FUNDAMENTAL STUDIES OF PASSIVE, ACTIVE AND SEMI-ACTIVE AUTOMOTIVE SUSPENSION SYSTEMS

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ABSTRACT

The fundamental properties of various automotive suspension systems are theoretically investigated on the basis of simple vehicle models subjected to realistic inputs chosen to represent road surfaces of different qualities. The vehicle response is evaluated through a performance index representing ride comfort, dynamic tyre load and suspension working space parameters, and interpreted in the light of these individual parameters together with the implications of the suspension design for attitude control and steering behaviour.

Linear analysis procedures are followed in studying the passive, active and slow-active suspension systems while suitable simulations are used for the non-linear semi-active suspension systems. Linear optimal control theory is used to determine the optimal parameters of the active and slow-active suspension systems. Semi-active suspension behaviours are evaluated on the basis of applying the optimal active parameters to each system, but the semi-active damper can only dissipate energy and switches off when external power would be needed for the system to follow the optimal active control law.

Results are generated and discussed for each of these types of system and their performance capabilities are compared with each other. Conclusions concerning the practical viability of each of the systems are drawn.

CONTENTS

LIST OF FIGURES

LIST OF TABLES

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TO THE NAME OF MY PARENTS

CHAPTER(1)

INTRODUCTION

Design and development of automotive suspension systems has been of great interest for nearly 100 years. Early systems were derived directly from horseless carriage practice. Complicated vibration problems have arisen as a result of the increase in vehicle speeds which directly affect both the ride comfort and the ride safety. The solution of these problems in general may be achieved either by the reduction of the excitation level which mainly comes from the road surface irregularities or by the design of good suspension systems capable of maintaining an acceptable level of comfort and ensuring the vehicle safety on existing tracks. The latter has been considered an important area of study and has been extensively investigated. The application of science to the problem has been increasing as time has passed.

The primary purpose of the suspension system is to provide a high level of ride quality and protect the vehicle structure from harmful stresses by performing good isolation from the road surface irregularities. This requires a soft suspension. It should also assure the lateral stability and controllability at various running conditions (road qualities, speeds, accelerating, braking, and manoeuvring), besides supporting the variable static loads. This requires a stiff suspension.

Although considerable theoretical and practical studies have been carried out in order to improve real suspension systems, most current road vehicles suffer this fundamental conflict and their suspension parameters still compromise between the requirements. Moreover, further improvement seems to be very difficult to achieve using only conventional elements. However, active and semi-active suspension systems offer new possibilities for improvement in vehicle behaviour, but the road vehicle industry is still cautious about their introduction, except in some racing and experimental cars, because of the cost and complexity implied.

Development in computer facilities, as well as advances in mathematical analysis, offer good opportunity for theoretical analysis to be used as an aid to good practical system design. It is advantageous to use a simple model containing few parameters, if it is possible to reasonably represent a real system with this restriction, in order to obtain economical and fast guidelines to the system design.

A good prediction of suspension system performance can be achieved by using these facilities based on accurate representation of the vehicle dynamics and the guideway inputs, and suitable performance criteria by which the system response can be evaluated.

A real vehicle however, is too complicated to be modelled in detail for the purpose of studying the fundamental properties of suspension systems, particularly when active and semi-active devices are introduced. The vertical motion of the full vehicle can be represented by the simple two degree of freedom quarter car model. This model contains sprung mass (quarter of the body mass), unsprung mass (wheel assembly), suspension elements (passive, active, semi-active, or combination of them), and linear spring to represent the tyre, as proposed by Hooker (1979). This simple model has been used in many suspension system studies and justified as representing passive, active, semi-active, and slow-active suspension systems sufficiently to allow effective study of their main design features.

Actual road surface measurements have been described statistically by simple spectral density representation in terms of the wave number (the inverse of the wave length). Dodds and Robson (1973) suggested the representation of these spectra analytically in a general form with road surfaces being classified according to their roughness. After processing many measured road profiles, they proposed the representation of the road surface as having a displacement spectral density function of the form

$$Dr(\nu)=B1\,\nu^{-n_1}$$

$$Dr(\nu)=B2\,\nu^{-n_2} \qquad \text{.................................} \quad (1.1)$$

the discontinuity in slope occurring at $\nu=1/2\pi$ m^{-1}. Suggested values for n1 and n2 were 2.9175, 3.075, 1.52, for $0.01 \leq \nu < 1/2\pi$, and 2.04, 2.16, 2.142, for $\nu > 1/2\pi$ for motorways, principal roads and minor roads respectively. However, Robson (1979) supported the idea that a single-slope spectrum is adequate for some purposes.

It is beneficial to judge the vehicle response through simple criteria representing the main requirements (ride comfort and ride safety). The ride comfort parameter is a simple indication of the complex vibration environment to which the vehicle passengers are subjected. Various methods for evaluating the ride quality have been discussed by Smith, McGehee, and Healey (1978). One method considered involves simply applying the ISO recommended weighting function to the vertical acceleration illustrated in Vries (1982). The ride safety can be evaluated in terms of the dynamic tyre load variations which indicate the road holding. Also the suspension working space is an important performance measure. The last of these is connected with the attitude changes under loading and in manoeuvring, and the handling behaviour of which the vehicle stability is some measure.

The validity of various vehicle models has been studied by Healey, Nathman, and Smith (1977), through extensive comparisons of measured and computed vehicle responses. They used linear representations of the full car, one side, and one corner subjected to measured road profiles in their theory. They showed good agreement between the behaviours of these models and suggested the correction of the input spectral density formula in order to represent the measured profiles accurately. Also, they showed good correlation between the measured responses and those of the linearised seven degree of freedom system for the frequency range up to 10 Hz. They attributed the differences of the higher frequency components to excitations by unbalanced wheels or resonances of the vehicle structure.

Passive suspension systems consist of conventional elements such as coil or leaf springs and viscous dampers. The basic limitation inherent in using these elements, as indicated by Sutton (1979), is that the static deflection varies as the inverse square of the body natural frequency (for linear springs) and the basic conflict between ride comfort and handling can not be completely solved even by using non-linear springs or additional cross coupling by such devices as anti-roll bars. There are other limitations, as demonstrated by Hedrick and Wormley (1975), attributed to the ability of these elements to only store or dissipate energy and to generate forces in response to local relative displacements and velocities. Self levelling mechanisms, by which the static deflection can be removed, are useful in commercial vehicles suffering relatively high static load changes. They can offer better body isolation if used with softer suspensions but this may lead to problems with attitude control in cornering and braking and lateral stability and control problems.

Passive suspension systems have been studied by Ryba (1974) who applied linear systems analysis to the well known quarter car model subjected to a white noise random velocity input. More recent information than Ryba had available however, suggests that more accurate response prediction can be achieved by using the road surface representation described by Robson (1979). Ryba also studied the possibility of improving the passive suspension system by adding dynamic absorbers in four different configurations. He concluded that the choice of the suspension parameters is a compromise between ride comfort and dynamic tyre load variations identifying the suspension working space as an important factor in the suspension system design. He also demonstrated the possibility of improving both ride comfort and dynamic tyre load by using dynamic absorbers of mass equal to the wheel mass, but realising an auxiliary mass equal to the unsprung mass seems to be impractical.

Active suspensions are closed loop control systems with feedback signals representing all or some of the system variables to control actuators in place of, or in addition to, the usual passive elements. In general they contain external power sources, actuators (hydraulic, pneumatic or electromechanical) as force generators, measuring and sensing instruments (accelerometers, force transducers and potentiometers), and conditioning and amplifying devices. Many active suspension studies have been conducted in a general ground vehicle context and the subject was reviewed by Hedrick and Wormley (1975) and recently by Goodall and Kortum (1983). Although both of these papers are mainly concerned with rail vehicle systems, some of the information included, concerning the analysis procedures, the optimisation techniques and the hardware development, is commonly applicable.

The best active suspension systems can be expected to be designed by applying optimisation techniques. These can allow the estimation of the best system parameters for any given combination of measured variables and control inputs. Some of these techniques are discussed by Hedrick and Wormley (1975) and are mentioned by Goodall and Kortum (1983). Thompson (1976) applied one form of optimal control theory to active suspension systems, assuming an integrated white noise random displacement input and a quadratic performance index, the optimal control being that (linear state variable feedback) control law which minimises the performance index. His scheme included the measurement of the height of the vehicle body above the road surface and his results showed that significant improvements can be gained by measuring the body velocity and the body displacement relative to the road as feedback signals to control the actuator fitted in parallel with the conventional spring and damper. The main disadvantage to applying this theory can be attributed to the difficulty of realising the height sensor as one of the measuring instruments.

Semi-active systems in principle can be defined as fully active systems which exclude the external power source, giving zero actuator force when the fully active system would require a power supply as indicated by Karnopp, Crosby, and Harwood (1974). According to this scheme, the semi-active system employs a dissipative device with very low power requirements, needing only to measure, process and amplify the system variables used to control the damper valve in order to produce a variable damping coefficient. Early investigation of this device by Karnopp, Crosby, and Harwood (1974), suggested that it has a capability for offering performance levels close to what can be achieved by a fully active system, but the evidence offered was by no means conclusive on the point.

The same conclusion has been suggested recently in Margolis' studies through applying semi-active control laws to various vehicle models including the single degree of freedom system in Margolis (1982-a), the two degree of freedom heave and pitch system in Margolis (1982-b), and the two degree of freedom body and wheel system in Margolis (1983). In Margolis (1982-b), a two degree of freedom heave and pitch semi-active suspension system was studied by testing each of two different control laws having the vertical velocity of the vehicle body mass centre and the complete state variables as the measured feedback signals. Margolis concluded that both of these systems are far superior to the conventional passive system, and approach fully active systems in performance capability, but the case made is far from complete. Also, he found that the first law provides better low frequency isolation than the second one and the converse for the high frequency response. The two degree of freedom body and wheel semi-active suspension system studied in (1983) has a control law which is a combination of the measured body and wheel velocities to control a semi-active damper fitted in parallel with the passive spring by which the body mass is supported. The shortcomings of these studies are associated with applying linearisation techniques of doubtful validity in generating results in the form of transmissibilities, and endeavoring to judge the quality of the suspension system on this basis, and with not apparently recognising the fundamental fact that ride quality can always be improved if greater working space is allowed, Ryba (1974), such that the judgements made are of questionable validity.

Sireteanu (1984) studied the two degree of freedom suspension system subjected to white noise input. He proposed a non-linear damping law based on an argument showing its desirability for sine wave motions, and used statistical linearisation for obtaining the approximate system response. He also added a dynamic absorber to the unsprung mass, making a three degree of freedom system, in order to control the wheel motion. He concluded that both ride comfort and dynamic tyre load control can be improved by using the three mass semi-active suspension system, but the calculations involved many approximations and the conclusions should be viewed sceptically until they are confirmed by more rigorous analysis.

Active suspension systems will be more attractive if their benefits can be achieved by using cheaper components. The entire cost of such systems may become tolerable if cheaper actuators can be realised, particularly since sensors and microprocessors are already showing signs of being inexpensive. The use of electro-mechanical actuators in rail vehicle suspension systems has been considered by Goodall, Williams, and Lawton (1979) in order to reduce the capital cost and the maintenance requirements inherent in using servo-valve hydraulic actuators. Goodall (1980) has studied the feasibility of using this system for rail lateral suspensions and demonstrated that good ride comfort can be gained, with reasonable working space, if the system parameters are correctly chosen.

Previous studies of vehicle suspension systems showed the main conflict to be between ride comfort and suspension working space. There is no limit for improving ride comfort by decreasing the spring stiffness, but that can not be acceptable according to the increase of the suspension working space and also the increase of the dynamic tyre load if the system is lightly damped. Further, soft suspensions imply large deflections under changes in static loading, which can be eliminated, at a cost, by a self-levelling system. Without self levelling much useful working space will be consumed under heavy static loading and will be unavailable for vibration isolation. The dynamic tyre load should be held as small as possible relative to the static force to minimise sensitivity of the steering responses to road irregularities, and in particular to avoid the extreme case in which wheels leave the road. The various choices of the characteristics of the suspension elements allows the generation of different systems having various responses. The spectral densities of the responses of these systems show body resonance at low frequency when using soft springs with damping ratio around 0.3 critical while, for the same system under the same conditions, the main problem area concerning the dynamic tyre load is around the resonance of the unsprung mass. The addition of a tuned dynamic absorber with mass equal to the unsprung mass is capable of reducing substantially the dynamic tyre load variations, particularly when connected to the unsprung mass.

The objective of this work is to identify the suspension system capabilities by means of studying a practical car model subjected to realistic road inputs.

The main activities included can be summarised as :-

1- Presenting a good understanding of passive suspension system design.

2- Evaluating the possibility of improving the passive system behaviour by using dynamic absorbers based on parameter optimisation, i.e. in some sense with optimal parameters.

3- Constructing optimal parameter designs for active suspension systems using practically realisable control laws.

4- Generating good semi-active suspension systems based on the optimal active parameters.

5- Studying the validity of introducing slow-active actuators, as cheaper devices, into the road vehicle suspension systems.

6- Constructing a general discussion including quantitative comparisons of all these systems in the context of a general purpose vehicle which must operate under a wide variety of different conditions.

In subsequent chapters, the optimal control theory used in determining the full and limited state feedback control laws by which the responses of the optimal active suspension systems are evaluated is discussed (chapter 2). The application of the theory to the quarter car model is included and the procedures followed are explained for each of the two cases through example solutions.

In chapter 3, the linear analysis followed for studying passive, active, and slow-active suspension systems and the non-linear procedures used for investigating the semi-active system behaviours are discussed. The road surface representations, used as input to the linear and non-linear vehicle suspension models, are described and the procedures followed for obtaining the system response in both cases are explained.

The study in chapter 4 involves investigating the performance and design properties of the two and three mass passive suspension systems. The linear analysis procedures are applied to the two and three mass suspension models using the road spectral density representations and the performance criteria described in chapter 3.

The behaviour of the full and limited state feedback active suspension systems are studied in chapter 5. The optimal control theory described in chapter 2 is used for obtaining the feedback gain parameters involved following which the performance properties of these systems are obtained through the linear analysis procedures described in chapter 3.

In chapter 6, three different types of semi-active suspension systems are studied. These types are based on the full and limited state feedback control laws calculated in chapter 5 and may have a passive damper in parallel with the passive spring in addition to the semi-active force generator. The non-linear analysis procedures and the time history of the road surface profile described in chapter 3 are followed for calculating the response of each of these types of systems.

In chapter 7, the validity of using a limited bandwidth actuator in the vehicle suspension systems (slow-active systems) is evaluated and studied for systems having 4 Hz bandwidth limit actuator. The optimal control theory described in chapter 2 is used for obtaining parameters and the linear analysis described in chapter 3 is used in calculating the responses of these systems.

The results obtained in chapters 4 to 7 are discussed in chapter 8. In this chapter, the relative performance capabilities of the systems studied are discussed and compared with each other on the basis of having the same suspension working space requirements. Also, the implications of the adjustable suspension systems, which are treated in detail in the appendix, are discussed as possible general purpose systems having the capacity to adapt to the different requirements of the situation as the running conditions vary.

CHAPTER(2)

OPTIMAL CONTROL THEORY

2.1 Introduction

Active suspension systems include actuators in place of or in addition to the conventional springs and dampers. These actuators are capable of generating forces by which the system dynamics can be well controlled with relatively high "intelligence" as compared with passive devices. Part of the active system design involves derivation of a suitable control law relating the control to all or some of the system state variables. Optimal control theory provides straightforward methods for obtaining the best possible control law, avoiding trial-and-error procedures which are difficult to follow to a conclusion for systems having many parameters.

Stochastic linear optimal control theory is a branch of optimal control theory appropriate for application to the current problem, in which the form of the input can be considered to be integrated or filtered white noise. It can be applied to systems in which the controlled variable is a linear combination of the state variables and in which a quadratic performance index is employed, representing in the current study, a ride comfort parameter, dynamic tyre load variations, and suspension working space. The optimal control is that which minimises a weighted sum of squares of variables relating to each of these aspects of performance.

In this chapter, the stochastic linear optimal control theory applicable is reviewed and discussed for cases with full state feedback, and when full state information is not available. Application of the theory to the quarter car model is considered. Example solutions are presented to illustrate the procedures.

2.2 General Theory

The theory reviewed in this section can be found in detail in Kwakernaak and Sivan (1972).

Consider the linear time-invariant system

$$\dot{x}(t)=Ax(t)+Bu(t)+v(t) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 2.1$$

and the controlled variable

$$z(t)=Cx(t) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 2.2$$

with n state variables x(t), p control inputs u(t), q controlled variables z(t), and white noise disturbance input v(t) satisfying

$$E[v(t)\ v^T(\tau)] = V\,\delta(t-\tau) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 2.3$$

where E is the expectation operator, δ is the Dirac delta function, and V is a non-negative definite symmetric matrix.

Consider the performance criterion

$$E\{\int_{t0}^{t1}[z^T(t)Qz(t)+u^T(t)Ru(t)]\,dt + x^T(t1)P1x(t1)\} \qquad \ldots\ldots \quad 2.4$$

where Q and R are positive definite symmetric matrices for $t0 < t < t1$ and $P1$ is a non-negative definite symmetric matrix. The first term in the performance index is to reduce the controlled variable z(t) as fast as possible. The second term is introduced in order to control the value of the control input u(t). The third term may be used if it is necessary to keep the terminal state x(t1) as close as possible to the zero state. The relative importance of the first and the second terms can be determined through the matrices Q and R.

It is required to obtain the optimal control input u(t) by which the performance criterion can be minimised for each t, $t0 < t < t1$. The case in which all the state variables are measurable (stochastic linear regulator problem) is discussed first. Then, the practical case in which all the state information is not available is considered.

2.2.1 Full State Feedback

The optimal solution of the stochastic linear regulator problem (in the steady-state case) is to determine the control law

$$u(t)=-F\,x(t) \qquad \text{.................} \quad 2.5$$

where

$$F = R^{-1}B^{T}P \qquad \text{.................} \quad 2.6$$

and P is the unique non-negative definite solution of the algebraic Riccati equation

$$A^T P + P A + C^T Q C - P B R^{-1} B^T P = 0 \quad \text{.................} \quad 2.7$$

which minimises the performance index

$$J = \lim_{t \to \infty} E[z^T(t) Q z(t) + u^T(t) R u(t)] \quad \text{.................} \quad 2.8$$

The necessary conditions under which the Riccati equation has a steady-state solution and under which the steady-state closed-loop system is stable are

(a) The pair (A,B) defines a stabilizable system.

(b) The pair (A,C) defines a detectable system.

It is well known that the system is stabilizable if all its uncontrollable modes are stable, and that the system is controllable (and therefore stabilizable) providing the matrix

$$[B, AB, \ldots\ldots A^{n-1} B] \quad \text{.................} \quad 2.9$$

has rank n. The reason why condition (a) is needed stems from the fact that the controllable modes of the system can be arbitrarily placed (in respect of eigenvalues) using linear state feedback, while the uncontrollable modes remain unaltered by the feedback.

The system is detectable if all its unstable modes are observable, and the system is observable (and therefore detectable) providing the matrix

$$[C^T, A^T C^T, \ldots\ldots\ldots (A^T)^{n-1} C^T] \quad \text{................} \quad 2.10$$

has rank n.

If the system is unobservable, it can be transformed to the form (note the omission of the disturbance input, which does not affect the argument)

$$\begin{bmatrix} \dot{x}1(t) \\ \dot{x}2(t) \end{bmatrix} = \begin{bmatrix} A11 & A12 \\ 0 & A22 \end{bmatrix} \begin{bmatrix} x1(t) \\ x2(t) \end{bmatrix} + \begin{bmatrix} B1 \\ B2 \end{bmatrix} u(t) \qquad \ldots\ldots\ldots\ldots\ldots\ldots \; 2.11$$

$$z(t) = [0 \quad C2] \begin{bmatrix} x1(t) \\ x2(t) \end{bmatrix} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \; 2.12$$

where the subsystem (A22, C2) is observable, and A11 contains the unobservable modes. It is then apparent that the performance index (2.8) is independent of x1(t), and it can easily be shown that the optimal feedback law only depends on x2(t). Thus if in this case, A11 contains any unstable modes, they are not stabilized by the feedback and it is clear that condition (b) is needed to ensure the stability of the closed-loop system. Once the transformation to the form (2.11) and (2.12) has been achieved, the control problem reduces from one of dimension n to one of dimension equal to that of the matrix A22, with A22, B2, and C2 replacing A, B, and C in the Riccati equation (2.7). However, although the feedback law in the coordinate system of (2.11) and (2.12) only involves x2(t), the feedback law in the original coordinate system may involve full state feedback.

If the system is uncontrollable it can be transformed into the form (again omitting the input)

$$\begin{bmatrix} \dot{x}1(t) \\ \dot{x}2(t) \end{bmatrix} = \begin{bmatrix} A11 & 0 \\ A21 & A22 \end{bmatrix} \begin{bmatrix} x1(t) \\ x2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ B2 \end{bmatrix} u(t) \qquad \text{................ 2.13}$$

$$z(t) = [C1 \quad C2] \begin{bmatrix} x1(t) \\ x2(t) \end{bmatrix} \qquad \text{................ 2.14}$$

where the subsystem (A22, B2) is controllable, and A11 contains the uncontrollable modes. In this case a simple piece of algebra shows that the optimal feedback control involves both x1(t) and x2(t), but that the uncontrollable modes in A11 remain unaltered. The uncontrollable modes of the system will contribute to the performance index (2.8) unless they are also unobservable.

2.2.2 Limited State Feedback

The costs associated with measuring all the state variables, particularly for systems subjected to external disturbances, can be avoided by using only limited state feedback. In this case, only the m state variables, denoted by g(t) and defined as below, can be measured,

$$g(t)=M\ x(t) \qquad \text{................ 2.15}$$

where M is an mxn measurement matrix, and m<n, the system order.

The control law can be chosen to be of the form

$$u(t)=-K\ g(t) \qquad \text{................ 2.16}$$

where K is a constant pxm matrix chosen to minimise the performance index (2.8) as before.

Clearly, a necessary condition for an optimal control system design to be satisfactory in practical terms is that, if any modes of the open loop system are unstable, they can be stabilised with the feedback available. Since the feedback defined by (2.15) and (2.16) can, at most, only alter the controllable and observable modes of the system (A,B,M), it is also clear that a necessary condition for the design to be stable is that the uncontrollable and/or unobservable modes of (A,B,M) must be stable. However, the detectability condition on (A,C) which for the full state feedback case is needed to guarantee the existence and uniqueness of the optimal control, is no longer applicable in this context for the limited state feedback case. Indeed it is well known that even if (A,B,C) is stabilizable and detectable, a minimising solution may not exist in the limited state feedback case. Therefore, in particular cases, the problem of ensuring closed loop stability is best dealt with numerically within the iteration process by which the control law is found (see section 2.4.2).

Substituting (2.15) and (2.16) in (2.1) gives the following closed loop system

$$\dot{x}(t)=\tilde{A}\,x(t)+v(t) \qquad \text{.................} \quad 2.17$$

where

$$\tilde{A}=A-BKM \qquad \text{.................} \quad 2.18$$

The performance index (2.8) may then be determined from

$$J = \mathrm{trace}[Wc(C^TQC+M^TK^TRKM)] = \mathrm{trace}[WoV] \qquad \ldots\ldots\ldots\ldots\ 2.19$$

where

$$\tilde{A}Wc+Wc\tilde{A}^T+V=0 \qquad \ldots\ldots\ldots\ldots\ 2.20$$

and

$$\tilde{A}^T Wo+Wo\tilde{A}+C^TQC+M^TK^TRKM=0 \qquad \ldots\ldots\ldots\ldots\ 2.21$$

Various methods for determining the optimum value of K have been proposed, for example by Levine and Athans (1970) and by Golub, Nash and van Loan (1979). In the particular application described in the next section as well as in all active suspension systems studied through the work, physical considerations can be used to choose the constant elements of K quite close to the optimum values. Hence, the approach taken in this work is to use a gradient search technique. For this purpose, the gradient of the performance index in (2.19) with respect to any element kij (i=1,2,....p; j=1,2,....m) of K can be found from

$$\frac{\partial J}{\partial K} = \begin{bmatrix} \frac{\partial J}{\partial k11} & \frac{\partial J}{\partial k12} & \cdots\cdots & \frac{\partial J}{\partial k1m} \\ \vdots & & & \\ \frac{\partial J}{\partial kp1} & \frac{\partial J}{\partial kp2} & \cdots\cdots & \frac{\partial J}{\partial kpm} \end{bmatrix}$$

$$= 2[RKMWcM^T - B^T WoWcM^T] \qquad \text{................} \quad 2.22$$

Hence, the basic algorithm uses the following steps:

(i) Guess an initial value for K

(ii) Solve (2.20) and (2.21) for Wc and Wo

(iii) Calculate J in (2.19) and $\partial J/\partial K$ in (2.22)

(iv) Update K using a gradient search routine

(v) Go to step (ii) until satisfactory convergence has been achieved.

It is perhaps worth noting that when all the state variables are available, M in (2.15) becomes the nxn identity matrix. Equating the derivatives in (2.22) to zero gives an expression for K which when substituted into (2.21) turns this last equation into the algebraic Riccati equation (2.7) with Wo=P.

2.3 The Quarter Car Problem

The describing equations of the quarter car model shown in Fig.(2.1) in their state space form are

$\dot{x}0(t)= \zeta(t)$

$\dot{x}1(t)=x3(t)$

$\dot{x}2(t)=x4(t)$

$\dot{x}3(t)=Kt/mwx0(t)-Kt/mwx1(t)-1/mwu(t)$

$\dot{x}4(t)=1/mbu(t)$ 2.23

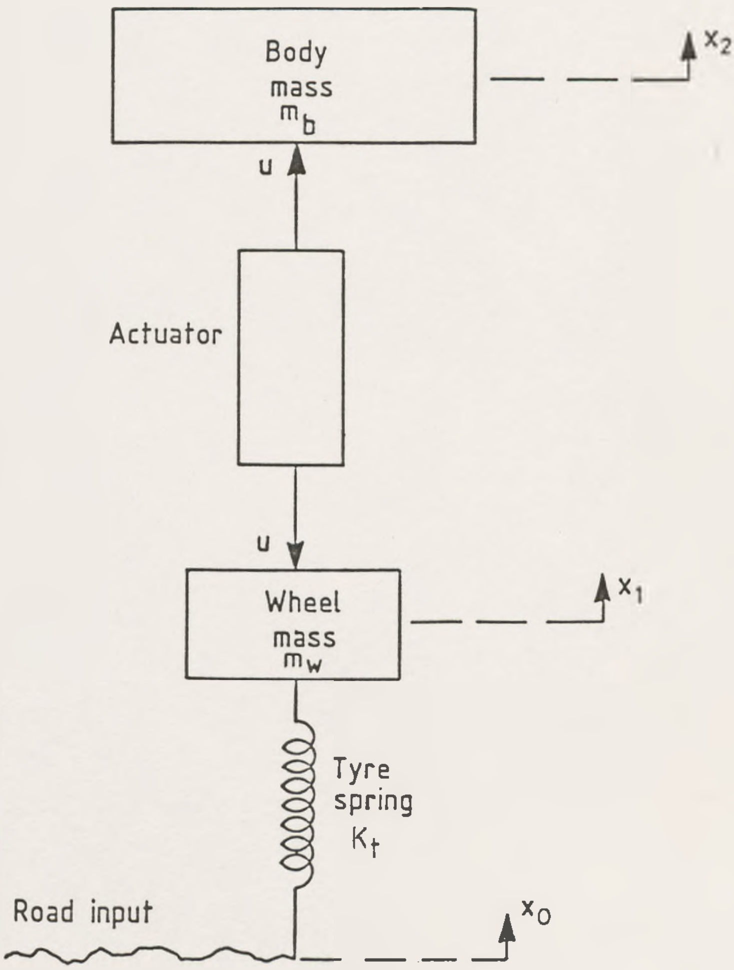

Fig. 2.1 Quarter car active suspension system

where mw, mb and Kt are the wheel mass, the body mass, and the tyre stiffness respectively.

Equations (2.23) in matrix form become

$$\begin{bmatrix} \dot{x}0(t) \\ \dot{x}1(t) \\ \dot{x}2(t) \\ \dot{x}3(t) \\ \dot{x}4(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ a & -a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x0(t) \\ x1(t) \\ x2(t) \\ x3(t) \\ x4(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1/mw \\ 1/mb \end{bmatrix} u(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \xi(t) \quad \ldots \quad 2.24$$

where a = Kt/mw

with the variables appearing in the quadratic performance index being related to the state variables by

$$[z(t)] = \begin{bmatrix} z1(t) \\ z2(t) \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x0(t) \\ x1(t) \\ x2(t) \\ x3(t) \\ x4(t) \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots \quad 2.25$$

$\xi(t)$ represents the single white noise disturbance input satisfying the requirement (2.3), implying that the road surface must have a displacement spectral density function of the form

$$Dr(\nu) = B1/\nu^2$$

where ν is the wave number and B1 is a constant, and that the vehicle speed is constant.

The performance index is

$$J = \int_0^{\infty} (z^T(t)Qz(t) + u^2(t))\, dt \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 2.26$$

in which $Q = \begin{bmatrix} q1 & 0 \\ 0 & q2 \end{bmatrix}$

consists of the constant weighting parameters which reflect the importances attached to the dynamic tyre load and to the suspension working space in comparison with the ride comfort parameter, represented by the control force u(t) to which the body acceleration is proportional. R = [1] is implied by this form of J.

Comparing (2.24) with (2.1) and (2.2)

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ a & -a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \dots\dots\dots\dots\dots\dots \quad 2.27$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1/mw \\ 1/mb \end{bmatrix} \qquad \dots\dots\dots\dots\dots\dots \quad 2.28$$

and

$$C = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \end{bmatrix} \qquad \dots\dots\dots\dots\dots\dots \quad 2.29$$

Testing the controllability of the system through applying (2.27) and (2.28) in the form of (2.9) gives

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -1/mw & 0 & a/mw & 0 \\ 0 & 1/mb & 0 & 0 & 0 \\ -1/mw & 0 & a/mw & 0 & -a\ /mw \\ 1/mb & 0 & 0 & 0 & 0 \end{bmatrix}$$

in which the 5th column is a multiple of the 3rd, so that the matrix is of rank 4. Thus the system has only one uncontrollable mode, and clearly this mode corresponds to the white noise input appearing in the first equation in (2.23), which is uncoupled from the other equations.

The observability test for (2.27) and (2.29) as in the form of (2.10) gives

$$\begin{bmatrix} -1 & 0 & 0 & 0 & a & -a & 0 & 0 & -a^2 & a^2 \\ 1 & -1 & 0 & 0 & -a & a & 0 & 0 & a^2 & -a^2 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & -a & a & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

with columns 5,6,9 and 10 linearly dependent on 1, and columns 7 and 8 linearly dependent on 3, so that the rank is again 4 and the system, has one unobservable mode.

2.3.1 Full State Information Available

The original state variables x(t) can be transformed into the new variables xs(t) according to the relation

$xs(t)=S\ x(t)$ where

$$S = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad S^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

this being the transformation of Thompson (1976), and the state equations become

$$\dot{x}s(t) = SAS^{-1}\, xs(t) + SBu(t) + Sv(t)$$

and

$$z(t) = CS^{-1}\, xs(t) \quad \text{in which}$$

$$SAS^{-1} = \left[\begin{array}{c|cccc} 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & -a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \qquad SB = \left[\begin{array}{c} 0 \\ \hline 0 \\ 0 \\ -1/mw \\ 1/mb \end{array}\right]$$

and

$$CS^{-1} = \left[\begin{array}{c|cccc} 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \end{array}\right]$$

and Q and R unchanged.

In the coordinate transformation, the uncontrollable mode associated with the road surface displacement $xs0(t)=x0(t)$, has also become unobservable. The optimal feedback law now only depends on $xs1(t)$------$xs4(t)$, irrespective of the fact that $xs0(t)$ is not bounded, and the sub-problem contained in the lower right partitions can be solved to yield the optimal control via the Riccati equation (2.7).

2.3.2 Limited State Feedback Available

In any real system, there are costs associated with the measurement of the state variables, and it will generally be of interest to examine the performances of optimal systems under different assumptions about the feedbacks available. In the present context, particular difficulties attach to the measurement of the height of the road surface, and systems which avoid the need for this measurement are of special interest.

With the system (2.24), the control $u(t)=-KMx(t)$, and the performance index (2.26), consider the possibility of finding a form of M which will imply (a) no need to measure the road height and (b) unobservability in the performance index of the uncontrollable neutrally stable mode associated with the input.

Transforming into the new variables $xs(t)$ as before, $u(t)$ becomes $-KMS^{-1}xs(t)$, and the performance index will not contain $xs0(t)$ if the first term in KMS^{-1} is zero. This is the condition that xs1(t) and xs2(t) contribute in equal and opposite degree to the control force, and if M is of the form

$$M = \left[\begin{array}{c|cccc} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right] \qquad \ldots\ldots\ldots\ldots\ldots \quad 2.30$$

both conditions (a) and (b) above are satisfied, and deriving the optimal control K by following the iterative procedure involving (2.19), (2.20), (2.21), and (2.22) while at each stage testing the closed loop system stability by finding the eigenvalues of $\tilde{A}$ will be straightforward.

If the fourth or fifth columns of M (or both) were to be altered, systems with reduced measurement and signal processing implications could be represented. The essential form of M for a well conditioned optimisation would be preserved however, so that such cases could be treated as above.

Hac (1985) preferred to describe the road surface displacement spectral density by

$$Dr(\nu) = B1/(\alpha^2+\nu^2)$$

where α is a constant implying that displacements remain finite for vanishingly small wave number.

With this description, the input xs0(t) is bounded and it is not now necessary to keep it from contributing to the performance index. An optimal control can be found for any form of M (having a first column of zeros). As α increases from zero, the optimal control law will increasingly differ from that deriving from the special form of M, i.e. the numerical values of the coefficients of xs1(t) and xs2(t) will differ increasingly, and it may be anticipated that if α is very small, numerical problems with this approach to the optimisation will occur. In practice however this does not appear to be a significant difficulty.

Supposing then that the control available gives a stable closed loop system, the optimal control can be determined using the algorithm described in section 2.2.2, with the stability being tested numerically.

2.4 Example Solutions

Example solutions will be shown for

mw=50 kg , mb=250 kg ,and Kt=120000 N/m

2.4.1 Full State Feedback

In the Riccati equation (2.7)

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ -1/mw \\ 1/mb \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad Q = \begin{bmatrix} q1 & 0 \\ 0 & q2 \end{bmatrix}$$

By choosing the performance index weighting parameters q1 and q2 in different ways, various mathematically optimal control laws can be determined. The values chosen for q1 and q2 govern the balance of the system performance qualities as between dynamic tyre load variations, reflecting the rough road manoeuvring capabilities, the suspension working space requirements, and the passenger discomfort levels.

To illustrate the technique, consider the case in which $q1=40.5\times10^{8}$ and $q2=3.35\times10^{8}$, when numerical solution of the Riccati equation by the negative exponential method described in Kuo (1975) gives the control

$$u(t)=30118xs1(t)-18303xs2(t)+1204xs3(t)-3170xs4(t)$$
$$=30118[x1(t)-x0(t)]-18303[(x2(t)-x0(t)]+1204\dot{x}1(t)-3170\dot{x}2(t)$$

Other examples can be found in Thompson (1976). By trial, combination of q1 and q2 values which give equal and opposite coefficients of xs1(t) and xs2(t) in the law can be found, in which case, realisation of the control is possible without measuring the height of the road as discussed by Thompson (1984).

2.4.2 Limited State Feedback

Using the same values of q1 and q2 as in the previous case, and taking M from (2.30), a reasonable initial estimate of the optimal control is

$$K = [\ 0 \quad -30000 \quad -1000 \quad 3000\ ]$$

the form of M causing the first term to be of no significance. Using this to form $\tilde{A}$ from (2.18) together with the transformation of coordinates as $SAS^{-1}-SBKM$ (A from (2.27) B from (2.28), its eigenvalues are found to be

$$-4.722 \pm 8.967 \quad \text{and} \quad -11.28 \pm 51.74$$

confirming that the closed loop system is stable, and following the procedure of section 2.2.2 a performance index

$J = 27588$ and gradients

$\partial J/\partial k2=-0.19404$, $\partial J/\partial k3=-0.38616$,and $\partial J/\partial k4=-1.7744$

are found for a road surface spectral density

$Dr(\nu) = 10^{-6}/\ \nu^2$ and $U = 20$ m/s.

Using the gradient information leads to an improved estimate of the optimal control

$$K = [\ 0 \quad -29865 \quad -1035 \quad 4243\ ]$$

and the cycle is repeated until the gradients are sufficiently small with

$$K = [\ 0 \quad -29820 \quad -1008 \quad 4457\ ]$$

implying the control

$$u(t)= 29820[xs1(t)-xs2(t)] +1008xs3(t) -4457xs4(t)$$

or $u(t)= 29820[x1(t)-x2(t)] +1008\dot{x}1(t) -4457\dot{x}2(t)$

Using the same values of q1 and q2 but assuming that only the difference between $\dot{x}1(t)$ and $\dot{x}2(t)$ is available and not their individual values

$$M = \left[\begin{array}{c|cccc} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

and following the same sequence as above, an optimal control

$$u(t)= 3785[x1(t)-x2(t)] + 1320[\dot{x}1(t)-\dot{x}2(t)]$$

results.

If the road surface spectral density is taken to be

$$Dr(\nu) = 10^{-6}(0.005^2 + \nu^2)$$

i.e. $\alpha = 0.005$, and

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

corresponding to the measurement of x1(t), x2(t), $\dot{x}1(t)$ and $\dot{x}2(t)$, then A comes from (2.27) with its first element changed to $-2\pi U\alpha$, B comes from (2.28) again, and $\tilde{A}$ from (2.18).

An initial estimate

$$K = [\ -30000 \quad 30000 \quad -1000 \quad 3000\ ]$$

and a repetition of the process described finally yields

$$u(t) = 29293x1(t) - 30530x2(t) + 998\dot{x}1(t) - 4487\dot{x}2(t)$$

as the control law required.

CHAPTER(3)

ROAD SURFACE INPUT AND SYSTEM RESPONSE

3.1 Introduction

Passive vehicle suspension systems contain inertial elements (masses), restoring elements (springs), and dissipative elements (dampers). The suspension system is passive in the sense that there is no external power added in order to control the system dynamics. The main feature of such systems is the cyclic interchange of kinetic and potential energy with some energy dissipation in damping devices. Active systems on the other hand essentially employ information feedback, involving the measurement of variables by sensors and the use of the signals to control actuators of some kind. The actuators require an energy supply. A mass, spring and damper system with a very slow-acting levelling device, needing a small power source, will be considered passive in the context of this work. If on the other hand, the device is more powerful, faster acting and responsive to feedback signals, but of limited bandwidth (about 4 Hz), such a system will be described as slow-active. A feedback controlled system in which the actuator has no power supply and is therefore a variable damper will be considered semi-active since the actuator can do no work on the suspension system. Clearly the power supply requirements of a semi-active system are very low.

According to the system behaviour, the suspension systems can be classified into linear systems (passive, active, and slow-active) and non-linear systems (semi-active).

The potential of the system analysis is to successfully predict the system response from an accurate definition of the input and adequate representation of the system characteristics. In road vehicles, the main source of disturbances is the road surface irregularities, which have a random nature. In the present study, the simple spectral density formula derived by Robson (1979) from an examination of much experimental data is taken to be a suitable description of the disturbance input. The physical properties of the system components (masses, spring stiffnesses, damping coefficients, and actuator and semi-active damper forces) can be combined together to define the mathematical model of the suspension system. The response of the model in hand can be expressed in terms of the ride comfort parameter, the dynamic tyre load variations, and the suspension working space.

In this chapter, the road surface irregularity is briefly described. The linear and non-linear analyses are discussed. The evaluation of the system response is considered.

3.2 Road Surface Description

The road surface irregularities have been represented statistically in the spectral density form

$$Dr(\nu) = B1/\nu^{n1} \qquad \text{................} \quad 3.1$$

as a function of the wave number ν.

Equation 3.1 as a function of the frequency f becomes

$$Dr(f) = B1.U^{n1-1}/f^{n1} \quad \text{................} \quad 3.2$$

where B1 is the road roughness constant and U is the vehicle speed.

This formula is suitable for application to linear systems in which the principle of superposition is applicable.

Non-linear system studies require representing the road surface profile in time history form. The time history can be generated by adding together l sine waves of different frequencies and amplitudes which can be chosen to represent properly the spectral density function in a chosen frequency range. The relationship to generate a single road profile can be specified as

$$Y(t) = \sum_{j=1}^{l} \sqrt{\frac{2\,B1\,U^{n1-1}}{f_j^{\,n1}}\,df}\ \sin(2\pi f_j\ t_k + \psi_j) \quad \text{.........} \quad 3.3$$

where,

Y(t) is the profile displacement, m

l is the number of sine waves

k is the number of time samples

f is the frequency in Hz.

df is the frequency interval in Hz.

ψ is the phase angle in rad.

t is the time in second, s

The phase angles can be determined by a random number generator which for any set of values gives the profile an approximately Gaussian probability distribution, provided l is sufficiently large, Newland (1984).

An alternative method to generate the road surface profile by simply applying the inverse Fourier transform to the amplitude and phase (frequency domain) data as demonstrated by Cebon and Newland (1983) is available.

Five time histories generated by using equation 3.3 are plotted in Fig. 3.1 as examples of these profiles. They are chosen to be of 4 seconds period. Five different sets of phase angles with the following additional parameters:

l = 60 sine waves

k = 3072 samples

f = 0.25 : 15 Hz.

df= 0.25 Hz.

t = 0 : 6 seconds

are used to represent a somewhat worse than average main road with roughness constant $B1=3.14\times10^{-6}$ traversed by a vehicle with speed U=20 m/s. The value of nl is taken as 2.5 as proposed by Robson (1979).

3.3 Linear Systems

The second order differential equations of the passive system can be derived by applying Newton's second law in the form

$$[MS]\,\ddot{x}(t) + [CS]\,\dot{x}(t) + [KS]\,x(t) = y(t) \qquad \text{................} \quad 3.4$$

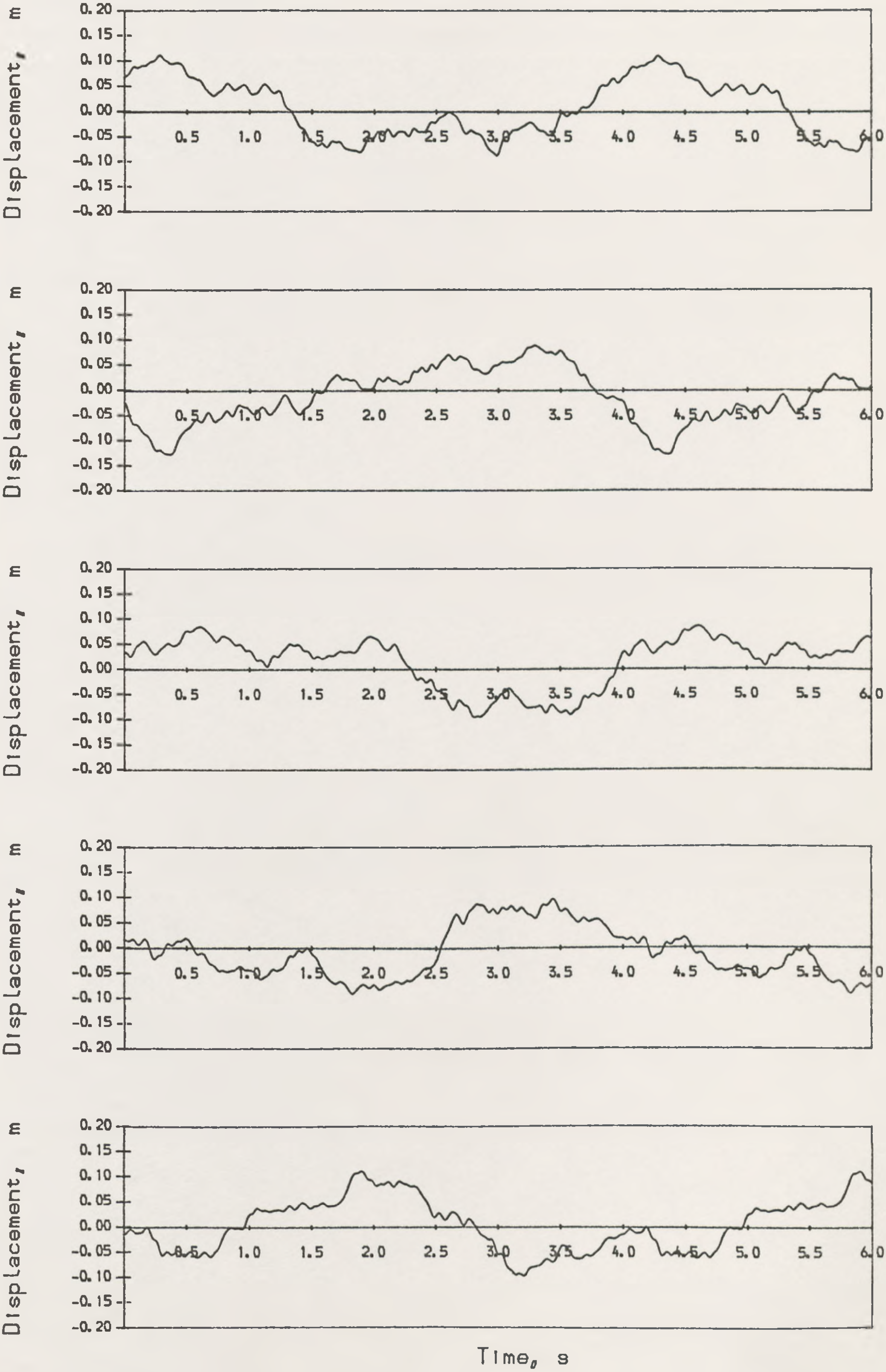

Fig. 3.1 Time histories of road surface profiles using different sets of phase angles

where y(t) is the input vector, x(t) is the output vector, [MS], [CS], and [KS] are square matrices of order r representing the masses, damping coefficients, and stiffnesses respectively, and r defines the number of equations (number of degrees of freedom). Also, $\dot{x}(t)$, and $\ddot{x}(t)$ are the first and second derivatives of x(t). The system is said to be linear if the separate outputs resulting from many inputs can be added together in order to correctly obtain the response to the combined excitations (the principle of superposition).

