Essays on the Macroeconomics of Migration

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Declaration

I, the author, confirm that the Thesis is my own work. I am aware of the University's Guidance on the Use of Unfair Means. This work has not been previously been presented for an award at this, or any other, university.

This dissertation contains approximately 59,000 words including appendices, references, footnotes, tables and equations and has 31 figures.

Arising from this thesis, I wrote a forthcoming book chapter, Barker (2020), which examines the brain waste and its economics. There is no replication of the theoretical or empirical models employed in chapter 3 of this thesis. I retain the full copyright of the content used in Barker (2020).

Emily Ruth Barker
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Abstract

This thesis consists of three papers to analyse the effect of migration dynamics on open economies. Each study examines a different topic of migration in a macroeconomic context using a dynamic stochastic general equilibrium (DSGE) model.

The first paper analyses the effect of an increase to migration on the macroeconomy and to native citizens. The analysis employs a structural vector autoregression (SVAR) and a DSGE model to analyse the effect of a migration shock on the macroeconomy and fiscal budget in per native terms. Analysis features a constructed dataset for the native population using population and net migration statistics for Germany. The DSGE model diverts from the standard per capita terms to have variables for the macroeconomy and native household in per native terms and the variables specific to the migrant household in per migrant terms.

The second paper features a DSGE model of a small open economy with asymmetric search and matching frictions to study brain waste and increased migration following a relaxation of migration policy. To show the gains from eliminating brain waste, the differences between natives and migrants are eliminated. A SVAR provides empirical analysis of the effects from a migration shock. These models use data from Canada, a country that has recognised brain waste on a microeconomic level.

The final paper analyses the effect of migration policies in a two-country DSGE model. The two countries are asymmetric, beginning with their profiles as an oil-producer country and an oil-consumer. The migration decision for agents in the resource cursed country is endogenous and depends on the job finding probabilities net of migration costs. The model uses occasionally binding migration constraints to evaluate the role of migration policies. The model is estimated using data from Venezuela and the United States.

The results presented in each of the papers show that immigration has a small but statistically significant positive effect for an economy empirically and theoretically. For the final paper, migration policies had negative effects in both countries by preventing the oil-consumer from maximising the labour force and the sending country by keeping unemployment high and draining fiscal resources.

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Chapter 1

Introduction

In the past twenty years, the number of international migrants residing in developed economies has increased significantly, creating economic and political discussions. The relevance of migration has opened up opportunities for research on the reasons and implications behind migration. Several theories behind the reasons for international economic migration are put forth by Massey et al. (1993) using neoclassical economics, dual labour markets, and world systems theory. The neoclassical macroeconomic theory on migration assumes that international economic migration is caused by: wage differentials, returns on human capital, labour markets being the main driver of international labour flows, and governments can control migration by influencing the labour markets. Whereas the neoclassical microeconomic side considers differentials in earnings, participation, and unemployment rates, the specifics of human capital characteristics, social conditions, expected net returns to migration, psychological factors, and that governments can control immigration using policies to influence expected earnings net of migration costs. Economic dualism arises from imperfect substitutability of workers in highly developed economies, where workers with a high degree of capital-skill complementarity hold relatively stable jobs, and the labour intensive sector is less secure. The associated characteristics with low-skilled work is less attractive to natives, therefore, employers turn to immigrants to fill the gaps. The driver of international migration in world systems is the location of markets that are local to the sources of capital and the exact directions can relate to former colonies. The wage and employment gaps between the host and sending country is most commonly used in these theories as to what drives international economic migration. However, there is no theory that can explain or help predict migration flows (Arango, 2000). The microeconomic research on economic, social, environmental, and political international migration is extensive while the literature examining the effects of migration in a macroeconomic context, particularly dynamic general equilibrium business cycles, is minimal.

This thesis answers some of the questions on migration that are unexplored within macroeconomics. The macroeconomic questions on *immigration* can only be answered by studying the different migration profiles of host countries. The first consideration is to analyse the relative (and absolute) size of net migration flows and the immigrant population. The United States of America has the highest stock of international migrants globally, however the foreign born residents account for less than 14% of their total population. In contrast, Australia, Canada, and New Zealand have excess of 25%. Annual net migration flows exceed 1% of the population in Canada and New Zealand but less than 0.4% in the United States. For countries with a higher relative size, net migration is of greater significance to the macroeconomy. Of the existing research, there is a great focus on the Mexico to United States path which can be traced back to Jerome (1926), in part due to its persistently large absolute size and migration policies dating back to the 1920s.

A second factor to profile, is the average age and qualification or human capital levels of the immigrant and how they compare to natives. Some countries such as the United States and Germany are host to a large number of low-skilled migrants whereas Australia, Canada, New Zealand, and the United Kingdom are host to high-skill migrants. This is not entirely due to the demand of a nation as resulting from shortages but the supply of migrants. Using Germany as an example, there is significant shortage of high-skilled workers but a large supply of low-skilled workers from Poland and the other Eastern European countries that are a part of the common labour market in Europe. The qualification level has significant effects on the participation and unemployment rates of migrants. This area extends to the substitutability of native and immigrant workers. The existing macroeconomic literature leans towards perfect substitution of workers whilst the microeconomic implies imperfect substitution (Ottaviano and Peri, 2012).

Thirdly, migration analysis should consider the requirements for the immigrant to work in the host country. International migration in a common labour market, such as the one in Europe, is much simpler than requiring a (temporary) visa. Visa systems differ too, for example a country's visa system can be dependent on securing employment before application. Research by Klein and Ventura (2009) looks at the role of barriers to labour mobility in output and welfare when productivity differs between countries. By considering an expansion of the European Union and creation of a hypothetical common labour market in North America, the authors found that there are large gains in both. Chassamboulli and Peri (2015) examine the labour market effects of reducing the number of illegal immigrants. Their research shows that legalising immigrants has a positive effect on the low-skilled labour market for natives and immigrants, whereas increasing deportations worsens the low-skilled labour market for both.

From a longer term perspective, what percentage of population growth is attributed to migration? For countries such as Germany and Japan, migration is the only source of population growth due to the long-term negative natural population change. Whilst in Canada, the forecast is expected to be the same by 2030. These are some of key factors in consideration for meaningful macroeconomic research applicable to a country and for linking it to the research in microeconomics. The existing theoretical macroeconomic literature of immigration in open economies is limited, even more so the estimation of the models of those focusing on immigration. The existing works on immigration include Canova and Ravn (1998), Canova and Ravn (2000), Ben-Gad (2008), Mandelman and Zlate (2012), Kiguchi and Mountford (2017), Braun and Weber (2016), Hauser and Seneca (2019), Lozej (2019), and Smith and Thoenissen (2019). The research generally gives a positive or insignificant results from immigration. Empirical macroeconomic research on immigration is somewhat limited due to the small number of countries who keep detailed net migration data over an extended period of time. Two papers to use quarterly net migration time series data are Furlanetto and Robstad (2019) and Smith and Thoenissen (2019) who examine Norway and New Zealand respectively. Kiguchi and Mountford (2017) and Weiske (2019) use constructed migration series for the United States. Weiske (2019) estimates a time series for migration flows using working age population using data based on the Current Population Survey. d'Albis et al. (2016) uses a constructed dataset from long-term residence permits in France.

When emigration is considered, if immigration is positive for the host country does that make it negative for the sending county? Immigration shocks can be identified as a relaxation of immigration policies, an expansion of a common labour market, or an unexpected change in migration such as a refugee influx, for which there is no comparable policy for emigration; beyond the limited scenario of entering into a common labour market with a developed country. Something is required to cause the change in emigration flows which is not as simple to model because there is not an opposite of relaxation of immigration restrictions; or at least one that is replicable across migrant sending countries. Massey et al. (1993) showed that the wage gap was one of the main factors of migration amongst many labour market factors. As migration occurs between highly developed nations, the neoclassical macroeconomic assumption that eliminating the wage gap will stop migration is unrealistic as the gap would be negligible or insignificant once migration costs are accounted for. The analysis of a positive or negative effect of emigration depends on the driver of the emigration decision, the condition of the sending economy, and for whom are the effects being considered? This thesis focuses on economic migration rather than environmental or political. The loss of human capital, or brain drain, and whether it is beneficial is analysed in Beine et al. (2001) and Beine et al. (2008). There is a gain for countries that have a low level of high skilled workers and low migration rates, however when there is a higher proportion of high-skilled workers and a high migration rate then there are losses. From their cross-country study (Beine et al., 2008), there are more losers than winners. The examination of refugees has been studied in the host country by Stähler (2017). A topic that became relevant following the European migration crisis of 2015-2016. There is research on labour mobility in currency unions, particularly the United States and less so the Eurozone, these works include Farhi and Werning (2014), Hauser (2014), Braun and Weber (2016), Hauser and Seneca (2019), and Bandeira et al. (2019). Two papers to examine migration in the European Union and removal of trade barriers are Iranzo and Peri (2009) and Kennan (2017). Iranzo and Peri (2009) analyse trade and migration flows in a two country model. Kennan (2017) looks at the effects of an enlargement of the European Union with the focus on the elimination of barriers to labour mobility. The results show that effects on real wages are small and there are significant gains to be experienced from the removal of migration restrictions. An area related to immigration and emigration that has been researched in the macroeconomics is remittances. The analysis of risk sharing using migration and remittances in sending countries by Mandelman and Zlate (2012) is a key contribution to the literature on migration and business cycles. Other studies include Durdu and Sayan (2010), Mandelman (2013), and Finkelstein Shapiro and Mandelman (2016). This pre-existing research on remittances, allows this thesis to focus on immigration and the macroeconomy of the host country and the introduction of migration policies from a country that remittances have not been a significant contribution to GDP (until recently).

This thesis presents three papers that examine the effects of migration on the macroeconomy by answering some of the questions related to immigration and emigration. The main contribution of each paper is the estimated dynamic stochastic general equilibrium (DSGE) model. The first two papers focus on the effects of immigration on the macroeconomy following a migration shock, each using a structural vector autoregression (SVAR) and a DSGE model. The third paper studies migration policies and the business cycles of a developing economy.

1.1 A Native Perspective of Immigration

Is migration more than population growth in the Solow model, and does it do more than dilute per capita stocks of assets in the short run? What is the effect of immigration on natives and the macroeconomy? This chapter is motivated by the research on the effect of immigration natives in the microeconomics which shows net immigration to be mainly positive or insignificant for natives. For instance, D'Amuri et al. (2010) shows the imperfect substitution of native and migrant workers in Western Germany and Glitz (2014) who concludes that segregation in present in the German labour market through ethnicity and most prevalent in low-skilled labour markets. Additionally, Dustmann et al. (2010) show that migrants are more likely to be made unemployed than natives following an economic shock. Depending on the country's type of migration, the long run effect on wages can be zero as in Kennan (2013). Further, the Solow model implies that if migration were simply an increase in population growth rate, this would have contractionary effects in per capita terms. As an increase in the population would dilute the per capita value of stocks making the capital less intensive without a change in the savings rate.

To answer these questions, chapter 2 uses an empirical and a theoretical model. The first part of the paper features a SVAR using data from Germany to assess the effects of a migration shock to the macroeconomy in per native and per capita terms. To contrast the effects of a per capita and per native immigration shock, chapter 2 employs a time series created using the working-age native population to allow analysis in per native terms and use working-age total population data for a per capita time series. Taking these population figures to a macroeconomic dataset using components of national accounts, including fiscal spending. The empirical analysis evaluates the effects of net immigration to the macroeconomy in per capita and per native terms.

The main contribution of this paper is the DSGE model that features heterogeneous households, perfectly competitive firms, and an active fiscal authority. One household consists of natives and long-term migrants, and the second is a collection of migrants. DSGE models usually have variables in per capita terms, however, to analyse whether migration is more than population growth, the model uses variables specific to the native household and macroeconomy in per native terms and variables specific to the migrant household in per migrant terms. The active fiscal authority uses countercyclical fiscal policy that includes: investment in public capital that is employed by the firms, consumption, provision of household transfers, and collection of taxes. Some of the model parameters along with the autoregressive parameters and standard deviation of shocks in the log-linearised model that are estimated with Bayesian methods, using the per native dataset.

The results from the SVAR shows positive results for the macroeconomy following a migration shock in per capita and in per native terms. Interestingly, the per capita and per native impulse responses give the same directional response but the persistence and peaks differ. The DSGE model shows positive responses for the macroeconomy, for the native household, fiscal authority, and negative effects for the migrant household. However, responses from both models are small and insignificant for some variables.

1.2 Brain Waste in the Canadian Macroeconomy

Why do immigrants experience different labour market conditions to natives and what effect does this difference, as well as skill-downgrading, have on the macroeconomy? How would the economy change if the labour market differences were eliminated and if immigrants were assimilated into the macroeconomy? The labour market differential between natives and immigrants, in particular wages and unemployment, has received a microeconomic focus but there are important implications for the macroeconomy. Differences in participation rates relate to the demographics of migrants relative to natives (Hilgenstock and Koczan, 2018). More productive workers offer greater returns, but if their skills are under-utilised, there is a loss of a potential gain to the economy. Lewis (2011) provides positive support the effect of immigration and degree of capital-skill complementarity in the United States is analysed using OLS and IV methods when immigration changes skill ratios. Brain waste occurs when (high-skilled) migrants face poorer labour market conditions then natives, which includes higher rates of involuntary unemployment, higher labour market frictions, or underemployment. Underemployment includes skill-downgrading and working part-time when full-time is desired. The Canadian government has recognised brain waste as an issue following the microeconomic research, however, there is a gap in the dynamic business cycle literature for analysis of brain waste.

Chapter 3 answers these questions, using data on participation rates, unemployment rates, and wage differences in a model of capital-skill complementarity with asymmetric search and matching frictions to analyse the role of brain waste to the macroeconomy. The recent Canadian policies to increase migration, and in particular a visa system that does not require the migrant to have a job to be granted a work visa, allows us to use this microeconomic data to calibrate this DSGE model. The model features three households, perfectly competitive firms, and a passive fiscal authority. To evaluate the potential gains to the macroeconomy from eliminating the labour market problems caused by brain waste, the model is adjusted so that high and low-skill migrants have the same labour market conditions as their native equivalents. This shows that there are statistically significant gains to be had from eliminating brain waste. Also presented is a small extension to the model that assimilates high-skilled immigrants into the macroeconomy by enabling them to invest in the firms. Using Bayesian estimation methods, the persistence parameters and standard deviations of the six model shocks, and three model parameters of the non-linear DSGE model are estimated. To complement the DSGE model, a SVAR is presented to analyse the empirics of a migration shock to Canada. The responses to the macroeconomic variables show that there are small but positive responses to migration.

1.3 The Resource Curse and Migration Policies

How do migration policies affect the host and sending economies? In developed countries that are migrant hosts, immigration is largely procyclical in relation to the business cycle. What about sending economies? It would be assumed that emigration is mostly countercyclical, and originates in the actual or expected wage premium and labour market conditions. However, if immigration is procyclical and immigration policies are imposed, what drives the business cycle of developing economies to follow this pattern of migration? To what extend do non-risk sharing shocks matter?

In this two-country model presented in chapter 4, with asymmetric search and matching frictions, one country is a developing resource cursed country and the other a developed country. The former is an oil-producer, and latter an oil-consumer. The resource curse refers to commodity dependent economies who have not, despite the abundant wealth that comes from oil especially, become a developed nation and links closely to macroeconomic populism. The agents in the resource cursed country have an endogenous migration choice that depends on the probability of finding a job, net of migration costs. The wage premium of the developed country sets migration of natives in the developed to the resource cursed country to zero. The model uses occasionally binding migration policies to reflect current migration policies that limit migration, the only example of OBCs being used in the migration literature is by Mehra (2017) who focuses on the effects of immigration in the host country. The model is taken to annual data from Venezuela and the United States for 1920-1995 which covers multiple business cycles. Venezuela has the largest proven reserves of oil globally, and its economy is strongly linked to the oil price, yet there is an absence of business cycle analysis. During this period, the majority of Venezuelan oil exports were to the United States, at approximately 75%. The results focus on migration and the effect of migration policies analyse the following an oil-price shock, which does not have an element of risk-sharing as some of the typical business cycle shocks do. The results show that (strong) migration policies have negative effects on both economies.

Developed countries usually employ immigration policies that ‘protect’ their labour markets from what they perceive to be too much immigration. In analysing the effect of migration policies, the analysis begins with a baseline model that has no barriers to labour mobility in the spirit of Iranzo and Peri (2009), who showed how a hypothetical NAFTA labour market would benefit the United States.

The first migration policy introduces a visa system whereby migrants have an endogenously determined probability of acquiring a work permit. If this is unsuccessful, they enter the labour market as an illegal immigrant. The second policy increases the efficiency of the border force and migration agency, which results in a lower probability of a successful employment match and increased migration costs. The final policy is by way of an exogenous job destruction, whereby the border force stops a new employment match and deports existing illegal immigrants. These policies reflect the ongoing situation faced by migrants. Kennan (2013) shows significant gains if there were opening up borders where wage differences result from productivity of workers and there is factor price equalisation.

The business cycle analysis contribution of the paper examines the role of the oil price and the endogenous demand and supply of oil. Using data on production in Venezuela and the elasticities of oil in production for production in the United States, the non-linear model is calibrated and estimated. Results show that there is a large contribution of oil price and production to the business cycle in Venezuela. Both theoretical and empirical research has tried to explain the business cycles of developing economies. Aguiar and Gopinath (2007) argued that non-stationary productivity shocks were the source of fluctuations whereas research by Garcia-Cicco et al. (2010) explain that stationary productivity shocks and interest rate shocks were the main drivers. For the developing countries dependent on commodities, terms of trade has been shown to have a significant impact on their economy and the dollarisation of external debt for the government. As a general fact on the resource curse, Sachs and Warner (1995) show that resource abundant countries usually have low growth rates in the analysis of 1971-1989; a period of high volatility including the energy crisis, oil crisis, and oil glut. Manzano and Rigobon (2001) extend this analysis with debt overhang as the resource wealthy countries borrowed highly which caused their debt crises in the 1980s following the fall in commodity prices rather than a resource curse stagnating their growth.

1.4 Organisation of this Thesis

The following three chapters employ DSGE models to analyse immigration and emigration. Chapter 2 introduces the topic of immigration as a political as well as an economic issue with a literature review followed by a discussion on immigration in Germany. The aforementioned SVARs and a DSGE model using data from Germany follow. The Bayesian estimation results of the persistence parameters and standard deviations of shocks, and selected model parameters of the log-linearised small open economy DSGE model follow. The results from Bayesian estimation are used in the analysis of an exogenous migration shock to present the results of the DSGE model. To finish, there is a conclusion to this analysis of immigration from a native perspective.

Chapter 3 begins with an introduction of brain waste as a concept that is applied to this macroeconomic model, followed by a review of relevant existing literature on immigration, and discussion on the importance of immigration in Canada. The analysis begins with the SVAR and setting out the DSGE model. The non-linear small open economy DSGE model is estimated using Bayesian methods. The results from the DSGE models with and without brain waste are presented, followed by a conclusion.

Chapter 4 includes an introduction to the resource curse and macroeconomic populism, followed by a discussion of the literature on commodities, emerging economies, and emigration. After setting out the DSGE model, the algorithms of the occasionally binding constraints used to simulate migration policies are detailed. The two country non-linear DSGE model is estimated using data from Venezuela and the United States, followed by analysis of the role of oil price and production in the business cycles of both countries. The next section presents the results from a positive and a negative oil price shock, to evaluate the asymmetries and effects of the migration policies. To conclude, there is a discussion of implications of migration policies, and those of business cycles in resource cursed countries.

Chapter 5 offers a conclusion to the thesis. This chapter summarises the results and implications of the three papers for economists and policy makers; an evaluation of the key findings and the economic interpretations; and a discussion of the limitations of each paper with opportunities for future research.

Chapter 2

A Native Perspective of Migration in a Small Open Economy

2.1 Introduction

Migration is a contentious topic that is economically and politically relevant. There has been an increase of international migrant stocks of 17% worldwide, and 18% in Western Europe between 2000-2017 (United Nations, 2017). The higher levels of migration in recent decades can, in part, be attributed to the increasing openness of economies and labour markets. This increase in the host economies has not been welcomed by all, indeed it has triggered a backlash on multiculturalism, a rise in support for nationalist political parties, as well as tensions between natives and recent immigrants.[1][2]

This paper analyses the business cycle effects of an exogenous migration shock in an estimated small open economy dynamic stochastic general equilibrium (DSGE) model from a native perspective.[3] This differs to the existing literature that presents variables in per capita terms. The model uses two households to represent natives and migrants, perfectly competitive firms, and an active fiscal authority. To compliment the DSGE model, we present empirical analysis in the form of a structural vector autoregression (SVAR). We question how a migration shock affects the macroeconomy in per native rather than per capita terms.

[1] See Ambrosini and Boccagni (2015) and Scholten and van Nispen (2015) for discussions on the politicisation of migration. Crawley and Skleparis (2018) discuss some of the issues raised following the 2015 migration crisis.

[2] Research suggests that immigrants who have resided in the host country for a significant period of time have similar views to natives on increasing immigration Braakmann et al. (2017)

[3] This shock is to simulate a relaxation in visa restrictions; in our applied case an expansion of freedom of movement areas in Europe. A direct comparison with the refugee crisis is not appropriate as the model assumes full employment.

The economic importance of migration is becoming more apparent to a number of host nations. Germany is home to the highest number of migrants in Europe, where migration flows are the only source of population growth due to the negative natural population change.[4] At the end of 1991, there were 5.9 million foreign-born residents in Germany or 7.3% of the population. By the end of 2018, these figures nearly doubled to 10.9 million or 13.1%. In terms of the working-age population (15-65 years old), migrants account for 14.5% at the end of 2018. The German economy has experienced four major population shocks since 1989: reunification of Germany, immigration following the collapse of the Soviet Union, the expansion of the European Union, and the (European) migration crisis of 2015.

We fit the model to Germany where immigration has become a sizeable political issue in addition to being an economic one. The refugee crisis of 2015 caused German net migration to exceed 1 million for the first time (figure 2.1) which is more than a twofold increase on the 2014 figure.[5] The disparagement by some Germans led to an increase in support for the anti-immigration Alternative für Deutschalnd (AfD) party, and a dent in Chancellor Angela Merkel's approval ratings.[6] Some of the anti-immigration arguments put forward are that there are negative effects on wages and jobs, and a drain on fiscal spending. To evaluate the contribution of migrants to the economy relative to natives we need to assess the profile of the average immigrant compared to native.

The results from the DSGE and SVAR models imply migration is positive for an economy, and the responses coincide where the significance of the results is the only major change. Migration has a small but statistically significant effect on per native GDP, private consumption, and private investment such that migration shocks matter but are not drivers of the business cycle.

Our paper offers a different way of looking at the economic responses of migration in a theoretical model by examining the macroeconomy in per native terms and the use of heterogenous households; and is supported with empirical analysis. The remainder of this paper is as follows: in section 2.2 we discuss relevant theoretical and empirical literature about effects of migration on the macroeconomy; section 2.3 elaborates on some of the migration data discussed in this section; we describe the empirical model and its results in section 2.4; we set out the DSGE model in

[4] The natural population change for West Germany since 1972 was negative and for Germany since reunification. Source Statistisches Bundesamt (Destatis) using natural population change defined as is births - deaths.

[5] See section 2.3 for a discussion on migration in Germany

[6] AfD is now the third largest party in the Bundestag, and received 12.6% of the votes in the September 2017 election. As of September 2019, the AfD hold 91 seats, where only the CDU/CSU and SPD hold more with 246 and 152 respectively.

section 2.5 and report the Bayesian estimation results of the model in section 2.6; we present the results of the model in section 2.7; and offer a conclusion in section 2.8.

2.2 Immigration and the Macroeconomy

There is a developing literature covering the macroeconomics of migration, particularly with regards to immigration in business cycle literature. A number of early works looked at the effects of migration following the collapse of the USSR on Israel such as Flug et al. (1994) and Hercowitz and Yashiv (2002). Others include Canova and Ravn (1998), Canova and Ravn (2000), and Ben-Gad (2004). More recent works include Baas and Brücker (2012), Mandelman and Zlate (2012), Burstein et al. (2017), Pandey and Ray Chaudhuri (2017), and Smith and Thoenissen (2019). Migration research incorporating search and matching fictions include Chassamboulli and Palivos (2013), Chassamboulli and Peri (2015), Kiguchi and Mountford (2017), and Iftikhar and Zaharieva (2019). Few papers model natives and migrants in separate households.

Dependent upon which migration path is being studied, the characteristics of migrant workers differ. In the literature, migrants are predominately considered to be low-skilled, at the very least they arrive into the low-skilled labour market.[7] When more than one skill level is considered, the effect on the low-skilled workforce usually has contrasting effects to the high-skill workers. As the low-skilled sector receives an increased workforce, typically higher skill sectors see positive effects on wages and employment. This can lead to labour market polarisation. However, as noted in microeconomic literature, natives and migrant workers are usually imperfect substitutes. We focus on the papers using full employment.

Canova and Ravn (2000) model the reunification of Germany as an inflow of low-skilled, non-Ricardian households (East Germans) to West Germany. The model includes a welfare state, the presence of which eliminates an investment boom and prolongs the recession with a consumption loss for the high-skilled agents. The government's purpose is to redistribute income between agents, balancing their budget every period. When the model includes a government, the low-skilled workers gain as wages increase. In the absence of a government, low-skilled workers lose as wages fall, there is no redistribution of income, and high-skill workers gain as return on capital increases, but their wages fall. The reunification process, or inflow of low-skilled workers, is modelled as an AR(1) process. The long run effects of unification show that there is still a fall in most variables, notably though skilled

[7] Partly due to the high frequency of studies for the Mexico-United States migration path.

income, and skilled consumption are still above steady state levels, with skilled labour hours below steady state. The authors note that the time needed to absorb the migration shock depends on the speed at which the migrants develop to have the same portion of skilled agents compared to natives.

Storesletten (2000) uses an overlapping generations model to analyse the effects of high and low-skilled migration on fiscal policy. The results show that young/working-age high and medium skilled immigrants have a positive impact for US taxpayers but a negative one in the case of low-skilled or older workers. The conclusion is that selective immigration policies should be towards individuals who are not of average age and skill composition of current immigrants and population. In the context of most countries where there is sizeable inward migration, such as countries in Western Europe, the US and Canada, who (in the absence of freedom of movement) grant visas to young workers.

Braun and Weber (2016) model immigration in West Germany following the end of World War II in which 8 million expellees returned, however, the distribution across two of the West German regions was different. Using search and matching frictions in a two-region model that fits the data on regional unemployment differential. There is small lost to native workers in the long-run in terms of lifetime labour income of 1.38%, but in the short-run there is a larger effect. Some of these results are applicable to the refugee inflow that Germany experienced in the 2015 European migration crisis. Stähler (2017) examines the effect of the refugee crisis in Germany using the Bundesbank's New Keynesian GEAR (Germany, Euro Area and Rest of the World) model. Refugees are initially receiving transfers for the first year following arrival. However, he notes the importance of fully incorporating the refugees into the labour market since GDP per capita falls whilst they have not transformed to a higher productivity worker. Fratzscher and Junker (2015) examine the effects of the refugee inflow into Germany. The paper finds that the effects can be negative in the short run, in terms of wages and output, but in the long run (5-10 years) the results are positive, particularly after the immigrants have entered the labour market, and especially for the younger immigrants, who have the potential to be more qualified than the average German native.

In terms of empirical research, Furlanetto and Robstad (2019) use a Bayesian VAR model to identify the results on aggregate macroeconomic variables from an immigration shock using quarterly Norwegian net immigration data. An immigration shock is treated as a labour market shock. The authors find that net immigration does not have negative effects on native employment and public finances, two of the key arguments used to oppose immigration, rather that a positive immigration shock increases employment among natives and migrant workers as well as a small positive effect on public finances. Immigration shocks are small but significant drivers of the business cycle, but immigration does not especially respond to the business cycle, rather labour market factors.

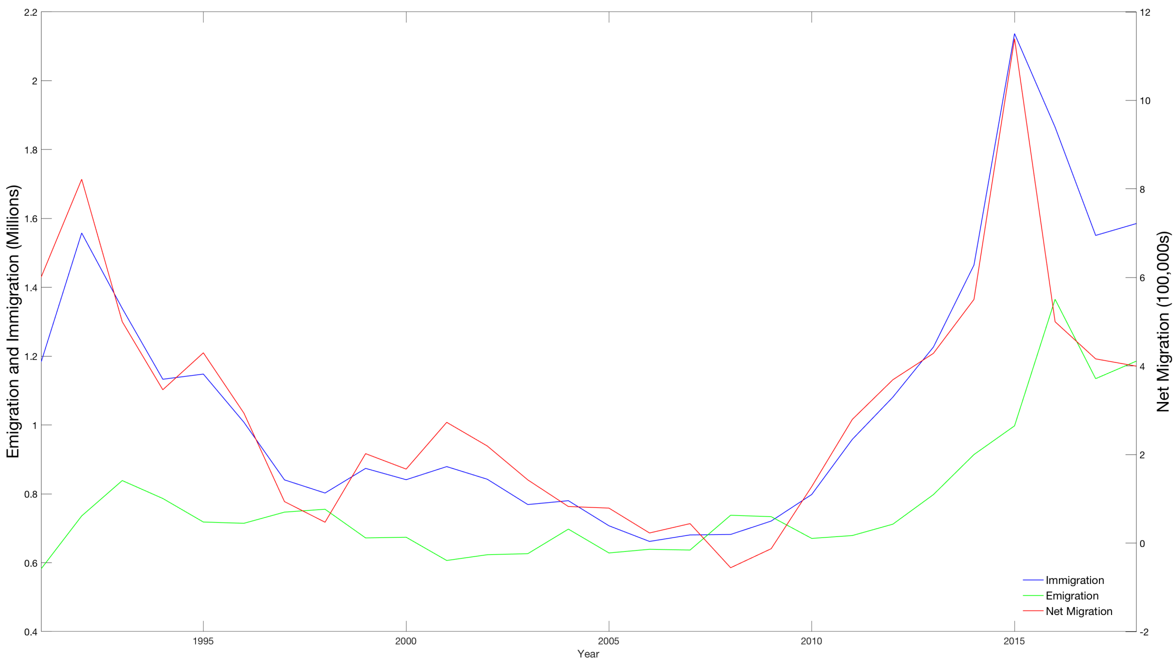
In both macroeconomics and microeconomics, there is an assumption of separate labour markets for natives and migrants which conforms with international evidence. D'Amuri et al. (2010) examined the wage and employment effects of immigration to Western Germany in the 1990s, where results showed that there were few effects on natives' wage or employment rather the negative effects were on previous immigrants employment, and to a smaller degree their wages. There are further topics examining the wage gap in Germany including wage setting in Brücker and Jahn (2011), and monopsonistic discrimination in Hirsch and Jahn (2015), where migrants supply labour which is less elastic than natives. Due to the imperfect substitutability of workers, the argument that migrants take natives' jobs is not necessarily the case. Ottaviano and Peri (2012) show that migrants and natives with the same skill and experience levels are imperfect substitutes and immigration has a positive effect on natives' wages in the short and long run. If native and migrant workers are not competing for the same job, immigrants, equivalently migration, cannot be blamed for any negative employment related problems that natives experience.

To summarise, there is a general analysis that native or higher skilled wages increase and migrant or lower skilled in the native country wages fall but only by a small amount. Natives experience more favourable labour market conditions. Migration has the effect of increasing output, as the economy is able to grow when there is an increased workforce and investment. Supply side effects cause corresponding increases in demand. Employment for highly skilled natives is positive, when migrants and natives compete for the same type of job there is the possibility that the natives experience a fall in employment and a decrease in wage. However, natives experience more favourable labour market conditions including a lower volatility of employment. Welfare increases for most types of households, with migrants seeing the largest change, any decrease in native welfare is small or insignificant.

2.3 Facts on Migration in Germany

Germany experiences the highest migration flows and stocks in Europe, and second only to the US worldwide. In the years since German reunification changes in net migration have been driven predominantly by volatile non-German immigration, rather than emigration. Annual migration data for Germany is available from 1991 shown in figure 2.1.

Fig. 2.1 Total Annual Migration Flows for Germany 1991-2018



Immigration and emigration are identified by the blue and green lines measure with the left axis (millions of people). Net migration is the red line measured on the right axis (100,000s people). Source: Statistisches Bundesamt (Destatis) - Federal Statistics Office Germany

Two of the greatest migration shocks to hit a reunified Germany were following the collapse of the Soviet Union and the refugee crisis of 2015. Of the people leaving Russia following the collapse of the USSR, Germany received 59%, Israel received 25%, and the United States 11% of 1.2 million who emigrated to countries outside the former Soviet Union between 1989-2002 (Heleniak, 2004). The largest number of migrants arrived in the period 1989-1993 during the separation of states.

The impact of the 2004 EU expansion, predominantly the arrival of the A8 countries, did not have a large shock to the migration figures in 2004Q2 because Germany did not allow free movement until 2011.[8] During the intervening period Germany issued a high number of work permits which staggered the increase. Due to the fall in numbers of Polish nationals in Germany in 2004, there is evidence to suggest that would be migrants from Poland changed their destination to other

[8]The A8 countries refer to the expansion of the May 2004 enlargement of the EU. The A8 countries include Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Slovakia and Slovenia. Ten countries joined in 2004, the A8 are distinguished from Cyprus and Malta due to the relatively low income per capita in comparison to the other EU nations.

countries that allowed free movement (Fihel et al., 2006).[9] The return migration of Eastern European migrants during 2008-2009, in particular Polish workers, was the reason for the negative net migration in 2008 and 2009. Workers returned due to more favourable economic conditions. GDP growth in Germany would have been 0.2% lower without the EU migrants in the years 2011-2016 (Clemens and Hart, 2018). Even though migrants earn approximately 20% less than their native equivalent after arrival and experience skill-downgrading (Beyer, 2016) where skill-downgrading is being employed in a job for which the worker is overqualified.

One reason that immigration is important can be found in a country's demographics. Germany has an ageing population, falling birth rate, negative natural population change, and an increased dependency ratio.[10] An increase in immigration counters these effects, since (in contrast to natives) immigrants typically have a higher birth rate, are younger (in demographic profile), and often return to their home country for retirement. The average age of migrants is 37.3 for men and 38 for women, compared to 45.7 for natives. Working-age migration helps to reduce the decrease in working-age population. Therefore, the significant changes in the employment can be partially attributed to migration.

As for the crude migration rate, shown in figure 2.2 for years 1990-2016, Germany has number of outliers, predominantly at the beginning and end of the sample.[11] However, in comparison to a number of other countries, this intervening period is relatively stable. Peripheral Eurozone countries experience large swings around the economic crisis, 2008 and 2009 are the only years that Germany experiences negative net migration.

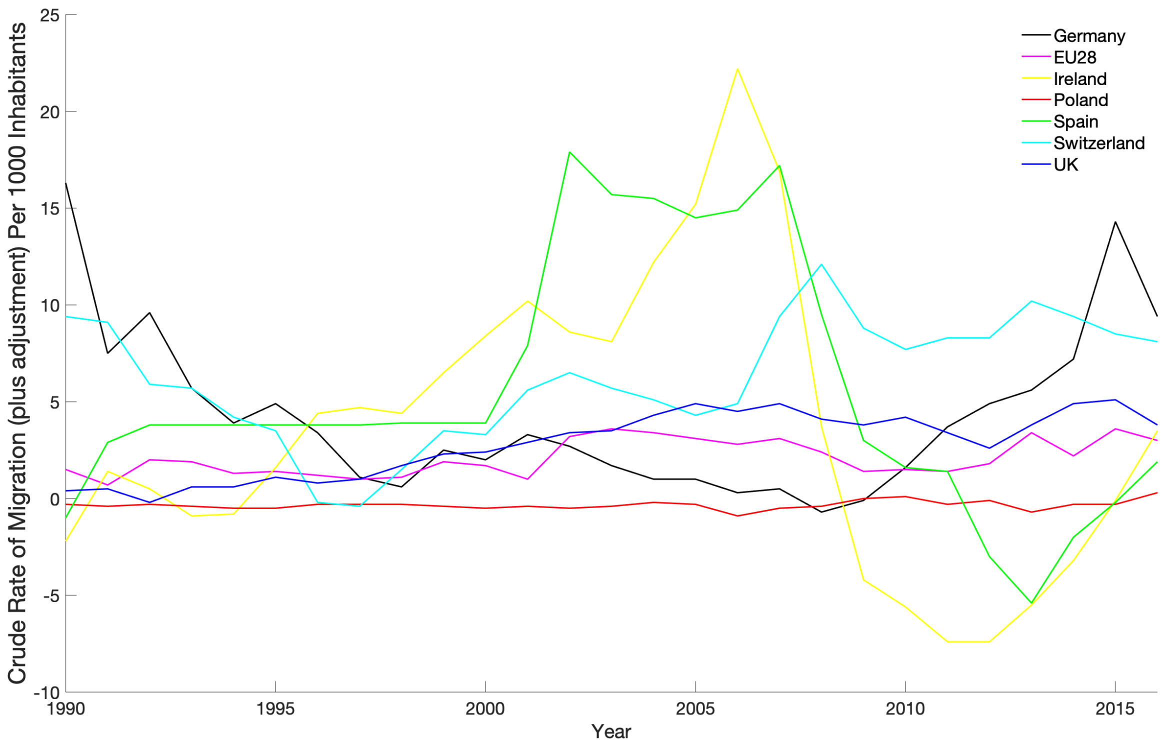
The other reason that migration is important is found in the labour market; there are an estimated 1.2 million unfilled vacancies in Germany. Even with an assumption of 200,000 migrant workers per annum, VBW (2015) estimate that there will be a shortage of 3 million skilled workers in 2030, rising to 3.9 million by 2040. Research by Vogler-Ludwig and Düll (2013) highlights the skills shortages faced by the labour market; predominantly caused by an ageing population, pre-existing skills gap, and a negative natural population change. A reduction in the availability of skilled workers

[9] Barrell et al. (2010) state that this may be due to the change in the data collection methods of foreign nationals. This change caused an overall fall in the number of foreign nationals residing in Germany.

[10] The dependency ratio is defined as the ratio of the number of dependants, aged 0 to 14 and 65+, to the total population, aged 15 to 64

[11] Crude rate of net migration is defined as the ratio of net migration (including statistical adjustment) during the year to the average population in that year (per 1000 persons). The net migration plus adjustment is calculated as the difference between the total change and the natural change of the population (Eurostat).

Fig. 2.2 Crude Rate of Net Migration for Selected Countries and EU28 1990-2016



Crude rate of net migration (plus adjustment) per 1000 inhabitants for selected countries and EU28. The crude rate of net migration is defined as the ratio of net migration (including statistical adjustment) during the year to the average population in that year (per 1000 persons). The net migration plus adjustment is calculated as the difference between the total change and the natural change of the population
Source: Eurostat

will cause firms to increase investment in human capital and longer working hours for those already employed.

To target reducing the skill gaps, in December 2018 the Bundestag passed a law *Fachkräftezuwanderungsgesetz*, which aimed to increase skilled migration from non-EU countries as there is a significant shortage in Germany and EU migrants.[12]

The ambiguity of response to migration in some features of the macroeconomy, particularly towards natives, absence of macroeconomic research for others, limited research in migration using a small open economy model, minimal use of migration data in macroeconomic research, and the stylised facts presented in sections 2.1 and 2.3 forms the motivation for our paper.

[12] Deutscher Bundestag (2019) http://dip21.bundestag.de/dip21/btd/19/082/1908285.pdf

2.4 Empirical Model

We begin the analysis of migration in Germany with a SVAR using the observables in the DSGE model and related data from the macroeconomy. Other VARs analysing the exogenous migration shock to the macroeconomy include Kiguchi and Mountford (2017), Smith and Thoenissen (2019) and Furlanetto and Robstad (2019) who use the United States of America, New Zealand, and Norway respectively.

The unrestricted VAR presented uses the same observables to be used in estimation of the DSGE model, and additional components from the government national accounts. As per Lütkepohl (2005), the VAR is specified by:

$$(y_t - \nu) = \Psi(y_{t-1} - \nu) + \epsilon_t \tag{2.1}$$

where y_t is a vector of endogenous variables, ν is the vector of expected values for the endogenous variables of y_t. The k x k matrix of coefficients is given by Ψ, and ϵ_t is a vector of N element white noise error terms with $\epsilon_t \sim N(0, \Sigma)$, where Σ is an k x k variance-covariance matrix. The vector y_t consists of net migration, government consumption and investment, labour hours, wages, private consumption and investment, tax receipts, government debt, and GDP.

2.4.1 Data

We use data for Germany, 2004Q1:2019Q3, which is seasonally adjusted, converted to real terms, transformed to working-age per capita and working-age per native (hereafter per capita and per native), logged and detrended using the Hamilton (2018) filter. The cyclical components of the data used in the SVAR include GDP, private consumption, private investment, labour hours, wage, government consumption, government investment, government tax receipts, and government debt which are subject to a migration shock.

We approximate a native population using data for total population, number of migrants, naturalisation of foreigners, and net migration data.[13] We use the two resulting datasets to produce a SVAR for per capita and per native then compare the results. The significance of comparing per capita and per native data is to show the role of migration in the economy and assimilation of migrants into the economy. The percentage of migrants of the population and of the working-age population has increased significantly over the time period studied. The only source of population growth is through migration. Whether the response of variables differs between per capita and per native terms, is dependent on the profile of the migrant and how well

[13] Details are given in section A.2.

the integration process occurs. In theory, the sign of the response has the potential to change.

Migration is identified as an AR(1) process using the autocorrelation and partial autocorrelation criteria. In selecting the lag length for the SVAR, Hannan-Quinn information criterion (HQIC) and Schwarz Bayesian information criterion (SBIC) suggest four lags and one lag respectively. The residuals are poorly behaved under four lags and well behaved under one lag, therefore, we proceed with one lag. Using Cholesky decomposition, we order net migration first as it is exogenous to the model. The remaining variables in order: government consumption and investment, labour hours, wages, private consumption and investment, tax receipts, government debt, and GDP for both datasets.

2.4.2 Results

Figure 2.3 shows the impulse responses from the SVAR using per capita data and figure 2.4 shows the SVAR using per native data. Our results show that the impact of migration on an economy does not have a negative effect on the majority of variables. The effects are mostly small, short lasting, and sometimes statistically insignificant on impact.

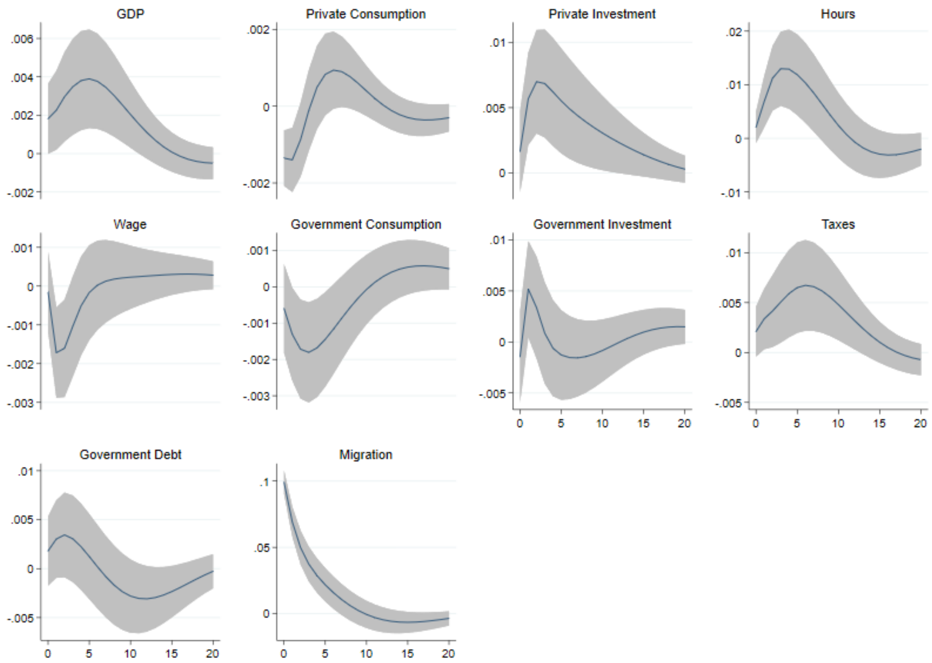
Firstly, there is a small and statistically significant positive response to GDP in both per capita and per native terms, with the peak arriving earlier in the case of per capita. This expansionary effect from migration is not extended to an increase in private consumption which negatively responds in both cases. On a household basis, migrants tend to have lower wages and in Germany be low-skilled. A large portion of the Eastern European and Turkish migrants send remittances which is a potential leakage from the economy.[14] The size of the response, while statistically significant, is quite small.

Private investment (excluding residential investment) has a positive response to migration of a size considerably larger than that of GDP or private consumption. Some of the decrease in consumption could be explained by a shift to investment. Labour hours increase in response to migration, more so in per capita terms than per native terms where the reason could be due to the fall in wage or expansionary effect from the increase in GDP.

The value for the wage is the gross value of the national concept. There is not a value available for native or migrant. However, migration to Germany is recognised as predominantly low-skilled. An increase in the number of low-skilled workers, and

[14]The two largest nationalities of migrants in Germany are Turkish and Polish (Source: Destatis via Ausländerzentralregister (Central Register of Foreigners)).

Fig. 2.3 Per Capita SVAR(1) Impulse Responses



A positive one standard deviation shock to working-age migration in Germany. The data used is in logged per capita terms and detrended using the Hamilton (2018) filter. The migration shock in the SVAR(1) is identified by a Cholesky decomposition where net migration is ordered first. The shaded areas are the 68% confidence intervals.

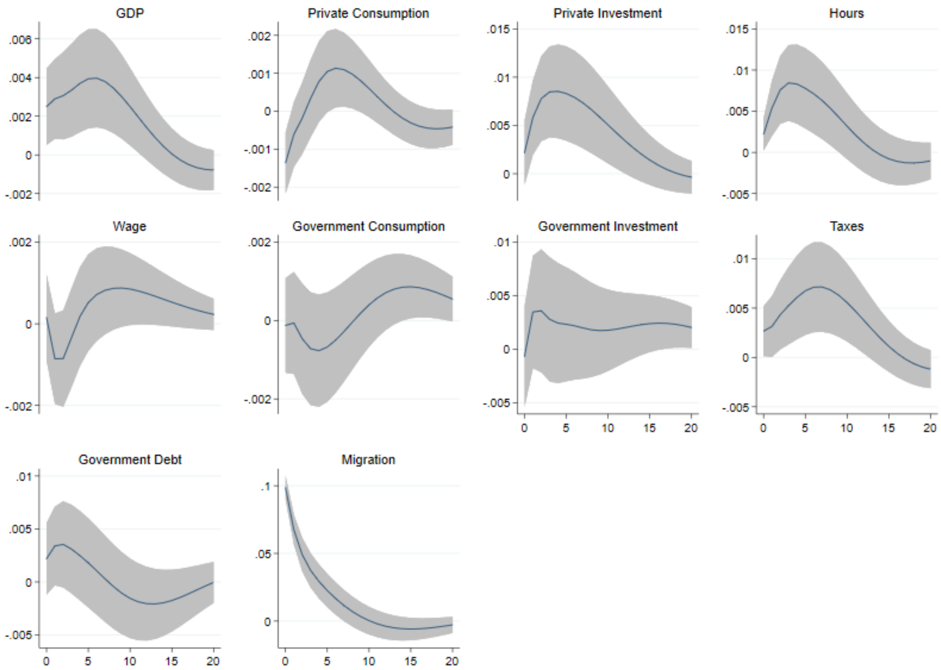
hence workers earning a lower wage, would reduce this figure. In the literature it is recognised that negative wage effects are evident with those workers that new migrants are directly competing for work with, which is typically migrants who have already arrived. That both impulse responses show they are statistically insignificant implies little effect overall.

The effects of government consumption and investment are mostly statistically insignificant. If there were negative effects on government spending, then it could be reasoned that due to the expansionary effect on GDP and the use of countercyclical fiscal policy which can reduce the government's debt.[15] The impulse response for household transfers is excluded due to the statistically insignificant responses.

The variable for taxes represents all taxes in the economy. In both per capita and per native responses there is an increase following a migration shock. We can deduce that the average migrant, directly and indirectly creates more tax revenue for the government. Indirectly due to the expansionary output effects which usually

[15] Germany has used countercyclical fiscal policy as analysed in Guerguil et al. (2017).

Fig. 2.4 Per Native SVAR(1) Impulse Responses



A positive one standard deviation shock to working-age migration in Germany. The data used is in logged per native terms and detrended using the Hamilton (2018) filter. The migration shock in the SVAR(1) is identified by a Cholesky decomposition where net migration is ordered first. The shaded areas are the 68% confidence intervals.

coincides with and increase to taxes through countercyclical fiscal policy. The effect on government debt is statistically insignificant in both data forms.

In conclusion, there is only a small difference in the responses between per capita and per native terms. The response is either the same in terms of positive or negative sign or one is statistically insignificant on impact. In neither per capita nor per native is the response to migration large. The responses in per native terms peak slightly later, and effects are longer lasting.

In the following section, we present a small open economy model with exogenous migration to evaluate these variables in a theoretical model.

2.5 A Model of Migration from a Native Perspective

We put forth a stochastic growth model of a small open economy fitting the data to Germany. The model features agents in infinitely lived heterogeneous households, perfectly competitive final good producing firms, and an active fiscal authority.

There are two types of households. The largest one consists of natives and long-term migrants, hereafter referred to as native households. The smaller household is an existing pool of recently arrived migrants. The households are imperfect substitutes. Both types of households consume, and provide labour to the firms. However, they differ by natives' ability to intertemporally optimise; own the firms; and financial market participation. Following the migration shock, the migrants first enter into the migrant households, and can acquire characteristics of native households to be classified as long-term residents. The relative size of the household plays a significant role in the economy, where a larger native household is preferable for the economy. Perfectly competitive firms employ a combination of labour, private capital, and public capital to produce the final good. The perfectly competitive and profit maximising nature ensures they pay only the marginal products for capital rental and labour services. To assess the effect of migration on government finances, we include a fiscal authority employing countercyclical fiscal policy. The government consumes, invests in public capital, provides household transfers, collects taxes, and issues one period debt.

The data implies that net migration is an AR(1) process, therefore we model it as exogenous. The shock is identified as the relaxation of migration policy with work permits, or an enlargement of the common labour market in Europe. Migration in Germany is driven by labour recruitment (de la Rica et al., 2013) therefore, these situations can be associated with full employment hence employment commences on arrival. We assume the process of migration is frictionless, such that there are no monetary or psychological costs to migration.

To consider the native perspective, and stochastic growth feature of the model, the variables specific to the native households, firms, government, and wider economy, are expressed in per native terms, and the variables specific to the migrant household are in per migrant terms.

2.5.1 Households

The small open economy features a continuum of two types of infinitely lived households. One features natives of relative size ϕ_h, and migrants of relative size $1-\phi_h$. Each household maximises their lifetime utility subject to a budget constraint. The households gain utility from consumption, c_t^j, and experience disutility from labour hours, h_t^j, where superscript $j \in [n, m]$ for native and migrant respectively. Workers within the same household are perfectly substitutable. Households consume units of the final consumption good, c_t, which consists of a domestically produced good, c_t^d, and an imported good, c_t^f

$$c_t = \left[v^{\frac{1}{\sigma}} (c_t^h)^{\frac{\sigma-1}{\sigma}} + (1-v)^{\frac{1}{\sigma}} (c_t^f)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

Where σ is the elasticity of substitution between home produced and imported intermediate good, and v is the share of home produced goods in the final consumption bundle or equivalently the home bias. The price index of the final good, P_t, is chosen to be numeraire. Consequently, all other prices are expressed relative to the final good.

2.5.1.1 Native Households

Firstly, we present the native households that consists of natives and longer-term residents of size N_t. For simplicity, referred to only as natives. They are intertemporally optimising agents whose utility function is

$$U_t = \beta^t E_t \sum_{t=0}^{\infty} \left(\psi_t^c \frac{(c_t^n - H_t^c)^{1-\theta}}{1-\theta} - \psi_t^h \frac{\phi_0 (h_t^n - H_t^h)^{1+\eta}}{1+\eta} \right) \tag{2.2}$$

There exists an internal habit formation on consumption, $H_t^c = \kappa^c c_{t-1}^n$, where κ^c measures the degree of habit persistence and on labour supply where $H_t^h = \kappa^h h_{t-1}^n$ and κ^h is the degree of habit persistence on labour supply. The standard utility parameters include: the coefficient of relative risk aversion is given by θ, ϕ_0 is the weight on disutility of labour, and η is the inverse Frisch elasticity of labour supply. The preferences on consumption and labour supply are subject to a shock, ψ_t^i where $i \in [c, h]$, which is an AR(1) process to allow for any unexpected changes in consumption or labour supply patterns. Where $\varepsilon_t^{\psi^i}$ is a i.i.d. shock with zero mean and a constant variance $\sigma_{\psi^i}^2$.

Consumption Preference Shock

$$\ln \psi_t^c = \rho^\psi \ln \psi_{t-1}^c + \varepsilon_t^{\psi^c} \tag{2.3}$$

Labour Supply Shock

$$\ln \psi_t^h = \rho^\psi \ln \psi_{t-1}^h + \varepsilon_t^{\psi^h} \tag{2.4}$$

Natives maximise utility subject to a budget constraint. Expenditures consist of units of the final consumption good, c_t^n, purchase of one-period risk-free international bonds, b_t, and government bonds b_t^g, where p_t^f is relative price of foreign produced good. Households receive income from labour services paid at a rate w_t^n per unit of labour, dividends from the firm, d_t, the repayment of the previous period's bond purchases at a real interest rate, r_t, and fiscal transfers z_t^n. The size of the native population at time t is N_t, the only way in which the size of the native population can change is when migrants gain the characteristics to be classified as native or long-term residents. Expressing the variables specific to the native household and macroeconomy in per native terms means that all the carried over stocks such as bonds are deflated by the inverse growth rate of the working-age native population, $n_t = \frac{N_t}{N_{t-1}}$.

Native Budget Constraint

$$(1+\tau^c)c_t^n+p_t^f b_t+b_t^g = w_t^n h_t^n(1-\tau^w)+d_t+p_t^f(1+r_{t-1})\frac{b_{t-1}}{n_t}+(1+r_{t-1})\frac{b_{t-1}^g}{n_t}+z_t^n \tag{2.5}$$

Consumption is taxed at a rate τ^c. Natives pay income tax that is proportional to their labour income at a rate τ^w.

Native Income Tax

$$tax_t^n = w_t^n h_t^n \tau^w \tag{2.6}$$

The natives maximise their utility subject to a budget constraint and make optimal bond market purchases.

$$\max_{c_t^n, h_t^n, b_t} \quad \beta^t E_t \sum_{t=0}^{\infty}\left(\psi_t^c \frac{(c_t^n - H_t^c)^{1-\theta}}{1-\theta} - \psi_t^h \frac{\phi_0 (h_t^n - H_t^h)^{1+\eta}}{1+\eta}\right)$$

$$(1+\tau^c)c_t^n+p_t^f b_t+b_t^g = w_t^n h_t^n(1-\tau^w)+d_t+p_t^f(1+r_{t-1})\frac{b_{t-1}}{n_t}+(1+r_{t-1})\frac{b_{t-1}^g}{n_t}+z_t^n$$

As a result, the optimality and first order conditions for consumption, labour supply, and international financial market bond purchases are as follows:

Marginal Utility of Consumption

$$\psi_t(c_t^n - \kappa^c c_{t-1}^n)^{-\theta} = \mu_t \tag{2.7}$$

Labour Supply

$$\phi_0 \psi_t (h_t^n - \kappa^h h_{t-1}^n)^{\eta} = \mu_t w_t^n (1-\tau^w) \tag{2.8}$$

Native Household Euler Equation

$$\mu_t p_t^f = \beta E_t p_{t+1}^f \mu_{t+1} \frac{(1+r_t)}{n_{t+1}} \tag{2.9}$$

2.5.1.2 Migrant Households

There is an existing pool of migrant households, of size M_t, which migrants first enter into. They differ from native households in their inability to access financial markets as they experience financial constraints similar to Canova and Ravn (2000). This type of household is a non-Ricardian or hand-to-mouth consumer.[16] Their inability to save prohibits consumption smoothing in the face of varying labour income, to the extent that any change in income is reflected by a change in consumption. We classify migrants in this way since they arrive with no capital, offer no means of firm investment, and are inactive in financial markets.

Migrants maximise period utility, which is derived from consumption, c_t^m, and leisure hours, l_t^m. Where the time constraint is defined as $1 = l_t^m + h_t^m$. The utility function uses consumption and labour hours, where ϕ_m is the weight on labour hours. The utility parameters θ^m, ϕ_m and η^m are specific to the migrant household.
Migrant Utility Function

$$U_t = \beta^t E_t \sum_{t=0}^{\infty} \left(\frac{(c_t^m)^{1-\theta^m}}{1-\theta^m} - \frac{\phi_m (h_t^m)^{1+\eta^m}}{1+\eta^m} \right) \tag{2.10}$$

They maximise utility subject to a budget constraint, expenditure on consumption is equal to the income they receive which consists of hourly labour income paid at a rate, w_t^m, and receipts of household transfers z_t^m.
Migrant Budget Constraint

$$(1+\tau^c)c_t^m = w_t^m h_t^m (1-\tau^w) + z_t^m \tag{2.11}$$

The income tax, tax_t^m, is directly proportional to labour income at a rate τ^w.
Migrant Income Tax

$$tax_t^m = w_t^m h_t^m \tau^w \tag{2.12}$$

We set up the optimisation problem for migrant households, where μ_t^m is the marginal utility of consumption for the migrant household.

$$\max_{c_t^m, h_t^m} \left(\frac{(c_t^m)^{1-\theta^m}}{1-\theta^m} - \frac{\phi_m (h_t^m)^{1+\eta^m}}{1+\eta^m} \right)$$

$$(1+\tau^c)c_t^m = w_t^m h_t^m (1-\tau^w) + z_t^m$$

The optimality conditions are given by:
Marginal Utility of Consumption

$$(c_t^m)^{-\theta^m} = \mu_t^m (1+\tau^c) \tag{2.13}$$

[16]The presence of hand-to-mouth consumers is documented in the wider economic literature, for instance Coenen and Straub (2005) and Forni et al. (2007).

Labour Supply

$$\phi_m(h_t^m)^{\eta^m} = \mu_t^m w_t^m(1-\tau^w) \tag{2.14}$$

Combining equations 2.13 and 2.14 to eliminate μ_t^m results in,

$$\phi_m(h_t^m)^{\eta^m} = \frac{w_t^m(1-\tau^w)}{c_t^m(1+\tau^c)} \tag{2.15}$$

2.5.2 Population Dynamics

The two types of households are of size N_t and M_t for natives and migrants respectively. The population in the next period $\mathrm{N_{t+1}}$ is a function of the natives at time t with a separation rate ρ^n (the emigration of natives or longer-term residents at time t) and stock of migrants. We assume no return migration of the most recently arrived migrants.

Total Population

$$\mathrm{N_{t+1}} = (1-\rho^n)N_t + M_t$$

We denote the population growth rates of natives and migrants as n_t and m_t respectively.

$$\frac{N_t}{N_{t-1}} = n_t \qquad \frac{M_t}{M_{t-1}} = m_t$$

The migration shock is reflected in the growth rate of the migrant household. To evaluate the growth rate of the migrant population, m_t, we use an AR(1) process, as supported in the data. The shock represents an increase in the pool of migrant labour, where ε_t^m is an i.i.d. shock with zero mean and a constant variance σ_m^2.

$$\frac{M_t}{M_{t-1}} = \ln(m_t) = \rho^m \ln(m_{t-1}) + \varepsilon_t^m \tag{2.16}$$

Overtime, migrants progressively gain the characteristics of natives to the extent that they are considered to be longer-term residents. This process causes an increase in the pool of native households.

$$\frac{N_t}{N_{t-1}} = n_t = \rho^{mn^0} m_t + \rho^{mn^1} m_{t-1} + \rho^{mn^2} m_{t-2}...$$

The parameter ρ^{mn^i} identifies the exogenously determined portion of migrants who become natives in that period, where $\sum_{i=0}^{n} \rho^{mn^i} = 1$ to ensure stationarity. We are interested in the ratio of migrants to natives, $\frac{m}{n}$. Using the growth rates, we can evaluate the rates of change.

$$\frac{m_t}{n_t} = \frac{M_t}{M_{t-1}} \frac{N_{t-1}}{N_t} = \frac{M_t}{N_t} \frac{N_{t-1}}{M_{t-1}}$$

Through the aggregation process, we establish a variable mn_t which is the rate of change for migrant with respect to native households. This shows that any change in the ratio of migrants to natives is dependent upon the growth rates of migrants and natives. Where $\frac{m}{n}$ is the ratio of migrants to natives.

$$\frac{M_t}{N_t} \frac{N_{t-1}}{M_{t-1}} = \frac{mn_t}{mn_{t-1}} \tag{2.17}$$

2.5.3 Aggregation

We aggregate variables for which there exist ones for natives and migrants. Total private consumption, c_t, effective labour hours, l_t, household transfers, z_t, total income tax, tax_t^w are given by a weighted average of the corresponding variables for each type of household, ϕ_h and $(1 - \phi_h)$. The aggregate variables are given in per native terms. In equation 2.18, we show the process by which mn_t is present. By transforming the variables to aggregate terms then dividing by the size of the by the native population, N_t, the ratio of migrants to natives features in the aggregate equations for consumption, labour hours, household transfers, and income tax.

