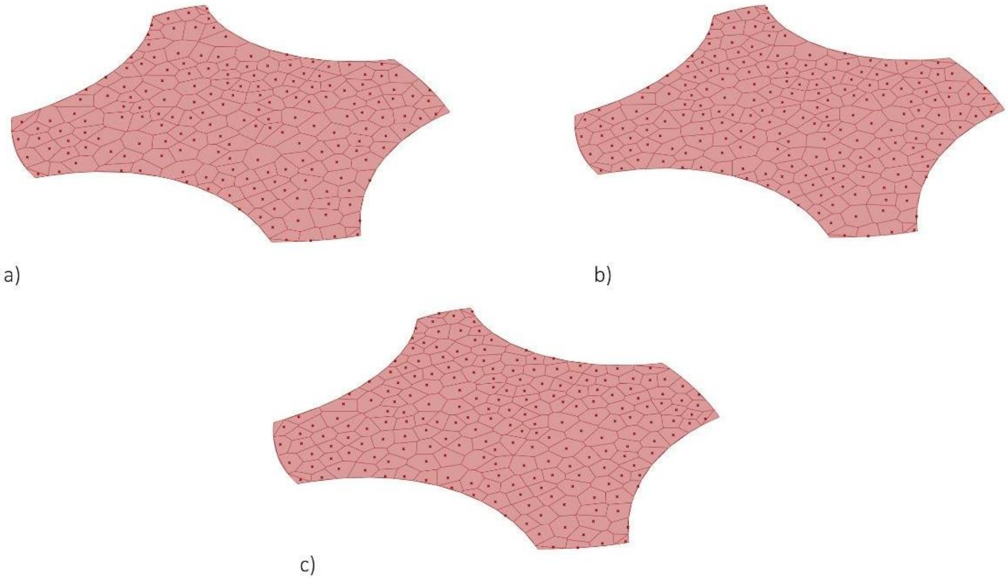


Figure 4.47 Different reduction factors for the population of the points: a) 180 b) 195 c) 210

Table 4.10 and 4.11 describe the whole definition to correctly implement the simulation, including the domain and the settings of Galapagos.

Genes	Domain
Reduction factor for points population	180 to 210
Vertical load factor	Lower limit: 0.01 Upper limit: 0.06

Table 4.10 Genes for the simulation

Variables for Evolutionary Algorithm	Value
Max Stagnant	100
Population	50
Initial Boost	3x
Mantain	5%
Imbreeding	50%

Table 4.11 Evolutionary Algorithm settings

Results

The simulation converged to a final solution after 101 generations. Figure 4.48 shows the simulation with all the genomes generated and the iterations. The total displacement is minimized to 2.45 cm and in the following table (Table 4.12) the results are summarized.

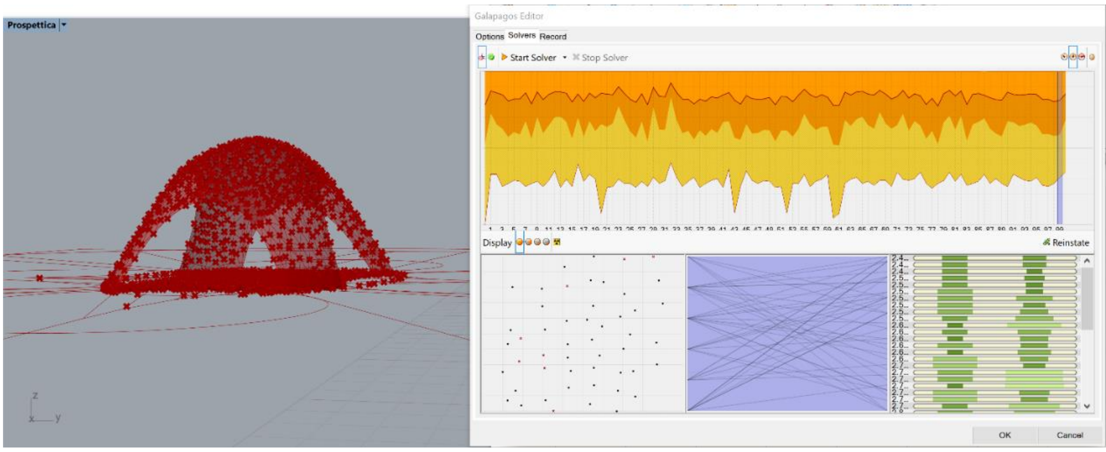


Figure 4.48 Iterations of the solver in Galapagos interface

For this case, the choice of the domain needed to be double-checked since some intervals for the population of the points yielded invalid meshes and therefore unreliable processes In order to obtain good results for the simulation it was necessary to test the domain before to carry out the form-finding process in order not to include into the optimisation invalid values. With a correct range of values, the optimisation has provided a good improvement of the shell in terms of displacement (Fig. 4.49).

Results	
Number of generations	101

Vertical load factor	0.05
Reduction factor	205
Max displacement (cm)	2.4552

Table 4.12 Results extracted from Galapagos

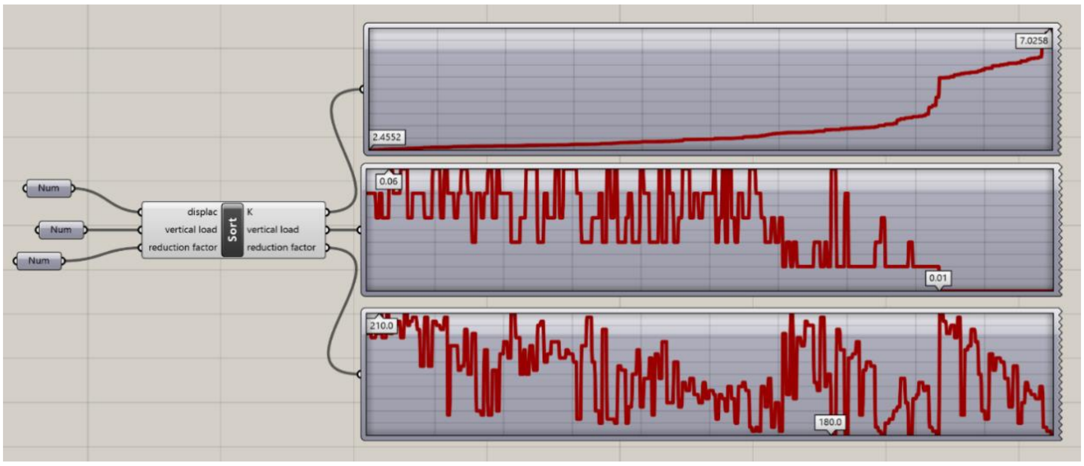
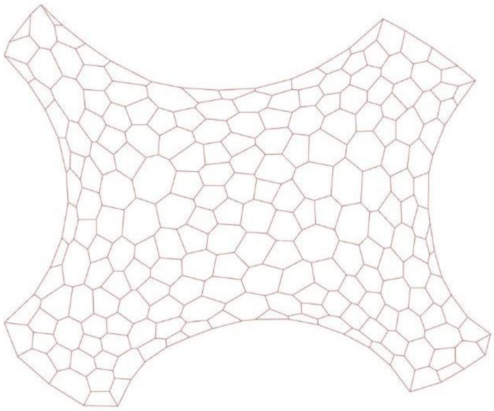
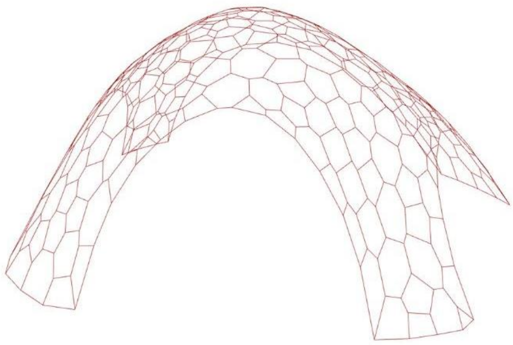


Figure 4.49 Graph showing the evolution of the displacement values and the two variables

As concerns as tessellation Figure 4.50 gives indications of the final subdivision and Table 4.13 shows the details of this subdivision, with a variety of different polygons.



a)



b)

Figure 4.50 Subdivision for the resulting shell: a) plan view and b) 3D view

Tessellation	**Total**
Number of polygons	205
Number of faces vertices n.3	3
Number of faces vertices n.4	14
Number of faces vertices n.5	77
Number of faces vertices n.6	69
Number of faces vertices n.7	35

Number of faces vertices n.8	7

Table 4.13 Tessellation results for the final solution

4.4.5 Comparison of the two cases

The two resulting shells have presented differences mainly from a geometric perspective: firstly, the overall height (Fig. 4.51) and the number of faces composing the shell, as highlighted in the previous tables, however it is possible to analyse such structures also from a structural perspective by taking into account the stresses generated on the shell. The structural models are implemented in Karamba3D. The geometrical inputs for the FEA are mesh elements while the parameters concerning material properties, load combinations and thickness are the same used in the previous analysis.

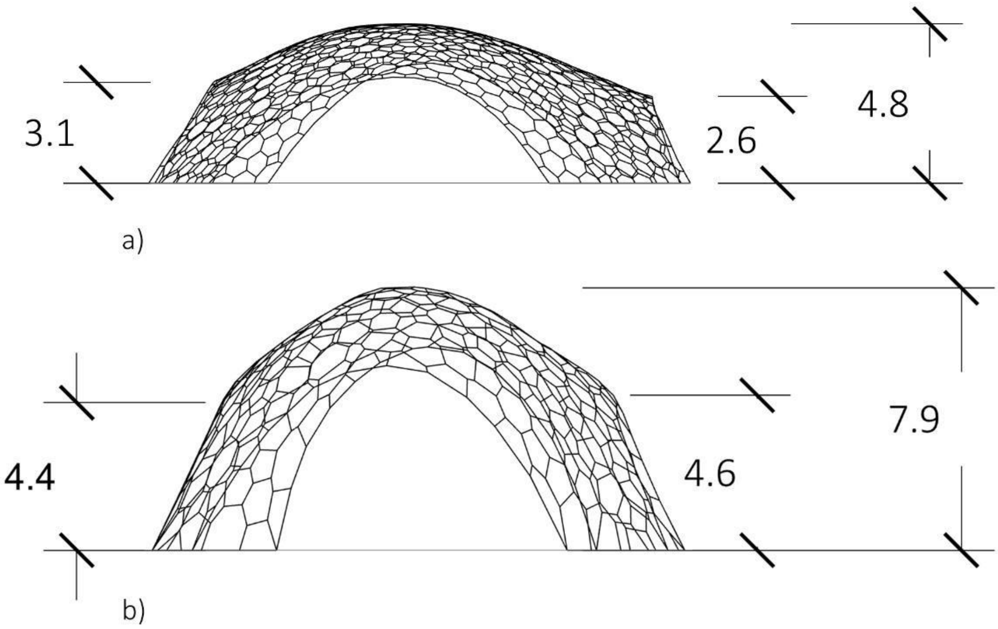


Figure 4.51 Height of the two shells (scale 1:200): a) hexagonal shell; b) Voronoi shell

The stresses generated on the mesh elements are predominantly in compression (in red), except for very few areas where tension is present (represented in blue) as displayed in Figure 4.52 and Figure 4.53. The graphs show values calculated by interpolating the element results from the centres to the nodes. The first case presents lower values than the second shell and a key-factor that contributes to such performance is the difference in height.

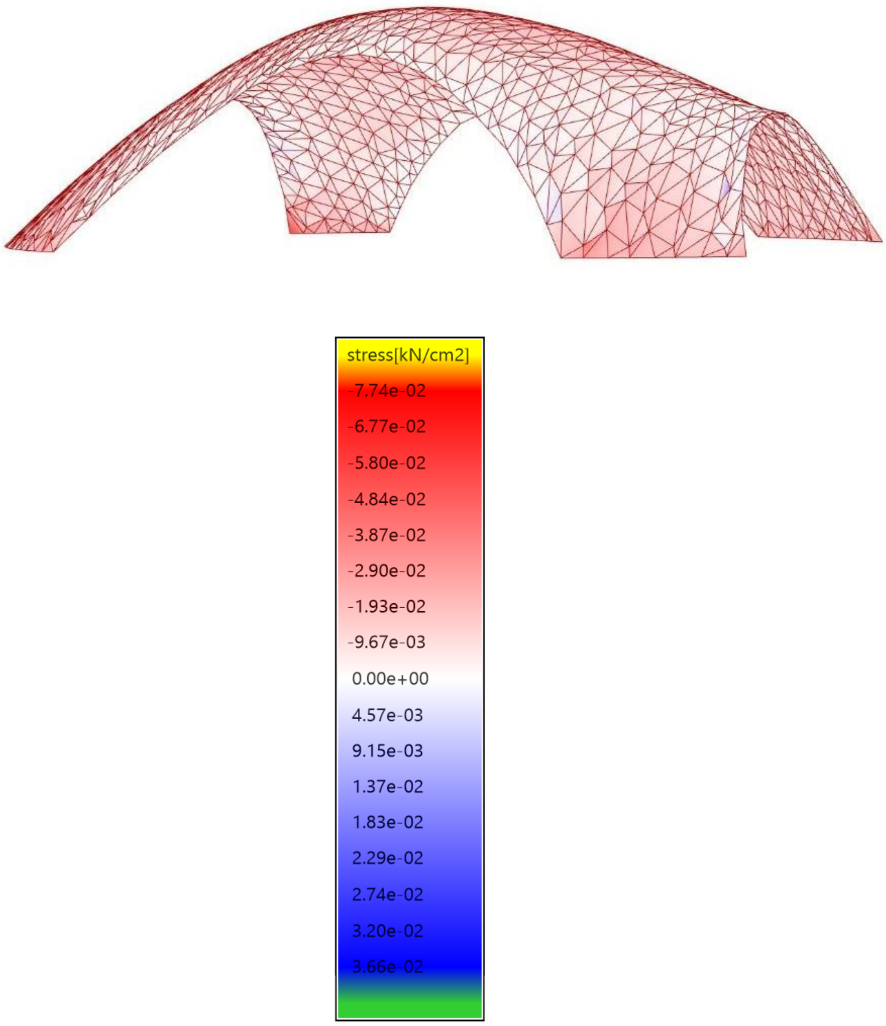
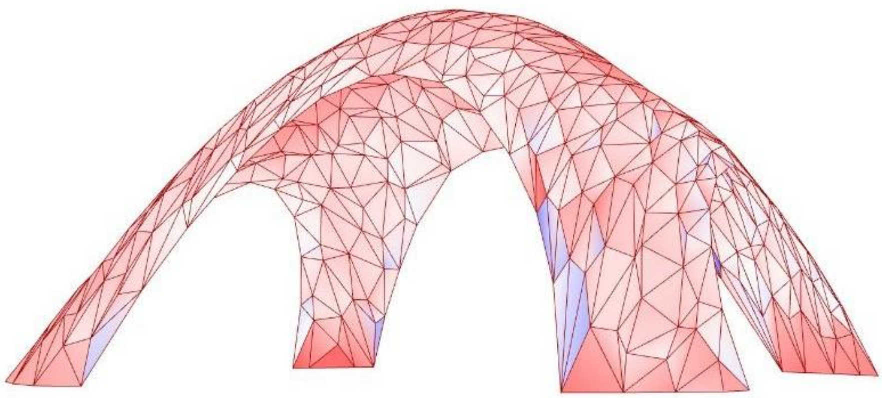


Figure 4.52 Stress visualization on the shell (red is for compression, blue for tension)



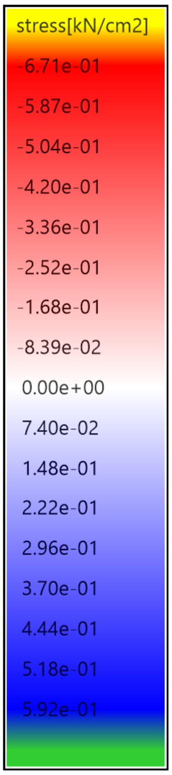


Figure 4.53 Stress visualization on the shell (red is for compression, blue for tension)

4.4.6 Summary

Starting from the same geometrical parameters the design of the two shells has been defined. Although spatial configuration relies on the same form-finding technique, the rationalization of their geometries is based on two different subdivisions. The subdivision has been the object of investigation in order to evaluate its impact in terms of structural efficiency. Moreover, the vertical load factor involved in the form-finding process has been analysed in order to control indirectly the height of the structure. This was possible thanks to the use of evolutionary solver that aimed to minimize the total displacement of the structure. The results have shown an effective improvement of the performance by controlling the density of the subdivision and the vertical load factor. Since the subdivision techniques are different as described in the design process, the variables result to be different but finalised to the same scope.

Evolutionary algorithms are undoubtedly a powerful tool in the design process but an accurate evaluation of the results obtained is compulsory, since they do not guarantee always good solutions and a good interaction between the user and the tool is required. When dealing with application in architecture, the final choices do not depend necessarily on the outcomes of the solvers, rather on a combination of factors including costs, realization and aesthetical/architectural features.

4.5 Design optimisation process for an unrealised shell

4.5.1 Introduction

The following section aims to give a further demonstration of the potentialities of Genetic Algorithms in the design of shell structures. This particular case investigates the search of a problem definition from a purely structural perspective: in fact, by using this optimisation, the structural efficiency improves to such an extent that the use of reinforcement throughout the shell is no longer necessary. Isler's shells are well known for their elegance in terms of aesthetical features and for the use of concrete and reinforcement in terms of structural behaviour. The main reason behind the reference of an unrealised Isler's shell lies in a new potential awareness dictated by the possibility to realize in the future his unrealised projects in a contemporary context that can make use of more efficient tools in the design.

The main inspiration has started from the expression "Potential unrealised", used by Chilton in his paper (2010) to highlight the gap between the shell design solutions by Isler and their realization. Isler showed a broad variety of possible solutions for shell structures in his emblematic contribution at the first conference of the International Association for Shell and Spatial Structures, in Madrid titled "New Shapes for Shells". His innovative form-finding techniques have paved the way to a new way of designing efficient structure overturning the relationship between architects and engineers. In such a way, engineers take control of the creation part of the design process, which relies on free-form principles rather than established geometric shapes. Isler made use of non-mathematical methods to obtain free-form shells and he was able to translate the outcomes of his physical methods into the definition of large-scale projects by measuring points of the model without the use of current technologies. Nowadays several groups have started to investigate the structural performance of the Isler's models (Chuang and Chilton 2016), by using 3D scanning and Finite Element Analysis (Borgart and Eigenraam, 2012). Despite the distrust of Isler in digital tools (justified by the technologies existing in the years he was active) more investigation and in depth research need to be carried out yet by exploiting the potentialities that digital tools can offer nowadays.

4.5.2 Investigation context

Although Isler developed a significant variety of shells in terms of form, most of those realized are attributable to the bubble shells he designed for industrial and commercial functions (Chilton, 2010). Above all the techniques, the reversed hanging cloth was the one allowing him to develop the most challenging projects such as the roofs of the N1 autobahn service station at Deitingen Süd in 1968 (Fig. 4.54) and the Sicli Shell in Geneva in 1970 (Fig. 4.55). However, this technique proved to be primarily suitable for small and medium scale projects, because it allowed proper prediction of structural stability, whereas for large scale, Isler adopted form-finding methods (Chilton, 2012).

Figure 4.54 the N1 autobahn service station at Deitingen Süd (Chilton, 2012)

Figure 4.55 Sicli Shell in Geneva (available from strucutrae.net)

Unfortunately, Isler did not publish technical details of his projects regarding design and construction, therefore it is not possible to follow exactly his modus operandi, but a good approximation can be obtained by considering some available sources.

For this work, the focus has been given to one of the unbuilt projects since the aim is to provide contributions especially to the “potential unrealised” and evaluates its feasibility by the use of modern computational tools. The project is the roof project for a dolphinarium at Parc Asterix in France in 1994 (Fig. 4.56). Three different solutions were proposed as Chilton points out in his research: two symmetrical shells with seven point supports and one asymmetrical shell with nine point supports. The drawing shows one of the symmetrical proposals although there is no mention about dimensions of the structure.

The starting point for the definition of the workflow is indeed such drawing and its geometrical peculiarities can be deducted after a first observation of the geometrical configuration.

The definition of the space has been elaborated by making use of symmetry and mirroring a portion of the space. The point supports are obtained through the intersection of a circumcircle with specific axes as displayed in the Figure. By defining these elements, it is possible to parametrize a set of factors to develop the form-finding phase.

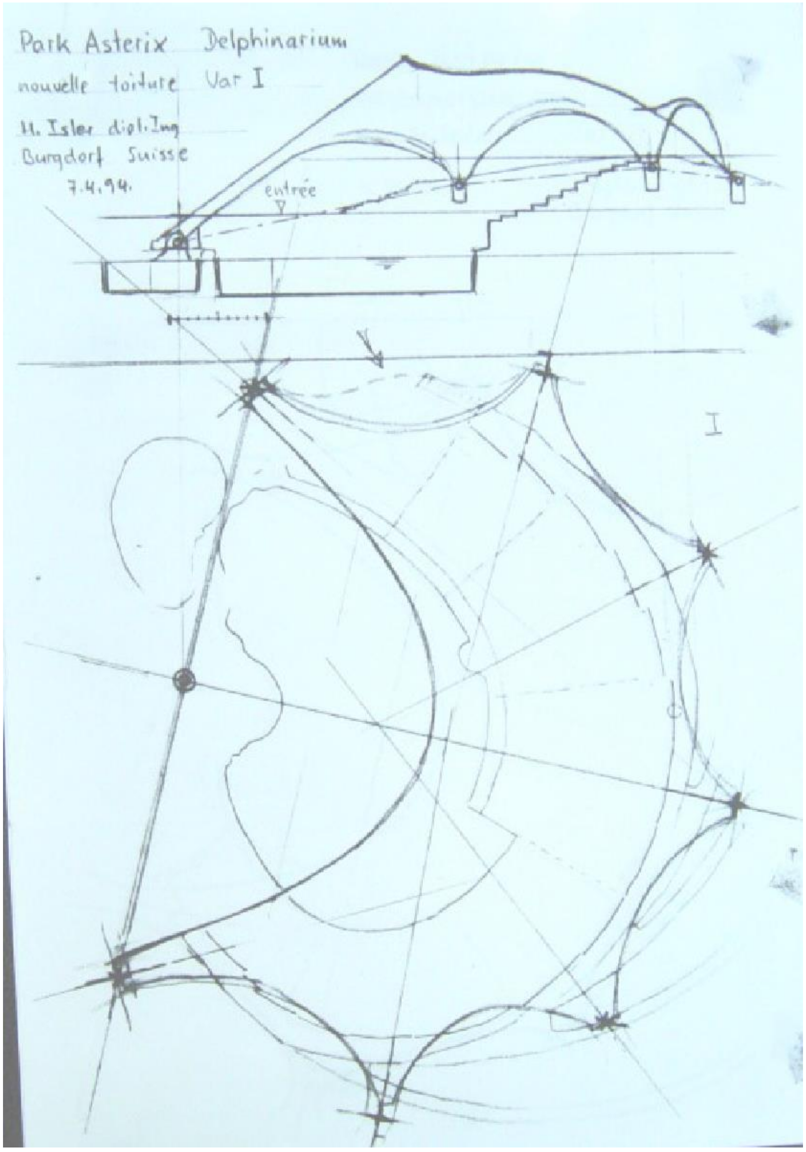


Figure 4.56 Symmetrical solution with seven point supports for the Dolphinarium (Chilton, 2010)

4.5.3 Design workflow

The proposed workflow (Fig. 4.57) aims to start with the definition of a space that refers to the unrealized project of the Dolphinarium and implement a design process in order to evaluate the structural optimisation by Genetic Algorithms.