If the input variable y(t) is assumed to be harmonic in the form

$$y(t) = \underline{Y}\, e^{j\omega t} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad 3.5$$

The steady-state response can be written as

$$x(t) = \underline{X}\, e^{j\omega t} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad 3.6$$

where $\underline{Y}$ is the excitation amplitude vector and $\underline{X}$ is the complex output amplitude vector. The vector $\underline{X}$ depends on the driving frequency ω and the system parameters.

Inserting equations 3.5 and 3.6 into 3.4 yields

$$[ZS(\omega)]\ \underline{X} = \underline{Y} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad 3.7$$

where

$$[ZS(\omega)] = -\omega^2[MS] + j\omega[CS] + [KS]$$

is the impedance matrix.

By inspection of equation 3.7 and using unity excitation amplitude vector, the frequency response function of the system becomes

$$\underline{X} = [ZS(\omega)]^{-1} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad 3.8$$

It can be noticed that the active control law is usually a function of the output vector x(t) and/or its derivatives $\dot{x}(t)$ and $\ddot{x}(t)$. Also, such an active law is linear. These aspects together allow the study of the active suspension systems in the same manner. The only difference is to add the coefficients of the active control law to the impedance matrix [ZS(ω)].

At this stage, attention should be transferred to the suspension system problem. This will be helpful in indicating how the system performance can be evaluated from the solution of the complex equation 3.8. For example, it is required to express the complex frequency response function of the suspension working space [Hs(ω)]. This function represents the difference between two of the output variables (wheel and body motions, x1(t) and x2(t)). It can be evaluated as

$$\left| Hs(\omega) \right| = \{[Real(X1-X2)]^2 + [Imag(X1-X2)]^2\}^{1/2} \ldots\ldots\ldots\ldots\ldots\ldots \quad 3.9$$

where X1 and X2 are the wheel and body amplitudes respectively.

Since the spectral density function represents an amplitude square relationship for both the input and the output, the system linearity allows multiplying the square of the frequency response function by the input spectral density function in order to obtain the output spectral density function for any value of the frequency f. More explanation can be found in Newland (1984) and Meirovitch (1975).

Writing the frequency response function 3.9 as a function of the frequency f, the output spectral density function can be formulated as

$$Ds(f) = Dr(f).\left|Hs(f)\right|^2 \qquad \text{............... } 3.10$$

Similarly, the spectral density functions of the body acceleration Da(f) and the dynamic tyre load variations Dd(f) can be expressed as

$$Da(f) = Dr(f).\left|Ha(f)\right|^2 \qquad \text{............... } 3.11$$

and $$Dd(f) = Dr(f).\left|Hd(f)\right|^2 \qquad \text{............... } 3.12$$

The ride comfort parameter can be evaluated by weighting the spectral density function of the body acceleration 3.11 according to the ISO standard weighting function shown in Fig. 3.2, Vries (1982).

The individual responses can be expressed in terms of the mean square values by numerically integrating the spectral density functions over the frequency range of interest. The root mean square value of the suspension working space, for example, is

$$SWS = \sqrt{\int_{fmin}^{fmax} Ds(f)\, df} \qquad \text{............... } 3.13$$

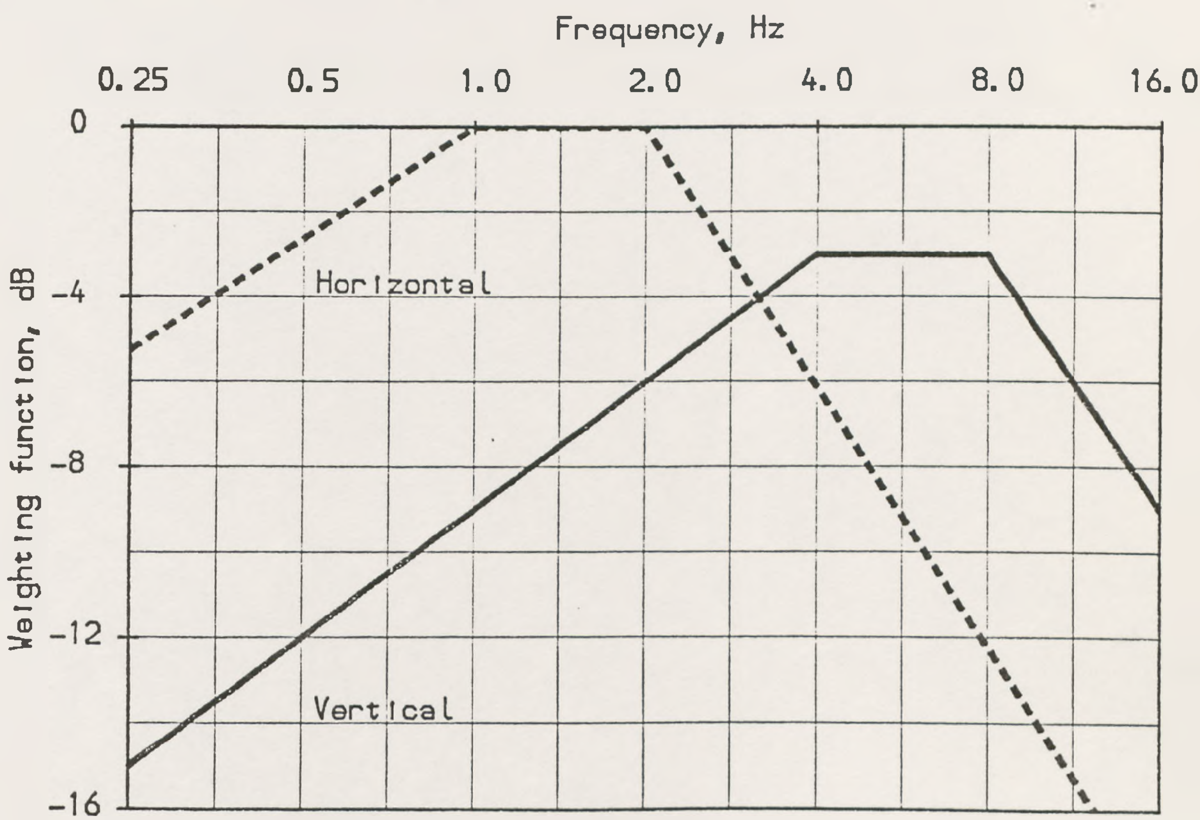

Fig. 3.2 ISO weighting functions for squared acceleration

The procedures for evaluating the linear system responses are illustrated in Fig. 3.3.

3.4 Non-Linear Analysis

Semi-active suspension systems are non-linear because of the switching dynamics of the semi-active damper. Simulations are necessary for studying these systems.

In this section, the time history of the system response is generated. Spectral density functions are derived in order to obtain results in standard form.

By inspection of equation 3.4, the equations of motion can be written in their state space form as

$$\dot{c}(t) = [L]\, c(t) + y(t) \qquad \ldots\ldots\ldots\ldots\ldots \quad 3.14$$

with

$$c(t) = \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}$$

where [L] is a coefficient matrix of order 2r (r is the number of degrees of freedom). It should be noted that the coefficients of the semi-active damper are included in the matrix [L].

Once the time history of the input function y(t) is known, the set of equations 3.14 can be numerically integrated in order to obtain the solution c(t). A suitable set of initial conditions c(0) are required. More explanation will be given in chapter 6.

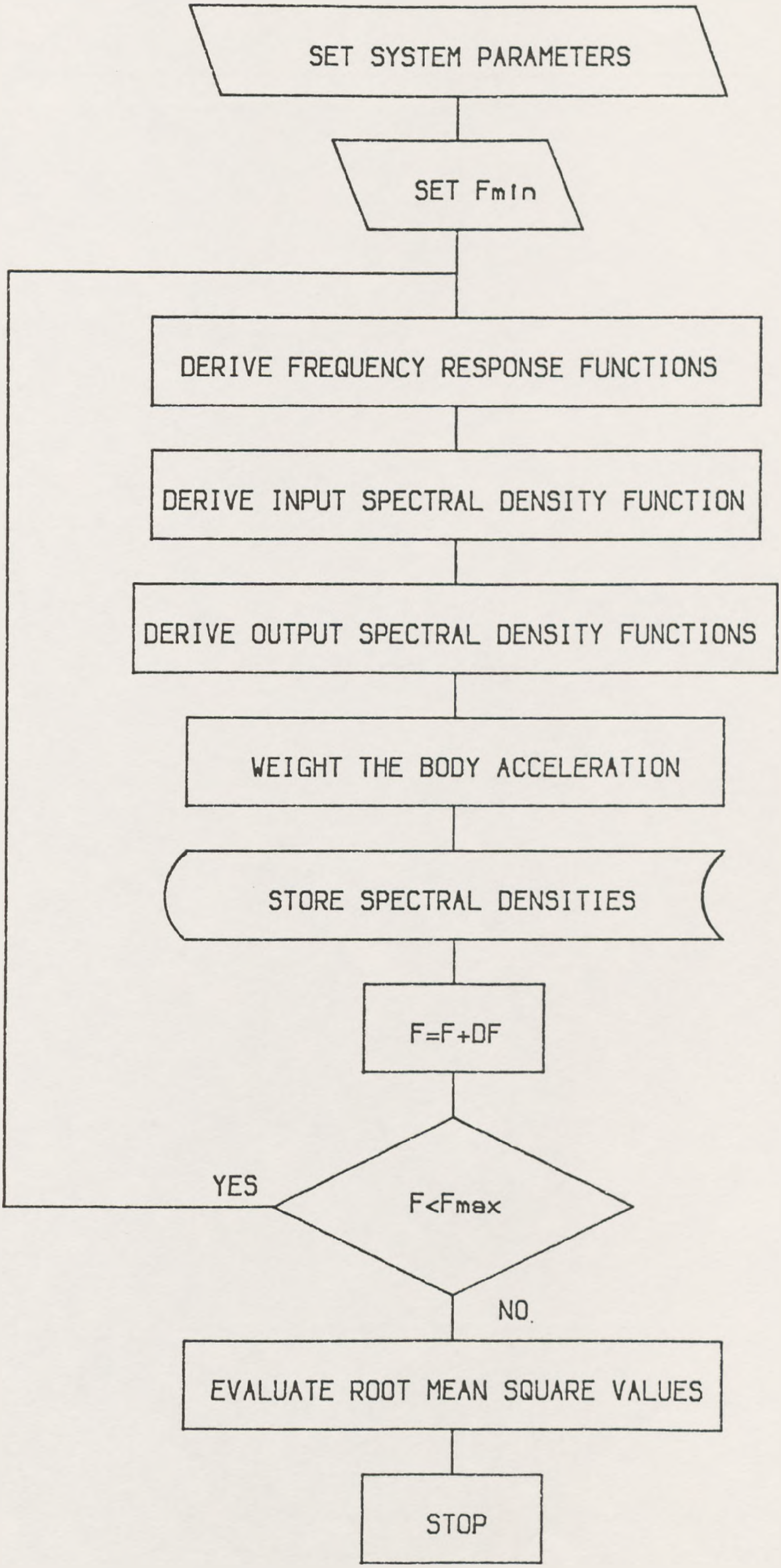

Fig. 3.3 Linear analysis

For the same example as in section 3.3, the time history of the suspension working space can be derived as

$$s.w.s.(t) = x1(t) - x2(t) \qquad \text{.............} \quad 3.15$$

The steady-state part of the time function 3.15 can be simply obtained by eliminating the transient region. The rest of the function can be transformed into spectral density form by using the Fast Fourier Transform. The arrangements of these transformations will be explained in chapter 6. The root mean square values of the individual responses can be obtained using the same procedures as described in section 3.3.

The simulation procedures are summarised in Fig. 3.4.

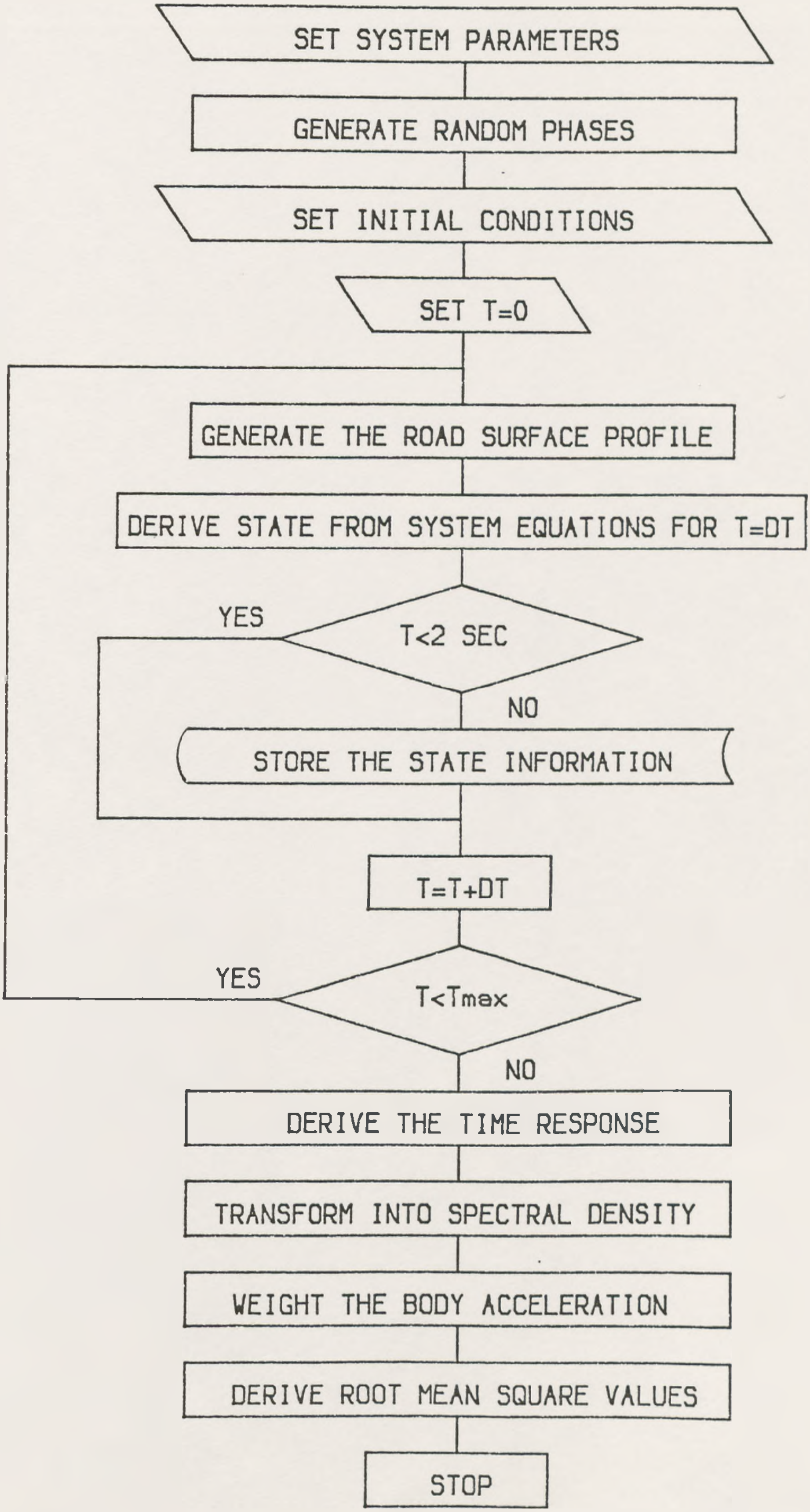

Fig. 3.4 Non-Linear analysis

CHAPTER(4)

PASSIVE SUSPENSION SYSTEM STUDIES

4.1 Introduction

Passive suspension systems of conventional elements (springs and dampers) have limitations in respect of completely controlling the vehicle dynamics. The difficulty comes from vehicles typically being operated over roads of different qualities at different speeds as well as requiring adequate attitude control with load changes and manoeuvring. Even for the same operating conditions, it is well known that the ride comfort parameter can be indefinitely improved (at the expense of the suspension working space) by softening the suspension spring. In practice, the working space must be restricted. Normal passive suspension parameter choices represent a compromise between the different requirements and are made according to the vehicle type and layout. Adjustable parameter passive suspension systems will be discussed in the appendix.

The study in this chapter is restricted to generating results by which the fundamental performance properties of passive suspensions can be understood. This understanding can help the designer to choose the appropriate parameters for different operating conditions. The study involves investigating the vehicle vertical behaviour by using the well known quarter car model subjected to realistic road roughness input. The possibility of improving the vehicle performance through adding a dynamic absorber to the two mass system is included. In both cases, the quality of the suspension system performance is assessed by passenger discomfort, dynamic tyre load, and suspension working space parameters. Different choices of the suspension parameters are made in order to map the two mass system properties while an optimisation process is used in order to obtain the three mass system parameters. Two different wheel to body mass ratios are used in both cases in order to cover the normal range of passenger cars. Different ratios of absorber to wheel mass are used in studying the three mass suspension systems.

4.2 Two Mass System

The two mass suspension system shown in Fig. 4.1 consists mainly of a quarter of the body mass (mb) and a wheel assembly mass (mw). The passive suspension elements are shown as a linear spring of stiffness Ks and a linear damper of coefficient Cs, connected between the body and wheel masses. A linear spring of stiffness Kt (representing the tyre vertical dynamics) is used to support the whole model.

Referring to Fig. 4.1, the equations of motion can be written as,

$$\begin{aligned} mw\ddot{x}1(t) &= -Ks[x1(t)-x2(t)]-Cs[\dot{x}1(t)-\dot{x}2(t)]+Kt[x0(t)-x1(t)] \\ mb\ddot{x}2(t) &= Ks[x1(t)-x2(t)]+Cs[\dot{x}1(t)-\dot{x}2(t)] \end{aligned} \quad \ldots \quad 4.1$$

where, $x0(t)$ is the road roughness displacement, $x1(t)$, $\dot{x}1(t)$ and $\ddot{x}1(t)$ are the wheel displacement, velocity and acceleration and

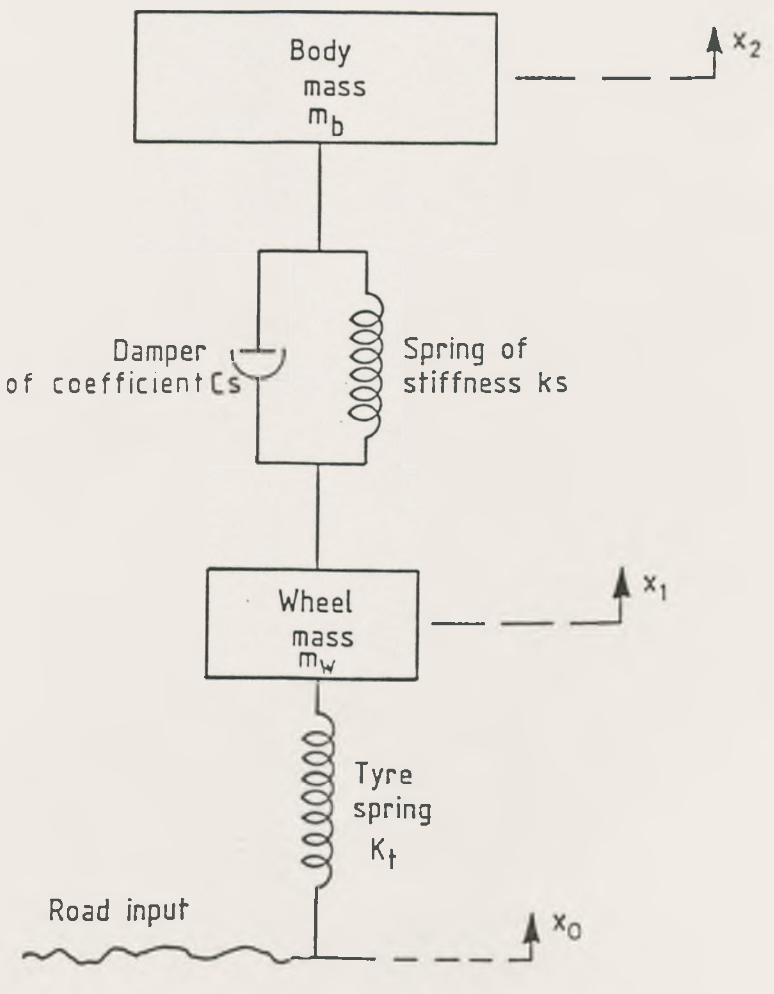

Fig. 4.1 Quarter car passive suspension system

$x2(t)$, $\dot{x}2(t)$ and $\ddot{x}2(t)$ are the body displacement, velocity and acceleration respectively.

Following the procedures described in section 3.3, equations 4.1 can be reduced to,

$$\begin{bmatrix} X1 \\ X2 \end{bmatrix} = \begin{bmatrix} Kt+Ks-\omega^2 mw+jCs\omega & -Ks-jCs\omega \\ -Ks-jCs\omega & Ks-\omega^2 mb+jCs\omega \end{bmatrix}^{-1} \begin{bmatrix} Kt \\ 0 \end{bmatrix} \quad \text{..........} \quad 4.2$$

The solution procedures described in Fig. 3.3 can be simply followed for each value of the frequency f which is chosen to be in the range from 0.25 to 15 Hz. The solution of the complex linear equations 4.2 has been obtained by using the Crout´s factorisation method available in the form of a NAG library subroutine. The frequency response function of the suspension working space [$Hs(\omega)$] can be derived in the form 3.9. Similarly, the frequency response functions of the body acceleration [$Ha(\omega)$] and dynamic tyre load variations [$Hd(\omega)$] can be written as,

$$\left| Ha(\omega) \right| = \omega^2 \{[Real(X2)]^2 + [Imag(X2)]^2\}^{1/2} \quad \text{..........} \quad 4.3$$

and

$$\left| Hd(\omega) \right| = Kt\{[Real(X1-X0)]^2 + [Imag(X1-X0)]^2\}^{1/2} \quad \text{..........} \quad 4.4$$

Using the road spectral density formula 3.2 and the frequency response functions 3.9, 4.3 and 4.4 as functions of the frequency f, the spectral density functions of the suspension working space Ds(f), the body acceleration Da(f), and the dynamic tyre load Dd(f) can be obtained as described in equations 3.10, 3.11 and 3.12 respectively. For each value of the frequency, the ISO weighting function shown in Fig. 3.2 can be applied to the spectral density function of the body acceleration in order to obtain the ride comfort parameter by using the following parameters:

$f/8$ for the frequency range from 0.25 to 4 Hz,

0.5 for the frequency range from 4 to 8 Hz, and

$32/f^2$ for the frequency range from 8 to 15 Hz.

Root mean square values are obtained by taking square roots of the trapezoidally integrated spectral density functions over the frequency range from 0.25 to 15 Hz.

The body mass mb is chosen to be 250 Kg. while two different values of the wheel mass mw (50 and 31.25 Kg) are used to represent wheel to body mass ratios (0.2 and 0.125) spanning the range normally found in passenger cars. The tyre stiffness Kt is chosen to be 120000 N/m to represent a reasonable value of the vertical stiffness of a rolling tyre as recorded by van Eldik Thieme (1981). The spectral density function 3.2 is used as input to the system with constants $B1=3.14\times10^{-6}$ and $n1=2.5$ and vehicle speed $U=20$ m/s.

The spring stiffness is described by the more familiar uncoupled undamped natural frequency of the body mass fn, while the damping is described by the damping as a proportion of critical of this same decoupled system γ, where,

$fn = 1/2\pi\sqrt{Ks/mb}$ and $\gamma = Cs/(2\sqrt{Ks\ mb})$

Values of fn from 0.3 to 2.0 Hz and of γ from 0.2 to 1.6 are used.

Results obtained for body to wheel mass ratio of 0.2 are shown in Fig. 4.2 and in Fig. 4.3 for 0.125 mass ratio. In both figures, the performance relationships have the same form. The greatest ride comfort can be obtained by using the softest spring possible with very little damping, but the high comfort is obtained at the expense of the suspension working space. This fact has been indicated by Ryba (1974) and by Sharp and Hassan (1984) and can be expected here if the solid curves ($\gamma = 0.2$) in figures 4.2 and 4.3 were extended towards a higher value of the suspension working space by generating more systems with lower spring stiffnesses.

The demand for working space can be assessed on the basis that the road surface is Gaussian, in which case each system output parameter will be Gaussian. Then the displacement of the wheel relative to the body measured from static equilibrium will exceed (in magnitude) twice the root mean square value for 4.6% of the time and three times this value for 0.27% of the time. If, for the sake of discussion, we regard the maximum tolerable r.m.s. wheel to body displacement to be 1.5, 2.0 or 2.5 cm, various systems, classified as groups (a), (b) and (c), can be obtained for each of these values. These groups, as depicted in figures 4.2 and 4.3, are summarised in table 4.1 for 0.2 mass ratio and in table 4.2 for 0.125 mass ratio. For ease of discussion, systems in each group are numbered from (1) to (5). Comparing the various systems in each group for each of these mass ratios, we can see that the best performing systems are systems (3) in which the best comfort and dynamic tyre load can be obtained for

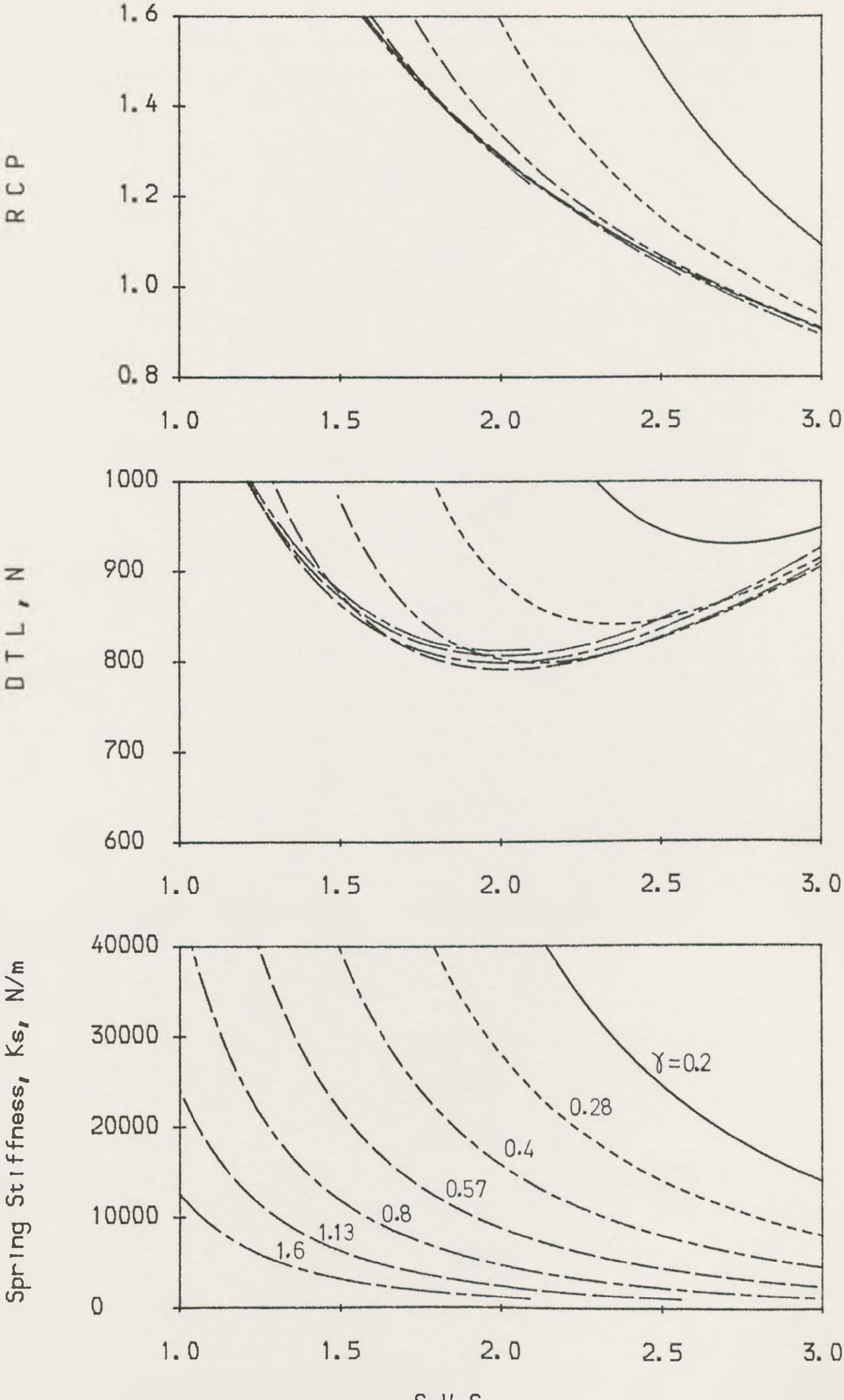

Fig. 4.2 Performance and design properties of two mass passive systems, mw/mb=0.200

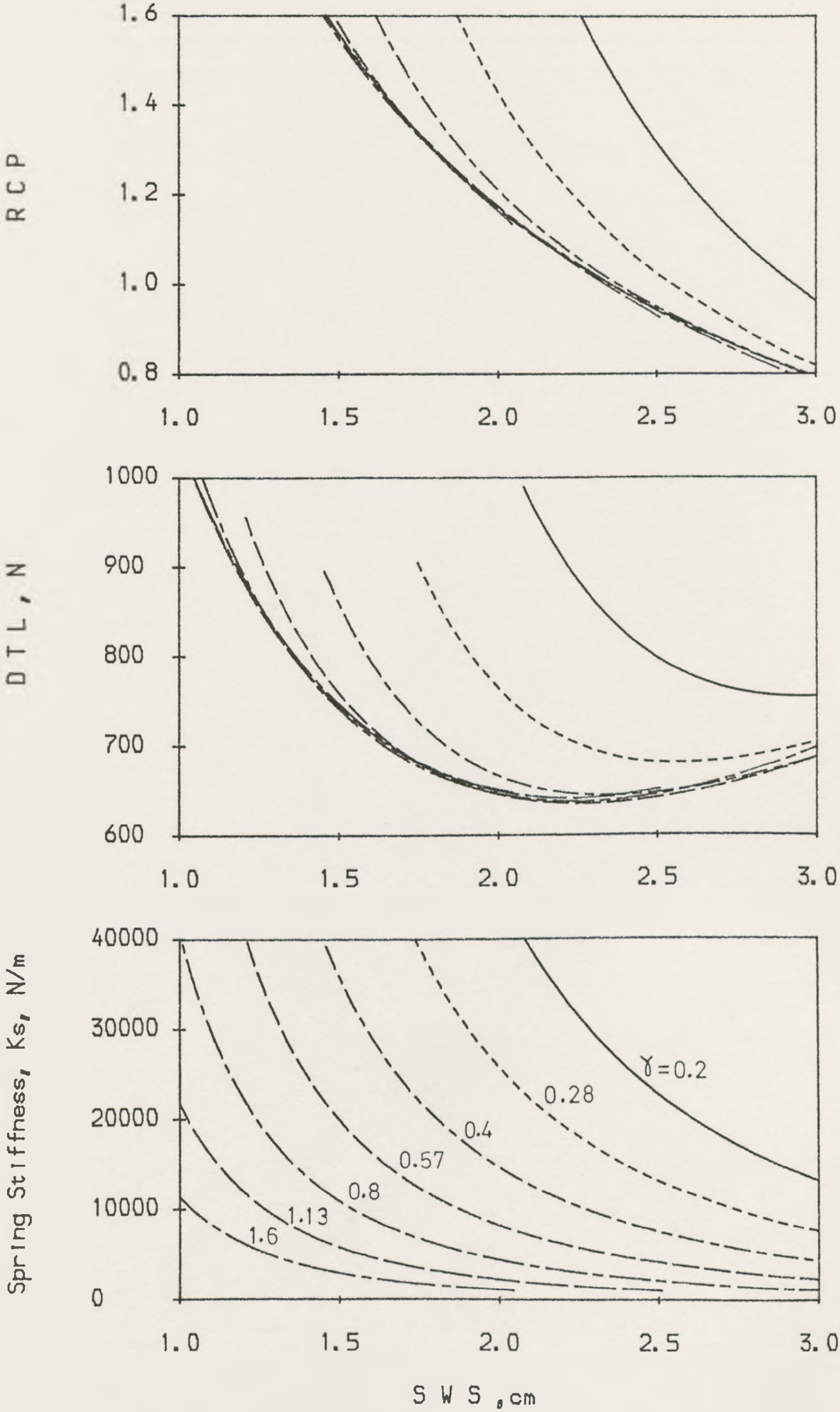

Fig. 4.3 Performance and design properties of two mass passive systems, mw/mb=0.125

Table 4.1 Design and performance properties of particular two mass passive systems having 0.200 wheel to body mass ratio

No	SWS(cm)	RCP	DTL(N)	fn(Hz)	γ
1		1.9549	978	1.997	0.400
2	(a)	1.7098	875	1.484	0.566
3	1.5	1.6662	864	1.097	0.800
4		1.6718	872	0.799	1.131
5		1.6759	878	0.569	1.600
1		1.5862	890	1.685	0.283
2	(b)	1.3387	802	1.267	0.400
3	2.0	1.2875	791	0.946	0.566
4		1.2895	798	0.694	0.800
5		1.2907	806	0.495	1.131
1		1.4760	945	1.579	0.200
2	(c)	1.1515	846	1.193	0.283
3	2.5	1.0678	825	0.902	0.400
4		1.0603	826	0.669	0.566
5		1.0584	835	0.478	0.800

Table 4.2 Design and performance properties of particular two mass passive systems having 0.125 wheel to body mass ratio

No	SWS(cm)	RCP	DTL(N)	fn(Hz)	γ
1		1.7854	859	1.904	0.400
2	(a)	1.5794	759	1.421	0.566
3	1.5	1.5478	742	1.053	0.800
4		1.5560	745	0.767	1.131
5		1.5591	747	0.546	1.600
1		1.4279	764	1.616	0.283
2	(b)	1.2111	666	1.220	0.400
3	2.0	1.1696	645	0.913	0.566
4		1.1735	646	0.670	0.800
5		1.1740	649	0.477	1.131
1		1.3166	798	1.520	0.200
2	(c)	1.0244	681	1.154	0.283
3	2.5	0.9500	647	0.875	0.400
4		0.9448	641	0.649	0.566
5		0.9419	645	0.463	0.800

1.5 and 2 cm suspension working space. For the 2.5 cm suspension working space, there are only small differences between ride comfort and dynamic tyre load for systems (3) and (4). As the value of fn is increased and the value of γ decreased, as compared with the parameters of system (3), in such a way as to keep the same value of the suspension working space, the comfort and dynamic tyre load for all groups are sacrificed. Although systems (4) and (5) behave as well as systems (3), they are practically of little interest because of the very soft springs employed as compared with systems (2) and (3).

The relative performances of each group in table 4.1 are shown in figures 4.4, 4.5 and 4.6 and in figures 4.7, 4.8 and 4.9 for the corresponding groups in table 4.2. Each of these figures indicates the frequency response functions and the mean square spectral densities of the weighted body acceleration, the dynamic tyre load and the suspension working space for systems (1), (3) and (5). In all figures, the high peaks of the weighted body acceleration and the dynamic tyre load in systems (1) at the body resonance can be seen. These peaks are responsible for the high r.m.s. values in both the ride comfort parameter and the dynamic tyre load in systems (1) as compared with systems (3) and (5). In systems (3) and (5), the peaks are significantly reduced with slightly higher values of these functions at the wheel resonance as a result of reducing the spring stiffness and increasing the damping coefficient. Practically, systems with stiffer springs may be preferred in the sense that they are better in controlling the ride height variations with load changes but on the other

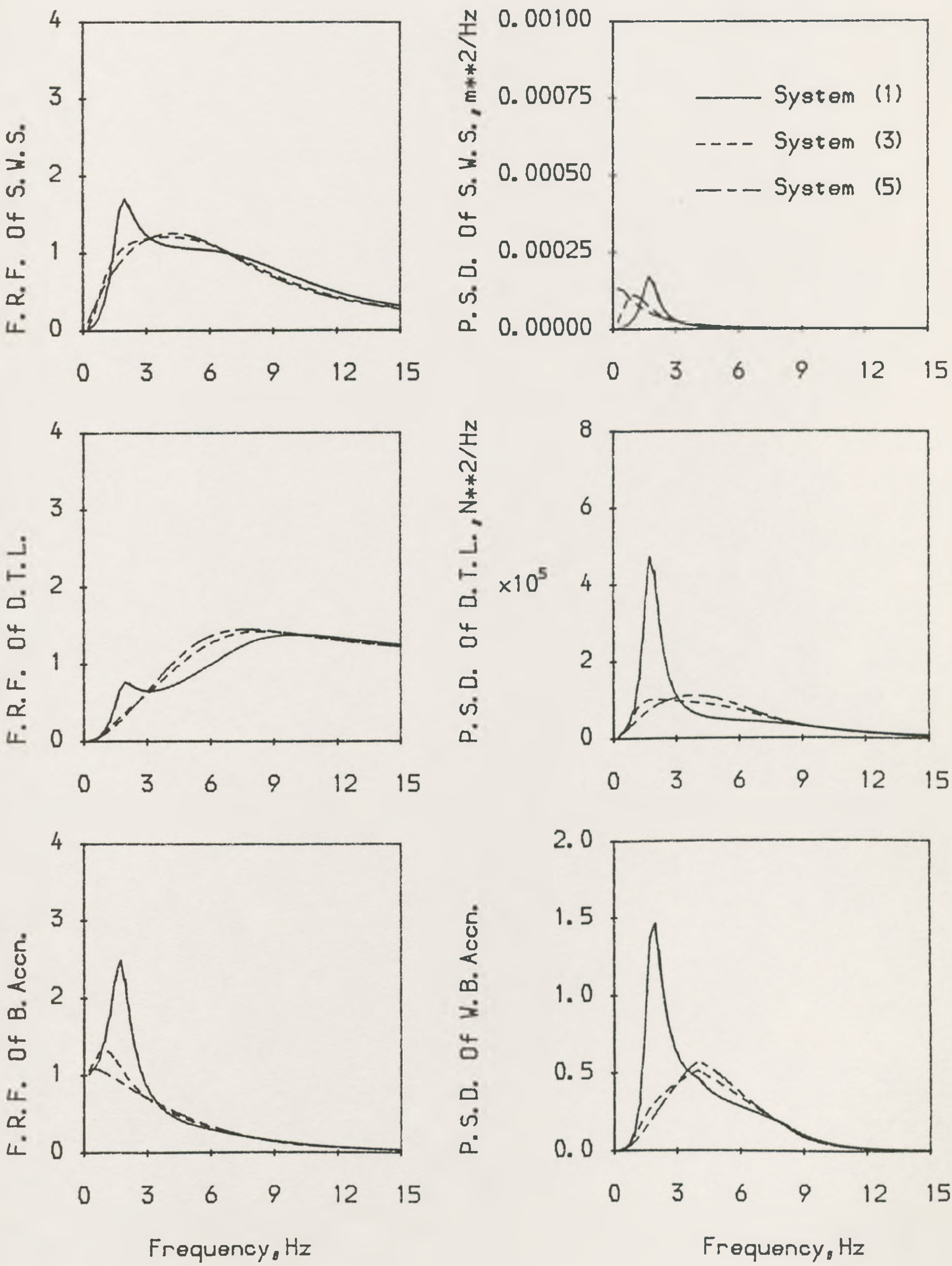

Fig. 4.4 Frequency responses and output mean square spectral densities of two mass passive systems, mw/mb=0.200
S W S = 1.5 cm

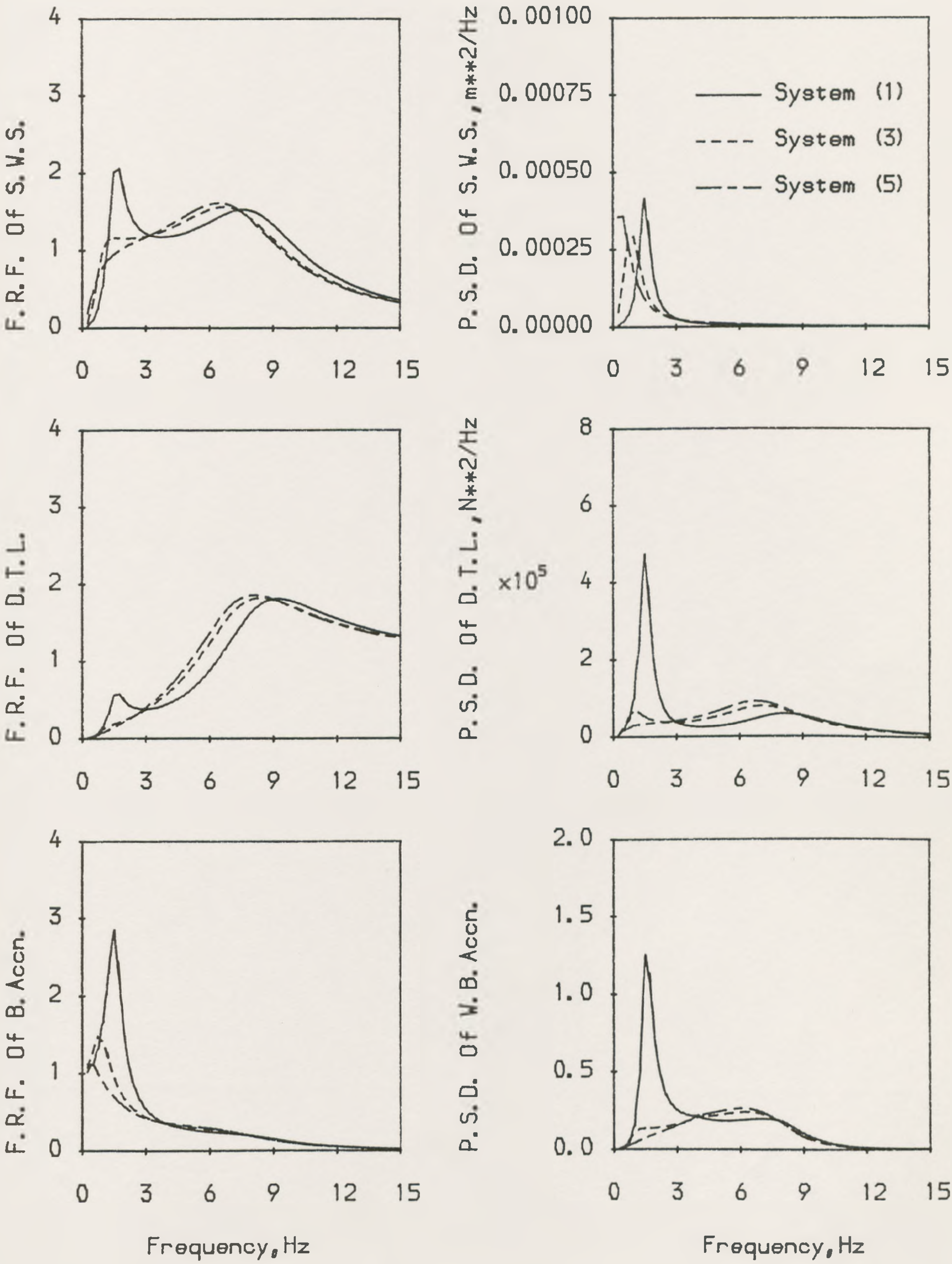

Fig. 4.5 Frequency responses and output mean square spectral densities of two mass passive systems, mw/mb=0.200
S W S = 2.0 cm

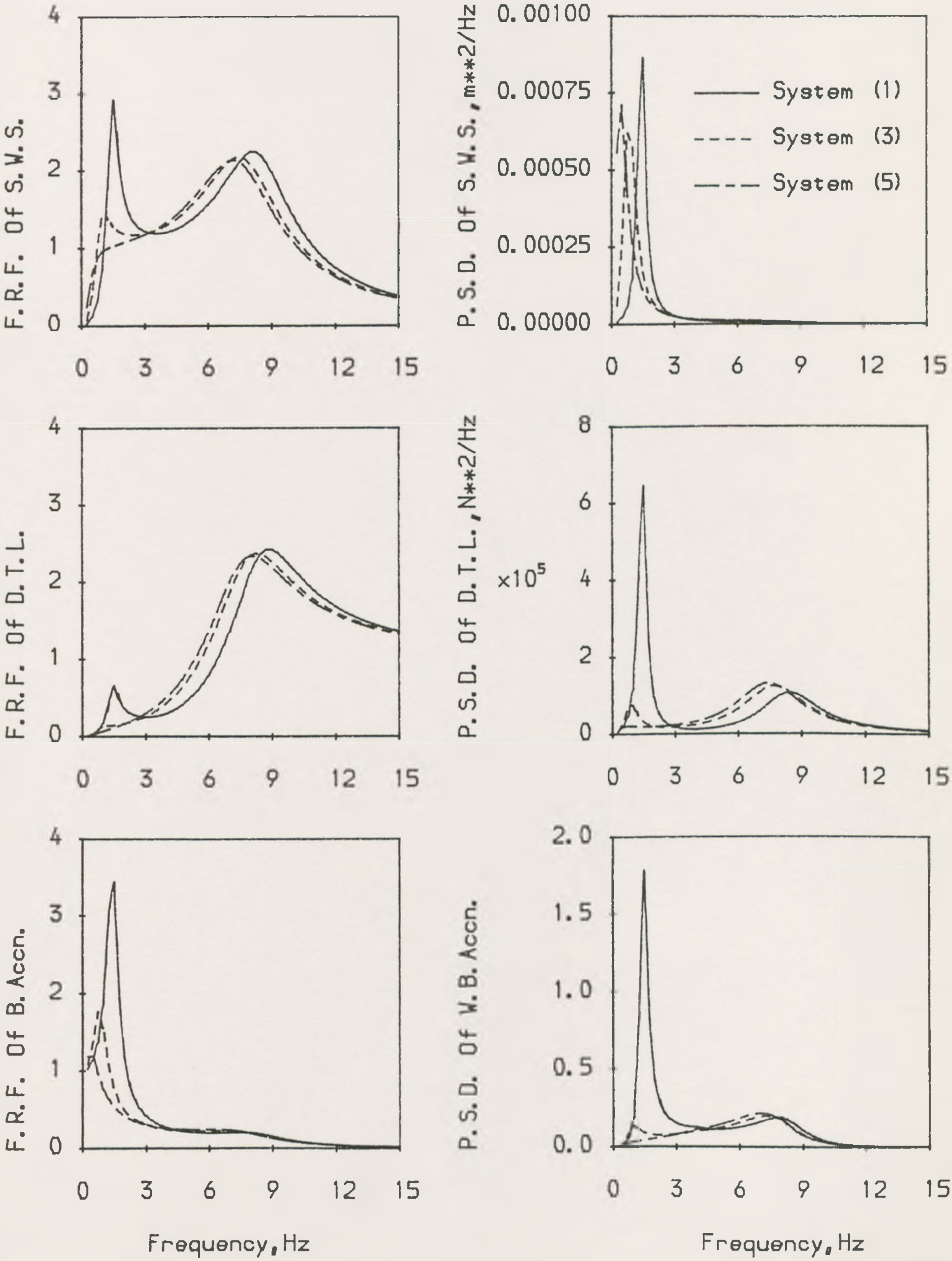

Fig. 4.6 Frequency responses and output mean square spectral densities of two mass passive systems, mw/mb=0.200