Total Private Consumption

$$c_t N_t = (c_t^n N_t)^{\phi_h} (c_t^m M_t)^{(1-\phi_h)}$$

$$c_t = (c_t^n)^{\phi_h} (mn_t c_t^m)^{(1-\phi_h)} \tag{2.18}$$

Total Labour Hours

$$l_t = (h_t^n)^{\phi_h} (mn_t h_t^m)^{(1-\phi_h)} \tag{2.19}$$

Household Transfers

$$z_t = (z_t^n)^{\phi_h} (mn_t z_t^m)^{(1-\phi_h)} \tag{2.20}$$

Total Income Tax

$$tax_t^w = (tax_t^n)^{\phi_h} (mn_t tax_t^m)^{(1-\phi_h)} \tag{2.21}$$

2.5.4 Firms

There is a continuum of perfectly competitive firms, each producing a variety of the traded consumption good. In production the firms employ labour, l_t, private capital,

k_t, and public capital, k_t^g where production is subject to a technological process a_t.[17]

Production Function

$$y_t = a_t k_t^{\alpha} l_t^{(1-\alpha)} (k_t^g)^{\alpha^g} \tag{2.22}$$

The share of private capital within a firm is given by α, with the share of labour given by $1-\alpha$. The firm utilises public capital, k_t^g, where α^g is the elasticity of output with respect to public capital indicating its productiveness.[18] Total factor productivity, a_t, follows an AR(1) process, where ρ^a is the coefficient on the previous period's value, and ε_t^a is an i.i.d. shock with zero mean and variance σ_a^2. A positive shock to total factor productivity, a_t, increases firm productivity.

Technological Process

$$\ln a_t = \rho^a \ln a_{t-1} + \varepsilon_t^a \tag{2.23}$$

Firms use a composite of native labour hours h_t^n and migrant labour hours h_t^m, in proportion to their relative size: ϕ_h for native, and $1-\phi_h$ for migrant. We formulate effective labour supply as in equation 2.19. As the firm's labour force is comprised from both households, the effective labour force needs to reflect the effect of a change in labour force size needs to resulting from a migration shock.

Firms employ private capital from the native households which follows the law of motion in equation 2.24. The capital accumulation constraint uses a Hayashi capital adjustment cost function. Where δ is depreciation rate of existing private capital and we assume that $s(.)$ has the following properties: $s'(.) > 0$, $s''(.) \geq 0$, $s'(\delta) = 0$, and $s''(\delta) = 0$.

Capital Accumulation

$$k_t = (1-\delta)\frac{k_{t-1}}{n_t} + s\left(\frac{x_t n_t}{k_{t-1}}\right)\frac{k_{t-1}}{n_t}\psi_t^x \tag{2.24}$$

The firm is subject to an investment shock, ψ_t^x, designed to reflect a situation where a firm's access to credit changes. It is an AR(1) process with an i.i.d. shock that has a zero mean and a constant variance $\sigma_{\psi^x}^2$, and ρ^{ψ^x} is the persistence parameter.

Investment Shock

$$\ln \psi_t^x = \rho^{\psi^x} \ln \psi_t^x + \varepsilon_t^{\psi^x} \tag{2.25}$$

Firms are profit maximisers, where firm profits are denoted by Π_t, and p_t^h is the relative price of domestically produced good.

[17] We introduce public capital due to the inclusion of fiscal policy and reasoning in Elekdag and Muir (2014).

[18] We can assume increasing returns to scale with respect to public capital, see Baxter and King (1993).

Firm Profit

$$\Pi_t = y_t p_t^h - w_t^n h_t^n - w_t^m h_t^m mn_t - x_t \tag{2.26}$$

Firms maximise their profits with respect to the budget constraint, to make optimal use of capital. Tobin's Q identifies the shadow price of capital. We set out their optimisation problem, where μ_t is the marginal utility of consumption of the native household, and q_t is Tobin's Q.

$$\max_{k_t, x_t, h_t^n, h_t^m} \quad y_t p_t^h - w_t^n h_t^n - w_t^m h_t^m mn_t - x_t$$

$$k_t = (1-\delta)\frac{k_{t-1}}{n_t} + s\left(\frac{x_t n_t}{k_{t-1}}\right)\frac{k_{t-1}}{n_t}\psi_t^x$$

Investment

$$q_t = \left[s'\left(\frac{x_t n_t}{k_{t-1}}\right)\frac{\psi_t^x}{n_t}\right]^{-1} \tag{2.27}$$

Tobin's Q

$$q_t = \beta\frac{\mu_{t+1}}{\mu_t}\left[\alpha\frac{y_{t+1}}{k_t} + (1-\delta)\frac{q_{t+1}}{n_{t+1}}\right] \tag{2.28}$$

The marginal product of capital is identified by ρ_t^k.

Marginal Product of Capital

$$\rho_t^k = \alpha\frac{y_t p_t^h}{k_{t-1}} \tag{2.29}$$

Firms employ workers, where wage is equal to the marginal product due to the frictionless labour market.

Native Wage

$$w_t^n = (1-\alpha)\phi_h\frac{y_t p_t^h}{h_t^n} \tag{2.30}$$

Migrant Wage

$$w_t^m = (1-\alpha)(1-\phi_h)\frac{y_t p_t^h}{h_t^m mn_t} \tag{2.31}$$

As a result from the per native evaluation, the variable mn_t is present for the migrant wage so that when a migration shock occurs, the migrant wage falls due to increased labour supply. This agrees with the literature that due to imperfect substitutability of workers, only those that are directly competing for the same type of employment are affected, in this case the migrants.[19] To compare the wage level between the two households, we introduce a variable denoting wage differential, w_t^{Δ}.

[19] See D'Amuri et al. (2010).

Wage Differential

$$w_t^{\Delta} = \frac{w_t^n}{w_t^m} \tag{2.32}$$

2.5.5 Fiscal Policy

There is a large literature dedicated to analysing varying aspects of monetary policy, yet fiscal policy remains relatively unexplored, especially in the setting of a small open economy. There are questions surrounding the use of fiscal policy as there are conflicting results from government expenditure changes on important macroeconomic variables stemming from an empirical or DSGE model. Empirical models mostly observe positive effects of government expenditure on the macroeconomic variables such as output, consumption, and employment. However, DSGE models offer alternative findings such as Coenen and Straub (2005) and Gali et al. (2007). To examine the effects of migration on government finances, we include a fiscal authority.

The existing literature on fiscal policy is predominantly in the context of a closed economy, featuring only one household. Studies featuring two households include Coenen and Straub (2005), Horvath (2009), and Stork (2011) who uses a small open economy model. Guerguil et al. (2017) analysis of countries' use of fiscal rules shows that Germany has used various fiscal policy rules including the balanced budget rule and cyclically adjusted balance rule. This supports the use of a countercyclical fiscal policy. The model features fiscal rules based on Coenen et al. (2013) and Gadatsch et al. (2016).

The fiscal authority collects taxes, consumes the final good, invests in public capital, provides household transfers, and finances any budget deficit by issuing one period debt.

2.5.5.1 Government Revenue

There are three types of taxes that the government collects, taxes from household income, tax_t^w, tax on consumption, tax_t^c, and corporation tax, tax_t^f. Labour income is taxed at a rate τ^w, consumption tax rate τ^c, and the corporation tax rate τ^f. We use proportional taxes.[20] Income tax is described in equation 2.21. Total consumption is a function of natives' consumption, c_t^n, and migrants' consumption, c_t^m, as shown in equation 2.18. Consumption is taxed at a rate τ^c.
Consumption Tax

$$tax_t^c = \tau^c c_t \tag{2.33}$$

[20] Canova and Ravn (1998) use implicit tax rates which enables the transfer of funds in a welfare state from skilled households to unskilled.

Firms are subject to corporation tax on profits at a rate τ^f.
Corporation Tax

$$tax_t^f = \tau^f \Pi_t \tag{2.34}$$

A variable tax_t provides the total government revenue from taxation.
Total Tax

$$tax_t = tax_t^w + tax_t^c + tax_t^f \tag{2.35}$$

2.5.5.2 Government Expenditure

Government expenditure consists of government consumption, government investment for which there is full home bias, and they provide household transfers. Following Coenen et al. (2013), fiscal instruments follow a countercyclical policy rule dependent upon output, government debt, and are subject to shocks.[21] Where ζ^{g,b^g} and $\zeta^{g,y}$ are the debt and output feedback coefficients and ϑ^g is the persistence parameter on government expenditure of form g where $g \in [c, x, z]$ for consumption, investment, and transfers.

Public capital k_t^g is adjusted through depreciation of the previous period's capital and government investment, x_t^g
Public Capital Accumulation

$$k_t^g = (1 - \delta^g)\frac{k_{t-1}^g}{n_t} + x_t^g \tag{2.36}$$

Public capital depreciates at a rate, δ^g. Government investment evolves according to equation 2.38 and subject to a shock which is an AR(1) process where ρ^{x^g} is the persistence parameter, and $\varepsilon_t^{x^g}$ is an i.i.d. shock with zero mean and variance $\sigma_{x^g}^2$.
Government Investment Shock

$$\ln x_t^g = \rho^{x^g} \ln x_{t-1}^g + \varepsilon_t^{x^g} \tag{2.37}$$

Government Investment

$$\widehat{x_t^g} = \vartheta^{x^g} \widehat{x_{t-1}^g} - \zeta^{x^g,y} \widehat{y_{t-1}} - \zeta^{x^g,b^g} \widehat{b_{t-1}^g} + \psi_t^{x^g} \tag{2.38}$$

Governments provide household transfers to the households, z_t^j. The fiscal rule for transfers is a function of government debt and output. In this section we present the log-linearised equations.

[21]We exclude the pre-announcement coefficients but include household transfers in the fiscal rules.

Household Transfers Rule

$$\widehat{z_t} = \vartheta^z \widehat{z_{t-1}} - \zeta^{z,y} \widehat{y_{t-1}} - \zeta^{z,b^g} \widehat{b^g_{t-1}} + \psi^z_t \tag{2.39}$$

Household transfers are subject to a shock, representing a policy change. The process follows an AR(1) form, where ρ^z is the coefficient on the previous period's value, and ε^z_t is an i.i.d. shock with zero mean and variance σ^2_z.
Household Transfers Shock

$$\ln \psi^z_t = \rho^{\psi^z} \ln \psi^z_{t-1} + \epsilon^{\psi^z}_t \tag{2.40}$$

We set $z^m_t = z_t$, therefore following equation 2.20, $z^{\phi_n}_t = (z^n_t)^{\phi_n}(mn_t)^{1-\phi_n}$ which simplifies to $z_t = z^m_t$ therefore

$$z_t = z^m_t (mn_t)^{\frac{1-\phi_n}{\phi_n}} \tag{2.41}$$

The final component of government expenditure is government consumption, c^g_t. The fiscal rule is countercyclical, and subject to a government consumption shock, $\psi^{c^g}_t$.
Government Consumption

$$\widehat{c^g_t} = \vartheta^{c^g} \widehat{c^g_{t-1}} - \zeta^{c^g,y} \widehat{y_{t-1}} - \zeta^{c^g,b^g} \widehat{b^g_{t-1}} + \psi^{c^g}_t \tag{2.42}$$

The shock is an AR(1) process where ρ^{c^g} is the coefficient on the previous period's value, and $\varepsilon^{\psi^{c^g}}_t$ is an i.i.d. shock with zero mean and variance $\sigma^2_{\psi^{c^g}}$.
Government Consumption Shock

$$\ln \psi^{c^g}_t = \rho^{\psi^{c^g}} \ln \psi^{c^g}_{t-1} + \epsilon^{\psi^{c^g}}_t \tag{2.43}$$

In summation, government expenditure is given as,
Government Expenditure

$$ge_t = x^g_t + z_t + c^g_t \tag{2.44}$$

2.5.5.3 Government Budget

The government's consumption is financed through taxation, and any deficits are financed by the issue of one-period risk-free bonds. These are repaid at the rate, r_t. The primary surplus is defined as government expenditures less tax revenues.[22]
Budget Surplus

$$surp_t = tax_t - ge_t \tag{2.45}$$

[22] We use government surplus rather than deficit due to the format of the data.

The government budget constraint comprises of primary budget and the government's borrowing and debt repayments.
Government Budget Constraint

$$ge_t + \frac{b^g_{t-1}(1 + r_{t-1})}{n_t} = tax_t + b^g_t \tag{2.46}$$

2.5.6 An Open Economy

To close the small open economy model, we analyse the market clearing conditions. We treat foreign demand for the domestic good, y^f_t, as an exogenous, or AR(1) process, where $\varepsilon^{y^f}_t$ is an i.i.d. shock with zero mean and a constant variance $\sigma^2_{y^f}$.
Foreign Demand

$$\ln y^f_t = \rho^{y^f} \ln y^f_{t-1} + \varepsilon^{y^f}_t \tag{2.47}$$

In closing the model, we require a market clearing equation to solve for terms of trade.
Market Clearing

$$y_t = v(p^h_t)^{-\sigma} \left(c_t + x_t + c^g_t + x^g_t\right) + v^f \left(\frac{rer_t}{p^h_t}\right)^{\sigma^f} y^f_t \tag{2.48}$$

The economy is defined by a current account which is a function of total home consumption, investment, and trading in the international financial market.
Current Account

$$y_t = c_t + x_t + p^f_t b_t - (1 + r_{t-1}) p^f_t \frac{b_{t-1}}{n_t} + c^g_t + x^g_t \tag{2.49}$$

To close the model, we introduce a debt-elastic interest rate premium as in Schmitt-Grohe and Uribe (2003). The premium allows deviations from the world interest rate when bond holdings deviate from their steady state levels. The agents active in international financial markets face an interest rate, r_t, which is a function of the world interest rate, r^*, and deviations of the country's level of debt away from its steady state. The world interest rate, r^*, is assumed to be constant.
Interest Rate

$$r_t = r^* \exp^{\left(-\phi_{bhc}\left(b_t - \bar{b}\right)\right)} \tag{2.50}$$

2.6 Bayesian Estimation

In order to solve the model, the constraints and first order conditions are log-linearised around the steady state. The log-linearised model is estimated using the Bayesian techniques as described in An and Schorfheide (2007).

2.6.1 Data

We fit the model to quarterly data for Germany. As the number of data series used in estimation cannot be greater than the number of shocks in the model, we therefore use nine data series including GDP, private consumption, private investment, household transfers, government consumption, government investment, total labour hours, net migration and US GDP for 2004Q1:2019Q3.[23] The economic time series have been collected in national currency and deflated using the corresponding deflator to convert the values to real terms. We use GDP data from the US to represent foreign output. The data series has been transformed to per working-age native terms, logged, and detrended using the Hamilton (2018) filter. Figure 2.5 plots the observables used in estimation.

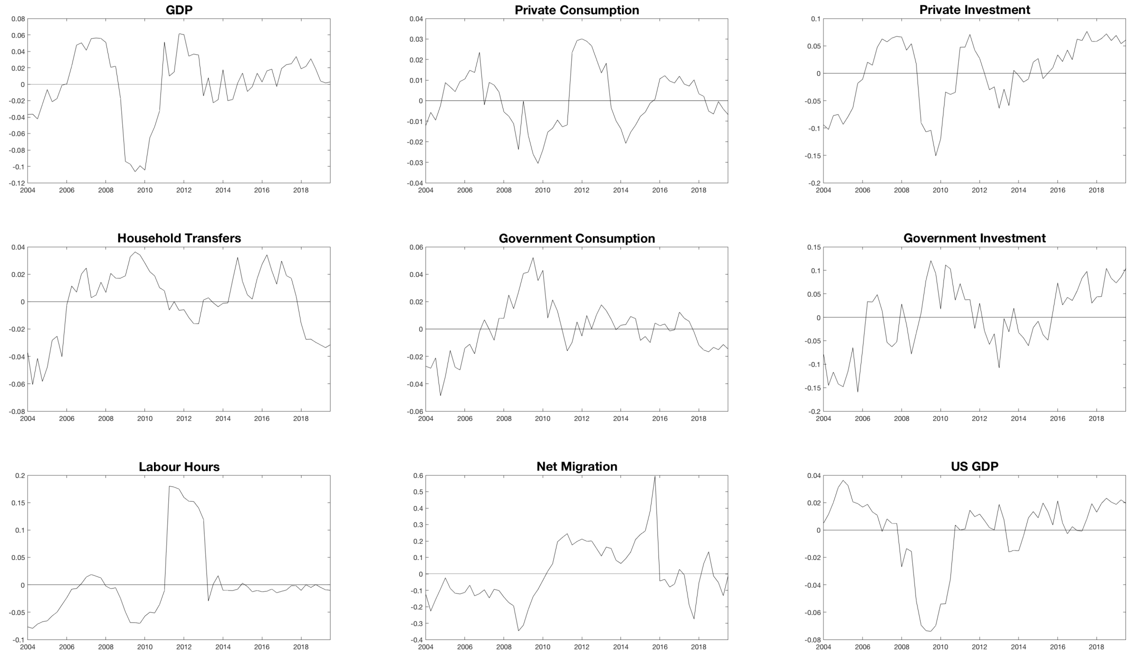
2.6.2 Calibration

We fix some parameters in the estimation process to address identification issues. The values are taken from literature or by matching long run targets in the data. The discount factor $\beta = \frac{1}{1.01}$ which equates to an annual real interest rate of approximately 4%. The values for a private capital depreciation rate $\delta = 0.025$ equivalent to 10% per year and public capital depreciation rate, $\delta^g = 0.01$ equivalent to 4% a year as per Elekdag and Muir (2014). The ratio of migrants to natives calibrated to $\frac{M}{N} = 0.1454$, and an openness to trade parameter $\gamma = 0.3$.[24] The value for the tax rates for consumption is taken from Gadatsch et al. (2016), corporation tax from Salgado (2011) and labour tax rates are determined by steady state calculations. For the international sector, we assume all relative prices and terms of trade to have a steady state value of 1. Net debt and bond holdings in steady stare are set to zero. We choose the transition rate ρ^{mn} ad hoc to due the lack of consensus or reliable data as in Stähler (2017).

[23] Data sources are detailed in table A.1.

[24] The average percentage of migrants in Germany during the sample is 12.7% (OECD data). If we change it to $\frac{M}{N}$ so $\frac{12.7}{100-12.7} = 0.1454$.

Fig. 2.5 Data Observables



This figure shows the Hamilton filtered series used in the estimation of this model. The vertical axis measures the percentage deviations from trend. The horizontal axis measures the quarterly period. The first eight variables are for Germany, and the final variable is GDP from the United States.

2.6.3 Estimation Results

We estimate nineteen model parameters, nine autoregressive parameters and the standard deviation of the nine shocks, the results of which are presented in table 2.2. The table shows the mean for the prior and posteriors, the standard deviation of the prior, and 5th and 95th percentiles of the posterior distribution (obtained after 2 million iterations). We choose standard priors.

Table 2.2 Priors and Posteriors

Parameter Description		**Prior**	**Mean**	90% HPD interval		**PDF**	**PstDev**
AR(1) Coefficients							
Technology	ρ^{a}	0.7000	0.8932	0.8418	0.9448	β	0.1000
Migration	ρ^{m}	0.7000	0.7835	0.6926	0.8773	β	0.1000
Foreign Output	ρ^{y^f}	0.7000	0.6651	0.5015	0.8255	β	0.1000
Consumption Preference	ρ^{ψ^c}	0.7000	0.8647	0.8056	0.9248	β	0.1000
Labour Supply Preference	ρ^{ψ^h}	0.7000	0.8155	0.7337	0.8962	β	0.1000
Private Investment	ρ^{ψ^x}	0.7000	0.9789	0.9670	0.9915	β	0.1000

Continued on next page

Parameter Description		**Prior**	**Mean**	90% HPD interval		**PDF**	**PstDev**
Government Investment	ρ^{x^g}	0.7000	0.6359	0.4830	0.7874	β	0.1000
Government Consumption	$\rho^{\psi^{g^c}}$	0.7000	0.7095	0.5897	0.8254	β	0.1000
Transfers	ρ^{ψ^z}	0.7000	0.6171	0.4312	0.7883	β	0.1000
Model Parameters							
Adjustment Costs	ac	2.0000	2.9239	2.3012	3.5738	Γ	0.5000
Elasticity of Substitution	σ	2.0000	1.6625	1.3427	1.9787	Γ	0.5000
Risk Aversion	θ	5.0000	4.6800	3.9842	5.3542	Γ	0.5000
Risk Aversion	θ^m	5.0000	4.9671	4.1685	5.7344	Γ	0.5000
Frisch Elasticity	η	5.0000	5.8816	5.2515	6.5395	Γ	0.5000
Frisch Elasticity	η^m	1.0000	1.2118	0.4730	1.9152	Γ	0.5000
Bond Holding Constraints	ϕ_{bhc}	0.1000	0.1076	0.0916	0.1222	β	0.0100
Consumption Habit Formation	κ^c	0.5000	0.5526	0.4367	0.4367	β	0.1000
Labour Hours Habit Formation	κ^h	0.5000	0.4277	0.2954	0.5572	β	0.1000
Openness to Trade	γ	0.3000	0.3012	0.2848	0.3164	β	0.0100
Persistence of Transfers	ϑ^z	0.7500	0.6667	0.5080	0.8393	β	0.1000
Debt Feedback on Transfers	$\zeta^{z,b}$	0.2000	0.2198	0.1384	0.3005	β	0.0500
Output Feedback on Transfers	$\zeta^{z,y}$	0.2000	0.2033	0.1262	0.2829	β	0.0500
Persistence of Gov Consumption	ϑ^{c^g}	0.7500	0.5360	0.3753	0.6995	β	0.1000
Debt Feedback on c^g	$\zeta^{c^g,b}$	0.2000	0.1640	0.1045	0.2220	β	0.0500
Output Feedback on c^g	$\zeta^{c^g,y}$	0.2000	0.2070	0.1361	0.2786	β	0.0500
Persistence of Gov Investment	ϑ^{x^g}	0.7500	0.7349	0.5947	0.8838	β	0.1000
Debt Feedback on x^g	$\zeta^{x^g,b}$	0.2000	0.1644	0.0994	0.2307	β	0.0500
Output Feedback on c^g	$\zeta^{x^g,y}$	0.2000	0.1930	0.1120	0.2708	β	0.0500

Continued on next page

Parameter Description		Prior	Mean	90% HPD interval		PDF	PstDev
Standard Deviations							
Technology	σ_a	1.0000	0.2013	0.1616	0.2374	Γ^{-1}	1.0000
Migration	σ_m	1.0000	0.1748	0.1564	0.1921	Γ^{-1}	1.0000
Foreign Output	σ_{y^f}	1.0000	0.1639	0.1564	0.1736	Γ^{-1}	1.0000
Consumption Preference	σ_{ψ^c}	1.0000	0.6354	0.4386	0.8254	Γ^{-1}	1.0000
Labour Supply	σ_{ψ^h}	1.0000	0.2883	0.2430	0.3324	Γ^{-1}	1.0000
Private Investment	σ_{ψ^x}	0.5000	0.1694	0.1406	0.1965	Γ^{-1}	0.2000
Government Investment	σ_{x^g}	1.0000	0.2375	0.2261	0.2512	Γ^{-1}	0.5000
Government Consumption	σ_{c^g}	0.5000	0.0882	0.0782	0.0975	Γ^{-1}	0.5000
Household Transfers	σ_z	0.5000	0.1998	0.1388	0.2633	Γ^{-1}	0.5000

Results from the Bayesian estimation after 2 million iterations. The first column describes the estimated parameter and column two shows the corresponding symbol. Column three gives the priors. Columns 4 to 6 show the mean, 5^{th} and 95^{th} percentiles of the posteriors. Column seven shows the distribution function and the final column shows the deviation of the posterior.
Distribution Functions: β Beta, Γ Gamma, and Γ^{-1} Inverse Gamma.

The posterior distributions obtained indicate that the majority of estimates are not prior driven, rather data driven. The main characteristics of the posterior distributions are shown in columns 4, 5, and 6 of table 2.2. Starting with the persistence parameters, all of the shocks have a large persistence with private investment the largest. In terms of the model parameters, given that some of the estimates are close to the prior means, they may not be well identified by the data. The habit formation of consumption is more persistent than labour supply. The persistence parameter of government consumption in the feedback rule is significantly lower than either of the other rules. However, the standard deviation is much lower than either transfers or investment. The feedback coefficients on debt for both government consumption and transfers are close to zero. In contrast, output feedback for both policy rules is significant. The size of the shocks larger than output are consumption preference, labour supply, and government investment.

2.7 Results

Using the results from estimation in section 2.6, we present the impulse responses following a one standard deviation migration shock to the small open economy in figure 2.6.

Table 2.1 Calibrated Parameters and Steady State Ratios

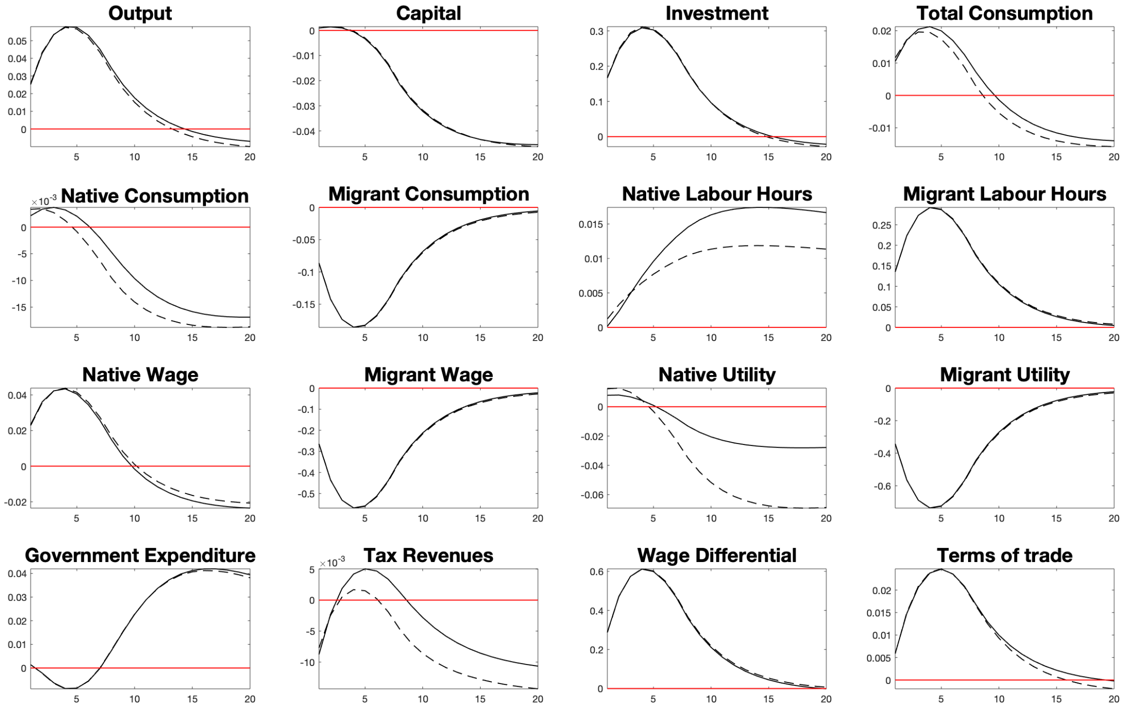
Description	Parameter	Value
Baseline Parameters		
Discount Factor	β	$\frac{1}{1.01}$
Depreciation Rate	δ	0.0250
Public Capital Depreciation Rate	δ^g	0.0100
Share of Capital	α	0.3300
Elasticity of Public Capital	α^g	0.0250
Labour Hours	h^n,h^m	0.4226
Migrant to Native Ratio	$\frac{M}{N}$	0.1454
Transition Rate	ρ^{mn}	0.1250
Share of Natives	ϕ_n	$\frac{1}{1+\frac{M}{N}}$
Tax Rates		
Consumption	τ^c	0.1830
Corporate	τ^f	0.2350
Steady State Ratios		
Private Consumption to GDP	$\frac{c}{y}$	0.6800
Private Investment to GDP	$\frac{x}{y}$	0.1300

This table shows the calibrated values of parameters and selected steady state ratios in the model.

We begin our analysis with the macroeconomy where the variables are in per native terms. The migration shock increases the population and hence the supply of labour. From the first period of landing, the native population size is effected. Due to this increase in the native population, per native capital stocks and bond holdings decrease ceteris paribus. The marginal product of capital increases due to the fall in capital stocks and increase in output. However, the terms of trade has insignificant effects. In response to a higher return on capital, there is a significant increase in investment in private capital.

As a result of the migration shock, there is a direct increase in supply of labour by the migrant household. This causes a decrease in the wage level they receive. However, as the majority of their income is from labour services, they continue with higher labour hours to offset the decrease in total labour income which results in a smaller response in consumption. The household transfers they receive do (eventually) increase but contribute only a small amount of their income. Both the decrease in consumption and increase in labour supply decreases utility of the household. In contrast, natives experience a higher wage level due to the increase in output but also have an increase in labour hours. The increase in labour supply is more than implied by the SVAR presented in figure 2.4.

Fig. 2.6 Impulse Responses to a Migration Shock



A migration shock to a small open economy following a 1 standard deviation shock to migration. The horizontal axis identifies the quarter following the migration shock. Vertical axis shows the deviation from steady state. The solid black line shows the impulse response when the native household experiences habits in labour and consumption. The dashed black line shows the impulse responses when the native household does not experience habit persistence $\kappa^c = \kappa^h = 0$.

Output increases due to the rise in labour supply from both natives and migrants, which outweighs the small decrease in public and private capital stocks. This increase in output is small but still greater than the increase implied by the SVAR. The higher level of demand is driven by investment more so than consumption or government expenditures.

Firms see a decrease in their profits due to higher levels of investment and a higher total labour expenses paid to natives. Whereas for migrants, the increase in labour hours is outweighed by the fall in wage, but the effect is lessened due to the larger household size. The wage differential (equation 2.32) is significantly in favour of natives. This difference is mostly down to the decrease in the wage level of migrants.

As recently arrived migrants gain native characteristics they transfer to be classified as native agents. The native household experiences an increase in wages levels due to the expansionary effect on the economy. Their increased labour income results in higher consumption initially, though the change is insignificant. The insignificant change in consumption and increase in labour hours reduces the utility for the household, however, the change is insignificant due to the small responses.

There is a different response amongst forms of tax receipts. Lower firm profits result in lower levels of corporation tax, yet increased aggregate consumption produces higher consumption tax. Total labour income tax increases due to the natives and increase in labour supply resulting from migration.

For fiscal expenditures, initially the increase in output outweighs the fall in government debt so each of the three types of government expenditure decreases slightly before the decrease in government debt outweighs in the increase in output which increases each of the three expenditures. The decrease in government debt stems from the migration shock which passes through to reduce the per native value of government debt. This is shown by an insignificant change in the surplus as the change in government spending is approximately equal to the change in tax receipts.

Aside from different parameter specifications, we introduced habit persistence in both labour supply and consumption for the native households to differ their utility function from the migrant household beyond parameter specification. The role of habits is designed to introduce persistence for the largest household. However, when we set the persistence parameters to zero $\kappa^c = \kappa^h = 0$, as identified on figure 2.6 by the dashed lines, we do not see a significant change in the impulse responses on the macroeconomy. The utility of the native household is understandable since persistence prohibits large changes. The most notable change lies with labour supply which continues in an upward trajectory in the presence of habits. Whereas consumption decreases slightly sooner without a change in magnitude. Understandably, the responses are small to begin with, but these results show that the direction of consumption and labour supply of natives do not change in response to migration.

2.8 Conclusion

The increasing levels of immigration, the contrasting views of natives, and the bearing it has within society posed a number of questions that related to macroeconomy in the countries that host migrants, and what immigration means for natives. All areas of society are, in one way or another, affected by migration. Immigration affects households in terms of consumption, income, investment, and labour supply. Firms are affected as an increase in output and labour supply as well as changing labour costs. In a wider environment, governments are affected in terms of their expenditure, tax receipts and borrowing requirements.

In this paper, we have presented an empirical model, and a DSGE model to show the effects of migration from a native perspective. The contribution of our paper to the literature of the macroeconomics of migration is the Bayesian estimation of a model in a small open economy with exogenous migration and an empirical evaluation of the impacts of migration on specific fiscal variables. The results from both models show that, on the whole, migration has a positive effect on the macroeconomy. The SVAR results show that output increases in per capita and per native terms. The distinction between per capita, per native, and aggregate is important because immigrant workers could be adding to the economy but less than another native. The speed of integration of immigrant workers into the native labour force is debated in the small microeconomic literature, we have used one of the slower, and smoother, timescales. As migration is exogenous, we do not assess the drivers of immigration, only the effects of immigration on the macroeconomy. The variables which experience the greatest impact are variables specific to the migrant household and total labour supply. This result matches with other literature which suggests that only sectors where there are migrants and natives competing in the same sector, are they affected. It is important for an economy to recognise migrants as an equivalent of natives to fully utilise their skills and contribution to society. We have used German data for this research but the interpretations are still valid for countries which host an increasing number migrants.

Appendix A

A.1 Model Derivations

International Relative Prices

This small open economy model accounts for domestic and foreign prices as follows. Household consumption and is comprised of domestic and foreign produced intermediate goods, c_t^h , c_t^f.

$$c_t = \left[v^{\frac{1}{\sigma}} (c_t^h)^{\frac{\sigma-1}{\sigma}} + (1-v)^{\frac{1}{\sigma}} (c_t^f)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

Where σ is the elasticity of substitution between home produced and imported intermediate good, and v is the share of home produced goods or equivalently the home bias. Where the corresponding price index is:

$$P = \left[v {P_H}^{1-\sigma} + (1-v) {P_F}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

The terms of trade is the ratio of import to export prices expressed in the same currency.

$$T = \frac{P_F}{SP_H^*}, \quad SP_H^* = P_H$$

Since we assume that the law of one price holds for individual goods, and all goods, we can rewrite the terms of trade expression as

$$T = \frac{P_F}{P_H}$$

The only way by which the consumer price based real exchange rate can deviate from purchasing power parity is by home bias.

A.2 Data

The sources of data used in the empirical model and in the estimation of the DSGE model are described in table A.1. Data that has been sourced which was not seasonally adjusted, has been seasonally adjusted using X-12-Arima in stata.

German migration data has been sourced from the German Federal Statistics Office at a monthly frequency which has been totalled to quarterly data and seasonally adjusted. The age group of migrants under 18, 18-65, and 65+ is available annually. The percentage of 18-65 year olds for each nationality of immigrants and emigrants is multiplied by the quarterly values of immigration to obtain working age migration per quarter.

In approximating the native population size, we use data for the total population by age and nationality which provides the number of migrants at the end of each year. The population change for the migrant household is calculated by adding net migration figures of non-Germans, less naturalisation of citizens. The native population size is calculated by net migration of working-age Germans plus naturalisation of citizens. The size of working age is calculated by the calculating percentage working-age of Germans and non-Germans of the total Germans and non-Germans then multiplied by the respective population.

To represent foreign output, we use macroeconomic data from the USA sourced from the OECD. This is deflated at the corresponding index for the USA. The USA is chosen since it is the world's largest economy, one of only three larger than Germany.

A.3 Further Details on Population Changes

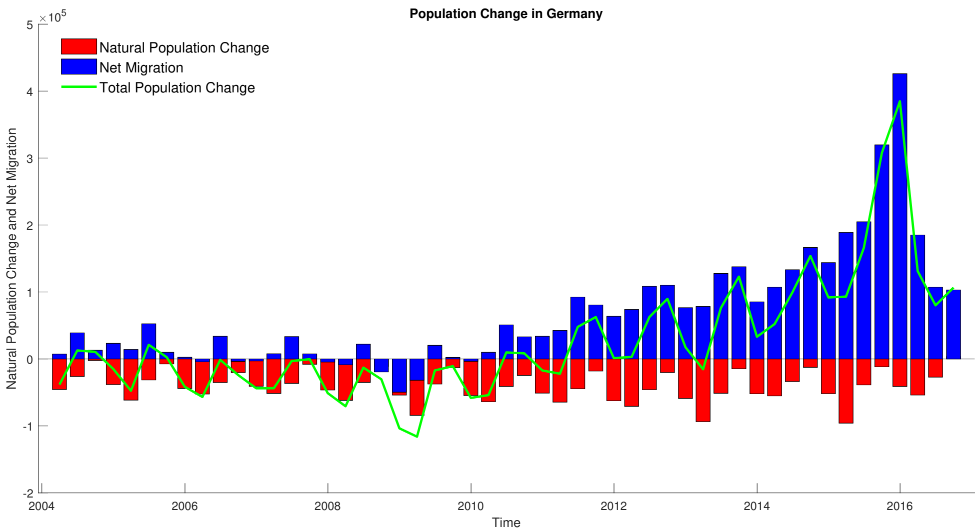
Following on from section 2.1 and section 2.3, here we discuss the evolution of natural population change and migration in Germany, that has resulted in the concerning demographic changes. Population change consists of natural population change (births minus deaths) and net migration. Figure A.1 shows the causes for population change in Germany. The natural population change is negative (apart from 2016Q3), deaths have continuously exceeded births. The total population change has only become consistently positive since 2011 which is driven by an increase in net migration. Notably, the strong upward trend of positive net migration began before the migration crisis of 2015 where net migration exceeded 1 million annually. Migration has significantly increased in recent years, caused by immigration of non-Germans. There are a group of countries which are frequent destinations/origins

Table A.1 Data Sources

Name	Source	Measure	Code
Migration to/from Germany	Destatis		12711-0008
Births in Germany	Destatis		12612-0001
Deaths in Germany	Destatis		12613-0002
Population by age and nationality	Destatis		12411-0006
Wages and Salaries (national concept)	Destatis	Euros per month	81000-0008
Hours worked by employee	Destatis		81000-0016
Government National Accounts	Destatis	Euros (1000)	81000-0032
Crude Rate of Net Migration	Eurostat	Per 1000 habitants	tsdde230
GDP	OECD	CARSA, DNBSA	B1-GE
Private Consumption	OECD	CARSA, DNBSA	P31S14-S1
Government Consumption	OECD	CARSA, DNBSA	P3S13
Gross Fixed Capital Formation	OECD	CARSA, DNBSA	P51
Private Non-Residential Investment	OECD	CARSA, DNBSA	
Public Investment	OECD	CARSA, DNBSA	
United States GDP	OECD	CARSA, DNBSA	B1-GE
Public Sector Debt	OECD	CARNSA	

The data sources used in calibration, estimation, and VAR. CARSA -National currency, current prices, annual levels, seasonally adjusted DNBSA - Deflator, national base year, seasonally adjusted.

Fig. A.1 Population Change in Germany



Net Migration is shown by blue bars, Natural Population Change by red bars, and Total Population Change by green line (10^5 - 100,000s of people)
Source: Statistisches Bundesamt (Destatis) - Federal Statistics Office Germany.

for migrants. Austria, Switzerland, France, Poland, Greece, Italy, and the United Kingdom have persistently high migration figures in the thousands.

A.4 Model Equations

There are 45 variables plus the utility of each household. In these sections we present the level equations, the log-linearised equations and steady state calculations. Time dependent log-linearised variables are shown with a hat, e.g. $\widehat{c_t^N}$ and the steady state variables are shown as c^N.

Level Equations

Native Households

Marginal Utility of Consumption

$$\psi_t^c (c_t^n - \kappa c_{t-1}^n)^{-\theta} = \mu_t \tag{A.4.1}$$

Labour Supply

$$\phi_0 \psi_t^h \psi_t (h_t^n)^\eta = \mu_t w_t^n \tag{A.4.2}$$

Native Household Euler Equation

$$\mu_t p_t^f = \beta E_t p_{t+1}^f \mu_{t+1} \frac{(1 + r_t)}{n_{t+1}} \tag{A.4.3}$$

Consumption Preference Shock

$$\ln \psi_t^c = \rho^{\psi^c} \ln \psi_{t-1}^c + \varepsilon_t^{\psi^c} \tag{A.4.4}$$

Labour Supply Shock

$$\ln \psi_t^h = \rho^{\psi^h} \ln \psi_{t-1}^h + \varepsilon_t^{\psi^h} \tag{A.4.5}$$

Migrant Households

Budget Constraint

$$c_t^m = w_t^m h_t^m (1 - \tau^w) + z_t^m \tag{A.4.6}$$

Labour Supply

$$\phi_0 (h_t^m)^\eta = w_t^m (c_t^m)^{-\theta} \tag{A.4.7}$$

Population Dynamics

Growth Rate Migrants

$$\frac{M_{t+1}}{M_t} = \ln(m_t) = \rho^m \ln(m_{t-1}) + \varepsilon_t^m \tag{A.4.8}$$

Rates of Change

$$\frac{m_t}{n_t} = \frac{mn_{t+1}}{mn_t} \tag{A.4.9}$$

Population Growth Rate

$$n_t = \rho^{mn^1} m_{t-j} + \rho^{mn^2} m_{t-(j+1)} + \rho^{mn^3} m_{t-(j+2)} \cdots \tag{A.4.10}$$

Aggregation

Total Household Consumption

$$c_t = (c_t^n)^{\phi_h} (mn_t c_t^m)^{(1-\phi_h)} \tag{A.4.11}$$

Native Income Tax

$$tax_t^n = \tau^w w_t^n h_t^n \tag{A.4.12}$$

Migrant Income Tax

$$tax_t^m = \tau^w w_t^m h_t^m \tag{A.4.13}$$

Total Income Tax

$$tax_t^w = (tax_t^n)^{\phi_h} (mn_t tax_t^m)^{(1-\phi_h)} \tag{A.4.14}$$

Household Transfers Aggregation

$$z_t = (z_t^n)^{\phi_n} (mn_t z_t^m)^{1-\phi_n} \tag{A.4.15}$$

Firms

Production Function

$$y_t = a_t k_{t-1}^{\alpha} l_t^{(1-\alpha)} (k_t^g)^{\alpha^g} \tag{A.4.16}$$

Technological Process

$$\ln a_t = \rho^a \ln a_{t-1} + \varepsilon_t^a \tag{A.4.17}$$

Effective Labour

$$l_t = (h_t^n)^{\phi_h} (mn_t h_t^m)^{(1-\phi_h)} \tag{A.4.18}$$

Capital Accumulation

$$k_t = (1-\delta)k_{t-1}\frac{N_{t-1}}{N_t} + s\left(\frac{x_t n_t}{k_{t-1}}\right)\frac{k_{t-1}}{n_t}\psi_t^x \tag{A.4.19}$$

Firm Investment Shock

$$\ln\psi_t^x = \rho^{\psi^x}\ln\psi_t^x + \varepsilon_t^{\psi^x} \tag{A.4.20}$$

Native Wage

$$w_t^n = (1-\alpha)\phi_h\frac{y_t p_t^h}{h_t^n} \tag{A.4.21}$$

Migrant Wage

$$w_t^m = (1-\alpha)(1-\phi_h)\frac{y_t p_t^h}{h_t^m m n_t} \tag{A.4.22}$$

Marginal Product of Capital

$$\rho_t^k = \alpha\frac{y_t p_t^h}{k_{t-1}} \tag{A.4.23}$$

Tobin's Q

$$q_t = \left[s'\left(\frac{x_t n_t}{k_{t-1}}\right)\frac{\psi_t^x}{n_t}\right]^{-1} \tag{A.4.24}$$

Firm Euler Equation

$$q_t = \beta\frac{\mu_{t+1}}{\mu_t}\left[\alpha\frac{y_{t+1}}{k_t} + (1-\delta)\frac{q_{t+1}}{n_{t+1}}\right] \tag{A.4.25}$$

Wage Differential

$$w_t^\Delta = \frac{w_t^n}{w_t^m} \tag{A.4.26}$$

Firm Profit

$$\Pi_t = p_t^h y_t - w_t^n h_t^n - w_t^m h_t^m m n_t - x_t \tag{A.4.27}$$

An Open Economy

Foreign Output

$$\ln y_t^f = \rho^{y^f}\ln y_{t-1}^f + \varepsilon_t^{y^f} \tag{A.4.28}$$

Current Account

$$y_t = c_t + x_t + p_t^f b_t - \frac{1}{\beta}p_t^f b_{t-1} + c_t^g + x_t^g \tag{A.4.29}$$

Market Clearing

$$y_t = v(p_t^h)^{-\sigma}\left(c_t + x_t + c_t^g + x_t^g\right) + v^f\left(\frac{rer_t}{p_t^h}\right)^{\sigma^f} y_t^f \tag{A.4.30}$$

Bonds

$$r_t = r^* \exp^{\left(-\phi_{bhc}\left(\frac{p_t^f b_t}{y_t} - \bar{b}\right)\right)} \tag{A.4.31}$$

Fiscal Policy

Government Capital Accumulation

$$k_t^g = (1-\delta^g)\frac{k_{t-1}^g}{n_t} + x_t^g \tag{A.4.32}$$

Government Investment

$$\widehat{x_t^g} = \vartheta^{x^g}\widehat{x_{t-1}^g} - \zeta^{x^g,y}\widehat{y_{t-1}} - \zeta^{x^g,b^g}\widehat{b_{t-1}^g} + \psi_t^{x^g} \tag{A.4.33}$$

Government Investment Shock

$$\ln\psi_t^{x^g} = \rho^{\psi^{x^g}}\ln\psi_{t-1}^{x^g} + \epsilon_t^{\psi^{x^g}} \tag{A.4.34}$$

Household Transfers

$$\widehat{z_t} = \vartheta^z\widehat{z_{t-1}} - \zeta^{z,y}\widehat{y_{t-1}} - \zeta^{z,b^g}\widehat{b_{t-1}^g} + \psi_t^{z^g} \tag{A.4.35}$$

Household Transfers Shock

$$\ln\psi_t^z = \rho^{\psi^z}\ln\psi_{t-1}^z + \epsilon_t^{\psi^z} \tag{A.4.36}$$

Native Household Transfers

$$z_t = (z_t^n)(mn_t)^{\frac{1-\phi_n}{\phi_n}} \tag{A.4.37}$$

Government Consumption

$$\widehat{c_t^g} = \vartheta^{c^g}\widehat{c_{t-1}^g} - \zeta^{c^g,y}\widehat{y_{t-1}} - \zeta^{c^g,b^g}\widehat{b_{t-1}^g} + \psi_t^{c^g} \tag{A.4.38}$$

Government Consumption Shock

$$\ln\psi_t^{c^g} = \rho^{\psi^{c^g}}\ln\psi_{t-1}^{c^g} + \epsilon_t^{\psi^{c^g}} \tag{A.4.39}$$

Consumption Tax

$$tax_t^c = \tau^c c_t \tag{A.4.40}$$

Corporation Tax

$$tax_t^f = \tau^f \left(y_t - w_t^n h_t^n - w_t^m h_t^m mn_t - x_t\right) \tag{A.4.41}$$

Government Revenue

$$tax_t = tax_t^w + tax_t^c + tax_t^f \tag{A.4.42}$$

Government Budget Constraint

$$ge_t + \frac{b_{t-1}^g r_t}{n_t} = tax_t + b_t^g \tag{A.4.43}$$

Government Expenditure

$$ge_t = x_t^g + z_t + c_t^g \tag{A.4.44}$$

Primary Budget Surplus

$$surp = tax_t - ge_t \tag{A.4.45}$$

Household Utility

Native Households Utility

$$U_t^N = \psi_t^c \frac{(c_t^n - H_t^c)^{1-\theta}}{|1-\theta|} - \psi_t^h \frac{\phi_0 (h_t^n - H_t^h)^{1+\eta}}{1+\eta} \tag{A.4.46}$$

Migrant Households Utility

$$U_t^M = \frac{(c_t^m)^{1-\theta^m}}{|1-\theta^m|} - \frac{\phi_m (h_t^m)^{1+\eta^m}}{1+\eta^m} \tag{A.4.47}$$

A.5 Log Linearised Equations

Native Households

Marginal Utility of Consumption

$$\frac{-\theta}{1-\kappa}\left(\widehat{c_t^n} - \widehat{c_{t-1}^n}\right) = \widehat{\mu_t} - \widehat{\psi_t^c} \tag{A.5.1}$$

Labour Supply

$$\eta\left(\widehat{h_t^n} - \kappa^h \widehat{h_{t-1}^n}\right) + \widehat{\psi_t^h} = \widehat{w_t^n} + \widehat{\mu_t} \tag{A.5.2}$$

Native Household Euler Equation

$$\widehat{\mu_t} - \widehat{\mu_{t+1}} + (1-\gamma)(\widehat{T_t} - \widehat{T_{t+1}}) + \widehat{n_t} = \widehat{r_t} \tag{A.5.3}$$

Consumption Preference Shock

$$\widehat{\psi_t^c} = \rho^{\psi^c}\widehat{\psi_{t-1}^c} + \varepsilon_t^{\psi^c} \tag{A.5.4}$$

Labour Supply Shock

$$\widehat{\psi_t^h} = \rho^{\psi^h}\widehat{\psi_{t-1}^h} + \varepsilon_t^{\psi^h} \tag{A.5.5}$$

Migrant Household

Labour Supply

$$\eta^m\widehat{h_t^m} = \widehat{w_t^m} - \theta^m\widehat{c_t^m} \tag{A.5.6}$$

Migrant Budget Constraint

$$c^m\widehat{c_t^m} = (1-\tau^w)h^m w^m\left(\widehat{h_t^m} + \widehat{w_t^m}\right) + z^m\widehat{z_t^m} \tag{A.5.7}$$

Population Dynamics

Migration Shock

$$\widehat{m_t} = \rho^m\widehat{m_{t-1}} + \varepsilon_t^m \tag{A.5.8}$$

Rates of Change

$$\widehat{m_t} - \frac{1}{\frac{m}{n}}\widehat{n_t} = \widehat{mn_t} - \widehat{mn_{t-1}} \tag{A.5.9}$$

Population Growth Rate

$$\widehat{n_t} = \frac{m}{n}(\rho^{mn^0}\widehat{m_t}+\rho^{mn^1}\widehat{m_{t-1}}+\rho^{mn^2}\widehat{m_{t-2}}+\rho^{mn^3}\widehat{m_{t-3}}+\rho^{mn^4}\widehat{m_{t-4}}+\rho^{mn^5}\widehat{m_{t-5}}+\rho^{mn^6}\widehat{m_{t-6}}+\rho^{mn^7}\widehat{m_{t-7}} \tag{A.5.10}$$

Aggregation

Consumption

$$\widehat{c_t} = \phi_h\widehat{c_t^n} + (1-\phi_h)(\widehat{c_t^m} + \widehat{mn_t}) \tag{A.5.11}$$

Native Income Tax

$$\widehat{tax_t^n} = \widehat{w_t^n} + \widehat{w_t^n} \tag{A.5.12}$$

Migrant Income Tax

$$\widehat{tax_t^m} = \widehat{w_t^m} + \widehat{h_t^m} \tag{A.5.13}$$

Income Tax

$$\widehat{tax_t^w} = \phi_h \widehat{tax_t^n} + (1-\phi_h)(\widehat{tax_t^m} + \widehat{mn_t}) \tag{A.5.14}$$

Aggregate Household Transfers

$$\widehat{z_t} = \phi_n \widehat{z_t^n} + (1-\phi_n)\left(\widehat{z_t^m}\widehat{mn_t}\right) \tag{A.5.15}$$

Firms

Output

$$\widehat{y_t} = \widehat{a_t} + \alpha \widehat{k_{t-1}} + (1-\alpha)\widehat{l_t} + \alpha^g \widehat{k_t^g} \tag{A.5.16}$$

Technology Shock

$$\widehat{a_t} = \rho_a \widehat{a_{t-1}} + \varepsilon_t^a \tag{A.5.17}$$

Effective Hours

$$\widehat{l_t} = \phi_h \widehat{h_t^n} + (1-\phi_h)(\widehat{h_t^m} + \widehat{mn_t}) \tag{A.5.18}$$

Capital Accumulation

$$\widehat{k_t} = (1-\delta)(\widehat{k_{t-1}} - \widehat{n_t}) + \delta(\widehat{x_t} + \widehat{\psi_t^x}) \tag{A.5.19}$$

Investment Shock

$$\widehat{\psi_t^x} = \rho^{\psi^x} \widehat{\psi_{t-1}^x} + \varepsilon_t^{\psi^x} \tag{A.5.20}$$

Native Wage

$$\widehat{w_t^n} = \widehat{y_t} - \widehat{h_t^n} - \gamma \widehat{T_t} \tag{A.5.21}$$

Migrant Wage

$$\widehat{w_t^m} = \widehat{y_t} - \widehat{h_t^m} - \widehat{mn_t} - \gamma \widehat{T_t} \tag{A.5.22}$$

Marginal Product of Capital

$$\widehat{\rho_t^K} = \widehat{y_t} - \widehat{k_{t-1}} - \gamma \widehat{T_t} \tag{A.5.23}$$

Tobin's Q

$$\widehat{q_t} + \widehat{\psi_t^x} = \delta(ac)\left(\widehat{x_t} + \widehat{n_t} - \widehat{k_{t-1}}\right) \tag{A.5.24}$$

Firm Euler Equation

$$\widehat{\mu_t} - \widehat{\mu_{t+1}} + \widehat{q_t} = \beta\alpha\frac{y}{k}\widehat{\rho_{t+1}^K} + \beta(1-\delta)(\widehat{q_{t+1}} - \widehat{n_{t+1}}) \tag{A.5.25}$$

Wage Differential

$$\widehat{w_t^{\Delta}} = \widehat{w_t^{n}} - \widehat{w_t^{m}} \tag{A.5.26}$$

Firm Profit

$$\widehat{\Pi}_t = \widehat{y}_t - w^m h^m \left(\widehat{w_t^m} + \widehat{h_t^m} + \widehat{mn}_t\right) - w^n h^n \left(\widehat{w_t^n} + \widehat{h_t^n}\right) - x\widehat{x_t} \tag{A.5.27}$$

Open Economy

Current Account

$$\widehat{y}_t = \frac{c}{y}\widehat{c}_t + \frac{x}{y}\widehat{x_t} + \frac{c^g}{y}\widehat{c_t^g} + \frac{x^g}{y}\widehat{x_t^g} + \widehat{b}_t - \frac{1}{\beta}\widehat{b_{t-1}} + (1-(1-\gamma))\widehat{T_t} \tag{A.5.28}$$

Market Clearing

$$\widehat{y}_t = (1-\gamma)\left(\frac{c}{y}\widehat{c}_t + \frac{x}{y}\widehat{x_t} + \frac{c^g}{y}\widehat{c_t^g} + \frac{x^g}{y}\widehat{x_t^g}\right) + \gamma\sigma(1-\gamma)\widehat{T_t} + \gamma\widehat{y_t^f} \tag{A.5.29}$$

Foreign Output

$$\widehat{y_t^f} = \rho^{y^f}\widehat{y_{t-1}^f} + \varepsilon y^f \tag{A.5.30}$$

Interest Rate

$$\widehat{r_t} = -bhc\widehat{b_t} \tag{A.5.31}$$

Fiscal Policy

Public Capital Accumulation

$$\widehat{k_t^g} = \delta^g\left(\widehat{k_{t-1}} - \widehat{n_t}\right) + \delta^g\widehat{x_t^g} \tag{A.5.32}$$

Government Investment

$$\widehat{x_t^g} = \vartheta^{x^g}\widehat{x_{t-1}^g} - \zeta^{x^g,y}\widehat{y_{t-1}} - \zeta^{x^g,b^g}\widehat{b_{t-1}^g} + \psi_t^{x^g} \tag{A.5.33}$$

Government Investment Shock

$$\widehat{\psi_t^{x^g}} = \rho^{\psi^{x^g}}\widehat{\psi_t^{x^g}} + \varepsilon_t^{\psi^z} + \varepsilon_t^{x^g} \tag{A.5.34}$$

Household Transfers

$$\widehat{z}_t = \vartheta^z\widehat{z_{t-1}} - \zeta^{z,y}\widehat{y_{t-1}} - \zeta^{z,b^g}\widehat{b_{t-1}^g} + \psi_t^z \tag{A.5.35}$$

Native Household Transfers

$$\widehat{z}_t = \widehat{z_t^n} + \frac{1-\phi_n}{\phi_n}\widehat{mn}_t \tag{A.5.36}$$

Transfers Shock

$$\widehat{\psi_t^z} = \rho^{\psi^z}\widehat{\psi_t^z} + \varepsilon_t^{\psi^z} \tag{A.5.37}$$

Government Consumption

$$\widehat{c_t^g} = \vartheta^{c^g}\widehat{c_{t-1}^g} - \zeta^{c^g,y}\widehat{y_{t-1}} - \zeta^{c^g,b^g}\widehat{b_{t-1}^g} + \psi_t^{c^g} \tag{A.5.38}$$

Government Consumption Shock

$$\widehat{c_t^g} = \rho^{c^g}\widehat{c_{t-1}^g} + \varepsilon_t^{c^g} \tag{A.5.39}$$

Consumption Tax

$$\widehat{tax_t^c} = \widehat{c}_t \tag{A.5.40}$$

Corporation Tax

$$\widehat{tax_t^f} = \widehat{\Pi}_t \tag{A.5.41}$$

Government Revenue

$$tax\widehat{tax}_t = tax^w\widehat{tax_t^w} + tax^c\widehat{tax_t^c} + y\widehat{tax_t^f} \tag{A.5.42}$$

Government Budget Constraint

$$\widehat{surp}_t = \frac{1}{\beta}\left(\widehat{b_{t-1}^g} + \widehat{r}_t - \widehat{n}_t\right) - \widehat{b}_t^g \tag{A.5.43}$$

Government Expenditure

$$ge\widehat{ge}_t = x^g\widehat{x_t^g} + c^g\widehat{c_t^g} + z\widehat{z}_t \tag{A.5.44}$$

Primary Budget Surplus

$$\widehat{surp}_t = \frac{tax}{y}\widehat{tax}_t - \frac{ge}{y}\widehat{ge}_t \tag{A.5.45}$$

Household Utility

Native Households Utility

$$\widehat{U_t^n}U^n = \widehat{\psi_t^c}\frac{(c^n - \kappa^c c^n)^{1-\theta}}{|1-\theta|} + \left(\widehat{c_t^n} - \kappa^c \widehat{c_{t-1}^n}\right)(c^n - \kappa^c c^n)^{1-\theta}$$
$$- \widehat{\psi_t^h}\frac{(h^n - \kappa^h h^n)^{1+\eta}}{1+\eta} - \phi_0\left(\widehat{h_t^n} - \kappa^h \widehat{h_{t-1}^n}\right)(h^n - \kappa^h h^n)^{1+\eta} \quad \text{(A.5.46)}$$

Migrant Households Utility

$$\widehat{U_t^m}U^m = \widehat{c_t^m}\frac{(c^m)^{1-\theta^m}}{|1-\theta^m|} - \widehat{h_t^m}\phi_m\frac{(h^m)^{1+\eta^m}}{1+\eta^m} \quad \text{(A.5.47)}$$

A.6 Steady State

$$l = 0.33 \quad \text{(A.6.1)}$$

$$mpk = \frac{(1-\beta)}{\beta} + \delta \quad \text{(A.6.2)}$$

$$mpk^g = \frac{(1-\beta)}{\beta} + \delta^g \quad \text{(A.6.3)}$$

By numerical solver for k and k^g

$$\alpha^g(k^\alpha)(l^{1-\alpha})(k^g)^{\alpha^g-1} = mpk^g \quad \text{(A.6.4)}$$

$$\alpha\left(\frac{l}{k}\right)^{1-\alpha}(k^g)^{\alpha^g} = mpk \quad \text{(A.6.5)}$$

$$y = k^\alpha l^{(1-\alpha)}(k^g)^{\alpha^g} \quad \text{(A.6.6)}$$

$$x = \delta k \quad \text{(A.6.7)}$$

$$x^g = \delta^g k^g \quad \text{(A.6.8)}$$

$$\frac{ge}{y} = 1 - \frac{c}{y} - \frac{x}{y} \quad \text{(A.6.9)}$$

$$ge = \frac{ge}{y}y \quad \text{(A.6.10)}$$

$$c^g = y - \frac{c}{y}y - x - x^g \quad \text{(A.6.11)}$$

$$z = ge - x^g - c^g \quad \text{(A.6.12)}$$

$$zm = z \quad \text{(A.6.13)}$$

$$zn = \frac{z}{mn^{\frac{1-\phi_n}{\phi_n}}} \tag{A.6.14}$$

$$n = \frac{l}{mn^{1-\phi_n}} \tag{A.6.15}$$

$$h^n = n \tag{A.6.16}$$

$$h^m = n \tag{A.6.17}$$

$$w^n = (1-\alpha)\phi_n \frac{y}{h^n} \tag{A.6.18}$$

$$w^m = (1-\alpha)(1-\phi_n)\frac{y}{h^m} \tag{A.6.19}$$

$$\frac{c^m}{y} = h^m \frac{w^m}{y} + \frac{z}{y} \tag{A.6.20}$$

$$\frac{c^n}{y} = \frac{c}{y} - \frac{c^m}{y} \tag{A.6.21}$$

$$c = \frac{c}{y} y \tag{A.6.22}$$

$$c^n = \frac{c^n}{y} y \tag{A.6.23}$$

$$c^m = \frac{c^m}{y} y \tag{A.6.24}$$

$$\Pi^f = 0 \tag{A.6.25}$$

$$tax = ge \tag{A.6.26}$$

$$tax^c = \tau^c (c^n)^{\phi_n} (c^m)^{1-\phi_n} \tag{A.6.27}$$

$$\tau^w = \frac{tax - tax^c}{h^n w^n + w^m h^m} \tag{A.6.28}$$

$$tax^w = (\tau^w h^n w^n) + (\tau^w w^m h^m) \tag{A.6.29}$$

$$tax = tax^c + tax^f + tax^w \tag{A.6.30}$$

Chapter 3

The Migrants' Plight: A Skill-Job Mismatch

Brain Waste in Canada

3.1 Introduction

In recent years, immigration has risen significantly in traditional immigrant host countries which has caused increased opposition leading to some governments to openly oppose (increased) immigration. An example of this is when countries voted against the Global Compact on Migration, including the United States, Hungary, the Czech Republic, Poland and Israel in December 2018.[1] A few countries have remained positive to increasing economic immigration, particularly those able to focus immigration on high-skilled and young workers through visa requirements. This focus is because of their human capital and the resulting demographic changes. Migrants on average are younger and higher skilled than natives, and they are most likely to be net fiscal contributors.[2]

Countries that can use tailored migration programmes typically have two types that are directly related to the labour market. Firstly, a high-skilled immigration programme which is a way for a country to increase its human capital. This would allow a country to aid development of high-skilled industries, which relates to long-

[1] Twelve countries abstained: Algeria, Australia, Austria, Bulgaria, Chile, Italy, Latvia, Libya, Liechtenstein, Romania, Singapore and Switzerland.

