Essays on Optimal Monetary Policy

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Abstract

The motivation for this thesis is to examine optimal monetary policy under alternative scenarios. Different models are applied, which incorporates both theoretical and practical methods. The thesis strives to address several questions in DSGE models: (1) Which instrument should central banks select, interest rate pegging, or constant growth rate of the money supply? (2) How should monetary and macroprudential policies be conducted when policy makers fear that their model is misspecified? (3) What is the welfare gain from a commitment when using unconventional monetary policy? Since financial frictions are being increasingly included in models that analyse monetary issues, the roles of macroprudential policy and model uncertainty are highly relevant today.

The research mainly consists of three essays. The first essay evaluates Poole (1970)'s analysis within a modified Smets and Wouters (2003) model. Previous works only incorporate limited features for central banks' monetary policy choices and instead of; most models are calibrated. I estimate a micro-founded model using Bayesian methods to assess the relative desirability of using the interest rate or the money supply as the monetary policy instrument. The result I find that monetary targeting fares better under fiscal shock when intertemporal substitution is high. Monetary targeting is also favoured under technology shock, preference shock, and labour supply shock.

The second essay analyses a DSGE model with financial frictions when when the macroprudential authority fears model misspecification. The robust control method is applied to a macroprudential policy to overcome model uncertainty. Under commitment to optimal macroprudential policy, I find that the robust central bank responds more aggressively than it in rational expectations case. The aggressiveness is mainly because the central bank fears that the impacts of shocks to economic variables are more persistent than they appear to be.

The third essay quantifies the welfare gains when unconventional monetary policy operates under commitment as opposed to under discretion. I find that the highest gain from commitment is obtained when credit intervention is small. However, the high intensity of the credit intervention policy generates the most stable consumption. The results imply that

the central bank has an incentive to discrete from a single interest rate policy instrument to the other policy instrument that combines with interest rate and credit intervention. This central bank's discretion behaviour would be more evident under financial shocks.

The results presented in each of the chapters suggest that both monetary and macro-prudential policies are important. However, their design and implementation should acknowledge the limitations that arise from the policy instrument being used, the awareness that models may be omitting important mechanisms and the fact that policy makers are generally regarded as unable to commit.

Declaration

I certify that the thesis I have submitted for examination for the award of a Ph.D. degree by the University of Sheffield is solely my own work, other than where I have clearly indicated otherwise (in which case the extent of any work produced jointly by myself and any other person is clearly identified in it).

I warrant that this authorisation does not, to the best of my knowledge, infringe the rights of any third party.

I certify that this thesis consists of approximately 44,649 words.

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Chapter 1

Introduction

1.1 Motivation and Objectives

This thesis mainly studies optimal conventional and unconventional monetary policy in different DSGE frameworks. There are three topics involved in the whole thesis. First, the Great Recession highlighted some of the limitations of using the interest rate as the policy instrument when faced with the effective lower bound (ELB). In this respect, the work of Poole (170) helped shape monetary operating procedures across many central banks. When faced with highly volatile velocity the work of Poole (1970) suggested moving towards interest rate targeting, whereas when the reverse was true the targeting of monetary aggregates was put forward as the preferable procedure. Even though most studies have ignored the monetary terms in the model specifications, the others have also argued the important role of monetary aggregates. The original Poole analysis was conducted using an IS-LM model that lacked micro-foundations. It is natural then to ask whether and how the conclusions of the original analysis carry over to a micro-founded model. Second, the Great Recession (GR) led central banks to implement macroprudential policy to buffer financial risks. However, given that the implementation of these instruments is fairly recent, their full implications may not be fully understood. Consequently, it would be analyse how a policy maker should behave when she fears that her model may be misspecified. I use robust control methods to model such fear of misspecification. Third, it is generally accepted that policy makers are unable to commit to and that this results in (discretionary) outcomes that are not first-best. However, the losses that arise from discretionary policy have not been considered in models that embody financial frictions and the final chapter sheds new light on this topic. To better understand the implementation of monetary policy with financial frictions, the welfare analysis of unconventional monetary policy with credit intervention allows us to examine the policy differences between commitment and discretion. As policymaking in the future is affected by the forward-looking firms and household's choices today, it is of importance to investigate the degree of welfare influences under commitment.

The structure of the thesis is as follows. Chapter 2 explores the appropriateness and impacts of a central bank's monetary policy instruments in a modified DSGE model. Chapter 3 uses the robust control method to analyse the macroprudential policy in a richer DSGE model within the housing market sector. Chapter 4 analyses the welfare gains from commitment under the optimal unconventional monetary policy. The remainder of this thesis provides an overview of each chapter of the essay in the thesis.

1.1.1 Chapter 2 Overview

The primary purpose of this chapter is to investigate Poole's analysis through a medium-scale New Keynesian model. According to the arguments of McCallum (1988) and Nelson (2003), monetary aggregates still play a fundamental role in the monetary policy. They argued that the monetary base instrument generated zero inflation in 1954-85 in the US and also dropped the volatility in nominal output. New Keynesian models that ignored the money terms are found to be inappropriate. Hoffmann and Kempa (2009) also investigated the DSGE model with an open economy to re-study Pool's analysis. They found some crucial results relating to monetary aggregates. The interest rate instrument was affected by lower bound issues, which make it challenging for policymakers to implement the interest rate instrument. Therefore, it is of our interest to re-study the alternative instruments in Poole's analysis to check for the optimal instrument.

In this chapter, we examine the optimal choice by using a modified Smets and Wouters (2003) model under nine structural shocks. Those nine shocks include three cost-push shocks (price mark-up shock, wage mark-up shock, and equity premium shock), three demand shocks (preference shock, investment shock, and government spending shock), preference shock, labour supply shock and monetary policy shock. The significant difference between ours and the Smets and Wouters (2003) model is that we apply a non-separable money-in-utility function by incorporating money demand shock. In this setup, the monetary term can affect the marginal cost, IS curve and then the Phillips curve. The main contribution to the Collard and Dellas (2005)'s research is that this chapter evaluates Poole's instrument ranking through the Smets and Wouters (2003) estimated model. Collard and Dellas (2005) compared the policy instruments through a calibrated model, whereas, we apply the welfare that is derived from the estimated model with full structural shocks.

Our main result shows that the ranking varies under different shocks. Constant money supply targeting fares better under technology shock, preference shock, and labour supply shock independent of the degree of risk aversion. Interest rate targeting is favoured when intertemporal substitution is low under fiscal shock and equity premium shock, and it is selected when intertemporal substitution is high under price mark-up shock. Besides, the performance of monetary targeting improves when risk aversion is high, and it is ambiguous when risk aversion is low. The interest rate policy instrument is chosen under wage mark-up shock regardless of the degree of intertemporal substitution. Although the interest rate targeting is still a more popular instrument, the analysis in this paper has found the importance of monetary aggregates under some disturbances.

1.1.2 Chapter 3 Overview

The macroprudential policy has been extensively studied by a large number of researchers in recent years. Most studies applied the DSGE model with financial frictions; however, few of them have considered whether the models are misspecified. Therefore, in this chapter, we use the robust control method to the Kannan et al. (2012) (KRS) model, which is a DSGE model that include macroprudential policy and the housing market booms.

Historical research on model uncertainty has mostly been carried out based on the Brainard (1967) Bayesian technique. However, this method requires that central banks know a prior probability distribution over models. The later minimax approach allows central banks to minimise the maximised worst-case scenario. There are also some other studies that investigate the robust control method, such as Hansen and Sargent (2001)and Onatski and Stock (2002). These approaches are too complicated to calculate as it has to be written in the state-space form. Through Dennis et al. (2009)'s structural form, the robust control method, the model is assumed to commit to the optimal monetary policy.

The reasons we choose the KRS model in for the primary model is because of the following reasons. First, the KRS model includes both durables and non-durables, borrowers and savers, different agents. These are fundamental for rational expectation models with forward-looking behaviours. Second, the model applied the DSGE framework with financial accelerator effects to capture macroprudential policies effects. Third, the KRS model performs well in macroeconomic stability under reasonably different parameter values. Fourth, the KRS result also suggested that it is crucial to identify the source of house price booms and financial conditions for the design of the monetary and macroprudential policy. Therefore, it is reasonable for us to dig further to ascertain if model uncertainty can affect the macroprudential policy model.

Our comparison under different scenarios yielded some exciting results. Under rational expectations, when the macroprudential policy is implemented, the central bank reacts less active than monetary policy alone as the new instrument could offset the financial shock when it is implemented. In addition, under the approximating equilibrium, the robust central bank applies the policy more aggressive than in the case of the rational expectation as it fears the response to the shocks is more persistent than it appears. Under the worst-case equilibrium, the central bank's responses are more volatile than the approximating equilibrium. Moreover, when the macroprudential policy is included in the model, it always offsets the influences of financial shocks, regardless of the robust control method.

Overall, when the robust control method is applied to the macroprudential policy, the central bank fears that shocks' impacts are more significant than it shows, therefore, responds with a more aggressive macroprudential policy.

1.1.3 Chapter 4 Overview

The unconventional monetary policy became a fundamental policy for central banks after the financial crisis of 2007-08. Most economists studied the different formats of unconventional monetary policy models, such as DSGE models, financial accelerator models, or DSGE models with some crucial characteristics from unconventional monetary policy. The importance of commitment was also extensively accepted by the public. One channel of transmission in unconventional monetary policy is called signalling channel, where central banks communicate and inform the public about its policy on short-term interest rates and other policies. This could affect public expectations about the economy. Therefore, it is essential to understand the commitment problems in the unconventional monetary policy model. In this third essay, we quantify the welfare gain from the commitment with the unconventional monetary policy model under various shocks.

For the unconventional monetary policy, we study the Gertler and Karadi (2011) model as this model is a pioneer study that explained the financial crisis and unconventional monetary policy through financial accelerators to various shocks. For the commitment analysis, we use the Dynare Ramsey method to calculate the first-order conditions to obtain welfare. On the other hand, discretionary analysis is more complicated. We choose the Covariance Matrix Adaptation Evolution Strategy (CMAES) approach to calculate the second-order approximations for the non-linear model. This method allows up to find optimal parameters of output and inflation coefficients, and the welfare loss under discretion as well. Similar to Dennis and Söderström (2006), we select two alternative measures that are applied to the model to distinguish the differences: one is the percentage gain welfare calculation, and the other is the consumption equivalence method.

There are some findings after the comparisons between different initial parameters and model comparisons. First for the financial accelerator model with credit intervention, the capital quality shock provides more substantial influences for the whole transmission mechanism than the impacts of other shocks. Second, the most affected variables are leverage ratio, net worth, investment, and asset price. Third, the intensity of the credit policy feedback parameter affects investment through leverage ratio, premium, and net worth. Fourth, the highest gain from commitment is obtained when credit intervention is small. However, to have the most stable consumption equivalent requires a higher intensity of credit intervention of around 50. The different levels of a diverted fraction of capital show a decreasing pattern in the gain from commitment as the fraction increases from 0.15 to 0.38. The outcome indicates that the central banks would always have an incentive to deviate from the single interest rate instrument to the policy with credit intervention, especially during the financial crisis period, when deterioration of the balance sheet occurs due to capital quality shock.

For the financial accelerator model, the capital quality shock is crucial in the economy as it affects financial frictions, such as interest rate spread, leverage ratio, and net worth. Under a discretionary regime, the shocks produce fewer effects on investment. The diverted fraction of capital and survival rate for intermediaries show similar patterns in the gains from commitment and consumption equivalent.

Chapter 2

Poole's Analysis in a Medium-scale DSGE Model

2.1 Introduction

A substantial number of researchers have attempted to unearth the optimal choice of monetary policy instruments: interest rate pegging or constant growth rate money supply. The issue was analysed early by Poole (1970) and recently updated with a calibrated model by Collard and Dellas (2005) (henceforth CD). However, this issue has been neglected in the past decade after the Taylor (1993) rule had become a more general policy rule in literature research. As suggested by McCallum (1988) and Nelson (2003), monetary aggregates are still crucial for monetary policy. Interest rate instruments are becoming more problematic when it can provide limited impacts on economics after the financial crisis. Therefore, it is fundamental to recapture Poole's analysis to check for the optimal instrument.

This paper will investigate Poole's analysis through a medium-scale New Keynesian model, as known as the Smets and Wouters (2003)(hereafter SW) model with non-separable money-in-utility form, with nine structural shocks. The SW model was chosen for the following reasons. First, modified SW includes a set of structural shocks: three cost-push shocks (price mark-up shock, wage mark-up shock, and equity premium shock), three "demand shocks" (preference shock, investment shock, and government spending shock), preference shock, labour supply shock and monetary policy shock. Second, the SW model uses the Bayesian framework, and it has been suggested that its estimation with structural disturbances obtain a good fit to the data as well as the conventional Vector Auto Regressions (VARs) method. Third, it has also been suggested that the SW model performs well for monetary policy analysis through welfare loss comparisons, since it is capable of capturing time-series data's properties. (Onatski et al. (2004)) Therefore, the adjusted SW model with more structural shocks could provide a well-performed analysis for alternative monetary policy instruments.

I extend the SW model to include real money balances in the utility function. In this case, money demand shock would impose an effect on the marginal utility of consumption and real wages. Then marginal cost, IS curve and the Phillips curve could also be affected. This had been suggested by Ireland (2001) who estimated the simple monetary business cycle model. Andrés et al. (2006) also examined the business cycle model for real balances under non-separability and separability in the utility function. The result indicates that real balances have impacts on output and inflation but plays a minor role in the model itself. It is for our interest to propose a more general model with non-separable money in a utility form to estimate the money demand shock since this may be an important factor when assessing the relative desirability of using a monetary aggregate as the policy instrument.

In contrast the Poole (1970) and Collard and Dellas (2005), I examine the relative performance of the money supply and the nominal interest rate as policy instruments using a model where the parameters are estimated. The comparison relies on the model-relevant welfare function. I derive welfare function to obtain the comparisons between not only from the volatility of output but also inflation, welfare and other variables. Under a technology shock, a constant money supply instrument is preferable as stability in output and consumption as well as in higher welfare. A preference shock favours a monetary targeting procedure as it stabilises output and generates higher output as well as consumption. Interest rate targeting fares better when intertemporal substitution is low under fiscal shock and an equity premium shock. Interest rate targeting is also preferable under wage mark-up shock independent of the degree of risk aversion, and when intertemporal substitution is high under price mark-up shock. Under the investment shock, monetary targeting fares better when risk aversion is high and is ambiguous when risk aversion is low. Therefore, the ranking and intertemporal substitution differ under some shocks.

The remainder of this paper is structured as follows—the second part reviews related literature. The third part presents the modified SW model. The fourth part demonstrates the data and methodology with the estimated results. The fifth part examines the welfare analysis for alternative policy instruments. Finally, a conclusion will come at the end.

2.2 Literature Review

For the study of optimal monetary policy instrument between the interest rate policy instrument and the money supply instrument, it has mainly split into the theoretical and empirical research. In terms of theoretical study, there is an abundance of related papers during the 1970s and 1980s. Initially, Poole (1970) had studied one of the most notable pieces in the literature for the optimal choice of the monetary policy instrument problem. Under their assumption, when the central bank targets the interest rate, the money supply is adjusted to clear the deviations of the interest rate. When the money supply is targeted, the growth rate of the money supply is constant, and then the interest rate adjusts in order to clear the movement of monetary aggregates. Poole (1970) analysed the stochastic model in a simple IS-LM framework and compared the output volatility, finding that money demand shocks favour the interest rate targeting procedure and monetary targeting is advocated when fiscal shocks dominate the economy. However, Poole's simple framework analysis has some limitations that were pointed out by later works.

Moore (1972) also gives an exposition to this instrument problem. Through the survey of earlier research on the values of lagged and behavioural parameters, the study readdressed Poole's simple IS-LM framework, including both dynamic and stochastic relationships. Turnovsky (1975) studied and extended the instrument issue with a simple linear dynamic model comprising of both disturbances and stochastic coefficients. The result suggested that if there are only output and interest rate stochastic parameters in the monetary sector, then the interest rate instrument is more appealing. Later in the 80s, more extensions on the instrument problem have been studied. Yoshikawa (1981) considers the issue in a stochastic Keynesian model. He found that the factors affecting the money supply instrument include elasticity of the money demand and the influences on the stability of the model. The paper points out that under uncertainty the main target of the monetary policy is to accommodate shocks by modifying the growth rate of money and making the variance of the interest rate independent of the elasticity of money demand. Under this assumption, the policy instrument instability of money supply variance could be possible, while its average should converge to some constant rate. From the other point of view, Driscoll and Ford (1982) showed that the optimal choice should also depend on the nature of the distribution of parameters and error terms, rather than model parameters and the covariance among the error terms. His research demonstrated that even if there is instability in the IS side, the money stock is not always the optimal policy instrument. However, Daniel (1986) investigated a model with rational expectations and permanent and temporary shocks to IS, LM and aggregate supply. The finding supports the money supply instrument when the monetary policy rules allow temporary deviations from the long-run money supply path to offset interest rate forecast errors. She also suggests that the interest rate would not be an optimal instrument if the source of disturbances were not clear. Moreover, Fair (1988) examined the optimal choice of the monetary policy instruments by applying stochastic simulation in different economic models. His study showed two aspects of results. On the one hand, the percentage differences between the variance under the money supply policy and the interest rate policy support the interest rate instrument. On the other hand, the model with rational expectation in the bond market provides support to the money supply policy.

In terms of the empirical study, early papers have attempted to research the optimal instrument issue with more empirical features or update it with an equilibrium model by incorporating different shocks. Canzoneri et al. (1983) restudied Poole's model by including imperfect information and rational expectations. The results demonstrated that households and monetary authorities could use incomplete current information to save the wage setter's indexing costs. Their findings confirmed Poole's analysis, which states that the real interest rate targeting is more desirable when the shocks of goods market are relatively small. Carlstrom and Fuerst (1995) applied a cash-in-advance real business cycle model, finding that interest rate pegging dominates the constant money supply policy. This is because the interest rate targeting provides a higher household's expected lifetime utility by allowing the household to react more effectively to fiscal shock and technology shock.

Additionally, Blanchard and Fischer (1989) used the aggregate supply shocks to show that the optimal choice depends on the slope of the IS curve. The volatility of output is greater under interest rate targeting when the multiplier is more significant than one as the demand curve becomes flatter. Friedman (1990) stressed that the appropriate definition of the policy maker's objective function is not merely the variance of output once inflation is included in the model. The optimal choice of instrument depends on the decision of the policy maker's monetary policy targets.

Conversely, there were many studies which implemented the rankings of policy instruments into the New Keynesian DSGE models but within different shocks. For instance, Ireland (2000) evaluated the central bank's actual policy from 1980 to 2000 in the DSGE model, implying that the central bank's real interest rate rule outperformed the constant money growth supply rule under money demand shocks. Moreover, Singh and Subramanian (2009) applied velocity and fiscal shocks to the DSGE model to compare the performance of monetary policy instruments with the Taylor rule. The results indicate that the monetary aggregates are superior to the interest rate targeting under fiscal shock; interest rate targeting is preferred under velocity shocks.

Recent related research concerning Poole's analysis within the DSGE model is mainly employed by which utilises supply, fiscal and money demand shocks. (Collard and Dellas (2005)) Their findings state that the money targeting procedure creates better welfare under money demand shocks regardless of the value of risk aversion. Further, the ranking of alternative targeting is determined by the value of risk aversion for supply and fiscal shocks. With respect to supply shocks, interest rate targeting is preferable when intertemporal substitution is high, which is a converse circumstance under fiscal shocks. Finally, the rankings are also different depending on some other factors, such as the degree of price stickiness and capital adjustment costs.

For optimal instrument investigations, there is some empirical evidence to endorse the monetary base instrument. First, the empirical study of a small macroeconomic model of Korea suggests that the money stock is a preferred instrument with autoregressive expectations unless the price variable weighs on a considerable level.Ahn and Jung (1985) This result stays valid even if the central bank's policy horizon is extended to infinity. Second, McCallum (1988) is also a supporter of the monetary base instrument. He believes that the monetary base growth rate could generate zero inflation in the 1954-85 US period and also reduce the fluctuations in nominal GNP. According to McCallum (1993), with a monetary base instrument and nominal income targets, the simulation results in the Japanese economy during 1972 and 1992 also perform better than the interest rate as the instrument variable. Hoffmann and Kempa (2009) evaluated a general two-country DSGE model indicating different results between the large economy and the small open economy. For the large economy model, a money supply instrument is preferred with the real shocks, and an interest rate instrument is favoured with liquidity shock. For the small open economy model, a money supply instrument performs better by applying the welfare measure. Besides, a money supply instrument is more attractive when shocks originate in foreign countries.

From previous reviews of the literature, we can sum up some fundamental points. First, the optimal instrument issue is more attractive in the 70s until the 90s. Even though the interest rate instrument has become the main rule in central banks, there were also some supporters for the monetary base instrument. Most of the theoretical analysis, however, my study concentrates on empirical estimation and evaluation. Second, later empirical research has attempted VAR models or simple DSGE models to compare the variations of output or other variables with alternative policy instruments. My paper would apply the

SW(2003) model with full structural shocks as we all apply the euro data in the DSGE models. It is helpful to compare the variations of estimation results between this work and the original one.

The works in both Smets and Wouters (2003) and Smets and Wouters (2007) have provided a fundamental structural study for the new Keynesian DSGE researchers. The SW model demonstrates a detailed economy that could generate the types of wedges we see in the data from primary and interpretable shocks. The model has a lot of micro-foundation features consisting of full structural shocks and parameters to react reasonably to monetary policy shocks. Smets and Wouters (2007) estimated a DSGE model for the US economy by applying a Bayesian technique. The estimated US model contains some crucial frictions, such as sticky nominal price and wage, habit information in consumption and investment adjustment costs. It also includes seven full structural shocks: total factor productivity shocks, wage and price mark-up shocks, intertemporal margin shocks (risk premium shocks and investment-specific technology shocks), exogenous spending and monetary policy shocks. This indicates that the intertemporal margin shocks and exogenous spending shocks can explain most short-run forecast variance in output.

There are some differences between this chapter and the models in Smets and Wouters (2003) and Smets and Wouters (2007). To begin with, the utility function is different from the SW model as I choose a non-separable utility function, including the real money balances. The reason I chose this format is that in comparing the relative performance of alternative monetary policy instruments, it is essential not to neglect the role of the demand for money. Although Ireland (2001) suggested that real balances play a minimal role in the monetary business cycle model, the importance of the real balances' role can not be ignored. Nelson (2003) examined the relationship between money and inflation and between money and aggregate demand. For inflation impacts, he showed that the monetary aggregates were closely linked to inflation in a few dimensions: First, inflation is always a monetary phenomenon in terms of previous models. Second, the quantity theory has not given a justification for the existence of explicit terms involving money in the model's price-setting relations. Third, the long-run relationship between inflation and money growth is fundamental for central banks. Furthermore, according to Andrés et al. (2003), econometric estimates provide less empirical support that utility function drops the money term. They find that the non-separable utility between consumption and real balances vary with habit formation and the New Keynesian model modified with adjustment costs for holding real balances. This implies that a forward-looking property of real money balances, conveying on money a significant role as a monetary policy indicator.

Furthermore, this paper has a different sample period compared to the SW(2003,2007). I have applied the quarterly data of the Euro area for the period between Q2 in 1984 and Q3 in 2007. The data involves a different period than the Smets and Wouters (2003) model, which used data over the period Q2 in 1980 to Q4 in 1999. The Smets and Wouters (2007) study used US data from the period between 1966-1979 and 1984-2004. My data was obtained from the Euro Area Business Cycle Network and described in Fagan et al. (2001). When the euro was created, influenced by the Bundesbank who adopted a two-pillar approach where the second pillar placed consisted of analysing monetary trends. In other words, one could think of the ECB paying attention to, at least in theory, monetary aggregates. Hence, using the euro area as the country in which to perform the Poole analysis seems appropriate.

There were several reasons that I chose a different region from a different period. For historical reasons, most countries started to establish their monetary targets in the middle of the 1970s. (Walsh (2017))The US, Germany and Switzerland announced monetary targets in 1975, the UK in 1976 and France in 1977. Most financial market innovations occurred in the 1980s have diminished the dependency on monetary targets, especially after 1985. Poole's analysis suggested that greater financial market instability that makes money demand more difficult to predict would reduce the benefits of an instrument operating procedure-oriented toward monetary aggregates. Therefore, it would be interesting

to check the data after 1984 and compare the result to the Smets and Wouters (2007) and original Poole's analysis. Second, as the consideration of the financial crisis, the data ended in 2007. The main targets of monetary policy have changed to overcome the financial crisis's adverse impacts. Hence, for policy consistency, the data applies from 1984 to 2007. Third, the data is selected in the Euro area rather than the US because the Fed has targeted the funds rate after the 1980s. (Walsh (2017)) The rule described by Taylor (1993) had been commonly applied by many central banks in the 1990s. The interest rate instrument was implemented before the financial crisis that occurred in 2007; therefore, it could be inconsistent with implementing the monetary aggregates as an instrument for the US data after the 80s. Using the same starting period in 1984 and the extended period from 2007 in the Euro area would be helpful for comparing the results from the Smets and Wouters (2007)model.
Finally, this paper applied two different monetary policy instruments to obtain the optimal choice between the interest rate and monetary aggregates.

2.3 Adjusted Smets Wouters (2003) model

The medium-scale model contains various features: external habit in consumption, sticky prices and wages, capital utilization, capital accumulation, the fixed cost in production, adjustment costs to investment and the indexation of prices and wages. Unlike the SW model leaves the money term and money demand equation, I have modified the model by allowing the real balances to enter the utility function.

2.3.1 Household Sector

The representative household of the economy maximizes the expected intertemporal utility function given by:

$$\mathbb{E}_t \beta^t U\{(C_t, \frac{M_t}{P_t}), l_t\} \tag{2.1}$$

$$U\{(C_t, \frac{M_t}{P_t}), l_t\} = \frac{1}{1-\sigma_c}(X_t)^{1-\sigma_c} - \frac{\epsilon_t^L}{1+\sigma_L}(l_t)^{1+\sigma_L} \tag{2.2}$$

where X_t shows a non-separable utility form that contains both consumption and real balances:

$$X_t = [(1-\eta)(C_t - H_t)^{1-\eta} + \eta\zeta_t(\frac{M_t}{P_t})^{1-\eta}]^{\frac{1}{1-\eta}} \tag{2.3}$$

In the above equations, β is the discount factor, σ_c is the degree of relative risk aversion or inverse of the intertemporal elasticity of substitution, σ_L is the inverse elasticity of labour supply, η gives the inverse elasticity of substitution between the consumption and real money balances, ϵ_t^L represents the labour supply shock, ζ_t shows the money demand shock. The external habit,

$$H_t = hC_{t-1} \tag{2.4}$$

where h is the proportion of past consumption in the external habit. When $h = 0$, the expression is transformed into the conventional forward-looking consumption with merely intertemporal elasticity of substitution. When h is close to 1, the effects of external habit on the real interest rate will vanish. I followed the external habit in the SW(2003) model as the habit formation explains well in some dynamic macroeconomic facts, such as output persistence, savings and growth, and consumption's response to monetary shocksAbel (1990). Apart from the preference shock, the utility function would be the same as in the SW model when $\eta = 0$. The change in money demand shock and real balances impacts on the marginal utility of consumption, and the total expected utility. A labour supply shock would only affect the household's total expected utility but not consumption and real balances.

The SW model applies a separable utility function; the real money balances would not enter any other structural equations. Therefore, the real money balances are ignored in the rest of the model. However, here I allow a non-separable utility so that the changes in the real quantity of money can influence the marginal utility of consumption. This would lead to two impacts on the model. First, the real money balance would emerge in the household's Euler condition. Second, to change real marginal cost with a measure of the output gap, the real wage now contains not only the marginal rate of substitute on between leisure and consumption but also the real money balances. Thereby the real wage would affect the Phillips curve and the rest of the structural equations.

The individual household confronts the budget constraint that takes the form:

$$\begin{aligned} M_t + b_t B_t = M_{t-1} + B_{t-1} + P_t(W_t l_t + r_t^k z_t K_{t-1} \\ -\Psi(z_t)K_{t-1}) - P_t C_t - P_t I_t - P_t T_t \end{aligned} \tag{2.5}$$

The equation illustrates that household's current financial wealth $(M_t + b_t B_t)$ is composite of past cash balances with bonds $(M_{t-1} + B_{t-1})$, (where $b_t = \frac{1}{1+R_t}$ gives the nominal rate of return on bonds and R_t is the nominal interest rate), total income $(w_t l_t + r_t^k z_t K_{t-1} - \Psi(z_t)K_{t-1})$, consumption C_t, investment in capital (I_t) and a lump-sum tax (T_t), r_t^k is the rental rate on past capital stock. The household's income received when they rent out capital services rely on both the level of capital installed last period and its utilization rate z_t. As in the SW(2003) and CEE(2001) model, $\Psi(z_t)$ is the utilization cost function and is assumed $\Psi(1) = 0$ when utilization rate is one in steady state. According to CEE(2001), most of time the output is produced by labour and capital has been ignored. The utilization rate of capital can differ once capital is involved, diminishing the effects of output variations on marginal cost and inflation to a monetary policy shock.

The maximization of the objective function (2.1) subject to a budget constraint (2.4), gives:

$$\begin{aligned} \mathcal{L} = \mathbb{E}\sum_{t=0}^{\infty}\beta^t\{U(C_t, \frac{M_t}{P_t}) - \frac{\epsilon_t^L}{1+\sigma_L}(l_t)^{1+\sigma_L}\} - \lambda_t[M_t + b_t B_t - M_{t-1} - B_{t-1} \\ -P_t(W_t l_t + r_t^k z_t K_{t-1} - \Psi(z_t)K_{t-1}) + P_t C_t + P_t I_t + P_t T_t] \end{aligned} \tag{2.6}$$

The first order conditions with respect to consumption, real balances, bond holdings and labour supply, which generates the following equations:

$$C_t : U_{ct} = \lambda_t P_t / \beta_t \tag{2.7}$$

$$\frac{M_t}{P_t} : U_{mt} = \frac{\lambda_t P_t - \mathbb{E}_t \lambda_{t+1} P_t}{\beta^t} \tag{2.8}$$

$$B_t : b_t = \frac{\mathbb{E}_t \lambda_{t+1}}{\lambda_t} = \frac{1}{1+R_t} \tag{2.9}$$

$$l_t : U_{lt} = -(\lambda_t W_t P_t)/\beta_t \tag{2.10}$$

Using equations (2.6) to (2.8) , the following equations can be derived,

$$\frac{U_{mt}}{U_{ct}} = \frac{\lambda_t - \mathbb{E}_t \lambda_{t+1}}{\lambda_t} = \frac{R_t}{1+R_t} \tag{2.11}$$

$$\frac{U_{lt}}{U_{ct}} = -W_t \tag{2.12}$$

$$\mathbb{E}_t[\beta \frac{\lambda_t}{\lambda_{t+1}} \frac{P_t}{P_{t+1}} \frac{U_{c_{t+1}}}{U_{c_t}}] = 1 \tag{2.13}$$

Equation (2.13) shows the marginal utility of consumption. Maximising the utility function (2.2) with respect to consumption, real balances, labour supply gives marginal utility of consumption, real money balances and marginal disutility of labour supply:

$$U_{ct} = (1-\eta) X_t^{\eta - \sigma_c} (C_t - hC_{t-1})^{-\eta} \tag{2.14}$$

$$U_{ct+1} = (1-\eta) X_{t+1}^{\eta - \sigma_c} (C_{t+1} - hC_t)^{-\eta} \tag{2.15}$$

$$U_{mt} = X_t^{\eta - \sigma_c} \eta \zeta_t (\frac{M_t}{P_t})^{-\eta} \tag{2.16}$$

$$U_{lt} = -\epsilon_t^L l_t^{\sigma_L} \tag{2.17}$$

Using equations (2.11), (2.14) and (2.16), the money demand is derived.

$$\frac{U_{mt}}{U_{ct}} = \zeta_t \frac{\eta}{1-\eta} [\frac{M_t}{P_t(C_t - hC_{t-1})}]^{-\eta} = \frac{R_t}{1+R_t} \tag{2.18}$$

Substituting marginal utility of money demand and marginal utility of consumption into equation (2.18), then, money demand becomes,

$$\frac{M_t}{P_t} = [\frac{\eta}{1-\eta}\zeta_t\frac{1+R_t}{R_t}]^{\frac{1}{\eta}}(C_t - hC_{t-1}) \tag{2.19}$$

It can be seen that real money balances are positively related to consumption with external habit and negatively correlated to the nominal interest rate.

2.3.2 Wage setting and labour supply

Flexible prices and wages

In the model we use two types of wages: flexible and sticky. Concerning equations (2.6), (2.9), (2.13) and (2.16),

$$\frac{U_{lt}}{U_{ct}} = -W_t \tag{2.20}$$

$$W_t = \epsilon_t^L l_t^{\sigma_L}\frac{X^{(\sigma_c-\eta)}(C_t - hC_{t-1})^{\eta}}{1-\eta} \tag{2.21}$$

Under flexible wages, the household's labour supply is positively related to consumption and real balances. It can be seen from this equation that the real balances enter flexible wages and then it would affect the marginal cost later.

Sticky prices and wages

As in the SW model, households are assumed to have monopoly power over wages. The wages of households that cannot be re-optimized according to the equations as follow.

$$W_t = (\frac{P_{t-1}}{P_{t-2}})^{\gamma_w} W_{t-1} \tag{2.22}$$

$$l_t = (\frac{W_{jt}}{W_t})^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} L_t \tag{2.23}$$

where γ_w is the degree of wage indexation. Aggregate nominal wage and labour demand are given:

$$L_t = [\int_0^1 (l_{jt})^{\frac{1}{1+\lambda_{w,t}}} dj]^{1+\lambda_{w,t}} \tag{2.24}$$

$$W_t = [\int_0^1 (W_{jt})^{\frac{1}{1+\lambda_{w,t}}} dj]^{1+\lambda_{w,t}} \tag{2.25}$$

According to equation (2.24), the aggregate wage index is provided by:

$$(W_t)^{-\frac{1}{\lambda_{w,t}}} = \xi(W_{t-1}(\frac{P_{t-1}}{P_{t-2}})^{\gamma_w})^{-\frac{1}{\lambda_{w,t}}} + (1-\xi)(\tilde{w_t})^{-\frac{1}{\lambda_{w,t}}} \tag{2.26}$$

where ξ_w is the probability that households can not re-adjust their wage level, and $1-\xi_w$ is thus the probability that households can optimize their wage. $\lambda_{w,t}$ is the wage mark-up shock that takes the form $\lambda_{w,t} = \lambda_w + v_t^w$. If households maximize the utility function (2.1) and subject to equation (2.21), then it gives:

$$\frac{\tilde{w}_t}{P_t}\mathbb{E}_t \sum \beta\xi_w \frac{(P_{t-1}/P_{t-2})^{\gamma_w})}{P_{t+i}/P_{T+I-1}} \frac{l_{t+i}U_{c_{t+1}}}{1+\lambda_{w,t+i}} = \mathbb{E}_t \sum \beta\xi_w l_{t+i} U_{t+i}^l \tag{2.27}$$

where $U_{c_{t+1}}$ is the marginal utility of consumption and U_{t+i}^l is the marginal disutility of labour. This equation illustrates that real wage would be mark-up over the ratio of marginal disutility of labour and marginal utility of consumption, when wages are completely flexible (ie. when $\xi_w = 0$). Calvo (1983) and Erceg et al. (2000) The linearized equation for the real wage is illustrated by equation (2.51). The equation demonstrates that real wage is influenced through some components: expected future real wages, past wages, expected and current inflation, and the difference between real wages and the marginal rate of substitution between the labour supply and the consumption level.