S W S = 2.5 cm

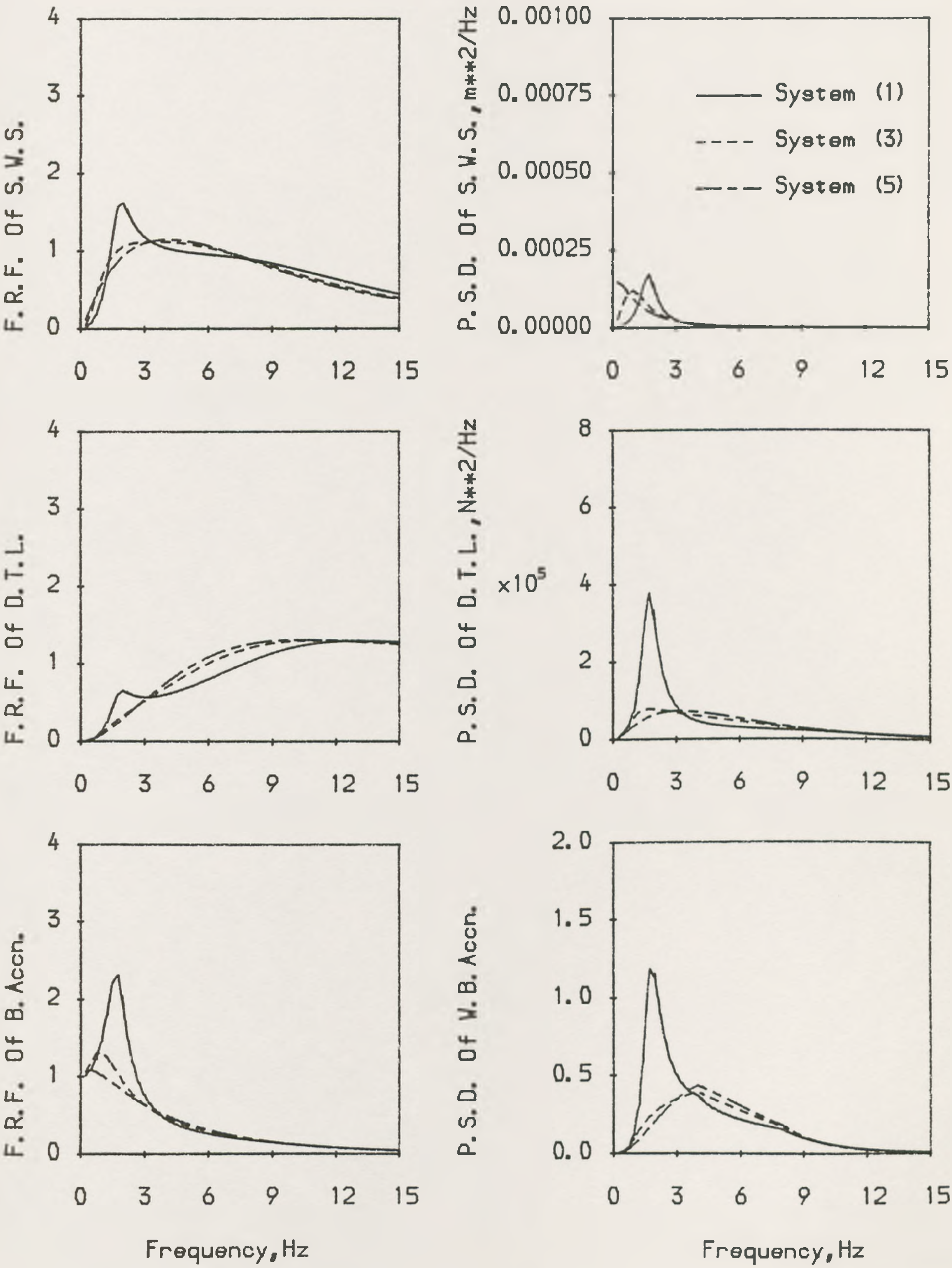

Fig. 4.7 Frequency responses and output mean square spectral densities of two mass passive systems, mw/mb=0.125

S W S = 1.5 cm

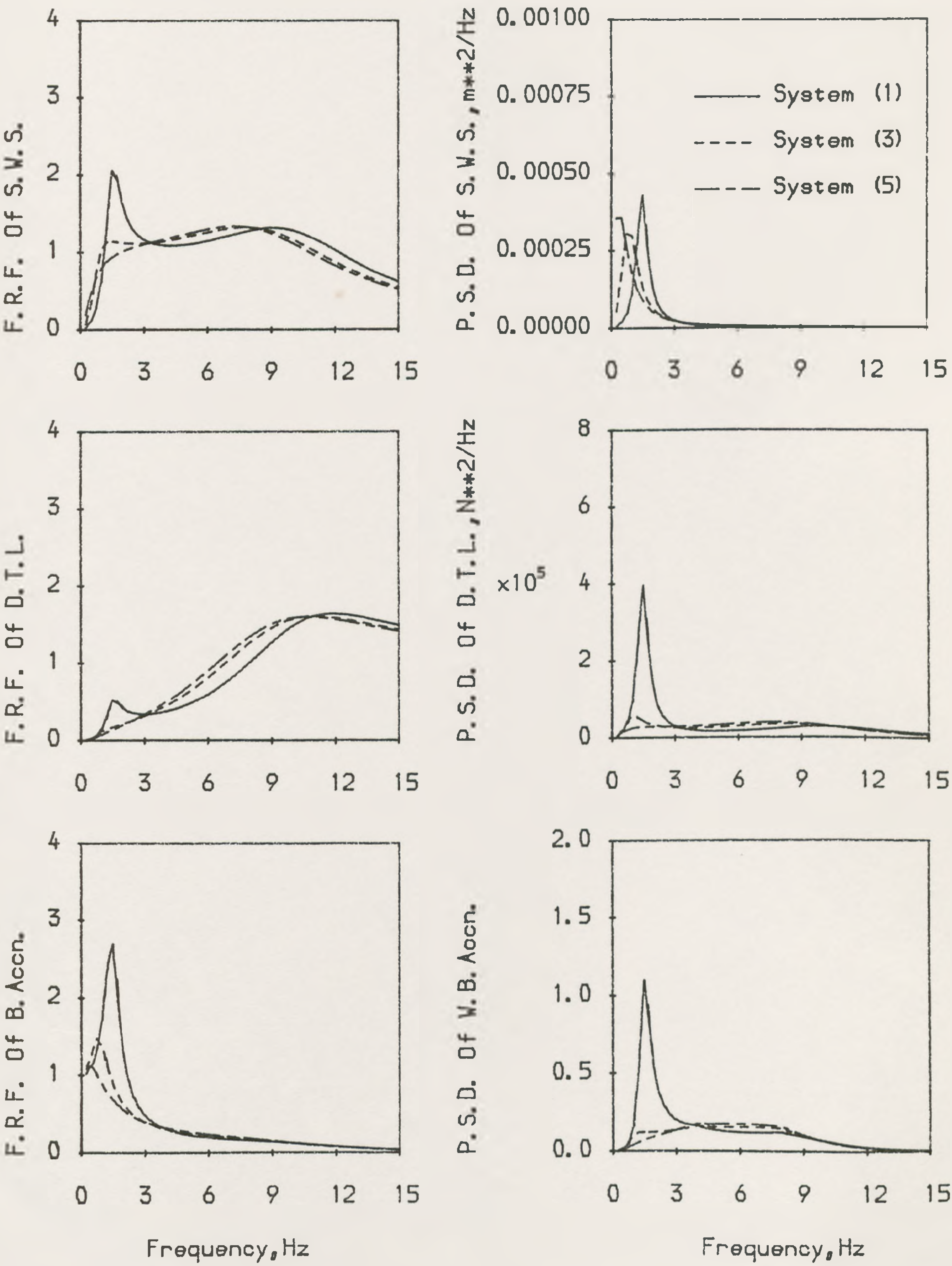

Fig. 4.8 Frequency responses and output mean square spectral densities of two mass passive systems, mw/mb=0.125
S W S = 2.0 cm

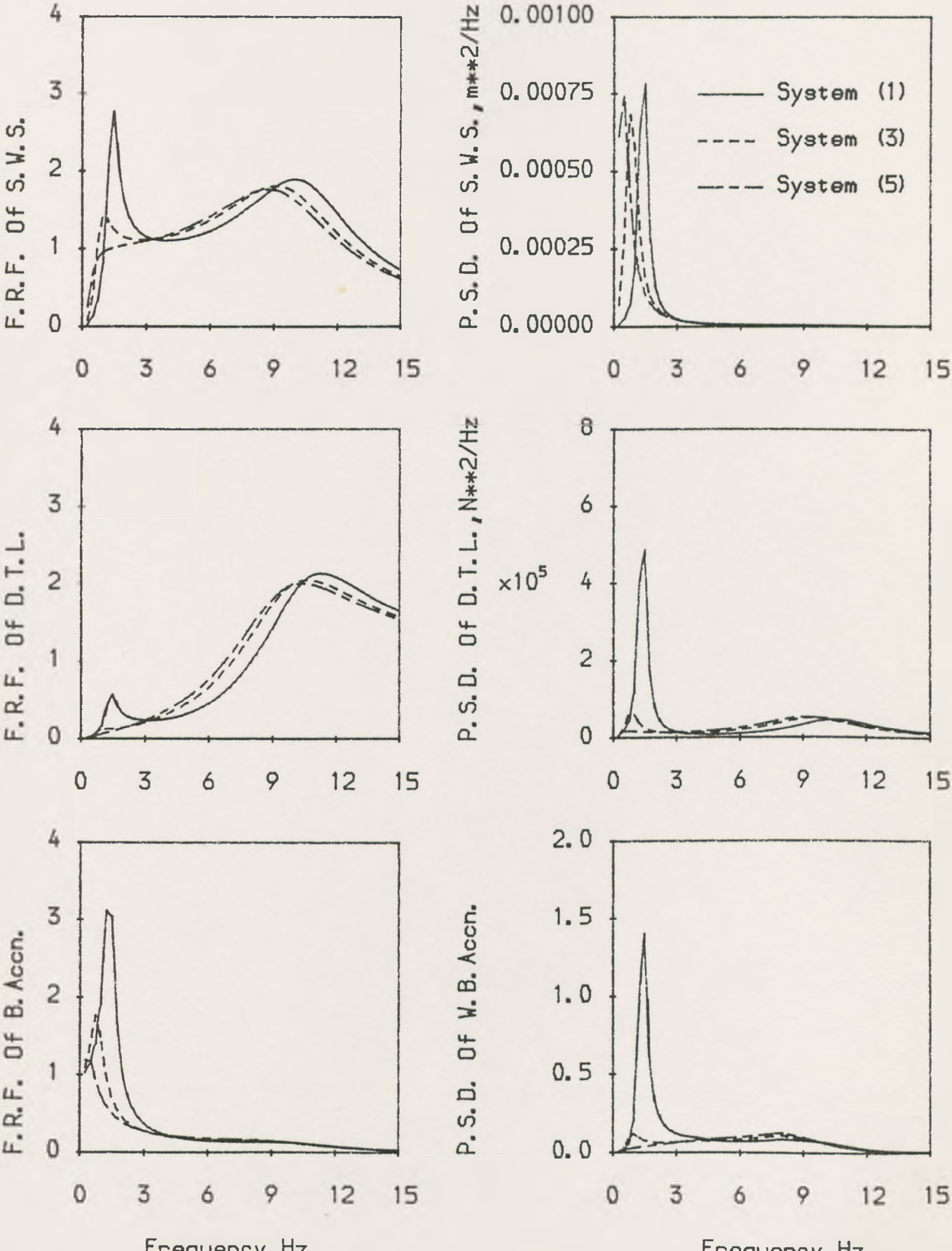

Fig. 4.9 Frequency responses and output mean square spectral densities of two mass passive systems, mw/mb=0.125
S W S = 2.5 cm

hand, systems with very stiff springs (systems (1)) perform very badly in terms of ride comfort and dynamic tyre load as compared with systems (3). Although systems (3) are a little better (about 3%) in both comfort and dynamic tyre load than systems (2), the spring stiffnesses of systems (2) are double those in systems (3).

If we compare the different groups, we find that systems (3) are the best performing systems. As the working space available increases from 1.5 to 2.5 cm, these best performing systems involve lowering spring stiffnesses and damping coefficients, implying that the "best" system design depends on the relationship between the road roughness and the working space available. The performance properties of systems in tables 4.1 and 4.2 are plotted in figures 4.10 and 4.11 respectively in order to map the relationship between ride comfort and dynamic tyre load variations for each value of the standard suspension working space. These figures indicate that better comfort can be gained if the suspension working space available is increased. Other results not included, together with an understanding of the problem, indicate that the returns diminish as the working space to road roughness parameter ratio increases. Better dynamic tyre load control is also obtained as the working space increases up to a point beyond which this trend is reversed. Variations in the optimum dynamic tyre load are small, however, and thus this is not a large issue.

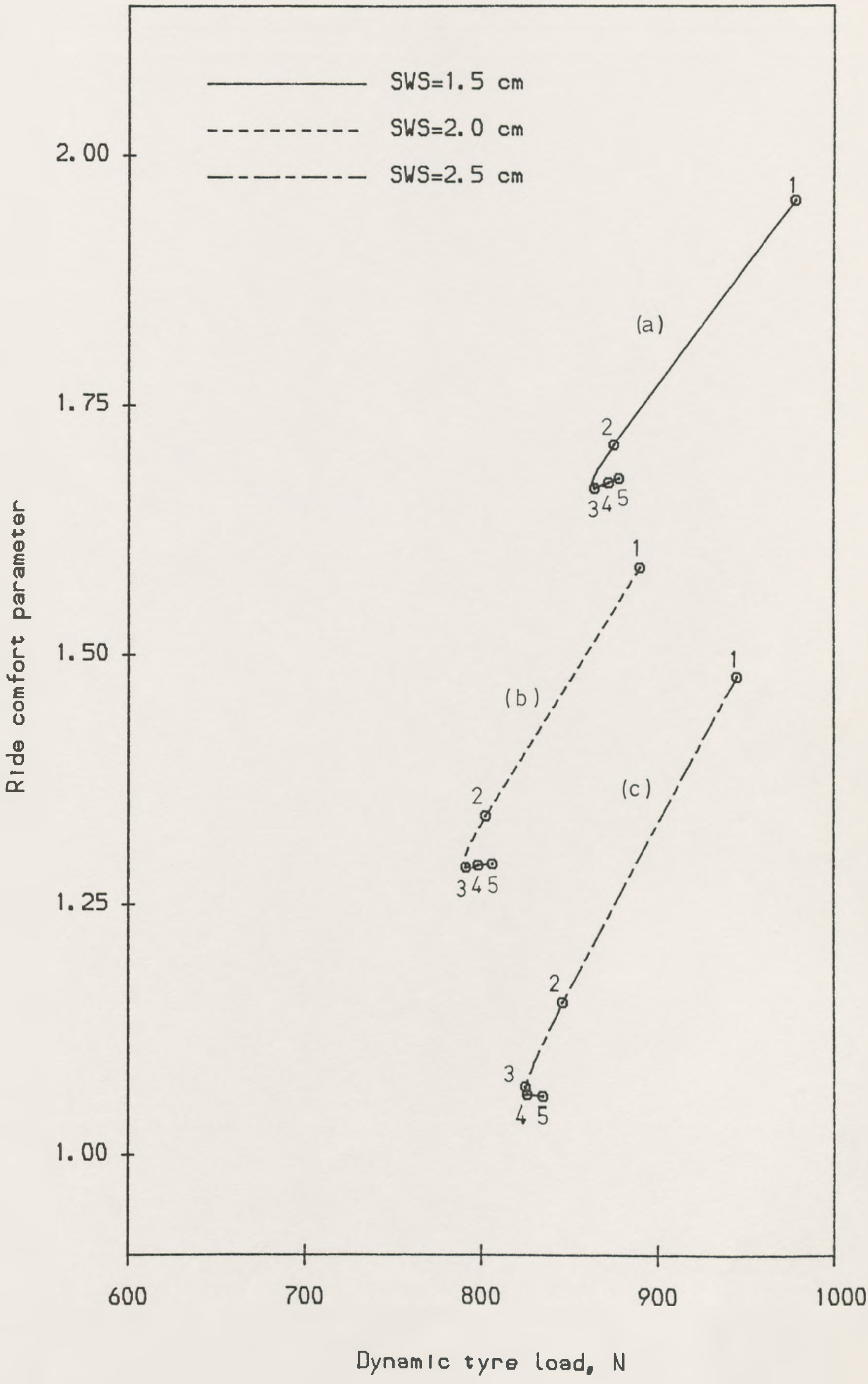

Fig. 4.10 Ride comfort and dynamic tyre load variation of two mass passive systems, mw/mb=0.200

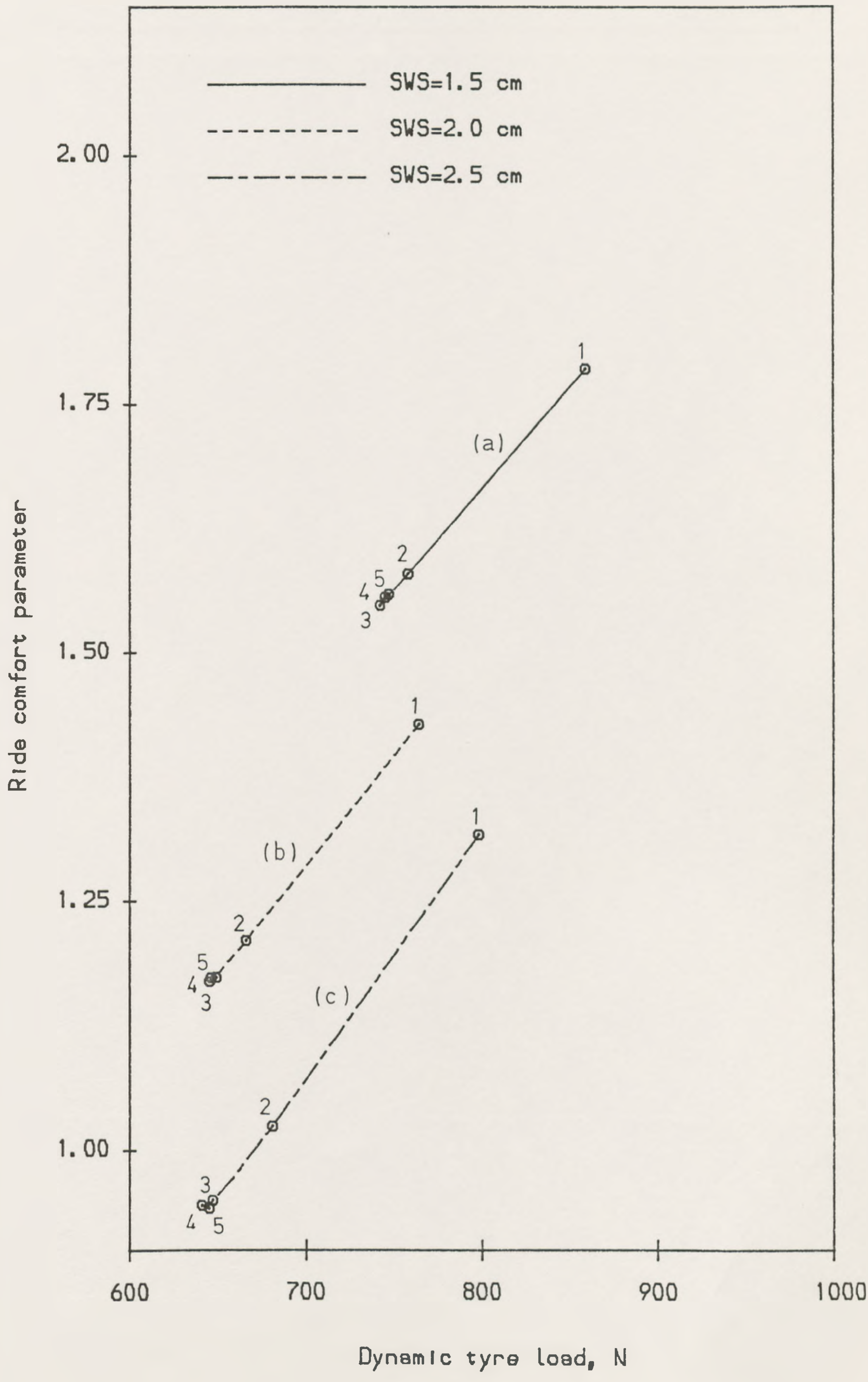

Fig. 4.11 Ride comfort and dynamic tyre load variation of two mass passive systems, mw/mb=0.125

The effect of using different wheel to body mass ratios can be seen by comparing systems of the same number in each group in tables 4.1 and 4.2. The comparison is based on calculating the percentage improvement in both comfort and dynamic tyre load when the wheel to body mass ratio is reduced from 0.2 to 0.125. Results, as shown in table 4.3, demonstrate that reasonable improvements in both comfort and dynamic tyre load are achieved accompanied by a slight reduction in the suspension spring stiffness for all systems in hand. As the working space increases, so the improvements are enhanced, and the spring stiffness changes needed are reduced.

Comparing systems within each group, the greatest improvements in comfort resulting from reduction of the unsprung mass are achieved from systems with the stiffest springs and lowest damping factors. The converse is the case for the dynamic tyre load.

4.3 Three Mass System

The three mass system shown in fig. 4.12 is developed from the quarter car model, shown in Fig. 4.1, by connecting an auxiliary mass (ma) to the wheel through a spring of stiffness Ka and a damper of coefficient Ca.

The describing equations of motion for this system can be written as,

Table 4.3 Changes in results of two mass passive systems obtained by reducing the mass ratio from 0.2 to 0.125

No	SWS(cm)	RCP	DTL(N)	fn(Hz)	γ
1		+ 9.5%	+13.9%	-4.9%	0.400
2	(a)	+ 8.3%	+15.3%	-4.4%	0.566
3	1.5	+ 7.7%	+16.4%	-4.2%	0.800
4		+ 7.4%	+17.0%	-4.2%	1.131
5		+ 7.5%	+17.5%	-4.2%	1.600
1		+11.1%	+16.5%	-4.3%	0.283
2	(b)	+10.5%	+20.4%	-3.9%	0.400
3	2.0	+10.1%	+22.6%	-3.6%	0.566
4		+ 9.9%	+23.5%	-3.6%	0.800
5		+ 9.9%	+24.2%	-3.8%	1.131
1		+12.1%	+18.4%	-3.9%	0.200
2	(c)	+12.4%	+24.2%	-3.4%	0.283
3	2.5	+12.4%	+27.5%	-3.1%	0.400
4		+12.2%	+28.9%	-3.1%	0.566
5		+12.4%	+29.5%	-3.2%	0.800

+ is the percentage improvement.

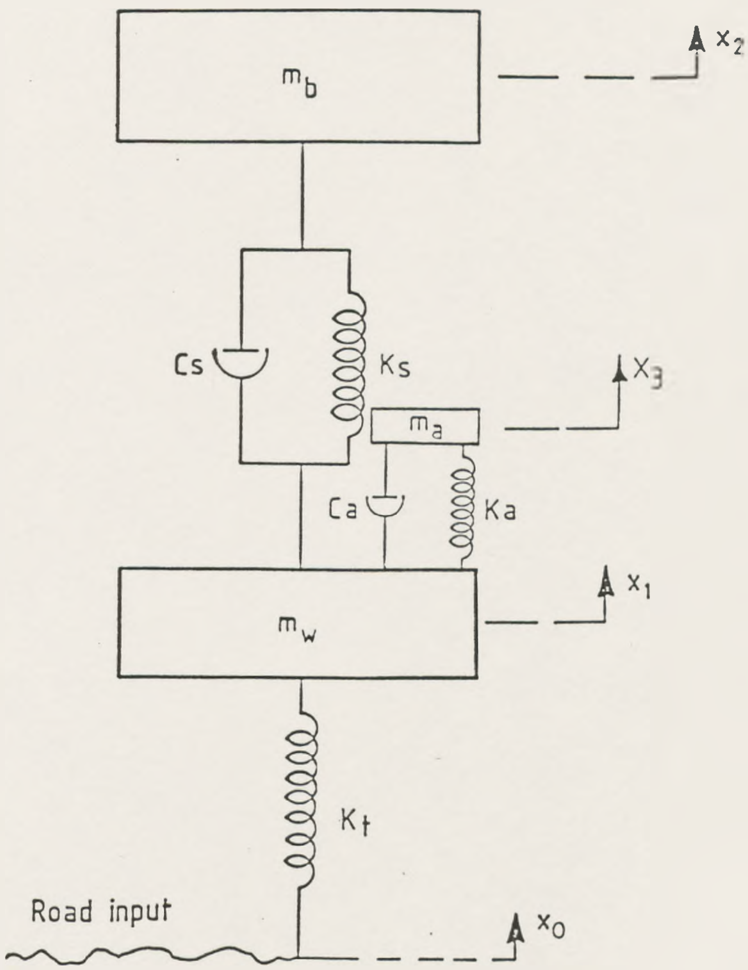

Fig. 4.12 Three mass passive suspension system

$$m_w\ddot{x}1(t) = -Ks[x1(t)-x2(t)] - Cs[\dot{x}1(t)-\dot{x}2(t)] - Ka[x1(t)-x3(t)]$$
$$- Ca[\dot{x}1(t)-\dot{x}3(t)] + Kt[x0(t)-x1(t)]$$
$$m_b\ddot{x}2(t) = Ks[x1(t)-x2(t)] + Cs[\dot{x}1(t)-\dot{x}2(t)]$$
$$m_a\ddot{x}3(t) = Ka[x1(t)-x3(t)] + Ca[\dot{x}1(t)-\dot{x}3(t)]$$

Following the procedures described in section 3.3 under the same assumptions as for equation 4.1, the complex amplitudes X1, X2 and X3 are given by

$$\begin{bmatrix} X1 \\ X2 \\ X3 \end{bmatrix} = \begin{bmatrix} Kt+Ks+Ka-\omega^2 mw+j(Cs+Ca)\omega & -Ks-jCs\omega & -Ka-jCa\omega \\ -Ks-jCs\omega & Ks-\omega^2 mb+jCs\omega & 0 \\ -Ka-jCa\omega & 0 & Ka-\omega^2 ma+jCa\omega \end{bmatrix}^{-1} \begin{bmatrix} Kt \\ 0 \\ 0 \end{bmatrix} \quad 4.5$$

In this section the effect of the absorber mass on the system performance is studied for both wheel to body mass ratios (0.2 and 0.125) using the same procedures as in section 4.2.

The suspension parameters (Ks and Cs) as well as the absorber parameters (Ka and Ca) by which the minimum dynamic tyre load can be obtained are calculated for each absorber mass ratio by using a Simplex minimisation routine available as a NAG library subroutine.

The absorber mass is chosen to vary from 0 to 1 relative to the wheel mass. The zero ratio, which represents a two mass system, is chosen to be an optimised system on the same basis. The mass ratio of unity is chosen to represent a three mass system with an unrealistically large absorber mass. Systems are studied over this range in order to identify the cost effectiveness of such systems.

The results, shown in figures 4.13 and 4.14, are obtained by solving the matrix equation 4.5 using the same procedures and the same assumptions as for the two mass system (section 4.2). In the figures for both wheel to body mass ratios (0.2 and 0.125), the relative performances are illustrated in terms of the r.m.s. value of the weighted body acceleration, the dynamic tyre load, and the suspension working space as functions of the absorber mass to wheel mass ratio. The optimal suspension and absorber parameters are included. A least square fitting process is applied to the output and parameter values in order to overcome the difficulty of obtaining a smooth distribution for these values directly from the search routine arising from the flatness of the dynamic tyre load function in the neighborhood of the minimum. Each function is approximated as a third degree polynomial by using the least-square fitting method available as a NAG library subroutine. The results in the figures show that the improvement in ride comfort as a result of increasing the absorber mass ratio is accompanied, for all ratios, by a higher value of the suspension working space without significant change in the minimum value of the dynamic tyre load.

Two particular systems are compared in order to indicate the actual gains available by using a realistic sized absorber mass to wheel mass ratio. The first system is chosen from table 4.2 to be system (3) as the best performing system with 2.5 cm suspension working space. The second one is chosen from Fig. 4.14 to be an optimized three mass system having the same value of the suspension working space. The results show that a 9%

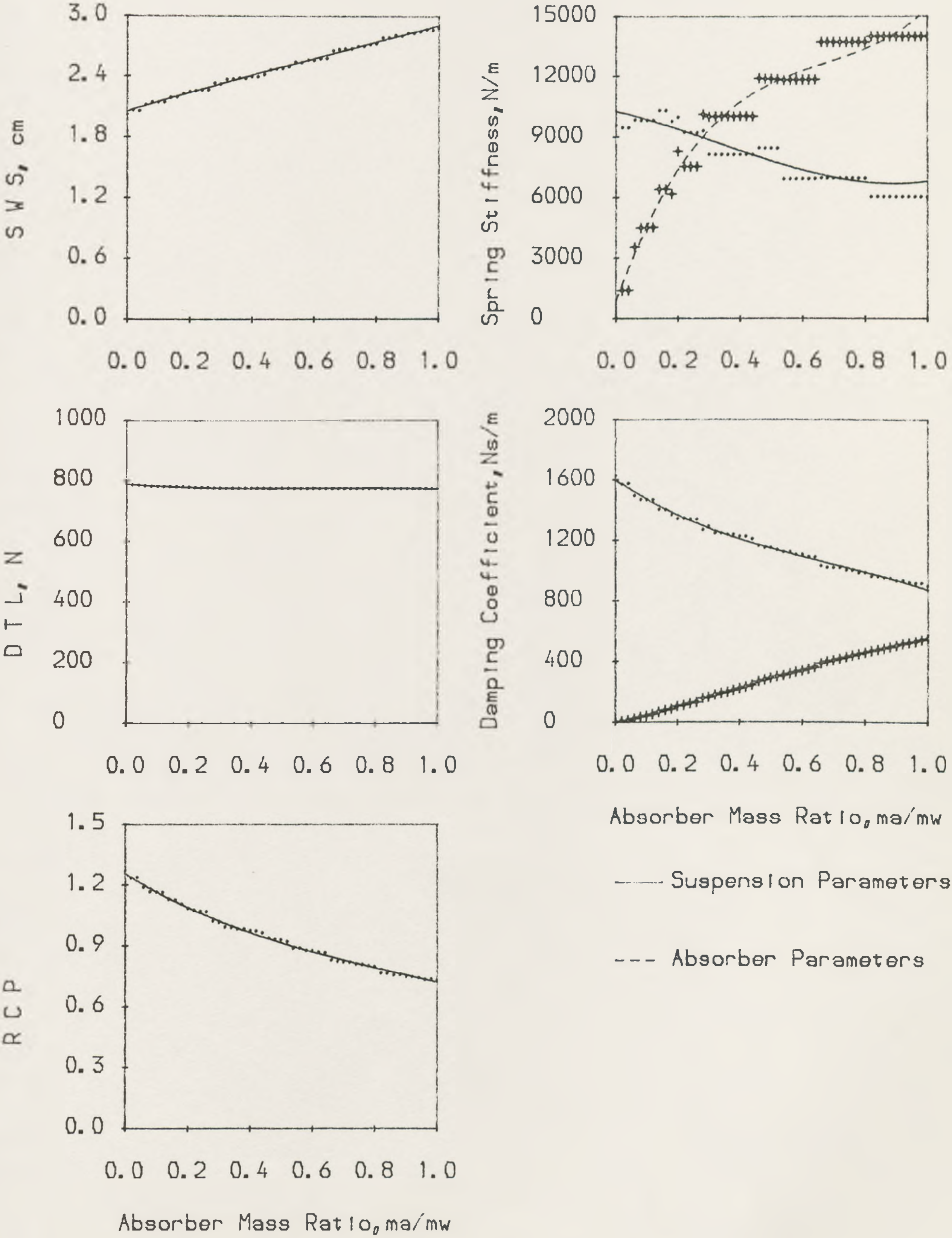

Fig. 4.13 Performance and design properties of three mass passive systems, mw/mb = 0.2

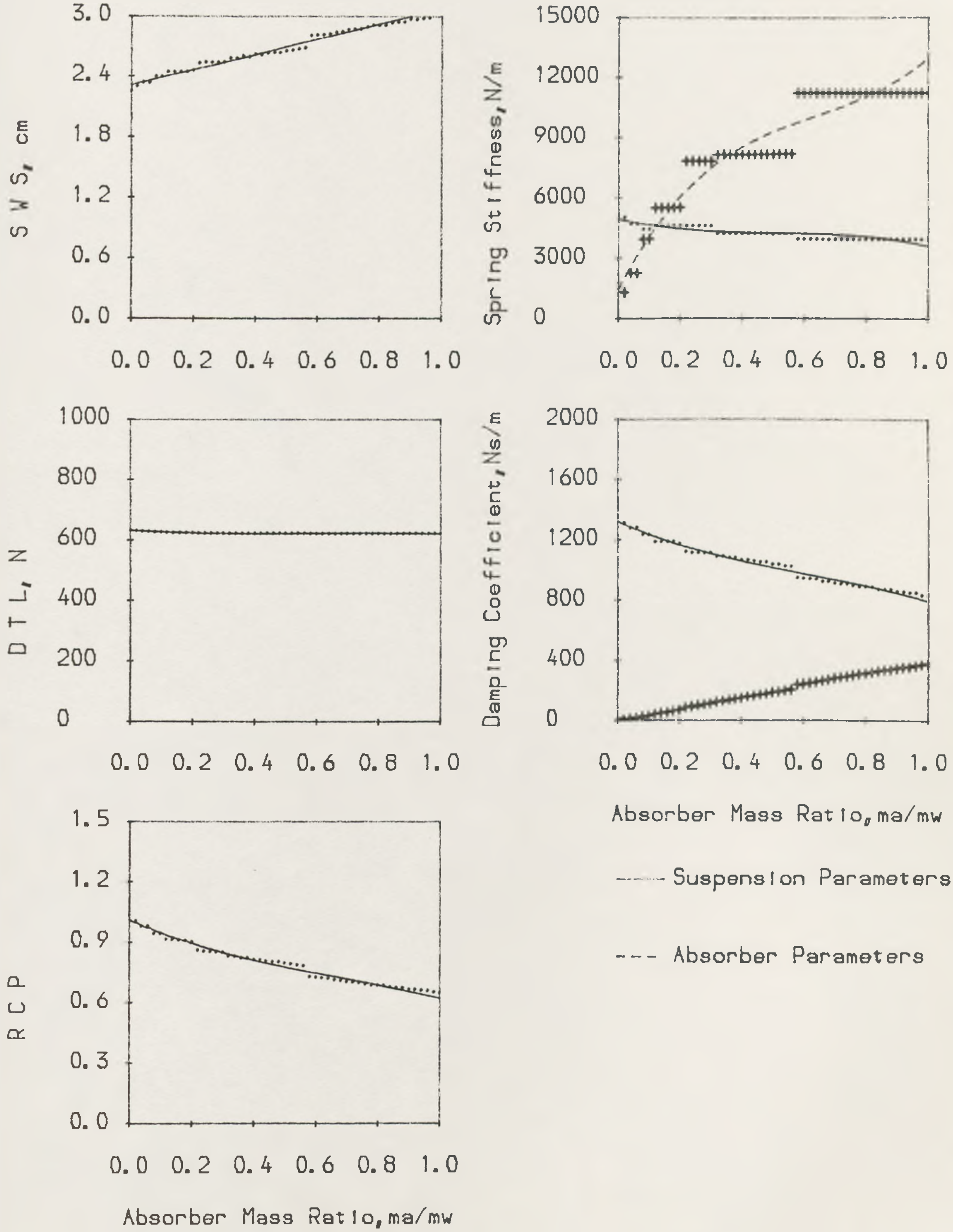

Fig. 4.14 Performance and design properties of three mass passive systems, mw/mb = 0.125

improvement in comfort and 4% improvement in dynamic tyre load can be gained if an absorber mass to wheel mass ratio of 0.20 is used (as compared with the two mass system).

The frequency response functions and the output mean square spectral density functions of the body acceleration, the dynamic tyre load and the suspension working space for these two systems are shown in Fig. 4.15. The spectral density functions of the suspension working space are identical with the 2.5 cm r.m.s. value. The high frequency peak in the dynamic tyre load spectral density (associated with the wheel resonance) is reduced implying that better control for the wheel dynamics can be achieved by using a dynamic absorber. The spectral density of the weighted body acceleration is badly affected due to the absorber resonance peak which appears at 4 Hz for this particular system. These results imply that the three mass system is better in controlling the wheel hop resonance, being particularly important when unbalanced wheels are used.

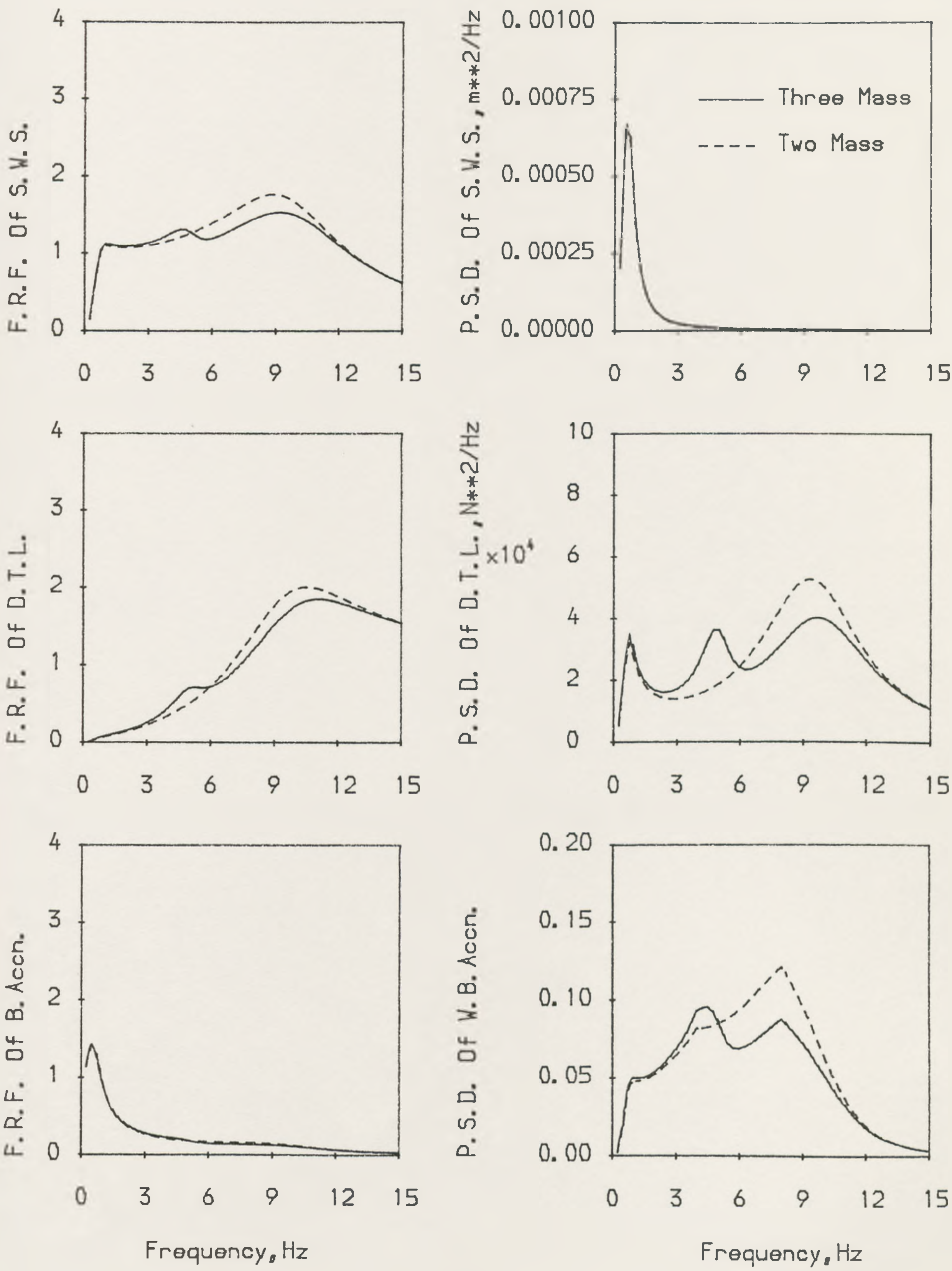

Fig. 4.15 Frequency responses and output mean square spectral densities of two and three mass passive systems, mw/mb=0.125, ma/mw=0.20
S W S = 2.5 cm

CHAPTER(5)

ACTIVE SUSPENSION SYSTEM STUDIES

5.1 Introduction

Active suspension systems are intelligent in the sense that they employ controllable elements (actuators of some kind). These elements, using force feedback, are cabable of generating forces which are linear combinations of measured state variables. Systems in which all the state variables are measurable will be defined as full state feedback active systems. If only some of the state variables can be measured, the systems will be defined as limited state feedback active systems. The optimal control theory discussed in chapter 2 can be applied to the quarter car model shown in Fig. 2.1 in order to obtain the optimal control law for either case. It is a feature of the optimisations however that the road surface input is treated as either an integrated or low-pass filtered white noise velocity signal. Also, no frequency weighting is applied to the vertical body acceleration in order to provide a measure of passenger discomfort, so that the resulting laws are optimal for conditions which are a little different from those of prime interest. Nevertheless parameter variations around the "optimal" points have shown these points to be very near optimal for the conditions of interest, and the control laws emerging from the optimal control theory have been used, as they are, in the subsequent frequency response and mean square value calculations.

These calculations, as described in chapter 3, allow the determination of ride comfort, dynamic tyre load and suspension working space parameters individually. Many optimal systems of different performance properties and parameter values can be obtained by using different weightings in the performance index. This process allows the identification of the control law and performance parameters of those systems with one of the three standard suspension working space requirements of special interest (1.5, 2.0 or 2.5 cm).

5.2 Calculations

The equations of motion of the active suspension system shown in Fig. 2.1 can be written as

$$mw\ddot{x}1(t) = -u(t) + Kt[x0(t)-x1(t)]$$
$$mb\ddot{x}2(t) = u(t) \qquad \text{..........} \quad 5.1$$

The control law u(t) of the full state feedback system can be derived as

$$u(t) = Kf1[x1(t)-x0(t)] + Kf2[x2(t)-x0(t)] + Kf3\dot{x}1(t) + Kf4\dot{x}2(t) \qquad \text{..........} \quad 5.2$$

Putting the control law 5.2 into equation 5.1, and following the procedures described in section 3.3 (assuming unity input amplitude X0), the output amplitudes X1 and X2 can be derived as

$$\begin{bmatrix} X1 \\ X2 \end{bmatrix} = \begin{bmatrix} Kt-\omega^2 mw & -\omega^2 mb \\ Kf1+jKf3\omega & Kf2-\omega^2 mb+jKf4\omega \end{bmatrix}^{-1} \begin{bmatrix} Kt \\ Kf1+Kf2 \end{bmatrix} \qquad \text{..........} \quad 5.3$$

The same procedures are used in the limited state feedback case using the control law in the form,

$$u(t) = K11x1(t) + K12x2(t) + K13\dot{x}1(t) + K14\dot{x}2(t) \quad \ldots\ldots\ldots\ldots \quad 5.4$$

in order to obtain the output amplitudes X1 and X2 ,where,

$$\begin{bmatrix} X1 \\ X2 \end{bmatrix} = \begin{bmatrix} Kt-\omega^2 mw & -\omega^2 mb \\ K11+jK13\omega & K12-\omega^2 mb+jK14\omega \end{bmatrix}^{-1} \begin{bmatrix} Kt \\ 0 \end{bmatrix} \quad \ldots\ldots\ldots\ldots \quad 5.5$$

In both cases, the procedures described in section 3.3 are followed in order to calculate the r.m.s. values of the weighted body acceleration, the dynamic tyre load and the suspension working space.

5.3 Active System Results

The results obtained in this section were generated using the same parameter values and the same road quality under the same assumptions as in chapter 4. The only difference is to replace the passive system's spring rate and damper coefficient by the four gains obtained from the optimisation process. Various gains are obtained as a result of changing the weighting parameters q1 and q2 in the matrix Q in equation 2.8.

