Economic Growth in Oil-Rich Countries:
A Theoretical Analysis with an Application to Saudi Arabia

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Abstract

The Saudi economy is faced with critical challenges due to excessive dependence on oil and the lack of other sources of income. The present thesis seeks to study the influence of these factors on the Saudi economy and examines the potential implications of introducing consumption and personal income taxes on economic growth. This thesis provides a theoretical analysis of economic growth, where the discrete-time of Barro (1990) model is extended in three chapters (Chapters 4, 5, and 6).

Chapter 4 aims to describe the Saudi economy before implementing the proposed fiscal policy reforms by the IMF. Since oil revenues solely finance productive government spending, Chapter 4 studies how the Saudi economy is affected by a negative shock in oil demand. Results of this chapter show that the growth rate of government spending is the growth rate of oil profits, g^2. It also shows that although the level of consumption is growing slowly due to negative shock, it is indeed increased relative to the previous trajectory. This suggests that there is some partially offsetting shift in the level of consumption, which may not be obvious.

In Chapter 5, the model presented in Chapter 4 is extended by introducing consumption tax to investigate the effectiveness of these taxes on economic growth. Oil revenues and consumption tax revenues in this chapter feed productive government spending. The results show that there are two types of steady-state, the exogenous and endogenous growth steady-state, which cannot exist for the same set of parameter values. This result demonstrates two main findings. The first is that if the g^2 is sufficiently high (low) for a given value of consumption tax, then the steady-state will be an exogenous (endogenous) growth steady-state. The second finding is that if we set g^2 constant and vary consumption tax, the economy at a certain value of consumption tax could move from one type of steady-state to another. In the latter case, the endogenous growth steady-state would be preferable because it would ensure a higher growth rate. Consequently, our results cast a new light on the possibility of switching regime as we change the policy parameter, and how consumption tax can compensate for any reduction in oil revenues.

Chapter 6 studies the possibility of introducing personal income tax in the Saudi economy and how its economic growth can be affected by this type of tax. Two different sources of government revenues finance productive government spending: oil and personal income tax revenues. The finding is like chapter 5, in which the two types of steady-state cannot exist for the same set of parameter values. However, the results of this chapter, contrary to the findings of chapter 5, show that two critical values allow the economy to switch from exogenous growth steady-state to endogenous growth steady-state and vice versa. These two critical values depend on the rate of personal income tax. The results also show that the personal income tax would temporarily reduce the growth rate under certain parameter values, which could be considered as a warning to a policymaker. This result indeed contrasts with the result in chapter 5, where the growth rate will unambiguously be temporarily increased by an increase in consumption tax.

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بسم الله الرحمن الرحيم

In the Name of Allah, the Most Beneficent, the Most Merciful

Dedication

To my parents, Saud and Hailah, for their prayers, encouragement, and wisdom

To my wife, Futun, for her love, support, and enthusiasm

To my twins, Saud and Ghala, for their patience

To my brothers and sisters

To all my friends in York, UK

Thank you all very much for your understanding and standing through my long journey

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Author's Declaration

I declare that this thesis is a presentation of original work, and I am the sole author. This work has not previously been presented for an award at this, or any other, University. All sources are acknowledged as References.

1 Chapter One: Introduction

1.1 Introduction

The economic growth of a country is strongly associated with high production capacity. Therefore, a country that produces abundant commodities and is well-endowed with natural resources is likely to attain significant economic growth. Understanding natural resources is an important aspect of understanding effects on income and demographic changes. Since political power can moderate the former, natural resource wealth can generate economic growth with strong governance and institutional policies. Among the natural resources, oil has been considered to be the most important for a long time. Its importance cannot be compared with any other commodity due to its unique properties and its diversity of uses in many areas. The importance of oil also lies in the high global demand for it to meet different human and production uses. Consequently, it plays a significant role in the economy, whether for producing or consuming countries.

For the oil-rich countries, however, oil revenues coupled with a weak government can damage economic growth. For example, corruption and lack of transparency would lead to inefficient use of economic resources. Although some people view oil revenues as a curse, effective management, well-designed fiscal policy, and an effective democratic approach can have significant implications for the economic prospects of oil-rich countries. In other words, the optimal and efficient use of oil could stimulate economic growth in the country.

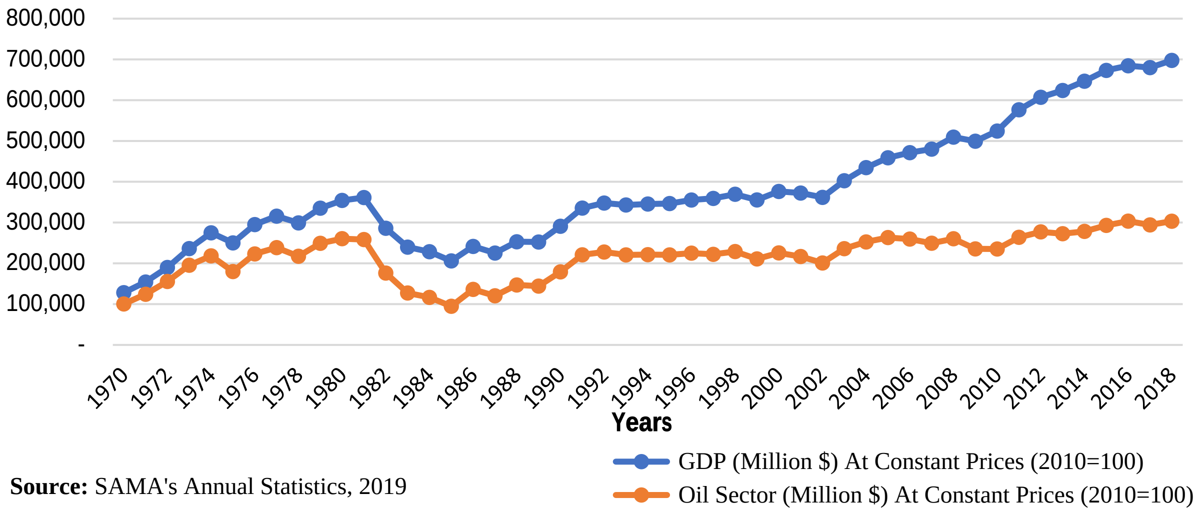
Saudi Arabia is one of those countries, which are abundant in oil resources. It heavily relies on revenues generated from oil to the extent that they determine the fate of the whole national economy. In a 2017 report, the Saudi Arabian Monetary Authority (SAMA)[1] stated that the highest oil revenues received by the country were 305.3 billion U.S. dollars in 2012. According to this report, oil revenues represented the most significant proportion of total revenues, compared with non-oil revenues. The oil revenues contributed to the state's total revenues by more than 80%, on average, during 1990-2017 (SAMA, 2018). Figure 1 displays how the oil sector[2] contributes to the real Gross Domestic Product (GDP), which shows the importance of oil to the Saudi economy during 1970-2018 (SAMA's Annual Statistics, 2019).

[1] The SAMA is the abbreviation for the central bank of Saudi Arabia.

[2] The oil sector includes crude oil, natural gas, and refined products.

Oil in the Saudi economy is the economy's main engine, and its revenues feed the public budget by a significant percentage. According to historical statistics, Saudi Arabia's GDP is closely related to oil, given that it is considered the most important economic sector in the country. In other words, the oil sector is one of the main GDP components in Saudi Arabia, where its average contribution to the GDP was about 79% during the period 1970-2018.

From the below figure, it is obvious that the GDP growth is greatly affected by the oil sector, where the GDP follows any changes in the oil sector. For instance, during the 1970s, oil prices doubled more than once[3], until they reached approximately $35 per barrel, which led to an increase in oil revenues. Thus, that ultimately was reflected in the rise in GDP, as shown in the below figure. During the 1980s and 1990s, oil prices continued to fluctuate between $12 and $34, affected by many economic and political events. These events led to a decline in oil revenues and therefore the GDP. However, post-1990s (*i.e.* at the beginning of 2008 and 2012), oil prices rose to unprecedented levels. During this period, Saudi Arabia's GDP benefited from this rise, bringing significant income from oil revenues. Since oil prices after that remained fluctuating due to the persistence of some global economic and political events, Saudi GDP also remained fluctuating due to its excessive sensitivity to global oil markets. As a result, the oil sector is seen as one of the main factors influencing the economic growth in Saudi Arabia.



Source: SAMA's Annual Statistics, 2019

Figure 1: Oil Sector and Real GDP for the period 1970-2018

[3] Historical oil prices (1970-2017) and associated economic and political events will be discussed and shown in the next chapter (Analysis of the Saudi Economy Chapter).

The Saudi economy consists of several sectors, the most important of which are the oil, public, and private sectors. However, all these sectors and their growth are closely related to the amount of financial returns from the oil export process, which is the primary engine of the economy (Al-Saeedi and Al-Otaibi, 2015). The fiscal policy in Saudi Arabia and its tools of government spending, fees, subsidies, and taxes are much more important than monetary policy in directing and managing the movement of the economy for some reasons. The most important is the size and role of government spending represented in the state budget in influencing the economy. Although fiscal policy generally includes taxes and public expenditures together, Saudi Arabia relied primarily on public spending, where taxes are very limited in the state budget. Still, public spending is the main component in economic growth, which is funded by oil revenues, as confirmed by Alrasheedy and Alrazyeg (2019), Alshammari and Aldkhail (2019), Al-Obaid (2004), and Barri (2001)[4]. The reason is that it has an impact on all economic activities through the support provided by the government for public services on the one hand and the support to the private sector on the other. The primary source of income in the Saudi economy is oil revenues. Due to the public ownership of oil, government spending has become a key role in injecting these revenues into all areas of the economy and leading the economic activity as a whole. As a result, there is no doubt to expect that the oil revenues and then the level of public spending and eventually the overall economic activity would have a substantial and direct impact due to volatility in the price of oil in global markets.

1.2 Importance of the Study

The significance of this study is related to the challenges facing the Saudi macroeconomy. Saudi Arabia's economy currently faces many difficulties due to oil price fluctuations. Among the challenges faced is that oil revenues are unstable because they rely on conditions in the global market. According to the International Monetary Fund (IMF) report of 2015, Saudi Arabia has experienced three painful periods due to low oil prices. Figure 2 shows that the first period was 1982-1986. It was a result of a general expansion in the oil sector and an attempt by powerful economies to prove themselves efficiently by cutting links with Saudi Arabia, thus paralysing the country's economy. The financial crisis of 1998-1999 in Asia and the Global financial crisis of 2008-2009 were the other two periods that derailed Saudi Arabia's financial system. Notably, the latter periods were characterised by a sharp contraction in demand for oil, which slowed down global economic activity.

[4] What these studies examined and found are discussed in the Literature Review Chapter, specifically in section 3.7.

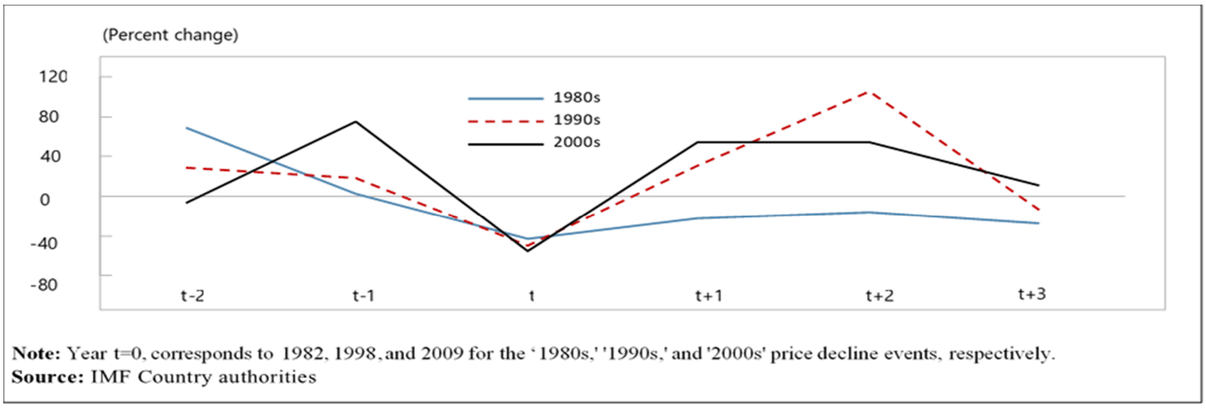


Figure 2: Oil Revenue Declines During the three Historical Episodes

The three periods were a learning experience as they showed the need for Saudi Arabia to consider its financial policies. The beginning fiscal situation was weak before the fall in oil revenues, with a GDP deficit of about 3% and an unresolved debt of about 70% as a proportion of GDP. This situation made the government undertake spending cuts during the oil crisis. In 2008-2009, oil prices further declined; consequently, Saudi Arabia and the Organization of the Petroleum Exporting Countries (OPEC)[5] had to cut production further. These three periods saw oil revenues plummet by 50%. Thus, drops in revenues and the contraction of global oil demand negatively affected Saudi Arabia's overall economic growth. The IMF has further indicated that the fiscal balances later declined, and credit money growth also slowed (IMF, 2015b). Moreover, the fall in oil revenues led to negatively affected credit growth. Consequently, government revenues fell despite a decrease in government spending to curb the problem.

Another challenge facing the country is that Saudi Arabia has no effective tax system and suffers from a lack of taxation to generate revenues away from oil[6]. The current taxes considered a tiny proportion of state revenues and therefore insufficient. Although the government is always required to spend more on education, health, and other social services, the Saudi government is currently facing a significant challenge regarding the stability of its revenues and its ability to meet the increased need for public spending. In 2015, the IMF released a report that pointed out a significant drop in oil prices had the potential to lead to weak growth in the coming period. They also mentioned uncertainty over future oil prices, stating that the massive decline in oil revenues and continued expenditure growth would lead to a substantial fiscal deficit over the medium term. As a result, the IMF provided a set of proposals to Saudi Arabia to reform its fiscal policy because

[5] The Organization of the Petroleum Exporting Countries (OPEC) was established in 1960 and comprises currently of 13 governments. OPEC members include Algeria, Angola, Equatorial Guinea, Gabon, Iran, Iraq, Kuwait, Libya, Nigeria, Republic of the Congo, Saudi Arabia, United Arab Emirates, and Venezuela (OPEC.org.2020).

[6] The details discussion of the tax system is available in chapter two.

the Saudi economy currently has a weak financial system to face the instability of the government's total revenues. It is, therefore, essential to focus on alternative revenue sources, such as taxes, as a way of ensuring that the economy is not hit by changes in oil prices in the future (IMF, 2015a). As a result, since the Saudi economy relies heavily on oil revenues to stimulate the economy, it is essential to study how the economic growth would be affected if there is a negative shock in demand for oil. Moreover, exploring the possibility of introducing new taxes, as suggested by the IMF, would be urgent for evaluating their effectiveness alongside oil revenues in the world's second-largest oil economy.

1.3 Research Questions

- What would be the impact of a negative shock in demand for oil on Saudi Arabia's economic growth?
- How would the main level variables[7] in Saudi Arabia's economy be affected if the level of government spending[8] reduces due to the fluctuations in oil revenues?
- How would Saudi Arabia's economic growth and its main level variables be influenced if consumption tax is introduced as per the IMF's suggestion?
- In what ways can the consumption tax compensate for any reduction in the level of government spending associated with a decrease in oil revenues?
- What would be the effect of introducing personal income tax on Saudi Arabia's economic growth and its main level variables?
- Which among consumption tax or personal income tax would be more suited for achieving economic stability in Saudi Arabia?

1.4 Objectives of the Study

Because of volatilities in oil prices witnessed in markets recently, oil can no longer be considered a consistent revenue source. The assessment of fiscal sustainability is identified as the main problem in developing countries because their structures of tax revenue are often not well developed. This situation applies particularly to resource-driven countries and especially oil-producing countries (Kia, 2008). Therefore, oil-producing countries have been motivated to reform their tax regimes in order to increase and diversify their tax revenues (Igberaese, 2013).

[7] We refer to level variables in this thesis as aggregate variables, *e.g.* the level of output, the level of capital stock, the level of consumption, etc.

[8] The growth rate in our model is assumed to be derived from the growth rate of government spending. Thus, although a level variable means a variable that does not represent a rate of growth, studying the changes in the government spending level is important not only for economic growth but also for other level variables in our economy.

According to Saudi statistics, most of the government's revenues in the country comes from the oil sector. Therefore, the importance of other revenues rather than oil revenues has become crucial due to the need for stimulating growth and sustainable economic development. Correspondingly, the IMF's recommendation to Saudi Arabia is to change its fiscal policy since it depends significantly on oil earnings. Among these recommendations is to reform the current tax system, which is the focus of this study.

This thesis is intended to examine how economic growth would be affected if there is a negative shock in the oil demand. It also aims to focus on the fiscal policy reforms and find out the effectiveness of implementing the proposals of IMF on Saudi economic growth for countering oil revenue fluctuations. More precisely, this study seeks mainly to examine the possibility of introducing consumption tax and personal income tax and discuss the effectiveness of these taxes along with oil revenues in economic growth. In addition, the present study pursues to investigate the appropriate amount of consumption tax that could compensate for potential reduction in oil revenues. In brief, this study is mainly based on two aspects. The first is a description of the Saudi economy and the impact on the economic growth of a negative shock in demand for oil. The second is the proposal of the IMF for reform of fiscal policy and the introduction of new taxes to compensate for any decrease in oil revenues. The reason behind this is that oil revenues are subject to fluctuation and decay. Therefore, stable additional revenues must be available to offset and cover recurrent costs in the country. This study will also simulate the impulse responses under two different scenarios: economic growth in Saudi Arabia before and after implementing the IMF's recommendations. The objective is to assess the effect of introducing new taxes on Saudi economic growth.

1.5 Motivations

The motivations for this study are based on three main aspects: the IMF recommendations to officials of Saudi Arabia, the role of fiscal policy in the Saudi economy in driving the economy, and the government's plans which can be summarised below.

1.5.1 International Monetary Fund (IMF) Recommendations

The IMF has issued recommendations that developing countries can adopt to improve their economic conditions and further increase their income. To do so, the IMF works with the governments of developing countries to develop their policies. A case in point is that of Saudi Arabia. The IMF has stipulated recommendations in its recent annual reports that are of significance as they contain guidelines necessary for boosting growth and extending stability among other integral issues. Their recommendations seek to ensure economic development and check risk spillovers in a country whose economy is dependent on oil revenues.

The IMF has recommended Saudi Arabia to reform its fiscal policy since the country is heavily reliant on revenues from oil. Low oil prices have led to financial instability risks in some periods. The fiscal consolidation measures suggested by the IMF to Saudi Arabia could be the best way to avoid such situations. The main reason why the IMF has proposed that Saudi Arabia reform its policy is that Saudi has a weak financial system, which can be attributed to the previous three periods, as shown in Figure 2. Among the IMF's suggestions is a proposal to reform the country's current tax system because Saudi Arabia cannot continue to rely exclusively on oil. In this regard, the IMF has recommended Saudi Arabia to consider and look for alternate revenue sources, such as implementing new taxes. The IMF has given high attention to the imposition of new taxes because it could generate significant revenues for the government's budget. These new taxes include consumption tax, particularly Value-Added Tax (VAT), and selective taxes on tobacco and energy drinks, as a way of offsetting the shocks that have previously affected the Saudi Arabian economy (IMF, 2016b).

The issue of limited domestic taxation was cited in the IMF as one of the most critical factors that restrict GDP growth in non-oil sectors. In that regard, the IMF suggests that Saudi Arabia should impose a consumption tax of VAT to cope with economic shocks resulting from dropping oil prices. The IMF recommendation was based on several reasons for implementing this type of tax. First, VAT has proven highly effective in increasing tax revenues, particularly revenue collection from the non-oil sector. In 2014, IMF staff estimated VAT revenue in Saudi Arabia and, assuming 90% of private consumption, found that if Saudi Arabia introduced a well-designed VAT at the rate of 3%, the potential revenue would be 0.9% of GDP. While if the country were to add VAT at a rate of 5%, then revenue would likely be about 1.5% of GDP. Second, Saudi Arabia has the ability to increase its revenues in all areas, whether through indirect taxes (like VAT) or direct taxes (like personal income tax) because they are both available tools. Another benefit of introducing a value-added tax would be that VAT administration is quite simple compared to different kinds of taxes. It does not also affect foreign direct investment (FDI) and exports. It would also reduce the need to introduce some other distortion taxes such as personal income tax (IMF, 2015c).

Moreover, the IMF has recently released a paper in 2020 called 'The Future of Oil and Fiscal Sustainability in the GCC[9] Region'. The IMF has urged the GCC countries, including Saudi Arabia, to help reduce future financial pressures on the budgets of these countries through additional income other than oil. In their paper, they have mentioned that the current financial reforms and plans are insufficient to meet the ongoing changes in the oil market. The main reason for this is that global oil demand is anticipated to reach its peak in the next two decades. As in previous papers, they have hinted that the reform of the current tax system as one of the main fiscal policy tools is essential for two main reasons. The first reason is to ensure revenue stability, while the second reason is due to the limited taxes in the state (IMF, 2020a). On the other hand, although Saudi's officials announced in 2020 that the personal income taxes would not apply[10], it could be soon implemented in the Saudi economy since the state budget is still tied to volatile oil prices, as indicated by the IMF and the statistics. The country, at the same time, tries to reduce its reliance on oil. As a result, it is a clear sign that the IMF is urging Saudi Arabia to work on major economic reforms. In the same vein, the article of Martin, Nereim, and El-Din in 2020 emphasised that the Saudi government is working to accelerate plans to sell some state-owned assets. It does not also rule out the possibility of imposing a personal income tax, as it seeks to strengthen the state treasury, which has been affected by the recent significant drop in oil prices. In this regard, the Saudi Minister of Finance stated that all possibilities are under review by the government to enhance its financial resources, and it will take a long time to plan the personal income tax. However, although there is currently no imminent plan to implement it, we do not rule out anything.

1.5.2 The Role of Fiscal Policy

Even though the fiscal and monetary policy are both essential tools for a government to sustain and promote the economy, the fiscal policy plays a crucial role more than monetary policy in stabilising the Saudi economy. Several reasons make us focus on fiscal policy and its reforms rather than monetary policy. First, the fiscal policy in Saudi Arabia is considered as the main stability performance criteria of the economy for the government because the Saudi currency (Riyal) is set at a fixed exchange rate regime to the U.S. dollar[11]. Thus, the Saudi Arabia economy remains relatively ineffective in monetary policy compared to the fiscal policy. Moreover, in most oil-exporting countries, the fiscal policy has changed in recent years as a result of high oil prices.

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[9] The GCC countries is the abbreviation for the Gulf Cooperation Council countries, which involve six nations: Bahrain, Kuwait, Oman, Qatar, Saudi Arabia, and the United Arab Emirates.

[10] According to the General Authority of Zakat & Tax (GAZT), Saudi Arabia does not have currently a personal income tax.

[11] The Saudi Riyal and US dollar exchange rate has been set at 3.75 Riyal/1 US dollars since 1986.

The fiscal expansion has increased inflation, and existing exchange-rate regimes have limited monetary policy in the fight against inflation. Simultaneously, the main tool for macroeconomic stability in these countries is the fiscal policy (Sturm et al., 2009). Second, the sole owner of the country's oil resources is the governmental Saudi Aramco; therefore, it contributes most of its profits to the government's budget. Consequently, Saudi Arabia's fiscal policy is essentially the only way to transfer revenues from the oil industry to a non-oil sector (Hasanov et al., 2020). Third, the Saudi economy is unique since personal income tax is not available. Thus, expansionary fiscal policy is carried out through the dependence of public expenditure on oil revenues. Fourth, the most important reforms advocated by the IMF, which we will be focused on in this thesis, are related to the fiscal policy.

1.5.3 The Government's Future Plans

Following the IMF's recommendations, Saudi Arabia has recently developed a new plan called *Vision 2030*, which basically maps out some of the suggestions of the IMF. It seeks to achieve several political, economic, and social objectives. The most significant economic priorities of the Vision are to reduce reliance on oil and diversify the sources of national income (Saudi Vision 2030, 2016). The main reason is that the Saudi government is looking for alternatives and new sources of revenue. Some of the policies which have been established to help achieve that goal are:

1.5.3.1 Increase non-oil exports:

Saudi Arabia intends to increase non-oil exports by building incentive programs to encourage exports, focusing on developing the export readiness of small and medium-sized enterprises. These programs include, for example, providing export credit financing, customs duty drawback and exemption on selected materials, equipment and machinery, as well as loan programs for public and private industrial investments. The aim is to find and provide opportunities for companies ready to export and work to improve the efficiency of the domestic export environment to international markets.

1.5.3.2 Diversification of income sources through the imposition of VAT:

As the IMF mentioned in their report of 2015 that Saudi Arabia has plenty of room for implementing new taxes, the country has responded to the advice of the IMF by introducing new tools that can boost revenues away from oil. Thus, Saudi Arabia recently announced the possibility of imposing taxes that could generate other revenues for the government's budget.

1.5.3.3 Increase local value by privatising some state-owned assets:

The government seeks to sell some of the national oil company's shares to the public. Since the company's capital is considered very large, the outlook says that the sale of a small percentage of the shares of the national company would generate a large amount of revenue. Therefore, Aramco Initial Public Offering (IPO) is the centre of attention in the Saudi Vision 2030. The main reason for selling these shares is to enhance transparency and look for investments away from oil.

As a result, the proposals from the IMF, the position of fiscal policy and the government's plan all together inspire us to study the economic growth in the Saudi economy and how it is affected by any changes in oil revenues and by introducing new taxes.

1.6 Contributions

Although our modelling approach presented in this thesis uses a similar approach to some applied in the previous literature, it aims to investigate economic growth in Saudi Arabia further and contribute in four main aspects to the field, as outlined below.

In this study, we model the oil sector as an exogenous and monopolistic sector. This approach is confirmed by the literature on the oil market structure, as we will see in the literature review chapter. The reason for modelling the oil sector in this fashion is that the economic growth of resource-rich countries in most research is considered to be affected, both positively and negatively, by the oil industries. However, the economic growth literature ignored to analyse the market power of oil. In other words, the economic market condition of the oil sector is not given enough and detailed attention in economic growth literature. In most economic growth studies, particularly concerning Saudi economic growth, the oil sector is treated as an exogenous factor, and they examine how the economic activities are only affected when a shock on this exogenous sector occurs. In other words, the oil sector is only considered like any other exogenous variable. However, due to the great importance and attention of this sector, modelling the oil sector in such a way, *i.e.* as a monopolistic sector, can help us to find out some aspects. For instance, what and how the government receives revenues from the sector, how this important exogenous source operates, and how it can affect other endogenous variables in growth models. Thus, we model the oil sector as a monopolistic sector and discover the government's net oil revenues. As yet, there has not been a definitive study that shows and investigates the economic growth and oil production of countries where the oil sector is treated as a monopolistic sector. Therefore, our first contribution is to model the oil sector as a monopolistic sector in an endogenous growth model, *i.e.* Barro (1990). The importance of the oil sector can be seen in its role in financing the productive government sector.

The second contribution in this thesis is to extend the original Barro (1990) model further by allowing for three different sources of government revenues to finance productive government spending. These sources are (i) only oil revenues, (ii) oil revenues with consumption tax, and (iii) oil revenues with personal income tax. (i), (ii), and (iii) would be studied separately in three different chapters. Thus, each chapter of the three main chapters would have a government sector financed by a different source of revenues. In the economic growth literature, there are many studies where the government sector is assumed to be an unproductive sector and therefore have treated government revenues as a pure waste of resources. However, a few models have considered the government sector as a productive sector, such as the Barro (1990) model. Also, as far as our knowledge goes, the only paper that examined consumption tax as an only source to finance the productive government sector in the Barro (1990) model is Bambi and Venditti's (2018) paper. Although we use the Barro (1990) type of production function and follow some of his assumptions, our models in each chapter hold some differences from his original model and Bambi and Venditti (2018) model. The main differences are as follows:

- ❖ The government spending in Barro (1990) model is financed by only a distorted tax, namely income tax. While in the primary model of Bambi and Venditti (2018), government spending is financed by a consumption tax. However, our models have different sources of government revenues to finance government spending in each chapter. More precisely, the government spending in chapter four will be fed by only oil revenues, while two different taxes, namely consumption and personal income tax, will be the source of government revenues along with oil revenues in chapters five and six, respectively.
- ❖ The Barro and Bambi and Venditti (2018) models are endogenous growth models. On the other hand, our models may be either exogenous growth models due to the exogenous oil revenues or endogenous growth model due to taxes. Thus, one type or two different types of balance growth paths could arise in some models.
- ❖ In the Barro (1990) and Bambi and Venditti (2018) models, there is no transitional dynamics, where the economy is always in the steady-state position. However, each model in our study would discuss the stability properties in more detail and represent transitional dynamics, examining each model separately.

Consequently, the second main contribution of this study can be summarised by adding oil revenues with different taxes to finance productive government spending in Barro (1990) model.

For the third contribution, as we will see in chapter three (literature review chapter), studies that have explored the best methods and strategies to deal with volatility in resource revenues has focused on offsetting volatility and shocks through Sovereign Wealth Funds (SWFs). However, the literature ignores the fact that tax reform could be one of the most important fiscal instruments to help oil-rich countries maintain stable economic growth, especially in countries that do not have many tax regimes. It is true that some oil-rich countries, such as Saudi Arabia and other GCC countries, suffer from a lack of taxation. Although these counties have some taxes, such as corporate tax and fees charged to foreigners, they are considered a tiny proportion of state revenues and therefore insufficient. Therefore, reforming the current fiscal policy would also be better than using SWFs which can be affected by several external factors. As will be discussed in the literature review chapter, these external factors may not be controlled by the countries that own them. As a result, the study is mainly aimed at investigating the introduction of new taxes in Saudi Arabia, as suggested by the IMF, and then to find out the amount of tax to compensate for a reduction in the government spending level associated with the decline in oil revenues. In other words, this study focuses on offsetting volatility and shocks through tax reforms to help oil-rich countries maintain stable economic growth in the event of negative shocks in oil demand. This is considered to be the third contribution to this study.

The fourth contribution is related to studying and analysing the role of taxes on Saudi economic growth. Although many studies examined Saudi economic growth, according to our research, no research has studied the taxation, consumption and personal income taxes, and economic growth of Saudi Arabia. In particular, how different taxes would affect Saudi economic growth and how the reforms of the tax system would be a solution to fluctuating oil revenues. Thus, since tax reforms in the Saudi economy have not been covered in detail before, this thesis aims to fill the large gap of economic growth studies in Saudi Arabia as the fourth contribution to this study. Consequently, this thesis analyses the possibility of introducing consumption and personal income taxes in the Saudi economy. It is also attempted to contribute a theoretical knowledge base on economic growth in Saudi Arabia with and without different types of taxes.

Finally, this study is also different from previous models developed for resource-rich countries that have focused on low-income countries. The current model attempts to fit the conditions of Saudi Arabia because it has much scope to implement new taxes, as the IMF mentioned. The unique characteristics of Saudi Arabia considered in this study[12] make the case study more attractive.

[12] Saudi Arabia has some characteristics that distinguish it from other countries. Thus, a separate chapter (Chapter Two) is devoted to highlight and analyse the Saudi economy.

1.7 Research Methodology

The methodology of this research is based on three foundations: a case study, a theoretical model, and a discrete-time method. Starting with a case study methodology, it provides the best approach to studying the country's economy. According to Lokke and Sorensen (2014), a case study is a positivist or interpretive approach used to enhance the refinement of areas under study, such as the economic status of a nation. A case study may also be inductive or deductive, and it may rely on qualitative or quantitative methods or mixed-method. Researchers apply it as a way of testing a theory. Case studies are valuable tools for testing theories that are usually involved in economic sectors. Case study methodologies facilitate understanding of criteria used to set economic decisions in a nation. Cases examine a theory by testing whether it corresponds to specific goals, analysed results, and generalizability of findings. It is, therefore, appropriate for evaluating and validating a particular exception in a country by revealing if the hypothesis is true or false (Lokke and Sorensen, 2014). Based on this, we study the case study of Saudi Arabia for two main reasons. The first reason is that it is chosen because it is considered one of the world's largest oil economies in terms of production and exports. The second reason is that it faces some challenges due to its massive dependence on oil revenues and the limitation of other sources of income.

Regarding the types of research papers in economics, there are three common types which are theoretical, empirical, and theoretical plus empirical papers together. The main difference among these papers is how the research question is approached. Theoretical papers are a complement to empirical research, but they are generally used to understand the deep underlying economic principles and mechanisms at work. Therefore, theoretical papers make reasonable assumptions about the world and the people involved and then determine what should potentially happen if the research problem scenario occurred. Even if data is available, economists may generalise how individuals or variables behave in certain situations using theory to predict what should occur. On the other hand, once data on a problem is available, empirical papers are a natural choice. In order to answer research questions, empirical papers use data collected by observation or experiment. Data refers to verifiable information, such as events of history, economic indicators, demographics. An empirical paper may explain the mutual occurrence of several events. Data analysis is a theory that is based on facts and not on opinions (Powers 2012). Our study is based on a theoretical framework about economic growth in Saudi Arabia. It is designed to fit the economy of Saudi Arabia. There are two key explanations for the implementation of a theoretical framework in this study. The first is to provide a theoretical knowledge base on Saudi economic growth, while the second is due to the lack of some data for this study, such as taxes. Thus, the theoretical framework is used to extend an existing neoclassical model, particularly the Barro (1990) model.

In the analyses of the theoretical models in macroeconomics, Macroeconomists usually use one of the two methods to describe 'state' variables in their models and to deal with time. These methods are discrete-time and continuous-time, where their models track variables relative to changes in time. Specifically, discrete models are used to analyse variables that occur at specific points in time, with the time treated as a countable or finite variable that is expressed in integer values. In contrast, continuous models conceptualise changes in variables as events that occur along a continuum of time (Brida, Lorenzo, and Yapor, 2017). The differences between presenting altered variables in discrete and continuous time are evident in explaining economic theories. For example, Brida, Lorenzo, and Yapor (2017) outlined that the continuous-time approach is mainly used to present changed variables through growth theory, while discrete-time expresses varying instances of business through a cycle theory. However, both reflect the occurrence of changes based on a specified time. Moreover, Romer (2012) explained in his book 'Advanced Macroeconomics' that the alternative of continuous-time is discrete-time, which uses the variables that are specified at specific dates (usually t=0,1,2,..). Typically, the choice between them is based on comfort. For instance, the Solow model has the same consequences in discrete time as in continuous time (Romer, 2012). Based on this, we use the discrete-time approach because we believe that it is elegant and more convenient to use.

In each economy, there are several determinants of economic growth. These determinants depend, in fact, on the economic structure. In Saudi Arabia, the public sector is one of the largest dominating sectors in the economy for an extended time, which is financed by oil revenues. According to statistics, government spending is one of the most influential sectors in the Saudi economy. The graph below shows the trend of real GDP and real government spending in Saudi Arabia from 2000 to 2018[13] (SAMA's Annual Statistics, 2019), where the government spending in 2018 was approximately 22% of GDP.

[13] To assess the relationship between these two variables, we run a regression to find out if a given correlation is statistically significant. Using the data provided by SAMA's Annual Statistics 2019, the results show that the government spending and real GDP variables have a significant (strong) positive relationship during (2000-2018), where is r(degree of freedom)=r(19)=0.976, P-value < 0.001.

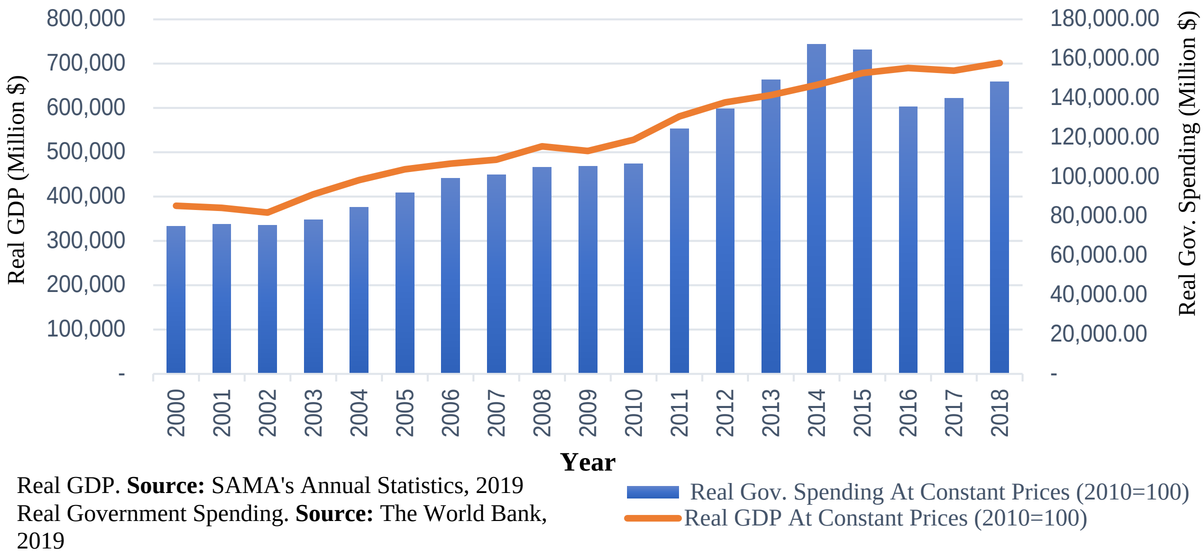

Figure 3: Real GDP and Government Spending Trend during (2000-2018)

There are a number of economic growth models, but we believe that the Barro (1990) model would fit the Saudi economy. The reason is that his model is designed to take much advantage of the important role of the government sector in the economy, as it is evident from the statistics of the Saudi economy. Furthermore, we understand now that the government sector plays a fundamental role as a productive sector in Saudi economic growth, as confirmed by Alrasheedy and Alrazyeg (2019), Alshammari and Aldkhail (2019), Al-Obaid (2004), and Barri (2001). Therefore, the government sector in the Saudi economy should not be treated as a sector that is a pure waste of resources. The objective here is also to use the Barro (1990) model in helping to understand as much as possible of the Saudi economic growth. Therefore, the government spending is productive and one of the main inputs in the production function, such that:

$$Y_t = A\,K_t^{\alpha}\,L_t^{1-\alpha}\,G_t^{1-\alpha} \qquad 0 < \alpha < 1$$

where :

Y_t: Level of output at time t;

A : Technology level in the economy;

K_t: Physical capital at time t;

L_t: Labour at time t;

G_t: Government spending at time t; and

α : The output elasticities of capital, labor, and government spending.

To have a constant return to scale, we apply the Barro (1990) type of production function by assuming that there is no population growth in the steady-state, where the aggregate labour is normalised to one ($L_t = 1$). Thus, the production function used in all our models takes the form:

$$Y_t = A\,K_t^{\alpha}\,G_t^{1-\alpha} \qquad 0 < \alpha < 1$$

The structure of the chapters in this thesis is sequential in one topic but comprises of different models. The thesis is divided into seven chapters, where the first chapter is the introduction, and the second is the analysis of the Saudi economy. The third chapter is the literature review. The fourth is modelling Saudi economic growth before the implementation of the proposed fiscal policy by the IMF. The fifth and sixth chapters are modelling Saudi economic growth separately with the consumption tax and the personal income tax, respectively. The last chapter includes the conclusion, policy recommendations, and limitations and future research. Thus, the three main chapters are the fourth, fifth, and sixth chapters[14].

Barro (1990) model will be extended in all three main chapters by adding the oil sector and taxes and modifying the government's budget constraint accordingly. Taxes will be added in the model of the fifth and sixth chapters, where the fourth chapter will only include the oil revenues in the government budget constraint. For all three main chapters, the key feature of our model is that the oil sector is modelled and treated as an exogenous and monopolistic sector, where the growth is basically led by a growing demand for oil. Moreover, by introducing taxes in our models, we aim to find out how effective the implementation of the proposals of the IMF would be on Saudi Arabia's economic growth to counter oil revenue fluctuations and identify the appropriate amount of consumption tax that could compensate for the potential reduction in oil revenues.

Given that there are three main chapters in this thesis, they are all theoretical and based on a standard neoclassical growth model along with Barro (1990) production function form. The essential difference in these three chapters is in the government budget constraints, where each chapter contains different sources of government revenues. However, they all have some similarities. One of the similarities is that we convert the main variables (*i.e.* level variables) to per government spending unit (PGSU) variables by dividing them by government spending to find the growth rate. For instance, we divide the capital stock, K_t, by government spending, G_t, to obtain the capital PGSU, $\hat{k}_t \equiv K_t / G_t$, and the same thing for the other level variables. Another similarity is that households and firms are assumed to not reach the international market, including financial markets[15]. Moreover, we assume that the government sector has a narrow dealing with the foreign sector. More precisely, the government sector just exchanges and uses all its revenues

[14] The detailed structure of the thesis will be presented at the end of this chapter.

[15] In some emerging markets, the government prohibits the domestic final goods sector to have an international trade or imposes high tariffs to protect domestic industries. In our model, however, we assume that just to simplify our model by avoiding international trade in the final goods sector. One more reason for this assumption is the fact that the oil exports in Saudi Arabia represented a significant percentage of total Saudi's exports. By excluding oil; therefore, the proportion of non-oil exports is a very small portion of total Saudi's exports (SAMA, 2018).

to purchase imported goods[16]. Thus, government revenues are equal to government spending every period. The imported goods are used as public goods, where the government provides them to the private sector (*e.g.* infrastructure, legal framework) to enhance the firm's production function. This represents the only international transactions with the government sector in our model of the fourth chapter. In the fifth and sixth chapters, besides this, the government sector exchanges its taxes revenues for imported goods. Thus, the balance of trade (BOT) in all chapters is assumed to be balanced, *i.e.* there is neither surplus nor deficits. Below are the details of the fourth, fifth, and sixth chapters in this thesis.

In the fourth chapter, we extend Barro's endogenous growth model and describe the Saudi economy before implementing the proposed fiscal policy by the IMF. We model the Saudi economy, where it is highly dependent on oil revenues. Thus, we simplify the theoretical model by setting the oil sector to maximise its profit as a monopolistic sector. Then, the government uses them to enhance the firms' production function. The primary assumption in this chapter is that we only have one exogenous source of revenue that finances government spending and determines the long-run growth. This exogenous source is the oil (monopolistic) sector which faces growing oil demand from abroad. Thus, the government budget constraint in this chapter can be written as,

$$G_t = \pi_t$$

where :
G_t: Government spending at time t; and
π_t: Oil revenues at time t.

Therefore, this chapter aims to answer two of the research questions. In the first research question, we examine how Saudi Arabia's economy would be theoretically affected if there is a negative shock to the demand for oil. Here the focus will be on the impact of reducing the growth rate of government spending on the whole economy. For the second research question, this chapter attempts to answer how the main level variables, such as the level of capital stock, output, and consumption, would be affected if the level of government spending changes due to the change in oil revenues. Despite our primary focus on growth, we also study the change of level variables as a special case of our model to determine the difference in the short and medium run.

[16] Although this is a strong assumption, the fact is that many oil-rich countries suffer from their excessive dependence on their foreign imports. The reason is due to the dependence of their economies on a single commodity for export, oil. Saudi Arabia is one of these countries, which relies on imports to meet the essential needs of goods and services, where most capital equipment are imported. According to SAMA's annual statistics report 2017, although oil exports declined significantly during the period from 2013 to 2016, imports continued to increase, where imports exceeded oil exports in two consecutive years 2015 and 2016. Thus, we assume in our model that all government’s revenues use in purchasing imported goods.

The fifth chapter is an extension of the main chapter, where we extend the model by introducing the consumption tax, as suggested by the IMF, and modify the model accordingly. We use the terminology of consumption tax in our model instead of sales tax or VAT[17] for three reasons. The first reason is that we set up this kind of tax in the model (*i.e.* when we give the assumption of households and firms initially) and treat it as a legal obligation to pay only by the households. Thus, the consumption tax is somewhat different from other types of taxes, such as sales tax which is supposed to impact firms' profit. Although there is no fundamental difference between the consumption tax and sales tax theoretically[18], there is an apparent difference between them. In reality, the consumption tax is usually on consumption goods, whereas the sales tax may be levied on both consumption and investment goods. Thus, the second reason for treating taxes in this chapter as consumption tax is that we ignore taxes on investment goods. The third reason is that the VAT affects intermediate goods because it imposes on every stage of production, but we do not have an intermediate sector in our model. As a result, there are, in this chapter, two sources of government revenues, oil revenues and consumption tax revenues, that finance government spending. The government budget constraint in this chapter can be written as,

$$G_t = \pi_t + \tau^c C_t$$

where :

τ^c: Consumption tax

Again, these two sources of government revenues will finance productive government spending. Similar to the previous chapter, all government revenues are used to enhance the firms' production function, as Barro (1990). To follow up the main economic policy in the country, such as an economic reform of the fiscal policy, this chapter aims to focus on four main aspects. The first aspect is to generate additional revenues for the government since the country does not have enough taxes by introducing consumption tax, as the IMF suggested. The second aspect is to analyse the real effects of introducing a new tax on the key level variables, such as the level of government spending, capital stock, and consumption. The third aspect is to investigate the impact of introducing consumption tax on economic growth. In fact, the second and third aspects are related to the third research question. The fourth aspect is to find the possible amount of consumption tax that can keep government revenues constant when there is a reduction in oil revenues. In other words, this chapter also seeks to examine the amount of consumption tax that

[17] In general, sales tax and VAT in a simple model without intermediate goods, such as our model, are very similar. The only minor difference between them is the initial model setup.

[18] Theoretically and particularly in a closed economy, consumption tax is equivalent to sales tax. However, there is a slight difference between them, which is in the initial model when we write down the equations. More precisely, this difference lies in firm sector, where sales tax, in general, reduce the firms' net profit.

could compensate for the decrease in the level of government spending associated with the decline in oil revenues. This analysis tries to answer the fourth research question.

The sixth chapter is also related to the previous chapters, but it now discusses introducing new taxes. A personal income tax will be presented as a proposal to reform the current fiscal policy. The motivation behind studying this type of tax is due to the fact that the state urgently needs to create other revenues away from oil. Thus, this chapter pursues to find out how a personal income tax can work and affect both the key level variables and the growth rate in the Saudi economy. The two main aims attempt to answer the fifth research question. In fact, this chapter is like chapter five in having two sources of government revenues to finance productive government spending, but it is different in the type of these sources. More precisely, the two sources of revenues in this chapter are oil revenues and personal income tax revenues. Thus, the government budget constraint in this chapter can be written as,

$$G_t = \pi_t + \tau^Y Y_t$$

where :

τ^Y : Income tax

In all three main chapters, the agents in the economy, equilibrium, analysis of the steady-state, local stability, transitional dynamics, and numerical simulation will be examined. In the concluding chapter, we summarise our previous chapters and our models, and we focus on our results. It also attempts to provide policy recommendations and possible future works.

1.8 Results

This section summarises our fundamental findings in the three main chapters, four, five, and six. In chapter four, although we used the Barro endogenous growth model and his type of production function, our model showed different conclusions from Barro's finding. More precisely, we found that the growth rate of government spending is the growth rate of oil profits, g^2. That means everything in the economy of this chapter grows at the rate of government spending, which is the exogenous growth rate, unlike Barro's conclusion. The findings of our analysis also indicated that if the growth rate of government spending declines, both capital PGSU and consumption PGSU would raise at the steady-state. Moreover, we obtained an interesting finding that although the level of consumption would grow at a slower rate due to the reduction in the government spending growth rate, the consumption PGSU would be higher. Higher consumption PGSU suggests that there would be partially offsetting of the effect of the reduction in the government spending growth rate. Thus, our model showed that there is some partial offsetting shift in the level of consumption, which may not be obvious.

Chapter five also extended Barro (1990) model, where two government revenues, oil revenues and consumption tax, finance productive government spending. Even though it is unlikely to have a steady-state in which the two sources of revenues grow at a different rate, thinking of an equilibrium in which one of these shares tends to zero, and the other tends to one could be a valid equilibrium to consider in this chapter. For this reason, we studied three possible types of steady-states. We then found that two of them involve that the growth rate of government spending tends to an exogenous growth rate (type (*I*) steady-state), while the third possible involves that the growth rate tends to an endogenous growth rate (type (*II*) steady-state). However, studying the existence of steady-states of both types showed that there is an unavoidable contradiction between two conditions related to type (*I*) steady-state and type (*II*) steady-state. Thus, we concluded that both types of steady-state growth rate could not exist for the same set of parameter values.

Based on the above finding, two results have been found in how and when the economy could move from a steady-state to another. The first finding was that if we set consumption tax, τ^c, constant and change g^2. The results showed that if g^2 is sufficiently high (low) for a given value of τ^c, then the steady-state will be an exogenous (endogenous) growth steady-state. The second finding was that if we set g^2 constant and vary τ^c. The outcomes indicated that at a certain value of τ^c, the economy would switch from one type of steady-state to another one. More precisely, the latter result implies that as we gradually increase τ^c, the economy eventually will pass from an exogenous growth steady-state to an endogenous growth steady-state. We could interpret the critical value of τ^c at which this switch occurs as being the 'take-off' point for the economy, *i.e.* the curve at that point becomes upsloping because high τ^c produces a high growth rate. Thus, as τ^c is increased, beyond this point, the steady-state growth rate of the economy is eventually 'freed' from the growth rate of oil profits; whereas, until it is reached, the steady-state growth rate is tied to the growth rate of oil profits. The switch to the endogenous growth steady-state would raise the rate of growth, as high taxes would help provide enough government spending to improve the production function of the firms.

This chapter also investigated how the consumption tax can compensate for any reduction in the level of government spending associated with a decrease in oil revenues. In this regard, we provided an exercise and derived a formula for consumption tax that allowed us to calculate the required tax rate that compensates for any reduction in oil revenues and keep, at the same time, the government spending unchanged. Our simple example of this exercise displayed that if the oil revenues declined by 10%, the required tax rate to compensate for this reduction is 6.21%.

Chapter six examined the possibility of introducing personal income tax in the Saudi economy, and how its economic growth can be affected by this type of tax. This chapter also extended the Barro (1990) model, where oil revenues and personal income tax finance productive government spending. Similar to chapter five, because there is a contradiction between two conditions for each type of growth models, the steady-state results confirmed that both types of steady-state cannot exist for the same set of parameter values. However, the findings of this chapter, contrary to the findings of chapter five, show that two critical values allow the economy to switch from exogenous growth steady-state to endogenous growth steady-state and vice versa. To examine under which conditions our economy would be in, we investigated two situations. The first situation was to change g^2 and fix τ^Y, whereas the second situation was the opposite. The result of changing g^2 and fixed τ^Y showed that if g^2 is sufficiently high (low) for a given value of τ^Y, then the steady-state will be an exogenous (endogenous) growth steady-state. The second situation is the opposite, *i.e.* keeping g^2 constant and varying τ^Y. The result of the second situation displayed two different positions for the steady-state growth rate. Specifically, if τ^Y is either (neither) sufficiently low or (nor) sufficiently high, then the steady-state will be an exogenous (endogenous) growth steady-state. These two patterns give us a clear picture of having two critical values for τ^Y. The first critical value is when the economy moves from exogenous growth steady-state to endogenous growth steady-state, while the second critical value is the opposite. These two critical values depend on the rate of personal income tax. Consequently, the main results of switching from one type of steady-state to another one, found in chapters five and six, are indeed unusual in most growth models.

Finally, the outcomes in chapter six also showed that the personal income tax would temporarily reduce the growth rate under certain parameter values, which could be considered as a warning to a policymaker. This outcome indeed contrasts with the result in chapter five, where the growth rate will unambiguously be temporarily increased by an increase in consumption tax. The intuition for why personal income taxes are found to harm growth whereas consumption taxes do not is that although consumption tax and personal income tax can provide additional revenues to the government, which implies that firms will receive more positive externalities, these taxes can have a different impact on other economic activities. For instance, consumption taxes would lower current consumption but savings, and therefore investment, would be increased, which implies high capital accumulation. Increased investment can increase labour productivity and wages, and therefore future output and consumption. On the other hand, personal income tax negatively affects private investment and labour supply. More precisely, it reduces capital accumulation and can encourage people to work less. Thus, personal income tax discourages capital accumulation and so decreases the growth rate in the economy. As a result, consumption taxes would stimulate the growth rate in the economy, unlike personal income tax.

Before we move to the next chapter, which gives an overview and analysis of the Saudi economy, the following diagram briefly outlines our research structure and chapters.

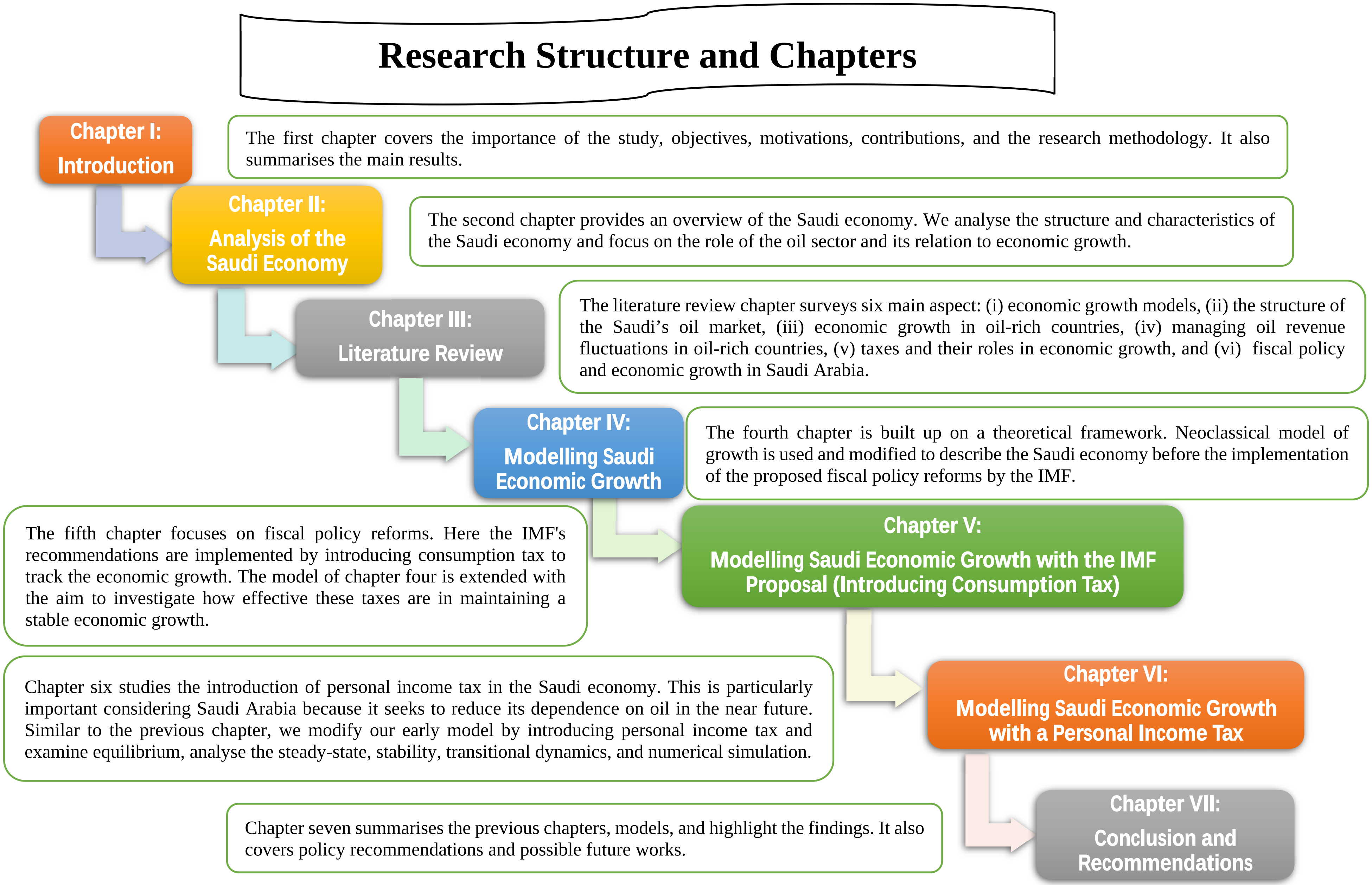

Diagram 1: The Structure of the Thesis

2 *Chapter Two: Analysis of the Saudi Economy*

2.1 Introduction

In this research, Saudi Arabia has been chosen as a case study for several reasons. The most important being the fact that it is the largest economy in the Middle East in terms of GDP, one of the emerging economies and a member of the Group of Twenty (G20), and one of the largest oil-producing and exporting countries in the world. However, it faces many problems, most of which have arisen because of structural economic imperfections. The most significant of these issues is the way to deal with the economy where it has been managed over the past decades through total dependence on oil, which is a challenge for the country due to fluctuating oil revenues. Furthermore, the private sector relies on direct and indirect government support, which is financed by oil revenues, to ensure their survival and continuity.

Based on these economic issues, the importance of deeply studying and analysing the Saudi economy has emerged. This chapter focuses on describing the characteristics of the Saudi economy, which distinguishes it from other economies. It also aims to highlight and analyse these economic problems by tracking economic performance for more than five decades. It includes five main sections covering basic information about Saudi Arabia, the Saudi Arabian tax system, development and growth in the Saudi economy, the oil sector and its role in the Saudi economy, and finally the development plans and current 'Vision' plan of Saudi Arabia. The role of the oil sector will be discussed in detail because of its importance to the Saudi economy. The fact is that the oil sector has been vital to the economic growth of the country. Thus, we also look at the role of the government sector, particularly government spending, in driving economic growth.

2.2 Basic Information about Saudi Arabia

Saudi Arabia is an essential gateway for the world, being at the centre of the three continents of Asia, Europe and Africa, as well as being surrounded by some of the most important water crossings. It locates in the far southwest of Asia and borders on the North by three countries, Jordan, Iraq and Kuwait, on the East by the Arabian Gulf[19], Bahrain, Qatar, and the United Arab Emirates, on the South by two counties, Oman and Yemen, on the West by only the Red Sea.

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[19] The Arabian Gulf is also called the Persian Gulf.

Saudi Arabia accounts for four-fifths of the Arabian Peninsula, with an area of around 2 million square kilometres. According to the Saudi General Authority for Statistics, the nation's population stands at 33.4 million with an estimated annual growth rate of 2.64%, males accounting for 57.6% and females accounting for 42.4% (General Authority for Statistics, 2018). The national currency is called Saudi Riyal (SAR), and the exchange rate between the Saudi Riyal and US Dollar has been fixed at 3.75 Riyal/1 US Dollar since 1986.

Source: (Worldatlas.com, 2017)

Figure 4: Saudi Arabia Map

2.3 Saudi Arabian Tax System

Taxation is generally one of the primary sources of revenues for a country to finance its activities and expenditures. In a national economy, it plays an essential role and is considered one of the key instruments for fiscal, political and social policy. The most important tax functions of a state can be summarised as financing state expenditures, achieving the social goal of redistributing income, finding a balance in financial policies by encouraging investment in certain domestic or foreign products. It also meets policy targets by providing exemptions or incentive measures to economic sectors that influence consumption, production and saving habits, and protecting local products by imposing higher taxes on imported products and reducing or abolishing domestic products. This section will give a historical overview of Saudi taxes and describe the current taxation system and how the government receives oil revenues, which constitute a significant part of the revenues in the country.

2.3.1 Historical Overview of Taxation

Islamic countries, including Saudi Arabia, have practised a system of social solidarity since the dawn of Islam through the imposition of Zakat[20] on Muslims, one of the pillars of Islam. Zakat encourages income distribution because it is imposed on those with wealth and is dedicated to helping and supporting poor and needy people. Regarding the role of Zakat as a financial tool, the rate of Zakat is fixed, *i.e.* it does not increase or decrease due to financial need. Also, Zakat does not go to the state treasury; instead, it goes directly to the poor and needy. However, due to the limited financial resources of states in the past, taxes were imposed along with Zakat to finance public expenditure and projects of the state (Alobaid and Atya, 1994).

According to an International Monetary Fund (IMF), they pointed out in their report of 2015 that the evolution of tax regimes has mainly been influenced by the role of oil exports and their revenues in financing the government budget. The introduction of taxes in Saudi Arabia goes back to 1950, when personal income tax, capital gain tax and corporate tax were imposed on all citizens and non-citizens. However, after a short period, the tax law was amended to exclude citizens, who only had to pay Zakat. In 1975, as oil revenues improved and the country's income increased, taxes on foreigners were temporarily paused because of the need to hire expatriates to help build infrastructure and develop the local economy. The IMF report also noted that one of the main characteristics of the Saudi tax system is the absence of consumption tax and personal income tax on citizens and non-citizens. Corporate tax is also very limited for non-citizens engaged in commercial or professional activities. However, Saudi Arabia imposed 2.5% of Zakat on real estate if it is held for speculative purposes (IMF, 2015c).

2.3.2 Current Taxation

Taxation in Saudi Arabia is very limited, meaning the contribution of its revenue to total government revenue is very small. The current main taxes in Saudi Arabia are corporate tax and value-added tax (VAT). Regarding corporate tax, according to the Saudi General Authority of Zakat and Tax, Saudi citizen investors pay Zakat while non-Saudi citizen investors are subject to corporate tax. The rate of corporate tax is 20% for any of the following: "(1) A resident capital company; (2) A non-Saudi resident natural person who conducts business; (3) A non-resident person who conducts business in the Kingdom through a permanent establishment" (General Authority of Zakat and Tax, 2004, Article 7). Regarding consumption tax, Saudi Arabia introduced a 5% VAT rate in 2018 on most countries' goods and services. Later, in 2020, the

[20] Zakat refers to Islamic alms and is an obligatory contribution of a particular portion of one's wealth (2.5%) in support of the poor, needy or other charitable purposes (The Free Dictionary, 2016).

government raised the VAT rate from 5% to 15% due to the COVID-19 pandemic, which has dramatically affected oil revenues. As a result, Saudi citizens are currently subject to only VAT and Zakat, whereas personal income tax is absent.

2.3.3 Oil Sector Taxation

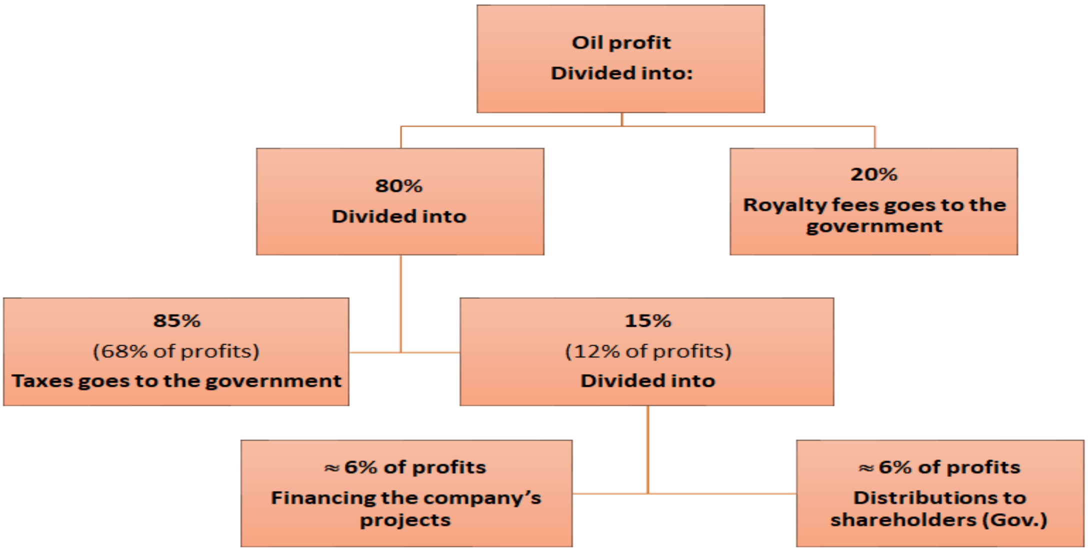
The government wholly owns the oil sector; however, the government does not receive 100% oil revenues. From this standpoint, our passion is to investigate the distribution of oil income in Saudi Arabia. This is because oil is the main engine of the Saudi economy, and oil revenues feed the public budget by a significant percentage. According to Abdul-Baqi, Saudi Aramco's[21] vice president of exploration (Susris 2004, para 36):

> we are a self-financing company. We sell our crude, for revenue, deduct our operating costs. Then pay the government taxes and royalties. The money remaining is used to finance our capital projects, and what is left is distributed as dividends to the shareholders, and we have been doing this since the company started in 1933.

Moreover, according to Nat Kern, the President of Foreign Reports Inc., Saudi Aramco pays 20% royalties and 85% of its oil profits as a tax rate (Mufson, 2016)[22]. Although no official source describes the exact percentage distribution of oil profits in Saudi Arabia, we have arrived at an approximate method for calculating the distribution of oil revenues and profits in Saudi Arabia. However, it can only be considered as the researcher's approximation of the distribution of oil income. The researcher estimates that the approximate percentage of oil revenues received by the government may be around 94% of total oil profits. The remainder goes to financing the company's capital projects, as shown in Diagram 2.

[21] Saudi Aramco is a national company and is one of the world's leading companies in terms of exploration, production, refining, marketing and exporting of oil.

[22] The source of this information is an unpublished report by Foreign Reports, Inc. Ideas were then deductively based on what has been reported by The Washington Post.



Source: Based on the researcher calculation

Diagram 2: Estimation of the Distribution of Oil Revenues

Even though oil revenues received by the government may account for around 94% of the country's budget revenues, oil revenues could represent almost 100% of Saudi's budget revenues. The reason is that most of the other revenues come from oil, too, such as petrochemicals and petroleum refining. Thus, these additional revenues vary as oil prices change.

2.4 Development and Growth in the Saudi Economy

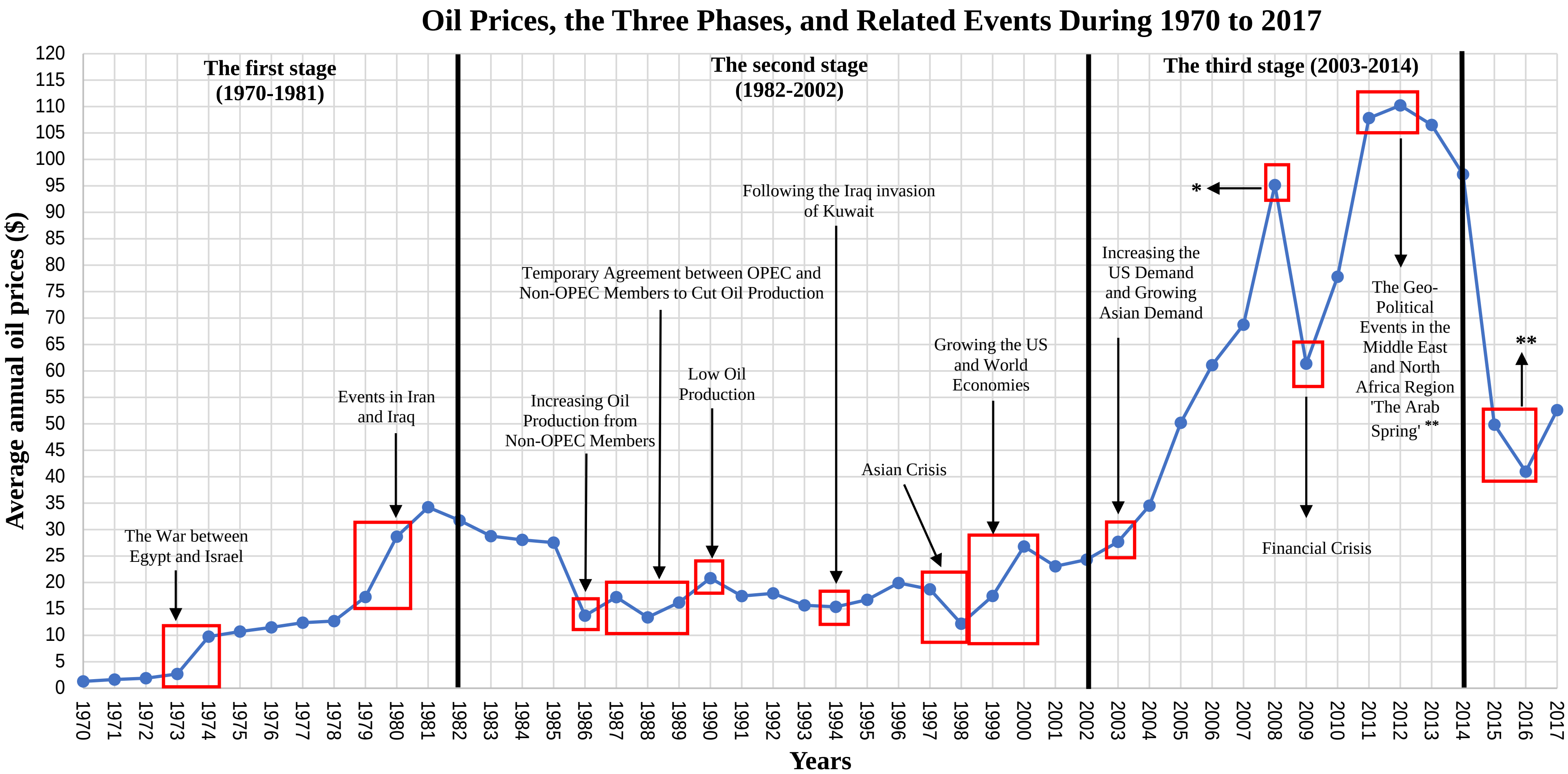
The Saudi economy has undergone various stages of development which differ significantly from other developing countries. It has some unique internal and external conditions, where the most important of these conditions are: (I) Saudi Arabia enjoys substantial oil reserves, which makes it highly ranked among the world's countries that export oil. This feature has undoubtedly provided sufficient resources for faster development than other developing countries. (II) It occupies importance in the heart of the Islamic world, with Makkah and Madinah being the two main holy cities for all Muslims. This has made it a commercial centre and a spiritual centre frequented by pilgrims from all over the world. (III) Saudi Arabia has not experienced any external colonisation, unlike many other developing countries. Thus, its wealth has not been looted, and it has not suffered cultural, religious and social distortion. This has helped it to achieve the social and political stability necessary for development. (IV) As development programs did not begin in Saudi Arabia until later, this gave it the advantage of being able to benefit from the experiences of others. Unlike some industrialised countries, it was able to obtain the latest technology from developed countries, helping it to achieve efficiency in performance and low costs (Alobaid and Atya, 1994).

2.4.1 Development Period Before 1970

Alobaid and Atya (1994) analysed the period before 1970 in Saudi Arabia, as the early 1970s was classified as an interval phase in the development of the Saudi economy. Since 1949, the development of Saudi society was influenced by oil due to significantly higher oil revenues. Despite the government not intervening in development during that period through a comprehensive plan, it was still based on an annual budget. During this period, the state's budget enjoyed surpluses for most years. However, the characteristics of a pre-planning phase can be identified in two main regards. The first was being the weak absorptive capacity of the Saudi economy, *i.e.* it was unable to exploit its resources fully to achieve a level of development commensurate with the level of the resources. The second was being the Saudi economy having the characteristics of a traditional developing country. These characteristics can be summarised as low levels of nutrition among individuals, poor education and health.

2.4.2 Development Period After 1970

This section includes a comprehensive summary of the most important phases of the development of the Saudi economy after 1970. SAMA's Annual Statistics (2018), AlThumairi (2016), Al-Kibsi et al. (2015), Suleiman (2013) have shown that Saudi Arabia went through three significant phases between 1970 and 2014: boom, recession and recovery. These stages witnessed substantial changes in the local economy due to fluctuations in oil revenues during that period and some important economic and political events. The figure below shows oil prices and charts the stages of development of the Saudi economy, as oil is the most important economic resource that has driven the development movement in Saudi Arabia. Also, this figure is generally indicative of the evolution of the price of oil at various stages. Due to the importance of the oil sector, its prices, and the revenues generated from it, the stages of growth of the Saudi economy have been positively and negatively affected by changes in oil revenues.



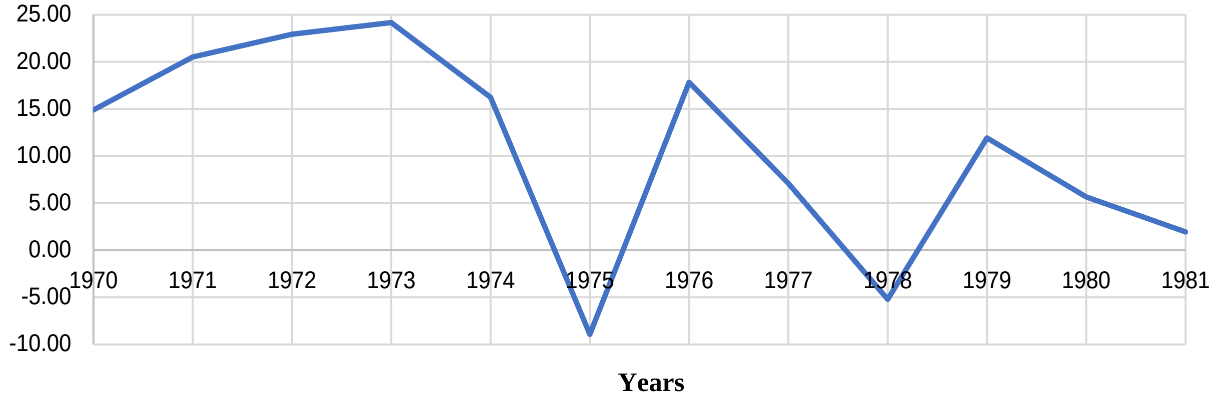
* Increasing oil demand from China and India, speculations in oil markets, disorder and violence in Nigeria, Venezuelan government tensions with foreign oil companies, and events in Iraq.
** Arab Spring is "a series of anti-government uprisings affecting Arab countries of North Africa and the Middle East beginning in 2010" (Merriam-Webster's Dictionary).
*** High oil production especially from the United States, declining world oil demand and rising the dollar exchange rate.

Sources: SAMA's Annual Statistics, 2018; AlThumairi, 2016; Al-Kibsi et al., 2015; Suleiman 2013; and the Researcher.

Figure 5: Average Annual Oil Prices, Economic and Political Events, and the Stages of the Development of the Saudi Economy

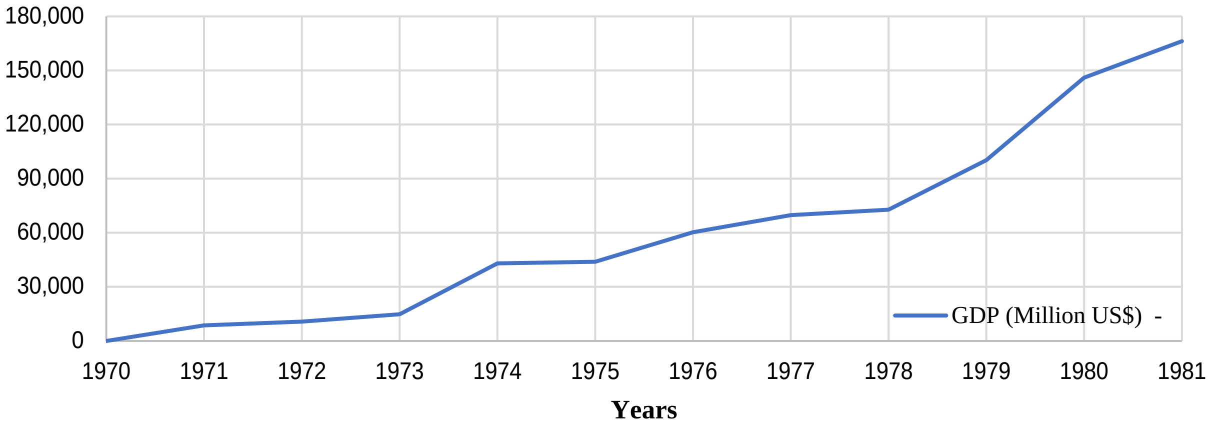
2.4.2.1 The First Phase (1970-1981) 'Boom Phase':

Economic indicators during this period showed an improvement in the financial and economic situation of Saudi Arabia. In the below figures, the average annual growth rate of real GDP during this period reached 10.75%. At the beginning of this phase, nominal GDP increased from $8,596 million to $166,231 million by 1981.



Source: SAMA's Annual Statistics, 2018

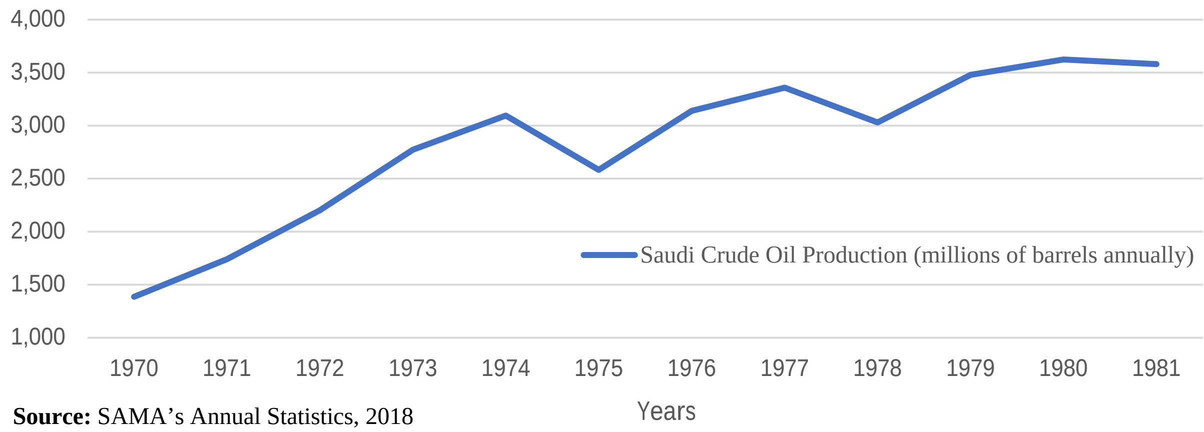
Figure 6: GDP Growth Rate at Constant Prices During the First Stage 1970-1981



Source: SAMA's Annual Statistics, 2018

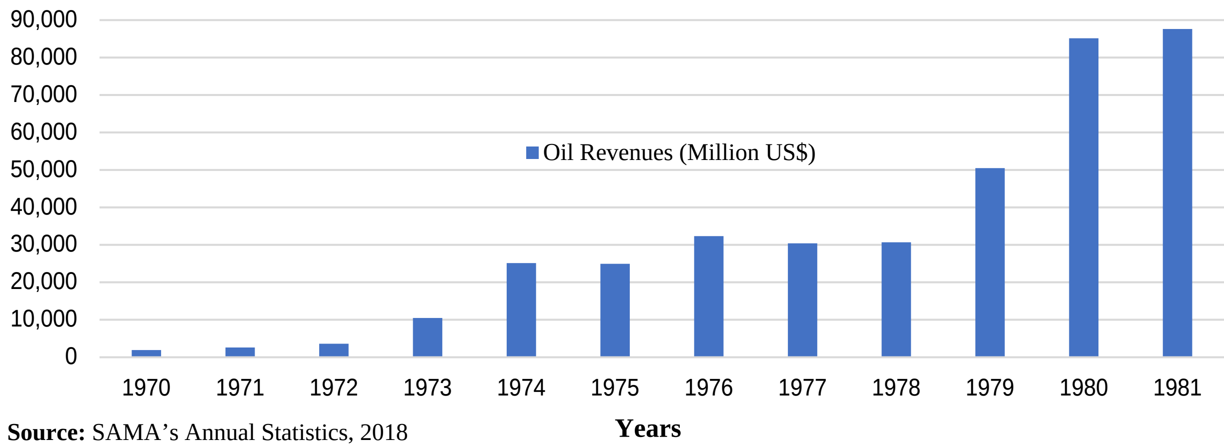
Figure 7: Gross Domestic Product During the First Stage 1970-1981

Saudi Arabia achieved a marked improvement from year to year due to the increase in oil production and its prices and revenues. Oil production rose from 1,386 million bpy at the beginning of 1970 to 3,579 million bpy at the end of 1981. There was no cut in production throughout this period other than in 1975, 1978, and 1981, where the decrease was 16.6%, 9.8% and 1.2%, respectively, see Figure 8. Oil prices increased significantly, rising from $1.3 per barrel in 1970 to $34.2 in 1981. The price of a barrel of oil doubled more than once, leading to a rise in oil revenues from $1,899 million in 1970 to $87,625 million in 1981. This period also witnessed two historical jumps in oil prices, the first occurring after the October War between Egypt and Israel in 1973 and the beginning of the transition of the oil pricing decision of international companies to OPEC, leading to a jump in price from $2.7 a barrel to $9.8 per barrel in 1974, an increase of 262%. The second jump was the result of an interruption of supplies from Iran during its revolution in 1979, and then its disconnection from Iraq and Iran after the outbreak of war between the two countries in 1980. Thus, oil prices rose by 65.9%, from $17.3 a barrel in 1979 to $28.7 per barrel in 1980. This reflected positively on the general budget of Saudi Arabia, where oil revenues rose in the last two years of the period to $85,148 and $87,625 million in 1980 and 1981, respectively, as shown in Figure 9.



Source: SAMA's Annual Statistics, 2018

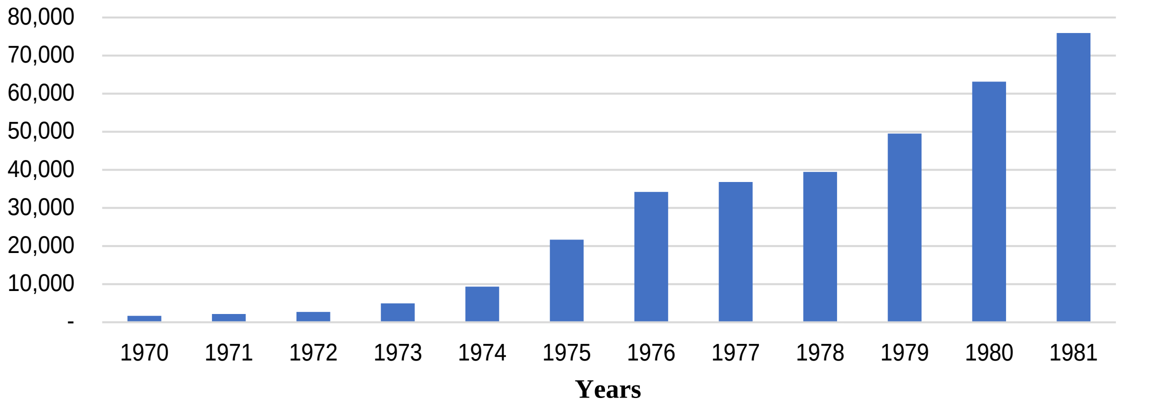
Figure 8: Saudi Crude Oil Production During the First Stage 1970-1981



Source: SAMA's Annual Statistics, 2018

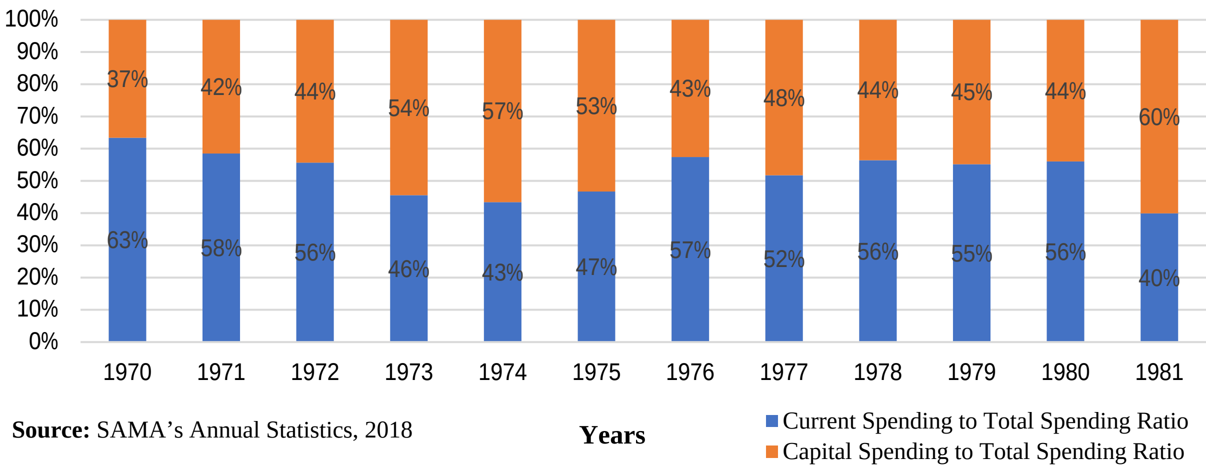
Figure 9: Oil Revenues During the First Stage 1970-1981

When comparing the first and last years of this stage, we find that the proportion of oil revenues rose very significantly during the ten years, reaching 4514%. This prompted the government to adopt successive increases in spending, rising from $1,678 million in 1970 to $75,906 million in 1981. Figure 10 displays how government spending sharply increased in this phase. Current government spending accounted for nearly 52%, while government investment spending was 48%, see Figure 11. The reason for this allocation of government spending and investment was the need to meet basic needs and obtain equipment such as defence, public administration, transport, communications, education and vocational training, and cultural affairs.



Source: SAMA's Annual Statistics, 2018

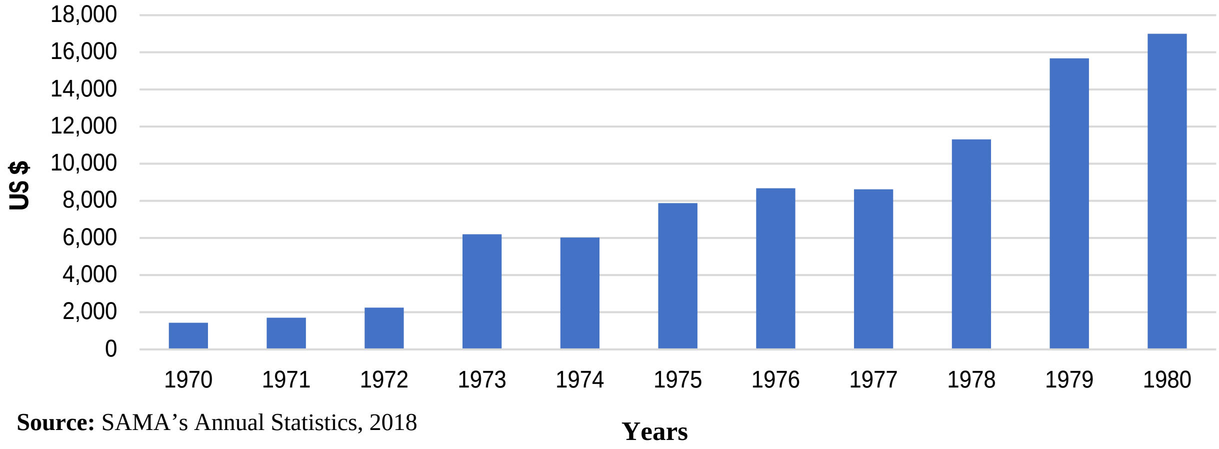
Figure 10: Nominal Government Spending During 1970-1981



Source: SAMA's Annual Statistics, 2018

Figure 11: Annual Current and Capital Government Spending: 1970-1981

The period also witnessed a significant increase in the average income per capita, which rose from $1,432 in 1971 to $16,997 in 1981, as shown in Figure 12, meaning the average rise in the average income during this period was 1087%. Thus, it is clear that the period 1970-1981 was one of the most critical stages witnessed by Saudi Arabia because of its prosperity, growth and high levels of welfare.

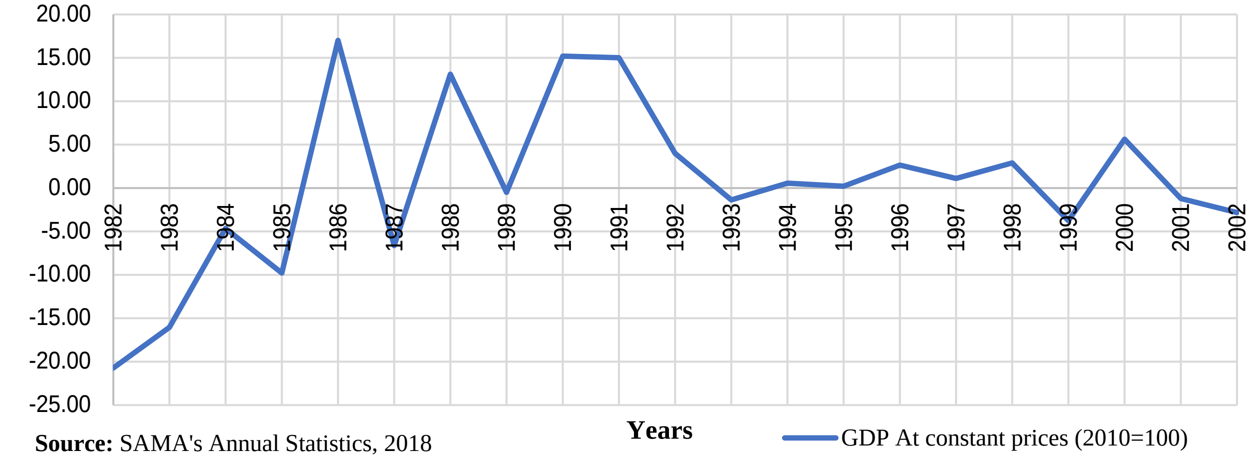


Source: SAMA’s Annual Statistics, 2018

Figure 12: Average Income Per Capita During the First Stage 1970-1981

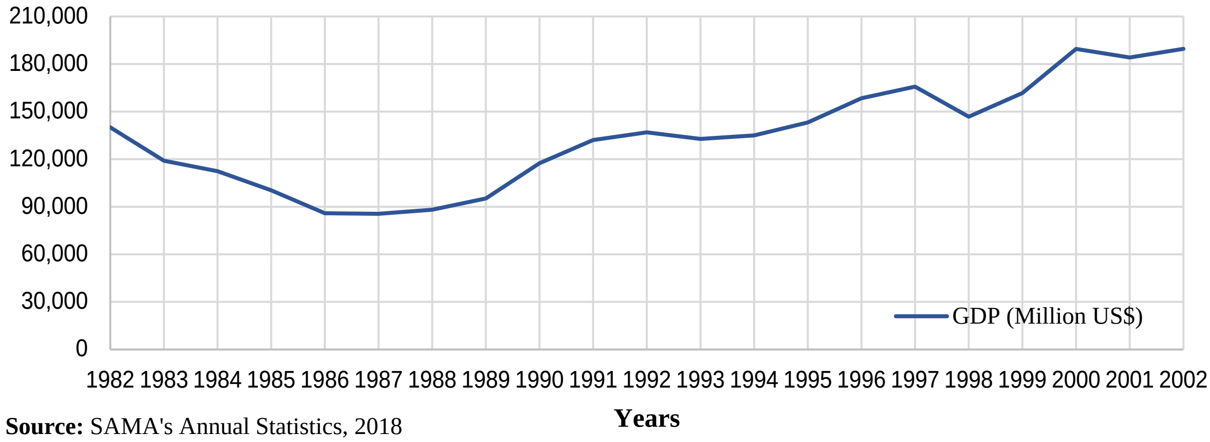
2.4.2.2 The Second Phase (1982-2002) ‘Recession Phase’:

The second period witnessed an economic recession. Figure 13 displays that the average annual real GDP growth rate dropped to 0.47% during the period, which can be considered very low compared to the previous boom phase. The nominal GDP fluctuated during this period, increasing and decreasing until it reached $189,605 million in 2002, where it was $140,089 million at the beginning of this phase, see Figure 14.



Source: SAMA's Annual Statistics, 2018

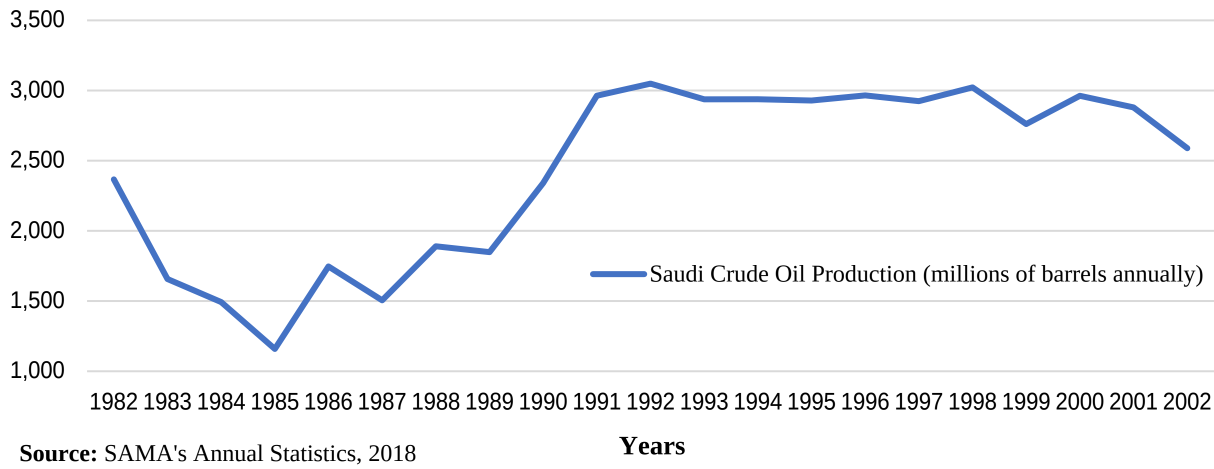
Figure 13: GDP Growth Rate at Constant Prices During the Second Stage 1982-2002



Source: SAMA's Annual Statistics, 2018

Figure 14: Gross Domestic Product During the Second Stage 1982-2002

During this phase, economic and political events significantly impacted oil prices (see Figure 5) and other macroeconomic variables in Saudi Arabia. During the period, oil prices fell from $27.5 a barrel in 1985 to $13.73, $17.23, and $13.40 per barrel in 1986, 1987, and 1988, respectively. The main reason for this was the temporary agreement to reduce production between OPEC and non-OPEC members. For this reason, Saudi Arabia's response was production reduction to a low of 3.2 million barrels per day (MMbbl/bpd) in 1985, equivalent to 1,159 MMbbl a year, as can be seen in Figure 15. As a result, oil revenues were negatively affected, reaching a low of 11,323, 17,974, and 12,905 million dollars during the period 1986-1988, see Figure 16. After that, oil prices remained relatively stable but fell again to a low of $12.2 per barrel during 1998, affected by some events such as the Iraq and Kuwait war and the Asian financial crisis.



