REORGANISING EXISTING FACILITY NETWORKS UNDER A BUDGET REDUCTION SCENARIO: MODELS, METHODS AND APPLICATIONS

By:

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"Verily, with hardship there is relief" (94:6)

This thesis is dedicated to my parents;
Zaharudin Abu Nasir
Kamsiah Zakaria

ABSTRACT

The suitable location of facilities is a key factor in achieving efficient supply systems both in the public and in the private sectors. Nowadays, most public and non-profit bodies offering essential services (such as healthcare or environmental management facilities) are suffering from severe funding limitations and budget cuts. In handling this scenario, the decision-makers must take any possible action to ensure facility networks can keep operating and providing a minimum required service level, even though, due to financial reasons, some facilities might be downsized (and their operating hours reduced) or, in extreme cases, closed down. Any reduction that is made might limit the service level, hence increasing the congestion level of the system. For essential services, this means an increase in demand's waiting times for server availability; as such, users could consider moving to another available facility or, at a certain point, leave the system.

This study aims to develop a mathematical model for reorganising the operations of existing facility networks which encounter budget reductions issues. Due to reorganisational actions, the network size might be reduced. Hence, this study is also concerned with the effect of the reorganisation, i.e. the congestion problems which might derive from the changes imposed onto the network. Limited studies were found in the area of reorganisation of facilities' operations, especially in a scenario of supply shortage problem. Moreover, no study considered congestion problem as part of reorganisation effect. Hence, this study proposed a dynamic mathematical model using a multi-period logic as the main approach to solve the reorganisation problem. The proposed model was adapted and used to solve two real-world case studies from the City of Sheffield (UK): the first one concerned with the rationalisation of Household Waste Recycling Centres (HWRC); the second one devoted to the organisation of General Practitioners (GP) Facilities. Both types of facilities are currently dealing with budget limitations issues.

The contribution of this thesis is twofold. First of all, the effect of reorganisational actions on the networks was also considered and integrated, through explicit consideration of congestion issues by means of a novel multi-period model which was proposed in order to solve facility networks reorganisation problems. As such, this work provides an enrichment of the literature related to reorganisation problems of existing facility networks; which not many authors have explored. Secondly, such model was applied to two real-world cases faced by local authorities and other planning bodies; through these implementations, the study also contributed to practical problem-solving issues.

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LIST OF ABBREVIATIONS

A&E	Accident and Emergency
BPR	Business Process Reengineering
CLSP	Capacitated Lot-Sizing Problem
DEFRA	Department of Environment, Food and Rural Affairs
GP	General Practice
HWRC	Household Waste Recycling Centre
LA	Local Authority
LSP	Lot-Sizing Problem
NHS	National Health Services

CHAPTER 1: INTRODUCTION

Facility location decisions are among the essential tasks faced by both private businesses and public service providers (Murray et al., 2010). Selection of an optimal location for a facility can improve service performance and minimise risk. However, performing these decisions is not an easy task (Bhatnagar & Sohal, 2005), also considering their long-term and strategic impact, and the difficulty in revising them once implemented. Therefore, it is important to examine a set of related issues prior to the location planning decision. For example, setup costs, traffic network condition, covering ability, and facility capacity are some of the elements that must be considered before choosing an appropriate facility location. Also, there are several elements that must be monitored after the decision, for example, potential changes in demand patterns, demand accessibility, and service performance. Managing and planning facility operations is not an easy task, especially when dealing with tight budget constraints. Even with a limited budget, any decisions must ensure that the system is effectively operated and at the same time ensure a decent level of user satisfaction (Bhaumik, 2010).

As time evolves, populations, environments, demand's needs, and trends might change. Some existing facilities may no longer be able to provide an adequate service (Sonmez & Lim, 2012); it is worth to mention that, nowadays, given the current climate of austerity, most existing services (especially those linked to non-profit and public-owned facilities) in Western countries are suffering from budget restriction issues (Bruno et al., 2016). Restrictions can have different impacts, which depend on the specific service provided by the facility. The impact of budget restrictions especially for facilities providing essential services (such as educational establishments, environmental services, healthcare, public toilets, libraries, or recreation centres) might be severe (Bruno et al., 2016). In addition, a further increment in the use of these facilities (public healthcare services, police departments, or waste recycling centres) might affect the residual capacity level and might lead to congestion issues.

For instance, the number of general practice (GP) consultations are growing at a 3.5% yearly rate on average, compared with the 2% average annual growth in GP staffing times, due to underfunding of the National Health System (Baird et al., 2016); all this is causing a severe congestion of the network of GP surgeries across the UK.

Also, other essential services, such as waste management, have been impacted by capacity reduction issues. By 2020, the UK is targeted to recycle at least 50% of household waste (DEFRA, 2018e). However, closure and reduction in opening hours of recycling centres nationwide (due to financial cuts to Local Authorities)[1] might go against this objective. This means that the service coverage for the public will be reduced and this will create congestion in the remaining centres. Also, this conflicts with the sustainability objectives.

Such examples clearly show that facility networks management problems (which include the location of the facility, its operational planning, and its scheduling) are an important matter, also for the existing facility networks. Therefore, more sophisticated approaches could be considered to cope with reorganisational problems arising in a context of supply reduction problem, yet minimising any possible risks.

1.1. Research Aims and Objectives

This study aims to develop an approach for reorganising existing facility networks in the presence of budget restrictions problem. In order to achieve such aims, the following three objectives are identified:

1. To identify the characteristics, parameters, and variables of models capable of solving decision-making problems in facility networks characterised by budget restrictions issues.
2. To develop mathematical models for existing facilities networks so as to solve facility location problems in the presence of resource reductions.
3. To apply the proposed a method to solve real-world case studies, in order to perform decisions such as the re-allocation, resizing, rescheduling or closure of existing facilities.

Our study contributes to the development of a mathematical model for reorganising existing facility networks, at the same time, enriching the literature in related fields. This is because only a very limited amount of literature has been found on the reorganisation of facility operations, especially when in shrinking or downsizing the network size. The model formulation is developed keeping into account the need to apply it to the real-world problems.

[1] See, for example, reports about closures in Oxfordshire (reported by Sproule, 2015) and Hampshire (reported by Neal, 2016) and the reduction in opening hours in North Yorkshire (reported by Prest, 2016).

1.2. The Developed Model: General Outlook

A brief review of the developed model is presented in order to ensure that readers are able to grasp its underlying fundamentals, especially the type of model and its characteristics, i.e. network type, demand and locations type.

The proposed model is a dynamic model that utilises a multi-time period logic. Therefore, the proposed solutions were varied from time period to time period, hence, assisting the decision-maker in creating an optimal schedule for each facility. Additionally, the model is able to predict the demand's circulation within the network for each time period. Any possible risks associated with the facilities' schedules is also highlighted by the model.

Multiple facilities within a network are considered. These facilities provide similar services and are related to each other. Thus, any action taken on one of these facilities will affect the network's operations. The proposed model employs the total cost to run the facility network as the main objective function; several constraints are included (for instance, the capacity of the facilities). This means the model is highly sensitive towards the costs and capacity level. The entire solution process was developed by using the mathematical programming solver software, CPLEX 12.6 on computer with a memory of 8.0 GB RAM, a 2.50 GHz processor and the Windows 10 operating system.

In general, the proposed model is able to deal with any service which is facing a supply reduction problem (such as budget or workforce reductions) and to reproduce other relevant characteristics. The main assumptions of the model are: (1) all facilities are managed by a single central authority; (2) all facilities are characterised by a given capacity constraint.

The model is suitable for existing facility networks, however, with several modifications and refinements, such as changing the set of existing facility locations to the set of potential facility locations, it can be used in planning for facility location and its operations in the future. Besides that, the proposed model in this thesis can be used for tactical and operational setting of facility networks to suit the current demand level. The real-life applications of the proposed model can be found later in this thesis.

1.3. Thesis Organisation

This thesis contains seven chapters and is arranged in three main parts that comprise: literature, methodology, and applications. The first two chapters deliver an overview of the dynamic facility location models and their related literature, especially in reorganising facility operations.

Chapter 3 highlights the development of the proposed multi-period model. Chapters 4 and 5 show the implementation of the proposed model on real-world case studies. Chapter 6 concludes and provides the future directions of our study.

Chapter 2 reviews the literature with the objective of assessing the body of knowledge related to problems dealing with the reorganisation of facility operations; contributions in this field are classified and analysed; also, facility location problems with congestion issues are investigated, as congestion can be seen as a side effect of supply reduction.

Chapter 3 focusses on the development of a multi-period model for the reorganisation of facility operations. A step-by-step construction process for the proposed model is presented and explained. Experimentations on the performance of the proposed model, focussing on sensitivity analysis and computational times, are presented.

Chapter 4 discusses the application of the proposed model to the first case study, which is related to the household waste recycling centre (HWRC) in Sheffield. A brief background on Sheffield, waste management systems and recycling are presented. Data collection processes are discussed and presented. The refinement of the proposed model is demonstrated and implemented in the case study. Key findings are discussed.

Chapter 5 focusses on the application of the proposed model to the primary healthcare service, i.e. GPs. This chapter introduces a new model to create a network of backup facilities as an initiative to reduce the network's congestion level, which is then utilised to provide users with alternative mechanisms for service provision.

In Chapter 6 some conclusions are drawn. The research objectives are recalled and discussed. The contributions and significance of the research are also evaluated. At the end of this chapter, possible areas for future work are presented.

CHAPTER 2: REORGANISING FACILITY NETWORKS WITH SUPPLY REDUCTION PROBLEM – A LITERATURE REVIEW

Reorganising facility networks is a common action in both public and private sectors (ReVelle et al., 2007). Changes in demand patterns, mergers between organisations, or financial restrictions may create pressure on facility operations and affect the spatial organisation of services, especially for non-competitive public sector facilities. Accessibility of user can be affected, and service quality can be reduced, causing over-utilisation and congestion of remaining facilities (Bruno et al., 2016). Therefore, the need for models and methods for reorganising facility networks in such a way to minimise the damage to the user has gained interest in the literature (Farahani et al., 2014). The tasks that need to be planned include, reschedule facility operating times, opening of new facilities, relocation, closure or downsizing of existing ones.

This chapter is dedicated to review the existing literature on reorganising facility networks, with special focus on studies which deal with supply reduction problem. In addition, a review of facility location problems dealing with congestion issues are examined too, in order to investigate how the literature has dealt with this issue which could be a potential consequence of rationalisation actions. Then, literature gaps are highlighted. At the end of this chapter, research philosophical that act as based to our model development is also underlined.

2.1 Coverage of The Literature Studies

To date, not many studies about the reorganisation or closedown of facilities can be found. However, in general, location models would also be applicable in choosing which existing locations are to be closed (ReVelle et al., 2007), with some modification in variable definitions and constraints, for example, limits in number of facilities to be closed. But these models are useful once per decision-making process. The "permanent and static" facilities concept does not resemble dynamic changes in the location network (Antunes & Peeters, 2000). Besides than the reduction in number of operating facilities, the reorganisation could also include: the reschedule the facility operating periods, and optimise the capacity of the facilities.

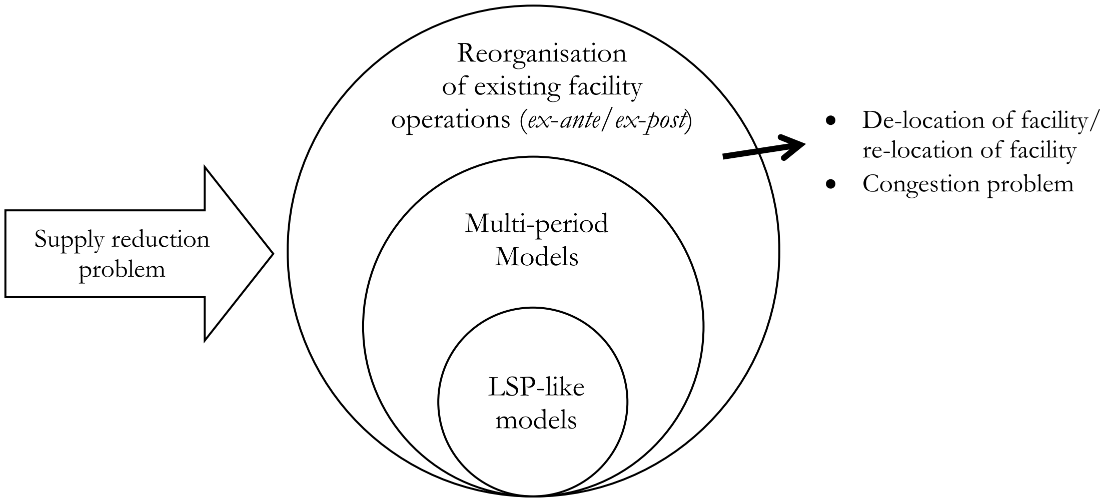


Figure 2-1: Coverage on the surveyed literature

Figure 2-1 highlights the related literature of facility location problem with reorganisation components in a concentric circles form. Specifically, our interest lies in dealing with the reorganisation of existing facility network; involved decisions might include closure, downsizing or variations of the opening schedule of considered facilities, which were caused by budget constraints or supply reduction problems. Therefore, the first stage of literature analysis starts with a review of past studies on reorganising facility location problems, which can be classified into *ex-ante* or *ex-post* models, which can be defined as follows:

- In *Ex-ante* models, decisions are made *before* changes occur; this deals with a need to undertake future planning exercises, specifically related to possible changes in the demand pattern which could lead to an increase or reduction of the size of the network (number of operating facilities) or of operating periods of a facility (or facilities).
- *Ex-post* models deal with decisions which are made *after* the changes have occurred, in order to respond to some demand/supply changes through the reduction of the network size or of the facilities' operating periods.

Any modifications (i.e. the reorganisational actions) made to the network will affect the entire facilities' operations. For essential services (such as recycling sites), the reorganisation of its operating times will affect the facilities' utilisation rate; this could potentially lead to congestion problems. Therefore, as shown in Figure 2-1, existing studies in facility location with congestion issues were also examined. This is to ensure that approaches employed to account for possible effects of reorganisation actions were seized.

Reorganisational actions (such as closing existing a facility and open a new facility location, variations of the opening schedule of considered facilities, or closing a facility completely); either ex-ante or ex-post implementations, can be described very well through the usage of multi-period models. Hence, as shown in the second layer of Figure 2-1, existing studies on dynamic multi-period models focussed on reorganising existing facility network were explored. However, limited literature can be retrieved in this area of interest.

Interestingly, the amount of demand being in the queue in a facility waiting to be served can be seen as 'inventory of a given item held by a facility', leading to a similarity with classical stock management problems. Due to the interest in the existing time-dependent based models, one of the renowned multi-period models, the Lot-Sizing Problem (LSP), was further explored in our study, as shown in the third layer of Figure 2-1. LSP is known for planning the inventory management in a manufacturing context, such as to find optimal quantities to be produced in order to satisfy the demand level. However, recently, a number of applied papers have been developed considering the application of the LSP model, and its variants, to a non-manufacturing context, where the model can be utilised for resource planning over a given time horizon.

Next section reviews the literature on reorganising facility network, using Figure 2-1 as guidance.

2.2 Organisation of Service Networks

Business Process Reengineering (BPR) was firstly introduced by Hammer (1990) to reorganise business operations in such a way to boost organisational efficiency and productivity. In business operations, reorganisational actions can include facility downsizing, relocation and reschedule (variations of the opening schedule) of the considered facilities. BPR is implemented in order to achieve optimisation of the entire business operations (Gerrits, 1994) which require a rationalisation of the organisation's workflow and increased process control (Esbenshade et al., 2016). The increasing interest in BPR was also determined by changes in the economic environment (Attaran, 2004) which led to a stronger focus on efficiency, profitability, and flexibility of the organisation (Majed et al., 2001). In order to achieve these goals, different approaches to BPR could be implemented (Doomun & Jungum, 2008). However, several criteria (such as the ones-based costs and on time measures) in BPR can be ambiguous. Hence, analytical (or mathematical) approaches should be employed for dealing with such criteria (Hofacker & Vetschera, 2001). Moreover, modelling could optimise the entire process performance through functions describing process flows through various components (Gerrits, 1994) with less time consuming.

Indeed, queuing time is one of the important tools in BPR (Gerrits, 1994) and it is hardly avoided. BPR is a process that involving customers-suppliers relationship (Scherr, 1993), which any actions taken will affect this relation. For example, any service operations reduced by the suppliers, the customers will be channelled to the remaining servers. Hence, increase the number of customers at the remaining servers. Sooner or later, increase the queue length and time taken to obtain the required service. Customers are powerful stakeholders (O'Neill & Sohal, 1999) where is related to the delay in service management problem; queueing problem. Queue can be translated into twofold; length and time, which later implemented to calculate the output; such as processing time, service time or server's workload performance. Buzacott (1996), Cheng et al., (2009) and Liu and Fan (2014) focussed on finding the total processing time of a task by using the queueing time. Similarly, Jiang and Giachetti (2008) adapted queueing network to estimate the total service time for patients. Meanwhile, Cho, Kim and Kim (1998) utilised the queue time of customers to calculate the workload allocation for each employee, to reduce the queueing problem. Additionally, through the utilisation of customers' queue length or queueing time, the number of employees in the organisation can be found. Such studies were proposed by Hao and Yifei (2011), Alotaibi and Liu (2013) and Rinaldi et al., (2015). Hao and Yifei (2011) and Rinaldi et al. (2015) focussed on reducing customer waiting time, while Alotaibi and Liu (2013) used the priority based-queue length to increase customer satisfactory level.

The existence of queueing problem within BPR process as discussed above showed the importance of this element in revamping entire organisation's operation. The queueing concept was adapted to improve customer's queue length, either through optimal allocation workload of employees or through improving the customer's service time. Meaning, the "extra" customers or demand were often acknowledged as additional time to the total processing time that happen within a single time-period only. Besides, the possibility of demand movement between inter-facilities (these facilities provide similar services) was not considered by any past studies. This means, this set of study did not dealt with inter-facility issues and ignored the spatial dimension of facility networks0

From the searches, past studies on BPR failed to explicitly consider the scenarios of budget reduction problem. Therefore, the next section focuses on reorganisation of facility network through the adaptation of analytical or mathematical models, focusing on the implementation process, either based on *ex-ante* or *ex-post* models.

2.3 Reorganising Facility Operations using Analytical Model

This section highlights the existing mathematical models that focused on reorganising facility operations, that divided into two parts; *ex-ante class* and *ex-post class.*

2.3.1 Ex-ante Class

Reorganisation actions can be performed by using *ex-ante* models; in this case, decisions are based on forecasted data, and are to be implemented in the future. In short, the decision about the opening, the closure, the downsizing or the expansion of a facility's operations in future periods is based a forecast developed starting from present/current usage or condition of the facility. This is similar to what happens in time-based models (for instance, in multi-period models). Re-location problems also fall in this category; in this type of problems, the decision to re-locate the facility is based on the present condition. Therefore, this section reviews the literature that focusses on decision-making by utilising time-based models.

Berman and Drezner (2008) injected uncertainty in the traditional p-median model developed by Hakimi (1964) in order to locate facilities for present and future usage. Uncertainty in this context limits the number of facilities to be located up to p facilities in the current period; however, it is possible to add up to q facilities in the future. Similarly, Sonmez and Lim (2012) introduced the Facility Location and Relocation Problem – under Uncertainty (FLRP-U); FLRP-U allows relocation in the future by closing some of the facilities that have been located initially and opening new ones.

A few studies dealing with this problem in a multi-period framework have been developed, such as the ones from van Roy and Erlenkotter (1982), Min (1988), Shulman (1991), Chardaire et al. (1996), Canel and Khumawala (1997), Melo et al. (2006) and Wilhelm et al. (2013).

Van Roy and Erlenkotter (1982) dealt with the simultaneous closing of existing facilities and opening of new ones. Authors also ignored capacity constraints assuming total flexibility. In contrast, Min (1998) utilised capacity as a constraint to expand, close and relocate public library facilities. A Fuzzy Goal Programming (FGP) model has been developed whereby, fuzzy values describe the qualitative-based variables such as facility accessibility and the relative distance between the 'old' and the 'new' facilities. Meanwhile, Shulman (1991) allowed capacity size to be flexible depending the on type of facilities in the network. Two situations have been considered: (1) various size of facilities and several types of facilities per node; (2) various size of facilities but only one type of facility per node. A similar approach was used by Chardaire et al. (1996) in a telecommunication network problem. Meanwhile, foreign manufacturing facility location was studied Canel and Khumawala (1997). They took into account trade agreements between countries, exchange rates, taxation schemes and operational cost, for several periods, such that total cost is minimised when choosing manufacturing facility location.

Melo et al. (2006) and Wilhelm et al. (2013) designed dynamic, multi-commodity supply chain networks to reconfigure the facilities over time as to react to changes in demand trends or to cope with the loss. Melo et al. (2006) did not allow facility's capacity to be transferred to the next period, but with reduced capacity. In contrast, Wilhelm et al. (2013) did not allow the same facility to be expanded and contracted over the planning horizon once opened (and before closure).

2.3.2 Ex-post Class

In contrast to *ex-ante*, *ex-post* model implementations focus on decision-making after changes (for instance, to demand patterns) have occurred. Precisely, this type of models looks to establish alterations in the facility's operations or in the network, which do not allow any further modification once the decision has been made. For example, a commonly used constraint in this kind of models does not allow any re-opening or re-closure of facilities once a decision has been made; this states the non-reversible nature of the changes that are going to be introduced in the facility network.

Wang et al. (2003) and Monteiro and Fontes, (2005) dealt with the restructuring of bank branches within a scenario of budget reductions. Wang et al. (2003) focussed on rationalising (by opening new branches and closing some existing ones) a bank network in an urban area. A median-like model was utilised to guarantee that the customers' travel distance to nearby branches is always minimised. However, the model was based on an uncapacitated framework. Therefore, Monteiro and Pontes (2005) added the capacity-like constraint to their model in order to deal also with resizing decisions.

The future characteristic is also dynamic. Changes in network structure can be made per period which could affect the system performance afterwards. ReVelle et al. (2007) considered capacities shrinkages in competitive and non-competitive service. Both models proposed by the authors aim to minimise the loss in demand: the first one by retaining a certain number of facilities; the second one was developed in a non-competitive setting (such as public based service, healthcare, schools) aims to minimise the number of people made worse-off when some of the facilities are closed to reduce costs. A similar approach has been used by Bhaumik (2010) in downsizing an organisation's distribution network. The number of distribution centres were reduced in order to cope with demand reduction. With the closure of some servers, service quality will be decreasing; total cost however is expected to be reduced. Among other studies in the *ex-post* class were the ones conducted by Loerch et al. (1996) and Dell (1998) in army sites closure. Dell (1998) created a multi-period model to maximise total savings for six years planning horizon.

A multi-period model in an *ex-post* fashion was introduced by Araya et al. (2012). Araya et al. (2012) dealt with reorganising a school system in a rural area in Chile due to insufficient students (demand) and teachers (supply). The authors allowed the system to operate under minimum quality, i.e. the distance between school and student accommodation is not the main concern, as it would be like in a median-like or centre-like problem.

Meanwhile, Bruno et al. (2016) dealt with the rationalisation of a higher education system in Italy; this model looked at the possibility of closing or downsizing facilities while minimising the cost for extra-capacities to be activated somewhere else in the network. It is important to mention that this study utilised individual preferences (based on a spatial interaction model) for dealing with the assignment of users to facilities.

These brief reviews of facility network rationalisation models are characterised by several common approaches, such as the possibility of performing capacity adjustment the long-term orientation in order to deal with changes in demand patterns, market flows, and/or financial restrictions. A major shortcoming of the current literature, however, seems to be represented by the fact that there is not an explicit consideration of congestion issues that might arise as an effect of the closure/downsizing of facilities. For this reason, next section is dedicated to review congestion issues within the general facility location literature, in order to understand how such situations have been modelled so far, and how they could be integrated in future rationalisation models.

2.4 Facility Location with Congestions Issues

Traditional location models assume that facilities are able to meet any demand in any instant (Hu et al., 2013). However, a more realistic situation is that demand is being elastic, an assumption meaning that customers can choose whether to stay or leave the system at any time (Marianov, 2003). Our interest lies within reduction of supply while amount of demand unchanged and non-reduced. Most literature, as mentioned in previous section, reorganisation for facility operations was based on capacity adjustments through reduction in number of operating facilities, and some suggested relocating facility to a much optimal place. However, in a case of supply reduction problem, having to relocate or add more capacity to the facility is not an option. Service provider most likely reduces either in the network size or the total operating periods. With a non-decreasing function of demand, from time to time, utilisation rate at facility will increase, and soon, creating congestion problem. In integrating congestion issues in reorganisation of facility location problem, many aspects need to be considered.

The review of the literature of facility location with congestion problem focusses on the existing methods used to solve and alleviate congestion problem, at the same time, identify for any possible reorganisation elements. Existing literature have dealt with *demand*, *network* and *area* congestion problem. *Demand* congestion occurs when the amount of demand exceeds the capacity and service rate of the facility. Typically, this is handled through queueing theory and queuing networks (for more details, see Appendix 2(C)), where the profiles of demand arrivals are assumed to be in a stochastic form. The queue could be translated as congestion costs, for example, opportunity costs incurred by a customer that wait in the queue for several hours. Meanwhile, *network* congestion occurs when related edges have to cope with more demand than allowed by their capacity. Traffic jams are one example; these may occur because of the presence of extra demand during certain time windows; this could create problem in reaching the targeted facilities within the network. For example, the US Bureau of Public Roads (BPR) developed a function to find the travel time between two points based on traffic volume and capacity. *Area* congestion occurs (mainly in competitive contexts) when there is a saturation of a region or no suitable place for a new facility in the presence of other facilities within the same region. It increases the travel distance between two points by the presence of prescribed areas (closed and bounded region) (Sarkar et al., 2005). Details on handling the network and area congestion problem as describe in Appendix 2(C).

The earliest facility location studies that dealt with demand congestion were: Berman and Larson (1982) - Stochastic Queuing Median (SQM) model; Berman (1985) - Stochastic

Expected Queueing Median (SEQM) model; Chiu (1986), Batta (1989); Brandeau and Chiu (1992) - Stochastic Queue Centre (SQC) and Stochastic Expected Queuing Centre (SEQC) location model. These studies tackled congestion problems through unpredictability in demand arrival, while later used as one of the elements in finding optimal facility location. Later, some studies were found deduce the queue as a cost. Such studies were by Melachrinoudis (1994); Vardar, et al. (2007); Farahani et al. (2014); Cho et al. (2014); and Zarrinpoor et al., (2017). These authors addressed the congestion occurred when there was a disruption in the system. For example, an organisation suffers with productivity loss (penalty cost) when a machine broke down and the technician (server) was fully occupied.

Some studies associated congestion with queue length. Queue length (amount of demand in the line) or waiting time for accessing the service are normally utilised for addressing congestion levels. Thus, through capacity expansion such as opening new facilities location or additional *k*-servers in the system network, congestion levels can be reduced. Studies that focussed on locating new facilities in the network, without increasing number of servers are Marianov and Serra (1998); Berman (1995); Wang et al. (2002); Shavandi and Mahlooji (2004); Galvão et al. (2005); Elhedhli (2006); Rodríguez et al. (2007); Romeijn et al. (2007); Sourirajan et al. (2007); Abouee-Mehrizi et al. (2011); Chambari et al. (2011); Kim (2012); Rahmaniani et al. (2014); Vidyarthi and Jayaswal (2014); Vidyarthi and Kuzgunkaya (2015) and Aboolian et al., (2016). Meanwhile, existing literature focussed on placing optimal number of *k*-servers are Marianov & Serra (2001); Marianov and Serra (2002); Marianov (2003); Dobson and Stavrulaki (2007); Vardar, et al. (2007); Aboolian et al. (2008); Baron et al. (2008); Marianov et al. (2008); Shavandi and Mahlooji (2008); Beraldi and Bruni (2009); Castillo et al. (2009), Zhang et al. (2009); Zhang et al. (2010); Seifbarghy et al. (2010); Marianov and Serra (2011); Aboolian et al. (2012); Hu et al. (2013); Davari et al. (2016) and Hajipour et al. (2016).

Customer waiting time is increased whenever congestion occur, hence, the concept of demand lost was introduced in the model. Such studies were conducted by Berman and Drezner (2006) and Berman et al. (2006). Therefore, in order to capture the 'lost' demand due to coverage distance and congestion, Berman et al. (2007) extended Berman et al. (2006) study by allowing demand to visit the second nearest facility provided that a visit has been made to the first facility. Demand 'loss' concept was also applied by Marianov et al. (2008) in expanding the multi-server immobile facilities network.

Taniguchi et al. (1999) and An et al. (2015) considered traffic and demand congestion problems. Both models used the queuing technique to deal with demand congestion and adapted the BPR formula to cope with traffic congestion. The BPR formula was commonly used to estimate the congestion costs whenever traffic capacity increased, like the study by Liu and Ralston (1989), Wang, et al. (2004), Bai et al. (2011), Hajibabai and Ouyang (2013) and Hajibabai et al. (2014). Besides the BPR formula, Desrochers et al. (1995), Wong and Sun (2001), Köksalan and Soylu (2010), Konur and Geunes (2011, 2012) and Hwang et al. (2016) tailored a specific equation to address traffic congestion problem, which later used as additional costs to the model. Meanwhile, Lee (2015) used GIS to address traffic congestion costs. A study by Smith (2010) linked two networks (i.e. queueing networks) in an attempt to study how to ease traffic congestion problem. However, the application of such models to facility location, planning and operations was not discussed.

Meanwhile, the congestion in an area was normally dealt through an additional penalty in the cost-based objective function. Such studies were performed by Braid (1991), Butt and Cavalier (1997), Sarkar et al. (2005), Date et al. (2014) and Saleh Farham et al. (2015).

Congestion problem could be gradually occurred, per se, from time to time. Considering time as the fundamental element to represent the congestion would help to reduce congestion problem. It seems that the "time" dimension is fundamental towards the representation of congestion; a good way to deal with the problem we're interested in (network rationalisation) could be the one which includes the "time" dimension. Therefore, existing literature on any time-dependent models in facility location with congestion problem were explored. However, only two articles have been found for the multi-period model; Jouzdani et al. (2013) and Atashi Khoei et al. (2017). Both models dealt with traffic congestion problem. Jouzdani et al. (2013) solved the traffic congestion problem through BPR function; the fuzzy logic concept was used for dealing with demand arrival uncertainties. The multi-period logic was used to keep track of investment costs, capacities and product conversion for each connected period *t*. Meanwhile, Atashi Kohei et al. (2017) used the variant in vehicle speeds per period to indicate traffic congestion, hence used this value to calculate the total fuel-emission cost per period.

One of the major gaps emerging from the literature that was surveyed on facility location problems with congestion issues seems to be represented by the fact that there is no study dealing, simultaneously, with the reorganisation of the facility network due to financial pressures, while explicitly taking into account congestion issues. Indeed, most of the research which has been produced so far only considers network expansion (that is to say, adding more servers to the facility network) as a way to deal with congestion; this is an option which cannot be pursued by any organisation which is dealing with financial problem. Meanwhile, the usage of queueing networks in solving the facility location with congestion problems was not discussed by any past study. Most past literatures focussed on one network and one type of facility only, hence neglecting the existence of interrelated and interconnected facilities operations. Therefore, in the next sub-section, existing studies concerned with the alleviation of the queuing and congestion problems without incurring in any additional costs to the provider, are explored.

2.5 The Quintessential Multi-Period Model: Lot-Sizing Problem (LSP) Approach

As mentioned previously, the time-dependent model or so-called the multi-period model, able to capture the demand mechanism in the network from time-to-time, including the 'extra' amount of demand. This concept is similar to the inventory management system; where demand in the queue could be considered as "inventory of goods". There are two types of inventory model which are single-period and multi-period. The multi-period allows a flexible re-sequencing of orders within a period (Sahling et al., 2009). Lot-Sizing Problem (LSP); one of the renowned multi-period models, has a constraint to seize items' movements throughout time and network. Even though the LSP is suitable for the industrial based problem, several modifications could be conducted to suit the problem that we worked on.

Therefore, this section described the LSP, from the aspect of modelling and its applications. Later the adaptations of LSP into the non-manufacturing-based problem is also outlined.

History of LSP starts from the Economic Ordering Quantity (EOQ) model. EOQ was introduced more than 100 years ago by Harris (1913). EOQ is used to find an optimal quantity of items needs to order or purchase to satisfy demand, at the minimum costs. Demand is assumed to be constant at all time. Literature on LSP and its improvements has been around for more than 100 years (Andriolo et al., 2014) and yet, many implementations could be done, especially outside the traditional concerns, i.e. manufacturing and industrial based studies (Bruno et al., 2014). One of the well-known extensions of the EOQ is the LSP.

LSP model was introduced by Wagner and Whitin (1958). LSP allows demand to vary per time (Drexl & Kimms, 1996; Bruno et al., 2014; Kang et al., 2018) that identify optimal production level and inventory level such that the total cost; setup costs, production costs and holding costs, are minimised. LSP involves in making an optimal decision-making of hierarchical planning process for either short-term, medium-term or long-term times (Drexl &

Kimms, 1996; Karimi et al., 2003; Clark et al., 2011). Karimi et al. (2003) and Brahimi et al. (2017) classified LSP based on several criteria or characteristics, but not limit to, such as: planning horizons (short-terms, medium-terms or long-terms), number of levels or production stages, number of products or items cater per problem, capacity nature (fixed or variable), demand type (deterministic or stochastic) and setup structure (continuous or discrete). Meanwhile, the variants of LSP can be seen through classical version – an enhancement of LSP through sets, variables definition or system level, or extended version – integrations of LSP with other decision-making problems (Glock et al., 2014).

Variants of LSP are large since it exists more than 50 years ago. Capacitated LSP (CLSP) is one of many extensions of LSP. CLSP limits the number of items produced through infusion of capacity constraint that heavily influence production-plan decision-making (Li & Meissner, 2011), at a minimum the sum of setup (ordering) and inventory carrying costs. CLSP is known for its complexity and classified as a *NP-Hard* problem by Florian et al. (1980) and Bitran and Yanasse (1982). The mathematical formulation of classical version of CLSP, a *single-item, single-facility* by Bruno et al. (2014) is:

System parameters:

τ_t = System capacity
ε_1 = Cost related to processing a unit of item
ε_2 = Cost related to holding a unit of item in inventory
C_t = Setup cost/ production cost

Decision variables:

y_t = 1 if item(s) produced, 0 otherwise
s_t = Inventory level (number of items held)
q_t = Production level (number of items produced)
x_t = Demand level (number of request items)

$$min\, Z = \sum_t (\varepsilon_1 q_t + \varepsilon_2 s_t + C_t y_t) \tag{2-1}$$

subject to;

$$s_t = s_{t-1} + q_t - x_t \qquad \forall t \in T \tag{2-2}$$

$$q_t \le \tau_t y_t \qquad \forall t \in T \tag{2-3}$$

$$s_t \ge 0; x_t \ge 0; \qquad \forall t \in T \tag{2-4}$$

$$y_t \in \{0, 1\} \qquad \forall t \in T \tag{2-5}$$

The objective function (2-1) is to minimise the system operational costs. Constraint (2-2) indicates the inventory level of item during period *t* (mass balance flow of items in the system) and production of items is limited at capacity, τ_t (as constraint (2-3)). (2-4) strictly indicates that inventory level and demand level must be a positive number. (2-5) indicates the binary condition for y_t.

Single-item means time-varying amount of demand for a single item over time-period *t* (Brahimi et al., 2017). Meanwhile, *single-facility* means the process take place in a single facility. One of the CSLP's variants is by Li and Meissner (2011). The authors added cost of capacity to the objective function while the rest of constraints were unchanged. The objective function:

$$min\, Z = \sum_t (\varepsilon_1 q_t + \varepsilon_2 s_t + C_t y_t) + capacity\ costs \qquad (2\text{-}6).$$

The cost of capacity could be the additional costs needed in acquiring extra capacity to fulfil demand level.

Meanwhile, formulation for *multi-item*, single-facility CLSP, as presented by Karimi et al., (2003), Jans and Degraeve (2008) and Buschkühl et al. (2010). A *multi-item* means more than one item are catered in a single facility's operation. Let J $(\forall j \in J)$ be the variations of item, thus formulation of *multi-item*, single-facility CLSP is:

$$min\, Z = \sum_t \left(\varepsilon_1 q_{j,t} + \varepsilon_2 s_{j,t} + C_t y_{j,t}\right) \qquad (2\text{-}7)$$

subject to;

$$s_{j,t} = s_{j,t-1} + q_{j,t} - x_{j,t} \qquad \forall t \in T;\ \forall j \in J \qquad (2\text{-}8)$$

$$q_{j,t} \leq \tau_{j,t} y_{j,t} \qquad \forall t \in T;\ \forall j \in J \qquad (2\text{-}9)$$

$$s_{j,t} \geq 0; x_{j,t} \geq 0; \qquad \forall t \in T;\ \forall j \in J \qquad (2\text{-}10)$$

$$y_{j,t} \in \{0, 1\} \qquad \forall t \in T;\ \forall j \in J \qquad (2\text{-}11)$$

(2-7) can be read as (2-1), which is to minimise the entire system operational costs. (2-8) is balance the inventory level of item *j* at time *t*, or known as mass balance constraint. (2-9) ensures the production of items *j* at capacity level. (2-10) strictly indicates that inventory level and demand level must be a positive number. (2-11) indicates the binary condition for y_t. Multi-items problem is a complex CLSP (Chung and Lin, 1988; Brahimi et al., 2017) and was proved a *NP-Hard* problem by Chen and Thizy (1990).

Besides two traditional CLSPs; the single- and multi-items, various extensions can be found. Each extension carries different complexities (Bruno et al., 2014). Additional features such as multi-level or multi-stage model (Van den broecke et al., 2008; Shim et al., 2011; Hu & Hu, 2016, 2018), demand uncertainty (Brandimarte, 2006; Guan & Liu, 2010; Zanjani et al., 2010; Tempelmeier, 2011; Helber et al., 2013), setup costs and/or times (Trigeiro et al., 1989; Haase, 1996; Bayley et al., 2018; Taş et al., 2019), linked lot sizes (Suerie & Stadtler, 2003; Gupta & Magnusson, 2005; Ramya et al., 2016; Mahdieh et al., 2017), allow backlogging (Agra & Constantino, 1999; San-José et al., 2014; San-José et al., 2017) and decay function/ lost sales (Absi & Kedad-Sidhoum, 2009; Absi et al., 2013).

Classical model of CLSP is flexible which become foundation and is integrated with other problems. Lot-sizing itself is implicitly related to scheduling problem (Zhu & Wilhelm, 2006) where lot-sizing is a decision-making process and produced results over finite or infinite planning time horizon. Such studies were conducted by Drexl and Haase (1995), Kovács et al., (2009), Toso et al. (2009), Ferreira et al. (2012), Meyr and Mann (2013) and Guimarães et al. (2014) for solving lot-sizing and scheduling problem simultaneously, while Shim et al., (2011) and Hu and Hu (2016, 2018) solved both problems by stages. Besides scheduling, studies on suppliers' or transportations' selection (as by Basnet and Leung (2005), and Choudhary and Shankar (2014), Ayhan and Kilic (2015) and Alfares and Turnadi (2018)) while integrations of CLSP with other problems are quite limited, such as network flow problem (Armentano et al., 1999), facility location problem (van Oudheusden & Singh, 1988) or both (Deleplanque et al., 2012). Extensions of CLSP were normally used to solve for production and manufacturing, either in planning or rescheduling system operations. Various approaches were also found, such as utilisation of fuzzy technique by Choudhary and Shankar (2014) and Ayhan and Kilic (2015). Most applications of CLSP are suitable for manufacturing operations (for examples, study by Haase, 1996; Meyr, 2002; Gupta & Magnusson, 2005; Fazlollahtabar et al., 2011; Delgoshaei et al., 2016; Hu and Hu, 2016). Some applied their model to real case studies, for example, production of photographic materials (Van Den Broecke et al., 2005; Van Den Broecke et al., 2008), production of pharmaceutical products (Sazvar et al., 2014; Ramya et al., 2019; Sahling & Hahn, 2019), automated teller machines (ATMs) networks (Chotayakul et al., 2013), automotive industry (Ayhan & Kilic, 2015), textiles industry (Miranda et al., 2018) and foods and beverages (Ferreira et al., 2012; Tanksale & Jha, 2016; Toscano et al., 2017). Review of lot-sizing integrated scheduling focussing on food industry by Stefansdottir et al. (2017).

2.5.1 Applications of LSP in Service Operation Management

CLSP is useful in planning system operations. Items' conservation flow that dynamically changed from time to time is useful in planning for organisations' operations (Bruno et al., 2014). Better adjustment allowing authority to fully utilised of limited capacity (Kang et al., 2018) and multi-period character of CLSP allows adjustments or planning made in certain time scale. However, not many focusses on the applications of CLSP outside production and manufacturing problem. Bruno et al., (2009) apprehended the traditional CLSP into new dimensions through redefined each of variables in CLSP without changing the whole concept. In contrast to the traditional CLSP; keeping inventory level at maximum (or optimal number), authors proposed a model aiming to reduce inventory level or demand queue length by finding the optimal number of servers at a minimum cost. The extension of this work was by Bruno, et al., (2012) with multiple end-nodes. Same idea was presented by Hassannayebi et al., (2017) for minimising demand waiting time for train service in Iran. Bruno and Genovese (2010) and Bruno et al., (2014) successfully highlighted the usefulness of looking at CLSP from another dimension. Bruno and Genovese (2010) applied their studies in finding optimal number of airport check-in counters, while Bruno et al., (2014) modified version of CLSP was applied at three different case studies: the departure schedule for a bus terminal; the management of a logistic cross-dock platform; and the optimisation of an airport check-in gates. The same concept of Bruno and Genovese (2010) and Bruno et al., (2014) was enhanced by Bruno et al. (2018) by considering number of operators that compatible to staff-schedule and work shift. On similar concept, a model by Güden and Süral (2014) found the optimal facility locations for borrow and waste facility, however, utilising the uncapacitated LSP model.

This section highlighted the existing literature on reorganising facility operations, without any extra costs, at the same time, alleviate and reduce congestion level. From these reviews, gaps of studies can be highlighted.

2.6 Gaps in the Current Literature

From the conducted reviews, most studies on facility location model with reorganisation did not considered the effect of reorganisation. Past literature on handling the queueing problem using BPR approach focussed on single facility problem on one-dimension only; neglecting the importance of spatial space in decision-making process. In the meantime, almost all studies reduce congestions problems by increasing number of servers, number of facilities and some even ignored the 'extra' demand. We were motivated by dynamic characteristics of demand;

where the concept of demand stay in the system (or being in the queue) resembles the inventory planning problem. Therefore, the researched was continued on the multi-period inventory model. One of the renowned models; CLSP was encountered. However, the adaptation of CLSP in non-industrial based problems are also difficult to find. In details, from the conducted reviewed, several research gaps can be highlighted:

1. Process of relocating (close-and-open), de-location or reduction in facility operations or facility network size has been applied in many areas, either for public or private organisation. However, from the review, clearly the reorganisation problem for public facility are quite limited. Assuming these facilities always have enough funds or supplies are unrealistic, hence making reorganisation critically important for any facilities that suffer with supply reduction problem. However, there are no studies found, especially in reorganisation of facility location problem that incorporates this issue.

2. Demand congestion has been successfully incorporated, for a while, in the facility location literature. This has been done mainly through probabilistic or stochastic dynamic models; queuing techniques are the most utilised technique in handling demand congestion. From the conducted review, however, it seems that most of the studies assume that excess demand will be ignored (according to Marianov and Serra (1998)). This assumption appears to be unrealistic when dealing with crucial services (such as healthcare).

3. Most of the papers suggested the location of new facilities and the optimal allocation of a number of new servers to alleviate congestion problems. However, for an organisation that suffers from budget restriction, capacity expansion or selecting a new location is not an option. As it has been shown, there is no availability of studies dealing with congestion issues arising from budget restrictions and the need for downsizing a facility network. In addition, no studies proposed congestion that could be occurring through reduction in supply that at the same time, have a non-decreasing demand pattern, which expansion is not an option.

4. In solving the reorganisation and facility location with congestion problem, most studies focus on locating one type of facility. The impacts of other facilities in the entire system were ignored. In reality, a network consists of more than one facility (multi-commodity) and these facilities are interrelated, interconnected and have an influence on each other, such as the impact on congestion levels. To have a better illustration, Figure 2-2 portrayed

as a general version of the healthcare system which consists of five facilities; NHS 111 Call centre, local pharmacist, GP, walk-in centre, and A&E.

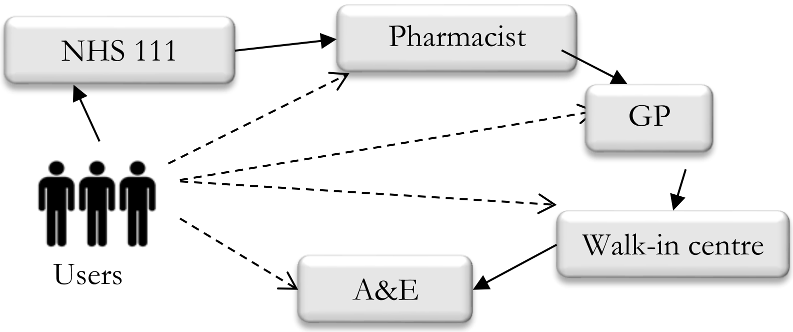


Figure 2-2: General version of healthcare system

The general hierarchical of healthcare service starts from NHS 111 Call centre, local pharmacist, GP, walk-in centre, and lastly, A&E; as represented by the line with an arrow. Meanwhile, the dashed line with an arrow indicates that patients also indirectly have access to all facilities. Patients can visit any of these four facilities based on needs and ignore the hierarchical order. For example, to see a GP, a patient has to wait for a certain amount of time; he/she can decide to leave the system or to visit another facility, such as A&E, which provides similar service. Indirectly, this will increase the amount of user in A&E. Hence, it is important to consider other facilities in the network in reorganising the existing facility network. However, as far as we concerned, no such studies considered the existence of interrelated and interconnected facilities in reorganisational problem.

5. The explicit representation of demand flows (for instance, demand arriving at the facility or demand served by the facility) is often missing. Multi-period models (such as the adaptations of CLSP) or queueing network theory could play a role here; however, these have not gained much attention in the literature so far.

2.7 Research Approach and Philosophical Review

This study proposed a mathematical model to solve some of the practical problems faced by organisations when they have to reorganise facility networks. Care is devoted to develop models that can be usable and beneficial in 'real world' practice (Jackson, 1993).

Consequently, the work falls within the Operational Research (OR) discipline. OR is a systematic approach for intervening on social systems in order to solve any problem (Jackson, 1993). The nature of social science is to develop a theory to represent a real-life problem, meanwhile, the nature of OR is more about practices or techniques to solve real-life problems. However, both are beneficial to each other (Jackson, 1993). Some OR studies belong to the science and technology domain, but some belong to the management or social sciences field, which reflect the main characteristic of OR, the one being an inter-disciplinary decision-making approach. Even though OR is defined as a combination of mathematical modelling and development of algorithmic solution approaches, its usage is strongly related to a management problem. OR techniques, indeed, should place a lot of emphasis on the model formulation (that should be strongly related to the problem situation), on solution methodologies and on the possibility of implementing and maintaining the solution for a certain period of time (Ackoff, 1979).

In relation to philosophical contexts, OR is a Natural Research that cannot strictly be applied in Positivist perspectives, but in a perspective known as Design Research (Manson, 2006). Manson (2006) described Design Research as a process of using knowledge to design and create useful abstract artefacts (mainly, models and methods). Therefore, it is important to know the underlying meta-theoretical assumptions for OR as a discipline. This includes classification of OR within ontological, epistemological, axiological and methodological perspectives. Vaishnavi and Kuechler (2004) distinguished between Positivist and Design Research in details, as shown in the following Table 2-1.

Table 2-1: Philosophical assumptions between positivist and design research (adapted from Vaishnavi & Kuechler (2004))

	Positivism	**Design Research**
Ontology	A single reality. Knowable, probabilistic.	Multiple, contextually situated alternative world states. Socio-technologically enabled
Epistemology	Objective, dispassionate. Detached observer of truth.	Knowing through making: objectively constrained construction within a context. Iterative circumscription reveals meaning.
Methodology	Observation, quantitative, statistical.	Developmental. Measure artefactual impacts on composite systems.
Axiology	Truth: universal and beautiful; prediction.	Control; creation; progress (i.e. improvement); understanding

2.8 Conclusion

Past studies on reorganisation of facility location were reviewed in this chapter; primarily on relocation, de-locations, and reduction on operations. Since supply reduction will eventually cause the congestion problem, this chapter also dealt with the representation of congestion conditions in facility location problems. However, prior studies also have failed to address the supply reduction that later caused congestion problem. When the facility operations being reduced, queue length will be increased from time to time. Similarly, this resembles the inventory planning problem. Motivated by the time-dependent changes, the searched was continued on the multi-period inventory problem. Meanwhile, the multi-period model, specifically the LSP, was explored. The LSP and its variant; the CLSP was reviewed. The literature applications of the CLSP in non-industrial based problems were scrutinised. It was found that only a few studies, but growing over time, has been applied the CLSP in the non-manufacturing-based problem. Through these reviews, several gaps were highlighted. At the end of this chapter, research approach and philosophical review of this study was explained. Next chapter describes the process to develop the state-of-the-art multi-period model on reorganising existing facility location, with supply reduction problem under the pressure of facility closure.

CHAPTER 3: CAPACITATED MULTI-PERIOD, MULTI-FACILITIES LOCATION MODEL FOR REORGANISING FACILITY OPERATION

The previous chapter reviewed the literature related to the models that were dealing with the reorganisation of facility network operations, with special emphasis on studies focussing on the supply reduction issues caused by budget constraints. Possible effects of such reorganisation (such as congestion problems) were also reviewed. As highlighted, most of the available studies do not explicitly consider congestion issues arising from the resource reduction problem. Therefore, this chapter focusses on proposing and solving a model aimed at reorganising an existing facility network which is experiencing a supply reduction problem, thus increasing the pressure to either close the facility or to downsize it. The entire procedure used to derive the model are reviewed; the computational results related to the model solution are also shown, and a sensitivity analysis is performed.

3.1 General Issues of the Problem of Interest

Today, due to the general climate of economic austerity, most profit and non-profit organisations supplying essential services are suffering from budget reduction problems. Even though the financial cuts affect the number of operating facilities or reduce total operating periods, service providers are often still obliged to serve as much demand as possible due to the nature of their service and contractual obligations. To reduce the impact of the cuts, appropriate actions need to be taken in order to perform downsizing and closure decisions in a way that minimises the damage or discomfort to the user. Also, side effects need to be taken into account; downsizing and closure decisions might lead to a capacity reduction, which in turn may lead to an increase in congestion within the supply system.

Most facilities operate in environments characterised by uneven congestion patterns where there is a general lack of predictability in the arrival of the demand. Additionally, some servers might have spare capacity during the same time windows in which others are fully utilised. Figure 3-1, adapted from Bruno et al. (2018), illustrates a good example of the uneven congestion concept. In this study, the authors focussed on finding the optimal number of check-in counters at an airport. From this figure, it can be seen there are times when counters are available but are not being used at full capacity (see grey shaded region), whilst there are other times during which there are insufficient counters to serve demand (see area between the blue and orange lines), leading to congestion.

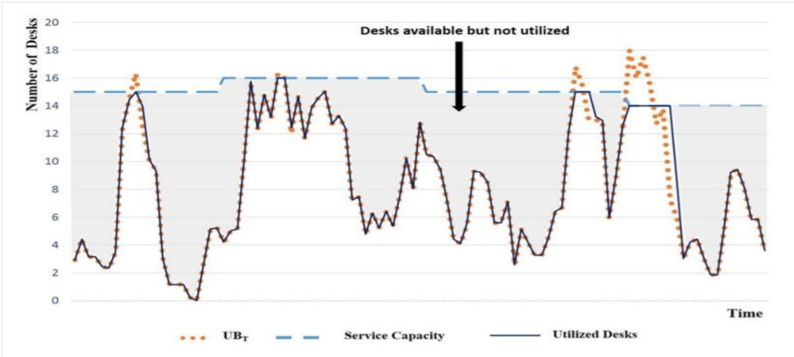


Figure 3-1: Typical pattern of utilisation of servers (Bruno et al., 2018)

Similarly, our interest lies in finding an optimal facility schedule capable of providing service to user when needed by additionally focussing on the possibility, of demand *circulating* within the facility network so as to allow as much demand as possible to be met.

Demand movements within a network can result in certain effects on each facility in the same network. Demand movement between facilities happens when the closest facility is either fully utilised (and is thus congested) or is not operating. Indeed, demand circulation inside the network (i.e. from a congested facility to another facility which is experiencing lower levels of congestion) might be related to additional cost (i.e. logistical and operational costs); also, demand originally assigned to the facility might have to wait for a long time to gain access to the required services. This scenario (demand circulation) will eventually increase the utilisation rate of another facility. This is portrayed through Figure 3-2. The movement of demand in this figure is based on the congestion at a given facility leading to demand choosing not to wait for the server to be available but moving away from the facility.

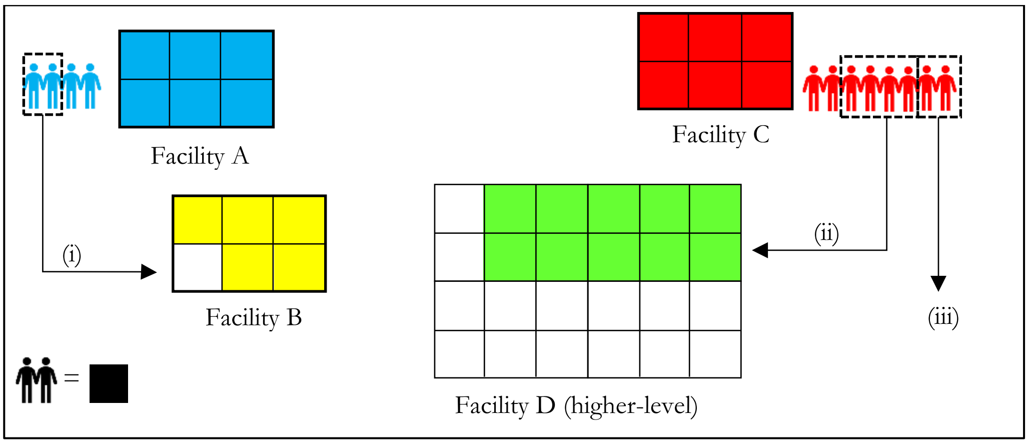


Figure 3-2: Demand moves to other facilities due to congestion

Figure 3-2 shows two types of demand movement: the demand moves to another facility (either to (i) 'same-level' facilities; or (ii) 'higher-level' facility), and the demand chooses to leave the entire system. The movement of demand shown in Figure 3-2 could be influenced by the movement level and the service type, as summarised in Figure 3-3.

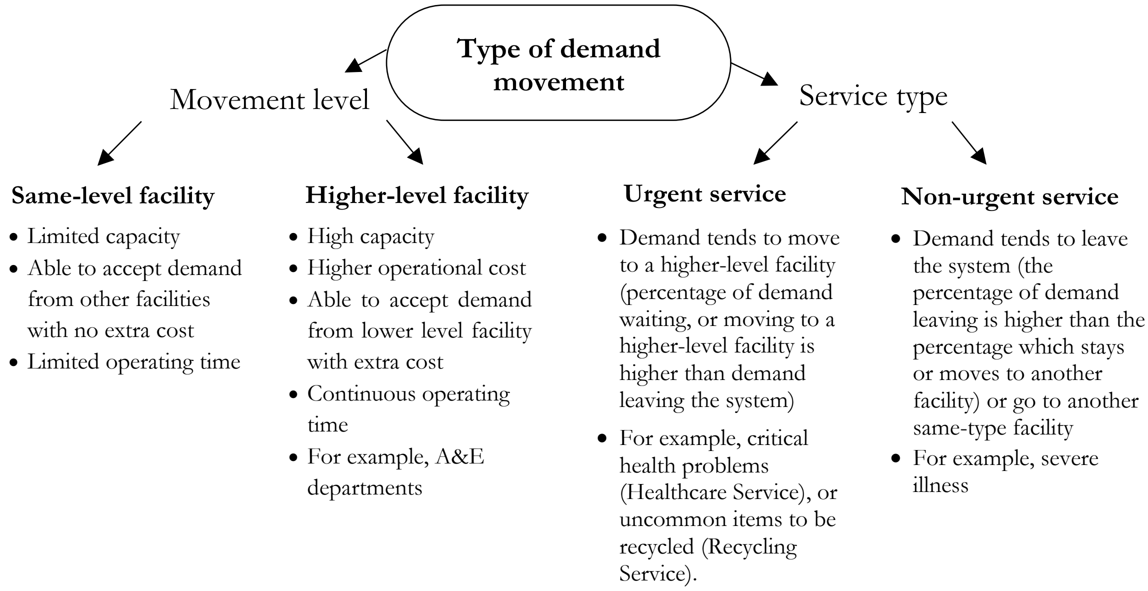


Figure 3-3: Demand movement

From Figure 3-3, '*same-level*' means that the facility has almost identical operational costs and capabilities to serve demand. Meanwhile, a '*higher-level*' facility refers to a facility with higher capacity and which operates all the time. A 'higher-level' facility could be the main service provider (headquarters); as such, it can be very costly to operate, and indeed extra costs will be required to deal with the extra demand. Figure 3-3 also classifies user movement as being related to service provision. If the demand is related to an *urgent* service provision, then it will likely go to a higher-level facility with a much higher capacity and is characterised by unlimited operating times. However, due to these characteristics, it is likely that such a facility will experience very high levels of congestion. If the demand requires a *non-urgent* service, then the tendency to leave can be higher than the one to stay or to move to other facilities. For example, given a demand for a healthcare service, i.e. a patient is suffering from a severe illness, he (or she) can choose to leave the general practitioner (GP) surgery once he (or she) cannot get an appointment with a doctor within a desired timeframe. The patient also has the choice to go to any other level of the facility, in this case a walk-in centre or an accident and emergency (A&E) department, to seek medical attention. However, the presence of such non-critical patients in walk-in centres or A&E departments might contribute to further congestion problems and increments in operating costs to the provider.

Besides the congestion problem (i.e. as portrayed in Figure 3-2), the movement of demand to another facility could also occur due to limited operating facilities or whenever one or more facilities is shut down completely. Figure 3-4 illustrates the effects arising when facility *B* is not in operation. Demand might choose to move to the next closest facility or to a higher-level facility, as in (i) and (ii). Meanwhile, when a facility becomes congested, demand either moves to a higher-level facility (iii) or leaves (iv). This will increase the higher-level facility utilisation by non-urgent demand (from facility *B* and *C*) and the waiting time for server availability in facility *D* might further increase.

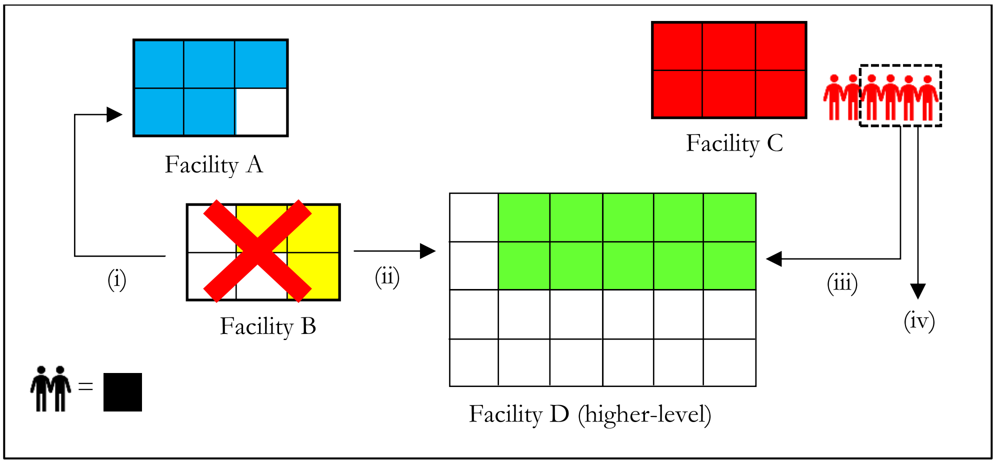


Figure 3-4: Demand movement due to facility closure or a facility not being in operation

As an example, reducing the number of household waste recycling centres (HWRCs) or reducing the operating hours of a HWRC will lead to congestion in the recycling system network. Some HWRCs do not have all the required recycling facilities and the closure of such a facility will also increase the congestion levels at a well-equipped HWRC. In this case, users can choose to go to another recycling centre or leave the system. When users lose interest in recycling, the question of where all these recyclable items go must be answered. An analogous problem is also experienced by the healthcare service.

A survey by the National Health Service (NHS) in July 2017 indicated that 20% of respondents had to wait more than a week to see a GP compared to 18.4% in July 2016 (NHS, 2017, p. 23). This report also noted that 14.6% of its 110,834 respondents decided not to have a check-up (leave), 5.7% of respondents moved to another healthcare service (such as, going to a pharmacy, or seeking a private healthcare facility) and 4.7% went to A&E to seek treatment. For any patients that decided to leave the system, their condition might get worse, or at worst, they might ultimately be hospitalised (Donnelly, 2017). Furthermore, if a patient chooses to go to A&E, extra costs in attending the patient will be incurred.

Service capacity or server availability is strongly affected by budget cuts. Readjustment of capacity to its optimal size is important as this helps increase servers' utilisation and increase system effectiveness. Any underutilised capacity must be adjusted, while the over-utilised or congested facility must be expanded in order to cater for the increased demand per unit time. For any existing facility, the need to reorganise is crucial. Operation of these facilities needs to be revised, even though the operating periods of these facilities are being reduced or indeed the facilities are being completely shut down. This also means an increase in the waiting time for the demand. Due to inaccessibility of the servers, the demand might lose interest (for any non-urgent service) or go to the higher-level facility. Clearly, this situation will affect the entire system operation. Hence, a solution which can minimise the damage or discomfort to the provider and the users, as a consequence of the need to reduce opening hours, is desired.

The problem that we are trying to solve is complex. The decision-maker might reduce the operational periods of some facilities, and/or other facilities might be closed completely. Additionally, the decision-maker might want to keep certain facilities open. Sometimes, the reorganisation options suggested by the decision-maker might also be insufficient to serve the area under analysis. Several reorganisation options could be delivered, especially when the decision-maker is dealing with multiple facilities in a network and so needs to consider the length of operational times for each facility. It is important that the decision-maker considers all of the options. Given the combinatorial nature of the problem, there could be a very high number of options to evaluate. As such, an enumerative process for obtaining an optimal solution would be time-consuming and tedious, thus leading to the decision-maker not considering all the available options. Moreover, to solve the reorganisation problem, several limitations must be considered, for instance, the capacity of a facility or facility performance level. Figure 3-5 illustrates four possible reorganisational actions using two facilities (denoted by j) and four time-periods (denoted by t), which are:

- Option 1 – All facilities are operating;
- Option 2 – Only Facility 2 is operating;
- Option 3 – Both facilities are operating for two periods, and
- Option 4 – Both facilities are operating at different total operating periods, and at different time t.

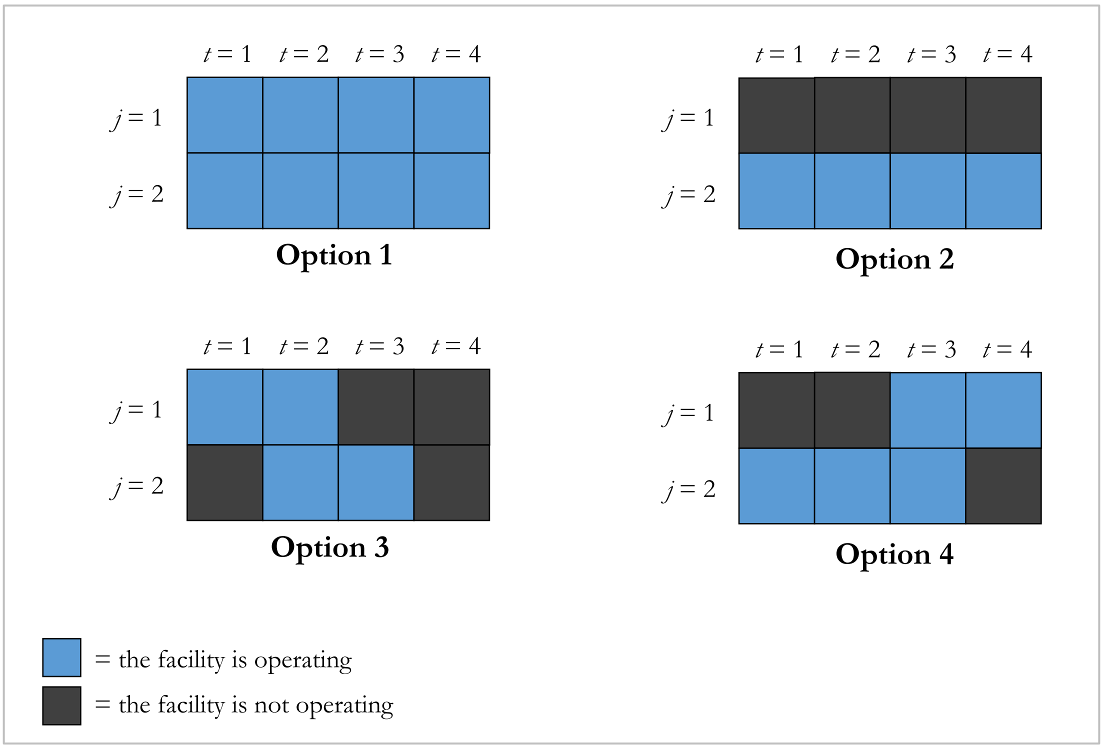


Figure 3-5: Several possible of reorganisation options

From Figure 3-5, options 1 and 2 show straightforward combinations, while options 3 and 4 clearly show the unique combination of facilities' operational periods. However, there are other options that could be conducted, for instance, only Facility 1 is operating, or both facilities to operate one, two, three or four three time-periods at different time *t*, and so on. From these options, clearly, there are millions of reorganisation options that need to be considered by the decision-maker; which is why an elegant method, i.e. a mathematical model to represent the reorganisational problem, must be proposed.

The following section discusses the model development process through the modification of several basic facility location models and the adaptation of the CLSP concept. The assumptions of the proposed model are also described.

3.2 A Mathematical Model to Reorganise Existing Facility Network

We propose a multi-period model aimed at dealing with the reorganisation of an existing facility network. Our study was inspired by variants of the multi-period models introduced in previous studies which adapted the CLSP to different problems arising in non-manufacturing environments, such as Bruno et al., (2009, 2012, 2018), Bruno and Genovese (2010), and Bruno et al. (2014). The proposed model is relevant to facility networks in which such facilities are interrelated and interconnected with each other. The explicit representation of the time dimension of the demand dynamics, along with reproduction of real-life options for demand (such as the possibility to move to other facilities or to leave the system) differentiates the proposed model from previous studies.

3.2.1 Adaptation of the CLSP by Variables Adjustment

Recall the so-called *mass balance constraint* (2-2) of the CLSP. In this equation, components of $flow_{in}$ consist of inventory levels from the previous period (t – 1) and of items produced during period t. Meanwhile, a component of $flow_{out}$ defines the number of items sold to customers at t. Hence the balance between $flow_{in}$ and $flow_{out}$ is the number of items held in storage, i.e., the inventory level at the end of period t.

$$\begin{aligned} s_t &= flow_{in} - flow_{out} \\ &= (s_{t-1} + q_t) - (x_t) \end{aligned} \qquad (2\text{-}2)$$

These quantities can be mapped to similar concepts in the CLSP by considering:

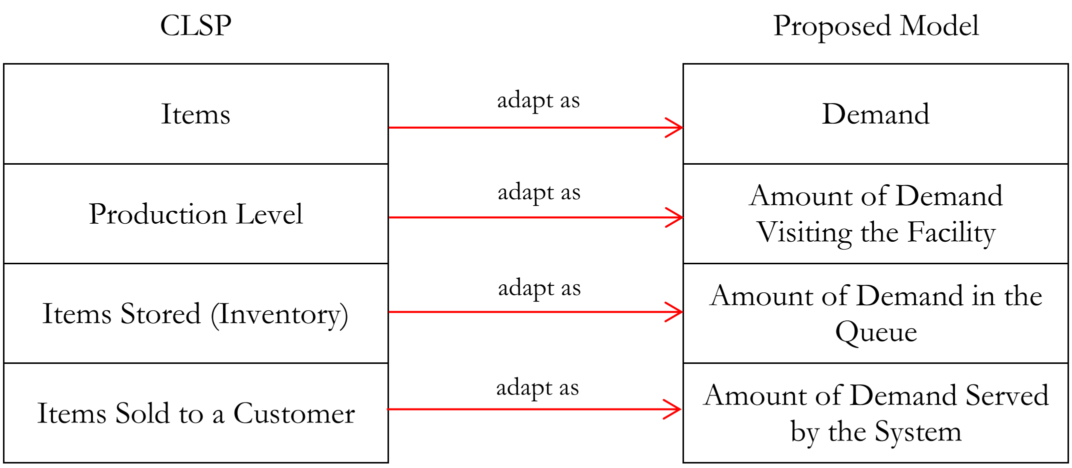


Figure 3-6: Adaptation of the CLSP by variables adjustment

In Figure 3-6, the adaptation of variables from the CLSP to the proposed model is shown. Besides adjusting the variables, the modification is constructed in a systematic process that is arranged into four stages:

Stage 1: Moving from the CLSP to a multi-period, single facility model.

Stage 2: Modelling a multi-period, single facility model with a *loss* variable.

Stage 3: A multi-period, *multi-facility* model with loss variable.

Stage 4: A multi-period, multi-facility model with loss variable and *demand movement* within interrelated facilities.

Stage 1: From the CLSP to a Multi-Period, a Single Facility Model

The CLSP is a renowned model in industrial- and supply chain-based studies, where the focus is on the production level and items (things or services). As mentioned, by looking at the problem from a different angle and perspective, it is clear that the definitions of $flow_{in}$ and $flow_{out}$ need to be modified to suit our purposes. The first step is to modify the definition of the parameters and decision variable terms. The adjusted definitions can then be given as follows:

System parameters:

Original			Adjusted		
τ_t	:	System capacity	τ_t	:	System capacity
ε_1	:	Cost related to producing a unit of an item	ε_1	:	Cost of serving a unit of demand
ε_2	:	Cost related to inventory level	ε_2	:	Cost of holding a unit of demand
C_t	:	Setup cost/ production cost	C_t	:	Setup cost/ operational cost/ cost related to operate the facility
			x_t	:	Demand level (amount of demand)

Decision variables:

Original			Adjusted		
y_t	:	1 if there is/are item(s) produced, 0 otherwise	y_t	:	1 if the facility/facilities is/are in operation, 0 otherwise
s_t	:	Inventory level (number of items held)	s_t	:	Holding level (amount of demand stay at the facility at the end of *t*)
q_t	:	Production level (number of items produced)	q_t	:	Processing level (amount of demand served)
x_t	:	Demand level (number of request items)			

From the adjusted variables, the components of $flow_{in}$ are the combination of the amount of demand from the previous period (s_{t-1}) and the amount of demand visiting the facility during *t* (x_t), whilst the $flow_{out}$ component is the amount of demand served during period *t*

(q_t). Therefore, the mass balance concept of the proposed model can be illustrated as per Figure 3-7.