[2] The majority of immigrant host countries have ageing populations, an increase in immigration shifts the balance somewhat away from the older generation who are retiring, receive a pension, and have increasing health costs to a younger generation who work, a number send remittances, and receive fewer fiscal benefits due to their migrant status. Oxford Economics (2018) show that EEA migrants, who are on average more highly skilled, contribute £2,300 more than the average UK citizen. Similar results in Lisenkova and Sanchez-Martinez (2016).

term migration. In the case of specific industry shortages, fast-track visas are occasionally available with federal programmes sometimes promoting these industries. Secondly, low-skill immigration programmes are predominantly used to ease labour market shortages and more likely to be temporary migrants. The gaps in the labour markets filled by these migrants helps to stimulate economic growth. However, (high-skilled) migrants are not always able to reach their full potential, a phenomenon known as brain waste. While brain waste is more akin to high-skill migrants, those with low-skill qualifications can experience it too, at a somewhat lesser extent.

The problem of brain waste occurs when (high-skilled) immigrants experience relatively higher rates of involuntarily unemployment, increased labour market frictions, or underemployment. Migrants are underemployed when they are working in jobs for which they are overqualified, receive lower wages (and associated benefits of employment e.g. bonuses) than natives, or part-time employment when full-time is desired. Underemployment in the form of a skill-job mismatch can be caused by a failure to recognise the qualifications gained outside the host nation. In some immigration systems, having a job is not a prerequisite to acquire a working visa.[3] The migration of workers causes brain drain in the sending country, which if utilised, results in brain gain for the host economy, and in absence of full utilisation creates brain waste.

To understand the macroeconomic effects of brain waste and migration we construct a dynamic stochastic general equilibrium (DSGE) model. We compliment the analysis with a structural vector autoregression (SVAR) to examine the effect of a migration shock on the macroeconomy. These models are tested on Canadian data; a country which has recognised the prominence at a microeconomic level.[4] Canada has used immigration systems to target both high-skill and low-skill migration, as well as visa systems that is not necessarily conditional on having employment.[5] [6] Canada has one of the highest proportions of foreign-born residents at 22% in 2019, compared to the United States that has a relative size of 15.1%, or the United Kingdom with 12.1%. In terms of foreign-born workers, this was 26.6% in 2016 which

[3]For example, Canada with *Express Entry* with two subcategories Canadian Experience Class and Federal Skilled Worker programme. There are common labour markets, such as *Single European Labour Market* which require no visa for those citizens in the participating countries. In contrast to countries such as the United States where work visas are based on a specific offer of employment.

[4]The prominence of brain waste of medical graduates is discussed in Lofters et al. (2014) and a study of skilled immigrant men in terms of mismatch and wage progression in Banerjee et al. (2019)

[5]Canada has employed a series of economic immigration programmes during the 21^{st} century, including Temporary Foreign Worker (TFW), Express Entry, Federal Skilled Worker Programme, and Federal Trades Worker Programme.

[6]See Clarke and Skuterud (2013), Green and Worswick (2012), and Gross and Schmitt (2012). Additional relevant research is discussed elsewhere in this paper.

is most comparable to countries such as Australia and New Zealand with ranges between 25 - 29%.[7]

In this research, we define brain waste as the underemployment of migrants which includes higher levels of involuntary unemployment, higher search and matching frictions, lower wage relative to natives, and skill-job mismatches within the labour market. These situations are repeated in the majority of migrant host countries. Microeconomic data identifies that on average, migrants are more highly qualified compared to natives. However, migrants experience a higher unemployment rate at each skill level, and participation rates of migrants are frequently lower.[8] Due to a lower rate of matches, migrants are evidenced as taking up employment for which they are overqualified - a form of skill-downgrading. These difficulties extend to the situation where a migrant is employed at the corresponding skill-level but are paid less than their native equivalent.

We analyse the consequences of brain waste in a DSGE model for a small open economy with asymmetric search and matching frictions. The model features three households: a high-skilled native, low-skilled native, and a migrant household; final good producing firms, and a fiscal authority. The model is calibrated and estimated using data from Canada. We analyse the effects of brain waste using the calibrated and estimated model, to reflect the differing labour market conditions between natives and migrants. Then compare the results with a situation where brain waste is absent. The model is subject to a migration shock, a representation of a relaxation of immigration constraints for economic migrants. The different calibrations allow analysis of the effects to an economy when migrants do not have the same recognition as natives. In addition, we empirically analyse the effect of a migration shock on several macroeconomic variables with a SVAR. The results from the DSGE model indicate that there is a small loss to the economy due to brain waste. In absence of brain waste, the macroeconomy would benefit in terms of output, consumption, and investment. Migrants would receive higher wages and hence be able to increase their consumption levels. There is a potential gain of 2.4% to output, 4.5% to private consumption, and 2.1% gain of total employment. The results from the SVAR are mostly small, for some insignificant, but positive for the economy in per capita values.

The remainder of this paper is as follows, in section 3.2, we discuss the relevant literature. Section 3.3 examines migration and the associated problems of brain waste in Canada. We present the empirical model and its results in section 3.4. The

[7]The difference between the residents and workers figures is due to residents including locally born children whereas workers only includes 15 years or older.

[8]Unemployment and participation rate are discussed in calibration, and data is presented in table B.2.

theoretical model is set out in section 3.5, with the calibration and estimation results are presented in section 3.6. The results of the model are analysed in section 3.7. Finally, we offer a conclusion in section 3.8.

3.2 Existing Migration Literature

Research in the macroeconomics of migration, particularly general equilibrium business cycle effects, is limited. Exceptions include Mandelman and Zlate (2012), which uses a two-country model to analyse international risk sharing via remittances, and Smith and Thoenissen (2019) who analyse the role of migrants' level of human capital relative to natives following an exogenous migration shock to a small open economy. A number of recent macroeconomic migration papers have incorporated search and matching frictions, including Chassamboulli and Palivos (2013), Chassamboulli and Peri (2015), and Kiguchi and Mountford (2017). The analysis of brain waste is mostly microeconomic. In terms of the macroeconomics, brain drain is discussed in Mountford (1997), Beine et al. (2001) and Docquier and Iftikhar (2019), and skill-downgrading is discussed in Muysken et al. (2015), with the focus is on medium and low-skill migration in a static general equilibrium framework.

Dungan et al. (2013) empirically examine the macroeconomic effects of migration in Canada, simulating a 100,000 increase in migrants.[9] [10] As a percentage, total annual net migration would reflect a 1% increase in the population. The results focus on GDP per capita, unemployment, investment, productivity; and government finances. The results are mostly positive, however, two cases are examined. The realistic case where migrants are paid less than natives, and an ideal scenario in which migrants are equivalent to natives. There is an increase in real GDP, but GDP per capita initially increases then falls in the baseline scenario. Unemployment is not affected. Investment and productivity increase. Government spending increases but less than the increase in tax receipts. Under a scenario where migrants are much more quickly integrated into the labour market, the results are more positive, and GDP per capita does not fall.

Fougère et al. (2011) examine how an increase in high-skilled workers would benefit workers in an applied general equilibrium model applied to Canada. The results show that highly skilled, and productive workers, increases human capital and economic productivity. The effects on real GDP and real GDP per capita are

[9]The authors use FOCUS (Forecasting and User Simulation). FOCUS is a model of the Canadian macroeconomy developed and maintained at the Policy and Economic Program within the Rotman School of Management at the University of Toronto.

[10]At 2013 levels, an increase of 100,000 immigrants would equate to 350,000 migrants per annum.

different mostly due to young high-skilled migrants having a lower participation rate and savings rate as they spend time acquiring skills in tertiary education relative to lower-skilled workers which has a negative effect on labour hours and capital stocks.

Ileri (2019) uses an overlapping generations model calibrated to Canada in 1981 to analyse the impacts of the selective migration policies on natives. The model uses endogenous education choice. The quantitative results show that skilled immigration is positive for the welfare in the economy. However, this result varies amongst the demographic groups. Lower rates of skilled migration reduces the welfare of the native born population. The white native-born university graduates benefit in terms of higher wages. This is not the case for non-graduates who experience deterioration in wages.

Tu, Jiong (2010) looks at the effects of the native born labour market, where no significant negative effects are found particularly on the native wage growth rate during the large increase in immigration during the 1990s.

Eckstein and Weiss (1998) analyse the wage growth of immigrants in Israel using human capital theory. The authors find that immigrants do not gain from the skills which they import. Rather, the value of skills increases the longer they stay. The occupational distribution of immigrants does converge to natives but not for wages. A higher return in wages to education occurs for high-skilled migrants but not as great natives.

Liu et al. (2017) examine the effects of skill heterogeneity and transfer of human capital in a dynamic search and matching model. The results show that both skilled and unskilled natives and migrants gain in terms of income and employment, and when migrants' human capital is utilised fully there is a further benefit to immigrants and natives. Borjas (1999) shows that natives gain when the productive endowments of immigrants differs from that of natives. However, the benefits for natives are not uniform across skill levels since the workers that are substitutable with migrants lose, while the natives who are complements of the productivity of migrants gain. In our model, we present migrant workers who are imperfect substitutes to natives. Dustmann et al. (2008) show that the skill downgraded high-skilled migrants have higher productivity than low-skilled native workers, but the wage level is driven by native productivity.

To summarise, there is a lack of research in the (dynamic) macroeconomics of brain waste and there are a number of aspects that require a fair representation of brain waste in a model. Brain waste consists of lower wages, imperfect substitution of workers, higher labour market frictions, increased (involuntary) unemployment and lower participation rates relative to natives. The participation and unemployment rates differ between migrant source country too.

3.3 Migration in Canada

Canada has been described as a nation of immigrants and views migration as a part of its national identity. Canada has one of the highest numbers of net migration flows and stock of migrants in the world. Figure 3.1 plots annual migration figures for Canada from 1952 to 2019. It shows how recent immigration trends have seen net migration surpass the previous highs from 1992, 1993, and 1957. Net migration closely follows immigration, especially from 1980 to 2010.

Fig. 3.1 Annual Migration in Canada 1952-2019

The graph plots immigration (dotted...), emigration (dashed-dot -.-) and net migration (solid -) 100,000s (10^5) of people. This is raw data 1952-2018. Net migration tracks the path of immigration as emigration appears relatively constant, particularly from 1980 onwards.
Source: Statistics Canada

Canada has one of the highest proportions of foreign born workers, 26.6% in 2016. The relative shortage of labour caused Canada to relax its immigration policy, and in 2017 committed to spending CA$440 million (£250 million) from 2017-2019 to increasing immigration over three years from 310,000 in 2018 to 340,000 in 2020; equivalent to 0.84% and 0.90% of the projected population respectively.[11] This programme has since been extended into 2020 and 2021. Immigration is predicted to be the only source of population growth by 2030. In 2017-2018, 80% of the

[11] Population predictions source UN: 2018: 37.0 million ; 2020: 37.6 million.

population growth in Canada was due to international migration (Hussen, 2018). Previous programmes in the 1990s and 2000s shifted the migration focus towards skilled migrants which is strongly related to long-term migration (Hou and Picot, 2016).

The main countries of origin for immigrants are the United Kingdom, France, Germany, and the United States, however, the share of immigration figures has considerably diminished since the beginning of the figures, approximately 60% to 8%, as immigration from predominantly Asian countries has increased. This is partly due to the immigration policy imposed by the government which until the early 1960s used a country preference system. In 1962 Canada changed to a skills-based system and by 1967 introduced a points based system to target immigrants with "desirable skills" (Aydemir and Borjas, 2007).

There is a consensus in the literature that immigrants experience problems integrating into the labour market. One aspect of this is labour earnings, for which theirs persistently lags behind those of natives even with similar skill levels.[12] Most problematic though, is that the ability to integrate into the labour market and into wider society is ever increasing. The most recent immigrants face the prospect of never expecting to match earnings of the equivalent native worker. Further, those most likely to be negatively affected is the workers that they are directly competing with for employment in terms of wage and employment status. In the case of migration, this is most frequently the existing immigrant worker who is a substitute. One reason, as suggested in Clarke and Skuterud (2013), as to why immigrants are worse off in countries such as Canada, compared to Australia, is the lack of focus on English proficiency in the selection system.[13] A language barrier is a significant obstacle to overcome, even if the immigrant is highly skilled.

In Quebec, there exists a rule *domaines de formation privilégié*, where full points for education are only attributed for degrees obtained in Quebec or equivalent to Quebec degrees.[14] A significant barrier for some migrants to overcome. There is a drive to increase the number of university students in Canada so that their skills should be directly comparable to natives. One of the most studied professions for

[12] See Aydemir and Skuterud (2008), Green and Worswick (2012), Fortin et al. (2016), Kaushal et al. (2016) and Green and Worswick (2017).

[13] English and French are the official languages of Canada, however, English is the most widely spoken. On the Express Entry visa application there is only a weak bias towards English language skills - exceptions apply for Quebec which is the only province to have French as its official language (New Brunswick has English and French). However, the majority of Canada has English as its first language. The census data in 2011 showed that English was the first language for approximately 58% of the Canadian population and French for nearly 22% which equated to 19.1 million and 7.2 million people respectively.

[14] Countries that are considered to be equivalent are The United States , Northern and Western Europe countries, Australia, New Zealand, Japan and Israel.

effects of brain waste is that of medical practitioners. Lofters et al. (2014) examine the case of brain waste for medical graduates from low-income and middle-income countries who were seeking a medical residency in Ontario, Canada. The results show that not only does brain waste have economic consequences but psychological too; some of those surveyed expressed feelings of "anger, shame, desperation, and regret". On a more general high-skill level, Banerjee et al. (2019), examine the wages of native-born and immigrant men over a six year period. The results show that a complete mismatch results in smaller initial wages, especially for racial minority immigrants, and that the wage gap does not completely close over the period.

The 2002 expansion of Canada's temporary foreign worker programme (TFW) to include low-skilled workers, which aimed to provide a short-term solution to labour shortages through immigration, in order to support the economic boom in particular provinces. However, the ease at which employers were able to hire this cheap labour had detrimental effects on the low-skilled labour market in certain areas of Canada. An increased cost to employers may have reduced the increase in immigrants and lessen the negative labour market effects (Gross and Schmitt, 2012). The relaxation of constraints led to a large increase in the TFW scheme, 101,000 immigrants in 1993 to 165,200 in 2007 (Clarke and Skuterud, 2013). By 2008, the number of foreign workers exceeded the permanent migrants. The temporary nature exacerbated the share of migrants being employed in low-skill employment (Finotelli, 2013). Such is the problem of immigrants' qualifications not being recognised, the Canadian government established the *Foreign Credentials Referral Office* in order to help these workers have their skills recognised. In addition, 2017 and 2018 saw an increase in the number of highly skilled American workers migrating to Canada. In particular, those who specialise in technology as Vancouver tries to create an alternative to Silicon Valley.

The ambiguity of response to migration in some variables particularly towards natives, absence of macroeconomic research for brain waste, limited research in migration using a small open economy model, minimal use of net international migration data in macroeconomic research, and the stylised facts presented in sections 3.1 and 3.3 forms the motivation for our paper.

3.4 Empirical Model

We begin our analysis of migration to a small open economy with a SVAR. The focus of the model is to analyse the response of macroeconomic variables to an increase in migration. As part of the analysis, we decide whether migration as an exogenous AR(1) process is a valid assumption.

The existing empirical macroeconomic literature is limited. The most notable being Furlanetto and Robstad (2019), who use a Bayesian VAR to analyse the macroeconomic impacts of migration in Norway. Additionally, papers featuring DSGE models and SVAR include Kiguchi and Mountford (2017) using data from the United States , and Smith and Thoenissen (2019) using data from New Zealand.

The unrestricted VAR presented uses the same observables, and additional labour market variables. As per Lütkepohl (2005), the VAR is specified as follows:

$$(y_t - \nu) = \Psi(y_{t-1} - \nu) + \epsilon_t \tag{3.4.1}$$

where y_t is vector of endogenous variables, ν is the vector of expected values for the endogenous variables of y_t. The k x k matrix of coefficients is given by Ψ, and ϵ_t is a vector of N element white noise error terms with $\epsilon_t \sim N(0, \Sigma)$, where Σ is an k x k variance-covariance matrix.

In the DSGE model we present a change to net migration as an exogenous AR(1) process, the empirical analysis of net migration flows confirms its exogeneity. The lag-length specification is dependent upon the criteria. While AIC identifies two lags, HQIC and SBIC identify one lag. The white noise processes are well behaved with one lag, we proceed the analysis with a VAR(1). We orthogonalise the shocks using Cholesky decomposition. In order, the vector y_t consists of net migration, unemployment rate, labour hours, government investment and consumption, government debt, private investment and consumption, wages, and GDP.

3.4.1 Data Description

The focus of the SVAR is to analyse the effect of a net migration shock on the macroeconomy. We use variables from the national accounts: GDP, private consumption, public consumption, private gross fixed capital formation (investment), government gross fixed capital formation (fiscal investment), general government net financial liabilities which are collected at a quarterly frequency, converted to real per capita values using the working-age population (15 years and older) and corresponding deflator. Further, wages are taken from Statistics Canada table 36-10-0114-01 (compensation of employees) at a seasonally adjusted quarterly frequency, and converted to real terms.[15] Net migration is seasonally adjusted and converted to per capita terms. Hours worked by employee, and the total unemployment rate are also included. United States GDP which is used as an observable in the estimation of the DSGE model is dropped from the dataset as it is exogenous to the small open economy. We

[15]Data sources and more detailed description is available in section B.1 of the appendix.

take the natural logarithm of trending series and then apply the Hamilton (2018) filter to compute the trends and cyclical components of the data series. Due to time series limitations, the duration of the time series is 1980Q1:2019Q1 inclusive.

3.4.2 Impulse Responses

Figure 3.2 shows the impulse response of the nine variables to a one standard deviation of 0.1870 shock to migration. The migration shock has a small effect on most of the variables, however, they are positive for the economy.

There is a short and statistically significant increase in Canadian GDP. This increase causes positive effects on the economy elsewhere. An increase in output results in an increase in wages for workers. This has the effect of increasing consumption for the workers (private consumption). The initial response of consumption is of approximately the same magnitude. However, private consumption does not increase any further while GDP continues to increase.

The real wage rises due to an increase in productivity and labour hours. The persistent increase in wage is longer than the increase in labour hours as the peak of the wage response is seven periods whereas for labour hours it is only four implying the lack of a wealth effect.

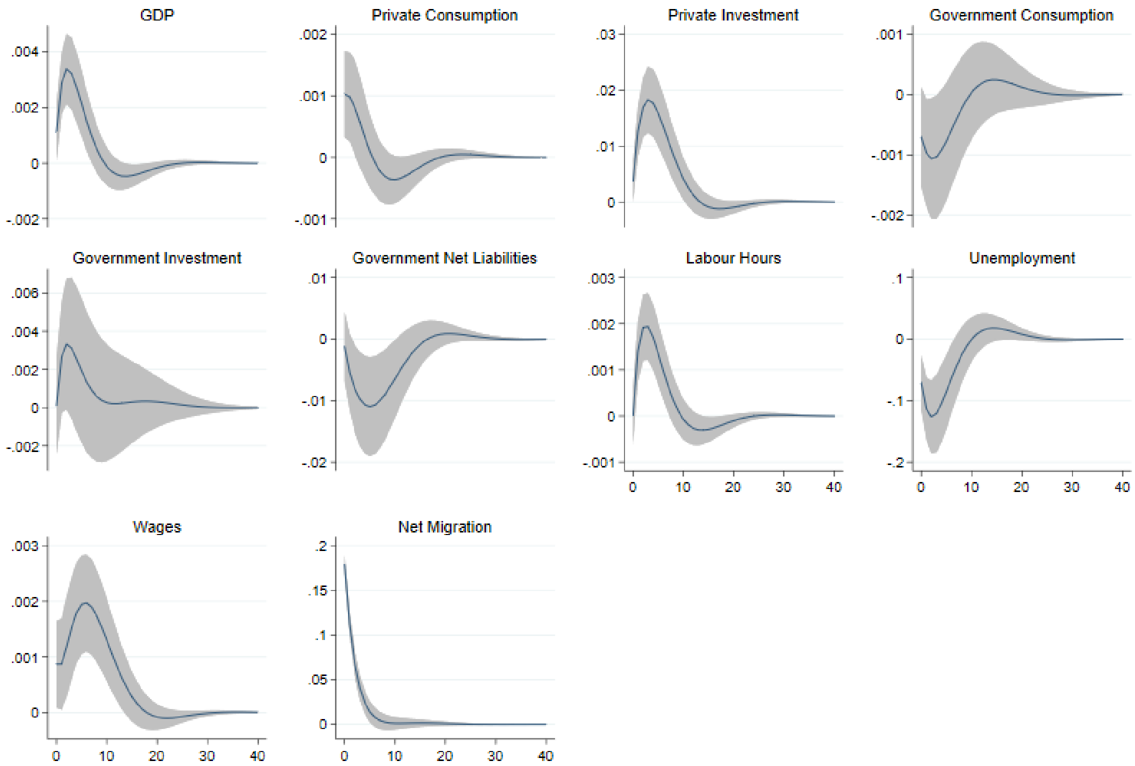
Aside from the shock to migration, the response of unemployment is of the largest magnitude. There is a fall in unemployment, a positive effect to the economy, that is of a relatively large size. Following the shock, the response is a continued fall in employment just as GDP continue to increase for three to four periods after the shock has occurred.

As the data is presented in per capita terms, the expansion of the economy is not only due to an increase in population but a per capita expansion.

The inflow of migrant workers and increase in economy results in a relatively large increase in private investment. An expansion of the economy will result in an increase in private investment as there are higher returns on capital due to the increased demand.

The responses of government consumption and investment are statistically insignificant on impact. However, government spending is often countercyclical which would partially explain a decrease. The Canadian government has persisted with a number of policies over the period of examination. The analysis of fiscal rules used by governments by Guerguil et al. (2017), indicate that Canada used the balanced budget rule (targets the budget balance often as a percent of GDP), debt rule (targets the level of borrowing), and expenditure rule (targets the level or growth rate of public spending).[16] Given that the response to the government's net liabilities is negative, one explanation is that the extra revenues from the taxes paid by migrants, as well as the reduction in government spending and investment is used to pay off liabilities.

Fig. 3.2 SVAR(1) Impulse Responses



An increase in per capita net migration in Canada. Data series are detrended using the Hamilton (2018) filter. The migration shock in the VAR(1) is identified by Cholesky decomposition where net migration is ordered first. The shaded areas are the 68% confidence intervals.

First we compare the results to those in Smith and Thoenissen (2019). We find the responses of total migration to Canada to be of a similar size to migration to New Zealand, however the responses for Canada are more statistically significant. The response of private consumption has a size of +0.001 compared to that of +0.003 for New Zealand. The size of the responses for private investment are of the same magnitude but not as long lasting in Canada. Real wages experience a small increase in Canada, however the response for real wages in New Zealand is insignificant. In comparison to the response for the United States in Kiguchi and Mountford (2017), the responses are much larger than for Canada. There is a decrease in unemployment, an increase in working hours, investment, consumption and GDP in Canada and

[16]The description in parentheses is the short description as defined by (Guerguil et al., 2017, p. 192).

the United States. However, the increase in real wages is statistically significant for Canada but not the United States.

3.5 Brain Waste in a Small Open Economy

We employ a DSGE model to analyse the effects of brain waste and migration in a small open economy. The stochastic growth model features heterogenous agents in native and migrant households, a series of perfectly competitive firms, and a fiscal authority. The model is augmented to the relaxation of visa restrictions in the introduction of *Express Entry* programme.

There are three households in this model, in the baseline case we assume there is no transition between the households.[17] The relative size of the household at time t is given by φ_t^i $i \in [H, L, M]$ where H is households of native agents classified as high-skill, L is households of native agents classified as low-skill, and M is the household of existing and (recently) landed migrants. There is a migration shock which increases the size of the migrant household on impact and decreases the *relative* size of the two native households. The agents in each household can be employed, unemployed, or enjoying leisure.

The model features asymmetric search and matching frictions in the labour market, and economy aggregates, as in Dolado et al. (2020) to reflect brain waste. The asymmetric search and matching frictions illustrate the increased difficulties for migrant and low-skilled native workers to successfully find employment relative to high-skilled natives. In the case of Canada, immigrants do not necessarily require a job to be granted a visa. Indeed of the recent *Express Entry* programme, workers are increasingly likely to be granted a visa without having found employment prior to arrival, which results in an increase of unemployed immigrant workers in addition to newly landed migrants who have secured employment. We assume that there are no costs to the migration process after arrival.

Perfectly competitive final good producing firms produce a homogeneous output by employing high and low-skilled labour from households and renting capital from the native high-skilled household. They post vacancies, at a cost, to hire workers of a specific type. Vacancy and employment decisions are subject to search and matching frictions. Wages are set by Nash bargaining.

There is a fiscal authority that runs a balanced budget consisting that finances exogenous government expenditures and unemployment insurance via tax collection.

[17] In a development of the model, we allow the household sizes to return to steady state following a migration shock. This is a process where the longest landed migrants transfer to the corresponding native household. However, this is not a choice by the household which is explicitly modelled.

3.5.1 Labour Market

The population has three types of household of size $\mathrm{N^i}$ where $i \in [H, L, M]$. The size of the population is normalised to one in steady state, i.e. $\Sigma\,\mathrm{N^i} = \mathrm{N} = 1$. A migration shock increases the size of the population which also changes the relative size of each household, denoted by φ_t^i. High-skilled natives are employed and search in market H, and similarly low-skilled natives in market L. The workers in the migrant household have three employment opportunities. There are two types of high-skill labour for migrants. The first, mH, is a high-skill level with low levels of brain waste. High-skill migrants experiencing high levels of brain waste are employed in sector mM. Low skill migrants find employment in one type of low-skill migrant labour, mL. The high-skill migrants strictly prefer sector mH to mM as it has lower levels of brain waste and utilises their skills to a greater extent. For each type of household, agents can be employed, n_t^j, (where $j \in [H, L, mM, mH, mL]$), unemployed, u_t^i or enjoying leisure, l_t^i.

$$1 = n^i + u^i + l^i \qquad i, j \in H, L \tag{3.5.1}$$

$$1 = n^{mH} + n^{mM} + n^{mL} + u^M + l^M \tag{3.5.2}$$

The aggregate measure of the employed and unemployed is $N_t^j = \varphi_t^j n_t^j$ and $U_t^j = \varphi_t^j u_t^j$.[18]

The firms post skill-specific vacancies, v_t^j, which are matched with unemployed workers, u_t^j. The parameter a^j reflects the matching efficiency for vacancy type j. The asymmetric matching technology allow this value to differ between employment type. A higher value makes for a more efficient process. The match elasticity with respect to unemployment is given by Γ^j. A match is formed with the following matching technology.

Matching Function

$$m_t^j = a^j \left(v_t^j\right)^{\Gamma^j} \left(U_t^j\right)^{\left(1-\Gamma^j\right)} \tag{3.5.3}$$

To analyse the labour market dynamics, we use the probability of filling a vacancy, Υ_t^j, probability of successfully searching for employment, Ω_t^j, and labour market tightness, θ_t^j, which are defined as follows.

Probability of Filling a Vacancy

$$\Upsilon_t^j = \frac{m_t^j}{v_t^j} \qquad j \in H, L, mH, mM, mL \tag{3.5.4}$$

[18]Following the relative household size given by φ_t^i, we separate φ_t^M to include φ_t^{mH}, φ_t^{mM}, and φ_t^{mL} to aid analysis.

Probability of Successfully Finding Employment

$$\Omega_t^j = \frac{m_t^j}{U_t^j} \qquad j \in H, L, mH, mM, mL \tag{3.5.5}$$

Labour Market Tightness

$$\theta_t^j = \frac{v_t^j}{U_t^j} \qquad j \in H, L, mH, mM, mL \tag{3.5.6}$$

The law of motion of employment follows that a number of individuals are made unemployed at an exogenous separation rate, ρ_v^j. These agents can find another job, remain unemployed, or exit the labour force. Since finding employment is dependent upon the labour market frictions, participation in the labour force can only be changed by the choice of u_t^j over l_t^i. Similarly, firms are only able to influence future employment by posting vacancies. Employment dynamics are as follows,
Law of Motion for Employment

$$N_{t+1}^j = (1 - \rho_n^j) N_t^j + m_t^j \qquad j \in H, L, mH, mM, mL \tag{3.5.7}$$

3.5.2 Households

The households share common features: they maximise lifetime utility with respect to consumption, c_t^i and leisure, l_t^i; employed individuals provide labour to the firms, n_t^j; and unemployed individuals, u_t^i, search for employment. We assume that each member within the household has equivalent preferences with respect to consumption and leisure such that households pool their income, so each member receives the same level of consumption. Households are endowed with an exogenously determined constant number of hours, $\overline{h^j}$, which they supply inelastically to the labour market.[19] Only the unemployed workers search for employment.[20] The matching process in the frictional labour markets determines the level of employment. Households pay lump-sum taxes, tax_t^j to the government. Carried over stocks, such as capital and bonds, are deflated by the inverse of the growth rate, $g_t^i = \frac{\mathrm{N_t^i}}{\mathrm{N_{t\text{-}1}^i}}$. Households are able to smooth consumption over time by trading in the bond markets. The high-skilled households own and invest in firms. The final consumption good, c_t, consists of a domestically produced good, c_t^h and an imported good, c_t^f such that the final good is

[19]Due to labour market frictions, and the collective nature of the household, we do not account for the intensive margin in the utility function.

[20]There is no on-the-job search.

defined as a constant elasticity of substitution (CES) aggregate:

$$c_t = \left[v^{\frac{1}{\Theta}} \left(c_t^h \right)^{\frac{\Theta-1}{\Theta}} + (1-v)^{\frac{1}{\Theta}} \left(c_t^f \right)^{\frac{\Theta-1}{\Theta}} \right]^{\frac{\Theta}{\Theta-1}}$$

Where Θ is the elasticity of substitution between the two types of goods, and v is the share of the domestically produced good in final consumption. Each household consumes c_t^j units of the final good. The price index of the final good, P_t, is chosen to be numeraire. Consequently, all other prices are expressed relative to the home final good, p_t^h is the relative price of the domestically produced good, and p_t^f is the relative price of the foreign produced good.

3.5.2.1 High-skilled Households

The household maximises utility from final good consumption, c_t^H, and leisure, l_t^H. The intertemporal preferences of the high-skilled household are given by:

$$U_t = \beta^t E_t \sum_{t=0}^{\infty} \left[\psi_t^H \frac{(c_t^H - \kappa c_{t-1}^H)^{1-\sigma_c}}{1-\sigma_c} + \Phi^H \frac{(l_t^H)^{1-\sigma_l}}{1-\sigma_l} \right] \tag{3.5.8}$$

The inverse of the intertemporal elasticity of substitution is denoted by σ_c, habit of consumption persistence by κ, Φ^H governs the weight of leisure specific to the high-skilled household, and σ_l is the inverse Frisch-elasticity of labour supply. The elasticities for consumption and labour, and habit persistence are household invariant.

Their utility is subject to a preference shock, ψ_t^H, to capture fluctuations of changes in consumption, which is an AR(1) process, where ε_t^H is an i.i.d. shock with zero mean and constant variance, σ_H^2.
Preference Shock

$$\ln \psi_t^H = \rho^H \ln \psi_{t-1}^H + \varepsilon_t^H \tag{3.5.9}$$

High-skilled households have the ability to smooth consumption over time by trading one-period bonds, b_t^H, that pay out in units of the foreign-produced goods. The interest rate, r_t is payable on the bonds. For each unit of labour provided to the firm, the household earns a wage of w_t^H. An unemployed member of the household is entitled to an unemployment absolute, or time invariant, insurance, ub^H. They pay lump-sum taxes, tax_t^H
High-skilled Household Budget Constraint

$$c_t^H + x_t \psi_t^x + p_t^f b_t^H + tax_t^H = w_t^H n_t^H + \frac{p_t^f (1 + r_{t-1}) b_{t-1}^H}{g_t^H} + u_t^H ub^H + r_t^k (z_t k_t) + d_t \tag{3.5.10}$$

The high-skilled households own the firms, investing x_t, and receive a share of profits as dividends, d_t, each period. The household rents out capital to the firm at a rate r_t^k. The intensity of capital utilisation, z_t, can increase returns on capital. However, can also affect the depreciation costs according to $\delta(z_t) = \delta z_t^{\phi}$, where $\phi > 1, \delta > 0$. The firm's capital stock evolves according to the capital law of motion which is formed from the previous period's capital stock less depreciation, current investment, and capital adjustment cost governed by ϕ_k.

Capital Accumulation

$$k_{t+1} = (1 - \delta(z_t))\frac{k_t}{g_{t+1}} + \frac{\phi_k}{2}\left(\frac{k_{t+1}}{k_t} - 1\right)^2 + x_t\psi_t^x \tag{3.5.11}$$

Investment is subject to a shock, ψ_t^x, an AR(1) process where ρ^x is the autoregressive parameter, and, ε_t^x is an i.i.d. shock with zero mean and constant variance, σ_a^2.

Investment Shock

$$\ln \psi_t^x = \rho^a \ln \psi_{t-1}^x + \varepsilon_t^x \tag{3.5.12}$$

Households maximise their utility subject to the budged constraint, and the laws of motions for employment and capital. We set up the optimisation problem:

$$\max_{\{c_t^H, n_{t+1}^H, u_t^H, b_t^H, k_{t+1}\}} \beta^t E_t\left[\psi_t^H \frac{(c_t^H - \kappa c_{t-1}^H)^{1-\sigma_c}}{1-\sigma_c} + \Phi^H \frac{(l_t^H)^{1-\sigma_l}}{1-\sigma_l}\right]$$

$$w_t^H n_t^H + \frac{p_t^f(1 + r_{t-1})b_{t-1}^H}{g_t^H} + u_t^H ub^H + r_t^k(z_t k_t) + d_t = c_t^H + x_t\psi_t^x + p_t^f b_t^H + tax_t^H$$

$$n_{t+1}^H = (1 - \rho_n^H)n_t^H + \Omega_t^H u_t^H$$

$$k_{t+1} = (1 - \delta(z_t))\frac{k_t}{g_{t+1}^H} + \frac{\phi_k}{2}\left(\frac{k_{t+1}}{k_t} - 1\right)^2 + x_t\psi_t^x$$

The marginal utility of consumption is identified by μ_t^H, and the multiplier on the law of motion of employment is given by Λ_t^H. We replace leisure using equation 3.5.1. The law of motion for employment as in the optimisation problem allows the household to account the effect of the unemployment decisions on matches rather than taking the number of matches as in equation 3.5.7.[21] Current employment can only be influenced in the next period due to the restrictions imposed by the labour market. The only adjustment in terms of participation that can be made today is through unemployment, u_t^H. The optimality and first order conditions are:

[21] This is only partial as the full effect would require equation 3.5.3 to be substituted into 3.5.7.

Marginal Utility of Consumption

$$\psi_t^H(c_t^H - \kappa c_{t-1}^H)^{-\sigma_c^H} = \mu_t^H \tag{3.5.13}$$

Employment

$$\Lambda_t^H = \beta E_t \left[(1-\rho_n^H)\Lambda_{t+1}^H + \mu_{t+1}^H w_{t+1}^H - \Phi^H \left(l_{t+1}^H\right)^{-\sigma_l}\right] \tag{3.5.14}$$

Unemployment

$$\Lambda_t^H = \frac{\Phi^H(l_t^H)^{-\sigma_L} - ub^H \mu_t^H}{\Omega_t^H} \tag{3.5.15}$$

Euler Equation

$$\mu_t^H p_t^f = \beta E_t p_{t+1}^f \mu_{t+1}^H \frac{(1+r_t)}{g_{t+1}^H} \tag{3.5.16}$$

Capital Holdings

$$\mu_t^H \left[1 + \phi_k \left(\frac{k_{t+1}}{k_t} - 1\right)\right] = \beta E_t \mu_t^H \left[r_{t+1}^k z_{t+1} + \left(\frac{(1-\delta(z_{t+1}))}{g_{t+2}^H} + \frac{\phi_k}{2}\left[\left(\frac{k_{t+2}}{k_{t+1}}\right)^2 - 1\right]\right)\right] \tag{3.5.17}$$

3.5.2.2 Low-skilled Households

Low-skilled households maximise lifetime utility subject to the consumption of a final good, c_t^L and leisure, l_t^L, where Φ^L is specific to the low-skill household.
Low-skilled Households Utility

$$U_t^L = \beta^t E_t \sum_{t=0}^{\infty} \left[\frac{(c_t^L - \kappa c_{t-1}^L)^{1-\sigma_c}}{1-\sigma_c} + \Phi^L \frac{(l_t^L)^{1-\sigma_l}}{1-\sigma_l}\right] \tag{3.5.18}$$

The household consumes, pays lump-sum taxes, trade in bonds which is financed by labour income and unemployment insurance.
Low-skilled Household Budget Constraint

$$c_t^L + tax_t^L + p_t^f b_t^L + \frac{\phi_{bhc^L}}{2}\left(p_t^f b_t^L - \overline{b^L}\right)^2 = w_t^L n_t^L + \frac{r_{t-1} p_t^f b_{t-1}^L}{g_t^L} + ub^L u_t^L + \Psi_{bhc}^L \tag{3.5.19}$$

There exists a bond holding cost, ϕ_{bhc^L} that the low-skilled household is subject to in the absence of financial autarky for the household, and deviation from the steady state holdings $\overline{b^L}$. Borrowing from the financial markets allows for consumption smoothing. To eliminate wealth effects, the costs are returned to the household as a lump-sum transfers, Ψ_{bhc}^L. As a result of utility maximisation, the household has the following first order conditions with respect to consumption, employment, and unemployment. The marginal utility of consumption is identified by μ_t^L, and the multiplier on the law of motion of employment is given by Λ_t^L. The maximisation problem for the household is given by

$$\max_{\{c_t^L, b_t^L, u_t^L, n_{t+1}^L\}} \beta^t E_t \sum_{t=0}^{\infty} \left[\frac{(c_t^L - \kappa c_{t-1}^L)^{1-\sigma_c}}{1-\sigma_c} + \Phi^L \frac{(l_t^L)^{1-\sigma_l}}{1-\sigma_l} \right]$$

$$c_t^L + tax_t^L + b_t^L p_t^f + \frac{\phi_{bhc^L}}{2} \left(b_t^L - \overline{b^L} \right)^2 = w_t^L n_t^L + \frac{r_{t-1} p_t^f b_{t-1}^L}{g_t^L} + ub^L u_t^L + \Psi_{bhc}^L$$

$$n_{t+1}^L = (1 - \rho_n^L) n_t^L + \Omega_t^L u_t^L$$

Marginal Utility of Consumption

$$\left(c_t^L - \kappa c_{t-1}^L \right)^{-\eta} = \mu_t^L \tag{3.5.20}$$

Employment

$$\Lambda_t^L = \beta_t E_t \left[\mu_{t+1}^L w_{t+1}^L + \left(1 - \rho_n^L \right) \Lambda_{t+1}^L - \Phi^L \left(l_{t+1}^L \right)^{-\sigma_l} \right] \tag{3.5.21}$$

Unemployment

$$\Lambda_t^L = \frac{\Phi^L (l_t^L)^{-\sigma_L} - ub^L \mu_t^L}{\Omega_t^L} \tag{3.5.22}$$

Euler Equation

$$\mu_t^L p_t^f \left[1 + \xi \left(b_t^L - \bar{b}^L \right) \right] = \beta_t E_t \mu_{t+1}^L \frac{(1 + r_t)}{g_{t+1}^L} p_{t+1}^f \tag{3.5.23}$$

Following the optimisation problem, the household eliminates the bond holding costs with the lump-sum transfers such that the budget constraint redefined as

$$c_t^L + tax_t^L + b_t^L p_t^f = w_t^L n_t^L + \frac{r_{t-1} b_{t-1}^L}{g_t^L} p_t^f + ub^L u_t^L \tag{3.5.24}$$

3.5.2.3 Migrant Households

Migrant households maximise lifetime utility from the final consumption good, c_t^M, and leisure l_t^M, and lose utility from searching for employment, s^k $k \in [H, M, L]$. The migrants experience this extra disutility because the job search is more difficult and can include multiple markets. The parameter Φ^M is specific to the migrant household, and Ψ^k is specific to each type of search intensity. The parameter σ_s governs the elasticity of search intensity.

Migrant Households Utility

$$U_t^M = \beta^t E_t \sum_{t=0}^{\infty} \left[\frac{(c_t^M - \kappa c_{t-1}^M)^{1-\sigma_c}}{1-\sigma_c} + \Phi^M \frac{(l_t^M)^{1-\sigma_l}}{1-\sigma_l} - \Psi^H \frac{(s_t^H)^{1-\sigma_s}}{1-\sigma_s} - \Psi^M \frac{(s_t^M)^{1-\sigma_s}}{1-\sigma_s} - \Psi^L \frac{(s_t^L)^{1-\sigma_s}}{1-\sigma_s} \right] \tag{3.5.25}$$

In the household, there are members that are high-skill and low-skill. Members of the household in high-skill employment experience low levels of brain waste, $j = mH$ earn a wage w_t^{mH} or experience high levels of brain waste in employment $j = mM$ who earn a wage level w_t^{mM}. The other sector of employment available to low-skill migrants and high-skill migrants who have skill-downgraded is the low skill sector, $j = mL$, who earn a wage w_t^{mL}. The remaining members of the household are searching for employment, u_t^M, or enjoying leisure, l_t^M.

$$1 = n_t^{mH} + n_t^{mM} + n_t^{mL} + u_t^M + l_t^M \tag{3.5.26}$$

At time t, a share of the unemployed migrants, s_t^H, are searching for employment in the labour market mH, the share s_t^M are searching in the labour market mM, and the remainder of unemployed migrants, s_t^L are searching in the low-skilled labour market, $j = mL$. The labour market mH is most desirable for high-skilled migrants as the degree of brain waste is lowest. However, due to the lack or lower levels of unemployment insurance, the unemployed high-skill migrants are willing to take up employment in the other sectors available to them; mM or mL the sectors of a skill-job mismatch. To participate in the low-skill labour market is a form of skill-downgrading, see Dustmann et al. (2013). We can define the unemployed migrants searching in the sector $j \in [mH, mM, mL]$ as $u_t^j = s_t^k u_t^M$, where $k \in [H, M, L]$. The search decision in each of the labour market is endogenous. Their budget constraint comprises from final good consumption, c_t^M, lump-sum taxes, tax_t^M, bond market purchases, b_t^M, labour income from the high-skill sectors, w_t^{mH} and w_t^{mM}, the low-skilled labour market w_t^{mL} and from unemployment insurance.

Migrant Household Budget Constraint

$$c_t^M + tax_t^M + b_t^M p_t^f + \frac{\phi_{bhc^M}}{2} \left(b_t^M - \overline{b^M} \right)^2 =$$
$$w_t^H n_t^{mH} + w_t^{mM} n_t^m + w_t^L n_t^{mL} + ub^{mH} u_t^{mH} + ub^{mM} u_t^{mM} + ub^{mL} u_t^{mL} + \frac{r_{t-1} b_{t-1}^M p_t^f}{g_t^M} + \Psi_{bhc}^M \tag{3.5.27}$$

There exists a bond holding cost, ϕ_{bhc^M} that the migrant household is subject to in the absence of financial autarky. To eliminate wealth effects, the costs are returned to the household as a lump-sum, Ψ_{bhc}^M.

The multiplier on the budget constraint, μ_t^M is the marginal utility of consumption for the household. In addition, they have three laws of motion of employment constraints, the multiplier on which is Λ_t^{mH}, Λ_t^{mM}, and Λ_t^{mL}. We setup the maximisation problem for the migrant household to find the choice variables.

$$\max_{\left\{c_t^M, b_t^M, u_t^M, n_{t+1}^j, s_t^k\right\}} \beta^t E_t \sum_{t=0}^{\infty} \left[\tfrac{(c_t^M - \kappa c_{t-1}^M)^{1-\sigma_c}}{1-\sigma_c} + \Phi^M \tfrac{(l_t^M)^{1-\sigma_l}}{1-\sigma_l} - \Psi^H \tfrac{(s_t^H)^{1-\sigma_s}}{1-\sigma_s} - \Psi^M \tfrac{(s_t^M)^{1-\sigma_s}}{1-\sigma_s} - \Psi^L \tfrac{(s_t^L)^{1-\sigma_s}}{1-\sigma_s}\right]$$

$$c_t^M + tax_t^M + b_t^M p_t^f + \frac{\phi_{bhc^M}}{2}\left(b_t^M - \overline{b^M}\right)^2 =$$

$$w_t^H n_t^{mH} + w_t^{mM} n_t^m + w_t^L n_t^{mL} + ub^{mH} u_t^{mH} + ub^{mM} u_t^{mM} + ub^{mL} u_t^{mL} + \frac{r_{t-1} b_{t-1}^M p_t^f}{g_t^M} + \Psi_{bhc}^M$$

$$n_{t+1}^j = (1-\rho_n^j) n_t^j + \Omega_t^j s_t^k u_t^m \quad j \in [mH, mM, mL] \quad k \in [H, M, L]$$

As a result, the optimality and first order conditions are:
Marginal Utility of Consumption

$$\left(c_t^M - \kappa c_{t-1}^M\right)^{-\eta} = \mu_t^M \tag{3.5.28}$$

Employment

$$\Lambda_t^j = \beta_t E_t \left[\mu_{t+1}^M w_{t+1}^j + \left(1 - \rho_n^j\right) \Lambda_{t+1}^j - \Phi^M \left(l_{t+1}^M\right)^{-\sigma_l}\right] \quad j \in mH, mM, mL \tag{3.5.29}$$

Unemployment

$$\Lambda_t^{mM} = \frac{\Phi^M \left(l_t^M\right)^{-\sigma_l} - ub^{mM} \mu_t^M}{\Omega_t^{mM}} \tag{3.5.30}$$

Search intensity

$$\Phi^k (s_t^k)^{-\sigma_s} = \mu_t^M u_t^M ub^j + \Lambda_t^j \Omega_t^j u_t^M \quad j \in mH, mM, mL \quad k \in H, M, L \tag{3.5.31}$$

Euler Equation

$$\mu_t^M p_{t+1}^f \left[1 + \xi \left(b_t^M - \overline{b}^M\right)\right] = \beta_t E_t \mu_{t+1}^M p_{t+1}^f \frac{(1 + r_t)}{g_{t+1}^M} \tag{3.5.32}$$

Once the bond holding costs are cancelled with the lump-sum transfers, the budget constraint is modified to

$$c_t^M + tax_t^M + b_t^M p_t^f = w_t^H n_t^{mH} + w_t^{mM} n_t^m + w_t^L n_t^{mL} + ub^{mH} u_t^{mH} + ub^{mM} u_t^{mM} + ub^{mL} u_t^{mL} + \frac{r_{t-1} b_{t-1}^M p_t^f}{g_t^M} \tag{3.5.33}$$

3.5.3 Firms

A continuum of perfectly competitive firms produce a final good, y_t, using high and low-skilled labour, L_t^H and L_t^L, and aggregate capital, K_t. Current employment levels are determined by the previous period's labour market conditions; hence the firms are only able to influence their next period value by respecting the constraints imposed by the frictions of the labour markets. The production function is a nested CES composite of production factors, where it is possible to control the elasticity of substitution between the different factors in production. We introduce capital-skill complementarity to reflect the higher productivity of high-skilled workers as in Mandelman and Zlate (2012).

Production Function

$$y_t = \psi_t^a \left[(\alpha^s)^{\frac{1}{\alpha}} \left[\left((\alpha^k)^{\frac{1}{\alpha^H}} K_t^{\frac{\alpha^H - 1}{\alpha^H}} + (1-\alpha^k)^{\frac{1}{\alpha^H}} (\xi L_t^H)^{\frac{\alpha^H - 1}{\alpha^H}} \right)^{\frac{\alpha^H}{\alpha^H - 1}} \right]^{\frac{\alpha - 1}{\alpha}} + (1-\alpha^s)^{\frac{1}{\alpha}} (L_t^L)^{\frac{\alpha - 1}{\alpha}} \right]^{\frac{\alpha}{\alpha - 1}} \tag{3.5.34}$$

The parameter α^s governs the skill intensity of total production, α^k controls the capital intensity of skilled production, and the elasticities of substitution between capital and high-skill labour and the elasticity of capital and low-skill labour are represented by α^H and α. Due to the format of the nested CES production function, the elasticity of substitution between high and low skill labour is the same as capital and low-skill labour. The parameter ξ governs the steady state relative productivity of high-skilled workers compared to low-skilled workers. Output is subject to a productivity shock, ψ_t^a, which is an AR(1) process. Where ρ^a is the autoregressive parameter, and, ε_t^a is an i.i.d. shock with zero mean and constant variance, σ_a^2.

Total Factor Productivity

$$\ln \psi_t^a = \rho^a \ln \psi_{t-1}^a + \varepsilon_t^a \tag{3.5.35}$$

Aggregate capital is a summation of capital provided by the households and its capacity utilisation, in the baseline model only the high-skill native households provide capital to the firm. Aggregate investment is similarly determined.

Aggregate Capital

$$K_t = \varphi_t^H z_t k_t \tag{3.5.36}$$

Aggregate Investment

$$X_t = \varphi_t^H x_t \tag{3.5.37}$$

High-skilled labour, L_t^H, is a CES aggregate of high skill labour from natives and migrants experiencing high and low levels of brain waste. There are two forms of high-skill employment that is filled by migrants which reflects the differing degrees of brain waste experienced by landed and more recently landed migrants. In the absence of brain waste, the only form of employment for high-skill migrants is N_t^{mH}, and there is no wage differential between natives and migrants. The migrants are able to experience the same benefits from employment as natives. The share of $j \in H, mH, mM$ workers in the CES function of high-skill labour is given by χ_j, and the elasticity of substitution between workers is given by θ^h.
High-skill Labour

$$L_t^H = \left[\chi_H^{\frac{1}{\theta^h}} (N_t^H)^{\frac{\theta^h - 1}{\theta^h}} + \chi_{mH}^{\frac{1}{\theta^h}} (N_t^{mH})^{\frac{\theta^h - 1}{\theta^h}} + (\chi_{mM})^{\frac{1}{\theta^h}} (N_t^{mM})^{\frac{\theta^h - 1}{\theta^h}}\right]^{\frac{\theta^h}{\theta^h - 1}} \tag{3.5.38}$$

Low-skilled labour, L_t^L, is a CES aggregate of low-skill labour from natives and migrants. The share of $j \in L, mL$ workers in the CES function of high-skill labour is given by χ_j and the elasticity of substitution between workers is given by θ^l.
Low-skill Labour

$$L_t^L = \left[\chi_l^{\frac{1}{\theta^l}} (N_t^l)^{\frac{\theta^l - 1}{\theta^l}} + (1 - \chi_l)^{\frac{1}{\theta^l}} (N_t^{mL})^{\frac{\theta^l - 1}{\theta^l}}\right]^{\frac{\theta^l}{\theta^l - 1}} \tag{3.5.39}$$

We set out the firms' profit maximisation problem, which is subject to the law of motion of employment for each type of employment. Vacancies are posted at a cost, κ^{v^j}. The multiplier on the laws of motion of employment is denoted by η_t^j.

$$\max_{\{K_t, N_{t+1}^j, v_t^j\}} p_t^h y_t - r_t^k K_t - w_t^j N_t^j - \kappa^{v^j} v_t^j$$

$$N_{t+1}^j = (1 - \rho_n^j) N_t^j + \Upsilon_t^j v_t^j \qquad j \in H, L, mH, mM, mL$$

The maximisation problem computes the first order conditions for capital, K_t, future employment, N_{t+1}^j and vacancies, v_t^j.
Price of Capital Rental

$$r_t^k = p_t^h mpk_t \tag{3.5.40}$$

Employment

$$\beta\eta^j_{t+1}\mu_{t+1}(1-\rho^j_n) = \mu_t\left[mpl^j_t - w^j_t - \eta^j_t\right] \tag{3.5.41}$$

Vacancies

$$\kappa^{v^j}\mu_t = \beta\eta^j_{t+1}\mu_{t+1}\Upsilon^j_t \tag{3.5.42}$$

By the envelope theorem, combining equations 3.5.41 and 3.5.42 to eliminate η^j.

$$\frac{\kappa^{v^j}}{\Upsilon^j_t} = \beta\frac{\mu_{t+1}}{\mu_t}\left[p^h_{t+1}mpl^j_{t+1} - w^j_{t+1} + (1-\rho^j_n)\frac{\kappa^{v^j}}{\Upsilon^j_{t+1}}\right] \tag{3.5.43}$$

3.5.4 Wage Bargaining

As a result of the asymmetries between labour markets, there are differing bargaining powers. One of the most studied areas of the labour market conditions for migrants is the difference with native which extends to bargaining power. Natives, and by extension high-skill, have the highest bargaining power, and the low-skill migrants the least. The primary reasons for migrants' lower bargaining power is that they are unlikely to be members of a union, and natives have higher job security.

Wages are determined by relative bargaining powers of the firm and the worker. The splitting rule is governed by $(1-\vartheta^j)sf^j_t = \vartheta^j sw^j_t$. Here, ϑ^j is the bargaining power of the firm. The worker has a surplus from the value of employment less the option of unemployment. First, we determine the surpluses for the worker, sw^j_t and for the firm, sf^j_t.

Worker Surplus

$$sw^j_t = \mu^i_t w^j_t - \Phi^i(l^i_t)^{-\sigma_l} + (1-\rho^j_n)\Lambda^j_t$$

Firm Surplus

$$sf^j_t = p^h_t mpl^j_t - w^j_t + (1-\rho^j_n)\frac{\kappa^j}{\Upsilon^j_t}$$

The final wage is a weighted average of the surplus from both bargainers. Firstly, the worker's marginal product of labour, and the continuation value of the match to the firm adjusted by the continuation value of the match to the household. The second term is the option of being unemployed corrected for the disutility from providing labour. A higher bargaining power for a worker is equivalent to a lower bargaining power for the firm. In this model, $\vartheta^H < \vartheta^{mH} < \vartheta^{mM}$ and for the low-skill labour market $\vartheta^L < \vartheta^{mL}$.

$$w^j_t = \left(1-\vartheta^j\right)\left[p^h_t mpl^j_t + \left(1-\rho^j_n\right)\frac{\kappa^j}{\Upsilon^j_t}\right] + \frac{\vartheta^j}{\mu^i_t}\left[\Phi^i\left(l^i_t\right)^{-\sigma_l} - \left(1-\rho^j_n\right)\Lambda^j_t\right] \tag{3.5.44}$$

3.5.5 Population dynamics and brain waste

To analyse brain waste, we focus our analysis on an increase to high-skill migration. A focus of the recent drive to increase immigration, by increasing permanent resident admissions to 350,000 by 2021 which is equivalent to 1% of the Canadian population.[22] While modelling migration as exogenous may seem a simplifying assumption, it is hard to predict when the changes in migration policy will occur.[23]

In this model, the high-skill migrants search for a job in the each of the high-skill sectors where the lower levels of brain waste is preferable. This is due to its higher wage, standing, and the ability to greater utilise their skillset. Migrants who have been granted a visa based on a particular job commence employment in the first period of arrival. Workers who have been granted a visa based only on their skills enter the labour market as unemployed. The comparison of brain waste is reflected in the asymmetric search and matching frictions presented in section 3.5.1, wage differential, unemployment rates, participation rates, and replacement rates. When the migrants have the same powers as natives, this partially reduces the effects of brain waste. In addition, the endogenous choice of employment search due to insufficient matches in the high-skill sector(s). The matching efficiency of the migrant households are significantly lower than that of natives. Unemployed migrants eventually tire after unsuccessful searches in the high-skill labour market with low levels of brain waste so search in the other labour markets. They are unable to utilise their higher skill in the low-skill job. They do not receive an increase in income either.

To enable calibration of different labour market statuses, we introduce the participation rate and the unemployment rate from which the disutility of labour and firm bargaining power can be calibrated.

$$\text{partic}_t^j \equiv \frac{N_t^j + U_t^j}{\varphi_t^j} \tag{3.5.45}$$

$$\text{unemp}_t^j \equiv \frac{U_t^j}{N_t^j + U_t^j} \tag{3.5.46}$$

High-skilled households experience higher participation rates and lower unemployment rates. The values for migrants can differ to what extent brain waste is experienced. The participation rates for migrants are on average lower than the native equivalent. The reasons for this agree with Hilgenstock and Koczan (2018) as it is dependent upon the demographics of migrants relative to natives, for example those who are

[22]This figure was 271,845 in 2015-16 fiscal year.
[23]Net migration is analysed as an AR(1) process in the data

caregivers to children or relatives are more likely to be out of the labour market than the native equivalents who try for a part-time employment. Similarly, those who are on family visas that may be ineligible to work. There is a significant difference in participation rates of females at age levels 15 to 24 years, 25 to 34 years, 35 to 44 years, and 45 to 54 years with up to 11%.[24]

Building on section 3.5.1, we put forth the distributional population dynamics to reflect the distribution of workers in the economy, and the effect of their skills being utilised fully. Where ζ_t^H is the share of high-skill employment of total employment, and ζ_t^L is the share of low-skill employment of total employment.

High-skill Employment

$$\zeta^H = \frac{N^H + N^{mH} + N_t^{mM}}{N^H + N^{mH} + N^{mM} + N^L + N^{mL}}$$

Low-skill Employment

$$\zeta^L = \frac{N^L + N^{mL}}{N^H + N^{mH} + N^M + N^L + N^{mL}}$$

Using the population dynamics set out in section 3.5.1, we introduce the migration shock that increases the growth rate of the migrant household. A migration shock in this stochastic growth model increases the population size which dilutes capital and financial asset stocks ceteris paribus. The migration shock increases the relative size of the migrant households and decreases the relative size of the native households. Hence there is a growth rate for each household which in steady state is equal to one.

Total Population

$$\mathrm{N_t} = \mathrm{N_t^H} + \mathrm{N_t^L} + \mathrm{N_t^M} \tag{3.5.47}$$

To evaluate the growth rates of the population,

$$g_t = \frac{\mathrm{N_t}}{\mathrm{N_{t\text{-}1}}} = \frac{\mathrm{N_t^H} + \mathrm{N_t^L} + \mathrm{N_t^M}}{\mathrm{N_{t\text{-}1}}}$$

$$g_t = \frac{\mathrm{N_t}}{\mathrm{N_{t\text{-}1}}} = \frac{\mathrm{N_t^H}}{\mathrm{N_{t\text{-}1}}}\frac{\mathrm{N_{t\text{-}1}^H}}{\mathrm{N_{t\text{-}1}^H}} + \frac{\mathrm{N_t^L}}{\mathrm{N_{t\text{-}1}}}\frac{\mathrm{N_{t\text{-}1}^L}}{\mathrm{N_{t\text{-}1}^L}} + \frac{\mathrm{N_t^M}}{\mathrm{N_{t\text{-}1}}}\frac{\mathrm{N_{t\text{-}1}^M}}{\mathrm{N_{t\text{-}1}^M}}$$

$$g_t = \frac{\mathrm{N_t}}{\mathrm{N_{t\text{-}1}}} = \sum_{i=H,L,M} \frac{\mathrm{N_t^i}}{\mathrm{N_{t\text{-}1}^i}}\frac{\mathrm{N_{t\text{-}1}^i}}{\mathrm{N_{t\text{-}1}}} = g_t^i \varphi_{t-1}^i \tag{3.5.48}$$

Where $\frac{\mathrm{N_t^i}}{\mathrm{N_{t\text{-}1}^i}} = g_t^i$ is the growth rate of household i, and $\frac{\mathrm{N_{t\text{-}1}^i}}{\mathrm{N_{t\text{-}1}}} = \varphi_{t-1}^i$ is the fraction of the household relative to the total population. The increased growth rate of

[24] The census provides different breakdowns of migration status. We define natives as "non-immigrants" and migrants is inclusive of "immigrants" and "non-permanent residents".

household j, g_t^j, will affect the total population growth rate, g_t. The growth rate of the total population is defined as

$$g_t = \sum_{i=H,L,M} g_t^i \varphi_{t-1}^i \tag{3.5.49}$$

The migration shock immediately affects the growth rate of the migrant household. In the baseline model, the relaxation of migration restrictions affects the most recently landed high-skill migrants, equivalently $j = mM$.

$$g_t^j = \rho^{g^m} g_{t-1}^j + \varepsilon_t^j \qquad j \in [mH, mM, mL] \tag{3.5.50}$$

Therefore, we replicate equation 3.5.49 for the growth rate of the migrant household.

$$g_t^M = \sum_{j=mH,mM,mL} g_t^j \varphi_{t-1}^j \tag{3.5.51}$$

We evaluated the share of each household size by combination of the relative household size, φ_t^j, and the growth rates. When household sizes return to steady state, $\rho^\varphi < 1$ but when there are permanent changes to the household size, $\rho^\varphi = 1$. Here ρ^φ is the persistence parameter on household size.[25]

$$\varphi_t^j = \left(\varphi^j\right)^{1-\rho^\varphi} \left(\frac{g_t^j}{g_t} \varphi_{t-1}^j\right)^{\rho^\varphi} \tag{3.5.52}$$

3.5.6 Fiscal Authority

There exists a fiscal authority that consumes, gc_t, provides unemployment insurance, ub^j, and receives lump-sum taxes, tax_t^j every period enabling the government to run a balanced budget. The government provides unemployment insurance which is determined by the replacement rates. Where Tax_t is the sum of taxes paid by each type of household, these are assumed to be equal.