Source: SAMA's Annual Statistics, 2018

Figure 15: Saudi Crude Oil Production During the Second Stage 1982-2002

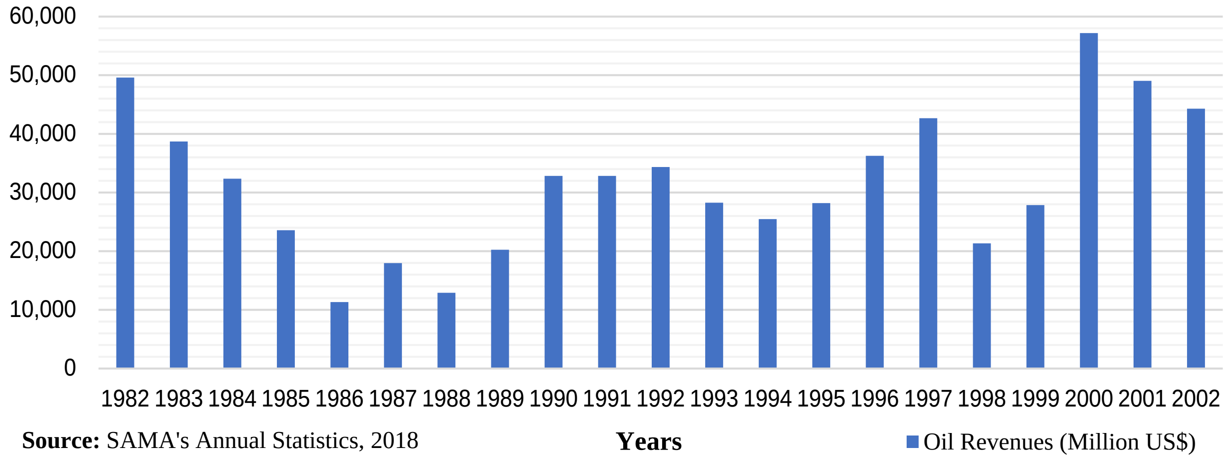


Figure 16: Oil Revenues During the Second Stage 1982-2002

Therefore, government spending during this period remained weak and low, between $36-68 million. However, in the early 1990s, government spending rose due to Iraq's invasion of Kuwait. Most of the government spending was in current government spending, reaching an average of 79% during this phase, as shown in Figure 18.

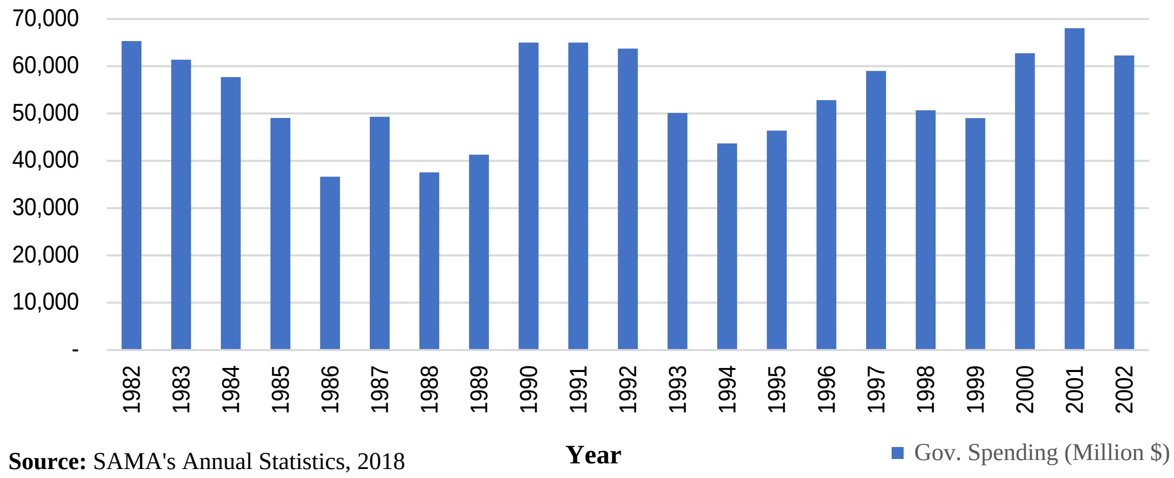


Figure 17: Nominal Government Spending During 1982-2002

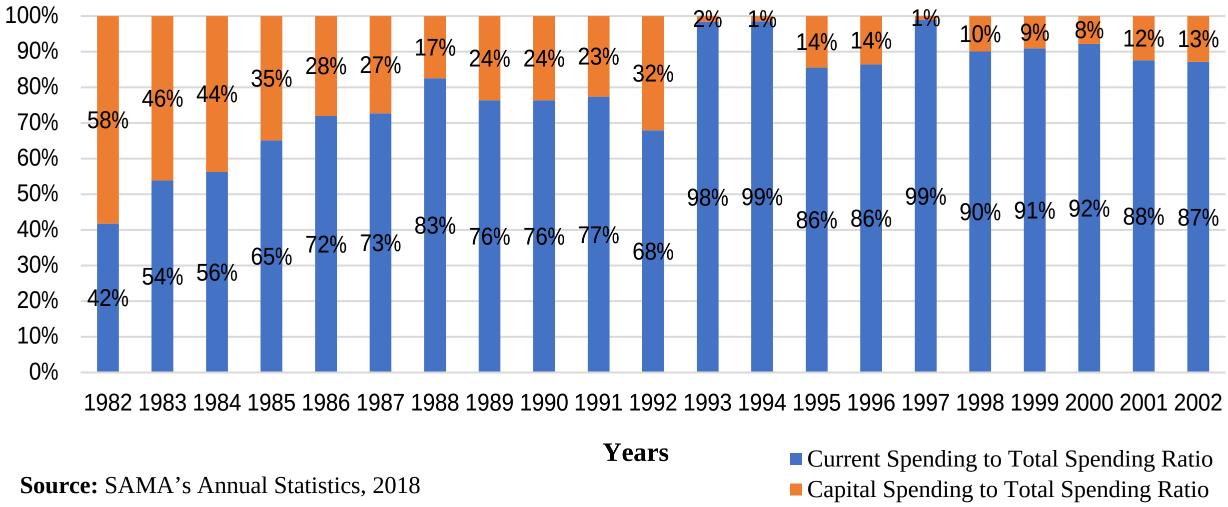


Figure 18: Annual Current and Capital Government Spending: 1982-2002

Besides, Figure 19 demonstrates how the average income per capita at this stage negatively impacted. It decreased from $16,997 in 1981 to $8,821 in 2002, a decrease of 48%. Although the last years of this phase witnessed a marked improvement in oil prices and consequently oil revenues due to the increase in American and Asian demand as well as the growth of the global economy, this stage led to much economic instability. The reason behind that is because of its total reliance on oil revenues, which was frequently fluctuated.

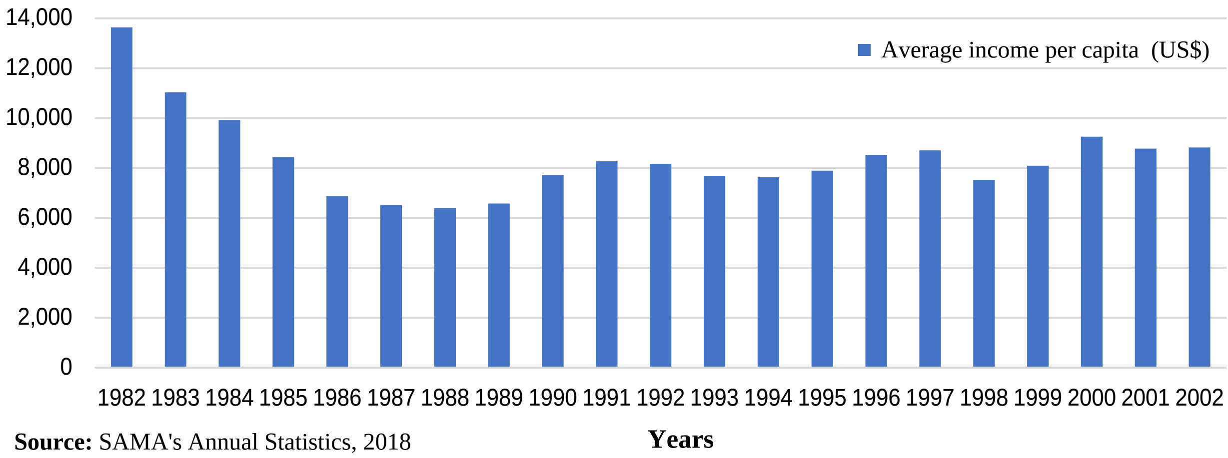
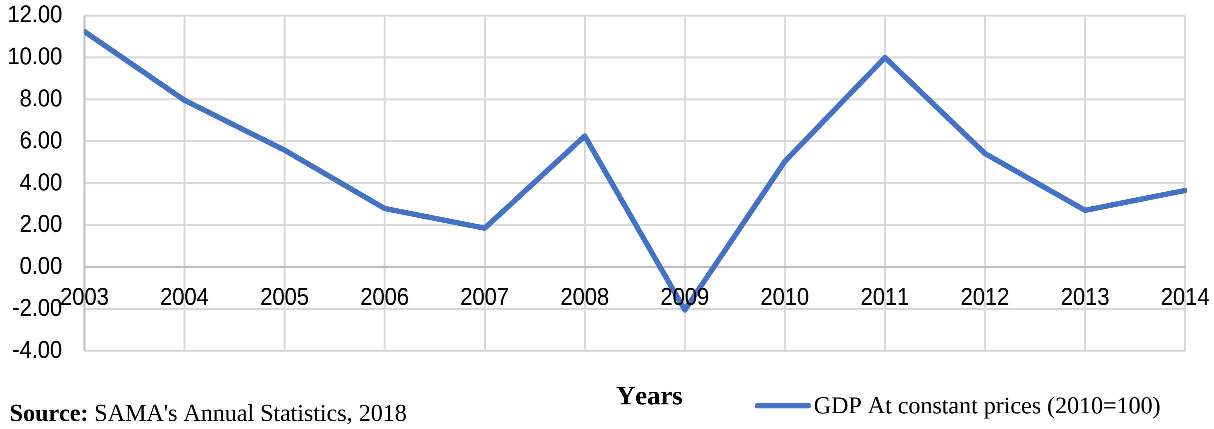


Figure 19: Average Income Per Capita During the Second Stage 1982-2002

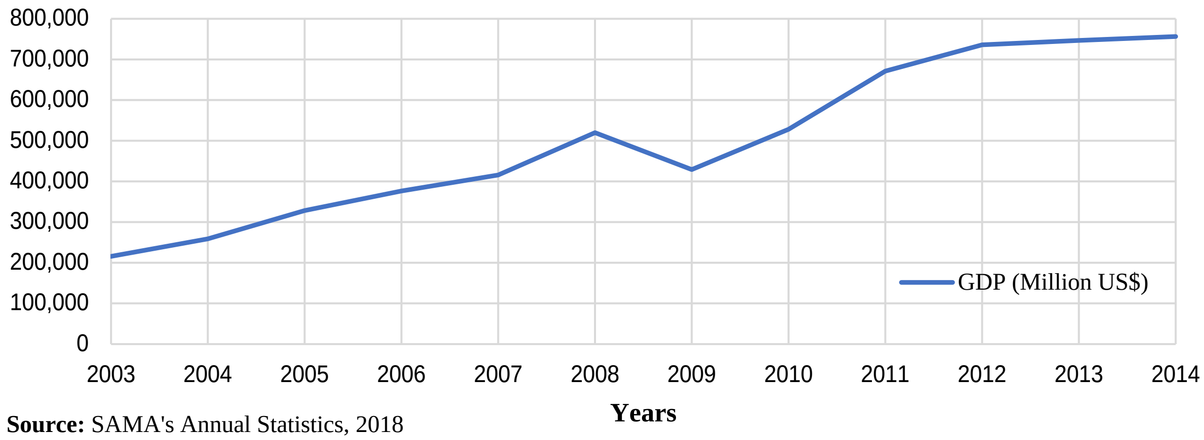
2.4.2.3 The Third Phase (2003-2014) 'Recovery Phase':

The third phase witnessed a recovery in the Saudi economy compared to the previous recession. Figure 20 displays that the average annual growth rate of real GDP in this phase was 5.03%. The nominal GDP increased to historical levels in this phase, reaching $756,350 million in 2014, see Figure 21.



Source: SAMA's Annual Statistics, 2018

Figure 20: GDP Growth Rate at Constant Prices During the Third Stage 2003-2014



Source: SAMA's Annual Statistics, 2018

Figure 21: Gross Domestic Product During the Third Stage 2003-2014

There were unprecedented increases in oil prices from 2003 to 2014, as shown in Figure 5. The average price of oil reached $31.11 a barrel as a result of increased US demand and growth of Asian demand as well. In 2008, oil prices rose to $95.16 a barrel because of many economic and political factors. For example, there was an increased oil demand from China and India, speculation in oil markets, disorder and violence in Nigeria, tensions between the Venezuelan government and foreign oil companies, and political events in Iraq. This was followed by the global financial crisis in 2008-2009, which caused the price of oil to decline by 35.6% to reach $61.3 a barrel. The consequence of geopolitical events in the Middle East and North Africa, the

'Arab Spring', was that oil prices rose again, but this time significantly, reaching $110.22 a barrel in 2012. Although Saudi oil production during this period remained relatively stable at an average daily output of about 9.06 million bpd, oil revenues were positively affected by the events that took place. From the beginning of this stage, oil revenues increased to reach $262,231 million in 2008 and then fell by 55.8% due to the global financial crisis. After that, oil revenues began to recover to a maximum of $305,284 million in 2012, as shown in Figure 23.

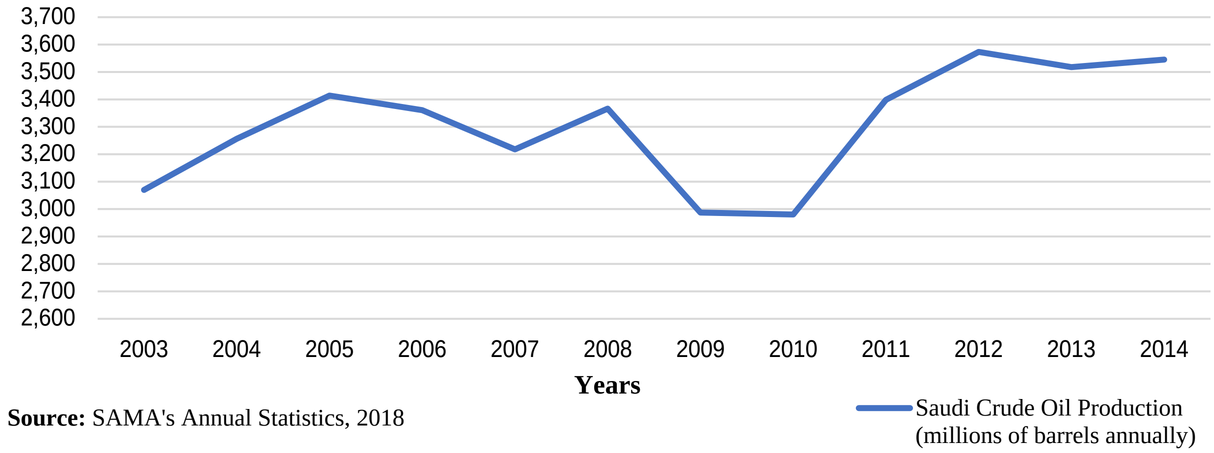


Figure 22: Saudi Crude Oil Production During the Third Stage 2003-2014

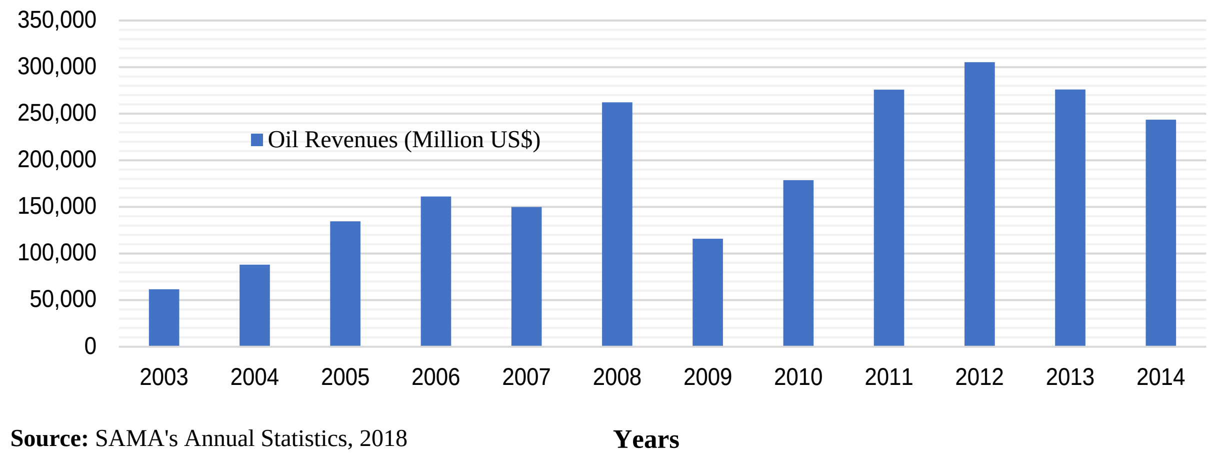


Figure 23: Oil Revenues During the Third Stage 2003-2014

The recent increase in oil revenues prompted the government to adopt increases in spending, rising from $68,533 million in 2003 to $304,160 million in 2014. The average current government spending was 77%, while the average government investment spending was 23%, see Figure 25. The increase in current government spending was due to the government's desire to raise standards of living, improve the population's quality of life, provide job opportunities for citizens, and expand quantitative and qualitative education, training, health and social services. The average income per capita increased by 157%, from $9,799 in 2003 to $25,214 in 2014, as shown in

Figure 26. Finally, compared to the previous stage, the Saudi economy tried to get out of the recession of the second phase, taking advantage of the high oil prices and oil revenues to finance government spending.

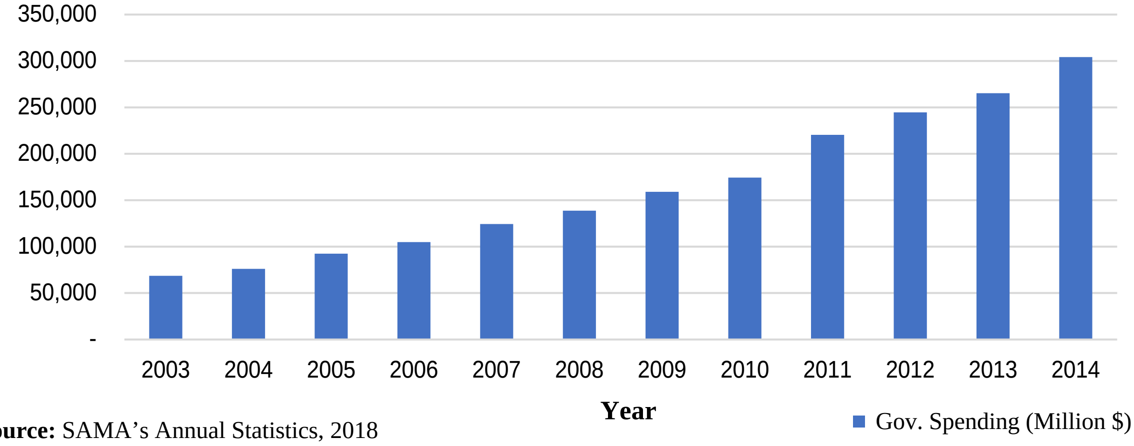


Figure 24: Nominal Government Spending During 2003-2014

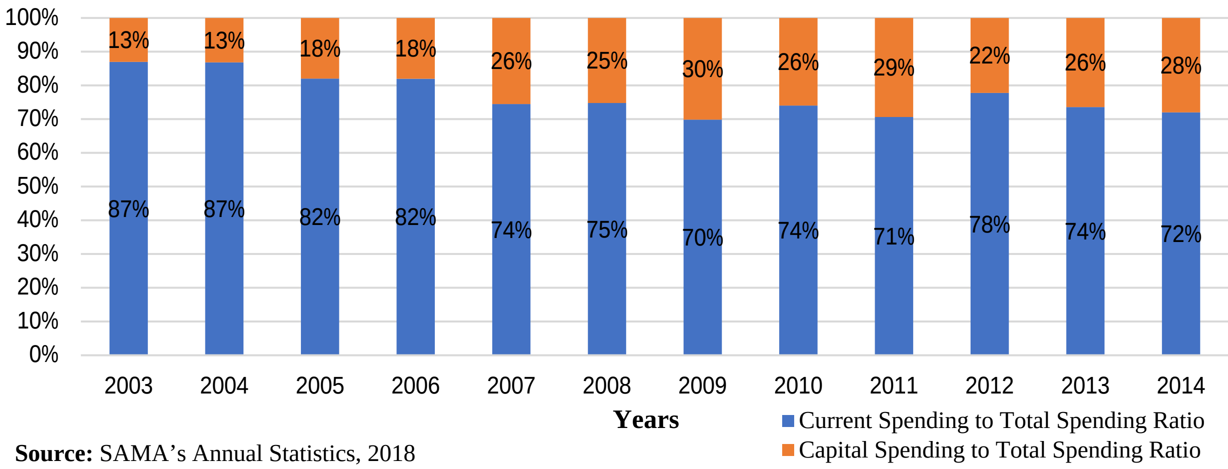


Figure 25: Annual Current and Capital Government Spending: 2003-2014

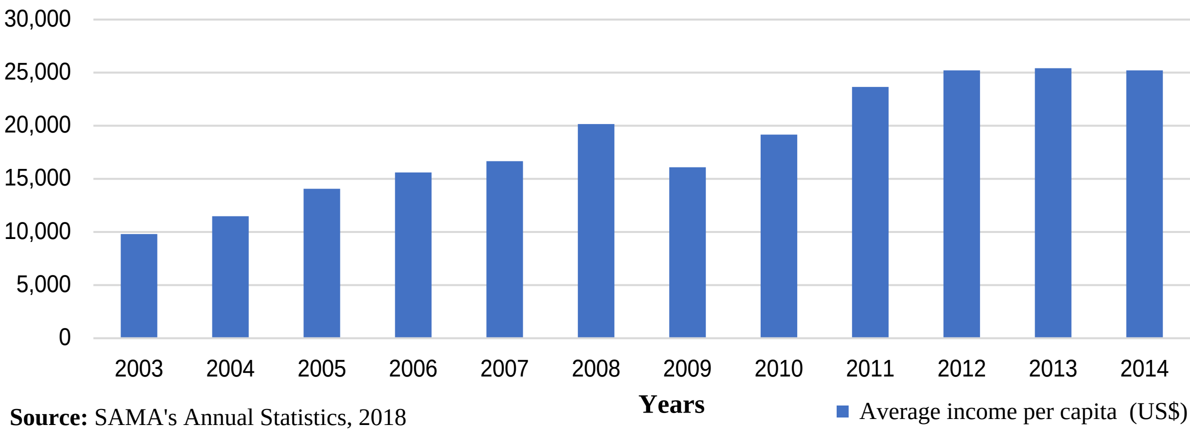


Figure 26: Average Income Per Capita During the Third Stage 2003-2014

2.5 The Oil Sector and Its Role in the Saudi Economy

Saudi Arabia is one of the oil-rich countries. It is considered the world's largest and most significant oil country in terms of oil reserves, output, exports, and refining capacity. Saudi Arabia's oil exports are primarily linked to Saudi production decisions because oil is the main commodity of the country that is exported abroad. According to OPEC, Saudi Arabia owns 18% of the proven oil reserves of the world. The oil and gas industry represents approximately 70% of export income. Moreover, Saudi Arabia in 2015 was one of the top three crude oil-producing countries (OPEC, 2018). Also, the oil sector's average contribution to the GDP was about 79% during the period 1970-2017 (SAMA's Annual Statistics, 2018). Thus, oil in the Saudi economy is the main driver of the economy, and its revenues feed the public budget to a significant extent. As a result, it is not surprising that the Saudi economy is described as a one-commodity economy.

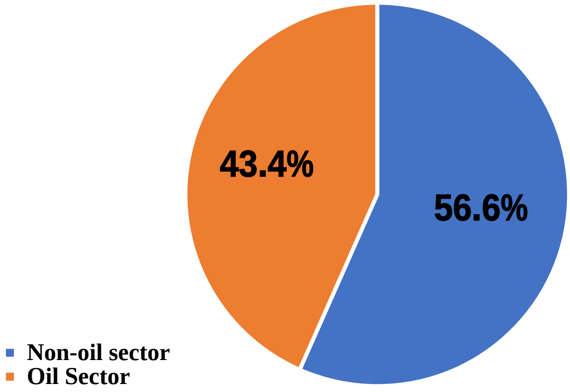
This section will be divided into four parts to discuss the significant role of the oil sector in the Saudi economy and the country's economic development programs. Briefly, it covers the following: the importance of the oil sector in the Saudi economy, the characteristics of oil production economics, Saudi Arabia's role in global oil markets, and finally the connection between global demand for oil and global economic growth.

2.5.1 The Importance of the Oil Sector to the Saudi Economy

There is no doubt that the oil sector in the Saudi economy has played a significant role in boosting growth and development. The massive financial revenues generated by its exports have contributed to the large government spending of Saudi Arabia and the financing of various development projects. The importance of the oil sector in the Saudi economy can be highlighted through the subsequent brief sections.

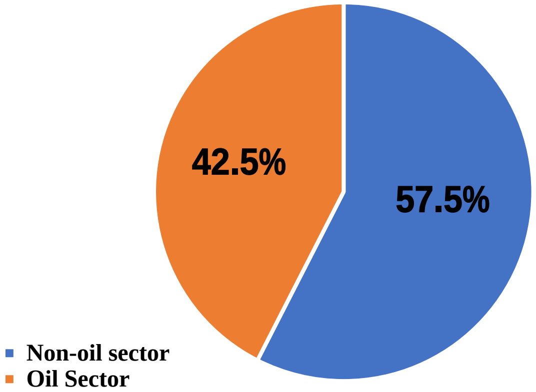
2.5.1.1 A Large Proportion of GDP is Produced in this Sector

The oil sector contributes a large share of GDP, with the average contribution rate between 1970-2000 being 43.4%. During the period 2001-2017, the average contribution to GDP was 42.5%. Although this percentage has gradually declined over the past five years as a natural result of efforts to diversify the local economy, the oil sector remains the dominant sector in the composition of the GDP of the Saudi economy.



Source: The First Annual Report of SAMA, 2015 & SAMA's Annual Statistics, 2018.

Figure 27: The Average Contribution of Oil and Non-Oil Sectors to GDP During 1970-2000

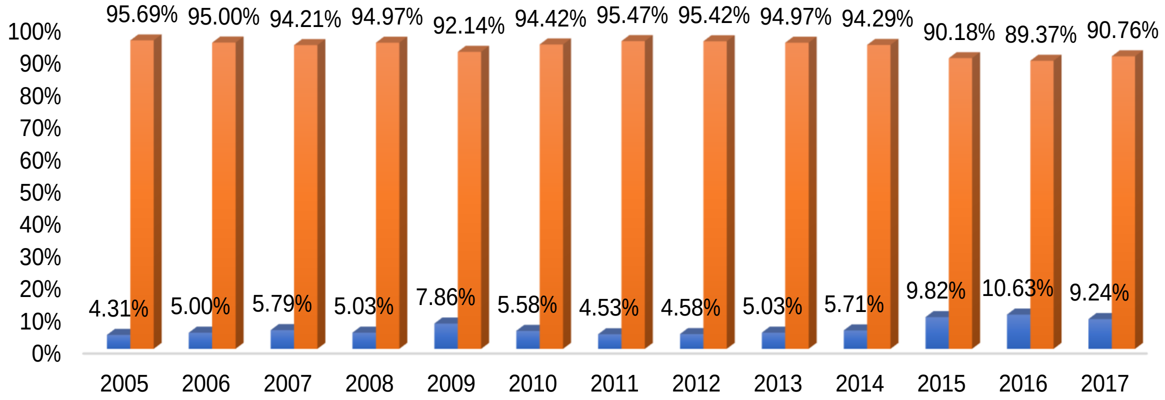


Source: SAMA's Annual Statistics, 2018.

Figure 28: The Average Contribution of Oil and Non-Oil Sectors to GDP During 2001-2017

2.5.1.2 Oil Exports and Derivatives Represent the Bulk of the Total Value of Exports

Saudi Arabia undeniable relies heavily on the export of oil and other petroleum products. The following figure clearly indicates the dominance of oil exports in the exports sector during the last 12 years. Looking at historical data of Saudi exports during the period 2005-2017, provided by the annual report of SAMA (2018), it is obvious that a huge part of exports comprises oil exports, with the average oil exports representing approximately 84%. However, this percentage does not reflect the real statistics since petrochemical exports are included as non-oil exports in official statistical reports. We believe that petrochemical exports should be included in oil exports for two reasons. The first reason is that a report by European Business states that the primary raw materials used in the manufacture of petrochemicals are typically obtained from oil and gas (European Business, 2006). The second reason is that the primary source of petrochemicals is crude oil, which is reused more than once to get petrochemical products. As long as petrochemical production come initially from oil, it should be treated under oil production. Therefore, if we add petrochemical exports with oil exports, then the average oil exports of Saudi Arabia during the same period, 2005-2017, would become approximately 94%, as shown in the below figure.



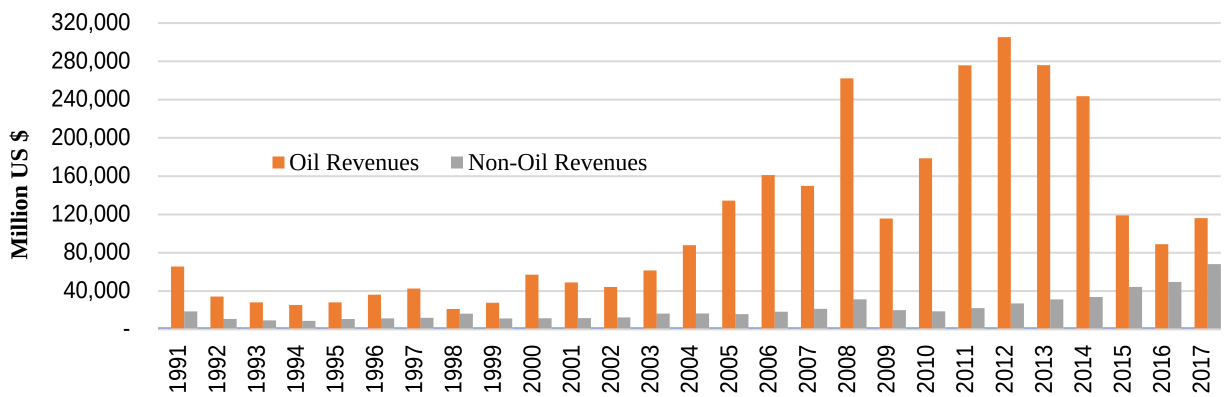
*** Petrochemical exports are included in oil exports.**
Source: SAMA's Annual Statistics, 2018.

Figure 29: Oil and Non-Oil Exports Percentage of Total Exports

2.5.1.3 The Oil Sector is Considered the Main Source of State Revenues

Saudi's dependence on the export of oil and other petroleum products is, therefore, a result of significant financial revenues. There is no doubt that these huge oil revenues have enabled Saudi Arabia to increase government spending and the financing of various development projects, putting the state in a good development situation compared to many developing countries. However, this exposes the economy to great fluctuations due to the instability of oil prices and hence its revenues.

The figure below shows that oil revenues represent most of the country's revenues. The average contribution of oil revenues accounted for more than 80% of the state's total revenues from 1990 to 2017. The increase in oil revenues resulted from the increase in global oil demand during the same period. Indeed, the average global oil demand rose from 66.53 million bpd in 1990 to 98.19 million bpd in 2017, an increase of 47.6%, which led to higher oil prices that reflected on Saudi oil revenues (BP Statistical Review of World Energy, 2018).



Source: SAMA's Annual Statistics, 2018.

Figure 30: Oil and Non-Oil Revenues 1991-2017

2.5.2 Characteristics of Oil Production Economics in Saudi Arabia

Saudi oil production is characterised by properties that distinguish it from other oil-producing countries, meaning it occupies a unique position in the global oil market, as outlined below. (I) The vast size of the proven oil reserves in Saudi Arabia, estimated in 2017 at 266.2 billion barrels, is equivalent to 16% of the world's reserves. This makes it the second country after Venezuela in terms of proven oil reserves (BP Statistical Review of World Energy, 2018). These substantial oil reserves and continuous production for a long time has made Saudi Arabia occupy a distinguished position in the global oil field. (II) The geographical location of Saudi Arabia makes it at the junction of most continents, see Figure 4. Thus, its location undoubtedly reduces the cost of transportation to various consumer markets in the world (Alobaid and Atya, 1994). (III) Lower production costs are common for Saudi oil compared to production costs in other regions for the following reasons: the availability of large reserves, advanced technology and the geological nature of Saudi oil wells, namely a few dry wells and abundant production and the decline of most deep wells (Alobaid and Atya, 1994). (IV) The diversity of oil production in Saudi Arabia means it produces various types of oil, such as Arabian Heavy, Arabian Medium, Arabian Light, Arabian Extra Light, and Arabian Super Light, which allows it to occupy a distinguished position in the oil market (Saudi Aramco, 2017). (V) It has the availability of infrastructure, production facilities, refining, transportation and distribution of crude oil and refined products, as well as the presence of integrated petrochemical plants (Saudi Aramco, 2017). These unique characteristics of Saudi oil production make it an essential global supplier of oil. They also enable Saudi Arabia to act as a prominent leader in the oil market, particularly in the OPEC.

2.5.3 The Role of Saudi Arabia in the Global Oil Market

Saudi Arabia plays a significant role in the global oil market to meet the increasing global oil demand. It has been able to increase production quickly due to its excess production capacity of more than 2.7 million bpd, representing over than half of the world's surplus capacity. The table below displays the total oil production in Saudi Arabia in 2017, at 11.950 million bpd, representing about 13% of the total daily world oil production. Despite the oil production in the United States exceeding Saudi Arabia for the first time in 2017, oil production in Saudi Arabia had the largest share of the daily output in previous years. Saudi Arabia is the second-largest country in terms of proven oil reserves, reaching 266.2 billion barrels, or 16% of the world's reserves, where Venezuela is the largest country in terms of proven oil reserves. The main reason for this goes back to 2008 when there were disputes between the Venezuelan government and foreign oil companies operating there, which led to reduced oil exports. As a result, Venezuela accumulated oil until it reached 303.2 billion barrels in 2017, equivalent to 18% of global reserves (BP Statistical Review of World Energy, 2018). By the provision of the necessary supply of oil, Saudi Arabia has been able to take a significant role in the global oil market and make a positive contribution to global economic stability. The following table shows a comparison between the most significant oil countries in terms of the size of production and proven oil reserves.

Table 1: The Biggest Oil Countries in Terms of Oil Production and Oil Proved Reserves in 2017

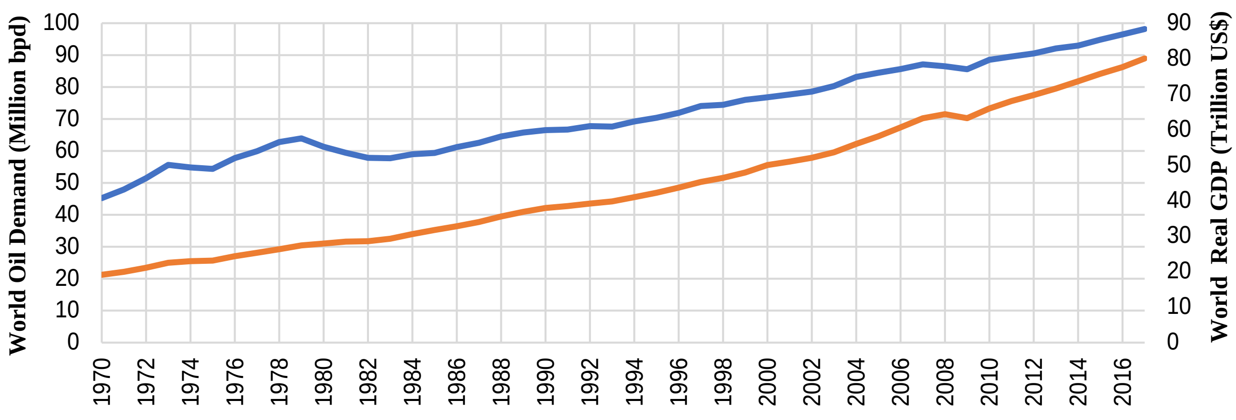
Country	Oil Production (Thousands Per Day)		Oil Proved Reserves (Thousands Million Barrels)	
	2017	%	2017	%
United States	13,056.99	14.1%	50.0	2.9%
Canada	4,830.63	5.2%	168.9	10.0%
Mexico	2,224.15	2.4%	7.2	0.4%
Brazil	2,733.99	3.0%	12.8	0.8%
Venezuela	2,110.20	2.3%	303.2	17.9%
Norway	1,968.87	2.1%	7.9	0.5%
United Kingdom	999.13	1.1%	2.3	0.1%
Russian Federation	11,257.26	12.2%	106.2	6.3%
Iran	4,981.68	5.4%	157.2	9.3%
Iraq	4,519.96	4.9%	148.8	8.8%
Kuwait	3,025.44	3.3%	101.5	6.0%
Qatar	1,915.83	2.1%	25.2	1.5%
Saudi Arabia	**11,950.84**	**12.9%**	**266.2**	**15.7%**
United Arab Emirates	3,935.27	4.2%	97.8	5.8%
Algeria	1,540.27	1.7%	12.2	0.7%
Angola	1,674.39	1.8%	9.5	0.6%
Libya	864.56	0.9%	48.4	2.9%
Nigeria	1,987.75	2.1%	37.5	2.2%
China	3,845.94	4.2%	25.7	1.5%
Other countries	13,225.46	14.3%	108.2	6.4%
Total World	**92,648.6**	**100.0%**	**1,696.6**	**100.0%**

Source: BP Statistical Review of World Energy, 2018

2.5.4 Global Demand for Oil and Its Relationship to Global Economic Growth

Looking at the data, it can be observed that there has been a growing demand for oil over time in recent years, which could be based on several factors. One possible reason for this increase in demand for oil over time could be economic growth in the rest of the world, leading to some countries demanding more oil. Developing countries, especially China and India, are growing rapidly due to industrialisation, which is one reason for increased oil demand globally (Kumhof and Muir, 2012). Several reports from OPEC emphasise that China has one of the highest demands for oil. The oil demand from China alone over the last ten years was 100.6 million bpd, accounting for 11.15% of the total world oil demand during the period 2008 to 2017 (Annual reports of OPEC, 2018, 2015, and 2011). Based on the BP Statistical Review of World Energy (2018), the US, China, Japan, and India were the main four countries in terms of oil consumption. They consumed almost 378 million bpd over the last ten years, representing 41.3% of total oil consumption in the world. On the other hand, the average GDP growth rates of these countries during 2008-2017 were 1.61%, 8.11%, 0.73% and 7.39%, respectively (BP Statistical Review of World Energy, 2018). Thus, it seems evident that there is a link between world oil demand and world real GDP, where their data need to be considered to understand if there is a significant correlation between these two variables.

According to the World Bank (2018), world real GDP (at constant 2010 US price) grew from \$19.11 trillion to \$80.08 trillion during the period 1970-2017, with an annual average growth rate of 3.14% (The World Bank, 2018). In the same direction, world oil demand grew from 45.23 million bpd in 1970 to 98.19 million bpd in 2017, representing a rise of 117% during the same period, with an annual average growth rate of 1.7% (BP Statistical Review of World Energy, 2018). From the available data, we show in Figure 31, the relationship between world oil demand and world real GDP during 1970-2017. To find out if the relationship between them is statistically significant or not, we run a regression to assess whether a given correlation is statistically significant. The data was obtained from the World Bank and BP statistical review of world energy for the period 1970-2017. The result displayed a significant (strong) positive relationship between world real GDP and world oil demand, where is r (degree of freedom) = r (46) = 0.989, P-value < 0.001. This result could be evidence that since the Saudi economy is highly dependent on oil revenues, it is also true that the economy would be affected by the world real GDP.



World GDP. **Source:** The World Bank, 2018
Total World Oil Demand.
Source: BP Statistical Review of World Energy, 2018

Figure 31: The Relationship Between World Oil Demand and World Real GDP

2.6 Development Plans and Vision 2030

This section will discuss two main phases of planning in Saudi Arabia: firstly, the ten five-year plans from 1970 to 2020 (Ministry of Economy and Planning, 2018; General Authority for Statistics, 2016) and secondly, the plan which is called *Vision 2030* (Saudi Vision 2030, 2016).

2.6.1 Development Plans 1970-2020

Saudi Arabia has recognised the importance of planning to achieve development performance and has embarked on comprehensive planning as a development method since 1970. Ten development plans have been approved from 1970 to 2020. Saudi Arabia has so far implemented these development plans, through which it has achieved distinguished experience in development projects with the major objectives and ambitions of coping with all economic and social change. Each plan contains five-years and has some economic and social goals to be achieved during specific years.

Even though Saudi Arabia has achieved much in building infrastructure, services, and productive sectors over the past fifty years of development, these plans were affected by all the phases that the Saudi economy went through, which has previously been analysed: boom, recession, and recovery. The boom period included the first and second five development plans. In this period, oil production and its revenues were grown fast; thus, its revenues were devoted to building the necessary equipment that the country needs. Most of the goals of these plans have been accomplished. The second phase was the recession, which involved the third five-year plan to the mid of the seventh five-year plan. Although ambitious strategic plans were put in place to be achieved during this period, the instability of oil revenues and their significant fluctuations limited the implementation of many of them. However, in the next period, *i.e.* the recovery phase, oil revenues increased unprecedentedly, which helped the decision-maker to draw applicable future policies and plans. Finally, all development plans were designed to a fundamental basis for economic development but relying solely on oil to finance these plans led to the postponement of some of the desired objectives to the following plans. As a result, it has emerged that serious research is necessary for other income sources that reduce oil dependency and assist the planner in drawing up plans to be executed within a defined period. Thus, an ambitious plan has been established and designed to achieve these goals in addition to other objectives.

2.6.2 Vision 2030

Although Saudi Arabia occupies a superior position among the world’s countries, it has earnestly sought to move the economy away from relying on oil revenues by adopting a vision of high ambition to be achieved by 2030. Thus, the Saudi Council of Ministers agreed in 2016 on Vision 2030, which has many economic, political and social objectives. On the economic side, the Vision was built on some of the IMF's recommendations to implement many reforms in the Saudi economy. Among these reforms are the following:

2.6.2.1 Increase non-oil exports:

Saudi Arabia intends to increase non-oil exports by building incentive programs to encourage exports, focusing on developing the export readiness of small and medium-sized enterprises. These programs include, for example, providing export credit financing, customs duty drawback and exemption on selected materials, equipment and machinery, as well as loan programs for public and private industrial investments. It also seeks to create a unique logistics platform, regional and international integration, and support for national companies. The aim is to increase the ratio of non-oil exports to non-oil GDP to 50%. It also aims to advance the rank of Saudi Arabia in the Logistics Performance Index from 49th to 25th globally and be the first regionally.

2.6.2.2 Diversification of income sources*:*

The country has responded to the advice of the IMF by introducing new tools that can boost revenues away from oil. Through diversifying sources of income by the imposition of a value-added tax and launching promising sectors, the country aims to increase the annual non-oil revenues from 163 billion riyals to one trillion riyals (*i.e.* from $43.5 to $266.7 billion). Moreover, it pursues to bolster small and medium-sized enterprises and productive families, in addition to the non-profit sector. The objective is to increase the participation of small and medium-sized business towards GDP from 20% to 35%, lower the unemployment rate from 11.6% to 7%, raise the savings rate from 6% to 10%, and raise the share of the non-profit sector in GDP to 5%.

2.6.2.3 Increase local value by privatising some state-owned assets*:*

The government seeks to maximise the investment capacity and allocation of some government services. For instance, it plans to sell a small part of the national oil company's shares (Saudi Aramco) to enhance transparency and look for investments away from oil. The goal is to raise the share of foreign direct investment to GDP from 3.8% to 5.7% and the value of Public Investment Fund[23] assets from 600 billion riyals to over 7 trillion Saudi riyals (*i.e.* from $160 billion to $1.87 trillion).

[23] The Public Investment Fund (PIF) is a government Sovereign Wealth Fund. It is owned by the Saudi government.

2.7 Conclusion

This chapter has described the Saudi economy during the period from 1970 to 2017. It is obvious from our analysis that the Saudi economy can be described as a single-commodity economy. As mentioned before, the economy relies significantly on oil, which is wholly owned by the government, where its revenues contribute by more than 80% of the total state revenues. These revenues are the primary source of public spending, and therefore most economic sectors, especially the private sector, depend on government support to ensure their survival and continuity. In fact, the Saudi economy, especially economic growth, is heavily influenced by government spending. The fluctuation rate of real GDP growth is linked to the fluctuations in the growth of government spending, and the latter is also related to fluctuations and instability of oil prices. Therefore, we can say that government spending is one of the most essential pillars of economic growth in the country. As a result, the two most significant challenges facing the economy are its primary reliance on oil revenues and the desire to diversify its income sources. Thus, the Vision 2030 plan has been presented as a road map towards achieving many economic and social reforms. The next chapter will be the literature review.

3 Chapter Three: Literature Review

3.1 Introduction

The overarching aim of this thesis is to develop an understanding of the factors that influence the economic growth in oil-rich countries, especially Saudi Arabia[24]. Towards this, the present chapter reviews the literature in this area under six main themes: (i) the economic growth models, (ii) the structure of the oil market: Saudi's oil sector, (iii) economic growth in oil-rich countries, (iv) managing oil revenue fluctuations in oil-rich countries, (v) taxes and their roles in economic growth, and finally (vi) fiscal policy and economic growth in Saudi Arabia. There are two main reasons behind searching in different themes. The first is due to the fact that the issues discussed in this thesis are overlapped. The second is to give us a deeper understanding of the works most related to what is being proposed in the present research.

3.2 Economic Growth Models

An analysis of neoclassical growth models shows that economic models could be classified into exogenous and endogenous growth models. In exogenous growth models, factors that affect the long-term growth rate are assumed to be determined outside the model, like technological progress and population growth. Solow and Ramsey's growth models were seen as prominent examples in the area of exogenous growth models. Ramsey's model is considered much more technically complex. However, the main difference between the two models is that agents and planners in the Solow model follow a simple linear rule of consumption and investment. While, the Ramsey's model, on the other hand, considers maximising the utility of agents and planners, and therefore choosing consumption and investment optimally (Angeletos, 2013). Hence, the objective of the Ramsey growth model is to maximise welfare. Further, exogenous growth models usually deal with a closed economy where households and firms are considered primary agents. Households are expected to maximise their lifetime utility based on an intertemporal budget restriction, while firms are anticipated to increase profits based on their factor accumulation constraints.

These exogenous growth models differ from endogenous growth models. The factors of long-term economic growth rate in endogenous growth models are set within the model. Endogenous growth models focus more on technology or knowledge, investment in human capital, research

[24] All chapters in this thesis have overlapping themes of one topic; therefore, we believe that combining the concerned literature review in one place would make this more convenient for the reader.

and development (R&D), and institutions promoting economic growth. There are some endogenous growth models that have been discussed in the literature, such as Rebelo (1991), Romer (1986), and Arrow (1962). Barro (1990) illustrated how decentralised choices could lead to sub-optimal and Pareto optimal saving and economic growth rates in their models. These models depended on constant returns[25] to private capital, where human and non-human capital are included. Along similar lines, Barro followed the aspects of this approach by introducing a public sector into a simple model and assuming constant returns of economic growth instead of diminishing returns.

Our study is based on the Robert Barro (1990) model; therefore, it is imperative to understand this model more deeply. In Barro's (1990) paper on, 'Government Spending in a Simple Model of Endogenous Growth', he developed a version of the Ramsey model. In 1990, Barro extended existing endogenous economic growth models to incorporate the government sector since production involves public services and private capital. The main idea of Barro's model is to introduce a public sector in a Cobb-Douglas production function. The reason behind using a Cobb-Douglas production function is that externalities and taxes which are linked with public expenditures arrive at productive efficiency. Barro argued that the reason for introducing public goods in the production function is that private inputs are different from public inputs. Therefore, it would be difficult, for example, for the private sector to undertake activities such as national defence rather than the public sector. In this setting, public goods are provided for each household producer. Barro clarified that public goods input into private production is likely to lead to a positive relationship between government and growth. Besides, capital and public goods together may show a constant return to scale. However, the absence of one of these inputs implied that production may represent decreasing returns to scale. Therefore, both private capital and government inputs have to be expanded in a similar way to avoid a decreasing return to scale (Barro, 1990).

The Barro model proved that public spending should be used for investment in public projects, such as schools, sanitation, and infrastructure. These projects are financed through income taxes and are used to accompany private investments. Since public investments are associated with the productivity of private investments, taxation can affect overall growth (Barro, 1990). However, it

[25] The rule of returns to scale applies to the long-term production analyses. In the long run, the output expansion can be accomplished across various variables, *i.e.* the production may be improved by adjusting all the variables by the same or different proportion. There are three types of returns to scale. First is called constant returns to scale (CRS), which means that the output increases with the same inputs proportion. Second is referred to increasing returns to scale (IRS), which implies that the output increases by more than the proportion increase in inputs. The third type is decreasing or diminishing returns to scale (DRS), which implies that the output increases by less than the inputs proportion increase (Koutsoyiannis, 1975).

is worth noting that government spending or public expenditure in the Barro model can be justified under the full depreciation of public capital. Therefore, government spending is equal to government investment (Novales, Fernández and Ruiz, 2014). Barro defined the current government spending as productive public spending (*i.e.* public spending increase the marginal productivity of private capital). The examples provided include property rights, public infrastructure, public transport systems, electricity networks, and fire and police departments.

Moreover, the Barro model focused on government taxation and how taxes are used to improve the economy through investment in public projects. It was assumed that the benefits are realised from productive government expenditures. Barro (1990) illustrated that growth and welfare could be maximised by a fraction of government expenditure and an income tax rate. In his model, the main hypothesis was that government expenditures improve efficiency in the economy's private sector. However, this study did not include government consumption expenditure or unproductive government spending. But, since the government expenditure has to be funded, distortionary taxation is necessary. The size of the government determines the effect of government expenditure and tax on the economy. If the size of government is big, the negative impact of taxation dominates the positive impact of government expenditure on private productivity and vice versa (Capolupo, 2009). In other words, productive public spending financed through income taxes tends to complement private investments. These, in turn, boost the private sectors' productivity, and thus higher taxes impact economic growth.

The key conclusions of the Barro model can be summarised in how changing productive expenditures and non-productive expenditures, and distorting taxes affect the long-term growth rates. In a more precise way, increased productive expenditures generate long-term economic growth, but increased non-productive expenditures have no impact on long-term growth. On the other hand, increased distorting taxes decreases the rate of growth if all other conditions being equal (Kudrin and Sokolov, 2017).

In addition to these two main types of models of economic growth, *i.e.* exogenous and endogenous growth models, semi-endogenous growth models are also proposed in the neoclassical growth model literature. However, semi-endogenous growth models are considered controversial. Charles I. Jones introduced a semi-endogenous growth model. In Jones' paper (1995), he emphasised that the debate was structured based on the scale of effects in the endogenous growth literature, *i.e.* the supported long-run growth by research and development (R&D) is either endogenous or semi-endogenous. Jones (1995) solved the issue of the scale effects, which implies a situation where the economic size of a country relates positively to growth. According to Jones (1995, p.779), the semi-endogenous growth model is:

> unlike the AK-style models and the R&D models of Romer/Grossman Helpman/Aghion-Howitt, this model predicts that the growth rate is determined by parameters that are typically viewed as invariant to policy manipulation. Growth in the economy is tied directly to growth in productivity, which in turn depends on the discovery of new designs through R&D. Individuals are the critical input into the discovery of new designs, and the growth rate of the economy depends crucially on the growth rate of the labor force, an exogenous variable.

Therefore, the semi-endogenous growth model can be defined as follows: exogenous population growth determines the long-run growth despite the technological change being endogenous. Thus, if growth is semi-endogenous, then a public policy like R&D subsidies does not define the long-run economy. On the other hand, if growth is endogenous, government actions affect growth.

Barcenilla-Visús, López-Pueyo, and Sanaú have illustrated the differences between the two close economic growth theories: the full endogenous growth theory and the semi-endogenous growth theory. They clarified that the full endogenous growth model theory supposes a constant return to scale in the production of knowledge. Under this assumption, the total factor productivity (TFP) depends on the growth rate of research intensity. To raise the TFP, it is important to increase the growth rate of the research intensity. On the other hand, the theory of a semi-endogenous growth model suggests a decreasing return to scale in the production of knowledge. This explains that, although there is continued growth in the technological input, the TFP growth is not grown fast. Under this assumption, the TFP is based on the population growth rate. Therefore, the population's growth rate must be increased to raise the TFP (Barcenilla-Visús et al., 2014).

Under this theme, we have scanned the three types of economic growth models for two reasons. The first is to understand the difference between the types of models and the factors determining each type. The second is due to the fact that our models, which will be presented in each chapter, would be based on different sources of government revenues to finance government spending. More precisely, the exogenous and endogenous sources of government revenues would be examined in our model. Although this study is based on the Barro (1990) model, our models are likely to end up being either exogenous or endogenous growth model. The reason behind that is due to the use of different sources of government revenues to finance productive government spending. As a result, our research must analyse these types of economic growth models to position this research in the right context.

3.3 The Structure of Oil Market: Saudi's Oil Sector

In this section, we focus on the literature on the structure or the behaviour of the oil market. In particular, we look at Saudi Arabia for two underlying reasons. The first is due to the importance of Saudi's oil sector in our study, where we need to understand the work of this sector for modelling it into an economic growth model. The second is due to the fact that there is disagreement among some economists, politicians and even the public about how the oil sector acts in general, and mainly in Saudi Arabia. Thus, there is a need to explore the oil market structure in Saudi Arabia (*i.e.* it works in perfect competition, imperfect competition, oligopoly, or monopoly market).

Most of the studies in this literature have analysed the behaviour of the Organization of the Petroleum Exporting Countries (OPEC), for which Saudi Arabia is one of its principal founders. In these studies, Saudi Arabia is often seen as a cartel that acts as a wealth-maximising monopolist. For example, Griffin (1985) studied the behaviour of the OPEC countries and found that the behaviour and structure of OPEC were both dominant and competitive. For Saudi Arabia, Al-Yousef (1998) examined Saudi behaviour in the oil market. She tested two economic models over the period 1970-1996. The two models investigated were the swing producer model (1975-1986) and the market sharing model (1987-1996). The results were positive, showing that Saudi Arabia acted during the first time frame of a swing product by adjusting output for price stability in the first model. On the contrary, in the second model, it was considered a producer of market share in terms of maximising its revenues.

In another approach, Alhajji and Huettner (2000) studied different models to verify the presence of a dominant producer in the oil market during 1973-1994. In their paper, they used three other models to examine OPEC and non-OPEC members. These models were the dominant firm model, the Cournot model[26], and the competitive model. Each model tested three variants of members, which are OPEC, Saudi Arabia, and the core countries of OPEC[27] as the dominant firms in the world oil market. All models' statistical findings only accepted the dominant firm model for Saudi Arabia. Thus, when examining only Saudi Arabia, they found that it acts as a dominant product in the market. Similarly, Kaufmann et al. (2008) and De Santis (2003) found similar results in concluding that Saudi Arabia does not show production sharing behaviour and acts as a dominant firm. Both studies came to assert the same concept about Saudi Arabic with reference to being an outstanding oil producer.

[26] The Cournot model is a model describing the situation that all firms produce a homogeneous product. According to Alhajji and Huettner (2000), based on the non-OPEC production in the previous period, the OPEC determines the production which maximises its profit.

[27] In Alhajji and Huettner (2000) paper, the core countries of OPEC include Saudi Arabia, Kuwait, UAE, and Qatar.

Although this literature is not very related to our main topic of economic growth, understanding how this sector works, particularly in Saudi Arabia, could be a starting point in our study. Moreover, many economic growth models focused on oil-rich countries have ignored the structure of the oil market. Thus, treating and modelling the oil sector as a monopolistic sector, as confirmed by many studies in the structure of the oil market literature, in an economic growth model would add a feature to our research.

3.4 Economic Growth in Oil-Rich Countries

The third important area reviewed in this research is closely related studies to the current study on economic growth in oil-rich countries. Although the literature in this area has been widely conducted in several studies as the majority focused on the influence of natural resources on economic growth, it is important for this study because it narrows the research scope and would help explain the relationship between economic growths in oil-rich countries. In this literature, two kinds of hypotheses will be examined about the possible effect of 1) natural resources are a curse or 2) natural resources are a blessing.

Let us begin with the hypothesis that natural resources are seen as a curse. In this regard, investigating natural resources has been the scope of many studies for being a curse. They have pointed out that natural resources can sometimes be a curse since they could slow the economic pace in certain economies like fiscal policy challenges, political issues, and bad governance. Gylfason (2001) conducted a study that found that economic growth continues to vary across wealthy nations, arguing that oil revenues appear to affect human capital and thus contribute towards slow economic growth. Other studies, such as Papyrakis and Gerlagh (2007) and Sachs and Warner (1995), have found that oil revenue, as a natural resource, is a significant negative determinant of growth.

Along these lines, Rodriguez and Sach (1999) conducted a study on why resource-rich countries show slow growth compared to resource-poor countries. Their theoretical model was used to illustrate how we expect an economy to respond to an influx of revenues from the export of a natural resource based on the Ramsey growth model. They introduced a new sector in their model called "the natural resource" (p.281) and considered it as an exogenous level of production. It was assumed that economies having abundant resources are living beyond their means and above a steady-state equilibrium. The reason behind their assumption of living beyond their means is that natural resource industries that basically depend on non-sustainable resources cannot expand their production at the same rate as the other industries due to limited natural resources.

Rodriguez and Sach (1999) applied a dynamic general equilibrium model, where the Venezuelan economy was used to analyse the impact of overshooting a steady-state point. In the model, it was assumed that the Venezuelan economy is a one-good economy, where its natural resources can be used as exports in exchange for imported investment goods. The country's current account was assumed to be balanced, with no foreign investment. The country's domestic resources were used as capital stock, and the natural resources used to cover the cost of importing investment goods. Due to a lack of foreign earnings, the Venezuelan economy has a balanced current account, and there are no surplus foreign earnings. It was further assumed that, in this economy, the available resources were either consumed or invested. In their study, the data on the country's growth performance was taken from 1972 to 1993. This study found that the natural resource production is going to tend towards zero in the steady-state. However, the natural resource during the transition to that steady-state would make the economy to enjoy exceptional possibilities for consumption. For this reason, the economy with natural resource would adjust to its steady-state not from below but from above. In the case of the Venezuelan economy, the study results showed that it was above a steady-state point which later converged to a steady-state optimal point. This was because, in a resource-abundant economy, consumption was high since it was dependent on domestic resources. During the transition, it showed on average negative rates of growth. Thus, in the long run, it would be challenging to sustain a high level of consumption because it would not be able to maintain its own capital stocks, which implies that a decline in output. In other words, although resource would lead to a temporarily boosting level of consumption and investment, the steady-state would return to its old steady-state. Consequently, the natural resources would cause a slow growth rate.

Notwithstanding, a small number of studies have examined how natural resources have been a blessing to some wealthy oil nations that have effective fiscal policies and good governance structural systems. On these lines, the study by Moneef (2006) suggested that "resource abundance is supposed to provide the economy with the investment capital and advanced technologies needed for the big push" (Moneef, 2006, p.15). Alkhathlan (2013) highlighted the likely effect of two variables, namely economic growth and oil revenues. Alkhathlan's study particularly examined the relationship between oil production and economic performance in Saudi Arabia during 1971-2010. This study underlined that a few studies indicated that an abundance of natural resources substantially positively influences economic growth. The economic growth of a wealthy nation with abundant natural resources potentially rises faster than less natural resource countries. The explanation for this is that the abundance of natural resources tends to reduce financial pressures and improves both income and purchasing power over imports. Thus, the abundance of natural resources is projected to increase investment and economic growth rate. Finally, the study findings showed that the effect of oil revenues on real GDP is positive.

Our literature review shows that studies on the possible impact of natural resources on economic growth are based on two hypotheses, curse or blessing of natural resources. However, a number of questions regarding how natural resources are managed and how their revenues are employed remain to be addressed. There are two main reasons for these questions to arise. The first is that some of these studies seem to deal with resource revenues as a pure waste of resources. Therefore, in this case, the natural resources represent a burden on the economy. Second, although some studies are empirical and based on historical data, they have not clearly clarified how the natural resources used. Thus, these studies did not provide clear evidence that natural resources are a blessing. Although our study would not directly contribute to this debate, we believe that our study would show that if government revenues are used to finance productive government spending, then the natural resources could stimulate economic growth and eventually it could be a blessing. Our thought is that if resource revenues are devoted to enhancing productivity instead of treating them as a pure waste of resources, they could have a favourable economic growth effect. However, a negative effect may also be possible; therefore, this study tries to study how natural resources, particularly oil, can be negatively affected, how this negative impact affects economic growth, and the possibility of compensating for this potential negative impact. For this reason, we next survey managing oil revenue in oil-rich countries.

3.5 Managing Oil Revenue Fluctuations in Oil-Rich Countries

This section will discuss the literature review on dealing with resource revenue fluctuations or the best strategy to deal with them. We examined the literature on managing oil revenue fluctuations in oil-rich countries because our study is mainly based on economic growth in the second-largest oil country, Saudi Arabia. Another reason behind surveying this kind of literature is because we are arguing that fiscal policy reforms could be a safe solution compared to other solutions provided by many previous studies on this topic, which will be discussed next.

Several neoclassical growth models have studied the impact of government operations on welfare, resource management, and economic growth. Some previous studies have suggested that stabilisation of funds, Sovereign Wealth Funds (SWFs)[28], are a solution to the volatility of oil revenues, for instance, Devarajan et al. (2017), Primus (2016), Richmond et al. (2015), and Berg

[28] The Sovereign Wealth Funds can be also called Investment Funds or Sovereign Investment Funds.

et al. (2013). Before discussing whether or not these funds are a suitable solution to the governments that own them, we need early understanding of what the SWFs are, who usually uses them, why some governments own them, and how these SWFs are financed.

First of all, understanding the Sovereign Wealth Fund (SWF) concept requires one to understand the reserves. Reserves refer to any funds set aside by the overarching authority in a country to increase investment targeting the economy and the citizens of that country. Consequently, the SWF refers to a state-owned investment fund that consists primarily of money drawn from the country's reserves. The resource-rich or oil-rich countries usually use these funds to save their financial surpluses represented by the revenues of these resources. The most common sources of the SWF include the surpluses in balance of payments, official operations/investments in foreign currency, and privatisation revenues. Additional sources involve government transfer payments as well as revenues from the exportation of resources. The SWFs serve different purposes that involve future generations savings, financing social and economic development, serving political strategy, and aiding in dissipating unwanted liquidity for monetary authorities (Beck and Fidora, 2008).

To start with the recent studies in managing resource revenues, Berg, Portillo, Yang, and Zanna (2013) studied public investment in developing countries with abundant resources. The CEMAC[29] region and Angola were the subjects of the study. They developed a dynamic stochastic general equilibrium (DSGE) model to examine the macroeconomic impacts of investing resource revenues. In their model, the main assumption was that the natural resource is exogenous and finances public investment. In their paper, Berg et al. (2013) proposed a sustainable investment approach combining public investment with a resource fund. The primary purpose of this approach is to address macroeconomic issues. Berg et al. (2013) emphasised that sustainable investing eliminates procyclical fiscal policy and decreases macroeconomic stability risk. By establishing a stabilisation fund, sustainable investing can also help disconnect recurring government spending from resource revenue streams; therefore, the economy becomes protected from instability due to resource income fluctuations.

Along the same lines, Richmond, Yackovlev, and Yang (2015) inspected the investing volatile resource revenues in capital-scarce economics. They used Angola as a case study to examine fiscal approaches to investing resource revenues. Their model introduced a stabilisation fund where revenues from the natural resource can be saved during good times and used to facilitate public

[29] The CEMAC is the abbreviation for the Central African Economic and Monetary Community, which involves Cameroon, Chad, the Central African Republic, Equatorial Guinea, Gabon, and the Republic of Congo. All these counties are abundant in natural resources and oil, except the Central African Republic (Akitoby and Coorey, 2012).

investment during hard times. Richmond et al. (2015) revealed that despite the overuse of the conservative investment approach[30] helping the economy to have a large stock of foreign assets, it could slow economic growth. Moreover, they emphasised that using the spend-as-you-go approach[31] could potentially harm the economy, causing boom-bust cycles. However, a gradual scaling-up approach[32] could be a better strategy to accomplish two main targets. The first is to encourage investment to foster growth, while the second is to achieve economic stability by a stabilisation fund. As a result, they concluded that the latter approach makes public investment more capable of diversifying economic growth.

Primus (2016) studied the fiscal rules for resource windfall allocation in a resource-rich developing country, Trinidad and Tobago. She employed different fiscal rules to identify the best way to allocate resource windfalls between spending now and saving in a sovereign wealth fund (SWF). The first rule is the full spending of resource windfall on consumption and investment. This rule would increase household welfare but would cause fiscal volatility. On the other hand, the second rule is the full saving of resource windfall in a SWF. This rule would help to reduce fiscal volatility but simultaneously affect household welfare. Primus (2016) found that as long as both household welfare and fiscal stability are the priority goals of the government to achieve, none of the two rules would be the best fiscal response to resource windfalls. Therefore, the government needs to save a part of the windfall in a SWF and spend the other on both consumption and investment to meet those goals.

Similarly, Devarajan, Dissou, Go, and Robinson (2017) examined the budget rules and resource booms and busts in low-income countries. They used a DSGE model and considered a small open economy. Their paper simulated the fluctuation impact of resource windfalls on long-term economic growth and welfare under resource price uncertainty. Three various scenarios have been examined: positive, negative, and periods of shocks on the resource price (*i.e.* positive and negative). Devarajan et al. (2017) found that the temporary positive shock in a resource price change would increase revenues. Therefore, saving the revenue windfall in a sovereign fund is the preferable strategy. During a negative temporary price change, public investment should be reduced. However, if there is a positive and a negative shock in all periods, combining the two

[30] According to Richmond et al. (2015, p.203), the conservative investing approach implies that:

> the government maintains constant ratios of public investment and government consumption to GDP at initial levels. As total GDP rises, public investment spending also rises, but there is no significant scaling-up. As a result, a resource fund can accumulate a large stock of external savings while public capital only grows slowly over time.

[31] The spend-as-you-go approach means that all resource windfall spends and does not save.

[32] The gradual scaling-up approach implies that "increases public investment gradually and then sustains it at a higher level" (Richmond et al.,2015, p.201).

previous strategies, *i.e.* public investment and saving abroad, could be the best strategy in this case. As a result, the three strategies would help the economy to achieve stable economic growth.

Although managing resource revenues to maintain economic growth has been examined in the literature, very few studies have considered the best methods and strategies to deal with volatile resource revenues. Devarajan et al. (2017), Primus (2016), Richmond et al. (2015), and Berg et al. (2013) assumed in their models that the resource (*e.g.* oil) output and the international resource prices follow an exogenous process, with the government taking revenues to finance its public investment. However, since the oil sector should be an essential sector when modelling oil-rich countries, these studies, as well as the economic growth literature, ignored to analyse the market power of oil. As confirmed by most research on the oil market structure, the oil sector in some oil-rich counties is treated as a monopolistic sector. Therefore, they have ignored some key questions that should be considered, particularly those related to oil-dependent countries. Furthermore, since resource (oil) revenues have been modelled in previous literature as an exogenous source, there is a necessity to investigate: (i) how this essential exogenous source operates and affects? (ii) what does a government get in revenues from that monopolistic source and how? and (iii) how can the growth rate of government spending be affected by that source? Although this source is exogenous and uncontrollable, the above questions are still worthwhile to investigate because the oil sector remains an important sector in many oil countries.

Furthermore, as Sovereign Wealth Funds (SWFs) supporters claim that they are an important strategy, they should not be considered a source of income diversification and much help for the domestic economy for several reasons. The most important reasons are that, instead of developing the domestic economy, SWFs may help develop the economies of foreign countries receiving these funds. In this regard, Beck and Fidora (2008) illustrated that many oil producers generally seek to acquire financial assets during the time in which they produce oil to prevent rapid fiscal policy changes when oil reserves are exhausted. Financial wealth is allocated to future generations, so the current general wealth of the country does not benefit from that and remains unchanged (Beck and Fidora, 2008). Besides, SWFs do not guarantee sustained revenues for a country that relies heavily on oil revenues to finance its persistent expenditures. For instance, if a country invests all or most of its resource revenues in foreign assets, these assets may incur losses due to the risks associated with some investments. In this regard, Park and van der Hoorn (2012) pointed out that many SWFs incurred significant financial losses on their investment as a consequence of the global financial crisis. For example, Malaysia, Ireland, Singapore (Temasek), Norway, and New Zealand funds lost about 35.7%, 30.4%, 30.0%, 23.3%, and 22.1%, respectively of their asset values in their funds during the financial crisis in 2008. Consequently, investing local wealth abroad can be adventurous and may expose the local economy to instability due to often uncertainty in the global markets.

Therefore, the idea of relying on SWFs may not be appropriate to support the continuation of economic growth for two reasons. The first reason is regarding the investment returns, where it is known that high (low) returns are associated with high (low) risks. Thus, if the SWFs invest in high returns investment, they may face high risks, implying that they may lose the local wealth anytime. If, on the other hand, the SWFs invest in low returns investment, then these returns may not be sufficient to meet the increased expenditures and support the economy. Especially if the economy was hit by a negative shock, whether related to oil or otherwise. The second reason is that SWFs are also potentially associated with specific periods of withdrawal, limiting the availability of cash at the time of need. As a result, the SWFs may not be able to provide enough or available funding to finance government spending in the case of negative shocks in demand for oil.

The shortfalls identified above are part of shortcomings associated with the SWFs on the economy. Although most studies in this regard have relied on SWFs as a solution to fluctuating oil revenues, the research in the reform of tax systems to deal with oil revenues fluctuation remains limited. As a result, there is a need to look at another method or policy that ensures the flow of state revenues and the maintenance of at least stable economic growth. For this reason, our study aims and seeks to contribute to how the reform of tax systems could be a solution to fluctuate resource revenues, in the case of one of the oil-rich countries such as Saudi Arabia. In brief, even though our study is similar to the previous studies in terms of studying possible solutions to the oil revenues fluctuations, it is different in terms of the method that can alleviate the fluctuations in these revenues and maintain stable economic growth.

3.6 Taxes and Their Roles in Economic Growth

We begin this theme by distinguishing first between taxes. In general, there are two types of taxes, direct and indirect taxes. The most common types of direct taxes involve corporate or company income tax, personal income tax, and property tax. Corporation tax is a tax imposed on the company's net income, while the personal income tax is a tax imposed on an individual's earnings of employment, pensions, savings. On the other side, the main indirect tax is a consumption tax, which includes some other types of taxes, such as sales tax, value-added tax (VAT), customs, and excise duty. These types of taxes are different from each other in their characteristics. More precisely, VAT is a consumption tax levied on services and products whose values have been introduced at various stages of a supply chain, *i.e.* from the production through to the final marketplace. In contrast, sales tax is charged on the turnover of the service or the product being sold, while excise duty is charged on the correct use of controlled products like Tobacco, Petroleum, or Alcohol. Although the sales tax and VAT seem similar, the main difference between them is concerned with how the government collects them. The sales tax is collected directly from consumers when they make a purchase. In contrast, the VAT is generally collected from producers instead, depending on the value they are adding up along the production chain (Stojkovic, Gasic, and Peric, 2013).

Taxes are considered a central feature of government, as they can shape the relationship between government and society. However, the concept of tax capacity has been the subject of controversy in public finance literature. Fiscal or tax capacity is the ability of the government to obtain revenues and provide public goods and other basic needs. It is also defined as the "capability of a governmental entity to finance its public services" (Advisory Commission on Intergovernmental Relations, 1982, p.2). However, George Chun-Yan Kuo (2000) stated that there is no single definition of tax capability. It can be defined either as the taxpayer's ability to pay or the government's ability to raise tax revenue. The first can include the income, expenditures, properties, and wealth of individuals. In contrast, the second is related to the government's willingness to manage and raise taxes in compliance with tax laws and regulations. Because tax capacity is a difficult concept, estimating the potential tax liability becomes even more complicated to undertake objectively. There are some examples in this regard, including the types of taxation that can be used, the tax bases and rates, whether the tax is progressive or not, issues of fiscal decentralisation, and comparing with similar economic situations in other nations (Chun-Yan Kuo, 2000).

Schneider (2006) clarified the role of tax capacity in public finance and its relation to social policy. He pointed out that tax capacity represents the relative scale and administrative capacity of the public sector. It also includes the power of the government to use private wealth for public purposes. On the other hand, the social policy represents a social contract that connects the public with the state. These two elements characterise the essence of public finance and the relationship between the public and the state. Along the same lines, Brautigam (2008) contended that taxation can play a significant role in developing and maintaining a state's power and shaping state-society relations. The political importance of taxation is that it aims to increase revenue and plays a vital role in state-building. He clarified this as follows:

> The state-building role of taxation can be seen in two principal areas: the rise of a social contract based on bargaining around tax, and the institution-building stimulus provided by the revenue imperative. Progress in the first area may foster representative democracy. Progress in the second area strengthens state capacity. Both have the potential to bolster the legitimacy of the state and enhance accountability between the state and its citizens. (Brautigam, 2008, p.1)

Regarding state institutions and tax capacity, Akanbi (2019) conducted an empirical analysis of the causality between the two. The author defined tax capacity as the ability to increase revenues, calculated by the tax revenue to GDP ratio. On the other hand, "state institutions (or institutions) —*i.e.*, rule of law, effectiveness of government, corruption control—are defined broadly as the traditions by which authority in a country is exercised" (Akanbi, 2019, p.4). Akanbi (2019) demonstrated that many developing countries need funding for vital social and development expenditure. Nevertheless, the question is whether improved state institutions lead to increased tax collection or whether increased tax collection enhances the improvement of state institutions. To examine the long-term causality between tax capacity and state institutions, Akanbi used a Panel Vector Error Correction Model (VECM) for 110 non-resource-rich countries during 1996-2017. The results showed that tax capacity and state institutions enhance each other, and that they should work together for the best outcomes, particularly in developing countries. For example, weak (strong) institutions could discourage (encourage) tax capacity, which would eventually lead to further weak (strong) institutions. In other words, any changes in tax capacity would alter the institutional structure and vice versa. As a result, it is vital that countries improve their institutions and simultaneously increase tax capacity because this will help obtain higher growth and development.

Besley and Persson (2013) mentioned that richer developed countries have more significant and powerful tax administrations and thus collect more revenue in the form of taxes than poorer developing countries. In fact, taxes differ in efficiency and ideal. In the optimal taxation theory, maximising efficiency is crucial for an ideal tax structure. There are two types of taxes: efficient and inefficient taxes. Efficiency taxes involve income and consumption taxes, whereas inefficient taxes involve corporate tax. Thus, richer developed countries depend on efficient taxes to support the productivity and redistribution in the country, while the opposite happens in developing countries.

Besley and Persson (2014) further emphasised that tax revenues as a proportion of GDP in developing countries are lower than in advanced countries. In developing countries, the tax bases are usually between 10-20% of GDP, compared to in advanced economies where they are on average 40% of GDP. Although economic growth is an important factor for widening the tax base, it is not guaranteed to be achieved with high tax. To benefit from growth and economic development, the government should reform the tax system and invest in it. According to Besley and Persson, low tax to GDP levels in developing countries is due to specific reasons. The first reason is the economic structure. In developing countries, the informal sector is broader than in developed countries and is hard to tax administratively. Also, developing countries depend on various tax regimes, and indirect taxes are much more important in terms of raising revenue than direct taxes. Another reason is dependence on aid, as this can limit incentives to raise taxes. A further reason is a political commitment and government action on tax reform. For example, weak political institutions, which are the result of a weak legislature and judiciary, discourage the government's power to perform tax measures, leading to increased tax evasion (Besley and Persson, 2014). As a result, economic growth is closely related to the government's ability to administer taxes.

To investigate the relationship between tax capacity and economic growth, Gaspar et al. (2016) examined whether there is a tipping point (*i.e.*, a minimum point) in tax to GDP levels that can accelerate the process of growth and development. They used a regression discontinuity design (RDD) to estimate the causal effect of a tipping point in tax-to-GDP ratios on GDP growth. Two different datasets were used: the first was a database of 139 countries between 1965 and 2011, while the second was based on a historical database for 30 developed economies for the period from 1800 to 1980. The results showed that the relationship between tax capacity and growth has a tipping point, where the minimum tax to GDP ratio is about 12.75%. This rate can significantly accelerate the process of growth and development. The findings also indicated that the GDP per capita rises dramatically when the tax-to GDP ratio exceeds the minimum rate (Gaspar et al., 2016).

Although our study is not closely related to the literature on optimal taxation, we will briefly highlight the main contributions to this literature. Starting with a simple definition of optimal taxation, it generally represents the interests between the competing society objectives of equity and economic efficiency, which maximises social wealth. More precisely, Mankiw et al. (2009) pointed out that "the standard theory of optimal taxation posits that a tax system should be chosen to maximise a social welfare function subject to a set of constraints" (p.148). Literature on optimal tax usually considers that the social planner is characteristic of a utilitarian, which means the function of social welfare focuses on individuals' utilities. The social planner seeks to select a tax regime designed to maximise the welfare of consumers. However, some researchers suggest that the social planner is only interested in average utility, proposing a linear social welfare function in individual utilities (Mankiw et al., 2009).

As the literature on optimal taxation is vast, we will only focus on the essential results, which are based on Ramsey (1927), Diamond and Mirrlees (1971), and Mirrlees (1971). In 1927, Frank Ramsey made the most significant early contribution to optimal tax theory. In his famous paper, 'A Contribution to the Theory of Taxation', Ramsey (1927) suggested that the planner collects revenues only in the form of commodities taxes. The main assumptions in his model were that there are no lump-sum taxes and considering linear taxes, not all commodities are taxed. Also, production prices are normalised to one. Government taxes are to increase total revenue and reduce the utility loss for agents in the economy. In his paper, he attempted to solve the issue of how to change tax rates on consumption under certain constraints. The purpose is to minimise the decrease of utility. Nevertheless, Mankiw et al. (2009) illustrated that literature on optimal taxation aims to provide the best tax system, where excluding some conceivable tax systems by assumption is an issue. Thus, the social planner should be allowed to consider all possible tax structures.

Ramsey (1927) also proposed that the inverse elasticity rule be used to determine optimal commodity taxation. The inverse elasticity rule suggests imposing a high tax rate on commodities with low elasticity of demand and a low tax rate on commodities with high elasticity of demand. In other words, the more elastic the demand, the lower the optimal tax. However, one of the criticisms of this rule is that the price elasticity of demand for necessity goods is low, while it is high for luxury goods (Aytaç, 2018).

Unlike Ramsey, Diamond and Mirrlees (1971) suggested a different proposition that allows the planner to take into account a variety of tax systems. In their paper, they discussed the issue of imperfect information stemming from the social planner and taxpayers. Mankiw et al. (2009) explained the basic version of Diamond and Mirrlees' model, in which individuals are different in their ability to earn income. Although the planner is allowed to observe income, they are unable to observe the ability or effort of an individual to gain income. Thus, if the planner tries to raise taxes on those with a high ability to earn an income, the opportunity to get the same income will be discouraged. As a result, the optimal tax problem is the lack of information among taxpayers and the social planner. More precisely, the planner wants to tax high ability individuals and transfer this to low ability individuals; however, the social planner also wants to ensure that the tax system should not lead to those with a high capacity to pretend that they are at a low level.

Diamond and Mirrlees (1971) reached two main conclusions, namely, "the demonstration of the desirability of aggregate production efficiency in a wide variety of circumstances provided that taxes are set at the optimal level; and an examination of that optimal tax structure" (p.8). They clarified that the efficiency of production is ideal while a full Pareto optimum cannot be obtained. Commodity taxes imply that marginal substitution rates are not the same as marginal transformation rates in the optimum position. On the other hand, if there are no lump-sum taxes, income distribution is not in the best situation. However, the existence of optimum commodity taxes would demonstrate that aggregate production efficiency is desirable.

In this regard, the marginal tax rates schedule is the centre of attention in the trade-off between equality and efficiency. In general, individuals' marginal tax rates increase when their income rises. This tax increase would have an efficiency cost on some individuals who are usually classified as lower-income earners. The reason for the efficiency cost is that high taxes would discourage individuals who earn a low income to work more. On the other hand, the tax change would not distort individuals who earn higher incomes. The reason is that although it would increase their average tax rate, it would not affect their marginal tax rate. In this case, if the marginal tax rate schedule is designed as the ability distribution, equality and efficiency would then be achieved.

James Mirrlees (1971) analysed optimal income taxation in his paper, 'An Exploration in the Theory of Optimum Income Taxation'. In the paper, he considered the optimal non-linear income tax that maximises a given social welfare function. Mirrlees (1971) discussed that the complete equality of marginal social utilities of income no longer becomes attractive. The reason is that the tax system derived from this will lead to discouraging unpleasant work. In his model, the main assumptions were that intertemporal problems, differences in tastes and family size, and migration are all excluded. The government is expected to have complete information about the people who work in the economy and their utilities. Also, the administration costs of the optimum tax schedule are supposed to be insignificant. Mirrlees (1971) showed that the optimal marginal rate could not be negative and that it should be below 100% due to the fact that there is no one going to want such a marginal rate. Income tax is seen as a less effective tool to decrease inequality. As a result, he concluded that taxes in addition to income tax should be used to avoid income tax problems. He further suggested applying a tax schedule that relies on time worked and labour income.

Numerous studies have investigated the effect of income tax and consumption tax on the economy. In this study, we examine both types of taxes, but with more attention to consumption tax. The reason is that consumption tax and its role in economic growth have received less attention, while income taxes, either personal or corporate taxes, are considered the most common types surveyed in economic growth literature.

Turnovsky (1996) evaluated the role played by consumption taxes on raising social welfare and economic growth. The author used an endogenous growth model to analyse this relationship. The study assumed that government expenditure causes a direct impact on the optimal consumption choices made by private agents. For an optimal economic outcome, the government must strike a balance between income and consumption taxes. The congestion level linked to the public good, and the government expenditure level affects the achievement of optimal economic growth. Similarly, the ratio of actual government spending to the estimated levels of optimal social spending also influences the first-best optimum for economic growth. Turnovsky (1996) confirmed that consumption taxes take a major role in shaping optimal fiscal performance. The author indicated that the United States (US) economy recorded low rates of savings (about 6.5%) and real gross national product (GNP) growth rate (about 2.6%) in the 1980s at a time when the government overlay relied on income tax. Under this tax regime, the US government ended up taxing both savings and consumption. The inherent weaknesses in this model prompted policymakers to debate the merits of using tax incentives to encourage growth. Consequently, they provided proposals for the adoption of a consumption tax regime as a partial or total replacement for income tax.

Furthermore, Turnovsky's (1996) study findings showed that the government could choose to either influence the production decisions of private agents in its economy or influence their consumption decisions. Two fiscal instruments can be chosen to achieve the first-best optimum outcome. According to the author, the first fiscal tool by the government should focus on eliminating any distortions in the decisions of the private agents. In contrast, the other fiscal tool should focus on providing finance for government expenditure. Besides the use of income tax, the government can use public debt appropriately, rely on consumption taxes, or use both fiscal strategies to attain optimal economic growth. However, if there is no consumption tax, then the government should attain equilibrium by positioning itself as a net lender instead of a net debtor to the private sector.

Similarly, Milesi-Ferretti and Roubini (1998) used a theoretical model to assess whether consumption and income taxation directly affect economic growth. Their model included three sectors, where the first sector is the production of final goods and physical capital. The second sector is the production of human capital, while the third sector produces a non-market good, such as leisure activity taking the form of home production. The physical outputs in the economy are produced with constant returns to scale (CRS) by using two inputs, human and physical capital. The authors sought to determine whether these taxes influence the accumulation of physical and human capital and consequently affect the rate of growth in an economy. They also discussed various channels by which taxes impact economic growth.

In their model, one common assumption behind the support for expenditure taxes is that expenditure taxes are a viable strategy for eliminating bias against saving. The current tax system in advanced economies like the United States causes double taxation of savings through its emphasis on taxing personal incomes. By shifting from personal income taxes, economies will experience increased accumulation of capital and raise the living standards of its future generations. Specific leisure activities cause unique effects on economic growth. The level of technology used to achieve the accumulation of human capital and the reliance of a market sector to produce human capital also produce unique moderating effects on the relationship between consumption taxes and economic growth. The study findings showed that consumption tax has only one essential distortion, which affects the option between work time and leisure time in favour of the latter. Consequently, it causes a reduction in economic growth. This option is also affected by income taxes, which causes other distortions as well. The distortions of income tax can be seen in the reduction of capital accumulation and growth. In general, consumption and income taxes have various impacts on growth and welfare in endogenous growth models; however, consumption taxes have less impact on growth compared to income taxes (Milesi-Ferretti and Roubini, 1998).

The effect of consumption tax on capital accumulation and productivity growth has been examined in a few studies. In this regard, Petrucci (2002) conducted a study using a one-sector endogenous growth with finite horizons. The author sought to determine the implications and effects of consumption taxes on capital growth/capital accumulation. The common assumption in this study was that the labour supply is inelastic to avoid the distortion of the intertemporal consumption-leisure caused by the consumption tax. Moreover, consumption taxes have the net effect of redistributing income from the current living generation to the future unborn generation. Another assumption considered in this study was that the consumption taxes are not supposed to cause any impact on the growth of the overall economy if the one-sector growth model is used.

Petrucci's study findings indicated that the consumption taxes raise the current levels of savings among households while reducing the aggregate consumption. In effect, consumption taxes stimulate capital accumulation and increases the growth rate in the economy. For instance, suppose the government chooses to use the resources obtained from consumption taxes to finance unproductive public expenditure, then the economy will not experience the consumption taxes. The results also showed that when the government increases consumption taxes and compensates for this move by reducing its public debt, it stimulates capital accumulation, increases the long-run growth rate, and lowers the consumption-output ratio. In other words, increasing consumption taxes and reducing debt implies that the government would have more money to spend on development, which in turn increases job and investment opportunities and provides additional income from taxes.

In a similar vein, El-Ganainy (2006) analysed theoretically and empirically the effect of value-added tax, as a consumption tax, on capital accumulation, productivity growth, and economic growth. Theoretically, the author applied a two-sector endogenous growth model. Her model depends on the overlapping generation model of Diamond's (1965). The model was extended by introducing consumption taxes to evaluate the effect of this tax on growth. It included three sectors: households, competitive firms, and the government. The results indicated that consumption taxes influence capital accumulation and productivity growth through savings, which in turns stimulate economic growth. However, these impacts were theoretically unclear because they rely on the relationship between the parameters of utility, interest rate, and tax structure. Empirically, El-Ganainy (2006) used the dynamic panel GMM-System estimators to estimate a panel of 14 European Union (EU) countries during 1961-1995. The main results found that the VAT negatively affects the aggregate consumption level and positively affects physical capital accumulation and, therefore, contributes to GDP growth. However, productivity growth is found to be statistically insignificant, meaning productivity growth through the VAT has less impact on economic growth.

Similarly, Nguyen, Onnis, and Rossi (2017) examined how the tax changes of income tax (personal and corporate taxes) and consumption tax can affect the macroeconomic variables[33]. Nguyen et al. (2017) empirically tested whether income tax and consumption tax shocks directly affect the UK's economy by using time-series data during 1973-2009. The study findings emphasised the significance of distinguishing the effects of direct taxes and indirect taxes when studying fiscal policies and their transmission mechanism. The results indicated that income tax shocks cause significant effects on investment, private consumption, and overall GDP in the short run. According to Nguyen et al. (2017), a reduction in the amount of consumption tax does not affect the GDP and the investment so much. They only cause a marginal expansionary effect on the levels of private consumption in the economy. Notably, consumption taxes produce less distortion in the economy when compared to income taxes.

[33] Nguyen et al. (2017) theoretically illustrated the reasoning of the impact of income tax and consumption tax. They pointed out that "taxing income (personal and corporate) depresses private investments and labour supply. Differently, taxing consumption would decrease current consumption, but it would increase savings and therefore investment. Higher investment would boost labour productivity, thus increasing wages, future output and consumption" (Nguyen, Onnis, and Rossi, 2017, p.2).

The role of taxes in reducing the rate of indeterminacy and lowering the chances for aggregate instability in the economy has been widely studied in the real business cycle literature. Although our study is in economic growth, understanding how taxes can overcome the fluctuations in the economy is also important to investigate in our study. In this literature, a balanced budget fiscal policy rule[34] is an ongoing theme of debate and critique. In this regard, Giannitsarou (2007) conducted a study on the role of taxes in balanced budget rules and aggregate instability. Her paper was based on Schmitt-Grohé and Uribe's (1997) work[35]. She argued that consumption taxation guarantees determinacy and should be favoured with respect to income taxation. Giannitsarou (2007) model was based on a standard neoclassical growth, the Ramsey model, with infinitely lived households, firm, and government. Her model was built on the absence of public spending externalities, and consumption taxes finance government spending.

Giannitsarou (2007) demonstrated that if endogenous consumption taxes only finance the government revenues, any steady-state is saddle-path stable, and no indeterminacy arises. The reason is that the consumption tax is not a distortions tax, unlike income tax, where the latter may generate the possibility of indeterminacy. The author further discussed the policy implications if income and consumption taxes are combined. The numerical investigation showed three main results. First, if both taxes are exogenous (fixed), and if income tax is exogenous and consumption tax is endogenous, indeterminacy never arises. Second, if income tax is endogenous but consumption tax is exogenous, then indeterminacy can arise. Third, if both taxes are endogenous, then the range of indeterminacy becomes smaller. As a result, Giannitsarou (2007) concluded that consumption taxation is a better channel for financing government expenditure compared to income taxes. The reason is that considering only consumption taxes ensures that indeterminacy does not arise. Moreover, endogenous consumption taxes can reduce the possible occurrence of aggregate instability caused by income taxes. These fluctuations can also be removed if endogenous consumption taxes replace income taxes.

Recently, Bambi and Venditti (2018) examined an endogenous framework to evaluate whether balanced-budget rules applied alongside endogenous taxes may cause aggregate instability in the economy. They used a neoclassical growth model, Barro (1990) endogenous growth model, with

[34] A balanced budget fiscal policy rule indicates that a country should not spend above its income. For instance, there should be a clear balance between the projected receipts and government expenditure. This policy refers to a pro-cyclical fiscal policy, which is the opposite approach of counter-cyclical. According to Schmitt-Grohé and Uribe (1997), the common narrative against the use of balanced-budget fiscal policy is linked to its tendency to increase aggregate demand (AD) through increased public expenditure and tax cuts during boom seasons, while reducing AD by taking fiscal contractions during recessions.

[35] Schmitt-Grobe and Uribe (1997) investigated that why a balanced budget rule can be destabilising. They demonstrated that if the government depends on only distortionary income taxes to finance its endogenous spending, it can face destabilising conditions when using a balanced-budget rule. In other words, if the balanced-budget rule determined the income tax, the indeterminate steady-state in the economy can arise.

the productive government. The government expenditures are fed by time-varying consumption taxes and a balanced budget rule. They assumed that consumption tax relies on de-trended consumption; therefore, it transforms from a control variable to a state variable. The reason for this assumption is to obtain a constant tax on a balanced growth path (BGP). They further assumed that the utility function is CRRA, and the labour supply is inelastic. Bambi and Venditti (2018) found the same result as Barro (1990), where there is no transitional dynamics regarding the unique BGP. Therefore, this is consistent with Giannitsarou's (2007) conclusion, in which endogenous fluctuations did not previously have room. They have further shown that the unique BGP is not a long-run solution. However, suppose in terms of consumption that the tax rule is counter-cyclical. In that case, there would be sunspot equilibria dependent on self-fulfilling expectations, which relies on the consumers' decision for a consumption path.

The literature on taxation and economic growth is central to our thesis because our study aims to contribute to this large literature. Although there are a large number of neoclassical growth models that investigate the effect of taxes on economic growth, many of them treated government revenues as a pure waste of resources. In other words, the government sector is assumed to be an unproductive sector. Moreover, a few studies in the literature examined only consumption tax in an economic growth model. However, Bambi and Venditti's (2018) paper is considered closely related to our study in terms of consumption tax that finance productive government spending. Their model used Barro's (1990) endogenous growth model and a consumption tax that finances the productive government sector. Even though we use a Barro (1990) type of production function, our models have some differences from the original Barro (1990) model as well as Bambi and Venditti (2018) model, particularly in the government budget constraint. In each chapter of our thesis, the productive government sector is financed by different sources of government revenues. More precisely, the government spending is financed by (i) only oil revenues, (ii) oil revenues and consumption tax, and (iii) oil revenues and personal income tax. Consequently, adding oil revenues with different taxes to finance productive government spending in Barro's (1990) model makes our study different from other studies.

3.7 Fiscal Policy and Economic Growth in Saudi Arabia

In general, from the standpoint of Keynesian Economics and the New Growth Theory, government spending is seen as having a prominent role and a driver of economic growth. Regarding the role of fiscal policy, Moreno-Dodson (2013) pointed out that the neoclassical growth models in the 1990s were focused on the rule of fiscal policy. Some of the key models include Ghosh and Mourmouras (2002), Devarajan, Swaroop, and Zou (1996), Futagami, Morita, and Shibata (1993), and Barro (1990). Thus, this section of the literature review addresses the link between fiscal policy, *i.e.* the government spending and taxes, and economic growth in Saudi Arabia. This literature is examined in our study for three reasons. First is due to the primary role played by fiscal policy, especially government spending, and highlight the lack of studies on taxes in the Saudi economy. The second is to understand the relationship between fiscal policy and economic growth in Saudi Arabia more accurately, which is the subject of this study. The third reason is that our study aims to examine the efficiency of introducing new taxes in the Saudi economy, as proposed by the IMF.

Starting with an early study about the government spending role in Saudi economic growth, Barri (2001) studied the link between government expenditure and economic growth during 1970-1998. His study intended to determine the proper size of government expenditure as part of their GDP in the short and long term. It was also aimed to measure the productivity of those expenditures. Barri (2001) used a simple theoretical model based on the neoclassical production function, which includes the government final consumption expenditure, gross fixed capital formation, and labour. 'Barro's rule' is used in his model, where the rule states that "the right size of government expenditure is where the value of the productivity of that expenditure is equal to the unit" (p.62).

In Barri's (2001) paper, he attempted to answer two main questions; is government expenditure in Saudi Arabia productive or unproductive? Is the size of government expenditure in Saudi Arabia appropriate, larger, or smaller than it should be? The results showed that government expenditure's productivity was positive and equal to 0.387, suggesting that the expenditure is productive. In the long term, the results showed that there was a correlation between the GDP and all government expenditure, gross fixed capital formation, and labour involved in the production. In the short term, the researcher found that government expenditure in Saudi Arabia is productive as the marginal productivity value of this spending is positive but less than one. According to 'Barro's rule', this result implies that government spending is larger than it should be, where its size is 29% of GDP, which is in line with almost the average world rate.