What is more, households' real wages also depend on the degree of wage indexation, γ_w. When $\gamma_w = 0$, past inflation has no impact on real wages. When $\gamma_w = 1$, past inflation exerts a strong effect on real wages. In addition, real wages are negatively affected by the component term regarding the probability of the adjustment in wages. If ξ_w, the degree of wage stickiness, is close to 0, real wages tend to become flexible and greater effects would be generated by the marginal disutility of labour and the marginal utility of consumption.

Investment and Capital

Households rent capital services to firms and choose capital accumulation given capital adjustment cost. The capital accumulation equation is given by:

$$K_t = (1-\delta)K_{t-1} + [1 - S(\frac{\epsilon_t^I I_t}{I_{t-1}})]I_t \tag{2.28}$$

where δ is the depreciation rate and S(.) is the investment adjustment cost function. In terms of the SW and CEE(2002), the function satisfies following conditions: $S(1) = S'(1) = 0$ and $S''(.) > 0$. ϵ_t^I denotes an investment shock that takes the form:

$$\epsilon_t^I = \rho_I \epsilon_{t-1}^I + v_t^I \tag{2.29}$$

When households maximize the objective function with capital accumulation with respect to capital stock, investment and the utilization rate, the new Lagrangian problem is given by:

$$\begin{aligned} \mathcal{L} = \mathbb{E}\sum_{t=0}^{\infty}\beta^t\{U(C_t, \frac{M_t}{P_t}, l_t) - \lambda_t[M_t + b_t B_t - M_{t-1} - B_{t-1} \\ -P_t(W_t l_t + r_t^k z_t K_{t-1} - \Psi(z_t)K_{t-1}) + P_t C_t + P_t I_t + P_t T_t \\ -q_t(K_t - (1-\delta)K_{t-1} + (1 - S(\frac{\epsilon_t^I I_t}{I_{t-1}}))I_t)]\} \end{aligned} \tag{2.30}$$

The first order conditions with respect to capital, investment and the degree of capital utilisation cost yields the following equations:

$$K_t : Q_t = \frac{q_t}{\lambda_t} = \beta\mathbb{E}_t[\frac{\lambda_{t+1}}{\lambda_t}((1-\delta)Q_{t+1} + z_{t+1}r_{t+1}^k - \Psi(z_{t+1}))] \tag{2.31}$$

$$I_t : Q_t S'(\frac{\epsilon_t^I I_t}{I_{t-1}})\frac{\epsilon_t^I I_t}{I_{t-1}} + \beta\mathbb{E}_t Q_{t+1}\frac{\lambda_{t+1}}{\lambda_t}S'(\frac{\epsilon_{t+1}^I I_{t+1}}{I_t})(\frac{\epsilon_{t+1}^I I_{t+1}}{I_t})\frac{I_{t+1}}{I_t} = 1 \tag{2.32}$$

$$z_t : r_t^k = \psi'(z_t) \tag{2.33}$$

The equation (2.31) implies that the current value of capital stock (Q_t) depends positively on its expected future value (E_tQ_{t+1}) considering the depreciation rate (δ). This equation is also affected by the expected future return as composed by the expected rental rate and expected capital utilization rate ($E_tr_{t+1}z_{t+1}$). Moreover, the equation (2.33) describes that the first-order condition of the utilization rate of capital equals the rental rate on capital. The rental rate of capital rises up to one point where the marginal benefit equals the marginal cost.

Price setting

The production function is given by:

$$y_{jt} = \epsilon_t^a \tilde{K}_{jt}^{\alpha} L_{jt}^{1-\alpha} - \Phi \tag{2.34}$$

where Φ is a fixed cost parameter and ϵ_t^a is the technology shock. The effective utilisation of capital stock $\tilde{K_{jt}}$ represents the effective utilization of capital stock and $\tilde{K_{jt}} = z_tK_{j,t-1}$, and $y_{jt} = (\frac{P_{jt}}{P_t})^{\frac{-(1+\lambda_{p,t})}{\lambda_{p,t}}} Y_t$. Total cost is given by $W_tl_t + r_t^k z_t K_{t-1}$. Minimising total cost subject to equation (2.34) yields:

$$W_t = (1-\alpha)\epsilon_t^a \psi_{jt} (\frac{\tilde{K_{jt}}}{L_{jt}})^{\alpha} \tag{2.35}$$

$$r_t^k = \psi_{jt}\alpha\epsilon_t^a (\frac{\tilde{K_{jt}}}{L_{jt}})^{(\alpha-1)} \tag{2.36}$$

$$\frac{W_tL_{jt}}{r_t^k\tilde{K_{jt}}} = \frac{1-\alpha}{\alpha} \tag{2.37}$$

Equation (2.35) implies that all firms face same capital labour ratio, and it is equal to the aggregate ratio of factors. This implies that all firms also must hold the same marginal cost as they hire according to the same capital labour ratio. According to equation (2.33) to (2.35), marginal cost is ψ_{jt} is given by:

$$\psi_{jt}(MC_t) = \frac{W_t}{(1-\alpha)\epsilon_t^a(\frac{\tilde{K_{jt}}}{L_{jt}})^\alpha} \tag{2.38}$$

$$= \frac{1}{\epsilon_t^a}(\frac{W_t}{1-\alpha})^{1-\alpha}(\frac{r_t^k}{\alpha})^\alpha \tag{2.39}$$

The equation (2.39) states that marginal cost is a function with wages and rental cost with technology shocks. ϵ_t^a follows an AR(1) form where,

$$\epsilon_t^a = \rho_a \epsilon_{t-1}^a + v_t^a \tag{2.40}$$

Since wages are linked with consumption and real balances with a money demand shock, then those in turn would also affect marginal cost. When a firm maximizes its profit,

$$\pi_{jt} = (p_{jt} - MC_t)(\frac{P_{jt}}{P_t})^{\frac{-(1+\lambda_{p,t})}{\lambda_{p,t}}} Y_t - MC\Phi \tag{2.41}$$

$$\mathbb{E}_t \sum \beta \xi_p^i \lambda_{t+i} y_{t+i}^i (\frac{\tilde{P_{jt}}}{P_t} \frac{(P_{t-1+i})^{\gamma_p}/P_{t-1}}{P_{t+i}/P_t}) - (1+\lambda_{p,t+i})mc_{t+i} = 0 \tag{2.42}$$

The price mark-up $\lambda_{p,t} = \lambda_p + v_t^p$, where the price mark-up shock v_t^p is an i.i.d. error term. The indexation cost of prices is given by γ_p. As in the SW model and Calvo (1983), a firm can only re-optimize prices with probability $(1-\xi_p)$, and prices are perfectly flexible when $\xi_p = 0$. Therefore, the price equation is given by:

$$P_t^{-\frac{1}{\lambda_{p,t}}} = \xi_p(P_{t-1}(\frac{P_{t-1}}{P_{t-2}})_p^\gamma)^{\frac{-1}{\lambda_{p,t}}} + (1-\xi_p)(\tilde{p_j t})^{\frac{-1}{\lambda_{p,t}}} \tag{2.43}$$

As described by Calvo (1983), firms could not change their prices unless they receive a signal. With probability at ξ_p that firms are unable to re-optimize and adjust their price according to past inflation. Firms can adjust their prices at probability $1-\xi_p$. Besides, in terms of Christiano et al. (2001), there is an indexation costγ_p to the previous period's inflation rate for non-optimized prices. In those conditions, the linearized form of the New Keynesian Phillips curve is derived by the equation (2.55). Inflation is determined by expected future inflation, past inflation, and weighted marginal cost function, which is a combination of the rental rate on capital, the real wage, a technology shock, $(\hat{\epsilon}_t^a)$ and a price mark-up shock, (η_t^p). When $\gamma_p = 0$, past inflation disappears, and equation (2.44) becomes a traditional Philip Curve. When $\gamma_p = 1$, there is a perfect indexation that relates current inflation to past inflation. In addition, when $1 - \xi_p = 0$, all prices are perfectly flexible, and no price mark-up shock exists.

Alternative Monetary Policy Instruments

In this section we present evaluations of alternative monetary policy instruments. The linearised money demand equation can be given by:

$$\hat{mp}_t = \frac{\sigma_c}{\sigma_m}(\hat{C}_t - h\hat{C}_{t-1}) - \frac{\hat{R}_t}{\sigma_m} + \epsilon_t^m \tag{2.44}$$

where $\hat{mp}_t = ln(\frac{MP_t}{MP})$, $MP_t = \frac{M_t}{P_t}$ and the ϵ_t^m is the money demand shock that follows the AR(1) process,

$$\epsilon_t^m = \rho_m \epsilon_{t-1}^m + v_t^m \tag{2.45}$$

, which only enters the money demand equation.

In practice, there is only one monetary policy instrument used by the central bank. When the central bank targets the nominal interest rate, the money supply adjusts in order to clear the market. To solve the indeterminacy problems, and as in Gali et al. (2004), the interest rate instrument follows the structure in the CD paper:

$$\hat{R}_t = \rho\hat{R}_{t-1} + (1-\rho)k_\pi\hat{\pi}_t \tag{2.46}$$

where the interest rate is pinned down by the past interest rate and adjusted current inflation, $\rho = 0.999$ and $k_\pi = 1.001$. According to Collard and Dellas (2005), this rule allows a long-run response to inflation that is slightly greater than one to ensure that indeterminacy would not appear in the rational-expectations equilibrium.

When the central bank targets money growth, the money supply is constant. Then

$M_t = (1 + \mu_t)M_{t-1}$, the linearized form is derived:

$$\hat{mp}_t - \hat{mp}_{t-1} = \mu_t - \hat{\pi}_t \tag{2.47}$$

where $\mu_t = m_t - m_{t-1=}\mu$ where μ is the steady state growth rate of money supply. The equation demonstrates that real balances are determined by the constant growth rate of money supply and inflation.

Market Equilibrium

The final goods market is in equilibrium if,

$$Y_t = C_t + G_t + I_t + \psi(z_t)K_{t-1} \tag{2.48}$$

output equals the demand for consumption, investment, government spending and capital with utilisation cost. The linearized equation is provided by (2.57). The output is expressed by an aggregate demand equation which is a function of consumption, investment and government spending. It is also a function of aggregate supply, including past capital stock, the rental on capital, the labour supply and a technology shock. In the equation (2.57), $c_y = 1 - \delta k_y - g_y$, denotes the steady-state consumption output ratio, k_y is the steady-state capital-output ratio, and g_y is the steady-state government spending-output ratio, finding Φ equals one plus the share of fixed costs in production.

2.3.3 The linearized Model

When nonseparable money is in utility function, where $\eta = 0$ and a preference shock ϵ_t^b is added into the expected utility, the model would be same as in the SW model. We describe the linearized equations below with zero-inflation steady state.

$$\hat{C}_t = \frac{h}{1+h}\hat{C}_{t-1} + \frac{1}{1+h}\mathbf{E_t}\hat{C}_{t+1}$$

$$-\frac{1-h}{(1+h)\sigma_c}(\hat{R}_t - \mathbf{E_t}\hat{\pi}_{t+1}) + \frac{1-h}{(1+h)\sigma_c}(\hat{\epsilon}_t^b - \mathbf{E_t}\hat{\epsilon}_{t+1}^b) \tag{2.49}$$

$$\hat{w}_t = \frac{\beta}{1+\beta}\mathbf{E_t}\hat{w}_{t+1} + \frac{1}{1+\beta}\hat{w}_{t-1} + \frac{\beta}{1+\beta}\mathbf{E_t}\hat{\pi}_{t+1} - \frac{1+\beta\gamma_w}{1+\beta}\hat{\pi}_t + \frac{\gamma_w}{1+\beta}\hat{\pi}_{t-1}$$

$$-\frac{1}{1+\beta}\frac{(1-\beta\xi_w)(1-\xi_w)}{(1+\frac{(1+\lambda_w)\sigma_L}{\lambda_w})\xi_w}[\hat{w}_t - \sigma_L\hat{L}_t - \frac{\sigma_c}{1-h}(\hat{C}_t - h\hat{C}_{t-1}) + \hat{\epsilon}_t^L - v_t^w] \tag{2.50}$$

$$\hat{I}_t = \frac{1}{1+\beta}\hat{I}_{t-1} + \frac{\beta}{1+\beta}\mathbf{E_t}\hat{I}_{t+1} + \frac{\phi}{1+\beta}\hat{Q}_t + \beta\mathbf{E_t}\hat{\epsilon}_{t+1}^I + \hat{\epsilon}_t^I \tag{2.51}$$

$$\hat{Q}_t = -(\hat{R}_t - \mathbf{E_t}\hat{\pi}_{t+1}) + \frac{1-\delta}{1-\delta+\bar{r}^k}\mathbf{E_t}\hat{Q}_{t+1} + \frac{\bar{r}^k}{1-\delta+\bar{r}^k}\mathbf{E_t}\hat{r}_{t+1}^k + v_t^Q \tag{2.52}$$

$$\hat{K}_t = (1-\delta)\hat{K}_{t-1} + \delta\hat{I}_{t-1}, \tag{2.53}$$

$$\begin{aligned}\hat{\pi}_t = &\frac{\beta}{1+\beta\gamma_p}\mathbf{E_t}\hat{\pi}_{t+1} + \frac{\gamma_p}{1+\beta\gamma_p}\hat{\pi}_{t-1} \\ &+ \frac{1}{1+\beta\gamma_p}\frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p}[\alpha\hat{r}_t^k + (1-\alpha)\hat{w}_t - \hat{\epsilon}_t^a + v_t^p]\end{aligned} \tag{2.54}$$

$$\hat{L}_t = -\hat{w}_t + (1+\psi)\hat{r}_t^k + \hat{K}_{t-1} \tag{2.55}$$

$$\hat{E}_t = \beta\hat{E_{t+1}} + \frac{(1-\beta\xi_e)(1-\xi_e)}{\xi_e}(\hat{L}_t - \hat{E}_t) \tag{2.56}$$

$$\begin{aligned}\hat{Y}_t &= (1-\delta k_y - g_y)\hat{C}_t + \delta k_y\hat{I}_t + g_y\epsilon_t^g \\ &= \phi\hat{\epsilon}_t^a + \phi\alpha\hat{K}_{t-1} + \phi\alpha\psi\hat{r}_t^k + \phi(1-\alpha)\hat{L}_t\end{aligned} \tag{2.57}$$

$$\hat{R}_t = \rho\hat{R}_{t-1} + (1-\rho)\{r_\pi\hat{\pi_{t-1}}_t + r_Y(\hat{Y}_t - \hat{Y_t^p}) + v_t^r \tag{2.58}$$

$$\hat{mp}_t = \frac{\sigma_c}{\sigma_m}(\hat{C}_t - h\hat{C}_{t-1}) - \frac{\hat{R}_t}{\sigma_m} + \epsilon_t^m \tag{2.59}$$

$$\hat{z}_t = \frac{1-\psi}{\psi}\hat{r}_t^k \tag{2.60}$$

$$\hat{R}_t = \rho\hat{R}_{t-1} + (1-\rho)k_\pi\hat{\pi}_t \tag{2.61}$$

$$\hat{mp}_t - \hat{mp}_{t-1} = \mu_t - \hat{\pi}_t \tag{2.62}$$

$$\epsilon_t^b = \rho_b\epsilon_{t-1}^b + v_t^b \tag{2.63}$$

$$\epsilon_t^L = \rho_l\epsilon_{t-1}^l + v_t^L \tag{2.64}$$

$$\epsilon_t^a = \rho_a\epsilon_{t-1}^a + v_t^a \tag{2.65}$$

$$\epsilon_t^w = \rho_w\epsilon_{t-1}^w + v_t^w \tag{2.66}$$

$$\epsilon_t^I = \rho_I\epsilon_{t-1}^I + v_t^I \tag{2.67}$$

$$\epsilon_t^g = \rho_g\epsilon_{t-1}^g + v_t^g \tag{2.68}$$

$$\epsilon_t^r = \rho_r\epsilon_{t-1}^r + v_t^r \tag{2.69}$$

$$\epsilon_t^p = \rho_p\epsilon_{t-1}^p + v_t^p \tag{2.70}$$

2.4 Estimation

In this section, we present the estimation results of the model described in section 3. The first subsection describes the data and Bayesian estimation method; the second subsection provides some calibrated parameters and prior specifications, and the third shows the estimation results and comparisons with the SW model.

2.4.1 Data and methodology

This paper applied the quarterly data of the Euro area for the period between Q2 in 1984 and Q3 in 2007. The data was obtained from the Euro Area Business Cycle Network and described in Fagan et al. (2001). There are nine shocks in total with seven observable variables: real output, real consumption, real investment, real wages, employment, inflation (the GDP deflator) and the nominal interest rate. It is assumed that every single shock is uncorrelated to overcome the identification issue caused by different structural shocks and observable variables. Four of the nine shocks, namely the wage mark-up shock, price mark-up shock, equity premium shock, and temporary interest rate shock, are standard i.i.d white noise processes. The remaining five shocks follow a persistent AR(1) process. All real variables are detrended by a linear trend, except that both the inflation and the nominal interest rate are detrended by the inflation's linear trend. The data construction method follows Fagan et al. (2001).

The Bayesian technique is implemented as an estimation method, the same as in the SW model. The Bayesian estimation technique is now commonly applied by many researchers recently. There are some benefits to applying Bayesian techniques. First, the prior has collected information either from microeconomics or macroeconomics studies, which makes a close connection with previously calibrated parameters. Second, as suggested in the Smets and Wouters (2003) model, " the usage of prior distributions makes the highly nonlinear optimization algorithm more stable". This is especially crucial when the size of the data sample is relatively small.

Table 2.1: Calibrated Parameters

Parameter	Description	Value
β	discount factor	0.99
α	Share of capital in production parameter	0.3
δ	quarterly depreciation rate	0.025
μ	average growth rate of M3 over period	0.02
λ_w	wage mark-up parameter	0.5
c_y	steady state consumption output ratio	0.53
k_y	steady state capital output ratio	8.54
g_y	steady state government spending output ratio	0.18

in our model, there were 500,000 draws executed by the Metropolis-Hastings sampling algorithm; the posterior mean of estimation result was obtained. 24% of the draws were removed, while there were 400,000 draws which were applied eventually. All priors were settled to achieve the optimal mode, which provided the numerical optimization of the posterior kernel.

Tabel (2.1) illustrates the calibrated parameters, which are similar to in the SW model and the CD paper. The average growth rate of M1 is, approximately 2%, calculated from Q2 in 1984 to Q3 in 2007 in the Euro area. Wage mark-up is assumed as 0.5, same as in the SW model. The depreciation rate, 0.025, indicates an annual depreciation of 10 per cent. The steady-state government spending ratio is presumed as 0.18; the other two steady-state parameters we calculated based on calibrated parameters.

2.4.2 Estimation results

This section will present estimation results and compare to the posteriors in the SW model as well as the calibrated values used in the CD paper and other related papers. Table (2.2) illustrates the mode and posterior mean for 28 estimated parameters. It provides the prior and posterior distributions of estimated parameters. The shocks' variance is inverted Gamma distributions and treated around 0.1 to 0.5 with a degree of freedom at 2. AR(1) persistent parameters follow Beta distributions and their means are assumed as

Table 2.2: Estimated Parameters

Parameter	description	type	mean	standard error	mode	posterior mean
ϵ_t^a	technology shock	Inv. Gamma	0.1	2	0.3022	0.3098
ϵ_t^b	preference shock	Inv. Gamma	0.1	2	0.1734	0.2574
ϵ_t^g	government spending shock	Inv. Gamma	0.1	2	1.6720	1.7026
η_t^Q	equity premium shock	Inv. Gamma	0.5	2	6.1060	6.5839
ϵ_t^L	labour supply shock	Inv. Gamma	0.4	2	2.0128	2.1500
ϵ_t^I	investment shock	Inv. Gamma	0.4	2	0.3940	0.5180
ϵ_t^R	interest rate shock	Inv. Gamma	0.1	2	0.0885	0.0909
ϵ_t^w	wage mark-up shock	Inv. Gamma	0.4	2	0.1898	0.5800
ϵ_t^p	price mark shock	Inv. Gamma	0.1	2	5.0000	4.7610
ρ_a	AR(1) for technology shock	Beta	0.85	0.1	0.9065	0.9039
ρ_b	AR(1) for preference shock	Beta	0.85	0.1	0.8118	0.7718
ρ_g	AR(1) for government spending shock	Beta	0.85	0.1	0.9745	0.9656
ρ_l	AR(1) for labour supply shock	Beta	0.85	0.1	0.9889	0.9805
ρ_I	AR(1) for investment shock	Beta	0.85	0.1	0.9121	0.9045
h	external habit for consumption	Beta	0.7	0.1	0.4219	0.4611
ξ_w	Calvo wage	Beta	0.75	0.1	0.3217	0.3391
ϕ	investment adjustment cost	Normal	4	0.15	5.9139	6.0835
ξ_p	Calvo price	Beta	0.75	0.1	0.8381	0.8324
ξ_I	Calvo employment	Beta	0.5	0.15	0.2882	0.2882
σ_c	risk aversion	Normal	1	0.375	1.4673	1.4806
σ_L	labour utility	Normal	2	0.75	3.1303	3.2276
γ_w	wage indexation	Beta	0.5	0.15	0.2599	0.2845
γ_p	price indexation	Beta	0.5	0.15	0.1037	0.1178
ψ	capital utility adjustment cost	Normal	0.3	0.175	0.1320	0.3416
Φ	production fixed cost	Normal	1.45	0.25	2.0475	2.0454
ρ	interest rate feedback	Beta	0.8	0.1	0.9063	0.9069
r_π	inflation feedback	Normal	1.5	0.35	1.1711	1.3763
r_y	output gap feedback	Normal	0.125	0.1	0.1521	0.1539

0.85, significantly distinguish from zero. Indexation of prices and wages are assumed at 0.5; the Calvo wage and price are set to be 0.75. Risk aversion is at the benchmark case of 1, and the habit parameter is 0.7, which is the same as in the SW model. Figure (2.9) to (2.12) in the Appendix depict the distribution of priors and posteriors.

The standard deviation of shocks is generally higher than the SW model, especially for the equity premium shock and price mark-up shock. Within the CD paper, the standard deviation of a technology shock, government spending shock and money demand shock are close to zero. For persistent AR(1) parameters, preference shock posterior mean is lower than the SW model (0.77:0.84), the higher in labour supply shock (0.98:0.88) and government spending shock (0.96:0.94). While the persistence shocks in the CD paper are generally more significant than the other two papers, as the parameters are between 0.95 to 0.97. Our estimated results are also similar as in Onatski et al. (2004)(OW, henceforth), where there was a very high value in equity premium shock (6.5839:7). There were also highly persistent AR(1) parameters in government spending and labour supply, 0.9656 and 0.9805 in this paper against 0.972 and 0.974 in OW's work.

Furthermore, there are some similarities and distinctions, among other parameters. First, the posterior mean of habit persistence is 0.46, which is lower than the SW model 0.59 and slightly higher than OW's result of 0.4. The common value of the relative risk aversion parameter is suggested between 1 to 4. Therefore, this paper's result is 1.48 against 1.39 in the SW model, while close to the benchmark case of 1.5 in the CD paper. Given the habit and the intertemporal elasticity of substitution parameters, it can be implied that a one per cent increase in ex-ante real interest rate would decrease approximately 25% in consumption. The posterior of investment adjustment cost in the other literature is advised between 0.12 to 0.28. Low value gives 0.13 in Onatski et al. (2004) and high value gives 0.28 in Christiano et al. (2001). In contrast, 0.166 and 0.148 in the SW model. The adjustment cost parameter suggests that investment would increase by 0.17 per cent if there is a 1 per cent increase in the current installed capital price.

Moreover, the mean of Calvo price re-optimisation is 0.84, lower than 0.91 in the SW model, but higher than 0.42 in Onatski et al. (2004). Given the discount factor, $\beta = 0.99$, then the weight on lagged inflation is 0.11, the expected future inflation is 0.88, and the current marginal cost is 0.03 when treating other variables constant. The mean of Calvo wages resetting is 0.34, and indexation parameter of wages is 0.28; compared to the value, 0.74 and 0.73, in the SW model. The Calvo wages parameter are also lower than 0.66 in Rotemberg and Woodford (1999). Furthermore, the weighted value is 0.64 on current inflation and 0.14 on past inflation. It can be derived that the average duration of price contracts is one and a half years, whereas the average duration of the wage contract is one quarter. This result is similar to the SW model where there was a higher stickiness in prices than real wages, except that both durations are shorter in this paper. Compared to the benchmark case in the CD paper, the probability of price resetting is 0.25, which indicates the average price contracts period is one year. The value of Calvo price stickiness is shown in the Euro area, as the results found in Galı et al. (2001) that applied the constant return to scale in the production function.

Finally, other values in fixed cost, capital utility adjustment cost, and the output gap feedback parameter are all higher than in the SW model. Both lagged interest rate feedback parameters are highly persistent but slightly lower in this paper (0.90:0.95). The inflation feedback is 1.38 against 1.68 in the SW model and 4 in the OW's work. In contrast, the output gap feedback parameter is higher than in the other the SW and OW papers, (0.062 and 0.099 respectively), compared to 0.15 in this one. Overall, although there were some differences with the SW model and OW model, the results are generally similar.

2.5 Welfare analysis under alternative instruments

The models under alternative policy instruments can be obtained through the estimated posteriors, except that the money demand shock is calibrated. The value is similar to that of Andrés et al. (2006), $\rho_m = 0.95, \sigma_{\epsilon_m} = 0.0046$. We can now derive the welfare loss function and then evaluate the monetary policy instruments under different shocks.

Conventional literature compares the monetary policy rules based on the linear model with simple quadratic loss functions and contrasts the volatility in output and inflation. Later researchers have attempted to apply non-linear second-order perturbation methods either in a simple ad hoc version or a utility-based welfare criterion of a welfare loss function. Such approaches allow the model to capture not only the variability in output and inflation but also others such as investment, wage inflation and employment. My research follows Levine et al. (2008) paper's welfare loss analysis, which is based on the Smets and Wouters (2003) model. The loss function is approximately given by, [1]

[1]see the detailed derivations in the Appendix

Table 2.3: Standard Deviations and Welfare($\sigma_c = 1.5$)

shocks	RT_y	MT_y	RT_c	MT_c	RT_π	MT_π	RT_r	MT_r	RT_w	MT_w
technology shock	0.62	0.53	0.23	0.21	0.13	0.17	-	0.05	0.52	0.58
preference shock	0.23	0.36	0.36	0.10	0.05	0.03	-	0.29	0.47	0.16
government spending shock	0.62	0.67	0.21	0.20	0.06	0.09	-	0.03	0.33	0.39
equity premium shock	0.33	0.35	0.08	0.07	0.018	0.022	-	0.007	0.15	0.17
labour supply shock	2.77	2.70	2.41	2.33	0.23	0.34	-	0.12	1.27	1.54
investment shock	0.28	0.32	0.14	0.13	0.03	0.04	-	0.01	0.14	0.18
wage mark-up shock	0.0067	0.005	0.004	0.003	0.0026	0.0028	-	0.0005	0.0507	0.0516
price mark-up shock	0.17	0.13	0.14	0.09	0.149	0.151	-	0.014	0.26	0.22
shocks	RT_{rk}	MT_{rk}	RT_{mc}	MT_{mc}	RT_l	MT_l	RT_i	MT_i	RT_{wel}	MT_{wel}
technology shock	0.38	0.35	0.79	0.89	0.40	0.12	2.28	1.97	-4.2830	-3.7317
preference shock	0.13	0.20	0.36	0.13	0.07	0.16	0.7	1.9	-0.7299	-0.3863
government spending shock	0.27	0.25	0.24	0.29	0.278	0.285	1.30	1.27	-1.69	-1.81
equity premium shock	0.13	0.14	0.13	0.14	0.13	0.14	1.67	1.72	-0.2768	-0.2989
labour supply shock	0.79	0.71	0.86	1.09	1.07	1.06	6.26	6.22	-44.7978	-44.7807
investment shock	0.22	0.23	0.11	0.15	0.11	0.12	1.53	1.68	-6.1764	-6.2115
wage mark-up shock	0.0095	0.0097	0.03827	0.03898	0.01143	0.01106	0.0197	0.0157	-0.0209	-0.0210
price mark-up shock	0.09	0.07	0.21	0.17	0.04	0.03	0.46	0.37	-0.9694	-0.9074

RT_y=output in the interest rate targeting; MT_y=output in the money supply targeting; the same applies for others; RT_{wel}=welfare in interest rate targeting; MT_{wel}= welfare in money supply targeting.

$$
\begin{aligned}
U_t &= w_c(c_t - hc_{t-1})^2 + w_l l^2 + w_\pi(\pi_t - \gamma_p \pi_{t-1})^2 + w_{\Delta w}(\Delta w_t - \gamma_w \Delta w_{t-1})^2 \\
&+ w_l k(l_t - k_{t-1} - z_t - \frac{1}{1-\alpha} a_t)^2 + w_z(z_t + \psi a_t)^2 - w_{al} a_t l_t - w_i(i_t - i_{t-1})^2
\end{aligned} \tag{2.71}
$$

where

$$w_c = \frac{\sigma}{(1-h)}(1-h)^{-\sigma}; \tag{2.72}$$

$$w_l = \frac{((1-h)^{-\sigma})(1-1/\eta)(1-\alpha)}{c_y}(\phi + \frac{1}{2}\phi + \frac{1}{2}\phi^2); \tag{2.73}$$

$$w_\pi = \frac{((1-h)^{-\sigma})(1-1/\eta)(1-\alpha)}{c_y}\frac{\zeta \xi_p}{(1-\beta\xi_p)(1-\xi_p)}; \tag{2.74}$$

$$w_\Delta w = \frac{((1-h)^{-\sigma})(1-1/\eta)(1-\alpha)}{c_y}\frac{\eta(1+\eta\phi)\xi_w}{(1-\beta\xi_w)(1-\xi_w)}; \tag{2.75}$$

$$w_l k = (1-\beta h)(1-h)^{-\sigma}\alpha(1-\alpha); \tag{2.76}$$

$$w_z = (1-\beta h)(1-h)^{-\sigma}\alpha/\phi; \tag{2.77}$$

$$w_i = (1-\beta h)(1-h)^{-\sigma}\delta k_y(1/\psi); \tag{2.78}$$

Table (2.3) illustrates the standard deviations of nine variables under alternative instruments when one shock dominates, and the others are treated as fixed. The interest

Table 2.4: Lower Volatility and Higher Welfare ($\sigma_c = 1.5$)

shocks	y	c	π	w	rk	mc	l	i	welfare
technology shock	MT	MT	RT	RT	MT	RT	MT	MT	MT
preference shock	RT	MT	MT	MT	RT	MT	RT	RT	MT
government spending shock	RT	MT	RT	RT	MT	RT	RT	MT	RT
equity premium shock	RT	MT	RT	RT	RT	RT	RT	RT	RT
labour supply shock	MT	MT	RT	RT	MT	RT	MT	MT	MT
investment shock	RT	MT	RT	RT	RT	RT	RT	RT	RT
wage mark-up shock	MT	MT	RT	RT	RT	RT	MT	MT	RT
price mark-up shock	MT	MT	RT	MT	MT	MT	MT	MT	MT

MT=money supply targeting; RT= interest rate targeting.

rate shock is eliminated in the monetary policy analysis as the nominal interest rate is exogenous in the interest rate targeting. Table (2.4) summaries the performance of the alternative monetary policy instrument with lower standard deviations and higher welfare. Like in the CD paper suggested, the rankings based on output stability and welfare do not coincide. However, both the CD and Poole's analysis have ignored the importance of inflation role. Therefore, the rankings in this paper will depend not only the volatility of output and inflation but also welfare. According to tables (2.3) and (2.4) , there are some interesting points worth noting:

(1) for both instruments, the labour supply shock generates the most volatility, while wage mark-up shock produces the most stability;

(2) for welfare, results are ambiguous under different disturbances;

(3) the money supply instrument is preferable in consumption stabilisation target regardless of any shock;

(4) interest rate pegging generates lower variability in all variables except consumption under both equity premium shock and investment shock;

(5) a money supply instrument delivers lower volatility for all variables except inflation under price mark-up shock.

In order to understand the results, it is necessary to analyse the alternative policy instruments under specific shock. Given the benchmark case ($\sigma_c = 1.5$), impulse response graphs are carried out. The figures below depict the impulse response exercises under nine

Table 2.5: Standard Deviations and Welfare ($\sigma_c = 0.5$)

shocks	RT_y	MT_y	RT_c	MT_c	RT_π	MT_π	RT_r	MT_r	RT_{wel}	MT_{wel}
technology shock	1.0737	0.7658	0.9212	0.7585	0.1014	0.1249	-	0.0355	-5.7969	-3.9563
preference shock	1.4633	0.2692	2.5206	0.6010	0.1057	0.0159	-	0.2962	-38.1536	-2.3075
government spending shock	1.2102	0.5594	2.0026	0.6413	0.0292	0.0557	-	0.0186	-24.0295	-2.6217
equity premium shock	0.3161	0.3383	0.1793	0.1568	0.0158	0.0199	-	0.0063	-0.2535	-0.2219
labour supply shock	3.4597	3.4079	4.0932	4.0199	0.1543	0.2349	-	0.0802	-104.4277	-101.3384
investment shock	0.2616	0.2990	0.3168	0.2874	0.0229	0.0338	-	0.0121	-0.6315	-0.5538
wage mark-up shock	0.0098	0.9640	0.011	0.7924	0.0024	0.1367	-	0.0386	-0.0097	-4.5317
price mark-up shock	0.2924	1.0767	0.3682	0.9055	0.1466	0.2030	-	0.0402	-1.1961	-6.1593

Table 2.6: Standard Deviations and Welfare ($\sigma_c = 3.5$)

shocks	RT_y	MT_y	RT_c	MT_c	RT_π	MT_π	RT_r	MT_r	RT_{wel}	MT_{wel}
technology shock	0.7831	0.5343	0.2012	0.2055	0.1387	0.1671	-	0.0512	-14.5464	-12.7364
preference shock	0.2611	0.3627	0.3982	0.1040	0.0531	0.0294	-	0.2929	-8.1358	-5.3291
government spending shock	0.1743	0.6749	0.2891	0.1957	0.0114	0.0876	-	0.0311	-3.9590	-15.1597
equity premium shock	0.3412	0.3557	0.0403	0.0364	0.0188	0.0226	-	0.0075	-3.3976	-3.6910
labour supply shock	2.2961	2.2368	1.3002	1.2427	0.2693	0.4019	-	0.1461	-219.5192	-216.4778
investment shock	0.2884	0.3277	0.0659	0.0604	0.0301	0.0419	-	0.0153	-2.6188	-3.3760
wage mark-up shock	0.0055	0.6846	0.002	0.1690	0.0026	0.1840	-	0.055	-0.0025	-18.0662
price mark-up shock	0.1343	0.7529	0.0606	0.1877	0.1501	0.2420	-	0.0552	-3.0532	-23.7152

structural shocks: the black solid line, RT, represents the interest rate targeting; the red dash line, MT, represents the money growth targeting.