Aggregate Taxation Receipts

$$Tax_t = \varphi_t^H tax_t^H + \varphi_t^L tax_t^L + \varphi_t^M tax_t^M \tag{3.5.53}$$

As per the balanced budget definition policy, government expenditure is equal to tax revenues.

Government Budget Constraint

$$ge_t = Tax_t \tag{3.5.54}$$

[25]Derivation of equation 3.5.52 is available in section B.3.

Total government expenditure, ge_t, is given by,[26]
Government Expenditure

$$ge_t = ub^H U_t^H + ub^L U_t^L + ub^{mH} U_t^{mH} + ub^{mM} U_t^{mM} + ub^{mL} U_t^{mL} + gc_t \tag{3.5.55}$$

Government consumption is exogenously determined and subject to a shock ε_t^{gc} Where ρ^{gc} is the autoregressive parameter, and, ε_t^{gc} is an i.i.d. shock with zero mean and constant variance, σ_a^2.
Government Consumption

$$\ln gc_t = (1 - \rho^{gc}) \ln \overline{(gc_t)} + \rho^{gc} \ln gc_{t-1} + \varepsilon_t^{gc} \tag{3.5.56}$$

3.5.7 An Open Economy

In this model of a small open economy, there is demand for the domestically produced good both at home and from abroad. We analyse this with export demand, ex_t.

Real Exchange Rate

The real exchange rate, rer_t, is defined in terms of the consumer price indices. Since the model is a *small* open economy, the relative size to the rest of the world is such that the domestic import price is approximately the consumption real exchange rate, $p_t^f \approx rer_t$.

$$1 = v \left(p_t^h\right)^{1-\Theta} + (1 - v) \left(p_t^f\right)^{1-\Theta} = v \left(p_t^h\right)^{1-\Theta} + (1 - v) \left(rer_t\right)^{1-\Theta} \tag{3.5.57}$$

Market clearing

Demand for the domestically produced good is a function of total home consumption, investment, vacancy costs, and export demand. To simplify analysis, we aggregate the household variables for consumption and bond holdings.
Aggregate Consumption

$$C_t = \varphi_t^H c_t^H + \varphi_t^L c_t^L + \varphi_t^M c_t^M \tag{3.5.58}$$

Aggregate Bond Holdings

$$b_t = \varphi_t^H b_t^H + \varphi_t^L b_t^L + \varphi_t^M b_t^M \tag{3.5.59}$$

[26]Unemployed low-skilled migrants receive no form of unemployment insurance and only a small fraction of those migrants experiencing high-levels of brain waste are eligible.

Market clearing for the domestically produced good is given by
Market clearing
$$y_t = v(p_t^h)^{-\Theta}\left(C_t + X_t + \kappa^j v_t^j + gc_t\right) + ex_t \tag{3.5.60}$$
The demand for exports is given by:
Exports
$$ex_t = v^f\left(\frac{rer_t}{p_t^h}\right)^{\Theta^f} y_t^f \tag{3.5.61}$$
Here, Θ^f is the foreign value for elasticity of substitution between the home and foreign goods and v^f is the share of the home produced good in the final consumption bundle. Total foreign demand for the domestic good, y_t^f, is modelled as an exogenous, or AR(1) process, where $\varepsilon_t^{y^f}$ is an i.i.d. shock with zero mean and a constant variance $\sigma^2_{y^f}$.
Foreign Demand
$$\ln y_t^f = \rho^{y^f}\ln y_{t-1}^f + \varepsilon_t^{y^f} \tag{3.5.62}$$
The economy only produces the final good, henceforth $gdp_t = y_t p_t^h$. We define the trade balance to GDP ratio to be
Trade Balance
$$tb_t = 1 - \frac{\left(C_t + X_t + \Sigma\kappa^j v_t^j + ge_t\right)}{gdp_t} \tag{3.5.63}$$
Hence the net asset position to be
Net Foreign Assets
$$b_t = r_t\frac{gdp_{t-1}}{gdp_t}\frac{rer_t}{rer_{t-1}}b_{t-1} + tb_t \tag{3.5.64}$$
We close the small open economy model with a debt elastic interest rate as put forth by Schmitt-Grohe and Uribe (2003). The households active in international financial markets face an interest rate, r_t, which is a function of the world interest rate, r^*, and is increasing in the country's average level of debt, denoted by b_t.
Interest Rate
$$r_t = r^*\exp^{\left(-\phi_{bhc}\left(b_t-\bar{b}\right)\right)} \tag{3.5.65}$$

3.6 Solving the model

We solve the model presented in section 3.5 with the estimation and calibration of a non-linear DSGE model that is subject to six stochastic shocks for which we use six observable variables. We describe the parameters calibrated and those taken from the literature in section 3.6.1. The data transformations are discussed in section

3.6.2 and the results of estimation in section 3.6.3. We use data from Canada, a country where brain waste is prominent as explained in earlier sections of this paper.

3.6.1 Calibration

Some parameters are fixed in estimation to address any identification issues raised by the complexity of the model. The values are taken from relevant literature or by matching long run targets in the data. Starting with mostly standardised parameters: the discount factor $\beta = 0.99$ which equates to an annual interest rate of approximately 4%, a capital depreciation rate $\delta = 0.0125$ equivalent to 5% per year, and an openness to trade parameter $\gamma = 0.3$. The values for the capital-skill complementarity production function are based on Mandelman and Zlate (2012) adjusted for the redefinition of high and low-skill labour. We calibrate the matching efficiencies, a^j and job destruction ρ_n^j such that the job-finding and vacancy probabilities are higher for the native worker at the same skill level. For the international sector, we assume all relative prices and terms of trade to have a steady state value of 1. The bargaining power is determined by the steady state computations. Vacancy posting costs for the migrants are taken from steady state calculations such that the difference in wage levels hold. To evaluate the change in wage at each skill level between migrants and natives, we take data from the 2016 census table 98-400-X2016280. The low-skill wage, Δ^{w^L} is taken from the average employment income of individuals that are employees (i.e. excluding self-employed workers) with qualifications in the classification "apprenticeship or trades certificate or diploma" (identifier 4), the native value is non-immigrants who gained qualifications inside Canada, and for the migrants is immigrants who gained qualifications outside of Canada.[27] For the wage difference of high-skill natives and high-skill migrants experiencing high levels of brain waste ($j = mM$), Δ^{w^M} is taken from category "university certificate, diploma or degree at bachelor level or above" (identifier 7) where the native wage is non-immigrants with qualifications gained inside Canada, and the mM wage is for migrants who gained qualifications outside of Canada. The difference between high-skill natives and high-skill migrants experiencing low levels of brain waste ($j = mH$), Δ^{w^H}, the wage is at skill level as in $\Delta^{w^{mM}}$ but the migrant wage is immigrants who gained their qualifications inside Canada. The values are described in table 3.1. The differences are a fraction of the wage received by natives, e.g. $w^{mL} = \Delta^{w^L} w^L$.

For the employment type, values for $\mathrm{partic}^{\mathrm{j}}$ and $\mathrm{unemp}^{\mathrm{j}}$ are taken from the 2016 census table 98-400-X2016198. We define high-skilled workers as those with a college

[27] There is no data for non-permanent residents, nor the year of migration.

Table 3.1 Calibrated Parameters

Description	**Param**	**Value**	**Description**	**Param**	**Value**
Discount Factor	β	0.9900	Matching Efficiency	a^H	0.8500
Depreciation Rate	δ	0.0125		a^L	0.3100
Openness to Trade	γ	0.3000		a^{mH}	0.6500
Share of K vs L^H	α^s	0.7100		a^{mM}	0.4500
Share of K and L^H vs L^L	α^k	0.2800		a^{mL}	0.2500
EoS K to L^H	α^H	0.5800	Matching Elasticity	Γ^j	0.5000
Eos K to L^L	α	0.6800	Job Destruction	ρ_n^H	0.0490
Productivity of L^H vs L^L	ξ	1.8500		ρ_n^L	0.0855
EoS	σ^c	2.0000		ρ_n^{mH}	0.0510
Inverse Frisch Elasticity	σ^l	3.0000		ρ_n^{mM}	0.0800
Elasticity of Search	σ_s	1.5000		ρ_n^{mL}	0.6280
Vacancy Costs	κ^H	0.1500	Bargaining Power	ϑ^H	0.2052
	κ^L	0.1400		ϑ^L	0.6695
	κ^{mH}	0.4226		ϑ^{mH}	0.5353
	κ^{mM}	0.5334		ϑ^{mM}	0.7242
	κ^{mL}	0.2429		ϑ^{mL}	0.7336
Wage Difference	Δ^{w^H}	0.9512	Weight on Search	Ψ^H	0.0104
	Δ^{w^M}	0.8032		Ψ^M	0.0258
	Δ^{w^L}	0.8855		Ψ^L	0.0321
Leisure Weight	Φ^H	0.0033	Replacement Rate	$\frac{ub^H}{w^H}$	0.2000
	Φ^L	1.0515		$\frac{ub^L}{w^L}$	0.3000
	Φ^M	0.6273		$\frac{ub^{mH}}{w^{mH}}$	0.1250
Bond Holding Constraint	ϕ_{bhc^M}	0.0010		$\frac{ub^{mM}}{w^{mM}}$	0.1000
	ϕ_{bhc^L}	0.0010		$\frac{ub^{mL}}{w^{mL}}$	0.0000

Calibrated parameters for the baseline small open economy model with brain waste. This includes the ones that are targeted specifically from the data and those from calculation of the steady state. EoS = elasticity of substitution.

certificate, university degree at bachelor and higher level degrees. To define high-skill migrants experiencing high levels of brain waste, labour market $j = mM$ is defined as high skill migrants that landed 2001-2016 and non-permanent residents. The labour market $j = mH$ is high skill migrants that arrived before 2001. The values for relative population size, participation, and unemployment rate for employment type j, are shown in table 3.2. The differing values within the migrant household supports the model specification of natives and migrants with two types of high skill migrants.

Table 3.2 Labour Market Values

Household	φ^j	partic	unemp
Native Low-skill (L)	0.4197	0.5856	0.1026
Native High-skill (H)	0.3141	0.7685	0.0503
Migrant (M)	*0.2662*		
mL	0.1255	0.4889	0.0907
mM	0.0676	0.7902	0.0902
mH	0.0731	0.6839	0.0527

The values for relative household size, participation rate, and unemployment rate of each labour market. The total size of the migrant household is given by $\varphi^M = \varphi^{mH} + \varphi^{mM} + \varphi^{mL}$.

3.6.2 Data

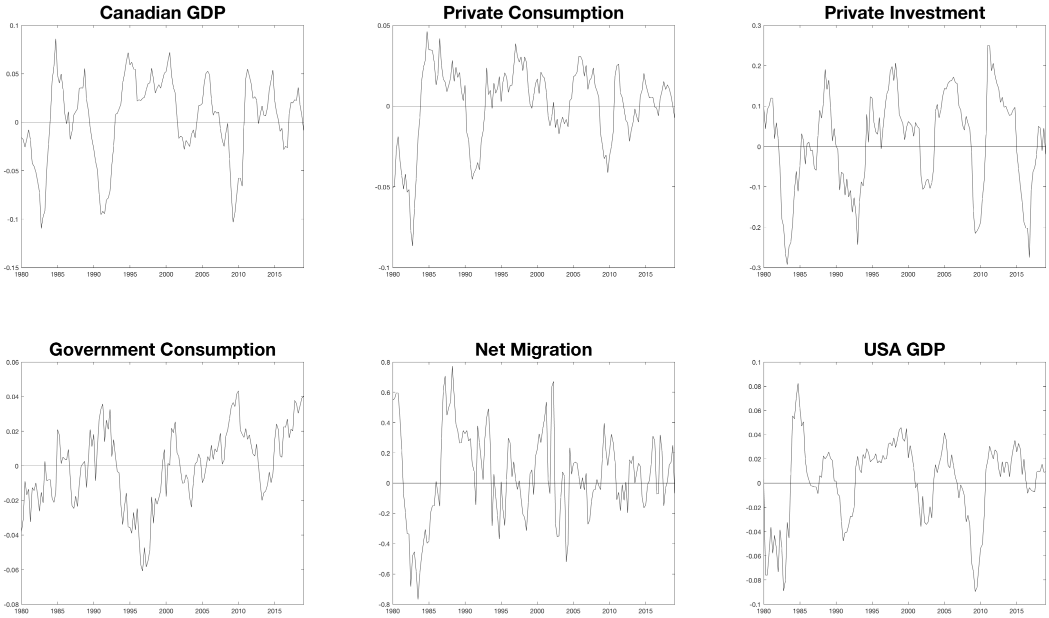
The number of data series used in the estimation cannot exceed the number of shocks in the model, therefore, estimation uses six data series. The six shocks of the model: home TFP, investment, consumption preference of high-skilled households, government consumption, foreign TFP, and net migration. We take data from the national accounts: GDP (Canada and the United States), private gross fixed capital formation (investment), aggregate private consumption, and government consumption. These time series have been collected from the OECD in national currency and deflated using the corresponding deflator to convert the values to real terms. Net migration data is collected from Statistics Canada. The national accounts and net migration data are transformed into per capita terms by dividing by the working-age (15 years and older) population. We take the natural logarithm of trending series and then apply the Hamilton (2018) filter to compute the trends and cyclical components of the data series. Figure 3.3 shows the cyclical component of each variable which is used in the measurement equations.

3.6.3 Estimation

We estimate the DSGE model using Bayesian methods by running a Markov Chain Monte Carlo (MCMC) algorithm. The results presented in table 3.3 shows the mean for the prior and posteriors, the standard deviation of the prior, and 5th and 95th percentiles of the posterior distribution (obtained after 1,500,000 iterations) for the baseline model of a small open economy with brain waste. We choose standard priors for the model parameters.

We introduce tight priors for the autoregressive coefficients. Notably, migration seems to be less persistent than in the SVAR and has a standard deviation slightly

Fig. 3.3 Data Observables



The six observables used in the Bayesian estimation of the baseline DSGE model as described in section 3.5. The data presented is the cyclical component of the Hamilton filter applied. The horizontal axis identifies the year of the observation. The series are at a quarterly frequency, with 157 observations, 1980Q1:2019Q1 inclusive. Measurement equations are $xobs_t = x_t - x_{ss}$ where $xobs$ is the observation of variable x, x_t is the actual value at time t, and x_{ss} is the steady state or expected value of variable x.

greater than output. The persistence parameter on investment is relatively small, however the standard deviation is very large. Government consumption has the same standard deviation as output but significantly different persistence parameter. We would have expected a difference with output. The preference shock has a high degree of persistence and has a higher standard deviation than output. Finally, foreign output has a similar level of standard deviation to home TFP but is less persistent. The model parameters deviated only slightly from their priors with capital adjustment costs above and the elasticity of substitution and elasticity of debt to the interest rate marginally lower than the respective priors.

3.7 Results

We now present the impulse responses from the migration shock with brain waste in the non-linear model presented in section 3.5 using the results of the Bayesian estimation in section 3.6. Then we present the model without brain waste that eliminates sector $j = mM$ and the labour market differences between natives and

Table 3.3 Bayesian Estimation: Priors and Posteriors

Parameter Description		Prior	Mean	90% HPD Interval		PDF	PstDev
Autoregressive Parameters							
TFP	ρ^a	0.7000	0.8092	0.7039	0.9192	β	0.1000
Migration	ρ^m	0.7000	0.5719	0.5067	0.6370	β	0.0500
Foreign Output	ρ^{y^f}	0.7000	0.7024	0.5493	0.8665	β	0.1000
Consumption Preference	ρ^{ψ^c}	0.7000	0.8934	0.8348	0.9571	β	0.1000
Investment	ρ^x	0.7000	0.4065	0.3569	0.4474	β	0.0500
Government Spending	ρ^g	0.7000	0.6081	0.4497	0.7705	β	0.1000
Model Parameters							
Adjustment Costs	ϕ_k	4.0000	4.1834	3.5959	4.7936	Γ	0.5000
Elasticity of Substitution	Θ	2.5000	2.2659	1.5677	2.9445	Γ	0.5000
Bond Holding Constraint	ϕ_{bhc}	0.0500	0.0354	0.0196	0.0508	β	0.0400
Standard Deviations							
TFP	σ_a	1.0000	0.2284	0.2261	0.2315	Γ^{-1}	0.5000
Migration	σ_m	1.0000	0.2465	0.2261	0.2635	Γ^{-1}	0.5000
Foreign Output	σ_{y^f}	1.0000	0.2283	0.2261	0.2313	Γ^{-1}	0.5000
Consumption Preference	σ_{ψ^c}	1.0000	0.3889	0.2949	0.4905	Γ^{-1}	0.5000
Investment	σ_x	1.0000	1.0933	0.9749	1.2097	Γ^{-1}	0.5000
Government Spending	σ_{x^g}	1.0000	0.2284	0.2261	0.2315	Γ^{-1}	0.5000

Results from the Bayesian estimation of model parameters after 1,500,000 iterations for the baseline model with brain waste. Probability Distribution Functions: β Beta, Γ Gamma, and Γ^{-1} Inverse Gamma.

migrants. Finally, we calculate the potential gains that could be achieved without brain waste.

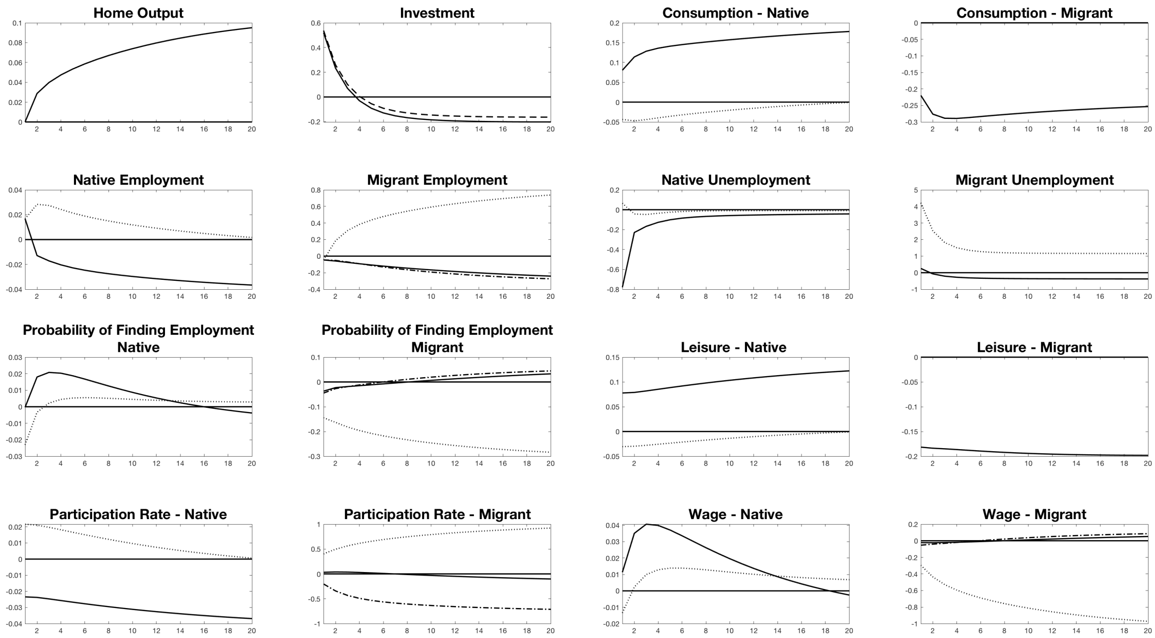
3.7.1 A Small Open Economy with Brain Waste

First, we present the model with brain waste in two scenarios, one with permanent changes to the relative size of the steady state household, and second a return to steady state of the relative household size.

Brain Waste with Permanent Changes to the Relative Household Size

The first model we present is the baseline case where brain waste is present and there is a permanent change to the household sizes of natives and migrants. There is no transfer between the migrant household to a native household. Hence the native households are relatively smaller following the migration shock. The impulse responses are shown in figure 3.4. The shock is to the growth rate of the high-skill migrants experiencing high levels of brain waste, mM. The migrant household that is high-skill and subject to high levels of brain waste is large however, they can search for employment in different sectors.

Fig. 3.4 A migration shock to an economy with brain waste without household transfer



Impulse responses to a positive one standard deviation shock to high-skill migration, that permanently changes the relative household size φ^j - there is no household transfer. For investment the solid line shows the economy aggregate, with the dashed line the investment by the high-skilled household. Subplots 3-16 show response to seven variables. In these subplots, the solid line represents the high-skill employment in the native household and mH employment in the migrant household. The dashed line represents the low-skill employment in the native household and mL in the migrant household. The dash-dotted line represents the migrants experiencing high levels of brain waste, mM. The impulse responses for employment and unemployment are for the household, n_t^j, u_t^j, with responses for aggregate, N_t^j, U_t^j are shown in in figure B.1 along with further impulse responses.

A migration shock increases the size of the population that results in a decreased the value of per capita carried over stocks including bonds and capital ceteris paribus. The decrease in aggregate capital stocks is in $K_t = k_t\varphi_t^H$, which the migration shock decreases φ_t^H on impact. The increase in population decreases the value of per capita lump-sum taxes. The decrease in capital stocks creates a higher return which stimulates investment from the high-skilled household. However, the increased investment does not return capital stocks to steady state levels quickly. High-skilled native households receive a large gain in income that they increase their consumption levels. This consumption increase outweighs the small decreases experienced by the low-skilled and migrant households to raise aggregate consumption. The relatively large increase in investment and aggregate consumption raises the demand side of the economy. The increase in firm output is due to higher productivity of all types of employment except the migrant low-skill. However, this is due to the significant increase in aggregate (N_t^{mL}) low-skill supply of labour.

The effect on labour market status are skill specific. For high-skilled natives, there is an initial increase in employment, a larger continuous decrease in unemployment, and increase in leisure. An increase in leisure is due to the wealth effect. The decrease in unemployment is initially relatively large compared to the increase in employment but quickly returns to small level. The increase in those enjoying leisure and decrease in unemployment results in a decrease in the participation and unemployment rates. There is a tightening of the labour market due to the increased probability of finding employment relative to the decrease of the firm successfully filling a vacancy. The tightening of the labour market creates upwards pressure on the wage level. The increase in leisure, and consumption results in an increase in utility for the high-skill native household. The decrease in per agent bonds for the native households are unaffected without household transfer. In contrast, the migrant households decrease the average holdings ceteris paribus.

For low-skilled natives, their effects occasionally contrast to high-skill natives. This is somewhat expected since the low-skill labour market is only indirectly affected by high-skill migration through skill-downgrading. Household labour, n_t^L increases, however, due to the decrease in relative household size, φ^L, aggregate labour decreases. Unemployment experiences a one period increase before some agents change to enjoy leisure. The increase in wage above steady state is not as great compared to high-skill natives which partially explains the dominance of the substitution effect. The low-skill labour market initially decreases before tightening which is explained by the one period increase in unemployment. There is a positive effect since there is a higher probability of successfully finding employment and higher wage. The overall decrease in agents enjoying leisure increases the participation rate. The change in consumption is small since the increases in employment and wage, less the reduction in unemployment insurance are of similar magnitudes. There is a very small increase in the probability of finding employment which comes from the decrease in unemployment. It is notable that all of these responses are statistically significant but would make little difference as the magnitudes are small. Both native households experience increases in labour income of a magnitude similar to other labour market responses.

Without household transfer, equivalently a very strict labour market experience for migrants, the labour market struggles to an extent as migrants are unable to fill gaps in the labour market left by high-skill natives.

For the migrants, the type of skill is even more divergent. This increase in high-skill migration reduces the number of agents employed in either of the high-skill sectors whereas, skill-downgrading leads to an increase in employment in the low-skill sector. One reason for the increase in unemployment as a household is the reduction in agents enjoying leisure, equivalently an increase in the overall participation rate of the migrants. Participation rates increase for all but the high-skilled migrant workers experiencing high levels of brain waste. It would be expected that the participation rates of new migrants would be high as they enter either unemployed or employed. The participation rate for high-skilled migrants experiencing low levels of brain waste eventually decreases, however the magnitude of response for this sector is significantly smaller than either of the other markets specific to migrants. The initial increase is due to some of the recently landed migrants switching their search to $j = mH$. The increase in the low-skilled participation rate is largely due to the higher number of unemployed agents searching in the low-skilled labour market but also the increase in low-skilled migrant employment. The increase of search in the low-skill labour market is a result of unsuccessful searchers in the high-skill, and lack of unemployment insurance causes the skill-downgrading. All of the migrant labour markets loosen on impact due to the increased number of unemployed agents relative to the vacancies posted. A decrease in labour market tightness puts a downward pressure on wages which is particularly true for the low-skilled migrant labour market. The eventual tightening of both high-skill sectors increases the wages and a result of the increase in marginal product of labour. As there is a reduction in labour income, the wealth effect dominates due to the reduction in leisure. A reduction in labour income, a reduction in taxes, decreased consumption results in an increase in borrowing to smooth consumption.

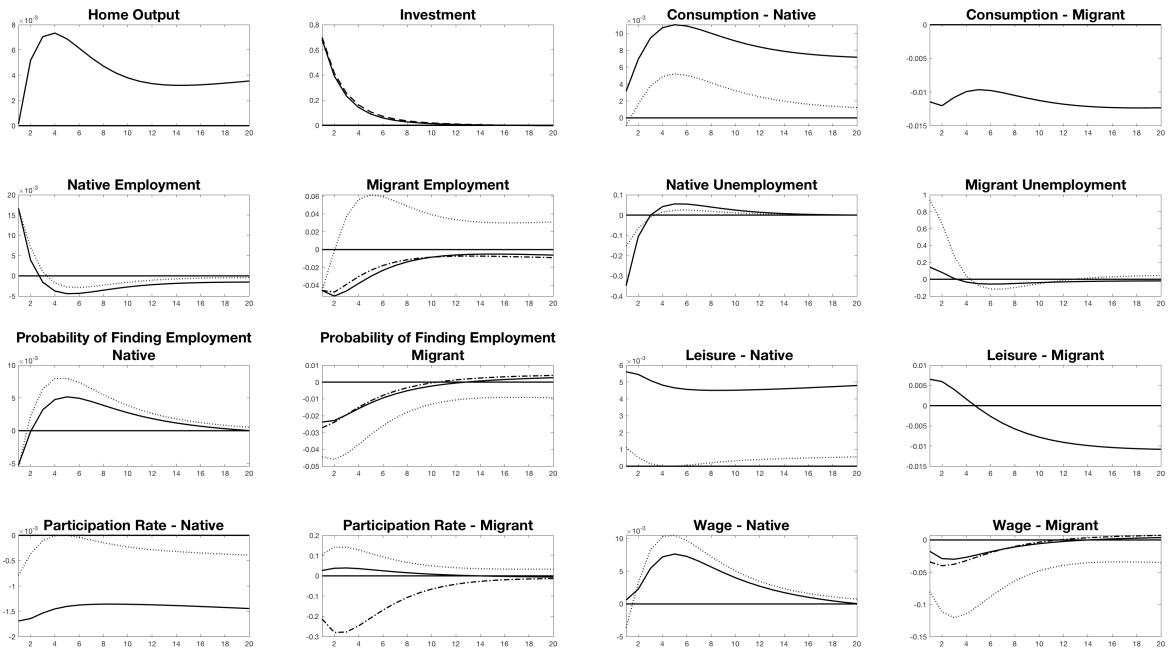
The results presented agree with the existing migration literature that migrants are affected most from an increase in migration and natives only experience small effects. As previously mentioned, the value of per capita taxes fall in this model. However, the only taxation in this model is lump-sum which is paid by the households to finance a constant government spending (in absence of a shock) and unemployment insurance to the households. Unemployment in the native household decreases who receive higher levels of the insurance than migrants, which counteracts the effect of increasing unemployment for migrants who receive lower or no unemployment insurance.

The responses on GDP, investment, and labour supply (hours) follow the SVAR. Aggregate private consumption increases in this model but only due to the high-skill native household. However, the model specification in the form of household budget constraints may explain this.

Brain Waste with Household Transfer

Using the same model, we now show the effect of brain waste when the relative household size returns to its steady state value.[28] This involves a form of household transfer, where some of the longest landed migrants acquire native characteristics to become either high or low-skilled natives respectively. Further, the high-level of brain waste migrants move to experiencing lower levels of brain waste, and some new migrants become low-skill migrants as they accept employment in a lower skilled sector. Household transfer is a way to eliminate brain waste for a small number of high and low-skill migrants by becoming classified as natives. The responses are shown in figure 3.5.

Fig. 3.5 A migration shock to an economy with brain waste with household transfer



Impulse responses to a positive one standard deviation shock to high-skill migration. All household sizes relative to the population, φ^j return to their steady state values - there is household transfer as the longest landed migrants transfer to the equivalent native household. For investment the solid line shows the economy aggregate, with the dashed line the investment by the high-skilled household. Subplots 3-16 show response to seven variables. In these subplots, the solid line represents the high-skill employment in the native household and mH employment in the migrant household. The dashed line represents the low-skill employment in the native household and mL in the migrant household. The dash-dotted line represents the migrants experiencing high levels of brain waste, mM. The impulse responses for employment and unemployment are for the household, n_t^j, u_t^j, with responses for aggregate, N_t^j, U_t^j are shown in figure B.2 along with further impulse responses.

Firstly, we consider output which has a smaller positive response and begins to return to steady state with the horizon presented (20 quarters, or 5 years). The

[28]Using equation 3.5.52, $\rho^{\varphi} < 1$.

response for investment on impact is larger and does not fall below steady state. Due to this, the impact on capital stocks is significantly smaller. The aggregate capital stocks were affected by the total population growth rate.

High-skilled natives continue to experience an increase in their wage level, however, it is smaller. This is due to the number of factors: less of an increase in the marginal product of labour, a smaller decrease in employment (even though the initial increase to employment is of the same magnitude), aggregate employment is converging to its steady state value, the initial decrease in unemployment is approximately half, a smaller increase in the number of agents enjoying leisure, and the labour market tightens slightly more. The magnitude of these effects are so small that they make little impact on the macroeconomy. That there are fewer agents enjoying leisure decreases utility of the household, however, the response to consumption is affected as the household increases their investment. Given both of these factors, there is a small increase in utility.

The impact on low-skilled natives' wage is relatively insignificant. As is the case of the high-skill natives, though the increase in marginal product of labour is lower. As before there is an increase in employment in the households, but due to the changing value of φ_t^L, the aggregate value of employment to steady state. The changes to the remainder of the labour market factors more closely matches that of high-skill natives than previously. Notably though, there is an increase, albeit insignificant, to those enjoying leisure which previously experienced a decrease. Similarly, for consumption there is a decrease on impact before increasing on steady state levels but still only a minimal change.

The wage for each of the migrant employment has a more persistent decrease than before, however, the effect on the low-skilled wage for migrants is significantly lower. Since some of the longest landed migrants move to be recognised as high or low-skill natives to fill gaps in those labour markets, the effects on the migrant labour markets is smaller. There are smaller negative impacts on both types of high-skill employment and this time the aggregate levels of employment increase. Importantly, there is a smaller reduction in labour market tightness which has less of a downward pressure on the wage. The increase in low-skill employment is smaller as there is less pressure on this market than before. The relative decrease in supply of workers after the shock, and through dispersion, creates some upward pressure on wages as the labour market begins to tighten. On impact there is a small increase in the number of agents enjoying leisure, which decreases as the longest landed migrants leave the household and some of the migrants experiencing high levels of brain waste switch to enjoy leisure as the substitution effect dominates. Participation rates of the other two migrant employment sectors dominate to reduce the number of migrants enjoying leisure. Their decrease in consumption levels is less than 10% of the previous model due to increases in labour income. The household still experiences a decrease in utility, however, it is relatively insignificant compared to the previous response which was considerable affected by the increase in searching for employment in the low-skilled labour market which has the highest disutility.

The decrease in the households' lump-sum taxation on impact is of the same magnitude as in the previous model. However, this value returns to steady state levels within the presented horizon. All forms of financial market interaction are smaller such that there is a decrease in overall interaction with the financial markets.

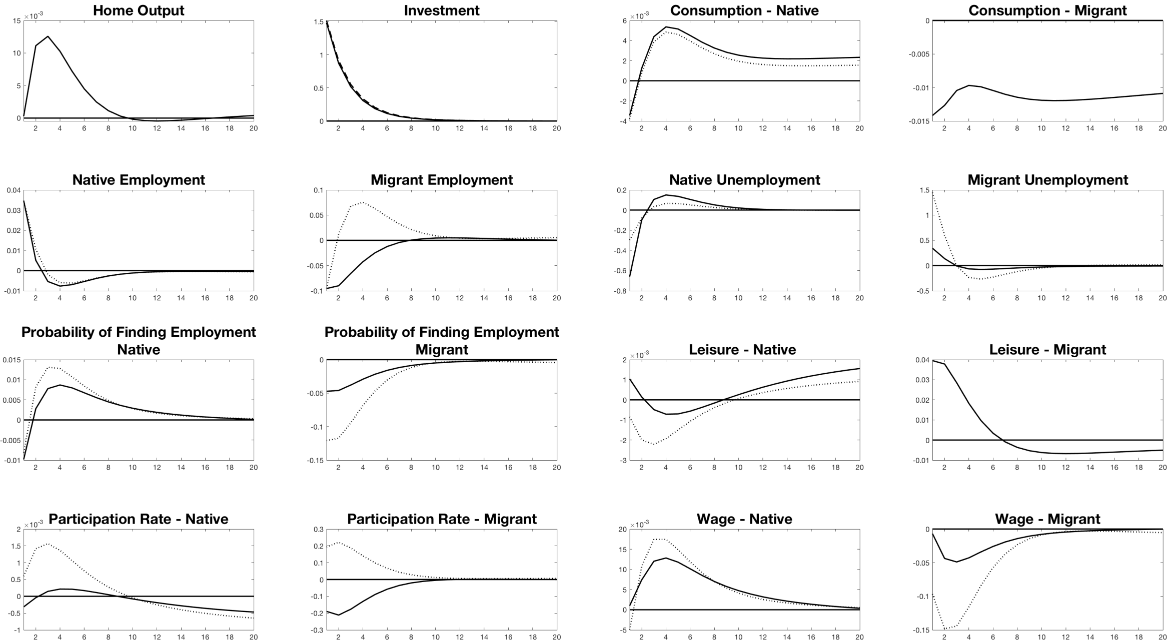
3.7.2 A Small Open Economy without Brain Waste

Having examined the case with brain waste, we now present analysis of a model where brain waste is absent. The results from eliminating brain waste in the labour market shown in figure 3.6. As there is only one high-skill employment option for migrants the shock is for the growth rate of high-skilled migrants g_t^{mH}. This begins with setting the labour market status, and probabilities equal between natives and migrants. Additionally, there is no longer a wage gap. As there is no brain waste, the labour market mM has been eliminated such that the remaining markets $j \in H, L, mH, mL$. For migrant households, they can only search for employment in two markets, therefore $k \in H, L$. We do not combine the households so that there would only be a high-skill and a low-skill as there is not a shock to the native household population size, and hence we can compare the effects on migrant households with other literature.

The response of output in figure 3.6 is greater than in figure 3.5 which has households returning to their steady state size. It is noteworthy that the steady state value of output is higher in the economy without brain waste in the labour market. There is a very small positive response to per capita output due to the increased productivity of native workers and increased low-skill labour. From the demand side, there is a large increase in investment due to the increased return on capital and a small decrease in aggregate consumption. There is an insignificant response to taxation which is higher in steady state as the increase in government spending is equivalent to the increase in output and there is now equivalent unemployment insurance for migrants at each skill level.

For high-skill natives, there is a small increase in employment to start with as the wage rate increases. However, then it does then fall slightly below steady state. This is due to the wealth effect, since wages have increased a number of high-skilled agents switch to enjoying leisure before returning to steady state. For the same

Fig. 3.6 A migration shock to an economy without brain waste



Impulse responses to a positive one standard deviation shock to high-skill migration. All household sizes relative to the population, φ^j return to their steady state values - there is household transfer as the longest landed migrants transfer to the equivalent native household. For investment the solid line shows the economy aggregate, with the dashed line the investment by the high-skilled household. Subplots 3-16 show response to seven variables. In these subplots, the solid line represents the high-skill employment in the native household and mH employment in the migrant household. The dashed line represents the low-skill employment in the native household and mL in the migrant household. The impulse responses for employment and unemployment are for the household, n_t^j, u_t^j, with responses for aggregate, N_t^j, U_t^j are shown in figure B.3 along with further impulse responses.

reason, there is a significant fall in the unemployed workers. As more are enjoying leisure, there is an initial fall in the participation rate and unemployment. Due to the increase in wage and employment levels, there is an increase in labour income.

The case for low-skill workers is similar in terms of wage and employment. However, their decrease in unemployment is smaller due to fewer agents switching to leisure as there is a slight tightening of the labour market. The participation rate decreases initially, as there are more agents enjoying leisure. Due to the increase in income, there is a higher level of consumption. For both high and low-skill natives, the effects are quite small that they are of little significance.

There now exists only two employment types for migrants, high and low-skill. The migrant employment in the high-skill sector decreases while the low-skill employment increases shortly after the initial decrease. The aggregate values both increase. Both types of unemployment increase, with unemployment of low-skill migrants experiencing the greatest increase. This is due to a lower probability of finding employment and increase in participation rate as unsuccessful searchers in the high-skill labour market switch to the low-skill labour market. Whereas the change in probability of finding high-skilled employment is relatively insignificant. There is a small increase in the number of migrants enjoying leisure. The wage level has fallen for both types of migrant labour particularly the low-skill migrants.

One reason for an increase in the participation rate of low-skill migrants is the greater decrease in wage, and skill-downgrading of high-skill migrants who would rather be employed in a low-skill job than unemployed or enjoying leisure. While the effect on the unemployment variables are greater than in figure 3.5, this is due to the reduction in sector of employment. The values return to steady state quicker without brain waste. The steady state analysis shows that there is a higher level of wages and consumption, higher participation rate, lower unemployment rate, higher employment, and probability of finding employment. Therefore, the negative responses are somewhat misleading. As the steady state analysis in table 3.4 shows, migrants are significantly better off than before.

The response of wages and (un)employment to the natives and migrants agrees with the general literature that they are only affected by those directly competing for that type of employment. The effects for natives are positive and often longer lasting than in the case of brain waste. As they make up the majority of the population that is important for an economy. The steady state values without brain waste have economic gains for both migrants and natives, which has positive overall effects for the economy.

From the results where household sizes return to steady state values, there are greater positive responses in the case without brain waste in addition to having higher steady state values.

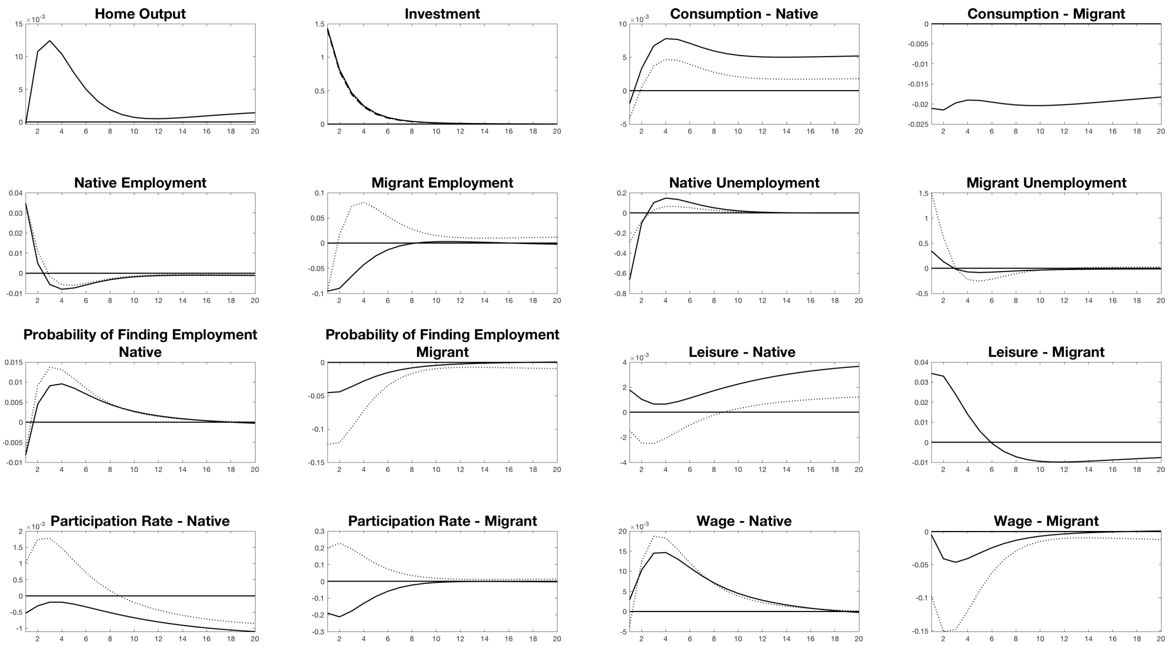
3.7.3 A Small Open Economy with Integration of Migrants

The analysis without brain waste in section 3.7.2 showed the responses to a migration shock if the steady state labour market differences between natives and migrants had been eliminated. However, the elimination of brain waste does not fully recognise the potential contribution of high-skill migrants. In the baseline model, native high-skill agents differ from native low-skill agents in their firm ownership, yet high-skill migrants do not differ from low-skill migrants. As a small extension, we introduce the ability for high-skill migrants to invest in the firms.[29] We introduce investment and capital stocks for the migrant household, therefore aggregate investment and

[29] Further description of this model extension is available in section B.4.

aggregate capital are modified to be $X_t = x_t\varphi_t^H + x_t^M\varphi_t^M$ and $K_t = k_t\varphi_t^H + k_t^M\varphi_t^M$. Figure 3.7 shows the impulse responses.

Fig. 3.7 A migration shock to an economy without brain waste with integration of migrants



Impulse responses to a positive one standard deviation shock to high-skill migration. All household sizes relative to the population, φ^j return to their steady state values - there is household transfer as the longest landed migrants transfer to the equivalent native household. For investment the solid line shows the economy aggregate, with the dashed line the investment by the high-skilled household. Subplots 3-16 show response to seven variables. In these subplots, the solid line represents the high-skill employment in the native household and mH employment in the migrant household. The dashed line represents the low-skill employment in the native household and mL in the migrant household. The impulse responses for employment and unemployment are for the household, n_t^j, u_t^j, with responses for aggregate, N_t^j, U_t^j are shown in in figure B.4 along with further impulse responses.

The results are similar to those as shown in the case where labour market brain waste has been eliminated, however, they further enhance the positive responses to the economy. There is an insignificant change the response in output, however aggregate capital increases initially and even though there is a small decrease below steady state levels, it is smaller than in the model that eliminates brain waste in the labour market. The results for the labour market are a slightly bigger response for both aggregate labour provided to the firm, L_t^H and L_t^L respectively. More capital allows for greater production with the same amount of labour. There are marginally higher increases in labour income for both the native households, and smaller decreases for the migrant household. More agents from the high-skilled household enjoy leisure and higher consumption as there are other agents who invest in the firms to maintain capital levels. Following the migration shock, a marginally lower return on capital then slows the amount of consumption exchanged for investment. Aggregate consumption increases in this model.

One distinct change between the two models without brain waste is the effects on capital stocks. When migrants invest in capital, after a migration shock, aggregate capital stocks increase. Once the effect of capital stock deflation due to the increase population size, the decrease of the aggregate capital is much smaller. In this model of capital-skill complementarity this results in a positive response to output with small effects on the labour market.

3.7.4 Gains from Eliminating Brain Waste

These models presented have shown that there are significant gains to be had in the case of full skill utilisation. To analyse the economic issues arising from brain waste, we present potential gains to the economy if brain waste was eliminated in table 3.4 using the steady states of the models with and without brain waste.

Table 3.4 Potential percentage gains to the economy from elimination of brain waste

Variable		Gain
Output	y_t	2.38
Total Consumption	C_t	4.48
Migrant Household Consumption	c_t^M	33.55
Labour Income for Migrant Household		27.42
Unemployment Migrant	u_t^M	-0.30
Total Migrant Employment	$1 - u_t^M - l_t^M$	11.13
Participation Rate High Skill Migrant	$\text{partic}_t^{\text{mH}}$	12.37
Participation Rate Low Skill Migrant	$\text{partic}_t^{\text{mL}}$	19.78
High-skill Labour	L_t^H	2.07
Low-skill Labour	L_t^L	3.68
Capital, Investment	K_t, X_t	-7.87

Steady state calculations between the models with and without brain waste. The gain of variable s, $\frac{s_{nobw}-s_{bw}}{s_{bw}}100$. The percentage change between the model without brain waste, and the model with brain waste.

Firstly, we consider output that based on the calculations, a 2.38% gain is small but statistically significant to the economy. The model has used capital-skill complementarity to assess the effects of high-skill migration. The increased labour supply reduces the requirement for as high of a level of capital to have the same steady state return, $r^k = \frac{1-\beta}{\beta} + \delta$, which explains the fall in capital and investment. This, along with lower vacancy posting costs allows higher consumption levels.

Low-skill natives marginally lose out when brain waste is eliminated, as their wages are lower using this model. This is due to the lower productivity of low-skill workers. Low-skill migrants gain as the lower steady state wage for low-skill natives in the model without brain waste, is higher than the wage differential caused by brain waste. High-skill natives experience a higher steady state wage and consumption levels.

Migrants gain utility from their increase in consumption by one third. The wage and employment rate equality with natives generates a higher labour income. Additionally, the migrant household now has equal unemployment insurance that marginally increases their income. The decrease in migrants enjoying leisure is due to the higher participation rates.[30] Of those that were previously enjoying leisure, the majority of them have become employed. This change is demonstrated by the very small decrease in unemployment and large increase in total employment.

From the results presented in this section we conclude that migration shocks do not have large effects on the economy. However, eliminating brain waste offers significant gains to the economy, and particularly the migrant household.

3.8 Conclusion

The increasing levels of immigration, the contrasting views of natives, and the bearing it has within society posed a number of questions that related to macroeconomic variations in the countries that host immigrants, and what migration means for natives. All areas of society are, in one way or another, affected by migration. Immigration affects households in terms of consumption, income, investment, and labour supplied. Firms are affected as an increase in output and labour supply as well as changing labour costs. Governments are affected by tax receipts and expenditure. The prominence of brain waste is negative for each sector of the economy, and elimination of it offers statistically significant gains.

This paper has shown the potential gains from eliminating brain waste in the labour market. However, we have not completely allowed the economy to experience gains should the high-skill migrants have the financial ability to match their native counterparts in investing in firms and wider participation. The migrant household consists of high and low-skill workers which have a balance of preferences as better demonstrated in labour preferences - low-skill workers place a higher weight on leisure, and more likely consume rather than invest. We included a basic extension to allow the migrant household to invest in physical capital, however, there is potential

[30] In the figures, the participation rate of $j = mM$ was higher than high-skill natives, but $j = mH$ was less than high-skill natives. The weighted average was less than natives.

to develop this further. One possibility is to separate the migrant household into a high and low-skill which would allow a high-skill migrant household to replicate investment decisions made by the high-skill natives.

The Canadian government has recognised that brain waste is an issue and has taken steps to eliminate it, which the government spending in the model does not reflect. This is an opportunity for future research. One issue with the current model is the number of agents entering as unemployed which is not reflective of the current statistics, further examination in methods of modelling this is required to match more immigration systems. The results from this paper support the policy of helping migrants gain recognition, and have equivalent standing to natives in the labour market.

Our paper has contributed to the literature using an estimated small open economy DSGE model, calibrated with recent data to present the effects of a migration shock with and without brain waste. The results presented in section 3.4 showed the effects of immigration on the macroeconomy and the first paper to use a SVAR with migration data from Canada. Section 3.7 highlighted the issues arising from migration and brain waste. The DSGE model allowed the illustration of brain waste and the different scenarios show the effects to the economy with and without brain waste. There were positive responses to output in the models with and without brain waste that matches with the SVAR. There were no long-run negative effects for the total economy and high-skill natives under the assumption of imperfect substitutes. The low-skill native household had a small loss, however, this could be due to model specification. While it is impossible for an economy to be perfectly responsive to the correct level of skilled immigration, our analysis shows there is a loss from the brain waste of migrant workers. The calculations in steady state have shown significant potential gains to the economy when migrants experience the same labour market conditions to natives. Overall, we conclude that migration has a positive effect on the macroeconomy and there are significant gains to be made when skills of migrants are fully recognised. The SVAR results show that when the responses are statistically significant, migration is beneficial to an economy. The results are given in per capita terms, the distinction between per capita and aggregate is important because immigrant workers could be adding to the economy but less than another native.

In this paper, we have modelled migration as exogenous, identifying shocks based on the relaxation of migration constraints. Thus we conclude that migration shocks and brain waste matter for the economy but only contribute a small amount to the business cycle, where the full utilisation and integration of migrants to the economy is beneficial.

Appendix B

B.1 Data Sources

Table B.1 lists the raw data used in analysis and the source.

B.2 Labour Market

Table B.2 shows the relative size and labour market statistics for the model of each qualification level in the 2016 census data. It is notable that the relative size of the households for those with university degrees is significantly higher for migrants than natives.

Table B.1 Data Source

Data	**Source**	**Code, Measure**
National Accounts		
GDP	OECD National Accounts	$B1\_GE$ CARSA, DNBSA
US GDP	OECD National Accounts	$B1\_GE$ - CARSA, DNBSA
Private final consumption expenditure	OECD National Accounts	$P31S14\_S15$ CARSA, DNBSA
General government final consumption expenditure	OECD National Accounts	P31S13 CARSA, DNBSA
Private non-residential gross fixed capital formation	OECD Economic Outlook	
Government gross fixed capital formation	OECD Economic Outlook	
General government net financial liabilities	OECD Economic Outlook	
Labour Market		
Migration	Statistics Canada	Table 17-10-0040-01 (formerly CANSIM 051-0037)
Compensation of Employees	Statistics Canada	Table 36-10-0114-01
Household wage	Statistics Canada	2016 Census 98-400-X2016280
Labour market status	Statistics Canada	2016 Census 98-400-X2016198
Population (15 years and older)	Statistics Canada	Labour force survey

The data sources used in calibration, estimation, and VAR.
CARSA -National currency, current prices, annual levels, seasonally adjusted DNBSA - Deflator, national base year, seasonally adjusted.

Table B.2 Labour Market Status

	Total		**Native**		**Migrant**	
	Relative	Cumulative	Relative	Cumulative	Relative	Cumulative
No certificate, diploma or degree	0.1829	0.1829	0.1878	0.1878	0.1738	0.1738
Secondary school diploma or equivalency certificate	0.2645	0.4474	0.2753	0.4630	0.2325	0.4063
Apprenticeship or trades certificate or diploma	0.0978	0.5452	0.1089	0.5720	0.0691	0.4754
College, CEGEP or other non-university certificate or diploma	0.1939	0.7391	0.2066	0.7786	0.1617	0.6371
University certificate or diploma below bachelor level	0.0284	0.7675	0.0249	0.8035	0.0375	0.6747
University certificate, diploma or degree at bachelor level or above	0.2325	1.0000	0.1965	1.0000	0.3253	1.0000
	Total		**Native**		**Migrant**	
	Partic	Unemp	Partic	Unemp	Partic	Unemp
Total	0.6519	0.0772	0.6639	0.0767	0.6210	0.0768
No certificate, diploma or degree	0.3832	0.1350	0.3978	0.1445	0.3408	0.1004
Secondary school diploma or equivalency certificate	0.6359	0.0946	0.6586	0.0954	0.5741	0.0880
Apprenticeship or trades certificate or diploma	0.7038	0.0784	0.7247	0.0794	0.6065	0.0724
College, CEGEP or other non-university certificate or diploma	0.7428	0.0601	0.7569	0.0579	0.6924	0.0665
University certificate or diploma below bachelor level	0.6789	0.0607	0.6724	0.0540	0.6933	0.0708
University certificate, diploma or degree at bachelor level or above	0.7805	0.0538	0.7929	0.0421	0.7634	0.0712

Relative size and labour market statistics for native and migrants by highest qualification

B.3 Equation Derivation

Firm Surplus

Here we derive equation 3.5.43. From equation 3.5.42 we solve for η^j_{t+1}

$$\eta^j_{t+1} = \frac{\kappa^{v^j}}{\beta a^j \Gamma^j} \frac{\mu_t}{\mu_{t+1}} \left(\frac{v^j_t}{u^j_t} \right)^{1-\Gamma^j}$$

Replace η^j_t and η^j_{t+1} respectively.

$$(1-\rho^j_n) \frac{\kappa^{v^j} \mu_t}{\beta \mu_{t+1} a^j \Gamma^j} \left(\frac{v^j_t}{u^j_t} \right)^{1-\Gamma^j} = w^j_t + \frac{\kappa^{v^j}}{\beta a^j \Gamma^j} \frac{\mu_{t-1}}{\mu_t} \left(\frac{v^j_t}{u^j_t} \right)^{1-\Gamma^j} g^j_t \tag{B.3.1}$$

Population Dynamics

The derivation of the distribution following the migration shock to the relative household size (equation 3.5.52) is as follows

$$\varphi^j_t = \frac{\mathrm{N^j_t}}{\mathrm{N_t}}$$

$$\varphi^j_t = \frac{\mathrm{N^j_t}}{\mathrm{N_t}} \frac{\mathrm{N^j_{t\text{-}1}}}{\mathrm{N^j_{t\text{-}1}}} \frac{\mathrm{N_{t\text{-}1}}}{\mathrm{N_{t\text{-}1}}}$$

$$\varphi^j_t = \frac{\mathrm{N^j_t}}{\mathrm{N^j_{t\text{-}1}}} \frac{\mathrm{N_{t\text{-}1}}}{\mathrm{N_t}} \frac{\mathrm{N^j_{t\text{-}1}}}{\mathrm{N_{t\text{-}1}}}$$

$$\varphi^j_t = g^j_t \frac{1}{g^T_t} \varphi^j_{t-1} = \frac{g^j_t}{g^T_t} \varphi^j_{t-1} \tag{B.3.2}$$

To ensure stationarity in the model, the relative household size returns to steady state, the equation is

$$\varphi^j_t = \left(\frac{g^j_t}{g^T_t} \varphi^j_{t-1} \right)^{\rho^\varphi} \left(\varphi^j \right)^{1-\rho^\varphi}$$

B.4 Model Extension - Full Integration of High-skill Migrants

In section 3.7.3, we presented the results to a model that fully integrates migrants into society. This built on the elimination of brain waste in the labour market as shown in 3.7.2. In the baseline model we assumed that the migrant household as a collective offered no form of firm investment. We eliminated brain waste from the labour market by equating the characteristics of natives to that for the migrants which reflected integration into the labour market. Using this methodology, we enable the migrant household to offer investment to the firms at the same per capita level as the native high-skill household.

Using the no brain waste model, we modify the migrant households' budget constraint to include investment and the return on capital.

$$c_t^M + tax_t^M + p_t^f b_t^M + x_t^M = \\ w_t^{mH} n_t^{mH} + w_t^{mL} n_t^{mL} + r_t p_t^f b_{t-1}^M + ub^{mH} u_t^{mH} + ub^{mL} u_t^{mL} + r_t^k (z_t k_t^M) \quad \text{(B.4.1)}$$

This adds two equations to the model, the first being capital accumulation and the second their choice of capital holdings. For simplicity, we model it in the same way as the native high-skilled household.

$$x_t^M = k_{t+1}^M - \left(1 - \delta z_t^{\phi}\right) k_t^M + \frac{\omega}{2}\left(\frac{k_{t+1}^M}{k_t^M} - 1\right)^2 k_t^M \quad \text{(B.4.2)}$$

$$\mu_t^M \psi_t^X \left[1 + \omega\left(\frac{k_{t+1}^M}{k_t^M} - 1\right)\right] = \beta E_t \mu_{t+1}^M \left\{ r_{t+1}^k z_{t+1} + \psi_{t+1}^X \left(1 - \delta\left(z_{t+1}\right) + \frac{\omega}{2}\left[\left(\frac{k_{t+2}^M}{k_{t+1}^M}\right)^2 - 1\right]\right)\right\} \quad \text{(B.4.3)}$$

Finally we modify the aggregate investment and capital stock.

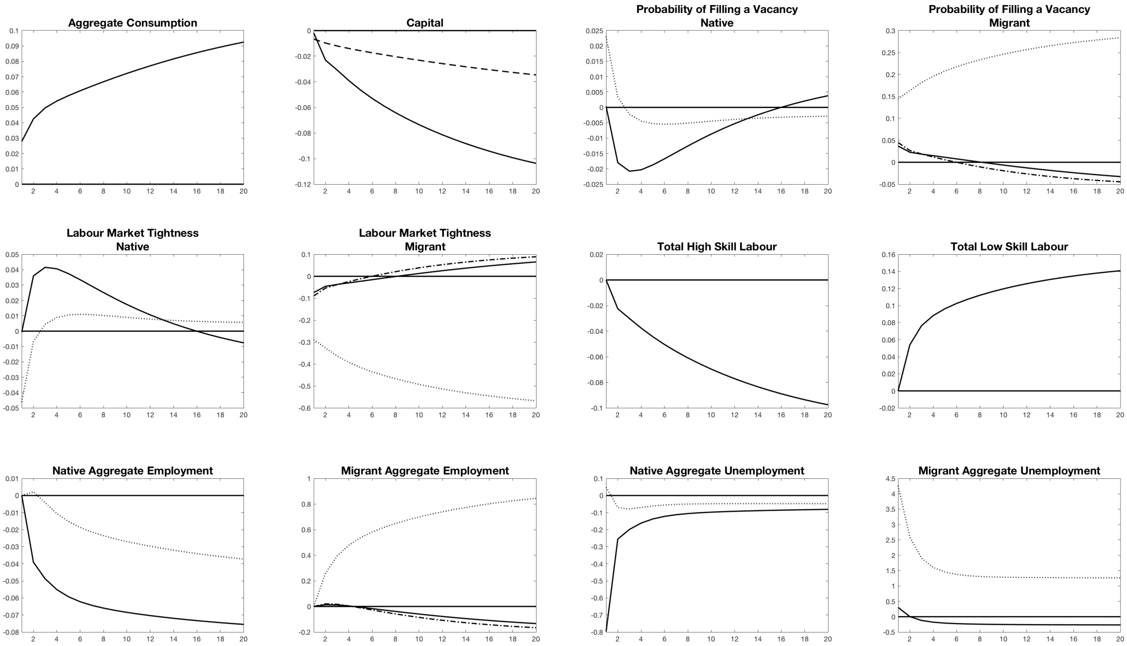
$$X_t = x_t \varphi_t^H + x_t^M \varphi_t^M \quad \text{(B.4.4)}$$

$$K_t = k_t \varphi_t^H + k_t^M \varphi_t^M \quad \text{(B.4.5)}$$

B.5 Additional Impulse Responses

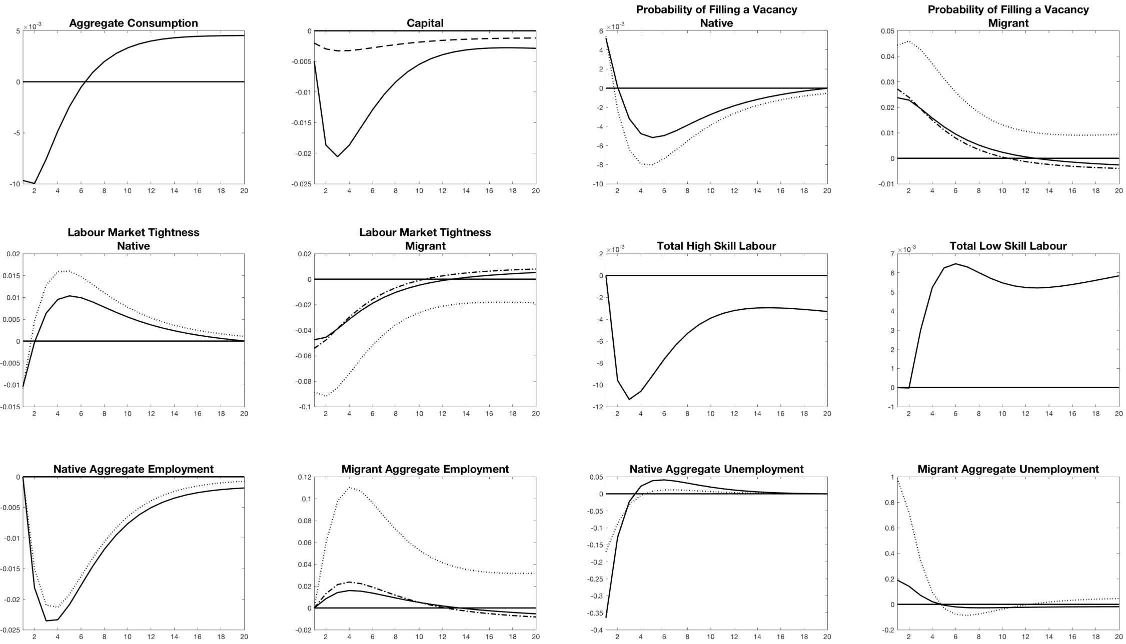
In section 3.7 we presented the impulse responses for household employment and unemployment. As aggregate values are defined as $N_t^j = \varphi_t^j n_t^j$ and φ_t^j is changing, these values differ. Additionally we present the impulse responses for labour market tightness, probability of filling a vacancy, total labour supply to the firms, aggregate consumption, capital stocks.