Results for full and limited state feedback active systems are shown in figures 5.1 and 5.2 respectively. These results are presented with the suspension working space as the abscissa. The ride comfort parameter and the dynamic tyre load as well as the four feedback gains are included. Using figures 5.1 and 5.2, the

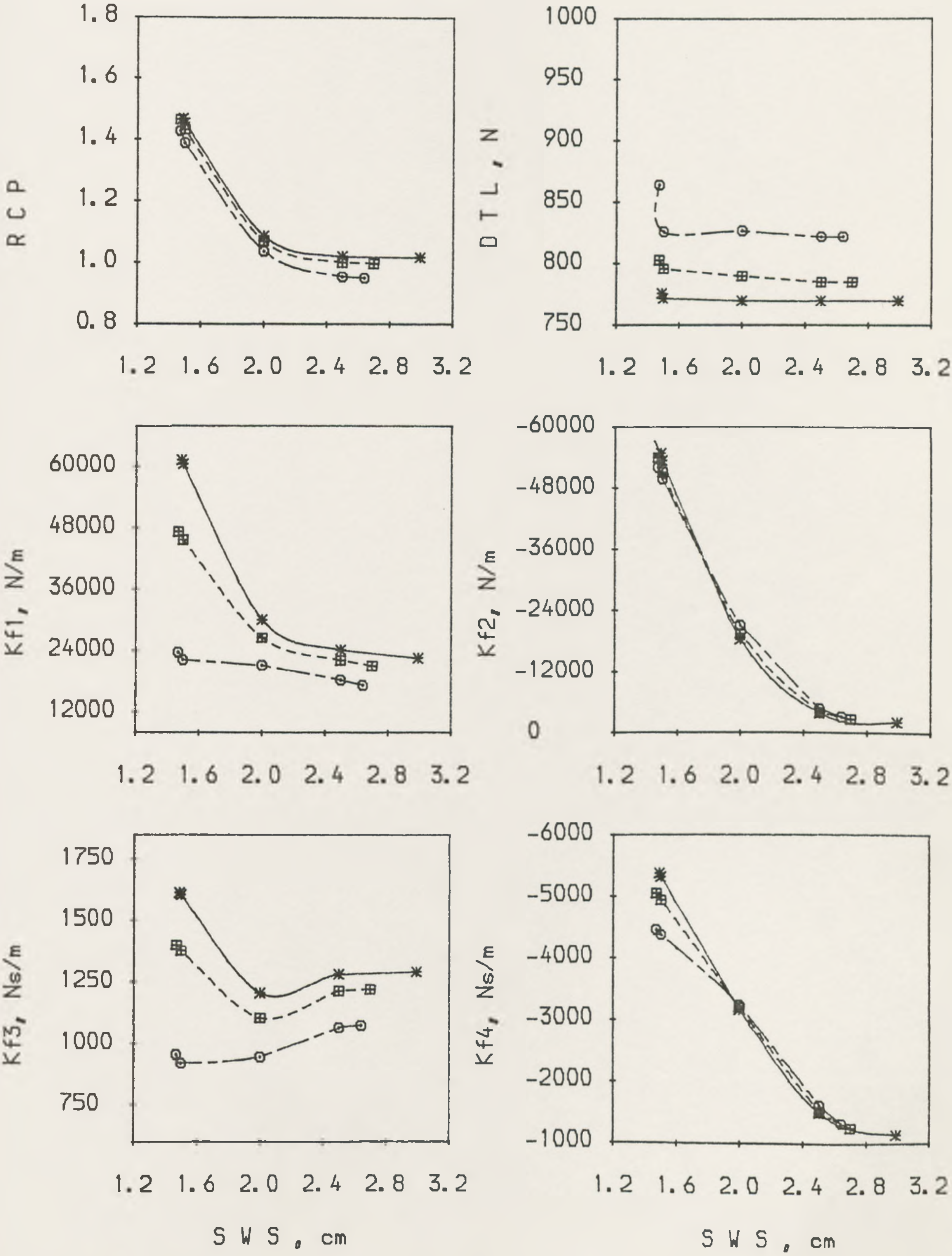

Fig. 5.1 Performance and design properties of full state feedback active systems

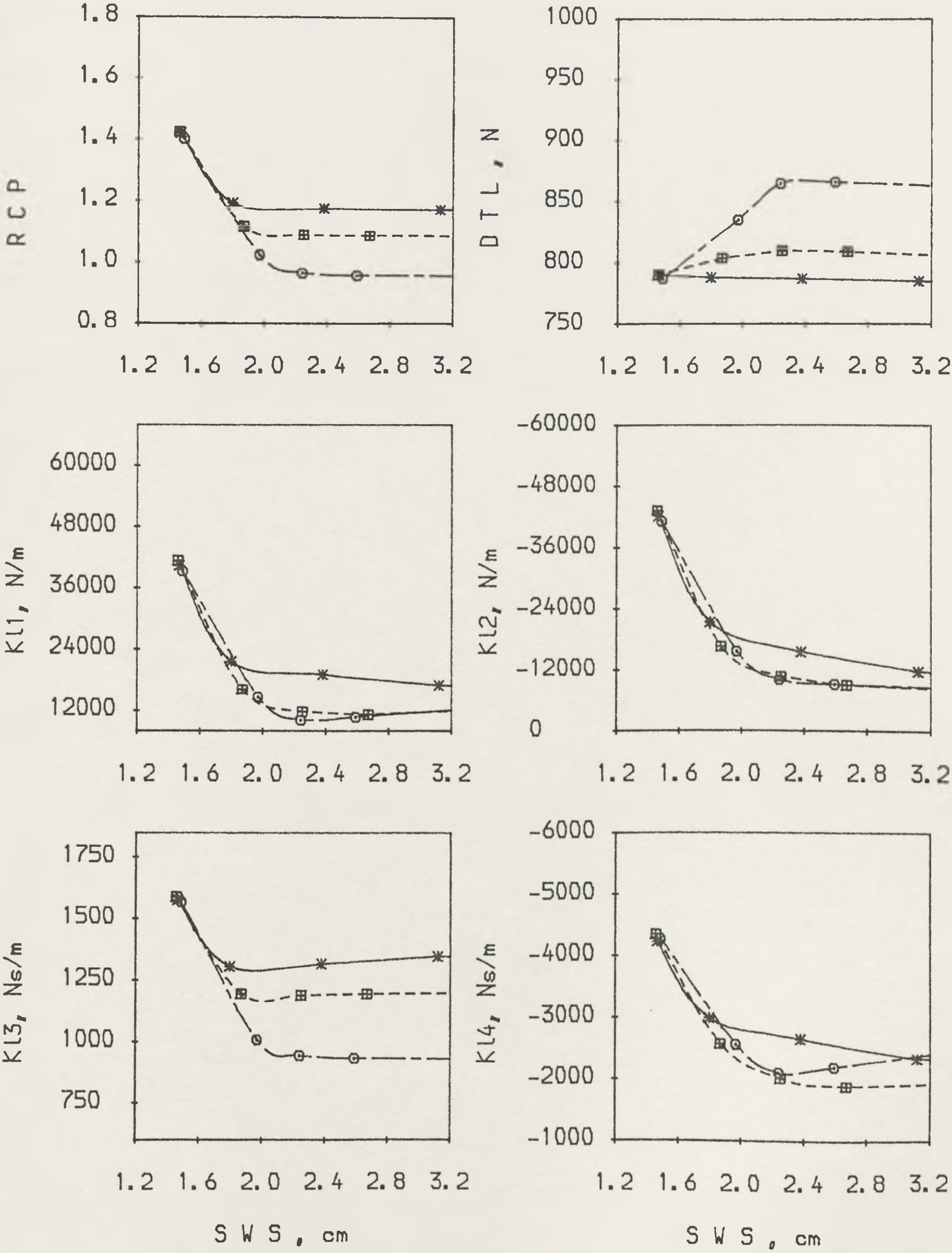

Fig. 5.2 Performance and design properties of limited state feedback active systems

design and performance parameters of the various types of system which have any one of the standard suspension working spaces (1.5, 2.0 and 2.5 cm) can be read. These systems are classified in each of three groups, in table 5.1 for the full state feedback case and in table 5.2 for the limited state feedback case. The frequency response functions and the spectral density functions for each group are presented in figures 5.3, 5.4 and 5.5 for full state feedback systems and in figures 5.6, 5.7 and 5.8 for limited state feedback systems refering to 1.5, 2.0 and 2.5 cm suspension working space respectively.

Results in the first group in table 5.1 show that the "best" dynamic tyre load control is obtained in system (1) as a result of using relatively higher values of the four constants Kf1, Kf2, Kf3 and Kf4. As these constants decrease, higher comfort can be gained accompanied by worsening of the dynamic tyre load variations. The "best" dynamic tyre load is obtained due to the low peak of the wheel resonance in the frequency response function and consequently in the spectral density function, without significant change in the peak of the body resonance as compared with systems (2) and (3), Fig. 5.3. The same sequence of performance is obtained in the second and third groups as can be seen in table 5.1 and in figures 5.4 and 5.5.

In the limited state feedback systems, table 5.2, there are no significant changes in the constants Kl1, Kl2, Kl3, and Kl4 in the first group, implying that the optimal control is the same for a wide range of weighting parameters q1 and q2 in the

Table 5.1 Design and performance properties of particular full state feedback active systems

No	SWS cm	RCP	DTL N	Kf1 N/m	Kf2 N/m	Kf3 Ns/m	Kf4 Ns/m
1	(a)	1.455	772	60526	-53339	1605	-5317
2	1.5	1.433	796	45769	-51015	1378	-4939
3		1.390	826	22280	-49749	920	-4380
1	(b)	1.087	770	30118	-18303	1205	-3170
2	2.0	1.069	790	26565	-19429	1103	-3208
3		1.037	827	21219	-21078	945	-3248
1	(c)	1.020	770	24205	-3873	1280	-1505
2	2.5	1.000	785	22164	-4000	1212	-1517
3		0.954	822	18316	-4743	1063	-1626

Table 5.2 Design and performance properties of particular limited state feedback active systems

No	SWS cm	RCP	DTL N	K11 N/m	K12 N/m	K13 Ns/m	K14 Ns/m
1	(a)	1.393	783	43341	-44990	1455	-4511
2	1.5	1.388	785	39815	-41553	1508	-4278
3		1.387	785	39176	-40980	1525	-4273
1	(b)	1.170	786	19386	-17895	1291	-2790
2	2.0	1.100	805	14210	-13947	1175	-2350
3		1.000	847	13333	-14386	960	-2500
1	(c)	1.173	785	18772	-14912	1309	-2588
2	2.5	1.086	809	12050	-10108	1186	-2017
3		0.956	867	10561	-9386	931	-2170

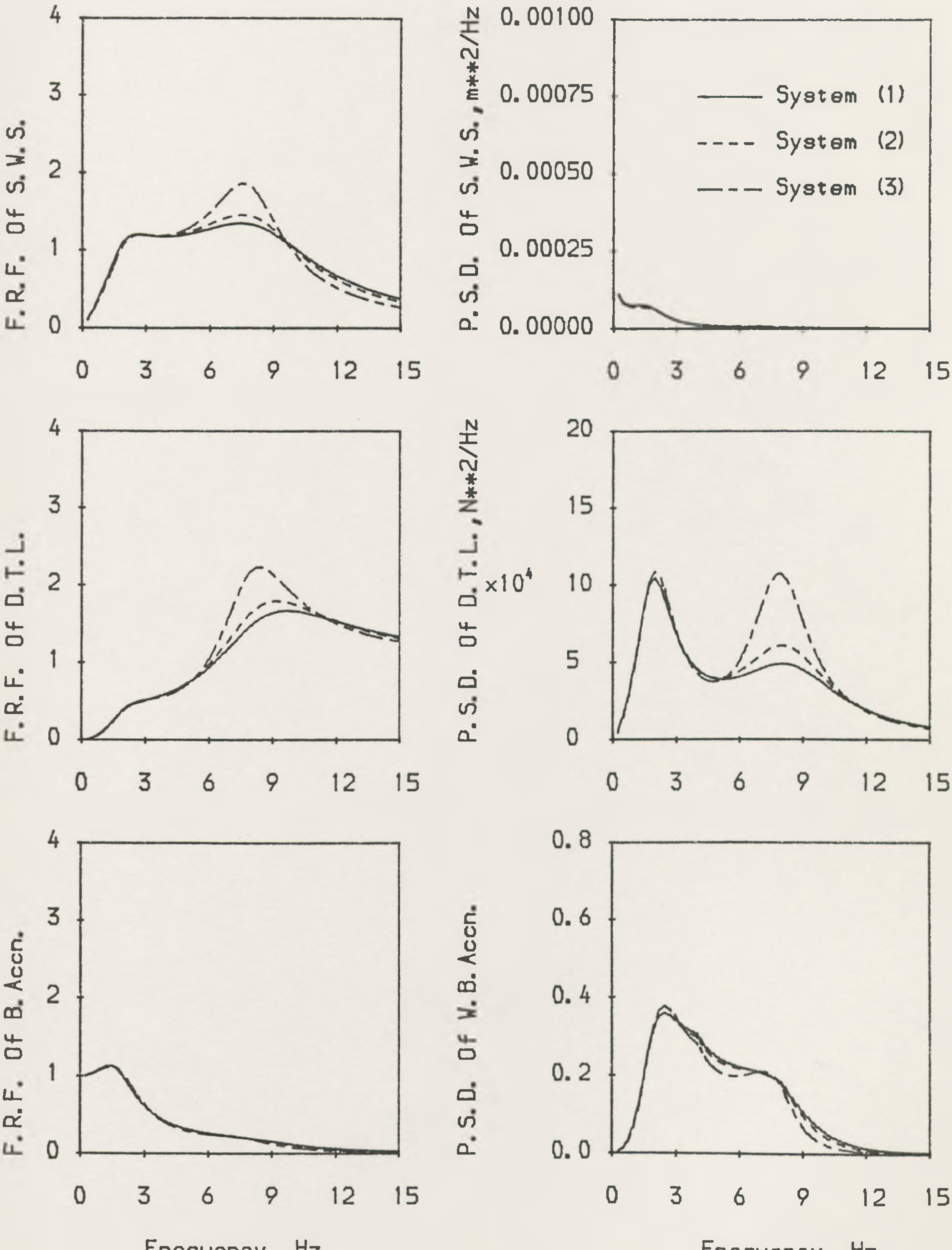

Fig. 5.3 Frequency responses and output mean square spectral densities of full state feedback active systems
S W S = 1.5 cm

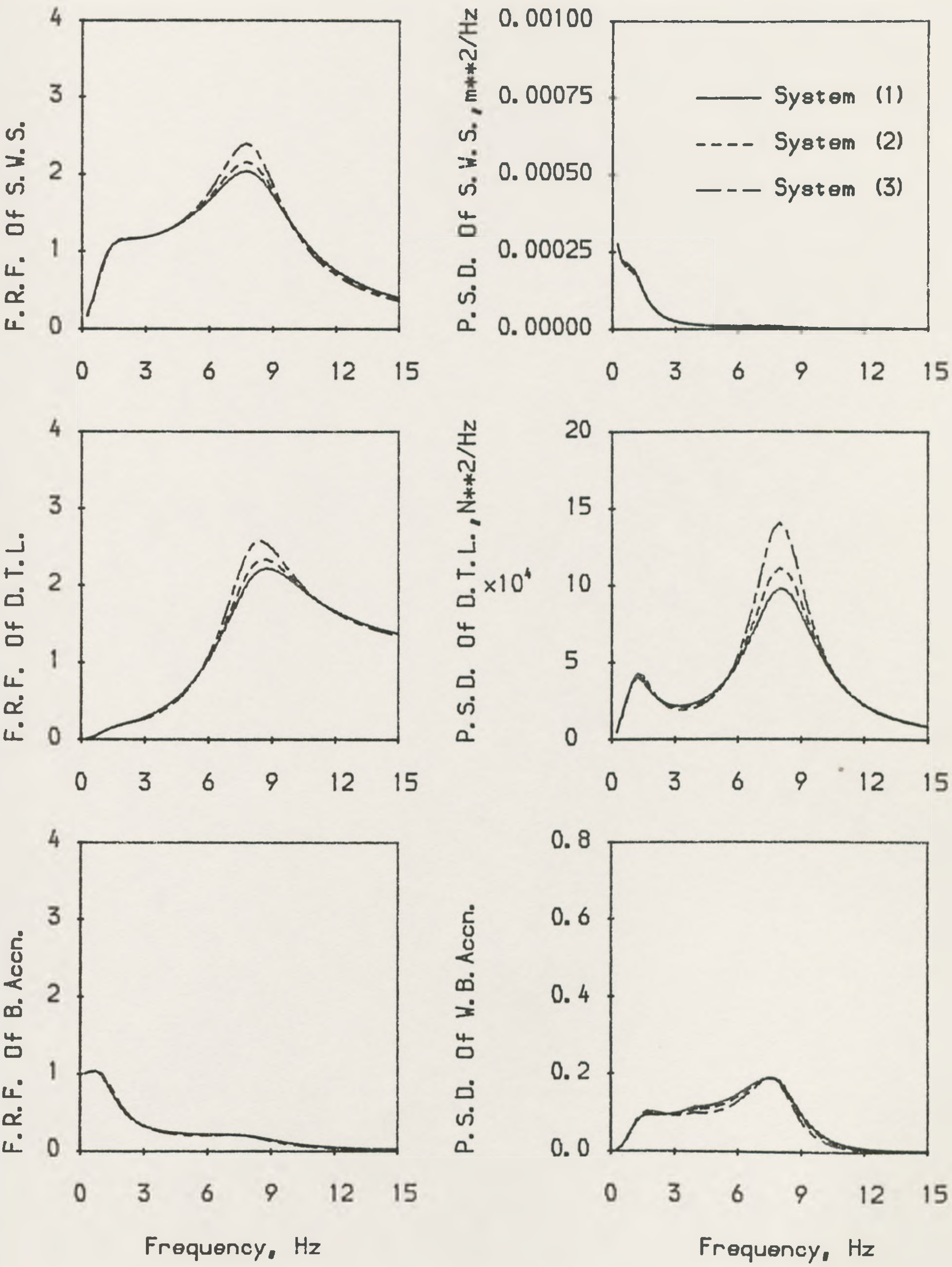

Fig. 5.4 Frequency responses and output mean square spectral densities of full state feedback active systems
S W S = 2.0 cm

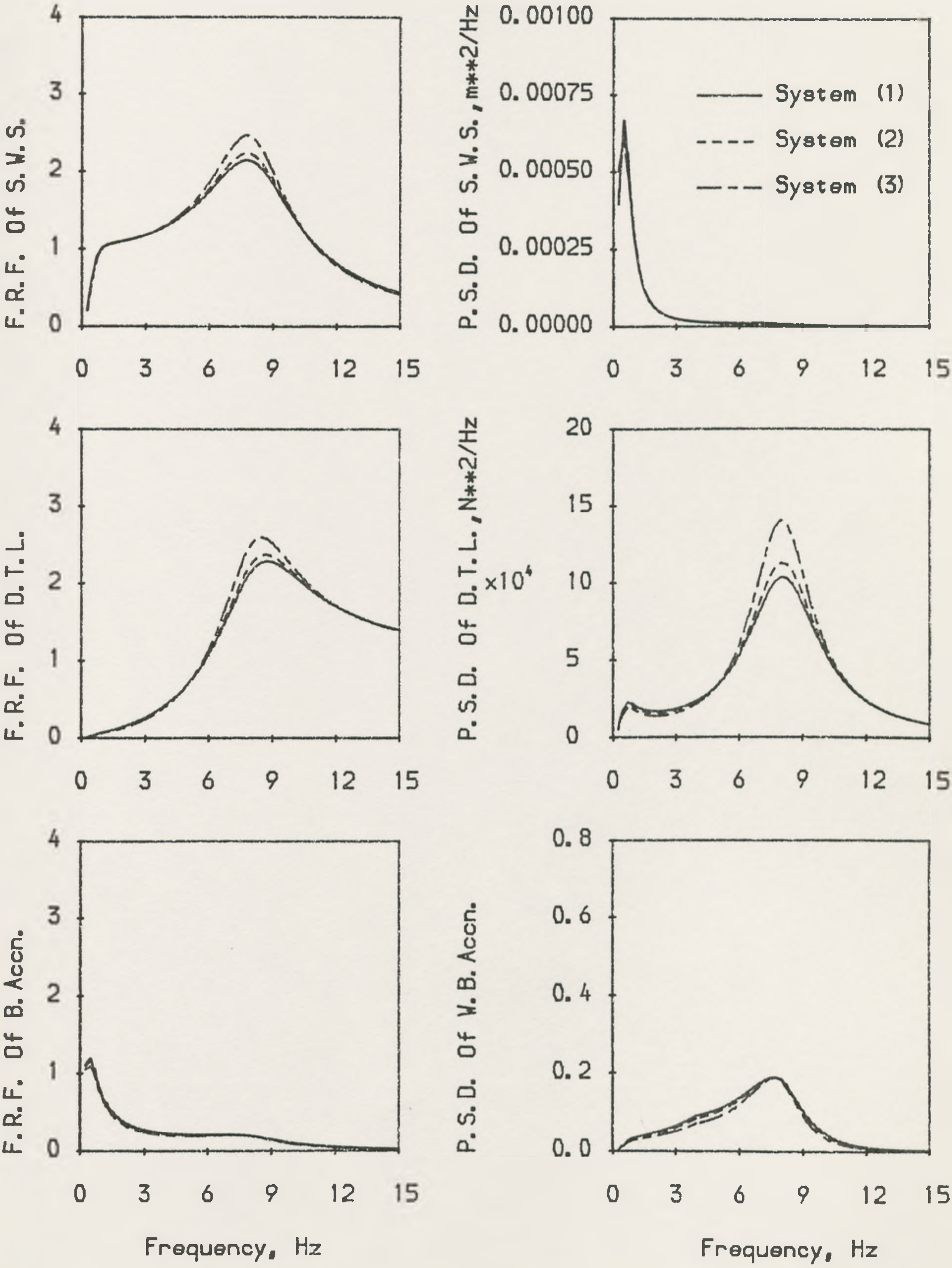

Fig. 5.5 Frequency responses and output mean square spectral densities of full state feedback active systems
S W S = 2.5 cm

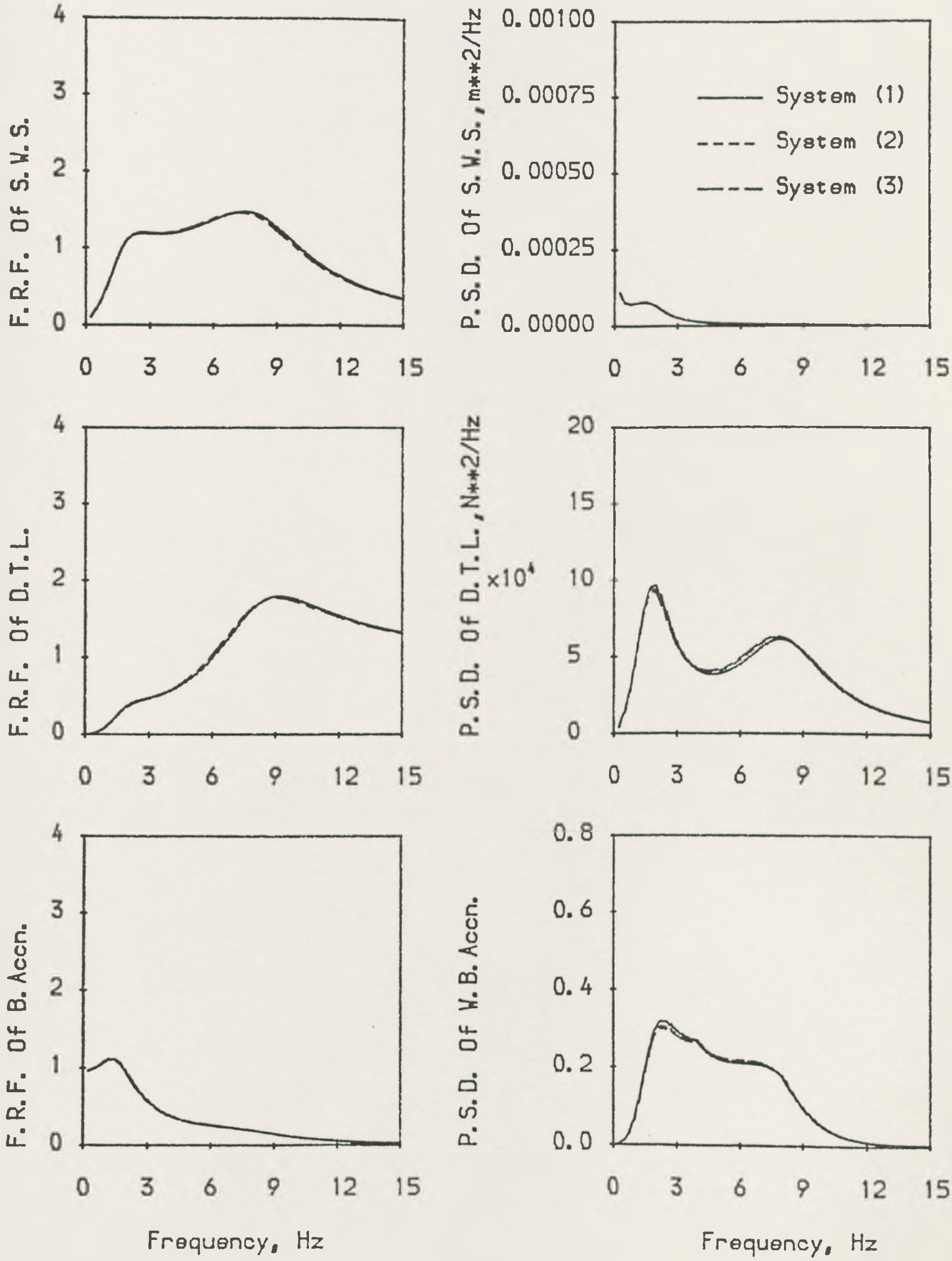

Fig. 5.6 Frequency responses and output mean square spectral densities of limited state feedback active systems
S W S = 1.5 cm

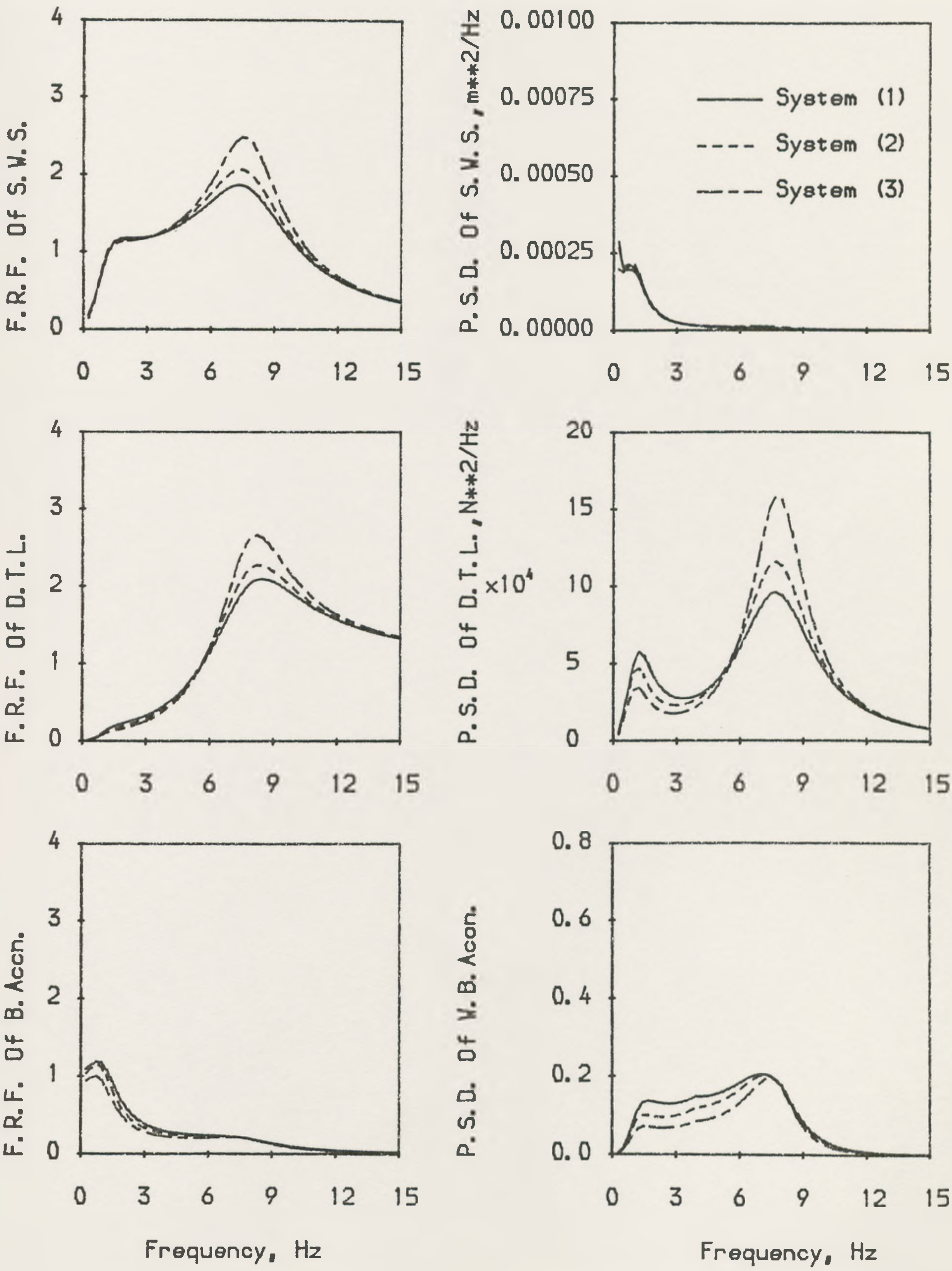

Fig. 5.7 Frequency responses and output mean square spectral densities of limited state feedback active systems
S W S = 2.0 cm

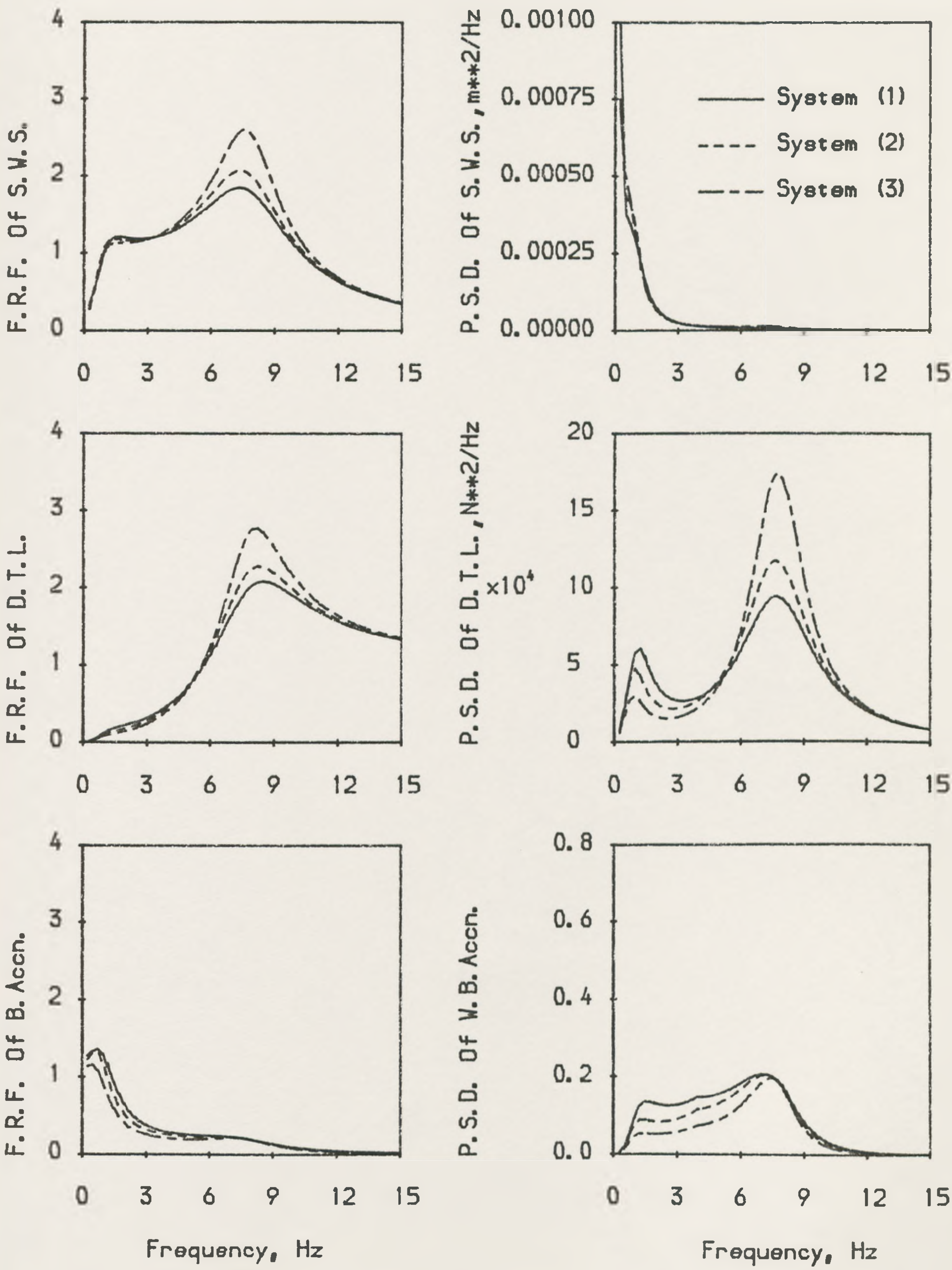

Fig. 5.8 Frequency responses and output mean square spectral densities of limited state feedback active systems
S W S = 2.5 cm

performance index. Consequently, these systems have almost the same performance properties as can be seen in Fig. 5.6. In the second and third groups in table 5.2, the changes in the four constants K11, K12, K13 and K14 as well as the change in the performance of these systems have the same behaviour as in the corresponding groups in the full state feedback case. The difference between the two cases is in the frequency response and spectral density functions for both ride comfort and dynamic tyre load variations. In the case of the dynamic tyre load we can see that, as we move from system (1) to system (3), better control for the body resonance is gained accompanied by worse control in the wheel resonance, implying higher r.m.s. value of the dynamic tyre load. At the same time, better comfort is gained as a result of the low spectrum of the weighted body acceleration in the frequency range from 0.25 Hz up to the wheel resonance.

The performance properties of the full and limited state feedback active systems are plotted in figures 5.9 and 5.10 respectively. These figures show the relative performances of these systems for the different values of the chosen standard suspension working space. In the full state feedback systems, Fig. 5.9, a significant improvement in ride comfort can be gained, with almost the same level of the dynamic tyre load control, if the r.m.s. value of the suspension working space is increased from 1.5 to 2.0 cm. A little further improvement can be gained if the suspension working space available is increased from 2.0 to 2.5 cm. For each value of the suspension working space, the optimal system design parameters can be varied over a wide range giving

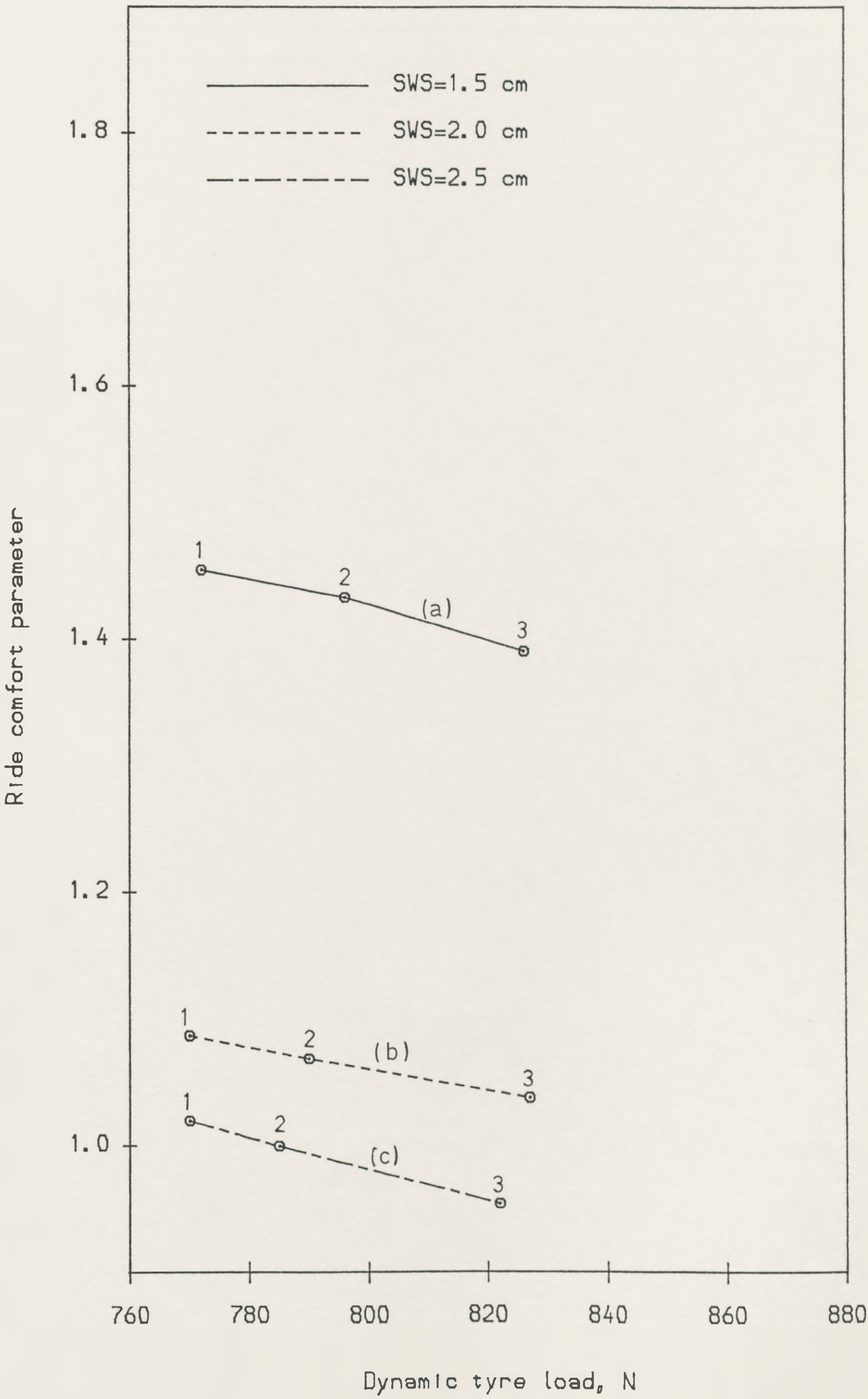

Fig. 5.9 Ride comfort and dynamic tyre load variation of full state feedback active systems

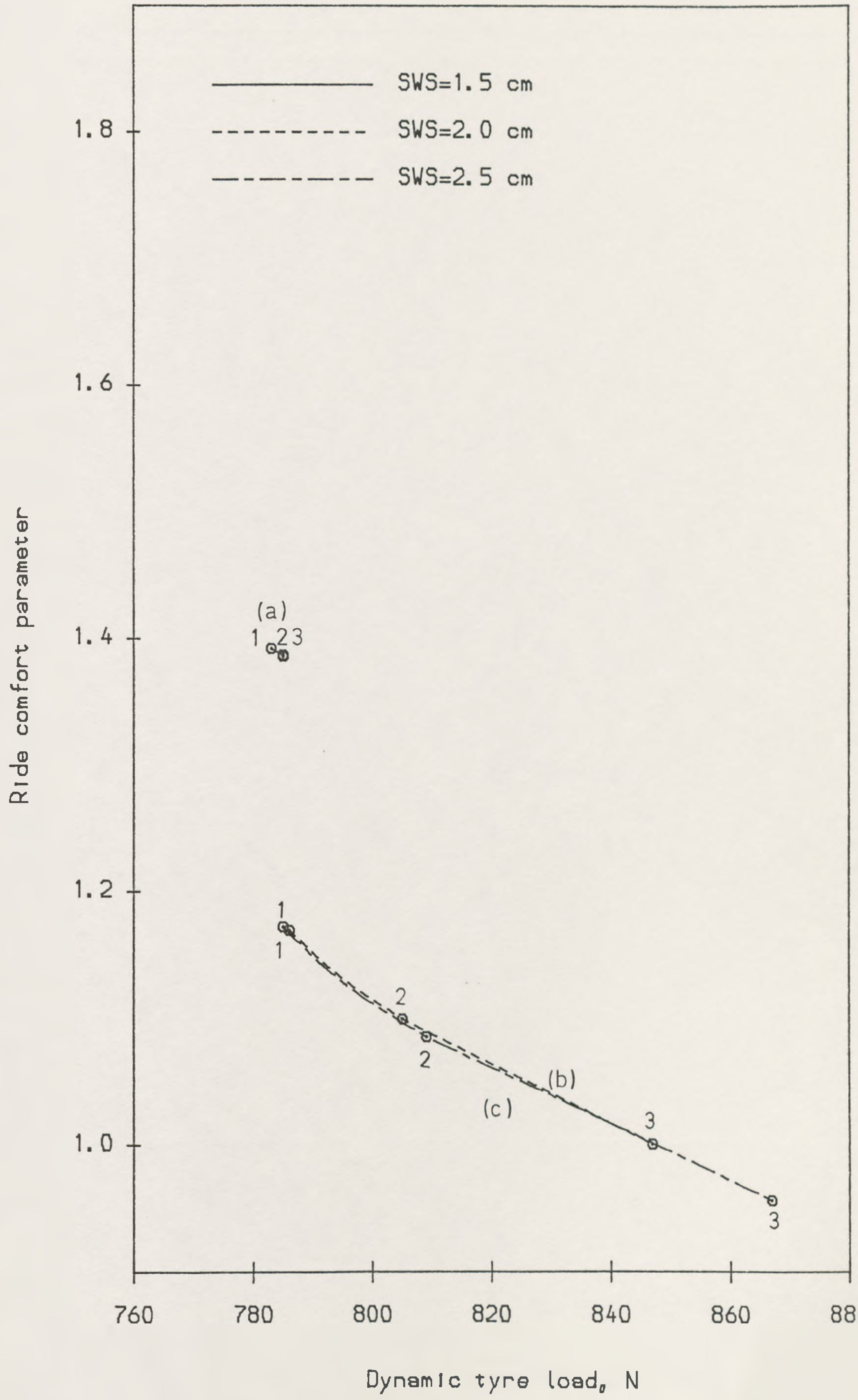

Fig. 5.10 Ride comfort and dynamic tyre load variation of limited state feedback active systems

the possibility of defining optimal systems with different performances.

In the limited state feedback case, Fig. 5.10, systems using 1.5 cm working space are all very similar to each other despite variations in the weighting parameters used in the derivation of the control laws. This is in contrast to the other cases. The general rule that better comfort can be obtained if more working space is made available is defied in the cases of the systems with 2.0 and 2.5 cm working spaces. No advantage is gained in the latter case, clearly associated with the road roughness and vehicle speed assumed. The manner in which the results scale for other roughnesses and speeds indicate that the 2.5 cm working space systems would show advantage in comfort if the road were rougher or the speed higher.

CHAPTER(6)

SEMI-ACTIVE SUSPENSION SYSTEM STUDIES

6.1 Introduction

Semi-active suspensions are feedback controlled systems in which the actuator is limited to providing energy dissipation. The main idea is to modulate the force of a damper according to some control policy based on measurements describing the instantaneous state of the system. The damper acts if energy dissipation is required. Otherwise it switches off. The behaviour of such systems is non-linear because of the switching process of the semi-active damper and a general method for predicting the performance properties in such cases is computer simulation. Such simulations are followed by frequency analyses in order to accurately represent the actual operation of such systems. Appropriate control laws must be derived for use in these simulations.

In this chapter, three different types of control law are used in studying semi-active suspension systems. The first type is based on the full state feedback active law while the other two are based on limited state feedback active laws with and without a passive damper in parallel with the semi-active one. Results are generated by incorporating each of these laws in the quarter car model subjected to a realistic road input. System performances are calculated as the root mean square values of the weighted body acceleration, the dynamic tyre load and the suspension working space. Results for each type are presented in the same manner as in chapter 5. From the many systems considered, those requiring each of the standard suspension working spaces (1.5, 2.0 and 2.5 cm) are identified for comparison with the other types of system, passive and active.

6.2 Calculations

The equations of motion of the quarter car model shown in Fig. 2.1 can be written in the state space form as,

$$\begin{aligned} \dot{x}1(t) &= x3(t) \\ \dot{x}2(t) &= x4(t) \\ \dot{x}3(t) &= Kt/mw\ x0(t) - Kt/mw\ x1(t) - 1.0/mw\ u(t) \\ \dot{x}4(t) &= 1.0/mb\ u(t) \end{aligned} \qquad \text{..........} \quad 6.1$$

The control force u(t) is constructed from the control laws described in chapter 5 to be in each of three different forms. The first form is obtained from the control law of the full state feedback active system 5.2 to be

$$\begin{aligned} u(t) = {} & Kf1[x1(t)-x2(t)] + Kf3\dot{x}1(t) + Kf4\dot{x}2(t) \\ & + (Kf1+Kf2)[x2(t)-x0(t)] \end{aligned} \qquad \text{..........} \quad 6.2$$

while in the case of limited state feedback, the control force is given by,

$$\begin{aligned} u(t) = {} & K11[x1(t)-x2(t)] + K13\dot{x}1(t) + K14\dot{x}2(t) \\ & + (K11+K12)x2(t) \end{aligned} \qquad \text{..........} \quad 6.3$$

for the semi-active system without passive damper and by

$$u(t) = K11[x1(t)-x2(t)] + K13[\dot{x}1(t)-\dot{x}2(t)] + (K11+K12)x2(t) + (K13+K14)\dot{x}2(t) \quad \ldots\ldots\ldots\ldots \; 6.4$$

for the semi-active system with a passive damper.

In each of these laws, the first term is realisable as a spring, of stiffness Kf1 in equation 6.2 and K11 in equations 6.3 and 6.4, by which the body mass can be supported. The second term in equation 6.4 represents a passive damper of coefficient K13 connected in parallel with the spring. The rest of each law represents a semi-active force (S.A.F.) which will be generated in the actuator according to the following policy:

The damper generates the force S.A.F. if :

$[\dot{x}1(t)-\dot{x}2(t)].[S.A.F.] > 0$,

but otherwise gives zero force.