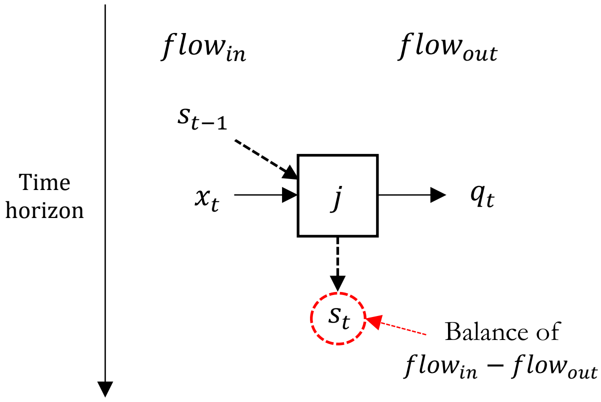


Figure 3-7: Mass balance concept of the proposed model

From the illustration, constraint (2-2) can be re-written as:

$$\begin{aligned} s_t &= flow_{in} - flow_{out} \\ &= (s_{t-1} + x_t) - (q_t) \end{aligned} \tag{3-1}$$

Constraint (3-1) describes the amount of demand in the queue at the end of period *t* is the balance between $flow_{in}$ and $flow_{out}$.

Stage 2: A Multi-period, a Single facility Model with Loss Variable

Demand is unpredictable, and each choice that is taken will affect the system's operation. Thus, leaving the system is an option for the demand with an additional cost either to demand itself or to the provider. For example, the act of leaving a recycling network might result in additional costs for the provider, where the lost demand may mean the disposal of recyclable materials in landfill, or worse, that it is fly-tipped. Let l_t be the demand leaving the system during the period *t*. Therefore, the objective function can be modified by adding the cost of demand leaving the system, denoted by ε_3. The objective function (3-1) of the model can thus be changed to:

$$min\, Z = \sum_t (\varepsilon_1 q_t + \varepsilon_2 s_t + \varepsilon_3 l_t + C_t y_t) \tag{3-2}$$

The mass balance definition of (3-1) is also changed due to the change of $flow_{out}$ components, which are the amount of demand processed or served (q_t), and the amount of demand leaving the system, during period *t* (l_t), respectively. Figure 3-8 further illustrates this change.

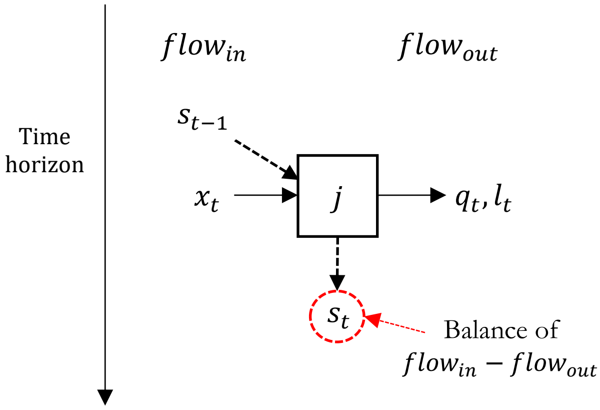


Figure 3-8: Mass balance concept of the proposed model

Hence, (3-1) can be re-written as:

$$\begin{aligned} s_t &= flow_{in} - flow_{out} \\ &= (s_{t-1} + x_t) - (q_t + l_t) \end{aligned} \tag{3-3}$$

where (3-3) now is a modified version of the CSLP that incorporates a loss variable.

Stage 3: A Multi-period, a Multi-facility Model with Loss Variable

Within a network, there is by definition more than one facility that provides an identical service and is managed by the same authorities; for example, branches of a bank or of a supermarket chain. Let J be a set of facilities ($j \in J$) to indicate a multi-facility network. We also assumed that the cost to operate a facility *j* (C_j^t) is the same for all time *t*. Thus, the modified CLSP is:

$$min\, Z = \sum_j \sum_t \left(\varepsilon_{1j} q_j^t + \varepsilon_{2j} s_j^t + \varepsilon_{3j} l_j^t + C_j y_j^t\right) \tag{3-4}$$

subject to;

$$s_j^t = \left(s_j^{t-1} + x_j^t\right) - \left(q_j^t + l_j^t\right) \qquad \forall t \in T; \forall j \in J \tag{3-5}$$

$$q_j^t \le \tau_j^t y_j^t \qquad \forall t \in T; \forall j \in J \tag{3-6}$$

$$s_j^t \ge 0; x_j^t \ge 0; \qquad \forall t \in T; \forall j \in J \tag{3-7}$$

$$y_t \in \{0, 1\} \qquad \forall t \in T; \forall j \in J \tag{3-8}$$

(3-4) – (3-8) represent the modified version of the CLSP that includes the modifications to the $flow_{in}$ and $flow_{out}$ components. Normally, demand will have more than one option when seeking the required service. The option provided is to move to another facility *j* at time *t*, assuming that the demand will be served at the second visited facility as soon as the demand leaves the first facility. Therefore, the next stage models allow for demand to move from one facility to another.

Stage 4: A Multi-period, a Multi-facility Model with Loss Variable and Demand Movement Within the Interrelated Facilities

In a non-dictatorial network with more than one facility, and where facilities are interrelated and interconnected, users can attempt to access the required service(s) at more than one facility. When demand moves from one facility to another, the components of $flow_{in}$ and $flow_{out}$ are changed. Let the second facility *j* that is visited by demand indexed by *k*, where $j, k \in J$ and $j \neq k$, forbidding demand from travelling back to the original facility *j*. The movement of demand from a facility *j* to a facility *k* is based on a predetermined binary integer denoted by u_{jk}^{t} (to allow the movement of demand from a facility *j* to a facility *k* during period *t*) or u_{kj}^{t} (to allow the movement of demand from a facility *k* to a facility *j* during period *t*). This binary integer could be represented through any numerical conditions such as a shortest distance (physically) or travel costs, or any arbitrary function, such as attractiveness movement from a facility *j* to a facility *k*. Let, the demand moving from facility *j* to facility *k* during the period *t* be represented by a decision variable S_{jk}^{t}, and the demand moving from facility *k* to facility *j* during the period *t* be represented by a decision variable S_{kj}^{t}. Thus, the product of $S_{jk}^{t}u_{jk}^{t}$ is the demand moving from facility *j* to facility *k* during the period *t*, as based on the prespecified characteristic. Figure 3-9 illustrates this concept further.

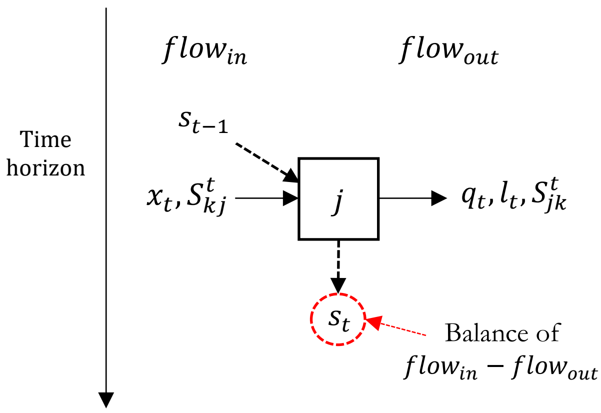


Figure 3-9: Mass balance concept of the proposed model

This movement comes with costs, however; for example, transportation costs to move from facility *j* to facility *k*, along with the time associated with this movement. Let ε_{4j} represent this cost; the final modified version of the model can thus be given as:

$$min\, Z = \sum_{j}\sum_{t}\left(\varepsilon_{1j}q_j^t + \varepsilon_{2j}s_j^t + \varepsilon_{3j}l_j^t + \ \varepsilon_{4j}\sum_{k,k\neq j} S_{jk}^t u_{jk}^t + \ C_j y_j^t\right) \tag{3-9}$$

Meanwhile, the mass balance concept of (3-5) can be re-written as:

$$\begin{aligned} s_t \quad &= flow_{in} - flow_{out} \\ &= \left(s_j^{t-1} + x_j^t + \sum_{k,k\neq j} S_{kj}^t u_{kj}^t\right) - \left(q_j^t + l_j^t + \sum_{k,k\neq j} S_{jk}^t u_{jk}^t\right) \end{aligned} \tag{3-10}$$

Figure 3-9 can be extended through the conservation of flow of demand in the system network between facilities and through time periods, as illustrated in Figure 3-10. This figure indicates the movement between interrelated same-level facilities. As can be seen, there could be users who are prepared to wait in a queue, whilst some users at a given facility *j* will move to facility *k*. Some users leave the network or go a higher-level facility. At the same time, there is demand arriving at facility *j* during the period *t*. Therefore, the amount of demand that moved into the network is either served or leave. For demand that in the queue will be served on the next period. Meanwhile, for demand that move to another facility is assumed to be served immediately at the facility that they moved into. This can be represented by introducing another constraint, which is:

$$\sum_{j}\sum_{t} x_j^t = \sum_{j}\sum_{t}\left(q_j^t + l_j^t\right); \tag{3-11}$$

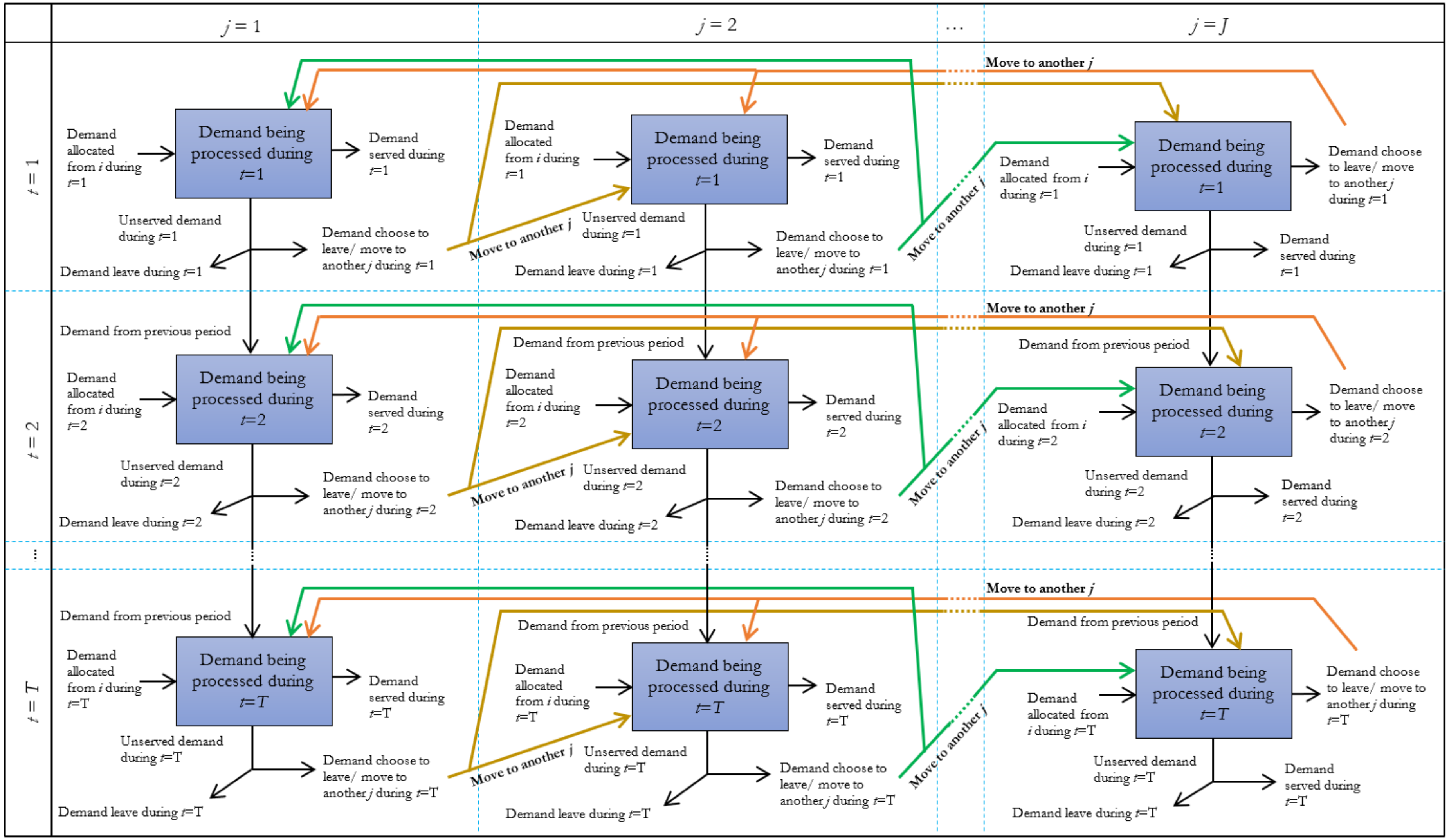


Figure 3-10: Demand movement between same-level facilities.

Besides the modifications on the mass balance constraints, additional constraints were proposed to suit the tackled problems. These constraints are as follows.

1. Let *B%* be the maximum amount of demand that is allowed to leave the facility network. This limit is imposed as part of entire amount of demand that move into the system, in order to ensure the service provider's performance within the required standard. Thus,

$$\sum_j \sum_t l_j^t \leq B\left(\sum_j \sum_t x_j^t\right); \qquad (3\text{-}12)$$

2. The movement of demand within the network and time-period must be restricted to the operated facility only. Hence, the proportion of demand that stay in queue (to be served on the next period) and the proportion of demand that move to another facility (to be served at another facility) have to be to the operating facility only.

$$\frac{s_j^t}{x_j^t} \leq y_j^{t+1}; \qquad \forall j \in J, \forall t \in T \qquad (3\text{-}13)$$

$$\frac{S_{jk}^t u_{jk}^t}{x_j^t} \leq y_k^t; \qquad \forall j \in J, \forall t \in T \qquad (3\text{-}14)$$

3. The operation of a facility is limited to several hours per day or several days per week. Hence, a constraint is needed to represent this limitation. Let δ_j be the maximum operating periods of a facility *j*. Hence, the total operating periods of a facility *j* is less than or equal to δ_j.

$$\sum_t y_j^t \leq \delta_j; \qquad \forall j \in J \qquad (3\text{-}15)$$

4. Once a facility operates, it must be operated for the entire day or week. Hence, a constraint forbids re-closure once the facility is operating is necessary.

$$y_j^{t-1} \leq y_j^t; \qquad \forall j \in J, \forall t \in T \qquad (3\text{-}16)$$

5. No queue formed at the beginning (at $t = 0$) and at the end of the time-period (at $t = T$) of a facility, hence

$$s_j^t = 0; \qquad \forall j \in J, t, T = 0 \qquad (3\text{-}17).$$

As highlighted earlier, the need to have an elegant method, i.e. a mathematical model to represent the reorganisational problem, is crucial. Thus, the next section presents the mathematical model for reorganising the facility network under the budget restriction problem.

3.3 A Multi-Period Model for Reorganising Multi-Facility Network Operations

This section introduces the proposed model as a mixed integer linear programming problem with a single objective function. Later, possible modifications to the proposed model are presented, including transforming a single objective model into a multi-component one. The parameter and decision variables for the model are as follows:

Sets		
J	=	the set of facility locations and index by *j*, *k* where $\forall j, k = \{1 \dots J' \mid j \neq k\}$
T	=	the set of time-periods and index by *t*, where $\forall t = \{1 \dots T'\}$

Parameters		
C_j	=	cost of operating facility *j*
$\varepsilon_{1j}, \varepsilon_{2j}, \varepsilon_{3j}, \varepsilon_{4j}$	=	assigned cost for each decision made where ε_{1j} indicates the cost of serving one unit of demand, ε_{2j} indicates the cost of transferring one unit of demand to the next period, ε_{3j} indicates the cost of losing one unit of demand and ε_{4j} indicates the cost of a unit of demand move to another facility *j*.
x_j^t	=	amount of demand at facility *j* during period *t*
δ_j	=	maximum period to operate a facility *j*
u_{jk}^t	=	predetermined binary number to indicate demand move facility *j* to facility *k* during period *t*
τ_j^t	=	capacity of facility *j* during period *t*
B	=	upper bound of amount of demand leaving system

Decision variables		
y_j^t	=	$\begin{cases} 1 & if\ facility\ j\ is\ operating\ at\ time\ t \\ 0 & otherwise \end{cases}$
s_j^t	=	amount of demand transferred to the following period at facility *j* at the end of period *t*
S_{jk}^t	=	amount of demand moved from facility *j* to facility *k* during period *t*
l_j^t	=	amount of demand chose to leave at each facility *j* during period *t*

q_j^t	=	amount of demand served at each facility *j* at the end of period *t*

$$Min\ Z = Min \sum_t \sum_j \left(C_j y_j^t + \varepsilon_{1j} q_j^t + \varepsilon_{2j} s_j^t + \varepsilon_{3j} l_j^t + \varepsilon_{4j} \sum_{k,k \neq j} S_{kj}^t u_{kj}^t \right) \tag{3-18}$$

subject to:

$$x_j^t + s_j^{t-1} + \sum_{k,k\neq j} S_{kj}^t u_{kj}^t = s_j^t + \sum_{k,k\neq j} S_{jk}^t u_{jk}^t + q_j^t + l_j^t; \qquad \forall j \in J, \forall t \in T \tag{3-19}$$

$$\sum_j \sum_t x_j^t = \sum_j \sum_t \left(q_j^t + l_j^t \right); \tag{3-20}$$

$$\sum_j \sum_t l_j^t \leq B \left(\sum_j \sum_t x_j^t \right); \tag{3-21}$$

$$q_j^t \leq \tau_j^t y_j^t; \qquad \forall j \in J, \forall t \in T \tag{3-22}$$

$$\frac{s_j^t}{x_j^t} \leq y_j^{t+1}; \qquad \forall j \in J, \forall t \in T \tag{3-23}$$

$$\frac{S_{jk}^t u_{jk}^t}{x_j^t} \leq y_k^t; \qquad \forall j \in J, \forall t \in T \tag{3-24}$$

$$\sum_t y_j^t \leq \delta_j; \qquad \forall j \in J \tag{3-25}$$

$$y_j^{t-1} \leq y_j^t; \qquad \forall j \in J, \forall t \in T \tag{3-26}$$

$$s_j^t = 0; \qquad \forall j \in J, t, T = 0 \tag{3-27}$$

$$q_j^t, l_j^t, s_j^t, S_{jk}^t \geq 0; \qquad \forall t \in T, \forall j \in J \tag{3-28}$$

$$y_j^t \in \{0, 1\}; \qquad \forall t \in T, \forall j \in J \tag{3-29}$$

The objective function of the model (3-18) is aimed at minimising the total operational cost of the whole network of facilities, which can be represented by the sum of four costs: service costs, queue costs, movement costs and leaving costs. The system constraints are given in (3-19) to (3-29). (3-19) was modified from the CSLP, as explained in the previous section. The constraint shows the flow of demand throughout the system, between facilities and periods. It shows the dynamic characteristic of demand where demand is able to enter and leave the system at any time *t* with an additional cost. (3-20) ensures that the amount of demand is either served or leaves the system at the end of the time horizon while (3-21) ensures the amount of unserved demand is limited at $B\%$ or rate of *B* (B is between 0 and 1). (3-22) guarantees the amount demand served is within the capacity level of the facility. (3-23) and (3-24) restrict the amount movement of demand to operating facilities only. Constraint (3-25) limits the total number of operating periods at each facility to at most δ_j and once the facility is closed (e.g., within a day), it will remain closed, as per (3-26). (3-26) can be changed to $y_j^{t-1} \geq y_j^t$, if the decision-maker allows late opening times and late closing times. For the analysis and the remaining computation for this model, (3-26) remains unchanged. (3-27) ensures that there is no demand in the queue at t = 0 and at the end of the period, $t = T'$. The decision variables q_j^t, l_j^t, s_j^t and S_{jk}^t are positive integers (3-28) and y_j^t is a binary variable (3-29).

(3-21) is closely related to (3-25). Ideally, δ_j of (3-21) can be set to T' to find the optimal schedule for a facility *j*. The decision-maker is able to control δ_j or the maximum operating period of each *j* based on his / her preferences. A different δ_j value for each facility *j* can identify the impact of the system reorganisation. Variation of δ_j may present several results, including the benefits or drawbacks of reducing the size of the facility network. For instance, setting δ_j to 0 (or maximum operating period is 0 time-unit) means the complete closure of a particular facility *j*. Meanwhile, by setting δ_j to any value for each *j*, means the facility *j* is only allowed to operate within limited periods of δ_j. The complete closure or limiting the total operational periods of a facility *j*, might cause more demand to leave, hence reducing the system performance; which is controlled by the *B* value of constraint (3-21).

The amount of demand at each facility *j* during time *t*, x_j^t can be calculated using:

$$x_j^t = \sum_i d_{ij}^t \qquad \forall t \in T, \forall j \in J \tag{3-30}$$

where locations of demand are indexed by *i*, $\forall i = \{1 ... I'\}$, d_i^t is the amount of demand from location *i* for each period *t*, and d_{ij}^t is demand from *i* allocated to *j* during period *t*. (3-30) indicates the allocated assignment of demand at *i* to a facility *j* at time *t*. The allocation can be *non-dictatorial* or *dictatorial*. Non-dictatorial allocation is allocation by choice, for instance, choosing a grocery shop, while dictatorial allocation is an allocation made by the respective authority, for instance, the healthcare service by GP is based on a patient's registration with the NHS.

Assuming that service provider has a say in this model, therefore it is important to measure both sides, i.e. provider and demand. This is because each action that demand takes will affect the cost of the system (i.e. the service provider) and will cost the demand too. For example, the act of leaving a recycling network might result in additional costs for the provider, where the lost demand may mean the disposal of recyclable materials in landfill, or fly-tipped. Hence, the model's objective function (3-18) can be further refined by considering the individual components in the demand side and the provider's side. If we let Z_1 be the cost on the provider's side and Z_2 be the cost on the demand side, then the objective is:

$$Min\ (Z_1, Z_2)$$

To solve the multi-component model, a scalarization technique is used. Let *n* be the number of components and α_n be the weight factor of each objective, where $\sum_{n=1} \alpha_n = 1$. Therefore, the refined objective function is:

$$Min\ (\alpha_1 Z_1 + \alpha_2 Z_2)$$

where;

$$\sum_n \alpha_n = 1 \tag{3-31}$$

Let the cost on the provider's side, or Z_1 consist of the operational costs $\left(C_j\right)$, the cost to serve a unit of demand $\left(\varepsilon_{1j}\right)$ and the cost when a unit of demand leave the system $\left(\varepsilon_{3j}\right)$. Let, the cost on the demand side, or Z_2 consists of the cost when a unit of demand be in the queue $\left(\varepsilon_{2j}\right)$ and the cost when a unit of demand move from facility *j* to facility *k* $\left(\varepsilon_{4j}\right)$. Therefore,

$$Z_1 = \sum_t \sum_j \left(C_j y_j^t + \varepsilon_{1j} q_j^t + \varepsilon_{3j} l_j^t\right) \tag{3-32}$$

$$Z_2 = \sum_t \sum_j \left(\varepsilon_{2j} s_j^t + \varepsilon_{4j} \sum_{k,k\neq j} S_{kj}^t u_{kj}^t\right) \tag{3-33}.$$

Thus, the modified objective function is:

$$\begin{aligned} Min \Bigg(\alpha_1 \sum_t \sum_j \left(C_j y_j^t + \varepsilon_{1j} q_j^t + \varepsilon_{3j} l_j^t\right) \\ + (1-\alpha_1) \sum_t \sum_j \left(\varepsilon_{2j} s_j^t + \varepsilon_{4j} \sum_{k,k\neq j} S_{kj}^t u_{kj}^t\right) \Bigg) \end{aligned} \tag{3-34}$$

Our model is inspired by the CLSP, which known for its complexity as shown by Florian et al. (1980) and Bitran and Yanasse (1982). Furthermore, from our previous illustration, see Figure 3-5, we could say our proposed model is as complex as the CLSP. The assumptions of the proposed model as described in the following section.

3.3.1 Assumptions of the Proposed Model

Generally, demand is uncertain and highly dynamic, thus modelling the problem is quite challenging. For an organisation that suffers from budget restrictions while simultaneously experiencing a non-decreasing amount of demand, a shortage of supply is likely to increase congestion levels. Meanwhile, it might be impractical to continue to operate a facility that has a shortage of demand, since it might be better for the allocated budget to be distributed to a highly congested facility. Therefore, in reorganising a multi-facility network, it is assumed that the reorganisation is strongly influenced by the allocation or distribution of demand among the facilities involved in the network. The general assumptions behind the model were:

1. **Facility** – The number of facilities is known and fixed. Only existing facilities that provide a non-profit service, or a public service industry are considered. The congestion distribution is uneven or inconsistent between facilities. Each facility is assumed to have a single server and provides a first-come, first-out (FIFO) service.

2. **Demand** – The locations of the demand of the facility are known and fixed. Demand also has access to any level of the facility (multi-flow) and might tend to go to a higher-level facility if a lower-level facility is congested or defunct. Even though demand can enter and leave the system at any given time, it is assumed that the demand has no knowledge about the congestion at the facility. Therefore, once a given user arrives experiencing high congestion levels, he/she can choose to stay in the queue, go to another facility (same-level or higher-level), or leave.

3. **Time-period** - The period length and the intervals between periods are known and fixed. The length and interval could adopt a daily, weekly, or monthly basis, or indeed any number that best suits the type of service provided, or as based on the preference of the decision-maker. The service time is assumed as independent and deterministic. For instance, consider a facility that operates 8-hour periods per day, and the interval between time-period is an hourly basis. Therefore, the service time is one hour. Meanwhile, if the focus on reorganising facility operations for a week, then the service

period is one day. In the meantime, the length of service time could be more than one-period. For example, to wait for GP availability, a patient must book an appointment day, i.e. waiting time. Assuming a patient has to wait for 2 days after booked an appointment for GP consultation. Given that the consultation of the GP is a day, hence the service times is 3 days (2 days of waiting time and a day of consultation time).

Besides the three assumptions discussed above, it is also assumed that each facility in a network has a different pattern of demand arrival. This means that the utilisation rates for the facilities are distinctive; indicating an uneven congestion level for the network.

The next section reports on the computational times required to solve the model using the single-objective model and the scalarising ones, the multi-component model.

3.4 Computational Experience

Computational experimentation is important to test the behaviour of the proposed model, especially in presence of larger problem sizes. Through this, the model behaviour in solving large instances within a reasonable computational timescale (in seconds) can be analysed.

3.4.1 Generating Testing Sets

Let I be the set demand locations, J the set of the facility locations and T the length of the time-horizon. The problem focussed on a **same-level** facility, where all facilities j are accessible by demand, though with some restrictions. The steps taken to generate the problem instances are:

1. Amount of demand requesting the service at each location i was distributed per time-period t (d_i^t) using a Poisson distribution so that each amount of demand generated is balanced and non-biased. The lambda (λ) of the Poisson distribution was set at 50 units for all generated data, or the highest amount of demand at each location i per time t was assumed to be 50 units of demand. The data was created and stored in an Excel spreadsheet.
2. Then, the allocation of demand at each location i per time t to each facility location j, or d_{ij}^t, was based on the **spatial interaction model**. Details of the model were given in the preceding chapter. In this instance, the general formulation was modified to:

$$d_{ij}^t = d_i^t \cdot \frac{Q_j \cdot \left(dist_{ij}\right)^{-2}}{\sum_j \left(Q_j \cdot \left(dist_{ij}\right)^{-2}\right)}$$

where *n* was set to 2. The distance between demand location *i* and facility location *j*, or $dist_{ij}$ was generated randomly using the appropriate built-in Excel function, where the range was set to be between 2 to 15 minutes of travel time. Similarly, the attractiveness value per facility *j*, or Q_j, was set between 0 and 1.

3. The amount of demand for all *i* that chose the *j* at time *t* was found by using the equation $x_j^t = \sum_i d_{ij}^t$, as (3-30). By holding *j* constant, the amount of demand from all locations *i* at each facility *j* at time *t* can be found. Figure 3-11 illustrates the process of generating the amount of demand at each facility *j* per time *t*.

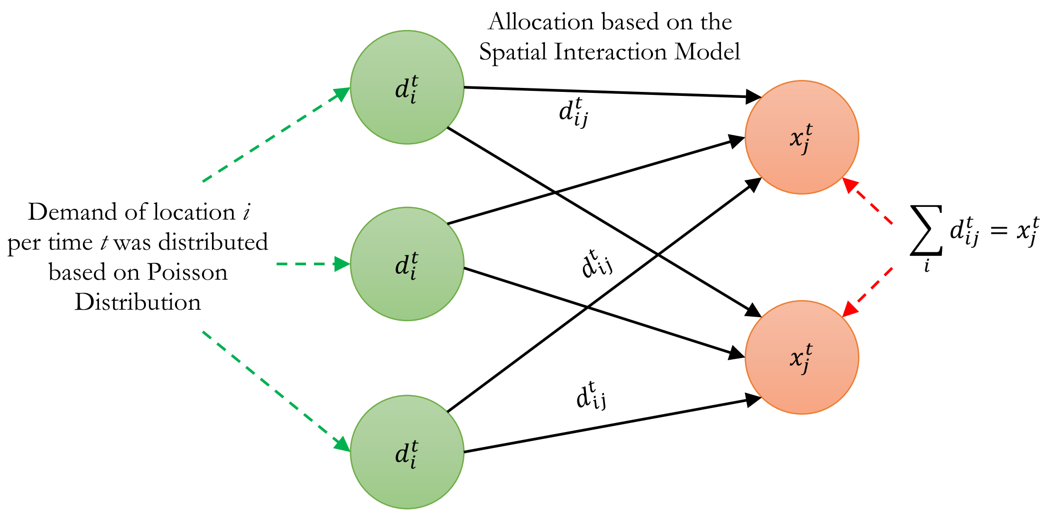


Figure 3-11: Flow to compute the amount of demand at each *j* per time *t*

4. It was assumed that the capacity level for each facility *j* (τ_j^t) was proportional to the attractiveness value, Q_j, through a factor, ω, i.e., $\tau_j^t = \omega \cdot Q_j$. To determine ω, the utilisation rate of each facility *j* $\left(\frac{\sum_t x_j^t}{\sum_t \tau_j^t}\right)$ was set between 30% - 120%, or on average the network utilisation rate is between 84% - 86%.

5. u_{jk}^t was set to be 1 if demand from facility *j* was allowed to move to facility *k* (this was possible if the travel time was less than 10 minutes), with the variable set to 0 otherwise. The distance between *j* and *k* was randomly generated using a range of 2 to 30 minutes. A minimum 2 minutes of travel time was chosen in order to indicate the existence of more than one facility in a given area. Similarly, a maximum of 30-minute travel time

was considered a realistic representation of the distance between facilities in two different areas.

Several combinations of $I = \{1 \dots I'\}$ where $(i \in I)$, $J = \{1 \dots J'\}$ where $(j \in J)$ and $T = \{1 \dots T'\}$ where $(t \in T)$ were tested; $|I|$ = 20, 50 and 100, $|J|$ = 20, 50 and 100, and $|T|$ = 20, 40, and 100. This set was chosen because of the nature of our network: first, the facilities were interrelated and interconnected, and second, in a given region there were several facilities providing similar services to a particular set of demand. Moreover, if we consider this network to be a managed single authority, then a small dataset would be sufficient to represent the real-life situation. T was restricted to a maximum of 100 units since we were considering hourly or weekly bases for facility operations. For instance, there are 28 wards in Sheffield ($|I|$ = 28) with five HWRCs ($|J|$ = 5). Some centres were operating seven days a week, with a minimum 8 hours of operating time each day ($|T|$ = 7 days x 8 hours per day = 56 hours per week).

Meanwhile, the costs of C_j, ε_{1j}, ε_{2j}, ε_{3j} and ε_{4j} were estimated based on the real-life situation. The operational costs, C_j, was set to be the largest since it is not realistic to assume a cheap operational cost. The arrangement $\varepsilon_{4j} < \varepsilon_{1j} < \varepsilon_{2j} < \varepsilon_{3j}$ was utilised using a pre-set ratio. Setting the cost of travelling between two facility locations (ε_{4j}) to be the cheapest indicated that demand always can go to another facility located within a reasonable travel distance. This was followed by ε_{1j}, which is the cost paid by the provider to serve a unit of demand. Then, the cost to be in the queue, or ε_{2j} was set to be an intermediate cost, where some demand might want to stay in the queue rather move to another facility. Lastly, the second-highest cost is the cost paid by the provider whenever a unit of demand leaves the system, or ε_{3j}. We assumed that whenever a unit of demand left the facility network that this means the demand will divert to a much more expensive facility (that is managed by the same provider) to obtain the service. For instance, due to unavailability of a GP, the demand might go to A&E instead (or leave the GP network), whereas we know A&E is very costly to operate in comparison to a GP visit. Thus, setting ε_{3j} as the second-highest cost after the operational costs, C_j, seems perfectly reasonable. For the multi-component model, α_1 and α_2 were set to 0.5 to balance the cost for both parties. Details of these costs are:

Costs	Value
C_j	500
ε_{1j}	7

ε_{2j}	15
ε_{3j}	30
ε_{4j}	4

CPLEX 12.6 was run on computer with a memory of 8.0 GB RAM, a 2.50 GHz processor and the Windows 10 operating system to perform the experiments. All datasets were created and stored using Excel™ 2016The results are presented in two forms: the single- and the multi-component model.

3.4.2 Results on Solution Times

The results show that the CPU time was influenced by the combination of $|I|$, $|J|$ and $|T|$. Table 3-1 shows the results obtained from the single-objective model, while Table 3-2 shows the results obtained from the multi-component model.

From Tables 3-1 and 3-2, as $|I|$ and $|T|$ increased, on average the number of iterations completed were increased. CPLEX solve Mixed Integer Programming by using Branch-and-Cut; i.e. a search tree that consisting of *nodes* (CPLEX, 2017). From both tables, the number of nodes explored by CPLEX are varies for all instances, probably due to the variation of dataset used. It can be seen that the times were increased as $|I|$ and $|T|$ increased. The increment of CPU time whenever I increased (even though allocation of demand from location i to each facility j at time t was computed separately from CPLEX) is mainly caused because of pre-processing time of the Excel file. The capacity distribution for all facilities was also influenced by the computational times since more options for J for each t can be found. The computational results show the model can be used to solve a larger instance, but would take an increased amount of time to solve. The solution found may depend on the problem characterises, i.e., the costs designated to each variable, the existing capacity, and the relation of distance between j and k; for the computational times, while the length of the time period also plays an important role. Meanwhile, for the five datasets used, on average, the gap is less than 1% (i.e. between 0.00% and 0.33%) showing the results proposed by the CPLEX is optimal. However, different datasets could produce different gaps; i.e. the gap could be optimal or not. Both tables also show when all $|I|$, $|J|$ and $|T|$ is large, the result was not found by the CPLEX i.e. out-of-memory. This clearly shows that the model might need to be solved by using tailored algorithms (such as meta-heuristics) for large instances.

Table 3-1: CPU times for several problem sizes – single-component model

$\lvert J \rvert$	$\lvert T \rvert$	$\lvert I \rvert$	# Constraints	# Binary Variables	# Iteration			Nodes			Optimality gap			CPU Times		
					Min	Average	Max	Min	Average	Max	Min	Average	Max	Min	Average	Max
		20			2,295	2,768	3,795	0	9	43	0.03%	0.15%	0.33%	0.95	1.28	1.77
	20	50	9,982	400	2,456	3,818	5,416	3	39	133	0.02%	0.06%	0.10%	1.05	1.52	1.88
		100			2,266	2,367	2,435	0	0	0	0.00%	0.00%	0.00%	1.49	1.94	2.08
		20			6,153	6,928	8,721	0	30	113	0.01%	0.06%	0.13%	3.30	3.48	3.95
20	40	50	19,982	800	5,490	6,994	9,084	6	20	45	0.05%	0.06%	0.07%	2.63	3.69	4.34
		100			5,301	5,640	5,924	0	0	0	0.00%	0.00%	0.01%	3.86	4.69	5.19
		20			17,283	20,807	27,971	16	58	178	0.01%	0.04%	0.07%	14.56	17.27	19.19
	100	50	49,982	2,000	14,521	18,520	23,823	5	63	212	0.01%	0.04%	0.09%	13.42	15.89	19.77
		100			14,588	14,894	15,479	0	0	0	0.00%	0.00%	0.01%	11.94	14.13	16.95
		20			9,653	12,561	15,536	0	0	0	0.01%	0.01%	0.02%	5.03	6.64	9.67
	20	50	54,952	1,000	11,780	26,542	47,424	0	260	1006	0.01%	0.06%	0.11%	16.72	20.12	24.34
		100			21,869	168,992	512,633	0	839	2176	0.01%	0.02%	0.04%	27.02	68.02	152.78
		20			19,933	22,119	28,994	0	0	0	0.00%	0.01%	0.02%	6.52	13.20	17.11
50	40	50	109,952	2,000	32,321	65,227	88,799	22	226	438	0.01%	0.04%	0.07%	31.64	50.18	66.66
		100			54,115	206,678	659,525	9	352	769	0.01%	0.03%	0.06%	91.86	170.85	321.53
		20			44,236	71,119	105,694	0	175	390	0.01%	0.01%	0.01%	49.00	74.78	114.27
	100	50	274,952	5,000	89,455	237,454	644,567	180	364	542	0.01%	0.02%	0.04%	206.95	298.64	495.34
		100			188,590	513,538	1,380,151	79	319	757	0.01%	0.02%	0.04%	663.84	1145.88	2113.94
		20			16,185	62,568	135,319	0	1043	2438	0.00%	0.01%	0.02%	16.06	59.22	151.81
	20	50	209,902	2,000	17,575	41,568	75,429	0	16	62	0.00%	0.01%	0.02%	22.89	105.40	331.41
		100			40,160	329,777	933,005	14	2501	9603	0.01%	0.02%	0.03%	76.30	365.89	902.61
		20			34,490	105,802	173,505	0	859	2662	0.00%	0.01%	0.01%	51.84	151.84	276.39
100	40	50	419,902	5,000	35,971	55,381	67,031	0	8	33	0.00%	0.01%	0.01%	51.80	112.79	194.47
		100			61,094	300,728	547,856	0	474	1574	0.01%	0.02%	0.03%	237.80	607.29	961.06
		20			88,693	89,001	89,308	0	0	0	0.01%	0.01%	0.01%	95.66	117.22	138.77
	100	50	1,049,902	10,000	out-of-memory											
		100			out-of-memory											

Table 3-2: CPU times for several problem sizes – multi-component model

$\|J\|$	$\|T\|$	$\|I\|$	# Constraints	# Binary Variables	# Iteration			Nodes			Optimality gap			CPU Times		
					Min	Average	Max	Min	Average	Max	Min	Average	Max	Min	Average	Max
20	20	20	9,982	400	2,325	2,965	3,581	0	13	54	0.00%	0.11%	0.32%	0.66	1.00	1.20
		50			2,858	3,492	4,405	3	30	98	0.01%	0.10%	0.16%	0.92	1.25	1.59
		100			2,288	2,354	2,400	0	0	0	0.00%	0.00%	0.00%	1.27	1.52	1.69
	40	20	19,982	800	5,901	7,097	8,328	8	34	100	0.01%	0.16%	0.31%	2.50	2.79	3.20
		50			5,736	6,496	7,900	8	14	30	0.00%	0.07%	0.17%	2.28	3.24	4.88
		100			5,484	5,981	6,202	0	0	0	0.00%	0.00%	0.01%	3.76	4.03	4.41
	100	20	49,982	2,000	12,443	18,435	22,850	0	46	106	0.01%	0.04%	0.09%	7.02	10.73	13.75
		50			14,260	16,750	21,774	7	34	67	0.00%	0.08%	0.12%	8.94	14.13	18.89
		100			14,407	14,784	15,099	0	0	0	0.00%	0.00%	0.01%	10.55	12.71	15.14
50	20	20	54,952	1,000	9,380	11,936	16,375	0	0	0	0.00%	0.01%	0.02%	5.51	6.48	8.19
		50			9,888	24,199	39,277	0	117	265	0.02%	0.07%	0.12%	12.01	15.54	20.00
		100			21,901	129,253	377,687	0	539	1371	0.00%	0.03%	0.06%	25.80	53.39	103.52
	40	20	109,952	2,000	16,111	22,964	30,357	0	7	35	0.00%	0.01%	0.02%	8.89	12.79	16.98
		50			28,638	63,277	99,356	35	231	454	0.02%	0.04%	0.05%	27.61	44.37	50.81
		100			73,099	310,478	1,043,455	30	465	1392	0.01%	0.02%	0.03%	86.56	203.48	530.78
	100	20	274,952	5,000	41,656	68,788	114,876	9	127	186	0.01%	0.01%	0.01%	35.14	60.70	87.19
		50			114,876	249,132	523,519	186	386	731	0.00%	0.02%	0.03%	146.39	231.69	353.03
		100			167,321	724,306	1,983,841	84	479	908	0.01%	0.03%	0.03%	617.06	1135.95	2095.72
100	20	20	209,902	2,000	19,645	61,936	151,361	0	989	2439	0.00%	0.01%	0.02%	13.41	51.32	138.19
		50			19,645	34,645	58,541	0	7	34	0.01%	0.01%	0.02%	21.89	48.08	80.73
		100			30,689	346,527	1,120,889	25	2881	11575	0.01%	0.02%	0.04%	81.70	349.79	976.80
	40	20	419,902	5,000	35,014	287,794	776,915	0	5354	22297	0.01%	0.01%	0.03%	31.05	240.01	542.42
		50			10,026	51,741	98,658	0	89	331	0.00%	0.01%	0.01%	50.88	158.37	261.16
		100			68,191	252,728	536,757	6	367	1094	0.01%	0.01%	0.02%	223.39	609.96	940.24
	100	20	1,049,902	10,000	87,655	145,703	203,750	0	0	0	0.01%	0.01%	0.01%	95.13	143.28	234.00
		50			out-of-memory											
		100			out-of-memory											

Meanwhile, cuts applied by CPLEX as presented in Figure 3-12 and 3-13. There are seven type of cuts used; flow cuts, mixed integer rounding cuts, flow path cuts, zero-half cuts, multi-commodity cuts, lift-and-project cuts and Gomory fractional cuts. Generally, for both figures, the number of cuts for all types of cut applied by the CPLEX are increasing as the dimension of sets increased. The highest number of cuts were recorded at i = 100, j = 50 and t = 100. This occurred probably due to the increment on the number of iterations i.e. more solutions need to be processed by the CPLEX. Meanwhile, no cut was shown for all out-of-memory outputs (when j = 100 and t = 100).

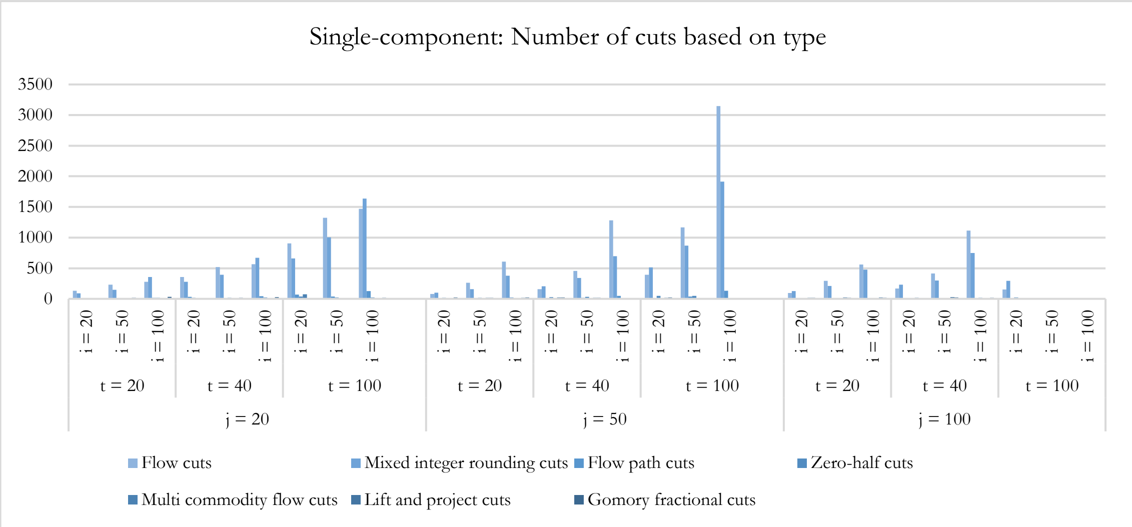


Figure 3-12: Number of cuts applied by the CPLEX for single-component model

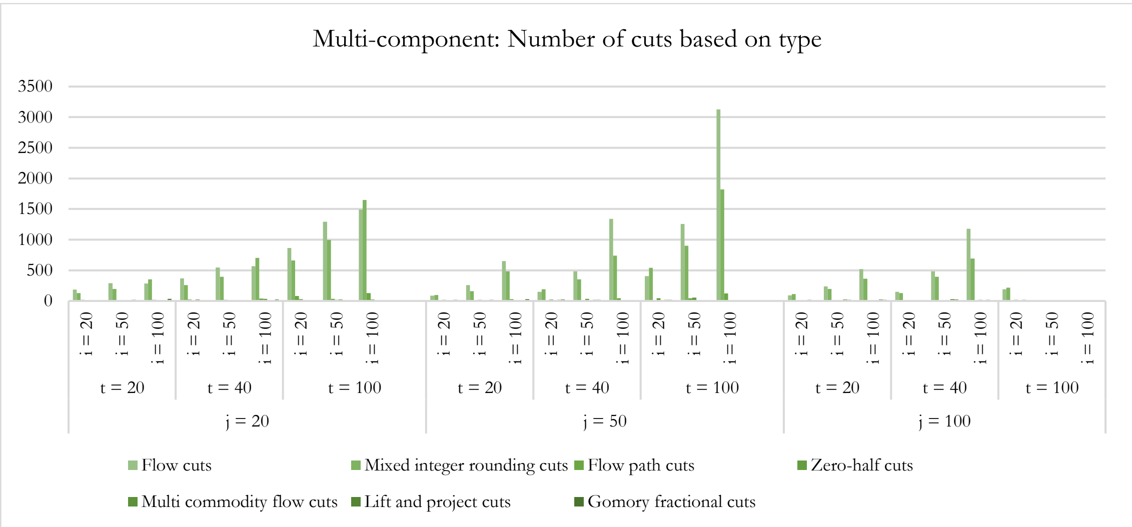


Figure 3-13: Number of cuts applied by the CPLEX for multi-component model

Both parties (provider and demand) are involved in the decision-making process. For instance, a provider locates a facility in a region so as to ensure more demand can be served at minimum cost. Meanwhile, demand visits a facility based on either the minimum travelling costs, good customer service or the minimum distance. These two parties are directly involved in determining the optimal facility location. For this reason, we are interested in considering the decision based on both sides. Therefore, only the sensitivity analyses for the multi-component model will be discussed on the following section.

3.5 Sensitivity Analyses

3.5.1 The Description of the Test Instances

Our intention is to determine the behaviour of demand towards parameter costs and the demand level for each *j* per time-period (x_j^t). Numerical analyses were carried out in order to assess the effects of varying the cost parameters C_j, ε_{1j}, ε_{2j}, ε_{3j} and ε_{4j}. The set of demand locations, *I* was not included in this section of analyses is because we are interested in looking at the behaviour of the demand at each *j* at time *t*, rather than focussing on the origin of the demand. It was assumed that the demand from *i* per time *t*, d_i^t was distributed uniformly across facilities. Hence, the amount of demand at each facility *j* was identical for all times *t*. Meanwhile, the number of facility location *j* was set at 4 and the time-period *t* was set at 5. A small problem size in terms of *J* and *T* eases the observation of the model's performance, especially the flow of demand in the network, either between time *t* or between facility *j*. Details on dataset used are as follows:

Table 3-3: Dataset used in each analysis

Parameters	**Range**
$\sum_t x_j^t$	= [550, 375, 325, 450]
$\sum_t \tau_j^t$	= [500, 500, 500, 500]
Utilisation rate $\left(\% \frac{\sum_t x_j^t}{\sum_t \tau_j^t}\right)$	= [110%, 75%, 65%, 90%]; average = 85%

The capacity level, τ_j^t, was set to 100 units for all facilities *j* at all times. The dataset used has a distinctive utilisation rate per facility *j* with a range of 65% - 110%, to show the uneven congestion in a network. Meanwhile, the weight for the provider's side, α_1, was varied from 0.1 until 0.9 parametrically in increments of 0.1 per step, while the associated α_2 was calculated as $1 - \alpha_1$. For example, when $\alpha_1 = 0.1$, $\alpha_2 = 1.0 - 0.1 = 0.9$, when $\alpha_1 = 0.2$, $\alpha_2 = 1.0 - 0.2 = 0.8$, and so on. The extreme values of 0.0 and 1.0 were excluded by assuming both parties were considered during the decision-making process. Four analyses were conducted:

Analysis 1: Variation of *B*.

To test the impact of *B* on the flow of demand, as this parameter controls the amount of demand leaving the facility network. *B* is also related to producing feasible results for the proposed model.

Analysis 2: Trade-off cost for each decision variable.

To test the impact of the cost of each decision variable on the flow of demand within the network.

Analysis 3: Variation of capacity level (τ_j).

To test the effect of capacity level in terms of system performance, especially on the flow of demand within the network.

Analysis 4: Variation of total operating periods per facility *j* (δ_j).

To test the effect of having similar and distinctive operating periods for each facility *j* in the network on the flow of demand.

For Analysis 1, values of α_1 and α_2 are kept at 0.5, as we are interested in observing the behaviour of the model when *B* is varied. The rest of the analyses focus on the impact of varying the values of α_n on system performance, especially on the objective function and the flow of demand within the network. The flow of demand for each analysis section is presented as a percentage using formulations as in Appendix 3(A).

3.5.2 Variation of B Values

The *B* value is important in terms of limiting the amount of demand that can leave the system, whilst at the same time it must be set at an appropriate level in order to produce feasible results. To test the effect of *B* on the objective function, the following costs for each parameter were used:

Table 3-4: Parameter costs

Parameters	**Values**
C_j	1000 units
$\varepsilon_{1j}, \varepsilon_{2j}, \varepsilon_{3j}, \varepsilon_{4j}$	50 units

The effect of *B* values on system performance is discussed using both α_1 and α_2 values set to 0.5, followed by a discussion on the relaxation of constraint (3-21) and the variation of ε_{3j} values. A value of 0.5 was chosen as the median of the α_1 and α_2 sets.

Table 3-5: Results on system performance by varying the *B*, using $\alpha_1 = \alpha_2 = 0.5$

$\boldsymbol{B}$	**Objective function (total cost)**	$\boldsymbol{Z_1}$	$\boldsymbol{Z_2}$	**Amount of demand left (%)**
0.00	53,750	105,000	2,500	0 (0%)
0.01	53,325	105,000	1,650	17 (1%)
0.02	52,900	105,000	800	34 (2%)
0.03	52,500	105,000	-	50 (3%)
0.04	52,500	105,000	-	68 (4%)
0.05	52,500	105,000	-	85 (5%)

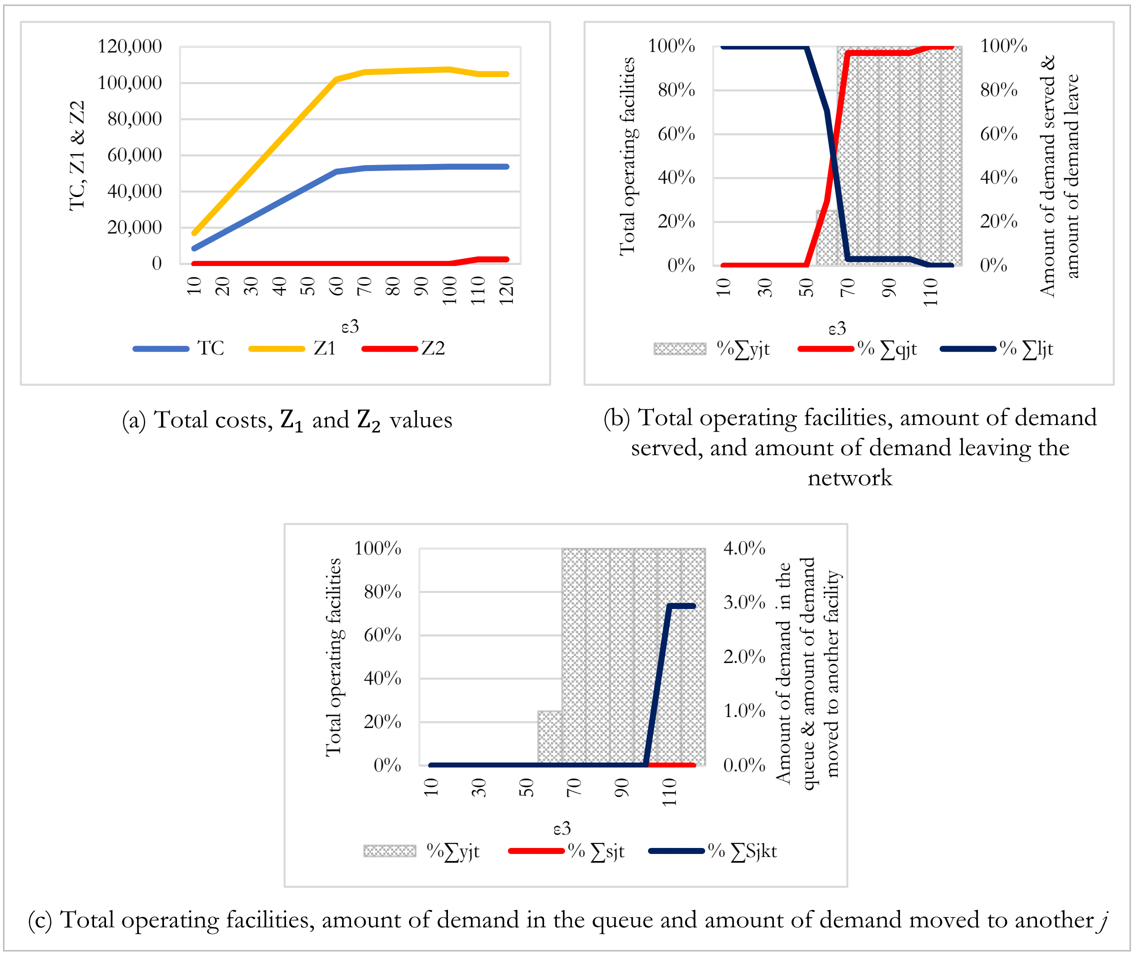
From Table 3-5, as *B* was increased, total costs were only slightly decreased. Total costs are a combination of both sides, the provider (or Z_1) and demand sides (or Z_2). The reduction in total costs was mainly affected by Z_2, as Z_2 was decreased to 0 unit when $B > 0.02$, or 2%. This was probably due to the limited availability of capacity on the network. The dataset used has a different utilisation rate per facility *j* in a network that may or may not be congested. Therefore, as *B* becomes smaller, more movement of demand can be found within the network. The direct relationship between *B* and the amount of demand left showed that it was cheaper to operate the system if more demand left. However, the amount of demand left in the network could also be influenced and controlled by the associated costs per unit of demand leaving the system, ε_{3j}.

To gain a better understanding of the effect of the *B* value, further experimentation was conducted. Recall constraint (3-21):

$$\sum_j \sum_t l_j^t \leq B\left(\sum_j \sum_t x_j^t\right)$$

This constraint limits the amount of demand leaving the system to a maximum of *B*. A further experiment was conducted where constraint (3-21) was relaxed. The experiments used values of ε_{3j} ranging from 10 until 120 units through increases of 10 units per permutation, and where the values of α_1 and α_2 were kept constant at 0.5 and the remainder of costs were kept at constant values. The results of this analysis are presented in Figure 3-14.

From Figure 3-14, it is clear that there are 'no limitations' on the amount demand leaving when constraint (3-21) was relaxed and when ε_{3j} was cheaper than the other costs. Generally, the proposed model 'pushes' all demand to leave since this is, ultimately, the cheapest solution. Similarly, when ε_{3j} is increased, the model 'forces' all demand to be served. Therefore, it is important to set a limit on the amount of demand leaving as we want to ensure that the service performance is maintained, even if this is at its minimum level.



(a) Total costs, Z_1 and Z_2 values

(b) Total operating facilities, amount of demand served, and amount of demand leaving the network

(c) Total operating facilities, amount of demand in the queue and amount of demand moved to another *j*

Figure 3-14: System operations when constraint related to *B* were relaxed, with variations in ε_{3j}

This analysis shows that the constraint related to *B* is important in order to control the service level of the network. Constraint (3-21) also bounds the proposed model from 'pushing' more demand to leave. The associated cost for a unit of demand leaving, ε_{3j} is also important so as to control the amount of demand kept within the network. The next discussion focusses on the trade-off between each cost associated with the decision variables, C_j, ε_{1j}, ε_{2j}, ε_{3j} and ε_{4j}, on the movement of the demand and system performance.

3.5.3 Trade-offs between different cost parameters

An initial value for *B* was determined to ensure feasible results were produced. The datasets, as reported in Table 3-3 (section 3.5.1), and costs, as reported in Table 3-4 (section 3.5.2), were used, and the following results were found.

Table 3-6: Feasibility results

$\boldsymbol{B}$%	Solution
0%	Feasible solution
1%	Feasible solution
5%	Feasible solution

From Table 3-6, it seems that *B* can be as low as 0%, and the system was still able to cater for all the demand. This is probably caused by the condition $\sum_j \sum_t x_j^t \leq \sum_j \sum_t \tau_j^t$ and the availability of extra capacity at nearby facilities *j*. Hence, for our study, 5% was chosen since the target is to service at least 95% of the demand.

Table 3-7 presents the costs' values and their variation for the analyses conducted in this section. As can be seen, the values of the other costs are fixed at one value.

Table 3-7: Parameters variation for each analysis

Case	Cost's values and its variation	Fixed costs values
I	C_j = 200, 400, 600, …, 5000	ε_{1j}= ε_{2j}= ε_{3j} = ε_{4j}= 50 units
II	ε_{1j} = 10, 20, 30, …, 150	C_j= 1000; ε_{2j}= ε_{3j} = ε_{4j}= 50 units
III	ε_{2j} = 10, 20, 30, …, 150	C_j= 1000; ε_{1j}= ε_{2j} = ε_{4j}= 50 units
IV	ε_{3j} = 10, 20, 30, …, 150	C_j= 1000; ε_{1j}= ε_{2j} = ε_{4j}= 50 units
V	ε_{4j} = 10, 20, 30, …, 150	C_j= 1000; ε_{1j}= ε_{2j} = ε_{3j}= 50 units

For Cases II – IV, the cost to operate each facility *j*, C_j, was always higher than all the other costs. At the same time, the weight for the provider's side, α_1, was varied between 0.1 and 0.9 parametrically in increments of 0.1 per step, while the associated weight for the demand's side, α_2, was calculated using $1 - \alpha_1$.

Five analyses were conducted, arranged according to the cost values for each decision variable. The focus for each case was to find the impact on the objective function (total cost), and costs on the provider (Z_1) and demand (Z_2) sides whenever the cost parameters and α_1 were varied. Besides these costs, each case focusses on:

- For **Case I** - C_j and total operating periods of the network $\left(\sum_j \sum_t y_j^t\right)$.
- For **Case II** - ε_{1j} and amount of demand served $\left(\sum_j \sum_t q_j^t\right)$.
- For **Case III** - ε_{2j} and amount of demand in the queue $\left(\sum_j \sum_t s_j^t\right)$.
- For **Case IV** - ε_{3j} and amount of demand left the system $\left(\sum_j \sum_t l_j^t\right)$. Even though the effect of *B* on system performance was discussed in Section 3.5.1, this section focusses on the demand circulation caused by variations in ε_{3j}, i.e., the cost the system has to trade off when a unit of demand leaves.
- For **Case V** - ε_{4j} and amount of demand moving to another facility *j* $\left(\sum_j \sum_t S_{jk}^t\right)$.

3.5.3.1 Varying C_j

C_j represents the cost to operate facility *j*. The effects of varying C_j and α_1 on the total costs, the total costs on the provider's side (Z_1), the total costs on the demand side (Z_2), and the total operating periods for all facilities in the network, $\sum_j \sum_t y_j^t$, are presented in Figure 3-15. The values for Case I were used in this section.

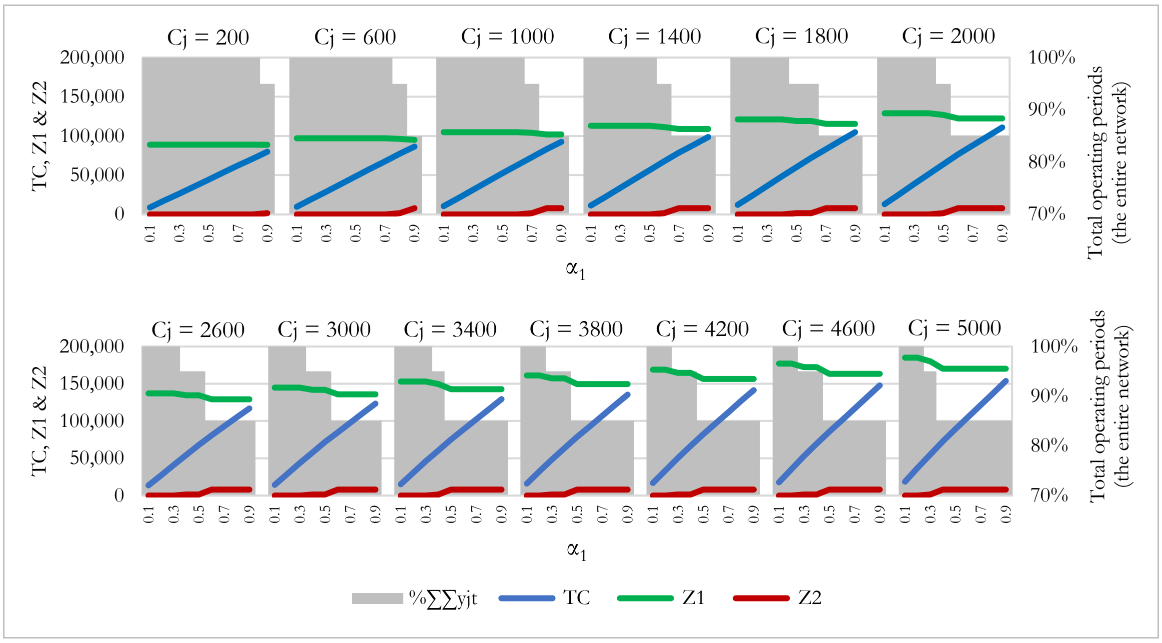


Figure 3-15: System performance, using Case I

From Figure 3-15, when C_j and α_1 were increased, the total operating facilities in the system were decreased. When the costs to operate the facility *j*, C_j, were cheaper, 100% of facilities were operating because certain facilities *j* were congested. As expected, the proposed model will only result in the closure of more facilities when C_j is expensive. Similarly, whenever the weighting on the provider's side, or α_1, was increased, the costs carried by the provider increased, hence the total operating facilities of the network began to reduce.

The total costs (or objective function, represented by the blue line) was increased whenever C_j and α_1 were increased. This is mainly caused by the increment in cost required to operate a single facility *j* becoming expensive, hence causing total costs to increase. The total costs on the provider's side, or Z_1 (the green line), were increased whenever C_j increased. An increase in C_j means an increase in costs to operate a facility *j* per time *t*. One of the components on the provider's side is C_j, hence an increment in provider's costs was mainly due to increments in C_j. However, as α_1 was increased, Z_1 was slightly reduced. The proposed model aims to minimise the total costs for the entire operation. Hence, as α_1 increased, the model effectively forced the dominance of the provider's side. This caused the model to reduce Z_1 in order to keep the model's total cost to a minimum. Meanwhile, total costs on the demand side, Z_2 (the red line), were slightly increased for all C_j and α_1. The increase was obvious whenever α_1 was increased. The increase in α_1 means that the total operating facilities in the network have been decreased, meaning less available capacity. Due to the limited capacity of the network and the need to maintain a 95% service level, more demand was expected to circulate within the network, hence increasing Z_2.

3.5.3.2 Varying ε_{1j}

ε_{1j} represents the cost to serve a unit of demand. The effects of variation in ε_{1j} and α_1 on total costs, total costs on the provider's side or Z_1, total costs on the demand side or Z_2, and the amount of demand served by the facility network, $\sum_j \sum_t q_j^t$, are presented in Figure 3-16. The values for Case II were used in this section.

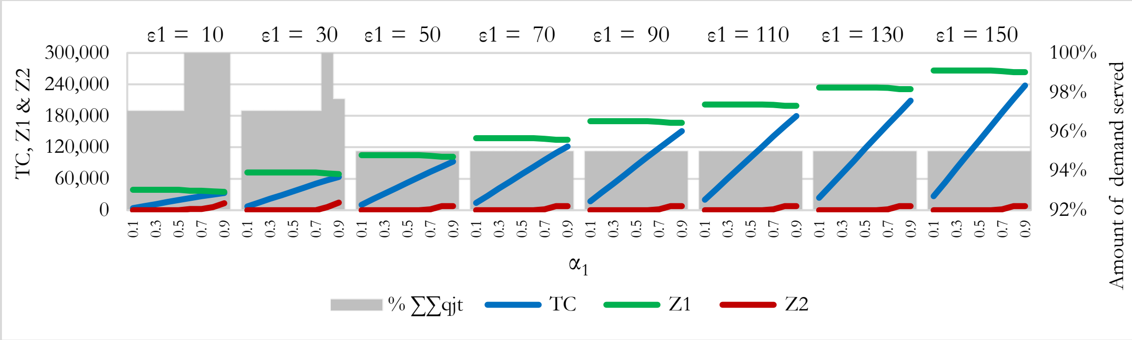


Figure 3-16: System performance, using Case II

When ε_{1j} and α_1 were increased, the amount of demand served by the facility network, $\sum_j \sum_t q_j^t$ (highlighted by grey bar in the chart), was reduced and remained unchanged when reaching 95%. Previously, the analysis of the constraint on the maximum amount of demand that can leave the system, or *B*, shows that the minimum amount of demand served by the network is highly dependent on *B*. For this analysis, *B* was set at 5%, which directly implies that the service level of the facility network must be a minimum of 95% (1 – *B*). For this reason, having a minimum 95% of demand served was predicted. Similarly, the system obliges in serving 95% of demand, causing the total costs (the blue line) to increase whenever ε_{1j} and α_1 were increased.

From Figure 3-16, the costs on the provider's side, Z_1 (the green line), were increased when ε_{1j} was increased, and Z_1 was slightly reduced when α_1 was increased. An increase in Z_1 was expected as one of the components of Z_1 is the cost to serve a unit of demand. As the system needs to serve at least 95% of demand, this causes Z_1 to increase directly. However, the reduction in Z_1 when α_1 was increased was mainly the result of the dominance of the provider in the decision process. As mentioned previously, the proposed model reduced Z_1 in order to minimize the model's total cost to the greatest extent possible. In the meantime, total costs on the demand side, or Z_2 (the red line), were on slightly changed when ε_{1j} was increased. Demand-side costs seemed to increase slightly when α_1 was increased. Whenever α_1 was increased, this directly implies that there is only a limited space to serve the demand. Since the demand must be served to a level of at least 95%, with the limited space it is thus cheaper to circulate the demand within the operating facilities, which causes Z_2 to increase.

3.5.3.3 Varying ε_{2j}

ε_{2j} represents the cost of a unit of demand being in the queue. The effects of varying ε_{2j} and α_1 on the total costs, the total costs on the provider's side or Z_1, the total costs on the demand side or Z_2, and the amount of demand in the queue in the facility network, $\sum_j \sum_t s_j^t$, are presented in Figure 3-17. The values for Case III were used in this section.

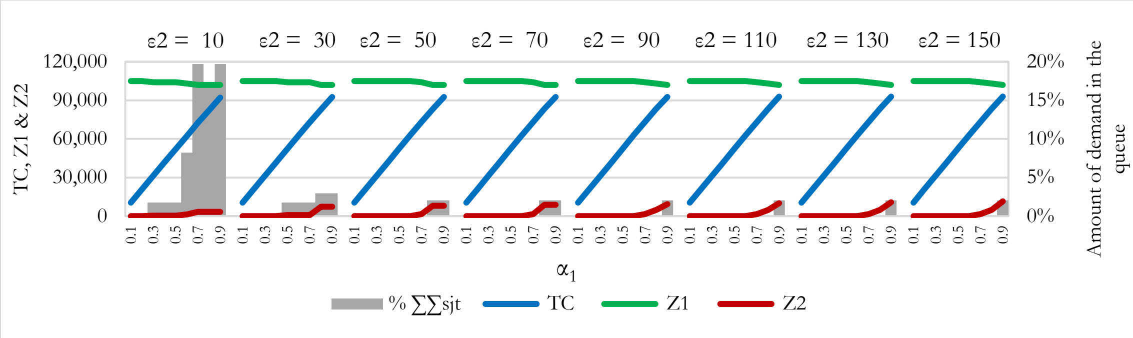


Figure 3-17: System performance, using Case III

Figure 3-17 shows the system performance when ε_{2j} and α_1 were increased. From this figure, it is obvious that there was more demand in the queue, $\sum_j \sum_t s_j^t$ (highlighted in grey bar chart), when the costs to be in the queue, ε_{2j} were low. The amount of demand in the queue, or the queue length, was drastically reduced even when ε_{2j} was increased to 20 units. Clearly, the queue length only reacted when ε_{2j} was cheapest. Similarly, the queue length was somewhat increased when α_1 was large. Therefore, even when ε_{2j} was expensive, the weight on the demand side was at its lowest, hence causing the queue length to increase slightly.

On the same figure, the total costs (the blue line) were increased when α_1 increased. However, for all values of ε_{2j} the total costs pattern remained unchanged. This is not surprising since less demand were in the queue. When ε_{2j} and α_1 were increased, the cost on the provider's side, or Z_1 (the green line), was reduced because of more demand being served in a limited space. Even though the amount of demand in the queue is not a component of Z_1, the increase in α_1 is directly associated with the costs. An increase in α_1 results in fewer facilities being operated, hence causing Z_1 to reduce. Correspondingly, for all values of ε_{2j} and α_1, Z_2 (the red line) was only slightly increased. The increment was noticeable when α_1 was a large number. This was solely due to the increase in queue length. Whenever weight on provider, or α_1, increased, this directly implied that the weight of the demand-related cost, or α_2, was reduced. Even when no queue was found (since ε_{2j} was expensive), the other components of Z_2, such as the costs for demand to move to another facility *j*, were at their cheapest. When having small α_2 values with cheaper movement costs, it is not surprising that Z_2 increased.

3.5.3.4 Varying ε_{3j}

ε_{3j} represents the cost of a unit of demand leaving the facility network. Whenever demand left the network, we assumed that it did not leave the system completely, but rather went to another 'expensive facility'. 'Expensive facility' refers to a facility that is managed by the same provider but it is more expensive to operate. For instance, visiting A&E is more costly (£160.00/patient per visit – as in NHS England, 2018c, p.5) than visiting a GP (an appointment costs on average £30/patient per visit – as in NHS England, 2019). The effects of varying of ε_{3j} and α_1 on the total costs, the total costs on the provider's side or Z_1, the total costs on the demand side or Z_2, and the amount of demand leaving the facility network, $\sum_j \sum_t l_j^t$, are presented in Figure 3-18. The parameter values from Case IV were used in this section. The amount of demand leaving the facility network, $\sum_j \sum_t l_j^t$ (represent in grey in the bar chart), dropped from 5% to 0% whenever ε_{3j} and α_1 were increased. The most demand that could leave, 5%, was bounded by the value of *B*.