Fig. B.1 A migration shock to an economy with brain waste



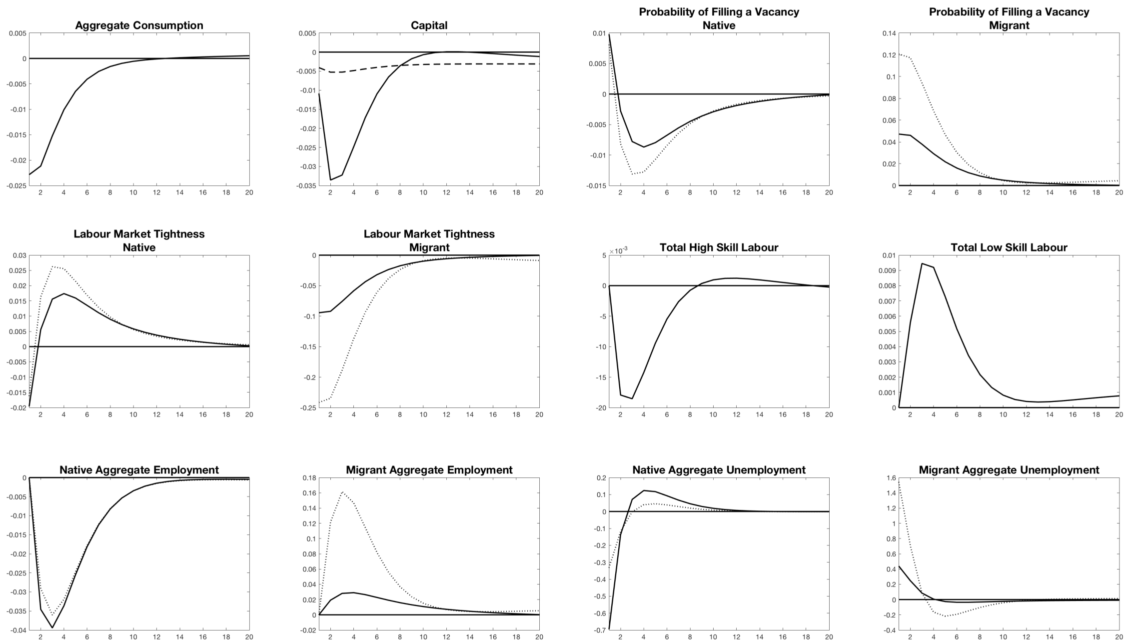
Impulse responses to a positive one standard deviation shock to high-skill migration, that permanently changes the relative household size φ^j - there is no household transfer. For capital the solid line shows the economy aggregate, with the dashed line the capital by the high-skilled household. In labour market subplots, the solid line represents the high-skill employment in the native household and mH employment in the migrant household. The dashed line represents the low-skill employment in the native household and mL in the migrant household. The dash-dotted line represents the migrants experiencing high levels of brain waste, mM. The impulse responses for employment and unemployment are aggregate N_t^j, U_t^j. The responses are a continuation of figure 3.4

Fig. B.2 A migration shock to an economy with brain waste with household transfer



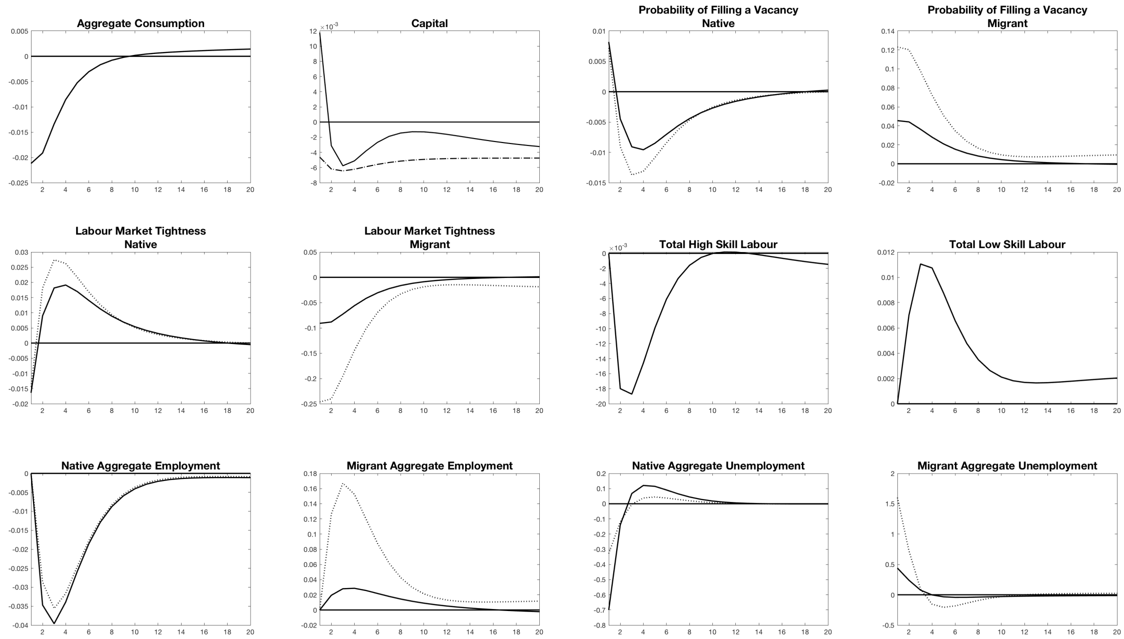
Impulse responses to a positive one standard deviation shock to high-skill migration. All household sizes relative to the population, φ^j return to their steady state values - there is household transfer as the longest landed migrants transfer to the equivalent native household. For capital the solid line shows the economy aggregate, with the dashed line the capital by the high-skilled household. In labour market subplots, the solid line represents the high-skill employment in the native household and mH employment in the migrant household. The dashed line represents the low-skill employment in the native household and mL in the migrant household. The dash-dotted line represents the migrants experiencing high levels of brain waste, mM. The impulse responses for employment and unemployment are aggregate N_t^j, U_t^j. The responses are a continuation of figure 3.5

Fig. B.3 A migration shock to an economy without brain waste



Impulse responses to a positive one standard deviation shock to high-skill migration. All household sizes relative to the population, φ^j return to their steady state values - there is household transfer as the longest landed migrants transfer to the equivalent native household. For capital the solid line shows the economy aggregate, with the dashed line the capital by the high-skilled household. In labour market subplots, the solid line represents the high-skill employment in the native household and mH employment in the migrant household. The dashed line represents the low-skill employment in the native household and mL in the migrant household. The impulse responses for employment and unemployment are aggregate N_t^j, U_t^j. The responses are a continuation of figure 3.6

Fig. B.4 A migration shock to an economy without brain waste with integration of migrants



Impulse responses to a positive one standard deviation shock to high-skill migration. All household sizes relative to the population, φ^j return to their steady state values - there is household transfer as the longest landed migrants transfer to the equivalent native household. For capital the solid line shows the economy aggregate, with the dashed line the capital by the high-skilled household, and dotted line for migrants household. In labour market subplots, the solid line represents the high-skill employment in the native household and mH employment in the migrant household. The dashed line represents the low-skill employment in the native household and mL in the migrant household. The impulse responses for employment and unemployment are aggregate N_t^j, U_t^j. The responses are a continuation of figure 3.7

B.6 Model Equations

The baseline DSGE model includes 133 variables, six of which are exogenous. The measurement equations in estimation are not presented.

Labour Market

$$m_t^H = a^h (v_t^H)^{\Gamma^H} (U_t^H)^{1-\Gamma^H} \tag{B.6.1}$$

$$m_t^L = a^l (v_t^l)^{\Gamma^L} (U_t^L)^{1-\Gamma^L} \tag{B.6.2}$$

$$m_t^M = a^m (v_t^M)^{\Gamma^M} (U_t^{mM})^{1-\Gamma^M} \tag{B.6.3}$$

$$m_t^{mH} = a^{mh} (v_t^{mH})^{\Gamma^H} (U_t^{mH})^{1-\Gamma^H} \tag{B.6.4}$$

$$m_t^{mL} = a^{ml} (v_t^{mL})^{\Gamma^L} (U_t^{mL})^{1-\Gamma^L} \tag{B.6.5}$$

$$\theta_t^H = \frac{v_t^H}{U_t^H} \tag{B.6.6}$$

$$\theta_t^L = \frac{v_t^l}{U_t^L} \tag{B.6.7}$$

$$\theta_t^M = \frac{v_t^M}{U_t^M} \tag{B.6.8}$$

$$\theta_t^{mH} = \frac{v_t^{mH}}{U_t^{mH}} \tag{B.6.9}$$

$$\theta_t^{mL} = \frac{v_t^{mL}}{U_t^{mL}} \tag{B.6.10}$$

$$\Upsilon_t^H = \frac{m_t^H}{v_t^H} \tag{B.6.11}$$

$$\Upsilon_t^L = \frac{m_t^L}{v_t^l} \tag{B.6.12}$$

$$\Upsilon_t^M = \frac{m_t^M}{v_t^M} \tag{B.6.13}$$

$$\Upsilon_t^{mH} = \frac{m_t^{mH}}{v_t^{mH}} \tag{B.6.14}$$

$$\Upsilon_t^{mL} = \frac{m_t^{mL}}{v_t^{mL}} \tag{B.6.15}$$

$$\Omega_t^H = \frac{m_t^H}{U_t^H} \tag{B.6.16}$$

$$\Omega_t^L = \frac{m_t^L}{U_t^L} \tag{B.6.17}$$

$$\Omega_t^M = \frac{m_t^M}{U_t^{mM}} \tag{B.6.18}$$

$$\Omega_t^{mH} = \frac{m_t^{mH}}{U_t^{mH}} \tag{B.6.19}$$

$$\Omega_t^{mL} = \frac{m_t^{mL}}{U_t^{mL}} \tag{B.6.20}$$

$$N_t^H = (1-\rho_n^H)N_{t-1}^H + m_{t-1}^H \tag{B.6.21}$$

$$N_t^L = (1-\rho_n^L)N_{t-1}^L + m_{t-1}^L \tag{B.6.22}$$

$$N_t^{mM} = (1-\rho_n^M)N_{t-1}^{mM} + m_{t-1}^M \tag{B.6.23}$$

$$N_t^{mH} = (1-\rho_n^{mH})N_{t-1}^{mH} + m_{t-1}^{mH} \tag{B.6.24}$$

$$N_t^{mL} = (1-\rho_n^{mL})N_{t-1}^{mL} + m_{t-1}^{mL} \tag{B.6.25}$$

High-Skilled Native Households

$$x_t = k_{t+1} - \left(1-\delta z_t^{\phi}\right)\frac{k_t}{g_{t+1}^H} + \frac{\omega}{2}\left(\frac{k_{t+1}}{k_t}-1\right)^2 k_t \tag{B.6.26}$$

$$\left(c_t^H - \kappa c_{t-1}^H\right)^{-\eta} = \mu_t^H \tag{B.6.27}$$

$$\mu_t^H \psi_t^X\left[1+\omega\left(\frac{k_{t+1}}{k_t}-1\right)\right] = \beta E_t \mu_{t+1}^H\left\{r_{t+1}^k z_{t+1} + \psi_{t+1}^X\left(\frac{(1-\delta(z_{t+1}))}{g_{t+2}^H} + \frac{\omega}{2}\left[\left(\frac{k_{t+2}}{k_{t+1}}\right)^2 - 1\right]\right)\right\} \tag{B.6.28}$$

$$\mu_t^H rer_t = \beta E_t \mu_{t+1}^H r_t rer_{t+1} \tag{B.6.29}$$

$$r_t^k = \psi_t^X \delta'(z_t) = \psi_t^X \delta\phi z_t^{\phi-1} \tag{B.6.30}$$

$$1 = n_t^H + u_t^H + l_t^H \tag{B.6.31}$$

$$\Lambda_t^H = \frac{\Phi^H\left(l_t^H\right)^{-\sigma_l} - ub^H\mu_t^H}{\Omega_t^H} \tag{B.6.32}$$

$$\Lambda_t^H = \beta E_t\left[\mu_{t+1}^M w_{t+1}^H + \left(1-\rho_n^H\right)\Lambda_{t+1}^H - \Phi^H\left(l_{t+1}^H\right)^{-\sigma_l}\right] \tag{B.6.33}$$

Low-Skilled Native Households

$$1 = n_t^L + u_t^L + l_t^L \tag{B.6.34}$$

$$c_t^L + tax_t^L + rer_t b_t^L = w_t^L n_t^L + rer_t r_t b_{t-1}^L + b^L u_t^L \tag{B.6.35}$$

$$\left(c_t^L - \kappa c_{t-1}^L\right)^{-\eta} = \mu_t^L \tag{B.6.36}$$

$$\mu_t^L\left[1+\xi\left(b_t^L - \bar{b}^L\right)\right] rer_t = \beta E_t \mu_{t+1}^L r_t rer_{t+1} \tag{B.6.37}$$

$$\Lambda_t^L = \frac{\Phi^L\left(l_t^L\right)^{-\sigma_l} - b^L\mu_t^L}{\Omega_t^L} \tag{B.6.38}$$

$$\Lambda_t^L = \beta E_t\left[\mu_{t+1}^L w_{t+1}^L + \left(1-\rho_n^L\right)\Lambda_{t+1}^L - \Phi^L\left(l_{t+1}^L\right)^{-\sigma_l}\right] \tag{B.6.39}$$

Migrant Households

$$1 = n_t^M + n_t^{mH} + u_t^M + l_t^M \tag{B.6.40}$$

$$c_t^M + tax_t^M + rer_t b_t^M = w_t^{mH} n_t^{mH} + w_t^M n_t^{mM} + w_t^{mL} n_t^{mL} + r_t rer_t b_{t-1}^M + ub^{mH} u_t^{mH} + ub^{mM} u_t^{mM} + u^{mL} u_t^{mL} \tag{B.6.41}$$

$$\left(c_t^M - \kappa c_{t-1}^M\right)^{-\eta} = \mu_t^M \tag{B.6.42}$$

$$\mu_t^M \left[1 + \xi \left(b_t^M - \bar{b}^H\right)\right] rer_t = \beta E_t \mu_{t+1}^M r_t rer_{t+1} \tag{B.6.43}$$

$$\Lambda_t^{mM} = \frac{\Phi^M \left(l_t^M\right)^{-\sigma_l} - ub^{mM} \mu_t^M}{\Omega_t^{mM}} \tag{B.6.44}$$

$$\Lambda_t^M = \beta E_t \left[\mu_{t+1}^M w_{t+1}^M + \left(1 - \rho_n^M\right) \Lambda_{t+1}^M - \Phi^M \left(l_{t+1}^M\right)^{-\sigma_l}\right] \tag{B.6.45}$$

$$\Lambda_t^{mH} = \beta E_t \left[\mu_{t+1}^M w_{t+1}^{mH} + \left(1 - \rho_n^{mH}\right) \Lambda_{t+1}^{mH} - \Phi^M \left(l_{t+1}^M\right)^{-\sigma_l}\right] \tag{B.6.46}$$

$$\Lambda_t^{mL} = \beta E_t \left[\mu_{t+1}^M w_{t+1}^{mL} + \left(1 - \rho_n^{mL}\right) \Lambda_{t+1}^{mL} - \Phi^M \left(l_{t+1}^M\right)^{-\sigma_l}\right] \tag{B.6.47}$$

$$\Phi^H (s_t^H)^{-\sigma_s} = \mu_t^M U_t^M ub^{mH} + \Lambda_t^{mH} \Omega_t^{mH} U_t^M \tag{B.6.48}$$

$$\Phi^M (s_t^M)^{-\sigma_s} = \mu_t^M U_t^M ub^{mM} + \Lambda_t^{mM} \Omega_t^{mM} U_t^M \tag{B.6.49}$$

$$1 = s_t^H + s_t^M + s_t^L \tag{B.6.50}$$

Firm

$$r_t^k = p_t^h mpk_t \tag{B.6.51}$$

$$\frac{\kappa^H}{\Upsilon_t^H} = \beta E_t \frac{\mu_{t+1}^H}{\mu_t^H} \left[p_{t+1}^h mpl_{t+1}^H - w_{t+1}^H + \left(1 - \rho_n^H\right) \frac{\kappa^H}{\Upsilon_{t+1}^H}\right] \tag{B.6.52}$$

$$\frac{\kappa^M}{\Upsilon_t^M} = \beta E_t \frac{\mu_{t+1}^H}{\mu_t^H} \left[p_{t+1}^h mpl_{t+1}^H - w_{t+1}^M + \left(1 - \rho_n^M\right) \frac{\kappa^M}{\Upsilon_{t+1}^M}\right] \tag{B.6.53}$$

$$\frac{\kappa^L}{\Upsilon_t^L} = \beta E_t \frac{\mu_{t+1}^H}{\mu_t^H} \left[p_{t+1}^h mpl_{t+1}^L - w_{t+1}^L + \left(1 - \rho_n^L\right) \frac{\kappa^L}{\Upsilon_{t+1}^L}\right] \tag{B.6.54}$$

$$\frac{\kappa^{mH}}{\Upsilon_t^{mH}} = \beta E_t \frac{\mu_{t+1}^H}{\mu_t^H} \left[p_{t+1}^h mpl_{t+1}^{mH} - w_{t+1}^{mH} + \left(1 - \rho_n^{mH}\right) \frac{\kappa^{mH}}{\Upsilon_{t+1}^{mH}}\right] \tag{B.6.55}$$

$$\frac{\kappa^{mL}}{\Upsilon_t^{mL}} = \beta E_t \frac{\mu_{t+1}^H}{\mu_t^H} \left[p_{t+1}^h mpl_{t+1}^{mL} - w_{t+1}^{mL} + \left(1 - \rho_n^{mL}\right) \frac{\kappa^{mL}}{\Upsilon_{t+1}^{mL}}\right] \tag{B.6.56}$$

$$y_t = \psi_t^a \left[(\alpha^s)^{\frac{1}{\alpha}} \left[\left((\alpha^k)^{\frac{1}{\alpha^H}} K_t^{\frac{\alpha^H - 1}{\alpha^H}} + (1 - \alpha^k)^{\frac{1}{\alpha^H}} (\xi L_t^H)^{\frac{\alpha^H - 1}{\alpha^H}}\right)^{\frac{\alpha^H}{\alpha^H - 1}}\right]^{\frac{\alpha - 1}{\alpha}} + (1 - \alpha^s)^{\frac{1}{\alpha}} (L_t^L)^{\frac{\alpha - 1}{\alpha}}\right]^{\frac{\alpha}{\alpha - 1}} \tag{B.6.57}$$

$$L_t^H = \left[\chi_h^{\frac{1}{\theta^h}} (N_t^H)^{\frac{\theta^h - 1}{\theta^h}} + \chi_m^{\frac{1}{\theta^h}} (N_t^{mH})^{\frac{\theta^h - 1}{\theta^h}} + (1 - \chi_h - \chi_m)^{\frac{1}{\theta^h}} (N_t^{mM})^{\frac{\theta^h - 1}{\theta^h}}\right]^{\frac{\theta^h}{\theta^h - 1}} \tag{B.6.58}$$

$$L_t^L = \left[\chi_l^{\frac{1}{\theta^h}} (N_t^l)^{\frac{\theta^h - 1}{\theta^h}} + (1 - \chi_l)^{\frac{1}{\theta^h}} (N_t^{mL})^{\frac{\theta^h - 1}{\theta^h}}\right]^{\frac{\theta^h}{\theta^h - 1}} \tag{B.6.59}$$

$$f(K,L^H)_t = (\alpha^k)^{\frac{1}{\alpha^H}} K_{t-1}^{\frac{\alpha^H-1}{\alpha^H}} + (1-\alpha^k)^{\frac{1}{\alpha^H}} (\xi L_t^H)^{\frac{\alpha^H-1}{\alpha^H}}$$

$$mpl_t^H = \chi_H^{\frac{1}{\theta^h}} (\alpha^s)^{\frac{1}{\alpha}} (1-\alpha^k) \xi^{\frac{\alpha^H-1}{\alpha^H}} \psi_t^A \left[f(K,L^H)_t^{\frac{\alpha^H(\alpha-1)}{(\alpha^H-1)\alpha}} + (1-\alpha^s)^{\frac{1}{\alpha}} (L_t^L)^{\frac{\alpha-1}{\alpha}} \right]^{\frac{\alpha}{\alpha-1}-1} (f(K,L^H)_t)^{\frac{\alpha^H(\alpha-1)}{(\alpha^H-1)\alpha}-1} (L_t^H)^{\frac{\alpha^H-1}{\alpha^H}-1} (N_t^H)^{\frac{-1}{\theta_h}} \quad \text{(B.6.60)}$$

$$mpl_t^{mH} = \chi_M^{\frac{1}{\theta^h}} (\alpha^s)^{\frac{1}{\alpha}} (1-\alpha^k) xi^{\frac{\alpha^H-1}{\alpha^H}} \psi_t^A \left[f(K,L^H)_t^{\frac{\alpha^H(\alpha-1)}{(\alpha^H-1)\alpha}} + (1-\alpha^s)^{\frac{1}{\alpha}} (L_t^L)^{\frac{\alpha-1}{\alpha}} \right]^{\frac{\alpha}{\alpha-1}-1} (f(K,L^H)_t)^{\frac{\alpha^H(\alpha-1)}{(\alpha^H-1)\alpha}-1} (L_t^H)^{\frac{\alpha^H-1}{\alpha^H}-1} (N_t^{mH})^{\frac{-1}{\theta_h}} \quad \text{(B.6.61)}$$

$$mpl_t^{mM} = (1-\chi_H-\chi_M)^{\frac{1}{\theta^h}} (\alpha^s)^{\frac{1}{\alpha}} (1-\alpha^k) xi^{\frac{\alpha^H-1}{\alpha^H}} \psi_t^A \left[f(K,L^H)_t^{\frac{\alpha^H(\alpha-1)}{(\alpha^H-1)\alpha}} + (1-\alpha^s)^{\frac{1}{\alpha}} (L_t^L)^{\frac{\alpha-1}{\alpha}} \right]^{\frac{\alpha}{\alpha-1}-1} (f(K,L^H)_t)^{\frac{\alpha^H(\alpha-1)}{(\alpha^H-1)\alpha}-1} (L_t^H)^{\frac{\alpha^H-1}{\alpha^H}-1} (N_t^{mM})^{\frac{-1}{\theta_h}} \quad \text{(B.6.62)}$$

$$mpl_t^L = \chi_L^{\frac{1}{\theta^h}} (1-\alpha^s)^{\frac{1}{\alpha}} \psi_t^A \left[f(K,L^H)_t^{\frac{\alpha^H(\alpha-1)}{(\alpha^H-1)\alpha}} + (1-\alpha^s)^{\frac{1}{\alpha}} (L_t^L)^{\frac{\alpha-1}{\alpha}} \right]^{\frac{\alpha}{\alpha-1}-1} (L_t^L)^{\frac{\alpha-1}{\alpha}-1} (N_t^L)^{\frac{-1}{\theta_h}} \quad \text{(B.6.63)}$$

$$mpl^{mL} = (1-\chi_L)^{\frac{1}{\theta^h}} (1-\alpha^s)^{\frac{1}{\alpha}} \psi_t^A \left[f(K,L^H)_t^{\frac{\alpha^H(\alpha-1)}{(\alpha^H-1)\alpha}} + (1-\alpha^s)^{\frac{1}{\alpha}} (L_t^L)^{\frac{\alpha-1}{\alpha}} \right]^{\frac{\alpha}{\alpha-1}-1} (L_t^L)^{\frac{\alpha-1}{\alpha}-1} (N_t^{mL})^{\frac{-1}{\theta_h}} \quad \text{(B.6.64)}$$

$$mpk_t = (\alpha^s)^{\frac{1}{\alpha}} (\alpha^k) \psi_t^A \left[f(K,L^H)_t^{\frac{\alpha^H(\alpha-1)}{(\alpha^H-1)\alpha}} + (1-\alpha^s)^{\frac{1}{\alpha}} (L_t^L)^{\frac{\alpha-1}{\alpha}} \right]^{\frac{\alpha}{\alpha-1}-1} (f(K,L^H)_t)^{\frac{\alpha^H(\alpha-1)}{(\alpha^H-1)\alpha}-1} K_{t-1}^{\frac{\alpha^H-1}{\alpha^H}-1} \quad \text{(B.6.65)}$$

$$w_t^H = \left(1-\vartheta^H\right) \left[p_t^h mpl_t^H + \left(1-\rho_n^H\right) \frac{\kappa^H}{\Upsilon_t^H} \right] + \frac{\vartheta^H}{\mu_t^H} \left[\Phi^H \left(l_t^H\right)^{-\sigma_l} - \left(1-\rho_n^H\right) \Lambda_t^H \right] \quad \text{(B.6.66)}$$

$$w_t^M = \left(1-\vartheta^M\right) \left[p_t^h mpl_t^M + \left(1-\rho_n^M\right) \frac{\kappa^M}{\Upsilon_t^M} \right] + \frac{\vartheta^M}{\mu_t^M} \left[\Phi^M \left(l_t^M\right)^{-\sigma_l} - \left(1-\rho_n^M\right) \Lambda_t^M \right] \quad \text{(B.6.67)}$$

$$w_t^L = \left(1-\vartheta^L\right) \left[p_t^h mpl_t^L + \left(1-\rho_n^L\right) \frac{\kappa^L}{\Upsilon_t^L} \right] + \frac{\vartheta^L}{\mu_t^L} \left[\Phi^L \left(l_t^L\right)^{-\sigma_l} - \left(1-\rho_n^L\right) \Lambda_t^L \right] \quad \text{(B.6.68)}$$

$$w_t^{mH} = \left(1-\vartheta^{mH}\right)\left[p_t^h mpl_t^{mH} + \left(1-\rho_n^{mH}\right)\frac{\kappa^{mH}}{\Upsilon_t^{mH}}\right] + \frac{\vartheta^{mH}}{\mu_t^M}\left[\Phi^M\left(l_t^M\right)^{-\sigma_l} - \left(1-\rho_n^{mH}\right)\Lambda_t^{mH}\right] \tag{B.6.69}$$

$$w_t^{mL} = \left(1-\vartheta^{mL}\right)\left[p_t^h mpl_t^{mL} + \left(1-\rho_n^{mL}\right)\frac{\kappa^{mL}}{\Upsilon_t^{mL}}\right] + \frac{\vartheta^{mL}}{\mu_t^M}\left[\Phi^M\left(l_t^M\right)^{-\sigma_l} - \left(1-\rho_n^{mL}\right)\Lambda_t^{mL}\right] \tag{B.6.70}$$

Government

$$T_t = ub^H U_t^H + ub^L U_t^L + G_t + ub^{mH} U_t^{mH} + ub^{mM} U_t^{mM} + ub^{mL} U_t^{mL} \tag{B.6.71}$$

$$T_t = \varphi^H tax_t^H + \varphi^M tax_t^M + \varphi^L tax_t^L \tag{B.6.72}$$

$$tax_t^M = T_t \tag{B.6.73}$$

$$tax_t^L = T_t \tag{B.6.74}$$

Aggregate variables

$$C_t = \varphi^H c_t^H + \varphi^M c_t^M + \varphi^L c_t^L \tag{B.6.75}$$

$$b_t = \varphi^H b_t^H + \varphi^M b_t^M + \varphi^L b_t^L \tag{B.6.76}$$

$$X_t = \varphi^H x_t \tag{B.6.77}$$

$$K_t = \varphi^H z_t k_t \tag{B.6.78}$$

Further Labour Market

$$U_t^H = \varphi^H u_t^H \tag{B.6.79}$$

$$U_t^L = \varphi^L u_t^L \tag{B.6.80}$$

$$U_t^{mM} = \varphi^{mM} u_t^{mM} \tag{B.6.81}$$

$$U_t^{mH} = \varphi^{mH} u_t^{mH} \tag{B.6.82}$$

$$U_t^{mL} = \varphi^{mL} u_t^{mL} \tag{B.6.83}$$

$$U_t^{mH} = u_t^M s_t^H \tag{B.6.84}$$

$$U_t^{mM} = u_t^M s_t^M \tag{B.6.85}$$

$$U_t^{mL} = u_t^M s_t^L \tag{B.6.86}$$

$$N_t^H = \varphi_t^H n_t^H \tag{B.6.87}$$

$$N_t^L = \varphi_t^L n_t^L \tag{B.6.88}$$

$$N_t^{mM} = \varphi_t^{mM} n_t^{mM} \tag{B.6.89}$$

$$N_t^{mH} = \varphi_t^{mH} n_t^{mH} \tag{B.6.90}$$

$$N_t^{mL} = \varphi_t^{mL} n_t^{mL} \tag{B.6.91}$$

$$\text{partic}_t^H \equiv \frac{N_t^H + U_t^H}{\varphi^H} \tag{B.6.92}$$

$$\text{partic}_t^L \equiv \frac{N_t^L + U_t^L}{\varphi^L} \tag{B.6.93}$$

$$\text{partic}_t^{mM} \equiv \frac{N_t^{mM} + U_t^{mM}}{\varphi^{mM}} \tag{B.6.94}$$

$$\text{partic}_t^{mH} \equiv \frac{N_t^{mH} + U_t^{mH}}{\varphi^{mH}} \tag{B.6.95}$$

$$\text{partic}_t^{mL} \equiv \frac{N_t^{mL} + U_t^{mL}}{\varphi^{mL}} \tag{B.6.96}$$

$$\text{unemp}_t^H \equiv \frac{U_t^H}{N_t^H + U_t^H} \tag{B.6.97}$$

$$\text{unemp}_t^L \equiv \frac{U_t^L}{N_t^L + U_t^L} \tag{B.6.98}$$

$$\text{unemp}_t^{mM} \equiv \frac{U_t^{mM}}{N_t^{mM} + U_t^{mM}} \tag{B.6.99}$$

$$\text{unemp}_t^{mH} \equiv \frac{U_t^{mH}}{N_t^{mH} + U_t^{mH}} \tag{B.6.100}$$

$$\text{unemp}_t^{mL} \equiv \frac{U_t^{mL}}{N_t^{mL} + U_t^{mL}} \tag{B.6.101}$$

$$\text{bw}_t^H \equiv \frac{ub^H}{w_t^H} \tag{B.6.102}$$

$$\text{bw}_t^L \equiv \frac{ub^L}{w_t^L} \tag{B.6.103}$$

$$\text{bw}_t^M \equiv \frac{ub^M}{w_t^M} \tag{B.6.104}$$

$$\text{bw}_t^{mH} \equiv \frac{ub^{mH}}{w_t^{mH}} \tag{B.6.105}$$

$$u_t^M = u_t^{mH} + u_t^{mM} + u_t^{mL} \tag{B.6.106}$$

Open Economy

$$r_t = r^* \exp(-\phi_{bhc}(b_t - \bar{b})) \tag{B.6.107}$$

$$v(p_t^h)^{1-\theta} = 1 - (1-v)(rer_t)^{1-\theta} \tag{B.6.108}$$

$$y_t = v(p_t^h)^{-\theta} \left(C_t + X_t + G_t + \kappa^H v_t^H + \kappa^{mH} v_t^H + \kappa^M v_t^M + \kappa^L v_t^L + \kappa^{mL} v_t^{mL} \right) + ex_t \tag{B.6.109}$$

$$ex_t = v^* \left(\frac{rer_t}{p_t^h} \right)^{\theta^*} y_t^* \tag{B.6.110}$$

$$b_t = r_{t-1} \frac{gdp_{t-1}}{gdp_t} \frac{rer_t}{rer_{t-1}} b_{t-1} + tb_t \tag{B.6.111}$$

$$tb_t = 1 - \frac{\left(C_t + \psi_t^x X_t + G_t + \kappa^H v_t^H + \kappa^{mH} v_t^{mH} + \kappa^M v_t^M + \kappa^L v_t^L + \kappa^{mL} v_t^{mL} \right)}{gdp_t} \tag{B.6.112}$$

$$gdp_t = p_t^h y_t \tag{B.6.113}$$

Exogenous Processes

$$\psi_t = \rho_\psi \psi_{t-1} + \varepsilon_t^\psi \tag{B.6.114}$$

$$\psi_t^a = \rho_{\psi^a} \psi_{t-1}^a + \varepsilon_t^a \tag{B.6.115}$$

$$G_t = (\overline{G})^{1-\rho_g} \left(G_{t-1}\right)^{\rho_g} \exp\left(\varepsilon_t^g\right) \tag{B.6.116}$$

$$\psi_t^x = \rho_{\psi^x} \psi_{t-1}^x + \varepsilon_t^x \tag{B.6.117}$$

$$y_t^* = \rho^{y^*} y_{t-1}^* + \varepsilon_t^{y^*} \tag{B.6.118}$$

Population Dynamics

$$g_t = \varphi_t^H g_t^H + \varphi_t^M g_t^M + \varphi_t^L g_t^L \tag{B.6.119}$$

$$g_t^H = g^H \tag{B.6.120}$$

$$g_t^L = g^L \tag{B.6.121}$$

$$g_t^{mM} = \rho^{g^m} g_{t-1}^{mM} + \varepsilon_t^{g^m} \tag{B.6.122}$$

$$g_t^{mH} = g^{mH} \tag{B.6.123}$$

$$g_t^{mL} = g^{mL} \tag{B.6.124}$$

$$\varphi_t^M g_t^M = \varphi_t^{mH} g_t^{mH} + \varphi_t^{mM} g_t^{mM} + \varphi_t^{mL} g_t^{mL} \tag{B.6.125}$$

$$\varphi_t^H = (\varphi^H)^{(1-\rho^\varphi)} \left(\frac{g_t^H}{g_t}\varphi_{t-1}^H\right)^{\rho^\varphi} \tag{B.6.126}$$

$$\varphi_t^L = (\varphi^L)^{(1-\rho^\varphi)} \left(\frac{g_t^L}{g_t}\varphi_{t-1}^L\right)^{\rho^\varphi} \tag{B.6.127}$$

$$\varphi_t^{mH} = (\varphi^{mH})^{(1-\rho^\varphi)} \left(\frac{g_t^{mH}}{g_t}\varphi_{t-1}^{mH}\right)^{\rho^\varphi} \tag{B.6.128}$$

$$\varphi_t^{mM} = (\varphi^{mM})^{(1-\rho^\varphi)} \left(\frac{g_t^{mM}}{g_t}\varphi_{t-1}^{mM}\right)^{\rho^\varphi} \tag{B.6.129}$$

$$\varphi_t^{mL} = (\varphi^{mL})^{(1-\rho^\varphi)} \left(\frac{g_t^{mL}}{g_t}\varphi_{t-1}^{mL}\right)^{\rho^\varphi} \tag{B.6.130}$$

$$\varphi_t^M = \varphi_t^{mH} + \varphi_t^{mM} + \varphi_t^{mL} \tag{B.6.131}$$

$$\zeta_t^H = \frac{N_t^H + N_t^{mH} + N_t^{mM}}{N_t^H + N_t^{mH} + N_t^{mM} + N_t^L + N_t^{mL}} \tag{B.6.132}$$

$$\zeta_t^L = \frac{N_t^L + +N_t^{mL}}{N_t^H + N_t^{mH} + N_t^{mM} + N_t^L + N_t^{mL}} \tag{B.6.133}$$

B.7 Steady State

There are 137 variables, excluding the observables. Of these, six of them are exogenous processes. We take the labour market variables, φ^j, partic^j, and unemp^j from the data.

$$U^j = \varphi^j partic^j unemp^j \tag{B.7.1}$$

$$N^j = \left(\frac{1}{unemp^j} - 1\right) U^j \tag{B.7.2}$$

$$m^j = \rho_n^j N^j \tag{B.7.3}$$

$$v^j = \left(\frac{m^j}{a^j (U^j)^{1-\Gamma^j}}\right)^{\frac{1}{\Gamma^j}} \tag{B.7.4}$$

$$\Omega^j = \frac{m^j}{U^j} \tag{B.7.5}$$

$$\Upsilon^j = \frac{m^j}{U^j} \tag{B.7.6}$$

$$\theta^j = \frac{v^j}{U^j} \tag{B.7.7}$$

$$u^j = \frac{U^j}{\varphi^j} \tag{B.7.8}$$

$$n^j = \frac{N^j}{\varphi^j} \tag{B.7.9}$$

$$l^i = 1 - n^j - u^j \qquad i, j \in [H, L] \tag{B.7.10}$$

$$l^M = 1 - n^{mH} - n^{mM} - n^{mL} - u^M \tag{B.7.11}$$

$$\xi^H = \frac{N^H}{N^H + N^{mH}} \tag{B.7.12}$$

$$\xi^L = \frac{N^L}{N^L + N^{mL}} \tag{B.7.13}$$

$$\zeta^H = \frac{N^H + N^{mH}}{N^H + N^{mH} + N^{mM} + N^L + N^{mL}} \tag{B.7.14}$$

$$\zeta^M = \frac{N^M}{N^H + N^{mH} + N^{mM} + N^L + N^{mL}} \tag{B.7.15}$$

$$\zeta^L = \frac{N^L + N^{mL}}{N^H + N^{mH} + N^{mM} + N^L + N^{mL}} \tag{B.7.16}$$

$$z = 1 \tag{B.7.17}$$

$$r^k = \frac{1 - \beta}{\beta} + \delta \tag{B.7.18}$$

$$r = \frac{1}{\beta} \tag{B.7.19}$$

$$r^* = \frac{1}{\beta} \tag{B.7.20}$$

$$b^i = 0 \tag{B.7.21}$$

$$mpk = r^k \tag{B.7.22}$$

$$L^H = \left[\chi_h^{\frac{1}{\theta^h}} (N^H)^{\frac{\theta^h - 1}{\theta^h}} + \chi_m^{\frac{1}{\theta^h}} (N^{mH})^{\frac{\theta^h - 1}{\theta^h}} + (1 - \chi_h - \chi_m)^{\frac{1}{\theta^h}} (N^{mM})^{\frac{\theta^h - 1}{\theta^h}}\right]^{\frac{\theta^h}{\theta^h - 1}} \tag{B.7.23}$$

$$L^L = \left[\chi_l^{\frac{1}{\theta^h}} (N^l)^{\frac{\theta^h - 1}{\theta^h}} + (1 - \chi_l)^{\frac{1}{\theta^h}} (N^{mL})^{\frac{\theta^h - 1}{\theta^h}}\right]^{\frac{\theta^h}{\theta^h - 1}} \tag{B.7.24}$$

Aggregate capital is determined by numerical solver only - no algebraic solution

$$K = MPK^{-1}\left(MPK, \Sigma N^j\right) \tag{B.7.25}$$

$$k = \frac{K}{z\varphi^H} \tag{B.7.26}$$

$$y = \psi^a \left[(\alpha^s)^{\frac{1}{\alpha}} \left[\left((\alpha^k)^{\frac{1}{\alpha^H}} K^{\frac{\alpha^H-1}{\alpha^H}} + (1-\alpha^k)^{\frac{1}{\alpha^H}} (\xi L^H)^{\frac{\alpha^H-1}{\alpha^H}} \right)^{\frac{\alpha^H}{\alpha^H-1}} \right]^{\frac{\alpha-1}{\alpha}} + (1-\alpha^s)^{\frac{1}{\alpha}} (L^L)^{\frac{\alpha-1}{\alpha}} \right]^{\frac{\alpha}{\alpha-1}} \tag{B.7.27}$$

$$f(K, L^H) = (\alpha^k)^{\frac{1}{\alpha^H}} K^{\frac{\alpha^H-1}{\alpha^H}} + (1-\alpha^k)^{\frac{1}{\alpha^H}} (\xi L^H)^{\frac{\alpha^H-1}{\alpha^H}}$$

$$\begin{aligned} mpl^H = \chi_H^{\frac{1}{\theta^h}} (\alpha^s)^{\frac{1}{\alpha}} (1-\alpha^k) \xi^{\frac{\alpha^H-1}{\alpha^H}} \psi^A \left[f(K, L^H)^{\frac{\alpha^H(\alpha-1)}{(\alpha^H-1)\alpha}} + (1-\alpha^s)^{\frac{1}{\alpha}} (L^L)^{\frac{\alpha-1}{\alpha}} \right]^{\frac{\alpha}{\alpha-1}-1} \\ (f(K, L^H))^{\frac{\alpha^H(\alpha-1)}{(\alpha^H-1)\alpha}-1} (L^H)^{\frac{\alpha^H-1}{\alpha^H}-1} (N^H)^{\frac{-1}{\theta_h}} \end{aligned} \tag{B.7.28}$$

$$\begin{aligned} mpl^{mH} = \chi_M^{\frac{1}{\theta^h}} (\alpha^s)^{\frac{1}{\alpha}} (1-\alpha^k) \xi^{\frac{\alpha^H-1}{\alpha^H}} \psi^A \left[f(K, L^H)^{\frac{\alpha^H(\alpha-1)}{(\alpha^H-1)\alpha}} + (1-\alpha^s)^{\frac{1}{\alpha}} (L^L)^{\frac{\alpha-1}{\alpha}} \right]^{\frac{\alpha}{\alpha-1}-1} \\ (f(K, L^H))^{\frac{\alpha^H(\alpha-1)}{(\alpha^H-1)\alpha}-1} (L^H)^{\frac{\alpha^H-1}{\alpha^H}-1} (N^{mH})^{\frac{-1}{\theta_h}} \end{aligned} \tag{B.7.29}$$

$$\begin{aligned} mpl^{mM} = (1-\chi_H-\chi_M)^{\frac{1}{\theta^h}} (\alpha^s)^{\frac{1}{\alpha}} (1-\alpha^k) \xi^{\frac{\alpha^H-1}{\alpha^H}} \psi^A \left[f(K, L^H)^{\frac{\alpha^H(\alpha-1)}{(\alpha^H-1)\alpha}} + (1-\alpha^s)^{\frac{1}{\alpha}} (L^L)^{\frac{\alpha-1}{\alpha}} \right]^{\frac{\alpha}{\alpha-1}-1} \\ (f(K, L^H))^{\frac{\alpha^H(\alpha-1)}{(\alpha^H-1)\alpha}-1} (L^H)^{\frac{\alpha^H-1}{\alpha^H}-1} (N^{mM})^{\frac{-1}{\theta_h}} \end{aligned} \tag{B.7.30}$$

$$mpl^L = \chi_L^{\frac{1}{\theta^h}} (1-\alpha^s)^{\frac{1}{\alpha}} \psi^A \left[f(K, L^H)^{\frac{\alpha^H(\alpha-1)}{(\alpha^H-1)\alpha}} + (1-\alpha^s)^{\frac{1}{\alpha}} (L^L)^{\frac{\alpha-1}{\alpha}} \right]^{\frac{\alpha}{\alpha-1}-1} (L^L)^{\frac{\alpha-1}{\alpha}-1} (N^L)^{\frac{-1}{\theta_h}} \tag{B.7.31}$$

$$mpl^{mL} = (1-\chi_L)^{\frac{1}{\theta^h}} 1-\alpha^s)^{\frac{1}{\alpha}} \psi^A \left[f(K, L^H)^{\frac{\alpha^H(\alpha-1)}{(\alpha^H-1)\alpha}} + (1-\alpha^s)^{\frac{1}{\alpha}} (L^L)^{\frac{\alpha-1}{\alpha}} \right]^{\frac{\alpha}{\alpha-1}-1} (L^L)^{\frac{\alpha-1}{\alpha}-1} (N^{mL})^{\frac{-1}{\theta_h}} \tag{B.7.32}$$

$$w^j = mpl^j + \left(1 - \rho_n^j - \frac{1}{\beta}\right) \frac{\kappa^j}{\Upsilon^j} j \in [H, L] \tag{B.7.33}$$

$$w^{mH} = w^H \Delta^H \tag{B.7.34}$$

$$w^{mM} = w^H \Delta^M \tag{B.7.35}$$

$$w^{mL} = w^L \Delta^L \tag{B.7.36}$$

$$ge = y \frac{g}{y} \tag{B.7.37}$$

$$Tax = \Sigma ub^j U^j + G \tag{B.7.38}$$

$$tax^M = T \tag{B.7.39}$$

$$tax^L = T \tag{B.7.40}$$

$$tax^H = \frac{T - \varphi^M tax^M - \varphi^L tax^L}{\varphi^H} \tag{B.7.41}$$

$$x = \delta k \tag{B.7.42}$$

$$X = \varphi^H x \tag{B.7.43}$$

$$ex = \frac{\overline{ex}}{y} y \tag{B.7.44}$$

$$gdp = p^h y \tag{B.7.45}$$

$$tb = 0 \tag{B.7.46}$$

$$C = (1 - tb)gdp - (X + ge + \Sigma\kappa^j v^j) \tag{B.7.47}$$

$$c^M = w^M n^{mM} + w^{mH} n^{mH} + w^{mL} n^{mL} + ub^j u^m - tax^M - b^M(1 - r) \tag{B.7.48}$$

$$c^L = w^L n^L + ub^L u^L - tax^L - b^L(1 - r) \tag{B.7.49}$$

$$c^H = \frac{C - \varphi^m c^M - \varphi^L c^L}{\varphi^H} \tag{B.7.50}$$

$$y^* = 1 \tag{B.7.51}$$

$$p^h = 1 \tag{B.7.52}$$

$$rer = 1 \tag{B.7.53}$$

$$\mu^i = \left((1 - \kappa^c)\, c^i\right)^{-\eta} \tag{B.7.54}$$

$$\Lambda^j = \frac{\mu^i (w^j - ub^j)}{\frac{1}{\beta} - (1 - \rho_n^j - \Omega^j)} \quad j \in [H, L, mH, mM, mL] \tag{B.7.55}$$

Chapter 4

The Resource Curse and Migration

4.1 Introduction

The migration profile of medium-sized developing economies varies as to whether they are net receivers or senders of migrants. In the case of Latin American countries, historically Argentina, Bolivarian Republic of Venezuela (hereafter Venezuela) and less so Brazil have been net hosts while Chile, Colombia, Ecuador, Paraguay, Peru, and Uruguay have been senders.[1] [2] A feature of a number of these emerging economies is that they are prone to severe economic crises, the likes rarely experienced by developed and diversified economies. These can be accompanied by a devaluation in currency and/or government default.[3] Often made worse by the dollarisation of government debt.

Both Argentina and Venezuela are examples of countries that suffer from a *resource curse* and experience *macroeconomic populism*.[4] The resource curse is experienced by commodity dependent non-advanced economies for which the effects can be political and economic.[5] If we dissect the OPEC member countries, none of

[1]The International Organisation for Migration provides migration flow estimates based on data such as visas granted. For a discussion on migration in Latin America see Zlotnik (1998)

[2]Venezuela was defined as an emerging economy by Morgan Stanley Capital International Emerging Market Index until 2006 when it was reclassified as a standalone economy. The UN defines all Latin American countries as developing.

[3]See Reinhart (2002), Reinhart et al. (2003), and Na et al. (2018). In Reinhart (2002), the unconditional probability of a large devaluation in any 2-year period is 17%. However, if there is a default in the 2-year period, the probability of a large devaluation is 84%. Bleaney (2005) for an analysis of currency collapse in emerging economies.

[4]The resource curse for oil economies is discussed in Humphreys et al. (2007).

[5]For a history of the Venezuelan economy, including fiscal and monetary policy see Restuccia (2018) and Bello et al. (2011). For Latin America political economy and economic development see Grilli (2005).

them would be described as advanced economies. In terms of politics, the resource wealth can have harmful effects including supporting authoritarian regimes, higher levels of corruption, and the incitement of violent conflict (Ross, 2015). The economic side is primarily a dependence on the sale of the commodity as an export, so focus is shifted away from domestic manufacturing industries or a developed service sector. When the price falls there is large macroeconomic instability that is transmitted through channels including the real exchange rate (Frankel, 2010). Including high unemployment and emigration as workers search for employment elsewhere. Without a developed industry that workers can transfer their skills to, and a reliance on imports resulting in shortages, the economic losses continue.

We apply the model to Venezuela due to the length of macroeconomic, oil specific, and migration data available. The research in this paper looks at the effects of the shocks in Venezuela, the policy implications and how they can be used to prevent other resource cursed, macroeconomic populist countries following their example.

Once a country of destination, Venezuela has become a sender of migrants in the last two decades. The change to migrant sender has been caused by the economic conditions brought on by the resource curse and macroeconomic populism under President Hugo Chávez and most recently President Nicolàs Maduro, has resulted in a deteriorating labour market, unstable currency, goods shortages, and increasingly hard to access services.[6] President Maduro's drive to keep Venezuela's international status by not defaulting on debt has led to a humanitarian crisis. The result of the economic crisis in Venezuela has shrunk GDP from a high of 554bn US$ in 2013 to 331bn US$ in 2018.[7] The accelerating factor is considered to be the 2014 fall in oil prices.[8] The stock of Venezuelans worldwide, increased 10-fold since 2004 (International Organisation for Migration, 2019). One reason for an increase in emigration was the country's participation is MECOSUR, an economic and political bloc providing a common labour market with Argentina, Brazil, Paraguay and Uruguay, and subsequently joined by Bolivia. Venezuela's membership was suspended in December 2016.[9] An estimated 2.9 million Venezuelan residents emigrated between April 2017 and March 2018 (Hausmann et al., 2018). The emigration of Venezuelans has grown exponentially since 2015, the extent to which

[6]A strict enforcement of currency controls have limited private imports which make the country dependent on its own produce.

[7]IMF, see table C.1

[8]As of December 2018, oil production is at the same level as 1930s. Nationalisation of PDVSA (Petróleos de Venezuela, S.A) in 1976 can partially be attributed to the fall in production. The price of Venezuelan crude oil in mid-2014 was just under \$100 per barrel, this had fallen to \$47 by the end of 2014, and by the end of 2015 was just \$29.

[9]Venezuela only gained full membership in 2012 but was eligible for many of the benefits prior to this.

the Venezuelan government created the *migration police* to limit emigration using increased border patrols, and the increased expense of a passport. The migration crisis is referred to as the *Venezuelan diaspora* (also as the *Bolivarian diaspora*). The migrants fleeing economic crises in particular, experience difficulties in attaining secure employment. Illegal migration is recognised in Colombia where, in 2018, the Colombian government estimated that 45,000 of the 870,093 Venezuelans in Colombia have expired visas or crossed the border illegally (International Organisation for Migration and Colombia Migration).[10] One significant issue for Venezuela is the flight of human and physical capital. The loss of high skilled workers through emigration, equivalently brain drain, has left gaps in the labour market.[11] Of those leaving, are a high number of business owners which due to the deteriorating economy leads to capital flight.

In this paper, we examine the role of economic contraction of an economy as the driver for emigration in an estimated two-country dynamic stochastic general equilibrium (DSGE) model using data from Venezuela and the United States. The cause is a negative shock to oil price. We use occasionally binding migration policies which reflect a difficulty in migration either legally or illegally. For legal migration it reflects increased challenges in attaining a working visa or means of working abroad legally. This can lead to an increase in illegal migration. The constraint on illegal migration is by way of increased efficiency of a border force, and finally the deportation of illegal migrants.

This paper differs from the existing literature on emigration with the inclusion of illegal and legal endogenous migration where the focus is on the effects of the sending economy, the use of occasionally binding migration policies, (asymmetric) labour market frictions, and the endogenous price and supply of oil decisions. We use a real business cycle (RBC) model, as Garcia-Cicco et al. (2010) demonstrate that RBC models with financial frictions can reflect the business cycles of emerging economies well. This is a basis for the commodity producing nation research by Drechsel and Tenreyro (2018). A considerable portion of the literature on Latin America focuses on Argentina and Mexico, and oil price shocks for oil exporting nations are predominantly focused elsewhere such as Norway. In terms of the commodity prices literature, additional contributions of this paper include that the price and supply of oil is endogenous, and labour market frictions.

[10] The top 10 destination countries for Venezuelan emigrants (2013): United States, Spain, Italy, Colombia, Portugal, the Dominican Republic, Ecuador, Canada, Mexico, Argentina (World Bank, 2016). In 2018, the top 4 countries were: Colombia, Peru, Spain and United States (International Organisation for Migration, 2019).

[11] See Arias and Guerra-Salas (2019) which discusses the high-skill immigration of Venezuelans and effects of immigration shock to an emerging economy in an OLG model.

The remainder of this paper is as follows, section 4.2 discusses existing literature on emigration, emerging economies, and commodities. Sections 4.3 and 4.4 set out the model and its constraints. Section 4.5 presents the calibration and estimation results. Section 4.6 analyses the results from the model, and finally section 4.7 concludes.

4.2 Related Literature

The research in this paper adds to the migration literature by examining emigration from a developing economy with legal and illegal migration, and the inclusion of occasionally binding migration policies. In this paper we are able to evaluate how *emigration* effects an economy, how much the economic contraction is a driver of emigration and the role of constraints in the emigration decision. Here we discuss some of the literature in commodities and emigration. The combination of emigration and commodities is unexplored in DSGE literature, including two-country models, and similarly the analysis of migration policies.

Much of the literature on commodity price shocks focuses on the purchaser where price rises have been shown to cause recessions (Kim and Loungani (1992), and e.g. United States 2001 Barsky and Kilian (2004)). However, there are a number of papers, including Bodenstein et al. (2018), that examine the role of the producing nation. A significant portion of the literature on commodity prices makes the price and/or supply exogenous. Where oil is the commodity, there are a select few papers including Bergholt et al. (2017) and Arora and Gomis-Porqueras (2011) using a New Keynesian and RBC approach respectively. Santos (2016) used a basic RBC to attempt to replicate the business cycle of Venezuela as an oil producing country while a number of papers have tried to examine to oil-consuming countries Kose et al. (2003), Barsky and Kilian (2004), Kilian (2009), and Bodenstein et al. (2011). For commodities in emerging economies, an RBC example includes Drechsel and Tenreyro (2018), and a New Keynesian model in Medina and Soto (2005) and Medina and Soto (2007) for Chile. Agnani and Iza (2011) discuss growth in an oil-abundant economy with an application to Venezuela. Aguiar and Gopinath (2007) compare business cycles, focusing on the Solow residual, in Mexico and Canada using a small open economy model and comparing the results of estimation. Their results show that emerging markets' business cycles are driven by shocks to the stochastic trend whereas developed economies have stable trends. There are two-country models in which one is an emerging economy and the other a developed such as Ozkan and Unsal (2017) who use South Korea and the United States in a study of the effects of a global financial crisis on an emerging economy.

Bandeira et al. (2018) examine emigration in peripheral Euro area countries and the focus on fiscal implications of emigration in a small open economy that employs debt consolidation. Migration effects are examined on the business cycle with responses for a total factor productivity (TFP), monetary policy, and a government spending shock. For fiscal consolidation, labour tax increases, cuts in unemployment benefits, and public spending cuts. There are no limits to emigration as the countries under focus are part of the European labour market. Bandeira et al. (2019) specifies the country to Greece. Our paper differs in that we use a developing economy that has a commodity sector shock as the driver rather than TFP and is not in a monetary union. The downturns of the developed economies are of a smaller severity to those under examination in this paper due to the diversification of industries. Additionally, this paper uses a shock to the oil price rather than those typically used in business cycle analysis.

Lazaretou (2016) empirically analyses the effects of emigration in Greece during the financial crisis with a focus on skilled emigration.[12] Of note, during 2013 alone, Greece lost 2.2% of its workforce through emigration. This was three times the levels of emigration experienced in 2008. The results are largely negative. The loss of the workforce has detrimental effects, however, there is an increase in remittances. Due to brain waste, remittances are not as high as they could be as the migrants are overqualified and underpaid in the host country.[13] There are positive responses on the return to education. As some of the emigrants were unemployed, on the one hand it decreased the burden on the social security system. However, as the majority of emigrants were of working-age, which negatively affects the birth rate, in the long run it increases the burden on the social security system. The full (macroeconomic) effects won't be fully realised for a number of years, particularly if the migrants do not return.

There is a large literature that studies the emigration of Mexicans to the United States. This includes Mandelman and Zlate (2012) who use a two-country model with an endogenous migration decision based on the wage premium however these two country models focus on the macroeconomics results of the host country. Kiguchi and Mountford (2017) analyse the United States using an empirical and DSGE model with search and matching frictions.

Mehra (2017) uses an occasionally binding constraint, in terms of imposing a cap to the number of foreign skilled workers that can be hired each period. She notes

[12] It is worthy to note that Greece is defined as an emerging economy by Morgan Stanley Capital International Emerging Market Index.

[13] Brain drain is the loss of skilled workers in an economy through emigration. Brain waste is the underemployment of migrants in the host economy.

that the cap imposed by the United States government is non-binding in periods such as recessions. Alternatively, the constraint is binding when demand for foreign labour is greater than the imposed quota. She offers an extension to her model that includes search and matching frictions in the migrant receiving economy to analyse the effects of immigration on (un)employment of natives whereas this paper uses search and matching frictions as core to our model. Other research using occasionally binding constraints in migration models focus on credit constraints, such as Bahadir et al. (2018). In their research, they analyse how the receipt of remittances affect the economy. When remittances are predominantly to the credit constrained households, the effects are contractionary for the economy. However, when the remittances go to mostly entrepreneur households, the effects are expansionary. The role of remittances in the informal economy is analysed in Chatterjee and Turnovsky (2018). Since informal employment has poorer labour market conditions than the formal sector, it has negative effects for the home economy. The effect of a temporary shock to remittance inflows results in depreciation of the real exchange rate and a contraction of GDP. Remittances can be shown to increase corruption, for example, governments getting a cut through exchange rate controls. However, Tyburski (2012) argues that remittances to Mexico actually reverse the resource curse.

A large portion of the literature analyses the effects of migration on the host country, with less examination of the sending country. There is limited use of occasionally binding constraints in the migration literature, and absence of analysis of oil prices in the Venezuelan business cycles which together forms motivation for our paper.

4.3 Baseline Model

We present a two-country model where each country is populated by infinitely lived households, perfectly competitive firms, and a fiscal authority. The primary country is referred to as Home which is a developing and resource cursed country. The second country, referred to as Foreign, is an oil-consuming developed economy. The asymmetries extend beyond the production versus consumption of oil. The countries differ in GDP per capita, relative population size, openness to trade, steady state technology, credit-worthiness, and whether they are migrant senders or hosts.

The households are intertemporal optimisers who provide labour services that are subject to search and matching frictions, participate in international financial markets and own the final-good producing firms. The agents in Home have domestic employment opportunities in a perfectly competitive final good producing and an oil-producing firm. A third source of employment requires migration to Foreign, where they can find employment in an oil-consuming firm. The migration decision for the household is endogenous based upon employment opportunity success, and it is subject to a migration cost which increases when there are constraints on migration. Their employment status in Foreign can be legal or illegal under migration constraints. The households in Foreign have a choice of being employed in the oil-consuming firm, or being unemployed. All agents are economically active.

There exists a series of perfectly competitive final good producing firms in Home that employ labour and capital services in production. They post vacancies at a cost and engage in wage bargaining with workers. There are oil-producing firms in Home that produces oil using capital and labour.[14] By introducing a second type of firm in the Home economy, we are allowing oil to be separate from the domestically produced final good. In Foreign, there are perfectly competitive oil-consuming firms that employ labour and capital services. The firms post vacancies at a cost and employ a composite of Foreign and migrant labour. The workers are imperfect substitutes, and due to brain waste, the migrants experience poorer labour market conditions.[15] The capital services consist of oil and physical capital. A contribution of this paper, by having an oil-consuming firm in Foreign, is the price, demand, and supply of oil is able to be determined by market forces in addition to a shock.

In each country, there is a government that consumes, collects taxes, and borrows from financial markets to finance deficits. Part of the governments' consumption includes defence spending which is proportional to the illegal migration rate.

In this paper, we introduce occasionally binding migration policies. The first constraint introduces a limit on legal migration, which makes a share of migrants illegal. This leads to negative labour market effects for the migrants. Their bargaining power, security in employment, and the wage level have all decreased. The reduction in wage level is inclusive of other employment benefits. The second constraint focuses on limiting migration altogether, legal and illegal. This further worsens the labour market conditions of migrants, by reducing migration more agents remain unemployed in Home. The final constraint causes a destruction of matches where illegal migrants are removed from employment in Foreign. The destruction occurs either when they have already been employed or during the migration process. The occasionally binding migration policies are described further in section 4.4.

[14]The oil sector is critical to representing exports for Venezuela as it accounts for approximately 96% of exports and in excess of 20% of GDP during the observed period. In 2016, the oil sector accounted for 12% of GDP, however, this was after the significant fall in oil price in 2014.

[15]Brain waste results from the migration constraints and a basic form in terms of higher frictions in the labour market.

4.3.1 Labour Market

There is a total number of native agents in country i, $\overline{\mathrm{N}^i}$, where $i \in [H.F]$ for Home and Foreign respectively. For Home, the agents are working or searching for employment both in Home and Foreign. The natives residing in Home, are denoted by N_t^H, and those in Foreign by m_t. Thus the native population is given by,
Total Home Population

$$\overline{\mathrm{N^H}} = N_t^H + m_t \tag{4.3.1}$$

All household members are in a type of employment, n_t^j or unemployed u_t^j. Employment is in one of the three types of firms. For Home agents, $j \in H, M$, where H is the final good producing firm in Home, and M is the oil-consuming firm in Foreign. The two types of firms in Home are modelled in a way that the oil producing, and final good producing firm are a collective such that employees are distributed between the two depending on labour demand. There is one wage level and the steady state marginal products of labour are equal. The effective labour hours that are dedicated to either the final good producing or oil-producing sector is dependent on the relative productiveness of the sector. Hence, n_t^H includes those working in the oil and final good producing sectors.[16]

Unemployed members of the households in Home can either search for employment in the Home labour market or in the Foreign labour market, specific to migrants due to the imperfect substitutability of native and migrant workers. A share of unemployed members, λ_t^H, are searching for employment in Home. The remainder, $1-\lambda_t^H$, search for employment in Foreign. We can define the agents residing in Home as
Effective Home Population

$$N_t^H = n_t^H + \lambda_t^H u_t^H \tag{4.3.2}$$

Therefore, the effective migrant stock, m_t, consists of the migrants who have already emigrated and those who are searching for employment in Foreign, $(1-\lambda_t^H)u_t^H$.
Effective Migrant Stock

$$m_t = n_t^M + (1-\lambda_t^H)u_t^H \tag{4.3.3}$$

We set immigration of Foreign agents to Home to zero, due to the wage premium, so that the total available workforce in Home is, N_t^H. Agents who become unemployed in Foreign, can remain in Foreign or return to Home to search for employment. When migration constraints are imposed, the employed migrants are either illegal or legal. The (relative) size varies depending on the migration constraints and business cycle

[16] By modelling this way, there is a choice of employment in Home and Foreign rather than Home, oil producing, and Foreign.

status since migration is driven by employment opportunities. Illegal migrants are denoted by n_t^{IL} and legal migrants by n_t^L.
Migrant Employment with Constraints

$$n_t^M = n_t^{IL} + n_t^L \tag{4.3.4}$$

The portion of migrants that are illegal is given by $\aleph_t$.[17]
Illegal Immigrant Share

$$\aleph_t = \frac{n_t^{IL}}{n_t^M} \tag{4.3.5}$$

In Foreign, there are a number of native agents, $\overline{\mathrm{N^F}}$, which is normalised to 1. These agents are either employed in the oil-consuming firms or unemployed in Home as migration from Foreign to Home by agents native to Foreign is zero due to the wage premium of Foreign over Home. For these agents, $j \in F$ where F is the identification of agents native to Foreign in the labour market.
Foreign Population

$$\overline{\mathrm{N^F}} = n_t^F + u_t^F \tag{4.3.6}$$

The effective population of Foreign, N_t^F is given by.
Effective Foreign Population

$$N_t^F = \overline{\mathrm{N^F}} + m_t \tag{4.3.7}$$

The number of unemployed workers, u_t^i, where $i \in H, F$, searching for a job at the start of the period is given by,

$$u_t^H = 1 - (1 - \rho_n^H) n_{t-1}^H - (1 - \rho_n^M) n_{t-1}^M \tag{4.3.8}$$

And in Foreign,

$$u_t^F = 1 - (1 - \rho_n^F) n_{t-1}^F \tag{4.3.9}$$

Where ρ_n^j is the exogenous periodic destruction rate of employment for type j. The unemployment rate is given by $\widetilde{u_t^i}$.
Home Unemployment Rate

$$\widetilde{u_t^H} = 1 - n_t^H - n_t^M \tag{4.3.10}$$

Foreign Unemployment Rate

$$\widetilde{u_t^F} = 1 - n_t^F \tag{4.3.11}$$

[17] While illegal migration does not feature in the baseline model, i.e. $\aleph = 0$, we define it here to aid explanation of the model.