Technology shock

Figure (2.1) illustrates the impacts of a positive technology shock. It can be seen that the movements generally share the same pattern in both monetary policy instruments, except that the interest rate is constant under RT. The improvement in technology increases output, consumption, and investment. However, higher productivity increases efficiency; this reduces the marginal cost and rental capital cost. It also declines employment and rises wages gradually. Under the constant growth rate of money supply instrument, the real interest rate declines. This leads to lower inflation, which reverts to its steady-state over time. Under interest rate targeting, the overall movements are slightly lower. The central bank increases the money supply when technology improves, then inflation decreases initially and then grows over time. This result is also similar to Ireland (2000).

A money supply instrument brings about stability in output, consumption, rental rate

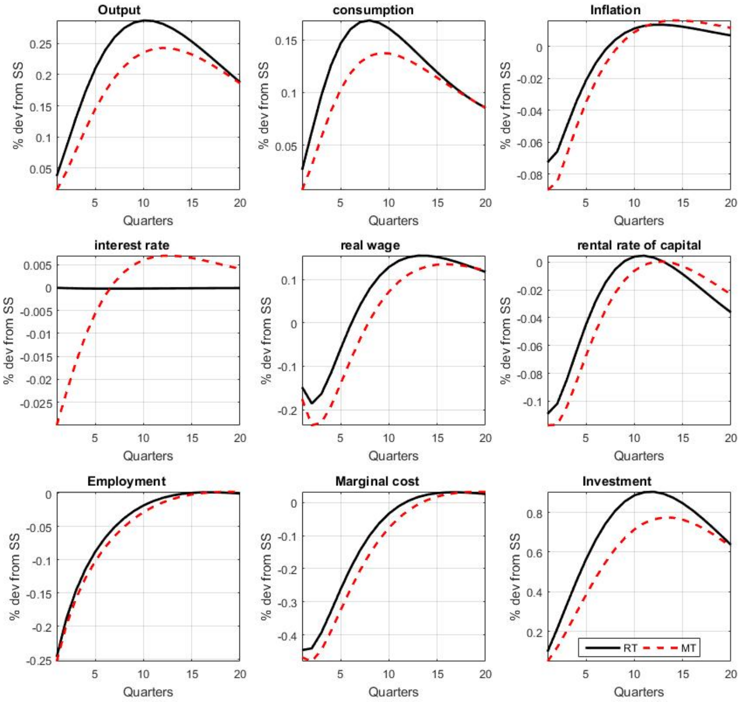

Figure 2.1: Impulse Response Function to a Techonology Shock
RT denotes the interest rate targeting; MT denotes the money supply targeting.

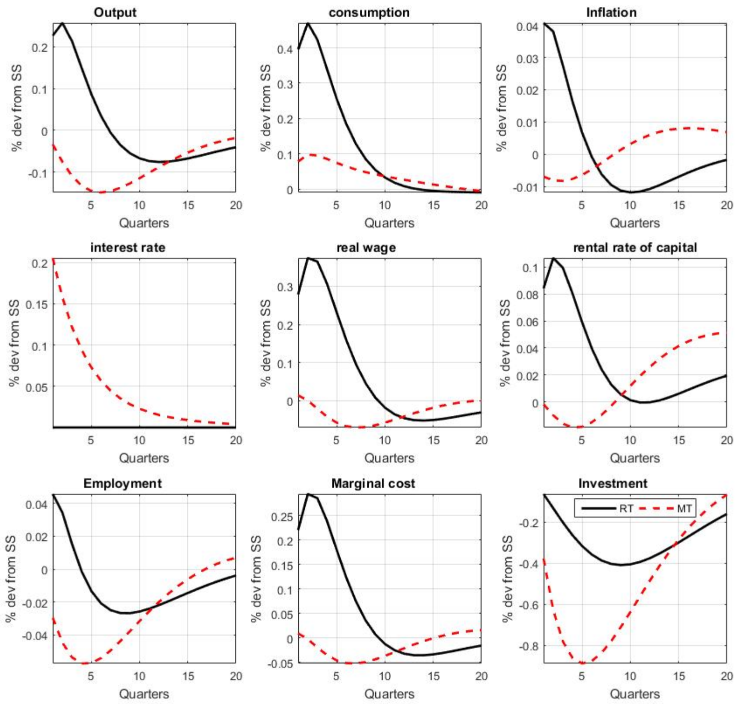

Figure 2.2: Impulse Response Function to a Preference Shock
RT and MT see notes in Figure 2.1

on capital, employment and investment. However, even interest rate pegging stabilise inflation and real wages, MT is better selected as it also generates higher welfare. Table (2.5) and (2.6) provide volatilities and welfare results under different degree of intertemporal substitution. It can be seen that higher intertemporal substitution ($\sigma_c = 0.5$) does not affect the volatility and welfare in terms of output and the inflation target. Therefore, the overall effects show that MT fares better under technology shock. This distinguishes from CD's result that RT fares better when intertemporal substitution is low, and MT is chosen when intertemporal substitution is high.

Preference shock

Figure (2.2) depicts the preference shock under two policy instruments. With interest rate pegging, preference shock raises the marginal utility of consumption and increases the output, employment. The central bank needs to reduce the money supply to prevent the raising of the interest rate. The accelerated output and consumption lead to a substantial crowding-out effect on investment. With a fixed interest rate, higher consumption leads to higher expected inflation; ultimately, a minor growth is delivered into inflation. On the other hand, money growth targeting generates different patterns of movements. Consumption still increases but in a much smaller magnitude. When monetary policy responds the interest rate declines, leading to lower output, in addition to real wages, and a larger decrease in investment.

The tables (2.3) to (2.5) show that monetary aggregates lead to higher volatility in output, the rental rate on capital, employment and investment, whereas the stability in output comes with a cost in the volatility of consumption, inflation and real wages. Higher welfare in monetary aggregates indicates that MT fares better. When intertemporal substitution is high ($\sigma_c = 0.5$), welfare is not changed, but the output is more stable in MT. Therefore, the overall effect is that MT fares better independent of the degree of intertemporal substitution.

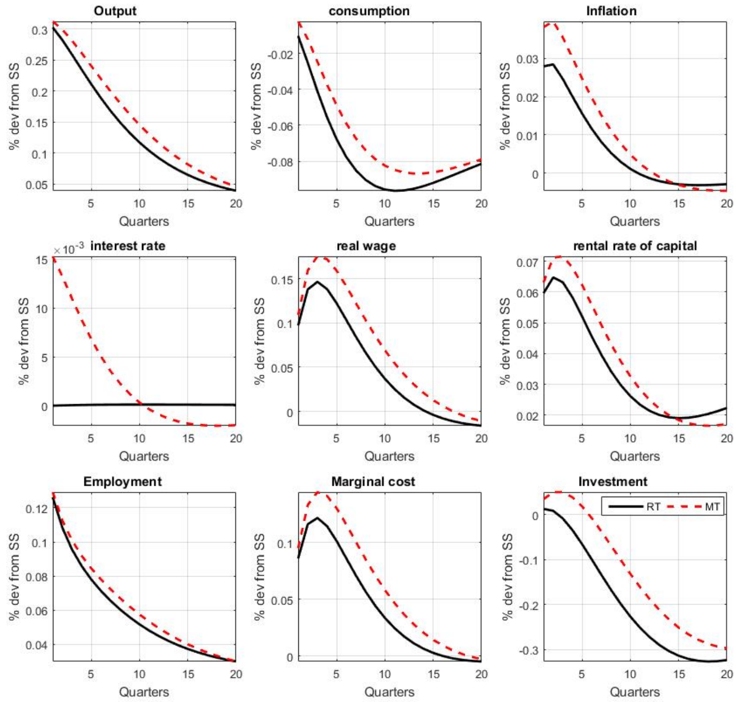

Figure 2.3: Impulse Response Function to a Government Spending Shock

RT and MT see notes in Figure 2.1

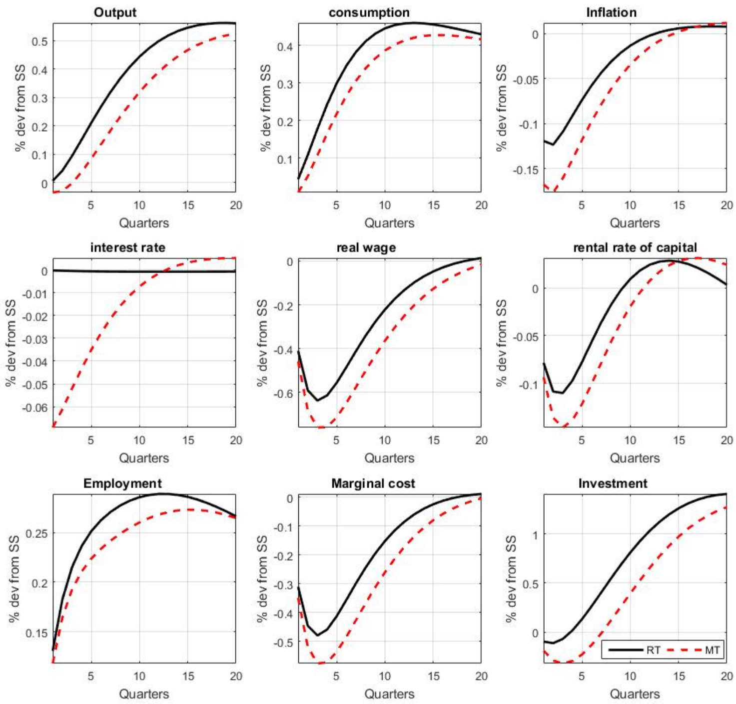

Figure 2.4: Impulse Response Function to a Labour Supply Shock

RT and MT see notes in Figure 2.1

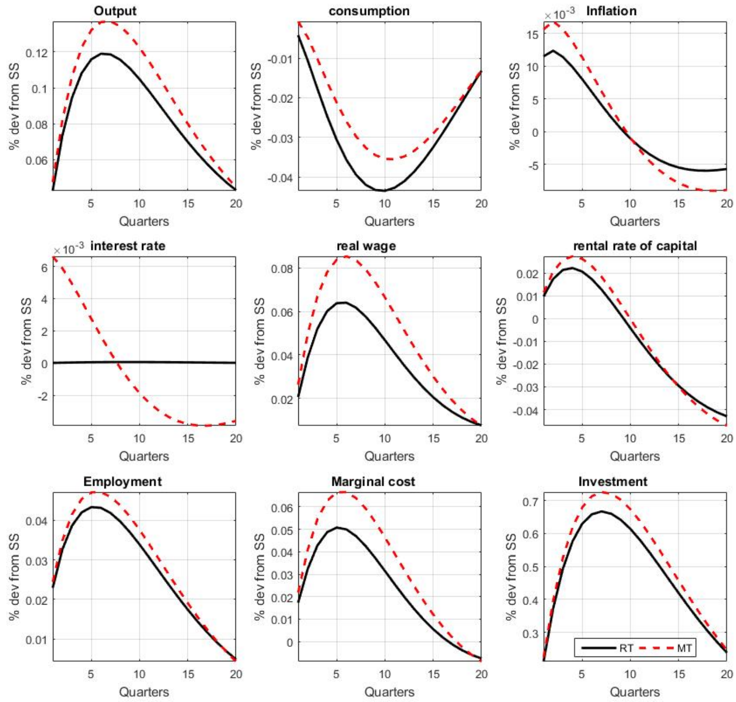

Figure 2.5: Impulse Response Function to a Investment Shock
RT and MT see notes in Figure 2.1

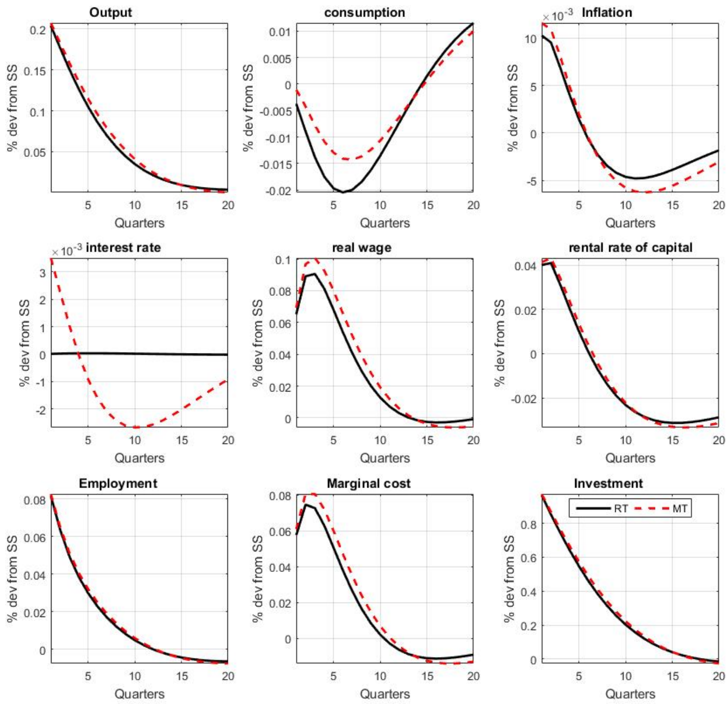

Figure 2.6: Impulse Response Function to an Equity Premium Shock
RT and MT see notes in Figure 2.1

Fiscal shock

As in the government spending shock shown in Figure (2.3) both instruments have similar patterns, but the monetary growth target has a slightly more substantial effect. With positive fiscal shock under the interest rate targeting, the higher output poses a significant crowding-out impact on consumption and investment. The central bank has to increase the money supply to prevent the interest rate from rising. Thus, creating higher inflation, the higher rental rate of capital, marginal cost, and real wages. In the other instrument, the money supply is constant, causing interest rates to rise. This also implies lower consumption and investment.

Moreover, Table (2.4) demonstrates that interest rate targeting generates higher stability in output, inflation, real wage, marginal cost and employment. However, higher stability in output leads to more fluctuations in consumption and investment due to the government spending crowding out effect. With higher welfare in interest rate targeting, RT fares better under the benchmark case. When intertemporal substitution is high, MT stabilise output and creates higher welfare; thus, MT fares better. This result is opposite to CD's result that MT is better off when intertemporal substitution is low.

Labour supply shock

For the labour supply shock in RT (Figure 2.4) output, employment, and consumption grows. Adversely, the real wage is reduced significantly but temporarily. This leads to a decline in marginal cost and inflation. MT shows a smaller response than RT. Given the fixed growth rate of money supply, the central bank reacts to the growth in labour supply by decreasing the interest rate, which poses the impacts in increasing consumption and investment.

According to the previous impulse responses and volatilities, the interest rate instrument produces higher stability in inflation, real wages and marginal cost, but more volatility in output, consumption and investment. Monetary targeting also generates higher welfare. Hence, the net effect is that the MT fares better. When risk aversion is low ($\sigma_c = 0.5$), the result does not change, which leads us to conclude that monetary targeting is better under labour supply shock.

Investment shock and Equity premium shock

An acceleration in investment increase output (Figure (2.5)) real wages and employment in both RT and MT. There exists significant crowding-out effect on consumption, while no substantial impact on inflation and interest rate were observed as they are close to the zero steady-state. Under the investment shock, the interest rate instrument is preferable for all variables excluding consumption. In terms of higher welfare in RT, the net effect shows that interest rate targeting fares better when risk aversion is high ($\sigma_c = 1.5$). The higher intertemporal substitution ($\sigma_c = 0.5$) alters the welfare result, leading to higher welfare in MT. While the RT still generates more stability in output and inflation, the result is ambiguous under a high degree of intertemporal substitution.

Figure (2.6) demonstrates the effects of equity premium shock. As this shock is not directly influenced by the structure of the economy, it can be seen that the overall effects to variables are limited and quite small. There is a rise of output, real wages, employment, marginal cost and investment, whereas there is a minor crowding out effect on consumption and nearly no influences on inflation and the interest rate. Table (2.4) shows that interest rate pegging generates higher stability in all variables, excluding that of consumption. Given higher welfare in RT, it is clear to show that RT fares better. When intertemporal substitution becomes higher, the output is more variable under RT. Since welfare is higher in MT, it is better off when intertemporal substitution is high.

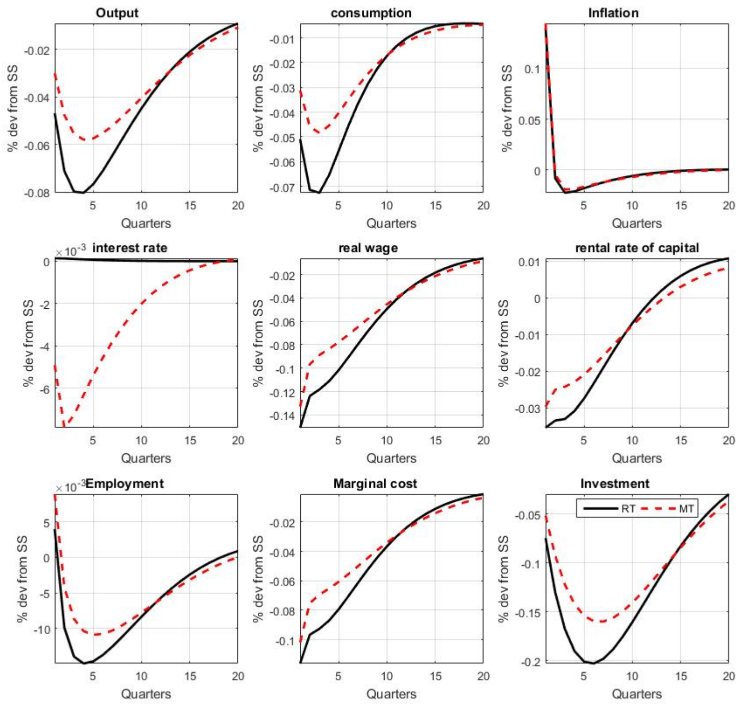

Figure 2.7: Impulse Response Function to a Price Mark-up Shock
RT and MT see notes in Figure 2.1

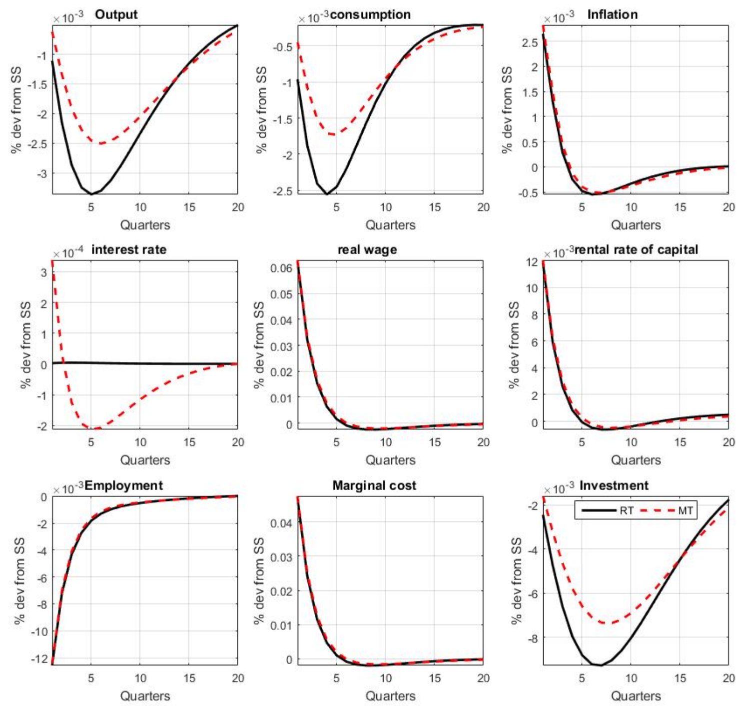

Figure 2.8: Impulse Response Function to a Wage Mark-up Shock
RT and MT see notes in Figure 2.1

Price and wage mark-up shock

Figure (2.7) plots the price mark-up shock, which leads to a higher inflation, and, contributes to a decline in output, consumption, real wages, and investment. To stabilize the price level, the central bank has to increase the interest rate under MT and reduce the money supply in RT, despite the impacts on interest rate and employment which are particularly small. Statically analysing a constant money supply instrument stabilizes all variables but comes at a cost with the volatility of inflation. With higher welfare in monetary targeting, the net effect shows that MT fares better. However, when intertemporal substitution is high ($\sigma_c = 0.5$), RT stabilize both output and inflation, and it also generates higher welfare. Therefore, RT is better off with low risk aversion.

Similarly, as with the price mark-up, the wage mark-up shock (Figure 2.8) produces lower output, consumption, employment and investment, whereas the higher real wages results in higher inflation and marginal cost. Those effects follow the close patterns in both RT and MT. However, the influences are overall too trivial. In addition, when wage mark-up shock dominates, money supply targeting stabilises output, consumption, employment and investment while interest rate targeting produces higher welfare and leads to less volatile in inflation. Therefore, the net effect implies that RT fares better. When higher intertemporal substitution is applied, RT stabilises both output and inflation, making the result unchanged. Thus, RT is better chosen under wage mark-up shock regardless of the degree of risk aversion.

Money demand shock

Since it is difficult to observe the money demand shock, the AR(1) coefficient and shock variance are calibrated from previous literature. In this paper, we calibrate the value of the money demand shock. The calibrated shock is then added into the estimation to obtain the final results. When the money demand increases, both output and consumption drop, the interest rate must be increased to offset the impacts on inflation. Over time, the interest rate reduces, leading to a growth of inflation. It can be seen that the influences merely enter into MT because the money demand shock is a function of real balance that is not related to other variables. In MT, real balance is a function of not only money demand shocks but also inflation. The result is the same as in the CD paper and in Ireland (2000), in spite of the magnitudes that are relatively small for inflation, interest rate and employment. As the money demand shock will have a small effect when the interest rate is the policy instrument, the model is estimated based on the calibrated value of the money demand shock.

2.6 Conclusion

In recent decades, most central banks have focused on the interest rate instrument. The Taylor rule is also extensively applied by most monetary policy studies. However, after the financial crisis of 2007-8, the zero-lower bond interest rate issue has guided people's insights towards reviewing Poole's analysis. How would full structural stochastic shocks affect the optimal choice problem? This paper evaluates the optimal choice of alternative policy instruments through a medium-scale New Keynesian DSGE model under nine structural shocks, followed by the Smets and Wouters (2003). The aim is to find the optimal selection of monetary policy instruments between the interest rate and monetary aggregates. Some apparent differences have been identified from the SW model. First, this paper has applied the non-separable money-in-utility function to recapture the importance of money supply in the DSGE model. Second, the monetary policy implements to peg either the growth rate of money supply or the interest rate to find the optimal instrument.

The main contribution of this chapter can be described in several ways: (1) This chapter applies a full structural estimated DSGE model into the Poole's analysis, rather than what the CD paper uses to calibrate the model. Therefore, the estimated model could provide more robust results for evaluation. (2) This paper implements welfare which is derived from the estimated model. The results based on the estimated welfare could also be more reasonable. (3) The previous studies evaluate the best policy instrument under mainly fiscal shocks, money demand shocks and supply shocks, while my study applies a set of full structural shocks. Having more stochastic disturbances are increasingly important as the macroeconomic environment has become more complicated, especially after the financial crisis. (4) The outcome suggests that the variations of other factors, such as inflation, consumption, and investment, also matter for the optimal choice of monetary policy. Therefore, the ranking in this chapter chooses the better-performed instrument based on both more stable factors and welfare.

This chapter sheds new light on the relative merits of using a monetary aggregate as opposed to a nominal interest rate as the monetary policy instrument. It does so by using an estimated DSGE model of the euro area (EA) to conduct an assessment in the spirit of Poole (1970). The chapter finds that the ranking varies under different disturbances. Monetary targeting fares better under technology shock, preference shock, and labour supply shock, independent of the degree of risk aversion. Interest rate targeting is favoured when intertemporal substitution is low under fiscal shock and equity premium shock. The interest rate instrument is also chosen when intertemporal substitution is high under price mark-up shock. Nevertheless, the performance of monetary targeting is improved when risk aversion is high and is ambiguous when risk aversion is low. Interest rate policy instruments are chosen under wage mark-up shock regardless of the degree of intertemporal substitution. Although the interest rate targeting is still a popular instrument, the analysis in this paper has found the importance of monetary aggregates under some disturbances. As Hoffmann and Kempa (2009) has suggested the exchange rate targeting outperforms the monetary targeting in the open economy. Testing whether Poole's analysis still holds for within other models could be interesting for future research.

2.7 Appendix

2.7.1 Welfare Loss Function Derivations

The utility function is approximately written as:

$$E_0 \sum_{t=0}^{\infty} \beta^t [\frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma} - \kappa \frac{L_t^{1+\phi}}{1+\phi}(1 + \frac{1}{2}(1+\phi)(\frac{Y_t}{F+Y_t}\zeta D_t^P + \eta(1+\eta\phi)D_t^W))]$$

The lower letters denote the proportional deviations for all variables, such as $c_t = \frac{C_t - C}{C}$. The first-order terms in this utility expansion are zero.

Proof:

$$dU = E_0 \sum_{t=0}^{\infty} \beta^t [(C - hC)^{-\sigma} \frac{(C_t - C) - h(C_{t-1} - C)}{C} C -$$

$$\kappa L^{\phi}(1 + \frac{1}{2}(1+\phi)(\frac{Y_t}{F+Y_t}\zeta D_t^P + \eta(1+\eta\phi)D_t^W))\frac{L_t - L}{L} L]$$

$$dU = E_0 \sum_{t=0}^{\infty} \beta^t [C^{1-\sigma}(1-h)^{-\sigma}(c_t - hc_{t-1}) - \kappa N^{1+\phi} l_t]$$

$$dU = E_0 \sum_{t=0}^{\infty} \beta^t [C^{1-\sigma}(1-h)^{-\sigma}(1-\beta h)c_t - \kappa N^{1+\phi} l_t]$$

where $N^{1+\phi} = L^{1+\phi}(1 + \frac{1}{2}(1+\phi)(\frac{Y_t}{F+Y_t}\zeta D_t^P + \eta(1+\eta\phi)D_t^W))$

Aggregate consumption is given by

$$C_t = A_t Z_t^{\alpha} L_t^{1-\alpha} K_{t-1}^{\alpha} - F - G_t - I_t(1 - S(\frac{I_t}{I_{t-1}})) - \psi(Z_t)K_{t-1}$$

Ignoring the second-order deviations in c_t, the first order deviations of the utility function become

$$dU = E_0 \sum_{t=0}^{\infty} \beta^t [C^{-\sigma}(1-h)^{-\sigma}(1-\beta h)(\alpha(Y+F)z_t + (1-\alpha)(Y+F)l_t$$

$$+(\frac{\alpha(Y+F)}{K}-\frac{1}{\beta}+1-\delta)Kk_{t-1}-\psi'(1)KZz_t)-\kappa N^{1+\phi}]$$

Proof:

Since that $c_t=\frac{C_t-C}{C}\approx\frac{lnC_t}{lnC}$, then

$$dU=E_0\sum_{t=0}^{\infty}\beta^t[C^{-\sigma}(1-h)^{-\sigma}(1-\beta h)C_t'-\kappa N^{1+\phi}l_t]$$

Take the first-order Taylor series expansion of Z,L,K:

$$AZ^{\alpha}L^{1-\alpha}K^{\alpha}=Y+F$$

$$\frac{\partial C_t}{\partial Z}=[\frac{\alpha(Y+F)}{Z}-\psi'(1)K]\frac{Z_t-Z}{Z}Z$$

$$\frac{\partial C_t}{\partial Z}=\alpha(Y+F)-\psi'(1)KZz_t$$

$$\frac{\partial C_t}{\partial L}=(1-\alpha)\frac{Y+F}{L}\frac{L_t-L}{L}L$$

$$\frac{\partial C_t}{\partial L}=(1-\alpha)(Y+F)l_t$$

$$\frac{\partial C_t}{\partial K}=[\alpha\frac{Y+F}{K}-\psi'(Z)]\frac{K_{t-1}-K}{K}K$$

$$\frac{\partial C_t}{\partial K}=[\alpha\frac{Y+F}{K}-\psi'(Z)]Kk_{t-1}$$

Since that

$$\psi'(Z)=\frac{1}{\beta}-1+\delta$$

then

$$\frac{\partial C_t}{\partial K}=[\alpha\frac{Y+F}{K}-\frac{1}{\beta}+1-\delta]Kk_{t-1}$$

Then, combining those terms,

$$dU=E_0\sum_{t=0}^{\infty}\beta^t[C^{-\sigma}(1-h)^{-\sigma}(1-\beta h)(\alpha(Y+F)z_t-\psi\prime(1)KZz_t+(1-\alpha)(Y+F)l_t$$

$$+(\frac{\alpha(Y+F)}{K}-\frac{1}{\beta}+1-\delta)Kk_{t-1})-\kappa N^{1+\phi}]$$

Those terms with l_t, z_t, k_{t-1} are equal to zero, and then the steady-state equations are given as below:

$$[(1-h)C]^{-\sigma}(1-\beta h)-\mu=0$$

$$[(1-h)C]^{-\sigma}(1-\beta h)=\mu$$

$$-1+(1-\delta)\beta+\alpha\beta AZ(\frac{L}{ZK})^{1-\alpha}-\beta\psi(Z)=0$$

$$-\frac{1}{\beta}+1-\delta+\alpha\frac{Y+F}{K}-\psi(Z)=0$$

$\psi(Z)=0$ as $Z=1$ from equation(77) to (81)

$$[\alpha\frac{Y+F}{K}-\frac{1}{\beta}-1+\delta]Kk_{t-1}=0$$

$$-\kappa L^{\phi}+(1-\alpha)A(\frac{ZK}{L})^{\alpha}\mu=0$$

$$-\kappa L^{\phi}+\mu(1-\alpha)\frac{Y+F}{L}=0$$

$$-\kappa L^{\phi}Ll_t+\mu(1-\alpha)\frac{Y+F}{L}Ll_t=0$$

$$-\kappa L^{1+\phi}l_t+\mu(1-\alpha)(Y+F)l_t=0$$

$$[(1-h)C]^{-\sigma}(1-\beta h)(1-\alpha)(Y+F)l_t-\kappa L^{1+\phi}l_t=0$$

$$\psi'(Z)-\alpha A(\frac{L}{ZK})^{1-\alpha}=0$$

$$\psi'(Z)-\alpha\frac{Y+F}{KZ}=0$$

$$\alpha(Y+F)z_t - \psi'(1)KZz_t = 0$$

Therefore, all first order terms in expansion are zero.

The welfare loss function is then the second-order terms in the Taylor-series expansion with two expression: the second-order terms in c_t, l_t and z_t, l_t, k_{t-1}.

The second-order Taylor series expansion of c_t, l_t are given respectively: For c_t

$$dU_c = (C - hC)^{-\sigma}$$

$$dU_{cc} = -\sigma(C-hC)^{-\sigma-1} = -\sigma C^{-1-\sigma}(1-h)^{-1-\sigma}$$

Second-order

$$dU_{cc}^2 = -\frac{1}{2}\sigma C^{-1-\sigma}(1-h)^{-1-\sigma}(\frac{(C_t - C) - h(C_{t-1} - C)}{C})^2 C^2$$

$$dU_{cc}^2 = -\frac{1}{2}\sigma C^{1-\sigma}(1-h)^{-1-\sigma}(c_t - hc_{t-1})^2$$

For l_t

$$dU_l = -\kappa L^{\phi}(1 + \frac{1}{2}(1+\phi)(\frac{Y}{F+Y}\zeta D_t^P + \eta(1+\eta\phi)D_t^W))$$

$$dU_{ll} = -\kappa\phi L^{\phi-1}(1 + \frac{1}{2}(1+\phi)(\frac{Y}{F+Y}\zeta D_t^P + \eta(1+\eta\phi)D_t^W))$$

$$dU_{ll}^2 = -\frac{1}{2}\kappa\phi L^{\phi-1}(\frac{L_t - L}{L})^2 L^2(1 + \frac{1}{2}(1+\phi)(\frac{Y}{F+Y}\zeta D_t^P + \eta(1+\eta\phi)D_t^W))$$

$$dU_{ll}^2 = -\frac{1}{2}\kappa\phi L^{\phi+1}l^2(1 + \frac{1}{2}(1+\phi)(\frac{Y}{F+Y}\zeta D_t^P + \eta(1+\eta\phi)D_t^W))$$

Since

$$D_t^P = \xi_{t-1}^p + \frac{\xi_p}{1-\xi_p}(\pi_t - \gamma_p\pi_{t-1})^2$$

$$D_t^P = \frac{\xi_p}{(1-\beta\xi_p)(1-\xi_p)}(\pi_t - \gamma_p\pi_{t-1})^2$$

$$D_t^W = \xi_w D_{t-1}^W + \frac{\xi_w}{1-\xi_w}(\Delta wr_t + \pi_t - \gamma_w(\Delta wr_{t-1} + \pi_{t-1}))^2$$

$$D_t^W = \frac{\xi_w}{(1-\beta\xi_w)(1-\xi_w)}(\Delta wr_t + \pi_t - \gamma_w(\Delta wr_{t-1} + \pi_{t-1}))^2$$

Then

$$dU_{ll}^2 = -\frac{1}{2}\kappa\phi L^{\phi+1} l^2 (1 + \frac{1}{2}(1+\phi)(\frac{Y}{F+Y}\frac{\zeta\xi_p}{(1-\beta\xi_p)(1-\xi_p)}(\pi_t - \gamma_p\pi_{t-1})^2$$

$$+\frac{\eta(1+\eta\phi)\xi_w}{(1-\beta\xi_w)(1-\xi_w)}(\Delta wr_t + \pi_t - \gamma_w(\Delta wr_{t-1} + \pi_{t-1}))^2)$$

$$dU_{ll}^2 = -\frac{1}{2}\kappa L^{\phi+1}(\phi l^2 + \frac{1}{2}\phi(1+\phi)l^2(\frac{Y}{F+Y}\frac{\zeta\xi_p}{(1-\beta\xi_p)(1-\xi_p)}(\pi_t - \gamma_p\pi_{t-1})^2$$

$$+\frac{\eta(1+\eta\phi)\xi_w}{(1-\beta\xi_w)(1-\xi_w)}(\Delta wr_t + \pi_t - \gamma_w(\Delta wr_{t-1} + \pi_{t-1}))^2)$$

Therefore,

$$dU^2 = -\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t[\sigma C^{1-\sigma}(1-h)^{-1-\sigma}(c_t - hc_{t-1})^2 + \kappa L^{\phi+1}(\phi l^2 + \frac{1}{2}\phi(1+\phi)l^2$$

$$(\frac{Y}{F+Y}\frac{\zeta\xi_p}{(1-\beta\xi_p)(1-\xi_p)}(\pi_t - \gamma_p\pi_{t-1})^2 + \frac{\eta(1+\eta\phi)\xi_w}{(1-\beta\xi_w)(1-\xi_w)}(\Delta wr_t + \pi_t - \gamma_w(\Delta wr_{t-1} + \pi_{t-1}))^2)]$$

According to the steady-state equations, $\kappa L^{1+\phi}$ can be replaced by the expression $C^{1-\sigma}(1-h)^{-\sigma}(1-\frac{1}{\eta})(1-\alpha)/c_y$ where $c_y = \frac{C}{Y+F}$, then utility based loss function is obtained:

$$dU^2 = -\frac{C^{1-\sigma}(1-h)^{-\sigma}}{2}E_0\sum_{t=0}^{\infty}\beta^t[(1-h)^{-1}\sigma(c_t - hc_{t-1})^2 + \frac{(1-\frac{1}{\eta})(1-\alpha)}{c_y}(\phi l^2 + \frac{1}{2}\phi(1+\phi)l^2$$

$$(\frac{Y}{F+Y}\frac{\zeta\xi_p}{(1-\beta\xi_p)(1-\xi_p)}(\pi_t - \gamma_p\pi_{t-1})^2 + \frac{\eta(1+\eta\phi)\xi_w}{(1-\beta\xi_w)(1-\xi_w)}(\Delta wr_t + \pi_t - \gamma_w(\Delta wr_{t-1} + \pi_{t-1}))^2)]$$

The second part of loss function is the second-order terms from constraints C.17, then the second-order Taylor series expansion of each variable is given below:
Recall that aggregate consumption,

$$C_t = A_t Z_t^{\alpha} L_t^{1-\alpha} K_{t-1}^{\alpha} - F - G_t - I_t(1 - S(\frac{I_t}{I_{t-1}})) - \psi(Z_t)K_{t-1}$$

For A,

$$\frac{\partial C_t}{\partial A} = Z^{\alpha} L^{1-\alpha} K^{\alpha}$$

$$\frac{\partial^2 C_t}{\partial A^2} = 0$$

The the second order terms of A is zero.