Similarly, Al-Obaid (2004) also studied the relationship between economic growth and government expenditure and its direction. He tested the validity of ‘Wagner's Law’ in Saudi Arabia. Wagner's law is suggested by Adolph Wagner, which states that public spending increases with economic growth. The results of examining Saudi Arabia during 1970-2001 by the co-integration test showed statistical support for Wagner's prediction in the relationship between government expenditure and GDP. Moreover, Alshahrani and Alsadiq (2014) empirically tested the economic growth and government spending in Saudi Arabia. They investigated the impact of seven varieties types of government spending on economic growth in the short and long run. These types of government spending include housing, education, defence, health care, current and capital expenditures, public investment, and the impact of total expenditure and domestic private investment. Three different econometric techniques, Vector Auto Regression (VAR), Cointegration, and Vector Error Correction Model (VECM), were used to estimate the impact of these expenditures on growth during 1969-2010. The results indicated that private domestic and public investments, as well as health care expenditure, have an impact on the long-run growth rate in Saudi Arabia. On the other hand, trade openness and spending on the housing sector enhance the short-run production.

Along the same lines, Alshammari and Aldkhail (2019) examined the effect of government spending on growth in Saudi Arabia. A simple econometric model was tested to measure the effect of government spending on growth during the period (1985-2017) using the Ordinary Least Squares (OLS) method. Economic growth is the dependent variable in their model, while three types of spending (consumption, investment and government spending) are independent variables. For government spending, the results showed that it has a favourable implication on economic growth, *i.e.* increasing government spending contributes to a higher rate of economic growth.

Similar to Al-Obaid (2004), Alrasheedy and Alrazyeg (2019) investigated the validity of the five various versions of Wagner's law and the Keynesian approach. The time-series data of Saudi Arabia during 1970-2017 was applied. The authors illustrated that the Keynesian approach showed that causality runs from government expenditure to economic growth (GDP), where Wagner's law suggested the opposite. Their paper analysed three tests, the stationary properties, Co-integration and Granger causality, to check the relationship between government spending and GDP. Alrasheedy and Alrazyeg (2019) used the autoregressive distributed lag (ARDL) approach of cointegration to verify the natural relationship between the variables in the long term. The results indicated that there is no evidence for the short-term effect of economic growth on government spending (Wagner's Law). However, all types of government spending as a share of income affect economic growth (Keynesian approach).

Although these studies confirmed the significance of government spending to stimulate Saudi economic growth, our study is different from them in some respects. First, we provide a theoretical framework built on Barro's (1990) work to model Saudi economic growth where the evidence from previous studies is based on applied studies. Second, the previous studies in this literature ignored the role of taxes in Saudi's economic growth models. Third, as per our research, no research has examined the possibility of compensating for any potential reduction in oil revenues by reforming the fiscal policy, *i.e.* current tax systems. Thus, this thesis aims to fill this gap in this literature by introducing consumption tax and personal income tax side by side with the source of oil revenues in a simple theoretical economic growth model that attempts to describe Saudi's economy. It also discusses how consumption tax, as suggested by the IMF, would compensate for any potential reduction in oil revenues.

3.8 Conclusion

In conclusion, we summarise all the themes discussed in this section. Starting with the economic growth models literature, we aimed to provide an overview of the different growth models. The fundamental reason for surveying this literature is due to the fact that our models in each chapter may represent a different growth model because of using different sources of government revenues. In the second theme, we focused on the literature on the structure of the oil market, particularly the oil sector in Saudi Arabia. Although this type of literature is not directly related to our study, we believe that this type of literature could give us a better understanding of how the oil sector works in order to link it with our growth models. For the third theme, which is the economic growth in oil-rich countries, we attempted to show the two opposite hypotheses on the possible impact of natural resources. Although our study does not intend to contribute to this debate in the literature, we believe that our study would show that if government revenues are used to finance productive government spending, then the natural resources could stimulate economic growth and eventually it could be a blessing.

The fourth theme presented a review of literature on managing oil revenue fluctuations in oil-rich countries. A series of recent studies in this area have indicated that the Sovereign Wealth Funds (SWFs) or saving financial surpluses from natural resources in an investment fund would be a solution to deal with fluctuations in resource revenues. However, the SWFs may suffer from some shortcomings, limiting or reducing the achievement of their primary objectives. Thus, we argue that reforming fiscal policy, in particular tax reforms, could be an ideal solution to offset or relieve the revenues fluctuations. Regarding the fifth theme, taxes and their roles in economic growth, it briefly addresses the public finance literature on tax capacity, the literature on optimal taxation stemming from Mirrlees, and the literature on institutions. However, it focuses more on the relationship between taxes and economic growth. In this regard, we found that only a few studies in the literature demonstrated consumption tax in an economic growth model. Besides, as far as we know, only Bambi and Venditti (2018) study has investigated the consumption tax in Barro (1990) endogenous growth model, where the government sector is productive. Thus, our study would also contribute to this literature by extending Barro's model (1990) by providing different sources of government revenues to finance the productive government sector. In other words, our study would be original in providing an understanding of financing the productive government sector by (i) only oil revenues as an exogenous source of government revenues, (ii) oil revenues and consumption tax, and (iii) oil revenues and personal income tax.

The last theme inspected the fiscal policy and economic growth literature in Saudi Arabia. A close look at this literature revealed that the previous studies have ignored the role of taxes in Saudi's economic growth models. To fill this gap in the literature, this thesis aims to introduce consumption and personal income taxes besides oil revenues in the Saudi economic growth. Furthermore, it investigates how consumption tax would mainly compensate for any potential reduction in oil revenues. Now, the next chapter will look at modelling the Saudi economy before the implementation of the proposed fiscal policy reforms by the IMF.

4 *Chapter Four: Modelling Saudi Economic Growth*

4.1 Introduction

This chapter is a theoretical paper about the economic growth in Saudi Arabia. We examine the case of the world's second-largest oil economy, Saudi Arabia, in terms of production and exports. As one of the oil-rich countries, the Saudi economy is highly dependent on oil revenues, which determine the fate of the whole national economy. Barro's endogenous growth model is extended to model the Saudi economy before implementing the proposed fiscal policy by the International Monetary Fund (IMF). The extension of the Barro model in this chapter is based on two things; (*i*) ignoring any taxes[36], and (*ii*) adding the oil sector and modified the budget constraint. The primary assumption in this chapter is that we only have one source of government revenues that finances government spending. This source is the oil sector profit, and the key feature of our model is that the oil sector is modelled and treated as an exogenous and monopolistic sector, where the growth rate is basically led by a growing demand for oil. We simplify the theoretical model by setting the oil sector maximises its profit, and then the government uses them to enhance the firms' production function. Despite the oil sector is an exogenous sector, it plays a central role in our economy because government spending is only fed by this sector. Therefore, it is worth mentioning that although we use Barro endogenous growth model with an exogenous source, our model could be represented as an exogenous or a semi-endogenous growth model where the growth is led by the growing demand for oil.

This chapter aims to answer the first two research questions in this thesis. It is mainly based on a description of the Saudi economy and the impact on the economic growth of a negative shock in demand for oil. We examine in this chapter how our economy can be affected if there is a negative shock to the demand for oil by focusing on the impact of reducing the growth rate of government spending on the whole economy. Thus, we work on an endogenous growth model with an exogenous source of finance which determines the long-run growth.

[36] Taxes will be introduced and investigated in the model of the next chapters.

4.2 Model Description

The model setup is organised to represent the Saudi Arabia economy where we assume that Saudi Arabia is a partially open economy and has four sectors: oil, government, households, and firms' sectors[37]. Oil and government sectors have limited access to deal with abroad. The oil sector simply exports oil abroad and receives revenues. It also maximises its profit as a monopolist, and then the government gets oil profits and purchases of imported goods. On the other hand, households and firms' sectors are assumed to be domestic agents where they both do not deal with abroad.

I. Oil Sector:

Although many studies in energy and non-renewable resources economics, starting from Hotelling's rule[38], have paid much attention to exploiting non-renewable resources optimally and distributing them among generations, the model in this study will not deal with it for several reasons. (*i*) The main purpose of this study is to treat the oil sector as an exogenous and monopolistic sector where it only maximises its profit in each period separately, and then the government sector collects that profit. (*ii*) One of the main objectives of the thesis is to examine how to compensate for any reduction in oil revenues by suggesting tax reforms to maintain the sustainable growth rate, which will be discussed in more detail in the next chapter. Thus, this study focuses on economic growth rather than the optimal extraction of non-renewable resources such as oil. (*iii*) Introducing the analysis of the optimal extraction oil between generations over time may make the model difficult and take it away from the purpose of this study. Therefore, what we basically want to study in this chapter is what happens in terms of growth if there is a negative shock to the demand for oil. Then, in the following chapters, we will introduce consumption tax and personal income tax alongside oil revenues to study the impact on economic growth and how consumption tax can compensate for any negative effect on the growth rate.

[37] We ignore the discussion of the foreign sector in our economy as a fifth sector. The reason is that this sector has only to deal in our model with the government sector, as we will see later.

[38] Hotelling's rule comes from Harold Hotelling's paper 'The Economics of Exhaustible Resources' who analysed the optional extraction between present and future. Hotelling's rule links the price of non-renewable natural resource, as a physical asset, with interest rate of financial asset, ignoring the extraction costs. It states that the optimal extraction path of a non-renewable resource is when the resource price increases at the interest rate. If the appreciation rate of non-renewable resources is greater (less) than the interest rate, the extraction of the non-renewable resources would (would not) be optimal (Hotelling, 1931).

Before studying the oil sector, it is worth looking at the oil demand data to have a clear picture of the most important determinant of oil demand. By looking at the data, we have observed in recent years that there was a more growing demand for oil over time, which could depend on many factors[39]. One possible reason that caused a positive trend in demand for oil over time could be due to the economic growth of the rest of the world, where some countries are demanding more oil. By using the available data provided by the World Bank data and BP statistical review of world energy, we have found a significant (strong) positive relationship between world real GDP and world oil demand[40].

In general, any change in the world real GDP growth would lead to changes in the world oil demand[41], which would lead to changes in oil profit. Changes in oil profit would lead to changes in the government's revenues, which would end up affecting government spending. Finally, this would affect the economic growth of a country that heavily depends on exporting oil, such as Saudi Arabia. Therefore, relying on oil revenues and then on public spending as a significant driver of economic activity has made the Saudi economy more sensitive to changes in the global oil markets. As a result, the profits of the oil sector are essential for our study because they are the only source of government revenues that are used to finance government spending.

II. Government Sector:

The government sector involves two sides: the revenue side and the spending side. In this chapter, there is only one source of government revenues which is oil revenues. Thus, the government receives oil profits from the oil sector and then spends it on purchasing imported goods from abroad. The transaction between the government and foreign sector is only based on exchanging oil revenues with imported goods. Then, the government sector provides public goods (*e.g.* infrastructure, legal framework) as positive externalities to the firms in order to enhance their production. Moreover, it is also assumed that the government has no access to international financial markets (lending and borrowing), *i.e.* it just spends all what it receives.

[39] As we understand from the principles of economics that there are several factors that affect the demand for any commodity. The main factors include the price of the commodity itself, the income of individuals, individuals' tastes and consumption habits, government policies of taxes and fees, prices of other goods, and other factors that have a direct or indirect impact on the quantities that individuals want and can purchase from goods or services. Regarding the oil, it is also a commodity, where its demand is also affected by such factors. The most important of these factors are the price of the oil itself, expected future oil prices (especially for speculators and hedgers), alternative commodity prices (*e.g.* coal, gas, renewable, nuclear, etc.), supplementary commodity prices (*e.g.* cars, plane tickets, tanker traffic, etc.), derivative prices (*e.g.* gasoline, diesel, petrochemicals, etc.), global economic activity (*e.g.* growth, contraction). There are also secondary factors such as taxes, nature fluctuations, political turmoil, and wars that also affect the demand for oil.

[40] The details of this relationship can be found in chapter two (section 2.5.4).

[41] For instance, due to the recent spread of Coronavirus that hits one of the large economies in the world, China, and other countries as well, many factories and companies have entirely or partially stopped working. Thus, some reports of international organizations, such as IMF (2020b) and IEA (2020), recently expect that the world economic growth and the world oil demand would slow down significantly due to this crisis.

III. Households Sector:

The households supply factors for production to the firms and demand goods and services from the firms. They maximise their utility over time. It is assumed that the households' utility is a function of only consumption, where labour supply in the model is completely inelastic.

IV. Firms Sector:

The firms' sector is assumed to be not dealing with abroad, and it is represented by the final non-oil goods sector. This sector receives positive externalities from the government to increase its production function. All firms act in a perfectly competitive market, and they maximise their profits at each date.

The below diagram shows a summary of the economy's structure for this chapter, where each arrow indicates what each sector in our economy provides and receives.

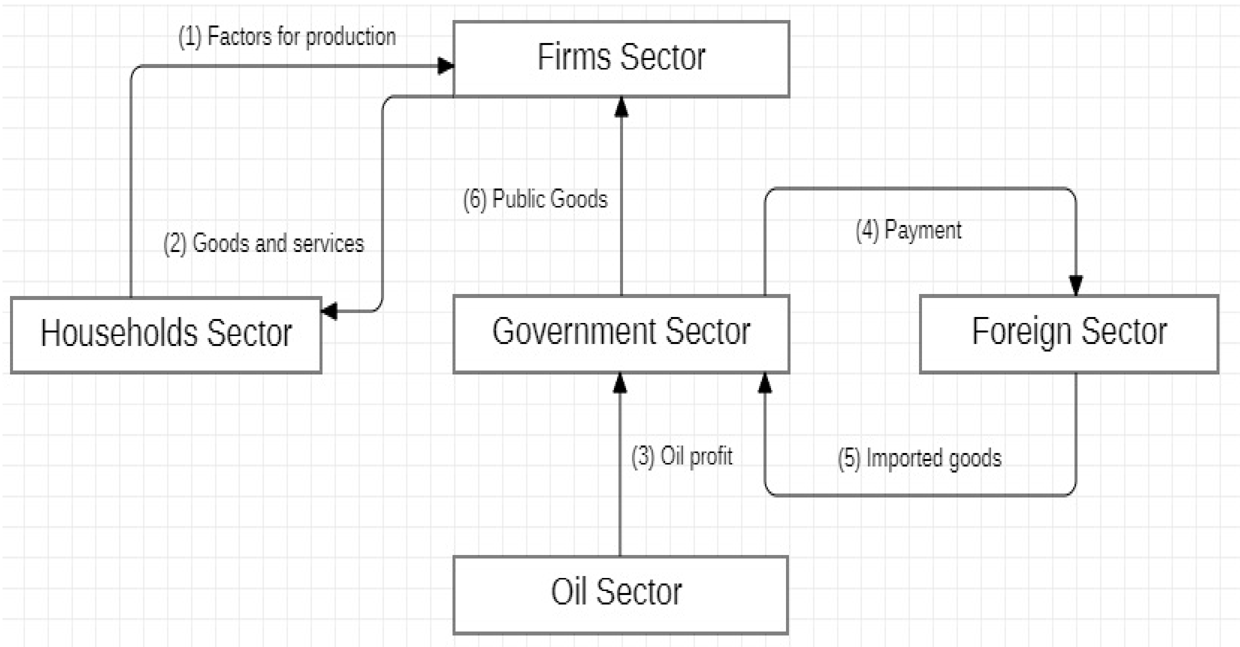


Diagram 3: The structure of the economy in chapter four

4.3 Model Setup

The main sections in our model of this chapter include the agents in the economy, equilibrium, analysis of the steady-state, comparison of the Ramsey model and our model, analysis of the stability, the transitional dynamics in our economy, numerical simulation, and finally a discussion of a special case of our model. Thus, let us begin with the details of the agents in the economy. The main agents in the economy are the oil, government, households, and firms' sectors. Each sector will be studied in more details.

4.3.1 Oil Sector (Monopolist Sector)

Oil is the only commodity exported abroad, which can be used to cover the cost of imported goods. Thus, the oil sector maximises its profit, and then the government takes that profit to finance its spending. We assume that the international capital flows (lending and borrowing) do not exist, so the current account has to be balanced (CA=0). The reason for making this assumption is to avoid an accumulation of Net Foreign Assets (NFA) and to simplify the model.

We also assume that the demand for oil is derived entirely from abroad and then is derived by world economic (GDP) growth. Therefore, the growth rate of the oil demand is considered here to be exogenously given. Moreover, the oil sector is considered in the model to have an oil demand growing potentially with a positive rate over time. Therefore, the oil sector profit grows over time. Thus, the dynamic demand function for oil is simply written as follows:

$$q_t = a_t - bp_t \qquad \forall t > 0 \tag{4.1}$$

$$a_t = ga_{t-1} \qquad \Rightarrow \qquad a_t = a_0 g^t \qquad where \;\; g > 1 \tag{4.2}$$

where g is the growth rate of a_t which is the global GDP growth rate, and a_0 is exogenously given.

In equation (4.1), q_t is the oil demand, b is the slope, and p_t is its relative price in terms of the final output, which is controlled by the monopolist. In equation (4.2), we assume that a_t is not constant, but it grows over time because we want to show that the oil demand grows over time and then to show the path of change in price and quantity.

Combining (4.1) and (4.2) leads to the following time-varying demand function:

$$q_t = a_0 g^t - bp_t$$

In general, the monopoly's profit is given by the following equation:

$$\pi = p \, . \, q - c(q)$$

In this formula, $p.q$ is the total revenue (TR), and $c(q)$ represents the total cost (TC). Profit is defined by π, and it is the difference between the total revenue and the total cost.

Although it is difficult in reality to obtain accurate data on production costs due to the difference from one country to another, the fact is that some oil-rich countries may have very low production costs depending on their oil characteristics[42]. In this study, we assume there is no intermediate input used in the oil sector; therefore, the cost function is assumed to be zero in the model, $c(q) = 0$.

Profit maximisation for the monopolist is a sequence of static problems. Each maximising profit at a single point in time takes the oil demand as given, so the monopolist solves,

$$\max_{p} \ (p_t q_t)$$

subject to

$$q_t = a_0 g^t - bp_t \tag{4.3}$$

Substituting the constraint (4.3) into the objective function (direct substitution method), and setting the derivative with respect to p_t equal to zero, we obtain the first-order condition for maximising net profit:

$$\frac{d\pi_t}{dp_t} = 0 \ \Rightarrow \ a_0 g^t - 2bp_t = 0$$

The optimality condition is that we must have marginal revenue (MR) equal to marginal cost (MC), to obtain the optimal price and quantity. Nevertheless, since the cost function in the model is assumed to be zero, then the marginal cost is zero too, MC=0. Thus, the optimal price is,

$$p_t^* = \frac{a_0 g^t}{2b} = \frac{a_0 g^t}{2b} \tag{4.4}$$

and the optimal quantity,

$$q_t^* = \frac{a_0 g^t}{2} \tag{4.5}$$

[42] In practice, when remaining oil reserves reduce, oil extraction becomes more expensive. The reason for the high costs in this case is due to the need to drill deeper wells or pump water or carbon dioxide to maintain the pressure inside those wells (Van Der Ploeg, 2010). However, in reality, oil production cost in some oil-rich countries such as Saudi Arabia is very low. According to Alobaid and Atya (1994), Saudi oil is characterized by lower production costs compared to production costs in other regions due to the following reasons: availability of large reserves, advanced technology and geological nature of Saudi oil wells, which are characterized by low dry wells, abundant production, and low most of the depth of wells.

Now we substitute the result of equation (4.4) and (4.5) inside the objective function. Then, we get the monopolist's profit (π_t):

$$\pi_t = TR - TC$$

$$where\ TR = P^* q^* \quad and \quad TC = 0$$

$$\pi_t = \frac{a_0^2 g^{2t}}{4b} \tag{4.6}$$

From equation (4.6), the growth rate of the oil demand q_t and the monopolist's profit π_t is g^2. The oil profit is exogenous because the growth rate that we treated as exogenous, in equation (4.2), is the growth rate of the intercept determined the demand for oil function. Thus, the growth rate of oil profits is not g, but it is g^2, which grows all the time (short run and long run). As a result, the oil profit is treated here as exogenous because the oil sector is independent of the rest of the economy.

4.3.2 Government Sector

The government budget contains two sides: revenues and spending. The revenues side of the government sector comes only from the profit of the oil sector, which is assumed to own, and they are used to purchase imported goods from abroad to enhance the production function of the final goods sector. Following the example of Barro's endogenous growth model, government spending is treated as current government spending[43]. Assuming that the government sector cannot access the international financial market, and there are no taxes[44], the government sector simply spends all the profits received by the oil sector in purchasing imported goods. So, the government budget is in balance and can be written as:

$$G_t = \pi_t$$

where G_t is the government spending and π_t is the oil sector profits.

$$G_t = \frac{a_0^2 g^{2t}}{4b} \tag{4.7}$$

[43] In spite of the fact that, in order to capture the idea of that government spending is on 'infrastructure' and that this has positive externalities, the government spending should be treated as capital spending, we follow Barro's endogenous growth model to consider them as current spending.

[44] In fact, the current tax system in Saudi Arabia is very limited. Thus, we do not model taxes in this chapter; however, new taxes, as suggested by IMF, will be considered and introduced in the model of the next chapters.

We define the growth rate of government spending as $\gamma_{G_t} \equiv G_{t+1}/G_t$, which can be written as,

$$\gamma_{G_t} \equiv \frac{G_{t+1}}{G_t} = \frac{\frac{a_0^2 g^{2t+2}}{4b}}{\frac{a_0^2 g^{2t}}{4b}} = \frac{g^{2t+2}}{g^{2t}} = g^2$$

We can now see that government spending grows at the rate of g^2 every period, including the short-run. As yet, the government has no instrument to react from a shock to government spending. Thus, in the case of having a negative shock on g, then the government revenues would be changed, affecting government spending and its growth rate. Once the government spending is affected, the public goods provided to the firms would also be affected. Finally, the economic growth rate would as well be impacted accordingly.

4.3.3 Households Sector

The households maximise their utility function, which is a function of consumption subject to the budget constraint, with preferences represented by a constant relative risk aversion (CRRA) utility function and parameter $\sigma > 0$. The utility of households depends only on consumption. So, the households supply completely inelastically their labour, which means that they supply at each date $L_t = 1$ to the firms. It is assumed that there is a unit measure of identical infinitely-lived households. From solving the household's problem, we find the households' capital accumulation equation and Euler equation.

The households have no access to international trade, where goods can be either consumed or invested domestically. Moreover, there is no transfer[45] in the model. So, the consumer's budget constraint at the time t is:

$$R_t K_t + w_t L_t = C_t + K_{t+1} - K_t + \delta K_t$$

$$C_t, K_{t+1} > 0 \quad \forall t \;; \quad K_0 > 0 \quad \textit{exogenously given}$$

Where $\beta, \delta \in (0,1)$ are the discount factor and the constant depreciation rate of capital, respectively. $R_t K_t$ is the gross capital income, $w_t L_t$ is the labour wage, C_t is the consumption, and $I_t = K_{t+1} - K_t + \delta K_t$ is the gross investment.

[45] Although we assume that there is no transfer to households in our model for simplicity, the households in Saudi Arabia receive subsidies in terms of reduced energy bills, where these subsidies are represented a very small percentage of the oil profit. However, the government has recently worked on energy price reform through the gradual cancellation of government subsidies on energy products as a goal to be achieved by 2030. The purpose of the energy price reform is to adjust pricing and focus on supporting needy citizens only (Ministry of Finance, 2017; Saudi Vision 2030, 2016).

So now, we can rewrite the capital accumulation as:

$$K_{t+1} = (R_t + 1 - \delta)\, K_t \; + \; w_t L_t \; - \; C_t \tag{4.8}$$

This represents the households' capital supply. Thus, the Lagrangian for this problem is:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[\left(\frac{C_t^{1-\sigma} - 1}{1 - \sigma} \right) + \; \lambda_t \left((R_t + 1 - \delta)\, K_t + w_t L_t \; - C_t \; - K_{t+1} \right) \right]$$

The first-order conditions:

$$\frac{d\mathcal{L}}{dC_t} \; = \; 0 \quad \Leftrightarrow \; \beta^t C_t^{-\sigma} \; = \; \beta^t \lambda_t \quad \Rightarrow \quad C_t^{-\sigma} \; = \; \lambda_t \tag{4.9}$$

$$\frac{d\mathcal{L}}{dK_{t+1}} \; = \; 0 \quad \Leftrightarrow \; -\beta^t \lambda_t + \; \beta^{t+1} \lambda_{t+1} \left[R_{t+1} + 1 - \delta \right] \; = \; 0 \tag{4.10}$$

$$\frac{d\mathcal{L}}{d\lambda_t} \; = \; 0 \quad \Leftrightarrow \quad K_{t+1} = (R_t + 1 - \delta)\, K_t + w_t L_t - C_t$$

Substitution (4.9) into (4.10) leads to the Euler equation:

$$\frac{C_{t+1}}{C_t} \; = \; \left[\beta \; \left(R_{t+1} + 1 - \delta \right) \right]^{\frac{1}{\sigma}} \tag{4.11}$$

4.3.4 Firms Sector

Following Barro (1990) by introducing positive externalities from the government's purchases of goods and services, the production function used in this economy is the Cobb-Douglas production function. The total output in period t, Y_t, is produced by firms using two inputs, namely, physical capital, K_t, and labour, L_t. Government spending, G_t, is positive externalities[46]. All firms are assumed to be identical, and they act in a perfectly competitive market. Thus, each firm produces output Y_t using the following Cobb-Douglas production function,

$$Y_t \; = \; A\, K_t^{\alpha} \; L_t^{1-\alpha} \; G_t^{1-\alpha} \qquad 0 \; < \; \alpha \; < \; 1 \tag{4.12}$$

For equation (4.12), there are two different interpretations, *i.e.* not two different versions of the production function, of the same production function.

[46] Cornes and Sandler (1996) clarified that the production of the private sector will be impacted by the activities of the government. If the output from a sector is increased due to activities of another sector, then it is a positive externality.

(i) For given A, the above equation shows (IRS) in K_t, L_t and G_t all together.

(ii) For given A and G_t, the aggerate level displays (CRS) in both private inputs of capital, K_t, and labour, L_t. Likewise, for given A and L_t, the production function also shows (CRS) in K_t and G_t together as in Barro's (1990) version.

The version of Barro's (1990) endogenous growth model with public spending assumed that both the aggregate labour supply and technology level are constant. The reason is that an increase in scale, L_t, raises the marginal product of capital and raises economic growth. In this case, the economy benefits from a greater scale. The reason is that the government service is assumed to be nonrival and can therefore be spread costlessly over additional users. Thus, Barro assumed zero population growth to study steady-state. Consequently, the above general production function, equation (4.12), with Barro's (1990) assumptions becomes such that:

$$Y_t = A\,K_t^{\alpha}\,G_t^{1-\alpha} \qquad\qquad 0 < \alpha < 1$$

However, we start from the general production function with government spending, equation (4.12),

$$Y_t = A\,K_t^{\alpha}\,L_t^{1-\alpha}\,G_t^{1-\alpha} \qquad\qquad 0 < \alpha < 1$$

The firm's problem of maximising profits is that they decide how much capital and labour to demand in order to maximise their profits by taking the rental rate of capital, R_t, and wage per unit of labour, w_t, as given, subject to the firms' production function:

$$\begin{aligned} &\underset{K_t,\,L}{Max}\; Y_t - R_t K_t - w_t L_t \\ &s.t \quad Y_t = A\,K_t^{\alpha}\,L_t^{1-\alpha}\,G_t^{1-\alpha} \end{aligned}$$

Substituting the production function into the firms' objective function (direct substitution method) gives the firms' problem:

$$\underset{\pi}{Max}\; A\,K_t^{\alpha}\,L_t^{1-\alpha}\,G_t^{1-\alpha} - R_t K_t - w_t L_t$$

The first-order conditions of the profit maximisation problem:

$$\frac{d\pi_t}{dK_t} = 0 \Rightarrow \alpha\, A\, K_t^{\alpha-1}\, L_t^{1-\alpha}\, G_t^{1-\alpha} - R_t = 0$$

$$\Rightarrow R_t = \alpha\, A\, K_t^{\alpha-1}\, L_t^{1-\alpha}\, G_t^{1-\alpha} = \alpha\, \frac{Y_t}{K_t} \tag{4.13}$$

$$\frac{d\pi_t}{dL_t} = 0 \Rightarrow (1-\alpha)\, A\, K_t^{\alpha}\, L_t^{-\alpha}\, G_t^{1-\alpha} - w_t = 0$$

$$\Rightarrow w_t = (1-\alpha)\, A\, K_t^{\alpha}\, L_t^{-\alpha}\, G_t^{1-\alpha} = (1-\alpha)\, \frac{Y_t}{L_t} \tag{4.14}$$

For the equilibrium purposes, we convert the variables in terms of per government spending units (PGSU), by dividing them by government spending, G_t. Thus, the output PGSU can be written as:

$$\frac{Y_t}{G_t} = \frac{A\, K_t^{\alpha}\, L_t^{1-\alpha}\, G_t^{1-\alpha}}{G_t} \Rightarrow \frac{Y_t}{G_t} = A\, K_t^{\alpha}\, L_t^{1-\alpha}\, G_t^{-\alpha} \Rightarrow \hat{y}_t = A\, \hat{k}_t^{\alpha}\, L_t^{1-\alpha} \tag{4.15}$$

where $\hat{y}_t \equiv Y_t / G_t$ is the output PGSU, and $\hat{k}_t \equiv K_t / G_t$ is the capital PGSU.

Since we have now obtained the output PGSU, it would be more appropriate for consistency to find the capital income PGSU and labour income PGSU.

Dividing both sides of the capital income, $R_t K_t$, by G_t to get capital income PGSU:

$$R_t \hat{k}_t = \alpha\, A\, \hat{k}_t^{\alpha}\, L_t^{1-\alpha} = \alpha\, \hat{y}_t \tag{4.16}$$

The share of capital income PGSU:

$$\frac{R_t \hat{k}_t}{\hat{y}_t} = \alpha$$

Similarly, the labour income PGSU can be obtained by dividing both sides of the labour income, $w_t L_t$, by G_t:

$$w_t \hat{l}_t = (1-\alpha)\, A\, \hat{k}_t^{\alpha}\, L_t^{1-\alpha} = (1-\alpha)\, \hat{y}_t \tag{4.17}$$

The share of labour income PGSU:

$$\frac{w_t \hat{l}_t}{\hat{y}_t} = (1-\alpha)$$

The firms' profit is zero and given by:

$$\hat{y}_t - R_t \hat{k}_t - w_t \hat{l}_t = \hat{y}_t - \alpha \hat{y}_t - (1-\alpha)\, \hat{y}_t = 0 \tag{4.18}$$

4.3.5 Equilibrium

Equilibrium occurs when the households' supply of inputs equals the firms' demand for inputs and when the households' demand for outputs equals the firms' supply of outputs at every point of time. According to the general equilibrium theory (Walrasian general equilibrium), the prices will adjust to match the supply from the households and the demand from the firms. In this regard, the prices will adjust in such a way that capital supply and capital demand are equalised, and that is the definition of capital market clearing. Furthermore, the total amount of production is completely used for consumption and investment purposes.

The market-clearing condition is that $Y_t = C_t + I_t$[47], where Y_t is the total resources in the economy that are used for consumption, C_t, and investment, I_t. However, the capital accumulation equation and Euler equation at the equilibrium are affected by boosting the government spending to the firm's sector. To see that, we start with the capital accumulation equation:

$$K_{t+1} = R_t K_t + w_t L_t - C_t + (1-\delta) K_t$$

We divide it by government spending, G_t, to get PGSU variables. As we defined earlier, the ratio of capital stock over government spending as $\hat{k}_t \equiv K_t / G_t$ and the ratio of labour over government spending as $\hat{l}_t \equiv L_t / G_t$, we now define the new ratio $\hat{c}_t \equiv C_t / G_t$ as a consumption PGSU. So, rewriting the capital accumulation gives us:

$$\frac{K_{t+1}}{G_t} = R_t \hat{k}_t + w_t \hat{l}_t - \hat{c}_t + (1-\delta) \hat{k}_t$$

Now, the right-hand side is only per government spending variables, but the left-hand side is not. Thus, we need to manipulate the left-hand side by multiplying and dividing it by G_{t+1} and simplify:

$$\frac{K_{t+1}}{G_t} \cdot \frac{G_{t+1}}{G_{t+1}} = R_t \hat{k}_t + w_t \hat{l}_t - \hat{c}_t + (1-\delta) \hat{k}_t$$

$$\frac{K_{t+1}}{G_{t+1}} \cdot \frac{G_{t+1}}{G_t} = R_t \hat{k}_t + w_t \hat{l}_t - \hat{c}_t + (1-\delta) \hat{k}_t$$

[47] The government spending, G_t, is not entered in the resource constraint of the economy because it is imported goods from abroad and is not purchased any goods from the domestic market, as we assumed in the government sector. In other words, home-goods are not used in government spending.

Considering that the profit of the firms is zero, equation (4.18), and assuming that the aggregate labour is $L_t = 1$, as Barro (1990) where there is no population growth in the steady-state, the capital accumulation at equilibrium can be rewritten as,

$$\hat{k}_{t+1} \cdot \gamma_{G_t} = \hat{y}_t + (1-\delta)\,\hat{k}_t - \hat{c}_t$$

$$\hat{k}_{t+1} = \frac{1}{\gamma_{G_t}} \left[\hat{y}_t + (1-\delta)\,\hat{k}_t - \hat{c}_t \right] \tag{4.19}$$

where $\hat{y}_t = A\hat{k}_t^{\alpha}$, and $\gamma_{G_t} = G_{t+1}/G_t = g^2$ is the growth rate of government spending.

The capital accumulation at equilibrium implies the law of motion for the stock of capital PGSU. It is a nonlinear function of the last period's capital PGSU and the current period of consumption PGSU.

On the other hand, we multiply both sides of the Euler equation, equation (4.11), by G_{t+1}/G_t and do some algebra. After that, the Euler equation at equilibrium becomes,

$$\frac{\hat{c}_{t+1}}{\hat{c}_t}\,\gamma_{G_t} = \left[\beta\,\left(R_{t+1} + 1 - \delta\right)\right]^{\frac{1}{\sigma}}$$

$$\frac{\hat{c}_{t+1}}{\hat{c}_t} = \frac{1}{\gamma_{G_t}} \left[\beta\left(\alpha A \hat{k}_{t+1}^{\alpha-1} + 1 - \delta\right)\right]^{\frac{1}{\sigma}} \tag{4.20}$$

which describes the optimal consumption PGSU choice of the household between today and the future period.

The transversality condition is:

$$\lim_{t\to\infty} \beta^t\,\lambda_t\,\hat{k}_{t+1} = 0$$

$$\lim_{t\to\infty} \beta^t\,\lambda_t \left(\frac{1}{\gamma_{G_t}} \left[\hat{y}_t + (1-\delta)\,\hat{k}_t - \hat{c}_t \right] \right) = 0$$

These two equations (4.19) and (4.20) plus transversality condition with K_0 given are the main equations that describe the market equilibrium of our economy.

Before moving on to the steady-state analysis, it is worthwhile to take stock of the accounting identities. In the model, there is one export commodity, oil, and imported goods purchased by the government. Thus, the balance of trade, exports and imports, is assumed to be balanced in the model, such that,

$$\text{Trade Balance} = \text{Exported Oil} - \text{Imported Goods} = 0$$

In this case, there is no chance of building up (NFA) credit[48]. As a result, the trade balance equation would be the same as the government budget constraint (recalling that government revenue equals the value of oil exports), where they are all in balance.

Moreover, to compare our model with Barro's model (1990), the tax rate is one of the main determinants in Barro's model. However, our model shows only one determinant of the growth rate, which is the exogenous shift rate of the oil demand. Consequently, our model differs from Barro's model in determining the growth rate, which means it is an exogenous growth model instead of being an endogenous growth model.

4.4 Analysis of the Steady-State

A steady-state of a sequence $\{X_t\}$ is defined as a point X^* such that, if $X_0 = X^*$, then $X_t = X^*$ for all $t \geq 1$. In words, if we start there, then we always stay there. Thus, in this section, we show that a steady-state in capital PGSU, $\hat{k}^*$, and consumption PGSU, $\hat{c}^*$, exist[49].

$$\hat{k}_t = \hat{k}_{t+1} = \hat{k}^*$$
$$\hat{c}_t = \hat{c}_{t+1} = \hat{c}^*$$

If the economy has $\hat{k}_0 = \hat{k}^*$ and $\hat{c}_0 = \hat{c}^*$, then moving forward in the time period will not change the capital stock and consumption. Thus, as in the Solow model, the steady-state can be seen as the long-run equilibrium of the economy where all the variables remain constant forever, which means that $\Delta\hat{k}_{t+1} = 0$, and there are no changes in consumption PGSU, $\Delta\hat{c}_{t+1} = 0$.

[48] Even though this assumption may not be realistic, but it is true that some emerging markets have some restrictions that prevent capital flows and outflows. Thus, we assume in the model that building up (NFA) does not exist for simplicity.

[49] The star * denotes steady-state. The steady-state $(\hat{k}^*, \hat{c}^*)$ is mathematically a fixed point of the dynamic system, equations (4.19) and (4.20) determining our economy.

4.4.1 Steady-States

A steady-state can be fund analytically using the fundamental two equations in the economy (4.19) and (4.20), taking into consideration that at the steady-state $\hat{k}_t = \hat{k}_{t+1} = \hat{k}^*$ and $\hat{c}_t = \hat{c}_{t+1} = \hat{c}^*$. Starting from the Euler equation,

$$\frac{\hat{c}_{t+1}}{\hat{c}_t} = \frac{1}{\gamma_{G_t}} \left[\beta\left(\alpha A \hat{k}_{t+1}^{\alpha-1} + 1 - \delta\right)\right]^{\frac{1}{\sigma}}$$

At the steady-state, this ratio $\hat{c}_{t+1} / \hat{c}_t$ will be equal to one. Then, we obtain:

$$\hat{k}^* = \left[\frac{1}{\alpha A}\left(\frac{\gamma_G^\sigma}{\beta} - 1 + \delta\right)\right]^{\frac{1}{\alpha-1}} \tag{4.21}$$

The capital accumulation equation is,

$$\hat{k}_{t+1} = \frac{1}{\gamma_{G_t}} \left[A\hat{k}_t^\alpha + (1-\delta)\,\hat{k}_t - \hat{c}_t\right]$$

At the steady-state, we have that $\hat{k}_t = \hat{k}_{t+1} = \hat{k}^*$. Therefore,

$$\hat{c}^* = A\hat{k}^{*\alpha} + (1-\delta-\gamma_G)\,\hat{k}^* \tag{4.22}$$

These expressions show the equations for the steady-state values $\hat{c}^*$ and $\hat{k}^*$, where everything in the economy grows at the rate of government spending, which is $\gamma_{G_t} = g^2$. This, in fact, represents an exogenous growth rate. Thus, this is a different conclusion from Barro's conclusion, where the main determinant of the economic growth in our model is the growth rate of oil profits, g^2.

In the steady-state, the level of capital and consumption must be growing if government spending is growing. To see that clearly, suppose that the economy has converged to a steady-state at period (t). This means that the capital PGSU cannot be expected to grow between the period (t) and period (t+1):

$$\hat{k}_{t+1} = \hat{k}_t \quad \Rightarrow \quad \frac{K_{t+1}}{G_{t+1}} = \frac{K_t}{G_t} \quad \Rightarrow \quad \frac{K_{t+1}}{K_t} = \frac{G_{t+1}}{G_t} \quad \Rightarrow \quad \gamma_{K_t} = \gamma_{G_t}$$

This ratio $\gamma_{K_t} = K_{t+1}/K_t$ is the gross growth rate of the capital stock. When we have arrived at the steady-state, this expression says that the growth rate of the capital stock must be equal to the growth rate of government spending, which is G_{t+1}/G_t. Therefore, in the steady-state in which capital PGSU is not growing, it must be the case that capital is growing at the same rate as government spending.

Regarding consumption, this ratio $\gamma_{C_t} = C_{t+1}/C_t$ is the gross growth rate of consumption. It would also be the same where consumption must also grow at the same rate of government spending:

$$\hat{c}_{t+1} = \hat{c}_t \quad \Rightarrow \quad \frac{C_{t+1}}{G_{t+1}} = \frac{C_t}{G_t} \quad \Rightarrow \quad \frac{C_{t+1}}{C_t} = \frac{G_{t+1}}{G_t} \quad \Rightarrow \quad \gamma_{C_t} = \gamma_{G_t}$$

4.4.2 The Effect on the Steady-State

This section analyses the effect on the steady-state when there is a negative shock of oil demand. It is assumed that the shock is a permanent reduction in the growth rate of government spending, γ_{G_t}.

We have the unique equilibrium path, which is the sequence of $\{\hat{c}_t\}_{t=0}^{\infty}$ and $\{\hat{k}_t\}_{t=0}^{\infty}$, which solves the system,

$$\hat{k}_{t+1} = \frac{1}{\gamma_{G_t}} \left[A\,\hat{k}_t^{\alpha} + (1-\delta)\,\hat{k}_t - \hat{c}_t \right]$$

$$\frac{\hat{c}_{t+1}}{\hat{c}_t} = \frac{1}{\gamma_{G_t}} \left[\beta\left(\alpha A \hat{k}_{t+1}^{\alpha-1} + 1 - \delta\right)\right]^{\frac{1}{\sigma}}$$

and the transversality condition $\lim_{t\to\infty} \beta^t \lambda_t \hat{k}_{t+1} = 0$, with K_0 given. Thus, when a negative shock on demand for oil occurs, the steady-state will be affected. To see that, we start with the steady-state equation (4.21):

$$\Delta\hat{c}_{t+1} = 0 \quad \Leftrightarrow \quad \hat{k}^{*} = \left[\frac{1}{\alpha A} \left(\frac{\gamma_G^{\sigma}}{\beta} - 1 + \delta \right)\right]^{\frac{1}{\alpha-1}}$$

From this equation, we can find a direct effect when γ_G changes. To see the change of γ_G at the steady-state, we take the derivative of $\hat{k}^*$ with respect to γ_G and obtain the following:

$$\frac{d\hat{k}^*}{d\gamma_G} = \underbrace{\frac{\sigma\,\gamma_G^{\sigma-1}}{(\alpha-1)\alpha A\beta}}_{(-)} \underbrace{\left(\frac{\gamma_G^{\sigma}}{\alpha A\beta} - \frac{1}{\alpha A} + \frac{\delta}{\alpha A}\right)^{\frac{2-\alpha}{\alpha-1}}}_{(+)} \tag{4.23}$$

Since $\alpha, \beta, \delta \in (0,1)$, $\sigma > 0$, and $\gamma_G \geq 1$[50], then it is clear that the sign of equation (4.23) is negative. Therefore, we conclude that there is a negative relationship between the capital PGSU, $\hat{k}^*$, and γ_G at the steady-state, meaning that if γ_G falls, then the capital PGSU, $\hat{k}^*$, rises.

On the other hand, we have already found the steady-state equation (4.22), which we now point it out as (4.22)' just to make it easier to follow,

$$\Delta\hat{k}_{t+1} = 0 \quad \Leftrightarrow \quad \hat{c}^* = A\hat{k}^{*\alpha} + (1-\delta-\gamma_G)\,\hat{k}^* \tag{4.22$'$}$$

We observe that $\hat{c}^*$does not only depend on γ_G but also depends on $\hat{k}^*$. Thus, we need to consider the total effect, which is not only the *'direct effect (DE)'* of $\hat{c}^*$ but also the *'indirect effect (IE)'* of $\hat{c}^*$ on $\hat{k}^*$. To look at the change of γ_G at the steady-state, we first substitute the expression for $\hat{k}^*$ into $\hat{c}^*$,

$$\hat{c}^* = A\left(\frac{\gamma_G^{\sigma}}{\alpha A\beta} - \frac{1}{\alpha A} + \frac{\delta}{\alpha A}\right)^{\frac{\alpha}{\alpha-1}} + \underbrace{(1-\delta-\gamma_G)}_{-}\left(\frac{\gamma_G^{\sigma}}{\alpha A\beta} - \frac{1}{\alpha A} + \frac{\delta}{\alpha A}\right)^{\frac{1}{\alpha-1}} \tag{4.22$''$}$$

$$\text{where } \hat{k}^* = \left(\frac{\gamma_G^{\sigma}}{\alpha A\beta} - \frac{1}{\alpha A} + \frac{\delta}{\alpha A}\right)^{\frac{1}{\alpha-1}}$$

and then we take the derivative of $\hat{c}^*$ with respect to γ_G:

$$\frac{d\hat{c}^*}{d\gamma_G} = \underbrace{\frac{\sigma\,\gamma_G^{\sigma-1}(1-\delta-\gamma_G)}{(\alpha-1)\alpha A\beta}}_{(+)} \underbrace{\left(\frac{\gamma_G^{\sigma}}{\alpha A\beta} - \frac{1}{\alpha A} + \frac{\delta}{\alpha A}\right)^{\frac{2-\alpha}{\alpha-1}}}_{(+)} + \underbrace{\left(\frac{\gamma_G^{\sigma}}{\alpha A\beta} - \frac{1}{A\alpha} + \frac{\delta}{A\alpha}\right)^{\frac{1}{\alpha-1}}}_{(+)} \underbrace{\left(\frac{\sigma\,\gamma_G^{\sigma-1}}{(\alpha-1)\beta}\right)}_{(-)}$$
$$- \underbrace{\left(\frac{\gamma_G^{\sigma}}{\alpha A\beta} - \frac{1}{A\alpha} + \frac{\delta}{A\alpha}\right)^{\frac{1}{\alpha-1}}}_{(+)} \tag{4.24}$$

---

[50] We set $\gamma_G \geq 1$ because we have assumed that there is growing in oil profits and therefore in government spending.

Looking at equation (4.22)', raising γ_G has three effects on $\hat{c}^*$: there is one *'direct effect (DE)'* at a given value of $\hat{k}^*$, and two *'indirect effects (IE)'*. The derivative of equation (4.22)' with respect to γ_G, holding $\hat{k}^*$ constant, gives us the *'direct effects (DE)'*, which is corresponded to the third expression in equation (4.24),

$$DE = -\hat{k}^* = -\left(\frac{\gamma_G^\sigma}{\alpha A\beta} - \frac{1}{\alpha A} + \frac{\delta}{\alpha A}\right)^{\frac{1}{\alpha-1}}$$

Since we have shown in equation (4.23) that $\left(\frac{\gamma_G^\sigma}{\alpha A\beta} - \frac{1}{\alpha A} + \frac{\delta}{\alpha A}\right)^{\frac{1}{\alpha-1}}$ is positive, then the *'direct effect (DE)'* must be here negative, $DE < 0$.

Regarding the two *'indirect effects (IE)'*, they operate through the fall in $\hat{k}^*$. Looking at equation (4.22)'', we can see that they work in the opposite direction and are also corresponded to the equation (4.24). To see that, $A\hat{k}^{*\alpha}$ in equation (4.22)' is corresponded to the second expression in equation (4.24), $\left(\frac{\gamma_G^\sigma}{\alpha A\beta} - \frac{1}{\alpha A} + \frac{\delta}{\alpha A}\right)^{\frac{1}{\alpha-1}}\left(\frac{\sigma\gamma_G^{\sigma-1}}{(\alpha-1)\beta}\right)$, which is negative because of $(\alpha-1)$. While $(1-\delta-\gamma_G)\hat{k}^*$ in equation (4.22)' is corresponded to the first expression in equation (4.24), $\left(\frac{\sigma\gamma_G^{\sigma-1}(1-\delta-\gamma_G)}{(\alpha-1)\alpha A\beta}\right)\left(\frac{\gamma_G^\sigma}{\alpha A\beta} - \frac{1}{\alpha A} + \frac{\delta}{\alpha A}\right)^{\frac{2-\alpha}{\alpha-1}}$, which is positive because $(1-\delta-\gamma_G)$ and $(\alpha-1)$ are both negative. In summary, the two *'indirect effects (IE)'* conflict with each other, so the sign of the *'net indirect effect (NIE)'* is unclear yet.

Let us now consider only the ambiguous sign of the *'net indirect effect (NIE)'* further. In this case, we can write the *'net indirect effect (NIE)'* and simplify it as follows:

$$NIE = \frac{\sigma\gamma_G^{\sigma-1}(1-\delta-\gamma_G)}{(\alpha-1)\alpha A\beta}\left(\frac{\gamma_G^\sigma}{\alpha A\beta} - \frac{1}{\alpha A} + \frac{\delta}{\alpha A}\right)^{\frac{2-\alpha}{\alpha-1}} + \left(\frac{\gamma_G^\sigma}{\alpha A\beta} - \frac{1}{A\alpha} + \frac{\delta}{A\alpha}\right)^{\frac{1}{\alpha-1}}\left(\frac{\sigma\gamma_G^{\sigma-1}}{(\alpha-1)\beta}\right)$$

$$NIE = \underbrace{\underbrace{\left[\underbrace{\left(\frac{\gamma_G^\sigma}{\alpha A\beta} - \frac{1}{A\alpha} + \frac{\delta}{A\alpha}\right)^{\frac{1}{\alpha-1}}}_{(+)}\underbrace{\left(\frac{\sigma\gamma_G^{\sigma-1}}{(\alpha-1)\beta}\right)}_{(-)}\right]}_{(-)}\left[\underbrace{\underbrace{\frac{(1-\delta-\gamma_G)}{\alpha A}\left(\frac{\gamma_G^\sigma}{\alpha A\beta} - \frac{1}{\alpha A} + \frac{\delta}{\alpha A}\right)^{-1}}_{(-)\text{ and less than 1 if }\sigma\text{ is sufficiently large}} + 1}_{(+)}\right]}_{(-)}$$

The first part of the *'net indirect effect (NIE)'* is negative because of $(\alpha - 1)$. For the second part of *(NIE)*, since $(1 - \delta - \gamma_G)$ is negative because of $\gamma_G \geq 1$, the $\left[\frac{(1-\delta-\gamma_G)}{\alpha A}\right]$ will be negative too. While $\left[\left(\frac{1}{\alpha A}\right)\left(\frac{\gamma_G^\sigma}{\beta} - 1 + \delta\right)\right]$ is positive as shown in equation (4.23), then $\left[\left(\frac{1}{\alpha A}\right)\left(\frac{\gamma_G^\sigma}{\beta} - 1 + \delta\right)\right]^{-1}$ will be positive. Thus, all this part $\left[\frac{(1-\delta-\gamma_G)}{\alpha A}\right]\left[\left(\frac{1}{\alpha A}\right)\left(\frac{\gamma_G^\sigma}{\beta} - 1 + \delta\right)\right]^{-1}$ will be negative. However, to determine whether it will be greater or less than one, it indeed depends on the coefficient of relative risk aversion, σ. Thus, if σ is sufficiently large, then this whole part $\left[\frac{(1-\delta-\gamma_G)}{\alpha A}\right]\left[\left(\frac{1}{\alpha A}\right)\left(\frac{\gamma_G^\sigma}{\beta} - 1 + \delta\right)\right]^{-1}$ is going to be negative and less than one. In this case, the *'net indirect effect (NIE)'* will end up with a negative sign. As a result, this proves that (provided $\gamma_G \geq 1$), the *'net indirect effect (NIE)'* of raising γ_G on $\hat{c}^*$ becomes unambiguously negative (recalling that raising γ_G causes falling $\hat{k}^*$). Given that the *'direct effect (DE)'* of raising γ_G on $\hat{c}^*$ is negative, it then follows that the overall effect is negative too.

We have shown in the analysis of the steady-state section that if the growth rate of government spending, γ_G, declines, then the growth rate of consumption, capital, and output will be declined too. Thus, if there is a permanent reduction in γ_G, the model predicts that the key variables in the new steady-state will grow at the new lower level of γ_G. Looking at the consumption as one of the primary endogenous variables, we have proved that $d\hat{c}^*/d\gamma_G$ is negative. The negative sign of $d\hat{c}^*/d\gamma_G$ means that although the consumption will grow at a lower rate, the consumption PGSU, $\hat{c}_t$, will be higher (recalling that consumption PGSU is $\hat{c}_t \equiv C_t/G_t$).

In reality, we are indeed indifferent about the PGSU variables. Thus, we have just used these variables and built our model to assist in analysing the model. However, what the policymakers and others may be interested in are the variables in terms of level. Therefore, we use the log scale, where a variable growing at a constant rate appears linear. The below diagram displays the exogenous change, government spending, G_t, in the solid line, which will initially grow before time $\tilde{t}$. Time $\tilde{t}$ refers to the time when the growth rate of government spending declined. In other words, it presents a time when new information and the implementation of a new regime occurs. Before time $\tilde{t}$, $(t > \tilde{t})$, it is assumed that the agents do not expect and know about shocks, but they, on the other hand, predict to carry on. However, when new information about a permanent reduction in γ_G comes along, the agents predict and know that a change in government spending is occurred at and after time $\tilde{t}$, $(t \leq \tilde{t})$. Then, they correctly anticipate that the growth rate will be lower forever. Therefore, the agents in our model are considered to be not perfect foresight but almost perfect foresight because their correct anticipation starts from the time $\tilde{t}$ and after, where they expect before the time $\tilde{t}$ that government spending is growing.

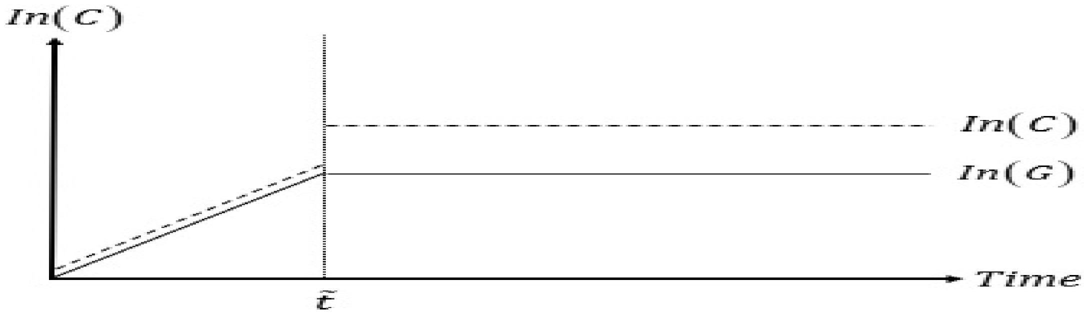


Diagram 4: The path of growing consumption when there is a permanent reduction in the growth rate of government spending

On the other hand, we know from the steady-state that the level of consumption grows at the same rate as government spending. Thus, since the government spending grows at a slower rate, if $\hat{c}^*$ did not change, the growth rate of consumption would just follow the solid line, which means there is no upward or downward jump. However, ignoring the transitional dynamics, the new steady-state at and after time $\tilde{t}$ will be like the dashed line, where it shows an upward step relative to the old steady-state. The reason is that $d\hat{c}^*/d\gamma_G$ is negative, which tells us that the new steady-state must be higher. High consumption PGSU implies that the consumption grows faster than the government spending along the transition path.

Two factors can explain the result in which a fall in γ_G raises $\hat{c}^*$. These two reasons can be seen from our previous discussion regarding the direct effect and the net indirect effect. The first reason comes from the direct effect, which means that a fall in γ_G reduces the investment needed, *i.e.* the investment needed to provide the new government spending units with the same amount of capital PGSU as the existing units of government spending. This can be seen in equation (4.22)', *i.e.* the term $\{-(\gamma_G \hat{k}^*)\}$. Thus, it leaves more output available for consumption in the steady-state. The second reason comes from the net indirect effect. We have found theoretically that if σ is sufficiently large (small), the net indirect effect is negative (positive). Thus, if σ is sufficiently large (small), the fall in γ_G raises (reduce) the steady-state value of capital PGSU. In turn, this increases (decreases) $\hat{c}^*$ even at a given value of γ_G, because the economy is on the upward-sloping (downward-sloping) part of the 'hill', which gives $\hat{c}^*$ as a function of $\hat{k}^*$ [51]. The reason for raising (reducing) $\hat{k}^*$ is intuitively due to that a fall in γ_G works in a similar way to a productivity improvement that raises (reduces) the marginal product of capital.

[51] The reason why σ matters for the sign of the net indirect effect is that it can alter the slope of the relationship between $\hat{c}^*$ and $\hat{k}^*$, *i.e.* the hill-shaped relationship.

Now, let us explain the situation in which the sigma is sufficiently large. Generally, in the case of raising γ_G, if agents save and invest more in period t (therefore sacrificing some consumption in period t), the extra output which this yields in period t+1 is 'diluted' because it has to be divided between a larger number of units of government spending. However, we are in the opposite situation, *i.e.* the case of lowering γ_G. In this case, a lower γ_G reduces this 'dilution' effect, meaning the reward to a sacrifice of consumption PGSU in period t is higher in terms of the gain which it enables in consumption PGSU in period t+1. In other words, the incentive to save and invest is increased, which can be seen in the Euler equations for consumption, where lower γ_G raises $\hat{c}_{t+1}/\hat{c}_t$.

Again, in reality, the slowdown of oil profits growth rate and then the growth rate of government spending are undesirable for Saudi's economy. However, although Saudi's economy is worst off due to the slow growth rate of oil profits, there is some kind of partially offsetting effect. In other words, although consumption is growing more slowly, it is indeed increased relative to its previous trajectory. As a result, the model rejects the thought that everything will remain in the same proportion to government spending and therefore the whole economy will just grow at a slow rate, but where the ratios to government spending are no different from what they were before the slowdown. The model, however, shows that there is some partially offsetting shift in consumption which may not be obvious.

Let us now link the analysis in this chapter and our case study in chapter two. By looking at the historical data presented in chapter two for Saudi Arabia, it seems that they are consistent with our model in this chapter. More precisely, our model demonstrates that the growth rate of government spending is the growth rate of oil profits. Thus, in the case of a negative shock on oil demand, government revenues would change, affecting government spending and, eventually, the economic growth rate. Both our results and the historical data confirm that oil revenues are closely related to Saudi's GDP. From the historical data, we can see clearly that the recession period of 1982-2002 witnessed a decline in GDP at current prices. This decline in GDP was associated with the oil crisis at that time, particularly the period between 1982-1989. During this period, GDP and oil revenues were moving in the same relative direction, where the negative impact on GDP was due to the sharp decline in oil revenues. Similarly, the Iraq and Kuwait war and the Asian financial crisis of the 1990s significantly affected Saudi oil revenues, which declined by half. This drop indeed had a significant impact on GDP, which fell by 11.4%. In addition, the global financial crisis of 2008-2009 caused the price of oil to decline by 35.6% and oil revenues by 55.8%, which in turn led to a fall in GDP of 17.45%. Consequently, our results in this chapter correspond to the historical data for Saudi Arabia, where oil revenues have a strong relationship with economic growth.

4.5 Comparison of the Ramsey Model and Our Model

This section studies the discrete-time Ramsey model in decreasing technological progress and compares it with our model if the oil profit growth (*i.e.* government spending growth rate) declines. We found that the two models seem similar[52]. Therefore, the comparison of the Ramsey model and our model can be highlighted as follows:

(i) In the Ramsey model, it is used a general Cobb-Douglas production function, which contains the total factor productivity, A_t, capital stock, K_t, and population or labour, L_t. However, in our model, we apply Barro's (1990) production function by assuming the constant technology level and introducing government spending as positive externalities to enhance the firms' production function. Both models display the same constant returns to scale (CRS).

(ii) In the Ramsey model, technological progress is not constant, but it grows over time. In the model, we assume that there is no population growth; therefore, the capital and consumption grow in the long run at the same rate of technological progress, γ_{A_t}. However, we follow Barro (1990) in our model by also assuming that there is no population growth in the steady-state because we normalise the aggregate labour (*i.e.* working population) to one $(L_t = 1)$. Moreover, we introduce the oil sector as a monopolistic and exogenous sector in our model, which essentially feeds the government sector. Thus, we have a government spending growth, which is basically led by a growing demand for oil. The government spending in our model grows at the rate of g^2 every period, $(\gamma_{G_t} \equiv g^2)$, where g^2 is the growth rate of oil profits.

[52] The comparison between the main equations in both models is in Appendix A.1.

4.6 Analysis of the Stability of Our Economy

The stability of the solution of a system relies on the characteristic equation. The characteristic equation of a square matrix is used to find the eigenvalues of that matrix, and the eigenvalues are important to check on stability. Thus, the characteristic equation takes the form,

$$\lambda^2 - Tr\,\lambda + Det = 0$$

where λ is called the eigenvalue of a matrix, the Tr is the trace of the matrix, which is the sum of a matrix diagonal elements, and the Det refers to the determinant of the matrix, which is the difference between the product of the two diagonal elements.

In discrete-time, an eigenvalue of a matrix is stable if the absolute value is less than one, *i.e.* lies inside the unit circle, such that $|\lambda| < 1$, and it is unstable if the absolute value is greater than one, *i.e.* lies outside the unit circle, such that $|\lambda| > 1$.

$$\begin{aligned} &If \quad |\lambda| < 1 \quad \Rightarrow \quad Stable \\ &If \quad |\lambda| > 1 \quad \Rightarrow \quad Unstable \end{aligned}$$

Thus, the characteristic equation has to be satisfied, and an eigenvalue to be stable should be lying inside the unit circle.

To analyse the stability in our economy, we begin to construct the linear approximation to the system around steady-state. In the steady-state, PGSU variables will stay constant $\hat{c}_t = \hat{c}^*, \hat{k}_t = \hat{k}^*, \hat{y}_t = \hat{y}^* \; \forall t$.

By taking these assumptions, we have found the steady-state in equations (4.21) and (4.22), respectively:

$$\hat{k}^* = \left[\frac{1}{\alpha A} \left(\frac{\gamma_G^{\sigma}}{\beta} - 1 + \delta \right) \right]^{\frac{1}{\alpha - 1}}$$

$$\hat{c}^* = A\hat{k}^{*\alpha} + (1 - \delta - \gamma_G)\, \hat{k}^*$$

We start with the capital accumulation equation (4.19) and find the first-order Taylor approximation around the steady-state:

$$\hat{k}_{t+1} = \frac{1}{\gamma_{G_t}} A\hat{k}_t^{\alpha} + \frac{1}{\gamma_G} (1-\delta)\, \hat{k}_t - \frac{1}{\gamma_G} \hat{c}_t$$

$$\hat{k}_{t+1} - \frac{1}{\gamma_{G_t}} A\hat{k}_t^{\alpha} - \frac{1}{\gamma_G} (1-\delta)\, \hat{k}_t + \frac{1}{\gamma_G} \hat{c}_t = 0$$

The first-order Taylor approximation of capital accumulation:

$$(\hat{k}_{t+1} - \hat{k}^*) - \frac{1}{\gamma_G} \left[\alpha A\hat{k}^{*\alpha\text{-}1} + 1 - \delta\right] (\hat{k}_t - \hat{k}^*) + \frac{1}{\gamma_G} (\hat{c}_t - \hat{c}^*) = 0$$

Then, we use the steady-state, which is the expression for $\hat{k}^*$ in equation (4.21). We substitute it into the first-order Taylor approximation of capital accumulation and rearrange it. Thus, we obtain:

$$(\hat{k}_{t+1} - \hat{k}^*) = \frac{\gamma_G^{\sigma-1}}{\beta} (\hat{k}_t - \hat{k}^*) - \frac{1}{\gamma_G} (\hat{c}_t - \hat{c}^*) \tag{4.25}$$

On the other hand, considering the Euler equation (4.20), we find the first-order Taylor approximation around the steady-state:

$$\hat{c}_{t+1} = \frac{\hat{c}_t}{\gamma_{G_t}} \left[\beta\left(\alpha A\hat{k}_{t+1}^{\alpha-1} + 1 - \delta\right)\right]^{\frac{1}{\sigma}}$$

$$(\hat{c}_{t+1} - \hat{c}^*) = \frac{\hat{c}^*}{\sigma\, \gamma_G} \left[\beta\left(\alpha A\hat{k}^{*\alpha\text{-}1} + 1 - \delta\right)\right]^{\frac{1-\sigma}{\sigma}} \left[(\alpha-1)\, \beta A\alpha\hat{k}^{*\alpha-2}\right] (\hat{k}_{t+1} - \hat{k}^*)$$
$$+ \frac{1}{\gamma_G} \left[\beta\left(\alpha A\hat{k}^{*\alpha\text{-}1} + 1 - \delta\right)\right]^{\frac{1}{\sigma}} (\hat{c}_t - \hat{c}^*)$$

$$(\hat{c}_{t+1} - \hat{c}^*) = \frac{\hat{c}^*}{\sigma\gamma_G} \Phi^{*\frac{1-\sigma}{\sigma}} \left[(\alpha-1)\, \beta A\alpha\hat{k}^{*\alpha-2}\right] (\hat{k}_{t+1} - \hat{k}^*) + \frac{1}{\gamma_G} \Phi^{*\frac{1}{\sigma}} (\hat{c}_t - \hat{c}^*)$$

Where Φ^* denotes the bracketed expression in (4.20), $\Phi^* = \beta\left(\alpha A\hat{k}^{*\alpha-1} + 1 - \delta\right)$. Particularising (4.20) at the steady-state, we obtain $\Phi^* = \gamma_G^\sigma$. Therefore,

$$(\hat{c}_{t+1} - \hat{c}^*) = \frac{\hat{c}^*}{\sigma\gamma_G^\sigma}\left[(\alpha-1)\,\beta\alpha A\hat{k}^{*\alpha-2}\right](\hat{k}_{t+1} - \hat{k}^*) + (\hat{c}_t - \hat{c}^*)$$

By substituting the approximation of capital accumulation equation (4.25), we obtain:

$$(\hat{c}_{t+1} - \hat{c}^*) = \frac{\hat{c}^*}{\sigma\gamma_G^\sigma}\left[(\alpha-1)\,\beta\alpha A\hat{k}^{*\alpha-2}\right]\left[\frac{\gamma_G^{\sigma-1}}{\beta}(\hat{k}_t - \hat{k}^*) - \frac{1}{\gamma_G}(\hat{c}_t - \hat{c}^*)\right] + (\hat{c}_t - \hat{c}^*)$$

$$(\hat{c}_{t+1} - \hat{c}^*) = -\frac{\chi}{\beta}(\hat{k}_t - \hat{k}^*) + \left(\frac{\chi}{\gamma_G^\sigma} + 1\right)(\hat{c}_t - \hat{c}^*) \qquad (4.26)$$

$$\textit{Where}\ \chi = -\frac{\hat{c}^*}{\sigma\gamma_G}\left[(\alpha-1)\,\beta\alpha A\hat{k}^{*\alpha-2}\right] > 0$$

The capital accumulation equation and the Euler equation describe the low of motion in this system. Thus, when we linearise them around the steady-state, as shown in (4.25) and (4.26), we realise that there will be two eigenvalues in this system. The linearised capital accumulation equation (4.25) and Euler equation (4.26) can be now expressed in matrix form as:

$$\begin{pmatrix} \hat{k}_{t+1} - \hat{k}^* \\ \\ \hat{c}_{t+1} - \hat{c}^* \end{pmatrix} = \underbrace{\begin{pmatrix} \dfrac{\gamma_G^{\sigma-1}}{\beta} & -\dfrac{1}{\gamma_G} \\ \\ -\dfrac{\chi}{\beta} & \left(\dfrac{\chi}{\gamma_G^\sigma} + 1\right) \end{pmatrix}}_{\textit{matrix } A} \begin{pmatrix} \hat{k}_t - \hat{k}^* \\ \\ \hat{c}_t - \hat{c}^* \end{pmatrix}$$

Since χ is positive, $\gamma_G \geq 1$, and $\beta \in (0,1)$, then the above coefficient matrix A has trace,

$$Tr = \underbrace{\frac{\gamma_G^{\sigma-1}}{\beta}}_{\substack{\text{This expression is} \\ \text{positive and greater than 1} \\ \text{if } \sigma \text{ is sufficiently large}}} + \underbrace{\frac{\chi}{\gamma_G^\sigma}}_{\text{This expression is positive}} + 1 > 2$$

and the determinant (Det) of the matrix A is

$$Det = \left(\frac{\gamma_G^{\sigma-1}}{\beta}\right)\left(\frac{\chi}{\gamma_G^\sigma} + 1\right) - \left(-\frac{1}{\gamma_G}\right)\left(-\frac{\chi}{\beta}\right) = \underbrace{\frac{\gamma_G^{\sigma-1}}{\beta}}_{\substack{\text{This expression is} \\ \text{positive and greater than 1} \\ \text{if } \sigma \text{ is sufficiently large}}} > 1$$

To check for the existence of a unique solution of a linear perfect-foresight model, Blanchard and Kahn (1980) showed three different propositions. These propositions state that (*i*) if the number of eigenvalues of a matrix outside the unit circle is equal to the number of non-predetermined variables, then a unique non-divergent solution exists. (*ii*) if the number of eigenvalues of a matrix outside the unit circle is greater than the number of non-predetermined variables, then a non-divergent solution to the system does not exist. (*iii*) if the number of eigenvalues of a matrix outside the unit circle is less than the number of non-predetermined variables, then an infinite number of non-divergent solutions exist.

The dynamic system has two different variables in our model: a predetermined variable and a non-predetermined variable. The predetermined variable is the capital PGSU, $\hat{k}_t$, because we know from the definition of capital PGSU that $\hat{k}_t \equiv \frac{K_t}{G_t} = \frac{K_t}{\pi_t}$, which implies that all variables in $\hat{k}_t$ cannot jump. On the other hand, the non-predetermined variable is the consumption PGSU, $\hat{c}_t$ because the definition of consumption PGSU shows that $\hat{c}_t \equiv \frac{C_t}{G_t} = \frac{C_t}{\pi_t}$, which contains only one jumper variable that is consumption. Thus, although G_t, which is equal to π_t, cannot jump, $\hat{c}_t$ becomes a non-predetermined variable.

The eigenvalues of the matrix are the solutions to the characteristic equation,

$$\lambda^2 - Tr\,\lambda + Det = 0$$

$$\lambda^2 - \left(\frac{\gamma_G^{\sigma-1}}{\beta} + \frac{\chi}{\gamma_G^{\sigma}} + 1 \right) \lambda + \frac{\gamma_G^{\sigma-1}}{\beta} = 0$$

The solution to this quadratic equation for λ is the two eigenvalues. We know that (*i*) the constant term equals the determinant (Det) of the matrix A (product of eigenvalues, $\lambda_1 . \lambda_2$) and (*ii*) the coefficient on λ equals minus the trace (Tr) of matrix A (the sum of eigenvalues, $\lambda_1 + \lambda_2$).

Rankin (2007)[53] provided an exposition of a test for the number of stable eigenvalues of the dynamical system to find saddle-point stability. He explained that for given particular restrictions on the signs and ranges of possible values of the model's coefficients, such tests might help to determine whether saddle-point stability holds. However, in discrete-time systems, a method of transforming the eigenvalues is applied because a stable eigenvalue is the one that lies inside the unit circle, unlike in continuous-time where a stable eigenvalue has a negative real part. This method goes beyond most of the mathematical economic textbooks, where they usually only provide conditions for all eigenvalues of a system to be stable.

[53] Neil Rankin, April 2007 (based on his handwritten note by David Currie from September 1982).

Now, we apply the test, explained by Rankin's (2007), to our characteristic equation to have 0, 1, or 2 stable eigenvalues. This test will equivalently produce conditions for 0, 1, or 2 of the original eigenvalues λ to lie inside the unit circle. The characteristic equation of a system takes the form,

$$a\lambda^2 + b\lambda + c = 0$$

where λ is the eigenvalue and a, b, c are the coefficients for the characteristic equation.

The below table shows the necessary and sufficient conditions for 0, 1, or 2 of the eigenvalues λ to lie inside the unit circle.

0	**1**	**2**
$\frac{a+b+c}{a-b+c} > 0$	$\frac{a+b+c}{a-b+c} < 0$	$\frac{a+b+c}{a-b+c} > 0$
$\frac{c-a}{a-b+c} > 0$		$\frac{c-a}{a-b+c} < 0$

The first row in the table presents how many eigenvalues that lie inside the unit circle. The second and third rows show the necessary and sufficient conditions from the characteristic equation that must be consistent with 0, 1, or 2 eigenvalue numbers.

Returning to our model, since we have one predetermined state variable in the model, $\hat{k}_t$, it should follow that we need one stable eigenvalue for saddle-point stability. Therefore, looking at the above table and converting the table's coefficients (a,b,c) to our coefficients from our characteristic equation[54], the necessary and sufficient condition is the one that is in the middle of the above table, such that,

$$\frac{a+b+c}{a-b+c} < 0 \Rightarrow \frac{1-\frac{\gamma_G^{\sigma-1}}{\beta}-\frac{\chi}{\gamma_G^{\sigma}}-1+\frac{\gamma_G^{\sigma-1}}{\beta}}{1+\frac{\gamma_G^{\sigma-1}}{\beta}+\frac{\chi}{\gamma_G^{\sigma}}+1+\frac{\gamma_G^{\sigma-1}}{\beta}} < 0 \Rightarrow -\frac{\left(\frac{\chi}{\gamma_G^{\sigma}}\right)}{2\left(\frac{\gamma_G^{\sigma-1}}{\beta}+\frac{\chi}{2\gamma_G^{\sigma}}+1\right)} < 0$$

In our model, this condition holds with no additional assumptions. Therefore, we can conclude that exactly one eigenvalue lies inside the unit circle. As a result, the steady-state $(\hat{k}^*, \hat{c}^*)$ is a saddle-point.

[54] By converting the table's coefficients (a,b,c) to our coefficients from our characteristic equation, we got that $a = 1$, $b = -\left(\frac{\gamma_G^{\sigma-1}}{\beta}+\frac{\chi}{\gamma_G^{\sigma-1}}+1\right)$, $c = \frac{\gamma_G^{\sigma-1}}{\beta}$.

4.7 The Transitional Dynamics in Our Economy

In the transitional dynamics, we use a phase diagram to show the characteristics of a dynamic system. In our case, the phase diagram contains two variables: the variable on the vertical axis is the consumption PGSU, $\hat{c}_t$, and the variable on the horizontal axis is the capital PGSU, $\hat{k}_t$. We analyse in this section the transitional dynamics before and after the negative shock for oil demand to occur.

4.7.1 The Transitional Dynamics Before the Negative Shock for Oil Demand

Before the negative shock of oil demand, we assume that our economy is at its steady-state level (E) in Diagram 6. In this diagram, the arrows indicate the dynamic behaviour of $\hat{c}_t$ and $\hat{k}_t$. The arrows change as the place of economy change in four different possible regions.

To analyse the dynamics of the model through a phase diagram, we first rewrite the key two equations of our economy in terms of $(\hat{k}_t, \hat{c}_t)$, starting with the capital accumulation equation,

$$\hat{k}_{t+1} = \frac{1}{\gamma_{G_t}} \left[A\hat{k}_t^{\alpha} + (1-\delta)\,\hat{k}_t - \hat{c}_t \right]$$

it follows that,

$$\Delta\hat{k}_{t+1} = \frac{1}{\gamma_{G_t}} \left[A\hat{k}_t^{\alpha} + (1-\delta)\,\hat{k}_t - \hat{c}_t \right] - \hat{k}_t = h\left(\hat{k}_t, \hat{c}_t\right) \tag{4.27}$$

and then the Euler equation,

$$\hat{c}_{t+1} = \frac{\hat{c}_t}{\gamma_{G_t}} \left[\beta\left(\alpha A \hat{k}_{t+1}^{\alpha-1} + 1 - \delta \right) \right]^{\frac{1}{\sigma}}$$

substituting $\hat{k}_{t+1}$ into the Euler equation, it follows that,

$$\Delta\hat{c}_{t+1} = \frac{\hat{c}_t}{\gamma_{G_t}} \left[\beta\left(\alpha A \left[\frac{1}{\gamma_{G_t}} \left[A\hat{k}_t^{\alpha} + (1-\delta)\,\hat{k}_t - \hat{c}_t \right] \right]^{\alpha-1} + 1 - \delta \right) \right]^{\frac{1}{\sigma}} - \hat{c}_t = g\left(\hat{k}_t, \hat{c}_t\right) \tag{4.28}$$

The phase diagram is built based on two curves relating $\hat{k}_t$ to $\hat{c}_t$. Each curve of them coincides with one of the two zero change cases: $\Delta\hat{k}_{t+1} = \hat{k}_{t+1} - \hat{k}_t = 0$ (capital accumulation), and $\Delta\hat{c}_{t+1} = \hat{c}_{t+1} - \hat{c}_t = 0$ (Euler equation).

First, when $\Delta\hat{k}_{t+1} = \hat{k}_{t+1} - \hat{k}_t = 0$, the equation (4.27) becomes,

$$\hat{c}_t = A\hat{k}_t^{\alpha} + (1-\delta-\gamma_{G_t})\,\hat{k}_t \tag{4.29}$$

We can rewrite equation (4.29) as:

$$\hat{c}_t = A\hat{k}_t^{\alpha} - (\gamma_{G_t}+\delta-1)\,\hat{k}_t$$

We can also draw $\hat{c}_t$ as a function of $\hat{k}_t$ (treating γ_G as given), as follows:

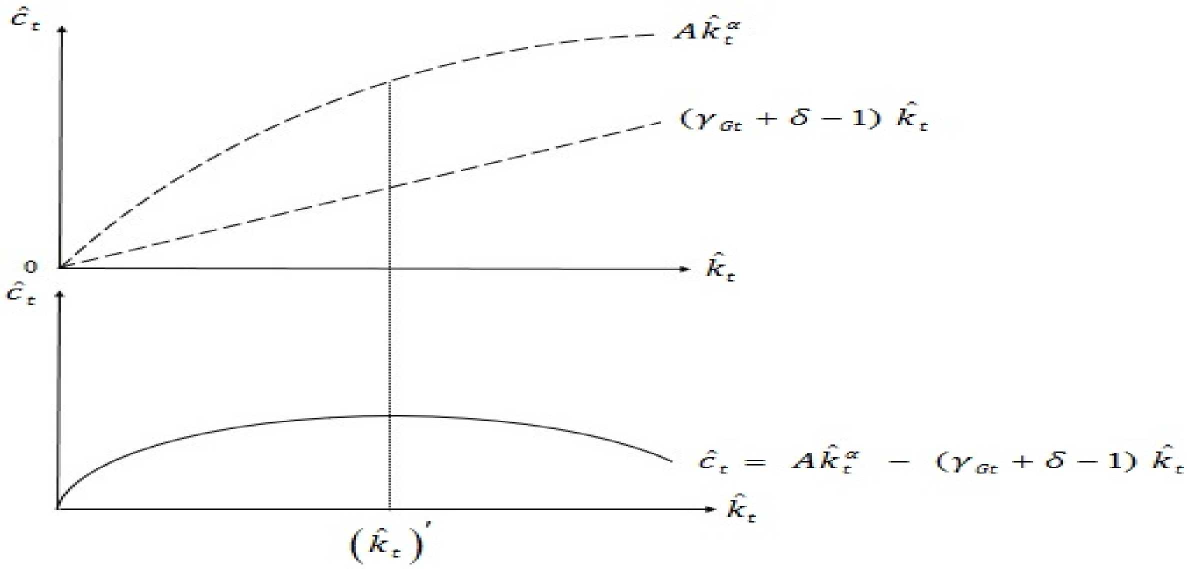


Diagram 5: Drawing equation $\hat{c}_t$ as a function of $\hat{k}_t$

The second diagram shows that $\hat{c}_t$ as a function of $\hat{k}_t$ is a 'hill shaped', where $(\hat{k}_t)'$ is the value of $\hat{k}_t$ which maximises $\hat{c}_t$, such that,

$$\frac{d\hat{c}_t}{d\hat{k}_t} = 0 \quad \Rightarrow \quad \alpha A\hat{k}_t^{\alpha-1} - (\gamma_{G_t}+\delta-1) = 0$$

For low values of $\hat{k}_t$, $d\hat{c}_t/d\hat{k}_t > 0$, while for high values of $\hat{k}_t$, $d\hat{c}_t/d\hat{k}_t < 0$. Therefore, equation (4.29) gives us a concave function (bell curve), as shown in the above diagram.

To track the directions of motion, we look at the $\Delta\hat{k}_{t+1} = \hat{k}_{t+1} - \hat{k}_t = 0$ curve. *Below* the $\Delta\hat{k}_{t+1} = 0$ curve described by (4.29), the consumption PGSU is *'lower'* at any point below the curve than it is on the curve, so that $\hat{c}_t < A\hat{k}_t^{\alpha} + (1-\delta-\gamma_{Gt})\hat{k}_t$, which, taken to the capital accumulation equation, it means: $\hat{k}_{t+1} > \hat{k}_t$ and the arrows point to *'right'*, showing the expected direction of capital PGSU in that area.

Above the $\Delta\hat{k}_{t+1} = 0$ curve, the consumption PGSU is *'higher'* at any point above the curve than it is on the curve, so that $\hat{c}_t > A\hat{k}_t^{\alpha} + (1 - \delta - \gamma_{G_t})\hat{k}_t$, which implies $\hat{k}_{t+1} < \hat{k}_t$ and the arrows indicate to *'left'*.

On the other hand, we set $\Delta\hat{c}_{t+1} = \hat{c}_{t+1} - \hat{c}_t = 0$ in equation (4.28) to express $\hat{c}_t$ as a function of $\hat{k}_t$ as,

$$\hat{c}_t = A\hat{k}_t^{\alpha} + (1-\delta)\,\hat{k}_t - \gamma_{Gt}\left[\frac{1}{\alpha A}\left(\frac{\gamma_{G_t}^{\sigma}}{\beta} - 1 + \delta\right)\right]^{\frac{1}{\alpha-1}} \tag{4.30}$$

We can further rewrite equation (4.30) as,

$$\alpha A\left[\frac{A\hat{k}_t^{\alpha}}{\gamma_{G_t}} + \frac{(1-\delta)\,\hat{k}_t}{\gamma_{G_t}} - \frac{\hat{c}_t}{\gamma_{G_t}}\right]^{\alpha-1} = \frac{\gamma_{G_t}^{\sigma}}{\beta} - 1 + \delta \qquad (4.30)'$$

This equation shows an explicit relationship between $\hat{k}_t$ and $\hat{c}_t$. If $\gamma_{G_t}^{\sigma} \geq 1$, then the right-hand side of the equation (4.30)' is greater than zero, $\gamma_{G_t}^{\sigma}/\beta - 1 + \delta > 0$. This is consistent with the fact that the marginal product of capital has to be positive for any positive capital stock.

The slope of the curve, of equation (4.30), is

$$\frac{d\hat{c}_t}{d\hat{k}_t} = \alpha A\hat{k}_t^{\alpha-1} + 1 - \delta \tag{4.31}$$

The slope is positive because $\alpha A\hat{k}_t^{\alpha-1} + 1 - \delta > 0$ for all $\hat{k}_t$. Therefore, the slope shows a positive relationship in the $(\hat{c}_t, \hat{k}_t)$ space.

This line (the slope) has a negative intercept with the vertical axis at $\hat{k}_t = 0$, since at that point we would have,

$$\alpha A\left[-\frac{\hat{c}_t}{\gamma_{G_t}}\right]^{\alpha-1} = \left(\frac{\gamma_{G_t}^{\sigma}}{\beta} - 1 + \delta\right) > 0 \quad ,$$

and the marginal product function is only defined over the positive real line.

As shown in (4.31), the curve has a positive slope, and along it, $\hat{c}_t \to \infty$ as $\hat{k}_t \to \infty$. Therefore, it will cross the horizontal axis one time.

From equation (4.28), it follows that $\Delta\hat{c}_{t+1} < 0$, the right of the $\Delta\hat{c}_{t+1} = 0$ curve implies that

$$\underbrace{\alpha A\left[\frac{A\hat{k}_t^{\alpha}}{\gamma_{G_t}} + \frac{(1-\delta)\,\hat{k}_t}{\gamma_{G_t}} - \frac{\hat{c}_t}{\gamma_{G_t}}\right]^{\alpha-1}}_{\text{Marginal Product}} < \frac{\gamma_{G_t}^{\sigma}}{\beta} - 1 + \delta$$

The capital PGSU, $\hat{k}_t$, is *'high'*, this implies that the marginal product will be *'small'*, and the consumption PGSU will be expected to *'decline'*, $\hat{c}_{t+1} < \hat{c}_t$. Thus, to the right of $\Delta\hat{c}_{t+1} = 0$ curve, the arrows indicate to *'down'*, displaying the expected direction of consumption PGSU in that region.

In the same way, from equation (4.28), it follows that $\Delta\hat{c}_{t+1} > 0$, the left of the $\Delta\hat{c}_{t+1} = 0$ curve implies that

$$\underbrace{\alpha A\left[\frac{A\hat{k}_t^{\alpha}}{\gamma_{G_t}} + \frac{(1-\delta)\,\hat{k}_t}{\gamma_{G_t}} - \frac{\hat{c}_t}{\gamma_{G_t}}\right]^{\alpha-1}}_{\text{Marginal Product}} > \frac{\gamma_{G_t}^{\sigma}}{\beta} - 1 + \delta$$

The capital PGSU, $\hat{k}_t$, is *'low'*, which means that the marginal product will be *'big'*, and the consumption PGSU will be expected to *'grow'*, $\hat{c}_{t+1} > \hat{c}_t$. Therefore, the arrows point to *'up'*.

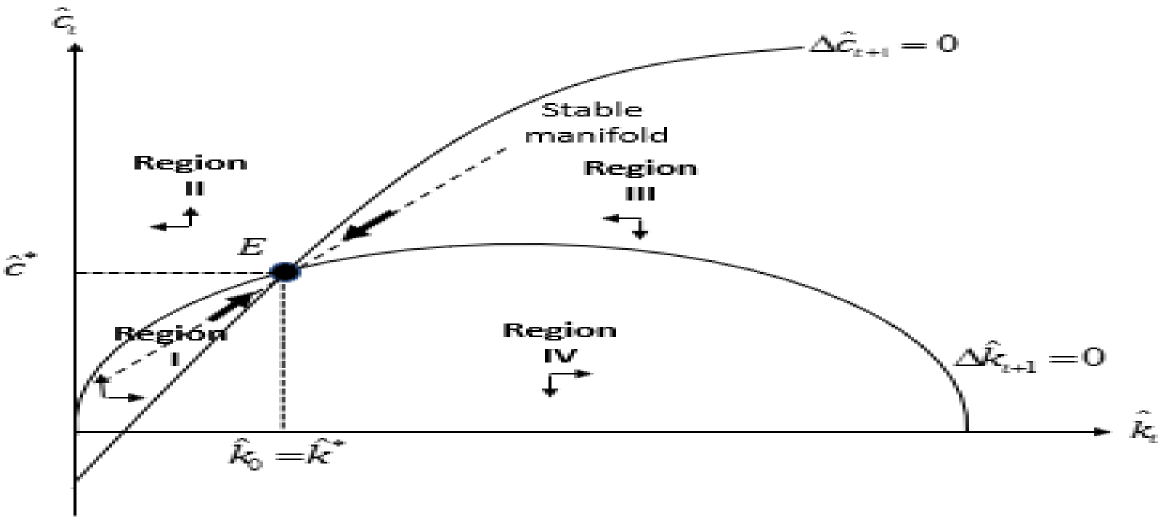


Diagram 6: Phase diagram of the transitional dynamics before the negative shock for oil demand

Diagram 6 shows the saddle path and some other unstable dynamic lines. The diagram shows four different regions *(I, II, III,* and *IV)*. The arrows point toward the steady-state when the economy is in region (*I)* or region (*III)*, where we have already discussed these two regions.

On the other hand, the reasons for the perfect foresight time path will not lie in the two regions (*II* and *IV*) are based on the following: picking any value of consumption PGSU, $\hat{c}_t$, in region (*II*), *above its saddle path*, would lead to zero capital PGSU, $\hat{k}_t$, and consumption PGSU, $\hat{c}_t$, would be infinite time. It is because the households would consume more over time by sacrificing capital. As a result, there will be no more capital at the next period where capital was already completely used in the previous period. In other words, $\hat{c}_t$ moves toward infinite with negative $\hat{k}_t$. For region (*IV*), picking any value of consumption PGSU, $\hat{c}_t$, in this region, *below its saddle path*, would lead the households to invest more over time by decreasing their consumption. Furthermore, they will keep doing that until the consumption goes to zero. In other words, this region leads to negative $\hat{c}_t$ and infinite $\hat{k}_t$.

There is a restriction in the model that capital stock and consumption have to be positive. According to that restriction, capital PGSU and consumption PGSU should follow the non-negatively constraint. Thus, if that restriction violated, then the path in both regions (*II* and *IV*) cannot be an equilibrium path. Consequently, for any given $\hat{k}_t$, $\hat{c}_t$ has to start either in the region (*I*) or region (*III*).

4.7.2 The Transitional Dynamics When a Negative Shock for Oil Demand Occurs

We want to investigate here how our economy will move from the old steady-state (E) to the new steady-state (E') if there is a permanent fall in γ_G. In this case, both the loci $\Delta\hat{k}_{t+1} = 0$ and $\Delta\hat{c}_{t+1} = 0$ will be shifted by the change in γ_G. To see how the negative shock on demand for oil will affect the transition dynamic of our economy, we look at the loci $\Delta\hat{k}_{t+1} = 0$ and $\Delta\hat{c}_{t+1} = 0$.

A- The locus $\Delta\hat{k}_{t+1} = 0$:

We have proved in the steady-state section that there is a negative relationship between the consumption PGSU, $\hat{c}_t$, and γ_G at the steady-state. Thus, if there is a negative shock on demand for oil, meaning γ_G decreases, the consumption PGSU will increase. However, this does not tell us the exact movement of $\Delta\hat{k}_{t+1} = 0$ curve. Thus, we looked back at equation (4.29) and found that it is obvious that holding $\hat{k}_t$ constant, declining γ_G must rise $\hat{c}_t$. Consequently, the $\Delta\hat{k}_{t+1} = 0$ locus shifts the bell curve up, as shown in Diagram 7.

B- The locus $\Delta\hat{c}_{t+1} = 0$:

Although we have found in the steady-state section that if γ_G falls, the capital PGSU, $\hat{k}_t$, will increase, that does not ensure that the $\Delta\hat{c}_{t+1} = 0$ locus will move to the right (*i.e.* move down). Thus, to determine the shift, whether the move to the right or left, we need to return to equation (4.30). Taking the derivative of $\hat{c}_t$ with respect to γ_G, holding $\hat{k}_t$ constant, gives us,

$$\frac{d\hat{c}_t}{d\gamma_G} = \underbrace{-\underbrace{\left(\frac{\gamma_G^\sigma}{\alpha A\beta} - \frac{1}{A\alpha} + \frac{\delta}{A\alpha}\right)^{\frac{1}{\alpha-1}}}_{(+)}}_{(-)} \left[1 + \underbrace{\underbrace{\left(\frac{\sigma\,\gamma_G^\sigma}{(\alpha-1)A\alpha\beta}\right)\left(\frac{\gamma_G^\sigma}{\alpha A\beta} - \frac{1}{A\alpha} + \frac{\delta}{A\alpha}\right)^{-1}}_{(-)\text{ and greater than 1 if }\sigma\text{ is sufficiently large}}}_{(-)} \right]$$

The first part of $d\hat{c}_t/d\gamma_G$ includes, $\left(\frac{\gamma_G^\sigma}{\alpha A\beta} - \frac{1}{\alpha A} + \frac{\delta}{\alpha A}\right)^{\frac{1}{\alpha-1}}$ which is positive, as shown in equation (4.23). The second part contains, $\left(\frac{\sigma\gamma_G^\sigma}{(\alpha-1)\alpha A\beta}\right)\left(\frac{\gamma_G^\sigma}{\alpha A\beta} - \frac{1}{\alpha A} + \frac{\delta}{\alpha A}\right)^{-1}$, which is negative because of $(\alpha-1)$, but it is still unclear if it is greater or less than one. The reason is that both $\left(\frac{\sigma\gamma_G^\sigma}{(\alpha-1)\alpha A\beta}\right)$ and $\left(\frac{\gamma_G^\sigma}{\alpha A\beta} - \frac{1}{\alpha A} + \frac{\delta}{\alpha A}\right)^{-1}$ depend on the coefficient of relative risk aversion, σ. Therefore, if σ is sufficiently large, then $\left(\frac{\sigma\gamma_G^\sigma}{(\alpha-1)\alpha A\beta}\right)\left(\frac{\gamma_G^\sigma}{\alpha A\beta} - \frac{1}{\alpha A} + \frac{\delta}{\alpha A}\right)^{-1}$ is negative and greater than one. As a result, holding $\hat{k}_t$ constant, then $d\hat{c}_t/d\gamma_G > 0$, which implies that the $\Delta\hat{c}_{t+1} = 0$ locus will move to the right (down) due to the reduction in γ_G, as shown in Diagram 7.

Due to the negative shock, both curves would change, which then lead to the new steady-state at point (E'), as shown in Diagram 7. However, we need to take into consideration the following points:

- At t=0, the initial stock of capital cannot be changed at this date, which implies that K_0 is predetermined.
- Since the negative shock of oil demand is a shock, *i.e.* it is not announced, the new curves and arrows will start to work immediately, whereas the old curves and arrows will disappear forever.
- The equilibrium path exists and is a unique path converging to (E').

From these points, the households at t=0 will adjust their initial consumption PGSU in order to be on their new optimal equilibrium path. However, the initial jump of consumption PGSU is not apparent yet, where the slope of the saddle path can be helpful to determine if the initial consumption PGSU jumps, whether up or down on impact. Thus, from our analysis of the stability, we can now write the slope of the saddle path equation[55] as,

$$\frac{\lambda_2 - m_{11}}{m_{12}} = \frac{\dfrac{\left(\dfrac{\gamma_G^{\sigma-1}}{\beta} + \dfrac{\chi}{\gamma_G^{\sigma}} + 1\right) - \sqrt{\left(\dfrac{\gamma_G^{\sigma-1}}{\beta} + \dfrac{\chi}{\gamma_G^{\sigma}} + 1\right)^2 - 4\left(\dfrac{\gamma_G^{\sigma-1}}{\beta}\right)}}{2} - \dfrac{\gamma_G^{\sigma-1}}{\beta}}{-\dfrac{1}{\gamma_G}} = -\underbrace{\frac{\gamma_G\left(\rho - \gamma_G^{\sigma-1}\right)}{2\beta}}_{(-)}$$

$$where\ \rho = \beta\left(\chi\gamma_G^{-\sigma} + 1 - \sqrt{\left(\frac{\gamma_G^{\sigma-1}}{\beta} + \frac{\chi}{\gamma_G^{\sigma}} + 1\right)^2 - 4\left(\frac{\gamma_G^{\sigma-1}}{\beta}\right)}\right)$$

λ_2 is a stable eigenvalue[56], while m_{11} and m_{12} are the first and second vectors in the first row of matrix A, respectively. The sign of the slope of the saddle path is positive because $\rho < \gamma_G^{\sigma-1}$.

