Mathematics Teachers' Appropriation of Digital and Non-Digital Resources and its Impact on Classroom Practices

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Dedication

Rev. Fr. Stephen Omojo Ogbe, OP
a brother and friend always

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Abstract

Mathematics teachers' appropriation of digital and non-digital resources for professional practices have become a research focus over the last decade. The aim of this doctoral research is to explore how mathematics teachers in English secondary schools access, adapt, create and use resources in-class and out-of-class, individually and collectively, from the practitioners' perspectives. This research also examines the genesis of a community of practice from teachers' collective work.

A qualitative case study approach and thematic data analysis were used. Data were collected through interviews, observations using the *Systematic Classroom Analysis Notation* (SCAN), screen capture and document collation. Seven teachers were purposively selected from three schools in England in which there exists a culture of resource use. Activity theoretic and 'documentational' approaches are frameworks used in exploring and discussing teachers' resource use and its impact on classroom practices.

The findings highlight the importance of digital and non-digital resources in teaching mathematics in England. They suggest that the widespread use of schemes of work, amongst other features, in England predisposes teachers to appropriate and use a variety of resources. The findings reveal that teachers undertake a range of formative assessments (FA) as an integral part to their teaching practices. The use of digital resources and the teachers' resources system enable these FA practices in a variety of ways. The research also indicates that the mathematics teachers' collectives exist in different forms. Indeed, teachers' collective participation occurs in a complex intersection of various loosely or tightly connected virtual and/or face-to-face networks with the potential to emerge into a mathematics teacher community of practice. The research found that teachers adopt the concepts of variation and differentiation and enact these ideas in distinctive ways.

This research has the potential to contribute to the discourse on mathematics teachers' work with resources from an English perspective. It also offers insights and recommendations that could benefit teachers' professional practices.

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List of Abbreviations

AfL- Assessment for Learning

AT- Activity Theory

CoP- Community of Practice

CPD- Continuous Professional Development

DAD- Documentational Approach to Didactics

DfE- Department for Education

GCSE- General Certificate of Secondary Education

ICMI- International Commission on Mathematical Instruction

ICT- Information Communication Technology

IWB -Interactive White Board

NCTM- National Council of Teachers of Mathematics

NRICH- Norwich, (The) Royal Institution, Cambridge (University) and Homerton (College)

OFSTED- The Office for Standards in Education, Children's Services and Skills

PISA- Programme for International Student Assessment.

QR- Quick Response Code

QTS- Qualified teacher status

SCAN-Systematic Classroom Analysis Notation

SIAM- National Society Statutory Inspection of Anglican and Methodist Schools

SKE- Subject Knowledge enhancement

SLG- Senior leadership Group

SSCE- Senior Secondary School Certificate Examination

STEM- Science, Technology, Engineering and Mathematics

TES- Times Education Supplement

TSST-Teacher Subject Specialist Training

VLE- Virtual Learning Environment

WAEC-West African Examination Council

Plickers- is a free, interactive tech tool that uses printable “paper clickers” instead of clicker devices.

CHAPTER 1

INTRODUCTION

In this chapter I present a general overview of this research. I began by setting the scene with regards to mathematics teachers' interactions with a wide range of resources and the consequences for their professional practices in the context of three schools in England. I also present the rationale and motives that influenced this research. This chapter also presents the purpose of the research and the research questions before, in the final subsection, I present a synopsis of the dissertation.

1.1 Setting the Scene for the Study

In the last decade, a field has emerged within mathematics education research dedicated to teachers' interactions and participation with resources from a practitioner's perspective (Gueudet, Pepin, & Trouche, 2012a, 2013a, 2012b; Gueudet & Trouche, 2009). This is a consequence of the widespread availability of the internet and a convergence between the fields of research on technology in mathematics education, on mathematics textbooks and on curriculum resources (Fan, Trouche, Qi, Rezat, & Visnovska, 2018).

The appropriation of technology, curricular and digital resources by mathematics teachers occurs in highly complex institutional and classroom contexts. The increasing variety of interconnected digital resources available to mathematics teachers and in the classroom, has upset traditional classroom practices and forms of task design with consequences for teachers' professional activity. There is also the emergence of new and multiple modes of using diverse resources in mathematics teaching and a shift away from relying on the use of a single textbook for teaching mathematics to a combination of several resources. Digitalisation and the changing roles of these varieties of technology and digital resources designed or adapted for mathematics teaching bring with them challenges, hiccups, and complexities and the processes of integration is arduously long (Clark-Wilson & Noss, 2015; Laborde, 2001; Monaghan, 2004). As Monaghan (2004, p. 352) observes, “The integration of technology into mathematics teachers’ classroom practice is a complex undertaking regardless of whether teachers find this an ‘easy’ transformation of practice or not”. The presence of technology and digital curricular resources changes the way mathematics teachers prepare lessons, the way they teach, the way they assess students and the way in which data are collected on students’ performance; teachers need to contend with the fast-changing context in their everyday practices.

The digitalisation of curricular and other mathematics teaching resources, in the context of England, seems to provide new opportunities for practicing teachers as well as presenting new problematic scenarios. In England, mathematics teachers have the freedom and responsibility to decide what teaching resources to select, adapt, create and use in their professional practices (Ruthven, 2013). The English context seems to present a devolved attitude in adapting, creating and sharing resources among teachers and teacher communities. A key influence on the school culture in the English context is the encouragement by education professionals and researchers, together with a government mandate, for teachers to collectively develop an organising document referred to as the ‘scheme of work’. Each scheme of work is a locally customised school-level interpretation and adaptation of the National Curriculum. It should be the vehicle through which the department’s vision, ethos and learning intentions are made clear to teachers. It will be developed through mutual discussion and sharing of ideas. It should ease the planning process for teachers, and it must support new, inexperienced and non-specialist staff. Such a scheme of work is never complete (Petty, 2009; Ruthven, 2016b).

In a sense the scheme of work is a ‘live’ document that is subject to ongoing revision, improvement and adaptation, especially in a period of transition into a new National Curriculum. As a result of this mandate and the support for taking up collective designing of the scheme of work, no particular resource is considered, within the context of secondary schools in England, as a sole basis for lesson preparation. Ruthven (2013) position on the scheme of work is further restated in Siedel and Stylianides (2018, pp. 126-127),

> the tradition of localized efforts in England represents the aims of the re-sourcing movement in that teachers are encouraged to use and develop a range of resources beyond conventional curriculum materials such as textbooks. However, although the perceived poor quality of textbooks in England made the scheme of work an attractive alternative, the quality of the locally developed schemes of work is highly variable.

In one of the key findings from the Office for Standards in Education, Children's Services and Skills (Ofsted), inspections of mathematics between April 2005 and December 2007 in 192 maintained schools in England reported that "Schemes of work in secondary schools were frequently poor, and were inadequate to support recently qualified and non-specialist teachers" (Ofsted, 2008, p. 6). In spite of the mixed nature of the evidence on the value of the scheme of work to mathematics teaching and what is observed in practice, its use is persistent in schools in England with official support. Hence, the Department for Education (DfE) recommends,

> All leaders have a key role in ensuring the availability of fully resourced collaboratively developed schemes of work. Once these are in place, and individual teachers understand the 'what' and 'why' of the curriculum, they can be freed to teach in a way that best suits their professional judgement and experience. (DfE, 2016, p. 5)

The cultural context of England reveals a very low reliance on the use of textbooks as basis for instruction (Mullis, Martin, Foy, & Arora, 2012), but there is an increased and early use of internet resources and multiple textbook alternatives (Siedel & Stylianides, 2018) inspired by what Ruthven (2016b, p. 76) termed the "'*re-sourcing movement'* - which has grown around the approaches in which teachers devise their own curriculum scheme through assembling, adapting and structuring material from a variety of sources". Now also, mathematics teachers in England have ever-growing access to a large range of resources not merely as users but as designers as well. This cultural context in England with regards to the mathematics teachers' liberty to select, adapt, create, draw from and use a vast range of resources for preparing lessons, for mathematics teaching and for assessment, serves as a good case study to better explore and understand teachers' interactions with resources from individual, collective and institutional perspectives. Hence, my study investigates the appropriation by mathematics teachers of resources and its impact on professional practices in selected schools in England. This qualitative case study examines the appropriation of technology, and curricular and digital resources by seven mathematics teachers for lesson preparation, delivery and assessment.

Teachers and researchers in England with long histories of engagement in the discourse on integration of digital technologies into mathematics teaching know all too well that the changing landscape requires a shift in emphasis towards a more holistic and multidisciplinary approach to unpacking teachers' practices with technology and the growing ubiquity of digital resources. Perhaps, the onus lies on mathematics education researchers and mathematics teachers to observe, analyse, identify, develop and recommend research-informed and theory-driven classroom practices with digital resources that will meet the associated challenges of appropriation. There is therefore an urgent need to research, understand and rethink teachers' appropriation of technology, and curricular and digital resources, its impacts on classroom practices and the implications for professional growth.

1.2 Personal Background and Motives for the Study

My passion for mathematics developed and was nurtured from an early age, 10, when I was selected for specialist training and mentoring for mathematically gifted children. Since then, mathematics and the sciences have always been fun: a source of excitement and fascination, pleasure and bewilderment. In my secondary school days in St. Kizito minor seminary, my class of 1995, under the tutelage of inspiring mathematics teachers, achieved a 100% national performance score in mathematics in the Senior School Certificate Examination (SSCE) conducted by the West African Examination Council (WAEC). These were the initial foundations of my love for mathematics and the vision that evolved into research in mathematics education with technology.

From 1996 to 2004 I undertook clerical training for the Catholic priesthood. During this period, I engaged in private tutoring and mentoring in mathematics alongside my studies in Philosophy (BA, University of Ibadan) and Theology (BTh, Pontifical Urban University). Providentially, after my ordination and eventual posting to my alma mater as Vice Rector, I resumed teaching mathematics voluntarily to the students during preps, break time and my spare time, though my official remit was to teach Christian Religious Knowledge. At the recommendation of the Rector and my late Bishop, I went back to the university to undertake a new undergraduate degree in

Mathematics and Computer Science culminating in a BSc (Ed) Mathematics from the University of Benin, Benin-City. During the study I investigated the factors responsible for students' poor performance in mathematics in the SSCE in the Idah local government area of Kogi State, Nigeria (Umameh, 2011). One of the findings suggests that the use of appropriate new technologies can impact on pedagogic practice and student learning. In the course of this study, I taught mathematics in several schools in Benin-City and Asaba in Nigeria. Upon completion of my study, I proceeded to the University of Bristol, UK, where I obtained an MSc Education, Technology and Society with a focus on teaching and learning mathematics education employing GeoGebra, under the guidance of Prof Rosamund Sutherland. My graduate research at the University of Bristol explored mathematics teachers' instrumental orchestration in the joint use of GeoGebra and the interactive white board (IWB) for teaching and learning straight-line graphs in the whole-class activity setting (Umameh, 2012).

It was through Prof Sutherland and the master's programme that I was introduced to Guy Brousseau's *Theory of Didactical Situations in Mathematics: Didactique des Mathématiques* (Brousseau, 1997), the metaphor and theory of *Instrumental Orchestration* (Trouche, 2004) and eventually to the *French Didactics of Mathematics*. Through these generous apprenticeships and my continuing interest in mathematics education, research into technologies and digital curricular resources has become a major focus with further support from my present supervisors and associates. These experiences and interactions are the major influences that have shaped my present research. In 2016 the School of Education, University of Leeds appointed me as the first education outreach fellow as part of the university's widening participation in raising the aspirations and attainment of people from backgrounds that are under-represented at university. This gave me an opportunity to interact, motivate and mentor young people as they make choices about their future in STEM education in selected schools in England. Recently, I undertook mathematics teacher subject-specialist training, which has enabled me to refresh my content knowledge, and deepen lesson planning and time management skills as well as my appreciation of the new National Curriculum in mathematics in the UK. This is a part of my initiation into the English education system where my research is located.

I place great importance on the integration of ICT tools and the mathematics teachers' appropriation of the growing varieties of new digital resources, with its potentials and affordances. I believe this integration and appropriation offer opportunities to improve mathematics teachers' practices in the areas of lesson planning, delivery, assessment and collection of data on students and thereby better enable students' performance. This is why it has become a focus of attention for my research. It is my firm belief that a research-informed and theory-driven appropriation of digital curricular resources by mathematics teachers could have a significant impact on their teaching practice and on students' learning. It is my hope that my research will add a new voice to the ongoing discourse in mathematics education research in the UK, in my home country, Nigeria, and globally. My ultimate career vision is to become a professor in mathematics education through a life-long dedication to research in the field.

I now present the purpose of my study and the four themed research questions.

1.3 The Purpose of the Study

The major aims of this study are as follows:

- To identify and analyse the processes of appropriation of digital resources by mathematics teachers
- To explore teachers' documentational and resource systems (These terms are later defined and discussed in subsection 2.2.3, p. 29.)
- To understand the implications of teachers' appropriation and documentations for their professional practices

Additional aims are also included:

- Contributing to the professional insights of the participating teachers
- Contributing to the discourse on teachers' appropriation of digital resources

I now present the research questions (RQ), which are grouped into themes and associated questions.

1.4 Research Questions

In this section, I present the research questions organised into four themes. The first theme relates to the mathematics teachers who are at the core of this research and define the activities which this research explores. The second and third themes involve associated research questions, which seek to examine the resources and tasks teachers engage with in their everyday professional practices. The fourth theme deals with the issues of mathematics teachers' collective participations through the available routes within in-school and out-of-school contexts. The various terms used in the research questions are defined in greater detail in the following subsections: resources in 2.1.2, p. 18; tasks in 2.1.3, p. 19; and collectives in 2.2.4, p. 33.

Theme 1. **The mathematics teachers**

RQ1.1 In what ways are mathematics teachers accessing, adapting and creating resources for classroom practices?

Theme 2. **The resources**

RQ 2.1 What resources do mathematics teachers access and use?

RQ 2.2 What constitutes the mathematics teachers' resource system?

Theme 3. **The tasks**

RQ 3.1 What tasks do mathematics teachers give to their students?

RQ 3.2 Where do these tasks come from?

RQ 3.3 Do they amend these tasks - if so, then how?

Theme 4. **The collectives**

RQ 4.1 What collectives do mathematics teachers participate in?

RQ 4.2 Which features of these collectives provide opportunities for the evolution of communities of mathematical teaching practice?

1.5 Overview of the Study

There are 11 chapters in this dissertation. Chapter 1 briefly sets the scene on the discourse of resources in the cultural context of mathematic teaching in three schools in England, my personal background with respect to mathematics, my purpose and motives, and the research questions and also presents an outline of the structure of the thesis.

Chapter 2 provides a review of relevant literature on the integration of technology and teacher appropriation of curricular and digital resources and the two theoretical perspectives that underpin this study. It takes into account the different perspectives of researchers, policymakers and practitioners regarding mathematics teachers' work with resources, especially in the context in England. The perspective of mathematics teachers' interactions with digital resources that the researcher subscribes to is also defined and presented.

Chapter 3 describes the research design and methodology. It provides the rationale for the use of qualitative case study, the choice of methods of data collection, and units of analysis and the method of analysis are elaborated. The researcher's role and reflexivity and credibility are also discussed along with the trustworthiness of the research and ethical issues.

Chapter 4 presents the datasets, processes of data management and analyses.

Chapter 5 prepares the reader for the case studies by presenting structuring themes drawn from the literature, research focus and initial data analysis.

In Chapters 6, 7 and 8 the case study descriptions of research findings are presented in the form of the embedded case studies of seven teachers drawn from three schools. The presentation is focused around the themes of school context and approach to mathematics teaching and learning. The individual cases are discussed around the mathematics teachers' profiles, roles, tasks, resources and participation in face-to-face and/or virtual collectives.

Chapter 9 presents a discussion by revisiting the research questions in the light of the emerging issues in the cases presented and as they stand in relation to the theoretical and research literatures.

Chapter 10 brings together the whole of the doctoral research, highlights three key findings among others and the contributions of the research to the field of mathematics education research and teacher practices. The three key findings discussed in Chapter 10 include the following: firstly, the range of formative assessment practices among the teachers and how digital resources support and enable innovative practices with formative assessment; secondly, the five ways in which teachers participate in collectives together with the geneses of these collectives becoming a communities practice; and finally, the findings regarding teachers' differing perceptions on variation and differentiation in relation to mathematics tasks.

Chapter 11 brings the thesis to a conclusion. The implication of the research for mathematics teachers' professional practices and mathematics education research are emphasised. The limitations of this research are identified, and issues are proposed for further research. A closing autobiographical reflection is presented.

CHAPTER 2
LITERATURE REVIEW AND THEORETICAL FRAMEWORKS

This chapter is divided into two broad parts. The first part, section 2.1, examines the literature on mathematics teachers' use of technology and digital resources. Here, I review relevant literatures on mathematics teachers' appropriation of technologies, digital tools and a range of resources for mathematics teaching. Firstly, I explore the mathematics teachers' professional activity against a broad spectrum of technologies and resources, the challenges, opportunities and impact of these interactions on the classroom practices. Secondly, I also consider some of the milestones and perspectives within the mathematics education research community and the priorities for future research. Thirdly, I evaluate mathematics tasks and the challenges teachers face in selecting and using technology-mediated tasks in the classroom.

The second part, section 2.2, introduces the conceptual framework that will be developed. I combine the activity theoretic approaches (AT) and the documentational approach to didactics (DAD) as tools for guiding the data collection and analysis and as models for interpreting and addressing the research questions. Finally, I examine the collective aspects of mathematics teachers' engagement with resources for their professional practices and how the collective emerges into a community of mathematical practice.

In the UK, there is already a considerable presence of digital technologies (Hinds, 2019) and resources to support[1] mathematics teaching and learning[2]. In the three schools used in my research, all the mathematics teachers and their classrooms have access to laptops, desktops or a trolley of iPads, data projectors and internet connectivity. Also available to most teachers are the interactive whiteboards (IWB) and virtual learning environment platforms (VLEs)[3]. This use of e-textbooks, videos,

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1 https://www.prometheanworld.com/gb/news-article-gb/promethean-tech-report-2019/
2 https://www.besa.org.uk/insights/technology-supporting-education-2018/
3 https://www.besa.org.uk/news/rise-rise-digital-technology-classroom/

and a variety of software and mathematics assessment tools is on the increase (Pepin, Gueudet, Yerushalmy, Trouche, & Chazan, 2016). However, despite government initiatives and investments, it has been reported that dynamic digital technologies are underutilised and there is a digital skills crisis[4] impacting on students and teachers in secondary mathematics classrooms in England at scale (Bretscher, 2014; Ofsted, 2012). The digital skills crisis includes not only shortages of key digital skills in the economy and across the school curriculum but also a shortage of qualified and confident ICT teachers. Hence, my starting point derives from this concern and an understanding of mathematics teachers as professionals with an ever-evolving context of practice saturated by a growing number of tools and the varieties of new digital resources (into mathematics classroom teaching and learning) with its potentials and complexities that have been investigated and recognised (Artigue, 2002; Clark-Wilson & Hoyles, 2017; Clark-Wilson & Noss, 2015; Hoyles & Lagrange, 2010; Hoyles, Noss, & Kent, 2004; Laborde, 2001; Monaghan, 2001, 2004; Monaghan, Trouche, & Borwein, 2016). In one study Clark-Wilson and Noss (2015, p. 95) argued that,

> the technology-enhanced classroom provides opportunities for additional student actions, such as the manipulation of on-screen objects and the ability to make a range of mathematical inputs, which places an additional demand on teachers as they strive to make sense of a diversity of student activity in real-time.

Most of the initial research on digital technologies had focused on either the teacher(s) and individual student(s)' specific ICT tools, on teachers' and students' beliefs and attitudes, or on institutional and political factors impacting integration. For instance, Clark-Wilson, Oldknow, and Sutherland (2011, p. 5) reported from their findings that

> The vast majority of young people are involved in creative production with digital technologies in their everyday lives, from uploading and editing photos to building and maintaining websites. They acquire many skills which will be relevant in their careers, but which are not drawn on during their time in school. They acquire new skills rapidly and share their knowledge with their peers – but rarely in an educational context.

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4 https://publications.parliament.uk/pa/cm201617/cmselect/cmsctech/270/27006.htm#footnote-062

One aspect of the context is the types of task design that would be a ‘good fit’ for technology integration. Task here refers to a particular piece of work a teacher has given the learner to complete; they are devices for initiating activity and for establishing a meeting place for teachers and learners (Johnston-Wilder & Mason, 2004). Several studies have been devoted to exploring the issues surrounding task design in mathematics education (Clark-Wilson & Timotheus, 2013; Fuglestad, 2007; Margolinas, 2013). Over time the emphasis has shifted from isolating specific components within the context of teaching and learning to a more systematic and holistic approach. Monaghan in his analysis advocates an holistic approach to the complex whole of teachers’ activities with technology: “digital technology is only one of the tools that a teacher may use and, when used, it does not act in isolation but impacts on, and is impacted on by, the use of other tools” (2004, p. 348). My research construct is in accord with this position and takes it further by investigating teacher collectives[5] (Gueudet et al., 2013a) and their potential to evolve into communities of practice (Wenger, 1998).

Monaghan (2001) investigated the difference in teachers’ interactions with learners in an ICT-based classroom compared with those in a non-ICT based classroom and the analysis revealed no significant changes. The 13 secondary mathematics teachers involved in his research had a range of ICT to choose from, yet most of the lessons were significantly teacher-led and the teaching style suggested the normal classroom technique was simply imported into the ICT-based format. In a similar study illustrating the processes of integrating Cabri Geometry, Laborde (2001) observed that the key factors at the heart of Cabri integration and interactions are teachers’ experiences and beliefs, the context of dynamic Cabri environment that affords continuous modification of solution strategies, and time. Laborde’s finding suggests that a more experienced teacher given sufficient time with technology could facilitate interaction and enhance the processes of integration. While most of the research on the integration of technology focused on high school mathematics teachers, Barnes and Sutherland (2007) focused on the use of spreadsheets in primary and secondary schools, identifying the potential of different tools for different purposes and understanding this is an important aspect of being resourceful. Several researchers

[5] The collectives are explored in subsequent subsection of this chapter.

have focused on various dimensions in the integration of various digital technologies (Drijvers, Tacoma, Besamusca, Doorman, & Boon, 2013; Hohenwarter, Preiner, & Yi, 2007; Umameh, 2012). The potential of technology to enable the implementation of interactive teaching has also been explored (Kennewell, Tanner, Jones, & Beauchamp, 2008). They found that although the software used in this lesson was highly interactive, it did not provide the type of constraints which would structure students' actions and reflection-in-action in a way that could lead to achieving the learning objectives. Sutherland et al. (2004) argued that such interactive teaching and learning are possible over time when ICT is embedded into everyday classroom practices towards harnessing the transforming potential of technology.

I turn to explore the "three grand challenges" (Joubert, 2013) used in an attempt to understand the issues around the integration of technology into the mathematics education research community.

2.1 Identifying the Grand Challenges

Joubert (2013) in her study developed three "grand challenges" as a framework for mapping the landscape of issues, interests and concerns of the mathematics education community with regards to the use of computers and other digital technologies in mathematics teaching and learning. These challenges are *connecting learners* (exploring diverse issues and questions that arise from the growing connectedness of learners); *orchestrating learning* (characterising the opportunities and challenges for teachers when technology is introduced into their classrooms); and *contextualising learning* (focusing on how the affordance and use of technology offers new and different contexts for teaching and learning). Each framework has the potential to reveal something new and interesting, but we should also be aware of the limits of what a framework may capture.

Even though it is generally believed that digital technologies have the potential to contribute significantly to the teaching and learning of mathematics, the overall uptake of these technologies is slow, complex, and disappointing and the grand visions of the early 1980s are not being achieved (Sutherland, 2007). For instance, a study for the Joint Mathematics Council in the UK (Clark-Wilson et al., 2011) reported that “technology within mathematics is underused and, where it is used, its potential is generally underexploited” (p. 6). In a related study Ruthven (2016a) identified three challenges as the reasons for the limited progress to date in integrating digital computational tools into school mathematics. The *ecological challenge* is the difficulty faced in adopting tools within the constraints of time, space and infrastructure of the everyday classroom practice of school mathematics. The *epistemological challenge* refers to the disciplinary and didactic knowledge needed to enable the adaptation of digital technologies into mathematics teaching and learning. Finally, the *existential challenge* comes to the fore in an effort to understand how the individual and collective representations, values and identities mediate the use (or non-use) of digital tools. One previous study also indicated that the integration of digital technologies into mathematics education has been marginal (Artigue, 2000). Even though the potential and affordances of digital technologies to transform teaching and learning have been highlighted, research evidence from different countries suggests that digital technologies will continue to play a marginal role in mathematics teaching and learning unless a range of certain factors are more thoroughly studied, such as providing professional development opportunities focused on teachers’ technology use, the impact of technology on students’ achievements, and the task design when technology is used.

For example, the study of Nigerian secondary school teachers by Tella, Tella, Toyobo, Adika, and Adeyinka (2007) identified a lack of technical support in the schools, and teachers’ lack of ICT expertise as the prominent factors hindering teachers’ readiness and confidence in using ICT. Some of the factors identified include the design of the technology, task design and lessons; the role of the teacher and continuous professional development; and the educational context with a broader view to include the out-of-school settings. The failure to adopt a more integrative theoretical framework and theorise the complexity of technology integration is another underlying factor that may hinder or promote the successful integration of digital technology into mathematics education. The lack of a unifying theoretical framework leaves the knowledge in the field fragmented and insular. In spite of these challenges, various research documents have started to generate a better understanding of these issues and, equally, show the potential to develop new knowledge that could inform and help to tackle these challenges (Drijvers, 2015; Drijvers, Doorman, Boon, Reed, & Gravemeijer, 2010; Hegedus et al., 2017; Hoyles et al., 2004).

Researchers and teachers continue to believe that in addressing the aforementioned factors affecting technology integration, the use of digital technologies in mathematics teachers' professional practices could deepen. It is hoped that an effective appropriation of digital technologies could contribute positively to the teaching and learning of mathematics (Chao, Chen, Star, & Dede, 2016).

2.1.1 From Optimistic Rhetoric to Research Evidence

Notwithstanding the great enthusiasm of recent decades, there has been continued disappointment regarding the limited impact and the lack of widespread uptake of technology and digital resources in mathematics teaching and learning (Trgalová, Clark-Wilson, & Weigand, 2018). Many questions remain unanswered.

In a survey concerning teachers' use of technology in secondary mathematics classrooms, Goos and Bennison (2008, p. 103) argue that

> Despite the early optimism for the future of technology integration in mathematics education, research in many countries has found that technology still plays a marginal role in mathematics classrooms and that educational policies, access to technology resources, and institutional support are insufficient conditions for ensuring effective integration of technology into teachers' everyday practice.

The appropriation of digital technologies and resources remains a contentious issue for many teachers, researchers and designers at the point of implementation (Sakonidis & Potari, 2014). Attitudes and beliefs as to whether, when, how and why it should be integrated into mathematics teaching and learning vary greatly and are highly nuanced. Over time the optimistic rhetoric of the past decades has gradually yielded ground to research-based evidence. In research there has been a shift towards developing a variety of practical and theoretical methods to further understanding of how and why teachers and students do, and do not, use digital technologies in mathematics, with a major focus on the role of the teacher. Laborde and Sträßer (2010, p. 130) reported that "the ICMI community has fully embraced the teacher as the most important actor in the introduction and use of technology, while the student learner has not been taken into account in the same way as the teacher". The critical role of the mathematics teacher in the complex undertaking of integrating digital technology into teaching practices has been acknowledged. Using Saxe's four parameter model (activity structures; social interactions; prior understandings; and conventions and artefacts) which is centred on emergent goals, Monaghan (2004) highlighted the crucial but complex role of the teacher in a technology-based classroom.

My research focus aligns with the above perspective and takes it further by investigating professional practices of mathematics teachers with digital resources/technologies and their impact in the context of English secondary schools. It is necessary to explore English mathematics teachers' perspectives on the appropriation of digital resources and to deepen the theoretical models in order to better understand and address the challenges and opportunities entailed in the whole weaving of teachers' professional practice together with digital technology practice in the English context. The exploration of the English mathematics teachers' work with digital resources using the *documentational approach to didactics* as a theoretical lens is still in its infancy. It is my hope that this research will contribute to deepening the understanding of how appropriation of digital resources takes place, how digitalisations extend the possibilities of collective activities among the teachers, and how these interactions shape the professional practices around the appropriation by mathematics teachers of digital resources for teaching. It is intended that this will be one of my major contribution to the literature.

Several reviews of the literature suggest that mathematics education researchers, policy makers, curriculum developers and mathematics teachers are developing coherent ideas about some issues in particular: what the challenges and opportunities are; what works or does not; and developing grand priorities for research into the appropriation of digital technology for mathematics classroom practices.

For instance:

- The 17th International Commission on Mathematical Instruction (ICMI) Study book, *Mathematics Education and Technology - Rethinking the Terrain* (Hoyles & Lagrange, 2010)

- The technology-related sections of the *Third International Handbook of Mathematics Education* (Clements, Bishop, Keitel-Kreidt, Kilpatrick, & Leung, 2012)

- *From text to 'Lived' Resources: Mathematics Curriculum Materials and Teacher Development* (Drijvers et al.; Gueudet et al., 2012b);

- *The International Journal of Technology in Mathematics Education* [19(4), 20(1) 2013]; *The Mathematics Teacher in the Digital Era* (Clark-Wilson, Robutti, & Sinclair, 2014)

- *Tools and Mathematics: Instruments for learning* (Monaghan et al., 2016)

- *Research on Mathematics Textbooks and Teachers' Resources* (Fan et al., 2018)

- *Emergent Practices and Material Conditions in Learning and Teaching with Technologies* (Pargman & Jahnke, 2019)

- *The 'Resource' Approach to Mathematics Education* (Trouche, Gueudet, & Pepin, 2019)

These studies taken together present remarkably similar emerging landscapes (for instance, a focus on digital resources and proliferation of technologies in the classroom) and overviews of current professional practices in teachers' uses of digital technologies in school and out of school. They also explore the possibilities for developing an understanding of more effective classroom practices, drawing on a wide range of research perspectives. These milestones in gathering together diverse studies and ongoing research efforts in understanding the opportunities and the challenges of integrating digital technologies into mathematics teaching are where my research finds its roots.

My present research efforts, therefore, in understanding the mathematics teachers' appropriation of digital technology sit with this focus on rethinking the theoretical, methodological and practitioner approaches in mathematics education research.

My review of the literature on the integration of digital technologies has identified some key factors that could hinder or promote a seamless, routine and effective appropriation of digital technologies into mathematics teaching. The mathematics teacher is challenged with an ever-growing number of different digital resources, challenged with appropriating and integrating these resources into mathematics classroom activity, and this in turn challenges the teacher's existing professional practices with digital resources. This is in line with a recent finding that "three factors emerge as decisive and crucial in the integration of digital technology: the design, the role of the teacher, and the educational context" (Drijvers, 2015). As a consequence, the appropriation of digital resources/technologies by mathematics teachers for professional activity in the nested contexts of practice in selected schools in the UK is the central focus of my research.

I now present the conceptualisation of resources in this research.

2.1.2 Digital Resources

The literature on the importance and relevance of the use of curricular and digital resources in mathematics teaching and learning has matured since the beginning of the millennium. The terms digital resources and technology have a wide range of applications and interpretations, though my understanding of resources aligns with Adler's reconceptualization of resources: "It is possible to think about resource as the verb re-source, to source again or differently" (2000, p. 7). Within educational settings, there are varying definitions of the tools for teaching: for instance, "curricular resources" (Stylianides, 2016); "curriculum material" (Remillard, 2005); and digital curriculum resources (Pepin, Choppin, Ruthven, & Sinclair, 2017). Taken together these include all the materials (digital or physical) that teachers appropriate in and for their teaching, with textbooks being the most dominant resource internationally. In the context of mathematics teaching and learning, Pepin, Gueudet, and Trouche (2013) define "mathematics teaching resources as all the resources which are developed and used by teachers (and pupils) in their interaction with mathematics in/for teaching and learning, inside and outside the classroom" (p. 929). In another publication, Monaghan et al. (2016) document the major milestones in the studies on teachers' integration of digital technology and the ongoing research efforts focusing on the appropriation of curricular and digital resources in the context of practice. I appropriate the above views in this research and consider mathematics teaching resources as including:

- Text resources, such as curriculum materials: mathematic textbooks, teacher curricular guides, teachers' and students' worksheets, spreadsheets, posters and syllabi

- ICT resources, such hardware and software: laptops, iPads, applets, e-textbooks, games, GeoGebra, blogs and learning platforms
- Digital curriculum resources
- Social resources (conversations, tweets, post on web/forum)
- Cognitive resources (like mathematical notation systems, symbols, formulae, bar models and charts)
- Other material resources, such as students' handheld white boards, manipulatives and tracing paper

The above suggest that the construct *resource* is understood in the context of mathematics teaching and learning as everything that supports and facilitates teachers' practices, but the practice takes place within a context and within a community that need to be considered in order to account for the actual use (and variations in use) of resources by teachers and students in the secondary school context in England. In the preparation for teaching, teachers select, use, combine and modify, bookmark and save a variety of resources over time into a structured and functional set of teacher's resources; this is referred to as the *teacher's resource system* (Ruthven, 2018). The teacher's resource system refers to the material resources – and different types of resource – in use in the classroom and to the ways in which their use – individually and collectively – is organised and made functional (Trouche, Gueudet, & Pepin, 2018). For Ruthven (2009) digital resources structure teachers' planning and classroom practices. He identifies five key features in the structuring process: *working environment, resource system, activity format, curriculum script* and *time economy*. My research takes this milieu into consideration, within which teachers' practices with mathematics teaching resources takes place. The milieu involving preparing to teach often includes people, and I consider people (face-to-face and online) as resources when they support teachers' practices.

2.1.3 Tasks

In my conception, a task is an element of an instructional sequence of activities. A tool/device for introducing, developing, practicing, consolidating, connecting and assessing specific learning progression in the mathematics curriculum. The *Professional Standards for Teaching Mathematics* (NCTM, 1991) state that it is the "central responsibility of teachers … to select and develop worthwhile tasks and materials that create opportunities for students to develop … mathematical understandings, competence, interests and dispositions" (p. 24). This document discusses professional mathematics teaching on the basis of the following expectations: mathematics teachers are crucial in changing the way mathematics is taught, learned and assessed in schools; and whatever proposed changes may require, mathematics teachers have sustained long-term support and adequate provision of resources. Such provision of support and resources could create a go-to pool of curriculum resources with a variety of tasks that could be used to cultivate different types of mathematical skills, strategies and thinking and to extend their application into new situations and contexts. One specific task could therefore assume a different conceptualisation based on students' needs, the teacher's intention and the point of use in the cycle of students' learning.

The ICMI study 22, *Task Design in Mathematics Education* (Watson et al., 2015) and a section on tasks in *Mathematics in the Digital Era* (Leung & Baccaglini-Frank, 2016) present the state-of-the-art literature on task and task design in mathematics education. Mathematical tasks play a central role in teaching. The type of task, the way it is crafted and used, has significant impact on student learning (Sullivan, Clarke, & Clarke, 2010). The ICMI Study 22 defines tasks as "anything that a teacher uses to demonstrate mathematics, to pursue interactively with students, or to ask students to do something" (Watson et al., 2015, p. 9). Tasks also refer to a wider range of 'things to do', including repetitive exercises, construction, exemplifying definitions, solving single-stage and multi-stage problems, deciding between two possibilities, or carrying out an experiment or investigation. Tasks are opportunities for students to act and learn. It has been argued that tasks and the associated classroom activities form the basis of interaction and create the 'meeting place' for initiating learners into an appropriate spectrum of mathematical activity through teachers' goal-directed actions (Christiansen & Walther, 1986). For Johnston-Wilder and Mason (2004), tasks are devices for initiating mathematically fruitful activity and opening up the dimensions of possible variations. The 'activity' here includes what the learner actually does, interaction with other learners, interaction with other resources and interaction with the teacher. John Monaghan and Luc. Trouche (2016) state that "Tasks refer to what the teachers plan and design for triggering and supporting learners' activity" (p. 391). Although there is no consensus on what the content of the activity, the device or the trigger could be, there is agreement on its aim: leaners' goal-directed activity. It is debatable whether this activity/interaction is meaningful or fruitful or whether the teacher's goal is eventually realised. In a study analysing the use of interactive technology to implement interactive teaching, Kennewell et al. (2008, p. 65) argued that a teacher's role extends beyond managing the set task:

> a teacher is not merely a manager of the activity which takes place. Their role can be seen as orchestrating the features so as to ensure that the activity proceeds fruitfully towards achievement of the planned learning objectives as well as completion of the task itself.

One key issue faced by mathematics teachers is the design of tasks. However, ordinarily, the tasks are considered as a *given* – in textbooks, worksheets or in resource banks – and more often integrated within the notion of the tool or classroom activity. As Monaghan (2004, p. 335) recorded in his observation of a non-technology lesson, "from the point of view of teachers, textbook tasks are 'safe' tasks, i.e., they are part of their established practice and teachers' activities around these tasks are justified through conforming to practice". But the mere presence of digital technologies and resources can completely change the dynamics of the classroom. For instance, YouTube videos could supplement teacher activities; e-assessment and facility for immediate feedback could alter what a teacher focuses on during a lesson; and animation could be employed to enable teacher's explanations to go into greater depth. Hence, in the context of technology-mediated lessons, design decisions and the mechanics of orchestrating the classroom features, the role of the mathematics teacher as task designer, is significant and pivotal.

As part of the complexity of teachers' practices with technology, we need to rethink what it means to be a task designer. Trgalová et al. (2018, p. 149) belief that "The term task designer has always held a broad definition to include: teachers; researchers; teacher educators; and technology developers", and as a result this expanded conception of the designer enriches the perspectives. Teachers transform curriculum ideals and lesson plans through mathematical tasks into real classroom activity. How and why a teacher chooses particular tasks or modifies them and how such technology-mediated tasks influence classroom activity are central to understanding teachers' professional engagement with technology/digital resources. Although this issue has been explored in the literature within a multi-representational technological environment (Clark-Wilson & Timotheus, 2013), my research looks at teachers as task designers in a context where there are a growing variety of different digital resources.

In the next subsection, I explore the theoretical framework underpinning this research.

2.2 Theoretical Frameworks

In this research, I combine an activity theoretic approach (AT); (Vygotsky, 1978) with the more recent 'documentational approach' (DAD); (Gueudet & Trouche, 2009) from the French didactics as theoretical frameworks. The online *Encarta World English Dictionary* defines framework "as a set of ideas, principles, agreements, or rules that provides the basis or the outline for something that is more fully developed at a later stage" (Soukhanov, 1999). This entails basic conventional assumptions, concepts, principles, and practices that organise a way of viewing reality. It provides the basis and justification for framing the research questions, data collection and analysis in my research. I draw on these approaches as an enabling lens for developing an understanding of teachers' appropriation and use of digital resources and for building up a coherent explanation for their impacts on classroom practices.

A body of well documented literature exists on the various theoretical perspectives and varied dimensions of use: theories with roots outside the field of mathematics education research adopted with particular attention to the social, political and cultural dimensions of mathematics teaching and learning (Jablonka, Wagner, & Walshaw, 2013); theories of and in mathematics education (Sriraman & English, 2010; Sriraman & Nardi, 2013); and theories dedicated to the exploration of issues on the integration of digital technologies and resources into mathematics education (Drijvers, 2011). Given the overwhelming arrays of theoretical perspectives, one wonders what theory or combination of constructs could be appropriate for one's context of research? What has a specific theory to offer? And what evidence exists in literature and research that could aid an informed choice?

The rationale and justification for using a theoretical framework align with the ideas of Hiebert and Grouws (2007, p. 373):

> Theories are useful because they direct researchers' attention to particular relationships, provide meaning for the phenomena being studied, rate the relative importance of the research questions being asked, and place findings from individual studies within a larger context. Theories suggest where to look when formulating the next research questions and provide an organizational scheme, or a story line, within which to accumulate and fit together individual sets of results.

Some of the considerations mentioned above are major motivations for selecting activity theory (AT) and the documentational approach to didactics (DAD; these are defined subsequently) as tools for illuminating the context of my research. On one hand, activity theory is a framework or an umbrella term for a line of eclectic social sciences theories and research. It considers an entire work/activity system beyond the single actor or user. It accounts for environment, personal history, culture, the role of the artefact, motivations, and the complexity of real-life activity. It bridges the gap between the individual subject and the social reality, through mediating activity (Nardi, 1996; Núñez, 2009). The increasing interest in and use of activity theory in education and especially mathematics education has been acknowledged and reported (Roth, 2014). On the other hand, the documentational approach developed in the context of mathematics education research is aimed at investigating teachers' work and professional learning through the lens of their interactions with the resources used in and for teaching. This approach is based on a dialectic between what a teacher is working on – the resources – and what a teacher is producing– a document, a mixed entity composed of organised resources and a pattern for their usage (Gueudet & Trouche, 2009; Trouche, Gitirana, Miyakawa, Pepin, & Wang, 2019). Others include my understanding of the role of theories in research, the conceptions on how the context of teachers' appropriation of digital resources could be analysed and understood, and my implicit epistemic assumptions.

AT brings specific insight and offers a variety of interconnected conceptual tools that can be employed to deepen the understanding of artefact-mediated human activity. DAD is compatible, sharing some common conceptual roots with AT, and has particular orientation towards studying mathematics teachers' resource-mediated and goal-directed activity in their social contexts. In combination, the frameworks enlarge the sets of conceptual tools used to develop the understanding and interpretation of mathematics teachers' appropriation of digital and curricular resources, especially in the complex socio-cultural contexts of professional practice (Gueudet et al., 2012a).

In the literature, the use of theoretical perspectives in tandem has been referred to as networking. In a paper by Kidron, Bosch, Monaghan, and Palmér (2018, p. 257), they argue that

> Motivation for networking[6] resulted from the fact that learning, especially in classrooms, is a complex phenomenon. Different theoretical approaches have, to some extent, arisen from a dominant focus on one or more aspects of this complex phenomenon.

There are great opportunities for deeper understanding and new ways of exploring, describing and explaining the complexity of classroom practices with technologies, when the intersects of complementary insights are offered and when collected data is analysed using different theories. As Radford (2008) explained, a theory can be seen as a way of producing understandings and ways of action based on a set of principles (P) that involves a set of methodologies (M) following a set of paradigmatic questions (Q). He uses this triptych (P, M, Q) to characterise a theory (Radford, 2008). Connections and integration between different theories can be drawn between the three parts: P, M and Q. The rich diversity of constructs from networking of theories as a research practice in mathematics education (Bikner-Ahsbahs & Prediger, 2014) offers the possibility for investigation at the micro and macro levels of the complex phenomenon of teacher appropriation of digital resources for mathematics classroom practices.

Here, I outline some of the basic principles and areas where AT and DAD complement each other in broadening the understanding of the appropriation of digital technologies/resources by mathematics teachers for professional practices, and then

[6] The networking of theories refers to a systematic way in mathematics education research practice for connecting theories of different traditions while valuing their identities. This is aimed at improving the theoretical basis of the scientific work of the community of mathematics educators by addressing the diversity of theories in mathematics education.

present my rationale for the theoretical framework and explore ways it might prove valuable to my research.

2.2.1 Activity Theory

The origins of activity theory can be traced to several philosophical sources, but pioneering influences are traced to Vygotsky (1978), and Leont'ev (1979), and a later expansion by Engeström (1987a).

Activity theory (AT) is a socio-cultural and cross-disciplinary theory with emphasis on the social and contextual nature of learning and practice. AT is focused on the historical developments of activity, the mediational means, the mediating role of artefacts in various forms of human practices, and the dialectical transformation of the individual and their community as a consequence of participation in a socially meaningful activity (Vandebrouck et al., 2013). AT fits into the categorisation found in the literature as a theory adopted and found appropriate for analysing issues of digital technology integration in the context of mathematics education research.

Engeström (2001) in his presentation of a historical overview, categorised the evolution of AT into three generations. The first generation centres on Vygotsky’s idea that human thought and activity are socially situated phenomena and are fundamentally mediated by various concrete and abstract mediational means. This conception has been classically represented by his mediational triangle of mediating artefact-subject-object. Leont'ev (1979) in the second generation further developed Vygotsky’s idea of social and cultural mediation by evolving an idea of a hierarchical structure of human activity. He further claims that thoughts and cognition are not only mediated by tools and signs but also by the structures of the activity in which they are embedded and the unity of interrelationships between activity, actions and operations. Activity, therefore, in the context of AT is composed of subject, object, actions and operations.

Learning contexts are often constituted by teaching and learning activities and when new mediational means are introduced, the activity changes as a result. AT provides concepts and notions for analysing, describing and interpreting such changes and can help assess how meaningful the tool-mediated activity is to the teacher and students.

It is stated that "The essence of Activity Theory is the dialectical transformations of individuals and their community as a result of involvement in an activity" (Vandebrouck et al., 2013, p. 127). Tool-mediated activity is a driver of transformation, innovation and appropriation in the context of teacher's professional practice. AT helps me to capture and take into account the teacher's resource-mediated activity, its complexity and the multiple factors impacting on it.

Sriraman and Nardi (2013) argue that theory is a tool for developing methodology, describing, interpreting, explaining, and justifying classroom observations of student and teacher activity and transforming practical problems into research problems. AT with its coherent system of concepts and notions and built-in language provides the rhetoric for analysing and describing the social, cultural and political dimensions of the context of my research.

2.2.2 Evolutions in Activity Theory

Engeström (2001), inspired by this development, expanded Vygotsky's mediational model to integrate Leont'ev's social and cultural dimensions of human-mediated activity. Engeström suggested an expansion of the basic mediational triangle into multiple forms of mediation to account for the socially distributed nature of human activity.

This expanded model of activity triangle incorporates the *Subjects, Object, Community, Tools, Rules* and the *Division of Labour* components of human activity as elaborated in Mwanza (2001).

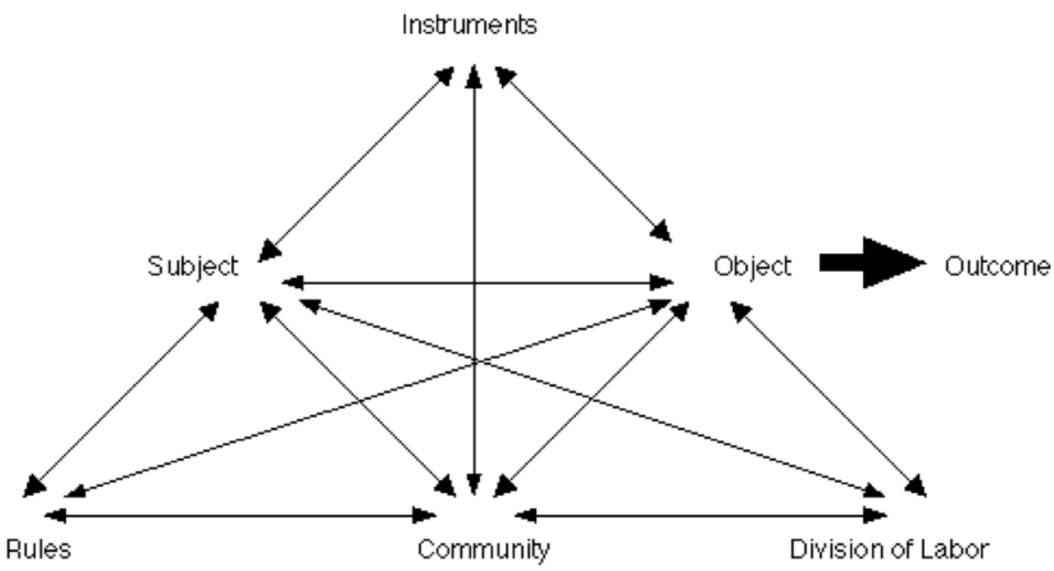

Figure 2-1. The structure of human activity system (Engeström, 1987b, p. 78)

Engeström (1999) further expands AT into its third generation of evolution, wherein he developed conceptual tools for understanding dialogues, multiple perspectives and the networks of interacting systems. This offers multiple perspectives and addresses issues relating to networks of interacting activity systems.

He proposed five tenets of activity theory: (a) activity systems are the units of analysis; (b) multi-voicedness; (c) historicity of activity; (d) contradictions as the driving force of change and development; and (e) expansive cycles as a form of transformation in activity. At the core of Engeström's idea is the 'activity system' as the prime unit of analysis. Activity is not just the sum of individual goal-oriented actions but includes a system of collective practice based on a division of labour. For Cole and Engeström, "Activity systems are complex formations in which tensions, contradictions, disturbances, and local innovations are the rule and the engine of change" (1993, p. 8). While it is held that the activity system is the unit of analysis, there exists a different understanding that lays emphasis on object-orientated activity as the unit of analysis.

Thus, Monaghan (2016, p. 198), drawing on the traditions of activity theoretic approaches, reiterates

> In AT 'object orientated activity' is the unit of analysis, that which preserves the essence of concrete practice. The 'object' here is not the object-thing but

> the object-raison d'être; indeed, if two individuals perform similar actions but have different objects, then it can be said that they are involved in different activities.

This activity system is the common and unifying node that guides the analysis across the various multidisciplinary fields of research. The emphasis on the person-artefact relationship is the central key attraction. Wertsch (1998) claims "mediated action is characterised by an irreducible bond between agent and mediational means" (p. 25), leading to a "reciprocal shaping" of both. This could aid the appropriation of tools by an agent or constrain action. From the AT theoretical approach, "the 'minimal meaningful context' for understanding human actions is the activity system, which includes the actor (subject) or actors (subgroups) whose agency is chosen as the point of view in the analysis and the acted upon (object) as well as the dynamic relations among both" (Barab, 2002, p. 533). The activity is not about 'merely doing' but 'doing' with a consequence of transforming or changing something. My research explores such productive activity by mathematics teachers with the use of resources.

In this research, I take as my unit of analysis, the mathematics teachers' nested activity contexts. These includes mathematics departments in the selected schools as the broad socio-cultural setting of teachers' professional activity and practice. This educational context consists of overlapping layers of interactions: the whole school environment, the mathematics department, classrooms, and curricular, non-digital and digital resources available to mathematics teachers for planning, for teaching and for assessment. Teaching is considered here as "a system of interacting features" (Hiebert & Grouws, 2007, p. 374) with a variety of mediating variables. Though the world of research does not come neatly divided into activity systems, the variety of interrelated conceptual tools that AT provides offers a form of flexibility that enables one to explore different directions and diverse possibilities of 'magnification' to help address the research questions and purpose.

Over the last three decades, the literature on the importance and relevance of the use of AT as a lens for addressing broader range of research questions and issues in mathematics education research has matured. The high frequency of the use of AT in the review of a number of PME papers by Jablonka et al. (2013) suggest its relevance in mathematics education research, though the review did not cover many non-English publications. Some recent studies have used the AT model and models with common socio-cultural roots to capture the complexity of the mathematics teaching context (Jaworski & Potari, 2009; Monaghan, 2004). Worthy of special mention are volumes 19(4) and 20(1) of the *International Journal of Technology in Mathematics Education* (IJTME, 2012, 2013, respectively), which were specially dedicated to studies using an activity theoretical framework to analyse the mathematics classroom and teachers' practices with various technologies: see, for example, Núñez (2009) and Abboud-Blanchard and Vandebrouck (2012). More recently, Monaghan (2016, pp. 197-218) has elaborated on the development of AT within the English-speaking world, the genesis of its influence in mathematics education research, and its nuanced appropriation.

My research is situated within this context and tradition.

2.2.3 Documentational Approach to Didactics

The *documentational approach to didactics* (DAD) is anchored in and draws from various interrelated contexts and the theoretical traditions of French European didactics of mathematics, resonating with international trends (Trouche, 2016a). DAD has been introduced by Ghislaine Gueudet and Luc Trouche (Gueudet & Trouche, 2009) and was further developed in collaboration with Birgit Pepin (Gueudet et al., 2012b). It is also acknowledged by these authors (Trouche, Gueudet, & Pepin, in press) that

> In addition to the French didactic tradition, the authors drew their inspirations from several main interrelated theoretical sources: the *field of technology use*, the *field of resources and curriculum design*, the *field of teacher professional development*, the *field of information architecture* and the *field of communities of practice.*

The concept of *resource* as a lens through which to analyse teachers' interpretations of and participations with resources in their professional practice is at the core of DAD. As a result of the proliferation of and greater access to texts and digital resources, teachers struggle to choose what is most pedagogically and qualitatively appropriate; hence, the study of mathematics teachers' work with resources has become a prominent area of research interest. This theoretical construct shares several similarities in concept and epistemology with the *instrumental approach* of didactics while also enlarging it (Guin & Trouche, 2002). It also incorporates elements from the Vygotskian idea of artefact-mediated activity (1978) and new concepts are also evolving. While AT is a well-established theory with intellectual roots outside of the fields of mathematics education and literature (Roth, 2014), DAD developed within the domain of mathematics education research in the unique context of the appropriation of digital resources; the literature is nascent and still evolving (Trouche, 2016b).

DAD takes Adler's reconceptualisation of resources as one starting point in focusing attention on resources in the context of practice (Adler, 2000). The documentational approach employs three key concepts adopted from Rabardel (1995): *instrumentation*, *instrumentalisation* and *instrumental genesis*. Within this *genesis* two interrelated processes take place as a teacher enacts a teaching activity with set of resources: the process of instrumentation, where the selected set of resources support, influence and shape the teacher's activity; and the process of instrumentalisation, where the teacher adapts, appropriates and shapes the resources for particular professional needs. Trouche, Gueudet, and Pepin (2020) consider that these processes include the design, re-design, or 'design-in-use practices' (whereby teachers change a document 'in the moment' and according to their classroom needs). I have referred to this phenomenon elsewhere as an *emergent (in lesson) task design* (Umameh, 2018); this is in line with the notion of teachers' use of mathematics curriculum resources as a design activity whereby teachers are active and creative designers before and in the course of enacting the lesson cycle (Pepin, Gueudet, & Trouche, 2017).

As a teacher draws on a variety of resources in the classroom and in professional engagement a *document* is created. In this case, a *document* refers not just to anything, but whatever is saturated with a teacher's intentions, experiences, choices and has become authoritative in the context of use. Pepin, Gueudet, et al. (2017, p. 802) explains it thus:

> This productive interaction between an individual teacher, or a group of teachers, and a set of resources, guided by a teaching goal, through successive stages of (re-) design and implementation in class, gives birth to a hybrid entity, a document: this consists of the resources adapted and re-combined; and the ways the teacher uses them … which include the stable organizations

> of associated activities and particular usages, and contain the ‘knowledge’ guiding the usages.

For Gueudet and Trouche (2009), “the document is much more than a list of exercises; it is saturated with the teachers’ experience, just as a word, for a given person, is saturated with sense in a Vygotskian perspective” (p. 205). This ongoing dialectical process wherein each appropriation of resources is an adaptation is referred to as *documentational genesis*.

Thus, “A documentational genesis develops in a field of interactions: interactions between teachers and resources (instrumentation vs. instrumentalization), interaction between teacher and students, interactions between teacher and colleagues” (Gueudet & Trouche, 2011, p. 4).

A formula advanced by Gueudet and Trouche reads

Document = Resources + scheme of utilisation

Vergnaud (1998) describes “scheme’ as an *invariant organisation of activity* for a given class of situation. This includes the knowledge and beliefs guiding and emerging from a set of professional situations and the stable organisation of action associated with a given objective. A scheme consists of the aim of an activity, rules of action, operational invariants (the cognitive structure guiding the action and the possibilities of inferences) and adaptation to the variety of situations (Trouche et al., in press). A mathematics teacher could enrich his or her scheme or new schemes of utilisation and usages employing a set of resources for the same class of professional situations across a variety of contexts over time. Gueudet and Trouche (2009, pp. 208-209) highlight the distinction between *utilisation* and *usages* thus: “We distinguish between *utilizations*: when a teacher draws on a resource once, or a few times, but without developing a stable behaviour for a given class of situations; and *usages*, which correspond to a stable organization of activity, and are part of a scheme”. They also offer a more precise formula for document:

Document = Resources + Usages + Operational Invariants

The document, therefore, consist of the resources, schemes of usages of a given set of resources, and the professional knowledge and beliefs guiding the usages. Documentational genesis as an ongoing process develops in a field of interactions at different levels and involves several actors. The digitisation of resources affords new opportunities for communicating and sharing resources among colleagues and offers ample opportunities for new forms of teacher interactions and engagements for collective projects in the production of shared resources (Gueudet & Trouche, 2011). Figure 2-2 below shows the schema of the documentation genesis.

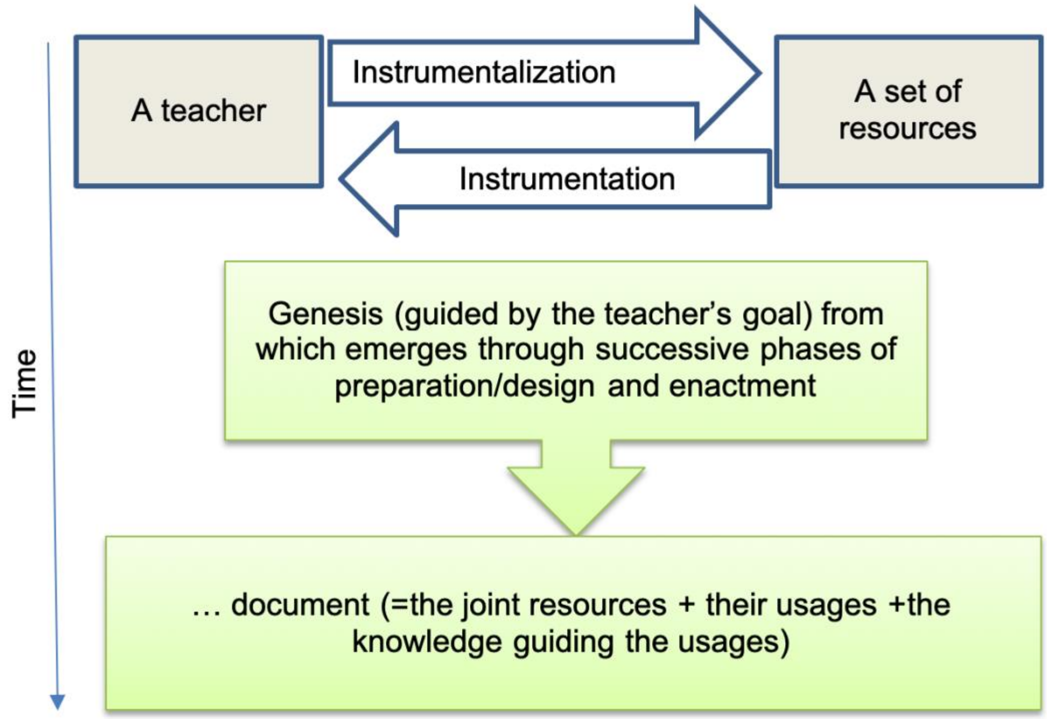

Figure 2-2. Schema of a documentation genesis (Pepin, Xu, Trouche, & Wang, 2017, p. 261)

Over time each teacher develops a structured documentation system (Gueudet & Trouche, 2009), which "is the set of all documents developed by the teacher; its structure corresponds to the structure of the teacher's activity. Since each document comprises particular resources, the documentation system encompasses the teacher's resource system: this is the resource part of the documentation system" (Gueudet, Buteau, Mesa, & Misfeldt, 2014, p. 142). The identification and analysis of the evolution of the teachers' documentation systems yields a new means of exploring teachers' professional growth.

This theorising of resources throws up a whole new set of opportunities and issues. For example, what dynamics are associated with a group of teachers or collective appropriating resources? In the anticipated emergence of interacting documentation systems or communities of practice, what will be the features of a community of mathematical practice?

'Collective' refers to a group of individuals working together as a unit. For Rocha and Trouche (2017, p. 2), "A collective is, a place designed by a teacher where interactions develop with other actors of her teaching". A collective could be a group of teaching professionals or a community of practice, providing space for collaboration, support, joint action and identity formation.

In the past decade the nature of resources has been changing with greater possibilities of teacher collaboration with and through curricular resources with other practitioners (Pepin et al., 2013). In line with the documentational approach, documentational geneses also takes place within the collectives. This emphasises two interlinked processes, on how the curricular resources are shaped, and how they shape teachers' work with resources. They indicate the creative dimension offered by resources used in teachers' work, including selecting, modifying, and creating new resources, in class and out of class. They then proposed to term this creative work *teacher documentation work*, and its outcomes *teacher documentation* (p. 1004).

2.2.4 Theorising the Collective Dimension

Gueudet and Trouche (2012b) extend the individual teacher's documentational genesis to include the social aspects of teachers' documentation work as well, since the teachers' work with resources is situated in the socio-cultural context of the institution. This collective perspective on teachers' work entails a group of people 'doing' mathematics together using diverse sets of resources. There are varieties of collectives that a teacher can be involved in; these could include colleagues in school and out of school, teacher educators and researchers in professional development, pupils, parents and pastoral caregivers, international exchanges, online fora, teams, communities and networks (Gueudet et al., 2013a; Krainer & Wood, 2008). These collectives exist at several overlapping levels. I distinguish different types of collectives; by context (institutional and prescribed), by access (open and voluntary), by mode of participation (online or face-to-face), by form of organisation (formal and informal) and other association could be restricted with a defined feature (for instance, a closed Facebook mathematics teacher group). A teacher's participation with and through resources always intersects with various groups. Pepin et al. (2013) argue that the collective dimensions play a crucial role in mathematics teachers' work with resources and in their professional learning and development and they offer opportunities for community building.

There are several similar, though not identical, constructs that address issues of 'community' from an adapted view point of the *Community of Practice* (Wenger, 1998*)*. For Wenger, "Communities of practice are groups of people who share a concern or a passion for something they do and learn how to do it better as they interact regularly" (Wenger, 2011, p. 1). At the core of the idea of CoP is that although individuals learn through participation in a community of practice, the generation of newer or deeper levels of knowledge through the sum of the group activity is more important. Wenger-Trayner and Wenger-Trayner further elaborated, saying that new technologies such as the Internet have extended the reach of our interactions beyond the geographical limitations of traditional communities and the growing range of digital technologies have expanded the possibilities for community and calls for new kinds of communities based on shared practice (Wenger-Trayner & Wenger-Trayner, 2015). In the mathematics education research community, Clark-Wilson (2017) has explored the issue from the perspective of the community of practice, investigating the possibility of transforming mathematics teaching with digital technologies.

Pepin et al. (2013) have also argued that a group of teachers could be considered as a potential community of practice (CoP). This is consistent with current research in mathematics teacher professional learning and growth: *Community of Interests* (Fischer, 2001); *Community of Inquiry* (Jaworski, 2008); *Theorizing Community of Practice and Community of Inquiry* (Goodchild, 2014); and *Community Documentational Genesis* (Visnovska & Cobb, 2013). Wenger (1998) indicated three dimensions of practice and two interrelated key processes. As to the three dimensions: *mutual engagement* - establishing norms, expectations, ways of working and collaborative relationships; *joint enterprise* – shared understanding of the enterprise, its aims and ideals; *shared repertoire* – objects in use, modes of usage and available resources. The two central processes of making meaning are *participation* and *reification*. Participation entails an involvement with the CoP, interacting, negotiating and taking part in its activities. Reification means “making into a thing… the process of giving form to our experience by producing objects that congeal this experience into thingness” (p. 58). Hence, Gueudet et al. (2013a, p. 309) state

> instead of reification, we coin the expression *community documentational genesis* for describing the process of gathering, creating and sharing resources to achieve the teaching goals of the community. The result of this process, the *community documentation*, is composed of shared repertoire of resources and shared associated knowledge.

Figure 2-3, below, captures the dynamic and inbound trajectories of these processes. In this figure, the *community genesis* – the development of mutual engagement and joint enterprise – and the *community documentational genesis* – creation of shared repertoire and joint building of common knowledge and dynamics among them – are presented. One could then argue that each teachers’ community of mathematical practice could also be considered a *community of documentation*, where “the documentation work leads to the production of temporary objects, as ‘lived’ resources, always engaged in new evolutions” (Gueudet, Pepin, & Trouche, 2013b, p. 320). Even though, there is evidence to indicate a collective dimension to teachers’ work, not all collectives of teachers could be described as a community of mathematical practice. The advent of digitalisation of resources has expanded the forms of interactions amongst the collectives, made the networks of interactions elusive, afforded unlimited teacher-resource interactions and opened up the opportunities for participation in multiple communities. This field of interactions presents and heightens the complexity of the processes of participation with resources and building up a stable community of mathematical practice.

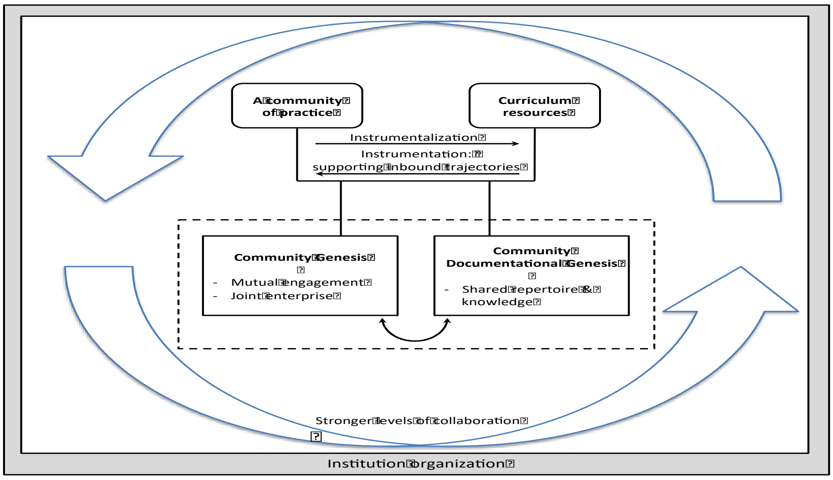

Figure 2-3. Schematic representation of a community documental genesis, based on Gueudet and Trouche (2012a, p. 308)

This theoretical combination of AT-DAD brings an added advantage to my research; while AT is socio-culturally orientated towards describing and analysing practices, DAD takes this further by analysing the specifics of digital resource integration, identifying 'good' practice and making informed recommendations for 'improvement'. This hybrid framework AT-DAD extends the interpretative potentials of both theories and brings together ideas and a range of shared assumptions and interrelated constructs that seem apart. My research seeks to understand the complexity of the appropriation by mathematics teachers of digital resources and taps into this promising analytical synergy. This framework will be used in order to understand the broader context of teachers' professional practices (national, regional and specific school contexts), and enable me to plan data collections methods, develop sets of guiding principles, and develop data collection questions and analysis. These elements are discussed in Chapters 3 and 4.

In summary, my decision to employ a combination of activity theory and documentational approach was borne out of a set of considerations. This framework provides me with robust arrays of parameters: practical educational issues in the integration of non-digital and digital resources could be formulated into research questions; a set of constructs to guide research design; and coherent multiple interpretative perspectives to be simultaneously considered in the processes of data collection, analysis and the discussion of the findings in relation to the appropriation of digital resources.

CHAPTER 3
RESEARCH DESIGN AND METHODOLOGY

This chapter presents the research design and methodology, further elaborating on the rationale for the use of the qualitative case study approach in my research. This chapter also reports on the unit of analysis, the pilot study and on the various methods of data collection before describing the various data collected and how these were condensed, organised and thematically analysed. Thematic analysis (Braun & Clarke, 2006) through repeated pattern recognition, code generation, constant comparison and integration of initial themes that cohere into categories is the primary analytical strategy in this study.

3.1 Research Design

In this study I adopt the qualitative case study approach (Cresswell & Poth, 2017; Creswell, 2013; Patton, 1990; Yin, 2009). Qualitative case study is a research approach that enables exploration of a complex phenomenon within its context in depth using multiple data sources. This ensures that an issue or area of interest is investigated through a variety of lenses, which allows for multiple aspects of the phenomenon under investigation to be probed, revealed and understood. The choice of method is dependent upon the nature of the research area. Thomas (2017) argues that the question of how appropriate a research design is derived from the nature of the social phenomena to be explored, its context and the sorts of evidence one seeks to address the research questions. My research is situated within a school context, wherein various levels of interactions amongst people, tools and institutional policies interact and mutually influence each other. Thus, Punch (2009, pp. 112-113) argues that "research design situates the researcher in the empirical world and connects the research questions to data, …the tools and procedures to be used for collecting and analysing empirical materials". A multiple case study strategy is the route through which I capture the data that enable a holistic overview of my context of exploration. This gives me the methodological tools and the procedures for collecting and analysing the required data.

3.1.1 Context of Study

My study seeks to understand the appropriation of digital resources by mathematics teachers in selected secondary schools in the Yorkshire region of England. The mathematics departments in the selected schools provide a broad setting since teachers usually undertake their practices within that collective context. This environment consists of overlapping layers of interactions: the whole school environment, the Mathematics Department, classrooms, and the curricular and digital resources available for mathematics teachers for planning, for teaching and for assessment, in the course of the school year 2015-2016. Schools were selected based on the following: the opportunity to have access; the growing encouragement to use technology in teaching mathematics; and the proximity of the schools. I made an average of 10 periodic whole-day visits to each of the three schools, respectively, to enable me collect data from the departments and from seven purposefully selected teachers in the real-life context of practice with digital resources. These teachers were contacted through email in the first stage. Then face-to-face meetings were scheduled and the details of what my research entails were discussed, and time was given to the teachers to reflect and respond. Out of the 12 teachers originally contacted, seven were eventually recruited to participate in the study. Some further factors were considered as criteria for selection of individual teachers: the context of the practice possibly yielding data relevant to addressing the research questions; the teacher willingly taking part in my research; the teacher using digital resources in the processes of lesson preparations, delivery and assessment; or the teacher being significantly involved with the collective of the department and other related communities outside the immediate context and culture of the department and classroom.

School visits and data collection came to a close towards the end of the school year as planned in my data collection schedule.

3.1.2 Qualitative Multiple Case Study

The vast majority of my data is qualitative in nature. It captures and describes what, why, when and how the mathematics teacher uses digital resource. Yin (2009, p. 18) defines a case study as "an empirical inquiry that - investigates a contemporary phenomenon in-depth and within its real-life context, especially when - the boundaries between phenomenon and context are not clearly evident". My study involves seven teachers from three different schools in the variously nested contexts in which they undertake their professional teaching activities. These contexts overlap and very often separating one context from another can be challenging. Even in cases of teachers who work in the same school, individual classroom contexts are always unique and so too are they ways digital resources are used in teaching mathematics to students. For Creswell (2013, p. 97), a qualitative case study approach "explores a real-life, contemporary bounded system (a case) or multiple bounded systems (cases) over time, through detailed, in-depth data collection involving multiple sources of information ... and reports a case description and case themes". Creswell (2013) highlights time as a factor as well as the variety of sources of data that helps develop themes that guide the narrative of the case description.

Qualitative research considers ever-changing, socially constructed, real-world settings as they unfold naturally, and it is discovery-oriented (Patton, 1990). In the earlier phase of my PhD research, I considered using both qualitative and quantitative approaches. After my pilot study (reported in subsection 3.2, p. 44), the insight developed during the pilot guided the eventual final framing of my research questions and the conclusive decision that qualitative strategies were more naturally suited to my research as it was focused on the professional activities of mathematics teachers with digital technologies and resources in the context of everyday practices. Cohen, Manion, and Morrison (2007, p. 416) state it thus:

> Qualitative research … draws the researcher into the phenomenological complexity of participants' worlds; here situations unfold, and connections, causes and correlations can be observed as they occur over time. The qualitative researcher aims to catch the dynamic nature of events, to see intentionality, to seek trends and patterns over time.

The qualitative approach employs a spectrum of data collection methods to access multiple data; this enabled me to build a richer picture and description of the professional activity of the mathematics teachers, especially as it relates to their use or neglect of digital resources. For Stake (1995), a qualitative design therefore weaves together "naturalistic, holistic, ethnographic, phenomenological, and biographic research methods to build a comprehensive case relevant in addressing the research questions". Patton (1990, p. 4) argues similarly, stating "Qualitative findings grow out of three kinds of data collection: (1) in-depth, open-ended interviews; (2) direct observation; and (3) written documents". Besides these, I employed a novel method for data collection: screen capture software[7] (Umameh & Monaghan, 2017). This implies that collection of multiple data and analysis methods are most appropriate in this study and is adopted in order to develop and comprehend the case under review as it is shaped by context and emerging data.

As noted in Yin and Creswell, this study aligns with the consensus that a case study approach is an empirical inquiry and exploration of a contemporary phenomenon within its real-life context over time; it involves multiple bounded system(s) and adopts multiple in-depth data collection methods involving multiple information sources geared towards analysing such datasets to capture the complexity of the context of the case. It is this conceptual framing that has guided this research undertaking in gathering data for addressing research questions.

In spite of the many advantages of using case study, Cohen et al. (2007) remark that researchers ought to be very vigilant since case study research is prone to selective reporting: picking only those pieces of evidence which support a particular conclusion, and as such misrepresenting the credibility of a case. Observer bias is addressed in this study through critical reflection and awareness on how my personal background and experiences could affect interpretation and through data triangulation

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7 https://snagit.en.softonic.com/

– a "method of cross-checking data from multiple sources to search for regularities in the research data" (O'Donoghue & Punch, 2003, p. 78). In these ways I hope to minimise the extent of observer bias. I address this in this study by sharing and checking with participating teachers some collected data and by triangulation of data from various sources.

3.1.3 Case Study in Mathematics Education Research

The case study approach has been used in mathematics education research for several decades; it is an established practice used in numerous studies on the integration of digital resources into mathematics teaching (Assude, 2005; Drijvers et al., 2010; Gueudet & Trouche, 2009; Lagrange & Monaghan, 2010). In adopting the case study as an approach for my research, I intend to be able to gather sufficient data and relevant contextual information to capture as much as possible the professional activity of mathematics teachers with digital resources and its impact on their practices by examining nested and overlapping contexts of practice not as isolated undertakings but as part of an integral interrelation with other contexts as well. The case study techniques are also useful in highlighting the emergent and intrinsic aspects of mathematics teachers' changing practice as they appropriate new digital resources into their everyday classroom activities. The disadvantages of case study have also been identified in research literature as well as professional literature: it is time-consuming and off-putting for novice researchers; there are the challenges of generalisability, reliability, difficulty of reporting the case, validity and trustworthiness; and associated ethical problems have been noted (Noor, 2008; Punch, 2009; Yin, 2009). These are further discussed in more detail in the subsequent subsection 3.5 (p. 68).

Different types of case study have been identified by various researchers. Stake (1995) identifies three types – the intrinsic case, the instrumental case, and the collective instrumental case – while Yin (2009) distinguishes between single, holistic case studies and multiple-case studies. Hyett, Kenny, and Dickson-Swift (2014, p. 3) develop a common understanding for the various types of case study thus:

> The intrinsic case is used to understand the particulars of a single case, rather than what it represents. An instrumental case study provides insight on an issue or is used to refine theory. The case is selected to advance understanding of the object of interest. A collective refers to an instrumental case which is studied as multiple, nested cases, observed in unison, parallel, or sequential order. More than one case can be simultaneously studied; however, each case study is a concentrated, single inquiry, studied holistically in its own entirety.

My study adopts a multiple case strategy which gives me the tools and procedures that will facilitate analysis within each mathematics teacher's setting and across settings towards building a comprehensive picture of the seven teachers' activities with digital resources and technology. I now discuss my unit of analysis in this research.

3.1.4 Unit of Analysis

There has been some ambiguity in the meaning of 'unit of analysis' and 'case' itself (Grünbaum, 2007). What is the proper unit of analysis? What then is the case? The unit of analysis is a key concept in case study research. In resolving the debate between 'case' and 'unit of analysis', Miles and Huberman (1994, p. 25) define 'case' as "a phenomenon of some sort occurring in a bounded context". The case is "in effect, your unit of analysis". For instance, a case could be a mathematics teacher using GeoGebra[8] to teach the construction of parabola in a whole classroom context or a group of students using a spreadsheet to compute a statistical analysis on the frequency of rainfall in a specified three-year period. Patton (1990, p. 447) affirms that the case is identical with the unit of analysis: "Cases are units of analysis. What constitutes a case or unit of analysis is usually determined during the design stage and becomes the basis for purposeful sampling". He argued further that "the key issues in selecting and making decisions about appropriate unit of analysis is to decide what it is you want to be able to say something about at the end of the study" (p. 229).

[8] GeoGebra is an interactive application intended for teaching and learning geometry, algebra, statistics and calculus in mathematics and science for all levels of schooling.

In terms of the Activity Theoretical perspective, an activity is the basic unit of analysis which enables the understanding of individual actions. As Monaghan (2016, p. 198) argued, “In AT ‘object orientated activity’ is the unit of analysis, that which preserves the essence of concrete practice.” Activity Theory, therefore, uses the tool-mediated activity as the unit of analysis, where the activity is subdivided into three analytical components of *subject*, *tool* and *object*. In this study the *subject* is the mathematics teacher under investigation, the *object* is the intended mathematical practice, and the *tool*(s) are the mediating digital and non-digital resources available with and through which mathematics teachers undertake and execute their teaching.

The consensus across a broad spectrum of literatures is that the case is identical with the unit of analysis: *objected orientated activity* in the light of the AT framework. A well-defined case constitutes a unit of analysis that is purposeful, holistic and context-sensitive. This principle has guided my delineation of the unit of analysis in this research.

The unit of analysis in my research therefore is as follows: *A mathematics teacher who is a member of a mathematics department in a school and is designated to teach in a context where digital resources are available for use.*

This implies that, in my research, the primary focus of data collection is on mathematics teachers’ use of digital resources and the impact on their teaching activities. Brewer and Hunter (1989) proposed six types of units of analysis: individuals, attributes of individuals, actions and interactions, residues and artefacts of behaviour, settings, incidents and events, and the collectives. Embedded designs (Yin, 2009) explore a number of nested subunits individually and findings from these units are re-combined to construct a holistic picture. This is undertaken through the use of six structuring themes (see subsection 5.1, p. 105) discussed in Chapter 5. The focus is the appropriation of digital resources by mathematics teachers; their actions and interactions with and through digital resources; the role of the context and various collectives the teachers are involved with; and the impact of these webs of interactions on teachers’ decisions to use digital resources.

3.2 Pilot Study

A pilot study is a small-scale study to assess the feasibility of research procedures, data collection methods and questions, to refine or modify research methodology, to try out sampling strategies, to gain mastery and confidence, and to refine other research techniques in preparation for a main study (Thabane et al., 2010). Towards the end of the first year of my research, I conducted a pilot study. This pilot study involved two mathematics teachers drawn from two secondary schools in the Yorkshire area. I had initially thought of using interviews, observation, document collection and a questionnaire at the outset of my research, but I needed some basic confidence and mastery in the use of these instruments and protocols developed before the main study. Hence, it was necessary to pilot the whole or some aspects of the data collection techniques and the development of confidence. Oppenheim (2000, p. 47) agrees on this when he states,

> we must allow a substantial period of time for the construction, revision and refinement of the questionnaire and any other data-collection techniques. This whole lengthy process of designing and trying out questions and procedures is usually referred to as pilot work.

During this period, I observed three lessons each from the two teachers where digital technology and resources were used in the classroom and computer lab. I also had two audio-recorded interview sessions and brief post-lesson discussions, several documents collected and, out of the 40 questionnaires distributed, 15 were returned. The questionnaires were designed to measure the variety of ICT tools in use by the mathematics teachers, to assess how often these ICT were used in lessons preparation and delivery, and to ascertain whether ICT was recommended by the school or teachers were at liberty to choose whatever ICT fitted their practice. The questionnaires were distributed to secondary school mathematics teachers of students in the age range of 11-16 years.