For Z,

$$\frac{\partial C_t}{\partial Z} = \alpha Z^{\alpha-1} A L^{1-\alpha} K^{\alpha} - \psi'(1)K$$

$$\frac{\partial^2 C_t}{\partial Z^2} = \alpha(\alpha-1)\frac{Y+F}{Z^2} - \psi''(1)K$$

The second order term is then,

$$dU_z = \frac{1}{2}[\alpha(1-\alpha)\frac{Y+F}{Z^2} - \psi''(1)K](\frac{Z_t - Z}{Z})^2 Z^2$$

$$dU_z = -\frac{1}{2}\alpha(1-\alpha)(Y+F)z_t^2 - \frac{1}{2}\psi''(1)KZ^2 z_t^2$$

For L,

$$\frac{\partial C_t}{\partial L} = (1-\alpha)AZ^{\alpha} L^{-\alpha} K^{\alpha}$$

$$\frac{\partial^2 C_t}{\partial L^2} = -\alpha(1-\alpha)\frac{Y+F}{L^2}$$

The second order term is then,

$$dU_l = \frac{1}{2}[-\alpha(1-\alpha)\frac{Y+F}{L^2}](\frac{L_t - L}{L})^2 L^2$$

$$dU_l = -\frac{1}{2}\alpha(1-\alpha)(Y+F)l_t^2$$

For K,

$$\frac{\partial C_t}{\partial K} = \alpha A Z^{\alpha} L^{1-\alpha} K^{\alpha-1} - \Psi(Z)$$

$$\frac{\partial^2 C_t}{\partial K^2} = \alpha(\alpha-1)\frac{Y+F}{K^2}$$

$$dU_k = \frac{1}{2}[-\alpha(1-\alpha)\frac{Y+F}{K^2}](\frac{K_{t-1}-K}{K})^2 K^2$$

$$dU_l = -\frac{1}{2}\alpha(1-\alpha)(Y+F)k_{t-1}^2$$

For I,

$$\frac{\partial C_t}{\partial I} = -(1 - S(\frac{I_t}{I_{t-1}})) + \frac{I_t}{I_{t-1}}S'(\frac{I_t}{I_{t-1}})$$

$$\frac{\partial^2 C_t}{\partial I^2} = S'(1)\frac{1}{I} + S''(1)\frac{1}{I^2}$$

Since $S'(1) = 0$, then,

$$\frac{\partial^2 C_t}{\partial I^2} = S''(1)\frac{1}{I^2}$$

$$dU_I = \frac{1}{2}S''(1)\frac{1}{I^2}(\frac{(I_t - I_t) - (I_{t-1} - I_t)}{I})^2 I^2$$

$$dU_I = \frac{1}{2}S''(1)(i_t - i_{t-1})^2$$

Then the partial derivatives of each variable are presented below:

For AZ,

$$\frac{\partial^2 C_t}{\partial A \partial Z} = \alpha Z^{\alpha-1} L^{1-\alpha} K^{\alpha}$$

$$dU_{AZ} = \alpha\frac{Y+F}{AZ}\frac{(A_t - A)(Z_t - Z)}{AZ}AZ = \alpha(Y+F)a_t z_t$$

For AL,

$$\frac{\partial^2 C_t}{\partial A \partial L} = (1-\alpha) Z^{\alpha} L^{-\alpha} K^{\alpha}$$

$$dU_{AL} = (1-\alpha)\frac{Y+F}{AL}\frac{(A_t - A)(L_t - L)}{AL}AL = (1-\alpha)(Y+F)a_t l_t$$

For AK,

$$\frac{\partial^2 C_t}{\partial A \partial K} = \alpha Z^{\alpha} L^{1-\alpha} K^{\alpha-1}$$

$$dU_{AK} = \alpha \frac{Y+F}{AK} \frac{(A_t - A)(K_{t-1} - K)}{AK} AK = \alpha(Y+F) a_t k_{t-1}$$

For ZL,

$$\frac{\partial^2 C_t}{\partial Z \partial L} = \alpha(1-\alpha) Z^{\alpha-1} L^{-\alpha} K^{\alpha}$$

$$dU_{ZL} = \alpha(1-\alpha) \frac{Y+F}{ZL} \frac{(Z_t - Z)(L_t - L)}{ZL} ZL = \alpha(1-\alpha)(Y+F) z_t l_t$$

For ZK,

$$\frac{\partial^2 C_t}{\partial Z \partial K} = \alpha^2 A Z^{\alpha-1} L^{-\alpha} K^{\alpha-1} - \psi'(1)$$

$$dU_{ZK} = [\alpha^2 \frac{Y+F}{ZK} - \psi'(1)] \frac{(Z_t - Z)(K_{t-1} - K)}{ZK} ZK = \alpha^2 (Y+F) z_t k_{t-1} - \psi'(1) KZ z_t k_{t-1}$$

For LK,

$$\frac{\partial^2 C_t}{\partial L \partial K} = \alpha(1-\alpha) A Z^{\alpha} L^{-\alpha} K^{\alpha-1}$$

$$dU_{LK} = \alpha(1-\alpha) \frac{Y+F}{LK} \frac{(L_t - L)(K_{t-1} - K)}{LK} LK = \alpha(1-\alpha)(Y+F) l_t k_{t-1}$$

Therefore, to sum those second terms up,

$$dU = -\frac{1}{2}\alpha(1-\alpha)(Y+F)z_t^2 - \frac{1}{2}\psi''(1)KZ^2 z_t^2 - \frac{1}{2}\alpha(1-\alpha)(Y+F)l_t^2 - \frac{1}{2}\alpha(1-\alpha)(Y+F)k_{t-1}^2$$

$$+\frac{1}{2}S''(1)(i_t - i_{t-1})^2 + \alpha(Y+F)a_t z_t + (1-\alpha)(Y+F)a_t l_t + \alpha(Y+F)a_t k_{t-1}$$

$$+\alpha(1-\alpha)(Y+F)z_t l_t + \alpha^2 (Y+F) z_t k_{t-1} - \psi'(1) KZ z_t k_{t-1} + \alpha(1-\alpha)(Y+F) l_t k_{t-1}$$

$$dU = -\frac{1}{2}\{(Y+F)(\alpha(1-\alpha)z_t^2 + [\alpha(1-\alpha)l_t^2 + \alpha(1-\alpha)k_{t-1}^2 - 2\alpha(1-\alpha)l_t k_{t-1}]$$

$$-2[(1-\alpha)a_t l_t + \alpha a_t k_{t-1} + \alpha(1-\alpha) z_t l_t + \alpha^2 z_t k_{t-1}] - \alpha a_t z_t) + KZ[2\psi'(1) z_t k_{t-1} + \psi''(1) Z z_t^2]$$

$$-S''(1)(i_t - i_{t-1})^2\}$$

$$dU = -\frac{1}{2}[(Y+F)(\alpha(1-\alpha)(l_t - k_{t-1})^2 - 2[(1-\alpha)l_t + a_t k_{t-1}](a_t + \alpha z_t) + \alpha(1-\alpha)z_t^2 - \alpha a_t z_t]$$

$$+KZ[2\psi'(1)z_t k_{t-1} + \psi''(1)Zz_t^2] - S''(1)(i_t - i_{t-1})^2]$$

Combining the rest terms first order terms and $Z = 1$, then the second-order term is,

$$dU = -\frac{1-\beta h}{2}C^{-\sigma}(1-h)^{-\sigma}E_0\sum_{t=0}^{\infty}\beta^t[(Y+F)(\alpha(1-\alpha)(l_t - k_{t-1})^2 - 2[(1-\alpha)l_t + a_t k_{t-1}]$$

$$(a_t + \alpha z_t) + \alpha(1-\alpha)z_t^2 - \alpha a_t z_t) + K[2\psi'(1)z_t k_{t-1} + \psi''(1)z_t^2] - S''(1)(i_t - i_{t-1})^2]$$

From the definition $\psi = \psi'(1)/\psi''(1)$, and the deterministic equilibrium conditions $\psi'(1) = R_k$ and $\alpha(Y+F) = R_k K$, $\psi'(1) = R_k = \frac{\alpha(Y+F)}{K}$, then the second component in utility based welfare loss function is,

$$dU = -\frac{1-\beta h}{2c_y}C^{1-\sigma}(1-h)^{-\sigma}E_0\sum_{t=0}^{\infty}\beta^t[(\alpha(1-\alpha)(l_t - k_{t-1} - z_t - \frac{1}{1-\alpha}a_t)^2$$

$$+\frac{\alpha}{\psi}(z_t + \psi a_t)^2 - 2a_t l_t - \frac{\delta K}{Y+F}S''(1)(i_t - i_{t-1})^2]$$

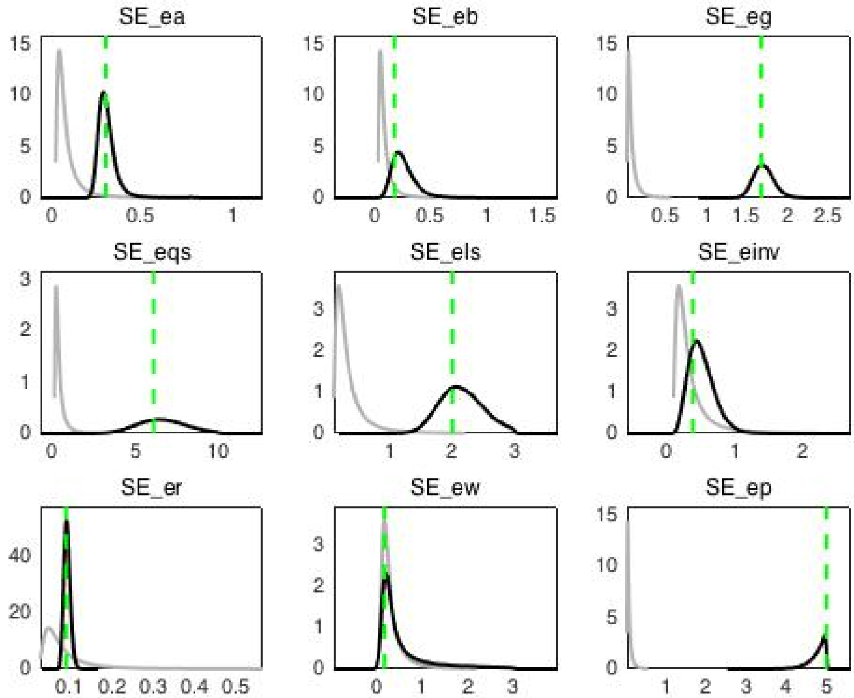

Figure 2.9: Prior and Posterior Distributions
Note: The figure shows the prior (grey lines) and posterior (dark lines) distributions for nine shocks.

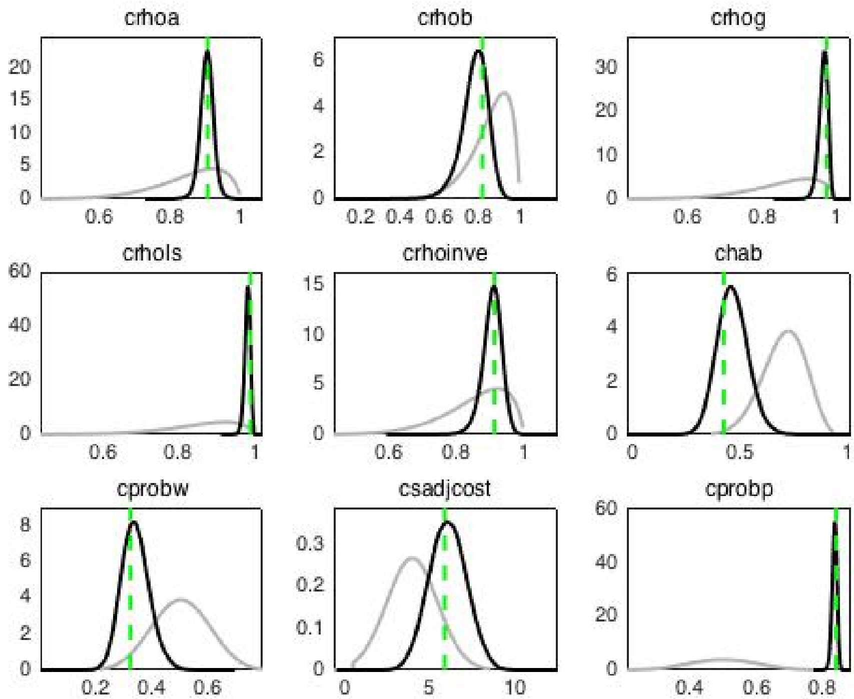

Figure 2.10: Prior and Posterior Distributions
Note: The figure shows the prior (grey lines) and posterior (dark lines) distributions for various parameters.

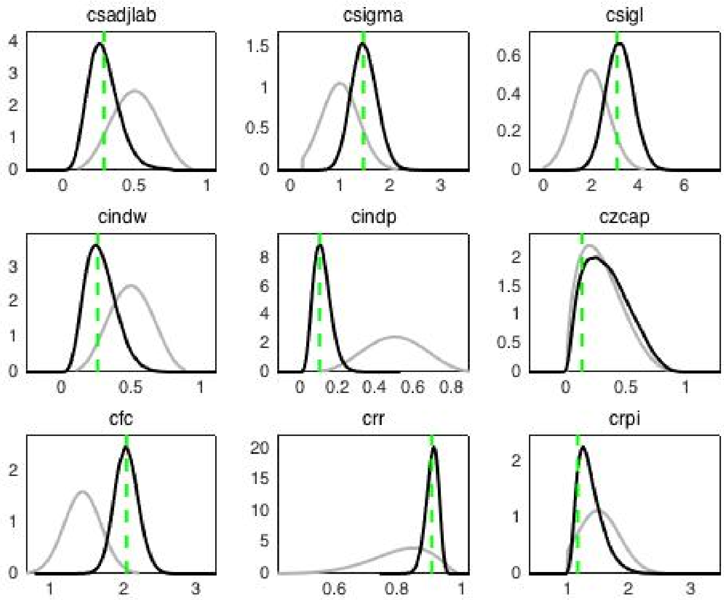

Figure 2.11: Prior and Posterior Distributions
Note: The figure shows the prior (grey lines) and posterior (dark lines) distributions for various output gap feedback parameters.

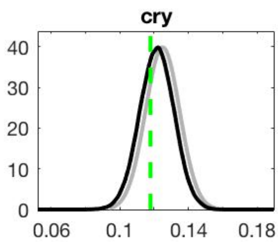

Figure 2.12: Prior and Posterior Distributions
Note: The figure shows the prior (grey lines) and posterior (dark lines) distributions for parameter.

Chapter 3

Optimal Monetary and Macroprudential Policy under Uncertainty

3.1 Introduction

In the past decade, the financial crisis of 2007-2008 has triggered a large body of research on the effects of macroprudential policy, and the interactions between monetary policy and macroprudential policy. Analysis in this area has involved modifying a standard DSGE model by including financial frictions, such as a moral hazard problem in firms' financing (Martinez-Miera and Repullo (2019)). A common finding in this literature is that macroprudential policy can help maintain financial stability (Agénor et al. (2013)). However, their implementation is relatively novel and so the full implication of macroprudential policies are yet to be determined. Against this backdrop, I study the implications of model uncertainty, in the form of a policy maker who fears that her model is misspecified, for optimal macroprudential policy.

There are different approaches to overcome the model misspecification. Given that models differ, sometimes markedly, in terms of their policy transmission and implications, those implementing policy would like to ensure against highly adverse outcomes. One approach to doing so involves robust control, as put forward by Hansen and Sargent (2001). This approach is desirable when decision makers are faced with uncertainty but cannot specify a probability distribution over possible specification errors. A key input in the application of robust control techniques is the reference model. Others like Cateau et al. (2006) apply the theory of robust control method and the minimax approach. He also compared different methods for multiple models when dealing with uncertainty.

In this chapter I analyse the implementation of macroprudential policy in a model developed by Kannan et al. (2012) (KRS hereafter). This is a model with financial frictions that contains a financial accelerator working through the housing sector. Developments in the housing market were at the centre of the crisis during the Great Recession, thus it is important to include this sector.

Crucially, given the lack of a long historical record with macroprudential policies, it is not clear whether any model captures the most important transmissions of fiscal and other shocks. Consequently, there is considerable interest in analysing how a policy maker, who fears that her model is misspecified, would operate. For this target, we apply the KRS model with robust macroprudential policy to analyse the central bank's behaviour. Through comparisons between the robust monetary policy and non-robust monetary policy with the macroprudential instrument, our result further confirms that the macroprudential instrument is a powerful tool to stabilize financial shocks. Moreover, the result suggests that it is beneficial to identify the source of shocks to ensure how a policy maker can implement the monetary policy with or without macroprudential policy instrument.

The remainder of the paper is structured as follows. Section 2 reviews the related literature. Section 3 presents a robust control method. Section 4 discusses and compares the results. Finally, we conclude in section 5.

3.2 Literature Overview

This section examines the development of macroprudential policy models, the applications of model uncertainty, and the linkage between robust control theory and macroprudential policy.

3.2.1 Understanding macroprudential policy

According to the various financial friction mechanisms below, several papers regarding macroprudential policy would be surveyed. First, some papers have used collateral constraint as the primary financial friction. For general studies, credit is modelled as a determinant of either physical capital accumulation or housing investment since there is existing asymmetric information in the financial market. Therefore, borrowers can issue credit only up to the value of their collateral. The availability and the cost of credit have an impact on aggregate demand, the output gap and inflation. We can, therefore, use such models to analyse how macroprudential policies may influence credit developments and price stability.

In Agénor et al. (2013)(AAPS), the macroeconomic stability is measured through the output gap and inflation, and financial stability is measured by three indicators: real house prices, the credit-to-GDP ratio and the loan spread. The AAPS paper develops a closed economy model with six types of agents and two alternative policy rules, which include a standard Taylor type interest rate rule augmented to account for additional variable credit growth and a countercyclical regulatory rule with capital requirements which are also connected to credit growth. There are two appealing outcomes. One is that the combination of a credit-augmented interest rate rule and a Basel III-type countercyclical capital requirement rule could be optimal for mitigating economic volatilities, even if monetary policy reacts strongly to inflation deviations from the target. The other shows that the sensitivity of the regulatory rule to credit growth gaps is related to the degree of policy interest rate smoothing and the level of policymakers' concern with financial stability. Unlike AAPS, we apply a model with committed optimal policy. The macroprudential policy is applied using the robust control method to analyze the impacts on the economy. By introducing technology shocks or financial shocks, Angelini et al. (2011) (ANP) researched the influences and outcomes of the macroprudential policies and their interactions with monetary policy. The research used the mechanism of financial frictions, including collateral constraint and LTV under two circumstances: co-operation and non-cooperation between monetary policy and macroprudential policy.

ANP applies an estimated DSGE model by incorporating the banking sector. For one of the financial friction mechanisms in this model, banks attempt to keep the capital-asset ratio close to a capital requirement target level. When loans increase, the capital-asset ratio declines below the capital requirement level, leading to growth in the lending rate. This then drops the demand for credit. For the LTV ratio, the collateral constraint is tightened when the housing market shrinks, and the borrowers' ability to finance consumption and housing investment is decreased. Moreover, ANP devises the monetary policy through a Taylor rule and macroprudential policy through two instruments. The interaction between monetary policy and macroprudential policy represents two situations: under the co-operation case, both macroprudential authority and the central bank minimize the same objective function; under the non-cooperation case, each authority minimizes their loss function.

As a result, when technology shocks are the main driving force for the economy, macroprudential policy has limited effect on macroeconomic stability. Additionally, the lack of co-operation between the macroprudential authority and the central bank could potentially create different policies, leading to excessive volatility. When financial shocks are the core driver of macroeconomic fluctuations, co-operation between macroprudential policy authorities and the central bank could mitigate volatilities in the economy, stabilizing output and the loans-to-output ratio. Finally, their studies recommend that macroprudential policy should be treated as a helpful complement to the traditional monetary policies for dealing with financial shocks. By contrast with ANP's model, we apply a DSGE model with housing market through the collateral constraint accelerator mechanism. However, we also consider model uncertainty by including robust control method when the central bank concern the model misspecification issues.

In contrast, Beau et al. (2012) (BCM) also investigates the interactions between monetary policy and macroprudential policy. By applying an estimated DSGE model for both the Euro area and the US data, they aim to identify whether monetary and macroprudential policies have mixed, neutral or conflicting influences on price stability. In terms of inflation dynamics, they compare different policy rules that depend on the monetary policy objectives and the financial stability objective within the implementation of macroprudential policy.

Brzoza-Brzezina et al. (2015) (BBKM) explored whether the macroprudential policy can contribute the stability in the peripheral countries in the Euro area. By building a two-country DSGE model, BBKM found that macroprudential policy is efficient only by applying individually for each region. Moreover, like the other research, macroprudential policy is found to be helpful under financial shocks and housing market-related shocks.

There are some more other papers have studied macroprudential policy in the Euro area as well, such as Pariès et al. (2011). They have examined an estimated DSGE model by incorporating some demand and supply credit frictions, applying the risk-sensitive capital requirements policy instruments to highlight the importance of the policy instruments.

There are also a large number of papers that have followed Bernanke et al. (1999)'s financial accelerator mechanism to explore the effects and interactions between monetary policy and macroprudential policies. Bailliu et al. (2015) studied the performance of different policies under financial imbalances. They showed that there would be welfare gain when policymakers respond to financial imbalances using both monetary policy and prudential instruments, especially in the presence of financial shocks. Other papers applied this type of financial friction, such as Kannan et al. (2012) , Quint and Rabanal (2013) and Suh (2012). This research applies the KRS model to examine prudential policy under uncertainty, which we discuss more in section (3). Unlike most papers that have concentrated on technology shocks, housing market-related shocks, or financial shocks, Lambertini et al. (2013) have investigated the effects of macroprudential policy under news-driven shocks. They distinguish the influences of both interest rate and LTV policies, implying that optimal policy for borrowers is satisfied by an LTV ratio responds counter-cyclically to credit growth as this could stabilize credit relative to output. The optimal policy for savers is characterized by a constant LTV ratio coupled with an interest-rate response to growth.

3.2.2 The applications of model uncertainty

As the robust control theory is one technique from the uncertainty study, it is fundamental to understand various model uncertainty theories before the application of the robust control method. Increasing researchers have extended the studies on the relations between model uncertainties and policy decisions. Cateau (2007) identified several approaches to deal with model uncertainty. He summarised them as the robust control method; the Bayesian approach; the worst-case model approach; and the trade-off approach. A robust control method is a powerful tool when policymakers have a good model of the economy. It is devised to select decision rules that perform well in a neighbourhood of a specific model. However, he also implies that the Bayesian approach, worst-case model approach and the trade-off method are more suitable tools when policymakers confront more than one reference model. This is because those approaches take into account the fact that policymakers could use models which are arbitrarily far from each other.

For the worst-case approach, policymakers' objective is to guarantee that the policy decision rule performs reasonably well regardless of which model of the economy is correct. Policymakers examine the policy choices and decide which model do not perform well under each of these choices. Then they choose to minimize the variability of choice irrespective of which model is the true model. For the trade-off method proposed by Cateau (2007), there are two assumptions. One is that policymakers can assign weights to the models in the decision sets, the other is that policymakers recognize their degree of aversion towards the across-model risk, which indicates the risk is associated with multiple models. Policymakers allow across-model risk by analysing the performance of a model both by its loss and the degree of aversion towards the across-model risk.

The main distinction between the Bayesian approach, worst-case approach and trade-off approach is the different assumptions concerning the attitude of policymakers toward model uncertainty. The Bayesian technique presumes that policymakers are across-model risk-neutral, meaning zero across-model risk aversion. They only care about the average performance of models. In contrast, the worst-case model approach presumes that policymakers are the across-model risk aversion infinity. This implies that policymakers only care about how the policy choice is. The trade-off method allows the aversion of policymakers to change between zero and infinity. Then the degree of policymakers' risk version determines how much policymakers' average trade-off performance is for robustness.

In a Bayesian technique, the planner applies his prior probability distribution over models, so the researcher needs to specify a prior over all possible models, which can quickly become problematic. The early representative work is from Brainard (1967)who considers the scenario where policymakers confront parameters uncertainty that is relevant for choosing an appropriate policy. While they still know the probability distribution in terms of which they are selected. If policymakers are informed of parameters' distribution, then they can obtain the expected losses resulting from different policies. Brainard found the smallest expected loss in solved policy and explained that when policymakers are sceptical about the impact on their moves, the best response to uncertainty is to restrain the extent to which they should react to the news they get.

3.2.3 The linkage between robust control theory and macroprudential policy

An alternative approach to model uncertainty is to consider a minimax rule with Wald (1950). It is difficult for policymakers to know probability distributions in different models, while the minimax method can avoid this problem. This method implies that policymakers should minimize the maximum loss of the worst-case under all potential scenarios. Therefore, the application of the minimax approach in macroeconomic decision problems is often considered as a robust control method and the policy that minimizes the worst scenario loss is referred to as a robust policy (Barlevy (2011)).

However, even approaches of model uncertainty are varied, early applications of robustness did have one result in common: In all of these applications, the robust policy tended to contradict the attenuation result in Brainard's model. It instead implied that uncertain policymakers should react more aggressively to news than they would in the absence of uncertainty. Examples of this result include Hansen and Sargent (1999), and Onatski and Stock (2002), Giannoni (2002), Onatski and Williams (2003), Hansen and Sargent (2001, 2008), Giordani and Söderlind (2004), Dennis et al. (2009), andDennis (2007), and Leitemo and Söderström (2008). These findings were sometimes interpreted to mean that concern for robustness leads to a more aggressive policy. In those papers, Hansen and Sargent (2008) have provided a full explanation of robust control theory, including both control and estimation issues, and both one-agent and multiple-agent settings in terms of model specification.

Furthermore, Giordani and Söderlind (2004) extend Hansen and Sargent's research by providing solutions for the robust control methods under the discretion and simple rules scenarios. By applying the robust control technique into the New Keynesian model, they find some typical result: robust monetary policy is more aggressive not only in the commitment solution but also in the discretionary case. Additionally, their result suggests that robustness can grow the inflation bias in the discretionary equilibrium. This outcome confirms Hansen and Sargent (2002) 's research but fear of misspecification increases the inflation bias in the DSGE model. At last, they recommend that robustness in the private sector increases the volatility of inflation and output, and this can provide an incentive for central banks to release transparent forecasts.

Moreover,Giannoni (2002) is closer to Hansen and Sargent in that the planner is solving for the minimax method. He assumes that policymakers have multiple priors concerning the probability distribution of the true model, and they are uncertainty risk-averse. He compares the robust optimal Taylor rule to the optimal monetary policy without model uncertainty. The result suggests that the central bank should allow a more aggressive policy to stabilize inflation and the output gap around their target values and ensures that welfare losses are included. Onatski and Stock (2002) apply the Rudebusch and Svensson (1999) (henceforward, the RS model) two-equation model with minimax approach of robust control theory. Their research concerns two main issues. One is to compare how robust monetary policies differ from policies without parameter uncertainty. The other one questions whether different perturbations in different formulations of the robust control is relevant for monetary policy applications. Then at the outcome, they confirm that the robust monetary policy is more aggressive under model uncertainty. For the second question, they indicate that the formulations of specific robust control rules vary depending on whether it is an error, model uncertainty, or both. What is more, for the same RS model application, Onatski and Williams (2003) examine specifically three types of uncertainty model by utilizing the "Model-Error Modelling" approach: parameters uncertainty of the reference model; the following correlation properties of shocks; and data quality uncertainty. Their result shows that aggressive monetary policy might be caused by overemphasizing uncertainty concerning economic dynamics at low frequencies.

Most studies discussed have applied a minimax approach within different models. However, many of these methods require that the reference model is described in a state-space form. This makes it difficult when some models cannot be written in that form, typically for medium-scale to large-scale models. By using Dennis et al. (2009) 's structural form methods, Leitemo and Söderström (2008) and Dennis et al. (2007) study a small open economy model which also shows that the central bank responds more aggressively to price markup shocks in the both domestic and import sector. In previous literature, most research is dealt with monetary policy and are incorporating different models based on the new Keynesian model. The study related to robust control on the macroprudential policy has been largely neglected. Bahaj et al. (2017) applies Brainard method to the case of macroprudential policy; while no other research has attempted to apply the robust control method for the macroprudential policy uncertainty issue. To analyze the macroprudential policy instruments' implementation and transmission, Bahaj et al. (2017) extends the Brainard model and suggests that learning about uncertain policy tools and concentrating on risk avoidance would corporate with more active policy-making. Additionally, uncertainty in the private sector over the financial stability objective, reaction function, and preferences might reduce the power of the signalling influence of macroprudential policy, demanding a more aggressive policy.

Therefore, it is of interest to explore the robust control approach with the macroprudential policy by using Dennis et al. (2009) structural form. Instead of the simple New Keynesian model, or the RS two-equation model, we put forth an optimal macroprudential policy with commitment in a more general DSGE model with financial accelerator effects.

3.2.4 The Kannan et al. (2012) model

The KRS model, which we apply in this paper, is modified from the standard New Keynesian model of Galí (2007) and closely related to Iacoviello (2005), Iacoviello and Neri (2010) and Monacelli (2009). The main aim of the KRS model is to examine how can monetary policy with macroprudential instruments reduce the volatility of housing booms under financial shocks. There are some fundamental features worth noting to introduce the KRS model.

Unlike the classic New Keynesian model, the households in this model comprise savers and borrowers who are consuming durables and nondurable goods. There are some basic assumptions. First, households choose the quantity of both investing in the housing market and consuming in nondurable goods. Second, the borrowers obtain funds from the savers through financial intermediaries, which charge the spread that depends on the net worth of borrowers. Third, the lending rate is dependent upon the macroprudential policy instrument, loan-to-value ratios, indicating that the markup is charged through the monetary policy rate, and the policy rate. Both the change of house prices and credit market conditions can contribute to the acceleration in investment, nondurable consumption and consumer prices. In other dimensions, the KRS model follows the standard New Keynesian principles. In both the durables and non-durables sectors, there exists monopolistic competition and nominal rigidities. The model contains other conventional assumptions, such as sticky prices, external habit, and adjustment costs. The detailed KRS model equations are shown in Appendix.

I choose the KRS model for several reasons. First, the KRS paper is part of the recent literature on macroprudential policies within DEGE models by incorporating financial accelerator effects. The KRS model finds out that macroprudential policies are an effective tool to stabilize financial shocks, especially with credit and asset price booms. Second, the KRS model implies that the macroprudential policy instrument performs well on improving macroeconomic stability under reasonable different parameter values. Third, they apply aggressive optimal monetary policy reaction parameters on stabilizing CPI inflation and the output gap. The simulation results indicate that the macroprudential instrument becomes increasingly fundamental when the policymakers become more aggressive and care about stabilizing real activities. Fourth, the KRS result also suggests that it is crucial to identify the source of house price booms and financial conditions for the design of the monetary and macroprudential policy.

The first two points show the critical role of macroprudential policy in the KRS model. The third point provides a link between the KRS model and the robust control method. In light of previous literature reviews, the robust monetary policy leads to more aggressive policies under commitment. It would be of our interest to evaluate how the robust control method on the KRS model would generate impacts on macroprudential policies. The final point indicates that the robust control method is appropriate for the model as the robust control are beneficial for policymakers concerning about model misspecification.

3.3 The Robust Control Method

3.3.1 The intuition behind robust control

The model equations in the last section estimated by the central bank are called reference model, which provides the best guess for the data generating process. When the central bank is concerned that the reference model is misspecified, it could apply the robust policy against the distortion generating the approximating model. The intuition can be explained through figure (3.1). Given that a central bank recognizes the presence of model misspecification, an unreliable approximating model f_{α_0} is surrounded by a set of unknown true data generating process. One unspecified element of true data generating process is model f. The central bank must base their decisions on the only specified model, f_{α_0}. The decision-maker is seeking a decision rule that would perform well as for the approximating model as for a set of models surrounding model f_{α_0} when $f \neq f_{\alpha_0}$. It is presumed that the specification errors function $I(\alpha_0, f)$ is bounded by a certain value η, and $I(\alpha_0, f) \leqslant \eta$. Therefore, the value η shows that how much the decision-maker believes that f_{α_0} is a good approximating model. A larger value of η indicates that the decision-maker believes the true model is further apart and considers a wider set of the model around f_{α_0} and vice versa.

3.3.2 The robust policy equations

The robust control approach was developed by Hansen and Sargent (2001) and extended by Giordani and Söderlind (2004). This paper applies the method that is modified by Dennis et al. (2009) who have proposed a structural-form approach to solving robust control problems.