We introduce asymmetric search and matching frictions to reflect the different difficulties in gaining employment in the different sectors; primarily in Foreign where migrants experience increased frictions in searching for employment. For each type of employment j, there is a matching function of the form:
Matching Technology

$$m_t^j = a^j (v_t^j)^{\Gamma^j} (u_t^j)^{(1-\Gamma^j)} \qquad j \in [H, F, M] \tag{4.3.12}$$

Where v_t^j is the vacancies posted by firms, a^j reflects the efficiency of the matching process, and Γ^j, the elasticity of the matching technology with respect to vacancies. The probability of a worker being hired, ζ_t^j, the probability of the firm filling a vacancy, γ_t^j, and labour market tightness θ_t^j are given by,
Probability of Finding Employment

$$\zeta_t^j = \frac{m_t^j}{u_t^j} \tag{4.3.13}$$

Probability of Filling a Vacancy

$$\gamma_t^j = \frac{m_t^j}{v_t^j} \tag{4.3.14}$$

Labour Market Tightness

$$\theta_t^j = \frac{v_t^j}{u_t^j} \tag{4.3.15}$$

Therefore, the law of motion of employment evolves according to
Employment Dynamics

$$n_t^j = (1 - \rho_n^j) n_{t-1}^j + m_t^j \tag{4.3.16}$$

4.3.2 Households in Home

The household has members working in Home or in Foreign. Those working in Foreign send remittances. In the household living in Home the income is pooled to allow the same consumption amongst all members irrespective of their employment status. The household experiences a utility function of the GHH form (Greenwood et al., 1988),
Home Household Utility Function

$$U_t = E_t \sum_{t=0}^{\infty} \beta^t \frac{\left(\psi_t c_t^H - \frac{\phi_0}{1+\phi} \left(h_t^H\right)^{1+\phi}\right)^{1-\sigma_c}}{1 - \sigma_c} \tag{4.3.17}$$

Where E_t is the expectations operator at time t, β is the intertemporal discount factor, c_t^H total household consumption in Home of the final good, h_t^H is the hours worked by the household, σ_c is the coefficient of relative risk aversion, ϕ_0 is the weight on labour hours, and ϕ is the inverse of labour supply elasticity. To reflect the high volatility of consumption in emerging economies, the household is subject to a preference shock on consumption, ψ_t, which is a first order autoregressive process. Where ρ^ψ is the autoregressive parameter, and ε_t^ψ is an i.i.d. shock with zero mean and a constant variance, σ_ψ^2.

Consumption Preference Shock

$$\ln \psi_t = \rho^\psi \ln \psi_{t-1} + \varepsilon_t^\psi \tag{4.3.18}$$

The household in Home consumes units a final consumption good, c_t^H and pays lump-sum taxes, Tax_t. The household smooths its consumption with bond purchases in the international financial market, d_t^* where $p_{H_t}^F$ is the price of the Foreign good in Home. Emigration costs are identified by Ω_t (see section 4.3.2.2). This is financed by labour earnings from Home, w_t^H, and remittances, Ξ_t, sent by household members working in Foreign. Additionally, households receive time invariant unemployment insurance, ub^H, paid to those searching in Home. The return paid on borrowing from the international markets are given by r_t. Carried over stocks of bonds are deflated by the gross growth rate of working-age population $\frac{N_t^H}{N_{t-1}^H} = g_t^H$ for Home. The households receive dividends from the firms, Π_t.

Budget Constraint in Home

$$c_t^H + Tax_t + p_{H_t}^F \frac{d_{t-1}^*}{g_t^H}(1+r_{t-1}) + \Omega_t = w_t^H n_t^H h_t^H + \lambda_t^H u_t^H ub + p_{H_t}^F d_t^* + \Pi_t + p_{H_t}^F \Xi_t \tag{4.3.19}$$

The agents working in Foreign are credit constrained so finance their consumption, lump-sum taxes. and remittances by the labour income, w_t^M. A number of migrants in Foreign are eligible for unemployment insurance, ub^M, which is adjusted using the replacement rate. The migrants who are working illegally in Foreign, receive a reduced income, the rate $\Delta^w \in (0, 1]$ is fraction that they earn of the wage paid to legal migrants.[18]

Budget Constraint of Migrants

$$c_t^M + \Xi_t + Tax_{M_t} = w_t^M n_t^M h_t^H (1 - \aleph_t) + w_t^M n_t^M h_t^H \aleph_t \Delta^w + ub^M u_t^M \tag{4.3.20}$$

[18] As well as loss of labour income, some additional employment benefits are lost. For simplicity, these are accounted for by Δ^w

The consolidated household budget constraint combines equations 4.3.19 and 4.3.20.
Consolidated Household Budget Constraint

$$c_t^H + Tax_t + p_{H_t}^F \frac{d_{t-1}^*}{g_t^H}(1+r_{t-1}) + \Omega_t + p_{H_t}^F c_t^M + p_{H_t}^F Tax_{M_t} =$$
$$w_t^H n_t^H h_t^H + \lambda_t^H u_t^H ub^H + p_{H_t}^F ub^M (1-\lambda_t^H) u_t^H + p_{H_t}^F d_t^* + p_{H_t}^F w_t^M n_t^M h_t^H (1-\aleph_t) + p_{H_t}^F w_t^M n_t^M h_t^H \aleph_t \Delta^w + \Pi_t \tag{4.3.21}$$

The final consumption bundle is formed of a Home produced good, c_t^h, and a Foreign produced good, c_t^f. The Home bias in the consumption bundle is identified by v, and θ is the elasticity of substitution between the Home and Foreign produced good.

$$c_t^H = \left[v^{\frac{1}{\theta}} \left(c_t^h\right)^{\frac{\theta-1}{\theta}} + (1-v)^{\frac{1}{\theta}} \left(c_t^f\right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{4.3.22}$$

The price index of the final good, P_t, is chosen to be numeraire. Consequently, all other prices are expressed relative to the Home final good.

The optimisation problem for the household is given by:

$$\max_{c_t^H, n_t^H, n_t^M, d_t^*} \beta^t \sum_{t=0}^{\infty} E_t \frac{\left(\psi_t c_t^H - \frac{\phi_0}{1+\phi}\left(h_t^H\right)^{1+\phi}\right)^{1-\sigma_c}}{1-\sigma_c}$$

$$w_t^H n_t^H h_t^H + \lambda_t^H u_t^H ub + p_{H_t}^F ub^M (1-\lambda_t^H) u_t^H + p_{H_t}^F d_t^* + p_{H_t}^F n_t^M h_t^H w_t^M ((1-\aleph_t) + \aleph_t \Delta^w) + \Pi_t$$
$$= c_t^H + Tax_t + p_{H_t}^F \frac{d_{t-1}^*}{g_t^H}(1+r_{t-1}) + \Omega_t + p_{H_t}^F c_t^M$$

$$n_t^H = (1-\rho_n^H) n_{t-1}^H + \zeta_t^H u_t^H \lambda_t^H$$
$$n_t^M = (1-\rho_n^M) n_{t-1}^M + \zeta_t^M u_t^H (1-\lambda_t^H)$$

Using the law of motion for employment as in the optimisation problem allows the household to account the effect of the unemployment decisions on matches rather than taking the number of matches as in equation 4.3.16.[19] Equation 4.3.13 is substituted into equation 4.3.16 to account for the employment opportunities.

The household maximises utility and the budget constraint with respect to consumption, employment, search intensity, and financial market investments. Marginal utility of consumption is denoted by μ_t^H, and η_t^j is the Lagrange multiplier on the law of motion of employment of type j.

[19] This is only partial as the full effect would require equation 4.3.12 to be substituted into 4.3.16.

Marginal Utility of Consumption

$$\mu_t^H = \psi_t \left(c_t^H - \frac{\phi_0}{1+\phi} (h_t^H)^{(1+\phi)} \right)^{-\sigma} \tag{4.3.23}$$

Euler equation

$$\mu_t^H p_{H_t}^F = \beta \frac{\mu_{t+1}^H p_{H_{t+1}}^F}{g_{t+1}^H} (1 + r_t) \tag{4.3.24}$$

Search intensity

$$\mu_t^H ub^H + \eta_t^H \zeta_t^H u_t^H = \mu_t^H ub^M + \mu_t^H \frac{\partial \Omega_t}{\partial \lambda_t^H} + \eta_t^M \zeta_t^M u_t^H \tag{4.3.25}$$

The surplus of a match to the household is given by the marginal value of employment less the marginal value of unemployment. By the envelope theorem, the worker surpluses for employment in Home and Foreign is given by

Home Worker Surplus

$$\eta_t^H = -\frac{\phi_0}{(1+\phi)} (h_t^H)^{1+\phi} + (w_t^H h_t^H - ub^h) + (1 - \rho_n^h) \beta \frac{\mu_{t+1}^H}{\mu_t^H} \eta_{t+1}^H (1 - \zeta_{t+1}^H) \tag{4.3.26}$$

Migrant Worker Surplus

$$\eta_t^M = -\frac{\phi_0}{(1+\phi)} (h_t^H)^{1+\phi} + (w_t^M h_t^H ((1 - \aleph_t) + \aleph_t \Delta^w) - ub^m) + (1 - \rho_n^m) \beta \frac{\mu_{t+1}^H}{\mu_t^H} \eta_{t+1}^H (1 - \zeta_{t+1}^M) - \frac{\partial \Omega_t}{\partial u_t^{HM}} \tag{4.3.27}$$

4.3.2.1 Remittances

The role of exchange rate provides an additional dimension to the role of the remittances. We follow the remittance rule as in Mandelman and Zlate (2012) and Finkelstein Shapiro and Mandelman (2016). Remittances are assumed to be countercyclical and follow the wage premium of Foreign over Home.[20]

Remittances

$$\Xi_t = \bar{\Xi} \varrho^{\Xi} \left(\frac{w_t^M}{w_t^H} \right)^{\xi} \qquad \xi > 0 \tag{4.3.28}$$

The size of remittances is proportional to the wage premium of Foreign over Home. Since the elasticity of remittances to wages $\xi > 0$, an increase in the wage premium will result in an increase in remittances.

[20] See section C.4 of the appendix for remittances received in Venezuela

4.3.2.2 Emigration Costs

When a member of the household decides to emigrate, there is a monetary cost incurred. The migration costs are captured by ω_1 and ω_2, where ω_1 is the scale parameter and ω_2 is the elasticity of the migration cost. This migration cost includes the search, visa application fee, and the relocation fee. The search cost to the household is given by
Emigration Costs

$$\Omega_t = \omega_1 \Big((1 - \lambda_t^H) u_t^H \Big)^{\omega_2} \tag{4.3.29}$$

An extended version, where there are additional costs for illegal migration, which is dependent on the efficiency of the border force, would modify equation 4.3.29 to

$$\Omega_t = \omega_1 \Big((1 - \lambda_t^H) u_t^H + b f_t \Big)^{\omega_2}$$

4.3.3 Households in Foreign

The household in Foreign gains utility from consumption, c_t^F and disutility from labour services, h_t^F. They provide labour to the oil-consuming firms in Foreign, n_t^F. Income is pooled to allow the same consumption amongst all members irrespective of their employment status. The household experiences a GHH utility function.
Foreign Household Utility Function

$$U_t^F = \beta^{F^t} E_t \sum_{t=0}^{\infty} \frac{\left(c_t^F - \frac{\phi_0^F}{1+\phi^F} \left(h_t^F \right)^{1+\phi^F} \right)^{1-\sigma_{c^F}}}{1 - \sigma_{c^F}} \tag{4.3.30}$$

There is a specific discount factor β^F to Foreign, the parameters for coefficient of relative risk aversion, σ_{c^F}, weight on hours, ϕ_0^F, and inverse of labour supply elasticity, ϕ^F are specific to the household in Foreign.

The final consumption bundle for agents in Foreign is formed of the Home produced good, c_t^h, and the Foreign produced good, c_t^f. The Home bias in the consumption bundle is identified by v^F, and θ^F is the elasticity of substitution between the Home and Foreign produced good.

$$c_t^F = \left[(v^f)^{\frac{1}{\theta^F}} \left(c_t^h \right)^{\frac{\theta^F - 1}{\theta^F}} + (1 - v^f)^{\frac{1}{\theta^F}} \left(c_t^f \right)^{\frac{\theta^F - 1}{\theta^F}} \right]^{\frac{\theta^F}{\theta^F - 1}} \tag{4.3.31}$$

The price index of the final good, P_t^F, is chosen to be numeraire. Consequently, all other prices are expressed relative to the Foreign final good.

They face a budget constraint, where expenditures including consumption, c_t^F, purchase of one-period bonds, d_t^*, and pay lump-sum taxes, Tax_{F_t} are financed by labour earnings, w_t^F, unemployment insurance, ub^F, return on the previous period's bond purchases and profits from the firm, Π_t^F. Carried over stocks of bonds are deflated by the gross growth rate of working-age population $\frac{N_t^F}{N_{t-1}^F} = g_t^F$ in Foreign.
Household in Foreign Budget Constraint

$$c_t^F + p_{F_t}^F d_t^* + Tax_{F_t} = w_t^F n_t^F h_t^F + u_t^F ub^F + p_{F_t}^F \frac{d_{t-1}^*(1+r_t^F)}{g_t^F} + \Pi_t^F \tag{4.3.32}$$

The household maximises utility with respect to consumption, bond holdings, and employment status. Marginal utility of consumption is denoted by μ_t^F, and η_t^F is the Lagrange multiplier on the law of motion of employment.

$$\max_{c_t^F, n_t^F, d_t^*} \beta^{F^t} \sum_{t=0}^{\infty} E_t \frac{\left(c_t^F - \frac{\phi_0}{1+\phi}\left(h_t^F\right)^{1+\phi}\right)^{1-\sigma_c}}{1-\sigma_c}$$

$$n_t^F w_t^F h_t^F + \Pi_t^F + p_{F_t}^F \frac{d_{t-1}^*}{g_t^F}(1+r_{t-1}^F) + u_t^F ub^F = c_t^F + p_{F_t}^F d_t^* + Tax_t^F$$

$$n_t^F = (1-\rho_n^F) n_{t-1}^F + \zeta_{t-1}^F u_t^F$$

Marginal Utility of Consumption

$$\mu_t^F = \left(c_t^F - \frac{\phi_0}{(1+\phi)}(h_t^F)^{(1+\phi)}\right)^{-\sigma_{c^F}} \tag{4.3.33}$$

Euler Equation

$$\frac{1}{1+r_t^f} = \beta^F \frac{\mu_{t+1}^F}{\mu_t^F} \frac{p_{F_{t+1}}^F}{p_{F_t}^F} \frac{1}{g_{t+1}^F} \tag{4.3.34}$$

The surplus of a match to the household is given by the marginal value of employment less the marginal value of unemployment.
Foreign Worker Surplus

$$\eta_t^F = -\frac{\phi_0}{(1+\phi)}(h_t^F)^{1+\phi} + (w_t^F h_t^F - ub^f) + (1-\rho_n^h)\beta^F \frac{\mu_{t+1}^F}{\mu_t^F} \eta_{t+1}^F (1-\zeta_{t+1}^F) \tag{4.3.35}$$

By applying the envelope theorem, this is formed from the first order conditions for employment and unemployment.

4.3.4 Firms

There are three types of firms: final good producers, oil-producers, and oil-consuming final good producers. The firms in Home that produce the final good and oil, employ capital and labour services. In Foreign, the oil-consuming firms use a composite of capital inputs consisting of oil and physical capital, along with labour to produce a final good. Current employment conditions in each sector is determined by the labour market conditions in the previous period, therefore the firms are only able to influence the next period value by respecting the constraints imposed by frictions of labour markets, and any migration constraints in the case of Foreign.

4.3.4.1 Final Good Producers

There is a continuum of perfectly competitive firms in Home. Output is denoted by y_t^H with a relative price, $p_{H_t}^H$. Production uses capital, k_t^H, and labour, l_t^H, and output is subject to a productivity shock, ψ_t^a.[21]
Home Final Good Production Function

$$y_t^H = \psi_t^a (k_{t-1}^H)^\alpha (l_t^H)^{1-\alpha} \tag{4.3.36}$$

Where the parameter α governs the intensity of capital services in production. Total factor productivity, ψ_t^a is a first order autoregressive process. Where ρ^a is the autoregressive parameter, and ε_t^a is an i.i.d. shock with zero mean and a constant variance, σ_a^2.
Home TFP

$$\ln \psi_t^a = \rho^a \ln \psi_{t-1}^a + \varepsilon_t^a \tag{4.3.37}$$

Current capital stocks are equal to the previous period's capital stock, depreciated at a rate δ, and current investment, x_t^H with conventional investment adjustment cost functions, $\iota(x_t^H, x_{t-1}^H)$.[22] Carried over capital stocks are deflated by the inverse growth rate of the population in Home, g_t^H.
Capital Accumulation

$$k_t^H = (1-\delta)\frac{k_{t-1}^H}{g_t^H} + \psi_t^x \iota(x_t^H, x_{t-1}^H) \tag{4.3.38}$$

[21]For simplicity, we do not use oil in the final good production. The majority of oil produced by the country is exported and price is determined largely by international market forces and shocks.
[22]The derivatives of the adjustment costs are given in equations C.7.1 and C.7.2 in section C.7.

Investment Adjustment Costs

$$\iota(x_t^H, x_{t-1}^H) = \kappa^x x_t^H \left(1 - \frac{\phi_x}{2}\left(\frac{x_t^H}{x_{t-1}^H} - 1\right)^2\right) \tag{4.3.39}$$

The process of posting vacancies has a variable cost, $\kappa(v_t^H, v_{t-1}^H)$ that depend on the rate at which vacancies are posted, and a fixed cost κ^H as in Pissarides (1985) who assumes the firm has to pay a fixed cost before the bargaining process can begin.
Vacancy Posting Adjustment Costs

$$\kappa(v_t^H, v_{t-1}^H) = \kappa_v^H v_t^H \left(1 + \frac{\phi_v^H}{2}\left(\frac{v_t^H}{v_{t-1}^H} - 1\right)^2\right) \tag{4.3.40}$$

Due to inefficiencies in the labour market, firms are constrained by the laws of motion of employment. The firm's law of motion of employment reflects how vacancies affect the number of matches by substituting equation 4.3.14 into equation 4.3.16.

$$n_t^H = (1 - \rho_n^H)n_{t-1}^H + \gamma_t^H v_t^H$$

The firm seeks to maximise profits subject to the capital accumulation constraint, the law of motion of employment, labour costs, and vacancy posting costs. There are two Lagranian multipliers: Υ_t^H on the law of motion of employment, and Tobin's q, tq_t on capital accumulation.

$$\max_{x_t^H, k_t^H, v_t^H, n_t^H} \beta^t \sum_{t=0}^{\infty} E_t \mu_t \left[p_{H_t}^H y_t^H - w_t^H l_t^H - x_t^H - \kappa(v_t^H, v_{t-1}^H) - \kappa^H \gamma_t^H\right]$$

$$k_t^H = (1-\delta)\frac{k_{t-1}^H}{g_t^H} + \iota(x_t^H, x_{t-1}^H)$$
$$n_t^H = (1 - \rho_n^H)n_{t-1}^H + \gamma_t^H v_t^H$$

The maximisation process results in the following first order conditions.
Investment

$$1 = tq_t \frac{\partial \iota(x_t^H, x_{t-1}^H)}{\partial x_t^H} + E_t \beta \frac{\mu_{t+1}}{\mu_t} tq_{t+1} \frac{\partial \iota(x_{t+1}^H, x_t^H)}{\partial x_t^H} \tag{4.3.41}$$

Tobin's Q

$$tq_t = \beta \frac{\mu_{t+1}}{\mu_t}\left[\alpha \frac{p_{H_t}^H y_{t+1}}{k_t^H} + (1-\delta)\frac{tq_{t+1}}{g_{t+1}^H}\right] \tag{4.3.42}$$

Vacancies

$$\gamma_t^H(\Upsilon_t^H - \overline{\kappa^H}) = \frac{\partial(v_t^H, v_{t-1}^H)}{\partial v_t^H} + \beta \frac{\mu_{t+1}^H}{\mu_t^H} \frac{\partial(v_{t+1}^H, v_t^H)}{\partial v_t^H} \tag{4.3.43}$$

Where the expected benefit from posting a vacancy, $\gamma_t^H(\Upsilon_t^H - \overline{\kappa^H})$ is equal to the marginal cost of posting a vacancy. The Lagrangian multiplier on employment, Υ_t^H, gives the value that the firm places on an additional unit of employment. Using the envelope theorem, the surplus of the firm per additional unit of employment is given by:

Home Firm Match Value

$$\Upsilon_t^H = \left((1-\alpha)\, p_{H_t}^H \frac{y_t^H}{l_t^H} - w_t^H\right) h_t^H + \left(1 - \rho_n^h\right) \beta \frac{\mu_{t+1}^H}{\mu_t^H} \Upsilon_{t+1}^H \tag{4.3.44}$$

The firm raises profits when the marginal product of labour exceeds the wage paid to workers. When the match survives, the firm experiences a continuation value.

4.3.4.2 Oil-consuming Firms

The perfectly competitive firms in Foreign are oil consumers, in addition they employ capital and a composite of labour from Foreign natives and migrants as described in the production for output, y_t^F.

Foreign Production Function

$$y_t^F = \psi_t^{a^F} \left(I_t^F\right)^{\alpha^F} \left(L_t^F\right)^{1-\alpha^F} \tag{4.3.45}$$

The parameter α^F governs the intensity of capital inputs in production. Total factor productivity, $\psi_t^{a^F}$ is a first order autoregressive process. Where ρ^{a^F} is the autoregressive parameter, and $\varepsilon_t^{a^F}$ is an i.i.d. shock with zero mean and a constant variance, $\sigma_{a^F}^2$.

Foreign TFP

$$\ln \psi_t^{a^F} = \rho^{a^F} \ln \psi_{t-1}^{a^F} + \varepsilon_t^{a^F} \tag{4.3.46}$$

Labour supply to the oil-consuming firm, L_t^F consists of labour services native to Foreign, n_t^F, and migrants, n_t^M; where $1-\chi$ is the share of migrants employed by the firm. The workers are imperfect substitutes. Therefore, the impact of immigration will have negligible effects on the labour market conditions for Foreign natives.

Labour Supply

$$L_t^F = \left(n_t^F h_t^F\right)^{\chi} \left(n_t^M h_t^H\right)^{1-\chi} \tag{4.3.47}$$

The capital services, I_t^F, oil and capital, are given in a CES function where τ^F is the elasticity of substitution between oil and capital and $\Phi^F = \frac{\tau^F - 1}{\tau^F}$. The share of oil in capital services is given by ν^F. The firms' demand for oil is given by $y_{F_t}^O$.

Production Inputs

$$I_t^F = \left[\nu^F (y_{F_t}^O)^{\Phi^F} + (1-\nu^F)(k_{t-1}^F)^{\Phi^F}\right]^{\frac{1}{\Phi^F}} \tag{4.3.48}$$

Current capital stocks are equal to the previous period's capital stock, depreciated at a rate δ^F, and current investment, x_t^F subject to investment adjustment costs, $\iota(x_t^F, x_{t-1}^F)$.[23] Carried over capital stocks are deflated by the inverse growth rate of the population in Foreign, g_t^F.

Capital Accumulation

$$k_t^F = (1-\delta^F)\frac{k_{t-1}^F}{g_t^F} + \iota(x_t^F, x_{t-1}^F) \tag{4.3.49}$$

Investment Adjustment Costs

$$\iota(x_t^F, x_{t-1}^F) = \kappa^{x^F} x_t^F \left(1 - \frac{\phi_{x^F}}{2}\left(\frac{x_t^F}{x_{t-1}^F} - 1\right)^2\right) \tag{4.3.50}$$

Vacancies are posted at a cost with a fixed component, κ^F and κ^M as well as a variable cost $\kappa(v_t^F, v_{t-1}^F)$ and $\kappa(v_t^M, v_{t-1}^M)$ for the Foreign natives and migrants respectively.

Vacancy Posting Adjustment Costs

$$\kappa(v_t^F, v_{t-1}^F) = \kappa_v^F v_t^F \left(1 + \frac{\phi_v^F}{2}\left(\frac{v_t^F}{v_{t-1}^F} - 1\right)^2\right) \tag{4.3.51}$$

$$\kappa(v_t^M, v_{t-1}^M) = \kappa_v^M v_t^M \left(1 + \frac{\phi_v^M}{2}\left(\frac{v_t^M}{v_{t-1}^M} - 1\right)^2\right) \tag{4.3.52}$$

The oil-consuming firm seeks to maximise profits subject to the capital accumulation constraint, the law of motion for employment, labour costs, oil consumption, and vacancy posting costs. There are Lagrangian multipliers on the law of motion of employment, Υ_t^F and Υ_t^M, and Tobin's q, tq_t^F on the capital accumulation constraint.

$$\max_{x_t^F, k_t^F, y_t^O, v_t^F, v_t^M, n_t^F, n_t^M} \beta^{F^t} \sum_{t=0}^{\infty} E_t \Big[p_{F_t}^F y_t^F - x_t^F - P_{F_t}^O y_t^O - w_t^F n_t^F h_t^F - w_t^F n_t^M h_t^H((1-\aleph_t)+\aleph_t \Delta^w)$$
$$- \kappa(v_t^F, v_{t-1}^F) - \kappa(v_t^M, v_{t-1}^M) - \kappa^F \gamma_t^F - \kappa^M \gamma_t^M\Big]$$

$$k_t^F = (1-\delta^F)\frac{k_{t-1}^F}{g_t^F} + \iota(x_t^F, x_{t-1}^F)$$

[23] See section C.7.1 of the appendix, equations C.7.3 and C.7.4 for the derivations

$$n_t^F = (1-\rho_n^F)n_{t-1}^F + \gamma_t^F v_t^F$$

$$n_t^M = (1-\rho_n^F)n_{t-1}^M + \gamma_t^M v_t^M$$

The first order conditions for investment, physical capital, vacancies posted, and oil are given by,

Investment

$$1 = tq_t^F \frac{\partial(x_t^F, x_{t-1}^F)}{\partial x_t^F} + \beta^F \frac{\mu_{t+1}^F}{\mu_t^F} tq_{t+1}^F \frac{\partial(x_{t+1}^F, x_t^F)}{\partial x_t^F} \tag{4.3.53}$$

Tobin's Q

$$tq_t^F = \beta^F \frac{\mu_{t+1}^F}{\mu_t^F}\left[\frac{\alpha^F(1-\nu^F)(k_{t-1})^{\Phi^F-1} y_{t+1}^F p_{F_{t+1}}^F}{(I_t^F)^{\Phi^F}} + (1-\delta^F)\frac{tq_{t+1}^F}{g_{t+1}^F}\right] \tag{4.3.54}$$

Oil Demand

$$mpo_t = P_{F_t}^O = \frac{\alpha^F(\nu^F)(y_{F_t}^O)^{\Phi^F-1} y_t^F p_{F_t}^F}{(I_{t-1}^F)^{\Phi^F}} p_{*_t}^O \tag{4.3.55}$$

Vacancies

$$\gamma_t^F(\Upsilon_t^F - \overline{\kappa^F}) = \frac{\partial(v_t^F, v_{t-1}^F)}{\partial v_t^H} + \beta^F \frac{\mu_{t+1}^F}{\mu_t^F}\frac{\partial(v_{t+1}^F, v_t^F)}{\partial v_t^F} \tag{4.3.56}$$

$$\gamma_t^M(\Upsilon_t^M - \overline{\kappa^M}) = \frac{\partial(v_t^M, v_{t-1}^M)}{\partial v_t^M} + \beta^F \frac{\mu_{t+1}^F}{\mu_t^F}\frac{\partial(v_{t+1}^M, v_t^M)}{\partial v_t^M} \tag{4.3.57}$$

Where the expected benefit from posting a vacancy, $\gamma_t^j(\Upsilon_t^j - \overline{\kappa^j}) j \in F, M$ is equal to the marginal cost of posting a vacancy. The Lagrangian multiplier on employment, Υ_t^j, gives the value that the firm places on an additional unit of employment for in labour market $j \in F, M$. Using the envelope theorem, the value to the firm per additional unit of employment is given by:

Foreign Firm Match Value of a Foreign Worker

$$\Upsilon_t^F = \left(\chi\left(1-\alpha^F\right)\frac{p_{F_t}^F y_t^F}{n_t^F h_t^F} - w_t^F\right)h_t^F + \left(1-\rho_n^f\right)\beta^F \frac{\mu_{t+1}^F}{\mu_t^F}\Upsilon_{t+1}^F \tag{4.3.58}$$

Foreign Firm Match Value of a Migrants Worker

$$\Upsilon_t^M = \left((1-\chi)\left(1-\alpha^F\right)\frac{p_{F_t}^F y_t^F}{n_t^M h_t^H} - w_t^M\left((1-\aleph_t) + \aleph_t\Delta^w\right)\right)h_t^H + (1-\rho_n^m)\beta^F \frac{\mu_{t+1}^F}{\mu_t^F}\Upsilon_{t+1}^M \tag{4.3.59}$$

The firm makes a profit when the marginal product of a worker exceeds the wage, and the firm experiences a continuation value.[24]

[24] When there is no illegal migration, $\aleph_t$ is equal to zero, therefore the term $((1-\aleph_t) + \aleph_t\Delta^w) = 1$ in equation 4.3.59. All migrant workers earn the same wage level.

4.3.4.3 Oil Producing Firms

There is an oil-producing firm in Home that employs labour and capital services in the production process.[25] [26] Following Arora and Gomis-Porqueras (2011), we endogenise the oil sector in terms of production and price. However, the international price of oil, P_F^O, is subject to a shock. There is a price at Home which is determined by the demand in Foreign. Oil production is given by

Oil Production Function

$$y_t^O = \psi_t^{a^O} (k_{t-1}^O)^{\alpha^O} (l_t^O)^{\beta^O} \tag{4.3.60}$$

The elasticities of capital and labour in production are given by α^O and β^O respectively.[27] Oil production is subject to a productivity shock, $\psi_t^{a^O}$. This is a first order autoregressive process, where ρ^{a^O} is the autoregressive parameter, and $\varepsilon_t^{a^O}$ is an i.i.d. shock with zero mean and a constant variance, $\sigma_{a^O}^2$.

Oil Production TFP

$$\ln \psi_t^{a^O} = \rho^{a^O} \ln \psi_{t-1}^{a^O} + \varepsilon_t^{a^O} \tag{4.3.61}$$

The capital stock of the oil-producing sector is subject to investment adjustment costs. Here δ^O is the depreciation rate of capital used in the oil producing sector.

Capital Accumulation

$$k_t^O = (1 - \delta^O) \frac{k_{t-1}^O}{g_t^H} + \iota(x_t^O, x_{t-1}^O) \tag{4.3.62}$$

The oil-producing firm faces investment adjustment costs.[28]

Investment Adjustment Costs

$$\iota(x_t^O, x_{t-1}^O) = \kappa^{x^O} x_t^O \left(1 - \frac{\phi_{x^O}}{2} \left(\frac{x_t^O}{x_{t-1}^O} - 1\right)^2\right) \tag{4.3.63}$$

[25] Venezuela shows evidence of oil dominance. In 1976, PDVSA was nationalised giving the government control of Venezuela's oil reserves. The highs of oil revenue to GDP were 26% (using 1968 as the base year), during the years 1929-1969 the ratio exceeded 20% every year and has not been above 20% since. In 2016, 98% of Venezuela's exports was oil, of which these exports account for 12% of GDP (Source: OPEC).

[26] We do not nationalise the oil industry because evidence shows that resource cursed countries, and definitely oil sectors, takes away from the endogenous supply and demand. With decisions no longer related to the business cycle, rather corruption. As our focus is on migration decisions and policies, we allow for full endogeneity of supply and price which the latter is determined internationally.

[27] Elasticity of labour is calibrated such that the marginal product of labour is the same as in the final good producing firm.

[28] For derivations see section C.7.1, equations C.7.5 and C.7.6.

The oil producing firm faces capital and labour adjustment costs of the form.
Capital Adjustment Costs

$$\xi(k_t^O) = k_t^O \frac{\xi(k^O)}{2}\left(\frac{k_t^O}{k_{t-1}^O} - 1\right)^2 \tag{4.3.64}$$

Labour Supply Adjustment Costs

$$\xi(l_t^O) = l_t^O \frac{\xi(l^O)}{2}\left(\frac{l_t^O}{l_{t-1}^O} - 1\right)^2 \tag{4.3.65}$$

Labour supply is determined in conjunction with the final good producing firm in Home and the adjustment costs. As a result, the oil-producing firm optimises for capital where $P_{H_t}^O$ is the price of oil in Home.
Tobin's Q

$$tq_t^O = \beta \frac{\mu_{t+1}^H}{\mu_t^H}\left[\alpha \frac{y_{t+1} p_{t+1}^{O,H}}{k_t^O} + (1-\delta^O)\frac{tq_{t+1}^O}{g_{t+1}^H}\right] \tag{4.3.66}$$

The shock to oil price, $p_{*_t}^O$, is a first order autoregressive process. Where ρ^{p^O} is the autoregressive parameter, and $\varepsilon_t^{p^O}$ is an i.i.d. shock with zero mean and a constant variance, σ_{POIL}^2.
Oil Price Shock

$$p_{*_t}^O = (1-\rho^{p^o})p_{*_{t-1}}^O + \varepsilon_t^{p^O} \tag{4.3.67}$$

4.3.4.4 Wage Bargaining

Due to labour market frictions the firms and workers engage in wage bargaining. In a continuation of asymmetric search and matching frictions, workers experience bargaining power of the differing levels. Particularly migrants relative to the native workers in Foreign. The bargaining process uses the surpluses of the match from the worker and the firm. The workers' surplus, η_t^j (equations 4.3.26, 4.3.27, and 4.3.35) is the marginal value for the household of being employed, and the firm surplus, Υ_t^j, (equations 4.3.44, 4.3.58, and 4.3.59) is the expected value for the firm of filling a vacancy. The general forms follow:
Worker Surplus

$$\eta_t^j = -\frac{\phi_0}{1+\phi}((h_t^i)^{1+\phi} + w_t^j h_t^j - ub^j) + (1-\rho_n^j)\beta \frac{\mu_{t+1}^i}{\mu_t^i}\eta_{t+1}^j(1-(\zeta_{t+1}^j)) \tag{4.3.68}$$

Firm Surplus

$$\Upsilon_t^j = (p_{i_t}^i mpl_t^j - w_t^j)h_t^j + (1-\rho_n^j)\beta \frac{\mu_{t+1}^i}{\mu_t^i}\Upsilon_{t+1}^j \tag{4.3.69}$$

The bargaining power of the worker is given by ϑ^j. The splitting rule is governed by:

$$\Upsilon_t^j = \frac{(1-\theta^j)}{\theta^j}\eta_t^j \tag{4.3.70}$$

4.3.5 Fiscal Authority

In both Home and Foreign there exists a fiscal authority that consumes, collects taxes, and borrows (lends) from (to) the international financial markets in the case of a primary deficit (surplus).

The fiscal authority collects taxes from households via time variant lump sum taxes, Tax_t^i. The government spending, G_t^i, has expenditures from defence spending, bf_t^i and the government consumption, gc_t^i that is modelled as exogenous.
Government Expenditure

$$G_t^i = gc_t^i + bf_t^i \tag{4.3.71}$$

The government in Home keeps $gc_t^H = \overline{gc^H}$ as long as it can access financial markets. Using this rationale, we set $gc_t^H = \overline{gc^H}$ and allow changes in government consumption in Home to be reflected by a shock.
Government Consumption

$$gc_t^H = (1-\rho^{gc})gc_{t-1}^H + \rho^{gc}\overline{gc}^H + \varepsilon_t^{gc^H} \tag{4.3.72}$$

Thus, the primary deficit, def_t^i is given by:
Government Deficit

$$def_t^i = G_t^i - Tax_t^i \tag{4.3.73}$$

The fiscal authority in Home borrows to finance any deficit such that the budget constraint is given as,
Home Government Budget Constraint

$$G_t^H + (1+r_{t-1}^H)p_{H_t}^F\frac{d_{t-1}^*}{g_t^H} = Tax_t^H + p_{H_t}^F d_t^* \tag{4.3.74}$$

In Foreign, the government collects taxes from households that are native to Foreign, and from migrants, Tax_t^F.
Foreign Government Budget Constraint

$$G_t^F + (1+r_{t-1}^F)p_{F_t}^F d_{t-1}^* = Tax_t^F + p_{F_t}^F d_t^* \tag{4.3.75}$$

The left side of equation 4.3.74 provides the expenditures: G_t is government spending and the real debt service on debt. The right-hand side represents the sources of financing of tax revenue and new debt.

The two fiscal authorities have one significant difference between them. The fiscal authority in Home is known to be a defaulter on its debt obligations and Foreign is assumed to be risk-free.

4.3.6 Open Economy

This is a two-country model where prices are chosen to be numeraire. In this section we cover the market clearing conditions, international financial markets, and the terms of trade.

4.3.6.1 Market clearing and trade

To simplify the analysis, we aggregate the capital stocks and investment for the Home economy.

Total Home Capital

$$k_t = k_t^H + k_t^O \tag{4.3.76}$$

Total Home Investment

$$x_t = x_t^H + x_t^O \tag{4.3.77}$$

Let n denote the size of Home relative to the size of Foreign. The notation on price level corresponds to equation 4.3.22 for consumption. Demand for the domestically produced good is a function of consumption, investment, vacancy costs, and demand from abroad.

Market Clearing Conditions

$$\begin{aligned} y_t^h + p_{H_t}^O y_{F_t}^O = {} & v(p_{H_t}^H)^{-\theta}(c_t^H + x_t + \kappa^H v_t^H + \gamma_t^H \overline{\kappa^H} + \xi(k_t^O) + \xi(l_t^O) + G_t^H) \\ & + \frac{1-n}{n} v^f (p_{F_t}^H)^{-\theta}(c_t^F + x_t^F + \kappa^F v_t^F + \kappa^M v_t^M + \gamma_t^F \overline{\kappa^F} + \gamma_t^M \overline{\kappa^M} + G_t^F + c_t^M) \end{aligned} \tag{4.3.78}$$

$$\begin{aligned} y_t^F = {} & (1-v^F)\left(\frac{p_{H_t}^F}{rer_t}\right)^{-\theta}(c_t^F + x_t^F + \kappa^F v_t^F + \kappa^M v_t^M + \kappa^M v_t^M + \gamma_t^F \overline{\kappa^F} + G_t^F + c_t^M) \\ & + \frac{n}{1-n}(1-v)(p_{H_t}^F)^{-\theta}(c_t^H + x_t + x_t^O + \kappa^H v_t^H + \gamma_t^H \overline{\kappa^H} + \xi(k_t^O) + \xi(l_t^O) + G_t^H) \end{aligned} \tag{4.3.79}$$

Oil is exported to Foreign, therefore the GDP of Home consists of the foreign demand for oil and the domestically produced good.
Home GDP

$$gdp_t = p^H_{H_t} y^h_t + p^O_{H_t} y^O_{F_t} \tag{4.3.80}$$

GDP in Foreign is a function of the demand for the Foreign produced good.
Foreign GDP

$$gdp^F_t = p^F_{F_t} y^F_t - p^O_{F_t} y^O_{F_t} \tag{4.3.81}$$

The ratio of trade balance to GDP, tb_t analyses the trade balance of final goods. In steady state we set this to zero.
Trade Balance

$$tb_t = 1 - \frac{c^H_t + x_t + \kappa^H v^H_t + G^H_t}{gdp_t} \tag{4.3.82}$$

4.3.6.2 International Financial Markets

In equation 4.3.82, we analysed the balance of the final good. Now we analyse the balance of financial assets. The net stock of financial assets is determined by the inverse growth of GDP, change in price of debt, and the trade balance.
Net Foreign Assets

$$d^*_t = (1 + r^H_{t-1}) \frac{gdp_{t-1}}{gdp_t} \frac{p^F_{H_t}}{p^F_{H_{t-1}}} \frac{d^*_{t-1}}{g^H_t} + tb_t \tag{4.3.83}$$

The interest rate payable on debt, r^H_t, following the small open economy nature of Home is dependent upon the world interest rate, r^F_t and a premium term. In addition to the standard method of closing a model with a debt elastic interest rate premium (Schmitt-Grohe and Uribe, 2003) we add two terms to the equation following Drechsel and Tenreyro (2018).
Interest Rate

$$1 + r^H_t = \left(1 + r^F_t + \psi^r_t\right) \left(\exp^{\phi_{d^*}(d^*_t - \overline{d^*})} - \phi_p(p^O_t - \overline{p^O})\right) \tag{4.3.84}$$

The premium term is a function of deviations of debt and oil price from steady state, and a shock to the default risk interest rate premium, ψ^r_t. The parameters $\phi_{d^*} > 0$ and $\phi_{p^O} > 0$ govern the sensitivity of the interest rate with respect to the deviations from the steady state debt and oil price respectively. The assumption that the interest rate responds to oil price changes is important in determining the effect of oil price fluctuations for the business cycle. The shock to the interest rate premium is due to exogenous changes in the country's interest rate which is considered one of the sources of fluctuations in the business cycle.[29] This is an exogenous process. Where ρ^r is the autoregressive parameter, and ε_t^r is an i.i.d. shock with zero mean and a constant variance, σ_r^2.

$$\psi_t^r = \rho^r \psi_{t-1}^r + (1-\rho^r)\overline{\psi^r} + \varepsilon_t^r \tag{4.3.85}$$

4.3.6.3 Terms of Trade

We assume that the consumption based price index corresponds to a CES aggregate of the Home and Foreign produced goods, where v is the share of Home-produced goods in the final consumption bundle and θ is the elasticity of substitution:

$$P^H = \left[v(P_H^H)^{1-\theta} + (1-v)(P_H^F)^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

Here terms of trade is defined as $tot = \frac{P_F}{P_H}$. In keeping with the two-country model, the price index in Foreign is given by

$$P^F = \left[v^f(P_F^H)^{1-\theta} + (1-v^f)(P_F^F)^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

Where v^f is the share of the Home produced good in final consumption.[30] Following these two definitions, we define the terms of trade as the price of the Home produced good in Foreign relative to the price of the Home produced good in Home, $tot_t = \frac{p_{F_t}^H}{p_{H_t}^H}$.[31] We assume that the law of one price holds for oil, where P_H^O and P_F^O is the price of oil at Home and Foreign respectively, such that

$$P_H^O = P_F^O rer_t \tag{4.3.86}$$

4.4 Migration Policies

In section 4.3, we set out the model where migration was unrestricted. However, in practice there are migration policies that are designed to restrict migration usually by

[29](Schmitt-Grohe and Uribe, 2017, p. 486) calculate the premium for Venezuela to be 9.23% for years not in default, and 9.24% for all years (table 13.4) meaning that there is not a significant impact from the default years (at least in this sample). Venezuela is considered a serial defaulter hence the high premium which is above the average for the sample given of 5.5% (and 4.5% for years not in default). The sample includes Argentina, Brazil, Chile, Colombia, Mexico, the Philippines, Turkey, and Venezuela who have all experienced on external default or debt restructuring in the years 1824-1999. The calculations of the averages 1994-2013 using the JP Morgan EMBI data.

[30]We link the relative size of a country and its openness to trade as described in De Paoli (2009). Due to the asymmetries of the countries, their openness to trade differs.

[31]Full definitions of the price levels are given in section C.7.2.

the host nation. To implement these migration policies we use occasionally binding migration policies to limit legal and illegal migration.[32] The first constraint on migration is limiting legal migration by the inclusion of a cap on working visas in Foreign. The second constraint is aimed to reduce illegal migration with an increase in policing to prevent illegal border crossings.[33] The final constraint causes an exogenous destruction of matches by border force agents.

4.4.1 Requirement for a Working Visa

In Foreign, there is an exogenously determined number of working visas available, V^*. There are certain periods of time that the government in Foreign wishes to limit the number of immigrants, including worsening domestic economic conditions. This limits the ability of a firm and a would-be migrant to successfully apply for a visa. This may be due to an economic downturn in Foreign or the limit of visas being reached. The visa application is then only successful with a probability, q_{OBC_t}.
Probability of a Successful Visa Application

$$q_{OBC_t} = \frac{V^*}{u_t^{HM}} \tag{4.4.1}$$

At the lower bound, $q_{OBC_t} = 0$, all new visa applications are rejected making the worker illegal as the visa cap is reached. At $q_{OBC_t} = 1$ all new visa applications are accepted, therefore all migrants are legal. The constraint is not binding when $0 < q_{OBC_t} < 1$. The introduction of this constraint does not stop new migrants but introduces illegal migration. An illegal migrant faces worse labour market conditions compared to those employed legally as they have access to a higher wage, employment benefits, and job security. When this constraint is imposed, it leads to a lower worker surplus, and a downward pressure on the wage.

4.4.2 Reducing Migration

Under this policy, the governments are aware that the migration rate is increasing to a level that they believe is too high. There is an emigration rate, γ that they deem to be acceptable since too much migration causes labour market problems in Home and Foreign. An argument based on a perceived shortage of workers in Home would hamper the planned economic recovery and 'protecting' the labour market

[32] To implement the occasionally binding migration policies, we use dynareOBC (Holden, 2019) as described in Holden (2016).
[33] Mandelman and Zlate (2012) include a border patrol force, however this a shock to the model rather than a constraint.

in Foreign. In Foreign there would be a swell of labour supply. Return migration depends on the job finding probability which could be slow. The enforcement of the migration restrictions affect the probability of a match being made, henceforth the probability of finding employment.
Probability of Permitted Migration

$$p_{OBC_t} = \frac{\gamma \bar{N}}{n_t^M} \tag{4.4.2}$$

When the constraint binds at $p_{OBC_t} = 1$, migration is unrestricted and legal migration is dependent upon the number of visas issued. When the constraint binds at the lower end, $p_{OBC_t} = 0$, the acceptable migration rate is set to zero and no further matches are made.
Matching Efficiency under Migration Policy Enforcement

$$m_t^M = p_{OBC_t} a^M (u_t^{HM})^{\Gamma^M} (v_t^M)^{1-\Gamma^M} \tag{4.4.3}$$

There exists a border force employed by the government, bf_t^i, which is mobilised to patrol the borders to prevent illegal crossings. The scale of deployment for migrants to overcome is given by
Border Force Deployment Efficiency

$$bf_t = (1 - p_{OBC_t}) \left(bf_t^H + bf_t^F \right) \tag{4.4.4}$$

When the constraint is binding at $p_{OBC_t} = 1$, there is no extra efficiency in deployment, i.e. $bf_t = 0$. However, when the constraint binds, to a varying extent, the border force deployment increases proportionally. The scale of deployment increases emigration costs.

4.4.3 Illegal Migration Constraints

So far the constraints have assumed that all illegal migrants would be employed. Under this policy, a share of illegal employed migrants are removed from employment. This occurs when the perceived level of illegal immigration is too high. The new definition of the employed migrant stock is[34]

$$n_t^M = n_t^L + n_t^{IN} + n_t^{IJ} \tag{4.4.5}$$

[34]This is different from the effective migrant stock which includes $(1 - \lambda_t^H) u_t^H$.

Where n_t^M is the total number of employed migrants, n_t^L is the number of migrants employed legally, n_t^{IN} is the migrants employed illegally, and n_t^{IJ} is the number of migrants who were previously illegal which the border force is in the process of deporting. This form of existing match destruction is a decision that is exogenous to the firm or worker and is different from the separation rate ρ_n^m. The number of illegal migrants who have pre-existing matches destroyed is determined by r_{OBC_t}.

$$r_{OBC_t} = \frac{\gamma^I \bar{N}}{n_t^M} \tag{4.4.6}$$

Where γ^I determines the proportion of steady state illegal immigrants that the government tolerates. When the number of illegal migrants increase, r_{OBC_t} decreases. Henceforth, the number of illegal migrants who have had their employment exogenously destroyed is given by

$$n_t^{IJ} = (1 - r_{OBC_t}) n_{t-1}^I \tag{4.4.7}$$

This has an effect on the firm in Foreign as their total workforce is now $n^M - n^{IJ}$. The border forces also prevent new crossings of a proportion of illegal matches. The destruction of new matches is determined by the proportion of illegal migrants $\aleph_t$.

New Matches Destroyed

$$m_t^D = m_t^M \aleph_t \tag{4.4.8}$$

The sum of migrants who have had old and new employment matches destroyed denoted by M_t^D.

Total Destroyed Matches

$$M_t^D = n_t^{IJ} + m_t^D \tag{4.4.9}$$

Following the match destruction, the would-be migrants return to Home as unemployed agents. The definition for the number of unemployed household members searching for a job at the beginning of the period described by equation 4.3.8 is modified to include the matches exogenously destroyed.

$$u_t^H = 1 - (1 - \rho_n^H) n_{t-1}^H - (1 - \rho_n^M) n_{t-1}^M + M_t^D \tag{4.4.10}$$

We have separated these two types of match destruction to allow deportation of employed migrants by the Foreign border force and destruction of new matches by both forces, to vary at a different rate. They are, however, along the same theme so presented jointly.

4.4.4 A Note On OBCs

The use of OBCs in applied models is minimal and one of the main contributions of this paper. The algorithms discussed so far in section 4.4 have introduced OBCs as probabilities. By definition, a probability is double bounded by 0 and 1. These constraints create profound changes in the labour market dynamics.

In the code, the double bounded constraint for equation 4.4.1 is written as

$$q_{OBC_t} = \max\left[0, \min\left[\frac{V^*}{u_t^{HM}}, 1\right]\right] \tag{4.4.11}$$

As the policy only categorises migration as legal and illegal, the effects are shown through $\aleph_t$ which affects the model as shown in section 4.3. Once migration is constrained by the second policy, as described in section 4.4.2, the probability, p_{OBC_t} is introduced by

$$p_{OBC_t} = \max\left[0, \min\left[\frac{\gamma\bar{N}}{n_t^{M}}, 1\right]\right] \tag{4.4.12}$$

This policy affects the job-finding probability for migrants (equation 4.3.13) where $j = M$. As there are now additional costs to emigration, p_t is introduced to equation 4.3.29. Finally we introduce the extension of match destruction by using r_{OBC_t}. Even though the introduction of these constraints to the model appears limited, there is a significant effect on the model dynamics.

$$r_{OBC_t} = \max\left[0, \min\left[\frac{\gamma^I\bar{N}}{n_t^{M}}, 1\right]\right] \tag{4.4.13}$$

4.5 Calibration and Estimation

Using the baseline model in section 4.3, we calibrate some model parameters and use Bayesian estimation techniques to study the business cycle characteristics of a resource cursed country, and an oil-consuming country. First, we present the calibration and estimation of the non-linear model where data is used for Venezuela for Home and the United States for Foreign. We estimate the baseline model without any constraints for accuracy purposes. In sections 4.5.4 we analyse the business cycle contributions.

4.5.1 Calibration

The parameters used in the model are shown in table 4.1. These include parameters that have been taken from the literature or calibrated, calculated to the steady state targets of values in the national accounts, and some parameters are implied by the steady state of the model.

The values for discount factor and depreciation are adjusted for an annual frequency as we take the model to annual data. The discount factor for Foreign is standard for a developed economy in the literature. For Home, the discount factor is calculated by adding the steady state premium as calculated in Schmitt-Grohe and Uribe (2017) of 9.23%, which the steady state value of the premium shock, ψ^r.[35] The capital elasticity of substitution in the final good producing firms in Home are taken from Drechsel and Tenreyro (2018), and for Foreign is standard from the literature. The values for inputs of the oil-consuming firms are taken from Arora and Gomis-Porqueras (2011). The depreciation rate for Home is taken from Garcia-Cicco et al. (2010) and for Foreign from Albonico et al. (2014). The adjustment costs for investment and vacancies are taken from Bodenstein et al. (2018). The difference in wages between legal and illegal migrants is taken from Borjas and Cassidy (2019) with a value of $\Delta^W = 0.65$. The share of illegal immigrants is calculated from data where an estimated 12.1 million (2014) (Baker, 2017) of the 42.4 million (United States Census Bureau) migrants are illegal, $\aleph = 0.285$. Due to asymmetries between the countries, they differ in size and openness to trade. The ratio of oil exports to oil production is an average of the barrels of oil exported to barrels of oil produced (in millions) during the observation years. We calibrate the openness to trade parameter for Home, γ^H to 0.3. Steady state calibration requires, $\gamma^H GDP^H = \gamma^F GDP^F$. In calibrating the model there are a number of steady state values that are targeted in the model which are presented in the middle panel of table 4.1 and the lower panel shows values that are determined by the steady state.

4.5.2 Data

We use annual data from Venezuela and the United States for the period 1920-1995. The data series are transformed to per capita real terms with 1968 and 2006 base years respectively, and detrended using the Hamilton (2018) filter to estimate the baseline model presented in section 4.3.[36] Estimation uses the cyclical component of eight data series. The data for Venezuela is sourced from Baptista (1997) includes GDP, oil production in millions of barrels, oil price, government consumption and private consumption. We use Venezuelan oil price rather than the world oil price as it directly relates to Venezuelan oil production. Data for the United States is sourced

[35] Equivalently $\beta = \frac{1}{1+r^F+\psi^r}$

[36] Data available is 1920-1995, in filtering the cyclical component of the business cycle the first three data points are lost.

Table 4.1 Calibrated Parameters

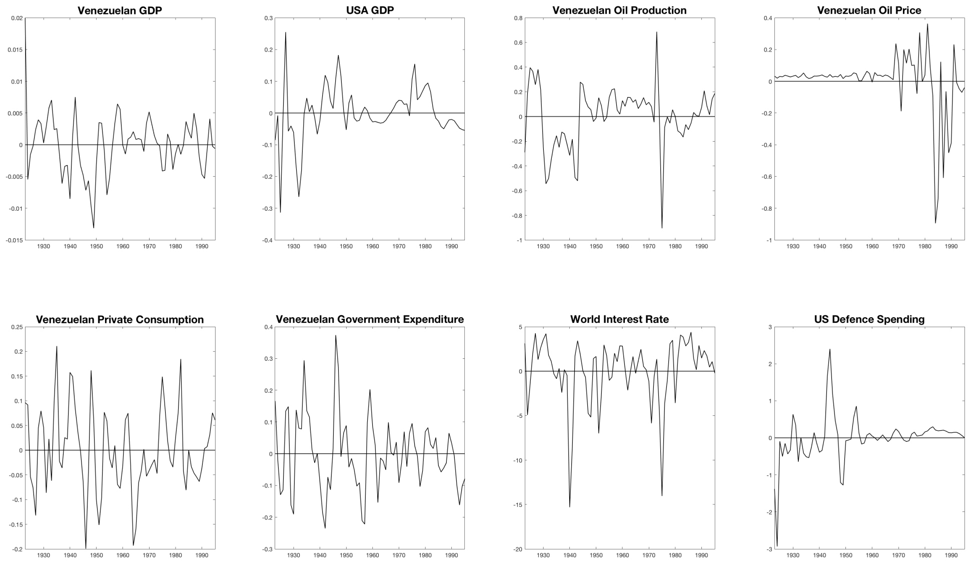
Parameter		**Value**	**Parameter**		**Value**
Model Parameters					
Home Discount Factor	β	0.8813	Intertemporal Elasticity	σ_{c^i}	1.0000
Foreign Discount Factor	β^F	0.9600	Inverse Frisch labour supply	ϕ^i	2.0000
Firm			*Labour Market*		
Production Function	α	0.3200	Job Destruction Rate	ρ_n^H	0.0500
	α^O	0.3600		ρ_n^F	0.0500
	α^F	0.3300		ρ_n^M	0.0700
Share of Oil in Production	ν^F	0.0500	Replacement Rate	$\frac{ub}{w}$	0.4000
EoS between Oil and Capital	τ^F	0.4400		$\frac{ub^F}{w^F}$	0.4000
Depreciation	δ^H	0.1255		$\frac{ub^M}{w^M}$	0.1000
	δ^O	0.1255	Remittances	ξ	0.9900
	δ^F	0.1170	EoS Open Economy	θ, θ^F	1.0100
Adjustment cost of capital	κ^x	1.0000	Relative size of Home	n	0.1500
Oil Sector Labour Elasticity	β^O	0.2350	Openness to Trade	γ^H	0.3000
Oil Sector Employment AC	ξ^{lo}	230	Matching Technology	Γ^j	0.5000
Oil Sector Investment AC	ξ^{ko}	230			
Vacancy Adjustment Costs	ϕ_v^j	3.0000	Wage Differential	Δ^w	0.6500
Emigration Costs	ω_1	0.5000	Emigration Costs	ω_2	2.0000
Steady State Targets					
Oil to GDP	$\frac{y^O P^O}{GDP}$	0.2800	Ratio of Oil Exported	$\frac{y_F^O}{y^O}$	0.9500
Interest Rate Premium	ψ^r	0.0930	GC to GDP	$\frac{G^H}{GDP}$	0.0700
			Foreign GC to GDP	$\frac{G^F}{GDP^F}$	0.0800
Implied by Steady State					
Scale on Hours	ϕ_0	0.7576	Remittances	ϱ^{Ξ}	0.0313
	ϕ_{0^F}	0.9482			
Bargaining Power	ϑ^H	0.9742	Matching Efficiency	a^H	0.8142
	ϑ^F	0.9680		a^F	0.8144
	ϑ^M	0.7766		a^M	0.7542

AC denotes adjustment costs; EoS denotes elasticity of substitution; GC denotes government consumption; and H = Home, F=Foreign, M=Migrant. The upper panel shows the model parameters, the middle panel shows the steady state targets (see appendix for data sources), and the lower panel shows the parameters implied by steady state.

from the Bureau of Economic Analysis and Balke and Gordon (1989) which includes GDP, and defence spending. The world interest rate uses the UK real interest rate.[37]

[37] The UK real interest rate is designed to represent the world interest rate and is calculated using the nominal interest rate published by the Bank of England minus the inflation rate as published by Office for National Statistics (ONS) as in Garcia-Cicco et al. (2010) and Drechsel and Tenreyro (2018).

Fig. 4.1 Data Observables



The eight observables used in the Bayesian estimation of the baseline DSGE model as described in section 4.3. The data presented is the cyclical component of the Hamilton filter applied. The horizontal axis identifies the year of the observation. The series have 73 observations at an annual frequency, 1923:1995 inclusive. The vertical axis shows deviation from trend. Measurement equations are $xobs_t = x_t - x_{ss}$ where $xobs$ is the observation of variable x, x_t is the actual value at time t, and x_{ss} is the steady state or expected value of variable x.

The time period 1920-1995 captures the rise and fall of Venezuela as explained by Bello et al. (2011) where in 1920 the ratio of Venezuelan GDP per capita to United States GDP per capita was approximately 20% to 90% in 1958. This has since returned to below 30% as shown in figure C.3. Figure 4.1 plots the observables used in estimation. Venezuelan GDP appears to have more volatile cycles that the USA. The cycle of Venezuelan oil production towards the end of the sample begins to be dependent less on the price and due to the nationalisation of PDVSA as explained in Restuccia (2018). We can see the volatility in particularly the oil price towards the end of the sample which relates to the world energy crisis of the 1970s was followed by the oil crises in the 1980s. Venezuelan private consumption is exhibits more cycles than GDP of either Venezuela or the United States which provides support for developing economies having highly volatile consumption relative to GDP. Venezuelan government consumption is quite high and procyclical as a consequence of macroeconomic populism and is explained in Moshiri (2015). The world interest rate is exogenous to the model. The effect of World War Two on United States defence spending causes the very distinct increase above trend.

4.5.3 Bayesian Estimation

We estimate eight persistence parameters and standard deviations of the shocks plus model parameters of the DSGE model with Bayesian methods, more specifically the MCMC algorithm, using the data presented in section 4.5.2.[38] Table 4.2 shows the mean values for priors and posteriors, 5^{th} and 95^{th} percentiles, prior shape, and deviation of the posterior (obtained after 3,000,000 iterations). We choose standard priors for the persistence parameters and standard deviations. The priors for sensitivity of debt deviations and the oil price deviations with respect to the interest rate used in the interest rate equation 4.3.84 are taken from Drechsel and Tenreyro (2018).