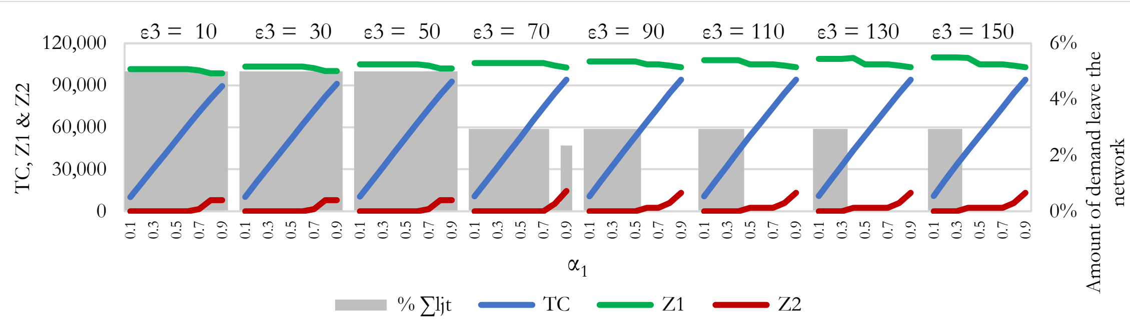


Figure 3-18: System performance, using Case IV

The total costs (the blue line) slightly increased when α_1 was increased. However, the pattern remained unchanged for all values of ε_{3j}. This is not surprising, as there is a high penalty cost for demand leaving the network. Costs on the provider's side, or Z_1 (the green line), were increased gradually when ε_{3j} was increased, but the pattern across the α_1 values shows the opposite effect. This was probably due to the increase in the components of Z_1, namely ε_{3j}. As the model limits the demand that can leave to 5%, this causes Z_1 to increase slowly. However, when α_1 was increased, no demand left, causing Z_1 to reduce slightly. As no demand left the network, 100% of the demand was still served. With limited capacity, the model 'forced' more demand to circulate between the operating facilities. This directly results in the costs on the demand side, or Z_2 (the red line), increasing.

3.5.3.5 Varying ε_{4j}

ε_{4j} represents the cost of a unit of demand moving to another facility. The effects of varying ε_{4j} and α_1 on the total costs, the total costs on the provider's side or Z_1, the total costs on the demand side or Z_2, and the amount of demand moving to another facility, $\sum_j \sum_t S_{jk}^t$, are presented in Figure 3-19. The parameter values from Case V were used in this section.

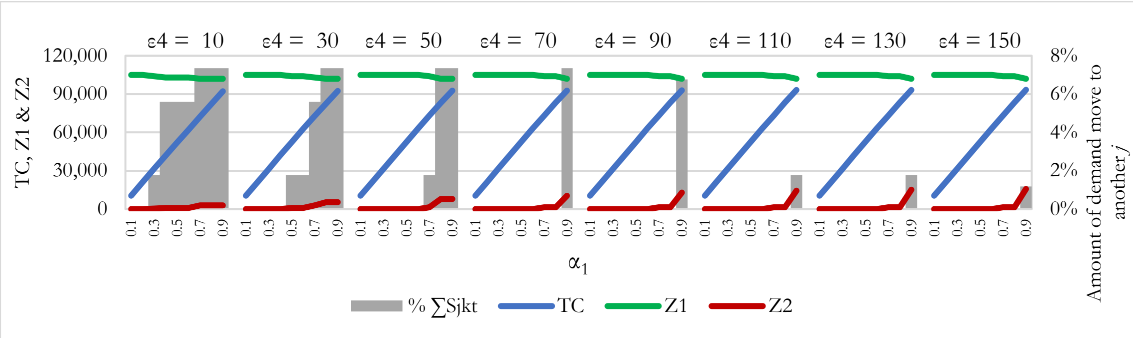


Figure 3-19: System performance, using Case V

Figure 3-19 presents the results for system performance whenever ε_{4j} and α_1 were varied. When both values (i.e. ε_{4j} and α_1) were increased, less demand was expected to move to another facility *j*, $\sum_j \sum_t S_{jk}^t$ (as highlight by the grey bar chart). As expected, when α_1 was increased, the amount of demand moving to another facility was increased. When the weight on the provider's side, or α_1, was increased, weight on the demand side, or α_2, was reduced. It was cheaper to have more demand move to another facility *j*.

Figure 3-19 also illustrates the total costs (the blue line) whenever ε_{4j} and α_1 were varied. Whenever the weight on the provider's side, or α_1, was increased, total costs were increased. However, an increase in ε_{4j} did not result in changes to the total costs of the system. This is surprising as less demand moves to another facility. The costs on the provider's side, or Z_1, were somewhat reduced when α_1 increased and slightly affected by all values of ε_{4j}. Even though the amount of demand moving to another facility *j* is not a component of Z_1, increasing α_1 can be directly associated with the costs. An increase in α_1 can result in less capacity being available, hence causing Z_1 to reduce. At the same time, Z_2 would begin to increase, especially when α_1 was increased. As ε_{4j} becomes more expensive, less demand moves to another facility *j*. Hence, it was cheaper to 'keep' demand within the facility; that is, in the queue, so as to maintain the 95% service level.

From experiments on the trade-off of each parameter's costs, it was found that the proposed model is sensitive to most costs, especially on the provider's side, i.e., the costs to operate facility *j* (C_j), the cost to serve a unit of demand (ε_{1j}) and the cost of a unit of demand leaving the network (ε_{3j}). Whenever one of these three costs is more expensive than the others, the proposed model will 'force' the demand to be in the network. Moreover, the service level needs to be maintained at least 95%. Therefore, whenever one of these costs is expensive, the total costs to operate the facility network will also increase. In addition, the multiplier for costs on the provider's side, i.e. α_1, also highly influences the mechanism of the proposed model. Whenever the α_1 values are increased, the associated costs (i.e., the C_j, ε_{1j} and ε_{3j}) will increase. Since the proposed model aims to minimise the total costs with 95% of service level, this situation forces the demand mechanism within operating facility *j* to vary. In the meantime, the cost of a unit of demand being in the queue (ε_{2j}) and the cost of a unit of demand moving to another facility *j* (ε_{4j}) contribute to an increased flow of the demand within the network. Associated with the demand's side are the α_2 values. For our analysis, only α_1 was utilised. An α_2 value is contradictory to α_1 (i.e., $1 - \alpha_1$). Hence, whenever α_1 increases, α_2 reduces. This small weight will make the associated costs on the demand's side – ε_{2j} and ε_{4j}, cheaper. Hence, causing more demand to circulate within the network. When the lower bound for the amount of demand leaving the system, or *B*, is modified, these two costs are particularly important to maintain the system performance.

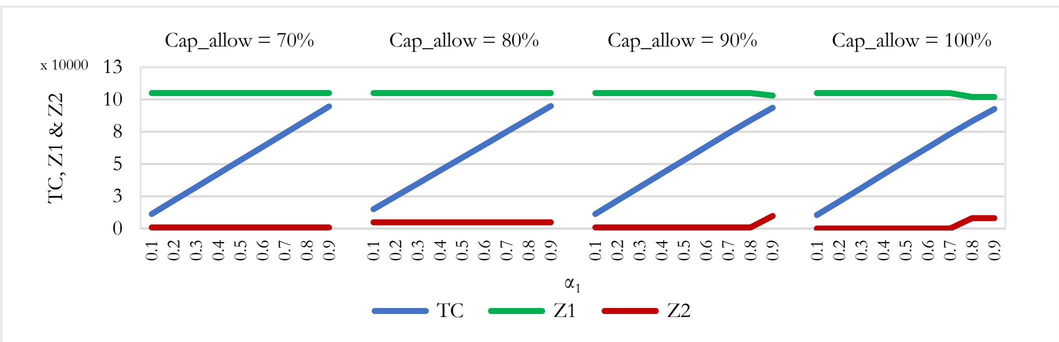
Further experiments were conducted by using the capacity level per facility *j* per *t* (τ_j^t) and limiting the operational periods per facility *j* (δ_j^t).

3.5.4 Capacity Level (τ_j^t)

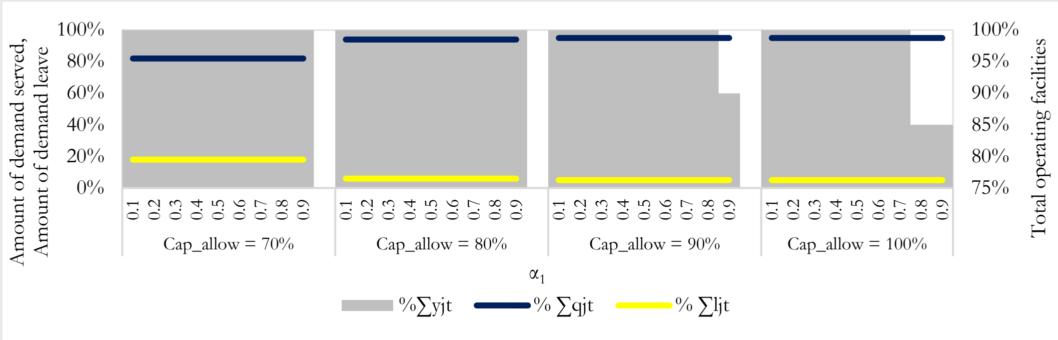
From our work in the previous section, the total number of operating facilities in the network determines the available capacity. The total number of operating facilities were controlled by the cost values set to each decision variable. Instead of focussing on costs, this analysis instead considers the effect of controlling the capacity at each facility *j* at time *t*, or τ_j^t, on system performance. Our interest lies in the flow of demand within the network, whenever the level of capacity or capacity allowance for each facility *j* varies.

Initially, for the experiments above, the capacity levels for all facilities *j* per *t* (τ_j^t) were set at 100 units, or no capacity restriction. This means that 100% of the capacity was allowed to process the demand. For this experiment, it was assumed that the capacity allowance, τ_j^t, was reduced by 10% at a time until a level of 70% was reached. Therefore, the tested capacity allowance, τ_j^t, of a facility *j* per time *t* was 100 units (100% of the capacity was allowed to process the demand), 90 units (90% of the capacity was allowed to process the demand), 80 units (80% of the capacity was allowed to process the demand) and 70 units (70% of the capacity was allowed to process the demand).

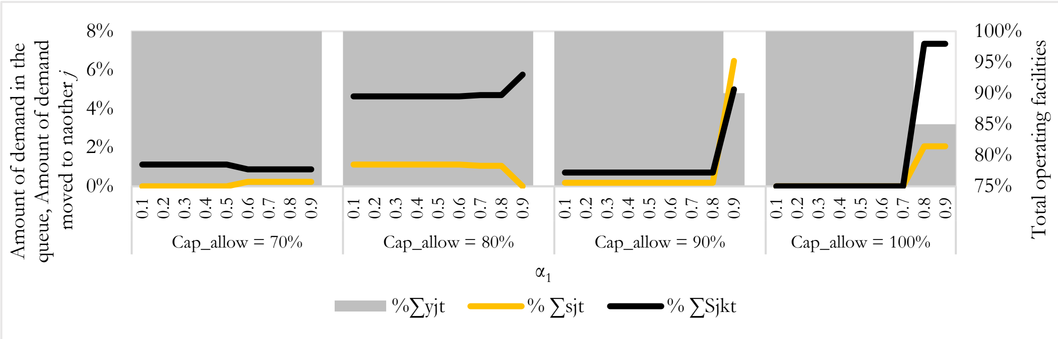
The reduction in the capacity allowance directly implies an increase in the network's average utilisation rate $\left(\frac{\sum_j \sum_t x_j^t}{\sum_j \sum_t \tau_j^t}\right)$. Hence, for each capacity allowance, the network's average utilisation rate was 85% for 100 units of capacity allowance all facilities *j* per *t*, 94% for 90 units of capacity allowance all facilities *j* per *t*, 106% for 80 units of capacity allowance all facilities *j* per *t*, and 120% for 70 units of capacity allowance all facilities *j* per *t*. Note that when the utilisation rate is more than 100%; the facility network is congested. To test for the capacity level's impact on demand performance, the dataset in Table 3-3 (section 3.5.1) and the costs in Table 3-4 (section 3.5.2) were used.



(a) Total costs, Z_1 and Z_2



(b) Total operating facilities, amount of demand served, and amount of demand leaving the network



(c) Total operating facilities, amount of demand in the queue and amount of demand moved to another facility

Figure 3-20: System performance and flow of demand with variations in capacity allowance (τ_j^t) and α_1

Figures 3-20(a) – (c) present the results of this analysis. As can be seen in Figure 3-20(a), when α_1 was increased, the total costs (the blue line) increased. Meanwhile, for all capacity allowances, the pattern of the total costs remained unchanged, probably due to the increase in the weight on the provider (α_1) resulting in more facilities being operated with lower capacities, and thus an increase in the total costs. On the same graph, the changes to the costs on the provider's side, or Z_1, and on the demand side, or Z_2, can be seen. Z_1 was slightly reduced, especially when the capacity allowance and α_1 were large. This was probably due to the facilities being allowed to operate with large capacities, resulting in fewer facilities operating. Hence, this caused Z_1 to reduce. Meanwhile, Z_2 was slightly increased when each facility *j* was allowed to operate at 80% capacity. This increment was probably the result of the amount of demand circulating within the network increasing due to the limited capacity. The flow of demand can be seen in Figures 3-20(b) and (c).

From both Figures 3-20(b) and (c), it can be seen that as the capacity allowance was increased, the total number of operating facilities within the network (grey bar chart) reduced. Obviously, when the capacity allowance for a facility *j* was limited, the proposed model 'forced' a larger number of facilities to operate to ensure that 95% of the demand was being served.

This claim can be clearly seen in Figure 3-20(b). The amount of demand leaving (yellow line) kept reducing as the capacity allowance increased. The highest amount of demand found to leave was 18%, whilst the lowest was 5%. Initially, *B* was set to its maximum of 5%; however, the results produced were not feasible since the capacity was insufficient to maintain the system performance at a 95% service level. Therefore, *B* was increased by 1% per step until feasible results were produced. Meaning, as the capacity allowance is increased, the amount of demand remaining was reduced. In contrast, the amount of demand served (dark blue line) was reduced due to the limited capacity and began to increase as the capacity increased.

From Figure 3-20(c), it can be seen that as the capacity allowance was increased, the queue length (black line) and amount of demand moving to another facility *j* (orange line), had no consistent patterns. It was found that when the capacity allowance was at 80%, more demand circulated within the facility network. This probably occurred due to more demand leaving the system whenever the capacity allowance was small, hence affecting the demand circulation. In addition, when α_1 was large, the total number of operating facilities was reduced. Therefore, due to limited operating facilities, the queue length and amount of demand moving to another facility *j* were significantly increased.

From the analysis given in this section, it was found that the model is obviously sensitive to the capacity allowance. A limited capacity allowance for each facility *j* will cause the model to 'push' more demand to leave. Even when all the facilities were operating, the capacity allowance was insufficient to achieve a 95% service level. With limited operating facilities, less demand circulated within the network (since most of the demand was 'forced' by the model to leave). The model was only responsive when α_1 was large. As the weight on the provider's side (α_1) increased in the decision-making process and at the same time the capacity allowance was increased, fewer facilities needed to operate since this represents a cheaper option for the model. Having fewer operating facilities will cause demand circulation to increase as the model needs to serve at least 95% of the demand. The following section is dedicated to a study of varying the total number of operating periods allowed per facility *j*.

3.5.5 Operation Periods (δ_j^t)

This section focusses on varying the total number of operating periods per facility *j* in the system and its significance for the flow of demand. The purpose of this experiment is a 'what-if' situation for when a decision-maker wants to reduce the number of operational periods, either for a specific facility j or for the entire system. This analysis is also useful in finding which facility is more 'suitable' for closure. The effect of the reduction can be understood, and further analysis can be conducted.

To achieve this, constraint (3-25) was modified: the limit on the total number of operating periods allowed at a facility j was changed (δ_j) to limit the total number of operating periods for all facilities j ($\sum_j \delta_j$). Recall constraint (3-25):

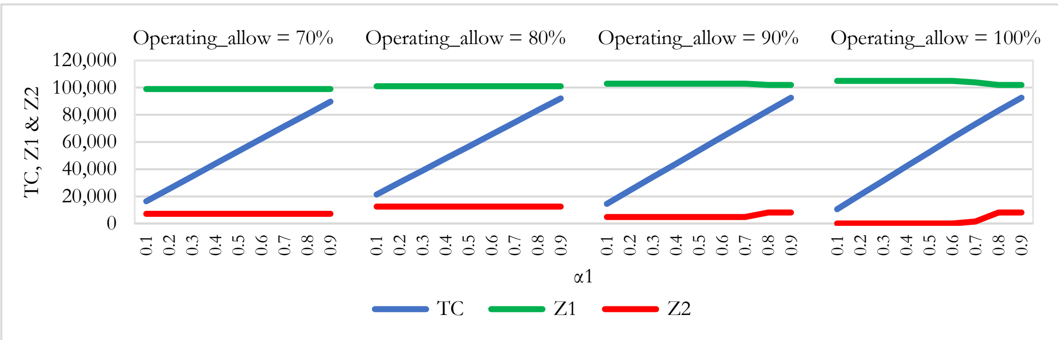
$$\sum_t y_j^t \leq \delta_j \; ; \qquad \forall j \in J \tag{3-25}$$

The modified version is:

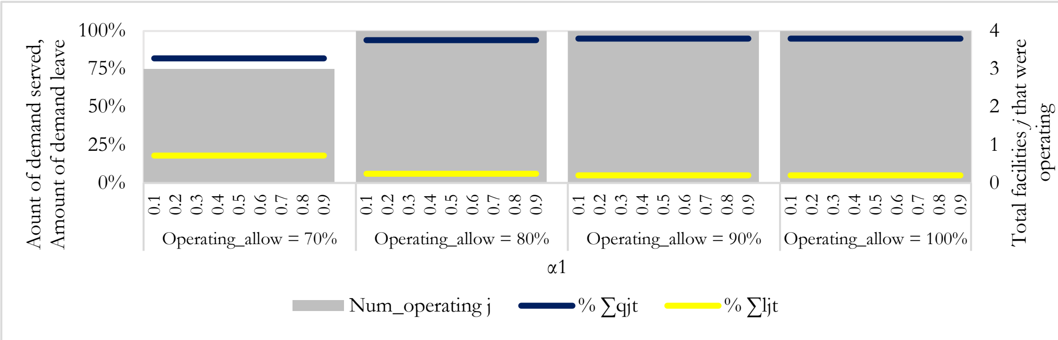
$$\sum_j \sum_t y_j^t \leq \sum_j \delta_j \; ; \tag{3-25*}$$

(3-25*) limits the total number of operating facilities for all facilities j to at most the sum of δ_j. The dataset in subsection 3.5.1 and the parameters from subsection 3.5.2 were used.

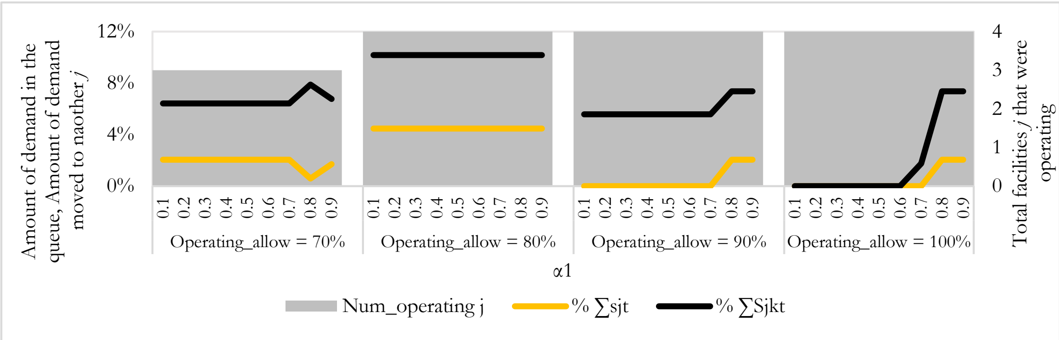
$\sum_j \delta_j$ was tested based on a percentage level, where 100% indicates 20 operating periods were allowed, 90% indicates 18 operating periods were allowed, and so on. Four percentage levels were tested which were 70%, 80%, 90% and 100%. For 70%, this means $\sum_j \delta_j = 14$ periods were allowed, 80% means $\sum_j \delta_j = 16$ periods were allowed, 90% means $\sum_j \delta_j = 18$ periods were allowed, and 100% means $\sum_j \delta_j = 20$ periods were allowed. Meanwhile, the α_1 values were varied between 0.1 and 0.9 in 0.1 step increments.



(a) Total costs, Z_1 and Z_2



(b) Number of operating *j*, amount of demand served, and amount of demand leaving the network



(c) Number of operating *j*, amount of demand in the queue and amount of demand moved to another facility

Figure 3-21: System performance and flow of demand with variations in total operating periods for all facility *j* ($\sum_j \delta_j$) and α_1

Figure 3-21(a) – (c) shows the system performance and demand mechanisms when the total operating periods for all facilities *j* ($\sum_j \delta_j$) and α_1 were varied. From Figure 3-21(a), the total costs did not show a distinctive pattern when the total number of operating periods was increased. The costs began to increase with the total operating facility *j* until 80%, then started to reduce slightly. Meanwhile, Z_1 was slightly increased when the total number of operating periods allowed for the system was greater than 80%. Z_2 was slightly increased when the total number of operating periods allowed were increased until 80%, after which it begun to reduce when the allowance was greater. Interestingly, the changes for the three costs (total costs, Z_1 and Z_2) were obvious when the allowance for the total operating periods was at 80%. Most likely, the system required 80% or more to ensure that the service level was maintained at 95%. A further explanation about this claim is given in Figure 3-21(b) and (c), where the total operating facilities (grey bar chart) were increased when the total operating periods allowed was greater than 80%. From Figure 3-21(b), when the total number of operating periods allowed was less than 80%, fewer facilities were operating. When fewer facilities were operating, the amount of demand served (dark blue line) was less than 95%. Hence, more demand could be expected to move within the network. This can be seen in Figure 3-21(c), where the pattern for movement of demand, i.e., the demand in the queue (black line) and demand moving to another facility *j* (orange line) were increased so as to ensure that 95% of the demand was served.

This section has highlighted the involvement of decision-makers in controlling the operating periods per facility *j* or for the entire system's operations. For example, certain facilities might want to be closed on certain days. If decision-makers want to reduce the total number of operating periods across the entire system, then performing this analysis is important to foresee the demand configuration within the system network. We can say that the model is highly sensitive to the upper bound of the total number of operating periods, but less so to α_1. The model only reacted when α_1 became large.

From the four analyses of this section (section 3.5), the numerical experiments were focussed on the relationships between the demand configurations that were affected by the value of *B*, the decision variable costs, the total capacity levels and the total number of operating facilities. A discussion of the effect of the upper bound for the amount of demand that can leave the system (i.e., *B*) on system performance, especially on the amount of demand served and the amount of demand leaving, is also included. It can be concluded that the proposed model is highly sensitive to the value of *B* and its particular constraint. Additionally, each parameter's costs are directly linked to the cost of the decision variables, for instance C_j is strongly related to the total number of operating facilities. Meanwhile, the capacity constraint is strongly related to demand circulation within the facility network, either through transfers between periods or through transfers between facilities. Details on the numerical results for these analyses can be found in Appendix 3(B). The model was also tested on different datasets. The results obtained were close to the ones we discussed in this section. Details of the numerical results can be found in Appendix 3(C). Meanwhile, the analyses for the single-objective model were also conducted, the results of which can be found in Appendix 3(D).

3.6 Conclusion

A mathematical model for reorganising interrelated and interconnected facilities that focussed on operations and demand circulation, was introduced in this chapter. The proposed model utilised the CLSP model and further modified into a multi-period model. A step-by-step derivation process was outlined. The illustration for solving the model was presented, and other possible solutions were also discussed. In general, the model could be solved using CPLEX 12.6. To test the model's capabilities and configurations, the steps taken to solve the proposed model were investigated through appropriate examples. The computational times required to solve the model using several datasets of varying sizes were analysed, and it was found that the computational times increased as J and T increased. We also analysed the model sensitivity analysis. It was found that the model was highly sensitive to the upper bound on the amount of demand leaving the network (or B). Besides B, the mechanism of demand within the facility network was controlled by the costs of the parameters, limitations on the amount of demand leaving the system and the capacity allowance.

Following on from this work, the application of the multi-period model developed is illustrated using two case studies with different demand characteristics and system operations.

CHAPTER 4: APPLICATION OF THE MULTI-PERIOD MODEL TO HOUSEHOLD WASTE RECYCLING CENTRES (HWRC) IN SHEFFIELD

As discussed in previous chapters, the location of facilities plays an important role in strategic planning activities (Bruno et al., 2014) in both public and private sectors. In some cases, external factors determining the locational choices for such facilities might change over time, such as the introduction of new policies, technology or the needs of society. As such, existing facilities might not be able to provide adequate services (Sonmez & Lim, 2012), thus affecting the optimality of the associated locations. In the previous chapter, a multi-period model was introduced; such model utilises concepts derived from a very well-established class of multi-period problems, in configuring the flow of demand across interrelated, interconnected facilities and time-periods. The proposed model can be used in order to solve real-life problems arising across a variety of sectors. This chapter focusses on the reorganisation of waste management facilities, with specific references to the Household Waste Recycling Centres (HWRC) managed by Sheffield City Council. Given the type of facility under investigation, this chapter also develops a spatial interaction model in order to deal with demand allocation to each recycling facility. Results relating to the reorganisation of HWRC operations are discussed, including benefits and potential risks; such results are compared to a reorganisation plan hypothesised by the decision-makers (Sheffield City Council).

4.1 Introduction to Waste Management

Changes in lifestyle, the increasing population living in urban areas, along with developments in industrial needs are just some of the factors contributing to the increase in the amount of waste produced. Proper management of solid waste is a legal requirement in many countries (Dai et al., 2015), and such management is a challenging process (Pepe, 2008). Types of waste have changed, as materials, chemicals and substances utilised in industrial production; therefore, the challenges posed by the waste management cycle have increased.

Figure 4-1 compares the five-year average for municipal waste generated per person in the EU countries between 2012 and 2016, to the same data for the UK. The average waste generated by the EU countries shows a decreasing trend between 2012 and 2014; however, it subsequently increased from 2015 onwards. The UK, on the other hand, has shown a consistent increasing trend since 2012, and by 2016 the average waste generated was on a par with that of the EU. Increase in municipal waste production is considered normal for an urban region due to the growth in population and in economic activity in such regions (Karak et al., 2012; Hoornweg et al., 2013, 2015). This indicates that the UK is a densely urbanised country with a growing economy, as evident by the increase in average waste generated on a yearly basis. This also calls for better waste treatment, in order to ensure a cleaner and sustainable environment and better air quality (Mendes et al., 2013; Laurent et al., 2014; Rodrigues et al., 2018).

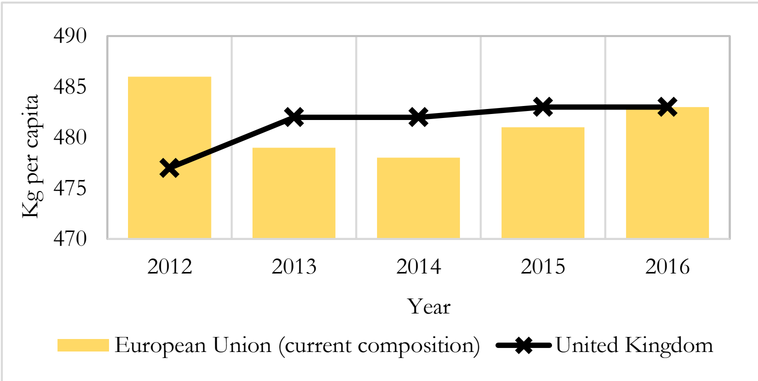


Figure 4-1: The average municipal waste generated per capita (in kilograms) in the EU countries in comparison to the waste generated per capita in the UK (Source: Eurostat, 2017)

4.1.1 Waste Management in the UK

Waste management is a key element in establishing a sustainable environment in the face of challenges such as population growth, increased affluence, and diminishing natural resources. Consequently, British Local Authorities are expected to achieve landfill diversion targets where at least 50% of waste (including paper, plastic, metal, textiles, biodegradable wastes and green wastes) can be re-used and recycled by 2020. In the UK, the Department of Environment, Food and Rural Affairs (DEFRA) bears the responsibility for waste management. This responsibility is further delegated to local authority's (LA) agencies. The waste collected by the LAs will either be recycled, sent to landfill, or incinerated. HWRCs are the facilities provided by councils for the management of special wastes that are not generally collected through the typical kerbside systems. Such facilities are pivotal in order to encourage the transition towards a circular economy, where virtually no waste is sent to landfill or incinerated. DEFRA classifies waste into four major categories: the Household waste, Commercial and Industrial (C&I) waste, Construction, Demolition and Excavation waste (CD&E), and other waste. The 'other' waste generally come from mining, agriculture, forestry and fishing sectors (DEFRA, 2018e). Figure 4-2 shows that in 2016, more than 50% of the waste collected was classified as CD&E, 14% was household waste, 14% C&I waste, and the remaining 13% from other sources.

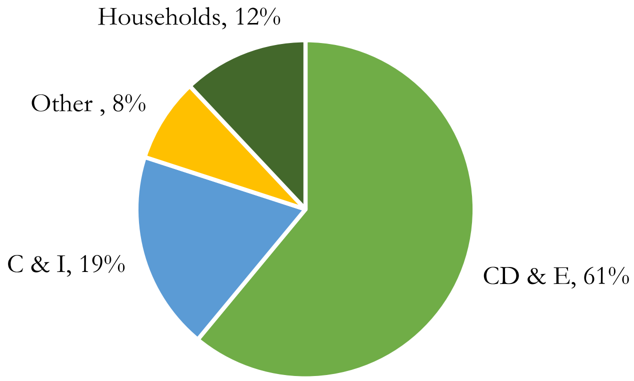


Figure 4-2: Percentages of waste collected in the UK in 2016 (Source: DEFRA, 2018d)

From the waste collected, 49% was recycled; 24% was sent to landfill; 13% was treated and released into water bodies; 8% were backfilled; 3% was recovered as energy and sent to incineration (DEFRA, 2018d).

Figure 4-3 illustrates the concept of a waste hierarchy was introduced by the EU Waste Framework Directive (Europa, 2008) and adapted by the Waste (England and Wales) Regulations in 2011 to minimise the amount of waste disposed.

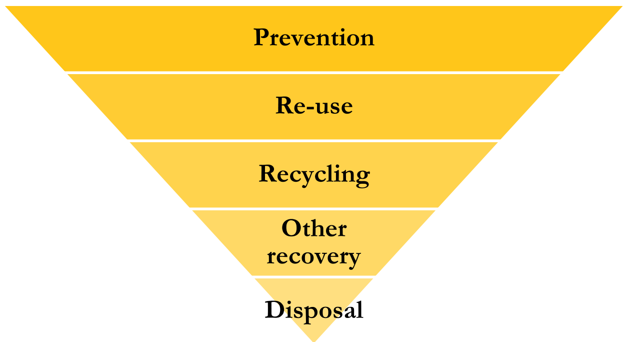


Figure 4-3: Waste disposal hierarchy (DEFRA, 2011)

Avoiding the creation of waste is the main priority for all (either for the industrial or for the public), followed by ensuring that the majority of waste generated can be re-used or repaired. For example, electrical items could be sent for repair rather than for disposal. If neither can be achieved, then attempts should be made to recycle waste, extracting useful resources which can be employed for the production of other goods. For example, lower grade paper could be generated by recycling paper waste.

However, the continuing cuts in funding to the public sector mean that the LAs are facing increasing challenges in terms of cost-effective provision of essential services (Widdowson et al., 2015; Smith & Bolton, 2018). The result is that in many LAs, HWRCs are facing the risk of closure. See, for example, reports about closures in Oxfordshire (reported by Sproule, 2015) and Hampshire (reported by Neal, 2016) and the reduction in opening hours in North Yorkshire (reported by Prest, 2016) and Buckinghamshire (Marino, 2018).

The following are some of the key questions that need to be answered when assessing the suitability of a HWRC network to current demand trends:

> "*What makes people visit a particular HWRC?*"
> "*Which of the HWRCs is the least visited?*"
> "*What will be the impact on the rest of the recycling network if recycling centre A is closed?*".

4.1.2 Treating Solid Wastes

England is predicted to have insufficient landfill capacity in the near future, and furthermore, the cost of dealing with the waste problem in the UK is estimated to reach £47 million per year (Liu et al., 2018). Solid waste management is a multi-billion dollar industry for most industrialised nations, hence it is crucial for any country to recycle and reuse (Gellynck et al., 2011). Recycling has become one of the main issues related to environmental conservation (Gamba & Oskamp, 1994), of which the recycling of household waste accounts for a huge proportion (Chu & Chiu, 2003). These days, public awareness in the importance of recycling has been raising due to the growth of environmental issues, such as global warming, carbon emission and sea pollution. Recycling is deemed to be the key to reducing the amount of waste sent to landfill. Indirectly, this will reduce environmental pollution and methane emissions (Abbott et al., 2011). Re-use of waste includes using second-hand items or remanufacturing (recycling). This has resulted in an eco-friendly method of waste handling, as well as generating income (Gellynck et al., 2011). Additionally, recycling is an important part of the circular economy that benefits both the economy and the environment (L. Smith & Bolton, 2018). The recycling and the reuse of disposed items is undertaken by the responsible authority, either by reselling it to the manufacturer, which contributes to the financial structure of the respective organisation, or as donations to charity shops.

Recycling is part of reverse logistics (Wright et al., 2011). Figure 4-4 illustrates how recycling activities are part of the system network that is also part of the supply chain process; which include the 'forward' and 'reverse' logistics.

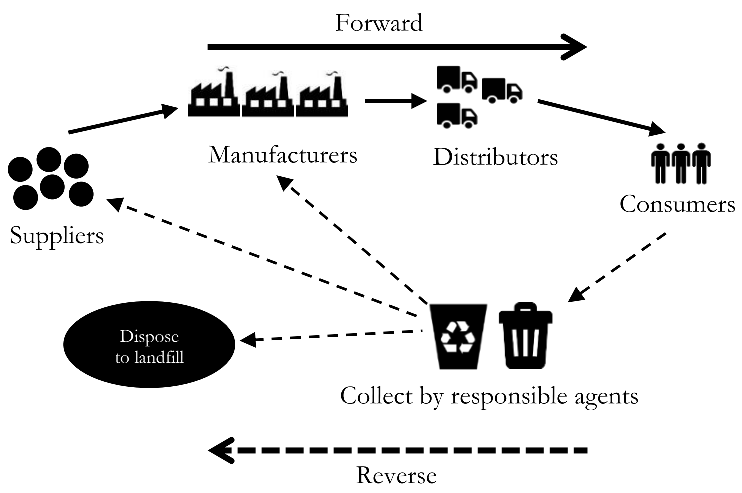


Figure 4-4: Distribution of items in the recycling network

Focussing on the 'reverse' logistics as shown represents by dashed arrow, refer to the activities happen after the sales stage (Reverse Logistics Association, n.d.). This involve the service provided to the customers after the sales, such as customer service, repairs and maintenance services. For instance, Sony Interactive Entertainment provides affordable after-sales service to repair and reuse the PlayStation consoles, even in case of expiry of the warranty date (DEFRA, 2018c).

4.2 Recycling in the UK

The UK government intends to move to a zero-waste economy, which means there should be no net production of waste (DEFRA, 2018a); in order to achieve this objective, the concepts of reduction, re-use and recycling are central. The desire for waste recycling can be generally considered from the perspectives of both economic and ecological factors, which are captured in relevant legislation (Fleischmann et al., 1997). In the UK, it is the LAs statutory duty to collect household waste (WRAP, 2016).

The management of household waste in the UK is based on a two-tier system, composed of Waste Collection Authorities (WCAs) and Waste Disposal Authorities (WDAs). Both tiers can be combined as Unitary Authorities (UAs). WCAs are usually under the control of a borough or a district council. They are responsible for the regular collection of household waste while WDAs are responsible for managing the waste collected by WCAs. However, not all items are disposed of or collected by WCAs. Through the 1967 Civic Amenities Act, local authorities must provide waste facilities known as Civic Amenity (CA) sites for the public to dispose of other forms of waste. These sites accept household waste delivered by the public, with limitations on the quantity and on the type of items that the authority is prepared to accept. The facilities are free to public i.e. no charge for any disposal of waste materials will be applied, as they are funded through local taxation (WRAP, 2016). The role of CA sites has changed over the years as more items can be recycled. Currently, such facilities are known as Household Waste Recycling Centres (HWRCs) – which mainly consist of drop-off areas equipped with waste disposal containers (Waite, 2013). HWRCs are provided by the WDAs for residents in larger regions, such as counties or wards, to dispose of their bulky recyclable waste, again without charge. Any waste that weights over 25 kg and/or waste that does not fit into the household bins provided (Controlled Waste Regulations, 1992) can be classified as "bulky item"; such items can include WEEEs, textiles, furniture (for example, mattresses) and also garden waste. These items may be prohibited from direct collection due to logistical difficulties. Statistically, about 65% of households disposed of bulky items over a 12-month period; in 56% of the cases, HWRCs were the preferred disposal method (Curran et al., 2007). Besides managing the HWRCs and managing the waste collected from WCAs, WDAs are also responsible to manage the waste disposal facilities such as landfill sites. Other differences between WCAs and WDAs are discussed by Abbott et al. (2011) and Cole (2014).

The total recycled waste that was collected in England in the 2017 was about 22.4 million tonnes, of which 44.4% (about 9.9 million tonnes) was household waste (DEFRA, 2018d). In 2016/17, household waste recycling rates across English local authorities ranged from 14% to 65% (L. Smith & Bolton, 2018). These variations were probably caused by the incentives promoted by the LAs themselves in promoting recycling among the public. For instance, Newham London Borough Council had the lowest 'household waste' recycling rate – 14%. This is caused by the dense population in the borough, high diversity level, and small amount of garden waste collected by the Council; all this contributes to the low recycling rates (Slow, 2017).

Household waste are collected either by 'private/voluntary' organisations, CA sites, or by other services managed by private organisations. The composition of this recyclable household waste is illustrated in Figure 4-6. The highest percentage of waste being recycled is that of paper and card (38.8%), followed by glass (19.6%), and other combinations of materials (15%). The items least-sent for recycling are textiles (at 2.5%); this is probably due to their high re-usage value (for example, they can be remade into other textile items or donated to charity).

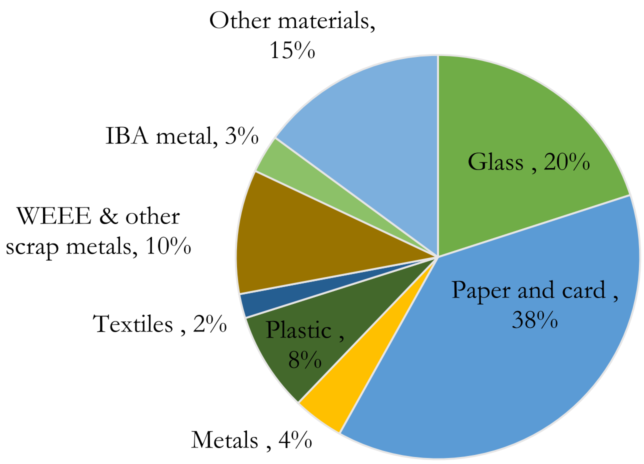


Figure 4-5: Composition of household waste materials (Source: DEFRA, 2018b)

Focussing on CA sites wastes collection, Figure 4-6 illustrates the fact that waste being deposited at CA sites continuously increased between 2013 and 2016, but slightly reduced in 2017 (DEFRA, 2018a). This shows that the awareness of the need to reduce, recycle and re-use has increased over time. Meanwhile, the reduction experienced in 2017 is probably related to the fact that fewer CAs have been in operation due to budget cuts; however, there are no official figures published by DEFRA related to this which can support this correlation.

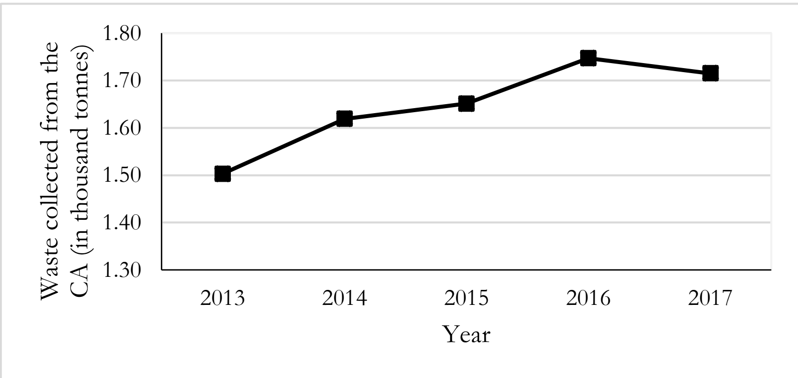


Figure 4-6: Total of household waste collected from CAs (or HWRCs) (Source: DEFRA, 2018a)

4.2.1 Problems Faced by Councils in Managing HWRCs

Household waste is collected from a number of places, including CAs. On average, household waste from CA sites generally shows a year on year increment, but in 2017 there was a slight reduction (Figure 4-5). Such decrease in 2017 was probably caused by the reduction in the overall opening hours of recycling facilities, i.e., closure of recycling points and several HWRCs. In England, as reported by WRAP (2012b) – in 2010/11, there were 734 sites; this number was reduced to 697 sites in 2013/14 (WRAP, 2016). There are no further updates on these figures; however, the trend does not seem to have been altered.

HWRCs are one of the public services provided by LAs for residents to properly manage waste, while at the same time increasing recycling rates. Recycling is an act of good practice that is significantly influenced by a willingness to participate (Barr et al., 2003; Barr, 2004) and accessibility to appropriate facilities to do so (Barr, 2007). At the same time, waste is a by-product of human activity (Periathamby, 2011) that contains a mixture of several types of materials. This makes the process of reorganising and separating the collected waste somewhat challenging. Thus, it is necessary for the public to receive adequate instructions to ensure that the waste they dispose of can be recycled (Bhat, 1996; Cole et al., 2011). The top two councils and boroughs; i.e. East Riding of Yorkshire and Rochford in Essex, in terms of recycling rates 2017 showed that through proper management and public participation, most household waste can in fact be recycled (Slow, 2017).

Recycling is an act involving both parties – the providers and users. Achieving efficient waste collection and separation involves not only site users but also other parties such as well-trained staff and the local authorities (Saphores et al., 2006; Sidique et al., 2010). Due to the financial pressure caused by budget cuts, LAs need to find more efficient ways to manage their HWRCs. However, the public is increasingly expecting more from HWRCs in terms of a broader range of materials accepted for recycling, well-trained staff and a more enjoyable site service (WRAP, 2016). Thus, good customer service is one of the crucial elements for ensuring an efficient and well-managed HWRC. Staff training is also needed to increase motivation. Increasing public awareness of recycling can also help to increase recycling rates.

The performance of a HWRC is measured through its recycling rates and site-user surveys (WRAP, 2016). Recycling rates are influenced by the materials accepted for recycling, the location and layout of the HWRC, and the assistance and service provided by their staff (WRAP, 2016). WRAP (2016) noted that diversified recycling portfolios attracted users to go to specific HWRCs, along with user-friendly, split-level designs, which also have a positive impact on recycling rates. Other factors that can boost recycling rates are ground-level access to deposit recycling materials, whilst clear signage with suitable pathways increases site accessibility and reduces disruptions. Cunningham and Conroy (2006) pointed out that vehicle movements and users' commercial permits are two major factors that need to be considered in the design of HWRCs. For example, in the case of Bristol Avenue HWRC in Blackpool, electronic signalling systems for detecting users' commercial permits have been introduced, making the site more attractive to potential users (Cunningham and Conroy, 2006).

Even though HWRCs are dedicated to household waste, a certain amount of commercial waste can nevertheless be found at HWRCs, generally varying from 4 to 9% (Woodard et al., 2004). This might increase the sites' congestion problem, take a lot of space and affect the recycling rates of these sites. Various programmes have been implemented to increase waste recycling rates. For example, a collaboration between major retailers and the Waste and Resources Action Programme in 2005 to produce and implement new packaging technologies (Jamasb & Nepal, 2010), as a result of which 70.2% of UK packaging waste can now be recycled and recovered (DEFRA, 2018e). Recycling or reuse rates of bulky waste from HWRCs strongly depends on the type and condition of the bulky waste. For an example, hard furniture re-use rates are between 9 – 54%, while soft furniture re-use rates are between 13 – 20% (Alexander et al., 2009). Almost 60% of users dispose of their bulky waste at HWRCs (Curran et al., 2007) and approximately 35% of this waste is reusable, either being in good condition or requiring only slight repairs (WRAP, 2012a). Reuse of bulky items is not only environmentally friendly but also brings social benefits through the community's or charity groups' participation in terms of employing volunteers, in addition to benefitting the recipients of such items (Sharp & Luckin, 2006).

The collection and transport of recyclable materials accounts for 75 – 80% of the solid waste management budget (Bhat, 1996). There is no monetary incentive for households to minimise the amount of waste they produce or to increase their recycling rates. Budget cuts contribute to low recycling rates (Smith & Bolton, 2018). Fiscal measures introduced to improve recycling performance have been directed at LAs rather than householders (Abbott et al., 2011). This includes managing centres with a suitable number of staff and the creation of an optimal schedule. Failure to properly manage HWRCs could lead to permanent site closure. The insufficient number of recycling facilities is one of the many reasons for the increase in the number in fly-tipping cases (Webb et al., 2006; Smith, 2018), which indirectly results in an increased cost of disposing of waste in landfills. Fly-tipping is the illegal dumping of waste, and which often causes environmental pollution. Fly-tipping has increased considerably over the years. Figure 4-7 shows the total incidents per year; the composition of the waste being fly-tipped over a period of five years from 2012 until 2017 is shown in Figure 4-8. The major contributor here is household waste (about 67%), which has increased by 8% since 2015/6.

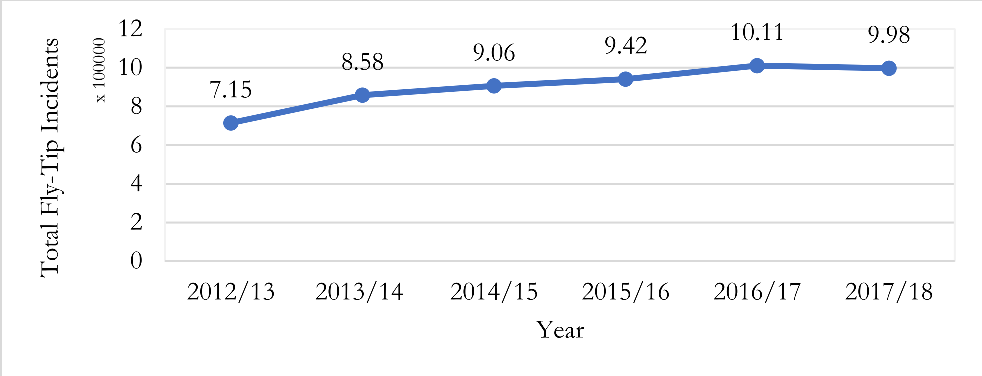


Figure 4-7: Total fly-tipping incidents reported per year (Source: DEFRA, 2018b)

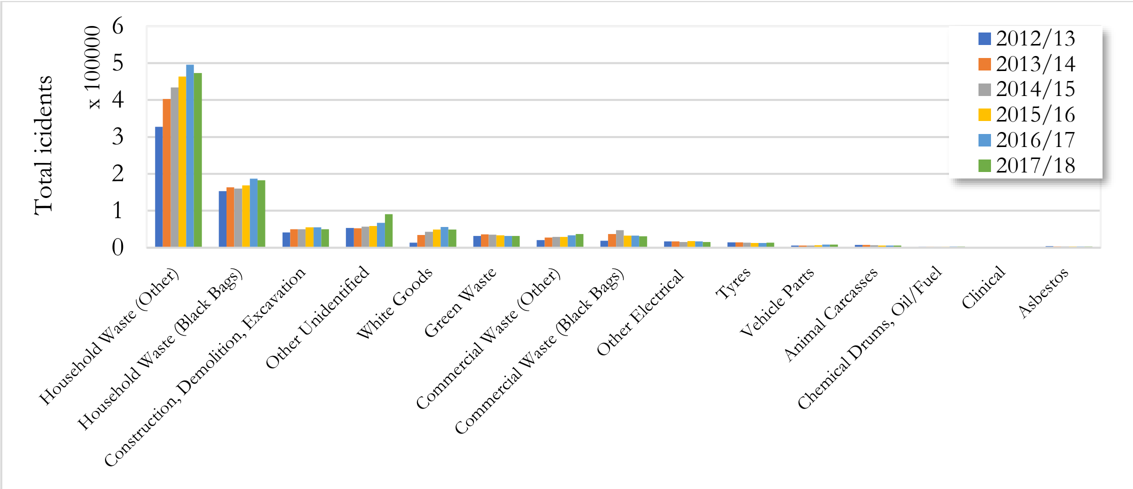


Figure 4-8: Composition of waste in fly-tipping incidents (Source: DEFRA, 2018b)

The highest total incidents were reported in 2016/17. In that year, DEFRA reported that the estimated total of cost related to the clearance of such incidents was £57.7 million (on average of £58 per incident) and the enforcement cost was £16 million (on average of £33.75 per enforcement) (DEFRA, 2017). In the same report, the most common enforcement action taken by local authorities is the one related to the issuing fines. The number of total fines for the 2016/17 year shows a decrease; however, the total amount of fines (£723,000 for 1,318 cases) represents an increase from 2015/16 figures (1,838 fines issued).

The proper management of HWRCs is crucial. These facilities not only accept and recycle the special discarded materials, but they also assist in the transition towards a circular economy. Even though many recycling site improvements can be made without incurring in financial cost, as highlighted by WRAP (2016) there are nevertheless some authorities who are considering closing HWRCs due to financial pressures. For example, in 2018, Buckinghamshire County Council planned to reduce their operating hours or completely close one or two of its recycling sites, even though the sites are well used (Marino, 2018). It is apparent that, in most of the cases, Local Authorities lack adequate planning tools in order to identify rationalisation plans which, if implemented, could return some financial savings without compromising the ability of the HWRC system to provide an adequate service level to users.

As such, in order to bridge this gap, and provide a useful tool to Local Authorities, the following section describes the adaptation and implementation of the multi-period model for facility networks rationalisation introduced in the previous chapter to the case of the HWRC network of Sheffield. Based on the current problems faced by the council in managing the HWRC network, our model is capable of reorganising facility network by ensuring that a required service level can be provided to users even in presence of budget cuts.

4.3 Sheffield – Brief Background and the Current HWRCs

Sheffield is a town in the Yorkshire and Humber region; specifically, it is in the county of South Yorkshire. It is the sixth-largest city in the UK and is known as 'Steel City' due to its previous focus on the steel industry, though it currently homes a diverse set of industries ranging from manufacturing to call centres. Details of Sheffield's demographic information and its area are given in Table 4-1 and Figure 4-9.

Table 4-1: Demographic information

Population		556,058
Census of age:	Under 25	231,755
	25 – 49	149,764
	50 – 64	88,841
	65 – 79	61,488
	80 and over	24,210
Number of wards*		28
Number of districts*		206
Number of households		229,922

Data gathered from National Statistics (2016) website (Office for National Statistics, 2016)
*Notes: as in Figure 4-9

Figure 4-9: Sheffield's wards (source: Sheffield 2016 City Council Ward and Polling District Maps)

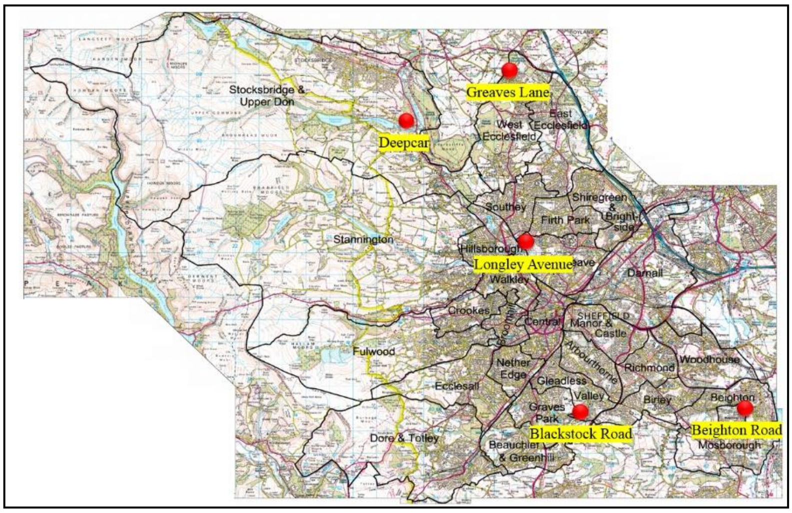
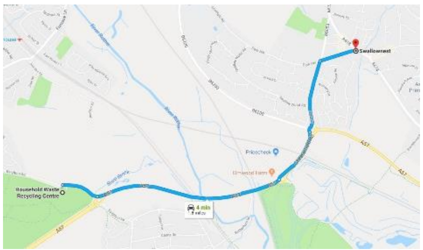


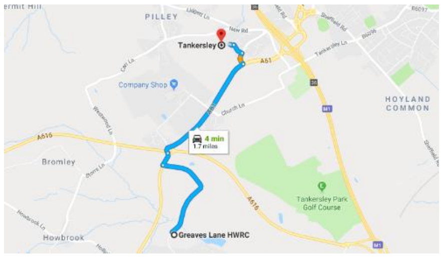
Figure 4-10: Map of Sheffield showing HWRC locations

Sheffield City Council is responsible for the management of the waste cycle in the city, in collaboration with its main contractor (the French MNE Veolia); it currently provides and manages five HWRCs. The site locations are as shown in Figure 4-10, and specifically are at Longley Avenue, Beighton Road, Blackstock Road, Deepcar and Greaves Lane. Besides catering for "normal" household waste, such facilities can be used to dispose of materials that are not accepted in kerbside collection or at neighbourhood recycling points; this can help in handling potentially hazardous and hard-to-treat waste.

From Figure 4-10, it can be seen that Longley Avenue is located towards the centre of the city council's area coverage, while the other four sites are located near to the council's borders. In particular, Greaves Lane, Deepcar, Blackstock Road and Beighton Road are near to the edge of the council's authority, and both are easily accessible by residents who live outside the Sheffield City Council area. This is illustrated in Figure 4-11(a) where, for example, Beighton Road is easily accessed by non-Sheffield City Council districts such as the Swallownest ward (which belongs to the Rotherham Council area), which is also only four minutes away. Likewise, some of the districts in the Tankersley ward (which is managed by Barnsley Council) are just four minutes away from Greaves Lane (as it can be seen Figure 4-11(b)).



(a) Distance between Beighton Road Recycling Centre and Swallownest



(b) Distance between Greaves Lane Recycling Centre and Tankersley

Figure 4-11: Distances between HWRCs and the areas that are non-authorised by Sheffield City Council.

The amount of waste (based on weight) received by the five HWRCs in Sheffield is provided in Table 4-2. The highest percentage of items deposited is represented by recyclable items; residuals and brick rubble are disposed of in similar amounts. The highest percentage of recyclable materials received by HWRCs was in 2010; but this reduced slightly subsequently. The percentage of brick rubble deposited continued to increase until 2011, but then reduced in 2012. In contrast, the percentage of non-recyclable waste (amount of residual) deposited by HWRCs' users continued to reduce until 2011, but in 2012 this figure subsequently increased; this shows that users need to be more aware of the items that can be deposited at an HWRC. The average composition of materials deposited at Sheffield's HWRCs in 2012 is illustrated in Figure 4-12.

Table 4-2: Percentage of different materials deposited at all HWRCs in Sheffield from 2008 – 2012 (source: Sheffield City Council, 2012)

Materials Received	2008	2009	2010	2011	2012	Average
Total Recyclables	56.5%	57.5%	58.0%	57.0%	55.1%	56.8%
Brick Rubble	24.6%	24.9%	25.7%	27.8%	25.4%	25.7%
Total Residuals	18.9%	17.5%	16.3%	15.2%	19.6%	17.5%

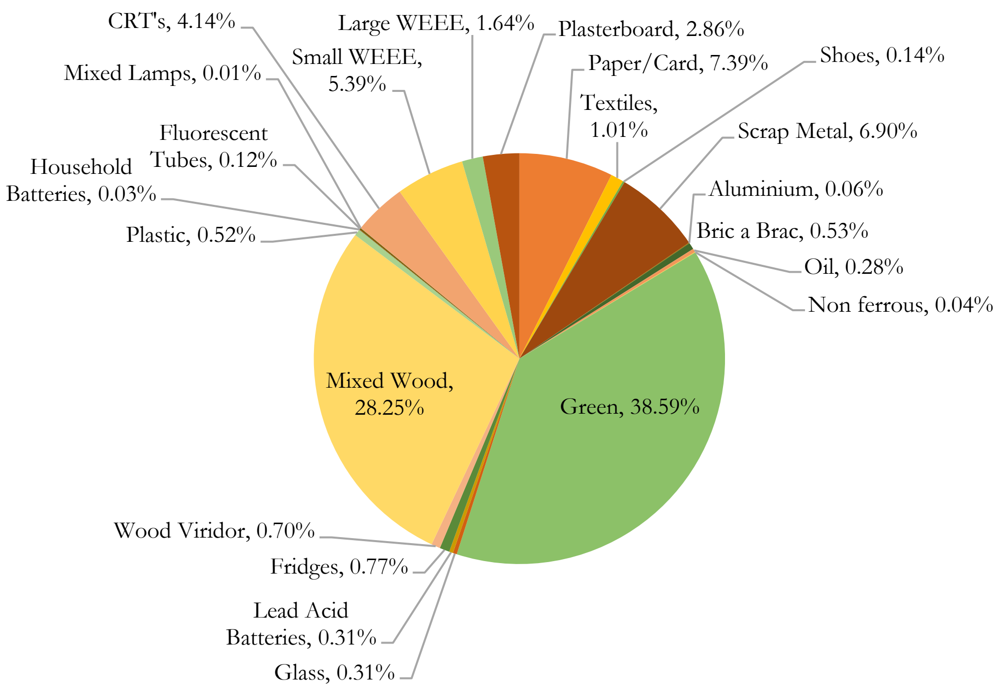


Figure 4-12: Composition of the recyclable of household waste received by all five HWRCs in Sheffield in 2012 source: Sheffield City Council, 2012)

The highest proportion of waste being recycled is represented by green waste (38.59%), followed by mixed woods (28.25%), whilst other recyclable materials being deposited account for less than 10% of the total. This is probably because the other materials besides greens and mixed woods are collected through the wheelie bins provided by the council or disposed of at local recycling points, hence explaining the small amount deposited in all HWRCs.

The operational days for each HWRC vary, while the operational hours are dependent on the season. The 'tick' symbol in Table 4-3 indicates that the facility in question operates on a particular day. Longley Avenue operates seven days a week, the Blackstock Road and Beighton Road centres operate six days a week, while Greaves Lane and Deepcar operate for five days a week. The schedule ensures that for each day, users can find at least two operational centres. The operating hours for all centres are between 10.00 a.m. and 6.00 p.m. in the summer and 10.00 a.m. until 4.00 p.m. in the winter.

Table 4-3: The schedule of HWRC

HWRC / Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Beighton Rd	√		√	√	√	√	√
Blackstock Rd	√	√		√	√	√	√
Deepcar	√			√	√	√	√
Greaves Lane	√	√			√	√	√
Longley Avenue	√	√	√	√	√	√	√

The recyclable items that can be received by each HWRC is vary. The recycling portfolio (types of materials accepted) at the current five waste recycling centres (as gathered from Sheffield City Council) is reported in Table 4-4.

Table 4-4: Recycling portfolio (types of materials accepted) for each HWRC

HWRC	Recycling portfolio
Beighton Road	23
Blackstock Road	26
Deepcar	23
Greaves Lane	24
Longley Avenue	22

These figures show that even though the Longley Avenue recycling centre operates seven days a week, the range of recyclable items it accepts is quite limited compared to the other centres. For instance, the only centre that can receive household chemicals is Longley Avenue, but this centre does not accept materials containing asbestos (VEOLIA, n.d.).

Table 4-5 and Figure 4-13 represent the user distribution per day and the proportion of users per hour for all the HWRCs in Sheffield. The distribution is based on data gathered from the city council, where on average the amount of users processed by all HWRCs per week is 14,069. The lowest amount of users is reported on a Wednesday, when only two recycling centres are open (yellow shaded box – see Table 4-5). Meanwhile, the highest amount of users is reported on a Sunday (blue shaded box – see Table 4-5). The preferred time for users to visit recycling centres is during the morning session; lower amounts of visits are reported towards noon, with an increase in the 2.00–3.00pm interval. The amount of users starts to decrease from 3.00pm until the facilities are closed.

Table 4-5: The average percentage of users visiting HWRCs per day per period (from 10 a.m. until 6 p.m.)

Hours	Time-period, t	Average percentage of visits per hour	Average percentage of visits per day							**TOTAL**
			Mon	Tues	Wed	Thurs	Fri	Sat	Sun	
			0.1615	0.1008	0.0916	0.1451	0.1552	0.1643	0.1815	**1.0000**
1000 – 1100	1	0.1441	0.0233	0.0145	0.0132	0.0209	0.0224	0.0237	**0.0262**	**0.1441**
1100 – 1200	2	0.1437	0.0232	0.0145	0.0132	0.0208	0.0223	0.0236	0.0261	**0.1437**
1200 – 1300	3	0.1393	0.0225	0.0140	0.0128	0.0202	0.0216	0.0229	0.0253	**0.1393**
1300 – 1400	4	0.1350	0.0218	0.0136	0.0124	0.0196	0.0210	0.0222	0.0245	**0.1350**
1400 – 1500	5	0.1399	0.0226	0.0141	0.0128	0.0203	0.0217	0.0230	0.0254	**0.1399**
1500 – 1600	6	0.1299	0.0210	0.0131	0.0119	0.0189	0.0202	0.0213	0.0236	**0.1299**
1600 – 1700	7	0.1002	0.0162	0.0101	0.0092	0.0145	0.0155	0.0165	0.0182	**0.1002**
1700 – 1800	8	0.0679	0.0110	0.0068	**0.0062**	0.0099	0.0105	0.0112	0.0123	**0.0679**
TOTAL		**1.0000**	**0.1615**	**0.1008**	**0.0916**	**0.1451**	**0.1552**	**0.1643**	**0.1815**	**1.0000**

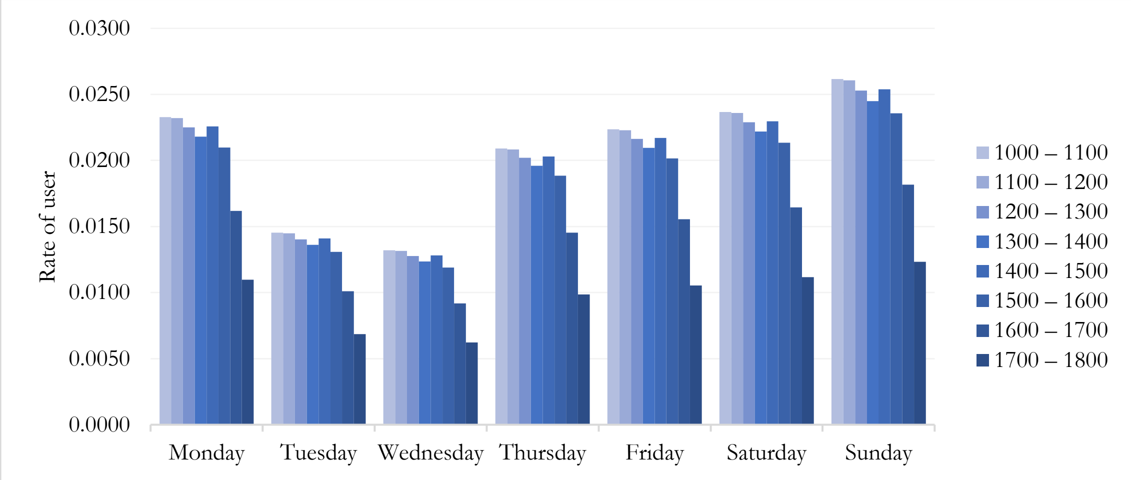
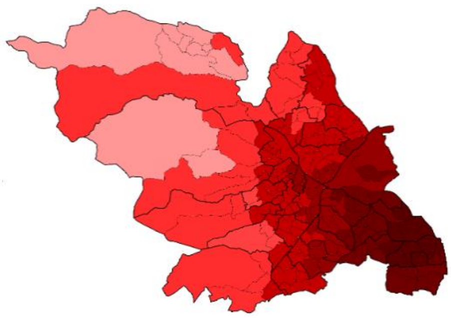
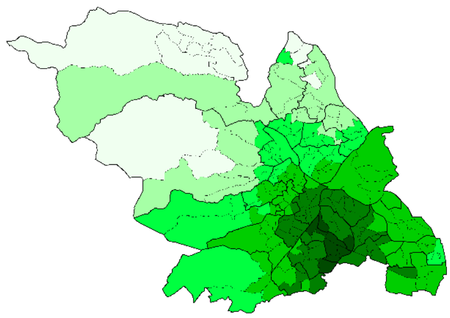


Figure 4-13: Rate of user per day

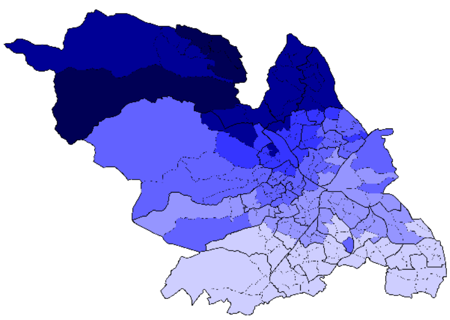
Figures 4-14(a) – 4-14(d) show the distance between each district and each HWRC and illustrate the reachability of each HWRC in each district. The data was provided by Sheffield City Council. The darker the shade, the greater the reachability of the recycling sites from the district. For example, from Figure 4-14(a), the HWRC in Beighton Road is located in the south-eastern area of Sheffield; hence the surrounding wards are darker red in colour whilst wards that are located further north are shown in lighter red. Travel times between each ward and each HWRC can be found in Appendix 4(A).



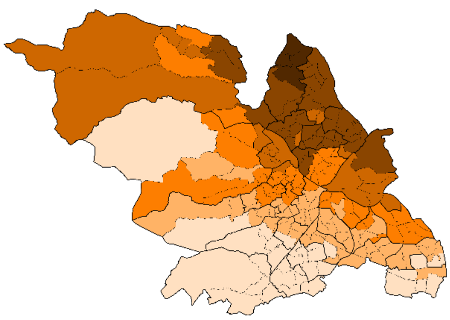
(a) Distance i to Beighton Rd



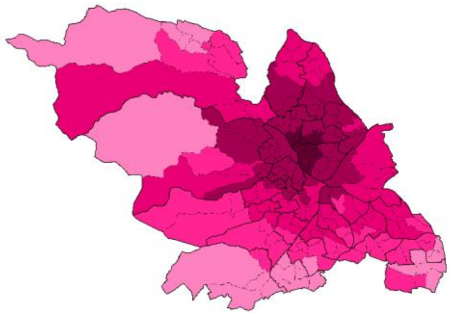
(b) Distance i to Blackstock Rd



(c) Distance i to Deepcar



(d) Distance i to Greaves Lane



(e) Distance *i* to Longley Avenue

Figure 4-14: Travel distance between each district and each given HWRC (Source: Sheffield City Council, 2012)

From Figure 4-14, hypothetically the users of each district have the access to the nearest HWRC. However, the allocation of users to each HWRC follows a non-dictatorial assignment; users are free to go to any centre that they prefer. Moreover, there is no mechanism for tracking access at sites, and determining the origin of users. As such, there is lack of information about the origin of users. For these reasons, before implementing the multi-period model described in Chapter 4, there is the need to formulate a mechanism for describing the process ruling the choice of the HWRC facility by users. The following section shows how the spatial interaction model can be utilised to capture user flow from a given ward to each HWRC.

In this chapter, the term 'user' was used to replace term 'demand' in the description of the model, in order to better describe the tackled problem.

4.4 Adapting the multi-period model to the HWRC Problem

4.4.1 Compatibility of HWRC Problems with the Proposed Model

Due to a lack in financial resources, Sheffield City Council intended to undertake a rationalisation of the HWRC facilities network operating within its boundaries. Possibly reducing operating hours, downsizing (or even closing) existing facilities, the remaining HWRCs would then be expected to provide sufficient service to Sheffield's residents. For this reason, our proposed multi-period model is a suitable choice for assisting the Council in the reorganisation of its HWRC operation. As explained previously, this section begins with the explanation of the mechanism employed for the allocation of users to each HWRC.

4.4.2 User allocation to facility

HWRCs the recycling facilities provided by the council for users to dispose of their recyclable materials. This service is free and available to anyone that resides within the Sheffield City Council authorised area. Users are free to visit any preferred facility, regardless of the area they live in. Hence, it is important to reproduce the mechanism which guides users' preferences in their choice of HWRC. Similarly to Baotai (2015), a short survey was conducted among Sheffield's residents to investigate the preferred criteria choosing the recycling centres; survey questions are reported in Appendix 4(B). Such survey will be utilised in order to calibrate the functioning the spatial interaction model. Details of the survey, of the designed spatial interaction model and of its reliability are discussed in the following.

4.4.2.1 Preference Level Using the Survey

The survey focussed on user satisfaction with their experience of using the recycling points and centres. The target respondents were approached via email through Sheffield University's volunteer lists. There were 504 respondents. Among the survey's questions, there were only two questions in which are relevant to this specific study, which are:

1. Respondent preference rankings for each HWRC;
2. Attractiveness factors for the selection of the preferred HWRC.

The first question is considered in this section, while the second was used in order to build the attractiveness function for the spatial interaction model.

The first question asked respondents who reside in any of the 28 wards in Sheffield to rank their preferences of HWRC using a ranking of between 1 and 5, where "1" represented the most preferred centre, and "5" the least preferred. Such responses were then utilised to compute the quota of demand originating from each ward for each HWRC (as also shown in Appendix 4(C)); this percentage can be interpreted as the probability that an user from a given ward will access one of the facilities. Results of this process are shown Table 4-6. On average, the highest percentage for the preferred centre was Blackstock Road (29.9%), followed by Longley Avenue (26.4%), Beighton Road (23.2%), Greaves Lane (11.1%) and lastly, Deepcar (9.4%). Derived from Table 4-6, the number of wards for which each HWRC is the most preferred option is indicated in Table 4-7.