In equation 6.2, the semi-active force required is governed by the wheel and body velocities and the relative displacement between the body and the road. In the limited state feedback cases, the semi-active force needed depends on the wheel velocity and the body velocity and displacement in equation 6.3, while in equation 6.4 it depends on the body velocity and displacement.

In order to avoid the unrealistic supposition that the semi-active damper can switch from on to off and conversely in an instantaneous manner, the switching action has been described in the simulation as being governed by first order lag dynamics with a break frequency of 16.3 Hz (time constant, 9.764×10^{-3} s), which also avoids the possibility that good performance predicted for the semi- active systems depends on very fast switching.

The simulation operates by taking the road profile data as described in section 3.2 and depicted in Fig. 3.1 and the control laws as described in equations 6.2, 6.3 and 6.4, and integrating the equations of motion 6.1 over a six second duration. The phase angles ψ in equation 3.3 are obtained by using a random number generator available in the form of a NAG library subroutine.

Equations 6.1, after manipulation, are integrated at each point of time, starting from zero time with zero initial conditions, by using the Runge-Kutta-Merson method available in the form of a NAG library subroutine. The solution for the first two seconds is then eliminated in order to remove the transient part of the system response, the remaining four seconds of output being one complete period of the system response to the periodic input. Body acceleration, dynamic tyre load and suspension working space responses are of special interest. The body acceleration is obtained in terms of the body force which is the summation of the semi-active damper force and the force generated by the passive elements. The latter force represents the spring force in the first two cases and the spring and passive damper forces in the third case. The body acceleration is determined as,

$$\text{b.Accn.} = (\text{S.A.F.} + \text{Passive force}) / mb \qquad \text{.........} \quad 6.5$$

The dynamic tyre load is calculated in terms of the relative displacement between the wheel and the road input as,

$$d.t.l. = Kt[x0(t)-x1(t)] \qquad \text{..........} \quad 6.6$$

and the suspension working space is calculated in terms of the relative displacement between the wheel and the body as,

$$s.w.s. = x1(t)-x2(t) \qquad \text{..........} \quad 6.7$$

The stored values representing the time functions 6.5, 6.6 and 6.7 are frequency analysed by using the Fast Fourier Transform algorithm. Each function is chosen to contain 2048 time samples with sampling frequency of 512 Hz. The number of samples is chosen large enough to allow a sufficient description of the exponential function representing the switching process. Once the spectral density functions are obtained, the procedures described in chapter 3 can be used to obtain the r.m.s. values of the system responses of interest.

For each semi-active system of interest, the whole process is repeated five times using the different road profiles shown in Fig. 3.1, and average values over the five ensembles are formed for use as system performance measures. This is to identify to what extent the transformed spectral density functions of the non-linear systems can be affected by changing the phase information used with the spectral density function in producing the time history of the road surface input. Limited variations of around 2% in r.m.s. values in the system responses were found making the average over five ensembles sufficiently representative.

6.3 Semi-active System Results

The same parameter values used in studying the active suspension systems are used again for obtaining the semi-active system results under the same considerations of road quality and vehicle speed. Also, results in this section are presented in the same way as in the previous chapter for ease of comparison. Results in Fig. 6.1 refer to semi-active suspension systems based on full state feedback optimal control laws with no passive damper, representing the performance and design properties of these systems. The results in figures 6.2 and 6.3 represent the performance and design properties of semi-active systems based on limited state feedback control laws without and with a passive damper. Results in tables 6.1, 6.2 and 6.3 are obtained from figures 6.1, 6.2 and 6.3 to represent different systems having the same requirement for suspension working space. Each table contains data on systems with one of the standard values of working space 1.5, 2.0 and 2.5 cm which are defined as groups (a), (b) and (c) respectively. The performance properties of the systems in tables 6.1, 6.2 and 6.3 are plotted in figures 6.4, 6.5 and 6.6 respectively. In Fig. 6.4, system 2(a) is found to be the "best" performing system in terms of controlling the dynamic tyre load variations for the 1.5 cm suspension working space and it contains a relatively stiff spring (Kf1), as

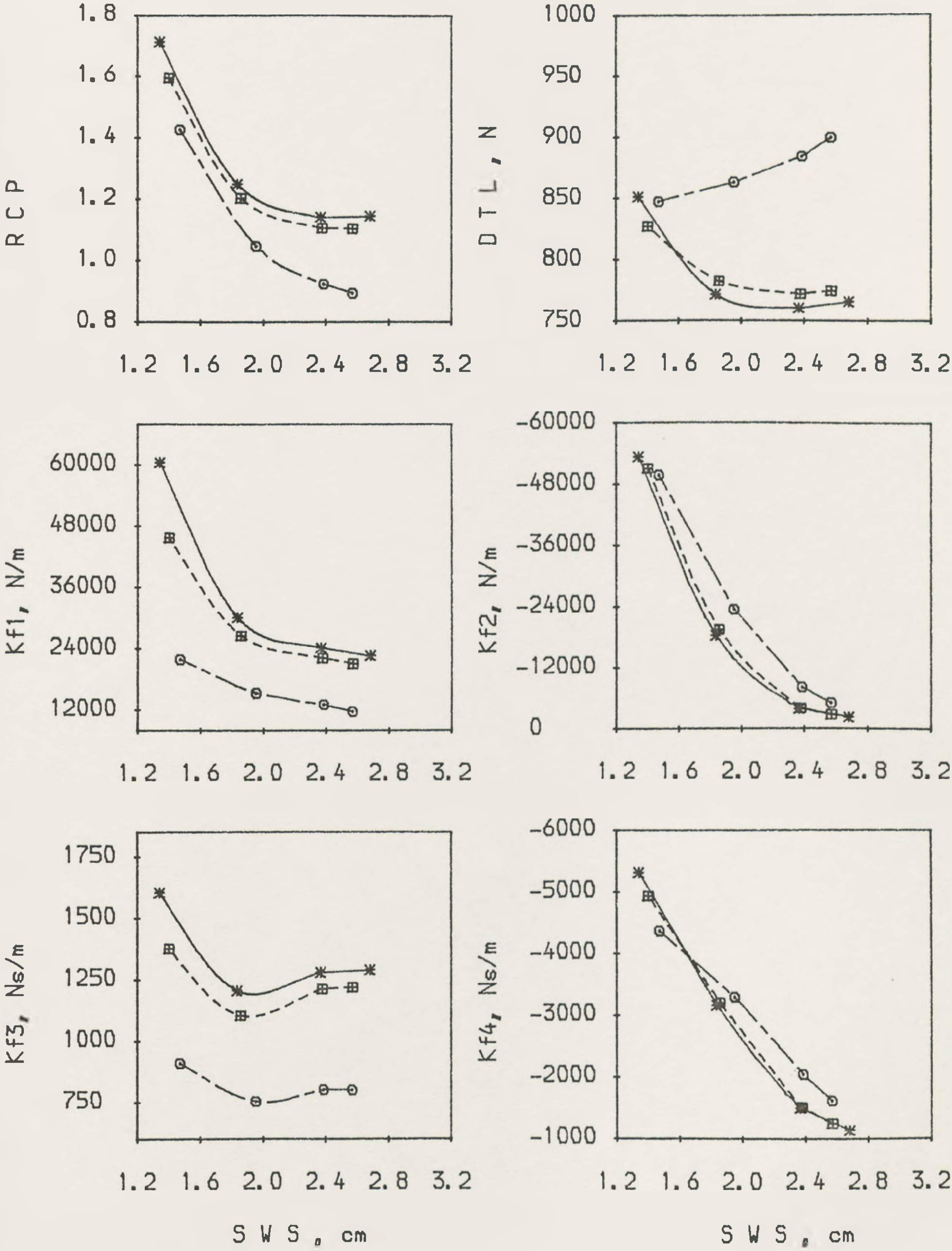

Fig. 6.1 Performance and design properties of semi-active systems based on full state feedback active control laws with no passive dampers

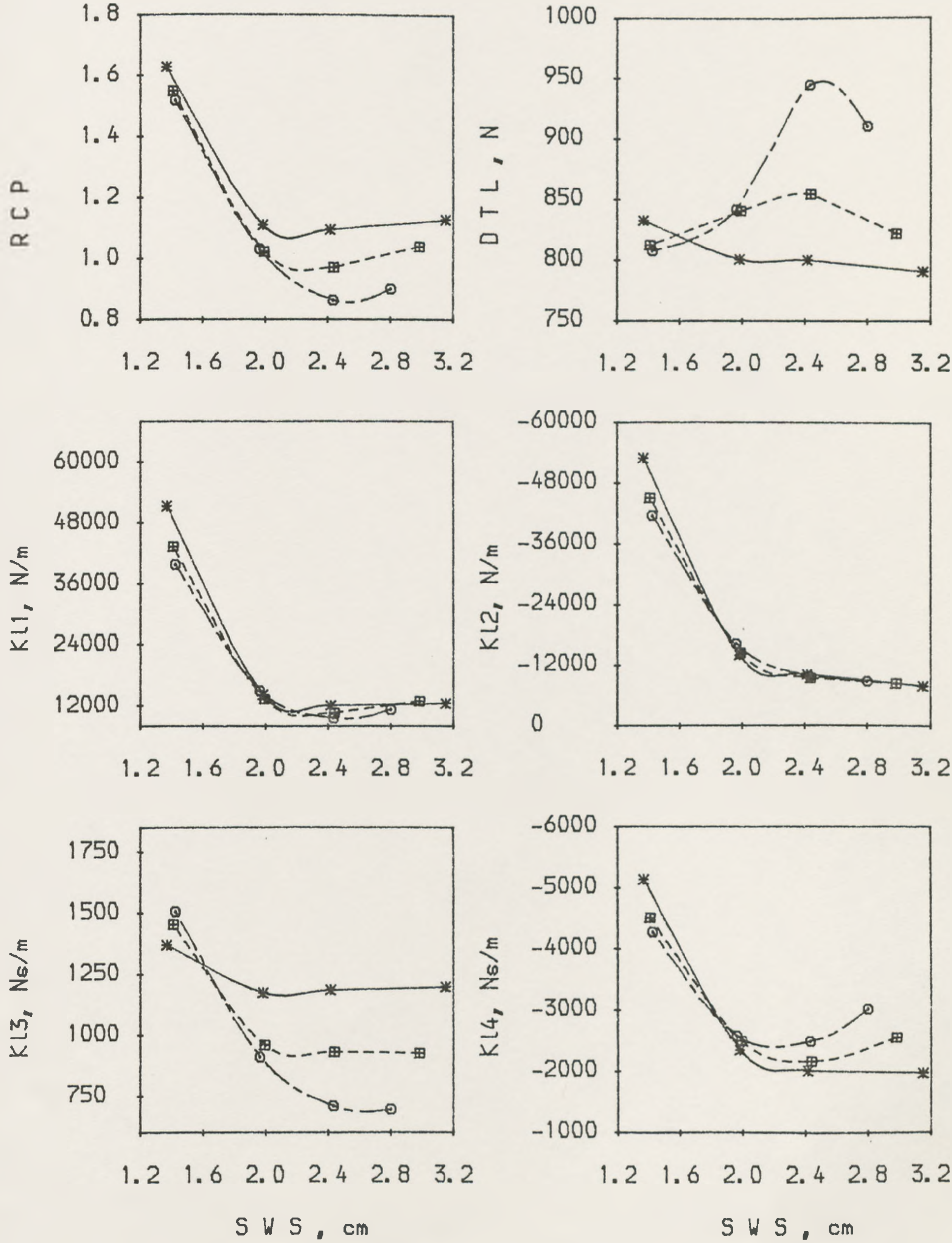

Fig. 6.2 Performance and design properties of semi-active systems based on limited state feedback active control laws with no passive dampers

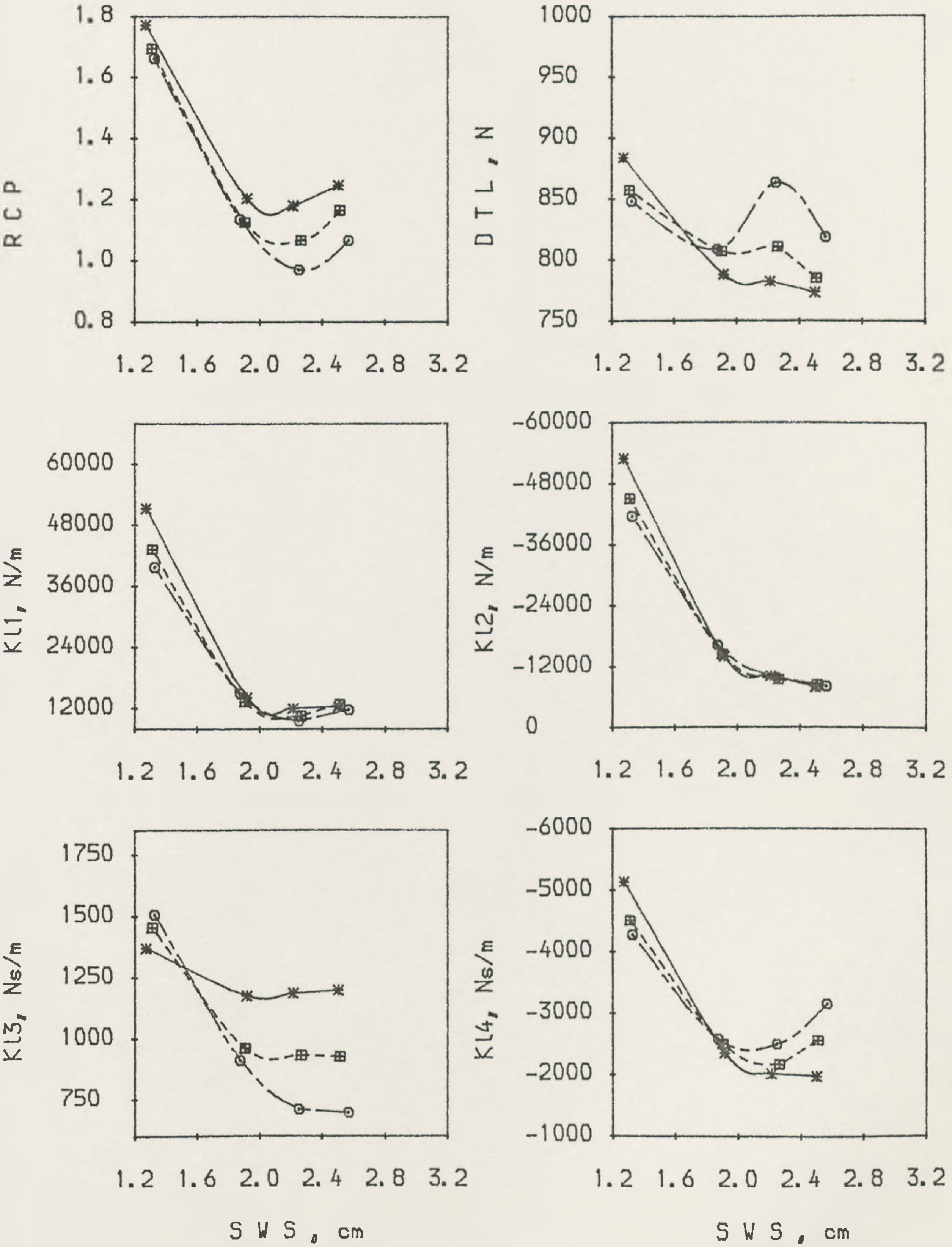

Fig. 6.3 Performance and design properties of semi-active systems based on limited state feedback active control laws with passive dampers

Table 6.1 Design and performance properties of particular semi-active systems based on full state feedback laws with no passive dampers

No	SWS cm	RCP	DTL N	Kf1 N/m	Kf2 N/m	Kf3 Ns/m	Kf4 Ns/m
1	(a)	1.541	819	47514	-39243	1416	-4595
2	1.5	1.486	814	39892	-41514	1290	-4515
3		1.400	849	21081	-47027	885	-4270
1	(b)	1.184	762	26595	-12000	1189	-2541
2	2.0	1.151	777	24000	-13946	1101	-2703
3		1.022	865	14919	-21730	750	-3122
1	(c)	1.130	761	23027	-1946	1290	-1216
2	2.5	1.103	773	21081	-1946	1203	-1324
3		0.903	894	12000	-6000	804	-1730

Table 6.2 Design and performance properties of particular semi-active systems based on limited state feedback laws with no passive dampers

No	SWS cm	RCP	DTL N	K11 N/m	K12 N/m	K13 Ns/m	K14 Ns/m
1	(a)	1.508	826	42714	-44000	1326	-4500
2	1.5	1.460	817	38286	-40000	1371	-4171
3		1.447	811	36000	-37786	1423	-4000
1	(b)	1.105	800	13778	-13778	1189	-2311
2	2.0	1.024	840	13333	-14556	956	-2489
3		1.000	852	13460	-15405	885	-2514
1	(c)	1.100	797	12000	-10000	1189	-2000
2	2.5	0.974	853	10555	-9111	911	-2178
3		0.856	948	9333	-9333	689	-2567

Table 6.3 Design and performance properties of particular semi-active systems based on limited state feedback laws with passive dampers

No	SWS cm	RCP	DTL N	K11 N/m	K12 N/m	K13 Ns/m	K14 Ns/m
1	(a)	1.568	850	38143	-39143	1300	-4158
2	1.5	1.505	841	33500	-34857	1291	-3842
3		1.493	832	31786	-33357	1314	-3697
1	(b)	1.159	780	11591	-10910	1182	-2136
2	2.0	1.077	804	10910	-11818	932	-2330
3		1.055	820	12273	-13636	795	-2455
1	(c)	1.245	772	12836	-7727	1182	-2000
2	2.5	1.159	786	12727	-8523	932	-2500
3		1.034	839	11023	-8410	693	-2932

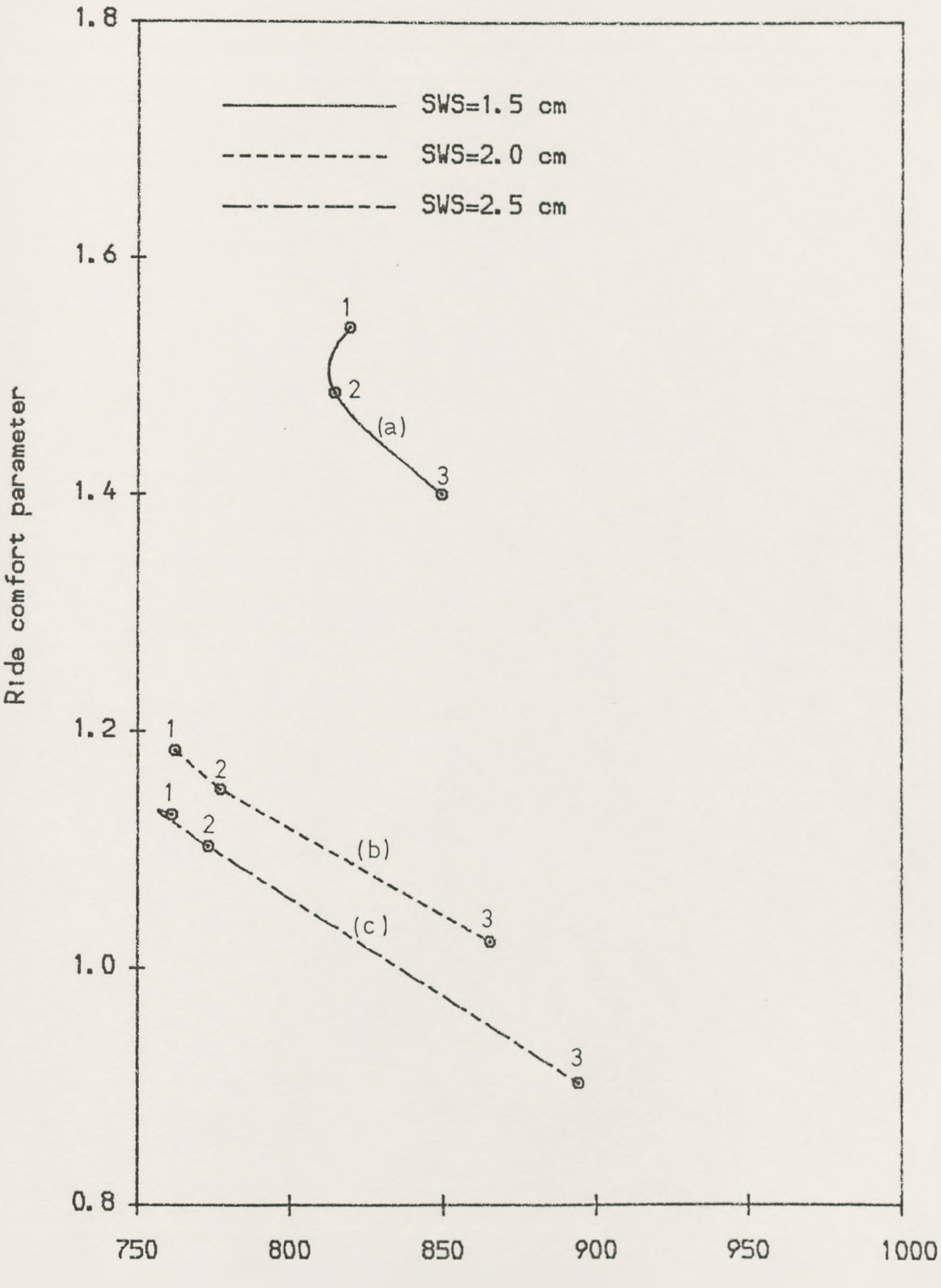

Fig. 6.4 Ride comfort and dynamic tyre load variation of semi-active systems based on full state feedback laws with no passive dampers

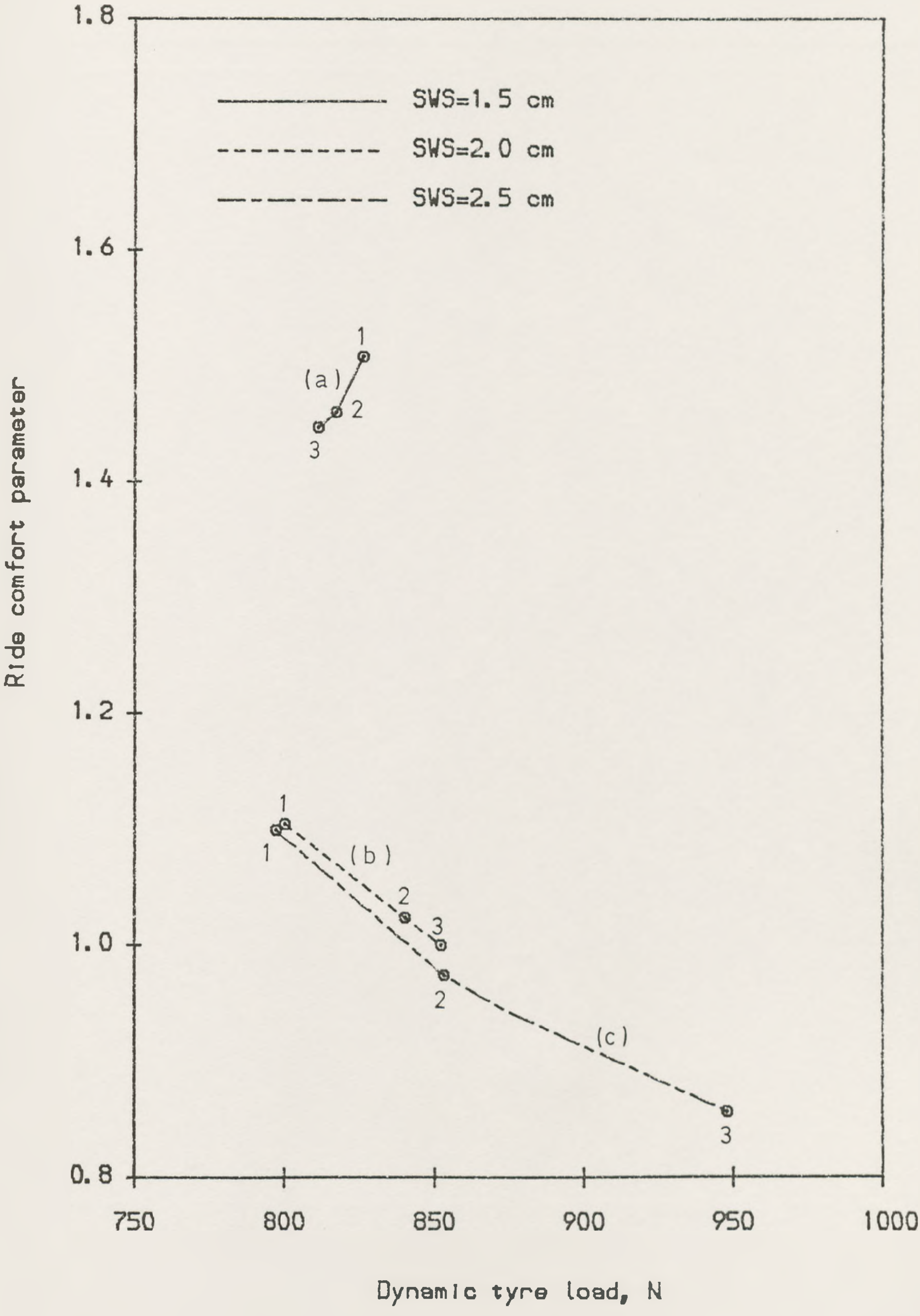

Fig. 6.5 Ride comfort and dynamic tyre load variation of semi-active systems based on limited state feedback laws with no passive dampers

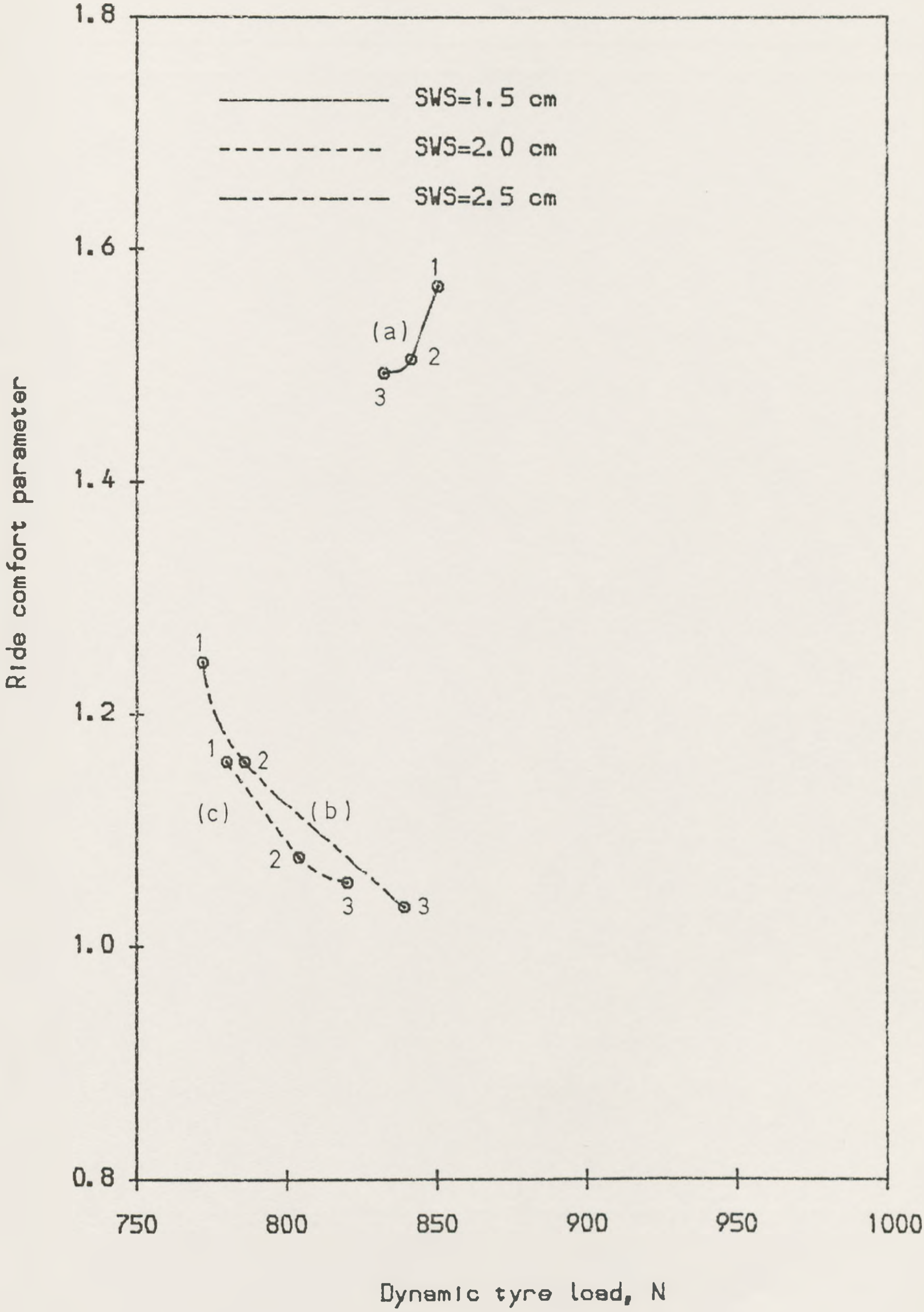

Fig. 6.6 Ride comfort and dynamic tyre load variation of semi-active systems based on limited state feedback laws with passive dampers

compared with convention, by which good handling and attitude control could be achieved. Moving towards system 3(a), the choice is open in the sense that comfort can be gained at the expense of the dynamic tyre load. In system 3(a), the "best" comfort can be obtained as a result of using a passive spring of stiffness very close to a conventional one, indicating that it would be acceptable from a handling viewpoint. Better comfort can be obtained in this case if more working space is made available, softer springs (but still in the conventional range) being needed as depicted by systems 2(b) and 2(c).

In the cases based on limited state feedback control laws without and with a passive damper, figures 6.5 and 6.6, systems with the same working space requirements are found to behave in nearly the same way in terms of ride comfort and dynamic tyre load variations. In both cases, system 3(a) is found to be the "best" performing system for the 1.5 cm suspension working space as a result of using a passive spring of stiffness 1.5 times the conventional one. Also, the ride comfort is significantly improved by increasing the suspension working space from 1.5 to 2.0 cm accompanied by softening the passive spring by a factor of three as compared with the systems requiring 1.5 cm suspension working space. Only small changes in the system performances and in the design parameters are obtained by increasing the suspension working space available from 2.0 to 2.5 cm.

The time histories of the motions of three semi-active systems based on the limited state feedback control law with no passive damper are shown in figures 6.7, 6.8 and 6.9 representing systems 3(a), 1(b) and 1(c) respectively. These systems are chosen to have nearly the same r.m.s. value of the dynamic tyre load particularly to show the nature of the switching process. In these plots, it can be seen that the semi-active damper rarely switches off, implying that it acts as a power dissipative element most of the time. It is implied that the performance will be very close to what can be achieved by using an active element. Figures 6.9, 6.10 and 6.11 can be used to compare the motions of systems obtained by using different control laws under the same suspension working space requirements (2.5 cm) and the same r.m.s. value of the dynamic tyre load variations. In Fig. 6.10, since a part of the power can be dissipated through the passive damper, it can be observed that the semi-active damper switches off most of the time implying greater power discontinuity as compared with the other two cases.

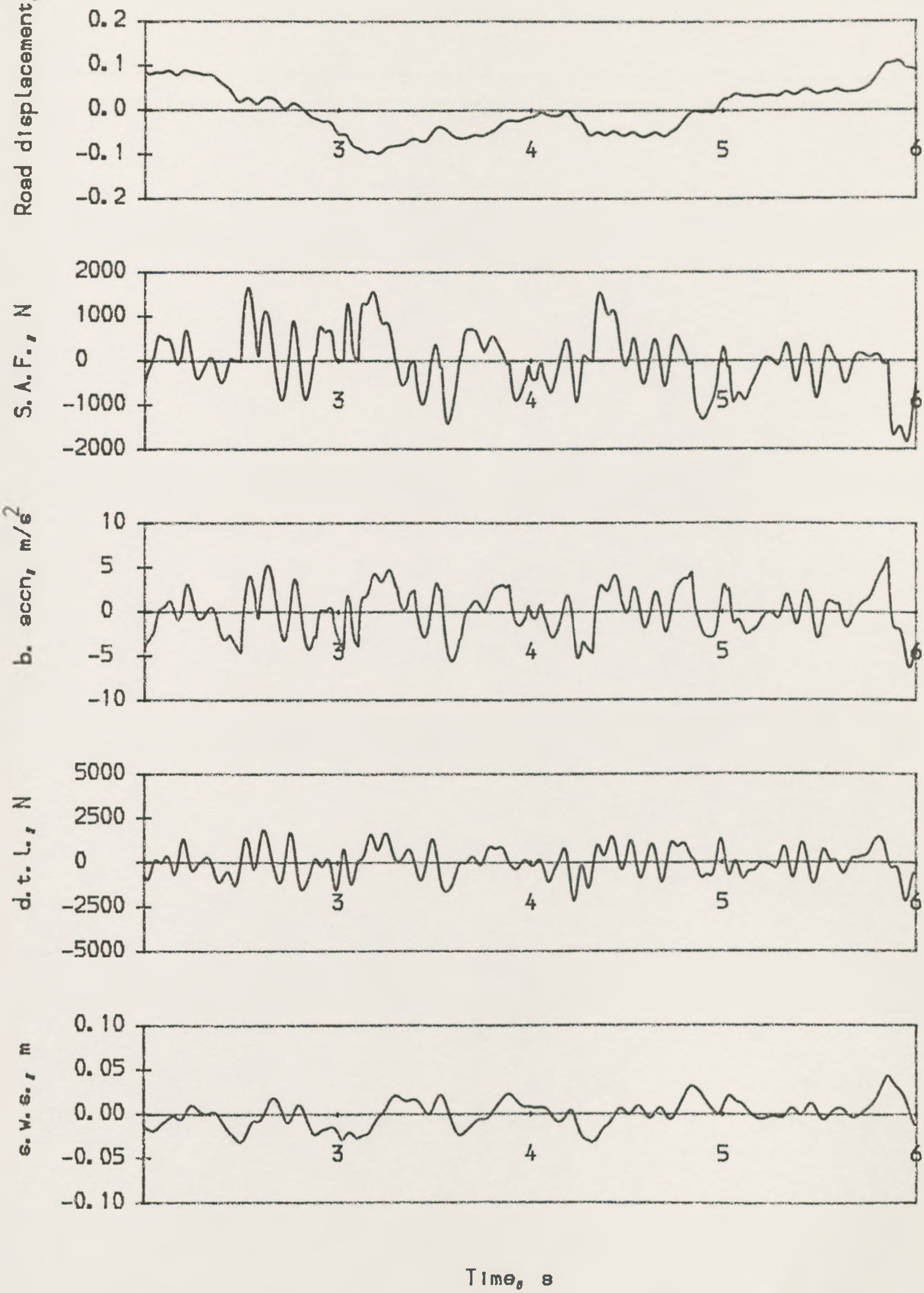

Fig. 6.7 Time history of the road input and the response of a semi-active system based on limited state feedback control law with no passive damper
S W S = 1.5 cm

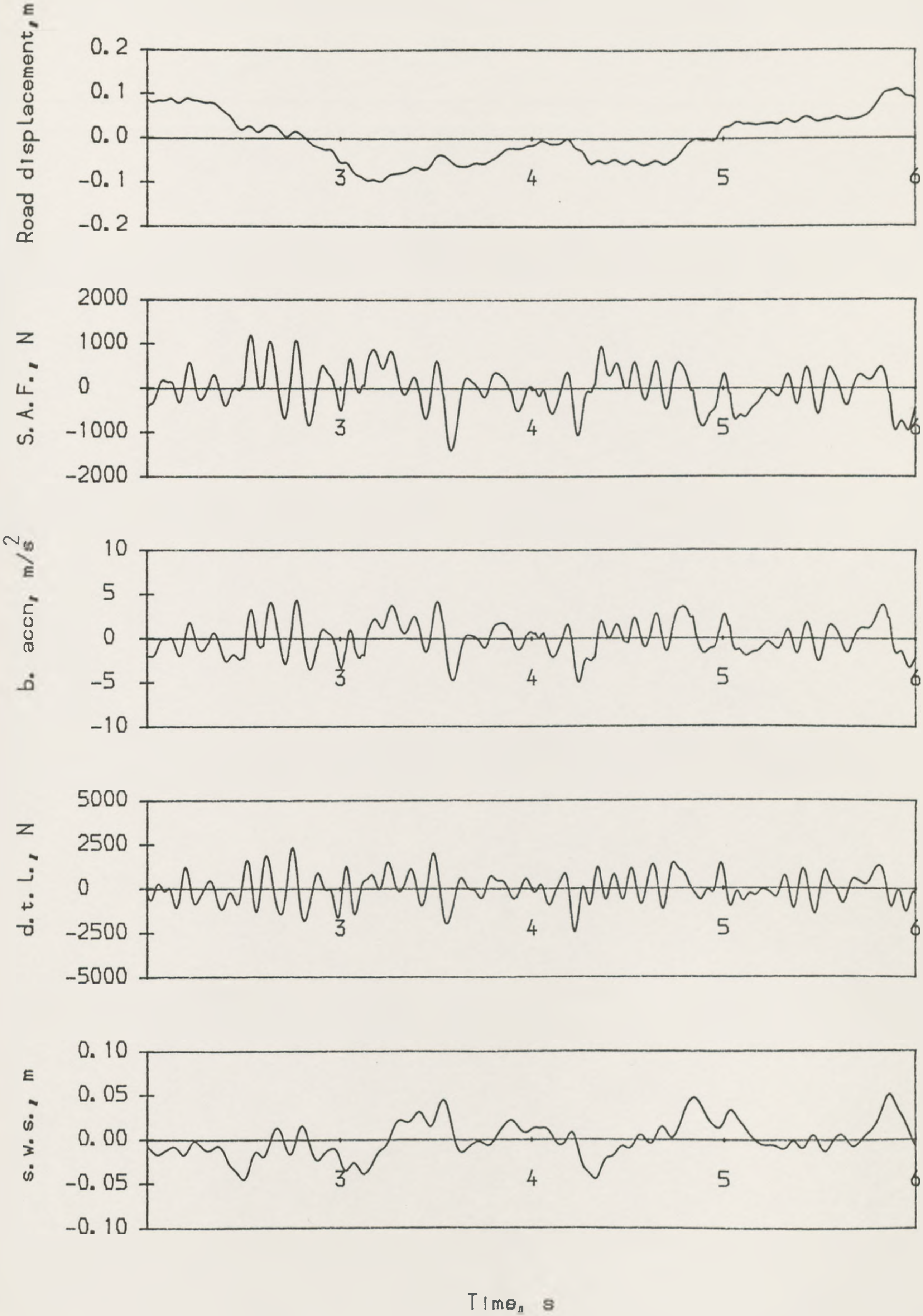

Fig. 6.8 Time history of the road input and the response of a semi-active system based on limited state feedback control law with no passive damper
S W S = 2.0 cm

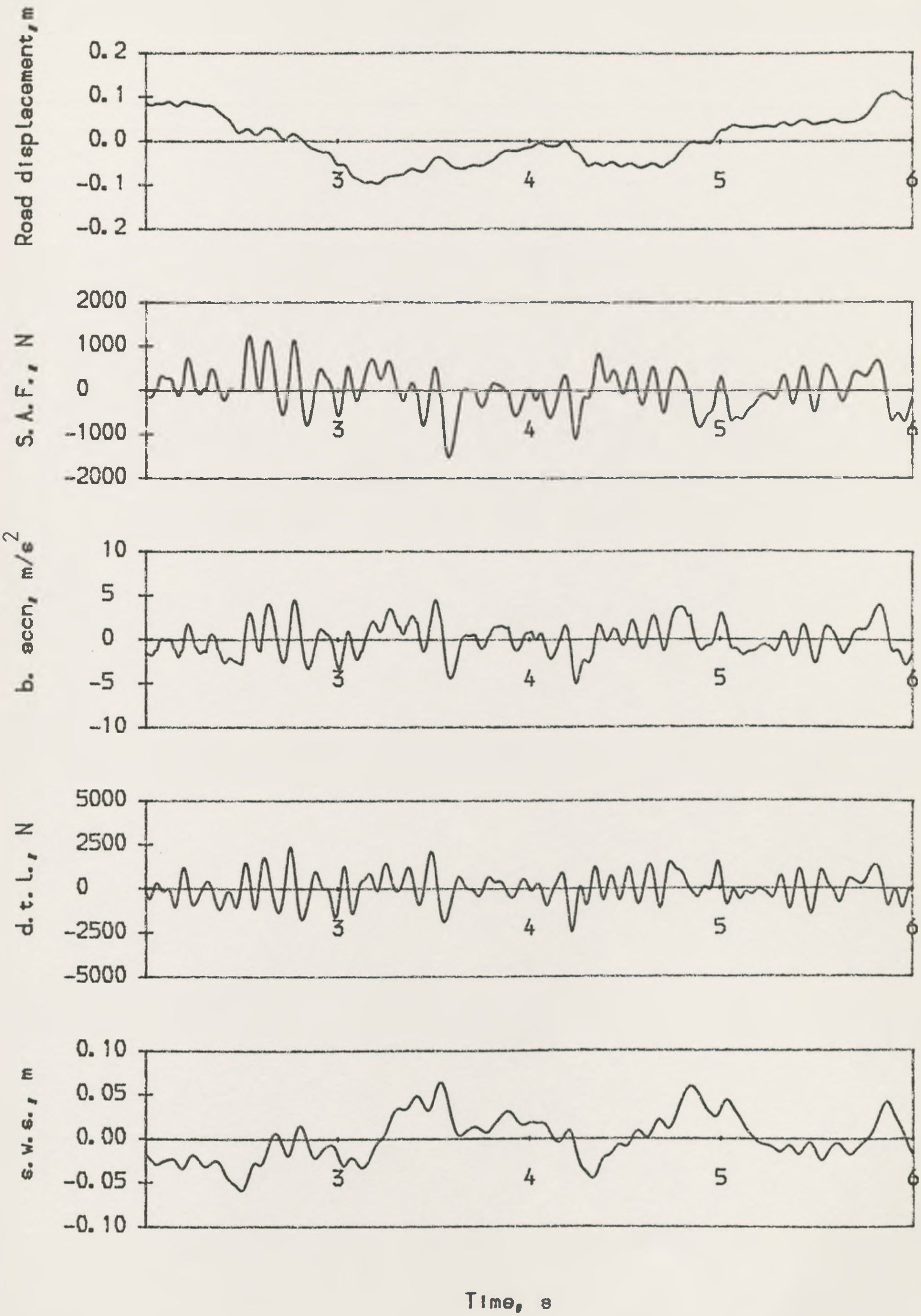

Fig. 6.9 Time history of the road input and the response of a semi-active system based on limited state feedback control law with no passive damper S W S = 2.5 cm

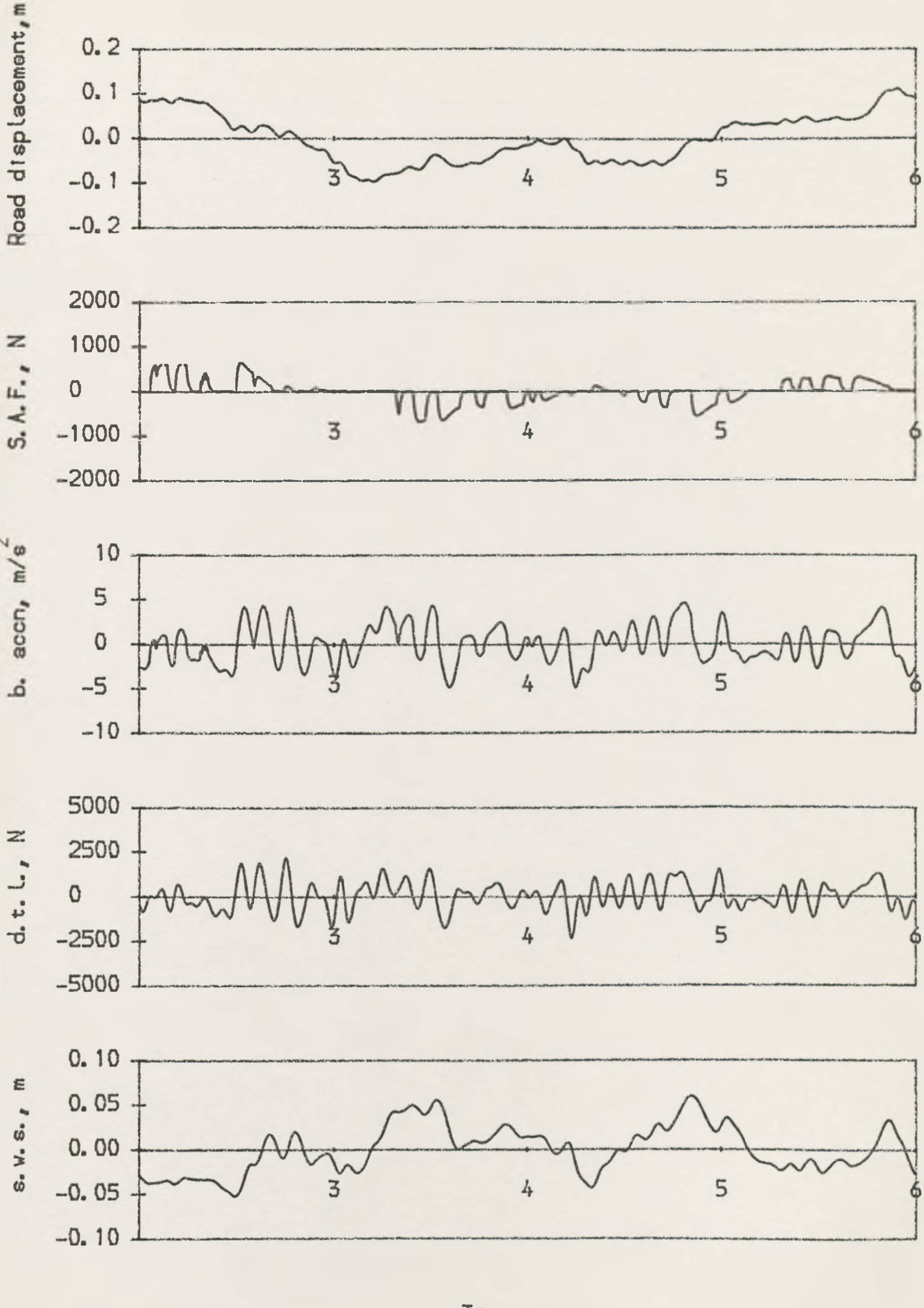

Fig. 6.10 Time history of the road input and the response of a semi-active system based on limited state feedback control law with a passive damper S W S = 2.5 cm

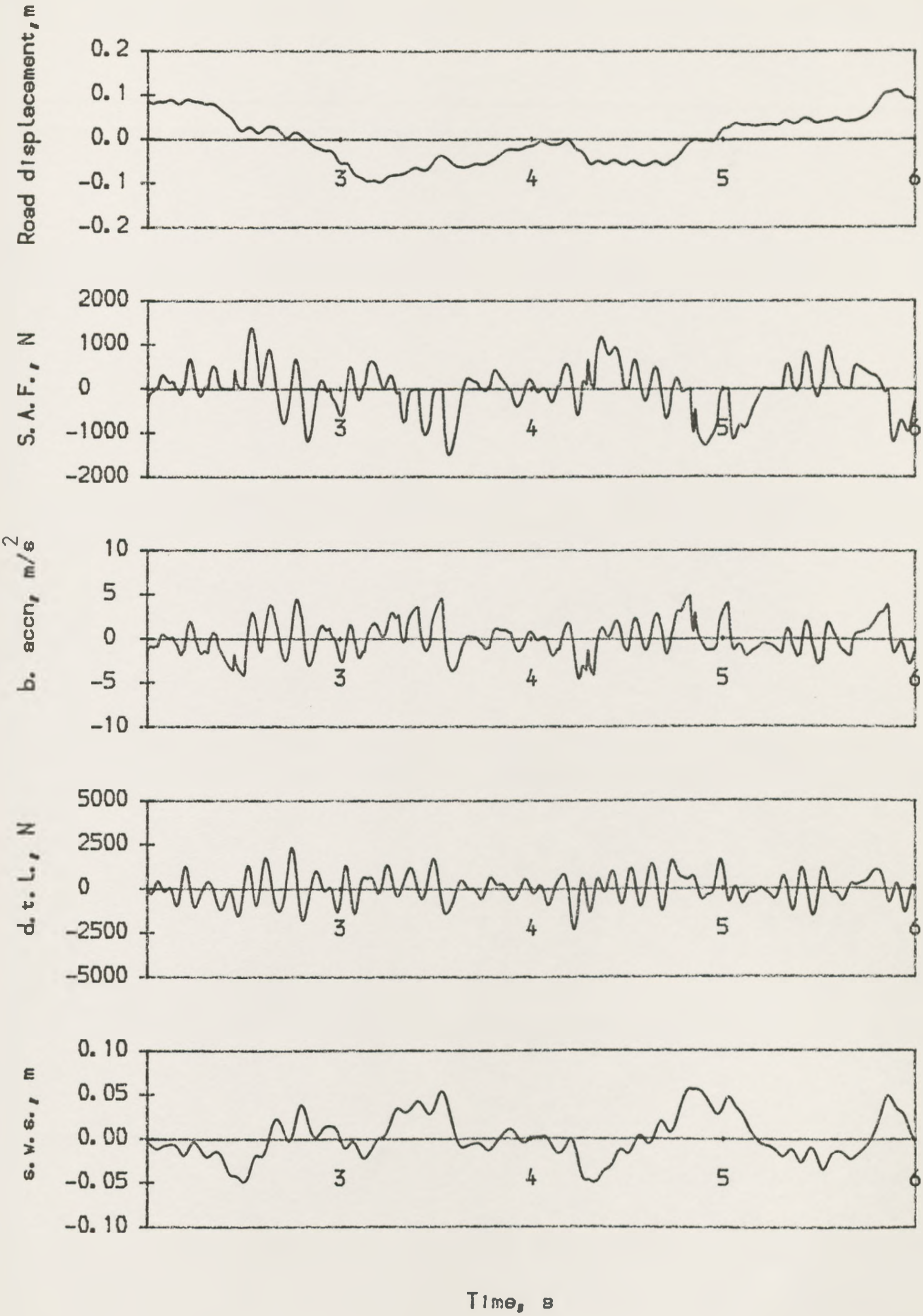

Fig. 6.11 Time history of the road input and the response of a semi-active system based on full state feedback control law with no passive damper
S W S = 2.5 cm

CHAPTER(7)

SLOW-ACTIVE SUSPENSION SYSTEM STUDIES

7.1 Introduction

In this chapter a class of systems in which feedback and actuators are used but in which the bandwidth of the active control loops is very limited, is discussed. The systems are referred to as slow-active. The main idea of such systems is to use the active device in controlling the system dynamics around the body resonance and to allow passive elements to exercise suitable control for the higher frequency components. The frequency band of the active device can be limited either by the use of a slowly responding actuator (a servo-motor for example) or by filtering out the high frequency components of the feedback signals. An inherent feature of this type of system is a passive spring connected in series with the actuator by virtue of which the system becomes essentially passive beyond the frequency limit of the actuator. A passive damper is needed to control the wheel dynamics.