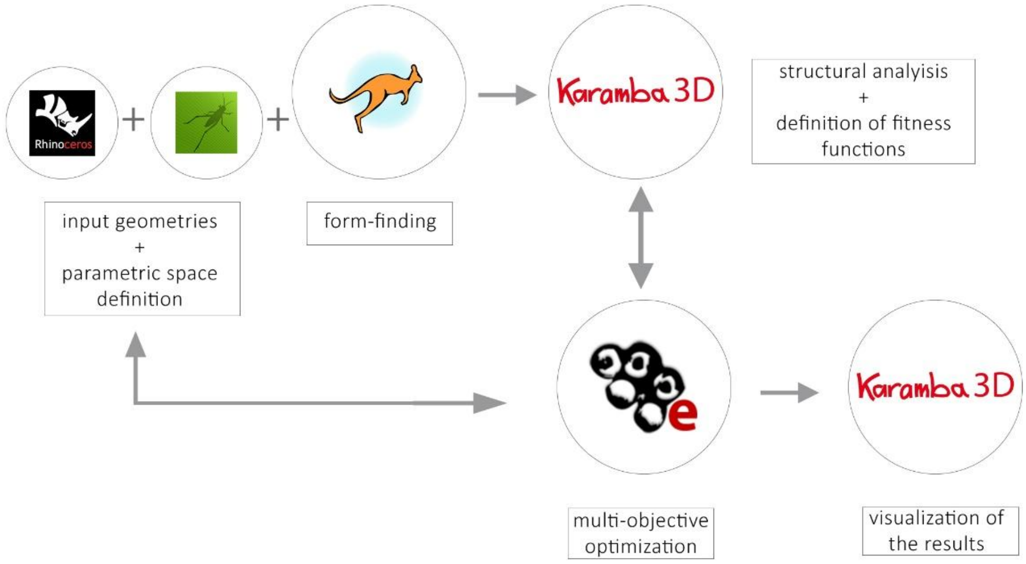


Figure 4.57 Design workflow highlighting the digital tools for each phase

The initial step concerns the definition of the geometry and its parametrization in order to obtain a form-found shape. Fig. 4.58 shows the inputs used to define input surface for the form-finding process:

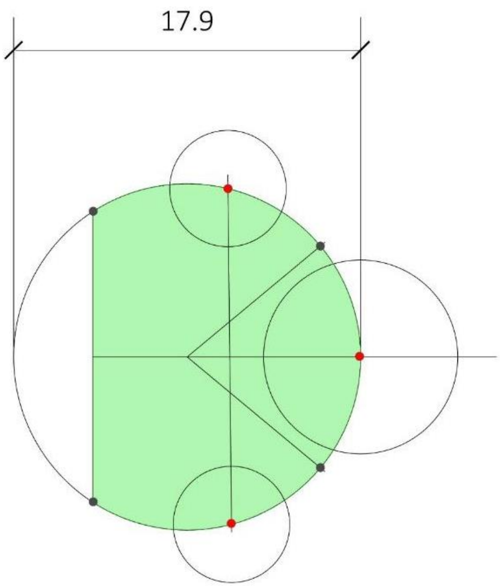
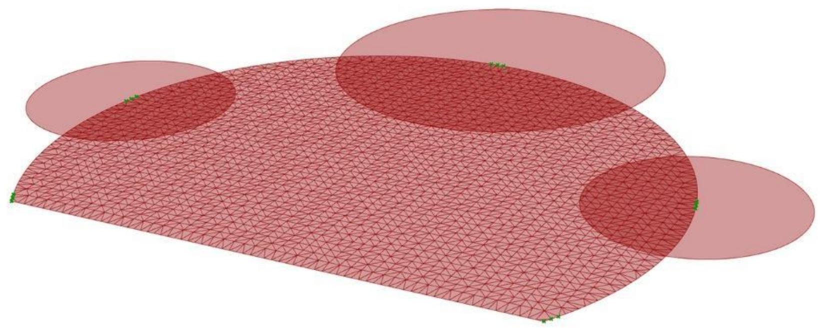


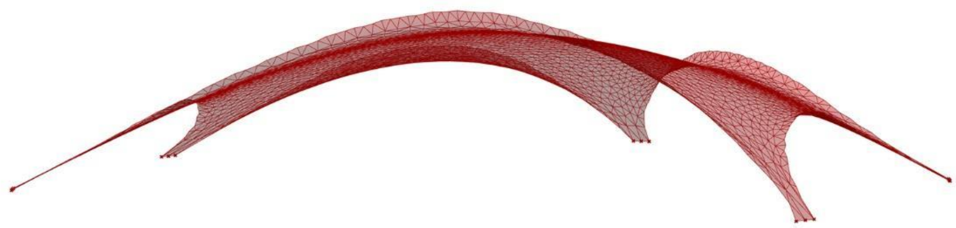
Figure 4.58 Initial setting of the geometry, of the supports and of the input surface

The black dots represent the fixed supports and the red dots are the supports that are object to the optimisation process explained in detail in the following point. The green area is the initial surface that undergoes the form-finding. Such a surface is then converted into a mesh, through an algorithm subdivision and by setting a target mesh size as well as an edge refinement factor in order to obtain an uniform mesh (Fig. 4.59a).

The initial setting is the symmetrical case where the supports lie in the centre of each small circle, starting from this definition the form-finding process is implemented (Fig. 4.59b) and the whole workflow can be applied. The structural model is then analysed through Karamba3D and connected to the multi-objective solver.



a)



b)

Figure 4.59 Form-finding process: a) starting mesh b) form-found shape

The structural model is analysed through Karamba3D considering self-weight load, gravity load and a live load of 2 kN/m^2. The material used is the same material used for the previous simulations (Concrete C30/37) while the cross-section is inserted as a range of values whose final values are selected by the optimisation tool of cross-section taking into account the performance of the structures. The range of value as input is from 8 cm to 36 cm.

4.5.4 Multi-objective Optimisation by GA

The domain of the design variables has a fundamental role in the optimisation process; its range influences the typology of results obtained, providing significant alterations of the project. The choice to use not all the supports in the simulation gives a certain amount of constraints in the final shape that is not completely free to change. The evaluation of the symmetry and asymmetry of the supports and how they change during the simulation was also considered for the purposes of this research. In detail, the domain is represented by the horizontal position of the supports, whose position is determined through a set of geometrical operations that allows the supports to move on the circumcircle. The boundaries of the domain are given by the two intersection points between the main circle and the smaller circles of the supports: this distance is then parametrized in the values ranging from 0.0 to 1.0 (Fig. 4.60).

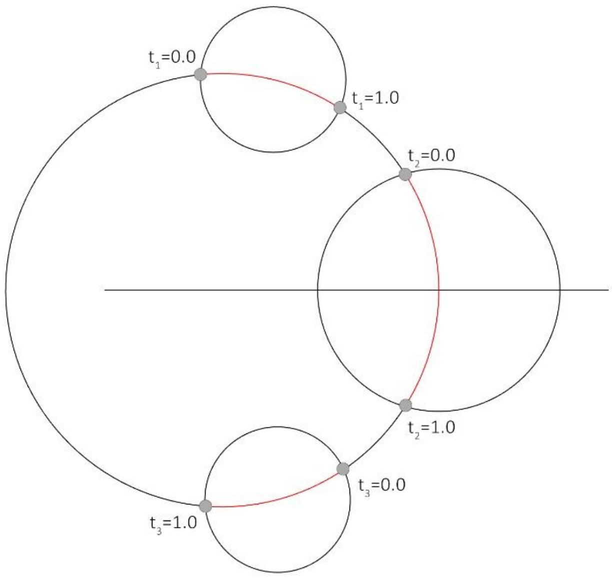


Figure 4.60 Graphical definition of the domain for the supports

The fitness function is in the form of multiple goals to satisfy, taking into account total displacement and mass of the structure. The aim is to optimize the structure not only structurally but also architecturally. In case of a single objective, the process results straightforward because the simulation can converge to a single solution: if such a solution is not considered acceptable the simulation can be repeated by changing the variables. When dealing with a multi-objective process the system does not converge to an "optimal" solution but to a set of solutions that considers the different objectives, representing a combination of acceptable results for the fitness functions. Hence, the designer can evaluate them and choose the most appropriate considering design and/or structural requirements. Digital optimisation is implemented in Octopus, which allows defining a set of fitness functions according to the Pareto principle for multiple goals. For this case, the variables are the position of supports and height; and the objective is to minimize displacements while minimizing mass. In particular: the supports are parametrized and free to move on a portion of the main circle, by setting boundaries of the domain, the height is varied by assigning a set of vertex loads. Vertex loads values affect indirectly the height and the process of form-finding.

The two fitness functions are defined as outputs of Karamba3D, e.g. mass and displacement and their minimization. The final goal of the simulation is to improve the efficiency of the structures obtaining acceptable results for a potential realization of the shell without the use of reinforcement or with local reinforcement in specific areas. Table 4.14 shows the setting used for the optimisation.

Population size	100
Elitism	0.500

Mutation Probability	0.200
Mutation Rate	0.90000
Crossover Rate	0.800

Table 4.14 Settings used in Octopus

4.5.5 Results

In a multi-objective simulation the role of the designer is represented by the possibility to decide which solution is the best one for the case at hand. A collection of outcomes is selected according to their best performance and then evaluated. As shown in Table 4.15 different values for displacement and mass are obtained. Before discussing the set of outcomes extracted, it was interesting to evaluate the two limit cases taken from the simulation:

The first limit solution (Fig. 4.61) describes the case with a high factor for the vertical load that translates into a taller structure, the support positions are approximately in the middle area and displacement is at the minimum, with a consequent rise in terms of mass of the structure. The structure works primarily in compression, tension has arisen only in the supports area and along the main unsupported edge although in small quantities.

- Strength 0.006
- Parameter t_1 0.4
- Parameter t_2 0.4
- Parameter t_3 0.5
- Mass 21168.041 kg
- Displacement 0.027 cm

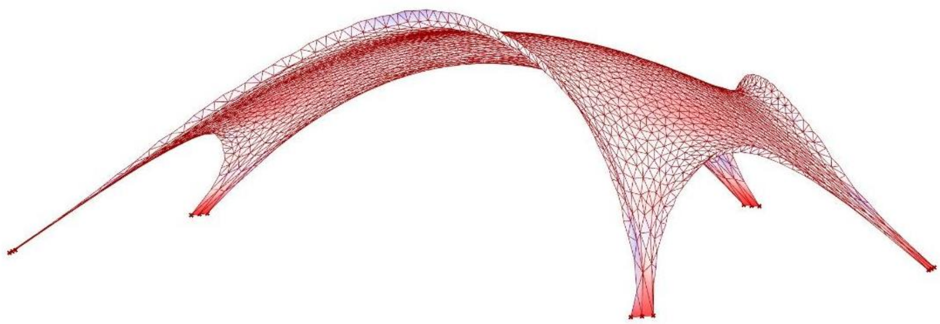


Figure 4.61 3D view of the first limit case

The second extreme of the interval of solutions is represented by an asymmetrical shell (Fig. 4.62) with a lower value for the vertical load. A shallower shell is defined and the two fitness functions are at the opposites: the mass value is minimum while the displacement is the maximum for the given simulation. However, it has been noted that tension stresses are less present than the first case providing an almost entirely compression behaviour to the structure.

- Strength 0.002
- Parameter t_1 1.0
- Parameter t_2 1.0
- Parameter t_3 0.0
- Mass 18256.737 kg
- Displacement 0.104 cm

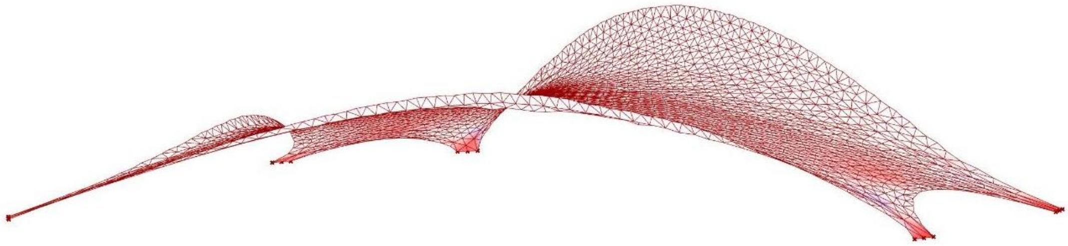
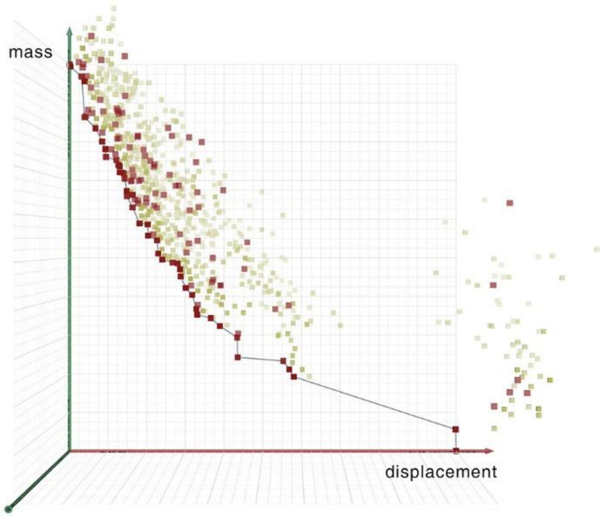
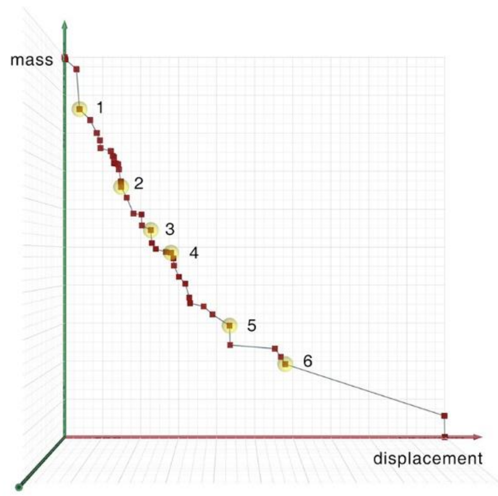


Figure 4.62 3D of the second limit case

Besides these two limited cases the simulation has yielded a set of results. Figure 4.63 shows all the solutions obtained (a), the Pareto-front isolated (b) and the selection of some of them in order to understand the trend of the two fitness functions (c).



a)



b)

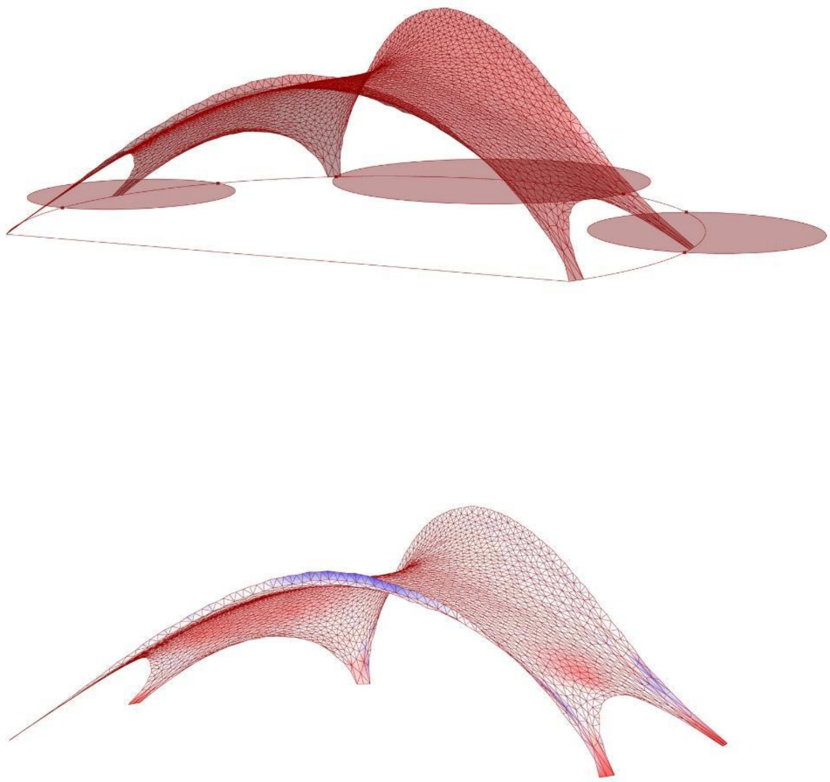
Figure 4.63 Results extracted from Octopus: a) the solutions obtained after the simulation; b) Pareto-front solutions and selections of a range of solutions

As shown in Table 4.15 the two fitness values are inversely proportional: the rise of a value implies the decrease of the other variable. Furthermore, the three parameters concerning the position of the supports yield outcomes that do not lie in the centre, providing asymmetrical shapes. As regards the choice of one solution within a simulation, the designer can operate according to the priorities planned for the design process.

Solutions	**Vertex load (strength)**	**Parameter t_1**	**Parameter t_2**	**Parameter t_3**	**Mass (kg)**	**Disp (cm)**
1	0.005	0.3	0.4	0.5	20770.793	0.030
2	0.005	0.0	0.4	0.7	20175.141	0.038
3	0.005	0.4	0.9	0.0	19843.587	0.044
4	0.005	0.7	1.0	0.0	19670.230	0.048
5	0.004	1.0	0.9	0.0	19110.996	0.060
6	0.003	0.8	1.0	0.0	18815.855	0.072

Table 4.15 Results for the selected solutions

For this simulation, it was considered interesting to refer to a result that considers values in the average of the range. For instance, taking as reference the case n.3 from Table 4.15, that is a not symmetrical shell (Fig. 4.64a) and its value for the vertical load is equal to other cases. Although it results in a good compromise in terms of mass and displacement values, tensile stresses are present in some areas of the structure with a thickness of only 8 cm (Fig. 4.64b). A potential increase in thickness in localized areas might improve the final performance of the structure.



a)

b)

Figure 4.64 Case n.3: a) design outcome; b) stress distributions

4.5.6 Summary

The work of Isler remains an important benchmark in the design of shell structures: his projects have given relevant contributions in the world of structural art. Unfortunately, most of his projects have remained only on drawings and sketches, without providing technical information. Nevertheless, they still represent an important inspiration for future projects that can be potentially realised, trying to gain understanding of these forms and to exploit their main peculiarities. Thanks to the powerful tools in the design for the shell structures, it is possible to take control of these processes and formulate workflows appropriate for these cases. This chapter aimed to follow this logic with the aid of multi-objective optimisation. It was interesting to highlight how the different solutions cannot give optimal results for all the parameters considered but rather they need a critical evaluation and a careful and balanced decision regarding the priorities of certain parameters over others. For each solution obtained, the results were different and not optimal for all the fitness functions. For instance, the "middle" case has shown a major appearance of tension in the structure compared to the two limit cases. Multi-objective optimisation may give more freedom to the designer in evaluating best solutions taking into account multiple requirements at the same time.

5. Digital fabrication for shell structures

5.1 Realization of case studies by digital fabrication techniques

5.1.1 Introduction

The previous chapters have highlighted the potential of the digital process and the infinite solutions in terms of shape, subdivisions and optimisation processes thanks to the techniques implemented with computational tools in terms of design and efficiency of the structures on different levels of complexity. Digital workflows within parametric modelling are flexible and able to fulfil a wide set of requirements, proving the high level of accuracy of the outcomes.

The following chapter focuses on the fabrication aspects connected with the design workflows, in order to develop models that can be realized without the use of mortar or any kind of reinforcement. For this scope, the shells described in this chapter have been investigated from structural and geometrical perspectives to implement volumetric tessellation. Moreover, the main aim of this work is to realize shells that can be realized with minimum use of formwork. The reduction of the formwork affects in a remarkable way the costs and the time for the realization of structures. The design of structures that require minimum formwork is beneficial for different reasons: firstly, its realization is simplified and it can be done quickly without the aid of qualified people, secondly, there is a significant economy on the total cost of the projects, thirdly, formwork cannot be reused, therefore waste material is consequently avoided.

In detail, two cases have been developed in order to implement this important aspect of the realization. The first case regards a previous tessellated shell which is completed by developing a puzzle-like connection system; the second case consists of a lightweight shell made where finger joints and an external connection system were developed.

5.1.2 Volumetric generation of the tessellation

The volumetric generation of a shell structure is a process composing of different stages (Fig. 5.1), each of which follows important assumptions for a correct design: as a matter of fact, a tessellation configuration that follows structural requirements can affect positively the stability of the structures. Besides the geometrical subdivision of a surface into panels, one of the main goals of the tessellation is the stability. Structures working in compression such as masonry structures have known an established building tradition handed down through the generations. Master builders from the past have always followed rules dictated by experience that create stable structures. Those traditional principles represent, even now, a fundamental benchmark and have been properly developed and embedded in innovative techniques in recent research (Rippmann et al., 2011) (Fallacara, 2009). Indeed, the innovation and potential of computational modelling has shorten the distance between designer and builder, formerly considered two distinctive entities. The designer, thanks to considerable computational resources, can develop a thorough design including fabrication and realization details in a more straightforward way that does not need any interpretation by the builder during the construction process. Moreover, parametric modelling has allowed the embedding in one workflow of all the required operations to develop a structure from the concept to the realization.