Table 4-6: User distribution percentages per HWRC - survey

i	Wards	Beighton Rd	Blackstock Rd	Deepcar	Greaves Lane	Longley Avenue
1	Arbourthone	22.50%	**56.30%**	11.30%	1.30%	8.80%
2	Beauchief	27.90%	**55.70%**	4.30%	1.40%	10.70%
3	Beighton	**51.10%**	30.00%	5.60%	1.10%	12.20%
4	Birley	**50.00%**	38.30%	1.70%	0.00%	10.00%
5	Broomhill	16.10%	32.60%	9.10%	5.70%	**36.50%**
6	Burngreave	0.00%	13.30%	3.30%	23.30%	**60.00%**
7	Central	29.10%	**45.50%**	8.20%	5.50%	11.80%
8	Crookes	14.30%	16.10%	9.60%	9.30%	**50.70%**
9	Darnall	**54.00%**	26.00%	2.00%	2.00%	16.00%
10	Dore and Totley	32.20%	**56.70%**	6.10%	2.80%	2.20%
11	East Ecclesfield	1.30%	0.00%	13.80%	**53.80%**	31.30%
12	Ecclesall	20.30%	**53.40%**	3.40%	2.90%	20.00%
13	Firth Park	8.00%	8.00%	6.00%	18.00%	**60.00%**
14	Fulwood	19.30%	25.70%	7.00%	8.30%	**39.70%**
15	Gleadless Valley	24.30%	**60.00%**	2.90%	5.00%	7.90%
16	Graves Park	21.30%	**53.80%**	3.80%	12.50%	8.80%
17	Hillsborough	11.50%	8.80%	20.00%	12.30%	**47.30%**
18	Manor e Castle	16.70%	**50.00%**	3.30%	0.00%	30.00%
19	Mosborough	**60.00%**	30.00%	6.30%	1.30%	2.50%
20	Nether Edge	17.00%	**53.50%**	11.00%	6.50%	12.00%
21	Richmond	**54.00%**	36.00%	2.00%	2.00%	6.00%
22	Shiregreen	16.70%	16.70%	7.80%	12.20%	**46.70%**
23	Southey	6.00%	24.00%	8.00%	14.00%	**48.00%**
24	Stannington	2.20%	2.20%	18.90%	23.30%	**53.30%**
25	Stocksbridge	2.70%	0.90%	**60.00%**	20.00%	16.40%
26	Walkley	15.00%	19.00%	10.00%	10.00%	**46.00%**
27	West Ecclesfield	0.00%	0.00%	16.00%	**54.00%**	30.00%
28	Woodhouse	**55.00%**	25.00%	3.30%	3.30%	13.30%
AVERAGE % OF EACH WARD PER HWRC		**23.20%**	**29.90%**	**9.40%**	**11.10%**	**26.40%**

Table 4-7: Frequency of users' HWRC preference

HWRC	Frequency
Longley Avenue	10
Blackstock Rd	9
Beighton Rd	6
Greaves Lane	2
Deepcar	1

Out of 28 wards, users from 10 wards choose Longley Avenue as their preferred place to dispose of recyclable items. This was followed by users of nine wards choosing Blackstock Road, six wards choosing Beighton Road and only two wards choosing Greaves Lane. The least preferred HWRC was Deepcar, with users of Stockbridge and Upper Don being the only wards choosing to use this facility. This is probably because the location of the recycling centre is far away from other wards and has poor accessibility. The estimated amount of users per HWRC using the survey is shown in Appendix 4(D).

4.4.2.2 Preference Levels Using Spatial Interaction Model

There are four important elements in the spatial interaction model: the attractiveness factor, the distance between each pair origin-destination, parameters to be calibrated and the demand generated by each origin.

The attractiveness of each recycling centre (Q_j) is based on several factors, hence let $Q_j = f(y_{kj})$, where y_{kj} is the normalised value of each attractiveness factor, and k is the factor's magnitude. An average mean formula is used to find Q_j.

$$f(y_{kj}) = \frac{y_{kj}}{max(y_{kj})}, \qquad Q_j = \frac{f(y_{kj})}{\sum_k f(y_{kj})}, \forall j \in J$$

As evidenced by the survey, the attractiveness factor for a HWRC was based on proximity to the potential user, centre organisation, operating hours, recycling portfolio and recycling rate. Proximity is part of the general formulation of a spatial interaction model; hence only two of the factors that influence an attractiveness level of a given *j*, namely each HWRC's recycling portfolio and its number of containers, are used in this study. Recycling rates and centre organisation were relaxed as these were considered part of the recycling portfolio. Meanwhile, using operating hours could introduce some logical problems, as the objective of the study is to redesign the network, possibly modifying the operating hours of each HWRC.

Table 4-8: The attractiveness score for each recycling centre *j*

HWRC (*j*)	y_k		Q_j
	Number of containers (*k* = 1)	**Recyclable materials (*k* = 2)**	
Beighton Road	0.92	0.88	0.90
Blackstock Road	1.00	1.00	1.00
Deepcar	0.50	0.88	0.69
Greaves Lane	0.58	0.92	0.75
Longley Avenue	0.83	0.85	0.84

The next variable is the travel time between each ward-recycling centre pair. The travel time between a given ward to each recycling centre was gathered thanks to data provided by Sheffield City Council, as indicated in Appendix 4(A).

It was assumed that the values of k_{ij}, α_j and β_j were constant over time (and equal to 1, as in most of standard applications). The allocation of users at each ward *i* per time *t* to each HWRC (or facility *j*), or d_{ij}^t is based on the spatial interaction model:

$$d_{ij}^t = d_i^t \cdot \frac{Q_j \cdot \left(dist_{ij}\right)^{-n}}{\sum_j (Q_j \cdot \left(dist_{ij}\right)^{-n})}$$

where the value of *n* can be calibrated based in such a way to minimise the difference between the actual and the estimated (survey-based) distribution of users at the HWRCs. The Excel was used to calculate the distribution of users from each *i* to a given facility *j*. The value of *n* was varied between 1.0 and 3.0 using a step of 0.01. To determine the optimal *n*, the absolute difference between the actual distribution of users that was gathered from the survey (values in Table 4-6), and the predicted distribution of users was used. Figure 4-15 highlights the absolute differences per *n*. From this calibration, the best value for *n*, corresponding to the smallest difference between distributions was when *n* = 1.59. Fixing *n* at 1.59, the percentage of demand from each ward attracted by each HWRC is reported in Table 4-9.

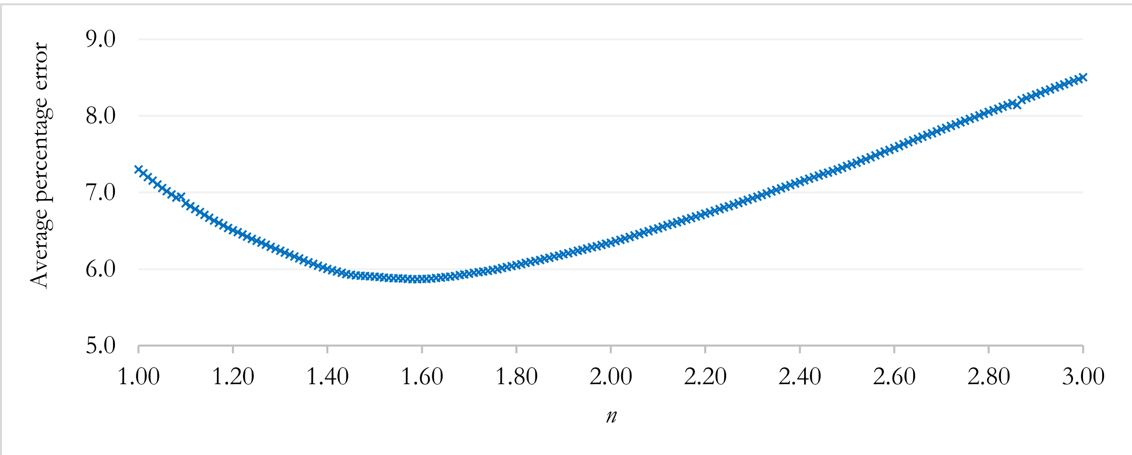


Figure 4-15: The change in absolute difference of the user distribution determined from the survey and the value estimated with the variation of *n* values

Table 4-9: Attractiveness score for user of ward *i* for each facility *j*

i	Wards	Beighton Rd	Blackstock Rd	Deepcar	Greaves Lane	Longley Avenue
1	Arbourthone	15.30%	**67.10%**	3.50%	4.00%	10.10%
2	Beauchief	20.00%	**55.30%**	5.60%	6.80%	12.40%
3	Beighton	**78.20%**	11.00%	2.30%	3.20%	5.40%
4	Birley	**42.30%**	41.10%	3.50%	4.60%	8.60%
5	Broomhill	15.40%	**32.50%**	9.70%	10.20%	32.10%
6	Burngreave	9.90%	12.30%	6.10%	8.10%	**63.70%**
7	Central	15.70%	**45.50%**	6.80%	7.40%	24.70%
8	Crookes	15.00%	25.90%	12.40%	12.30%	**34.40%**
9	Darnall	**31.30%**	25.00%	7.80%	11.40%	24.50%
10	Dore and Totley	20.40%	**44.60%**	8.20%	9.20%	17.60%
11	East Ecclesfield	9.00%	7.90%	15.40%	**36.80%**	30.80%
12	Ecclesall	18.50%	**38.80%**	9.40%	9.90%	23.40%
13	Firth Park	7.80%	8.10%	8.00%	12.10%	**64.00%**
14	Fulwood	17.60%	27.40%	13.40%	11.80%	**29.80%**
15	Gleadless Valley	8.60%	**77.70%**	2.80%	3.10%	7.80%
16	Graves Park	16.60%	**58.80%**	5.40%	5.80%	13.30%
17	Hillsborough	11.20%	12.60%	17.30%	12.90%	**46.00%**
18	Manor e Castle	29.30%	**37.40%**	5.70%	7.70%	19.80%
19	Mosborough	**63.10%**	20.00%	3.80%	4.90%	8.20%
20	Nether Edge	17.00%	**44.40%**	8.00%	8.40%	22.20%
21	Richmond	**42.50%**	34.90%	4.30%	6.00%	12.30%
22	Shiregreen	13.80%	12.80%	12.60%	19.90%	**40.90%**
23	Southey	6.40%	7.10%	11.60%	17.60%	**57.30%**
24	Stannington	13.40%	15.10%	17.40%	14.30%	**39.90%**
25	Stocksbridge	4.70%	4.80%	**64.90%**	15.00%	10.70%
26	Walkley	14.90%	19.70%	11.00%	11.20%	**43.20%**
27	West Ecclesfield	3.70%	3.80%	12.50%	**67.30%**	12.60%
28	Woodhouse	**70.20%**	13.00%	3.40%	4.80%	8.60%
	AVERAGE % OF EACH WARD PER HWRC	**23.20%**	**22.56%**	**28.74%**	**10.46%**	**12.38%**

Table 4-9 highlights the percentage distribution of users from each ward *i* to each facility *j*. The maximum preference value for each ward is indicated in bold; also, the last row of the table reports the overall distribution of users across the facilities. The most preferred recycling facility was Blackstock Road (28.74%), which is the preferred site for ten wards (Arbourthone, Beauchief, Broomhill, Central, Dore and Totley, Ecclesall, Gleadless Valley, Graves Park, Manor Castle and Nether Edge). The next preferred recycling facility was Longley Avenue (strongly attracting residents from Burngreave, Crookes, Firth Park, Fulwood, Hillsborough, Shiregreen, Southey, Stannington and Walkley). Blackstock Road and Longley Avenue were probably preferred due to their accessibility from all wards. The third-most preferred recycling facility was Beighton Road, which residents of Beighton, Birley, Darnall, Mosborough, Richmond and Woodhouse preferred to use. Only two wards preferred to use the HWRC in

Greaves Lane, which were East Ecclesfield and West Ecclesfield, and lastly only one ward's residents, those of Stocksbridge, used Deepcar as their preferred recycling facility. From these figures, it was concluded that residents of each ward prefer to use recycling facilities that are within a reasonable distance; for example, Blackstock Road and Longley Avenue are located in the middle of Sheffield and are clearly easily accessible to the surrounding wards. The estimated amount of users per HWRC using the spatial interaction model as in Appendix 4(D).

The following section focusses on a comparison of preferences found using actual data (from the survey) to those predicted using the spatial interaction model.

4.4.3 Results comparison: Survey vs Spatial Interaction Model

The results are arranged and discussed in three parts:

- Difference between the actual percentage and predicted percentage (Figure 4-16).
- Composition of users *i* at each facility *j* (Figures 4-17 to 4-21) and the differences of each composition per HWRC using actual and predicted percentage (Figure 4-22).
- The average difference between the actual and the predicted amount of users in each HWRC (Table 4-10 and Figure 4-23).

4.4.3.1 Distribution of Users to Each Ward i for each Facility j

The difference between the surveyed and the predicted distribution of users for each ward for each HWRC is shown Figure 4-16. This was obtained simply as the difference between the expected preference level (as reported in Table 4-6) and the actual preference level (as reported in Table 4-9). The average of the absolute differences between the distributions was found at 5.86%, which shows that the spatial interaction model was able to predict user preferences in terms of their choice of recycling facility with reasonable accuracy. For example, the spatial interaction model over predicted the users of the Beighton Road facility by 27.06%. The relatively good accessibility that Beighton users had to other recycling facilities, for example Blackstock Road and Longley Avenue, could have contributed to the over prediction. The largest under-prediction was for users from Darnall going to the Beighton Road recycling facility, at 22.07%. Probably high of accessibility in terms of Beighton Road's operating hours (operates 6-days a week) and distance between Darnall and HWRC of Beighton Road (i.e. less than 15 minutes) causing Darnall's users to visit this centre instead.

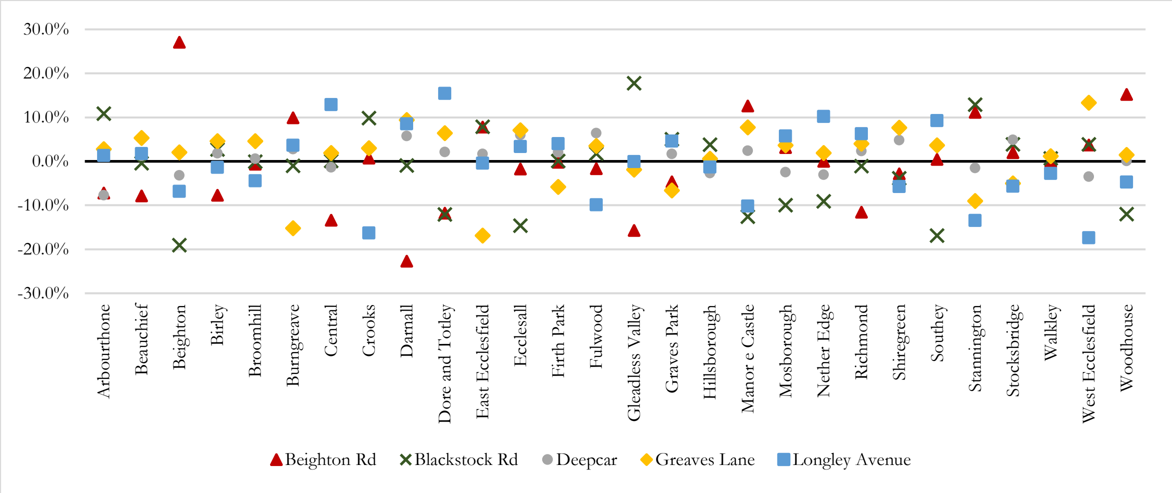


Figure 4-16: Difference between the surveyed and predicted user distributions for each HWRC

4.4.3.2 Composition of Users i at Each Facility j

Figures 4-17 – 4-22 highlight the users' residency at each HWRC. Most of Beighton Road's users are from Beighton (11.8%), followed by Woodhouse (10.7%) and Mosborough (9.8%). However, most of Blackstock Road's users are from Gleadless Valley (9.8%). However, from the chart in Figure 4-17, the percentage of users from several wards in this centre are very similar, which means users of Blackstock Road are accessible from every part of Sheffield. This is probably because the centre is in the central area and accordingly, is accessible by a large number of wards. The highest percentage of Deepcar users were from Stocksbridge and Upper Don ward, at 22.4%. This is to be expected as Deepcar is located within this ward. Greaves Lane, on the other hand, is accessible to the residents of West Ecclesfield (19.3%) and East Ecclesfield (10.7%). Thus, these figures are predictable. The percentage of users attending Greaves Lane from other wards was, on average, less than 5% – this was probably due to the time required to reach this centre being longer than for other centres. Lastly, the majority of users of Longley Avenue were from Burngreave (9.6%). However, the associated user distribution is similar to that for Blackstock Road, which indicates that this centre is easily accessible from any users in Sheffied.

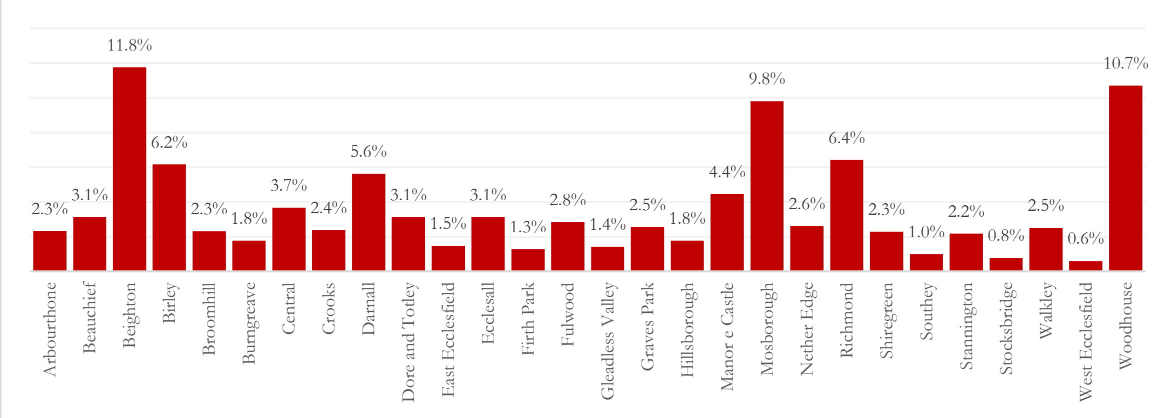


Figure 4-17: Beighton Rd predicted users

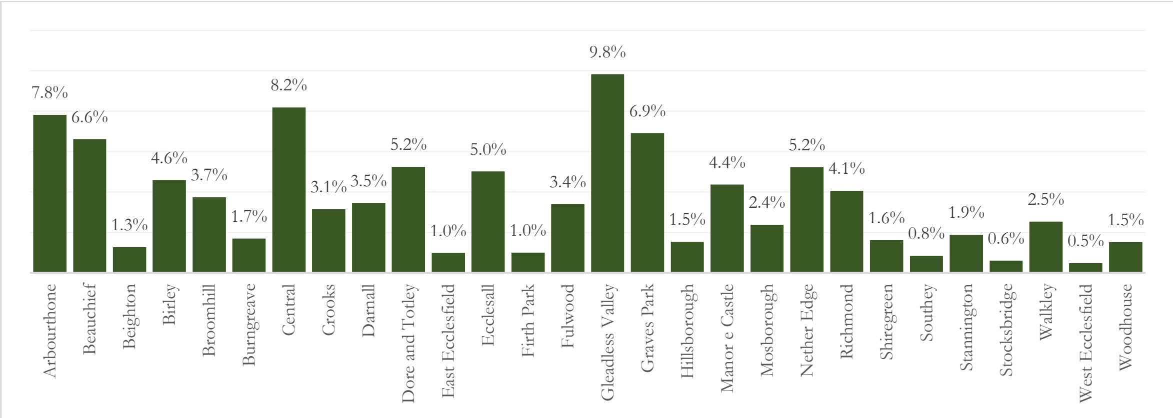


Figure 4-18: Blackstock Rd predicted users

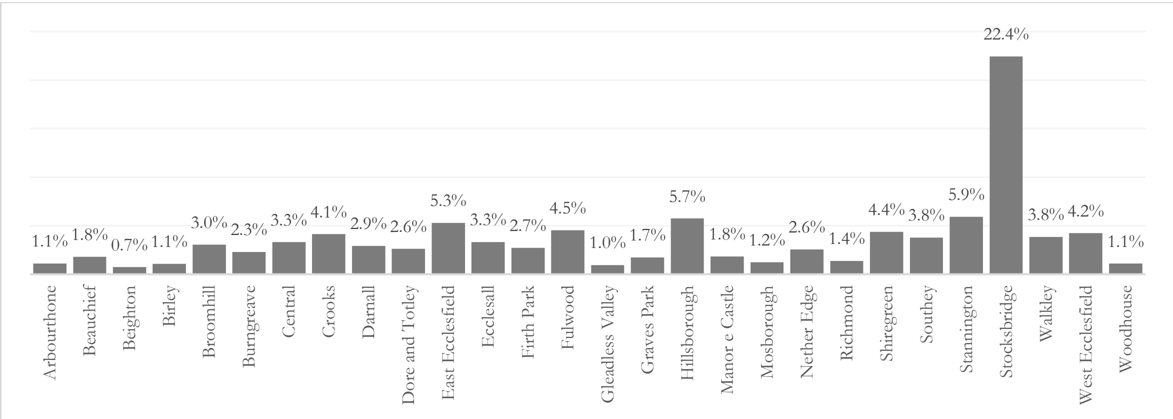


Figure 4-19: Deepcar predicted users

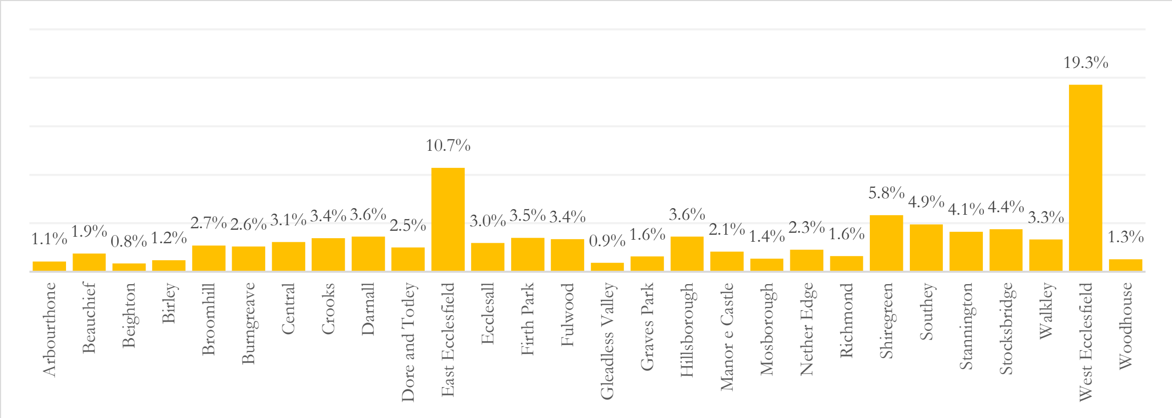


Figure 4-20: Greaves Lane predicted users

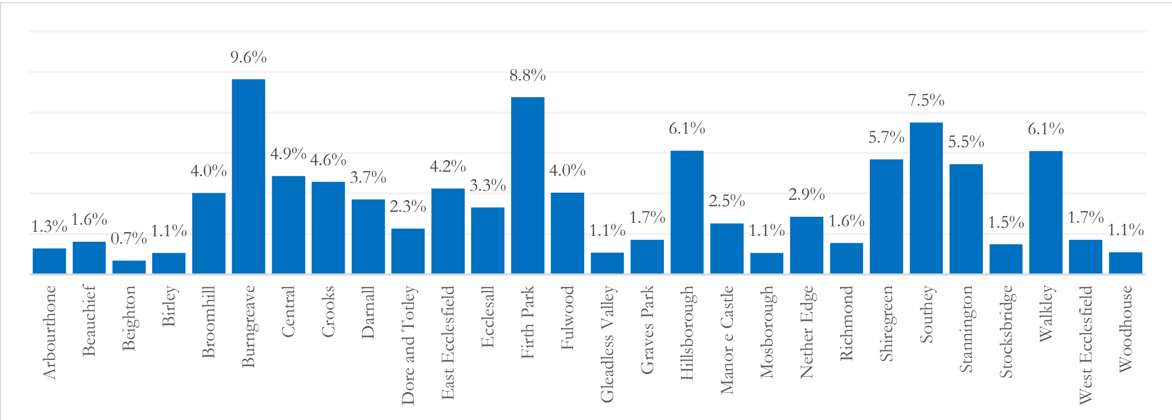


Figure 4-21: Longley Avenue predicted users.

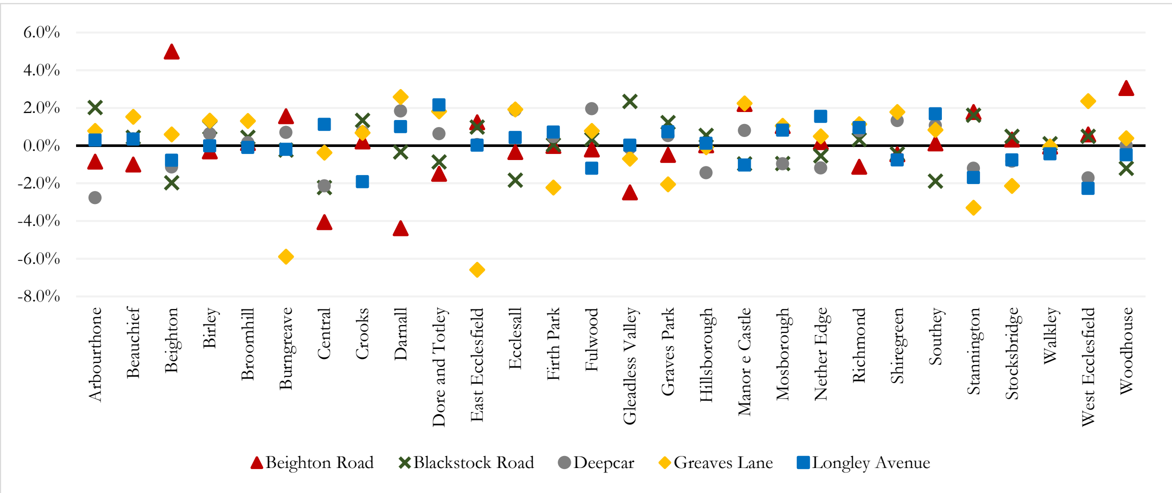


Figure 4-22: Difference between actual and predicted at all five HWRCs

Meanwhile, the average difference between actual (surveyed) and predicted (spatial interaction model) of user compositions at all HWRCs was found as 1.14%. The difference for users' residency at each HWRC is illustrated in Figure 4-22. From this figure, the highest overestimate is 5%; for the distribution of users of Beighton ward to Beighton Road recycling centre. Even through the HWRC at Beighton Road is located in Beighton, users' preference for the use of this HWRC was less than expected due to the nearby recycling facility located at Blackstock Road, which can also be easily reached by users from Beighton. The spatial interaction model underestimates the number of people from East Ecclesfield using the Greaves Lane facility by 6.6%. This is predictable since East Ecclesfield is one of West Ecclesfield's neighbouring wards (Greaves Lane facility location) and is located a long way from other recycling facilities.

4.4.3.3 Comparison of Average User Distributions at each HWRC

Table 4-10 gives the average percentage user distribution at each HWRC. The average difference between the survey and the model was found as 0.9%. The spatial interaction model overestimates users at Blackstock Road by only 1.2% and Longley Avenue by only 0.5%, which was probably because these two centres are highly accessible from the majority of wards, and so there may be respondents that reside more than the threshold distance away but who nevertheless prefer to use these centres. Meanwhile, the spatial interaction model slightly underestimates the amount of users at Beighton Road (0.5%), Deepcar (1.0%) and Graves Lane (1.3%). Results show that, overall, the spatial interaction model reproduces in a very accurate way the user preference ranking for the HWRCs, and the overall user attracted by each centre. The first preference is Blackstock Road, followed by Longley Avenue, Beighton Road, Greaves Land and, lastly, Deepcar. Figure 4-23 compares the actual and predicted distribution per HWRC.

Table 4-10: The difference between the actual and predicted distributions at each HWRC

HWRC	**Percentage distribution of users using survey (*A*)**	**Percentage distribution of users using SIM* (*B*)**	**Difference = $B-A$**	**Absolute difference = $\lvert B-A \rvert$**	**Average of absolute difference**
Beighton Rd	22.6%	23.1%	-0.5%	0.5%	**0.9%**
Blackstock Rd	28.7%	29.9%	1.2%	1.2%	
Deepcar	10.5%	9.5%	-1.0%	1.0%	
Greaves Lane	12.4%	11.1%	-1.3%	1.3%	
Longley Avenue	25.9%	26.4%	0.5%	0.5%	

*spatial interaction model

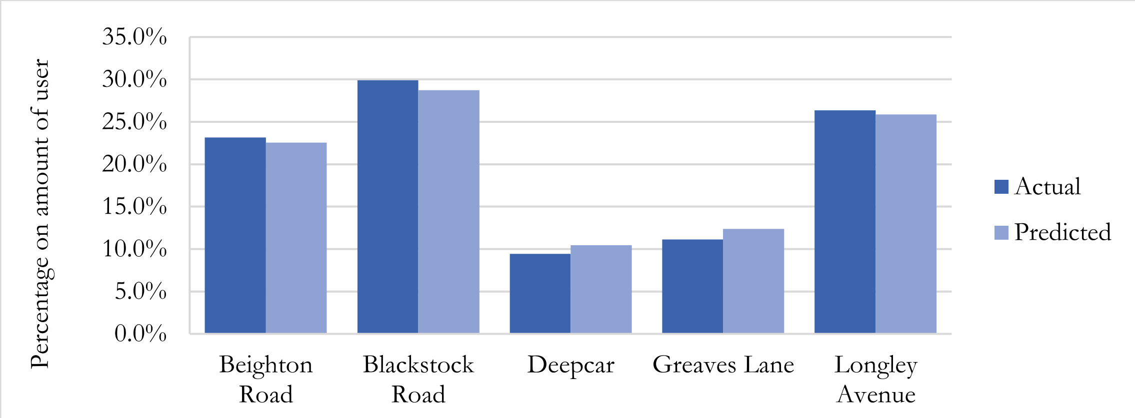


Figure 4-23: Percentage distribution of users at each HWRC, based on actual (survey) and predicted numbers

Coherently to the spatial interaction model, it can be seen that users choose HWRC facilities not just because of the distance factor, but also because based on an attractiveness factor (based, in the model, on the number of waste containers and the recycling portfolio provided at each HWRC). Values obtained from the spatial interaction model for the user allocations will be implemented as d_{ij} in the proposed model in order to generate the demand for HWRC services from each electoral ward within the facility network. The next section focusses on refinement of the proposed model to suit the reorganisation of HWRC.

4.5 Multi-Period Model Refinements

The constraints and the objective function of the model proposed in Chapter 4 were modified in order to apply the model to the reorganisation of HWRC operations. Before conducting any modification, the assumptions used for the HWRC case study are given below:

1. The operating periods for all HWRCs are on a weekly basis, seven days a week.
2. The time periods for each HWRC are on an hourly basis and operated from 10 a.m. - 6 p.m., per day. By 6 p.m., all facilities are closed, and all users need to be served.

From both points, it is necessary to divide the overall time period T in several time-intervals, to ensure the operation per facility j is performed on a daily basis.

4.5.1 Refinement 1: Creating a Range within T

Let the set of time-periods for T be divided into macro and micro periods. Figure 4-24 illustrates this concept, where the macro-period represents the number of days per week, and the micro-period represents the number of operating periods per day.

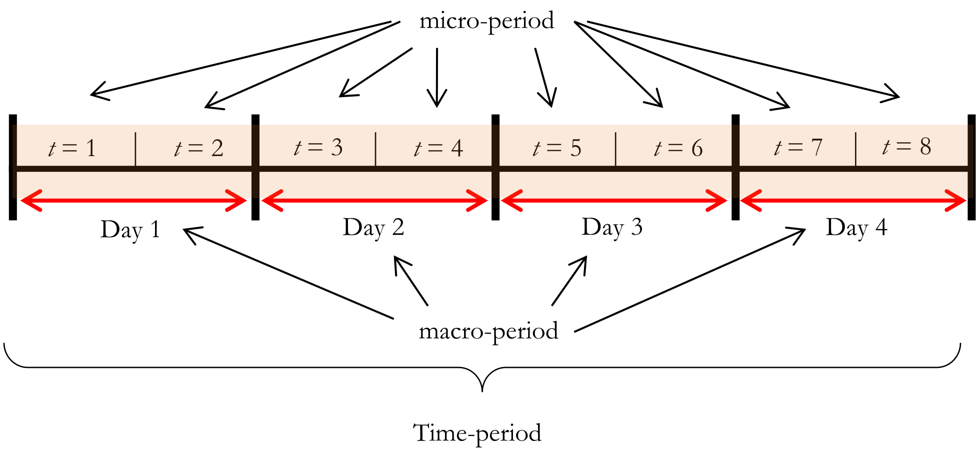


Figure 4-24: Illustration of macro- and micro-periods

From Figure 4-24, a general picture of the situation can be demonstrated, which is presented in Figure 4-25. Let, W be the set number of macro-periods, indexed by $w = \{1...W'\}$, and H be the length per macro-period or number of micro-periods per macro-period. It is assumed that the length of each macro-period is identical. The general concept of macro-periods (w) and micro-periods (H) is presented in the Figure 4-25.

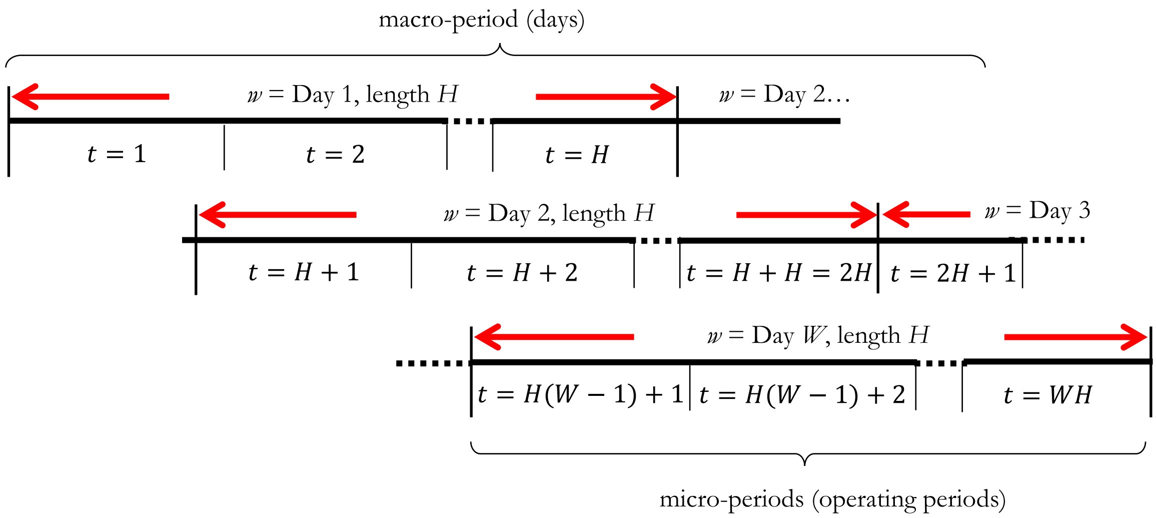


Figure 4-25: Illustration of macro- and micro-periods in H and w

Therefore, the range for operations per day is from $H(w - 1) + 1$ until wH. Using this range, further refinements to constraints can be introduced.

1. No user can be transferred between days.
2. No facility can be reopened if it has been closed during a day.
3. A minimum number of operating periods per day and per week.

4.5.2 *Refinement 2: No User can Transfer Between Days*

We considered the operation of a given facility *j* is a daily basis. This condition resembles the real-life situation, where at the end of the day there is no user allowed to wait in a queue. Therefore, the initial and final value of users waiting in a queue for each daily operation, s_j^t (as introduced in Chapter 4) must be equal to 0 units. Therefore, in general:

$$s_j^t = 0;\ \forall j \in J, t = 1 + H(w-1), t = wH$$

4.5.3 *Refinement 3: Reopening of the Facility is Restricted to a Daily Basis Only*

To ensure reopening is restricted to a daily basis only, the range for constraint $y_j^{t-1} \leq y_j^t$ was refined. This only allows facility *j* to reopen on the following day once they have been closed. The modified version of $y_j^{t-1} \leq\ y_j^t; \forall j \in J, \forall t \in T$ is:

$$y_j^{t-1} \geq\ y_j^t; \quad \forall j \in J, t = (1 + H(w-1), Hw], \forall w \in W$$

The constraint indicates that once facility *j* is closed during a given day, it will remain closed for the rest of a given day *w*. Originally, the notation of "≤" in $y_j^{t-1} \leq y_j^t$ is changed to "≥" (i.e., $y_j^{t-1} \geq\ y_j^t$) to set an early opening time and early closing time for each facility *j*.

4.5.4 *Refinement 4: Minimum Operating Periods Per Day and Per Week*

In order to obtain realistic optimal schedules, minimum operating periods per day and per week are imposed for a facility *j*. This is done in order to ensure that results suggested by the model are feasible and compatible with real-world scenarios, in which, due to fixed-cost reasons, facilities need to observe minimum standard operating times; for instance, schedules suggesting the facility to be opened just one or two hours per day might not be feasible due to constraints on human resources. Recall constraint (4-19) from chapter 4:

$$\sum_t y_j^t \leq\ \delta_j\ ;\ \forall j \in J$$

where this constraint limits the number of operating periods per facility *j* at δ_j. Let $\delta_{min\,j}$ be the minimum number of operational periods per week for a given facility *j*. Constraint (4-19) is then changed to:

$$\sum_t y_j^t = \begin{cases} \sum_t y_j^t & if \quad \delta_{min_j} \leq \sum_t y_j^t \leq \delta_j \\ 0 & otherwise \end{cases} \quad ; \quad \forall j \in J$$

where the total operating periods for a given facility *j*, is either at a minimum of δ_{min_j} or 0.

Further refinements to limit the operational periods per day for a given facility *j* were determined. To do this, constraint (4-19) was modified to:

$$\sum_{t=1+H(w-1)}^{Hw} y_j^t \leq \sum_w \delta_{jw} \, ; \qquad \forall j \in J, \forall w \in W$$

which indicates the total operational periods per day for a given facility *j* is limited to $\sum_w \delta_{jw}$. Similar reasoning can be given in creating a limit for the total number of operational periods per week, and the equation above can thus be further modified. Let $\delta_{min_{jw}}$ be the minimum number of operational periods per day, or else complete closure for a day.

$$\sum_{t=1+H(w-1)}^{Hw} y_j^t = \begin{cases} \sum_{t=1+H(w-1)}^{Hw} y_j^t & if \quad \delta_{min_{jw}} \leq \sum_{t=1+H(w-1)}^{Hw} y_j^t \leq \delta_{jw} \\ 0 & otherwise \end{cases} \quad ; \quad \forall j \in J, \forall w \in W.$$

4.6 Multi-period Model for HWRC Facility

For this case study, the sets, parameter and decision variables for the model are as follows:

Sets		
J	=	the set of facility locations, indexed by *j* and *k*, where $\forall \mathrm{j}, \mathrm{k} = \{1 \ldots \mathrm{J}' \mid \mathrm{j} \neq \mathrm{k}\}$
T	=	the set of time-periods, indexed by *t*, where $\forall t = \{1 \ldots T'\}$

Parameters		
C_j	=	cost of operating a facility *j* for a single period *t* (which can be assumed constant)
$\varepsilon_{1j}, \varepsilon_{2j}, \varepsilon_{3j}, \varepsilon_{4j}$	=	assigned cost for each decision made, where ε_{1j} indicates the cost of serving one user, ε_{2j} indicates the cost of transferring one user to the next period, ε_{3j} indicates the cost of losing one user and ε_{4j} indicates the cost of serving extra amount of users from other facilities.
α_1	=	weights on provider's side
τ_j^t	=	capacity level of facility at location *j* during a period *t*

x_j^t	=	amount of users requiring a service at facility *j* during a period *t*
u_{jk}^t	=	predetermined binary integer to indicate the possibility for users to move from facility *j* to facility *k* during period *t*
δ_j	=	maximum operating periods per *j*
$\delta_{min\,j}$	=	minimum number of operational periods per *j* per week
$\delta_{min\,jw}$	=	minimum number of operational periods per *j* per day
B	=	upper bound of amount of users leaving system

Decision variables		
y_j^t	=	$\begin{cases} 1 & if\ facility\ j\ is\ operating\ during\ a\ period\ t \\ 0 & otherwise \end{cases}$
s_j^t	=	non-negative decision variable representing the user transferred to the next period at a facility at location *j* at the end of a period *t*
S_{jk}^t	=	user transferred between *j* and *k* during a period *t*
l_j^t	=	user chosen to leave at each facility at location *j* during a period *t*
q_j^t	=	user served at each facility at location *j* during a period *t*

For the HWRC case study, the multi-component model was used by assuming the cost on the provider's side, or Z_1 consist of the operational costs (C_j), the cost to serve a unit of demand (ε_{1j}) and the cost when a unit of demand leave the system (ε_{3j}). Assume the cost on the demand side, or Z_2 consists of the cost when a unit of demand be in the queue (ε_{2j}) and the cost when a unit of demand move from facility *j* to facility *k* (ε_{4j}). Therefore,

$$Z_1 = \sum_t \sum_j \left(C_j y_j^t + \varepsilon_{1j} q_j^t + \varepsilon_{3j} l_j^t\right)$$

$$Z_2 = \sum_t \sum_j \left(\varepsilon_{2j} s_j^t + \varepsilon_{4j} \sum_{k,k\neq j} S_{kj}^t u_{kj}^t\right)$$

Thus, the modified objective function is:

$$Min\left(\alpha_1 \sum_t \sum_j \left(C_j y_j^t + \varepsilon_{1j} q_j^t + \varepsilon_{3j} l_j^t\right) + (1-\alpha_1) \sum_t \sum_j \left(\varepsilon_{2j} s_j^t + \varepsilon_{4j} \sum_{k,k\neq j} S_{kj}^t u_{kj}^t\right)\right) \tag{4-1}$$

subject to:

$$x_j^t + s_j^{t-1} + \sum_{k,k\neq j} S_{kj}^t u_{kj}^t = s_j^t + \sum_{k,k\neq j} S_{jk}^t u_{jk}^t + q_j^t + l_j^t; \qquad \forall j \in J, \forall t \in T \tag{4-2}$$

$$\sum_j \sum_t x_j^t = \sum_j \sum_t \left(q_j^t + l_j^t\right); \tag{4-3}$$

$$\sum_j \sum_t l_j^t \leq B\left(\sum_j \sum_t x_j^t\right); \tag{4-4}$$

$$q_j^t \leq \tau_j^t y_j^t; \qquad \forall j \in J, \forall t \in T \tag{4-5}$$

$$\frac{s_j^t}{x_j^t} \leq y_j^{t+1}; \qquad \forall j \in J, \forall t \in T \tag{4-6}$$

$$\frac{S_{jk}^t u_{jk}^t}{x_j^t} \leq y_k^t; \qquad \forall j \in J, \forall t \in T \tag{4-7}$$

$$\sum_t y_j^t = \begin{cases} \sum_t y_j^t & if \quad \delta_{min_j} \leq \sum_t y_j^t \leq \delta_j \\ 0 & otherwise \end{cases}; \qquad \forall j \in J \tag{4-8}$$

$$\sum_{t=1+H(w-1)}^{Hw} y_j^t = \begin{cases} \sum_{t=1+H(w-1)}^{Hw} y_j^t \; if \; \delta_{min_{jw}} \leq \sum_{t=1+H(w-1)}^{Hw} y_j^t \leq \delta_{jw} \\ 0 \qquad otherwise \end{cases}; \qquad \forall j \in J, \forall w \in W, \tag{4-9}$$

$$y_j^{t-1} \geq y_j^t; \qquad \forall j \in J, \forall w \in W, \; t = (1 + H(w-1), Hw] \tag{4-10}$$

$$s_j^t = 0; \qquad \forall j \in J, \; t = 1 + H(w-1), t = Hw \tag{4-11}$$

$$q_j^t, l_j^t, s_j^t, S_{jk}^t \geq 0; \qquad \forall t \in T, \forall j \in J \tag{4-12}$$

$$y_t \in \{0, 1\}; \qquad \forall t \in T, \forall j \in J \tag{4-13}$$

The objective function (5-1) indicates the total operational costs for the entire system, concerning costs on both sides: the provider's and the users'. (5-2) is the mass balance constraint. Constraint (5-3) ensures that each user is either served or leaves the system at the end of the time-period, while (5-4) ensures that the unserved users are limited to *B*%; this constraint expresses the required service level which the provider wants to achieve. (5-5) guarantees that the amount of users served is within the capacity of the facility. Constraints

(5-6) and (5-7) limit users' movements to operational facilities only. Constraints (5-8) and (5-9) ensure the total operating periods for a given facility *j* are within an acceptable range. (5-10) restricts reopening of a given *j*; once the facility is closed (e.g., within a day), it will remain closed. (5-11) ensures no user in the queue at the beginning and at the end of the day. Decision variables q_j^t, l_j^t, s_j^t and S_{jk}^t are positive integers (5-12) and y_j^t is a binary variable (5-13).

For this chapter, the allocation of users from ward *i* to each facility *j* was based on the previously described spatial interaction model, where *i* is the index of the set of user locations, $\forall i = \{1...I'\}$. The results obtained in section 5.4.3.2 were used to determine the amount of users trying to access each HWRC.

The following section tests the modified model by varying the associated parameters. All the calculations used to solve the model, including the sensitivity analysis and application to the HWRC problem, were conducted using CPLEX 12.6 on computer with a memory of 8.0 GB RAM, a 2.50 GHz processor and the Windows 10 operating system.

4.7 Sensitivity Analyses

4.7.1 Description of Test Instances

Numerical analyses were carried out to assess the effects of varying the cost parameters C_j, ε_{1j}, ε_{2j}, ε_{3j} and ε_{4j}. The size of the problem was set at J = 4, T = 15, W = 3 and H = 5. Users at each *j* per *t*, x_j^t were set to be a discrete uniform distribution function, but each *j* has a distinctive utilisation rate, $\frac{\sum_t x_j^t}{\sum_t \tau_j^t}$. The capacity level, τ_j^t was set to 10 units for all *j* at all times. Details of the dataset used were:

Table 4-11: Datasets used in each analysis

Parameters	Range
$\sum_t x_j^t$	= [195, 105, 75, 135]
$\sum_t \tau_j^t$	= [150, 150, 150, 150]
Utilisation rate $\left(\% \frac{\sum_t x_j^t}{\sum_t \tau_j^t}\right)$	= [130%, 70%, 50%, 90%], average = 85%

For this analysis, travel times from a given facility *j* to given facility *k* were:

$$dist_{jk} = \begin{pmatrix} 1000 & 7 & 11 & 7 \\ 7 & 1000 & 6 & 13 \\ 11 & 6 & 1000 & 9 \\ 7 & 13 & 9 & 1000 \end{pmatrix}$$

where rows represent the initial facility *j,* and each column represents the distance from a given facility *j* to a given facility *k*. 1000 units were set to forbid movement to the same facility *j*. From the distance data, u_{jk}^{t} were generated. It was assumed that u_{jk}^{t} is 1 if the travel time between two facility *j* is less than 10 minutes, and 0 otherwise. As result, users at j_1 can go to j_2 and j_4, users at j_2 can go to j_1 and j_3, users at j_3 can go to j_2 and j_4, and users are j_4 can go to j_1 and j_3. It was assumed that the movement was identical for all times *t*. In the meantime, the weight for the provider's side, α_1, was varied parametrically from 0.1 until 0.9 in increments of 0.1 per step while the associated α_2 were calculated as $1 - \alpha_1$.

The initial *B* value was determined through the feasibility of the results. To do this, let ε_{1j}, ε_{2j}, ε_{3j} and ε_{4j} be 10 units, C_j be 100 units and α_1 be 0.5. The results were as presented in Table 4-12, and this dataset was used to cater for all users (*B* = 0%). This probably was due to $\sum_j \sum_t x_j^t \leq \sum_j \sum_t \tau_j^t$, which the user will always be able to find at least one available server.

Table 4-12: Feasibility results

B **%**	**Solution**
0%	Feasible solution
1%	Feasible solution
5%	Feasible solution

Therefore, for all experiments, *B* is set to 0.05 to ensure a 95% minimum service level. Two analyses were conducted:

Analysis 1: Relaxation of constraints (5-8) and (5-9)

To test the impact of restricting the minimum number of operating periods per day and per week for each facility *j*.

Analysis 2: Trade-off in each variable's costs

To test the impact of each cost of decision variables towards flow of users within the network.

Percentage of users served, left, queue length and moved from the primary to the backup facility were calculated using the following formulations as in Appendix 3(A).

4.7.2 Analysis 1: Effect of δ_{min_j} and $\delta_{min_{jw}}$ on System Performance

Recall constraints (5-8) and (5-9).

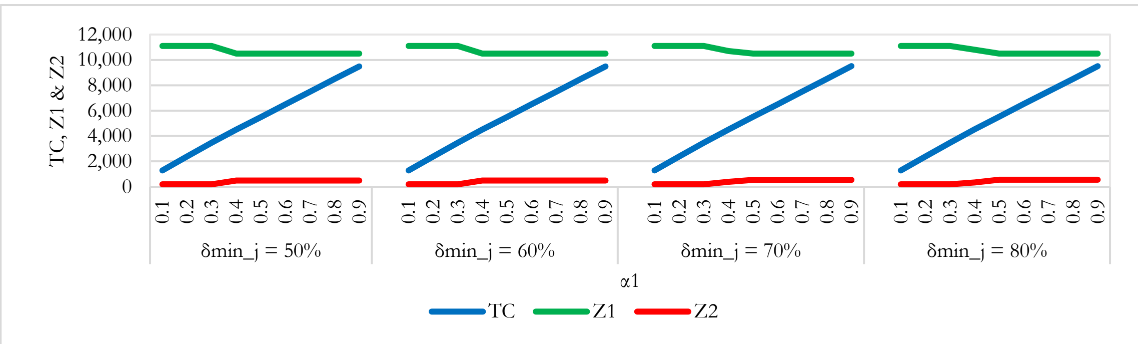
$$\sum_t y_j^t = \begin{cases} \sum_t y_j^t & if \quad \delta_{min_j} \le \sum_t y_j^t \le \delta_j; \\ 0 & otherwise \end{cases} \qquad \forall j \in J \qquad (5\text{-}8)$$

$$\sum_{t=1+H(w-1)}^{Hw} y_j^t = \begin{cases} \sum_{t=1+H(w-1)}^{Hw} y_j^t \; if \; \delta_{min_{jw}} \le \sum_{t=1+H(w-1)}^{Hw} y_j^t \le \delta_{jw}; \\ 0 \qquad otherwise \end{cases} \qquad \forall j \in J, \forall w \in W \qquad (5\text{-}9)$$

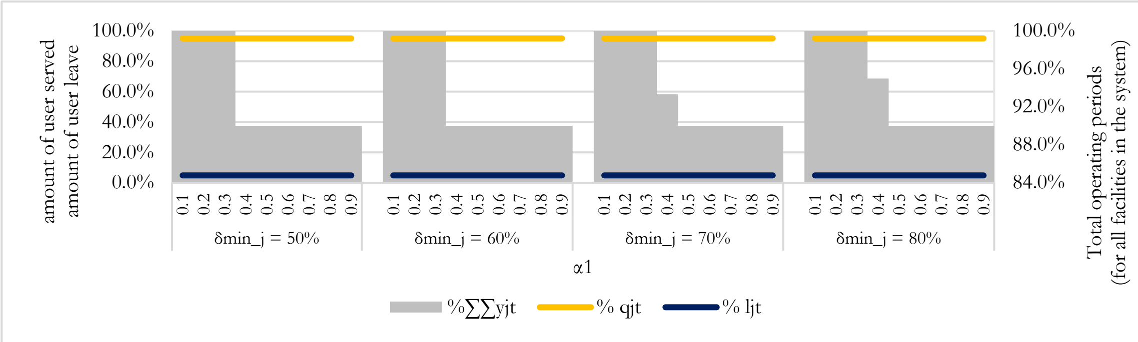
The purpose of this analysis is to test the behaviour of the constraints, especially on system performance. To achieve this, constraint (5-8) was relaxed while $\delta_{min_{jw}}$, or minimum operating periods per day, was varies between 50% - 80%. A similar procedure was used to test constraint (5-9).

4.7.2.1 Results: Variations of δ_{min_j} (minimum operating periods per week)

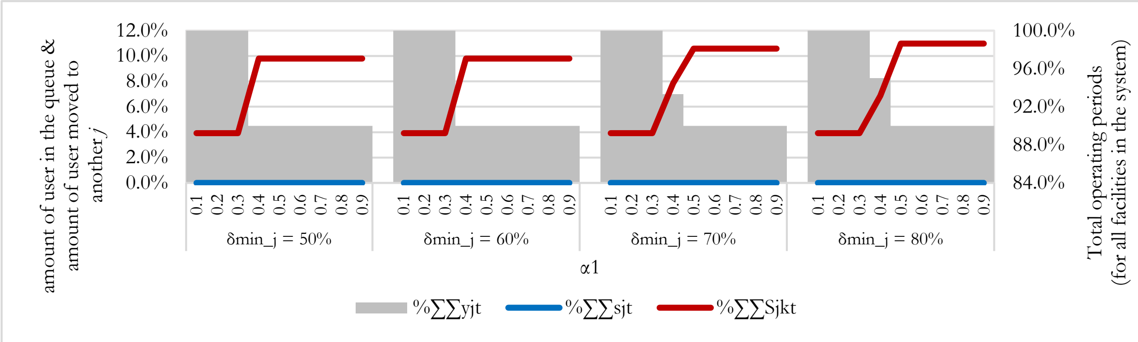
For this analysis, constraint (5-9) was relaxed. This is to determine the effect of the minimum operating period per week of a facility *j* being increased on system performance. The results in terms of total costs, on total costs, total costs on the provider's side or Z_1, total costs on the user side or Z_2, are as presented in Figures 4-26(a) – 4-27(c).



(a) Total costs, Z_1 and Z_2 values



(b) Total operating facilities, amount of users served, and amount of users leave the facility network



(c) Total operating facilities, amount of users in the queue and amount of users moved to another *j*

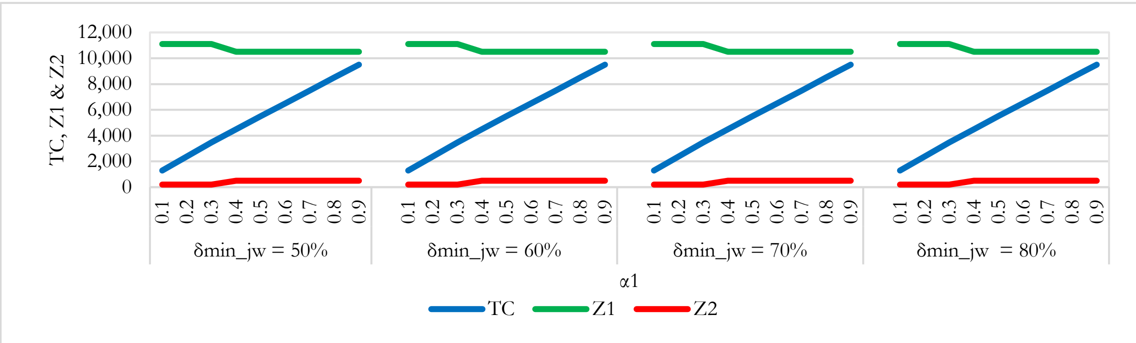
Figure 4-26: System performance with variations of α_1 and minimum of total operating facilities *j* per week

Figure 4-26(a) shows the total costs were increased when α_1 was increased. However, the increment in δ_{min_j} or minimum number of operating periods for a facility *j* per week did not affect the pattern of total costs. On the same graph, costs on the provider's side (or Z_1) and users' side (or Z_2) can be found. For all δ_{min_j} values, the Z_1 showed to decrease when α_1 was increased. In contrast, the Z_2 only showed an increase when α_1 was increased.

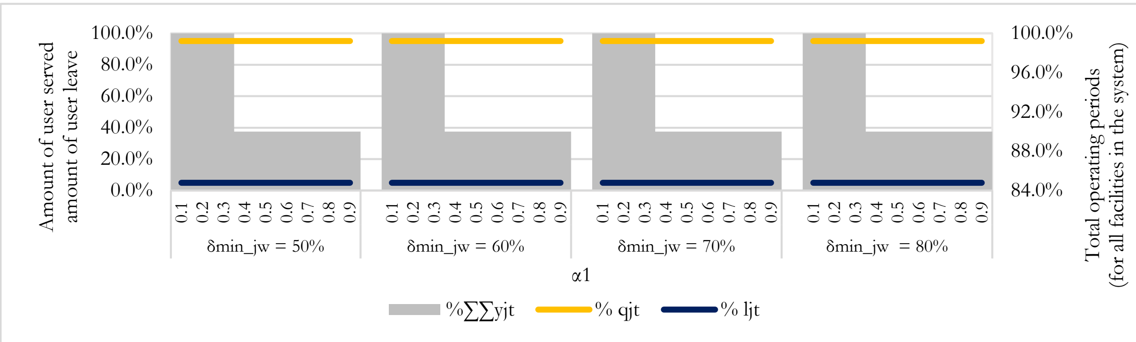
Figure 4-26(b) presents the system performance when minimum the number of operating periods for a facility *j* per week and α_1 were increased. As α_1 was increased, fewer facilities were in operation. However, incremental in δ_{min_j}, or in the minimum number of operating periods for a facility *j* per week, did not affect the system significantly. On the same graph, for all α_1 and δ_{min_j}, even fewer facilities were operating, the amount of users served remained at 95%. Therefore, as seen in Figure 4-26(c), flows of users were increased, but only for users moved to another facility. As α_1 increases, no queue was expected since the total operating *j* in the network was reduced. Having fewer operating *j*, the proposed model 'forces' more users to move to another less congested *j* instead. This is also happened probably due to the average utilisation rate of all *j* being 85%; meaning there is more residual capacity within the network. Hence, further capacity reduction can be made.

4.7.2.2 Results: Variations in $\delta_{min\,jw}$ (minimum operating periods per day)

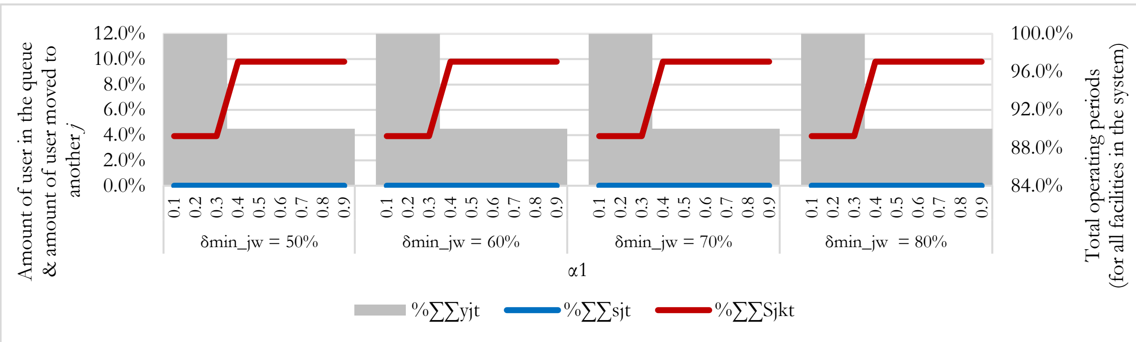
For this analysis, constraint (4-8) was relaxed. This was to determine the effect of the minimum operating period per day of a facility *j* being increased on system performance. The results in terms of total costs, on total costs, total costs on the provider's side or Z_1, total costs on the user's side or Z_2, are as presented in Figures 4-27(a) – 4-27(c).



(a) Total costs, Z_1 and Z_2 values



(b) Total operating facilities, amount of users served, and amount of users leave the facility network



(c) Total operating facilities, amount of users in the queue and amount of users moved to another j

Figure 4-27: System performance, with variations of α_1 and minimum of total operating facilities *j* per day

Figure 4-27(a) shows the total costs were increased when α_1 was increased. However, an increment in $\delta_{min_{jw}}$ or reduction in minimum operating period per day of a facility *j* was not found to affect the pattern of total costs. From the same figure, as α_1 was increased, costs on the provider's side (or Z_1) were reduced. Pattern of Z_1 however did not changed when $\delta_{min_{jw}}$ were increased. In contrast, costs on the user's side (or Z_2) were slightly increased when α_1 was increased. Similarly, pattern of Z_2 however did not changed when $\delta_{min_{jw}}$ were increased.

Figure 4-27(b) shows the flow of user and system operation whenever α_1 and $\delta_{min_{jw}}$ were increased. Results for total operating facilities in the network remained unchanged for all α_1 and $\delta_{min_{jw}}$. In general, when α_1 was increased, fewer facilities were operating and when $\delta_{min_{jw}}$ was increased, the number of operating facility *j* was unaffected. On the same figure, the amount of users served remained at 95% for all α_1 and $\delta_{min_{jw}}$. A similar situation as that for the previous experiment was found in that when α_1 and $\delta_{min_{jw}}$ were increased, no queue formed, and more user move to another facility *j* due to the existence of residual capacities in the network.

Using the dataset in section 5.7.1, constraints (5-9) and (5-10) were not found to affect system performance. This was most likely caused by $\sum_j \sum_t x_j^t \leq \sum_j \sum_t \tau_j^t$, i.e. the user will always be able to find at least one available server. The limitation in *B* also contribute to the results obtained, which required that at least 95% of amount of users were served; while limitations to users' movements, i.e., via u_{jk}^t, could also have been contributing to this result. The following analysis concentrates on the trade-off between cost and system performance.

4.7.3 Analysis 2: Trade-off in Each Variable's Costs

Five costs (C_j, ε_{1j}, ε_{2j}, ε_{3j} and ε_{4j}) and α_1 were varied for this experiment where Table 4-13 reports these variations. For cases II – IV, we assumed the fixed total costs to operate each facility *j*, C_j, were always higher than all other costs. For all experiments, *B* was set to 5% and minimum operating periods per week $\left(\delta_{min_j}\right)$ and minimum operating periods per day $\left(\delta_{min_{jw}}\right)$ was set at 50%.

Table 4-13: Parameter variations for each analysis

Case	Parameter and its variation	Fixed parameter costs
I	C_j = 20, 40, 60, …, 200	ε_{1j}= ε_{2j}= ε_{3j} = ε_{4j}= 10 units
II	ε_{1j} = 1, 2, 3, …, 10	C_j= 100; ε_{2j}= ε_{3j} = ε_{4j}= 10 units
III	ε_{2j} = 1, 2, 3, …, 10	C_j= 100; ε_{1j}= ε_{2j} = ε_{4j}= 10 units
IV	ε_{3j} = 1, 2, 3, …, 10	C_j= 100; ε_{1j}= ε_{2j} = ε_{4j}= 10 units
V	ε_{4j} = 1, 2, 3, …, 10	C_j= 100; ε_{1j}= ε_{2j} = ε_{3j}= 10 units

4.7.3.1 Varying C_j

C_j represents the cost to operate at facility *j*. The effects of variation of C_j and α_1 on total costs, total costs on the provider's side or Z_1, total costs on the user's side or Z_2, and the total operating periods for all facilities in the network, $\sum_j \sum_t y_j^t$, are presented in Figure 4-28. The values for Case I were used in this section.

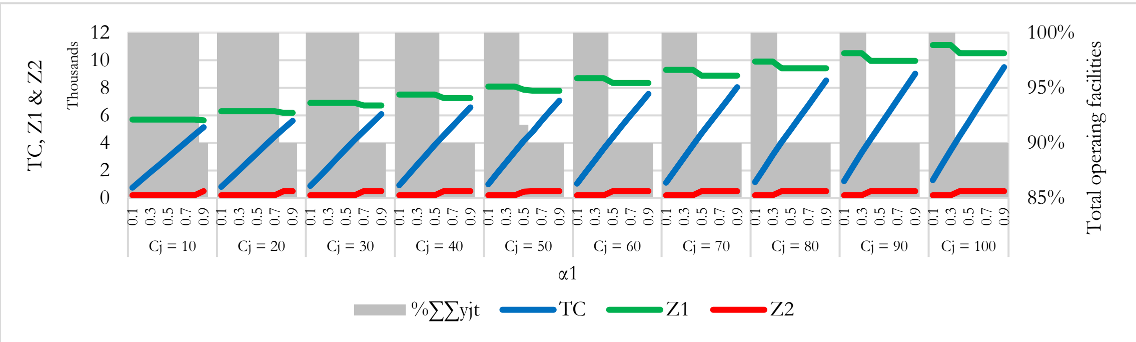


Figure 4-28: System performance using Case I

Figure 4-28 shows the changes in total costs for the system as α_1 was increased. As α_1 and C_j were increased, the total costs were found to increased rapidly, especially when α_1 and C_j were large. Costs on the provider's side, or Z_1 increased rapidly as C_j increased. However, the increase was not a result of the increment in α_1 values but was probably the result of the increment in C_j itself. Figure 4-28 also shows costs on the users' side, or Z_2. When C_j and α_1 were increased, Z_2 increased. However, the increment was influenced by the α_1 value. The increment probably contributed to a reduction in total operating facility *j* in the network whenever C_j and α_1 were increased, as shown in in grey shaded region. Hence in maintaining a 95% service level, more users could be expected to circulate within the network.

4.7.3.2 Varying ε_{1j}

ε_{1j} represents the cost to serve a unit of user. The effects of variation in ε_{1j} and α_1 on total costs, total costs on the provider's side or Z_1, total costs on the user's side or Z_2, and the amount of users served by the facility network, $\sum_j \sum_t q_j^t$, are presented in Figure 4-29. The values for Case II were used in this section.

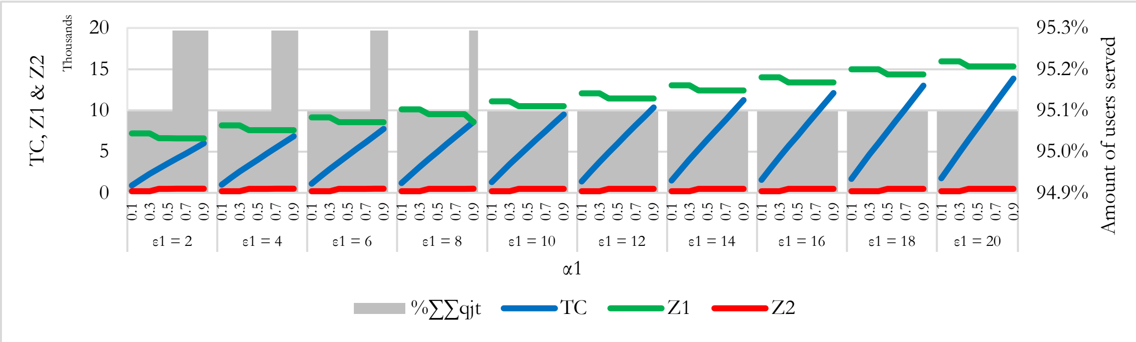


Figure 4-29: System performance using Case II

Figure 4-29 shows the total cost on the provider's side, or Z_1, increased when ε_{1j} increased. However, α_1 seems not to contribute to the increment. Since ε_{1j} is part of the Z_1 component, this explains the increment. From Figure 4-29, when ε_{1j} and α_1 were increased, the costs on the users' side, or Z_2 was found to barely change. This showed that there was a greater circulation of users within the network when α_1 was increased. Meanwhile, the amount of users served by the operating facilities in the network is shown in grey shaded region. For each α_1 value, whenever ε_{1j} increased, the amount of user served was at least 95%. This was probably the amount of user served was bounded by *B* value.

4.7.3.3 Varying ε_{2j}

ε_{2j} represents the cost of a unit of user being in the queue. The effects of variation of ε_{2j} and α_1 on total costs, total costs on the provider's side or Z_1, total costs on the user's side or Z_2, and the amount of user in the queue in the facility network, $\sum_j \sum_t s_j^t$, are presented in Figure 4-30. The values for Case III were used in this section.

From Figure 4-30, it can be seen that the total costs increased when α_1 was increased. ε_{2j} did not affect the total costs at all. Similarly, the cost on the provider's side, or Z_1, were reduced when α_1 was increased. In contrast, the cost on the users' side, or Z_2, were slightly increased when α_1 was increased. From this figure, no queue was formed for all α_1 and ε_{2j}

values. This is because the dataset used in this analysis consisted of only one congested facility j, while the other three did not, implies that the residual capacity was always present in this dataset, i.e., $\sum_j \sum_t x_j^t \leq \sum_j \sum_t \tau_j^t$. From this analysis, it was clear that neither the smallest or largest values of ε_{2j} contributed to flow of demand in the network.

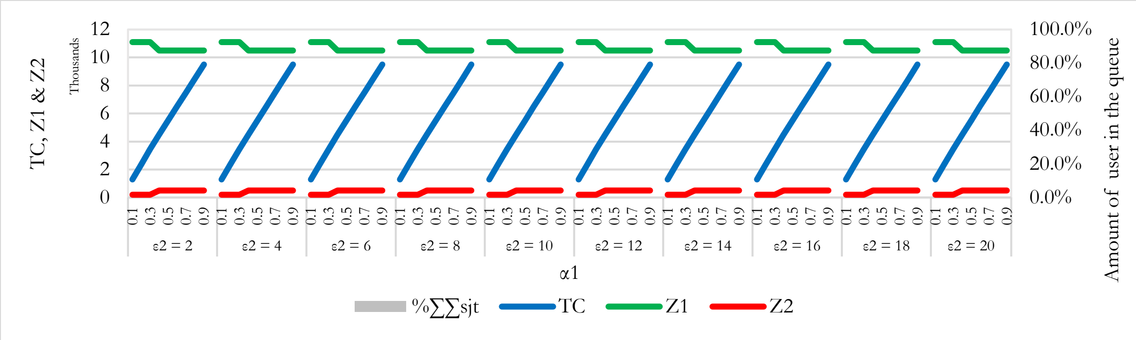


Figure 4-30: System performance using Case III

4.7.3.4 *Varying* ε_{3j}

ε_{3j} represents the cost for a unit of user to leave the facility network. For this chapter, whenever users leave the network, we assumed that demand will not recycle, or fly-tip. For not recycling, is a loss to the provider as they have to provide extra cost in handling the fly-tipping cases. The effects of variation of ε_{3j} and α_1 on total costs, total costs on the provider's side or Z_1, total costs on the user's side or Z_2, and the amount of user to leave the facility network, $\sum_j \sum_t l_j^t$, are presented in Figure 4-31. Value of Case IV were used in this section.

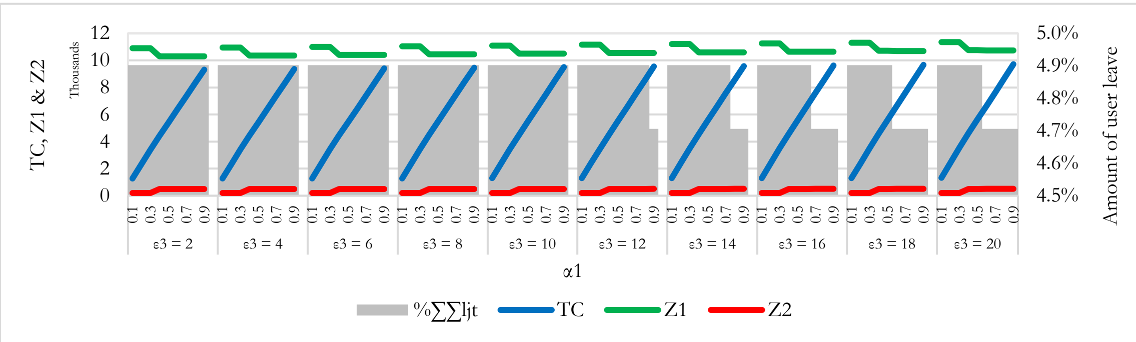


Figure 4-31: System performance using Case IV

Figure 4-31 illustrates the fact that the total cost was increased when α_1 was increased. Increment in ε_{3j} also increased the total costs, but not that obviously. Meanwhile, costs on the provider's side, or Z_1 were remained unchanged when ε_{3j} increased. However, Z_1 were decreased with increments in α_1, probably due to reduction in operating facilities. The reduction in operating facilities was caused by the increment in cost that paid by the provider. From Figure 4-31, the costs on users' side, or Z_2 was unchanged whenever ε_{3j} was greater than other costs. Z_2 was affected by the weight, in which Z_2 was decreased when α_1 was increased. Figure 4-31 also shows the amount of user leaving the network, which has been reduced when ε_{3j} and α_1 were increased. The maximum amount of user that could leave was 4.9%, as bounded by *B*.