The effectiveness of such systems is evaluated by generating results based on the quarter car model subjected to realistic road input. The study involves calculating the optimal parameters for each system as explained in chapter 2, after which the frequency response and spectral density functions can be calculated as described in chapter 3. The system performances are identified in terms of the ride comfort, the dynamic tyre load and the suspension working space parameters as before. The effect of limiting the actuator bandwidth is examined first, and then further results are generated using a reasonable value of the actuator frequency response limit. Among these results, systems with each one of the standard suspension working spaces (1.5, 2.0 and 2.5 cm) are identified for comparison with each other.

7.2 System Model and Calculations

The slow-active suspension model used is shown in Fig. 7.1. In this model, the actuator is connected in series with a passive spring of stiffness Ks. A passive damper of coefficient Cs is connected between the body mass mb and the wheel mass mw. The high frequency components of the feedback signal are filtered out by using a second order low-pass filter, the actuator being treated as ideal for those frequencies passed by the filter. The frequency responses of filters having different values of cut-off frequency and a damping ratio of 0.707 are shown in Fig. 7.2.

The differential equations of motion of the model shown in Fig. 7.1 can be written as,

$$\begin{aligned} mw\ddot{x}1(t) &= -Ks[x1(t)-x3(t)] - Cs[\dot{x}1(t)-\dot{x}2(t)] + Kt[x0(t)-x1(t)] \\ mb\ddot{x}2(t) &= Ks[x1(t)-x3(t)] + Cs[\dot{x}1(t)-\dot{x}2(t)] \qquad \dots\dots\dots\dots \quad 7.1 \\ x2(t) - x3(t) &= Cm\, u'(t) \end{aligned}$$

where, x3(t) is the actuator displacement, $u'(t)$ can be thought of as a displacement demand signal to be obtained by judicious combination of measured system states and Cm is the actuator displacement coefficient.

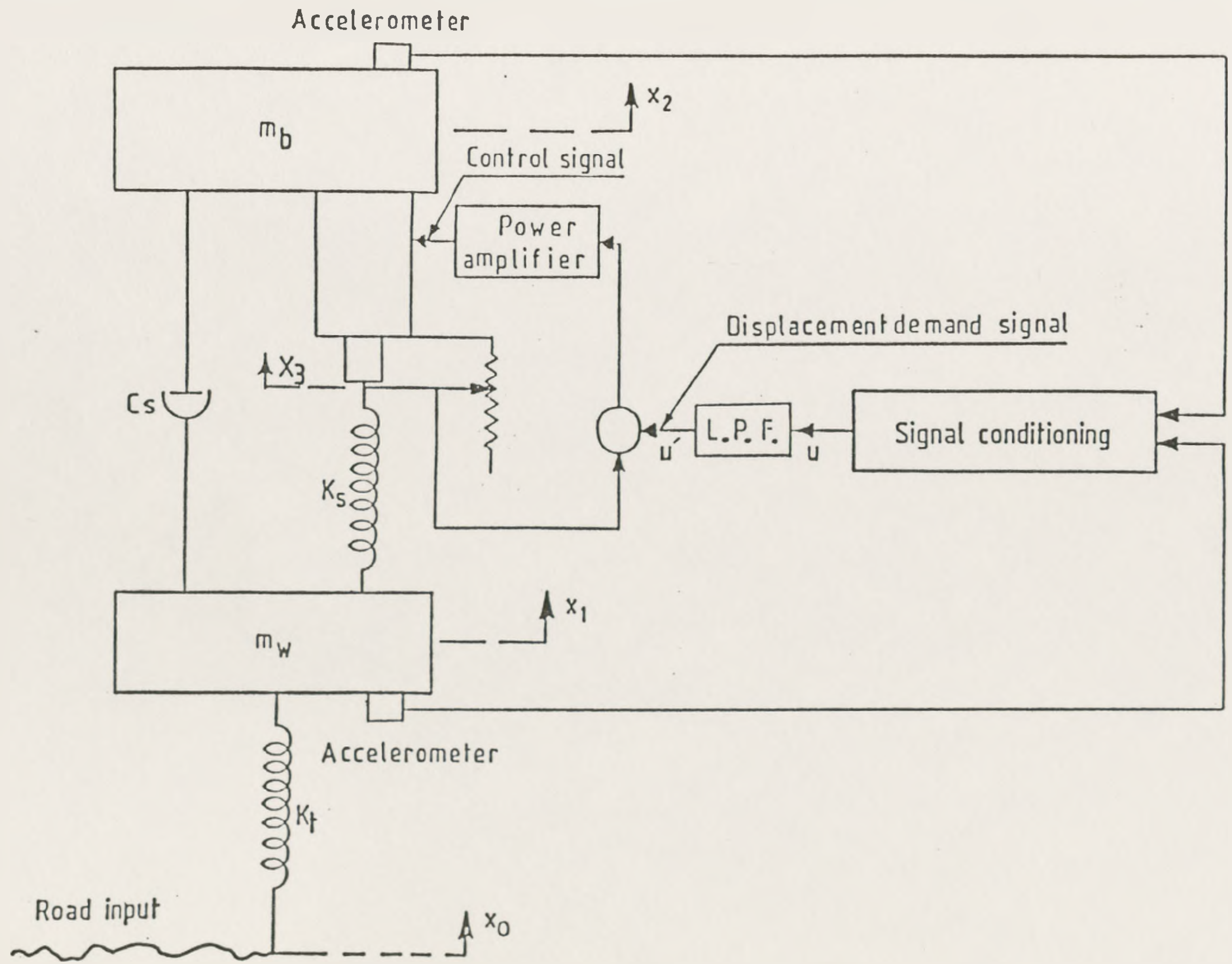

Fig. 7.1 Quarter car slow-active suspension system

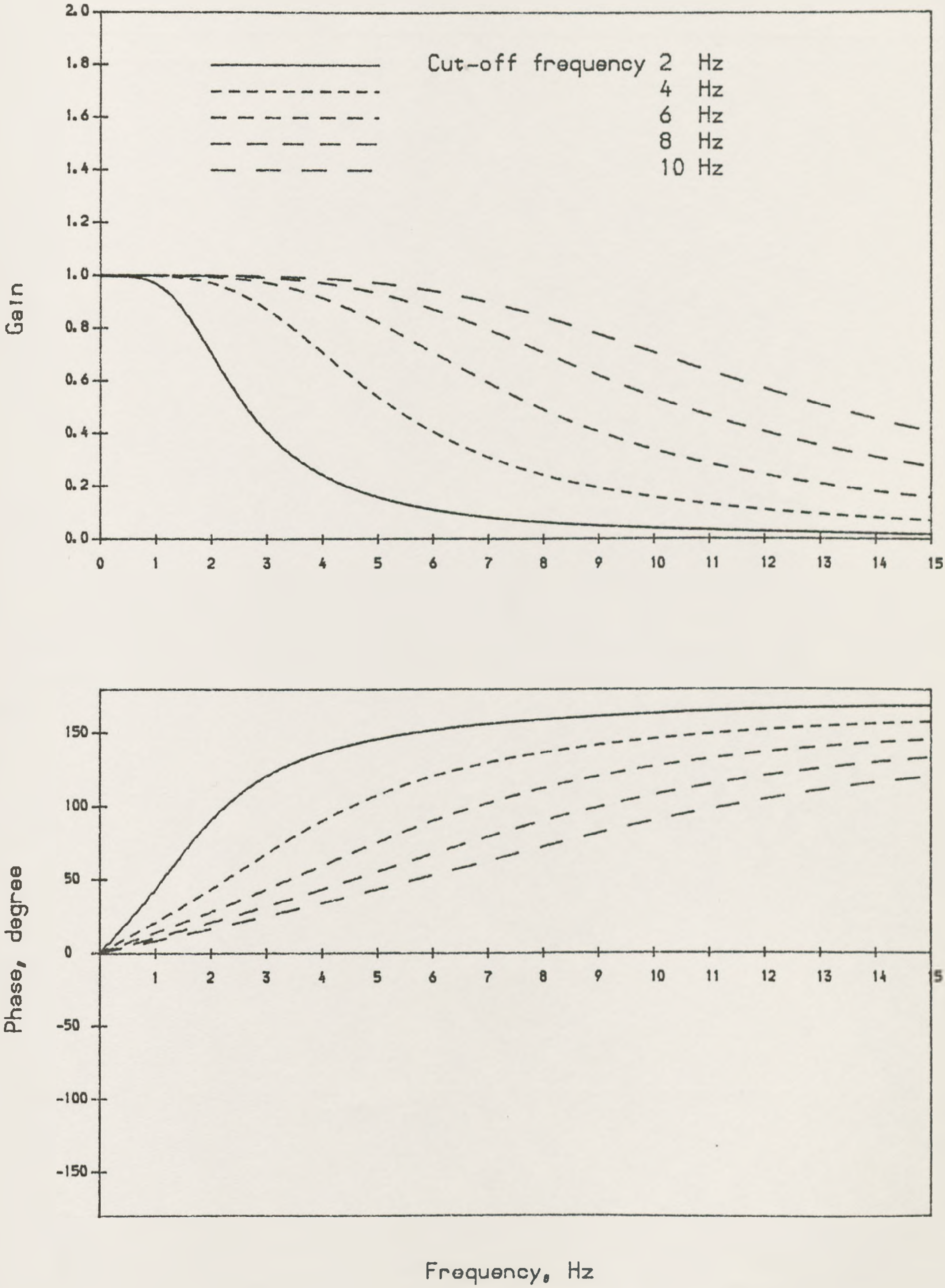

Fig. 7.2 Frequency responses of second order low-pass filters

The filter characteristics can be expressed as,

$$u'(t) = u(t)wf^2 / (s^2 + 2\gamma wfs + wf^2) \quad \text{..........} \quad 7.2$$

where wf and γ are the filter cut-off frequency and damping ratio and

$$u(t) = Ks1x1(t) + Ks2x2(t) + Ks3\dot{x}1(t) + Ks4\dot{x}2(t)$$

Ks1, Ks2, Ks3 and Ks4 constituting the control law.

Equations 7.1 and 7.2 in their state space form become,

$$\dot{x}1(t) = x3(t)$$

$$\dot{x}2(t) = x4(t)$$

$$\dot{x}3(t) = -Ks/mwx1(t) + Ks/mwx2(t) - Kswf^2/mwx5(t) - Cs/mwx3(t) + Cs/mwx4(t) - Kt/mwx1(t) + Kt/mwx0(t)$$

$$\dot{x}4(t) = Ks/mbx1(t) - Ks/mbx2(t) + Kswf^2/mbx5(t) + Cs/mbx3(t) - Cs/mbx4(t)$$

$$\dot{x}5(t) = x6(t)$$

$$\dot{x}6(t) = -2\gamma wfx6(t) - wf^2x5(t) + u(t) \quad \text{..........} \quad 7.3$$

where,

$$x5(t) = u(t) / (s^2 + 2\gamma wfs + wf^2)$$

The optimisation process employed necessitates the description of the road surface input as a filtered integrated white noise velocity signal (see chapter 2). This can be derived from the white noise process $x0(t) = \xi(t)$ (in equation 2.23) by passing the signal through an integrator followed by a first order high-pass filter in order to remove the infinite power at zero frequency from the corresponding displacement spectral density function, which will now take the form $Dr(\nu)=B1/(\alpha^2+\nu^2)$. The road surface input can then be formulated as one of the system states being described by

$$\dot{x}0(t) = -(1.0/\tau)x0(t) + \xi(t) \qquad \text{.......... } 7.4$$

where, τ is the time constant of the high-pass filter, $\xi(t)$ is the white noise process, and x0(t) is the displacement input to the system model.

The weighting matrix Q in equation 2.4 is

$$Q = \begin{bmatrix} q1 & 0 & 0 \\ 0 & q2 & 0 \\ 0 & 0 & q3 \end{bmatrix} \qquad \text{.......... } 7.5$$

to represent the weighting parameters q1, q2 and q3 by which the relative weights of the dynamic tyre load, the suspension working space and the ride comfort parameters in the performance index are specified.

The matrix C in equation 2.2 relating the controlled variables (which contribute to the performance index) to the state variables is formulated as,

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ Ks & -Ks & Cs & -Cs & Kswf^2 & 0 & 0 \end{bmatrix} \qquad \text{.......... } 7.6$$

The form of the matrix indicates that the three contributions to the performance index are the dynamic tyre load as represented by the difference between wheel and road displacements, the suspension working space as revealed by the difference between the wheel and body displacements and the body acceleration as given by the sum of the forces acting on the body mass.

The optimisation procedures described in chapter 2 are applied to the system equations 7.3 and 7.4 and the relationships in equations 7.5 and 7.6 are used in order to obtain the optimal parameters Ks1, Ks2, Ks3 and Ks4 by which the performance index in equation 2.19 can be minimised. A stability test is included in order to calculate the eigenvalues of the closed loop system by using the QR method available in the form of a NAG library subroutine, in particular to make sure that all of them have negative real parts.

The optimal parameters are then used in the system equations 7.1 and 7.2 in order to calculate the frequency response functions and the output mean square spectral density functions of the ride comfort parameter, the dynamic tyre load and the suspension working space as described in chapter 3, using the input spectral density function 3.2.

7.3 Slow-active System Results

The base parameters of the system (body mass, wheel mass and tyre stiffness) are chosen to be the same as those used in studying the previous systems under the same road conditions and vehicle speed. Different values of the spring stiffness Ks and the damping coefficient Cs are used in generating results by which the idea of limiting the actuator bandwidth can be judged, following which an appropriate cut-off frequency for the low-pass filter can be chosen.

Results in figures 7.3, 7.4 and 7.5 are obtained using springs of stiffnesses 16000, 10000 and 4000 N/m respectively to represent a conventional and softer than conventional springs. In each of these figures, two different values of the damping ratio, 0.2 and 0.4 of critical are employed. The cut-off frequency of the low-pass filter in each figure is varied from zero, representing a passive system, to a value of 30 Hz, representing a constrained optimal active system, the constraint deriving from the fact that full state variable information is not available to the controller. It can be observed from these figures that the system performance properties are significantly improved by using a limited bandwidth actuator with a cut-off frequency around 4 Hz, as compared with passive systems with the same passive elements, and the benefits gained by increasing the bandwidth beyond this limit are of no significance. By using the limited actuator and conventional spring, improvements in ride comfort, dynamic tyre load variations and suspension working space parameters can be gained if the system is lightly damped. As the damping ratio increased from 0.2 to 0.4 with the same spring, the performance improvement is almost all in respect of the suspension working space. In systems of soft spring and

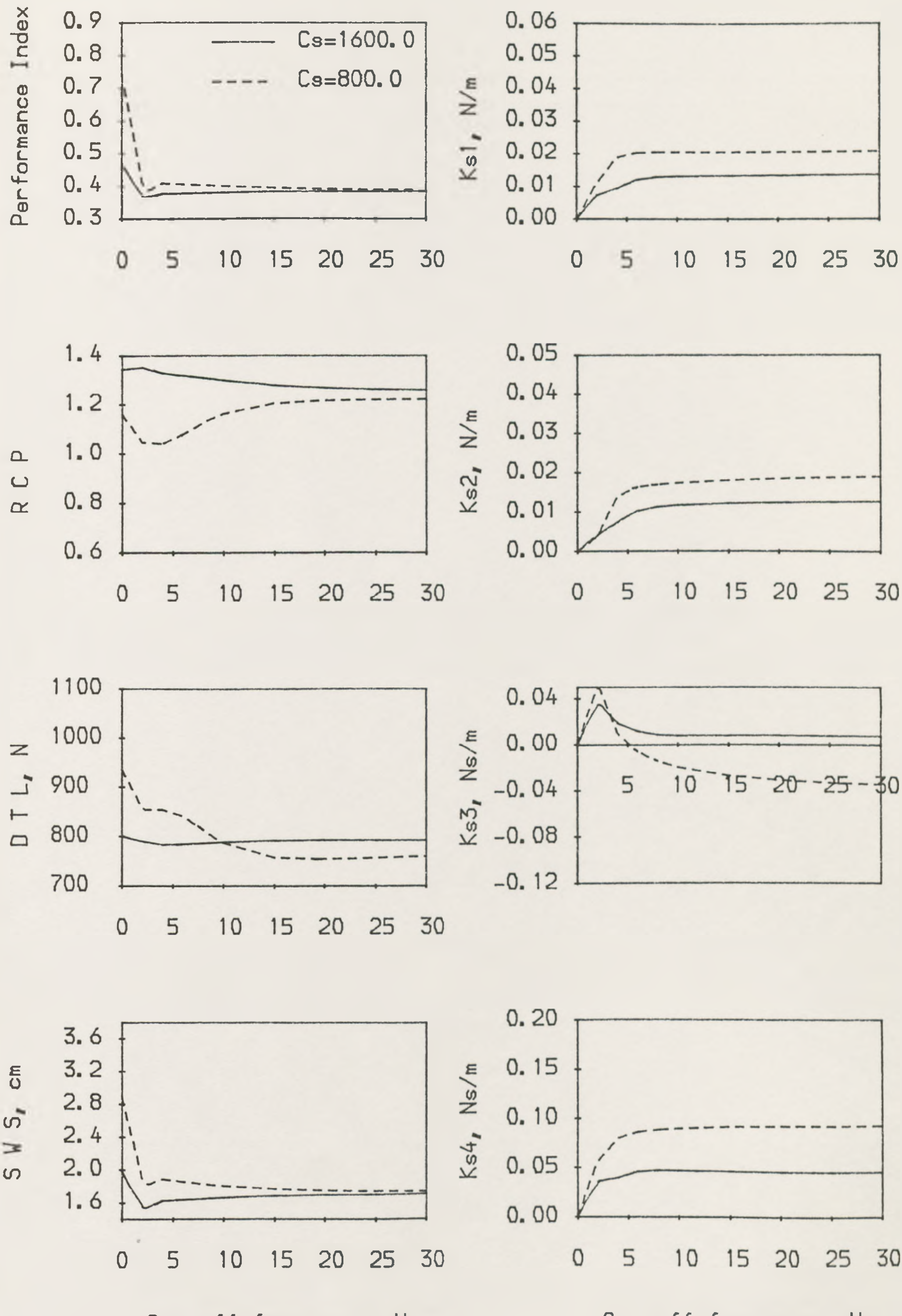

Fig. 7.3 Performance and design properties of slow-active systems
Spring stiffness = 16000.0 N/m

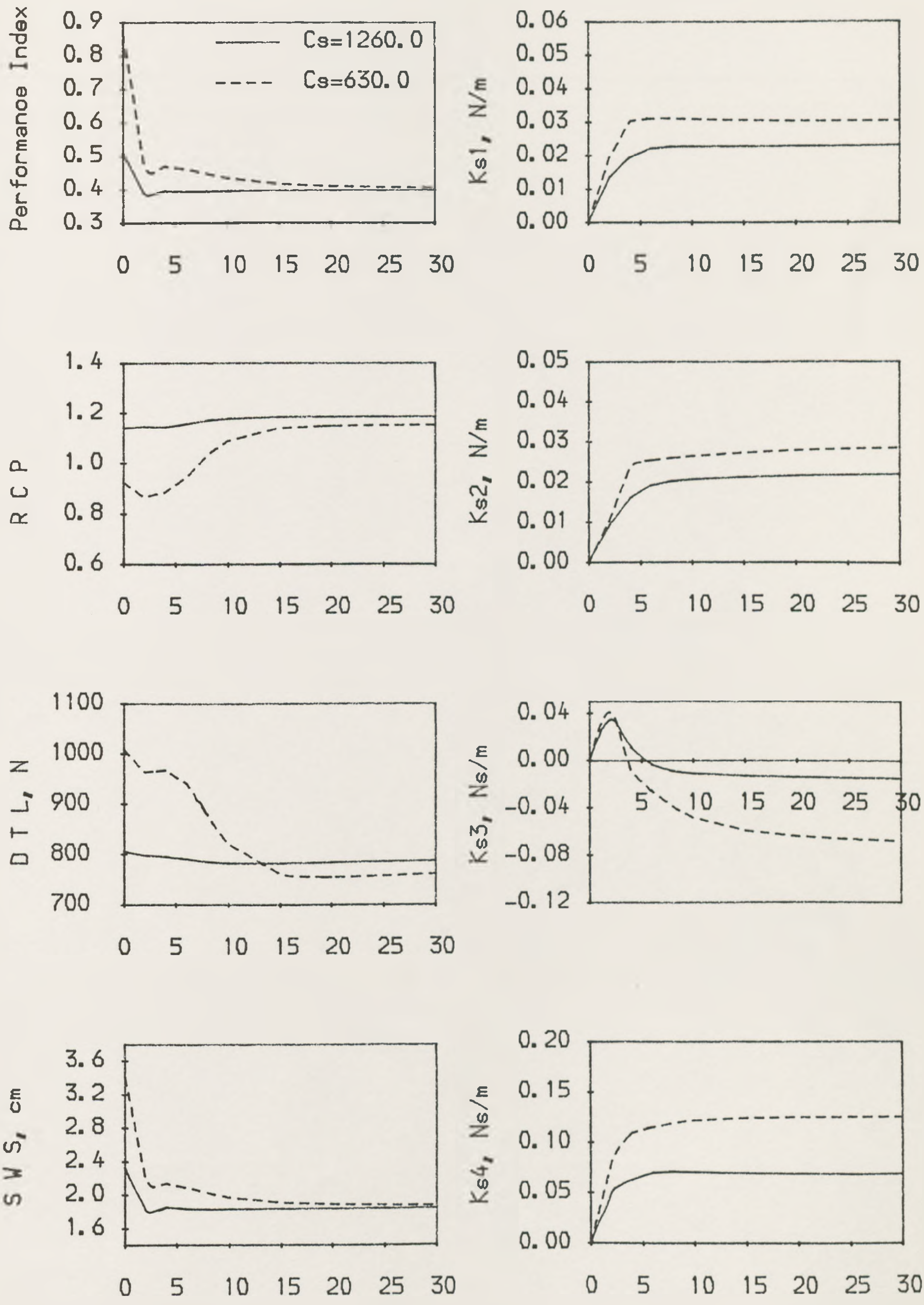

Fig. 7.4 Performance and design properties of slow-active systems
Spring stiffness = 10000.0 N/m

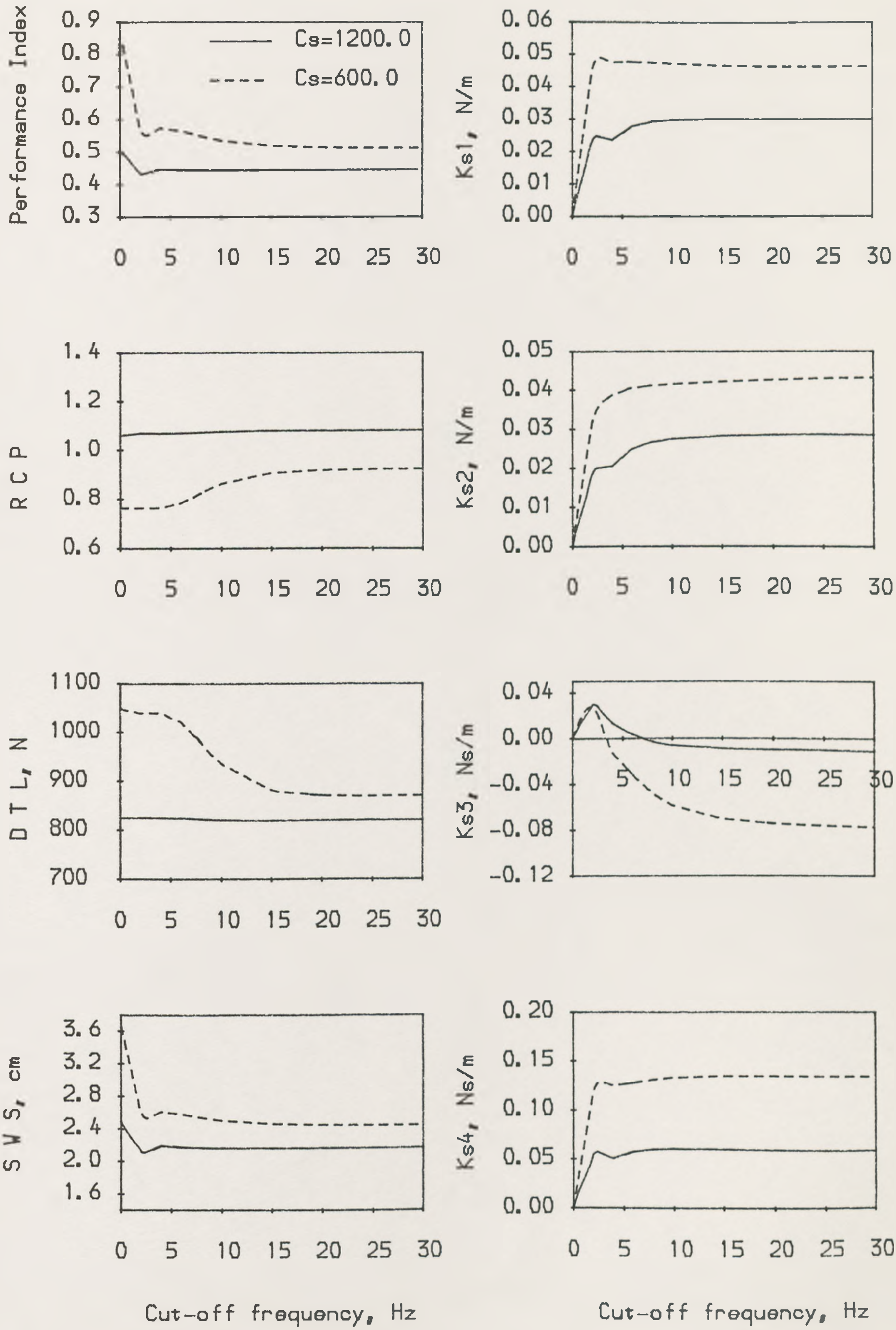

Fig. 7.5 Performance and design properties of slow-active systems
Spring stiffness = 4000.0 N/m

light damping, improvement in dynamic tyre load accompanied by worse ride comfort is obtained if the cut-off frequency of the low-pass filter is increased to values higher than 7 Hz.

Systems having actuators of 4 Hz bandwidth limit are plotted in Fig. 7.6. These results indicate the performance properties of interest and the four feedback gains all plotted against the suspension working space, to allow the identification of those systems with each one of the standard suspension working spaces. The systems so identified are classified as groups (a), (b) and (c) in table 7.1 to represent systems of 1.5, 2.0 and 2.5 cm suspension working space respectively.

The frequency response and spectral density functions of these particular systems are plotted in figures 7.7, 7.8 and 7.9 for groups (a), (b) and (c) respectively. The ride comfort parameter is plotted against the dynamic tyre load for each of these groups in Fig. 7.10, in which system 1(a) is found to be the "best" performing system in terms of ride comfort and dynamic tyre load as compared with systems 2(a) and 3(a). As the spring stiffness and the damping coefficient in group (a) are decreased, systems have higher body and wheel resonance peaks in the dynamic tyre load response functions and an increasing body resonance peak in the weighted body acceleration function, as can be seen in Fig. 7.7, indicating worse comfort and dynamic tyre load control. The major weaknesses of these systems are the high levels of body acceleration amplitudes at low frequencies for which human sensitivity to the vibration is high and of the

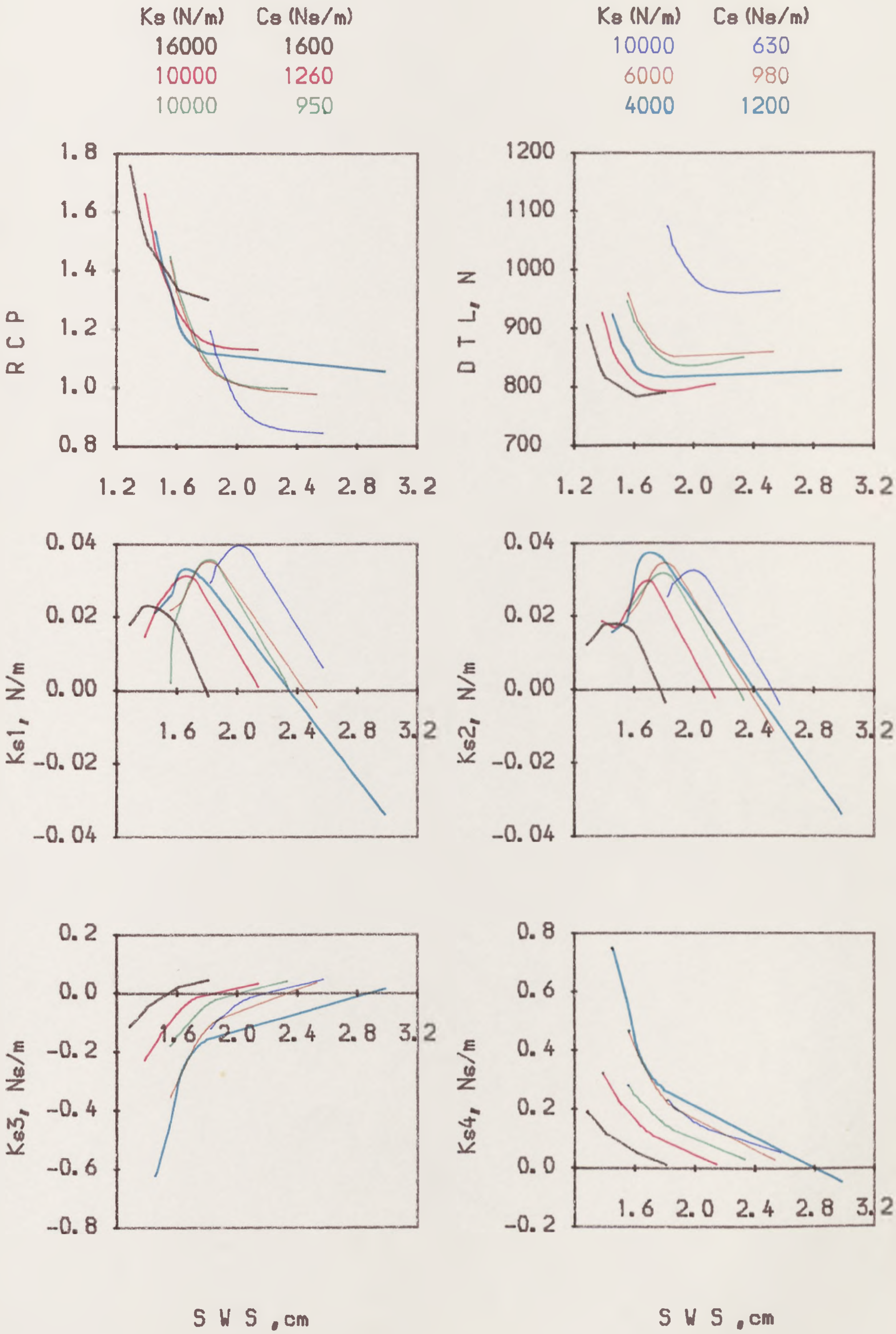

Fig. 7.6 Performance and design properties of slow-active systems of 4 Hz bandwidth

Table 7.1 Design and performance properties of particular slow-active systems

No	SWS cm	RCP	DTL N	Ks N/m	Cs Ns/m	Ks1 N/m	Ks2 N/m	Ks3 Ns/m	Ks4 Ns/m
1	(a)	1.400	795	16000	1600	0.021	0.017	-0.011	0.091
2	1.5	1.400	840	10000	1260	0.025	0.017	-0.141	0.223
3		1.420	884	4000	1200	0.024	0.016	-0.532	0.641
1	(b)	1.136	800	10000	1260	0.010	0.008	0.009	0.041
2	2.0	1.021	834	10000	950	0.024	0.020	0.000	0.100
3		0.950	982	10000	630	0.040	0.040	-0.041	0.209
1	(c)	1.086	823	4000	1200	-0.008	-0.006	-0.059	0.077
2	2.5	0.986	860	6000	980	-0.003	-0.010	0.025	0.036
3		0.857	961	10000	630	0.010	0.001	0.034	0.059

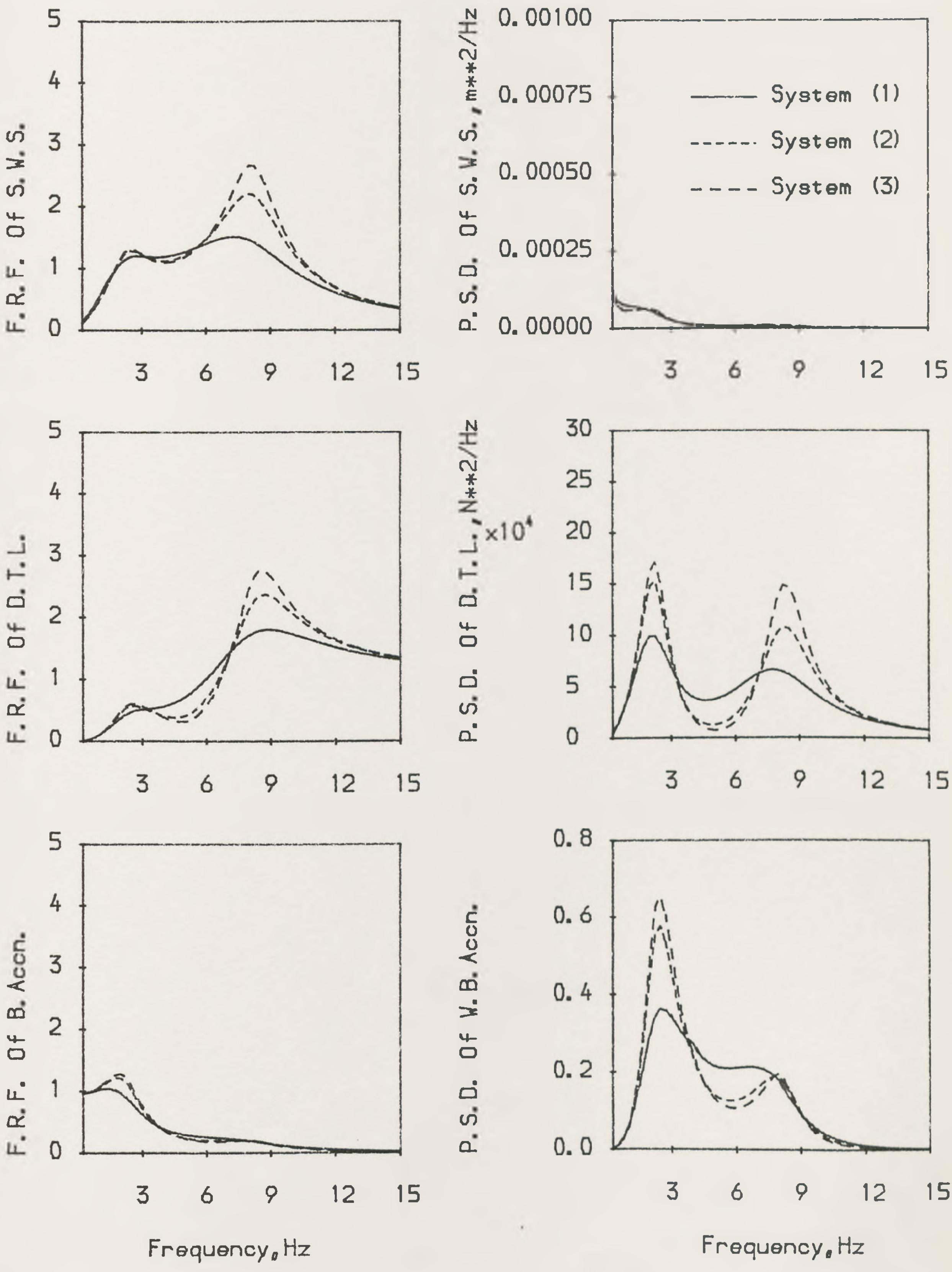

Fig. 7.7 Frequency responses and output mean square spectral densities of slow-active systems, S W S = 1.5 cm

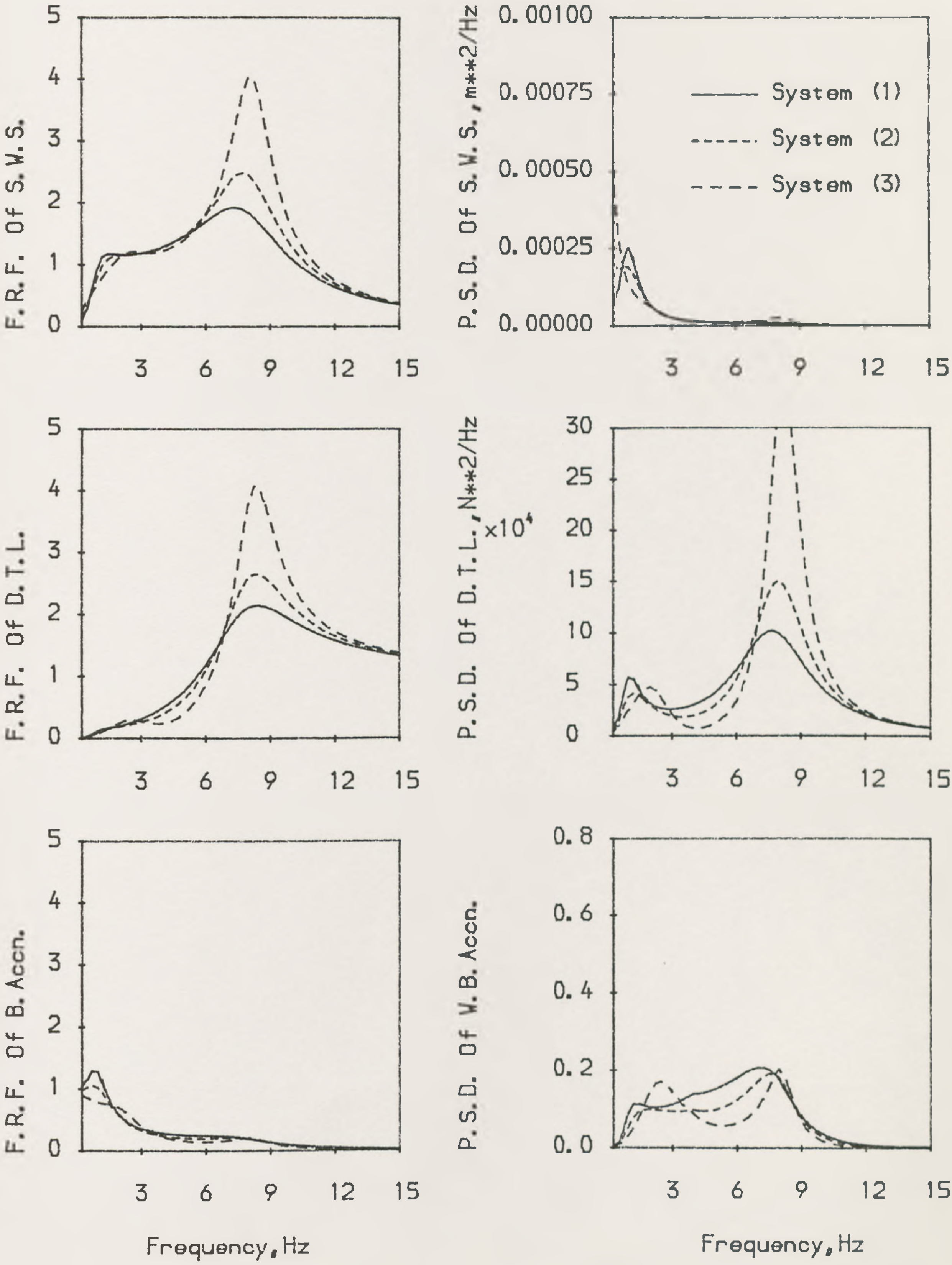

Fig. 7.8 Frequency responses and output mean square spectral densities of slow-active systems, S W S = 2.0 cm