The pilot study was designed to serve these objectives:

- To enable me to gain some experience as a researcher and have a first-hand overview of the practicalities involved in undertaking interviews and observing lessons. Also, to check for the feasibility of the theoretical and methodological framework and effect any necessary adjustments.
- To get some indication from trying out the set of interview questions if the wordings of the questions are clear, straightforward and concise. To test too if the order of asking the interview questions and interview guide are structured to create a logical flow that will elicit detailed responses.
- To further develop my research questions, theoretical frameworks and methodology in the light of my pilot study experience.
- To provide opportunity to rehearse the context of the data collection, and practice the use of prompts and probes in clarifying ambiguities and misunderstandings.

Critical feedback from the teachers and my supervisors on the pilot study were an invaluable means of seeking further guidance on how to deal with specific practical and methodological issues emerging from the exercise as well as the modifications required for the main study.

I transcribed and analysed the data. The preliminary thematic analysis of the interviews, observation notes and questionnaires revealed a range of ICTs in use across schools and amongst teachers with various teachers at differing degrees of ICT integration. Also identified was the demanding and time-consuming nature of undertaking qualitative study and the benefits of a pilot study as an important tool for a novice researcher for framing a main study afterwards. These findings from the pilot study contributed to the seminar[9] presented as part of the Centre for Studies in Science and Mathematics Education Seminar Series at the School of Education, University of Leeds.

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9 https://pvac-sites.leeds.ac.uk/cssme/2015/05/06/the-mathematics-teachers-appropriation-of-digital-resources-understanding-the-implications-for-professional-development-and-classroom-practices/

Lessons, insights and critical feedback from the pilot study later informed the various revisions, deletion and modifications of my research design for the main study. Key among these decisions were the refinement of the research questions and grouping them into foci areas and dropping a question relating to boundary objects, which though of great interest to me, merited a separate PhD study to investigate adequately. I decided too to shelve the idea of using a questionnaire since it was not going to be a good fit in yielding the qualitatively relevant data this study requires. As a consequence of the challenges of ethical approval to conduct a video-recorded classroom observation, I opted to adopt and adapt the *systematic classroom analysis notation for mathematics lessons* (SCAN) developed by Beeby, Burkhardt, and Fraser (1979) and also added screen capture software to enable me to collect data relating to teachers' lesson preparation. On the whole, the pilot study was an invaluable tool in reformulating and refocusing my main study appropriately.

Purposive Sampling

Given the unit of analysis for my research and with insights from the pilot study, a deliberate or purposive sampling was used in selecting the participating mathematics teachers from whom I intended to gather relevant data for the study. Patton (1990, p. 230) clarifies this understanding thus: "purposive sampling focuses on selecting information-rich cases whose study will illuminate the questions under study". One reason for selecting the particular mathematics teachers used in this study is that they made regular and consistent use of digital resources. It is not always possible to study everything and collect data from everyone in the context of research; a selection process and criteria are always required. In this study I adopt a purposive sampling which involves selecting mathematics teachers in the identified schools who use or intend to use digital resources. This identifies a specific, evidence-rich sample that manifests consistently the sustained appropriation and use of digital resources for professional mathematics teaching practices which is needed to explore the unit of analysis. Seven mathematics teachers from three schools were purposively selected for this study based on the reasons previously mentioned in subsection 3.1.1, p. 38. My rationale for this sample size of seven was that it will give me a range of mathematics teachers' experiences and practices with digital resources and a research depth that allows for credibility. Furthermore, it is small enough for the time and resources available for undertaking this study. The final rationale is that it will afford me the opportunity for cross-case analysis: across and within schools, and to compare and contrast (Patton, 1990). Now, I turn to the data collection methods used.

3.3 Methods of Data Collection

One of the hallmarks of case study research is the use of multiple data sources and multiple methods of data collection, a strategy which, it is argued, enhances rigour, accuracy and data credibility (Frels, Sharma, Onwuegbuzie, Leech, & Stark, 2011; Patton, 1990; Punch, 2009). This attempt to investigate and analyse the activity of mathematics teachers with digital resources in preparing, enacting and assessing students requires me to take account of the range of resources teachers appropriate; the variety of social, institutional and collective collaborations; the real-time effect of new tools on professional practices; and the impact they have on teachers' work. Hence, this research adopts multiple methods. In collecting data from multiple sources my target is to achieve data triangulation (Patton, 1990), wherein data collected are wide-ranging and relevant in addressing the research question. Also, datasets were cross-checked against similar data collected using different methods to ensure consistency in the hope of enhancing credibility. For instance, the data collected through classroom observation were compared with those collected through the use of screen capture, to enable me to double-check what the teacher was doing in class and what was reported during the planning phase.

Yin (2009) identified six methods of data collection that can be used in case study strategy: documents; archival records; interviews; direct observation; participant-observation; and physical artefacts. Data in this study were collected during the 2015-2016 school year through scheduled periodic whole-day school visits. They were collected from a range of sources: audio-recorded semi-structured interviews; classroom observations using an adapted systematic classroom analysis notation for mathematics lessons (SCAN; (Beeby et al., 1979; Monaghan, 2001); screen capture recordings of teachers planning lessons and accessing digital resources, enabled by screen capture software (SnagIt); the researcher's field notes; and the collation of documents to which the teachers made reference. The data collation and analysis processes are discussed in this chapter.

I now discuss guiding principles in undertaking the data collection.

Guiding Principles for Data Collection

My data collection spanned an academic school year. Data were collected through a periodic in-class and out-of-class contexts. The various classrooms constitute the settings where majority of the data were collected. However, most of teachers' preparatory work takes place outside of the class. In my research, teachers' preparatory work takes place at home, in the staffroom, in the computer workstations, with colleagues and at workshops. This *year-long periodic in-class and out-of-class continuous follow-up* enabled me to capture these dimensions of mathematics teachers' engagement with curricular resources that may never appear in-class and the various collectives that influence teachers' choices and decisions. There was also a *wide-ranging collection of material resources*: for instance, internet links, web addresses of online resources, worksheets, spreadsheets, poster and lesson plans. This aided me in documenting the sources of tasks, in assessing mathematics-related internet sites, resource banks like *Resourceaholic.com*, and social networking platforms, and to map out the various ways through which teachers access resources. I also used a talk-aloud[10] process with the teachers with the screen capture recording the corresponding activity on the computer screen. This talk-aloud process also included a retrospective reflection. Krogstie (2009, p. 418) defines *retrospective reflection* as "a conscious collaborative effort to systematically re-examine a process in order to learn from it". This is a collaborative form of reflection-on-action (Schön, 1987, p. 31) that involves a "dialogue of thinking and doing through which I become more skilful". This is a critical practitioners' self-assessment of their practice in order

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10 A data-gathering method that involves participants only describing their actions and choices.

to make thinking more visible and uncover the processes and knowledge used in a particular situation, by analysing and interpreting the information collaboratively recalled. This, I hoped would enable teachers using digital resources to gain greater insights into their own thinking and the processes involved in appropriating digital resources. Below is one example of a teacher/researcher retrospective mapping of variety of resources used for lesson planning and enactment.

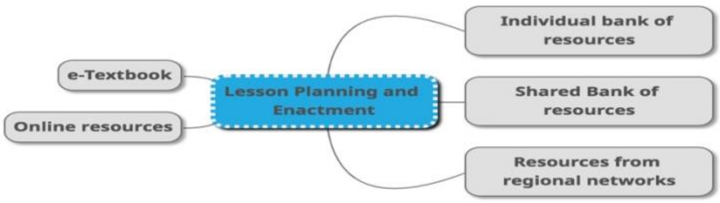

Figure 3-1. A Retrospective mapping of teacher resources

In my study, screen capture software was used in tandem with the mathematics teachers to reconstruct, retrace and capture the paths of teachers' lesson-planning processes. Figure 3-1 is a sample of how the resources were mapped, which I was able to do by identifying the resources cited, sites visited, and the banks of resources materials were drawn from. This retrospective reflection is collaborative (teacher and researcher).

The various data collection methods guided by these principles are now discussed below.

3.3.1 Interviews

Interview is a common data collection method in qualitative research. Yin (2009) sees the interview as a "guided conversation" with someone or a group of persons with the intent of eliciting relevant information on attitudes, opinions, definitions of meaning, rationale for choices and the construction of what reality means in a mathematics teacher's professional context. Cannell and Kahn (1968, p. 526) consider an interview as "a two-person conversation initiated by the interviewer for the specific purpose of obtaining research-relevant information and focused by him on content specified by research objectives of systematic description, prediction, or explanation". In the context of my research, the interview was a theme-guided conversation with a purpose focused on obtaining relevant information towards addressing research questions on appropriation of resources by mathematics teachers for their classroom activity. My initial step, informed by the insights and experiences from the pilot study, was to schedule an initial visit to chat with participating teachers, build some rapport and develop a sense of the context, and cultivate an acceptable manner in which interviews question would be asked.

My intent was to capture from the conversation the repertoire of teacher resources; how they are used, how they are accessed and their impact on teacher practice and rationale for the choices of digital resources and how digitisation impact on resources, task and the collective engagements. I aimed also to understand and analyse teachers' participations in collective interactions. A semi-structured form of interview was adopted with an initial set of questions guided by the literature and then from the initial sets of classroom observations. Below is a set of semi-structured interview questions according to the four foci with the associated rationale.

The Mathematics Teacher

Rationale: The teacher is at the core of my research and as such the questions seek to elicit the choices, the resources and sources of those resources and the impact on teachers' teaching activities.

a. What resources do you use in your teaching activities?
b. In what ways do you access these resources for lesson preparations and classroom practices?
c. How do you choose the specific ICT/digital resources used in your lesson?
d. Are there specific digital resources recommended by the school or department?
e. Has any student recommended any ICT/Digital resources for use? Any suggested by colleagues or friends?
f. Do you use the same ICT/digital resources for lesson preparation and delivery or assessment?
g. What are the resources that have become a permanent part of your lesson preparation, teaching and professional practice?

The Resources

Rationale: Resources facilitate teaching. From both the theoretical and practitioner perspectives, these resources shape and are shaped by the teacher. This transactional and dynamic relationship is the focus for the questions.

a. What are digital and non-digital resources that have remain in use in your teaching over the years? And can you recall those you no longer use and why?
b. Are these there because they work for you, for your students or on school recommendation?
c. Do you re-use, update, modify and combine previous resources for a new lesson? In what ways?
d. What are the impacts/effects of the integration of ICT on professional activity and classroom practice?
e. How does the use of ICT/digital resources impact on your classroom activity format?
f. In what ways does it affect the way you teach, manage the class and structure your lessons?

The Tasks

Rationale: A task is a unit of goal-directed activity and a mechanism for attaining a specific mathematical objective. Here, I explore from the teacher's point of view the tasks in use, their sources and how teachers amend them or not.

a. What type of tasks do you give your students?
b. What are the sources of these tasks?
c. Do you amend these tasks, and if yes, how?

The Collectives

Rationale: Teachers are involved in a web of interactions and the advent of digitalisation has extended the possibilities of such interactions and exchanges. I explore the context of teacher participation with and in such groupings.

a. Do you belong to any mathematics teachers' groups? Are they formal or informal? Are there any online groups?
b. Do these mathematics teachers' group influence your use of resources?
c. In what way do you support each other through the groups?
d. Any other benefit of belonging to the mathematics teachers' groups?
e. Is there any other thing you want to share in relation to the various groups you belong to?

The choice of the audio-recorded, semi-structured, open-ended interview offers sufficient flexibility to approach different participating teachers differently while still covering the same research foci of data collection. The interviews were audio-recorded to ensure an accurate reporting of the conversations and avoid data loss. Each interview was labelled with the name (pseudonyms were used), date of interview and the specific school. The interview questions are grouped under the four foci of my research. As stated in the research questions, there is a core general statement in the questions, then a sequence of sub-questions and alternative questions geared towards possible further probing and prompting as the interviews unfold. For instance, on the focus of 'Task', the general question was *What mathematics tasks do you give your students?* and the sub-questions were *Do you amend these tasks? If so, how?* and some further probing question, *Please could you explain a bit further the idea of plenary tasks you mentioned in passing?*

I was guided by the hierarchical focused interview strategy of P. Tomlinson (1989, p. 165): "That is, the researcher asks an initial question at the highest level of generality and seeks further elaboration and development of anything that emerges, noting which aspects of the agenda are covered as the interview proceeds". These sets of planned questions and probing tactics were intended to achieve a satisfying level of completeness from the interviewee's perspectives. Moving from the more general to the more specific helped clear up misunderstandings and ambiguities or allowed the interviewee to go deeper into their explanations, deepen rapport and flesh out emerging themes, and showed the uniqueness of the particular teacher's experience. These audio-recordings were transcribed and the preliminary analysis commenced in the process. I started the preliminary analysis by highlighting related phrases, sentences and words (such as mastery, formative assessment, and scheme of work) that were emerging more frequently across the datasets. As the research advanced, I progressively focused on the emerging themes (such as formative assessment, and task types) from the datasets from the mathematics-teaching contexts with digital resources and the uniqueness of each case through data reduction (Namey, Guest, Thairu, & Johnson, 2008) - sorting and noting patterns, themes, commonalities, regularities and uniqueness across the different teachers and schools. These included the various digital recourses teachers were using, number line and bar modelling, and how each teacher is unique in their classroom practices.

3.3.2 Observations

Observation has been widely used by education researchers in naturally occurring, real-life events in school settings (Foster, 1996; Punch, 2009). Some distinguishing features of observation as a method of data collection include that it offers researchers the opportunity to gather 'live' data and 'go native' to obtain an insider's perspective by immersing oneself into naturally occurring social situations *in situ* (Cohen et al., 2007). Observation can be relatively or totally unstructured, but for Gray (2013, p. 397), "Observation involves the systematic viewing of people's actions and the recording, analysis and interpretation of their behaviour". I tend to favour Gray's approach because it is systematic and structured to capture and gather data on the features of the context of the research.

In the course of the 2015-2016 school year, periodic lesson observations were undertaken with a focus on the teacher's use of digital resources in the lesson. The central goal was to observe and record the stream of actions, activity and interactions as they unfolded in the context of the mathematics lessons where curricular and digital resources are in use. The observation schedule *Systematic Classroom Analysis Notation* (SCAN) is used in my research. I previously used SCAN in the pilot study and I subsequently made further adjustments, refinements and adaptations to fit into the reality of my main study context. Similar use of adapted SCAN for capturing and analysing teachers' use of technology in secondary mathematics classes has been noted in literature (Monaghan, 2001; Schoenfeld, 2013).

3.3.2.1 Systematic Classroom Analysis Notation (SCAN)

In the year-long data collection, I adapted SCAN as a tool for recording the lesson observations. SCAN affords the observer a framework for recording the essence of dialogue in the mathematics lessons, teacher's objectives, pupil work and the use of resources (Beeby et al., 1979). There exists a range of classroom observation schedules constructed by researchers and professionals for recording and analysing classroom interactions. Notable among them are *UTeach Teacher Observation*

Protocol (Marder & Walkington, 2012), *TRU Math* (Schoenfeld, 2013), *The Reformed Teaching Observation Protocol* (RTOP; (Sawada et al., 2002) and *Mathematical Quality of Instruction* (MQI; (Hill, 2014). There is a comparative study that has identified the specific focus of each tool, its features and how it might serve the purposes of researchers (Boston, Bostic, Lesseig, & Sherman, 2015; Ko & Sammons, 2013). None of the above-mentioned schedules focus on the use of technology. As a consequence, I chose an adapted version of SCAN since it is focused in clear ways on the major categories of classroom actions (mathematics teachers' and students' actions with technology and tasks) and the features of classroom activities that are at the core of my research: the teacher, the resources, the task and how these connect with the teacher's participation in the collective.

My lesson observation sheet is in two parts: the first part is the background, which consists of class profile and topic of the lesson.

Observer:	Class year:	
Observation date:	Observation starts:	
School:	Observations ends:	
Teacher (Anonymised name):		
Number of students:	Boys:	Girls:
Teacher's stated goal:		
Physical arrangement:		
Technology resources:		
Topic:		

Table 3-1. Background and class profile

The second part is my adapted SCAN descriptions schedule for recording the observation. This is set in a table below.

5-minute Timescale	Resources	Activity	Episodes	Events of interest	Comments
Post-lesson reflections					

Table 3-2. SCAN observation record sheet

My aim is to observe and record the teacher's use of resources – hardware, software, and digital and printed materials, Whiteboard, IWB, classroom activity and 'events of interest'[11]. Through my pilot study, I adapted SCAN to highlight and capture the variety of teacher-made materials, digital resources and, on a moment-by-moment basis, the activity structures with these digital technology and resources, teacher routines and decisions, the dynamics of classroom interactions and the mathematics topic taught.

These questions were considered in the course of adapting SCAN to fit the present research efforts:

- What do I want to capture in the lessons?
- Which research question(s) does the observation data address align to?
- What are the criteria for identifying natural units of events of interest?
- How do I capture the task(s) in the lesson?

The key research questions observation data would address the following:

- In what ways are mathematics teachers accessing resources for classroom practices and teacher education?
- What resources do mathematics teachers access and use?

11 Events of Interest – refers to key moments of appropriating digital resources into the lesson or moments when digital technology or internet-based curricular materials is used support the task.

- What constitutes the essentials of the mathematics teachers' resource systems?
- What tasks do mathematics teachers give to their students?

I now describe the adapted SCAN system of notations and how it is used in this research. This includes subdividing the lessons into interconnected series of teacher-students and student-student activities with and through the use or non-use of digital resources. The observational notes using the schedules were taken at successive intervals of five minutes to enable me to analyse and trace the path of transformation with the set of resources and frequencies of use across the period of observation. The SCAN analysis is inputted into my interpretation of data, by noting the digital resources used and at what point in the lesson, by exploring the frequency of the task types and teachers' actions.

The SCAN notations and descriptors focused on the whole-class teacher-student activity with and through digital resources in their engagement with mathematics tasks. The SCAN descriptors consist of letters in upper- and lower-case abbreviations and of numbers indicating the interactions, dialogues and student groupings. The activity structure is made up of series of episodes and events of interests. These include Resources: IWB (interactive whiteboard in place of BB, blackboard in the original version); TM – teacher-produced materials; and Web – Web-based resources. Activities include Ce - whole class exposition; Dn – dialogue, between teacher and a number (n) of students; $D_{t\text{-}s}$ captures the one-to-one teacher to student dialogue; and $D_{s\text{-}s}$ for students to student dialogue or in groups of three. Episodes focus on the teacher's actions: setting up, initiating lessons, facilitating or explaining technical issues of mathematical ideas. Table 3-3 below shows the key to understanding the SCAN schedule descriptors. Most of the SCAN analysis (subsection 4.1.5, p. 81) is inputted into my interpretation of the data and the result in the subsequent Chapters 6 to 8.

In Table 3-3, the SCAN notations and descriptors are shown as an aid to understanding what the classroom observation revealed about the nature and content of the lessons observed. These enable me to capture and code in real time mathematics teachers' activity structures.

Resources	Activity level	Episode Level
TMw - teacher-produced materials	**Ce -** whole-class exposition	I - initiating lesson
Web - websites/teacher-dedicated online resources.	$\mathbf{D_n}$ - dialogue	**Pex -** practice exercise
TMD - digital/electronic material	$\mathbf{D_{t\text{-}s}}$(n) - teacher-student(s)	$\mathbf{C_o}$ - coaching, checking the logic of student's ideas/eliciting reasons
C - computer	**Ds-s**(n) - student-students	$\mathbf{E_T}$ - explaining technical ideas
WB - whiteboard	**O -** observation, circulating	$\mathbf{E_i}$ - explaining mathematical ideas
IWB - interactive whiteboard	$\mathbf{AwT_t}$ - teacher action with ICT	$\mathbf{F_i}$ - facilitating mathematical ideas
Wbh - handheld white board	$\mathbf{AwT_s}$ - students' action with ICT	$\mathbf{F_i}$ - facilitating mathematical idea
Cal. - calculator	$\mathbf{W_n}$ - student work in groups n>1	$\mathbf{F_T}$ - facilitating technology
Tks - mathematics task	$\mathbf{W_o}$ - student working alone	**D -** defining
iP – iPad	**M** - student movement	**R -** revising
	L - listening	$\mathbf{S_S}$ - student setting up
		$\mathbf{S_T}$ - teacher setting up

Table 3-3. My adapted SCAN notations and descriptors

These consist of subdivisions into related sequences of logical 'episodes, events, dialogues, resources and decision-making' indicating the cohesive series of classroom activity.

Researcher's Role and Reflexivity

It has been argued that all qualitative research is in some form a participant observation since the observer shares, at varying degrees, in the life and activities of the context while attaining an insider's perspective on habits, norms, practices, rituals and patterns of interaction (Patton, 1990). I do acknowledge that my interpretative lens is influenced by my background, beliefs and values, and the research context has had subtle effects on me over time. In spite of this, my overall intent is to have first-hand experience of mathematics teachers' uses of digital resources in their everyday practices since the impression and feeling of the observer could be useful in understanding and interpreting the setting and its dynamics. The nature of observation and level of participation could oscillate along a continuum of possibilities. Patton (1990, p. 268) argued that "the challenge is to combine participation and observation so as to become capable of understanding the setting as an insider while describing it to and for outsiders". Hence, Gold (1958) proposed a well-known cross-classification of researchers' roles in observation that reflected that it changes over the course of research in varying degrees along a continuum. At one end is the *researcher as complete participant,* in the middle is the researcher as *participant-as-observer*, then the researcher as *observer-as-participant*, and at the other end of the continuum is the researcher as a *complete observer*. Robson (1993) refers to the *participant-as-observer role* as one observing through participating in activities, where the observer can ask for explanations and trust is key. Intermittently, I was involved in helping the teachers and students set up, assisting in troubleshooting frozen iPads, handing out worksheets, and responding to students' queries. From time-to-time, the teacher spoke to me in class, explaining or bringing me up to speed on some aspects of the lesson that was a follow-up from previous work. The effect of my participation, in my opinion, was minimal since the three schools are regional teacher training and 'research schools' and the students were familiar with having researchers in the lessons over time.

I position my role as a researcher at the midpoint between *participant-as-observer* and *observer-as-participant,* whereby I am a participant in and an observer of the context/situation (Punch, 2009). This blending of roles is aimed at achieving a balance against any intrusion that could artificially affect the data. This framing enabled me to be aware of the possible consequences of my presence in the context and what effects this could have on the data. These schools of research have had long traditions of hosting researchers and as such students and teachers seem attuned to my presence; it had no observable obstructive impact on the students' behaviour, since students and teachers acted as far as I can ascertain, naturally.

The immediate challenges for me were the issues of reliability and validity. As part of my initial training for effective use of observation protocol and to achieve an acceptable level of inter-coder reliability (ICR), I had (with my primary supervisor) an independent observation of a lesson for the PGCE students at the University of Leeds. Afterwards we compared our observation notes and codes, and eventually arrived at 80-percent agreement in the inter-coder reliability in coding the observation. One other way I have endeavoured to reduce the challenges of reliability is in adopting a more structured observation schedule process using SCAN as a notation system to capture the data. Another way to improve validity and reliability is in the process of triangulation through the use of multiple methods of data collection.

My level of participation varied across the period of data collection. On various occasions, I conducted short post-lesson conversations to seek further clarification or ascertain if actions and events I had noted were what I understood them to be. SCAN, a structured observation schedule, was my main instrument for recording, describing, analysing and making ongoing preliminary interpretations of what had been observed. The observations in the departmental and other group meetings were recorded in my field notes and used to supplement data from the interviews and screen capture recordings.

3.3.3 Documentary Data

'Document' is understood here in the light of a documentational approach to didactics (Gueudet & Trouche, 2009). It is conceived as whatever is saturated with a teacher's intentions, experiences and choices and has become authoritative in the context of use (Gueudet & Trouche, 2012b). Documentary evidence is a frequently used unobtrusive data source. Documents, both historical and contemporary, are a rich source of data for educational research. They include a wide range of organisational, institutional records and digital archives (Gray, 2013; Punch, 2009). The array of documentary evidence in this research includes, in many cases, independent data sources in their own right and in other cases were collected in conjunction with the interviews, observations and screen captures as complementary data. School and professional teaching practice records gathered were routinely compiled and stored as both soft and hard copies. These are workbooks, worksheets, lesson plans, textbook extracts, posters, summaries of curriculum, guide-books, internet links and the e-textbooks consulted. Hammersley and Atkinson (2007, pp. 132-133) suggest relevant questions that could guide a systematic examination of documentary evidence

> How are documents written? How are they read? Who writes them? Who reads them? For what purposes? On what occasions? With what outcomes? What is recorded, and how? What is omitted? What does the writer seem to take for granted about the reader(s)? What do readers need to know in order to make sense of them?

Punch (2009) indicates the aspect of *social production* – how a document comes into existence – is a possible analytical theme. Drawing on DAD and my research questions, various documents in practice collected from mathematics teachers were scrutinised to reconstruct systematically the developments or transformations that are traceable through the documentary trail.

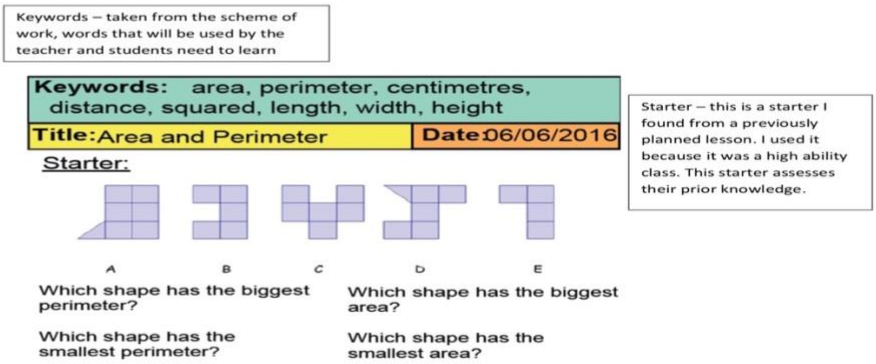

Figure 3-2. Sample of document with teacher annotations

Figure 3-2 shows the teacher's (Richelle) self-annotated lesson planning note and tasks. The tasks include keywords to define in preparation for the lesson and starter tasks Richelle intends to use in assessing prior knowledge before the topic of the day. This is one sample of the documents collected.

3.3.4 Screen Capture Software

There is an increasing use of audio-visual screen capture recording as an approach for qualitative data collection (Bhatt & de Roock, 2013; Chaney, Barry, Chaney, Stellefson, & Webb, 2013). Screen capture software is used in this study as an approach for data collection. A great deal of the mathematics teachers' preparations for teaching involves searching for resources for lessons and exchanges that inform teachers' professional practice and that could enhance students' achievements; they are conducted outside of the classroom and most times individually on the teachers' computers. By using screen capture to record the trajectories of teachers' searches for resources and interactions, I hope to re-construct, map and analyse these trajectories and identify how these define the teachers' practices with resources.

My aim is to gain insights into the moment-by-moment and *in situ*, out-of-class lesson-planning practices of mathematics teachers. I also intend to provide evidence in developing an argument that teachers, individually and collectively, access, use, modify and create a variety of digital resources for their professional engagements.

SnagIt[12] is a comprehensive and powerful screen capture software that captures video display and audio output, offering tools for organising, simplifying, and editing (Chastain, 2007). It was created and distributed by *TechSmith* and first launched in 1990. This application allows you to trim, crop and rearrange captured footage and add images as well as adjust picture settings. This screen capture software is accessible and easy to use. The toolbar has a cache of tools (as shown in Figure 3-3) that facilitates the capturing and editing of a variety of content like images, text and video; it also has a range of toolbar add-ins that allows annotations and the facility to mark up, organise, convert, edit and share captured content.

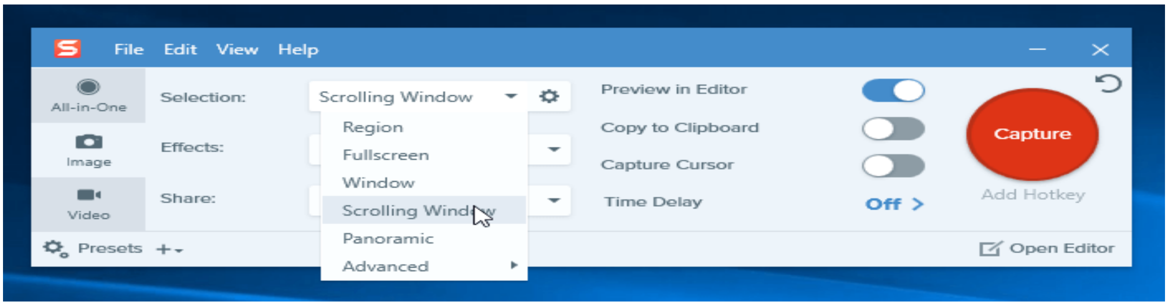

Figure 3-3. Screen shot of SnagIt toolbar

SnagIt can be used to take snapshots or record anything on the computer screen, desktop, window, application, region, hidden window, entire webpage, or scrolling area. Figure 3-3 shows the various toolbars: the image capture and video-recording tabs in the left-hand section; then a drop-down scrolling window to enable specific area selection or Fullscreen; and the function buttons on the right-hand section to activate the selected command.

Two techniques were used in eliciting information from the teachers: think-aloud interviewing and verbal probing. *Think-aloud* is a research technique in which participants speak their thoughts aloud as they complete a task (Charters, 2003). In this research, this refers to teachers voicing out their thoughts as they prepare activities

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12 https://www.techsmith.com/screen-capture.html

with digital resources for mathematics lessons on-screen. For Collins (2003, p. 235) "in the think-aloud approach the respondent is asked to 'think-aloud' as she or he answers the question …, whereas the probing method involves the interviewer asking specific questions or probes". Series of questions were drafted to elicit more detailed information on aspects of the lesson preparation that needed further clarification. SnagIt concurrently captured all the computer on-screen activities of the teachers – text inputs, video output, webpages accessed, documents opened, screen changes, modifications – while simultaneously recording audio.

In spite of the many advantages of using screen capture software, there are some limitations as well. It is unavoidably invasive and raises privacy concerns on how data could be used as a detailed record of teachers' interactions on their computers are collected. One way this was addressed was that the teachers were made aware of the possibility of recording all opened pages on screen and as such teachers logged out of emails and social media accounts while the recording took place. The other was to delete data that were personal and anonymise data that, though personal, was relevant to the study. Throughout, the consent was always requested from the teachers. The next major challenge was how to transcribe and analyse the screen capture data. I decided to use a matrix of data display and analysis that consists of text, images and analytical codes of relevant sections of the data that enabled me to address the research questions.

Text	**Images**	**Codes**

Table 3-4. Matrix of Screen Capture Analysis

Table 3-4 shows the matrix for analysing the screen capture data. The table is divided into three parts for the text of what is recorded, the corresponding images and my codes as the next step in the analysis.

In summary, in the above section, I presented the rationale for the choice of data collection methods, making references to the insight from my pilot study that helped refine and further develop my data collections skills and schemes. I also highlighted my role as researcher, acknowledging how self-perception in relation to research constitutes an aspect of the research process and the possible impact this could have on the research outcome, before presenting ways through which I have tried to minimise selective data entry and its effects.

Now I turn to the processes of thematic data analysis.

3.4 Thematic Data Analysis

Thematic analysis involves an iterative and reflexive search for and extraction of general patterns discovered in the data through multiple readings of the dataset (Boyatzis, 1998; Patton, 1990). Fereday and Muir-Cochrane (2006, p. 82) defined thematic analysis as "a form of pattern recognition within the data, where emerging themes become the categories for analysis". For Braun and Clarke (2006, p. 79), "Thematic analysis is a method for identifying, analysing and reporting patterns (themes) within data. It minimally organizes and describes your data set in (rich) detail". Thematic analysis consists of five steps: data familiarisation, code generation, theme search, theme revision, and theme definition (Braun & Clarke, 2006).

On the question of what counts as a *pattern* or *theme*, Braun and Clarke (2006) explain that a theme captures and sums up important elements in the data in relation to the overall research foci and questions, and represents repeated patterns of meaning within and across the dataset. Patton (1990) argues further that the term *pattern* denotes a descriptive data item, while *theme* refers to a more topical or categorical conceptualisation of such items.

Below the phases of thematic analysis are shown as conceived by Braun and Clarke (2006) and, alongside other relevant literatures, this has influenced how thematic analysis is used in this study.

Phases	Description of the process
Familiarizing yourself with your data	Transcribing data (if necessary), reading and re-reading the data, noting down initial ideas
Generating initial codes	Coding interesting features of the data in a systematic fashion across the entire dataset, collating data relevant to each code.
Searching for themes	Collating codes into potential themes, gathering all data relevant to each potential theme.
Reviewing themes	Checking if the themes work in relation to the coded extracts (Level 1) and the entire dataset (Level 2), generating a thematic 'map' of the analysis.
Defining and naming themes	Ongoing analysis to refine the specifics of each theme, and the overall story the analysis tells, generating clear definitions and names for each theme.
Producing the report	The final opportunity for analysis. Selection of vivid, compelling extract examples, final analysis of selected extracts, relating back of the analysis to the research question and literature, producing a scholarly report of the analysis.

Table 3-5. Phases of thematic analysis (Braun & Clarke, 2006, p. 87)

For instance, five out of the seven participating teachers reported a *pattern* of assessing students at various points during lessons and at the midpoint of a lesson cycle. These repeated patterns made 'formative assessment' a major *theme* in my investigation of mathematics teachers' use of digital resources for teaching. Illustrative examples are disused in the case reports in Chapters 6- 8.

In the process of thematic analysis, the researcher plays an active role in identifying and selecting patterns/themes/categories that are central to the description of the phenomenon under investigation, as informed by the research foci and questions. To determine what a theme is in this study, I collated the emerging codes into potential themes that could enable me to address the research question. (This is discussed in further detail below; see for instance subsection 4.1.7, pp. 86-91.) I also took note of unanticipated nuances which were unique and of interest in my judgement. For example, in terms of unanticipated nuance of teachers' use of digital resources, I found that one of the teachers created YouTube videos for his students' use; through this, Gray encourages a form of flipped classroom (Alvarez, 2012) ambience and equally makes the YouTube lessons available to the wider public as well.

3.4.1 Analytical Coding Processes

Given the size of my *data corpus* (i.e. all the data collected for this study), open coding was my initial phase in the process of data organisation and data reduction or condensation. For Miles, Huberman, and Saldana (2014, p. 12), "Data condensation refers to the process of selecting, focusing, simplifying, abstracting, and/or transforming the data that appear in the full corpus (body) of written-up field notes, interview transcripts, documents, and other empirical materials". In the literature this is also referred to as data reduction. In my research, data were condensed and organised through coding and highlighting meaningful and relevant segments that showed the potential to address the research question. For instance, only segments of the interviews and screen capture relevant (data that enabled me to address the research questions) to the research were transcribed, read and re-read. Additionally, an iterative process took place of fracturing the condensed data word-by-word, line-by-line, incident to incident and sticking closely to the data. This employs the processes of constant comparison. These initial process generated lots of codes while integrating them into potential themes or categories. (Table 3-6 shows the definition of the terms used.)

Data Corpus	This consists of all the data collected for this research: interviews, SCAN, documents and screen capture.
Dataset	All the selected data from the data corpus relevant in addressing the research questions and emergent issues in the course of the analysis
Data Item	Each subunit of descriptive information from a dataset

Data Extract	An individual coded chunk of data

Table 3-6. Definition of terms used

This coding specifically enabled me to reduce the enormous amount of raw data and focus on manageable datasets that are relevant to the research questions. Codes are tags, labels or names given to an idea or meaning in a data extract. For Charmaz (2006, p. 43), "Coding means categorizing segments of data with a short name that simultaneously summarizes and accounts for each piece of data". These codes were then synthesised, thematically mapped, then repeatedly grouped, refined and regrouped into themes/categories.

Data extracts	Initial Code	Potential theme
Quite often I go onto our shared resources within the department which people have collated, some have created themselves, some have researched on the Internet or some they have just picked up from previous schools and we collate those into topics	Sources of shared resources	Shared resource bank
for example, on Monday night diagnosticsquestions.com was demonstrated to us and that was shared and am trying to share that with the rest of the department	Resource sharing practices	

Figure 3-4. Data extract, with initial coding process

Figure 3-4 shows the data extract, the initial codes and the potential theme as constituting the processes of thematic data analysis. The sample data extracts were taken from the interview datasets.

There were four basic stages in the development from initial coding to themes and categories: (i) initial coding by the researcher; (ii) discussion and refinement of the initial coding by the researcher and one of the supervisors; (iii) an informal inter-coder reliability session between the researcher and a teacher from a non-study school which also produced a slight refinement of the coding; and (iv) a second meeting of the researcher and the supervisor for a final regrouping of themes/categories achieving 80-percent intercoder reliability.

Miles et al. (2014) suggest, besides data condensation, a thick and rich description of the condensed data provides a form of qualitative analysis and reporting. The process includes drawing conclusions from data and offering explanations; the inferences are linked to the literature and supported by the data. Through these processes, a holistic picture of the complex phenomenon of the appropriation of digital resources by mathematics teachers and its impact on their everyday practices could be interpreted, illuminated and understood.

3.5 Credibility and Trustworthiness of Study

Over the years, qualitative research methodology in education has been criticised for lacking rigour, credibility and trustworthiness (Cohen et al., 2007; Punch, 2009). However, what constitutes quality and rigour in qualitative research remains contentious. Schwandt, Lincoln, and Guba (2007) consider *dependability*, *credibility*, t*ransferability* and *confirmability* as trustworthiness criteria that could ensure the integrity of qualitative research findings. In an effort to enhance the quality and credibility of qualitative research, Patton (1990) proposes three distinct but interrelated factors as crucial. These elements are *rigorous methods*, *credibility of researcher* and *philosophical belief in the value of qualitative inquiry*. My research effort at ensuring rigour and quality aligns with these elements.

In terms of *rigorous methods*, time-tested qualitative data collection methods – SCAN observation protocol, interview guide and screen capture software – were piloted and adapted for the specifics of my research. This was to ensure that high-quality data that are systematically analysable were collected and occasions for bias minimised as much as possible. In safeguarding the rigour and integrity of this study, I adopted triangulation, intercoder-reliability checks, thick description (see next subsection) and the consideration of the possible impact of my role of researcher in the context of the phenomenon under investigation. Through the use of triangulation of data sources (where data from interview and screen capture are compared with those from observation for consistency over time), I hope I can reduce and compensate for the intrinsic biases of the individual methods and take advantage of their respective strengths.

Intercoder-reliability checks were undertaken involving the researcher and two other coders who are familiar with the issues of the mathematics teacher's appropriation. Intercoder reliability of 80 percent was obtained and further refinement made to the codes. In addition, the lessons learned from the pilot study, the outcomes of which enabled modification and improvement to the data collection strategies, data organisation and analysis (Yin, 2009) has supported my research quality.

The *credibility of the researcher* refers to the training, experience, track record, intellectual rigour, professional integrity and methodological competence the researcher is equipped with. In the course of my study, I have had a considerable number of supervisory sessions with my supervisors and it has been a very fruitful research apprenticeship for me in honing my own skills. My professional background in Philosophy (BA, 2000), Theology (B.Th., 2004), Mathematics Education (BSc. Ed., 2011) and Education, Technology and Society (MSc, 2012) and associated experiences have prepared me for the multi-disciplinary nature and demand of research in Mathematics Education. Over the years I have worked in various academic and pastoral institutions in Nigeria and the UK. I was the First Education outreach fellow (2016) for the School of Education, University of Leeds, as part of the University's widening participation activity. This includes engaging with students as they decided about their future and in inspiring the next generation to take on mathematics and science education. I have had opportunities to attend several methodology workshops, training seminars and conferences wherein I have had useful corrective and formative feedback that have enabled me to deepen my research outlook. This too has reinforced my philosophical belief in the value of qualitative research strategy as the most suitable for my research objectives. Patton (1990, p. 566) argues further that the researcher is the instrument in qualitative exploration and as such it is important to ask, "What experience, training, and perspective does the researcher bring to the field? What previous knowledge did the researcher bring to the research topic and to the study site?" The principle is to report personal and professional information that could have affected, either negatively or positively, the processes of data collection, analysis and interpretation. Becoming a proficient qualitative researcher is a time-consuming, complex process. The opportunities to engage with more experienced researchers and an extended period of supervision formed part of my own research apprenticeship and a series of graduate research training sessions assisted me in becoming a reflective, reflexive and professional researcher, which has been invaluable in the process of conducting this research.

Thick Description and Rich Setting

'Thick description' has been described as a strategy for enhancing the dependability and transferability of qualitative research (Robson, 1993; Schwandt et al., 2007). For Patton (1990, p. 437), "Thick, rich description provides the foundation for qualitative analysis and reporting. Good description takes the reader into the setting being described". Thick description refers to the detailed account of a researcher's field experiences in which the patterns of cultural and social relationships in the real-life context are made explicit (Holloway, 1997).

Deciding what constitutes 'good, rich and relevant thick description' of mathematics teachers' appropriation of digital resources for professional practices was not an easy undertaking nor a picture-perfect task for me. However, I do hope that all the evidence presented in the report and those included in the appendices capture and convey holistically, comprehensively and in context the 'total phenomena of the particular mathematics teacher's professional participation with digital resources.' I have endeavoured to highlight, as best as I could, the design and constructs of this research. It is hoped that this could enable other researchers to replicate the study methodologically. And I believe these rich, detailed and concrete descriptions of the mathematics teachers' profiles, contexts of practices, resources and interactions provide sufficient illumination for the cases in this study. Fusch and Ness (2015, p. 1409) argued for data saturation in terms of rich and thick data thus:

> The easiest way to differentiate between rich and thick data is to think of <u>rich as quality</u> and <u>thick as quantity</u>. Thick data is a lot of data; rich data is many-layered, intricate, detailed, nuanced, and more. One can have a lot of thick

> data that is not rich; conversely, one can have rich data but not a lot of it. The trick, if you will, is to have **both.**

My research reports the professional activities of mathematics teachers in sufficient detail and in such a way that the interpretations and conclusions drawn are transferable to other times, contexts and teachers.

3.6 Ethical Consideration

In this subsection I address some of the key ethical considerations associated with this study. Central amongst the considerations are the issues of informed consent, anonymity of participants and confidentiality, negotiating access and privacy (Cohen et al., 2007; Patton, 1990). At the onset of the study, the purpose of the study and intended methods of data collection were explained to the participating teachers in the consent letter I drafted (which was approved by the University of Leeds faculty research ethics committee). These were signed by the teachers as evidence of an agreement to participate in the research and also to withdraw before the end of the data collection phase. A promise of confidentiality was given and data anonymised in the reports. In taking these measures, I was informed and guided by the University of Leeds research ethics policy[13], the guidelines on ethical research practice for students and the ethical principles of the British Educational Research Association (Association, 2018). Due and thorough reflection were given to these ethical issues as briefly discussed below. The teachers were informed also of their right to withdraw from the research at any point in the research. The ethical consent protocol is attached in Appendix A.

3.6.1 Informed consent

Informed consent is a voluntary agreement to participate in research (Cohen et al., 2007). It provides participants with appropriately detailed information on the purpose

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13 http://ris.leeds.ac.uk/ris/info/70/ethics

of the study to enable potential participants make an informed, voluntary and rational decision to participate. In accordance with required policy regulations[14], an approved written informed consent was obtained from each of the seven participants. All the participants in the research were sufficiently informed of the process in which they are to be engaged, including the data to be collected and its methods of collection and why their participation is necessary. How and to whom the research findings will be reported and disseminated were made clear.

3.6.2 Anonymity and Confidentiality

To ensure confidentiality, all participating mathematics teachers in both the pilot and the main study were ensured anonymity. The essence of anonymising participants is that information provided by the teachers should in no way reveal their identity (Robson, 1993). Hence, none of the teachers were identified by their real names in the conference papers and published report of the research. Pseudonyms were used. As pseudonyms are aliases, pennames or any alternative and fictitious names that mask the teachers' and schools' true identities, in my report teachers are identified by such pseudonyms and schools identified by letters of the alphabet.

3.6.3 Key to Interviews and Screen Capture Extracts

I now briefly present a key to understanding how the interview and screen capture extracts are presented in Chapters 6-8:

(a) 2intK:#3 means the second (**2**) interview with **K**itty and the extracts is from the third minute of the audio-recording.

(b) RScrncpt: #5 means **R**ichelle's **Scr**een capture recording and extracts are from the fifth minute of the recording.

(c) Pre- and post-lesson interviews are identified as such and further explanations are given in the associated footnotes.

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14 Approval to undertake this study in line with the aforementioned guidelines is attached in Appendix A

CHAPTER 4
AN OVERVIEW OF DATASETS, DATA DESCRIPTIONS AND ANALYSES

In this chapter, I present an overview of the datasets that informed the case studies. I begin (section 4.1) by presenting the methodological processes of the data collection, a description of the datasets and their analyses. These datasets include the interview data, observation notes using SCAN, screen captures, documentary evidence and field notes. This chapter also describes (section 4.2) the emergence of the task types and the processes leading to their classification.

4.1 Processes of Data Collection, Datasets and Analysis

In the following sections and subsections (4.1.1-4.1.8), I present and describe the data collection and management processes, the various datasets and examples of how each data was analysed and the themes developed.

4.1.1 Interview Data

In Chapter 3, I discussed my rationale for using interview as one of my data collection methods (section 3.3). This was backed by the relevant literature. I also highlighted that the interview was guided by a hierarchical-focused interview strategy: that is, beginning the initial question from the highest level of generality leading to more specific questions that are aimed at addressing the research questions directly. The semi-structured interview questions are structured to match the four focusing themes of this study previously outlined on pages 7-8. On pages 50-52, I presented a set of semi-structured interview questions/probes used and the associated rationales behind these questions.

Table 4-1 below shows a description of the frequencies of the one-to-one interview datasets of the seven teachers from the three schools: A, B and C. This consists of semi-structured and pre- and post-lesson interviews.

Descriptions		School A				School B		School C
		Kitty	Emilia	Jimmy	Jose	Gray	Gavin	Richelle
One-to-one Interviews	Semi-structured	2	2	2	1	2	1	2
	Pre- and post-lesson	7	5	3	3	5	4	5

Table 4-1. Frequency of the one-to-one interviews with the teachers

I also provide a second table (Table 4-2) below showing a cumulative summary of the interview dataset for the three schools. In all, 44 interviews were carried out in the course of the study. In school A, I had a total number of 25 interviews with the four participating teachers; in school B, I had 12 with two teachers, and 7 in school C with one participating teacher. The difference in the number is a result of the availability of teacher-time, since most of the teachers had, besides their regular teaching duty, other pastoral and external educational activities: for instance, involvement with the maths hub in school A.

	Number of one-on-one interviews		
	School A	School B	School C
For each school	25	12	7
Total for A, B and C	44		

Table 4-2. Summary of interview datasets

In Table 4-1, two forms of the one-to-one interview are presented. The first row of the table lists the one-to-one semi-structured interviews, which lasted between 15 to 35 minutes each. For instance, one of Kitty’s semi-structured interviews lasted for 35 minutes; one of Richelle’s interviews lasted for 20 minutes. These interviews were audio-recorded and in those cases where I had two full interviews from a teacher, the first interview was fully transcribed. Subsequently, only segments I considered relevant to addressing the research questions were transcribed in the second interview. One interview each from the teachers were fully transcribed and annotated as one of the initial phases of the process of data analysis. The semi-structured interview across the three schools took place in a designated office where interruption or noise were minimal.

The second form of interview were the pre- and post-lesson interviews that lasted between 3-5 minutes. These mostly took place as I walked with the teacher to begin the lesson and mention is made of the lesson for the day, topic and digital technology that the teacher intends to use in the course of the lesson. Typically, it is at this point the teacher shares with me worksheets and relevant material that they intend to use during the lesson. The post-lesson interviews were the conversations I had with the teachers at the end of the lesson, where I sought clarification on some of the activities observed in the class and sought further explanation as they relate to the specific lesson observed. Records were made in my field notes of the teachers' responses as soon as time and space allowed. These field notes were subsequently updated at the end of the day when scheduled research activities were completed.

4.1.2 Interview Timeline

The initial visit to the three schools took place in September 2015. Research activities and data collection began in October 2015. Table 4-3 shows the interview timeline.

Year/		2015				2016					
Months		Sept	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun
Schools/Teachers											
A	Kitty		•αo		αo	o	o	o•			
	Emilia		αo	•α	o		o		•		
	Jimmy		α	•o			•	o			
	Jose				αo	•		o			
B	Gray				•		αo	o•	α		
	Gavin					•αo			αo		
C	Richelle		•	αo		o	α		•		o

Table 4-3. Timeline of interviews with teachers in schools A, B and C.

The meaning of the symbols in the table is explained thus:

• = Semi-structured interview
α = Pre-lesson interview
ο = Post-lesson interview

And in terms of the analysis, thematic data analysis is employed, and the iterative and reflexive processes applied are discussed with examples in subsection 3.4.

I now turn to the classroom and teachers' collective work datasets.

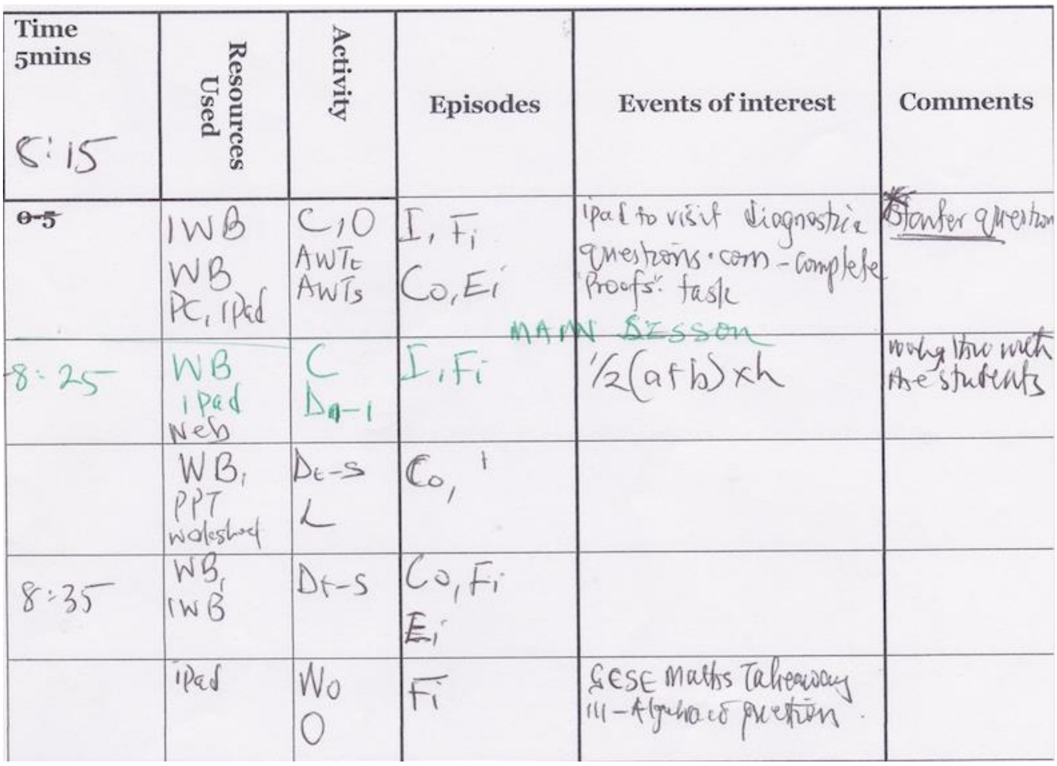
4.1.3 Classroom Observation and Teachers' Collective Work Dataset

In this subsection, I describe the processes of recording the lessons and teachers' collective work in the various instances where they formally and informally collaborate. These collective works take place in the mathematics departments, seminars, maths hub sessions and other mathematics-themed groups. These also include online fora for collaboration such as Facebook, Twitter and online professional development sessions.

4.1.4 Classroom Observation

In recording the lesson observation, I adapted and used the systematic classroom analysis notation (SCAN). Amongst several lesson observation schedules, I preferred SCAN since it focused on the major categories of teachers' and students' actions, and the features of the classroom activities that are central to my research. My rationale for observation, for my choice of SCAN and for its adapted form are discussed in more detail on page 53.

To address the research questions, SCAN was adapted in such a way as to enable me to address RQs 1.1, 2.1, 2.2, 3.1, 3.2, and 3.3 (presented on page 7). Table 3-3, p. 57 shows the SCAN shorthand codes for understanding the recording of the observation. Below I include a scanned sample of the actual classroom observation as a means of understanding how the codes were used and collated. The observation notes were taken roughly every 10 minutes during the lesson. Figure 4-1 is a sample of the record of a lesson taught by Kitty to Year 11 on ‘Algebraic expressions and proofs’ observed on 20/11/2015.



Time 5mins 8:15	Resources Used	Activity	Episodes	Events of interest	Comments
~~0-5~~	IWB WB PC, iPad	C, O AWTt AWTs	I, Fi Co, Ei	iPad to visit diagnostic questions.com - complete proofs: task	Starter question
			MAIN SESSION		
8:25	WB iPad Web	C Da-1	I, Fi	½(a+b)xh	working thro with the students
	WB, PPT worksheet	Dt-S L	Co,		
8:35	WB, IWB	Dt-S	Co, Fi Ei		
	iPad	Wo O	Fi	GCSE Maths Takeaway 11-Algebraic question	

Figure 4-1. Scanned sample of a SCAN schedule in use

Figure 4-1 above shows a sample of SCAN schedule divided into six rows indicating time interval for recording the observation, resources used by teachers or students in the lesson, activity, episodes of interest and my immediate comments on the lessons. The comment columns enabled me to commence initial descriptive coding while collecting the data. There are also five columns of data item focused on.

The mathematics teachers employed a variety of classroom seating arrangements. These included the whole class facing the board, students sitting in a horseshoe shape facing each other and the board to the side, and other instances of students all sat in pairs and groups. Usually, the set-up of the class informed where I sat to observe the lesson. My working principle in the lesson observations was to take a position where I could observe the entire class with minimal obstruction to lessons while maximising the opportunities to capture as much as possible of the lesson activities.

Having mastered the SCAN shorthand codes, I took notes as the lesson went on, trying to record as much as I could to observe. In the *resources used* column, I recorded all the digital and non-digital resources observed during the lessons and in a few instances where I was not sure what the resources were or the sources, the post-lesson interview was the opportunity to seek clarification. Whenever a conversation was not possible because the teacher is moving on to another duty, email exchanges were used to supplement the data. The data elicited through this column were used, in most part, to address the question on teacher's resources and this was triangulated by the other methods of data collections used.

The *Activity* and *Episode*s columns capture the layers of the actual ongoing interactions, the whole of the lesson delivery and students' engagement with tasks. For instance, an *activity level* in this sense could be **C** – that is, the teacher's exposition to the whole class, or **O** – teacher circulates in the class. An *episode* is a specific subunit of the classroom activities; this is an incident occurring that forms a part of the sequence of lesson activities. Episodes referred to what the teachers were doing during the lesson, such as facilitating or explaining. I defined mathematical and technological emphases in the episodes. Hence, **Fi**, refers to 'facilitating mathematical ideas' while **Ft** refers to 'facilitating technology' and **Ei** denotes 'explaining' mathematical ideas. Another included in the episode descriptors is **I** – meaning teacher initiating the lesson or a task. These various parsing of the lesson helps to create an evidence-based narrative for the case reports in the subsequent Chapters 6-8.

The *events of interest* are those teachers' or students' actions or interactions which I considered of interest to the research purpose and that could enable me to address the research questions. These could be incidents that reinforce an emerging theme or an outlier to the regular instances of lesson observation. For example, I considered students' use of an *iPad with the diagnostic questions* as an event of interest and elsewhere I highlighted *students taking a timed test with background music* as an event of interest. These events provide me with the opportunity to further seek clarifications from a specific teacher – say, Kitty or Emilia – as to why music was used during the test. In Table 4-4 below, I show the frequencies of the activities and episodes.

Teachers	Resources[15]	Lesson Activity Levels								Episodes							
		C	L	Wo	O	D_{2-2}	D_{t-1}	AwTt	AwTs	SS/ST	Co	Fi	Ei	Ft	D	R	I
Kitty (13)[16]	18	44	9	32	28	16	11	18	8	13	40	28	34	3	5	12	20
Emilia (9)	10	39	3	35	25	11	8	11	4	9	28	18	19	0	3	14	10
Jimmy (8)	8	35	7	17	11	10	13	12	9	8	28	17	17	0	2	8	7
Jose (7)	9	29	11	16	13	4	9	8	5	7	16	13	9	0	0	7	10
Gray (8)	11	37	19	19	20	10	15	10	4	8	32	22	24	1	6	9	11
Gavin (8)	12	32	16	20	22	8	14	12	3	8	26	20	23	1	7	9	13
Richelle (8)	8	34	15	23	19	10	8	13	7	8	36	18	29	4	2	12	9

Table 4-4. Frequencies of the pre-defined SCAN descriptors for the seven teachers

The codes are explained in more detail in the following pages. The activity levels relate to what the teachers and students where observed to be doing during the lessons. The episodes focus on teacher-led activities.

[15] This shows the number of distinct digital/ non-digital resources identified during the lesson observations.
[16] This indicates the number of times Kitty's classroom activities were observed (13 times).

4.1.5 Analysis of SCAN Datasets

The adapted SCAN schedule, in a sense, plays a double role, as an *instrument for data collection* and concurrently as a *form of data analysis*. Thus, as data is collected, a first level of data analysis takes place. The frequencies of resources used in class enabled me to identify distinct digital and non-digital resources used in the course of the lessons (e.g. Kitty - 18; Gray - 11; Richelle – 8, as shown in Table 4-4). I was able to match these resources with those the teachers had mentioned in their interviews and accessed on the screen capture data. In collating and classifying the resources in the case study report in Chapters 6-8, these identified resources in the lesson formed a large part of the teacher's resource system, the set of resources drawn from the collective and how the teachers modify them. This observation confirms, on several levels, the teacher's claim on resources as discussed in the case study reports. I found a range of different digital resources, non-digital resources and tools in classrooms in all three school contexts. All classrooms were equipped with interactive whiteboards and teachers had regular access to computers. The IWB has become a common feature in most UK secondary school classrooms (Kearney, Schuck, Aubusson, & Burke, 2018; Umameh, 2012). All teachers also had access to a range of resources available through the Internet and dedicated sites. This in part aided me in addressing one aspect of RQ 1.1 (*In what ways are mathematics teachers accessing, adapting and creating resources for classroom practices?*) and RQ 2.1 (*What resources do mathematics teachers access and use?*)

In terms of structuring the analysis of the observation dataset, the pre-defined SCAN descriptor codes which are applied, enabled me to organise the dataset in such a way as to make closer examination and analysis possible in a structured manner: for example, by examining relationships between SCAN descriptor codes as shown in Table 4-4 (**AwTt** - teachers' activity with technology and **AwTs** - students activity with technology during the lessons). I deduce from the frequencies of the SCAN descriptors (AwTt vs AwTs) that technologies were used more often by the teachers than the students. For instance, as shown in Table 4-4 above, while Kitty was observed using the digital resources 18 times, her students were observed using similar resources eight times. This approach was used for all the teachers and their students.

With regards to the forms of interactions under the activity level columns (**D_{2-2}** – students in pairs/group and **D_{t-1}** - student-teacher interaction), Table 4-4 shows that all seven teachers regularly used these forms of interaction with their students: Emilia (8), Gray (15) and Richelle (8 times).

In terms of the episode level, what the teacher was seen doing (**Co**, **Ei**, **Fi**), there is a substantial amount of evidence in the data to argue that the lessons were largely teacher-led, though a counter-argument could be that the frequencies are so because the research is primarily focused on the mathematics teachers and not so much on the students' activity. The episode levels enabled me to address RQ 3.1 (*What tasks do mathematics teachers give to their students?*) and to build a plausible narrative in the classification and presentation of the tasks as presented in the context of the lessons. How the tasks were identified using SCAN and the logic of tasks classifications are discussed in a subsequent subsection, 4.2.3.

I now present a table of the number of observations made in the three schools (schools A=37 B=16 and C= 8) and the cumulative number of observations (61) in this study.

	Number of Classroom Observations		
	School A	School B	School C
For each school	37	16	8
Total for A, B and C	61		

Table 4-5. Total number of classroom observations

Table 4-5 shows the individual total for each school and the total number of observations undertaken between October 2015 and June 2016.

4.1.6 Screen Capture Dataset (SnagIt)

In Chapter 3, subsection 3.3.4, I discussed the rationale for the use of screen capture as a tool for data collection. I also presented how it was used in my study, the limitations and ethical challenges and how these were addressed. In analysing the data collected using screen capture, I developed and used a matrix of data display and analysis using text, images and analytical codes shown in the aforementioned subsection and pages. This is further explained with an example in the subsequent pages in this chapter.

In this study the screen capture datasets are used to supplement the observation and interview datasets by providing video-recorded evidence of teachers' claims.

	Number of screen capture data		
	School A	School B	School C
For each school	6	3	2
Total for A, B and C	11		

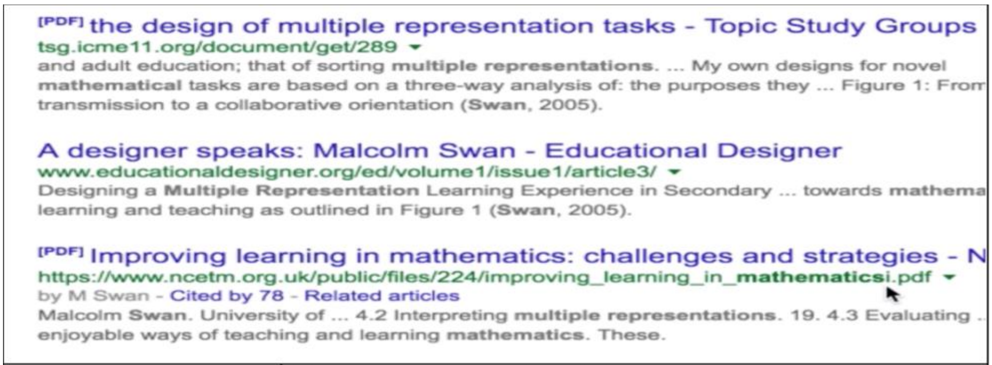
Table 4-6. Total number of times screen capture data was collected

Table 4-6 shows the number of times screen capture data was collected across the three schools. I now describe the matrix used for analysing the relevant section of the screen capture.

Table 4-7 below shows a sample of how the relevant sections of the screen capture data were analysed.

Matrix for screen capture analysis	
Topic	**Text**
Representations of graph	"I was looking for something that has different representations for the same data. Maybe, I will look for multiple representation." (Gray, GScrncpt 1: #4)
	Search: via google.co.uk

Screen shot



In vivo codes	'Representations of graphs' 'Different representations for the same data' 'Multiple representations'
Analytic code	Multiple representational task

Table 4-7. Matrix for screen capture analysis

Table 4-7 above shows the matrix I used in analysing the screen capture. The left row indicates that Gray was preparing a mathematics lesson on the *Topic*: *Representation Graph*. The adjacent cell *Text* is a transcript of the thinking aloud comment by Gray while searching the Internet for the relevant resources for the lesson preparation. www.google.co.uk was the search engine used in the process. The *screen shot* box displays the three resources Gray consulted and used in preparing the lesson. Sample tasks were obtained from these resources and some were modified to fit the needs of the class. The *in vivo codes* row represents the process of my thematic analysis using terms extracted from the resources and the teacher's lesson preparation sheet. In combination with the interview data and documents analysis, I arrived at defining the analytical code for this group of tasks as multiple representational tasks, and this is supported by evidence in the data, as Table 4-7 shows.

I collected a large volume of screen capture data across the seven teachers. Due to the sheer volume and multimodal nature of data contained in these screen capture recordings (the teachers' screen-captured activities, the mouse clicks, typing, editing, cutting and pasting, accessing online sites, audio etc.), my first reflective query and panic was how do I code, analyse and interpret the datasets in a reasonable and manageable manner. How do I ensure rigour in the processes without undermining the validity and trustworthiness of the study?

My first decision was to refer back to my theoretical framework (Activity Theoretic approaches and the documentational approach to didactics, as discussed on pp. 22-33) as it is useful in focusing my attention on the teachers' technology-mediated nested activities, actions and operations. I also considered the research questions of this study and what particular data from the screen capture could enable me to address specific research questions. This decision enabled me to focus on the precise selection of relevant segments of the vast amount of screen capture data I have addressing the research questions. For instance, Table 4-7 already provides in part pertinent information for addressing the research questions RQ 3.1, 3.2, 3.3 and 2.1 on page 7. It shows how the teachers access tasks and where they get their tasks from (i.e. NCETM, Malcom Swan resources, etc.). The tasks are modified using a snipping tool, and the cut-and-paste features of the Word document.

I outline the sequence of decisions and actions for the screen capture data management and analysis.

- Selected identified relevant segments for addressing particular research questions
- Transcribed the audio part and annotated the associated screen shots
- Coded the applicable data intended to inform the research questions
- Undertook a retrospective analysis and reconstruction of the teachers' on-screen activities, actions and operations

Through this sequence of decisions, I was able to manage the screen capture dataset and analyse relevant data towards developing case study reports for each of the seven mathematics teachers in the subsequent chapters. This was then complemented with supporting data from the observational notes and interview datasets in the course of the analysis and mapping of the processes of lesson planning and preparations.

The analysis of the screen capture data revealed the relevant features of teachers' lesson planning and structuring. Most often these aspects were invisible in the actual classroom lesson observations: the lesson planning processes, the traces of teacher decisions and choice-making captured by the actions and operations on the screen, sources of mathematics tasks, how these tasks are amended and modified (cutting and pasting, using snipping tools and editing), teacher design capability and the teachers' awareness of freely available depositories of mathematics resources.

In the next subsection, I describe the dataset on teachers' collective work.

4.1.7 Group Meetings/Teachers' Collective Work

In this study, I also focus on the collective perspective of teachers' collaborative work, their interactions regarding resources in their documentation work, and the consequences of these interactions and the resources for the teachers' community of mathematical practice. There are two main sources of data: firstly, the teachers' self-report on their face-to-face and online collective participation in the interviews; and secondly, my field notes taken during my observation as the teachers engaged in collaborative practices in diverse groups, as indicated in Table 4-9.

In Table 4-8 below, I show the frequencies of the collective work of teachers in which I was a participant observer.

	Number of Group Meetings		
	School A	School B	School C
For each school	8	3	4
Total for A, B and C	15		

Table 4-8. Total number of collective/group meetings I was present at

I also present Table 4-9, which indicates the various groups (face-to-face and online) that teachers participated in. Table 4-8 and Table 4-9 show a range of teacher collectives and form the basis for the descriptive analysis that follows in the subsequent Chapters 6-9 in the individual case reports.

	Teachers in Schools: A				**B**		**C**
Collectives	Kitty	Emilia	Jimmy	Jose	Gray	Gavin	Richelle
CPD	✓	✓	✓	✓	✓	✓	✓
TeachMeet	✓	✓	✓	✓	✓	✓	
Twitter	✓		✓	✓	✓		✓
England-China exchange	✓		✓				
Facebook			✓				✓
Department/staffroom	✓	✓	✓	✓	✓	✓	✓
Maths Hubs	✓	✓	✓				
Maths/English					✓	✓	✓
Maths Networks							✓
Web Conference						✓	
Group Leadership	✓	✓			✓	✓	✓
Mathematics Association			✓	✓			

Table 4-9. The collectives teachers participate in. Ticks indicate teacher participation in the corresponding collectives.

These collectives are discussed in more detail in Chapters 6-8. I now describe the frequencies in Table 4-8 and Table 4-9. In Table 4-8, for school A, my participation were in TeachMeet (2[17]), maths hub (2), Staffroom[18] (10), Subject knowledge enhancement course (1), Mathematics Association (1) and Group leadership meetings (1). School B included Staffroom (5), Faculty meeting (1) and Teacher-teaching-teachers session (1). School C included Staffroom (6), formal meeting (2) and feedback session on the Sheffield Network of mathematics teachers meeting (1). I have recorded the collectives' 'staffroom informal meeting' as one form of collective

17 The number in parentheses indicate the frequency of participation.

18 I record my staffroom informal participation as one form of collective participation.

participation. Even though in school A, I was present on 10 occasions, school B five times and school C six times, I consider these as one form of participation. Hence, it is counted as 1.

In Table 4-9 above I present the list of the various collectives I observed the teachers engaged in and other collectives the teachers reported that they are involved in individually. I have also indicated for each individual teacher the number of collectives they collaborate in within the three schools and the collectives that cut across the schools as well. For example, CPDs and departmental meetings are common features in all three schools; however, participation in the maths hub is peculiar to school A only. TeachMeet exists only in schools A and B, while Maths/English teachers meeting occurred in schools B and C. Five teachers out of the seven are on Twitter. These individual and school-level participations are discussed in further detail in Chapters 6-8.

I now turn to the processes of data analysis that led to the emergence of a rationale for regrouping the collectives into analytic themes.

In Table 4-10 below I present a phase of the data analysis process leading up to the various thematic groups in which I place the collectives. These grouping of the mathematics teachers' collectives exist:

- *By Context* (institutional and prescribed): CPD sessions, *TeachMeet*, maths hub, England-China exchange and formal departmental meeting and trainings
- *By Access* (open and voluntary): Maths network and Mathematical Association[19]
- *By Mode of Participation* (face-to-face and/or online): Twitter, Facebook and Web Conferencing

[19] The Mathematical Association is a professional society concerned with mathematics education in the UK.

- *By Form of Organisation* (formal and informal): Maths/English and lead teacher groups that teachers could belong to formally or informally
- *By Geographical Region* (Yorkshire and Derbyshire)

The first process in the data analysis was to familiarise myself with the datasets, especially the segments of the interview dataset relating to teacher collectives. I then manually highlighted segments that had potential to address the research questions. Subsequently, data were extracted from the dataset and initial codes identified.

Table 4-10 is a sample of the processes which led to the first theme: *By context* (institution and prescribed).

Phase of data analysis towards understanding the collectives	
Raw data extracts from the interviews	Initial codes
We do have professional development sessions. Last year for example when we were doing a lot of work on the Singapore bar modelling, we had a CPD session every week 1IntK: #15	Professional development Singapore bar modelling CPD session every week
We are actually the lead school for the White Rose Maths Hub across the country. I myself actually take part in delivering CPD to external Maths teachers. 1IntK: #17	Lead school for maths hub CPD to external Maths teachers Personal involvement
Yes definitely, we have the formal, like when we have departmental meetings and training sessions. We talk in more professional manner and do delicate tasks. 2intR: #14	Formal Departmental meetings and training We talk in more professional manner
For instance, yesterday, I gave a presentation to all the members of staff, we are doing Teach-meet, where members of staff from all over the school, share all the different experiences they have and to demonstrate their ideas and many of those are technology based.	All the members of staff 'Teach-meet' Share all the different experiences To demonstrate their ideas Technology

1intJ: #25	
We have faculty meetings fairly regularly, maybe every 6 or 8 weeks or so, and that is very much a case where we are able to share good practice, share experiences. 1intGr: #21	Faculty meetings every 6 or 8 weeks To share good practice, Share experiences

Table 4-10. A phase of data analysis towards understanding the collectives

The left-hand column of Table 4-10 shows in verbatim the relevant meaning units from the interview datasets. The right-hand column shows the emerging initial codes. These 16 initial codes were reviewed, refined and regrouped into thematic maps. Figure 4-2 below, shows the analysis for school A and the eventual thematic mappings out of which the final theme emerged.

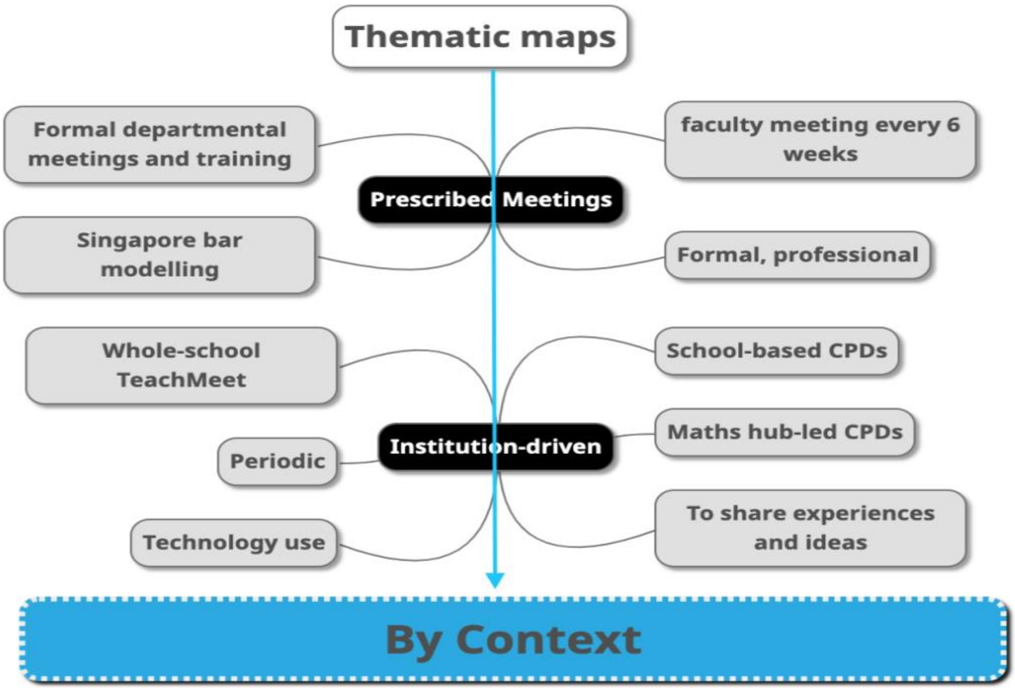


Figure 4-2. Thematic maps leading to the emergence of a theme

Figure 4-2 shows the thematic mapping of the collectives of school A. From the *18 initial codes* reviewed, refined and sorted into *10 potential themes* (in the grey boxes in Figure 4-2). These potential themes were further regrouped around *two broad sub-themes*: *prescribed meetings* and *institution-driven meetings*. The prescribed meetings had a specific departmental focus, occurred in the mix of formal and informal settings and took place at set intervals. These meetings were predominantly organised by the teachers within the mathematics department and tailored to focus on mathematics specifically. The *institution-driven meetings,* in my opinion, were mostly generic; these dealt with issues of pedagogy, new technology resources being introduced to the teachers and creating fora for teachers across school to share and exchange ideas and experiences. Except for the case of the maths hub, which is specific to mathematics, the meetings were always driven by the institutions, but in the case of school A, the school's and the maths-hub leaders took the initiative. Hence, my choice of sub-theme. The overall organising theme is: *By Context.*

For instance, the theme *by context*, refers to all the collectives/group meeting driven by institutional prescriptions. This also refers to the context of the department and the internal activities and professional goals and aspirations. It also takes into consideration prevailing external educational changes and policy: such as, the adoption of the new curriculum and use of new digital resources. These are the groups that teachers belong to and attend by virtue of their professional practice, and their existence is solely dependent on the school or maths hub recommendations. In most of the cases, the collective engagement is periodic, formal and professional. Further elaborations are made in the case study report in the subsequent chapters on these themes, on the working of the collectives, and the advantages the mathematics teachers claim they bring to their professional and classroom practices.

In the next subsection, I provide a description of the documentary datasets and present how they were used in the presentation of the case study reports.

4.1.8 Documentary Data

In Chapter 3, subsection 3.3.3, p. 60, I discussed documentary data and my rationale for using this type of data in this study. The documents are complementary to the interviews, observations and screen capture datasets. A range of documents were collected and collated for each mathematics teacher. These documents were analysed and grouped by their content and forms of availability (such as hard copy, soft copy, internet links etc.).

In this study collected documents were analysed by content and subdivided into two broad groups. First, the *official documents* (for instance, school-based documents like the curriculum, school ethos), government policy documents (OfSTED) and documents from educational bodies (for example, the National Centre for Excellence in the Teaching of Mathematics (NCETM), maths hub). This group of documents relate to a specific educational policy-issuing authority within and outside of the school. The second subdivision of the documents are defined by their *mathematics content*; these are the teachers' lesson plans, students' worksheets, excerpts from textbooks, past questions and schemes of work. These documents exist online and in hard copies. In several instances, teachers indicated documents used in lesson planning and the sources of the tasks, but upon review and analysis of these documents, teachers adapted and implemented the mathematics tasks in their lessons. These observations are discussed in more detail in the context of tasks analysis and classification later in this chapter. Subsequently, in the discussion in Chapter 10, I appraise how official pronouncements and recommendations are interpreted and implemented in teachers' professional practice or not.

4.1.9 Summary

In the section above, I presented descriptions and clarifications on the methodological processes of the data collection, rationale and decisions made in the selection of relevant datasets toward addressing the research questions. I used tables, figures and diagrams to present, manage and display the datasets and show representative samples of the data coding and analysis strategies across the various data types. In doing this, I drew on various datasets that I collected. I described in this chapter the processes of my data collection, data management, the various datasets (interview, observation, SCAN, screen capture and documentary data) and how these were analysed. I used several representative examples to indicate the processes and the rationale for the choices I made.

In Table 4-11 below, I present a summary of all the data collected for this study.

Data collection methods	**Teachers of School A**			
	Kitty	Emilia	Jimmy	Jose
Observation	13	9	8	7
1-1 Interviews	2(7)[20]	2(5)	2(3)	1(3)
SnagIt (Screen capture)	2	2	1	1
Documents[21]				
Field notes	For these group of teachers			
Group meetings[22]	8			
	Teachers of School B			
	Gray		Gavin	
Observation	8		8	
1-1 Interviews	2(5)		1(4)	
SnagIt (Screen capture)	2		1	
Documents				
Field notes	For these group of teachers			
Group meetings	3			
	Teacher of School C			
	Richelle			
Observation	8			

20 The number in the parentheses refers to the number of times I had a pre- or post-lesson conversation with the teacher and these were recorded in my field notes.

21 The number of documents has not been indicated; this is because they consist of hard and soft copies, and links to the huge online resources used. Relevant resources are noted and discussed in the case study report in the following chapters.

22 This entails my participation in teacher collectives like TeachMeet, the maths hub sessions or sessions where a teacher teaches colleagues a new topic in the curriculum, i.e. sort algorithms in school B. Staying in the staff room in between observation times is not counted here.

1-1 Interviews	2(5)
SnagIt (Screen capture)	2
Documents	
Field notes	Recorded for Richelle
Group meetings	4

Table 4-11. Summary of all the data collected for the seven teachers

In Table 4-11, the number in the parentheses refers to the number of times I had a pre or post lesson conversation with the teacher, and these were recorded in my field notes: for instance, **2(5)** means, I had two semi-structured interviews and five pre- or post-lesson conversations with Emilia, Gray and Richelle, respectively. The number of documents has not been indicated; this is because they consist of hard and soft copies, and links to the huge online resources used. The resources observed in use and mentioned by the teachers are noted and discussed in the case study reports in Chapters 6-9.

The number shown against the group meeting entails the frequency of my participation in teacher collectives like TeachMeet, maths hub sessions or sessions where a teacher teaches colleagues a new topic in the curriculum, i.e. sort algorithms in school B.

I now turn to the description and classification of mathematics tasks.

4.2 The Emergence of Task Types and Classifications

In my conception, a task is an element of an instructional sequence of mathematics activities. In Chapter 2, subsection 2.1.3, p.19, I presented a literature review on mathematics tasks and associated issues in mathematics education research with emphasis on technology use and task design. In this section, I review the emergence of the task types and their classifications using a structuring strategy informed by prevailing education policy context, the datasets and its analyses. I begin in subsection 4.2.1 by identifying the influences on the mathematics teachers' lesson structuring and practices with tasks and continue with subsections 4.2.2 and 4.2.3 in describing the emergence of task types and their classifications.