4.7.3.5 Varying ε_{4j}

ε_{4j} represents the cost of a unit of demand move to another facility. The effects of variation of ε_{4j} and α_1 on total costs, total costs on the provider's side or Z_1, total costs on the demand side or Z_2, and the amount of demand moving to another facility, $\sum_j \sum_t S_{jk}^t$, are presented in Figure 4-32. The values of Case V were used in this section.

From Figure 4-32, the total costs were increased when α_1 was increased, but the pattern remained unchanged when ε_{4j} was increased. Costs on provider's side, or Z_1, were increased when ε_{4j} was increased but, as α_1 increased, Z_1 were reduced. Meanwhile the costs on the users' side, or Z_2 increased rapidly when ε_{4j} and α_1 increased. Figure 4-33 also illustrates the amount of user moving to another facility *j* for all ε_{4j} and α_1. Whenever ε_{4j} was large and α_1 small, fewer users moved to another facility *j*. With increasing α_1, the percentage of users moving to another facility *j* was increased regardless of the value of ε_{4j}.

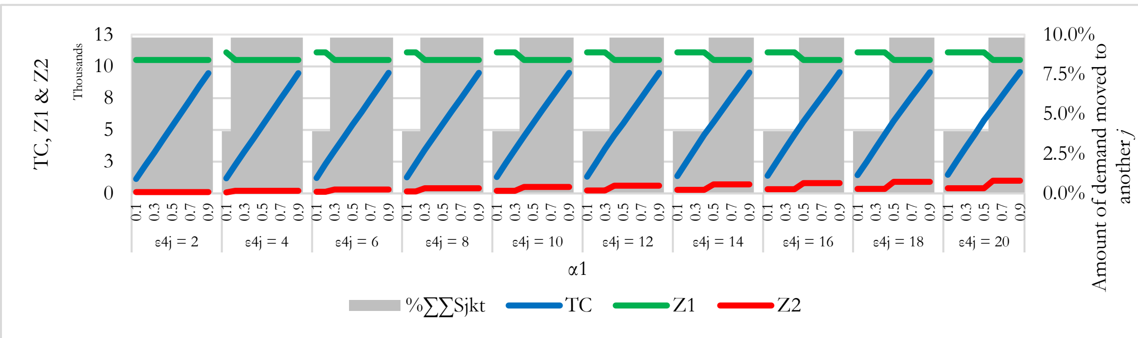


Figure 4-32: System performance using Case V

From the analyses, similar to our previous sensitivity analysis, *B* plays an important role in keeping the service level to a minimum. Due to this, even the costs on the provider's side were increased; more users would be served, hence increasing the total costs of operating the network. At the same time, using the test dataset, minimum operating periods per week

$\left(\delta_{min_j}\right)$ and minimum operating periods per day $\left(\delta_{min_{jw}}\right)$ were found not to affect the reorganisation significantly. This is because the dataset for this experiment had an average 85% utilisation rate, whilst at the same time the proposed model forced the network to serve 95% of its users. The entire numerical results for this section (i.e., the sensitivity analyses) as presented in Appendix 4(E). The following section considers the implementation of the proposed model to examine the HWRC problem in Sheffield City Council area.

4.8 Data Collection and Parameter Setting

There are five HWRCs, therefore, $J = 5$. Assuming each HWRC operates seven days a week, eight hours per day, hence $W = 7$, length $H = 8$ and $T = 56$. The remaining parameters set as shown in the following sub-paragraphs.

4.8.1 Costs value

4.8.1.1 Cost to Operate Facility j (C_j)

Staffing costs and **operation costs** were utilised to calculate the value of C_j. The costs to operate facility *j* were as indicated in Table 4-14, where these figures were obtained directly from Sheffield City Council. From this table, the most expensive facility to operate is Longley Avenue. This may be due to the facility operating seven days a week, and thus increasing overall operating costs.

Table 4-14: The staffing and operational costs for each HWRC

HWRC	Staffing cost	Operation cost	C_j
Beighton Road	60	136	196
Blackstock Road	110	136	246
Deepcar	48	107	155
Greaves Lane	60	107	167
Longley Avenue	88	167	255

4.8.1.2 Costs to Serve a Unit of User (ε_{1j})

The cost of ε_{1j} is defined as the cost of **processing** a unit of users in a facility *j*. Table 4-15 provides the price of recycling various materials given by the council.

Table 4-15: Cost of processing various materials

Materials	Price (£)/ tonne
Paper/Card	38.61
Textiles	38.61
Green	38.61
Glass	38.61
Wood Viridor and Mixed Wood	50.70
Plastic	38.61
Plasterboard	108.49
Shoes	38.61
TOTAL	**390.85**

From Table 4-15, the total cost required to recycle these eight materials is £390.85 or an average of £48.86 per tonne per material. To find the cost of **processing or serving a unit of users** at each *j*, the following formulation was used:

$$\varepsilon_{1j} \quad = \quad \frac{cost\ (£)}{weight\ of\ recyclables} \times \frac{weight\ of\ recyclables}{user}$$

To find the average recyclables per user, a combination of the number of visits and the collected data on weights of recyclable items received by the Council was used. Table 4-16 provides information on total recyclables for 2012/13.

Table 4-16: Recyclable materials collected at each HWRC for 2012/13

HWRC	Total Recyclables (tonnes/ year)
Beighton Road	2,750.37
Blackstock Road	2,746.64
Deepcar	1,146.34
Greaves Lane	1,269.62
Longley Avenue	3,517.11
TOTAL	**11,430.08**

Therefore, the total amount of recyclables received per year was further modified, in order to obtain the total amount of total recyclables per week as:

$$\frac{Total\ weight\ per\ year}{12 \times 4} = \frac{11430.08}{48} = 238.13\ tonnes\ per\ week$$

by assuming 48 weeks per year (4 weeks per month). Since the average number of visits for Sheffield's HWRCs is **14,069** users per week, thus the total weight of recyclables received from a single user per week are:

$$\frac{238.13\ tonnes}{week} \div \frac{14069\ users}{week} = 0.02\ tonnes\ per\ user\ per\ week$$

Therefore, the ε_{1j} can be determined as:

$$\begin{aligned}\varepsilon_{1j} &= \frac{cost\ (£)}{weight\ of\ recycables} \times \frac{weight\ of\ recycables}{user} \\ &= \frac{£\ 390.85}{tonne} \times \frac{0.02\ tonnes}{user} = \ £\ 6.62\ per\ user\end{aligned}$$

4.8.1.3 Cost of a Unit of Users Waiting in the Queue (ε_{2j})

The cost of for a single user waiting in the queue, or ε_{2j}, is measured as an hourly **opportunity cost**. An opportunity cost is the profit gained or lost if another alternative is taken. For example, the cost of waiting in a queue could be set equal to one hour of salary. In the UK, the minimum hourly salary is standardised at £7.83 (source: HMSO, 2018). Figure 4-33 shows the opportunity costs for increasing amounts of time, as a step function.

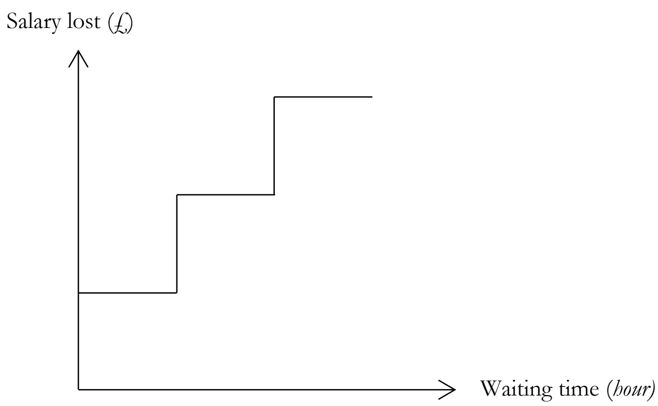


Figure 4-33: The prediction of the salary loss with respect to increases in waiting time.

4.8.1.4 Cost of a Unit of Users Leaving the System (ε_{3j})

For this chapter, the cost of ε_{3j} is the defined as cost faced by the council for a customer leaving the system, and not properly recycling their waste; in this case, it is assumed that they are illegally dumping this waste or **fly-tipping**. As mentioned, an insufficient number of recycling facilities, and non-optimal facility locations or schedules may contribute to this problem. There are two costs involved in solving fly-tipping; clearance costs and enforcement costs. Besides service provider, users also face the consequences of not recycling, for example, paying a fine. This also shows both parties suffer due to fly-tipping. Hence, the cost of ε_{3j} is assumed to be a combination of **clearance costs**, **enforcement costs**, **and fines paid** per incident, which is:

ε_{3j} = Clearance costs per incident + enforcement cost per action – fine paid per offender

As mentioned, 1,002,000 incidents of fly-tipping were reported in England in 2016/17, at a cost of £57.7 million to clear the associated waste (DEFRA, 2017). Enforcement costs total £16 million for almost 500,000 actions taken. It is assumed that both costs applied to Sheffield as well. Therefore:

$$\text{Clearance costs} = \frac{£\,57{,}667{,}483}{1{,}002{,}154\ incidents} = £\,57.52\ per\ incident$$

$$\text{Enforcement costs} = \frac{£\,16{,}029{,}265}{474{,}460\ actions} = £\,33.78\ per\ action$$

Finding penalty fines for fly-tipping incidents is tricky since it is depending on incident locations, level of violence and Council's fees. Sheffield City Council set a minimum fly-tipping penalty of £80 per incident. Thus, the final cost of ε_{3j} can be determined as:

ε_{3j} = Clearance costs per incident + enforcement cost per action – fine paid per offender

$= £\,57.52 + £\,33.78 - £\,80$

$= £\,11.30$ per user

4.8.1.5 Cost of a Unit of Users Moving to Another Facility (ε_{4j})

ε_{4j} cost is assumed to be a combination of **opportunity** and **transportation cost** for a unit of users to move to another facility. Each movement is assumed to be identical and the journey costs 15 minutes of time per user. As mentioned previously, each user that moves to another facility *j* will be served within the same period.

Opportunity cost is the loss suffered by the user if they move to another facility, due to the waste of time this involves. This cost is assumed to be £7.83 per hour. This cost is then divided by 4 to obtain the cost for a 15-minute time-frame. Therefore, user suffers a loss of about £1.96 per movement.

Transportation costs are assumed to be a combination of current unleaded fuel prices (as of 2018), distance covered per litre of fuel (constant coverage) at a constant speed. We use a medium sedan car and fuel prices from the 'PetrolPrices' website as our reference (PetrolPrices.com, 2018). A medium sedan car consumes 6 litres per 100 kilometres or 0.10 litres per mile at a constant speed at 60 miles per hour. Fuel prices are taken to be 129.5 pence per litre. Therefore, the transportation cost can be determined as:

$$\frac{£1.295}{l} \times \frac{0.10\, l}{miles} \times \frac{60\, miles}{hour} = \frac{£\,7.77}{hour}\ or\ \frac{£\,1.94}{15\, minutes}$$

From the opportunity and transportation costs, the ε_{4j} is £1.96 + £1.94 = £3.90 per movement.

4.8.2 *Capacity Level (τ_j^t)*

The capacity of a HWRC per time period is based on the maximum amount of user visiting an HWRC per hour. This data was provided by the Council and is based on 15-minute time-windows. Since we are using *t* on a per hour basis, hence the data is multiplied by 4, and the results are presented in row 2 of Table 4-17. From this table, Beighton Road is capable of serving 3 users per minute while the others are able to serve 2 users per minute.

Table 4-17: Capacity level for each HWRC per hour

HWRC	Beighton Rd	Blackstock Rd	Deepcar	Greaves Lane	Longley Avenue
Maximum capacity per facility (per hour)	164	136	108	100	124
Maximum capacity per facility (per 15 minutes)	41	34	27	25	31
Capability to serve users per minute	3	2	2	2	2

4.8.3 *Reachability of Facility j and other (u_{jk}^t)*

Let u_{jk} be defined as $u_{jk} = \begin{cases} 1 & if \quad dist_{jk} \leq 15\, minutes \\ 0 & \quad otherwise \end{cases}$ for all time *t*. Table 4-18 shows the distance between each HWRC ($dist_{jk}$) in minutes while Table 4-19 shows the computed u_{jk} values.

Table 4-18: Distance between facility *j* in minutes (Source: Sheffield City Council).

HWRC		*k*				
		Beighton Rd	**Blackstock Rd**	**Deepcar**	**Greaves Lane**	**Longley Avenue**
j	**Beighton Rd**		15	30	30	20
	Blackstock Rd	15		30	30	15
	Deepcar	30	30		10	17
	Greaves Lane	30	30	10		17
	Longley Avenue	20	15	17	17	

Table 4-19: The u_{jk} values

HWRC		*k*				
		Beighton Rd	**Blackstock Rd**	**Deepcar**	**Greaves Lane**	**Longley Avenue**
j	**Beighton Rd**		1	0	0	0
	Blackstock Rd	1		0	0	1
	Deepcar	0	0		1	0
	Greaves Lane	0	0	1		0
	Longley Avenue	0	1	0	0	

Table 4-19 shows value of 1 if the second recycling centre that is visited by users (indexed by *k*), 0 otherwise. The users of Beighton Road and Longley Avenue will go to is Blackstock Road, and vice versa. Meanwhile, Deepcar users will move to Greaves Lane, and vice versa. It is assumed that each movement is identical at all times. For example, if Beighton Road is closed, users will always go to either Blackstock Road or Longley Avenue to dispose of their recyclable items.

4.8.4 Minimum Operation Periods Per Day and Per Week for a Given Facility j

δ_{min_j} and $\delta_{min_{jw}}$ is set at 50% for both lower bounds. It is assumed that the minimum operating periods per day are 50% of *T* (about 28 hours per week) and minimum operating periods per day are 50% of *H* (about 4 hours per day).

4.9 Results

The results section will be discussed in three distinct sections:

1. New operational times for each HWRC
2. Improvement of current schedule for each HWRC
3. What-if analysis –Sheffield City Council as a decision-maker.

For these results, parameter B is set to 0.05 and α_1 parameterised between 0.1 and 0.9 in increments of 0.1 per step. In terms of the considered facilities, y_1 is Beighton Road, y_2 is Blackstock Road, y_3 is Deepcar, y_4 is Greaves Lane and y_5 is Longley Avenue and arranged as $[y_1, y_2, y_3, y_4, y_5]$.

4.9.1 *Result 1: New Operational Times for Each HWRC*

To ensure optimal operation of a given HWRC, it is assumed that each HWRC is able to operate from Monday to Sunday, from 10:00 am until 6:00 pm. Initial δ_j for all facility *j* is [56, 56, 56, 56, 56] showing no restriction on maximum operating periods per facility *j*. Figures 4-34 – 4-36 presents the associated results.

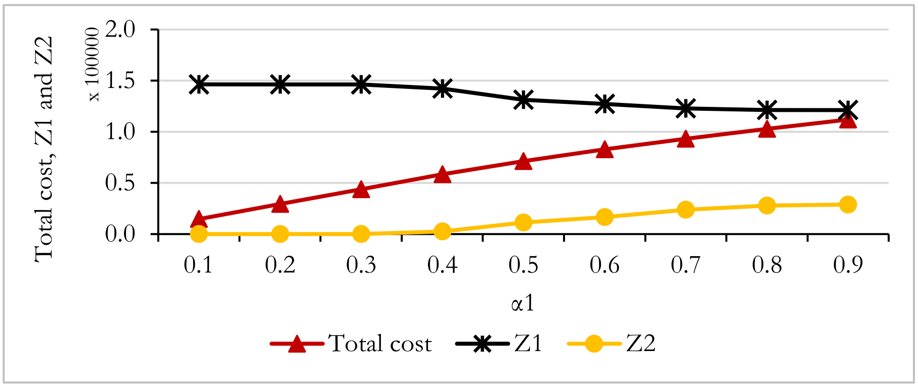


Figure 4-34: Changes in objective function (total cost) values for the Z_1 and Z_2 based on variation in α_1

From Figure 4-34, the total costs to operate the HWRC network increase as the weight on the provider, α_1, increases. Even so, the costs borne by the provider (Z_1) are slightly reduced compared to costs on the users' side (Z_2). Details on the flow of users are shown in Figures 4-35 and 4-36.

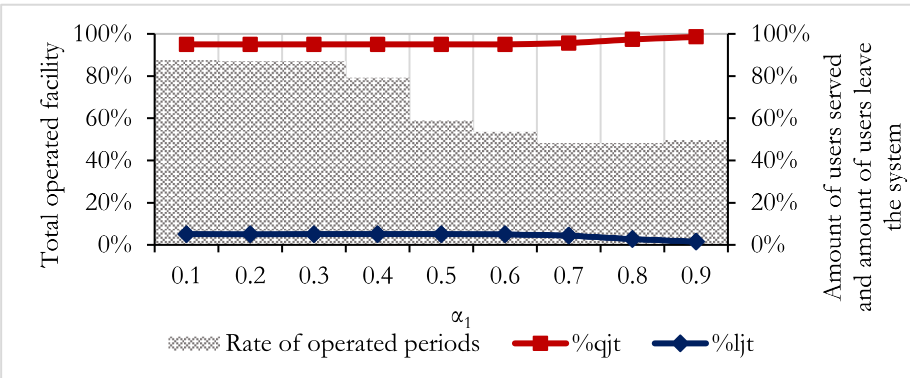


Figure 4-35: The effect of variation in α_1 values as a percentage of total operating periods, the amount of users served, or q_j^t, and the amount of users leaving, or l_j^t.

From Figure 4-35, it was found that as α_1 increases, the percentage of the total number of operating periods is reduced. Components on the provider's side, q_j^t and l_j^t, are slightly altered as α_1 increases, where a minimum of 95% of the total number of HWRC users are expected to be served, even though the operating facilities are quite limited. In contrast, amount of users left were reduced as α_1 increased. Both changes indirectly indicate that users circulation between existing facilities was increased. This is further confirmed in Figure 4-36.

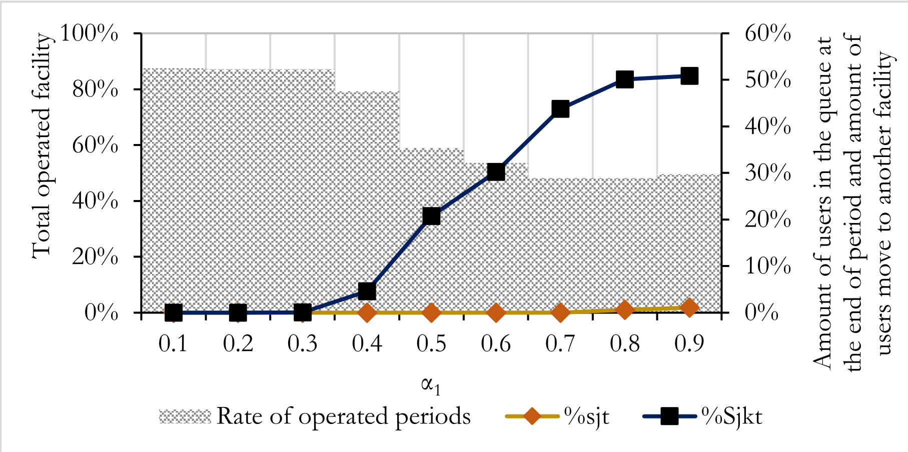


Figure 4-36: The effect of variation in α_1 on the amount of users in the queue at the end of *t* (s_j^t) and amount of users that move to another facility *j* (S_{jk}^t)

Clearly, from Figure 4-36, the amount of users moving to another *j* increases drastically when $\alpha_1 \geq 0.3$ units and reaches approximately 50% at 0.9 units. Meanwhile, the amount of users in the queue is not as large as we expected. This could be caused by ε_{4j}, which is the lowest costs compared to others, hence having more users moving to another *j* is possible. Meanwhile, the cost related to queue (ε_{2j}) was the second highest compared to the other costs, hence prevent users to be in the queue. In addition, the operating periods per network were reduced when α_1 was increase, meaning there was less available capacity for users to wait in the queue and to be served on the next period.

Details of all the permutations examined are presented in Table 4-20. From this table, when α_1 is between 0.5 and 0.7, one HWRC is completely closed by the model. When $\alpha_1 \geq 0.8$, only three HWRCs are in operation. Within the α_1 values, the minimum number of operating HWRCs occurs when $\alpha_1 = 0.8$, hence an optimal weekly schedule for the HWRC network can be formed – as in Table 4-2. Meanwhile, Figure 4-37 summarised the user movement within the HWRC network when $\alpha_1 = 0.8$.

Table 4-21 presents the new optimal schedule for the HWRC network. This table indicates that the operation of all HWRCs when Greaves Lane and Longley Avenue are completely closed. From the results of the spatial interaction model, Longley Avenue is the most preferred HWRC. However, the use of our model suggests this site should be closed completely due to high operational costs and reachability of Longley Avenue compared to the next preferred recycling sites; i.e., Beighton Road and Blackstock Road. For the same reasons, our model indicated that the Greaves Lane site should also be closed. Greaves Lane and Deepcar are located within 15 minutes of each other, and it is much more expensive to operate Greaves Lane. Hence, our model chose Greaves Lane for closure, and that most of this site's users can instead visit Deepcar to do their recycling. Due to there being only three operating HWRCs, a large percentage of S_{jk}^{t} is expected. Table 4-21 also shows only two HWRCs are in operation on Tuesdays and Wednesdays, which are Blackstock Road and Deepcar. Blackstock Road is completely open for the entire week, and for the entire periods. With three HWRCs, namely Beighton Road, Deepcar and Blackstock Road, about 97% of Sheffield's residents can be covered in terms of recycling. More than 50% of Sheffield's residents would be expected to move to the next nearest recycling site. However, these users might also leave the system entirely. Details on users movement as in Figure 4-37.

Table 4-20: System performance

α_1	α_2	Total cost (objective function)	Total cost provider's side (Z_1)	Total cost user's side (Z_2)	δ_j	$\% \sum_j \sum_t y_j^t$ (Total Operating Facilities)	$\% \sum_j \sum_t q_j^t$ (Amount of Users Served)	$\% \sum_j \sum_t l_j^t$ (Amount of Users Leaving the System)	$\% \sum_j \sum_t s_j^t$ (Amount of Users were in the Queue)	$\% \sum_j \sum_t S_{jk}^t$ (Amount of Users Moved to Another Facility)
0.1	0.9	14,634	146,339	-	[54, 54, 41, 45, 51]	88%	95%	5%	0%	0%
0.2	0.8	29,268	146,338	-	[53, 54, 39, 45, 53]	87%	95%	5%	0%	0%
0.3	0.7	43,899	146,284	20	[54, 54, 39, 45, 52]	87%	95%	5%	0%	0%
0.4	0.6	58,322	142,062	2,496	[50, 54, 31, 41, 46]	79%	95%	5%	0%	5%
0.5	0.5	71,264	131,217	11,310	[40, 54, 0, 41, 30]	59%	95%	5%	0%	21%
0.6	0.4	82,985	127,329	16,470	[37, 39, 0, 45, 29]	54%	95%	5%	0%	30%
0.7	0.3	93,187	122,909	23,833	[29, 31, 47, 0, 28]	48%	96%	4%	0%	44%
0.8	**0.2**	**102,688**	**121,370**	**27,958**	**[31, 56, 48, 0, 0]**	**48%**	**97%**	**3%**	**1%**	**50%**
0.9	0.1	111,958	121,189	28,876	[31, 56, 52, 0, 0]	50%	99%	1%	1%	51%

Table 4-21: Weekly Schedule for HWRCs using $\alpha_1 = 0.8$

Day	HWRC	Time							
		10:00 - 11:00	11:00 - 12:00	12:00 - 13:00	13:00 - 14:00	14:00 - 15:00	15:00 - 16:00	16:00 - 17:00	17:00 - 18:00
Monday	Beighton Road								
	Blackstock Road								
	Deepcar								
	Greaves Lane								
	Longley Avenue								
Tuesday	Beighton Road								
	Blackstock Road								
	Deepcar								
	Greaves Lane								
	Longley Avenue								
Wednesday	Beighton Road								
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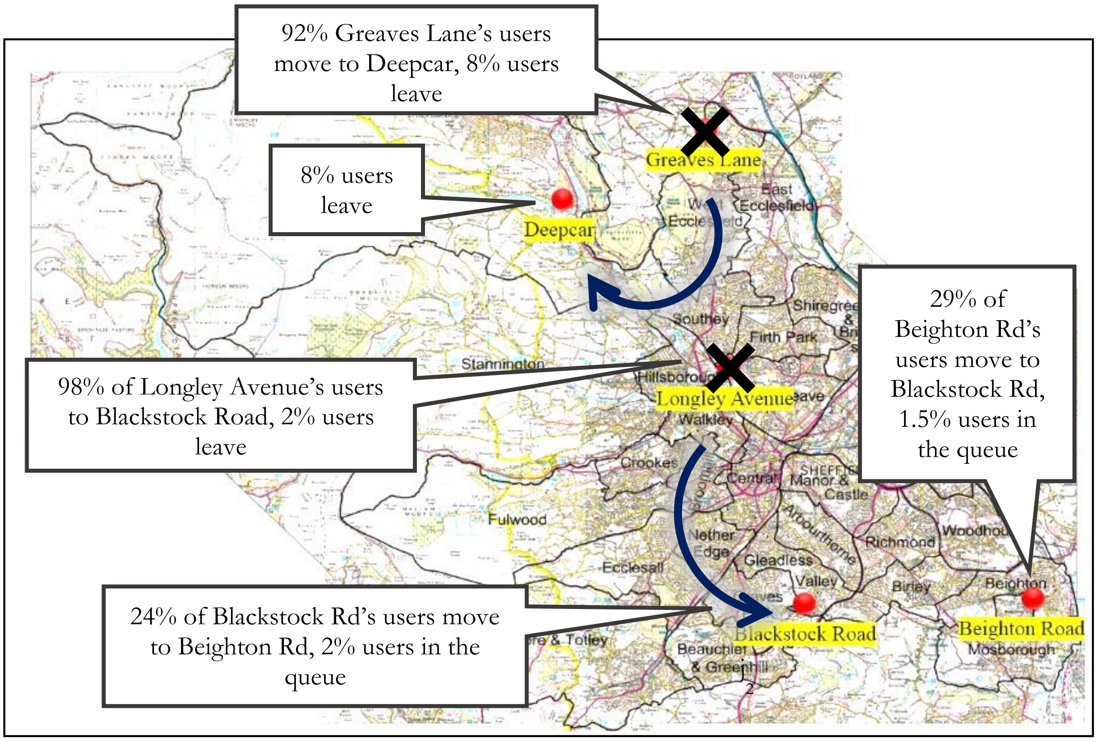
	Greaves Lane								
	Longley Avenue								



Note: System performance is at 95%, with α_1=0.8.

Figure 4-37: HWRC's user movement within the network

Figure 4-37 presents the movement of users after two HWRCs, i.e. Greaves Lane and Longley Avenue, were closed completely by the proposed model. When Greaves Lane site was shut down completely, 92% of its users were expected to use the Deepcar site, while the remaining left the network. Meanwhile, 98% of Longley Avenue users were expected to move to Blackstock Road and 2% were expected for not recycling. Additionally, the remaining HWRCs were expected to have overflow of users; causing some users left the network (not recycling), moved to another HWRC, or being in the queue. For instance, some users of Deepcar left due to increase in amount of user that were from Greaves Lane. Meanwhile, some of users of Beighton Road moved to Blackstock Road whenever the site is congested, and vice versa.

Next section presents the result when the original workload of each HWRC were improve.

4.9.2 *Result 2: Improvement in the current operations of each HWRC*

Given that decision-makers refuse to increase the workload of any HWRC, the current schedule can be improved using the proposed model. The current schedules for each HWRC are:

Table 4-22: Current schedule of all HWRCs

HWRC / Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Beighton Road	√		√	√	√	√	√
Blackstock Road	√	√		√	√	√	√
Deepcar	√			√	√	√	√
Greaves Lane	√	√			√	√	√
Longley Avenue	√	√	√	√	√	√	√

Therefore, the initial $\delta_j = [48, 48, 40, 40, 56]$. The results are presented in Figures 4-38 – 4-41 and Tables 4-22 and 4-23.

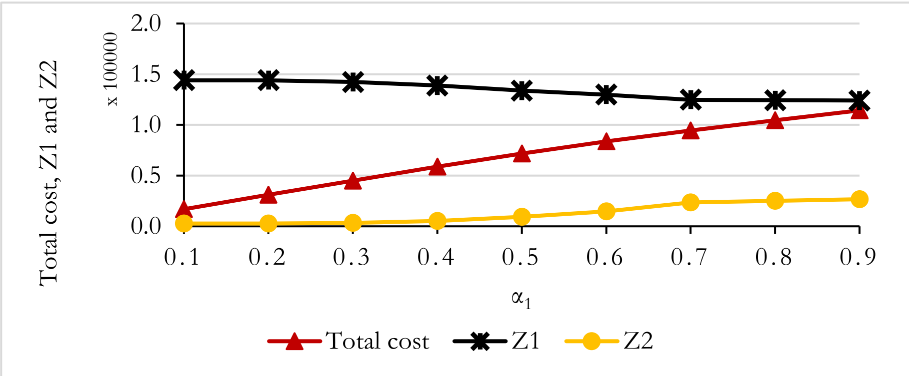


Figure 4-38: Changes in objective function values; provider's side (Z_1) and users' side (Z_2) based on variation in α_1 values

From Figure 4-38, when α_1 increases, the total costs for the system network increase. Similar to result 1, when α_1 increases, Z_1 decreases and Z_2 increases. This occurs because the weight on the provider's side is increased, and hence fewer facilities would be expected to operate. Further confirmation is provided in Figures 4-39 and 4-40, which show the flow of users within the network.

Figure 4-39 shows the amount of users served by the operating HWRCs and amount of user left in the network. The percentage seems unchanged for all α_1; even though the total number of operating HWRCs is decreased. This implies an increment in user circulation between the operating HWRCs, as shown in Figure 4-40.

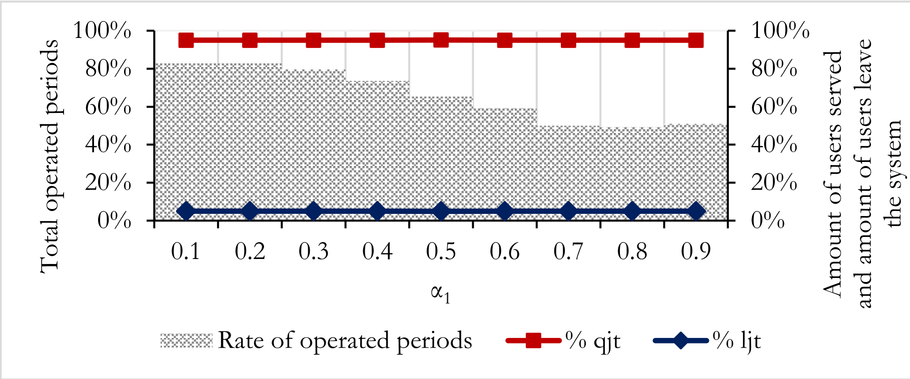


Figure 4-39: The effect of different α_1 values as a percentage of total operating periods, the amount of users served, or q_j^t, and amount of users leaving, or l_j^t.

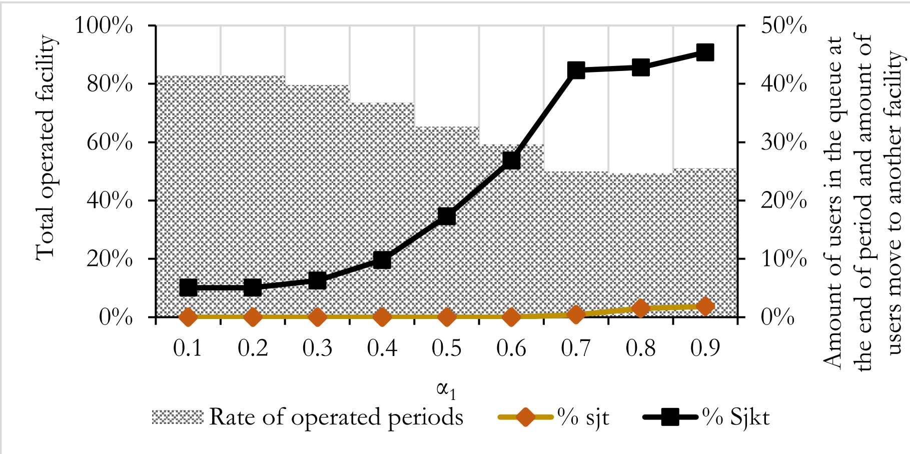


Figure 4-40: The effect of different in α_1 on the amount of users in the queue at the end of t (s_j^t) and amount of users that move to another facility j (S_{jk}^t)

From Figure 4-40, the amount of user moving to another facility *j*, or S_{jk}^t show a rapid increasing trend until α_1 = 0.6. At this value of α_1, the amount of user in the queue shows a slight increment. Again, this is because ε_{4j} is at its lowest, i.e., the cheapest cost to the user.

Table 4-23 presents the entire set of results, which when $\alpha_1 \geq 0.7$, one HWRC should be closed completely. However, as α_1 increases to greater than 0.7, the number HWRCs closed by the proposed model remained unchanged. The lowest percentage of total operating periods for the network occurs when α_1 = 0.8. At this point, only 49% of HWRCs are operating for the entire week but without further increases in the amount of user served or users leaving the network. Compared to other α_1 values, focussing on such values as when one HWRC is closed, i.e., when α_1 = 0.8, the system is operating at its minimum capacity, and hence an improved weekly schedule for HWRC can be generated.

Table 4-23: The operation periods for facility *j* as based on the current schedule

α_1	α_2	Total cost (objective function)	Total cost provider's side (Z_1)	Total cost user's side (Z_2)	δ_j	$\% \sum_j \sum_t y_j^t$ (Total Operating Facilities)	$\% \sum_j \sum_t q_j^t$ (Amount of Users Served)	$\% \sum_j \sum_t l_j^t$ (Amount of Users Leaving the System)	$\% \sum_j \sum_t s_j^t$ (Amount of Users were in the Queue)	$\% \sum_j \sum_t S_{jk}^t$ (Amount of Users Moved to Another Facility)
0.1	0.9	16,872	143,972	2,750	[48, 48, 40, 40, 56]	83%	95%	5%	0%	5%
0.2	0.8	30,994	143,972	2,750	[48, 48, 40, 40, 56]	83%	95%	5%	0%	5%
0.3	0.7	45,059	142,233	3,413	[47, 47, 36, 39, 54]	80%	95%	5%	0%	6%
0.4	0.6	58,711	138,768	5,339	[45, 47, 32, 35, 47]	74%	95%	5%	0%	10%
0.5	0.5	71,631	133,855	9,407	[42, 48, 28, 30, 35]	65%	95%	5%	0%	17%
0.6	0.4	83,737	129,824	14,606	[42, 40, 28, 28, 28]	59%	95%	5%	0%	27%
0.7	0.3	94,390	124,766	23,511	[38, 32, 40, 0, 30]	50%	95%	5%	0%	42%
0.8	**0.2**	**104,446**	**124,315**	**24,970**	**[37, 32, 40, 0, 29]**	**49%**	**95%**	**5%**	**2%**	**43%**
0.9	0.1	114,330	124,064	26,723	[39, 48, 28, 28, 0]	51%	95%	5%	2%	45%

Table 4-24: Improved weekly schedule for HWRCs based on $\alpha_1 = 0.8$

Day	HWRC	Time							
		10:00 - 11:00	11:00 - 12:00	12:00 - 13:00	13:00 - 14:00	14:00 - 15:00	15:00 - 16:00	16:00 - 17:00	17:00 - 18:00
Monday	Beighton Road								
	Blackstock Road								
	Deepcar								
	Greaves Lane								
	Longley Avenue								
Tuesday	Beighton Road								
	Blackstock Road								
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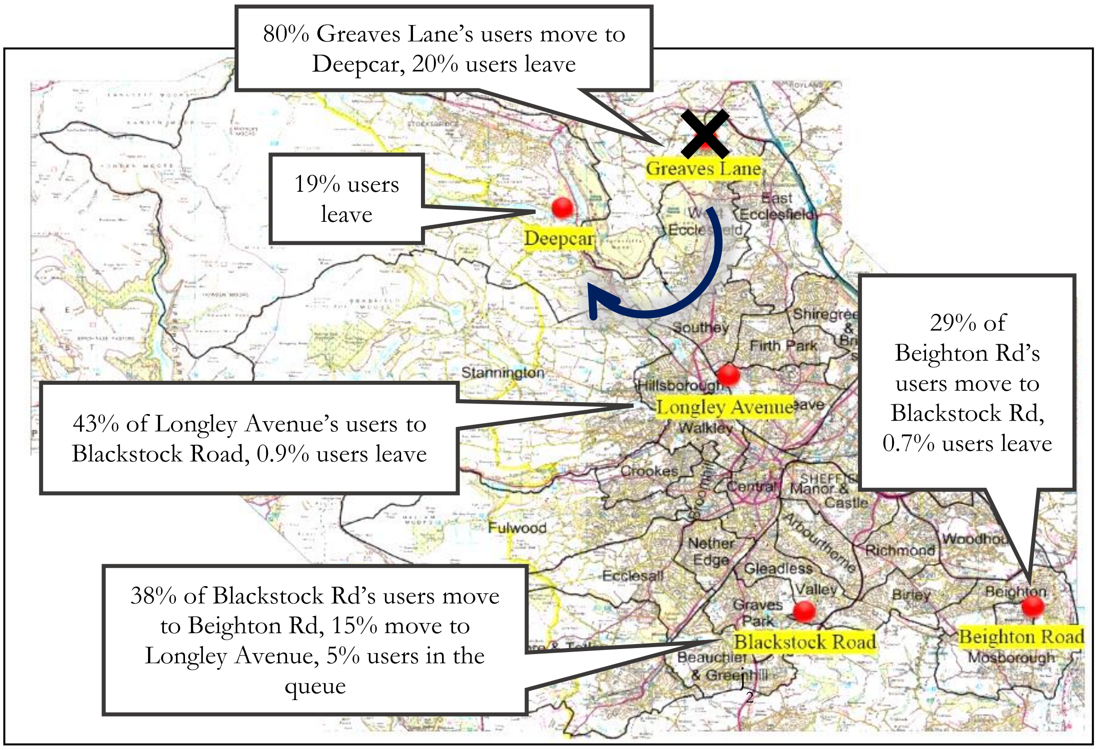
Saturday	Beighton Road								
	Blackstock Road								
	Deepcar								
	Greaves Lane								
	Longley Avenue								
Sunday	Beighton Road								
	Blackstock Road								
	Deepcar								
	Greaves Lane								
	Longley Avenue								

Table 4-24 presents the improved schedule for each HWRC using α_1 = 0.8. Only one HWRC is closed, Greaves Lane; probably closed by the proposed model due to its high operational costs and reachability compared to Deepcar. Users of Greaves Lane would move to Deepcar for their recycling, hence explaining the large percentage of users moving to another HWRC. On Tuesdays, only Blackstock Road site is in operation, and this site is completely closed over the weekend. Meanwhile, on Wednesday, two sites, Beighton Road and Longley Avenue, are in operation. For the remainder of the week, three sites are in operation. Based on the results obtained, if the Council want to improve, or reduced the HWRC's current schedule, then four HWRCs are needed to ensure at least 95% of Sheffield's residents are served. Having fewer facilities means having more users move to another facility *j*. This will directly increase the risk of users not recycling, and ultimately might result in increased fly-tipping.



Note: System performance is at 95%, with α_1=0.8.

Figure 4-41: HWRC's user movement within the network

Figure 4-41 illustrates on the users' mechanism for α_1 = 0.8. 80% of users from Greaves Lane moved to Deepcar when the site was not operating anymore, and the remaining were expected to leave. Due to overflow of users from Greaves Lane, about 19% of Deepcar users were expected for not recycling anymore. Since Longley Avenue were only operating 29 hours per week, hence about 49% of its users were expected to move to Blackstock Road and less than 1% were expected to leave the network (not recycling). At the same time, Blackstock Road were also expected to experience users from Beighton Road (about 29%), as the Beighton Road operation hours has been reduced from 48 hours to 37 hours. Similarly, some users of Blackstock Road were expected to move to either Longley Avenue or Beighton Road, as its operating hours were reduced to 32 hours per week. This also causing the Blackstock Road to have more users in the queue.

4.9.3 Result 3: What-if Analysis: The Decision-Maker - Sheffield City Council

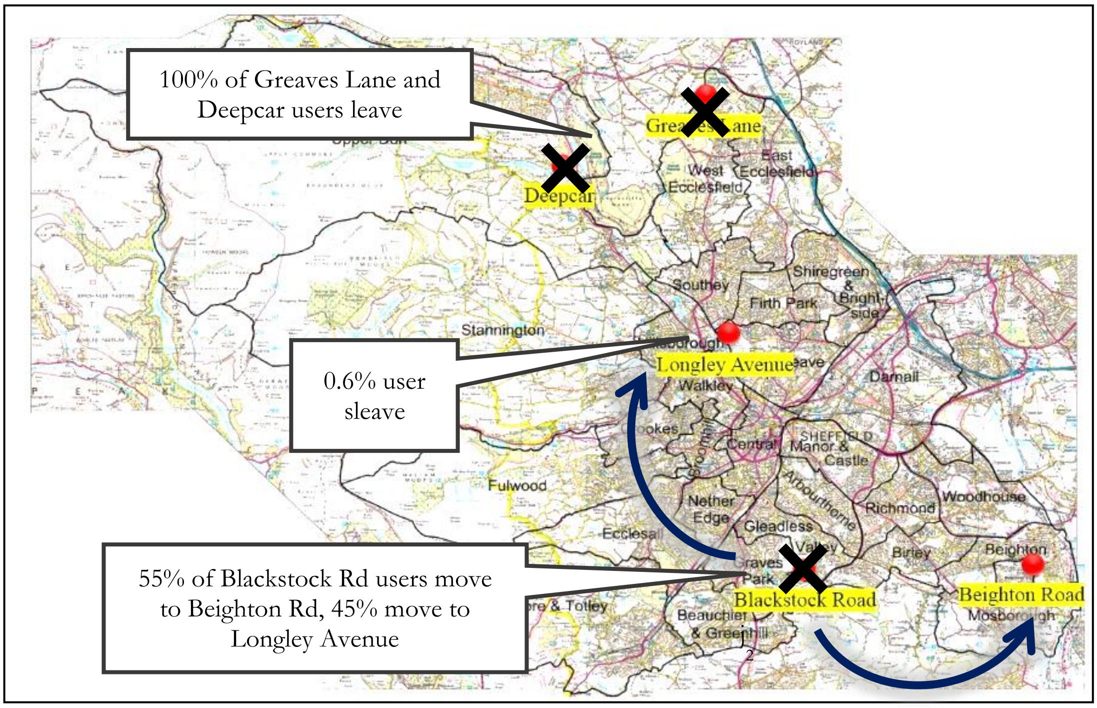
A what-if analysis gives a specific scenario that is controlled by the decision-maker. For this section, the decision-maker is Sheffield City Council. They have requested an analysis based on two operating HWRCs, i.e., Longley Avenue and Beighton Road. The initial δ_j for this case is $[56, 0, 0, 0, 56]$, the results for which are presented in Figure 4-42 and Table 4-25.

From Table 4-25, the user mechanism is unchanged when α_1 is increased, except when $\alpha_1 \geq 0.3$, where the percentage of total operating periods for the network is reduced by 1%. Only 76% of Sheffield's residents are expected to recycle. The 24% of users leaving the network can be gained through an increment of *B* by 0.1 per step. Having less than this, insufficient results are produced by CPLEX. These 24% are the users of the Greaves Lane and Deepcar sites. This would mean that users of both these sites would not be recycling at all. Only 24% of Sheffield's resident would be expected to move to either Longley Avenue or Beighton Road. In this case, Blackstock Road users would be expected to move either to Beighton Road or to Longley Avenue, since both sites are located within 15 minutes travel time of the Blackstock Road site. Again, these users might choose to leave the network entirely, since both facilities are limited in capacity and not all users are committed to recycling. Besides having their preferred recycling sites closed, traveling an extra mile to these alternative sites might be sufficient to discourage them from recycling. In fact, the distance between the recycling centre and resident sites plays an important role in encouraging this activity (Rousta et al., 2015; Struk, 2017). The Council is also expected to face an associated increase in fly-tipping. Figure 4-43 illustrates the user mechanism within the network using $\alpha_1 = 0.8$.

Table 4-25: Changes in user circulation in the system network when there are only two operating facilities

α_1	α_2	**Total cost (objective function)**	**Total cost provider's side (Z_1)**	**Total cost user's side (Z_2)**	δ_j	$\%\sum_j\sum_t y_j^t$ (Total Operating Facilities)	$\%\sum_j\sum_t q_j^t$ (Amount of Users Served)	$\%\sum_j\sum_t l_j^t$ (Amount of Users Leaving the System)	$\%\sum_j\sum_t s_j^t$ (Amount of Users were in the Queue)	$\%\sum_j\sum_t S_{jk}^t$ (Amount of Users Moved to Another Facility)
0.1	0.9	38,929	133,276	15,423	[56, 0, 0, 0, 56]	40%	76%	24%	0%	28%
0.2	0.8	50,706	133,021	15,428	[56, 0, 0, 0, 55]	40%	76%	24%	0%	28%
0.3	0.7	62,465	133,021	15,428	[56, 0, 0, 0, 55]	40%	76%	24%	0%	28%
0.4	0.6	74,224	133,011	15,436	[56, 0, 0, 0, 55]	40%	76%	24%	0%	28%
0.5	0.5	85,981	133,011	15,436	[56, 0, 0, 0, 55]	40%	76%	24%	0%	28%
0.6	0.4	97,739	133,011	15,436	[56, 0, 0, 0, 55]	40%	76%	24%	0%	28%
0.7	0.3	109,496	133,011	15,436	[56, 0, 0, 0, 55]	40%	76%	24%	0%	28%
0.8	0.2	121,254	133,011	15,436	[56, 0, 0, 0, 55]	40%	76%	24%	0%	28%
0.9	0.1	133,011	133,011	15,436	[56, 0, 0, 0, 55]	40%	76%	24%	0%	28%

From Figure 4-43, 100% of users previously assigned to Greaves Lane and Deepcar were expected to leave the system (or not to undertake any recycling). The closure of Blackstock Road caused 55% of its users to move to Beighton Road and the remaining ones to Longley Avenue.



Note: System performance is at 76%, with α_1=0.8.

Figure 4-42: HWRC's user movement within the network

From this analysis, two HWRCs would be unable to provide a sufficient service for Sheffield's residents. The Council needs at least three HWRCs to maintain a 95% service level. In order to have only two operating HWRCs, the size of these HWRCs would have to be increased; however, this would also incur greater ongoing costs.

4.10 Conclusion

This chapter has highlighted the application of the proposed model in reorganising HWRC operation in Sheffield. The model has been modified to include division of *T'* into macro-periods and micro-periods. This range was suitable for the consideration of a weekly schedule, with the computation performed on a daily basis. Additional constraints in limiting total operating periods per day and per week were introduced in the refined model. Instead of using traditional allocation using distance, we incorporated the spatial interaction model to allocate users to each HWRC. This chapter also shows the implementation of HWRC data into the modified model. As a result, it was found that three HWRCs, Blackstock Road, Beighton Road and Deepcar would be able to cater for the majority of Sheffield's residents. Due to the Council's plan, only two sites, i.e., Beighton Road and Longley Avenue, will be operated in the future. However, through our analysis using the proposed model, these two sites alone would be insufficient to maintain a 95% service level. It was determined that in this instance, 24% of Sheffield's residents might not continue to recycle, and 28% would be expected to change site to recycle at the two remaining HWRCs. This might also prevent the UK from achieving its target of 50% of waste being recycled by 2020, and worse would likely increase fly-tipping incidents.

The following chapter focusses on implementing the proposed model in the reorganisation of the healthcare network in Sheffield.

CHAPTER 5: REORGANISING HEALTHCARE SERVICES USING THE PROPOSED MODEL

In the previous chapter, the proposed model was adapted in order to reorganise Sheffield's HWRC network. HWRC facilities are available to all users at all times (within the bound of their operating hours). Users are served on the same day and are generally willing to stay in a queue or move to another recycling centre, while the provided service cannot be classified as dealing with *urgent* necessities. In contrast, healthcare services are crucial to all users, and this service normally requires a particular urgency in dealing with its users. The UK primary healthcare service requires patients to book an appointment to meet their general practitioner (GP). With an appointment system, limited slots are available. If these slots are fully booked for the week, it is expected that an increased number of patients will be left unattended, and some of them might instead visit an accident and emergency (A&E) if they need same-day attention. Hence, this is one of the reasons why most healthcare facility location studies suggest adding more services or expanding the size of the network. However, the current context of limited financial allocation for healthcare services cannot be neglected; in this scenario, it might be impossible for planning authorities to enlarge the current network service.

In this chapter, we will look at the problem from an alternative perspective. In order to deal with the need for extra capacity, facilities within the same network might be *paired* in order to provide some form of back-up coverage. For example, consider a network where both facilities X and Y are providing a similar type of service. X gains the 'extra' capacity by transferring some of its demand to Y, provided this situation is permissible under several prespecified conditions such as availability and accessibility of Y (which, for instance, might be experiencing lower levels of demand). This concept was introduced in our proposed model, in which extra capacity is gained by considering the reallocation of demand to other interrelated and interconnected facilities within a given network. In a similar manner to HWRCs, healthcare service facilities suffer from tight budget allocations, and the facilities in such a network experience different levels of congestion. For this reason, the proposed model will be modified to reorganise the entire primary healthcare facility network, focussing on GP services. The concept of a backup GP has been introduced to channel any unattended demand, and might directly reduce the demand's appointment waiting times and congestion levels within the healthcare network.

This chapter starts with background studies on the UK healthcare system, followed by problems faced by planning authorities. Then, a description of the refinements to the proposed model will be given and, finally, an analysis of the results obtained for the GP service will be performed and discussed.

5.1 Background - the Healthcare Service in the UK

In the UK, the National Health Service (NHS) is responsible for managing and providing the healthcare services to the UK residents. The NHS deals with over 1 million patients every 36 hours (NHS England, 2013). The NHS serves patients through primary healthcare facilities. Primary healthcare services or primary care is an essential tool that allows for contact between individuals, facilities and/or communities and the healthcare system itself, and guidance advice on the use of this service is illustrated in Figure 5-1. It is advisable for patients to attempt self-care for any common illness or call the nationwide healthcare operator service (111) for any questions. Pharmacists also provide advice and treatment for most common illnesses. Patients could book an appointment at the registered GP surgery for more serious cases, such as prolonged illnesses. For any urgent case, a patient can attend an accident and emergency (A&E) service.

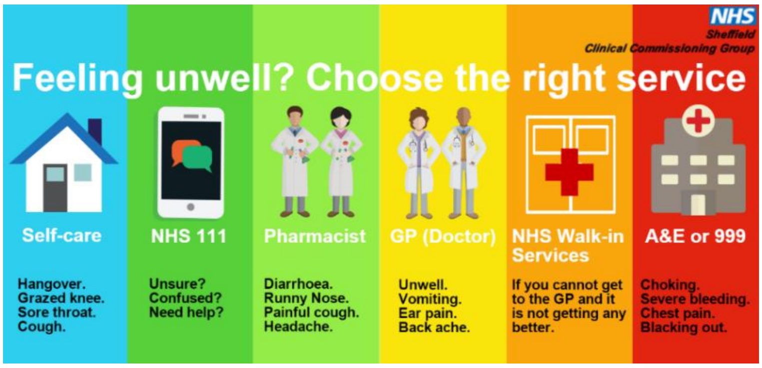


Figure 5-1: Healthcare services hierarchy (source: NHS Sheffield CCG, n.d.)

GP surgeries are one of the principal components of the primary healthcare services (NHS England, 2013). The services they provide, also known as General Medical Services (GMS), which are funded by the NHS based on yearly contracts, are free to UK residents. For each city or region in the UK, GPs are gathered under Clinical Commissioning Groups (CCGs). The CCGs were created following the Health and Social Care Act in 2012 and replaced Primary Care Trusts on 1 April 2013. The CCG is responsible for planning and managing the health care services for its local area. This means that each GP surgery under the CCG scheme can employ their own staff, either for non-clinical admin posts or for the clinical. The practitioners at GP surgeries are either GP partners or sessional GPs. GP partners are responsible for running the practice while the sessional GPs are practitioners that work based on consultation sessions. Sessional GPs are either full-time salaried GPs, locum GPs, or part-time practitioners, such as GP retainers. As of September 2018, there were about 7,086 GP surgeries in England with 42,445 practitioners (reported by NHS England, 2018).

GP surgeries do not have any standardised operating hours. Normally, GP opening times are based on reception times and appointment times. Reception times represent time periods when the surgery is open but for non-clinical services, such as when a patient comes to pick up his/her prescription. Appointment times are when a practitioner consults a patient, given that the patient has made an appointment beforehand. For an *impromptu* consultation, the patient must call the surgery; patients may see a practitioner if there are cancelled appointments. For out-of-hours GP operating times, the patient is advised to go to a walk-in centre, or, for any emergency cases, the patient can go to an A&E.

According to the report by QResearch, the average number of consultations and consultation rates performed by GP at the 465 participating surgeries in England for year 2007 were 12,118,622 consultations out of 3,654,441 registered patients, or 3.32 consultations per person per year (Hippisley-Cox & Govind, 2008). Baird et al., (2016) published the latest consultation rate (4.91 consultations per person per year), which has grown by about 62.6% between 2010/11 and 2014/15. In much recent figures were published by the NHS England (2018a) for GP services in England, the total 308 million of appointments were booked for GP service, which 89.4% (about 275 million consultations) of the appointments were attended. However, the average consultation rates per person per year for England were not provided by the NHS England as the number of participating GP surgeries and patients demographic information were unavailable. Clearly, from both sources (QResearch and NHS England), the number of consultations has been increased. This increase is probably due to the increase in the use of technology and changes in the telecommunications system to consult the patients, which is now much more convenient to use.

5.1.1 Measuring the GP System Performance

A heated debate about the measurement of healthcare performance (especially focussing on primary care) is ongoing (Sliwa & O'Kane, 2011); such measurement represents a complex process. One of the attempts of the NHS to measure the performance of the GP service is through a GP patient survey. This survey is managed and controlled by Ipsos MORI, an independent research agency, on behalf of the NHS. Through this survey, patients can provide their feedback on their experiences of using the GP service. The survey was published in 2007, and in 2011 the survey's questions were improved based on the suggestions made by the Department of Health to ensure the questions remained relevant to the current health system (NHS England & Ipsos MORI, 2018, pp. 1–2). There are a total of 63 questions in the survey, where the questionnaires are categorised into eight sections; six sections on GP services, one section on NHS Dentistry and one section that requests demographic information. The sections are:

1. Your local GP services
2. Making an appointment
3. Your last appointment
4. Overall experience
5. Your health
6. When your GP practice is closed

GP practice survey

7. **NHS Dentistry**
8. **Some questions about you**

The results of the survey are accessible through a dedicated GP survey website. In 2018, about 84% of respondents claimed to be satisfied with the GP services, and 70% noted that their GP is highly accessible by phone. About 69% were satisfied with the process of making an appointment, with 5.8% being unable to book an appointment at their GP. Of the patients who were unable to book appointments, about 28% 'did not see or speak to anyone', 22% contacted the practice at another time, and 14% booked an appointment on a different day. Meanwhile, 11% searched for information online, 11% went to another NHS services, 11% went to an A&E, 11% spoke to family or friends, 10% spoke to pharmacists, and 7% spoke to the NHS helpline. The overall results of this survey (extracted from the GP Patients Survey website) are reported in Appendix 5(A).

5.1.2 The Problems Arising in GP Services

In general, problems faced by GP surgeries and their demand are:

1. An imbalance between numbers of practitioners and total registered patients;
2. The increment of unnecessary attendance at A&E.

Even though only 5.8% of patients were unattended, the ratio between registered patients and the number of GPs shows that this percentage might well increase over time. Figure 5-2 shows the trends of numbers of full-time practitioners and total patients registered between 2016 and 2018.

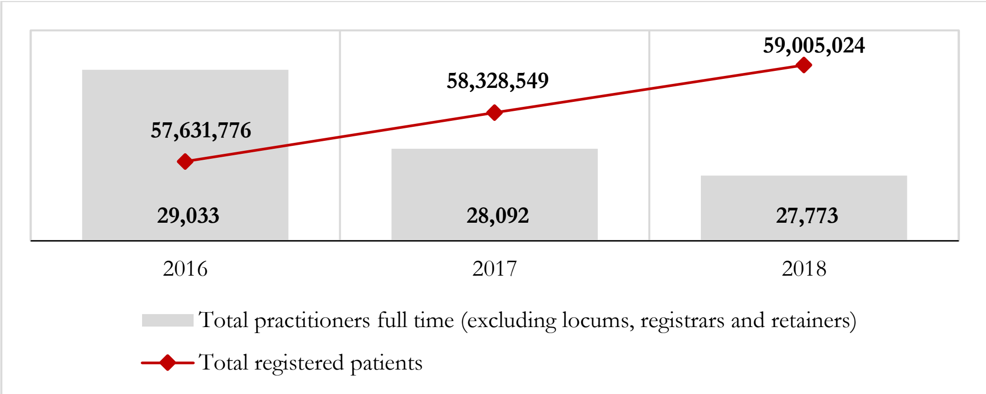


Figure 5-2: Total full-time practitioners and total registered patients at all CCGs in the UK for 2016-2018 (Source: NHS England, 2018b)

From Figure 5-2, the total number of practitioners has experienced a reduction, while the total number of registered patients is increasing. The ratio of practitioners to total registered patients has grown from 1:1985 in 2016 to 1:2125 in 2018. The workload for a practitioner has increased over the years, and the chance of unattended patients is increasing too. Professor Helen Stokes-Lampard, chair of the Royal College of GPs, said that as waiting times increase, the risk of a non-urgent case becoming urgent well might increase (BBC News, 2016). Besides, having fewer practitioners and more patients will affect appointment waiting times. The appointment waiting time or waiting time is measured in days, and is defined the time elapsing between the day when a patient books their appointment until the day of the actual appointment itself. In general, there is no specific maximum waiting time set by the NHS. However, more than 30% of the appointments booked have to wait more than a week to see the GP (NHS England, 2018a). Due to the long waiting times, these patients might go to a walk-in centre or to A&E.

The capacity of walk-in centres is very small, and it just caters for non-urgent cases that need immediate attention. However, in 2010 about 53 walk-in centres were completely closed (ITV Report, 2013). Another possible alternative for a patient to get immediate medical attention is through visiting an A&E. However, the provision of A&E services is generally expensive; also, A&E services generally suffer from overcrowding problems. The minimum waiting time at A&Es has increased over the years, above the national standard of 4 hours per patient (Anandaciva & Thompson, 2017; O'Keeffe et al., 2018). This situation is likely to become worse if the number of patients visiting A&E services increases; this can also be exacerbated by unnecessary visits. Professor Willett, the Director for Acute Care for NHS England said, "*about 15% to 30% of patients in A&E could be treated at GP surgeries*" (Moore, 2013). Such patients' attendance at A&E might be considered unnecessary.

The Centre for Urgent and Emergency Care (CURE) has conducted research into unnecessary attendance at the A&E for Yorkshire and Humberside. Mason et al. (2017, p. 12) defined unnecessary attendance as any one of the following instances:

- not investigated in the A&E (except by urinalysis, dental or pregnancy test)
- not treated in the A&E (except by a prescription, dental, recording of vital signs or guidance or advice)
- discharged completely from care in the A&E or referred to their GP

or as: "*First attendance with some recorded treatments or investigations all of which may have been reasonably provided by a GP, followed by discharge home or to GP care*". CURE reported that the A&E received an overall 1,693,203 non-ambulance arrivals in 2014. Of these figures, about 216,439 arrivals (13%) were classified as unnecessary attendances. It has been reported that 69% (149,053) of unnecessary attendees are adults, while the remainder (31% or 67,386) are child patients. Table 5-1 provides details about arrival times of unnecessary attendances.

Table 5-1: Total unnecessary attendance at A&E according to arrival times

Arrival time-periods	**Total per arrival times (%)**	
	Children	**Adults**
Weekday in Hours (08:00 – 18:00)	25,842 (38%)	63,868 (43%)
Weekday Out-of-Hours (18:00 – 08:00)	16,294 (24%)	33,064 (22%)
Weekend Out-of-Hours (18:00 Friday – 08:00 Monday)	16,294 (37%)	33,064 (35%)
TOTAL	**67,386**	**149,053**

Overall, for both age categories, the highest number of unnecessary attendees was recorded on weekdays during normal hours, followed by weekend out-of-hours and weekday out-of-hours. CURE reported that unnecessary attendees come to the A&E since it is very convenient, and the attendees want immediate responses to their health problems. The highest percentage of unnecessary attendance at A&E was during GP operating hours, i.e. weekdays in hours, indicating the unavailability of their GP at their preferred times.

Since 2015, the NHS has estimated the portion of unnecessary attendance in England by adapting the definition given by CURE. About 16.5% of attendances in 2015/16 were classed as unnecessary, about 16.1% in 2016/17 and about 15.7% in 2017/18 (NHS England, 2018e), which represents a decrease of 0.4% per year. Perhaps the incentives introduced by the NHS Five Year Forward View in October 2014 have contributed to this reduction. One of the key points of the NHS Five Year Forward View was to increase the accessibility of GPs at evenings and weekends (NHS England, 2014). However, the reduction is small in comparison to the growth of ratio of practitioners to total registered patients (as in Figure 5-2). Moreover, even though the NHS Five Year Forward View plan improved healthcare services in the UK, its implementation has been costly.

The following section describes the organisation of GP services in Sheffield, which we will later use as our case study.

5.2 Healthcare – Sheffield

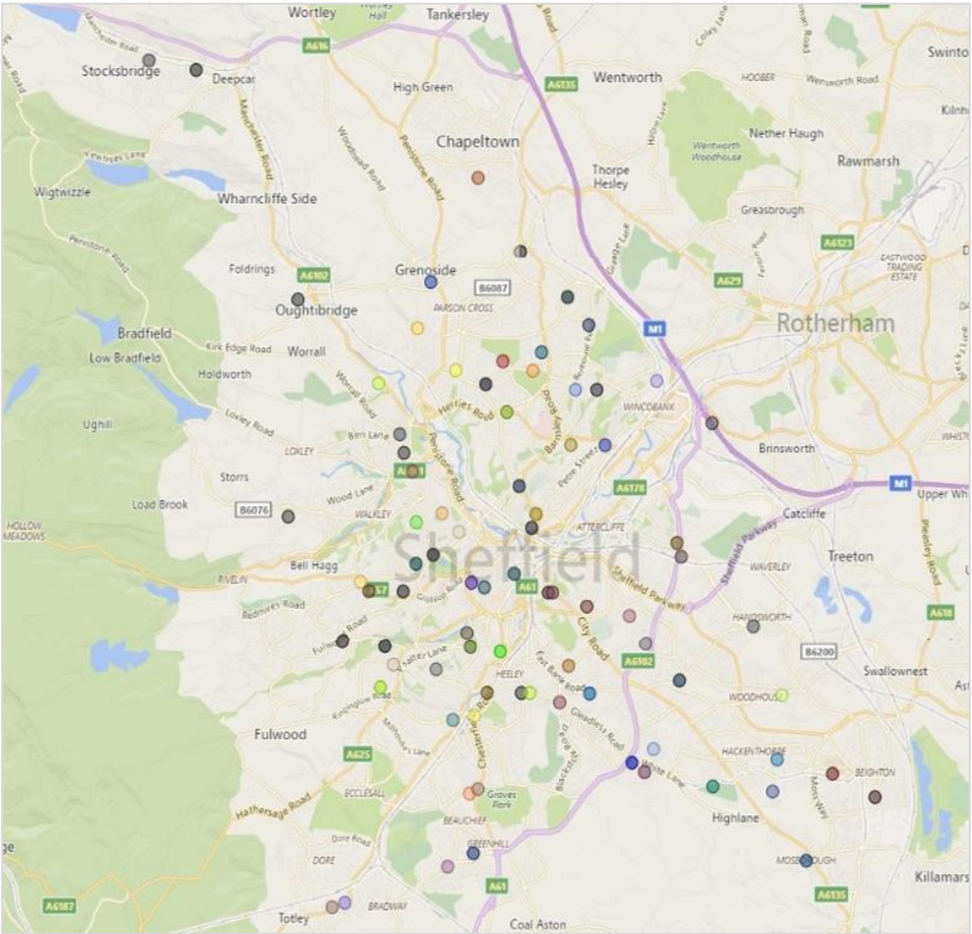


Figure 5-3: Location of CCG GPs in Sheffield (source: NHS England, 2018b)

In Sheffield, there are 82 GP surgeries under the NHS CCGs scheme; the locations of these GPs are given in Figure 5-3. In 2018, there were 339 total full-time practitioners, and the total number of patients registered at GPs in Sheffield was 600,274. Slightly more than half of the registered patients were male (304,742, or 51%) while the remainder were female (296,236, or 49%).

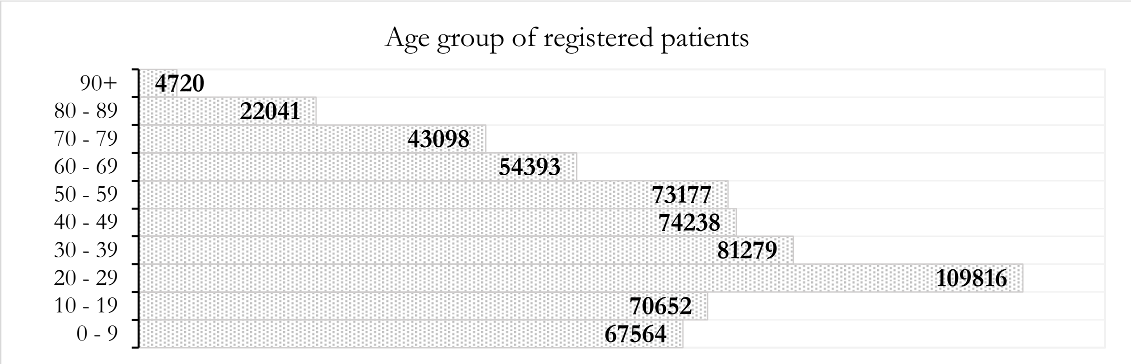


Figure 5-4: Number of registered patients in Sheffield (source: NHS England, 2018c)

Figure 5-4 categorises the total registered patients according to age group. The highest number of registered patients belongs to the 20-29 years old banding; this is probably due to the large student population in Sheffield, which is linked to the presence of two major universities (University of Sheffield and Sheffield Hallam University). Table 5-2 presents details on each GP, including the names, the number of full-time practitioners, the total registered patients and the expected number of consultations (this was estimated using Baird et al., (2016) where a patient go to the GP about 4.91 times per year).

Table 5-2: The GP surgeries in Sheffield, with total number of practitioners, total registered patients and the estimated number of patients per day

j	General Practice	Number of full-time practitioners	Total registered patients	Number of potential patients per day [2,3]
1	Abbey Lane Surgery	2	3,129	59
2	Avenue Medical Practice	3	7,130	135
3	Barnsley Road Surgery	2	2,653	50
4	Baslow Rd, Shoreham St & York Rd Surgeries	5	12,642	239
5	Birley Health Centre	5	8,502	161
6	Broomhill Surgery	5	9,633	182
7	Buchanan Road Surgery	4	4,703	89
8	Burngreave Surgery	3	6,726	127
9	Carrfield Medical Centre	1	1,260	24
10	Carterknowle & Dore Medical Practice	6	12,380	234
11	Chapelgreen Practice	13	15,452	292

[2] Estimated number of consultations at a GP surgery per year = Total registered patients x 4.91. The 4.91 represents the average consultation rate per patient per year, i.e., by Baird et al., (2016).
[3] Assumes the GP is operating 52 weeks per year and five days per week.

12	Charnock Health Primary Care Centre	3	5,381	102
13	Clover City Practice	2	4,395	83
14	Clover Group Practice	7	16,394	310
15	Crookes Practice	4	7,962	150
16	Crookes Valley Medical Centre	1	2,317	44
17	Crystal Peaks Medical Centre	5	6,598	125
18	Darnall Health Centre (Mehrotra)	1	3,415	64
19	Deepcar Medical Centre	3	5,200	98
20	Devonshire Green Medical Centre	5	6,959	131
21	Dovercourt Group Practice	6	8,338	157
22	Duke Medical Centre	3	6,966	132
23	Dunninc Road Surgery	2	2,983	56
24	Dykes Hall Medical Centre	6	9,735	184
25	East Bank Medical Centre	4	5,608	106
26	Ecclesfield Group Practice	5	8,177	154
27	Elm Lane Surgery	4	5,185	98
28	Falkland House Surgery	2	3,790	72
29	Far Lane Medical Centre	3	7,249	137
30	Firth Park Surgery	8	9,884	187
31	Foxhill Medical Centre	4	6,189	117
32	Gleadless Medical Centre	6	8,865	167
33	Grenoside Surgery	4	7,391	140
34	Greystones Medical Centre	1	3,732	70
35	Hackenthorpe Medical Centre	5	6,715	127
36	Handsworth Medical Practice	4	9,850	186
37	Harold Street Medical Centre	1	1,282	24
38	Heeley Green Surgery	4	5,886	111
39	Hollies Medical Centre	4	9,034	171
40	Jaunty Springs Health Centre	2	3,630	69
41	Manchester Road Surgery	3	4,691	89
42	Manor Park Medical Centre	3	4,416	83
43	Meadowgreen Health Centre	10	9,841	186
44	Mill Road Surgery	5	5,264	99
45	Mosborough Health Centre	6	6,590	124
46	Nethergreen Surgery	4	9,286	175
47	Norfolk Park Health Centre	2	4,418	83
48	Norwood Medical Centre	4	7,971	151
49	Oughtibridge Surgery	3	5,848	110
50	Owlthorpe Medical Centre	2	4,583	87
51	Page Hall Medical Centre	5	7,586	143
52	Park Health Centre	3	5,103	96
53	Pitsmoor Surgery	9	9,401	178
54	Porter Brook Medical Centre	12	28,820	544
55	Richmond Medical Centre	6	8,806	166
56	Rustlings Road Medical Centre	2	4,591	87
57	Selborne Road Medical Centre	1	2,730	52
58	Sharrow Lane Medical Centre	2	3,883	73
59	Sheffield Medical Centre	1	1,700	32
60	Shiregreen Medical Centre	8	7,834	148
61	Sloan Medical Centre	6	12,964	245
62	Sothall & Beighton Health Centres	4	10,180	192
63	Southey Green Medical Centre	2	2,996	57
64	Stannington Medical Centre	1	3,198	60
65	Stonecroft Medical Centre	2	4,101	77
66	The Flowers Health Centre	2	4,885	92
67	The Health Care Surgery	2	5,027	95
68	The Mathews Practice Belgrave	5	8,722	165
69	The Medical Centre Dr Okorie	1	1,183	22
70	Totley Rise Medical Centre	2	3,442	65
71	Tramways and Middlewood Medical Centres	5	10,604	200
72	Tramways Medical Centre (O'Connell)	4	8,553	162
73	University Health Service Health Centre	8	32,891	621
74	Upperthorpe Medical Centre	6	11,466	217
75	Upwell Street Surgery	3	4,769	90
76	Valley Medical Centre	7	9,628	182
77	Veritas Health Centre	1	1,462	28
78	Walkley House Medical Centre	5	11,749	222
79	White House Surgery	4	6,363	120
80	Wincobank Medical Centre	4	7,649	144
81	Woodhouse Medical Centre	10	12,117	229
82	Woodseats Medical Centre	6	9,643	182
	TOTAL	**339**	**600,274**	**11,336**

From Table 5-2, the highest number of full-time practitioners is at the Chapelgreen Practice, with 13 practitioners. This practice has more than 15,000 registered patients and is estimated to receive 292 patients per day. Even though this practice has the highest number of practitioners, the highest numbers of registered patients is reported at the University Health Service (UHS) Health Centre, at 32,891. The UHS Health Centre has only got eight full-time practitioners; it also has four GP associates and four nurse practitioners to ease the large estimated number of consultations. On average, each surgery has 7,320 registered patients and each practitioner is responsible for 1,771 registered patients. Meanwhile, the total estimated number of consultations in Sheffield is 2,947,345 per year[4], or on average 11,336 consultations per day. Each GP in Sheffield is expected to consult an average of 138 patients per day.