The reference model above can be written in structural form:

$$A_0 y_t = A_1 y_{t-1} + A_2 E_t y_{t+1} + A_3 u_t + A_4 \epsilon_t + A_5 \epsilon_{t+1} \tag{3.1}$$

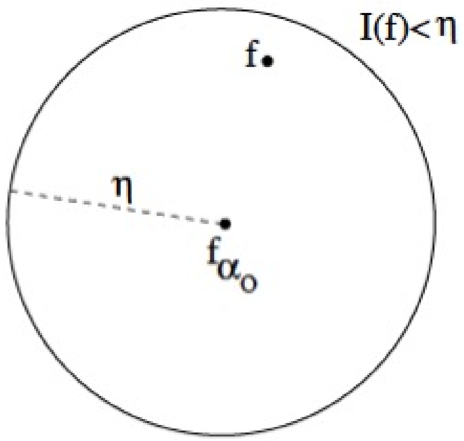

Figure 3.1: Robust decision making

Notes: A decision-maker specifies an approximating model f_{α_0} doubts that the true data generated by a nearby model f, which cannot be specified. α_0 denotes the specification error, and η denotes the constraint, which summarises the policymaker's confidence level. $I(\alpha_0, f) \leqslant \eta$. (Hansen and Sargent (2008): ch.1, P10; Cateau et al. (2006))

where y_t is a vector of all endogenous variables; u_t is a vector of policy instruments; ϵ_t denotes a vector of all shocks; $A_0, A_1, A_2, A_3, A_4, A_5$ must have dimensions consistent with y_t, u_t and ϵ_t; A_0 is assumed to be non-singular. It is presumed that the shocks, s_t, follows the process

$$s_t = \Phi s_{t-1} + \Omega \epsilon_t \tag{3.2}$$

where $|\Phi| < 1$ and the shocks are distributed by a normal distribution where ϵ_t is i.i.d.[0,1]. There are two circumstances under assumptions. One states that agents make decisions after observing shocks s_t. Then the timing of the equation (3.2) moves forward by one period and s_t is included within equation with y_{t-1}; then $A_4 = 0$, and $A_5 = [\Omega' \;\; 0']'$. The other assumes that agents make decisions before observing the shocks s_t, the timing of the equation above keeps the same form and s_{t-1} is included within y_{t-1}; then $A_4 = [\Omega' \;\; 0']'$; $A_5 = 0$. This only considers the previous situation.

To deal with the concern of the central bank for the model misspecification problems, the specification errors are introduced to obtain the distorted model. It is assumed that one evil agent chooses the distorted errors to maximize the loss function. By assuming the expectational errors, $\epsilon_{yt+1} = y_{t+1} - E_t y_{t+1}$ is a linear function of the shocks in equilibrium, $\epsilon_{yt+1} = C\epsilon_{t+1}$, then equation (3.1) is modified as,

$$A_0 y_t = A_1 y_{t-1} + A_2 y_{t+1} + A_3 u_t + A_4 \epsilon_t + (A_5 - A_2 C)\epsilon_{t+1} \tag{3.3}$$

where the matrix C will be determined later. Then, equation (3.3) can be expressed with specification errors as follows:

$$A_0 y_t = A_1 y_{t-1} + A_2 y_{t+1} + A_3 u_t + A_4(\epsilon_t + v_t) + (A_5 - A_2 C)(v_{t+1} + \epsilon_{t+1}) \tag{3.4}$$

where v_t denotes the sequence of specification errors. When distorted specification error term is eliminated, it becomes the reference model (3.1). In addition, v_t, is assumed to satisfy the constraint,

$$E_0 \sum_{t=0}^{\infty} \beta^t v_t' v_t \leq \eta \tag{3.5}$$

where $\eta \in [0, \bar{\eta})$ implies the whole budget is subject to misspecification.

The central bank's loss function is presumed to be quadratic and illustrated as below:

$$E_0 \sum_{t=0}^{\infty} \beta^t [y_t' W y_t + u_t' R u_t] \tag{3.6}$$

When the central bank formulates policy to against the distorted model, the policymaker chooses to minimize $\{u_t\}_0^{\infty}$, and an evil agent chooses to maximize $\{v_t\}_0^{\infty}$ in terms of the updated function,

$$E_0 \sum_{t=0}^{\infty} \beta^t [y_t' W y_t + u_t' R u_t - \theta v_t' v_t] \tag{3.7}$$

where θ, the multiplier is inversely related to the misspecification constraint η. When θ converges to infinity, the specification errors tend to become smaller and smaller, until the robust decision issue becomes a non-robust decision problem. The W and R are matrices containing policy weights and are assumed to be symmetric and positive semi-definite and symmetric and positive definite, respectively.

The problem for the robust decision can be solved by the Lagrangian method,

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \{ y_t' W y_t + u_t' R u_t - \theta v_t' v_t + 2\lambda_t [A_1 y_{t-1} + A_2 y_{t+1} + A_3 u_t \\ + A_4(\epsilon_t + v_t) + (A_5 - A_2 C)(v_{t+1} + \epsilon_{t+1}) - A_0 y_t] \} \tag{3.8}$$

where λ_t includes all the Lagrange multipliers. When decisions are made after observing the shocks, s_t is included the equation y_{t-1} and $A_4 = 0$, the decision-maker and private agents are fully convinced in their knowledge of the current state variables. Given the first order conditions with respect to λ_t, y_t, u_t, v_t,

$$\frac{\partial L}{\partial \lambda_t} : A_1 y_{t-1} + A_2 E_t y_{t+1} + A_3 u_t + (A_5 - A_2 C) v_{t+1} + A_5 \epsilon_{t+1} - A_0 y_t = 0; \tag{3.9}$$

$$\frac{\partial L}{\partial y_t} : W y_t + \beta A_1' E_t \lambda_{t+1} + \beta^{-1} A_2' \lambda_{t-1} - A_0' \lambda_t = 0; \tag{3.10}$$

$$\frac{\partial L}{\partial u_t} : R u_t + A_3' \lambda_t = 0; \tag{3.11}$$

$$\frac{\partial L}{\partial v_{t+1}} : -\beta \theta v_{t+1} + (A_5 - A_2 C)' \lambda_t = 0; \tag{3.12}$$

the above four equations can be solved:

$$\lambda_t = M_{\lambda\lambda}^W \lambda_{t-1} + M_{\lambda y}^W y_{t-1} \tag{3.13}$$

$$y_t = M_{y\lambda}^W \lambda_{t-1} + M_{yy}^w y_{t-1} + N_y^W \epsilon_{t+1} \tag{3.14}$$

$$u_t = F_\lambda^W \lambda_{t-1} + F_y^W y_{t-1} \tag{3.15}$$

$$v_{t+1} = K_\lambda^W \lambda_{t-1} + K_y^W y_{t-1} \tag{3.16}$$

To obtain the worst-case equilibrium, C is updated according to C $\longleftarrow M_{yy}^W S$ where S is the selection matrix from the sides of M_{yy}^W associated with the shocks and iterate through equations (3.9) to (3.16) until a fix-point is reached. As the shocks s_t enters the equation y_{t-1}, all of the variables in equations (3.13) to (3.16) respond to s_t therefore to ϵ_t. Now the worst-case equilibrium can be presented,

$$z_t = M^W z_{t-1} + N^W \epsilon_{t+1} \tag{3.17}$$

$$u_t = F_z z_{t-1} \tag{3.18}$$

$$v_{t+1} = K_z z_{t-1} \tag{3.19}$$

where $z_t \equiv [\lambda_t' \ \ y_t']'$. Therefore, once the worst-case equilibrium is achieved, the approximating equilibrium is derived from equation (3.1) (with$A_4 = 0$) jointly with equations (3.13) to (3.16), which gives the form,

$$z_t = M^A z_{t-1} + N^A \epsilon_{t+1} \tag{3.20}$$

$$u_t = F_z z_{t-1} \tag{3.21}$$

3.3.3 Detection-error probabilities

The multiplier θ is calibrated through a detection-error probability method. It shows the probability when the econometricians make inaccurate inferences whether the worst-case equilibrium or approximating equilibrium generated the data. When the value of θ is small, larger distinctions between the distorted model and the reference model occur, these can be easily detected. For instance, model A shows the approximating model and model W denotes the worst-case model. With the assumption of equal weight attached to each model, then the probability detection error is given by

$$p(\theta) = \frac{prob(A|W) + prob(W|A)}{2} \tag{3.22}$$

where $prob(A|W)$ illustrates the probability that econometricians erroneously choose model A when in fact the data is generated by the model W. The same explanation applies for the $prob(W|A)$. $p(\theta)$ provides a lower bound on the probability of making a detection error. It is presumed that the econometricians' choice over model selection is based on the likelihood ratio principle. $\{z_t^W\}_1^T$ specifies a finite sequence of results generated in terms of the worst-case equilibrium, model W. The likelihood related to the models A and W are denoted by L_{AW} and L_{WW} , respectively. When $log(\frac{L_{WW}}{L_{AW}}) < 0$, the econometricians select model A over model W. According to this, there are M independent sequences $\{z_t^W\}_1^T$ generated through the process, the probability $prob(A|W)$ can be achieved through the equation below,

$$prob(A|W) \approx \frac{1}{M}\sum_{m=1}^{M} I[log(\frac{L_{WW}^m}{L_{AW}^m}) < 0] \tag{3.23}$$

where the function $I[log(\frac{L_{WW}^m}{L_{AW}^m}) < 0]$ is an indicator function belongs to $[0 \;\; 1]$. The function equals one if its statement is satisfied and equals zero if it is not satisfied. The probability $prob(W|A)$ is obtained applying draws generated from the approximating model. In the likelihood function to generate probability $prob(W|A)$ and $prob(A|W)$, the shocks are assumed to be normally distributed.

To achieve detection error probabilities and take into account of the distortions to both conditional means and the conditional variances of the shocks, provided that

$$z_t^A = M^A z_{t-1}^A + N^A \epsilon_t \tag{3.24}$$

$$z_t^W = M^W z_{t-1}^W + N^W \epsilon_t \tag{3.25}$$

determine equilibrium results under the approximating equilibrium and the worst-case equilibrium, respectively. When $N^A \neq N^W$, the detection error probability $p(\theta)$ is calculated as follows: First, it allows for the stochastic singularity that generally characterizes equilibrium; second, it accounts suitable Jacobian transformation that is included the likelihood function. Through the QR decomposition, N^A is decomposed according to $N^A = Q_A R_A$ and N^W is decomposed according to $N^W = Q_W R_W$. Q_A and Q_W are orthogonal matrices and R_A and R_W are upper triangular. Let the equation

$$\hat{\epsilon}_t^{i|j} = R_i^{-1} Q_i (z_t^j - M^i z_{t-1}^j), \{i, j\} \in \{A, W\} \tag{3.26}$$

show the inferred shocks in period t when model i is fitted to data $\{z_t^j\}_1^T$ that are obtained in terms of model j; and let $\hat{\sum}^{i|j}$ be the related estimates of the shock variance-covariance matrices. Then

$$log(\frac{L_{AA}}{L_{WA}}) = log|R_A^{-1}| - log|R_W^{-1}| + \frac{1}{2}tr(\hat{\sum}^{W|A} - \hat{\sum}^{A|A}) \tag{3.27}$$

$$log(\frac{L_{WW}}{L_{AW}}) = log|R_W^{-1}| - log|R_A^{-1}| + \frac{1}{2}tr(\hat{\sum}^{A|W} - \hat{\sum}^{W|W}) \tag{3.28}$$

where tr is the trace operator.

With equations (3.27) and (3.28), equation (3.23) is used to estimate prob(A—W) and prob(W—A) that are required to construct the detection error probability, as in equation (3.22) . The multiplier, θ is then specified by choosing a detection-error probability and inverting equation (3.22). Svensson (2000) stresses the issue to choose θ as the robust control method's disadvantage. The choice of θ is therefore fundamental as the planner's policy function differs with it. According to Giordani and Söderlind (2004), zero robustness is compatible with a detection-error probability of 50%. Hansen and Sargent (2000) recommends the range lies between 10% and 20%. Normally, 150 observations correspond to a detection-error probability of 20%. The larger the sample, the higher θ, the uncertainty around the reference model becomes smaller. In this paper, the detection-error probability θ is calibrated as 20%.

3.4 Robust Control Macroprudential Policy in the KRS model

The structural form robust control method would be applied to the KRS model in this section. The model is solved under full commitment. We presume that the monetary policy is aimed to stabilise the annualised CPI inflation ($\bar{\pi}_t$), the level of the output gap (x_t) and the annualised nominal interest rate ($\tilde{r}_t$). Therefore, the central bank's objective functions are assumed as follows,

$$E_0 \sum_{t=0}^{\infty} \beta^t [\bar{\pi_t}^2 + \lambda_y x_t^2 + \lambda_r \tilde{r_t}^2] \tag{3.29}$$

where $\bar{\pi_t} = 4\pi_t$, $\tilde{r_t} = 4r_t$, $x_t = y_t - y^*$. The variable y^* is the level of output in the flexible-price equilibrium. The weighted parameters show the central bank's preferences for output gap and interest rate: $\lambda_y = 0.5$, $\lambda_r = 0.2$, $\beta = 0.99$.

Table (3.1) gives the calibrated parameters for the model. The calibrated parameters are

Table 3.1: Calibrated Parameters

Parameter	Description	Value
β	savers' discount factor	0.99
β^B	borrowers' discount factor	0.98
δ	depreciation rate	0.025
λ	share of savers	0.5
χ	down payment rate $(1 - LTV)$	0.2
$\frac{\sigma}{\sigma-1}$	average markup	1.1
l_L	labour disutility of switching sectors	1
φ	inverse Frisch elasticity of labour supply	1
ϵ	external habit proportionate parameter	0.8
η	adjustment cost residential investment	0.5
κ	elasticity of spread with respect to net worth	0.05
α	Share of nonduables in production parameter	0.9
θ_c	Calvo lottery nondurable	0.75
θ_d	Calvo lottery durable	0.75
ϕ_c	backward looking behaviour in nondurable section	1
ϕ_d	backward looking behaviour in durable section	1
τ	macroprudential coeffiecient on credit growth	0.3
ρ_a	AR(1) coefficient on technology shock	0.98
ρ_v	AR(1) coefficient on financial shock	0.95
ρ_d	AR(1) coefficient on housing demand shock	0.95
γ_π	Taylor rule coefficient on inflation	1.3
γ_y	Taylor rule coefficient on output gap	0.125
γ_b	Augmented Taylor rule with macroprudential policy coefficient	0.7
γ_r	Augmented Taylor rule coefficient on credit growth	0.7
σ_a	standard deviation in technology shock	1.5
σ_v	standard deviation in financial shock	0.125
σ_d	standard deviation in housing demand shock	2.5

Note: The calibrated parameters are cited from the Kannan et al. (2012) model

cited from the KRS model, which are based on Iacoviello and Neri (2010). The parameters are adjusted to match the relevant second moments in the data. It can be seen that savers are more patient than borrowers ($\beta > \beta^B$); therefore, they have a higher discount factor. The parameter κ is calibrated to be 0.05, measuring the size of the financial accelerator effect. Most crucially, τ is the macroprudential policy coefficient, which is assumed to be 0.3. A higher value of this policy coefficient would imply that policymakers care more about financial stability. As mentioned in the KRS model, the optimal weight on the macroprudential policy instrument drops to 0.16, when the policymaker care similarly between the CPI inflation and the output gap. Since the main target of central banks are inflation stabilization, it is reasonable to apply 0.3 on macroprudential instrument. All shocks are assumed to be persistent in high values.

In the next part, we discuss and compare the model with and without the macroprudential policy in the KRS model. We show the three results from robust control method: the worst-case model, the approximating model, and the rational expectations model. We assume that the policy is set after observing the shocks, then concentrating on the circumstance where a monetary policy with or without macroprudential policy and the specification errors are selected under commitment. The robust control method is then applied to devise the robust monetary and macroprudential policy that guard against distortions to the reference model.

Outcomes would be shown in four dimensions. The first part shows the benchmark case, where the outcomes without robustness but with or without macroprudential policy. The second section gives a robust model with interest rate rule but without macroprudential policy. The third section provides a robust model with interest rate rule and with exogenous macroprudential policy. The fourth part shows the robust model with both interest rate rule and macroprudential policy.

3.4.1 Benchmark: The non-robust model (Rational Expectations)

When there is no robust control method applied to the model, the central bank implements the monetary policy with or without the macroprudential instrument to stabilize the economy. Figure (3.2) presents the benchmark case where the KRS model without robustness when the monetary policy is applied with and without macroprudential policy. In all illustrated graphs later in this chapter, the first column shows the impulse responses under a one standard deviation technology shock (ϵ_A). The second and the third depict the outcomes under a one standard deviation housing demand shock (ϵ_D) and financial shock (ϵ_v), respectively.

In terms of Figure (3.2), the impulse responses under the non-robust policy without a macroprudential policy are illustrated by the solid lines. First, under positive technology shock, the innovation of technology lowers marginal cost and CPI inflation. As for the response, the central bank lowers the nominal policy rate to offset the fall in inflation. All variables go back to the steady-state after overshooting process. Second, under housing demand shock, increasing demand for housing generates higher residential investment and housing prices. This leads to higher output. With constant potential production, therefore, causing a higher output gap. To respond to those influences, the central bank lifts policy rate to calm the economy down and reduce the output gap. Thirdly, the financial shock gives the most different responses between the model with and without the macroprudential policy. When financial shocks hit the economy, to respond to an increase in housing prices, the central bank increases the policy rate. As collateral values soar, credit market also grows; therefore, banks are assumed to lower the lending rates.

On the other aspect of Figure (3.2), the dashed lines present the non-robust model with macroprudential policy. The most distinctive differences can be seen from the impulse response of financial shocks. The macroprudential policy has offset the impacts on the financial shocks, as all those variables stay at the steady-state level. Furthermore, under technology shock, both the output gap and inflation are more stable than they are without a macroprudential policy case. Under housing demand shocks, the central bank responds less when using the policy rate, which generates lower volatility in the output gap. However, the nominal debt and the lending rate both grow significantly more than before.

Overall, impulse responses for the non-robust policy show that the central bank actively implements interest rate or macroprudential policy to stabilise the economy. The macroprudential policy generates the most significant impacts on financial shocks.

3.4.2 The robust model without the macroprudential policy ($\hat{\tau}_t = 0$)

Without the macroprudential policy, the mechanism is mainly driven by the interest rate instrument. Comparing the KRS model with standard Taylor rule, the case of the rational expectation under commitment with an optimal policy is more stable for the interest rate and CPI inflation. This shows the advantage that the central bank has when it is committed to optimal policy rules. In this section, we present the results where the KRS model is applied with robust control technique but without macroprudential policy. Figure (3.3) presents the responses of three shocks between the approximating equilibrium (continuous solid lines), the worst-case scenario (dashed dot lines), and the rational expectations equilibrium (dashed lines). The policy is set after observing the shocks. When robustness is introduced into the model, the central bank is always concerned that the economy will fluctuate more in response to the shocks. The detailed explanations come next.

To comprehend the influences of robustness on monetary policy, we first study the worst-case model for inflation, output gap, policy interest rate, nominal debt growth and lending rate. This equilibrium is appealing since it depicts the central bank's worst fears of model misspecification, thus, contribute to our understanding for the device of the robust policy. Under positive technology shock, there is one immediate increase in output. Also, households consumption, residential investment, housing price all rise. Higher productivity improves efficiency, therefore leading a decline in CPI goods inflation. Since the growth of output is lower than flexible-level output, generating a negative output gap at the beginning. As a response of the central bank, monetary policy is expanded first by reducing the interest rate from the beginning to seven periods. This leads to higher output and the positive output gap, consumption, residential investment, and credit growth. The CPI inflation also increases and return to the steady-state level. To offset the boom in inflation and decrease the output gap, therefore, the central bank then lifts the interest rate, leading to a drop in real GDP, consumption, and investment.

Under housing demand shocks, the worst-case monetary policy responses aggressively for output gap, growth in debt, both the policy interest rate and the lending rate. When housing demand increases, the central bank fears that the CPI inflation would accelerate more than it appears, leading a more active policy reaction in the interest policy rate and lending rate. Under financial shocks, variables fluctuate as the interest rate instrument plays an important role. The central bank fears that the worst-case financial shocks hit the economy, then increasing the interest rate gradually more than it is in the approximating model. This turns out that the intermediaries lower lending rate more and are reluctant to lend to firms. To overcome adverse impacts, the central bank must drop the policy rate later to boost the economy.

Furthermore, we find that the approximating equilibriums are more volatile than the case of rational expectation. After recognizing when the technology shock hits the economy, policymakers fear that the influences would be larger due to the existence of the model distortions. With the booming economy, there are increases in output, residential investment, and credit growth, and the central bank becomes concerned that the impacts would be more persistent than rational expectation cases. Therefore, the corresponding interest rate drops deeper at the beginning of a few periods. Moreover, the figure illustrates that the largest impacts from the robust monetary policy occur in areas concerning the lending rate, housing price inflation, and nominal debt growth. With more active reaction on those variables, the interest rate is tightened more than it would under the rational expectation model. Therefore, this brings about a contraction in consumption, investment, and real GDP. The cost to ensure against the model misspecification comes in the form of higher volatility in the inflation and output gap.

3.4.3 The robust model with the exogenous macroprudential policy ($\hat{\tau}_t = 0.3$)

This section would introduce the robust model with the exogenous macroprudential policy when the policy coefficient is equal to 0.3 as in the KRS model. Figure (3.4) illustrates the worst-case equilibrium against the approximating equilibrium and rational expectations under commitment to the optimal policy. In contrast with approximating equilibrium, the figure shows similar movements in terms of the worst-case equilibrium. There are some crucial points that can be drawn. First, the worst-case equilibrium is also more volatile than the approximating equilibrium for all variables. This is since the robust policymakers fear the specification errors are as damaging as possible that shocks have a larger impact on those variables, so acting with more aggressive policy response. Second, consumption, residential investment and output have larger magnitude impacts compared to the other variables under technology shock. Third, the CPI inflation declines at the beginning and grows continuously after three periods. To respond the growth of productivity, the robust central bank fears larger and persistent volatility in consumption, residential investment, output, housing price inflation and credit growth, responding with a more aggressive expansionary policy, leading larger growth in output gap and inflation.

When housing demand shocks hit the economy, there are some immediate impacts: increasing in housing price inflation, output, residential investment, and output gap; and declining in total consumption and CPI inflation. Durable goods consumption grows as the preference for investing more in the housing market. However, private nondurable goods decrease more, generate an overall decline in total consumption. After policymakers observe the housing demand shock, the booming housing market corresponds with tightened monetary policy. The higher policy rate leads to a contraction in consumption, investment, real GDP. Moreover, reducing housing price inflation is correlated with a reduction in nominal credit growth as the willingness of borrowing cools down.

In addition, the robust policymakers should respond with more aggressive monetary policy. Given the drop in investment, housing price, and lending rate after five periods, the central bank fears that impacts of shrinking housing demand would be persistent and larger than expected, therefore generating a gentle expansionary monetary policy under the approximating equilibrium. It can be seen that the overall impacts between the approximating equilibrium and the rational expectation case are similar. This is since the committed optimal policy coordinates with the shocks. However, the magnitudes under housing demand shock are actually quite trivial for inflation, lending rate and the monetary policy interest rate. The result implies that the monetary policy itself has a limited response in terms of the housing demand shock.

What about the worst-case scenario? This indeed brings about more significant impacts after five periods of time between the approximating equilibrium and the worst-case equilibrium. As before, the worst-case scenario provides more volatile movements than the other. The central bank's fear over model distortions generates more aggressive monetary policy. Nonetheless, the magnitudes are still small for interest rate, CPI inflation and lending rate. Therefore, even with the worst-case equilibrium, the monetary policy does not respond to the housing demand shock.

Similar to in the KRS model, the financial shock can be seen as an increasing perceived lending risk, bringing about an immediate drop in housing price inflation and residential investment. The impact on housing price is actually extremely small. Under optimal policy commitment, it is assumed that the banks lower the lending rate. Thus, higher collateral constraint allows borrowers to take out more loans. This works through the credit accelerator mechanism. Moreover, higher interest rate contracts the demand for nondurable consumption goods by borrowers, leading to a fall in total consumption. With higher interest rate monetary policy, the influence on CPI inflation is not significant, but the output gap indeed declines.

Moreover, after the central bank observes the financial shock, it formulates a tighter monetary policy that reduces the CPI inflation, real GDP and output gap. The impulse responses illustrate that the largest gap between the approximating equilibrium and the rational expectation model is shown in the output gap and real GDP. The other variables do not make big differences even when the central bank fears the model misspecification.

Finally, as for the worst-case scenario, the central bank fears the shock would pose many large impacts and then devises interest rate more volatile than approximating equilibrium after overshooting period. In terms of the worst-case equilibrium, the central banks fear the model misspecification is as damaging as possible. Therefore, there are larger reactions on consumption, output and output gap. To respond to those reactions, the central bank increases the interest rate more aggressively than the approximating equilibrium. Consequently, the monetary policy generates a dramatic drop in the output gap, but the overall impacts on CPI inflation are still trivial.

Under commitment, the central bank clings to the optimal policy. It can be summarised that the interest rate policy with robustness can be more easily detected under the technology shock as the larger impacts under the approximating equilibrium and the worst-case equilibrium. Contrast that with the result from the KRS models, when the model is based on the simple Taylor interest rate rule, and there are still some distinctions. First, the clearest advantage is that the robust monetary policy under commitment is less volatile, leading to lower welfare losses. Second, monetary policy instrument generates an opposite direction between inflation and the output gap under commitment rather than the same direction under the Taylor rule. Therefore, the main target of the central bank matters more in the commitment case with robustness.

3.4.4 The Robust model with endogenous macroprudential policy

This section analyses the results for the KRS model that include the macroprudential policy. The impulse responses are shown in Figure (3.5). The economy is affected by both monetary policy and macroprudential policy instrument when the macroprudential policy is implemented.

We begin the analysis with the financial shock when the macroprudential policy is included in the rational expectations model as the second policy instrument. It has been shown that the results are quite different from the previous one that was without macroprudential policy. When the macroprudential policy instrument is applied, the responses of all variables to financial shock are zero. This implies that the new instrument itself or coordination with monetary policy together is able to fully offset the financial shock.

Furthermore, the increased productivity leads to a housing market boom: output and the demand for credit all rise immediately. The lower housing price decreases the collateral constraint, limiting borrowers' credit growth. After the central bank observes financial shock, it drops the interest rate, leading an increase in CPI inflation and the output gap. The growing lending rate brings about a drop in real credit growth due to the increasing borrowing cost. When the lending rate goes back to the steady-state, it generates a widespread acceleration in credit growth. Interestingly, with the macroprudential instrument under commitment, the central bank does not have to react strongly as it recognizes the new instrument is also implemented to stabilize the output gap and inflation. Therefore, rather than sticking to the Taylor rule, the benefits of the central bank commitment can be seen that there are fewer volatilities in CPI inflation and the output gap.

To contrast with the approximating equilibrium and rational expectations model in Figure (3.5), when the preference for robustness is introduced into the model with two instruments, the central bank fears that the economy would fluctuate more by corresponding to the shocks as well as to the policy feedback. For the technology shock, the movements are manifested compared to the rational expectations outcomes. The central bank is concerned by much larger movements in the output gap, inflation and consumption by then respond a more aggressive shift in the interest rate. The monetary policy is more expansionary and persistent after it observes the decline in CPI inflation and growth in the output gap. Notably, the largest volatility occurs in credit growth and the lending rate after the robust control method is applied, corresponding to the concern about volatile housing price.

In comparison with the previous rational expectation case without macroprudential policy, there are some significant points to note. First, with the macroprudential policy instrument, the nominal debt growth, lending rate and housing price inflation are more volatile than before under the circumstance without macroprudential policy. This makes sense as the variables can have direct impacts on the new instruments. Second, the model, within the macroprudential policy, mitigates the volatility of CPI inflation and the output gap. Without the macroprudential policy, the CPI inflation is shift around the region between -0.04 to 0.18, and the output gap moves between 0.18 to 0.6. However, with macroprudential policy instrument, the CPI inflation shifts between -0.03 to 0, the output gap moves between 0 and 0.51. Therefore, the macroprudential policy instrument with monetary policy together could stabilize the output gap and CPI inflation under technology shock. Finally, the robust control method generates more volatility in credit growth and housing price inflation with macroprudential policy. However, the interest rate shifts less than before. The results imply that the monetary policy takes fewer actions when it observes the macroprudential policy generates larger impacts on the economy.

For the worst-case scenario, the results are similar as before: the worst-case shows more volatile movements than the approximating equilibrium. The largest impacts are on consumption, output, investment, and the output gap. The central bank is concerned that the impacts of technology shock would be more persistent; therefore, the interest rate decreases more.

With housing demand shock, higher housing price comes with higher residential investment, output and credit growth. The durable goods consumption increases as borrowers are more willing to invest in the housing market with higher collateral constraint. However, nondurable goods decline more than durable goods consumption, generating an overall drop in total consumption. The impacts on CPI inflation and interest rate are trivial and close to zero. On the other hand, after the housing demand shock, the robust central bank has a larger movement in the interest rate by shifting it to the opposite direction. However, the magnitude of the movement is actually trivial compared to the technology shock. Moreover, the influences on CPI inflation are also insignificant. This is probably since the effects of macroprudential policy eased the most shock under changes in the demand for housing. Contrast with the case of rational expectation; the approximating equilibrium generates slightly more volatile results. The monetary policy also corresponds with larger impacts under the technology shock.

Furthermore, it is noticeable to find out that all variables have similar movements between the outcomes of approximating equilibrium and the worst-case equilibrium. The greatest difference is made from consumption, output and output gap. As before, the worst-case model is more volatile than the approximating equilibrium. Under both circumstances, inflation is more stable than the output gap.

In addition, compared to the KRS model with Taylor rule, our model is assumed to commit with optimal policy. The advantages of commitment are manifest to be shown through the graphs. Under housing demand shock, the CPI inflation under commitment moves a similar pattern with the Taylor rule plus macroprudential policy instrument. However, the output gap is much more stable. The monetary policy is less expansionary than in the Taylor rule-based model, regarding the reaction with macroprudential policy.

3.4.5 Comparing Results with Others

After the demonstration of robust control method over different policy regimes, it is now fundamental to evaluate the differences between this paper and the others. Some implications can be drawn through the comparisons with the KRS and others.

First, the robust control method has limited impacts on the macroprudential policy under financial shocks. When the robust KRS model is applied without macroprudential policy, (Figure (3.3)), most variables show volatilities under three shocks. However, when the exogenous macroprudential policy is added into the robust monetary policy, the shocks under both housing demand shocks and financial shocks are stabilized. When the policymaker implements the endogenous macroprudential policy, the macroprudential policy instrument even shows more powerful effects to stabilize all variables under financial shocks. This result confirms the outcomes in the KRS model, which indicates that macroprudential policy allows more macroeconomic stabilization. As in the ranking of optimal policy rules, augmented Taylor rule with macroprudential instrument shows strong preferences when the policymaker is even aggressive (both 'hawk' and 'super hawk'). Our result further indicates that macroprudential policy is a powerful tool even under the robust control model.

Second, the central bank fears the uncertainties of model misspecification; the policy responses are more aggressive under the endogenous macroprudential policy regime with monetary policy. There appear to be more volatilities for variables under technology shocks, and housing demand shocks when the macroprudential policy is endogenous. (Figure 3.4 & 3.5) This result seems reasonable, to some extent, that the endogenous macroprudential policy (two policy instruments) has increased more uncertainties than the exogenous macroprudential policy ($\tau = 0.3$) (one policy instrument). Policymakers now face policy-making under both monetary and macroprudential policies. The uncertainty could be reduced if policymakers can identify the sources of shocks. This is also consistent with the KRS outcome. If the shocks originate from the financial market, both monetary and macroprudential policy can be applied to stabilize the CPI inflation. If shocks come from the housing demand side, then monetary policy rule alone (Figure 3.3) can stabilize the CPI inflation. It is illustrated that the worst-case scenario generates most volatilities under housing demand shocks.

Third, the optimal policy ranking has something familiar with technology shocks as well. The policymakers prefer the robust model without a macroprudential instrument to stabilize the CPI inflation, (3.3) as the robust monetary policy shows the most aggressive response. This is also consistent with the KRS result, which implies that the policymakers prefer the augmented Taylor rule regime the most.

3.5 Conclusion

The research on the macroprudential policy has been popular after the financial crisis; however, policymakers still confront model misspecification and policy-making issues. There has also been abundant of literature digging out a robust control method on various DSGE models. However, the research of robust control methods applies to macroprudential policy has been limited. Bahaj et al. (2017) uses the Brinard approach to the macroprudential policy. In comparison, we use the robust control method to the KRS model, where the primary mechanism is more about the collateral constraint channel. Besides, the extension allows us to explore the uncertainty of the model when specification errors might be founded with macroprudential policy studies. This brings more advantages than the Taylor rule as they show more stable outcomes.

There are some exciting outcomes through the comparisons between a robust monetary policy with and without macroprudential policy instrument. First, the robust model can offset financial shocks completely when policymakers implement the endogenous macroprudential policy. This suggests the importance of macroprudential policy instrument if the policymaker can identify the source of financial shock. Second, policymakers are more aggressive when they apply both monetary and macroprudential instruments under the robust control model. Generally, under the approximating equilibrium, the robust central bank applies the policy more aggressively than in the case of the rational expectation since it fears that the response to the shocks is more persistent. Under the worst-case equilibrium, the impulse response functions illustrate more volatile results than the approximating equilibrium. Third, the robust monetary policy under technology shock contributes most on stabilizing the output gap and CPI inflation. This implies that the robust control method generates most impacts under technology shocks. All these results are also consistent with the KRS outcomes where macroprudential policy instrument is significant under financial shocks.

We can conclude that a robust control method is a helpful tool for the central bank; it helps to inform policy decisions in the face of uncertainty when macroprudential policy instrument is included with monetary policy. My robust control method outcome further confirms the importance of macroprudential policy instrument even under the worst-case scenario. Besides, by identifying the source of disturbances are also helpful for policymakers to deal with uncertainties under the robust control model. Finally, as Bahaj et al. (2017) suggested, increasing transparency of policy decisions to the public can also reduce uncertainties. This paper has applied the calibrated model with macroprudential policy instrument; an estimated version would also be a worthwhile topic for further study.

Appendix

3.6 The KRS Model

Savers' optimal decisions

For households, there is a fraction λ of households are savers and $1-\lambda$ of the remaining households are borrowers. In equilibrium, the savers would lend to borrowers. For savers, individual j maximises the utility function:

$$E_0\{\sum_{t=0}^{\infty}\beta^t[\gamma log(C_t^j-\epsilon C_{t-1})+(1-\gamma)\xi_t^D log(D_t^j)-\frac{(L_t^j)^{1+\psi}}{1+\psi}]\} \tag{3.30}$$

where β is the discount factor, C_t^j provides the consumption of nondurable goods, D_t^j shows the consumption of durable goods, and L_t^j denotes individual j's total working hours. The savers' budget constraint in nominal terms is provided by,

$$P_t^C C_t^j + P_t^D I_t^j + B_t^j \leq R_{t-1}B_{t-1}^j + W_t^C L_t^{C,j} + W_t^D L_t^{D,j} + \Pi_t^j \tag{3.30-1}$$

where $q_t = \frac{p_t^D}{p_t^C}$ is the relative price of durables in terms of nondurables. ϵ denotes the external habit stock parameter, which measures the influence of past aggregate nondurable consumption c_{t-1}. c_t shows consumption of nondurable goods, i_t gives residential investment, η denotes adjustment costs to residential investment. ξ_t^D shows the housing demand shock, and d_t denotes consumption of durable goods, δ denotes the depreciation rate.