Now, the equation for the saddle path can be written as,

$$\left(\hat{c}_0 - \hat{c}^*\right) = -\frac{\gamma_G\left(\rho - \gamma_G^{\sigma-1}\right)}{2\beta}\left(\hat{k}_0 - \hat{k}^*\right) \tag{4.32}$$

The saddle path equation (4.32) combines the initial value of $\hat{k}_t$ with the initial value of $\hat{c}_t$, where the slope determines the original value of $\hat{k}_t$ either below, above, or exactly at the original value of $\hat{c}_t$. Thus, to find the initial jump in consumption PGSU, the short-run consumption PGSU, we take the derivative of the initial consumption PGSU with respect to the growth rate of government spending, γ_G,

$$\frac{d\hat{c}_0}{d\gamma_G} = \underbrace{\frac{d\hat{c}^*}{d\gamma_G}}_{\substack{(-)\\ \text{if } \sigma \text{ is sufficiently large}}} + \underbrace{\underbrace{\frac{\gamma_G\left(\rho - \gamma_G^{\sigma-1}\right)}{2\beta}}_{(-)} \cdot \underbrace{\frac{d\hat{k}^*}{d\gamma_G}}_{(-)}}_{(+)} \tag{4.32$'$}$$

[55] A full proof of the saddle path and its slope can be found in Appendix A.2.

[56] The general formula for λ is $\lambda = \frac{Tr \pm \sqrt{(Tr)^2 - 4(Det)}}{2}$. As a long as we have shown in section 4.6 that if σ is sufficiently large, then $Tr > 2$ and $Det > 1$, therefore the stable eigenvalue is $\lambda = \frac{Tr - \sqrt{(Tr)^2 - 4(Det)}}{2}$, which is $|\lambda| < 1$.

It should be noted first that although it is true that the coefficient on $\left(\hat{k}_0 - \hat{k}^*\right)$ in (4.32) also depends on γ_G, in fact the derivative of this coefficient w.r.t γ_G will drop out of the overall formula for $d\hat{c}_0/d\gamma_G$, because we will evaluate this overall formula in an initial situation where $\hat{k}_0 - \hat{k}^* = 0$ (*i.e.* we will evaluate it in the pre-shock steady-state), which means that the derivative of the coefficient w.r.t. γ_G will be multiplied by zero. For (4.32)', we have already calculated both the $d\hat{k}^*/d\gamma_G$ and $d\hat{c}^*/d\gamma_G$ in the steady-state section, finding that the sign of $d\hat{k}^*/d\gamma_G$ was clearly negative, and the sign of $d\hat{c}^*/d\gamma_G$ was also negative, but if σ is sufficiently large. Thus, rewriting (4.32)' and simplifying it further, we get:

$$\frac{d\hat{c}_0}{d\gamma_G} = \underbrace{\underbrace{\left(\frac{\gamma_G^{\sigma}}{\alpha A\beta} - \frac{1}{A\alpha} + \frac{\delta}{A\alpha}\right)^{\frac{1}{\alpha-1}}}_{(+)} \left[\underbrace{\underbrace{\left(\frac{\sigma\,\gamma_G^{\sigma-1}}{(\alpha-1)\beta}\right)}_{(-)} - 1}_{(-)}\right]}_{(-)}$$

$$+ \underbrace{\left[\underbrace{\underbrace{\left(\frac{\sigma\,\gamma_G^{\sigma-1}}{(\alpha-1)\alpha A\beta}\right)}_{(-)} \underbrace{\left(\frac{\gamma_G^{\sigma}}{\alpha A\beta} - \frac{1}{\alpha A} + \frac{\delta}{\alpha A}\right)^{\frac{2-\alpha}{\alpha-1}}}_{(+)}}_{(-)}\right] \left[\underbrace{\underbrace{\frac{\gamma_G\left(\rho - \gamma_G^{\sigma-1}\right)}{2\beta}}_{(-)} + \underbrace{\left(1-\delta-\gamma_G\right)}_{(-)}}_{(-)}\right]}_{(+)} \qquad (4.32)''$$

The sign of (4.32)'' is theoretically ambiguous because its expression becomes complicated. Thus, the initial jump, whether up or down, in consumption PGSU appear to be possible in the phase diagram. As a result, given the ambiguity over whether $\hat{c}_0$ falls or rises on impact, we will proceed to investigate further using numerical simulation in the next section.

Although the initial consumption PGSU is possible to jump whether up or down, we only consider in the below diagrams the case in which the consumption PGSU initially jumps down on impact. Thus, Diagram 7 and Diagram 8 below display the transitional dynamics and the impulse responses that follow the dynamic responses of consumption PGSU, capital PGSU, and output PGSU to the negative shock on demand for oil.

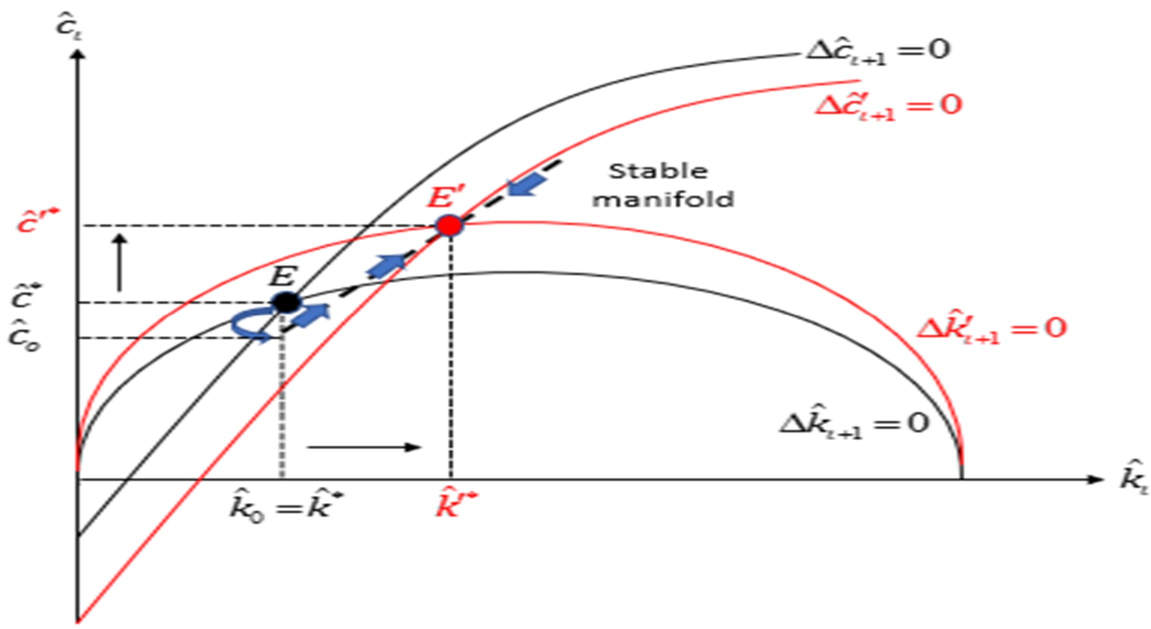


Diagram 7: Phase diagram of the transition dynamic when a negative shock for oil demand occurs

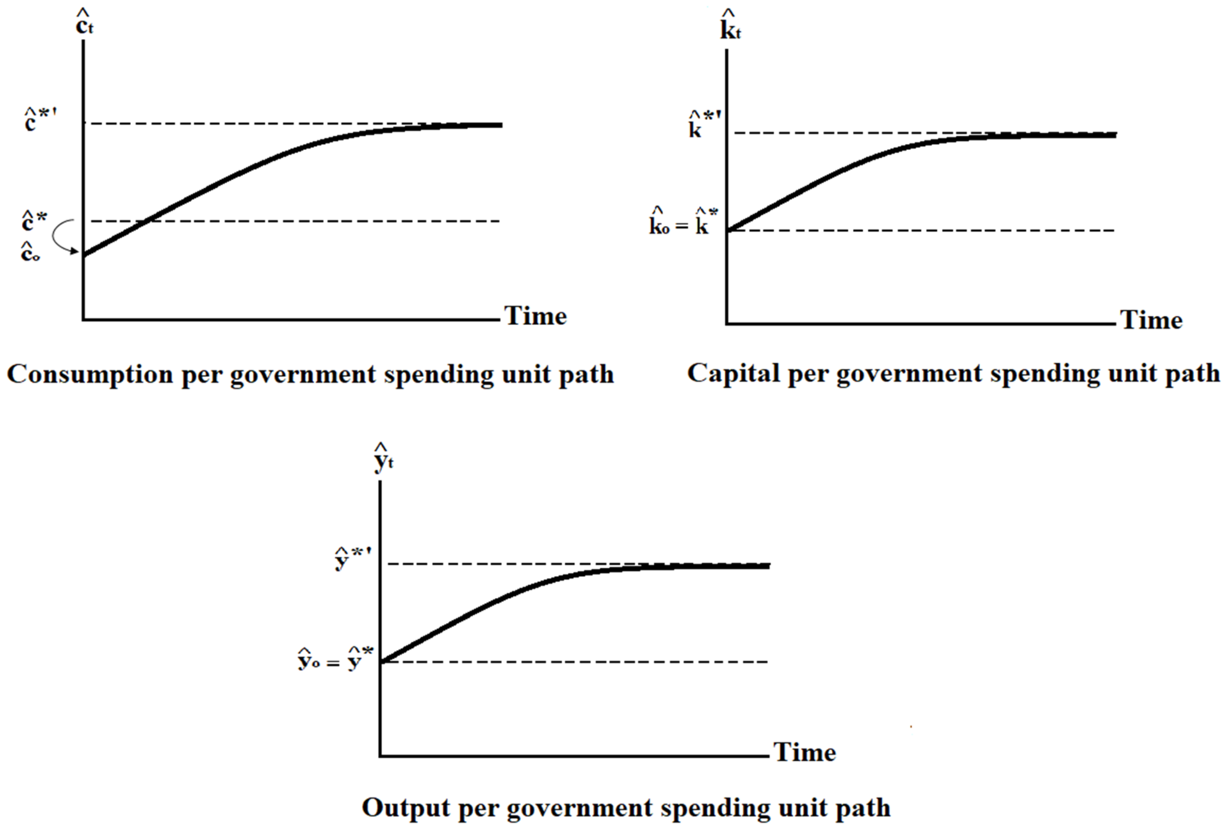


Diagram 8: The impulse responses to the negative shock on demand for oil

4.8 Parameterisation and Solution

We now investigate how a permanent shock in demand for oil would affect the economy. Suppose we predict that in a particular time, there will be a drop in the growth rate of government spending γ_{G_t}. In this case, the economy will grow more slowly for many decades because the oil demand would not carry on growing at the same rate as before. Thus, it is essential to look at an expected future change due to that decline. Consequently, we use in this section a numerical solution to link our previous theoretical results in the steady-state and the phase diagram analysis with the numerical solution.

In fact, there is occasionally no analytical solution for some dynamic and stochastic general equilibrium (DSGE) models. Thus, numerical solutions are applied to clarify the theoretical properties of each model and discuss the impacts of interventions in economic policy (Novales, Fernández and Ruiz, 2014). If a model is complicated and not possible to have an analytical solution, then numerical solutions become an alternative solution. In our model, although we have used the analytical solution, we are also using the numerical solution to find the magnitude of the steady-state and match the numerical solution with the analytical solution, which has been shown in the steady-state analysis section.

Dynare software is employed to simulate the main two nonlinear equations, capital PGSU and consumption PGSU, and other conditions in the model, under standard parameter values. We apply a 5% decline in the growth rate of government spending, which is caused by the negative shock on oil demand. All agents are almost perfect foresight in the model, where they correctly anticipate the lower growth rate forever when and after the shock occurs. As a result, the economy will move to a new steady-state, as will be shown in the solution section.

4.8.1 Parameterisation

The simulation takes the parameter values that are standard and commonly used in the literature, where they can be reasonable yearly. The value of the depreciation rate, the discount factor, the coefficient of relative risk aversion, and the production function parameter are summarised below.

Table 2: Parameter values

Notation	Description	Value
δ	Depreciation Rate	0.1
β	Discount Factor	0.95
σ	Coefficient of Relative Risk Aversion	2 - 0.1
α	Production Function Parameter	0.33

4.8.2 Solution

We assume that there is a permanent negative shock in demand for oil, where the growth rate of government spending, γ_{G_t}, decreases by 5%, *e.g.* from 1.1 to 1.05. However, we have found in section 4.7.2 that it was theoretically ambiguous to determine the initial jump of consumption PGSU. Thus, we assume here two different values of the coefficient of relative risk aversion, σ, because we believe that σ would be critical to determine whether the initial consumption PGSU falls or rises on impact. By using Dynare software to simulate the model, the solution can be seen in Figure 32 and Figure 33. These figures display the dynamic responses of several PGSU variables in the economy to a permanent shock under two assumptions regarding the value of σ. The first assumption is to set σ sufficiently large, as shown in Figure 32, while the second is to be sufficiently small, as shown in Figure 33.

Under the calibration considered here with a sufficiently large value of σ $(i.e.\ \sigma \geq 0.5)$[57], the simulation in Figure 32 shows under this assumption that consumption PGSU would first jump down in the short-run on impact. The economic explanation is that the initial jump in consumption is affected by the intertemporal elasticity of substitution (IES). The IES is one of the main parameters for household consumption and saving, where it is important to determine policy and welfare evaluations (Okubo, 2011). It defines in discrete time, in the case of the utility function with constant relative risk aversion (CRRA), as $1/\sigma$. If σ is high, then the households have a lower intertemporal elasticity of substitution for consumption. It implies that the households would not have a desire over time to bear a significant difference in consumption level. On the other hand, if σ is sufficiently small $(i.e.\ \sigma < 0.5)$, the consumption PGSU would jump up on impact, as shown in Figure 33. Consequently, the transitional dynamics in these figures show the movement from the old steady-state to a new steady-state under two assumptions of σ, where all PGSU variables in the economy have been affected. In other words, changing the value of σ causes the steady-state to change. Thus, the impact of the steady-state values on PGSU variables are summarised in the following tables and figures:

[57] According to our model and the parameter values used, we have calculated the value of σ and found that the value of 0.5 is a turning point for the initial consumption PGSU to jump either up or down.

The dynamic responses of a number of PGSU variables in the economy to a permanent shock (with, *e.g.* $\sigma = 2$):

Table 3: The steady-state responses of a number of PGSU variables in the economy to a permanent negative shock in demand for oil (with e.g. $\sigma = 2$):

Old Steady-State Values		New Steady-State Values	
Variables	**Value**	**Variables**	**Value**
The Growth Rate of Government Spending	1.10	The Growth Rate of Government Spending	1.05
Output PGSU	0.940605	Output PGSU	1.12348
Capital PGSU	0.830647	Capital PGSU	1.42307
Consumption PGSU	0.774476	Consumption PGSU	0.910018
Interest Rate	0.373684	Interest Rate	0.260526

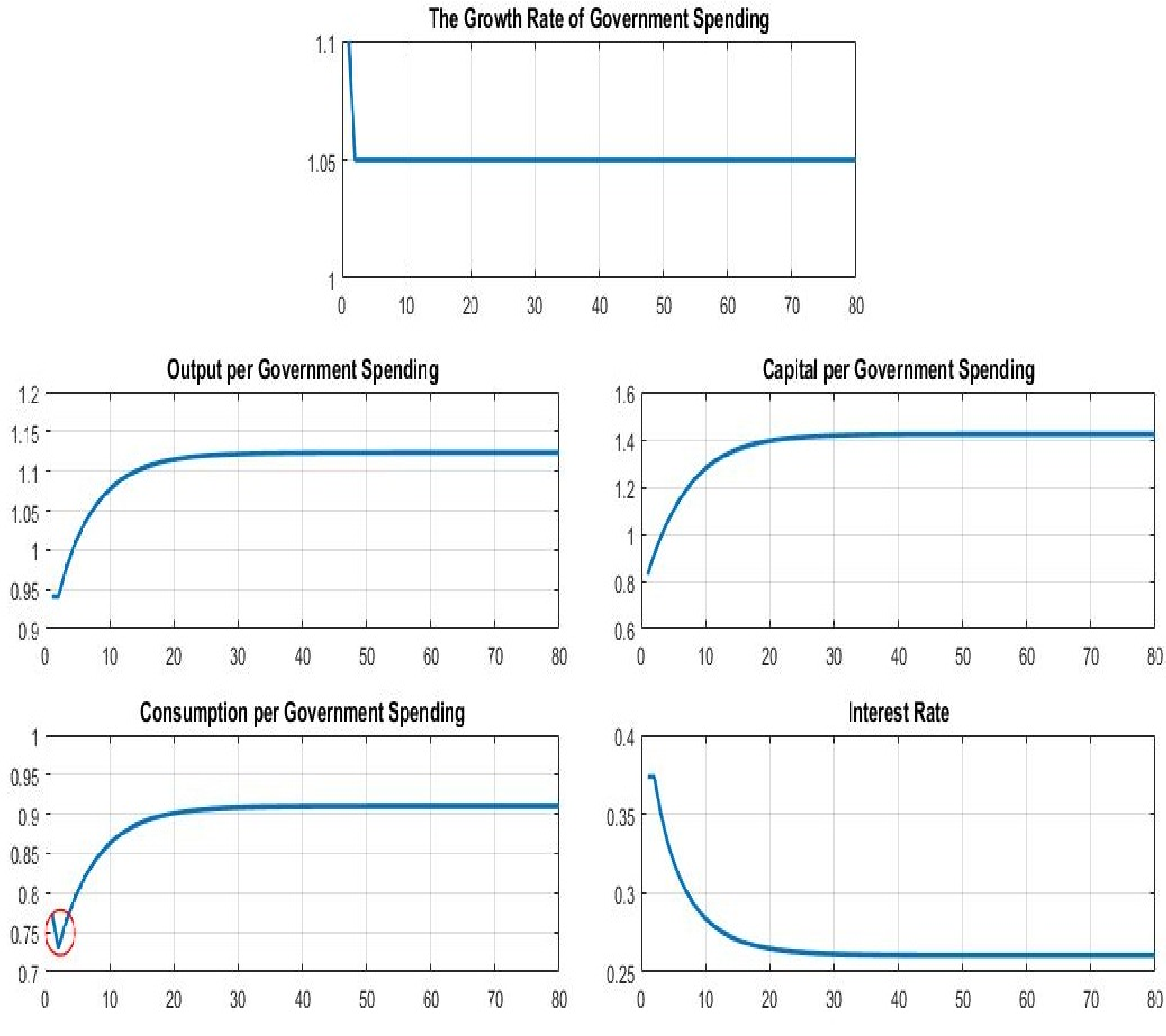


Figure 32: The dynamic responses of a number of variables in the economy to a permanent shock, with σ=2

The dynamic responses of a number of PGSU variables in the economy to a permanent shock (with, *e.g.* $\sigma = 0.1$):

Table 4: The steady-state responses of a number of PGSU variables in the economy to a permanent negative shock in demand for oil (with e.g. σ=0.1):

Old Steady-State Values		**New Steady-State Values**	
Variables	**Value**	**Variables**	**Value**
The Growth Rate of Government Spending	1.10	The Growth Rate of Government Spending	1.05
Output PGSU	1.41663	Output PGSU	1.43826
Capital PGSU	2.87309	Capital PGSU	3.00815
Consumption PGSU	0.842008	Consumption PGSU	0.987043
Interest Rate	0.162712	Interest Rate	0.15778

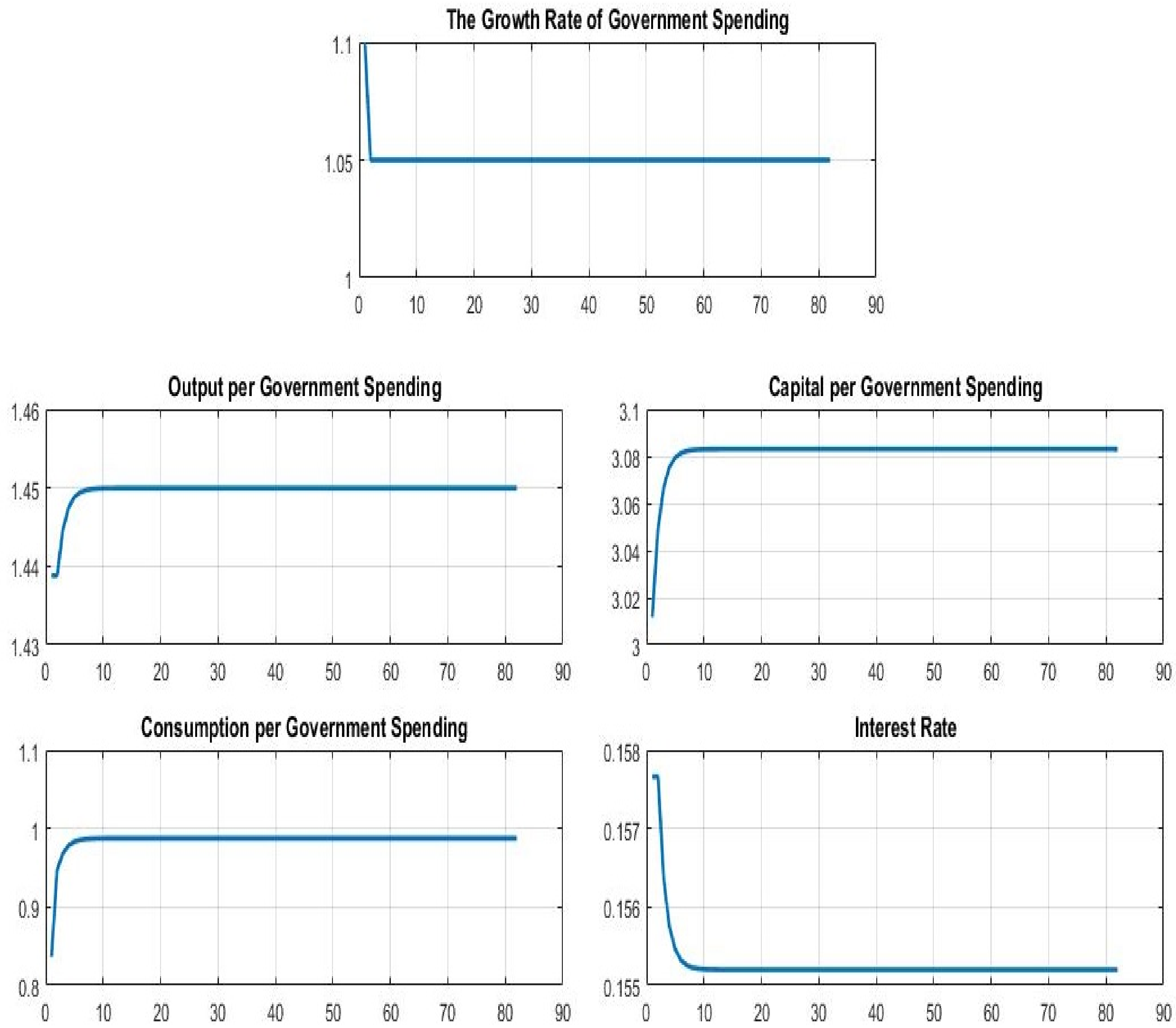


Figure 33: The dynamic responses of a number of variables in the economy to a permanent shock, with σ=0.1

4.9 A Special Case of Our Model

This section discusses the key variables in the model in level variables as a special case, unlike the variables of PGSU. The reason is that it would be interesting for the policymakers to examine the impact of government spending on other level variables, which have meaning in contrast with the variables of PGSU. Thus, we convert our key variables in the model to be in the level term by setting the growth rate of government spending in our previous setup equals one (*i.e.* $\gamma_{G_t} = 1$) and then find the steady-state equations in each level variables. To move from our previous model to this particular case, we use the same setup as before, and we just set $\gamma_{G_t} = 1$. The government spending is treated in this section as an exogenous variable and assumed to be hold constant over time $(G_t = G, \forall t)$, but it enters into the firms' production function as positive externalities. Then, we investigate what would happen for the level variables if the government spending decline, which may be due to the shock of oil demand.

In this special case, we only show the equilibrium and steady-state equations and how the steady-state equations are affected when we change the level of government spending. Moreover, we provide a numerical solution for this special case.

4.9.1 Equilibrium

The key equations at equilibrium are the capital accumulation equation and Euler equation, respectively,

$$K_{t+1} = A\,K_t^{\alpha}\,G_t^{1-\alpha} - C_t + (1-\delta)K_t \tag{4.33}$$

where $Y_t = A\,K_t^{\alpha} G_t^{1-\alpha}$, and

$$\frac{C_{t+1}}{C_t} = \left[\beta\left(\alpha A K_{t+1}^{\alpha-1} G_{t+1}^{1-\alpha} + 1 - \delta\right)\right]^{\frac{1}{\sigma}} \tag{4.34}$$

The transversality condition is:

$$\lim_{t\to\infty} \beta^t\,\lambda_t\,K_{t+1} = 0$$
$$\lim_{t\to\infty} \beta^t\,\lambda_t\,\left(A\,K_t^{\alpha}\,G_t^{1-\alpha} - C_t + (1-\delta)K_t\right) = 0$$

These two equations (4.33) and (4.34) plus transversality condition with K_0 given are the main equations that describe the market equilibrium of our special case economy.

4.9.2 Analysis of the Steady-State

We find in this particular case the steady-state equations of the two key equations in the economy (4.33) and (4.34) as well as the output, where at the steady-state, $K_t = K_{t+1} = K^*$, $Y_t = Y_{t+1} = Y^*$, $C_t = C_{t+1} = C^*$, and $G_t = G_{t+1} = G^*$.

From the Euler equation (4.34), we obtain:

$$K^* = \left[\left(\frac{1}{\alpha A}\right)\left(\frac{1}{\beta} - 1 + \delta\right)\right]^{\frac{1}{\alpha-1}} G^* \tag{4.35}$$

The output at the steady-state:

$$Y^* = A\left[\left(\frac{1}{\alpha A}\right)\left(\frac{1}{\beta} - 1 + \delta\right)\right]^{\frac{\alpha}{\alpha-1}} G^* \tag{4.36}$$

From the capital accumulation equation (4.33), we get:

$$C^* = AK^{*\alpha}G^{*1-\alpha} - \delta K^* \tag{4.37}$$

$$C^* = G^*\left[\left(\frac{1}{\alpha A}\right)\left(\frac{1}{\beta} - 1 + \delta\right)\right]^{\frac{1}{\alpha-1}}\left[\left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta} - 1 + \delta\right) - \delta\right] \tag{4.37$'$}$$

These expressions show the equations for the steady-state values in level K^*, $Y^*$and C^*.

4.9.2.1 The impact of changing government spending on the steady-state of capital stock:

Starting with the steady-state equation (4.35), we can find a direct effect when government spending, G^*, changes. To see that, we take the derivative of K^* with respect to G^* and obtain the following:

$$\frac{dK^*}{dG^*} = \underbrace{\left[\left(\frac{1}{\alpha A}\right)\left(\frac{1}{\beta} - 1 + \delta\right)\right]^{\frac{1}{\alpha-1}}}_{(+)} \tag{4.38}$$

Since $\alpha, \beta, \delta \in (0,1)$, A is constant at 1, then it is clear that the sign of dK^*/dG^* is positive. Therefore, we conclude that there is a positive relationship between the level of capital stock, K^*, and the level of government spending, G^*, at the steady-state, meaning that if G^* falls, then the K^* will also fall.

4.9.2.2 The impact of changing government spending on the steady-state of output:

Similarly, from equation (4.36), we examine how the output changes when the government spending changes,

$$\frac{dY^*}{dG^*} = \underbrace{A\left[\left(\frac{1}{\alpha A}\right)\left(\frac{1}{\beta} - 1 + \delta\right)\right]^{\frac{\alpha}{\alpha-1}}}_{(+)} \tag{4.39}$$

We can see that the sign of dY^*/dG^* is positive, meaning that if the level of government spending declined, then the output level should decrease too.

4.9.2.3 The impact of changing government spending on the steady-state of consumption:

Although we have already found the steady-state equation (4.37), we notice that C^* does not only depend on G^* but also depends on K^*. Thus, we need to consider the total effect, which is not only the *'direct effect (DE)'* of C^* but also the *'indirect effect (IE)'* of C^* on K^*, as shown in equation (4.37)'.

To look at the change of G^* at the steady-state, we take the derivative of equation (4.37)' with respect to G^*:

$$\frac{dC^*}{dG^*} = \underbrace{\left[\left(\frac{1}{\alpha A}\right)\left(\frac{1}{\beta} - 1 + \delta\right)\right]^{\frac{1}{\alpha-1}}}_{(+)} \underbrace{\left[\left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta} - 1 + \delta\right) - \delta\right]}_{(+)} \tag{4.40}$$

Looking at equation (4.40), we found that $dC^*/dG^* > 0$. The reason is that the first part of the bracketed expression is positive, as we have shown in (4.38), and the second part is also positive because $(1/\alpha)(1/\beta - 1 + \delta) > \delta$. Therefore, the whole bracketed expression is positive. As a result, we can conclude that there is a positive relationship between the level of consumption, C^*, and the level of government spending, G^*, at the steady-state, meaning that if G^* falls, then C^* will also fall.

In brief, changing government spending would change all the level variables in the same path. This fact is consistent with what we mentioned early that government spending is the economy's main engine in our model.

4.9.3 Parameterisation and Solution

In this section, we simulate this special case by using Dynare software. We apply a 5% decline in the level of government spending. As before, all agents are almost perfect foresight. They correctly anticipate the reduction in government spending once and after it happens. As a result, the economy in this special case will also move from an old to a new steady-state, as it will be shown in the solution section.

4.9.3.1 Parameterisation

The parameter values used here are the same as before. We also assume two different coefficients of relative risk aversion, σ. The first one is to be sufficiently large, while the second is to be sufficiently small.

4.9.3.2 Solution

We assume that there is a permanent reduction in the level of government spending, where government spending, G_t, declined by 5%, *i.e.* from 2 to 1.90. By using Dynare software to simulate the model, the solution can be seen in Figure 34 and Figure 35. These figures display the dynamic responses of a number of level variables in the economy to a permanent shock under two assumptions regarding the value of the coefficient of relative risk aversion, σ. The first assumption is to set σ to be sufficiently large (*e.g.* $\sigma = 2$), as shown in Figure 34, while the second is to be sufficiently small (*e.g.* $\sigma = 0.1$), as shown in Figure 35.

The steady-state responses of level variables, Table 5 and Table 6, can confirm our theoretical results, in which all level variables would decrease when the level of government spending decreases. Moreover, when we compared our simulation results with the two different coefficients of relative risk aversion, $\sigma = 2$ and $\sigma = 0.1$, we found that all the values of steady-state variables are identical in the long run because σ does not enter into the steady-state equations. However, changing σ only affects the short-run level of consumption because it enters into the Euler equation, *i.e.* equation (4.34). The impact is that when σ is sufficiently large, the short-run level of consumption would jump down on impact. On the other hand, if σ is sufficiently small, the short-run level of consumption would jump up on impact. Figure 34 and Figure 35 show the dynamic responses in both the short and long run.

The dynamic responses of a number of level variables in the economy to a permanent shock (with, *e.g.* $\sigma = 2$)**:**

Table 5: The steady-state responses of a number of level variables in the economy to a permanent negative shock in government spending (with e.g. σ=2):

Old Steady-State Values		New Steady-State Values	
Variables	**Value**	**Variables**	**Value**
Level of Government Spending	2.0	Level of Government Spending	1.90
Level of Output	2.92392	Level of Output	2.77772
Level of Capital	6.32172	Level of Capital	6.00563
Level of Consumption	2.29175	Level of Consumption	2.17716
Interest Rate	0.152632	Interest Rate	0.152632
Investment	0.632172	Investment	0.600563

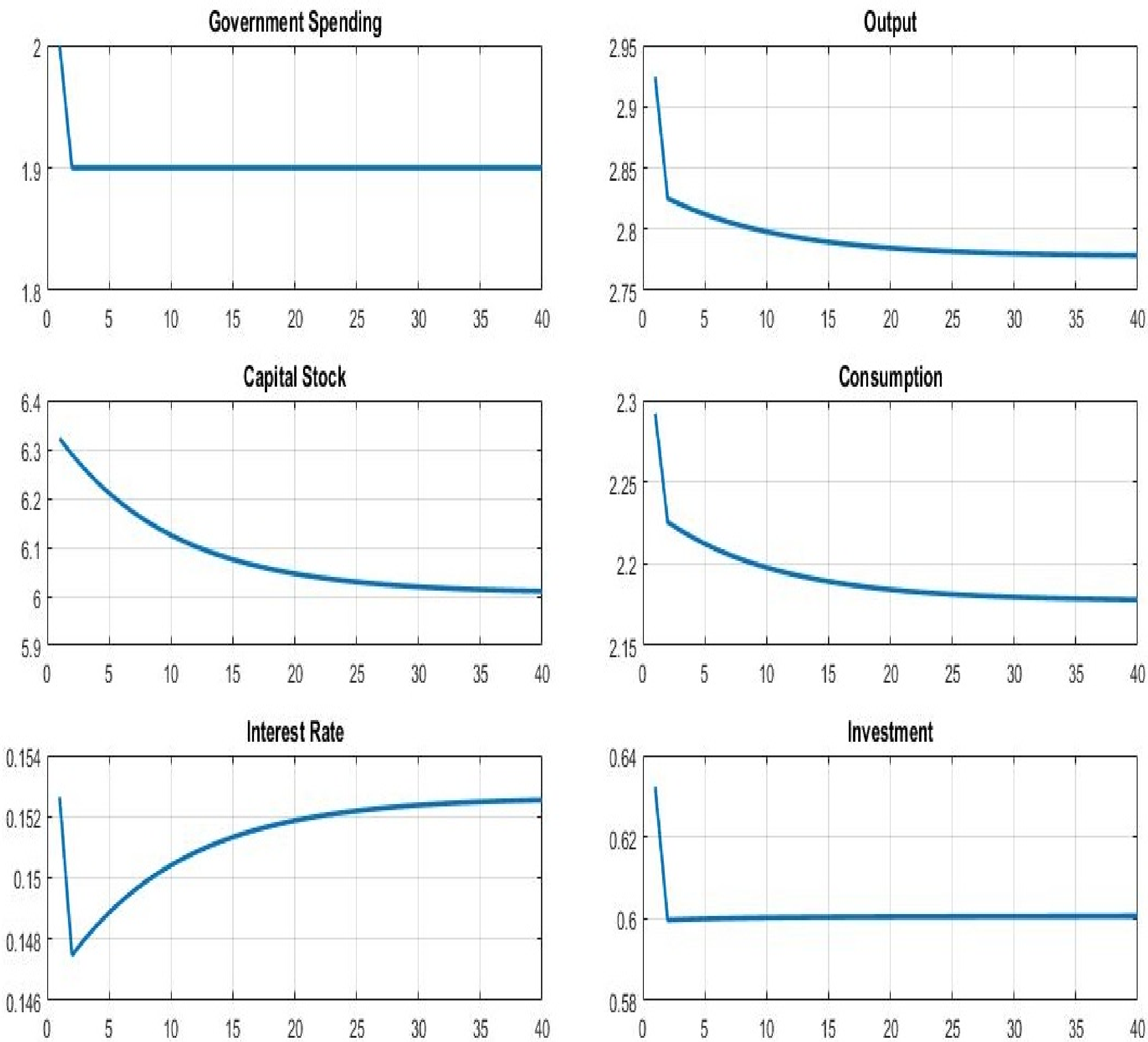


Figure 34: The dynamic responses of a number of level variables in the economy to a permanent reduction in government spending, with σ=2

The dynamic responses of a number of level variables in the economy to a permanent shock (with, *e.g.* $\sigma = 0.1$)**:**

Table 6: The steady-state responses of a number of level variables in the economy to a permanent negative shock in government spending (with e.g. σ=0.1):

Old Steady-State Values		**New Steady-State Values**	
Variables	**Value**	**Variables**	**Value**
Level of Government Spending	2.0	Level of Government Spending	1.90
Level of Output	2.92392	Level of Output	2.77772
Level of Capital	6.32172	Level of Capital	6.00563
Level of Consumption	2.29175	Level of Consumption	2.17716
Interest Rate	0.152632	Interest Rate	0.152632
Investment	0.632172	Investment	0.600563

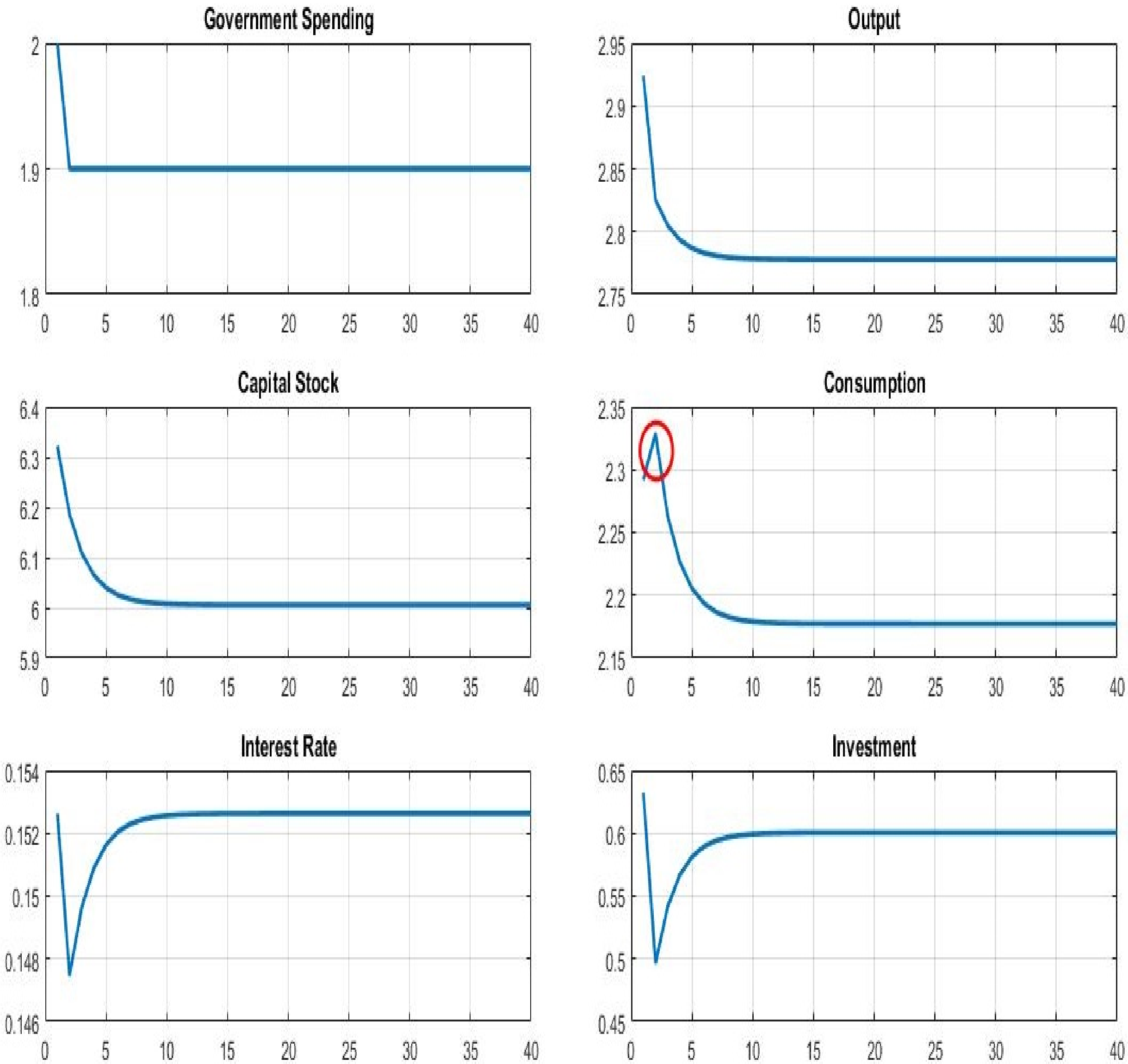


Figure 35: The dynamic responses of a number of level variables in the economy to a permanent reduction in government spending, with σ=0.1

4.10 Conclusion

In summary, this chapter is built up a theoretical framework to study the economic growth in Saudi Arabia. The model is derived from Barro's endogenous growth model, but without taxes, where the government has only one source of revenues: oil revenues. The oil sector is one of the main sectors in our economy, where the growth rate is led by a growing demand for oil. This sector maximises its profits as a monopolistic and then transfers them to the government sector because it wholly owns it. We assumed that the government provides public goods (*e.g.* infrastructure, legal framework) as positive externalities to the firms to enhance their production function instead of supplying that profits as a transfer to households. For simplicity, we also assumed both households and firms to have no excess to the international market, including financial markets.

In the model setup, we have found that the growth rate of government spending is the growth rate of oil profits, g^2. That means everything in the economy of this chapter grows at the rate of government spending, which represents an exogenous growth rate. Therefore, this conclusion is different from Barro's conclusion, where the main determinant of the economic growth in our model is the growth rate of oil profits, which is g^2. The main objective of this chapter is to examine how our economy can be affected if there is a negative shock to the demand for oil. In other words, how the economic growth in our model would be affected if the growth rate of government spending declines, $(i.e.\ \downarrow \gamma_G \equiv \downarrow g^2)$. For this reason, we have studied the steady-state properties and how a shock (*i.e.* a permanent reduction) in the growth rate of government spending would affect the steady-state position. We have found that if γ_G falls, then both the capital PGSU, $\hat{k}^*$, and the consumption PGSU, $\hat{c}^*$, at the steady-state would raise. Moreover, we have arrived at an interesting result that although the level of consumption would grow at a slower rate due to the reduction in γ_G, the consumption PGSU would be higher. Higher consumption PGSU would give us an indication that there would be partially offsetting the effect of the reduction in γ_G. In other words, although the level of consumption is now growing slowly, it is indeed increased relative to the previous trajectory. Thus, our model has shown that there is some partially offsetting shift in the level of consumption, which may not be obvious.

In the stability section, we realised that there would be two eigenvalues in this system by linearising the capital accumulation equation and the Euler equation. Moreover, the dynamics system has one predetermined variable, which is capital PGSU, $\hat{k}^*$, and one non-predetermined variable, which is consumption PGSU, $\hat{c}_t$. By applying Blanchard and Kahn (1980) conditions and the test explained by Rankin (2007), we concluded that we need one stable eigenvalue and one unstable eigenvalue. Then, we have found that the steady-state $(\hat{k}^*, \hat{c}^*)$ is saddle-point. Regarding the transitional dynamics in our model, we have built up a phase diagram to show the characteristics of a dynamic system before and after the negative shock for oil demand to happen. Once the shock occurs, *i.e.* reduction in γ_G, it was essential to study the saddle path properties to find out its slope. The reason for studying the slope of the saddle path is that it can help us identify the initial consumption PGSU, *i.e.* short-run of consumption PGSU, to jump up or down on impact. After all, we have discovered that the initial jump of consumption PGSU is theoretically ambiguous, meaning that both situations appear to be possible in the phase diagram. Therefore, using numerical simulation would be an appropriate method to apply in this case.

Furthermore, we have provided a numerical solution to simulate the model under a permanent negative shock in the government spending growth rate. We assumed that the growth rate of government spending declined by 5%. Due to the theoretical ambiguity over whether the initial consumption PGSU jumps down or up on impact, we assumed that the initial consumption PGSU could be determined by the coefficient of relative risk aversion, σ . Thus, we set two values of σ: one is sufficiently large $(e.g.\ \sigma = 2)$, and one is sufficiently small $(e.g.\ \sigma = 0.1)$. The simulation results have shown in both cases of σ that all PGSU variables would increase when the shock occurs. However, when σ is sufficiently large (small), the short run of consumption PGSU would jump down (up) on impact.

Finally, we have studied the level variables as a special case. Even though our focus in the whole thesis is on economic growth, examining the level variables could be interesting for policymakers to understand the impact of government spending on other level variables. Therefore, we have found that government spending would have a significant effect on other level variables. The simple reason is that government spending, which is fed by oil profits, is the main engine in our model. Moreover, in this special case, we have also shown a numerical solution that simulates this case when there is a reduction in the level of government spending. Like the simulation in PGSU variables, we set two values for σ, sufficiently large and small values. The simulation results of this special case display that all level variables decline when the government spending decline. Furthermore, when we compare the results of this case under the two assumptions of σ, we have found that all values of steady-state variables are identical in the long run. The reason is simply due to that σ does enter in their equations. However, the value of σ only changes the short run of the level of consumption, where the large (small) value of σ causes the short-run of the consumption level to jump down (up). In the next chapter, we will model Saudi economic growth with a consumption tax.

5 Chapter Five: Modelling Saudi Economic Growth with Consumption Tax

5.1 Introduction

Chapter five is based on a strong motivation regarding actual policy discussion in fiscal policy in Saudi Arabia. Thus, this chapter is considered an extension of the fourth chapter. We extend the model by introducing consumption tax, as suggested by the International Monetary Fund (IMF), and modifying the model accordingly. In this chapter, there are two sources of revenues, oil revenues and consumption tax, that finance government spending. The primary purposes of this chapter are to generate additional revenues to the government away from oil since the country does not have enough taxes, examine the impact of introducing a consumption tax on economic growth, analyse the real effects of this tax on the key level variables, and find the amount of this tax that can keep government revenues constant when there is a reduction in oil revenues. In other words, this chapter seeks to examine what would be the amount of consumption tax that could compensate for the decrease in oil revenues associated with the case of negative shocks in demand for oil. We study in this chapter the main agents in the economy, equilibrium, analysis of the steady-state, stability, transitional dynamics, and numerical solution.

5.2 Model Description

The economy has five sectors named households, firms, oil, government, and foreign sectors. Households sector own factors of production, namely capital stock and labour. They provide factor services to firms and buy the final good, and they also pay consumption tax to the government. Moreover, the households sector has no access to international markets. The oil sector is like in chapter four, where it is an exogenous and monopolistic sector. It only maximises its profit[58] each period of time separately, and then the government receives its profit because the government owns this sector. Thus, the government sector collects the oil profit from the oil sector plus consumption tax from the households sector. Then, the government spends all its revenues on purchasing imported goods, and it has limited deals with international markets. The transaction between the government sector and the foreign sector is that the government sector exports oil and the home-produced goods as a consumption tax and then exchanges them with imported goods. Finally, these goods are used to enhance the firms by entering into their production function. Thus,

[58] To distinguish here between the two terms, the oil profits and revenues, the oil profits term is used to refer to the oil sector itself, while the oil revenues term is used to refer to what the government sector receives from oil sector.

the firms' sector benefits from these goods as positive externalities. Diagram 9 below summarises the circular flow and the structure of the economy in this chapter.

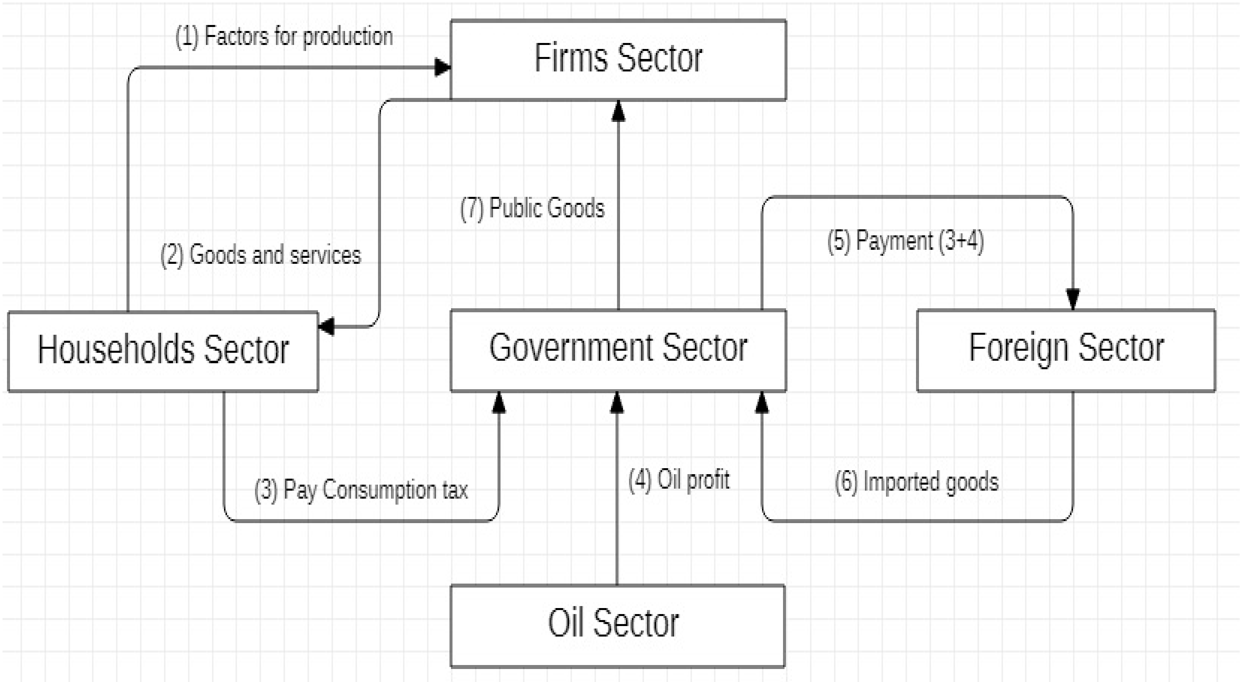


Diagram 9: The structure of the economy in chapter five

The current chapter is different from chapter four in three aspects. The first aspect concerns the accounting identity and government budget constraint, while the second and third aspects are about the balance of trade and level variables, respectively.

For the accounting identity (national level) and government budget constraint, the government now levies a tax on consumption where the households hand over some resources, which are domestically produced resources, to the government. Thus, the households give this fraction to the government sector as a source of tax revenue. Then, the government converts that into imported goods, which is physically different goods, by selling them in the international market. Thus, the government is not only selling oil abroad, but it has to sell as well some part of the domestic produced output in the international market, namely the part which is obtained in tax revenue from the domestic households. Similar to the fourth chapter, we assumed that the government spends all its revenues on purchasing imported goods.

Moreover, looking at the trade conditions, there will be a relative price of domestic goods to imported goods. Thus, we treat this relative price as being exogenous and fixed in world markets. As a result, the market-clearing condition would be different from the one in the fourth chapter. The accounting identity[59] would be now like,

$$Y_t = C_t + I_t + X_t$$

where $X_t = \tau^c C_t = \text{exports}$

Rewriting the resource constraint of the economy gives us,

$$Y_t = \left(1+\tau^c\right)C_t + I_t$$

That means the government now exports the domestically-produced good. Therefore, if we assume that the relative price of the imported good in terms of domestic good is exogenous and normalised to one[60], then the government budget constraint can be written as,

$$G_t = \pi_t + X_t$$

Regarding the balance of trade, we had in chapter four a single type of export: oil revenues and a single type of import, which is the government purchasing goods. However, we now have two types of exports: oil revenues and some fraction of the domestically produced output received through the consumption tax, and one type of import, which is the government purchasing goods. Thus, the trade balance can be written as,

$$\text{Trade Balance} = \text{Exports} - \text{Imported Goods} = 0$$

where exports = $\pi_t + \tau^c C_t$, which is identical to the government budget constraint.

The third aspect is regarding the level variables, which will be discussed in section 5.5.

[59] We should note that the reason for not including government spending in the resource constraint is that the economy imports goods. Thus, in terms of physical commodities, they are imported goods not domestically produced goods. It is also worth noting that the notation Y_t does not stand in the following equation for GDP, it stands for the output of one sector. This is one sector of GDP, which represents the non-oil sector.

[60] However, if the relative price is assumed to be exogenous but fixed at some value other than one, PC_t, this would introduce some new notation. Thus, we set it to be one for simplicity.

5.3 Model Setup

The main agents in the economy are the government, households, and firms sectors. However, the model will include the oil sector revenues, which have been discussed and found in the fourth chapter.

5.3.1 Government Sector

The government budget contains two sides: revenues and spending. The revenues side includes two sources of revenues that are revenues of the oil sector and consumption tax. The revenues are used to purchase imported goods from abroad, and then the government provides public goods (*e.g.* infrastructure, legal framework) to enhance the production function of the final goods sector. The government sector is simply assumed to spend all the revenues on purchasing imported goods. Moreover, it cannot access the international financial market (lending and borrowing). So, the government budget is in balance and can be written as:

$$G_t = \pi_t + \tau^c C_t \tag{5.1}$$

where G_t is government spending, π_t is the revenues from the oil sector, and τ^c is a constant consumption tax.

From equation (5.1), we define the growth rate of government spending as $\gamma_{G_t} \equiv G_{t+1}/G_t$,

$$\gamma_{G_t} \equiv \frac{G_{t+1}}{G_t} = \frac{\pi_{t+1} + \tau^c C_{t+1}}{\pi_t + \tau^c C_t} \tag{5.2}$$

where $\pi_{t+1}/\pi_t = g^2$ is the growth rate of the oil profits, which we have found in the fourth chapter, and can also be written as,

$$\frac{\pi_{t+1}}{\pi_t} = g^2 \quad \Rightarrow \quad \pi_{t+1} = g^2\, \pi_t$$

and $C_{t+1}/C_t = \gamma_{c_t}$ is the growth rate of consumption, which can also be written as,

$$\frac{C_{t+1}}{C_t} = \gamma_{c_t} \quad \Rightarrow \quad C_{t+1} = \gamma_{c_t} C_t$$

5.3.2 Households Sector

The households maximise their utility function, which is a function of consumption subject to the budget constraint, with preferences represented by a constant relative risk aversion (CRRA) utility function and parameter $\sigma > 0$. The utility of households depends only on consumption. So, the households supply completely inelastically their labour, which means that they supply at each date $L_t = 1$ to the firms. The consumer's budget constraint at a time t is:

$$R_t K_t + w_t L_t = \left(1+\tau^c\right) C_t + K_{t+1} - K_t + \delta K_t$$

$$C_t,\, K_{t+1} > 0 \quad \forall t \;; \quad K_0 > 0 \quad \textit{exogenously given}$$

where $\beta, \delta \in (0,1)$ are the discount factor and depreciation rate, respectively. $R_t K_t$ is the gross capital income, $w_t L_t$ is the labour wage, C_t is the consumption, τ^c is the consumption tax, and $I_t = K_{t+1} - K_t + \delta K_t$ is the gross investment. So now, we can rewrite the capital accumulation as:

$$K_{t+1} = (R_t + 1 - \delta)\, K_t + w_t L_t - \left(1+\tau^c\right) C_t \tag{5.3}$$

Equation (5.3) represents the households' capital supply. Thus, the Lagrangian for this problem is:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[\left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} \right) + \lambda_t \left((R_t + 1 - \delta)\, K_t + w_t L_t - \left(1+\tau^c\right) C_t - K_{t+1} \right) \right]$$

The first-order conditions:

$$\frac{d\mathcal{L}}{dC_t} = 0 \;\Leftrightarrow\; \beta^t C_t^{-\sigma} = \beta^t \lambda_t \left(1+\tau^c\right) \quad \Rightarrow \quad \frac{C_t^{-\sigma}}{\left(1+\tau^c\right)} = \lambda_t \tag{5.4}$$

$$\frac{d\mathcal{L}}{dK_{t+1}} = 0 \;\Leftrightarrow\; -\beta^t \lambda_t + \beta^{t+1} \lambda_{t+1} \left[R_{t+1} + 1 - \delta \right] = 0 \tag{5.5}$$

$$\frac{d\mathcal{L}}{d\lambda_t} = 0 \quad \Leftrightarrow \quad K_{t+1} = (R_t + 1 - \delta)\, K_t + w_t L_t - \left(1+\tau^c\right) C_t$$

Substitution (5.4) into (5.5) gives us the Euler equation:

$$\frac{C_{t+1}}{C_t} = \left[\beta \left(R_{t+1} + 1 - \delta \right) \right]^{\frac{1}{\sigma}} \tag{5.6}$$

5.3.3 Firms Sector

Following Barro (1990) by introducing positive externalities from the government's purchases of goods and services, the production function used in this economy is the Cobb-Douglas production function. The total output in period t, Y_t, is produced by firms using two inputs, namely, physical capital, K_t, and labour, L_t. Government spending, G_t, is a positive externality. All firms are assumed to be identical, and they act in a perfectly competitive market. Thus, each firm produces output Y_t using the following Cobb-Douglas production function.

$$Y_t = A\,K_t^{\alpha}\,L_t^{1-\alpha}\,G_t^{1-\alpha} \qquad 0 < \alpha < 1 \tag{5.7}$$

where A is the constant and exogenously given technology level in the economy, and α is the output elasticities of capital, labour, and government spending.

The firm's problem is to maximise their profits by taking the rental rate of capital, R_t, and wage per unit of labour, w_t, as given, subject to the firms' production function:

$$\underset{K_t,\,L}{Max}\; A\,K_t^{\alpha}\,L_t^{1-\alpha}\,G_t^{1-\alpha} - R_t K_t - w_t L_t$$

The first-order conditions of the profit maximisation problem:

$$\frac{d\pi_t}{dK_t} = 0 \;\Rightarrow\; \alpha\,A\,K_t^{\alpha-1}\,L_t^{1-\alpha}\,G_t^{1-\alpha} - R_t = 0$$

$$\Rightarrow\; R_t = \alpha\,A\,K_t^{\alpha-1}\,L_t^{1-\alpha}\,G_t^{1-\alpha} = \alpha\,\frac{Y_t}{K_t} \tag{5.8}$$

$$\frac{d\pi_t}{dL_t} = 0 \;\Rightarrow\; (1-\alpha)\,A\,K_t^{\alpha}\,L_t^{-\alpha}\,G_t^{1-\alpha} - w_t = 0$$

$$\Rightarrow\; w_t = (1-\alpha)\,A\,K_t^{\alpha}\,L_t^{-\alpha}\,G_t^{1-\alpha} = (1-\alpha)\,\frac{Y_t}{L_t} \tag{5.9}$$

For equilibrium purposes, we convert the variables in terms of per government spending units (PGSU). Thus, the output PGSU can be written as:

$$\frac{Y_t}{G_t} = \frac{A\,K_t^{\alpha}\,L_t^{1-\alpha}\,G_t^{1-\alpha}}{G_t} \;\Rightarrow\; \frac{Y_t}{G_t} = A\,K_t^{\alpha}\,L_t^{1-\alpha}\,G_t^{-\alpha} \;\Rightarrow\; \hat{y}_t = A\,\hat{k}_t^{\alpha}\,L_t^{1-\alpha} \tag{5.10}$$

where $\hat{y}_t \equiv Y_t / G_t$ is the output PGSU, and $\hat{k}_t \equiv K_t / G_t$ is the capital PGSU. The firms' profit is zero and given by:

$$\hat{y}_t - R_t\hat{k}_t - w_t\hat{l}_t = \hat{y}_t - \alpha\hat{y}_t - (1-\alpha)\,\hat{y}_t = 0 \tag{5.11}$$

5.3.4 Equilibrium

The capital accumulation equation and Euler equation at the equilibrium are affected by boosting the government spending to the firm's sector. To see that, we start with the capital accumulation equation:

$$K_{t+1} = R_t K_t + w_t L_t - \left(1+\tau^c\right) C_t + \left(1-\delta\right) K_t$$

We divide it by the government spending G_t to get PGSU variables. As we defined earlier, the ratio of capital stock over government spending as $\hat{k}_t \equiv K_t / G_t$ and the ratio of labour over government spending as $\hat{l}_t \equiv L_t / G_t$, we now define the new ratio $\hat{c}_t \equiv C_t / G_t$ as a consumption PGSU. After some algebra, the capital accumulation becomes,

$$\frac{K_{t+1}}{G_{t+1}} \cdot \frac{G_{t+1}}{G_t} = R_t \hat{k}_t + w_t \hat{l}_t - \left(1+\tau^c\right) \hat{c}_t + \left(1-\delta\right) \hat{k}_t$$

Considering that the profit of the firms is zero, equation (5.11), and assuming that the aggregate labour is $L_t = 1$, as Barro (1990) where there is no population growth, the capital accumulation at equilibrium can be rewritten as,

$$\hat{k}_{t+1} = \frac{1}{\gamma_{G_t}} \left[\hat{y}_t + \left(1-\delta\right) \hat{k}_t - \left(1+\tau^c\right) \hat{c}_t \right] \tag{5.12}$$

where $\hat{y}_t = A \hat{k}_t^{\alpha}$.

We divide both sides of equation (5.12) by $\hat{k}_t$ to get the growth rate of capital PGSU,

$$\frac{\hat{k}_{t+1}}{\hat{k}_t} = \frac{1}{\gamma_{G_t}} \left[A \hat{k}_t^{\alpha-1} + \left(1-\delta\right) - \left(1+\tau^c\right) \frac{\hat{c}_t}{\hat{k}_t} \right] \tag{5.13}$$

On the other hand, we multiply both sides of the Euler equation, equation (5.6), by G_{t+1} / G_t and do some algebra. After that, the Euler equation at equilibrium becomes,

$$\frac{\hat{c}_{t+1}}{\hat{c}_t} = \frac{1}{\gamma_{G_t}} \left[\beta \left(\alpha A \hat{k}_{t+1}^{\alpha-1} + 1 - \delta \right) \right]^{\frac{1}{\sigma}} \tag{5.14}$$

The transversality condition is:

$$\lim_{t\to\infty} \beta^t \lambda_t \hat{k}_{t+1} = 0$$

$$\lim_{t\to\infty} \beta^t \lambda_t \left(\frac{1}{\gamma_{Gt}} \left[\hat{y}_t + \left(1-\delta\right) \hat{k}_t - \left(1+\tau^c\right) \hat{c}_t \right] \right) = 0$$

These two equations (5.12) and (5.14) plus transversality condition with K_0 given are the main equations that describe the market equilibrium of our economy in terms of PGSU variables.

5.4 Analysis of the Steady-State

Since our model contains two sources of government revenues, namely oil revenues and consumption tax revenues, we write the growth rate of government spending as[61],

$$\gamma_{G_t} \equiv \frac{G_{t+1}}{G_t} = \frac{\pi_{t+1} + \tau^c C_{t+1}}{\pi_t + \tau^c C_t} = g^2 \frac{\left(1-\tau^c \hat{c}_t\right)}{\left(1-\tau^c \hat{c}_{t+1}\right)} \tag{5.15}$$

At the steady-state, we have $\hat{k}_t = \hat{k}_{t+1} = \hat{k}^*$, $\hat{c}_t = \hat{c}_{t+1} = \hat{c}^*$. Therefore, the steady-state equations can be then written as,

$$\hat{k}^* = \left[\frac{1}{\alpha A}\left(\frac{\gamma_G^\sigma}{\beta} - 1 + \delta\right)\right]^{\frac{1}{\alpha-1}} \tag{5.16}$$

$$\hat{c}^* = \frac{1}{\left(1+\tau^c\right)}\left(A\hat{k}^{*\alpha} + \left(1-\delta-\gamma_G\right)\hat{k}^*\right) \tag{5.17}$$

From the growth rate of government spending, equation (5.2), we know that the growth rate of the oil profits is g^2. However, consumption is also growing. Thus, it is worth noting that for any model where the government is financed by two or more sources, in order for the steady-state to not degenerate into one in which one of these sources ends up being negligible as proportion, it has to be in the steady-state that all these sources of revenues grow at a common rate. In our model, the reason is that either the share of oil revenue or consumption tax revenue in the financing of the government spending would tend to zero or would tend to one over time.

Even though it is unlikely to have a steady-state in which the two sources of revenues grow at a different rate, thinking of an equilibrium in which one of these shares tends to zero, and the other tends to one could be a valid equilibrium to consider. Thus, we will next discuss the possibilities for the steady-state by combining the two sources of government revenues.

5.4.1 The Possibilities for Steady-State

Suppose we only consider the government budget constraint, equation (5.1). In that case, we can identify three logically possible ways in which the growth rate of government spending, equation (5.2), could reach a steady-state value.

[61] In Appendix B.1, we show how we arrived at, equation (5.15), the growth rate of government spending when we combine the two sources of government revenues together.

We start with rewriting equation (5.2) as,

$$\gamma_{G_t} \equiv \frac{G_{t+1}}{G_t} = \frac{\pi_{t+1}}{\pi_t}\,\frac{1+\tau^c\left(C_{t+1}/\pi_{t+1}\right)}{1+\tau^c\left(C_t/\pi_t\right)} = g^2\,\frac{1+\tau^c\left(C_{t+1}/\pi_{t+1}\right)}{1+\tau^c\left(C_t/\pi_t\right)} \qquad (5.2)'$$

This expression, equation (5.2)', shows that but with an exception[62] to be mentioned below, we need C_t/π_t to be constant over time in order for γ_{G_t} to be constant over time. Thus, we can see that as $t \to \infty$, there are two ways we can imagine how C_t/π_t could tend to be constant over time:

(i) *The share of consumption relative to oil revenues tends to zero, as time tends to infinity, such that:*

$$C_t/\pi_t \to 0 \quad as \quad t \to \infty$$

(ii) *The share of consumption relative to oil revenues tends to some finite value and strictly positive value, as time tends to infinity, such that:*

$$C_t/\pi_t \;\to\; \textit{some finite value, strictly } (+)\textit{ve}, \quad as \quad t \to \infty$$

If either *(i)* or *(ii)* hold, then the growth rate of government spending tends to the *exogenous growth rate*, as time tends to infinity, such that:

$$\gamma_{G_t} \to g^2 \quad as \quad t \to \infty$$

Nevertheless, there are differences between the steady-states implied by *(i)* and *(ii)*. *(i)* could occur if C_t grows asymptotically at any rate lower than g^2, whereas in order for *(ii)* to occur, C_t needs to grow asymptotically at exactly the rate of g^2.

Looking at these two possibilities, it seems intuitive that the economy would be unlikely to tend to a steady-state of type *(i)*. The reason is that this steady-state would imply that the economy would not be able to take advantage of its growing oil revenues.

Moreover, there is a third possibility for how γ_{G_t} could tend to a time-invariant value. We can also rewrite the growth rate of government spending, γ_{G_t}, equation (5.2), as:

$$\gamma_{G_t} \equiv \frac{G_{t+1}}{G_t} = \frac{C_{t+1}}{C_t}\,\frac{\left(\pi_{t+1}/C_{t+1}\right)+\tau^c}{\left(\pi_t/C_t\right)+\tau^c} = \gamma_{C_t}\,\frac{\left(\pi_{t+1}/C_{t+1}\right)+\tau^c}{\left(\pi_t/C_t\right)+\tau^c} \qquad (5.2)''$$

[62] The exception here implies that we do not always need that C_t/π_t to be constant over time, as we will see.

Looking at the expression in equation (5.2)”, and then considering a situation where:

(iii) *The share of oil revenues relative to consumption tends to zero, as time tends to infinity. Moreover, the growth rate of consumption tends to some finite value and strictly positive value, as time tends to infinity, such that:*

$$\pi_t / C_t \rightarrow 0 \quad as \quad t \rightarrow \infty \quad \left(i.e.\ C_t / \pi_t \ \rightarrow \ \infty\right) \ and$$

$$\gamma_{C_t} = C_{t+1} / C_t \ \rightarrow \ some\ finite\ value,\ \ strictly\ (+)ve, \quad as \quad t \ \rightarrow \infty$$

If *(iii)* holds, then the growth rate of government spending tends to the *endogenous growth rate*, as time tends to infinity, such that:

$$\gamma_{G_t} \rightarrow \gamma_c \quad as \quad t \rightarrow \infty$$

It is clear that for this situation, *(iii)*, to arise, we need that $\gamma_c > g^2$, (*i.e.* consumption grows asymptotically faster than the oil revenues).

In brief, we consider the government budget constraint alone and then show three possible types of steady-states. Two of these steady-states involve that government spending, G_t, grows at the rate of g^2. One of these two steady-states involves that consumption grows at a slower rate than g^2, while the other involves that consumption grows at the rate g^2. The third possible type of steady-state involves that government spending, G_t, tends to the endogenous growth rate when consumption grows asymptotically fast compared to oil revenues.

5.4.2 Two Types of the Steady-States

The first type is that the consumption grows at the same rate as the oil revenues, which means everything grows at the rate of g^2 (*i.e.* $\gamma_{G_t} = g^2$). The second type is that the consumption grows faster than oil revenues, which means everything, in this type, grows at the rate of γ_{c_t} (*i.e.* $\gamma_{G_t} = \gamma_{c_t}$). In both types, we will find the steady-state equations.

5.4.2.1 Type One of Steady-State ($\gamma_{G_t} = g^2$):

In this type, we set the consumption, C_t, grows at the same rate of oil revenues, π_t, such that:

$$\frac{\pi_{t+1}}{\pi_t} = g^2 \quad \Rightarrow \quad \pi_{t+1} = g^2\, \pi_t$$

$$\frac{C_{t+1}}{C_t} = g^2 \quad \Rightarrow \quad C_{t+1} = g^2\, C_t$$

The growth rate of government spending, in this type, is written as,

$$\gamma_{G_t} \equiv \frac{G_{t+1}}{G_t} = \frac{g^2\pi_t + \tau^c g^2 C_t}{\pi_t + \tau^c C_t} = g^2$$

As a result, everything in the economy of this type grows at the rate of g^2, which represents *an exogenous growth rate,* and the steady-state equations can be now written as,

$$\hat{k}^* = \left[\frac{1}{\alpha A}\left(\frac{\left(g^2\right)^\sigma}{\beta} - 1 + \delta\right)\right]^{\frac{1}{\alpha-1}} \tag{5.18}$$

$$\hat{c}^* = \frac{1}{\left(1+\tau^c\right)}\left(A\hat{k}^{*\alpha} + (1-\delta-g^2)\hat{k}^*\right) \tag{5.19}$$

From the steady-state equations, (5.18) and (5.19), we can conclude that if the consumption grows at the same rate of g^2 and the consumption tax is time-invariant, then the steady-state is one where all variables grow at the rate of g^2.

Before we move to type two of the steady-state, it is worth noting that we used Barro's endogenous growth model[63] with only consumption tax, and we thought at the beginning that our model would be an endogenous growth model. However, our model is still an exogenous growth model if we make the consumption grow as oil. The reason is that even though we introduced an exogenous consumption tax, we came to the result that there is only one determinant of the growth rate, meaning all variables grow at the rate of g^2, which is exogenous, as in chapter four.

[63] Barro model (1990) studies income tax as a one source of government revenues in an endogenous growth model, but we study in this chapter only consumption tax instead.

5.4.2.2 Type Two of Steady-State ($\gamma_{G_t} = \gamma_{c_t}$):

In this type, the growth rate of government spending, γ_{G_t}, is equal to the growth rate of consumption, γ_{c_t}. In other words, we look at the case where the consumption grows faster than oil revenues. The growth rate of consumption is,

$$\frac{C_{t+1}}{C_t} = \gamma_{c_t} \quad \Rightarrow \quad C_{t+1} = \gamma_{c_t} C_t$$

In this type, the growth rate of government spending can be written,

$$\gamma_{G_t} \equiv \frac{G_{t+1}}{G_t} = \frac{C_{t+1}}{C_t} \frac{(\pi_{t+1}/C_{t+1}) + \tau^c}{(\pi_t/C_t) + \tau^c} = \gamma_{c_t} \frac{(\pi_{t+1}/C_{t+1}) + \tau^c}{(\pi_t/C_t) + \tau^c}$$

Since the consumption grows faster than oil revenues, then $\pi_t/C_t \to 0$ as $t \to \infty$. In other words, oil revenues, π_t, becomes infinitesimal in comparison to consumption, C_t. Therefore, the growth rate of government spending in this type would end up being $\gamma_{G_t} = \gamma_{c_t}$. As a consequence of that, everything in the economy of this type grows at the rate of γ_{c_t}, which represents *an endogenous growth rate*, and the steady-state equations can be then written as,

$$A\hat{k}^{*\alpha} + (1-\delta-\gamma_c)\,\hat{k}^* - \frac{1+\tau^c}{\tau^c} = 0 \tag{5.20}$$

$$\gamma_c = \left[\beta\left(\alpha A\hat{k}^{*\alpha-1} + 1 - \delta\right)\right]^{\frac{1}{\sigma}} \tag{5.21}$$

The steady-state equations (5.20) and (5.21) confirm that the steady-state in this type is the one where all variables grow at the rate of γ_c. In the next section, we examine two different special cases of the model to find out if one or both of them is/are possible to occur and what would happen in the economy accordingly.

5.4.3 Two Special Cases of the Model

Let us now consider two different special cases for the growth rate. Since we have shown in equation (5.1) that we have two sources of revenues in our model, we want in this section to study them separately in order to investigate what our growth model would represent. The first case is when we have only one source of government revenues, namely oil revenues. The growth rate is g^2, which represents *an exogenous growth model*. The second case is when we have a pure consumption tax revenue as the only government revenue source. The growth rate is γ_{c_t}, which represents *an endogenous growth model*.

In fact, we want to examine these two different cases for two reasons. The first reason is to check if it is possible or not to have $\gamma_c > 1$ because if it is not possible (*i.e.* we only have $\gamma_c < 1$), then we would have a model of *'endogenous shrinkage'* of the economy, not of *'endogenous growth'*. The second reason is to compare these two cases (*i.e. if* $g^2 > \gamma_c$ *or* $g^2 < \gamma_c$) because we want to find out whether a case two steady-state can really exist in our model and whether or not our model would have multiple steady-states.

More precisely, if we are in a situation in which we can only have $g^2 > \gamma_c$, then the steady-state would be the one which we have found in chapter four. On the other hand, if we can have a situation where $g^2 < \gamma_c$, then a case two steady-state would seem possible because π_t / C_t will tend to zero asymptotically, as discussed in situation *(iii)* in section 5.4.1. We believe that one could be a preferable steady-state, while the other could be an undesirable steady-state. The preferable steady-state in this situation is the endogenous growth rate one, while the undesirable steady-state is the exogenous growth rate one. Now, we discuss the two cases of growth in more details.

5.4.3.1 First Case of Growth Rate (*only oil revenues as a source of government revenues*):

The first case is when we have only one source of government revenues, namely oil revenues. This case indeed has been done in chapter four, where we have found that the growth rate of government spending is g^2, which represents *an exogenous growth model*[64]. Moreover, the stability and the transitional dynamics of this case have been previously analysed. Consequently, since the steady-state of this case has been discussed in more details in chapter four, we focus now on the second case.

5.4.3.2 Second Case of Growth Rate (*only consumption tax as a source of government revenues*):

The second case has only a pure consumption tax revenue as one source of government revenues, representing *an endogenous growth model*. This case needs to be considered in more details and understood better. In addition, we seek to study in the second case the stability and the transitional dynamics, if possible, in section 5.6. Thus, the government budget constraint and the growth rate of government spending, in this case, can be written as,

$$G_t = \tau^c C_t \quad \Rightarrow \quad \gamma_{Gt} = \frac{C_{t+1}}{C_t} = \gamma_{c_t}$$

[64] The government budget constraint and the growth rate of government spending in the first case are: $G_t = \pi_t \;\Rightarrow\; \gamma_{G_t} \equiv G_{t+1}/G_t = \pi_{t+1}/\pi_t = g^2$.

The capital accumulation, in this case,

$$\hat{k}_{t+1} = \frac{1}{\gamma_{c_t}} \left[A\hat{k}_t^{\alpha} + (1-\delta)\,\hat{k}_t - (1+\tau^c)\,\hat{c}_t \right] \tag{5.22}$$

where $\hat{k}_t = K_t / G_t \;\Rightarrow\; \hat{k}_t = K_t / \tau^c C_t$ *and* $\hat{c}_t = C_t / G_t \;\Rightarrow\; \hat{c}_t = 1/\tau^c$

We can note that $\hat{c}_t$ now turns out only to depend on τ^c, *i.e.* moving from endogenous to exogenous variable. As we will see later, this conversion is beneficial to solving this particular case of the model.

On the other hand, the Euler equation,

$$\frac{\hat{c}_{t+1}}{\hat{c}_t} = \frac{1}{\gamma_{c_t}} \left[\beta\left(\alpha A \hat{k}_{t+1}^{\alpha-1} + 1 - \delta \right) \right]^{\frac{1}{\sigma}} \tag{5.23}$$

At the steady-state, we have $\hat{k}_t = \hat{k}_{t+1} = \hat{k}^*$ and $\hat{c}_t = \hat{c}_{t+1} = \hat{c}^*$; therefore, the capital accumulation equation (5.22) and the Euler equation (5.23) in a steady-state can be written respectively as,

$$A\hat{k}^{*\alpha} + \left(1-\delta-\gamma_c\right)\,\hat{k}^* - \left(1+\frac{1}{\tau^c}\right) = 0 \tag{5.24}$$

$$\gamma_c = \left[\beta\left(\alpha A \hat{k}^{*\alpha-1} + 1 - \delta \right) \right]^{\frac{1}{\sigma}} \tag{5.25}$$

Equations (5.24) and (5.25) together constitute a pair of simultaneous non-linear steady-state equations in the endogenous variables $\left(\hat{k}^*, \gamma_c\right)$. Therefore, they lie down the steady-state equilibrium of this case.

We now try to characterise how the two endogenous variables, $\left(\hat{k}^*, \gamma_c\right)$, behave in response to changes in exogenous parameters. More precisely, we want to understand how an increase in consumption tax, τ^c, affects γ_c. Although equations (5.24) and (5.25) are non-linear in $\left(\hat{k}^*, \gamma_c\right)$ and hence we cannot obtain an explicit solution for $\left(\hat{k}^*, \gamma_c\right)$, we can try to rewrite the equation to understand how γ_c responses to a change in τ^c.

We need at the beginning to simplify the algebra by restricting some parameters. The reason is that without restricting some parameters, we cannot find an explicit solution (*i.e.* solving for $\hat{k}^*$ as a function of a set of only parameters). For this reason, we set $\sigma = 1$, and then (5.25) becomes:

$$\gamma_c = \beta\left(\alpha A\hat{k}^{*\alpha-1} + 1 - \delta\right) \qquad (5.25)'$$

We substitute (5.25)' into (5.24) in order to reduce the system to a single equation in $\hat{k}^*$ alone and to eliminate γ_c:

$$A\hat{k}^{*\alpha}(1-\beta\alpha) + (1-\delta-\beta+\beta\delta)\,\hat{k}^* - \left(1 + \frac{1}{\tau^c}\right) = 0 \qquad (5.26)$$

Although we still cannot solve this equation, we can try to solve how $\hat{k}^*$ behaves as a function of τ^c by using a diagram.

Rearranging (5.26) gives us,

$$A\hat{k}^{*\alpha}(1-\beta\alpha) = \left(1 + \frac{1}{\tau^c}\right) - (1-\delta-\beta+\beta\delta)\,\hat{k}^* \qquad (5.26)'$$

Now we can sketch (5.26)', where the left-hand side *(LHS)* has the same shape as the '*intensive form*' production function. The right-hand side *(RHS)* has a decreasing linear form, $-(1-\delta-\beta+\beta\delta)\,\hat{k}^*$, with a vertical intercept of $1 + (1/\tau^c)$. Thus, $\hat{k}^*$ can be depicted in Diagram 10 as the horizontal coordinate of the point of intersection of the following two curves.

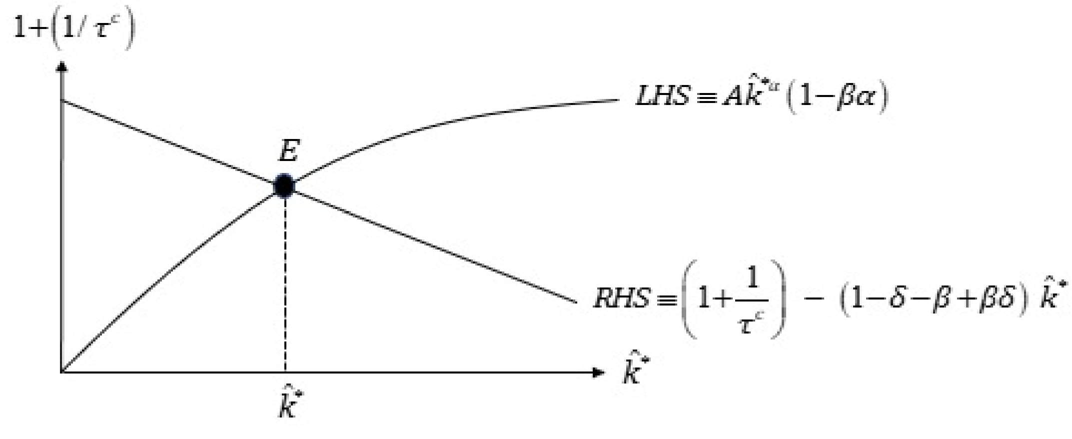


Diagram 10: Drawing equation (5.26)'

Let us suppose that τ^c increases. In this case, the vertical intercept of the *RHS* function shifts down, and the slope is unchanged, as shown in Diagram 11.

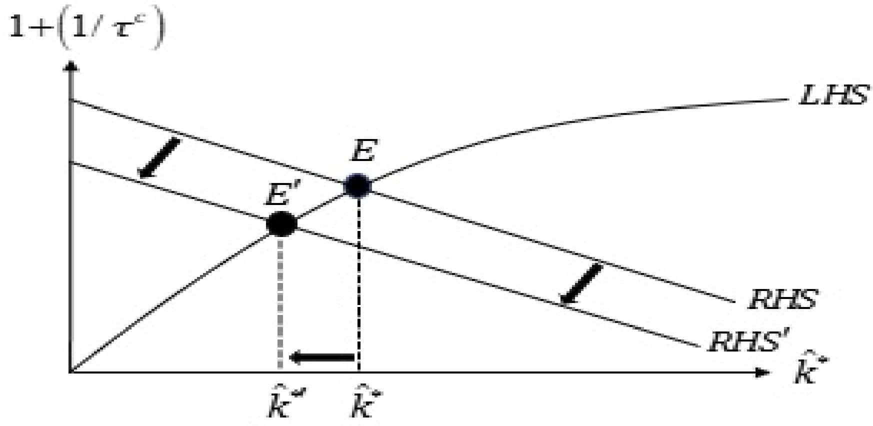


Diagram 11: Drawing the change of consumption tax in equation (5.26)'

Therefore, it is unambiguous from Diagram 11 that the intersection moves to the left due to increasing τ^c, so that $\hat{k}^*$ falls, as shown above.

Referring back to (5.25)', a fall in $\hat{k}^*$ clearly implies a rise in γ_c. As a result, we can unambiguously conclude that:

$$\frac{d\gamma_c}{d\tau^c} > 0 \qquad \textit{meaning} \qquad \uparrow \tau^c \;\Rightarrow\; \downarrow \hat{k}^* \;\Rightarrow\; \uparrow \gamma_c$$

Before carrying on our analysis, it is worth considering the maximum value for consumption tax, τ^c. In general, the consumption tax is different from income tax in some characteristics and functions. In terms of their rates, income tax takes the rate between zero and one, where the value that maximises the growth rate can be found as in Barro (1990). The value that maximises government revenues can also be seen in the 'Laffer's curve' theory[65]. On the other side, consumption tax could be between zero and one in some situations (*e.g.* sales tax), while it could be more than one (*i.e.* >100%) in some other situations (*e.g.* taxes on soft and energy drinks, Tobacco and Alcohol, and oil and its derivatives). Thus, mathematically, it is obvious that the maximum value for the consumption tax could be infinite ($\tau^c = \infty$), which is unlike income tax.

Regarding our model, the maximum value for the consumption tax would produce the most significant possible reduction in the vertical intercept of the *RHS* function, lowering it to one, since $1/\tau^c$ then equals zero. This would lead to the lower possible value of $\hat{k}^*$, and thus (via (5.25)') to the highest possible value of γ_c. However, it should be noted that $\tau^c = \infty$ implies in our model

[65] The Laffer curve demonstrates a theoretical relationship between tax rates and government revenue levels as a result. The main idea of the Laffer curve is that at a certain tax rate, government revenues will be at their maximum value, and if the tax rate increases then government revenues will decrease.

$\hat{c} = 0$, since $\hat{c} = 1/\tau^c$. In other words, the ratio of consumption PGSU, C_t/G_t, is driven to zero, which means that at the growth maximising steady-state, there is no consumption. All of the output is devoted either to investment or government spending (in the latter case, via consumption tax revenues being exchanged on the international market by the government for government spending, such that $Y_t = (1 + \tau^c)C_t + I_t$). As a result, the maximum value for the consumption tax in our model could not correspond to welfare-maximisation.

In fact, the previous analysis has not shown clearly that if it is possible or not to have $\gamma_c > 1$. Thus, we now try to investigate that in more details because knowing either $\gamma_c > 1$ or $\gamma_c < 1$ would help us to understand the possibility of having endogenous growth in our model.

By looking at (5.25)', we can see that, in the limit as $\beta \to 1$ and $\delta = 0$, we may have $\gamma_c > 1$, unless $\alpha A\hat{k}^{*\alpha-1} \to 0$. However, since $\hat{k}^*$ is an endogenous variable and will, therefore, change as $\beta \to 1$ the possibility that $\alpha A\hat{k}^{*\alpha-1} \to 0$ needs to be considered.

To see what happens to $\hat{k}^*$, and thus to $\alpha A\hat{k}^{*\alpha-1}$ as $\beta \to 1$, we consider again (5.26).

Assuming $\beta \to 1$ in (5.26) gives us:

$$A\hat{k}^{*\alpha}(1-\alpha) = \frac{1+\tau^c}{\tau^c}$$

Therefore,

$$\hat{k}^* = \left(\frac{1+\tau^c}{A(1-\alpha)\tau^c}\right)^{\frac{1}{\alpha}}$$

On the diagram, this can be depicted as the special case in which the *RHS* line is horizontal. Hence, it is obvious from the diagram that there is no discontinuity in the behaviour of $\hat{k}^*$ as $\beta \to 1$. Therefore, we conclude that, for $\beta < 1$, but arbitrarily close to one, $\hat{k}^*$ will be arbitrarily close to the value in the above expression.

We deduce that, as $\beta \to 1$, $\alpha A\hat{k}^{*\alpha-1}$ tends to

$$\alpha A\hat{k}^{*\alpha-1} = \alpha A^{\frac{1}{\alpha}}\left(\frac{(1-\alpha)\tau^c}{1+\tau^c}\right)^{\frac{1-\alpha}{\alpha}}$$

and thus that γ_c , equation (5.25)’, tends to

$$\gamma_c = \alpha A^{\frac{1}{\alpha}}\left(\frac{(1-\alpha)\tau^c}{1+\tau^c}\right)^{\frac{1-\alpha}{\alpha}} + 1 - \delta \qquad (5.27)$$

Although we cannot just set $\beta = 1$ in the model because this makes households' lifetime utility infinity even in a steady-state, we can make β arbitrarily close to one, which is still being less than one. This means we can make γ_c arbitrarily close to the value in the above formula.

However, by looking at the above formula of γ_c, equation (5.27), we found that it depends on the value of δ and τ^c. As a result, by making β arbitrarily close to one, we obtain two possible situations for γ_c. The first possible situation is that if δ is close to one and τ^c is very small (*i.e.* τ^c is close to zero, so that $1/\tau^c = \infty$), then $\alpha A^{1/\alpha}[(1-\alpha)\tau^c/(1+\tau^c)]^{1-\alpha/\alpha} \rightarrow 0$, meaning that $\alpha A^{1/\alpha}[(1-\alpha)\tau^c/(1+\tau^c)]^{1-\alpha/\alpha} < \delta$, then we obtain $\gamma_c < 1$. The second possible situation, on the other hand, is that if δ is zero and τ^c is any positive value, or if $\alpha A^{1/\alpha}[(1-\alpha)\tau^c/(1+\tau^c)]^{1-\alpha/\alpha} > \delta$, then we must have $\gamma_c > 1$. Therefore, we now can see that $\gamma_c > 1$ is possible to obtain in our model, which in turn implies to have the possibility of $\gamma_c > g^2$, because g^2 is an exogenous parameter, and we can choose a value of g^2, which is greater than one and less than γ_c.

What we have discussed previously is the two special cases of the model. Thus, we now return to the general situation, *i.e.* we go back to our entire model. As long as both cases, $\gamma_c > g^2$ and $\gamma_c < g^2$ would be possible to occur, we need now to understand what would happen in the economy if one of these cases occurs. Let us start with the case where $\gamma_c > g^2$. In this case, although it starts out a situation where oil revenue is a significant share of government revenues, if the economy converges to the γ_c steady-state, then oil revenue would shrink over time as a share of the government budget. The reason is that the long-run endogenous growth rate, to which the economy's path is tending, is greater than g^2. That means the share of consumption tax revenues in total government revenues must become 100%, and the share of oil revenues must tend to zero. Thus, this case would produce long-run behaviour that is different from the case where $\gamma_c < g^2$ because the latter case would show the opposite since the economy at the steady-state would converge on g^2. The reason behind that is because the g^2 steady-state, in this case, would dominate since oil revenues remain very important, and that is unlike the consumption tax revenues, which would tend to zero over time.

Now, there is one fundamental question that needs to be investigated. The question is, what would happen in the dynamics if we are in a situation where $\gamma_c > g^2$ [66]. In fact, this is an essential question because we know that if we consider this case, it seems possible that we will not converge to a steady-state where the growth rate is g^2, but will converge to a steady-state where the growth

[66] The dynamics in the situation where $\gamma_c < g^2$ is similar to what we have done in chapter four; thus, it will not be discussed here.

rate is γ_c since it is simply higher than g^2. As a result, what happens during the transition, if any, of this case is a key important thing that needs to be checked and studied. Thus, section 5.6 attempts to answer this question. However, before we move on to the stability and transitional dynamics in the pure consumption tax case, it is crucial to study whether both types of steady-state can exist for the same set of parameter values or whether there is just one.

5.4.4 Existence of Steady-States of Both Types

This section investigates if both types of steady-states can exist for the same set of parameter values. The first type of steady-state (*I*) is the *exogenous growth steady-state*, where C_t/π_t tends to a finite, non-zero, value, and the growth rate is g^2. The second type of steady-state (*II*) is the *endogenous growth steady-state*, where C_t/π_t tends to infinity, and the growth rate is γ_{c_t} where $\gamma_{c_t} > g^2$, and γ_{c_t} is a function of τ^c amongst other parameters.

We attempt in this section to draw a diagram (not a phase diagram), but in the space of $\hat{c}^*$ and $\hat{k}^*$. This diagram contains some curves in which if they have two points of intersection that represents two simultaneous steady-states but of different types. However, after all[67], we have found a contradiction, in which two conditions completely conflict with each other. These conditions are (*i*) C_t/π_t cannot be negative and (*ii*) $\gamma_{c_t} > g^2$, where if one of them holds, then the other cannot hold and vice versa. Therefore, since there is an unavoidable contradiction between the two conditions, we conclude that type (*I*) and type (*II*) steady-states cannot exist for the same set of parameter values.

What we have indeed found is that for a given set of parameter values, the steady-state will be either of type (*I*) or type (*II*), *i.e.* there will be a single type of steady-state. In more precise detail,

- *If g^2 is sufficiently high (for a given value of τ^c), then the steady-state will be of type (I), i.e. an exogenous growth steady-state.*

- *If g^2 is sufficiently low (for a given value of τ^c), then the steady-state will be of type (II), i.e. an endogenous growth steady-state.*

As a result, there can still be two steady-states, but they cannot exist for the same set of parameter values. In other words, for one set of parameter values, either the steady-state will be the exogenous growth steady-state and will not exist any other type, or for a different set of parameter values, instead, the only type of steady-state will be the endogenous growth steady-state.

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[67] A detailed proof is given in Appendix B.2 to show that both types of steady-states cannot exist for the same set of parameter values.

What we have shown previously is that we hold consumption tax, τ^c, constant and set different values of g^2 to find out that the steady-state will be either of type *(I)* steady-state, *i.e. exogenous growth steady-state*, or type *(II)* steady-state, *i.e. endogenous growth steady-state*. Since this perception is just about the steady-state, leaving aside stability and transitional dynamics, we can further imagine another way to think about it. Thus, we would now study the properties of the economy and how it responds to an increase in consumption tax, contrary to what we have done before. In other words, we hold g^2 constant and discuss at length the different values of τ^c. The main reason for investigating consumption tax is due to the fact that it is a policy parameter, where the authority cannot set g^2 because it is exogenous and out of their control, but they can change consumption tax instead. Thus, we aim now to discover how and when the economy could move from a steady-state to another if consumption tax changes.

We start with making g^2 at a certain fixed value, and we consider different values of τ^c. Suppose that, initially, the economy is in a steady-state with no consumption tax, *i.e.* $\tau^c = 0$. Then, its growth rate will be clearly type *(I)* steady-state, which is g^2. Now, suppose that the government introduces a consumption tax, but at first sets at a low level, *i.e.* the value is positive but still close to zero. If it allows the economy to settle into a new steady-state, then that steady-state will again be one in which the growth rate is g^2. Then, suppose the government increases the consumption tax, but also still quite close to zero. Likewise, the economy will once move to settle into a type *(I)* steady-state, which is still a growth rate of g^2. This cycle of events could then be repeated several times. Although the economy will have each time some disturbance, adjustment, and transitional dynamics, it will settle into type *(I)* steady-state. The reason behind that is due to the fact that g^2 may still be sufficiently high compared to γ_c, as discussed previously. However, at a certain value of τ^c, (let us call this value a critical value of τ^c, such as $\bar{\tau}^c$), the economy will now turn out to type *(II)* steady-state, in which the growth rate becomes now $\gamma_c > g^2$. In other words, at this point, instead of returning the economy to a growth rate of g^2, it switches now to a growth rate of γ_c. This represents a complete switch of the regime, where the economy converges to a different type of steady-state, which is a higher growth rate, *i.e.* permanent higher growth rate, which depends now on τ^c. The explanation is that taxes would enable to provide sufficient government spending to maintain growth through the firms' production function.

In the diagram below, we attempt to show how and when the economy could move from one steady-state to another if we change consumption tax. We sketch a consumption tax, τ^c, in the horizontal axis and the steady-state growth rate in the vertical axis. The flat part in the diagram represents the type *(I)* steady-state for the lower values of τ^c. However, at a certain value of τ^c, *i.e.* at the critical value of τ^c, we move to type *(II)* steady-state. In other words, as we gradually increase τ^c, eventually the economy will pass from an exogenous growth steady-state to an endogenous growth steady-state. We could interpret the critical value of τ^c at which this switch occurs as being the 'take-off' point for the economy, *i.e.* the curve at that point becomes upsloping because high τ^c produces a high growth rate. Thus, as τ^c is increased, beyond this point, the steady-state growth rate of the economy is eventually 'freed' from the growth rate of oil profits; whereas, until it is reached, the steady-state growth rate is tied to the growth rate of oil profits. The below diagram displays the switching regime due to changing τ^c.