GP operating hours vary from day to day. Some practices operate during the weekend, and some might do clinical services. The operating hours (for weekday only) per GP are included in Appendix 5(B).

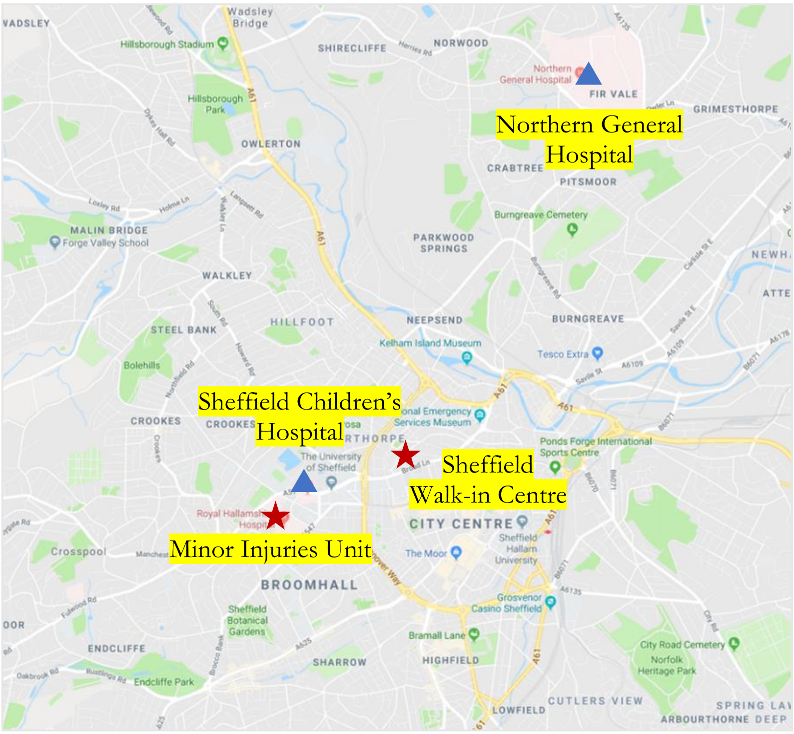


Figure 5-5: Walk-in centre locations (star shapes) and A&E locations (triangle shapes)

[4] Total estimated number of consultations in Sheffield = Total registered patients in Sheffield (600,274) x 4.91. The 4.91 represents the average consultation rate per patient per year, i.e., by Baird et al., (2016).

Figure 5-5 presents the locations of hospitals and walk-in centres in Sheffield. Currently, there are two walk-in centres and two A&Es in Sheffield. The two walk-in centres in Sheffield are the Sheffield Walk-in Centre and the Minor Injuries Unit at the Royal Hallamshire Hospital. Both walk-in centres are operating 7 days a week. The two EDs in Sheffield are managed by the Sheffield Teaching Hospitals NHS Foundation Trust, which are located in the Northern General Hospital and the Sheffield Children's Hospital. The Northern General Hospital caters for the emergency needs of all age groups, whilst emergency cases for patients under 19 years old can go to the Sheffield Children's Hospital. The Problems Faced by Healthcare Providers in Sheffield

In 2017, the NHS revealed its plans to close both walk-in centres and relocate them to a single location at the Northern General Hospital (Torr, 2017). However, the plan was postponed for two years after criticism and opposition from the public (Hayes, 2018), but the 'risk' of these centres being closed remains.

The percentage of unnecessary attendance at the Sheffield Teaching Hospitals NHS Foundation Trust in 2015/16 was 12.8%. This percentage increased by 0.2% in 2016/17. Table 5-3 provides information on non-ambulance arrivals for 2016/17.

Table 5-3: Non-ambulance arrivals for 2016/17 (source: (NHS England, 2017)

Provider Description	Total
Sheffield Children's NHS Foundation Trust	52,232
Sheffield Teaching Hospitals NHS Foundation Trust	100,827

From the table, Sheffield Teaching Hospitals received 100,827 non-ambulance arrivals in 2016/17. About 13% or 13,108 arrivals in 2016/17, or 1,092 arrivals per month[5], were considered unnecessary. Meanwhile, the Children's Hospital estimated that he received 6,790 unnecessary attendances in 2016/17 or more than 566 unnecessary attendees per month[6]. In total, the A&E in Sheffield are struggling with more than 1,500 unnecessary arrivals in a month, or an 'extra' of 50 unnecessary patients per day[6]. The NHS highlights the fact that young adults or university students are the main contributors to this figure, probably due to patients

[5] The figure is calculated using the formulation of $\frac{Total\ unnecessary\ attandance\ per\ year}{12\ months}$.

[6] We assume there are 30 days per month. Hence, this figure is obtained by $\frac{Total\ unnecessary\ attandance\ per\ month}{30\ days}$.

registered at the UHS Health Centre being restricted to certain time-windows to visit their GPs, hence forcing them to go to A&E instead for medical attention.

The closure of a healthcare facility in the network would almost certainly affect the other interrelated and interconnected facilities. For instance, if a patient is unable to book an appointment at his/her GP surgery, some might not do anything. However, some might visit the walk-in centre or A&E in the attempt to gain treatment. The NHS plans to close its walk-in centres completely. Even though this plan has been postponed for two years, sufficient action must be taken to ensure the remaining facilities are sufficient to serve their patients. As far as we can discern, there are no official records for walk-in centre arrivals. If patients are unable to gain treatment from their GP or walk-in centre, they might visit A&E; hence, this can result in unnecessary attendance. Through an adaptation of the model developed in Chapter 3, the following section provides a suggestion to reduce unnecessary attendance at A&E by taking advantage of interrelated facility operations within the same network, by developing *clusters* of GP surgeries which might provide back-up coverage during closure hours.

5.3 Model Refinements

This section presents the adaptations to the model proposed in Chapter 3, including an introduction to the backup facility model, while the following section will present the entire refined model for the healthcare facility network.

Unlike the HWRC case, this chapter concentrates on reorganising GP operations through optimal use of the existing resources, especially increased network capacity without, or at minimal, cost. Increasing GP accessibility might alleviate the burden of A&E visits (Cecil et al., 2016). Having 'extra' capacity might reduce unnecessary attendances at the walk-in and A&E; hence, we focus on increasing the 'extra' capacity within the GP network, which means that it will not be suggested that any GPs should be shut down completely; however, in the current scenario of budget cuts to the NHS, it is also unrealistic foreseeing a network expansion by extending opening hours. As such, opportunities for 'extra' capacity will be sought by re-allocating demand across facility, in order to better utilise existing capacity and diminish users' waiting times.

Within this context, we relax the constraints on y_j^t as one of the decision variables. This means that the variable y_j^t, defining operating periods for the facility *j* at time *t*, is treated here as an input parameter, defining the operating schedule of each facility (which won't be altered by the model). Besides y_j^t, two further refinements have been made:

1. Backup model to find a backup for a GP,
2. Refinements to the model's objective function and constraints that are related to y_j^t, and
3. Increase patient waiting time up to *G* periods.

From this section onwards, please note that the term 'demand' was used to indicate 'patients seeking for medical attention at a GP surgery'.

5.3.1 Refinement 1: Creating a Backup for a GP

Figure 5-6 illustrates expected forward movement of demand accessing the healthcare facility network, assuming, for the sake of simplicity, that demand would not otherwise intend to visit a private healthcare facility.

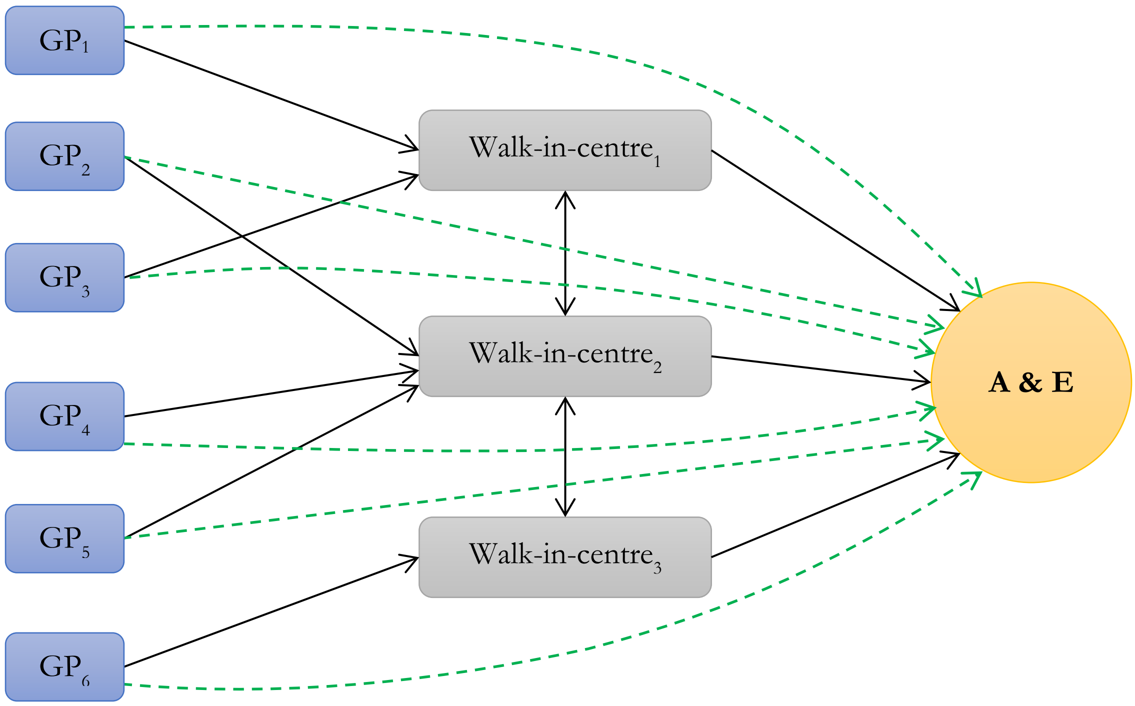


Figure 5-6: Expected demand movements within the healthcare system

The black lines in this figure represent such movement. For example, demand will attempt to book an appointment at their GP, but if they fail, they will go to a walk-in centre. If the walk-in centre is full, then they will go to A&E. In the same figure, green dashed lines represent a demand moving directly from a GP to A&E. A&E is intended for emergency cases but is accessible by anyone. As such, typical forward movements within this network are represented by the following demand flows: GP → walk-in centre → A&E; GP → A&E. The forward movement also indicates lack of alternative in healthcare pathways; this directly increases the unnecessary attendance in A&E (Agarwal et al., 2012).

These three healthcare facilities are interrelated and interconnected because they all provide a similar service. Moreover, the walk-in centres and A&E are accessible by anyone. Each resident in the UK have the right to choose their preferred GP, however, once registered, the patient is not allowed to go to another GP, reflecting a *dictatorial* assignment scheme. In this case, whenever demand is unable to book a GP appointment at their preferred time, they might either go to the walk-in centre or to A&E, which indirectly increases the number of unnecessary demand attending walk-in centres or the A&E. In order to reduce the number of unnecessary demand or attendances at these two facilities, it is best to *circulate demand* within the GP network only. Moreover, from GP Patient Survey, half of the respondents (52.2%) are willingly to speak to any practitioner, no matter whether this is the one assigned to them (NHS, 2017, p.15); this clearly indicates the rationalisation opportunities which could be obtained by means of having backup facility arrangements for each GP surgery.

Therefore, we introduced a **Backup Facility** in a sense to offer provide 'extra' capacity within the network. A backup facility offers an option for patients to see a GP at another facility, whenever the GP that they are registered for, is not operating or operating at full capacity. In essence, patients requiring to see a GP and not being able to do so at their usual surgery, might be offered an appointment at the backup facility. The underlying hypothesis of this mechanism is that, in this way, additional demand could be served by the existing GP surgeries network, without investments, just slightly "relaxing" the *dictatorial* user-surgery assignment which allows patients just to visit the GP where they are registered. Therefore, the backup facility that was introduced is able to contribute in two ways:

- reduction in amount of demand leaving to attend A&E, and
- reduction in amount of demand waiting for long periods either at the GP itself or at the interconnected healthcare facilities.

Through the backup facility model, bottlenecks within the GP network could be reduced. Figure 5-7 shows the effect of having a backup facility on the healthcare network.

The grey shaded region in Figure 5-7 shows two clusters that are formed using several pre-set criteria. Cluster 1 contains more than one GP: GP_1, GP_2, GP_3, and GP_4, while Cluster 2 consists of two GPs: GP_5 and GP_6. Within a cluster, the red lines represent demand movement to the backup facility. For example, in Cluster 1, the backup for GP_1 is GP_2, the backup for GP_2 is GP_4, the backup for GP_4 is GP_3, and the backup for GP_3 is GP_1 (in short: $GP_1 \rightarrow GP_2 \rightarrow GP_4 \rightarrow GP_3 \rightarrow GP_1$). In Cluster 2, however, each GP is the other's backup, i.e., GP_5 and GP_6 or $GP_5 \leftrightarrow GP_6$. Creating a backup facility does not drastically reduce the rate of unnecessary attendances in A&E or walk-in centres, hence the dashed grey line in Figure 6-7.

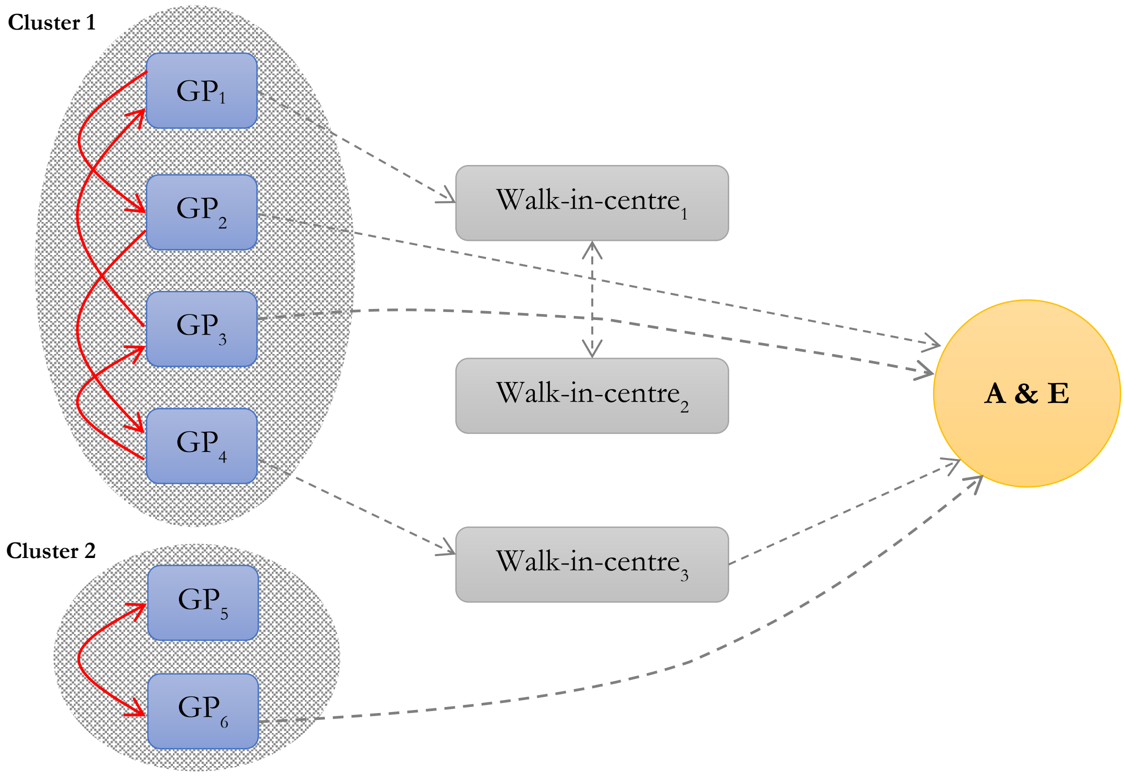


Figure 5-7: Expected demand movements when creating backup services

Let h_{jk}, be the variable that creates the 'link' from one GP to another, defining a *backup* arrangement. The concept is that the backup facility needs to be operating if the main facility is not. In other words, we want to find *pairs* of GP surgeries which can present the minimum overlap in their operating schedules. We define sets, parameters and decision variables for the backup model as per the following:

Sets		
J	=	the set of facility locations ($\forall j, k \in= \{1...J' \mid j \neq k\}$
T	=	the set of time-periods ($\forall t \in T$)

Parameters		
y_j^t	=	operating periods for facility *j* during period *t*
$dist_{jk}$	=	distance between facility *j* and facility *k*
D	=	maximum distance between facilities

Decision variables		
h_{jk}	=	$\begin{cases} 1 & if\ k\ is\ backup\ facility\ for\ j \\ 0 & otherwise \end{cases}$

The backup model is:

$$Max \sum_{t} \sum_{j,k \mid j \neq k} h_{jk} \left| y_j^t - y_k^t \right| \tag{5-1}$$

subject to:

$$\sum_{j} h_{j,k} \leq 1; \qquad \forall k = 1 \dots J' \tag{5-2}$$

$$\sum_{k} h_{j,k} = 1; \qquad \forall j = 1 \dots J' \tag{5-3}$$

$$h_{j,k} \cdot dist(j,k) \leq D; \qquad \forall j,k = 1 \dots J' \mid j \neq k \tag{5-4}$$

$$h_{j,k} \in \{0,1\}; \qquad \forall j,k = 1 \dots J' \mid j \neq k \tag{5-5}$$

The objective function of the backup model (5-1) is to ensure the combination of the operating periods of two GPs are at a maximum. The constraints of the model are (5-2) – (5-4). Constraint (5-2) indicates each facility *k* can be a *backup* for at most one facility *j*. (5-3) strictly ensures each facility *j* needs to have a backup. (5-4) maintains the distance between the primary and its backup as less than a distance *D*. $h_{j,k}$ is a binary variable (5-5).

The combination of the objective function (5-1) and (5-5) expresses the possibility that the back-up facility *k* for a given facility *j* might not necessarily be the closest facility to *j* itself; indeed, the model will seek to combine facilities based on their operation times, in such a way to minimise their overlaps. Of course, distance plays a role, as constraint (5-4) stipulates that a maximum distance threshold needs to be respected in forming such pairs.

Further refinement of the backup model can be conducted to ensure both GPs are each other's backup facilities. An additional constraint is introduced:

$$h_{j,k} = h_{k,j} \; ; \qquad \forall j,k = 1 \dots J' \mid j \neq k \tag{5-6}$$

where (5-6) ensures that both facilities are each other's backup facilities. This refinement is only suitable for a set with an even number of facilities. Since we are interested in creating a set or cluster of backup GP facilities, therefore, from this point onwards, constraint (5-6) will be excluded from our implementation.

The h_{jk} is adapted into our proposed model, per se, within the definition of u_{jk}^t. h_{jk} is extended to u_{jk}^t by assuming the backup facility remains open for all time *t*. Therefore, the new definition of u_{jk}^t is:

$$u_{jk}^t = \begin{cases} 1 & if\ h_{jk} = 1 \\ 0 & otherwise \end{cases} \; ; \qquad \forall j,k = 1 \dots J' \mid j \neq k, \forall t = T$$

5.3.2 *Refinement 2: Modification of Equations Related to* y_j^t

In this chapter, y_j^t is changed to a parameter, hence any related equations in the proposed multi-period model are relaxed. Therefore, constraints (3-18), (3-19) and (3-22) are relaxed. In the meantime, the objective function of the model:

$$Min\left(\alpha_1 \sum_t \sum_j \left(C_j y_j^t + \varepsilon_{1j} q_j^t + \varepsilon_{3j} l_j^t\right) + (1-\alpha_1) \sum_t \sum_j \left(\varepsilon_{2j} s_j^t + \varepsilon_{4j} \sum_{k,k\neq j} S_{kj}^t u_{kj}^t\right)\right)$$

is modified. Both the C_j and y_j^t are relaxed from the objective function. We also exclude the decision for q_j^t since allocating demand at operating facility *j* is no longer a priority in this chapter. Hence, both ε_{1j} and q_j^t are also relaxed. The refined objective function is thus:

$$Min\left(\alpha_1\sum_t\sum_j\left(\varepsilon_{3j}l_j^t\right)+(1-\alpha_1)\sum_t\sum_j\left(\varepsilon_{2j}s_j^t+\varepsilon_{4j}\sum_{k,k\neq j}S_{kj}^t u_{kj}^t\right)\right).$$

The above objective function focusses on minimising the total costs of the demand of leaving, queue length and a move to the backup facility. The provider's side is focussed on limiting the amount of demand of leaving the network, while the demand's side is focussed on queue length and amount of demand the move to another facility.

5.3.3 Refinement 3: Increment of Demand Waiting Period

To resemble the GP network problem, let *G* be the **amount of time of a patient waiting** for GP consultation day. By which, for this case study, we assume **the amount of waiting time** as **service time**, i.e. the length of time between the booking day until the consultation day. For instance, consider a GP with 8-hours operation times per day and the interval between time-periods is assumed to be an hourly basis. A patient has booked for a GP appointment and the waiting times is 2 days or 16 hours. Therefore, the service times is 2 days or 16 hours. We also assumed that the patient will be served at the end of service times, i.e. after period *G*.

Previously, the mass balance constraint considered the demand to wait for one period in the queue. The constraint is:

$$x_j^t+s_j^{t-1}+\sum_{k,k\neq j}S_{kj}^t u_{kj}^t=s_j^t+\sum_{k,k\neq j}S_{jk}^t u_{jk}^t+q_j^t+l_j^t;\qquad \forall j\in J,\forall t\in T \qquad \text{(3-12).}$$

The s_j^{t-1} indicates the demand from the previous period as part of the left hand-side component, i.e., the $flow_{in}$. This means the demand has to wait one period to be served by the facility. Since we have assumed the waiting times or service times as *G* periods, therefore, the s_j^{t-1} is changed to s_j^{t-G}, meaning the demand has waited from *G* periods. The modified mass balance constraint is:

$$x_j^t+s_j^{t-G}+\sum_{k,k\neq j}S_{kj}^t u_{kj}^t=s_j^t+\sum_{k,k\neq j}S_{jk}^t u_{jk}^t+q_j^t+l_j^t;\qquad \begin{array}{l}\forall j\in J,\forall t\in T,\\ \forall (t-G)>0\end{array} \qquad \text{(3-12)*}$$

The constraint above indicates the $flow_{in}$ components are demand move into facility *j* at time *t* $\left(x_j^t\right)$, the demand from *G* periods $\left(s_j^{t-G}\right)$ and amount of demand from other facility *j*s $\left(S_{kj}^t\right)$.

$s_j^{t-G} = 0$ for $t - G \leq 0$. Due to changes in the mass balance constraint, three related constraints, i.e., (3-13), (3-16) and (3-20) are also changed.

5.3.3.1 Amount of demand on Flow for All j and t

The balance of $flow_{in} - flow_{out}$ is the demand in the queue at the end of period *t*, i.e., s_j^t. The demand will be in the queue for G periods and will be served at $(t + G)$ periods. Since the length of set T is until T', a certain amount of demand are still in the queue at the end of T', i.e., $s_j^{T'} \neq 0$ which will be served after T'. Thus, the constraint:

$$\sum_j \sum_t x_j^t = \sum_j \sum_t (q_j^t + l_j^t) \tag{3-13}$$

is modified and the constraint:

$$s_j^t = 0; \qquad \forall j \in J; \;\; t, T = 0 \tag{3-20}$$

is relaxed. The derivation of the unserved amount of demand (or demand that are still in the queue at the end of T') is given in Appendix 5(C). The modified form of (3-13) is:

$$\sum_j \sum_t x_j^t = \sum_j \sum_t q_j^t + \sum_j \sum_t l_j^t + \sum_j \sum_{(T-G)+1}^{T} s_j^t \tag{3-13*}$$

5.3.3.2 Regulations for Demand to be in the Queue

Previously, constraint (3-16) restricted a demand to be in the queue if the facility *j* was operating on the next period, i.e.:

$$\frac{s_j^t}{x_j^t} \leq y_j^{t+1}; \qquad \forall j \in J, \forall t \in T \tag{3-16}$$

For the healthcare system, there are no limitations to waiting period or space since demand are not physically queued. Since we limit the waiting period to G periods, demand can book an appointment or be in the queue if facility *j* is operating for $(t + G)$ periods.

$$\frac{s_j^t}{x_j^t} \leq y_j^{t+G}; \qquad \forall t \in T, (t + G) \leq T, \forall j \in J \tag{3-16*}$$

The following section presents the entire refined multi-period model for the healthcare facility network.

5.4 The Modified Model for the Healthcare System

Based on the refinements discussed above, the modified multi-period model for reorganising the GP network can be formulated. The parameters and decision variables for the model are as follows:

Sets		
J	=	the set of facility locations and index by *j*, *k* where $\forall j, k = \{1...J \mid j \neq k\}$
T	=	the set of time-periods and index by *t*, where $\forall t = \{1...T\}$

Parameters		
C_j	=	cost of operating the facility *j*
$\varepsilon_{2j}, \varepsilon_{3j}, \varepsilon_{4j}$	=	assigned cost for each decision made ε_{2j} indicates the cost of one unit of demand waiting in the queue, ε_{3j} indicates the cost of losing one unit of demand and ε_{4j} indicates the cost of serving extra units of demand from other facilities.
y_j^t	=	operating periods for facility *j* during a period *t*
u_{jk}^t	=	predetermined binary integer to indicate demand to move from *j* to *k* during a period *t*
τ_j^t	=	capacity level of the facility at location *j* during a period *t*
x_j^t	=	amount of demand at a facility *j* during a period *t*
G	=	waiting periods, i.e., $G \leq T$'
B	=	upper bound of amount of demand leaving the system
α_1	=	weights on provider's side

Decision variables		
s_j^t	=	non-negative decision variable representing amount of demand transferred to the next period at a facility at location *j* at the end of a period *t*
S_{jk}^t	=	amount of demand transferred between facility *j* and facility *k* during a period *t*
l_j^t	=	amount of demand choosing to leave at each facility at a location *j* during a period *t*

q_j^t	=	amount of demand served at each facility at a location *j* during a period *t*

For the healthcare facility network case study, the multi-component model was used by assuming the cost on the provider's side, or Z_1 consists of the cost when a unit of demand leave the system (ε_{3j}), meanwhile, the cost on the demand side, or Z_2 consists of the cost when a unit of demand be in the queue (ε_{2j}) and the cost when a unit of demand move from facility *j* to facility *k* (ε_{4j}). Therefore,

$$Z_1 = \sum_t \sum_j \left(\varepsilon_{3j} l_j^t\right)$$

$$Z_2 = \sum_t \sum_j \left(\varepsilon_{2j} s_j^t + \varepsilon_{4j} \sum_{k,k \neq j} S_{kj}^t u_{kj}^t\right)$$

Thus, the objective function of the modified model is:

$$Min \left(\alpha_1 \sum_t \sum_j \left(\varepsilon_3 l_j^t\right) + (1-\alpha_1) \sum_t \sum_j \left(\varepsilon_2 s_j^t + \varepsilon_4 \sum_{k,k \neq j} S_{kj}^t u_{kj}^t\right)\right) \tag{5-7}$$

subject to:

$$x_j^t + s_j^{t-G} + \sum_{k,k \neq j} S_{kj}^t u_{kj}^t = s_j^t + \sum_{k,k \neq j} S_{jk}^t u_{jk}^t + q_j^t + l_j^t \; ; \quad \forall j,k \in J, j \neq k, \; \forall t \in T, (t-G) > 0 \tag{5-8}$$

$$q_j^t \leq \tau_j^t y_j^t \; ; \quad \forall j \in J, \forall t \in T \tag{5-9}$$

$$\sum_j \sum_t x_j^t = \sum_j \left(\sum_t \left(q_j^t + l_j^t\right) + \sum_{(T-G)+1}^{T} s_j^t\right) \tag{5-10}$$

$$\frac{s_j^t}{x_j^t} \leq y_j^{t+G} \quad \forall j \in J, \forall t \in T, \; (t+G) \leq T \tag{5-11}$$

$$\frac{S_{jk}^t}{x_j^t} \cdot u_{jk}^t \leq y_k^t \quad \forall j,k \in J, \forall t \in T \tag{5-12}$$

$$\sum_j \sum_t l_j^t \leq B \left(\sum_j \sum_t x_j^t\right) \tag{5-13}$$

$$s_j^{t-G} = 0; \quad \forall j \in J, (t-G) \leq 0 \tag{5-14}$$

$$q_j^t, l_j^t, s_j^t, S_{jk}^t \geq 0 \quad \forall t \in T, \forall j \in J \tag{5-15}$$

(5-7) presents the objective function of the refined multi-period model. The component on provider's side are the total costs of demand left the network while the component on the demand's side is the total costs of queuing or moving to the backup facility. The aim is to minimise the total cost of a facility, where this is defined by the consumption on the provider's side and demand's side. The constraints are (5-8) to (5-15). (5-8) is the mass balance constraint, indicating the amount of demand flow in is equal to the amount of demand flow out of the system. (5-9) guarantees that the amount of demand served is within the facility's capacity. (5-10) ensures the amount of demand of the system over all time are either served, leave the system or are still in the queue. (5-11) restricts the demand in the queue if facility *j* is operating at a $t + G$ period. Constraint (5-12) rules the demand movement towards the operating backup facility. (5-13) limits the maximum amount of unserved demand across all facilities to B%. (5-14) ensures no demand in the queue at $t - G \leq 0$, while (5-15) restricts variables q_j^t, l_j^t, s_j^t and S_{jk}^t to positive integers.

The main improvement in this chapter is that of providing extra capacity through the backup facility model. The backup model is implemented within the multi-period model to act as an extension to an interrelated and interconnected facility's network. The refined multi-period model focusses on expanding the network's size without incurring additional costs to the authority. The following section focusses on testing and analysing the refined model by varying the variables' costs and parameters.

5.5 Sensitivity Analyses

Analysis of the model is divided into two parts:

1. testing the backup facility model,
2. testing the multi-period model, which includes:
    i. varying the variables' costs and parameter values' costs;
    ii. varying the waiting periods, *G*.

5.5.1 Backup Service for Each Facility j

Two datasets are created, namely, the *even* dataset and *odd* dataset. The *even* dataset contains an even number of facilities and the *odd* dataset consists of an odd number of facilities. Both datasets are useful in illustrating the flexibility and capabilities of the backup model to create a backup facility for any size of facility network. The *even* dataset, which consists of 4x9, is used where *J'* = 4 and *T'* = 9. For each *T*, it is assumed that the length per day is 3. Meanwhile, the *odd* dataset consists of a 5x9, where *J'* = 5 and *T'* = 9. Details for each dataset are:

Even dataset

$$dist_{jk} = \begin{pmatrix} 1000 & 7 & 11 & 7 \\ 7 & 1000 & 6 & 13 \\ 11 & 6 & 1000 & 9 \\ 7 & 13 & 9 & 1000 \end{pmatrix}$$

$$y_j^t = \left(\begin{array}{ccc|ccc|ccc} 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \end{array}\right)$$

Odd dataset

$$dist_{jk} = \begin{pmatrix} 1000 & 7 & 11 & 7 & 2 \\ 7 & 1000 & 6 & 13 & 6 \\ 11 & 6 & 1000 & 9 & 11 \\ 7 & 13 & 9 & 1000 & 3 \\ 2 & 6 & 11 & 3 & 1000 \end{pmatrix}$$

$$y_j^t = \left(\begin{array}{ccc|ccc|ccc} 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{array}\right)$$

For all analysis in this section, no mutual backup arrangements are foreseen; for example, if GP_2 is a backup for GP_1, GP_1 is not necessarily the backup facility for GP_2. *D* values are varied in order to create a backup facility for each facility *j*. *D* is varied parametrically between 1 unit and 30 units in increments of one unit per step.

Figure 5-8 shows the objective function values based on the variation of *D* values. For both datasets, when *D* = 13 units, the objective function reached its maximum value. This could also be interpreted as the maximum distance between a given facility *j*, and its backup should be at most 13 units. On this same figure, it was found that the maximum combination of total operating periods for *even* datasets is 18 units, whereas for *odd* datasets it is 22 units.

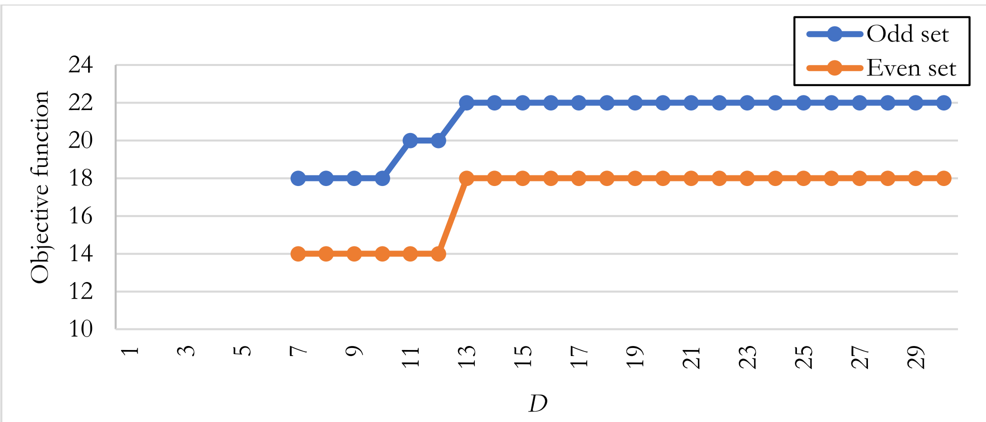


Figure 5-8: Variation of *D* and the objective function values for both sets.

Figure 5-9 shows the h_{jk} results for both datasets. For the *even* dataset, j_1 is the backup for j_3 and vice versa, and j_2 is the backup for j_4 and vice versa, even though we do not include the restriction that each two *j*s are each other's backup facilities (constraint 5-5). For the *odd* dataset, j_1 and j_3 are each other's backup facilities, while j_2 is the backup for j_4, j_4 is the backup for j_5 and j_5 is the backup for j_2, or $j_2 \rightarrow j_4 \rightarrow j_5 \rightarrow j_2$.

<u>*Even*</u> <u>dataset</u>

$$h_{jk} = \quad j\left\{\overbrace{\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}}^{k}\right.$$

<u>*Odd*</u> <u>dataset</u>

$$h_{jk} = \quad j\left\{\overbrace{\begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}}^{k}\right.$$

Figure 5-9: Each facility *j* and its backup facility for both datasets.

From the analysis, the backup model allows a backup facility for each facility *j* to be assigned when there are either an *even* or an *odd* number of facilities in the network. Therefore, for the remaining analyses, only an *even* number of facilities are considered.

5.5.2 Multi-period Model for Reorganising the Facilities Network

5.5.2.1 The Description of the Test Instances

The refined multi-period model for case study 2 consisted of three decision variables and is defined through a multi-component model. The size of the problem is set at *J'* = 4 and *T'* = 9. The demand at each facility *j* per period *t*, x_j^t are distributed as a discrete uniform function, but each facility *j* have a distinctive utilisation rate, $\frac{\sum_t x_j^t}{\sum_t \tau_j^t}$. The capacity level, τ_j is set at 15 units for all facility *j* at all times. The details on the dataset used are:

Table 5-4: Datasets used in each analysis

Parameters	**Range**
$\sum_t x_j^t$	= [126, 99, 90, 153]
$\sum_t \tau_j^t$	= [135, 135, 135, 135]
Utilisation rate (%)	= [93%, 73%, 67%, 113%], average = 87%

The weight for the provider's side, α_1, ranges from 0.1 until 0.9 in increments of 0.1 per step while α_2 is calculated in each case as $1 - \alpha_1$. The initial *B* value is determined through the feasibility of the results. To do this, let ε_{2j}, ε_{3j} and ε_{4j} be 10 units and α_1 be 0.5. The waiting periods *G* is set to 3, or a day, to book a consultation. The results gained from the previous section are used to set the backup facility for each facility *j*. The results of the above are presented in Table 5-5.

Table 5-5: Feasibility results

$\boldsymbol{B}$%	**Solution**
0%	Infeasible solution, relaxed solution
1%	Infeasible solution, relaxed solution
2%	Feasible solution

Even for $\sum_j \sum_t x_j^t \leq \sum_j \sum_t \tau_j^t$, the feasible solution starts after 2%. This is probably influenced by the *G* parameter, where demand must wait in the queue for some time before being served by the facility *j*. However, for all experiments, *B* is set to 0.05 to ensure a 95% minimum service level. Two analyses were conducted:

Analysis 1: Variation of costs and α_1 on system performance.

Analysis 1 tests the impact of costs on a related decision variable, ε_{2j}, and amount of demand in the queue, ε_{3j}, and amount of demand leaving the network, ε_{4j}, and amount of demand to move to a backup facility. This test also focussed the impact of α_1 values on system performance.

Analysis 2: Variation of the duration of waiting times, *G*.

To test the impact of having longer *G* on flow of demand in the network, i.e., queue length, amount of demand goes to the backup facility and amount of demand go to A&E.

Percentage of demand leaving, queue length and demand moving from the primary to the backup facility are calculated using the following formulations in Appendix 3(A). Meanwhile, the percentage of the difference between the current objective function and the new objective function can be calculated as:

$$\%\ changes = \frac{Total\ cost_{new} - Total\ cost_{old}}{Total\ cost_{old}} \times 100\%$$

5.5.2.2 Results: Analysis 1 – Variation of Costs on System Performances.

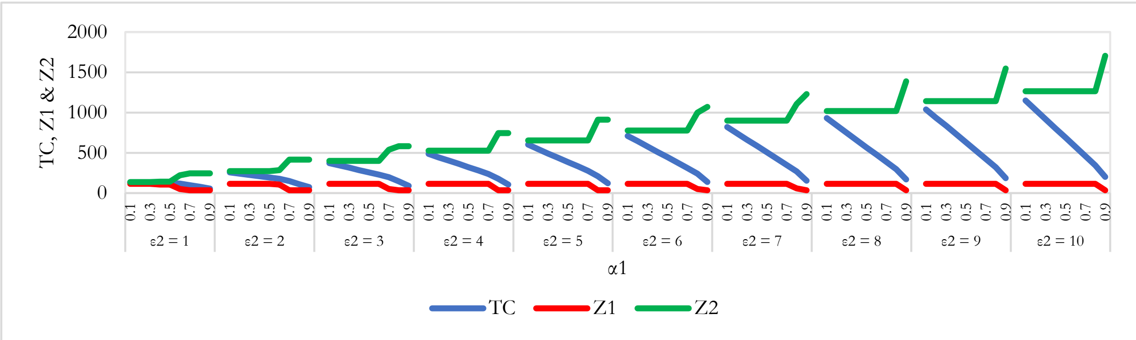
This section focusses on testing the effect of parameter costs on the flow of demand within the network. The variations used are given in Table 5-6.

Table 5-6: Parameter variations for each analysis

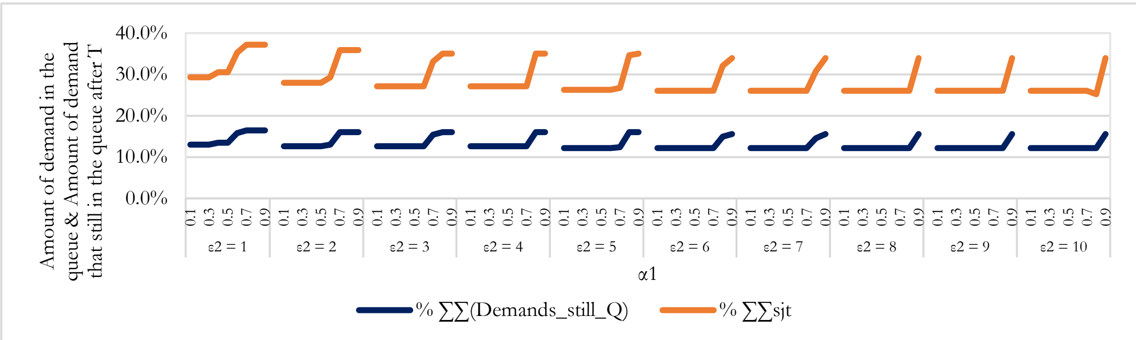
Case	Parameter and its variation	Fixed parameter costs
I	ε_{2j} = 1, 2, 3, …, 10	ε_{3j} = ε_{4j}= 5 units
II	ε_{3j} = 1, 2, 3, …, 10	ε_{2j} = ε_{4j}= 5 units
III	ε_{4j} = 1, 2, 3, …, 10	ε_{2j} = ε_{3}= 5 units

1. Results: Variation of ε_{2j} Cost

ε_{2j} represents the cost of a unit of demand wait for the appointment with the GP. The effects of variation in ε_{2j} and α_1 on total costs, total costs on provider's side or Z_1, total costs on demand's side or Z_2, and the amount of demand in the queue in the GP network, $\sum_j \sum_t s_j^t$ and amount of demand that are still in the queue at the end of period *T*, $\sum_{(T-G)+1}^{T} s_j^t$, as presented in Figures 5-10(a) and 5-10(b). Value of Case I were used in this section.



(a) Total costs, Z_1 and Z_2



(b) Amount of demand that were still in the queue at the end of *T* and amount of demand in the queue

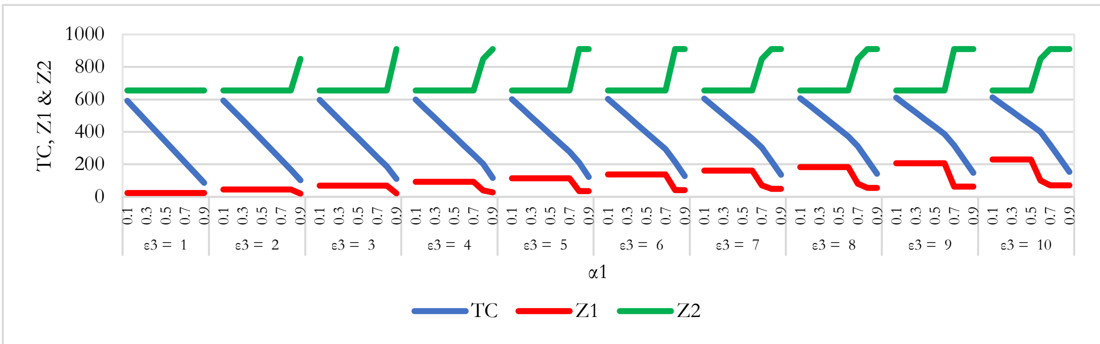
Figure 5-10: System performance with variations of α_1 and ε_{2j} (Case I)

Figure 5-10(a) shows the total cost function increased when ε_{2j} increased. However, when α_1 was increased, the total cost fell slowly. From the figure, it seems the increment in ε_{2j} and α_1 reduced the cost on provider's side or, Z_1 values. Meanwhile, the cost on demand's side, or Z_2 for each α_1 was slightly reduced when ε_{2j} was increased. However, Z_2 gradually increased for all combinations of increases in ε_{2j} and α_1.

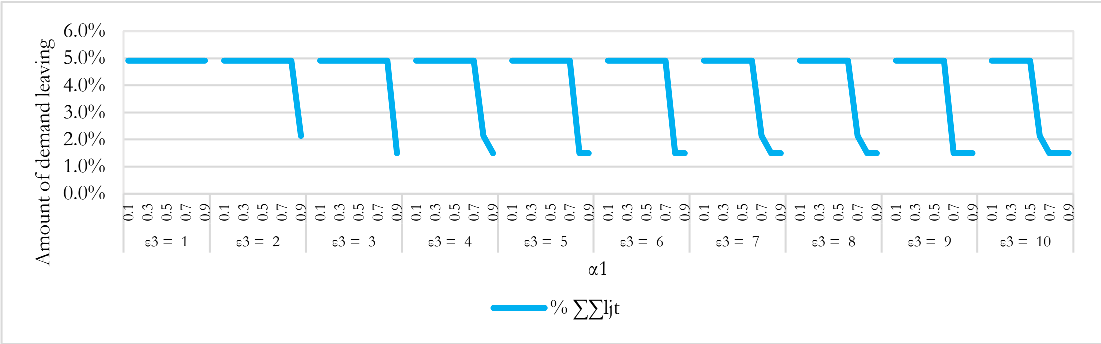
In addition, from Figure 5-10(b), for all values of ε_{2j}, the percentage of demand that are still in the queue at the end of period *T* and the overall amount of demand in the queue, have almost similar patterns. Both lines show that when $\alpha_1 \geq 0.5$ and when ε_{2j} is cheaper than other costs, the percentage of demand is gradually increased. From this experiment, both ε_{2j} and α_1 do not affect the queue length unless they take an extreme value, i.e., ε_{2j} is at a minimum and α_1 is at a maximum.

2. *Results: Variation of ε_{3j} Cost*

ε_{3j} represent the cost for a unit of demand to leave the facility network. Whenever demand leaves the GP, we assumed that demand did not leave the healthcare system completely, but rather went to another 'expensive facility', i.e., the A&E. Previously, ε_{3j} played an important role in controlling the amount of user leave the system, as did the *B* value. Value of Case II were used in this section. The results are presented in Figure 5-11.



(a) Total costs, Z_1 and Z_2



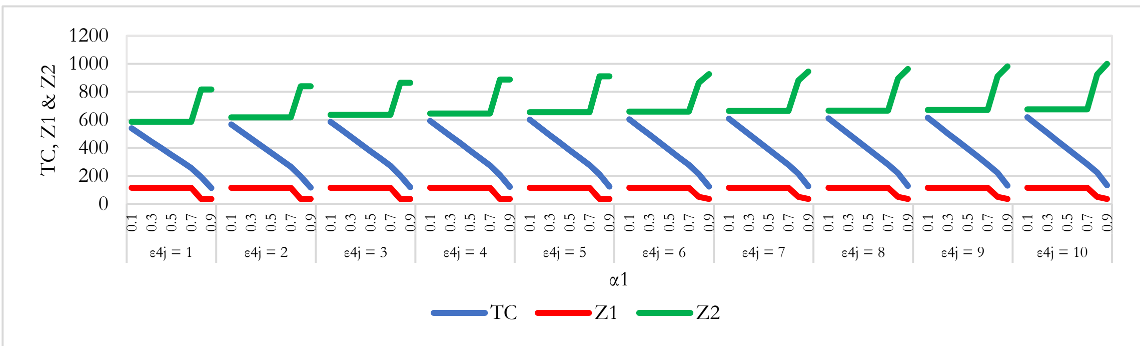
(b) Amount of demand leaving the system

Figure 5-11: System performance with variations of α_1 and ε_{3j} (Case II)

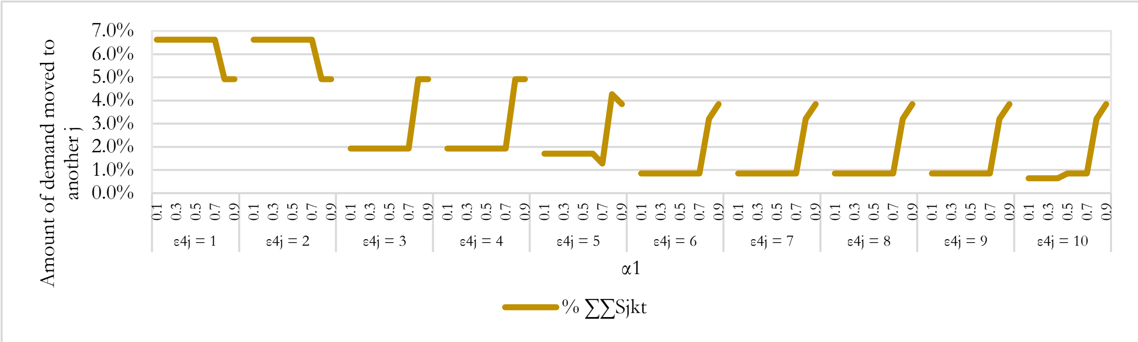
As can be seen in Figure 5-11(a), when ε_{3j} were increased, the total costs increased. However, for all ε_{3j} values, as α_1 increases, the total costs were reduced. Meanwhile, both Z_1 and Z_2 were increased as ε_{3j} costs increased, however, as α_1 increased, both costs showed the opposite trend. Figure 5-11(b) illustrates the percentage of demand that left the network when α_1 and ε_{3j} were increased. In general, when ε_{3j} increases, the percentage of demand left remains stable. When α_1 grew, the percentage of demand left was reduced. This experiment shows α_1 is dominant over ε_{3j} in the refined model, especially when α_1 > 0.5. The proposed model only slowly reacted when ε_{3j} was increased.

3. Results: Variation of ε_{4j} Cost

ε_{4j} represents the cost of a unit of demand be in the queue or cost for demand deterred in the system. The effects of variation in ε_{4j} and α_1 on total costs, total costs on provider's side or Z_1, total costs on demand's side or Z_2, and the amount of demand move to the backup GP, $\sum_j \sum_t S_{jk}^t$ as presented in Figure 5-12. Value of Case III were used in this section.



(a) Total costs, Z_1 and Z_2



(b) Amount of demand move to another facility

Figure 5-12: Variations of α_1 and ε_{4j} on system performance

From Figure 5-12(a), when the ε_{4j} cost was increased, the total cost itself only increased slightly. However, when α_1 was increased, the total cost was found to gradually reduce. Total costs on the provider's side (Z_1) were slightly increased. However, when α_1 was increased, Z_1 reduced drastically. In contrast, for all ε_{4j} costs, costs on demand's side, or Z_2 showed only trivial increments. When α_1 was increased, Z_2 was found to increase.

Figure 5-12(b) presents the effect on the percentage of demand moving to another facility *j* when α_1 and ε_{4j} increased. As expected, when ε_{4j} increased, the amount of demand moving to the backup facility started to reduce, especially after $\varepsilon_{4j} \geq 3$ units. However, when α_1 was increased, the percentage increased drastically. This is because of the weight on the demand's side (α_2) had reduced, and hence the costs to the user to move to another facility *j* became cheaper. As a conclusion to this experiment, clearly α_1 and ε_{4j} do not affect the total costs of the network. Even so, reducing the ε_{4j} costs 'forces' more demand move to another operating facility *j,* which indirectly contributes to more demand being served.

5.5.2.3 Result: Analysis 2 – Variations in Minimum Waiting Periods, G

The purpose of this test was to verify the involvement of *G* in controlling the amount of demand in the queue. For this section, we set $\varepsilon_{2j} = \varepsilon_{3j} = \varepsilon_{4j} = 5$ units and α_1 was set to 0.5 units. In this sub-section, *B* was set to 0.05 for all experiments. *G* was tested for five values: 2, 4, 6, 8, and 10 units. The results are presented in Table 5-7.

Table 5-7: Variations in *G* and implication on demand configuration

G	Total cost	Z_1	Z_2	$\% \sum_j \sum_t q_j^t$ (Amount of Demand Served)	$\% \sum_j \sum_t l_j^t$ (Amount of Demand Leaving the System)	$\% \sum_j \sum_{t>T-G}^{T} s_j^t$ (Amount of Demand That Still in The Queue After Period *T*)	$\% \sum_j \sum_t s_j^t$ (Amount of Demand in The Queue at The End of *T*)	$\% \sum_j \sum_t S_{jk}^t$ (Amount of Demand Moved to Another Facility)
2	518	115	920	84%	5%	11%	33%	7%
4	403	115	690	79%	5%	16%	22%	8%
6	338	115	560	80%	5%	15%	22%	2%
8	328	115	540	77%	5%	18%	18%	5%
10	328	115	540	77%	5%	18%	18%	5%

Table 5-7 shows total costs, Z_1, Z_2, and percentage of the demand configuration whenever the value of *G* was increased by 2 units per iteration. As result, when *G* increased, total costs and Z_2 were reduced while Z_1 remained unchanged. This was due to the percentage of demand remaining the same for all values of *G*. The percentage of the flow of demand per iteration showed various patterns. When *G* = 4 units, both the percentage of demand served and the percentage of queue length were decreased, while the percentage of demand still in the queue at the end of *T* and percentage of demand moving to another facility *j* showed slight increments. When *G* = 6, the percentage of demand served was slightly increased, while the percentage of demand still in the queue at the end of *T* and percentage of demand moving to another facility *j* showed minor reductions. When *G* = 8, the percentage of demand served and percentage of queue length were reduced; in contrast, the percentage of demand still in the queue at the end of *T'* and percentage of demand moving to another facility *j* were increased. Meanwhile, for *G* = 10, the values remained unchanged.

Having longer waiting periods also shortens the queue length since the optimal solution in such an instance is to 'push' more demand to leave the system. This also means demand have to pay extra to wait in the queue; the longer the waiting period, the greater the costs incurred by a demand. This is somehow true, looking into the current scenario, that longer waiting times will increase the costs to users, encouraging them to look for (more expensive) alternatives. For example, buying more medicine or for a casual worker, having to take a few days off from work, meaning, no pay for the rest day. Having a longer *G* would also reduce the possibility of demand going to the backup facility. This can be seen from Table 5-7, where amount of demand moving to a backup facility continued to reduce as *G* increased. Most probably the optimal solution to the modified model would be achieved when the user stays in the system and waits for the appointment day instead of going to the appropriate backup facility. From the table, the percentage of demand leaving the network (i.e. go to the A&E for medical attention) were unchanged for all tested *G* values. This is because the limitation imposed by *B*, i.e. 5%.

This section provides information on the sensitivity analysis of the refined model. We also analysed a backup facility model in this section, intended for use as part of the component to create the 'link' between one facility *j* and another facility. It can be said that the refined model is highly sensitive to *B*, similar to both the main multi-period model and the refined model for the HWRC problem. *B* is important to ensure that a given solution is feasible. We tested the refined model by varying α_1 and parameter costs for each decision variable, and the length of the waiting periods, *G*. It was found that α_1 influences the demand circulation by controlling the costs for each provider's and the demand's side. However, the effect was more obvious if α_1 was set to greater than 0.5 units. But this surely depending on costs set to each decision variable. Since y_j^t is set as one of the parameters, its costs, i.e., C_j, is also relaxed. Costs to serve a unit of demand are also relaxed as it is assumed to be part of the cost to operate a given facility *j* and a system. Since C_j and ε_1 are not in the refined model, therefore it can be said that the remaining parameter costs will be fairly influential. As can be seen from Table 5-7, flow of demand is similar. This could plausibly be caused by the limitation imposed on *B*. Even when *G* is changing, *B* has to be estimated to produce feasible solutions. *B* is somehow affecting flow of demand in the system. This was demonstrated in Chapter 4, in which *B* was found to be responsible for maintaining or pushing demand in the network, either from facility *j* to another facility *j* or from *t* to the next *t*. Another important aspect that needs to be considered is τ_j^t or the capacity of facility *j* at a time *t*. Perhaps limited capacity also contributes to the flow of demand within the network.

The numerical results for the analyses for this section as presented in Appendix 5(D). The following section presents the implementation of the refined model to the case study; which includes the dataset for healthcare network facility in Sheffield and its related costs.

5.6 Implementation within the GP System Network

This section introduces the parameters used to solve the GP facility network problem using the refined model. There are six parameters used, namely waiting costs, costs of a unit of demand leaving the system, costs due to a demand moving to the backup facility, capacity of each GP and, finally, waiting periods.

There are 82 GPs in Sheffield, therefore, let *J* represent the set of these facilities. Details on each GP's name and representation in *j* are given in Appendix 5(B). It is assumed that each GP operates five days a week, from Monday until Friday, and are open for consultation from 07:00 until 20:00, or 14 hours a day (H = 14). Therefore, for five days (W' = 5), the total operating periods are 5x14 = 70 periods per week or T' = 70. Mathematically, the sets are $j \in J$ or $j = \{1, \dots, J'\}$, $t \in T$ or $t = \{1, \dots, T'\}$ and $w \in W$ or $w = \{1, \dots, W'\}$. The backup facility is part of *J*, hence $j, k = \{1, \dots, J' \mid j \neq k\}$. The remaining parameters can be described as follows:

5.6.1 *Cost of waiting for the appointment day for an user* (ε_{2j})

This cost is assumed to be a combination of **minor ailment costs** and **opportunity costs** per hour during the waiting periods. It is assumed that demand will affect self-care for a minor ailment. The cost of a minor ailment is assumed to be £2.63 (Baqir et al., 2011). Besides getting their medications, the demand is also assumed to absent from work because of sickness and not productive. Therefore, it was assumed that the opportunity cost is the amount of salary loss for a casual worker if the person is absent because of sickness. According to the HM Website (HMSO, 2018), an individual is allowed to work a maximum of 48 hours a week on average. It is also assumed that this cost is based on opportunity costs based on minimum salary per hour per person; £7.83. For this study, $\boldsymbol{G}$ is assumed to be the duration to from the day on which the booking is made until the day of the appointment. It was assumed the **waiting time** to be **2 days**, and that demand will be served on the third day. This is because the second-highest appointment waiting periods from the GP Patient Survey (see Appendix 5(A)) are 'a few days later'. Even though the national standard for a waiting period is a maximum of 13 days, our aims are to reduce demand waiting times. Hence, for the initial implementation in this analysis, G was set to 2 days. Therefore, for two days, total estimated salary loss for not being able to work is:

$$\frac{48}{7\text{ days}} \times 2\text{ days} \times £7.83 = £107.38$$

Therefore, the entire cost for a demand to wait for his/her appointment day is:

$$\varepsilon_{2j} = £2.63 + £107.38 = \textbf{£110.01 per person}.$$

5.6.2 *Cost of a Unit of Demand Leaving the System* (ε_{3j})

This cost is assumed to be the cost when a demand **goes to the A&E**. It is reported that have to spend an extra **£160.00** to attend such a demand (NHS England, 2018c, p.5).

5.6.3 *Cost of a Unit of Demand Moving to Backup Facility* (ε_{4j})

The ε_{4j} cost is assumed to be the **transportation cost** for a demand to go to the backup facility, and it is assumed that the cost is evenly distributed for each movement. We use similar transportation costs as in the previous chapter, which are £7.77/ hour. It is assumed that a unit of demand will require 15 minutes to go to the backup facility, and therefore, the cost for a demand to go to the backup facility is **£1.94/15 minutes**. We did not consider any opportunity costs for a demand due to the type of service needed.

5.6.4 *Waiting Periods, G*

As mentioned previously on cost for a demand to wait for an appointment day, *G* was assumed to be **2 days**, i.e. demand will be served (t+G) period. Currently, we assumed that *G* was a constant.

5.6.5 *Capacity level* (τ_j^t)

Capacity level of a GP per time *t*, or τ_j^t, focusses on the capability of a full-time time doctor to serve a demand. Average consultation times in general practice in England was 8.86 minutes in 2014 (Baird et al., 2016); however, a safe environment requires each consultation to be completed within 15 minutes (BMA, 2016, p. 2). Therefore, the consultation capacity per GP is calculated based on the assumptions that:

- consultation rates are 15 minutes per user;
- the average capacity of a practitioner is four (4) consultations per hour.

The capacity per facility *j* is assumed to remain unchanged for all *t* and is independent of the particular operating periods of a given facility *j* (y_j^t). The following shows the calculation that gives the capacity of a given doctor.

$$\begin{aligned} \tau_j^t &= \text{1 practitioner} \times \text{4 sessions per hour} \\ &= \text{4 demand per hour or 56 potential capacities per day.} \end{aligned}$$

The values that we use are estimated and gathered mostly from online articles and the official NHS website. The remaining datasets, especially on GP operating periods and amount of demand per GP, are given in Appendix 5(B). The following section focusses on the results of reorganising the GP facility network using the backup facility as an option.

5.7 Results

The backup model and the refined model were solved using CPLEX 12.6, on a computer with an Intel® Core™ i5-7200 CPU, 2.50 GHz and 8G RAM. The results were further categorised into two parts:

1. Creating a backup facility for a GP,
2. Solving the GP facility network and comparing with a system with no backup available.

The first part was to create a backup for a GP in order to give maximum coverage to the demand. We focussed only on a cluster-like GP backup, meaning both *j*s are not necessarily each other's backups. The second set of results discussed the results obtained with the backup facility in terms of demand circulation and configuration. The results obtained were then compared the results obtained in the second part, namely a system without a backup (i.e., the current configuration of the GP network).

5.7.1 Cluster-like Backup Facility

Figure 5-13 shows the objective function values as D is varied. It was found that as D was increased to greater than or equal to 14 units, the objective function reached a maximum, 668 units. From the result, several pair of facility j and its backup with travel distance at maximum; 14 minutes. Details on the cluster formed when including a backup facility as reported in Table 5-8 and Figure 5-14.

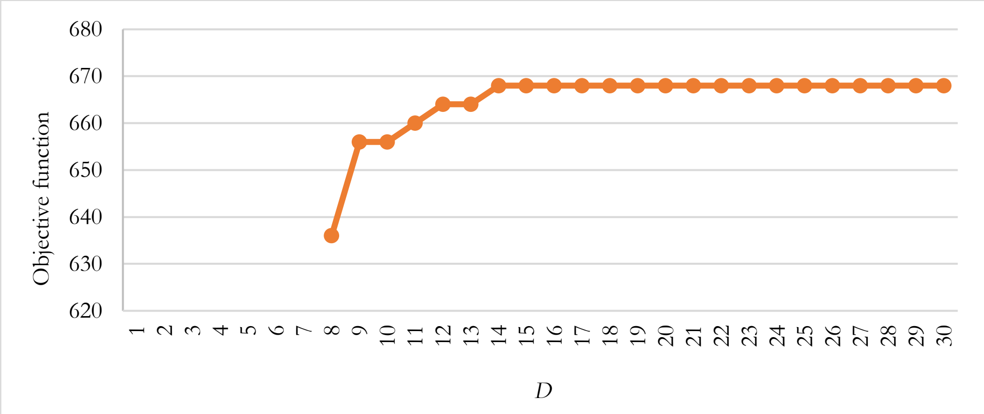


Figure 5-13: *D* values and objective function produced

Details on GPs and their backup facilities are given in Table 5-8. Consider a 'set' as being when a set of facility j and their backup facilities form a cluster-like shape. For example, as reported in Table 5-8, in Set or Cluster 1, j_1 is backed up by j_{81}, j_{81} is backed up by j_{79}, j_{79} is backed up by j_5 and j_5 is backed up by j_1. Through this, demand can move to the backup facility when there is no availability at their primary GP. The movement, somehow restricted if only the facility is operating and it is assumed that demand will move only once. Illustration of the cluster formed as in Figure 5-14.

Table 5-8: GPs and their backups.

Set	*j* and its backup *j*
1	1 – 81 – 79 – 5 – 1
2	2 – 77 – 38 – 9 – 54 – 74 – 61 – 65 – 82 – 47 – 43 – 70 – 4 – 57 – 28 – 20 – 2
3	10 – 45 – 30 – 36 – 21 – 10
4	19 – 33 – 76 – 49 – 19
5	25 – 42 – 25
6	34 – 41 – 34
7	39 – 58 – 39
8	40 – 69 – 40
9	3 – 53 – 75 – 14 – 16 – 31 – 63 – 11 – 29 – 44 – 60 – 13 – 6 – 22 – 24 – 78 – 32 – 72 – 67 – 27 – 71 – 64 – 46 – 73 – 37 – 56 – 15 – 68 – 17 – 12 – 50 – 62 – 52 – 51 – 7 – 80 – 26 – 23 – 66 – 35 – 8 – 55 – 59 – 18 – 48 – 3

Figure 5-14 illustrates all the 'Sets' or 'Clusters' that were formed using the Backup model. A close-up view of Cluster 1 as illustrated in Figure 5-15. This cluster is a good example to show the mechanism of the Backup model.

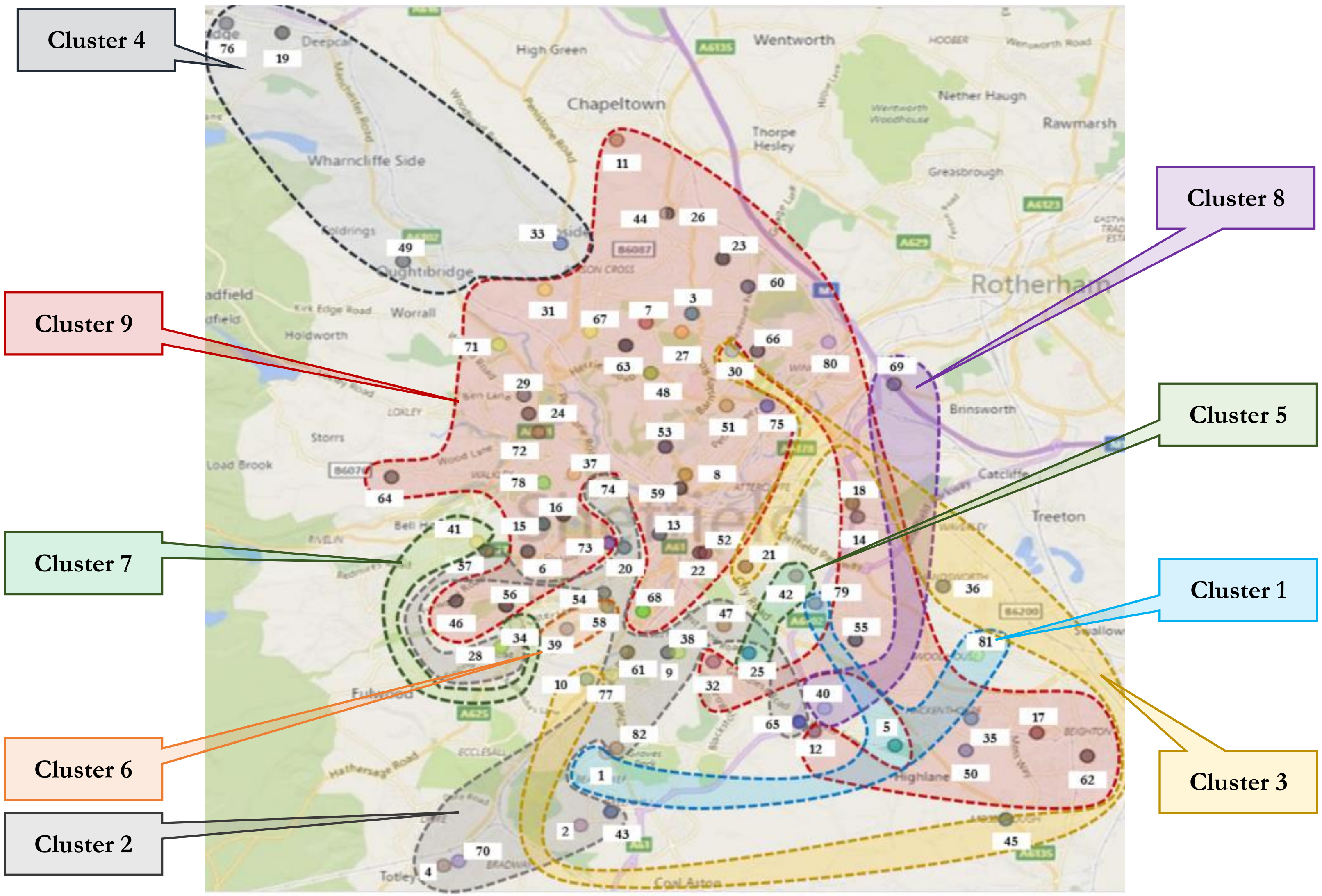


Figure 5-14: The Sets or Clusters Formed by the Backup Model

Figure 5-15: The close-up view of Cluster 1 and the GPs in this cluster.

From Figure 5-15, Cluster 1 consists of j_1, j_{81}, j_{79} and j_5. Backup of j_1 is j_{81}, which j_{81} is the optimal backup facility that able to suit the operation time facility for j_1. Even though there are several facilities that are located closest to j_1, such as j_{82}, however, the combination of operating time for both facilities (the j_1 and j_{82}) probably not the maximum ones. In addition, travel times from j_1 to j_{81} is not more than 15 minutes. Clearly, the backup model not just focusses on finding the maximum combination of operating time between two GPs, but also ensure the travel times between these facilities are within a reasonable range. The remaining close-up view of the remaining clusters as in Appendix 5(E).

From the backup facility created in Table 5-8, the travel times between facility j and its backup facility j can be found. The details of travel times between js are given in Table 5-9. It was found that the longest travel time between a facility j and its backup is 14 minutes and the shortest is 2 minutes. Meanwhile, the average travel times between a given facility j and its backup is about 10 minutes.

Table 5-9: Travel time between a given facility j and its backup facility

j	*k*	**Travel time between *j-k***	*j*	*k*	**Travel time between *j-k***	*j*	*k*	**Travel time between *j-k***
1	81	11	29	44	12	57	28	9
2	77	9	30	36	11	58	39	4
3	53	7	31	63	6	59	18	13
4	57	11	32	72	11	60	13	11
5	1	13	33	76	14	61	65	12
6	22	13	34	41	8	62	52	14
7	80	11	35	8	14	63	11	12
8	55	12	36	21	11	64	46	12
9	54	8	37	56	14	65	82	12
10	45	11	38	9	2	66	35	11
11	29	14	39	58	4	67	27	7
12	50	8	40	69	11	68	17	11
13	6	7	41	34	8	69	40	11
14	16	13	42	25	8	70	4	3

15	68	10
16	31	14
17	12	12
18	48	13
19	33	14
20	2	11
21	10	13
22	24	11
23	66	7
24	78	8
25	42	8
26	23	5
27	71	12
28	20	12
43	70	10
44	60	6
45	30	3
46	73	10
47	43	12
48	3	4
49	19	8
50	62	7
51	7	8
52	51	12
53	75	9
54	74	8
55	59	12
56	15	8
71	64	14
72	67	10
73	37	12
74	61	11
75	14	8
76	49	12
77	38	6
78	32	14
79	5	11
80	26	12
81	79	10
82	47	12
AVERAGE = 10.04 minutes		

As mentioned earlier (as in section 6.6.3), the time windows for a demand to move to the backup facility *j* was assumed to be 15 minutes. Through the backup facility system, it was guaranteed that the demand is able to move to the backup facility in a time less than or equal to 15 minutes. Using the created clusters, h_{jk} is then converted into a variable u_{jk}^{t} ruling the transfer of demand from a facility to its backup.

5.7.2 *Reorganising the GP Network with the Backup Facility System*

The previous subsection focusses on creating a 'link', or finding a backup facility, for each GP. Therefore, this section focusses on the solution, which consists of the variation of α_1 and the optimal results so produced. Before we proceed with the results and discussion, the minimum value for *B* will be determined based on the feasibility of the results. To do this, we set $\alpha_1 =$ 0.5. As a result, a feasible solution was found when $B \geq 0.24$, as shown in Table 5-10. This is because the current operation of the GP network (without the backup service) is unable to cater for the number of registered patients; i.e., $\sum_j \sum_t x_j^t \geq \sum_j \sum_t \tau_j^t$ (amount of demand is more than capacity level). Therefore, the value of $B = 0.24$ was used to determine the optimal solution for the GP facility network.