The savers' inter-temporal optimization problem generates to the consumption of nondurable goods Euler equation,

$$\epsilon\Delta c_t = E_t\Delta c_{t+1} - (1-\epsilon)(r_t - E_t\Delta p_{t+1}^c) \tag{3.31}$$

Equations (3.32) and (3.33) imply the optimal labour supply conditions to nondurable and durable sectors, respectively.

$$\frac{c_t - \epsilon c_{t-1}}{1-\epsilon} + [(\psi - l_L)\alpha + l_L]l_t^C + (\psi - l_L)(1-\alpha)l_t^D = w_t^C \tag{3.32}$$

$$\frac{c_t - \epsilon c_{t-1}}{1-\epsilon} + [(\psi - l_L)(1-\alpha) + l_L]l_t^D + (\psi - l_L)\alpha l_t^C = w_t^D \tag{3.33}$$

where ψ denotes inverse Frisch elasticity of labour supply, l_L denotes labour disutility of switching sectors, l_t^C and l_t^D denote labour supply in nondurable and durable sectors, respectively. w_t^C and w_t^D show the real wages in each sector.

Borrowers' optimal decisions

It is assumed that borrowers share similar conditions as the savers. All variables with superscript B denote borrowers' variables. The borrowers' utility function is given as,

$$E_0\{\sum_{t=0}^{\infty}\beta^{B,t}[\gamma log(C_t^{B,j} - \epsilon C_{t-1}^B) + (1-\gamma)\xi_t^D log(D_t^{B,j}) - \frac{(L_t^{B,j})^{1+\psi}}{1+\psi}]\} \tag{3.34}$$

The borrowers' log-linearized budget constraint is given as below:

$$\begin{aligned} C^B c_t^B + I^B(q_t + i_t^B) + R^L B^B(r_{t-1}^L + b_{t-1}^B - \Delta p_t^c) \\ = B^B b_t^B + \alpha W L^B(w_t^C + l_t^{C,B}) + (1-\alpha)W L^B(w_t^D + l_t^{D,B}) \end{aligned} \tag{3.35}$$

where b_t^B denotes the real value of credit in nondurable consumption units. Borrowers consume nondurables and invest in the housing stock, and supply labour to both sectors. Given the budget constraint, borrowers' log-linearized optimal conditions are illustrated below,

$$q_t - \frac{c_t^B - \epsilon c_{t-1}^B}{1-\epsilon} + \eta(i_t^B - i_{t-1}^B) = \mu_t^B + \beta^B\eta(E_t i_{t+1}^B - i_t^B) \tag{3.36}$$

$$[1 - \beta^B(1-\delta)](\xi_t^D - d_t^B) = \mu_t^B - \beta(1-\delta)E_t\mu_{t+1}^B \tag{3.37}$$

$$\epsilon\Delta c_t^B = E_t\Delta c_{t+1}^B - (1-\epsilon)(r_t^L - E_t\Delta p_{t+1}^c) \tag{3.38}$$

$$\frac{c_t^B - \epsilon c_{t-1}^B}{1-\epsilon} + [(\psi - l_L)\alpha + l_L]l_t^{B,C} + (\psi - l_L)(1-\alpha)l_t^{B,D} = w_t^C \tag{3.39}$$

$$\frac{c_t^B - \epsilon c_{t-1}^B}{1-\epsilon} + [(\psi - l_L)(1-\alpha) + l_L]l_t^{B,D} + (\psi - l_L)\alpha l_t^{B,C} = w_t^D \tag{3.40}$$

One main distinction is that savers invest to achieve deposit rate r_t and borrowers obtain loans from financial intermediaries at a lending rate of r_t^L.

Financial Intermediaries

For financial intermediaries, the functional form is assumed as follows,

$$\frac{R_t^L}{R_t} = v_t F(\frac{B_t^B}{P_t^D D_t^B})\tau_t \tag{3.41}$$

Equation (3.14) is the most fundamental element of the model. The spread between the lending and the deposit rates is affected by a financial shock (v_t), an increasing function of the leverage of borrowers (F) and a macroprudential instrument (τ_t). The equation describes the behaviour of the lending rate. It not only shows the determination of the spread between the policy rate and the lending rate but also indicates where the macroprudential instrument enters into the model.

In this paper, we discuss two forms of functions of macroprudential rules. On one hand, as presumed in the KRS model, the macroprudential rule implies the reaction of a macroprudential instrument to lagged nominal credit changes,

$$\tau_t = \tau \frac{B_{t-1}^B}{B_{t-2}^B} \tag{3.42}$$

When equation (3.42) is substituted into (3.41), the log-linearized steady-state form is derived:

$$r_t^L = r_t + \kappa(b_t^B - d_t^B - q_t) - v_t + \tau(b_{t-1}^B - b_{t-2}^B + \Delta p_{t-1}^C) \tag{3.43}$$

When $\tau = 0$, this means that macroprudential policy instrument is not operational. Under this scenario, the spread between the lending rate and the deposit rate is just positively related to endogenous component depending on the LTV ratio for borrowers and is negatively related to a financial shock. When τ is positive, it increases the costs of the lending proportional to nominal credit growth. However, when macroprudential instrument becomes an endogenous variable, the equation is shown as follows,

$$r_t^L = r_t + \kappa(b_t^B - d_t^B - q_t) + v_t + \hat{\tau}_t \tag{3.44}$$

Since the macroprudential instrument influences the lending rates, central bank could change the regulations for different levels of capital requirements as asset prices fluctuate, to some extent, which offsets volatility in collateral values and financial shocks.
The relative housing price evolves as given,

$$q_t = q_{t-1} + \Delta p_t^D - \Delta p_t^C \tag{3.45}$$

Production function

$$y_c^C = a_t^C + l_t^{C,tot} \tag{3.46}$$

$$y_t^D = l_t^{D,tot} \tag{3.47}$$

Equation (3.46) and (3.47) show the production functions. The equations imply that intermediate goods in both sectors are produced using only labour. The production function in the nondurable sector is affected by a technology shock.

CPI equations

$$\Delta p_t^C - \psi_C \Delta p_{t-1}^C = \beta E_t(\Delta p_{t+1}^C - \psi_C \Delta p_t^C) + \kappa^C(w_t^C - a_t^C) \tag{3.48}$$

$$\Delta p_t^D - \psi_D \Delta p_{t-1}^D = \beta E_t(\Delta p_{t+1}^D - \psi_D \Delta p_t^D) + \kappa^D(w_t^D - q_t) \tag{3.49}$$

where $\kappa^C = \frac{(1-\theta_C)(1-\beta\theta_C)}{\theta_C}$ and $\kappa^D = \frac{(1-\theta_D)(1-\beta\theta_D)}{\theta_D}$.

Equations 3.48 and 3.49 describe both sectors' pricing decisions. Firms' CPI price decisions are distinct, not only as various Calvo parameters are applied but also since that technology shocks enter into the non-durable sector only.

Market clearing conditions

The market-clearing conditions are given according to aggregate quantities. In the non-durable sector, total production is equal to total consumption that includes savers and borrowers. In the durable sector, total production equals aggregate residential investment. Aggregate real GDP gives the combined output of the durable and non-durable sectors. The equations are given by:

$$y_t^C = \frac{\lambda C c_t + (1-\lambda) C^B c_t^B}{\lambda C + (1-\lambda) C^B} \tag{3.50}$$

$$y_t^D = \frac{\lambda \delta D i_t + (1-\lambda) \delta D^B i_t^B}{\lambda \delta D + (1-\lambda) \delta D^B} \tag{3.51}$$

$$y_t = \alpha y_t^C + (1-\alpha) y_t^D \tag{3.52}$$

The law of motion of two types of housing stocks for savers and borrowers are provided by,

$$d_t = (1-\delta) d_{t-1} + \delta i_t \tag{3.53}$$

$$d_t^B = (1-\delta) d_{t-1}^B + \delta i_t^B \tag{3.54}$$

Total hours worked equal labour supply in each sector:

$$l_t^{C,tot} = \frac{\lambda L l_t^C + (1-\lambda) L^B l_t^{B,C}}{\lambda L + (1-\lambda) L^B} \tag{3.55}$$

$$l_t^{D,tot} = \frac{\lambda L l_t^D + (1-\lambda) L^B l_t^{B,D}}{\lambda L + (1-\lambda) L^B} \tag{3.56}$$

Shocks equations

All shocks follow AR(1) process, the shocks featured are technology shock, housing demand shock, and financial shock.

$$a_t^C = \rho_a a_{t-1}^C + \epsilon_t^a \tag{3.57}$$

$$\xi_t^D = \rho_d \xi_{t-1}^D + \epsilon_t^d \tag{3.58}$$

$$v_t = \rho_v v_{t-1} + \epsilon_t^v \tag{3.59}$$

3.7 The Linearized Model Equations

The appendix summarizes all the log-linearized equations that has been implemented in the model. All the equations are written in structural form which shows in equation (3.1).

$$q_t - \frac{c_t - \epsilon c_{t-1}}{1-\epsilon} + \eta(i_t - i_{t-1}) = \mu_t + \beta\eta(E_t i_{t+1} - i_t) \tag{3.60}$$

$$[1 - \beta(1-\delta)](\xi_t^D - d_t) = \mu_t - \beta(1-\delta)E_t\mu_{t+1} \tag{3.61}$$

$$\epsilon\Delta c_t = E_t\Delta c_{t+1} - (1-\epsilon)(r_t - E_t\Delta p_{t+1}^c) \tag{3.62}$$

$$\frac{c_t - \epsilon c_{t-1}}{1-\epsilon} + [(\psi - l_L)\alpha + l_L]l_t^C + (\psi - l_L)(1-\alpha)l_t^D = w_t^C \tag{3.63}$$

$$\frac{c_t - \epsilon c_{t-1}}{1-\epsilon} + [(\psi - l_L)(1-\alpha) + l_L]l_t^D + (\psi - l_L)\alpha l_t^C = w_t^D \tag{3.64}$$

$$\begin{aligned} C^B c_t^B + I^B(q_t + i_t^B) + R^L B^B(r_{t-1}^L + b_{t-1}^B - \Delta p_t^c) \\ = B^B b_t^B + \alpha W L^B(w_t^C + l_t^{C,B}) + (1-\alpha)W L^B(w_t^D + l_t^{D,B}) \end{aligned} \tag{3.65}$$

$$q_t - \frac{c_t^B - \epsilon c_{t-1}^B}{1-\epsilon} + \eta(i_t^B - i_{t-1}^B) = \mu_t^B + \beta^B\eta(E_t i_{t+1}^B - i_t^B) \tag{3.66}$$

$$[1 - \beta^B(1-\delta)](\xi_t^D - d_t^B) = \mu_t^B - \beta(1-\delta)E_t\mu_{t+1}^B \tag{3.67}$$

$$\epsilon\Delta c_t^B = E_t\Delta c_{t+1}^B - (1-\epsilon)(r_t^L - E_t\Delta p_{t+1}^c) \tag{3.68}$$

$$\frac{c_t^B - \epsilon c_{t-1}^B}{1-\epsilon} + [(\psi - l_L)\alpha + l_L]l_t^{B,C} + (\psi - l_L)(1-\alpha)l_t^{B,D} = w_t^C \tag{3.69}$$

$$\frac{c_t^B - \epsilon c_{t-1}^B}{1-\epsilon} + [(\psi - l_L)(1-\alpha) + l_L]l_t^{B,D} + (\psi - l_L)\alpha l_t^{B,C} = w_t^D \tag{3.70}$$

$$r_t^L = r_t + \kappa(b_t^B - d_t^B - q_t) - v_t + \tau(b_{t-1}^B - b_{t-2}^B + \Delta p_{t-1}^C) \tag{3.71}$$

$$\Delta p_t = \gamma \Delta p_t^C + (1-\gamma)\Delta p_t^D \tag{3.72}$$

$$q_t = q_{t-1} + \Delta p_t^D - \Delta p_t^C \tag{3.73}$$

$$y_c^C = a_t^C + l_t^{C,tot} \tag{3.74}$$

$$y_t^D = l_t^{D,tot} \tag{3.75}$$

$$\Delta p_t^C - \psi_C \Delta p_{t-1}^C = \beta E_t(\Delta p_{t+1}^C - \psi_C \Delta p_t^C) + \kappa^C(w_t^C - a_t^C) \tag{3.76}$$

$$\Delta p_t^D - \psi_D \Delta p_{t-1}^D = \beta E_t(\Delta p_{t+1}^D - \psi_D \Delta p_t^D) + \kappa^D(w_t^D - q_t) \tag{3.77}$$

$$y_t^C = \frac{\lambda C c_t + (1-\lambda)C^B c_t^B}{\lambda C + (1-\lambda)C^B} \tag{3.78}$$

$$y_t^D = \frac{\lambda \delta D i_t + (1-\lambda)\delta D^B i_t^B}{\lambda \delta D + (1-\lambda)\delta D^B} \tag{3.79}$$

$$y_t = \alpha y_t^C + (1-\alpha)y_t^D \tag{3.80}$$

$$d_t = (1-\delta)d_{t-1} + \delta i_t \tag{3.81}$$

$$d_t^B = (1-\delta)d_{t-1}^B + \delta i_t^B \tag{3.82}$$

$$l_t^{C,tot} = \frac{\lambda L l_t^C + (1-\lambda)L^B l_t^{B,C}}{\lambda L + (1-\lambda)L^B} \tag{3.83}$$

$$l_t^{D,tot} = \frac{\lambda L l_t^D + (1-\lambda)L^B l_t^{B,D}}{\lambda L + (1-\lambda)L^B} \tag{3.84}$$

$$a_t^C = \rho_a a_{t-1}^C + \epsilon_t^a \tag{3.85}$$

$$\xi_t^D = \rho_d \xi_{t-1}^D + \epsilon_t^d \tag{3.86}$$

$$v_t = \rho_v v_{t-1} + \epsilon_t^v \tag{3.87}$$

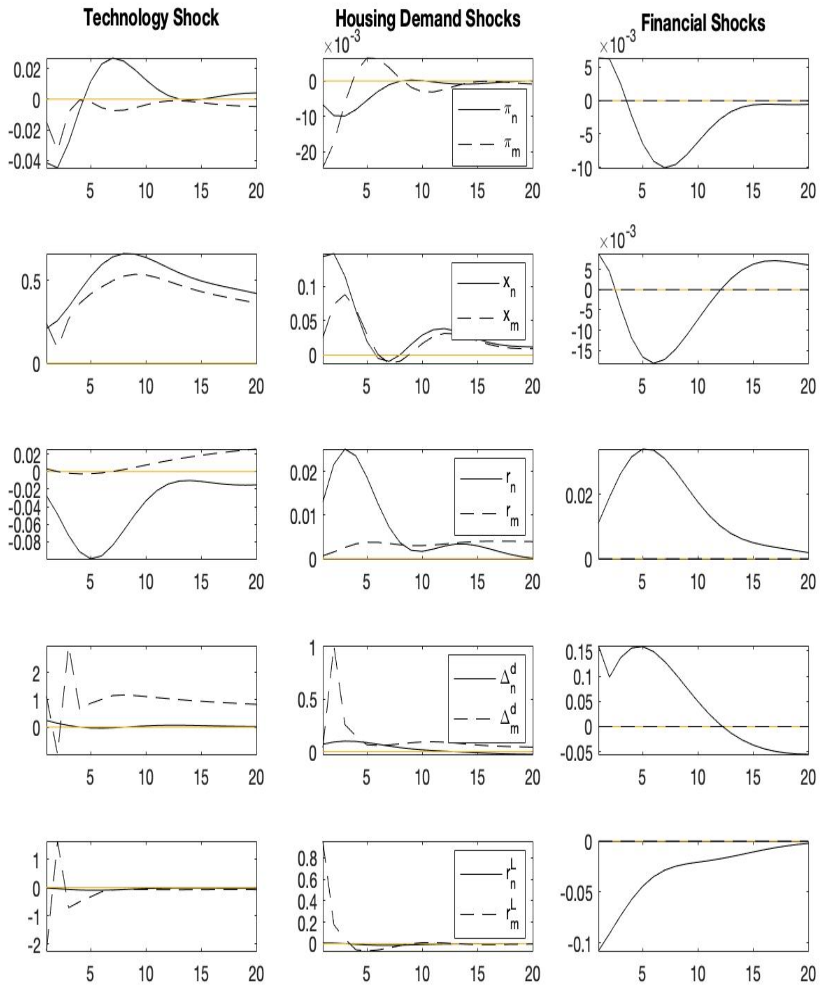

Figure 3.2: Impulse responses for the non-robust model with vs without macroprudential policy under commitment

Notes: This figure shows impulse response functions to one-standard-deviation shocks in the non-robust policy model with or without the macroprudential policy under commitment. The continuous solid lines that subscripted with 'n' mean the non-robust model without macroprudential policy; the dashed lines that subscripted with 'm' mean the non-robust model with macroprudential policy; π_t denotes CPI inflation; x_t denotes the output gap; r_t denotes the policy rate; Δ_t^d shows the nominal debt growth; r_t^L shows the lending rate.

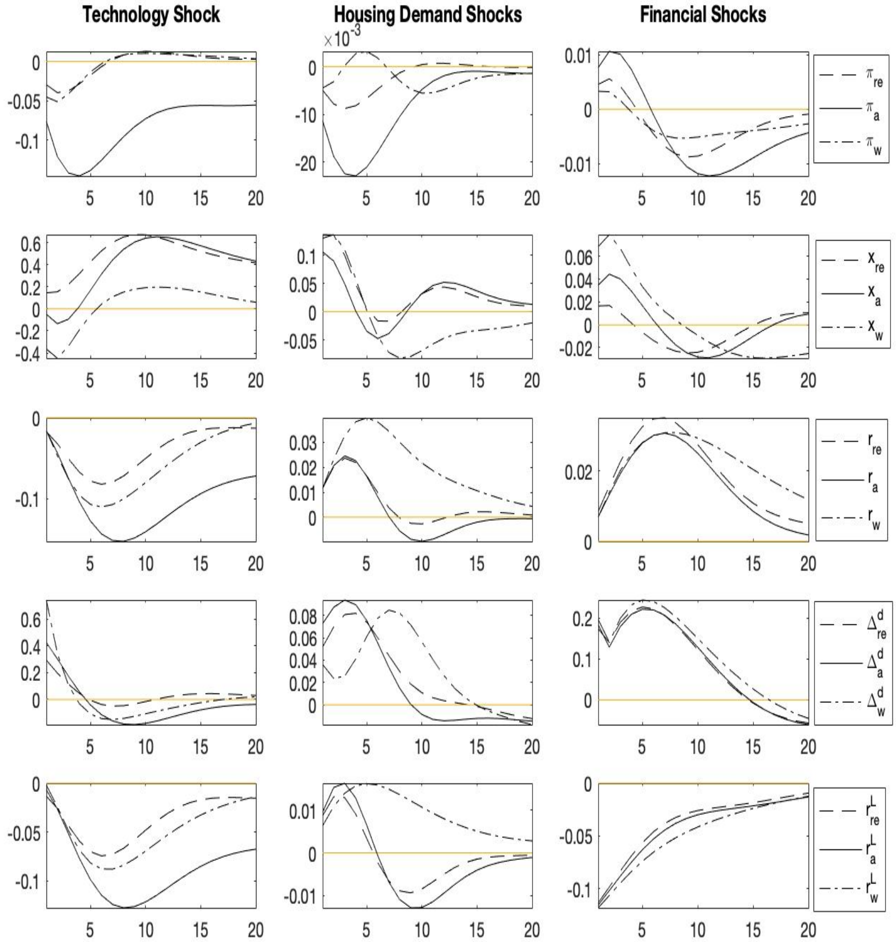

Figure 3.3: Impulse responses for the robust model without macroprudential policy under commitment

Notes: This figure shows that the impulse response functions to one-standard-deviation shocks in the robust policy model without macroprudential policy, when the model is set with commitment. The dashed lines that subscripted with 're' mean the outcome with rational expectations with non-robust monetary policy; the continuous solid lines that subscripted with 'a' indicate the outcome in the approximating equilibrium with robust monetary policy; the dash dot lines that subscripted with 'w' represent the outcome with the worst-case equilibrium in robust monetary policy. π_t denotes CPI inflation; x_t denotes the output gap; r_t denotes the policy rate; Δ_t^d shows the nominal debt growth; r_t^L shows the lending rate.

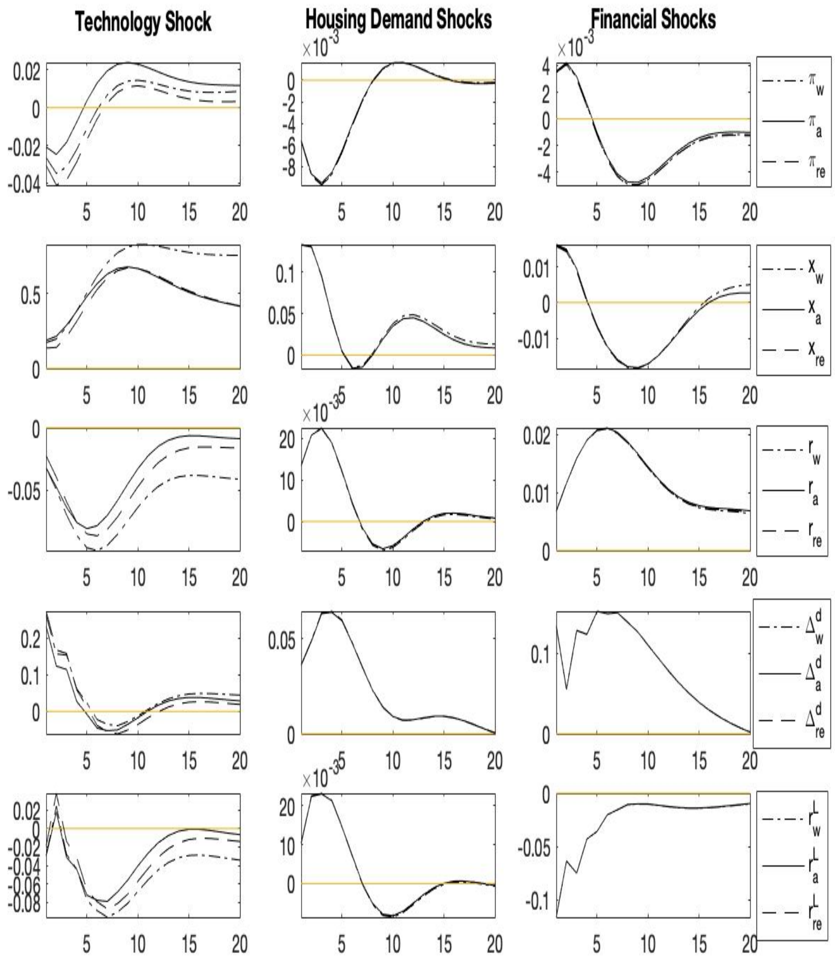

Figure 3.4: Impulse responses for the robust model with exogenous macroprudential policy under commitment $\hat{\tau}_t = 0.3$

Notes: This figure shows the impulse responses of main variables to three shocks with approximating model, the worst-case equilibrium and rational expectations. The dashed lines that subscripted with 're' mean the outcome with rational expectations with non-robust monetary policy; the continuous solid lines that subscripted with 'a' indicate the outcome in the approximating equilibrium with robust monetary policy; the dash dot lines that subscripted with 'w' represent the outcome with the worst-case equilibrium in robust monetary policy.π_t denotes CPI inflation; x_t denotes the output gap; r_t denotes the policy rate; Δ_t^d shows the nominal debt growth; r_t^L shows the lending rate.

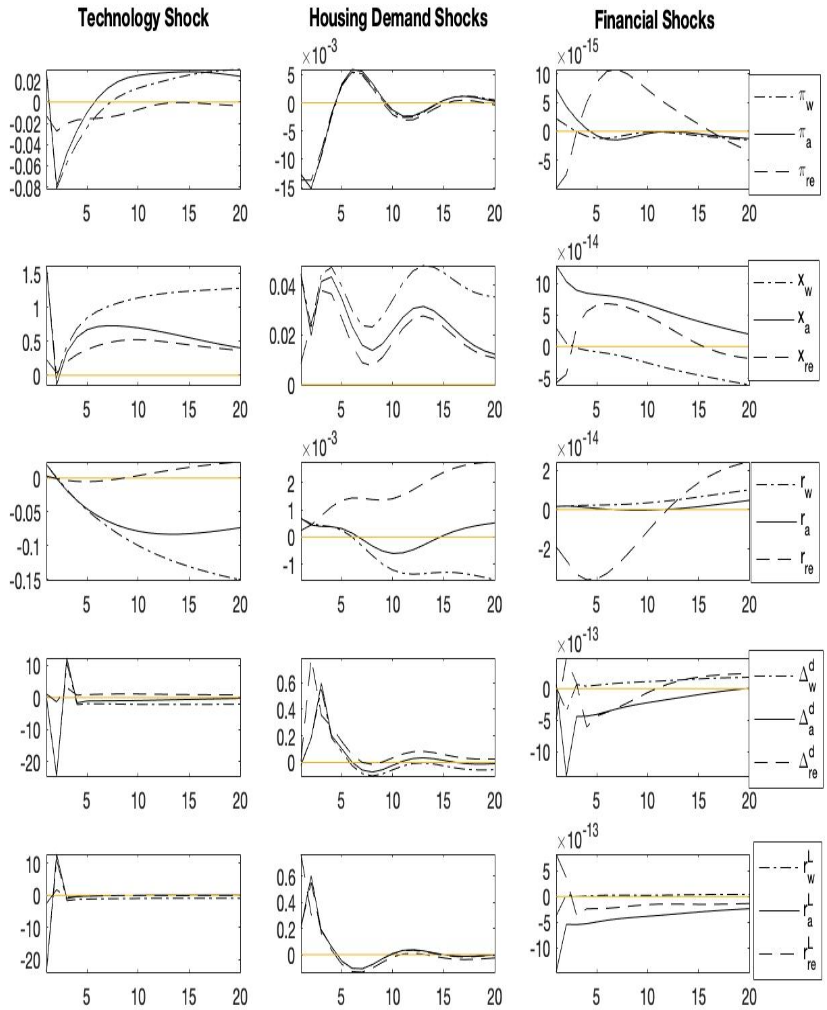

Figure 3.5: Impulse responses for the robust model with endogenous macroprudential policy under commitment

Note: This figure shows the impulse responses with rational expectations and the approximating model. The dashed lines that subscripted with 're' mean the outcome with rational expectations with non-robust monetary policy; the continuous solid lines that subscripted with 'a' indicate the outcome in the approximating equilibrium with robust monetary policy; the dash dot lines that subscripted with 'w' represent the outcome with the worst-case equilibrium in robust monetary policy.π_t denotes CPI inflation; x_t denotes the output gap; r_t denotes the policy rate; Δ_t^d shows the nominal debt growth; r_t^L shows the lending rate.

Chapter 4

Welfare Gains from Commitment under Unconventional Monetary Policy

4.1 Introduction

The seminal contributions by Kydland and Prescott (1977) and Barro and Gordon (1983) highlighted the importance of reputation and credibility for monetary policymakers. A key insight from the literature these works spawned is that there are substantial macroeconomic gains in formulating policy by committing to a policy rule. In the original literature the lack of commitment reflected itself in an inflationary bias, where average inflation is above its desired value. The New Keynesian literature with more dynamic features, such as Clarida et al. (1999) the absence of commitment results not in an inflation bias but in a stabilisation bias, which occurs even in the absence of the inflation bias. In these instances the macroeconomic losses that arise occur due to the discretionary policymaker stabilising inflation relatively too little compared to the output gap. Much of the literature in this area has aimed to quantify the losses from lack of commitment (McAdam et al. (2007)) as well as in the design of central banks, through institutional arrangements, that replicate commitment Paez-Farrell (2012).

Nonetheless, the analysis comparing commitment to discretion has typically relied on New Keynesian models that ignore financial frictions. Therefore, this chapter is interested in obtaining the welfare differential of the unconventional monetary policy model under commitment and discretion. There are three shocks in the model: technology shock, capital quality shock, and net worth shock. Moreover, to analyse the crucial function of credit intervention, we compare three types of models, which are financial accelerator plus credit policy, financial frictions model, and the DSGE model without credit intervention. There are some notable results worth mentioning. First, the loss under commitment is always smaller than the loss under discretion for all types of models. Second, welfare gain from commitment varies as the financial sector parameters differ. Central banks commit to keeping low credit intervention intensities for highest welfare gains under commitment, while they also have incentives to deviate to intervene more to achieve more stable consumption. Third, credit policy plays a fundamental role in changing leverage-ratio and net worth. Fourth, capital quality shock matters more than other shocks in the models with financial frictions.

Our research is similar to Dennis and Söderström (2006) (hereafter, DS), in calculating the welfare losses that arise from discretion we use the approach by the DS paper. The remainder of this paper consists of four sections. The first section shows the GK model under commitment and the comparisons to the GK results. Section 2 describes the other part demonstrates the GK model under discretion case and the comparisons with the two, the last part would conclude the analysis.

4.2 Literature Review

Research interests relating to unconventional monetary policy studies have mostly been in the area of its impacts on different areas, the different channels, and incorporating various financial frictions in a DSGE model. However, there is sparse research correlated to the optimal unconventional monetary policy under commitment and discretion scenarios. For conducting optimal policy under commitment and discretion, the most closely related papers are Woodford and Cúrdia (2008), Carlstrom et al. (2010), Brendon et al. (2011), and Fiore and Tristani (2012). However, we will focus on the optimal unconventional monetary policy based on the GK model. In this section, we will first review the crucial concept of unconventional monetary policy, and later find the related literature concerning the important role of commitment in unconventional monetary policy. At last, we find the related research on how to quantify the commitment under the monetary policy.

4.2.1 Understanding Unconventional Monetary Policy

In normal times, the conventional monetary policy indicates that central banks steer official interest rates. Policymakers make decisions and announcements, which are then transmitted to the interbank market. However, during the financial crisis in 2007, the transmission mechanism was severely disrupted due to the following reasons. First, the channel from official interest rates to money market rates was severely affected due to the volatility of demand for reserves and the limited redistribution of reserves. Second, interruptions in other segments of the financial market, such as asset prices, bank rates, exchange rate and money credit may also hinder the transmission channels. Third, the zero lower bound of official interest rates may be binding for central banks' monetary policy decisions. (Cecioni et al. (2011)) Therefore, unconventional monetary policy was carried out by most countries to limit the damage in case of the financial crisis to occur. Instead of affecting the economy through financial intermediaries, central banks can implement some policies directly to alter the financial market situations. Most existing literature classifies three powerful tools for unconventional monetary policy: directly injected liquidity to dysfunctional markets (credit easing); or implement purchased long-term government bonds and mortgage-backed securities to reduce credit cost, increase asset prices and inject more energy into the economy (quantitative easing); or provide communication about future interest rates (forward guidance).

According to Cecioni et al. (2011), unconventional monetary policy can be theoretically classified into two channels of transmission. One is the signalling channel, implying that central banks communicate and inform the public about its future evolution of short-term interest rates, the purchase of financial assets and the implementation of other policies to save the dysfunctional market. The signalling channel depends on whether central banks can provide credibility to keep low-interest rates and the extent to which private expectations and confidence influence macroeconomic and financial market conditions. Therefore, it is necessary to analyse the optimal unconventional monetary policy under commitment and discretion scenarios.

For the signalling channel, there are different views of the research.Krugman et al. (1998) suggests one "irresponsibility principle". In his study, to escape from the zero lower bound, central banks should convince the market that they would allow prices to grow so to increase inflationary expectations. As he mentioned that central banks might confront the credibility crisis if they cannot overcome the liquidity trap through monetary policy. Moreover, Eggertsson et al. (2003) evaluates a simple optimised model of the monetary transmission mechanism; they summarise that the signalling channel seems to be the only effective channel to save the liquidity trap. They also indicate that optimal monetary policy is possible under credible commitment. Furthermore, Woodford (2012) argues that central banks should commit to a target criterion that is used to determine an appropriate timing to inform the changes in interest rates. He claimed that the most effective form of forwarding guidance relies on whether central banks can provide an advanced commitment to definite criteria for future policy decisions.

The second channel is the portfolio-balance channel, which implies the purchase of securities and supports credit flows. In this channel, central banks implement different tools to affect both the private sector and central banks' balance sheet. Central banks can make operations such as the large-scale purchase of securities, asset swaps and direct liquidity injections. There are a large number of papers which examine unconventional monetary policy through this channel. Vayanos and Vila (2009) built a formal preferred-habitat model for the interest rate tern structure. Changing the supply of the asset of a given maturity can affect the yields of the assets when investors have specific preferences over maturities. Besides, many others have applied such preferred-habitat preferences into DSGE models. Chen et al. (2012) investigated a calibrated DSGE model to assess the effects of Large Scale of Asset Purchases (LSAPs). Their result suggests that the portfolio balance channel has less powerful effects than the signalling channel on the macroeconomy. Other groups of researchers derive unconventional monetary policy models from the agent issue between households and financial intermediaries, such as Gertler and Kiyotaki (2010),Gertler and Karadi (2011) and Gertler and Karadi (2018). Those models all apply financial intermediation frictions and heterogeneous private sectors through the financial accelerator mechanism. Gertler and Karadi (2018) investigated the critical role of LSAPs and suggested that LSAPs perform well in the model even when the zero-lower-bond is binding.

Besides, Quint and Tristani (2018) provide a quantitative assessment of the central bank's liquidity provision effect on macroeconomic variables. They first investigate the connection between the interbank market intentions and the economic recession in a structural VAR model to obtain theoretical restrictions. Then they apply the Smets and Wouters (2003) DSGE model with main characteristics from the Gertler and Kiyotaki (2010) model to capture the transmission mechanism of liquidity shocks to the macroeconomy. After the investigation, they find that structural liquidity shocks originate in the interbank market and lead to an increasing in bank lending spreads and a decrease in private investment. The results also further confirm the benefits of the central bank's liquidity provisions effects. However, Bauer and Rudebusch (2013) argued that the signalling channel accounts for 30-65% of the total impact under the unconventional monetary policy. They suggested that bond investors could change their expectations of long-term interest rates if the large-scale purchase of QE signal to market participants that central banks have changed their views over the economic outlook or policy preference.

4.2.2 Commitment under Unconventional Monetary Policy

Since the 1970s, Kydland and Prescott (1977) indicate the essential role of rational expectations for policymakers. The enormous amount of research has recommended that commitment to a policy rule can make the monetary policy optimal. However, such policy rule is time-inconsistent, and a time-consistent policy was suggested as a suboptimal policy. Later, Barro and Gordon (1983) show that time-consistent policy could generate high inflation, known as inflation bias. Under discretion, the inflation bias could occur for two reasons. One is that the central bank has an incentive to inflate once the private sector's expectations are settled. The other is that the central bank could not pre-commit to a zero average inflation rate. Although challenging, it is fundamental for policymakers to commit to obtain an optimal monetary policy. In this paper, we mainly concentrate on the importance of commitment under the unconventional monetary policy.

As the previous section has mentioned, for the forward guidance to be efficient, commitment to the low-interest rates is crucial for central banks. Dell'Ariccia et al. (2018) study unconventional monetary policies in the UK, Japan and the Euro area, they suggest that unconventional monetary policy could be more effective if the central bank is credibly committed to providing sustained monetary accommodation. The example implies that the Bank of Japan's commitment to delivering sustain inflation between 2010 and 2012 has been undermined by long-term deflation. Moreover, Cúrdia and Woodford (2009) extend the New Keynesian model allowing variations of the central bank's balance sheet and the interest rate paid on reserves. The research studies the model with zero lower bond issue and suggests that commitment is crucial to maintain policy accommodation in mitigating the binding effects. The best method to formulate this commitment is to identify with certainty in advance in terms of targets must be met.