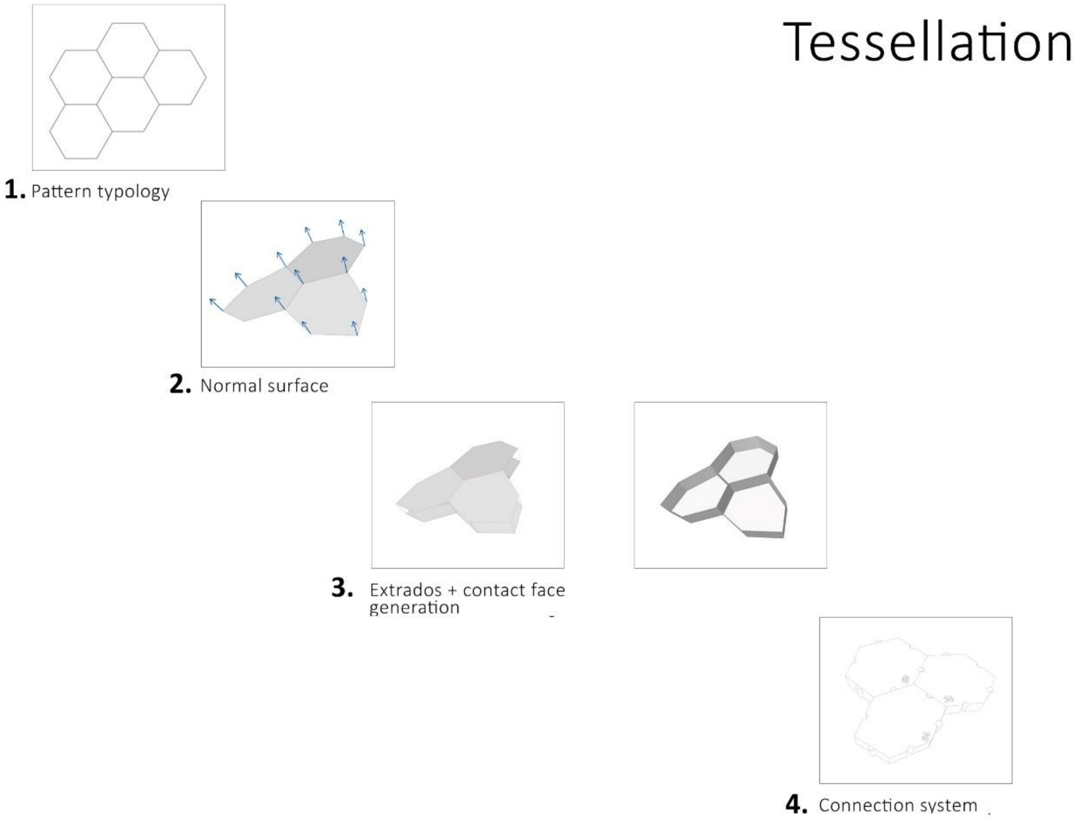


Figure 5.1 Framework for tessellation process

In particular, research by Rippmann and Block (2013) has given important assumptions for designing shell structures within computational tools and a strict correlation between theory and practice, so that the Armadillo Vault (Rippmann e al., 2016) is the proof of these cutting-edge works in particular in terms of fabrication of the components and realization of shells. Such assumptions, which have provided guidelines for the next steps in this work, concern structural and fabrication requirements, explained as follows:

- suitable thickness to assure safety of the structure;
- the tessellation has to be perpendicular to the thrust surface;
- staggered tessellation.

In the development of a volumetric definition for a shell structure, namely for the volumetric definition of its panels, it is fundamental to provide an appropriate thickness capable of withstanding stresses generated throughout the structures as well as buckling that can cause collapse. As already noted, the thickness is not a parameter included in the form-finding process where the form-found shell is a 2D surface with a 3D spatial configuration. Thickness comes into play when structural analysis is performed and load combinations are applied to the structure. Another pivotal requirement regards the extrusion direction of the tessellation. Although Block and Rippmann (2012) refer to the thrust surface obtained from the TNA this principle can be applied in general to structures working mainly in compression as long as the membrane of the shell is retrievable and it can provide information regarding the forces acting on the shell. The surface generated from the form-finding contains the force vector field, therefore a perpendicular orientation of the tessellation to the surface allows the structure to prevent from sliding, representing a fundamental assumption for unreinforced structures working in compression.

Staggered tessellation does not allow the alignment of their elements that may cause instability in the structures, representing another fundamental requirement for a correct tessellation process. As regards the contact faces between the volumetric elements, they are geometrically dependent on the perpendicular vectors obtained from the surface. Thus, such contact faces are ruled faces by definition as generated by lines. The typology of contact faces depend on the complexity of geometries in terms of curvature - they can be flat, singly curved or doubly curved.

Geometrically this work has dealt with two-dimensional elements and the consideration of a volumetric shell has been done only in the structural analysis with the requirement to add thickness to the structure. The form-finding with the input geometries, subdivision techniques and definition of the spatial configuration handle surfaces, meshes and systems of lines as already proved; the planarization concerns the intrados of the future volumetric as well as any sort of optimisation applied to the form-found shell. Therefore, a volumetric definition starting from the resulting shape needs to be implemented.

5.1.3 Physical model of design case #1: symmetrical shell

The symmetrical shell is part of the workflow defined in Paragraph 3.2.3 where the pre-rationalization approach has been applied. The pre-rationalization has relied on a hexagonal subdivision since it represents an appropriate typology of subdivision for structural and aesthetical reasons. Fig. 5.2 shows the framework of such a symmetrical shell including all the phases.

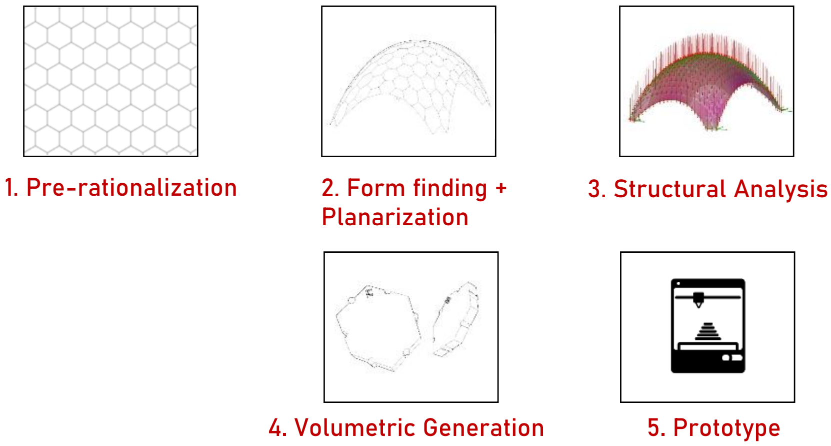


Figure 5.2 Workflow for the whole design process

In this chapter, the focus is on the stages following the form-finding/planarization and structural analysis since they have already been described beforehand. Nevertheless, for better clarity the outcome from such workflow is displayed in Figure 5.3:

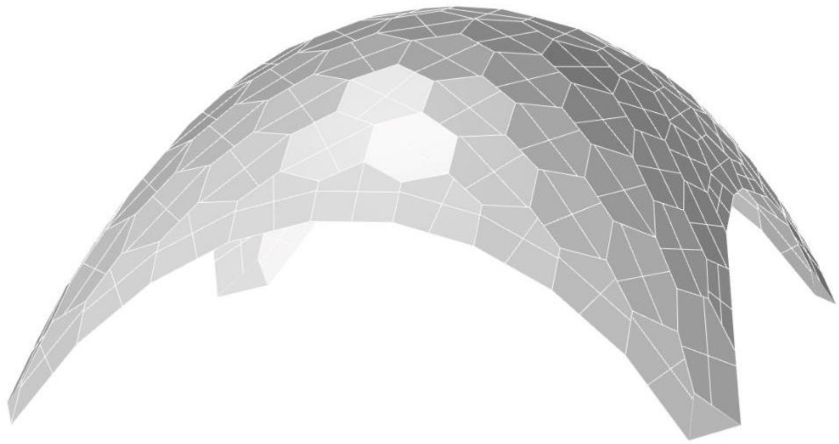


Figure 5.3 Symmetrical shell with its planar hexagonal panels

Starting from these conditions, a geometrical process has been carried out in order to generate 3D hexagonal elements. As referred to in the principles underpinning correct tessellation, the continuous membrane of the shell is necessary to evaluate the normal vectors of the surface. Such normal vectors, as displayed in Fig. 5.4 represent the vectors normal to the surface.

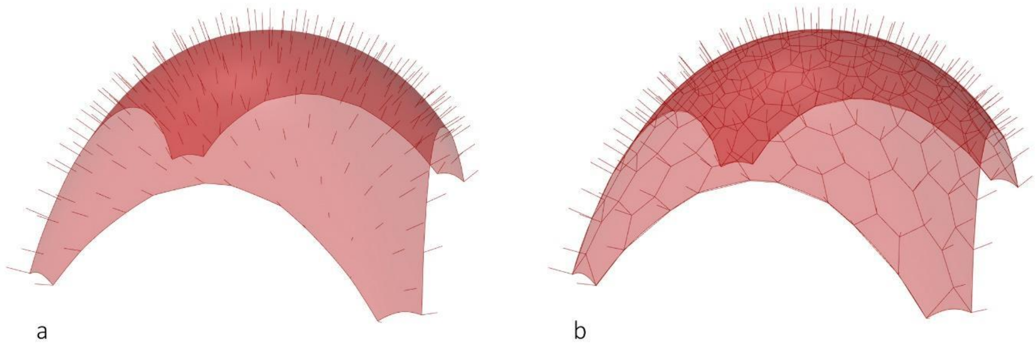


Figure 5.4 Normal vectors: a) on the surface; b) on the vertices

The evaluation of the surface through normal vectors takes place in Grasshopper® by means of an analysis tool which reparametrizes the surface and is able to obtain local surface properties. In detail, the normal vectors are required in the vertices of the planar hexagons so that the correct direction can be defined.

Therefore, before extruding the intradoses, normal vectors are computed in the vertices of the polygons, in this way extradoses are generated according to the value of the thickness.

The process is repeated for all the elements, guaranteeing parallelism between corresponding faces. The surface generation for the extrados (Fig. 5.5) takes into account the end points of the normal vectors which are also the vertices of the future extrados. Such vertices lie on the same plane but a non-planarization condition can occur in certain areas of the shell.

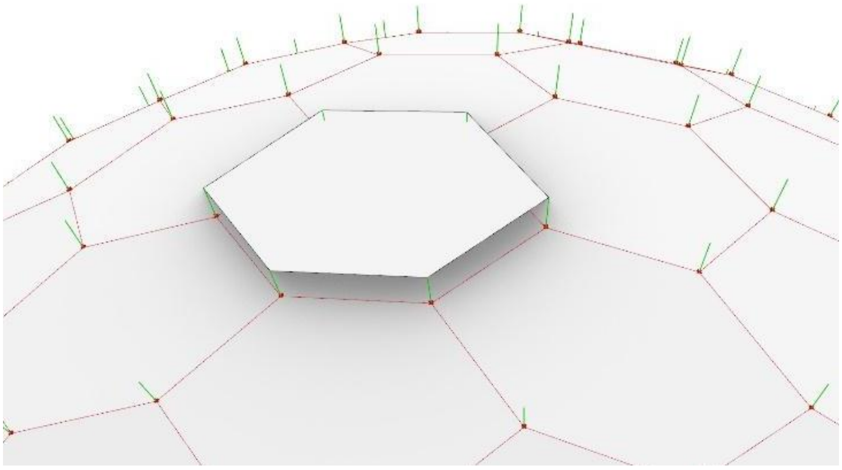


Figure 5.5 Extrados generation

Finally, contact faces are obtained by using the normal vectors as generators (Fig. 5.6). This operation creates ruled surfaces that, by definition, are composed of a set of straight lines. In such a case, the contact faces are single curved surfaces as proved by the curvature analysis (Fig. 5.7).

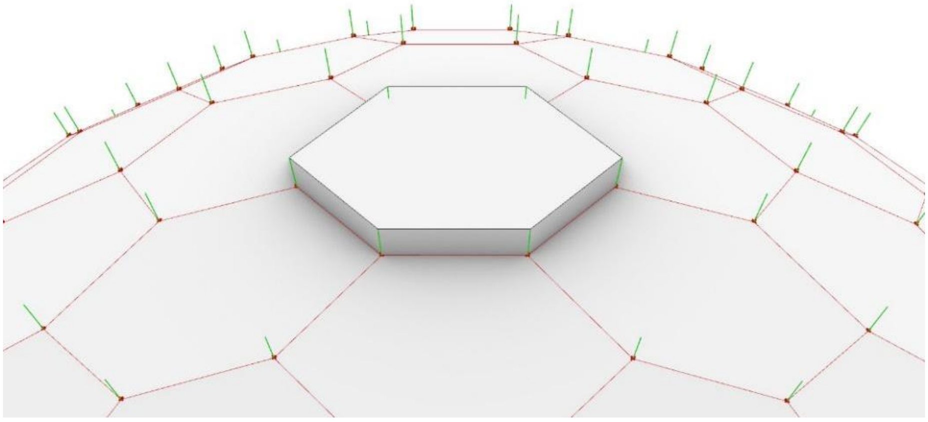


Figure 5.6 Contact face generation

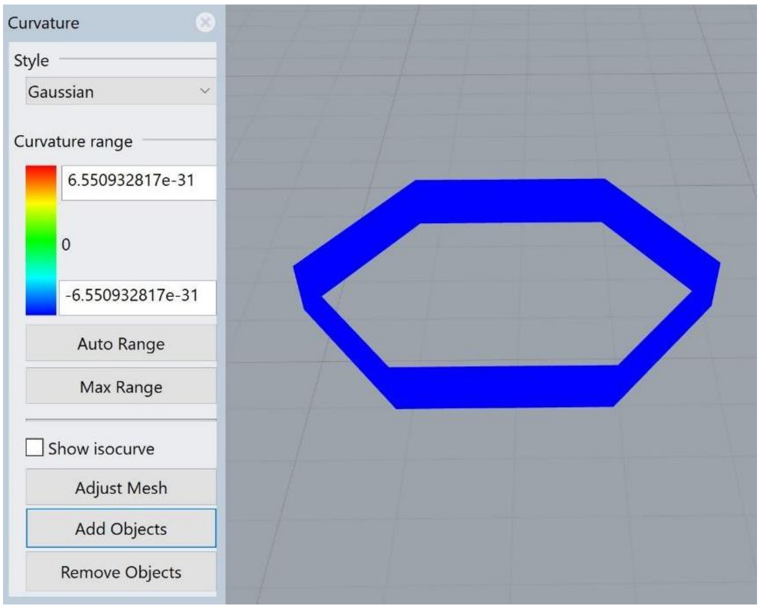


Figure 5.7 Curvature analysis for the side faces

Close to the supports, where there is a significant difference between the directions of the vectors of adjacent elements, non-planar extrados are present (Fig. 5.8). In case of non-planarity, some adjustments regarding the vertical position of the vertices are required to fulfil parallelism, meaning that all the vertices must lie in a common plane. This operation is carried out for all the non-planar extradoses.

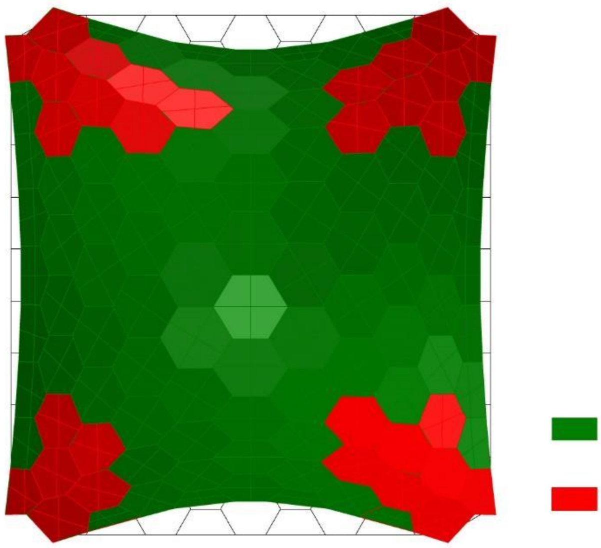


Figure 5.8 Plan view of the shell with indication of planar and non-planar cells (in green planar cells, in red non-planar cells)

2-step structural analysis

This symmetrical shell has been the object of analysis regarding mechanical behaviour in two stages. The first stage has concerned the preliminary structural analysis consisting of a macro model approach with the goal to evaluate the performance of the membrane under prescribed loading conditions. The simulation refers to the conditions prescribed in Paragraph 3.2.4 in terms of loads, material properties and support positions. However, the cross section has been selected to 12 cm according to the reduced dimensions of this model (5.8 x 4m). The results extracted from FEA (Fig. 5.9) have shown the presence of tensile stresses generated by the wind load as described in Table 5.1, in detail in-plane tensile stresses rather than compressive stress are present in the edge zones. The second stage represents a further investigation that takes into account discretized elements and therefore no longer a continuous membrane. By carrying out a straightforward exercise of graphic static on the transversal cross section of the shell (Fig. 5.10), a rigid block analysis has been done with the same assumptions used in the first stage analysis.

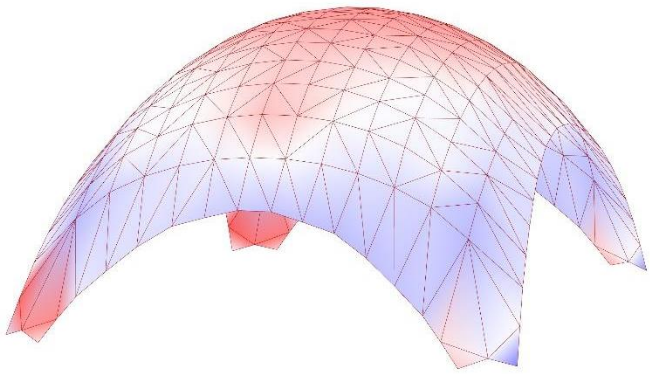


Figure 5.9 Stress visualization on the symmetrical shell

	Case 1
Max comp. stress N/mm^2	0.24
Max tensile stress N/mm^2	0.19
Max Displacement (mm)	0.0014

Table 5.1 Results extracted from FEA

In order to have an arch in static equilibrium, the thrust line has to lie within the thickness of the shell. This is a fundamental condition in order to guarantee that the structure works uniquely in compression (Heyman 1966).

Figure 5.11 shows the graphic static applied to the section of the case study for the load conditions already described in the preliminary structural analysis (refer to Table 4.2). The graph demonstrates that the thrust line does not always lie within the section, which means that the shell does not work only in compression, as already established. Graphic statics proves to be an efficient tool to evaluate the behaviour of the cross section taken.

In conclusions, both analytical and graphical analysis have shown and proved the behaviour not purely in compression of such a shell, meaning that local instability may occur in the areas where tension values are present.

In the hypothesis to pursue an only compression behaviour, adjustments would be necessary in order to have the thrust line inside the cross section (e.g. increasing locally thickness).

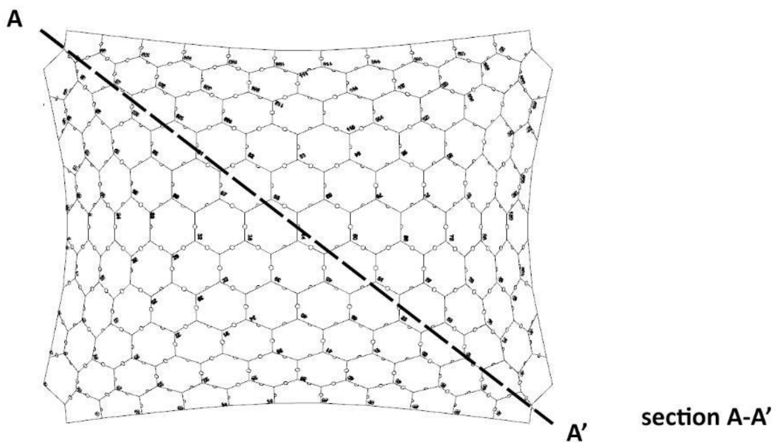


Figure 5.10 Cross section of the vault to analyse by graphic statics

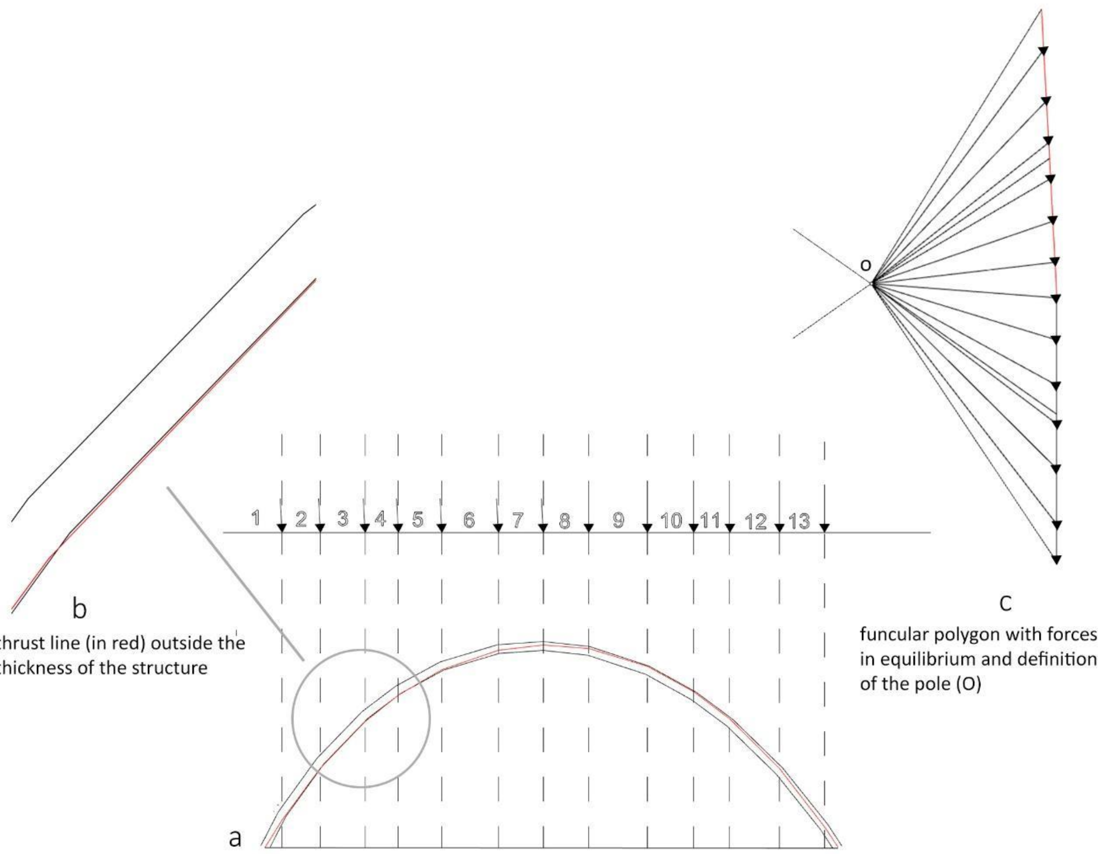


Figure 5.11 Graphic statics process for the cross section of the shell: a) in red the resulting thrust line; b) detail showing the thrust line outside the section; c) funicular polygon

Connection system

The next step within the tessellation process regards the design of a connection system. Taking inspiration from topological interlocking (TI) in architecture (Weizmann et al., 2017), a puzzle-like approach has been considered.