Table 5-10: Results of feasibility tests

B	Solution
0.05	Relaxed, infeasible solution
0.10	Relaxed, infeasible solution
0.15	Relaxed, infeasible solution
0.24	Feasible solution
0.25	Feasible solution

Table 5-11: Results of each component when reorganising the GP facility network

α_1	Total cost	Z_1	Z_2	$\%\sum_j \sum_t q_j^t$ (Amount of Demand Served)	$\%\sum_j \sum_t l_j^t$ (Amount of Demand Leaving the System)	$\%\sum_j \sum_{t>T-G}^{T} s_j^t$ (Amount of Demand That Still in the Queue After Period *T*)	$\%\sum_j \sum_t s_j^t$ (Amount of Demand in the Queue At The End of *T*)	$\%\sum_j \sum_t S_{jk}^t$ (Amount of Demand Moved to Another Facility)
0.1	393,428	2,177,280	195,222	73.4%	24.0%	2.6%	3.0%	4.4%
0.2	591,633	2,177,280	195,222	73.4%	24.0%	2.6%	3.0%	4.4%
0.3	789,839	2,177,280	195,222	73.4%	24.0%	2.6%	3.0%	4.4%
0.4	988,045	2,177,280	195,222	73.4%	24.0%	2.6%	3.0%	4.4%
0.5	1,186,251	2,177,280	195,222	73.4%	24.0%	2.6%	3.0%	4.4%
0.6	1,384,324	2,173,120	201,130	73.4%	24.0%	2.6%	3.1%	4.5%
0.7	1,580,708	2,160,800	227,159	73.4%	23.8%	2.8%	3.5%	4.5%
0.8	1,774,072	2,160,800	227,159	73.4%	23.8%	2.8%	3.5%	4.5%
0.9	1,967,436	2,160,800	227,159	73.4%	23.8%	2.8%	3.5%	4.5%

For case study 2, the remaining parameters were as outlined in the previous section. α_1 was parameterised as between 0.1 and 0.9. Table 5-11 shows the results for case study 2 using the refined multi-period model with an integrated backup facility system. The cost on the provider and demand's side remained unchanged until α_1 = 0.7. Even when $\alpha_1 \geq$ 0.7, the solution began to change only slightly, with about a 0.2% reduction in amount of demand leaving the system, an increment of 0.4% in amount of demand in the queue and 0.2% in the amount of demand after G periods. The amount of demand served remained unchanged for all values of α_1. The results obtained were then compared with the GP facility network with no backup facility system available.

5.7.3 *Comparison with no Backup Facility*

To test our multi-period with backup model and demonstrate its benefits, the results obtained in the previous section were compared to the current GP facility network operations, i.e., with no 'link' between GP facilities, and where the demand movement being restricted to remaining in the queue or going to A&E. To ensure no relation between a facility *j* and other facilities, we simply set $u_{jk}^t = 0$ for all times *t* for all facilities *j*. Results for the GP network without any backup facility system are reported in Table 5-12.

Table 5-12: Results of not having backup facility system, with variation of α_1

α_1	Total cost	Z_1	Z_2	$\%\sum_j\sum_t q$ (Amount of Demand Served)	$\%\sum_j\sum_t l_j^t$ (Demand Leaving the System)	$\%\sum_j\sum_{t>T-G}^{T} s_j^t$ (Demand still in the Queue After Period *T*)	$\%\sum_j\sum_t s_j^t$ (Demand in the Queue at the End of *T*)
0.1	504,113	2,358,720	298,045	69.8%	26.0%	4.2%	4.7%
0.2	710,180	2,358,720	298,045	69.8%	26.0%	4.2%	4.7%
0.3	916,248	2,358,720	298,045	69.8%	26.0%	4.2%	4.7%
0.4	1,122,315	2,358,720	298,045	69.8%	26.0%	4.2%	4.7%
0.5	1,328,383	2,358,720	298,045	69.8%	26.0%	4.2%	4.7%
0.6	1,533,720	2,338,880	325,980	69.8%	25.8%	4.4%	5.1%
0.7	1,733,650	2,318,400	369,234	69.8%	25.6%	4.7%	5.8%
0.8	1,928,567	2,318,400	369,234	69.8%	25.6%	4.7%	5.8%
0.9	2,123,483	2,318,400	369,234	69.8%	25.6%	4.7%	5.8%

From Table 5-12, it was found that the total costs to operate the system increased with increasing α_1. This shows if provider's intended to increase their weight or credence in the planning, without the backup system more demand will leave the system or, in this case, go to A&E. This is clearly shown in the sixth column of Table 5-12. The expected percentage of demand that would end up at the A&E is more than 25%. However, if the demand is not going to A&E, the potential of not receiving any proper treatment might pose risk to users themselves. Interestingly, fewer demand would be in the queue, i.e., waiting for the appointment days, even though the pre-set waiting time is two days only. The worst would be expected in terms of the proportion of demand that would leave the GP network. From our findings, even if the authority increases their influence in the decision-making process, especially in making plans to suit its budget, demand flow within the healthcare network system would not change that much. Demand is not going to be in the queue since most appointment slots have been filled.

The comparison between the results for the GP facility network with a backup system and the GP network without can be found Table 5-13. To calculate the data in the comparison table, we use an actual figure gained from both implementations. For example, the percentage of demand served, without and with the backup facility, at $\alpha_1 = 0.1$ can be found as:

$$= \frac{Total\ patients\ served\ (with\ backup)\ -\ Total\ patients\ served (without\ backup)}{Total\ patients\ served\ (without\ backup)} \times 100\%$$

$$= \frac{41{,}618 - 39{,}561}{39{,}561} \times 100\% = 5.2\%$$

As in Table 5-13, the first three columns show the difference in total cost (objective function) and both the provider and demand's cost side of having a backup facility for the GP network. All percentage differences in costs show negative values, meaning a reduction in operating costs, and the obvious reduction is from the demand's side, as the highest figure obtained was a reduction of 38.5%. Meaning, cost on demand's side was reduced, directly implies on benefits attained by the demand. Our model allowed for demand to move to another facility *j*, i.e., in the instance of the GP network to move to the backup facility and therefore increase circulation within the GP network. An increment of 5.2% for amount of demand served indicates that demand has another option beside the primary registered GP, to get medical attention. The amount of demand leaving the GP network was also found to be reduced by at least 7.7%. Even if the provider, i.e., the NHS increases their weight in their decision, the percentage of demand that are expected to leave is not significantly reduced. By introducing the backup system, from our dataset, more demand would be able to get an appointment day and wait to be served.

Table 5-13: Comparison between percentage differences with α_1*

α_1	Total cost	Z_1	Z_2	$\sum_j \sum_t q_j^t$ (Amount of Demand Served)	$\sum_j \sum_t l_j^t$ (Amount of Demand Leaving the System)	$\sum_j \sum_{t>T-G}^{T} s_j^t$ (Amount of Demand that Still in the Queue After Period *T*)	$\sum_j \sum_t s_j^t$ (Amount of Demand in the Queue At the End of *T*)
0.1	-22.0%	-7.7%	-34.5%	5.2%	-7.7%	-36.1%	-38.5%
0.2	-16.7%	-7.7%	-34.5%	5.2%	-7.7%	-36.1%	-38.5%
0.3	-13.8%	-7.7%	-34.5%	5.2%	-7.7%	-36.1%	-38.5%
0.4	-12.0%	-7.7%	-34.5%	5.2%	-7.7%	-36.1%	-38.5%
0.5	-10.7%	-7.7%	-34.5%	5.2%	-7.7%	-36.1%	-38.5%
0.6	-9.7%	-7.1%	-38.3%	5.2%	-7.1%	-39.8%	-40.5%
0.7	-8.8%	-6.8%	-38.5%	5.2%	-6.8%	-39.8%	-40.5%
0.8	-8.0%	-6.8%	-38.5%	5.2%	-6.8%	-39.8%	-40.5%
0.9	-7.3%	-6.8%	-38.5%	5.2%	-6.8%	-39.8%	-40.5%

*Formulation used = $\frac{X(with\ backup) - X(without\ backup)}{X\ (without\ backup)} \times 100\%$

From our findings, more demand would be served with the introduction of the backup facility. This can be seen from the changes in percentage demand leaving and percentage of demand remaining in the queue. To explain this, $\alpha_1 = 0.1$ can be used as an example. Recall constraint (6-9) that indicates the distribution of demand in the system network:

$$\sum_j \sum_t x_j^t = \sum_j \left(\sum_t (q_j^t + l_j^t) + \sum_{T-G}^{T} s_j^t \right)$$

Constraint (6-9) strictly controls demand circulation within the system for all times *t*. This constraint also indicates that all demand in the system are either served, leave, or wait in the queue. It is assumed that demand that wait in the queue will always be served after a period *T'*. To gain a better understanding, we illustrate the movement of demand using this constraint.

<u>Without backup system:</u> 100% = 69.8% $\boxed{+\ 26\%}$ $\boxed{+\ 4.2\%}$

-2% -1.6%

<u>With backup system:</u> 100% = 73.4% + 24% + 2.6%

+2% +1.6%

The blue line represents demand leaving the system while the orange line represents demand that are still in the queue. From the illustration, for a system with a backup facility, more demand will be served instead of leaving or remaining in the queue. Through this improvement, problems such as being unable to get an appointment, or increases in unnecessary attendance in A&E, may be reduced. As mentioned, the backup facility should be able to reduce the effect across the entire system's operations. From this comparison, with the backup model, more demand are expected to be served, less demand waiting for the server availability and fewer would be expected to leave the GP system.

5.8 Conclusion

From the proposed model in Chapter 4, a modified version of the multi-period model embedded with backup facility system was introduced in this chapter. Our main purpose was to consider and provide an alternative system by means of which reducing congestion within GP networks, and, indirectly at A&E facilities themselves. Looking at this problem, we believe inaccessibility of the GP contributes to the overcrowding problems in A&E. Hence, a potential alternative in order to increase accessibility and provide 'extra' capacity through incremental demand circulation within the GP network itself was introduced, i.e., the backup model. This backup model was used as a form of 'link' between different GPs. As the 'link' is part of our multi-period model, the refinement of related variables could be achieved. Another refinement was that of changing the variable y_j^t to be one of the parameters; instead of focussing on reorganising the facility operations (such as operating times or which facility needs to be closed), we had shifted our focus to increase server accessibility and optimise system operations.

This chapter focussed on applying the multi-period model with the backup system to a GP facility network. A comparison between a system with and a system without the backup facility was conducted, from which it was found that with the backup system more demand can be served. It was also found that fewer demand leave the system and fewer demand are waiting for their assigned appointment day. Hence, a better healthcare system could be provided in which medical treatment would be available quickly and could indirectly reduce the expenses of an already highly expensive facility, i.e., A&E.

The following chapter will give an overall discussion on our work on reorganising facility networks. A summary of the research and thesis will be provided. At the end of the chapter, several of the contributions made by this thesis will also be highlighted.

CHAPTER 6: DISCUSSIONS AND CONCLUSIONS

Within the planning of facilities networks, demand arrival patterns, supplies, and technologies, can change after some time. For this reason, the configuration of facility networks might become obsolete. These facilities might as well have reduction in financial allocation. This reduction might affect the network size, for example, there may be a reduction in total operating hours, or reduced in operating facilities. Due to the reduction, system utilisation rates will increase, and hence directly contribute to congestion problems as demand only have a limited choice of services. For essential services, this means an increase in waiting times before servers become available. Some demand could be moved to another facility or, at a certain point, leave the system entirely. In handling this scenario, the decision-maker must take any possible action to ensure facility networks can keep operating and that they provide a minimum required service level, even though, due to financial reasons, some facilities might have to be downsized (and their operating hours reduced) or, in extreme cases, closed down entirely.

This chapter focusses on summarising the work in the previous chapters, i.e. chapters 2 to 6. The contributions of the research are discussed thereafter, followed by the limitations of this research. Lastly, future research directions are highlighted.

6.1 Summary of Chapters

The focus of this thesis is to reorganise existing facility operations with due consideration for supply reduction. In solving this problem, the related problems arising from the reorganisation, i.e., congestion, was also investigated.

The first three chapters provided an overview of dynamic facility location models. This included the investigation of the problem of interest, conducting a literature survey, and highlighting the gap of the studies. The problem of interest emphasised the need to reorganise existing facility networks, especially with regards to a context of reduction of financial allocations (as in Chapter 1). Chapter 2 discussed the foundation for facility location model. In Chapter 3, the existing studies in the field of facility location were discussed, especially with regards to reorganisation that involved facility closures. However, very few studies have focussed on this area, hence another perspective was explored, based on the usage of multi-period models. One of the renowned multi-period models is capacitated lot-sizing problem (CLSP). CLSP is a model that considers the mass balance concept; all items in a facility must be thoroughly considered.

Chapter 4 focussed on adapting the concept of the CLSP model to solve the facility network reorganisation problem. Even though CLSP model was adapted to develop our multi-period model, both models are entirely distinctive, especially in terms of their outputs. CLSP is used to determine the optimal amount of stock so that overall costs are minimised. The proposed multi-period model focussed on finding the optimal facility network plan in order to ensure that most demand would be served, even if at the minimum required level. In addition, the developed model is able to propose optimal operating periods. The major improvement realised by the proposed multi-period model is relating one facility to another. This includes the movement of demand to another facility, i.e., being further enriched, as described in chapter 5 and chapter 6. The enrichments include providing 'extra' capacity by creating a 'link' between two facilities. Therefore, for any facility that is 'forced' to be closed by the model, the designated demand move or go to another less congested facility or move to a facility with residual capacity. Due to reorganisation, facility network might encounter congestion problems. Thus, using the proposed model, the congestion problem arises from the reorganisation can be reduced.

Two chapters of the thesis describe the application of the proposed model by solving two case studies. These case studies were arranged into two chapters: Chapter 5 and 6. Chapter 5 implemented the proposed model within the HWRC problem. The proposed model was refined in terms of micro- and macro-periods. As a result, two out of five HWRCs were suggested for complete closure; in addition, operating periods for the remaining HWRCs were also proposed. Chapter 6 examined a solution to the healthcare facility problem. The proposed model was modified to reduce congestion level at the A&E. This was achieved by creating a 'link' between GPs to increase demand circulation, indirectly resulting in fewer demand waiting for their appointment day and reducing A&E congestion problems.

All of the six chapters play an important role in signifying the contributions of our study, especially in solving real-life problem, which is highlighted in detail in the following section.

6.2 Contributions of the Research

This study focussed specifically on the reorganisation of existing facility operations, as caused by the reduced financial allocations. This situation might lead to facility closure or reduced in facility operating periods. The congestion problem caused by the reorganisation is also considered in the solution to this problem. The contributions of our study are threefold – first, the enrichment of the area of study, second, through the proposed multi-period model itself, and lastly, the application of the model to two public case studies.

6.2.1 Contribution to the Area of Study

Looking into the reorganisation study, the contributions can be seen from the identification of problem of interest and the gap of study. The problem of interest focussed on the need to reorganise the existing facility operations due to reductions in supply. An extended problem definition was looked into, namely that of considering the congestion that can occur because of the supply reduction problem. There are very few studies in the literature that have considered this area, as highlighted in chapter 3, especially the concept of interrelated and interconnected facilities. Our study acknowledged the effects of interrelation and interconnection between the facilities within the same network. After the reorganisation took place, number of operating facilities are limited and with non-decreasing demand in the network, congestion problem will took place. Hence, our study also contributes in refining the reorganisation concept of the interrelated and interconnected facilities within the network.

6.2.2 Contribution to Model Development

Our study focusses on the operation of facilities within a network which are interrelated and interconnected; and which provide similar services to the users. Some of these facilities might suffer from the congestion problem, while others might have residual capacities. Therefore, by changing operations in one of the facilities, the entire network could be affected. For this reason, we investigated the flow of demand and mechanism within the network. A multi-period model was developed in order to minimise the damage or discomfort to the provider and the demand, as a consequence of the need to reduce opening hours. A demand mechanism taking into account the interrelated and interconnected nature of facilities is considered in this model. Through such mechanism, the optimal operation schedule for each facility can be found, also taking into account the possibility of closure and downsizing of certain facilities. The development of this model was inspired by the mass balance concept in the CLSP. This concept represents the state-of-the-art in terms of applying the model to a non-manufacturing problem. The model also used interrelated and interconnected facilities to gain extra capacity, with or without a minimum cost. Indirectly, the congestion problem can be reduced by allowing extra demand to be shifted to another facility that has extra capacity. An optimal schedule was produced since the model is multi-period based. At the same time, the risk of users leaving the system can be measured.

The concept of interrelated and interconnected facility is similar to the queueing network problem. But, as far as we know, no past studies that work on facility location with congestion problem and the BPR used the queueing network problem. Therefore, our study marks an important phase of developing a leading model for reorganising the operations of interrelated and interconnected facilities.

6.2.3 Contribution to Practice

Considering the findings of the literature review (i.e. chapter 3) of this thesis, only a few studies in the literature have utilised non-industrial or public facilities as case studies. The developed model was applied to two case studies, as addressed in chapters 5 and 6. Chapter 5 focussed on reorganising HWRC operations. We allocated demand at these facilities using a spatial interaction model. This contributed to the flexibility of the proposed model, which could be further refined and adapted to any related formulation. We solved the HWRC problem using real data, and as a result it was found that only three recycling centres are needed to cover the Sheffield residential area. As far as we have been able to determine, there are no studies in the literature which have made the attempt to reorganise recycling waste centres or indeed any other waste management facility location problem.

The second case study used real data from primary healthcare centres. Instead of closing or reducing facility operation periods, chapter 6 focussed on providing the network with 'extra' capacity. As we know, healthcare facilities are an important aspect in any country. However, they also suffer from overcrowding problems, either in the GP surgery or in A&E. Both facilities; the GP and A&E are interrelated and interconnected. Due to high inaccessibility of GP facilities, the demand might go to the walk-in centre or the A&E. This situation will cause the unnecessary attendees in both facilities (i.e., the walk-in centre and the A&E) to increase; and causing congestion problem. We attempt to solve the congestion in GP surgeries and the A&E through providing the 'extra' capacity for the GP network; without additional cost to the provider. This 'extra' capacity is introduced in the form of a new mathematical model to create a backup facility for each GP. Hence, demand will circulate within this network, instead going straight to the walk-in centre or to the A&E. In chapter 6, our contributions were twofold; first, creating a backup facility system, and second, reducing demand waiting times.

Through these applications, our model showed flexibility and capability in handling any public-related facility location problem. Similarly, the proposed model can be further modified to suit any practice in reorganising their existing facilities due to financial reduction problem. For instance, two large supermarkets in the UK; Sainsbury and Asda were planned to merge – this could lead to stores closure (Simpson, 2018). The proposed model, with some modifications, could be implemented to overview and identify the store(s) that is(are) suitable to be closed or retained. For instance, the average sales per period could be used to estimate the amount of demand, and the queue length per counter could be used to estimate waiting times of demand in the facility. In addition, using the proposed model, the organisation could improve the service level of the remaining stores, for instance, redesigning operating times in a more efficient way. Next section highlights the limitation of the studies. We also induce the possible actions that can be taken in the future.

6.3 Limitations and Future Research

Our study has several limitations; however, these could be used as the bases for future development of related research. These limitations are:

1. The deterministic nature of the proposed model. Even we have got reliable demand forecast for the case studies; however, in the future, stochastic variants of the model could be developed in order to take into account demand uncertainty scenarios.
2. In addition, we used the discrete demand type to represent the demand level. The reason we use discrete demand is that we are focussing on developing the model for the reorganisation problem.
3. Each server was assumed to be independent and deterministic in terms of service time and service rate. This due to set an initial mark on the research area where lack of existing studies that focused on the reorganisation of facility operations can be found. We also focused only on developing and testing the proposed model, where further improvement of the model, including consideration of facility service time could be implemented in the future research.

4. Our model utilises the set of demand that move to another facility through operating periods (as for backup facility) and distance, whereas there are other, additional reasons that a user might to move to another facility, for instance its capacity.
5. The data we used to represent the parameter costs was based on real data, either was given by the Council (for HWRC case study) or gained through healthcare metadata provided by the NHS Official website (for GP case study). However, some data is unavailable (not collected by the organisation). Thus, for any unavailable data, we estimate the value through information provided by reports delivered by other research institutes, such as from Centre for Urgent and Emergency Care Research (CURE), and Ipsos MORI).

In the future, we hope to address all the limitations above by:

1. Refinement of the backup model by considering capacity. In addition to capacity, demand preference in moving to another facility j could also be considered.
2. Stochastic data on demand arrival and server processing levels (service times) can be applied in order to improve results, especially for demand served by the facility. Also, even the introduced model is an adaptation of deterministic CLSP, the queueing network problem could be implemented in the future.
3. Enhance the proposed model to the multi-level facility network. The interrelated facilities might have different level of services provided. For instance, the recycling portfolio of each HWRC is different – indicating existence of multi-level service.
4. It is also interesting to see that the proposed model can be applied at any public facility network, such as the schooling system, and also in the private sector, such as the merging of private bus operators.
5. Since we are not dealing with a large dataset or network size, it would be interesting to consider the behaviour of the model when used to solve problems with large datasets; it would also be interesting to consider the use of a heuristic-based algorithm to solve the model.

REFERENCES

Abbott, A., Nandeibam, S., & O'Shea, L. (2011). Explaining the variation in household recycling rates across the UK. *Ecological Economics*, *70*(11), 2214–2223. https://doi.org/10.1016/j.ecolecon.2011.06.028

Aboolian, R, Berman, O., & Verter, V. (2016). Maximal accessibility network design in the public sector. *Transportation Science*, *50*(1), 336–347. https://doi.org/10.1287/trsc.2015.0595

Aboolian, Robert, Berman, O., & Drezner, Z. (2008). Location and allocation of service units on a congested network. *IIE Transactions*, *40*(4), 422–433. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-39049167784&partnerID=tZOtx3y1

Aboolian, Robert, Sun, Y., & Leu, J. (2012). Deploying a Zone-Based Massively Multiplayer Online Game on a Congested Network. *International Journal of Information Systems and Supply Chain Management*, *5*(1), 38–57. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-84873812066&partnerID=tZOtx3y1

Abouee-Mehrizi, H., Babri, S., Berman, O., & Shavandi, H. (2011). Optimizing capacity, pricing and location decisions on a congested network with balking. *Mathematical Methods of Operations Research*, *74*(2), 233–255. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-80054920201&partnerID=tZOtx3y1

Absi, N., Detienne, B., & Dauzère-Pérès, S. (2013). Heuristics for the multi-item capacitated lot-sizing problem with lost sales. *Computers and Operations Research*, *40*(1), 264–272. https://doi.org/10.1016/j.cor.2012.06.010

Absi, N., & Kedad-Sidhoum, S. (2009). The multi-item capacitated lot-sizing problem with safety stocks and demand shortage costs. *Computers and Operations Research*, *36*(11), 2926–2936. https://doi.org/10.1016/j.cor.2009.01.007

Ackoff, R. (1979). The Future of Operational Research is Past. *Journal of the Operational Research Society*, *30*(2), 93–104.

Agarwal, S., Banerjee, J., Baker, R., Conroy, S., Hsu, R., Rashid, A., … Habiba, M. (2012). Potentially avoidable emergency department attendance: interview study of patients; reasons for attendance. *Emergency Medicine Journal*, *29*(12). https://doi.org/10.1136/emermed-2011-200585

Agra, A., & Constantino, M. (1999). Lotsizing with backlogging and start-ups: The case of Wagner-Whitin costs. *Operations Research Letters*, *25*(2), 81–88. https://doi.org/10.1016/S0167-6377(99)00030-9

Alexander, C., Curran, A., Smaje, C., & Williams, I. (2009). Evaluation of bulky waste and reuse schemes in England. In *Proceedings of the Institution of Civil Engineers-Waste and Resource Management* (Vol. 162, pp. 141–150). Thomas Telford Ltd.

Alfares, H. K., & Turnadi, R. (2018). Lot sizing and supplier selection with multiple items, multiple periods, quantity discounts, and backordering. *Computers and Industrial Engineering*, *116*(August 2017), 59–71. https://doi.org/10.1016/j.cie.2017.12.019

Alotaibi, Y., & Liu, F. (2013). Average waiting time of customers in a new queue system with different classes. *Business Process Management Journal*, *19*(1), 146–168.

https://doi.org/10.1108/14637151311294903

An, S., Cui, N., Bai, Y., Xie, W., Chen, M., & Ouyang, Y. (2015). Reliable emergency service facility location under facility disruption, en-route congestion and in-facility queuing. *Transportation Research Part E: Logistics and Transportation Review*, *82*, 199–216. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-84941266151&partnerID=tZOtx3y1

Anandaciva, S., & Thompson, J. (2017). What is happening to waiting times in the NHS? Retrieved from https://www.kingsfund.org.uk/publications/articles/nhs-waiting-times

Andriolo, A., Battini, D., Grubbström, R. W., Persona, A., & Sgarbossa, F. (2014). A century of evolution from Harris's basic lot size model: Survey and research agenda. *International Journal of Production Economics*, *155*, 16–38. https://doi.org/10.1016/j.ijpe.2014.01.013

Antunes, A., & Peeters, D. (2000). The dynamic expansion and relocation of capacitated public facilities: A multi-objective approach. *Computers and Operations Research*, *34*, 101–120. Retrieved from http://linkinghub.elsevier.com/retrieve/pii/0305054888900378

Arabani, A. B., & Farahani, R. Z. (2012). Facility location dynamics: An overview of classifications and applications. *Computers & Industrial Engineering*, *62*(1), 408–420.

Araya, F., Dell, R., Donoso, P., Marianov, V., Martínez, F., & Weintraub, A. (2012). Optimizing location and size of rural schools in Chile. *International Transactions in Operational Research*, *19*(5), 695–710. Retrieved from http://doi.wiley.com/10.1111/j.1475-3995.2012.00843.x

Armentano, V. A., França, P. M., & De Toledo, F. M. B. (1999). A network flow model for the capacitated lot-sizing problem. *Omega*, *27*(2), 275–284. https://doi.org/10.1016/S0305-0483(98)00045-0

Atashi Khoei, A., Süral, H., & Tural, M. K. (2017). Time-dependent green Weber problem. *Computers & Operations Research*, *88*, 316–323. https://doi.org/10.1016/j.cor.2017.04.010

Attaran, M. (2004). Exploring the relationship between information technology and business process reengineering. *Information & Management*, *41*(5), 585–596.

Ayhan, M. B., & Kilic, H. S. (2015). A two stage approach for supplier selection problem in multi-item/multi-supplier environment with quantity discounts. *Computers and Industrial Engineering*, *85*, 1–12. https://doi.org/10.1016/j.cie.2015.02.026

Bai, Y., Hwang, T., Kang, S., & Ouyang, Y. (2011). Biofuel refinery location and supply chain planning under traffic congestion. *Transportation Research Part B: Methodological*, *45*(1), 162–175. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-78149500409&partnerID=tZOtx3y1

Baird, B, Charles, A., Honeyman, M., Maguire, D., & Das, P. (2016). Understanding pressures in general practice. *The King's Fund*, (May), 97. Retrieved from https://www.kingsfund.org.uk/sites/default/files/field/field_publication_file/Understanding-GP-pressures-Kings-Fund-May-2016.pdf%0Ahttps://www.kingsfund.org.uk/publications/pressures-in-general-practice%0Ahttps://www.kingsfund.org.uk/sites/files/kf/field/

Baird, Beccy, Charles, A., Honeyman, M., Maguire, D., & Das, P. (2016). *Understanding pressures in general practice*.

Baotai, Q. (2015). *Waste Management in Current Sheffield Recycling Sites : A Study on Utilisation Rate and Accessibility. Unpublished Thesis.*

Baqir, W., Learoyd, T., Sim, A., & Todd, A. (2011). Cost analysis of a community pharmacy “minor ailment scheme” across three primary care trusts in the North East of England. *Journal of Public Health*, *33*(4), 551–555. https://doi.org/10.1093/pubmed/fdr012

Barbati, M., Bruno, G., & Genovese, A. (2012). Applications of agent-based models for optimization problems: A literature review. *Expert Systems with Applications*, *39*(5), 6020–6028.

Baron, O., Berman, O., & Krass, D. (2008). Facility Location with Stochastic Demand and Constraints on Waiting Time. *Manufacturing & Service Operations Management*, *10*(3), 484–505. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-61349156124&partnerID=tZOtx3y1

Barr, S. (2004). What we buy, what we throw away and how we use our voice. Sustainable household waste management in the UK. *Sustainable Development*, *12*(1), 32–44. https://doi.org/10.1002/sd.226

Barr, S. (2007). Factors Influencing Environmental Attitudes and Behaviors: A U.K. Case Study of Household Waste Management. *Environment and Behavior*, *39*(4), 435–473. https://doi.org/10.1177/0013916505283421

Barr, S., Ford, N. J., & Gilg, A. W. (2003). Attitudes towards recycling household waste in Exeter, Devon: quantitative and qualitative approaches. *Local Environment*, *8*(4), 407–421.

Basnet, C., & Leung, J. M. Y. (2005). Inventory lot-sizing with supplier selection. *Computers and Operations Research*, *32*(1), 1–14. https://doi.org/10.1016/S0305-0548(03)00199-0

Batta, R. (1989). A queueing-location model with expected service time dependent queueing disciplines. *European Journal of Operational Research*, *39*(2), 192–205. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-0024621032&partnerID=tZOtx3y1

Bayley, T., Süral, H., & Bookbinder, J. H. (2018). A hybrid Benders approach for coordinated capacitated lot-sizing of multiple product families with set-up times. *International Journal of Production Research*, *56*(3), 1326–1344. https://doi.org/10.1080/00207543.2017.1338778

BBC News. (2016). Senior GP Helen Stokes-Lampard voices fears for services. Retrieved from https://www.bbc.co.uk/news/health-38453534

Beraldi, P., & Bruni, M. E. (2009). A probabilistic model applied to emergency service vehicle location. *European Journal of Operational Research*, *196*(1), 323–331. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-56749160634&partnerID=tZOtx3y1

Berman, O. (1985). Locating a facility on a congested network with random lengths. *Networks*, *15*(3), 275–293. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-0022129102&partnerID=tZOtx3y1

Berman, O. (1995). The maximizing market size discretionary facility location problem with congestion. *Socio-Economic Planning Sciences*, *29*(1), 39–46. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-0029532549&partnerID=tZOtx3y1

Berman, O., & Drezner, Z. (2006). Location of congested capacitated facilities with distance-sensitive demand1. *IIE Transactions*, *38*(3), 213–221. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-32144445293&partnerID=tZOtx3y1

Berman, O., & Drezner, Z. (2008). The p-median problem under uncertainty. *European Journal* *of Operational Research*, *189*(1), 19–30.

Berman, O., Huang, R., Kim, S., & Menezes, M. B. C. (2007). Locating capacitated facilities to maximize captured demand. *IIE Transactions*, *39*(11), 1015–1029. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-34548666771&partnerID=tZOtx3y1

Berman, O., Krass, D., & Wang, J. (2006). Locating service facilities to reduce lost demand. *IIE Transactions*, *38*(11), 933–946. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-33748581976&partnerID=tZOtx3y1

Berman, O., & Larson, R. C. (1982). The median problem with congestion. *Computers & Operations Research*, *9*(2), 119–126. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-0019898247&partnerID=tZOtx3y1

Bhat, V. N. (1996). A model for the optimal allocation of trucks for solid waste management. *Waste Management & Research*, *14*(1), 87–96.

Bhatnagar, R., & Sohal, A. S. (2005). Supply chain competitiveness: measuring the impact of location factors, uncertainty and manufacturing practices. *Technovation*, *25*(5), 443–456.

Bhaumik, P. K. (2010). Optimal shrinking of the distribution chain: the facilities delocation decision. *International Journal of Systems Science*, *41*(3), 271–280. Retrieved from http://www.tandfonline.com/doi/abs/10.1080/00207720903326860

Bitran, G. R., & Yanasse, H. H. . (1982). Computational Complexity of the Capacitated Lot Size Problem. *Management Science*, *28*(10), 1174–1186. Retrieved from http://www.jstor.org/stable/2630946

BMA. (2016). Safe working in general practice. *British Medical Association*, 1–14. Retrieved from https://www.bma.org.uk/collective-voice/committees/general-practitioners-committee/gpc-current-issues/safe-working-in-general-practice

Brahimi, N., Absi, N., Dauzère-Pérès, S., & Nordli, A. (2017). Single-item dynamic lot-sizing problems: An updated survey. *European Journal of Operational Research*, *263*(3), 838–863. https://doi.org/10.1016/j.ejor.2017.05.008

Braid, R. M. (1991). The locations of congestible facilities in adjacent jurisdictions. *Regional Science and Urban Economics*, *21*(4), 617–626. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-0042337932&partnerID=tZOtx3y1

Brandeau, M. L., & Chiu, S. S. (1992). A center location problem with congestion. *Annals of Operations Research*, *40*(1), 17–32. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-21144480294&partnerID=tZOtx3y1

Brandimarte, P. (2006). Multi-item capacitated lot-sizing with demand uncertainty. *International Journal of Production Research*, *44*(15), 2997–3022. https://doi.org/10.1080/00207540500435116

Bruno, G., Diglio, A., Genovese, A., & Piccolo, C. (2018). A decision support system to improve performances of airport check-in services. *Soft Computing*, 1–10. https://doi.org/10.1007/s00500-018-3301-z

Bruno, G., & Genovese, A. (2010). A mathematical model for the optimization of the airport check-in service problem. *Electronic Notes in Discrete Mathematics*, *36*(C), 703–710.

https://doi.org/10.1016/j.endm.2010.05.089

Bruno, G., & Genovese, A. (2012). A Spatial Interaction Model for the Representation of the Mobility of University Students on the Italian Territory. *Networks and Spatial Economics*, *12*(1), 41–57. https://doi.org/10.1007/s11067-010-9142-7

Bruno, G., Genovese, A., & Improta, G. (2014). A historical perspective on location problems. *BSHM Bulletin: Journal of the British Society for the History of Mathematics*, *29*(2), 83–97. Retrieved from http://dx.doi.org/10.1080/17498430.2014.889266

Bruno, G., Genovese, A., & Piccolo, C. (2014). The capacitated Lot Sizing model: A powerful tool for logistics decision making. *International Journal of Production Economics*, *155*, 380–390. https://doi.org/10.1016/j.ijpe.2014.03.008

Bruno, G., Genovese, A., & Piccolo, C. (2016). Capacity management in public service facility networks: a model, computational tests and a case study. *Optimization Letters*, 975–995. Retrieved from http://link.springer.com/10.1007/s11590-015-0923-2

Bruno, G., Genovese, A., Piccolo, C., & Sterle, C. (2014). A Location Model for the Reorganization of a School System: The Italian Case Study. *Procedia - Social and Behavioral Sciences*, *108*, 96–105. Retrieved from http://linkinghub.elsevier.com/retrieve/pii/S1877042813054633

Bruno, G., Genovese, A., & Sgalambro, A. (2012). An extension of the schedule optimization problem at a public transit terminal to the multiple destinations case. *Public Transport*, *3*(3), 189–198. https://doi.org/10.1007/s12469-011-0039-8

Bruno, G., Improta, G., & Sgalambro, A. (2009). Models for the schedule optimization problem at a public transit terminal. *OR Spectrum*, *31*(3), 465–481. https://doi.org/10.1007/s00291-008-0161-4

Burke, E. K., Gendreau, M., Hyde, M., Ochoa, G., Kendall, G., Özcan, E., & Qu, R. (2013). Hyper-heuristics : A Survey of the State of the Art. *Journal of the Operational Research Society*, 1695–1724.

Buschkühl, L., Sahling, F., Helber, S., & Tempelmeier, H. (2010). *Dynamic capacitated lot-sizing problems: A classification and review of solution approaches*. *OR Spectrum* (Vol. 32). https://doi.org/10.1007/s00291-008-0150-7

Butt, S. E., & Cavalier, T. M. (1997). Facility location in the presence of congested regions with the rectilinear distance metric. *Socio-Economic Planning Sciences*, *31*(2), 103–113. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-0030851801&partnerID=tZOtx3y1

Buzacott, J. A. (1996). Commonalities in reengineered business processes: models and issues. *Management Science*, *42*(5), 768–782.

Canel, C., & Khumawala, B. M. (1997). Multi-period international facilities location: An algorithm and application. *International Journal of Production Research*, *35*(7), 1891–1910. Retrieved from http://www.tandfonline.com/doi/abs/10.1080/002075497194976

Castillo, I., Ingolfsson, A., & Sim, T. (2009). Social Optimal Location of Facilities with Fixed Servers, Stochastic Demand, and Congestion. *Production and Operations Management*, *18*(6), 721–736. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-73349133687&partnerID=tZOtx3y1

Cecil, E., Bottle, A., Cowling, T. E., Majeed, A., Wolfe, I., & Saxena, S. (2016). Primary Care Access, Emergency Department Visits, and Unplanned Short Hospitalizations in the UK. *Pediatrics*, *137*(2). https://doi.org/10.1542/peds.2015-1492

Chambari, A., Rahmaty, S. H., Hajipour, V., & Karimi, A. (2011). A bi-objective model for location-allocation problem within queuing framework. *World Academy of Science, Engineering and Technology*, *78*, 138–145. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-84855243949&partnerID=tZOtx3y1

Chardaire, P., Sutter, A., & Costa, M.-C. (1996). Solving the dynamic facility location problem. *Networks*, *28*(2), 117–124. Retrieved from http://doi.wiley.com/10.1002/(SICI)1097-0037(199609)28:2%3C117::AID-NET5%3E3.0.CO;2-H

Chen, W.H; Thizy, J. . (1990). ANALYSIS OF RELAXATIONS FOR THE MULTI-ITEM CAPACITATED LOT-SIZING PROBLEM. *Annals of Operations Research*, *26*, 29–72.

Cheng, M. Y., Tsai, H. C., & Lai, Y. Y. (2009). Construction management process reengineering performance measurements. *Automation in Construction*, *18*(2), 183–193. https://doi.org/10.1016/j.autcon.2008.07.005

Chiu, S. S. (1986). Optimal M/G/1 server location on a tree network with continuous link demands. *Computers & Operations Research*, *13*(6), 653–669. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-0022883214&partnerID=tZOtx3y1

Cho, S.-H., Jang, H., Lee, T., & Turner, J. (2014). Simultaneous Location of Trauma Centers and Helicopters for Emergency Medical Service Planning. *Operations Research*, *62*(4), 751–771. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-84940262885&partnerID=tZOtx3y1

Cho, Y. H., Kim, J. K., & Kim, S. H. (1998). Role-based approach to business process simulation modeling and analysis. *Computers and Industrial Engineering*, *35*(1–2), 343–346. https://doi.org/10.1016/S0360-8352(98)00090-4

Chotayakul, S., Charnsetthikul, P., Pichitlamken, J., & Kobza, J. (2013). An Optimization-Based Heuristic for a Capacitated Lot-Sizing Model in an Automated Teller Machines Network. *Journal of Mathematics and Statistics*, *9*(4), 283–288. https://doi.org/10.3844/jmssp.2013.283.288

Choudhary, D., & Shankar, R. (2014). A goal programming model for joint decision making of inventory lot-size, supplier selection and carrier selection. *Computers and Industrial Engineering*, *71*(1), 1–9. https://doi.org/10.1016/j.cie.2014.02.003

Chu, P.-Y., & Chiu, J.-F. (2003). Factors Influencing Household Waste Recycling Behavior: Test of an integrated Model. *Journal of Applied Social Psychology*, *33*(3), 604–626. https://doi.org/10.1111/j.1559-1816.2003.tb01915.x

Chung, C.-S., & Lin, C.-H. M. (1988). An O(T2) Algorithm for the NI/G/NI/ND Capacitated Lot Size Problem. *Management Science*, *34*(3), 420–426. https://doi.org/10.1287/mnsc.34.3.420

Church, R., & ReVelle, C. (1974). The maximal covering location problem. *Papers of the Regional Science Association*, *32*(1), 101–118. https://doi.org/10.1007/BF01942293

Clark, A., Almada-Lobo, B., & Almeder, C. (2011). Lot sizing and scheduling: industrial extensions and research opportunities. *International Journal of Production Research*, *49*(9), 2457–2461. https://doi.org/10.1080/00207543.2010.532908

Cole, C. (2014). *Developing sustainable household waste management-a Local Authority approach to zero waste*. © Christine Cole.

Cole, C., Osmani, M., Quddus, M. A., Wheatley, A. D., & Kay, K. (2011). Household waste management in the UK: current practices and challenges. In *Proceedings of the First International Conference on Wastes: Solutions, Treatments and Opportunities* (pp. 56–61). Guimaraes, Portugal: WASTES: Solutions, Treatments and Opportunities.

CPLEX, I. I. (2017). 12.6. 1, User Manual.

Cunningham, L. S., & Conroy, A. (2006). New household waste recycling centre in Blackpool, UK. In *Proceedings of the Institution of Civil Engineers-Waste and Resource Management* (Vol. 159, pp. 7–11). Thomas Telford Ltd.

Curran, A., Williams, I. D., & Heaven, S. (2007). Management of household bulky waste in England. *Resources, Conservation and Recycling*, *51*(1), 78–92. https://doi.org/10.1016/j.resconrec.2006.08.003

Dai, Y. C., Lin, Z. Y., Gordon, M. P. R., Robinson, N. K. L., Woodard, R., Ye, J. Y., … Harder, M. K. (2015). Why doorstepping can increase household waste recycling. *Resources, Conservation and Recycling*, *102*, 9–19. https://doi.org/10.1016/j.resconrec.2015.06.004

Daskin, M. S. (2008). What you should know about location modeling. *Naval Research Logistics*, *55*(4), 283–294. Retrieved from http://doi.wiley.com/10.1002/nav.20284

Date, K., Makked, S., & Nagi, R. (2014). Dominance rules for the optimal placement of a finite-size facility in an existing layout. *Computers & Operations Research*, *51*, 182–189. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-84902688973&partnerID=tZOtx3y1

Davari, S., Kilic, K., & Naderi, S. (2016). A heuristic approach to solve the preventive health care problem with budget and congestion constraints. *Applied Mathematics and Computation*, *276*, 442–453. https://doi.org/10.1016/j.amc.2015.11.073

DEFRA. (2011). *Guidance on applying the Waste Hierarchy*. *Department for Environment, Food & Rural Affairs*.

DEFRA. (2017). *Fly-tipping statistics for England, 2016/17*. *Department for Environment, Food & Rural Affairs*. Retrieved from https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/652958/Flytipping_201617_statistical_release_FINAL.pdf

DEFRA. (2018a). *ENV18 - Local authority collected waste: annual results tables*. *Department for Environment, Food & Rural Affairs*. Retrieved from https://www.gov.uk/government/statistical-data-sets/env18-local-authority-collected-waste-annual-results-tables

DEFRA. (2018b). *Fly-tipping statistics for England, 2017/18*. *Department for Environment, Food & Rural Affairs*. Retrieved from https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/756306/FlyTipping_201718_Statistical_Release_rev.pdf

DEFRA. (2018c). *Our Waste, Our Resources : A Strategy for England* (Vol. 146). Retrieved from https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/765914/resources-waste-strategy-dec-2018.pdf

DEFRA. (2018d). *Statistics on waste managed by local authorities in England in 2017/18*. *Department for Environment, Food & Rural Affairs*. Retrieved from https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/763191/LACW_mgt_annual_Stats_Notice_Dec_2018.pdf

DEFRA. (2018e). *UK Statistics on Waste*. *Department for Environment, Food & Rural Affairs*. United Kingdom.

Deleplanque, S., Quilliot, A., & Kedad-Sidhoum, S. (2012). *Decomposing a multi-plant, multi-item Lot-Sizing problem with transfer costs/capacities into facility location and flow sub-problems. IFAC Proceedings Volumes (IFAC-PapersOnline)* (Vol. 14). IFAC. https://doi.org/10.3182/20120523-3-RO-2023.00062

Delgoshaei, A., Ali, A., Ariffin, M. K. A., & Gomes, C. (2016). A multi-period scheduling of dynamic cellular manufacturing systems in the presence of cost uncertainty. *Computers and Industrial Engineering*, *100*, 110–132. https://doi.org/10.1016/j.cie.2016.08.010

Dell, R. F. (1998). Optimizing army base realignment and closure. *Interfaces*, *28*(6), 1–18.

Denning, P. J., & Buzen, J. P. (1978). The Operational Analysis of Queueing Network Models. *ACM Computing Surveys*, *10*(3), 225–261. https://doi.org/10.1145/356733.356735

Desrochers, M., Marcotte, P., & Stan, M. (1995). The congested facility location problem. *Location Science*, *3*(1), 9–23. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-0029521129&partnerID=tZOtx3y1

Dobson, G., & Stavrulaki, E. (2007). Simultaneous price, location, and capacity decisions on a line of time-sensitive customers. *Naval Research Logistics*, *54*(1), 1–10. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-33846660010&partnerID=tZOtx3y1

Donnelly, L. (2017). One million patients a week cannot get a GP appointment, statistics show, (July 2017).

Doomun, R., & Jungum, N. V. (2008). Business process modelling, simulation and reengineering: Call centres. *Business Process Management Journal*, *14*(6), 838–848. https://doi.org/10.1108/14637150810916017

Drexl, Andreas; Kimms, A. (1996). Lot sizing and scheduling: Survey and extensions Manuskripte. *Manuskripte Aus Den Instituten Für Betriebswirtschaftslehre Der Universität Kiel, 421*.

Drexl, A., & Haase, K. (1995). Proportional lotsizing and scheduling. *International Journal of Production Economics*, *40*(1), 73–87. https://doi.org/https://doi.org/10.1016/0925-5273(95)00040-U

Drezner, T., & Drezner, Z. (2001). A Note on Applying the Gravity Rule to the Airline Hub Problem. *Journal of Regional Science*, *41*(1), 67–72. https://doi.org/10.1111/0022-4146.00207

Eiselt, H. A., Laporte, G., & Thisse, J.-F. (1993). Competitive Location Models: A Framework and Bibliography. *Transportation Science*, *27*(1), 44–54. https://doi.org/10.1287/trsc.27.1.44

Eiselt, H. A., & Marianov, V. (2009). A conditional p-hub location problem with attraction functions. *Computers and Operations Research*, *36*(12), 3128–3135. https://doi.org/10.1016/j.cor.2008.11.014

Elhedhli, S. (2006). Service System Design with Immobile Servers, Stochastic Demand, and Congestion. *Manufacturing & Service Operations Management*, *8*(1), 92–97. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-33645145492&partnerID=tZOtx3y1

Elloumi, S., Labbé, M., & Pochet, Y. (2004). A New Formulation and Resolution Method for the p-Center Problem. *INFORMS Journal on Computing*, *16*(1), 84–94. https://doi.org/10.1287/ijoc.1030.0028

Erlang, A. K. (1909). The theory of probabilities and telephone conversations. *Nyt Tidsskrift for Matematik B*, *20*(33–39), 16.

Esbenshade, J., Vidal, M., Fascilla, G., & Ono, M. (2016). Customer-driven management models for choiceless clientele? Business process reengineering in a California welfare agency. *Work, Employment & Society*, *30*(1), 77–96. https://doi.org/10.1177/0950017015604109

Europa. (2008). Directive 2008/98/EC on waste (Waste Framework Directive). Retrieved from http://ec.europa.eu/environment/waste/framework/

Eurostat. (2017). Municipal waste statistics. Retrieved from https://ec.europa.eu/eurostat/statistics-explained/index.php/Municipal_waste_statistics#Municipal_waste_generation

Farahani, R. Z., Hassani, A., Mousavi, S. M., & Baygi, M. B. (2014). A hybrid artificial bee colony for disruption in a hierarchical maximal covering location problem. *Computers & Industrial Engineering*, *75*(1), 129–141. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-84904562305&partnerID=tZOtx3y1

Farahani, R. Z., Hekmatfar, M., Fahimnia, B., & Kazemzadeh, N. (2014). Hierarchical facility location problem: Models, classifications, techniques, and applications. *Computers & Industrial Engineering*, *68*(1), 104–117. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-84893752576&partnerID=tZOtx3y1

Fazlollahtabar, H., Mahdavi, I., Ashoori, M. T., Kaviani, S., & Mahdavi-Amiri, N. (2011). A multi-objective decision-making process of supplier selection and order allocation for multi-period scheduling in an electronic market. *International Journal of Advanced Manufacturing Technology*, *52*(9–12), 1039–1052. https://doi.org/10.1007/s00170-010-2800-6

Ferreira, D., Clark, A. R., Almada-Lobo, B., & Morabito, R. (2012). Single-stage formulations for synchronised two-stage lot sizing and scheduling in soft drink production. *International Journal of Production Economics*, *136*(2), 255–265. https://doi.org/10.1016/j.ijpe.2011.11.028

Fleischmann, M., Bloemhof-Ruwaard, J. M., Dekker, R., Van Der Laan, E., Van Nunen, J. A. E. E., & Van Wassenhove, L. N. (1997). EUROPEAN JOURNAL OF OPERATIONAL RESEARCH Quantitative models for reverse logistics: A review. *European Journal of Operational Rcsearch*, *103*(97), 1–17.

Florian, M., Lenstra, J. K., & Rinnooy Kan, A. H. G. (1980). Deterministic Production Planning: Algorithms and Complexity. *Management Science*, *26*(7), 669–679. https://doi.org/10.1098/ROYAL/

Fotheringham, A. S. (1983). Some theoretical aspects of destination choice and their relevance to production-constrained gravity models. *Environment and Planning A*, *15*(8), 1121–1132. https://doi.org/10.1068/a151121

Fotheringham, A. S., & O'Kelly, M. E. (1989). *Spatial interaction models: formulations and applications* (Vol. 1). Kluwer Academic Publishers Dordrecht.

Galvão, R. D., Chiyoshi, F. Y., & Morabito, R. (2005). Towards unified formulations and extensions of two classical probabilistic location models. *Computers & Operations Research*, *32*(1), 15–33. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-4043082606&partnerID=tZOtx3y1

Gamba, R. J., & Oskamp, S. (1994). Factors Influencing Community Residents' Participation in Commingled Curbside Recycling Programs. *Environment and Behavior*, *26*(5), 587–612. https://doi.org/10.1177/0013916594265001

Gellynck, X., Jacobsen, R., & Verhelst, P. (2011). Identifying the key factors in increasing recycling and reducing residual household waste: A case study of the Flemish region of Belgium. *Journal of Environmental Management*, *92*(10), 2683–2690. https://doi.org/10.1016/j.jenvman.2011.06.006

Gerrits, H. (1994). Business modelling based on logistics to support business process re-engineering.

Glock, C. H., Grosse, E. H., & Ries, J. M. (2014). The lot sizing problem: A tertiary study. *International Journal of Production Economics*, *155*, 39–51. https://doi.org/10.1016/j.ijpe.2013.12.009

Goodman, J. ., & Massey, W. . (1984). The Non-Ergodic Jackson Network, *21*(4), 860–869.

Guan, Y., & Liu, T. (2010). Stochastic lot-sizing problem with inventory-bounds and constant order-capacities. *European Journal of Operational Research*, *207*(3), 1398–1409. https://doi.org/10.1016/j.ejor.2010.07.003

Güden, H., & Süral, H. (2014). Locating mobile facilities in railway construction management. *Omega*, *45*, 71–79. https://doi.org/10.1016/j.omega.2014.01.001

Guimarães, L., Klabjan, D., & Almada-Lobo, B. (2014). Modeling lotsizing and scheduling problems with sequence dependent setups. *European Journal of Operational Research*, *239*(3), 644–662. https://doi.org/10.1016/j.ejor.2014.05.018

Gupta, D., & Magnusson, T. (2005). The capacitated lot-sizing and scheduling problem with sequence-dependent setup costs and setup times. *Computers and Operations Research*, *32*(4), 727–747. https://doi.org/10.1016/j.cor.2003.08.014

Haase, K. (1996). Capacitated lot-sizing with sequence dependent setup costs. *OR Spectrum*, *18*(1), 51–59. https://doi.org/10.1007/BF01539882

Hajibabai, L., Bai, Y., & Ouyang, Y. (2014). Joint optimization of freight facility location and pavement infrastructure rehabilitation under network traffic equilibrium. *Transportation Research Part B: Methodological*, *63*, 38–52. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-84897836858&partnerID=tZOtx3y1

Hajibabai, L., & Ouyang, Y. (2013). Integrated Planning of Supply Chain Networks and Multimodal Transportation Infrastructure Expansion: Model Development and Application to the Biofuel Industry. *Computer-Aided Civil and Infrastructure Engineering*, *28*(4), 247–259. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-84874741231&partnerID=tZOtx3y1

Hajipour, V., Farahani, R. Z., & Fattahi, P. (2016). Bi-objective vibration damping optimization for congested location-pricing problem, *70*, 87–100. https://doi.org/10.1016/j.cor.2016.01.001

Hakimi, S. L. (1964). Optimum Locations of Switching Centers and the Absolute Centers and Medians of a Graph. *Oper. Res.*, *12*(3), 450–459. https://doi.org/10.1287/opre.12.3.450

Hakimi, S. L. (1965). Optimum Distribution of Switching Centers in a Communication Network and Some Related Graph Theoretic Problems. *Operations Research*, *13*(3), 462–475. https://doi.org/10.1287/opre.13.3.462

Hammer, M. (1990). Reengineering work: don't automate, obliterate. *Harvard Business Review*, *68*(4), 104–112.

Hao, T., & Yifei, T. (2011). Study on queuing system optimization of bank based on BPR. *Procedia Environmental Sciences*, *10*(PART A), 640–646. https://doi.org/10.1016/j.proenv.2011.09.103

Harris, F. W. (1913). How many parts to make at once.

Hassannayebi, E., Zegordi, S. H., Yaghini, M., & Amin-Naseri, M. R. (2017). Timetable optimization models and methods for minimizing passenger waiting time at public transit terminals. *Transportation Planning and Technology*, *40*(3), 278–304. https://doi.org/10.1080/03081060.2017.1283156

Hayes, D. (2018). Sheffield health bosses rethink walk-in centre and minor injuries unit closure after public outcry. Retrieved from https://www.thestar.co.uk/news/sheffield-health-bosses-rethink-walk-in-centre-and-minor-injuries-unit-closure-after-public-outcry-1-9359885

Haynes, K. E., & Fotheringham, A. S. (1984). *Gravity and spatial interaction models* (Vol. 2). Sage Beverly Hills, CA.

Helber, S., Sahling, F., & Schimmelpfeng, K. (2013). Dynamic capacitated lot sizing with random demand and dynamic safety stocks. *OR Spectrum*, *35*(1), 75–105. https://doi.org/10.1007/s00291-012-0283-6

Hippisley-Cox, J., & Govind, J. (2008). *Qresearch Report on Trends in Consultation Rates in General Practices - UK, 1995-2008*.

HMSO. (2018). National Minimum Wage and National Living Wage rates. Retrieved September 1, 2018, from https://www.gov.uk/national-minimum-wage-rates

Hofacker, I., & Vetschera, R. (2001). Algorithmical approaches to business process design. *Computers & Operations Research*, *28*(13), 1253–1275. https://doi.org/https://doi.org/10.1016/S0305-0548(00)00038-1

Hoornweg, D., Bhada-Tata, P., & Kennedy, C. (2013). Environment: Waste production must peak this century. *Nature News*, *502*(7473), 615.

Hoornweg, D., Bhada-Tata, P., & Kennedy, C. (2015). Peak waste: When is it likely to occur? *Journal of Industrial Ecology*, *19*(1), 117–128.

Hu, D., Liu, Z.-W., & Hu, W. (2013). Congestion Service Facilities Location Problem with Promise of Response Time. *Mathematical Problems in Engineering*, *2013*, 1–11. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-84890053785&partnerID=tZOtx3y1

Hu, Z., & Hu, G. (2016). A two-stage stochastic programming model for lot-sizing and scheduling under uncertainty. *International Journal of Production Economics*, *180*, 198–207. https://doi.org/10.1016/j.ijpe.2016.07.027

Hu, Z., & Hu, G. (2018). A multi-stage stochastic programming for lot-sizing and scheduling under demand uncertainty. *Computers and Industrial Engineering*, *119*(February), 157–166. https://doi.org/10.1016/j.cie.2018.03.033

Hwang, T., Lee, M., Lee, C., & Kang, S. (2016). Meta-heuristic approach for high-demand facility locations considering traffic congestion and greenhouse gas emission. *Journal of Environmental Engineering and Landscape Management*, *24*(4), 233–244. https://doi.org/10.3846/16486897.2016.1198261

ITV Report. (2013). How many NHS walk-in centres closed in your area? Retrieved from https://www.itv.com/news/2013-11-11/the-nhs-walk-in-centres-closed-in-your-area/

J. R. Beaumont. (1980). Spatial Interaction Models and the Location-Allocation Problem. *Journal of Regional Science*, *20*(1), 37–50.

Jackson, M. C. (1993). Social Theory and Operational Research Practice. *The Journal of the Operational Research Society*, *44*(6), 563–577. Retrieved from http://www.jstor.org/stable/2584513

Jamasb, T., & Nepal, R. (2010). Issues and options in waste management: A social cost–benefit analysis of waste-to-energy in the UK. *Resources, Conservation and Recycling*, *54*(12), 1341–1352.

Jans, R., & Degraeve, Z. (2008). Modeling industrial lot sizing problems: A review. *International Journal of Production Research*, *46*(6), 1619–1643. https://doi.org/10.1080/00207540600902262

Jiang, L., & Giachetti, R. E. (2008). A queueing network model to analyze the impact of parallelization of care on patient cycle time. *Health Care Management Science*, *11*(3), 248–261. https://doi.org/10.1007/s10729-007-9040-9

Jouzdani, J., Sadjadi, S. J., & Fathian, M. (2013). Dynamic dairy facility location and supply chain planning under traffic congestion and demand uncertainty: A case study of Tehran. *Applied Mathematical Modelling*, *37*(18–19), 8467–8483. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-84883464846&partnerID=tZOtx3y1

Kang, Y., Albey, E., & Uzsoy, R. (2018). Rounding heuristics for multiple product dynamic lot-sizing in the presence of queueing behavior. *Computers and Operations Research*, *100*, 54–65. https://doi.org/10.1016/j.cor.2018.07.019

Karak, T., Bhagat, R. M., & Bhattacharyya, P. (2012). Municipal Solid Waste Generation, Composition, and Management: The World Scenario. *Critical Reviews in Environmental Science and Technology*, *42*(15), 1509–1630. https://doi.org/10.1080/10643389.2011.569871

Karimi, B., Fatemi Ghomi, S. M. T., & Wilson, J. M. (2003). The capacitated lot sizing problem: A review of models and algorithms. *Omega*, *31*(5), 365–378. https://doi.org/10.1016/S0305-0483(03)00059-8

Kendall, D. G. (1953). Stochastic processes occurring in the theory of queues and their analysis by the method of the imbedded Markov chain. *The Annals of Mathematical Statistics*, 338–354.

Khoban, Z., & Ghadimi, S. (2009). Appendix : Metaheuristic Methods. *Facility Location: Concepts, Models, Algorithms and Case Studies*, (1975), 535–544.

Kim, S. (2012). A column generation heuristic for congested facility location problem with clearing functions. *Journal of the Operational Research Society*, *64*(12), 1780–1789. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-84888867187&partnerID=tZOtx3y1

Klose, A., & Drexl, A. (2005). Facility location models for distribution system design. *European Journal of Operational Research*, *162*(1), 4–29.

Köksalan, M., & Soylu, B. (2010). Bicriteria p -Hub Location Problems and Evolutionary Algorithms. *INFORMS Journal on Computing*, *22*(4), 528–542. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-78649340826&partnerID=tZOtx3y1

Konur, D., & Geunes, J. (2011). Analysis of traffic congestion costs in a competitive supply chain. *Transportation Research Part E: Logistics and Transportation Review*, *47*(1), 1–17. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-77957020795&partnerID=tZOtx3y1

Konur, D., & Geunes, J. (2012). Competitive multi-facility location games with non-identical firms and convex traffic congestion costs. *Transportation Research Part E: Logistics and Transportation Review*, *48*(1), 373–385. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-80054091472&partnerID=tZOtx3y1

Kovács, A., Brown, K. N., & Tarim, S. A. (2009). An efficient MIP model for the capacitated lot-sizing and scheduling problem with sequence-dependent setups. *International Journal of Production Economics*, *118*(1), 282–291. https://doi.org/10.1016/j.ijpe.2008.08.033

Laporte, G., & Nickel, S. (2015). *Location Science*. (G. Laporte & S. Nickel, Eds.). Switzerland: Springer International Publishing.

Laporte, G., Nickel, S., & da Gama, F. S. (2015). Introduction to Location Science. In G. Laporte, S. Nickel, & F. da Gama (Eds.), *Location Science* (pp. 1–18). Cham: Springer International Publishing. Retrieved from http://dx.doi.org/10.1007/978-3-319-13111-5_1

Laurent, A., Bakas, I., Clavreul, J., Bernstad, A., Niero, M., Gentil, E., … Christensen, T. H. (2014). Review of LCA studies of solid waste management systems – Part I : Lessons learned and perspectives. *Waste Management*, *34*(3), 573–588. https://doi.org/10.1016/j.wasman.2013.10.045

Lee, E. (2015). Spatial analysis for an intermodal terminal to support agricultural logistics. *Management Research Review*, *38*(3), 299–319. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-84924807647&partnerID=tZOtx3y1

Li, H., & Meissner, J. (2011). Competition under capacitated dynamic lot-sizing with capacity acquisition. *International Journal of Production Economics*, *131*(2), 535–544. https://doi.org/10.1016/j.ijpe.2011.01.022

Liu, C., & Ralston, B. A. (1989). Maximizing accessibility on congested networks. *Geographical Analysis*. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-0024786019&partnerID=tZOtx3y1

Liu, S., & Fan, Y. (2014). Workflow performance analysis and simulation based on multidimensional workflow net. *Computers in Industry*, *65*(2), 333–344. https://doi.org/10.1016/j.compind.2013.12.002

Liu, Y., Kong, F., & Santibanez Gonzalez, E. D. R. (2018). Dumping, waste management and ecological security: Evidence from England. *Journal of Cleaner Production*, *167*, 1425–1437. https://doi.org/10.1016/j.jclepro.2016.12.097

Loerch, A., Boland, N., Johnson, E., & Nemhauser, G. (1996). Finding an optimal stationing policy for the US Army in Europe after the force draw down. *Military Operations Research*, *2*(4), 39–51.

Mahdieh, M., Clark, A., & Bijari, M. (2017). A novel flexible model for lot sizing and scheduling with non-triangular, period overlapping and carryover setups in different machine configurations. *Flexible Services and Manufacturing Journal*, 1–40. https://doi.org/10.1007/s10696-017-9279-5

Majed, A., Irani, Z., & Zairi, M. (2001). Business process reengineering: a survey of international experience. *Business Process Management Journal*, *7*(5), 437–455. https://doi.org/10.1108/14637150110406812

Manson, N. J. (2006). Is operations research really research? *ORiON*, *22*(2), 155–180. Retrieved from http://www.ajol.info/index.php/orion/article/view/34262

Manzini, R., & Gebennini, E. (2008). Optimization models for the dynamic facility location and allocation problem. *International Journal of Production Research*, *46*(8), 2061–2086. https://doi.org/10.1080/00207540600847418

Marianov, V., & Serra, D. (2011). Location of Multiple-Server Common Service Centers or Facilities, for Minimizing General Congestion and Travel Cost Functions. *International Regional Science Review*, *34*(3), 323–338. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-79958697362&partnerID=tZOtx3y1

Marianov, V, Boffey, T. B., & Galvão, R. D. (2008). Optimal location of multi-server congestible facilities operating as M/Er/m/N queues. *Journal of the Operational Research Society*, *60*(5), 674–684. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-64849108775&partnerID=tZOtx3y1

Marianov, Vladimir. (2003). Location of Multiple-Server Congestible Facilities for Maximizing Expected Demand, when Services are Non-Essential. *Annals of Operations Research*, *123*(1–4), 125–141. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-0347756771&partnerID=tZOtx3y1

Marianov, Vladimir, & Serra, D. (1998). Probabilistic, maximal covering location-allocation models from congested systems. *Journal of Regional Science*, *38*(3), 401–424. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-0031785297&partnerID=tZOtx3y1

Marianov, Vladimir, & Serra, D. (2001). Hierarchical location–allocation models for congested systems. *European Journal of Operational Research*, *135*(1), 195–208. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-0035900283&partnerID=tZOtx3y1

Marianov, Vladimir, & Serra, D. (2002). Location-allocation of multiple-server service centers with constrained queues or waiting times. *Annals of Operations Research*, *111*(1–4), 35–50. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-0036424626&partnerID=tZOtx3y1

Marino, D. (2018). Funding cuts prompt recycling centre closure proposals. Retrieved from https://www.mrw.co.uk/latest/funding-cuts-prompt-recycling-centre-closure-proposals/10034664.article

Martí, R., & Reinelt, G. (2011). Heuristic Methods. In *The Linear Ordering Problem: Exact and Heuristic Methods in Combinatorial Optimization* (pp. 17–40). Berlin, Heidelberg: Springer Berlin Heidelberg. Retrieved from http://dx.doi.org/10.1007/978-3-642-16729-4_2

Mason, S., O'Keeffe, C., Jacques, R., Rimmer, M., & Ablard, S. (2017). *Perspectives on the reasons for Emergency Department attendances across Yorkshire and the Humber: Final Report*. Retrieved from www.sheffield.ac.uk/scharr/sections/hsr/cure

Melachrinoudis, E. (1994). Discrete location assignment problem with congestion. *IIE Transactions*, *26*(6), 83–89. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-

0028548403&partnerID=tZOtx3y1

Melo, M. T., Nickel, S., & Saldanha da Gama, F. (2005). Dynamic multi-commodity capacitated facility location: a mathematical modeling framework for strategic supply chain planning. *Computers & Operations Research*, *33*(1), 181–208.

Mendes, P., Carina, A., Miguel, L., & Ribau, M. (2013). Evaluating municipal solid waste management performance in regions with strong seasonal variability. *Ecological Indicators*, *30*, 170–177. https://doi.org/10.1016/j.ecolind.2013.02.017

Meyr, H. (2002). Simultaneous lotsizing and scheduling on parallel machines. *European Journal of Operational Research*, *139*(2), 277–292. https://doi.org/10.1016/S0377-2217(01)00373-3

Meyr, H., & Mann, M. (2013). A decomposition approach for the General Lotsizing and Scheduling Problem for Parallel production Lines. *European Journal of Operational Research*, *229*(3), 718–731. https://doi.org/10.1016/j.ejor.2013.03.036

Min, H. (1988). The dynamic expansion and relocation of capacitated public facilities: A multi-objective approach. *Computers & Operations Research*, *15*(3), 243–252. Retrieved from http://linkinghub.elsevier.com/retrieve/pii/0305054888900378

Miranda, P. L., Morabito, R., & Ferreira, D. (2018). Optimization model for a production, inventory, distribution and routing problem in small furniture companies. *Top*, *26*(1), 30–67. https://doi.org/10.1007/s11750-017-0448-1

Monteiro, M. S. R., & Fontes, D. (2005). Locating and Sizing Bank-Branches by Opening, Closing or Maintaining Facilities. *Operations Research Proceedings 2005, Selected Papers of the Annual International Conference of the German Operations Research Society (GOR)*, *2005*(i), 303–308. Retrieved from http://www.liaad.up.pt/pub/2005/MF05a

Moore, T. (2013). Millions "Should Not Be In A&E." Retrieved from https://news.sky.com/story/millions-should-not-be-in-ae-exclusive-10435122

Murray, A. T., Tong, D., & Kim, K. (2010). Enhancing Classic Coverage Location Models. *International Regional Science Review*, *33*(2), 115–133.

Neal, C. (2016, March). Recycling centres in Rushmoor and Hart could close due to Hampshire County Council cuts. Retrieved March 30, 2016, from http://www.gethampshire.co.uk/news/local-news/recycling-centres-rushmoor-hart-could-11094023

NHS. (2017). *GP Patient Survey - National Report. National Health Service*. Retrieved from https://www.england.nhs.uk/statistics/statistical-work-areas/gp-patient-survey/

NHS England. (2013). *Guide to the Healthcare System in England. National Health Service*. https://doi.org/10.1074/jbc.M114.570952

NHS England. (2014). *Five Year Forward View*. Retrieved from https://www.england.nhs.uk/five-year-forward-view/next-steps-on-the-nhs-five-year-forward-view/primary-care/

NHS England. (2017). A&E Attendances and Emergency Admissions. Retrieved December 30, 2018, from https://www.england.nhs.uk/statistics/statistical-work-areas/ae-waiting-times-and-activity/

NHS England. (2018a). Appointments in General Practice. Retrieved from https://digital.nhs.uk/data-and-information/publications/statistical/appointments-in-general-practice/december-2018

NHS England. (2018b). GP Workforce. Retrieved from https://digital.nhs.uk/data-and-information/publications/statistical/general-and-personal-medical-services/final-30-september-2018-experimental-statistics

NHS England. (2018c). Patients registered at a GP practice. Retrieved January 7, 2018, from https://digital.nhs.uk/data-and-information/data-tools-and-services/data-services/general-practice-data-hub/patients-registered-at-a-gp-practice

NHS England. (2018d). *Reference costs 2017/18 : highlights, analysis and introduction to the data. NHS Improvement.*

NHS England. (2018e). Unnecessary A and E attendances. Retrieved from https://digital.nhs.uk/data-and-information/data-tools-and-services/data-services/innovative-uses-of-data/demand-on-healthcare/unnecessary-a-and-e-attendances

NHS England. (2019). Missed GP appointments costing NHS millions. Retrieved February 1, 2019, from https://www.england.nhs.uk/2019/01/missed-gp-appointments-costing-nhs-millions/

NHS England, & Ipsos MORI. (2018). *GP patient survey*. Retrieved from https://www.england.nhs.uk/statistics/statistical-work-areas/gp-patient-survey/

NHS Sheffield CCG. (n.d.). Feeling unwell? Choose the right service. Retrieved September 1, 2018, from https://www.sheffieldccg.nhs.uk/Your-Health/choose-well-guide.htm

O'Keeffe, C., Mason, S., Jacques, R., & Nicholl, J. (2018). Characterising non-urgent users of the emergency department (ED): A retrospective analysis of routine ED data. *PloS One*, *13*(2).

O'Neill, P., & Sohal, A. S. (1999). Business Process Reengineering A review of recent literature. *Technovation*, *19*(9), 571–581. https://doi.org/https://doi.org/10.1016/S0166-4972(99)00059-0

Office for National Statistics. (2016). Sheffield demographic information. Retrieved October 31, 2016, from https://www.ons.gov.uk/

Owen, S. H., & Daskin, M. S. (1998). Strategic facility location: A review. *European Journal of Operational Research*, *111*, 423–447.