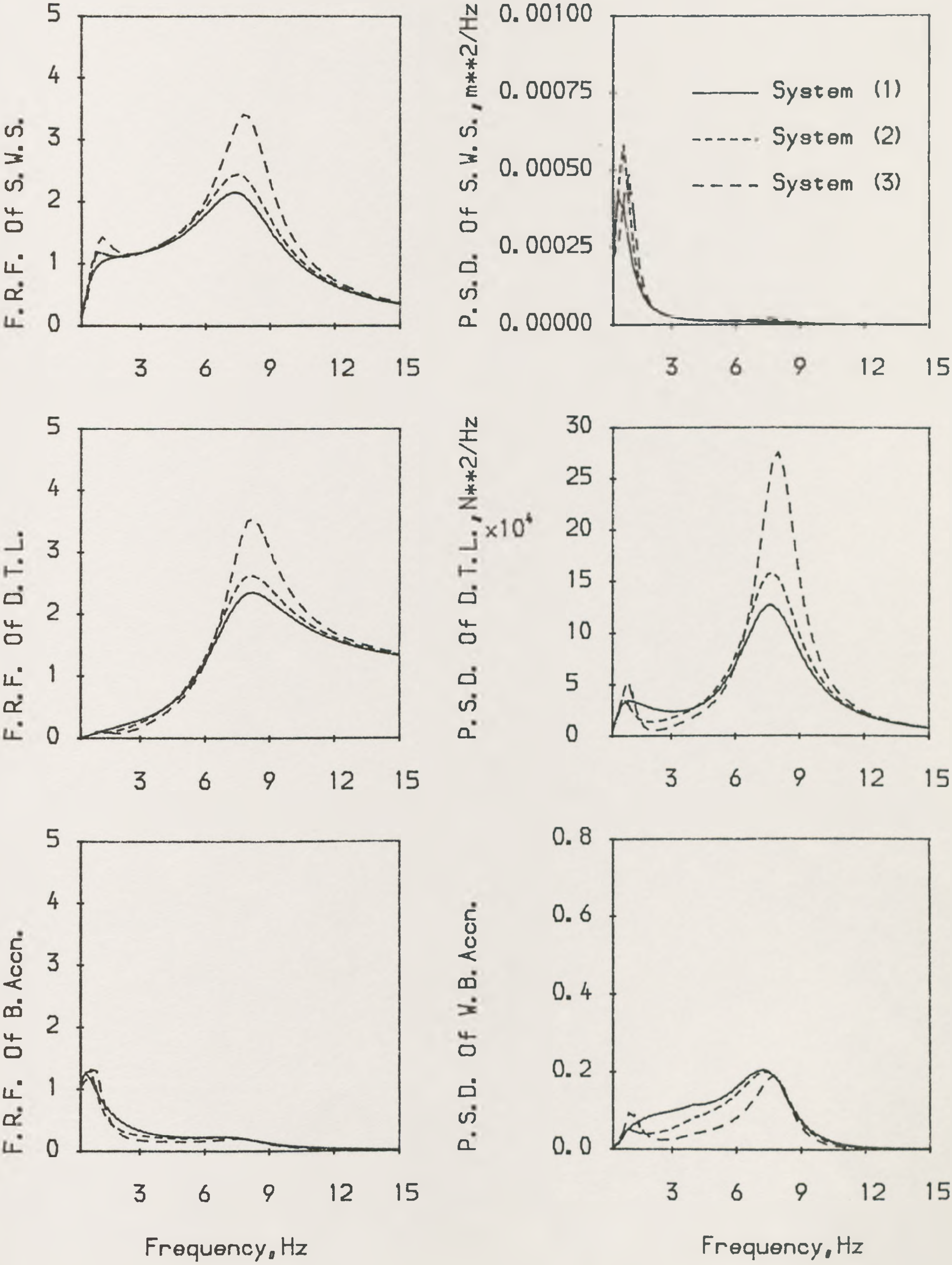

Fig. 7.9 Frequency responses and output mean square spectral densities of slow-active systems, S W S = 2.5 cm

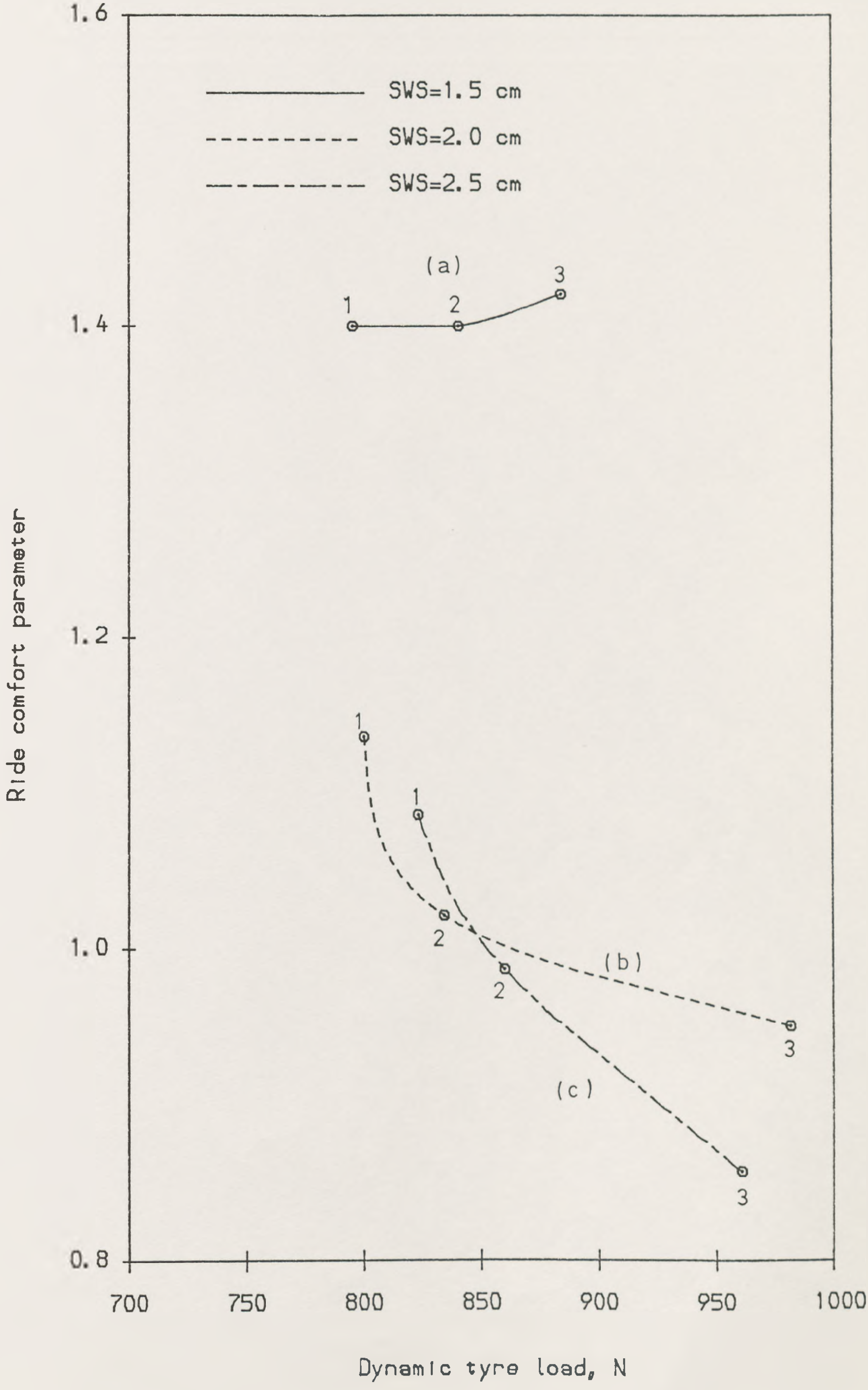

Fig. 7.10 Ride comfort and dynamic tyre load variation of slow-active systems

dynamic tyre load near to the wheel resonance where difficulties of controlling the wheel hop motion may arise particularly when unbalanced wheels are used. Improvement in the ride comfort parameter accompanied by worse dynamic tyre load control is found in group (b) as a result of decreasing the damping coefficient and in group (c) as a result of decreasing the damping coefficient and increasing the spring stiffness. The frequency response plots for these groups, figures 7.8 and 7.9, show that the higher peaks of the wheel resonance in systems 3(b) and 3(c) are responsible for the higher r.m.s. values of the dynamic tyre load variations, implying that the light damping in these systems causes big problems to the dynamic tyre load control even if stiff springs are used.

The possible advantages of using an adaptable suspension damper may be judged to some extent by making comparisons between some of these results. Systems 1(b), 2(b), and 3(b) in table 7.1, in which a spring of stiffness 10000 N/m is used, show that a variety of systems with different values of ride comfort parameter and dynamic tyre load variations can be obtained for the same suspension working space requirements (2.0 cm in these cases) by using the damping coefficients 1260, 950 and 630 Ns/m. It can also be seen that by using the same spring stiffness with each of the damping coefficients above, systems 2(a), 2(b) and 3(c) are obtained needing 1.5, 2.0 and 2.5 cm r.m.s. values of the suspension working space, which suggest that very good performance under a variety of operating conditions could be obtained from systems which combine the slow actuator in series with a spring of fixed rate with an adjustable damper.

A comparison of the frequency response functions and the mean square spectral density functions between a conventional passive system and two slow-active systems, is made in Fig. 7.11, in order to identify, in terms of the system performances, the benefits of introducing the limited bandwidth actuator into the existing passive system. As can be seen from the figure, significant reductions in the spectral density functions of the weighted body acceleration, the dynamic tyre load and the suspension working space at the body resonance peaks have been achieved without change at the wheel resonance. Percentage reductions of 13%, 8% and 30% in the r.m.s. values of the ride comfort parameter, the dynamic tyre load variations and the suspension working space for the first slow-active system are obtained and corresponding reductions are 3%, 7% and 39% for the second one as compared with the passive system. Although these comparisons involve only one road surface quality and vehicle speed and involve systems which differ in respect of all three performance parameters, it is clear that, since they all improve in going from passive to active that the slow-active systems have some potential. If the active system parameters were altered to provide comparisons on an equal working space basis, the advantages in respect of ride comfort and dynamic tyre load would be greater than those above. Further discussion of these results and their implications appears in chapter 8.

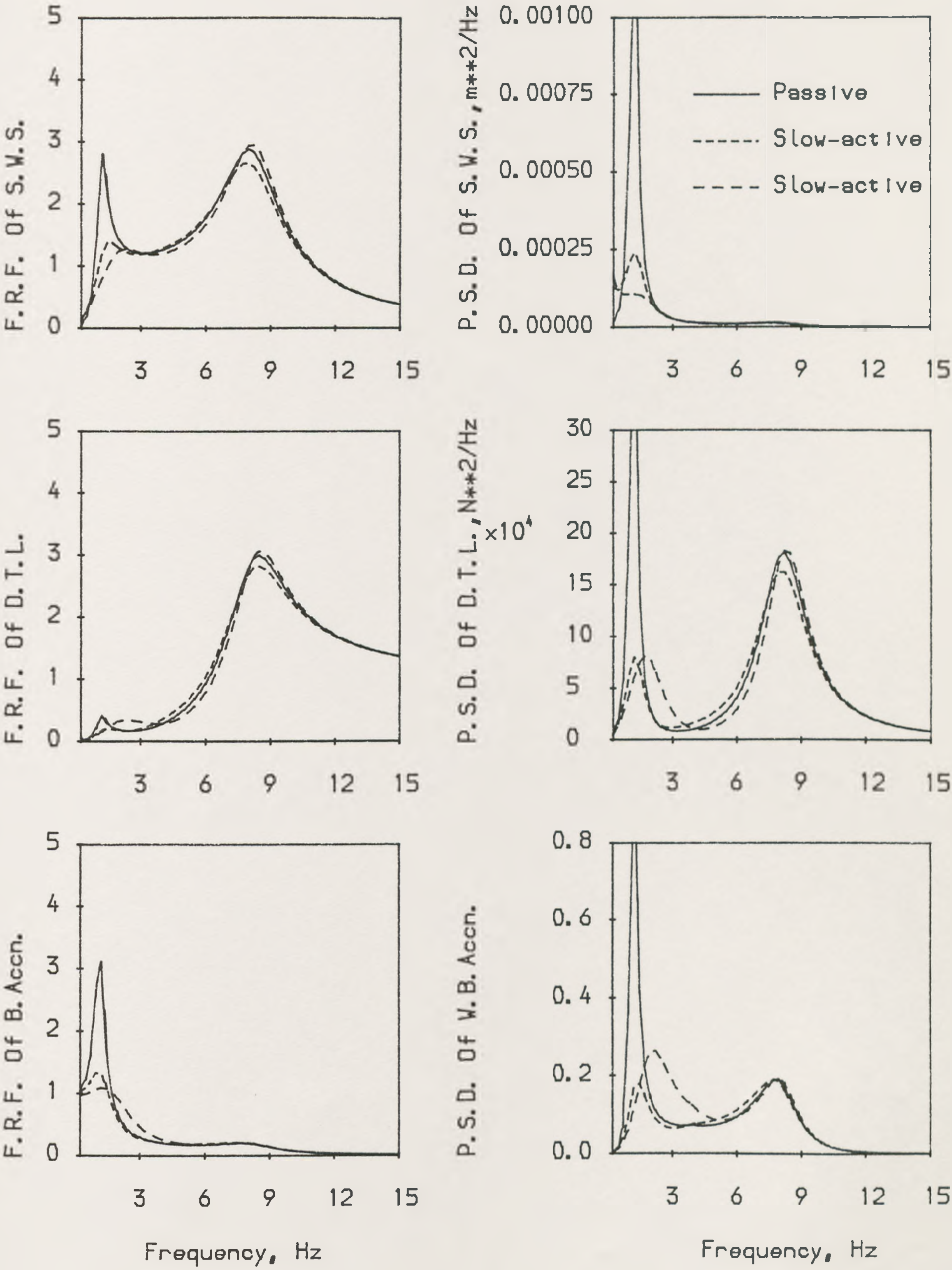

Fig. 7.11 Frequency responses and output mean square spectral densities of slow-active and passive systems

CHAPTER(8)

DISCUSSION OF RESULTS

The results presented relate directly to the problem of designing a vehicle suspension system subjected to specific operating conditions (road surface quality and vehicle speed). These results show the performance and design properties of passive, active, semi-active and slow-active suspension systems as generated in chapters 4, 5, 6 and 7 respectively. The passive suspension results include systems with 0.125 and 0.200 wheel to body mass ratios representing two different types of vehicle and systems each having a dynamic absorber mass added to the wheel mass. Two types of optimal active suspension systems, based on full and limited state feedback control laws, and three different types of semi-active suspension system, based on full and limited state feedback control laws with and without a passive damper, are included. Slow-active suspension systems with limited bandwidth actuator (4 Hz) are also covered. In each of these types, the system performance has been assessed in terms of ride comfort, dynamic tyre load and suspension working space parameters and the results have been plotted as r.m.s. values of the ride comfort against dynamic tyre load in figures 8.1, 8.2 and 8.3 for 2.5, 2.0 and 1.5 cm suspension working space respectively.

The responses of all the systems, even the semi-active ones, are proportional to the inputs, so that the performance parameters of any of the systems studied for smoother or rougher

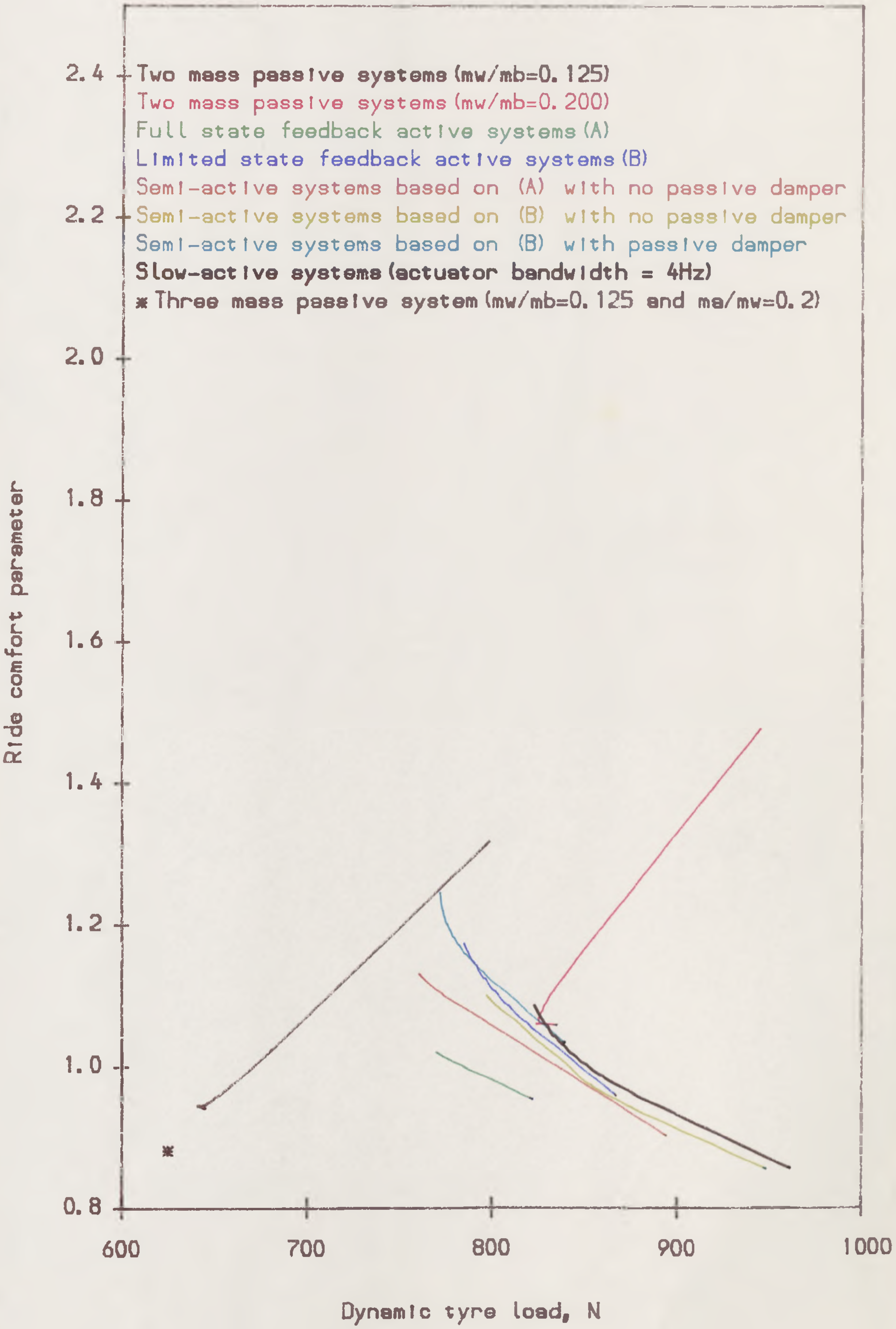

Fig. 8.1 Ride comfort and dynamic tyre load variation of different suspension systems, S W S = 2.5 cm

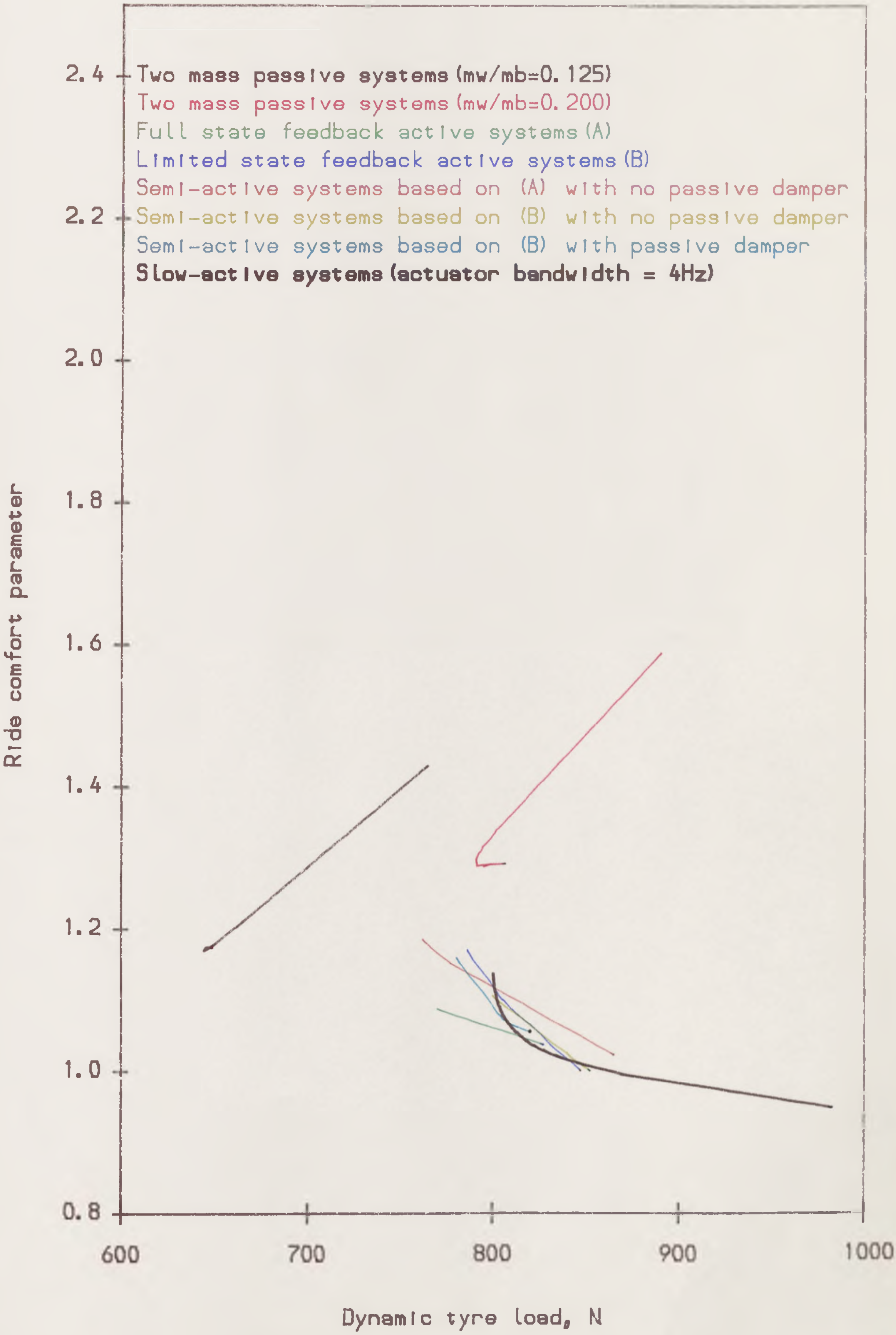

Fig. 8.2 Ride comfort and dynamic tyre load variation of different suspension systems, S W S = 2.0 cm

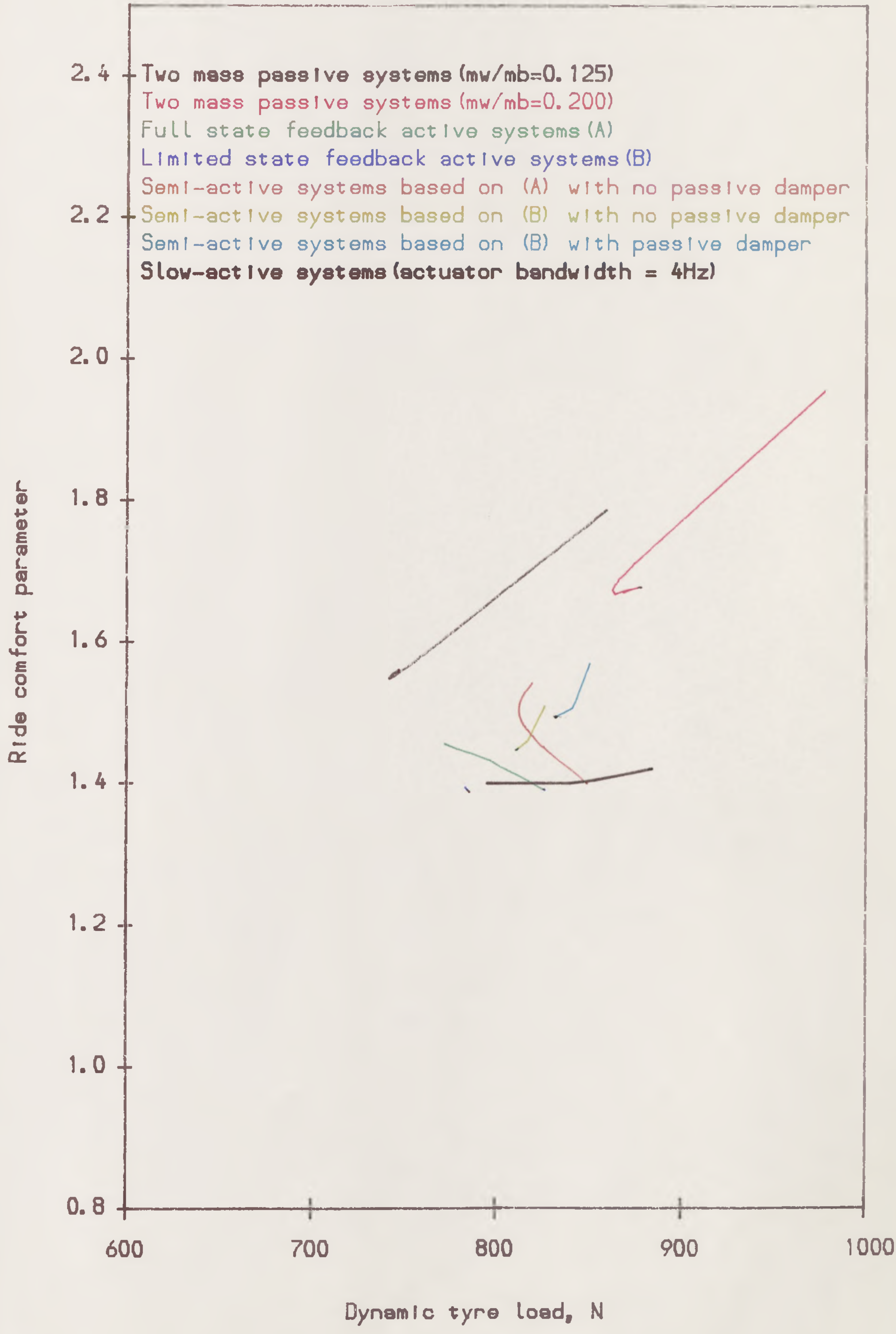

Fig. 8.3 Ride comfort and dynamic tyre load variation of different suspension systems, S W S = 1.5 cm

road surface/vehicle speed combinations than that assumed in equation 3.2 are scaled down or scaled up versions of those plotted in figures 8.1, 8.2 and 8.3. For example, if the effective road roughness constant Bl in equation 3.2, describing mean square values, were multiplied by 4, all the performance parameters would be multiplied by 2, since they are r.m.s. values. This includes the suspension working space parameter, so that the standard working spaces for which results are plotted would be 5, 4 and 3 cm for the new conditions.

The results in figures 8.1, 8.2 and 8.3 show that the benefits which can be gained by reducing the wheel mass of the passive system or by using the feedback controlled suspension elements (actuators or semi-active dampers), and the relative performances of these systems to be strongly dependent on the suspension working space available. Since the results scale in the manner described, the significant parameter is the ratio of the r.m.s. working space to $\sqrt{B1U^{1.5}}$ in which Bl is the road roughness spectral density constant and U is the vehicle speed. This ratio has the values 1.49, 1.19, and 0.895 for figures 8.1, 8.2 and 8.3 respectively.

At the high end of that range, Fig. 8.1, the passive suspension systems with 0.125 wheel to body mass ratio perform better than all the other types of system, having wheel to body mass ratio of 0.2, and further performance improvements are gained by connecting a dynamic absorber of 0.2 absorber mass to wheel mass ratio to the wheel assembly. The "best" passive system with 0.2 wheel to body mass ratio (system 3(c) in table 4.1) is found to perform as well as the active, semi-active and slow-active suspension systems, the best of which requires a vehicle body mounted height sensor and will give a 10% improvement in ride comfort with some gain in "road-holding". The semi-active systems perform nearly as well as the active systems on which they are based, and the simulation time histories in chapter 6 show that they rarely switch the damper off. The performance properties of the slow-active suspension systems are very close to those obtained from the limited state feedback active systems implying that the passive suspension elements (springs and dampers) are adequate for controlling the higher frequency components of the system response.

Moving towards the other end of the range, the differences between passive, active, semi-active and slow-active systems increase while the performance properties of the two passive systems (with different mass ratios) come closer to each other. The value of the height sensor decreases since the limited state feedback systems become as good as those with full state feedback. The addition of the passive damper to the semi-active suspension systems seems to be of no value in terms of the system performance, and the idea should be rejected altogether because of space and cost considerations.

Static deflections occurring as a result of varying the payload may be too great for the softer passive and semi-active suspension systems to be practicable without self-levelling, while attitude changes in manoeuvring and deterioration of the vehicle handling dynamics by encouragement of vehicle roll in cornering may also be excessive with the least stiff systems. An interesting and possibly useful feature of the active, semi-active and some of the slow-active systems, which can be deduced from tables 5.1, 5.2, 6.1, 6.2, 6.3 and 7.1, is that for any given requirement for working space under specific running conditions, they are much stiffer statically than corresponding passive systems (tables 4.1 and 4.2) giving good ride comfort. Thus it may well be appropriate to compare the performances of semi-active and slow-active suspension systems with passive systems having substantially greater stiffness and substantially less damping than those which perform best according to figures 8.1, 8.2 and 8.3, since the "best performing" passive systems will often be impracticable because of their low stiffness. The active systems can, in principle, be made self-levelling, and be made to adjust the control laws in manoeuvring and hard braking to obtain low steady state pitch and roll angles and good handling.

Almost invariably in reality an automobile suspension system must be designed to operate over a wide range of road surface roughnesses. The data obtained from Robson (1979) show that the ratio of worst to best mean square spectral density constants, B1, for European roads, to be 1000. This variation is compensated by the reasonable expectation that the highest vehicle speeds will not be needed on the worst road surfaces and the best roads are probably so good that vehicle speeds much higher than those possible now would be needed to give a suspension problem of any significance when traversing them. Bearing these things in mind, it seems reasonable to suppose that a road vehicle suspension system should operate effectively over a 30 to 1 range of effective mean square values, and the limitations of fixed parameter passive systems, slow-active systems, semi-active systems with fixed spring stiffness, and of active systems which employ fixed characteristic passive elements in their realisation become very apparent when figures 4.1, 4.2, 5.1, 5.2, 6.1, 6.2, 6.3, and 7.1 are examined. It is clear that obtaining good performance for a given road surface but for different suspension working spaces requires wide variations in suspension parameters, and obtaining good performance for many different road surfaces with a fixed working space is an exactly equivalent problem. Thus the major weakness of systems which contain suspension elements of fixed parameters (springs and dampers) is that they can be ideal for only one of the many different conditions under which they operate, and will probably be far from ideal in conditions differing much from this one. It is anticipated however that systems with adjustable elements will be capable, with a suitable control stratagem, of performing well in many different running conditions.

Among existing practical systems are some with adjustable suspension elements. Some of these systems are included in the review by Goodall and Kortum (1983) and descriptions of others have appeared later. The adjustable rate damper has been standard technology for some time. Remote adjustment of the damping electrically or pneumatically has been shown to be feasible [Yokoya, Asami, Hamajima, and Nakashima (1984) and Mizuguchi, Suda, Chikamori, and Koboyashi (1984)], and in either case it can be controlled by microprocessor. Changes in stiffness appear more difficult to realise than changes in damping. Stepwise changes can be effected by using valves to add volume to an air spring system or by using a steel spring in parallel with a controllable air spring as described by Mizuguchi, Suda, Chikamori, and Koboyashi (1984). A limited bandwidth active suspension system (slow-active system) is available from Automotive Products Ltd, a component manufacturer. This system has the capability of improving the ride comfort as well as controlling the attitude changes in cornering and braking through the use of mechanically controlled hydropneumatic struts, see Goodall and Kortum (1983). The electronically controlled hydraulic damper (semi-active suspension system) has been tested and promoted by Lucas Aerospace Ltd. It has been used for improving ride comfort by using the error signal generated by comparing the actual vehicle´s motions with corresponding signals from an ideal vehicle, existing in mathematical model form only, to control the hydraulic suspension damper, see Goodall and Kortum (1983). A fully active suspension system has been developed by Lotus Cars Ltd. In this system servo-valve controlled hydraulic actuators take the place of the conventional springs and dampers, and inputs to the servo-valves derive from an elaborate measurement and signal processing system, Wright and Williams (1984).

It can be seen from the results generated in the appendix that a suspension system incorporating slowly variable damping will be no different from a conventional fixed parameter system in respect of its requirements for spring stiffness. The damping should be adjustable over a range from a little less than conventional to perhaps two and half times conventional, and judicious adjustment will give much improved ride comfort on rough roads, a gain in high speed stability, and improvements in transient handling responses, but otherwise will give conventional performance. A suspension which allows switching between a conventional spring rate and half of that (with self-levelling) in which a variable rate damper is employed, offers 25 to 30% improvement in ride comfort under normal conditions with greater gains on rough roads (compared with convention). Increasing the range of stiffness variation possible will provide only small additional benefits. The slow-active suspension results show that if the 4 Hz bandwidth actuator were to be electronically controlled, by measuring and processing the body and the wheel displacements and velocities, good performance gains would be achievable particularly for a vehicle running on rough roads. However, further work is required for precisely determining these gains for vehicles running on smooth roads. The passive spring in series with the actuator may be chosen softer than the conventional one since the system is inherently capable of self-levelling and dealing effectively with the handling problems associated with soft springs.

CHAPTER (9)

CONCLUSIONS

The fundamental conflict between ride comfort and suspension working space requirements is the main problem in passive suspension system design. It is well known that the best comfort is obtainable by the use of very soft springs ~~with little damping~~ which would require impractically large suspension working space for travelling at normal speeds on normal roads, and would suffer unacceptablly large riding height changes with load changes, and attitude changes in manoeuvring and hard braking. The suspension working space available is therefore the starting point for a design, and best performances are obtained at values of suspension spring stiffness at which riding height changes with loading in normal circumstances use up a very significant proportion of the available suspension working space. This last factor is liable to dictate the choice of a spring rate higher than that for optimum performance accompanied by lower damping than that of the best dynamic system. Self-levelling removes the riding height change constraint, and is clearly particularly valuable for high ratios of laden to unladen weight. With the introduction of self-levelling, a reduction in spring stiffness and an increase in damping will normally bring substantial improvements in comfort and tyre load control for a given suspension working space.

Slight performance gains in ride comfort and somewhat more in dynamic tyre load control can be obtained by reducing the unsprung mass, but in view of the practical difficulty of doing so to any substantial degree beyond the point at which contemporary vehicles stand, the benefits do not appear sufficient to warrant extreme efforts in this direction. Although some performance gains in both ride comfort and dynamic tyre load control are achievable by adding a dynamic absorber to the unsprung mass, these benefits become very small when a practically dimensioned one is used. This limits the cost/effectiveness of such systems.

The performance limitations inherent in using conventional fixed parameter passive systems in vehicles running on different road qualities* can be to some extent overcome by the use of adjustable passive elements. Substantial improvements in ride comfort and in dynamic tyre load control, as compared with convention, can be achieved for a vehicle running on rough roads if the damping coefficient is increased to perhaps two and a half times conventional with a fixed spring of conventional stiffness. Again in comparison with a conventional system, roughly 30% improvement in ride comfort can be gained in normal running conditions if the spring stiffness is switched to a half of the conventional one (with self-levelling) with slightly less than conventional damping, with greater gains on rough roads if the damping is set to two and a half times convention.

Active suspension systems, employing actuators in place of the conventional suspension elements, are capable of generating forces intelligently by which performance gains over conventional

* This work on different surfaces is described in the appendix.

systems are obtainable. As the suspension working space becomes more restricted, the performance advantages of active over passive suspension systems increase, and it becomes very valuable if only a very limited suspension working space is available. Obtaining the best performance from any particular type of suspension over a variety of operating conditions, involving road roughness and vehicle speed variations and a fixed working space, requires the adjustment of suspension parameters through wide ranges. Fully active systems are capable, in principle, of such adjustment, and through avoiding the need to compromise can be made much better than fixed parameter passive systems. In addition to the performance gains, the active devices, with suitable measurements, can be used effectively for controlling the attitude changes due to static load changes and in manoeuvring, accelerating and braking. There are no big performance differences between full and limited state feedback active suspension systems which suggests that the value of the vehicle body to road surface distance sensor is not great, unless it can be used to preview the road surface.

Semi-active suspension systems employing passive springs to support the body mass and rapidly variable dampers can perform, if suitable control laws are used, better than fixed parameter passive systems. Semi-active suspension systems can be made, with some practical difficulties, to perform very well in all operating conditions, like fully active ones, if the semi-active damper is accompanied by a widely variable spring stiffness. In view of the relatively high static stiffness of the best performing semi-active systems, it may also be possible to obtain good performance over a wide variety of operating conditions with such types having fixed spring rates. Appropriate results are needed for evaluating the performance gains of such systems, with regard to the many different running conditions to which they are subjected.

With a limited bandwidth actuator fitted in series with a passive spring, and a passive damper between the body and wheel masses (slow-active suspension system), remarkable performance gains in ride comfort, dynamic tyre load control and suspension working space can be achieved, as compared with passive arrangements, if the actuator bandwidth is limited to 4 Hz, and no significant gains occur beyond this limit. The performance improvements are also dependent on the suspension working space available with the highest improvements at the smallest working space available following the same pattern as obtained in respect of active and semi-active systems (see figures 8.1, 8.2 and 8.3). If the 4 Hz bandwidth limit actuator is used with a conventional spring and damper, typical performance improvements of 13%, 8% and 30% in the r.m.s. values of ride comfort, dynamic tyre load and suspension working space parameters can be made, and clearly greater ride comfort improvements could be obtained if both systems were designed to have the same suspension working space. It can be anticipated that the use of an adaptable damper will help in obtaining good performances over many running conditions, but appropriate results to confirm the actual gains have not yet been generated.

It can be expected that fully active suspension systems will be very expensive as compared with the other systems because of the high frequency response servo-hydraulic actuators needed. Since recent advances in electronic technology make the measuring and processing instrumentation quite cheap, the entire cost of the suspension system will be tolerable if cheaper suspension elements are available. The adjustable damper passive system has the opportunity in terms of cost/effectiveness because of the relatively minor extensions to existing technology needed for its realisation. It is anticipated that greater capital and running costs will be incurred if the adjustable spring rate is added. The semi-active suspension system is an attractive alternative since no power supply is needed for its operation. Slow-active suspension systems are also expected to be economically viable because of the relatively small and inexpensive actuators needed and the relatively low power source required for their operation.

In view of the results presented, further attention may be needed to some unresolved points. Additional understanding will come from these new results. The points of interest include;

1- The use of a fixed conventional spring in conjunction with the semi-active damper seems to be of valuable practical interest because of the simple modifications needed in respect of the existing dampers. The actual performance gains and the suitable laws required for controlling the dynamics of such systems over the wide range of running conditions to which they are subjected are not yet known.

2- The use of a variable rate damper in parallel with a limited bandwidth actuator. The actuator in series with a fixed rate conventional spring with a controllable damper may provide excellent performance and reasonable costs. Suitable results are needed for evaluating the performance gains and the control policy required for controlling such systems.

3- The use of a servo-motor as a limited bandwidth actuator in the slow-active suspension system. More studies are likely to be required for determining the motor size, weight and cost and for evaluating the power consumption required for its operation.

4- The justification of the theory used in studying the simple quarter car models to be applicable to more complicated models. Further theoretical studies are required for understanding the behaviour of the two sided and full car models.

5- The execution of laboratory work in order to assist the theoretical results obtained, to practically test the actual performance ability of these systems, to confirm ideas about the specifications of the hardware required in their realisation, and to provide guidance in the design of prototype vehicles.

NOMENCLATURE

Symbol	Description
A, $\tilde{A}$, B, C, CS, F, K, KS, M, MS, P, P1, Q, R, S, V, Wc, Wo, ZS	matrices defined in the text
B1, B2	road surface roughness constants
Ca	dynamic absorber damping coefficient
Cs	suspension damper coefficient
Da	mean square spectral density of body acceleration
Dd	mean square spectral density of dynamic tyre load
Dr	mean square spectral density of the road roughness displacement input
Ds	mean square spectral density of suspension working space
DTL	r.m.s. value of dynamic tyre load
E	expected value of
Ha	frequency response function of body acceleration
Hd	frequency response function of dynamic tyre load variations
Hs	frequency response function of suspension working space
J	performance index
Ka	dynamic absorber spring stiffness

Kf1,Kf2,Kf3,Kf4	full state feedback gains
Kl1,Kl2,Kl3,Kl4	limited state feedback gains
Ks	suspension spring stiffness
Ks1,Ks2,Ks3,Ks4	slow-active feedback gains
Kt	tyre spring stiffness
RCP	r.m.s. value of weighted body acceleration
SWS	r.m.s. value of suspension working space
U	vehicle speed
$\underline{X}$	output amplitude vector
$\underline{Y}$	input amplitude vector
Y(t)	single road profile displacement
b.accn.	time history of body acceleration
d.t.l.	time history of dynamic tyre load
df	frequency interval
dt	time interval
f	frequency, Hz
fn	uncoupled natural frequency
g(t)	vector of measurable states
k	number of time samples
l	number of sine waves
m	number of measurable states
ma	dynamic absorber mass
mb	car model body mass
mw	car model wheel mass
n	system order
n1,n2	constants defining the slope of the road displacement spectrum
p	number of control inputs

q	number of controlled variables
r	number of degrees of freedom
s	Laplace operator
s.w.s.	time history of suspension working space
t, τ	time
u(t)	control vector
$u'(t)$	filtered control vector
v(t)	white noise input vector
wf	cut-off frequency
x(t)	state vector
xs(t)	transformed state vector
y(t)	sinusoidal input vector
z(t)	controlled state vector
α	filter constant
γ	damping as a proportion of critical
δ	Dirac delta function
ψ	phase angle
ν	wave number
$\xi(t)$	Gaussian random variable of "white noise" form
ω	frequency, rad/s

Suffices

B.accn.	body acceleration
D.T.L.	dynamic tyre load
F.R.F.	frequency response function
P.S.D.	power spectral density

R.C.P.	ride comfort parameter
r.m.s.	root mean square value
S.A.F.	semi-active force
S.W.S.	suspension working space
W.B.Accn.	weighted body acceleration

REFERENCES

CEBON D. and NEWLAND D.E. (1983), "The artificial generation of road surface topography by the inverse FFT method.", Proceedings of the 8th International Association for Vehicle Dynamics Symposium on The Dynamics of Vehicles on Roads and on Railway Tracks 1983 ed. J. Karl Hedrick (Swets and Zeitlinger, Lisse) 29-42.

CROSBY M.J. and KARNOPP D.C. (1973), "The active damper - a new concept for shock and vibration control.", Shock and Vibration Bulletin, 43, part H.

DODDS C.J. and ROBSON J.D. (1973), "The description of road surface roughness.", Journal of Sound and Vibration, 31, 175-183.

van ELDIK THIEME H. (1981), "Mechanics of pneumatic tires.", S.K.Clark (ed) U.S. Department of Transportation, National Highway Traffic Safety Administration, Chapter 8, 546-658.

GOLUB G.H., NASH S. and van LOAN C. (1979), "A Hessenberg-Schur method for the problem ax + xb = c.", IEEE Transactions on Automatic Control, Vol.AC-24, No.6, 909-913.

GOODALL R.M. (1980), "Feasibility of using electro-mechanical actuator for an active lateral suspension.", DOS Note 248.

GOODALL R.M. and KORTUM W. (1983), "Active controls in ground transportation - a review of the state-of-the-art and future potential.", Vehicle System Dynamics, 12, 225-257.

GOODALL R.M., WILLIAMS R.A. and LAWTON A. (1979), "Practical applications of actively controlled suspensions to railway vehicles.", A.S.M.E. Winter Conference paper, New York, 7th Dec.

HAC A. (1985), "Suspension optimisation of a 2-DOF vehicle model using a stochastic optimal control technique.", Journal of Sound and Vibration, 100, 343-357.

HEALEY A.J., NATHMAN E. and SMITH C.C. (1979), "An analytical and experimental study of automobile dynamics with random roadway inputs.", Trans. A.S.M.E., Journal of Dynamic Systems, Measurement and Control, 99, 284-292.

HEDRICK J.K. and WORMLEY D.N. (1975), "Active suspensions for ground transport vehicles - a state of the art review.", Mechanics of Transportation Suspension Systems, eds B. Paul et al, A.S.M.E., AMD-Vol 15, 21-40.

HOOKER R.J. (1979), "A model for the radial dynamic behaviour of pneumatic tyres.", International Journal of Vehicle Design, 1, 361-372.

KARNOPP D.C., CROSBY M.J. and HARWOOD R.A. (1974), "Vibration control using semi-active force generators.", Trans. A.S.M.E., Journal of Engineering for Industry, 96, 619-626.

KUO B.C. (1975), "Automatic control systems." Prentice-Hall.

KWAKERNAAK H. and SIVAN R. (1972), "Linear optimal control systems.", Wiley-Interscience.

LEVINE W.S. and ATHANS M. (1970), "On the determination of the optimal constant output feedback gains for linear multivariable systems.", IEEE Transactions on Automatic control, Vol.AC-15, No.1, 44-48.

MARGOLIS D.L. (1982-a), "The response of active and semi-active suspensions to realistic feedback signals.", Vehicle System Dynamics, 11, 267-282.

MARGOLIS D.L. (1982-b), "Semi-active heave and pitch control for ground vehicles.", Vehicle System Dynamics, 11, 31-42.

MARGOLIS D.L. (1983), "Semi-active control of wheel hop in ground vehicles.", Vehicle System Dynamics, 12, 317-330.

MEIROVITCH L. (1975), "Elements of vibration analysis.", McGraw-Hill Kogakusha, Tokyo.

MIZUGUCHI M., SUDA T., CHIKAMORA S. and KOBOYASHI K. (1984), "Chassis electronic control systems for the Mitsubishi 1984 Galant.", Society of Automotive Engineers paper 840258.

NEWLAND D.E. (1984) "An introduction to random vibrations and spectral analysis.", 2nd edition, Longman Group.

ROBSON J.D. (1979), "Road surface description and vehicle response.", International Journal of Vehicle Design, 1, 25-35

RYBA D. (1974), "Improvements in dynamic characteristics of automobile suspension systems.", Vehicle System Dynamics, 3, Part 1(17-46), Part 2(55-98).

SHARP R.S. and HASSAN S.A. (1984), "The fundamentals of passive automotive suspension system design.", Proceedings of the Society of Environmental Engineers Conference on Dynamics in Automotive Engineering, Society of Environmental Engineers, Buntingford.