There have been several studies which attempt to quantify the welfare of commitment under various models. Those papers choose either to quantify inflation bias or stabilisation bias. McCallum and Nelson (2000) review the differences between the timeless perspective and discretionary models to confirm the vital feature of forward-looking expectations. They calculated the welfare loss through various values of inflation and output gap volatility. The outcome implies that the welfare loss is more significant when the model specifications show that the inflation rate is persistent and more weight is on output-gap variability. As McAdam et al. (2007) (LMP) argued, the previous literature has mainly utilised simple New Keynesian models with the representative consumer's utility in quadratic approximation form to compare variances of the output gap and inflation. Those models have some drawbacks, such as missing consumers and firms' forward-looking information. McAdam et al. (2007) applied the Smets and Wouters (2003) model with micro-foundations to quantify the stabilisation gains from commitment in the light of household welfare. Dennis and Söderström (2006) (DS) quantified the welfare gains between commitment and discretionary monetary policy by applying "the permanent deviation of inflation from the target that in welfare terms is equivalent to moving from discretion to precommitment". They evaluate social welfare through two measures, "percentage gain in welfare" and "inflation equivalent", within three stylised models of optimal discretionary rules and optimal commitment rules. One result suggests that the monetary policy regime, which focuses on output stabilisation more than inflation stabilisation and leads to the increasing influence of precommitment to welfare.

In this chapter, there are some similarities and variations in contrast to Dennis and Söderström (2006) 's. First, as the LMP paper argued that many econometric models, who have missed micro-foundations using a quadratic approximation of the representative consumer's utility as welfare loss function, are inappropriate. This is because that forward-looking behaviour is fundamental for rational expectations so that would affect policymakers' decisions. However, the DS considers the representative agent's discounted life-time utility as a social welfare loss function. Therefore, we apply the welfare loss function that is based on the GK model's utility function. One vital feature of the GK model is that it demonstrates heterogeneous agents constrain financial intermediaries. Second, this chapter follows the DS paper by quantifying two welfare differentials between commitment and discretion. Two methods can provide results for easy comparisons between this work and others. Third, we quantify the welfare gains from the GK unconventional monetary policy model; this is the main contribution of this chapter. As previous literature has focused on either simple New Keynesian models or three-equation models, those models bring limited impact to practical studies after the financial crisis.

4.3 The Unconventional Monetary Policy Model

The GK model is closely related to conventional monetary policy models with financial accelerators. Financial intermediaries might be subject to endogenously determining balance sheet constraints, and the central bank can lend directly to private credit markets.

There are some pioneering contributions of the GK model, which have become one fundamental literature in unconventional DSGE models. First, the GK model provides a quantitative unconventional DSGE model with financial accelerators confronting a balance sheet constraint. Second, the GK model quantified the effects of the financial crisis and explained the impacts of credit policy on the economy. Third, the GK model also considers the zero lower bound constraint and suggest that the unconventional monetary policy effects will mitigate after the financial crisis is over. We will recapture the important parts from the GK model in the next section before we quantify the welfare gains of committed unconventional monetary policy.

4.3.1 Model Analysis

The model follows the conventional monetary DSGE model framework with nominal rigidities. There is a continuum of identical households with measure unity. Within each household, there are two types of members: workers and bankers. Workers supply labour, and each banker manages a financial intermediary. They both give the earnings back to the households. However, households make deposits in financial intermediaries that they do not own. There exists perfect consumption insurance in the family. At any point, there exists a fraction of household $1 - f$ who are workers, and the remainder f, are bankers. The two types of workers can also switch over time. Therefore, it is supposed that $(1-\theta)f$ of bankers exit and become workers in every period. The relative portion of each type is fixed because a similar size of workers can randomly become bankers. The households preferences function is given by

$$maxE_t \sum_{i=0}^{\infty} \beta^i [ln(C_{t+i} - hC_{T+I-1}) - \frac{\chi}{1+\psi} L_{t+i}^{1+\psi}] \tag{4.1}$$

where the discount factor $0 < \beta < 1$, habit parameter $0 < h < 1$, ψ and χ are both positive. The budget constraint is given by

$$C_t = w_t L_t + \Pi_t + T_t + R_t B_t - B_{t+1} \tag{4.2}$$

where w_t is the real wage, Π_t is the net payouts to the household from both ownership, T_t gives the lump-sum taxes, B_{t+1} shows the total quantity of short term debt the household acquires, R_t is the gross real return of one period real bonds from time t-1 to t.
Financial intermediaries lend money obtained from households to firms. The balance sheet of a financial intermediary is given by

$$Q_t S_{jt} = N_{jt} + B_{jt+1} \tag{4.3}$$

where N_{jt} shows the amount of the wealth or net worth at the end of period t; B_{jt+1} gives the deposits that intermediaries obtain from households can also be thought as the intermediaries' debt; S_{jt} shows the number of financial claims on non-financial firms; Q_t is the relative price of each claim. Therefore, the amount that intermediaries can lend is equal to the number of their equities and the deposits from households.
Banker's equity capital evolves

$$N_{jt+1} = R_{kt+1} Q_t S_{jt} - R_{t+1} B_{jt+1} = (R_{kt+1} - R_{t+1}) Q_t S_{jt} + R_{t+1} N_{jt} \tag{4.4}$$

where R_t is the gross real return on intermediary deposits and government debt from period t-1 to t, R_{kt+1} is the stochastic return on intermediary assets over period t+1. The difference between banker's equity and the riskless return depends on the premium $(R_{kt+1} - R_{t+1})$ and his total quantity of assets $Q_t S_{jt}$.

The agency problem limits the intermediaries' ability to borrow. It is assumed that the banker can divert λ fraction of available funds and the depositors can force intermediaries into bankruptcy and recover the remaining fraction $(1 - \lambda)$ of assets. The total demand for assets is given by,

$$Q_t S_t = \phi_t N_t \tag{4.5}$$

where $\phi_t = \frac{\eta_t}{\lambda - v_t}$ is the leverage ratio that is equal to the ratio of intermediate assets to equity. v_t is interpreted as the expected discounted marginal gain to the banker of expanding assets one more unit Q_tS_t by holding N_{jt} constant. The variable η_t is explained as the expected discounted value of increasing one more unit of N_{jt} by holding Q_tS_{jt} constant. S_t provides the aggregate quantity of intermediary assets, N_t shows the aggregate intermediary capital stock. Therefore, the variation in total assets demand (Q_tS_t) is influenced by the leverage ratio (ϕ_t) and total capital stock (N_t).

When the central bank is willing to fund a fraction ψ of intermediated assets, total value of intermediated assets is given by,

$$Q_tS_t = \phi_t N_t + \psi_t Q_t S_t = \phi_{ct} N_t \tag{4.6}$$

where $\phi_{ct} = \frac{1}{1-\psi}\phi_t$ is the leverage ratio total intermediated funds and depends positively on the intensity of credit policy ψ_t.
In normal times, the central bank is assumed to follow a Taylor rule with interest-rate persistence, which is given by

$$i_t = (1-\rho)[i + \kappa_\pi \pi_t + \kappa_y(logY_t - logY_t^*)] + \rho i_{t-1} + \epsilon_t \tag{4.7}$$

where i_t is the nominal interest rate, which is the instrument of monetary policy in normal times. The interest rate smoothing parameter ρ belongs zero to one, κ_π denotes the inflation coefficient of the Taylor rule, κ_y shows the output gap coefficient of the Taylor rule. During the 2007-8 financial crisis, the monetary policy also involves credit intervention. The central bank injects credit into the market in response to movements in credit spreads, in terms of the feedback rule,

$$\psi_t = \psi + vE_t[(logR_{kt+1} - logR_{t+1}) - (logR_k - logR)] \tag{4.8}$$

$logR_k - logR$ is the steady-state premium,v is the credit policy feedback parameter. When the crisis happens, the central bank expands credit as the interest rate premium increase relative to its steady-state value.

The second-order form of the whole model equations is in the Appendix. All the variables are solved in the exponential form in the second-order approximation. According to the GK model, we classify the welfare results into three scenarios with the different equations:
(1) The financial accelerator model with credit policy intervention (FA_{CP}). This model includes all 42 equations.
(2) The financial accelerator model without credit policy (FA) model. This model will omit equations (4.43), (4.47) and (4.53). Those equations show the credit policy intervention.

4.3.2 Calibration

Table (4.1) provides the calibrated parameters cited from the GK model. As the GK model mentions that there are fifteen conventional parameters and three particular parameters in the model. Most of the values come from the estimates in Primiceri et al. (2006), who applied the Bayesian method to US data. Their model results show competitive to the others' research, such as Smets and Wouters (2003). The cited values include the proportionate habit parameter, the elasticity of marginal depreciation with respect to the utilisation rate, the relative utility weight on labour, price indexing and price rigidity parameters, the Frisch elasticity of labour supply, and the inverse elasticity of the net investment to the price of capital. Other parameters are used in common values, such as the discount rate, the depreciation rate, the capital share, the elasticity of substitution between goods, and the steady-state government expenditures share.

The specific parameters in the GK model are the divertable fraction of capital assets ($\lambda = 0.381$), the proportional transfer to the entering bankers ($\omega = 0.002$), and the survival rate of the bankers ($\theta = 0.972$). Those parameters were achieved to meet the targets that include a one hundred basis point of a steady-state interest rate spread, a

Table 4.1: Calibrated Parameters

β	0.990	Discount rate
σ	1.000	Intertemporal elasticity of substitution
h	0.815	Habit parameter
χ	3.409	Relative utility weight of labour
φ	0.276	Inverse Frisch elasticity of labour supply
ζ	7.2	Elasticity of marginal depreciation with respect to utilization rate
λ	0.381	Starting value divertable fraction of capital assets
ω	0.002	Proportional transfer to the entering bankers
θ	0.972	Survival rate of the bankers
α	0.330	Capital share
δ	0.025	Depreciation rate
η_i	1.728	Inverse elasticity of net investment to the price of capital
ϵ	4.167	Elasticity of substitution
γ	0.779	Probability of keeping the price constant
γ_P	0.241	Price indexation parameter
$\frac{G}{Y}$	0.200	Steady state proportion of government expenditures
ρ_i	0.000	Interest rate smoothing parameter (under FA and FA_{CP} model)
ρ_i	0.8	Interest rate smoothing parameter (under DSGE model)
κ_π	1.500	Inflation coefficient in the Taylor rule
κ_y	0.125	Output gap coefficient in the Taylor rule
σ_ξ	0.05	Size of the capital quality shock
ρ_ξ	0.66	Persistence of the capital quality shock
σ_a	0.01	Size of the TFP shock
ρ_a	0.95	Persistence of the TFP shock
σ_g	0.01	Size of the government expenditure shock
ρ_g	0.95	Persistence of the government expenditure shock
σ_{Ne}	0.01	Volatility to the wealth shock
σ_i	0.01	Volatility to the Taylor rule shock
ρ_{ϵ_ψ}	0.66	Persistence of the CP shock
σ_ψ	0.072	Size of the CP shock

Note: The calibrated parameters are cited from the Gertler and Karadi (2011) model steady-state leverage ratio of four and an average horizon of a decade for bankers. In terms of calibration, the model assumes that the banker can choose to divert 38.1 per cent of available funds at the beginning period and transfer them back to the household. The banker's cost is that depositors can force the intermediary to recover the rest of 61.9 per cent of assets. In every period, 97.2 per cent of bankers can survive to be bankers in the next period. However, $2.8f$ per cent of bankers would have to exit and become workers. f is the fraction of bankers. Therefore, the household transfers the fraction $\frac{\omega}{1-\theta} = 71.4$ per cent of total final period assets to the entering bankers each period.

4.3.3 Welfare Analysis Approach

This chapter follows the welfare analysis approach used by Dennis and Söderström (2006), while the difference is that we are using the non-linear model with a recursive utility function. The utility function is given by

$$\Omega_t = U(C_t, L_t) + \beta E_t \Omega_{t+1} \tag{4.9}$$

where β is the discount factor. As McAdam et al. (2007) argue that many studies apply simple New Keynesian models with the standard ad hoc loss functions without micro-foundations. These loss functions either include the standard deviation of the output gap and inflation, or the variances of the output gap: output and inflation. Even though the quadratic approximation of the representative consumer's utility function coincides with the ad hoc loss functions, those models, including forward-looking behaviour, are inappropriate as the important sources of time-inconsistency are missing. Therefore, we apply the welfare approach based on the unconventional monetary policy model with a heterogeneous agent and financial intermediaries. The welfare-based utility function is solved by a second-order approximation about the steady-state first. Then we take a second-order approximation of the whole model for the steady-state and apply this approximation as the objective function to the predetermined variables and shocks of the system.

To obtain the optimal policy rules under commitment and discretion, we use two different methods from Dynare. For the commitment case, we use the Dynare Ramsey approach to calculate the first-order conditions of the Ramsey planner' objective function subject to the non-linear constraints that are the first-order conditions of the private economy. Then we take approximates of the first-order conditions of the planner's objective function to first order. The central bank commits to the interest rate as the instrument to obtain the optimal commitment policy. On the other hand, the discretionary case is more complicated. In the linear quadratic loss function format model, the approximation of optimal policy under discretion can be solved by LQ solver algorithm, which is described by Dennis (2007). However, the unconventional model we apply here is non-linear; the conventional discretionary method is not applicable here. Therefore, we choose the Covariance Matrix Adaptation Evolution Strategy (CMAES) to solve the second-order approximation problem for the discretionary case. The CMAES is a second-order method estimating a positive definite matrix within a covariance matrix procedure. This method takes advantages of being feasible on non-separable and conditioned problems. It has been proved to be a reliable and highly competitive algorithm for both local optimisation and global optimisation. (Hansen et al. (2003) and Ros and Hansen (2008)). Therefore, the CMAES can calculate the optimal parameter of κ_y and κ_π with corresponding welfare loss in terms of different policy rules.

There are two alternative measures that can be applied to analyse the differences between commitment and discretionary monetary policy. One is the percentage gain in welfare-related with moving from the discretionary monetary policy to commitment scenario:

$$\Omega = 100 * [1 - \frac{L_C}{L_D}] \tag{4.10}$$

where L_C and L_D are the social loss function under commitment and discretion, respectively. Equation (4.10) captures the welfare gains over commitment under the discretionary case. In this equation, L_C shows the welfare outcome when the central bank can commit to the optimal interest rate rule; L_D demonstrates the scenario when the central bank deviates from its promise and attempts to optimise the policy rule in the time-consistent equilibrium.

The welfare results vary under different policy rule settings. First, for the FA_{CP} model, the central bank must commit to the low-interest rate with credit intervention without interest rate smoothing ($\rho_i = 0$) to respond to the capital quality shock for a single crisis period. However, as the GK model mentioned that this not an on-going sequence, the central bank might discrete from the optimal policy rules once it considers the financial crisis is over. Second, for the FA model, the central bank commits to the optimal interest rate rule without credit intervention and interest rate smoothing. This is the situation during normal times when the central bank implements policy rules to take into account financial frictions. The central banks might re-optimise the future interest rates by deviating from original promises. Third, for the DSGE model without financial frictions, the central bank simply commits to the Taylor rule with interest rate smoothing ($\rho_i = 0.8$). The welfare loss L_D is achieved when the central bank cannot keep its promise by smoothing interest rates.

The other approach is followed by Jensen (2002) that obtains a permanent deviation of consumption, in welfare terms is equivalent to moving from the discretion to commitment scenario. Differentiated with Dennis and Söderström (2006) inflation equivalent, we are using the “consumption equivalent” term as the recursive utility function is applied. The “consumption equivalent” is given by

$$\hat{CE} = \sqrt{L_D - L_C} F \tag{4.11}$$

Through equations (4.9) and (4.10), the welfare between commitment and discretion can be easily discussed.

Table 4.2: Welfare Gain from Commit in FA_{CP} Model (Benchmark Case)

κ_π	κ_y	**Loss under commitment**	**Loss under the discretion**	**Gain from commitment**	**Consumption Equivalent**
1.099	0.0435	296.6207	452.8033	34.4924	12.4973

NOTES: This table shows the optimal interest rate rule parameters (κ_{pi} and . κ_y), the value of the central bank loss under commitment (L_C) and discretion (L_D), the percentage gain from commitment (Ω) and the consumption equivalent ($\hat{CE}$).

4.4 Unconventional Monetary Policy with Credit Intervention (FA_{CP} Model)

This section presents results for the optimal welfare analysis under the financial accelerator model with credit intervention. Table (4.2) provides the results from optimal welfare outcomes under commitment and discretion with optimized parameter κ_π and κ_y. According to the Matlab CMAES optimiser, optimal parameters suggested were 1.099 for inflation coefficient and 0.0435 for the output gap coefficient. The optimal parameters are slightly lower than the calibrated value in the GK model, which applies 1.5 for inflation and 0.125 for the output gap. For the calibrated parameters $v = 10$, $\lambda = 0.381$, $\theta = 0.9715$, the welfare loss is 296.6207 under commitment and 452.8033 under the discretionary monetary policy. The welfare loss is much smaller under the commitment case than the discretionary monetary policy. By sticking to the committed policy rule, the central bank can obtain benefits of approximately 34 per cent. The consumption equivalent is 12.4973.

We will also manipulate some fundamental parameters, while keeping others constant, to identify the variation in the welfare benefits from commitment and consumption equivalent. Tables (4.3) to (4.5) illustrate the variations of welfare under different levels of parameters. The first parameter we considered was v in Table (4.3) that shows with the credit intervention intensities parameter. When $v = 10$, the situation is close to the real financial crisis scenario, as suggested in the GK model. $v = 100$ is seen as an aggressive intervention and close to the optimum scenario. The differences between commitment and discretion are shown from level v=10 to v=100. The loss under commitment is always lower than under discretion. All results show a decreasing and then increasing pattern when the central bank intervention intensities expand.

On the one hand, when the central bank follows the committed optimal policy, the welfare loss declines first and grows later. The lowest loss is obtained when $v = 30$. The highest gain from commitment is when $v = 10$. The most stable consumption level is obtained when $v = 50$. On the other hand, under the discretionary monetary policy, there are no certain sequences when the credit feedback parameters increase. The loss gets bigger at the beginning, drops and then grows after that. Moreover, the inefficiency of discretionary policy is equivalent to a permanent deviation of consumption from a target that is between 10.0937 and 12.5186. In terms of Table (4.3), there are some other interesting phenomena as well. When the central bank is committed to the credit intervention at level $v = 10$, it has the incentive to deviate from the current level and intervene more, because level $v = 30$ has a lower loss under two scenarios and has more stable consumption. The same applies to the other levels; the central bank always has the incentive to deviate from the current level of credit intervention. However, the central bank should choose to stick to at level $v = 30$ due to lower loss under commitment; this would generate growth in the central bank's balance sheet, equal to approximately ten per cent of the value of the capital stock.

Table (4.4) provides the different specifications of the parameter λ, the fraction of capital that can be diverted by financial intermediaries. This parameter aims to pin down the steady-state leverage ratio. As calibrated at 38 per cent of assets the bankers can divert, the steady-state leverage ratio is low at four. Gertler and Kiyotaki (2010) recommended that the fraction can be much lower for the leverage ratio to be higher in sectors that are investing. Therefore, we consider 0.381 as the highest fraction of capital that can be diverted; the variation levels begin from $\lambda = 0.15$. Under commitment, the loss

Table 4.3: Gain from commitment under FA_{CP} Model with Different Credit Intervention Intensities

v	**Welfare under commitment**	**Loss under discretion**	**Gain from commitment**	**Consumption Equivalent**
10	296.0892	452.8033	34.6098	12.5186
30	296.067	405.7637	27.0346	10.4736
50	296.0699	397.9521	25.6016	10.0937
70	296.0803	398.3557	25.6744	10.1131
100	296.0997	402.8214	26.4936	10.3306

NOTES: This table shows the value of the central bank loss under commitment (L_C) and discretion (L_D), the percentage gain from commitment (Ω) and the consumption equivalent ($\hat{CE}$), for different parameters with survival rate of bankers (v).

varies from 294.4079 to 296.0892; interestingly, the lowest loss occurs when $\lambda = 0.15$. The gain from commitment varies from 34.6 per cent to 65.5 per cent. The percentage gain declines as the diverted fraction of capital increases. Under the discretionary case, the loss decreases as the fraction grows. The same pattern applies to the consumption equivalent. The most stable consumption occurs when $\lambda = 0.381$. The central bank should commit to the low level of a diverted fraction of capital stock as the losses are all smaller than the case under discretion, the percentage gain is also high when the fraction is low. However, the central bank would always want to deviate from the current level as the discretionary case illustrate a decreasing pattern and consumption equivalent also implies an increasing stable path.

Table (4.5) shows the results of different levels of survival rate of banker θ. All results under commitment are smaller than those under discretion, implying that the central bank is better off by committing the optimal monetary policy. Under commitment, the loss drops as θ increases from 0.7 to 0.9715. However, other results do not show a clear pattern. Under discretion, the loss declines first, increases to a level of 0.9 and then decreases to the level of 0.9715. The same applies to the percentage gain from commitment and consumption equivalent. The overall results suggest that θ can be chosen at

Table 4.4: Gain from commitment under FA_{CP} Model with Different Diverted Fraction of Capital

λ	**Loss under commitment**	**Loss under discretion**	**Gain from commitment**	**Consumption Equivalent**
0.15	293.8769	851.3653	65.4817	23.6112
0.2	294.4079	776.0162	62.0616	21.9456
0.25	294.8995	726.9587	59.4338	20.7860
0.3	295.3655	690.6028	57.2308	19.8806
0.381	296.0892	452.8033	34.6098	12.5186

NOTES: This table shows the value of the central bank loss under commitment (L_C) and discretion (L_D), the percentage gain from commitment (Ω) and the consumption equivalent ($\hat{CE}$), for different parameters financial intermediaries' fraction of capital that can be diverted (λ).

Table 4.5: Gain from Commit under FA_{CP} Model with Different Survival Rate of Bankers

θ	**Loss under commitment**	**Loss under discretion**	**Gain from commitment**	**Consumption Equivalent**
0.7	312.4634	353.7438	11.6696	6.4250
0.8	308.2312	343.1127	10.1662	5.9061
0.85	305.8565	335.2020	8.7546	5.4171
0.9	303.0573	646.3183	53.1102	18.5273
0.9715	296.0892	452.8033	34.6098	12.5186

NOTES: This table shows the value of the central bank loss under commitment (L_C) and discretion (L_D), the percentage gain from commitment (Ω) and the consumption equivalent ($\hat{CE}$), for different parameters with the central bank intervention intensities (θ).

the level of 0.85 that would generate a relative less loss and more stable consumption level.

4.4.1 Impulse Responses

With credit policy, the GK model only shows the impulse response of capital quality shock. There are four shocks that will be discussed in this section, including technology shock, shock to capital quality, shock to the net worth, and credit policy shock. The solid black lines in the figures show the response of the variables under optimal commitment policy; the red dashed lines represent the outcomes under the optimal discretionary policy.

The technology shock is set as a negative one per cent innovation with a quarterly autoregressive factor of 0.95. The impulse responses are depicted in Figure (4.1) with twelve variables. Under optimal commitment policy, when the economy is hit by negative technology shock, the most striking impact on variables is the 3 per cent drop of investment. The decline in investment leads to a slight reduction in asset price by 1 per cent and net worth by 1.5 per cent. The decreased balance sheet leads to a growth of premium by 0.02 per cent. Due to the increasing cost of capital, capital declines, then pushing investment and asset prices further down. Since the central bank is committed to the optimal policy, the interest rate goes down to prevent an economic downturn. The inflation rate is strictly negatively correlated with the nominal interest rate. Even though credit intervention is also implemented, the changes in inflation, credit intervention, premium, and optimal leverage are quite small.

4.4.2 Technology shock

Under the optimal discretionary policy, the responses are more volatile than those under optimal commitment policy. The most substantial impacts are still depicted in investment, output, and capital. The central bank makes decisions that are optimal in each period without committing itself to any actions, implementing less change in interest rate but

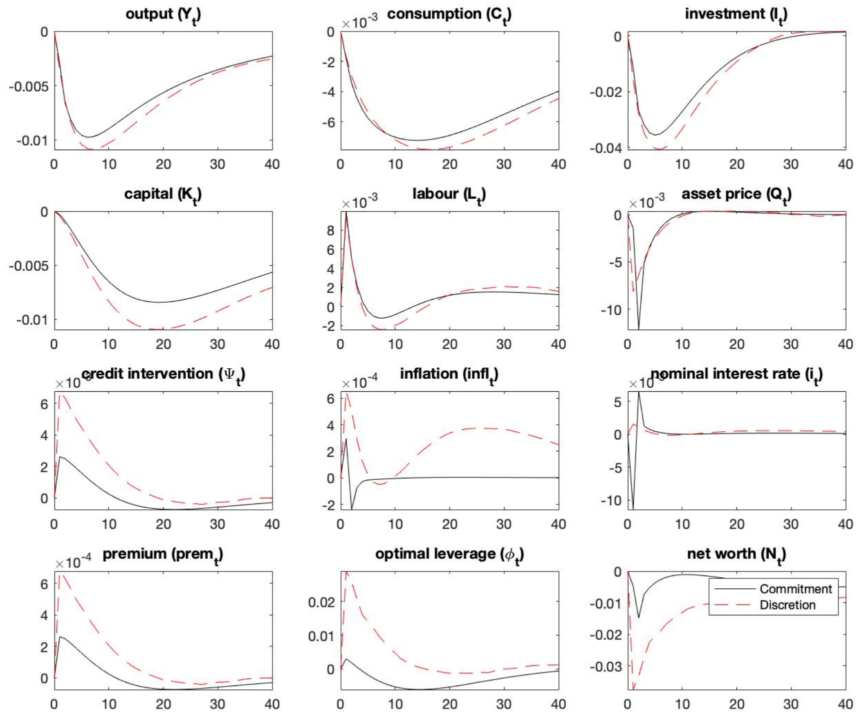

Figure 4.1: Impulse Responses in the FA_{CP} Model to a negative technology Shock

Note: This figure shows the impulse responses to a negative technology shock in the financial accelerator model with credit interventions under commitment (solid lines) and discretion (dashed lines) more credit intervention. With stronger credit intervention, the inflation bias also occurred when inflation is more persistently above the zero levels steady-state.

4.4.3 Capital Quality Shock

The FA_{CP} Model ($v = 10$)

Figure (4.2) describes the impulse responses of capital quality shock under commitment and discretion. The level of credit intervention is selected as $v = 10$. Under optimal commitment policy, the central bank is committed to the interest rate instrument only. This can be shown in Figure (4.2) that the nominal interest rate is around zero under the discretion, and credit intervention is also around zero steady-states under commitment. When the quality of capital declines by 5 per cent, the deterioration of intermediary capital would disrupt lending and borrowing in the intermediary sector, then the net worth falls by approximately 15 per cent. The reduced effective quality of capital in the production function also leads to a decline in output by 1.6 per cent. After that, the consumption by 4.4 per cent, investment by 3.7 per cent. With interest rate instruments, premium and leverage also contracted significantly to a level around zero. The effect on the inflation rate is also small and back to zero steady-states once the policy's influence has vanished.

On the other hand, when the central bank discrete from the interest rate instrument and implements the credit intervention, the responses are more volatile than those under commitment. With the reduced 5 per cent capital quality, the net worth drops approximately 50 per cent; leverage increases around 37 per cent, making funding for firms much more difficult than before. Therefore, investment also declines by around 18 per cent. Stronger influences also occur for output, consumption, and capital. If the central bank implements the moderate credit intervention ($v = 10$), this will generate an expansion of the central bank balance sheet, increasing to around seven per cent of the value of the capital stock. This result is the same as in the GK model.

Moreover, the central bank's intermediation dampens the lift in the premium and optimal leverage, inducing growth in investment, net worth, output, consumption, and capital. Besides, the inflation rate under discretion has recovered, overshot from the zero steady-state

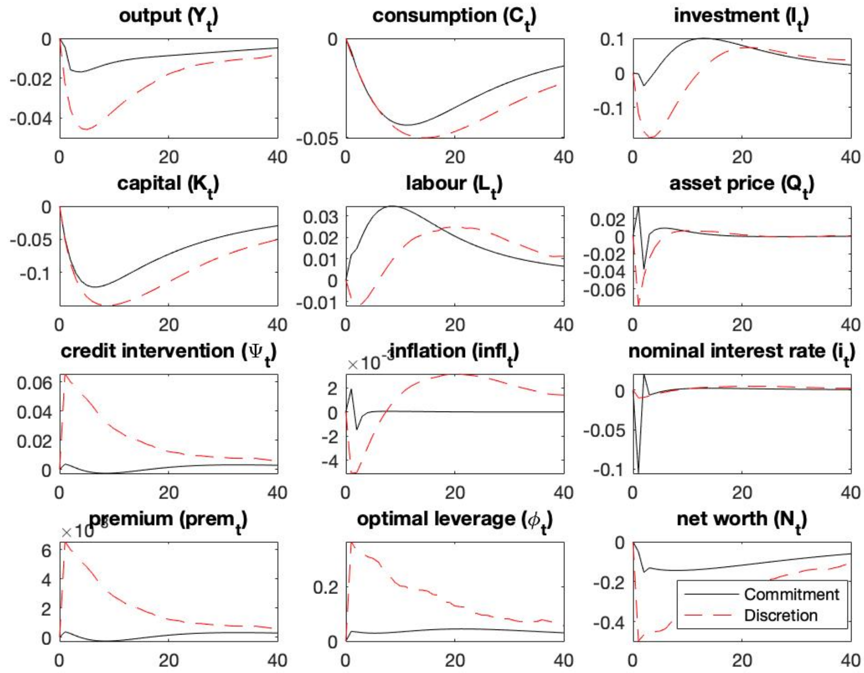

Figure 4.2: Impulse Responses in the FA_{CP} Model to Capital Quality Shock ($v = 10$)
Note: This figure shows the impulse responses to a capital quality shock in the financial accelerator model under commitment (solid lines) and discretion (dashed lines), when there is a low level of credit intervention (v=10).

and increased at a higher level by 0.1 per cent permanently; even it is a small amount. This is the stabilisation bias due to the central bank not being able to commit and leads to over-stabilised output at the cost of greater inflation variability.

The FA_{CP} Model ($v = 100$)

In terms of the GK model, the above results give a moderate credit intervention that has occurred in practice. This part will access the results when the feedback parameter is increased to 100, indicating an aggressive credit intervention. The aggressive policy intervention produces a lower intensity of the response.

Under an optimal commitment policy, the central bank commits to the interest rate rule to minimise the unconditional expectation of the welfare loss function. According to Figure (4.3), the responses under commitment have similar movements compared to the committed moderate credit intervention. This is because they both commit to the interest rate rule; the change in credit intervention intensity does not matter.

Moreover, under the discretionary case, the central bank implements an aggressive intervention, requiring that lending grows to approximately 15 per cent of the capital stock. This result is the same as in the GK model. The whole process has been moderated significantly by reducing the rise in the premium and the leverage. The premium drops approximately 0.5 per cent, and the leverage decreases by approximately 7 per cent. Therefore, this leads to a 10 per cent reduction in net worth. The drop in investment is moderated from 18 per cent to 11 per cent. The other significant moderation occurs in asset price, which decreases from approximate 7.9 per cent to 5.7 per cent. Nevertheless, output, consumption, and capital all drop less and tend to recover at a faster pace.

The FA_{CP} Model ($v = 0$)

When the feedback parameter (v) is zero, the model becomes the FA model without credit intervention. Under commitment, the main differences of responses appear in the leverage

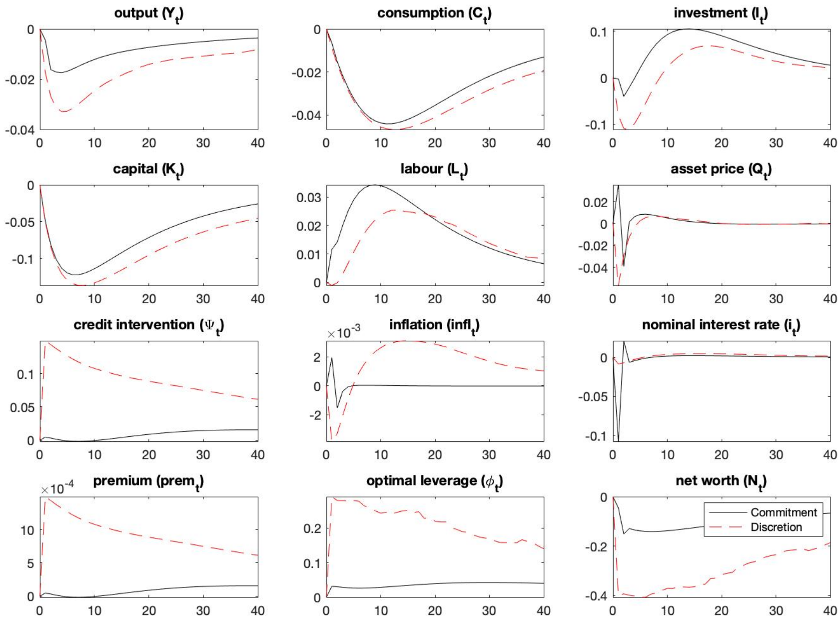

Figure 4.3: Impulse Responses in the FA_{CP} Model to Capital Quality Shock at Level ($v = 100$)

Note: This figure shows the impulse responses to a capital quality shock in the financial accelerator model under commitment (solid lines) and discretion (dashed lines), when there is a high level of credit intervention (v=100).

ratio and net worth. When the capital quality deteriorates, leverage ratio increases by 5 per cent, which is 1 per cent higher when $v = 10$ and 2 per cent higher when $v = 100$. Net worth drops by 17 per cent, that is 2 per cent more than the other two scenarios. The other variables show similar results. This implies that leverage and net worth are different under capital quality shock, even though the credit intervention plays no role under the commitment scenario.

In the discretionary case, the FA model without credit intervention illustrates many volatile responses than the previous two cases. In the beginning, the leverage ratio increases by 49 per cent. This is 16 per cent higher than the moderate credit intervention case and 20 per cent higher than the aggressive credit intervention case. Therefore, the net worth declines by 66 per cent, which is much higher than 50 per cent ($v = 10$) and 40 per cent ($v = 100$). Investment also drops by 27 per cent, much higher than before. The contraction is even worse in output, consumption, and asset price. In contract to the three levels, we can find that the credit intervention has more significant impacts on macroeconomic variables to capital quality shock.