4.2.1 Influences on Teachers' Practices with Tasks

During my research, there were three broad innovations in mathematics education that were impacting on schools and mathematics teachers' practices as observed and reported by participating teachers. First, there was the introduction of the new National Curriculum, which became statutory in September 2015. Secondly, the regional *maths hubs* were established across the UK, and thirdly of the England-China Mathematics Education Innovation Research Project emerged (Boylan, Maxwell, Wolstenholme, Jay, & Demack, 2018). These were mentioned previously in the literature review in Chapter 2. Here, I consider them in relation to teachers' practices with tasks and their influences on teachers' choices. One significant observation is the wave of renewed interest in *Mastery approach to teaching mathematics and the associated lesson structuring templates*. This development forms a basis for understanding in part how the participating teachers structured their lessons, and conceived and employed a variety of tasks.

4.2.2 Identifying the Distinct Types of Tasks

In the course of exploring the datasets with a view to addressing the research questions on tasks on p.7, 15 distinct task types were identified. These 15 distinct task types are presented in Table 4-12 below and what follows is the descriptions and rationales for identifying these task types.

Tasks Types and Associated Abbreviations						
	Abbreviations	**Task Types**			**Abbrev.**	**Task Types**
1	**ST**	Starter		9	**MOD**	Modelling
2	**SA**	Skill audit		10	**CO**	Consolidation
3	**D**	Definition		11	**MIX**	Mixed ability
4	**IN**	Interactive		12	**EM**	Emergent
5	**DS**	Diagnostics		13	**FA**	Formative
6	**D/V**	Differentiations/ Variations		14	**PS**	Peer/self-assessment

7	**EX**	Extension		15	**PL**	Plenary
8	**MR**	Multiple representation				

Table 4-12. Task types and associated abbreviations

Here, I will explain the processes of task identification, states what makes them distinct and point out where this overlaps.

These rationales and principles guided my identification and classification of the tasks.

- The teacher-specific labelling of the tasks on the board, in the lesson worksheet, and collected in the interview/post-lesson conversations. For instance, starter tasks, formative assessment tasks, definitions, differentiation/variation, modelling etc. were indicated as such by the teachers themselves.

- Tasks identified in textbooks, the schemes of work and online resources used by the teachers. For example, extension tasks, multiple representation tasks and modelling belong to this classification.

- Tasks I have identified through the analysis of the datasets. Within the groups of tasks, I tagged these as *skill audit tasks;* these are tasks like multiplication tables, quizzes, and addition and subtraction activities. The goal of these tasks, as the teachers indicated, was to enable the student to become confident in recall and use of basic number operations. Also, in this category are the 'emergent' and mixed-ability task types.

- Finally, I only included such tasks that were observed in the lessons and teachers reported they had used. Where there are overlaps in the task types, I have provided further explanations in the case study reports. (There are instances were starter tasks for one teacher are plenary tasks or formative assessment tasks for another.)

Table 4-12 above shows the 15 task types and I have also included related abbreviations for ease of in-text referencing. These tasks are listed from the SCAN observation notes, screen capture data, mathematics-specific documents and interview transcripts. A more detailed description of each type of task is given, and the teacher-stated purpose of each task type is presented in the case study report in the subsequent Chapters 6-8. Here, my focus is on presenting the datasets on tasks, the emergence of these tasks, and my considerations in the process of classifying these tasks.

I also present Table 4-13, which indicates the task type by teachers across the three schools.

Tasks Types	**Teachers in Schools A, B and C**						
	Kitty	Emilia	Jimmy	Jose	Gray	Gavin	Richelle
ST	•	•	•	•	•	•	•
SA	•	•					
D					•	•	•
IN	•	•					
DS	•	•		•		•	
D/V	•	•					
EX					•	•	•
MR	•				•		
MOD	•	•					•
CO							•
MIX	•	•					
EM			•				•
FA	•	•	•	•	•	•	•
PS	•	•			•		
PL						•	•

Table 4-13 Task types by individual teachers

Table 4-13 above shows the task types as used by teachers in their lessons. My initial thought was to look to the three-part lesson plan for explanation, but on a closer examination, the teachers were using different approaches to their lesson structures. Since all the teachers included *starter tasks*, and two teachers included *plenary* as elements in the sequence of their lesson activity. *Starter* and *plenary* are constructs drawn from the three-part lesson plan discussed in the literature review.

However, I realised the teachers were not consistent in all their lessons with this form of lesson structuring. Upon further analysis of the official and mathematics-related documents like the scheme of work, it emerged from the documents that there are several underlying influences (shown in Table 4-14 below) that were impacting on the way the teachers structured their lessons and consequently the types of tasks they pose in the sequence of lesson activity.

At the core of the varied influences mentioned in the opening paragraph of this subsection and shared across the three schools is the 'mastery teaching phenomenon'. Recent research has argued that the mastery approach to teaching mathematics is central to current policy in mathematics education in England, influenced by East Asian success in transnational assessments (Boylan et al., 2018).
In Table 4-14 below, I present a recommended lesson structure drawn from the central resources used by each of the three schools. While none of the teachers was observed using any of these lesson plan structures in full, some elements were present in their lesson structure.

	Mastery Teaching Approaches		
	School A	School B	School C
Influences	UK-China exchange/ Maths hub[23]	Pearson curriculum[24]	GCSE Mastery Scheme of work[25]
Lesson structure	Starter tasks I do (learn from me) You do (on your own) We do (whole class) We do (group) We do (pairs)	Prior knowledge Master (teacher-led) Problem solving Check (Short FA) *Strengthen or extend* Test	Starter Investigation Extension Assessment

Table 4-14 Mastery teaching approaches, influences and associated lesson structure.

23 http://www.mathshubs.org.uk/what-maths-hubs-are-doing/teaching-for-mastery/
24 https://www.pearsonschoolsandfecolleges.co.uk/secondary/Mathematics/11-16/GreatMathsTeaching/maths-5-year-curriculum-11-16.aspx
25 Available on school website and screen shot used in the case description.

For example, in school C, Richelle uses only the terms ‘starter’ and ‘extension’ explicitly in her worksheet. In school B, while tasks under lesson structure provided by the Pearson curriculum designer were used, Gray’s and Gavin’s lessons were structured differently. In my opinion, teachers in the three schools were using the official resources as one of the many resources they can draw tasks and inspiration from.

Teachers in school A adopt various maths-hub-recommended key elements of the Chinese variant of the mastery-teaching approach for secondary schools in England. This was considered alongside the uptake of the new curriculum. Here, I outline some of these elements:

- Lesson designed to have a high level of teacher-student and student-student interactions
- Teachers should keep the whole class learning together
- The use of differentiation, variation and multiple representations
- Recommendation of intelligent practice and regular use of formative assessments
- Number facts using multiplication tables and formulae to avoid cognitive overload[26]

The lesson structure in school A column is what is to be promoted in the class and in the training of other teachers through the maths hub led by school A.
In school B, the *Pearson ActiveTeach and ActiveLearn* digital learning and teaching platform is the central resources for mathematics teachers’ activities. (This is discussed in the case study report for school B in Chapter 7.) The Pearson website stated this with regards to mastery approach: “Mastering mathematics involves all students achieving a true depth of understanding of mathematical concepts. Our UK-built approach to teaching for mastery underpins our 11-16 schemes, KS3 Maths

26 https://www.ncetm.org.uk/public/files/19990433/Developing_mastery_in_mathematics_october_2014.pdf

Progress and Edexcel GCSE (9-1) Mathematics[27]". The Pearson resources designers seem to argue for a "UK-built approach to teaching for mastery" and in the second columns for school B in Table 4-14, the e-textbook and related scheme of work recommend the above five-part lesson structure and strategy for implementing the Pearson variant of mastery teaching.

School C subscribed to *Collins Connect*, an educational publishing company which provides resources for school C. They claim that "based on the successful maths programme delivered in Shanghai, these comprehensive resources provide authentic mastery practice adapted for the English curriculum and we help your class to achieve mastery in maths with The Shanghai Maths Project"[28]. As such they make available to mathematics teachers in school C a so-called 'GCSE mastery scheme of work' and in the third column under school C is how Richelle adapted and implemented this in her lesson structure.

In spite of the renewed wave of interest in the mastery teaching approach and associated lesson design, there is still a high level of uncertainty among researchers as to whether this will be a 'seasonal vogue' or lead to lasting changes in practices and, if changes are effected, whether they will improve learners' outcomes (Boylan et al., 2018). From the vantage point of my research observation and datasets, the three schools and the seven teachers in this study seem to be 'enthusiastic' about the prospect of their particular version of the mastery teaching approach.

The mastery teaching approach subscribed to by each school varies considerably from school to school. From the data, each teacher seems to have interpreted, adapted and implemented their personal idea of mastery and what they deemed fit for their teaching purposes. Also, the teachers, I argue, relying on the evidence from my data,

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27 https://www.pearsonschoolsandfecolleges.co.uk/secondary/Mathematics/11-16/GreatMathsTeaching/maths-5-year-curriculum-11-16.aspx
28 https://collins.co.uk/pages/secondary-maths-the-shanghai-maths-project

have structured and presented their tasks in a sort of 'mix and match' from previous knowledge of the three-part lesson structure and the emerging mastery teaching strategy. All seven teachers retained the use of elements that reflect the three-part lesson plan (starter, plenary, extension) and merging these with elements associated with the mastery approach (such as differentiation and variation). This extended background provides the context and the subtle influences through which I identified the 15 distinct tasks and their classifications.

4.2.3 Logic of Task Classifications

Following from the above background to the emergence of task types and the diverse manner in which teachers implemented their lessons, using their lesson structures to categorise the tasks and to understand task types mentioned in the interviews proved to be problematic.

I went back to the SCAN datasets and decided to explore closely using a rough estimate of 10-minute time intervals of observation (as shown in Figure 4-1, p. 77) across the typical 60-minute-long lessons. I then collated, across the seven teachers, lists of all the tasks used in the first 10 minutes of their lessons. Below is a table showing a sample of the first process in my task classifications.

	Tasks posed by teachers in the first 10 minutes of lessons
Kitty	Starter tasks, check 20 tests Multiplication tables, quizzes, and definitions
Emilia	Starter task, addition and subtractions tasks, multiplication tables and maths games.
Jimmy	Starter tasks, check 20, maths challenge and multiplication tables
Jose	Starter tasks, timed test/peer marking, multiplication tables and 5-a-day tasks
Gray	Starter tasks, definitions and introduction of new concepts
Gavin	Fluency/warm up, definitions, quizzes, maths games and starter tasks
Richelle	Starter tasks, 5-a-day tasks, and keywords and definitions

Table 4-15. Tasks posed by teachers in the first 10 minutes of lessons

As shown in Table 4-15, all teachers used *starter* tasks. The use of *multiplication tables* is typical of school A at the time of the study. Three task types are used by teachers across the three schools: *5-a-day tasks* (Jose and Richelle), *maths game tasks* (Emilia and Gavin), and *definition tasks* (Kitty, Gray, Gavin and Richelle).

My next consideration was to find a way of grouping all these tasks under a unifying theme. Drawing again on one of the SCAN descriptors on what the teacher was doing in class (**I**- Initiating the lesson), I felt this was sufficient in classifying all these tasks under a common theme. Hence, these task types occurring within roughly the first 10 minutes of the lesson were categorised as ***Tasks Initiating Lesson***. This same logic was used in the classifications of the rest of the tasks with the mindset of keeping the narrative of the case report as close as possible to the sequence of the lesson activities. Building on the time interval analysis, I developed four categories as a structuring framework (shown in Table 4-16) for classifying all the tasks and presenting them in the case study report in the subsequent chapters.

Structuring Framework for Task Categorisation	
Categories	**Task Types**
Tasks Initiating Lesson	Starter, skill audit[29] and definition
Posing the Main Mathematics Tasks	Interactive, diagnostics, differentiations/ variations, multiple representations, modelling and mixed ability tasks
Demonstration of Mathematics Learning	Peer/self-assessment and formative assessment and emergent tasks
Concluding the Lesson	Extension, consolidation and plenary tasks

Table 4-16. Structuring framework for task categorisation

Table 4-16 shows the structuring framework for the task categories and the task types under each category. Sample tasks and details of what these tasks demonstrate are presented in fuller detail in the case study reports.

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29 Multiplication tables, check 20 and 5-a-day tasks belong together as skill audit tasks.

The next question I addressed is the issue of selecting a representative sample task for each teacher. Two working principles guided my choice.

1. That such a task occurred in half or more of the lessons observed (For instance, in the eight lessons observed for Richelle, a task that occurred in four or more of the lessons is selected as representative of the teacher's practice.)
2. Tasks that I considered unique and relevant to the central purpose of study are also selected, especially tasks with digital technology (such as emergent tasks and formative assessment tasks using Plickers).

4.2.4 Summary

In this section, using concepts drawn from the literature and the theoretical framework, I examined the relevant documents collected. I describe the prevailing education policy context for the emergence of a variety of tasks used by the teachers. I also presented the decisions and rationales leading up to task categorisations and how tasks representative of each teacher's practices was defined and selected.

CHAPTER 5
A STRUCTURE FOR THE CASE STUDIES

This chapter prepares the reader for the case studies in Chapters 6-8, which provide data and information that will allow me to address the research questions (RQs) in Chapter 9. My intention is to illuminate the context in which the mathematics teachers work and how they appropriate digital and non-digital resources for their professional practices. This chapter also presents an overall organising structure for the qualitative analysis and reporting through concrete case descriptions (Creswell, 2013; Patton, 1990; Yin, 2009). The case descriptions, analysis and interpretations of mathematics teachers' engagement with and through resources and the collectives are presented from the perspectives of the teachers. My impressions, interpretations and meanings, inferred from the data, are also presented. Here, I consider that "the case story is itself the result" (Flyvbjerg, 2006, p. 238). The structure grounds my findings in many illustrative examples, tables and excerpts that depict the diverse aspects of mathematics teachers' professional practices with digital resources in their everyday in-school and out-of-school activities. My goal is to provide an understanding of how the everchanging context of mathematics teachers' use of digital and non-digital resources fit together, either meaningfully or not.

Recall that the RQs are presented (Chapter 1) in four foci: (i) the mathematics teachers; (ii) the resources; (iii) the tasks; and (iv) the collectives. Focus 1 has a single RQ:

RQ 1 In what ways are mathematics teachers accessing, adapting and creating resources for classroom practices?

RQ 1 is at the core of this thesis. In the remainder of this chapter I present and discuss six structuring themes drawn from the foci and themes that emerged from the coding. These themes discussed below will form the organising basis for presenting the case studies.

5.1 Structuring Themes

A well-defined case constitutes a unit of analysis that is purposeful, holistic and context-sensitive. It is this principle that has guided my delineation of the unit of analysis in this research. The unit of analysis presented in Chapter 3 of this research is restated again:

A mathematics teacher who is a member of mathematics department in a school and is designated to teach in a context where digital resources are available for use.

Brewer and Hunter (1989) proposed six types of units of analysis: individuals, attributes of individuals, actions and interactions, residues and artefacts of behaviour, settings, incidents and events, and the collective. My research adopts an embedded unit of analysis to capture the nested components of interactions and networks that influence the mathematics teachers' professional activities with digital resources. The intent is to achieve an in-depth description of the real-life settings and events of mathematics teachers' engagement with resources and people. As Yin (2009) argues, in understanding real-life phenomenon in depth, contextual conditions are indispensable. I list below the structuring themes based on the four foci of this research and the subthemes that supplements the first focus. I now discuss them in turn as they guide the case descriptions.

- The mathematics teacher
    - The school context
    - The profile of the mathematics teacher
    - The role in the department/school
- The resources of the teacher
- The tasks of the teacher
- The collectives

5.2 The Mathematics Teacher

The mathematics teacher is at the core of this research and other themes are related to what the teacher does in class, with tasks and resources and the teacher's engagement with the collectives. With regards to the mathematics teacher, I now present the context of the school wherein teachers work, the profile of the teacher and the role in the department or school.

5.2.1 The School Context

Schools exist in a socially meaningful and goal-orientated environment and are often influenced by several factors. I recall here the general context of English secondary school with focus on mathematics teaching and learning, as previously discussed in Chapter 1. It is vital therefore for the researcher to be sensitive to and capture the political, social and cultural contexts of the school. *The Oxford English Dictionary* defines context as "The circumstances that form the setting for an event, statement, or idea, and in terms of which it can be fully understood"[30]. This qualitative research is intended to enable an understanding of the mathematics teachers' experiences with digital resources and the context which makes their everyday engagements meaningful. For the most part, the mathematics teachers' local contexts act as 'givens' that could constrain or enable their everyday practices and the ways in which they can access resources and opportunities for professional practice and growth. Context shapes the way an activity is interpreted and understood by those involved in a particular practice. For instance, government teaching standards, availability of teaching resources and a particular school ethos influence the way teachers undertake their practice. Morgan (2006) distinguishes two dimensions of context: the *context of situation,* in which an individual is immediately embedded (for instance, a student or teacher at school in the immediate situation of the classroom, the activity with digital resources, within a current engagement with an educational experience), and the *context of culture*, which includes the context of situation and the overall setting,

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30 https://en.oxforddictionaries.com/definition/context

school institution and the world outside the school. The context of culture could be argued to approximate the combination of the in-school and out-of-school contexts (Sutherland, 2007).

In recent years, mathematics education research has paid increasing attention to the issue of context. Kynigos and Psycharis (2009, p. 269) describe this growing sensitivity of the mathematics education research community towards context thus:

> shift of the focus of investigation in the field of mathematics education community to study context not as a backdrop of the teachers'/students' activities but rather as an integral part of the teaching/learning process that influences the research objectives, the followed approach as well as the analysis of the results.

I believe that local contextual complexity and the peculiarity of individual school contexts impact and shape teachers' professional activities. This includes the *context of situation* and *context of culture*, which surround and weave together the whole use of digital and non-digital resources by mathematics teachers. Context, therefore, in my estimation has a structuring and transforming impact on teachers' practices.

I now discuss the profile of the mathematics teacher.

5.2.2 The Profile of the Mathematics Teacher

This research takes into consideration the possible effects of teachers' demographic profiles (gender, age, education and years of teaching experience) on their professional practices. Inan and Lowther (2010) argued that teachers' demographic characteristics (years of teaching and age) and their years of teaching have significant positive effects on their readiness to embrace technology integration, while age was non-significant. In related previous research, the findings suggest that female teachers may be less confident than males about using technology, while with regards to the years of teaching the study suggested no significant effect (Pierce & Ball, 2009). This, in my opinion, indicates that the effects of teachers' demographics profiles on their professional practices may vary from teacher to teacher and from context to context.

The mathematics teachers' demographic profiles provided data on basic characteristics which are of interest in this research and are informative elements in the analysis. The teacher demographic profile consists of self-reported demographic characteristics elicited through a form[31]. This provided data on the teacher's age, previous work experience (if there is any), educational qualifications and specialisation, years of teaching, year groups taught and their role in the department or school. Although the teacher profile is not a central focus of my research, it provides background information which may be of use in my effort to understand the appropriation and use of resources in their mathematics teaching practices.

5.2.3 Role in the Department/School

Teachers in the UK take on a wide range of duties besides teaching in the department and school leadership (Day, Hopkins, Harris, & Ahtaridou, 2009). Scholars suggest that leadership has significant impacts on the quality of school organisation and on students' learning, and serves as a catalyst for unleashing the potential capacities that already exist (Leithwood, Harris, & Hopkins, 2008). The mathematics teacher's role in the school or department could be an important factor in shaping their work, how teaching is done in the schools and how they access resources and support collaboration. The teacher as a leader has a responsibility to help promote teaching and learning preconditions such as working habits, attitudes, knowledge, motivation and collaboration among colleagues.

In the literature, teacher leaders have been designated variously as coordinator, coach, specialist, lead teacher, department chair, mentor teacher, and teacher trainer just to mention a few (Mangin & Stoelinga, 2008). Neumerski (2013, p. 320) argued that "there is little consensus around what constitutes 'teacher leadership' … it tends to be an umbrella term referring to a myriad of work". For Lovett (2018), leadership is fluid space where multiple leaders engage in leadership practices with the intent of

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31 This form is attached in Appendix E.

leadership roles connected to students being distributed in order to enhance their learning. However, this is subject to what those in formal leadership positions permit and constrain through expected compliance standards. Although leadership style is not a focus of this study, it forms anecdotal evidence worthy of mention since it impacts on teachers' roles and activities in the department and school.

In one study, three types of leadership practices emerged from the findings: strategic, operational and distributive leadership (Maughan, Teeman, & Wilson, 2012). With *strategic leadership*, they claim, the leadership sets out a clear and realistic vision adapted to the local context of the particular school where teachers are inspired and allowed to take ownership of the change expected in their teaching practices. *Operational leaders* refer to the leader's ability to create a culture and the right climate of change through the provision of periodic learning and sharing opportunities. A third practice, *distributed leadership*, entails the mutual confidence in sharing of leadership responsibilities across the department and school. I believe that a teacher's role and leadership could have a significant influence on their appropriation of resources, and that they in turn could act as change drivers for the use of digital and non-digital resources in their teaching practices. In the subsequent chapters (6-8), the specific role(s) of each teacher is discussed.

5.3 The Resources of the Teacher

In this subsection, I describe how teacher resources are classified and presented in the case study. I then illustrate this with an example of a table of such classifications.
As discussed earlier in Chapter 2, mathematics teaching resources in this study include everything that supports and facilitates teachers' practices. This includes text resources; ICT resources; other material resources (e.g. handheld students' whiteboards); human resources (people and collectives – face-to-face and virtual); digital curriculum resources; and social and cognitive resources. A crucial undertaking for teachers is their interactions with curriculum materials, with other resources and with colleagues. In their work with resources they search, select, create,

modify, and adapt them to support their professional activities and also co-design and share with their colleagues. With the advent of digitisation, there is a growing proliferation of resources that has profoundly changed the dynamics of sharing knowledge and offers new forms of communication, association and networking amongst teachers.

In the process of the analysis on resources, first, I systematically noted the resources I observed in use in the lesson and at what point each was used: resources cited and mentioned during the interviews; resources referred to during the screen capture dialogue; and resources inferred from the documents. Second, in order to give a meaningful order to the array of resources, they were logically classified into manageable divisions. This is also reported in Umameh and Monaghan (2017). The dataset relevant to addressing RQ 2. 1 (*What resource do mathematics teachers access and use?*) on page 7 were coded and thematically mapped, and repeatedly grouped and regrouped into categories (the term 'category' is used for a set of codes). The coding process produced many codes/categories. The four stages in the development from initial coding/categories to the classification in Table 5-1 below are described in detail in Chapter 3, p. 37.

I then drew on the 'law of excluded middle' (A or not A) from classical mathematics to categorise the resources logically into 'human' and 'non-human'. This provides a as one way of partitioning all the resources a teacher may use. Taking this logical division further, I partitioned human resources into those where there is 'physical contact' and those where there is 'not physical contact'; similarly, non-human resources can be partitioned into those which are 'electronic' and those which are 'non-electronic'. The final division is to partition electronic resources into 'hardware' and 'not hardware' (notice that 'hardware' and 'software' is not a logical partition) and non-electronic resources into those created by the individual teacher under consideration ('individual', e.g. Kitty, who is one of the teachers in school A and her profile is discussed in subsequently in Chapter 6) and those which were not created by the individual teacher under consideration ('not individual'). Note that a worksheet created by Kitty and used by Kitty and Emilia would be coded 'individual' for teacher

Kitty but 'not individual' for Emilia. (Emilia is one of the four teachers in school A and her profile is discussed in the next chapter.)

I do acknowledge that further divisions are possible. For instance, 'human, physical contact' could be partitioned into 'formal' (e.g. within a scheduled meeting that has an agenda) or 'informal' but I found the classification provided in Table 5-1 below as sufficient to accommodate all of the codings developed, and also manageable.

Table 5-1 below is the table of classification of mathematics teacher resources.

Human		Non-Human			
Physical contact	Not Physical Contact	Electronic		Non-Electronic	
		Hardware	Not Hardware	Individual	Not Individual

Table 5-1. Classification of resources

Table 5-1 above is then used to classify the resources of the seven mathematics teachers in this research in the case studies in Chapters 6-8. Full details of the resources indicated by individual mathematics teachers and those identified by the researcher are discussed in the case study chapters. Here, I present the classification of Kitty's resources as an example of how individual teacher's resources are classified.

Human		**Non-Human**			
Physical contact	Not Physical Contact	Electronics		Non-Electronics	
Personnel CPD TeachMeet Chinese teacher Maths trail	Podcast Twitter YouTube Khan Academy	Hardware	Not Hardware	Individual	Not Individual
		iPads IWB iWB	Resource banks MangaHigh Socrative Task-spec websites	Paper-based resources Workbooks	Resource banks Paper-based

			applets Gcsepod.com Resourceaholic TES.com Music FrogOS Diagnosticsquestions.c om	Workshee ts Posters	resource s Textboo k Tracing paper

Table 5-2. Classification of Kitty's resources

Table 5-2 above shows the classification of Kitty's resources. In the subsequent chapters on the case study presentation, this classification is undertaken for each teacher. In the classification of the resources, two broad themes and six sub-themes emerged as a way of classifying these resources. For instance, in the case of Kitty above, I considered the resources under these two broad themes of human and non-human. An additional subdivision under the human resources was human resources with physical contact and not with physical contacts. In Kitty's case, human resources with physical contact include continuing professional development (CPD), TeachMeet, the Chinese teacher, and mathematics trail activities where human contact is involved. In the human but non-physical contact resources category, Podcast, Twitter, YouTube, and Khan Academy are identified. These resources involve human actors, but no direct physical contact takes place in the usage.

In the second broad theme, non-human, a further subdivision into electronics and non-electronics follows. In the case of Kitty above, under the electronics category are the hardware (i.e., iPads, IWB and iWB-handheld whiteboards) and non-hardware, which refers to those resources that are electronics but are not hardware (e.g., Mangahigh and Socrative). Under the non-electronics sub-category are individual and not individual resources. The individual resources refer to all the resources designed and used by a specific teacher (e.g., worksheets) and the not individual are resources that are collectively owned by all the teachers in the department; usually these resources are held in the 'shared bank of resources' (a folder which contains all the resources that are considered ‘good’ by the teachers and are accessible to all the teachers in the department).

5.4 The Tasks of the Teacher

In this subsection, I restate the understanding of tasks and how it is used in this study. Mathematical tasks play a central role in teaching and learning, This has been previously explored in Chapter 2, where it is argued that the type of task, the way it is crafted and used, has significant impact on student learning (Sullivan et al., 2010). Recall that for John Monaghan and Luc. Trouche (2016, p. 391), “Tasks refer to what the teachers plan and design for triggering and supporting learners’ activity”. Kaiser (2006) believes that the choice of tasks reflects mathematics teachers’ beliefs about instructional goals, while the way the tasks are formulated or modified reflects teachers’ beliefs about learning and teaching.

In the three schools in the study and the seven teachers focused on in this research, there were a range of lesson structures in use. The teachers were observed to use a variety of adopted, modified, online and textbook tasks in their teaching activities for a range of purposes, as already mentioned in Chapter 4. Reynolds and Muijs (1999, pp. 280-281) argued that “this well-structured mix of whole-class, group and individual work not only made the lesson more manageable, it also established a climate and a common language for talking about mathematics which benefited more children for more of the time”. For the UK department for education and skills,

> An effective lesson will be organized into a sequence of distinct learning episodes with a beginning (teacher input), a middle (activity for pupils) and then a quick check for understanding before moving to the next episode – until the end of the lesson, at which there might be a longer review time. (DfES, 2004, p. 9)

In the analysis of datasets related to teachers’ tasks, a four-part framework emerged through which teachers’ tasks are classified (see Chapter 4, Table 4-16). This framework will be used in presenting and discussing the tasks in the case studies in the subsequent Chapters 6-8. The task will be discussed under these subheadings:

- Tasks Initiating Lesson
- Posing the Main Mathematics Tasks
- Demonstration of Mathematics Learning
- Synopsis of the Lesson

Some of the tasks have been categorised in a flexible manner: for instance, diagnostic tasks and peer/self-assessment tasks were used by some of the teachers at various points of the lesson depending on the objective of the lesson and the needs of the students. Thus, peer assessment tasks could be used as *task initiating the lesson* and/or as part of the *synopsis of the lesson*. In the subsequent chapters (6-8), I elaborate on each teachers' tasks and how they were used.

5.5 The Collectives

I focus on the collective perspective of teachers' collective work and interactions with and through resources in their documentation work, and its implication for the teachers' community of mathematical practice. This collective work develops in diverse ways: sometimes, the mathematics teachers of a department of a given school meet at intervals; sometimes, communities of teachers are set up within a training cohort. On other occasions, mathematics teachers meet regularly aiming to collectively design teaching materials. In many instances, in the schools in this research, teachers meet formally and informally most often. This will be discussed in more detail in the subsequent chapters.

Grangeat and Gray (2008) conceptualise teaching as a collective work consisting of a wide range of interactions. Grangeat and Gray outline these connected multiple interactions as including curricular structure; student learning trajectories; day-to-day events; and cooperation with professional others and school organisations. I now discuss briefly these collective interactions in relation to the three schools in this research, as already indicated in Chapter 4.

- Curricular structure – teachers have specific roles in the organisation of the curriculum. This is the observed case in England, where secondary mathematics teaching is usually structured through their department-level ‘schemes of work’, which teachers collectively interpret, modify or adapt for use with their own classes.

- Student learning trajectories[32] – teachers participate with students in building on their prior knowledge arranging the available resources in manner that could facilitate new learning as they progress through the curriculum.

- Day-to-day events in school – teachers constantly react to the expectations and needs of learners and colleagues as they arise within the school.

- Cooperation with professional others – Many professionals like social workers, health, teacher trainees, researchers, teacher educators and technology specialists often collaborate with the teachers in the context of practice. For instance, the maths hub is equally a hub of collaborations and collective engagements amongst teachers and other professionals.

- School organisation – teachers have regular scheduled formal and spontaneous informal meetings as part of the professional school practice.

However, my research puts the mathematics teacher and the digital resources appropriated at the centre of these multi-level collective interactions. I specifically focus in on how these interactions relate to the mathematics teachers’ work with resources and on how these interactions transform both the collective and the work teachers do (Pepin et al., 2013). The notion of *documentation work* includes all these interactions. In subsequent chapters (6-8), the collective contexts of mathematics teachers’ practices in the three different schools A, B and C are discussed in detail.

In summary, this chapter has outlined and explored six structuring themes for the case studies presented in the subsequent chapters (6-8). The structuring themes provide the framework and rationale for organising the succeeding three chapters. The six structuring themes discussed above emerged from the research foci and categories

[32] Student learning trajectory refers to content-specific learning path and progression. For instance, one of the study teachers building on students’ learning of fraction to advance to the idea of decimal numbers.

from the initial analysis of data. These are the school contexts, teacher profiles, roles in the department/school, resources, tasks and the collectives.

This chapter also described the processes of a logical classification of resources and presented a four-part framework for the presentation of the tasks in the case studies. It recognised the collective aspect of teachers' work with resources. Teachers' work with digital resources has social, institutional and cultural dimensions. These themes discussed above will guide the case study presentation in Chapters 6-8.

CHAPTER 6

CASE STUDY REPORT: SCHOOL A

This chapter offers a thick description of the case studies of four teachers in school A. My intent is to give a detailed account of the contexts of practices, content and approach, resources, school ethos and interactions and develop these with concrete data items and extracts. The individual mathematics teachers' case studies are then addressed along the six themes discussed in Chapter 5. The first and last themes (the mathematics teacher and the collectives) are considered as a whole for the four teachers at the beginning and end of the chapters. The four case studies each address the four middle themes. The broad approach is interpretative, through which I hope to provide sufficient illumination and draw a comprehensive picture of the four cases. This chapter is organised in two sections. The first section considers issues of school context, teacher practices, grouping and assessment for learning and school ethos and sets the scene for the four case studies. The second section explores the individual cases of the four teachers in school A. The four mathematics teachers in school A will be subsequently referred to as Kitty, Emilia, Jimmy and Jose, respectively. The chapter closes with a consideration of the collectives in school A.

6.1 The Context of School A

School A is located in the West Yorkshire. The school is a Church of England trust for students aged 11 to 19 years with over 1676 students. The school is reported to have an excellent reputation and the school's vision is to be 'outstanding within outstanding'. The OfSTED report of 2013[33] judged the academy outstanding in the four areas of inspection: achievement of pupils, quality of teaching, behaviour and safety of pupils, and leadership and management. OfSTED also reports how students' achievement is high, with students enjoying great success in crucial areas such as

33 https://reports.ofsted.gov.uk/inspection-reports/find-inspection-report/provider/ELS/136094

mathematics and literacy. In 2015, the most recent results available, 72 percent of pupils at school A gained at least 5 A* to C grade GCSEs including English and Mathematics. In 2017, the Progress 8[34] score stood at +0.65, which puts school A in the top 3 percent of schools nationally and means that students, on average, achieve over half a grade better in eight subjects than their peers in other schools. The Progress 8 score is a new rating for English secondary schools which was introduced in the 2016/2017 academic school year. It is an average score for all students at a school, showing how well they have progressed academically from the end of primary school to when they finish in secondary education. Students' results at a school are compared to those from children at other schools who had attained similar academic results at the end of their primary education. The principal[35] describes School A:

> As one of the country's leading academies we are in the vanguard of school improvement. We are the flagship academy in the Multi Academy Trust, a national sponsor (which means we support other schools), a designated teaching school (Teaching School Alliance) and a Maths Hub of excellence. We have received two outstanding judgments since opening.

School A is designated as a National Support School due to its work in improving other schools as well as a lead in the teaching school alliance across many authorities. It hosts one of the White Rose Maths Hubs[36] (a collection of national networks of school initiatives) where the use of digital resources and the Shanghai teacher exchange programme are encouraged and supported. The school hosts many visitors in the course of the data collection year including mathematics teachers from China as part of the UK-Shanghai exchange programme. The students are taught in mathematics mixed-ability groupings.

School A was rebuilt in 2000 as part of the UK government's £45-billion 'Building Schools for the Future' (BSF) investment programme. It was aimed at refurbishing or

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34 https://www.gov.uk/government/publications/progress-8-accountability-measure-schools-responses
35 On school website page
36 http://www.mathshubs.org.uk/find-your-hub/white-rose-maths-hub/

rebuilding all 3500 secondary schools in England by 2020. The purpose was to remedy the decline in the overall state of school buildings and making them fit for purpose in meeting the educational demands of teaching and learning in the twenty-first century (Mahony & Hextall, 2013). A UK study argues that capital investment in school buildings has a strong influence on students' performance, on teacher motivation and effective learning time (CABE, 2006). However, another study contended that no such link exist, arguing instead that further research is needed to illuminate the connections between school building and institutional effectiveness (Mahony & Hextall, 2017).

The architecture of the school took into cognisance the nexus between pedagogy, technology and the design of the learning space for the school of the twenty-first century. There are plenty of open spaces and corridors lined with desktop computers and study desks. School A has dedicated a mathematics department, a common working room for all mathematics teachers with a library of textbooks, a computer suite for students and teachers and mathematics-related texts, and a trolley of handheld resources. The mathematics staffroom is equipped with desktop computers for every teacher. This context encourages and supports collective work. The departmental ethos expects and facilitates a continuous atmosphere of discussion, asking and sharing, and whatever resources the majority deems useful are always added to the shared departmental bank of resources.

6.1.1 Content and Approach

The central ongoing innovation in mathematics in school A is the move towards the Singapore/Shanghai mastery-type approaches to teaching mathematics including the use of digital resources. UK mathematics education reform[37] is currently focused on how the UK's mathematics performance compares to the highest-attaining mathematics education systems internationally. There is a government-backed

37 https://www.gov.uk/government/collections/national-curriculum

mandate to explore, adapt and embed the Singapore/Shanghai model of mastery teaching and assessment for learning approaches across England and Wales (You & Morris, 2016). “Mastery learning can be described as a set of group-based, individualized, teaching and learning strategies based on the premise that virtually all students can and will, in time, learn what the school has to teach” (Anderson, 1975, p. 4). This has been the major influence on the mathematics teachers in School A in adopting the key elements of feedback, mathematical fluency and enrichment as tools towards enabling higher levels of achievement, deep understanding and confidence among students. The use of a ‘concrete, pictorial and abstract’ approach of mathematics teaching for mastery, and assessment (Guskey, 2010) for learning has become the regular means of teaching for understanding. It is reported in *The Guardian* that 30 Shanghai mathematics teachers were flown in by the Department for Education in the hope they can raise flagging standards in schools across England (Weale, 2015). However, a study raised some doubt on the current enthusiasm in the West, especially in England, for copying East Asian mathematics teaching methods. The finding shows that second-generation immigrants of East Asian descent outperform their native Australian peers in mathematics by more than 100 PISA test points – roughly the equivalent of two and a half years of schooling. In England also, children of Chinese origin have the highest GCSE scores of any ethnic group – in 2014, 78 percent gained at least five A*-C GCSEs, compared with the UK national average of 60 percent. Jerrim (2015) argued that the decisive factor is culture not curriculum, suggesting that widespread cultural factors beyond school also play a part in their success.

This adoption of the mastery approach has had a major influence on the appropriation of resources by mathematics teachers for lesson preparation and classroom practices. In the course of my data collection, two invited Chinese mathematics teachers were there to train the resident mathematics teachers and helped them adapt to the mastery approach of teaching. Kitty reported in an interview on the role of the Chinese teachers in relation with CPD:

> We do have professional development sessions. Last year for example when we were doing a lot of work on the Singapore bar modelling, we had a CPD

> session every week where we were talking about teaching and learning how to deal with it using the bar modelling methods, yes, we did last year. This year it is not as frequent; however, we have mentioned as a department that we are learning a lot from the Chinese teachers every minute who are helping us, and they actually meet every day after the lessons. We were thinking maybe we should have more meetings like that where you are maybe observed and filmed, and you looked back. All you are looking at is the pedagogy behind that lesson to improve that particular thing you are questioning and students' learning that comes from that. (1intK:15#).

The teachers in School A have regular formal and informal CPDs on pedagogy, the new curriculum, on available resources and on how to assess, select and adapt resources for use in teaching specific mathematical concepts. There are rich varieties of sources of resources for the teachers. This is shown in the subsequent subsection of the case study reports.

6.1.2 Grouping and Assessment for Learning

In all the classes observed in school A, students are taught in mixed-ability groups. Students' progress is closely monitored on a regular basis, with the use of a variety of digital resources – Plickers, QR code, Socrative (explained in more detail in section 6.2.3, p. 138) – that provides instant marking and indication of areas of difficulty or identifies students who are struggling and need further support. An interesting emergent focus lies at the intersection of teachers' resource systems for mastery teaching and their appropriation of digital tools for formative assessment (FA). FA is also referred to in the literature as assessment for learning (AfL) where student seat-work is instantly analysed during lessons which allows the teacher to enact changes in the tasks. Heritage (2018, p. 52) considers that

> Assessment for learning (AfL) is integral to teaching and learning and has as its central foci (i) pedagogical intervention in the immediacy of student learning, and (ii) the students' agency in the learning and assessment process. The role that students adopt in AfL is consistent with the idea of self-regulated learning, which involves students as metacognitively, motivationally, and behaviourally active agents in their own learning.

Self-regulated learning through self- and peer-assessment by learners is a regular aspect of formative assessment in the mathematics classrooms observed. The use of 'traffic lights', pasted in bold typeface into the backs of learners' notebooks, serve as a simple means for students to communicate judgements about their understanding of an ongoing task. Learners raise up a coloured square to indicate to call teacher's attention to where they are. Figure 5.1 below is a sample of the traffic light colour device.

Figure 6-1. Traffic light colour coded device

Figure 6-1 shows the system for assessing students' progress and understanding based on a 'traffic light colour code' approach, denoting the level of complexity – green, amber and red – whereby children are encouraged to self-assess their work. Black and Harrison (2001, p. 46) state that "one advantage of the traffic lighting device is that the teacher can identify at a glance the main learning difficulties that have arisen, without lengthy interrogation of each student individually. It also facilitates communication between students". This device allows the teacher to quickly identify situations that require intervention.

6.1.3 School Ethos

School A has a very well-established, rich Christian ethos built around four core values: empathy, honesty, respect and responsibility. Its admissions policy welcomes all students irrespective of their religious backgrounds or affiliations. This Christian character and vision are intended 'to ensure that young people in our care fulfil their potential'. SIAMS[38], the body responsible for inspecting provision in church-based schools, has also judged the impact of the Christian ethos in creating an atmosphere of purposeful learning and aspiration throughout the academy as outstanding[39]. The curriculum review policy stipulated

> Monitoring of standards, systems and teaching and learning across the Academy is the responsibility of the Senior Leadership Group (SLG). Monitoring of the quality of lessons on a daily basis is the responsibility of the Curriculum Leadership Team. Each subject has ongoing Quality Assurance throughout the year via learning walks, lesson observations, work scrutiny, student voice and weekly SLG link meetings[40].

The curriculum is targeted at a model of the mastery teaching approaches in the hope that this will impact positively on students' learning and progress, working towards developing students as well-rounded global citizens of the twenty-first century both academically and socially.

Having described the context of school A, I now turn to consider the individual case study report for the four participating teachers in this school.

The case study report is presented in the following sections for each of the four teachers. The profile of the teachers, their roles in the department, their tasks deployed in their lessons observed, their resources and the collectives the four teachers participate in are explored and presented.

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38 National Society Statutory Inspection of Anglican and Methodist Schools

39 School website

40 School A curriculum policy document

6.2 Kitty's Profile

Kitty is a 26-year-old mathematics teacher who started her teaching post over five years ago and still teaches in school A. She is one of the lead mathematics teachers and teaches groups in Years 7-11 between the ages of 11 and 16years old. These belong to Key Stages 3 and 4 of the National Curriculum for secondary education in England. The average number of students in her class is 30.

She is actively involved in the *White Rose Maths Hub* and one of the training providers for the *Mathematics Teacher Subject Specialists Training*[41] (TSST). The maths hub provides opportunities for teachers to develop skills, devising schemes of work to enable teachers to adopt the teaching for mastery approach and offering a range of specialist training and tools to enable mathematics teachers to introduce and teach crucial topics effectively, and to boost mathematics confidence while making mathematics fun. In the West Yorkshire mathematics teaching community, she is well respected and regarded as a successful teacher.

> We are actually the lead school for the White Rose Maths Hub across the country. I myself actually take part in delivering CPD to external Maths teachers in terms of increasing their subject knowledge, so we are involved with the Maths hub and that I think has a massive effect on how we teach things: for example, the CPD that went on Monday about the diagnostic questions was organised with the Maths hub's help. I won't have necessarily have known about that without our affiliations to the hub. We are also involved with the teaching school; we are teaching school. And we train; we are actually training three Maths teachers at the minute, and they are provided with subject knowledge sessions by one of our more experienced teachers, on a weekly basis and are involved in delivering lessons with Maths teachers in the department as well. (1intK:18#[42])

Kitty uses a broad range of resources in the course of her lesson preparation and teaching. She believes too that the use of ICT and other resources could have positive

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41 https://www.gov.uk/guidance/teacher-subject-specialism-training-courses

42 This shorthand is used subsequently. It refers to the first interview with Kitty, on the 18th minute of the recording.

impacts on students' engagement with mathematics and on how teachers undertake their practices.

She regularly works with other mathematics teachers in the school and other mathematical communities and alliances, in providing subject knowledge enhancement (SKE) sessions for primary and secondary teachers, selecting resources, planning teaching and assessment. She has also participated in the Shanghai teaching for mastery exchange programme and actively adopts this in her teaching as well. This was confirmed by Kitty, who said, "We have done a lot of research recently into Shanghai methods and Singapore bar modelling and variations within questions". She presents a broad outlook of the purpose of students' learning, seeing it as more than the understanding and mastery of mathematics and including a preparation beyond school, a preparation for the development of life-long skills. This excerpt from an interview indicates her belief:

> In terms of students' learning it is important that they have access to all those different types of learning because you need to prepare them for that change when they leave school. We are not just preparing them for Maths alone; we are preparing them for life really. Technology is a big part of that, being quiet and working; being focused is a big part of that. Even up to the university those skills are really useful, and it is important to incorporate a lot of things really. (1IntK: 23:00#)

This mindset is consistent with the UK's idea of building school for the future and school A's curriculum policy drive. Schools are not just preparatory ground for examination success but inculcate an outlook addressing the skills needed for life in a future driven by technological innovation and digital resources.

6.2.1 Role in the Department: Lead Teacher-Mathematics

Lead teachers in mathematics are experienced educators who work with other teachers, administrators, regional networks and parents to help both teachers and students have a positive and productive school experience. They are also referred to as lead practitioners and considered to be excellent teachers who achieve the very highest standards of classroom practice, leadership, and whose key roles are to share skills and experience with other teachers. Lead teachers usually take on further responsibilities in decision-making beyond the administrative team in a district or school. The *Glossary of education reform*[43] describes the role of a lead teacher as including

- Serving in leadership teams
- Advocating innovative assessment strategies
- Facilitating stronger lesson plans and classroom management
- Coordinating the professional learning community or other teacher groups
- Leading efforts to modify content-area-curriculum
- Develop external partnerships that bring additional resources and learning opportunities

In school A's job description[44] for lead mathematics teachers, the key expectations were as below:

- To inspire imaginative and effective approaches to learning and teaching across the Academy
- To support other teachers to improve their effectiveness, modelling excellent practice and providing professional mentoring
- Analysing national, local and academy data, research and inspection findings to inform curriculum area policies and practices
- Be committed to the use of new technologies to improve teaching and learning

In this role, Kitty is a model of the mathematics teacher the academy expects, the teacher's voice (values, opinions, beliefs, perspectives, and cultural background) and

43 http://edglossary.org/teacher-leader/
44 In the website for school A

the type of leadership it anticipates. Kitty's professionalism in educational practices and her specialism in mathematics is expected to have a school-wide positive impact on students' attainment and make a major contribution by raising the quality of teaching in mathematics across the whole school and beyond.

Kitty describes this role as a motivating factor in her conscious effort to reflect on her teaching, choices and to help address her own professional development needs and encourage her colleagues to share with her as well. In another interview she stated

> … if you are thinking, how do I teach this? am not quite sure of this or maybe it's your first time you've taught it, you are thinking, what do I need to be aware of, in terms of pupil misconceptions? In your preparation of lessons, you might do a lot of research and quiet often as well, you talk to a teacher, who I know has taught that topic before. (1IntK: 10:40#)

Besides the reflection on her teaching and professional self-development, she is actively involved in CPD. As mentioned earlier, she is a lead teacher in the maths hubs mathematics teacher-training programme, and she provides support for teaching assistants. She reported that her earlier work experience included working as a supermarket cashier and a language assistant in France.

6.2.2 Kitty's Tasks

In Chapter 4's subsection on tasks, I identified 15 task types (see Table 4-12, p. 96) across the seven teachers and then organised them into four time-intervals in the lessons (as shown in Table 4-16, p. 102). In this subsection, I present and discuss in detail the various tasks used by Kitty in her teaching, the sources of these tasks, the rationale behind their modification and how they are modified. The exploration here draws from the interviews, observation notes, screen-capture screen shots, supportive worksheets and the analyses of tasks in Chapter 4. In the following subsections, I present the Kitty's classroom routine, her resources and her tasks, in terms of the four themes.

Classroom Tasks Routine

Kitty has a classroom task-routine strategy that underlines the mode of working, beginning with a teacher-led phase and then monitored independent practice and group work. From observation it seems to me that Kitty plans her lessons in such a way that students are engaged and empowered to be owners of their own learning and support each other in their progress. This pinned-up class poster below in Figure 6-2 sums this attitude and learning strategy: *I do*, *you do*, *we do* (group work), *We do* (pairs), and *We do* (together as a class). This strategy for engaged and self-directed learning is referred to in the literature as the gradual release of responsibility (Fisher & Frey, 2013). The gradual release of responsibility (GRR), Fisher and Frey (2013) states, is anchored in purposefully planned lessons that integrate four interrelated instructional phases. The first is a *focused modelled instruction that is teacher-led* (I do it/you watch).

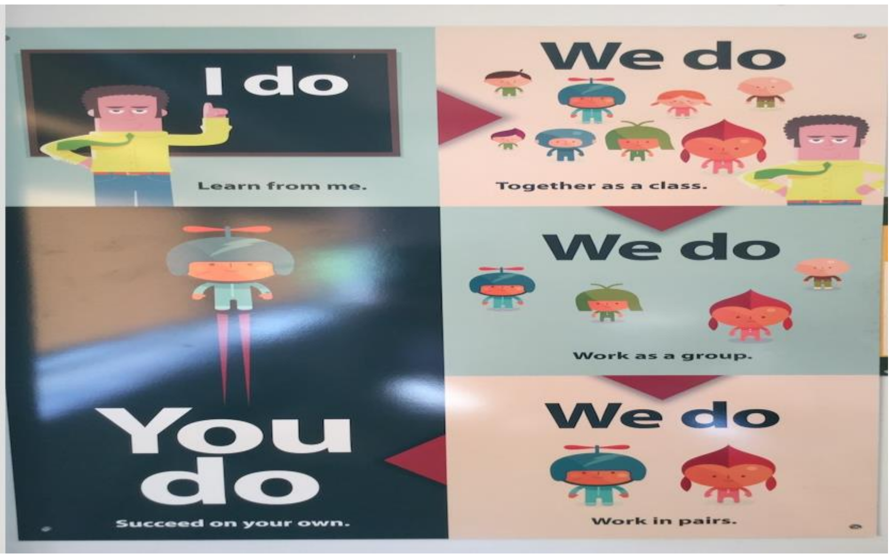

Figure 6-2. Gradual release of responsibility model (Fisher & Frey, 2013)

The second phase involves *shared instruction* – we do it (together-whole class work), we do it (in pairs), and we do it (group work); a third phase is based on *guided practice*, you do it (I watch and guide) and the fourth phase is *independent practice* – you do it alone (reflect). The GRR and a reverse form of it drives most of Kitty's instructional practices in tasks, from my observation, and this large classroom poster that Kitty regularly points students to is a reminder of the routine of activities. The frequency of these types of instructional activity levels is captured in the SCAN dataset in Table 4-4, p.80. I now present the tasks under the four-part framework discussed in subsection 4.2.3, pp. 101-103.

6.2.2.1 Tasks Initiating the Lesson

In this subsection, I present the tasks Kitty uses in initiating her lessons. I follow the four themes for discussing the tasks in subsection 5.4, p.113. One of the task types Kitty uses for initiating her lessons is a range of tasks she refers to as starter tasks. In the context of this case study, I present them collectively as tasks initiating her lessons. The purpose of the starter tasks, Kitty claims, is to instill mathematical fluency in multiplication, mastering the order of operations, mastering the addition of numbers of different sizes, developing ability to recall instantly and being able to write out integers in words. At a different level, this is also to enable the teacher to correct misconceptions and misperceptions since the same questions are presented in varied forms. At the beginning of Kitty's lessons, a paper form or online version of *Times Table Rock Stars*[45](also known as Ttrockstars) is used. This is a carefully sequenced daily schedule of multiplication and division practice over the course of 20 weeks. Each week concentrates on a different times table, with a recommended consolidation week for rehearsing the tables that have recently been practiced every third week. Students have three minutes to complete a worksheet with a set timer and music playing in the background. The screenshot Figure 6-3 is the Ttrockstars worksheet for week 13 as taken by Year 10.

45 https://ttrockstars.com/

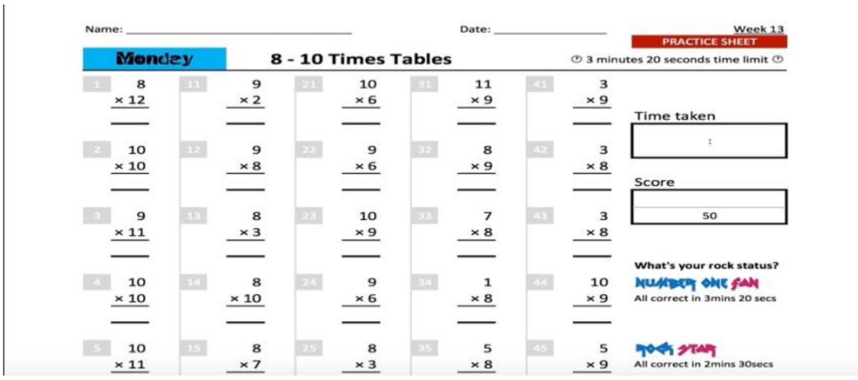

Figure 6-3. Ttrockstars worksheet week 13

The paper version is a worksheet with 50-60 questions (time limit of three minutes) taken at the beginning of every mathematics lesson. In the last lesson of the week, usually Fridays, students calculate their total time and score for the week and these are recorded on the website.

In another lesson for Year 11 on Vectors, the teacher-designated starter task was

Task 1. Solve $2x^2$ –$9x$ + 4 = $(2x - 1)2$

In a lesson for Year 8 on multiplying a fraction by an integer, the starter task given was

Task 2. (1) $\frac{1}{7}+\frac{3}{7}$

(2) $\frac{2}{5}+\frac{6}{15}$

(3) $\frac{4}{12}+\frac{1}{6}$

The repeated practices afforded by the starter tasks for Kitty is intended as a means of reinforcing computational procedure and accuracy. Strong emphasis is placed on speed and ease of recall, correct answers and not on the heuristic element. Students' participation and engagement with such tasks, in my opinion, was usually high and with minimal mental demand for interpretation or application. There were also starter tasks that were used further by the students.

In the BIDMAS word search in Figure 6-4 below,

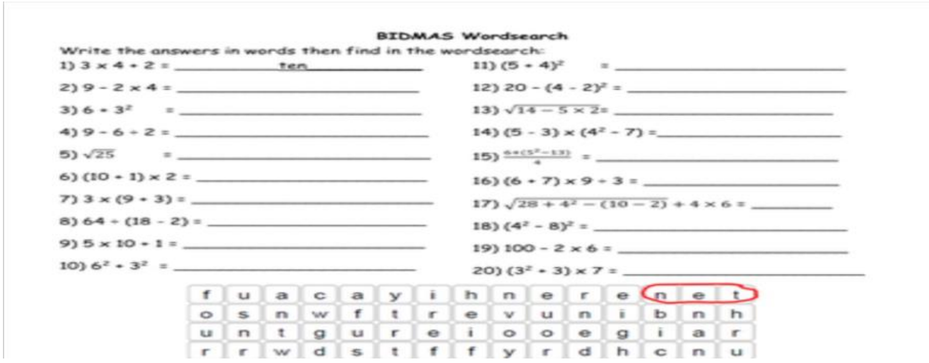

Figure 6-4. BIDMAS word search task

Kitty stated that her intention was for the student to become confident and achieve mathematical fluency in the areas of multiplications, addition, and division, to be familiar with the order of operations, and write out their answers in words.

On several occasions these starter tasks were taken online using the *diagnostics questions*, Mathsbox and the TES websites using iPads. There are also *5-a-day*[46] videos, worksheets and practice papers.

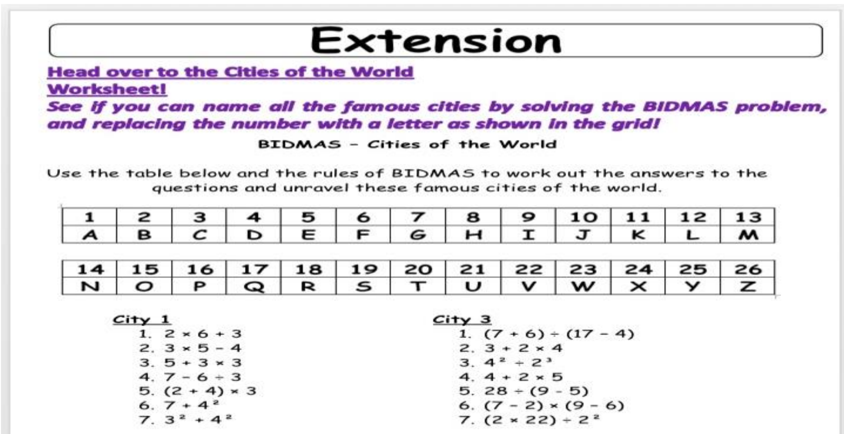

Figure 6-5. Extension tasks on BIDMAS

Kitty suggested that the starter tasks equally provide opportunity for self- and peer-reviews. For instance, the set of starter tasks in Figure 6-5 was used for extension

46 https://corbettmaths.com/

activities. Although the starter tasks are most often individual work, sometimes the teacher ask the students to peer-review each other's work and correct answers are called out. Kitty in an interview referred to this practice thus:

> I quite often use a website called Mathsbox for a starter; it is a quick skills test. If you like you can give out to students, it really being efficient in terms of your planning. (3intK: #2)[47]

Kitty conceives of the starter as a device for skill testing to assess the student's ability to solve a given task. This also enables her to plan ahead during the course of the lesson. Informed by the outcome of the starter tasks, she could anticipate where students may struggle and actively prepare to help them resolve that difficulty.

In summary, there is a range of tasks Kitty used to initiate the lessons, as shown above. Kitty claims these tasks are devices for developing factual and procedural fluency. They are primarily for introducing new concepts and skills, for practicing, recalling and reinforcing previously learned skills, for review and assessment. These tasks place minimal mental demands on the students. These starters equally serve as tools for motivation and elicit high student engagement with mathematics at the entry level.

In the following subsection, I present how Kitty poses the main mathematics tasks for the class after the lesson has been initiated.

6.2.2.2 Posing the Main Mathematics Tasks

Here, using illustrative examples drawn from Kitty's lessons, I present the various ways in which Kitty poses the main mathematics tasks in the lessons. I use three different examples of tasks (multiple representational, differentiation/variation and mixed ability group tasks) that occurred more than six times during the 13 lessons observed in Kitty's classes.

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47 Post-lesson interview, 21/04/2016

*Multiple representational task*s

In mathematics, teachers use many different representations when solving problems, such as tables, graphs, diagrams, algebra and the dynamic possibilities brought about by digitalisation. In these tasks, Kitty encourages students to use a variety of different representations to gain valuable insights into the mathematics being taught. For instance, in Year 11 on the topic of Vectors, GeoGebra software was used in class as a tool to demonstrate the dynamic and simultaneous relationship between the algebra and graphic representations of vectors. Figure 6-6 below is a screenshot from Kitty's action on vectors with the GeoGebra software.

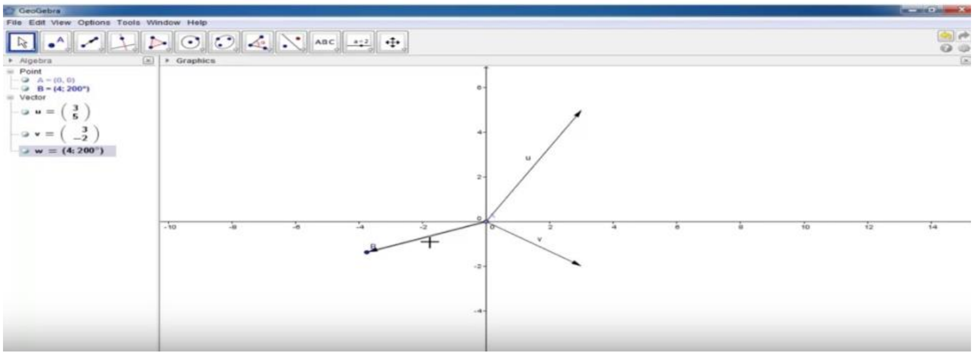

Figure 6-6. GeoGebra screenshot on Vectors

While Kitty worked on the IWB, the students worked on their iPads, given the opportunity to interact with the images and observe the instant feedback on the graphic view as they vary the parameters on the algebra view.

Other multiple representational tasks were drawn from the Khan Academy websites, TES and Mangahigh. Further tasks were then given to students to work on independently. These tasks include finding the equation of a line $y - y_1 = m(x - x_1)$

a. $5x - 4y + 3 = 0$

b. $y - 3x + 9 = 0$

c. $2x - 3y = 12$

In another lesson on fractions, the idea of a fraction was presented in a variety of ways. In figure 6-7 below, Kitty presented this task and asks the student to complete the open task using the different ways in making their understanding explicit.

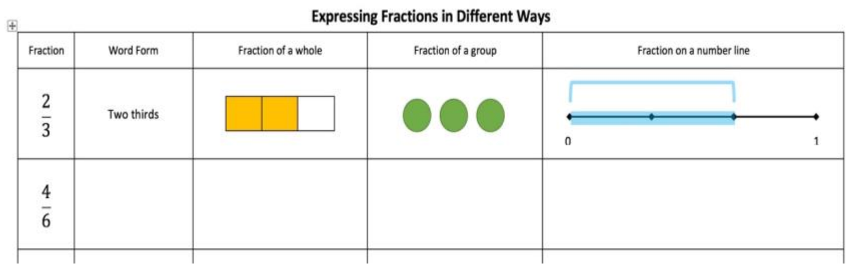

Figure 6-7. Multiple representations of fractions

Kitty believes that multiple representational tasks and making connections between such representations play a crucial role in students' understanding of mathematics. The use of a variety of digital resources and hardware technology in the classrooms appears to influence students' learning in a positive way. Since they could access multiple representations, Kitty mentioned during a post-lesson conversation that, through her use of the dynamic and static multiple representations of the tasks, students could develop a richer concept image and broader understanding of mathematical relations and objects.

I now turn to the second task type through which Kitty poses her main lesson tasks.

Tasks for differentiation/variations

The idea of differentiation and variation[48] and how the teachers in this research applied it in their lesson is discussed in section 10.7, p. 302. For the purpose of this subsection, I define both terms briefly. Differentiation in teaching refers to the practice of tailoring your lessons for students with individual needs. This involves changing the content, delivery, or methods of learning to ensure every child learns in a way that's suitable for them, while variation is a way of analysing and planning teaching and learning

48

activities which focuses on what changes and what stays the same and the effects this might have.

Here, I focus on Kitty's presentation of the tasks in her lessons. Kitty has been influenced by Singapore/Shanghai mathematics teachers starting to use variation in teaching students who are at different points in their learning. She reported thus,

> We have done a lot of research recently into Shanghai methods and Singapore bar modelling and variations within questions. To give you a simple example, 10 questions: 3 add 5, 2 add 4, 6 add 4. In order to vary that, you might have a blank ⊡+ 6 =10. A lot of textbooks we have in the UK don't necessarily do that, so we do create a lot of our resources or look elsewhere. (2intK:#14)

This was discussed in relation to the use and content of English mathematics textbooks, bar modelling and variation within questions. Kitty reported that using a variety of questions and different ways of presenting them was useful to the students. The third form of task presentation lies with the mixed ability grouping tasks.

Mixed ability grouping tasks

There two types of grouping in the class. The first is the teacher's own grouping and pairing of students to work together on a similar topic but at different pace. The second was a grouping based on the levels of difficulty, whereby students progressed from the green-coloured questions, which Kitty considered the simpler, to the gold-coloured questions, which are considered the most difficult. Each student group tried to advance from the green to gold while the teacher circulated and offered feedback. She also spoke in an interview about tasks she designated as interactive and engaging tasks:

> I quite like tasks that get the students up and moving around the room, so there are some great, like Maths Trail, we use, like treasure hunt activities and things like that, or sometimes relay activities, which is really interesting. And I think you got to have a variation of resources just so that it keeps students engaged. (1intK: #2)

On the question of the modification of tasks Kitty stated,

> For the majority of the time I have to modify them. I might quite often have to use the snipping tool on computers to select and then create my worksheet from

that; again, it saves time not going to create your own questions, but you are still tailoring it to the need of the class. And quite often you might have two worksheets on the go or more. In terms of differentiating into different learners within the group because some students might get it straight away, some might need more practice, some might think we need to go back a step and just work on multiplication before we do work on algebra. So, I think the majority of the time I do tend to cut and paste, if you like. (post-lesson interview [49] 2IntK: #3)

Kitty demonstrates an awareness of where her students are in their learning. Her task design intention is to tailor the tasks to meet them, using multiple worksheets focused on supporting the learners in their learning trajectory. The use of a snipping tool, and the cutting and pasting mentioned in the interview, was demonstrated in the retrospective reflection on lesson planning using screen capture. Figure 6-8 below is a screen capture recording which lays credence to this practice.

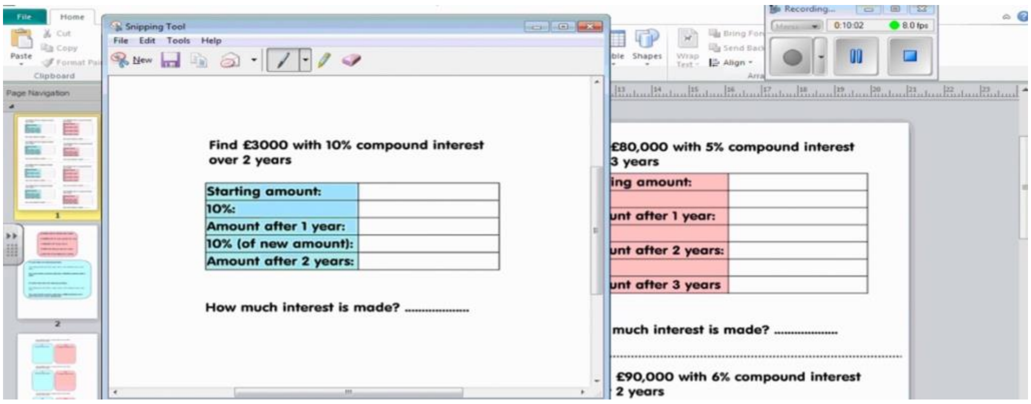

Figure 6-8. Kitty's task design and modification using a snipping tool

Figure 6-8 above shows Kitty designing and modifying a worksheet with a design intention targeted at the needs of the class and the direction she wants their learning to go.

[49] Post-lesson interview on 13/11/2015

6.2.2.3 Demonstration of Mathematics Learning

In the last 15 minutes of the lessons, Kitty begins to refocus the lesson towards self-assessment and to check on the students' progress in their learning through self-reporting of what they have understood and learnt in the class. One of the strategies was giving the students a number of self-assessment tasks that resembles the learning tasks treated earlier.

For example, in a lesson on *Standard Form*[50], the students were asked to work initially in pairs on a series of tasks.
Solve the following and leave the answers in standard form:

1. a^3x a^{-4}x a^5
2. 4 x 10^5- 2 x 10^2
3. 81 900 000 000 000

Then, after a while of working on these tasks, the students were asked to share and discuss their strategies in solving them. In five of the lessons observed, a student was asked to demonstrate their solution strategy on the IWB for the whole class. In some other instances, students explained their difficulties and their solution to the task from their handheld white board, when this involves individual tasks.

Kitty reported at the end of one of the post-lesson conversations[51] that through this she intends to promote a sense of ownership of learning and for the students to think independently about their strategies as they engage with these mathematics activities.

I now turn to the synopsis of the lesson.

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50 Standard form is a way of writing down very large or very small numbers easily.
51 On 11/12/2015

6.2.2.4 Concluding the Lesson

The conclusion of the lesson refers to how the teachers bring the lesson to a close. In the case of Kitty, she brings the lesson to a close by first asking the whole class a general question about their learning. This question is framed in many different ways: ‘What have you learned today?’ or ‘Who can tell us two things you have learnt in today’s lesson?’ After the students have listened to a couple of their classmates’ responses, Kitty goes on to give a general review of activity in the light of the lesson’s objective and highlights the progress the students have made in that specific lesson. This usually takes place in the last five minutes of the lesson and concludes the lesson as well.

In the next subsection, I present Kitty’s mathematics teaching resources.

6.2.3 Kitty’s Resources

Table 6-1 shows Kitty’s resources logically categorised into various types.

Human		**Non-Human**			
Physical contact	Not Physical Contact	Electronics		Non-Electronics	
		Hardware	Not Hardware	Individual	Not Individual
Personnel CPD TeachMeet Chinese teacher Maths trail	Podcast Social networking Twitter YouTube Khan Academy	iPads IWB HWB	Resource banks Mangahigh Socrative Task-spec websites applets Gcsepod.com Resourceaholic TES.com Music FrogOS Diagnosticquestions.com	Paper-based resources Workbooks Worksheets Posters	Resource banks Paper-based resources Textbook Tracing paper

Table 6-1. Kitty's resources

6.2.3.1 Human and Non-human Resources[52]

In this presentation, Table 6-1 above shows the broad classification of Kitty's resources into human and non-human resources as earlier explained in subsection 5.3, p. 109. The construct *resource* is understood in the context of mathematics teaching and learning as everything that supports and facilitates teachers' practices.

Human resources in the context of my research refer to all the human beings working and supporting the mathematics learning by their physical presence, through a video recording or in a virtual presentation. In this research this includes the teachers, support staff, TAs, exchange staff and CPD interactions. In Kitty's case these include physical contact ranging from a TeachMeet whole-school teacher session to the not-physical contact engagement through Khan Academy videos. This range of human resources, as Kitty reported, bring several advantages to her teaching efforts and support to improving the teaching and learning of mathematics.

In terms of the processes of her using resources in preparation, Kitty acknowledged that "each department has its own resource sharing folder, to share resources within the department" and she buttressed this fact when addressing the question of the sources of her resources for learning preparation.

> What I would do, I get my learning objectives, and then I decide on my resources. I might speak to a teacher and they might recommend the resources they have used in the lesson or they might have seen. Quite often I go onto our shared resources within the department which people have collated, some have created themselves, some have researched on the internet or some they have just picked up from previous schools and we collate those into topics and that is usually my first point of call.[53] (1intK: #2)

Kitty acknowledges that the use of digital resources makes planning time quicker, and she bookmarks places 'to go' in terms of searching for resources saves time as well.

52A more elaborate exploration is reported in Umameh, M., & Monaghan, J. (2017). A classification of resources used by mathematics teachers in an English high school. *Technology in Mathematics Teaching ICTMT 13 Ecole Normale Supérieure de Lyon/Université Claude Bernard Lyon 1 3 to 6 July, 2017*, 276.

53SCAN/23/11/15/yr10c3, ScrCpt#4:30

> In terms of my planning, it makes my planning a lot quicker, I am now in my fifth year of my teaching, I kind of have my favourite places to go, I know places that works. For example, I quite often use the staff shared resources on our shared computer network, I quite often use a website called MathsBox for starters. (1intK: #4)

In the case of the non-human resources, these are tangible mathematics teaching resources which are developed or adapted and used by teachers (and pupils) in their interaction with mathematics in/for teaching and learning, inside and outside the classroom. The non-human resources and the associated subdivisions of the types of resources in that category constitute the major sources of Kitty's resources.

There are several electronic resources (hardware and not hardware) accessed by Kitty: some are free-to-use resources-worksheets, video demonstrations, pedagogic insights and fora for interaction among users. In Kitty's resource system, the TES[54], Resourceaholic[55], Mathsbox[56], Corbettmaths[57] and 10ticks[58] feature often. A few of these are commercial sites and need to be subscribed to by the school or individual department. The Mathsbox, Corbettmaths and 10ticks are specifically mathematics-dedicated sites.

Some of the hardware resources act as a collective space for whole-class interaction or a means to organise access to learning materials from diverse online location. Kitty and her class frequently use an IWB, iPads, laptops, and calculators. Regarding the use of iPads, she explained

> When I use iPads in my lesson it's quite often for assessment for learning, getting the information from the students in a quicker and more efficient way so I can react to that within my lesson and also promote engagement with the students which I mentioned at the start. (2IntK: #6)

54 https://www.tes.com/teaching-resources
55 http://www.resourceaholic.com/
56 http://www.mathsbox.org.uk/index1.php
57 https://corbettmaths.com/about/
58 http://www.10ticks.co.uk/

Reporting on how the availability of iPads influenced her teaching practice and classroom dynamics, Kitty said,

> Over time as well, there are more websites out there and we have got more iPads to use among the students, so it is much easier now to use iPads with the full class. Before, it was between two; you'd have to use group tasks. It might be more research; it was a bit harder than to track individual progress. I think now quite often it's more independent tasks rather than a group. (2intK: #10)

The increasing frequency of iPad use in the lesson and how this facilitates both groups and individuals to engage with their learning is further explored in the discussion.

I now turn to present the case of Emilia in the next section.

6.3 Emilia's Profile

Emilia, 26, is a lead teacher of mathematics who has taught for four years in school A and takes on leadership roles in various capacities. She teaches Years 9 to 11, with an age range of 13 to 16 years old. These are mixed-ability classes and the average number of students in her class is 25.

She is actively involved in the *White Rose Maths Hub* as one of its training and professional development providers.

In terms of her use of resources, Emilia relies very largely on her collections of resources over her years of teaching, creates new ones when needed and intermittently goes to electronic resources when it is necessary.

> … so, the majority of the stuff that I use is saved over the years from when I have been teaching, so I don't go back to the websites. [*So it is a build of resources then*?] Yes, then it's just saved on my memory stick. (1IntE:03:54#)

Other sources of her resources mentioned include ideas from informal conversations with colleagues, her CPD engagements, shared resource folders and those recommended by the department. Emilia believes good classroom management and teacher consistency are the central thing for her teaching, that the ICT and resources are fundamental and very useful but not necessary in achieving the overall purposes of learning.

> But I think it is by building a good relationship with your class and being consistent is the main thing for me. Because if you can't control your class and you get bad behaviours, you can’t really get any learning done. It’s great to have all these ICT and all these resources, but if they are not listening to you, then you can’t really do much. (1IntE: 13#)

In her overview on the use of ICT and various resources she stated,

> I think definitely I don't know what I will do without the IWB, like all my resources, have got my sticks and can get the lesson on to the computer. I don't know what I will do if don't have these. It is fundamental, to be honest. I think like laptop and stuff is good to have them and I definitely appreciate them, but I don't think it is necessary. (1IntE: 13:39#)

She uses the laptops and iPads provided by the school but not very often. The IWB, the shared resources and the collected resources in her memory stick form the major pillars around which most of her lesson planning and teaching revolve. The other resources play auxiliary roles when they are necessary for the needs of the class and specific topics or practices.

6.3.1 Role in the Department: Lead Teacher-Mathematics

As a lead teacher in mathematics, she takes on similar roles as Kitty above.

> I am part of the lead teacher team. We have sort of meeting every two weeks, where we might discuss teaching and learning. I do tend to pick up ideas from there though, but not on ICT stuffs. [*What is your role as lead teacher*?] I do go into other schools, and every other Wednesday I deliver a training session to primary teachers by helping improve their Maths knowledge, so that they can teach it to their students. I use the smart notebook for that. (1IntE: 10:56#)

Besides providing leadership within the school, Emilia also belongs to a team of teachers in the region providing CPD for primary school teachers and other colleagues in the department.

She engages in peer-lesson observation and reviews, helping the other teachers to improve on their teaching and asking others to observe her and give her feedback as well. One other outcome of this is the sharing of mathematics and teaching related resources. In an interview she refers to this practice thus,

> We do observations, other teachers will come to observe your lessons and give you suggestions of how to improve. We also have a shared resource where we put in my worksheets and lessons, so that we can share really good resources and we also use email to send them out to each other, say this is a very good resource you want to use in your classes. I also go to observe other teachers' lessons and pick up good ideas from them. It does help and also just talking to each other and asking if there is anything they have got that is good. (1intE: 7#)

The shared resource bank is a central means of assessing resources in school A as well as sharing via email and word of mouth, pointing each other to websites and worksheets that may be useful.

6.3.2 Emilia's Tasks

The idea of tasks variation, Emilia reports, is central to her lesson planning and delivery. She acknowledges in the screen-capture retrospective reflection and post-lesson interviews that many mathematics teachers in England, including herself, are excited by and adopting a Shanghai/Singapore mastery approach to mathematics teaching with task variation and designing tasks and exercises to support their students' learning. These ideas have been taken into consideration. This growing adoption of the mastery approach to mathematics teaching is one of the underlying principles in Emilia's task designs and selections. In this subsection, I use illustrative examples from the observed lesson and data from the screen captures as an organising frame to present the tasks. I now turn to the tasks initiating the lesson.

6.3.2.1 Tasks Initiating Lesson

In seven out of the nine lessons observed in Emilia's class, sets of starter tasks were used as tasks for initiating the lessons. In five lessons, the starter task was preceded by times table, addition and subtraction exercises. Here, I focus on the starter tasks as the primary tasks for initiating the lesson, given the frequency of their use by Emilia.

Starter Tasks

Emilia begins her lesson plan and task design by looking at the 'scheme of work[59]'. The scheme of work is designated as a GCSE mastery scheme of work, with listed topics and units in an ordered progression. Figure 6-9 below shows the layout of the scheme of work.

GCSE Mastery Scheme of Work

UNIT	TOPIC
0	Number Skills 1
1	Fractions 1
2	Area, Perimeter and Volume
3	Fractions 2
4	Manipulating Algebra 1
5	Solving Linear Equations and Inequalities
6	Number Skills 2: Ratio and Proportion
7	Number Skills 3: Percentages
8	Substitution and Nth Term
9	Linear Graphs
10	Angles
11	Pythagoras and Trigonometry
12	Circles and Cylinder
13	Simultaneous Equations
14	Manipulating Algebra 2
15	Non-Linear Functions
16	Transformations
17	Surface Area and Volume
18	Number Skills 4: Factors, Multiples and Primes (introduction to Venn diagrams)
19	Probability 1 - Intro to probability, 2-Way tables & Venn diagrams
20	Probability 2 – Combined Events

Figure 6-9. GCSE mastery scheme of work

Each topic, when expanded, opens up a drop-down tab with more detailed guidance and instructions on the objectives of the topic, the class band, the resources (textbooks) and suggested starter tasks. Figure 6-10 provides evidence of this.

Unit	Topic	Objectives	Higher Objectives (10A1 and Y11 Higher classes)	Recourses	Suggested starter topics (spaced repetition)
0	**Number Skills 1**	a. Add, subtract, multiply and divide and positive integers **(N2)** b. Add, subtract, multiply and divide and *negative* integers **(N2)** c. Use the order of operations (BIDMAS) in conjunction with a and b **(N3)** d. Multiply and divide integers and decimals by 10, 100 and 1000 **(N2)** e. Perform short division **(N2)**		Collins Foundation Chapter 1 Collins Foundation Chapter 6 Collins Foundation Chapter 7.1	

Figure 6-10. Mastery scheme of work on number skills for Years 10 and 11

The Collins Foundation textbook[60] Chapters 1, 6 and 7 are recommended as resources to use. Starters are not specifically suggested, but Emilia reported that individual

59 In the UK, a scheme of work is a guideline that defines the structure and content of work to be done by students in the classroom and homework. It maps out resources, allocates time for each aspect of tasks and assessment strategies to be used towards achieving the learning objectives.

60 GCSE Maths Edexcel Foundation Student Book (Collins GCSE Maths) by Michael Kent, Brian Speed, Kevin Evans, Keith Gordon (2015)

teachers, with the knowledge of her students, are left to decide on the starter tasks to use. The topic for Year 11 was mixed numbers and improper fractions. Emilia begins with basic definitions and concept clarifications with a colourful table of definition. This is shown in Figure 6-11 below.