Topological interlocking (Fig. 5.12) is a system of connections, which assures that the elements are held together by global and local constraints depending on their shape and arrangement (Estrin et al. 2011). Different methods were implemented for the design of topological interlocking as well as their application in a parametric environment (Weizmann 2016). Their potential use in freeform architecture is rather promising although it is still under investigation. This case deals with the design of a connection that is not a topological interlocking but exploits the principle of interlocking interpreted as self-stability. The puzzle connection designed intends to be an interlocked system by tapered section which constrains the bottom part of the tessellation. This specific feature could potentially help the assemble phase to be easier and quicker.

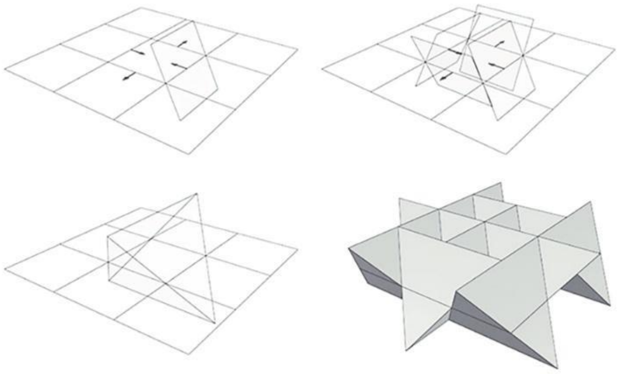


Figure 5.12 Method to develop interlocked components based on a square grid (Estrin et al. 2011)

A parametric definition has been developed to create the connections between panels. Figure 5.13 shows the application to three shell panels.

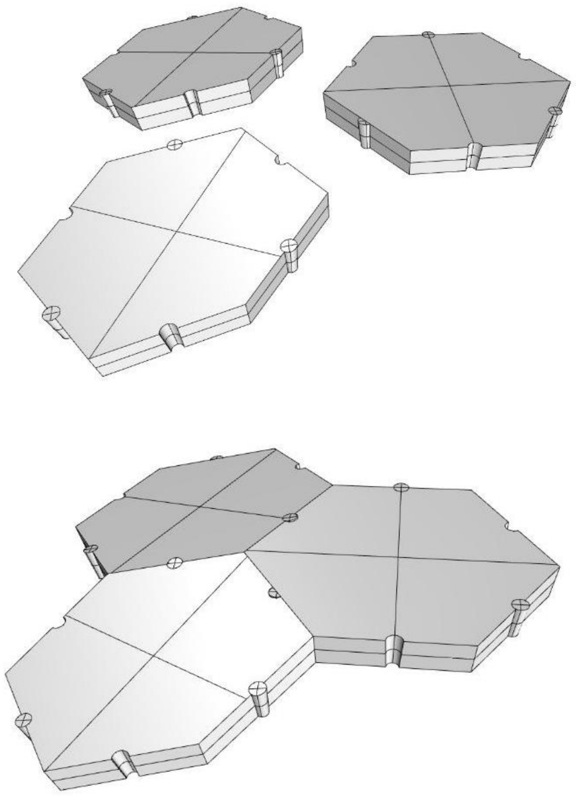


Figure 5.13 Connection system 1 applied on three hexagonal panels

Planarity is a necessary requirement in the design of the connection system since the circular sections of the top and bottom faces can be generated only if the latter are planar. In particular, two sections of different diameters and the successive surface between the two sections are generated, in order to create the interlocking behaviour composed of female-male system.

Experimental scaled models

The use of scale models have always been a fundamental tool in architectural practice; an evaluation tool for aesthetic features but also for a better comprehension of mechanical features. This is particularly valid for structures working in compression (Calvo Barrentin et al., 2017) where the mechanism behaviour can be investigated through scale models according to the principle of scalability by extracting important insights. Indeed, by testing scale models it is possible to compare results from physical tests and digital simulation. In case of unreinforced masonry structures, this principle is further demonstrated by the stability that depends on the geometry rather than the material properties as described by Heyman (1996). This important principle provides a consistent reliability in case of physical models whose scalability does not affect the feasibility of the results obtained.

The models realized have the scope to provide insights regarding the tessellation, the connection system and the stability.

The first experiment regards a portion of the shell structure constructed at a scale of 1:10, as shown in Fig 5.14.

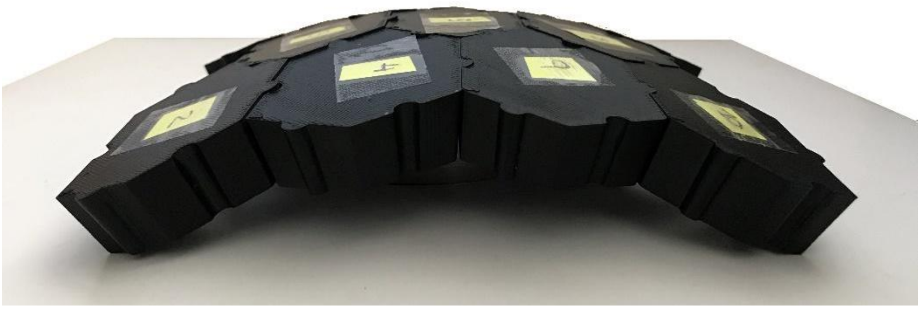


Figure 5.14 First test on a portion

The scale model has been realized through additive manufacturing by using the printer Stratasys F370 FDM in order to produce the prototype quickly and economically, minimizing the use of material and fabrication time. The prototype is made of PC-ABS (polycarbonate-acrylonitrile butadiene styrene), a material widely used in FDM (fused deposition modelling) processes, thanks to its strength and heat resistance. The total time to fabricate this prototype was

6 hours for 11 units. In particular, the scope of this portion was to analyse the connection system and the orientation of the contact faces. This first model demonstrated that the connections efficiently allowed the interlocking of the tessellation. This represents the first test for the connection that assesses the feasibility of such a system.

After evaluating the feasibility of the system, a new scaled model of a symmetrical shell was designed and digitally fabricated in its entirety. A base for an ad hoc support system as well as volumetric foam have been developed to help with the assembly of the scale model. Moreover, the foam can be removed by lowering it with a pump system once the model is set in place (Fig. 5.15).

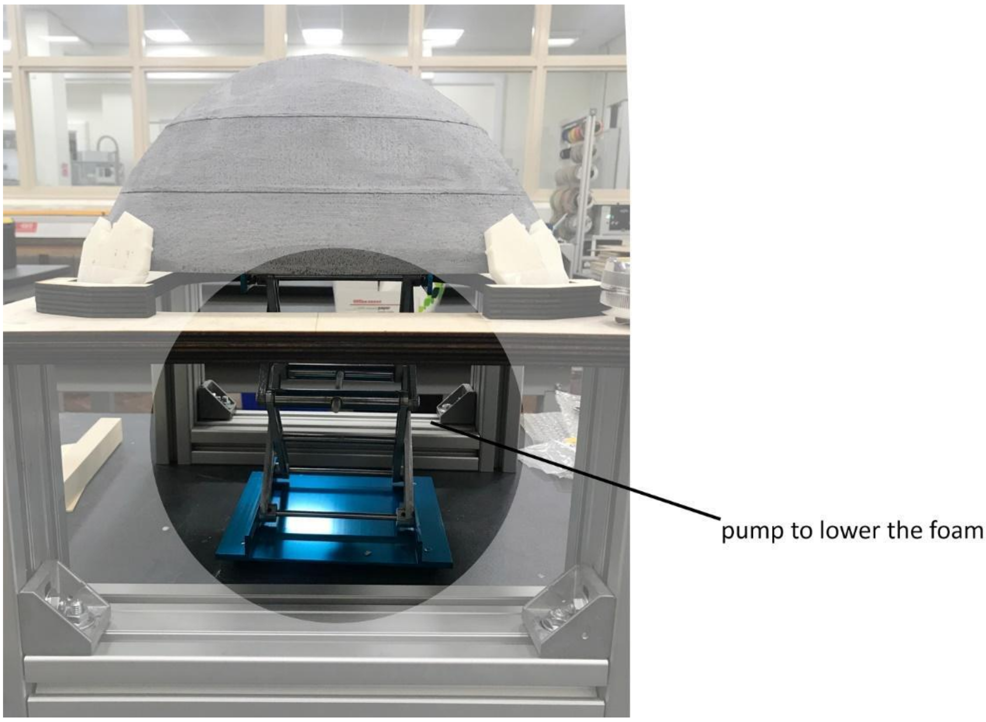


Figure 5.15 Pump system to lower foam once the shell is assembled

The model is in scale 1:15 and is realized in PC-ABS. The construction of this model has contributed to a better understanding of its strengths and weaknesses.

The aim of this model was to demonstrate the efficiency of the interlocking system and capacity of the shell to resist its own weight without the necessity to add any extra material (the equivalent of mortar in a masonry shell). However, the experimental model (Fig. 5.16-5.17) has shown the limitations of the developed shell. It indeed demonstrated that a shape which is non-funicular, therefore not working only in compression, as demonstrated by the results of the structural analysis and graphic statics, struggle to take its own shape. In particular, the stability can be undermined along the unsupported edges. Indeed, the four edge arches were not able to develop the arch effect at the same time, which caused the failure of the model.

However, the model demonstrated that an improvement of the shape or an improvement of the connection system could allow the shell to prevent local collapse. Moreover, a redesign of the connection system could also allow the shell to be assembled without support (i.e. foam).

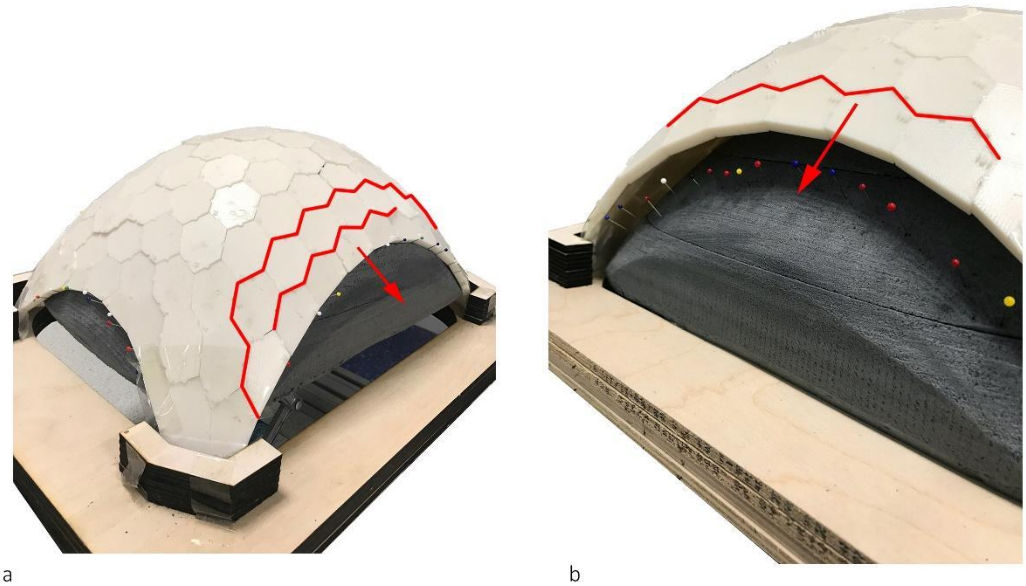


Figure 5.16 Scaled model of the symmetrical shells (a and b), the red lines show the free edges that tend to collapse

Figure 5.17 Local collapse along unsupported edge after the assemble

Collapse mechanism of the shell

The feasibility of the model has been evaluated through two approaches: a numerical approach based on the development of FEA of the shell as a membrane (macromodel), and an experimental approach consisting of a scaled physical model that was assembled by using a connection system. In fact, the macromodel and experimental approaches delivered results which were used to formulate conclusions regarding the structural efficiency of the shell but the lack of a high accuracy of such results may provide some limitations in the assessment of the model. At this stage of the research, following the definition the volumetric tessellation of the shell, it was possible to carry out a more in depth analysis regarding the collapse mechanism under prescribed loading conditions. Rigid block limit analysis has been a key-approach for the assessment of masonry buildings, in particular for the preservation of historic buildings (Gilbert and Melbourne, 1994) (Block et al, 2006). Such a methodology is generally based on the application of limit analysis principles, taking into account the discretization of the structure, imagining that there is no tension and friction on the contact faces.

An innovative software tool developed at the University of Naples Federico II, LiABlock_3D, implements rigid block modelling based on limit analysis and it is able to define load collapse factor, collapse load and the process time or CPU time in order to predict the behaviour of the shell (Cascini et al, 2018). Its potentialities have been extensively demonstrated within the analysis of masonry structures (Gagliardo et al. 2019). The LiABlock is a modelling approach based on MATLAB scripts and it is composed of three main stages: the model definition, the solver process and the output as described below (Fig. 5.18). The process has been analysed by referencing to the symmetrical shell object of investigation in this chapter. LiABlock_3D can analyse structures made of 3D polyhedral blocks by importing the relevant information from an Excel document. The first step to take regards the definition of the attributes related to each piece of the shell and to do so the geometry is modelled as a set of AutoCAD blocks with the following defined attributes: the Cartesian coordinates of the vertices of the 3D hexagons by using the object “points”, their centroid, and the definition of the contact surfaces which in this case represent the side faces of the geometries, the volume and a final label which define the number of edges of the blocks (Fig. 5.19).

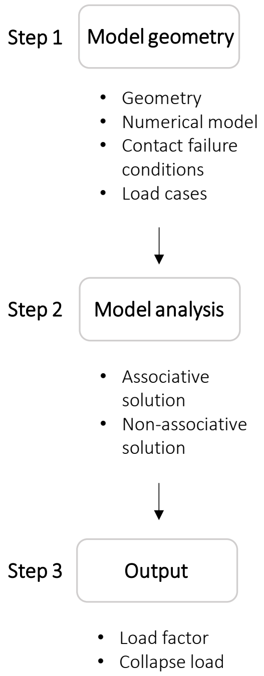


Figure 5.18 LiaBlock_3D workflow

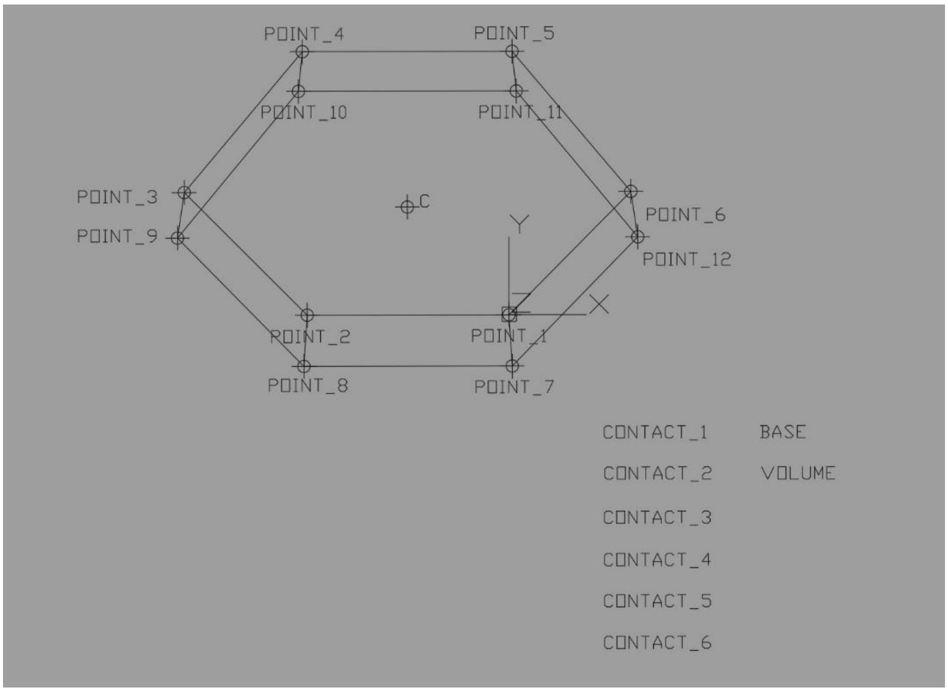


Figure 5.19 Definition of the attributes for a an hexagonal block

After the definition of the labels, the model data are ready to be imported in the software in order to generate the numerical model. Such labels are exported from AutoCAD as an Excel document (Fig. 5.20) which generates the model within LiaBlock_3D. The analysis relies on a few parameters, which are the friction coefficient μ and weight per unit volume ρ as well as the boundary and loading conditions.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	Count	Name	BASE	C	CONTACT_1	CONTACT_2	CONTACT_3	CONTACT_4	CONTACT_5	CONTACT_6	POINT_1	POINT_2	POINT_3	POINT_4
2	1	BLOCK10	5	-0.599, 0.826, 1.386	1,2,8,7	2,3,9,8	3,4,10,9	4,5,11,10	5,6,12,11	1,6,12,7	-0.491, 0.604, 1.032	-0.734, 0.595, 1.251	-0.872, 0.814, 1.539	-0.778, 1.104, 1.655
3	1	BLOCK100	5	-4.304, 1.368, 1.742	1,2,8,7	2,3,9,8	3,4,10,9	4,5,11,10	5,6,12,11	6,1,7,12	-4.171, 1.104, 1.653	-4.411, 1.121, 1.441	-4.487, 1.426, 1.514	-4.386, 1.672, 1.742
4	1	BLOCK101	5	-4.267, 1.944, 1.986	1,2,8,7	2,3,9,8	3,4,10,9	4,5,11,10	5,6,12,11	6,1,7,12	-4.130, 1.657, 1.951	-4.366, 1.672, 1.742	-4.441, 1.981, 1.755	-4.332, 2.252, 1.926
5	1	BLOCK102	5	-4.243, 2.544, 2.110	1,2,8,7	2,3,9,8	3,4,10,9	4,5,11,10	5,6,12,11	6,1,7,12	-4.099, 2.243, 2.133	-4.332, 2.252, 1.926	-4.413, 2.557, 1.878	-4.318, 2.850, 1.987
6	1	BLOCK103	5	-4.241, 3.154, 2.109	1,2,8,7	2,3,9,8	3,4,10,9	4,5,11,10	5,6,12,11	6,1,7,12	-4.086, 2.848, 2.194	-4.318, 2.850, 1.987	-4.409, 3.141, 1.878	-4.328, 3.448, 1.923
7	1	BLOCK104	5	-4.261, 3.757, 1.983	1,2,8,7	2,3,9,8	3,4,10,9	4,5,11,10	5,6,12,11	6,1,7,12	-4.095, 3.455, 2.131	-4.328, 3.448, 1.923	-4.434, 3.719, 1.755	-4.361, 4.030, 1.736
8	1	BLOCK106	5	-4.298, 4.337, 1.786	1,2,8,7	2,3,9,8	3,4,10,9	4,5,11,10	5,6,12,11	6,1,7,12	-4.124, 4.045, 1.945	-4.361, 4.030, 1.736	-4.478, 4.277, 1.513	-4.407, 4.585, 1.432
9	1	BLOCK107	5	-4.342, 4.881, 1.375	1,2,8,7	2,3,9,8	3,4,10,9	4,5,11,10	5,6,12,11	6,1,7,12	-4.166, 4.602, 1.644	-4.407, 4.585, 1.432	-4.532, 4.807, 1.159	-4.455, 5.105, 1.018
10	1	BLOCK11	5	-0.639, 1.366, 1.743	1,2,8,7	2,3,9,8	3,4,10,9	4,5,11,10	5,6,12,11	6,1,7,12	-0.583, 1.119, 1.443	-0.778, 1.104, 1.655	-0.906, 1.349, 1.893	-0.813, 1.656, 1.952

Figure 5.20 Data extracted from AutoCAD in Excel

The proposed approach takes into account the internal forces, which are related to each contact points, in detail the normal force n_k and shear forces t_{1k} and t_{2k}. Two different contact failure modes are assumed: opening and sliding of interfaces at contact points, governed by the following conditions with the assumption that the shear failure is governed by a Coulomb type criterion:

$$-n_k \leq 0 \tag{19}$$

$$\sqrt{t_{1k}^2 + t_{2k}^2} - \mu n_k \leq 0 \tag{20}$$

A load condition f_i *is* applied to the centroid of each block. It represents the sum of the dead load f_{Di} and of the live load f_{Li}, increased by an unknown scalar multiplier α:

$$f_i = f_{Di} + \alpha \times f_{Li} \tag{21}$$

The self-weight of each block is calculated according to the unit weight of the material ρ and the volume attribute V_i.

It is possible to define the directions of the live loads along the three global coordinate axes, such loads can be applied on a group of the block or to the whole geometry.

The model is composed of 138 blocks with a total of 1516 contact points.

Figure 5.21 and table 5.2 show the parameters defined for the friction coefficient μ, the weight per volume ρ and the load directions.

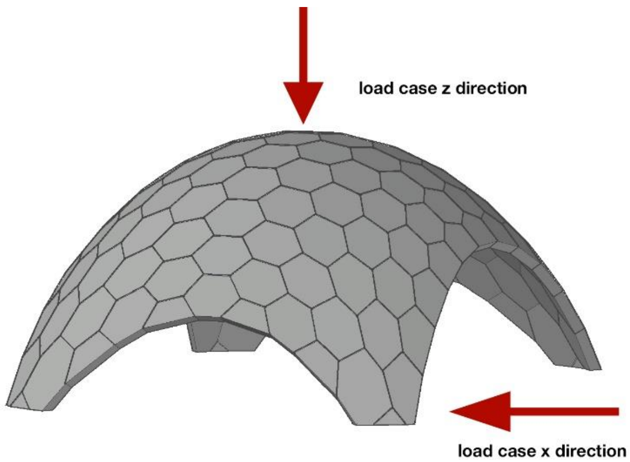


Figure 5.21 Directions used for the load conditions

friction coefficient μ	weight per volume ρ	load conditions
0,7 0,75 0,8 0,85 0,9	19	Case 1: load x direction (whole shell) Case 2: load z direction (portion of shell)

Table 5.2 Parameters and load conditions for the simulation

The outcome is represented by a collapse mechanism which defines the load factor and the collapse load computed in the case of associative friction model. The associative model assumes dilatancy when there is sliding at a contact point, representing an upper bound value for the load factor (Portioli et al, 2014).