4.4.4 Net Worth Shock

Figure (4.5) response to the shock to the intermediary net worth under commitment and discretion. It can be seen that the influences of net worth shock are trivial except for optimal leverage. Under optimal commitment policy, a decline in the intermediary net worth leads to the growth of leverage and premium, leading to higher funding costs for firms. A 2 per cent increase in leverage ratio produces a drop in asset price, output, and investment of 0.1 per cent, 0.4 per cent, and 0.9 per cent. When the central bank is committed to interest rate, and credit intervention, leverage, and premium all move downward, leading to a lift in asset price, investment and output. When the central bank discrete from the interest rate instrument and implements only credit interventions, a 1 per cent drop in net worth produces a 1 per cent reduction in leverage and 0.03 per cent fall in premium. With credit intervention, a 0.4 per cent increase in the value of capital

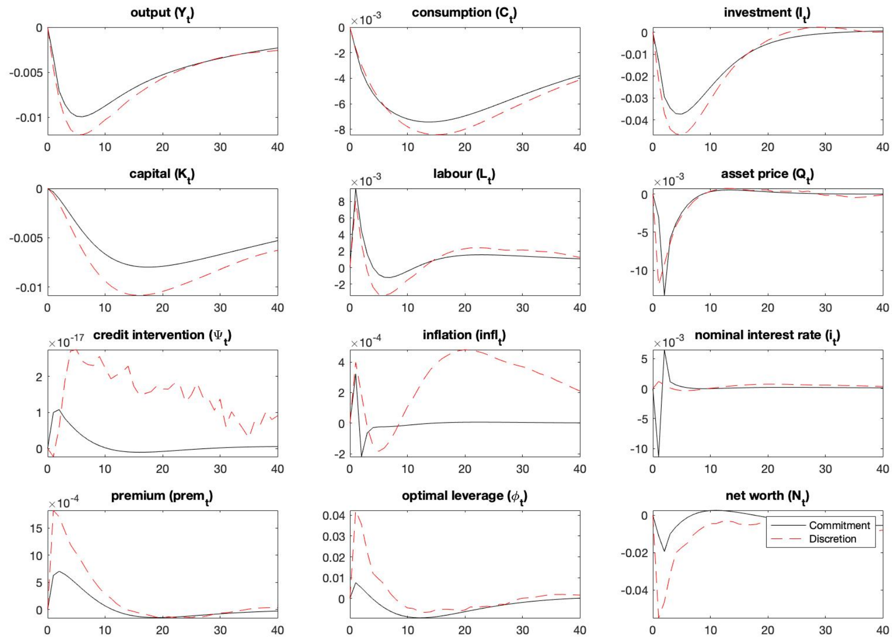

Figure 4.4: Impulse Responses in the FA_{CP} Model to Capital Quality Shock at Level ($v = 0$)
Note: This figure shows the impulse responses to a capital quality shock in the financial accelerator model under commitment (solid lines) and discretion (dashed lines), when there is no credit intervention (v=0).

stock leads to a contraction in both interest rate spread and leverage. After that, the eased cost for funding recovers the investment, asset price, and output.

4.4.5 Credit policy shock

Figure (4.6) plots the responses of credit policy shock under commitment and discretion. The credit intervention concerns the situation that the central bank intervenes in the market directly through injecting more credit during the financial crisis period. Under optimal commitment policy, an approximately two per cent increase in credit policy intervention would induce a 2 per cent growth in net worth through a 1.6 per cent drop in leverage and a 0.6 per cent decline in premium. This, in turn, leads to an acceleration of investment by 0.6 per cent. Therefore, more investment implies higher output, consumption, and asset price. Since the central bank is committed to the interest rate instrument, it observes the credit policy shock by changing only slightly the nominal interest rate during the first few periods. The inflation rate is negatively related to the interest rate and goes to zero steady-state once the effect has vanished.

Under the optimal discretionary policy, the central bank discrete from interest rate instrument by applying credit policy only. The subplots show that the overall responses are more volatile than those under commitment. Without the influence of interest rate, leverage is dragged down by 3.3 per cent. This, in turn, leads to an increase of net worth by 4.6 per cent. Also, the lower cost of funding again drives investment higher by 1.3 per cent, which is more than double it was under commitment. The growth of output, consumption, and asset price comes after higher investment. However, the inflation bias still occurs with discretion behaviour as it varies dramatically around a steady-state. Luckily, the sacrifice of inflation rate variation is not insignificant intensity.

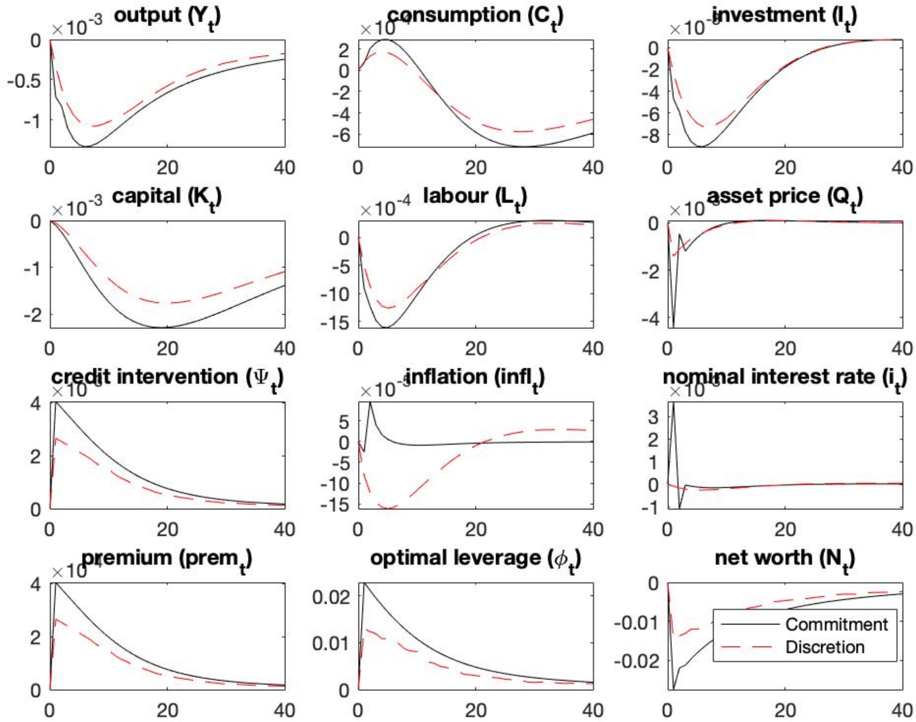

Figure 4.5: Impulse Responses in the FA_{CP} Model to Intermediary Net Worth Shock
Note: This figure shows the impulse responses to a net worth shock in the financial accelerator model with credit intervention under commitment (solid lines) and discretion (dashed lines).

4.5 Financial Accelerator (FA) Model under Commitment and Discretion

In this section, we apply the commitment and discretion scenario into the financial accelerator model, which is analysed in the GK model. Different from the previous part, there is no credit intervention; thus, this study of the financial frictions model can be compared to previous research. We then can identify the influences of credit intervention in the unconventional monetary policy. Three shocks will be discussed in the following.

4.5.1 Technology Shock

Figure (4.7) illustrates the responses of technology shock to the financial frictions model under optimal commitment and discretion. One per cent drop in technology shock would generate a reduction in investment by 3.7 per cent under commitment. The contraction of investment is a result of the rise in the premium, leverage ratio and a decline in the net worth of intermediaries. The unanticipated fall in investment decreases output, capital, and asset prices, which in turn generates a deterioration of banks' balance sheets. This increases the premium and leverage, diminishing borrowing capacity for firms, and further reduces aggregate demand, which generates lower output, consumption, and investment. The main responses of the technology shock under the optimal commitment policy is to produce leverage, net worth, investment, and output. Since the central bank is committed to the nominal interest rate instrument, it drops the nominal interest rate by approximately 1 per cent to observe the technology shock. This would reduce the leverage ratio and premium, in turn, increasing the investment and output. The central bank lifts the nominal interest rate after a short period once it observes the growth of the inflation rate.

When the central bank discrete from the nominal interest rate, the nominal interest rate is approximately around the zero steady-state. The responses are more volatile under the discretionary case. The leverage ratio increases approximately by 4 per cent, much higher than the 1.3 per cent under commitment. The investment also shrinks further to 4.98 per cent, which is a result of a 0.17 per cent increase in premium and a 5.5 per cent reduction in net worth. The inflation bias occurs when the inflation rate overshoots the zero steady-state and goes up to a higher level permanently.

4.5.2 Shock to Capital Quality

Figure (4.8) provides the plots of the responses for the capital quality shock under optimal commitment and discretionary policy. The overall movements of those variables are similar to the previous credit intervention model, except that the responses are slightly more volatile. For the commitment scenario, the leverage increases by 5.26 per cent in the financial accelerator model, while it grows 3.35 per cent in the credit intervention model. The net worth falls by 16.55 per cent, relative to 15.28 per cent, respectively. The biggest gap occurs in the premium, which goes up 0.2 per cent in this financial frictions model and 0.037 per cent in the previous model. For the discretionary case, the leverage ratio increases more than 13 per cent under the financial frictions model. The net worth drops by more than 20 per cent compared to the credit intervention model. The other dramatic difference is drawn in the investment, which decreases more than 10 per cent in the financial frictions model. In addition, without credit intervention, the impacts on the contraction under capital quality shock vanish much faster. For most variables, the influences of credit intervention diminish around period 20 and return quickly to the steady-state.

4.5.3 Net Worth Shock

The responses of net worth shock with the financial accelerator model is plotted in Figure (4.9). The movements of variables are similar as before, except that the model without credit intervention shows more volatile movements. Under the optimal commitment policy, the leverage ratio increases by 5.25 per cent in the financial frictions model and 2.29 per cent in the credit intervention model. The most significant difference is seen in invest-

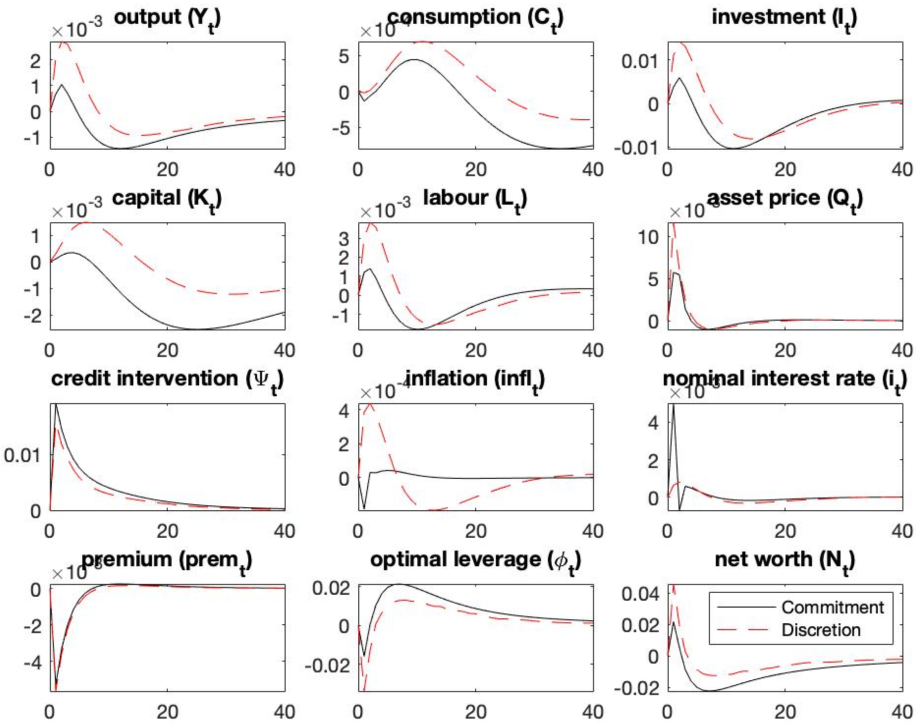

Figure 4.6: Impulse Responses in the FA_{CP} Model to Credit Policy Shock

Note: This figure shows the impulse responses to a credit policy shock in the financial accelerator model with credit intervention under commitment (solid lines) and discretion (dashed lines).

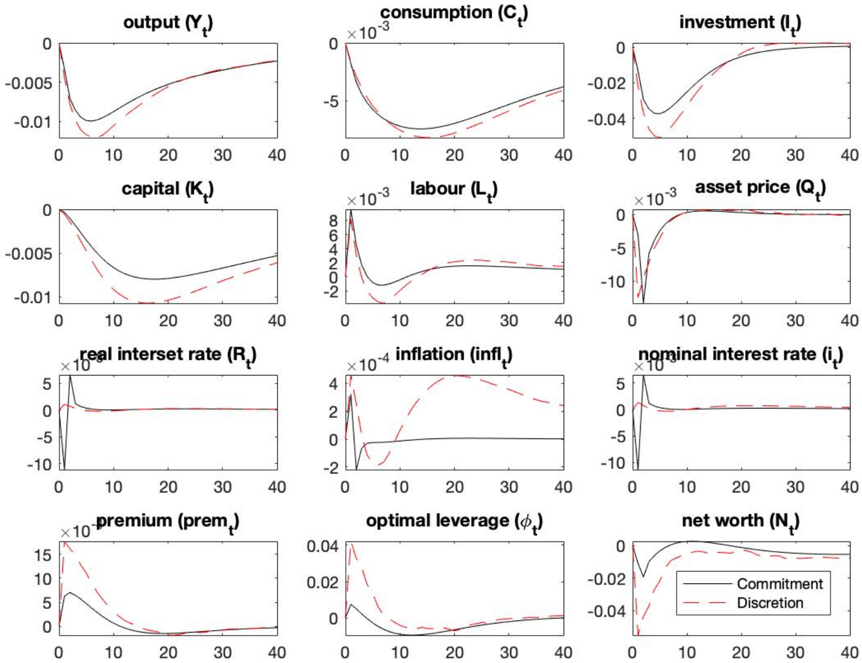

Figure 4.7: Impulse Responses in the FA Model to a negative technology shock
Note: This figure shows the impulse responses to a negative technology shock in the financial accelerator model under commitment (solid lines) and discretion (dashed lines).

ment, which drops 0.9 per cent in the credit intervention model and 2.3 per cent without the credit intermediation. Other variables have a small impact on the net worth shock in both models. Under the discretionary scenario, the gap between those two models is smaller than it is in the commitment case.

4.5.4 Assessing the Results of the Financial Accelerator Model

Table 4.6: Results under optimal monetary policy with Financial Accelerator Model

κ_π	κ_y	**Loss under commitment**	**Loss under discretion**	**Gain from commitment**	**Consumption Equivalent**
1.1995	0.0244	296.0766	367.2076	19.3708	8.4339

NOTES: This table shows the optimal interest rate rule parameters (κ_{pi} and κ_y), the value of the central bank loss under commitment (L_C) and discretion (L_D), the percentage gain from commitment (Ω) and the consumption equivalent ($\hat{CE}$)

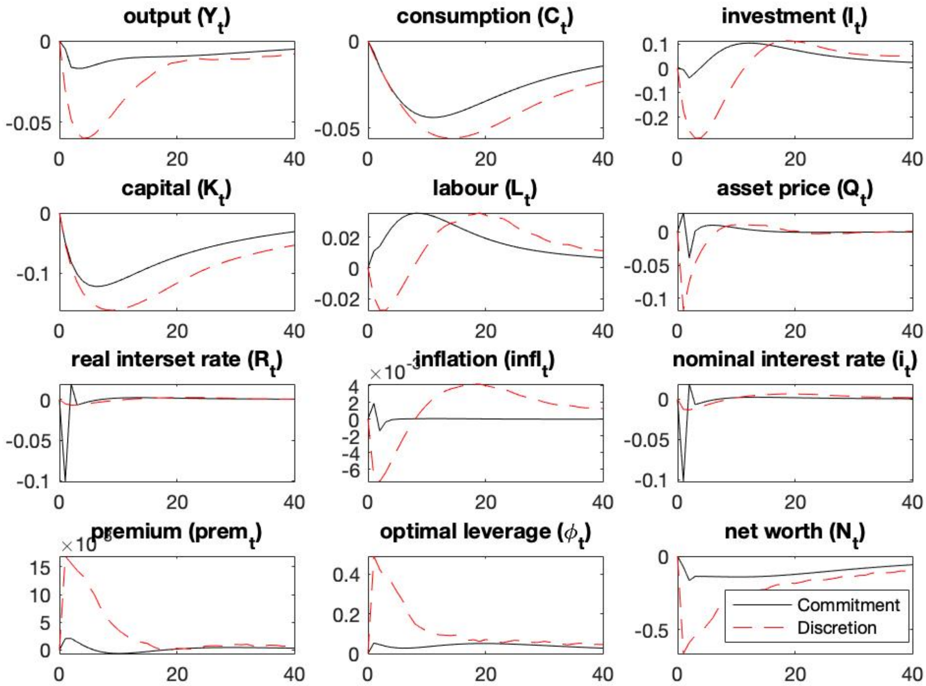

Figure 4.8: Impulse Responses in the FA Model to Capital Quality Shock
Note: This figure shows the impulse responses to a capital quality shock in the financial accelerator model under commitment (solid lines) and discretion (dashed lines).

In this part, we will discuss the welfare gain from the financial accelerator model. Table (4.6) provides the results of welfare gain under commitment and consumption equivalent. The best parameters using the interest rule are 1.1995 for the inflation rate and 0.0244 for the output gap. Same as before, the loss under commitment is still smaller than the loss under discretion. In addition, the loss and welfare gain under commitment in the financial accelerator model is smaller than it is in the previous model. However, the consumption equivalent shows by 8.4339, which is more stable than the previous model by 12.4973.

Table 4.7: Results under optimal monetary policy with different diverted fraction of capital

λ	**Loss under commitment**	**Loss under discretion**	**Gain from commitment**	**Consumption Equivalent**
0.15	293.9258	547.9779	46.3617	15.9390
0.2	294.4478	490.7053	39.9950	14.0092
0.25	294.9273	459.5828	35.8272	12.8318
0.3	295.379	440.8087	32.9916	12.0594
0.381	296.0766	422.9360	29.9949	11.2632

NOTES: This table shows the value of the central bank loss under commitment (L_C) and discretion (L_D), the percentage gain from commitment (Ω) and the consumption equivalent ($\hat{CE}$), for different parameters financial intermediaries' fraction of capital that can be diverted (λ).

When the diverted fraction of capital varies in the financial accelerator model, the results in Table (4.7) show a similar pattern with the FA_{CP} model. First, the loss under commitment raises as the fraction of capital level increases from 0.15 to 0.381. At the same time, the loss under discretion goes down as the fraction of capital lifts. Second, the gain from commitment and consumption equivalent also follows the decreasing pattern. Third, compared to the previous credit intervention model, the loss under commitment provides similar figures. However, the loss under discretion is much bigger in the credit intervention model. The gain from commitment varies from 46.3617 to 29.9949, which is smaller than the model before (65.4817 to 34.6098). The consumption equivalent illustrates a more stable result than before varying 15.9390 to 11.2632 compared to 23.6112 to 12.5186. Therefore, there is one trade-off between commitment and stabilisation of consumption when implementing the credit intervention policy.

Table 4.8: Results under optimal monetary policy with different survival rate of bankers

θ	**Loss under commitment**	**Loss under discretion**	**Gain from commitment**	**Consumption Equivalent**
0.7	312.3917	631.9192	50.5646	17.8753
0.8	308.1775	699.4026	55.9370	19.7794
0.85	305.8165	442.2415	30.8485	11.6801
0.9	303.0392	422.9360	28.3487	10.9497
0.9715	296.0766	367.2076	19.3708	8.4339

NOTES: This table shows the value of the central bank loss under commitment (L_C) and discretion (L_D), the percentage gain from commitment (Ω) and the consumption equivalent ($\hat{CE}$), for different parameters with the central bank intervention intensities (θ).

Table (4.8) gives the results under different levels of survival rate for bankers in the financial frictions model. For the loss under commitment, the result is more or less similar as before. However, the loss under discretion shows dramatic differences. In the financial accelerator model, the highest loss is 699.4026 when $\theta = 0.8$ and lowest loss is 367.2076 when $\theta = 0.9715$. In the previous model, the highest loss generated when $\theta = 0.9$ and lowest loss produces when $\theta = 0.8$. The loss without credit intervention is overall larger.

Moreover, there is also a trade-off between the gain of commitment and consumption equivalent in these two models. In the financial frictions model, the gain varies from 19.3708 to 55.9370, but the previous model varies from 8.7546 to 53.1102. The consumption equivalent, on the other hands, shows less stable results that vary from 8.4339 to 19.7794 in the financial accelerator model and 5.4171 to 18.5273 in the credit intervention model.

4.5.5 Results Analysis for the FA_{CP} and FA Model

In terms of the previous comparisons and impulse responses, we can summarise some fundamental points in the FA_{CP} and FA Model under commitment and discretion. First, the central bank benefits welfare gains around 24 per cent by committing to the optimal interest rate policy rule during the financial crisis period. Second, the variations of credit intervention intensities (v) and the divertable fraction of capital stock (λ) in un-

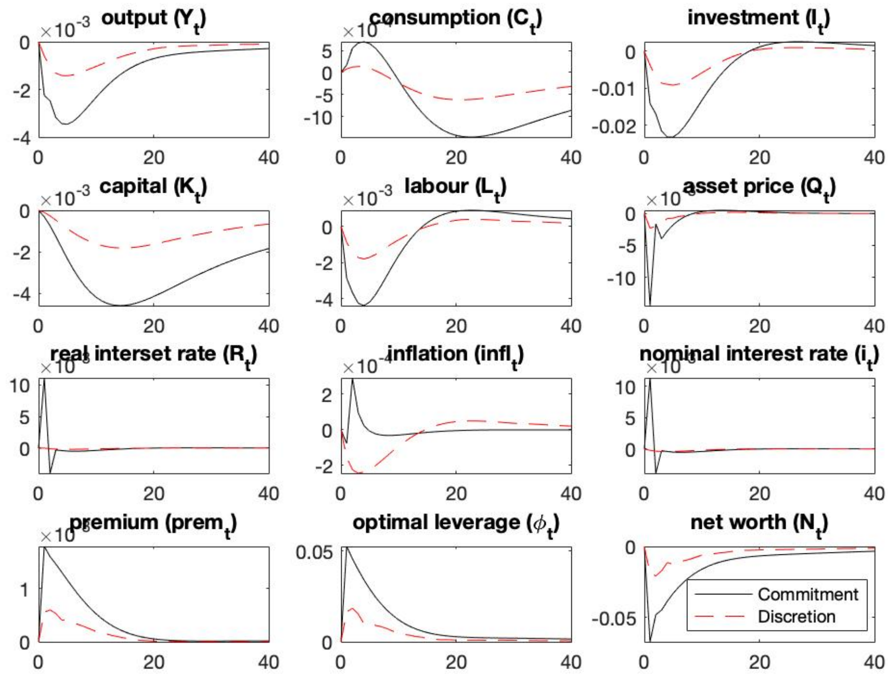

Figure 4.9: Impulse Responses in the FA Model to Net Worth shock
Note: This figure shows the impulse responses to a net worth shock in the financial accelerator model under commitment (solid lines) and discretion (dashed lines).

conventional monetary policy matters for welfare-based gains and consumption volatility. Higher credit interventions could lead to more stable consumption during the financial crisis period. However, the central bank is committed to keeping the intervention level at v=10 to achieve highest welfare gains. Besides, a higher leverage ratio leads to more consumption volatilities and lower welfare gains. The central bank has an incentive to keep a high divertable fraction of capital value, meaning a low leverage ratio, to achieve the most stable consumption. Third, we quantitatively confirm that the financial accelerator model with credit interventions have crucial effects on the economy to the capital quality shock. It is fundamental for central banks to commit to optimal policy rules to achieve stable consumption and higher welfare.

4.6 Conclusion

There has been a large amount of literature to study the unconventional monetary policy, either with DSGE models or with an interest rate lower bound. The common research has confirmed the importance of central banks to commit to a low-interest rate with other credit interventions. In this paper, we have quantified the welfare gain from the commitment to the unconventional monetary policy model. We implemented the simple Taylor rule to minimise the non-linear loss function. When central banks commit to the optimal interest rate instrument, credit intervention plays a fundamental rule in the mechanism. In a discretionary regime, central banks optimise in every period, taking as given the current state of the economy and private sector expectations. Since the public recognises that central banks would optimise each period, any promise that central banks make about future policy would not be credible.

For the financial accelerator model with credit policy, the capital quality shock shows more fundamental impacts for the whole transmission mechanism than the other shocks. The most affected variables are leverage ratio, net worth, investment, and asset price. Moreover, the intensity of the credit policy feedback parameter affects investment through leverage ratio, premium, and net worth. The highest gain from commitment was obtained when credit intervention is small. However, to have the most stable consumption equivalent requires a higher intensity of credit intervention around 50. If central banks can commit to a low level of intervention, then they can obtain the highest welfare gains. However, central banks might also deviate to a higher level of interventions to achieve more stable consumption. The different levels of a diverted fraction of capital show a decreasing pattern in the gain from commitment as the fraction increases from 0.15 to 0.38. In contrast, consumption equivalent provides the most stable consumption with the highest fraction of divertable capital value or low leverage ratio. For the survival rate of intermediaries, the highest gain from commitment was 0.9, but the most stable consumption equivalent was 0.85. From the results, we can see that central banks would always have the incentive to deviate from the single interest rate instrument to credit intervention, especially for the financial crisis period with a deterioration of balance sheet due to capital quality shock.

For the financial accelerator model, capital quality shock still plays a crucial role in the economy as it influences financial frictions, such as interest rate spread, leverage ratio, and net worth. Under a discretionary regime, the shocks produce fewer effects on investment. The diverted fraction of capital and survival rate of intermediaries show similar patterns in the gain from commitment and consumption is equivalent to the results with credit intervention policy. The main purpose of this chapter is to quantify the welfare gain under commitment for the unconventional monetary policy. Compared to the welfare gains of McAdam et al. (2007) to ours, the outcome shows that the welfare gains from commitment are significantly larger than McAdam et al. (2007)'s. The gains of consumption equivalent under commitment are approximately 10% higher in financial accelerator model with credit intervention than McAdam et al. (2007)'s model without financial frictions. The gains of consumption equivalent under commitment are also approximately 8% higher in our financial accelerator model without credit intervention than McAdam et al. (2007)'s model. Therefore, given the large gains from the model with financial frictions under commitment, policymakers have massive incentive to commit to keeping a good reputation.

Appendix

4.7 The Unconventional Monetary Policy Model

$$\rho = (C_t - hC_{t-1})^{-\sigma} - \beta h E_t (C_{t+1} - hC_t)^{-\sigma} \tag{4.12}$$

$$\beta E_t R \Lambda_{t,t+1} = 1 \tag{4.13}$$

$$\Lambda_{t,t+1} = \frac{\rho_{t+1}}{\rho_t} \tag{4.14}$$

$$\chi L_t^{\phi} = \rho P_{mt}(1-\alpha)\frac{Y_t}{U_t} \tag{4.15}$$

$$\nu_t = E_t\{(1-\theta)\beta\Lambda_{t+1}(R_{kt+1} - R_{t+1}) + \beta\Lambda_{t,t+1}\theta x_{t,t+1}\nu_{t,t+1}\} \tag{4.16}$$

$$\eta = E_t\{(1-\theta) + \beta\Lambda_{t,t+1}\theta z_{t,t+1}\eta_{t+1}\} \tag{4.17}$$

$$\phi_t = \frac{1}{1-\psi}\frac{\eta_t}{\lambda - \nu_t} \tag{4.18}$$

$$z_t = (R_{kt} - R_{t-1})(1-\psi_{t-1})\phi_{t-1} + R_{t-1} \tag{4.19}$$

$$x_t = \frac{\phi_t(1-\psi)}{\phi_{t-1}(1-\psi_{t-1}} z_t \tag{4.20}$$

$$Q_t K_t = \phi_t N_t \tag{4.21}$$

$$N_t = N_{et} + N_{nt} \tag{4.22}$$

$$N_{et}t = \theta z N_{t-1} e_{N_e} \tag{4.23}$$

$$N_{nt} = \xi_t \omega Q_t (1-\psi_{t-1})K_{t-1} \tag{4.24}$$

$$R_{kt} = \frac{(P_m \alpha \frac{Y_m}{K_{t-1}} + \xi_t Q_t - \delta)}{Q_{t-1}} \tag{4.25}$$

$$Y_m = A_t(\xi_t U_t K_{t-1})^{\alpha} L_t^{1-\alpha} \tag{4.26}$$

$$\begin{aligned} Q_t = 1 + \frac{\eta_i}{2}\left(\frac{I_{nt}+I_{ss}}{I_{nt-1}+I_{ss}} - 1\right)^2 + \eta_i\left(\frac{I_{nt}+I_{ss}}{I_{nt-1}+I_{ss}} - 1\right)\left(\frac{I_{nt}+I_{ss}}{I_{nt-1}+I_{ss}}\right) \\ -\beta\Lambda_{t,t+1}\eta_i\left(\frac{I_{nt+1}+I_{ss}}{I_{nt+1}+I_{ss}} - 1\right)\left(\frac{I_{nt+1}+I_{ss}}{I_{nt}+I_{ss}}\right)^2 \end{aligned} \tag{4.27}$$

$$\delta = \delta_c + \frac{b}{1+\zeta} U_t^{1+\zeta} \tag{4.28}$$

$$P_{mt}\alpha \frac{Y_{mt}}{U_t} = bU_t^{\zeta}\xi_t K_{t-1} \tag{4.29}$$

$$I_{nt} = I_t - \delta\xi_t K_{t-1} \tag{4.30}$$

$$K_t = \xi_t K_{t-1} + I_{nt} \tag{4.31}$$

$$G_t = G_{ss} g_t \tag{4.32}$$

$$Y_t = C_t + G_t + I_t + \frac{\eta_i}{2}(\frac{I_{nt} + I_{ss}}{I_{nt-1} + I_{ss}})^2 (I_{nt} + I_{ss}) + \tau\psi K \tag{4.33}$$

$$Y_{mt} = Y_t P_t \tag{4.34}$$

$$P_t = \gamma P_{t-1} \pi_{t-1}^{-\gamma_p \epsilon} \pi^{\epsilon} + (1-\gamma)(1 - \gamma \pi_{t-1}^{\gamma_p (1-\gamma)} \pi_t^{\frac{\gamma-1}{1-\gamma}}) \tag{4.35}$$

$$X_t = 1/P_{mt} \tag{4.36}$$

$$F_t = Y_t P_{mt} + \beta\gamma\Lambda t + 1) * \pi_{t+1}^{\epsilon} * (\pi_t)^(- \epsilon\gamma_P) F_{t+1}) \tag{4.37}$$

$$Z_t = Y_t + \beta\gamma\Lambda_{t+1})\pi_{t+1})^{\epsilon-1}\pi)^{(\gamma_P * (1-\epsilon))} * Z_{t+1} \tag{4.38}$$

$$\pi^* = \frac{\epsilon}{\epsilon - 1} \frac{F_t}{Z_t} \pi_t \tag{4.39}$$

$$\pi_t^{1-\epsilon} = \gamma \pi_{t-1}^{\gamma_p (1-\epsilon)} + (1-\gamma)(\pi^*)^{1-\epsilon} \tag{4.40}$$

$$i_t = R_t \pi_{t+1} \tag{4.41}$$

$$i_t = (i_{t-1})^{\rho_i} (\frac{1}{\beta} \pi^{\kappa_\pi} (\frac{X}{\epsilon/(\epsilon-1)})^{\kappa_y})^{1-\rho} \epsilon_i \tag{4.42}$$

$$\psi_t = \kappa(R_{kt+1} - R_t - R_{km} R_{ss}) + \epsilon_\psi \tag{4.43}$$

$$a_t = \rho_a a_{t-1} - e_a \tag{4.44}$$

$$\xi_t = \rho_\xi \xi_{t-1} - e_\xi \tag{4.45}$$

$$g_t = \rho_g g_{t-1} - e_g \tag{4.46}$$

$$\epsilon_{\psi_t} = \rho_{\epsilon_\psi} \epsilon_{\psi_{t-1}} + e_\psi \tag{4.47}$$

$$K_{eff,t} = \xi_t K_{t-1} \tag{4.48}$$

$$w_t = P_{mt}(1-\alpha)\frac{Y_t}{L_t} \tag{4.49}$$

$$MPK_t = P_{mt}\alpha\frac{Y_t}{\xi_t}K_{t-1} \tag{4.50}$$

$$W_{wel,t} = (C_t - hC_{t-1}) - \frac{\chi L_t^{1+\phi}}{1+\phi} + \beta W_{wel,t+1} \tag{4.51}$$

$$prem_t = \frac{R_{kt+1}}{R_t} \tag{4.52}$$

$$QK_t = \psi Q_t \frac{K_t}{4Y_t} \tag{4.53}$$

Chapter 5

Conclusion

This thesis attempts to answer three questions:
(1) What is the optimal choice for the central bank in alternative monetary policy instruments: interest rate pegging, or constant money supply?
(2) What happens if the macroprudential policy is model misspecified?
(3) What are the gains from the commitment under the optimal unconventional monetary policy?
To answer the above questions, we apply various DSGE models under different circumstances. In the first essay, the main contribution of our research is to apply an estimated Smets and Wouters (2003) model into Collard and Dellas (2005) to examine Poole's analysis. Through changing the utility function to keep the real balance, the welfare function is derived from comparing those alternative policy instruments. We utilize the quarterly data of the Euro area for the period between Q2 and Q3 in 2007; the results are appealing under different shocks.
The result shows that under a technology shock, the constant money supply instrument is preferable as stability in output and consumption as well as in higher welfare. Second, the preference shock prefers a monetary targeting procedure as it stabilizes output and generates higher consumption. Third, interest rate targeting fares better when intertemporal substitution is low under fiscal shock and an equity premium shock. Besides, interest rate targeting is also favoured under wage mark-up shock regarding the degree of risk aversion, and when intertemporal substitution is high under price mark-up shock. Fourth, under-investment shock, monetary targeting fares better when risk aversion is high and is ambiguous when risk aversion is low. Therefore, the ranking differs under various shocks and intertemporal substitution matters under some shocks.
Further, the second essay is aimed to apply Dennis et al. (2009) robust control method into macroprudential policy. Since the prolonged adverse financial crisis effects, central banks have carried out diverse macroprudential policies to buffer financial risks in the future. However, policymakers do not know if their models with

macroprudential is a true DGP or not. The robust control method by Dennis et al. (2009) can simplify the state-space form problem from the minimax approach and prior distribution issues in the Bayesian method. By applying the robust control method in the KRS model with the housing market boom, we find that macroprudential policy is a strong instrument to offset financial shocks. The robust macroprudential policymakers response more aggressive in the worst-case scenario than it in the rational expectations and the approximating case.

Finally, the third essay quantifies the welfare differences between commitment and discretion for the optimal unconventional monetary policy. We implement the simple Taylor rule to minimize the non-linear loss function. In the discretionary regime, central banks optimize its policy strategy in every period, taking as given the current state of the economy and private sector expectations. Since the public recognizes that central banks would optimize each period, any promises that central banks make about future policy would not be credible. The outcomes show that capital quality shock plays a more important role than other shocks for the financial accelerator model with credit intervention. Further, the intensity of the credit policy feedback parameter influences investment through leverage ratio, premium, and net worth. The highest gain from commitment is obtained when credit intervention is small. However, to have the most stable consumption equivalent requires a higher intensity of credit intervention around 50.

Even though there are some crucial findings that have carried out in the thesis, there are still some limitations. To begin with, money demand shock is not included in the first essay. This could be one important factor when the real money balance is kept in the DSGE model. Furthermore, the KRS model applied in the second essay is calibrated. Those parameters' value might be more practical through further estimation. Finally, the unconventional model in the third essay is also not estimated for empirical research. The welfare analysis might be worth to dig out more other approaches.

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