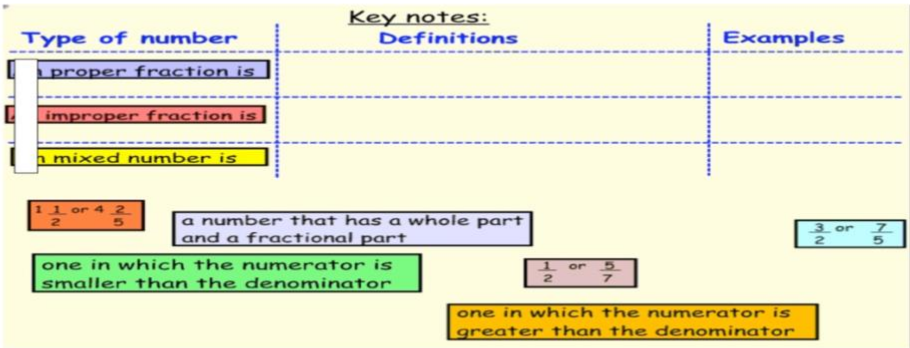

Figure 6-11. Table of key notes

When she felt the students have understood the concept, she gave the following starter tasks as shown in Figure 6-12 on a variety of fraction tasks.

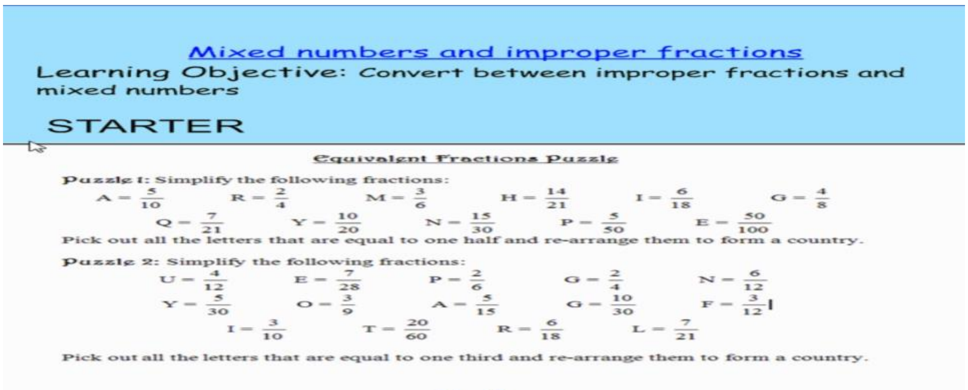

Figure 6-12. Starter tasks A on fractions

Then other starter tasks timed with an online clock were given to the students (Figure 6-13).

Figure 6-13. Timed starter tasks B

Emilia stated that starter tasks A and B are given as drill, practice and warm-up exercises and to develop speed and fluency in calculations and undertaking mathematical operations. Starter tasks A ask the student to find equivalent fractions and simplify the second segments of the tasks. Included also is a word problem of re-arranging the letters to identify the name of a country.

6.3.2.2 Posing the Main Mathematics Tasks

In making the choice for tasks, resources and decisions on how to teach a specific topic, Emilia begins with looking at the scheme of work and the associated textbook resources recommended. In Figure 6-10 above, on number skills, the fifth column of the table suggest the textbook *Collins Foundation* Chapters 1, 6 and 7.1 as the base resource. Similarly, in her preparation for the lesson she consults an array of other resources from which she creates, adopts and adapts the tasks that align with the topic of the lesson and the needs of her students. For examples of the tasks given during the main part of the lessons, see Figure 6-14 below.

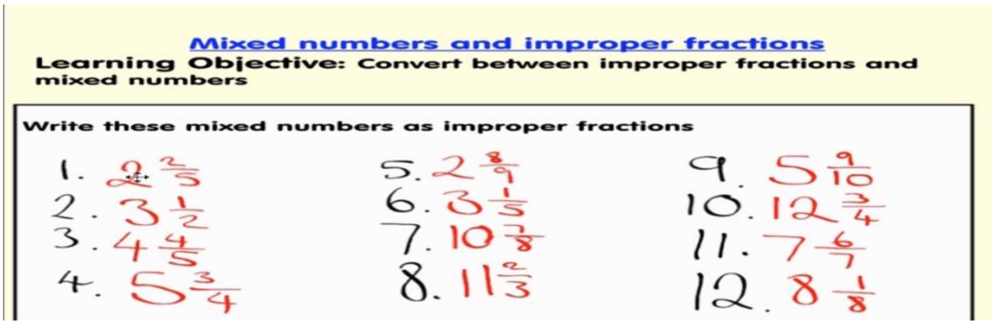

Figure 6-14. Main lesson activity tasks

The main lesson tasks in Figure 6-14, Emilia emphasises, are driven by the mastery scheme of work and the need for variation[61] to promote students' engagement and deepen their understanding of what improper fractions are and how they relate to mixed numbers through multiple examples. These were largely teacher-led sections of the lesson. The next set of questions were student-led in group and pairs.

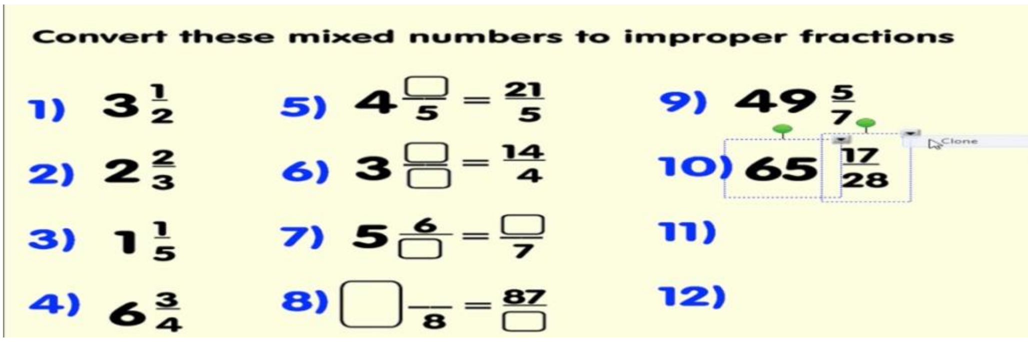

Figure 6-15. Main lesson student-led tasks

The students using various groupings and pairing formations explored and converted mixed numbers to improper fractions and vice versa. In a post-lesson interview, Emilia explained that her intent was to help the students understand the concepts of mixed numbers and improper fractions and enable them to deal confidently with various mathematical operations as they are presented in the tasks. Another focus was to help students practice multiplication skills of two-digit numbers as they exist within a context of mathematical tasks. Tasks 1-5 in Figure 6-15 show all the elements of the

61 Variation is further explored in the discussion chapter.

fractions are provided, and students can work directly with the numbers. In tasks 5-8, Emilia used blanks to vary the numbers and student are asked to discern and enter the number in the blank boxes working backward from the answers. In tasks 9-10, larger two-digit number co-efficients are used. These sets of tasks were designated with three colours: amber, red and green. Emilia explains the meaning of the colour coding of the tasks thus:

> Red, amber and green. It's a sort of differentiation. I do worked examples with them. If they feel they are 100% comfortable with that and they feel they totally get it, they can go to the green questions. These are slightly different from the worked example. I push them a little bit and get them thinking a little bit more. Whereas the amber one is quite similar to the worked example, sort of repeating but changing the number a bit. And then the red one is sort of a step back and they are a bit easier altogether. The students choose where they want to go. Generally, they go to the amber or green ones, but obviously if they are really struggling, they start on the red ones. The green is the hardest. (2IntE: #14)

In these ways, from teacher-led examples to student-led tasks using variation and differentiation (these terms are explored in detail in the discussion chapter) as the principles behind the task designs, Emilia directs her students progressively and systematically on the mathematical content and what she wants them to learn.

When she felt they were confident enough, she went on to plenary/independent tasks using bar modelling.

6.3.2.3 Demonstration of Mathematics Learning

The previous tasks (see Figure 6-14) were teacher-led, but tasks in Figure 6-15 were student-led with Emilia circulating and giving guidance. In the plenary, tasks were independent, and students worked individually on what was given using bar models.

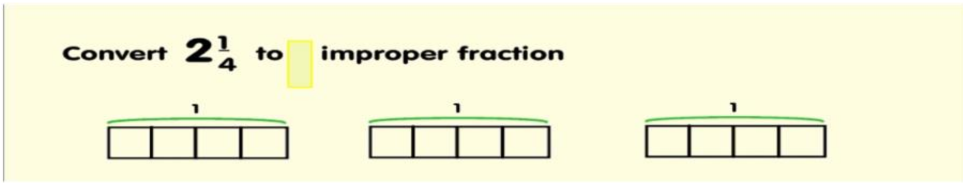

Figure 6-16. Convert to improper fractions using bar model

Emilia uses the bar model, which students are familiar with from previous lessons. The renewed use of bar model has been encouraged by the Chinese teacher's visit. Emilia worked through the example with the whole class as a reminder, and then the following tasks were given to the students to work on individually.

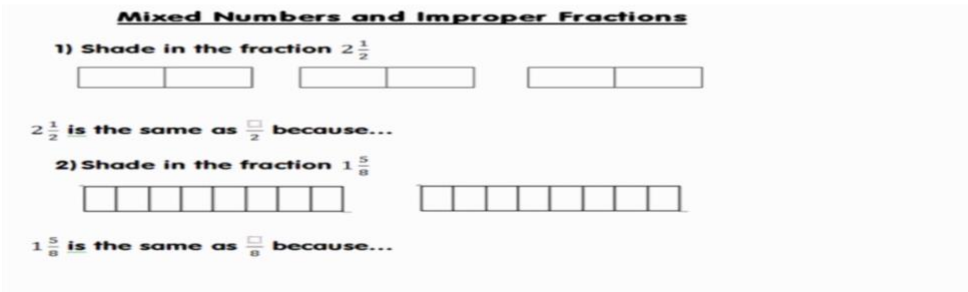

Figure 6-17. Bar modelling tasks with variation

In Emilia's planning, designing of tasks and delivery, she reported that the insight from the mastery approach is a key factor in her choices. This can be seen in the independent tasks above, asking the students to offer explanations for their answers and demonstrating the answers by shading the bars as well.

When I asked why there seem to be a lot of tasks for this class, Emilia's responded that the class consists of high-ability students and that all they need is more challenges to develop mastery through repeated exercises, practice and drills.

In the next subsection I present how Emilia concludes her lesson.

6.3.2.4 Concluding the Lesson

In closing the lesson, Emilia predominantly uses two strategies: the use of mathematics game; and activity review questions. In terms of the game, two mathematics games *Who wants to be a Millionaire?*[62] and *King of Maths*[63] are used. Emilia then moved to her next strategy by saying, “Tell the person next to you one or two things you have learned today”. The students in turn report to the whole class what their peer has shared and what they have learnt individually.

Emilia eventually pulls the main points of the lesson together linking these with what the students have shared. In some of the lessons, homework is then given.

6.3.3 Emilia’s Resources

In the exploration of the resources of Emilia, it is pertinent to note at the onset that much of the hardware, resource banks, school-subscribed websites and human resource people are similar to those reported in the case of Kitty above. I will explore those resources that are peculiar to Emilia and highlight the specific ways her usages of the common resources are different from those of the others.

As Emilia reported previously, she has a collection of personal and collective resources put together over her years of teaching and which she continues to update as newer resources are made available. Below, Figure 6-18 is a screenshot depicting Emilia’s personal resources collected over a four-year period (2012-2016).

---

62 http://www.math-play.com/math-millionaire.html
63 https://www.mathgames.com/play/kingofmath.html

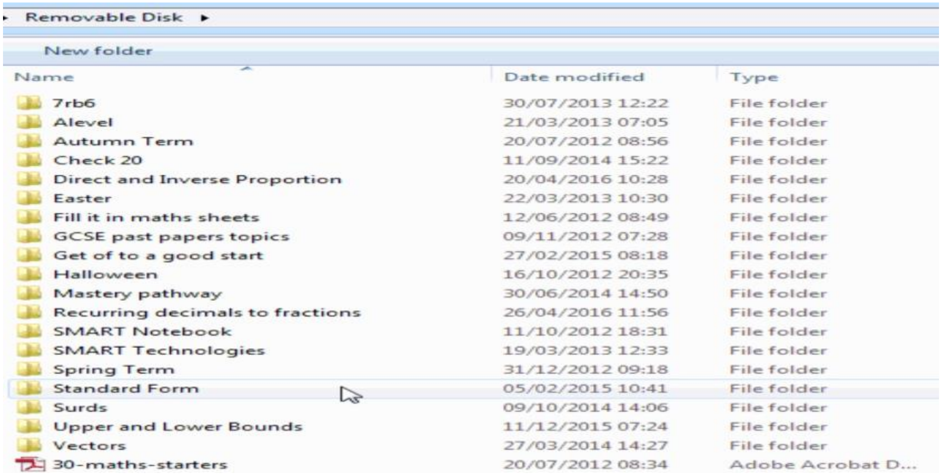

Figure 6-18. Emilia's personal resource bank from 2012-2016

Figure 6-18 shows the materials in Emilia’s personal resource bank are organised and designated by topic, class, term and reference to SMART technology use as well. Regarding the collective resource bank and how she sources her resources from the colleagues, she reported

> We also have a shared resource where we put in my worksheets and lessons, so that we can share really good resources and we also use email to send them out to each other, say this is a very good resource you want use in your classes. I also go to observe other teachers' lessons and pick up good ideas from them. It does help and also just talking to each other and asking if there is anything they have got that is good. (1intE: #7)

Emilia, as per the interview, lesson observations and screen capture recall, consistently makes references to her use of her personal collection of resources on her memory stick, laptop and the collective shared resources as places to anchor her lesson preparations. The shared resource folders are organised yearly. These also include homework resources and departmental documents that are accessible to all teachers. The shared resources folder shows in more detail how content is organised and used by the department according to topics and recommended resources.

The new shared resource folder (at the time of data collection) covered various topics for two year groups and this had been collected between February and June 2016. For Emilia, curricular resources for lesson preparations are adapted from the websites, personal and collective resources and from what she picks up from observing colleagues. Emilia's resources have been categorised using the logical classification mentioned earlier. Table 6-2 is Emilia's resources categorised into various types.

Emilia's Resources

Human		**Non-Human**			
Physical contact	Not Physical Contact	Electronics		Non-Electronics	
CPD Teach-meet Teaching Assistant (TA)		Hardware	Not Hardware	Individual	Not Individual
		IWB HWB iPad Calculator Laptop ICT Suit	QR code Spreadsheet Code buster Who wants to be a millionaire[64] Word Wall[65] Ttrockstars[66] King of Maths[67] Maths box[68] music Keshmaths[69] Corbettmaths	Worksheet Bar-modelling Number line Microsoft Word Emails	Shared resources folder

Table 6-2. Emilia's resources

6.3.3.1 Human and Non-human Resources

In the category of the human resources in Table 6-2, Emilia reported three physical contact resources: CPD, TeachMeet and TA. TeachMeet is a periodic school-wide teacher meeting to discuss common issues on education and how to improve teaching. This is mandatory for all teachers in school A, and Emilia reported that the gathering

64 http://www.math-play.com/math-millionaire.html
65 http://wordwall.co.uk/
66 https://www.ttrockstars.com/
67https://www.mathgames.com/play/kingofmath.html
68 http://www.mathsbox.org.uk/index1.php
69 https://keshmaths.com/

has been beneficial in introducing the teachers in the schools to new websites and resources that are useful and supportive of their teaching efforts.

In terms of the non-human resources, Emilia presents a wide range of hardware, websites and applications she uses in lesson preparation and delivery. These were specific and particular additions to her collection of resources: *King of Maths, Code buster* and *Who wants to be a millionaire* are interactive mathematical games. These mathematical games have a single-player feature and a multi-player option. This enables her to set mathematics contest between groups and pairs so as to promote student engagement and as a means of reviewing important mathematics concepts and operations. Her intent is to make mathematics an enjoyable experience for her students and to enrich their learning with fun embedded with mathematics. The *King of Maths* is a fast-paced mathematics game set in a medieval environment where you climb the social ladder by answering mathematics questions, solving puzzles and matching mathematics images. Players collect stars, get medals, compete and compare their scores against friends. The one who solves most of the mathematics tasks wins and becomes a knight or king of mathematics for that day game. *Word Wall* consist of diagrams, words and images pasted in the classroom and e-version displayed at the beginning of lesson as aid for teaching and learning of mathematical vocabulary. *Keshmaths*[70] are online banks of resources, these consist of Foundation GCSE Mathematics papers by year, video solutions to the tasks and marking schemes provided to all the questions by the Edexcel examination board. While topics are taught in class, students can have a follow-up using the video and the marking scheme to ensure they do not drop marks by carelessly skipping steps in the solutions.

70 KESH is an acronym for King Edward VI Sheldon Heath Academy. (These resources are produced in that school.)

6.4 Jimmy's Profile

Jimmy is 26 years old and has a BSc in Mathematics and Music, and a MMus in Composition. He taught for a year in a previous school before coming to school A and this is his second year of teaching as a qualified mathematics teacher. His teaching engagements spans Years 7-11, with an average class size of 25. He displayed a relatively high level of confidence in using various technologies to teach mathematics. The statistics regarding ICT and digital resources identified are shown later in the table of resources.

Jimmy is keen on the use of technology in his teaching and reported that it was one of the reasons he took up teaching in this school. In the first interview he made elaborate reference to this

> This is my second year teaching here and one of the reasons I did come to the school is because they have that already here; they have the IT room, they have the laptop, this school, the Maths department kind of already leads the way in results and in terms of using technology because we have our own set of iPads. We are the only department in school that does. I don't think that is common in other schools for just the Maths department to have technology; I think a lot of it has come from the leadership team being aware of tech being in use outside of school and saying it's clearly having an impact we should get involved. (1IntJ: #4)

Jimmy was enthusiastic about exploring the affordances of digital technologies in enhancing his search for resources, planning, delivery of lessons and assessment. He was also interested in exploring the potential of the digital resources, especially how these resources could motivate and enrich students' conceptual understanding of mathematics.

In the first interview Jimmy indicated his central motivation when teaching:

> I want to teach them for understanding. Do those online questions assess their understanding? So that's what am looking for, whether the questions are appropriate for their level, whether the students can actually do it after I have instructed them. (1IntJ:22:34#)

The emphasis on teaching for understanding is a mindset adopted and encouraged by the entire school. This was firmly restated in a formal session involving all the mathematics teachers and mentioned by Jimmy in the course of the interview.

> We want to teach for understanding using concrete, pictorial and visual methods. Let's do something on this. They were led by Paola (pseudonym), one of the assistant principals, and we just all talked about methods, and we got together, saying how could you improve this method? How could you explain this better to students? (1intJ:#25)

Jimmy's overall attitude is to 'teach for understanding using concrete, pictorial and visual methods' supported by technology use.

6.4.1 Role in the Department

Jimmy has been teaching for two years as at the time of the research. He reported that he is very interested in the use of technology in teaching mathematics. As a result of his interest and skills in the use of various digital and non-digital resources, he is one of the leading persons supporting other teachers in the department in terms of technology use. He is involved in a school-wide CPD programme.

> Yesterday, I gave a presentation to all the members of staff. We are doing TeachMeet, where members of staff from all over the school, all the different experiences, have to demonstrate their idea and many of those are technology-based and I did one on a website called diagnostic questions. Basically, it is a really, really big improvement on multiple choice questioning. Another one was done by a member of staff talking about an app called Plickers – again technology-based and makes life easier. It's again multiple choice; it scans every single student's answers immediately and it saves them. (2intJ: #7)

So, besides the regular teaching duties, Jimmy is involved in the school-based CPD programmes for the teachers and in the maths hub as well.

I now turn to discuss Jimmy's tasks.

6.4.2 Jimmy's Tasks

Jimmy's tasks are drawn from a wide range of online resource depositories, personal collation of resources, departmental shared folder, go-to bookmarked websites, CPDs and mathematics-dedicated social media groups.

In the preparation of a lesson Jimmy stated that

> The first thing I will do is to look at our school's scheme of work and look through what the next thing I need to teach is. (Screen capture: #00:06)

I now present Jimmy's tasks under four themes. The illustrative tasks in the presentation are based on the frequency of use in the observed lessons.

6.4.2.1 Tasks Initiating Lesson

As is the practice with school A, most of Jimmy's lessons begin with a starter (six out of the eight lessons observed had starter tasks). This is seen as one of the tasks used in initiating the lessons. These starter tasks are drawn from an online resource depository. One such depository is *flash maths*, shown below indicating various genres of starter tasks.

Figure 6-19. Starter tasks depository

Several of Jimmy's starter tasks were drawn from this site[71].

71 https://flashmaths.co.uk/menu.php?type=allStarters

I describe a Year 10 lesson observed on converting decimals to fractions and another lesson with Year 11 on factorisation to explore the range of tasks Jimmy uses in initiating his lessons.

In a lesson[72] to Year 10b4 on *Converting Decimals to Fractions*, these starter tasks were given to the students:

1. Write $\frac{64}{10}$ as decimal
2. Write $\frac{31}{100}$ as decimal
3. Write $\frac{24}{300}$ as decimal
4. Write $\frac{82}{200}$ as decimal
5. Write $\frac{24}{40}$ as decimal
6. Write $\frac{31}{30}$ as decimal

In a second lesson[73] to Year 11c1on *factorising*, the starter tasks were

1. Factorise $x^2 - 5x - 6$
2. Factorise $5x^2 + 16\text{x} + 3$

In each of the lessons, after the starter tasks the main lesson activity tasks were grouped into *Bronze*, *Silver and Gold* tasks in progressive order of difficulty. Using a QR code, the questions were usually scanned into an individual student's iPad or, when they worked in pairs or groups of three, the iPads were shared. Jimmy alerted me from the beginning on the regularity of his use of the QR codes and iPads in his lessons:

> I use QR code all the time, so you saw in my in my lesson earlier I had the students scanning for different work, which means they can all have their own work in front of them on their own iPads. They can zoom in; they can look at

[72] Year 10b4 lesson on 03/10/2015

[73] Year 11c1 lesson on 11/12/2015

> specific questions. I have often in the past had other QR codes to support, so if you don't get this worksheet, perhaps you should go to this website. They would have the QR code, and they can scan that and they can do it independently, and they have got support there. (2intJ: #5)

Jimmy seems to favour students working in a variety of ways, especially independently with support from the teacher when needed. Students take responsibility for their own learning with available support from the teacher. The iPad in conjunction with QR codes has become a stable teaching practice routine for Jimmy and his students as the following subsections reveal.

6.4.2.2 Posing the Main Mathematics Tasks

Jimmy uses iPads or some forms of digital resources in every one of his lessons. Tasks were given in three different stages – bronze, silver and gold. Figure 6-20 below shows the bronze tasks.

BRONZE

Write these fractions as decimals.

1) $\frac{13}{100}$ 5) $\frac{83}{100}$

2) $\frac{3}{100}$ 6) $\frac{99}{100}$

3) $\frac{26}{100}$ 7) $\frac{9}{100}$

4) $\frac{32}{100}$ 8) $\frac{10}{100}$

Figure 6-20. Bronze tasks for main lesson

When the QR code above is scanned, it automatically links up with the questions and these are displayed on the iPads for the students to solve. The bronze tasks are at the first level of difficulty, Jimmy explained. Similarly, the silver and gold questions (Figure 6-21) are progressively more difficult. The QR codes for the silver and gold questions are displayed below and the entire sets of associated questions are in Appendix D.

SILVER GOLD

Figure 6-21. QR code for the Silver and Gold questions

In commenting further on the use of bronze, silver and gold, Jimmy observes

> Once the technology is there, they have the standalone questions for the bronze, silver and gold, the Mangahigh tasks too, so they can have a bit more time on the Mangahigh task and they can really show me how well they have understood it, and those results will now feed back into my understanding of where I need to go next. (1intJ: #32)

The *Mangahigh* app has a huge collection of questions that are automatically generated as students engage with tasks and the level of difficulty varies as the students make progress. Mangahigh has a feature that enables automatic marking and grading, and these give analytics of students' overall performance, which informs Jimmy's future lesson planning.

6.4.2.3 Demonstration of Mathematics Learning

Jimmy uses two approaches in guiding the student to demonstrate their mathematic learning in the course of the lesson. These are through the extension and formative assessment tasks. In the following, using selected examples from Jimmy's lessons. I present how Jimmy guides the students to demonstrate their learning through these tasks.

Extension Tasks

These tasks are extensions of the tasks already learnt and used to develop a firm mastery of the topic. The QR code and associated extension tasks are shown in Figure 6-22.

MANGA HIGH

Assigned task: Changing Decimals to Fractions

a. Write the following in decimal number
   1. $\frac{71}{100}$
   2. $\frac{34}{50}$
   3. $\frac{186}{200}$

b. Write the following decimals as fraction in its simplest form.
   i. 0.92
   ii. 0.3
   iii. 0.45

Figure 6-22. Mangahigh based extension tasks

The extension tasks were often seen as part of a student's effort to demonstrate their learning and for the teacher to identify where intervention was required. As students work on their iPads, their answers are sent via the QR code and are instantly saved. Jimmy e-analyses[74] the feedback to inform in-lesson actions and future lesson plans. In the second lesson for Year 11c1, a similar QR code links the students to the tasks in Jimmy's *one drive* collection. Figure 6-23 below shows some of the questions drawn from a worksheet used in the lesson.

DIRT questions

2 Factorising quadratics

a) $x^2 - 2x - 15$

b) $x^2 + 5x - 14$

c) $x^2 - 49$

d) $x^2 - 25x + 156$

Figure 6-23. QR code for Year 11 tasks and some of the associated questions

These questions are stored in Jimmy's one drive collection, and students can easily retrieve them using iPads or a regular phone with an QR code scanner. The second approach Jimmy uses to elicit an idea of the students' understanding was through the formative assessment tasks and practices.

74 This is a feature in apps like Plickers and Socrative that instantly collate students' responses, analyse them and provide an instant graph or colour-coded view of each individual student's performance. This offers the teacher an opportunity for a real-time intervention in the lesson.

Formative Assessment Tasks

In the processes of formative assessment, students get an iPad each and scan in designated sets of tasks (as shown in Figure 6-22 and Figure 6-23) from select banks of questions via a QR code (Quick Response Code), which provides students with links to the task material that instantly pops open on their iPads. They then work on tasks individually for five minutes, followed by the teacher's whole-class intervention with constructive feedback.

The students' responses are collected using the iPad and Plickers and instantly e-analysed and colour-coded on a spreadsheet. The figure below shows the spreadsheet.

First name of the person	Question 1	Question 2	Question 3	Question 4	Question 5	Question 6	Question 7	Question 8	Question 9	Question 10	Question 11	Question 12	Question 13	Question 14	Question 15
	0	0	0	0	1	0	0	0	0	1	1	1	0	0	0
	0	0	0	1	1	1	0	0	0	1	0	0	0	0	0
	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0
	1	0	0	1	1	1	0	0	1	1	1	1	0	0	0
	1	1	0	1	0	0	0	0	1	1	1	1	0	0	0
	0	0	0	0	1	1	0	0	1	1	0	1	0	0	0
	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0
	1	1	1	1	1	1	1	0	1	1	1	0	0	0	0
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
	1	1	1	1	1	1	1	0	1	1	1	0	0	0	0
	1	0	1	0	1	1	0	0	0	1	1	1	0	0	0
	0	0	0	0	1	0	0	0	0	1	1	0	1	0	0
	1	1	0	1	1	1	0	0	0	0	1	1	1	0	0
	1	1	0	1	1	0	1	0	0	1	1	1	0	0	0
	1	1	0	1	1	0	0	0	0	1	1	0	0	0	0
	1	1	0	1	1	0	1	0	1	0	0	0	1	0	0
	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
	1	0	0	1	1	0	1	0	0	1	1	1	0	0	0

Figure 6-24. E-analysed formative assessment tasks

Jimmy explained in detail his use of this tool as an aid to improving the lesson, getting students engaged and demonstrating their learning:

> We did a mini-assessment just before the holidays. You saw on the slide I had a google form, I converted it into a spreadsheet for me to see and I then shared that spread with them. I took the names off, because I don't think it was appropriate for them to know who got what but I show them exactly what I was getting. I feel that's incredibly powerful, because suddenly the results are there for me and for the student; they can now understand exactly why I have to teach them a topic they were not necessarily getting before. They know they have been taught it; they don't really understand it, so they would say wow! we are doing this again. Because in all the tests you all got these questions wrong, so we have to do these again, and I think that is incredibly visual and incredibly powerful for them. (2intJ:#29)

As shown in Figure 6-24, the e-analysed and colour-coded whole-class performance is displayed on the interactive whiteboard (IWB), this triggers class discussion and formative intervention, and then further explanations or tasks are given to students in pairs and tasks done on mini-whiteboards while the teacher moves around the class. This lasts for about five minutes. Nominated/volunteer students share their solutions and peer-checking takes place. Students' e-analysed and mini-whiteboard feedback informs the teacher's 'emergent lesson planning'. Jimmy remarks on this:

> I think I can now feed that back into my next lesson. So, rather than being a standalone task, the results I have obtained from using that technology have given me something for the next lesson; I probably won't go back to that same format by saying you can do these bronze questions, these silver questions or these gold questions. I probably won't go back to that same format because it is very, very specific errors many of them were making, which only became apparent when I started marking the books, unfortunately. (2intJ: #31)

In the course of teaching, Jimmy always had several alternative QR code for tasks at various levels of difficulty and depending on the feedback from the e-analysed task. He goes on with his lesson as planned or he takes a step backward – redesigns the tasks moment of the lesson – and builds the knowledge for the ongoing topic.

6.4.2.4 Concluding the Lesson

In the case of Jimmy, the synopsis of the lesson involves extra tasks to further illustrate what the students have learnt and provide a summary of the key facts and ideas in the lesson: for instance, in the closing of a lesson for his Year 11c1 class, as a way of summarising key facts, checking progress and rounding off the lesson.
These related tasks are presented below.

1. $x^2 + 12\text{x} + 35 = 0$

$(x + 7)(x + 5) = 0$

so, x =___ or x = ___

2. $x^2 - 3\text{x} - 28 = 0$
   $(x - 7)(x + 4) = 0$
   so, x =___ or x =___

These two tasks were undertaken by all the students and random answers were taken and when the teacher was satisfied that the students had finally grasped the idea and topic, the class came to a close.

To further understand the role of digital resources and hardware technology in Jimmy's planning and delivery of lessons using various tasks, I now turn to explore in the next subsection on Jimmy's resources.

6.4.3 Jimmy's Resources

While exploring Jimmy's resources, it is relevant to note that the human resources (physical), hardware, non-electronics (individual and collective resource banks) and school-subscribed applets are similar for all the four teachers in school A. I will here focus on those resources that are peculiar to Jimmy and in the subsequent subsection on discussion, I also highlight and further examine the resources that Jimmy reported to use more frequently. Jimmy's resources are shown in Table 6-3.

Human		**Non-Human**			
Physical contact	Not Physical Contact	Electronics		Non-Electronics	
		Hardware	Not Hardware	Individual	Not Individual
Personnel Colleagues Chinese teacher CPD	Facebook Twitter (#mathschat) Blogs[75]	Calculator iPad Laptop IWB HWB	GeoGebra Autograph Desmas Socrative Virtual manipulatives mathsbox.org.uk[76] m4ths.com Mathspad.co.uk[77] Corbettmaths UKMT maths challenge[78] Diagnosticquestions.com	Workbooks Worksheets Poster Google form	Resources bank Shared Folder

75 http://donsteward.blogspot.co.uk/
76 http://www.mathsbox.org.uk/index1.php
77 http://www.mathspad.co.uk/
78 https://www.ukmt.org.uk/

			www.resourceaholic.com[79] Plickers[80] QR code studymaths.co.uk[81](GCSE maths revisions only). http://flashmaths.co.uk/[82] Dreambox.com[83] Mangahigh.com Mathscentre.ac.uk[84] Resourceholic.com TES STEM website		

Table 6-3. Jimmy's resources

I now discuss some of these resources.

6.4.3.1 Human and Non-human Resources

In the human resources category, Jimmy reported the CPD, Chinese exchange teachers, Facebook, Twitter and blogs, as shown in the table above. These human resources besides Facebook, Twitter and the blogs are unique to Jimmy; the other resources are reported by the other teachers in school A.

Here, I focus more on two (adaptive learning resources and STEM centre websites) of Jimmy's reported non-human resources because of the emphasis Jimmy placed on these and how they share a central place in his teaching practice.

Adaptive learning technologies are emerging in educational settings. They use computer algorithms to orchestrate interaction with learners and deliver customised resources and learning activities to address the unique needs of each student (Walkington, 2013). In Jimmy's lessons and assessments, he uses the

79 http://www.resourceaholic.com/
80 https://www.plickers.com/
81 http://studymaths.co.uk/
82 http://flashmaths.co.uk/
83 https://www.dreambox.com/why-dreambox
84 http://www.mathcentre.ac.uk/about/

Mathspad.co.uk, *Dreambox.com*, and *Mangahigh.com*; these resources share adaptive learning characteristics. They consist of a growing collection of mathematics resources aligned with the National Curriculum. Take *Mathspad.co.uk*, for instance: Figure 6-25 below shows its various features. Mathspad has a collection of worksheets and interactive tools, where students can manipulate objects and diagrams.

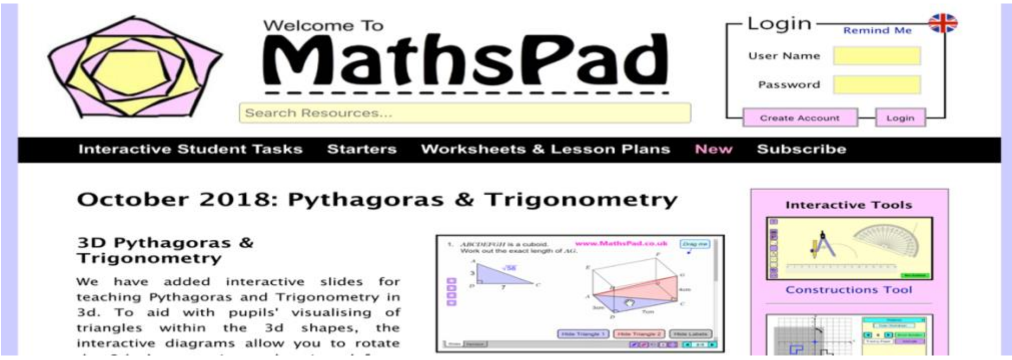

Figure 6-25. Mathspad adaptive and interactive resources

There are tasks, worksheets, and features for personalising documentation and tracking and reporting progress on individual students and on the whole class. The computer algorithm enables the technology to modify the presentation of tasks in response to a student's performance and adjust to the student's pace. Jimmy attested to the use of these in the school:

> This department is brilliant at sharing things. With such a big department I don't think we will work very well together if we didn't share. Mangahigh for instance is a subscription website; the department had to get involved with that altogether. I know Mr. Geoffrey[85] came across it, before I was here; this year they have really gone into it. It is a paid service ... we need to make most of it. *Studymaths.co.uk* is something I have discovered personally. I have not really shared it around much. It's great for marking and great for short, sharp one-mark questions; it is not brilliant for developing the understanding, simply for demonstrating it. Whereas Mangahigh, it helps to develop it because it offers hints; it changes the questions. If they can do the easy one, it gives them a harder one. (2intJ: #8:31)

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85 All names used in the interviews and in the thesis are pseudonyms.

These adaptive learning resources have authoring tools that enable students to take on constructions of tasks interactively online. It has an embedded continuous-assessment facility that builds up records of performance for the students over time. *DreamBox* and *Mangahigh.com* share similar adaptive learning features and Jimmy uses them to complement classroom activities, adapt to students' actions, and personalise instruction, which together promote student decision-making and strategy development.

A sample of the authoring pad with tools and an interactive task are shown in Figure 6-26 below.

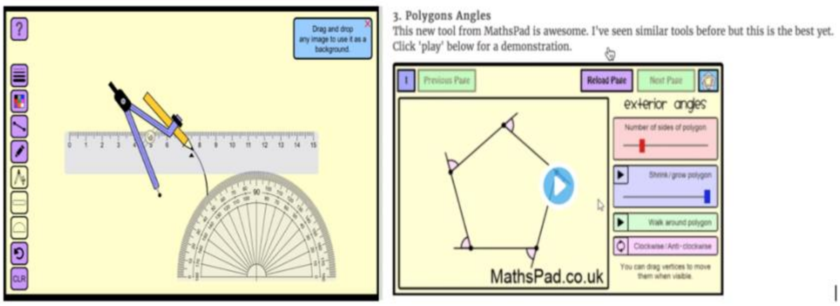

Figure 6-26. Features of adaptive and interactive learning resources

STEM Centre Website

The STEM website houses thousands of curriculum-linked STEM resources created and updated by a virtual STEM community. The quality of the resources is assured through the gatekeeping function of members. It also provides opportunities for CPD and bursaries for teacher training and doubles as a research dissemination centre. Jimmy identified this centre as one of his go-to areas in lesson preparation and delivery:

> I personally have a list of websites, which I've bookmarked. So, the first is the national STEM centre website because there are some fantastic worksheets, questions, manipulatives and everything I want from there. You can see I have got absolutely loads and am happy to send you a list ... Every time I find something useful online, I tend to bookmark that website and go back to it. (3intJ: #3)

In summary then, Jimmy is a technology and digital resource enthusiast and the case description points to this claim. Jimmy presents an assessment of how he feels about the place of digital resource and technology in society, and within teaching and learning. This is worth quoting in its entirety.

> I think technology is proving to be incredibly powerful and incredibly engaging for the students in the UK because they are surrounded by it. They are surrounded by technology whether we like it or not. And it's the way they're gonna live. I got my mobile phone in my pocket; I can get it out when I'm at work in case I might go to Twitter and look at some maths questions. I got my iPad; I can check my work in it. I use it all the time regardless of whether I'm using it for tasks or not and that is clearly having an impact in the UK where the children grow up. They have so much more online presence t than before technology, before it was so widely available, before there was IWB in classroom. When I was in school, and I still remember the first lessons we had on IWB, and I'd say what is the point? All you are doing, you are writing on the board and you can rub it out, which you can do on a normal white board, just a bit less mucky. So, I think the uses of technology can make learning incredibly more immersive, but everything can be done without technology. There are certain things, of course, technology can do far better; you could not do the calculation with precision ... without technology behind it. In terms of learning, in terms of learning the fundamentals, I think the curriculum is built around knowledge which you can get without tech; tech can enhance the learning and make it much deeper and make much better mathematicians and much better people. I don't think anyone should say we can't learn without it. It is not ultimately necessary, but when I think of planning now, I have become so used to this technology, so used to this ability to delve into topics in so much more detail, with the dream tools, virtual manipulatives. I can zoom in on a number line; I could never do that before. I could only get a whole number line and draw on the floor; think about getting 30 kids around it – you couldn't do that. I think it would be absolute chaos and the students would not get as much out of it. It can be incredibly powerful, but it is not wholly necessary. (1intJ: #35)

Jimmy acknowledges the positive role of digital resources in society and for learning and the impact on his professional practice, yet he believes it is not 'ultimately necessary' or 'wholly necessary' all the same.

I now explore the case of Jose, the last of the four teachers in school A.

6.5 Jose's Profile

The data on which Jose case description is based comprises of lesson observations, self-reported profile sheet, interviews, screen capture recordings and collated documents.

Jose, 29 years old, teaches mathematics to year groups 7-13, in the age range of 11-18. This is the second year of teaching since he attained qualified teacher status (QTS). The average number of students across the classes he teaches is 24. As reported earlier (subsection 6.1, p. 117) in school A, the use of technology and digital resources is encouraged and supported. In his two years of teaching, Jose seems to have been under both informal and formal mentorship and beginning to develop his own identity and confidence as a mathematics teacher. He explains in this interview extract how he came to start using technology and digital resources in his teaching practice:

> Before I started teaching, I was not tech savvy at all. But what made me start using iPads was the lead teacher in the department at that time. He was in charge of the iPads. In a few CPD events where he introduced the iPads apps you can use in the classroom. And simply because I was told what to do, I felt I was confident enough to try it. And what I really like about how he introduced it to me, it was about, you only use it when it makes it easier for you. Technology saves time, rather than creating more complications. (#1intJs1:00)

For Jose the introduction to using technology was based on the influence of the lead teacher and CPD training. Jose reports he eventually started using the digital resources following the instruction and guidance from the lead teacher.

The data indicates three teachers as the dominant influences on Jose's mathematics teaching and his use of digital resources. In one interview extract, Jose restates the influence of two teachers in the department and one via Twitter on his own professional growth as a teacher.

> The mathematics department has a huge, huge influence. Every single thing I do I can point to a different teacher that has influenced me. For example, in

> technology, *Mr. Mill*[86] completely; he is the only reason I use iPads, he is the only reason I use laptops. And then again *Mr. Stan* is the only reason I use IWB regularly and assessment tools. It is not through anything formal, just occasionally seeing him teach, in passing, talking to them about something but sometimes it can be CPD events. (#1intJs9: 40)

This extract shows Jose gradually becoming immersed in the culture of technology and digital resource use. This is influenced by the other teachers through formal and informal routes that are available and create a condition wherein Jose can pick up the practice himself. The other influence mentioned was Mr. Barton, through Twitter. *Mr Barton* is a mathematics teacher who has a major influence in the mathematics-teacher Twitter community in England. This will be further explored in the subsequent subsection on Jose's resources.

From the self-report and all the data collected on Jose there is no indication that he has any specific leadership role in the school or mathematics department. I now turn to explore his tasks.

6.5.1 Jose's Tasks

Jose's appropriation and use of tasks is driven by a belief and a mindset that is unique among the teachers in school A. He believes in self-created tasks that are primarily tailored towards the needs of his class, and he seems to argue that no external task designer can create a task that could fit into a learning cycle and could anticipate where his students are in their learning and where they need to be. Jose states the reason and the underlying belief in creating tasks, and this forms the background to the tasks that are identified and discussed subsequently.

> In terms of getting resources together, nearly all teachers try going on TES and downloading resources from there, **but I cannot stand downloading those because it is so difficult to find something appropriate for your class**. And I think if I find something that was good, **I will be doing my class a disservice because I will be trying to wangle their learning towards that**

86 All names used in the interviews and in the thesis are pseudonyms.

resource rather than what they needed. So, **I try to create my resources myself.** (#1intJs 2:50; emphases are mine)

In more practical terms, Jose presents the initial steps in his lesson preparation and how he finds inspiration to create tasks for his students.

> Say I am just starting a new topic and I want to create my own resources. I will start with the school scheme of work on the system; look at the typical questions they ask. I might then look online, just to see roughly where the questions are going. Maybe I might look at online textbooks rather than something like TES, which is a compilation of teacher resources. I will look at that and then tailor to my class. (#1intJs 3:54)

This resonates with a case mentioned earlier in that it is typical among English secondary school mathematic teachers to begin lesson planning by seeking guidance from the scheme of work. In Figure 6-27, below, Jose accesses the scheme of work in the staff's departmental documents folder for mathematics. This is where the direction for implementing the local adaptation of the National Curriculum is provided. The scheme of work most often suggests possible resources to draw from if the teacher wishes. There is a high level of teacher freedom to choose from any resources elsewhere as long as the objectives of the topic are met, and students are able to achieve the desired learning targets.

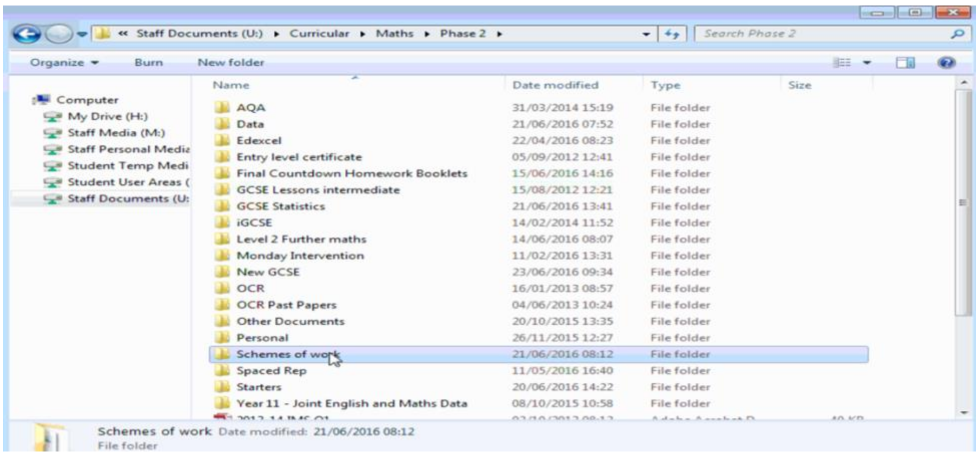

Figure 6-27. Jose accesses the scheme of work

Jose's use of schemes of work reinforces the earlier finding on the use of schemes of work. There is also some evidence to suggest that while Jose acknowledges the value of TES as a compilation of teacher resources and '*look at them for inspiration'*, he does not believe the resources address his class's needs. This mindset seems applicable to his interaction with and appropriation of digital resources. I now take Jose's lessons on *fractions* for Year 7 as a case in point in order to explore the variety of tasks he creates and uses.

6.5.1.1 Tasks Initiating Lesson

In Jose's case, the tasks initiating his lesson include starter tasks, bar modelling and number lines. These are now presented in turn.

The starter tasks are often the basic opening activity of most lessons in school A. Joe had starter tasks to begin all seven lessons I observed. These are the sets of starter tasks given at the beginning of one of the lessons on fractions.

Starter 1.1 Division (Fraction Revision)

1. $\frac{1}{2} \div \frac{3}{5}$
2. $\frac{2}{3} \div \frac{3}{4}$
3. $\frac{2}{5} \div \frac{1}{4}$
4. $\frac{3}{4} \div \frac{7}{10}$
5. $\frac{2}{5} \div \frac{7}{8}$

These starter tasks were drawn from the departmental folder and Jose's personal collection of resources. Besides the above set of starter tasks, Jose gave another form of starter task: a word problem. The screen capture below shows the problem-solving starter task, which is geared towards understanding what a fraction is, as Jose stated in a post-lesson conversation. In Figure 6-28, Jose sets a variety of questions that point to how fractions exist in the real-life context using time, weight and money to illustrate what a fraction is to his students.

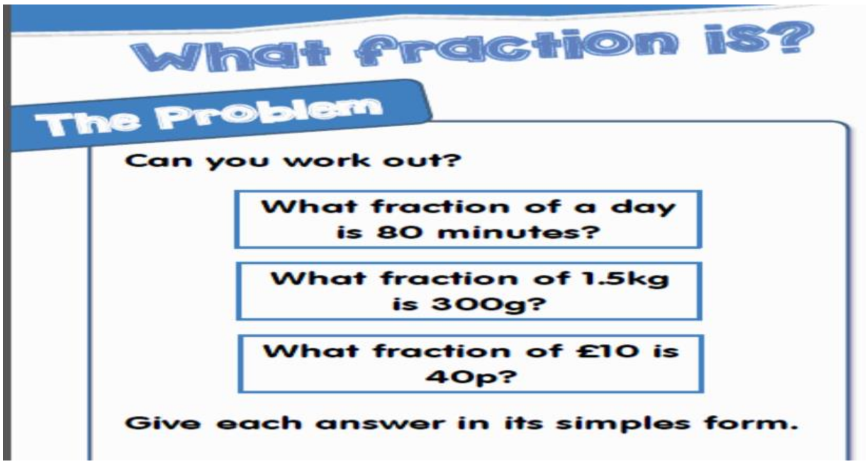

Figure 6-28. Problem-solving starter tasks

Joe asked the students to work on these three tasks. Then, a few answers were reported and feedback given as well.

From the observation too, Jose uses bar modelling and number lines to demonstrate the meaning of fractions at the beginning of his lessons. In school A there is a renewed and increased use of bar modelling and the use of number lines inspired by the visiting Chinese teachers, who deployed this in their own sample lessons in school A. Here, Jose appropriated the same bar model and number line in teaching fractions. Figure 6-29, below, is an example of one of the bar models used in the class.

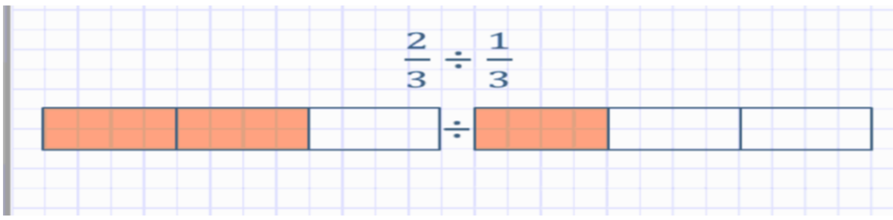

Figure 6-29. Bar model on fractions

The students replicated this on their hand-held whiteboards as they engaged with the tasks. Each student then raised their handheld board up as instructed by Jose and he moved around assessing how the students had responded to the tasks. Volunteer students explained their solution strategy to the whole class with Jose adding emphasis.

This bar modelling taken from Singapore-style mastery mathematics allows students to draw, visualise and make mathematics concrete, and it is frequently used by mathematics teachers in school A. Jose's students were able to replicate this on a paper with grids, adapting and working their way using the bar to arrive at the possible answer. Another similar strategy is Jose's use of number lines.

Figure 6-30 shows an instance of Jose's use of the number line in teaching fractions.

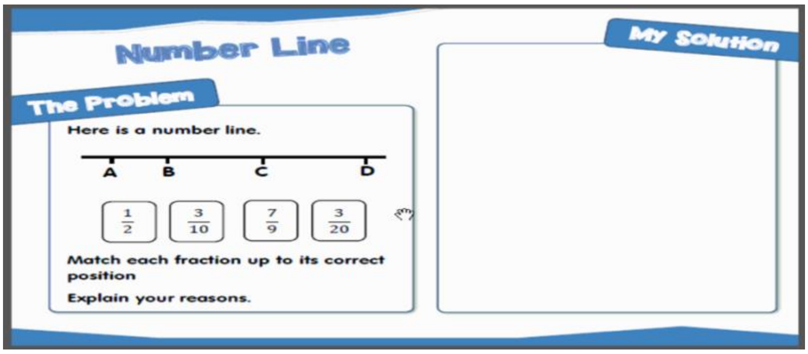

Figure 6-30. Number lines in teaching fractions

The number line shown above is a straight line with letters placed at segments or intervals along its length. This number line could extend infinitely in any direction. This is used by Jose to illustrate mathematical operations with fractions. Students in the class were asked to provide solutions and give reasons for their answers. Apart from identifying the starter tasks, Jose did not clearly state if his lesson follows any specific lesson structure. Beyond the starter questions, Jose had a series of other tasks with varying levels of difficulty. These tasks were colour-coded into bronze, silver and gold.

6.5.1.2 Posing the Main Mathematics Tasks

In posing the main mathematics tasks, Jose uses colour-coded differentiated sets. He explains that this is to enable him to take care of the wide range of student abilities and groupings. The format of task differentiations and grouping within Jose's classroom is identified by the colours bronze, silver and gold. Jose believes, "*I think it is Archimedes, or some sort of Greek thinker used that to teach his pupils: bronze, silver and gold. Bronze will be the easiest; silver will be more difficult; gold most difficult"* (#1intJs15:35). He says this allows students to access the fractions topic at a level where they feel most confident. In explaining the differentiation tasks further, he gave an example of a Year 10 class on the topic of *expanding the brackets* and how this leads to task modifications.

> What I do, I will have one resource for expanding the brackets for Year 10. It will be perfect. It starts where they need to start. Then I will now use it next year. I might have middle set Year 10, what I will do, I will keep the slides, add bronze questions, add silver questions, maybe I will add more gold questions, so that my new Year 10 can start about the halfway down. stress themselves to get the gold questions. Take a resource, expand a little bit so they fit the class, a little less work than the previous year. (1intJs: #14:33)

For Jose the lesson slides with tasks differentiated into three groups are kept and continually modified each year to fit the needs of the class and the sets. The *middle set* referred to in the interview extract is a form of ability grouping[87] found in many English secondary schools, whereby students are grouped by ability in a specific subject. The 'setting' consists of lower, middle and top sets. For him, this iteration of modifications is to align the tasks with the ability group of his class. Being able to modify and reuse previous resources with a new class means less work, since he does not need to start creating tasks again. Going back to the tasks on fractions, Jose's silver questions and gold questions on dividing fractions by fractions for the middle and top sets ability groups, respectively, are shown below.

Silver Questions

1. $\frac{4}{3} \div \frac{3}{5}$
2. $\frac{7}{3} \div \frac{9}{7}$

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87 See Ireson, J., Hallam, S., Hack, S., Clark, H., & Plewis, I. (2002). Ability grouping in English secondary schools: effects on attainment in English, mathematics and science. *Educational Research and Evaluation*, *8*(3), 299-318.

3. $\frac{6}{7} \div \frac{1}{10}$

4. $\frac{11}{5} \div \frac{1}{9}$

5. $\frac{1}{6} \div \frac{10}{11}$

Below are some gold questions selected from Jose's bank of practice questions.

Gold Questions

1. $1\frac{4}{5} \div \frac{2}{5}$
2. $2\frac{6}{7} \div \frac{3}{7}$
3. $\frac{3}{5} \div 1\frac{7}{5}$
4. $9\frac{4}{5} \div 2\frac{9}{10}$
5. $3\frac{2}{5} \div \frac{7}{10}$

In the selected silver and gold questions, one might observe the gradual variation of the elements in the fractions and, as Jose stated, the difficulty level as perceived by students changes from the silver to the gold questions. Many of these tasks were also used as assessment tasks.

Now, I turn to present how students demonstrate their learning in Jose's lessons.

6.5.1.3 Demonstration of Mathematics Learning

Jose uses formative assessment tasks to elicit demonstrations of their learning from the students in the lesson. These formative assessment tasks are drawn from a bank of differentiated GCSE examination-type practice questions, *Check 20*[88]. For instance, Figure 6-31 is a sample task used in the lesson.

88 Top 20 GCSE differentiated question sets for practice tests

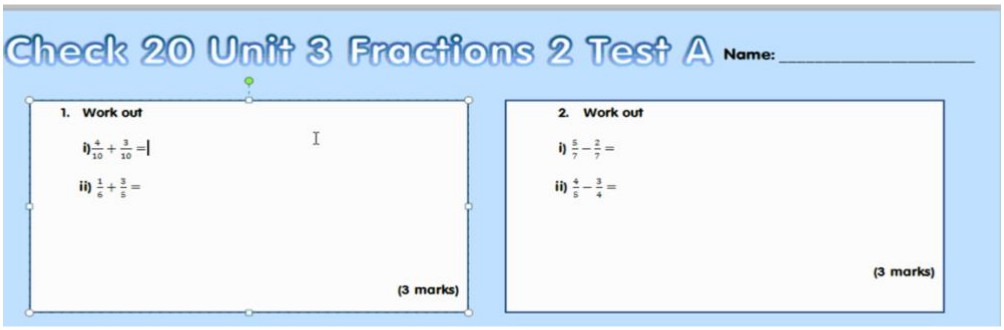

Figure 6-31. Assessment tasks from Check 20

The assessment tasks were then given to students towards the end of the lesson. Jose was checking their learning progression and finding out which varieties of fraction tasks he had taught they are still finding it difficult to solve. Jose's assessment was formative in nature. He used the tasks to assess where the students are in their learning, where they need to go and how best to get there. He made this clear in this interview extract:

> I will close that down and I will think of the steps I will need to get from where the students are to where they need to be. (#1Js4:35)

For Jose, these assessments could encourage students to work harder when they are able to see how they are doing in their learning, how their friends are getting on and where they need to get to. In four of Jose's lessons some form of assessment were carried out towards the end of the lessons.

6.5.1.4 Concluding the Lesson

In Jose's case, concluding the lesson begins with asking students to mention two or three key ideas they think everyone in class should have learned. Then, a couple of students are selected to verbally share their responses with the whole class. The lesson then comes to a close with a homework activity worksheet given out.

In the next subsection, I look at Jose's resources.

6.5.2 Jose’s Resources

Table 6-4 below shows an inventory of the resources used by Jose and the logical classifications of them into two broad categories of human and non-human resources. These resources were identified and listed primarily from the interviews and observation data. Other resources, while not mentioned in the lessons, were recorded from the screen capture when they were seen in use by Jose, like QR codes and spreadsheets. The table reflects Jose’s belief, stated earlier, about his preference to create his own resources and mostly look at online resources only for inspiration.

Human		**Non-Human**			
Physical contact	Not Physical Contact	Electronics		Non-Electronics	
CPD TeachMeet (Whole School) Weekly 10-15mins departmental meet	Twitter[89] MathsWatch[90]	Hardware	Not Hardware	Individual	Not Individual
		Calculator iPads Laptop IWB iWB	Socrative (apps) Plickers QR code Spreadsheet Online textbooks* Mr. Barton maths[91] 10ticks[92] *iKnow My Class Survey*[93] TES	Workbook Worksheet Bar modelling Number line	Resources bank

Table 6-4. Jose's resources

The table shows the range of Joe’s resources and their classification.

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89 https://twitter.com/mrbartonmaths
90 http://mathswatch.co.uk/
91 https://www.mrbartonmaths.com/
92 http://www.10ticks.co.uk/
93 http://svsurveys.corwin.com/?loc=US

6.5.2.1 Human and Non-human Resources

Jose's human resources include all those that are typical of school A with Jose reporting, additionally, a weekly departmental meeting. My emphasis here will be on the digital resources, since a major focus of this research is on teachers' use of digital and non-digital resources and those applications specific to mathematics.

The IWB, iPads and laptop are the central hardware Jose uses. In school A, these are commonly used by the teachers who participated in this research. This hardware plays an organising function for the teacher and students together with other digital resources. Analysis of Jose's lessons show a frequent use of this hardware as a standalone resource and/or in combination with other digital resources.

> iPad and various iPad apps: Plickers, Socrative. I use this thing called *iKnow my own class survey*, which are online surveys for students to fill in; they are mainly to do with asking students their opinion on how they feel about their learning, what they like about mathematics and what they find difficult. I have used laptops for MathsWatch, which is just series of revision videos for usually Year 10 and 11 to watch while they revise their specific topic. (#1intJs1:30)

Plickers, Socrative and *iKnow my own class survey*[94] are data collection and analysis resources used for assessment and for giving the students the opportunity to contribute to the lessons. By using spreadsheets, Jose is able to analyse, visualise and assess student's achievements formatively and to find out the overall feeling and mood with regards to the mathematics that is taught and what students feel about the teaching. Closely connected to this is the departmental resource bank where resources considered the 'best' are stored and shared. The departmental resource banks are managed by the head of department and the deputy.

Jose explains how the departmental group operates:

> We meet 10-15 minutes a week. We often put resources into a shared area in a computer system so that everyone can access them. They are supposed to be

94 http://svsurveys.corwin.com/?loc=US

> the best resources that can be used by everyone and shared by the department. (#1intJs10:30)

In school A, there is a reported culture of collaboration, and this practice is encouraged and supported at both formal and informal levels.

In the screen capture recording of lesson preparation, Jose is seen using the resources that have been collated over time in the shared resources area. The two screen shots below show the range of the resources in the shared area. First, Figure 6-32 shows the shared folder resources and the second, Figure 6-33, shows the content of a sub-folder where Jose sourced resources used in the observed lessons on fractions.

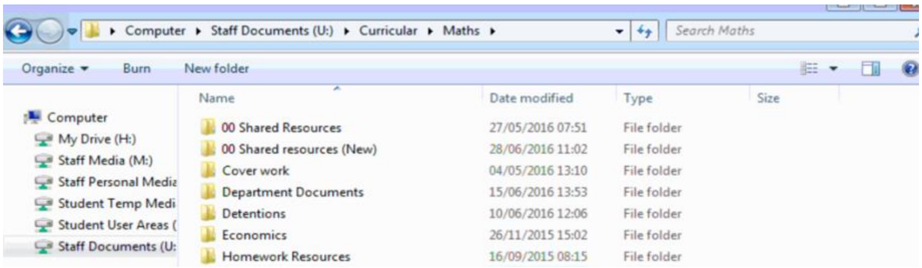

Figure 6-32. Departmental shared resource folder in school A

Figure 6-32 shows the various collated resources that are accessible to all teachers. In the interview, Jose indicates that “the lead teacher has the responsibility to make sure the resources are looked after and well ordered. The head of department and the deputy as well” (#1intJs11:21).

The sub-folder on fractions in Figure 6-33 shows the range of resources collected on fractions, variation of activities and formative assessment questions available for use.

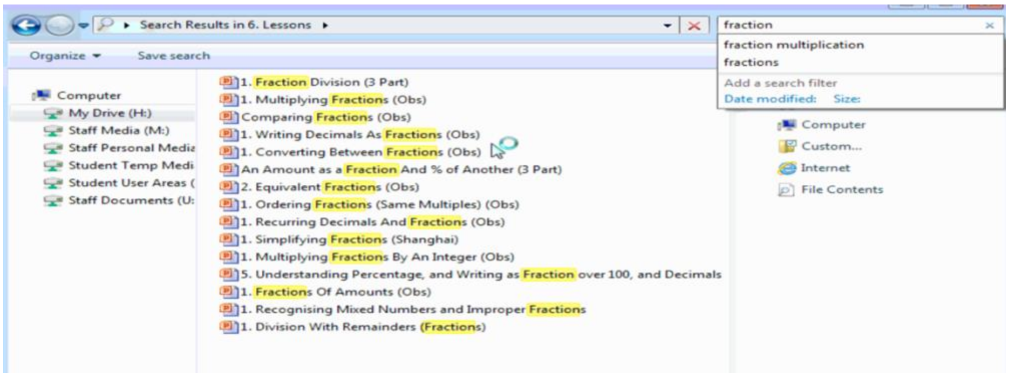

Figure 6-33. Departmental sub-folder on fractions

Figure 6-33 shows the accumulation of the resources on fractions from six lessons that are deposited in the departmental shared folder. This folder functions for the teacher as an available 'go-to' place for lesson preparation, delivery and assessment.

One observation worth stating is that there are only four resources that are specific to doing mathematics across Jose's table of classified resources: *Mr. Barton Twitter page*, *10ticks*, calculator *and resources bank*, though online textbook is mentioned but only in a generic sense. The MathsWatch videos are designed for students to use as revision support and an independent learning resource. These resources cut across the various categories: the not physical contact, hardware, not hardware and not individual classifications. This may be as a result of Jose's stance that resources are often not well-suited to his class's needs. As he stated earlier in the interview extract above (p.119), "I do not want *to wangle their learning towards that resources rather than what they need. So, I try to create all my resources myself*". This mindset, I believe, influences Jose's appropriation and use of digital mathematics resources. Jose affirms this in this interview extract:

> I just remembered something I use quite a lot, using Twitter and following Mr Barton. Because Mr Barton is a well-known mathematics teacher and he tends to retweet anything that is actually good on TES. I usually follow him on Twitter and pick up resources that he is tweeting from TES. (#1intJs12:34)

Jose belongs to the mathematics teacher Twitter community and in particular follows Mr Barton, who is a TES adviser and at the time of writing has posted nearly 11,000 tweets with over 28,000 followers online. He has been on Twitter since September 2010. The screen shot below in Figure 6-34 shows the extent of his activities and following.

Figure 6-34. Mr. Barton Twitter dashboard

Mr Barton's Twitter page is the only resource Jose explicitly states he picks resources from. Although Jose claimed that he 'cannot stand downloading resources' from TES, he acknowledges how widely used TES is in the mathematics teacher community.

> TES is heavily relied on nowadays. People definitely use them a lot. I think I am slightly more or probably an exception. I tend to use things I have created, which creates a lot more work. But what it means is that I can re-use them year after year because they are good enough. (#1intJs 13:04)

Jose is an exception in his belief about the personal value of resources on TES in his teaching. He presumably uses the Mr Barton Twitter page as a quality filter or a gatekeeper for making judgements on the resources. He explains, "I *usually follow him on Twitter and pick up resources that he is tweeting from TES".* In an indirect way, Jose uses resources from TES through a third party. Perhaps this is a good case for a boundary object. Does Mr Barton tweeting TES resources make it less of a TES resources? This is a good case for examining a *boundary object*, though this is beyond the remit of my present study. The value of the digital resources for Jose is that they are sources of inspiration and guidance for framing his questions: "*I look at them for inspiration. Roughly, what kind of questions I need to be aiming towards".*[95]

I now present the collective practice among the mathematics teachers in school A.

6.6 The Collectives of School A

The teachers in school A participate in a variety of teacher collectives. In terms of continuous professional development (CPD), there is a monthly TeachMeet, an informal but well-organised opportunity for teachers (from all the departments in the school) to meet and share 'good practice', introduce a generic aspect of the curriculum (e.g. Progress 8), share resources and build up collegial rapport.

The national *maths hubs* provides opportunities for the collective co-production of resources, collective design of lessons, mathematics subject-specialist training and teacher-researcher interactions, which are the norm. The core aim is to collaboratively develop and spread excellent practice for the benefit of teachers, students and other practitioners through various working groups.

The four teachers considered above from school A belong to various online communities. They all belong to the *TES*[96] online community wherein educational professionals, teachers and school leaders share and exchange resources and there is a live forum where conversations take place and support is provided. On Twitter, the hashtags #SLTchat[97] and #educhat[98] are dedicated online Twitter communities of teachers, and two of the teachers belong to this group. One of the teachers also belongs to a closed Facebook group: closed in the sense that membership is restricted to a specific class of a previous set of NQTs (newly qualified teachers). The *maths hub* also provides a Facebook page and twitter handle for dissemination of information,

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95 (#1intJs4: 35).
96 https://community.tes.com/
97 https://twitter.com/SLTchat
98 https://twitter.com/hashtag/educhat?src=hash

sharing resources and advertising upcoming opportunities.

Two of the teachers (Jimmy and Jose) are members of the Mathematical Association (MA). The MA is a professional society concerned with mathematics education in the UK. The association's website states the core aim of the group:

> The Mathematical Association exists to support and promote confidence and enjoyment in mathematics for all, and especially young people. We do this through interacting with teachers and others, including young people themselves, via our publications and resources, workshops, conferences, professional development provision, nation-wide branches, and interactions with the media. We work to influence mathematics education policy in evidence-based ways that support the development of a mathematically enabled confident and interested population.[99]

The central aim of the MA is to provide support for mathematics education professionals, teachers and young people and review matters and policies concerning the teaching and learning of mathematics. There are also regular branch meetings and an annual national conference.

In summary, school A provides a varied landscape of opportunities for collective mathematics teachers' work. The school-recommended platforms are the continuing professional development (CPD) sessions, TeachMeet[100] and the *maths hub*. The departmental staffroom is open to many spontaneous and informal collaborations and periodic formal meetings to plan a specific topic and discuss modes of assessment. The online collectives are usually voluntary, and the teachers here are committed to their membership and participation.

6.6.1 In Summary

In Chapter 6, I presented the case descriptions of the four participating teachers in school A. The teachers share similarities and show individual uniqueness in the way

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99 https://www.m-a.org.uk/who-we-are

100 http://schoolsweek.co.uk/why-teachers-are-catching-the-teachmeet-bug/

they teach. These similarities and differences exist in their lesson planning and delivery practices, the roles they play in the school, the resources they use and how they participate in various face-to-face and online collectives. The school is a leading academy and in the vanguard for school improvement through the maths hub initiative and UK-Shanghai exchange programme. The initiative and programme influence the approach to mathematics teaching and how the teachers engage with digital resources. The four teachers have different background experiences that they bring to bear on their teaching practices. There is a culture of collective work and mutual sharing of resources, and the use of technology is supported.

In the selection and use of tasks, the four teachers share many similarities and individual differences. The similarities, it appears, are influenced by the commonly available resources in the shared bank of resources and technology: for instance, Socrative and the iPads. The difference comes from the unique way teachers appropriate the resources and technology, the specific needs of their classes and how they modify their resources in the context of everyday practices.

Each teacher has several digital resources stored on their iPads or computers and they draw on several online resource depositories like Resourceaholic when they plan and deliver their lessons.

The four teachers, together with others in the department and school, demonstrate a culture of working together as a community and as such collective practice in various overlapping groups exists. The teachers also participate in different voluntary online collectives like Facebook and Twitter. Subsequently, in Chapters 9, 10 and 11, I discuss the findings in relations to school A.

CHAPTER 7

CASE STUDY REPORT: SCHOOL B

This chapter presents the contextual information on school B and on the two participating teachers there. The chapter is divided into three broad sections. The first section, 7.1, explores the context of school B as a background to the case studies. The second section considers the cases of the two teachers (7.2 and 7.3) and is organised into the following subsections: teacher profile drawn from the first interview and from observation notes, their role in the department, their tasks and their resources. The third section (7.5.1) sums up with a comparison of the two cases explored.

7.1 The Context of School B

School B is a specialist sport college[101] and one of the largest secondary schools in the North-East Derbyshire district with a large body of students, as well as one of the largest sixth forms[102] in the county. The welcoming statement of the headteacher presents the vision and mission of school B:

> At school B students and staff benefit from first rate facilities, ensuring that learning environments are engaging and well-equipped. ICT provision throughout the school is excellent, enabling teaching and learning to be at the cutting edge of innovation and development for our students. As an experienced and highly effective Sports College, we are proud of our students' achievements and the communities that we serve.[103]

With regards to mathematics it states,

> The Mathematics Faculty at school B is extremely proud of the achievements

101 Sports Colleges were introduced in 1997 as part of the Specialist Schools Programme in the UK. The programme enabled secondary schools to specialise and gain qualifications in certain fields, like physical education, sports and dance.

102 In the education systems of England, Northern Ireland and Wales, the sixth form represents two years of post-GCSE (General Certificate of Secondary Education) academic education, wherein students (typically between 16 and 18 years of age) prepare for their A-level (or equivalent) examinations.

103 From the school prospectus

> of its students. A 100% pass rate at A2 Level Mathematics has been achieved for the last three years, due to the commitment and hard work demonstrated by both students and staff alike. Since the introduction of A* grade three years ago, 12 students have achieved this prestigious grade across Mathematics and Further Mathematics. Additionally, each year a number of our students take part in the UKMT[104] Mathematics Challenges – individual and team events – and have the opportunity to progress to national level competitions, according to their ability.[105]

School B has two adjacent mathematics staff rooms: a smaller one occupied by the head of mathematics and the director of mathematics, and another, larger office space for the rest of the mathematics teachers. These offices provide opportunities for exchanges of ideas and for discussing mathematics related issues face-to-face within and between the offices.

I now describe the overall content and approach to teaching mathematics in school B.

7.1.1 Content and Approach

The new National Curriculum took effect from September 2014, which advocates a mathematics mastery curriculum. A mastery curriculum breaks down each mathematics subject area into interlinked units with clearly specified objectives which are pursued until they are mastered[106]. Students work on the units through series of sequential steps that include tasks, assessments and interventions.

Figure 7-1 shows the theta 1 scheme of work for Year 7 evidencing this.

Topics	Year Curriculum	How you can support your child's learning at home E.g. Books, Websites, Family learning through visits
Theta 1 Scheme of Work (Year 7)		All topics can be supported by using:
Unit Title	Year 7 Programme of study	www.pearsonactivelearn.com
1 Analysing and displaying data	describe, interpret and compare observed distributions of a single variable through: appropriate measures of central tendency (mean, mode, median) and appropriate measures of spread (range, consideration of outliers) construct and interpret vertical line (or bar) charts for ungrouped and grouped data	www.mathswatchvle.com www.ttrockstars.com All students will be given login details

Figure 7-1. Theta scheme of work for Year 7

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104 United Kingdom Mathematical Trust (UKMT, https://www.ukmt.org.uk/).

105 From the school prospectus.

106 https://educationendowmentfoundation.org.uk/evidence-summaries/teaching-learning-toolkit/mastery-learning/

The mathematics curriculum adopts a three tiered (delta, pi and theta) scheme of work and three online resources are recommended to support the students' learning: www.pearsonactivelearn.com, www.mathswatchvle.com, and www.ttrockstars.com. The tiered scheme is a dedicated scheme of work designed to attend to students' learning needs. Through this tiered scheme, students are encouraged to work in different ways to help build their own mathematical confidence at a pace appropriate to them using a mastery approach.

In school B, the mastery approach to the new curriculum and GCSE is a major focus of the department. The Pearson ActiveTeach[107] website, MathsWatch[108] revision videos and Ttrockstars[109] skill practice and audit website are the three formal central resources hubs that all the planning, delivery, data gathering, and assessment practices of teachers and students revolve around.

It is against this background that the cases of the two teachers (Gray and Gavin) are presented.

7.2 Gray's Profile

Gray was 45 years of age at the time of research. He is the head of the mathematics department in School B. He has taught in the school for six years. He teaches Years 7 to 13. He was a marketing manager in manufacturing previously in his career.

As the head of the mathematics department, Gray is responsible for the running, development and improvement of the department, providing the required support for all mathematics teachers in their varied departmental responsibilities and professional practices. He also guides, as appropriate, the work of the other mathematics teachers

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107 https://www.pearsonschoolsandfecolleges.co.uk/secondary/Secondary.aspx
108 http://mathswatch.co.uk/
109 https://ttrockstars.com/

to ensure successful and effective teaching and learning. His responsibilities revolve around administrative, academic, pastoral and safeguarding duties.

Gray stated that the school's mastery curriculum is hinged on the ActiveTeach and ActiveLearn platforms[110].

> Our scheme of work is based around a package we have bought in from a company called Pearson or the owner of Edexcel. They provide a lot of materials, both papers-based, as well as an electronic version of the textbook that we can use. That is called ActiveTeach. Yes, ActiveTeach. There is also something called ActiveLearn, that the students can use, that again is also electronic; it is all based on the Internet. We can set them homework assignments, they can practice things, practice the skills and they can also click on little video clips to be able to help them. And we are using that with Years 7, 8, 9 and 10; they are the ones that are doing the new GCSE and following the mastery curriculum. (1intG. 1#42)

These platforms are learning tools from Pearson designed with the intention to help mathematics teachers and educators conduct classes by combining innovative technology and learning resources with a pedagogically sound and comprehensive school curriculum. These are repositories of activities where the mathematics teachers and students are instantly connected and data on students are constantly collected and assessed.

7.2.1 Role in the Department: Head of the Mathematics Department

Gray leads and manages mathematics curriculum development and assessment, ensuring the schemes of work are broad-based and offer a relevant and differentiated curriculum, aimed at raising standards of attainment and achievement for all students. The head of mathematics also has the duties of managing and deploying teaching/support staff, finances and resources within the department. He actively encourages and assists members of the department in their own professional development, creates opportunities for recognising their training needs and provides those resources that enhance the growth of best practice.

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110 https://www.pearsonelt.com/tools/digital/activeteach.html

In the general school leadership, he fully takes part in the life of the school community to support its unique vision and ethos and to encourage and ensure other staff and students follow this example.

Gray is a committed mathematics teacher and enthusiastic about the use of technology in teaching mathematics. He advocates the use of the everyday digital tools – phones, YouTube – and encourages students to apply these technologies to learning mathematics as well.

He is active on Twitter and creates YouTube[111] video resources for his students and the mathematics teacher community as well. In the interview, he acknowledges the purpose of the videos:

> Personally, I also make YouTube videos, but lots of teachers make YouTube videos, so, we will also suggest to students to look on there, find a tutorial that will help them. (1IntG: # 3)

He continues,

> We let the students know through social media really, once they are aware there is a bank of videos available, then they can go themselves, it is very much independent study. I regularly go around classes and remind students, don't forget: if you get stuck, you can watch my videos or anybody else's. There are a lot of things out there. (1intG: #3:41)

Gray takes on a supervisory role, ensuring students are accessing the available learning support for their independent study, using their phones at home and outside of the class.

In the next subsection, I present a sample[112] of the tasks that Gray gives to his students.

7.2.2 Gray's Tasks

The tasks Gray gave in lessons I observed were drawn from the formally recommended resources mentioned earlier in the preceding subsection, from self-created resources and from dedicated online resource banks. A lesson to Year 9 on

111 https://www.youtube.com/watch?v=OllblvF1ksc

112 The sample tasks presented in Gray's case were selected based, first, on 10-minute intervals across each lesson observed and, second, on Gray's self-reported typical lesson.

Distance Time Graphs began with the lesson's objectives and success criteria (shown in Figure 7-2 in bronze, silver, gold and literacy) and this is part of the classroom practice for every lesson, as Gray attested.

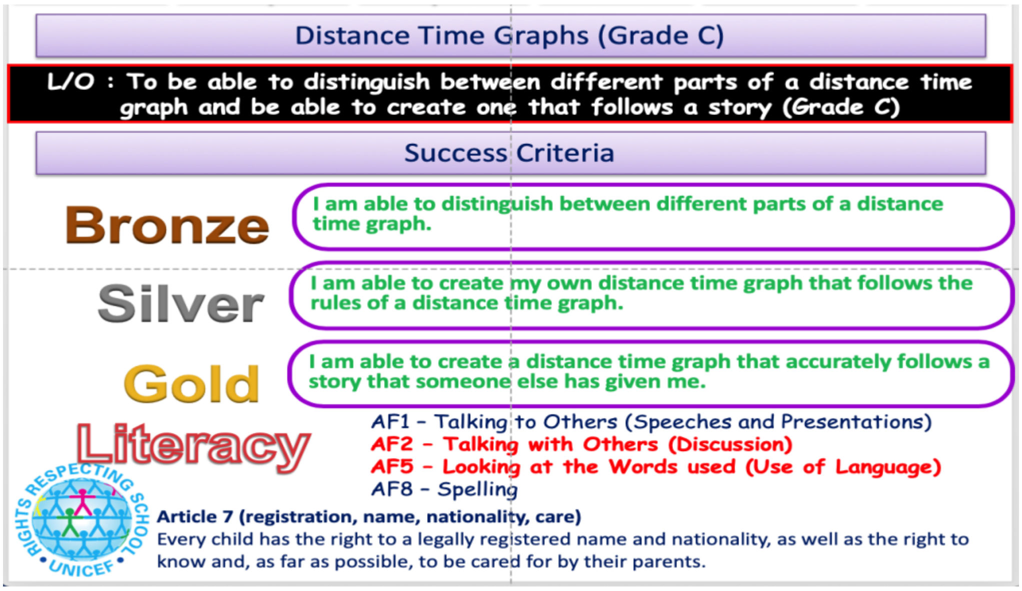

Figure 7-2. Lesson objectives and success criteria

Figure 7-2 shows the various shades of mastery Gray expected of the students. The objective of this lesson is stated at the onset, followed by the three-tiered success criteria together with a literacy in mathematics vocabulary as another success criterion. This was followed by the starter tasks as one of the task types for initiating the lessons, as presented in the next subsection.

7.2.2.1 Tasks Initiating Lesson

The eight lessons observed in Gray's class began with starter tasks and three out of the eight had terms definition tasks. Here, I focus on the starter tasks as an example of the tasks initiating the lessons.

The initial starter task on a lesson on travel graphs is shown below in Figure 7-3.

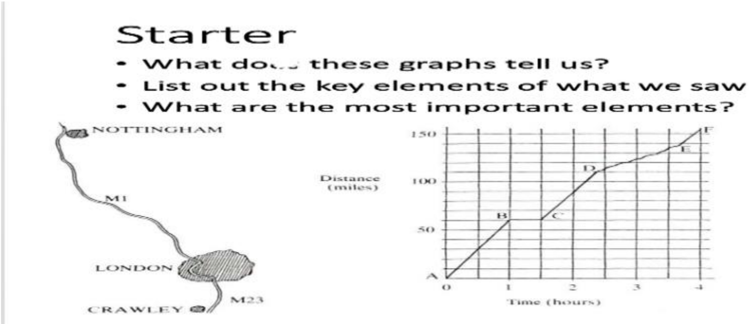

Figure 7-3. Whole-class starter task showing a graph of a journey to Crawley

Five people were asked the same question: how well did you sleep last night? (You have 3 minutes to try to match up each story to a graph and to discuss why you think they match.)

James: I slept really well until a pair of cats decided to have a fight in our back garden. Then I couldn't get back to sleep.
Libby: Great! I got to sleep straight away and slept 'till morning.
Karen: I was wide awake! By the time I got to sleep it was nearly morning!
Mike: I dozed in and out of sleep for ages before dropping off.
Nasir: I kept waking up and then going back to sleep again. Must have been the cheese!