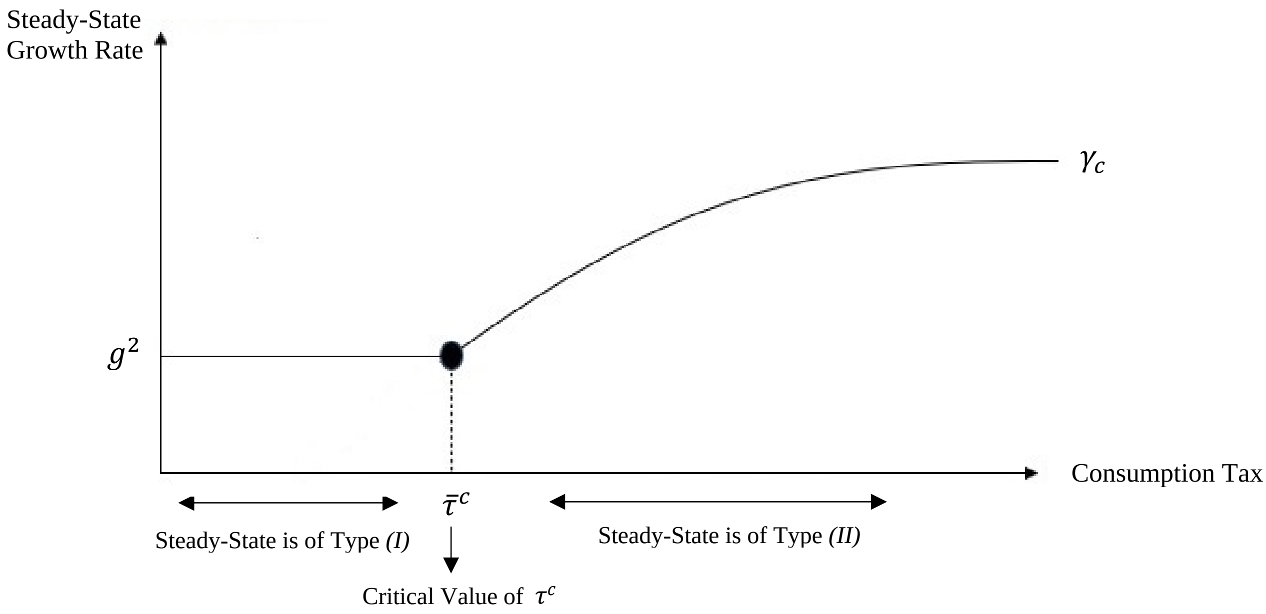

Diagram 12: How and when the economy could move from one steady-state to another one when consumption tax changes

Let us now imagine the story to be a possible scenario for the Saudi economy. First of all, suppose the authority in Saudi Arabia follows the advice by the IMF to implement a very low consumption tax, *e.g.* $\tau^c = 5\%$, in order to see what would happen in the economy by this change. In fact, according to our model, a bit of thing would undoubtedly occur in the economy. For instance, there will be a shock, the steady-state will shift, and also the level of variables would somewhat change. However, the steady-state type will still be type *(I)*, meaning we still remain in the low range of consumption tax, as shown in the above diagram. Similarly, if the authority decides to raise consumption tax further, but still at a lower level, *e.g.* $\tau^c = 10\%$. Although this slight increase in τ^c would cause some adjustment and transitional dynamics in the economy, the economy, as explained previously, would still in the growth rate of g^2. However, there exists a critical value of τ^c, and let us suppose this critical value is $\bar{\tau}^c = 25\%$. Thus, the increase in τ^c would not help the economy to move to another steady-state unless it passes this critical value. More precisely, if the Saudi authority decides again to raise the consumption tax, *e.g.* from 20% to 30%, then they may believe at the beginning that they will be at the type *(I)* steady-state, but since they have already passed the critical value, $\bar{\tau}^c = 25\%$, then the economy will be at the type *(II)* steady-state, which is the endogenous growth one. Moreover, if the authority raises more the consumption tax, *e.g.* $\tau^c = 35\%$, then the growth rate would increase as well because the economy now in type *(II)* steady-state, where we know that within type *(II)* steady-state range, an increase in τ^c increases the growth rate indeed.

The motivation behind studying this aspect is our conclusion in section 5.4.3.2, which discussed the case in which we have only consumption tax as a source of government revenues to show how endogenous variables respond to change in consumption tax. In that section, after simplifying the key equations, we have arrived at a result that increasing consumption tax at steady-state would fall the capital PGSU, $\hat{k}^*$, and would eventually increase the endogenous growth rate, γ_c. Thus, we attempted to link this conclusion with the first case, which is considered oil revenues as the only source of government revenues. After all, it is obvious that if the consumption tax increases, it implies that the endogenous growth rate would increase too. This result ensures that $\gamma_c > 1$, which is the condition to have $\gamma_c > g^2$. However, if the consumption tax decreases, the endogenous growth rate would decline as well. In this case, γ_c would not be guaranteed to be greater than one. In other words, γ_c would be most likely less than one, $\gamma_c < 1$. This scenario implies that the economy would return to type *(I)* steady-state, which is the exogenous growth steady-state. This type of steady-state is now higher than type *(II)* steady-state because g^2 is assumed to be $g^2 \geq 1$. As a result, low rates of consumption tax in our model implies that $g^2 > \gamma_c$, while high consumption tax rates imply that $g^2 < \gamma_c$. In between of them, there is a critical value of consumption tax, $\bar{\tau}^c$, that could determine when the economy could turn out from one type of steady-state to another one.

To summarise our perception of this policy implications, if consumption tax sets to be at low values, then nothing very significant would happen. Indeed, something in the economy would change, but not a huge change would happen. However, at a certain point of consumption tax, the economy would have significant change, where it moves from the *exogenous growth steady-state* to the *endogenous growth steady-state*. The latter type would be a better situation because it is higher than the *exogenous growth steady-state*.

5.5 Analysis of the Effect of Consumption Tax on the Level Variables in a Type *(I)* Steady-State

Like the previous chapter, any level variable, discussed in this section and henceforth, refers to a variable that is not a 'growth rate'[68]. Since each level variable is constantly growing, even in the steady-state, we cannot attempt to solve for a single, time-invariant, value of it, such as consumption, C^*. However, what we can do instead, for each level variables, is to calculate how increasing consumption tax, $\uparrow \tau^c$, affect its steady-state value relative to oil revenues, π_t. Comparing its value with π_t is helpful because we know that the growth path of π_t is unaffected by $\uparrow \tau^c$.

In this section, we want to understand how the level variables respond when we change the consumption tax. From our analysis in the first case where we set both consumption and oil revenues to grow at the same rate as g^2 [69], we know that changing the consumption tax, τ^c, does not affect the steady-state growth rate, which remains at g^2. It is also apparent that from equations (5.18) and (5.19) that changing consumption tax does not affect the capital PGSU steady-state, $\hat{k}^*$, because it does not enter in equation (5.18). However, it has an impact on the consumption of PGSU steady-state, $\hat{c}^*$. Nevertheless, changing the consumption tax, τ^c, will generally affect the level variables, and this is an effect which could not be consisted in chapter four.

Therefore, let us now consider and study the key endogenous level variables when changing the consumption tax. In other words, we want to examine for any steady-state time path it whether lies above or below the level which it would have had at the previous consumption tax.

To do that, we first analyse the effect of changing the consumption tax, τ^c, on the capital PGSU steady-state and consumption PGSU steady-state. For simplicity, let us define $\omega^* = A\hat{k}^{*\alpha} + (1 - \delta - \gamma_G)\,\hat{k}^*$. As long as the steady-state of capital PGSU, $\hat{k}^*$, is independent of consumption tax, then ω^* would be constant. However, if consumption tax changes, the consumption PGSU steady-state would be negatively affected, such that,

$$\frac{d\hat{c}^*}{d\tau^{c*}} = -\frac{\omega^*}{\left(1+\tau^c\right)^2} < 0 \tag{5.19$'$}$$

[68] In chapter four, we have discussed the level variables as a special case.

[69] That means $\gamma_{G_t} = g^2$.

Equation (5.19)' shows that increasing consumption tax, τ^c, reduces the consumption PGSU, $\hat{c}^*$. So now, we want to find out how increasing the consumption tax would affect $\tau^c\hat{c}^*$. The reason is that knowing how $\tau^c\hat{c}^*$ is affected by changing τ^c would help us in analysing the level variables because all of them would contain $\tau^c\hat{c}^*$, as we will see.

Substituting equation (5.19) into $\tau^c\hat{c}^*$, gives us,

$$\tau^c\hat{c}^* \;\Rightarrow\; \frac{\tau^c}{\left(1+\tau^c\right)}\,\omega^* \tag{5.28}$$

Taking the derivative of (5.28) *w.r.t.* τ^c,

$$\frac{\omega^*}{\left(1+\tau^c\right)^2} > 0 \tag{5.28$'$}$$

The sign is positive, which implies that if τ^c increases, then $\tau^c\hat{c}^*$ would also increase, such as:

$$\uparrow\tau^c \;\Rightarrow\; \downarrow\hat{c}^* \;\Rightarrow\; \uparrow\tau^c\hat{c}^*$$

5.5.1 Level of Government Spending

We start with the level of government spending, G_t, because other variables, such as consumption and capital stock, are determined relative to, G_t, by determining $\hat{c}_t$ and $\hat{k}_t$. So, we divide the government budget constraint by G_t and then rearrange it, to have,

$$\frac{G_t}{\pi_t} = \frac{1}{1-\tau^c\hat{c}_t} \tag{5.29}$$

Substituting the steady-state of consumption PGSU in equation (5.29) gives us,

$$\frac{G_t}{\pi_t} = \frac{1}{1-\dfrac{\tau^c}{1+\tau^c}\,\omega^*} \tag{5.29$'$}$$

Taking the derivative of (5.29)' *w.r.t.* τ^c, gives us

$$\frac{d\left(G_t/\pi_t\right)}{d\tau^c} = \frac{\omega^*}{\left(1+\tau^c-\tau^c\omega^*\right)^2} > 0 \tag{5.29$''$}$$

Since ω^* is not affected by changing consumption tax, τ^c, and we do not have endogenous labour supply in our model, there is a positive effect on output working through government spending. Thus, if consumption tax increases, then the level of government spending relative to the π_t would increase too. The diagram below shows the time path of government spending relative to the π_t.

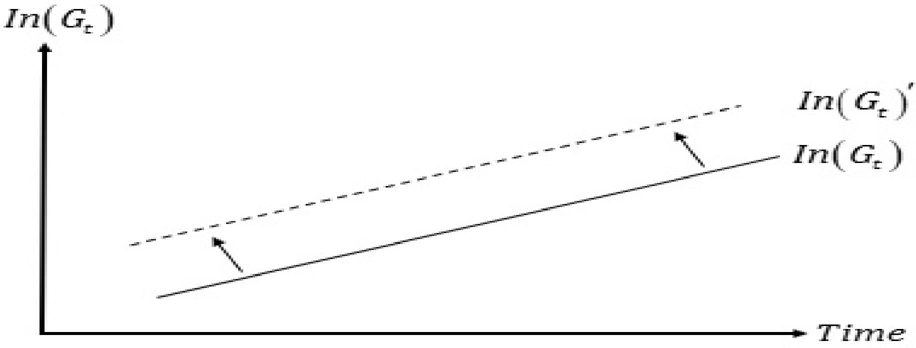

Diagram 13: The time path of government spending relative to the π_t

The intuition behind this result is that introducing a consumption tax would increase government revenues which would, in turn, increase government spending[70].

5.5.2 Level of Capital Stock

The capital PGSU can be written as,

$$\hat{k}_t = \frac{K_t}{G_t} \quad \Rightarrow \quad K_t = \hat{k}_t G_t$$

Dividing both sides by π_t,

$$\frac{K_t}{\pi_t} = \hat{k}_t \frac{G_t}{\pi_t} \tag{5.30}$$

We know that the capital PGSU at steady-state, $\hat{k}^*$, equation (5.18), is constant over time and independent of consumption tax, but the level of government spending relative to the π_t is affected by changing the consumption tax. Thus, increasing the consumption tax would increase G_t/π_t , as shown before, and then increase the level of capital relative to the π_t, by the same proportion. The reason for increasing capital stock can be explained by increasing government spending, which in turn encourages firms to have more capital. The diagram below shows the time path of capital relative to the π_t.

[70] Since it was assumed in our model that the government sector spends all its revenues, then the government revenues must equal government spending.

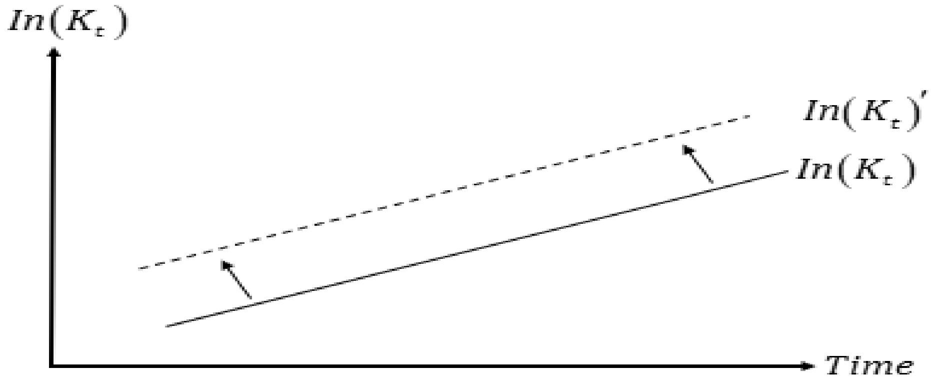

Diagram 14: The time path of capital stock relative to the π_t

To compare our results in the level of capital stock with the standard Ramsey growth model, we discover that the steady-state level of capital stock is not affected by a consumption tax in the Ramsey model with both income and consumption taxes. Thus, the long-run process of capital accumulation is not affected by taxing consumption (Novales, Ruiz and Fernández, 2014). However, in our model, the consumption tax would positively impact the level of capital stock. The reason is that increasing the consumption tax would increase government revenues, which is reflected in increased government spending. High government spending implies more positive externalities enters in firm's production function. This, in turn, enhances the firms' production function and leads to an increase in their level of capital and output.

5.5.3 Level of Consumption

We know that the consumption PGSU and the government budget constraint are written, respectively as $\hat{c}_t = C_t / G_t$ and $G_t = \pi_t + \tau^c C_t$, then the level of consumption relative to π_t can be written as,

$$\frac{C_t}{\pi_t} = \frac{\hat{c}_t}{1-\tau^c \hat{c}_t} \tag{5.31}$$

Substituting the steady-state of consumption PGSU in equation (5.31) gives us,

$$\frac{C_t}{\pi_t} = \frac{\omega^*}{1+\tau^c - \tau^c \omega^*} \tag{5.31$'$}$$

Equation (5.31)' shows that the change in the time path of consumption relative to the π_t is ambiguous when we change the consumption tax. Therefore, let us take the derivative of (5.31)' *w.r.t.* τ^c,

$$\frac{d(C_t/\pi_t)}{d\tau^c} = \frac{\omega^*(\omega^*-1)}{(1+\tau^c-\tau^c\omega^*)^2} \tag{5.31$''$}$$

The sign of $d(C_t/\pi_t)/d\tau^c$ is still ambiguous because the value of ω^* is crucial here. ω^*, in fact, depends on many parameters. To see that,

$$\omega^* = A\hat{k}^{*\alpha} + (1-\delta-\gamma_G)\,\hat{k}^*$$

Let us now investigate ω^* further. Thus, substituting the steady-state of capital PGSU into this gives us,

$$\omega^* = A\left[\frac{1}{\alpha A}\left(\frac{\gamma_G^\sigma}{\beta} - 1 + \delta\right)\right]^{\frac{\alpha}{\alpha-1}} + (1-\delta-\gamma_G)\left[\frac{1}{\alpha A}\left(\frac{\gamma_G^\sigma}{\beta} - 1 + \delta\right)\right]^{\frac{1}{\alpha-1}} \tag{5.32}$$

where the parameters and their conditions are $\alpha, \beta, \delta \in (0,1)$, $\gamma_G \geq 1$, $\sigma > 0$. Let us now try to find out the possible values of ω^*. Thus, if we assume that $A, \gamma_G, \sigma = 1$ and $\delta = 0$, then,

$$\omega^* = \left[\frac{\alpha\beta}{(1-\beta)}\right]^{\frac{\alpha}{1-\alpha}}$$

If we assume further that $\alpha = 0.5$, then ω^* becomes,

$$\omega^* = \frac{\beta}{2(1-\beta)}$$

So now, ω^* depends on the value of the discount factor, β, which measures the patience to consume now or in the future. In this case, if β is sufficiently close to zero, meaning we are not very patient to consume more in the future, then $\omega^* < 1$. However, if β is sufficiently close to one, meaning we are very patient to consume more in the future, then $\omega^* > 1$. As a result, if the value of ω^* is greater than one, $\omega^* > 1$, that means increasing consumption tax increases the time path of consumption relative to the π_t, as shown in the below diagram, $ln(C_t)'$. If, however, the value of ω^* is less than one, $\omega^* < 1$, that means increasing consumption tax decreases the time path of consumption relative to the π_t, as shown in the below diagram, $ln(C_t)''$ [71].

[71] In Appendix B.3, we attempt to study what happens not just as β changes, but also as $\alpha, A, \gamma_G, \sigma$ and δ change.

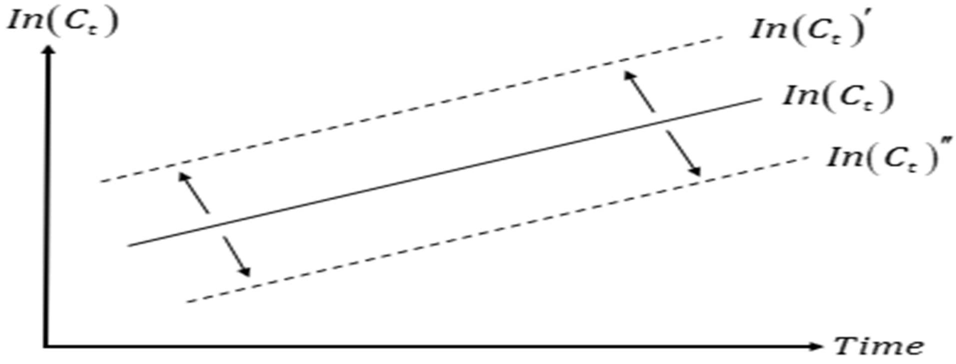

Diagram 15: The time path of consumption relative to the π_t

The reason for increasing the level of consumption when we increase consumption tax may be due to the fact that the households benefit from increased government spending through the firms. That means the firms could somehow pass on some of the positive externalities received through the government to the households. Examples of this may be high wages or a high rental rate of capital.

5.6 Are There Any Transitional Dynamics in Pure Consumption Tax Case, Type *(II)* Steady-State?

Sections 5.4 and 5.5 have discussed the steady-state. Thus, we move now to study the dynamics of our model. In this section, we go back to the question of the stability and the transitional dynamics in the pure consumption tax case, *i.e.* type *(II)* steady-state. Our purpose here is to find out if there are any transitional dynamics in this case, *i.e.* the 'pure consumption tax case'.

To analyse that, we start with the capital accumulation equation (5.22) and Euler equation (5.23). In this case, we get from the definition of consumption PGSU that $\hat{c}_t = 1/\tau^c$. Thus, since the consumption tax, τ^c, is exogenous, then $\hat{c}_t$ is exogenous here too. The capital accumulation equation becomes,

$$\hat{k}_{t+1} = \frac{1}{\gamma_{c_t}} \left[A\hat{k}_t^{\alpha} + (1-\delta)\,\hat{k}_t - \frac{1+\tau^c}{\tau^c} \right] \tag{5.22'}$$

On the other hand, from the Euler equation and the definition of consumption PGSU, we obtain $\hat{c}_{t+1}/\hat{c}_t = 1$. That is true even if the economy is not in a steady-state. Hence, Euler equation (5.23) can be now written as,

$$\gamma_{c_t} = \left[\beta\left(\alpha A\hat{k}_{t+1}^{\alpha-1} + 1 - \delta \right) \right]^{\frac{1}{\sigma}} \tag{5.23'}$$

If we combine equations (5.22)' and (5.23)' to eliminate γ_{c_t}, we obtain,

$$\hat{k}_{t+1} = \frac{A\hat{k}_t^{\alpha} + (1-\delta)\,\hat{k}_t - \dfrac{1+\tau^c}{\tau^c}}{\left[\beta\left(\alpha A\hat{k}_{t+1}^{\alpha-1} + 1 - \delta\right)\right]^{\frac{1}{\sigma}}} \tag{5.33}$$

Equation (5.33) is an implicit first-order difference equation in $\hat{k}_t$. From this equation, we can see that there are no other endogenous variables. Thus, this equation in principle should be sufficient, by itself, to determine the time path of $\hat{k}_t$.

However, we know that the definition of consumption PGSU and capital PGSU, in this case, imply that,

$$\hat{c}_t = \frac{C_t}{G_t} = \frac{C_t}{\tau^c C_t} = \frac{1}{\tau^c} \qquad ; \qquad \hat{k}_t = \frac{K_t}{G_t} = \frac{K_t}{\tau^c C_t}$$

Thus, it is clear from the definition of consumption PGSU that there is no dynamics in $\hat{c}_t$. The reason is that the consumption tax, τ^c, in our model, is time-invariant. On the other hand, from the definition of capital PGSU, we understand that capital stock, K_t, is undoubtedly a predetermined variable while consumption, C_t, is generally a non-predetermined variable. Thus, if C_t can jump, when something unexpected happens, then G_t must also jump. Therefore, $\hat{k}_t$ becomes a non-predetermined variable because for $\hat{k}_t$ to be a predetermined variable, all of the variables which comprise it, must be predetermined variables (*i.e.* as soon as one of the components of $\hat{k}_t$ can jump, it must be the case that $\hat{k}_t$ itself can jump as well).

We know from Blanchard and Kahn (B-K) (1980) that if the number of eigenvalues of a matrix outside the unit circle is equal to the number of non-predetermined variables, then a unique non-divergent solution exists. In the case of pure consumption tax, $\hat{k}_t$ is a non-predetermined (jump) variable. We have also shown that there is only a single first-order different equation, equation (5.33), which is the law of motion for the capital PGSU. That means that it will produce only one eigenvalue in this system. However, the question now is to investigate whether the implicit differential equation, equation (5.33), for $\hat{k}_t$ is stable or unstable. After all[72], we found that this single eigenvalue is unstable. Thus, when we apply the conditions of B-K, we found that it is also satisfied since we have one unstable (outside the unit circle) eigenvalue, and $\hat{k}_t$ is a non-predetermined variable. As a result, because of $\hat{k}_t$ is a non-predetermined variable in period t, it

[72] In Appendix B.4, we prove that equation (5.33) is unstable.

can jump to its steady-state. In general, the equation of $\hat{k}_{t+1}$, equation (5.33), will not determine a time-varying path for $\hat{k}_t$. As a result, there is no transitional dynamics in this case, according to B-K conditions. So, the economy jumps to a new steady-state, following any shock.

Even though we have found in the pure consumption tax case the same result as in the Barro model (1990) that there is no transitional dynamics in both models, there are some apparent differences between our case and the Barro model (1990). One of them is that Barro is assuming income tax in his original model, while we are assuming, in this case, consumption tax instead. Another difference is regarding the method of proof. In Barro, the reason for the absence of the transitional dynamics is that the Euler equation is the only difference equation, where the interest rate in his model, r, is exogenous, which depends on a set of other exogenous parameters. Thus, since the growth rate of consumption in his model does not depend on consumption, the growth rate does not change over time. Therefore, there is no transitional dynamics[73], where the growth rate of consumption, capital, and output all equal the same constant. On the other hand, the only difference equation in our model is equation (5.33), which has explained previously.

5.7 Analysis of the Stability of a Type *(I)* Steady-State

Since we have found in section 5.6 that the steady-state is unstable and there is no transitional dynamics in type *(II)* steady-state, we now analyse the local stability of a type *(I)* steady-state. This section combines the two sources of government revenues, oil revenues and consumption tax revenues. If both sources of government revenues are simultaneously considered, then the growth rate of government spending can be written as,

$$\gamma_{G_t} \equiv \frac{G_{t+1}}{G_t} = \frac{\pi_{t+1} + \tau^c C_{t+1}}{\pi_t + \tau^c C_t} = g^2 \frac{\left(1-\tau^c \hat{c}_t\right)}{\left(1-\tau^c \hat{c}_{t+1}\right)}$$

To analyse the stability in our model, we construct the linear approximation to the system around the steady-state, where the PGSU variables in the steady-state will stay constant all the time. So now, we start with the capital accumulation equation,

$$\hat{k}_{t+1} = \left[\frac{\left(1-\tau^c \hat{c}_{t+1}\right)}{g^2\left(1-\tau^c \hat{c}_t\right)}\right]\left[A\hat{k}_t^{\alpha} + (1-\delta)\,\hat{k}_t - \left(1+\tau^c\right)\hat{c}_t\right]$$

[73] In the AK model, where Barro model (1990) is a version of it, consumption is a non-predetermined (jump) variable. Thus, there is no transitional dynamics of consumption.

and then find the first-order Taylor approximation of it around the steady-state, and then evaluate the coefficients in the steady-state[74],

$$(\hat{k}_{t+1}-\hat{k}^*) = \frac{\left(g^2\right)^{\sigma-1}}{\beta}(\hat{k}_t-\hat{k}^*) - \left[\frac{1+\tau^c}{g^2}-\frac{\tau^c\hat{k}^*}{1-\tau^c\hat{c}^*}\right](\hat{c}_t-\hat{c}^*) - \frac{\tau^c\hat{k}^*}{1-\tau^c\hat{c}^*}(\hat{c}_{t+1}-\hat{c}^*) \quad (5.34)$$

Then, we consider the Euler equation,

$$\hat{c}_{t+1} = \left[\frac{\hat{c}_t\left(1-\tau^c\hat{c}_{t+1}\right)}{g^2\left(1-\tau^c\hat{c}_t\right)}\right]\left[\beta\left(\alpha A\hat{k}_{t+1}^{\alpha-1}+1-\delta\right)\right]^{\frac{1}{\sigma}}$$

Rearrange it,

$$\hat{c}_{t+1} = \left[\frac{\hat{c}_t\left[\beta\left(\alpha A\hat{k}_{t+1}^{\alpha-1}+1-\delta\right)\right]^{\frac{1}{\sigma}}}{g^2-g^2\tau^c\hat{c}_t+\tau^c\hat{c}_t\left[\beta\left(\alpha A\hat{k}_{t+1}^{\alpha-1}+1-\delta\right)\right]^{\frac{1}{\sigma}}}\right]$$

So now, we find the first-order Taylor approximation of the Euler equation and then evaluate its coefficients in the steady-state obtaining:

$$(\hat{c}_{t+1}-\hat{c}^*) = (\hat{c}_t-\hat{c}^*) + \left[\frac{(\alpha-1)\alpha\hat{c}^*A\beta\left(1-\tau^c\hat{c}^*\right)\hat{k}^{*\alpha-2}}{\sigma\left(g^2\right)^{\sigma}}\right](\hat{k}_{t+1}-\hat{k}^*) \quad (5.35)$$

We first substitute the approximation of Euler equation (5.35) into (5.34) and then substitute again what we obtained into (5.35). After some algebra, we get the two linearised equations of the system around the steady-state, as functions of $(\hat{k}_t, \hat{c}_t)$.

$$(\hat{k}_{t+1}-\hat{k}^*) = \frac{\left(g^2\right)^{\sigma-1}}{\beta(1+Z)}(\hat{k}_t-\hat{k}^*) - \frac{1+\tau^c}{g^2(1+Z)}(\hat{c}_t-\hat{c}^*) \quad (5.34)'$$

$$(\hat{c}_{t+1}-\hat{c}^*) = \left[1-\frac{Z\left(1-\tau^c\hat{c}^*\right)\left(1+\tau^c\right)}{g^2\tau^c\hat{k}^*(1+Z)}\right](\hat{c}_t-\hat{c}^*) + \left[\frac{Z\left(g^2\right)^{\sigma-1}\left(1-\tau^c\hat{c}^*\right)}{\beta\tau^c\hat{k}^*(1+Z)}\right](\hat{k}_t-\hat{k}^*) \quad (5.35)'$$

where $Z = \dfrac{(\alpha-1)\alpha\hat{c}^*A\beta\hat{k}^{*\alpha-1}\tau^c}{\sigma\left(g^2\right)^{\sigma}} < 0$

[74] The steady-state of capital accumulation equation and Euler equation can be written respectively as, $\hat{k}^* = \frac{1}{g^2}\left[A\hat{k}^{*\alpha} + (1-\delta)\hat{k}^* - (1+\tau^c)\hat{c}^*\right]$ and $g^2 = \left[\beta\left(\alpha A\hat{k}^{*\alpha-1} + 1 - \delta\right)\right]^{\frac{1}{\sigma}}$.

The two pair of equations $\hat{k}_{t+1}$ and $\hat{c}_{t+1}$ allow us to move from knowing $\hat{k}_t$ and $\hat{c}_t$ to knowing $\hat{k}_{t+1}$ and $\hat{c}_{t+1}$. These two equations describe the law of motion in this system and define the dynamics in $\hat{k}_t$ and $\hat{c}_t$. Thus, by linearising these two equations, as shown in equations (5.34)' and (5.35)', we understand that there will be two eigenvalues in this system.

The linearised capital accumulation equation (5.34)’ and Euler equation (5.35)’ can be now expressed in matrix form as:

$$\begin{pmatrix} \hat{k}_{t+1} - \hat{k}^* \\ \\ \hat{c}_{t+1} - \hat{c}^* \end{pmatrix} = \underbrace{\begin{pmatrix} \dfrac{\left(g^2\right)^{\sigma-1}}{\beta(1+\mathrm{Z})} & -\dfrac{1+\tau^c}{g^2(1+\mathrm{Z})} \\ \\ \left[\dfrac{\mathrm{Z}\left(g^2\right)^{\sigma-1}\left(1-\tau^c\hat{c}^*\right)}{\beta\tau^c\hat{k}^*(1+\mathrm{Z})}\right] & \left[1-\dfrac{\mathrm{Z}\left(1-\tau^c\hat{c}^*\right)\left(1+\tau^c\right)}{g^2\tau^c\hat{k}^*(1+\mathrm{Z})}\right] \end{pmatrix}}_{matrix\ A} \begin{pmatrix} \hat{k}_t - \hat{k}^* \\ \\ \hat{c}_t - \hat{c}^* \end{pmatrix}$$

The above coefficient matrix A has a trace,

$$Tr = \frac{\left(g^2\right)^{\sigma-1}}{\beta(1+\mathrm{Z})} + 1 - \frac{\mathrm{Z}\left(1-\tau^c\hat{c}^*\right)\left(1+\tau^c\right)}{g^2\tau^c\hat{k}^*(1+\mathrm{Z})}$$

We know that $g^2 \geq 1$ and $Z < 0$. Thus, if Z is between zero and one[75], then $Tr > 1$.

The determinant (Det) of the matrix A is,

$$Det = \left(\frac{\left(g^2\right)^{\sigma-1}}{\beta(1+\mathrm{Z})}\right)\left(1-\frac{\mathrm{Z}\left(1-\tau^c\hat{c}^*\right)\left(1+\tau^c\right)}{g^2\tau^c\hat{k}^*(1+\mathrm{Z})}\right)-\left(-\frac{1+\tau^c}{g^2(1+\mathrm{Z})}\right)\left(\frac{\mathrm{Z}\left(g^2\right)^{\sigma-1}\left(1-\tau^c\hat{c}^*\right)}{\beta\tau^c\hat{k}^*(1+\mathrm{Z})}\right)=\frac{\left(g^2\right)^{\sigma-1}}{\beta(1+\mathrm{Z})}>1$$

The eigenvalues of the matrix are the solutions to the characteristic equation,

$$\lambda^2 - Tr\,\lambda + Det = 0$$

$$\lambda^2 - \left(\frac{\left(g^2\right)^{\sigma-1}}{\beta(1+\mathrm{Z})} + 1 - \frac{\mathrm{Z}\left(1-\tau^c\hat{c}^*\right)\left(1+\tau^c\right)}{g^2\tau^c\hat{k}^*(1+\mathrm{Z})}\right)\lambda + \frac{\left(g^2\right)^{\sigma-1}}{\beta(1+\mathrm{Z})} = 0$$

[75] The value of Z will be verified later.

To apply Blanchard and Kahn (1980) test for a unique non-divergent rational expectation solution to exist, if we have two state variables that are non-predetermined, then we would need two unstable (outside of the unit circle) eigenvalues for the test to be satisfied. However, although both $\hat{k}_t$ and $\hat{c}_t$ in our case are non-predetermined variables, both do not move or jump freely[76]. The reason for not having the freedom of movement is that there is an implicit relationship, or some restrictions, between $\hat{k}_t$ and $\hat{c}_t$ that must hold in the period of the shock. Due to this relationship between them, we need one stable eigenvalue and one unstable eigenvalue. The nature of this relationship will be discussed in more details in the transitional dynamics section.

So now, we put into practice the test, explained by Rankin (2007), for our characteristic equation to have 0, 1, or 2 stable eigenvalues[77]. This test will equivalently produce conditions for 0, 1, or 2 of the original eigenvalues λ to lie inside the unit circle.

Looking at the table explained in chapter four and converting the table's coefficients (a,b,c) to our coefficients from our characteristic equation, the necessary and sufficient condition is,

$$\frac{a+b+c}{a-b+c} \Rightarrow \frac{\dfrac{Z\left(1-\tau^c\hat{c}^*\right)\left(1+\tau^c\right)}{g^2\tau^c\hat{k}^*\left(1+Z\right)}}{2+2\dfrac{\left(g^2\right)^{\sigma-1}}{\beta\left(1+Z\right)}-\dfrac{Z\left(1-\tau^c\hat{c}^*\right)\left(1+\tau^c\right)}{g^2\tau^c\hat{k}^*\left(1+Z\right)}}$$

After simplifying the above expression, we obtain:

$$\frac{Z\beta\left(1-\tau^c\hat{c}^*\right)\left(1+\tau^c\right)}{\left(2g^2\tau^c\hat{k}^*\right)\left[\beta\left(1+Z\right)+\left(g^2\right)^{\sigma-1}\right]-Z\beta\left(1-\tau^c\hat{c}^*\right)\left(1+\tau^c\right)}$$

The necessary and sufficient condition for stability in our model requires that the above ratio should be negative and between zero and one. The sign of the numerator is clearly negative because of the sign of Z; however, the sign of the denominator is still ambiguous since the value of Z is crucial here (*i.e.* it is unclear at first sight if it is greater or less than one in absolute value). Thus, we consider the expression for Z again.

[76] In general models, if all variables are non-predetermined variables, then they all jump freely, which means they all jump to the steady-state when an unexpected shock happens. However, we believe in our case that they cannot jump immediately to the new steady-state.

[77] This test has explained in detail in chapter four.

The sign of the expression for Z is obviously negative because of $\alpha < 1$, and it contains two values of steady-state variables, $\hat{k}^*$ and $\hat{c}^*$, which are endogenous variables. These two steady-state variables depend in fact on the value of g^2, τ^c, and other exogenous parameters, as shown in equations (5.18) and (5.19). In other words, they both may change as we change g^2 and τ^c. Looking at these two steady-state equations, we have earlier found that if g^2 is sufficiently high (for a given value of τ^c), then we would have an exogenous growth steady-state. Besides, the steady-state of $\hat{k}^*$ shows that if g^2 tends to infinity $(g^2 \rightarrow \infty)$, then $\hat{k}^*$ tends to zero $(\hat{k}^* \rightarrow 0)$. That, in turn, makes the second steady-state of $\hat{c}^*$ smaller because it is a function of $\hat{k}^*$, where $\hat{k}^*$ shrinks as g^2 getting higher. Therefore, we can conclude that if g^2 is set to be high enough, then both $\hat{k}^*$and $\hat{c}^*$ would be less than one. Now, the expression for Z is clearer, which is $-Z < 1$ [78]. Consequently, if g^2 is sufficiently high, then the whole expression for Z is negative and between zero and one.

According to that, the above ratio becomes unambiguously negative and between zero and one, because of $Z\beta(1 - \tau^c\hat{c}^*)(1 + \tau^c)$ becomes positive in the denominator. It then implies that one eigenvalue lies inside the unit circle. As a result, the steady-state $(\hat{k}^*, \hat{c}^*)$ is a saddle-point in this type of steady-state, *i.e.* type *(I)* steady-state. That indeed confirms our early speculation that there is one stable eigenvalue and one unstable eigenvalue.

[78] We prove algebraically in Appendix B.5 that Z is negative and less than one.

5.8 The Effect of Consumption Tax on the Transition Path of Variables in the Neighbourhood of a Type *(I)* Steady-State

This section studies the transitional dynamics in type *(I)* steady-state by using a phase diagram. The purpose of this section is to find out what happens in the short and medium terms when consumption tax, τ^c, is raised. We divide this section into two parts: the first part is the transitional dynamics before changing the consumption tax, while the second part is the transitional dynamics when we change consumption tax. The reason for that is to find out how the two variables, $\hat{k}_t$ and $\hat{c}_t$, in phase diagram move. Moreover, we study the relationship between $\hat{k}_t$ and $\hat{c}_t$.

5.8.1 The Transitional Dynamics Before Changing Consumption Tax

To analyse the dynamics of the model through a phase diagram, we first rewrite the two fundamental equations of our economy in terms of $\left(\hat{k}_t, \hat{c}_t\right)$, starting with the capital accumulation equation,

$$\hat{k}_{t+1} = \frac{\left(1-\tau^c\hat{c}_{t+1}\right)}{g^2\left(1-\tau^c\hat{c}_t\right)}\left[A\hat{k}_t^\alpha + (1-\delta)\,\hat{k}_t - \left(1+\tau^c\right)\hat{c}_t\right]$$

it follows that,

$$\Delta\hat{k}_{t+1} = \frac{\left(1-\tau^c\hat{c}_{t+1}\right)}{g^2\left(1-\tau^c\hat{c}_t\right)}\left[A\hat{k}_t^\alpha + (1-\delta)\,\hat{k}_t - \left(1+\tau^c\right)\hat{c}_t\right] - \hat{k}_t = h'\left(\hat{k}_t, \hat{c}_t, \hat{c}_{t+1}\right) \qquad (5.36)$$

and then the Euler equation,

$$\hat{c}_{t+1} = \frac{\left(1-\tau^c\hat{c}_{t+1}\right)\hat{c}_t}{g^2\left(1-\tau^c\hat{c}_t\right)}\left[\beta\left(\alpha A\hat{k}_{t+1}^{\alpha-1} + 1-\delta\right)\right]^{\frac{1}{\sigma}}$$

substituting $\hat{k}_{t+1}$ into the Euler equation, it follows that,

$$\Delta\hat{c}_{t+1} = \frac{\left(1-\tau^c\hat{c}_{t+1}\right)\hat{c}_t}{g^2\left(1-\tau^c\hat{c}_t\right)}\left[\beta\left(\alpha A\left[\frac{\left(1-\tau^c\hat{c}_{t+1}\right)}{g^2\left(1-\tau^c\hat{c}_t\right)}\left[A\hat{k}_t^\alpha + (1-\delta)\,\hat{k}_t - \left(1+\tau^c\right)\hat{c}_t\right]\right]^{\alpha-1} + 1-\delta\right)\right]^{\frac{1}{\sigma}} - \hat{c}_t = g'\left(\hat{k}_t, \hat{c}_t, \hat{c}_{t+1}\right) \quad (5.37)$$

The phase diagram is built on the basis of two curves relating $\hat{k}_t$ to $\hat{c}_t$. Each curve of them coincides with one of the two zero change cases: $\Delta\hat{k}_{t+1} = \hat{k}_{t+1} - \hat{k}_t = 0$ (capital accumulation), $\Delta\hat{c}_{t+1} = \hat{c}_{t+1} - \hat{c}_t = 0$ (Euler equation). However, the two key equations of our economy, (5.36) and (5.37), are both functions of $(\hat{k}_t, \hat{c}_t, \hat{c}_{t+1})$. Thus, to find the stationary locus for $\hat{k}_t$, we need to substitute out $\hat{c}_{t+1}$[79] from the RHS of equation (5.36), then we impose that $\hat{k}_{t+1} = \hat{k}_t$. Therefore, we got after arrangement,

$$\hat{c}_t = \frac{A\hat{k}_t^{\alpha} - (g^2 - 1 + \delta)\,\hat{k}_t}{(1+\tau^c) + \tau^c\left[\left(\beta\left(\alpha A\hat{k}_t^{\alpha-1} + 1 - \delta\right)\right)^{\frac{1}{\sigma}} - g^2\right]\hat{k}_t} \tag{5.36$'$}$$

We can further draw $\hat{c}_t$ as a function of $\hat{k}_t$ (treating g^2 as given and τ^c as sufficiently close to zero), as follows:

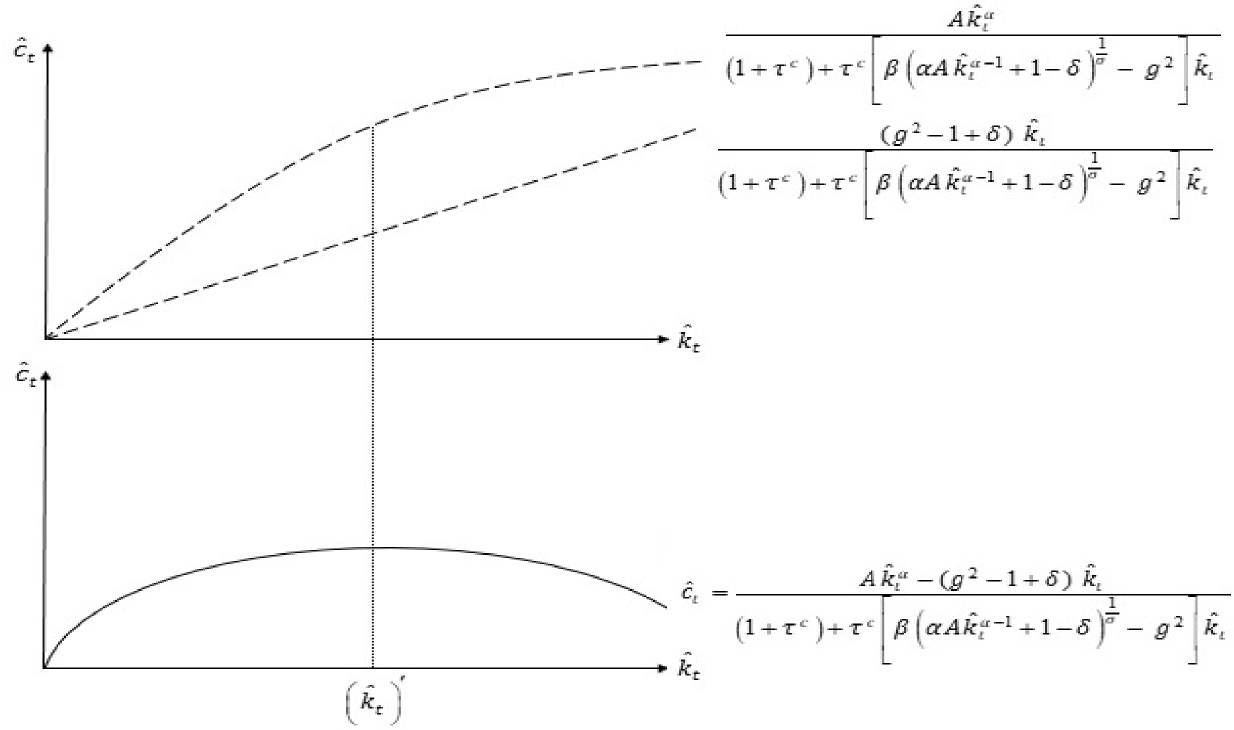

Diagram 16: Drawing equation $\hat{c}_t$ as a function of $\hat{k}_t$

[79] We have found in section 5.7 that $\hat{c}_{t+1} = \left[\frac{\hat{c}_t\left[\beta\left(\alpha A\hat{k}_{t+1}^{\alpha-1}+1-\delta\right)\right]^{1/\sigma}}{g^2 - g^2\tau^c\hat{c}_t + \tau^c\hat{c}_t\left[\beta\left(\alpha A\hat{k}_{t+1}^{\alpha-1}+1-\delta\right)\right]^{1/\sigma}}\right]$.

If g^2 is given and τ^c is sufficiently close to zero, then the second diagram shows that $\hat{c}_t$ as a function of $\hat{k}_t$ takes approximately a 'hill shaped' [80], where $\left(\hat{k}_t\right)'$ is the value of $\hat{k}_t$ which maximises $\hat{c}_t$, such that,

$$\frac{d\hat{c}_t}{d\hat{k}_t} = 0 \;\Rightarrow\; \frac{\alpha A\hat{k}_t^{\alpha-1} - g^2 - \delta + 1}{1+\tau^c+\tau^c\left(\left(\beta\left(\alpha A\hat{k}_t^{\alpha-1}+1-\delta\right)\right)^{\frac{1}{\sigma}} - g^2\right)\hat{k}_t}$$

$$-\frac{\left[A\hat{k}_t^{\alpha} - (g^2+\delta-1)\,\hat{k}_t\right]\left[\tau^c\left(\left(\beta\left(\alpha A\hat{k}_t^{\alpha-1}+1-\delta\right)\right)^{\frac{1}{\sigma}} - g^2\right) + \frac{(\alpha-1)\alpha A\tau^c\hat{k}_t^{\alpha-1}\left(\beta\left(\alpha A\hat{k}_t^{\alpha-1}+1-\delta\right)\right)^{\frac{1}{\sigma}}}{\sigma\left(\alpha A\hat{k}_t^{\alpha-1}+1-\delta\right)}\right]}{\left[1+\tau^c+\tau^c\left[\left(\beta\left(\alpha A\hat{k}_t^{\alpha-1}+1-\delta\right)\right)^{\frac{1}{\sigma}} - g^2\right]\hat{k}_t\right]^2} = 0$$

For low values of $\hat{k}_t$, $d\hat{c}_t/d\hat{k}_t > 0$, while for high values of $\hat{k}_t$, $d\hat{c}_t/d\hat{k}_t < 0$. Therefore, equation (5.36)’ gives us a concave function (bell curve), as shown in the above diagram.

Below the $\Delta\hat{k}_{t+1} = 0$ curve described by (5.36)’, the consumption PGSU is *‘lower’* at any point below the curve than it is on the curve, so that $\hat{c}_t < \frac{A\hat{k}_t^{\alpha}-(g^2-1+\delta)\hat{k}_t}{(1+\tau^c)+\tau^c\left[\left(\beta\left(\alpha A\hat{k}_t^{\alpha-1}+1-\delta\right)\right)^{1/\sigma} - g^2\right]\hat{k}_t}$, which, taken to the capital accumulation equation, it means: $\hat{k}_{t+1} > \hat{k}_t$ and the arrows point to *'right'*, showing the expected direction of capital PGSU in that area.

Above the $\Delta\hat{k}_{t+1} = 0$ curve, the consumption PGSU is *‘higher’* at any point above the curve than it is on the curve, so that $\hat{c}_t > \frac{A\hat{k}_t^{\alpha}-(g^2-1+\delta)\hat{k}_t}{(1+\tau^c)+\tau^c\left[\left(\beta\left(\alpha A\hat{k}_t^{\alpha-1}+1-\delta\right)\right)^{1/\sigma} - g^2\right]\hat{k}_t}$ which implies $\hat{k}_{t+1} < \hat{k}_t$ and the arrows indicate to *'left'*.

On the other hand, we set $\Delta\hat{c}_{t+1} = \hat{c}_{t+1} - \hat{c}_t = 0$ in equation (5.37) to express $\hat{c}_t$ as a function of $\hat{k}_t$ as,

$$\hat{c}_t = \left(\frac{1}{1+\tau^c}\right)\left[A\hat{k}_t^{\alpha} + (1-\delta)\,\hat{k}_t - g^2\left[\frac{1}{\alpha A}\left(\frac{\left(g^2\right)^{\sigma}}{\beta} - 1 + \delta\right)\right]^{\frac{1}{\alpha-1}}\right] \qquad (5.37)'$$

[80] It is because if $\tau^c = 0$, then $d\hat{c}_t/d\hat{k}_t = 0 \Rightarrow \alpha A\hat{k}_t^{\alpha-1} - g^2 - \delta + 1 = 0$, which is similar to chapter four.

We can further rewrite equation (5.37)’,

$$\alpha A\left[\frac{A\hat{k}_t^{\alpha}}{g^2}+\frac{(1-\delta)\,\hat{k}_t}{g^2}-\frac{\left(1+\tau^c\right)\,\hat{c}_t}{g^2}\right]^{\alpha-1}=\frac{\left(g^2\right)^{\sigma}}{\beta}-1+\delta \qquad (5.37)''$$

This equation shows an explicit relationship between $\hat{k}_t$ and $\hat{c}_t$. If $(g^2)^\sigma \geq 1$, then the right-hand side of the equation (5.37)" is greater than zero, $(g^2)^\sigma/\beta - 1 + \delta > 0$. This is consistent with the fact that the marginal product of capital has to be positive for any positive capital stock.

The slope of the curve, of equation (5.37)’, is

$$\frac{d\hat{c}_t}{d\hat{k}_t}=\frac{\alpha A\hat{k}_t^{\alpha-1}+1-\delta}{\left(1+\tau^c\right)} \qquad (5.38)$$

The slope is positive because $\alpha A\hat{k}_t^{\alpha-1}+1-\delta > 0$ for all $\hat{k}_t$. Therefore, the slope shows a positive relationship in the $\left(\hat{c}_t, \hat{k}_t\right)$ space.

This line (the slope) has a negative intercept with the vertical axis at $\hat{k}_t = 0$, since at that point we would have,

$$\alpha A\left[-\frac{\left(1+\tau^c\right)\,\hat{c}_t}{g^2}\right]^{\alpha-1}=\left(\frac{\left(g^2\right)^{\sigma}}{\beta}-1+\delta\right)>0 \quad ,$$

and the marginal product function is only defined over the positive real line.

As shown in (5.38), the curve has a positive slope, and along it, $\hat{c}_t \to \infty$ as $\hat{k}_t \to \infty$. Therefore, it will cross the horizontal axis one time.

From equation (5.37), it follows that $\Delta\hat{c}_{t+1} < 0$, the right of the $\Delta\hat{c}_{t+1} = 0$ curve, implies that

$$\underbrace{\alpha A\left[\frac{A\hat{k}_t^{\alpha}}{g^2}+\frac{(1-\delta)\,\hat{k}_t}{g^2}-\frac{\left(1+\tau^c\right)\,\hat{c}_t}{g^2}\right]^{\alpha-1}}_{\text{Marginal Product}}<\frac{\left(g^2\right)^{\sigma}}{\beta}-1+\delta$$

The capital PGSU, $\hat{k}_t$, is ‘*high*’, this means that the marginal product will be *'small'*, and the consumption PGSU will be expected to *'decline'*, $\hat{c}_{t+1} < \hat{c}_t$. Thus, to the right of $\Delta\hat{c}_{t+1} = 0$ curve, the arrows indicate to *'down'*, displaying the expected direction of consumption PGSU in that region.

In the same way, from equation (5.37), it follows that $\Delta\hat{c}_{t+1} > 0$, the left of the $\Delta\hat{c}_{t+1} = 0$ curve, implies that

$$\underbrace{\alpha A\left[\frac{A\hat{k}_t^{\alpha}}{g^2} + \frac{(1-\delta)\,\hat{k}_t}{g^2} - \frac{\left(1+\tau^c\right)\,\hat{c}_t}{g^2}\right]^{\alpha-1}}_{\text{Marginal Product}} > \frac{\left(g^2\right)^{\sigma}}{\beta} - 1 + \delta$$

The capital PGSU, $\hat{k}_t$, is *'low'*, which means that the marginal product will be *'big'*, and the consumption PGSU will be expected to *'grow'*, $\hat{c}_{t+1} > \hat{c}_t$. Therefore, the arrows point to *'up'*.

5.8.2 The Transitional Dynamics When We Change Consumption Tax

In this section, we want to study how changing consumption tax would affect the transitional dynamic, moving from the old steady-state to a new one. In particular, we examine how the $\Delta\hat{k}_{t+1} = 0$ and $\Delta\hat{c}_{t+1} = 0$ loci would change when we increase the consumption tax.

A- The locus $\Delta\hat{k}_{t+1} = 0$:

Equation (5.36)' shows that consumption tax affects the consumption PGSU. Moreover, we have demonstrated at the steady-state that the consumption PGSU is negatively affected by changing consumption tax. Thus, taking the derivative of equation (5.36)' *w.r.t.* consumption tax gives us,

$$\frac{d\hat{c}_t}{d\tau^c} = -\frac{\left[A\hat{k}_t^{\alpha} + (1-\delta-g^2)\,\hat{k}_t\right]\left[\left(\left(\beta\left(\alpha A\hat{k}_t^{\alpha-1}+1-\delta\right)\right)^{\frac{1}{\sigma}} - g^2\right)\hat{k}_t + 1\right]}{\left[1+\tau^c+\tau^c\left(\left(\beta\left(\alpha A\hat{k}_t^{\alpha-1}+1-\delta\right)\right)^{\frac{1}{\sigma}} - g^2\right)\hat{k}_t\right]^2} \tag{5.39}$$

If we use the fact that at the steady-state $g^2 = \left(\beta\left(\alpha A\hat{k}^{*\alpha-1} + 1 - \delta\right)\right)^{1/\sigma}$, (5.39) can be simplified to,

$$\frac{d\hat{c}_t}{d\tau^c} = -\frac{A\hat{k}_t^{\alpha} + (1-\delta-g^2)\,\hat{k}_t}{\left(1+\tau^c\right)^2} < 0 \tag{5.39$'$}$$

Thus, if we increase the consumption tax, the consumption PGSU would decline. As a result, $\Delta\hat{k}_{t+1} = 0$ locus would shift the bell curve down.

B- The locus $\Delta\hat{c}_{t+1} = 0$:

Although we have shown in the steady-state section that capital PGSU is independent of consumption tax, $\Delta\hat{c}_{t+1} = 0$ locus would be determined by taking the derivative of equation (5.37)' *w.r.t.* consumption tax,

$$\frac{d\hat{c}_t}{d\tau^c} = -\frac{\left[A\hat{k}_t^{\alpha} + (1-\delta)\,\hat{k}_t - g^2\left[\frac{1}{\alpha A}\left(\frac{\left(g^2\right)^{\sigma}}{\beta} - 1 + \delta\right)\right]^{\frac{1}{\alpha\text{-}1}}\right]}{\left(1+\tau^c\right)^2} \tag{5.40}$$

The sign is negative in (5.40); therefore, we can conclude that $d\hat{c}_t/d\tau^c < 0$ implies that the $\Delta\hat{c}_{t+1} = 0$ locus will move down due to the increase in consumption tax.

Before we construct a phase diagram for this system, we consider the phase diagram in the Ramsey model[81]. In his model, we know that $\hat{k}_t$ is a predetermined variable, while $\hat{c}_t$ is a non-predetermined variable. Thus, when an exogenous shock (*i.e.* a permanent shock) occurs, $\hat{k}_t$ cannot jump, but $\hat{c}_t$ can jump. Consequently, in the Ramsey model and ours in chapter four, the economy would move initially to the point that is vertically above or below the old steady-state. Thus, it jumps to that point, and then it will converge to the new steady-state. This indeed represents the conventional Blanchard and Kahn (B-K) type solution. However, what would happen in our case is slightly different from the conventional B-K type solution. The reason is that although $\hat{k}_t$ and $\hat{c}_t$ in our model are completely non-predetermined variables, they both cannot jump freely because there may be some relationship between them, which has to be held.

The relationship between $\hat{k}_t$ and $\hat{c}_t$ can be found both geometrically and analytically. Let us begin with the geometrical approach, the relationship between them can be described in a new locus in the phase diagram. This locus is equivalent to the vertical line through $\hat{k}_0$ in Ramsey's phase diagram and ours in chapter four. Thus, once the shock happens, the economy will still have to jump on the saddle-point, but there is a predetermined locus (a new diagonal line) along which $\hat{k}_t$ and $\hat{c}_t$ have to jump. In the conventional case of Ramsey, this locus is a vertical line through the initial point, and $\hat{k}_t$ does not jump, but $\hat{c}_t$ jumps. In our case, both $\hat{k}_t$ and $\hat{c}_t$ jump, but they cannot jump anywhere, they both have to jump to somewhere on that new locus (*i.e.* the diagonal locus).

[81] Our phase diagram in chapter four is quite similar to the one in the Ramsey model, where the differences are in terms of variables and type of exogenous shock (see our phase diagram in chapter four).

In other words, $\hat{k}_t$ and $\hat{c}_t$ both have to jump in some relationship, and this relationship given by the diagonal line, which is slightly different from the conventional case. Thus, we need to study more the exact relationship between them, particularly the new diagonal line.

Since we have found that the system is saddle-point stable, then if $\hat{k}_t$ and $\hat{c}_t$ have to jump somewhere on the diagonal line, there is only one point to which they can jump, such subsequence the economy will converge and that where the diagonal line intersects the saddle path, at point (A) in the below phase diagram. Then, the local stability properties for the steady-state would remain as usual (*i.e.* the economy at this point will proceed along the saddle path as standard).

To prove our anticipation that both $\hat{k}_t$ and $\hat{c}_t$ can jump but cannot jump at the moment of impact freely, we need to study the relationship between them using an analytical approach. In fact, the new thing relative to this relationship is that $\hat{k}_t$ can jump, but once $\hat{k}_t$ jumps, then $\hat{c}_t$ has to jump in a certain ratio. This ratio has to be underline some restrictions, which need to be explored more fully. Thus, this is a sort of conventional of B-K type solution, where the principle of B-K still applies in our model (*i.e.* it is just a new jump locus).

To show this relationship more clearly, we start with the government budget constraint and the definition of both capital PGSU and consumption PGSU,

$$G_t = \pi_t + \tau^c C_t$$

$$\hat{k}_t \equiv \frac{K_t}{G_t} = \frac{K_t}{\pi_t + \tau^c C_t} \qquad ; \qquad \hat{c}_t \equiv \frac{C_t}{G_t} = \frac{C_t}{\pi_t + \tau^c C_t}$$

We can see from the definition of capital PGSU that G_t can jump because C_t can jump, but the numerator, K_t, cannot jump. Also, from the definition of consumption PGSU, it is obvious that both numerator and denominator can jump. Although $\hat{k}_t$ and $\hat{c}_t$ can jump, the ratio in which they jump is not entirely free. That means that they will jump according to some restrictions on how they can jump, which will implicitly define some kind of locus once we linearise it. Moreover, since $\hat{c}_t$ can jump, then that will determine how $\hat{k}_t$ and $\hat{c}_t$ also jump. In other words, we cannot have independence jump in $\hat{k}_t$ and $\hat{c}_t$, because they have to jump in a certain way consistently. Thus, in algebra, where there are not linearised yet, we can find this relationship.

First, rearranging the definition of the capital PGSU as,

$$\pi_t + \tau^c C_t = \frac{K_t}{\hat{k}_t}$$

Solving for C_t:

$$C_t = \frac{1}{\tau^c}\left[\frac{K_t}{\hat{k}_t} - \pi_t\right] \tag{5.41}$$

Then, by dividing the numerator and denominator of consumption PGSU by C_t, we get:

$$\hat{c}_t = \frac{1}{\left(\pi_t / C_t\right) + \tau^c} \tag{5.42}$$

Now, we substitute equation (5.41) into (5.42) and then simplify it to obtain:

$$\hat{c}_t = \frac{K_t - \pi_t \hat{k}_t}{\tau^c K_t} \tag{5.43}$$

Equation (5.43) shows that $\hat{c}_t$ is expressed as a function of $\hat{k}_t$, where the endogenous variables are $\hat{k}_t$, $\hat{c}_t$, and K_t, while the exogenous variables are τ^c and π_t. This equation is not linear, and it is the equation of the new diagonal line in the below phase diagram, which we call it locus *(D)*. Moreover, equation (5.43) tells us that when the shock occurs, there is some relationship between $\hat{k}_t$ and $\hat{c}_t$, which is described by this equation.

To know the slope of this new diagonal line, we take the derivative of equation (5.43) w.r.t. $\hat{k}_t$,

$$\frac{d\hat{c}_t}{d\hat{k}_t} = -\frac{\pi_t}{\tau^c K_t} \tag{5.44}$$

The slope tells us that there is a negative relationship between $\hat{k}_t$ and $\hat{c}_t$ (*i.e.* the diagonal line moves from left to right, downward slope).

The new diagonal locus, (*D*), should pass below the old steady-state, (E), *i.e.* it should be shifted down. The reason for this is due to the fact that consumption tax, τ^c, *i.e.* the shock variable, directly enters into equation (5.43). Thus, when τ^c increases, even if $\hat{k}_t$ remained at its old steady-state, $\hat{c}_t$ would need to fall to satisfy equation (5.43).

Moreover, although the new steady-state should lie vertically below the old one[82], there is still a key question that is whether the jump locus, *i.e.* locus (*D*), passes below or above the new steady-state. To discover that, we need first to differentiate equation (5.43) w.r.t. τ^c and hold $\hat{k}_t$ constant at its initial steady-state value.

$$\frac{d\hat{c}_t}{d\tau^c} = - \frac{K_t - \pi_t \hat{k}_t}{K_t \left(\tau^c\right)^2} \tag{5.43$'$}$$

Equation (5.43)' tells us how much the (*D*) locus moves down when τ^c increases. However, the question to determine whether the jump locus passes below or above the new steady-state has not been answered yet. Thus, we need to compare equations (5.43)' and (5.19)' [83] to see which is larger.

When we compare (5.43)' and (5.19)', the algebraic calculation in Appendix B.6 shows that the (*D*) locus in the phase diagram should shift downwards by more than the steady-state shifts. The reason is that increasing the consumption tax would have a larger impact on (5.43)', compared to (5.19)'. As a result, the economy must jump down from its initial steady-state position (E) to the intersection of the saddle path with the jump locus (*D*) and then converge to its new steady-state (E') from below.

The below phase diagram combines the steady-state values before and after the shock (*i.e.* changing consumption tax). The black loci display the economy before the shock, at point (E). The red loci show the economy after the shock, at point (E'), where $\Delta\hat{k}_{t+1} = 0$ locus shifts the bell curve down, and $\Delta\hat{c}_{t+1} = 0$ locus moves down, as a consequence of the shock. The green locus, locus *(D)*, is the diagonal line that describes the relationship between $\hat{k}_t$ and $\hat{c}_t$, as we have discussed previously. The blue dotted line is the saddle path, where the intersection between the diagonal line and the saddle path at point (A) is the saddle-point stable. Besides that, the blue arrows indicate the dynamic behaviour of $\hat{k}_t$ and $\hat{c}_t$. Thus, since the shock in our model is permanent shock, these arrows will start to work immediately, which means that a unique path converging to (E') exists.

[82] We know from section 5.5 that the consumption tax does not affect the capital PGSU, but it affects the consumption PGSU negatively. For this reason, we understand that increasing consumption tax implies that the new steady-state, (E'), should be vertically below the old steady-state, (E).

[83] Equation (5.19)' is found in section 5.5, and it tells us how much the steady-state point in our phase diagram moves down when τ^c increases.

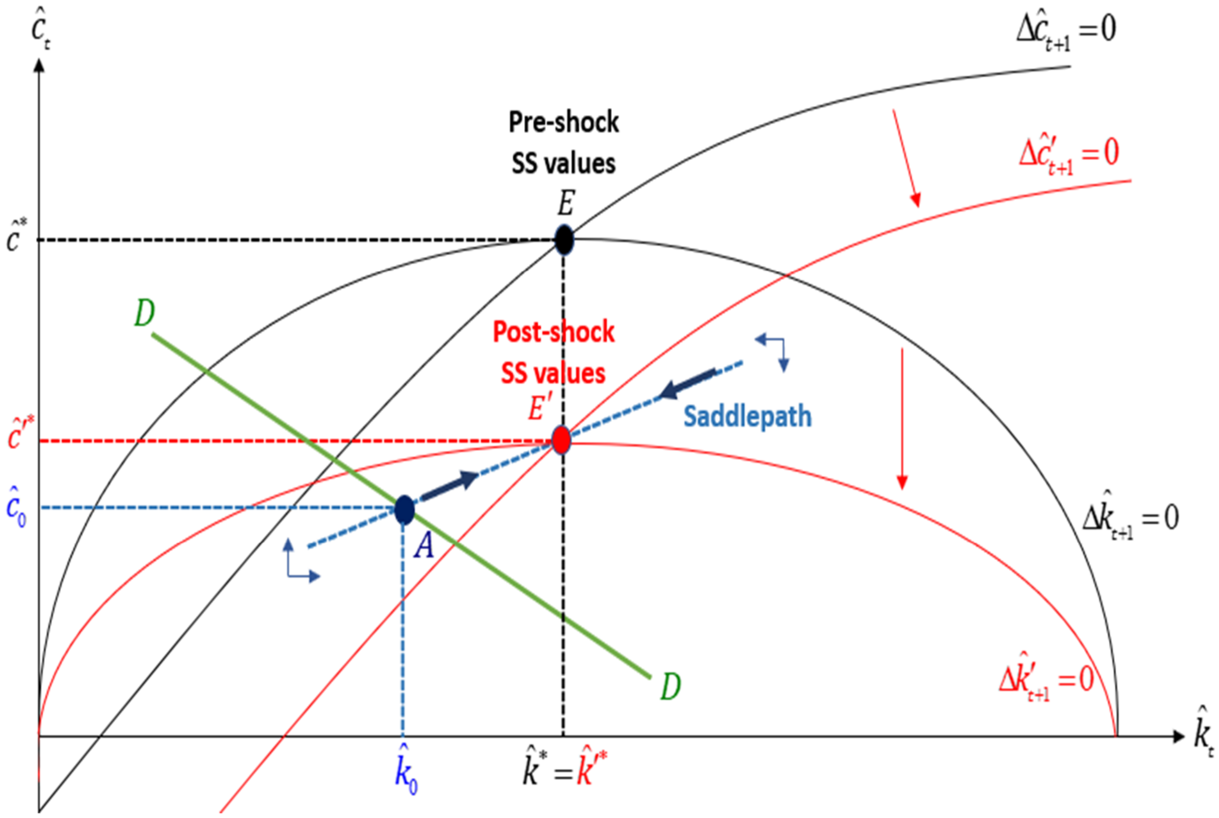

Diagram 17: Phase diagram of the transition dynamic when a consumption tax increases

Consequently, our result can now confirm our anticipation that there is a limit or a restriction on how $\hat{k}_t$ and $\hat{c}_t$ can jump relative to each other. In other words, given one of them jump, the other one must also jump because if the other one does not follow that, the equation (5.43) is then violated. Thus, equation (5.43) can describe the relationship between $\hat{k}_t$ and $\hat{c}_t$, which represents the diagonal line in the phase diagram. Moreover, according to the conventional B-K condition, if there are two non-predetermined variables, it should be the case that we need two unstable eigenvalues. However, we have confirmed in our model that although we have two jump variables, we do need, after all, one stable eigenvalue and one unstable eigenvalue.

5.9 Parameterisation and Solution

In this section, we use a numerical simulation under standard and commonly used parameters in literature[84]. Similar to chapter four, we use Dynare software to simulate the model in three parts. The first part deals with PGSU variables, while the second and third parts study the level variables. In the first part, we want to examine whether the PGSU variables would change when we introduce a consumption tax, *e.g.* $\tau^c = 5\%$. In other words, this part aims to show the transitional dynamics for the PGSU variables. Also, we know from our previous analysis in existence steady-state that there will be an exogenous or an endogenous growth steady-state under a particular set of parameter values. Thus, we pick out a particular set of parameter values to determine what type of steady-state we must be in. This part also makes some sensitive analysis in our simulations regarding the consumption tax thresholds.

In the second part, we investigate what would happen to the level variables when introducing the consumption tax and keeping the oil revenues unchanged. Moreover, we would examine two cases of introducing a 5% consumption tax rate in level variables in this part. The first case is that when β is sufficiently close to one, the second is when we set β to be sufficiently close to zero. The reason behind doing this exercise is to verify our theoretical result in the level variables section and particularly in the level of consumption since it has shown up an interesting result. The third part provides a numerical exercise to study the possibility of lowering oil revenues in our economy and how the consumption tax can compensate for that reduction. We also try to find out the required rate of consumption tax not only to compensate for any potential decline in oil revenues but also to guarantee the level of government spending unchanged. In fact, we need to fix the level of government spending to ensure that economic growth is not affected. In other words, we play in this part with the two sources of government revenues by finding how one source of revenues, *i.e.* consumption tax, can offset any reduction in the other source, *i.e.* oil revenues.

5.9.1 Part One: Introducing a Consumption Tax and What Type of Steady-State We Are in (PGSU Variables)

In this part, we introduce consumption tax in our model to see how the PGSU variables can change, *i.e.* the dynamic responses of the PGSU variables. This part also pursues to find out what type of steady-state we are in under a given set of parameter values. The simulation results in Table 7 and Figure 36 show the steady-state and the dynamic responses of the main PGSU variables in the economy.

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[84] The parameter values used here are the same as in the fourth chapter, where we set $A = 1, \alpha = 0.33, \beta = 0.95, \delta = 0.1, \sigma = 2$.

Table 7: The steady-state responses of a number of PGSU variables in the economy:

Old Steady-State Values		**New Steady-State Values**	
Variables	**Value**	**Variables**	**Value**
The Growth Rate of Government Spending	1.1	The Growth Rate of Government Spending	1.1
Consumption Tax	0	Consumption Tax	0.05
Output PGSU	0.9406	Output PGSU	0.9406
Capital PGSU	0.8306	Capital PGSU	0.8306
Consumption PGSU	0.7744	Consumption PGSU	0.7375
Interest Rate	0.3736	Interest Rate	0.3736

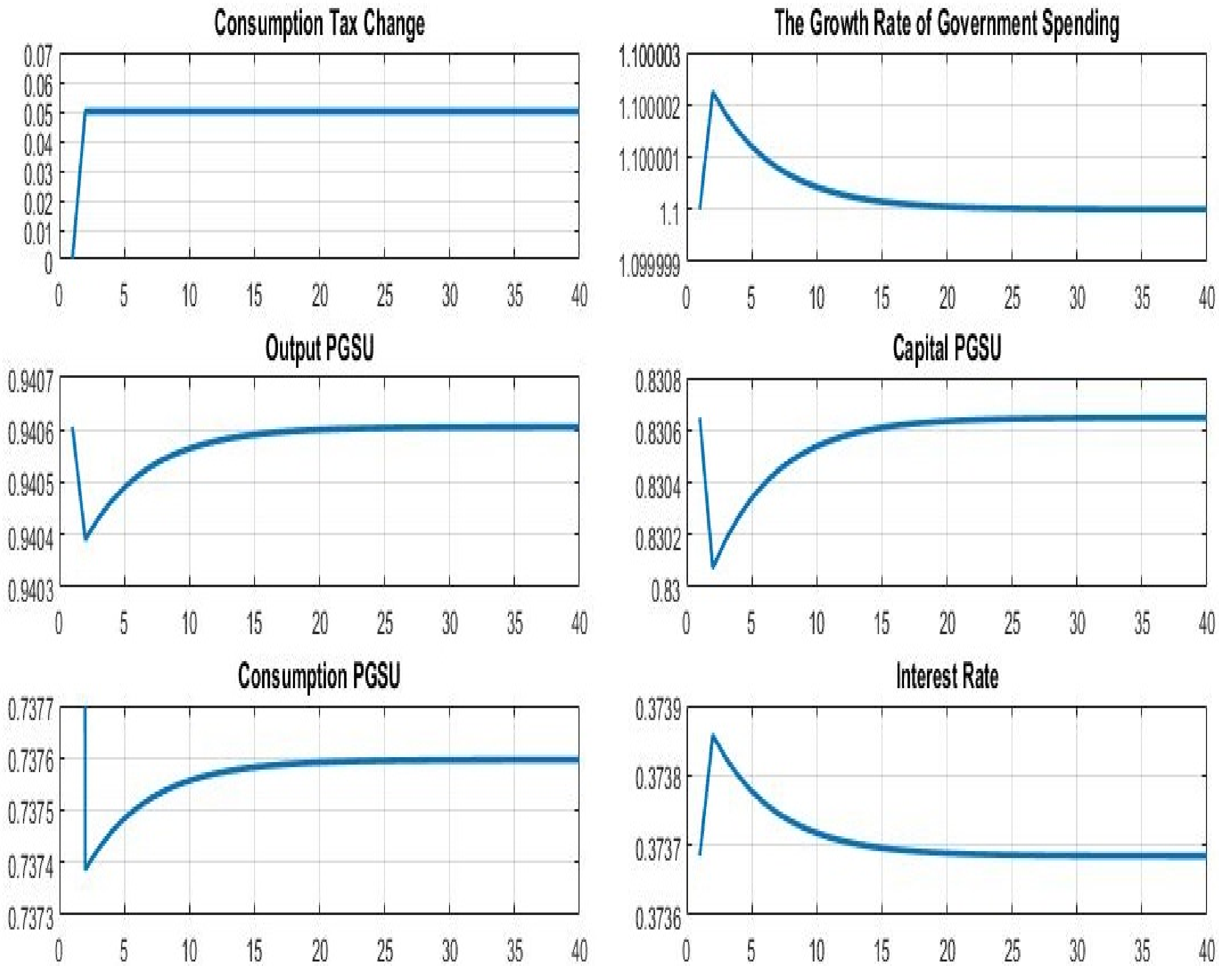

Figure 36: The dynamic responses of a number of PGSU variables in the economy

To compare the above table with our conclusion in the steady-state section, we find that they are consistent with each other. More precisely, the capital PGSU, output PGSU, and interest rate are not affected by introducing the consumption tax since their equations are independent of consumption tax. However, the consumption PGSU is affected, where introducing a 5% consumption tax rate would lead to a decline in the consumption PGSU by 4.76%.

The figures above are also consistent with our phase diagram. Specifically, introducing the consumption tax in our model would generate additional government revenues and then raise government spending. This, in turn, would raise the growth rate of GDP, even though only temporarily. As we can see from the above figures, the growth rate increases only in the short and medium run and then tends back to its steady-state asymptotically. Although we have not carefully studied how the growth rate of government spending itself behaves during the transition, it is obvious that if the government spending suddenly rises in the impact period, since the level of capital, K_t, is predetermined, then the capital PGSU, $\hat{k}_t = K_t / G_t$, should suddenly drop. Moreover, if $\hat{k}_t$ drops on impact, then it will rise over time, meaning the level of capital and output, K_t and Y_t, grow faster than the level of government spending along the transition path. The reason behind this fact is the temporary boost in growth. Therefore, the transitional dynamics for $\hat{k}_t$ and $\hat{y}_t$ show that they initially fall and then return to their old steady-state. The reason is that the consumption tax, τ^c, does not affect the steady-state of both the capital PGSU and the output PGSU, but it affects the level of government spending. Regarding consumption PGSU, $\hat{c}_t$, a consumption tax would decrease the consumption PGSU, moving from one steady-state to another one. However, during the transition path, consumption PGSU jumps initially downwards in the impact period and then tends to its new steady-state, which is less than the old one. Therefore, the initial jump of $\hat{c}_t$ is unlike the one of $\hat{k}_t$, where the latter adjusts gradually downwards over time.

On the other hand, we have found theoretically in the existence of steady-state of both types that the type of steady-state depends on g^2 and τ^c. For g^2, if it is sufficiently high, then the steady-state will be an exogenous growth steady-state (*i.e.* $g^2 > \gamma_c$). On the contrary, if g^2 is sufficiently low, then the steady-state will be an endogenous growth steady-state (*i.e.* $g^2 < \gamma_c$). However, since g^2 is out of the authority's control, we focus on varying τ^c instead, since it is a policy parameter.

We know that for a different set of parameter values, but under some restrictions in parameter values, γ_c would be greater than one, *i.e.* to ensure that we have endogenous growth instead of endogenous shrinkage. These restrictions, as shown in equation (5.27), are that β should be close to one, $\sigma = 1$, and $\delta = 0$. By using these parameter value restrictions as well as other standard parameter values in equation (5.27), we can verify that γ_c is greater than one (*i.e.* $\gamma_c = 1.0003$). Consequently, under these parameter value restrictions, we can also confirm that γ_c would be $\gamma_c > g^2$ too. In other words, we could have endogenous growth steady-state under these restrictions because if it is otherwise, then γ_c is turning out below g^2 and then the actual steady-state becomes the exogenous growth steady-state. Therefore, if we compare γ_c under its restrictions with two values of g^2, (one is high, *e.g.* $g^2 = 1.01$ and one is low, *e.g.* $g^2 = 0.1$), we can conclude that when g^2 is high, then the actual steady-state is the exogenous growth steady-state because $\gamma_c < g^2 \Rightarrow 1.0003 < 1.01$. However, if g^2 is low, then the actual steady-state in this case would be the endogenous growth steady-state because $\gamma_c > g^2 \Rightarrow 1.0003 > 0.1$.

We can now also provide a simple numerical exercise to show when the economy could move from one type of steady-state to another one if we hold g^2 constant and change consumption tax. In this exercise, we attempt to find out the critical value that allows the economy to change its steady-state position. We assume that g^2 is fixed at 1.01, and then we increase τ^c using the formula of γ_c in equation (5.27) to find its possible values. The below table displays our calculation for γ_c and the potential type of steady-state for each value of τ^c.

Table 8: The calculation for $\boldsymbol{\gamma_c}$ and the potential type of steady-state for each value of $\boldsymbol{\tau^c}$:

τ^c	γ_c	g^2	***Steady-State Type***	
10%	*1.0011*	*1.01*	$g^2 > \gamma_c$	*Exogenous Growth Steady-State*
20%	*1.0038*	*1.01*	$g^2 > \gamma_c$	*Exogenous Growth Steady-State*
30%	*1.0074*	*1.01*	$g^2 > \gamma_c$	*Exogenous Growth Steady-State*
36.4%	*1.0100*	*1.01*	$g^2 = \gamma_c$	***Critical Value***
40%	*1.0115*	*1.01*	$g^2 < \gamma_c$	*Endogenous Growth Steady-State*
50%	*1.0157*	*1.01*	$g^2 < \gamma_c$	*Endogenous Growth Steady-State*
60%	*1.0199*	*1.01*	$g^2 < \gamma_c$	*Endogenous Growth Steady-State*

The above table shows that the critical value is 36.4%, which implies that if the authority sets the consumption tax at $\tau^c < 36.4\%$, then the economy would have exogenous growth steady-state. On the other hand, if the authority sets the consumption tax at $\tau^c > 36.4\%$, then the economy would move to the endogenous growth steady-state. Thus, according to our formula and simplification, the above table clarifies that it is possible to discover the critical value. However, it should be noted that even though our simple numerical example does not reflect exactly the real world because of our simplifying and some parameter restrictions, it is helpful to get a better understanding regarding the idea of how to achieve the critical value and how the economy could turn out from one type of steady-state to another one.

Let us now make some sensitive analysis in our simulations regarding the consumption tax thresholds and the parameters. The critical value discussed in our model can be defined as the tax threshold or the value of the switching regime, *i.e.*, from exogenous to endogenous growth steady-state or vice versa. Thus, to determine which variables affect the consumption tax thresholds in our model, we look at equation (5.27) and solve it for consumption tax. By doing so, equation (5.27) can now be rearranged as follows:

$$\tau^c = \frac{\left(\frac{A^{-\frac{1}{\alpha}}(\gamma_c - 1 + \delta)}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}}{1 - \alpha - \left(\frac{A^{-\frac{1}{\alpha}}(\gamma_c - 1 + \delta)}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}} \qquad (5.27)'$$

At the critical value, we understand that γ_c is equal to g^2. Thus, it is obvious that the consumption tax threshold, equation (5.27)', depends on four parameters, namely g^2, α, δ, and A. However, their effects on the tax thresholds seem to be theoretically ambiguous since equation (5.27)' is a non-linear equation. Thus, it would be better to use a numerical solution to find out the tax thresholds within different sets of parameter values.

Before we start our experiment, it is worth mentioning that there are plausible parameters used in the literature. According to Mitra, Hosny, Abajyan, and Fischer, it is assumed that the value of the production function parameter, α, is within the range of 0.40 to 0.67 for oil-exporting countries and 0.25 to 0.40 for oil-importing countries. On the other hand, the physical capital depreciation rate, δ, is also estimated to be between 0.05 and 0.15 for these countries (Mitra et al., 2015). Similarly, the IMF's selected issues report regarding Saudi Arabia empirically showed that the share of physical capital in emerging and developing economies is assumed to be 0.67 because the capital in these countries has a high rate of return due to its scarcity (IMF, 2013). Moreover, Aljebrin (2013) specifically estimated Saudi's production function for GDP for 1984-2011, finding that the elasticity of output relative to capital and labour were 0.67 and 0.57, respectively.

To see how the tax thresholds change, we start this experiment by varying α and keeping other parameter values constant. As mentioned previously, the values of α will be within the range referred to in the literature as empirical evidence. We then keep making the same calculations for all other parameters. We provide figures that display how the consumption tax thresholds vary when we change the other parameter values in each experiment. We then discuss at the end of this section these tax thresholds.

Let us begin by varying the values of α and holding other parameters fixed at $g^2 = 1.01$, $\delta = 0.05$, and $A = 1$.

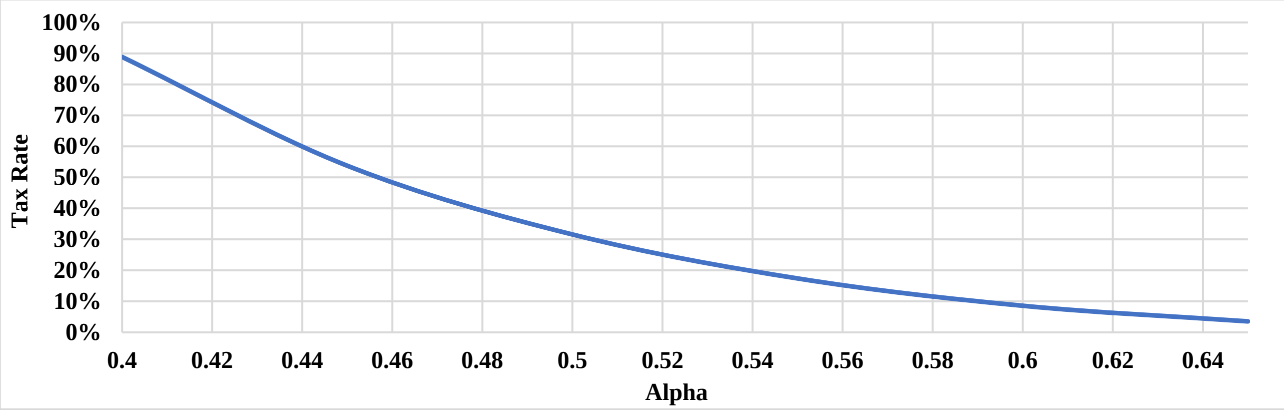

Figure 37: The consumption tax thresholds response to changes in α

Next, we change the value of A and keep all other parameters constant.

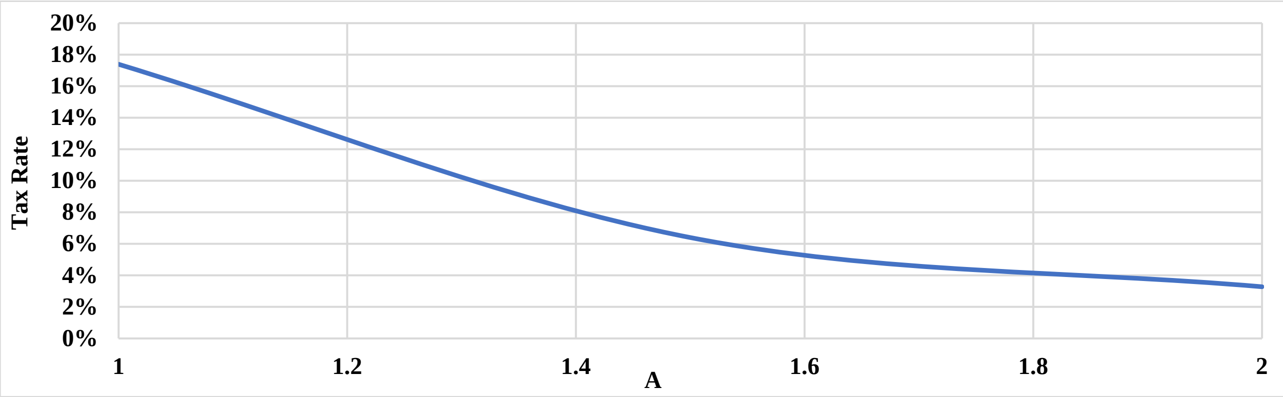

Figure 38: The consumption tax thresholds response to changes in A

Similarly, we alter the values of g^2 and fix all other parameters constant.

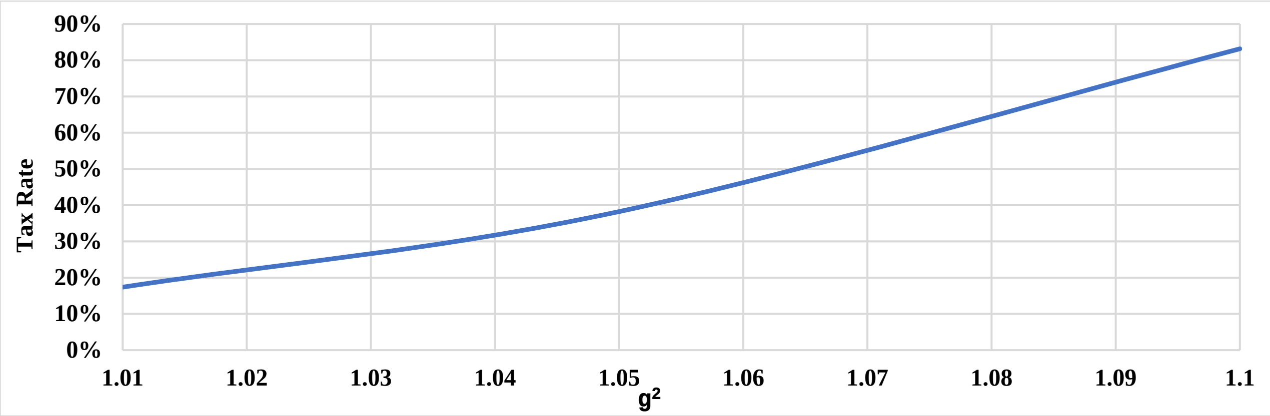

Figure 39: The consumption tax thresholds response to changes in g^2

Finally, we vary the values of δ and hold all other parameters constant.

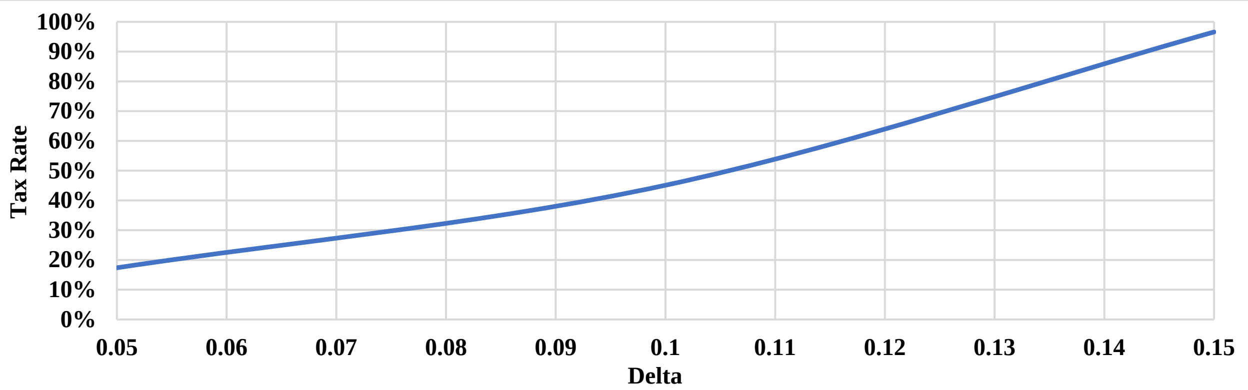

Figure 40: The consumption tax thresholds response to changes in δ

As illustrated in the above figures, our numerical solutions show that higher α implies lower consumption tax thresholds. Similarly, higher A provides lower consumption tax thresholds. However, if either g^2 or δ is higher, the consumption tax thresholds would then be higher too.

5.9.2 Part Two: Keeping Oil Revenues Unchanged and Introduce Consumption Tax (Level Variables)

This part seeks to explore how the key endogenous level variables in our model, such as the level of consumption, capital stock, government spending, and output, can be changed if we introduce a consumption tax, keeping both the growth rate of our economy[85] and oil revenues unchanged. The purpose of studying this part is to find out whether this simulation matches our results in the theoretical section of level variables. According to our model, if the consumption tax is introduced, then the level variables in our economy will be changed accordingly. However, we have found that the value of ω^* is crucial in our model, which depends on the value of the discount factor, β Thus, determining the value of β and then the value of ω^* is, in fact, more important to track the time path of consumption relative to the π_t, where the other level variables are less sensitive to the value of ω^*. Thus, we discuss in this part two different cases to find out how the main level variables respond to the change in the consumption tax.

[85] As we have shown in the level variables section, the growth rate of government spending, $\gamma_G = g^2$, enters into the equation of w^*, see equation (5.32); thus, we set $g^2 = 1$.

5.9.2.1 Case One: Introducing 5% consumption tax and keeping oil revenues unchanged: β is sufficiently close to one (*e.g.* β=0.90):

In the first case, we set β to be sufficiently close to one, so that the value of ω^* will be greater than one, $\omega^* > 1$. Therefore, introducing a consumption tax rate of 5% would increase all endogenous variables in our economy. More precisely, the level of consumption would increase by 3.68%, while the level of capital stock, government spending, and output would all increase by almost the same percentage, 8.86%. The reason for increasing these variables is that government spending plays a crucial role in our economy. Thus, introducing a consumption tax would boost government revenues, which is reflected in increased government spending. High government spending implies more positive externalities enters in firm's production function. This, in turn, enhances the firms by increasing their level of capital stock and output. Since the firms receive some positive externalities, they may somehow transfer them to the households in terms of goods and services.

Moreover, this case also shows that the level of investment would also increase by 8.89% due to the introduction of a 5% consumption tax rate. The increase in the level of investment can be easily seen from its equation[86]. We understand from the neoclassical theory[87] that increasing the level of consumption would decrease the level of investment. However, our model shows that although the level of consumption increases due to consumption tax, the level of investment would also increase. The reason is that the level of output changes, and its increase is higher than the increase in the level of consumption[88].

The intuition behind increasing the level of investment is that introducing a consumption tax leads to an increase in the government spending level. That implies that the firms would receive more positive externalities; therefore, the level of investment would increase. The higher investment would encourage the firms to create more employment. Thus, starting from introducing consumption tax to increase the investment level would eventually benefit the economy. Consequently, according to our model, if we introduce a 5% consumption tax rate, set β sufficiently high, and keep the oil revenues constant, then the economy will benefit positively from that. The table below shows the steady-state responses of a number of level variables in the economy to a change in consumption tax.

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[86] We know that the investment PGSU equation can be written at steady-state as $I/G = (g^2 - 1 + \delta)\hat{k}^*$, which allows us to rewrite the level of investment as $I = (g^2 - 1 + \delta)\hat{k}^*.G$. Thus, since rising consumption tax does not affect $\hat{k}^*$, but it increases G, then it should eventually raise the level of investment.

[87] We know in the neoclassical growth models that the resource constraint of a closed economy is $Y_t = C_t + I_t$ and $I_t = savings$. Thus, if Y_t is unchanged, then increasing consumption decreases investment/savings.

[88] Note that, in our model, $Y_t = (1 + \tau^c)C_t + I_t$, which can be also written as $I_t = Y_t - (1 + \tau^c)C_t$.

Table 9: The steady-state responses of a number of level variables in the economy when we introduce a 5% consumption tax rate and keeping oil revenues unchanged, where β=0.90:

Consumption Tax = 0		**Consumption Tax = 5%**	
Variables	**Value**	**Variables**	**Value**
Level of Government Spending	2	Level of Government Spending	2.1772
Level of Output	3.6753	Level of Output	4.0010
Level of Capital	1.2824	Level of Capital	1.3960
Level of Consumption	3.4188	Level of Consumption	3.5445
Investment	0.2564	Investment	0.2792
Interest Rate	0.4444	Interest Rate	0.4444

5.9.2.2 Case Two: Introducing 5% consumption tax, and keeping oil revenues unchanged: β is sufficiently close to zero (*e.g.* β=0.10):

In the second case, we set β to be sufficiently close to zero, *i.e.* $\beta = 0.1$, so that the value of ω^* will be less than one, $\omega^* < 1$. Introducing a 5% consumption tax rate would increase all the main level variables in our economy except the level of consumption. More precisely, the level of capital stock, government spending, and output would all increase by almost the same percentage, 0.94%. In contrast, the level of consumption would decline by 3.86%. The table below shows the steady-state responses of a number of level variables in the economy to a change in consumption tax, under the assumption of setting β sufficiently close to zero.

Table 10: The steady-state responses of a number of level variables in the economy when we introduce a 5% consumption tax rate and keeping oil revenues unchanged, where β=0.10:

Consumption Tax = 0		**Consumption Tax = 5%**	
Variables	**Value**	**Variables**	**Value**
Level of Government Spending	2	Level of Government Spending	2.01887
Level of Output	0.394614	Level of Output	0.398336
Level of Capital	0.010385	Level of Capital	0.010482
Level of Consumption	0.392537	Level of Consumption	0.377371
Investment	0.002077	Investment	0.002096
Interest Rate	11.2	Interest Rate	11.2

If we compare case one, section 5.9.2.1, and case two, section 5.9.2.2, we can confirm our previous result discussed in section 5.5.3, which can be summarised in the below table:

Table 11: The impact of τ^c on C_t/π_t when β is sufficiently close to one and sufficiently close to zero

The value of β	**The value of ω^***	**The impact of τ^c on C_t/π_t**
If β is sufficiently close to one	$\omega^* > 1$	↑
If β is sufficiently close to zero	$\omega^* < 1$	↓

5.9.3 Part Three: Decreasing Oil Revenues and Changing Consumption Tax (Level Variables)

Since we understand that government spending plays an essential role in our economy, it is then apparent that any reduction in government spending would cause a negative impact on economic growth and the whole economy. In more precisely, we know that before introducing the consumption tax, the government budget constraint is $G_t = \pi_t$, while after introducing the consumption tax, it becomes like, $G_t = \pi_t + \tau^c C_t$. Therefore, if oil revenues decline, government spending will be negatively affected, whether with or without consumption tax revenues, since the oil revenues remain one of the main sources of government revenues. Thus, this part aims to study how the consumption tax can compensate for any possible reduction in oil revenues and the rate of consumption tax that can guarantee the government spending to be unchanged. In other words, what the amount of consumption tax that can maintain stable government revenue and equally stable government spending when the oil revenues decline by a certain percentage. The purpose of making government spending fixed in this part is because we want to keep the economic growth not affected by changing government spending.

To see that, we begin with the government budget constraint and derive a formula for the consumption tax, which ensures the government spending unchanged, such that,

$$\tilde{\tau}^c = \frac{\bar{G} - \pi_t}{C_t} \tag{5.45}$$

$\tilde{\tau}^c$ is the required consumption tax rate that compensates for any reduction in oil revenues. $\bar{G}$ is the target government spending, which is assumed to be constant all the time in this exercise, *i.e.* we assume that $g^2 = 1$. π_t and C_t are the oil revenues and the level of consumption, respectively.

We start the numerical exercise when there is no consumption tax $(\tau^c = 0)$; therefore, from the government budget constraint $(G_t = \pi_t + \tau^c C_t)$, we know that the government spending will be equal to the oil revenues at the target value $(G_t = \pi_t = \bar{G})$. We assume that the target government spending in this exercise equals 2, $(\bar{G} = 2)$.