SHARP R.S. and HASSAN S.A. (1985-a), "The relative performance capabilities of passive, active, and semi-active car suspension systems.", Proc. I. Mech. E., Part D Transportation, in printing.

SHARP R.S. and HASSAN S.A. (1985-b), "Performance of passive, active, and semi-active suspension systems.", A report prepared for The Ford Motor Company.

SHARP R.S. and HASSAN S.A. (1986), "An evaluation of passive automotive suspension systems with variable stiffness and damping parameters.", Vehicle System Dynamics, in refereeing.

SIRETEANU T. (1984), "The effect of sequential damping on ride comfort improvement.", Vehicle Noise and Vibration, Mech. Eng. Publications, London, 77-82.

SMITH C.C., McGEHEE D.Y. and HEALEY A.J. (1978), "The prediction of passenger riding comfort from acceleration data.", Trans. A.S.M.E., Journal of Dynamic Systems, Measurement and Control, 100, 34-41.

SUTTON H.B. (1979), "The potential for active suspensions systems.", Automotive Engineer, April/May, 21-24.

THOMPSON A.G. (1976), "An active suspension with optimal linear state feedback.", Vehicle System Dynamics, 5, 187-203.

THOMPSON A.G. (1984-a), "Optimal and sub-optimal linear active suspensions for road vehicles.", Vehicle System Dynamics, 13, 61-72.

THOMPSON A.G., DAVIS B.R. and SALZBORN F.J.M. (1984-b), "Active suspensions with vibration absorbers and optimal output feedback control.", Society of Automotive Engineers paper 841253.

VRIES E.G.M.J. (1982), "Development of measuring systems.", Delft Progress Report, 7, 171-185.

WILSON D.A., SHARP R.S. and HASSAN S.A. (1985), "The application of linear optimal control theory to the design of active automotive suspensions.", Vehicle System Dynamics, in printing.

WRIGHT P.G. and WILLIAMS D.A., "The application of active suspension to high performance road vehicles.", I. Mech. E., C239, 1984, 23-28

YOKOYA Y., ASAMI K., HAMAGIMA T. and NAKASHIMA N. (1984), "Toyota electronic modulated suspension system for the 1983 Soarer.", Society of Automotive Engineers paper 840341.

APPENDIX

ADJUSTABLE PASSIVE SUSPENSION SYSTEMS

The results presented in chapter 4 show that the chief limitation of conventional fixed parameter passive suspension systems arises from the need to compromise in the choice of parameters between the demands of smooth and rough surfaces, vehicle attitude control with load changes and manoeuvring, and high speed vehicle handling quality. The results offered here have been obtained as a result of collaboration with a car manufacturing company from which the design parameters of an existing passenger car have been provided. It should be noted however that, it is not possible to include such results in chapter 4 because they are not easily compared with the results presented in chapters 4 to 7.

For many years, it has been practically feasible to vary the suspension damping under manual control, and recently two-state dampers under automatic control (with manual over-ride) were introduced on some production cars as demonstrated by Yokoya, Asami, Hamagima and Nakashima (1984). All that is necessary in principle, is that the damper orifice sizes be externally controllable. Also for many years, pneumatic and hydropneumatic springing systems which pump the vehicle to the same riding height regardless of the load carried, and stiffen the suspension for increasing load, have been in use, but the stiffness changes, although useful, are small. More recently, systems with stiffnesses more widely variable by virtue of having steel and controllable air springs in parallel as demonstrated by Mizuguchi, Suda, Chikamori and Koboyasha (1984), were introduced. It is therefore quite reasonable at this time to consider a vehicle with passive suspension systems containing variable spring and damping elements to be commercially feasible provided its performance is sufficiently good in comparison with that of other known systems, in particular fixed parameter, conventional systems.

Using vehicle parameters deriving from a part laden current production car, and disturbance inputs describing an average main road traversed at 70 m.p.h. (31.11 m/s), an average minor road traversed at 45 m.p.h. (20 m/s), and a very rough road traversed at 30 m.p.h. (13.33 m/s), those combinations of suspension spring stiffness and suspension damping parameter which imply a particular (realistic) root mean square value of the suspension working space are identified. Then, for each of these combinations, passenger discomfort and dynamic tyre load parameters are determined, and those designs which offer the best performance for each condition (of road roughness and speed) are identified. The performance properties of these designs operating in the other two "off-design" conditions are then examined, and the rationale for conventional fixed parameter systems is considered. The performance differences between purpose designed and compromise systems are then scrutinised, and the advantage which can be obtained by varying parameters, and the extent of the required parameter variations are regarded.

The quarter car model used in generating these results is shown in Fig. A.1 with base parameter values, as indicated in the figure, which differ somewhat from those used in studying the suspension systems in the previous chapters.
The equations of motion for the model shown in Fig. A.1 can be written as,

$$
\begin{aligned}
mw\ddot{x}1(t) &= -Ks[x1(t)-x2(t)] - Cs[\dot{x}1(t)-\dot{x}2(t)] + Kt[x0(t)-x1(t)] \\
&\quad + Ct[\dot{x}0(t)-\dot{x}1(t)] \\
mb\ddot{x}2(t) &= Ks[x1(t)-x2(t)] + Cs[\dot{x}1(t)-\dot{x}2(t)] \qquad \ldots\ldots\ldots\ldots \quad A.1
\end{aligned}
$$

The linear analysis discussed in chapter 3 is applied to the system equations A.1 using the mean square spectral density function $D(f)=B1U^{1.5}/f^{2.5}$ to represent the different road surface qualities and the different vehicle speeds. Values of road constants (B1) are taken from Robson (1979) to be $0.5x10^{-6}$, $5.0x10^{-6}$ and $30x10^{-6}$ to represent the main road, the minor road and the very rough road respectively. The performance parameters representing passenger discomfort, suspension working space and tyre load fluctuations are calculated as described in chapter 3.

The working space standard deviation of particular interest is judged to be 4 cm on the following basis. When the vehicle is in its static laden condition, the wheel can travel about 8 cm relative to the body before either bump or rebound stop is contacted. Thus, if the standard deviation of the wheel to body displacement were 4 cm, bump or rebound stop contact would occur for only 4.55% of the running time according to the linear system

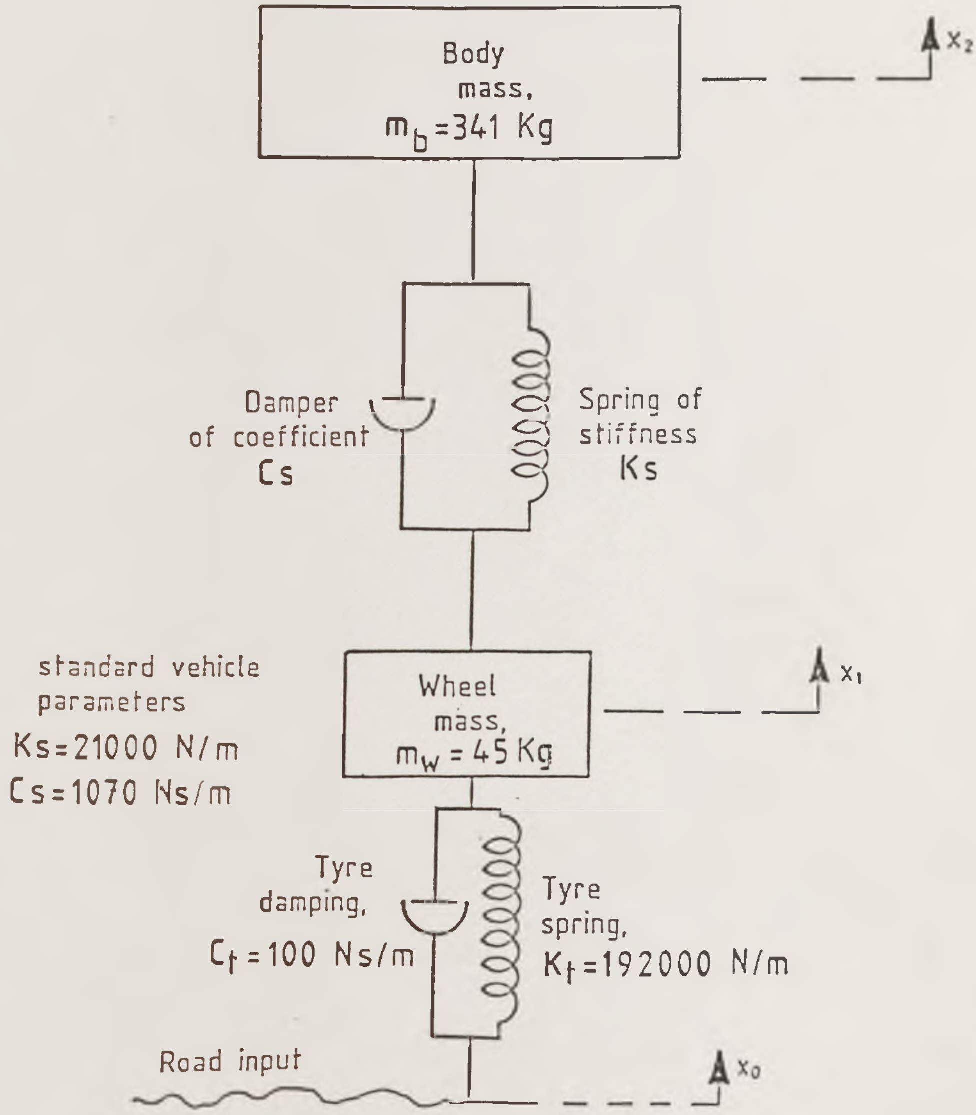

Fig. A.1 Quarter car model with base parameter values calculations and the idea that the road surface is Gaussian. With the choice of 4 cm, the full working space available will be well utilised, while the linear system calculations will remain reasonably accurate.

For ease of reference, let the condition $B1=0.5x10^{-6}$, U=31.11 m/s be called (a), the condition $B1=5.0x10^{-6}$, U=20.0 m/s be called (b) and the condition $B1=30x10^{-6}$, U=13.33 m/s be called (c). Results for condition (a) are shown in figures A.2 and A.3, those for (b) in figures A.4 and A.5 and those for (c) in figures A.6 and A.7. In each case the range of stiffness and damping values are chosen so that the results span the suspension working space of special interest, 4 cm. The performance and design parameters of those systems giving the 4 cm suspension working space value under the conditions (a), (b) and (c) are identified in table A.1. The performance properties of systems with stiffnesses of 21000 and 10500 N/m, the former representing conventional design, as functions of damping are shown in Fig. A.8.

Results relating to the systems 5(a), 4(b) and 3(c), which give what can arguably be described as the best performances under conditions (a), (b) and (c) respectively, are extracted from figures A.2 to A.8 and scaled to appear in table A.2.

The results presented relate directly to the vehicle with specific loading travelling in a substantially straight path at constant speed. In reality, variations in load and accelerating, braking and cornering occur, and these components of the

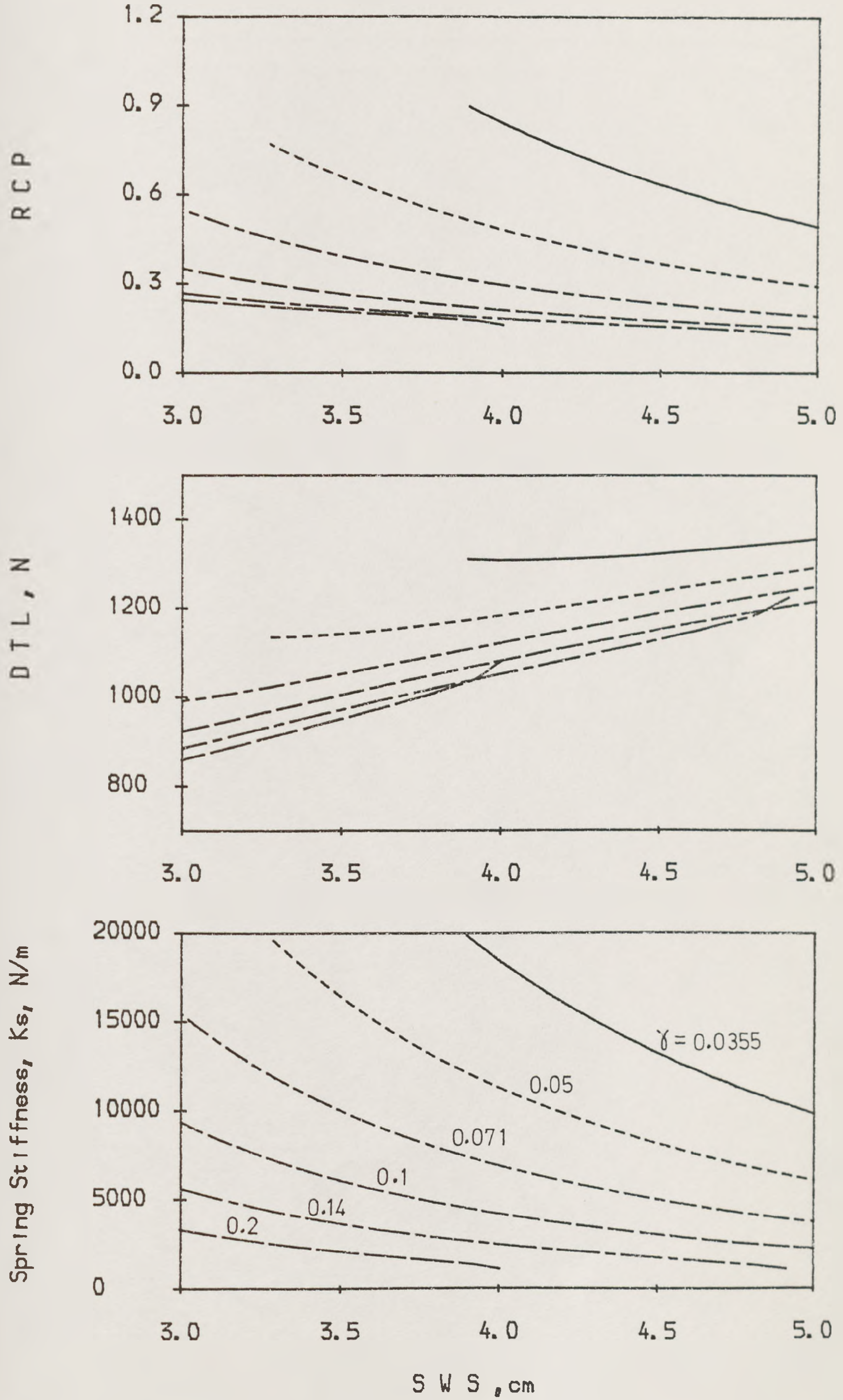

Fig. A.2 Performance properties of passive systems as functions of stiffness and damping for condition (a)

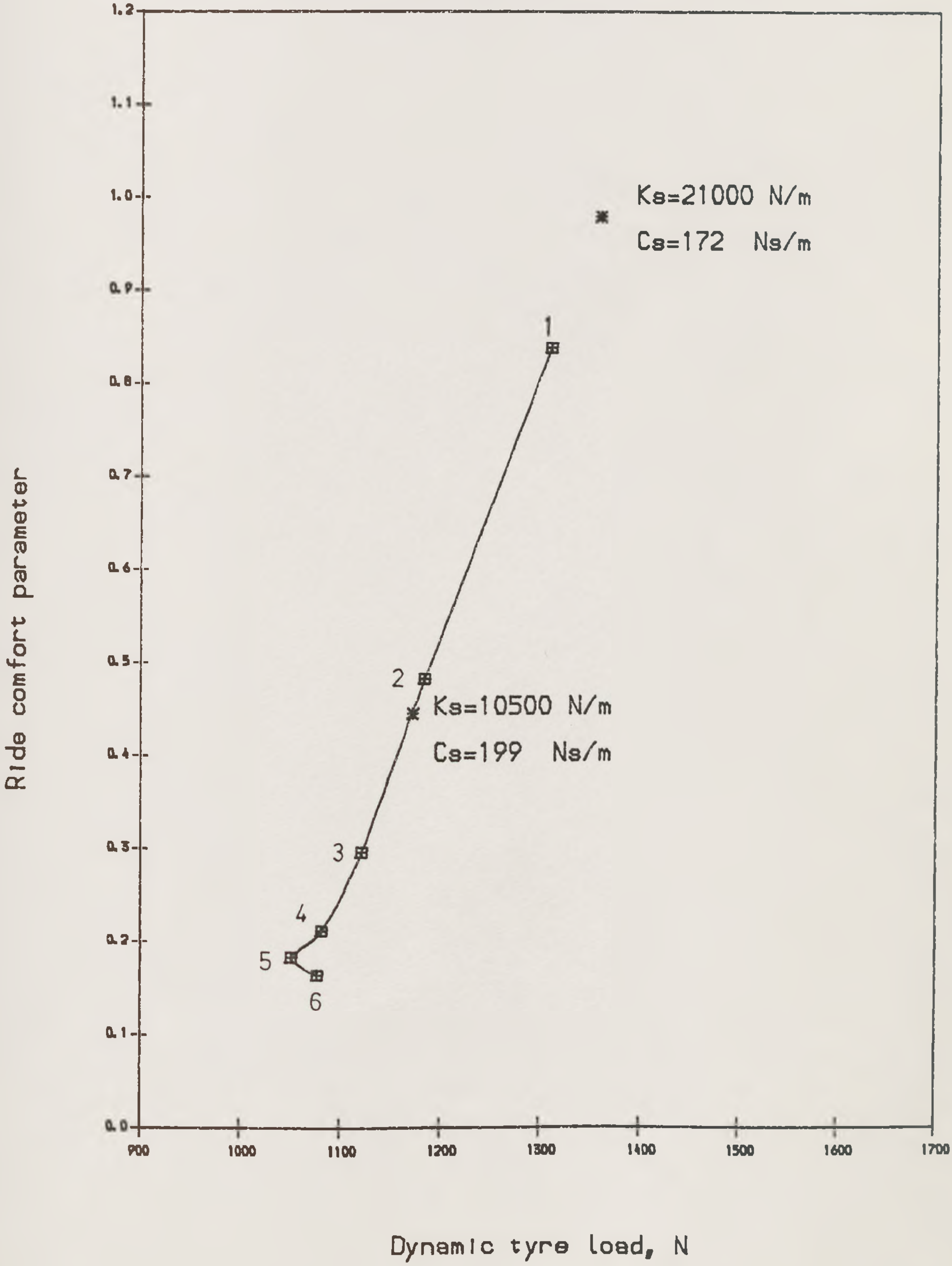

Fig. A.3 Performance and design properties of special passive systems having SWS=4 cm for condition (a)

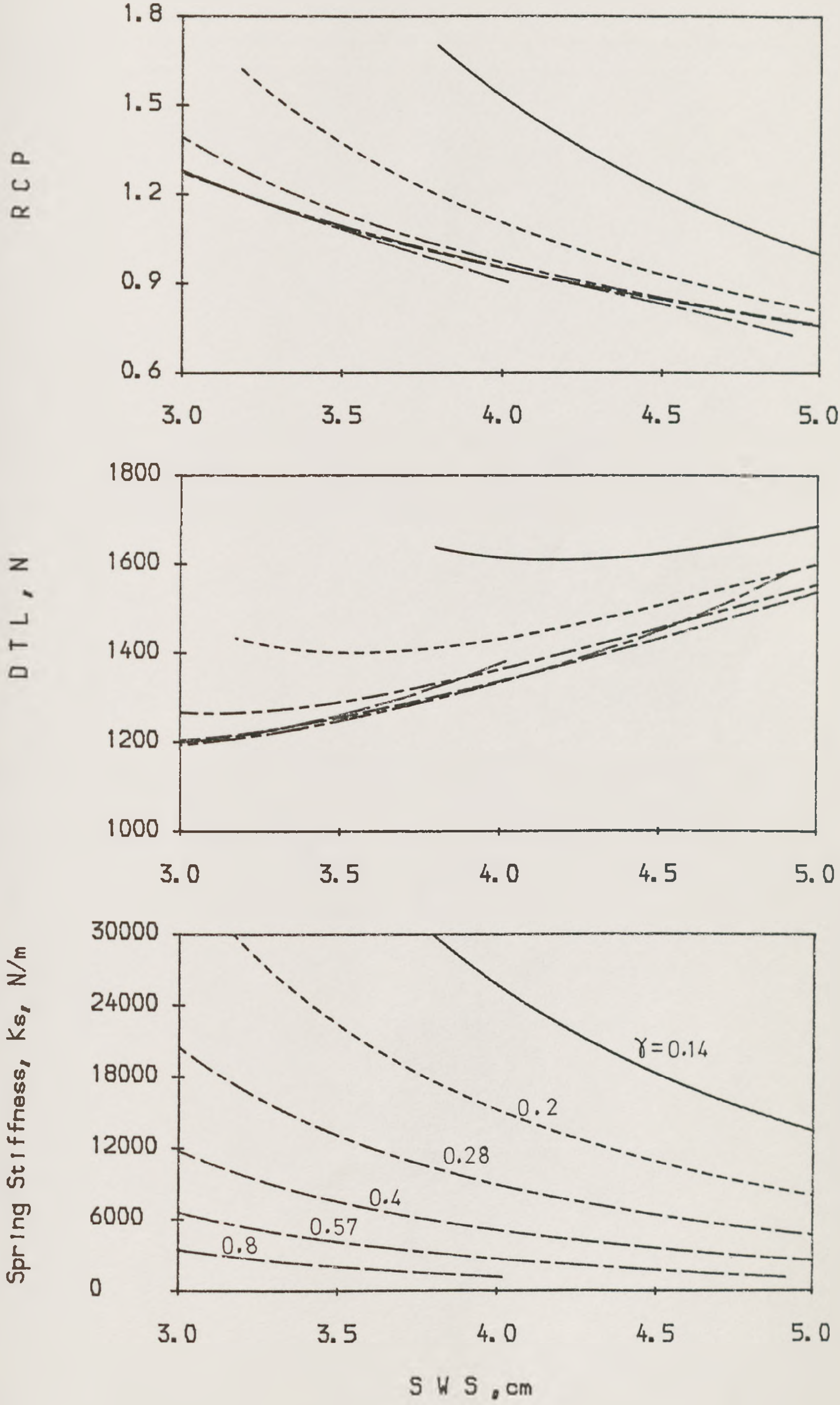

Fig. A.4 Performance properties of passive systems as functions of stiffness and damping for condition (b)

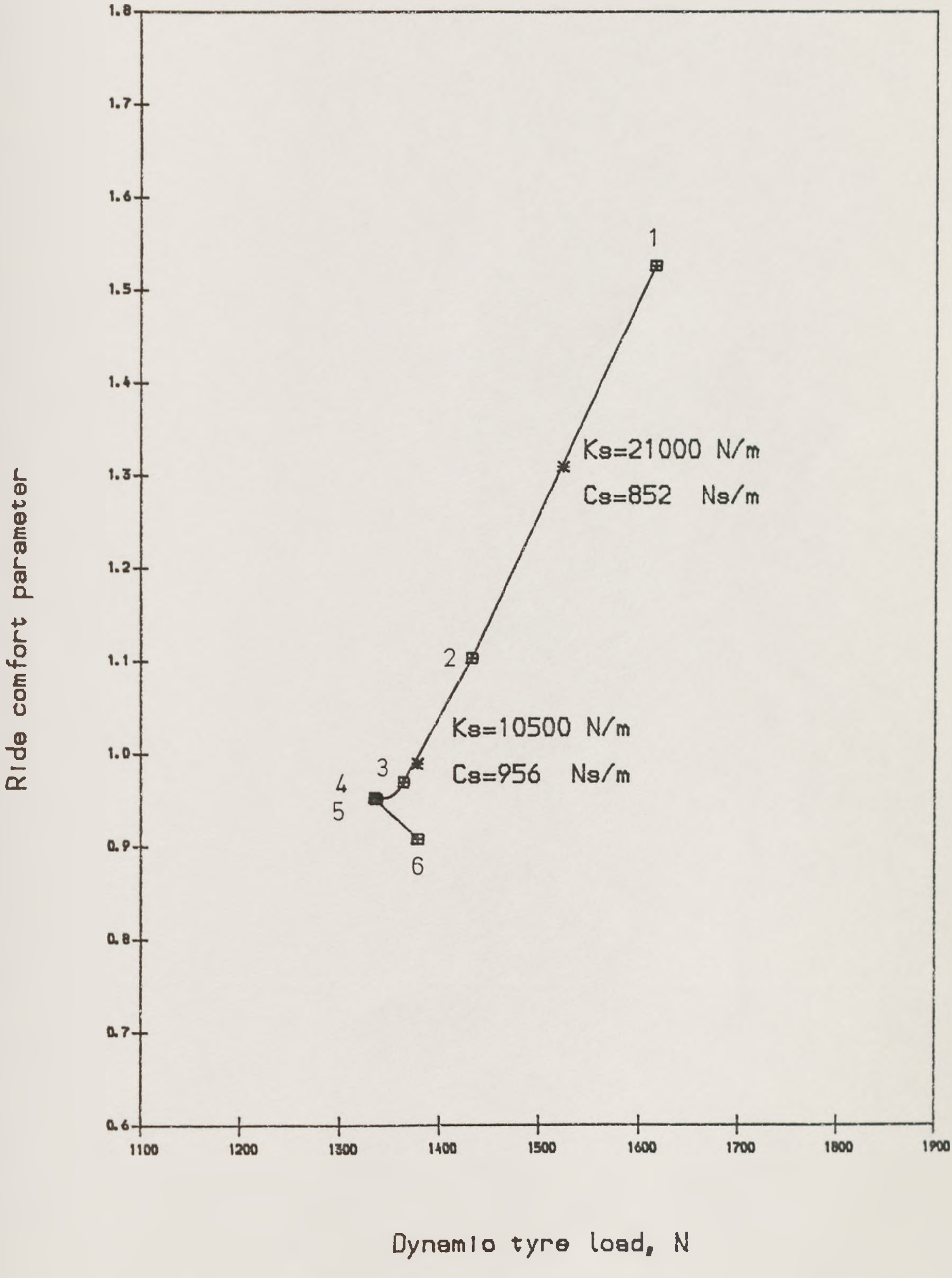

Fig. A.5 Performance and design properties of special passive systems having SWS=4 cm for condition (b)

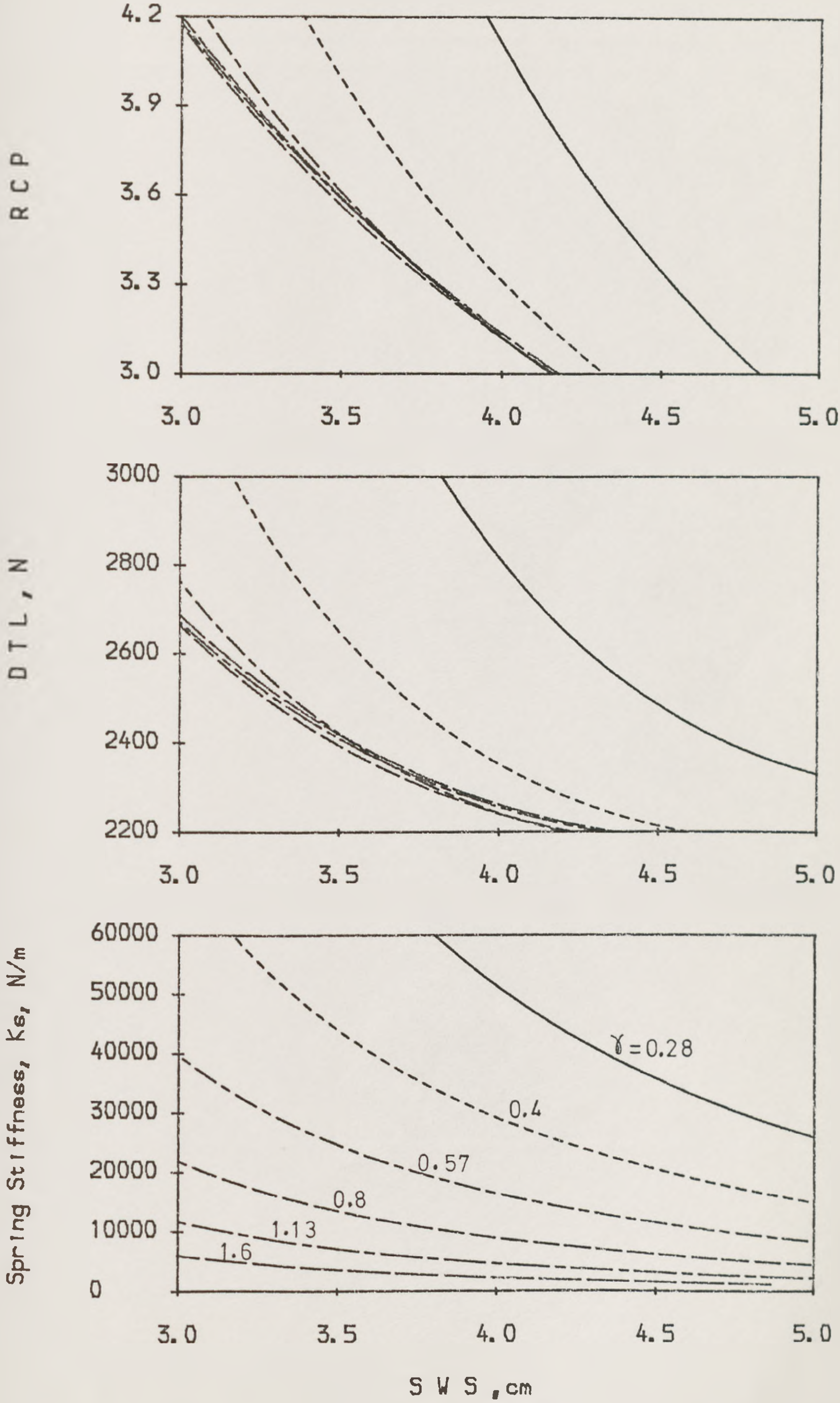

Fig. A.6 Performance properties of passive systems as functions of stiffness and damping for condition (c)

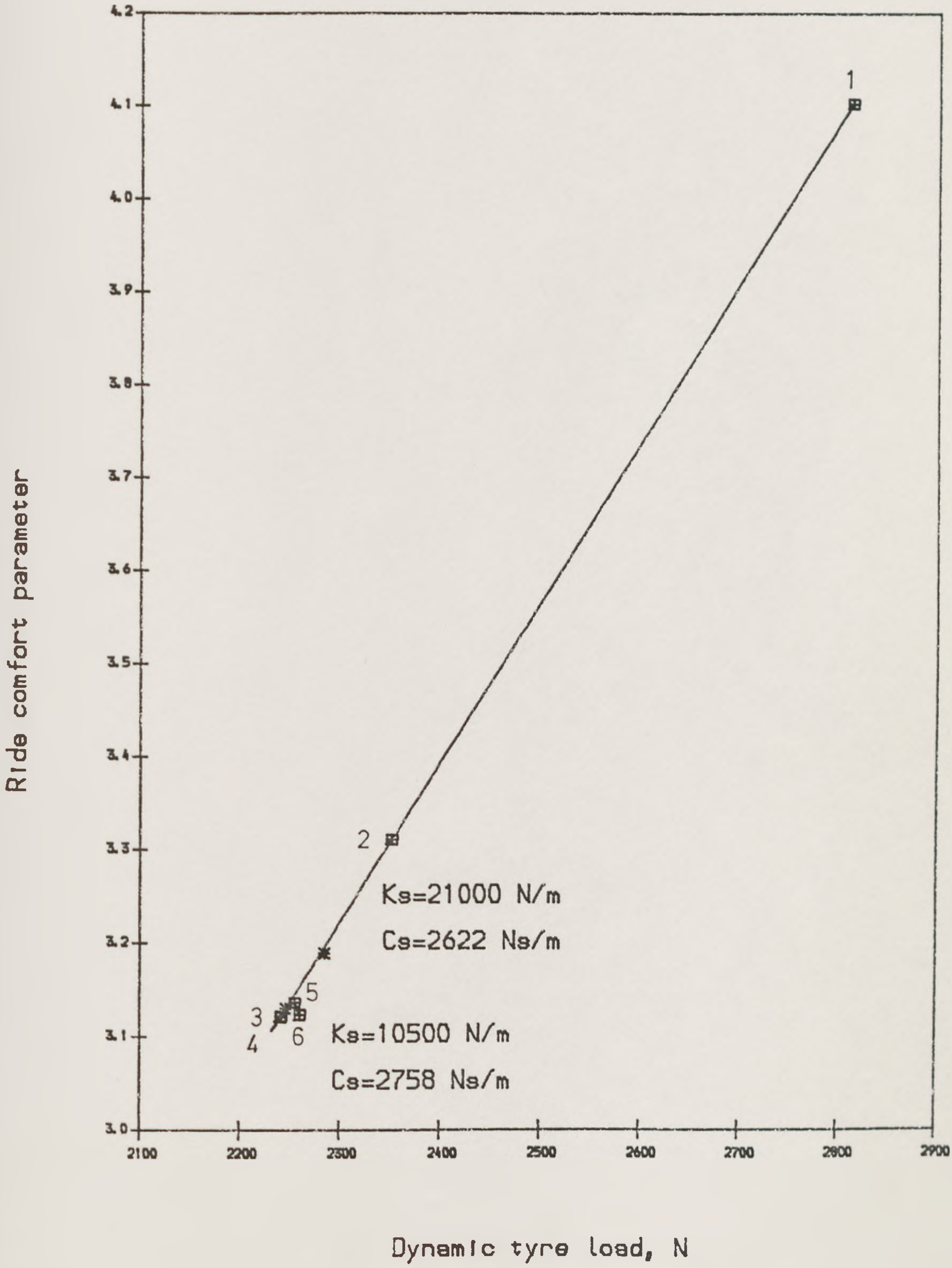

Fig. A.7 Performance and design properties of special passive systems having SWS=4 cm for condition (c)

Table A.1 Design and performance properties of particular passive systems needing 4 cm suspension working space

No	Con.	RCP	DTL N	Ks N/m	Cs Ns/m
1	(a)	0.8386	1309	18350	177
2		0.4824	1184	11286	196
3		0.2962	1122	6906	217
4		0.2119	1082	4196	239
5		0.1834	1052	2490	261
6		0.1643	1078	1132	248
1	(b)	1.5265	1615	25680	834
2		1.1038	1432	15203	911
3		0.9699	1364	8917	987
4		0.9518	1337	5094	1054
5		0.9532	1335	2697	1086
6		0.9089	1378	1172	1011
1	(c)	4.1030	2813	51207	2365
2		3.3114	2352	29167	2523
3		3.1218	2242	16419	2679
4		3.1211	2242	8968	2780
5		3.1366	2256	4654	2850
6		3.1236	2261	2304	2836

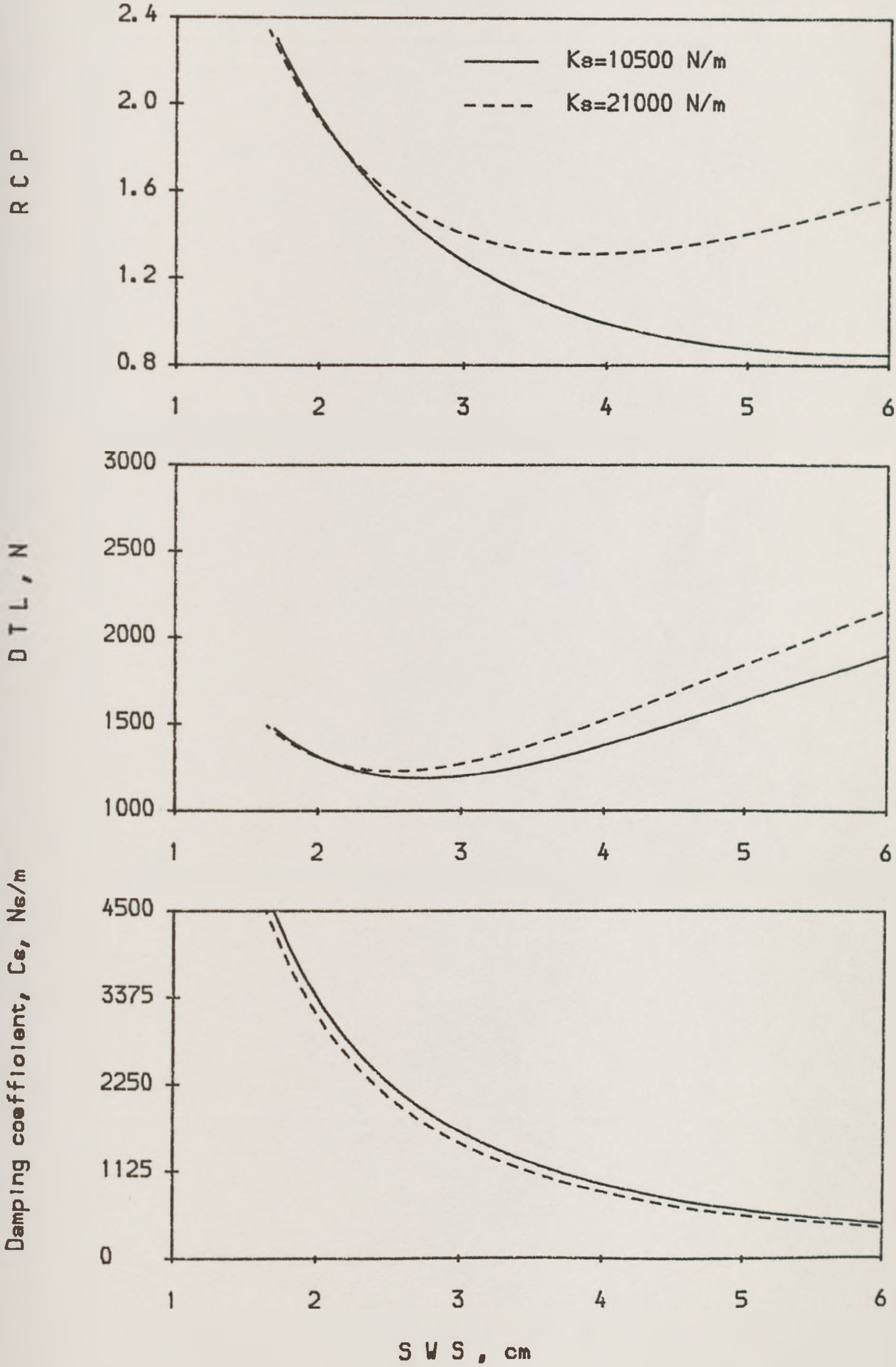

Fig. A.8 Performance properties as functions of suspension damping for passive systems having stiffness of 21000 and 10500 N/m

Table A.2 Main performance properties of passive systems of interest.

Condition / System	(a)	(b)	(c)	Parameter
5(a)	0.1834	0.4164	0.7525	RCP
	1052.0	2388.0	4316.0	DTL(N)
	4.0	9.08 *	16.4 *	SWS(cm)
4(b)	0.4190	0.9518	1.7200	RCP
	589.0	1337.0	2416.0	DTL(N)
	1.76	4.0	7.23 *	SWS(cm)
3(c)	0.7610	1.7270	3.1220	RCP
	546.0	1241.0	2242.0	DTL(N)
	0.97	2.21	4.0	SWS(cm)
Standard vehicle	0.5818	1.3210	2.3870	RCP
	616.0	1399.0	2528.0	DTL(N)
	1.573	3.571	6.45 *	SWS(cm)

* indicates a requirement for more working space than is available and the consequent inaccuracy of the calculated values of RCP AND DTL.

operational spectrum must be remembered in interpreting the results.

Consider a suspension specifically designed for condition (a). Of those systems considered the best performing has a spring stiffness of 2490 N/m (decoupled undamped natural frequency fn =0.43 Hz) and a damping factor γ = 0.14. This system (system 5(a), Fig. A.3 and table A.2) gives a ride comfort parameter RCP= 0.183 and a dynamic tyre load parameter DTL= 1052 N under the condition (a). If it were operating under condition (b), it would require much more working space between bump and rebound stops than it in fact has, in order for it to contact the stops sufficiently rarely for the linear calculations to be accurate. That is to say that the linear calculations can no longer tell us, with some precision, how the system as modelled would behave. However, the effect of hitting the stops is to some extent equivalent to stiffening the suspension spring, and as the system becomes effectively stiffer, its damping factor decreases (since Cs must increase as Ks does to maintain γ constant), and the equivalent linear system is too stiff and too lightly damped to perform well in comparison with a purpose designed system. If the system were to operate in condition (c), the same argument applies, but more strongly. Further its static stiffness is so low that even very small load changes would use up much of the suspension bump movement available (in the absence of self-levelling) and, in any event, attitude changes in manoeuvring would be excessive and high speed stability would be problematical.

For condition (b), the system with $\gamma=0.4$ and fn=0.615 Hz (system 4(b), Fig. A.5 and table A.2) performs well, having RCP=0.952 and DTL=1337 N. If this system were operating in condition (a), scaling of these results and the 4 cm working space parameter yields the conclusion that the values of RCP, DTL and SWS would be 0.419, 589 N, and 1.76 cm respectively. Compared with system 5(a), which is purpose designed for condition (a), this performance is very bad in terms of ride comfort, but good in terms of wheel load control. If system 4(b) were to be used in condition (c), the situation would be similar to that involving system 5(a) in condition (b). The limit stops would be contacted with sufficient frequency for the system to become equivalent to a linear one which is too stiff and insufficiently damped for good performance, Fig. A.5. The static stiffness of this system [4(b)] is also inadequate in respect of loading variations, manoeuvring, and handling dynamics, self-levelling alleviating only the first of these difficulties.

Near to ideal for condition (c) is a design having fn=1.1 Hz, $\gamma=0.57$ (system 3(c), Fig. A.7 and table .2) but operating under conditions (b) or (a), this system uses only a little of the working space and gives very poor comfort in comparison with the purpose designed systems. It is however a good system under all conditions from the tyre load control viewpoint.

The standard vehicle design is representative of current practice for fixed parameter systems and reference to table A.2 will reveal that it is good in terms of tyre load control, but could, in principle, be improved in terms of comfort out of all recognition under conditions (a) and (b), and significantly under condition (c) by appropriate parameter adjustments (remembering that under condition (c) the standard vehicle will use the limit stops excessively and perform much like one of the system having too stiff a spring and insufficient damping depicted in Fig. A.7). The choice of the parameters of the standard vehicle is clearly dictated by the need for attitude control, the spring stiffness being too high for best comfort performance under all conditions. Using a self-levelling system improves matters a little from a ride comfort point of view by allowing a lower spring stiffness to be employed, but this may exacerbate high speed stability and cornering attitude problems depending on other aspects of the vehicle design.

Suppose now that the design of a vehicle in which the damping can be controlled while the stiffness is fixed is being considered. The stiffness must be substantially that of a conventional system for satisfactory attitude control, while best performance would appear to result from controlling the damping in the following manner. In condition (a), the damping constant would be set at about 950 Ns/m since this gives the best comfort, Fig. A.8, notwithstanding that the full working space available is not used (SWS = 1.68 cm) in this case. In initiating a manoeuvre, as sensed by an appropriate combination of steering wheel angle and vehicle speed signals [Yokoya, Asami, Hamagima, and Nakashima (1984)], the damping constant would be raised to about 2050 Ns/m since this value gives the best wheel load control, and raising the damping would improve the steering responses. The same setting (950 Ns/m) would be employed for any road roughness up to condition (b), when the working space would be fully used. Consequently, for roughness greater than (b), the damping would be increased from 950 Ns/m as necessary to limit the working space used, since increasing the damping to limit working space is better than increasing stiffness, figures A.5 and A.7, the effective results of letting the bump and rebound stops come into play. The damping needed could be decided from continuously updated averaged wheel to body displacement information or by sensing the frequency of bump and rebound stop contact. Again manoeuvring would demand high damping, perhaps 2500 Ns/m being the maximum coefficient attainable.

The advantages of this system over one of fixed design are that the vehicle handling, particularly at high speed, and/or on rougher surfaces, would be improved, and the ride comfort on rough roads would be improved quite substantially, but for most of the time, the system would be set up much like a conventional one.

If we now imagine that the stiffness will be controllable as well as the damping. To simplify the discussion and to take account of practicalities which suggest that changes in stiffness will occur stepwise, let us assume that stiffnesses of 21000 N/m and 10500 N/m are obtainable. Reference to Fig. A.8 will reveal that for Ks=10500 N/m, least discomfort occurs for Cs=378 Ns/m provided that the running conditions are not severe enough to cause significant bump and rebound stop contact. With this configuration SWS<4cm for roughnesses somewhat worse than (a), so that for smoother surfaces the suspension would be set up with these parameters. Note that the more flexible the spring is, the greater is the separation between the values of damping which are best for ride comfort and for wheel load control, so this arrangement is not good from the latter point of view. Since straight running is being contemplated, this would matter only in respect of braking and accelerating, and increasing the damping to give γ=0.5 in response to suitable brake pedal position, or throttle position and gear engagement signals, would improve this aspect. As the road roughness increases, the best policy for comfort is to leave the spring stiffness at the lower value and to increase the damping up to a likely maximum approaching 3000 Ns/m sufficient to limit the demand for working space to what is available with only infrequent stop contact. By scaling the results in Fig. A.8, it can be deduced that in condition (a), The discomfort parameter would be about 70% of that for the standard configuration shown in table A.2, while in condition (b), it would be about 75% of the standard (table A.2). Again scaling Fig. A.8 data, in condition (c), RCP would be about 50% of the standard based on an estimate of the standard vehicle performance (with frequent limit stop contact) in this condition.

At high speed on smooth roads, the low stiffness and light damping suggested above may be such that the directional stability is poor, and if so, increases in damping should be made first, followed by switching to the high stiffness (if needed) to correct matters, ride comfort being sacrificed. As before, on initiating a steering manoeuvre of significance, switching immediately to the high stiffness mode and possibly also to maximum damping for a period of one or two seconds would be an appropriate policy, providing better handling behaviour than that available from a fixed parameter, compromise design.

If the spring stiffness were further reducible to say one quarter of that of a conventional car, further fairly small gains in comfort with a straightforward extension of the parameter adjustment stratagem described would be achieveable. The required range of the damping coefficient would not be much affected.