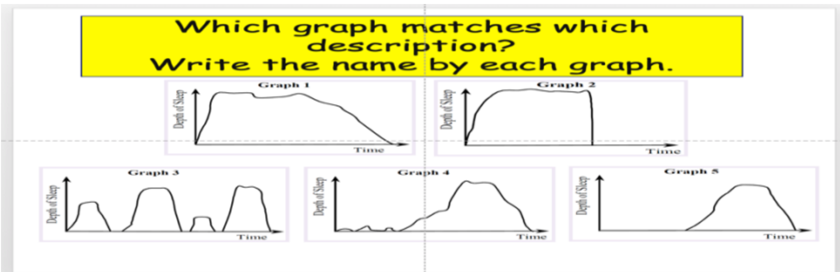

Figure 7-4. A group starter task showing graphs depicting depth of sleep

This starter task in Figure 7-4 exemplifies what Gray said in a post-lesson interview, that he wants them to be able to remember, understand, apply, analyse and evaluate. This group of starter tasks is geared towards strengthening that learning practice.

The second type of task Gray uses to initiate lessons is the task on the definition of key terms to be used. For instance, in a lesson to A-level students on sorting algorithms, the task initiating the lesson was to share their understanding of the terms, *shell, quick* and *bubble* sorts.

In all of Gray's lessons, the tasks initiating the lessons were directly linked with the main topic of the day.

7.2.2.2 Posing the Main Mathematics Tasks

When the tasks initiating the lesson were discussed and most students were confident, Gray moved on to series of main lesson tasks consisting of distance-time and speed-time graphs. Figure 7-5 is one of the main lesson tasks.

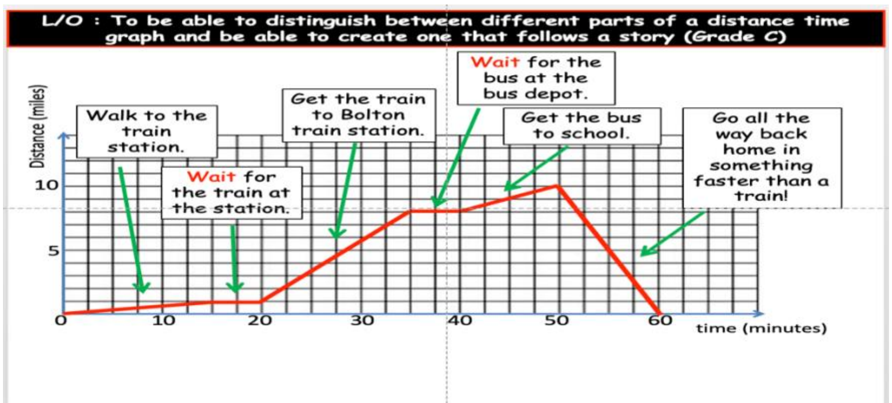

Figure 7-5. Main lesson tasks

The main lesson tasks were more teacher-led with input elicited from students every now and then. Gray then asked the students to work in pairs to identify and distinguish different parts of the graphs and create a story that follows the graph. This and the associated progress check tasks took about 25 minutes of the lesson time. The progress check task was given to the students to self-assess themselves and their understanding. With this task set-up there was a whole class discussion, followed by interactions in pairs with Gray circulating and making interventions for various pairs of students. At the end of this session with the pairs and groups, a new task was given for students to solve independently. This is the pattern that runs through Gray's lessons.

7.2.2.3 Demonstration of Mathematics Learning

Gray's approach to asking the students to demonstrate their learning is through sets of independent tasks. This task is similar to the various tasks undertaken during the main section of the lesson, and students are given the opportunity to develop mastery, boost confidence and assess each other on their performance. Gray then gives students time to verbalise what they are thinking and what they understand. They share their ideas with the whole class and ask questions looking for further explanation and clarification in areas they were still finding difficult. This was closely followed by a peer-assessment activity.

Figure 7-6 below is an example of the independent tasks given to students to enable them to demonstrate their understanding and learning of the topic.

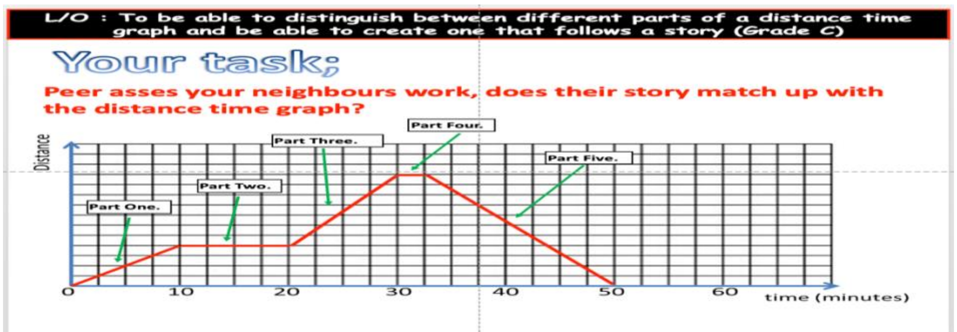

Figure 7-6. Independent task with peer assessment

This independent task was followed by a consolidation task. Students undertook these tasks independently and Gray only circulated, looking through what they were doing. Afterwards, Gray recommended that students should look online for videos to support their learning whenever they feel stuck. Figure 7-7 below shows the consolidation tasks for the topic.

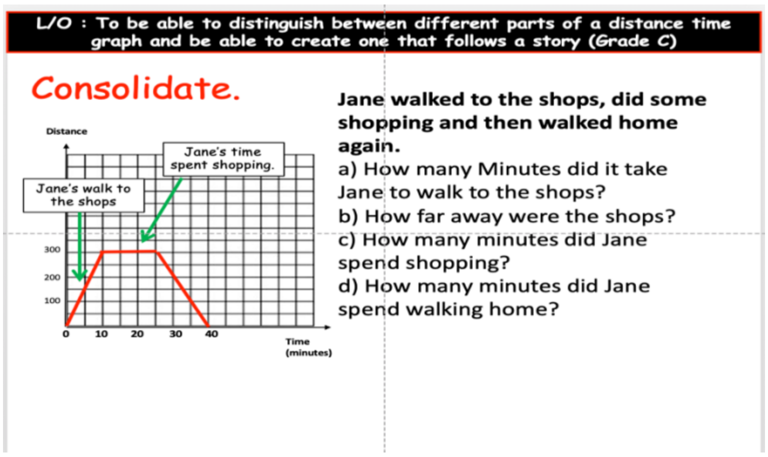

Figure 7-7. Consolidation tasks

7.2.2.4 Concluding the Lesson

At the conclusion of the lesson, Gray provides a colour-coded guide for students to reflect on where they are in their learning. This chart became a guiding point for further self-assessment and closing the lesson. The guided self-assessment chart is shown below.

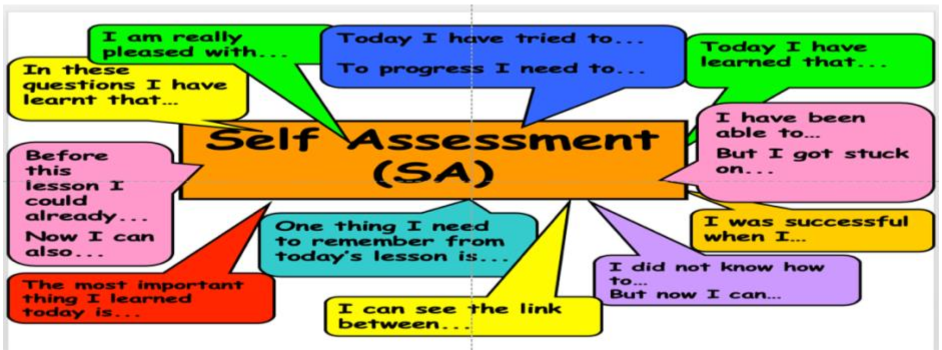

Figure 7-8. Guided self-assessment chart

Gray then randomly selects students to respond and verbalise what they have understood, beginning with any of the phrases displayed in the chart in Figure 7-8. Students take turns in making personal statements of where they are in their learning.

Most of Gray's lessons I observed followed this pattern: a lesson objective/success criterion, tasks initiating the lessons that are drawn from school recommended resources (ActiveTeach and Ttrockstars and others), a variety of differentiated (bronze, silver, gold and literacy aspect) of main lesson tasks, a progress check activity, independent tasks and a consolidation task at the end of each lesson. Students too were always reminded of available resources on *ActiveLearn* and elsewhere on the Internet. As mentioned earlier, these tasks are drawn from different sources; I now examine those resources that Gray reported as the sources of his classroom activities.

7.2.3 Gray's Resources

In the interviews, observation notes and screen capture recording, Gray explains the various resources available to him as a teacher and how these resources are accessed and used. In the interview, he stated the various sources of the resources.

There is a set of school-recommended resources to support mathematics teaching and learning as shown in the scheme of work in Figure 6.1 above. In this first category, there are three sources for the resources. First, the *Pearson* subscription package (*ActiveTeach/ActiveLearn*) of a textbook, e-textbook, associated online materials and videos, and features to enable teachers to monitor students' achievements and learning progress. Second, the *MathsWatch* bank of video clips that are tailored towards students' independent learning and revision exercises. Third, the *Ttrockstars* applets for practice and developing mastery in mathematical operations. These provide the mathematics teachers worksheets and various tasks aligned with the curriculum.

In terms of hardware resources, Gray explained,

> Every classroom teacher has a laptop, every class we teach in has a PC in the room connected to an interactive whiteboard. So, we have all got interactive whiteboards with computers; we've all got laptops. I have also got an iPad but that is not something everybody has. Over and above that, everybody has a mobile phone as well, and most people have their mobile phone connected to the school's email system … Some teachers have also a visualiser in their classroom. (1intG: #8:45-11)

Gray reported that his resources are drawn from those sites subscribed to by the school and from online sources. In the interview he stated that recommended resources are prescribed for Years 7 to 10. For Year 11 the teachers source materials independently.

> Our Year 11 are a little bit different because they are less prescribed. Our Year 11 teachers will be finding resources from wherever they can. Typically, trusted resources will be from something like the TES, where they can look and download things, and people have their own favourite places to find them. I, as Head of Maths, would not tell them where to get things from. I do not really check; I trust their professional judgment that this is something suitable for their class. (1intG: #2)

On probing him further on favourite places he finds his resources, Gray stated,

> There are three or four places I will go to straight away. Do you want to know where they are? [Yes]. There is a website called Resourceaholic, lots of resources for Key Stage 3 and Key Stage 4. A-level, I will go to a website called Douis.net and I know the resources there are same spec that we use… The TES is very good, and again from an A-level point of view, individual persons like SRwhitehouse, brilliant resources and you know that the quality is good. (1int G: #7-8)

Commenting further on the resources available to the teachers, Gray reported,

> It is all driven from two sources; independently, they are finding things on the Internet or because we bought into something. We are not dictating really, but the resources are there, use them when you feel. (2int G: #3:41)

Although Gray claims the head and lead teachers in the mathematics department do not dictate to teachers which resources to use, at times they might give them a more specific direction with regards to the *MathsWatch* videos for students.

Gray believes that the ideal for him is for teachers to create their own resources, but this is only possible in an ideal world, while time constraints force them to adopt the ones already created.

> Quite often it is easier to spend 15 or 20 minutes trying to find something someone has done rather than spend two hours making it yourself because we don't have the time, fundamentally. In an ideal world we all want to make our own resources for everything, but there isn't time. (2intG: #6:21-)

From the interview extracts, my observation notes and sites visited on the screen capture, a list of Gray’s resources was made and categorised as follows.

Human		**Non-Human**			
Physical contact	Not Physical Contact	Electronics		Non-Electronics	
CPD Dept. Meet Maths-English collaboration	Twitter[113] YouTube (Self-created)[114]	Hardware	Not Hardware	Individual	Not Individual
		IWB HWb Calculator Laptop Digital Pen[115] Visualiser[116] Mobile phone iPad	E-Textbooks Maths Watch[117] SRwhitehouse[118] Resourceaholic Tes.com Pearson ActiveLearn/Teach[119] ActivInspire studio[120] Shell Centre for mathematics education[121] Whiteboard Maths[122] GeoGebra Douis.net[123]	Workbook Worksheet	Edexcel Hub[124]

Table 7-1. Gray's Resources

I now present Gray’s resources.

113 https://twitter.com/mr_g_walton
114 https://www.youtube.com/user/MrWaltongr
115 https://www.livescribe.com/uk/smartpen/ls3/
116 https://www.educationsupplies.co.uk/ict-and-audio-visual/visualisers/u200l-usb-visualiser
117 http://mathswatch.co.uk/
118 https://www.tes.com/teaching-resources/shop/SRWhitehouse
119 https://www.pearsonactivelearn.com/
120 https://support.prometheanworld.com/product/activinspire
121 http://www.mathshell.com/
122 http://www.whiteboardmaths.com/
123 http://www.douis.net/
124 https://qualifications.pearson.com/en/home.html

7.2.3.1 Human and Non-Human Resources

In presenting the categories of resources, *Twitter* (a social networking site) and *ActiveLearn/ActiveTeach* (a digital learning platform) each represent the human and non-human resources, respectively. I will give more space to these resources, because both resources featured more in Gray's teaching activities and, in the interviews, he put more emphasis on their usefulness to his teaching activity than he did any other resources he identified. I begin with Twitter.

Social Network: Twitter

The social networking sites, especially *Twitter*, have become a means of connecting teachers, exchanging resources, sharing good practices and building an online mathematics community. Gray describes the place of Twitter in his teaching thus:

> The maths community on Twitter is like a big staffroom and so you are able to call upon people across the country to be able to help you with things. At the moment I am looking at providing different feedback to students and I know I can ask a range of people for their opinion. And it is not just from my school, it cuts across the country. (1intG: #15:13)

Elsewhere in the interview Gray sees Twitter as a more professional online space.

> I think the social media has gone down two, three or four routes. And Twitter seems to be very professional, a place that professional people can discuss and share information, and once you get into who to follow and who is dealing with it, it tends to be same 500 people. It is an online community; it is like a big maths staffroom. People share advice and opinion about things and sometimes people argue passionately about things they believe in. (1int G: #22:25)

Figure 7-9 shows Jo Morgan's Twitter page with over 36,000 tweets and 22,000 followers.

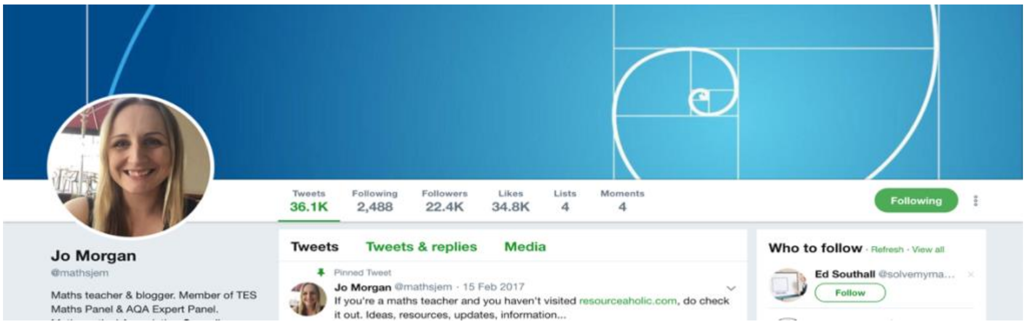

Figure 7-9. Twitter page of Jo Morgan

For Gray Twitter is a big online maths staffroom that is professional and replicates informally with large-scale interactions and exchanges that would usually take place in the departmental office and staffroom.

In terms of the benefits of these resources and technology, Gray believes that

> Most of the resources are of much higher quality and more consistent from one teacher to the next teacher. Everybody is on similar level in terms of resources they give to the students. The data we have on the students enable us to identify weaknesses, provide more specific interventions, which we were never able to do before. I think that improves overall grades. (1intG: #28:25)

With the availability of freely available resources Gray suggested, "*I think we are at a point at the moment where paper and pen are in a bit of a transition*" and as such he advocates a more cautious use of technology and digital resources.

> What we also say is that there are times when technology fails us. And we also must be able to teach, just with pen, a voice and the board. And that is something the new teachers find very difficult because they have become very reliant, then on the technology. (1intG: #12:41)

For Gray while he acknowledges the advantages digital resources and technology bring for both teachers and students, he advocates that the traditional means of teaching with pen and voice should not be totally abandoned; this is a safeguard for those times when technology fails.

ActiveLearn and ActiveTeach

The ActiveTeach/ActiveLearn digital learning platform offers the teachers resources to help them plan, teach, track and assess their students. It adopts a mastery approach[125] to learning, aligned with the UK 2014 National Curriculum. It has a range of front-of-class teaching resources, online homework and practice tasks, as well as planning and resources for the assessment of student learning progression. There is a huge bank of explanatory videos, thousands of differentiated mathematics activities and exercises tailored to the individual's student's progress, pace and level.

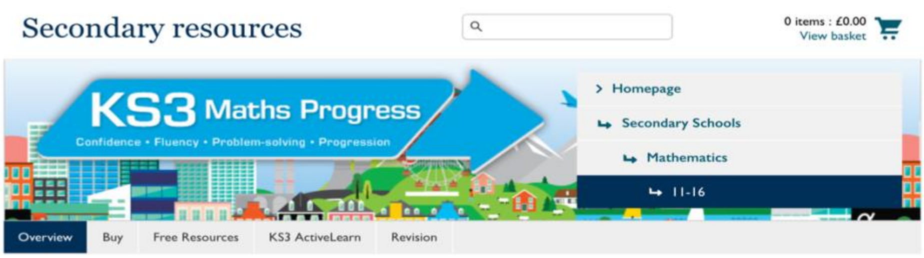

Figure 7-10. ActiveLearn secondary resources

Figure 7-10 above shows Gray's home page for secondary mathematics resources. There are embedded 'ActiveBooks' accessed by a thumbnail, which are the e-book equivalent of the students' textbooks, workbooks, various guides and complimentary worksheets. There are interactive on-screen pop-up hints, instant targeted feedback and progress tracking that could motivate learners to work independently. Below, Figure 7-11 is the sample screenshot page of an ActiveBook.

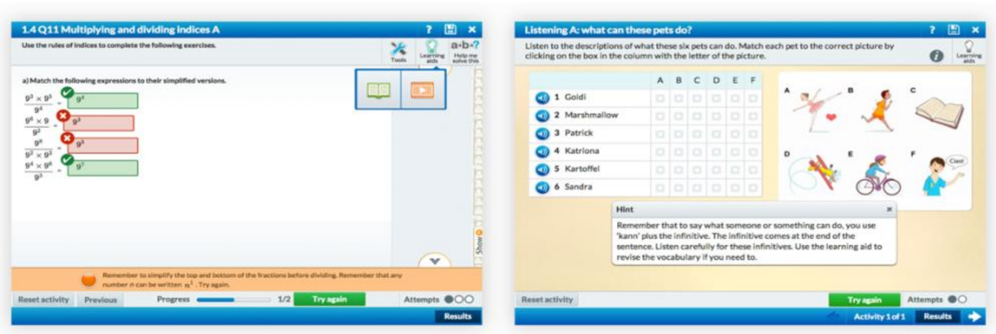

Figure 7-11. ActiveBook page

The designers of the service claim a UK-built approach to teaching for mastery underpins the schemes of work for 11- to 16-year-olds, Key Stage 3 Mathematics

125 https://www.pearsonschoolsandfecolleges.co.uk/AssetsLibrary/SECTORS/Secondary/SUBJECT/Mathematics/11-16-maths-story/w384-dps-6-7.pdf

Progress and Edexcel GCSE (9-1) Mathematics. The designers claim that teaching for the mastery[126] approach is an underlying design principle of the resources. This package is the central resource in the department and everything else in terms of resources are built around this.

In the next section, I discuss the case of Gavin the second teacher in school B.

7.3 Gavin's Profile

Gavin, 51, is a mathematics teacher and a senior lead practitioner who has taught for 13 years. At the time of research, he was completing his term as a senior lead practitioner of mathematics and was going to take on a new role later in the year as director of mathematics. In the extract from one of the interviews, he stated,

> My position last year, was what you call senior lead practitioner; so, I work within maths, but my responsibility was to improve teaching and learning across the whole school. And I was allocated staff to work with to improve their teaching. Maybe some of them weren't getting a consistently good judgement by the school leaders, so I work with them. And occasionally we have to lead certain parts of the whole school CPD. (1intGn: #20:21)

This role is additional to the regular mathematics teaching engagements in the school**.** Gavin has had previous professional experience in the industry. In school B he teaches mathematics as a primary professional duty with an added leadership role as a senior lead practitioner.

7.3.1 Role in School: Senior Lead Practitioner

A senior lead practitioner of mathematics is one who is known to be consistently good or an outstanding mathematics teacher over years of professional practice, ambitious and highly motivated to lead others. Gavin as a lead practitioner of mathematics is

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126 https://www.pearsonschoolsandfecolleges.co.uk/Secondary/Mathematics/1116/GreatMathsTeaching/Mastery/maths-mastery.aspx

focused on whole-school improvement in the key areas of teaching and learning, and assessment. In addition to the professional responsibilities that are common to all teachers, the lead practitioners' handbook[127] listed some of the key roles:

- Developing a high-quality school ethos of learning amongst students based on high academic expectations
- Leading and taking part in the demonstration of 'model' lessons to support colleagues' professional development and reviewing support programmes
- Delivering high quality coaching, mentoring, and responding to individual teachers' requests for support through shared lesson observations, targeted learning walks, incorporating innovation into schemes of work and making accurate judgements of lessons, providing appropriate and timely feedback

Furthermore, Gavin is expected to analyse and interpret data generated and collated through ActiveLearn and the MathsWatch digital resources, appropriate the new National Curriculum in a way that addresses the school's needs and to inform future practice, expectations and mastery teaching methods. This constitutes the broad background of Gavin's role in the school.

I now turn to explore Gavin's tasks.

7.3.2 Gavin's Tasks

In terms of the task given, Gavin uses the ActiveLearn textbook as a central resource around which other tasks are crafted and modified to fit in with students' needs. In the following extract from the interview, he made this more explicit and commented further on the quality of the textbook:

> In terms of resources for the lesson, my first point of call is to look at the textbook because the people who write the textbook also write the assessment. It is important that students are familiar with the language of the textbook, so that they are familiar with the language of assessment. The textbook itself is really, really very good especially with the videos embedded in it. (1intGn: #00:27)

127 http://woodlandsschool.org/wp/wp-content/uploads/2015/07/Lead-Practitioner-handbook-2017-2018.pdf

The ActiveLearn textbook's tasks have a specific lesson-plan design. First, the scheme of work associated with the textbook categorises students into three ability groups (Pi – lower ability, Theta 2 – middle ability, and Delta 2 – higher ability). In commenting on the ability groupings, Gavin stated that

> The new curriculum is encouraging us to be more specific in what we teach. it is looking at the ability of the group and make sure we teach to the ability. (1int G: #15)

Second, each topic is subdivided into various steps of addressing the tasks:

- Master (confidence, fluency and definition of concepts)
- Warm-up exercises (then from worked examples leading up to investigation, problem solving, exploration and reflection)
- Check-up tasks (midpoint assessment exercises)
- Strengthen your understanding with practice tasks (tasks that are more difficult)
- Extend your understanding with problem solving (tasks set in real-life contexts)
- Tests (series of formative assessment tasks)

Although these were prescribed in the textbook, Gavin did not strictly follow this lesson structure. Here, I use the themes derived from my analysis of the tasks to make sense of the lesson plan and structure.

I now begin with how Gavin initiates his lessons.

7.3.2.1 Tasks Initiating Lessons

In initiating the lessons, Gavin reports that he is guided by the lesson plan as advocated by the ActiveTeach learning platform, the primary resources of school B. The master and check tasks belong together in the category of tasks for initiating lessons.

For instance, in a lesson on surds (Figure 7-12), Gavin shows the top of the textbook with the expected progression path in teaching and learning the surd using the ActiveLearn textbook.

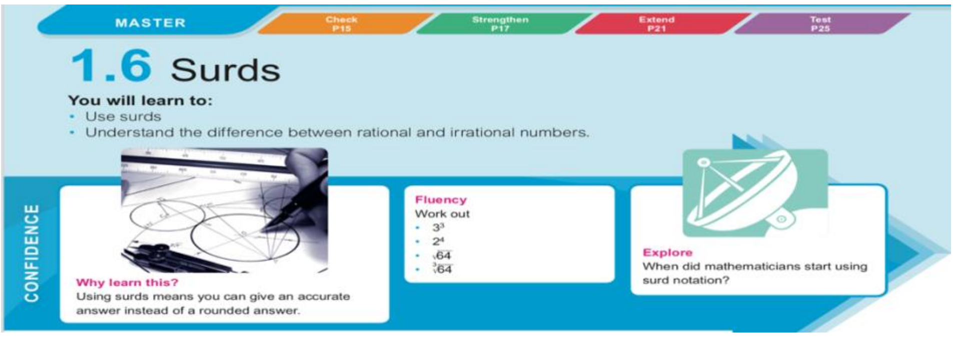


Figure 7-12. Screenshot of ActiveLearn textbook on surds

This is then followed up by warm up exercises.

Work out

a) $\sqrt{64}$ b) $\sqrt{121}$

c) $\sqrt{\frac{1}{4}}$ d) $\sqrt{\frac{4}{9}}$

Gavin followed this up by displaying a worked example in the textbook and other examples drawn from *MyMaths* online resources. Figure 7-13 is the worked example.

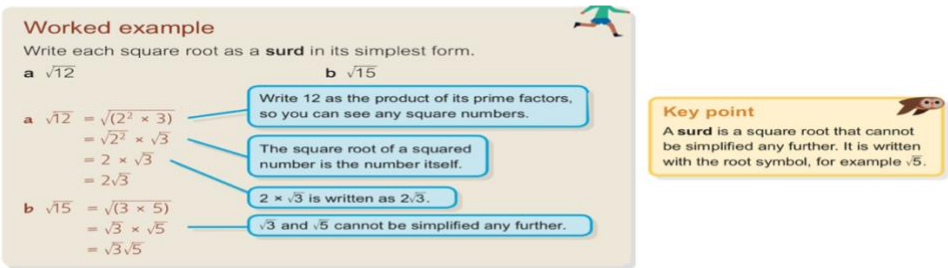


Figure 7-13. Worked example from the textbook

After the worked example there were series of tasks drawn from the hyperlinked dashboard through which Gavin plans, structures and gives himself cues to go to an online resource or bring up exploration tasks. The dashboard planner has multiple hyperlinks to other school recommended resources, teacher-selected resource depositories online and video clips that demonstrate what surds are in many different ways. Gavin's activity dashboard is shown in Figure 7-14.

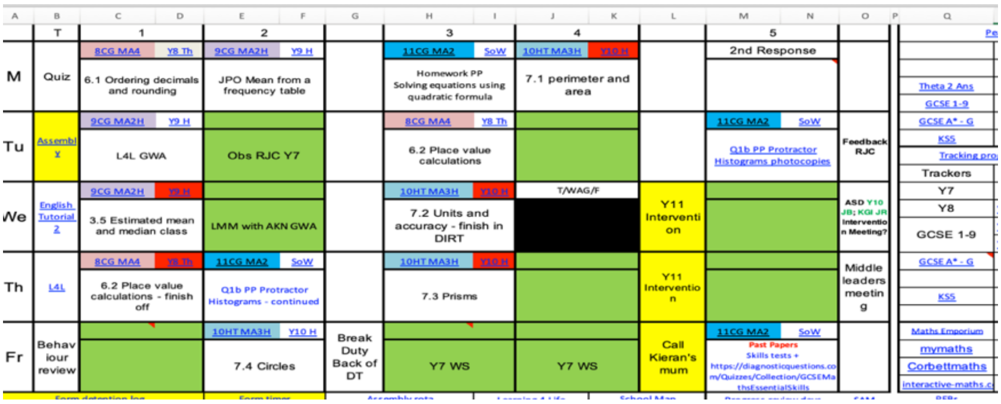


Figure 7-14. Gavin's dashboard showing hyperlinks to resources

This dashboard is a weekly planner including classroom activities with hyperlinks to associated online resources, lesson plans for various classes, past papers, student data and progress reviews. The three rows on the bottom right of the dashboard show a collation of go-to sites for resources and teaching ideas. In the interview, Gavin made this reference:

> The dashboard I use has hyperlinks all over the place; it has a significant benefit to me. I think I use ICT different to other people. I think a lot of teachers are still using their planners for planning. (1intGn: #10:40)

The dashboard Gavin mentions plays an organising role for his school and classroom interactions. Each class and each topic has associate cells on the dashboard with related hyperlinks connecting other resources for the class.

7.3.2.2 Posing the Main Mathematics Tasks

The next series of tasks are given to strengthen the students' learning through differentiation and multiple practices. Figure 6-15 is one of the main lesson tasks.

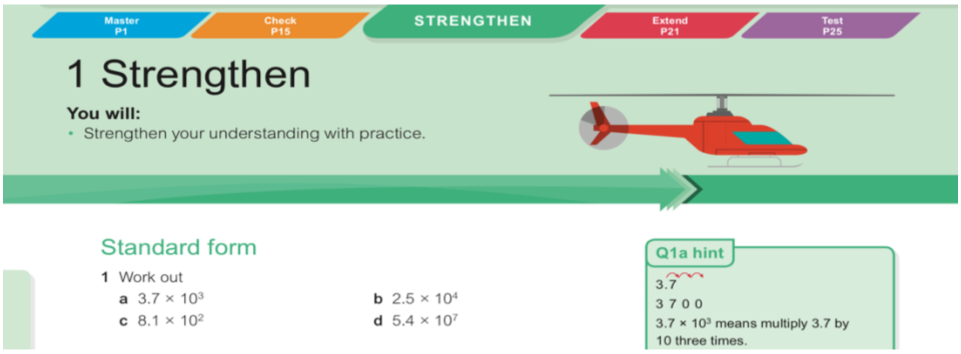


Figure 7-15. Tasks for strengthening understanding

The aim of these groups of tasks is to enable students to improve their understanding through repeated practice and hints are given as a reminder and cue for students to make progress in their learning. Similar tasks were taken from *Corbettmaths*:

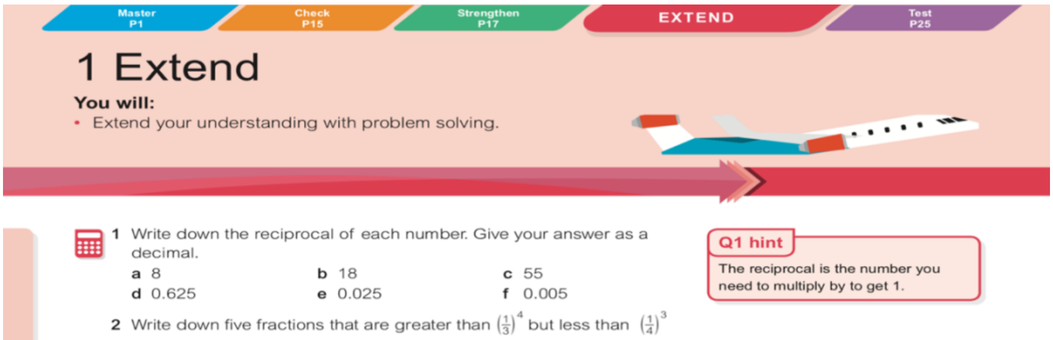
Write the following in number in standard form

1. 40000
2. 5600
3. 41200000
4. 0.00000008
5. 0.000345

Students are asked not to use calculators for these tasks. They work first on their handheld whiteboards and when they are sure about their answers transfer them to their workbook. Gavin circulates, checking and chatting to the students and providing intervention when students call to him. When most of the students were done with these tasks, further to enable them demonstrate their learning were given.

7.3.2.3 Demonstration of Mathematics Learning

Gavin uses extension tasks and series of questions drawn from the *diagnosticquestions* app to enable the students demonstrate what they have learnt. Figure 7-16 below shows sample questions drawn from the ActiveTeach platform and these questions are often supplemented with tasks from the *diagnosticquestions* bank of resources.



Master P1 | Check P15 | Strengthen P17 | EXTEND | Test P25

1 Extend

You will:

- Extend your understanding with problem solving.

1 Write down the reciprocal of each number. Give your answer as a decimal.

a 8 b 18 c 55

d 0.625 e 0.025 f 0.005

2 Write down five fractions that are greater than $\left(\frac{1}{3}\right)^4$ but less than $\left(\frac{1}{4}\right)^3$

Q1 hint

The reciprocal is the number you need to multiply by to get 1.

Figure 7-16. Extension tasks from the textbooks.

In the extension tasks students worked in pairs and were allowed to use calculators to explore the answers and explain to the rest of the class when they were confident of the answers they were getting. This exercise lasted for about 10 minutes of the lesson.

7.3.2.4 Concluding the Lesson

In the final stage of the lesson, students are asked to individually reflect on their learning and share what they have learnt or the difficulties they experienced in the lesson. These are some of the example questions for reflection:

Reflect *In this unit you have learned a lot of new vocabulary.*
Write a list of all the new vocabulary you have used.
Write, in your own words, a definition for each.
Compare your definitions with your classmates.
Did you all learn the same thing?

A few students volunteered to share what they have learnt during the lessons and Gavin asked them to take the questions as homework to report back the next day.
In the next subsection, I examine Gavin's resources for lesson planning, delivery and assessment.

7.4 Gavin's Resources

In school B three central resources are recommended: the *Pearson ActivInspire* package, *MathsWatch* virtual learning environment (vle) revision videos and *Ttrockstars*. For Gavin, the ActiveLearn textbook and associated teacher guide, videos and worksheets form the basis of his usage of other resources.

> Within the department and individually we don't have a central bank of resources where information is, it is what you find, see what you find is suitable. It is a brand-new scheme, it's a brand-new specification, so there is not a great deal out there tailor-made for that yet. More and more are coming onstream. (1intGn: #1:20)

As a result of this, Gavin depends largely on the school resources and those appropriated and modified from online sources to align with the new curriculum and students' needs. There are other resources that students focused on and used in the collection of data that informs teachers' interventions and lesson planning. Gavin commented on this thus:

> So, we have the mathswatchvle.com, which has a brand-new spec built within it, that is a web-based site that students have got their own access to with their own log-in, which is very useful. And from the administration point of view, we can look and see how active different students are on it. To see who is using it, who isn't and who needs to get a finger on it and work harder. (1intGn: #1:44)

From the interviews with Gavin, my observation notes and screen capture records, a list of Gavin's resources was drawn up and categorised as shown in the table below.

<table>
<tr><th colspan="2">Human</th><th colspan="4">Non-Human</th></tr>
<tr><td>Physical contact</td><td>Not Physical Contact</td><td colspan="2">Electronics</td><td colspan="2">Non-Electronics</td></tr>
<tr><td rowspan="2">CPD
Quarterly
Faculty
Meeting</td><td rowspan="2">Maths
Watch[1]
Web
Conferencing</td><td>Hardware</td><td>Not Hardware</td><td>Individual</td><td>Not Individual</td></tr>
<tr><td>IWB
HWb
Calculator
Laptop
Digital
Pen[2]</td><td>TES
GeoGebra
MyMaths[3]
Maths Box[4]
Code Breaker
Activities[5]</td><td>Workbook
Worksheet</td><td>Edexcel
Hub[9]</td></tr>
</table>

1 http://mathswatch.co.uk/

2https://www.livescribe.com/uk/smartpen/ls3/

3 https://www.mymaths.co.uk/

4 http://www.mathsbox.org.uk/index1.php

5 https://schoolcodebreaking.com/code-breaking-resources/

9https://qualifications.pearson.com/en/home.html

			Don Steward Blog[6] Pearson ActiveLearn/Teach[7] ActivInspire studio[8] Corbettmaths Diagnosticquestions Mathsloops Maths emporium Socrative		

Table 7-2. Gavin's resources

7.4.1 Human and Non-Human Resources

In this subsection, I present the Gavin's human and non-human resources. The human resources consist of the periodic departmental and faculty meetings and occasional web conferencing as reported by Gavin. In this presentation more attention is given to two non-human resources – *MathsWatch* and *MyMaths* – which alongside the *ActiveTeach* platform form the central resources for Gavin.

MathsWatch is a virtual learning environment consisting of video clips and worksheets that helps students practice and rehearse their mathematics at home and outside of the classroom. Figure 7-17 shows statistics on the users, number of questions and how many videos were watched as of 1 January 2017.

6 http://donsteward.blogspot.co.uk/
7 https://www.pearsonactivelearn.com/
8https://support.prometheanworld.com/product/activinspire

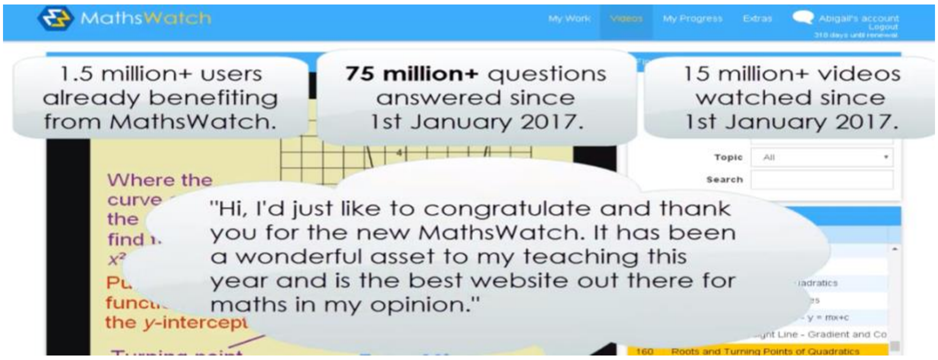


Figure 7-17. Statistics on MathsWatch usage and resources

MathsWatch contains videos tailored to specific topics and aligned to the curriculum to remind students of the tasks that have been taught in class and then problem-solving activities that could help extend their understanding.

MathsWatch has a bank of thousands of examination quality questions and banks of interactive videos for students and millions of people are subscribed to it across the UK and abroad as shown in Figure 7-17. *MathsWatch* has a feature that automatically marks both the working out and the answers like an examiner and provides interventions that are tailored towards individual students. It tracks the progress of individual students as shown in Figure 7-18.

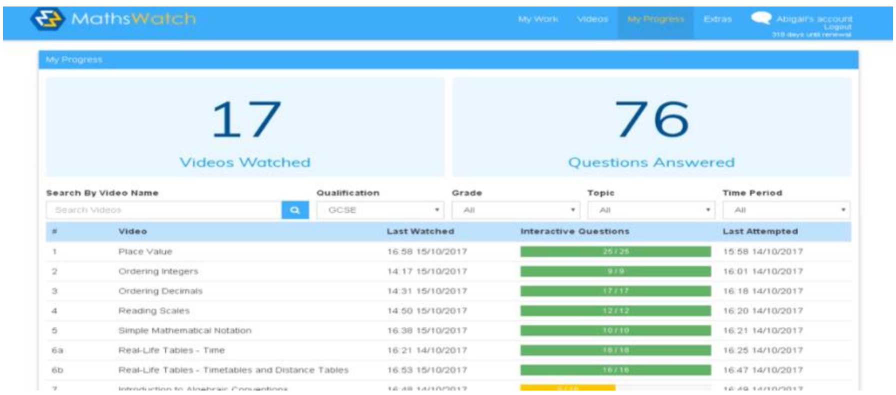


Figure 7-18. MathsWatch data on a student’s activities

The *MathsWatch* functionality includes colour codes on a spreadsheet of the whole-class interaction showing the tasks and assignments and documenting their progress.

Figure 7-19. Whole-class colour-coded spreadsheet data

Figure 7-19 is Gavin’s whole-class spreadsheet data on students’ activities. This shows the data from the whole class and their performances in a given mathematics tasks. The various shades of green show students who have attained 70 percent and above in their tasks. The yellow shows percentage performances between 65 and 59 while the brown colour code shows students performing below 45 percent. The red colour code shows 0 percent and white indicates absence and not engaging with the activities.

As Gavin said in the interview extract above, this enables the teacher to see who is using the site, who needs intervention and who needs to be encouraged to work harder. Besides this resource, Gavin also compares the MathsWatch to another resource, MyMaths:

> There is also MyMaths, which is used predominantly with Year 11, an older sort of web-based site. The benefit of MathsWatch is that there are videos out there that explain how to do the topic, whereas MyMaths, they have to work through a step at a time, do this, do this, do this; it is not verbal. It is more reading and you need to be scholarly to be able to do that and a lot of our students don’t cope with that as easy as listening. There are the main two. (1intGn: #2)

In addition to MathsWatch and MyMaths, Gavin made mention of other resources and the list of bookmarked places and sites that he goes to in his teaching activities and interactions with students.

> TES has a fantastic range of resources. Other software and websites, there is a guy called Don Steward; he has some great, great creative ideas. They are

the main ones that spring to mind. I have more on my dashboard, so I just click and look for different ideas. (1intGn: #2:54)

Figure 7-20 is a section of Gavin's dashboard showing a variety of resources hyperlinked to his lesson plan.

The dashboard is Gavin's school and lesson activities organiser. It shows two adjacent columns: the first, on the left-hand side is a colour-coded section which Gavin labelled as "progress review days". This consists of activities for Year 11 with associated schemes of work (SoW), past papers, skill tests and the respective web addresses (diagnostics questions, quizzes, GCSE Maths Essential Skills collections). Also included are notes to call a parent and two meetings scheduled.

The second, on the right-hand side, includes four columns divided into several rows. The first three columns of this second section show the "Pearson Active-Teach-Learn" tiered schemes for Years 7-13 and sixth form (excluding Year 11). At the bottom of the first three columns are various mathematics resources and their associated web addresses (for instance, MyMaths, Corbettmaths and Mathsloops). These resources have been previously presented in Table 7-2, p. 209 and subsequently described. The fourth column in the second section, shows varieties of hyperlinks to documents, handbooks, agenda and minutes.

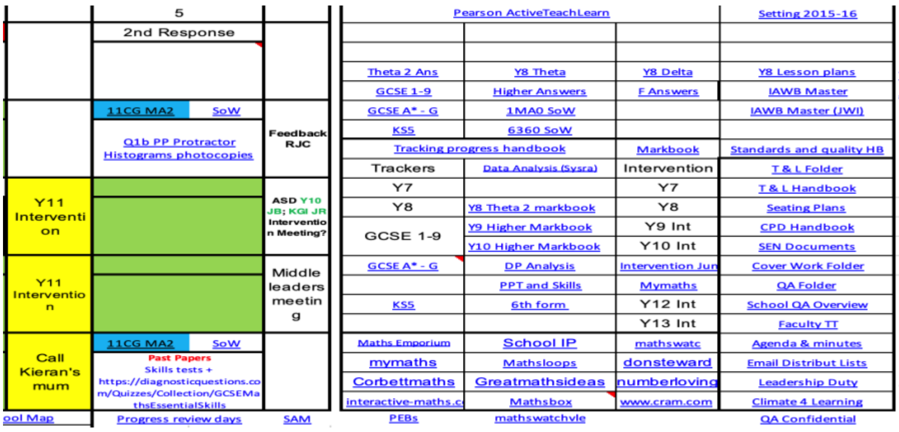


Figure 7-20. Section of Gavin's dashboard showing his resources.

Gavin mentioned too, that in spite of the huge number of available resources, time is a great challenge for him: "I think that part of the problem of teaching at the moment, you don’t have time to go away and explore. If I have time, that’s what I would like to pursue” (1intGn: #3:56).

Following from the ideas of tasks and resources is the question of the collective in school B. The exploration that follows is drawn from the data relating to Gray and Gavin and the observation made in the context of the research.

7.5 The Collectives of School B

In terms of professional development activities, there is a faculty meeting every six weeks, wherein teachers share good practices, experiences and challenges. There is periodic mathematics-topic teaching by teachers for teachers, whereby novices, trainee teachers and advanced beginners learn from more experienced and proficient teachers, and pedagogic styles and classroom management skills are shared.

There is a collaborative interaction between the mathematics and English departments, toward synchronising teaching time in a way that maximises opportunities for students. The TES and school website are platforms allowing teachers to access and download mathematics teaching resources purchased by the school, and all other official government policy documents and teacher guides. One of the teachers produces videos that are made freely available to students and teachers alike on YouTube[137].

School B offers its mathematics teachers opportunities to work collectively on a face-to-face basis and with online platforms as well. In terms of resources, the school purchased *ActiveLearn* and *ActiveTeach*[138] as an integrated hub for lesson planning, delivery, designing new resources and aligning teaching to the new curriculum.

Besides the mandatory CPD events and faculty meetings, there are no other formal mathematics teachers’ meetings and Gray and Gavin gave likely reasons for this. For Gavin, referring to collectives within the school, he claims,

137 https://www.youtube.com/watch?v=cvGs-eJyfSw

138 https://www.pearsonschoolsandfecolleges.co.uk/ActiveTeach/Maths/MathsActiveTeach.aspx

> I think that, that has possibly declined because of the fact that social media means I can interact with other people without being part of a formal group and then meet up. I can see the benefit of doing it, that's because I do it any way in more informal electronic way. (1int G: #21:40)

Gray is very committed to his use of Twitter and refers to it throughout the data collection period. He believes the advent of social media like Twitter has led to the decline of formal face-to-face teacher collective meetings. For him, social media – especially Twitter – has become a 'big, live mathematics staff room' where conversations and sharing of ideas and resources now takes place. Gavin has a slightly more different take on the mathematics teacher collectives.

> I follow few blogs, Mary Rayner[139], she is OFSTED, leading learner[140], and head guru[141] teacher. I follow their blogs, but I don't actively participate within them, but I am trying to keep an ear to the ground to find out what is happening within the industry, what is moving and what OFSTED is looking for. (1int Gn: #19)

Gavin engages with various blogs and web conferencing by listening in and gathering information but does not actively participate in what goes on within these blogs.

The difference between Gray and Gavin in their informal (active and listening in) online participation, respectively, is that while Gray participates in a Twitter group that is focused on mathematics teaching and resources sharing, Gavin's blogging group deal with generic pedagogy issues that are not specific to mathematics teaching. One reason comes to mind: while Gray is head of the mathematics department with focus on mathematics itself, Gavin is a senior leading practitioner looking out for resources that could enable the improvement of whole-school teaching practice.

7.5.1 In Summary

With respect to the two teachers (Gray and Gavin) in school B, the case descriptions enabled me to begin to construct an inventory of the assortment of resources available to these teachers in their professional practices. Both teachers have leadership roles

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139 https://www.cmatrust.co.uk/staff/mary-rayner/

140 https://leadinglearner.me/about/

141 https://teacherhead.com/mentions/headguruteacher/

in the mathematics department and across the school and as such are key stakeholders in encouraging and supporting use of digital resources in the school for teaching and learning.

While no specific shared bank of resources exists in the department, the *ActiveTeach* package bought by the school seems to take on the role as a collective resources repository from which all the teachers are expected to draw resources and this can be supplemented by the wide range of mathematics-specific and generic teaching resources that are available online. There is a multiplicity of mathematics teaching resources between the two teachers considered in school B, and these large variations in the types and forms of resources possibly point to the mathematics teachers' confidence and competence in merging diverse resources together with the hope of offering students a range of opportunities to learn from and opening up more ways in which students could meaningfully engage with and deepen their mathematical understanding. While there are several schools in England where there are no prescribed or mandated textbooks or resources used by all teachers, the case of school B holds a useful lesson, that an apparently well-designed textbook and e-textbook supplement could be a dependable resource for improving mathematics teaching practice and in improving mathematics lessons. Siedel and Stylianides (2018) caution that we cannot underestimate the value of a well-designed textbook for teachers and students. The adoption of *ActiveTeach* package strengthens this argument.

This study indicates that digital resources and mathematics teachers mutually influence each other. While teachers modify, adapt and revise resources for particular needs, the resources have a way of influencing the mathematics teaching practices of teachers. The two teachers in school B, enabled by the abundance of digital resources, are beginning to cultivate the collective of teachers in the department into a community. Table 7-1 and Table 7-2 show the ranges of the two teachers' generic and mathematics-specific resources. Changes and transition to the new National Curriculum with the introduction of new mathematics topics teachers are unfamiliar with, and the availability of *ActiveTeach* with embedded videos and hyperlinks to other resources, seems to have become launchpads for community building among the teachers. At the time of observation, the mathematics teachers of school B, who previously acted as a group of individuals, are beginning to meet and undertake peer-to-peer teaching and sharing of resources.

It is pertinent to note that *documentational geneses* and *community geneses* go hand in hand as discussed in the literature review in subsection 2.2.3, p. 29. The setting of school B is undergoing changes and the mathematics teaching practices have been impacted on as well. There is a likelihood that tensions may arise since teachers at school B who are used to professional autonomy are beginning to build a collective that could have implications for individual teachers' preferences. One could only anticipate that the community evolution enabled by the abundance of digital resources will promote discussion about resource selection, task modification and mathematics teaching practice that could eventually provide support for community building and best practice with digital resources for all the teachers. My interpretation is there is ample opportunity for teachers' professional growth from being groups of individuals into becoming a community of mathematics teaching practitioners.

CHAPTER 8

CASE STUDY REPORT: SCHOOL C

In this chapter I present the context of school C and the case of Richelle. The chapter is divided into three broad sections. The first section explores the context of school C, and the content and approach to mathematics teaching as the background to the case study. The second section considers the case of the mathematics teacher Richelle under the following subsections: a teacher profile drawn from the first interview and from observation notes; their role in the department; their tasks as teacher; the resources; and the collective. The third section is a summary of the case of Richelle in school C.

Although one case may not be sufficient to draw a viable conclusion, there is anecdotal evidence that points to the underlying dynamics at work among mathematics teachers in school C which I consider valuable when taken together with the other cases in this research.

8.1 The Context of School C

School C is one of 24 E-ACT[142] academies. E-ACT is a multi-academy trust in England: "a leading, independent academy sponsor whose principal purpose is to create centres of excellence for all by establishing, maintaining, managing and developing academies and Free Schools[143]". They set to achieve this through five regional clusters of schools. The regional education teams provide tailored support for the particular needs of the communities that their academies serve. They also provide a professional learning programme for the teachers. This offers opportunity for collaboration among teachers. School C has a new state-of-the-art building that has been specifically designed to facilitate students' learning and provide an

142 https://www.e-act.org.uk/
143 https://www.e-act.org.uk/how-we-work/

innovative educational experience for students. It equally boasts of an extensive sports facility which includes a brand-new fitness suite and outdoor tennis courts. As a consequence, students participate in a wide range of sporting activities as support to their extra-curricular activities. The school online prospectus claims

> Most importantly by being part of E-ACT our students feel part of something much bigger. By encouraging our students to think big and believe that anything is possible, by showing them the importance of team spirit and working together to achieve, and by teaching them to carefully think through their decisions to do the right thing, we know that every one of our students can realise their full potential.[144]

School C has a central focus on excellence for all students through a personalised student experience towards achieving fullest potentials. The context and approach described in the next subsection relate to mathematics teaching and learning.

8.1.1 Content and Approach

In the 2018 OFSTED[145] report, the inspectors praised school C for the 'consistently strong teaching' at the academy, with improved standards in mathematics and it is now in the top 20 percent of academies and schools in terms of student progress. The mathematics teaching approach, as reported by the participating teacher and insights drawn from school documents, is built around two sets of central teaching practice. First is the *sequencing of the lesson and consistency of teaching*: inspiration and guidance for teaching and learning mathematics are primarily sourced from the *Collins Connect digital resources for schools*[146], *GCSE Maths Textbook Range*[147] - *CGP Books* and a *central bank of mathematics resources* in the department. These are associated with the new National Curriculum and underlie the scheme of work. Second is the *task differentiation* and *assessment for learning* practices. Students in school C are taught in ability groups where instructions are tailored to meet individual students' needs. The mathematics teachers differentiate the tasks with appropriate questions, flexible grouping and pace, use of digital resources and interactive

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144 From school website
145 From school website
146 https://connect.collins.co.uk/school/portal.aspx
147 https://www.cgpbooks.co.uk/

activities to help students deepen and extend their mathematics learning. As Richelle states, "*I want to make sure I was pitching my lesson correctly for the right ability*[148]". Closely related to the differentiation of tasks is the use of ongoing assessment for learning, where through regular assessment and feedback teachers adapt their teaching and are informed of where students are in their levels of understanding in the moment of the classroom. The central bank of departmental resources is a collation of worksheets, lesson plans and best teaching practice ideas that are available to all the teachers as a 'go-to place' to draw on resources. It is within the context of the school's personalised mathematics-teaching approach that Richelle undertakes her teaching.

8.2 Richelle's Profile

Richelle, 27, is the head of the mathematics department and has seven years of teaching experience. She has taught for six years in school C and recently rose from being the director of mathematics to become head of department. She teaches year-groups 7-11. Richelle acknowledges the value of digital resources and uses them in her lesson planning, delivery and assessments. She uses manipulatives and 3D wooden blocks for low-ability students. In terms of collaborations, she is involved in the regional networks of mathematics teachers and plays a mentoring role for the teaching assistants (TAs) and the newly qualified teachers (NQTs). She became the head of the mathematics department during the last phase of my data collection.

8.2.1 Role in the Department: Head of Mathematics

As head of mathematics, her core roles and expectations include providing strong leadership in the department that might enable best practice in teaching, initiating and fostering innovative approaches to mathematics that will motivate all students towards higher achievements, and facilitating partnerships whereby others with wider expertise support the mathematics curriculum and enrich the holistic learning experiences of the students and the wider community. In a self-report she outlined her main responsibilities thus:

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148 Screen capture conversation 16/06/2016

- Leader of the mathematics department
- Accountable for performance management of staff and achievement of all KS3 and KS4 students in mathematics
- Monitoring and analysing progress and attainment data for each year group and cohort
- Quality assurance of teaching and learning, marking and feedback, behaviour for learning, attendance and punctuality etc.
- Designing and implementing suitable KS3 and KS4 scheme of work and curriculum in mathematics
- Leadership of KS3 Catch Up programmes and interventions
- Contribution to the academy raising achievement panels to ensure all students have personalised learning plans
- Set and use robust assessment procedures including gap analyses, to monitor and track student progress

Richelle promotes and relies on the departmental scheme of work – a written guideline detailing the topic to be taught, the contents, structure, activities, resources and strategies for delivering the lesson. The printed textbook and its e-text equivalent are linked with other digital resources, exercises and ready-to-use materials that are mapped to the scheme of work and form the organising hub for her lesson preparation, delivery and assessment. She is also responsible for developing schemes of work and lesson plans in line with the curriculum objectives of the school.

In terms of professional development, she provides regular support for in-service trainee teachers and teaching assistants (TAs) in the course of their professional training. She also coordinates the mathematics department's interaction with other communities of mathematical practice.

My analysis of the observation and interviews (Chapter 4) revealed that she values and adopts several new digital resources alongside the old or traditional technologies, such as the use of 3D blocks and the animation of the same blocks and the use of traditional printed textbooks and e-textbook equivalents. She believes this complements and reinforces the learning and attends to the differentiated levels in her class. I now explore Richelle's tasks.

8.2.2 Richelle's Tasks

Richelle reported that most of her resources are primarily drawn from the *Collins Connect digital resources, CGP Maths textbooks, the departmental central bank of mathematics resources* and *TES* websites. Richelle explains the processes of her lesson planning with various annotations to identify where the tasks and resources were taken from.:

> The first thing I will do when planning my lesson, I will go to my department's scheme of work, the current topic we are looking at is length, perimeter, area and volumes and that is going to be over the next couple of weeks. And then what I will do is go to a matching document – this document is an overview of that whole topic and I will choose appropriate grades for the class I'm teaching. I then use this outcome to decide the sequencing of learning. Once I know what my outcome will be, I then need some resources to fill it out. (1intR: #1:4)

In the lesson on *area and perimeter*, Richelle follows the pattern explained in the interview and offers me an annotated lesson plan. Figure 8-1 below shows a screenshot of the outlined learning outcomes. (In the various figures are annotations added by Richelle.)

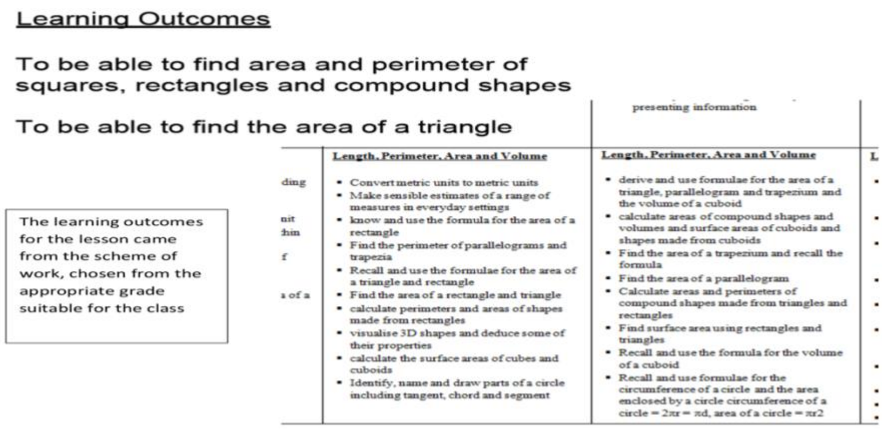

Figure 8-1. Learning outcomes

The learning outcomes are clear and specific statements detail the knowledge and skills the students are expected to achieve at the end of a lesson cycle. The screenshot in Figure 8-1 shows the learning outcome for a lesson on area and perimeter. The annotations and comments in the square and rectangular boxes are those Richelle made on her actual lesson plan. This set the 'pitch' for the lesson.

8.2.2.1 Tasks Initiating the Lesson

Richelle initiates her lessons with starter tasks alongside the definition of keywords or terms used in the lesson that students may not be familiar with or need reminding about.

A set of starter tasks (Figure 8-2) related to the day's topic are presented at the outset as warm-up tasks and to also assess where the students are with their learning and prior knowledge. The figure below shows some examples of the tasks and key words given to help build the students' mathematical vocabulary required for the topic. Richelle's annotations state her aims for the use of this tasks.

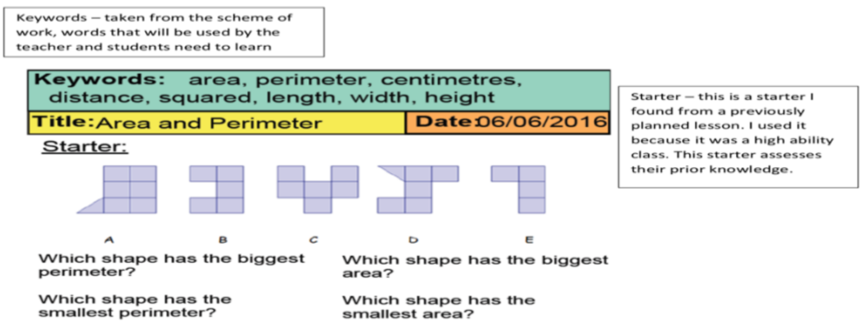

Figure 8-2. Starter tasks

Students engage with these starter tasks and definitions of the terms in the first 10 mins of the lesson. Richelle reports that this is her typical pattern of initiating her lessons.

When Richelle feels satisfied with the students' understanding, she goes on to give investigation tasks to the students in pairs.

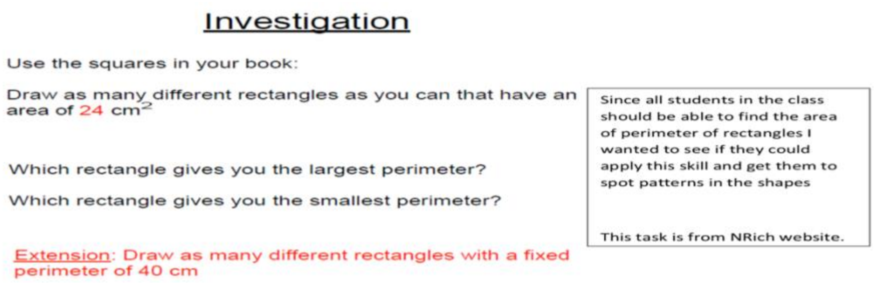

Figure 8-3. Investigation tasks

These tasks, as Richelle comments in the right-hand box, are geared towards application of knowledge in spotting patterns in rectangular shapes and finding out their properties. The tasks in Figure 8-3 were taken from the *NRICH* website (https://nrich.maths.org/). NRICH is a UK-based project aimed at providing resources and professional development for teachers wishing to embed rich mathematical tasks into everyday classroom practice and to enrich the mathematical experiences of all learners.

8.2.2.2 Posing the Main Mathematics Tasks

When Richelle feels students are confident with the preceding tasks that initiated the lesson, she moves on to compound shapes made of rectangles using square grids as a visual and mental aid to enable students to engage with the tasks as shown in Figure 8-4.

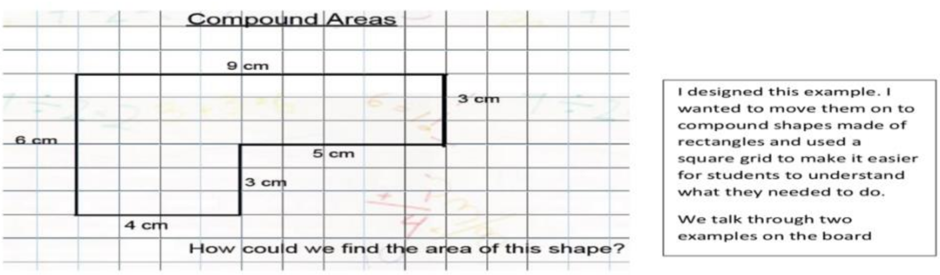

Figure 8-4. Teacher-led task examples

This first example was largely teacher-led with students adding passing comments on the go. The task was created by Richelle as part of the entry task to a set of more complex compound shapes. In the box on the right-hand side of Figure 8-4, Richelle made an annotation, stating that her use of the square grid is to make it easier for the students to understand what is asked of them. Prior to giving this task, she referred to related examples on the board. My interpretation is that she hope students will be able to work on later tasks independently.

I now consider the sample task through which students are asked to demonstrate their mathematics learning.

8.2.2.3 Demonstration of Mathematics Learning

When requesting the students to demonstrate their mathematics learning, Richelle uses tasks drawn from the textbook. She indicates that she wants the students to practice more of what they have learnt and to show that they understood what has been taught.

Figure 8-5 shows sample tasks, with two extra tasks that are labelled as extension tasks in the textbook. Richelle reported that the extension tasks are slightly more difficult than the first set of tasks.

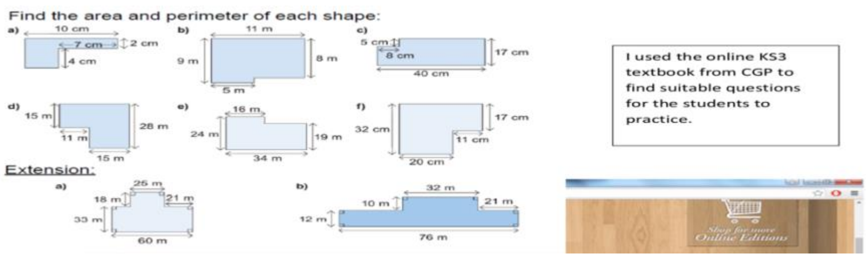

Figure 8-5. Extension tasks from the CGP e-textbook

The extension task, Richelle explains, is to help students consolidate and deepen their understanding of the topic using a variety of shapes and orientations. Students work in pairs on these tasks and report back to the whole class with their answers to each of the tasks given.

When Richelle feels most of the students are able to find the areas and perimeters of the compound shapes, she moves the class on towards the close of the lesson with formative assessment tasks as part of the concluding activities.

8.2.2.4 Concluding the Lesson

Formative assessment features as one of the cardinal approaches to mathematics teaching in school C. Richelle presents three assessment tasks to the students: a past examination question from a problem-solving worksheet, a second task from an online *Boardworks*[149] *mathematics* presentation and a third task on triangles from the Centre for Innovation in Mathematics Teaching (CIMT[150]) website. Figure 8-6 is the assessment task taken from the worksheet saved in Richelle's personal bank of resources as she states in the annotation in the box. Her stated aim is to assess what the students have learnt using past examination questions drawn from the collection on problem-solving in her own saved resources.

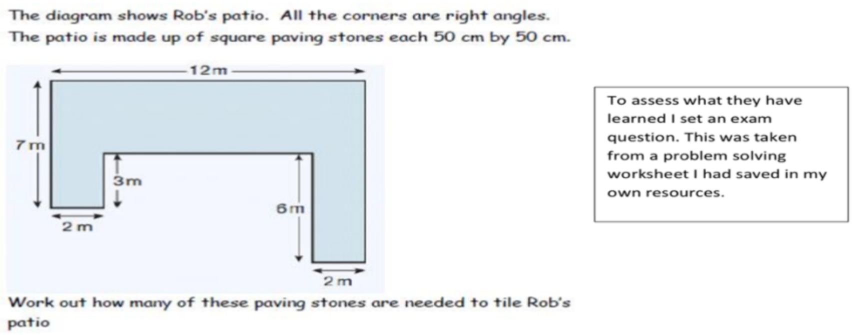

Figure 8-6. Assessment task from the worksheet

In all the tasks given to the students, technology was mainly used solely by the teacher to create, source, modify and display tasks for students. 3D wooden blocks and digital animation of shapes were also used by students. Most of the technology available in the class was used by Richelle.

I now turn to consider Richelle's resources.

149 http://www.boardworks.co.uk/maths_71
150 https://www.cimt.org.uk/menus/resources.htm

8.2.3 Richelle's Resources

From the exploration of Richelle's tasks, it appears that her resources are intertwined with her lesson plans, classroom interactions and assessment practices. A list was made of all the digital resources mentioned, used and observed in the lesson and Table 8-1 displays all of Richelle's resources.

Human		**Non-Human**			
Physical contact	Not Physical Contact	Electronics		Non-Electronics	
		Hardware	Not Hardware	Individual	Not Individual
CPD departmental meet Teaching Assistant (TA) Sheffield Maths teacher network Mr Kelly quiz E-ACT Academies Chain[151]	Twitter[152] MathsWatch YouTube	IWB iPad Calculator Laptop Visualiser[153] Netbook[154]	GeoGebra Spreadsheet Corbettmaths[155] mathsbox.org.uk TES nrich.maths.org Centralised resource folder Collins Connect[156] digital resources Resourceaholic	Workbooks Worksheets	CGP books[157] Edexcel Hub[158]

Table 8-1. Richelle's resources

The table shows the classification of Richelle's resources into human and non-human resources and the further subdivisions. I now explore those resources Richelle claimed she used more frequently in her teaching activities.

151 http://www.e-act.org.uk/about-e-act/who-are-e-act/
152 https://twitter.com/mrbartonmaths
153 https://www.educationsupplies.co.uk/ict-and-audio-visual/visualisers/u200l-usb-visualiser
154 http://whatis.techtarget.com/definition/netbook
155 https://corbettmaths.com/
156 https://connect.collins.co.uk/school/portal.aspx#
157 https://www.cgpbooks.co.uk/whoAreYou?page=books
158 https://qualifications.pearson.com/en/home.html

8.2.3.1 Human and Non-Human Resources

In the classification of Richelle's resources in Table 8-1, there are several human resources available to Richelle, but most of her emphasis lies on three non-human resources that appear more frequently in her teaching engagements. These three resources are Textbooks (*Collins Connect* and *CGP e-textbooks),* resource banks and websites (TES and NRICH). Hence, my presentation highlights these frequently used resources.

The *Collins Connect* and *CGP e-textbooks* are central to Richelle's resources. At the time of research, secondary teachers were at the midpoint of transition to the new curriculum; therefore, Richelle had to rely largely on the two e-textbooks as a guide and the primary sources of understanding the curriculum and adapt it to the class's needs at the time of transition.

Richelle explains the process of her lesson planning:

> As a department we all have access to online kind of PDFs for textbooks, so there is CGP books; we've Key Stage 3 and Key Stage 4 books, which we can get questions from and examples from, exercises on there with answers. We also have the Collins Connect one and then we also have copies of the textbooks; that means we can also get them online. And there are matching, homework tasks, quizzes and videos you can embed in your lessons as well, so we use those. Any textbooks we have got in the department, we have got online PDF versions, so that we can cut and paste into our lessons. (1int R: #00:28)

In the screen capture recorded interview, Richelle also made references to the e-textbooks as she demonstrates her lesson planning activities

> The other thing I will do: we've got access to Collins Connect, which has basically all the Collins textbooks on PDF versions. I can get access to them, then look through and search for the relevant topic. And look up the questions and use that to fill out my PowerPoints… The other we use is the CGP, we get access from our computers, just online books with answers, but then it gives us plenty access to different resources, particularly looking for problem-solving things, because the new curriculum gives that sense we need to put in more problem-solving questions. (RScrncpt: #5)

The CGP books and Collins Connect are two collections of different resources aligned with the new National Curriculum whereby teachers and students could undertake activities with mathematics on the go by simply logging in. The CGP e-textbooks and textbooks contain a wide range of examination-style questions for every topic, with fully worked answers and marking schemes. It boasts tens of thousands of realistic practice questions, workbooks, revision notes, video tutorials for every topic, challenges, and complete revision resources for the new GCSE Mathematics. For each of the materials there are online versions for PC, Mac, tablets and phones. In this way, the CGP resources have a very high practice and examination focused tones.

The design of both the Collins Connect mathematics textbook and e-textbook is based on the achieving mastery in mathematics with the Shanghai mathematics project[159]. Teaching for mastery (the different conceptualisations of mastery are addressed later on in the discussion, see subsection 10.7.1, p. 302) with Collins Connect means enabling students to achieve two related things: first, competence – the students' ability to carry out mathematics tasks with fluency and accuracy; and second, confidence – the students' ability to take on unfamiliar problems and apply mathematics skills in problem-solving.

The Collins Mathematics KS3 and GCSE resources aim to support the achievement of mastery through these paths:

- Set high expectations for all.
- Build student knowledge to ensure continuity and progression with a carefully designed curriculum and well-planned lessons.
- Develop fluency and confidence with plenty of practice and consolidation.
- Identify where students have not grasped a concept or procedure and remedy with quick and early intervention workbooks.[160]

The dashboard of Collins Connect at-a-glance is presented in Figure 8-7. The dashboard shows a range of services and mathematics education policies that are

159 https://collins.co.uk/pages/secondary-maths-the-shanghai-maths-project
160 https://collins.co.uk/pages/secondary-maths-teaching-for-mastery

driving the resources created by Collins, including the Shanghai maths project, GCSE Maths for post-16 students, and more, as shown in the figure. Collins Connect provides a collection of mathematics resources for the levels of mathematics education at the secondary and post-16 year groups.

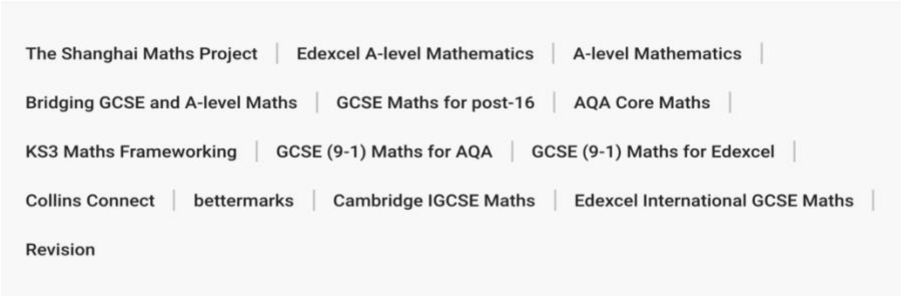

Figure 8-7. Online dashboard of Collins Connect

Richelle reported that the Collins Connect website is one place she finds guidance for the new curriculum and she uses it as a means of re-sourcing and updating her personal bank of resources to reflect the ongoing curriculum change. This textbook, available both in text and electronic versions, offers flexibility and saves Richelle, as she said herself, from ‘re-inventing the wheel’ in planning new lessons from afresh. She can easily modify, cut and paste and adapt them to her class’s needs, which she remarks on in another interview.

> The textbooks give you a lot more flexibility because you’ve got foundation textbooks and the higher textbooks for the grade you are looking for. They are tailored a bit specifically to different abilities. Before I went onto the TES, I would do a search through my old lessons and see if there is anything appropriate. Most of the time I can find a basic structure, but I might look to update the questions I have got because with the new curriculum a lot of things are going to be outdated. (1intR: #7)

In a time of curriculum transition, Richelle seems to have developed a heightened awareness of the implications of this change for her personal bank of resources and the departmental central resources folder. She is actively making efforts to update and add resources so she is able to reflect the demand of including problem-solving much more in her lessons as prescribed by the new curriculum. There is a sense of co-emergence of the use of novel resources and an ongoing adaptation to the new curriculum.

The *resource banks* feature largely in Richelle's self-report as the source of resources. There are two forms of resource banks available to Richelle: her personal bank of resources stored on her iPad and laptop, and the departmental central folder. In a second interview extract she observed,

> I have been here for six years, so much stuff I used a couple of years but no good anymore; we have to build a bank of resources that you use again and again and again. And again, we are very open in this department, so if someone has something good then we get them to send it to us. Everyone is required to send their lessons to myself or the head of department and every week we can review people's lessons and we send out good ones and everyone would have a copy. We also have the school network itself, have a folder called Central Resource and there is a Maths folder, and then everyone can put all their resources in there that they think are useful so that everyone can get access, so it is not just for one teacher. (2intR: #4)

Richelle speaks of the constant addition and deletion of resources collectively adjudged as 'good' or 'not good anymore'. In the face of transiting into a new curriculum, the older resources in both her personal and departmental resource banks are undergoing a new form of updating, eliminating what is no longer suitable for the curricular requirements and adding new materials as they emerge. There is equally a 'gatekeeping quality assessment' undertaken by the head of mathematics and director of mathematics to ensure consistency, as Richelle puts it,

> It means for us as a team there is consistency across the department because they will use the same format … but also then to have a bank of things ready, so that if that was too hard or too easy that you could bring something out more quicker than, I think, if they need to go and find a textbook and flip through the pages, it would all be there much faster. (2intR: #5)

The collegial vetting of resources for mathematics teaching promotes consistency in the structure, content and quality of mathematics that are tailored to the group's ability and to enable the teacher to react in a timely manner when there is need to make unplanned changes when occasioned by emerging developments in the course of the lesson delivery. Besides the resource's banks, websites feature as well in Richelle's sets of resources; I now turn to these.

Websites

Two free web-based resources (TES and NRICH) featured frequently in the resources that Richelle mentions unprompted. In all the interviews and screen-capture snapshots, the TES and NRICH websites are her go-to places after her personal bank of resources, as mentioned in the exploration of her tasks above. Her use of these two sites is consistent with what has been reported by the other teachers previously. TES and NRICH are sites for the aggregation of useful resources and because both are curated by trusted and well-known individuals in the professional practice community, the resources hubs are frequently referenced. Another area of interest building from the idea of a resource hub is the teacher collectives. In spite of the possible advantages the use of digital resources might offer, it creates a dilemma for teachers. Richelle commented thus:

> Yeah, I mean, I would never, I hope I will never give them something that wasn't going to benefit them. I think the main use for worksheets and questions on the board is to enable them to practice and consolidate what we have been learning; I know a few teachers like to use things rather than a pile of questions, use of cards, some things to manipulate, so they are getting a bit more discussion and problem-solving, definitely feel resources nowadays its getting more and more advanced people just come up with new ideas and they get shared around, so it is definitely for the benefit of the students.
> The only thing I will say is that they tend to be more tailored towards exams now than they used to be, because especially with the new curriculum everything is very, very content-heavy and it is about being able to read and answer exam questions, whereas before you could look in more detail at the concepts behind it and the actual understanding of the topic, whereas now you have to be able to answer questions, and that is a bit of a shame. The exams… And you do need them. (2intR: #10)

Richelle's focus is always on students' needs and how the digital resources benefit their learning. The abundance of resources and new National Curriculum regulations come with their own challenges. Digital resource is content-heavy, and a lot is tailored towards examination.

8.3 The Collectives of School C

The mathematics teachers in school C have a staff room/departmental office space set-up. Teachers have one shared scheme of work and a matching document provides further details. The head and deputy head of the department peer-review lesson plans and create a shared central resources folder wherein materials adjudged to be excellent are stored and accessible to every teacher. There are also monthly formal and periodic informal departmental meetings and training sessions across the school year. Richelle reported a ‘culture of support and culture of sharing’ enhanced by the use of technology. Regular meetings between the mathematics and English departments take place to ‘share professional difficulty’, thereby helping students achieve excellence. The layout has two adjacent offices: one as the main staffroom, the second from my observation was used mainly when a peer or group of teachers want to work on some tasks quietly. These offices open onto a large workspace, which houses an industrial-size printer and sets of desktop computers used by students and teachers. The space provides room for students to undertake out-of-class activities and teachers attend to students who have brought in queries.

Continuing professional development (CPD) takes place for the teachers through the regional secondary mathematics networks of the local teaching school alliance (TSA). This alliance is a set of partnerships between Sheffield University and Sheffield TSA, and Sheffield Hallam University and Hallam TSA, which offers a collaborative school-based training with the universities and a varied network of partnered schools across Sheffield. Two strands of training programmes are available: academic study and classroom practice[161]. There is also a subject support network accessible to mathematics teachers in the region.

Another opportunity for professional learning is the termly meeting of the Sheffield network of mathematics teachers. Mathematics teachers get together to discuss the new curriculum, how to implement it in school, review and share ideas for assessment, and collectively prepare quizzes, YouTube support videos and lessons on various topics that are distributed across schools. There is also sustained participation in the activities of the TES online community, teach first[162] subject mentoring and an active NQT teacher training Facebook-friendship group.

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161 http://www.sheffieldtsa.org/

162 https://community.teachfirst.org.uk/user/login?destination=homepage#region-content

School C offers diverse opportunities for teachers to work collectively in-school: the departmental culture of friendliness and openness to sharing, and interdepartmental interactions. In relation to professional development, Sheffield TSA provides alliances and networks for the professional learning of the mathematics teachers. At the individual level, TES, Twitter and Facebook groups are the main online communities.

8.3.1 In Summary

In this chapter, I explored a mathematics teacher's (Richelle) work with resources in and for mathematics teaching and her professional practices through the lens of a documentational approach of didactics and activity theoretical approaches. The context of school C where Richelle teaches is underlined by two general pedagogic dispositions: teaching mathematics with *sequencing of lessons and consistency of teaching*, and employing *tasks differentiation and assessment for learning*. Richelle stated her central goal thus: "*I want to make sure I was pitching my lesson correctly for the right ability*". These general and specific approaches are the basis on which this discussion is undertaken.

In terms of mathematics teaching resources, Richelle has accumulated in her personal bank a wide range of resources over her seven years of teaching. As head of department, she organises and manages the collective shared bank of resources of the department. School C also subscribes to two e-textbook packages: the *Collins Connect* and *CGP-GCSE Mathematics e-textbooks* and its supplementary links to other resources. These three banks of resources are the core places from which Richelle draws several resources for professional practices. She also participates in ongoing school- and regional-based mathematics teachers' collectives working with resources. These nested contexts situate Richelle's practices with digital resources in a network of activity systems (Núñez, 2009).

Having stated the overall context of Richelle's work with digital resources as she undertakes her professional teaching practices, it is possible to distinguish the stages of her individual and collective documentation work (Trouche, Gitirana, Miyakawa, Pepin, & Wang, 2018). In Richelle's individual preparation for lessons, these phases of documentation work could be reconstructed from her practices as evident in the case description.