Table 5.3 displays the results obtained from the analysis with a different value of friction coefficient. As concerns load case 1, the collapse of the shell is local when a lower friction coefficient is used with a consequent loss of cohesion (Fig. 5.22); with the friction coefficient of 0,8 (Fig. 5.23) the collapse starts to become global, meaning that the shell behaves accordingly to the lateral load condition.

Load condition	Associative solution		CPU time (s)
Friction coefficient 0,7	α	αF_L(kN)	
Case 1	0,019911	-1,7175	0,2598
Case 2	0,081295	-0,4294	0,1981
Friction coefficient 0,75			
Case 1	0,15412	-13,2947	0,2763
Case 2	2,0272	-10,7078	0,2181
Friction coefficient 0,8			
Case 1	0,23946	-20,6555	0,2487
Case 2	3,15	-16,6385	0,2029
Friction coefficient 0,85			
Case 1	0,29785	-25,6924	0,2004
Case 2	3,8934	-20,565	0,1860
Friction coefficient 0,9			
Case 1	0,34421	-29,6915	0,2523
Case 2	4,6227	-24,417	0,1902

Table 5.3 Results extracted from the analysis

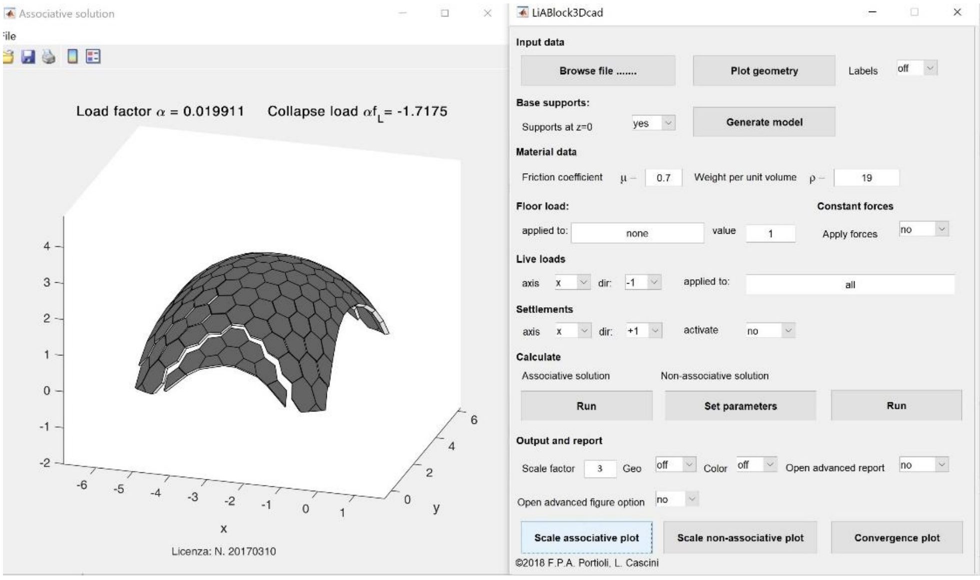


Figure 5.22 Collapse mechanism for the x-load on the central portion of the shell (μ=0,7)

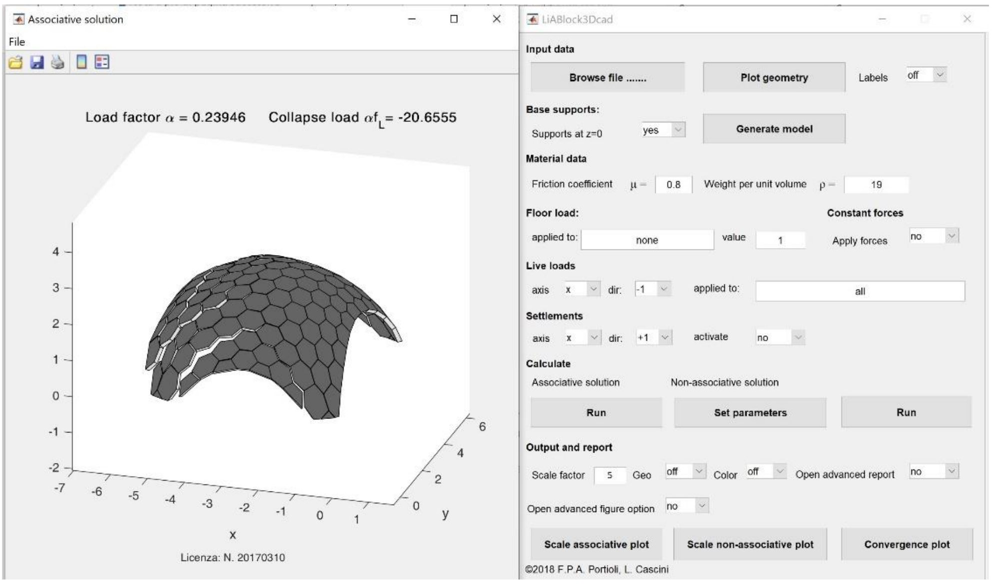


Figure 5.23 Collapse mechanism for the x-load on the central portion of the shell (μ=0,8)

The load applied on the central portion (red area) of the shell triggers a global mechanism on the shell, however a lower value of friction coefficient may lead to a collapse of macro-portions of the shell especially along the edges. Figure 4.24 shows the collapse mechanism for the middle value (μ=0,8).

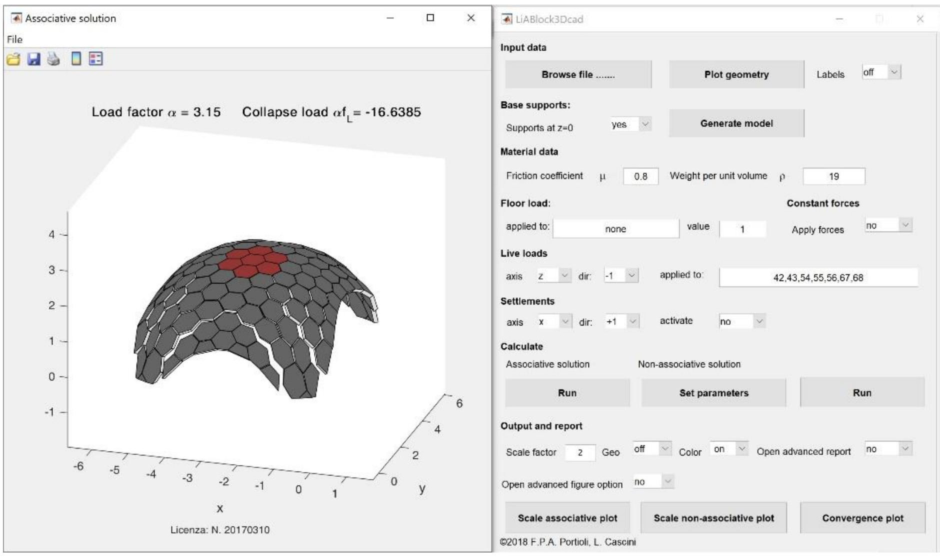


Figure 5.24 Collapse mechanism for the z-load on the central portion of the shell (μ=0,8)

The analysis has showed some critical issues with lower values of friction, mainly due to geometry of the shell that play a key-role in the overall stability. The thickness (12 cm) if incremented may provide better results even with low values of friction, otherwise the increment of the friction coefficient (μ=0,8-0,85) has led to a better performance. The use of a connection system redesigned according to such requirements can help to improve the stability of the shell.

5.1.4 Physical model of design case #2: Echo Pavilion

The second case developed with the need for minimum formwork consists of a shell made of a thin material. It differs from the first case for several reasons: it is not a shell obtained from a form-finding process, therefore it presents a different structural behaviour with a material that does not work in compression and the tessellation process is subject to a different set of requirements. However, its design and realization represent a significant part of such a research from both the digital and physical perspectives. Operations of rationalization and planarization have been implemented in parametric modelling with successful outcomes and the whole construction was realized by digital fabrication techniques. Moreover, the connection system proves to be in this case pivotal for the use of minimum formwork and for the whole stability.

Therefore, the development of this design process allowed it to demonstrate the potentialities of the connection system.

ECHO is a lightweight wooden shell, result of an interdisciplinary teamwork carried out in occasion of a competition for innovative pavilions at the 2019 IASS Symposium. It is made of 94 CNC hexagonal panels with a thickness of only 6 mm made of timber plywood, connected together by 144 ad hoc 3D printed connections. The initial surface was developed in Rhinoceros®, the surface was subdivided into hexagonal cells by using LunchBox plugin and a planarization process was carried out in order to obtain flat panels. The whole design process was implemented in Grasshopper® (Fig. 5.25). At this stage, there was no need to focus on the digital process, since the rationalization and planarization have been implemented with a straightforward approach described in paragraph 4.1.6. The panels were extruded along their normal vertices according to the required thickness value.

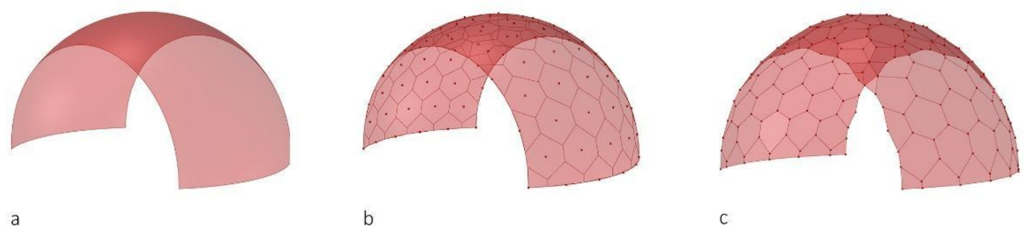


Figure 5.25 Design process for the pavilion: a) shape generation; b) hexagonal tessellation; c) planarization

Connection system

Such a structure does not work in pure compression from a geometrical point of view further evidenced by the choice of the material as well, therefore a connection system was required in order to prevent buckling for panels. The panels use finger-jointing (Fig. 5.26) along their profiles in order to have the right orientation and an initial interlocking but an additional connection system was necessary at this stage to provide more stability.

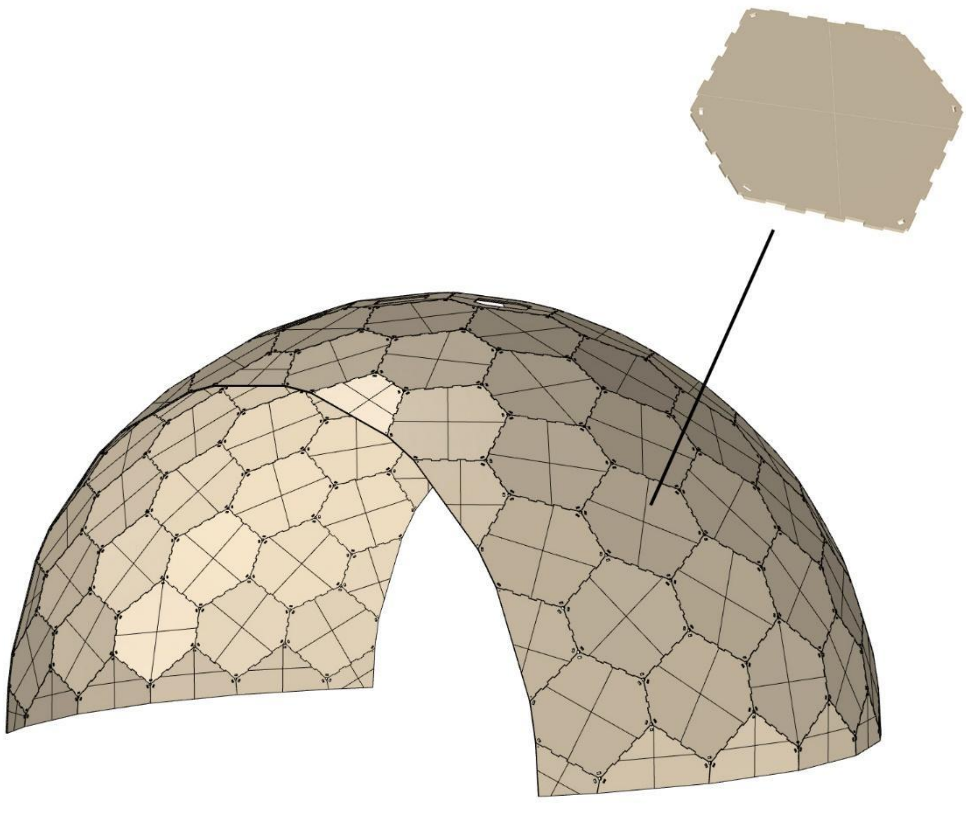


Figure 5.26 Finger joints applied on the panels with a detail of a panel

The connections were developed using Autodesk® Fusion360 first and then parametrized within Grasshopper® (Fig. 5.27). The final design was 3D printed as a 2-part body that will interlock 3 panels at time. Careful selection of material, infill value and layer height were done to reduce the amount of material and minimize the printing time.

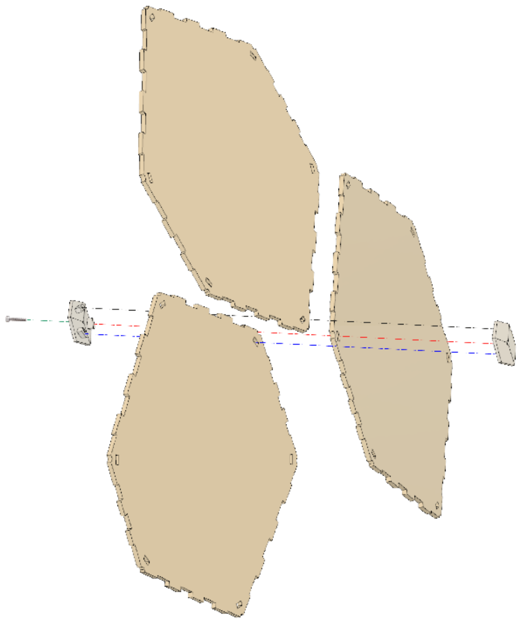


Figure 5.27 Connection system

Experimental physical tests and full-scale model

The connection system was tested through the fabrication of a portion of the shell in order to check their stability. In detail, three panels were fabricated demonstrating the feasibility of the system (Fig. 5.28). Moreover, it was asserted that they are able to preserve the angle between them with very good degree of accuracy.

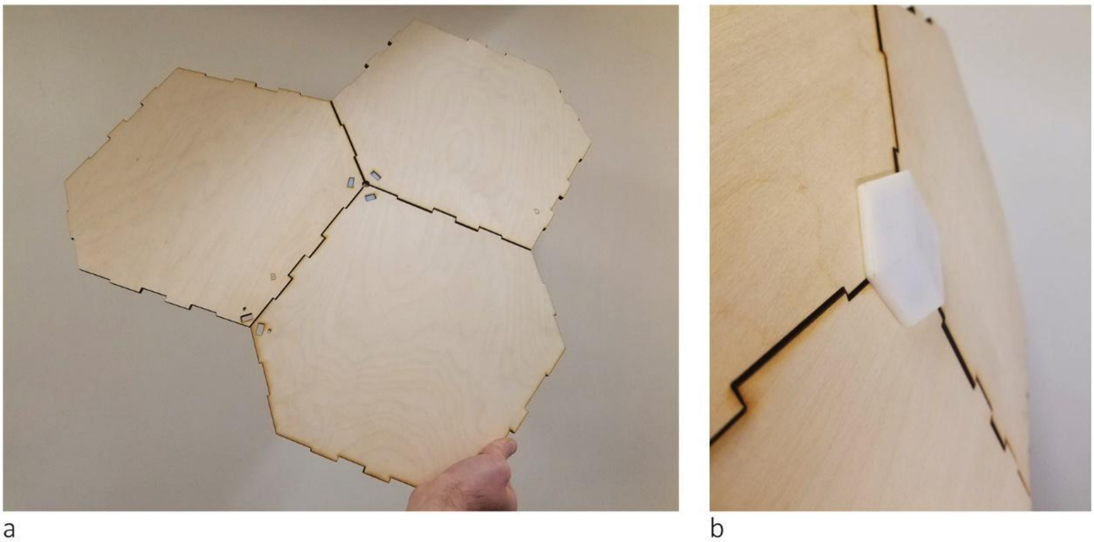


Figure 5.28 Details of the panels: a) three panels assembled together; b) top part of the connection system

ECHO was realized by using different digital fabrication techniques: the connections were produced by 3D printing and the plywood panels were realized by laser-cut. The construction of a full-model scale started with a fixing system for the supports by using a continuous element (Fig. 5.29a).

The full-scale model was assembled with the use of minimum formwork (Fig. 5.29b). The whole construction took half a day to be completed with a maximum of three people working simultaneously (Fig. 5.30).

This process of construction was simplified thanks to the finger joints of the panels that provide more stability as well as the clamping force exerted by the connections.

a

b

Figure 5.29 Realization phase of the shell: a) fixing system for the supports; b) construction process of the shell

Figure 5.30 Full-scale model of ECHO (3.36 x 2.46 m), height max 2.10 m

5.1.5 Summary

In this chapter, the realization process has been carried out in order to validate digital processes implemented in Grasshopper®, by using digital fabrication techniques. In detail, the tessellation has taken a step forward with the design of a volumetric tessellation and its materialization. Two main cases were developed in order to demonstrate that a) a correct design process can deliver optimized outcomes that can be digitally fabricated; and b) it is possible to realize stable structures that make use of minimum formwork by the use of a connection system. It was important to define two different cases in order to cover a wider set of requirements: a shell working in compression with an integrated interlocking system and a lightweight wooden shell with an external system.

In the first case, a puzzle-like connection system has been developed and a scaled model realized. However, the results extracted from the several experimental tests demonstrated that the structure did not result stable along the unsupported edges. Therefore, a rigid block analysis was introduced to deliver more accurate results. The results have proved the reliability of the scaled model that represents a valid tool for a preliminary structural evaluation. The analysis confirmed some critical issues in terms of stability but with a higher value of friction the problems can be overcome. Therefore, the connection system can be redesigned in the future in order to guarantee a stronger interlocking behaviour together with the increase of the thickness.

The second shell was realized with wooden panels connected by an external 3D printed connection system composed of two parts that interlock the panels in an efficient way. The finger joints on the panels allow the right orientation and an additional aid during assembly. The realization of a full-scale prototype has shown that minimum formwork is required, without compromising the stability of the structure.

Both structures were entirely fabricated by using standard digital fabrication techniques, without the need to customise fabrication setups, thanks to the optimisation processes implemented in parametric modellin

6. Conclusions and recommendation

6.1 Overview

This thesis addresses the investigation of shell structures by the definition of design workflows. Geometrical and structural requirements are embedded in the early design phases in order to optimize the design process. The literature review has defined key-aspects and limitations in this field, especially valid in the practical implementation of strategies within architectural practice. Parametric design for shell structures represents an established and well-known topic, physical models were a fundamental medium to deal with the form-finding of the shell design and successively the digitalisation of such techniques have contributed to extend their implementation. However, the design process is not largely investigated yet in architectural practice where many problems regarding geometrical subdivisions, planarization and optimisation are addressed on a theoretical basis with few practical implementations within digital computation. At this stage, a reformulation of the design process in a straightforward way implemented with parametric software has been carried out in order to promote their use in practice. In order to address this gap that concerns a practical application, a range of problems together with potential solutions was defined for each case study: this was possible through the definition of symmetrical and asymmetrical structures in order to have more flexibility in terms of shape definition, without limitations in the formal aspects.

This work drew particular attention in the formulation of practical indications for the design of shell structures aiming to create connections between academic and professional activities, between research and practice, in order to give support to students and architects that want to explore the design of shell structures. For this reason, design guidance covering all aspects of shell design from conceptualization to fabrication have been proposed. The design process has been carried out within Grasshopper® environment and by the use of commercial plug-ins.

Such a work has dealt with two main fields of interest: the digital process and the fabrication process.

6.2 Digital Process

The digital process is a core part of the thesis aiming to provide design guides for the parametric design of shell structure. In detail, workflows regarding the subdivision of the input surfaces, form finding, planarization, structural optimisation, tessellation have been elaborated with different degrees of complexity, depending on the set of constraints applied in the process.

The thesis has defined design workflows for symmetrical and asymmetrical shells. A first investigation considered the influence of the input geometries in the form-finding, providing an in-depth analysis of the main differences between curves and meshes used in the simulation. Strategies regarding triangular, quadrangular and hexagonal subdivisions, including mesh subdivisions techniques, were presented in order to deal with input geometries involved in the form-finding.

Planarization represents another key-aspect of the design process and it was implemented taking into account different geometries and a range of constraints, starting from simple geometries (domes and other symmetrical shells) to complex shapes (asymmetrical shell). The set of constraints was defined according to the kind of geometry in order to overcome potential limitations, such as geometrical deformations. In the case of planarization of hexagonal shells, two different strategies have been compared, pre-rationalization and post rationalization approaches. Considering the rationalization of the shell before or after the form-finding simulation can provide important remarks in terms of geometrical optimisation and structural efficiency. At this stage, form-finding and planarization problems have concerned symmetrical shells, however a further investigation has dealt with asymmetrical shells in order to extend the potentialities of these parametric workflows to a wider set of cases. The asymmetrical shells were analysed taking into account the possibility to use different subdivisions for asymmetrical geometries: hexagonal and Voronoi. They followed the same workflow, consisting of the mesh subdivision, form-finding and planarization, delivering promising results. Furthermore, for the case of asymmetrical shells, the optimisation process has investigated the correlation between tessellation and structural performance with the goal to optimize the structural behaviour by controlling the density of the tessellation and minimizing the total displacement. In regards to the structural optimisation, the problem was identified from another perspective as well: in detail, an asymmetrical case which took reference from Isler's unrealised shells has been subject to structural multi-objective optimisation in order to extend its study in the design of shell structures but in this case the correlation between support positions and structural efficiency was investigated.

6.3 Fabrication Process

The fabrication process was implemented taking into account two different shells, proving to be a fundamental tool of evaluation in the design process.

The first case is a symmetrical shell subject to a form-finding process, hexagonal tessellation and planarization and a connection system. The fabrication process was developed in order to assemble the shell with minimum formwork. For this purpose, the tessellation and the connection system have undergone a refinement process and a puzzle-like connection was developed in order to ensure stability, minimizing supporting structures in the realization phase.

In detail, the tessellation was investigated taking into account architectural and structural requirements and volumetric tessellation was carried out generating planarized extradoses and ruled contact faces. The puzzle-connection was developed parametrically and successively tested on a scaled model. The scaled model was digitally fabricated including a supporting structure realised by digital fabrication as well. Since this first attempt did not provide stability throughout the shell, the shell was investigated by rigid block analysis, taking into account the discretized structure. Such an analysis has provided results that are consistent with the scaled physical model, highlighting the importance to improve the clamping force of the connections and to increase the thickness of the shell.