Table 4.2 Bayesian Estimation: Priors and Posteriors

Parameter Description		**Prior**	**Mean**	**90% HPD Interval**		**PDF**	**PstDev**
Autoregressive Parameters							
Home TFP	ρ^{a}	0.8500	0.9814	0.9679	0.9958	β	0.1000
Foreign TFP	ρ^{a^f}	0.8500	0.9967	0.9937	0.9997	β	0.1000
Oil TFP	ρ^{a^o}	0.8500	0.7239	0.5598	0.8865	β	0.1000
Interest Rate	ρ^{ψ^r}	0.7000	0.7209	0.6380	0.8024	β	0.1000
Defence Spending	ρ^{bff}	0.7000	0.6430	0.5235	0.7632	β	0.1000
Oil Price	ρ^{p^o}	0.8500	0.2584	0.2138	0.3046	β	0.1000
Government Consumption Home	ρ^{g^c}	0.7000	0.7334	0.5772	0.8800	β	0.1000
Consumption Preference	ρ^{ψ}	0.7000	0.7173	0.6687	0.7679	β	0.1000
Model Parameters							
Oil Price Elasticity	ϕ_o	0.1990	1.4943	1.4869	1.5000	Γ	0.1500
Debt Elasticity	ϕ_{d^*}	1.5000	1.4380	1.3846	1.4931	Γ	0.5000
Standard deviation of shocks							
Home TFP	σ_a	1.0000	11.3660	9.8281	12.9209	Γ^{-1}	0.5000
Foreign TFP	σ_{a^f}	1.0000	3.7339	3.2124	4.2312	Γ^{-1}	0.5000
Oil TFP	σ_{a^o}	1.0000	1.0247	0.8820	1.1676	Γ^{-1}	0.5000
Oil Price	σ_{p^o}	1.0000	14.9607	14.9099	15.0000	Γ^{-1}	0.5000
Defence Spending	σ_{g^c}	1.0000	0.4272	0.3267	0.5230	Γ^{-1}	0.5000
Interest Rate	σ_{ψ^r}	1.0000	4.6107	3.6559	5.5298	Γ^{-1}	0.5000
Government Consumption Home	σ_{bff}	0.5000	0.3924	0.3379	0.4451	Γ^{-1}	0.5000
Consumption Preference	σ_{ψ}	1.0000	29.1494	28.0767	30.0000	Γ^{-1}	0.5000

Results from the Bayesian estimation after 3,000,000 iterations. The first column describes the estimated parameter and column two shows the corresponding symbol. Column three gives the mean priors. Columns four to six show the mean, 5^{th} and 95^{th} percentiles of the posteriors. Column six shows the distribution function and the final column shows the deviation of the posterior. Distribution Functions: β Beta, Γ Gamma, and Γ^{-1} Inverse Gamma.

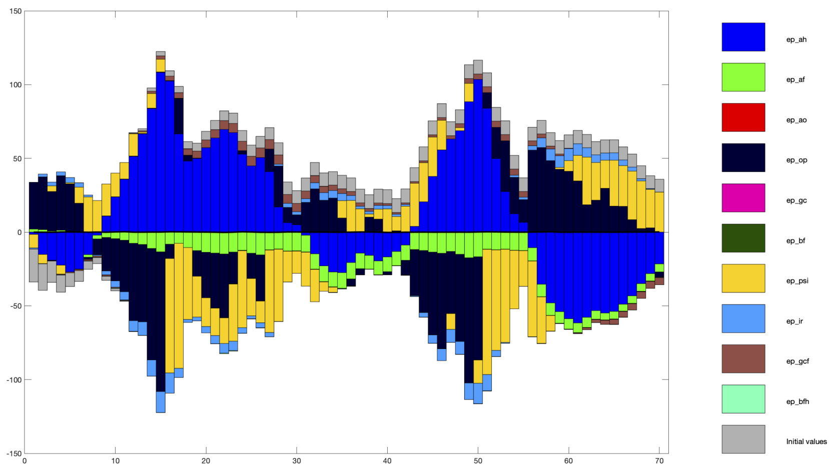
[38] We do not estimate the persistence parameters or standard deviation of defence spending in Home or government consumption in Foreign.

The autoregressive parameters differ from the estimates which suggests they are data driven. The interest rate is reactive to the existing deviations from steady state debt levels and the oil price. The standard deviations for the variables apart from government consumption and defence spending is sizeable. The literature on macroeconomics of emerging/developing economies acknowledges that private consumption is very volatile relative to output which the results from estimation are quite large. The size of the interest rate deviation is significantly large which is expected given the observable plot.

4.5.4 Historical Decomposition

In figures 4.2 and 4.3 we see the historical decomposition of private consumption and GDP in Home.

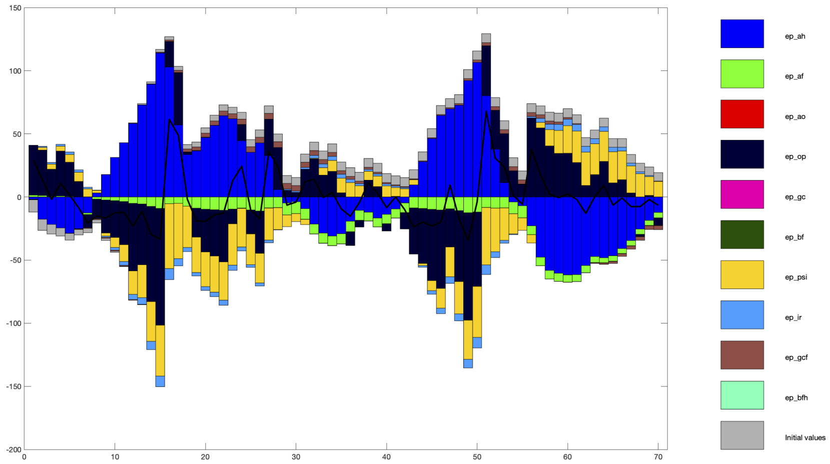
Fig. 4.2 Historical Decomposition of Private Consumption in Venezuela



The line shows the cyclical component of real price of Venezuelan oil per capita. The bars show the contribution of each of the shocks to the changes of the series at a given time. These estimates are a result of the estimation of the baseline model. Blue - Home TFP, Light green - Foreign TFP, Red - Oil TFP, Dark blue - Oil Price, Pink - Home Government Consumption, Dark green - Foreign Defence Spending, Yellow - Home Consumption Preference, Light blue - Interest Rate, Brown - Foreign Government Consumption, and Turquoise - Home Defence Spending.

The changes in consumption are driven mostly by productivity in Home, preference shocks, oil price, and productivity in Foreign. It is recognised that consumption is highly volatile in Home. For GDP, productivity in Home is expectedly the biggest source but also private consumption is a large driver. Of significance is also Foreign productivity and oil price. The effect of interest rate and government expenditure is minimal. In terms of the economics, consumption is procyclical albeit in developing economies highly volatile relative to GDP. Since oil price is a driver of the business

Fig. 4.3 Historical Decomposition of Venezuelan GDP

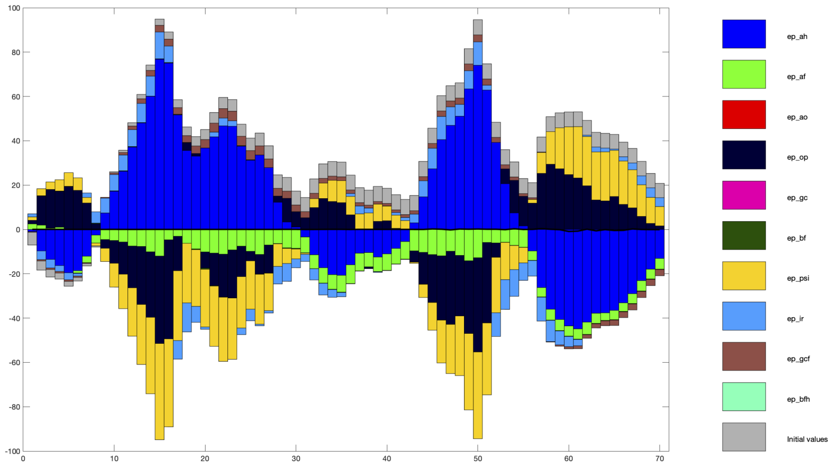


The line shows the cyclical component of real price of Venezuelan oil per capita. The bars show the contribution of each of the shocks to the changes of the series at a given time. These estimates are a result of the estimation of the baseline model. Blue - Home TFP, Light green - Foreign TFP, Red - Oil TFP, Dark blue - Oil Price, Pink - Home Government Consumption, Dark green - Foreign Defence Spending, Yellow - Home Consumption Preference, Light blue - Interest Rate, Brown - Foreign Government Consumption, and Turquoise - Home Defence Spending.

cycle, it follows that oil price is also a driver of private consumption. The relationship between Foreign TFP follows the same reasoning as Foreign is the larger economy and hence their demand for the domestically produced good. Oil production has minimal effects on either. Labour income is mostly from the Home final good producing firm and labour supply easily switches between oil production and final good production. The majority of oil is consumed in Foreign. The effect of the interest rate is perhaps small due to the model specification and that only three periods of sovereign debt defaults are experienced during the sample (1983-1988, 1990, and 1995-1997, 1998). The sample ends at 1995, and all of the sovereign defaults are towards the end of the sample.

The historical distribution of oil price is shown in figure 4.4 . Productivity in Home is a large contributor but interestingly, productivity in the oil-producing firm is not. This suggests a demand driven market. The interest rate has a more predominant presence than in either private consumption or production in Home which is similar to the findings in Drechsel and Tenreyro (2018). This can be due to the fact only a small proportion of oil is consumed in Home and that the price of oil in Home is determined by the marginal product of oil in Foreign and the real exchange rate. The relation to the national economy and Foreign TFP can change the demand for oil. These figures have shown that government expenditure is not a

Fig. 4.4 Historical Decomposition of the Venezuelan Oil Price



The line shows the cyclical component of real price of Venezuelan oil per capita. The bars show the contribution of each of the shocks to the changes of the series at a given time. These estimates are a result of the estimation of the baseline model. Blue - Home TFP, Light green - Foreign TFP, Red - Oil TFP, Dark blue - Oil Price, Pink - Home Government Consumption, Dark green - Foreign Defence Spending, Yellow - Home Consumption Preference, Light blue - Interest Rate, Brown - Foreign Government Consumption, and Turquoise - Home Defence Spending.

large driver in the business cycle but productivities of Home and Foreign are however, the model specification limits full analysis.

The results suggest that oil prices should be taken account of in the analysis of developing oil-producing nations. It provides a significant contribution to each of the variables presented.

4.6 Results

We take the values from calibration and estimation to evaluate the role of oil prices on the macroeconomy and the labour market in the baseline two-country model. We then introduce the migration policies in subsequent sections to show the effects. In section 4.6.1 we discuss the economics of a positive and negative oil price shock to the baseline model in Home and Foreign and the asymmetries. In sections 4.6.2 to 4.6.4 we show the results from a positive and negative oil price shock and the economic effects of the migration restrictions.

The use of migration constraints enables us to separate the effects of emigration from those of a negative oil price shock. The impulse responses plot the responses of the models with and without constraints. In sections 4.6.3 and 4.6.4, the responses of illegal and legal migration are compared to the model with constraints on legal migration (section 4.6.2). For each model, the presentation of the positive and negative shocks enable the illustration of the effects from the double bounded constraints.

4.6.1 Baseline Model

We begin our analysis with the baseline model where there are no constraints on migration. The impulse responses to a one standard deviation to the oil price are shown in figure 4.5. The upper panel shows the response to a positive shock to the oil price, and the lower panel a negative shock to the oil price.

In the baseline model all migration is legal. Those who want to migrate to find employment are free to. From the upper panel, a positive shock to the oil price increases output in the final good producing and oil-producing sectors in Home. In comparison to a TFP shock, where there would be a degree of risk sharing that would have positive effects for Foreign, there is a contractionary effect in Foreign. The price of the input for the oil-consuming firm has increased, which reduces demand and hence decreases output as per Kim and Loungani (1992). Oil production increases since higher price entices the firm to supply more, however as oil demand in Foreign falls, this results in higher domestic consumption. The change in output for Foreign is much smaller than GDP due to the increase in price of the oil imports and weaker real exchange rate. In Home, there is a greater increase in the production of the final good than the production of oil due to the fall in demand.

As a result of the final good producers in Home and the oil-producing firms increasing output, there is an increase in investment in capital stocks which raises the demand side of the economy also. Due to the expansionary effects being greater in the final good producing firm, the increase in labour supply shifts there. The firms post more vacancies to meet labour demand. There is a tightening of the labour market due to the increase number of vacancies and there are fewer unemployed workers native to Home. As a result of the tightening of the labour market, there is a higher probability of finding employment for the worker and lower probability for filling a vacancy for the firm in Home. This puts upwards pressure on wages.

For residents in Home, there is an increase in employment at Home including the number of hours supplied to the firms. Due to the favourable employment conditions in Home, there is a reduction in migration flows and the search intensity switches to Home.

Resulting from the narrowing wage premium of Foreign over Home, and fewer migrants being employed in Foreign, there is a decrease in remittances. However, as labour income has increased sufficiently in Home, there is still an increase in consumption levels. As a result of lower wages and labour hours by migrants, there is a decrease in labour income, and even though there is a decrease in remittances, there is a fall of consumption by migrants. The increase in labour market tightness is caused by the reduction in number of searchers. An increase in labour market tightness puts an upward pressure on the wage level. There is a sharp decrease of household taxes on impact of the price shock however this increases soon after. This is due to the increase in debt levels and price of debt in the government budget constraint. The overall debt dynamics concur with Drechsel and Tenreyro (2018).

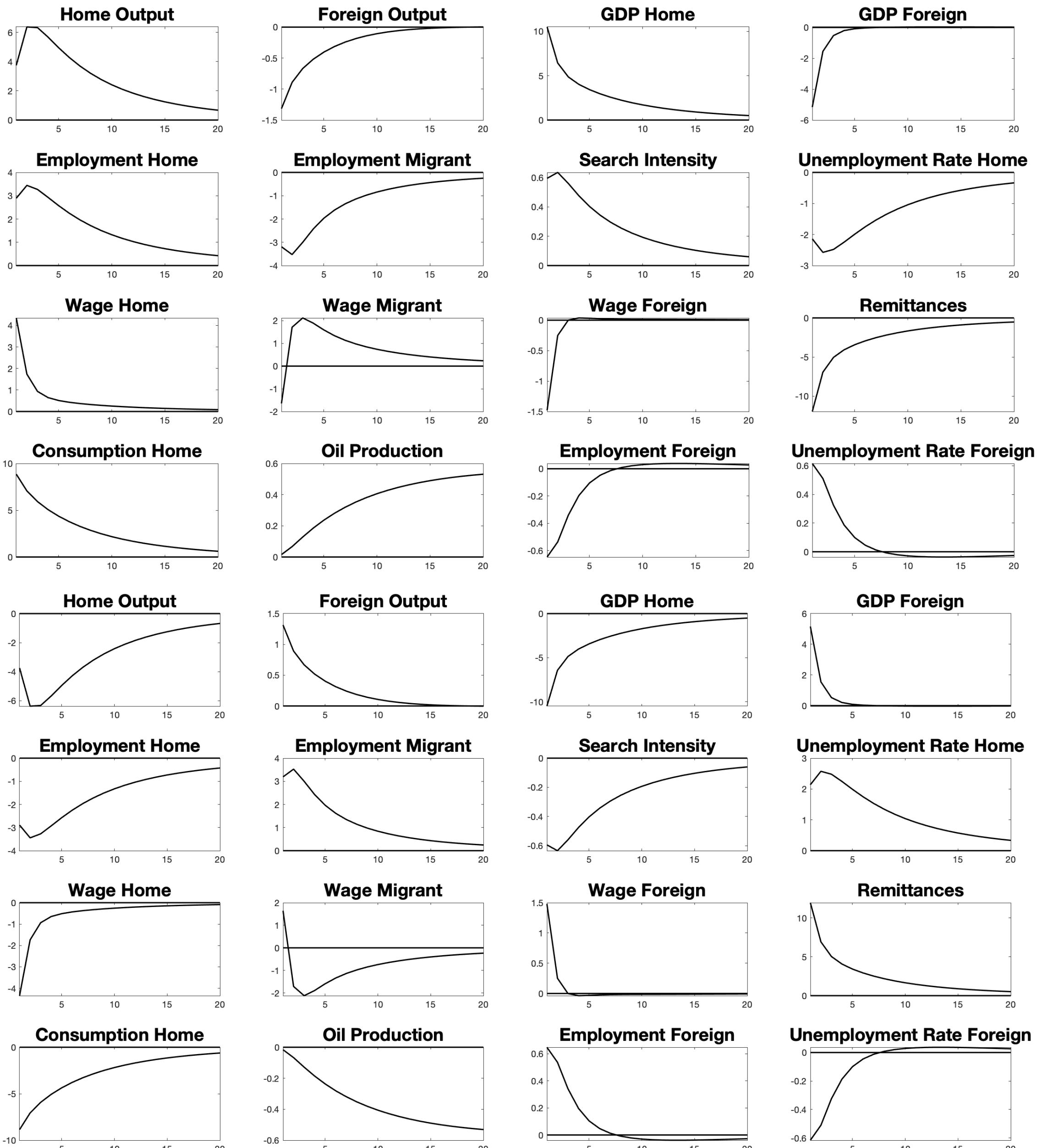
Even though the effects on wages and labour market tightness are similar for migrants and the workers in Home, the causes differ. Initially the productivity for Home workers decrease due to the higher labour supply, while for migrants the decrease is cause by the decrease in output. The following increase in Home is caused by more positions being filled by workers, thus allowing a decrease in hours per worker. For migrants, the increase is due to a lower labour supply which increases their individual productivity and from fewer vacancies being posted.

For households in Foreign, the negative effects on the economy reduce employment levels and labour hours supplied to the firm. Their labour market status is either employed or unemployed in Foreign. The reduction in labour market tightness is due to the increased number of searchers. Natives and migrants are imperfect substitutes, therefore the two labour markets are only weakly linked. The number of vacancies posted by the firm for natives and migrants both decrease, but migrants are able to switch their search to Home. The reduction in wage is also due to the lower marginal product of labour. Notably, the wage is only affected for a short period. Due to the lower labour income for Foreign households, they reduce their final good consumption. On impact there is large increase in taxation, but this quickly falls to below steady state levels. The government in Foreign is unaffected by the weakening of the real exchange rate as the debt is denominated in the domestic price level.

Due to the contractionary effect of the positive oil price shock on production in Foreign, there is also a reduction in their investment levels. The decrease is significant but short lasting. There is only a small reduction in per capita capital stocks, in part due to the stochastic growth. Fewer residents in Foreign (residents in Foreign include migrants), increase the value of per capita stocks ceteris paribus. The fall in labour demand reduces the labour supply to the firm.

Using the endogenous quantity of oil produced via supply and demand channel and endogenous price shows the effect on Home. As the price increases, there is a significant decrease in oil demanded from Foreign, hence only a small increase in production. Though the decrease in demand is small relative to the change in price. The demand side is dominant because the change in production is minimal. Home consumption increases to meet the excess supply.

Fig. 4.5 A model without constraints



IRFs for a 1SD shock to oil price in the model with no constraints or conditions on migration. The top panel shows a positive oil price shock, the bottom panel a negative oil price shock.

The results are mostly mirrored in the lower panel of figure 4.5. However, due to the low level of diversification of Home the negative effects are greater than the positive.[39] There is still a greater effect on the Home economy in terms of GDP and the final good producing sector than in Foreign.

As a result of the reduction in oil price, there is an increase in demand from abroad but a small reduction in supply. The increased Foreign demand reduces domestic consumption.

On the supply side, there is a reduction in demand for the Home final produced good. There is a decrease in employment and labour hours by the household. There is a significant increase in unemployment with a large shift of search intensity to Foreign. There is a reduction in labour market tightness which puts downwards pressure on wages, even though there is a small initial increase in the productivity of labour. There are significantly fewer vacancies posted.

For migrants there is a resulting wage fall due to the reduction in labour market tightness and labour productivity. The initial increase in wage is due to the expansionary effect of the economy before migration increases. The reduction in wage in Home is greater than for migrants and the employment prospects allow for an increased wage premium and hence remittances.

Workers native to Foreign gain in employment, and an increase in wages due to labour market dynamics and increased productivity. Increased productivity is smaller than the wage but is more persistent. There is an increase in labour market tightness resulting in an increase in probability of finding employment and reduction in filling a vacancy. There are fewer searchers and an increased number of vacancies to help fill the labour demand. Due to higher levels of labour income the households increase consumption.

Asymmetries of an oil price shock

As acknowledged by Hamilton (2003), oil price shocks are not symmetrical. Rather positive oil price shocks have greater economic effects than negative ones in oil consuming countries. From this we expect that the negative price shocks have a smaller effect on Foreign rather than a positive oil price shock. The effects on the oil producing nation is not researched to the same extent. However, Venezuela, and other developing oil-exporting nations experience asymmetries following oil price shocks as shown by Moshiri (2015) who uses data from 1970-2010. Negative oil price

[39] As there is no heteroskedasticity in oil price shocks in the DSGE model, the mirroring of results of magnitude can be expected.

shocks lead to prolonged contractions in the economy while oil price increases do not see prolonged expansions.[40]

We analyse here the asymmetries in the baseline model and then describe any changes when the migration policies are implemented in the following sections. The asymmetries for both oil-consuming and exporting nations is based on the size and length of the effects of oil price shocks. The oil producer experiences greater effects when there is a shock to the oil price because of how much the country is dependent on it. Where shocks are risk-sharing, the shocks in Home do not affect Foreign significantly due to the size asymmetry. However, risk-sharing shocks in Foreign can have a significant effect in Home.

While the asymmetry between a positive and negative shock is small, there are considerable asymmetries between Home and Foreign. For instance, the responses to GDP on both the demand and supply sides, as well as labour market effects are much greater for Home than Foreign. On the labour market, this includes migrants. The imperfect substitutability of workers is noticeable in Foreign as the oil price shock affects each differently. The effect of the shock on final good production, and GDP, has a prolonged effect on Home than Foreign. This is due to the path of oil production which has not started to return to steady state.

The asymmetries become more apparent the stricter migration policies become. However, a double bounded constraint is somewhat misleading, as there are no restrictions on return migration, or migration of Foreign natives to Home.

4.6.2 Legal and Illegal Migration

In this section we present the results which introduces illegal migration to the model. As the constraint is a probability, the lower bound is zero, and upper bound is one as described in equation 4.4.1. Illegal migration is only used to classify migrants and introduce the classification to the model. When the upper bound is reached, all new migrants are legal and conversely a lower bound implies all new migrants are illegal. As there was no illegal and legal migration in figure 4.5, there is only a solid line in these sub-plots.

The impulse responses to a positive oil price shock are shown in the upper panel of figure 4.6. The core analysis is the same as in the model without constraints. Of the responses presented, we now include the responses for legal and illegal migration in additional to total migration. Following the positive oil price shock, the decrease in employment of migrants is comparable to the baseline model, only insignificantly

[40] However, the asymmetries would only be more apparent if the parameters were to re-estimated following a negative oil price shock or the use of a GARCH model to estimate the difference in volatilities.

smaller decrease in migration. Though, when this is split into legal and illegal migration the sizes and dynamics differ slightly. There is a greater decrease in the number of illegal migrants. A lower migrant stock enables more migrants to be legal, the reason why illegal migration reduces the most, which is unsurprising as their wage is even lower. This reduces the fraction of new illegal migrants to zero when the lower bound is reached. When there are favourable employment opportunities at Home the net payoff is greater to remain. Legal migration, however, ends up increasing for a short time as illegal migration is zero.

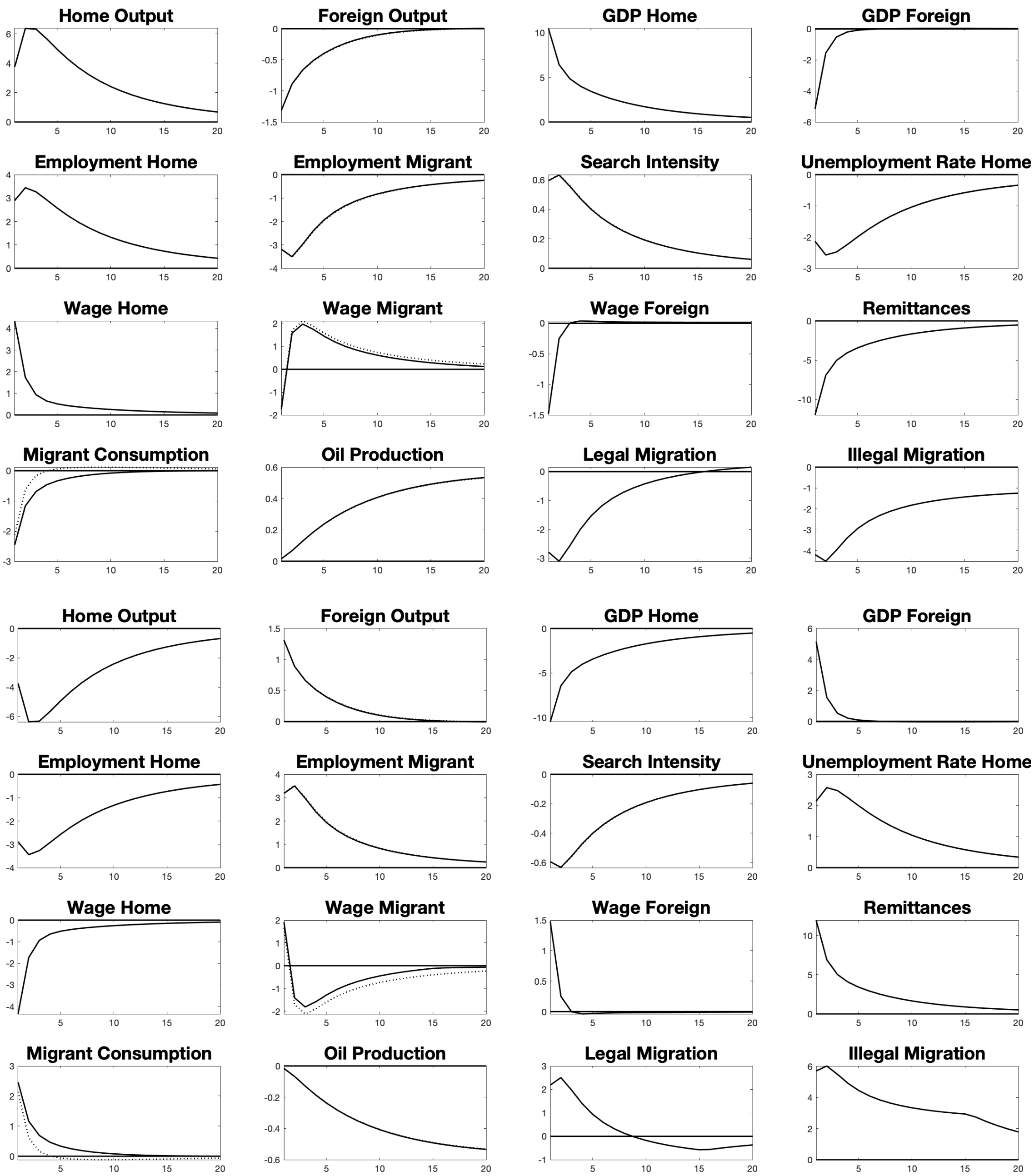
The noticeable change on migration comes when the migrants know that there are now negative effects with illegal migration. Once the wage difference is introduced to 4.3.27, the incentive to migrate is reduced. In the baseline model, and this model with illegal migration, the increase to the migrant wage decreases initially following the lower productivity but then increases as the reduction in number of workers outweighs the reduction in firm productivity. The eventual increase in the productivity is insignificantly less in the model with constraints. When there is illegal migration, the weighted average wage is lower. The response presented is to w_t^M, not weighted by the share of illegal migrants, $\aleph_t$, but firms make their employment decisions knowing the wage differential and net costs which is lower, hence more vacancies are posted.

Following a negative price shock, the main business cycle results from the model continue. In terms of the labour market for migrants, there is an initial increase determined in wage by the increase in productivity of the firm in Foreign. However, this effect lasts only a short time before turning negative and a smaller decrease relative to the baseline model. This then reduces the number of vacancies posted by the firm relative to the baseline model. In contrast, the wage for workers native to Foreign, increases by the same amount and does not differ from the baseline model. These results show support for the difference in wage as a factor in the decision of the migration process.

As in the positive shock, the change to migration is insignificantly less than the baseline model and the exact paths of legal and illegal migration differ. This is determined by the large increase in migration which the new migrants are increasingly likely to be illegal. The significance of this increase of illegal migrants illustrate how even a lower wage for illegal is still an incentive to migrant over the labour market conditions in Home.

The introduction of a constraint shows the first distinctive asymmetries between a positive and negative oil price shock, The response of illegal migration is greater following a negative oil price shock and legal migration follows a different path depending on when the bounds are reached. The response of the migrant wage differs.

Fig. 4.6 A model with legal and illegal migration



IRFs for a 1SD shock to oil price.
This model introduces the requirement of a visa for employment. Migrants are either illegal or legal which depends on the probability of attaining a visa q_{OBC_t}. At $q_{OBC} = 1$, all migrants are legal and at $q_{OBC} = 0$, all migrants are illegal.
The top panel shows a positive oil price shock, the bottom panel a negative oil price shock. The solid line shows the model with constraints, the dashed line without constraints.

Following a positive shock, there is only a small decrease to the baseline model, relatively insignificant, however, after a negative shock, the decrease is smaller. The distinct asymmetry of legal and illegal migration between a positive and negative oil price shock occurs due to the constraints reaching the bound and differences in labour flows. The response of illegal migration is larger following a negative oil price shock, however, it is slightly less persistent, so returns to approximately the same deviation after 20 periods. For legal migration, the response is marginally small, and returns to steady state levels sooner. As migration increases, and the number of visas are taken up, more migrants are illegal. Illegal workers receive a lower wage, which is more favourable to the hiring firm. As a result of this constraint, legal migration actually decreases due to the dominance of illegal migration. This form of migration policy is not supporting migration constraints, only showing the effect of lower wages, and the decisions of the firm based on labour demand.

4.6.3 Deterring Migration

We introduce the second constraint on migration that reduces the probability of finding employment and increases the migration costs. The impulse responses are shown in figure 4.7. In steady state, a match is determined by the destruction rate, ρ_n^j, and the steady state level of employment. There is a steady state reduction in the matching efficiency and hence probability of finding employment when this constraint is imposed and less than one. The effects of this constraint on migration are more pronounced than the one that introduced illegal migration. There is also an increase in emigration costs due to an increase in the efficiency of border force patrols. Of note, a one standard deviation shock to the oil price does not reach either bound for p_{OBC_t}.

First, we consider the response to a positive shock to the oil price. The reduction of final good production in Foreign is less under the model with migration restrictions and returns to steady state levels quicker. There is no significant difference in the production of the final good in Home. As a result of the smaller reduction in output for Foreign, the wage for workers native to Foreign returns to steady state slightly quicker, though the initial response is unchanged. Additionally, there is less of a reduction in labour market tightness which eases the downward pressure on wages, and soon returns to a tightening of the labour market which contributes to the upward pressure on wages. There is a smaller reduction in employment and smaller increase in unemployment.

The labour market implications for migrants is different. The increase in employment in Home is insignificantly lower under the model with constraints, with the unemployment rate decreasing marginally more. However, due to the constraints on migration and increased migration costs, the change in search intensity is lower. On impact the response is approximately two thirds of the baseline model. There is a significantly smaller decrease in the number of migrants. Fewer vacancies are posted initially, however, due to the cheaper labour, firms eventually post more vacancies relative to the baseline model. The wages are significantly lower than the model without constraints, or presence of illegal migration. As migration does not reduce as much, migrant labour hours increase, which means that their productivity decreases (rather than increases) and hence the decrease in wage. The migrants experience an increased job finding probability relative to the baseline model as a result of the size of increase in vacancies being posted relative to the decrease in unemployed workers searching in Foreign. As a result from fewer migration opportunities, there is an increased number of workers searching for employment in Home. Since the decrease in illegal migration is large, there is an increase in legal migration as the remaining migrants are permitted visas.

The effect of the constraints on migration actually have a small positive effect on Foreign output which is often why migration constraints are tightened during periods of economic downturn. As the emigration, and employment, of Home agents has fallen, there is a smaller reduction in output. There is an increase in the number of labour hours dedicated to the oil producing firm, as labour hours for the final good producing firm increase less in the model with constraints.

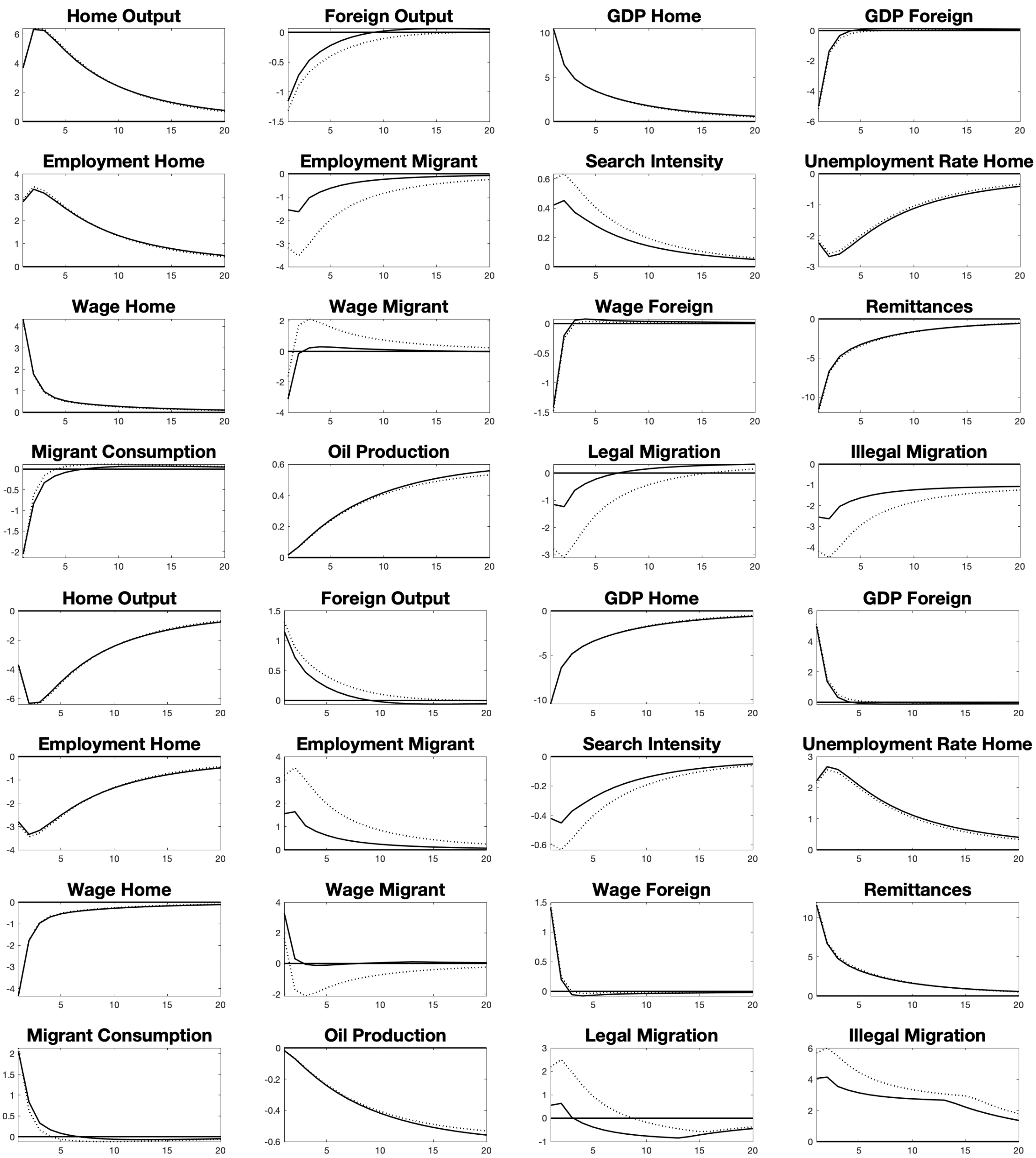
The response to both legal and illegal migration are smaller than the previous version of the model, as was total migration. The decrease in illegal migration will be greater than legal, as the number of searchers in Foreign reduces, more migrants will be legal. Legal migration increases when illegal has decreased enough, and the labour market conditions are favourable again.

Interestingly, the constraints imposed in Foreign begins to impact on Home. Even though the changes are insignificant, the increase in employment in Home is lower, though this is compensated by an insignificantly higher number of working hours. There is an insignificant effect on the final good producers in Home, with total unemployed workers native to Home increasing.

Following a negative oil price shock, the positive effects for the Foreign economy are less, for the firms and the household. For the households native to Foreign, they experience a lower level of increase to consumption and employment which negatively affects their utility. For the oil-consuming firm, they lose out in both labour supply and insignificantly lower levels of capital inputs.

For migrants, there are overall negative effects caused by the constraints. One positive is that there is an increase in wage, however, this is due to an increase in their productivity from fewer migrants being employed and their individual hours. In the baseline model, migration resulted in a decrease in the marginal product which

Fig. 4.7 Constraints on Migration



IRFs for a 1SD shock to oil price. There are constraints on legal and illegal migration. Migrants are either illegal or legal which depends on the probability of attaining a visa q_{OBC_t}. At $q_{OBC} = 1$, all migrants are legal and at $q_{OBC} = 0$, all migrants are illegal. When $p_{OBC} = 1$ there is no effect on the matching efficiency or increased emigration costs. At $p_{OBC} = 0$ there are no new matches. The top panel shows a positive oil price shock, the bottom panel a negative oil price shock. The solid line shows the model with constraints, the dashed line without constraints.

now increases. Due to fewer searchers in the labour market, and a lower probability of the firm filling a vacancy, there is less of a downward pressure on the wage as demonstrated by the labour market tightness loosening less. Additionally, as the wage of migrants has increased, the costs associated with employing that worker have increased. The negative effects include a lower probability of finding employment. There is a lower increase in illegal migrants and legal migrants. The increase in legal migration is only short lasting, it eventually falls below steady state due to the constraints on migration which reach the bounds.

There is an increase in total unemployment for Home because there are fewer employed migrants. The change in Home employment is insignificantly less than in the case of no constraints. It is labour hours that increase in Home with a large effective loss of labour hours by migrants compared to the baseline model, approximately an 80% reduction in the peak increase.

The asymmetries are further highlighted in this model, most notably in the paths of legal and illegal migration, the labour market dynamics for migrants, and the economy in Foreign. The gain from a negative oil price shock is smaller without the replicable increase in migrant labour supply. The migration decision is impacted by the efficiency of border force deployment. The results from a negative oil price shock presented imply negative effects as a result of migration policies in Foreign, and no noticeable gain.

4.6.4 Match Destruction

Until now, it is assumed that illegal migrants have been able to take up employment only that the number of matches is lower and illegal migrants experience poorer labour market conditions. Under this policy, prior to employment commencing or after a period of time, some illegal migrants are removed from employment as described in section 4.4.3. This introduces profound changes to the labour market for migrants. For the oil-consuming firm in Foreign, the labour they are supplied with by migrants, l_t^M, is reduced by the exogenous destruction of the previous period's illegal migrants and some of the new matches. The effects of the final constraint are shown in figure 4.8. In this model following the negative oil price shock, the gains for Foreign are smaller. There is an increase in destruction of matches, and an increase in the border patrols which increases the cost of emigration further.

The models presented so far in this section have had only small differences for some variables, now they are more pronounced. One reason for this, in Foreign particularly, is that there is a lower steady state due to the marginally smaller total labour force. The steady states for the case with jailed migrants results in lower wages for migrants and lower output for the oil-consuming firm. There is also a significant impact on Home.

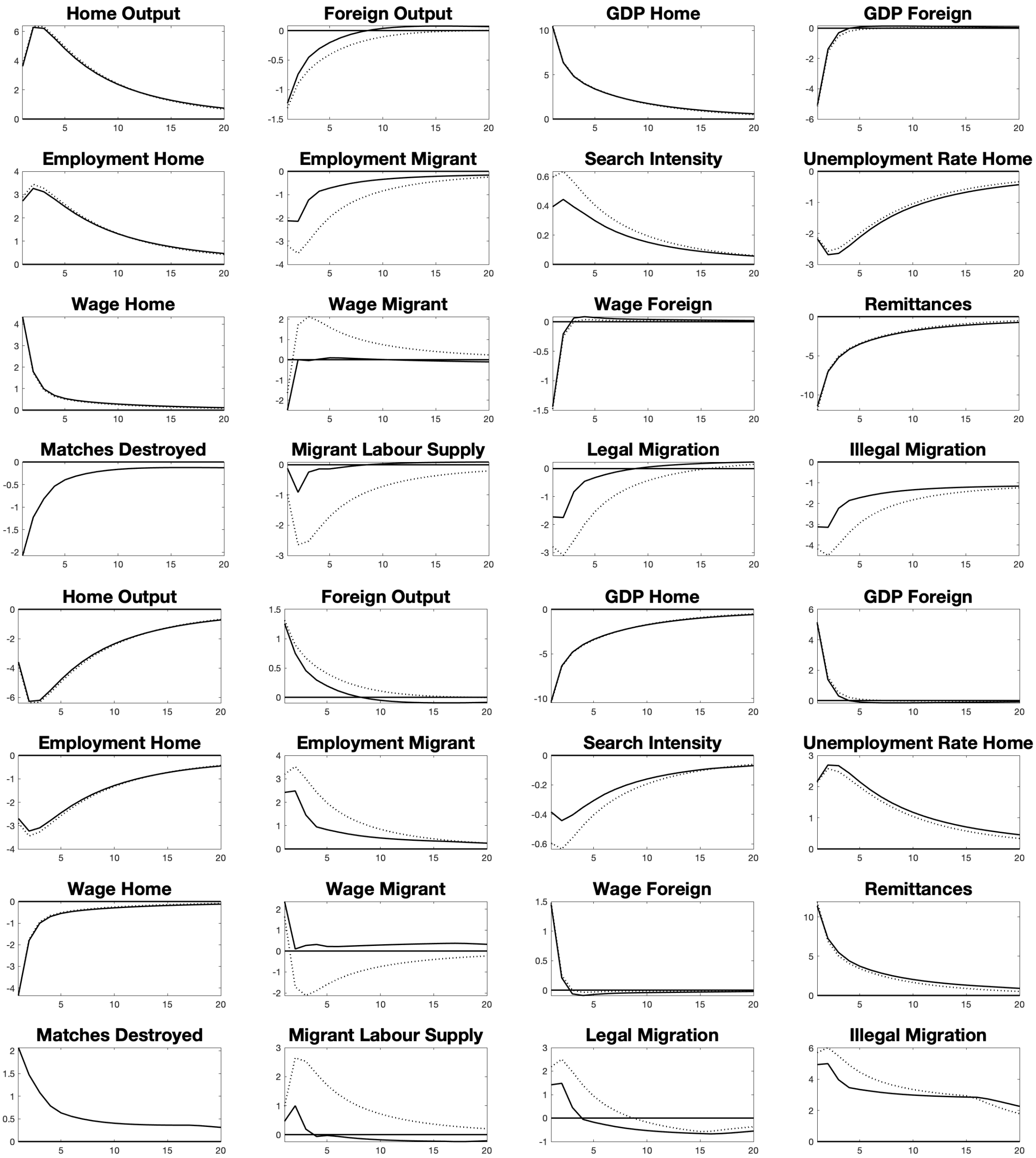
Firstly we consider the results from a positive oil price shock. Largely the results do not differ to those under the previous model, however, migrants experience more issues due to the exogenous destruction of matches. The decrease in migrant employment is approximately 40% smaller on impact due to fewer matches being made. Hence the decrease in labour hours, $(n_t^M - n_t^{IJ})h_t^H$, is a small fraction of the model without constraints. There is a greater decrease to illegal migration than legal, as the reduction in migration enables more migrants to be legal. There is a further decrease to the wage level which also negatively affects remittances. Migrants' consumption levels show a larger decrease. There is a smaller increase to labour market tightness, which together with lower changes to productivity, keeps the wage below steady state levels, and firms end up placing fewer vacancies.

As the labour market in Home tightens, there is an upward pressure on the wage. The increase in employment is marginally less because there are fewer illegal migrants from the previous period that had employment or matches destroyed. The economy in Foreign returns to steady state levels slightly quicker than in the previous models.

The consequences of this constraint are more significant following a negative oil price shock. For the firms and workers native to Foreign, the gains from the negative oil price shock in production and consumption are reduced to the point that they fall below steady state values. Firms have to continue posting vacancies until the labour demand is met. As the vacancies are unfilled due to poorer matching efficiency, the firm has to continue with the vacancy posting costs which diverts financial resources. On the demand side, there are lower levels of consumption, investment and henceforth capital stocks. Though the responses to employment and unemployment are small for the workers native to Foreign, employment falls below its steady state value. As a result of the smaller increase to output, less oil is demanded.

For migrants, employment increases but is significantly lower relative to the baseline model. The migrant labour hours are most important to the firm in Foreign. As a result of the destruction of some new and existing matches plus the number of individual working hours, the labour supply by migrants actually falls. This is costly to an economy that pays to post vacancies, only to have them destroyed. As labour hours per migrant worker decrease, the marginal product of labour increases, beyond that of the increased firm productivity. The effect of increased productivity, increases the wage. Such is the change in the labour market for migrants, that the probability of filling a vacancy is reduced immediately after the shock. There is still a decrease in labour market tightness, but this is significantly less than the baseline model. Workers native to Foreign see an insignificant decrease in their wage. Since employment does not increase as much in this model, there is a decrease in labour income which causes lower consumption levels.

Fig. 4.8 Match Destruction



IRFs for a 1SD shock to oil price. There are constraints on legal and illegal migration. Migrants are either illegal or legal which depends on the probability of attaining a visa q_{OBC_t}. At $q_{OBC} = 1$, all migrants are legal and at $q_{OBC} = 0$, all migrants are illegal. When $p_{OBC} = 1$ there is no effect on the matching efficiency or increased emigration costs. At $p_{OBC} = 0$ there are no new matches. Previous illegal migrants remain in employment with probability r_{OBC_t}, and new matches made by the firm are destroyed by based on the share of illegal migrants and efficiency of the border force deployment. The top panel shows a positive oil price shock, the bottom panel a negative oil price shock. The solid line shows the model with constraints, the dashed line without constraints.

The implications on the labour market for workers in Home has been relatively insignificant under the previous constraints, but now there is negative effects. The unemployment rate includes migrants jailed, this shows a large increase in the number of unemployed agents. As there are more searchers in Home, there is an increased probability of the firm filling a vacancy, and a greater decrease in labour market tightness which puts downward pressure on the wage. By removing the migrants from employment, there are wasted labour hours, $n_t^{IJ} h_t^H$, from the Home household, causing disutility without any gain.

The issue with legal migration decreasing for a while is due to the bounds being reached. For a short time, there are no more new legal matches. It is not until migration reduces and the number of searchers in Foreign decrease.

Figures 4.6, 4.7 and 4.8 have compared the impulse responses of a positive and negative shock to the oil price to the baseline scenario of no constraints as shown in figure 4.5. As a general conclusion, there are positive effects on the Home economy following a positive oil price shock and negative effects following a negative oil price shock. However, in Foreign, a positive oil price shock causes negative economic effects, and negative price shock has positive effects. Migration and oil prices are related to the business cycle. In particular, the oil price is a strong driver of the business cycle to the Home economy. The deviation in oil price from its steady state does affect the interest rate.

From the results we have shown the impact of migration policies following a non-risk sharing shock. When there is a positive oil price shock, the restrictions on migration, excluding match destruction, can have positive effects for both countries. However, these effects are limited, and in some cases statistically insignificant. For instance, employment and output return to steady state levels quicker in the model with weak immigration policies. The most significant change is the reduction in illegal migration and wage of migrants, which is increasingly negative due to the role of heterogenous labour markets.

After a negative oil price shock, the policies are harmful for Foreign especially, with migrants experiencing the extremes. The tougher the migration policies, the more noticeable the negative effects on Home are. Mostly, when migrants are unable to find employment in Foreign, they must remain unemployed in Home. The scenario of match destruction reflects extremes of migration policy can be harmful to the economies.

The results show (weak) support for migration policies when there are negative economic conditions in Foreign, as it provides an additional deterrent to migrants. But the constraints imposed in Foreign during positive economic conditions, that negatively affect the probability of finding employment for migrants, do not have support. Though migration policies are defended as being important to protect the economy, they can be counterproductive.

4.7 Conclusion

In this paper we have put forth a model with endogenous migration from a resource cursed country. The migration decision is based on the job finding probability which relates to the business cycle and migration constraints. To resemble migration policies, we have used three occasionally binding migration policies to reflect the current challenges that migrants face. This, and the endogenously determined demand and supply of oil are the contributions of this paper to the literature. To assess the role of emigration, we have used a negative oil price shock. Our model allows the diversion from a standard two-country model and migration by use of a non-risk sharing shock.

The use of constraints allows us to (partially) distinguish the effects of migration from those of the oil price. The results presented in this paper show that there are mixed effects to the Home economy from emigration. Following a negative oil price shock, the supply of oil falls as does the production of the domestically produced good which leads to a large fall in GDP due to the lower levels of investment, hence capital stocks. There is an increase in migration which reduces the increase in unemployment. Remittances allow consumption smoothing for the household in Home due to their countercyclical profile and provide an opportunity for risk sharing. Lower levels of migration reduce total remittance flows and increase unemployment in Home. With the migration constraints in place, unemployment in Home increased. The reduction in household income through labour income and remittances kept private consumption below equilibrium for longer and hence prolonged the fall in output and GDP of Home.

For the government, they receive lower levels of per capita taxes, but because of the increased interest rate, the price of debt increases. As their access to financial markets reduces, the only way to maintain the desired government consumption levels is to increase taxes. The debt levels increase in per capita terms ceteris paribus as there is emigration of workers. The one period form of debt does not allow full replication of the problems that the government would face. However, increased payments on debt and lower taxes from households, would increase the likelihood of default.

For Foreign there is an increase in production, employment, wages, and investment resulting from a less expensive capital input of oil. In addition, terms of trade become favourable. The migration restrictions reduce migration flows, which reduces total labour supply to the firm.

The role of the migration policies are significant, as they decrease migration and reduce the income of migrant workers. We have presented results to a positive and negative oil price shock to demonstrate the effects of the upper and lower bounds on the constraints and business cycle as a driver of emigration.

First, we imposed a limit on visas issued which introduced illegal migration to the model. This constraint decreased the number of legal migrants. However, the overall employment of migrants remained relatively unchanged.

The second constraint is based on reducing immigration by ways of increased efficiency of the border force and a reduction in the job finding probability. This began having a detrimental effect of the economy in Foreign. A reduced labour supply and the effects were worse in Home due to the increased number of unemployed agents. The overall levels of migration significantly decreased, as too has the search intensity in Foreign.

The final constraint imposed, created destruction of matches of new migrants and existing illegal ones. This had a greater significant negative effect on the economy of both Home and Foreign relative to the model without constraints. A decrease in migration results in a decreased labour supply to Foreign, a decrease in remittances and an increase in unemployment in Home. Due to the imperfect substitutability of migrant and native workers to Foreign, not all employment can be recovered from the Foreign workers.

There are policy implications for migration and business cycles. From a migration perspective we have seen that migration policies during an economic contraction in Home have negative effects as unemployment remains high, and there are lower levels of total remittances. During the economic expansion in Foreign, these migration policies restricts additional labour supply which in turn prevents as large of an economic expansion as is possible. Not only do these policies create expenditures that could be used for government consumption or lower taxes. They create illegal migration that makes migrants worse off, generates lower tax revenue for the government in Foreign, and makes Foreign natives no better off as the workers are imperfect substitutes. For countries which are heavily dependent upon commodities as a main source of exports and an industry that is sizeable for the economy is of a concern due to the high volatility and significant effect on the economy. Whereas a Home productivity shock is much less volatile. A shift to diversified exports is required as the reliance on non-renewables needs to change. Without diversification, the high volatility will continue which has detrimental effects on growth, investment, income distribution, and educational attainment (van der Ploeg (2011), Aizenman and Marion (1999), and Flug et al. (1998)). Reducing the dependency on commodities allows for progression and towards a more macroeconomically stable and sustainable economy. In addition, the reduction of the resource curse can promote democracy peacefully.

The role of macroeconomic populism is significant in the resource curse and in our analysis. For instance, Norway is a developed economy (non-OPEC nation) with oil production contributing to 20-25% of GDP and around 50% of exports (Bergholt et al., 2017). However, in contrast to Venezuela the non-oil economy of Norway is highly developed. Crucially, the revenues from oil are saved into a sovereign wealth fund so that future generations can gain from the wealth, the economy is protected during price fluctuations, and helps support fiscal spending during economic contractions which is the opposite to macroeconomic populist countries.[41] If Venezuela had the same features as Norway, the economic and political problems that they currently experience would unlikely be happening.

The research in this paper has introduced occasionally binding constraints into the emigration literature to create occasionally binding migration policies. The effects of the constraints on migration are negative for both the host and sending economies after a negative oil price shock. Migration was a way of reducing the economic burden of unemployed workers in Home and making them productive as a worker in Foreign. As many Venezuelan emigrants plan to return to Venezuela in the future, or be a temporary rather a than permanent migrant, the restrictions are making the economic problems worse in the short term. While Venezuela is facing a number of other problems at the moment, this model has enabled us to illustrate the effects of migration and migration policies on the business cycle. We have not analysed the role of sovereign default as the current debt dynamics do not allow for full representation, an area for future research.

[41] The Government Pension Fund Global is a sovereign wealth fund that invests in international assets and does not have any pension liabilities https://www.nbim.no/en/the-fund/about-the-fund/ .

Appendix C

C.1 Data Sources

Table C.1 lists the raw data used in analysis and the source. We use annual data for two reasons. Firstly, quarterly data is not available for some variables, but most importantly, the data that is available does not cover the full length of a business cycle. The data used in the estimation covers a time when politics was less influential.

Table C.1 Data Source

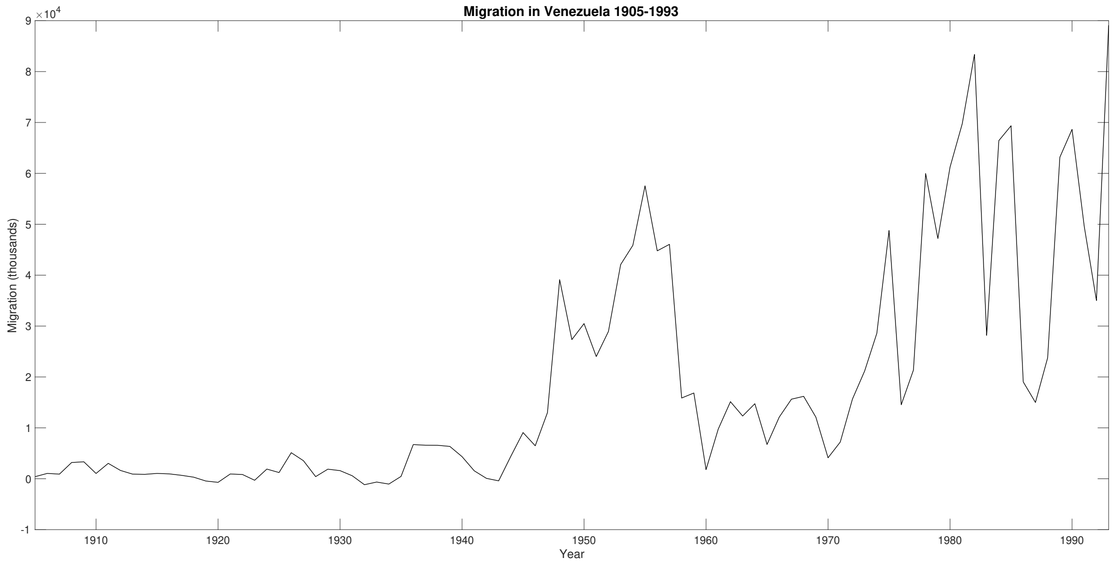
Data	**Source**
Oil Price	US Department of Energy
US CPI	OECD
Remittances in Venezuela	World Bank (GFDD.OI.13)
Venezuela-US Migration	Department of Homeland Security
	Office of Immigration Statistics
Venezuela-Latin America Migration	International Organisation for Migration
US Population	US Census Bureau
	National Intercensal Tables
US National Income Account	Bureau of Economic Analysis
US GDP Pre 1929	Balke and Gordon (1989)
US Defence Spending Pre 1929	United States. Bureau of the Census (1975)
Venezuela Economic Data pre 1995	Baptista (1997)
Venezuela GDP post 2000	World Economic
	Outlook Database

C.2 Migration in Venezuela 1905-1993

In section 4.1, we described recent trends of migration of Venezuela. Venezuela was long a host of migrants due to the attraction of employment within the oil sector.

The change in migration has been driven by the a large number of migrants returning to their home nation. Venezuelans who have migrated have long since been regarded as highly skilled, in addition, a large number of (recent) Venezuelans emigrants moving to Latin American nations move to countries with similar socialist tendencies, hence the seemingly disproportionate migration flow to Bolivia. The emigration to Colombia is largely due to its location. As a part of its history of a migrant host, in figure C.1 we show migration *to* Venezuela. The data before 1960 is the estimated migratory movement and the data from 1960 is the number of residences granted by the migration department.[1] Migration has been very volatile in Venezuela, and is heavily related to the business cycle. As shown in the data, and model, the oil prices is one of the main drivers of the business cycle and hence a driver of migration. The predictions of future migration is modelled by Bahar and Barrios (2018) using variables oil price, oil production, and foreign aid/remittances shows the relationship of oil price and production to migration.[2] In the fiscal year for 2018, Venezuelans

Fig. C.1 Migration in Venezuela



Annual migration in Venezuela (10,000s of people). For the years before 1960, the figures are for migratory movement. From 1960 to 1993, the figures are the number of residences granted by the migration department.
Source: Baptista (1997)

[1]These figures have been taken from Baptista (1997). Annual data is not available for intervening years

[2]https://www.brookings.edu/blog/up-front/2018/12/10/how-many-more-migrants-and-refugees-can-we-expect-out-of-venezuela/

submitted the highest number of affirmative asylum applications 28,485 in the USA. This is ahead of Guatamala (10,177), and El Salvador (9157).

For the fiscal years 2015 to 2018, Venezuela experienced some of the highest visa refusal rates which was comparative to those that were under travel bans imposed by the current administration. At 74%, this was the same as Libya and behind only Somalia, Iran, Yemen, and Syria.

C.3 Default and devaluation

We take data from each of Venezuela's four most recent defaults periods and the corresponding devaluation date as shown in table C.2.[3] In addition to the extraction of the table presented in Schmitt-Grohe and Uribe (2017), we note the devaluation date and the oil price.

Table C.2 Sovereign Default Episodes (1980-present)

Start of Default	**End of Default**	**Data Source**	**Devaluation**	**Oil Price****
1983	1988	BC, table 6		$58,51,55,52,23,31,26
1990	1990	BC, table 6		$35
1995	1998	BC, table 4		$22,27,23,14
2005	2005	CG, table 2	03/2005*	$56
2017			08/2018***	$ 44.09, 58.39

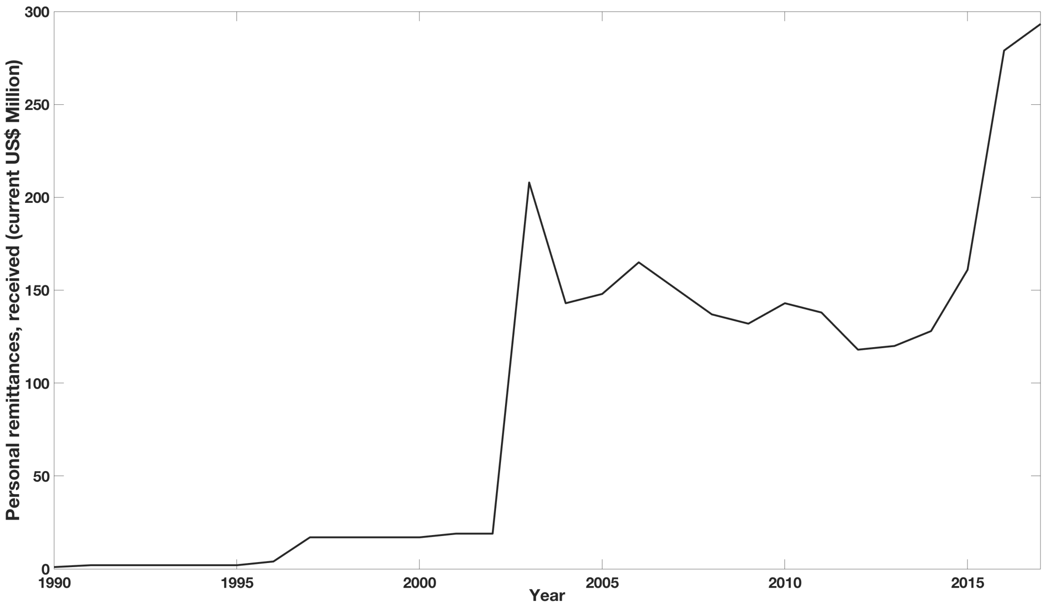
Data source: BC: Beers and Chambers (2006) ; CG, Chambers and Gurwitz (2014)
∗ In real terms. See figure C.4, own calculations
∗∗ On January 1, 2008, Venezuela introduced a new currency that eliminated three zeros from the national currency. The Bolivar was replaced with the Bolivar Fuerte.
∗ ∗ ∗ On August 20, 2018, Venezuela replaced the Bolivar Fuerte with the Bolivar soberano which was a 96% devaluation.