Phase 1: Choosing a mathematics topic according to year-long teaching plan
Phase 2: Sequencing the lesson using scheme of work, learning outcome guide and the National Curriculum
Phase 3: Accessing, selecting and modifying a range of mathematics teaching resources to prepare a lesson that is '*pitched correctly for the right ability*'
Phase 4: Taking into account task differentiation and assessment for learning activities
Phase 5: Anticipating misconceptions and preparing for emergent mid-lesson task design
Phase 6: Reflecting on lessons and sharing with the collective's shared bank of resources

This is one possible reconstruction of Richelle's documentational geneses. As her resources undergo transformation through lesson planning, delivery and assessment, her professional teaching practices are impacted upon as well. With the growing use of technology and the convergence of diverse practitioners of mathematics teaching it enables, Richelle's work with resources is enriched by her participation, as she reported. The departmental collective and regional networks of mathematics teachers, beyond sharing resources and best practices, have become communities of mathematical teaching practices where teachers might establish a long-term professional relationship as documentation-working colleagues (Wang, 2018) with mutual influences on each other's professional growth and practices. In the collective of mathematics teachers facilitated by the use of digital resources, opportunities to develop common forms of addressing and making sense of resources are extended (Gueudet et al., 2013a).

CHAPTER 9

REVISITING THE RESEARCH QUESTIONS

In this chapter, I revisit the research questions. The discussion is organised around the four themes of the research. In section 0, I address RQ1 by highlighting the ways teachers access resources and the ways they adapt and create new ones. In section 9.2, I explore the question of which resources mathematics teachers access and use and also their associated resource systems. Section 0 examines the tasks teachers give to their students, the sources of these tasks and how they been modified or not in teachers' practices. In the last section, 0, I address the question of the collectives in which teachers participate and explore the features of the collectives that could lead to the formation of a community of practice. This chapter concludes with a summary.

9.1 The Mathematics Teachers

RQ1 *In what ways are mathematics teachers accessing, adapting and creating resources for classroom practices?*

The first research question explores, from the mathematics teachers' perspectives, the ways in which teachers access, adapt and create resources for their practices. This section discusses this question in two parts. Firstly, it looks at the ways through which teachers access resources. With regards to RQ1, the findings revealed three ways that teachers access resources: namely, school and departmental recommendations, banks of resources, and the use of various applications and social media. Secondly, the section examines ways in which teachers adapt and create resources for their particular classroom needs.

9.1.1 The Ways Teachers Access Resources

The findings reveal three ways that mathematics teachers in this research access resources for their everyday professional practices. Several researchers have investigated mathematics teachers' engagement with resources, as shown in the literature review in Chapter 2. One gap in the literature which the findings address is how mathematics teachers access these resources. Previous studies have looked at mathematics teachers' practices with 'resources-on-offer'. For example, the

Cornerstone Maths project in England (Clark-Wilson, 2017) was introduced by a team of researchers; Sésamath in France (Gueudet, Pepin, Restrepo, Sabra, & Trouche, 2016) was introduced for mathematics use by an association of teachers and researchers. There is also an investigation into the trends in design, development, and use of digital curriculum materials/resources from multiple perspectives (Choppin & Borys, 2017). More recently, a case of collaborative design of digital resources in mathematics has been examined (Kynigos, Essonnier, & Trgalova, 2020). It is in the context of the ongoing development in how resources are used in mathematics teaching that the findings are discussed. I now discuss the three ways through which mathematics teachers in the three schools in the study access resources for their professional practices.

School and Departmental Recommendations

In the three schools in the research, there exists at least one school/department-subscribed resource that teachers are encouraged to use. For instance, in school A (see 6.2.3, p. 138; 6.3.3, p. 150;6.4.3, p. 163 and 6.5.2, p. 177), Mangahigh, Mathsbox and Socrative were employed by teachers. In school B (see 7.2.3, p. 195 and 7.4, p. 207), the central resources bought by the school for teachers and students were ActiveTeach/ActiveLearn and MathsWatch. In school C (see 8.2.3, p. 226), the Collins Connect and CGP books and associated online resources were accessed.

Annual paid subscriptions are paid to access these resources, and teachers and students register and create their user identities. These resources have added value for teachers since they enable them to provide immediate feedback to students and collect data on students' progress over time through a regular collection of data on a measured test (6.4.2.3, p. 159and 7.4.1, p. 209). Teachers also indicated the interactive game-based potential of these resources for motivating students in learning. The growing use of Mangahigh, Mathsbox and MathsWatch for collecting data on students is reported in several studies (Morrison & Lee, 2019; Siedel & Stylianides, 2018). This growing amount of collected data may pose a challenge to teachers since it may have an impact on teacher’s time. The teachers raised this issue: "*we do not have time*" (Gray, 7.2.3, p. 195) and "*the problem of teaching at the moment, you do not have* *time*" (Gavin, 7.4, p. 207), and such excessive data collection appears in these instances to be counterproductive.

Banks of Resources

The findings revealed that the banks of resources accessed by teachers exist in three different forms: a personal bank of resources, a shared departmental bank of resources, and online depositories of resources (in schools A, 6.6, p. 182, B 7.5, p. 213 and C 8.3, p. 231 respectively).

The first is the personal bank of resources. All the teachers reported having a personal bank of resources they had accumulated over the years of teaching that is still growing. This consists of a range of classroom resources, PowerPoint presentations, lesson plans, worksheets and teaching ideas that have been created, used and reviewed by teachers. These resources are collected and collated into categories: for instance, by topics, by class and by intended purpose (for example, starter and assessment tasks). The teachers also reported that as they adapt to the new curriculum and students' needs, they update, delete and re-use the resources year on year. For the teachers, this personal bank of resources is a starting point when planning for the lesson (for instance, Kitty, 6.2.3.1, p. 139). It is often held on the teachers' iPad, phone, computer or USB memory sticks.

The second is the shared bank of resources. The shared bank of resources is a collectively created departmental resource bank that is accessible to all the teachers in the department. The head of the department or a designated teacher oversees the updating of the resources and points colleagues to any newly available resource they think could be useful. The resources in this shared bank are drawn from those found online, individual teachers’ creations and those recommended from seminars, training sessions and departmental discussions. The shared bank of resources, it appears, is an indication of the emergence of a shared belief, a collective system of thinking and a community of mathematical practice. The resources in the shared bank are ordered by class, topics and other categories, including teaching suggestions and tips and web links to other resources. All three schools in the study reported the existence of a shared bank of resources as a collectively accessible folder kept within the departmental office or staffroom.

The third is the online depositories of resources, which include TES, NRICH and Resourceaholic. These depositories offer teachers free lesson plans, education articles, teaching tips, recommendations and professional development resources. For instance, the TES has an extensive collection of teaching ideas, worksheets, editable schemes of work, classroom resources and information regarding teacher training and job vacancies. The TES[163] claims to provide innovative services and access to over 900,000 teacher-made resources to help teachers succeed in the classroom. It also reported it gives particular support for the English National Curriculum and other English-language curricula including International Baccalaureate and Cambridge International in the UK and international schools. NRICH and Resourceaholic share similar roles and missions but with a primary focus on primary and secondary mathematics teaching and learning. The development of open resource banks has been explored from teachers' and teacher educators' perspectives (Hassler, Hennessy, Knight, & Connolly, 2014). When Hassler and colleague focused on open education resources[164] (OER), the research findings revealed the use of both OER and non-OER resources by teachers. The findings add to our understanding of teachers' practices with these depositories and the growing literature on how teachers access resources and their sharing practices in the context of secondary school mathematics teaching. The constructs "open content" and "open educational resources" refer to any copyrightable material (traditionally excluding software, which is described by other terms like "open source") that is licensed in a manner that provides users with free and perpetual permission to engage in the 5R activities: *retain, reuse, revise, remix* and *redistribute*. These resources can be found around a virtual community of teachers, teacher educators and researchers, where everyone is an equal partner in developing, adding and using quality resources with restrictions prohibiting only

163 https://www.tes.com/about-us

164 Open educational resources are freely accessible, openly licensed text, media, and other digital assets that are useful for teaching, learning, and assessing as well as for research purposes.

commercial use (Wiley, 1998). They also function as collaborative platforms by bringing teachers, educators and researchers together online to share expertise and keep up to date with the latest education news, research and analysis. These depositories also add a social layer, so that the resources can be evaluated, and comments and recommendations left, and offer mostly free subscriptions of resources that are legally free to access, copy, distribute, use, adapt, or modify together with several paid subscription resources.

Social Media and Applications

The findings also show that teachers access resources from social media (Facebook and Twitter) and applications (mobile and desktop apps). A wide range of mathematics-based and generic resources are available and accessed by the teachers through these means.

In terms of social media use as means of accessing resources, Facebook and Twitter are mentioned by the teachers. For instance, Jimmy (Table 6-3, p. 164), Gray (Table 7-1, p. 197) and Richelle (Table 8-1, p. 226) cited their Facebook use and regarding Twitter, Kitty (Table 6-1, p. 138), Jimmy, Jose (Table 6-4, p. 177), Gray and Richelle belong to various Twitter groups that share resources and post links to worksheets, quizzes and seminars. In the review of literature on teachers' use of social media, Van Den Beemt, Thurlings, and Willems (2020) report that it enables the teachers to gain knowledge, receive feedback and support, while simultaneously sharing teachers' knowledge and expertise. The findings on teachers' use of social media is consistent with this report and takes it further by highlighting the fact that social media like Facebook and Twitter also enable the teachers to evolve into an online community wherein sharing of resources and mutual support is valued. Goodyear, Parker, and Casey (2019) report that online professional learning communities enabled by the use of social media have the potential to impact on teachers' practices.

Mobile and desktop applications featured in the case reports as ways of accessing resources by teachers. Applications (commonly known as apps) are software that bundles together certain electronic and digital features in a way that is accessible to a user. Mobile devices offer one of the easiest and fastest ways to access educational content (Drigas & Angelidakis, 2017). There are several apps installed on mobile devices (like phones or iPads) offering services and support to the teachers in their professional activities. For example, the Khan Academy app is freely available for both students and teachers. Khan Academy offers a range of practice exercises, instructional videos, and a personalised learning dashboard that empower learners to study at their own pace in and outside of the classroom. In the context of this research, it was used to find educational resources for mathematics teaching and learning (see Kitty, p.132 and Gavin, p. 203).

Socrative is another teacher app which offers an interactive environment for students and teacher to share their learning. It was used by five out of the seven teachers in this research (see Table 9-3, p. 248). The teachers reported that Socrative is easy-to-use and fun to implement. Teachers used this app to set questions, conduct polls, and conduct assessments with the real-time data on display during the session. This app was used as part of the tools for formative assessment practices. Wright, Clark, and Tiplady (2018) argue that mobile and desktop applications are tools in teachers' toolkits that help with design for formative assessment. Mathematics teachers in this research used social media and applications to access, re-source, collaborate and improve their teaching practices.

9.1.2 The Ways Teachers Adapt and Create Resources

The seven mathematics teachers in this study are continually (as shown Table 9-1244 and Table 9-2, p. 247) appropriating and transforming resources for their teaching purposes. Adler (2000) uses "re-sourcing" as a construct to describe the dynamic nature of teachers' work with resources. The teachers in the schools in the study often access resources from a wide range of places (9.1.1, p. 235); they interpret, adapt, modify and enhance the visual presentation by adding colour and dynamic images and sounds. This continuous process of interpreting and transforming resources is described by Gueudet and Trouche (2009) as documentational genesis. The mathematics teachers in this research use and re-use previous resources, adding and deleting elements over time. This practice is informed by their experiences, inputs from colleagues and students' feedback as they use the resources in classroom practice; thus, the documentational genesis of these teachers' work with resources is both a dialectic and a dynamic one. Remillard (2012) argues that teachers do not merely pick up resources and use them; rather, there are participatory and transactional perspectives to teacher-resource interactions, wherein teachers actively partner with designers and each other and in collectives to transform resources, and these resources transform their professional practices as well. The evidence from this research suggests that the teachers are not merely making superficial changes or modifications to the resources they appropriate; instead, they are becoming designers of resources for themselves and others. For instance, Kitty and Jose in school A create resources that are suitable for their class needs (pp. 127 and 169 respectively); Gray in school B designs and creates YouTube videos that are used by his students and are made freely available for the broader mathematics teaching community (7.2.2, p. 189). Richelle also reported how she designs resources and uses a square grid to make it easier for students to understand (8.2.2.2, p. 223). These teachers shape and transform resources and they, in turn, together with their practices are shaped and transformed by these resources. This mutual shaping resonates with the intertwined teacher-resources dialectics of *instrumentation* and *instrumentalisation*: instrumentation involves how the digital resources shape the thinking and practice of the mathematics teacher using resources, and *instrumentalisation* includes the ways the mathematics teachers adapt, revise, reorganise and appropriate the resources in the terms discussed by Trouche (2004). They will periodically reorganise and adapt resources to a novel situation of students' needs or curriculum change, and these teacher-resource interactions led Gueudet, Pepin, and Trouche (2011, p. 358) to reach this conclusion about the nature of teachers' resources: "We now suggest viewing them as 'living resources' emphasising their present and continuous use in teachers' work". The evidence supports these findings and extends our understanding by highlighting specific ways in which mathematics teachers adapt and create resources in the context of schools in England.

England provides a 'rich' context for studying the growing phenomenon of teachers creating and designing their own resources (Siedel & Stylianides, 2018). Mathematics teachers in England have the encouragement and support (guided by the National Curriculum and schemes of work) to freely decide what mathematics teaching resources to select, adapt and use. The seven teachers in this research transform resources, using the *cut-copy-and-paste* function on a Word document to revise, reorganise, delete or add to the resources they are modifying. The *Microsoft snipping tool* as in the case reported by Kitty and *screen capture tool* (*prtscrn*) feature on the computer system used by all the teachers (for instance, Emilia, Jose and Gavin) when preparing their resources for teaching. Several functionalities – *colour annotations, hyperlinks to embed videos and link other resources to the central resources used in teaching, freeware used to generate QR codes* – of the laptop, iPad and IWB provide the teachers with these facilities to enable them to modify, adapt, and mashup various resources. One of the teachers, Gray, creates YouTube videos by self-recording (Gray has a dedicated YouTube channel for these videos) his teaching and posting them online for students and other teachers to use. In these ways, the teachers in my study reported how they appropriate digital resources for their students' needs.

Teachers' creative work with resources which include selecting, modifying, and creating new resources, in-class and out-of-class, is termed *documentation work*. Siedel and Stylianides (2018, p. 121) refer to the inventory of resource choices available to teachers as a resource "pool of possibilities". Teachers, guided by specific aims and intentions as stipulated in the scheme of work, collate the relevant resources into a well-structured and organised collection as part of the mathematics *documentation system*. The documentation system consists of all the *documents* developed by these mathematics teachers with a structure that corresponds to the teachers' planned classroom activity (Gueudet et al., 2014). The cataloguing of the resources of the seven selected mathematics teachers and mapping them against the growing abundance of digital resources could illuminate teachers' reactions to the proliferation of resources and point new teachers and others to where they could begin in their documentation work. Teachers can be challenged by the abundance of resources and find difficulty in selecting the right resources for their class need, as was the case with Jose (6.5.1, p. 169). I now explore the theme of the teachers' resources further.

9.2 The Resources

RQ 2.1 *What resources do mathematics teachers access and use?*
RQ2.2 *What constitutes the mathematics teachers' resource system?*

The findings with regards to the resources mathematics teachers access and use (RQ 2.1) reveal various commonly used resources and a range of resources unique to individual teachers. At one level, the answer to RQ2.1 is straight forward and the resources of each teacher are shown in the various tables of classification of resources: Kitty (Table 6-1, p. 138); Emilia (Table 6-2, p. 152); Jimmy (Table 6-3, p. 164); Jose (Table 6-4, p. 177); Gray (Table 7-1, p. 197); Gavin (Table 7-2, p. 209) and Richelle (Table 8-1, p. 226).

In all, 187 distinct resources are reported by the seven teachers. In school A, the four teachers mentioned 108 specific resources (Kitty, 26; Emilia, 23; Jimmy, 35 and Jose, 24 resources, respectively). In school B, 51 resources were mentioned (Gray, 27 and Gavin, 24). In school C, Richelle identifies 28 distinct resources in her practice. Table 9-1 below shows 15 of the resources mentioned by three or more teachers. The diversity, it appears, shows the autonomy of mathematics teachers' practices in England. In the context of schools in England, there is the freedom to choose and use resources, a shared national practice of using schemes of work and growing belief in the use and value of resources like IWB, iPads, laptops, TES and the prevalent practice of having shared banks of resources. These shared resources may hold some hints as to how mathematics teachers in England evolve in their professional learning and practices as mathematics teachers.

	Teachers in Schools: A				B		C
Resources	Kitty	Emilia	Jimmy	Jose	Gray	Gavin	Richelle
iPads	√	√	√	√	√		√
Laptops	√	√	√	√	√	√	√
Shared bank	√	√	√	√	√		√
IWB	√	√	√	√	√	√	√
Calculator		√	√	√	√	√	√
iWB	√	√	√	√	√	√	
Resourceaholic	√		√		√		√
Corbettmaths		√	√			√	√
Mathsbox		√	√			√	√

MathsWatch				√	√	√	√
CPD	√	√	√	√	√	√	√
Twitter	√		√	√	√		√
TES		√	√	√	√	√	√
GeoGebra			√		√	√	√
YouTube	√				√		√

Table 9-1. Resources used by three or more teachers

Covering the three schools and seven teachers, Table 9-1 shows the resources mentioned by three or more teachers. In addition, I consider continuing professional development (CPD) as a resource and a resourcing opportunity for the teachers in terms of the material resources and teaching ideas. All seven teachers are actively committed to school-based CPD and those organised within the local region. All seven teachers reported regular use of IWB and laptops and six of them also reported the use of iPads, TES, iWB, and calculators. Of the 15 resources n used by three or more teachers, Richelle identifies 14 of them, and Gray and Jimmy 13 each; although my data does not point to the reason why this is so, one could probably infer (from the data on the teachers' profiles) that this is because Gray and Richelle were both heads of the mathematics department at the time of the research and Jimmy considers himself a 'technology enthusiast'.

In the classification of the resources, one piece of hardware, the iPad, plays a significant role in teachers' classroom activities. It is through the iPad that teachers and students are able to access most of the applications and websites (Resourceaholic, Corbettmaths, Mathsbox, MathsWatch, GeoGebra and YouTube) in the classroom. The iPad also plays a central role in the assessment practices, discussed in Chapter 10. Therefore, I highlight the use of iPads here.

Hardware: iPads in Schools

The findings with regards to the use of iPads suggest a growing amount of use of them among mathematics teachers and that there is an impact on their pedagogy. Geer, White, Zeegers, Au, and Barnes (2017) report that the use of iPads led to changes in the way teachers teach and collaborate. In formative assessment, iPads have been reported to enhance opportunities for feedback and student learning, potentially

(Dalby & Swan, 2019). My findings with regards to iPad use reveal further roles that iPads play, primarily in the mathematics classroom. (This is further discussed in Chapter 10.1, p. 255). Table 9-1 shows that an iPad is used by six out of seven teachers. In school A, the teachers are encouraged to use the class sets of iPads together with the laptops and IWB as the central hardware in their teaching and students' learning. iPads have features similar to those of mobile phones and most of the students have had previous exposure to iPads at home, and as such the iPad was relatively easy to use in the classroom setting. It appears that iPads provide space for student's autonomy and privacy since they can work individually on iPads and respond and submit solutions to tasks using the dynamic features of the iPad. Whenever students work in pairs or as a group, this became a collective space for working together. In the three schools, teachers used iPads for formative assessments, as part of the set of resources for teaching, for retrieving, and working on and submitting tasks using QR codes. iPads are seen in school A as having a motivating effect on students, supporting collaborative and personalised learning, and the touch-sensitive features of iPads makes it appear relatively easy to learn and use. The usefulness of iPads in engaging with tasks and functioning as portable hardware through which teachers and students access other resources (like Mathsbox and MathsWatch) seems to be a decisive factor in the whole class appropriation of iPads as the first tools of choice. This practice with iPads cuts across the three schools but in a variety of ways. In schools B and C, the iPads were mainly used by the teachers, while in school A it was both teachers and students. The significance of iPad use in the three schools is further discussed in Chapter 10. I now turn to the question of teachers' resource systems (RQ 2.2).

Teachers' Resource Systems

In this subsection, I address the question of resource systems. Ruthven (2012) introduced the notion of *a resource system* as one of the structural features of teachers' activity. The resource system, in Ruthven's conceptualisation, implies an a priori structured and organised collection of mathematical tools and materials aligned with educational goals, and appropriate to the intended students. For Gueudet and Trouche (2012b, p. 27), "The *resource system* of the teacher constitutes the 'resources' part of her *documentation system* (i.e. without the scheme part of the documents)". Although neither Gueudet and Trouche nor Ruthven have entirely resolved the varied use of the metaphor 'resource system', it indicates in a way the divergence in national educational systems (England and France) and professional practices. In England, mathematics teachers use the scheme of work to structure their lesson cycle, organise and pace their teaching and align appropriate resources for their lessons. This is not the case in France. In this research, I use the notion of a resource system as advocated by Ruthven (2012).

The teachers' resource systems are organised in an effort to achieve specific teaching goals. In addressing RQ 2.2, two instances of teachers' resource systems for mathematics and for formative assessments and the associated resources are explored, respectively.

Table 9-2 shows the resource systems used by individual teachers for mathematics across the three schools. In this consideration, I refer to the digital resources such as Mangahigh, Mathsbox, Mathsloops, MathsWatch, Mathspad and Coberttmaths, which are specifically designed as resources for mathematics. These resources are structured and organised using iPads, desktop computers or IWB to enable the teachers to teach and students to engage with mathematics tasks in an organised manner. The mathematics-themed digital resources and applications are used together and organised by the teachers in order to achieve the intended teaching objectives. The resources which I considered as resources for doing mathematics are those that enable and support engagement with mathematics: GeoGebra, Calculator, Autograph and Desmos belong to this category of resources.

		Resource Systems for (Doing) Mathematics
A	Kitty	Mangahigh, Mathstrail, Khan Academy and iPad
	Emilia	Kings of Maths, Mathsbox, Keshmaths, Coberttmaths, Calculator and iPad
	Jimmy	GeoGebra, Autograph, Desmos, Mathsbox, Calculator m4ths, Mathspad, Coberttmaths, Studymaths. flashmaths, Dreambox, Mangahigh, Mathscentre, virtual manipulatives and iPad
	Jose	E-textbooks, Mr Barton Maths, iPad and MathsWatch
B	Gray	E-textbook, GeoGebra, iPad, Whiteboard maths, Douise.net, SRWhitehouse, Calculator
	Gavin	GeoGebra, MyMaths, Coberttmaths, Mathsloops,

		Mathsemporium and Calculator
C	Richelle	GeoGebra, Coberttmaths, Mathsbox, Nrich.maths, MathsWatch, E-textbook, iPad and Calculator

Table 9-2. Resource systems for mathematics

Table 9-2 shows evidence of the growth both availability of digital resources and teachers' confidence in their use. The range of interconnected non-digital and digital resources are organised in a systematic way, using the scheme of work and teacher professional expertise in undertaking their professional activities in and outside of the classroom. It is this ordered assembly of different resources by the teachers that enables the students to undertake mathematics tasks that constitute the resource system. For instance, Kitty's lesson on vectors (Figure 6-6, p. 133) employs GeoGebra software (displayed on IWB and students' iPads), and the tasks are scanned in from the Mangahigh website using iPads via QR codes posted on the wall. Further examples were given drawn from a worksheet downloaded from the Khan Academy collections. It is this structured and organised collection of digital resources, hardware, QR codes and worksheets aligned to the lesson goals on a vector that constitutes the resource system for engaging with mathematics, the set formed by all the resources used by the teacher. Such appropriated resources are stored and bookmarked into standardised and systematic resource collections (a personal bank of resources). This organised system of resources with the scheme of usage is known as a teacher *documentation system* (Gueudet & Trouche, 2012a).

This research also found that a teacher could have multiple resource systems for different educational goals. An example is the resource system for formative assessment.

Resource Systems for Formative Assessment

The evidence (10.1.2, p. 259) from this research suggest that iPads play a central role in formative assessment practices. The use of iPads was consistent with the findings by Beauchamp, Burden, and Abbinett (2015), who looked at teachers in Scotland and Wales learning to use the iPad in their instructional practices. The use of a variety of apps enables the teacher to control the sequence of assessment activities, provide instant feedback and keep records of learning attainment. Although different devices can do this, the added advantage of using iPads is that it becomes a 'digital hub' (digital hub in this study is conceived as an effective link from and through which students' activities with tasks and interactions with other students, teacher and learning resources are initiated, stored and shared) for different sets of resources organised in one space, accommodating videos, audio and the versatility of use across curricular and extracurricular activities.

Table 9-3 shows the resources that constitute each teachers' resource systems for formative assessment.

		Resource Systems for Formative Assessment
A	Kitty	Socrative, FrogOs, Diagnosticquestions and iPad
	Emilia	Code buster, Ttrockstars, Spreadsheet, Socrative and iPad
	Jimmy	Socrative, Diagnosticquestions, Plickers and iPad
	Jose	Socrative, Plickers, Spreadsheet, 10ticks, I know my class survey and an iPad.
B	Gray	Activinspire studio, Digital pen
	Gavin	Activinspire studio, Socrative, Digital pen, Diagnosticquestions
C	Richelle	Spreadsheet, iPad, and Resourceaholic

Table 9-3. Resource system for formative assessment

A variety of resources were identified that have the potential to enhance FA. The findings show that the set of digital and non-digital resources intentionally mapped onto the learning sequence for FA includes a resource system for that reason. The findings regarding the resource system for FA is further discussed in Chapter 10.

In spite of the growing evidence of the potential of digital and non-digital resources enabling formative assessment practices, the challenges of the 'complexity' (Monaghan, 2004) and the 'hiccups' (Clark-Wilson & Noss, 2015) arising from the integration of resources could be the catalysts for further professional development of teachers' assessment literacy using digital and non-digital resources.

The next section explores the question of tasks.

9.3 The Tasks

RQ 3.1 *Which tasks do mathematics teachers give to their students?*
RQ 3.2 *Where do these tasks come from?*
RQ 3.3 *Do they amend these tasks? If so, then how?*

In this section, I address the questions on mathematics tasks: the tasks teachers give to their students, selection and task modification. With regards to RQ 3.1, in the findings, I identified 15 distinct task-types presented in Chapter 4 (4.2.2, p. 95). These task-types are listed once more: starter, skill audit, definition, interactive, diagnostic, differentiation/variation, extension, multiple representations, modelling, consolidation, mixed ability, emergent, formative, peer-/self-assessment and plenary tasks. In the analysis of these tasks, I developed a four-part structuring framework (see 4.2.3, p.101 and 5.4, p. 113) for presenting these task-types in Chapters 6-8.

Watson et al. (2015) refer to tasks as anything that mathematics teachers use to demonstrate mathematics, to pursue mathematically meaningful interactions amongst students, or to ask students to do something. There are several ways the seven teachers of this research appropriate and modify tasks to fit into the lesson objectives. For instance, there are tasks for initiating the lessons, formative assessment tasks and extension tasks.

Regarding the sources of these tasks (RQ 3.2), all the mathematics teachers reported the tasks are usually drawn from a wide range of areas, as shown in Table 9-2. In school A, the teachers select, create and adapt tasks for their students' needs from the personal and collective banks of resources stored over the years of teaching and from online resources depositories. In school B, mathematics tasks are drawn from the *ActiveTeach* Pearson mathematics teaching package and a vast range of digital mathematics teaching resources. For school C, two sets of e-textbooks, personal and collective resources banks, and regional teacher networks are the critical sources for tasks, amongst others.

With regards to RQ3.3, the findings show that the seven mathematics teachers in the three schools actively select, create, modify and adapt tasks regularly. This practice of task modification is grounded in the teachers' belief that there is always the daily need to use, adapt and modify available resources to meet students' particular needs. *Documentational geneses* in the context of 'resource adaptation' and 'task modification' typically takes place among teachers working with resources. The mathematics teachers showed confidence in amending tasks and linking the tasks to the objectives of the National Curriculum using the scheme of work. The mathematics teachers in this research acknowledged the reality of task modification. For example, Kitty reported thus: "*for the majority of the time; I have to modify them. I might quite often have to use the snipping tool on computers to select and then create my worksheet from that"* (2IntK:#3). Jose too stated, *"I try to create my resources myself*" (1IntJs:#2.50). On other occasions, tasks are used in the way they were presented in the worksheet.

Tasks are given to students in modified forms, and the features of technology make it possible for teachers to engage in feedback from peers and responses from students. The availability of a wide range of task resources enabling teachers, a more extensive choice of tasks and the adoption of mastery teaching are some of the motives that encourage task modification. The role of digital tools and resources, and technological environment in the design of mathematical tasks and their execution have been identified and investigated (Clark-Wilson & Timotheus, 2013; John Monaghan & Luc Trouche, 2016) and the emerging role of practising teachers designing digitally mediated mathematics tasks has been acknowledged in the literature (Trgalová et al., 2018).

9.4 The Collectives

RQ 4.1 *Which collectives do mathematics teachers participate in?*
RQ 4.2 *Which features of these collectives offer opportunities for the evolution of communities of mathematical teaching practice?*

A collective is a group of individuals whose interactions are motivated by at least a common interest and who work together as a unit towards achieving a shared goal. There is a growing research interest in the collective perspective on mathematics teachers' work and interaction with resources, their use and transformation (Gueudet et al., 2013a; Pepin et al., 2013; Sabra & Trouche, 2017). Taken together, these authors argue that collectives play an essential role in teachers' documentation work, as well as the collectives providing teachers with resources and opportunities for professional learning.

The three schools considered in this research have dedicated mathematics departments with established professional learning groups where sharing resources and best practice is commonplace. There is an atmosphere that encourages communication, sharing of expertise and resources, and working together more collectively. There is also a 'school culture' that encourages collaborations and supports the use of non-digital and digital resources. For Fullan (2007), school culture can be defined as the guiding beliefs, perceptions, attitudes, rules and values that shape and influence everyday activity and is evident in the way a school operates. The school culture of collaboration and working together as members of the mathematics department is typical in England (school A6.1.1, p. 119, school B 7.1.1, p. 186 and school C 8.1.1, p. 218) and was observed in all three schools. Table 9-4 shows the various collectives teachers participate in.

Six out of the seven teachers take on specific leadership (Kitty, 6.2.1, p. 125, Emilia 6.3.1, p. 142, Jimmy, 6.4.1, p. 155, Gray, 7.2.1, p. 188, Gavin, 7.3.1, p. 201 and Richelle, 8.2.1, p. 219) roles relating to mathematics, whole-school leadership roles and roles relating to a group associated with the mathematics department. In school A, there is a regular TeachMeet that involves the teachers in the whole school meeting to share ideas, share behaviour management strategy and learn about the use of generic tools – for instance, Plickers and QR codes – that could be adapted to any subject area and support teaching and learning. Social media – Twitter and Facebook – were indicated by Gray, who states, "*Twitter is like a big mathematics staffroom*" (1intG: #22:25). The teachers see Twitter as an online mathematics community where their search for resources and teaching inspiration is always supported.

In school A the maths hubs group the teachers into a learning community across the region with mathematics is at the centre. In school B, there exist various groups – particularly, the Mathematics and English teachers' periodic meetings – set up to explore common concerns that impact on teaching and students' learning. In school C, the regional mathematics network of teachers serves a similar purpose in enabling teachers to meet, share ideas, share resources and organise exchanges and visits to each other's departments (int. ref).

	Teachers in Schools: A				**B**		**C**
Collectives	Kitty	Emilia	Jimmy	Jose	Gray	Gavin	Richelle
CPD	√	√	√	√	√	√	√
TeachMeet	√	√	√	√	√	√	
Twitter	√		√	√	√		√
Chinese Teacher	√		√				
Facebook			√				√
Department	√	√	√	√	√	√	√
Maths Hubs	√	√	√				
Maths/English					√	√	√
Maths Networks							√
Web Conference						√	
Group Leadership	√	√			√	√	√

Table 9-4. Mathematics teachers' collectives

These collectives of mathematics teachers, as shown in Table 9-4, interact (face-to-face and online) through resources that exist at several overlapping levels. In the analysis of the findings with regards to mathematics teachers' collectives, I identified five different ways (4.1.7, p. 86) of being in the teacher collectives: by context (institutional and prescribed); by access (open and voluntary); by mode of participation (face-to-face and online); by forms of organisation (formal and informal) and by geographical region (Yorkshire and Derbyshire). Collectives are discussed further in Chapter 10 (10.4, p. 280).

Krainer and Wood (2008) point to a collective as a *cultivated community of teachers*. In terms of the collective dimension of teacher documentation work, Gueudet and Trouche (2012a) argue that the processes through which the collective of teachers gather, create and share resources in order achieve a shared teaching goal are a form of *community documentational genesis* leading up to *community documentation*, which could lead to an emergence of a mathematics teachers' *community of documentation*. The evidence (school A 6.6182, school B 7.5, p. 213 and school C

8.3, p. 231) reveals that these consciously *cultivated* and sustained face-to-face and online *communities of documentation* exist across the schools and that mathematics teachers actively participate in them.

With regards to RQ 2.1, there are several features of these collectives that offer the potential for the evolution of a community of practice. Firstly, they successfully meet the terms of the three dimensions of a community of practice (Wenger, 1998): *mutual engagement* in the face-to-face and online collaborative practices; *joint enterprise* in developing best practice in engaging with mathematics using resources and having the performance of the students as one central goal; and a *shared repertoire* in the variety of shared resources collected over the years of teaching. Secondly, other features seem to constitute these groups as communities of practice, over time, like the departmental micro-culture of collaboratively designing the scheme of work (6.1.3, p. 122), regional initiatives and networks that bring teachers together periodically (p. 231), and the growing availability of digital resources, web conferences and social media platforms that enable online meetings (p. 207). My findings share similarities with those of Clark-Wilson (2017), whose large-scale multi-year study centred on transforming mathematics teaching with digital technologies set in the context of *Cornerstone Maths* (CM) and involved four mathematics teachers. Further implications of these findings are discussed in Chapter 10 (10.5, p. 284).

9.5 In Summary

In this chapter, I revisited the research questions organised around four themes. These themes include the questions around the mathematics teachers' appropriation of resources, and the resources teachers use and the associated resource systems for meeting specific teaching goals. The two other themes deal with the question of teachers' tasks. There are a variety of tasks used by teachers, drawn from a range of websites and banks of resources. These tasks are modified and reused over time. This chapter also addressed the theme of teachers' collectives, identified the ways in which teachers participate in the collectives and highlighted the features that appear to transform the collectives into communities of practice. The findings have the potential to contribute new insights into these areas of research and add to the growing literature on the opportunities and implications of teachers' interactions with digital and non-digital resources.

CHAPTER 10
DISCUSSION OF THREE KEY FINDINGS

In this chapter I discuss three findings that I consider have the potential to contribute to mathematics education research and suggest new insights into the mathematics teachers' practices. In the first part, section 10.1 to10.3, I explore a range of formative assessment (FA) practices among the mathematics teachers, their FA practices with digital resources, and the implications for classroom practice. In the second part, section 10.4 to 10.6, I identify and discuss five ways that mathematics teachers participate in the collectives through the idea of the community of practice. Finally, in section 10.7 to 10.9, I examine the mathematics teachers' divergent perceptions of variation and differentiation and highlight the specific ways teachers in this research enacted these ideas in their classrooms.

I now begin with the first part on formative assessment practices.

10.1 Formative Assessment Practices of English Secondary Mathematics Teachers as Supported by their Resource Systems

In this section, I discuss the findings regarding the mathematics teachers' formative assessment practices using the notion of resource systems as a window to explore these practices. The discussion is structured as follows: firstly, I provide a background on formative assessment as a unifying basis for the discussion (since formative assessment was unanticipated in the research); secondly, I examine teachers' resource systems for formative assessment; thirdly, I discuss students' responses and how teachers elicit these responses through a range of formative assessment practices (which include diagnostic assessment, self-assessment, students' peer assessment, group-based assessment and whole-class discourse) using a number of diverse digital and non-digital resources; and fourthly, I discuss teachers' formative feedback and how this informs the teachers' emergent lesson planning. I begin by presenting a unifying basis for discussing these findings with regards to formative assessment practices.

10.1.1 Unifying Basis for the Discussion

In the course of this research, I repeatedly observed formative assessment (henceforth referred to as FA) practices with digital and non-digital resources in lessons. The mathematics teachers also reported these formative assessment practices as a regular feature of their teaching strategy (evidence of these formative assessment practices is cited subsequently). Several studies claim that FA is an effective means of achieving the goal of high performance and providing learners with knowledge and skills for lifelong learning (Black & Wiliam, 1998; Heritage, 2018; Wiliam & Thompson, 2007).

Black and Wiliam (2009, p. 9) describe FA as follows:

> Practice in a classroom is formative to the extent that evidence about students' achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited.

This definition by Black and William takes into consideration the idea of feedback, evidence-based decision-making and the role of the three agents of FA (teachers, learners and their peers) together with the three key processes in teaching and learning described by Ramaprasad (1983):

- Establishing where the learners are in their learning
- Establishing where they are going
- Establishing what needs to be done to get them there

By convention, the teacher is considered as responsible for these three key processes of teaching and learning, but following the above definition by Black and Wiliam (2009), it becomes crucial to take into account the part the learners and their peers play in the FA processes. Since both teachers and learners jointly share the responsibility for learning in the mathematics classroom, FA is understood in this light as a joint process of decision-making towards learning that could change the way teachers plan, organise and deliver their lessons, and the way learners actively contribute to their learning. At the core of the FA processes lies the feedback, the intention of guiding the learning towards an intended goal and assisting learning activities. This is the reason why FA is also referred to as 'assessment for learning' (Heritage, 2018).

Wiliam and Thompson (2007) draw on the three FA processes advocated by Ramaprasad (1983) and the role of the three agents of FA (teachers, learners and their peers) to suggest five key strategies that underlie FA practices:

1. Clarifying and sharing learning intentions and criteria for success
2. Engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding
3. Providing feedback that moves learners forward
4. Activating students as instructional resources for one another
5. Activating students as the owners of their learning

This framework highlights the fundamental elements of effective FA practices. Within this view, the *instruction* (organisation of the learning environment to create learning*)*, *intention* (of the teachers in gathering the evidence, interpreting them and using the evidence) and *action* (in adjusting and matching their teaching to the need of the learners) are central to effective FA practices.

In the last decade, to develop effective systems for FA, different research efforts and projects have focused on the roles of digital and non-digital resources in fostering FA, especially in mathematics teaching and learning. For instance, the FaSMEd[165] European project is aimed at investigating the use of technology in formative assessment classroom practices in ways that allow teachers to respond to the emerging needs of low-achieving learners in mathematics and science so that they are better motivated in their learning of these important subjects (Wright, Clark, & Tiplady, 2015). Dalby and Swan (2019) report on the use of a specific resource, the iPad. They

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165 FaSMEd (Improving Progress through Formative Assessment in Science and Mathematics Education) https://microsites.ncl.ac.uk/fasmedtoolkit/

explore how iPads are used within formative assessment processes by six mathematics teachers and their classes in two comprehensive[166] secondary schools in the Midlands of England. Another example of technology for FA in the mathematics classroom is the so-called *connected classroom technology* (CCT), which refers to networked systems of personal computers and handheld devices designed to be used for interactive classroom practices (Cusi, Morselli, & Sabena, 2017; Shirley & Irving, 2015). The TI-Navigator is an example of a *networked graphing calculator system* in the CCT category. Clark-Wilson (2010) explores teachers' practices with the TI-Nspire Navigator and highlights the opportunities offered to teachers and students as they engage in a range of FA practices. She further reports that the use of TI-Nspire provides teachers with additional insights into their students' sense-making processes; promotes purposeful classroom discourse prompted by shared responses and screens; and helps develop strategies for students' peer- and self-assessment. My research appears to be the first study in the English context to report findings with regards to FA practices as investigated through the lens of the concept of the mathematics teachers' resource systems.

Against this background, the next subsections cover four areas. Firstly, I describe the teachers' resource system for formative assessment practices. Secondly, I discuss a 'question and discussion cycle' as a mechanism at the core of the five identified dominant (in terms of FA) ways by which teachers elicit responses and evidence of students' understanding, and then the five dominant FA practices are discussed, namely diagnostic assessment, self-assessment and student peer assessment, group-based assessment and whole-class discourse. Thirdly, I highlight the findings on teachers' formative assessment feedback. Finally, I discuss the emergent lesson planning as a consequence of the formative assessment practices with the use of digital and non-digital resources in the classroom. These findings were also

166 A comprehensive school is a school type in the United Kingdom. It is a school for secondary aged (11 to 16 or 18) children that does not select its intake on the basis of academic achievement or aptitude, in contrast to the selective school system where admission is restricted on the basis of selection criteria. It takes children of all abilities and provides a wide range of secondary education for most of the children.

triangulated with the associated lesson observation, interviews and screen capture datasets (see subsection 4.1.9, p. 92).

I now turn to describe my findings regarding the mathematics teachers' resource systems for formative assessment practices. The discussion on resource system sets the scene for understanding how the concept of the resource system is used as a lens to explore the teachers' formative assessment practices with emphasis on the digital and non-digital resources used.

10.1.2 Resource Systems for Formative Assessment

My first findings with regards to the FA practices shows that teachers create a *living resource system* for their FA practices. This idea of a living resource systems corresponds to how Trouche, Gueudet, et al. (2018, p. 17) describe resource system as a "living entity". The notion of living resource systems highlights the teachers' reported practise of continually updating, deleting, adding and reorganising their resources as the needs arise. The evidence shows (see Kitty, p. 138; Emilia, p.150; Jimmy, p. 163; Jose, p. 177; Gray, p. 195; Gavin, p. 207 and Richelle, p. 226) that mathematics teachers build up a set of resources and engage in resource orchestration (Trouche, 2004): that is, the teachers' intentional and systematic organisation and use of resources for achieving specific learning goals, and in this context for FA practices. This is consistent with what has been found in the previous study as Gueudet and Trouche (2009, p. 205) state, "A resource is never isolated; it belongs to a set of resources". In the context of teachers in France as reported by Gueudet and Trouche, the teachers' activities were built around a central software resource 'Mathenpoche', in contrast with my research set in the context of teachers in England where several resources were considered. My finding, therefore, throws new light on the differences in practice among teachers in France and in England in their use of resources. The construct of resource systems is an evolving construct from two contemporary theoretical frameworks: Structuring Features of Classroom Practice (SFCP; (Ruthven, 2009) and the Documentational Approach to Didactics (Gueudet & Trouche, 2009). In the context of DAD as discussed in the literature (Chapter 3), the *resource systems* (the visible part of the documentation system) is a set of resources organised towards accomplishing an intended goal or set of goals (Gueudet & Trouche, 2009; Ruthven, 2018). The idea of a resource system in this sense is expansive and considers individual teachers over the whole span of their professional activity. However, for Ruthven (2009), a resource system refers specifically to the diverse mathematical resources and curriculum materials in use in the classroom, especially in the way these resources and materials are organised and made functional by the teacher for a particular focus. Even though the ideas of resource systems differ considerably, there are overlaps as well. In this discussion, I subscribe to Ruthven's idea of resource systems, since my research focuses on current activity of mathematics teacher in the classroom context. In particular, teachers in this research develop and use resource systems for FA practices that have become an integral aspect (pp. 119, 186 and 219) of their teaching strategies.

Table 10-1 below shows the range of resources (these resources have been described in the various case study reports: Table 6-1, Table 6-4, Table 7-2 and Table 8-1) in the teachers' resource systems used for FA practices. Teachers in school A, a lead school in the region and at the forefront of promoting the integration of technologies in the mathematics classroom, have a more extensive range of resources in use.

		Teachers' Resource Systems for FA
A	Kitty	Socrative, FrogOs, Diagnostic questions[167], iPad
	Emilia	Code buster, Ttrockstars, Spreadsheet, iPads
	Jimmy	Socrative, diagnostic questions, Plickers, iPad, QR codes and colour-coded calling cards
	Jose	Socrative, Plickers, spreadsheets, 10ticks, iKnowMyClass survey and iPads
B	Gray	Activinspire studio, Digital pen, iPad, iPhone
	Gavin	Activinspire studio, Digital pen, Diagnostic questions
C	Richelle	Spreadsheet, worksheet and iPad

Table 10-1. Teachers' resource systems for FA

[167] 'Diagnostic questions' is a web-based assessment tool that provides insights into student understanding in a fraction of the time and shows how each student's answer reveals a specific, and different mistake or misconception (https://diagnosticquestions.com/).

Using hardware like iPads and software/applications like Socrative and diagnostic questions are mandatory for teachers in school A. Schools B and C also have several digital and non-digital resources in use, as shown in Table 10-1 above. The table shows the range of resources that constitute each teacher's resource system for FA practices. One possible explanation for the increase in the number of resources teachers use is that there appears to be growing confidence and competence in the use of digital resources in their lessons and the readiness to take on the challenges of integration of resources into their teaching.

The Scheme of Utilisation and Usages

I provide here further evidence for the existence of such resource systems, *the scheme of utilisation* and *usages* of this resource system for FA by highlighting Jimmy's case (see subsection 6.5.1.3, p. 175) as an 'organising case study' when discussing the findings. I chose Jimmy's case because his two-hour lesson cycle on the topic of fractions, in my opinion, provides a sense of completeness for discussing the case. As one aim of the research is to gain further insight into the mathematics teachers' classroom practices, what follows is a description and discussion of one teacher's FA practices as reported by the teacher and as documented during the lesson observations. As discussed in the literature in subsection 2.2.3, p. 29, the *scheme of utilisation* of a set of resources has both an observable part and an invisible aspect. A scheme is comprised of goals (for example, Jimmy's lesson on *converting decimals into fractions* on 6.4.2.1, p. 156); subgoals (the idea of *improper fractions*); anticipation; rules of action; and information gathering (the FA practices). The invisible aspect of a scheme refers to the *operational invariant*: in this context, the cognitive structure guiding Jimmy's FA practices with resources, a sort of mental map by which he orchestrates the various features in his resources system. The observable part matches the regularities of Jimmy's FA practices with resources for the same class of situations through different contexts. My evidence for the regularity is that this practice was observed in six out of eight of Jimmy's lessons and was also seen in one screen capture demonstration. The six other teachers in this study reported similar use of a set of resources for FA, as shown in Table 10-1, p. 260. In terms of *utilisation* and *usages*, *utilisation* refers to when a teacher draws on a resource one or more times without developing a stable behaviour for a given class of situations, while *usages* indicate a stable organisation of activity. The analysis of my observation datasets and the evidence mentioned above permit me to propose a reconstruction and offer a possible explanation for Jimmy's stable organisation of FA practices with resources. It is worth repeating the description of Jimmy's FA practices for clarity here:

- Jimmy's lesson objective: Convert decimals into fractions
- The students get an iPad each and scan in designated sets of tasks from select banks of questions via a QR code. This provides students with links to the mathematics tasks that instantly pop up on their iPad devices.
- The students work on the tasks (individually for five minutes)
- The teacher, using the Plickers app on the iPad, scans the students' responses from customised cards, and collates the results in real-time (e-analysed, and colour-coded on a spreadsheet).
- Then, the e-analysed, anonymised and colour-coded whole-class performance are displayed on the interactive whiteboard (IWB), which triggers class discussion, and formative intervention. Extension tasks are given to students in pairs and tasks are done on mini-whiteboards while the teacher moves around the class. This lasts for about 10 minutes.
- Nominated/volunteer students share their solutions and peer-checking takes place.
- The students' e-analysed responses and mini-whiteboard feedback inform the teacher's 'emergent lesson planning' (see p. 161 and Figure 7-19).

Jimmy's resources system consists of a set of digital/non-digital resources. These are categorised into four groups with regards to their function during the FA processes:

- Visualisation resources: IWB and iPads
- Data-capture tools: QR codes, Plickers
- Data-handling and display: spreadsheets
- Resource banks

With regards to the visualisation resources, the departmental heads in the three schools in the study recommended the use of IWB and laptops as a matter of policy, and in all the classrooms there is a whiteboard adjacent to the IWB supplemented by students’ handheld mini-whiteboards. The four teachers (Jimmy inclusive) in school A are encouraged to use the class sets of iPads with the laptops and IWB as the central visualisation and presentation resources to enhance their teaching and students’ learning. My finding is broadly consistent with the previous report by De Vita, Verschaffel, and Elen (2017) on the potential of IWB to stimulate mathematics learning through a range of interactive features, but it goes beyond this report to highlight the fact that the IWB is a catalyst that enables the sharing, display and re-use of other mathematical resources and belongs in a system of resources orchestrated by the teachers for FA purposes. Several researchers have pointed out that the IWB has become the central resource for teachers in most of the secondary schools in England, especially in supporting classroom dialogues and mathematics learning (De Vita et al., 2017; Hennessy, 2011; Umameh, 2012). My finding throws new light on the aspect of IWB being a part of connected resources systems that enable teachers to work towards a set goal, and how the IWB, alongside other resources in the resource system, enhances FA practice. The teachers in my research reported (as shown in the table of resources mentioned previously and also documented in my observational notes) that they rely on the features of IWB (*drag and drop; hide and reveal; highlighting and animation*) that seem to enhance their FA practices. Through these features, learners are afforded a 'global view' of an anonymised whole-class performance. This expands the role of IWB as a catalyst in helping to trigger whole-class discussion about particular mathematics concepts and ideas that the class is aiming to achieve mastery over. In all the lessons observed, the IWB forms the space from which most classroom discussion begins. Six out of the seven teachers reported that IWB use supports the idea of IWB as a 'collective shared learning space' where a 'social production of knowledge' takes place. With FA practices, the IWB display of e-analysed students’ responses revealed the students seem to experience heightened interest and the tendency to participate actively in the classroom discussion. As one of the teachers (Emilia), emphasising on how relevant the IWB is to her practice, stated, “*I think definitely I don’t know what I will do without the IWB*". In a survey, Glover, Miller, Averis, and Door (2005) shows similar findings that IWBs can enable teachers to stimulate and motivate students to discuss their mathematical thinking and problem-solving openly.

In summary, I have argued here that the evidence from the data shows that the use of IWB seems to promote classroom dialogue, teachers' immediate formative intervention and visualisation of FA data around mathematics teaching and learning.

With regards to the use of iPads in FA practices, six out of seven (see Table 10-1, p. 260) of the teachers engage with iPads. One of the six teachers alternates between his mobile phone (iPhone) and the iPad since they have similar shared features. The difference between the schools is that in school A the students have individual access to iPads while in school B and C, the iPad was solely in the hands of the teachers for formative assessment processes. Evidence for this exists, for instance, in the case study subsections for Kitty (p.124), Jimmy (p. 154) and Jose (p. 168). This finding is consistent with the recent literature by Dalby and Swan (2019) on the growing use of iPads within formative assessment processes. My finding with regards to the use of iPads in FA processes demonstrates three things. First, the iPads are used by students in school A to access the designated mathematics tasks via a QR code. Secondly, the students engage with the tasks on their iPads, and thirdly, the teacher uses Plickers to scan in the students' responses and automatically e-analyse and colour-code them. In doing this, I believe teachers are allowing students to take ownership of their learning and the chain of learning processes, including the formative aspect of mathematics learning. With the use of QR codes, the student can access various mathematics tasks with different levels of difficulty differentiated into gold, silver or bronze (subsection 6.4.2.2, p. 158). The evidence of the differentiated tasks exists as reported on pages 132, 146, 173 and 206. (The teachers' understandings of 'differentiation' noted in the interview datasets and classroom observation notes, differ widely, and this will be discussed in the subsequent section in this chapter.) With the use of iPads and QR codes and access to banks of resources (diagnostic questions, teachers' banks of resources, and use of Socrative), the mathematics teachers in school were able to assign differentiated tasks to an individual, a pair or the whole class of students as suitable. In several of the lessons (see subsections: 6.2.2, p. 127; 6.4.2, p. 156; 6.5.1, p. 169), the students chose a starting point for themselves where they think they are in their learning and progressively work towards where they want to be, with the teacher providing timely interventions. My finding regarding teachers targeting appropriate differentiated support to students confirms what has been in reported previous research by Clark-Wilson (2010); however, the evidence (for instance, subsections 6.4.2.3, p. 159; 7.2.2.3, p. 193) from my finding highlights the role of students' self-assessment and how they choose a starting point for themselves in engaging with the tasks.

I have discussed in this subsection how the resources system, especially concerning iPad use, provides opportunities for students to take ownership of their learning, offer teachers occasions for task differentiation and offer real-time feedback to students during the lesson.

10.1.3 Students' Responses and Teachers' Formative Feedback

In this section I discuss how the mathematics teachers in this research elicit evidence of students' understanding and progress or do not (Black & Wiliam, 2009); establish where they are in their learning (Ramaprasad, 1983); and provide feedback that moves that learning forward (Wiliam & Thompson, 2007). The analysis (the associated evidence will be presented subsequently) looks at the evolving instructional practices with regards to FA that the mathematics teachers developed using what I consider here as their resource systems. This revealed the following themes, which the mathematics teachers reported that their sets of resources (resource systems) enabled:

- developing a 'question and discussion cycle'
- undertaking a range of formative assessment practices in the classroom, such as diagnostic assessment, self-assessment and peer assessment, and group-based assessment and whole-class discourse
- provision of teachers' formative feedback
- 'emergent lesson planning' informed by students' responses

The subsequent subsections focus on these four themes with the associated sub-headings. I now discuss them in turn.

10.1.4 Developing a ‘Question and Discussion Cycle’

The findings reported below shed new light on how the mathematics teachers involved in this research develop a 'question and discussion cycle' and integrate a range of FA practices (which includes diagnostic assessment, self-assessment, student peer assessment, group-based assessment and whole-class discourse) into the teaching and learning activities. Teachers' use of their resource systems has afforded viable opportunities for the students to switch when needed from working individually, in pairs, or in small groups to working as a whole class. I begin with three actions: firstly, discussing what I refer to here as the 'question and discussion cycle'; secondly, throwing more light on the teacher-reported FA practices; and thirdly, highlighting the implication of these findings for the classroom and teachers’ professional practices.

The ‘question and discussion cycle’ refers to the teachers’ reported and observed practice where there is a built-in iteration of teacher-student dialogue, mostly guided by the teachers. All the teachers in school A indicated that this is an adapted aspect of the gradual release of responsibility (GRR) model (Fisher & Frey, 2013) of class routine promoted by the department as previously highlighted on page 94. The strategy is framed thus: *I do* (learn from me); *We do* (together as a class); *We do* (work as a group); *We do* (work in pairs), and *You do* (succeed on your own). Within this framework lies the intermittent insertion of the ‘question and discussion cycle’ as a focusing tool for undertaking a specific FA goal as the lesson sequence unfolds (see Kitty, p. 137; Jose, p. 175 and Richelle, Figure 7-2, p. 190). The question and discussion cycle was used to elicit FA data (as in the case of all the teachers, especially see pp. 148 and 159) and to prompt students to verbalise their mathematical thinking or provoke a whole-class discourse. A related finding on how questioning is used in FA practice has been reported by Clark-Wilson (2010) in her exploration of emergent teaching practices following the introduction of a TI-Nspire networked system in the classroom.

Students also call for teachers’ intervention and discussion using the ‘traffic light coded device’ (Figure 6-1, p. 122). Students use this 'traffic light device' as a means of communicating their own assessment of their understanding of an ongoing task to the teacher and their peers. In all the schools the teachers retained the traditional call for attention of the student raising their hand; however, there was also the unexpected (to me) observation of students raising and showing their work on the handheld mini-whiteboards or the teacher taking up a couple of the mini-whiteboards as a means to start discussion or compare the variety of methods used by students while engaging with a specific task.

The general idea of using question and answer as part of the instructional activity is not new (Reay, Li, & Bao, 2008). What appears new in my finding is how the various forms of digital and non-digital resources that constitute the teachers' resource systems help to create, facilitate and extend what seems like a well-structured and coherent classroom environment. Enabled by the resource systems and internet links to online repositories of tasks, the teachers could instantly retrieve and present a range of differentiated tasks to the students. The access to short video clips through *MathsWatch* enabled Jimmy, it appears, to refocus the question and discussion, which ordinarily would not have been possible in the absence of these digital resources.

> I have become so used to this technology, so used to this ability to delve into a topic in so much more detail, with the dream tools, virtual manipulatives. I can zoom in on a number line; ***I could never do that before***. I could only get a whole number line and draw on the floor; think about getting 30 kids around it, ***you couldn't do that***. I think it would be absolute chaos and the students would not get as much out of it. It can be incredibly powerful. (1intJ: #35)

In the context of the definition of FA by Black and Wiliam (2009) and the first of the three key processes of teaching proposed by Ramaprasad (1983), it is my understanding that the 'question and discussion cycle' constitutes a key mechanism by which teachers in this research establish where students are in their learning and elicit evidence about students' achievements towards a formative intervention. The import of the 'question and discussion cycle', it appears to me, is that while it is essential for teachers that students get the right answer to a task, the development of a sort of 'habit of the mind' is much more important: a heuristic, that is applicable in a wide range of related mathematics contexts. For instance, students are given the space at first to discover a solution to a task independently, and then if they are stuck, to ask another student, and then they compare their methods. The teacher was always a last resort when all these options fail. In addition, I argue that the ‘question and discussion cycle’ is at the core of the reported range of FA practices in the classrooms observed.

In the next section, I discuss my findings on the range of teacher-reported FA practices.

10.2 A Range of Formative Assessment Practices in the Classroom

Having established that the ‘question and discussion cycle’ device is an essential aspect of the teachers’ reported FA practices, this section considers the reported and observed ways the teachers in this research elicit students’ responses towards providing formative feedback. This research (in terms of FA) identifies five dominant ways by which teachers elicit responses and evidence of students’ understanding: namely, through diagnostic assessment, self-assessment, peer assessment, group-based assessment and whole-class discourse. It also discusses the role of the sets of digital and non-digital resources. Findings on the diverse approaches to FA practices enabled by technology are still recent and growing. While previous studies like those of Clark-Wilson (2010) and Swan and Foster (2018) lay emphasis on self- and peer-assessment practices in the classroom, the results of this research confirm and extend their findings by highlighting the growing range of innovative FA practices (like the diagnostic assessment, group-based assessment and whole-class discourse) and the roles of the teachers’ resource systems in the context of schools in England.

10.2.1 Diagnostic Assessment

The findings concerning the range of FA practices among teachers in this research appear more complex and varied than anticipated. There are several pieces of evidence to show this. With respect to *diagnostic assessment* practice, it seems this approach to FA is used to identify individual strengths in ‘generic aspects of mathematics’ (such as types of number, multiplication/division or forms in which quadrilaterals could exist) and students' readiness for the following mathematics topic (see tasks initiating lessons, pp. 143, 156 and 190). Diagnostic assessment, as reported by the teachers in this research, was also used as a means of identifying that wrong answers pinpoint a specific misconception and inform teachers' awareness. In school A (as shown in these tables: Table 6-1, p. 138; Table 6-4, p. 177 and Table 7-2, p. 209), the diagnostic questions[168], Socrative[169] and Ttrockstars[170] apps were the commonly used digital resources for the diagnostic assessment practices and in school B, the diagnostic questions and ActiveLearn resources were used. This diagnostic assessment practice was not reported in school C. Even though the six teachers identified using diagnostic assessment practices used a range of digital resources, three themes emerged in my analysis (subsection 4.2.3, p. 101): diagnostic assessment practice is a part of the activities initiating the lessons; the practice is technology-driven; and that diagnostic questions focus on the 'generic aspect of mathematics' that students are presumed to have learnt previously. For instance, in terms of the order of mathematical operation: how to engage with the tasks in the correct order is shown in Figure 6-4, p. 131, and also highlighting the difference between rational and irrational numbers (Figure 6-13, p. 146). Kitty refers to the diagnostic assessment as a "quick skill test". Out of the 61 lessons observed (Table 4-5, p. 82), 30 instances of diagnostic assessment practices were seen. A likely explanation for the frequency of this practice in the classroom is that the teachers found it a useful method for supporting their students' learning and for "pitching" the lesson right, as Richelle claimed (p. 218).

In a previous study, Kemp and Scaife (2012) argue that the term diagnostic assessment has declined in use in recent educational discourse. On the contrary, my findings point instead to a renewed and growing interest in diagnostic assessment, as the evidence shows. This is possibly as a result of the availability of a range of digital resources like the web-based assessment tools (diagnostic questions, Socrative, 10ticks and Ttrockstars) and the opportunities these digital resources offer to teachers and students in their teaching and learning, respectively. Over the years too, there has been

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168 https://diagnosticquestions.com/
169 https://socrative.com/
170 https://ttrockstars.com/

inconsistency in the understanding of the term diagnostic assessment. For Kemp and Scaife, diagnostic assessment is a distinct concept different from FA, while Black, Harrison, and Lee (2003) argue that diagnostic assessment is an aspect of FA. My understanding of diagnostic assessment aligns with those of Black, Harrison, and Lee and goes further to highlight the nature of the diagnostic assessment practice as enhanced by digital resources. FA assesses students' achievements in the mathematics topic that the teacher has been teaching and looks forward to considering the next steps in making it 'better', while diagnostic assessment looks back to assess the students' current understanding of previously learnt mathematical units and seeks to remedy any residual misconceptions in such a way as not to interfere with subsequent learning.

I now turn to consider the finding with regards to students' self-assessment and peer assessment practices.

10.2.2 Self-Assessment and Peer Assessment Practices

The findings of this research also contribute to the current literature on self- and peer-assessments by highlighting the evolving varieties of self- and peer-assessment practices enabled by the resource systems among selected mathematics teachers in England. The findings confirm those of Clark-Wilson (2010) and Cusi et al. (2017) which, taken together, suggest that the use of technologies could support the development and promotion of strategies for students' self- and peer-assessment practices in the mathematics classroom.

Within the context of the five key strategies of FA suggested by Wiliam and Thompson (2007), self-assessment (SA) approximates to activating students as the owners of their learning and peer-assessment (PA) to activating students as instructional resources for one another. Several teachers reported instances of SA and PA that illustrate both forms of assessment practices. All of the teachers cited at least one SA and PA strategy including ones like 'traffic lights' (p. 122); 'timed mathematics tasks' (p. 146); 'single and multiplayer maths games'(p.150); 'students personalised QR coded cards'(p. 159); 'guided self-assessment chart' (p. 194); and the use of handheld mini-whiteboards and several mathematics applications with facilities that enable SA and PA practices.

In the traffic light strategies (and similar devices using the tags gold, silver and bronze), red, yellow and green coloured cards are pasted in the students' workbooks. The students are asked to display these cards at key points in the lesson to indicate where they are in their understanding. (Green implies – *I understand fully*. *I am okay without help*; yellow – *I'm not quite sure. I need a little help. And I have asked the person next to me*, and red – *I' m stuck. I need some extra help*.) Similarly, the 'guided self-assessment chart' has a catalogue of 'starter' statements that students are expected to complete as a way of indicating their level of understanding, such as *I have been able to… but I got stuck on…*; or *I did not know how to… but now I can….* In addition, teachers also selected students to share their understanding with a fellow student or the whole class using their mini-whiteboard. The teachers reported (see demonstrating mathematics learning, pp. 159, 175 and 193) that using one or more of these strategies facilitated the quality of mathematical discourse (discussion about mathematics in a manner that articulates students' understanding of concepts) in the lessons. The iPads have played a central role in the evolving SA and PA practices, as previously discussed in this chapter. The capacity to gather and process data on a large scale and at speed empowers both teachers and students and increase their opportunities for varying the assessment possibilities. From my understanding, the peer assessment was used to confirm a peer's working on a task, suggest alternate methods, correct errors or call out to the teachers when both peers are stuck. Whatever the outcome of the peer-assessment of each other's work, the central purpose is served of supporting each other to take the learning forward.

By using this variety of strategies supported by digital resources, the teachers reported that they promote engagements (Kitty, p. 139 and Emilia, p. 152), make learning incredibly engaging, enhance learning and make it much deeper (Jimmy, p. 164), and enable teachers to identify weakness and provide more specific interventions (Gray, p. 198). This finding is consistent with the results of a study (Andersson & Palm, 2018) which indicates that there is ample empirical evidence to suggest that SA and PA strategies have the power to enhance student achievements and also the use of technologies (like iPads, plickers, QR codes etc.) could multiply the opportunities for enhancing students' learning (Dalby & Swan, 2019). My findings lead to similar conclusions and underline the specific practices among selected teachers in England. While this research was not designed to seek evidence for the effect of SA and PA practices on students' achievements in the mathematics lesson, anecdotal evidence exists (for two examples, subsections p. 161 and 8.2.2.4, p. 225) from teachers' reports that point to productive learning engagements and participation in the classroom.

10.2.3 Group-Based Assessment and Whole-Class Discourse

This research provides evidence that the selected mathematics teachers in this research have embraced group-based assessment practice and whole-class discourse as part of their repertoire of FA practices. This practice, in my estimation, is driven by the nature of the schemes of work in use and the features of the various sets of digital and non-digital resources available in the class and to the teachers. These schemes of work suggest a list of resources that could support the teachers in the group-based assessment practice and whole-class discourse. There is a substantial amount of evidence in the finding to make these claims. In school A, students are put into 'mixed ability groups' (subsection 6.1.2, p. 121), and the adopted classroom tasks routine also recommend working in small groups (Figure 6-2, p. 128). Students are also grouped by the level of difficulty of the tasks (gold, silver and bronze) and teachers' own specific grouping. In school B, the scheme of work associated with *ActiveLearn* resources used by the two teachers categorised students into three tiers (Pi-lower, Theta 2-middle and Delta 2-higher) of ability groups (subsection 7.1.1, p. 186). A teacher in school B (Gavin) stated, "*The new curriculum is encouraging us… to teach to the ability*". The teacher in school C, even though she did not report any form of group-based assessment practice, said, "*I want to make sure I was pitching my lesson correctly to the right ability*". Although the contexts of schools A and B encouraged teaching in groups and the consequent group-based assessment, this practice was not always observed in all the lessons.

In agreement with previous studies (Swan, 2005; Weurlander, Söderberg, Scheja, Hult, & Wernerson, 2012), the contributions of my findings on group-based assessment and whole-class discourse in mathematics have the potential of providing necessary reinforcement for the students in their learning. These findings also provide the teachers with insights into their students' understanding of the mathematics tasks and also acted as tools to facilitate collaborative learning where students strive to explain and justify their mathematical thinking to each other. Given the time-consuming nature of SA and PA (in terms of providing detailed formative feedback to individual students in a class of 25-30), group-based assessment and whole-class discourse could afford the teachers opportunities to circulate, listen and elicit responses from groups of students that may be used to address specific misconceptions or/and to refocus teaching. The teachers e-analysed students' responses (Jimmy, p.159 and Gavin, p. 211) in order to trigger whole-class discourse, formative intervention and re-teaching. The grouping of students appears to provide a secure setting (being among a small group of classmates) in which students across the various ability groups can feel confident enough to contribute to the discussion. Through the group-based assessment, therefore, students can compare their understanding and knowledge with the other students, and this, arguably, gives them insight into *how they are doing in their learning, how their friends are getting on and where they need to get to*, as Jose stated in one of the interviews (p. 178). These insights could stimulate self-reflection on their own learning and help students become more engaged and motivated.

This research reveals two ways through which whole-class discourse takes place. Firstly, whole-class discourse takes place after the work in small groups, usually towards the end of the lesson (Table 4-4, p. 80 and p. 175). Secondly, it also happens midway during a period of group work or work in pairs when something arises that requires the attention of the whole class (see pp. 161, 176 and 224). In the whole-class discourse, students present and report on the work done. The teachers ask a representative from the group to describe the tasks they have undertaken and the answer they have obtained and then guide the student to use the mathematically appropriate vocabulary. Through this practice, the students fine-tune their mathematical thinking through appropriate language use. The teachers facilitate mathematical ideas by guiding students in the way they used mathematical vocabularies to express themselves and check the logic of their presentations. In the whole-class discourse session, the teacher invites students to show their working on the chalkboard or to explain them from the mini-white board, highlighting for the entire class the variety of methods available for engaging with a particular task. For instance, Kitty summed up the various ways to represent a fraction (p. 132) and then the students' ideas were compared and evaluated by the whole class. In this way, the teacher uses the various student-generated approaches from the group work to facilitate the whole-class discourse towards advancing critical mathematical ideas. In a previous study, Stein, Engle, Smith, and Hughes (2008) report a similar finding, that a teacher-orchestrated whole-class discourse using the students' responses could help students clear up a misunderstanding and draw new connections between mathematical ideas reflected in the various approaches and representations that they use. In my research, teachers purposively select a common area of difficulty for most of the students using the e-analysed whole-class responses (pp. 161 and 211). The use of digital and non-digital resources (IWB, visualisers, YouTube, e-analysed students' responses on display, iPads) bring an added advantage, since the teacher can have a 'global and specific feel' of the error the students are making and sequence the whole-class discourse in such a way as to enable them to clear up the basis of the misconception.

In the whole-class discourse, the teacher functioned as a *moderator*. In this role, the teachers recognise and value points of emphasis and highlight for the whole class a crucial moment in the students' working, thereby alerting them of a possible error. The teachers moderated the flow of the discourse (pp. 137, 193 and 224) allowing each student an opportunity to participate, stopping others from interrupting when a student makes a presentation, and supporting the students to clarify their mathematics ideas using their own words. Previous research acknowledged that making students' thinking public could enable the teacher to guide the students in a more mathematically sound direction (Stein et al., 2008). Also, by moderating and guiding the whole-class discourse, teachers create a classroom norm that allows the students to feel that their contributions are listened to and valued as useful instructional resources for each other (Wiliam & Thompson, 2007; Yackel & Cobb, 1996). For instance, Gavin gave a series of leading questions to guide the discourse. (Write a list of all the new vocabulary you have used. Write in your own words, a definition for each. Compare your definition with your classmates, p. 207.) My finding with regards to the whole-class discourse resonates with those of Radford (2016), who argues that whole-class discourse is about communication that actively engages students' ownership and their mechanisms of knowledge production, allowing them to construct new meanings and understandings of mathematics for themselves.

Another finding of this research with regards to group-based assessment and whole-class discourse is in line with and supports the idea that new technology can make students more active in the class discussion and enable the teachers to monitor the whole-class progress instantly and provide appropriate intervention at speed that is not possible without the aid of digital technology (Cusi et al., 2017). In the specific case of this research, the teachers' resource systems offer such possibilities to the teachers. The use of these sets of digital resources simultaneously provides real-time FA and anonymity to students. The capacity of the technology to instantly analyse and provide the teacher with an overview of whole-class and individual performance informs students on where they are in their learning and can drive productive discourse in the hope of taking the learning forward. The availability of the resource systems, the findings suggest, could enhance the teachers' efforts in line with the ideas of Wiliam and Thompson (2007) in engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding.

10.2.4 The Provision of Teachers' Formative Feedback

This research has shown that teachers provided real-time formative feedback and corrections to students' work during their lessons. The teachers' use of digital resources also appears to have the effect of enhancing the provision of immediate formative feedback and impact on the way teachers teach. These findings correspond in part with the results of a previous review (Hattie & Timperley, 2007) and add new insights to the literature. Hattie and Timperley emphasised that the provision of feedback is one of the most powerful influences on learning and achievement. Similarly, the teachers in my research, using the information elicited from the students and aided by digital resources, sought to support the growth of students' understanding and move the students forward in their learning following the three steps considered below.

Firstly, encouraging increased effort, motivation and promoting engagements (as discussed in subsection 10.1.4, p. 266, through mechanisms such as the ‘question and discussion cycles’), the range of formative assessment practices and the use of digital resources during the lessons appear to foster attitudes that are relevant for learning. Central to formative feedback is the information elicited from students, and its effectiveness in moving the students’ learning forward depends on how timely and appropriate the intervention is and whether students act on it or not. It would, of course, be naive to assume that all kinds of feedback to students about their work are equally effective. The effectiveness depends on the skills of the teachers in making relevant interpretations of the information, teaching skills and the ability to incorporate this information confidently in the complex mix of the lesson. Additionally, all the teachers made the learning objectives/success criteria (for instance, Gray on page 190) explicit by writing them on the board at the beginning of the lesson. This has the possibility of encouraging students to increased their effort and thereby achieving success.

Secondly, the teachers offer verbal corrections and validate students’ responses in the course of engaging with a given task. The analysis of the SCAN datasets and the evidence of the high frequency of teachers coaching (Co), teachers facilitating mathematics ideas (Fi) and teachers explaining mathematics ideas (Ei) are taken together as aspects of teachers' formative feedback (p. 80). These provide evidence for this finding. In the UK, policy requires that teachers give pupils regular feedback, both orally and through accurate marking (Micklewright et al., 2014). Previous research suggests that oral feedback is more effective than written feedback (Boulet, Simard, & De Melo, 1990). This practice of providing real-time verbal correctives and validation of students' solutions in real time is enabled by the use of digital resources like iPads, Plickers and Socrative as previously discussed. It is possible that oral feedback might not always be the most appropriate strategy and may need to be supplemented by other formative feedback strategies.

Thirdly, teachers provide formative feedback by indicating examples and providing alternative strategies to finding a solution to the task(s): for instance, in the lessons on fractions across various schools and teachers. In Kitty's lesson on fractions, she gave alternative strategies, and multiple ways of representing fractions (p. 134); in Emilia's lesson bar modelling was used (p. 148); in Jose's lesson, the number line was provided (p. 173); Gray used YouTube video examples (p. 188); and in Richelle's case, she provided the students with extension tasks (p. 224).

Teachers identified that formative feedback works on two levels: firstly, if appropriate and provided in a timely fashion, it has the potential to move students' learning forward; and secondly, formative feedback may require the teachers to make a substantial change to their teaching itself. While most of the research on formative feedback focuses mainly on its impact and effectiveness with regard to how students improved their learning, my research highlights how it informs the teachers in making substantial modification in their teaching strategy.

It is to this resultant 'emergent lesson planning' that I now turn.

10.3 Emergent Lesson Planning Informed by Students' Responses

Another finding of this research suggests that formative feedback, besides moving the students' learning forward, also gave rise to an 'emergent' (in the lesson) lesson planning afforded by access to multiple digital and non-digital resources. Bennett (2011) points out that the centrality of formative assessment (and associated formative feedback) becomes clear when we consider the distinctions between errors, slips, misconceptions, and lack of understanding. Each of these (errors, slips, misconceptions or lack of understanding) entails a different instructional action on the part of the teachers: from reminder (for errors) and minimal feedback (for slips) to re-teaching (for lack of understanding) and the significant investment required to engineer a deeper cognitive shift (for misconceptions). The teachers did not explicitly make any conceptual distinction amongst errors, slips, misconceptions or lack of understanding but one could infer from the lesson observations that the teachers addressed errors and slips by activating the students as instructional resources for one another. For example, teachers asked students to (i) peer review each other's work, (p. 137); (ii) work in pairs, (pp. 159, 193); and (iii) compare strategies and solutions with each other (p. 207). This remediation is done by engaging the students to work in pairs and groups. Teachers appeared also to address lack of understanding by reteaching and providing further explanations, by providing examples or by asking students to give reasons and justifications for the tasks they have addressed correctly, and the other students can share from their knowledge. Instances exist (as noted in the field notes) of the teachers asking a volunteer/nominated student to share their strategies in solving a task on the adjacent whiteboard; on other occasions the teachers gave the students extension and consolidation tasks. The teachers also randomly selected the mini-whiteboard from a pair of students, displayed it to the whole class and asked the pair to explain and justify their solutions as well. This practice arguably deepens students' sense of ownership of their learning, confidence that their inputs count and a feeling of belonging to a community of mathematics learners where everyone is valued.

Also significant in my estimation is how the teachers addressed misconceptions by students. In several cases (Kitty, p. 137; Jose, p. 175; Gray, p. 193 and Richelle, p. 224) and more specifically (in the case of Kitty, p. 122) most of the students communicated *'I am stuck, I need some extra help'* by using the formative assessment call card. In other instances, the teachers' analyses of whole-class responses and mini-whiteboard feedback showed that most of the students are struggling with the tasks; these gave rise to teachers' 'emergent lesson planning'. This 'emergent (in-lesson) lesson planning' seems to approximate what Bennett (2011) referred to as *the significant investment required to engineer a deeper cognitive shift* (for the misconception). The emergent lesson planning was a result of the available and instantly accessible digital resources in the teachers' resource systems since the teachers drew from several digital resources like *Mangahigh*, a *personal bank of resourc*e and *Resourceaholic*. I also consider that this intricate undertaking of emergent lesson planning seems to point to the teachers' experiences in lesson planning and classroom management skills. The teachers' ability to act on the spur of the moment could be associated with what Gueudet and Trouche (2009) call the *operational invariant*, a sort of mental map with which the teacher orchestrates the various features of the classroom and the lesson. For teachers working with digital and non-digital resources, several of the operational invariants come from socio-historical aspects of their professional practice and hence can be shared by all mathematics teachers, while others are particular to an individual teacher as a result of their background and experiences. The interactions with digital and non-digital resources can contribute to the development of both kinds of operational invariants. For instance, mathematics teachers in England, as reported in this study, begin their lesson planning by reviewing the departmental scheme of work, then consult the designated bank of resources or textbook for associated tasks, revisit previous resources stored on their laptop or flash drive, talk to a colleague or visit a mathematics resources website. The context of the classroom reveals the individual teacher’s operational invariant: for instance, Kitty always begins her lesson with a timed multiplication-table exercise with background music. In the case of Gray, self-created YouTube videos are complimentary resources he encourages his students to always refer to if they face difficulties. Gruson, Gueudet, Le Hénaff, and Lebaud (2018) report similar findings relating to mathematics and English teachers’ operational invariants and how to trace these documentational trajectories.

10.3.1 Conclusion of Formative Assessment

In this section, I have discussed that in many of the lessons observed, classroom practices are centred on formative assessment. These formative assessment practices form a range of diverse activities with a common goal of taking the students learning forward and also enabling the teachers to improve their teaching. The information elicited from the students, the provision of formative feedback by the teachers are supported and enhanced by the availability of teachers' resource systems (and the constituting digital and non-digital resources). The combination of the teachers' pedagogical skills and the features of their digital resources have the potential for activating innovative formative assessment practices that privilege the intricate undertaking of emergent lesson planning.

10.4 Geneses of Mathematics Teachers' Collectives, the Community Documentation and Professional Geneses

In this section, I discuss the findings regarding the mathematics teachers' collectives in the light of their interactions with and through a variety of digital and non-digital resources. Firstly, in subsection 10.4.1, I discuss the five identified ways mathematics teachers participate in the collectives and highlight the theoretical contributions these findings make to the documentational approach to didactics (DAD). Secondly, in section 10.5, I examine the mathematics teachers' collectives in schools A, B and C and present the associated stages of their respective community geneses from the perspective of the concept of the community of practice (CoP). Finally, in section 10.6, I consider the community documentation system as revealed in this research.

10.4.1 Ways of Participating in Mathematics Teachers' Collectives

The findings reveal that a teacher's participation with and through resources always intersects in various loosely or tightly connected networks. The evidence exists in the analysis of the datasets in subsection 4.1.7, pp. 86-91, and in the teachers' self-reports on their participations in the collectives: in school A (section 6.6, p. 182); in school B (section 7.5, p. 213) and in school C (section 8.3, p. 231), respectively. This discussion draws on the analyses of the reported practices by the teachers and a year-long direct observation of teachers' collective work by the researcher.

A collective is a group of individuals whose interactions are motivated by a common interest, at least, and who work together as a unit towards achieving a shared goal. This is previously explored in the literature (subsection 2.2.4, p. 33). The findings of my research with regards to mathematics teacher collectives reveal that teacher collectives are made manifest by the spaces they occupy, spaces developed for teachers by teachers and other educational professionals where series of dynamic interactions with educators and diverse resources take place in order to improve the teaching and learning of mathematics. This collective of mathematics teachers' interactions through resources exists at several overlapping levels. In the analysis, I identify five different themes (subsection 4.1.7, p. 86) to highlight the ways of being in the teacher collectives and discuss them in turn.