The second case has regarded a full-scale prototype developed for an international competition that can be assembled and disassembled with minimum formwork. The shell was subdivided into hexagonal cells and then planarized. Such a shell has a different structural behaviour from the first case since it is not generated from a form-finding process, it is made of wooden material and a connection system was developed in order to prevent buckling. In this case, the connection system is an external body composed of two parts. It consists of top and bottom sections that interlock the panels and ensure stability during assembly. The assembly was also facilitated by the introduction of finger joints between the panels that allow the panels to accommodate different orientations. The whole design process and fabrication was validated with the realization of the shell in full scale.

6.4 Contributions to the digital process

Although new methods have not been developed for the design of shells, this thesis has provided guidance for the parametric design of shell structures covering geometrical and structural aspects through an in-depth investigation. In detail, the main contributions are:

- The investigation and implementation of discretization techniques for input geometries by using several strategies, and indication of their potentialities within design process. Their evaluation proved to be fundamental, since they represent one of the first inputs of the form-finding process. The discretization of the surface was carried out by curves and meshes, and both delivered successful results. The study has proved that the discretization in curves has the advantage to define in the early phase the kind of subdivision to implement in the realization phase. As regards the mesh subdivision techniques, the one implemented by Dynamic Remeshing represents the most appropriate in terms of smoothness of the geometry, while the quadrangulation is efficient only for untrimmed surfaces, as shown by the final 3D shapes.

- The investigation and implementation of the form-finding and planarization problems integrated within the same digital workflows. Form-finding and planarization were developed for a symmetrical shape by following different processes: form-finding of a quadrangular mesh and successively subjected to hexagonal tessellation and planarization, form-finding of a triangular mesh and successively subdivision and planarization oh hexagonal cells, form-finding and planarization of quadrangular shell, form-finding and planarization of hexagonal shell. Each case presents a set of constraints justified by the differences in terms of geometric inputs and subdivisions. The diversified sets of constraints have allowed planarization of the shapes to be achieved. In order to investigate the relationship between form-finding, planarization and structural efficiency, the final shapes were analysed by FEA integrated in Grasshopper®. It was demonstrated that when there is a greater number of constraints applied in the simulation, the shapes assume a less "funicular" configuration, higher stress values are generated throughout the shell. Tensile stress values ranging from 0,14 N/mm^2 for the funicular shape to 0,37 N/mm^2 for the case with more constraints.

- Development of two main workflows regarding pre-rationalization and post-rationalization approaches was applied on a symmetrical hexagonal shell. The pre-rationalization approach showed a good solution in terms of geometrical limitations, overcoming potential deformations occurring at the support areas, the post-rationalization approach follows simulation that provides funicular shapes, therefore with a better structural behaviour when dealing with materials working in compression. However, the latter does not take into account the planarization issue, therefore both approaches are beneficial for different reasons, depending on the requirements to fulfil.

- The development of hexagonal and Voronoi subdivisions and their planarization, which have been addressed by considering different shapes in order to cover a wide range of cases and providing solutions by making use of different sets of constraints according to the complexity of the problem. The outcomes were successful in the case of planarized funicular asymmetrical shapes.

- The application of Genetic Algorithms (GA) as a structural tool for asymmetrical shells in order to improve their efficiency and integrate it within the design workflow. Since the shape is the result of the form-finding process, minimum intervention on the shape was done, by formulating the problem from the perspective of the tessellation aspect: the geometrical subdivisions was parametrised and used as variable in the simulation by controlling the density of its grid. Form-finding and optimisation were used as design tools in the same workflow with promising results. The results extracted from the simulation have showed that for both hexagonal and Voronoi shell, a more dense tessellation is connected to a minimization of the displacement showing a similar value (2,49 cm for the Hexagonal shell and 2,45 cm for the Voronoi shell).The results in terms of height have shown different behaviour. The hexagonal tessellated shell presents a shallower shape (4.8m) than Voronoi tessellated shell (7.9m). Another problem that has considered a different set of variables was formulated in order to further evaluate the effectiveness of GA within the design process of shell structure and this was done by using a multi-objective solver. In detail, supports position and the overall height were parametrized in order to minimize mass and displacement. In case of multi-objective simulations, a collection of outcomes is provided and the decision-maker role of the architect in this process is fundamental in order to select the best outcome according to the requirements to prioritise. In this case, the best results were selected showing different values for the fitness functions, ranging from the first case where displacement is optimized over mass to the last case with the optimisation of the mass over the displacement. The middle case (n.3) presents an average minimization values of mass and displacement.

- The development of guidelines for digital design of shell structures that include tessellation, planarization and structural optimisation. To this end, the appendix covers all the different topics addressed in the thesis from a purely technical and digital perspective with the aim to support designers when dealing with these matters. The whole definitions are described, divided by categories and are easy to follow even for designers with little experience in parametric modelling. Such digital workflow are useful guidelines in order to carry out digital operations within the design process.

The second core part of the thesis regards the fabrication process. Taking into account geometrical and structural requirements are fulfilled in order to implement a volumetric generation of two shells in particular.

6.5 Contributions to the fabrication process

- Development of a scaled physical model that shows great reliability in the evaluation of the structural behaviour in a preliminary analysis validated through the comparison of the results with rigid block analysis.

- Development of a connection that is integrated in the symmetrical shell and is based on the interlocking behaviour of a puzzle like system. Hexagonal volumetric pieces are composed of planar intrados and extrados and contact ruled faces in order to follows structural and geometrical requirements for fabrication. Connection system proves to be an important strategy in the design process of shell structures to overcome instability and assemble problems. However, a further improvement of such a system is required.

- Development of a full-scale model of a second shell that can be assembled with minimum formwork and can be assembled and disassembled multiple times. The shell is digitally fabricated and realized with lightweight material.

- Development of a connection system applied on the full-scale shell, designed as a two-part body, able to provide stability to the whole structure with minimum use of the formwork.

6.6 Critical review of the digital tools

The digital tools have played a key role in this research. The implementation of all the strategies investigated was possible thanks to their implementation in powerful and intuitive digital environments. In order to gain a complete understanding especially related to an architectural perspective such tools have been reviewed by taking into account potential limitations. When dealing with different phases integrated in one workflow technical limitations may occur. Since form-finding techniques implemented in Kangaroo provides as output a mesh, while subdivision techniques deal with surfaces, a problem of compatibility between different geometrical entities arises when implementing these procedures, therefore a reconversion from mesh to surface needs to be done by retrieving a surface that approximates the form-found mesh. Regarding the structural analysis implemented in Karamba, some limitations regards the type of approach taken: Karamba allows to analyse the structures via a macro-model approach, which can be e an efficient tool for architects that want to run preliminary analysis. More advanced tools instead are typical of structural engineering. At this stage, it is important to point out that the design guides provided in this work represent a beneficial tool for architect especially related to the form-finding and planarization aspects thanks to the high reliability of the outcomes. However refined structural analysis need to be performed in order to realize safe structures. The analysis run in this work represents a reliable starting point as demonstrated but the support of structural engineers is a mandatory step for their realization.

6.7 Recommendations for future works

The digital designs have been performed by using Particle-Spring Systems. It is established that the use of a constraint-based approach limits the freedom in terms of architectural and aesthetic features, leading to outcomes that are rather similar to each other in terms of design features. By definition, the form-finding process does not give a significant field of action to designers. The investigation of surface rationalization has concerned two typologies: hexagonal and Voronoi subdivision, since they present many potentialities not only for aesthetic features but also for the definition of efficient structural models However, a further investigation regarding quadrangular panels could be carried out in the future. Another aspect regards the material used in the digital workflows. Assumptions were given in order to use materials working in compression, without the use of any kind of reinforcement. The scope was to simulate the behaviour of stone-like material and in this regards mechanical properties comparable to stone were considered. However, the identification of a specific material in the future might be beneficial for research purposes.

A symmetrical shell was evaluated and tested by using different approaches to assess the feasibility of the initial connection system and of the whole geometry. Future work regards mainly the improvement of a connection system, which is able to provide support and stability to the structures, facilitating also the assembly process. Therefore, the next step regards the realization of the scaled model with the application of the improved connection system. Together with the evaluation of the connection system, tests on the structural efficiency will be carried out by applying punctual loads on critical areas of the shell as simulated by the analysis.

Since the work dealt with the design process of shell structures, future works could include the realization of real scale models with the use of standard digital fabrication techniques. The shell structure could be realized with minimum use of the formwork as demonstrated through simulation and small-scale prototype. One of the hypotheses may regard the use of material that can be processed by cutting machine without the need to design bespoke setups. The tessellation process can implement a further optimisation in case a planarization of the contact faces results necessary.

Appendix

In this appendix, design workflows implemented in Grasshopper® are presented referring to the design cases carried out in this thesis. In detail, portions of various processes are described with the aim to show their parametric definitions.

The topics covered in this appendix are the following:

- Mesh subdivisions techniques;
- Form-finding;
- Planarization;
- Surface rationalization;
- Optimisation by GA;
- Connection systems.

Mesh Subdivisions

1. Triangulation Dynamic Remeshing

Step 1. Surface as geometrical input (**Srf)**

Step 2. First triangulation (**Mesh+Tri**)

Step 3. Setting parameters for remeshing (**MeshMachine**): fix Vertices (**DeBrep**) and fix naked edges (**Edges**) and setting target edge lenght, setting flip option to false by **Boolean toggle** and **button** to reset simulation.

Step 4. Remeshing simulation

Step 5. Output (**PMesh+Mesh**)

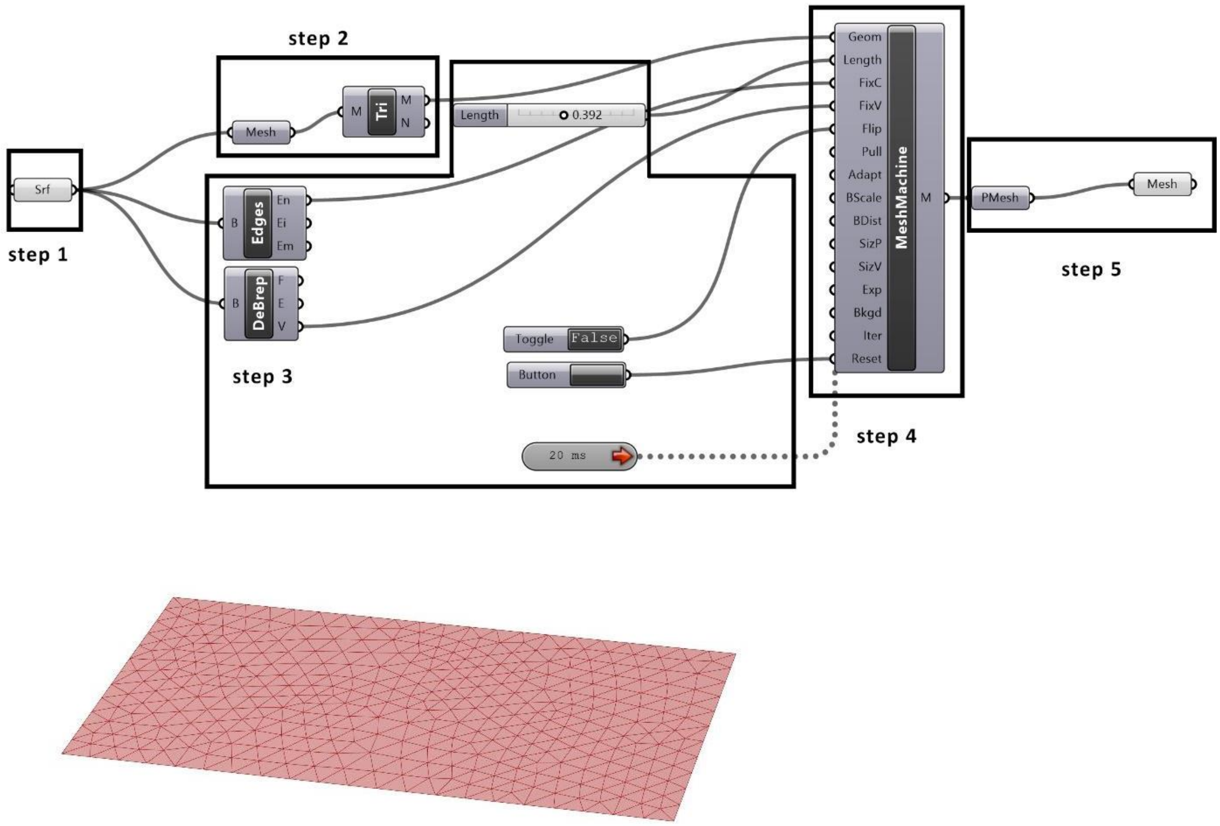


Figure 1 Triangulation of the mesh by Dynamic Remeshing

2. Delaunay triangulation with constraint to avoid skinny triangles

Step 1. Curve input geometry (**Crv**) and then transformed in Surface (**Boundary**)

Step 2. Definition of point on boundary curves (**CP+Crv CP+PopGeo**)

Step 3. Population of the points for the Delaunay triangulation (**Pt+Del**)

Step 4. Definition of mesh faces, in case some faces need to be eliminated such as skinny triangles (**FaceN+InCurve+Equals+CullF**)

Step 5. Output (**Mesh**)

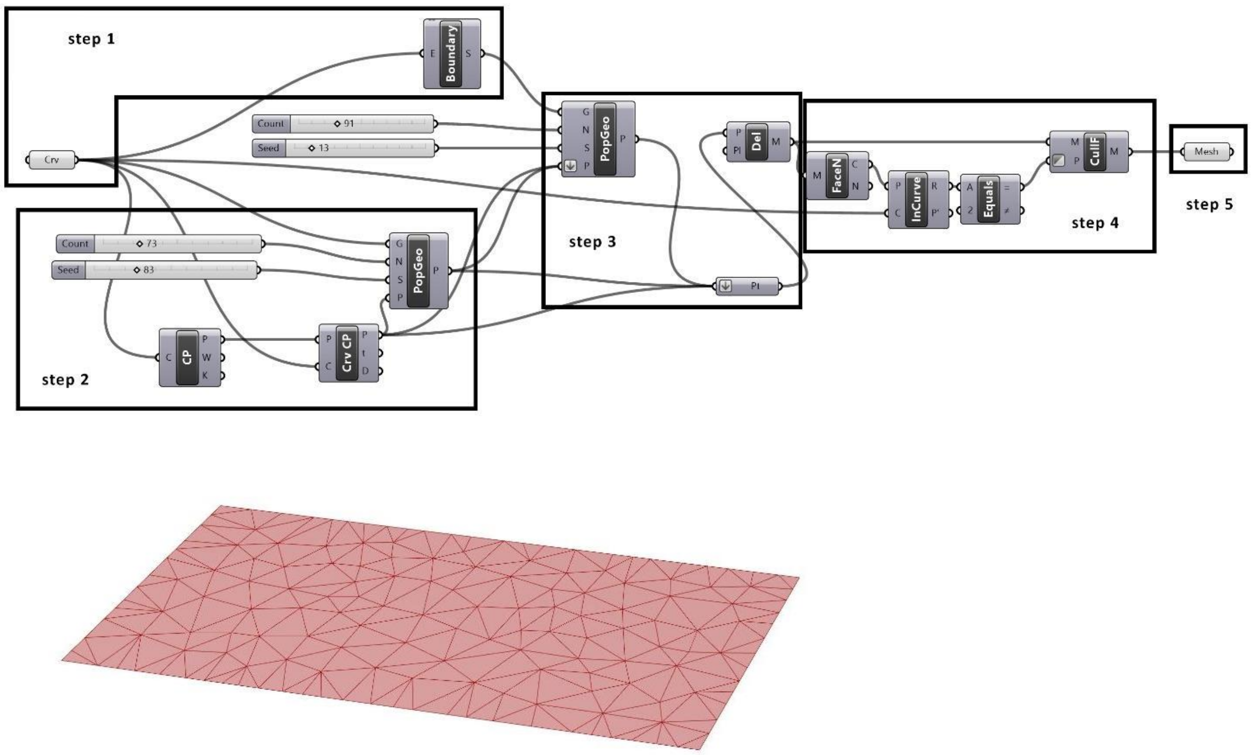


Figure 2 Delaunay triangulation for a mesh

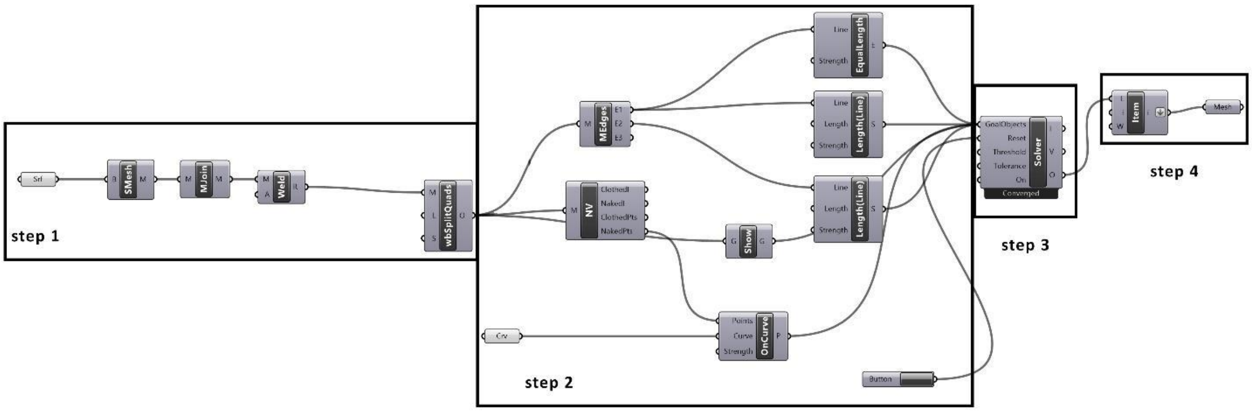
3. Quadrangulation by coarse mesh and relaxation

Step 1. Coarse surface input geometry transformed into a quadrangular mesh (**Srf+SMesh+MJoin+Weld+wbSplitQuads**)

Step 2. Setting parameters for mesh relaxation process with reference curve (**MEdge+MV+EqualLength+Length(Line)+Oncurve**)

Step 3. Simulation solver (**Solver**)

Step 4. Output (**Item+Mesh**)



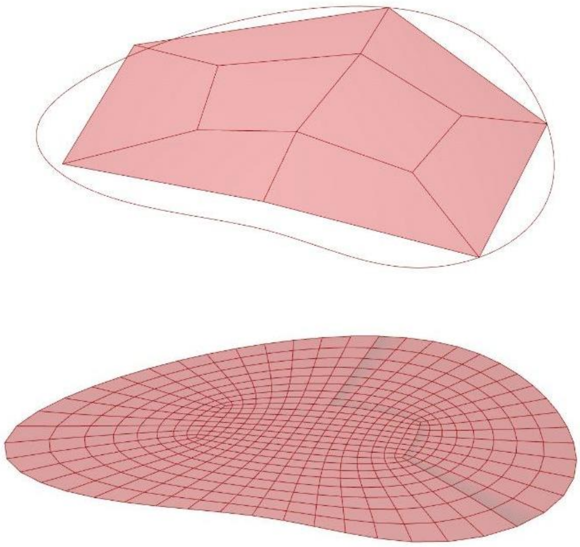


Figure 3 At the top: a) coarse mesh definition and division in quads; b) final quadrangulation with relaxation

Form-finding

4. Form finding for pre-rationalization approach (with planarization)

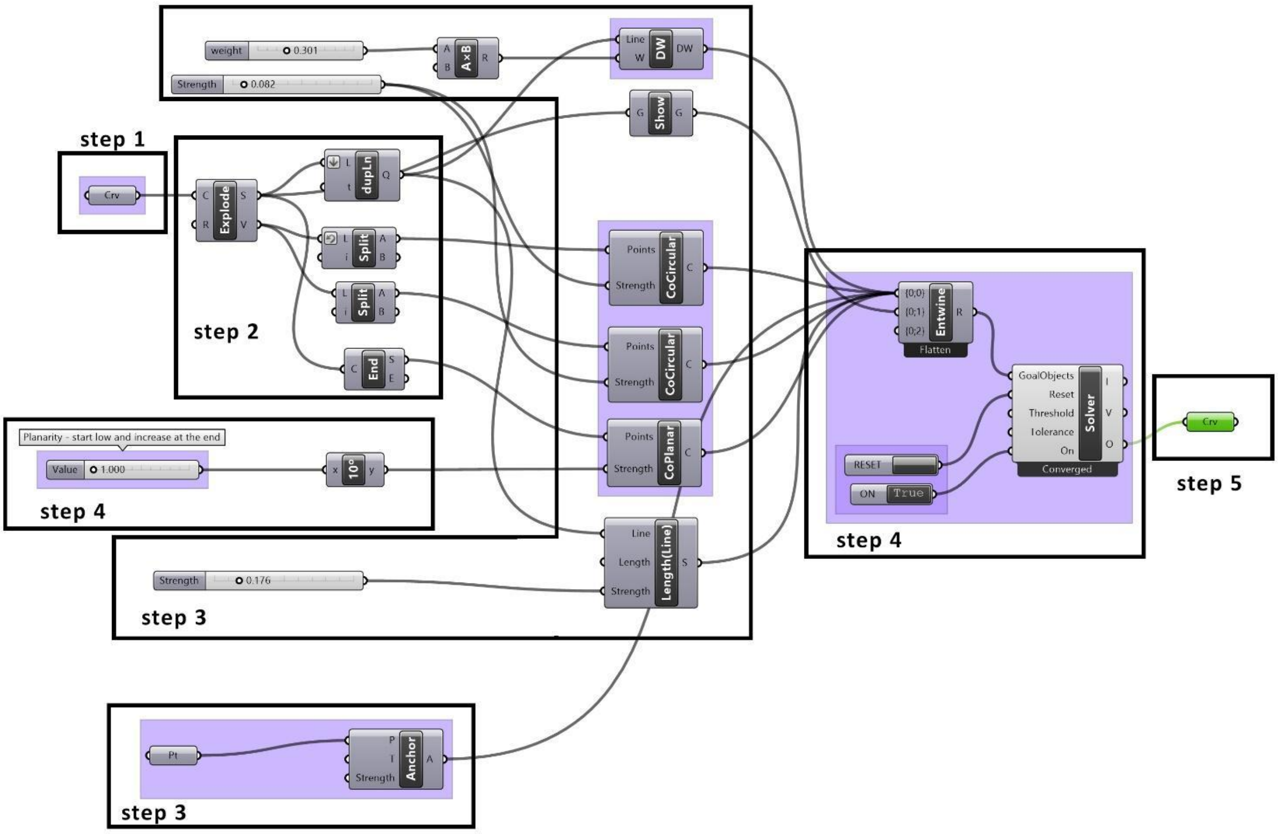
Step 1. Curve input surface divided into hexagons (refer to Fig. 4 for clarification) (**Crv**)