Equation (5.45) depends on the level of consumption, which can be found in equation (5.31)' and can also be written as:

$$C_t = \frac{\pi_t \ \omega^*}{1 + \tau^c - \tau^c \omega^*}$$

Therefore, substituting the above equation into equation (5.45) and simplifying it gives us,

$$\tilde{\tau}^c = \frac{\bar{G} - \pi_t}{\pi_t + \bar{G}\left(\omega^* - 1\right)} \tag{5.46}$$

Equation (5.46) depends on the target government spending, $\bar{G}$, oil revenues, π_t, and ω^*. However, the value of ω^* is crucial[89] to determine the required tax rate that can compensate for any reduction in oil revenues. In more precisely, if ω^* tends to infinity $(\omega^* \rightarrow \infty)$, then the required tax rate tends to zero $(\tau^c \rightarrow 0)$. In other words, a high value of ω^* implies a less required consumption tax rate to offset any fall in oil revenues.

According to our calculation, under our used parameter values, the value of $\omega^*$is 1.71[90]. We keep this value of ω^*unchanged and vary the oil revenues. We initially decline the oil revenues by 1% and find the required tax rate that can compensate for this reduction. We then carry on declining oil revenues to determine the appropriate tax rate for each decline in oil revenues. To be more precise, we give an example of how this exercise works. Suppose first that the target government spending is 2, which can maintain the economic growth rate unchanged[91]. Thus, if the oil revenues declined by 5% (from 2 to 1.90), the required tax rate to compensate for this reduction is 3.01%. As a result, based on our formula in equation (5.46), this tax rate can ensure that not only it compensates for the 5% reduction in oil revenues, but also the government spending, and eventually the economic growth rate would remain unchanged at 2.

Table 12: A numerical exercise on how different tax rates can compensate for different declining in oil revenues to keep the government spending stable and unchanged at 2:

Target $\bar{G}$ (Value)	***Oil Revenues Declining (Rate)***	***Oil Revenues (Value)***	***Required Consumption Tax (Rate)***
2	0%	2.00	0.00%
2	1%	1.98	0.59%
2	2%	1.96	1.18%
2	3%	1.94	1.79%
2	4%	1.92	2.40%
2	5%	1.90	3.01%
2	10%	1.80	6.21%
2	20%	1.60	13.25%
2	30%	1.40	21.28%
2	40%	1.20	30.53%
2	50%	1.00	41.32%

[89] The value of ω^* is the crucial one here because both $\bar{G}$ and π_t do not change the tax rate (*i.e.* if $\omega^* = 0$, then $\tau^c = \frac{\bar{G}-\pi_t}{\pi_t-\bar{G}} = -1$).

[90] We obtained the value of ω^* by using its simplified equation in section 5.5.3.

[91] On the other hand, the oil revenues also initially set to be at 2, because if there is no consumption tax, then $G_t = \pi_t = \bar{G} = 2$.

According to our model and depending on the parameter values chosen, the above table shows all possible required rates of consumption tax that can compensate for any reduction in oil revenues and keep the government spending unchanged.

5.10 Conclusion

In conclusion, we summarise our model and highlight the main results we obtained in each section of this chapter. First of all, the model of this chapter is built upon chapter four, where we extended the Barro model (1990) by introducing a consumption tax as recommended by the IMF. One fundamental difference from chapter four is in the government budget constraint. In chapter four, we only have one source of government revenues, namely oil revenues, whereas we included in the current chapter a consumption tax as an additional source of government revenues. Thus, the government budget constraint in this chapter has two sources of revenues: oil revenues and consumption tax revenues.

Since we have two different sources of government revenues, we came to a result that we cannot have a steady-state in our model in which one of these sources of government revenues grows at a different rate from the other. Thus, we discussed three possible types of steady-state. Two of these steady-states involved that government spending, G_t, grows at the rate of g^2. One of these two steady-states involved that consumption also grows at the rate g^2, while the other involved that consumption grows at a slower rate than g^2. The third possible type of steady-state involved that the consumption grows faster than oil revenues.

However, at the beginning of analysing the steady-state, the growth model in this chapter was not obvious because the two sources of government revenues did not tell us what our economy would eventually represents. Therefore, we studied these two sources separately (*i.e.* two cases of growth rate). The first case is when we have only one source of government revenues, namely oil revenues, and the growth rate of this case represents an *exogenous growth model*. The second case is when we have a pure consumption tax revenue as an only source of government revenues, and the growth rate of this case represents an *endogenous growth model*. Because of the first case discussed in more detail in chapter four, we focused on the second case (*i.e.* pure consumption tax case). By studying the second case, the *endogenous growth model*, we found that there are some restrictions in parameter values (*i.e.* the discount factor, β, should be close to one, $\sigma = 1$, and $\delta = 0$) that must hold, in order for γ_c to be greater than one. This result can ensure us to have *endogenous growth steady-state* in our model. Consequently, since γ_c is possible to be $\gamma_c > 1$ (of course with the above restrictions), then it would also be possible to obtain $\gamma_c > g^2$, because g^2 is an exogenous source and can be less than γ_c.

Besides that, we examined if both types of steady-state, the *exogenous and the endogenous growth steady-state*, exist for the same set of parameter values. Our proof showed that there is an unavoidable contradiction between two conditions. These two conditions are: for the *exogenous growth steady-state* to exist, this ratio C_t/π_t cannot be negative, and for the *endogenous growth steady-state* to exist, we need $\gamma_c > g^2$. These two conditions are indeed essential for each steady-state to exist. Therefore, we came to a result that both types of steady-state growth rate cannot exist for the same set of parameter values. We also found that if g^2 is sufficiently high (low) for a given value of τ^c, then the steady-state will be an *exogenous growth steady-state* (an *endogenous growth steady-state*).

We have also studied how and when the economy could move from a steady-state to another one when we set g^2 constant and vary consumption tax. The reason behind investigating this further was due to the fact that g^2 is exogenous and out of control. Thus, the policymaker would be interested in focusing on the policy parameter. Then, we have concluded that the economy will switch from the *exogenous growth steady-state* to the *endogenous growth steady-state* at a certain value of consumption tax. This transformation would provide a higher growth rate because high taxes would help provide sufficient government spending to enhance the firms' production function. Thus, we have shown the possibility of a switching regime as we change the policy parameter. This is a crucial result in our model because the switch from one type of steady-state to another is unusual in most growth models. Regarding the transitional dynamics in a pure consumption tax case, we found that there is only one eigenvalue that is unstable. We then conclude that there is no transitional dynamics in this case, which has also confirmed by Bambi and Venditti (2018).

In the level variables, we studied these variables with the two sources of government revenues. The primary reason for studying the level variables in our model is understanding how our level variables respond to a change in consumption tax. Our results showed that if we increase consumption tax, then the level of government spending and the capital stock would increase. The intuition behind increasing the level of government spending is simply due to the increase in government revenues with a consumption tax, while increasing the capital stock level is because firms receive positive externalities from the government sector. Regarding the level of consumption, increasing consumption tax would generally cause a decrease in consumption level because people would save more when consumption tax is high since their purchasing power decreases. However, we found an interesting result that although consumption tax is increased, the level of consumption in our model may increase or decrease, depending on the value of the discount factor, β. In this regard, we discovered that if β is sufficiently close to zero, then increasing consumption tax would decrease the level of consumption. If, however, β is sufficiently close to one, then increasing consumption tax would increase the level of consumption. The intuition behind the latter result may be due to the fact that the households get an advantage from increasing government spending through the firms. More precisely, the firms could somehow pass on some of the positive externalities, which received through the government, to the households. Examples of this may be high wages or a high rental rate of capital.

Regarding the stability and transitional dynamics in our economy, we at the beginning speculated that there are one stable eigenvalue and one unstable eigenvalue due to the relationship between $\hat{k}_t$ and $\hat{c}_t$. Then, we confirmed that is true by applying the test provided by Rankin (2007). Subsequently, we concluded in this section that the steady-state is a saddle-point, if g^2 is sufficiently high. On the other hand, we drew a phase diagram to show the characteristics of a dynamic system and compared it with the one in the Ramsey model and chapter four. In the Ramsey model, $\hat{k}_t$ is a predetermined variable, while $\hat{c}_t$ is a non-predetermined variable. In our current model, however, both $\hat{k}_t$ and $\hat{c}_t$ are completely non-predetermined variables. Although both variables are jump variables, they cannot jump freely, unlike the conventional Blanchard and Kahn type solution, because we believed that there might be some relationship between them, which has to be held. We then proved the relationship between $\hat{k}_t$ and $\hat{c}_t$ both geometrically and analytically. Geometrically, it represents the new locus (*i.e.* the diagonal line, *D*) in the phase diagram, which is somewhat different from the one in the Ramsey model and ours in chapter four. Analytically, we derived equation (5.43), which represents the diagonal line as a non-linear equation. We also found that the slope of the diagonal line is downward sloping. As a result, we confirmed our anticipation that there is a limit or a restriction on how $\hat{k}_t$ and $\hat{c}_t$ can jump relative to each other.

Finally, we use a numerical simulation under standard and commonly used parameters in the literature to simulate the model in three main parts. In the first part, we introduce consumption tax to see how the PGSU variables can change, and we also examine what type of steady-state we should be in under a given set of parameter values. Our simulation results indicated that the growth rate of government spending in the impact period would be temporarily high due to the consumption tax. During the transition path, $\hat{k}_t$ and hence $\hat{y}_t$ would initially drop in the impact period because of the sudden rise in government spending. Then, they return to their old steady-state because the consumption tax does not have an impact on their steady-state equations. On the other hand, $\hat{c}_t$ falls initially in the impact period and then tends to its new steady-state, which is less than the old one. The reason for not turning back to its old steady-state is due to the fact that consumption tax directly affects $\hat{c}^*$. Regarding what type of steady-state we must be in, we found that under some restriction in parameter values, γ_c can be greater than one. We compared γ_c under its restrictions with two values of g^2, then we concluded that if g^2 is set to be high (*e.g.* $g^2 = 1.01$), then the actual steady-state is the *exogenous growth steady-state,* because $\gamma_c < g^2$. If, on the other hand, g^2 is set to be close to one (*e.g.* $g^2 = 0.1$), then the actual steady-state in this case would be the *endogenous growth steady-state*, because $\gamma_c > g^2$. Moreover, we presented a simple numerical exercise to calculate when the economy could move from a steady-state to another if we hold g^2 constant and change consumption tax. Based on the parameter values chosen in our model, if the authority sets the policy parameter at $\tau^C < 36.4\%$ ($\tau^C > 36.4\%$), then the economy would have the *exogenous (endogenous) growth steady-state*.

In part two, we investigated what happens to the key endogenous level variables if the consumption tax is added by simulating these level variables under two different cases. The first case is that when β is sufficiently close to one (*e.g.* $\beta = 0.9$), while the second case is that when β is sufficiently close to zero (*e.g.* $\beta = 0.1$). Our simulation results are consistent with our findings in all level variables, where the value of β is only crucial in our model to determine the time path of the level of consumption. Nevertheless, the value of β did not substantially impact other level variables (*i.e.* the level of government spending, capital stock, and output all increased in both cases of β, and that increase was due to increased consumption tax). In other words, the value of the discount factor played an essential role in determining the time path of the level of consumption.

In the third part of our simulation section, we worked on a numerical exercise to determine the amount of different consumption tax that can compensate for any reduction in oil revenues. We derived a formula for consumption tax, equation (5.46), that allowed us to calculate the required tax rate that compensates for any reduction in oil revenues and keeps, at the same time, the government spending unchanged. Then, we provided a table, *i.e.* Table 12, that displayed how different tax rates could compensate for different declining in oil revenues to keep the government spending stable and unchanged at 2. In the next chapter, we extend our model further by introducing personal income tax.

6 Chapter Six: Modelling Saudi Economic Growth with Personal Income Tax

6.1 Introduction

This chapter is related to the previous chapters, where we would now have more discussion about introducing a new type of taxes in our model. This type of tax is a personal income tax. In particular, this chapter aims to find out how a new fiscal policy tool can work and affect the growth rate and the key level variables in the Saudi economy. Although Saudi Arabia does not currently have this type of tax, there are four motivations to study personal income tax in our economy. *(i)* One of the important primary goals of Saudi's Vision to be achieved in 2030 is to generate revenues away from oil through diversifying sources of income, where this type of tax could be one of these sources. Thus, personal income tax would be applied in the near future in the Saudi economy since the country now seeks to reduce its dependence on oil. *(ii)* The International Monetary Fund (IMF) has mentioned early in their report 'Tax Policy Reforms in the GCC Countries: Now and How?' in 2015 that there is plenty of scope for Saudi Arabia to implement new taxes. Furthermore, the IMF has recently[92] urged the GCC, including Saudi Arabia, to diversify their sources of income. The IMF has confirmed that additional revenues other than oil may contribute to alleviating future financial pressure on the budgets of these countries. For two main reasons, they have indicated that the reform of the current tax system as a key tool in fiscal policy is important. The first reason is to ensure revenue stability, while the second reason is due to the limited taxes in the state. *(iii)* As per our knowledge, there is no previous research in the literature that examine introducing personal income tax and its impact on Saudi economic growth. *(iv)* The last motivation for studying this type of tax is our desire to compare the two fiscal policy tools: consumption and personal income taxes, in our model[93]. The main objective of making this comparison is to find out the suitable reform in the fiscal policy for the Saudi economy. Consequently, this chapter will be focused on introducing personal income tax as the second source of government revenues along with oil revenues. Like previous chapters, we will build a theoretical framework based on the Barro (1990) model to examine the agents in the economy, equilibrium, analysis of the steady-state, stability, transitional dynamics, and numerical simulation.

[92] The IMF published a paper on February 6, 2020, called 'The Future of Oil and Fiscal Sustainability in the GCC Region'.

[93] It could be possible to study three sources of government revenues, the consumption and personal income taxes along with oil revenues, together; however, that would make our theoretical model more complicated. Thus, we examine them separately and then compare between them.

6.2 Model Description

Like chapter five, the economy in this chapter has five sectors: households, firms, oil, government, and external sectors. The households sector receives personal income through wages and salaries from firms, and purchase goods and services provided by firms. They also pay income tax to the government sector. We assume that households cannot access the international market and financial sector. The firms' sector hires the factor of production and pays wages and rent. As we set it up previously, the oil sector is monopolistic and exogenous. It maximises its profits and then passes them on to the government because the government entirely owns it. The government sector receives personal income taxes from the households sector and oil profit from the oil sector. Then, it exchanges all these revenues to imported goods from the external sector. In other words, the government exports all its revenues and imports goods. Thus, the external sector in our model deals only with the government sector from abroad. Finally, once the government receive imported goods from abroad, it then provides public goods to the firms' sector instead of supplying social transfers to households. These public goods enter into the firms' production function as positive externalities. The below diagram elaborates the structure of our economy in this chapter.

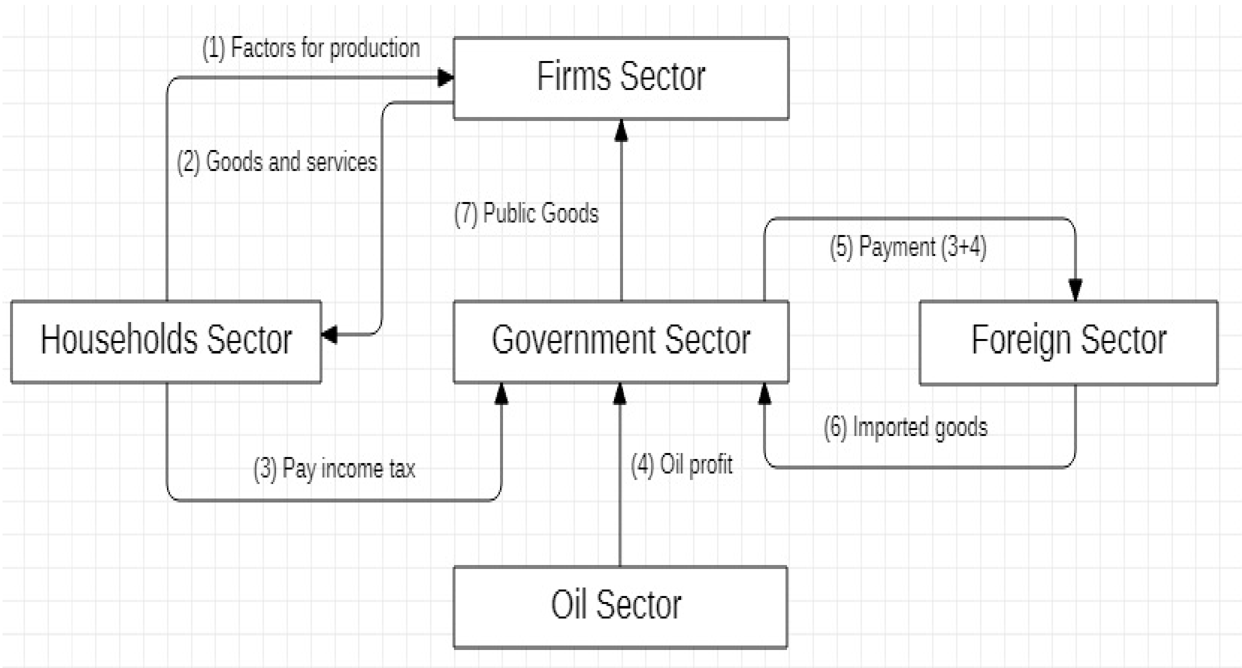

Diagram 18: The structure of the economy in chapter six

In this chapter, there will be some fundamental differences from previous chapters due to the introduction of personal income tax. The differences are in the accounting identity and government budget constraint, balance of trade, and the PGSU and level variables.

Let us begin with the accounting identity and government budget constraint. Regarding the accounting identity, it is indeed like the case of introducing only consumption tax in the fifth chapter, where the government converts all its revenues, oil profits plus tax, into imported goods. However, the market-clearing condition[94] becomes now different and takes the form:

$$Y_t = C_t + I_t + X_t'$$

where $X_t' = \tau^Y Y_t = \text{exports}$[95].

Rewriting the resource constraint of the economy give us,

$$Y_t = \frac{C_t + I_t}{\left(1-\tau^Y\right)}$$

Similar to chapter five, we assume that the relative price of the imported goods in terms of domestic good is exogenous and normalised to one. Thus, the government budget constraint can be written as,

$$G_t = \pi_t + X_t'$$

Regarding the balance of trade in this chapter, there are two types of exports, oil revenues and what the government received through the personal income tax, and one type of import, which is the government purchasing goods.

$$\text{Trade Balance} = \text{Exports} - \text{Imported Goods} = 0$$

where exports = $\pi_t + \tau^Y Y_t$, which is identical to the government budget constraint.

For the PGSU variables, the capital PGSU and output PGSU steady-state would be changed when introducing personal income tax because it enters into them, unlike the case when we introduce only consumption tax. Moreover, the consumption of PGSU steady-state would also be affected by this new tax. In this regard, we will study in section 6.5.1 of this chapter how personal income tax would impact the PGSU steady-state variables. On the other hand, introducing personal income tax would also have an impact on other key level variables in our model. Thus, we will discuss that as well in more details in section 6.5.2.

[94] Similar to the previous chapter, the notation Y_t does not represent the totality of GDP in our economy. It indeed represents the non-oil goods.

[95] The different from chapter five is that X_t in chapter five represents the revenues of consumption tax (*i.e.* $X_t = \tau^c C_t$).

6.3 Model Setup

There are three main agents in the economy of this chapter: the government, households, and firms sectors. The two other sectors, oil and external sectors, are not considered in this chapter. The reason is that the oil sector is exogenous and has been discussed in detail in chapter four. Thus, we include the oil revenues as a source of government revenues along with the personal income tax. On the other hand, the reason for not studying the external sector here is because what it does is like the previous chapters, where it only deals with the government sector, as explained before.

6.3.1 Government Sector

The government sector has two sources of government revenues: *(i)* the profit of the oil sector and *(ii)* personal income tax from the households sector. As before, the government revenues are assumed to be spent all on purchasing imported goods from abroad. The public goods (*e.g.* infrastructure, legal framework) will be provided to the firm's sector in order to enhance their production function. As a result, the government budget can be written as:

$$G_t = \pi_t + \tau^Y Y_t \tag{6.1}$$

where G_t is government spending, π_t is the oil revenues, and τ^Y is a constant personal income tax.

From equation (6.1), we define the growth rate of government spending as $\gamma_{G_t} \equiv G_{t+1}/G_t$.

$$\gamma_{G_t} \equiv \frac{G_{t+1}}{G_t} = \frac{\pi_{t+1} + \tau^Y Y_{t+1}}{\pi_t + \tau^Y Y_t} \tag{6.2}$$

where $\pi_{t+1}/\pi_t = g^2$ is the growth rate of the oil profits, which we have found it early.

$$\frac{\pi_{t+1}}{\pi_t} = g^2 \quad \Rightarrow \quad \pi_{t+1} = g^2 \; \pi_t$$

while $Y_{t+1}/Y_t = \gamma_{Y_t}$ is the growth rate of output, which can also be written as,

$$\frac{Y_{t+1}}{Y_t} = \gamma_{Y_t} \quad \Rightarrow \quad Y_{t+1} = \gamma_{Y_t} Y_t$$

6.3.2 Households Sector

It is similar to the problem of the households defined in the previous chapters, where the utility function is CRRA, and only a function of consumption with parameter $\sigma > 0$. Thus, the households supply is also inelastic. However, personal income tax is now added and modified the consumer's budget constraint. Thus, the consumer's budget constraint at a time (t) is written as:

$$\left(1-\tau^{Y}\right)\left(R_t K_t + w_t L_t\right) = C_t + K_{t+1} - K_t + \delta K_t$$
$$C_t,\, K_{t+1} > 0 \quad \forall t \;; \quad K_0 > 0 \quad \textit{exogenously given}$$

Where $\beta, \delta \in (0,1)$ are the discount factor and the constant depreciation rate of capital, respectively. $R_t K_t$ is the gross capital income, $w_t L_t$ is the labour wage, C_t is the consumption, τ^Y is the personal income tax, and $I_t = K_{t+1} - K_t + \delta K_t$ is the gross investment. So now, we can rewrite the capital accumulation as:

$$K_{t+1} = \left(1-\tau^{Y}\right)\left(R_t K_t + w_t L_t\right) + \left(1-\delta\right) K_t - C_t \tag{6.3}$$

This represents the households' capital supply. Thus, the Lagrangian for this problem is:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[\left(\frac{C_t^{1-\sigma} - 1}{1-\sigma}\right) + \lambda_t\left(\left(1-\tau^{Y}\right)\left(R_t K_t + w_t L_t\right) + \left(1-\delta\right) K_t - C_t - K_{t+1}\right)\right]$$

The first-order conditions:

$$\frac{d\mathcal{L}}{dC_t} = 0 \;\Leftrightarrow\; \beta^t C_t^{-\sigma} = \beta^t \lambda_t \;\Rightarrow\; C_t^{-\sigma} = \lambda_t \tag{6.4}$$

$$\frac{d\mathcal{L}}{dK_{t+1}} = 0 \;\Leftrightarrow\; -\beta^t \lambda_t + \beta^{t+1}\lambda_{t+1}\left[\left(1-\tau^{Y}\right) R_{t+1} + 1 - \delta\right] = 0 \tag{6.5}$$

$$\frac{d\mathcal{L}}{d\lambda_t} = 0 \;\Leftrightarrow\; K_{t+1} = \left(1-\tau^{Y}\right)\left(R_t K_t + w_t L_t\right) + \left(1-\delta\right) K_t - C_t$$

Substitution (6.4) into (6.5) gives us the Euler equation:

$$\frac{C_{t+1}}{C_t} = \left[\beta\left(\left(1-\tau^{Y}\right) R_{t+1} + 1 - \delta\right)\right]^{\frac{1}{\sigma}} \tag{6.6}$$

6.3.3 Firms Sector

The firms' problem is the same as we set it up in previous chapters, where the firms' production function is assumed to be a Barro-type (1990) production function. In this type of production function, government spending acts as positive externalities to enhance the firms' production.

$$Y_t = A\, K_t^{\alpha}\, L_t^{1-\alpha}\, G_t^{1-\alpha} \qquad 0 < \alpha < 1$$

where :

Y_t: Level of output at time t;
A : Technology level in the economy;
K_t: Physical capital at time t;
L_t: Labour at time t;
G_t: Government externalities at time t; and
α : The output elasticities of capital, labor, and government externalities.

The firms maximise their profits by taking the rental rate of capital, R_t, and wage per unit of labour, w_t, as given, subject to their production function:

$$\underset{K_t,\, L}{Max}\; A\, K_t^{\alpha}\, L_t^{1-\alpha}\, G_t^{1-\alpha} - R_t K_t - w_t L_t$$

In the same way as before, we divide the firms' production function by government spending, G_t, to get pre government spending unit (PGSU) variables,

$$\hat{y}_t = A\, \hat{k}_t^{\alpha}\, L_t^{1-\alpha}$$

where now :

$\hat{y}_t$: *The output PGSU at time t; and*
$\hat{k}_t$: *The capital PGSU at time t.*

Assuming that the aggregate labour is $L_t = 1$, as Barro (1990), where there is no population growth. Then, the firms' production function becomes like the AK model, as Barro (1990) demonstrated, but now in PGSU variables,

$$\hat{y}_t = A\hat{k}_t^{\alpha}$$

The first-order condition of the profit maximisation problem gives us the rental rate of capital,

$$R_t = \alpha A \hat{k}_t^{\alpha-1}$$

6.3.4 Equilibrium

Like previous chapters, we divide the capital accumulation equation and the Euler equation at equilibrium by government spending, G_t, to obtain PGSU variables. We start rewriting the capital accumulation at equilibrium as,

$$\hat{k}_{t+1} = \frac{1}{\gamma_{G_t}} \left[\left(1-\tau^{Y}\right)\hat{y}_t + (1-\delta)\,\hat{k}_t - \hat{c}_t\right] \tag{6.7}$$

where $\hat{y}_t = A\hat{k}_t^{\alpha}$, and γ_{G_t} is the growth rate of government spending.

On the other hand, the Euler equation at equilibrium becomes,

$$\frac{\hat{c}_{t+1}}{\hat{c}_t} = \frac{1}{\gamma_{G_t}} \left[\beta\left(\left(1-\tau^{Y}\right)\alpha A\hat{k}_{t+1}^{\alpha-1} + 1-\delta\right)\right]^{\frac{1}{\sigma}} \tag{6.8}$$

where $\hat{c}_t \equiv C_t / G_t$ is the consumption PGSU at time t.

The transversality condition is:

$$\lim_{t\to\infty} \beta^t \lambda_t \hat{k}_{t+1} = 0$$

$$\lim_{t\to\infty} \beta^t \lambda_t \left(\frac{1}{\gamma_{G_t}} \left[\left(1-\tau^{Y}\right)\hat{y}_t + (1-\delta)\,\hat{k}_t - \hat{c}_t\right]\right) = 0$$

These two equations (6.7) and (6.8) plus transversality condition with K_0 given are the main equations that describe the market equilibrium of our economy in terms of PGSU variables with personal income tax.

6.4 Analysis of the Steady-State

In this model, government revenues come from two revenue sources: oil revenues and personal income tax revenues. Thus, the growth rate of government spending can be written as[96],

$$\gamma_{G_t} \equiv \frac{G_{t+1}}{G_t} = \frac{\pi_{t+1} + \tau^Y Y_{t+1}}{\pi_t + \tau^Y Y_t} = g^2 \frac{\left(1-\tau^Y \hat{y}_t\right)}{\left(1-\tau^Y \hat{y}_{t+1}\right)} \tag{6.9}$$

A steady-state can be fund analytically using the two fundamental equations in the economy, equations (6.7) and (6.8), taking into consideration that at the steady-state $\hat{k}_t = \hat{k}_{t+1} = \hat{k}^*$ and $\hat{c}_t = \hat{c}_{t+1} = \hat{c}^*$. Thus, we can write the steady-state of capital PGSU, output PGSU, and consumption PGSU equations, respectively, as:

$$\hat{k}^* = \left[\left(\frac{1}{\alpha A\left(1-\tau^Y\right)}\right)\left(\frac{\gamma_G^\sigma}{\beta} - 1 + \delta\right)\right]^{\frac{1}{\alpha-1}} \tag{6.10}$$

$$\hat{y}^* = A\left[\left(\frac{1}{\alpha A\left(1-\tau^Y\right)}\right)\left(\frac{\gamma_G^\sigma}{\beta} - 1 + \delta\right)\right]^{\frac{\alpha}{\alpha-1}} \tag{6.11}$$

$$\hat{c}^* = \left(1-\tau^Y\right)A\hat{k}^{*\alpha} + (1-\delta-\gamma_G)\,\hat{k}^* \tag{6.12}$$

These expressions show the equations for the steady-state values $\hat{k}^*$, $\hat{y}^*$, and $\hat{c}^*$.

Since we have in this current chapter two sources of government revenues, as in chapter five, it should be the case that in order to avoid that one of these sources tends to be negligible as a proportion, all these sources need to grow at a common rate in the steady-state. Thus, we next try to understand the possibilities for the steady-state in our model when both sources of revenues are considered together.

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[96] Following the same procedures that we have shown in Appendix B.1 of chapter five, we can reach such a simplification of equation (6.9).

6.4.1 The Possibilities for Steady-State

Given the government budget constraint, equation (6.1), three logically possible ways could be defined to achieve a steady-state value in the growth rate of government spending, *i.e.* equation (6.2). We, therefore, begin to rewrite the equation (6.2) as,

$$\gamma_{G_t} \equiv \frac{G_{t+1}}{G_t} = \frac{\pi_{t+1}}{\pi_t}\,\frac{1+\tau^Y\left(Y_{t+1}/\pi_{t+1}\right)}{1+\tau^Y\left(Y_t/\pi_t\right)} = g^2\,\frac{1+\tau^Y\left(Y_{t+1}/\pi_{t+1}\right)}{1+\tau^Y\left(Y_t/\pi_t\right)} \qquad (6.2)'$$

With an exception to be mentioned below, equation (6.2)' illustrates that we have to get Y_t/π_t constant over time, so that γ_{G_t} is constant over time. Therefore, we can see that as $t \to \infty$, two different ways exist in which we can imagine Y_t/π_t tending to be constant over time:

(i) *The share of output relative to oil revenues tends to zero, as time tends to infinity, such that:*

$$Y_t/\pi_t \to 0 \quad as \quad t \to \infty$$

(ii) *The share of output relative to oil revenues tends to some finite value and strictly positive value, as time tends to infinity, such that:*

$$Y_t/\pi_t \;\to\; some\ finite\ value,\ strictly\ (+)ve, \quad as \quad t \to \infty$$

If either *(i)* or *(ii)* hold, then the growth rate of government spending tends to the *exogenous growth rate*, as time tends to infinity, such that:

$$\gamma_{G_t} \to g^2 \quad as \quad t \to \infty$$

There are, however, variations between the steady-states indicated by *(i)* and *(ii)*. The difference lies in the following: *(i)* could occur if π_t grows asymptotically at any rate higher than Y_t, whereas in order for *(ii)* to occur, Y_t needs to grow asymptotically at exactly the rate of g^2.

Looking at these possibilities, it seems obvious that the economy is unable to move towards a steady-state of type *(i)*. The explanation is that this situation means that the economy would not benefit from rising oil revenues.

There is one more possibility for how γ_{G_t} could tend to a time-invariant value. We can also rewrite the growth rate of government spending, γ_{G_t}, equation (6.2), as:

$$\gamma_{G_t} \equiv \frac{G_{t+1}}{G_t} = \frac{Y_{t+1}}{Y_t}\,\frac{\left(\pi_{t+1}/Y_{t+1}\right)+\tau^Y}{\left(\pi_t/Y_t\right)+\tau^Y} = \gamma_{Y_t}\,\frac{\left(\pi_{t+1}/Y_{t+1}\right)+\tau^Y}{\left(\pi_t/Y_t\right)+\tau^Y} \qquad (6.2)''$$

Looking at the expression in equation (6.2)", and then considering a situation where:

(iii) *The share of oil revenues relative to output tends to zero, as time tends to infinity. Moreover, the growth rate of output tends to some finite value and strictly positive value, as time tends to infinity, such that:*

$$\pi_t / Y_t \rightarrow 0 \quad as \quad t \rightarrow \infty \quad \left(i.e.\ Y_t / \pi_t \rightarrow \infty \right)\ and$$

$$\gamma_{Y_t} = Y_{t+1} / Y_t \rightarrow \text{ some finite value, strictly } (+)ve, \quad as \quad t \rightarrow \infty$$

If *(iii)* holds, then the growth rate of government spending tends to the *endogenous growth rate*, as time tends to infinity, such that:

$$\gamma_{G_t} \rightarrow \gamma_Y \quad as \quad t \rightarrow \infty$$

It is also evident that in order to have this situation, *(iii)*, we need that $\gamma_Y > g^2$, (*i.e.* output grows asymptotically faster than the oil revenues). Next, we investigate two types of steady-states in our model.

6.4.2 Two Types of the Steady-States

The first type is that the output grows at the same rate as the oil revenues, which means everything grows at the rate of oil revenues, that is g^2 (*i.e.* $\gamma_{G_t} = g^2$). The second type is that the output grows faster than oil revenues, which means that everything in this type grows at the rate of γ_{Y_t} (*i.e.* $\gamma_{G_t} = \gamma_{Y_t}$). Thus, we will find the steady-state equations in both types.

6.4.2.1 Type One of Steady-State ($\boldsymbol{\gamma_{G_t} = g^2}$)

In this type, we set the output, Y_t, grows at the same rate of the oil revenues, π_t, such that:

$$\frac{\pi_{t+1}}{\pi_t} = g^2 \quad \Rightarrow \quad \pi_{t+1} = g^2\ \pi_t$$

$$\frac{Y_{t+1}}{Y_t} = g^2 \quad \Rightarrow \quad Y_{t+1} = g^2\ Y_t$$

The growth rate of government spending, in this type, is written as,

$$\gamma_{G_t} \equiv \frac{G_{t+1}}{G_t} = \frac{g^2 \pi_t + \tau^Y g^2 Y_t}{\pi_t + \tau^Y Y_t} = g^2$$

As a result, everything in the economy of this type grows at the rate of g^2, which clearly represents *an exogenous growth rate*, and the steady-state equations can be then written as,

$$\hat{k}^* = \left[\left(\frac{1}{\alpha A\left(1-\tau^Y\right)}\right)\left(\frac{\left(g^2\right)^\sigma}{\beta} - 1 + \delta\right)\right]^{\frac{1}{\alpha-1}} \quad (6.13)$$

$$\hat{y}^* = A\left[\left(\frac{1}{\alpha A\left(1-\tau^Y\right)}\right)\left(\frac{\left(g^2\right)^\sigma}{\beta} - 1 + \delta\right)\right]^{\frac{\alpha}{\alpha-1}} \quad (6.14)$$

$$\hat{c}^* = \left(1-\tau^Y\right)A\hat{k}^{*\alpha} + (1-\delta-g^2)\,\hat{k}^* \quad (6.15)$$

The steady-state equations (6.13), (6.14), and (6.15)[97] show that if the output grows at the same rate of g^2 and the personal income tax is time-invariant, then the steady-state is one where all variables grow at the rate of g^2.

6.4.2.2 Type Two of Steady-State ($\gamma_{G_t} = \gamma_{Y_t}$):

In this type, we consider the case in which the output grows faster than oil revenues. Thus, the growth rate of government spending becomes,

$$\gamma_{G_t} \equiv \frac{G_{t+1}}{G_t} = \gamma_{Y_t}\,\frac{\left(\pi_{t+1}/Y_{t+1}\right)+\tau^Y}{\left(\pi_t/Y_t\right)+\tau^Y}$$

As long as the output grows faster than oil revenues, it is obvious that $\pi_t/Y_t \to 0$ as $t \to 0$. The reason is that oil revenues turn out to be extremely small compared to output. Then, the government spending growth rate would become $\gamma_{G_t} = \gamma_{Y_t}$. As a result, everything in the economy of this type grows at the rate of γ_{Y_t}, which represents *an endogenous growth rate*. The steady-state equations can be then written as,

$$\hat{k}^* = \left[\left(\frac{1}{\alpha A\left(1-\tau^Y\right)}\right)\left(\frac{\gamma_Y^\sigma}{\beta} - 1 + \delta\right)\right]^{\frac{1}{\alpha-1}} \quad (6.16)$$

$$\hat{y}^* = A\left[\left(\frac{1}{\alpha A\left(1-\tau^Y\right)}\right)\left(\frac{\gamma_Y^\sigma}{\beta} - 1 + \delta\right)\right]^{\frac{\alpha}{\alpha-1}} \quad (6.17)$$

$$\hat{c}^* = \left(1-\tau^Y\right)A\hat{k}^{*\alpha} + (1-\delta-\gamma_Y)\,\hat{k}^* \quad (6.18)$$

[97] We can note that these three equations are identical to equation (6.10), (6.11), and (6.12) if and only if we have case one, $\gamma_{G_t} = g^2$.

The steady-state equations (6.16), (6.17), and (6.18)[98] confirm that the steady-state in this type is the one where all variables grow at the rate of γ_{Y_t}.

In summary, type one of steady-state (*i.e.* type (*I*)) requires that the output grows at the same rate as the oil revenues, while in order for type two of steady-state (*i.e.* type (*II*)) to occur, the output should grow faster than oil revenues. Thus, these two types, (*I*) and (*II*), are visible and can also be achieved. In the next section, we examine two different special cases of the model to find out if one or both is/are possible to occur and what would happen in the economy accordingly.

6.4.3 Two Special Cases of the Model

As mentioned previously, our model in this chapter contains two revenue sources. Thus, we study in this section two different special cases of the model separately. The first case is when we have only one source of government revenues, namely oil revenues. This case represents an *exogenous growth model*, where the growth rate is g^2. On the other hand, the second case is when we have personal income tax revenues as the only source of government revenues. The growth rate, in this case, is an *endogenous growth model*, which is γ_{Y_t}. The justification for analysing the two cases of growth rate separately is to compare between them, (*i.e. if* $g^2 > \gamma_Y$ *or* $g^2 < \gamma_Y$), since we want to see if the case two steady-state can occur in our model.

More specifically, if we are in a situation in which we can only have $g^2 > \gamma_Y$, then the steady-state would be the one which we have found in chapter four. Thus, this situation represents *an exogenous growth model*. On the other hand, if we can have a situation where $g^2 < \gamma_Y$, we would expect to see *an endogenous* growth model in our model. The reason is that π_t / Y_t will tend to zero asymptotically, as shown in situation *(iii)*. Thus, we next discuss in more depth the two cases of growth.

6.4.3.1 First Case of Growth Rate (*only oil revenues as a source of government revenues*):

The first case is that we only have one type of government revenues, that is, oil revenues. In fact, this case was achieved in chapter four, where we found that government spending grows at g^2, *i.e.* it represents an *exogenous growth model*. Besides, the stability and transitional dynamics, in this case, were already analysed previously. Since the steady-state has been addressed in depth prior, we are therefore focusing on the second case.

[98] Analogously, these three equations are identical to equation (6.10), (6.11), and (6.12) if and only if we have case two, $\gamma_{G_t} = \gamma_{Y_t}$.

6.4.3.2 Second Case of Growth Rate (*only personal income tax as a source of government revenues*):

The second case is that personal income tax is the only source of government revenue, representing an *endogenous growth model*. This case indeed represents Barro (1990) endogenous growth model. In this case, the government budget constraint and the growth rate of government spending can be written as,

$$G_t = \tau^Y Y_t \quad \Rightarrow \quad \gamma_{Gt} = \frac{Y_{t+1}}{Y_t} = \gamma_{Y_t}$$

The capital accumulation, in this case,

$$\hat{k}_{t+1} = \frac{1}{\gamma_{Y_t}} \left[\left(1-\tau^Y\right) \hat{y}_t + (1-\delta)\, \hat{k}_t - \hat{c}_t \right] \tag{6.19}$$

where

$\hat{k}_t = K_t / G_t \quad \Rightarrow \quad \hat{k}_t = K_t / \tau^Y Y_t$;

$\hat{c}_t = C_t / G_t \quad \Rightarrow \quad \hat{c}_t = C_t / \tau^Y Y_t$; *and*

$\hat{y}_t = Y_t / G_t \quad \Rightarrow \quad \hat{y}_t = 1 / \tau^Y$

It is evident that the output PGSU, $\hat{y}_t$, depends now on only personal income tax. This switching would be helpful to solve this case, as we will see after equation (6.20).

On the other hand, the Euler equation,

$$\frac{\hat{c}_{t+1}}{\hat{c}_t} = \frac{1}{\gamma_{Y_t}} \left[\beta \left(\left(1-\tau^Y\right) \alpha A \hat{k}_{t+1}^{\alpha-1} + 1 - \delta \right) \right]^{\frac{1}{\sigma}} \tag{6.20}$$

From the definition of output PGSU, we know that:

$$\hat{y}_t = A\hat{k}_t^{\alpha} = 1/\tau^Y$$

This is helpful to find $\hat{k}_t$,

$$\hat{k}_t = \left(\frac{\hat{y}_t}{A} \right)^{\frac{1}{\alpha}} = \left(\frac{1}{A\tau^Y} \right)^{\frac{1}{\alpha}}$$

Thus, we can say that the output PGSU, $\hat{y}_t$, and capital PGSU, $\hat{k}_t$, in this case, are both constant. Now we can rewrite the capital accumulation, equation (6.19), and the Euler equation, equation (6.20), respectively as,

$$\hat{k}_{t+1} = \frac{1}{\gamma_{Y_t}}\left[\frac{1-\tau^{Y}}{\tau^{Y}} + (1-\delta)\left(A\tau^{Y}\right)^{-\frac{1}{\alpha}} - \hat{c}_t\right] \tag{6.19)'$$

$$\frac{\hat{c}_{t+1}}{\hat{c}_t} = \frac{1}{\gamma_{Y_t}}\left[\beta\left(\frac{\alpha\left(1-\tau^{Y}\right)}{\tau^{Y}}\left(A\tau^{Y}\right)^{\frac{1}{\alpha}} + 1-\delta\right)\right]^{\frac{1}{\sigma}} \tag{6.20)'$$

The Euler equation (6.20)' exhibits that the growth rate of consumption is constant over time because it depends on only exogenous parameters. Moreover, since government spending is also considered proportional to the capital stock, both inputs in the firms' production would have the same growth rate.

At the steady-state, we have $\hat{k}_t = \hat{k}_{t+1} = \hat{k}^*$, $\hat{c}_t = \hat{c}_{t+1} = \hat{c}^*$, and $\hat{y}_t = \hat{y}_{t+1} = \hat{y}^*$; therefore, the capital accumulation equation (6.19)' and the Euler equation (6.20)' in a steady-state can be written respectively as,

$$\hat{c} = \frac{1-\tau^{Y}}{\tau^{Y}} + \left(1-\delta-\gamma_{Y}\right)\left(A\tau^{Y}\right)^{-\frac{1}{\alpha}} \tag{6.21}$$

$$\gamma_Y = \left[\beta\left(\frac{\alpha\left(1-\tau^{Y}\right)}{\tau^{Y}}\left(A\tau^{Y}\right)^{\frac{1}{\alpha}} + 1-\delta\right)\right]^{\frac{1}{\sigma}} \tag{6.22}$$

By making β sufficiently high and rising τ^Y, we can get $\gamma_Y > 1$ under one possible situation. This situation is if $\left(\frac{\alpha(1-\tau^Y)}{\tau^Y}(A\tau^Y)^{\frac{1}{\alpha}}\right) > \delta$, which could be obtained when we set $A > 1$.

As mentioned in chapter five, one difference between income and consumption taxes is regarding their rates. Personal income tax must be less than one (*i.e. <100%*), while consumption tax could exceed one (*i.e. >100%*) on some products. Thus, it can be proved that there is a tax rate that maximises the economic growth rate. To see that, we take the derivative of equation (6.22) w.r.t. the personal income tax,

$$\frac{d\gamma_Y}{d\tau^Y} = \left[\beta\left(\frac{\alpha\left(1-\tau^{Y}\right)}{\tau^{Y}}\left(A\tau^{Y}\right)^{\frac{1}{\alpha}} + 1-\delta\right)\right]^{\frac{1-\sigma}{\sigma}} \cdot \left[\frac{\beta(A)^{\frac{1}{\alpha}}\left(\tau^{Y}\right)^{\frac{1-\alpha}{\alpha}}}{\sigma}\right] \cdot \left[(1-\alpha)\left(\tau^{Y}\right)^{-1} - 1\right] \tag{6.22)'$$

According to Barro[99], the above derivative, equation (6.22)', means that the maximum growth rate can be obtained when a tax rate is set to be $\tau^Y = (1-\alpha)$. Thus, if the tax rate is $\tau^Y > (1-\alpha)$, then economic growth will be negatively affected, and vice versa. The intuition behind the negative impact on growth is that there is a crowding-out effect between government spending and private investment. With too much taxation, the disincentive of households to invest in the capital will slow down the accumulation of capital so that the public good partially offsets the positive impact on the production process. As Barro (1990) illustrated, the growth rate of consumption, capital, and output all equal the same constant, with all PGSU variables growing at the same rate all the time. As a result, there is no transitional dynamics in this type of steady-state, *i.e.* type *(II)* steady-state. So now, we examine, as in the previous chapter, if whether both types of steady-state can exist for the same set of parameter values or whether there is just one.

6.4.4 Existence of Steady-States of Both Types

We extend our study in this section by examining that if both types of steady-states can exist for the same set of parameter values. The first type of steady-state *(I)* is the *exogenous growth steady-state*, while the second type of steady-state *(II)* is the *endogenous growth steady-state*. In the *exogenous growth steady-state*, Y_t/π_t tends to a finite, non-zero, value, and the growth rate is g^2. In the *endogenous growth steady-state*, on the contrary, Y_t/π_t tends to infinity, and the growth rate is γ_{Y_t} where $\gamma_Y > g^2$ and γ_{Y_t} is a function of τ^Y amongst other parameters.

We have found a similar result as in chapter five that there is a contradiction[100]. This contradiction originates from the two conditions, (*i*) Y_t/π_t cannot be negative and (*ii*) $\gamma_Y > g^2$. As mentioned before, if one of these condition holds, then the other condition cannot hold, and vice versa. However, we need these two conditions for each steady-state to exist. In more precisely, the first condition, (*i*), is required for type *(I)* steady-state to exist, while the second condition, (*ii*), is necessary for type *(II)* steady-state to occur. Therefore, we can conclude that there can still be two steady-states, but the two types of steady-state cannot exist for the same set of parameter values. In other words, for a given set of parameter values, the steady-state will be either of exogenous or endogenous steady-state. In fact, it depends on the value of g^2. If g^2 is sufficiently high (for a given value of τ^Y), then the steady-state will be of type *(I), i.e. an exogenous growth steady-state*. If, on the other hand, g^2 is sufficiently low (for a given value of τ^Y), then the steady-state will be of type *(II), i.e. an endogenous growth steady-state*.

[99] In Barro (1990) paper, 'Government Spending in a Simple Model of Endogenous Growth', the analysis was done in continuous-time. However, since our models in all chapters are analysed in discrete-time, we show in Appendix C.1 how Barro (1990) arrived at the growth rate formula in discrete-time.

[100] The contradiction is quite similar to what we have found in chapter five, and the only difference is that we have now output, Y_t, instead of consumption, C_t. Thus, we do not show here the full proof of that since chapter five has comparable proof.

If we apply here the same analysis as in chapter five, *i.e.* keeping g^2 constant at a certain value and changing τ^Y, we could have a different result because of the different type of taxes. Thus, it is possible to have a similar discussion as the previous chapter regarding the shift of steady-state type. However, before we study how and when the two types of steady-state can switch, it is worth mentioning that we have early found that increasing personal income tax would have a positive and negative impact on the growth rate. More precisely, the formula of γ_Y, equation (6.22), shows that increasing τ^Y would increase the growth rate. However, at a certain value of τ^Y, the growth rate would decline. Thus, the relationship between the personal income tax and the growth rate can take a 'hill shape' [101].

Returning now back to the two types of steady-state, we consider that g^2 is fixed at a certain value, but the personal income tax, τ^Y, can vary. To see that, suppose at the beginning that there is no personal income tax, $\tau^Y = 0$, then the economy would be a type *(I)* steady-state, which is g^2. The reason is that g^2 is the only steady-state that the economy can stay in. This situation represents what we have early found in chapter four when there are no taxes at all, and the only source of government revenues is the oil revenues. Now, if the government introduces a low rate of personal income tax, *e.g.* $\tau^Y = 5\%$, as we know, this would increase the growth rate of γ_Y, but it would be less than g^2. Thus, the economy would grow at the rate of g^2, meaning that it is going to stay at type *(I)* steady-state. Suppose the government increases τ^Y, but also still at a lower rate, *e.g.* $\tau^Y = 10\%$, then although the growth rate of γ_Y would be higher now, and the economy would have some changes in its level variables, it would be still at g^2 because the tax rate is low yet and then $g^2 > \gamma_Y$. However, at a certain value of τ^Y, *e.g.* $\tau^Y = 20\%$, γ_Y would become greater than g^2, *i.e.* $g^2 < \gamma_Y$. In this case, the economy would now move to type *(II)* steady-state, which is γ_Y. Increasing τ^Y further, *e.g.* $\tau^Y = 30\%$, would also increase the growth rate, and the economy would be in a higher position at the growth rate γ_Y. However, if the authority decides to raise τ^Y significantly, *e.g.* $\tau^Y = 60\%$, then γ_Y would decline because this tax rate passes its maximum rate, but the economy would still settle into type *(II)* steady-state. When the tax rate increases more, *e.g.* $\tau^Y = 70\%$, although the policymaker may think this scenario would stimulate the economy, γ_Y indeed declines more and becomes now less than g^2. In this case, the economy would return to type *(I)* steady-state because the growth rate is now $g^2 > \gamma_Y$.

[101] This relationship is indeed what has been confirmed by Barro (1990), where there is a value of income tax that maximises the growth rate.

Therefore, contrary to the result of chapter five, we now have two critical values for τ^Y. As increasing the tax rate, the first one, $\bar{\tau}_1^Y$, is when the economy turns from type *(I)* steady-state to type *(II)* steady-state*, i.e.* from *exogenous growth steady-state* to *endogenous growth steady-state*, while the second one, $\bar{\tau}_2^Y$, is the opposite. The below diagram shows the two critical values when the economy switches from one type of steady-state to another one, as well as the maximum tax rate, $\dot{\tau}^Y$.

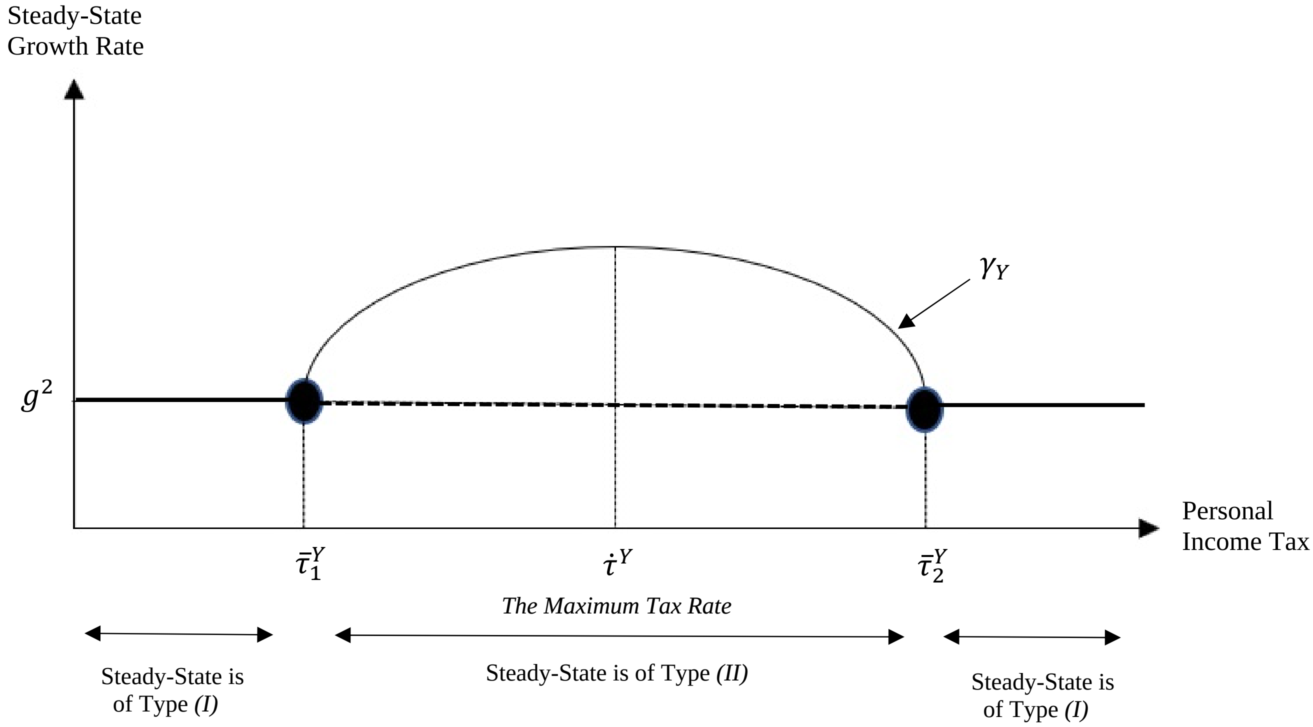

Diagram 19: How and when the economy could move from one steady-state to another one when personal income tax changes

6.5 Analysis of the Effect of Personal Income Tax on PGSU and Level Variables in a Type (*I*) Steady-State

The analysis of the personal income tax in type *(I)* steady-state[102] will be discussed in two sections. The first section studies the impact on the steady-state PGSU variables, whereas the second section is devoted to analysing the impact on the level variables.

6.5.1 The Impact of the Personal Income Tax on the Steady-State PGSU Variables

In this section, we want to know how changing personal income tax would affect the type *(I)* steady-state of capital PGSU, output PGSU and consumption PGSU to find out the relationship between them in our model.

6.5.1.1 The Impact of Personal Income Tax on the Steady-State of Capital PGSU

We take the derivative of the steady-state of capital PGSU, equation (6.13), with respect to income tax,

$$\frac{d\hat{k}^*}{d\tau^Y} = \underbrace{\left(\frac{1}{(\alpha-1)(1-\tau^Y)}\right)}_{(-)} \underbrace{\left(\frac{\frac{(g^2)^\sigma}{\beta} - 1 + \delta}{\alpha A(1-\tau^Y)}\right)^{\frac{1}{\alpha-1}}}_{(+)} < 0 \qquad (6.13)'$$

The sign is negative because of $(\alpha - 1)$. That means increasing personal income tax would decline the steady-state of capital PGSU and vice versa.

6.5.1.2 The Impact of Personal Income Tax on the Steady-State of Output PGSU:

We take the derivative of the steady-state of output PGSU, equation (6.14), with respect to income tax,

$$\frac{d\hat{y}^*}{d\tau^Y} = \underbrace{\left(\frac{\alpha A}{(\alpha-1)(1-\tau^Y)}\right)}_{(-)} \underbrace{\left(\frac{\frac{(g^2)^\sigma}{\beta} - 1 + \delta}{\alpha A(1-\tau^Y)}\right)^{\frac{\alpha}{\alpha-1}}}_{(+)} < 0 \qquad (6.14)'$$

Like (6.13)', the sign is negative because of $(\alpha - 1)$. That means increasing personal income tax would decline the steady-state of output PGSU and vice versa.

[102] Note that type *(I)* steady-state is $\gamma_{G_t} = g^2$.

6.5.1.3 The Impact of Personal Income Tax on the Steady-State of Consumption PGSU:

We first substitute equation (6.13) into equation (6.15),

$$\hat{c}^* = \left(1-\tau^Y\right)A\left[\left(\frac{1}{\alpha A\left(1-\tau^Y\right)}\right)\left(\frac{\left(g^2\right)^\sigma}{\beta} - 1 + \delta\right)\right]^{\frac{\alpha}{\alpha-1}} + (1-\delta-g^2)\left[\left(\frac{1}{\alpha A\left(1-\tau^Y\right)}\right)\left(\frac{\left(g^2\right)^\sigma}{\beta} - 1 + \delta\right)\right]^{\frac{1}{\alpha-1}}$$

and then take the derivative of the steady-state of consumption PGSU, with respect to personal income tax,

$$\frac{d\hat{c}^*}{d\tau^Y} = \frac{A\left(\tau^Y-1\right)\left(\dfrac{\dfrac{\left(g^2\right)^\sigma}{\beta} - 1 + \delta}{\alpha A\left(1-\tau^Y\right)}\right)^{\frac{\alpha}{\alpha-1}} + \left(\delta+g^2-1\right)\left(\dfrac{\dfrac{\left(g^2\right)^\sigma}{\beta} - 1 + \delta}{\alpha A\left(1-\tau^Y\right)}\right)^{\frac{1}{\alpha-1}}}{\left(\alpha-1\right)\left(\tau^Y-1\right)} \tag{6.15$'$}$$

If we restrict some parameter values under their conditions by assuming that $A, g^2, \sigma = 1$ and $\delta = 0$, then the $d\hat{c}^*/d\tau^Y$ can be simplified as,

$$\frac{d\hat{c}^*}{d\tau^Y} = \frac{\left(\dfrac{\alpha\beta\left(1-\tau^Y\right)}{\left(1-\beta\right)}\right)^{\frac{\alpha}{1-\alpha}}}{\left(\alpha-1\right)} < 0 \tag{6.15$''$}$$

The numerator is positive, but the denominator is negative. Thus, $d\hat{c}^*/d\tau^Y < 0$, which means that increasing personal income tax would also decline the steady-state of consumption PGSU, and vice versa.

6.5.2 The Impact of the Personal Income Tax on the Level Variables

This section examines the key level variables in the model as a special case, unlike the variables of PGSU. Similar to the previous chapter, a level variable implies a variable that is not a growth rate. Thus, we investigate here how the key endogenous level variables, *i.e.* the level of government spending, G_t, consumption, C_t, capital stock, K_t, and output, Y_t, would be affected when we change the personal income tax, τ^Y. In more precisely, we seek in this section to study how increasing, τ^Y, would affect the steady-state value relative to oil revenues, π_t, of each level variables. This calculation would be helpful since we know that the growth path of π_t cannot be affected by increasing τ^Y.

We have found in the steady-state section that personal income tax, τ^Y, has a negative impact on all the steady-state of capital PGSU, $\hat{k}^*$, consumption PGSU, $\hat{c}^*$, and output PGSU, $\hat{y}^*$, such that,

$$\uparrow \tau^Y \begin{cases} \downarrow \hat{k}^* \\ \downarrow \hat{c}^* \\ \downarrow \hat{y}^* \end{cases}$$

Therefore, compared to chapter five, we attempt here to calculate how changing personal income tax would affect the steady-state value of each level variable relative to oil revenues, π_t. However, before we start studying these effects, it is helpful to understand first how $\tau^Y \hat{y}^*$ can be affected when personal income tax changes. Knowing how τ^Y affects $\tau^Y \hat{y}^*$ would help solve the level variables, as we will see later. To do that, we substitute equation (6.14) into $\tau^Y \hat{y}^*$.

$$\tau^Y \hat{y}^* \Rightarrow \tau^Y A\left[\left(\frac{1}{\alpha A\left(1-\tau^Y\right)}\right)\left(\frac{\left(g^2\right)^\sigma}{\beta} - 1 + \delta\right)\right]^{\frac{\alpha}{\alpha-1}} \tag{6.23}$$

Then, we take the derivative of (6.23) w.r.t. τ^Y, and simplify it to obtain:

$$\frac{d\left(\tau^Y \hat{y}^*\right)}{d\tau^Y} = \frac{A\left[\left(\frac{1}{\alpha A\left(1-\tau^Y\right)}\right)\left(\frac{\left(g^2\right)^\sigma}{\beta} - 1 + \delta\right)\right]^{\frac{\alpha}{\alpha-1}}\left[1-\tau^Y-\alpha\right]}{(\alpha-1)\left(\tau^Y-1\right)} \tag{6.23$'$}$$

The first expression in the numerator, $A[((g^2)^\sigma/\beta - 1 + \delta)/\alpha A(1-\tau^Y)]^{\frac{\alpha}{\alpha-1}}$, is positive, and the denominator is clearly positive too because $\alpha, \tau^Y \in (0{,}1)$. However, the crucial part is $[1-\tau^Y-\alpha]$, which can determine the sign of the (6.23)'. More precisely, if τ^Y and α are sufficiently high, then $[1-\tau^Y-\alpha]$ will be negative and therefore, the sign of the (6.23)' will be negative too. If, on the other hand, both τ^Y and α are sufficiently low (*e.g.* $\alpha, \tau^Y < 0.5$), the sign of the (6.23)' will end up being positive because $1 > \tau^Y + \alpha$. However, the value of τ^Y and α may not be all together sufficiently high or low. Thus, we know that the value of α can take any value between zero and one mathematically, but its standard and common value in the economic literature is 0.33. Thus, if we consider that, then the value of τ^Y would be mainly determined the sign of (6.23)'. As a result, we can conclude that if the value of τ^Y is sufficiently high (*i.e.* $\tau^Y > 0.67$), then the sign of (6.23)' will be negative. If τ^Y is sufficiently low (*i.e.* $\tau^Y < 0.67$), then the sign of (6.23)' is going to be positive.

6.5.2.1 Level of Government Spending:

We begin with the level of government spending because it would help us to determine the other level variables. To do so, we divide the government budget constraint, equation (6.1), by G_t and then rearrange it, to have,

$$\frac{G_t}{\pi_t} = \frac{1}{1-\tau^Y \hat{y}_t} \tag{6.24}$$

Substituting equation (6.14) into (6.24) gives us,

$$\frac{G_t}{\pi_t} = \frac{1}{1-\tau^Y A\left[\left(\frac{1}{\alpha A\left(1-\tau^Y\right)}\right)\left(\frac{\left(g^2\right)^{\sigma}}{\beta} - 1 + \delta\right)\right]^{\frac{\alpha}{\alpha-1}}} \tag{6.24)$'$}$$

Taking the derivative of (6.24)' w.r.t. the personal income tax and simplifying it, we obtain:

$$\frac{d\left(G_t/\pi_t\right)}{d\tau^Y} = \frac{A\,\Xi^{\frac{\alpha}{\alpha-1}}\left[1-\tau^Y-\alpha\right]}{\left(\alpha-1\right)\left(\tau^Y-1\right)\left[A\,\tau^Y\;\Xi^{\frac{\alpha}{\alpha-1}}-1\right]^2} \tag{6.24)$''$}$$

$$where\;\; \Xi = \left[\left(\frac{1}{\alpha A\left(1-\tau^Y\right)}\right)\left(\frac{\left(g^2\right)^{\sigma}}{\beta}-1+\delta\right)\right] > 0$$

The denominator of (6.24)" is clearly positive because of $(\alpha-1)(\tau^Y-1)$. However, the numerator depends on the rate of personal income tax, τ^Y. Like the previous section, if τ^Y is sufficiently low, then $d(G_t/\pi_t)/d\tau^Y$ will be positive. That meaning that the level of government spending relative to π_t would increase, as shown in the below diagram, $(ln\, G_t)'$. If, on the other hand, τ^Y is sufficiently high, then $d(G_t/\pi_t)/d\tau^Y$ will be negative. This implies that the level of government spending relative to π_t would decrease, as can be seen in the below diagram, $(ln\, G_t)''$.

τ^Y	G_t/π_t
Sufficiently Low	$(+)\uparrow$
Sufficiently High	$(-)\downarrow$

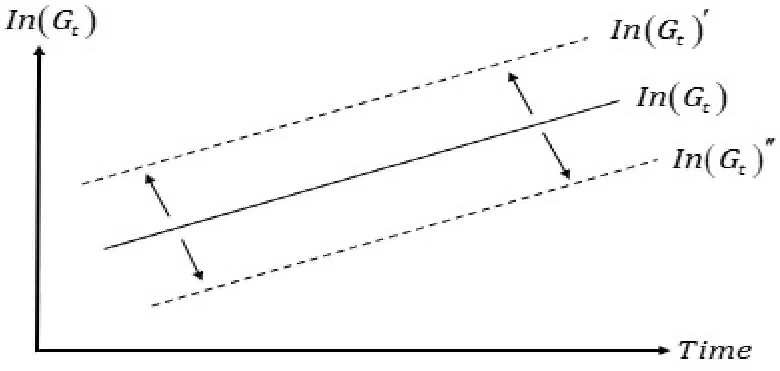

Diagram 20: The time path of the level of government spending relative to the π_t

The result of the government spending level relative to oil revenues, π_t, could be intuitively consistent with the 'Laffer curve' theory. The government revenues in our model are equal to the government spending (*i.e.* as we assumed that the government spends all its revenues; thus, they are equal). Although high personal income tax generally implies high revenues that the government can receive and use for spending purposes, the activities in the economy (*e.g.* work and investment/savings) would be negatively affected, as illustrated by the 'Laffer curve'. Subsequently, if the taxes are very high, government tax revenues would decline due to that effect. In contrast, if the taxes are very low, then that would generate additional revenues and would at the same time encourage economic activities and eventually would raise tax revenues. The 'Laffer curve' is shown in the below diagram[103].

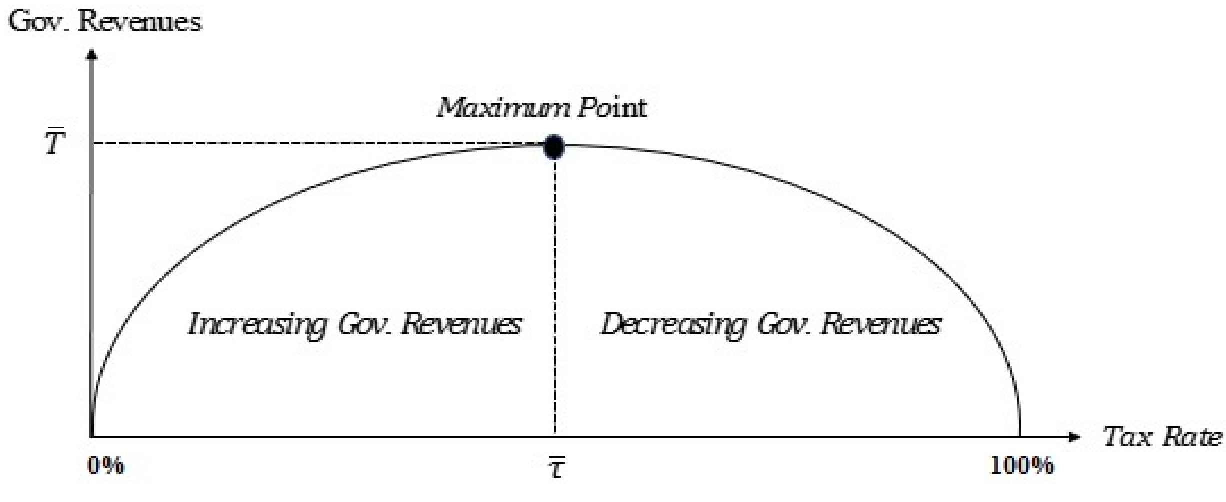

Diagram 21: Laffer Curve

[103] We also provide in Appendix C.2 a simple numerical exercise by using the formula in (6.24)' to show a similar result to the 'Laffer curve'.

6.5.2.2 Level of Consumption:

The level of consumption can be obtained from the definition of consumption PGSU as,

$$\hat{c}_t = \frac{C_t}{G_t} \quad \Rightarrow \quad C_t = \hat{c}_t G_t$$

Dividing both sides by π_t,

$$\frac{C_t}{\pi_t} = \hat{c}_t \frac{G_t}{\pi_t} \tag{6.25}$$

Substituting equation (6.24) into (6.25) gives us,

$$\frac{C_t}{\pi_t} = \frac{\hat{c}_t}{1-\tau^Y \hat{y}_t}$$

Now, we use the steady-state equations (6.13), (6.14), and (6.15) to have,

$$\frac{C_t}{\pi_t} = \frac{\left(1-\tau^Y\right)A\left[\left(\frac{1}{\alpha A\left(1-\tau^Y\right)}\right)\left(\frac{\left(g^2\right)^{\sigma}}{\beta}-1+\delta\right)\right]^{\frac{\alpha}{\alpha-1}} + (1-\delta-g^2)\left[\left(\frac{1}{\alpha A\left(1-\tau^Y\right)}\right)\left(\frac{\left(g^2\right)^{\sigma}}{\beta}-1+\delta\right)\right]^{\frac{1}{\alpha-1}}}{1-\tau^Y A\left[\left(\frac{1}{\alpha A\left(1-\tau^Y\right)}\right)\left(\frac{\left(g^2\right)^{\sigma}}{\beta}-1+\delta\right)\right]^{\frac{\alpha}{\alpha-1}}} \tag{6.25$'$}$$

We take the derivative of (6.25)' w.r.t. τ^Y and simplify it as,

$$\frac{d\left(C_t/\pi_t\right)}{d\tau^Y} = \frac{\left[A\left(\tau^Y-1\right)\Xi^{\frac{\alpha}{\alpha-1}} + (\delta+g^2-1)\,\Xi^{\frac{1}{\alpha-1}}\right]\left[\left(\alpha-1\right)A\,\Xi^{\frac{\alpha}{\alpha-1}}+1\right]}{\left(\alpha-1\right)\left(\tau^Y-1\right)\left[A\tau^Y\,\Xi^{\frac{\alpha}{\alpha-1}}-1\right]^2} \tag{6.25$''$}$$

We need now to find out the sign of (6.25)", in order to understand how changing personal income tax, τ^Y, would affect the level of consumption relative to oil revenues, π_t. However, the sign of (6.25)" seems algebraically ambiguous because it depends on several parameters. The source of this ambiguity comes basically from the 'Laffer curve' effect, where its impact on the level of government spending also causes some impact on other variables. Among these variables is the level of consumption. To see that, equation (6.25) contains two components, namely the consumption PGSU, $\hat{c}_t$, and the level of government spending relative to oil revenues, G_t/π_t. For $\hat{c}_t$, we know from the steady-state equation, equation (6.15)", that it is decreasing with increasing personal income tax, τ^Y. However, the second component, G_t/π_t, increases (decreases) if τ^Y is sufficiently low (high). Thus, these two components work in the opposite direction if only the τ^Y is sufficiently low, which means that the effect is partially ambiguous. As a result, we resort to a numerical solution because of this theoretical ambiguity, which would be an appropriate method in this case[104]. For this reason, we use the standard and commonly used parameters values, as in previous chapters, which are the following: $A = 1$, $\alpha = 0.33$, $\beta = 0.95$, $\delta = 0.1$, $g^2 = 1.1$, and $\sigma = 2$.

In the below table, we provide a numerical exercise to show how the steady-state variables and the level variables respond when the personal income tax changes. Calculating first the steady-state variables would help us to determine the numerical values of the level of government spending relative to oil revenues, G_t/π_t, and then the level of consumption relative to oil revenues, C_t/π_t [105].

Table 13: The numerical values for the steady-state, G_t/π_t, and C_t/π_t variables when τ^Y changes

τ^Y	$\hat{k}^*$	$\hat{c}^*$	$\hat{y}^*$	(G_t/π_t)	$\Delta(G_t/\pi_t)$	(C_t/π_t)	$\Delta(C_t/\pi_t)$
5%	0.7694	0.7174	0.9171	1.0480	−	0.7518	−
10%	0.7098	0.6618	0.8930	1.0980	↑	0.7266	↓
20%	0.5954	0.5551	0.8427	1.2027	↑	0.6676	↓
40%	0.3875	0.3613	0.7314	1.4135	↑	0.5107	↓
60%	0.2116	0.1973	0.5990	1.5610	↑	0.3080	↓
80%	0.0752	0.0701	0.4257	1.5164	↓	0.1063	↓

The numerical exercise shows for the particular parameter values, as above, that there is a negative relationship between the personal income tax and the level of consumption relative to π_t. Thus, the diagram below shows the time path of the level of consumption relative to π_t.

[104] The effects on the other variables are sequence partly of the ambiguity in effect on the level of government spending (*i.e.* 'Laffer curve' effect). Thus, when we theoretically studied the level of consumption, capital stock, and output, we found that they are ambiguous and complicated to determine how τ^Y affects them. The reason is that there are many effects on these level variables. Some of these effects are due to changes in the level of government spending, while others are due to some additional effects. Thus, we resort to a numerical solution to provide values for all steady-states and level variables when the tax rate changes.

[105] This is also useful to find out the values of other level variables, such as K_t/π_t and Y_t/π_t, as will be seen next.

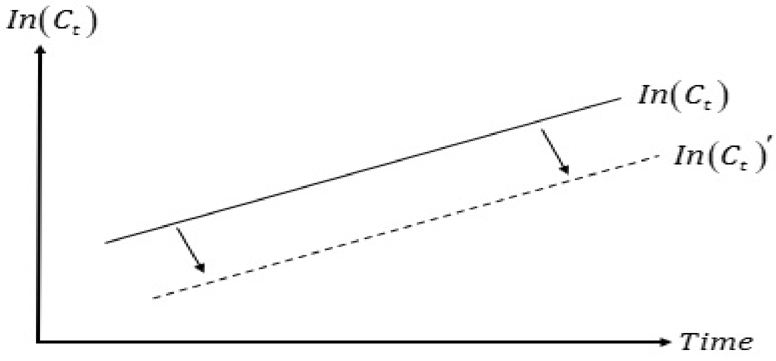

Diagram 22: The time path of the level of consumption to the π_t

The intuition of this result is returned to the fact that personal income tax would decline the households' disposable income. That implies that the households would have less possibility to spend on additional consumption goods. Thus, the level of consumption is generally decreased.

6.5.2.3 Level of Capital Stock:

The level of capital stock can be written as,

$$\hat{k}_t = \frac{K_t}{G_t} \quad \Rightarrow \quad K_t = \hat{k}_t G_t$$

Dividing both sides by π_t,

$$\frac{K_t}{\pi_t} = \hat{k}_t \frac{G_t}{\pi_t} \tag{6.26}$$

Substituting equation (6.24) into (6.26) gives us,

$$\frac{K_t}{\pi_t} = \frac{\hat{k}_t}{1-\tau^Y \hat{y}_t}$$

Similar to the level of consumption, the 'Laffer curve' effect also impacts the level of capital relative to oil revenues. This effect can be seen in equation (6.26). Thus, by applying the same parameters values, the table below shows how the level of capital changes when personal income tax changes.

Table 14: The numerical values for the K_t/π_t variable when τ^Y changes

τ^Y	(G_t/π_t)	$\Delta(G_t/\pi_t)$	(K_t/π_t)	$\Delta(K_t/\pi_t)$
5%	1.0480	–	0.8063	–
10%	1.0980	↑	0.7794	↓
20%	1.2027	↑	0.7161	↓
40%	1.4135	↑	0.5477	↓
60%	1.5610	↑	0.3303	↓
80%	1.5164	↓	0.1140	↓

Based on the particular parameter values chosen, the above table shows that increasing personal income tax decreases the level of the capital stock relative to π_t. Thus, the diagram below displays the time path of the level of the capital stock relative to π_t.

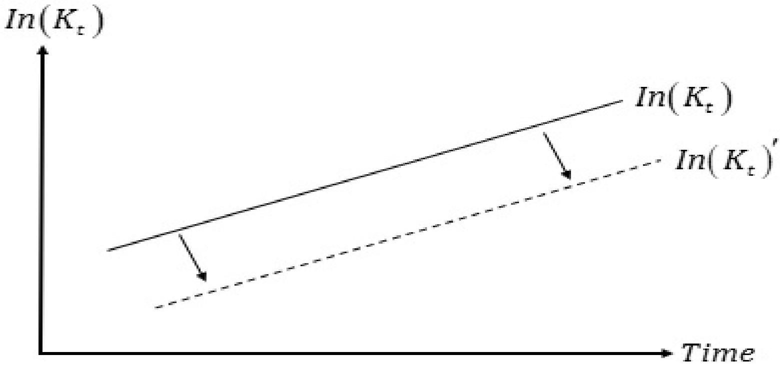

Diagram 23: The time path of the level of capital stock relative to the π_t

Intuitively, increasing personal income tax in our model has two effects, positive and negative impact, on the level of the capital stock relative to oil revenues. The positive effect comes from government spending. Specifically, we know that increasing personal income tax increases government revenues and then increase government spending. Increasing government spending implies that the firms would receive more positive externalities from the government to enhance their productivity. On the other hand, the negative effect comes from households. In more precisely, increasing personal income tax would decline the households' disposable income. In turn, this leads to a decrease in the households' demand from firms and then discourages business from hiring and investing more. As a result, the particular parameter values, which we have chosen in this numerical exercise, show that the negative effect in our model would dominate the positive one because the level of the capital stock is shown to be negatively affected by increasing personal income tax.

6.5.2.4 Level of Output:

The level of output can be written as,

$$\hat{y}_t = \frac{Y_t}{G_t} \quad \Rightarrow \quad Y_t = \hat{y}_t G_t$$

Dividing both sides by π_t,

$$\frac{Y_t}{\pi_t} = \hat{y}_t \frac{G_t}{\pi_t} \tag{6.27}$$

Substituting equation (6.24) into (6.27) gives us,

$$\frac{Y_t}{\pi_t} = \frac{\hat{y}_t}{1-\tau^Y \hat{y}_t}$$

The level of output is also affected by the effect on government spending, as shown in equation (6.27). As before, we use the same parameter values in the table below to understand the relationship between the level of output and the personal income tax.

Table 15: The numerical values for the Y_t/π_t variable when τ^Y changes

τ^Y	(G_t/π_t)	$\Delta(G_t/\pi_t)$	(Y_t/π_t)	$\Delta(Y_t/\pi_t)$
5%	1.0480	−	0.9611	−
10%	1.0980	↑	0.9805	↑
20%	1.2027	↑	1.0135	↑
40%	1.4135	↑	1.0338	↑
60%	1.5610	↑	0.9350	↓
80%	1.5164	↓	0.6455	↓

The numerical exercise clarifies that the personal income tax changes the level of output. More precisely, if τ^Y is low (high), then (Y_t/π_t) increases (decreases). The diagram below shows two different time paths of the output relative to π_t. The first time path is $(ln\,Y_t)'$, which is when personal income tax is low, while the second time path is $(ln\,Y_t)''$, which is when personal income tax is high.

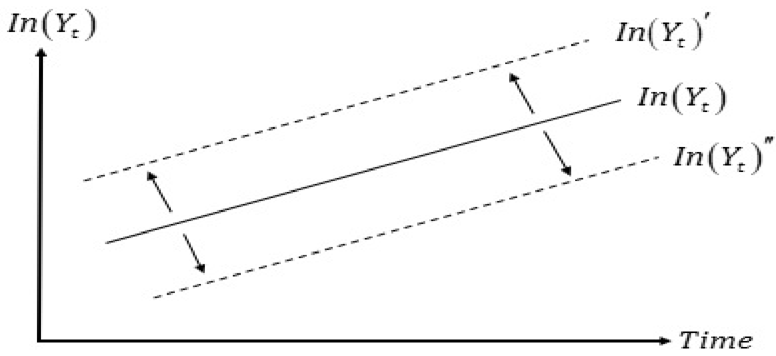

Diagram 24: The time path of the level of output relative to the π_t

The economic interpretation for this result is that low personal income tax would help the output to increase, benefiting from an increased flow of government spending. However, high personal income tax would discourage savings, investment, and work, and it would also reduce the households' disposable income. Moreover, since we have found that the level of consumption and capital stock relative to oil revenues would be negatively affected, such negative impacts would dominate the output level relative to oil revenues if the tax rate is high.

The steady-state was addressed in sections (6.4) and (6.5); thus, we next focus on studying the dynamics of our model.

6.6 Analysis of the Stability of Type (*I*) Steady-State

As long as we understand now that there is no transitional dynamics in type *(II)* steady-state, we examine in this section the local stability of type *(I)* steady-state. Throughout this section, we merge the two sources of government revenue, oil revenues and personal income tax revenues. If all streams of government revenues are viewed together, the growth rate of government spending can be expressed as follows:

$$\gamma_{G_t} \equiv \frac{G_{t+1}}{G_t} = \frac{\pi_{t+1} + \tau^Y Y_{t+1}}{\pi_t + \tau^Y Y_t} = g^2 \frac{\left(1-\tau^Y \hat{y}_t\right)}{\left(1-\tau^Y \hat{y}_{t+1}\right)}$$

To study the stability in our model, we need first to apply the linearisation method to evaluate the local stability of our system. Thus, we construct the linear approximation to the system around the steady-state, where the PGSU variables in the steady-state will stay constant all the time. Starting with the capital accumulation equation,

$$\hat{k}_{t+1} = \left[\frac{\left(1-\tau^Y A\hat{k}_{t+1}^{\alpha}\right)}{g^2\left(1-\tau^Y A\hat{k}_t^{\alpha}\right)}\right]\left[\left(1-\tau^Y\right)A\hat{k}_t^{\alpha} + (1-\delta)\,\hat{k}_t - \hat{c}_t\right]$$

where $\hat{y}_t = A\hat{k}_t^{\alpha}$ *and* $\hat{y}_{t+1} = A\hat{k}_{t+1}^{\alpha}$

We can see that the right-hand side of this equation is a function of $\left(\hat{k}_t, \hat{k}_{t+1}, \hat{c}_t\right)$. Thus, we find the first-order Taylor approximation of it around the steady-state and evaluate the coefficients in the steady-state[106],

$$(\hat{k}_{t+1} - \hat{k}^*) = \frac{\left(g^2\right)^{\sigma-1}\Psi + \alpha\beta\left(1-\Psi\right)}{\beta\left(\alpha+\Psi\left(1-\alpha\right)\right)}(\hat{k}_t - \hat{k}^*) - \frac{\Psi}{g^2\left(\alpha+\Psi\left(1-\alpha\right)\right)}(\hat{c}_t - \hat{c}^*) \qquad (6.28)$$

where $\Psi = 1-\tau^Y A\hat{k}^{*\alpha}$

Then, we consider the Euler equation,

$$\hat{c}_{t+1} = \left[\frac{\hat{c}_t\left(1-\tau^Y A\hat{k}_{t+1}^{\alpha}\right)}{g^2\left(1-\tau^Y A\hat{k}_t^{\alpha}\right)}\right]\left[\beta\left(\left(1-\tau^Y\right)\alpha A\hat{k}_{t+1}^{\alpha-1} + 1-\delta\right)\right]^{\frac{1}{\sigma}}$$

---

[106] The steady-state of capital accumulation equation and Euler equation can be written respectively as,

$\hat{k}^* = \frac{1}{g^2}\left[(1-\tau^Y)A\hat{k}^{*\alpha} + (1-\delta)\hat{k}^* - \hat{c}^*\right]$ and $g^2 = \left[\beta\left((1-\tau^Y)\alpha A\hat{k}^{*\alpha-1} + 1-\delta\right)\right]^{\frac{1}{\sigma}}$.

The right-hand side of the Euler equation is also a function of $(\hat{k}_t, \hat{k}_{t+1}, \hat{c}_t)$. Therefore, we find the first-order Taylor approximation of the Euler equation and evaluate its coefficients in the steady-state,

$$(\hat{c}_{t+1} - \hat{c}^*) = \frac{\Upsilon \hat{k}^* \Psi - \alpha \hat{c}^* (1-\Psi)}{\hat{k}^* \Psi} (\hat{k}_{t+1} - \hat{k}^*) + \frac{\alpha \hat{c}^* (1-\Psi)}{\hat{k}^* \Psi} (\hat{k}_t - \hat{k}^*) + (\hat{c}_t - \hat{c}^*) \qquad (6.29)$$

$$where \ \Upsilon = \frac{(\alpha - 1)\alpha \hat{c}^* A \beta (1-\tau^Y) \hat{k}^{*\alpha-2}}{\sigma (g^2)^\sigma} < 0$$

Υ is negative because of $(\alpha - 1)$. Thus, by substituting the approximation of the capital accumulation equation, equation (6.28), into the approximation of the Euler equation, equation (6.29), we obtain:

$$(\hat{c}_{t+1} - \hat{c}^*) = \frac{\left[\Upsilon \hat{k}^* \Psi - \alpha \hat{c}^* (1-\Psi)\right]\left[(g^2)^{\sigma-1} \Psi + \alpha\beta(1-\Psi)\right] + \left[\alpha \hat{c}^* (1-\Psi)\beta(\alpha + \Psi(1-\alpha))\right]}{\hat{k}^* \Psi \beta (\alpha + \Psi(1-\alpha))} (\hat{k}_t - \hat{k}^*)$$

$$+ \frac{\left[\hat{k}^* g^2 (\alpha + \Psi(1-\alpha))\right] - \left[\Upsilon \hat{k}^* \Psi - \alpha \hat{c}^* (1-\Psi)\right]}{\hat{k}^* g^2 (\alpha + \Psi(1-\alpha))} (\hat{c}_t - \hat{c}^*) \qquad (6.29)'$$

Now the above two equations describe the law of motion in this system and define the dynamics in $\hat{k}_t$ and $\hat{c}_t$. Thus, by linearising these two equations, as shown in equations (6.28) and (6.29)', we understand that there will be two eigenvalues in this system.

The linearised capital accumulation equation (6.28) and Euler equation (6.29)' can also be now expressed in matrix form as:

$$\begin{pmatrix} \hat{k}_{t+1} - \hat{k} \\ \hat{c}_{t+1} - \hat{c} \end{pmatrix} = \underbrace{\begin{pmatrix} \dfrac{(g^2)^{\sigma-1} \Psi + \alpha\beta(1-\Psi)}{\beta(\alpha + \Psi(1-\alpha))} & -\dfrac{\Psi}{g^2 (\alpha + \Psi(1-\alpha))} \\ \dfrac{\left[\Upsilon \hat{k} \Psi - \alpha \hat{c} (1-\Psi)\right]\left[(g^2)^{\sigma-1} \Psi + \alpha\beta(1-\Psi)\right] + \left[\alpha \hat{c} (1-\Psi)\beta(\alpha + \Psi(1-\alpha))\right]}{\hat{k} \Psi \beta (\alpha + \Psi(1-\alpha))} & \dfrac{\left[\hat{k} g^2 (\alpha + \Psi(1-\alpha))\right] - \left[\Upsilon \hat{k} \Psi - \alpha \hat{c} (1-\Psi)\right]}{\hat{k} g^2 (\alpha + \Psi(1-\alpha))} \end{pmatrix}}_{matrix\ A} \begin{pmatrix} \hat{k}_t - \hat{k} \\ \hat{c}_t - \hat{c} \end{pmatrix}$$

The above coefficient matrix A has a trace,

$$Tr = \left(\frac{\hat{k}^{*}\Psi\left(g^{2}\right)^{\sigma} + \hat{k}^{*}\beta\left[g^{2}\alpha\left(1-\Psi\right) + g^{2}\left(\alpha+\Psi\left(1-\alpha\right)\right) - \Upsilon\Psi\right] + \beta\alpha\hat{c}^{*}\left(1-\Psi\right)}{\beta\hat{k}^{*}g^{2}\left(\alpha+\Psi\left(1-\alpha\right)\right)} \right)$$

The determinant (Det) of the matrix A is,

$$Det = \left(\frac{\left(g^{2}\right)^{\sigma}\Psi\hat{k}^{*} + \alpha\beta\left(1-\Psi\right)\left[\hat{k}^{*}g^{2} + \hat{c}^{*}\right]}{\beta\hat{k}^{*}g^{2}\left(\alpha+\Psi\left(1-\alpha\right)\right)} \right)$$

The eigenvalues of the matrix are the solutions to the characteristic equation,

$$\lambda^{2} - Tr\,\lambda + Det = 0$$

$$\lambda^{2} - \left(\frac{\hat{k}^{*}\Psi\left(g^{2}\right)^{\sigma} + \hat{k}^{*}\beta\left[g^{2}\alpha\left(1-\Psi\right) + g^{2}\left(\alpha+\Psi\left(1-\alpha\right)\right) - \Upsilon\Psi\right] + \beta\alpha\hat{c}^{*}\left(1-\Psi\right)}{\beta\hat{k}^{*}g^{2}\left(\alpha+\Psi\left(1-\alpha\right)\right)} \right)\lambda + \left(\frac{\left(g^{2}\right)^{\sigma}\Psi\hat{k}^{*} + \alpha\beta\left(1-\Psi\right)\left[\hat{k}^{*}g^{2} + \hat{c}^{*}\right]}{\beta\hat{k}^{*}g^{2}\left(\alpha+\Psi\left(1-\alpha\right)\right)} \right) = 0$$

Since we have identified two eigenvalues in this system, we need to examine if there are stable, unstable, or one of them is stable, and the other is not. To do that, we go back to the government budget constraint equation and the definition of both the capital PGSU and consumption PGSU equations. These equations are helpful to find out if $\hat{k}_t$ and $\hat{c}_t$ are predetermined or non-predetermined variables.

$$G_t = \pi_t + \tau^{Y}Y_t$$

$$\hat{k}_t = \frac{K_t}{G_t} = \frac{K_t}{\pi_t + \tau^{Y}Y_t} \qquad ; \qquad \hat{c}_t = \frac{C_t}{G_t} = \frac{C_t}{\pi_t + \tau^{Y}Y_t}$$

Let us begin with the definition of capital PGSU, $\hat{k}_t$. K_t cannot jump and G_t cannot also jump because both π_t and Y_t cannot jump[107]. Thus, we can definitely say that $\hat{k}_t$ here is a predetermined variable. On the other hand, we see from the definition of consumption PGSU, $\hat{c}_t$, that the numerator, C_t, can jump, but the denominator cannot jump. Since one of the components of $\hat{c}_t$

[107] Determining whether Y_t can or cannot jump may not be obvious at first sight. Thus, we need to elaborate it more. First of all, we know that Y_t depends on G_t through the production function, *i.e.* $Y_t = AK_t^{\alpha}G_t^{1-\alpha}$. Thus, if G_t cannot jump, then Y_t undoubtedly cannot jump too, because K_t is already a predetermined variable. However, G_t depends partly on Y_t through the government budget constraint, *i.e.* $G_t = \pi_t + \tau^{Y}Y_t$. Therefore, if we substitute G_t out of the production function, we obtain that $Y_t = AK_t^{\alpha}(\pi_t + \tau^{Y}Y_t)^{1-\alpha}$, where Y_t enters on both sides. This equation determines Y_t as an implicit function of K_t and π_t. As a result, since K_t and π_t cannot jump, it follows that Y_t cannot jump either.

can jump, it must be the case that $\hat{c}_t$ itself can jump. Therefore, $\hat{c}_t$ here is a non-predetermined variable. According to Blanchard and Kahn (1980) test, we need, in this case, one stable eigenvalue and one unstable eigenvalue for the test to be satisfied.

Now, we apply the test explained by Rankin (2007) for our characteristic equation to have 0, 1, or 2 stable eigenvalues. This test will equivalently produce conditions for 0, 1, or 2 of the original eigenvalues λ to lie inside the unit circle[108]. Therefore, applying the table provided in chapter four and converting the table's coefficients (a,b,c) to our coefficients from our characteristic equation gives us the necessary and sufficient condition,

$$\frac{a+b+c}{a-b+c} \Rightarrow \frac{\hat{k}^{*}\beta\,\Upsilon\Psi}{2\left[\beta\hat{k}^{*}g^{2}\left(\alpha+\Psi(1-\alpha)\right)+\hat{k}^{*}\Psi\left(g^{2}\right)^{\sigma}+\hat{k}^{*}\beta g^{2}\alpha(1-\Psi)+\beta\alpha\hat{c}^{*}(1-\Psi)\right]-\hat{k}^{*}\beta\,\Upsilon\Psi}$$

Since $\Upsilon < 0$, the above ratio shows that the numerator is negative while the denominator is positive and greater than the numerator. Now, it is clear that the value of this ratio is negative and between zero and one. Therefore, there is one eigenvalue that lies inside the unit circle. As a consequence of this, we can conclude that the steady-state $(\hat{k}^{*}, \hat{c}^{*})$ is a saddle-point in this type of steady-state, *i.e.* type *(I)* steady-state[109].

[108] Chapter four clarified in detail the method explained by Rankin (2007).

[109] The detailed procedure for analysing this system, starting from the law of motion equations to the necessary and sufficient condition, is available in Appendix C.3.

6.7 The Effect of Personal Income Tax on the Transition Path of Variables in the Neighbourhood of a Type (*I*) Steady-State

This section analyses the transitional dynamics in type *(I)* steady-state before and after a permanent shock, *i.e.* a personal income tax. This section aims to examine the short-term and medium-term effects, through a phase diagram, when a personal income tax, τ^Y, is increased.