10.4.1.1 Mathematics teachers' collectives by context (institutional and prescribed)

Mathematics teachers' collectives by context refers to all the teachers' groups prescribed by an institution that operate within that context. This also refers to the context of the department and the internal activities geared towards the professional goals and aspirations. The findings reveal a range of teacher collectives by context, that are periodic, formal and professional in nature (p. 86). Pepin et al. (2013) report a similar finding that one way the collective work of teachers is organised is through formal and school-organised teams. While Pepin and colleagues highlight the organisation based on their immediate school context, my finding extends that understanding to include the context of prevailing external educational changes and policy, such as the adoption of the new curriculum, the institutional context of the maths hubs, the use of new digital resources and the England-Shanghai exchange programmes. Siedel and Stylianides (2018) argue that collegiality is important to teachers and reflects the national characteristics of schools in England. In spite of this, there exists a tension between the prescribed formal practice of collective work and individual teachers' preferences for working alone: for instance, in school B (section 7.5, p. 213).

10.4.1.2 Mathematics teachers' collectives by access (open and voluntary)

The ordinary everyday practice of mathematics teachers consists of many collaborative and collective interactions geared towards improving specific practices as an aspect of professional development or career growth. My findings indicate that the mathematics teachers in this research participate in collectives which are open and voluntary. For instance, in school A, two of the teachers (p.87 and p.182) participate in the Mathematical Association[171] (MA). The association welcomes everyone with an interest in and enthusiasm for mathematics. In school C, there is a network (p. 231) that is open and voluntary for any mathematics teachers. A similar finding has been reported in the literature (Miyakawa & Xu, 2019). This open and voluntary access to the collectives provides opportunity for teachers to take the initiative as individuals in assessing their professional needs and take action for growth within a collaborative context. This finding is in line with the result of a previous study, wherein Joubert, Back, De Geest, Hirst, and Sutherland (2010) argue that it is important for teachers to have time away from school for thinking, discussing and sharing their experiences of teaching practice.

10.4.1.3 Mathematics teachers' collectives by mode of participation (face-to-face and/or online)

The finding of this research reveals that teachers belong to and participate in multiple mathematics-teacher collectives across several contexts. I argue here that one way the teachers manage the challenge of participation and benefiting from the available professional development resources is the available mode of participation. Most of the CPD sessions reported by the teachers were in face-to-face mode. In other instances, the teachers took a Facebook collective (p.124); Gray's mathematics collectives on Twitter (p.198), and Gavin's web conferencing (p. 207). Trouche, Gitirana, et al. (2018) argue that digitalisation supports both the emergence of online communities and ways of disseminating their resources. In the context of my research, the online route supplements the face-to-face collectives' mode of participation and teachers can engage solely through online participation in a way that is flexible and manageable.

171 https://www.m-a.org.uk/organisation

10.4.1.4 Mathematics teachers' collectives by forms of organisation (formal and informal)

Mathematics teachers' collectives exist on both the formal and informal level of organisations. The prevalence of staff rooms and departmental offices in the context of English secondary schools offers teachers the opportunity for informal and spontaneous engagements and productive interaction with each other. In the three schools, these were the more regular form of participation in the collectives. Richelle reported this ongoing practice in school C (p. 231), where the attitude of sharing best practice, resources and professional difficulties is commonplace. Several mathematics collectives exist in which teachers participate formally, for example the TeachMeet, the England-Shanghai exchange programme and the formal CPDs of the various schools.

10.4.1.5 Mathematics teachers' collectives by geographical region (Yorkshire and Derbyshire)

Another way in which teachers in this research reported to have participated in mathematics teachers' collectives is through membership of associations within the geographical regions. In school A, it is the maths hub initiative (p.117) that promotes best practices and brings mathematics teachers together to share and learn. In school B (p. 213), the teachers engage with the regional North-East Derbyshire mathematics community through Twitter and web-conferencing. In school C, the regional mathematics network of teachers serves a similar purpose in enabling teachers to meet, share ideas, share resources and organise exchanges and visits to each other's departments. Krainer and Wood (2008) point to this as a *cultivated community of teachers*. In terms of the collective dimension to teacher documentation work, Gueudet and Trouche (2012a) argue that the processes through which the collective of teachers gather, create and share resources in an attempt to achieve a shared teaching goal is a form of *community documentational genesis* leading up to *community documentation* and which could lead to an emergence of a mathematics teachers' *community of documentation*. (This notion of the mathematics teachers' community of documentation is more fully discussed in the subsequent section 10.6, p. 299.)

In subsection 10.4.1 above, I have discussed the five ways through which the teachers of this research participated in collectives. In addition, I highlighted how these findings contribute to broadening our understanding of mathematics teachers' collectives.

I now turn to examine the dynamics of the mathematics teachers' collectives in schools A, B and C and argue, from the perspective of the concept of CoP, for the existence of different stages of their respective community geneses.

10.5 Community Geneses of the Collectives

This section is divided into two parts. In the first part (10.5), I discuss the collectives of the three schools with particular reference to the idea of community of practice and highlight the community genesis of each school, respectively. In the second part (10.6, p. 299), I discuss the phenomenon of community documentations and professional geneses as evolving together in the contexts of teachers' practice.

10.5.1 The Collectives in School A

In school A, there is an institutional provision of departmental office space (also referred to as the mathematics staffroom). This is both a social and professional space where interactions are frequently taking place. Here social and professional identity and practices are constructed and continually negotiated: the day-to-day running of the mathematics department, professional dialogue amongst staff, sharing information, lesson planning and official daily and periodic meetings, and informal spontaneous peer and collective work happen. The architecture of the department in school A includes a large rectangular office space with individual desks and desktop computers, wall shelves of mathematics textbooks and other mathematics related materials, posters, timetables and schedules of assigned classes and use of rooms and technologies (i.e. the class set of iPads, computer room etc.), and a storage trolley for important documents and personal effects. The teachers reported the departmental ethos of collaboration as strong and well-resourced, where the opportunity to work with friendly professional colleagues is the norm; my periodic observations confirm this (see subsections 4.1.3, p. 76 and 6.1.3, p. 122).

The mathematics department in school A is open to all the teachers, including the newly qualified teachers, teacher trainees, teachers from the England-Shanghai mathematics exchange programme (the visiting Chinese mathematics teacher) and researchers (like myself), respectively. The mathematics department architecture enables teachers to associate with each other, promoting multiple form of interactions with a view to a collective professional-practice micro-culture. From teacher self-reports and my observations and participation with the teachers in the department, two broad patterns of social and professional interactions have been identified: teacher to teacher; and teacher to collective patterns of interactions. These interactions occur with and through the use of digital and non-digital resources. There exist many patterns of teacher interactions in the collective, as observed and noted in the interviews, ranging from professional dialogue on teaching and learning to giving reassurances and support, as is the case with teachers Jose (p. 168) and Jimmy (p. 154). There are also clusters of collegiate interaction with shared lesson planning, mutual peer lesson observation and feedback, and informal 'mentoring and coaching' of newly qualified teachers and teaching assistants explaining classroom practices as reported in all three schools in section 6.6, p. 182. The findings on the overlapping patterns of interactions in my research highlight and confirm in a new way (taking into consideration the role of digital resources) that the context has an impact on teacher activities. As Kynigos and Psycharis (2009) argue, there is a growing focus on context not as a backdrop for teachers' and students' activities but rather an integral part of the teaching and learning process and that the context influences professional activities.

This friendly (in my opinion) atmosphere of the collective of mathematics teachers provides the opportunity for the more experienced teachers to share their expertise and the new teachers to develop their professional learning, construct their professional identity and seize the peer-mentoring chances that this space offers. These face-to-face knowledge-building and -sharing practices from direct observation (see sections 6.6, p. 182; 7.5, p. 213 and 8.3, p. 231), in my opinion, are central to the teachers' professional learning. The department also provides a forum to plan peer lesson observation and share constructive critiques of each other's practices. As Lefebvre argues, "Social space is a social product. Space thus produced also serves as a tool of thought and of action. In addition to being a means of production, it is also a means of control, and hence domination, of power" (1991, p. 26). The staffroom is considered here as a collective social and professional space borne of complex interactions and negotiations. These interactions enable and support the construction of mathematical thoughts and the creation of curricular resources and professional practice. As a means of control, the departmental ethos (as evidenced in subsections 6.1.3, p. 122, 7.2.1, p. 188 and 8.2.1, p. 219) enables and constrains the behaviour and practice of teachers towards expected and acceptable classroom management techniques.

The above-mentioned elements of the collectives of school A are similar to those found in a related study (Pepin et al., 2013), whereby particular resources (such as iPads, the TES website and Facebook) have the potential of enabling the mathematics teachers' collective work. My research extends this finding by showing the specific ways these collectives are formed and collaborate in three schools in England. The departmental micro-culture, multiple patterns of interaction and a shared bank of resources play pivotal roles in shaping the collectives of the teachers into communities of mathematical teaching practice. My findings (school A) are consistent with the idea of *communities of practice* (CoP) as proposed by Lave and Wenger (1991). The notion of CoP has been previously defined and explored in the literature review (subsection 2.2.4, pp. 33-37) and the three essential conditions necessary for a CoP to exist (Wenger, 1998) – mutual engagement, joint enterprise and shared repertoire – are found in the teacher collectives of school A. For instance, in terms of *mutual engagement*, there is are established norms and a departmental ethos of mathematical practice and collaborative interactions (see sections 6.1, p.117 and 6.1.3, p. 122). Regarding *joint enterprise,* there is a shared understanding of their common professional purpose and goals as evidenced in their adoption of the mastery approach to teach mathematics, the use of schemes of work and the promotion of digital resources (subsection 6.1.1, p.119). Looking at *shared repertoire*, there is the collective production of resources and shared ownership as evident in the shared bank of resources (section 6.6, p. 182) and the sharing of best practice through the maths hub, the Shanghai-England exchange and peer-observation and feedback (p. 142) among the teachers. As a consequence of this finding, I argue, in line with (Engeström, 1987a; Lave, 1996) that the mathematics teacher collectives in school A represent a community of learning in practice. Thus, each community of mathematical teaching practice is a community for personal and collective professional learning in the context of everyday practice.

In the extended model of the documentational approach, Gueudet and Trouche (2011) adopt the idea of a teachers CoP in connection with the set of resources teachers create towards achieving shared professional objectives. They propose a dual construct in relation to the processes of teachers' participation and documentation. Understanding these in terms of geneses: firstly, the *community genesis* (the mutual engagement with mathematics and its teaching, enabling the evolution of a community of mathematical practice) is shown in the collective activities of the maths hub, the TeachMeet sessions (see Table 4-9), the presence of a shared bank of resources and a collectively developed school ethos (p.122). Secondly, the *community documentational genesis* is seen as a means of defining the processes through which the collectives of mathematics teachers search, collate, create and share resources for mathematics teaching, validating best practice and accomplishing the teaching objectives. This process, as a consequence, facilitates the creation of a shared repertoire of curricular resources and building up of shared knowledge. In school A, the existence of a micro-culture, patterned interactions, a shared bank of resources and use of a scheme of work (see Figure 6-9 and Figure 6-27) are the tangible and symbolic indications of the *community documentation*. Gueudet et al. (2013a) stated that "the conditions for *the development of teachers' collective documentation work* are likely to correspond to the conditions of the *emergence of teachers*' CoPs". I make a similar argument for the findings of my research regarding the existence of a community of mathematical teaching practice in school A. This collective documentation work enables the mathematics teaching practices to evolve over time as seen in the cases of Jimmy (p.154) and Jose (p.168).The exploration of community documentation work could provide a useful template for developing a programme of continuing professional development. There is, it appears, several moments of convergence of teacher documentation work and collective documentation. In schools A (p. 182) and C (p. 231) where there exists a ‘living shared bank of resources’, instances exist where resources found in Emilia’s and Richelle’s personal resource systems share great similarity with those in the shared folder. I believe it is possible that either they have ‘contributed’ the specific resources to the pool or they made additions to their personal collections from the shared folder, and a similar finding has been reported by (Sabra & Trouche, 2017).

The four teachers in school A participate in various mathematics teachers’ collectives and digitalisation has multiplied the diversity of the forms of collectives and participation. There are the quarterly ‘TeachMeet’ sessions, ongoing *math hub* initiatives, and several online communities. Borba and Gadanidis (2008) reported that the growing use of technology and digitalisation offers new routes for teachers’ collective work in virtual communities of practicing mathematics teachers. A related study indicated similar findings for virtual communities involving prospective teachers and NQTs (Llinares & Olivero, 2008). The pattern of mathematics teachers’ participation on Twitter, Facebook, NRICH and TES communities in school A are consistent with the findings of these studies.

My finding also revealed teachers belonging to a complex intersection of collectives and the resources in use. In school A, two of the teachers (Kitty in subsection 6.2.1, p. 125 and Emilia in subsection 6.3.1, p. 142) belong to these collectives: the lead teacher teams – providing mathematics specialist training for newly recruited teachers in the region; the White Rose Maths Hub; membership of a Facebook group of an initial teacher training (ITT) set; the TES online community; and the school department community. The other two teachers (Jimmy in subsection 6.4.1, p. 155 and Jose in subsection 6.5.2, p. 177) participate actively in the Twitter community (#mathschat) and blog groups. These show several levels of overlapping complexities: the complexity of intersecting face-to-face communities, various forms of virtual collectives, and the abundant production of 'mashup[172] materials' and 'lived' resources always evolving into newer hybrids (Gueudet & Trouche, 2011). From analysis of the interview and observation notes (pages 73 and 81, respectively), these virtual collectives feed the teacher's individual professional growth, and the teacher collectives of the mathematics department where the everyday professional practice of the teacher is effectively situated.

In summary, for school A the departmental micro-culture, ethos promoting and resourcing of mathematical teaching practice, the goal-oriented patterns of interactions with resources, and the co-production of an ever-evolving shared bank of resources are arguably the manifest features of an established and thriving community of mathematical practice. These contribute to the number of other identified features of the teacher collectives that offer opportunity for community genesis.

10.5.1.1 School A Community Genesis: Stewardship and Transformation

In exploring the documentation work of mathematics teachers in a collective and the various stages of evolution into a community of mathematical teaching practice, like Gueudet and Trouche (2012a), I adopt the five possible steps of *community genesis* suggested by Wenger, McDermott, and Snyder (2002). Even though the mathematics teachers' collective is constantly evolving, Wenger et al. (2002, p. 68) proposed five steps in the lifecycle of a community evolution: *potential, coalescing, maturing, stewardship, and transformation*. Using these constructs of CoP, I now consider the stages of *community genesis* of the mathematics teacher collectives in school A.

Three underlying characteristics mark a community of practice: *domain*; *community*; and *practice* (Wenger et al., 2002; Wenger, White, & Smith, 2009). A domain points to the fundamental shared interest of a collective, what inspires participation and gives meaning to their activities. The idea of a community crafts the social fabric, where

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172 Mashups are a combination of multiple data sources into a single representation. This typically consist of graphics, texts, and audio-visual clips that have been sourced from various media such as blogs, wikis, YouTube, social media, Google Maps, etc., being combined into a new hybrid resource.

relationships of trust mutual engagement, leadership roles, interactions and willingness to share ideas are developed. The practice entails the specific focus around which the community develops, shares and maintains its common body of knowledge. I consider school A, based on evidence (section 6.6, p. 182) from the findings, to be at these two stages of its evolution as a community of mathematical teaching practice. As Wenger et al. (2002) comment, once a CoP has matured, its central focus becomes how to sustain its momentum in the face of shifts in its practice, membership, technology and context of practice. Mathematics teachers in school A are involved in a number of diverse collectives – the mathematics department collective, the weekly whole-school TeachMeet, the Mathematics Association – and they host the regional maths hub. In this context, community documentational and professional geneses are taking place concurrently (Gueudet & Trouche, 2012a). In the course of the research, lead teachers in the school (Kitty and Emilia) provide training for primary school teachers in the region (p. 117). Wenger et al. (2002, p. 104) believe that "the key practice issue for communities at the stewardship stage is to keep the community at the cutting edge". In that regard, Hustad (2010) argues that creating a knowledge-networking infrastructure to facilitate knowledge sharing, creation of resources and activities, renewing interest through the inclusion of technological means, education of novices, establishment of a formal institution and winning influence are some of the ways of keeping the community at the cutting edge and sustaining it. With the establishment of the maths hub, involvement in the Shanghai exchange programme and offering of regional training opportunities for teachers across boundaries, school A shows the elements of a community of stewardship and transformation.

For the community at the transformation stage three alternatives are possible: maintaining the status quo; a radical transformation; or death of the community. As tension rises between 'ownership and 'openness', when a CoP widens its scope of engagement as school A does, Wenger et al. (2002, p. 109) state that "when the community widens its boundaries, it risks diluting its focus. New members feel less ownership of the community's topic, practices and processes". One way such a community could be sustained is to be 'transformed into a centre of excellence … and maintain a particular competence linked to the community'. School A's array of regional and international engagements (China exchange) and appropriation of a wide range of digital resources could be considered as indicating a radical transformation. In this, the availability of a variety of digital resources and the encouragement to use them play a great role. Digital resources (such as Facebook, Twitter, web conferencing, resources sharing) could facilitate the convergence of content and networks of people, enabling the intertwining of digital resources and community where each shapes the other and creates new possibilities for communities to develop, grow and evolve (Wenger et al., 2002). The nested contexts of school A, I believe hold these possibilities.

In summary, in this subsection, I have argued that the notion of communities of practice in the light of the *documentational approach of mathematics didactics* is useful to capture the dynamics of mathematics teachers' collectives sharing resources and supporting each other in the context of practice. The conditions for the evolution of a community of mathematical teaching practice could be *spontaneous* as well as *cultivated* (Wenger et al., 2002). Relying on the evidence from the findings of this research, I believe the mathematics teachers' collectives in school A in this study are at the stewardship and transformation stages in their evolution as communities of mathematical teaching practice.

10.5.2 The Collectives in School B

In school B, there are two adjacent mathematics department office spaces; a larger room with most of the teachers, setup of individual desks and computers, wall shelves of books and teaching materials, facilities for refreshment and storage. This is the formal staffroom. The adjacent smaller office space is used more frequently by the head of the mathematics department and a 'senior lead practitioner' who participated in this research. It also houses the mathematics-related supplies, textbooks, calculators, workbooks, worksheets, stationery and laptops.

ActiveTeach is designed to simulate the regular textbook with added digital features that extend its possibilities. Among the teachers observed, it increased the dimensions of possible routes of interacting with each other and with the resources. Mathematical teaching and learning communities can be enabled and built through and around the use of these e-resources by teachers and learners: for instance, the communities enabled by the shared use of TES, Resourceaholic and ActiveTeach (section 7.5 p.213). This can provide an organised sequence of ideas and activities matched to the curriculum and easily adapted to the scheme of work for the teachers. Comparable findings about the potential for the digital resources to nurture a professional community of mathematical practice were reported in the research on the *Cornerstone Maths* project in England (Clark-Wilson, 2017) and *Sésamath* in France (Gueudet, Pepin, Sabra, & Trouche, 2016).

In school B, the face-to-face staffroom collective participates together through the use of this shared common e-resources. Face-to-face collectives run simultaneously with a virtual component (see 7.2.3, p. 195 and 7.4, p. 207). This ‘connected learning, planning and teaching’ between and across groups of individuals, teachers, or students resonates with the idea of “the connectivity of a mathematics digital resource, [its] connecting potential for a given user (student or teacher) both practically as well as cognitively” (Gueudet, Pepin, Restrepo, et al., 2016, p. 6). The availability of a common staffroom, common e-resources and its text components and collaborative opportunities to plan lessons together, to share difficulties and challenges are indications of an evolution of a stable community of mathematical practice. From a practical perspective, *ActiveTeach* becomes a ‘living’ e-resource bank – under continuous negotiation and transformation through collaborative redesign by the teachers in response to input from other teacher users and students’ assessment feedbacks and emerging with a collectively acceptable best practice. Individual teachers also develop personalised resource systems from these interactions with and through the e-resources.

One observation is the temptation of being *alone together* as stated by Turkle (2017), whereby teachers absorbed in the use of these e-resources become isolated from each other as observed in school B (section 7.5, p. 213). In four instances (subsection 4.1.7, p. 86), sitting in the staffroom, I observed several extended periods that teachers were immersed in their digital resources and were barely interacting with each other. The use of technology to build a collective could become a mere simulation of such a community. As in school B, social media, and specifically, Twitter, plays a role in the collective. One of the participating teachers, Gray referred to Twitter as a "live, big mathematics staffroom" where mathematics teachers and mathematics teacher educators are continuously engaging with each other through mathematics and mathematics-related resources. In a *ZDM Special Issue* (Borba & Llinares, 2012), Goos and Geiger (2012) drew attention to how exploration of resource-use is linked to online teacher interactions, teacher education and virtual mathematics communities The interactions here belong on the overlapping and intersecting continuum of the community of mathematics teaching practice.

Three other periodic face-to-face mathematics teacher collectives in school B exist as well. There is a quarterly faculty meeting and periodic mathematics topic-teaching meeting for teachers by teachers – this involved a scheduled time for selected mathematic topics teaching, as preparation for teachers of the same year group, usually with one teacher leading the session.

There also exists an interdepartmental mathematics and English teachers' collaboration. This cross-curricular collaboration between mathematics and English teachers is a sort of networking referred to in literature in terms of "boundary crossing" and boundary objects" (Akkerman & Bakker, 2011; Star & Griesemer, 1989), where new connections across communities are made, and it opens up new possibilities of learning and sharing to help improving teaching and learning in respective subject areas.

Both cultural-historical activity theory on expansive learning (Engeström, 1987a) and situated learning theory on communities of practice (Wenger, 1998) have highlighted how boundaries carry potential for learning, change and development. In the context of school B, Gray reported the practice of the English and mathematics departments meeting and working together (p. 197) on how to improve the language ability of the students in relations to their mathematics learning since they argued that students' language ability impacted on their understanding of mathematics.

Language proficiency has been acknowledged as having implications for mathematics teaching and learning (Beal, Adams, & Cohen, 2010). In today's multi-lingual classrooms, non-native English speakers are at some disadvantage in mathematics achievement due to their low English-language proficiency levels (Henry, 2013). The ongoing interdepartmental collaboration in school B could be a platform to explore the specific language features that cause difficulty for this particular group of students when learning mathematics. Teachers could then develop a collaborative framework on how to address this challenge. The teacher collectives offer opportunities for building up a community of mathematical practices, where resources, teaching strategies, classroom managements tips, innovations and professional peer support are available, networked with other communities, and provoke change.

10.5.2.1 School B Community Genesis: Potential and Coalescing

My approach here is characterised by two claims: first, that secondary schools in England have departmental offices where teachers work and have opportunities to collaborate; and secondly, that such mathematics teacher collectives in the department are at some stage of *community genesis*. Wenger et al. (2002, p. 82) believe that "the key domain issues of the coalescing stage is to establish the value of sharing knowledge about the domain" – in this context the domain of practice in school B.

For instance, School B seems to be at the stage of *coalescing*, which refers to a stage of evolution where the collective is developing its practice, exploring the joint enterprise, adapting to changing circumstances and renewing their commitment to the common practice, recognising each other's potential and engaging in mutual learning activities (Hustad, 2010). There are several examples that give credence to the claim that school B is at the coalescing stage. First, its practice is developing through participation in a variety of face-to-face and online professional mathematics platforms like Twitter, web conferencing and YouTube sharing (section 7.5, p. 213). Second, it is exploring joint enterprise and renewed commitment to a common practice through the use of a shared scheme of work based on the *ActiveLearn/ActiveTeach* learning platform (subsection 7.1.1, p. 186). Third, teachers recognise each other's potential and engage in mutual learning activities through peer-teaching and resources sharing (see subsections 7.2.1, p. 188 and 7.3.1, p. 201).

Although, school B is not new, collaborations and cooperation between teachers seem few and far between. As Gavin reported,

> Within the department and individually, we don't have a central bank of resources where information is; it is what you find, see if what you find is suitable. It is a brand-new scheme, it's a brand-new specification, so there is not a great deal out their tailor-made for that yet. More and more are coming on stream. (1intGn: #1:20)

The teachers in school B were transiting to the use of the new National Curriculum and *a brand-new scheme, a brand-new specification*. These circumstances, Gray stated, are the sources of the new impetus for the growing sense of community. The teachers in school B were now beginning to build cooperation, subject knowledge training workshops on new topics were taking place weekly (see collective of school B, p.213). One of such workshops I was invited to observe was on *sorting algorithms* and taught by two of the teachers (section 7.5, p. 213). In the course of the lesson, documents were shared and teachers were divided into working groups who would take up sub-topics (i.e. *shuttle sort, shell sort, bubble sort and quick sort*) and pointers (i.e. weblinks, web address) to whom other resources were shared. This finding suggests that each teacher has an opportunity to develop themselves personally as well as professionally in the context of the community. This practice of mutual peer-teaching resonates with the claim of Grangeat and Gray (2008) that collective activities are not incidental since they contribute to enhancing individual competencies and practices.

School B has no shared central resources but by sharing documents, teaching techniques and classroom management strategies; collating resources associated with the topic; and documenting the practice with *sorting algorithms*, collectively produced resources were beginning to emerge. The ongoing documentation in school B is evidence for the evolution of a community of practice. Gueudet and Trouche (2012a, p. 320) argue that "each community of practice is a *community of documentation*, which means that community geneses and documentation geneses act in concert". My findings with regards to school B lead to a similar conclusion and extend Gueudet and Trouche's work by highlighting the specific context of a school in England. In terms of conceptualisation (Engeström (2001), the *tools*

(*ActiveTeach/ActiveLearn*); *rules* (new National Curriculum, scheme of work and departmental objectives) and *division of labour* (shared peer-teaching, form of school leaderships; *(Maughan et al., 2012)* provide opportunities for periodic learning and sharing enabling the emergence of a community of mathematical practice in school B. As Wenger et al. (2002, p. 29) state, “practice is a set of frameworks, ideas, tools, information, styles, language, stories and documents that community members share”. Gueudet and Trouche (2012a) argue further that *documenting is collaborating* and each teachers’ community of practice is a *community of documentation*, where documentation work leads to the production of ‘lived’ resources and in the case of school B, shared collectively produced resources. My interpretation is that the mathematics teachers' collective of school B is at the coalescing stage in its genesis into a community of mathematical teaching practice.

10.5.3 The Collectives in School C

In school C, two central factors facilitate the mathematics teachers collective work with digital resources. Firstly, there exists a common departmental office, where each teacher is allocated a workspace, a desktop and various stationery available on the shelves. Through this arrangement, where teachers stay in close proximity to one another, they are able to offer support and facilitate regular informal and formal meetings and collaborations with each other. Secondly, mathematics teachers in school C have several collective activities, a shared resource bank and periodic after-school meetings with newly qualified teachers and teaching assistants. The findings (section 8.3, p. 231) reveal that these two factors offer opportunities for teacher-collective work. A study investigating the collective dimension of teachers’ work in their ordinary daily practice (Gueudet et al., 2013a) similarly indicated that in Norway, mathematics teachers have a staff room where they meet at break time and for plenary meetings, while in England there is a departmental office with teacher-allocated workspaces, where teachers actually do their work in preparation for lessons. In spite of the differences, both contexts enable collective work with resources. One direct observation of teacher-collective work in school C included watching the teachers plan and craft a new scheme of work that would align with the new National Curriculum. Each teacher shared ideas and resources on how to adapt and resource their teaching for the ongoing transition.

The existence of a shared central resources folder facilitated discussion with other teachers and gave the teacher a starting point for collaboration and collective work with resources. Grangeat and Gray (2008) explain that such collective work around a shared resource could enhance individual competencies and practices. The collectives' work with shared resources enables teachers to work with each other and learn from each other's 'best practice', thereby enabling professional growth. Richelle reported this 'culture of support and culture of sharing' as being enhanced by the use of technology.

Krainer and Wood (2008) claim that collective work by teachers is most evident in teacher education programmes. In school C, the regional secondary mathematics networks of the teaching school alliance provide occasions for continuous professional development. This mathematics teaching school alliance offers school-based CPD for enhancing classroom practices. Linked to this, the mathematic subject support network supports new teachers and more experienced teachers who need some 'refreshing' as a new curriculum comes into place. This finding shows, in my opinion, the existence of a community of practice, as Wenger states "a community of practice exists because it produces a shared practice as members engage in a collective process of learning" (Wenger, 1998, p. 4).

School C has a number of newly qualified teachers and teaching assistants, which has given rise to what Grangeat and Gray (2008, p. 179) call "micro-collectives formed by mentors and new teachers", where the experienced teachers mentor the new ones, developing a shared project through this practice, like the bank of resources. On several occasions Richelle's lessons were observed by a number of new teachers and she in turn observed some of the new teachers' lessons and offered feedback. I believe this could deepen the rapport between a new teacher and the others and enable a stronger bond among the teacher collectives of school C. These 'productive' collectives (Pepin et al., 2013) could offer opportunities for community building and the evolution of mathematics teachers' professional practices with digital resources.

In summary, school C presents several diverse opportunities for teachers' collectives working with resources and with each other. The departmental space, shared resource bank and common scheme of work, regional networks and the micro-collective of mentors and new teachers point to community and teacher professional geneses. These opportunities are enhanced by the presence of digital resources and social-networking platforms like Twitter and Facebook. I now discuss school C in its community genesis.

10.5.3.1 School C Community Genesis: Maturing

The maturing stage in the evolution of a CoP is the phase where members begin to appreciate each other's contributions and perspectives, set standards, define learning agenda, develop joint activities, create resources, take charge of practice and grow (Hustad, 2010). In this context they act more as a collective rather than as a group of individuals. The evidence in this study and my interpretation of the indicators place school C at the maturing stage of its evolution. For example, Richelle reported a *culture of support and a culture of sharing* aided by the use of digital technology.

> We talk in a more professional manner and on delicate tasks, but day to day because of a lot of the staff have come in and been new teachers and have had to be supported quite a lot, our culture is to ask anyone and they will stop and support you. Everyone is very willing to send lessons and resources to each other… might see a worksheet on the side, some done … and I want that and they will happily give it to you. So the culture of sharing here is extremely good, and I suppose technology makes that faster because you can just photocopy or send an email. (2intR:#16)

She stated further in the interview,

> We have to build a bank of resources that you use again and again and again. And again, we are very open in this department, so if someone has something good then we get them to send to us. Everyone is required to send their lessons to myself or the head of department and every week we can review people's lessons and we send out good ones and everyone would have a copy. (2intR: #4:15)

As Hustad indicates, there is an ongoing appreciation of each other's contributions, a developing commitment to share and organise the departmental practice knowledge signified by the bank of resources managed by the leaders in the department which every teacher has access to. Wenger et al. (2002, p. 97) state that at the maturing stage "the key practice issue at this point shifts from simply sharing knowledge and insights into organising the community's knowledge, taking stewardship seriously… and its relationship to other domains". School C's building up of a resource bank, cultivating a culture of support and sharing, and taking advantage of technology, point to the maturing stage of their evolution into a community of mathematical teaching practice. Wenger et al. (2002) further state that another characteristic is the collective beginning to link their practices across the departments and geographical area or by connecting practitioners in related disciplines and creating knowledge-sharing spaces. Mathematics teachers in school C, as reported (section 8.3, p. 231) by Richelle in this study, belong to a regional secondary mathematics network in a teaching school alliance and a local mathematics teachers' partnership where sharing knowledge, practices, resources and support are at the core. Leaning on the constructs by Wenger (1998), a *joint enterprise* of working and meeting together towards improving teaching practice is already evident, *mutual engagement* is established and a *shared resources bank* already exists as the evidence in the findings shows. Having stated the evidence and the findings, I argue that school C's teacher collective has the features of a maturing community of mathematical teaching practice supported by the use of digital and non-digital resources.

I now examine the findings on community documentation and professional geneses across the three schools in the research.

10.6 Community Documentation and Professional Geneses

This research suggests that community documentational and teachers' professional geneses develop simultaneously, and this has an influence on the individual teacher's professional growth. These findings are consistent with those reported in previous literature (Gueudet, Pepin, Sabra, et al., 2016; Gueudet & Trouche, 2009, 2012b), as together they argue that documentation is an outcome of participation – though participation in a community does not imply homogeneity of practice. The diversity of individual teaching practices reported in the case studies (pp. 117, 185 and 217) show how the community supports the individual teacher at varying degrees in developing the trajectories of their practice and in creating individual resources for their particular teaching and classroom needs.

Gueudet and Trouche (2009, p. 211) argue that the "*documentation system* and the *teacher's professional practice* evolve together" since the community documentation is composed of the shared repertoire of knowledge and shared associated resources (see Table 6-1; Table 6-2; Table 6-3; Table 6-4; Table 7-1; Table 7-2 and Table 8-1). For examples of shared repertoire, in school A (section 6.1), this is evidenced by the growing use of bar modelling and number lines, and the adoption of the mastery approaches and the maths hub initiatives, and in school B (section 7.1) by the adoption of the ActiveTeach learning platform and peer-teaching. The evidence mentioned above from this research permits further possible interpretations, that teachers' professional practice 'co-evolves' with community documentation and that each mutually influences the other. I also argue that the individual teacher's professional practice also co-evolves with the community's professional practices. For instance, in the case of Jose (p.168), who has been teaching for two years, he made a point earlier in the case study that is worth repeating here:

> The mathematics department has a huge, huge influence. Every single thing I do I can point to a different teacher that has influenced me. For example, in technology, *Mr. Mill* completely; he is the only reason I use iPads; he is the only reason I use laptops. And then again *Mr. Stan* is the only reason I use the IWB regularly and assessment tools. It is not through anything formal, just occasionally seeing him teach, in passing, talking to them about something, but sometimes it can be CPD events. (#1intJs9: 40)

Jose's documentation work involves several iterations of professional evolution inspired and supported by his participation in the informal and formal activities of the department. First, there is an evolution in terms of his appropriation of the use of technology, iPads, IWB and assessment tools. This arguably leads to the evolution of his individual teaching practice. Gueudet and Trouche (2012a, p. 308) indicated this: "the evolution of the community goes with the evolution of its members' identities" where "identity in practice arises out of an interplay of participation and reification. And as such, it is not a static object that emerges but a constant becoming" (Wenger, 1998, pp. 153-154). For Wenger (1998, p. 215), identity "serves as a pivot between the social and the individual, so that each can be talked about in terms of the other". From the perspective of *documentational genesis,* Jaworski (2012, p. 343) argues "*genesis* means becoming: becoming a mathematics teacher; becoming a professional user of resources; becoming a knowledgeable professional". The evidence exists to claim that the seven teachers in this study and the various collectives they participate in (as shown in subsection 4.1.7, p. 86) are contexts for the co-evolution of teachers' professional practice and collective professional practice, where both mutually enrich each other. As Wenger et al. (2009, p. 9) state, "community acts as a social container" oriented towards the members' learning experiences and develops in them the capabilities and competencies that serve the professional context of mathematics teaching and learning, as in the case of this research.

10.6.1 In Summary

In the above discussion, I argued that mathematics teachers participate in a variety of overlapping teacher collectives, enabled by the availability of a range of digital and non-digital resources. I then discussed the five ways that mathematics teachers participate in the collectives: namely, by context, by access, by mode of participation, by form of organisation and by geographical region. These findings make a theoretical contribution by identifying ways of participation in the collectives and highlighting the different forms the collectives take, thereby throwing more light on the notion of collectives in the context of the documentational approach to didactics (DAD).

I also examined at length the specific contexts and dynamics of the mathematics teachers' collectives in the three respective schools under research. Gueudet et al. (2013a) argue that the conditions for the development of teachers' collectives and their documentation work are likely to correspond to the conditions for the emergence of the teachers' community of practice. I have argued similarly above.

Furthermore, relying on the idea of CoP and on the evidence from this research, I considered that the three schools are each at different stages in their emergence as teachers' CoP, respectively (Schools A: *stewardship and transformation*; school B: *potential and coalescing*; and school C: *maturing*). Lastly, I discussed how community documentation and teachers' professional geneses co-evolve and mutually influence each other toward serving the context of the mathematics teaching practice.

I now discuss the findings on teachers' perception of variation and differentiation.

10.7 Teachers' Perceptions of Variation and Differentiation

In this subsection, I discuss the mathematics teachers' divergent perceptions of variation and differentiation in relation to the mathematics tasks they select and use in their lessons. I begin by stating that the adoption of the mastery approaches in mathematics teaching in England has provoked a chain of activities relating to tasks design and teaching strategies. Secondly, I examine variation, considered as one of the five big ideas of teaching for mastery and how it is understood by the teachers. Finally, I discuss how the teachers have (in divergent ways) appropriated the underlying principles of variation and differentiation and setting/ability grouping.
I begin by stating the context of the renewed and growing use of the ideas of variation and differentiation.

10.7.1 The English Mathematics 'Teaching for Mastery' Approach

The findings of this research reveal that there is growing interest and ongoing adoption of mastery teaching approaches amongst mathematics teachers in secondary schools in England. Teachers in the three schools in the research adopted different loosely defined strands of the mastery approach and conflicting enactments in the lessons (see Table 4-14, p. 98). What my findings highlight, first, is that the term 'mastery' from the practitioners' perspectives is used to refer to an array of teaching, task design and task selection strategies. Second, in the effort to adopt mastery approaches, the teachers 'pick-and-mix' from the different but related mastery pedagogical approaches and innovations informed by the Shanghai exchange initiatives. Third, the teachers are locally adapting the mastery approaches to their specific classes' needs and objectives. This 'blending' of mastery adoption practices, consequently, has influenced the task types and teaching strategies enacted by the teachers (see, for instance, subsection 4.2.1, p. 95 and 6.2, p. 124).

In order to account for the sources of the divergent adoption of the mastery teaching approaches by teachers, it is relevant to situate the development in the context of the ongoing mathematics education reform in England.

The mastery approaches became central to the content and principles that formed the foundation of the 2014 mathematics curriculum in England. The ongoing appropriation of mastery approaches is influenced by East Asian success in transnational assessments like PISA (Boylan et al., 2018). The NCETM report with regards to mastery approaches and the new National Curriculum in England states,

> The content and principles underpinning the 2014 mathematics curriculum reflect those found in high performing education systems internationally, particularly those of east and south-east Asian countries such as Singapore, Japan, South Korea and China[173].

This led to the establishment of the Mathematics Teacher Exchange (MTE) – an element of the mastery innovation – involving teachers visiting Shanghai and then in turn hosting Shanghai teachers in their schools. School A belongs to one of the first cohorts of MTE schools, and there was a visiting Chinese teacher at the time of this study. An exploration of the literature on the mastery approaches and associated tasks design principles have been presented (see subsections 2.1.3, p. 19; 4.2.1, p. 95 and 6.1.1, pp. 119-121). The term 'mastery' is a contested and problematic notion to define and the outlook on mastery in current English educational discourse and classroom practices amongst teachers is broadly a Western appropriation of mathematics teaching practices in the Shanghai and Singapore education systems. As a consequence, Townsend argued that "'mastery' is a nebulous concept and therefore is not consistently understood and applied in schools" (Townsend, 2015, p. 95).

The mastery innovation consists of a number of overlapping initiatives, projects, programmes and exchanges promoted by a variety of private and state-funded agencies (Boylan et al., 2019; Hodgen, Monaghan, Shen, & Staneff, 2014; Umameh,

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173 https://www.ncetm.org.uk/public/files/19990433

2017). The National Centre for Excellence in Teaching Mathematics (NCETM) is the most prominent government-funded organisation operating with and through the 35 regional networks of maths hubs. NCETM/maths hubs organise and promote professional development and mathematics improvement programmes that are informed by the Shanghai and Singapore pedagogy. The NCETM/maths hubs use the term ‘teaching for mastery’ as its preferred designation for its adaptation. These efforts at developing a unified idea of an English mathematics ‘teaching for mastery’ approach, the findings suggest, is complex and in practice, inconsistent and conflicting in the observed classroom implementations (see subsections 6.2.2, 6.3.2, 7.2.2 and 8.2.2).

To illustrate these findings, school A hosts one of the maths hubs and the school mathematics teaching and learning outlook is influenced by the hubs’ activities and programmes (see subsection 6.1.1, p. 119). School B subscribes to the *Pearson ActiveTeach and ActiveLearn* digital learning and teaching platform with its associated five-part lesson plan template. The Pearson curriculum website boasts of a “UK-built approach to teaching for mastery[174]”. School C uses the *Collins Connect* resources; the publisher states “based on the successful maths programme delivered in Shanghai, these comprehensive resources provide authentic mastery practice adapted for the English curriculum and we help your class to achieve mastery in maths with the Shanghai Maths Project[175]”. While each school claims to subscribe to a variation of mastery teaching and the associated lesson-plan templates, the observed classroom lesson delivery (Table 4-14, p. 98) appeared different from the mastery teaching approaches the scheme of work and curriculum documents showed. The evidence for this will be provided shortly. As a result, in a similar finding, Jerrim (2015) expressed doubts on the efficacy of mastery teaching in its current form. Furthermore, there is also a question of whether this uptake of teaching for mastery is a ‘seasonal vogue’ or one that could lead to a lasting change in practices (Boylan et al., 2018). The question of whether the current enthusiasm around mastery approaches

174 https://www.pearsonschoolsandfecolleges.co.uk/secondary/Mathematics/11-
175 https://collins.co.uk/pages/secondary-maths-the-shanghai-maths-project

will improve students' attainment is beyond the remit of my research, this is a likely area for further research.

It is within this current context that the use of variation and differentiation have emerged.

10.8 Mathematics Teaching with Variation

A significant finding is the divergent notions of variation in use among the mathematics teachers. Two themes capture the way teachers applied the concept of variation in their classroom practices. Firstly, *variation of tasks* (see p. 134), with the teachers using alternate phrasing as well: such as, 'variation within question', 'variety of questions' and 'gradual variation'. Secondly, *variation of resources* (see pp. 135, 214 and 226). In the teachers' pedagogic use of the theory of variation, there was no explicit mention or distinction in terms of 'procedural' and 'conceptual' variations. I now turn briefly to the idea of variation in the context of teaching for mastery and how the teachers in this research are appropriating the idea (variation of tasks and variation of resources) in their classroom practices.

One aspect of teaching for mastery within the context of the ongoing reform in mathematics education in England is the promotion of variation theory (Askew, Bishop, & Christie, 2015; Jerrim et al., 2015). The NCETM/maths hubs programmes to develop mastery specialists identified five big ideas underpinning teaching for mastery. These ideas include coherence, representation and structure, mathematical thinking, fluency and variation[176]. In the context of the NCETM/maths hubs programmes, variation is conceived as follows.

> Variation is twofold. It is firstly about how the teacher represents the concept being taught, often in more than one way, to draw attention to critical aspects, and to develop deep and holistic understanding. It is also about the sequencing of the episodes, activities and exercises used within a lesson and follow up practice, paying attention to what is kept the same and what changes, to

176 https://www.ncetm.org.uk/resources/50042

> connect the mathematics and draw attention to mathematical relationships and structure[177].

Kullberg, Kempe, and Marton (2017) argued that both forms of variation (conceptual and procedural) rely on a careful choice of mixed examples or tasks which enables students' learning more than the use of multiple examples of the same type of tasks. For Boylan et al. (2018), conceptual variation on one hand focuses on varying representations or exemplifications to allow for the mathematical structures and meaning to be highlighted. Procedural variation on the other hand is concerned with the application of procedures or algorithms in a purposeful way so that the choice of tasks similarly allows for an understanding of the relationships between procedures and mathematical meaning. The Shanghai mathematics pedagogy adopted by teachers in England favours a teaching strategy that integrates the growth of conceptual understanding, problem-solving and a proficiency in routine skills through conceptual and procedural variation (Watson & Mason, 2006; Yuan & Huang, 2019).

The idea of variation as presented by Watson and Mason (2006) predates the one described in the Shanghai mastery teaching focus. Both ideas of variation seem similar but with one noticeable difference. There is an emphasis on the distinction between procedural and conceptual variation in Watson and Mason, but it appears to me that in the Western interpretations of Shanghai mastery teaching this distinction regarding variation is unclear. The Shanghai mastery with variation focuses more on what is to be learnt, and how it is going to be learnt through experiencing, or generating, and reflecting on a varied set of examples, problem-solving or questions. It is within this context of ongoing reform that teachers' adoption and application of the variation theory to their task designs is located.

Now I return to the two themes (variation of tasks and variation of resources) that, in my understanding, describe the teachers' use of variation in their classroom practices.

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177 https://www.ncetm.org.uk/resources/50042

These themes go beyond the distinction between procedural and conceptual variations.

10.8.1 Variation of Tasks

In the classroom observation, the teachers present the students with sets of mixed tasks and examples: for instance, the two sets of tasks given by Emilia (Figure 6-14 and Figure 6-15, p.147, respectively) on mixed numbers and improper fractions. In the first set of tasks (Figure 6-14), the sense of variation, in my opinion, lies with the aspects that are varied in the composition of the tasks. I purposively select some examples to illustrate the variation of tasks in the exercises given, to convert mixed numbers into improper fractions:

1. $2\frac{2}{5}$ 2. $3\frac{1}{2}$ 3. $5\frac{3}{4}$ 4. $10\frac{7}{8}$ 5. $5\frac{9}{10}$

Emilia shows aspects of what can be varied in this set of tasks. The five selected examples show that all the aspects that constitute a mixed number can be varied, ranging from a single digit to a double-digit number as coefficient, numerator or denominator. This is similar to what Watson and Mason (2006, p. 98) described:

> what aspects are fixed, what is varied (different aspects constitute different *dimensions of possible variation*), and how it is varied (which indicates a *range of permissible change*) throughout the exercise, and what is thus available for discernment by the learners.

Emilia, like the other teachers, hopes that the students are able, first, to learn about the variety of forms of mixed numbers (*the different dimensions of possible variation*), then second, to discern in engaging with the tasks how the aspects of the mixed numbers vary, how the various numbers are interdependent and what is fixed (*a range of permissible change*). The students gradually discerned that the denominator remains always unchanged in the conversion processes and forms the basis for the relationship between the mixed number and the corresponding improper fraction. This finding is similar to that reported by Watson[178] (2016), that variation is used to

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178 Anne Watson https://www.atm.org.uk/write/MediaUploads/Journals/MT252/MT252-16-04.pdf

demonstrate an underlying mathematical structure of equivalence and a ‘dependency relation’; that is, when one variable changes, another one also changes in ways decided by their relation or structure.

Gibson[179] (2016) argued that it is possible to find worksheets online that have plenty of variation but the purpose is unidentifiable. The variation of tasks neither aids the development of fluency nor enables deeper understanding of a specific mathematical operation. Hence, he advocates ‘progressive exercises’: a sequence of progressively harder versions of the similar set of tasks with the intention that by engaging with these tasks, students will be enabled to tackle tasks successfully that they would not have been willing or able to tackle initially. Gibson’s position on the variation tasks with unidentifiable purpose is contestable, since the intentions behind sets of tasks are always contextual and sit with the teacher/task designer, specific to the needs of the class. However, his idea of ‘progressive exercises’ is useful in discussing the next set of variation tasks given by Emilia.

This second set of tasks was purposively selected (see Figure 6-15) to illustrate the sense of ‘progressive exercises’ within the context of variation of tasks.

$$1.\ \ 3\frac{1}{2} \quad 2.\ 4\frac{\ }{5} = \frac{21}{5} \quad 3.\ 3\frac{\ }{\ } = \frac{14}{4} \quad 4.\ 5\frac{6}{\ } = \frac{\ }{7} \quad 5.\ \square\frac{\ }{8} = \frac{87}{\ }$$

In the selected examples above, I argued that Emilia offers the students a ‘sequence of progressively harder versions’ of similar tasks to Figure 6-14. Her intentions were, firstly, that students will reinforce their ability to discern the aspects that are fixed (like the denominator, the dependency relation among the aspects of the number and equivalence of mixed numbers and improper fractions) and aspects that vary (coefficient and numerator); and secondly, to enable the students to develop in confidence and fluency with multiplications. In the interview she states, “*I push them a little bit and get them thinking a little bit more*” *(*2IntE:#14).

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179 Michael Gibson https://www.atm.org.uk/write/MediaUploads/Journals/MT252/MT252-16-04.pdf

It is argued that a carefully selected choice of mixed tasks (Kullberg et al., 2017), a sequence of progressively harder versions of similar sets of tasks with identifiable learning intentions (Gibson, 2016) are elements that appear important in the ways the teachers in this research have appropriated and enacted the idea of variation in tasks. The significant finding, therefore, is that the teachers have adapted variation theory in a context-specific and unique manner and that the distinction between procedural and conceptual variation becomes unclear. What appears important is what the students are to learn and how the teachers, leaning on the underlying principles of variation theory, set up tasks to offer students diverse opportunities to engage with the tasks in mathematically meaningful ways.

In the next subsection, I turn to discuss how the teachers conceptualise resources from the perspective of variation.

10.8.2 Variation of Resources

Another finding in relation to variation theory is teachers identifying diverse resources in use as a 'variation of resources' alongside the idea of variation of tasks (see pp. 134-135). For instance, in one interview Kitty stated, "*you've got to have a variation of resources just so that it keeps students engaged*" (1int K:#2). My understanding from the lesson observations and interviews is that the teachers associate sets of tasks with corresponding sets of resources intended to support the students' learning. In the lessons on fractions, Kitty uses multiple representations (Figure 6-7,p. 134) to support students learning. For Emilia, bar models (Figure 6-17, p. 149) and number lines (Figure 6-30, p. 173) are corresponding resources to support students' engagements with the tasks. The teachers match different resources with different sets of tasks in order to support students' learning. Some of the digital resources (such as GeoGebra, Mathsbox, Mathspad and virtual manipulatives) do offer students the opportunity to access mathematical concepts in dynamic, visually exciting ways that can engage and motivate them. As Kullberg et al. (2017) argue, when different types of tasks are mixed, learners are forced to distinguish between them and thus become better at making sense of the mathematics. Al-Murani, Kilhamn, Morgan, and Watson (2019) contend that if differences in the examples used are too difficult to align, it can be less beneficial to students' learning. The teachers in this research appear to support this practice of the use of variation of tasks by making available a variety of resources to enhance the students' engagement with the tasks.

Kullberg et al. (2017, p. 560) argue that "learning, from a variation point of view, implies differentiation rather than accumulation". It is to the idea of differentiation from the perspective of ongoing use of the underlying principles of variation theory that I now turn.

10.9 Mathematics Teaching with Differentiation

In the findings with regards to differentiation three themes are identified that captured how teachers have adapted the idea for their classroom practices. Differentiation appears to be undertaken by the teachers through differentiation by tasks, differentiation by the levels of support and differentiation by flexible groups and tiers. In the *Glossary of Education Reform*,

> Differentiation refers to a wide variety of teaching techniques and lesson adaptations that educators use to instruct a diverse group of students, with diverse learning needs, in the same course, classroom, or learning environment. Differentiation is commonly used in "heterogeneous grouping"—an educational strategy in which students of different abilities, learning needs, and levels of academic achievement are grouped together.[180]

In this sense, differentiation is a flexible way of proactively adjusting teaching and learning, and assessment practices prior to classroom delivery and/or in real time in response to differing students' interests, learning needs and preferences, regardless of differences in ability. C. A. Tomlinson (2004) also refers to differentiation as "differentiated instruction". She defines differentiated instruction as a way of "ensuring that what a student learns, how he/she learns it, and how the student demonstrates what he/she has learned is a match for that student's readiness level, interests, and preferred mode of learning" (p. 188). It is also argued that there are

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180 https://www.edglossary.org/differentiation/

teaching models or strategies in which differentiated instruction has a central place. One well-known example is group-based *mastery learning* (Smale-Jacobse, Meijer, Helms-Lorenz, & Maulana, 2019).

A considerable amount of literature has been published on the notion of differentiation (Smale-Jacobse et al., 2019; Subban, 2006; Taylor, 2017; C. A. Tomlinson et al., 2003). These studies taken together provide important insights into the various dimensions of differentiation as entailing modifications to *practice* (how teachers deliver instruction to students), *process* (how the lesson is designed for students), *products* (the kinds of work products students will be asked to complete), *content* (the specific readings, research, or materials students will study), *assessmen*t (how teachers measure what students have learned), and *grouping* (how students are arranged in the classroom or paired up with other students). Differentiation techniques may also be based on specific student attributes, including *interest* (what subjects inspire students to learn), *readiness* (what students have learned and still need to learn), or *learning style* (the ways in which students tend to learn material best).

This construct is not advocated by all in the mathematics education research community. Taylor (2017) argues that theoretical perspectives, frameworks and strategies for the implementation of differentiation have become conflated because of the contested nature of the terms and techniques in classroom practices. This has led to increased inequality in the classroom, which is the opposite result of its intended purpose. Taylor suggests the need for more research-led strategies targeted at closing the attainment gap among students, especially where differentiation fails to challenge existing learning approaches.

I now turn to discuss the three themes (differentiation by tasks, differentiation by level of support, differentiation by flexible groups and tiers) that capture the way the teachers in this research, in my opinion, understood, interpreted and implemented the principles of differentiation in their lessons.

10.9.1 Differentiation by Tasks

The findings suggest that teachers used a variety of strategies and codes in differentiating the tasks that are given to their students. These strategies include colour coding and the use of precious-metal identifiers. The tasks are coded in such a way that they are progressively more difficult. For instance, a bronze-coded task is easier than a gold-coded task, as the teachers claimed.

From examining the findings and analysis (see sections 4.2, pp. 94-101 and 10.8.1, p. 307), it is evident that the mathematics teachers in this research undertook differentiation by tasks in their lessons. For instance, in school A, tasks are differentiated using colour-coding (red, amber and green) and identifying the tasks by precious metals (bronze, silver and gold). Kitty (p.132) and Emilia ("*Red, amber and green. It is a sort of differentiation*", p. 146) use colour coding in differentiation for their classroom tasks, while Jimmy (p. 156) and Jose (p. 173) prefer the use of the precious-metal codes. Similarly, in school B, only Gray used the precious-metal code, but as a form of self-assessment and success criteria (p. 189). In the case of school C, even though Richelle referred to differentiation of task in terms of the content and approach (p. 218) to teaching in the school, differentiation was not observed in the lesson, neither was it reported in the subsequent interviews.

In this approach to differentiation by tasks, the teachers gave students some choice in the mathematics activities they undertake. Through the colour coding and use of precious-metal identification, students were asked to choose to progress from tasks their teacher defined as easier tasks to a more challenging and then a very challenging set of tasks. This practice of differentiation by tasks has been reported in previous studies (Swan, 2005; Taylor, 2017) and this resonates with the idea of content (the tasks students engage with) that is quantitatively and qualitatively different based on students' readiness. This is a set of tasks (content) at an appropriate level of challenge.

In my observations, few students started from the so-called easier task (red or bronze); most preferred a challenge (amber or silver). In the differentiation by tasks approach, there are three assumptions, in my opinion, in the minds of the teachers. Firstly, that students are able to make a realistic assessment of their own ability to engage with the tasks chosen. From my observation, this was not always so. In several instances (Kitty, p. 137; Jimmy p. 159; Gray, p. 193; and Richelle, p. 224), students were not able to solve the problems they thought they could, initially. Secondly, it assumes that the mathematics teacher can anticipate the attainment of each student accurately and that there is also a bank of suitable mathematics tasks or related activities that may be drawn on. The practice of formative assessments gives teachers a possibility of knowing where most of the students are in their learning, as already reported in this research (p. 255). The results of another study support my findings when it reported that two types of knowledge are essential for being able to differentiate: the teachers need to have knowledge about their students, and subject-matter knowledge (van Geel et al., 2019). Knowing the pedagogical needs of the students, students' interests, peer relations, how to motivate each of them, and the kind of problem-solving strategies they will understand is central to a meaningful effort at differentiation. The teachers of my research have demonstrated these by making formative assessments practices an integral part of their lessons as discussed previously (section 10.1, p. 255) and the self-reported profile of the teachers indicated that five out of the seven teachers are lead teachers (like Kitty, p.124; Gray, p. 187; and Richelle, p. 219). Furthermore, the availability of digital banks of adaptive tasks (p. 230) enables teachers to easily draw on a range of tasks. The evidence (for instance, the incident of the emergent lesson planning discussed in section 10.3, p. 277) suggests that teachers can accurately anticipate students' readiness and the level of challenge they could cope with. In spite of the many possibilities for supporting mathematics learning that differentiation offers, in a systemic review of research evidence, Smale-Jacobse et al. (2019) argued that the empirical evidence on the effectiveness of differentiated instruction in secondary education is limited. My findings therefore add to the growing literature.

10.9.2 Differentiation by the Levels of Support

In terms of differentiation by levels of support, the findings reveal that teachers give different levels of support to students according to their needs. In the analysis, I identified two forms of differentiation by levels of support: first, in the assistance and support given to students by the teaching assistant (TA) in the class; and second, by the multiple representations used in the lessons. In the literature, this has also been referred to as differentiation by deepening and support (Boylan et al., 2018, p. 15). In schools A and C, teaching assistants (pp. 139, 152 and 227) were seen in several of the lessons observed providing one-to-one support to individual students and then supporting groups during group tasks. In the lessons, all students are given similar mathematics tasks, but struggling individuals are offered further personal support by the TA while the teacher attends to other students. Swan (2005) reports a similar practice where he indicated that 'hint cards' were used to provide further help without giving too much away. In my research, the TA supported the students by verbal means and discussion. Furthermore, Boylan et al. (2018) suggest that in the light of the East-Asian inspired conception of mastery teaching, if appropriate resources, support, time and teaching are provided, every student can succeed mathematically. In my estimation it is these extra resources, support, and teaching time that the TA offers the students.

The second form of differentiation by levels of support is the use of multiple representations. Evidence exists that teachers use multiple representations as a device for differentiation, such as Kitty's multiple representation of fractions (p. 134), Jimmy's use of interactive learning resources with dynamic and multiple representations (p. 166) and the growing use of bar models and number lines across the three schools. The teachers reported that this enables students to engage with the tasks in a more meaningful way and helps to reinforce their learning (Kitty, p. 127 and Emilia, p. 143) By using multiple representations, I believe, students are offered the chance to access the tasks from a diverse range of perspectives with the possibility of enhancing their learning. In the context of mathematics teaching, previous research maintained that the use of multiple representations helps increase students' engagement, enables them to make connections among the various ways of representing an idea and maximises the learning for all students (Parsons, Dodman, & Burrowbridge, 2013). In the light of my finding, I argue there is a high likelihood that over time this support will make significant difference in students' learning.

10.9.3 Differentiation by Groups and Tiers

The findings of my research revealed that the mathematics teachers differentiate the students into flexible ability groups and tiers in the course of their lessons. In schools A and C, the mathematics teachers reported that the students are taught in flexible ability groupings (see subsections 6.1.2, p. 121 and 8.2.2, p. 221 for schools A and C respectively), while in school B, tiering into delta, pi and theta is indicated (see subsection 7.1.1, p. 186). The concept of ability as a strategy for making sense of students' cognitive capabilities and differing achievements and segmenting them in an effort to meet their needs is not new (Levy, 2008; Ruthven, 1987; Smale-Jacobse et al., 2019). Taken together, these researchers argue that, first, in ability groupings the teacher clusters students into different homogenous groups based on their needs, abilities, readiness, interest or learning styles in order to provide support. What is new in my findings is that in schools A and C, following the implementation of differentiation by ability groups, the groups were not fixed but flexible; as students in a particular group improved, they moved on to another group with a little more challenging set of tasks. These findings are in line with the work of Parsons et al. (2018), who found adaptive teaching in all phases of instruction, during planning, in the midst of teaching, and when reflecting on their instruction. In this context teachers engage more high-performing peers to teach and share with the lower-performing students and bring them up to where the class is in the learning. Here the use of digital resources plays a significant role: for example, the use of Socrative, where there is a readily available bank of resources the teachers can access during the lesson. Also, the use of QR codes by students to engage with the specific set of tasks they feel they are capable of undertaking. In school A, bar modelling, number lines and online mathematics applications like Socrative, Mathspad and Mathsbox were regularly used to support the flexible ability grouping practices.

Secondly, tiering refers to using the same curriculum materials for all students while maintaining flexibility and adjusting the depth of content and learning activity process to students' readiness, interests or learning styles (Levy, 2008; Richards & Omdal, 2007). The tiered instruction was designed to match with high attainer, middle attainer, or low attainer (in school B: *Pi, Theta and Delta*[181]) background knowledge in mathematics. The scheme of work indicated the tasks and supporting material for each tier and provided guidance to the teachers as well. In school B, where this was on offer to the students, the teachers often combined tiering with ability groupings and interchanged this practice in the various lessons observed. In some other instances the teachers taught the lessons in a flexible ability grouping but gave homework in a way to match the tiering arrangement. In Figure 7-20, p. 212, Gavin makes use of a variety of graphic organisers and reading materials, matching resources at different levels of complexity for the different tiers. While the practice of differentiation by tiering has been observed in the previous studies cited above, my finding reveals a new practice where the teachers use digital resources to support the flexible ability grouping and tiering, especially in the area of homework and extension tasks. This practice is possible for the teachers, in my opinion, as a result of the available digital resources from which the teacher can quickly select a variety of tasks and mathematics activities. For example, the availability of digital resources like Mangahigh, Resourceaholic, 10ticks and Mathspad offer the teachers a range of mathematics tasks they can readily and easily access for the various groups or tiers. The students too can scan in designated tasks using their iPads and work at their own pace while engaging with the tasks. Without the enabling features of these digital resources, these would not have been possible.

In summary, once the teachers in my research were confident of where the students are in their learning (through frequent cycles of FA), reasonably identified individual students' differences and where they want them to be, differentiation became the obvious response to individual students' differences. Differentiation, therefore, is the teachers' deliberate effort at knowing the needs of their students, the level of challenge they can confidently take on and the available resources to support and enable the

181 Differentiated materials catering for students of all abilities. The scheme is structured around three tiers: Pi, Theta and Delta. In order to support teachers in moving over to the new 2014 UK KS3 National Curriculum, the content is mapped back to previous national curriculum levels for teacher reference in the Teacher Guides (https://www.pearsonglobalschools.com).

students to attain the next level up of learning. In this subsection, I have discussed three strategies through which the teachers operationalised differentiation in their classrooms. One question this research did not address is how effective is differentiation in improving students' attainment? This is a possible start for further research in future.

CHAPTER 11

CONCLUSIONS AND RECOMMENDATIONS

This chapter is divided into six sections. The first section brings together the whole of the dissertation, highlights the key findings in addressing the research questions. The second section highlights the contributions of this research to the field of mathematics education research and the resource movement. In the third and fourth sections, I make recommendations for mathematics teachers' practice and proposes issues for further research respectively. In the fifth section, I present the limitations of this research. The final section presents an autobiographical reflections as one background of my research journey.

11.1 The Main Aims of the Research

The central aims of this doctoral research include the following:

- To explore how mathematics teachers' access, adapt, create and use resources in-class and out-of-class, individually and collectively, in lesson planning, delivery and assessment and the impact on classroom practices
- To examine the features of teachers' collective work with resources that could lead to the evolution of a community of mathematical teaching practice
- To contribute to the discourse on teachers' appropriation of digital resources and its implication for professional practices

The main aims of the research were stated previously in section 1.3, p. 6.

The research explored the complex and interactive ways mathematics teachers work with resources. Three schools in England were identified and purposively selected as fertile research settings for exploring emergent questions on mathematics teachers' interactions with digital and non-digital resources and their collective work with resources, especially in the context of secondary school mathematics teaching. This is because mathematics teachers in England have the liberty to search, select, adapt and use a variety of resources for their professional practices (Ruthven, 2013; Siedel & Stylianides, 2018). Through the review of research and theoretical literature, this study set out with the aforementioned central aims to get an understanding of the mathematics teachers' interactions with resources and the impact on practice in the cultural context of secondary schools in England. Seven teachers from three schools in England participated in this study accessed, adapted, created, shared and used a variety of technologies and digital resources for professional practices with reported impact on their classroom practices. The findings from this research are highlighted in this chapter. (Three key findings that have the potential to contribute to the literature and teachers' practice were discussed in Chapter 10.)

The combined use of activity theoretic approaches and the documentational approach of didactics of mathematics guided the entire research process. The activity theoretic perspectives enabled me take into account cultural and institutional influences on mathematics teachers, the motivations to undertake teaching activity using digital resources in order to achieve the teaching goal. Seven secondary school mathematics teachers in the nested contexts of various activity systems formed my units of analysis. The documentational approach, on the other hand, focused on how the mathematics teachers appropriate and transform resources for professional practices. The use of these frameworks aided the analysis, understanding and provision of the explanation for the phenomenon of mathematics teachers' interactions with digital resources. I now discuss the main findings of this doctoral research.

Firstly, one of the main findings of this research suggests that mathematics teachers in selected schools in England, guided by a collectively designed scheme of work, are at liberty to draw from an assortment of resources to design new resources, adapt pre-existing materials and tailor them to curricular objectives and specific class needs, and share them across departments and teacher communities. The scheme of work is one key driver that influences their selection of mathematics teaching resources. This "active and eclectic re-sourcing and diffusion of digital mathematics teaching resources" (Ruthven, 2016b) has been extended by digitisation and offers mathematics teachers greater understanding and more opportunities to use resources selectively and effectively. It is possible to also suggest that just as the availability of a wide range of mathematics teaching resources could offer the teacher a chance to vary their teaching styles and resources, so the use of multiple resources appears to provide diverse learning opportunities for students as well. By the same token, other features like the adoption of Singapore/Shanghai mastery teaching, the transition to a new National Curriculum and the role of the maths hubs in the mathematic teaching context influence and support mathematics teachers in England to appropriate a variety of resources for their teaching practice. The evidence from several studies, Gueudet and Trouche (2009), Ruthven (2013), and more recently Siedel and Stylianides (2018) have identified and constructed mathematics teachers' 'predisposition taxonomy' that characterises the complex set of drivers influencing teachers' selection of resources. This thesis similarly identifies features that stimulate the appropriation of resources by mathematics teachers but in contrast suggests a different set of features influence that appropriation. These include (i) the mathematics hubs (section 6.1, p. 117), (ii) the prevalent use of a variety of nuanced lesson plans (section 4.2.195), (iii) the adoption of the Shanghai/Singapore mastery teaching approach (subsection 6.1.1, p. 119), (iv) the ongoing transition to a new National Curriculum (pp. 119, 186 and 218) and (v) the prevalent use of the government-approved and locally designed school-based schemes of work (see subsections 6.3.2, p. 143; 6.4.2, p. 156; 7.1.1, p. 186 and 8.2.2, p. 221). This set of features appears to be a key influence on mathematics teachers in their appropriation of resources and building up of their documentation systems. These were presented and explored in Chapters 6, 7 and 8.

Secondly, in relation to mathematics teachers' resource systems, it emerged that there exists an ongoing co-evolution of an individual teacher's documentation system and the collective documentation system of the teachers in the mathematics department (section 10.6, p. 299). There exists a mutual interaction and reinforcement of varied subsystems on both documentation systems. As teachers (influenced by the recommendation of the school and scheme of work, changes in the National Curriculum, the ongoing proliferation of digital resources, the teacher collectives they participate in and the students' needs) search, select, adapt and store resources in their individual banks of resources, they also feed the shared bank of resources and this collectively stored resource also aids the renewal of the individual teacher's resource system. The two documentation systems exert mutual selection pressure on each other (pp. 289, 294, 298 and 299). This process of mutual selection is cumulative, rather than final, leading to constantly evolving and transforming teacher and collective documentation systems, wherein "*equilibrium is an exception and tensions, disturbances and local innovations are the rule and the engine of chang*e" (Cole & Engeström, 1993, p. 8). These local innovations (driven by the maths hub, the introduction of a new National Curriculum and the growing availability of digital resources) in mathematics teacher's practice (Engeström, 1987a; Núñez, 2009) could possibly illuminate opportunities for improvements and change in teacher's individual work and the collective work with resources. In a way, there comes a point where a subsystem (i.e. resources on fractions) of a teacher's documentation system is 'in sync' with a related topical subsystem in the collective documentation system of the department. This research suggests that there is a co-evolution of two intersecting documentation systems (p. 299) and their interactions seem to sustain and reinforce the teachers' autonomy that appears to be characteristic of schools in England (Siedel & Stylianides, 2018) and the professional and government-approved encouragement to work collaboratively. What this further suggests about teachers' underlying beliefs about mathematics is that growth in mathematical knowledge and teaching practice is both an individual responsibility and a collective pursuit in a context of knowledge-building communities of mathematics practitioners supported by their use of technology.

Thirdly, research findings reveal instances of 'emergent (in lesson) task design and planning' afforded by access to multiple resources (10.3, p. 277). Mathematics teachers in this study use various categories of resources: visualisation tools; data-capture tools; data-handling/display; and personal/collective resource banks to provide a variation of tasks and give real-time feedback during the lesson. The e-analysed feedback appears to support the teacher in making informed choices and decisions and enables the teacher to modify the instructional practices and adjust the level of difficulty of the tasks as experienced by students. The research suggests that digital resources could enhance formative assessment practices, a possible extension of this might be in enabling the teachers to adjust tasks and modify the learning sequence mid-lesson.

Fourthly, the use of social media[182] for mathematical teaching practices, especially Facebook and Twitter, appear to be growing (subsection 7.2.3.1, p. 198). The evidence from the study shows that social media creates a collaborative and sharing space where mathematics teachers develop a sense of being in a "massive live staff room". While previous research (Goos & Geiger, 2012) identified resources enabling online teacher interactions and collaboration, my study extends that finding by suggesting that Facebook and Twitter not only enable collaboration but offer opportunities for face-to-face and online partnerships that enable the building up of a community of mathematical practice where the appropriation and use of digital resources are encouraged.

11.2 Contributions

In Chapter 10, three findings were discussed with the potential of making theoretical contributions to the literature on DAD and contribute to our understanding of the mathematics teachers' formative assessment practices and how they operationalise the ideas of variation and differentiation in the classroom.

First, the findings reveal that teachers undertake a range of formative assessment (FA) practices including diagnostic assessment, self-assessment, students' peer assessment, group-based assessment and whole-class discourse (section 10.1, p. 255). The mathematics teachers' FA practices are supported and enabled by a growing availability and use of digital and non-digital resources, and banks of freely available mathematics tasks and worksheets. In this, the teacher's resource system also plays a vital role as they strive to make FA practices an integral part of their teaching strategy.

182 https://www.ncetm.org.uk/resources/53170

Second, the research shows that mathematics teachers' collectives exist at several overlapping levels (subsection 10.4.1, p. 280). The teachers' collective is constituted *by context* (institutional and prescribed), *by access* (open and voluntary), *by mode of participation* (face-to-face and online), *by forms of organisation* (formal and informal) and *by geographical region* (Yorkshire and Derbyshire). Teachers' participation with and through resources occurs in a complex intersection of various loosely or tightly connected virtual and face-to-face networks. It appears that the departmental micro-culture, ethos promoted and resourcing of mathematical teaching practice play significant roles in the existence of the collective. There is also the goal-oriented patterns of teachers' interactions with resources and the collective collation of an ever-evolving shared bank of resources, which are features of an established and thriving community of mathematical practice. These findings contribute to the understanding of the development of the DAD theoretical framework from the context of mathematics teachers in England. Central to DAD is the idea of teacher's individual and collective work with resources and how this might impact on their practices. My findings broaden the understanding of how teachers constitute themselves into collectives and highlight the significant role that digital resources play in sustaining the collectives and their genesis into communities of practice.

Third, the research also found that there are different ways in which the mathematics teachers appropriated and implemented the ideas of variation and differentiation in their classrooms practices (section 10.7, p. 302). In terms of variation, two themes capture the way teachers applied the concept of variation in their classroom practices: *the variation of tasks* and *the variation of resources*. For the teachers, the variation of mathematics tasks demonstrates an underlying mathematical structure of equivalence and a dependency relation, that is, when one variable changes, another one also changes in ways decided by their relationship or structure. In terms of the variation of resources, through the use of multiple representations, resources are matched with tasks to aid students' engagement.

Differentiation, as a teaching strategy, featured in the case study reports. For the teachers, differentiation is a flexible way of proactively adjusting teaching and assessment practices in response to differing students' needs or interests. The teachers sought to achieve this in three ways: through differentiation by tasks, differentiation by the level of support, and differentiation by ability groups and tiers (section 10.9, p. 310). These findings contribute to the literature on variation and differentiation, especially in highlighting the differences that exist between the theory and how teachers operationalise these ideas in the context of everyday practice.

11.3 Recommendations for Mathematics Teachers' Practice

On a practical level, I believe this research has raised the awareness of a wide-ranging set of resources that are freely available to mathematics teachers in schools and online, the educational value of these resources and what they could mean for their pedagogic practices. This awareness might suggest to mathematics teachers in England and elsewhere that they develop a reflective mindset and quality assessment criteria in their selection of resources that could present a desirable opportunity for students learning quality mathematics. An area of further research is to explore how to develop the quality assessment criteria that teachers need to rely on in the selection of resources and examine the impacts of these resources on students' performance.

This research offers an inventory of resources and the factors that encourage their use. This could form a valuable starting point for teachers across schools and contexts to share best practice from each other and develop possible ongoing collaborative interactions that could benefit their pedagogic practices and improve students' academic performance.

The regular modification, creation and designing and re-designing of resources by mathematics teachers illuminate the opportunities available to enhance the creative capacity of both individuals and collectives over time. Mathematics teachers' ability to create resources has become one available skill as they continue to individually work on resources and share in the expertise with the collective in which they participate. This expertise in designing resources appears useful in the community where the teacher belongs.

This bring me to the pressure of time in this context, wherein teachers are fast becoming the key designers of their teaching resources. The information from this study on the creation of collaborative and collective resources could offer teachers an opportunity and patterns of working together and sharing suitable resources to enable them cut down on time and energy spent recreating resources that are already available and accessible. As Kitty stated, "*it is saving time not going to create your own questions",* and Jose similarly reported, "*technology saves time, rather than creating more complications"*. Gray stated, "*in an ideal world we all want to make our own resources for everything, but there isn't time*". This is consistent with previous research on how the presence of digital resources could free up or constrain teachers' time (Assude, 2005; Ruthven, 2009).

11.4 Recommendations for Further Research

This doctoral research, as stated earlier (section 1.3, p. 6), is geared towards understanding in a holistic manner the mathematics teachers' appropriation of digital resources, the impacts on their classroom practices and the implications for their professional growth. Furthermore, the other goal of my research is to contribute to the ongoing discourse on issues and challenges surrounding the appropriation of digital resources. Unfortunately, the qualitative nature of my data did not allow me to generalise the findings of this research over a wider population of mathematics teachers, but it offers a possible 'working hypothesis' for understanding other similar cases in England and elsewhere.

There are interesting implications concerning the mathematics teacher's interaction with digital resources and teacher education. First, this study identified ways in which resources shape and are shaped by mathematics teachers' classroom practice. Complementary study is required to investigate mathematics teachers' interactions with multiple resources and the consequences for students' achievements. There is a need for a more focused study that could further illuminate whether teachers' objectives for students' learning outcomes are being met. Such a study would investigate the impact of the proliferation of resources on students' performance.

Secondly, further study is required to investigate the extent to which mathematics teachers' practices with a wide range of digital resources influence, or do not influence, a teacher's professional development. Certainly, at one level they do.

Third, one of the findings of this study shows English secondary schools will incorporate the adoption by mathematics teachers of the Shanghai/Singapore mastery teaching approach in a school context where technology and digital resources use is supported. The extent to which the specific English context and customary practice modifies and affects the mastery approach and uptake could be an area of interesting investigation.

11.5 Limitations of Study

My research has certain limitations that arise due to methodological and design issues. The case selection was done following purposive sampling, which implies that cases selected were based on certain characteristics and availability of the teachers to participate. For instance, the teachers selected are in schools where technology and digital resources use were encouraged and supported. The purposive sampling of the teachers may have introduced some biases. In particular, six out of the seven teachers held leadership positions (as head of department or lead teacher in the school) and the research took place in two geographical areas of England (Yorkshire and Derbyshire). It therefore appears to me that these factors do limit the generalisability of the findings, but not their validity.

Insight into students' use of resources could provide a means of comprehending students' own ways of learning mathematics. The way teachers intend and instruct students to use resources may not be the ways students perceive and use them. For example, in school B, it appears students prefer video learning support (Maths Watch) to using textbooks or paper-based study (see subsections 7.2.3, p. 195 and 7.4.1, p. 209). I do believe that mathematics teachers' knowledge about students' use of resources is an important aspect of teachers' awareness and professional knowledge. Unfortunately, the focus of this study did not cover teachers' knowledge of students' use of resources nor did the nature of my data allow me to determine whether this is the case in the context of this research.

11.6 Autobiographical Reflections

Undertaking this doctoral study has been an invaluable research apprenticeship and wide-ranging learning experience. I have gained and deepened my understanding of the nature of research and of the recurring nature of the research processes. It has expanded my knowledge of various theoretical, methodological and practitioner perspectives on mathematics, technology and digital resources across various schools of thought. I have learned, for instance, that research designs and plans seem only to fit perfectly on paper at the planning stages but that things do not fit neatly into categories during the actual engagement with the research and that research can be frustrating, takes unpredictably turns, and that unplanned events can disrupt set targets and timelines and that it is sometimes tedious, yet at other times immensely rewarding, enjoyable and even exhilarating.

This doctoral study and the opportunities to work with my supervisors and several teachers in the context of their everyday practices have also provided some key ideas, which have helped me examine my own professional values, background, beliefs and guidelines for possible changes to my own envisioned future research plans and practice. I hope to be able to adopt several key recommendations I have picked up from my supervisors, colleagues and others in the mathematics education research communities in England and Europe as added impetus to explore further the impact of mathematics teachers' use of digital resources in teacher education and professional development programmes I will be involved in over the coming years.

The doctoral research process has also encouraged me to look back home to Nigeria where my interest in mathematics education research originated. It is hoped that a research partnership can be found whereby I can share experiences and my growing expertise and some sets of transferable skills developed with other teachers and trainees. With my knowledge of the growing phenomenon of freely available resources, I hope to be able to point teachers and researchers towards those resources in the hope that they will be of value to their practices. Above all, it is my hope, that these research findings might contribute to enhancing the ongoing campaign to adopt school-based mathematics teacher education and professional growth practices.

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APPENDICES

Appendix A: Ethical Consent Protocols

CONSENT TO TAKE PART IN MY STUDY
PHD STUDENT, SCHOOL OF EDUCATION, UNIVERSITY OF LEEDS

	Add your initials next to the statements you agree with
I confirm that I have read and understand the information sheet/ letter dated 02/03/2015 explaining the above pilot study and I have had the opportunity to ask questions about the study.	
I agree for the data collected from me to be used in this study.	
I agree to take part in the above research study and also aware of right to withdraw should the need arise.	

Name of participant	
Participant's signature	
Date	
Researcher	
Signature	
Date*	

*To be signed and dated in the presence of the participant.