Step 2. Geometrical operations, such as definition of points, removal duplicate lines and division of vertices into two lists (**dupLn, Split, End)**

Step 3. Setting for form-finding simulation by vertical load (**DW**) and by lenght (**Length(Line)**) and setting for planarization process **(CoCircular** and **CoPlanar**)

Step 4. Form finding simulation first (by setting planarization force to 0) and planarization (by increasing planarization value), **Solver** with **reset** button and **Boolean toggle**

Step 5. Output (**Crv**)



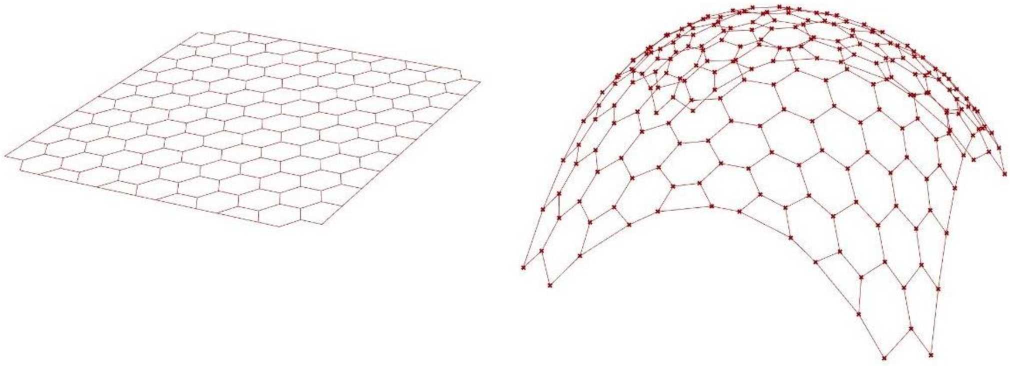


Figure 4 From the left: a) hexagonal subdivision for the geometry as input of the process; b) form-finding and planarization

5. Form-finding for post rationalization

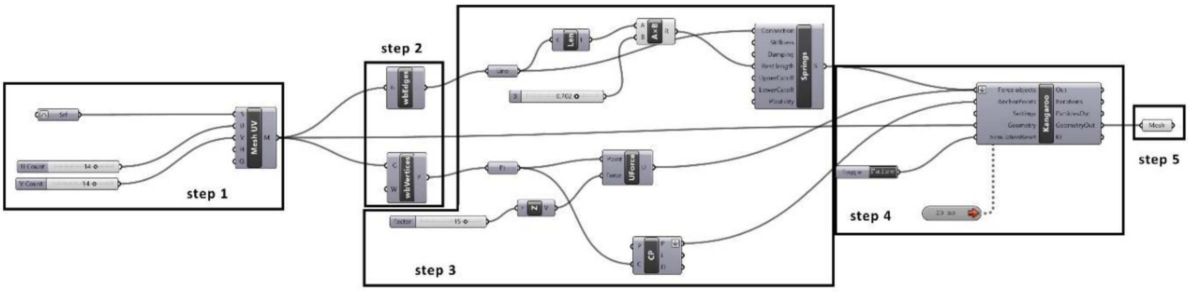
Step 1. Input surface geometry and transformation in mesh (**Srf+Mesh UV**)

Step 2. Discretization of the mesh (**wbEDGES** and **wbVertices**)

Step 3. Settings for form-finding simulation such as definition of springs with rest length (**Springs**), vertical force (**Uforce**) and supports by defining closest points on the mesh (**CP**)

Step 4. Simulation (**Kangaroo**) with **Boolean toggle** and **timer**

Step 5. Output (**mesh**)



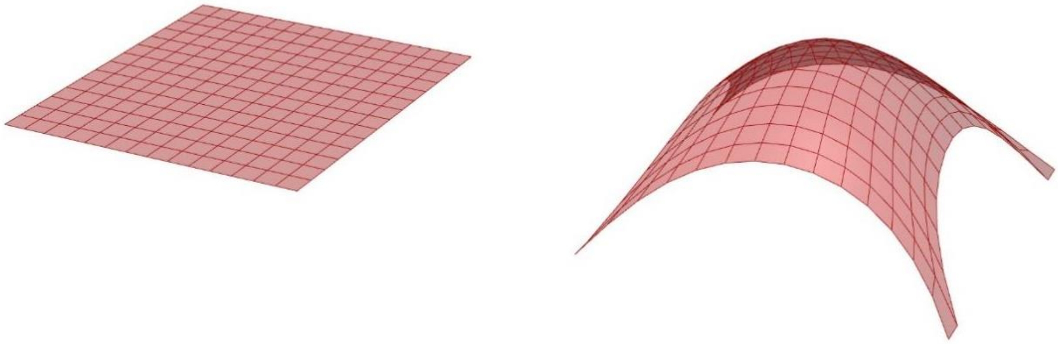


Figure 5 From the left: a) input surface converted into a quad mesh; b) form-finding result

Surface rationalization for asymmetrical shells

6. Hexagonal subdivision based on hexagonal pattern and its projection

APPENDIX

Step 1. Definition of the geometry and hexagonal subdivision (**Crv+Cir+RDiff+Boundary+Hex**)

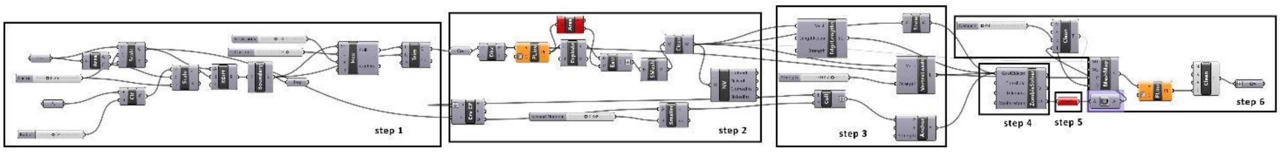
Step 2. Conversion into a mesh extrusion of the polylines according to their centers and mesh (**PLine+Area+Extr+SMesh**)

Step 3. Settings for form-finding (**EdgeLengths, VertexLoads, Anchor**)

Step 4. Form-finding simulation (**Zombiesolver**)

Step 5. Output (**Mesh**)

Step 6. Projection of the hexagonal cell on the final mesh (**MeshMap+Pline+Clean+Crv**)



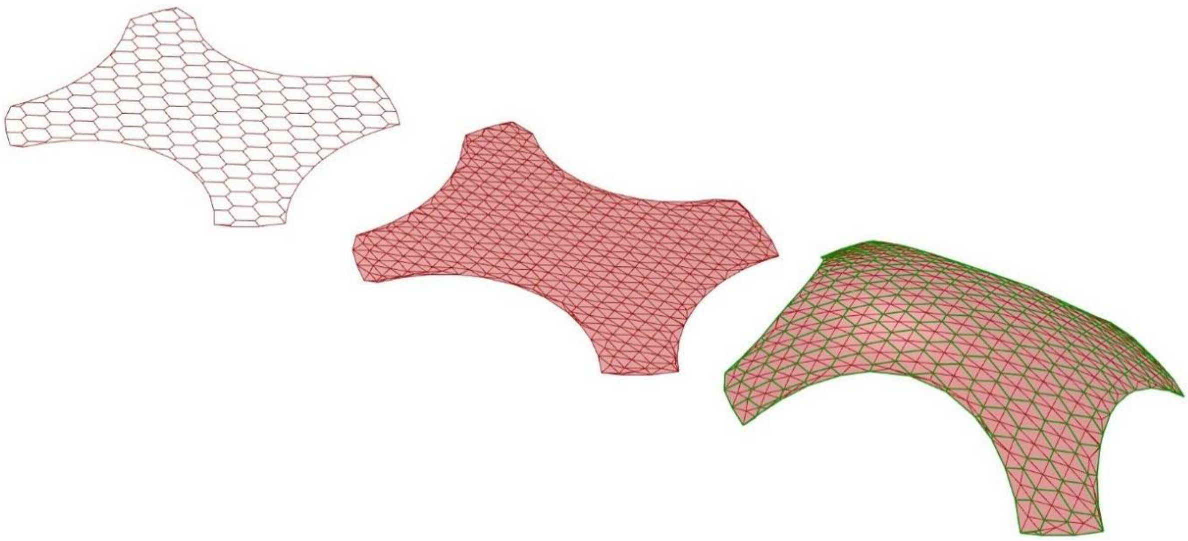


Figure 6 From the left: a) input geometry definition; b) conversion in mesh; c) form-finding and hexagonal projection

7. Hexagonal subdivison by triangular mesh and dual based on incircles centers

Step 1. Definition of the geometry, surface and brep to subdivide (**Crv+Cir+RDiff+Boundary+Brep**)

Step 2. Setting for mesh subdivision (**Edges, DeBrep, Target edge length, Boolean toggle** for flip and reset)

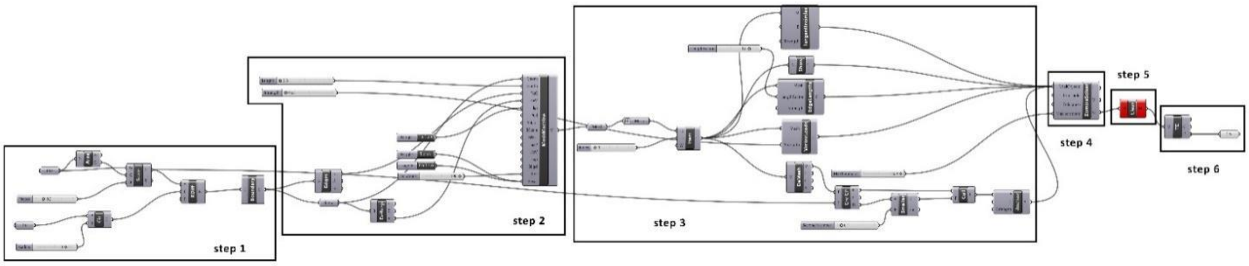
Step 3. Setting for form-finding with an additional constraint that allow to inscribe circles in the mesh (**TangentInCircles, EdgeLengths, VertexLoads**), definition of supports by defining closest points on the supports area (**DeMesh+CrvCP+Cull+Anchor**)

Step 4. Form-finding simulation (**Zombiesolver**)

Step 5. Output (**Mesh**)

Step 6. Hexagonal subdivision by dual based on incircles (**TC**)

APPENDIX



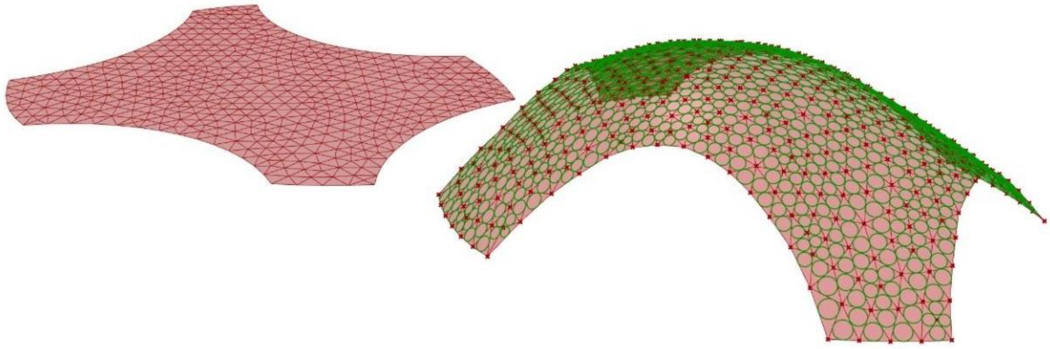


Figure 7 From the left: a) triangulation mesh dy Dynamic Remeshing; b) final shape with incircles

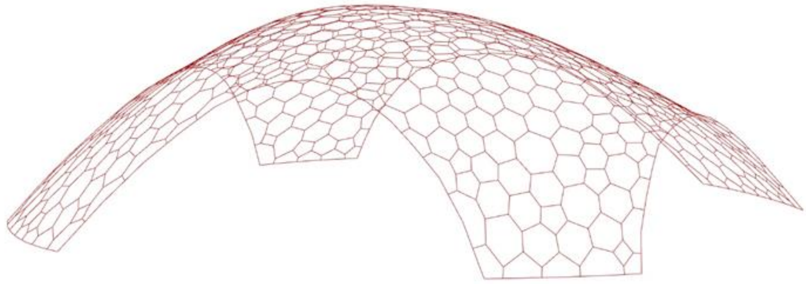


Figure 8 Final shape with hexagonal subdivision

8. Voronoi subdivision based on Voronoi pattern and its projection

Step 1. Definition of the geometry and Voronoi subdivision (**Crv+Cir+RDiff+Boundary+PopGeo+Voronoi**)

Step 2. Conversion into a mesh extrusion of the polylines according to their centers and mesh (**PLine+Area+Extr+SMesh**)

Step 3. Settings for form-finding (**EdgeLengths, VertexLoads, Anchor**)

Step 4. Form-finding simulation (**Zombiesolver**)

Step 5. Output (**Mesh**)

Step 6. Projection of the Voronoi cells on the final mesh (**MeshMap+Pline**)

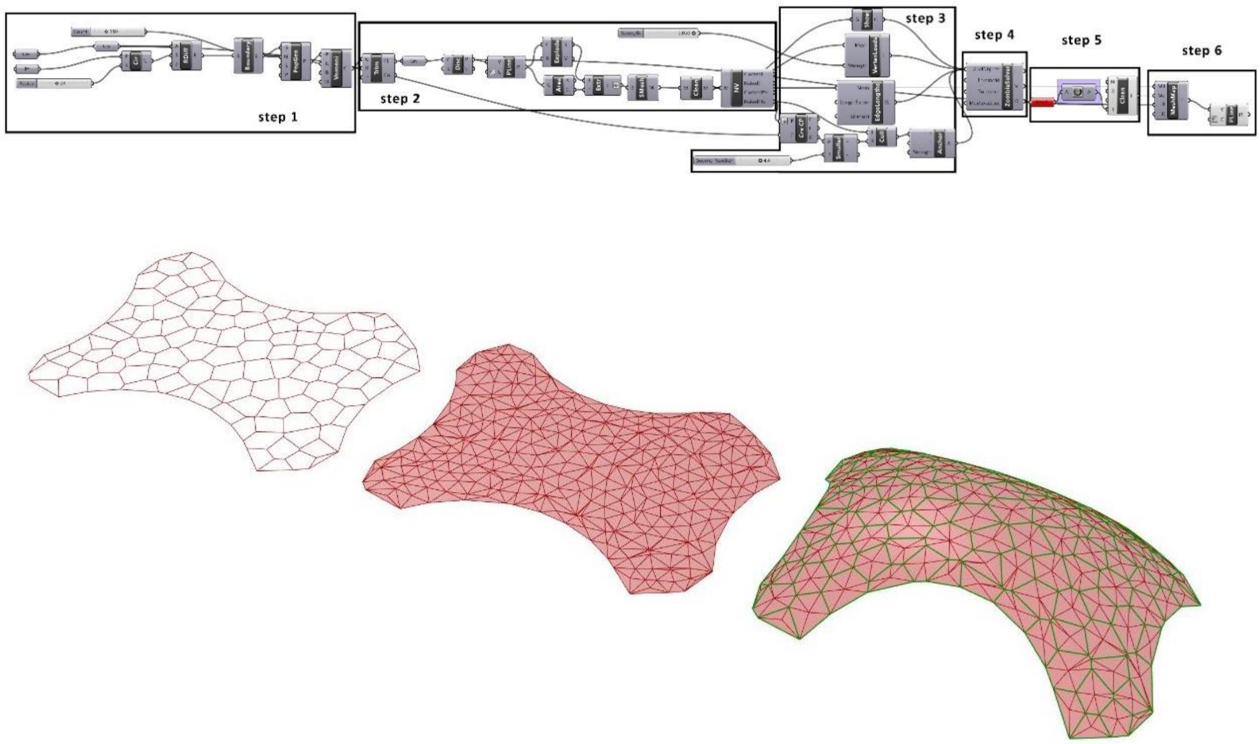


Figure 9 From the left: a) input geometry with Voronoi subdivision; b) conversion in mesh, final shape with Voronoi projection

Planarization

9. Method by equalizing diagonals, clamping force and constraining points on the mesh (for quad panels)

APPENDIX

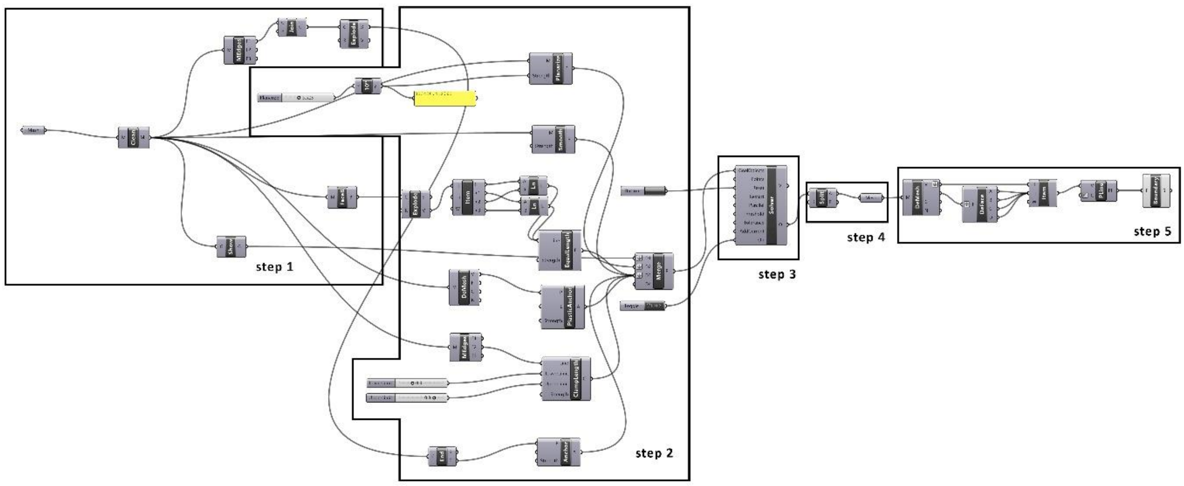
Step 1. Definition of the mesh as input geometry (**Mesh, Clean+MEdges and FaceB)**

Step 2. Settings for the planarization simulation: planarization force (**Planarize**), equalization of diagonal length (explode+item+ln/lm+EqualLength), fix points (PlasticAnchor), clamping force (ClampLength), anchor points (**Anchor**)

Step 3. Planarization solver (Solver)

Step 4. Output (**Mesh**)

Step 5. Generation of the planar surfaces (**DeMesh+DeFace+Pline+Boundary**)



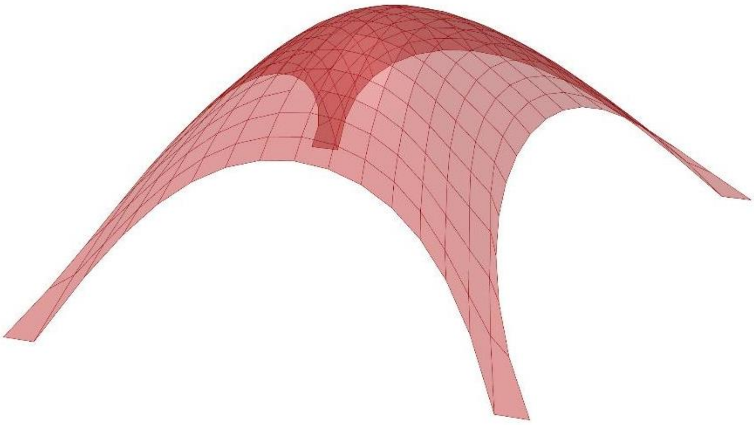


Figure 10 Planarized quads

10.Method by conformal hexagons

For further explanation, please refer to **Form finding for pre-rationalization approach (with planarization)**

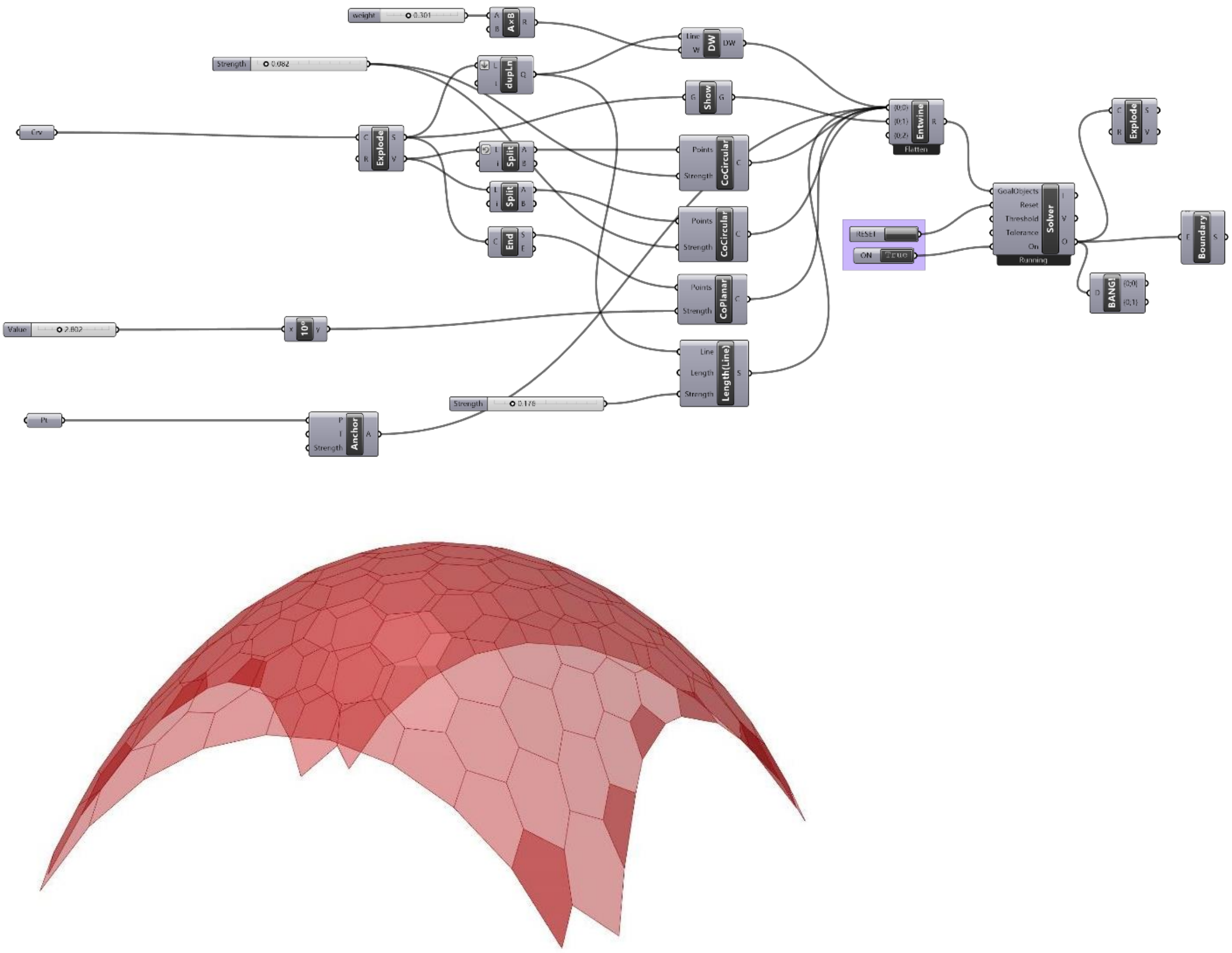


Figure 11 Planarized hexagons

11. Method by constraining points on mesh and length control

APPENDIX

Section A is the same workflow of "**Hexagonal subdivision based on hexagonal pattern and its projection**"