6.7.1 The Transitional Dynamics Before the Shock

The dynamics analysis of the model through a phase diagram requires rewriting the two fundamental equations in our economy in terms of $(\hat{k}_t, \hat{c}_t)$. In other words, the phase diagram is based only on two variables, $\hat{k}_t$ and $\hat{c}_t$, where each curve corresponds with one of the two zero change cases, *i.e.* $\Delta\hat{k}_{t+1} = \hat{k}_{t+1} - \hat{k}_t = 0$ and $\Delta\hat{c}_{t+1} = \hat{c}_{t+1} - \hat{c}_t = 0$. Thus, we start with the capital accumulation equation,

$$\hat{k}_{t+1} = \frac{\left(1-\tau^Y A\hat{k}_{t+1}^{\alpha}\right)}{g^2\left(1-\tau^Y A\hat{k}_t^{\alpha}\right)} \left[\left(1-\tau^Y\right)A\hat{k}_t^{\alpha} + (1-\delta)\,\hat{k}_t - \hat{c}_t\right]$$

$$\textit{where} \quad \gamma_{G_t} = \frac{g^2\left(1-\tau^Y \hat{y}_t\right)}{\left(1-\tau^Y \hat{y}_{t+1}\right)}, \quad \hat{y}_t = A\hat{k}_t^{\alpha}\ , \quad \textit{and} \quad \hat{y}_{t+1} = A\hat{k}_{t+1}^{\alpha}$$

We subtract $\hat{k}_t$ from both sides of the capital accumulation equation,

$$\Delta\hat{k}_{t+1} = \left[\frac{\left(1-\tau^Y A\hat{k}_{t+1}^{\alpha}\right)}{g^2\left(1-\tau^Y A\hat{k}_t^{\alpha}\right)}\right] \left[\left(1-\tau^Y\right)A\hat{k}_t^{\alpha} + (1-\delta)\,\hat{k}_t - \hat{c}_t\right] - \hat{k}_t \tag{6.30}$$

Then, we set $\Delta\hat{k}_{t+1} = \hat{k}_{t+1} - \hat{k}_t = 0$, and then simplify it to obtain:

$$\tau^Y A\hat{k}_t^{\alpha} = \frac{\left(1-\tau^Y\right)A\hat{k}_t^{\alpha} + (1-\delta)\,\hat{k}_t - \hat{c}_t - \hat{k}_t g^2\left(1-\tau^Y A\hat{k}_t^{\alpha}\right)}{\left(1-\tau^Y\right)A\hat{k}_t^{\alpha} + (1-\delta)\,\hat{k}_t - \hat{c}_t}$$

Equation (6.30) becomes now,

$$\hat{c}_t = \left(1-\tau^Y\right)A\hat{k}_t^{\alpha} + (1-\delta-g^2)\,\hat{k}_t \tag{6.31}$$

which can also be written as:

$$\hat{c}_t = \left(1-\tau^Y\right)A\hat{k}_t^{\alpha} - (g^2+\delta-1)\,\hat{k}_t$$

We can also draw $\hat{c}_t$ as a function of $\hat{k}_t$ (treating g^2 as given), as follows:

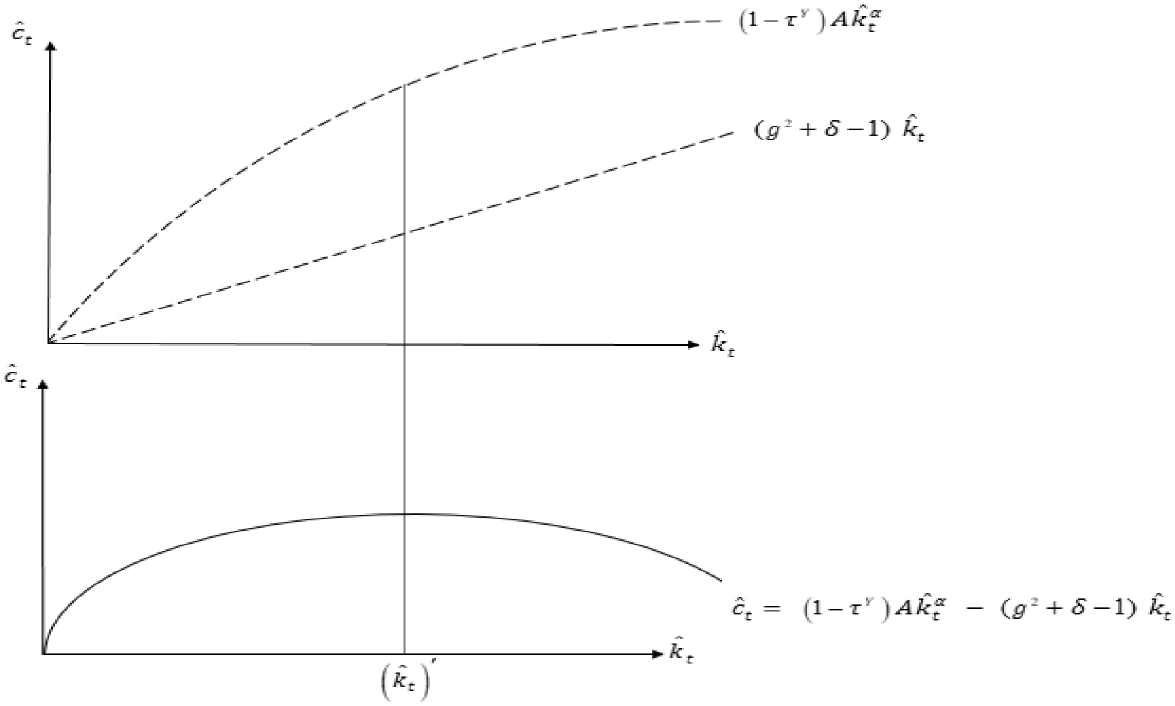

Diagram 25: Drawing equation $\hat{c}_t$ as a function of $\hat{k}_t$

The second diagram shows that $\hat{c}_t$ as a function of $\hat{k}_t$ is a 'hill shaped', where $(\hat{k}_t)'$ is the value of $\hat{k}_t$ which maximises $\hat{c}_t$, such that,

$$\frac{d\hat{c}_t}{d\hat{k}_t} = 0 \quad \Rightarrow \quad (1-\tau^Y)\alpha A\hat{k}_t^{\alpha-1} - g^2 - \delta + 1 = 0$$

For low values of $\hat{k}_t$, $d\hat{c}_t / d\hat{k}_t > 0$, while for high values of $\hat{k}_t$, $d\hat{c}_t / d\hat{k}_t < 0$. Therefore, equation (6.31) gives us a concave function (bell curve), as shown in the above diagram.

Below the $\Delta\hat{k}_{t+1} = 0$ curve described by (6.31), the consumption PGSU is *'lower'* at any point below the curve than it is on the curve, so that $\hat{c}_t < (1-\tau^Y)A\hat{k}_t^\alpha + (1-\delta-g^2)\hat{k}_t$ which, taken to the capital accumulation equation, it means: $\hat{k}_{t+1} > \hat{k}_t$ and the arrows point to *'right'*, showing the expected direction of capital PGSU in that area.

Above the $\Delta\hat{k}_{t+1} = 0$ curve, the consumption PGSU is *'higher'* at any point above the curve than it is on the curve, so that $\hat{c}_t > (1-\tau^Y)A\hat{k}_t^\alpha + (1-\delta-g^2)\hat{k}_t$, which implies $\hat{k}_{t+1} < \hat{k}_t$ and the arrows indicate to *'left'*.

On the other hand, the Euler equation is written as follows:

$$\hat{c}_{t+1} = \frac{\hat{c}_t\left(1-\tau^Y A\hat{k}_{t+1}^{\alpha}\right)}{g^2\left(1-\tau^Y A\hat{k}_t^{\alpha}\right)}\left[\beta\left(\left(1-\tau^Y\right)\alpha A\hat{k}_{t+1}^{\alpha-1}+1-\delta\right)\right]^{\frac{1}{\sigma}}$$

By subtracting $\hat{c}_t$ from both sides, setting $\Delta\hat{c}_{t+1} = \hat{c}_{t+1} - \hat{c}_t = 0$, and then simplifying it, we can rewrite it as follows:

$$g^2\left(1-\tau^Y A\hat{k}_t^{\alpha}\right) - \left(1-\tau^Y A\hat{k}_{t+1}^{\alpha}\right)\left[\beta\left(\left(1-\tau^Y\right)\alpha A\hat{k}_{t+1}^{\alpha-1}+1-\delta\right)\right]^{\frac{1}{\sigma}} = 0 \qquad (6.32)$$

where $\hat{k}_{t+1}$ is an implicit function of $\left(\hat{k}_t, \hat{c}_t\right)$ given by:

$$\frac{\hat{k}_{t+1}}{\left(1-\tau^Y A\hat{k}_{t+1}^{\alpha}\right)} = \frac{1}{g^2\left(1-\tau^Y A\hat{k}_t^{\alpha}\right)}\left[\left(1-\tau^Y\right)A\hat{k}_t^{\alpha} + (1-\delta)\,\hat{k}_t - \hat{c}_t\right]$$

The above two equations can be schematically written as:

$$\text{Equation (6.32)} \Rightarrow h\left(\underbrace{\hat{k}_t}_{h_1}, \underbrace{\hat{k}_{t+1}}_{h_2}, \underbrace{\tau^Y}_{h_3}\right) = 0, \quad \text{where } \hat{k}_{t+1} = j\left(\underbrace{\hat{k}_t}_{j_1}, \underbrace{\hat{c}_t}_{j_2}, \underbrace{\tau^Y}_{j_3}\right)$$

Therefore, the equation of $\Delta\hat{c}_{t+1} = 0$, equation (6.32), is implicitly given by:

$$h\left(\underbrace{\hat{k}_t}_{h_1}, \underbrace{j\left(\underbrace{\hat{k}_t}_{j_1}, \underbrace{\hat{c}_t}_{j_2}, \underbrace{\tau^Y}_{j_3}\right)}_{h_2}, \underbrace{\tau^Y}_{h_3}\right) = 0$$

Now, we can try to find out its slope and how it shifts when τ^Y increases. To do that, we totally differentiate it w.r.t. $\left(\hat{k}_t, \hat{c}_t, \tau^Y\right)$, which then gives us:

$$\left[h_1 + h_2 j_1\right].d\hat{k}_t + \left[h_2 j_2\right].d\hat{c}_t + \left[h_2 j_3 + h_3\right].d\tau^Y = 0$$

where j_1, j_2, and j_3 denote the partial derivate of $j(.)$ w.r.t. their arguments $\hat{k}_t, \hat{c}_t$, and τ^Y respectively.

Now, we need to evaluate h_1, h_2, h_3, j_1, j_2, and j_3[110]. Thus, they can be written as:

$$[h_1 + h_2 j_1].d\hat{k}_t \; + [h_2 j_2].d\hat{c}_t \; + [h_2 j_3 + h_3].d\tau^Y \; = 0$$

$$\left\{ \left[-\alpha g^2 \tau^Y A \hat{k}^{*\alpha-1} \right] + \left[\frac{\sigma\alpha\tau^Y A\hat{k}^{*\alpha-1}\left(g^2\right)^{\sigma} - \left(1-\tau^Y A\hat{k}^{*\alpha}\right)(\alpha-1)\alpha A\left(1-\tau^Y\right)\hat{k}^{*\alpha-2}\beta}{\sigma\left(g^2\right)^{\sigma-1}} \right] \left[\frac{\left(g^2\right)^{\sigma-1} + \tau^Y A\hat{k}^{*\alpha}\left(\alpha\beta - \left(g^2\right)^{\sigma-1}\right)}{\beta\left[1+(\alpha-1)\tau^Y A\hat{k}^{*\alpha}\right]} \right] \right\} .d\hat{k}_t$$

$$-\left\{ \left[\frac{\sigma\alpha\tau^Y A\hat{k}^{*\alpha-1}\left(g^2\right)^{\sigma} - \left(1-\tau^Y A\hat{k}^{*\alpha}\right)(\alpha-1)\alpha A\left(1-\tau^Y\right)\hat{k}^{*\alpha-2}\beta}{\sigma\left(g^2\right)^{\sigma-1}} \right] \left[\frac{\left(1-\tau^Y A\hat{k}^{*\alpha}\right)}{g^2\left[1+(\alpha-1)\tau^Y A\hat{k}^{*\alpha}\right]} \right] \right\} .d\hat{c}_t$$

$$+\left\{ \left[\frac{\sigma\alpha\tau^Y A\hat{k}^{*\alpha-1}\left(g^2\right)^{\sigma} - \left(1-\tau^Y A\hat{k}^{*\alpha}\right)(\alpha-1)\alpha A\left(1-\tau^Y\right)\hat{k}^{*\alpha-2}\beta}{\sigma\left(g^2\right)^{\sigma-1}} \right] \left[-\frac{A\hat{k}^{*\alpha}\left(1-\tau^Y A\hat{k}^{*\alpha}\right)}{g^2\left[1+(\alpha-1)\tau^Y A\hat{k}^{*\alpha}\right]} \right] + \left[\frac{\left(1-\tau^Y A\hat{k}^{*\alpha}\right)\alpha A\hat{k}^{*\alpha-1}\beta}{\sigma\left(g^2\right)^{\sigma-1}} \right] \right\} .d\tau^Y$$

$$= 0 \tag{6.33}$$

The slope of the equation of $\Delta\hat{c}_{t+1} = 0$ is,

$$\frac{d\hat{c}_t}{d\hat{k}_t} = -\frac{h_1 + h_2 j_1}{h_2 j_2}$$

$$= \frac{\left\{ \left[-\alpha g^2 \tau^Y A \hat{k}^{*\alpha-1} \right] + \left[\frac{\sigma\alpha\tau^Y A\hat{k}^{*\alpha-1}\left(g^2\right)^{\sigma} - \left(1-\tau^Y A\hat{k}^{*\alpha}\right)(\alpha-1)\alpha A\left(1-\tau^Y\right)\hat{k}^{*\alpha-2}\beta}{\sigma\left(g^2\right)^{\sigma-1}} \right] \left[\frac{\left(g^2\right)^{\sigma-1} + \tau^Y A\hat{k}^{*\alpha}\left(\alpha\beta - \left(g^2\right)^{\sigma-1}\right)}{\beta\left[1+(\alpha-1)\tau^Y A\hat{k}^{*\alpha}\right]} \right] \right\}}{\left\{ \left[\frac{\sigma\alpha\tau^Y A\hat{k}^{*\alpha-1}\left(g^2\right)^{\sigma} - \left(1-\tau^Y A\hat{k}^{*\alpha}\right)(\alpha-1)\alpha A\left(1-\tau^Y\right)\hat{k}^{*\alpha-2}\beta}{\sigma\left(g^2\right)^{\sigma-1}} \right] \left[\frac{\left(1-\tau^Y A\hat{k}^{*\alpha}\right)}{g^2\left[1+(\alpha-1)\tau^Y A\hat{k}^{*\alpha}\right]} \right] \right\}} \tag{6.34}$$

The sign of the slope of $\Delta\hat{c}_{t+1} = 0$ locus is ambiguous due to multiple terms in (6.34), making it difficult to determine its sign. However, we know that if the personal income tax sets to be zero, $\tau^Y = 0$, then we would return to the situation in chapter four model, where the slope of the stationary locus for $\hat{c}_t$ is positive. Nevertheless, τ^Y in this chapter is not zero, $\tau^Y \neq 0$; therefore, we can consider it to be sufficiently close to zero. The reason for this is to be able to draw a phase diagram because otherwise, it may be difficult to draw it for very general values of τ^Y. Thus, we draw a phase diagram for a value of τ^Y which is close to zero. Consequently, with a positive slope of $\Delta\hat{c}_{t+1} = 0$, we then understand that the saddle path to the new steady-state would be upward-sloping, *i.e.* like the one in chapter four model.

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[110] The procedures to evaluate h_1, h_2, h_3, j_1, j_2, and j_3 is available in Appendix C.4.

6.7.2 The Transitional Dynamics After the Shock

We study in this section how the shock, a change in personal income tax, would affect the transitional dynamics. In other words, how the two curves in the phase diagram, $\Delta\hat{k}_{t+1} = 0$ and $\Delta\hat{c}_{t+1} = 0$, would move when the personal income tax changes.

A- The locus $\Delta\hat{k}_{t+1} = 0$:

Equation (6.31) shows that personal income tax affects consumption PGSU. Thus, taking the derivative of equation (6.31) w.r.t. personal income tax gives us,

$$\frac{d\hat{c}_t}{d\tau^Y} = -A\hat{k}_t^{\alpha} \tag{6.35}$$

The sign of (6.35) is negative, meaning that holding $\hat{k}_t$ constant, rising τ^Y must fall $\hat{c}_t$. As a result, we can conclude that $\Delta\hat{k}_{t+1} = 0$ locus would shift the bell curve down when personal income tax is increased.

B- The locus $\Delta\hat{c}_{t+1} = 0$:

The vertical shift of $\Delta\hat{c}_{t+1} = 0$, when personal income tax changes, is,

$$\frac{d\hat{c}_t}{d\tau^Y} = -\frac{h_2 j_3 + h_3}{h_2 j_2}$$

$$= \frac{\left\{\left[\frac{\sigma\alpha\tau^Y A\hat{k}^{*\alpha-1}\left(g^2\right)^{\sigma} - \left(1-\tau^Y A\hat{k}^{*\alpha}\right)(\alpha-1)\alpha A\left(1-\tau^Y\right)\hat{k}^{*\alpha-2}\beta}{\sigma\left(g^2\right)^{\sigma-1}}\right]\left[-\frac{A\hat{k}^{*\alpha}\left(1-\tau^Y A\hat{k}^{*\alpha}\right)}{g^2\left[1+(\alpha-1)\tau^Y A\hat{k}^{*\alpha}\right]}\right]+\left[\frac{\left(1-\tau^Y A\hat{k}^{*\alpha}\right)\alpha A\hat{k}^{*\alpha-1}\beta}{\sigma\left(g^2\right)^{\sigma-1}}\right]\right\}}{\left\{\left[\frac{\sigma\alpha\tau^Y A\hat{k}^{*\alpha-1}\left(g^2\right)^{\sigma} - \left(1-\tau^Y A\hat{k}^{*\alpha}\right)(\alpha-1)\alpha A\left(1-\tau^Y\right)\hat{k}^{*\alpha-2}\beta}{\sigma\left(g^2\right)^{\sigma-1}}\right]\left[\frac{\left(1-\tau^Y A\hat{k}^{*\alpha}\right)}{g^2\left[1+(\alpha-1)\tau^Y A\hat{k}^{*\alpha}\right]}\right]\right\}} \tag{6.36}$$

The sign of the vertical shift of $\Delta\hat{c}_{t+1} = 0$, equation (6.36), is ambiguous. However, what we need to understand instead is how $\hat{k}_t$ and $\hat{c}_t$ behave as the economy moves along the transition path to the new steady-state. For this purpose, we know from section 6.5.1 that the personal income tax affects both the capital PGSU and the consumption PGSU negatively, as shown in (6.13)' and (6.15)''. Thus, higher τ^Y means that the new steady-state, (E'), should be below the old steady-state, (E), in the phase diagram. In other words, both $\hat{k}_t$ and $\hat{c}_t$ fall in the new steady-state. We also know that the saddle path to the new steady-state would be upward-sloping if we consider the case where τ^Y remains close to zero. However, the fall in the new steady-state does not inform us about the initial jump of the capital PGSU. For this reason, we need to examine two main questions regarding the effect on $\hat{k}_t$ when the shock occurs, *i.e.* the tax rate changes. The first question is that would $\hat{k}_t$ jump on the impact period? The second is that if it can jump, would it jump up or down? In fact, answering these questions would help us to understand the transition path of $\hat{k}_t$ at the moment of the impact period.

Before trying to answer these two questions, it is worth mentioning that in the analysis of the local stability, section 6.6, we showed that $\hat{k}_t$ is a predetermined variable while $\hat{c}_t$ is a non-predetermined variable. However, when we now study the transition path for the case of chaining τ^Y, $\hat{k}_t$ turns to be a non-predetermined variable, like $\hat{c}_t$. The reason is that if we shock τ^Y, that means G_t would be shocked (*i.e.* immediately change G_t even though G_t is given) because the effect is simultaneous on revenues. More precisely, the sudden change in G_t, when τ^Y changes, can be seen from the government budget constraint (*i.e.* $G_t = \pi_t + \tau^Y Y_t$). Then, from the definition of capital PGSU, $\hat{k}_t \equiv K_t / G_t$, we know that K_t cannot jump, but G_t can now jump. Therefore, since G_t can now jump, it must be the case that $\hat{k}_t$ itself would immediately change as well. As a result, changing τ^Y implies that $\hat{k}_t$, as $\hat{c}_t$, would jump at the moment of impact, where it could not jump before the shock.

As long as $\hat{k}_t$ and $\hat{c}_t$ are both now non-predetermined variables, we can say that our model in this chapter is different from the phase diagram in the Ramsey model (*i.e.* the one in chapter four). In other words, when a permanent shock happens, $\hat{k}_t$ and $\hat{c}_t$ can both jump, but $\hat{k}_t$ cannot jump in chapter four. Therefore, the economy in this chapter would not move initially to the point that is vertically above or below the old steady-state. Since we understood that $\hat{k}_t$ and $\hat{c}_t$ can both jump, we need next to answer the second question, which is regarding how $\hat{k}_t$ would jump on the impact period. To do that, let us investigate further the definition of the capital PGSU, which can be written as:

$$\hat{k}_t \equiv \frac{K_t}{G_t} = \frac{K_t}{\pi_t + \tau^Y Y_t}$$

We can see that there is a direct effect of higher τ^Y on $\hat{k}_t$ even if Y_t did not change. In addition to that, when τ^Y is unexpectedly increased, K_t cannot jump, but Y_t can jump because τ^Y enters directly in Y_t through the production function. To see how the rise in τ^Y makes Y_t jumps, we need to use the production function, Y_t, which is implicitly determined by:

$$Y_t = AK_t^{\alpha}\left(\pi_t + \tau^Y Y_t\right)^{1-\alpha}$$

Differentiating the production function w.r.t τ^Y would help us to determine the initial jump in $\hat{k}_t$. Thus, using the implicit differentiation method and simplifying it gives us:

$$\frac{dY_t}{d\tau^Y} = \frac{(1-\alpha)(Y_t)^2}{\pi_t + \alpha\tau^Y Y_t} > 0$$

To find out now how $\hat{k}_t$ would jump, we need to find $d\hat{k}_0 / d\tau^Y$, which requires some differentiation due to the implicit function of Y_t. Appendix C.5. provides the full steps and differentiations to arrive at $d\hat{k}_0 / d\tau^Y$ and its sign. Therefore, after simplifying it, we have:

$$\frac{d\hat{k}_0}{d\tau^Y} = -\frac{\hat{k}_t}{\underbrace{\left(1/A\hat{k}_t^{\alpha}\right) - (1-\alpha)\tau^Y}_{(+)}} < 0 \tag{6.37}$$

The sign of equation (6.37) is negative, which means that $\hat{k}_t$ would jump initially down in the impact period. In other words, equation (6.37) tells us how much the initial jump in $\hat{k}_t$ moves down when τ^Y increases. The reason behind jumping down is that the sudden rise in government spending, caused by receiving additional government revenues, makes $\hat{k}_t$ to suddenly drop in the impact period[111]. Thus, we need now to find out whether the initial downwards jump in $\hat{k}_t$, is larger or smaller than the fall in the steady-state value of $\hat{k}_t$. The purpose of this comparison is to understand how the economy must jump in the impact period. In other words, the jump locus lies, whether to the left or the right, of the new steady-state. To see that, we compare the initial downwards jump in $\hat{k}_t$, *i.e.* (6.37), and the fall in the steady-state value of $\hat{k}_t$, *i.e.* (6.13)'.

For equation (6.13)', it is found in section 6.5.1.1. Thus, we simplify it more to be able to compare it with (6.37), as[112]:

$$\frac{d\hat{k}^*}{d\tau^Y} = -\frac{\hat{k}^*}{\underbrace{(1-\alpha) - (1-\alpha)\tau^Y}_{(+)}} \tag{6.13$''$}$$

[111] When G_t unexpectedly rises, the sudden drop in $\hat{k}_t$ and $\hat{c}_t$ can be seen obviously from both the definition of the capital PGSU, $\hat{k}_t \equiv K_t / G_t$, and the consumption PGSU, $\hat{c}_t \equiv C_t / G_t$.

[112] It should be noted that both derivatives of (6.13)' and (6.37) are evaluated in the initial steady-state.

The comparison between (6.13)'' and (6.37), *i.e.* $d\hat{k}^*/d\tau^Y$ and $d\hat{k}_0/d\tau^Y$, becomes now obviously between $(1-\alpha)$ and $\left(1/A\hat{k}_t^\alpha\right)$. Since $\alpha \in (0{,}1)$, then we know that $0 < 1-\alpha < 1$. On the other hand, we also know that $\left(1/A\hat{k}_t^\alpha\right) = (G_t/Y_t) = (1/\hat{y}_t)$. Thus, we evaluate it in the steady-state by using the solution for $\hat{k}^*$ in the steady-state to have:

$$\frac{1}{A\hat{k}^{*\alpha}} = A^{\frac{1}{\alpha-1}} \left(\frac{\frac{\left(g^2\right)^\sigma}{\beta} - 1 + \delta}{\alpha\left(1-\tau^Y\right)} \right)^{\frac{\alpha}{1-\alpha}} \tag{6.38}$$

It seems clear that, depending on parameters, $\left(1/A\hat{k}^{*\alpha}\right)$ could take any values between zero and infinity, meaning that $\left(1/A\hat{k}^{*\alpha}\right)$ could be greater than or less than $(1-\alpha)$. In other words, for the condition $\left(1/A\hat{k}^{*\alpha}\right) > (1-\alpha)$ or $\left(1/A\hat{k}^{*\alpha}\right) < (1-\alpha)$ to hold, it depends on the parameter values chosen. Thus, if $\left(1/A\hat{k}^{*\alpha}\right)$ is greater than $(1-\alpha)$, it then implies that (6.13)'' > (6.37), *i.e.* $d\hat{k}^*/d\tau^Y > d\hat{k}_0/d\tau^Y$, and the opposite is true.

To investigate more the possible values for $\left(1/A\hat{k}^{*\alpha}\right)$, we need to relate the parameter condition in $\left(1/A\hat{k}^{*\alpha}\right)$ to the parameter conditions under which a type (*I*) steady-state exists. For type (*I*) steady-state to exist, we know that some conditions should be held. For example, if g^2 is sufficiently high for a given value of τ^Y, if g^2 is fixed at a certain value and τ^Y is too high, and if g^2 is fixed at a certain value and τ^Y is too low. Looking at g^2 in equation (6.38), it is obvious that the higher in g^2 implies the higher in $\left(1/A\hat{k}^{*\alpha}\right)$, which would ensure that $\left(1/A\hat{k}^{*\alpha}\right) > (1-\alpha)$ because $(1-\alpha) \in (0{,}1)$. Now, we can say that if g^2 is high, then $\left(1/A\hat{k}^{*\alpha}\right) > (1-\alpha)$, *i.e.* (6.13)'' > (6.37). In other words, the fall in the steady-state value of $\hat{k}_t$ is greater than the initial downwards jump in $\hat{k}_t$, *i.e.* $d\hat{k}^*/d\tau^Y > d\hat{k}_0/d\tau^Y$. However, the opposite is also true, meaning if g^2 is low, then $d\hat{k}^*/d\tau^Y < d\hat{k}_0/d\tau^Y$. In words, the fall in the steady-state value of $\hat{k}_t$ is less than the initial downwards jump in $\hat{k}_t$. As a result, we can conclude that the two situations, $\left(1/A\hat{k}^{*\alpha}\right) > (1-\alpha)$ or $\left(1/A\hat{k}^{*\alpha}\right) < (1-\alpha)$, are both theoretical possible to occur, depending on the parameter values.

The below phase diagrams display the steady-state values before and after the shock (*i.e.* changing personal income tax). The black loci show the economy before the shock happens, at point (E), while the red loci represent the economy after the shock, at point (E'). After the shock occurs, the $\Delta\hat{k}_{t+1} = 0$ locus would shift the bell curve down, whereas the $\Delta\hat{c}_{t+1} = 0$ locus would move to up (*i.e.* to the left). The blue dotted line is the saddle path, where the intersection between the initial jumps of both $\hat{k}_t$ and $\hat{c}_t$ at point (A) is the saddle-point stable. The phase diagrams also show blue arrows, which indicate the dynamic behaviour of $\hat{k}_t$ and $\hat{c}_t$. As long as the personal income tax is a permanent shock in our model, then these arrows will start to work immediately, which means that the economy will move toward the new steady-state, (E') in both phase diagrams.

As we found previously, there are two possible situations to construct a phase diagram. Thus, we will discuss these two possible situations in two separate phase diagrams. In the first situation, where g^2 sets to be high, the jump locus lies to the right of the new steady-state, as shown in Diagram 26, where $\hat{k}_t$ and $\hat{c}_t$ would intersect at a point such A. The intersection point, A, is higher than the new steady-state (E'). At this point, $\hat{k}_t$ and $\hat{c}_t$ would fall gradually over time, which means that both the level of capital stock and consumption would grow slower than the level of government spending along the transition path. This situation would show that the growth rate is temporarily declined. As a result, the transitional dynamics for $\hat{k}_t$ and $\hat{c}_t$ would initially fall and intersect at a point such A. Then, the economy at point A will converge along the saddle path toward the new steady-state, as usual, *i.e.* it converges on $\hat{k}^*$ from above. On the other hand, the second situation can be occurred if g^2 is low. In this case, the jump locus lies to the left of the new steady-state, $\hat{k}_t$ and $\hat{c}_t$ would intersect at a point such A, as shown in Diagram 27. At this point, $\hat{k}_t$ and $\hat{c}_t$ would rise over time, as the economy converges along the saddle path to the new steady-state, *i.e.* it converges on $\hat{k}^*$ from below. Contrary to the first situation, this situation would exhibit a temporarily higher growth rate because $\hat{k}_t$ (and hence $\hat{y}_t$) is growing over time.

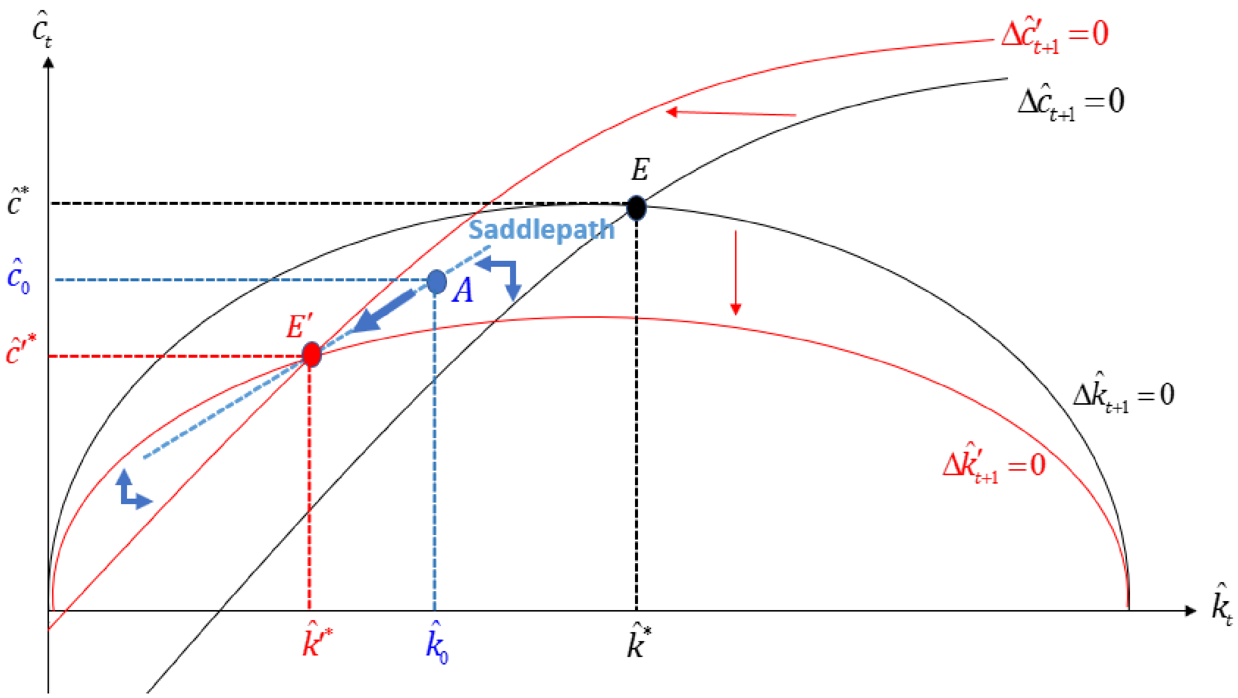

Diagram 26: Phase diagram of the transition dynamic when a personal income tax increases (the first situation)

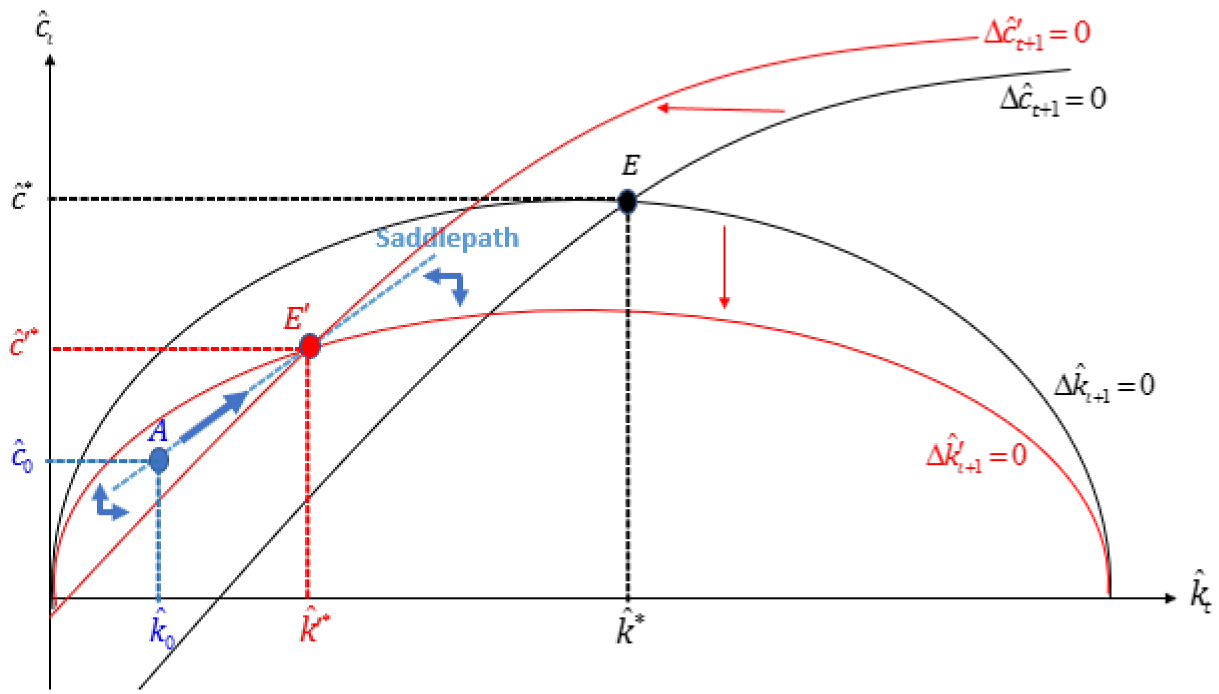

Diagram 27: Phase diagram of the transition dynamic when a personal income tax increases (the second situation)

6.8 Parameterisation and Solution

Similar to chapter five, we apply in this section a numerical simulation under standard and commonly used parameters in literature[113]. Dynare software is used to simulate the model in two parts. The first part deals with PGSU variables, where we investigate how introducing personal income tax, *e.g.* $\tau^Y = 10\%$[114], would affect all PGSU variables, *i.e.* the steady-state responses and the transitional dynamics. This part aims to explore the full range of possible qualitative results in our model (*e.g.* whether $\hat{k}^*$ falls by more or less than $\hat{k}_t$ when τ^Y increases). In the previous section, we found theoretically that changing g^2 would help us to determine if $\hat{k}^*$ falls by more or less than $\hat{k}_t$. However, this result is not clear enough; thus, we would investigate that further numerically. Specifically, we would provide two simulations under two different set of parameter values to explore how $\hat{k}^*$ and $\hat{k}_t$ behave in both cases. The second part pursues to numerically determine what type of steady-state, an *exogenous* or an *endogenous growth steady-state*, that we should be in, under a particular set of parameters values. This part also makes some sensitive analysis in our simulations regarding the personal income tax thresholds. Finally, it should be noted that this section would not discuss the level variables, *i.e.* G_t, C_t, K_t, and Y_t, numerically. The reason is that section 6.5.2 has already examined what would happen to the level variables when the personal income tax changes by providing some tables to show the numerical values for these variables.

6.8.1 Part One: Introducing a Personal Income Tax (PGSU Variables)

In this part, we introduce personal income tax in our model to see how the PGSU variables can change and calculate the dynamic time paths of PGSU variables. We found theoretically that the if g^2 is high (low), then the fall in the steady-state value of $\hat{k}_t$ is greater (less) than the initial downwards jump in $\hat{k}_t$. Thus, we provide two simulations by setting g^2 in the first simulation as a high, *i.e.* $g^2 = 1.1$, while setting g^2 in the second simulation as a low, *i.e.* $g^2 = 1.01$. The tables below summarise the impact of the steady-state values on PGSU variables when a 10% personal income tax rate is introduced under two values of g^2. On the other hand, the graphs display the dynamic responses of a number of PGSU variables in the economy when personal tax is also added, where g^2 is high as in Figure 41 and low as in Figure 42.

[113] The parameter values used here are the same as in the previous chapters, where we set $A = 1, \alpha = 0.33, \beta = 0.95, \delta = 0.1$, and $\sigma = 2$.

[114] We previously, *i.e.* in chapter five, introduced a 5% tax rate, but we now introduce a 10% tax rate. The reason is that introducing a 5% tax rate in this chapter does not show us clearly the transition paths of all variables.

Table 16: The steady-state responses of a number of PGSU variables in the economy when the value of g^2 is 1.10:

Old Steady-State Values		**New Steady-State Values**	
Variables	**Value**	**Variables**	**Value**
Personal Income Tax	0	Personal Income Tax	0.10
Output PGSU	0.9406	Output PGSU	0.8930
Capital PGSU	0.8306	Capital PGSU	0.7098
Consumption PGSU	0.7745	Consumption PGSU	0.6618
Interest Rate	0.3737	Interest Rate	0.4152

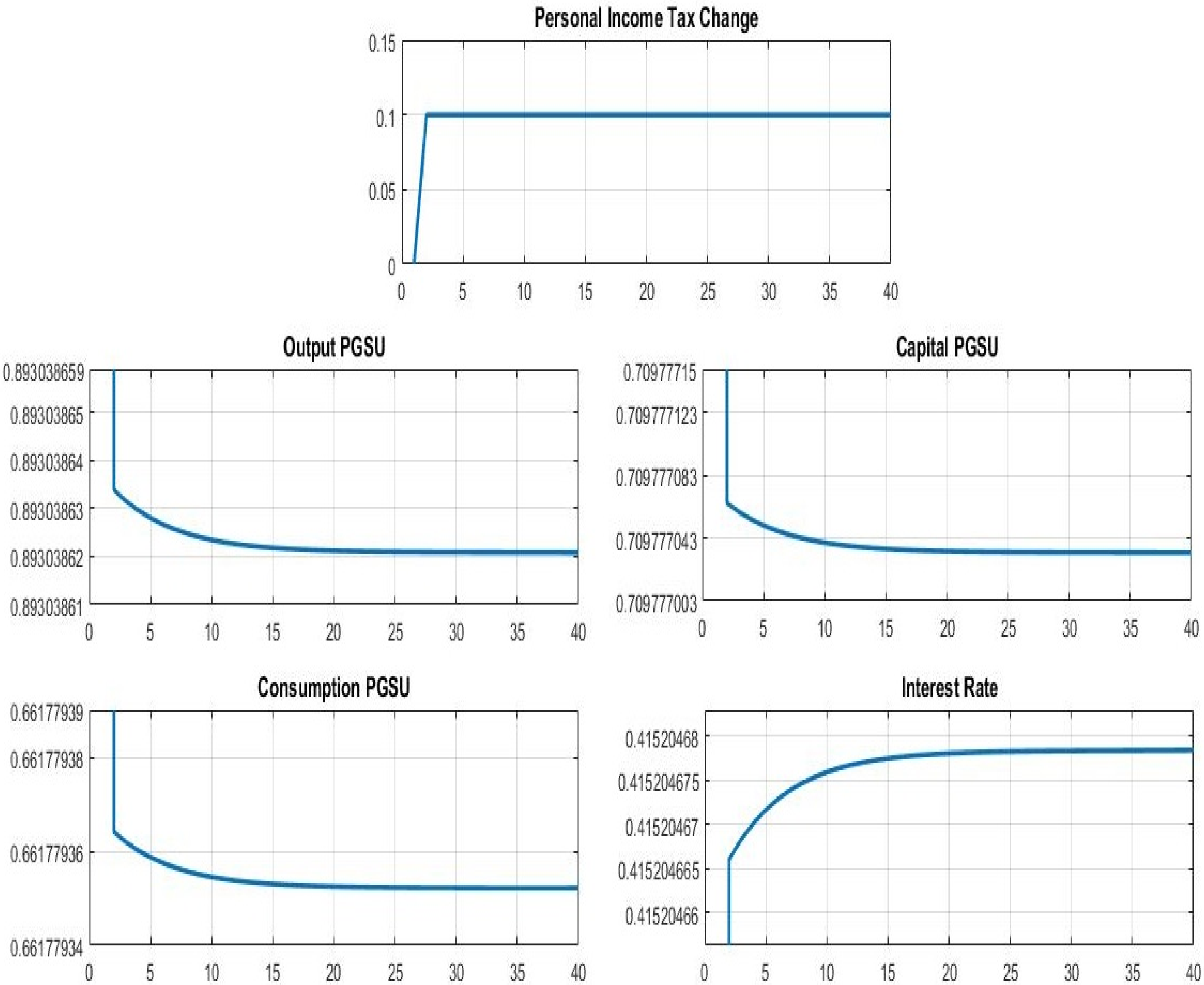

Figure 41: The dynamic responses of a number of variables in the economy to a permanent shock when g^2 = 1.10

Table 17: The steady-state responses of a number of PGSU variables in the economy when the value of g^2 is 1.01:

Old Steady-State Values		**New Steady-State Values**	
Variables	**Value**	**Variables**	**Value**
Personal Income Tax	0	Personal Income Tax	0.10
Output PGSU	1.3714	Output PGSU	1.3021
Capital PGSU	2.6041	Capital PGSU	2.2252
Consumption PGSU	1.0850	Consumption PGSU	0.9271
Interest Rate	0.1738	Interest Rate	0.1931

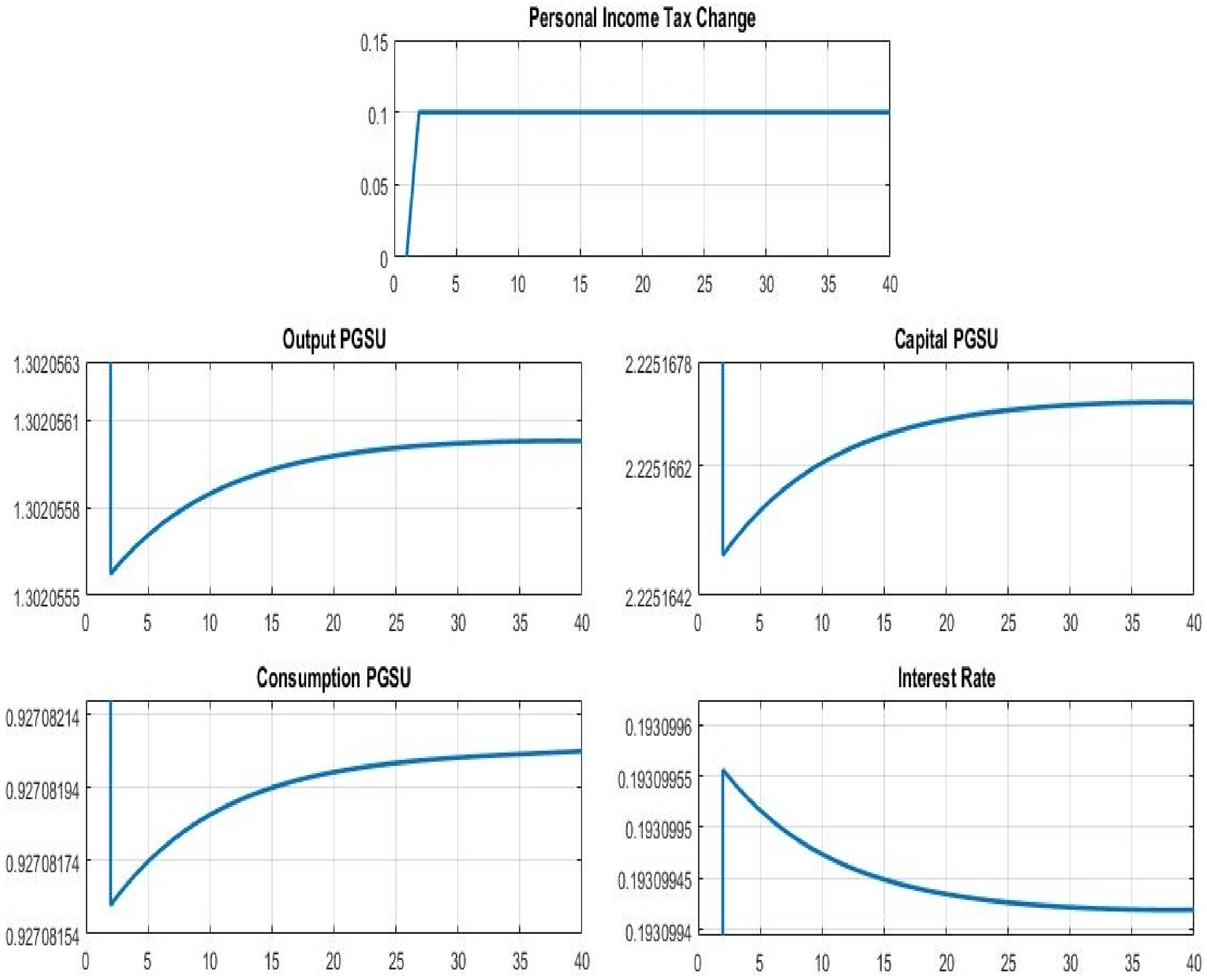

Figure 42: The dynamic responses of a number of variables in the economy to a permanent shock when g^2 = 1.01

When we compare the above tables and graphs with our theoretical conclusion in the steady-state and the transitional dynamics sections, we find that they are consistent with each other. However, to ensure that the two simulations are valid, we need also to verify that the condition in type (*I*) steady-state is satisfied for both simulations. We understand from analysing the steady-state that if the growth rate in type (*I*) steady-state is greater (less) than the growth rate in type (*II*) steady-state, for the same set of parameter values, then the correct steady-state will be type (*I*) steady-state, *i.e.* exogenous growth steady-state (type (*II*) steady-state, *i.e.* endogenous growth steady-state). Thus, using the same set of parameter values with a tax rate at 10%, the corresponding value for the growth rate in the endogenous growth steady-state (*i.e.* γ_Y) is less than the growth rate in the exogenous growth steady-state (*i.e.* g^2)[115]. As a result, we can say that these two simulations are under type (*I*) steady-state. In other words, both simulations confirm that the condition in type (*I*) steady-state is satisfied.

Returning to our simulations, although we set two different values of g^2, introducing personal income tax decreases the steady-state values of all PGSU variables, as shown in Table 16 and Table 17. More precisely, a 10% personal income tax rate would reduce, in both cases of g^2, the output PGSU by 5.06%, and both the capital PGSU and the consumption PGSU by 14.55%. For the transitional dynamics, the dynamic responses shown in the above figures are also consistent with our two phases diagrams in section 6.7.2. More precisely, with an additional income source, namely personal income tax, the government revenues increase, which in turns is reflecting in the increase in government spending. However, when the g^2 is high, *i.e.* $g^2 = 1.10$, the growth rate of government spending declines in the short run and medium run[116], and it then returns asymptotically to its initial steady-state in the long run. The reason is that $\hat{k}_t$ (and hence $\hat{y}_t$) is falling over time, which causes a temporary decline in the growth rate. Moreover, a shock on τ^Y makes the consumption PGSU, capital PGSU, and output PGSU all initially jump down at the moment of impact period, and then they fall over time. On the contrary, if the g^2 is low, *i.e.* $g^2 = 1.01$, the growth rate of government spending in this case is temporarily boosted because $\hat{k}_t$ (and hence $\hat{y}_t$) is rising over time. Thus, once the shock happens, the consumption PGSU, capital PGSU, and output PGSU all initially jump down at the moment of the impact period, and then they rise over time. However, the new steady-state of all PGSU variables would be still less than the old steady-state due to the negative impact caused by the tax.

[115] We used the formula (6.22) with the same set of parameter values in the two simulations. Then, we found that $g^2 > \gamma_Y$.

[116] Although introducing personal income tax would intuitively generate extra government revenues and then increases the government spending, which would also affect the growth rate positively, the growth rate in this situation would temporarily decline. To see that clearly, Appendix C.6 discusses the reason for the temporary decline in the growth rate of government spending when g^2 is high.

6.8.2 Part Two: What Type of Steady-State We Should Be in (PGSU Variables)

We understand from our discussion in the existence steady-state section that the two types of steady-state, *i.e.* type *(I)* steady-state or type *(II)* steady-state, cannot exist for the same set of parameter values. We have also shown that under some restrictions in parameter values, γ_Y would be greater than one in order to have endogenous growth. These restrictions include that β should be sufficiently high, high personal income tax, and $A > 1$. By using these parameter value restrictions, as well as other standard parameter values in equation (6.22), we can ensure that γ_Y is greater than one[117]. Consequently, under these parameter value restrictions, we can also confirm that γ_Y could be greater than g^2, *i.e.* $\gamma_Y > g^2$, because γ_Y would increase with rising tax until it reaches its maximum value. Thus, we could have endogenous growth steady-state when the tax rate increases, but at a certain tax rate, γ_Y may turn out below g^2 and then the actual steady-state becomes the exogenous growth steady-state.

To see that, we provide a simple numerical exercise to show when the economy could move from a steady-state to another, if we hold g^2 constant and change the personal income tax. In this exercise, we attempt to find out the critical values that allow the economy to change its steady-state position. To find the possible values for γ_Y and compare it with g^2, we first assume that g^2 is fixed at 1.1, and then we increase τ^Y using the formula of γ_Y in equation (6.22) and applying the same set of parameter values as before, except that. The below table displays our calculation for γ_Y and the potential type of steady-state for each value of τ^Y.

Table 18: The calculation for γ_Y and the potential type of steady-state for each value of τ^Y:

τ^Y	γ_Y	g^2	***Steady-State Type***	
10%	*0.936*	*1.100*	$g^2 > \gamma_Y$	*Exogenous Growth Steady-State*
20%	*0.966*	*1.100*	$g^2 > \gamma_Y$	*Exogenous Growth Steady-State*
30%	*1.005*	*1.100*	$g^2 > \gamma_Y$	*Exogenous Growth Steady-State*
40%	*1.046*	*1.100*	$g^2 > \gamma_Y$	*Exogenous Growth Steady-State*
50%	*1.080*	*1.100*	$g^2 > \gamma_Y$	*Exogenous Growth Steady-State*
57.8%	***1.100***	***1.100***	$\boldsymbol{g^2 = \gamma_Y}$	***First Critical Value***
60%	*1.104*	*1.100*	$g^2 < \gamma_Y$	*Endogenous Growth Steady-State*
70%	*1.108*	*1.100*	$g^2 < \gamma_Y$	*Endogenous Growth Steady-State*
75.4%	***1.100***	***1.100***	$\boldsymbol{g^2 = \gamma_Y}$	***Second Critical Value***
80%	*1.087*	*1.100*	$g^2 > \gamma_Y$	*Exogenous Growth Steady-State*
90%	*1.030*	*1.100*	$g^2 > \gamma_Y$	*Exogenous Growth Steady-State*

[117] Specifically, using the same set of parameter values as before, setting $A = 2$, and $0.28 < \tau^Y < 0.94$ would ensure that $\gamma_Y > 1$.

The above exercise shows that there are two critical values for τ^Y. The first critical value is when $\tau^Y = 57.8\%$, while the second critical value is when $\tau^Y = 75.4\%$. That implies that if the authority sets the personal income tax at $\tau^Y < 57.8\%$ or $\tau^Y > 75.4\%$, the economy would have the exogenous growth steady-state because of $g^2 > \gamma_Y$. On the other hand, if the authority sets the personal income tax at $57.8\% < \tau^y < 75.4\%$, then the economy would move from exogenous growth steady-state to the endogenous one because of $g^2 < \gamma_Y$. Thus, based on the parameter values chosen in this exercise, the above table clarifies that it is possible to discover the two critical values. However, it should be noted that even though our simple numerical example does not reflect exactly the real world because of our simplifying and some parameter restrictions, it is helpful to get a better understanding regarding the idea of how to achieve these two critical values and how the economy could turn out from one type of steady-state to another one. For this reason, we will next investigate more the personal income tax thresholds, like the analysis of chapter five.

For equation (6.22), it seems not possible to provide an analytical solution for personal income tax thresholds. The reason is that it cannot be solved algebraically, since it is a non-linear equation. For this reason, we resort to a numerical solution, which would be a suitable method in this situation, as shown in the previous chapter. However, it should be noted that the personal income tax in our model, contrary to the consumption tax, has two thresholds (*i.e.*, minimum and maximum values[118]) for each set of parameter values.

Although equation (6.22) cannot be explicitly solved in our model for personal income tax, it is clear that the tax thresholds depend on six parameters. These parameters include $\gamma_Y = g^2$, β, α, A, δ, and σ. Thus, we will next examine these parameters separately to identify the possible values of the tax thresholds.

Similar to our numerical solution presented in chapter five, we use the same range of parameter values for α and δ, as empirical evidence, where $\alpha \in [0.40, 0.69]$ and $\delta \in [0.05, 0.15]$. Moreover, the coefficient of relative risk aversion, σ, is assumed to be at 2, as considered for oil-exporting countries by Cherif and Hasanov (2012) and for Saudi Arabia by Pierru and Matar (2014). Therefore, to find the tax thresholds, we begin by varying the values of α and keeping the other parameter values constant, *e.g.*, $g^2 = 1.1$, $\beta = 0.99$, $\delta = 0.05$, $A = 2$, and $\sigma = 2$.

[118] The minimum and maximum values are the two critical values discussed in section 6.4.4.

Table 19: The personal income tax thresholds response to changes in α

Parameter	**Income Tax Thresholds**	
$\boldsymbol{\alpha}$	**min values**	**max values**
0.40	31.30%	84.52%
0.45	23.37%	83.94%
0.50	16.25%	83.75%
0.55	10.37%	83.78%
0.60	5.92%	83.94%
0.65	2.89%	84.18%

As we can see from the above table, a higher α would lower the minimum tax thresholds. However, higher α would lower (higher) the maximum tax thresholds for the range $\alpha \in (0.40, 0.50]$ $\left(\alpha \in [0.55, 065)\right)$.

Now, we change the values of g^2 and hold the other parameter values fixed to see how the tax thresholds would respond.

Table 20: The personal income tax thresholds response to changes in g^2

Parameter	**Income Tax Thresholds**	
$\boldsymbol{g^2}$	**min values**	**max values**
1.05	5.20%	90.88%
1.10	10.37%	83.78%
1.15	17.61%	74.76%
1.20	29.64%	61.04%

The above table shows that if g^2 is higher, the minimum tax thresholds become higher too, while the maximum tax thresholds become lower. Next, we vary the values of δ and hold the other parameter values fixed.

Table 21: The personal income tax thresholds response to changes in δ

Parameter	**Income Tax Thresholds**	
$\boldsymbol{\delta}$	**min values**	**max values**
0.05	10.37%	83.78%
0.08	12.06%	81.60%
0.12	14.54%	78.49%
0.15	16.60%	75.97%

Similar to changing the values of g^2, if the δ is higher, the minimum tax thresholds are higher, whereas the maximum tax thresholds are lower. Finally, we alter A values and keep the other parameter values constant.

Table 22: The personal income tax thresholds response to changes in A

Parameter	**Income Tax Thresholds**	
A	**min values**	**max values**
2	10.37%	83.78%
2.5	5.96%	89.78%
3	3.87%	92.87%
3.5	2.71%	94.69%

In contrast to changing the values of both g^2 and δ, higher A implies lower minimum tax thresholds and higher maximum tax thresholds.

6.9 Why Do Taxes Matter in Saudi Arabia?

The way in which the Saudi economy is characterised by limited taxes was covered in the first and second chapters. Thus, the current taxes in Saudi Arabia predominantly comprise the corporate tax on only foreign companies and the value-added tax (VAT). Saudi citizens' companies are not subject to corporate tax but are subject to Zakat, which is 2.5% on wealth. A 5% VAT rate was recently introduced in 2018. Therefore, a typical Saudi citizen is currently covered by the VAT and Zakat and no personal income tax. This tax structure is considered one of our main motivations to study taxes in the Saudi economy. It gives also rise to three important questions: (i) are taxes (direct or indirect), appropriate for the Saudi economy? (ii) what are the advantages and disadvantages of the taxes for the Saudi economy? (iii) what type of tax would be more suitable for the Saudi economy? Thus, this section is devoted to discussions on these three important questions related to the Saudi Arabian tax policy.

Before answering the first question, it is important to mention that the International Monetary Fund (IMF) indicated in its 2015 report that there is a large space for new taxes in the Saudi economy due to the limited taxes directed in the current tax policies (IMF, 2015c). This leads us to the discussion on the first question regarding the importance of taxes, whether direct taxes (like personal income tax) or indirect taxes (like consumption tax), as additional sources of revenues for the Saudi Arabian economy. In fact, there is no doubt that economic diversification through diversification of income sources is one of the most critical challenges facing countries that rely heavily on oil revenues. According to the IMF report in 2016, the decline in oil prices in 2014 led to many oil-exporting countries facing a substantial reduction in government revenues and public spending in addition to their international reserves. Thus, low oil revenues negatively affected domestic consumption in oil-dependent countries, where a lot of jobs in these countries are directly or indirectly related to the productivity of the oil sector. This negative impact is represented in the loss of many job seekers from obtaining a job due to the recession, which means less income and therefore less consumption (IMF, 2016a). For this reason, diversifying sources of income and then reducing reliance on oil revenues would be an appropriate way to stabilise the economy. As long as taxes would generate additional revenues to the government, the Saudi government should be aware that diversifying the sources of income by reforming the tax system would minimise its significant reliance on oil revenues.

In Saudi Arabia, all economic activities, *i.e.* the government, private sector, exports and, imports, depend heavily on oil. This means that the Saudi economy is directly influenced by the shocks to which the oil markets are usually exposed, hence impacting the economy. Therefore, economic diversification would help overcome or even reduce the negative effects of temporary and permanent changes in oil markets. One source of income diversification is tax revenues, which may stimulate non-oil GDP growth. In fact, these additional revenues may help to separate government expenditures from oil revenues, which in turn helps to reduce the exposure of the Saudi economy to the oil markets risks, *i.e.* not transferring oil price fluctuations to the Saudi economy. Therefore, using taxes as one of the fiscal policy tools could potentially be suitable and valuable for the Saudi economy in diversifying income sources and reducing its dependence on oil revenues. Moreover, the importance of reforming the tax system can be seen from the role of fiscal policy in the Saudi economy. The fiscal policy in the state and its tools of government spending, fees, subsidies, and taxes are much more important than monetary policy in directing and managing the economy for several reasons. These reasons were already discussed previously, but the most important one is related to the size and role of government spending represented in the state budget in influencing the movement of the economy. Before implementing VAT in 2018, the fiscal policy was fundamentally limited on government spending, which was significantly supported by oil revenues and some other fees, but the tax instrument did not have much significance in the Saudi financial policy system. As a result, because of the important role played by the government sector in the economy, the tax reform would not only be appropriate for the Saudi economy, but it would also be necessary to deal with the fluctuations in oil revenues.

Answering the first question leads us to discuss the second question on the advantages and disadvantages of taxes for the Saudi economy. In general, introducing taxes in countries that depend on government spending for boosting their economy (such as Saudi Arabia) are likely to have a significant positive impact on the economic growth of those countries. These countries are mostly developing or emerging countries, where government spending has a prominent role in leading the local economy. However, the situation may differ in developed countries, where the private sector may consider to be one of the most critical drivers of economic growth. Thus, taxes in these countries may discourage the private sector and therefore the economic growth. The best example is what happened recently (*i.e.* in 2020) in Saudi Arabia and the UK to change the VAT rates. With the COVID-19 pandemic, all the world's economies have undoubtedly been negatively affected, which made these countries seek earnestly to offer various incentives to support their economy from collapse. For example, Saudi Arabia decided to raise the VAT rate from 5% to 15%, while the UK reduced the tax rate of VAT from 20% to 5%. Both countries seek to support its economy by alleviating the negative effects of the COVID-19 pandemic through fiscal policy tool. However, a simple question, that may arise, is who among them does the right thing to stimulate its economy. Although the accurate answer to such a question requires a detailed study, they all have their own expectations regarding stimulating their economy.

According to our model in chapter five, the Saudi economy could benefit from raising taxes. The main reason is that with these taxes, government revenues and then government spending would increase. This, in turn, would confirm that the government's support for the private sector would continue despite the existence of this pandemic. Thus, it can be said that the introduction or increase of taxes in an economy such as the Saudi economy could cause potentially stimulation for economic growth or even limits the growth shrinking in the worst cases such as this pandemic situation or the case of low oil revenues. On the other hand, taxes in countries that depend on the private sector to lead the economy, such as the UK, reducing taxes would be appropriate to stimulate economic growth. To see that, we understand from the theory that the tax cut would have two main effects on the demand and supply sides. For the demand side, the tax cut would help to increase spending on consumption and then increase aggregate demand. On the supply side, the tax cut would enhance people to work more, which in turn would be reflected in the increase in productivity. Therefore, reducing the tax rate in developed countries could boost their private sectors and then economic growth.

From the above example, we understand that the recent high tax rate by decision-makers in Saudi Arabia gives evidence that our use of the Barro model (1990) may be suitable for the Saudi economy, even if it is simplified. The reason returns to our findings in chapter five, where consumption tax would stimulate the economy in the short and long terms. Specifically, if consumption tax sets to be low, then it would enhance the economy in the short run. On the other hand, if it is high, then economic growth would be boosted. Thus, reforming the tax system may be the most appropriate rather than investing in SWFs, as we argued previously, to offset potential declines in oil revenues and support economic growth (through government spending). As a result, the consumption tax would be unlikely to increase the hardships on the Saudi economy, given that the situation in Saudi Arabia is somewhat different from others.

Despite taxes, whether personal income or consumption tax, generating additional revenues for the state treasury, there are many differences among them, and therefore each of them has advantages and disadvantages. One of the main differences between them is the type of money imposed on tax. Personal income tax is imposed on earning money, while consumption tax is imposed on spending money. Therefore, the consumption tax generally encourages savings and reduces consumption, unlike the personal income tax. Thus, tax evasion or fraud may be less than the personal income tax. However, consumption tax is considered less fair than the personal income tax because it negatively affects poor people, who generally spend a more significant part of their income compared to rich people.

Taxes generally play an essential role in all economies in affecting production, consumption, savings, prices, and achieving economic stability. Given the state of the Saudi economy, although the Saudi economy may have benefited from taxes, the consumption tax, unlike the personal income tax, may have many advantages that make it fit more the Saudi economy for several reasons. In terms of production, the presence of revenues other than oil may help the Saudi economy improve natural resources use in sustainable ways. In other words, with additional income sources such as taxes, excessive dependence on natural resources could be reduced and hence achieved the optimal use of these resources, which makes future generations benefit from it as well. Even though both taxes, personal income tax or consumption tax, could achieve this goal, consumption tax could be less distorted than personal income tax on other economic activities. For instance, a consumption tax would not harm foreign direct investment (FDI) and exports. In terms of consumption and savings, some oil-rich countries, including Saudi Arabia, are characterised by high average income per capita, leading to higher consumption of luxury goods. Therefore, consumption tax, more than personal income tax, may help Saudi citizens to spend less money on 'unnecessary goods' and thus change consumers behaviour or re-evaluate their spending habits. The consumption tax also encourages the consumers to save money and reduce financial waste, which ends up enhancing the ability of society in the future to invest more and then improve income levels and offset the effects of the tax.

Regarding the general level of prices and economic stability, taxes generally aim to achieve economic stability by addressing the state of recession or inflation. For inflation, taxes absorb excessive purchasing power by reducing total demand. This can be accomplished by raising current taxes or introducing new taxes. In the event of a recession, taxes are generally reduced to raise total demand to increase and create purchasing power. Also, taxes may achieve financial sustainability and adopt sound economic policies targeting non-oil GDP growth. Thus, they help improve the quality of public services offered to citizens and the private sector through continued government spending and addressing the negative effects of low global oil prices. In short, consumption taxes would have less distortion and more advantages for the Saudi economy than personal income taxes.

We need now to discuss what type of taxes would be more suitable for the Saudi economy by linking that with our findings in chapters five and six. Intuitively, personal income tax would raise the growth rate temporarily because it would provide additional revenues to the government sector. However, our model showed that the growth rate for certain parameter values would fall[119]. This result could be, in fact, the complete opposite of the intention of the policy. Thus, let us compare our main results in chapters five and six. Chapter five concluded that the growth rate would go up when we raise the consumption tax. In chapter six, we found that the growth rate would depend on the parameter values chosen, but there is a situation in which if we raise the personal income tax, the growth rate would temporarily go down. The latter result cannot happen in chapter five; thus, it could be a warning to a policymaker that personal income tax could reduce the growth rate. As a result, we believe that the consumption tax would be more suited for achieving economic stability in Saudi Arabia.

6.10 Conclusion

The primary aim of this chapter is to study the possibility of introducing a new type of tax, namely, personal income tax, in the Saudi economy. This chapter is based on a theoretical framework similar to previous chapters, where the Barro model (1990) is also extended. The main difference from the previous chapter, *i.e.* chapter five, is that the tax whose effects are studied is now personal income tax, rather than consumption tax, alongside the oil revenues. Thus, there are only two sources of government revenues to finance government spending. We then investigated these two sources separately in two cases. The first case is when oil revenues can only finance government spending, and the growth rate of this case is exogenous. This case was already discussed in more details in chapter four. The second case is when personal income tax can only feed the government revenues and then government spending, where the growth rate of this case is endogenous. The growth rate formula, in the latter case, was found in (6.22), which is a similar result as the Barro model (1990), but it showed that we could have $\gamma_{Y_t} > 1$ if we set $A > 1$.

[119] In spite of the fact that the parameter values used would not be very realistic (*i.e.* not match the exact real world), they are still theoretically possible.

We then checked if both types of steady-state, the exogenous and the endogenous growth steady-state, exist for the same set of parameter values. The results showed that there is a contradiction between two conditions, which are similar but not identical to chapter five. These two conditions are: *(i)* the ratio Y_t/π_t can not be negative for *exogenous growth steady-state* to occur, and *(ii)* we need the condition $\gamma_{Y_t} > g^2$ for an *endogenous growth steady-state* to exist. Thus, we concluded that both types of steady-state growth rate could not exist for the same set of parameter values .

From this conclusion, we understood that the steady-state would be either type *(I)* steady-state, *i.e. exogenous growth steady-state*, or type *(II)* steady-state, *i.e. endogenous growth steady-state*. Thus, like chapter five, we examined two situations that are changing g^2 and keeping τ^Y constant, and the opposite. The first results showed that if g^2 is sufficiently high (low) for a given value of τ^Y, then the steady-state will be an *exogenous growth steady-state* (*endogenous growth steady-state*). The second results showed that keeping g^2 fixed at a particular value and varying τ^Y would have a different result than chapter five. The difference came from using a different type of taxes. More precisely, changing personal income tax, unlike consumption tax, would positively and negatively impact the growth rate, as shown in equation (6.22). Thus, keeping g^2 constant and changing τ^Y clarified two positions for the steady-state growth rate. The first position is that when τ^Y sets to be either sufficiently low or sufficiently high, then the steady-state growth rate is exogenous. The second position is that if τ^Y sets to be at a certain value, that is neither sufficiently low nor sufficiently high, the steady-state growth rate is endogenous. As a result, the main conclusion drawn in this chapter is that we have two critical values for personal income tax. The first is when the economy moves from type *(I)* steady-state to type *(II)* steady-state, *i.e.* from *exogenous growth steady-state* to *endogenous growth steady-state*, while the second is the opposite. These results indeed illustrated in Diagram 19.

This chapter also studied how the personal income tax would affect the main level variables, *i.e.*, government spending, consumption, capital stock, and output. The findings showed that if personal income tax is sufficiently low (high), then the level of government spending would increase (decrease). This result is consistent with the 'Laffer curve' theory, where the government revenues in our model are assumed to be equal to government spending. However, the effect on government spending caused a lot of theoretical ambiguity in the effects on other level variables. For this reason, we resorted to a numerical solution, which would be an appropriate method in this case. Based on the particular parameter values chosen, the numerical solution results showed a negative relationship between the personal income tax and both the level of consumption and capital stock. These findings mean that raising the tax rate implies a fall in both the level of consumption and capital stock. Regarding the level of output, the results indicated that if personal income tax is sufficiently low (high), then the level of output would increase (decrease).

In the analysis of local stability of type *(I)* and type *(II)* steady-states and the transitional dynamics, we understood from type *(II)* steady-state that there is no transitional dynamics because the growth rate of consumption PGSU, capital PGSU, and output PGSU all equal the same rate all the time, as illustrated by Barro (1990), unlike type *(I)* steady-state. Although there is transitional dynamics in type *(I)* steady-state, it is different from chapter five in terms of predetermined and non-predetermined variables. In chapter five, we confirmed that both $\hat{k}_t$ and $\hat{c}_t$ are non-predetermined variables, while in the current chapter, $\hat{k}_t$ is a predetermined variable and $\hat{c}_t$ is a non-predetermined variable. Thus, we applied the stability properties for this steady-state type, which tells us that one stable eigenvalue is required for the Blanchard and Kahn (1980) test to be satisfied. Then, we concluded that steady-state of type *(I)*, $\left(\hat{k}^*, \hat{c}^*\right)$, is a saddle-point.

In the transitional dynamics of this type, $\hat{k}_t$ turned to be a non-predetermined variable in the event of the shock. The reason behind this switching is that changing personal income tax would change the government budget constraint and then alter $\hat{k}_t$. Thus, both $\hat{k}_t$ and $\hat{c}_t$ can now jump at the moment of impact. However, in spite of the fact that $\hat{k}_t$ is now a non-predetermined variable as $\hat{c}_t$, they are still different from chapter five. Briefly, the difference between them in the two chapters is in $\hat{k}_t$ jumping, which is not immediate to observe. The transformation of $\hat{k}_t$ in this chapter made us see not only a jump in the consumption PGSU but also in the capital PGSU and the output PGSU in the impact period. However, when we attempted to study the transitional dynamics in a phase diagram, it was worthwhile to explore how $\hat{k}_t$ would jump on impact period. The reason is that it would help us to understand the transition path of $\hat{k}_t$ on impact period. After all, we found that $\hat{k}_t$ would jump down at the moment of impact, but the jump was found to be either right or left of the new steady-state, depending on the parameter values chosen. We then showed that both situations are theoretically possible to occur. Specifically, the first situation, which happens when g^2 is high, showed that the growth rate is temporarily declined and then returned asymptotically to its initial steady-state. The reason behind a temporarily fall in growth rate is due to that $\hat{k}_t$ is falling over time. The second situation, which happens when g^2 is low, displayed that the growth rate is temporarily increased and then went back to its old steady-state. The reason is that $\hat{k}_t$ is raising over time.

We then presented two numerical simulations by using Dynare software. The first simulation looked at the transitional dynamics near a type (*I*) steady-state. The time paths showed that there are consistent with our previous analysis in section 6.7.2. More precisely, the implementation of personal income tax would provide extra public revenues and thus boost government spending. However, depending on the parameter values, the growth rate of government spending would go down or up in the short run and medium run and then tends back to its steady-state asymptotically. The second simulation also looked at steady-state, but it displayed when the economy could move from one type of steady-state to another one when we keep g^2 constant and vary the tax rate. According to our parameter values, the simple numerical exercise showed that there are two critical values for the tax rate, τ^Y = 57.8% and τ^Y = 75.4%, which represent the possibility of a regime switch. If the tax rate sets to be τ^Y < 57.8% or τ^Y > 75.4%, then the economy would have an exogenous growth steady-state. If, on the other hand, tax rate sets to be 57.8% < τ^Y < 75.4%, then the growth steady-state would be endogenous.

Lastly, we briefly discussed why taxes matter in the Saudi economy. This question includes some sub-questions that all together attempt to answer one of the main research questions in this thesis. Thus, we compared in this section between consumption tax and personal income tax, and what among them would be appropriate for the Saudi economy. We concluded that consumption tax seems to be more suitable than the personal income tax for several reasons. The consumption tax would be a less distortion tax compared to personal income tax. It would also play a vital role in the Saudi economy by influencing production, consumption, savings, prices, and achieving economic stability more than personal income tax. Moreover, consumption tax was found in our model to simulate the economy in the short-run and the growth rate, unlike the personal income tax, which would discourage the growth rate temporarily under certain parameter values. Now, we move to the last chapter, which is regarding conclusions, policy recommendations, limitations and future research.

7 Chapter Seven: Conclusions, Policy Recommendations, Limitations and Future Research

7.1 Conclusion

In conclusion, we summarise our contributions, models, and main results. This chapter also provides policy recommendations and highlights limitations and future research. Broadly, this thesis attempts to fit and describe the Saudi economy with a practical focus on Saudi economic growth. The thesis is divided into seven chapters: (i) introduction, (ii) analysis of the Saudi economy, (iii) literature review, (iv) modelling Saudi economic growth, (v) modelling Saudi economic growth with the International Monetary Fund (IMF) proposal (introducing consumption tax), (vi) modelling Saudi economic growth with personal income tax, and finally (vii) the conclusion. The significance of this research is related to the two main challenges facing the Saudi macroeconomy, namely, continuing to the significant reliance on oil to finance economic activities and the suffering from a lack of taxation to generate revenues away from oil. The primary objective of this thesis involves studying how economic growth would be affected if there is a negative shock in the oil demand. This thesis also aimed to focus on the fiscal policy reforms to investigate the effectiveness of implementing the proposals of the IMF on Saudi economic growth for countering oil revenue fluctuations. We sought to examine the possibility of introducing consumption tax and personal income tax and discussed the effectiveness of these taxes along with oil revenues in economic growth. Besides, this study pursued investigating the appropriate amount of consumption tax that could compensate for potential reduction in oil revenues. Finally, it discusses the suitability of the consumption tax and personal income tax for the Saudi economy.

This thesis aims to contribute to four main aspects of the field. The first contribution is to extend the Barro (1990) model by adding the monopolistic oil sector. The key feature of our model is that the oil sector is treated as an exogenous and monopolistic sector, where the growth is basically led by a growing demand for oil. The importance of the oil sector can be seen in its role in financing the productive government sector. The reason for modelling the oil sector in this way is that the economic growth of resource-rich countries in most research is considered to be affected, both positively and negatively, by the oil industries. However, the economic growth literature ignored to analyse the market power of oil. In other words, the oil sector is not given enough and detailed attention to economic growth literature. Therefore, modelling the oil sector as a monopolistic sector could provide a better understanding of economic growth in a country like Saudi Arabia.

The second contribution is that our models are different from the original Barro (1990) model and Bambi and Venditti's (2018) paper on the government budget constraint. More precisely, the government budget constraint in each model in our study has been modified to include oil revenues and other taxes, meaning that there are different sources of government revenues. These sources are (i) only oil revenues, (ii) oil revenues with consumption tax, and (iii) oil revenues with personal income tax. Consequently, the contribution of this study can be summarised by adding the oil, *i.e.* monopolistic sector, revenues with different taxes to finance productive government spending in Barro (1990) model.

The third contribution is regarding the other possible methods or strategies to deal with oil fluctuations and its impact on economic growth. A few studies, such as Devarajan et al. (2017), Primu (2016), Richmond et al. (2015), and Berg et al. (2013), have explored the best methods and strategies to deal with volatility in resource revenues. More precisely, they have focused on offsetting volatility and shocks through Sovereign Wealth Funds (SWFs). However, they did not consider the fact that tax reform could be one of the fiscal instruments to help oil-rich countries maintain stable economic growth in the event of negative shocks in oil demand. We argue that tax reform would be a suitable strategy, especially in countries that suffer from a lack of taxes, as in the case of Saudi Arabia. The main reason for our argument is that there are many countries, which lack natural resources, rely on only taxes as revenues to finance their public spending. Consequently, the priority for oil-rich counties is to be able to reform their tax system, even if they have abundant resources, to maintain stable economic growth. Therefore, reforming the current fiscal policy could be a solution to reduce the negative effects of fluctuations in natural resource revenues. It could also be a better strategy than using SWFs, which can be affected by several external factors, as mentioned before. These external factors may not be under the control of the countries that own them. As a result, our thesis would contribute to investigating the introduction of consumption tax and personal income tax in Saudi Arabia. It would also seek to find out the amount of consumption tax to compensate for a reduction in the level of government spending associated with the decline in oil revenues.

The fourth contribution is based on the absence of a study that examines taxation, *i.e.* consumption and personal income taxes, and their effects on the economic growth of Saudi Arabia. Moreover, this study is also different from previous models developed for resource-rich countries that have focused on low-income countries. The current model attempts to fit the conditions of Saudi Arabia, as the world's second-largest oil economy, which also has much scope to implement new taxes, as mentioned by the IMF.

This thesis aims to provide a theoretical framework based on Barro's (1990) neoclassical growth model to model Saudi economic growth. There are a number of studies, such as Alrasheedy and Alrazyeg (2019), Alshammari and Aldkhail (2019), Al-Obaid (2004), and Barri (2001), that confirmed the significance of government spending to stimulate economic growth in Saudi Arabia. From this point, we believe that the Barro (1990) model is one of the most appropriate models to represent countries that depend on government spending to finance economic activities. Thus, the seminal contribution of Barro (1990) in economic growth literature can be extended to fit our case study. For this reason, we extended his original model by modifying the government budget constraints in three models of chapter four, five, and six. Each chapter has a different budget constraint, where all sources of revenues in the three main chapters finance productive government spending. In all models, the government is assumed to spend all its revenues on purchasing imported goods; therefore, the government revenues in all our models are equal to government spending. For this reason, government spending is not entered in the resource constraint of the economy because the home-goods are assumed to be not used in government spending. In addition, for the growth rate purposes, we converted the main level variables to per government spending unit (PGSU) variables by dividing them by government spending; thus, we obtained the capital PGSU, consumption PGSU, and output PGSU.

Chapter two presented an overview of the Saudi economy. It also analysed the structure and characteristics of the Saudi economy and focused on the role of the oil sector and its relation to economic growth. After studying the Saudi economy, we found that it faces two significant challenges: its primary dependence on oil revenues and the need to diversify its income sources. The main reason is that oil revenues are the primary source of public spending. Therefore, relying on oil revenues and then on government spending as a significant driver of economic activity has made the Saudi economy insecure and more sensitive to fluctuations in the global oil markets. As proposed by the IMF, the reform of the current tax system would provide additional revenues and help reduce the continued volatility in oil revenues. On the other hand, chapter three is the literature review, where we examine six main themes. They include the economic growth models, the structure of the oil market: Saudi's oil sector, economic growth in oil-rich countries, managing oil revenue fluctuations in oil-rich countries, taxes and their roles in economic growth, and finally fiscal policy and economic growth in Saudi Arabia. The main reason behind searching in different literature is to give us a better perception of the most related works to ours.

Chapter four extended the Barro (1990) model to describe the Saudi economy before implementing the proposed fiscal policy by the IMF. We modelled the Saudi economy, where government spending is financed by only one government revenue source, namely oil revenues. Although we used Barro endogenous growth model and his type of production function, our model showed different conclusions from Barro's finding. More precisely, we found that the growth rate of government spending is the growth rate of oil profits, g^2. That means everything in the economy of this chapter grows at the rate of government spending, which is the exogenous growth rate, unlike Barro's conclusion.

Besides, in this chapter, we studied how a shock, a permanent reduction in the growth rate of government spending, would affect the steady-state position. The findings indicated that if the growth rate of government spending declines, both capital PGSU and consumption PGSU would raise at the steady-state. Moreover, we obtained an interesting result that although the level of consumption would grow at a slower rate due to the reduction in the government spending growth rate, the consumption PGSU would be higher. Higher consumption PGSU suggests that there would be partially offsetting of the effect of the reduction in the government spending growth rate. Thus, our model showed that there is some partial offsetting shift in the level of consumption, which may not be obvious. Away from the growth rate, the results also displayed that the level of government spending would significantly impact other level variables in the economy. More precisely, if the level of government spending falls, then the level of output, capital stock, and consumption would also fall. The reason is that government spending is the main engine in our model, which in fact reflects the situation of the Saudi economy. For this reason, we understood that the Barro (1990) model could be an appropriate theoretical model for the Saudi economy.

Chapter five focuses on fiscal policy reforms and follows the IMF's recommendations by introducing a consumption tax, τ^c. We extended the model of chapter four and aimed to investigate how introducing these taxes affect economic growth and how effective these taxes are in maintaining stable economic growth if there is a negative shock to oil demand. The government budget constraint in this chapter has two sources of revenues: oil revenues and consumption tax revenues. As long as it is unlikely to have a steady-state in which the two sources of revenues grow at a different rate, thinking of an equilibrium in which one of these shares tends to zero, and the other tends to one could be a valid equilibrium to consider in this chapter. For this reason, we studied three possible types of steady-states. We found that two of them involve that the growth rate of government spending tends to an exogenous growth rate (type (*I*) steady-state), while the third possible involves that the growth rate tends to an endogenous growth rate (type (*II*) steady-state).

Furthermore, our proof showed an unavoidable contradiction between two conditions related to type (*I*), *i.e.* the exogenous steady-state, and type (*II*), *i.e.* the endogenous steady-state, of the growth model. Thus, we concluded that both types of steady-state growth rate could not exist for the same set of parameter values. In other words, for one set of parameter values, either the steady-state will be the exogenous growth steady-state and will not exist any other type, or for a different set of parameter values, instead, the only type of steady-state will be the endogenous growth steady-state. In this regard, two main results have been found in how and when the economy could move from a steady-state to another. The first finding was that if we set consumption tax, τ^c, constant and change g^2. The results showed that if g^2 is sufficiently high (low) for a given value of τ^c, then the steady-state will be an exogenous (endogenous) growth steady-state. The second finding was that if we set g^2 constant and vary τ^c. The results indicated that at a certain value of τ^c, the economy would switch from the exogenous growth steady-state to the endogenous growth steady-state. The switch to the endogenous growth steady-state would raise the rate of growth, as high taxes would help provide enough government spending to improve the production function of the firms.

To explain the latter result further, as we gradually increase τ^c, eventually the economy will pass from an exogenous growth steady-state to an endogenous growth steady-state. Thus, there is a critical value, which can be interpreted as the critical value of τ^c at which this switch occurs as being the 'take-off' point for the economy. In other words, as τ^c is increased, beyond this point, the steady-state growth rate of the economy is eventually 'freed' from the growth rate of oil profits; whereas, until it is reached, the steady-state growth rate is tied to the growth rate of oil profits. Consequently, the result of switching from one type of steady-state to another one is indeed unusual in most growth models.

Continuing in the steady-state analysis, we investigated in this chapter how the key level variables respond to a change in consumption tax. Our results showed that if we increase consumption tax, then the level of government spending and the capital stock would increase. The intuition behind increasing the level of government spending is simply due to the increase in government revenues with a consumption tax. While increasing the capital stock level is because firms receive positive externalities from the government sector, which would promote their production. For the level of consumption, increasing consumption tax would generally cause a decrease in consumption level because people would save more when consumption tax is high since their purchasing power decreases. However, we found that although consumption tax is increased, the level of consumption in our model may increase or decrease, depending on the value of the discount factor. In this regard, we theoretically discovered that if the discount factor is sufficiently close to one (close to zero), then increasing consumption tax would increase (decrease) the level of consumption. The intuition behind increasing the level of consumption could be due to that high consumption tax provides additional revenues to the government, which in turns allows the firms to receive more positive externalities from the government. Thus, firms may somehow pass some of these positive externalities to the households, *e.g.* through high wages or high rental rate of capital. This would end up being the cause of high consumption.

Regarding the transitional dynamics, we studied first a pure consumption tax case, *i.e.* type (*II*) steady-state. In this case, there was only one eigenvalue, which was then found unstable. We also concluded that there was no transitional dynamics in this case, which is a similar result as Bambi and Venditti (2018). We then studied the effect of consumption tax on the transition path of variables in the neighbourhood of type (*I*) steady-state. Our analysis showed that there is one stable eigenvalue and one unstable eigenvalue. Then, we found that the steady-state is a saddle point if g^2 is sufficiently high. In the transitional dynamics, we illustrated that our phase diagram is somewhat different from the one in the Ramsey model. The main difference lies in the jump locus. More precisely, the economy in the Ramsey model would jump initially on a point that is vertically above or below the old steady-state. The reason is that the capital PGSU is a predetermined variable, while consumption PGSU is a non-predetermined variable. However, our economy at the moment of the shock would jump initially on a new locus. This new locus originated from the relationship between capital PGSU and consumption PGSU, which are both non-predetermined variables. We then studied this relationship carefully and found that this locus is a diagonal line with a downward slope. Our result also showed that the growth rate would unambiguously be temporarily increased by the increase in τ^c.

Finally, we provided in this chapter a numerical exercise to find out the amount of different consumption tax that can compensate for any reduction in oil revenues. We derived a formula for consumption tax, equation (5.46), that allowed us to calculate the required tax rate that compensates for any reduction in oil revenues and keep, at the same time, the government spending unchanged. The reason behind holding the government spending fixed is to avoid at least having low growth, *i.e.* $\gamma_G = g^2 = 1$. Our simple example of this exercise showed that if the target government spending is at 2, which can maintain the economic growth rate unchanged and the oil revenues declined by 10%, the required tax rate to compensate for this reduction is 6.21%. As a result, based on our derived formula and our exercise, this tax rate can ensure that it compensates for not only the 10% reduction in oil revenues but also the government spending, and the economic growth rate would remain unchanged.

Chapter six is also related to the previous chapters, where it studied another type of tax, personal income tax, τ^{Y}. This chapter extended the Barro (1990) model by allowing for two sources of government revenues to finance productive government spending. These two sources are oil revenues and personal income tax revenues. Although Saudi Arabia does not have a personal income tax currently, four motivations encouraged us to study this type of tax in this research. The first is that the country seeks to generate revenues away from oil through diversifying sources of income. The second is based on the IMF recommendation, which hinted several times that Saudi Arabia has plenty of space to implement new taxes. The third is because no prior study has been conducted exploring the implementation and effect of personal income tax on the Saudi economy. The final motivation is to check on the IMF recommendations of introducing new taxes by investigating which among consumption tax or personal income tax would be more suited for the Saudi economy.

As long as we have two sources of government revenues, our analysis found a similar result to the previous chapter, where the two sources cannot grow at a different rate. Thus, studying these two sources separately showed that we could have an exogenous or endogenous growth model. Moreover, the results showed that both types of steady-state, *i.e.* the exogenous and endogenous growth steady-state, cannot exist for the same set of parameter values. The reason is that there is a contradiction between two conditions for each type of growth models. For this reason, we realised that our model would present two types of steady-state, as in chapter five. Type (*I*) steady-state is the exogenous growth steady-state, where the growth rate of government spending is equal to the oil profits growth rate, $\gamma_{G_t} = g^2$. Correspondingly, type (*II*) steady-state is the endogenous growth steady-state, where the growth rate of government spending is equal to the growth rate of output, $\gamma_{G_t} = \gamma_{Y_t}$. Thus, to examine under which conditions our economy would be in, we investigated two situations. The first situation was to change g^2 and fix τ^Y, whereas the second situation was the opposite. The result of changing g^2 and fixed τ^Y showed that if g^2 is sufficiently high (low) for a given value of τ^Y, then the steady-state will be an exogenous (endogenous) growth steady-state. The result of the second situation, *i.e.* keeping g^2 constant and varying τ^Y, displayed two different positions for the steady-state growth rate. Specifically, if τ^Y is either (neither) sufficiently low or (nor) sufficiently high, then the steady-state will be an exogenous (endogenous) growth steady-state. These two positions give us a clear picture of having two critical values for τ^Y in our model. As increasing τ^Y, the first critical value is when the economy moves from exogenous growth steady-state to endogenous growth steady-state, while the second critical value is the opposite. The diagram in section 6.4.4 illustrated how and when the economy could move from one type of steady-state to another when personal income tax changes.

Like previous chapters, we also studied in this chapter the impact of personal income tax on all level variables, *i.e.* the level of government spending, consumption, capital stock, and output. We found that if personal income tax is sufficiently low (high), then the level of government spending would increase (decrease) in response to an increase in the tax rate. This result was similar to the 'Laffer curve' result, but it caused a lot of theoretical ambiguity on the other level variables. Thus, to overcome this ambiguity, we used a numerical solution as a suitable method in this case. The results then showed that rising personal income tax would decline both the level of consumption and capital stock. However, the level of output would change depending on the tax rate, *i.e.* if the tax rate is sufficiently low (high), then the output level would increase (decrease).

Regarding the analysis of local stability, we found in type (*II*) steady-state a similar result as Barro (1990), where there is no transitional dynamics. The reason is due to the fact that the growth rate of all PGSU variables are equal to the same rate all the time. However, our analysis of the stability properties in type (*I*) steady-state showed that there is one stable eigenvalue for the Blanchard and Kahn (1980) test to be satisfied. We then found that the steady-state of this type is a saddle point. Even though there was transitional dynamics in type (*I*) steady-state, studying the transition paths of the economy revealed that there is a switch in the capital PGSU variable. More precisely, the consumption PGSU was a non-predetermined variable, while the capital PGSU was a predetermined variable before the shock occurred, *i.e.* when we studied the local stability of this type of steady-state. However, the capital PGSU, at the moment of the shock, turned to be a non-predetermined variable, as the consumption PGSU. The switching of the capital PGSU was due to the fact of changing the tax rate, which also caused government spending to change. Thus, this transformation allowed all PGSU variables to jump initially at the moment of the impact period. Although, in the beginning, we could not draw a phase diagram for very general values of τ^Y, we attempted to draw it for a value of τ^Y that is close to zero. This indeed helped us to understand better how the two stationary loci for capital PGSU and consumption PGSU would take place before and after the shock. Also, it told us that the saddle path toward the new steady-state would be upward-sloping, if τ^Y is close to zero.

Moreover, we found that $\hat{k}_t$ would jump down at the moment of impact, but the jump locus was found to be depending on the parameter values chosen. Thus, we showed two situations that are theoretically possible to happen. In the first situation, we set g^2 to be high, while g^2 to be low in the second situation. The result of the first situation indicated that the growth rate is temporarily declined and then returned asymptotically to its initial steady-state. The reason behind a temporarily fall in growth rate is due to that $\hat{k}_t$ and (hence $\hat{y}_t$) is falling over time. The result of the second situation displayed that the growth rate is temporarily increased and then went back to its old steady-state. The reason is that $\hat{k}_t$ and (hence $\hat{y}_t$) is raising over time.

We subsequently provided two numerical simulations by using Dynare software. The two numerical simulations studied how changing the personal income tax would affect the transitional dynamics and steady-states. We found an interesting feature of this chapter regarding the impact of rising the personal income tax on the growth rate. As we know, raising the tax rate would intuitively generate additional government revenues, but we found that, depending on parameter values chosen, a rise in the tax rate could cause a temporary lower growth rate. The reason behind this decline is that the capital PGSU is falling over time. In fact, this result is the opposite of the fifth chapter result, *i.e.* in the case of consumption tax, where an increase in consumption tax would lead to a temporary increase in the growth rate.

Finally, this chapter also discussed what among consumption tax and personal income tax would be suitable for the Saudi economy. It then concluded that consumption tax could be more effective, less distortion, and growth stimulator, compared to personal income tax, for the Saudi economy. This conclusion was based on some of the positive effects of the consumption tax on production, consumption, and economic stability, which seem to dominate the effect of personal income tax.

7.2 Policy Recommendations

Although this thesis is based on a simplified theoretical approach, we still believe that it would provide a great deal of visualisation and expectations of economic growth paths to policymakers. It may also potentially help in providing some necessary and appropriate information about the fiscal policy reforms and the role of different taxes in the Saudi economy. The study results showed that any effect of oil revenues would affect the economy in the long term. In other words, if the primary determinant of economic growth in the country is the government spending, which is supported by solely oil revenues, it undoubtedly would create many economic problems due to the instability of oil revenues. With continued oil revenue fluctuations, the government has to search and move quickly towards diversifying the sources of economic income, which the IMF has recommended. Thus, there is an urgent need to increase non-oil revenues because the level of spending will continue to be associated with fluctuations in oil revenues, which will inevitably affect economic growth. Diversifying sources of economic income, such as reforming the current tax system, may help to alleviate these fluctuations. In other words, additional revenues may be used to offset a likely continuous decline in oil revenues and reduce excessive dependence on oil revenues to finance other economic activities. As a result, tax revenues may move the oil revenues from a primary to a secondary source in the economy, which would relieve the volatility of the economy.

Even though taxes would create additional revenues to the government sector and would at the same time cause some economic distortions, consumption tax, as found in our model, would be the best choice since it would have a positive impact on economic growth. As shown in chapter five, if we only consider consumption tax as a revenue source, it would enhance economic growth. However, if consumption tax and oil revenues are together considered, economic growth will be determined based on the consumption tax rate. Specifically, as our result showed, low (high) consumption tax implies that the economic growth would be exogenous (endogenous), where the endogenous growth steady-state is higher than the exogenous one. Thus, as a policy recommendation, a lower consumption tax may not be sufficient to boost the growth rate. However, a high tax rate would significantly change and push the economy to be determined by the endogenous growth steady-state, which ensures a higher growth rate, according to our model.

Regarding personal income tax, although it would also provide extra revenues to the state's budget, it is still a distortion tax in some other economic activities. The economic impact of introducing personal income tax may have a different effect than consumption taxes. Our model showed that if personal income tax and oil revenues are the only sources of government revenues, then personal income tax would stimulate economic growth in quite limited cases. More precisely, if the tax rate is set to be between a certain range, *i.e.* neither sufficiently low nor sufficiently high, the economic growth would be endogenous. If otherwise, *i.e.* the tax rate sets to be sufficiently low or sufficiently high, then the economic growth would be in this case exogenous. In contrasts to consumption tax, based on particular parameter values chosen, a rise in the personal income tax could result in a temporary lower growth rate due to the decline in the capital PGSU over time. Therefore, the personal income tax may be undesirable compared to consumption tax because the latter could be sufficient to address the negative side of oil dependence and avoid the negative effects of the personal income tax. However, suppose that the application of personal income tax becomes necessary to finance government spending. In that case, it should be applied with caution because their rate, based on our model, may switch the rate of economic growth from one position to another. Also, it would cause a temporary reduction in the growth rate under certain parameter values. As a result, our findings in chapters five and six confirm that consumption tax would be a good choice for the Saudi economy, compared to personal income tax.

7.3 Limitations and Future Research

We have learned from this thesis to build a base of knowledge of the fiscal policy reforms of the Saudi economy by extending an existing theoretical work. As per our knowledge, this kind of analysis is being done for the first time in Saudi Arabia. However, the models in this thesis have been greatly simplified and have governed some limitations. The theoretical framework used in this study is assumed to reflect the Saudi economy; thus, our results in this thesis are considered to be limited. However, it can be generalised to only limited countries. For instance, the characteristics and the structure of the economics of some GCC countries are like the Saudi economy[120]. Consequently, our study and our results can be generalised and applied to these countries.

Even though the theoretical approach used in this thesis aimed to provide a theoretical knowledge base on economic growth in Saudi Arabia, the lack of available data regarding taxes limits the scope of our analysis. Thus, there are two priority research agendas for the future. The first is to use an empirical approach to attempt to provide evidence to justify our results in this thesis and provide broad 'stylised facts' about our findings. Developing an empirical model to verify our model and predictions of our theoretical results would be our priority to convert these predictions to precise numerical outcomes. The second research agenda is to work on our main research results in chapter five and six regarding the possibility of switching regimes as we change the policy parameters. They are important results; thus, we intend to develop and confirm them further as a research agenda for the future. The reason for examining these results is that they are unusual in most growth models; thus, we believe that it is worth exploring them more.

If we were able to redesign our models in this thesis, we would make a number of modifications. Most importantly, we would extend the model in each chapter by allowing for each sector to be more realistic. Starting with the oil sector, it was assumed in our model to be derived by the global GDP growth. Although this is the most influential factor based on historical oil statistics, some other factors may affect oil demand as well. A number of examples of factors affecting oil demand were mentioned in the fourth chapter. Thus, considering and modelling some of these factors would add many features to a model in economic growth literature, especially for modelling the economic growth of oil-rich countries like Saudi Arabia. For the government sector, we make it simple by avoiding lending and borrowing, transfer to households, and other revenues. Therefore, allowing to access the international financial market and adding households' transfer

[120] Some GCC countries, such as Kuwait, United Arab Emirates, and Qatar, are rich in natural resources. They rely heavily on natural resources to finance their expenditures and have also limited tax regimes.

and other non-oil revenues could make the model closer to reality. Regarding the household's sector, we assumed that the labour supply is inelastic, and that consumption tax would stimulate the economy. Although assuming elastic labour supply would be affected by reducing the real value of net earnings, we believe that the result would not be much affected. The reason is that the government used these tax revenues in our models to enhance the firms' production, which in turn would somehow reflect in the household sector.