Appendix B: Mathematics Teachers Interviews Transcripts

Kitty: First Interview Transcripts
As a mathematics teacher how do you access the resources you use for your teaching? #00:00:22.2#

Great question, eeee mmm I think the most important thing is that the students like using it, they are responding, it feels like they are learning, and they are on task. Then that's is an indication that it is a good resource to use. Eeee mmm [are you talking in terms of technology or just in general? In general] seem I like, I think it is important to use a range of resources, So you know I quite, I mean most lessons I will have some worksheet with task on, set out there obviously a worksheet can be reviewed and refined as you going through, I like to use iPads or laptops maybe once a week depends on the learning need of the group, and if there is a suitable task on there for them to use. I quite like task that get the students up and moving around the room, so there is some great, like Maths trail we use, like a treasure hunt activities and things like that, or sometimes relay activities which is really interesting. And I think You got to have a variations of resources just so that it keep students engaged, because if you using the same thing all the time, they get fed up with it, you get fed up with it, sometimes a worksheet, some time it more suitable on iPad so you can check the learning with the students. So, I think it is important to have a variety #00:01:56.9#

MU: How do you get these resources? #00:02:06.3#

KITTY: It depends really. what I would do, I get my learning objectives, and then I decide on my resources, I might speak to a teacher and they might recommend the resources they have used in the lesson or they might have seen. Quite often I go onto our shared resources within the department which people have collated, some have created themselves, some have researched on the internet or some they have just picked up from previous schools and we collate those into topics and that is usually my first point of call. If at all in the lesson before there is something that's worthwhile, I will do that. Quiet often I use the TES website, which is got resources from teachers across the country, I will use those in my lesson. Sometimes I will show a video, use a YouTube just to start a lesson and get the students engaged. Sometimes if I can't find anything in the first 5 or 10mins of talking to teachers or searching the internet I tend to create my own which is suitable for students learning. #00:03:11.1#
MU: Are the usages recommended by department/ #00:03:44.0#

KITTY: Sometimes its talking to teachers who recommended it. Sometimes it being on CPD class, for example on Monday night diagnostics questions.com was demonstrated to us and that was shared and am trying to share that with the rest of the department. so, it really depends, and it is quite a range of factors. #00:04:16.5#

MU: How has the resources affected your teaching? #00:04:30.5#

KITTY: I think as a teacher you always struggle is a never-ending job. If you could find resources that are ready made and are suitable for your students, or that you just have to adapt, it's going to make your life a lot easier when you got 5, 6 lessons to plan. In terms of my planning, it makes my planning a lot quicker, I am now in my 5th year of my teaching, I kind of have my favourite places to go, I know places that works. For example, I quiet often use the staff shared resources on our shared computer network, I quiet often use a website called Maths box for starter, it is a quick skills test, if you like you can give out to student it really being efficient in term of your planning. That's side but also, I think the impact, it is important that your resources are suitable for your students, it got to have a good impact on students, there are times I use resources and I will say am not going to use that resources again because it didn't work. I think it is that practice really, and because I have been teaching a bit longer then or less long that's why I have a go to websites. #00:06:03.3#

MU: How does your use of range of resources affects the delivery of the lesson? #00:06:23.9#

KITTY: I think, I tend to use a worksheet when I want students to do a lot of practice. If the student has not grasp something that when I use the worksheet tied to resources as my final line up and we do sharing. When I use iPad in my lesson its quiet often for assessment for learning, getting the information from the student in a quicker and more efficient way so I can react to that within my lesson and also promote engagement with the students which Mentioned at the start. If I know we have had a couple of worksheets lessons and you can tell the students are getting bit fed up, they might still not be getting it, I will change the structure of my lesson to maybe include more cooperative learning and get them moving around. It does have different impact definitely. #00:07:54.2#

MU: Can recall some resources that has become places of constant reference? #00:08:08.4#

KITTY: When I first started teaching, I always really feel the pressure of time and I always use to search on the website for resources. In my first year as a teacher I was bold enough to get the resources and be used to them. As time go on, I think now, you know I have got those resources myself, on my own on USB drive and you can go to those and in the meantime, we might want to adapt. Quiet often I have a PowerPoint that have all spice girls, just need to update to make it a little bit more relevant to the time. I know over time the use of technology has increased massively just within the last 5 years, maybe it's because am not use it. I know I have a lot on my iPad now than it used to be. I feel it is really important the students get those technologies in their lessons. We come across websites like Mangahigh which I use with my year 10s quiet frequently on Friday lesson because they are tired and just having something different and something that is more animated keep them engaged a bit longer while I can still assess the learning. Over time as well, there are more websites out there and we have got more iPad to use among the students, so it much easier now to use iPad with the full class before it was between 2, you have to use group task, it might be more research, it was a bit harder than to track individual progress. I think now quiet often its more independent tasks rather than a group. laptop we always had, but again access to it has definitely increased with had a new building recently and every corridor has got a laptop banks for staff to use and you see staff using them quiet frequently to support the planning and also to support the assessments of the students. #00:10:13.0#

MU: Resources for planning and resources for delivery how do they interface? #00:10:38.6#

KITTY: TES is a shared bank of resources you won't necessarily see that in your lessons. Sometimes you research the pedagogy of what you are going to teach. I don't if you have heard
of the websites Khan academy which has little video clips, so that if you are thinking how do I teach this am not quite sure of this or maybe it’s your first time you've taught it, you are thinking what do I need to be aware of, in terms of pupil misconceptions. In your preparation of lessons you might do a lot of research and quiet often as well you talk to a teacher who I know has taught that topic before, that's will be the first thing I will do to see what they say, they might give me the answer, but if I didn't get the answers that I need, then I probably tend to like use the internet or quiet often the textbooks within school as well, just to practice myself and get use to that new topic and just think about the questions that might come up for the students on that topic. [Probe: It seems textbooks are not in the class?] #00:11:37.6# KITTY: ..e e e m..I think it depends on the school, I do use textbook in my lessons, I use them yesterday with my year 11s, the lesson you observed today was a worksheet lesson, but we do use them, I think because quiet often the textbooks are a little bit restrictive and we have done a lot of research recently into Shanghai methods and Singapore bar modelling and variations within questions. To give you a simple example, 10 questions; 3 add 5, 2 add 4, 6 add 4 in order to varied that, you might have a blank 6 + =10. A lot of textbooks we have in the UK don't necessarily do that, so we do create a lot of our resources or look elsewhere. WE want to make sure that those resources maximize the students' learning. Sometimes the textbook can be restrictive. However, the textbook can be really good, quiet often I will find a couple of good questions from the textbooks, we definitely do have them but not necessarily always used. #00:12:53.5#

MU: Do you modify the tasks? #00:13:21.8#
KITTY: For the majority of the time I have to modify them. I might quiet often have to use the snipping tool on computers to select and then create my worksheet from that, again its saving time not going to create your own questions, but you are still tailoring it to the need of the class. And quiet often you might have two worksheets on the go or more. In terms of differentiating into different learners within the group because some students might get it straight away, some might need more practice, some might think we need to go back a step and just work on multiplication before we do work on algebra. So, I think the majority of the time I do tend to cut and paste if you like. #00:14:03.9#

MU: Within some of the lessons, I have observed that the task is differentiated into Bronze, silver and gold medal or green, amber and red what is the idea behind it?

KITTY: It is great for the teachers, because you can see exactly the student who is performing the best and who need additional assistance. It motivates the students. You saw them today wanting to get about 5 on the questions we were doing. My year 10s as well we are on Mangahigh and they do medals even when they have got a bronze it gives them that energy to try and improve really increased the depth of their knowledge. It is not just finished when you get a bronze medal you keep going, there is that challenge there.

MU: #00:15:10.7# Do you have formal fora to share on teaching?

KITTY: We do have professional development sessions. Last year for example when we were doing a lot of work on the Singapore bar modelling, we had a CPD session every week where we were talking about teaching and learning how to deal with it using the bar modelling methods, yes, we did last year. This year it is not as frequent however we have mentioned as a department that we are learning a lot from the Chinese teachers every minute who are helping us, and they actually meet every day after the lessons. We were thinking maybe we should have more meetings like that were you are maybe observed and filmed, and you looked back. All you are looking at is the pedagogy behind that lesson to improve that particular thing you are questioning and students' learning that comes from that. At the minute we do have it but probably it is more informal chats in the Maths office, how do you teach these, you got any idea, rather than a really formal weekly meeting. I think probably everyday those discussions are taking place but not formally. #00:17:01.1#

MU: Do you have informal/formal Maths groups?
KITTY: We are actually the lead school for the White Rose Maths Hub across the country. I myself actually take part in delivering CPD to external Maths teachers in terms of increasing their subject knowledge, so we are involved with the Maths hub

and that I think has a massive effects on how we teach things for example the CPD that went on Monday about the diagnostic questions was organized with the Maths hubs help, I won't have necessarily have known about that without our affiliations to the hub. We are also involved with the teaching school; we are teaching school. And we train, we are actually training three Maths teachers at the minutes and they are provided with subject knowledge session by one of our more experienced teachers, on a weekly basis and are involved in delivering lessons with Maths teachers in the department as well. #00:18:14.5#

MU: Do you have a Facebook group or twitter?

KITTY: We are on twitter and that is actually massive really in terms of engaging the Maths hub with the wider school. WE do send a lot of emails to the schools about things. Having twitter provides swift, social network to make people aware what's Maths hub is doing, we can post pictures, so they can see what they are learning, what they CPD sessions are like for teachers. Most of it as well, I am on twitter and a lot of teachers that use twitters actually, you see on a Sunday night, they have this EDCHAT#, SLTCHAT and things like that. I think twitter is massive in terms of the teaching world, I think a lot of teachers are on it and I use it as well for my resources. But Facebook not so much, we don't have a Facebook.

MU: How do you exchange with other departments within the school?
KITTY: It is such a large school and we quiet have large departments, we probably don't interact as much, we do have weekly CPD sessions together probably discuss generally kind of teaching methods rather than Maths specifically. We do share ideas. In terms of sharing resources, we might share websites potentially, sometimes we have world Maths day, world English day, world science day, something that we do jointly in the past. But I think it's much harder to share resources because it is perhaps quite unique to Maths, quite unique to English but all the subject does use similar websites like the TES, use the iPad and each department has its own resource sharing folder, to share resources within the department. #00:20:44.9#

MU: What else do you recommend? #00:21:37.4#

KITTY: I think it really important, the collaborations between teachers is absolutely crucial in terms of developing new teachers, from that perspective and then you can share the resources that really work for certain groups and you can share that pedagogic knowledge between staff. I think you see us all in the Maths office working on the computer all the time, technology is a massive thing in terms of support and with resources and I think it is also important that you communicate with other schools through things like the Maths hub and being involved in training teachers because a lot of new teachers have new ideas and it is important that you get those new ideas in and share their experiences. Having meetings is something we need to do more here.

In terms of students' learning it important that they have access to all those different types of learning because you need to prepare them for that change when they leave school, we are not just preparing them for Maths alone, we are preparing them for life really. Technology is a big part of that, being quiet and working, being focused is a big part of that even up to the university those skills are really useful, and it is important to incorporate a lot of things really. #00:23:00.0#

Emilia: First Interview Transcripts

MU: How do you access the resources you use in your teaching? #00:00:39.1#

EMILIA: I use smart notebook for most of lessons, that's is provided by the school and you encouraged to put your lesson on. Other resources I use if it like a worksheet I sometimes tend to use Microsoft word or make it myself or sometimes get them from websites, I google whatever topic and use the worksheet that are on the websites. [Do you have specific websites?] I tend to use like GCSE websites, like Coberttmaths, Keshmaths, maths ed. The majority are the stuffs I do on smart notebook and I do it myself. [Are the other software recommended by the school?] We got iPad, laptops, ICT room and we book an ICT room and we can actually go to that room and use the computers with the students. #00:02:09.7#

MU: Are the other ICT resources you use that are chosen by you? #00:02:54.1#

EMILIA: No not really not at all. #00:03:06.0#

MU: are they particular go to site or resources you go back to frequently? #00:02:53.9#

EMILIA: Just the ones I have mentioned, the Coberttmaths, Keshmaths, maths ed. But I do tend to if I find a good worksheet, I would just save it and I don't have to go back to the websites. So, it is preserved. so, the majority of the stuffs that I use is saved over the years from when I have been teaching, so I don't go back to the websites [so it is a build of resources then] then it just saved on my memory stick #00:03:54.5#

MU: Some of the task do you usually modify them? #00:04:08.6#

EMILIA: If I can but they do tend to be pdf. So, you can't really modify them that is why I do tend to make my own, but If I can modify them, I will probably do to make them suitable for my class. But it does tend that you can't. #00:04:29.1#

MU: What is the interaction within the department what impact has it on your teaching? #00:04:58.9#

EMILIA: We do observations, other teachers will come to observe your lessons and give you suggestions of how to improve. We also have a shared resource where we put in my worksheets and lessons, so that we can share really good resources and we also use email to send them out to each other, say this is a very good resources, you want use in your classes. I also go to observe other teachers' lesson and picking up good ideas from them. It does help and also just talking to each other and asking if there is anything, they have got that is good. [Is there a formal forum to plan together?] No, we don't do that. you just do planning on your own. No, we don't do that, no. #00:05:50.8#

MU: Do you use social networks do they play any role? #00:07:44.4#

EMILIA: No, I don't, have twitter, so I don't go on twitter. #00:07:02.7#

MU: The class categorised? #00:07:44.4#

EMILIA: The are the A band, the ones that do the languages, whereas the B and C don't do the languages. This is according to their ability. #00:07:44.4#

MU: In your class you had a lot of exercises for the students, why? #00:07:51.0#

EMILIA: Just getting them practicing and they are more likely to remember. The idea is that they remember easily hopefully. #00:08:59.3#

MU: Before the informal meeting, do you have an exchange with another department? #00:09:27.4#

EMILIA: I am part of the lead teacher team, we have sort of meeting every two weeks, where we might discuss teaching and learning, I do intend to pick up ideas though not ICT stuffs. [What is your role as lead teacher?] I do go into other schools, and every other Wednesday I deliver a training session to primary teachers by helping improve their maths knowledge, so that they can teach it to their students. I use the smart notebook for that. #00:10:56.8#

MU: #00:11:04.4# How often do us the laptop?

EMILIA: I don't use the laptop that much because, by the time you get them out it takes quite a while to log on, open them, they intend to behave, and it get a little bit worse. I do sometime use them but not very often. #00:11:27.4#

MU: Anything else #00:12:11.4#

EMILIA: You are doing the right thing; you need to get experience in school. But I think it is by building good relationship with your class and been consistent is the main thing for me. Because if you can't control your class and you get bad behaviour you can really get any learning done. It’s great to have all these ICT and all these resources but if they are not listening to you, then you can’t really do much. I like them to be getting on with their work. #00:13:00.4#

MU: What’s the opinion with use of ICT? #00:13:09.9#

ETAF: I think definitely I don't know what I will do without the IWB, like all my resources, have got my sticks and can get the lesson on to the computer, I don't know what I will do if don't have these. It is fundamental, to be honest. I think like laptop and stuff is good to have them and I definitely appreciate them, but I don't think it is necessary. #00:13:39.5#

MU:The colour coding what does it mean? #00:14:01.0#

EMILIA: Red, amber and green. It’s a sort of differentiation. I do worked example with them, if the feel they 100% comfortable with that and they feel totally get it they can go to the green questions. They slightly different from the worked example. I push them a little bit and get them thinking a little bit more. whereas the amber one is quite similar to the worked example sort of repeating but changing the number a bit. And then the red one is sort of a step back and they are a bit easier altogether. The students choose where they want to go. Generally, they go to the amber or green ones, but obviously if they are really struggling, they start on the red ones. The green is the hardest. What I usually do, with the planner they got red, amber and green pages, so I will get them to show me red or green, if they are showing me green then I will go straight to the green questions, they are the hardest ones. If the amber, then I will go to the amber questions. that is just differentiation and pushing them

Jimmy: First Interview Transcripts

0-10mins

MU: In what ways do you access resources for use in class and the preparation of your lessons?

JIMMY: We are really lucky that we have the iPads, I do all of research online, on google for resources for instance, if am looking at GCSE questions, exams questions and things I want the kids to be used to, I will always have GCSE exams questions in my search have a lot of go-to websites that I have bookmarked , so I have a lot of places I automatically go to if I want resources off line, because i pretty always refer to the computer in planning, a number NNNN will just be worksheet or series of

questions or very very rarely some slides that i can easily nab… eee mm but a lot of the time it will be ways in which I can manipulate things on the board. I don't want to do a question and say this is how it work out, I want to add something showing why it works, virtual manipulative is a big thing I research on line.
MU [probe: so, of the go-to websites are they specially dedicated teacher sites?] In fact, rather than, there is such a huge number of places I can go, Caroline our deputy head uses twitter a lot, and share resources with teachers from other schools, I personally have a list of websites, which I bookmarked, so the first is the national STEM centre websites because there are some fantastic worksheets, questions, manipulative and everything I want from there #2:52#You can see I have got absolutely loads and am happy to send you a list rather than….. Every time I find something useful online, I tend to bookmark that website and go back to it.

Besides some of these worksheets and materials you use, the digital or ICT themselves how do you come to use them is the school deciding, we are buying iPad or you recommending or making your choice, how do you arrive at the choice? #00:03:17.2# #00:03:29.3#

JIMMY: This is my second year teaching here and one of the reasons I did come to the school is because they have that already here, they have the IT room, they have the laptop, this school, the Maths department kind of already leads the way in results and in terms of using technology because we have our own set of iPad we are the only department in school that does , I don't think that is common in other school for just Maths department to have technology, I think a lot of it has come from the leadership team being aware of tech being use outside of school and saying it's clearly having an impact we should get involved, and this what a lot of schools has done, where they have said if we can get the funding this sounds the great way we should go, great way we can go #00:04:35.2# i think the iPads were a late addition and the advantages of iPad far outweighs the laptops in my opinion. #00:04:49.1# You have a range of ICT tools but it up to the teacher to use a particular technology how do you make that choice? #00:04:54.9#

JIMMY: I often book the iPads if i want to do work that available online, i prefer the iPad because it is less time wasting than grabbing the laptop waiting for them to log on, students tend to take a bit more pride and respect #00:05:19.2# ,that have a bit more responsibility because they understand iPad are very expensive tools than laptops. They can often arrive in the students' hands, they have a bit more respect for iPad. also the great of thing i do a lot of is rather than saying students go to this websites here is URL, you need to type this in, I use QR code all the time #00:05:56.6# so you saw in my in my lesson earlier i had the student scanning for different work, which means they can all have their own work in front of them on their own iPad, they can zoom in, the can look at specific questions, I have often in the past had other QR code to support, so if you don't get this worksheet perhaps you should go to this website #00:06:04.7# they would have the QR code and they can scan that and they can do it independently, and they have got support there. Personally, I prefer to use the iPad. if it is the case of iPad or paperwork, if it is seemed quick question where ones I don't need them to do lots of work, I don't need then to do rim and rim of working hour, I often use the iPad.... For them to do the questions without me having to intervene too much, we use Mangahigh quite a bit. I also use a website called Studymaths.co.uk, again its 10 questions on a topic and the students can answer themselves the questions are randomly generated so they are not gonna be able to copy off each other, it marks it, so the student knows whether they are getting it right or wrong. So, there is plenty of ways I can use the iPad and its advantage over paperwork. I use paperwork if they need to do that working out, if they need to practice, for exams techniques and legibility #00:07:10.0#

Are the sites self-discovered or shared among yourselves? #00:07:45.2#

JIMMY: This department is brilliant at sharing things. With such a big department I don't think we will work very well together if we didn't share. Mangahigh for instance is a subscription website the department had to get involved with that altogether, I know Mr. Geoffrey came across it, before I was here, this year they have really gone into it, it is a paid service.... we need to make most of it. study Maths is something I have discovered personally, I have not really shared around much, it’s great for marking and great for short sharp one-mark question it is not brilliant for developing the understanding, simply than demonstrating it. Whereas Mangahigh it helps to develop it because it offers hints, it changes the questions, if they can do the easy one it gives them a harder one. So, a lot of the technology resources we do share, there are certain ones that individually used. #00:08:31.9# #00:08

MU: What are the impacts of the digital resources on you're planning and teaching? #00:09:03.6#

JIMMY: Planning can be made a lot shorter. If for instance I feel I need to make my own questions that will often lead to me doing some book work. It can shorten planning for you to use technology because the questions are often already made. For instance, going back to Mangahigh and Studymaths those questions are already there, and there are quality questions. The Mangahigh is fantastic website for adjusting to the level of understanding of the student. So those questions are varied, they are level appropriate and I don't have to plan and I don't have to create those questions, I have to make those questions accessible that is the teaching part, that is the teacher-led part but in terms of assessing whether they understand it, I feel those tools are far better than me simply having a series of questions. So, it can really impact on planning time, eem I think it also make the feedback loop much shorter #00:10:08.4#, rather than saying here is homework you going to do am going to make it in the next three weeks and then you find out if they didn't understand it. Within the lesson I can assess them, I can find out what their weaknesses are, I can help them with those weaknesses, I can then assess them again and see if they have understood it. And I can proof if they have made that progress all within one lesson #00:10:35.2#

10:35- 20:35mins

MU: Some of the resources from these dedicated sites, do you modify some of the task? #00:10:48.9# #00:11:18.3#

JIMMY: A lot of the tasks on Mangahigh, I can adjust the difficulty of the questions, it just so automatic. I only use online materials if am comfortable. a lot of online materials you can't adjust, you can't change the questions. you have to be happy with those question before you ever consider using them. We use an apps called Socrative a huge amount. Which is entirely based on you creating questions, You can create a series of questions, different types, you can do multiple choice, you can do short worded answers, you can do true or false, there is a huge variations of questions you can do, and you have complete control over what those the questions are, but it is incredibly powerful, but it is more planning time. so, you do have that quiz often ready before you even come to the class, but it is incredibly powerful because as with the Mangahigh its marks, it marks everything instantaneously, so if the vast majority of the class are getting certain questions mark. I can immediately respond to that... #00:12:18.1#

MU: With ICT and some of the digital resources what has changed for you as a person and your teaching? #00:12:42.0#

JIMMY: When I first started teaching, I started my ITTA in two Septembers ago, 2013 that’s when I started teaching, the first school I was didn't have access to iPad, or laptops for the students, there was computer room, but three in the whole school. The only instance of technology was IWB for the teachers to use. I think Starting there, made really obvious how I can use IWB better than standard typical IWB, so I got to using that very quickly , but it did mean all the work we were doing was as a class or if we were doing individual paperwork, I think that can sometime mean that all the work must be more engaging and interesting, I can think straight back to two lessons in particular, perhaps the most engaging lessons I have ever done, on how many more teachers could we fit into this room, just a volume lesson, where they have to actually get out of their seats to measure things, again you could use technology could aid that but I don't think technology can make that lesson any better. #00:13:56.9# Another lesson I can think that technology perhaps wouldn't have help

was a lesson on bearing, it was a flight plan , you left at certain time, you have to get from London Gatwick to Brussels and you have to do basically a loop around Europe, they have to draw their path, accurate, then the speed, distance, time measurement based on what distances the are travelling, that was great, but I don't think technology would have helped. In that case when I started the lessons in some cases took a lot more planning to make it. To seat down do lot of questions. do a lot of activities something to make it worthwhile. I think a lot of technology now, allows for me to change the way I teach things, so for instance there is website I came across earlier which I will be using- dream-box, dream box is all about visual representations, in fact the task that I came across was aimed, I believe third grade, an American website, about identifying decimal on a number line seem pretty simple you could do that on a white board or you could do that on a piece of paper. But what was brilliant about the way the website did it, was you could zoom in and out, you could move left and right, so immediately what happens is that they give you a number line which shows between 90 and 900, and they say put 3.3 on the number line, so immediately they say I can fit it on there, whereas if I have do that on the whiteboard, it will be lets rub that and start again. This is actually manipulating that on screen, so you can move the number line along and say this number continue forever, let’s find the number that is appropriate for us and you can zoom in a bit and say this what am interested in now. whereas If I do that on a WB I will have to start again, draw another line, which takes away from the understanding. so In terms of my approach I am looking to use a technology which makes my explanation clearer , eeeem and that’s what am close to and push for now, in terms of planning, that is going to be easier if I can find things like that, in terms of changing the way I teach, my planning is changed completely, I don't want to give them a series of questions, I want to give them something to put in focus, I want to find tools which does it better than my immediate expertise #00:16:36.1#
MU: Task, examples, exercises different names for the same thing? #00:19:16.8#

JIMMY: Personally, I will say an example is something that is more teacher led or something that students can refer back to and say this how it is done, I want to use this if I want to do a question similar to it. Task I think can be varied, the task could be measured, the task could be do these questions. I would say, exercise personally for me refers more to that very thing, doing these series of questions, exercise textbook 15A answer questions 1-5 its always what is called in the textbook. A task can have lots more variations in what are asked to do. Example is something that demonstrate how something is done. #00:20:11.1#
MU: Do you modify your task before use with digital technology in lessons?
JIMMY: I think I always plan the lessons ahead. Is it going to make my life easier as a teacher? Is any of these questions appropriate? I will not use technology if I don't feel like they will benefit from it. I think the way I use technology I have to assess whether, first are the questions am finding quality, are they good quality, could I make

these questions better; secondly of are the students going to be given feedback as quickly as possible. And I think often with the resources I use, yes, it is almost instantaneous or a lot quicker at least than if they are textbooks to mark. And I can mark a lot more. Finally, will they know the material after the instruction or even prior to instruction will they know the material that is coming up. The online questions I don't want to teach, follow them to do online questions, I want to teach them for understanding, do those online questions assess their understanding, so that what am looking for, whether the questions are appropriate for their level, whether the students can actually do it after I have instructed them. #00:22:34.8#

MU: Do you belong to any formal Maths teacher community? #00:23:04.6#

JIMMY: Informally I follow a lot of people on twitter who share fantastic resources and others in the department do as well, and in fact when they find a question that they think is quality they email around the whole department. This worth using. Again not a formal community by any means but the students who did TT training with me and my colleagues who are now in other schools we have Facebook group, we talk about things we are going through, perhaps even share things, if there is someone panicking , they can just write help, I have this lesson on this topic and I don't know how to approach it has anyone got any activity or any material that one can use to explain it, there is a bit of sharing there but no forma, no formal... #00:24:00.1# ....[Probe: Within the department besides meeting in the department, is there a meeting on a chosen topic or?] #00:24:23.9#

JIMMY: The school has always run a CPD program; but Last year we did have sessions where it was in the department rather than the whole school, so rather than saying everyone let’s all talk about assessment for learning or everyone let’s talk about marking and feedback, Maths then got together, we want to teach for understanding using concrete, pictorial and visual method, let’s do some seeing on this, there were led by Paul Rowlandson, one of the assistant principal, and we just all talked about methods, and we got together saying how could you improve this method, how could you explain this better to students, so yeah we had something you could have as a formal meeting within the department once a week, we use to have that, but now there an informal conversations, I need to work on this how would you explain it, it is always questions within the Maths department, you got anything on this but we share a lot informally through email... #00:25:39.0#

MU: Besides what happens within the Maths department is there a sharing across departments? #00:25:53.2#

JIMMY: We do weekly CPD, where it could be a general focus on let’s say assessment for learning, marking, it is not always technology based.... However for instance yesterday, I gave a presentation to all the members of staff, we are doing teach-meet, where members of staff from all over the school, all the different experiences have to demonstrate their idea and many of those a technology based and I did one on a website called diagnostic questions basically is a really really big improvement on multiple choice questioning, another one was done by member of staff talking about an apps called; Plickers again technology based and makes life easier, its again multiple choice, it scans every single students answers immediately and it saves them, once they have done their ABC cards and they put them down, you haven't just forgotten them you got them in front of you saved, that is an incredible tool. But there none, specifically saying let's just meet and talk about technology. Last year there was one CPD session which was on using technology. No specific technology but just to give and hear a lot of teachers' views on technology #00:27:26.1#

MU: #00:28:05.5# ON THE LESSON OF TODAY

JIMMY: The first thing that was really important to me, was getting the kids to understand how I was using that technology. We did a mini assessment just before the holidays. You saw on the slide I had a google form, I converted it into a spreadsheet for me to see and I then shared that spread with them, I took the names off, because I don't think it was appropriate for them to know who got what but i shoe them exactly what I was getting, I feel that's incredibly powerful, because suddenly the results are there for me and for the student, they can now understand exactly why I have to teach them a topic they won't necessarily getting before they know they have been taught it they don't really understand it, so they would say whooah we are doing this again, because all the tests you all got these questions wrong, so we have to do these again, and I think that incredibly visual and incredibly powerful for them #00:29:01.0#, you saw I had them identified which topics should we not teach again, because I don't have to teach you the ones you know already, that was incredibly powerful. A lot use of QR code today, different tasks. And the students I think engage a bit more, rather than having, if I was to have those question on the board, I could have done, I could have squeezed them easily on the board but they would have just switched off, they wouldn't engaged with the fact that there is now 15 to 20 questions, on the same thing on the board. I think by having them understand that if they can get this correct, they can then move up, which means we are now working on something more difficult and there is an improvement. #00:29:45.9#

MU: How does the outcome of this lesson impact on the next in your planning?

JIMMY: Writing decimal as a fraction. In terms of the results I can now look back into their books, I can mark that, that's why I always mark towards the end of the lesson, just to get an idea at where to go. They also towards the end did a Mangahigh task which will then on the teacher's VLE, it will tell me who really understood it and who has done enough practice on it, some of them may have a couple of minutes but I can tell from the questions they have answer correctly on there, who has made fundamental error, how that should feedback into my next lesson, so if there is any glaring error that many of them made. For instance a lot of them wrote that 600th, was not 0.6 because they took it to mean that once the denominator was 100, they would just write that number, which is a fundamental misunderstanding which I think I have ingrained in them now but from using the technology in Mangahigh, from me marking this book, and more work on different things, I think I can now feed that back into my next lesson. So Rather than being stand-alone task, the results I have obtained from using that technology has given me something for the next lesson; I probably won't go back to that same format by saying you can do these bronze questions, this silver questions or this gold questions, I probably won't go back to that same format because it is very very specific errors many of them were making which only became apparent when I started marking the books unfortunately #00:31:48.8# Once the technology is there has stand alone for the bronze, gold; the Mangahigh task so they can have a bit more time of the Mangahigh task and they can really show me how well they have understood it and those results will now feedback into my understanding of where I need to go next #00:32:06.7#

MU: Anything else to add? #00:32:34.8#

JIMMY: I think technology is proving to be incredibly powerful and incredibly engaging for the students in the UK because they are surrounded by it. They are surrounded by technology whether we like it or not. And its they way the gonna live, I got my mobile phone in pockets, I can get it out when am at work in case I might go to twitter and look some Maths questions, I got my iPad I can check my work i it, I use it all the time regardless whether am using for task or not and that's is clearly having an impact in the UK where the children grow up. They have so much more online presence but then before technology, before it was so widely available, before there was IWB in classroom, when I was in school and I still remember the first lessons we had on IWB and I say what's is the point, all you are doing you are writing on the board and you can rub it out which you can do on a normal white board just a bit less mucky, so I think the uses of technology can make learning incredibly more immersive but everything can be done without technology, there are certain things of course technology can do far better, you could not do the calculation with precision ...without technology behind it. In terms of learning, in terms of learning the fundamentals, I think the curriculum is built around knowledge which you can get without tech, tech can enhance the learning and make it much deeper and make much better mathematician and much better people. I don't think anyone should say we can't learn without it. It is not ultimately necessary but when I think of planning now, I have become so used to this technology, so used to this ability to delve into topic in

so much more details, with the dream tools, virtual manipulatives, I can zoom in on a number line, I could never do that before, I can only get a whole number line and draw on the floor, think about getting 30 kids around it, you couldn't do that I think it will be absolute chaos and the students would not get as much out of it. It can be incredibly powerful, but it is not wholly necessary. #00:35:38.3#
Thanks

Jose: Extracts from First Interview (*in the order of appearance in Thesis***)**

Before I started teaching, I was not tech savvy at all. But what made me to start using iPads was the lead teacher in the department at that time. He was in charge of the iPads. In a few CPD events where he introduced the iPads apps you can use in the classroom. And simply because I was told what to do, I felt I was confident enough to try it. And what I really like about how he introduced it to me, it was about, you only use it when it makes it easier for you. Technology saves time, rather than creating more complications (#1intJs1:00).

The mathematics department has a huge, huge influence. Every single thing I do I can point to a different teacher that has influenced me. For example, in technology, *Mr. Mill* completely; he is the only reason I use iPads, he is the only reason I use laptops. And then again *Mr. Stan* is the only reason I use IWB regularly and assessment tools. It is not through anything formal, just occasionally seeing him teach, in passing, talking to them about something but sometimes it can be CPD events. (#1intJs9: 40).

In terms of getting resources together, nearly all teachers try going on TES and downloading resources from there, but I cannot stand downloading those because it is so difficult to find something appropriate for your class. And I think if I find something that was good, I will be doing my class a disservice because I will be trying to wangle their learning towards that resources rather than what they needed. So, I try to create my resources myself (#1intJs 2:50).

Say, I am just starting a new topic and I want to create my own resources. I will start with the school scheme of work on the system, look at the typical questions they ask. I might then look online just to see roughly where the questions are going, maybe I might look at online textbooks rather than something like TES, which is a compilation of teacher resources. I will look at that and then tailor to my class (#1intJs 3:54).

What I do, I will have one resources for expanding the brackets for year 10. It will be perfect. It starts where they need to start. Then I will now use it next year. I might have middle set year 10, what I will do, I will keep the slides, add bronze questions, add silver questions, maybe I will add more gold questions, so that my new year 10 can start about the half-way down. stress themselves to get the gold questions. Take a resource, expand a little bit so they fit the class, a little less work than the previous year (#1intJs14:33).

I will close that down and I will think of the steps I will need to get from where the students are to where they need to be (#1Js4:35).

I just remembered something I use quiet a lot, using twitter and following Mr. Barton. Because Mr. Barton is a well-known mathematics teacher and he tends to retweet anything that is actually good on TES. I usually follow him on twitter and pick up resources that he is tweeting from TES (#1intJs12:34)

TES is heavily relied on nowadays. People definitely use them a lot. I think I am slightly more or probably an exception. I tend to use things I have created, which creates a lot more work. But what it means is that I can re-use them year after year because they are good enough (#1intJs 13:04).

iPad and various iPad's apps; Plickers, Socrative. I use this thing called iKnow my own class survey which are online surveys for students to fill in, they are mainly to do with asking students their opinion on how they feel about their learning, what they like about mathematics and what they find difficult. I have used laptops for Maths Watch which is just series of revision videos for usually year 10 and 11 to watch while they revise their specific topic (#1intJs1:30)

We meet 10-15minutes a week. We often put resources into a shared area in a computer system so that everyone can access them. There are supposed to be the best resources that can be used by everyone and shared by the department (#1intJs10:30)

Interviews Transcript from School B
Gray: First Interview Selected Extracts

Our scheme of work is based around a package we have bought in from a company called Pearson or the owner of Edexcel who the exams board are. They provide a lot of materials, both paper-based as well as electronic version of the textbook that we can use. That is called Active teach. Yes, active teach. There is also something called active learn, that the students can use, that again is also electronic, it is all based on the internet. We can set them homework assignments, they can practice things, practice the skills and they can also click on little video clips to be able to help them. And we using that with years 7, 8, 9 and 10, they are the ones that are doing the new GCSE and following the mastery curriculum. (Graham #1: 00:42-1:48).

Personally, I also make YouTube videos, but lots of teachers make YouTube videos, so, we will also suggest to students to look on there find a tutorial that will help them.

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We let the students know through social media really, once they are aware there is a bank of videos available, then they can go themselves, it is very much independent study. I regularly go around classes and remind students, don't forget if you get stuck, you can watch my videos or anybody else's. There are a lot of things out there (1intG: #3:41).

Every classroom teacher has a laptop, every class we teach in has a PC in the room connected to interactive whiteboard. So, we have all got interactive whiteboards with computer, we 've all got laptops. I have also got iPad but that is not something everybody has. Over and above that, everybody has a mobile phone as well, and most people have their mobile phone connected to the school's email system…. Some teachers have also visualizer in their classroom (1intG: #8:45-11)

There are three or four places I will go to straight away. Do you want to know where they are? [Yes]. There is a website called Resourceaholic, lots of resources for key stage 3 and key stage 4. A level, I will go to a website called Douis.net and I know the resources there are same spec that we use… The TES is very good, and again from an A level point of view, individual persons like SRwhitehouse, brilliant resources and you know that the quality is good (1int G: #7-8).

Gavin: First Interview Selected Extracts

My position last year, was what you call senior lead practitioner, so, I work within Maths, but my responsibility was to improve teaching and learning across the whole school. And I was allocated staff to work with to improve their teaching. Maybe some of them weren't getting a consistently good judgement by the school leaders, so I work with them. And occasionally we have to lead certain parts of the whole school CPD. (1intGn: #20:21).

In terms of resources for the lesson, my first point of call is to look at the textbook. Because the people who write the textbook also write the assessment. It is important that students are familiar with the language of the textbook, so that they are familiar with the language of assessment. The textbook itself is really really very good especially with the videos embedded in it (1intGn: #00:27).

The dashboard I use has hyperlinks all over the place, it has a significant benefit to me. I think I use ICT different to other people. I think a lot of teachers are still using their planners for planning (1intGn: #10:40)

Within the department and individually we don't have a central bank of resources where information is, it is what you find, see what you find is suitable. It is a brand-new scheme, it's a brand-new specification, so there is not a great deal out their tailor-made for that yet. More and more are coming on stream (1intGn: #1:20).

Richelle: Second Interview Transcripts

MU: In what ways as Maths teacher do you access ICT or digital resources for your teaching, preparation? Is it recommended by the school?

Richelle: so how I use to plan my lesson and teaching lessons. The school provides every single teacher with a laptop, hmmm so we have that resource available to us and then because we 've got IWB smart notebook in every single classroom, the laptops is connected to smart boards and there is a software package that is install in the computers and so that we can plan lessons on that software and it come up in the IWB

I think though in maths, the department use the MS software, I know in school most people still feel safe with using things like power point, we use this smart notebook because there is a specific maths tools in it, which have got things like protractors, shapes, rulers, graphs, loads of diff tins so we can manipulate things better than power point.

1:20 MU: beyond that are there other resources apart from what is recommended by the school or by the teachers you feel need to bring in by yourself personally to help

Richelle: I personally have started with brought in using iPad with maths apps on, so we have got a couple of TAs who do interventions for some low ability student in the morning and go round pick up students and seat with the iPad and do similar work on there, it makes it quicker the have planning, take the topic and go thru on the iPad, they are quiet good, we have also brought in visualizers, which we use a lot for the exams questions, at the moment we been using to do transformation because as a teacher rather than demonstrate everything on the board where the technology might be complicated they can just have a piece of paper in front of them and the visualizer just watches what teacher is doing on the paper and puts on the board for the whole class to see, so if you answering question or using a compass, ruler or protractor to constructing something this whole class can watch you do it in big instead of gathering the whole class round the table see what the teacher is doing we find them really useful, we got two of those and we actually trying to get a third because we find them so useful.

2:48 MU: the use of these apart from the one coming from the school, the Visualizers, you are using as colleagues within the department, have there been any suggestion?

Richelle: visualizers were our idea, we did not have it originally, we found it and put it, in terms of digital resources we have not thought of anything else

3:20 MU: Some of the ICT/digital resources you have, how do you it? For delivery or what part does it play in your preparation?

Richelle: we use internet a lot for preparation, putting the plan on to the smart note book, a lot of resources from the internet, look at different ways of teaching a certain topic or to find if someone else's done resources in which…… we are not always starting from the scratch. In terms of emails, 4 if someone finds a good resource our practice …to send an email to say that MATHS department – the title, and they will email to the whole department and everyone can have a copy, and in case they would want to use it in their lesson, that what we do quite a lot.

4:15 MU: Do you re-use previous materials?

Richelle: I have been here for 6 years, so many stuffs I use couple of years but not good anymore, we have to build a bank of resources that you use again and again and again. And again, we are very open in this department, so if someone has something good then we get then to send to us. Everyone is required to send their lessons to myself or the head of department and every week we can review people's lessons and we send out good ones and everyone would have a copy. We also have the school network itself, have a folder called central resource and there is a Maths folder, and then everyone can put all their resources in there that they think are useful so that everyone can get access, so it is not just for one teacher.

5 MU: what is the impact of using the ICT/digital resources in lessons, how does it affect professional practice and classroom practices?

Richelle #5:27: if definitely make it more effective, because I do some time wonder how people use to teach Maths without the resources, because it means it get a structure, it definitely helps to give a structure throughout the lesson, it means for us as a team their consistency across the department because they will use the same format, everyone has the same them and things on, their first slide of the lesson and everyone has to include certain things in their lesson which means the consistency is maintained. It make it easier if you just have power points, to know what going to come next, but also then to have a bank of things ready, so that if that was too hard or too easy that you could bring something out more quicker than, I think, if they need to go and find a textbook and flip through the pages, it would all be there much faster.

6:33 MU: When you use ICT/digital resources does it change the class setting? What becomes different, from the class where ICT is not been use

Richelle: To be honest I have never not used any technology, I don't think, except for one week when the whole network was down, and nobody could use it, it was interesting. I definitely think it is better because the students are more engaged especially if you got something interactive on the 6 screen that they themselves could come up and have a go at, because they are more willing to listen and get involved themselves, it makes it better for them in terms of their participation, an also if you have some animation they quite often find that quiet interesting or want to know how you did it, meanwhile from 7 the engagement side, it's a lot better, Eeeem same with the iPad, not necessarily in teaching always, but if you have those, they are much more willing to engage with it because they like working with technology, they are always on their phone all the time, any chance they got to use technology is always good.

7:33 MU: you are a traditional user of technology now, using technology has become a norm, ICT, textbooks and whatever online resources… over the 6 years of your teaching what do you think has become very constant in your teaching?

Richelle: There is still, because there is an internet site called TES and everyone uploads things in TES and has always been there, in terms of the textbook, although we don't use textbook so much , the main difference is that it is now available electronically, so the textbook is still there , it is rather no more physically in front of students, you can cut and paste the bits you want from the PDF format into a lesson or into a worksheet and print them off and photocopy for the student. For student don't necessarily have the physical resources anymore, emmmm it is still the same resources, that has not change, we just actually had a meeting yesterday where a AQA 9 sales person came in to sell us some textbooks but then said we can get them all electronically, you don't actually need the physically copy, so it just the case that ….everyone can access it. They are the same people want to have something for students to get on with, so everyone you see at photocopy, being used multiple times.

9:45 MU: Do you use most of the resources because they make your work as a teacher better or because it is good for the students?

Richelle: Yeah, I mean, I would never, I hope I will never give them something that wasn't going to benefit them. I think the main use for worksheet and questions on the

board is to enable them practice and consolidate what we have been learning; I know a few teachers like to use things rather than a pile of questions, use of cards somethings to manipulate, so there are getting a bit more discussions and problem solving, definitely feel resources nowadays its getting more and more advanced people just come up with new ideas and they get shared around, so it is definitely for the benefit of the students.
The only thing I will say is that they tend to be more tailored towards exams now than they use to be, 10: 49 because especially with the new curriculum everything is very very content-heavy and it is about being able to read and answer, exams questions, whereas before you could look in more details the concepts behind it and the actual understanding of the topic, whereas now you have to be able to answer questions, and that is a bit of a shame. The exams…. And you do need them…..

11: 15 MU: The task; with the use of ICT what is different about the task, when you design the task with a particular technology?

Richelle: I think it is more more visual, it is really good for the top students we have, some quiet often are coming from the lower level of academic level, and don't have the same Maths ability, so we try and pair up the concept with the visual image, so the technology allows you to put visual image on the board and manipulate it in ways to show the students how to understand what is going on, so it's not just the case of following a method or following a teacher's model but you can give them an image to begin with and that conceptual understanding first before they then go away and have a go at some questions. And having the white board particularly and the smart note book is much easier to manipulate things you could not necessarily do with WB and a pen, which we have to do before, it does enable that a lot better.

12:32 MU: With the use of Technology how will you describe your class arrangement?

Richelle: I have got the main groups in the group of four, I have so that they can have discussion between themselves about what they are doing. But they need to be able to see the board that is obviously important, they have all to be able to turn round and face the board. The WB is like the centre of the class; that's why everyone ought to see that's where I stand to do my delivery, I walk around the classroom during tasks but whenever I am doing explanations I need to be there at the board because that is kind of safety for a lot of people to have that there.

13:23 MU: Technology sometime there could be glitch here and there?

Richelle: it could be very interesting when you begin a lesson and it crashes and freezes, but it is important that quiet often we say your first activity needs to be maybe

on paper or on a desk ready for the students. Because our policy in our department they have to write down the date, the title and do a starter in the first 5 minutes of any of the lesson, lets teacher stand at the door, meet and greet all the student and take a register because there is something there for them to do.
So if you completely rely on WB and something happens you are stuck in the first 5 to 10 minutes, it depends, some teachers are good at thinking on their feet and we do have the WB normal, WBs as well, so it is the case of quickly writing something up there, put on a different task for as long as there is something, you can usual fix things, 14:22 we have got a very good IT department, they come up and fix things, if we have got a serious problem, if it's just a case of log in problem someone will come and help you.
14: 44 MU: The collective: Does the Maths department belong to an informal group or some formal teacher association, where they can share resources, how does this help in professional development?
Richelle: Yes definitely, we have the formal, like when we have departmental meetings and training sessions. We talk in more professional manner and delicate tasks, but day to day because of a lot of the staff have come in and being new teachers and have had to be supported quite a lot, our culture is to ask anyone and they will stop and support you, everyone is very willing to send lessons and resources to each other… might see a worksheet on the side, some done ….and oooh I want that and they will happily give it to you, 16:00 so the culture of sharing here is extremely good, and I suppose Technology make that faster, because you can just photocopy or send an email.
16:30 MU: what about belonging to a formal Mathematics teacher association outside of the school?
Richelle: In XXX we have the…. There is a network, they have meetings every term and they tend to have sent out some resources that is shared across by lots of Maths course, there is one guy in particular, called Mr. Kelly I think in Fair-val. And he makes an end of the term quiz which he sends out to all the schools, YouTube, IWB, he is really good, he seems to be a wizard in technology, there are loads of animations and different things, so also in case you can meet, 17:17 people get together and discuss the new curriculum, how they can implement it in school, discuss how we gonna grade it, there is a Maths group we tend to keep up to date with and go when we can.

17:29 MU: which is more helpful?
Richelle: I like that our department is a bit more informal. ... the HOD of Maths is obviously not like this autocratic leader that makes you do everything, we try to make everyone feel equal, on the same level, we can ask the lowest member of the department for help on something because they might have strength in different area, it is about using everybody's strength to make the department better. For example we have got Lee, who is fantastic with Non-English students or Mary who is brilliant

with the top set of students; that is how we have who teaches what class and then if you have got a problem in your class you know who to go to and everyone have different strength able to help when you have problem.
It is important we can easily ………………….. We don’t have anything wrong about it.

18:40 MU: Anything that comes to your mind differently?
Richelle: Have you asked about Calculator as resources or ICT or not? Yes, I have a scientific calculator we use it just as you would support things. I have just started teaching year 12 this year and they have to use the calculator for certain things, when I was in school, we had the graphic calculator, if people are using those; that is something to look at. I know you can do graphs plotting and things on those, it might be something to look at.

Appendix C: Systematic Classroom Analysis Notation

Kitty:

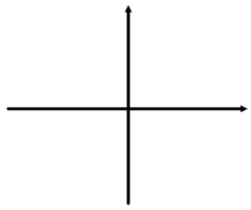
Date	Yrs.	Num.	Resources	Activity	Episodes	Events of Interests	Topic
23/11/15	11a1	30	IWB, Wb, PC, textbook, countdown	4C, 4L, 3Wo, 2O	SS, ST, 3Tks, 3Co, 2Ei, 3Fi, D, R	2timed tasks with music. IWB & textbook	Function notation
23/11/15	10c3	23	IWBb, Tmw, paper-folding	7C, L, 6Wo, 6O, 2D2-2,	SS, ST, 4Co, 5Ei, 4Fi, 2Tks, D	3TAs supporting in the lesson with paper-folding activities.	Fraction -Division -Addition -Multiply
20/11/15	7	22	IWBb, www.10ticks.co.uk, Tmw, tracing paper, online stopwatch	4C, 2Wo, O	SS, ST, Co, I, Fi, Ei	Testing understanding through tasks	Shapes
20/11/15	11a1	31	IWBb, iPad, PC, ppt, wksheet, https://www.diagnosticquestions.com/	4C, AwTt, AwTs, 3D1-1, 2Dt-s, 7O, 4Wo, L, M,	2I, 7Fi, 6Ei, 8Co, R,	iPad/diagnQues Quizzes, Seated in groups of 3s &4s	Algebraic expression -complete proof
13/11/15	10y	20	IWBb, PC, mathsbox.org.uk, Corbettmaths, TMw, Pdf,	2C, AwTt, 3AwTs, 3L, 4Wo, 3L, D2-2, 2W2,	SS, ST, I, 5Co, 3Fi, 3Ei, R,	-Student solving at the WB -working in pairs -working with teacher	Equation -Forming -Solving
11/12/15	10b1	19	IWBb, PC, FrogOS, Calc., iPad, Coberttmaths, 5-a-day practice questions, videos, music by Maria Carey	C, L, 2O, 2Wo, D2-2, D1-1, W1-1	3Co, 3Tks, 2Fi, 2Ei, R	-Task with music -varieties of practice	Ratio -Simplify -As fraction -with unknown
11/12/15	10a1	27	IWBb, PC, Coberttmaths, Calc., TMw, iPad, Mangahigh,	8C, 3Wo, 3O, Dt-s, 2W1-1, 3L,	SS, ST, I, 5R, 4TKs, 2Fi, 3Ei, 2Co,	-Lesson 4 of 4 -Revision for exams -working in 2, 3 and 4s -Teacher attending to individuals	Rectangular shapes -square -rhombus -rectangle -kite
11/01/16	8	23	IWBb, PC, Musci, Mangahigh, TMw,	2C, 3Wo, 3O, 3W1-1, 3D1-1, L	ST, SSI, 5TKs, 5Co, 2Ei, 3Fi, Dt-1	-Task with music -Collaborating	Fraction -Multiply by integer

1/02/16	11a1	30	TMD, IWBb, PC, Textbook,	4C, 3D1-1, D2-2, Wo, O, 2AWTt,	SS, ST, I, 4Co,5Ei, 2Tks, 2R, 2D	Revisions of previous lesson	Gradient -Equation of line Sketching graph
3/03/16	10c3	22	IWBb, PC,	4C, 3Wo, L	I, 2Co, 2Ei, Fi, 4Tks, R		Operation -Order -Mixed
21/03/16	11a1	31	IWBb, Laptops, Mangahigh, PC	4C, AwTt, 4AwTs, L, 3O, 3Wo, 2D1-1	D, I, 4TKs, 3Ei, 2Fi, 3Co R, SS, ST	Starter Series of tasks	Vectors -Direction -Magnitude
21/03/16			Introducing gcsepod.com to the teachers and students. https://www.gcsepod.com/				
21/04/16	10c3	23	Revision, practice questions, coaching and further explanations.				

Appendix D: Mathematics Tasks and Lesson Plans

Kitty

Name: ______________________________ www.corbettmaths.com

January 1st **5-a-day**	**Core 1**
Solve $2x^2 - 13x + 21 > 0$	
L_1 has equation $x + 3y + 1 = 0$ L_2 is parallel to L_1 and passes through (8, 3) Find the equation of L_2	
Sketch $y = (x - 3)(x + 1)(4 - x)$	
In an arithmetic sequence $U_6 = 20$ $U_{12} = 38$ Find the common difference	Find the first term
Calculate the sum of the first 20 terms.	

Extension

Head over to the Cities of the World Worksheet!

See if you can name all the famous cities by solving the BIDMAS problem, and replacing the number with a letter as shown in the grid!

BIDMAS – Cities of the World

Use the table below and the rules of BIDMAS to work out the answers to the questions and unravel these famous cities of the world.

1	2	3	4	5	6	7	8	9	10	11	12	13
A	B	C	D	E	F	G	H	I	J	K	L	M

14	15	16	17	18	19	20	21	22	23	24	25	26
N	O	P	Q	R	S	T	U	V	W	X	Y	Z

City 1

1. $2 \times 6 + 3$
2. $3 \times 5 - 4$
3. $5 + 3 \times 3$
4. $7 - 6 \div 3$
5. $(2 + 4) \times 3$
6. $7 + 4^2$
7. $3^2 + 4^2$

City 3

1. $(7 + 6) \div (17 - 4)$
2. $3 + 2 \times 4$
3. $4^2 \div 2^3$
4. $4 + 2 \times 5$
5. $28 \div (9 - 5)$
6. $(7 - 2) \times (9 - 6)$
7. $(2 \times 22) \div 2^2$

Name: ______________________ Date: ______________ Week 13

PRACTICE SHEET

Monday **8 - 10 Times Tables** 3 minutes 20 seconds time limit

1	8 × 12	11	9 × 2	21	10 × 6	31	11 × 9	41	3 × 9
2	10 × 10	12	9 × 8	22	9 × 6	32	8 × 9	42	3 × 8
3	9 × 11	13	8 × 3	23	10 × 9	33	7 × 8	43	3 × 8
4	10 × 10	14	8 × 10	24	9 × 6	34	1 × 8	44	10 × 9
5	10 × 11	15	8 × 7	25	8 × 3	35	5 × 8	45	5 × 9

Time taken

:

Score

50

What's your rock status?

NUMBER ONE FAN
All correct in 3mins 20 secs

ROCK STAR
All correct in 2mins 30secs

Emilia

Name : ______________________ Score : __________

Teacher : ______________________ Date : __________

Graphing Inequalities

1) $2.5 > d$

-3 -2 -1 0 1 2 3

2) $-p \leq -3$

-7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7

3) $c \geq 6$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

4) $-1 > -h$

-7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7

5) $0.5 \geq -h$

-3 -2 -1 0 1 2 3

6) $5 \geq s$

-7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7

7) $-v \geq 4$

-7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7

8) $q < -1$

-3 -2 -1 0 1 2 3

9) $2 < c$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

10) $-s < -6$

-7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7

11) $0.5 > -r$

-3 -2 -1 0 1 2 3

12) $-0.5 < m$

-3 -2 -1 0 1 2 3

13) $f \leq 12$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

14) $3 \geq n$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

15) $r \leq -1.5$

-3 -2 -1 0 1 2 3

16) $12 < s$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

17) $-2.5 \leq h$

-3 -2 -1 0 1 2 3

18) $j \leq 6$

-7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7

19) $-g > -1$

-7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7

20) $h > 4$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

Substituting Into Algebra Fill It In

Work out each question and colour the answers in the grid. What is the answer to 11513 ÷ 397?

40	14	8	17	35	100	25	12	7
1	26	34	22	36	6	15	38	37
150	19	41	16	42	13	43	24	21
33	60	50	2	27	11	32	144	29
70	20	80	44	46	3	39	18	5
23	10	48	9	31	49	30	4	28

$\boldsymbol{a} = \mathbf{2}$, $\boldsymbol{b} = \mathbf{3}$, $\boldsymbol{c} = \mathbf{4}$, $\boldsymbol{d} = \mathbf{5}$, $\boldsymbol{e} = \mathbf{6}$

1) $3a$
2) $5c$
3) ac
4) de
5) $2ab$
6) $2b + 5$
7) c^2
8) b^2
9) $cd - e$
10) abc
11) $2a + 2b$
12) $de - bc$
13) $3b + 2c$
14) $e^2 + a$
15) $2d^2$
16) $(2d)^2$
17) $(3c)^2$
18) $3c^2$
19) $2(d + e)$
20) $(b + c)^2$
21) $4(d - c)$
22) $5 + 2c$
23) $bc + 5a^2$
24) d^2

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6) $2b + 5$
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8) b^2
9) $cd - e$
10) abc
11) $2a + 2b$
12) $de - bc$
13) $3b + 2c$
14) $e^2 + a$
15) $2d^2$
16) $(2d)^2$
17) $(3c)^2$
18) $3c^2$
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33	60	50	2	27	11	32	144	29
70	20	80	44	46	3	39	18	5
23	10	48	9	31	49	30	4	28

$a = 2, \quad b = 3, \quad c = 4, \quad d = 5, \quad e = 6$

1) $3a$
2) $5c$
3) ac
4) de
5) $2ab$
6) $2b + 5$
7) c^2
8) b^2
9) $cd - e$
10) abc
11) $2a + 2b$
12) $de - bc$
13) $3b + 2c$
14) $e^2 + a$
15) $2d^2$
16) $(2d)^2$
17) $(3c)^2$
18) $3c^2$
19) $2(d + e)$
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33	60	50	2	27	11	32	144	29
70	20	80	44	46	3	39	18	5
23	10	48	9	31	49	30	4	28

$\boldsymbol{a = 2, \quad b = 3, \quad c = 4, \quad d = 5, \quad e = 6}$

1) $\boldsymbol{3a}$
2) $\boldsymbol{5c}$
3) $\boldsymbol{ac}$
4) $\boldsymbol{de}$
5) $\boldsymbol{2ab}$
6) $\boldsymbol{2b + 5}$
7) $\boldsymbol{c^2}$
8) $\boldsymbol{b^2}$
9) $\boldsymbol{cd - e}$
10) $\boldsymbol{abc}$
11) $\boldsymbol{2a + 2b}$
12) $\boldsymbol{de - bc}$
13) $\boldsymbol{3b + 2c}$
14) $\boldsymbol{e^2 + a}$
15) $\boldsymbol{2d^2}$
16) $\boldsymbol{(2d)^2}$
17) $\boldsymbol{(3c)^2}$
18) $\boldsymbol{3c^2}$
19) $\boldsymbol{2(d + e)}$
20) $\boldsymbol{(b + c)^2}$
21) $\boldsymbol{4(d - c)}$
22) $\boldsymbol{5 + 2c}$
23) $\boldsymbol{bc + 5a^2}$
24) $\boldsymbol{d^2}$

Substituting Into Algebra Fill It In - ANSWERS

Work out each question and colour the answers in the grid. What is the answer to 11513 ÷ 397?

40	**14**	**8**	**17**	35	**100**	**25**	**12**	7
1	26	34	**22**	36	**6**	15	**38**	37
150	19	41	**16**	42	**13**	43	**24**	21
33	60	**50**	2	27	**11**	**32**	**144**	29
70	**20**	80	44	46	3	39	**18**	5
23	**10**	**48**	**9**	31	**49**	**30**	**4**	28

$a = 2, \quad b = 3, \quad c = 4, \quad d = 5, \quad e = 6$

1)	$3a$	6	7)	c^2	16	13)	$3b + 2c$	17	19)	$2(d + e)$	22
2)	$5c$	20	8)	b^2	9	14)	$e^2 + a$	38	20)	$(b + c)^2$	49
3)	ac	8	9)	$cd - e$	14	15)	$2d^2$	50	21)	$4(d - c)$	4
4)	de	30	10)	abc	24	16)	$(2d)^2$	100	22)	$5 + 2c$	13
5)	$2ab$	12	11)	$2a + 2b$	10	17)	$(3c)^2$	144	23)	$bc + 5a^2$	32
6)	$2b + 5$	11	12)	$de - bc$	18	18)	$3c^2$	48	24)	d^2	25

Jimmy

1) Write $\frac{58}{100}$ as a decimal

2) Write $\frac{47}{50}$ as a decimal

3) Write $\frac{9}{20}$ as a decimal

4) $-11 + (-3)^2$

5) Round 36.242 tp 1 significant figure

6) Simplify $\frac{39}{63}$

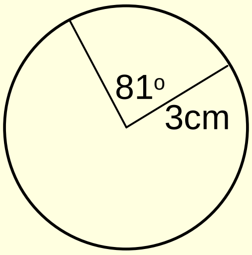
1) Factorise

$x^2 - 5x - 6$

2) Factorise

$5x^2 + 16x + 3$

3) Find the perimeter of the sector.



Write your answer in terms of π

DIRT

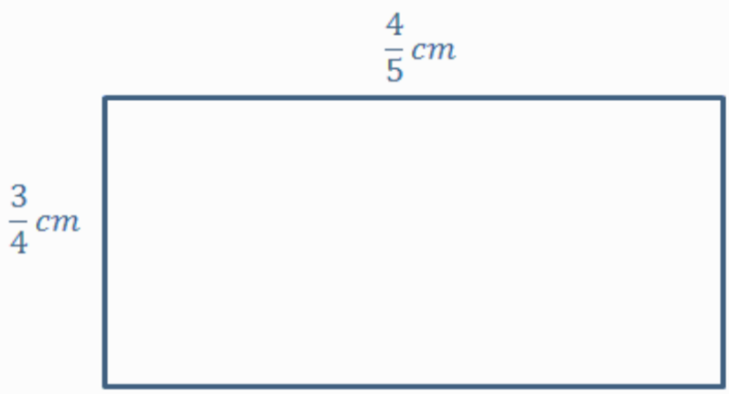
Jose

Master Of Fractions

	Multiplication		Division		Addition & Subtraction
1.	$\frac{1}{2} \times \frac{3}{4}$	8.	$\frac{4}{3} \div \frac{3}{5}$	15.	$\frac{5}{2} + \frac{7}{4}$
2.	$\frac{2}{3} \times \frac{4}{9}$	9.	$\frac{7}{3} \div \frac{9}{7}$	16.	$\frac{34}{9} + \frac{16}{3}$
3.	$\frac{1}{5} \times \frac{7}{10}$	10.	$\frac{6}{7} \div \frac{1}{10}$	17.	$\frac{27}{4} + \frac{25}{9}$
4.	$\frac{3}{4} \times \frac{7}{8}$	11.	$\frac{3}{2} \div \frac{7}{5}$	18.	$\frac{49}{5} - \frac{29}{10}$
5.	$\frac{2}{5} \times \frac{4}{5}$	12.	$\frac{2}{5} \div \frac{1}{7}$	19.	$\frac{47}{6} - \frac{34}{7}$
6.	$\frac{1}{6} \times \frac{1}{2}$	13.	$\frac{11}{6} \div \frac{1}{5}$	20.	$\frac{9}{4} + \frac{9}{4}$
7.	$\frac{1}{5} \times \frac{9}{10}$	14.	$\frac{11}{5} \div \frac{1}{9}$	21.	$\frac{39}{10} + \frac{57}{10}$

Key Question:

Find the perimeter of this shape.



Gray

Travel Graphs

Name:	Class:	Date:

Mark	/ 14	%

1) Julia drove from her home in Derby to London. She went shopping and then drove back home.

Find

a) the distance from Derby to London.

b) the time she spent shopping in London.

c) the average speed on her journey home.

Leave answers to nearest whole number where necessary.

[1]

2) Hazel went on a cycle ride. The travel graph shows Hazel's distance from home on this cycle ride.

Find

a) how far Hazel cycled after 10 minutes?

b) how long she took a rest?

c) how far she cycled in total on her ride?

Leave answers to nearest whole number where necessary.

[1]

Coordinates geometry

Straight lines

$= \frac{-}{-}$

$- = (-)$

$= +$

$+ =$

$= + \qquad = +$

- $=$
- $\times \quad = -$

$= - \text{—}$

$\left(\frac{+}{\ } \ \frac{+}{\ } \right)$

$= = \sqrt{- + -}$

Gavin

MASTER | Check P15 | Strengthen P17 | Extend P21 | Test P25

1.6 Surds

You will learn to:
- Use surds
- Understand the difference between rational and irrational numbers.

CONFIDENCE

Why learn this?
Using surds means you can give an accurate answer instead of a rounded answer.

Fluency
Work out
- 3^3
- 2^4
- $\sqrt{64}$
- $\sqrt[3]{64}$

Explore
When did mathematicians start using surd notation?

Exercise 1.6

Warm up

1 Work out
   a $\sqrt{64}$ b $\sqrt{121}$
   c $\sqrt{\frac{1}{4}}$ d $\sqrt{\frac{4}{9}}$

2 Write each number as the product of its prime factors.
   a 32   b 50
   c 120   d 225

Worked example

Write each square root as a **surd** in its simplest form.

a $\sqrt{12}$ b $\sqrt{15}$

a $\sqrt{12} = \sqrt{(2^2 \times 3)}$ — Write 12 as the product of its prime factors, so you can see any square numbers.
$= \sqrt{2^2} \times \sqrt{3}$ — The square root of a squared number is the number itself.
$= 2 \times \sqrt{3}$
$= 2\sqrt{3}$ — $2 \times \sqrt{3}$ is written as $2\sqrt{3}$.

b $\sqrt{15} = \sqrt{(3 \times 5)}$
$= \sqrt{3} \times \sqrt{5}$ — $\sqrt{3}$ and $\sqrt{5}$ cannot be simplified any further.
$= \sqrt{3}\sqrt{5}$

Key point
A **surd** is a square root that cannot be simplified any further. It is written with the root symbol, for example $\sqrt{5}$.

3 Write these as surds in their simplest form.
   a $\sqrt{20}$ b $\sqrt{48}$
   c $\sqrt{100}$ d $\sqrt{3}\sqrt{6}$
   e $\sqrt{2}\sqrt{10}$ f $\sqrt{3}\sqrt{60}$

Q3d hint
$\sqrt{3}\sqrt{6} = \sqrt{3} \times (\sqrt{3} \times \sqrt{2})$

9 Fractions and percentages

MASTER | Check P201 | Strengthen P203 | Extend P207 | Test P211

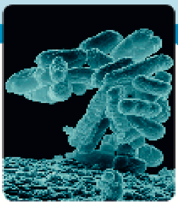
9.1 Comparing fractions

You will learn to:

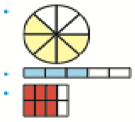
- Compare fractions
- Simplify fractions
- Identify equivalent fractions.

CONFIDENCE

Why learn this? Scientists use fractions when investigating how bacteria reproduce and survive.



Fluency
What fraction is shaded?



Explore
How can you tell whether a fraction is bigger than, smaller than or equal to another fraction?

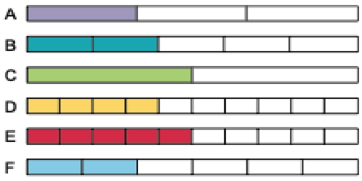
Warm up

Exercise 9.1

1 Write > or < between each pair of fractions.

a $\frac{4}{7} \square \frac{2}{7}$ b $\frac{1}{2} \square \frac{1}{4}$ c $\frac{1}{8} \square \frac{1}{5}$ d $\frac{1}{10} \square \frac{1}{9}$

2 Match pairs of equivalent fractions.



3 Copy and complete. Simplify each fraction.

a $\frac{15}{20} = \frac{\square}{\square}$ (÷5) b $\frac{16}{20} = \frac{\square}{\square}$ (÷4) c $\frac{18}{21} = \frac{\square}{\square}$ (÷3) d $\frac{30}{36} = \frac{\square}{\square}$ (÷6)

4 List the common factors of

a 18 and 24 b 10 and 30.

5 Write these fractions under the headings 'more than $\frac{1}{2}$' and 'less than $\frac{1}{2}$'.

$\frac{7}{10}$ $\frac{5}{9}$ $\frac{2}{7}$ $\frac{2}{5}$ $\frac{11}{20}$ $\frac{9}{20}$ $\frac{4}{11}$ $\frac{6}{11}$

Q5 hint
Is the numerator more or less than half of the denominator?

Topic links: Multiplication, Division, HCF, Equivalent fractions

Richelle

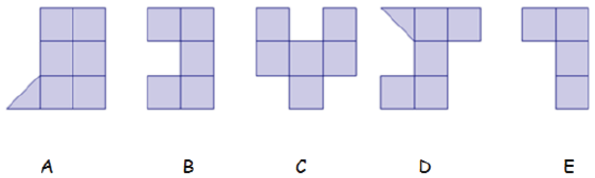
Keywords – taken from the scheme of work, words that will be used by the teacher and students need to learn

Keywords: area, perimeter, centimetres, distance, squared, length, width, height

Title: Area and Perimeter **Date:** 06/06/2016

Starter – this is a starter I found from a previously planned lesson. I used it because it was a high ability class. This starter assesses their prior knowledge.

Starter:



Which shape has the biggest perimeter?

Which shape has the smallest perimeter?

Which shape has the biggest area?

Which shape has the smallest area?

Learning Outcomes

To be able to find area and perimeter of squares, rectangles and compound shapes

To be able to find the area of a triangle

The learning outcomes for the lesson came from the scheme of work, chosen from the appropriate grade suitable for the class

		presenting information	
	Length, Perimeter, Area and Volume	**Length, Perimeter, Area and Volume**	**L**
ding nit hin f a of a	▪ Convert metric units to metric units ▪ Make sensible estimates of a range of measures in everyday settings ▪ know and use the formula for the area of a rectangle ▪ Find the perimeter of parallelograms and trapezia ▪ Recall and use the formulae for the area of a triangle and rectangle ▪ Find the area of a rectangle and triangle ▪ calculate perimeters and areas of shapes made from rectangles ▪ visualise 3D shapes and deduce some of their properties ▪ calculate the surface areas of cubes and cuboids ▪ Identify, name and draw parts of a circle including tangent, chord and segment	▪ derive and use formulae for the area of a triangle, parallelogram and trapezium and the volume of a cuboid ▪ calculate areas of compound shapes and volumes and surface areas of cuboids and shapes made from cuboids ▪ Find the area of a trapezium and recall the formula ▪ Find the area of a parallelogram ▪ Calculate areas and perimeters of compound shapes made from triangles and rectangles ▪ Find surface area using rectangles and triangles ▪ Recall and use the formula for the volume of a cuboid ▪ Recall and use formulae for the circumference of a circle and the area enclosed by a circle circumference of a circle = $2\pi r = \pi d$, area of a circle = $\pi r2$	▪ ▪ ▪ ▪ ▪ ▪ ▪ ▪ ▪ ▪

Investigation

Use the squares in your book:

Draw as many different rectangles as you can that have an area of 24 cm^2

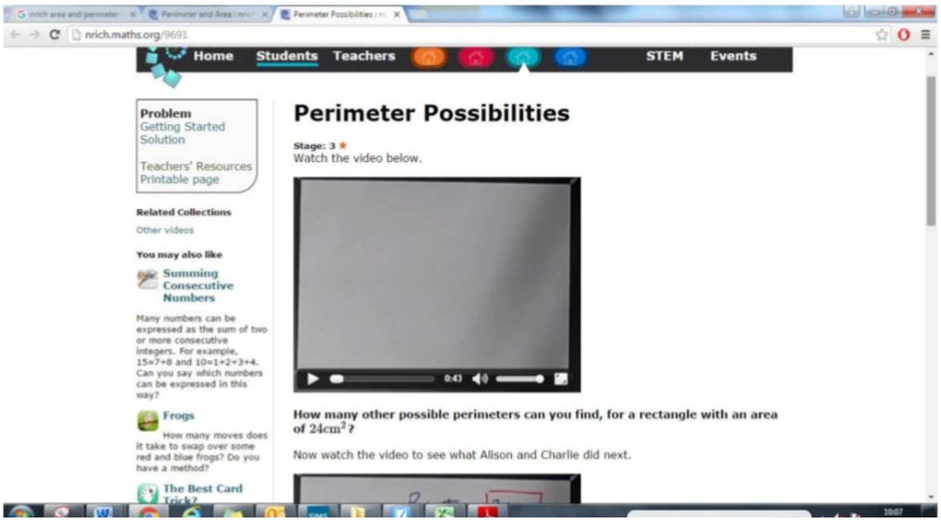
Which rectangle gives you the largest perimeter?

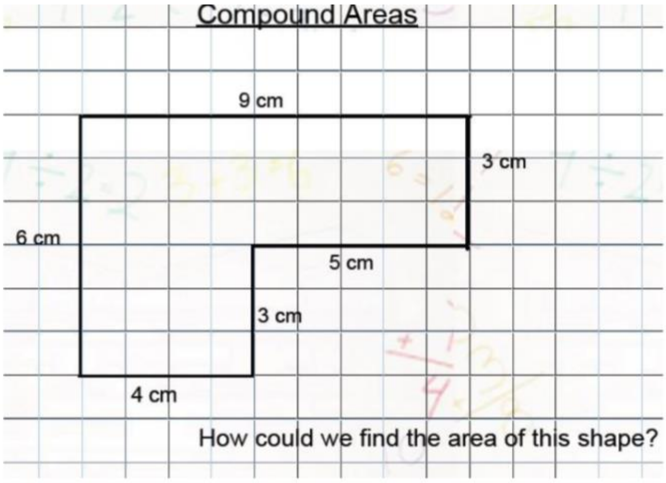
Which rectangle gives you the smallest perimeter?

Extension: Draw as many different rectangles with a fixed perimeter of 40 cm

Since all students in the class should be able to find the area of perimeter of rectangles I wanted to see if they could apply this skill and get them to spot patterns in the shapes

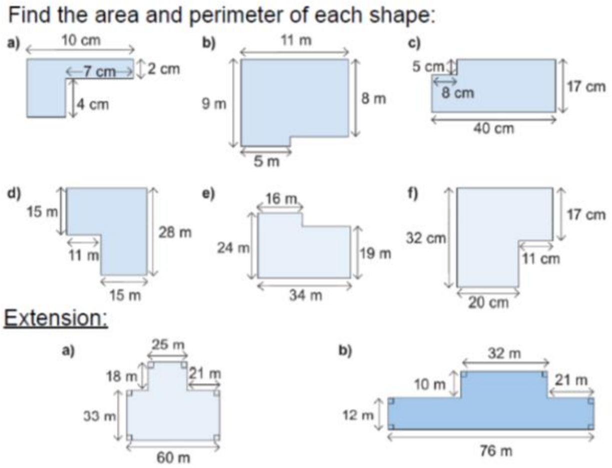
This task is from NRich website.



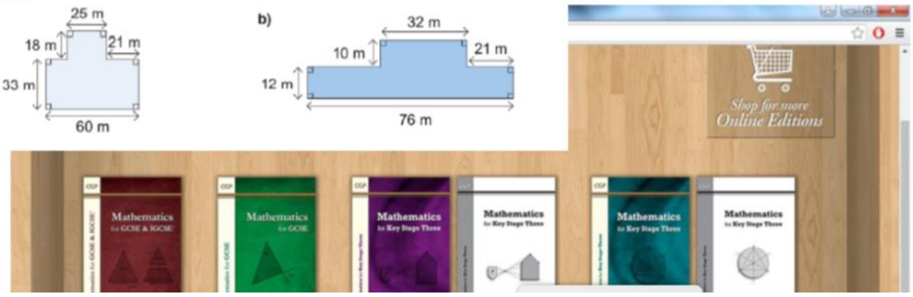


I designed this example. I wanted to move them on to compound shapes made of rectangles and used a square grid to make it easier for students to understand what they needed to do.

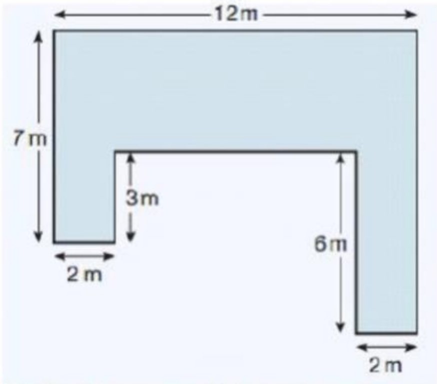
We talk through two examples on the board



I used the online KS3 textbook from CGP to find suitable questions for the students to practice.



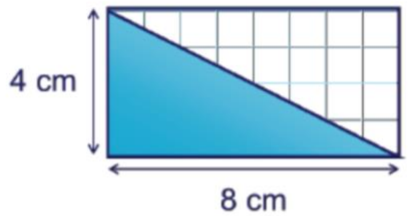
The diagram shows Rob's patio. All the corners are right angles.
The patio is made up of square paving stones each 50 cm by 50 cm.



Work out how many of these paving stones are needed to tile Rob's patio

To assess what they have learned I set an exam question. This was taken from a problem solving worksheet I had saved in my own resources.

What proportion of this rectangle has been shaded?



What is the shape of the shaded part?

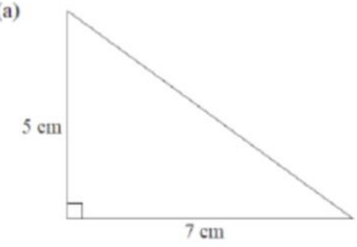
What is the area of this right-angled triangle?

This part was taken from a Boardworks Maths presentation that you can get online.

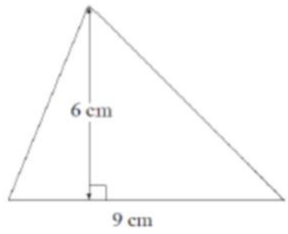
Examples: Area = ½ x b x h

Find the area of each of these triangles:

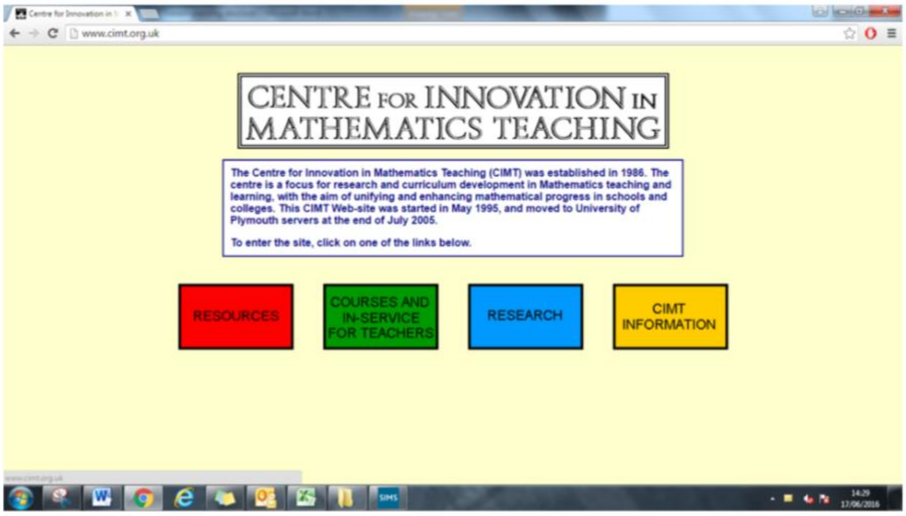
(a)



(b)



These examples were taken from a CIMT worksheet that can be found online on the CIMT website



Appendix E: Teachers' self-reported profile

NAME (This will be used anonymously in the research)			
YEARS OF TEACHING			
OTHER EXPEREINCES, e.g Work in non-teaching field/ previous school.			
ROLE IN DEPARTMENT			
CLASS/YEAR GROUP TAUGHT			
AGE RANGE	**20-29**	**30-39**	**40+**

Appendix F: List of Websites and Applications

http://donsteward.blogspot.co.uk/
http://flashmaths.co.uk/
http://mathswatch.co.uk/
http://studymaths.co.uk/
http://svsurveys.corwin.com/?loc=US
http://wordwall.co.uk/
http://www.10ticks.co.uk/
http://www.boardworks.co.uk/maths_71
http://www.douis.net/
http://www.math-play.com/math-millionaire.html
http://www.mathcentre.ac.uk/about/
http://www.mathsbox.org.uk/index1.php
http://www.mathshell.com/
http://www.mathspad.co.uk/
http://www.resourceaholic.com/
http://www.whiteboardmaths.com/
https://community.tes.com/
https://connect.collins.co.uk/school/portal.aspx#
https://corbettmaths.com/
https://keshmaths.com/
https://microsites.ncl.ac.uk/fasmedtoolkit/
https://schoolcodebreaking.com/code-breaking-resources/
https://support.prometheanworld.com/product/activinspire
https://ttrockstars.com/
https://twitter.com/hashtag/educhat?src=hash
https://twitter.com/mrbartonmaths
https://twitter.com/SLTchat
https://www.cgpbooks.co.uk/
https://www.cimt.org.uk/menus/resources.htm
https://www.dreambox.com
https://www.e-act.org.uk/
https://www.m-a.org.uk/who-we-are
https://www.mathgames.com/play/kingofmath.html
https://www.mrbartonmaths.com/
https://www.mymaths.co.uk/
https://www.pearsonactivelearn.com/
https://www.plickers.com/
https://www.tes.com/teaching-resources
https://www.ttrockstars.com/
https://www.ukmt.org.uk