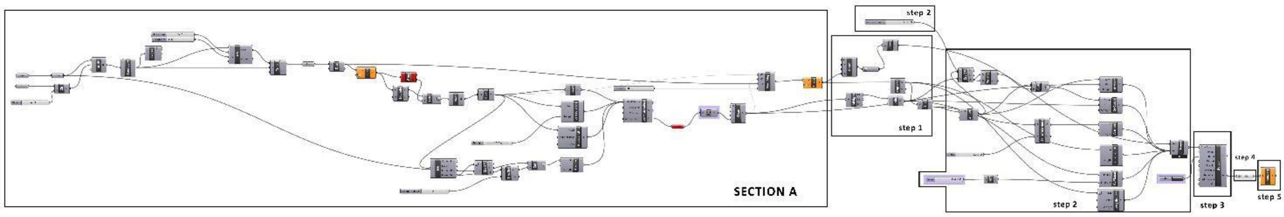
Step 1. Definition of the geometries involved in the process: mesh, polylines, points (**Pline, Medges,End**)

Step 2. Settings for the planarization simulation: planarization force (**CoPlanar**), constraint on the edges of the mesh (**OnMesh**), constraints on the curves (**OnCurve**), anchor points (**SplitAtCorners+Anchor**), constraint for the final length (**EqualLength and Length(Line)**)

Step 3. Planarization solver (Solver)

Step 4. Output (**Curves**)

Step 5. Generation of the planar surfaces (**Boundary**)



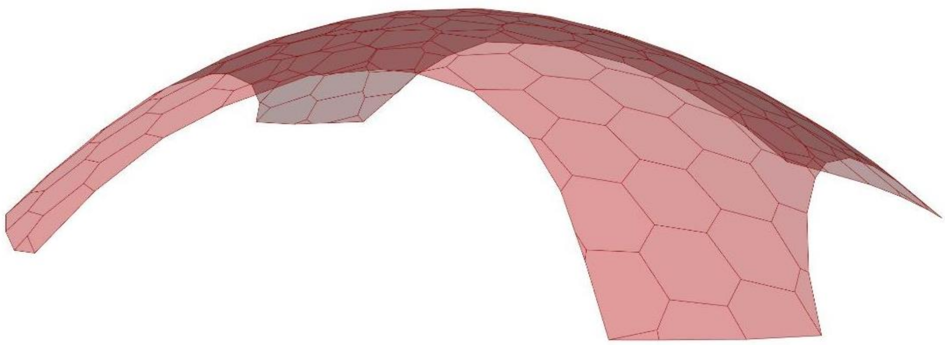


Figure 12 Planarization result for asymmetrical shell

Optimisation by GA

12. Single-objective optimisation by controlling hex grid density and the height

Section A is the same workflow of **Hexagonal subdivision by triangular mesh and dual based on incircles centers**

Creation of structural model by Karamba

Step 1. Definition of the inputs for structural analysis such as conversion of the lines into spring elements (**LinetoBeam**), conversion of mesh into a shell element (**MeshtoShell**). Spring elements were used to simulate hinged connections.

Step 2. Definition of material (**MatSelect**), cross-section (**Cross Section**), load conditions (**loads typologies: Gravity-Point-Point**), definition of supports **(Supp)**

Step 3. Assemble for the structural model to be analysed **(Assemble)**

Step 4. Analysis (**Analyze**)

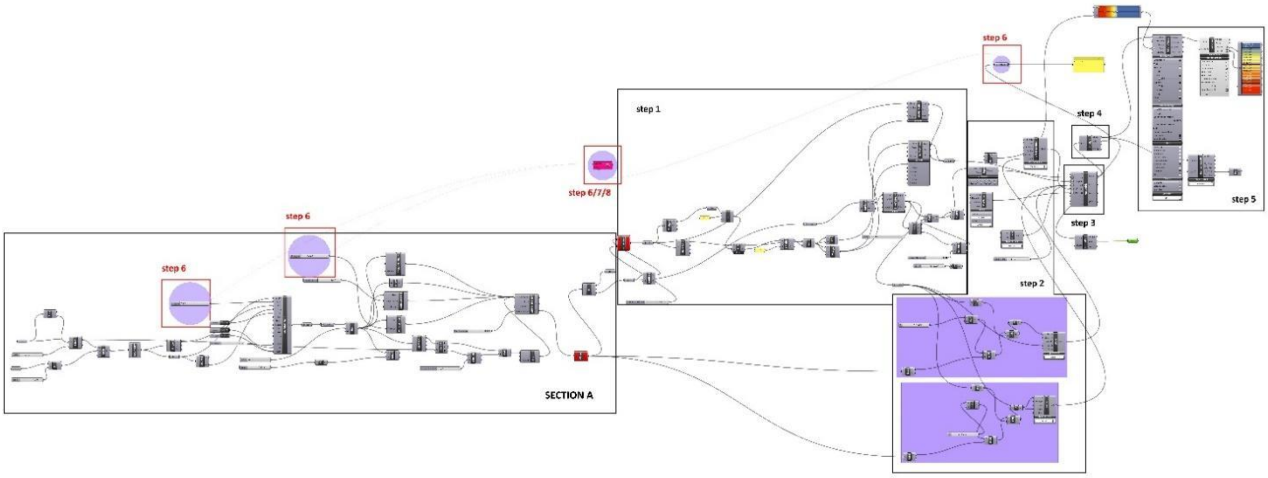
Step 5. Extract results to visualize displacement, stress (**Model View+Shell View**)

Definition of objectives and fitness function for Galapagos

Step 6. Galapagos Evolutionary Solver: connect number sliders of **Vertex load** and target edge length from **MeshMachine** to **Genome** and **Number** component of Displacement from **Analyze** to **Fitness**.

Step 7. Double click on **Galapagos Evolutionary Solver** and set "Minimize" in Fitness category

Step 8. Run the simulation (**Solvers-Start Solver**)



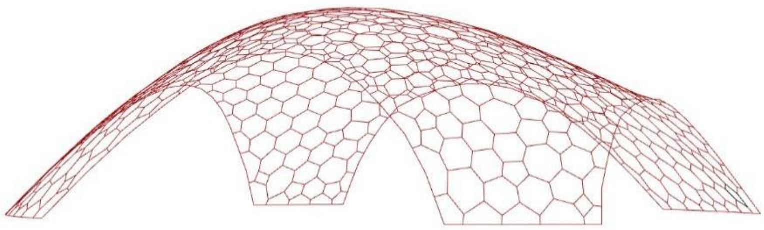


Figure 13 Final shape after optimisation process

13. Single-objective optimisation by controlling Voronoi grid density and heigh

Section A is the same workflow of **Voronoi subdivision based on Voronoi pattern and its projectionCreation of structural model by Karamba**

(Please note that the different part regards the definition of the reduction factor to use in the Galapagos simulation as showed in the image)

To define reduction factor:

Connect **PopGeo** to the total number of the points, to a substraction number and a reduction factor (**Lng+A-B+Reduce**). The resultant point will be used to generate Voronoi (**CP+Voronoi**)

Starting from the outputs of the form-finding (Mesh and Pline):

Step 1. Definition of the inputs for structural analysis such as conversion of the lines into spring elements (**LinetoBeam**), conversion of mesh into a shell element (**MeshtoShell**). Spring elements were used to simulate hinged connections.

Step 2. Definition of material (**MatSelect**), cross-section (**Cross Section**), load conditions (**loads typologies: Gravity-Point-Point**), definition of supports **(Supp)**

Step 3. Assemble for the structural model to be analysed **(Assemble)**

Step 4. Analysis (**Analyze**)

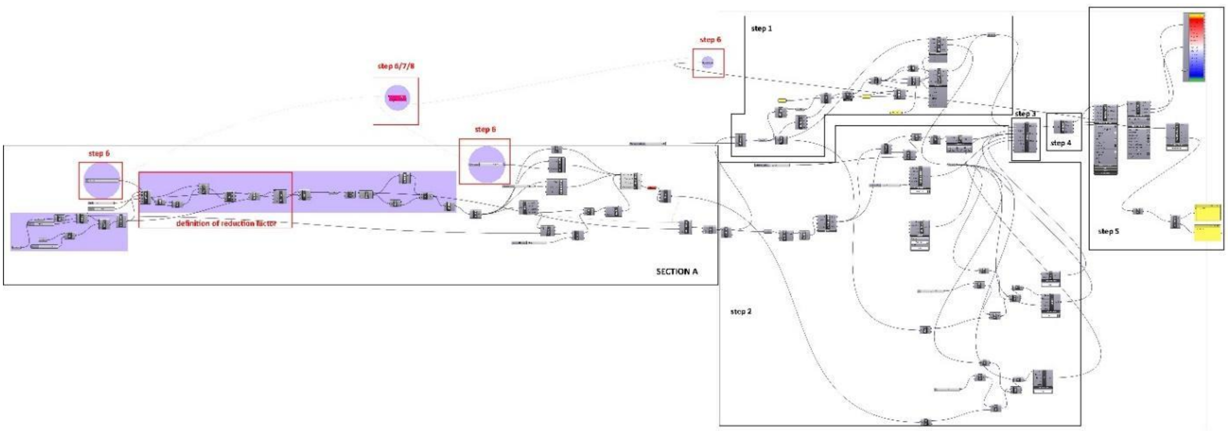
Step 5. Extract results to visualize displacement, stress (**Model View+Shell View**)

Definition of genomes and fitness function for Galapagos

Step 6. **Galapagos Evolutionary Solver:** connect number sliders of **Vertex load** and number slider of **subtraction** (A-B) linked to **Reduce** to **Genome** and **Number** component of Displacement from **Analyze** to **Fitness**.

Step 7. Double click on **Galapagos Evolutionary Solver** and set "Minimize" in Fitness category

Step 8. Run the simulation (**Solvers-Start Solver**)



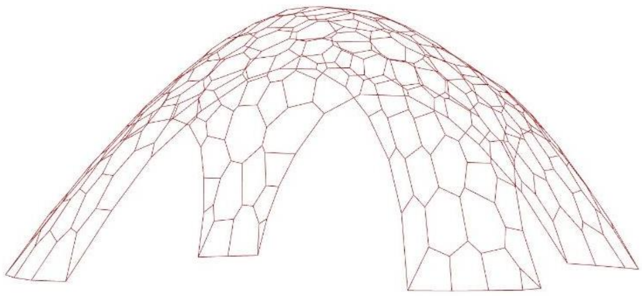


Figure 14 Final shape after optimisation process

14. Multi-objective optimisation by controlling the supports position

Step 1. Definition of the input geometry (**Srf**) and the variable position of supports (**Boundary+Split+Item+Eval**)

Step 2. Conversion of the surface into a mesh (MeshBreps)

Step 3. Setting for form-finding (**VertexLoads, EdgeLenghts**), definition of anchor points (**NV+CPs+Anchor**)

Step 4. Form-finding simulation (**Zombiesolver**)

Step 5. Output (**mesh**)

Definition of the structural model by Karamba

Step 6. Conversion of mesh into a shell element (**MeshtoShell**). Definition of material (**MatSelect**), cross-section (**Cross Section**), load conditions (**loads typologies: Gravity-Mesh load**), definition of supports **(Supp)**

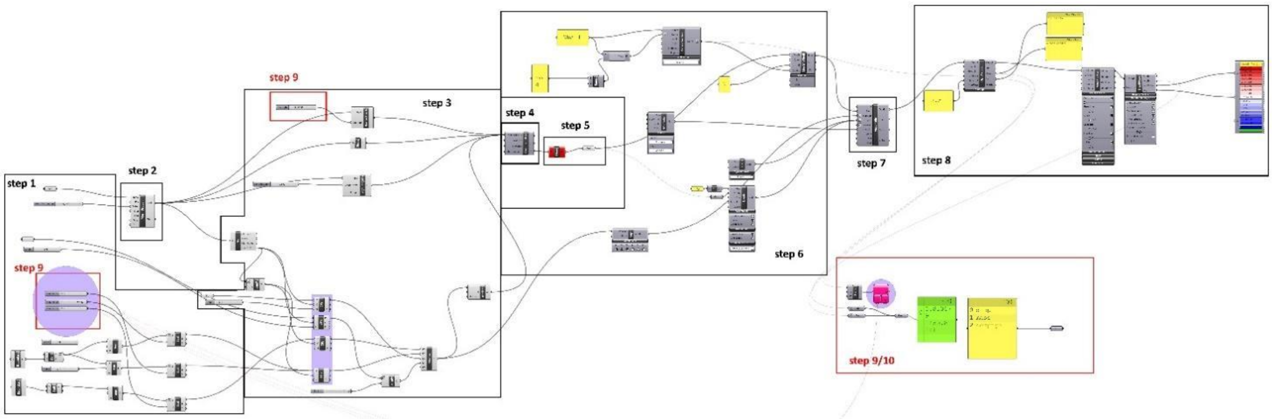
Step 7. Assemble for the structural model to be analysed **(Assemble)**

Step 8. Extract results to visualize displacement, stress (**OptiCroSec**, **Model View+Shell View**)

Definition of objectives and fitness functions for Octopus

Step 9. Octopus Solver: connect number sliders of **Vertex load** and number sliders of Evaluate Curve (**Eval**) to **Genome (G)**. Connect Mass and Displacement from **OptiCroSec** to Phenotypes (**O**).

Step 10. Double click on **Octopus** and run the simulation (**Start**). To visualize the best results tick on Pareto front.



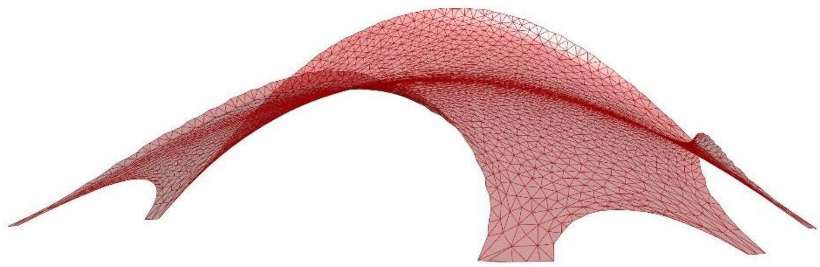


Figure 15 One of the result extracted by the multi-objective optimisation

Connection system

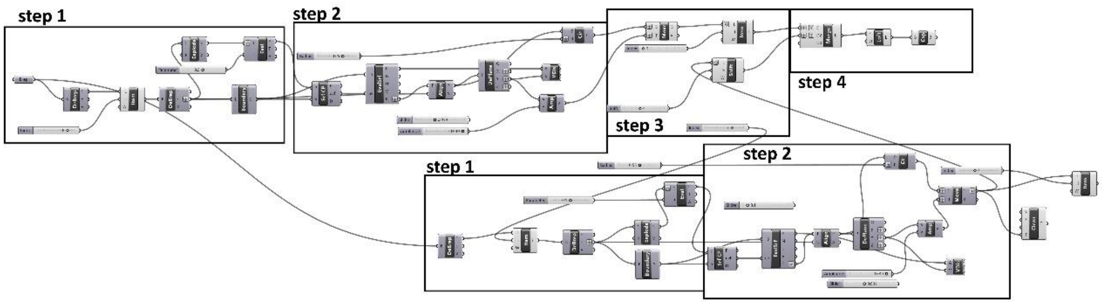
15.Connection system

Step 1. Selection the solids in Grasshopper with the top face and the bottom face (they must be planar) (**Brep+DeBrep+Item+DeBrep+Boundary**)

Step 2. Definition of the plane contained on the top and bottom surface and circle with different radius to create tapered geometry (EvalSrf+Align+Cir)

Step 3. Move the geometry along the *y* vector of the plane to create interlocking (positive and negative values for female and male conntections) (**DePlane+Amp+Move**)

Step 4. Selection of the connections (**Item**) and creation of the solid (**Loft+Cap**)



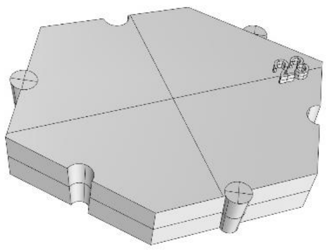


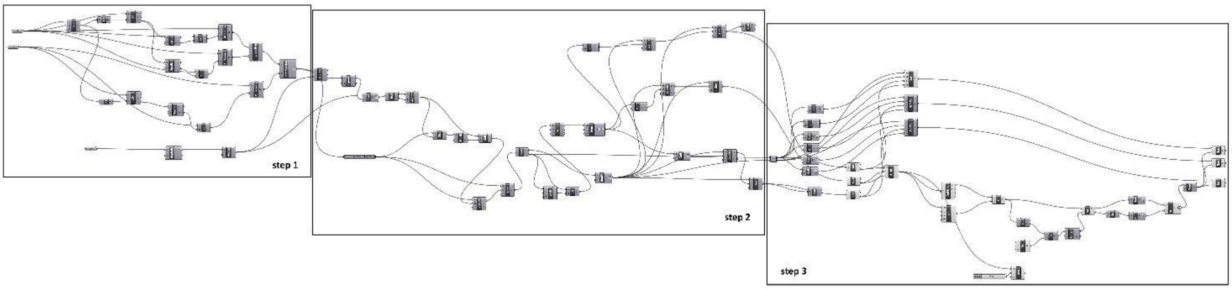
Figure 16 Puzzle connection for a hexagonal panel

16. Connection system (ECHO)

Step 1. Starting from the planar panels, definition of the vertices (the end sides are not included) (**CP+Difference+Intersections**)

Step 2. Definition of the intersection lines to define polylines to extrude surfaces for the top part and bottom. Three triangle portions are defined by end points of the lines (**CrvCP+Cull+End+Vec2Pt+Ln+Shift+Ln+Divide+Cull+Vec2Pt+Line**)

Step 3. Surface generation and extrusion for the top and bottom part (**Srf4Pt+Move+Extr**)



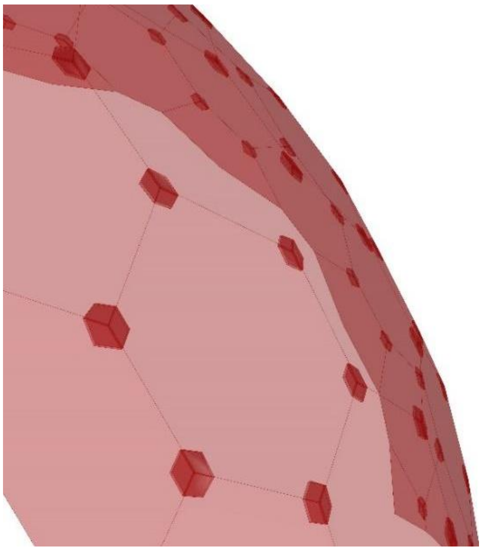


Figure 17 Connections on the ECHO shell

17. Finger joints-ECHO

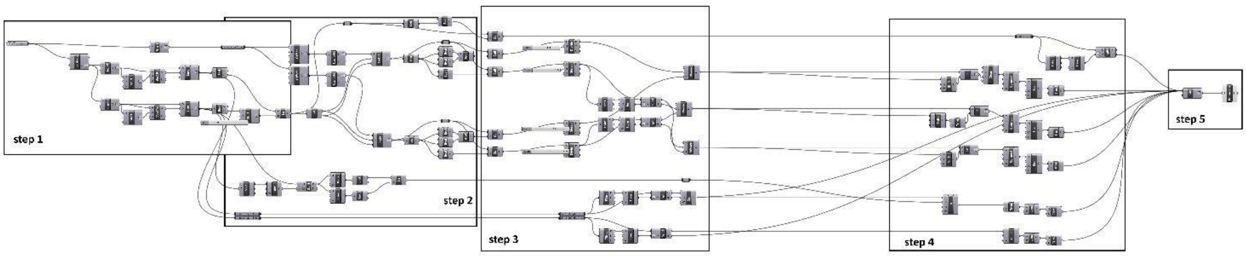
Step 1. Definition of the edges excluding the end parts for the connections (**Crv+Vec2Pt+Line+End+Shift+Ln**)

Step 2. Offset of the edges according to the required thickness (**Offset+Ln4Pt**)

Step 3. Division of the offset lines in a set of points to create pattern (**Cull+Divide+Split+Weave**)

Step 4. Generation of lines passing through the points to create finger joint (**Shift+Ln+Ext+Trim**)

Step 5. Join of the edges of the panels with the finger joint and creation of the surface (**Join+Boundary**)



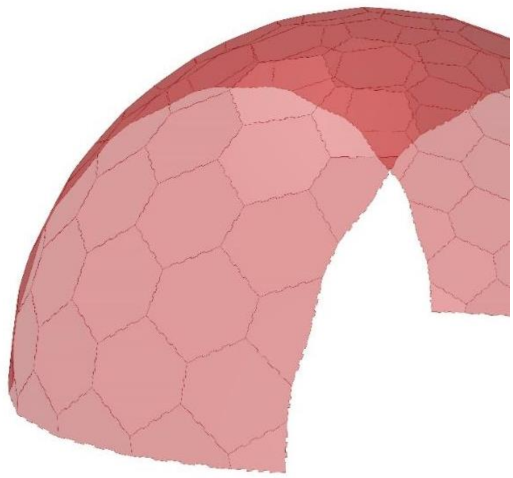


Figure 18 Finger joints designed on the shell

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