C.4 Remittances

Data suggests that remittances sent to Venezuela are countercyclical, indicating that in times of economic struggle there is an increase in remittances. Particularly illustrated by the sizeable increase since 2000, shown in figure C.2. Prior to 2000, the economy had been fairly strong, a change in the migration profile of Venezuela from host to sender significantly increased the size of remittances received.

[3] Included in the table is the current default period.

Fig. C.2 Remittances received in Venezuela 1990-2017



Remittances received by households in Venezuela for years 1990-2017 inclusive (current US$ Millions). There is a significant rise in the 18 years with two notable peaks. First in 2003, in an economic downturn and the political unrest including the coup and reinstatement of the then President Hugo Chavez. PDVSA workers went on strike, eventually the 18,000 of the strikers were sacked. These workers were replaced but the strike had shrunk production. The recent increase of 2015-present is due to the ongoing economic and political crisis previously discussed.
Data source: World Bank

C.5 Relative GDP Per Capita

As discussed in section 4.5.2 and elsewhere in this paper, the economic state of Venezuela was at times quite good. Figure C.3 shows the relative GDP of Venezuela to the United States of America.

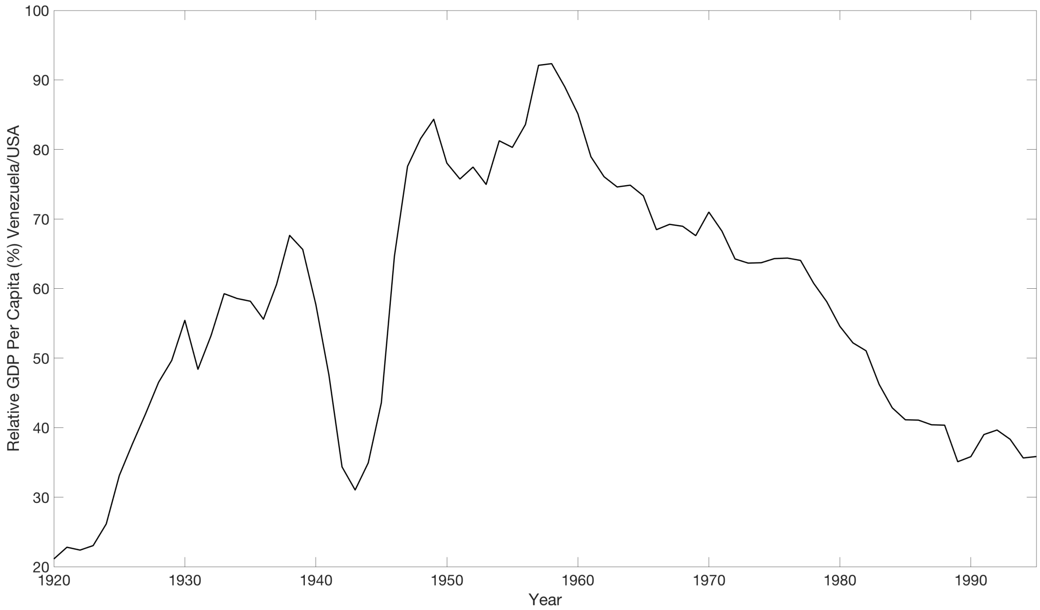
C.6 Oil Prices and Production

Venezuela is a member of OPEC (Organisation of the Petroleum Exporting Countries). In figure C.4, we show the Crude Oil price in US$ per barrel in real and nominal terms. The price has been deflated by the US CPI.

Due to the nationalisation of the oil industry, PDVSA, oil production has been driven by government interests rather than market forces. Figure C.5, shows Venezuelan oil production, vs OPEC oil production (less Venezuela).

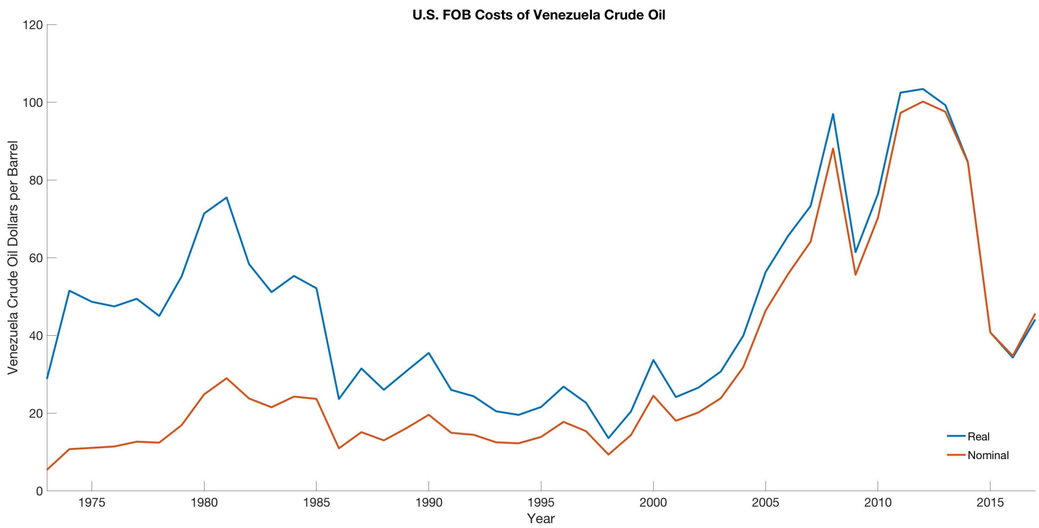
The gap between the two blue lines is relatively constant from 1970 onwards. The reason for the large decline in the 1980s was the oil glut following the energy crisis

Fig. C.3 Relative GDP Per Capita - Venezuela to the USA



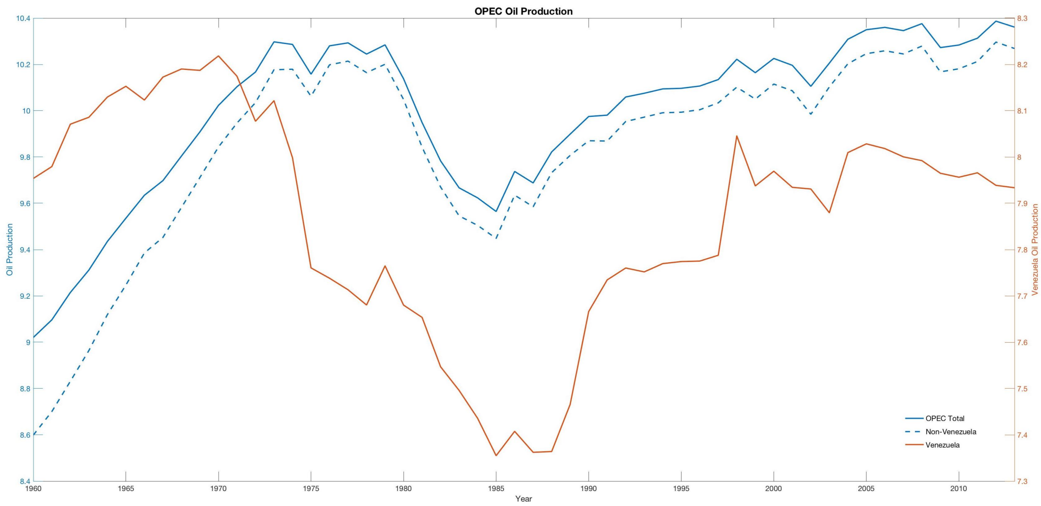
The relative GDP per capita of Venezuela to the USA during the period of observation 1920-1995. GDP per capita is measured in 1990 International Geary-Khamis dollars as calculated by Bolt and Zanden (2014), the relative figures are the author's calculations using the data for Venezuela and the USA.

Fig. C.4 U.S. FOB Costs of Venezuela Crude Oil Dollars per Barrel



The blue line shows the real pice of Venezuelan crude oil in US$ per barrel. The original line shows the nominal price. "Free on Board" in this case is when the buyer is responsible for transporting and insuring the good (oil) and the seller makes it delivers it to a given port for a given price. Source: Oil Price: US Department of Energy ; CPI: OECD

Fig. C.5 Crude Oil Production Daily Average



Logged daily average of crude oil production for OECD, OECD non-Venezuela, and Venezuela. The left hand axis is the scale for production of oil by OPEC countries. The solid line is for all OPEC countries, and the dashed line is all the OPEC countries excluding Venezuela. The orange line is scaled on the right hand axis is the logged production of oil production in Venezuela.
Date source: OPEC

in the 1970s. There is a weakly similar pattern in production of the solid orange, and dashed blue line. However, this has diverged in the beginning of the century in particular. The nationalisation finalised in 1976, however the process had started many years previous, causes a pronounced decline from 1970. The decline by other OECD countries doesn't begin until 1979.

C.7 Additional Model Equations

C.7.1 Adjustment Costs

The investment costs for the final good producing firm,

$$\frac{\partial\iota(x_t, x_{t-1})}{\partial x_t} = \frac{\iota(x_t, x_{t-1})}{x_t} - \kappa^x \phi_x \left(\frac{x_t}{x_{t-1}} - 1\right)\left(\frac{x_t}{x_{t-1}}\right) \tag{C.7.1}$$

$$\frac{\partial\iota(x_{t+1}, x_t)}{\partial x_t} = \kappa^x \phi_x \left(\frac{x_t}{x_{t-1}} - 1\right)\left(\frac{x_t}{x_{t-1}}\right)^2 \tag{C.7.2}$$

The investment costs for the oil-consuming firm

$$\frac{\partial\iota(x^F_t, x^F_{t-1})}{\partial x^F_t} = \frac{\iota(x^F_t, x^F_{t-1})}{x^F_t} - \kappa^{x^F} \phi_{x^F} \left(\frac{x^F_t}{x^F_{t-1}} - 1\right)\left(\frac{x^F_t}{x^F_{t-1}}\right) \tag{C.7.3}$$

$$\frac{\partial\iota(x^F_{t+1}, x^F_t)}{\partial x^F_t} = \kappa^{x^F}\phi_{x^F}\left(\frac{x^F_t}{x^F_{t-1}} - 1\right)\left(\frac{x^F_t}{x^F_{t-1}}\right)^2 \tag{C.7.4}$$

The investment costs for the oil-producing firm.

$$\frac{\partial\iota(x^O_t, x^O_{t-1})}{\partial x^O_t} = \frac{\iota(x^O_t, x^O_{t-1})}{x^O_t} - \kappa^{x^O}\phi_{x^O}\left(\frac{x^O_t}{x^O_{t-1}} - 1\right)\left(\frac{x^O_t}{x^O_{t-1}}\right) \tag{C.7.5}$$

$$\frac{\partial\iota(x^O_{t+1}, x^O_t)}{\partial x^O_t} = \kappa^{x^O}\phi_{x^O}\left(\frac{x^O_t}{x^O_{t-1}} - 1\right)\left(\frac{x^O_t}{x^O_{t-1}}\right)^2 \tag{C.7.6}$$

C.7.2 Terms of Trade

$$(p^H_{H_t})^{\theta-1} = v + (1-v)(tot_t)^{1-\theta} \tag{C.7.7}$$

$$(p^F_{H_t})^{\theta-1} = v(tot_t)^{\theta-1} + (1-v) \tag{C.7.8}$$

$$rer_t^{1-\theta} = \frac{(v^f + (1-v^f)(tot_t)^{1-\theta})}{(v + (1-v)tot_t^{1-\theta})} \tag{C.7.9}$$

$$(p^H_{F_t})^{\theta-1} = v^f + (1-v^f)(tot_t)^{1-\theta} \tag{C.7.10}$$

$$(p^F_{F_t})^{\theta-1} = v^f(tot_t)^{\theta-1} + (1-v^f) \tag{C.7.11}$$

Using C.7.9, we define the real exchange rate as:

$$rer_t^{1-\theta} = \frac{p^H_{F_t})^{\theta-1}}{(p^H_{H_t})^{\theta-1}}$$

$$rer_t^{1-\theta} = \left(\frac{p^H_{H_t}}{p^H_{F_t}}\right)^{1-\theta}$$

$$rer_t = \frac{p^H_{H_t}}{p^H_{F_t}}$$

C.8 Baseline DSGE Model

C.8.1 Home Household

$$\mu^H_t = \left(\psi^H_t c^H_t - \frac{\phi_0}{1+\phi}(h^H_t)^{(1+\phi)}\right)^{-\sigma} \tag{C.8.1}$$

$$ww^H_t = \phi_0(h^H_t)^{\phi} \tag{C.8.2}$$

$$hrs^H_t = h^H_t n^H_t \tag{C.8.3}$$

$$hrs^H_t = l^H_t + l^O_t + l^M_t \tag{C.8.4}$$

$$l_t^M = hrs_t^H n_t^M \tag{C.8.5}$$

C.8.2 Foreign Household

$$\mu_t^F = \left((c_t^F) - \frac{\phi_0}{(1+\phi)}(h_t^F)^{(1+\phi)}\right)^{-\sigma} \tag{C.8.6}$$

$$mpl_t^F = \phi_0 (h_t^F)^{\phi} \tag{C.8.7}$$

$$hrs_t^F = h_t^F n_t^F \tag{C.8.8}$$

C.8.3 Labour Market

$$m_t^H = a^H (u_t^{HH})^{\Gamma^H} (v_t^H)^{(1-\Gamma^H)} \tag{C.8.9}$$

$$m_t^F = a^F (u_t^F)^{\Gamma^F} (v_t^F)^{(1-\Gamma^F)} \tag{C.8.10}$$

$$m_t^M = a^M (u_t^{HM})^{\Gamma^M} (v_t^M)^{1-\Gamma^M} \tag{C.8.11}$$

$$n_t^H = (1-\rho_n^h) n_{t-1}^H + m_t^H \tag{C.8.12}$$

$$n_t^F = (1-\rho_n^f) n_{t-1}^F + m_t^F \tag{C.8.13}$$

$$n_t^M = (1-\rho_n^m) n_{t-1}^m + m_t^M \tag{C.8.14}$$

$$u_t^H = 1 - (1-\rho_n^H) n_{t-1}^H - (1-\rho_n^M) n_{t-1}^M \tag{C.8.15}$$

$$u_t^{HH} = u_t^H \lambda_t^H \tag{C.8.16}$$

$$u_t^{HM} = u_t^H (1-\lambda_t^H) \tag{C.8.17}$$

$$\widetilde{u_t^H} = 1 - n_t^H - n_t^M \tag{C.8.18}$$

$$\widetilde{u_t^{HH}} = \widetilde{u_t^H} \lambda_t^H \tag{C.8.19}$$

$$\widetilde{u_t^{HM}} = \widetilde{u_t^H} (1-\lambda_t^H) \tag{C.8.20}$$

$$u_t^F = 1 - (1-\rho_n^F) n_{t-1}^F \tag{C.8.21}$$

$$\widetilde{u_t^F} = 1 - n_t^F \tag{C.8.22}$$

$$\gamma_t^H = \frac{m_t^H}{v_t^H} \tag{C.8.23}$$

$$\gamma_t^F = \frac{m_t^F}{v_t^F} \tag{C.8.24}$$

$$\gamma_t^M = \frac{m_t^M}{v_t^M} \tag{C.8.25}$$

$$\zeta_t^H = \frac{m_t^H}{u_t^{HH}} \tag{C.8.26}$$

$$\zeta_t^F = \frac{m_t^F}{u_t^F} \tag{C.8.27}$$

$$\zeta_t^M = \frac{m_t^M}{u_t^{HM}} \tag{C.8.28}$$

$$\theta_t^H = \frac{v_t^H}{u_t^{HH}} \tag{C.8.29}$$

$$\theta_t^F = \frac{v_t^F}{u_t^F} \tag{C.8.30}$$

$$\theta_t^M = \frac{v_t^M}{u_t^{HM}} \tag{C.8.31}$$

$$\Upsilon_t^H = \frac{1-\vartheta^h}{\vartheta^h}\eta_t^H \tag{C.8.32}$$

$$\Upsilon_t^F = \frac{1-\vartheta^f}{\vartheta^f}\eta_t^F \tag{C.8.33}$$

$$\Upsilon_t^F = \frac{1-\vartheta^m}{\vartheta^m}\eta_t^M \tag{C.8.34}$$

$$\Upsilon_t^H = (ww_t^H - w_t^H)hrs_t^H + (1-\rho_n^h)\beta\frac{\mu_{t+1}^H}{\mu_t^H}\Upsilon_{t+1}^H \tag{C.8.35}$$

$$\Upsilon_t^F = (ww_t^F - w_t^F)hrs_t^F + (1-\rho_n^f)\beta^f\frac{mu_{t+1}^F}{\mu_t^F}\Upsilon_{t+1}^F \tag{C.8.36}$$

$$\Upsilon_t^M = (ww_t^M - w_t^M)hrs_t^H + (1-\rho_n^m)\beta^f\frac{\mu_{t+1}^F}{\mu_t^F}\Upsilon_{t+1}^M \tag{C.8.37}$$

$$\eta_t^H = -\frac{\phi_0}{(1+\phi)}(h_t^H)^{1+\phi} + (w_t^H h_t^H - ub^h) + (1-\rho_n^h)\beta\frac{\mu_{t+1}^H}{\mu_t^H}\eta_{t+1}^H(1-\zeta_{t+1}^H) \tag{C.8.38}$$

$$\eta_t^F = -\frac{\phi_0}{(1+\phi)}(h_t^F)^{1+\phi} + (w_t^F hrs_t^F - ub^f) + (1-\rho_n^h)\beta^f\frac{\mu_{t+1}^F}{\mu_t^F}\eta_{t+1}^F(1-\zeta_{t+1}^F) \tag{C.8.39}$$

$$\eta_t^M = -\frac{\phi_0}{(1+\phi)}(h_t^H)^{1+\phi} + (w_t^M h_t^H - ub^m) + (1-\rho_n^m)\beta\frac{\mu_{t+1}^H}{\mu_t^H}\eta_{t+1}^H(1-\zeta_{t+1}^M) \tag{C.8.40}$$

$$\gamma_t^H(\Upsilon_t^H - \overline{\kappa^H}) = \frac{\partial(v_t^H, v_{t-1}^H)}{\partial v_t^H} + \beta\frac{\mu_{t+1}^H}{\mu_t^H}\frac{\partial(v_{t+1}^H, v_t^H)}{\partial v_t^H} \tag{C.8.41}$$

$$\gamma_t^F(\Upsilon_t^F - \overline{\kappa^F}) = \frac{\partial(v_t^F, v_{t-1}^F)}{\partial v_t^H} + \beta^f \frac{\mu_{t+1}^F}{\mu_t^F} \frac{\partial(v_{t+1}^F, v_t^F)}{\partial v_t^F} \tag{C.8.42}$$

$$\gamma_t^M(\Upsilon_t^M - \overline{\kappa^M}) = \frac{\partial(v_t^M, v_{t-1}^M)}{\partial v_t^M} + \beta^f \frac{\mu_{t+1}^M}{\mu_t^M} \frac{\partial(v_{t+1}^M, v_t^M)}{\partial v_t^M} \tag{C.8.43}$$

C.8.4 Home Firm

$$y_t^H = a_t (k_{t-1}^H)^{\alpha^H} (l_t^H)^{1-\alpha^H} \tag{C.8.44}$$

$$k_t^H = (1-\delta)\frac{k_{t-1}^H}{g_t^H} + \iota(x_t, x_{t-1}) \tag{C.8.45}$$

$$tq_t = \beta \frac{\mu_{t+1}^H}{\mu_t^H} \left(\alpha^H p_{H_{t+1}}^H \frac{y_{t+1}^H}{k_t} + (1-\delta) \frac{tq_{t+1}}{g_{t+1}^H} \right) \tag{C.8.46}$$

$$1 = tq_t \frac{\partial(x_t, x_{t-1})}{\partial x_t} + \beta \frac{\mu_{t+1}^H}{\mu_t^H} tq_{t+1} \frac{\partial(x_{t+1}, x_t)}{\partial x_t} \tag{C.8.47}$$

$$mpl_t^H = (1-\alpha^H) p_{H_t}^H \frac{y_t^H}{n_t^H} \tag{C.8.48}$$

$$p_{H_t}^H mpl_t^H = ww_t^H \tag{C.8.49}$$

C.8.5 Foreign Firm

$$y_t^F = a_t^F (I_t^F)^{\alpha^F} (l_t^F)^{1-\alpha^F} \tag{C.8.50}$$

$$I_t^F = \left[\nu^F (y_{F_t}^O)^{\Phi^F} + (1-\nu^F)(k_{t-1}^F)^{\Phi^F} \right]^{\frac{1}{\Phi^F}} \tag{C.8.51}$$

$$k_t^F = (1-\delta^F)\frac{k_{t-1}^F}{g_t^F} + \iota(x_t^F, x_{t-1}^F) \tag{C.8.52}$$

$$tq_t^F = \beta^f \frac{\mu_{t+1}^F}{\mu_t^F} \left(\alpha^F p_{F_{t+1}}^F \frac{y_{t+1}^F}{k_t^f} + (1-\delta^F) \frac{tq_{t+1}^F}{g_{t+1}^F} \right) \tag{C.8.53}$$

$$1 = tq_t^F \frac{\partial(x_t^F, x_{t-1}^F)}{\partial x_t^F} + \beta^f \frac{\mu_{t+1}^F}{\mu_t^F} tq_{t+1}^F \frac{\partial(x_{t+1}^F, x_t^F)}{\partial x_t^F} \tag{C.8.54}$$

$$mpk_t^F = \frac{\alpha^f (1-\nu^f) p_{F_t}^F y_t^F (k_{t-1}^F)^{\phi^f - 1}}{(I_t^F)^{\phi^f}} \tag{C.8.55}$$

$$mpl_t^F = (1-\alpha^F)\chi^L p_{F_t}^F \frac{y_t^F}{h_t^F} \tag{C.8.56}$$

$$mpl_t^M = (1-\alpha^F)(1-\chi^L) p_{F_t}^F \frac{y_t^F}{l_t^M} \tag{C.8.57}$$

$$p_{F_t}^F mpl_t^F = ww_t^F \tag{C.8.58}$$

$$p^F_{F_t} mpl^M_t = ww^M_t \tag{C.8.59}$$

$$L^F_t = (h^F_t)^{\chi^L}(l^M_t)^{1-\chi^L} \tag{C.8.60}$$

$$p^O_{F_t} = p^O_{*_t}\frac{\alpha^f(\nu^f)p^F_{F_t}y^F_t(y^O_{F_t})^{\phi^f-1}}{(I^F_t)^{\phi^f}} \tag{C.8.61}$$

$$y^O_{F_t} = \left(\frac{p^O_{F_t}(I^F_t)^{\phi^f}}{\nu^f\alpha^f y^F_t p^F_{F_t}}\right)^{\frac{1}{\phi^f-1}} \tag{C.8.62}$$

$$y^O_t = y^O_{h_t} + y^O_{F_t} \tag{C.8.63}$$

C.8.6 Oil Producer

$$y^O_t = a^O_t(k^O_t)^{\alpha^O}(l^O_t)^{\beta^O} \tag{C.8.64}$$

$$mpl^O_t = \beta^O\frac{y^O_t}{l^O_t} \tag{C.8.65}$$

$$mpk^O_t = \alpha^O\frac{y^O_t}{k^O_{t-1}} \tag{C.8.66}$$

$$p^O_{H_t}mpl^O_t = ww^H_t + \xi(l^O_t)_{diff} + \beta\frac{\mu^H_{t+1}}{\mu^H_t}\xi(l^O_{t+1})_{difflag+1} \tag{C.8.67}$$

$$p^O_{H_t}mpk^O_t = r^O_{k_t} + \xi(k^O_t)_{diff} + \beta\frac{\mu^H_{t+1}}{\mu^H_t}\xi(k^O_{t+1})_{difflag+1} \tag{C.8.68}$$

$$k^O_t = (1-\delta^O)\frac{k^O_{t-1}}{g^H_t} + x^O_t \tag{C.8.69}$$

$$p^O_{H_t} = rer_t p^O_{F_t} \tag{C.8.70}$$

$$\xi(l^O_t) = l^O_t\frac{\xi(l^O)}{2}\left(\frac{l^O_t}{l^O_{t-1}}-1\right)^2 \tag{C.8.71}$$

$$\xi(l^O_t)_{diff} = \frac{\xi(l^O_t)}{l^O_t} + \xi(l^O)\left(\frac{l^O_t}{l^O_{t-1}}-1\right)\frac{l^O_t}{l^O_{t-1}} \tag{C.8.72}$$

$$\xi(l^O_t)_{difflag} = -\xi(l^O)\left(\frac{l^O_t}{l^O_{t-1}}-1\right)\left(\frac{l^O_t}{l^O_{t-1}}\right)^2 \tag{C.8.73}$$

$$\xi(k^O_t) = k^O_t\frac{\xi(k^O)}{2}\left(\frac{k^O_t}{k^O_{t-1}}-1\right)^2 \tag{C.8.74}$$

$$\xi(k^O_t)_{diff} = \frac{\xi(k^O_t)}{k^O_t} + \xi(k^O)\left(\frac{k^O_t}{k^O_{t-1}}-1\right)\frac{k^O_t}{k^O_{t-1}} \tag{C.8.75}$$

$$\xi(k^O_t)_{difflag} = -\xi(k^O)\left(\frac{k^O_t}{k^O_{t-1}}-1\right)\left(\frac{k^O_t}{k^O_{t-1}}\right)^2 \tag{C.8.76}$$

C.8.7 Adjustment cost functions

$$\iota(x_t, x_{t-1}) = \kappa^x x_t \left(1 - \frac{\phi_x}{2}\left(\frac{x_t}{x_{t-1}} - 1\right)^2\right) \tag{C.8.77}$$

$$\frac{\partial(x_t, x_{t-1})}{\partial x_t} = \frac{\iota(x_t, x_{t-1})}{x_t} - \kappa^x \phi_x \left(\frac{x_t}{x_{t-1}} - 1\right)\left(\frac{x_t}{x_{t-1}}\right) \tag{C.8.78}$$

$$\frac{\partial(x_{t+1}, x_t)}{\partial x_t} = \kappa^x \phi_x \left(\frac{x_t}{x_{t-1}} - 1\right)\left(\frac{x_t}{x_{t-1}}\right)^2 \tag{C.8.79}$$

$$\iota(x_t^F, x_{t-1}^F) = \kappa^x x_t^f \left(1 - \frac{\phi_x}{2}\left(\left(\frac{x_t^f}{x_{t-1}^f}\right) - 1\right)^2\right) \tag{C.8.80}$$

$$\frac{\partial(x_t^F, x_{t-1}^F)}{\partial x_t^F} = \frac{\iota(x_t^F, x_{t-1}^F)}{x_t^f} - \kappa^x \phi_x \left(\frac{x_t^f}{x_{t-1}^f} - 1\right)\left(\frac{x_t^f}{x_{t-1}^f}\right) \tag{C.8.81}$$

$$\frac{\partial(x_{t+1}^F, x_t^F)}{\partial x_t^F} = \kappa^x \phi_x \left(\frac{x_t^f}{x_{t-1}^f} - 1\right)\left(\frac{x_t^f}{x_{t-1}^f}\right)^2 \tag{C.8.82}$$

$$\kappa(v_t^H, v_{t-1}^H) = \kappa_v^H v_t^H \left(1 + \frac{\phi_v^H}{2}\left(\frac{v_t^H}{v_{t-1}^H} - 1\right)^2\right) \tag{C.8.83}$$

$$\frac{\partial(v_t^H, v_{t-1}^H)}{\partial v_t^H} = \frac{\kappa(v_t^H, v_{t-1}^H)}{v_t^H} + \kappa_v^H \phi_v^H \left(\frac{v_t^H}{v_{t-1}^H} - 1\right)\frac{v_t^H}{v_{t-1}^H} \tag{C.8.84}$$

$$\frac{\partial(v_{t+1}^H, v_t^H)}{\partial v_t^H} = -\kappa_v^H \phi_v^H \left(\frac{v_t^H}{v_{t-1}^H} - 1\right)\left(\frac{v_t^H}{v_{t-1}^H}\right)^2 \tag{C.8.85}$$

$$\kappa(v_t^F, v_{t-1}^F) = \kappa_v^F v_t^F \left(1 + \frac{\phi_v^F}{2}\left(\frac{v_t^F}{v_{t-1}^F} - 1\right)^2\right) \tag{C.8.86}$$

$$\frac{\partial(v_t^F, v_{t-1}^F)}{\partial v_t^F} = \frac{\kappa(v_t^F, v_{t-1}^F)}{v_t^F} + \kappa_v^F \phi_v^F \left(\frac{v_t^F}{v_{t-1}^F} - 1\right)\frac{v_t^F}{v_{t-1}^F} \tag{C.8.87}$$

$$\frac{\partial(v_{t+1}^F, v_t^F)}{\partial v_t^H} = -\kappa_v^F \phi_v^F \left(\frac{v_t^F}{v_{t-1}^F} - 1\right)\left(\frac{v_t^F}{v_{t-1}^F}\right)^2 \tag{C.8.88}$$

$$\kappa(v_t^M, v_{t-1}^M) = \kappa_v^M v_t^M \left(1 + \frac{\phi_v^M}{2}\left(\frac{v_t^M}{v_{t-1}^M} - 1\right)^2\right) \tag{C.8.89}$$

$$\frac{\partial(v_t^M, v_{t-1}^M)}{\partial v_t^M} = \frac{\kappa(v_t^M, v_{t-1}^M)}{v_t^M} + \kappa_v^M \phi_v^M \left(\frac{v_t^M}{v_{t-1}^M} - 1\right)\frac{v_t^M}{v_{t-1}^M} \tag{C.8.90}$$

$$\frac{\partial(v_{t+1}^M, v_t^M)}{\partial v_t^M} = -\kappa_v^M \phi_v^M \left(\frac{v_t^M}{v_{t-1}^M} - 1\right)\left(\frac{v_t^M}{v_{t-1}^M}\right)^2 \tag{C.8.91}$$

C.8.8 Adding Up

$$k_t = k_t^H + k_t^O \tag{C.8.92}$$

$$x_t = x_t^H + x_t^O \tag{C.8.93}$$

$$r_t^K = \alpha^H p_{H_t}^H \frac{y_t^H}{k_{t-1}^H} \tag{C.8.94}$$

C.8.9 Open Economy

$$\frac{1}{(1+r_t)} = \beta \frac{\mu_{t+1}^H}{\mu_t^H} \frac{p_{H_{t+1}}^F}{p_{H_t}^F} \frac{1}{g_t^H} \tag{C.8.95}$$

$$\frac{1}{(1+r_t^f)} = \beta \frac{\mu_{t+1}^F}{\mu_t^F} \frac{p_{F_{t+1}}^F}{p_{F_t}^F} \frac{1}{g_t^F} \tag{C.8.96}$$

$$(p_{H_t}^H)^{\theta-1} = v + (1-v)(tot_t)^{1-\theta} \tag{C.8.97}$$

$$(p_{H_t}^F)^{\theta-1} = v(tot_t)^{\theta-1} + (1-v) \tag{C.8.98}$$

$$rer_t^{1-\theta} = \frac{(v^f + (1-v^f)(tot_t)^{1-\theta})}{(v + (1-v)tot_t^{1-\theta})} \tag{C.8.99}$$

$$(p_{F_t}^H)^{\theta-1} = v^f + (1-v^f)(tot_t)^{1-\theta} \tag{C.8.100}$$

$$(p_{F_t}^F)^{\theta-1} = v^f(tot_t)^{\theta-1} + (1-v^f) \tag{C.8.101}$$

$$\begin{aligned} y_t^H + y_{H_t}^O &= v^h (p_{H_t}^H)^{-\theta} (c_t^H + x_t + \overline{\kappa}(\gamma_t^H) + \kappa(v_t^H, v_{t-1}^H) + \xi(l_t^O) + \xi(k_t^O) + g_{e_t}^H) \\ &+ \frac{(1-n)}{n} v^f (p_{F_t}^H)^{-\theta} (c_t^F + x_t^f + \overline{\kappa^F}(\gamma_t^F) + \kappa(v_t^F, v_{t-1}^F) + \overline{\kappa^M}(\gamma_t^M) + \kappa(v_t^M, v_{t-1}^M) + c_t^M + g_{e_t}^F) \end{aligned} \tag{C.8.102}$$

$$\begin{aligned} y_t^F &= (1-v^f) \left(\frac{p_{H_t}^F}{rer_t} \right)^{-\theta} (c_t^F + x_t^f + \overline{\kappa^F}(\gamma_t^F) + \kappa(v_t^F, v_{t-1}^F) + \overline{\kappa^M}(\gamma_t^M) + \kappa(v_t^M, v_{t-1}^M) + c_t^M + g_{e_t}^F) \\ &+ \frac{n}{1-n}(1-v^h)(p_{H_t}^H)^{-\theta}(c_t^H + x_t + \overline{\kappa}(\gamma_t^H) + \kappa(v_t^H, v_{t-1}^H) + \xi(l_t^O) + \xi(k_t^O) + g_{e_t}^H) \end{aligned} \tag{C.8.103}$$

$$d_t^* = (1 + r_{t-1}) \frac{gdp_{t-1}}{gdp_t} \frac{p_{H_t}^F}{p_{H_{t-1}}^F} \frac{d_{t-1}^*}{g_t} + tbal_t \tag{C.8.104}$$

$$1 + r_t = (1 + r_t^f + \psi_t^r) \left(\exp\left(\phi_b (d_t^* - \overline{d^*}) \right) - \phi_o \left(p_{F_t}^O - p_o^F \right) \right) \tag{C.8.105}$$

$$gdp_t = p_{H_t}^H y_t^H + p_{H_t}^O y_{H_t}^O p_{*_t}^O \tag{C.8.106}$$

$$gdp_t^F = p_{F_t}^F y_t^F - p_{F_t}^O y_{F_t}^O p_{*t}^O \tag{C.8.107}$$

$$tbal_t = 1 - \frac{c_t^H + x_t + \overline{\kappa}\gamma_t^H + \kappa(v_t^H, v_{t-1}^H) + \xi(l_t^O) + \xi(k_t^O) + g_{e_t}^H}{gdp_t^H} \tag{C.8.108}$$

C.8.10 Miscellaneous

$$\zeta_t^{\frac{M}{H}} = \frac{\zeta_t^M}{\zeta_t^H} \tag{C.8.109}$$

$$\lambda_t = \frac{m_t^H}{m_t^H + m_t^M} \tag{C.8.110}$$

$$\Xi_t = \overline{\Xi}\varrho^{\Xi}\left(\frac{w_t^M}{w_t^H}\right)^{\rho^{\Xi}} \tag{C.8.111}$$

$$c_t^m + \Xi_t + Tax_{M_t} = w_t^m l_t^m \tag{C.8.112}$$

C.8.11 Fiscal Authority

$$g_{e_t}^H = g_{c_t}^H + bf_t^H + p_{H_t}^O y_{H_t}^O \tag{C.8.113}$$

$$bf_t^H = (\overline{bf^H})^{1-\rho^{bf}}(bf_{t-1}^H)^{\rho^{bf}} + \varepsilon_t^{bf} \tag{C.8.114}$$

$$g_{c_t}^H = (\overline{g_c^H})^{1-\rho^{g^c}}(g_{c_{t-1}}^H)^{\rho^{g^c}} + \varepsilon_t^{g_c^H} \tag{C.8.115}$$

$$g_{e_t}^H + (1 + r_{t-1}^F)p_{H_t}^F d_{t-1}^* = tax_t + p_{H_t}^F d_t^* \tag{C.8.116}$$

$$g_{e_t}^F = g_{c_t}^F + bf_t^F \tag{C.8.117}$$

$$bf_t^F = (\overline{bf^F})^{1-\rho^{bf}}(bf_{t-1}^F)^{\rho^{bf}} + \varepsilon_t^{bf} \tag{C.8.118}$$

$$g_{c_t}^F = (\overline{g_c^F})^{1-\rho^{g^c}}(g_{c_{t-1}}^F)^{\rho^{g^c}} + \varepsilon_t^{g_c^F} \tag{C.8.119}$$

$$g_{e_t}^F + (1 + r_{t-1}^F)p_{F_t}^F d_{t-1}^* = tax_t^F + p_{F_t}^F d_t^* \tag{C.8.120}$$

C.8.12 Shocks and Population Growth Rates

$$a_t^H = \rho^{a^H} a_{t-1}^H + \varepsilon_t^{a^H} \tag{C.8.121}$$

$$a_t^F = \rho^{a^F} a_{t-1}^F + \varepsilon_t^{a^F} \tag{C.8.122}$$

$$\psi_t^H = \rho^{\psi^H}\psi_{t-1}^H + \varepsilon_t^{\psi^H} \tag{C.8.123}$$

$$g_t^H = \frac{n_t^H + u_t^{HH}}{n_{t-1}^H + u_{t-1}^{HH}} \tag{C.8.124}$$

$$g_t^F = \frac{n_t^F + u_t^F + n_t^M + u_t^{HM}}{n_{t-1}^F + u_{t-1}^F + n_{t-1}^M + u_{t-1}^{HM}} \tag{C.8.125}$$

$$a_t^O = \rho^{a^O} a_{t-1}^O + \varepsilon_t^{a^O} \tag{C.8.126}$$

$$p_{*_t}^O = \rho^{p_*^O} p_{*_{t-1}}^O + \varepsilon_t^{p_*^O} \tag{C.8.127}$$

$$\psi_t^r = \rho^r \psi_{t-1}^r + (1-\rho^r)\overline{\psi^r} + \varepsilon_t^r \tag{C.8.128}$$

End of baseline model

C.9 Steady State

$$h^H = 1.00 \tag{C.9.1}$$

$$h^F = 1.00 \tag{C.9.2}$$

$$r^F = \frac{1}{\beta^f} - 1 \tag{C.9.3}$$

$$\psi^r = 0.093 \tag{C.9.4}$$

$$r^H = r^F + \psi^r \tag{C.9.5}$$

$$\beta = \frac{1}{1 + r^H} \tag{C.9.6}$$

$$g^H = 1 \tag{C.9.7}$$

$$g^F = 1 \tag{C.9.8}$$

$$n^H = 0.85 \tag{C.9.9}$$

$$n^M = 0.10 \tag{C.9.10}$$

$$n^F = 0.95 \tag{C.9.11}$$

$$u^H = 1 - n^H - n^M \tag{C.9.12}$$

$$u^F = 1 - n^H \tag{C.9.13}$$

$$m^H = \rho_n^H n^H \tag{C.9.14}$$

$$m^F = \rho_n^F n^F \tag{C.9.15}$$

$$m^M = \rho_n^M n^M \tag{C.9.16}$$

$$\gamma^j = \overline{\gamma^j} \tag{C.9.17}$$

$$\lambda^H = \frac{m^H}{m^H + m^M} \tag{C.9.18}$$

$$u^{HH} = u^H \lambda^H \tag{C.9.19}$$

$$u^{HM} = u^H (1 - \lambda^H) \tag{C.9.20}$$

$$\widetilde{u^H} = 1 - n^H - n^M \tag{C.9.21}$$

$$\widetilde{u^{HH}} = \widetilde{u^H}\lambda^H \tag{C.9.22}$$

$$\widetilde{u^{HM}} = \widetilde{u^H}(1 - \lambda^H) \tag{C.9.23}$$

$$\widetilde{u^F} = 1 - n^F \tag{C.9.24}$$

$$v^H = \frac{m^H}{\gamma^H} \tag{C.9.25}$$

$$v^F = \frac{m^F}{\gamma^F} \tag{C.9.26}$$

$$v^M = \frac{m^M}{\gamma^M} \tag{C.9.27}$$

$$\zeta^H = \frac{m^H}{u^{HH}} \tag{C.9.28}$$

$$\zeta^F = \frac{m^F}{u^F} \tag{C.9.29}$$

$$\zeta^M = \frac{m^M}{u^{HM}} \tag{C.9.30}$$

$$hrs^H = (n^H + n^M)h^H \tag{C.9.31}$$

$$hrs^F = n^F h^F \tag{C.9.32}$$

$$a^H = 1 \tag{C.9.33}$$

$$\psi^H = 1 \tag{C.9.34}$$

$$tq^H = \frac{1}{\kappa_x^H} \tag{C.9.35}$$

$$tq^F = \frac{1}{\kappa_x^F} \tag{C.9.36}$$

$$l^H = \left(1 - \frac{l^M}{l} - \frac{l^O}{l}\right) hrs^H \tag{C.9.37}$$

$$l^O = \frac{l^O}{l} hrs^H \tag{C.9.38}$$

$$l^M = n^M h^H \tag{C.9.39}$$

$$L^f = (hrs^F)^\chi (l^M)^{1-\chi} \tag{C.9.40}$$

$$r^k = \frac{1}{\beta} - (1 - \delta^H) \tag{C.9.41}$$

$$k^H = \left(\frac{r^k}{\alpha(l^H)^{1-\alpha}}\right)^{\frac{1}{\alpha-1}} \tag{C.9.42}$$

$$y^H = a^H(k^H)^\alpha(l^H)^{1-\alpha} \tag{C.9.43}$$

$$mpl^H = (1-\alpha)\frac{y^H}{l^H} \tag{C.9.44}$$

$$y^O = \overline{\frac{y^O}{y^H}}y^H \tag{C.9.45}$$

$$k^O = \frac{\alpha^O}{\alpha^H}\frac{y^O}{y^H}k^H \tag{C.9.46}$$

$$\beta^O = mpl^H\frac{l^O}{y^O} \tag{C.9.47}$$

$$k = k^H + k^O \tag{C.9.48}$$

$$gdp = \left(\overline{\frac{y_F^O}{y^O}}\frac{y^O}{y^H} + 1\right)y^H \tag{C.9.49}$$

$$y_F^O = \overline{\frac{y_F^O}{y^O}}y^O \tag{C.9.50}$$

$$y_H^O = y^O - y_F^O \tag{C.9.51}$$

$$r_F^K = \frac{1}{\beta^F} - (1-\delta^F) \tag{C.9.52}$$

Numerical solver for a^F and k^F for for $p_F^O = 1$ and $mpk^F = r_k^F$.

$$p_F^O = \frac{\alpha^f(\nu^f)p_F^F y^F(y_F^O)^{\phi^f-1}}{\left[\nu^F(y_F^O)^{\Phi^F} + (1-\nu^F)(k^F)^{\Phi^F}\right]} \tag{C.9.53}$$

$$mpk^F = \frac{\alpha^f(1-\nu^f)p_F^F y^F(k^F)^{\phi^f-1}}{\left[\nu^F(y_F^O)^{\Phi^F} + (1-\nu^F)(k^F)^{\Phi^F}\right]} \tag{C.9.54}$$

Where

$$y^F = a^F\left(\nu^F(y_F^O)^{\Phi^F} + (1-\nu^F)(k^F)^{\Phi^F}\right)^{\frac{\alpha^F}{\phi^F}}(L^F)^{1-\alpha^F}$$

$$\Upsilon^j = \frac{\kappa_v^j}{\gamma^j} + \overline{\kappa^j} \tag{C.9.55}$$

$$w^j = ww^j - \left(1 - \left(1-\rho_n^j\right)\beta^i\right)\frac{\Upsilon^j}{h^i} \tag{C.9.56}$$

$$\eta^j = \frac{w^j - h^j - ub^j - \frac{\phi_0}{1+\phi}(h^j)^{1+\phi}}{1-(1-\rho_n^j)(1-\zeta^j)\beta^i} \tag{C.9.57}$$

$$\vartheta^j = \frac{\eta^j}{\Upsilon^j + \eta^j} \tag{C.9.58}$$

$$\xi(l_t^O) = 0 \tag{C.9.59}$$

$$\xi(l_t^O)_{diff} = 0 \tag{C.9.60}$$

$$\xi(l_t^O)_{difflag} = 0 \tag{C.9.61}$$

$$\xi(k_t^O) = 0 \tag{C.9.62}$$

$$\xi(k_t^O)_{diff} = 0 \tag{C.9.63}$$

$$\xi(k_t^O)_{difflag} = 0 \tag{C.9.64}$$

$$x^H = \delta^H k^H \tag{C.9.65}$$

$$x^O = \delta^O k^O \tag{C.9.66}$$

$$x^F = \delta^F k^F \tag{C.9.67}$$

$$x = x^H + x^O \tag{C.9.68}$$

$$k = k^H + k^O \tag{C.9.69}$$

$$rer = p_H^H = p_H^F = P_F^F = P_F^H = tot = 1 \tag{C.9.70}$$

$$d^* = 0 \tag{C.9.71}$$

$$tbal = d^* - (1 + r^H)d^* \tag{C.9.72}$$

$$ge^H = \frac{ge^H}{gdp} gdp \tag{C.9.73}$$

$$bf^H = \frac{bf^H}{ge^H} ge^H \tag{C.9.74}$$

$$gc^H = ge^H - bf^H \tag{C.9.75}$$

$$ge^F = \frac{ge^F}{gdp^F} gdp^F \tag{C.9.76}$$

$$bf^F = \frac{bf^F}{ge^F} ge^F \tag{C.9.77}$$

$$gc^F = ge^F - bf^F \tag{C.9.78}$$

$$tax = ge \tag{C.9.79}$$

$$tax^F = ge^F \tag{C.9.80}$$

$$tax_F = tax^F \frac{1}{1 + n^M + u^{HM}} \tag{C.9.81}$$

$$tax_M = tax^F \frac{n^M + u^{HM}}{1 + n^M + u^{HM}} \tag{C.9.82}$$

$$\Xi = \overline{\Xi} \varrho^{\Xi} \left(\frac{w^M}{w^H} \right)^{\rho^{\Xi}} \tag{C.9.83}$$

$$c^H = (1 - tbal)gdp - x - \overline{\kappa^H}\gamma^H - \kappa(v_t^H, v_{t-1}^H) - G^H - \xi(k^O) - \xi(l^O) \tag{C.9.84}$$

$$c^F = (1-tbal)(gdp^F - y_F^O) - x^f - \overline{\kappa^F}\gamma^F - \kappa(v_t^F, v_{t-1}^F) - \overline{\kappa^M}\gamma^M - \kappa(v_t^M, v_{t-1}^M) - G^F - c^M \tag{C.9.85}$$

$$\mu^H = \left(\psi c^H - \frac{\phi_0}{1+\phi} (h^H)^{(1+\phi)} \right)^{-\sigma} \tag{C.9.86}$$

$$\mu^F = \left(c^F - \frac{\phi_0^f}{1+\phi^f} (h^F)^{(1+\phi^F)} \right)^{-\sigma^F} \tag{C.9.87}$$

$$\iota(x_t, x_{t-1}) = \kappa^x x^h \tag{C.9.88}$$

$$\frac{\partial(x_t, x_{t-1})}{\partial x} = \kappa^x \tag{C.9.89}$$

$$\frac{\partial(x_{t+1}, x_t)}{\partial x} = 0 \tag{C.9.90}$$

$$\iota(x_t^F, x_{t-1}^F) = \kappa^x x^f \tag{C.9.91}$$

$$\frac{\partial(x_t^F, x_{t-1}^F)}{\partial x} = \kappa^x \tag{C.9.92}$$

$$\frac{\partial(x_{t+1}^F, x_t^F)}{\partial x^F} = 0 \tag{C.9.93}$$

Chapter 5

Conclusion

Motivated by the increasing size, economic and political importance of migration, this thesis has presented three, but separate essays related by the topic of migration in open economies. To fill key gaps in the literature, this thesis has answered some questions that are untouched or relatively unexplored in empirical and theoretical macroeconomics. What are the effects of migration on natives, how does an exogenous migration shock affect the macroeconomy, how does brain waste affect the macroeconomy, and what are the implications of migration policies on a host and a sending country?

Chapters 2 and 3 focused on the effects of immigration following a shock to simulate the relaxation of immigration policies using theoretical and empirical models. Chapter 4 examined the effects of immigration, emigration, the emigration decision and migration policies with the use of occasionally binding constraints. The empirical models employed in chapters 2 and 3 introduced two new net migration time series into the literature. These analysed the effects of increases in net migration to the macroeconomy in Germany and Canada respectively. The datasets were used in the estimation of the DSGE models. The DSGE models put forth were of small open economies, however, they contrasted firstly in their profile of per native and per migrant in chapter 2 to per capita terms in chapter 3, and secondly in the use of skill levels and of search and matching frictions in chapter 3 . The final essay presented a two-country model using search and matching frictions, with an endogenous migration decision dependent on the labour market conditions. The emigration decision was affected by migration policies introduced by the host country. The model used in chapter 4 extended the analysis to include the business cycle characteristics of a resource cursed country and use an extensive dataset for Venezuela and the United States that is unused in business cycle analysis.

There are a series of general policy implications that can be taken from the three papers. Firstly, immigration has a positive effect on the macroeconomy. The models have presented native and migrant workers as imperfect substitutes or heterogeneous agents, to match with the microeconomic literature. When there is household transfer, or transfer between labour markets, the process shows how migrants fill gaps in the labour market. The positive effects are experienced in the theoretical and empirical models. For both Germany and Canada, a migration shock is small but (statistically) significant to the business cycle. The models do not find evidence of negative effects for natives in relation to immigration that is suggested by anti-immigration politicians and journalists. The modelling of workers as imperfect substitutes could be one of the reasons in the theoretical model. However, the empirical analysis of the wage level included in the national accounts is either positive or statistically insignificant on impact. The gains from immigration are only fully realised when immigrants are employed to their full skill level as demonstrated in chapter 3. When this does not occur, there are still positive effects since the size of the labour force has increased. The final paper of this thesis examined the migration process as an endogenous decision made based on the labour market conditions, which showed that migration policies had negative effects for both the host and sending economies. The host country suffered due to unfilled vacancies, and the sending country by having a higher number of unemployed workers and a lower level of remittances. Overall, emigration has positive and negative effects on the macroeconomy and is largely dependent on the type of shock to the business cycle - risk sharing or otherwise.

5.1 Summary of Findings and Policy Implications

Chapter 2 presented a novel model where variables were either in per native or per migrant terms rather than standard per capita business cycle literature and two SVARs to use migration and macroeconomic data from Germany. The results from the empirical models showed that migration has a positive effect on the macroeconomy in per capita and per native terms, which emphasises that migration is positive to the macroeconomy for natives and total working-age population. The responses using the different datasets were in the same direction and of similar magnitude, however, in per native terms the peak was slightly later. The response to migration in variables for which the response is statistically significant is generally positive, including an expansion on output. The effects of fiscal policy are interesting as the rules employed can offer different explanations. If fiscal policy is assumed to be countercyclical, as in the DSGE model, government spending would decrease as output increases. Taxes would go up rather than saying migration decreases government spending.

The insignificant response of government debt requires more research. The results from the DSGE model matched those from the SVAR; though it is not possible to compare the wage level with the SVAR due to the two types of wage level. The model showed that migration was more than an increase to population growth rate, as depicted in the Solow model. Migration increased the growth rate of the migrant households, then the native household resulted in expansionary output effects which resulted from an increase in labour supply. Both households experience a decrease in utility due to the increase in labour hours; for natives the change is insignificant. Natives first experience an increase due to the increase in consumption, before it is outweighed by the increase in labour hours. Interestingly, the absence of habit formation on consumption and labour created a greater change in utility, due to the period changes rather than the small two-period change. From these models, there are some policy implications. The size of the migration shock is relatively small, such that there is not a significant impact on the business cycle and no support is found for negative effects on natives following an increase to migration in the short or long-run. The design of the model has limited analysis to two labour markets, however, as both models show expansionary effects from increase in migration, and given Germany's ageing population, migration can be viewed as a positive necessity as alluded to in VBW (2015).

Chapter 3 examined immigration and the effect of brain waste in a migrant host nation. The DSGE model put forth is of a small open economy with capital-skill complementarity. The baseline model of brain waste featured three types of households: a high-skill native, low-skill native, and a migrant household that comprised of high and low-skill migrants. The high-skill migrants experienced two levels of brain waste designed to represent the most recently landed and landed migrants. The model included an option for skill-downgrading, rather than being unemployed as the migrants have less access to unemployment insurance than natives. The use of models with and without brain waste that enabled calculations of potential gains to the macroeconomy if brain waste was to be eliminated. The results from the scenario with brain waste show a small positive response to the economy following the increase to migration for the macroeconomy. The SVAR showed similar positive responses to the macroeconomy. However, the purpose of the DSGE model was to examine the potential gains from eliminating brain waste. Using steady state calculations and the impulse responses, in the absence of brain waste there is a significant gain to be found. Particularly for migrants, who have higher levels of labour income, job security, unemployment insurance, and consumption levels. Even though a high-skill migration shock creates some negative labour market responses, there is an overall gain due to the higher employment levels in steady state which originates from the participation and unemployment rates. From this model, policy implication show that migration has a positive effect to the economy but the brain waste of migrants limits the positive effects. The results support the fiscal programmes to have migrants' status fully recognised and that targeted migration programmes are most efficient when migrants have their skills fully recognised. Under the assumption of imperfect substitutability of migrant and native workers, there are positive or insignificant impacts from migration for natives. The modelling of capital-skill complementarity has a role in analysing the effects of high-skill migration as industries advance with technology (Lewis, 2011). Given that migration is predicted to be the only source of population growth by 2030, migration can be viewed as necessary to continue economic growth and maintain the working-age population.

Chapter 4, the final essay in this thesis looked at emigration from a developing economy using a two-country model with occasionally binding constraints on migration. This was the only model to use endogenous migration decision of the three essays, which enabled the analysis the effects of emigration and of a migration decision due to labour market factors net of migration costs. It used a unique dataset in this area of research, as a time series of Venezuela has not been used in the estimation of DSGE models. Two sets of policy implications can be taken from this model. Concerning migration, the policies implemented have shown to hamper both economies in the case of a non-risking sharing shock. If the migration decision is based on the labour market conditions, not just wage, deviation from steady state, then migration is likely to be temporary. Indeed, the number of Venezuelans who consider their migration to be temporary is significant so imposing these barriers has negative effects for both. If applied to the current situation in Venezuela, a greater distribution of migrants across the rest of Latin America will help ease the burden on Colombia, in particular, as their economy cannot quickly absorb all the new workers. Whereas the introduction of migration barriers will prevent Venezuelan residents leaving. This will keep unemployment persistently high and make the already relatively scarce resources even more so. Migrants would be able to send remittances, which already features consumption goods rather than money.

The use of real business cycles in developing economies has been questioned, however, in this model it allows the focus to be on migration and the business cycle of oil production. If the business cycles of commodity dependent countries are considered, which is relevant since there are a number of Latin American countries as well as other developing economies worldwide, then policy makers can use the analysis to help protect their economy from the booms and busts originating from commodity prices. The resource curse and macroeconomic populism tied in, and the results from the business cycle analysis show that dependence on the economy needs to shift away from goods that have highly volatile prices, and a development of the domestic economy to reduce the dependence on imports. The dollarisation of debt worsens their economy, though this model has not fully replicated the sovereign debt issue as the model uses one period debt. Lessons can be taken from the case of Norway, which is a highly developed economy where oil revenues are one of the biggest contributors to GDP, during the transition. Instead of using the revenues when the commodity prices are high to enhance the booms, governments should invest the revenues to support the economy when there are negative price shocks, the economy can be supported with these funds.

A further general conclusion that can be taken from each of the chapters is that it is beneficial to the host economy to fully incorporate migrants into the labour market, and the macroeconomy by allowing them equivalent financial market interaction as natives. When the migrants have the same labour market status as natives, there are macroeconomic gains to be found including GDP, consumption, investment, and for the fiscal budget. For developed economies, migration is or in the near future be the only source of population growth, migration will be a vital source of filling gaps in the labour market.

5.2 Limitations and Future Research

The three papers presented in this thesis have made their own contribution to the literature, however, each has its limitations and potential for further development.

For the purpose of analysing from a per native perspective, the data used an approximated native population in chapter 2. However, the time series of a native population can be argued as questionable since what exactly defines a long-term resident? The naturalisation of citizens was used to represent the process, but there are a number of other European labour market members resident in Germany that would consider themselves as long-term residents. This would suggest that by the paper's definition of native households, is that the migrant population is larger, and the native plus long-term residents is smaller than it should be. Fiscal policy was based on the countercyclical rules used in the Bundesbank's GEAR model; however, there was no a robustness teast of those or any of the other policy rules that Germany has previously used. One drawback is the relatively short migration time series that is available. While it is detailed, there is not a significant number of years that precedes the arrival of the A8 countries, even though the opening of the labour market was staggered with the use of work permits. Future research could expand research into the specifics of fiscal policy of migration across countries.

Chapter 2 used data from Germany that recently has predominantly low-skilled migrants, or those working in low-skill occupations. Future research could contrast this with a country that had predominantly high-skill migration. As outlined in the introduction to this thesis, the effects of migration can be country specific due to relative size of net migration and demographics of the average immigrant. High-skilled migrants are likely to assume native and labour market characteristics much quicker and other socio-economic characteristics. This problem for the process of assimilation is highlighted by Stähler (2017) following the European migration crisis of 2015-16 that had largely low-skilled immigrants. One of the increases in population that has risen in recent years is Germans and non-Germans migrating to Germany due to the United Kingdom's vote to leave the European Union. These migrants are likely to be high-skilled. This will be of a relatively small size, but closer examination on this topic would be of interest as it is replicable to a number of other European countries. The DSGE model shows the process to when migrants become long-term residents, however, there is not much evidence on this, or research in Germany. Based on the limitation of fiscal policy, an opportunity would be to test the effects of the other policy rules. We have used a countercyclical fiscal policy rule that are dependent on government debt and output, would the effects of a migration shock be replicable in another model and the rule could be more tailored to Germany.

Chapter 3 presented an extensive model of search and matching frictions, capital-skill complementarity, skill-downgrading, and fiscal policy running a balanced budget. The primary aim of the paper was to demonstrate the effects of brain waste on an economy however, the model did not use a proactive form of fiscal policy that reflected attempts by the government to reduce the effects of brain waste or illustrate the loses to the government from lower level of taxes and social security contributions from migrant workers. Taxation was lump-sum and household invariant. The government budget consisted of unemployment insurance to households and constant government spending that is subject to a shock. To incorporate fiscal policy that tackles brain waste, it could be modelled by way of vacancy subsidies to encourage firms to support migrant workers similar to the fiscal stimulus as suggested by Campolmi et al. (2011). The fiscal effects of brain waste were not fully explored, which is a notable factor since lower wages for firms imply lower social security contributions, taxes and other sources of fiscal revenues. Further opportunities for research include enabling migrants to gain qualifications in the host country via on-the-job-learning. In the paper, the short extension presented allowed migrants to invest in physical capital for use by the firms. However, this does not fully reflect a high-skilled household as the migrant household features a significant proportion of low-skilled workers whose labour supply and investment decisions are different from those of high-skill workers.

One of the limitations of chapter 4 is that the model assumes no migration to the Home country other than return migration. During the oil boom years, Venezuela indeed received many migrants from other Latin American countries and Spain. However, adding a third country to incorporate this dimension would introduce complexities beyond the scope of the paper and making the Home country the destination of migrants would not match the use for migration policies. The policies are rather narrow and depend only on the labour market, net of migration costs which in current format are insignificant. On the business cycle aspect of chapter 4, limitations exist on the government debt analysis due to existing limited research in the areas of liability dollarisation for governments in DSGE literature. There are a considerable number of political implications and problems that are difficult to model for example nationalisation of the oil industry and changing policies. However, this has shown a contrast to the emigration literature with a shock that is not a traditional business cycle form. There is an absence of risk sharing following an oil price shock which is the significant difference. Migration policies allowed us to introduce occasionally binding constraints for the first time in this area of the literature.

Chapter 4 has the possibility to be extended in several ways for both migration and business cycles. Future research could examine the effects of fiscal policy in a resource cursed country. The fiscal policy is used in a limited capacity, with primarily exogenous expenditures and lump-sum taxes. However, as described by the role of a resource curse, there are governments who use the high commodity prices to increase government spending. In which there is an opportunity to evaluate whether fiscal expenditures dependent on the oil price and contribution of oil rents, is a valid assumption. Further, show whether the implementation of a sovereign wealth fund model as in Bergholt et al. (2017) for emerging economies, with a focus on government consumption, investment and transfers, would be beneficial during the transition to a diversified economy. This would help to further explore the debt dynamics. In terms of migration research, debt bonded migration and liquidity constrained migrants which is an explored in the microeconomics (for example, Djajic and Vinogradova (2013) and Djajic and Vinogradova (2014)). There are a number of possibilities which the business cycle perspective of emerging or developing economies could come from this paper. Including the analysis of "sudden stops" and effect of foreign direct investment. Whereby making capital mobile between the developed and developing economy which would be interesting due to Latin America's extensive use of foreign capital as evidenced in Taylor (1999). This would provide a connection to migration as immobile capital is what keeps the developed economies further advanced (and drive migration) under the world systems theory. Further financial frictions could be explored, which closely associate developing economies with RBC models as shown in Garcia-Cicco et al. (2010) for Argentina and Mexico, and Tatan (2013) for Turkey.

The model examined the asymmetries introduced by positive and negative oil price shocks however, oil price exhibits heteroskedasticity which has not been recognised in the DSGE model. This would have been more suited to a model where the focus was on the oil price dynamics rather than migration as is the case in chapter 4. Nevertheless, using other econometric techniques to analyse the effects of a positive versus a negative oil price and supply shock would provide opportunity for future research in this area. Fernández et al. (2017) use an annual dataset for 1977-2014 from the World Economic Outlook for Venezuela in analysing terms of trade, world shocks and business cycles across 138 countries. This is a shorter sample, but covers more of the downturn in the economy, the results show that for oil exporting countries, (27 in the sample), the share of the variance in trade balance explained by world shocks is 28% compared to 15% for oil importers. It would be interesting to compare the results using the more recent dataset.

Each of these papers have shown that it is important for the host economy to full incorporate migrants into the labour market and to recognise their skills. In doing so, the macroeconomy gains. An additional conclusion is that natives are not negatively affected by immigration which agrees with the wider literature on migration, both empirical and theoretical. This chapter has shown that there are more areas to be explored in the macroeconomics of migration.

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