As far as taxes are concerned, we studied consumption and personal income taxes separately along with oil revenues in the Barro (1990) model in order to compare the effectiveness of different fiscal policy instruments in the Saudi economy. However, the possibility of combining all three sources to finance productive government spending requires further investigation. With rapid transformations and the orientation of many reforms in the Saudi economy, we believe that personal income tax may be one of the potential options for the decision-maker in the future. Thus, it would be interesting for future work to explore and study all these sources together as sources of government revenues. Finally, although these points could make our theoretical models more complicated, they are still worth to be considered in future works.

A. Appendix to Chapter 4

A.1 The Key Equations in Ramsey Model and Our Model

The below table illustrates the fundamental equations in the two models. In the Ramsey model, we show how decreasing technological progress would affect the whole economy and compare it with our model when the growth rate of oil profit declines. The key equations in both models are summarised as:

	The discrete-time Ramsey model with decreasing technological progress	**Our model with decreasing the growth rate of government spending**
Equilibrium Equations (The Capital Accumulation and the Euler Equations)	$\hat{k}_{t+1} = \frac{1}{\left(1+\gamma_{A_t}\right)} \left[\hat{y}_t + (1-\delta)\,\hat{k}_t - \hat{c}_t \right]$ $where \quad Y_t = \left(A_t L_t\right)^{1-\alpha} K_t^{\alpha}, \quad L_t = 1 \;\Rightarrow\; \hat{y}_t = \hat{k}_t^{\alpha}$ $\hat{y}_t = \frac{Y_t}{A_t}, \quad \hat{k}_t = \frac{K_t}{A_t}, \quad \hat{c}_t = \frac{C_t}{A_t}$ $\frac{\hat{c}_{t+1}}{\hat{c}_t} = \frac{1}{\left(1+\gamma_{A_t}\right)} \left[\beta\left(\alpha \hat{k}_{t+1}^{\alpha-1} + 1 - \delta\right)\right]^{\frac{1}{\sigma}}$	$\hat{k}_{t+1} = \frac{1}{\gamma_{G_t}} \left[\hat{y}_t + (1-\delta)\,\hat{k}_t - \hat{c}_t \right]$ $where \quad Y_t = A K_t^{\alpha} L_t^{1-\alpha} G_t^{1-\alpha}, \quad L_t = 1 \;\Rightarrow\; \hat{y}_t = A\hat{k}_t^{\alpha}$ $\hat{y}_t = \frac{Y_t}{G_t}, \quad \hat{k}_t = \frac{K_t}{G_t}, \quad \hat{c}_t = \frac{C_t}{G_t}$ $\frac{\hat{c}_{t+1}}{\hat{c}_t} = \frac{1}{\gamma_{G_t}} \left[\beta\left(\alpha A \hat{k}_{t+1}^{\alpha-1} + 1 - \delta\right)\right]^{\frac{1}{\sigma}}$
The Growth Rate	$A_{t+1} = \left(1+\gamma_{A_t}\right) A_t \;\Rightarrow\; \frac{A_{t+1}}{A_t} = \left(1+\gamma_{A_t}\right)$	$\gamma_{G_t} \equiv \frac{G_{t+1}}{G_t} = g^2$
Steady-State Equations	$\hat{k}^* = \left[\frac{1}{\alpha}\left(\frac{\left(1+\gamma_A\right)^{\sigma}}{\beta} - 1 + \delta\right)\right]^{\frac{1}{\alpha-1}}$ $\hat{c}^* = \hat{k}^{*\alpha} - (\gamma_A + \delta)\,\hat{k}^*$	$\hat{k}^* = \left[\frac{1}{\alpha A}\left(\frac{\gamma_G^{\sigma}}{\beta} - 1 + \delta\right)\right]^{\frac{1}{\alpha-1}}$ $\hat{c}^* = A\hat{k}^{*\alpha} + (1-\delta-\gamma_G)\,\hat{k}^*$
The Effect on the Steady-State	Since $\left(1+\gamma_A\right) \geq 1$, then $\frac{d\hat{k}^*}{d\gamma_A} < 0$ $\frac{d\hat{c}^*}{d\gamma_A} < 0$	If $\gamma_G \geq 1$, then $\frac{d\hat{k}^*}{d\gamma_G} < 0$ $\frac{d\hat{c}^*}{d\gamma_G} < 0$
Results	The results show that: $\downarrow \gamma_A \;\Rightarrow\; \uparrow \hat{k}^* \; and \; \uparrow \hat{c}^*$	The results show that: $\downarrow \gamma_G \;\Rightarrow\; \uparrow \hat{k}^* \; and \; \uparrow \hat{c}^*$

A.2 The Saddle Path and Its Slope in Our Model

The linearised capital accumulation equation (4.25) and Euler equation (4.26) can be now expressed in matrix form as:

$$\begin{pmatrix} \hat{k}_{t+1} - \hat{k}^* \\ \hat{c}_{t+1} - \hat{c}^* \end{pmatrix} = \underbrace{A}_{2x2} \begin{pmatrix} \hat{k}_t - \hat{k}^* \\ \hat{c}_t - \hat{c}^* \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} \hat{k}_{t-1} - \hat{k}^* \\ \hat{c}_{t-1} - \hat{c}^* \end{pmatrix} \qquad \text{(I)}$$

where the 2x2 matrix of coefficients, $A = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$, has a characteristic equation,

$$\lambda^2 - (m_{11} + m_{22})\,\lambda + (m_{11}m_{22} - m_{12}m_{21}) = 0 ,$$

where the trace is $Tr = m_{11} + m_{22}$, and the determinant is $Det = (m_{11}m_{22} - m_{12}m_{21})$, with eigenvalues,

$$\lambda = \frac{(m_{11} + m_{22}) \pm \sqrt{(m_{11} + m_{22})^2 - 4(m_{11}m_{22} - m_{12}m_{21})}}{2}$$

For the dynamics of the solution, we apply the spectral decomposition of A to express the matrix of the eigenvectors and its inverse,

$$A = Q\Lambda Q^{-1}$$

where Λ is a diagonal matrix with λ_1 and λ_2 on its diagonal, $Q = \begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \end{pmatrix}$ which includes the eigenvectors of A, and its inverse is $Q^{-1} = \begin{pmatrix} u_1 & v_1 \\ u_2 & v_2 \end{pmatrix}$. Thus, the dynamics of the solution can be expressed as,

$$\begin{pmatrix} \hat{k}_t - \hat{k}^* \\ \hat{c}_t - \hat{c}^* \end{pmatrix} = Q\Lambda Q^{-1} \begin{pmatrix} \hat{k}_{t-1} - \hat{k}^* \\ \hat{c}_{t-1} - \hat{c}^* \end{pmatrix}$$

$$= \begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \end{pmatrix} \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} u_1 & v_1 \\ u_2 & v_2 \end{pmatrix} \begin{pmatrix} \hat{k}_{t-1} - \hat{k}^* \\ \hat{c}_{t-1} - \hat{c}^* \end{pmatrix} \qquad \text{(II)}$$

Now, we need to find the parameter values of (II). The eigenvector corresponding to λ_1 eigenvalue is the vector (x_1, x_2), which is satisfied,

$$\begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \lambda_1 \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

that is,

$$m_{11}x_1 + m_{12}x_2 = \lambda_1 x_1$$

$$m_{21}x_1 + m_{22}x_2 = \lambda_1 x_2$$

which lead to,

$$x_2 = \frac{\lambda_1 - m_{11}}{m_{12}} x_1 \qquad (A)$$

$$x_2 = \frac{m_{21}}{\lambda_1 - m_{22}} x_1 \qquad (B)$$

Since λ_1 is an eigenvalue of the characteristic equation, equations (A) and (B) would be the same. Thus, by normalising $x_1 = 1$, we obtain the eigenvector,

$$x_1 = \begin{pmatrix} x_1 \\ \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ \dfrac{\lambda_1 - m_{11}}{m_{12}} \end{pmatrix}$$

Similarly, by normalising $y_1 = 1$, the eigenvector associated with the λ_2 eigenvalue would be,

$$y_1 = \begin{pmatrix} y_1 \\ \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 \\ \dfrac{\lambda_2 - m_{11}}{m_{12}} \end{pmatrix}$$

For the parameters u_1, v_1, u_2 and v_2, we know that they are the inverse of Q matrix.

Rewriting Q matrix as,

$$Q = \begin{pmatrix} x_1 & y_1 \\ \\ x_2 & y_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ \dfrac{\lambda_1 - m_{11}}{m_{12}} & \dfrac{\lambda_2 - m_{11}}{m_{12}} \end{pmatrix}$$

Then, the inverse of Q matrix is,

$$Q^{-1} = \begin{pmatrix} u_1 & v_1 \\ u_2 & v_2 \end{pmatrix} = \frac{1}{|Q|}\begin{pmatrix} \frac{\lambda_2 - m_{11}}{m_{12}} & -1 \\ -\frac{\lambda_1 - m_{11}}{m_{12}} & 1 \end{pmatrix} = \begin{pmatrix} \frac{\lambda_2 - m_{11}}{\lambda_2 - \lambda_1} & -\frac{m_{12}}{\lambda_2 - \lambda_1} \\ -\frac{\lambda_1 - m_{11}}{\lambda_2 - \lambda_1} & \frac{m_{12}}{\lambda_2 - \lambda_1} \end{pmatrix}$$

$$\textit{where } |Q| = (1)\left(\frac{\lambda_2 - m_{11}}{m_{12}}\right) - (1)\left(\frac{\lambda_1 - m_{11}}{m_{12}}\right) = \frac{\lambda_2 - \lambda_1}{m_{12}}$$

Thus, the parameter values are the following:

$$x_1 = 1 \quad , \quad y_1 = 1 \quad , \quad u_1 = \frac{\lambda_2 - m_{11}}{\lambda_2 - \lambda_1} \quad , \quad v_1 = -\frac{m_{12}}{\lambda_2 - \lambda_1}$$

$$x_2 = \frac{\lambda_1 - m_{11}}{m_{12}} \quad , \quad y_2 = \frac{\lambda_2 - m_{11}}{m_{12}} \quad , \quad u_2 = -\frac{\lambda_1 - m_{11}}{\lambda_2 - \lambda_1} \quad , \quad v_2 = \frac{m_{12}}{\lambda_2 - \lambda_1}$$

By iterating over time, we obtain the full trajectory from starting value $\hat{k}_0, \hat{c}_0$,

$$\begin{aligned}
\begin{pmatrix} \hat{k}_t - \hat{k}^* \\ \hat{c}_t - \hat{c}^* \end{pmatrix} &= Q\Lambda^t Q^{-1} \begin{pmatrix} \hat{k}_0 - \hat{k}^* \\ \hat{c}_0 - \hat{c}^* \end{pmatrix} \\
&= \begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \end{pmatrix} \begin{pmatrix} \lambda_1^t & 0 \\ 0 & \lambda_2^t \end{pmatrix} \begin{pmatrix} u_1 & v_1 \\ u_2 & v_2 \end{pmatrix} \begin{pmatrix} \hat{k}_0 - \hat{k}^* \\ \hat{c}_0 - \hat{c}^* \end{pmatrix} \\
&= \begin{pmatrix} \left(x_1\lambda_1^t u_1 + y_1\lambda_2^t u_2\right)\left(\hat{k}_0 - \hat{k}^*\right) + \left(x_1\lambda_1^t v_1 + y_1\lambda_2^t v_2\right)\left(\hat{c}_0 - \hat{c}^*\right) \\ \left(x_2\lambda_1^t u_1 + y_2\lambda_2^t u_2\right)\left(\hat{k}_0 - \hat{k}^*\right) + \left(x_2\lambda_1^t v_1 + y_2\lambda_2^t v_2\right)\left(\hat{c}_0 - \hat{c}^*\right) \end{pmatrix}
\end{aligned}$$

Since we have two eigenvalues in this system, we assume that λ_1 is an unstable eigenvalue, while λ_2 is a stable eigenvalue. Thus, the previous matrix can be expressed and simplified as,

$$\hat{k}_t - \hat{k}^* = x_1\, \lambda_1^t \left[u_1\left(\hat{k}_0 - \hat{k}^*\right) + v_1\left(\hat{c}_0 - \hat{c}^*\right)\right] + y_1\, \lambda_2^t \left[u_2\left(\hat{k}_0 - \hat{k}^*\right) + v_2\left(\hat{c}_0 - \hat{c}^*\right)\right]$$

$$\hat{c}_t - \hat{c}^* = x_2\, \lambda_1^t \left[u_1\left(\hat{k}_0 - \hat{k}^*\right) + v_1\left(\hat{c}_0 - \hat{c}^*\right)\right] + y_2\, \lambda_2^t \left[u_2\left(\hat{k}_0 - \hat{k}^*\right) + v_2\left(\hat{c}_0 - \hat{c}^*\right)\right]$$

As long as λ_1 is an unstable eigenvalue, the only way to avoid a solution in which $\left(\hat{k}_t, \hat{c}_t\right)$ tend to positive or negative infinity is if the coefficient of unstable eigenvalue, λ_1, sets to be zero. Moreover, because of $x_1 = 1$, the bracketed term associating with λ_1^t must be zero in the capital

PGSU equation, $(\hat{k}_t - \hat{k}^*)$. Since x_2 also relies on the value of the structural parameters, the same condition must hold for the consumption PGSU equation, $(\hat{c}_t - \hat{c}^*)$.

$$u_1\left(\hat{k}_0 - \hat{k}^*\right) + v_1\left(\hat{c}_0 - \hat{c}^*\right) = 0 \tag{III}$$

Therefore, for the stability requirement, the initial consumption, $\hat{c}_0$, should be chosen by,

$$\left(\hat{c}_0 - \hat{c}^*\right) = -\frac{u_1}{v_1}\left(\hat{k}_0 - \hat{k}^*\right) = \frac{\lambda_2 - m_{11}}{m_{12}}\left(\hat{k}_0 - \hat{k}^*\right) \tag{IV}$$

This equation, (IV), is the equation for the saddle path, which combines the initial value of $\hat{k}_t$ with the initial value of $\hat{c}_t$. The slope of the saddle path equation is $(\lambda_2 - m_{11})/m_{12}$, which determines the cross of the original value of $\hat{k}_t$ either below, above, or exactly at the original value of $\hat{c}_t$.

Again if λ_1 is an unstable eigenvalue, then the dynamics of the system is specified by,

$$\hat{k}_t - \hat{k}^* = y_1\, \lambda_2^t \left[u_2\left(\hat{k}_0 - \hat{k}^*\right) + v_2\left(\hat{c}_0 - \hat{c}^*\right)\right] \tag{V}$$

$$\hat{c}_t - \hat{c}^* = y_2\, \lambda_2^t \left[u_2\left(\hat{k}_0 - \hat{k}^*\right) + v_2\left(\hat{c}_0 - \hat{c}^*\right)\right] \tag{VI}$$

Dividing (VI) by (V), we get the consumption PGSU equation $(\hat{c}_t - \hat{c}^*)$,

$$\hat{c}_t - \hat{c}^* = \frac{y_2}{y_1}\left(\hat{k}_t - \hat{k}^*\right) = \frac{\lambda_2 - m_{11}}{m_{12}}\left(\hat{k}_t - \hat{k}^*\right) \tag{VII}$$

Moreover, it can be seen that conditions (V) and (VI) will hold at the time (t=0) by plugging the initial consumption PGSU, (IV), into (V) and (VI). As a result, the dynamics of the system can be written as,

$$\hat{k}_t - \hat{k}^* = \lambda_2^t \left[y_1\left(u_2\left(\hat{k}_0 - \hat{k}^*\right) + v_2\left(\hat{c}_0 - \hat{c}^*\right)\right)\right] = \lambda_2^t \left(\hat{k}_0 - \hat{k}^*\right) \tag{VIII}$$

$$\hat{c}_t - \hat{c}^* = \frac{\lambda_2 - m_{11}}{m_{12}}\, \lambda_2^t \left(\hat{k}_0 - \hat{k}^*\right) = \lambda_2^t \left(\hat{c}_0 - \hat{c}^*\right) \tag{IX}$$

As a result, these two linear approximations, (VIII) and (IX), show that both capital PGSU and consumption PGSU converge to their steady-state levels from their initial values.

B. Appendix to Chapter 5

B.1 The Growth Rate of Government Spending When We Combine the Two Sources of Government Revenues, Oil and Consumption Tax Revenues

The government budget constraint is,

$$G_t = \pi_t + \tau^c C_t \tag{A}$$

The growth rate of government spending is,

$$\gamma_{G_t} \equiv \frac{G_{t+1}}{G_t} = \frac{\pi_{t+1} + \tau^c C_{t+1}}{\pi_t + \tau^c C_t} \tag{B}$$

which can be writing as[121],

$$\gamma_{G_t} = \frac{\pi_{t+1}}{\pi_t} \frac{1+\tau^c \left(C_{t+1}/\pi_{t+1}\right)}{1+\tau^c \left(C_t/\pi_t\right)} = g^2 \frac{1+\tau^c \left(C_{t+1}/\pi_{t+1}\right)}{1+\tau^c \left(C_t/\pi_t\right)} \tag{C}$$

Substituting equation (*A*) into the definition of consumption PGSU, $\hat{c}_t = C_t/G_t$, and simplify it, gives us,

$$\frac{C_t}{\pi_t} = \frac{\hat{c}_t}{1-\tau^c \hat{c}_t} \tag{D}$$

Finally, by substituting (*D*) into (*C*), we obtain:

$$\gamma_{G_t} = g^2 \frac{\left(1-\tau^c \hat{c}_t\right)}{\left(1-\tau^c \hat{c}_{t+1}\right)}$$

[121] Equation (*C*) can be also written as $\gamma_{G_t} = \gamma_{C_t} \left[\frac{\tau^c + (\pi_{t+1}/C_{t+1})}{\tau^c + (\pi_t/C_t)}\right]$. However, if we follow the same steps as above, we arrive at a result that $\gamma_{G_t} = 1$.

B.2 Existence of Steady-States of Both Types

We have considered the steady-states equilibria of two types, that are:

(I) When C_t/π_t tends to a finite, non-zero value (*i.e.* that means both revenues are non-zero), and the growth rate is g^2 (calling this type *an exogenous growth steady-state*).

(II) When C_t/π_t tends to infinity (*i.e.* this occurs when C_t is very large relative to oil revenues), and the growth rate is γ_{c_t} where $\gamma_{c_t} > g^2$ and γ_{c_t} is a function of τ^c amongst other parameters (calling this type *an endogenous growth steady-state*).

Although these steady-states equilibria are of different types, they both have the feature that PGSU variables (*i.e.* $\hat{y}_t$, $\hat{c}_t$, $\hat{k}_t$) are constant over-time. That means all variables in both types grow at the same rate of government spending, but the way of growth is different. For this reason, let us now distinguish between the two systems:

***- System (I)** is the difference equation when $\pi_t/G_t > 0$[122] (and hence $\hat{c}_t < 1/\tau^c$[123])*

Here, we want the ratio of π_t/G_t to remain strictly positive because, in the 'pure consumption tax case', this ratio goes to zero. Moreover, we understand from the formula $(1-\tau^c\hat{c}_{t+1})/g^2(1-\tau^c\hat{c}_t)$ that $\hat{c}_t$ should be strictly less than $1/\tau^c$. It could be however that $\hat{c}_t > 1/\tau^c$, but this would imply that $(1-\tau^c\hat{c}_{t+1})/g^2(1-\tau^c\hat{c}_t)$ is a negative value which becomes meaningless. Therefore, we cannot possibly have $\hat{c}_t > 1/\tau^c$ because of the negative value. The two main equations that describe system *(I)* are the following:

$$\hat{k}_{t+1} = \underbrace{\frac{\left(1-\tau^c\hat{c}_{t+1}\right)}{g^2\left(1-\tau^c\hat{c}_t\right)}}_{1/\gamma_{G_t}} \left[A\hat{k}_t^{\alpha} + (1-\delta)\,\hat{k}_t - \left(1+\tau^c\right)\,\hat{c}_t\right]$$

$$\frac{\hat{c}_{t+1}}{\hat{c}_t} = \frac{\left(1-\tau^c\hat{c}_{t+1}\right)}{g^2\left(1-\tau^c\hat{c}_t\right)} \left[\beta\left(\alpha A\hat{k}_{t+1}^{\alpha-1} + 1-\delta\right)\right]^{\frac{1}{\sigma}}$$

[122] We have shown (C_t/π_t) in the first type steady-state equilibrium (*i.e.* type *I*), where the ratio (π_t/G_t) is the same thing but in terms of a different variable. To see that clearly, the level of government spending relative to oil revenues in our model can be written as, $G_t/\pi_t = 1/(1-\tau^c\hat{c}_t)$, *i.e.* $\pi_t/G_t = 1-\tau^c\hat{c}_t$, while the level of consumption relative to oil revenues can be written as, $C_t/\pi_t = \hat{c}_t/(1-\tau^c\hat{c}_t)$. We also know the definition of consumption PGSU is $\hat{c}_t = C_t/G_t$. Thus, by using all these facts together, we obtained that $C_t/\pi_t = (C_t/G_t)/(\pi_t/G_t)$.

[123] This ensures $(\pi_t/G_t) > 0$ since $\pi_t/G_t = 1-\tau^c\hat{c}_t$.

- System (II) *is the difference equation when* $\pi_t / G_t = 0$ *(and hence* $\hat{c}_t = 1/\tau^c, \gamma_{G_t} = \gamma_{c_t}, \forall t$*)*

Here, the ratio of π_t / G_t becomes zero because in the 'pure consumption tax case', G_t becomes very large and then $\hat{c}_t = 1/\tau^c$ unchanged over-time. The two main equations that describe this system are the following:

$$\hat{k}_{t+1} = \frac{1}{\gamma_{c_t}} \left[A\hat{k}_t^{\alpha} + (1-\delta)\,\hat{k}_t - \frac{1+\tau^c}{\tau^c} \right]$$

$$\gamma_{c_t} = \left[\beta\left(\alpha A \hat{k}_{t+1}^{\alpha-1} + 1 - \delta \right) \right]^{\frac{1}{\sigma}}$$

These two systems are similar but not formally identical. The reason is that if we set $\hat{c}_t = \hat{c}_{t+1} = 1/\tau^c$ in the system (*I*), it does not resolve itself into the system (*II*) in a simple way. The ratio $(1 - \tau^c \hat{c}_{t+1})/(1 - \tau^c \hat{c}_t)$ is $0/0$ in this case, and so undefined. Instead, we can take the limit of $(1 - \tau^c \hat{c}_{t+1})/(1 - \tau^c \hat{c}_t)$ as $\hat{c}_t, \hat{c}_{t+1} \to 1/\tau^c$. In certain circumstances, one can find this limit using L'Hôpital's rule, but it is unclear how to apply it here. Hence, we will continue to treat these two systems as similar but not identical for the time being.

Now, we consider the steady-state versions of these two systems[124]:

$$\left.\begin{aligned} \hat{k}^* &= \frac{1}{g^2} \left[A\hat{k}^{*\alpha} + (1-\delta)\,\hat{k}^* - \left(1+\tau^c\right)\hat{c}^* \right] \\ 1 &= \frac{1}{g^2} \left[\beta\left(\alpha A \hat{k}^{*\alpha-1} + 1 - \delta \right) \right]^{\frac{1}{\sigma}} \end{aligned}\right\} \quad \textit{System } (I)\textit{: The exogenous case when } \hat{c} < 1/\tau^c$$

$$\left.\begin{aligned} \hat{k}^* &= \frac{1}{\gamma_c} \left[A\hat{k}^{*\alpha} + (1-\delta)\,\hat{k}^* - \frac{1+\tau^c}{\tau^c} \right] \\ 1 &= \frac{1}{\gamma_c} \left[\beta\left(\alpha A \hat{k}^{*\alpha-1} + 1 - \delta \right) \right]^{\frac{1}{\sigma}} \end{aligned}\right\} \quad \textit{System } (II)\textit{: The endogenous case when } \hat{c} = 1/\tau^c$$

[124] We have already found them, but we now rewrite them in order to compare between the two systems.

The differences between the two systems at the steady-state are the following: In the first system, $\hat{c}^*$ is not exogenously imposed to equal $1/\tau^c$, whereas γ_c is exogenously imposed to equal g^2. In the second system, $\hat{c}^*$ is exogenously imposed to equal $1/\tau^c$, while γ_c is not exogenously imposed to equal g^2. In other words, which variables are treated as exogenous and endogenous has been switched around. In more precisely, $\hat{k}^*$ and $\hat{c}^*$ are endogenous variables in the first system, while $\hat{k}^*$ and γ_c become endogenous variables in the second system. So now, from the two steady-state systems, we can derive the third system as,

$$\left.\begin{aligned} \hat{k}^* &= \frac{1}{\gamma_c}\left[A\hat{k}^{*\alpha} + (1-\delta)\,\hat{k}^* - \left(1+\tau^c\right)\hat{c}^*\right] \\ 1 &= \frac{1}{\gamma_c}\left[\beta\left(\alpha A\hat{k}^{*\alpha-1} + 1-\delta\right)\right]^{\frac{1}{\sigma}} \end{aligned}\right\} \qquad System\ (III)$$

In system (*III*), we neither set $\hat{c}^* = 1/\tau^c$ nor $\gamma_c = g^2$, and this pair of equation holds in both systems (*I*) and (*II*). However, it is, by itself, not sufficient to determine the steady-state equilibrium values of variables because we have three endogenous variables (namely: $\hat{k}^*, \hat{c}^*, \gamma_c$), but only in two equations. Then, the system (*III*) is not a complete description. Therefore, to complete the system, one needs either to add the assumption that $\gamma_c = g^2$ (which converts it into the system (*I*)), or to add the assumption that $\hat{c}^* = 1/\tau^c$ (which converts it into the system (*II*)). In other words, depending on the two additional assumptions which we add, we either go from the system (*III*) to the system (*I*) or from the system (*III*) to the system (*II*). However, system (*III*) can be applied for both types.

System (*III*) is nevertheless helpful because it enables us to see what system (*I*) and system (*II*) have in common. We can try to simplify the system (*III*) by combining the two equations to eliminate γ_c:

$$\hat{k}^* = \frac{A\hat{k}^{*\alpha} + (1-\delta)\,\hat{k}^* - \left(1+\tau^c\right)\hat{c}^*}{\left[\beta\left(\alpha A\hat{k}^{*\alpha-1} + 1-\delta\right)\right]^{\frac{1}{\sigma}}}$$

This yields an implicit relationship between $\hat{k}^*$and $\hat{c}^*$. Moreover, this relationship holds in both system (*I*) and system (*II*). We can try to use it to understand better the relationship between type (*I*) steady-state and type (*II*) steady-state. In particular, we want to know whether, for a given set of exogenous parameter values, both types of steady-state can exist in the model, or whether only one type of steady-state will exist for a particular set of parameter values.

Let us now simplify the system (*III*) by assumption $\sigma = 1$. Then, we can rewrite the equation as:

$$\beta\alpha A\hat{k}^{*\alpha} + \beta(1-\delta)\hat{k}^{*} = A\hat{k}^{*\alpha} + (1-\delta)\hat{k}^{*} - (1+\tau^{c})\hat{c}^{*}$$

$$(1-\alpha\beta)A\hat{k}^{*\alpha} + (1-\beta)(1-\delta)\hat{k}^{*} = (1+\tau^{c})\hat{c}^{*}$$

$$\hat{c}^{*} = \frac{1}{1+\tau^{c}}\left[(1-\alpha\beta)A\hat{k}^{*\alpha} + (1-\beta)(1-\delta)\hat{k}^{*}\right]$$

We can also draw $\hat{c}^{*}$ as a function of $\hat{k}^{*}$, as follows:

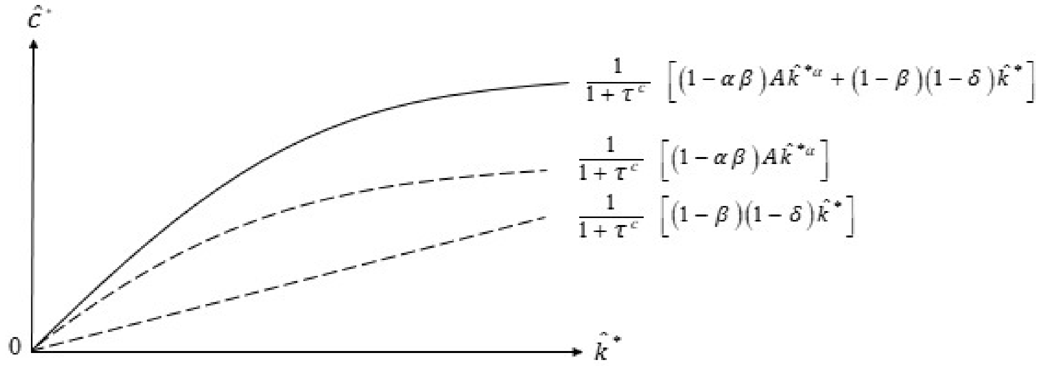

Diagram 28: Drawing equation $\hat{c}_t$ as a function of $\hat{k}_t$

From Diagram 28, we can see that the relationship between $\hat{c}^{*}$ and $\hat{k}^{*}$ is increasing and concave, which passes through the original case. It is indeed common for both steady-states. Thus, whether type (*I*) or type (*II*) steady-state, such a steady-state must lie somewhere on this locus.

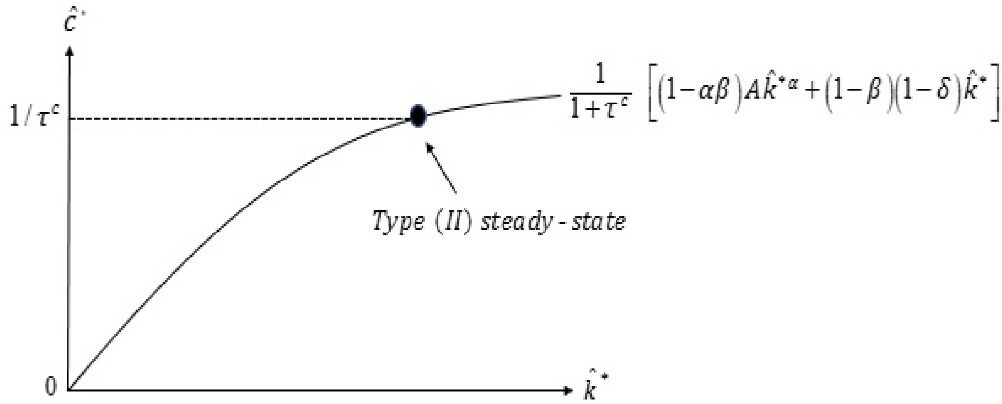

Diagram 29: The location of type (II) steady-state

Type (*II*) steady-state can be easily located on Diagram 29. To locate the type (*I*) steady-state on the same diagram, note that, in the type (*I*) steady-state, we know that (with $\sigma = 1$):

$$g^2 = \beta\left(\alpha A \hat{k}^{*\alpha-1} + 1 - \delta\right)$$

This can be re-arranged to give the value of $\hat{k}^*$ in the type (*I*) steady-state as a function of g^2:

$$\hat{k}^* = \left[\left(\frac{1}{\alpha A}\right)\left(\frac{g^2}{\beta} - 1 + \delta\right)\right]^{\frac{1}{\alpha-1}}$$

So, by knowing g^2, we can identify that $\hat{k}^*$ is associated with the type (*I*) steady-state. This shows that the higher in g^2, the lower in $\hat{k}^*$ (since $\alpha < 1$). Given g^2, this, therefore, locates a point on the curve which corresponds to a type (*I*) steady-state.

Now, we try to know whether the type (*I*) steady-state lies to the left or the right of the type (*II*) steady-state. Since we must have $\hat{c}^* < 1/\tau^c$ (this is necessary to be consistent with $\pi_t/G_t > 0$), it then follows that type (*I*) steady-state must be to the left (below) the type (*II*) steady-state, as shown in Diagram 30 below:

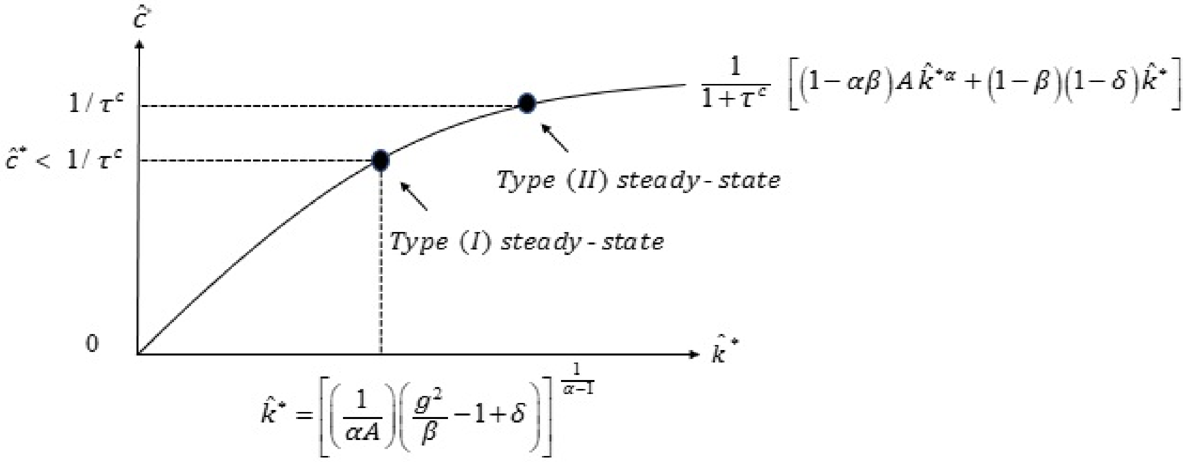

Diagram 30: The location of both type (I) and (II) steady-states

In the diagram, we cannot have $\hat{c}^* > 1/\tau^c$, so we cannot have the type (*I*) steady-state, which is the right (above) the type (*II*) steady-state because it will violate its condition, as shown in system (*I*). However, if the type (*I*) steady-state exists for the same set of parameter values, it will be the left of type (*II*) steady-state because that implies $\hat{c}^* < 1/\tau^c$. Thus, if it is possible to have two types of steady-states for the same set of parameter values, they must be ordered this way around as in Diagram 30, where they could not be the other way around. Therefore, for the type (*I*) steady-state to lie to the left of the type (*II*) steady-state, g^2 is indeed what determines this position.

Meaning that higher g^2 is more to the left will be the position of this steady-state, and higher τ^c is more down. In brief, for both types of steady-states to be that way around, as shown in Diagram 30, certain conditions will need to hold about g^2, τ^c, and other parameters.

It is now clear that for the type (*I*) steady-state to lie to the left of the type (*II*) steady-state, g^2 must be 'sufficiently high' and τ^c must be 'sufficiently low'. On the other hand, if the opposite is true, when g^2 is 'sufficiently low' and τ^c is 'sufficiently high', then they will be the other way around, where the type (*I*) steady-state lies to the right of the type (*II*) steady-state, as shown in Diagram 31 [125] below.

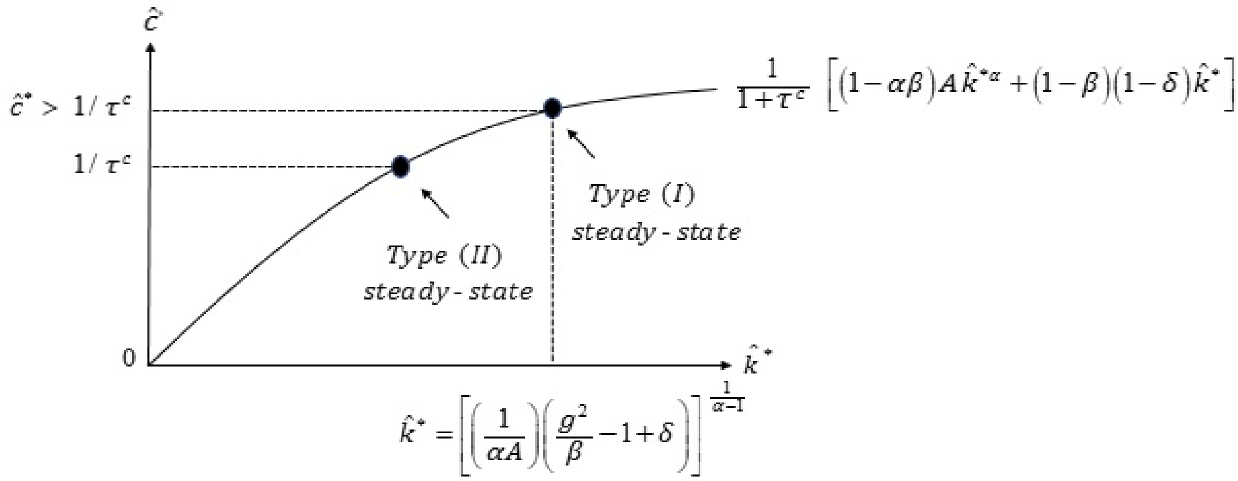

Diagram 31: The reverse location of both type (I) and (II) steady-states

However, we now have a contradiction because there is another condition linked to these underline parameter values, and that is for type (*II*) steady-state to exist, we need $\gamma_c > g^2$. This condition is violated if this is a way around, as shown in Diagram 31[126].

We have just found that, for a type (*I*) steady-state to exist for the same set of parameter values as a type (*II*) steady-state, we need g^2 to be 'sufficiently high' and τ^c to be 'sufficiently low'. However, we also know from our previous discussion that a type (*II*) steady-state to exist, we need $\gamma_c > g^2$. This latter condition ensures that oil revenues, π_t, grows slower than consumption, C_t, and government spending, G_t, so that $\pi_t / C_t \to 0$ as $t \to \infty$, *i.e.* oil revenues become insignificant. This means we need g^2 to be 'sufficiently low' not 'sufficiently high'. In fact, the condition $\gamma_c > g^2$ is the exact opposite of the requirement that the type (*I*) steady-state should lie to the left of the type

[125] Although the situation shown in the Diagram 31 is not possible to happen in this case because it is violated the condition of π_t / G_t being negative, we just draw this diagram to show the opposite situation of type (*I*) and type (*II*) steady-state.

[126] The two conditions are: π_t / G_t cannot be negative and $\gamma_c > g^2$. These two conditions completely conflict with each other, where if one holds, then the other cannot hold and vice versa.

(*II*) steady-state on the diagram. To see this, note that in a type (*II*) steady-state, the growth rate, γ_c, is decreasing in $\hat{k}^*$ (see equations of system (*III*)) and hence high γ_c requires that $\hat{k}^*$ to be lower.

In conclusion, since we have seen that there is an unavoidable contradiction between the two conditions for both type (*I*) and type (*II*) steady-states to exist for the same set of parameter values, we conclude that type (*I*) and type (*II*) steady-states cannot exist for the same set of parameter values. The corollary is that for a given set of parameter values, the steady-state will be either of type (*I*) or type (*II*), *i.e.* there will be a single type of steady-state. In more precise detail,

- If g^2 is sufficiently high (for a given value of τ^c), then the steady-state will be of type (*I*), *i.e. an exogenous growth steady-state*.

- If g^2 is sufficiently low (for a given value of τ^c), then the steady-state will be of type (*II*), *i.e. an endogenous growth steady-state*.

The critical value of g^2, for these purposes, is whatever the endogenous growth rate associated with a particular value of τ^c, in the pure consumption tax version of the model. (In general, a simple algebraic expression for this critical value of g^2 cannot be written down: it whatever makes the type (*I*) steady-state lie to the right of the type (*II*) steady-state).

B.3 Possible Parameter Values

Looking at equation (5.32), we can see that ω^*depends on a set of parameters. Thus, we try in this section to show what happens not just as β changes but also as other parameters change by using numerical exercise. To do that, we vary each parameter separately and keep the others constant.

From the previous calculation, we found that if $1 > \beta \geq 0.67$, then that insures $\omega^* > 1$. Thus, we consider now two values of β (*i.e.* the large value, $\beta = 0.99$, and the small value, $\beta = 0.67$). We do that because we want to understand what happens as all parameters change if we set β with these ranges of values.

We start finding the possible values of γ_G and keep other parameter values constant by setting $\beta = 0.99$ and $\beta = 0.67$, in order to guarantee that $\omega^* > 1$. Then, we use the possible values of γ_G, which we got, to determine the possible values of δ, when also $\beta = 0.99$ and $\beta = 0.67$. After that, we employ both values of γ_G and δ to find the possible values of α for the same set of β values. We then carry on using the same method to determine the possible values of all other parameters.

B.3.1 Possible values of γ_G:

$$\textit{If} \quad \beta = 0.99 \ , \ \alpha = 0.5 \ , \ \delta = 0 \ , \ A = 1 \ , \ \sigma = 1 \ , \ \textit{then}$$

$$\omega^* = \frac{1.02\ \gamma_G\ -\ 1}{\left(2.02\ \gamma_G\ -\ 2\right)^2}$$

To ensure that $\omega^* > 1$, then γ_G should be $1 \leq \gamma_G < 1.25$.

$$\textit{If} \quad \beta = 0.67 \ , \ \alpha = 0.5 \ , \ \delta = 0 \ , \ A = 1 \ , \ \sigma = 1 \ , \ \textit{then}$$

$$\omega^* = \frac{1.985\ \gamma_G\ -\ 1}{\left(2.985\ \gamma_G\ -\ 2\right)^2}$$

To ensure that $\omega^* > 1$, then γ_G should be $0.55 < \gamma_G \leq 1$.

B.3.2 Possible values of δ:

$$\textit{If} \quad \beta = 0.99 \ , \ \alpha = 0.5 \ , \ \gamma_G = 1.24 \ , \ A = 1 \ , \ \sigma = 1 \ , \ \textit{then}$$

$$\omega^* = \frac{0.265\ +\ \delta}{\left(0.5\ +\ 2\delta\right)^2}$$

To ensure that $\omega^* > 1$, then δ should be $\delta < 0.015$.

$$If \quad \beta = 0.67 \ , \ \alpha = 0.5 \ , \ \gamma_G = 1 \ , \ A = 1 \ , \ \sigma = 1 \ , \ then$$

$$\omega^* = \frac{0.985 + \delta}{\left(0.985 + 2\delta\right)^2}$$

To ensure that $\omega^* > 1$, then δ should be $\delta < 0.005$.

B.3.3 Possible values of α:

$$If \quad \beta = 0.99 \ , \ \delta = 0.009 \ , \ \gamma_G = 1.24 \ , \ A = 1 \ , \ \sigma = 1 \ , \ then$$

$$\omega^* = \left(\frac{0.26152}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \left(\frac{0.26152}{\alpha}\right)^{\frac{1}{\alpha-1}}\left(0.249\right)$$

To ensure that $\omega^* > 1$, then α should be $\alpha \geq 0.5$.

$$If \quad \beta = 0.67 \ , \ \delta = 0.004 \ , \ \gamma_G = 1 \ , \ A = 1 \ , \ \sigma = 1 \ , \ then$$

$$\omega^* = \left(\frac{0.496}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \left(\frac{0.496}{\alpha}\right)^{\frac{1}{\alpha-1}}\left(0.004\right)$$

To ensure that $\omega^* > 1$, then α should be $\alpha \geq 0.5$.

B.3.4 Possible values of A:

$$If \quad \beta = 0.99 \ , \ \delta = 0.009 \ , \ \gamma_G = 1.24 \ , \ \alpha = 0.5 \ , \ \sigma = 1 \ , \ then$$

$$\omega^* = \left(1.002\right)A^2$$

To ensure that $\omega^* > 1$, then A should be $A \geq 1$.

$$If \quad \beta = 0.67 \ , \ \delta = 0.004 \ , \ \gamma_G = 1 \ , \ \alpha = 0.5 \ , \ \sigma = 1 \ , \ then$$

$$\omega^* = \left(1.003\right)A^2$$

To ensure that $\omega^* > 1$, then A should be $A \geq 1$.

B.3.5 Possible values of σ:

$$If \quad \beta = 0.99 \;\;,\;\; \delta = 0.009 \;\;,\;\; \gamma_G = 1.24 \;\;,\;\; \alpha = 0.5 \;\;,\;\; A = 1 \;\;,\; then$$

To ensure that $\omega^* > 1$, then σ should be $0.9 < \sigma \leq 1.015$.

$$If \quad \beta = 0.67 \;\;,\;\; \delta = 0.004 \;\;,\;\; \gamma_G = 0.56 \;\;,\;\; \alpha = 0.5 \;\;,\;\; A = 1 \;\;,\; then$$

To ensure that $\omega^* > 1$, then σ should be $\sigma \leq 1$.

The table below summarises the possible values of γ_G, δ, α, A, and σ parameters in ω^* which ensure $\omega^* > 1$.

Table 23: The possible values of all parameters in ω^* which insure $\omega^* > 1$

Parameters	**Possible Range of Values**	
	$\boldsymbol{If \;\; \beta = 0.99}$	$\boldsymbol{If \;\; \beta = 0.67}$
$\boldsymbol{\gamma_G}$	$1 \leq \gamma_G < 1.25$	$0.55 < \gamma_G \leq 1$
$\boldsymbol{\delta}$	$\delta < 0.015$	$\delta < 0.005$
$\boldsymbol{\alpha}$	$\alpha \geq 0.5$	$\alpha \geq 0.5$
$\boldsymbol{A}$	$A \geq 1$	$A \geq 1$
$\boldsymbol{\sigma}$	$0.90 < \sigma \leq 1.015$	$\sigma \leq 1$

Although we have shown all parameters in ω^* which insure $\omega^* > 1$, it would also be possible to have $\omega^* < 1$ if the values of the above parameters are different from the mentioned ranges. Therefore, both scenarios are possible to happen, depending on the parameters' values. To see now whether $\omega^* > 1$ or $\omega^* < 1$ is more likely, we compare between the two possible scenarios. Let us just look at the possible values when β is large, $\beta = 0.99$. According to our calculations, it seems that $\omega^* < 1$ would be more likely to occur for some reasons. The first reason is that γ_G needs to be $\gamma_G \in [1, 1.25)$ to guarantee that $\omega^* > 1$. However, if $\gamma_G \geq 1.25$ that means $\omega^* < 1$. Thus, the possible range of γ_G values to ensure $\omega^* < 1$ is greater than the ones that satisfy $\omega^* > 1$. The second reason is that $\omega^* > 1$ requires the δ to be close to zero, but for $\omega^* < 1$ to occur, it should be $\delta > 0.015$. Again, it is clear that the possible range of δ values to ensure $\omega^* < 1$ is greater than the ones that satisfy $\omega^* > 1$. Another reason is that the range value for σ to guarantee $\omega^* > 1$ should be $0.90 < \sigma \leq 1.015$, which implies very limited range values compared to $\omega^* < 1$. As a result, $\omega^* < 1$ seems likely to occur because it allows for the most parameters values taking a broad range of values more than $\omega^* > 1$.

B.4 The Proof of Equation (5.33) Is Unstable

The single first-order different equation in the case of pure consumption tax is,

$$\hat{k}_{t+1} = \frac{A\hat{k}_t^{\alpha} + (1-\delta)\,\hat{k}_t - \dfrac{1+\tau^c}{\tau^c}}{\left[\beta\left(\alpha A\hat{k}_{t+1}^{\alpha-1} + 1-\delta\right)\right]^{\frac{1}{\sigma}}} \tag{5.33}$$

From equation (5.33), we can see that although we cannot write $\hat{k}_{t+1}$ as an explicit function of $\hat{k}_t$, we can simplify the equation somewhat if we assume $\sigma = 1$. In that case, we can rewrite it as:

$$\beta\left(\alpha A\hat{k}_{t+1}^{\alpha} + (1-\delta)\hat{k}_{t+1}\right) = A\hat{k}_t^{\alpha} + (1-\delta)\,\hat{k}_t - \frac{1+\tau^c}{\tau^c}$$

To make the consideration of stability even more accessible, let us assume for the moment that $\delta = 1$, then

$$A\hat{k}_{t+1}^{\alpha} = \frac{1}{\alpha\beta}\left[A\hat{k}_t^{\alpha} - \frac{1+\tau^c}{\tau^c}\right]$$

In fact, we can replace $A\hat{k}_t^{\alpha}$ by $\hat{y}_t$, using the production function, thus

$$\hat{y}_{t+1} = \frac{1}{\alpha\beta}\left[\hat{y}_t - \frac{1+\tau^c}{\tau^c}\right]$$

Given that $0 < \alpha\beta < 1$, so that $(1/\alpha\beta) > 1$. It is now clear that this first-order, linear, difference equation in $\hat{y}_t$ is unstable. To see that, we sketch the above equation in Diagram 32,

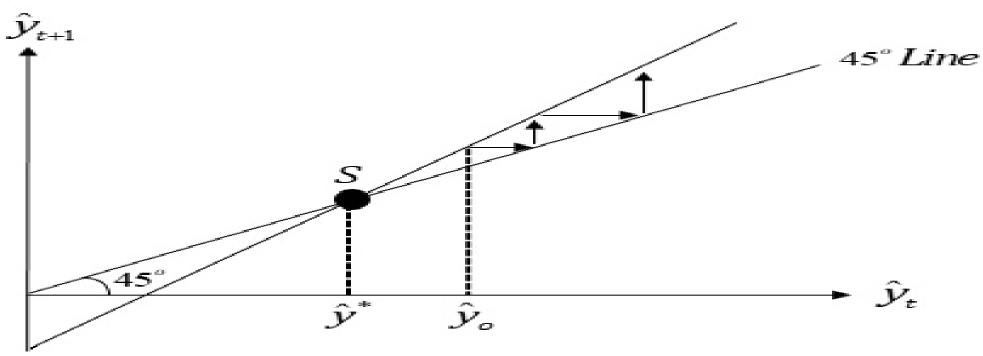

Diagram 32: Drawing the equation $\hat{y}_{t+1}$

The 'phase line' has a slope greater than one and a negative intercept. As a result, we can conclude that the steady-state at (S) is unstable.

B.5 The Proof of $-Z < 1$

$$Z = \frac{(\alpha - 1)\alpha\hat{c}^{*} A\beta\hat{k}^{*\alpha-1}\tau^{c}}{\sigma\left(g^{2}\right)^{\sigma}}$$

We know that Z is negative because of $(\alpha - 1)$, but it is still unclear if its value is greater or less than one. Knowing that would help us to determine the necessary and sufficient condition for stability in our model.

In the above Z ratio, the numerator contains $\hat{k}^{*\alpha-1}$, and as $\hat{k}^{*} \to 0$, then $\hat{k}^{*\alpha-1} \to \infty$. Also, we know from equation (5.18) that

$$\hat{k}^{*} = \left[\frac{1}{\alpha A}\left(\frac{\left(g^{2}\right)^{\sigma}}{\beta} - 1 + \delta\right)\right]^{\frac{1}{\alpha-1}}$$

which can also be written as,

$$\alpha A\hat{k}^{*\alpha-1} = \frac{\left(g^{2}\right)^{\sigma}}{\beta} - 1 + \delta$$

Substituting this into Z ratio and simplifying it, we get:

$$Z = \frac{(\alpha - 1)\left[\left(g^{2}\right)^{\sigma} - \beta + \beta\delta\right]\hat{c}^{*}\tau^{c}}{\sigma\left(g^{2}\right)^{\sigma}}$$

Assuming now that $\sigma = 1$ and $g^{2} \geq 1$, and using the fact explained in the system *(I)* in Appendix B.2 that $\hat{c}_{t}\tau^{c} < 1$, we can ensure that the numerator is negative and less than one, while the denominator is positive and either greater than (or equal to) one. As a result, we can conclude that $-Z < 1$.

B.6 The Comparison Between (5.19)' and (5.43)'

The purpose of this algebraic calculation is to compare between (5.19)' and the differentiation of (5.43) w.r.t. consumption tax in order to see which is larger. This would help us determine whether the (*D*) locus in the phase diagram shifts downwards by more or less than the steady-state shifts.

Rewriting (5.19)' and (5.43)',

$$\frac{d\hat{c}^*}{d\tau^{c*}} = -\frac{\omega^*}{\left(1+\tau^c\right)^2} \tag{5.19)$'$(}$$

$$\frac{d\hat{c}_t}{d\tau^c} = -\frac{K_t - \pi_t \hat{k}_t}{K_t\left(\tau^c\right)^2} \tag{5.43)$'$(}$$

From (5.19)', we know that $\omega^* = A\hat{k}^{*\alpha} + (1-\delta-\gamma_G)\,\hat{k}^*$ and $\hat{c}^* = \omega^*/(1+\tau^c)$. Then, $\omega^* = (1+\tau^c)\,\hat{c}^*$. Thus, (5.19)' can be now written as:

$$\frac{d\hat{c}^*}{d\tau^{c*}} = -\frac{\hat{c}^*}{\left(1+\tau^c\right)} \tag{5.19)$''$(}$$

On the other hand, using the government budget constraint, $G_t = \pi_t + \tau^c C_t$, the definition of capital PGSU, $\hat{k}_t \equiv K_t/G_t$, and consumption PGSU, $\hat{c}_t \equiv C_t/G_t$, allow us to rewrite equation (5.43)' as:

$$\frac{d\hat{c}_t}{d\tau^c} = -\frac{\hat{c}_t}{\tau^c} \tag{5.43)$''$(}$$

Now, we can compare between (5.19)" and (5.43)", where increasing τ^c seems to have more impact on (5.43)" compared to (5.19)". Therefore, the (*D*) locus should shifts downwards by more than the steady-state shifts downwards.

C. Appendix to Chapter 6

C.1 Barro Endogenous Growth Model in Discrete-Time

In Barro (1990) paper, 'Government Spending in a Simple Model of Endogenous Growth', he developed the variation of the Ramsey model by introducing productive government spending in an endogenous growth model. In his model (discrete-time version), where the PGSU variables are absent, and the utility function is given by $u(c_t) = log\ c_t$, the government uses the tax revenue to finance its spending; thus, the government budget constraint is

$$G_t = \tau^Y R_t k_t \quad \Rightarrow \quad R_t = \frac{G_t}{\tau^Y k_t} \tag{A}$$

The government provides public goods to the firms to enhance their production function,

$$y_t = A k_t^{\alpha} G_t^{1-\alpha} \tag{B}$$

The firms' rental rate of capital is,

$$R_t = \alpha A k_t^{\alpha-1} G_t^{1-\alpha} \tag{C}$$

By substituting the later rental rate of capital, *i.e.* (C), into the government budget constraint, *i.e.* (A), and solving for G_t, we get:

$$G_t = \left(\tau^Y \alpha A\right)^{\frac{1}{\alpha}} k_t \tag{D}$$

Then, substituting back the last expression for G_t, *i.e.* (D), into the rental rate of capital, *i.e.* (C), we obtain:

$$R_t = \left(\alpha A\right)^{\frac{1}{\alpha}} \left(\tau^Y\right)^{\frac{1-\alpha}{\alpha}} \tag{E}$$

Finally, the growth rate formula in the Barro (1990) model in discrete-time is,

$$\gamma_c = \frac{c_{t+1}}{c_t} = \beta\left(\frac{\left(1-\tau^Y\right)}{\tau^Y}\left(\alpha A \tau^Y\right)^{\frac{1}{\alpha}} + 1 - \delta\right) \tag{F}$$

which is slightly different from ours in α, as shown in equation (6.22). The tax rate that maximises the economic growth rate,

$$\frac{d\gamma_c}{d\tau^Y} = \left[\beta\left(A\right)^{\frac{1}{\alpha}}\left(\alpha\tau^Y\right)^{\frac{1-\alpha}{\alpha}}\right] \cdot \left[\left(1-\alpha\right)\left(\tau^Y\right)^{-1} - 1\right] \tag{F}'$$

$(F)'$ also varies somewhat from ours shown in (6.22)' because of the use of different utility function, *i.e.* above is used log utility function, while we use in our model a CRRA utility function with $\sigma > 0$.

C.2 A Simple Numerical Exercise to Show How the Personal Income Tax Affects the Level of Government Spending Relative to Oil Revenues, 'Laffer Curve'

According to the formula in (6.24)', we can show a similar result to the 'Laffer curve'. We use the same set of parameters values as before. Then, we start setting the personal income tax rate at 0% and then increases it by 5% to find out how the level of government spending relative to oil revenues responds on impact.

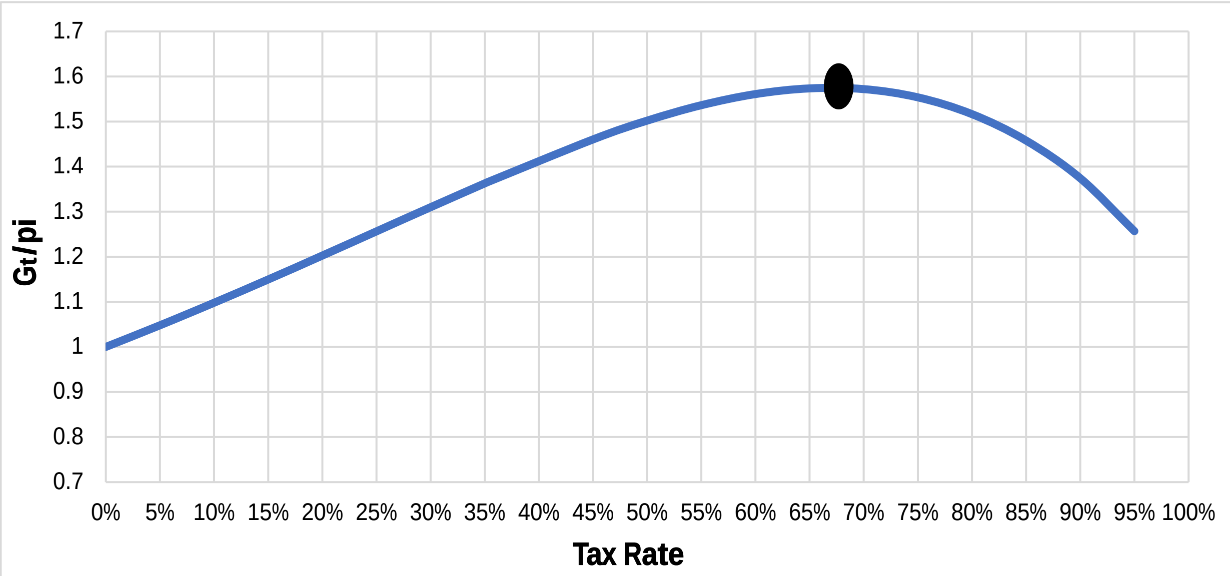

Figure 43: How the personal income tax affects the level of government spending relative to π_t

As discussed in section 6.5.2.1, when personal income tax is low, the level of government spending relative to oil revenues increases, and vice versa. According to our model and parameters values used, our simple numerical exercise shows in the above figure that the maximum value of personal income tax is $\tau^Y = 67\%$. That means that if $\tau^Y < 67\%$ ($\tau^Y > 67\%$), then the level of government spending relative to π_t would respond positively (negatively).

C.3 The Detailed Procedure for Analysing the Stability of Type *(I)* Steady-State

The capital accumulation equation can be written as,

$$\hat{k}_{t+1} - \left[\frac{\left(1-\tau^{Y} A\hat{k}_{t+1}^{\alpha}\right)}{g^{2}\left(1-\tau^{Y} A\hat{k}_{t}^{\alpha}\right)}\right]\left[\left(1-\tau^{Y}\right)A\hat{k}_{t}^{\alpha} + (1-\delta)\,\hat{k}_{t} - \hat{c}_{t}\right] = 0$$

where $\hat{y}_{t} = A\hat{k}_{t}^{\alpha}$ *and* $\hat{y}_{t+1} = A\hat{k}_{t+1}^{\alpha}$

Differentiation of the capital accumulation equation w.r.t. $\hat{k}_{t+1}, \hat{k}_{t}$, and $\hat{c}_{t}$, and evaluate them in the steady-state and then simplify them give us the first-order Taylor approximation of the capital accumulation equation around the steady-state,

$$(\hat{k}_{t+1} - \hat{k}^{*}) = \frac{\left(g^{2}\right)^{\sigma-1}\Psi + \alpha\beta\left(1-\Psi\right)}{\beta\left(\alpha+\Psi\left(1-\alpha\right)\right)}\,(\hat{k}_{t} - \hat{k}^{*}) - \frac{\Psi}{g^{2}\left(\alpha+\Psi\left(1-\alpha\right)\right)}\,(\hat{c}_{t} - \hat{c}^{*}) \qquad (6.28)$$

where $\Psi = 1-\tau^{Y} A\hat{k}^{*\alpha}$

On the other hand, the Euler equation can be written as,

$$\hat{c}_{t+1} = \left[\frac{\hat{c}_{t}\left(1-\tau^{Y} A\hat{k}_{t+1}^{\alpha}\right)}{g^{2}\left(1-\tau^{Y} A\hat{k}_{t}^{\alpha}\right)}\right]\left[\beta\left(\left(1-\tau^{Y}\right)\alpha A\hat{k}_{t+1}^{\alpha-1} + 1-\delta\right)\right]^{\frac{1}{\sigma}}$$

Similarly, differentiation the Euler equation w.r.t. $\hat{k}_{t+1}, \hat{k}_{t}$, and $\hat{c}_{t}$, and evaluate them in the steady-state and then simplify them give us the first-order Taylor approximation of the Euler equation around the steady-state,

$$(\hat{c}_{t+1} - \hat{c}^{*}) = \frac{\Upsilon\hat{k}^{*}\Psi - \alpha\hat{c}^{*}\left(1-\Psi\right)}{\hat{k}^{*}\Psi}\,(\hat{k}_{t+1} - \hat{k}^{*}) + \frac{\alpha\hat{c}^{*}\left(1-\Psi\right)}{\hat{k}^{*}\Psi}\,(\hat{k}_{t} - \hat{k}^{*}) + (\hat{c}_{t} - \hat{c}^{*}) \qquad (6.29)$$

where $\Upsilon = \dfrac{\left(\alpha-1\right)\alpha\hat{c}^{*}A\beta\left(1-\tau^{Y}\right)\hat{k}^{*\alpha-2}}{\sigma\left(g^{2}\right)^{\sigma}} < 0$

By substituting the approximation of the capital accumulation equation, equation (6.28), into the approximation of the Euler equation, equation (6.29), we obtain:

$$(\hat{c}_{t+1} - \hat{c}^*) = \frac{\left[\Upsilon\hat{k}^*\Psi - \alpha\hat{c}^*(1-\Psi)\right]\left[\left(g^2\right)^{\sigma-1}\Psi + \alpha\beta(1-\Psi)\right] + \left[\alpha\hat{c}^*(1-\Psi)\beta\left(\alpha+\Psi(1-\alpha)\right)\right]}{\hat{k}^*\Psi\beta\left(\alpha+\Psi(1-\alpha)\right)}(\hat{k}_t - \hat{k}^*)$$

$$+ \frac{\left[\hat{k}^* g^2\left(\alpha+\Psi(1-\alpha)\right)\right] - \left[\Upsilon\hat{k}^*\Psi - \alpha\hat{c}^*(1-\Psi)\right]}{\hat{k}^* g^2\left(\alpha+\Psi(1-\alpha)\right)}(\hat{c}_t - \hat{c}^*) \qquad (6.29)'$$

The linearised capital accumulation equation (6.28) and Euler equation (6.29)' can also be now expressed in matrix form as:

$$\begin{pmatrix} \hat{k}_{t+1} - \hat{k}^* \\ \\ \hat{c}_{t+1} - \hat{c}^* \end{pmatrix} = \underbrace{\begin{pmatrix} \dfrac{\left(g^2\right)^{\sigma-1}\Psi + \alpha\beta(1-\Psi)}{\beta\left(\alpha+\Psi(1-\alpha)\right)} & -\dfrac{\Psi}{g^2\left(\alpha+\Psi(1-\alpha)\right)} \\ \\ \dfrac{\left[\Upsilon\hat{k}^*\Psi - \alpha\hat{c}^*(1-\Psi)\right]\left[\left(g^2\right)^{\sigma-1}\Psi + \alpha\beta(1-\Psi)\right] + \left[\alpha\hat{c}^*(1-\Psi)\beta\left(\alpha+\Psi(1-\alpha)\right)\right]}{\hat{k}^*\Psi\beta\left(\alpha+\Psi(1-\alpha)\right)} & \dfrac{\left[\hat{k}^* g^2\left(\alpha+\Psi(1-\alpha)\right)\right] - \left[\Upsilon\hat{k}^*\Psi - \alpha\hat{c}^*(1-\Psi)\right]}{\hat{k}^* g^2\left(\alpha+\Psi(1-\alpha)\right)} \end{pmatrix}}_{matrix\,A} \begin{pmatrix} \hat{k}_t - \hat{k}^* \\ \\ \hat{c}_t - \hat{c}^* \end{pmatrix}$$

The determinant (*Det*) and the trace (*Tr*) of matrix A are the following:

$$Det = \left(\frac{\left(g^2\right)^{\sigma-1}\Psi + \alpha\beta(1-\Psi)}{\beta\left(\alpha+\Psi(1-\alpha)\right)}\right)\left(\frac{\left[\hat{k}^* g^2\left(\alpha+\Psi(1-\alpha)\right)\right] - \left[\Upsilon\hat{k}^*\Psi - \alpha\hat{c}^*(1-\Psi)\right]}{\hat{k}^* g^2\left(\alpha+\Psi(1-\alpha)\right)}\right)$$

$$-\left(-\frac{\Psi}{g^2\left(\alpha+\Psi(1-\alpha)\right)}\right)\left(\frac{\left[\Upsilon\hat{k}^*\Psi - \alpha\hat{c}^*(1-\Psi)\right]\left[\left(g^2\right)^{\sigma-1}\Psi + \alpha\beta(1-\Psi)\right] + \left[\alpha\hat{c}^*(1-\Psi)\beta\left(\alpha+\Psi(1-\alpha)\right)\right]}{\hat{k}^*\Psi\beta\left(\alpha+\Psi(1-\alpha)\right)}\right)$$

$$Det = \left(\frac{\left[\left(g^2\right)^{\sigma-1}\Psi + \alpha\beta(1-\Psi)\right]\left[\left[\hat{k}^* g^2\left(\alpha+\Psi(1-\alpha)\right)\right] - \left[\Upsilon\hat{k}^*\Psi - \alpha\hat{c}^*(1-\Psi)\right]\right]}{\beta\hat{k}^* g^2\left(\alpha+\Psi(1-\alpha)\right)^2}\right)$$

$$+\left(\frac{\left[\Upsilon\hat{k}^*\Psi - \alpha\hat{c}^*(1-\Psi)\right]\left[\left(g^2\right)^{\sigma-1}\Psi + \alpha\beta(1-\Psi)\right] + \left[\alpha\hat{c}^*(1-\Psi)\beta\left(\alpha+\Psi(1-\alpha)\right)\right]}{\beta\hat{k}^* g^2\left(\alpha+\Psi(1-\alpha)\right)^2}\right)$$

$$= \left(\frac{\left[\left(g^2\right)^{\sigma-1}\Psi + \alpha\beta(1-\Psi)\right]\left[\hat{k}^* g^2\left(\alpha+\Psi(1-\alpha)\right)\right] + \left[\alpha\hat{c}^*(1-\Psi)\beta\left(\alpha+\Psi(1-\alpha)\right)\right]}{\beta\hat{k}^* g^2\left(\alpha+\Psi(1-\alpha)\right)^2}\right)$$

$$= \left(\frac{\left(g^2\right)^{\sigma}\Psi\hat{k}^* + \alpha\beta(1-\Psi)\left[\hat{k}^* g^2 + \hat{c}^*\right]}{\beta\hat{k}^* g^2\left(\alpha+\Psi(1-\alpha)\right)}\right)$$

$$Tr = \frac{\left(g^2\right)^{\sigma-1}\Psi + \alpha\beta\left(1-\Psi\right)}{\beta\left(\alpha+\Psi\left(1-\alpha\right)\right)} + \frac{\left[\hat{k}^* g^2\left(\alpha+\Psi\left(1-\alpha\right)\right)\right] - \left[\Upsilon\hat{k}^*\Psi - \alpha\hat{c}^*\left(1-\Psi\right)\right]}{\hat{k}^* g^2\left(\alpha+\Psi\left(1-\alpha\right)\right)}$$

$$Tr = \frac{\hat{k}^*\Psi\left(g^2\right)^{\sigma} + \hat{k}^*\beta\left[g^2\alpha\left(1-\Psi\right)+g^2\left(\alpha+\Psi\left(1-\alpha\right)\right)-\Upsilon\Psi\right] + \beta\alpha\hat{c}^*\left(1-\Psi\right)}{\beta\hat{k}^* g^2\left(\alpha+\Psi\left(1-\alpha\right)\right)}$$

The eigenvalues of the matrix are the solutions to the characteristic equation,

$$\lambda^2 - Tr\,\lambda + Det = 0$$

$$\lambda^2 - \left(\frac{\hat{k}^*\Psi\left(g^2\right)^{\sigma} + \hat{k}^*\beta\left[g^2\alpha\left(1-\Psi\right)+g^2\left(\alpha+\Psi\left(1-\alpha\right)\right)-\Upsilon\Psi\right] + \beta\alpha\hat{c}^*\left(1-\Psi\right)}{\beta\hat{k}^* g^2\left(\alpha+\Psi\left(1-\alpha\right)\right)}\right)\lambda + \left(\frac{\left(g^2\right)^{\sigma}\Psi\hat{k}^* + \alpha\beta\left(1-\Psi\right)\left[\hat{k}^* g^2 + \hat{c}^*\right]}{\beta\hat{k}^* g^2\left(\alpha+\Psi\left(1-\alpha\right)\right)}\right) = 0$$

The necessary and sufficient condition for this system is:

$$\frac{a+b+c}{a-b+c}$$

$$= \frac{1-\left(\frac{\hat{k}^*\Psi\left(g^2\right)^{\sigma} + \hat{k}^*\beta\left[g^2\alpha\left(1-\Psi\right)+g^2\left(\alpha+\Psi\left(1-\alpha\right)\right)-\Upsilon\Psi\right] + \beta\alpha\hat{c}^*\left(1-\Psi\right)}{\beta\hat{k}^* g^2\left(\alpha+\Psi\left(1-\alpha\right)\right)}\right) + \left(\frac{\left(g^2\right)^{\sigma}\Psi\hat{k}^* + \alpha\beta\left(1-\Psi\right)\left[\hat{k}^* g^2 + \hat{c}^*\right]}{\beta\hat{k}^* g^2\left(\alpha+\Psi\left(1-\alpha\right)\right)}\right)}{1+\left(\frac{\hat{k}^*\Psi\left(g^2\right)^{\sigma} + \hat{k}^*\beta\left[g^2\alpha\left(1-\Psi\right)+g^2\left(\alpha+\Psi\left(1-\alpha\right)\right)-\Upsilon\Psi\right] + \beta\alpha\hat{c}^*\left(1-\Psi\right)}{\beta\hat{k}^* g^2\left(\alpha+\Psi\left(1-\alpha\right)\right)}\right) + \left(\frac{\left(g^2\right)^{\sigma}\Psi\hat{k}^* + \alpha\beta\left(1-\Psi\right)\left[\hat{k}^* g^2 + \hat{c}^*\right]}{\beta\hat{k}^* g^2\left(\alpha+\Psi\left(1-\alpha\right)\right)}\right)}$$

$$= \frac{\hat{k}^*\beta\,\Upsilon\Psi}{2\left[\beta\hat{k}^* g^2\left(\alpha+\Psi\left(1-\alpha\right)\right) + \hat{k}^*\Psi\left(g^2\right)^{\sigma} + \hat{k}^*\beta g^2\alpha\left(1-\Psi\right) + \beta\alpha\hat{c}^*\left(1-\Psi\right)\right] - \hat{k}^*\beta\,\Upsilon\Psi}$$

Since $\Upsilon < 0$, then the numerator is negative and less than the positive denominator.

C.4 Evaluating of h_1, h_2, h_3, j_1, j_2, and j_3

Starting with the capital accumulation equation, $\hat{k}_{t+1}$, which is implicitly determined by,

$$\frac{\hat{k}_{t+1}}{\left(1-\tau^{Y} A\hat{k}_{t+1}^{\alpha}\right)} = \frac{1}{g^2\left(1-\tau^{Y} A\hat{k}_{t}^{\alpha}\right)}\left[\left(1-\tau^{Y}\right)A\hat{k}_{t}^{\alpha} + (1-\delta)\,\hat{k}_{t} - \hat{c}_{t}\right]$$

we totally differentiate it:

$$\left\{\frac{\left(1-\tau^{Y} A\hat{k}_{t+1}^{\alpha}\right)+\alpha\tau^{Y} A\hat{k}_{t+1}^{\alpha}}{\left(1-\tau^{Y} A\hat{k}_{t+1}^{\alpha}\right)^{2}}\right\}d\hat{k}_{t+1} = \left\{-\frac{1}{g^2\left(1-\tau^{Y} A\hat{k}_{t}^{\alpha}\right)}\right\}.d\hat{c}_{t}$$

$$+\left\{\frac{\left(1-\tau^{Y} A\hat{k}_{t}^{\alpha}\right)\left[\left(1-\tau^{Y}\right)\alpha A\hat{k}_{t}^{\alpha-1}+1-\delta\right]+\alpha\tau^{Y} A\hat{k}_{t}^{\alpha-1}\left[\left(1-\tau^{Y}\right)A\hat{k}_{t}^{\alpha}+(1-\delta)\,\hat{k}_{t}-\hat{c}_{t}\right]}{g^2\left(1-\tau^{Y} A\hat{k}_{t}^{\alpha}\right)^{2}}\right\}.d\hat{k}_{t}$$

$$+\left\{\frac{A\hat{k}_{t}^{\alpha}\left[\left[\left(1-\tau^{Y}\right)A\hat{k}_{t}^{\alpha}+(1-\delta)\,\hat{k}_{t}-\hat{c}_{t}\right]-\left(1-\tau^{Y} A\hat{k}_{t}^{\alpha}\right)\right]}{g^2\left(1-\tau^{Y} A\hat{k}_{t}^{\alpha}\right)^{2}} - \frac{A\hat{k}_{t+1}^{\alpha+1}}{\left(1-\tau^{Y} A\hat{k}_{t+1}^{\alpha}\right)^{2}}\right\}.d\tau^{Y}$$

We know that $\hat{k}_{t+1} = j\left(\underbrace{\hat{k}_t}_{j_1}, \underbrace{\hat{c}_t}_{j_2}, \underbrace{\tau^Y}_{j_3}\right)$. Thus, we evaluate j_1, j_2, j_3 in the steady-state [127]:

$$j_1 \equiv \frac{d\hat{k}_{t+1}}{d\hat{k}_{t}} = \frac{\dfrac{\left(1-\tau^{Y} A\hat{k}_{t}^{\alpha}\right)\left[\left(1-\tau^{Y}\right)\alpha A\hat{k}_{t}^{\alpha-1}+1-\delta\right]+\alpha\tau^{Y} A\hat{k}_{t}^{\alpha-1}\left[\left(1-\tau^{Y}\right)A\hat{k}_{t}^{\alpha}+(1-\delta)\,\hat{k}_{t}-\hat{c}_{t}\right]}{g^2\left(1-\tau^{Y} A\hat{k}_{t}^{\alpha}\right)^{2}}}{\dfrac{\left(1-\tau^{Y} A\hat{k}_{t+1}^{\alpha}\right)+\alpha\tau^{Y} A\hat{k}_{t+1}^{\alpha}}{\left(1-\tau^{Y} A\hat{k}_{t+1}^{\alpha}\right)^{2}}}$$

$$j_1 \equiv \frac{d\hat{k}_{t+1}}{d\hat{k}_{t}} = \underbrace{\frac{\left(g^2\right)^{\sigma-1}+\tau^{Y} A\hat{k}^{*\alpha}\left(\alpha\beta-\left(g^2\right)^{\sigma-1}\right)}{\beta\left[1+(\alpha-1)\tau^{Y} A\hat{k}^{*\alpha}\right]}}_{\textit{In the steady-state}}$$

---

[127] At the steady-state, we know that $\hat{k}_t = \hat{k}_{t+1} = \hat{k}^*$, $\hat{k}^* = \frac{1}{g^2}\left[(1-\tau^Y)A\hat{k}^{\alpha} + (1-\delta)\hat{k}^* - \hat{c}^*\right]$ and $g^2 = \left[\beta\left((1-\tau^Y)\alpha A\hat{k}^{\alpha-1} + 1 - \delta\right)\right]^{\frac{1}{\sigma}}$.

$$j_2 \equiv \frac{d\hat{k}_{t+1}}{d\hat{c}_t} = \frac{-\dfrac{1}{g^2\left(1-\tau^Y A\hat{k}_t^\alpha\right)}}{\dfrac{\left(1-\tau^Y A\hat{k}_{t+1}^\alpha\right)+\alpha\tau^Y A\hat{k}_{t+1}^\alpha}{\left(1-\tau^Y A\hat{k}_{t+1}^\alpha\right)^2}} = \underbrace{-\frac{\left(1-\tau^Y A\hat{k}^{*\alpha}\right)}{g^2\left[1+(\alpha-1)\tau^Y A\hat{k}^{*\alpha}\right]}}_{\text{In the steady-state}}$$

$$j_3 \equiv \frac{d\hat{k}_{t+1}}{d\tau^Y} = \frac{\dfrac{A\hat{k}_t^\alpha\left[\left[\left(1-\tau^Y\right)A\hat{k}_t^\alpha+(1-\delta)\,\hat{k}_t-\hat{c}_t\right]-\left(1-\tau^Y A\hat{k}_t^\alpha\right)\right]}{g^2\left(1-\tau^Y A\hat{k}_t^\alpha\right)^2}-\dfrac{A\hat{k}_{t+1}^{\alpha+1}}{\left(1-\tau^Y A\hat{k}_{t+1}^\alpha\right)^2}}{\dfrac{\left(1-\tau^Y A\hat{k}_{t+1}^\alpha\right)+\alpha\tau^Y A\hat{k}_{t+1}^\alpha}{\left(1-\tau^Y A\hat{k}_{t+1}^\alpha\right)^2}} = \underbrace{-\frac{A\hat{k}^{*\alpha}\left(1-\tau^Y A\hat{k}^{*\alpha}\right)}{g^2\left[1+(\alpha-1)\tau^Y A\hat{k}^{*\alpha}\right]}}_{\text{In the steady-state}}$$

On the other hand, we totally differentiate equation (6.32) to find h_1, h_2, h_3. Then, we evaluate them in the steady-state:

$$g^2\left(1-\tau^Y A\hat{k}_t^\alpha\right)-\left(1-\tau^Y A\hat{k}_{t+1}^\alpha\right)\left[\beta\left(\left(1-\tau^Y\right)\alpha A\hat{k}_{t+1}^{\alpha-1}+1-\delta\right)\right]^{\frac{1}{\sigma}} = 0$$

$$h_1:\ \left\{-\alpha g^2\tau^Y A\hat{k}_t^{\alpha-1}\right\}.d\hat{k}_t = \underbrace{-\alpha g^2\tau^Y A\hat{k}^{*\alpha-1}}_{\text{In the steady-state}}$$

$$h_2:\ \left\{\begin{array}{c}\alpha\tau^Y A\hat{k}_{t+1}^{\alpha-1}\left[\beta\left(\left(1-\tau^Y\right)\alpha A\hat{k}_{t+1}^{\alpha-1}+1-\delta\right)\right]^{\frac{1}{\sigma}}\\ -\dfrac{\left(1-\tau^Y A\hat{k}_{t+1}^\alpha\right)(\alpha-1)\alpha A\left(1-\tau^Y\right)\hat{k}_{t+1}^{\alpha-2}\left[\beta\left(\left(1-\tau^Y\right)\alpha A\hat{k}_{t+1}^{\alpha-1}+1-\delta\right)\right]^{\frac{1}{\sigma}}}{\sigma\left(\left(1-\tau^Y\right)\alpha A\hat{k}_{t+1}^{\alpha-1}+1-\delta\right)}\end{array}\right\}.d\hat{k}_{t+1}$$

$$= \underbrace{\frac{\sigma\alpha\tau^Y A\hat{k}^{*\alpha-1}\left(g^2\right)^\sigma-\left(1-\tau^Y A\hat{k}^{*\alpha}\right)(\alpha-1)\alpha A\left(1-\tau^Y\right)\hat{k}^{*\alpha-2}\beta}{\sigma\left(g^2\right)^{\sigma-1}}}_{\text{In the steady-state}}$$

$$h_3:\ \left\{\begin{array}{c}\dfrac{\left(1-\tau^Y A\hat{k}_{t+1}^\alpha\right)\alpha A\hat{k}_{t+1}^{\alpha-1}\left[\beta\left(\left(1-\tau^Y\right)\alpha A\hat{k}_{t+1}^{\alpha-1}+1-\delta\right)\right]^{\frac{1}{\sigma}}}{\sigma\left(\left(1-\tau^Y\right)\alpha A\hat{k}_{t+1}^{\alpha-1}+1-\delta\right)}\\ +A\hat{k}_{t+1}^\alpha\left[\beta\left(\left(1-\tau^Y\right)\alpha A\hat{k}_{t+1}^{\alpha-1}+1-\delta\right)\right]^{\frac{1}{\sigma}}-g^2A\hat{k}_t^\alpha\end{array}\right\}.d\tau^Y = \underbrace{\frac{\left(1-\tau^Y A\hat{k}^{*\alpha}\right)\alpha A\hat{k}^{*\alpha-1}\beta}{\sigma\left(g^2\right)^{\sigma-1}}}_{\text{In the steady-state}}$$

C.5 How $\hat{k}_t$ Would Jump on Impact Period?

The definition of the capital PGSU can be written as,

$$\hat{k}_t \equiv \frac{K_t}{G_t} = \frac{K_t}{\pi_t + \tau^Y Y_t} \qquad (A)$$

where Y_t is implicitly determined by:

$$Y_t = AK_t^{\alpha}\left(\pi_t + \tau^Y Y_t\right)^{1-\alpha} \qquad (B)$$

From (B), we apply the implicit differentiation method to get $dY_t/d\tau^Y$:

$$\frac{dY_t}{d\tau^Y} = \frac{(1-\alpha)AK_t^{\alpha}Y_t}{\left(\pi_t + \tau^Y Y_t\right)^{\alpha} - (1-\alpha)AK_t^{\alpha}\tau^Y} \qquad (C)$$

To determine the sign of $dY_t/d\tau^Y$, we first multiply the numerator and denominator of equation (*C*) by $(\pi_t + \tau^Y Y_t)^{-\alpha}$:

$$\frac{dY_t}{d\tau^Y} = \frac{(1-\alpha)AK_t^{\alpha}Y_t\left(\pi_t + \tau^Y Y_t\right)^{-\alpha}}{1 - (1-\alpha)AK_t^{\alpha}\tau^Y\left(\pi_t + \tau^Y Y_t\right)^{-\alpha}}$$

Then, when substituting out $(\pi_t + \tau^Y Y_t)^{-\alpha}$ as $Y_t/AK_t^{\alpha}(\pi_t + \tau^Y Y_t)$ by using (*B*) and simplifying it, we obtain,

$$\frac{dY_t}{d\tau^Y} = \frac{(1-\alpha)(Y_t)^2}{\pi_t + \alpha\tau^Y Y_t} > 0 \qquad (C)'$$

By using the product rule, we obtain:

$$\frac{d\left(\tau^Y Y_t\right)}{d\tau^Y} = \frac{d\tau^Y}{d\tau^Y}Y_t + \tau^Y\frac{dY_t}{d\tau^Y} = Y_t + \tau^Y \underbrace{\frac{dY_t}{d\tau^Y}}_{=(C)'} \qquad (D)$$

Now, we use the expression of $\hat{k}_t$ in equation *(A)* to get its derivative w.r.t. τ^Y (*i.e.* $d\hat{k}_0/d\tau^Y$). However, we should note that Y_t also depends on τ^Y as shown in the implicit function. Thus, when we take the derivative of the expression of $\hat{k}_t$, we need to take the derivative of the product $(\tau^Y Y_t)$. Then,

$$\frac{d\hat{k}_0}{d\tau^Y} = \frac{d}{d\tau^Y} \cdot \left(\frac{K_t}{\pi_t + \tau^Y Y_t}\right) = -\frac{K_t}{\left(\pi_t + \tau^Y Y_t\right)^2} \cdot \underbrace{\frac{d\left(\tau^Y Y_t\right)}{d\tau^Y}}_{=(D)}$$

$$\frac{d\hat{k}_0}{d\tau^Y} = -\frac{K_t Y_t}{\left(\pi_t + \tau^Y Y_t\right)\left(\pi_t + \alpha\tau^Y Y_t\right)} \tag{E}$$

By using the government budget constraint, $G_t = \pi_t + \tau^Y Y_t$, the definition of capital PGSU, $\hat{k}_t = \frac{K_t}{G_t}$, the production function, $Y_t = AK_t^{\alpha} G_t^{1-\alpha}$, and the fact that $\frac{\pi_t}{G_t} = 1 - \tau^Y \hat{y}_t = 1 - \tau^Y A\hat{k}_t^{\alpha}$, we can rewrite ($E$) after simplifying it as:

$$\frac{d\hat{k}_0}{d\tau^Y} = -\frac{\hat{k}_t}{\underbrace{\left(1/A\hat{k}_t^{\alpha}\right) - \left(1-\alpha\right)\tau^Y}_{(+)}} \tag{E)$'$($}$$

As a result, since $dY_t/d\tau^Y > 0$ and then $d(\tau^Y Y_t)/d\tau^Y > 0$, we conclude that $d\hat{k}_0/d\tau^Y < 0$.

C.6 Why the Growth Rate of Government Spending Drops on Impact in the Case of Rising the Personal Income Tax

This section discusses the reason behind the low growth rate at the moment of impact period of rising personal income tax. When we set g^2 to be high (*e.g.* $g^2 = 1.1$), the initial downwards jump in $\hat{k}_t$ falls by less than the steady-state value on the impact period. Thus, to see the impact on the growth rate of government spending, we look closely at its equation,

$$\gamma_{G_t} \equiv \frac{G_{t+1}}{G_t} = \underbrace{\frac{\pi_{t+1} + \tau^Y Y_{t+1}}{\pi_t + \tau^Y Y_t}}_{Formula\ (A)} = \underbrace{g^2 \frac{\left(1-\tau^Y \hat{y}_t\right)}{\left(1-\tau^Y \hat{y}_{t+1}\right)}}_{Formula\ (B)} ,$$

which has two correct formulas that can describe it on the transition path to a type (*I*) steady-state. However, the growth rate at the moment of the shock is not correctly represented by these formulas. To see that, let us begin with the formula (A).

Formula (A) came from the government budget constraint, $G_t = \pi_t + \tau^Y Y_t$, and we know that π_t grows at the constant rate, g^2, which is here $g^2 = 1.1$. Now, suppose that the impact period in which the tax rate increased is t=2. Then,

$$\frac{G_2}{G_1} = \frac{\pi_2 + \tau^Y Y_2}{\pi_1} > \frac{\pi_2}{\pi_1} \left(= g^2\right) \qquad (A)'$$

This is true because, by assumption, there is no tax in period one, t=1. Therefore, by this reasoning, the growth rate of government spending in the impact period should be greater than g^2 (*i.e.* not less than g^2). However, it should be emphasised that formula (A) does not tell us anything about the time path because it is only about the impact period. As a result, the growth rate of government spending should increase in the impact period.

On the other hand, formula (B) is consistent with our simulation, shown in Figure 41. In fact, the consistency could not be obvious until we look at the transition path for $\hat{k}_t$. In the simulation, $\hat{k}_t$ (and hence $\hat{y}_t$) is falling over time. This implies that $\hat{y}_t > \hat{y}_{t+1}$, and then:

$$\gamma_{G_t} = g^2 \frac{\left(1\text{-}\tau^Y \hat{y}_t\right)}{\left(1\text{-}\tau^Y \hat{y}_{t+1}\right)} < g^2 \qquad (B)'$$

Thus, formula (B) tells us that the growth rate of government spending should decrease in the impact period, such that,

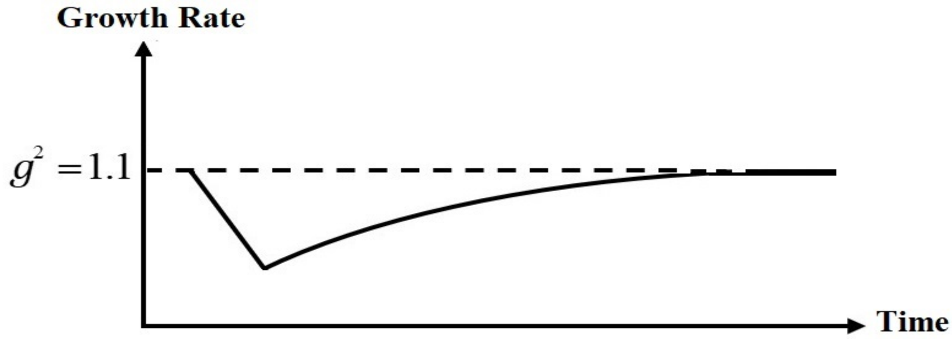

Diagram 33: The time path of the growth rate of government spending, using formula (B)

Now, we are facing a mystery regarding the effect on the growth rate of government spending at the moment of the impact period. Thus, we need to understand carefully the above two formulas, (A) and (B), in the impact period.

It should be noted that the definition of γ_{G_t}, given in formula (B)', is forward-looking. It means that γ_{G_t} measures the growth rate of government spending between period (t) and period (t+1), *i.e.* not between period (t-1) and period (t). Therefore, the first instance of γ_{G_t} to deviate from g^2, given that the tax rate increases in t=2, is not γ_{G_2}, but γ_{G_1}. The reason can be seen in formula (A)', *i.e.* $\gamma_{G_2} \equiv G_2/G_1 = [(\pi_2 + \tau^Y Y_2)/\pi_1] > g^2$. As a result, the graph of the time path for government spending should be like:

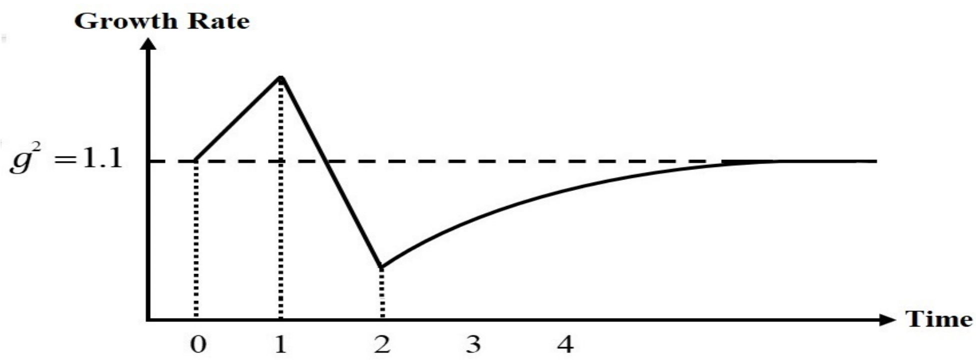

Diagram 34: The time path of the growth rate of government spending

However, it was difficult to make Dynare correctly produce the growth rate of government spending in the impact period, like Diagram 34, because it showed that there is a missing data point in our simulation. The reason for the missing data point is due to that we used formula (B), which could not show the start of the time path because it assumes perfect foresight. Thus, there is in the impact period a one-period departure from perfect foresight, which could be recognised as the missing data point. To solve this issue, we calculated the growth rate of government spending in the impact period by using formula (A), and then used the value obtained, with the remaining values for the growth rate generated by Dynare[128], to plot the below graph for the growth rate of government spending.

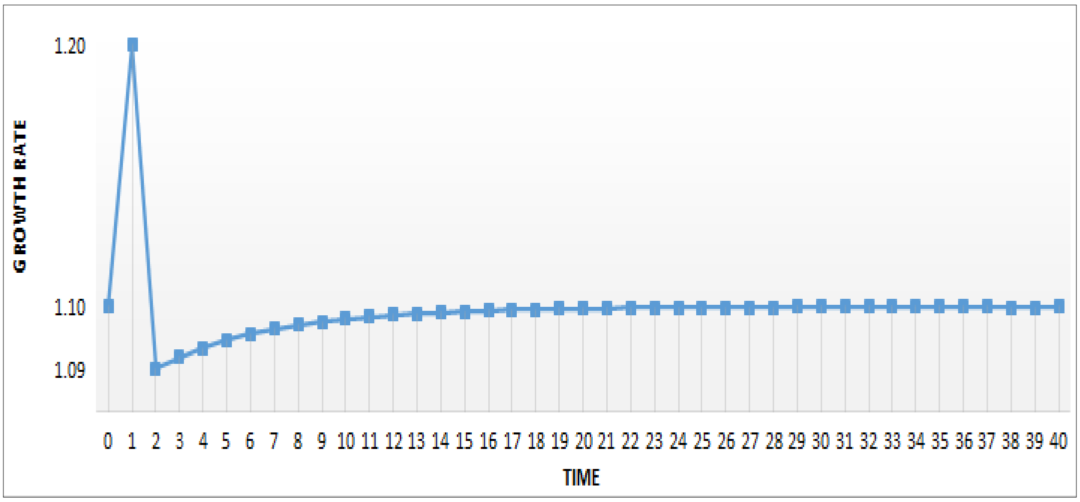

Figure 44: The simulation for the path of the growth rate of government spending

[128] We used formula (B) in Dynare to obtain these values.

Abbreviations

AD	Aggregate Demand
ARDL	Autoregressive Distributed Lag
B-K	Blanchard and Kahn
BOT	Balance of Trade
BP	The British Petroleum Company
BPD	Barrels Per Day
BPY	Barrels Per Year
CA	Current Account
CEMAC	The Central African Economic and Monetary Community
CRRA	Constant Relative Risk Aversion
CRS	Constant Returns to Scale
DE	Direct Effect
Det	The determinant of a Matrix
DRS	Decreasing Returns to Scale
DSGE	Dynamic Stochastic General Equilibrium
FDI	Foreign Direct Investment
GAZT	The General Authority of Zakat & Tax
GCC	The Gulf Cooperation Council
GDP	Gross Domestic Product
GNP	Gross National Product
IE	Indirect Effect
IES	Intertemporal Elasticity of Substitution
IMF	The International Monetary Fund
IPO	Initial Public Offering
IRS	Increasing Returns to Scale
LHS	Left-Hand Side

Mbbl	One Thousand Barrels
MC	Marginal Cost
MMbbl	One Million Barrels
MR	Marginal Revenue
NFA	Net Foreign Assets
NIE	Net Indirect Effect
OLS	Ordinary Least Squares
OPEC	The Organization of the Petroleum Exporting Countries
PGSU	Per Government Spending Unit
PIF	The Public Investment Fund
R&D	Research and Development
RHS	Right-Hand Side
ROW	Rest of the World
SAMA	The Saudi Arabian Monetary Authority
SWF	Sovereign Wealth Fund
TC	Total Cost
TFP	Total Factor Productivity
TR	Total Revenue
Tr	Trace of a Matrix
VAR	Vector Auto Regression
VAT	Value-Added Tax
VECM	Vector Error Correction Model

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