Pepe, F. (2008). CHAPTER THIRTEEN - ENVIRONMENTAL IMPACT OF THE DISPOSAL OF SOLID BY-PRODUCTS FROM MUNICIPAL SOLID WASTE INCINERATION PROCESSES. In B. D. E. VIVO, H. E. BELKIN, & A. LIMA (Eds.), *Environmental Geochemistry* (pp. 317–332). Amsterdam: Elsevier. https://doi.org/https://doi.org/10.1016/B978-0-444-53159-9.00013-9

Periathamby, A. (2011). Chapter 8 - Municipal Waste Management. In T. M. Letcher & D. A. B. T.-W. Vallero (Eds.) (pp. 109–125). Boston: Academic Press. https://doi.org/https://doi.org/10.1016/B978-0-12-381475-3.10008-7

PetrolPrices.com. (2018). Historical Fuel Price. Retrieved August 20, 2006, from https://www.petrolprices.com/

Plastria, F. (2001). Static competitive facility location: An overview of optimization approaches. *European Journal of Operational Research*, *129*(3), 461–470. https://doi.org/10.1016/S0377-2217(00)00169-7

Prest, V. (2016, March). Opening times to be cut next week at 20 local waste centres. *The Press*. Retrieved from http://www.yorkpress.co.uk/news/14373260.Opening_times_to_be_cut_next_week_at_20_local_waste_centres/?ref=mac

Rahmaniani, R., Rahmaniani, G., & Jabbarzadeh, A. (2014). Variable neighborhood search based evolutionary algorithm and several approximations for balanced location–allocation design problem. *The International Journal of Advanced Manufacturing Technology*, *72*(1–4), 145–159. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-84903268904&partnerID=tZOtx3y1

Rajagopalan, H. K., Saydam, C., & Xiao, J. (2008). A multiperiod set covering location model for dynamic redeployment of ambulances. *Computers & Operations Research*, *35*(3), 814–826.

Ramya, R., Rajendran, C., & Ziegler, H. (2016). Capacitated lot-sizing problem with production carry-over and set-up splitting: Mathematical models. *International Journal of Production Research*, *54*(8), 2332–2344. https://doi.org/10.1080/00207543.2015.1076942

Ramya, R., Rajendran, C., Ziegler, H., Mohapatra, S., & Ganesh, K. (2019). CLSP: Real Life Applications and Motivation to Study Lot Sizing Problems in Process Industries. In *Capacitated Lot Sizing Problems in Process Industries* (pp. 33–45). Cham: Springer International Publishing. https://doi.org/10.1007/978-3-030-01222-9_2

ReVelle, C., Murray, A. T., & Serra, D. (2007). Location models for ceding market share and shrinking services. *Omega*, *35*(5), 533–540. Retrieved from http://linkinghub.elsevier.com/retrieve/pii/S0305048305001398

ReVelle, C. S., Eiselt, H. A., & Daskin, M. S. (2008). A bibliography for some fundamental problem categories in discrete location science. *European Journal of Operational Research*, *184*(3), 817–848.

Reverse Logistics Association. (n.d.). What is Reverse Logistics? Retrieved February 25, 2019, from http://www.reverselogisticstrends.com/reverse-logistics.php

Rinaldi, M., Montanari, R., & Bottani, E. (2015). *Improving the efficiency of public administrations through business process reengineering and simulation A case study*. *Business Process Management Journal* (Vol. 21). https://doi.org/10.1108/BPMJ-06-2014-0054

Rodrigues, A. P., Fernandes, M. L., Rodrigues, M. F. F., Bortoluzzi, S. C., Gouvea, S. E., & Lima, E. P. De. (2018). Developing criteria for performance assessment in municipal solid waste management. *Journal of Cleaner Production*, *186*, 748–757. https://doi.org/10.1016/j.jclepro.2018.03.067

Rodríguez, V., Alvarez, M. J., & Barcos, L. (2007). Hub location under capacity constraints. *Transportation Research Part E: Logistics and Transportation Review*, *43*(5), 495–505. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-34347250492&partnerID=tZOtx3y1

Romeijn, H. E., Shu, J., & Teo, C.-P. (2007). Designing two-echelon supply networks. *European Journal of Operational Research*, *178*(2), 449–462. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-33751011340&partnerID=tZOtx3y1

Rousta, K., Bolton, K., Lundin, M., & Dahlén, L. (2015). Quantitative assessment of distance to collection point and improved sorting information on source separation of household waste. *Waste Management*, *40*, 22–30. https://doi.org/10.1016/j.wasman.2015.03.005

Roy, J. R., & Thill, J.-C. (2004). Spatial interaction modelling. *Papers in Regional Science*, *83*(1), 339–361. https://doi.org/10.1007/s10110-003-0189-4

Sahling, F., Buschkühl, L., Tempelmeier, H., & Helber, S. (2009). Solving a multi-level capacitated lot sizing problem with multi-period setup carry-over via a fix-and-optimize heuristic. *Computers and Operations Research*, *36*(9), 2546–2553. https://doi.org/10.1016/j.cor.2008.10.009

Sahling, F., & Hahn, G. J. (2019). Dynamic lot sizing in biopharmaceutical manufacturing. *International Journal of Production Economics*, *207*, 96–106. https://doi.org/https://doi.org/10.1016/j.ijpe.2018.11.006

Saleh Farham, M., Süral, H., & Iyigun, C. (2015). The Weber problem in congested regions with entry and exit points, *62*, 177–183. https://doi.org/10.1016/j.cor.2014.10.014

San-José, L. A., Sicilia, J., & García-Laguna, J. (2014). Optimal lot size for a production-inventory system with partial backlogging and mixture of dispatching policies. *International Journal of Production Economics*, *155*, 194–203. https://doi.org/10.1016/j.ijpe.2013.08.017

San-José, L. A., Sicilia, J., González-De-la-Rosa, M., & Febles-Acosta, J. (2017). Optimal inventory policy under power demand pattern and partial backlogging. *Applied Mathematical Modelling*, *46*, 618–630. https://doi.org/10.1016/j.apm.2017.01.082

Saphores, J.-D. M., Nixon, H., Ogunseitan, O. A., & Shapiro, A. A. (2006). Household Willingness to Recycle Electronic Waste: An Application to California. *Environment and Behavior*, *38*(2), 183–208. https://doi.org/10.1177/0013916505279045

Sarkar, A., Batta, R., & Nagi, R. (2005). Planar area location/layout problem in the presence of generalized congested regions with the rectilinear distance metric. *IIE Transactions*, *37*(1), 35–50. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-11944252425&partnerID=tZOtx3y1

Sazvar, Z., Mirzapour Al-E-Hashem, S. M. J., Baboli, A., & Akbari Jokar, M. R. (2014). A bi-objective stochastic programming model for a centralized green supply chain with deteriorating products. *International Journal of Production Economics*, *150*, 140–154. https://doi.org/10.1016/j.ijpe.2013.12.023

Scherr, A. L. (1993). A new approach to business processes. *IBM Systems Journal*, *32*(1), 80–98.

Seifbarghy, M., Rabieian, R., & Karimi, F. (2010). Locating service centers optimizing customers' perspective criteria. *The International Journal of Advanced Manufacturing Technology*, *54*(5–8), 811–819. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-79955673436&partnerID=tZOtx3y1

Sen, A., & Smith, T. E. (2012). *Gravity models of spatial interaction behavior*. Springer Science & Business Media.

Sharp, L., & Luckin, D. (2006). The community waste sector and waste services in the UK: current state and future prospects. *Resources, Conservation and Recycling*, *47*(3), 277–294.

Shavandi, H., & Mahlooji, H. (2008). Fuzzy hierarchical queueing models for the location set covering problem in congested systems. *Scientia Iranica*, *15*(3), 378–388. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-47949115806&partnerID=tZOtx3y1

Shavandi, Hassan, & Mahlooji, H. (2004). Fuzzy queuing location-allocation models for congested systems. *International Journal of Industrial Engineering : Theory Applications and Practice*, *11*(4), 364–376. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-10344265507&partnerID=tZOtx3y1

Sheffield City Council. (2012). *Sheffield City Council*.

Shim, I. S., Kim, H. C., Doh, H. H., & Lee, D. H. (2011). A two-stage heuristic for single machine capacitated lot-sizing and scheduling with sequence-dependent setup costs. *Computers and Industrial Engineering*, *61*(4), 920–929. https://doi.org/10.1016/j.cie.2011.06.002

Shulman, A. (1991). An Algorithm for Solving Dynamic Capacitated Plant Location Problems with Discrete Expansion Sizes. *Operations Research*. Retrieved from http://pubsonline.informs.org/doi/abs/10.1287/opre.39.3.423

Sidique, S. F., Lupi, F., & Joshi, S. V. (2010). The effects of behavior and attitudes on drop-off recycling activities. *Resources, Conservation and Recycling*, *54*(3), 163–170. https://doi.org/https://doi.org/10.1016/j.resconrec.2009.07.012

Simpson, E. (2018). Sainsbury's-Asda may "have to sell at least 73 shops." Retrieved January 10, 2019, from https://www.bbc.co.uk/news/business-44072796

Sliwa, M., & O'Kane, J. (2011). Service quality measurement: appointment systems in UK GP practices. *International Journal of Health Care Quality Assurance*, *Volume 24*(Issue 6), 441–452. https://doi.org/10.1108/09526861111150707

Slow, E. (2017). East Riding 'chuffed' over recycling result. Retrieved October 10, 2018, from https://www.letsrecycle.com/news/latest-news/east-riding-chuffed-over-recycling-result/

Smith, J. M. (2010). Robustness of state-dependent queues and material handling systems, *48*(16), 4631–4663. https://doi.org/10.1080/00207540903049373

Smith, L. (2018). *BRIEFING PAPER: Fly-tipping - the illegal dumping of waste*. Retrieved from http://researchbriefings.files.parliament.uk/documents/SN05672/SN05672.pdf

Smith, L., & Bolton, P. (2018). *BRIEFING PAPER: Household recycling in the UK. BRIEFING PAPER*. https://doi.org/10.1016/j.ics.2005.03.225

Sonmez, A. D., & Lim, G. J. (2012). A decomposition approach for facility location and relocation problem with uncertain number of future facilities. *European Journal of Operational Research*, *218*(2), 327–338.

Sourirajan, K., Ozsen, L., & Uzsoy, R. (2007). A single-product network design model with lead time and safety stock considerations. *IIE Transactions*, *39*(5), 411–424. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-33847775973&partnerID=tZOtx3y1

Sproule, L. (2015, August). All change at the tips with fewer hours and closures on the cards. *Oxford Mail*. Retrieved from http://www.oxfordmail.co.uk/news/13586305.All_change_at_the_tips_with_fewer_hours_and_closures_on_the_cards/?ref=mr&lp=8

Stefansdottir, B., Grunow, M., & Akkerman, R. (2017). Classifying and modeling setups and cleanings in lot sizing and scheduling. *European Journal of Operational Research*, *261*(3), 849–865. https://doi.org/10.1016/j.ejor.2017.03.023

Struk, M. (2017). Distance and incentives matter: The separation of recyclable municipal waste. *Resources, Conservation and Recycling*, *122*, 155–162. https://doi.org/10.1016/j.resconrec.2017.01.023

Suerie, C., & Stadtler, H. (2003). The Capacitated Lot-Sizing Problem with Linked Lot Sizes. *Management Science*, *49*(8), 1039–1054. https://doi.org/10.1287/mnsc.49.8.1039.16406

Taniguchi, E., Noritake, M., Yamada, T., & Izumitani, T. (1999). Optimal size and location planning of public logistics terminals. *Transportation Research Part E: Logistics and*

Transportation Review, *35*(3), 207–222. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-0032783892&partnerID=tZOtx3y1

Tanksale, A., & Jha, J. K. (2016). An effective heuristic for multi-period multi-foodgrain inventory transportation problem in India. *Infor*, *54*(4), 355–379. https://doi.org/10.1080/03155986.2016.1231498

Tansel, B. C., Francis, R. L., & Lowe, T. J. (1983). Location on Networks: A Ssurvey. Part I: The p-Center and p-Median Problems. *Management Science*, *29*(4), 482–497.

Taş, D., Gendreau, M., Jabali, O., & Jans, R. (2018). *A capacitated lot sizing problem with stochastic setup times and overtime*. *European Journal of Operational Research*. https://doi.org/10.1016/j.ejor.2018.07.032

Taş, D., Gendreau, M., Jabali, O., & Jans, R. (2019). A capacitated lot sizing problem with stochastic setup times and overtime. *European Journal of Operational Research*, *273*(1), 146–159. https://doi.org/10.1016/J.EJOR.2018.07.032

Tempelmeier, H. (2011). A column generation heuristic for dynamic capacitated lot sizing with random demand under a fill rate constraint. *Omega*, *39*(6), 627–633. https://doi.org/10.1016/j.omega.2011.01.003

The Controlled Waste Regulations. (1992). *The Controlled Waste Regulations 1992. London: Her Majesty's Stationary Office*. Retrieved from http://www.legislation.gov.uk/uksi/1992/588/contents/made

Toregas, C., Swain, R., ReVelle, C., & Bergman, L. (1971). The Location of Emergency Service Facilities. *Operations Research*, *19*(6), 1363–1373. https://doi.org/10.1287/opre.19.6.1363

Torr, G. (2017). EXCLUSIVE : NHS bosses reveal plans to close Sheffield minor injuries unit and city walk-in centre. Retrieved from https://www.thestar.co.uk/news/exclusive-nhs-bosses-reveal-plans-to-close-sheffield-minor-injuries-unit-and-city-walk-in-centre-1-8772307

Toscano, A., Ferreira, D., & Morabito, R. (2017). A decomposition heuristic to solve the two-stage lot sizing and scheduling problem with temporal cleaning. *Flexible Services and Manufacturing Journal*, 1–32. https://doi.org/10.1007/s10696-017-9303-9

Toso, E. A. V., Morabito, R., & Clark, A. R. (2009). Lot sizing and sequencing optimisation at an animal-feed plant. *Computers and Industrial Engineering*, *57*(3), 813–821. https://doi.org/10.1016/j.cie.2009.02.011

Transportation Research Board. (2000). *Highway capacity manual*. *Environmental Protection*. United States of America: National Academy of Sciences.

Trigeiro, W. W., Thomas, L. J., & McClain, J. O. (1989). Capacitated Lot Sizing with Setup Times. *Management Science*, *35*(3), 353–366. https://doi.org/10.1287/mnsc.35.3.353

Vaishnavi, V., & Kuechler, W. (2004). *Design Research in Information Systems*.

Van den broecke, F., Van Landeghem, H., & Aghezzaf, E. H. (2008). Implementing a near-optimal solution for the multi-stage, multi-product capacitated lot-sizing problem by rolling out a cyclical production plan. *International Journal of Production Economics*, *112*(1), 121–137. https://doi.org/10.1016/j.ijpe.2006.08.029

Van Den Broecke, F., Van Landeghem, H., & Aghezzaf, E. H. (2005). An application of cyclical master production scheduling in a multi-stage, multi-product environment. *Production Planning and Control*, *16*(8 SPEC. ISS.), 796–809.

https://doi.org/10.1080/09537280500181061

van Oudheusden, D. L., & Singh, K. N. (1988). Dynamic lot sizing and facility location. *Engineering Costs and Production Economics*, *14*(4), 267–273. https://doi.org/10.1016/0167-188X(88)90031-6

van Roy, T. J., & Erlenkotter, D. (1982). A Dual-Based Procedure for Dynamic Facility Location. *Management Science*, *28*(10), 1091–1105. Retrieved from http://search.ebscohost.com/login.aspx?direct=true&db=buh&AN=7350563&site=ehost-live

Vardar, C., Gel, E. S., & Fowler, J. W. (2007). A framework for evaluating remote diagnostics investment decisions for semiconductor equipment suppliers. *European Journal of Operational Research*, *180*(3), 1411–1426. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-33847034111&partnerID=tZOtx3y1

VEOLIA. (n.d.). Household Waste Recycling Centres. Retrieved April 30, 2017, from https://www.veolia.co.uk/sheffield/recycling-sites/recycling-sites/household-waste-recycling-centres

Verter, V. (2011). Foundations of Location Analysis. *Foundations of Location Analysis*, *155*, 25–38. Retrieved from http://www.springerlink.com/index/10.1007/978-1-4419-7572-0

Vidyarthi, N, & Kuzgunkaya, O. (2015). The impact of directed choice on the design of preventive healthcare facility network under congestion. *Health Care Management Science*, *18*(4), 459–474. https://doi.org/10.1007/s10729-014-9274-2

Vidyarthi, Navneet, & Jayaswal, S. (2014). Efficient solution of a class of location–allocation problems with stochastic demand and congestion. *Computers & Operations Research*, *48*, 20–30. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-84897429930&partnerID=tZOtx3y1

Wagner, H. M. ., & Whitin, T. M. . (1958). Dynamic Version of the Economic Lot Size Model. *Management Science*, *5*(1), 89–96. Retrieved from http://www.jstor.org/stable/2626974

Waite, R. (2013). *Household waste recycling*. Routledge.

Wang, J. Y. T., Yang, H., & Lindsey, R. (2004). Locating and pricing park-and-ride facilities in a linear monocentric city with deterministic mode choice. *Transportation Research Part B: Methodological*, *38*(8), 709–731. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-2942618493&partnerID=tZOtx3y1

Wang, Q., Batta, R., Bhadury, J., & M. Rump, C. (2003). Budget constrained location problem with opening and closing of facilities. *Computers & Operations Research*, *30*(13), 2047–2069.

Wang, Q., Batta, R., & Rump, C. M. (2002). Algorithms for a facility location problem with stochastic customer demand and immobile servers. *Annals of Operations Research*, *111*(1–4), 17–34. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-0036421121&partnerID=tZOtx3y1

Webb, B., Marshall, B., Czarnomski, S., & Tilley, N. (2006). *Fly-tipping: causes, incentives and solutions. DEFRA.*

Wesolowsky, G. O. (1973). Dynamic facility location. *Management Science*, *19*(11), 1241–1248.

Widdowson, S., Sankey, H., & McElearney, R. (2015). *Waste on the Front Line – Challenges and Innovations*. England.

Wilhelm, W., Han, X., & Lee, C. (2013). Computational comparison of two formulations for dynamic supply chain reconfiguration with capacity expansion and contraction. *Computers & Operations Research*, *40*(10), 2340–2356.

Wilson, A. G. (1971). A Family of Spatial Interaction Models, and Associated Developments. *Environment and Planning A: Economy and Space*, *3*(1), 1–32. https://doi.org/10.1068/a030001

Wong, S. C., & Sun, S. H. (2001). A combined distribution and assignment model for continuous facility location problem. *Annals of Regional Science*, *35*(2), 267–281. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-1842765988&partnerID=tZOtx3y1

Woodard, R., Bench, M., Harder, M. K., & Stantzos, N. (2004). The optimisation of household waste recycling centres for increased recycling—a case study in Sussex, UK. *Resources, Conservation and Recycling*, *43*(1), 75–93.

WRAP. (2012a). *Composition and re-use potential of household bulky waste in the UK*. Retrieved from http://www.wrap.org.uk/content/study-re-use-potential-household-bulky-waste

WRAP. (2012b). Household Waste Recycling Centre (HWRC) Guide, (October).

WRAP. (2016). *Household Waste Recycling Centre (HWRC) Guide*. Retrieved from http://www.wrap.org.uk/sites/files/wrap/HWRC_Guidance_2018_4.pdf

Wright, R. E., Richey, R. G., Tokman, M., & Palmer, J. C. (2011). Recycling and reverse logistics. *Journal of Applied Business and Economics*, *12*(5), 9–20.

Zanjani, M. K., Nourelfath, M., & Ait-Kadi, D. (2010). A multi-stage stochastic programming approach for production planning with uncertainty in the quality of raw materials and demand. *International Journal of Production Research*, *48*(16), 4701–4723. https://doi.org/10.1080/00207540903055727

Zarinbal, M. (2009). Distance Functions in Location Problems. In R. Zanjirani Farahani & M. Hekmatfar (Eds.), *Facility Location: Concepts, Models, Algorithms and Case Studies* (pp. 5–17). Heidelberg: Physica-Verlag HD. Retrieved from http://dx.doi.org/10.1007/978-3-7908-2151-2_1

Zarrinpoor, N., Fallahnezhad, M. S., & Pishvaee, M. S. (2017). Design of a reliable facility location model for health service networks. *International Journal of Engineering, Transactions A: Basics*, *30*(1), 75–84. https://doi.org/10.5829/idosi.ije.2017.30.01a.10

Zhang, Y., Berman, O., Marcotte, P., & Verter, V. (2010). A bilevel model for preventive healthcare facility network design with congestion. *IIE Transactions*, *42*(12), 865–880. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-77957892648&partnerID=tZOtx3y1

Zhang, Y., Berman, O., & Verter, V. (2009). Incorporating congestion in preventive healthcare facility network design. *European Journal of Operational Research*, *198*(3), 922–935. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-64049108678&partnerID=tZOtx3y1

Zhu, X., & Wilhelm, W. E. (2006). Scheduling and lot sizing with sequence-dependent setup: A literature review. *IIE Transactions (Institute of Industrial Engineers)*, *38*(11), 987–1007. https://doi.org/10.1080/07408170600559706

APPENDICES

APPENDIX 1: CPLEX CODES

This section provides the codes used in CPLEX to solve the proposed model. The codes for the refined models are also presented.

A. Main Model (Single-objective model)

```
int J=...;
int T=...;

range facility=1..J;
range time_period=1..T;

float cost=...;
float E1=...;
float E2=...;
float E3=...;
float E4=...;
float B=...;
int M[facility][time_period]=...;
int x[facility][time_period]=...;
int past_dmd[facility]=...;

int num_period[facility]=...;
int u[facility][1..((J*T))]=...;
int u1[j in facility][k in facility][t in time_period] =u[j][t+T*(k-1)];

dvar boolean y[facility][time_period];
dvar int+ s[facility][0..T];
dvar int+ loss[facility][time_period];
dvar int+ q[facility][time_period];
dvar int+ S[facility][facility][time_period];

//the model
minimize sum (j in facility, t in time_period) (cost*y[j][t] + E1*q[j][t] + E3*loss[j][t] +
E2*s[j][t] + E4*sum(k in facility)S[j][k][t]*u1[j][k][t]);

subject to{

forall (j in facility, t in time_period : t=0 && t=T) s[j][t] == past_dmd[j];

forall (j in facility, t in time_period : t>1) s[j][t] == x[j][t] + s[j][t-1] + sum (k in
facility) S[k][j][t]*u1[k][j][t] - sum (k in facility) S[j][k][t]*u1[j][k][t] - q[j][t] -
loss[j][t];

sum (j in facility, t in time_period) x[j][t] == sum(j in facility, t in time_period) (q[j][t]
+ loss[j][t]);

forall (j in facility, t in time_period) q[j][t] <= M[j][t]*y[j][t];

forall (j in facility, t in time_period: t<T) (s[j][t]/x[j][t]) <= y[j][t+1];

forall (j,k in facility, t in time_period: j!=k) ((S[j][k][t]*u1[j][k][t])/x[j][t]) <= y[k][t];

forall (j in facility) sum (t in time_period) y[j][t] <= num_period[j];

forall (j in facility, t in time_period: t>1) y[j][t-1] <= y[j][t];

};
```

B. Main Model (Multi-component model)

```
int J=...;
int T=...;


range facility=1..J;
range time_period=1..T;

//parameter: data from .DAT
float cost=...;
float E1=...;
float E2=...;
float E3=...;
float E4=...;
int M[facility][time_period]=...;
int x[facility][time_period]=...;
int past_dmd[facility]=...;
float B=...;
int num_period[facility]=...;

int u[facility][1..((J*T))]=...;
int u1[j in facility][k in facility][t in time_period] =u[j][t+T*(k-1)];

float R = ...;

//variable
dvar boolean y[facility][time_period];
dvar int+ s[facility][0..T];
dvar int+ loss[facility][time_period];
dvar int+ q[facility][time_period];
dvar int+ S[facility][facility][time_period];

dexpr float Z1 = sum (j in facility, t in time_period) (cost*y[j][t] + E1*q[j][t]+
E3*loss[j][t]);
dexpr float Z2 = sum (j in facility, t in time_period) (E2*s[j][t] + E4*sum(k in
facility)S[j][k][t]*u1[j][k][t]);

//objective function
minimize (R*Z1 + (1-R)*Z2);

//constraints
subject to{

forall (j in facility, t in time_period){
               s[j][T] == past_dmd[j];
               s[j][0] == past_dmd[j];
       }

forall (j in facility, t in time_period)
s[j][t] == x[j][t] + s[j][t-1] + sum (k in facility) S[k][j][t]*u1[j][k][t] - sum (k in
facility) S[j][k][t]*u1[j][k][t] - q[j][t] - loss[j][t];

sum (j in facility, t in time_period) x[j][t] == sum(j in facility, t in time_period)
(q[j][t] + loss[j][t]);

sum (j in facility, t in time_period) loss[j][t] <= B* sum (j in facility, t in
time_period) (x[j][t]);

forall (j in facility, t in time_period) q[j][t] <= M[j][t]*y[j][t];

forall (j in facility, t in time_period: t<T) (s[j][t]/x[j][t]) <= y[j][t+1];

forall (j,k in facility, t in time_period: j!=k) ((S[j][k][t]*u1[j][k][t])/x[j][t]) <=
y[k][t];

forall (j in facility) sum (t in time_period) y[j][t] <= num_period[j];

forall (j in facility, t in time_period: t>1) y[j][t-1] <= y[j][t];

};
```

C. Modified Model (Case Study: the HWRC)

```
int J=...;
int T=...;
int W=...;

range facility=1..J;
range time_period=1..T;
range macro_period=1..W;

float cost[facility]=...;
float E1[facility]=...;
float E2[facility]=...;
float E3[facility]=...;
float E4[facility]=...;
int M[facility][time_period]=...;
int x[facility][time_period]=...;
int past_dmd[facility]=...;
float B=...;
int H=...;
int num_period2[facility][macro_period]=...;
int num_period[facility]=...;
float R=...;
int u[facility][1..((J*T))]=...;
int u1[j in facility][k in facility][t in time_period] =u[j][t+T*(k-1)];

dvar boolean y[facility][time_period];
dvar int+ s[facility][0..T];
dvar int+ loss[facility][time_period];
dvar int+ q[facility][time_period];
dvar int+ S[facility][facility][time_period];

dexpr float Z1 = sum (j in facility, t in time_period) (cost[j]*y[j][t] + E1[j]*q[j][t]
+ E3[j]*loss[j][t]);
dexpr float Z2 = sum (j in facility, t in time_period) (E2[j]*s[j][t] + E4[j]*sum(k in
facility)S[k][j][t]*u1[k][j][t]);

//objective function
minimize (R*Z1 + (1-R)*Z2);

subject to{

forall (j in facility, w in macro_period){
               s[j][w*H] == past_dmd[j];
               s[j][1+H*(w-1)] == past_dmd[j];}

forall (j in facility, t in time_period) s[j][t] == x[j][t] + s[j][t-1] + sum (k in
facility) S[k][j][t]*u1[k][j][t] - sum (k in facility) S[j][k][t]*u1[j][k][t] - q[j][t]
- loss[j][t];

forall (j in facility, t in time_period) q[j][t] <= M[j][t]*y[j][t];

sum (j in facility, t in time_period) x[j][t] == sum(j in facility, t in time_period)
(q[j][t] + loss[j][t]);

sum (j in facility, t in time_period) loss[j][t] <= B* sum (j in facility, t in
time_period) (x[j][t]);

forall (j in facility) sum (t in time_period) y[j][t] <= 0 || 0.5*T <= sum (t in
time_period) y[j][t] <= num_period[j];

forall (j in facility, w in macro_period) sum (t in time_period: 1+H*(w-1)<=t<=H*w)
y[j][t] <= 0 || 0.5*num_period2[j][w] <= sum (t in time_period: 1+H*(w-1)<=t<=H*w)
y[j][t] <= num_period2[j][w];

forall (j in facility, t in time_period: t<T) (s[j][t]/x[j][t]) <= y[j][t+1];

forall (j in facility, t in time_period, w in macro_period: 1+H*(w-1)<t<=H*w)
       y[j][t-1] >= y[j][t];

forall (j,k in facility, t in time_period) (S[j][k][t]/x[j][t])*u1[j][k][t] <= y[k][t];

};
```

D. Modified Model (Case Study: the healthcare facility network)

1. The backup model

```
int J=...;
int T=...;
int D=...;

range facility=1..J;
range time_period=1..T;
int y[facility][time_period]=...;
int dist[facility][facility]=...;
dvar boolean h[facility][facility];

//objective function
maximize sum (j,k in facility, t in time_period) (h[j][k]*(abs(y[j][t]-y[k][t])));

//constraints
subject to{

forall (k in facility) sum(j in facility: j!=k) h[j][k] <= 1;

forall (j in facility) sum(k in facility: k!=j) h[j][k] == 1;

forall (j,k in facility) h[j][k]*dist[j][k] <= D;
}
```

2. The multi-period model

```
int J=...;
int T=...;
int G=...;

range facility=1..J;
range time_period=1..T;

float E2[facility]=...;
float E3[facility]=...;
float E4[facility]=...;
int M[facility][time_period]=...;
int x[facility][time_period]=...;
float B=...;
float R=...;
int u[facility][1..((J*T))]=...;
int u1[j in facility][k in facility][t in time_period] =u[j][t+T*(k-1)];
int y[facility][1..T+G]=...;

//variable
dvar int+ s[facility][1-G..T];
dvar int+ loss[facility][t in time_period];
dvar int+ q[facility][t in time_period];
dvar int+ S[facility][facility][t in time_period];

dexpr float Z1 = sum (j in facility, t in time_period) (E3[j]*loss[j][t]);
dexpr float Z2 = sum (j in facility, t in time_period) (E2[j]*s[j][t] + E4[j]*sum(k in
facility)S[k][j][t]*u1[k][j][t]);

//objective function
minimize (R*Z1 + (1-R)*Z2);

//constraints
subject to{

forall (j in facility) sum(t in 1-G..0)s[j][t] == 0;

forall (j in facility, t in time_period) x[j][t] + s[j][t-G] + sum (k in facility)
S[k][j][t]*u1[k][j][t] == s[j][t] + sum (k in facility) S[j][k][t]*u1[j][k][t] + q[j][t] +
loss[j][t];

forall (j in facility, t in time_period) q[j][t] <= M[j][t]*y[j][t];

sum (j in facility, t in time_period) x[j][t] == sum(j in facility, t in time_period)(q[j][t] +
loss[j][t]) + sum(j in facility,t in time_period: T-G<t<=T) s[j][t];

sum (j in facility, t in time_period) loss[j][t] <= B* sum (j in facility, t in time_period)
(x[j][t]);

forall (j in facility, t in time_period) (s[j][t]/x[j][t]) <= y[j][t+G];

forall (j,k in facility, t in time_period: j!=k) (S[j][k][t]*u1[j][k][t]/x[j][t]) <= y[k][t];

};
```

APPENDIX 2: LOCATION SCIENCE

This section provides information related to Chapter 3, which includes the outlines the general elements that are needed in developing a facility location model. This includes the locations' and demand's spaces, the type of facilities, the way to compute distance, and objective function of the model itself. The systematic review past studies on facility location models with congestion issues were also discussed.

Location Science is an interdisciplinary field of study that combines mathematics, economics, geography and computer science (Laporte & Nickel, 2015). Studies in location science involve in locating facilities in a network or space; this field of study provides important tools for decision-making in management operations. The interest in this area (also known as facility location problem) has significantly grown over time; this can be observed by looking at the evolution of the problem focus and the involved elements, applications, and solution techniques.

The foundation of location theory can be traced back to seminal work from the French mathematician Pierre de Fermat in 1600s, who provided some advances to the Euclidean theory of distance; known as Fermat's Problem. John Heinrich von Thünen (1783 – 1850) and Alfred Weber (1868 – 1958) are among the earliest scholars who developed the fundamentals of location theory. Thünen's work focussed on agricultural location and land usage meanwhile Weber's model focussed on an industrial location problem.

Solving a facility location problem is a complex and challenging task, which has attracted a wide interest from scholars in the last centuries; this is testified by the large amount of theoretical and applied developments which can be found in the literature. As such, facility location is not a new study area and its historical evolution was discussed extensively by Bruno et al., (2014) and Laporte et al., (2015); however, the steady growth of the academic interest towards facility location problems can be seen through the constant development of new elements, applications, and solution techniques.

A. Elements in Location Models

In general, there are five *essential* components that must be considered in developing models for location problems. These are the location space, the demand space and its characteristics, the metrics (distances), the type of facilities, and the objective functions. This section also briefly introduces some mechanisms which can be used in order to allocate user or demand to facilities. This due to the fact that the allocation of user to each facility is also part of the elements needed in constructing a location model.

Location Spaces

The location space can be a geographical area (e.g. a region or a city) or not; for example, the concept can be utilised for positioning a company in a market described as a virtual *space* in a set of economic variables generally corresponds to a space where facilities are to be located. There are three types of location spaces known as *discrete*, *continuous* or *network* spaces. Figure 0-1, 0-2 and 0-3 portray the differences between these spaces.

- A *discrete location space* indicates that only an enumerable number of potential facility locations is available. Facilities can be located anywhere in this space, apart from the presence of pre-specified restrictions.

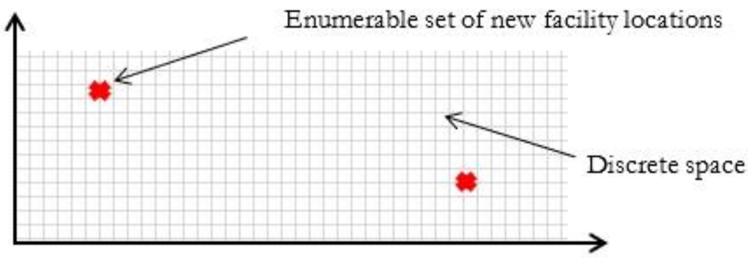


Figure 0-1: Discrete location space

- A *continuous location space* deals with a non-enumerable number of potential facility locations, which can be placed anywhere in the area.

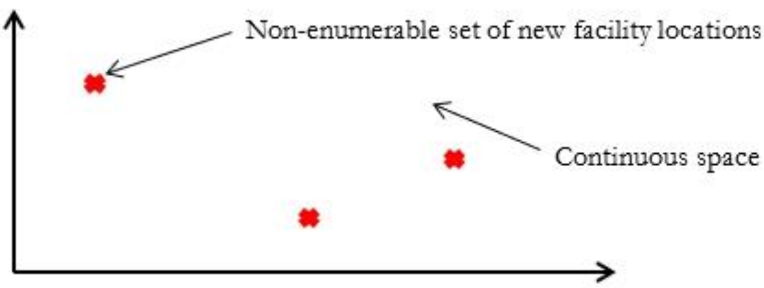


Figure 0-2: Continuous location space

- A *network location space* is a location space that has a network structure where the facility can be located only on the network structure, whether on its edges or on its nodes.

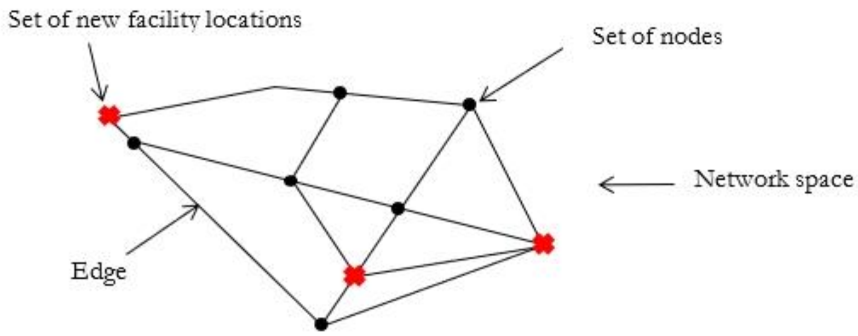


Figure 0-3: Network location space

Demand Spaces

In location problems, demand is one of the fundamental factors in influencing facility location choices. It is defined based on its application: for instance, in a health-care context, the demand for a given service can be expressed through patients; in a public transportation context, passengers represent the actors who are requiring the service. Demand may be generated anywhere within the area of interest. There are three types of demand space: *discrete demand space*, *continuous demand space,* and *network demand space*.

- A *discrete* demand space indicates that the demand is concentrated at a set of pre-specified areas (sometimes categorised as a zone, a cluster or a centroid). Demand that is distributed within the pre-specified area is normally treated as located in a single demand point (normally, the area centroid).
- A *continuous* demand space indicates that the demand is distributed anywhere over a continuous portion of space.
- In a *network* demand space, the demand is distributed at the nodes or at any point in the edges of a graph structure.

The nature of the demand can be classified into *deterministic* or *probabilistic*. When dealing with a *deterministic demand* type, the amount of demand is known in advance, for example, the number of patients per day at a GP surgery based on a booking service. In contrast, *probabilistic demand* deals with uncertainty about the amount of demand; for example, the unpredictability about the number of patients visiting an A&E department over a given time period.

Distance (Metric)

Another important element in locating a facility is represented by the distance between the potential facility and set of demand points. For all the described scenarios, distance computations can vary; the most common ones are those based on *Euclidean* and *Manhattan* metrics. Figures 0-4 and 0-5 illustrate these distances.

In the Euclidean metric, as in Figure 0-4, the distance between point A and B can be calculated by using $D(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

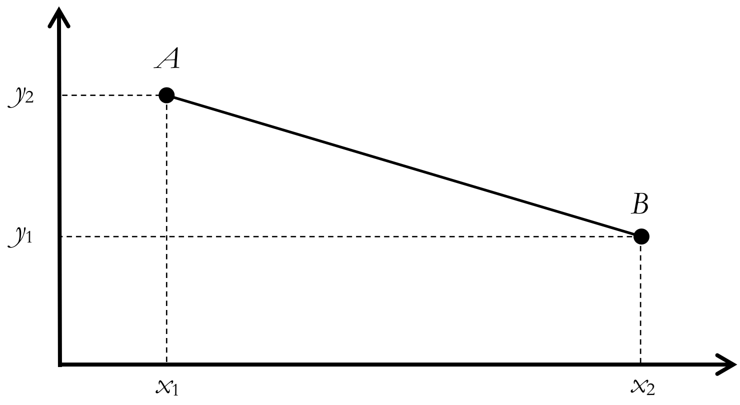


Figure 0-4: Euclidean metric

The Manhattan metric (also known as Taxicab or rectilinear distance) calculates the distance between two points using paths that are orthogonal or perpendicular to each other, as illustrated in Figure 0-5. This distance can be calculated using the following formula: $D(A,B) = |x_1 - x_2| + |y_1 - y_2|$.

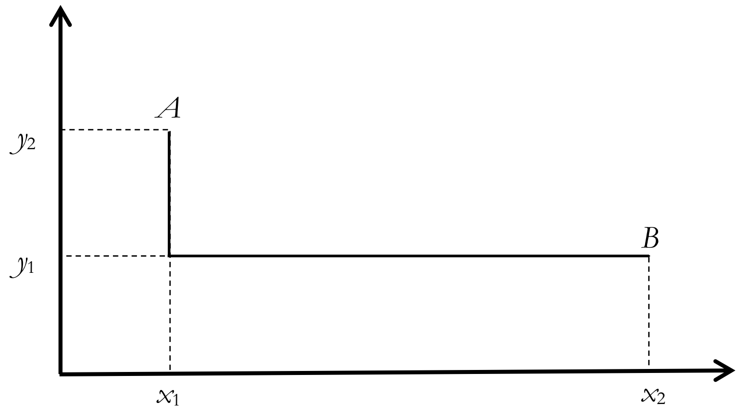


Figure 0-5: Manhattan metric

For *discrete* and *network* location model, distance could be translated into a given function such as travelling costs or time.

Facilities

The decision-making process underlying the location of facilities is usually based on specific features, such as the number of facilities to be located, the type of facility, associated operational costs and the capacity of the facility.

- Theoretically, the number of facilities that can be located in a given location space can be *infinite* or *finite*. If only a single facility needs to be located, the respective problem is then called a *single-facility* problem; *multi-facility* problems indicate that more than one facility has to be located in a location space. The number of facilities to be located can be stated *a priori* or can be a result of the decision-making process.
- Facilities can also be classified into *static* or *mobile* ones. *Static* facilities indicate that demand is travelling to the desired facility in order to get access to a product or service. In contrast, *mobile* facilities require the service to travel to the demand nodes. For example, ambulance services or food delivery service.
- *Fixed costs* and *variable costs* are among the associated costs found in the facility location problem. *Fixed costs* are mainly representing the costs related to facility opening and its basic functioning. Meanwhile, *variable costs* are mainly related to the allocation of the demand to the specific facility.
- The capacity of a facility can be unlimited (*uncapacitated*) or limited (*capacitated*). An *uncapacitated facility* indicates a facility that can be located without any consideration of budgetary, technological, or physical restrictions (Verter, 2011). This also means that the facility is capable of serving an unlimited amount of demand. A more realistic approach is provided by *capacitated facilities*, whereby an upper limit for facility service capacity or budget is introduced.
- Location problems can be further classified on more general outlook, *competitive* and *non-competitive* setting. A *competitive* setting refers to a new facility being integrated into an existing location space or network, having to compete for its market share (Plastria, 2001). A *non-competitive* location problem deals with facilities being normally managed by a single central authority, whose objective is to provide an equitable access to the service to users; as such, these models are quite common in dealing with non-profit sectors.

Objective Functions

In locating a facility, various objective functions can be utilised for optimising the whole system performance. Such objective functions can include single or multiple objectives. Four general classifications of objective functions were established for this study:

- *Financial* – the objective function is based on monetary considerations, such as minimising the total cost for opening a new facility or maximising the total profit.
- *Physical* – the objective function is based on aspects such as distances, capacities, number of facilities. For example, minimising the maximum total distance between demand points and facilities.
- *Time-based* – the objective function employs time as the main consideration; for example, minimising the maximum travel time between a customer and a facility or minimising the average waiting time for the demand.
- *Demand-based* – the objective function is developed using demand-related considerations; for example, maximising demand or population covered.

Using either one or more from the four objective functions described before, the location models can also be described as *median*-like models, *covering*-like models, and *centre*-like models.

The *median*-like model is a distance-based objective function. One of the classical facility location models that use distance as the main concern was the *p*-median model by Hakimi (1964). The *covering-like* models are based on *demand*-based objective function. The aim of this location model is to maximise the amount of demand that can be covered within pre-specified distances (or travel times) between facilities and demand points. Two renowned basic covering-like models are Location Set Covering Model (LSCM) by Toregas et al. (1971) and the Maximal Covering Location Problem (MCLP) by Church and ReVelle (1974). Meanwhile, a *centre-like* model can be classified as a minimax model (Tansel et al., 1983). A *centre-like* model ensures that a facility can be reached by the most disadvantaged customer within an acceptable travel distance (Hakimi, 1964, 1965); as such, such models seek to minimise the maximum travel distance faced by the most disadvantaged customer when visiting the facility. A weighted version of the problem (where the weight of the demand point i (w_i) could represent the total demand concentrated in that point) could be developed (Elloumi et al., 2004).

Allocation of demand to facilities

Besides the five elements needed in formulating a facility location model, a further enrichment can be performed, for example, by adding an *allocation* element to the problem. Allocation, in the facility location problem literature, is defined as the process of assigning demand to each facility (Manzini & Gebennini, 2008).

Such allocation could be based on distance or personal interest (or attractiveness level of a given facility). Figure 0-6 illustrates the flow of demand from *i* to facility *j*. The figure on the left-hand side shows demand allocation to facility *j* based on distance, meanwhile on the right-hand sight shows the demand allocation using the attractiveness level.

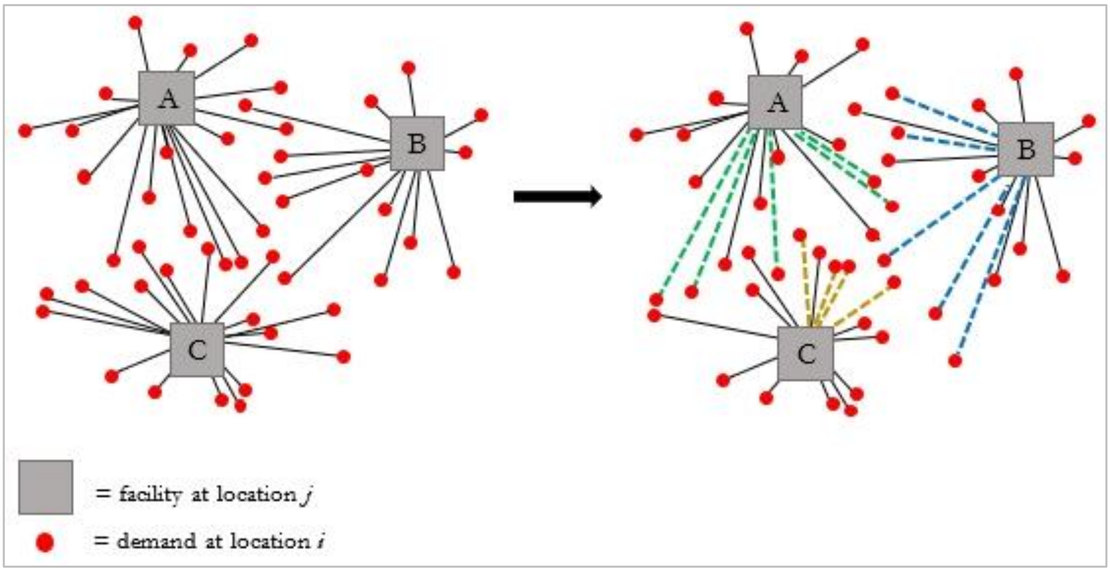


Figure 0-6: Illustrative demand allocation

In this figure, facilities *A*, *B* and *C* provide a similar service; users have to travel to either one of these facilities to obtain service. From Figure 0-6, on the left-hand side, the assignment of demand points to each facility is *distance-based*; each user is allocated to the closest facility. On the same figure, the right-hand side shows there is some demand which, while being allocated to facility A, would be closer to facility *C* (as shown through the green dashed line). This is probably caused by the *attractiveness* level of facility *A* compared to the one of *C*. Further explanation on this is in provided in the next section.

Let *I* be the set of customer locations $(i \in I)$ and *J* be the set of potential facility locations $(j \in J)$. As mentioned previously, there are two types of services: (i) *static* and (ii) *mobile*. When dealing with *static* facilities, customer *i* needs to travel to facility *j* in order to obtain the service. The travel of customer *i* to facility *j* could be based on shortest distance (physically), costs or attractiveness level. Meanwhile, when dealing with *mobile* facilities, the given facility j *travels* to the generic customer i in order to provide the service. The provider at facility j normally chooses the customer location i based on shortest distance or shortest travel times. In short, the allocation of customer i to facility j is based on distance between customer at location i and facility at location j, or attractiveness of facility at location j.

Distance-based allocations

Distance could be further interpreted as physical distance, transportation costs or travel time between two points. In most location models, the allocation of demand to each facility is conducted on a distance-based mechanism. As we know, distance is one of the influencing factors in consumer choices (Eiselt et al., 1993). The earliest models (median-like, covering-like and centre-like location models) utilised distance as the essence in finding the optimal facility location.

Allocation of customer i to facility j or facility j to customer i that is solely based on distance, might be the best option for mobile facilities. However, for static facility, such as recycling facilities or a retail shops, distance may be just one of the factors in locating the facility. The existence of other factors (such as facility layout, customer service, personal preferences staff assistance) are totally neglected in distance-based allocation.

Preference-based allocations

The *spatial interaction model* is one of the possible formulations that include distance as one of the factors for the allocation process; however, this class of models also considers the attractiveness level of the facility j towards the customer i (Haynes & Fotheringham, 1984). The attractiveness level is based on one or more factors, such as capacity level of facility j, or customer service provided by facility j. This mathematical expression is used to represent the consumers' choice among a set of available alternatives (Bruno & Genovese, 2012). The general formulation and description of the spatial interaction models were discussed in Fotheringham and O'Kelly (1989) and Sen and Smith (2012), article by Wilson (1971), Beaumont (1980), Fotheringham (1983), and Roy and Thill (2004) are amongst others.

In general, the model assumes the probability that customer i chooses facility j is based on facility j's attractiveness value and inversely proportional to a power of the distance between customer i and facility j. The attractiveness factors can be either numerical (for example, the congestion level during certain time-windows) or non-numerical (for example, the satisfaction for the service provided). Non-numerical ones can be transformed into a numerical form by using existing techniques such as weightage systems or fuzzy numbers. Meanwhile, the distance is generally implemented in a physical sense i.e. kilometric distance or travel times. A spatial interaction model is constructed by first considering a set *I* of origin nodes (representing the location of potential demand's points) and a set *J* of destination nodes (representing the location of facilities). The general formulation is given as:

$$G_{ij} = k_{ij} \cdot (P_i)^{\alpha_i} \cdot (Q_j)^{\beta_j} \cdot f(d_{ij}) \tag{0-1}$$

where G_{ij} is the flow of demand from origin *i* to facility location *j*. The flow is dependent on a generation factor (P_i) associated with the origin *i*, an attractiveness factor (Q_j) due to the features of the destination *j* and the "impedance" between *i* and *j* measured as a function $f(d_{ij})$ of the distance from *i* to *j*; k_{ij}, α_i and β_j represent calibration parameters. The meaning of the factors P_i and Q_j can vary. The deterrence (also known as impedance or distance decay) variables are usually assumed to be an exponential or a power function, representing the effect of distance on spatial interaction. Many versions of the deterrence function can be found in different applications. However, in general, exponential functions are more appropriate for analysing short distance interactions, such as those that take place within urban areas; on the other hand, power functions are generally more appropriate for analysing longer distance interactions, such as migration flows (Fotheringham & O'Kelly 1989). A frequently used expression for the power form of the impedance function is:

$$f(d_{ij}) = (d_{ij})^{-n} \tag{0-2}$$

where in most cases, $1 \le n \le 3$ (Haynes & Fotheringham, 1984). Assumed $\alpha_j = \beta_j = 1$, $k_{ij} = k_{ji} = k$, and, hence, the flow G_{ij} in (0-1) is:

$$G_{ij} = k \cdot P_i \cdot Q_j \cdot f(d_{ij}) \tag{0-3}$$

The definition of the attractiveness factors for each facility *j* (Q_j), of the distances d_{ij} (*i, j* =1, …, *N*) between each pair of *i-j* and of the calibration parameters of the model (k_{ij}, α_j, β_j, n) represent the necessary steps required for the implementation for this model (Bruno & Genovese, 2012). The attractiveness factors of each facility *j* should represent the capability of attracting a given demand; which hugely depend on the specific application.

In some cases, there are constraints about the sum of total flow emanating from origins or entering at destinations. If the total flows emanating from origins (O_i) are known, the model is called "origin constrained" and:

$$\sum_j G_{ij} = k \cdot P_i \cdot \sum_j (Q_j \cdot (d_{ij})^{-n}) = O_i \tag{0-4}$$

Dividing the two last expressions, we obtain:

$$\frac{G_{ij}}{\sum_j G_{ij}} = \frac{Q_j \cdot (d_{ij})^{-n}}{\sum_j (Q_j \cdot (d_{ij})^{-n})} \tag{0-5}$$

From which, it derives:

$$G_{ij} = O_i \cdot \frac{Q_j \cdot (d_{ij})^{-n}}{\sum_j (Q_j \cdot (d_{ij})^{-n})} \tag{0-6}$$

There are various applications of the spatial interaction model, such as Drezner and Drezner (2001) and Eiselt and Marianov (2009). Both studies dealt with allocation of traffic to airlines hubs.

This section outlined elements needed for structuring facility location models. Based on these elements, the next section will provide a classification of location models.

B. Classification of Location Models

In general, location models can be classified into *discrete*, *continuous* and *network* models based on location and demand spaces (Daskin, 2008; ReVelle et al., 2008; Zarinbal, 2009). Arabani and Farahani (2012) further classified location models into two general types: static and dynamic models (see Figure 0-7). Dynamic models can be divided into two categories; probabilistic or *stochastic* models and *time-period* models.

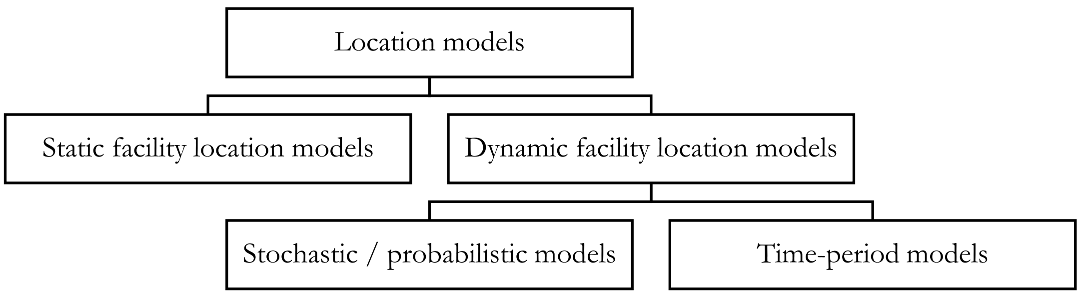


Figure 0-7: Further classifications on facility location models

Static Models

Static models focus on a one-time decision and a single solution is found at a time (Owen & Daskin, 1998). The solution found might not be suitable for usage in the future as having static variables and parameters could not be a suitable choice, since it is highly likely that they will change over time (Wesolowsky, 1973; Klose & Drexl, 2005; Arabani & Farahani, 2012). This constitutes a major drawback in utilising these models for solving problems involving a longer time horizon (Wesolowsky, 1973). A static model could also be modified by adding any dynamic elements, such as, time.

Dynamic Models

Dynamic models are useful in capturing time-varying data over time, such as cost, amount of demand, and capacities (Klose & Drexl, 2005). Dynamic models are able to enhance quality in decision-making due to their capability of capturing variations of demand or any other parameters across a given time and space horizon (Rajagopalan et al., 2008). Such models can reproduce by varying capacity level or varying facility operations during a planning horizon. Therefore, for this study, models that capture dynamic features may belong to the following classes: *probabilistic* (or *stochastic*) models and *time-dependent* models.

- *Probabilistic/stochastic models*: probabilistic models deal with random and independent variables and parameters. Usually, demand is considered as being characterised by a random arrival rate. *Queueing* techniques are normally used to handle this and used to determine the probability of a server being busy (for further discussion of queuing theory, refer to Appendix 2(A)).
- *Time-period models*: time-period based models are categorised into two sub-categories, namely *single* period models and *multi-period* models. Figure 2-8 illustrates the time-period models. From the figure, a single period model deals with a single time period in which the result obtained is only for period *t* and is useful for that particular period only. At the same figure, a multi-period model is a combination of several single period models, linked together by a 'connection' variable. This 'connection' variable is used to transfer current results into the next period (for example, from period *t* to period *t+1*). A multi-period model is also able to capture or cope with any repeated processes until the end of period *T*.

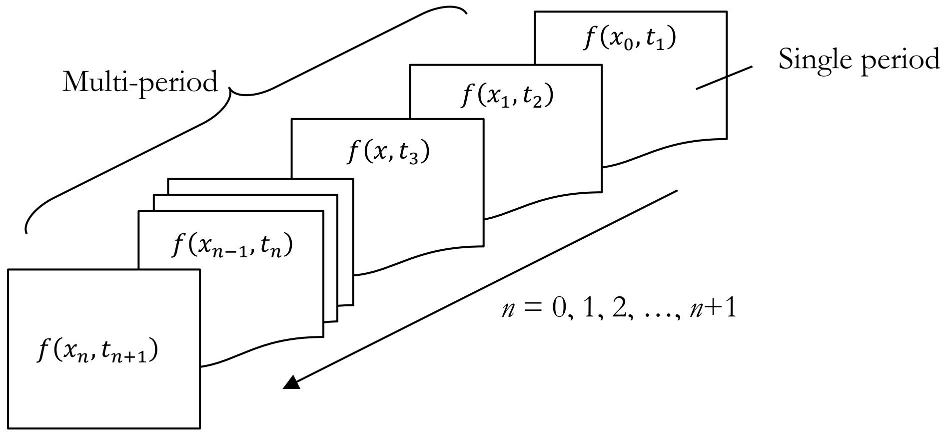


Figure 0-8: Multi-period model concept, in general

C. Facility location models with congestion issues

The scope of the review consists of existing technique in handling congestion, solution approaches and validation approaches. First, the general elements in handling congestion problems is describes, followed by review on each study, focussing on the general elements and lastly, a detail on each study are reviewed, which is presented using a table.

Techniques for handling congestion

In handling congestion issues, two techniques are mainly used: queueing approaches and congestion functions.

1. Queuing theory

The queuing theory originated as a technique for studying systems characterised by waiting in lines or *queues*. It was introduced by Agner Krarup Erlang (1909). A commonly utilised used notation in classifying queueing systems was provided by Kendall (1953); this is based on the following six components:

{arrival process} / {service distribution} / {number of servers} / {system's capacity} / {population size} / {queue discipline}

The meaning of each of the components can be explained as follows:

- Arrival process: Arrivals may originate from one or several sources (referred to as the calling population). Number of arrivals can be '*single arrival*' or '*batch arrivals*', '*fixed arrival*' or '*random arrival*', '*limited*' or '*unlimited*'.
- Service distribution: The service mechanism of a queuing system is specified by the number of servers. Each server can have its own queue or a common queue; also, a probability distribution for customers' service time is specified.
- Number of servers: Servers are tasked with satisfying demand. A system can have one (single) server or several (more than one) servers.
- System's capacity: This notation is used to indicate the maximum amount of demand that can be catered per server (in a given time period); any additional demand will be in the line or waiting. The capacity of the server can be finite or infinite.
- Population size: The size of the population represents the amount of demand of a system which can be limited or unlimited.
- Queue discipline: The discipline of a queuing system identifies the rule that a server uses to select the next customer from the queue (if any) when the server completes the service of the current customer. Commonly used queue disciplines are:

    a. FIFO - Customers are served according to a first-in-first-out basis
    b. LIFO - Customers are served according to a last-in-first-out manner
    c. Priority - Customers are served in order of their importance on the basis of their service requirements.
    d. Random Service – customers are selected randomly.

For instance, a system of *M/m/1/1/20/FIFO* indicates:

- *M* – Poisson arrival distribution/ exponential arrival distribution;
- *m* – Poisson service distribution/ exponential service distribution;
- *1* – the system is a single server system;
- *1* – the server capacity is limited to one user per time;
- *20* – the system has in total of 20 potential users;
- *FIFO* – the users are served according to a first-in-first-out basis.

In general, combination of multiple facilities with multi-servers formed a complex network. This could be seen, for instance, in manufacturing networks, a product is assembled in a factory that have gone several process and stations (multi-servers) and have to be transported to several locations before reach the customers (retailer). Besides manufacturing, similar complex networks could be found in telecommunications and transportations. Delay in one of the servers (or more) could affected the entire operation. Queue will be formed at stations or locations and customer might re-routed as this network is connected or the customer leave the network entirely. This behaviour is known as queueing networks, which has established in 1950s (Denning & Buzen, 1978) and Jackson and Gorden-Newell Queueing Network are among the earliest theorems published. Jackson network is a collection of several M/M/1 queues, i.e. 'M' indicates the users' arrival and service rate are based on Poisson distribution and independent while '1' means single server (Goodman & Massey, 1984). In Jackson network, the users are allowed to enter, move to the next network or exit anywhere (Denning and Buzen, 1978). Each movement (enter, exit or moves to next network) is based on probabilistic values, non-revisable (or forward movement) and open system. Jackson network is then extended by restricting the movement of users only within the network, i.e. closed system known as Gorden-Newell network. The closed system, where users are not allowed to enter and exit the system, while the open system does allow the users to enter and exit anywhere.

2. *Congestion as a function*

Some studies represent congestion through a function. For example, the US Bureau of Public Roads (BPR) developed a function that is used to find the travel time between two points based on traffic volume and capacity. The value was later modified to represent congestion costs (specifically, traffic congestion cost). The formula that was derived in the late 1960s by BPR as follows (Transportation Research Board, 2000, pp. 30-39):

$$T_{ij} = TF_{ij}\left(1 + \alpha\left(\frac{V_{ij}}{C_{ij}}\right)^{\beta}\right)$$

where, T_{ij} is travel time of customer *i* to facility *j*, TF_{ij} is free flow travel time of customer *i* to facility *j*, C_{ij} is the traffic capacity of customer *i* to facility *j*, α and β is a pre-set parameter that is based on characteristic of the network and V_{ij} indicates the traffic volume of customer *i* to facility *j*.

In some cases, congestion is added to the objective function as penalty to represent the cost incurred for passing through a congested area. To have a better understanding, for any vehicle that travels through any congested area, a penalty of γ value will be added to the objective function value. The value of γ varies according to the congestion level. Figure 0-9 portrays the basic idea of (Butt & Cavalier, 1997).

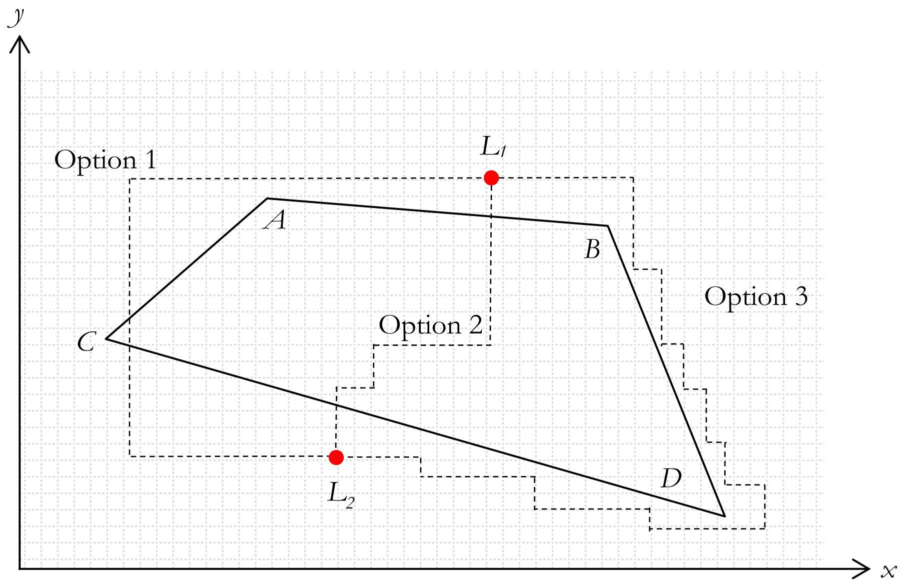


Figure 0-9: Planar in a region with rectilinear distance

Assuming that ABCD is a congested region where distance between L_1 and L_2 are computed as rectilinear (or Manhattan) distances, using L_1 and L_2 as a facility-demand pair (as one of the infinite facility-location pairs in this continuous space), three classes of paths could be followed by users in L_2 for accessing the facility in L_1, as reported in the following Table 0-1.

Table 0-1: Three classes of paths using L_1 and L_2

Option	Descriptions
1	Partially passing through the congested region
2	Mostly passing through the congested region
3	Not passing through the congested region at all

For 1 and 2, travelling through the congested region, a penalty value of γ will be added for each unit of distance travelled in the congested region. It can be noticed that, even though travelling through option 3 allows users not to incur any penalty γ, the total distance travelled may be larger than option 1 and 2.

Solution approaches

The optimisation model can be solved using optimisation methods such as *exact* or *heuristic* methods.

- *Exact* methods are constructed to guarantee optimal solutions in a finite amount of time. However, these methods might be only useful for problems with a small number of instances since as instances increase, the time taken to produce optimal solution increases as well.
- *Heuristic* approaches are helpful in producing a reasonably good solution within acceptable computational times. Heuristic approaches can be classified as follows (Martí & Reinelt, 2011):
- *Simple heuristics* are built to find a solution based on the specific problem or problem-dependent approach; for example, graph colouring and Dijkstra algorithm.
- *Meta-heuristics* find better quality solutions compared to the heuristic approach (Khoban & Ghadimi, 2009). Using a candidate solution, *meta-heuristic* approaches (for example steepest descent, simulated annealing, tabu search, ant colony algorithms and genetic algorithms) search through a given neighbourhood to find a "better" solution.
- *Hyper-heuristics* refers to a search method or automated methodology for selecting the most appropriate heuristic from a set of meta-heuristics (Burke et al., 2013).

Validation approaches

Validation ensures the developed model (or a given solution approach) is usable and makes sense. Barbati, et al. (2012) categorised validation types into six categories: ad-hoc built instances; comparison with heuristic techniques; comparison with previous scenarios; comparison with an exact algorithm; comparison with several approaches; lack of formal validation.

D. Facility location model with congestion issues: A systematic review

Congestion issues are among the issues considered in solving the facility location problem whereby selecting an optimal location that is able to resist congestion may contribute to a better future planning. In order to do a systematic review for this study, *Scopus* has been used as the literature search engine, using 'facility location' AND 'congest' syntax.

Keywords have been searched within the article title, abstract and keywords. Then, Mendeley was used as the reference software to avoid any repeated articles from *Scopus*. Articles explicitly involving congestion issues in decision variables and/ or parameters in facility location model were selected. Besides that, only articles that were published in international peer-reviewed journals were selected, and as result, 88 articles were found.

88 articles were retrieved from 1982 until 2019, as shown in Figure 0-10. From this figure, the number of articles published between the year 2001 and 2019 had slightly increased compared to previous years. However, the slight increment shows that the congestion issues are not the main interest of location science researchers.

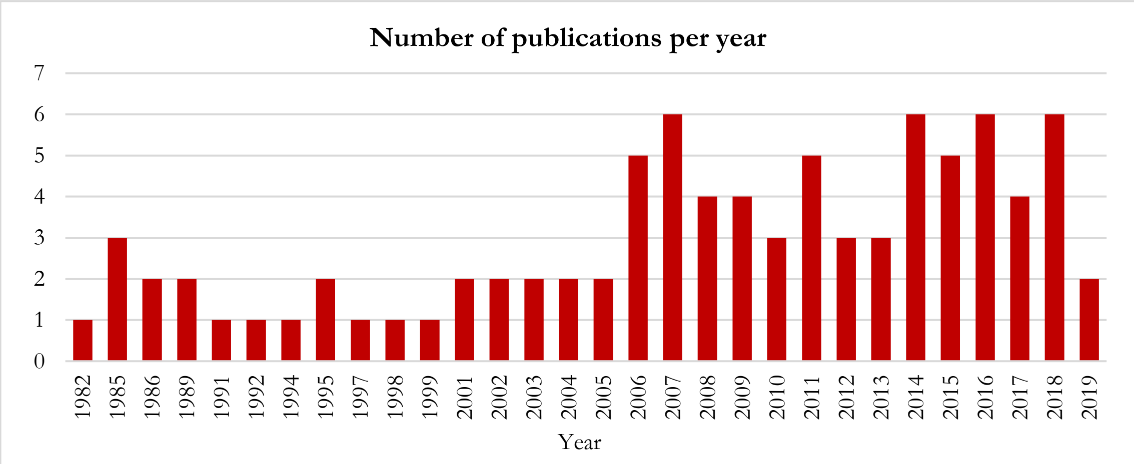


Figure 0-10: Number of publications per year

The number of articles published by each journal is indicated in Table 0-2. 60 articles out of 89 articles were published in the journal with at least 2 articles. Most of the articles were published in *Computers & Operations Research* (11), followed by *European Journal of Operational Research* and *IIE Transactions* (8 each). From this table, it is clearly shown that most of the articles for facility location with congestion issues were published in operations research and transportation research-based journals. This indicates that this field of study is characterised by some extent of interdisciplinary.

Table 0-2: Journal names that published with at least two articles.

Journal	Number of publications
Computers & Operations Research	11
European Journal of Operational Research	8
IIE Transactions	8

Annals of Operations Research	6
Transportation Research Part E: Logistics and Transportation Review	5
Transportation Research Part B: Methodological	3
Operations Research	3
Applied Mathematics & Computation	2
Journal of Operational Research Society	2
Manufacturing & Service Operations Management	2
Scientia Iranica	2
Socio-Economic Planning Sciences	2
The International Journal of Advanced Manufacturing Technology	2
Computer-aided Civil and Infrastructure Engineering	2
Naval Research Logistics	2
Total	60

Among the 89 articles found, only four articles that focussed on solving existing location model using new solution approach, as addressed in Table 0-3. Therefore, from this point onwards, the remaining 76 articles are further discussed in relation to its elements.

Table 0-3: Focus of each study

Article focus	Number of articles
Introduced/ Developed new optimisation model	76
Solved existing model using new solution approach (using new heuristic approach)	4

Table 0-4 classifies papers according to the number of objectives. 66 articles presented involved a single objective study. The remaining 6 articles proposed a multi-component function.

Table 0-4: Number of objectives

Number of objectives	Papers
Single objective	66
Multi-component	6

Table 0-5 shows 35 studies focussing on financial as the objective function. This is true since opening a new, closing an existing or re-organising facility networks is a costly operation. 16 studies were using demand considerations in other objective function, followed by a time-based objective function with 14 studies. Five studies dealt with physical issues. From six multi-component model, only two have two objectives. One that focus on physical and demand based objective function, and one focus on time and financial-based objective function.

Table 0-5: Objective functions used in facility location models with congestion issues

Objective functions	Number of articles
Financial-based	35
Demand-based	16
Time-based	14
Physical-based	5
Demand- and physical- based	1
Time- and financial- based	1

Table 0-6 presents the type of congestion issue dealt with in the 72 articles. There are 53 articles focussing on demand congestion, 12 on network congestion, five articles on area congestion; two articles coped with both demand and network congestion.

Table 0-6: Congestion issues

Congestion issues	Number of articles
Demand	53
Network	12
Area	5
Demand and traffic	2

Table 0-7 displays the list of techniques used to solve congestion issues in the mentioned papers. There are 43 articles that used queueing technique and 29 articles treated congestion as a function.

Table 0-7: Technique to solve congestion issues

Technique used	Number of articles
Queueing techniques	43
Congestion as function	29

Table 0-8 shows further classification on a model developed. Out of 72 articles, 51 utilised a stochastic based model, only two developed a multi-period model and 19 introduced a static model.

Table 0-8: Type of model developed

Model		Number of articles
Static model		19
Dynamic model	Stochastic based model	51
	Multi-period model	2

From the 72 articles, 43 used heuristic approach in solving the developed model; as shown in Table 0-9. Most location models are classified as an *NP-Hard* model (Owen & Daskin, 1998) thus, this figure is predictable. There are 29 articles that applied exact approaches.

Table 0-9: Solution approaches

Solution approach	Papers
Heuristic approach	43
Exact solution	29

Table 0-10 indicates that 42 studies focusses on numerical testing as their validation approach; 16 studies validated their model using comparison with heuristic techniques, seven studies were compared with previous studies, and seven articles used a comparison with exact algorithm.

Table 0-10: Validation types

Validation type	Papers
Numerical testing only	42
Comparison with heuristic techniques	16
Comparison with the previous study	7
Comparison with exact algorithm	7

From 89 articles, there is no study that opted for reorganising the entire service network. All articles focussed on expanding facility network at minimum financial allocation (details as in Table 0-11).

Evolution of the Field of Study

This section focusses on the review of the selected 72 articles. Two sections can be found: firstly, a discussion of early developments in facility location models with congestion issues; then, a review of the entire 72 articles based on several criteria.

1. Classification of facility location models with congestion issue

In dealing with congestion issues, most of the researchers are using dynamic techniques. There are also several studies that employed static techniques, which indirectly refer to planar versions problem.

Table 0-11 classifies 76 articles across some of the characteristics discussed before. This table also indicates into decision-maker approach: single and multi-components model, congestion type: demand, traffic and area; model types: static and dynamic type of model; space type: network, continuous and discrete; and decision-maker's action towards coping with facility location problem with congestion problem. In each article, financial-based objective function, hierarchical structure existence, fuzzy set theory implementation, multi-period model and facility network expansion's actions, are further observed.

Table 0-11: Classification of selected articles on developed model

Authors (year)	DM approach		Congestion			Model	Dynamic model		Space type			Solution technique		Validation approach				Application to real life problem	DM's action
	Single objective	Multi-components	Demand	Traffic	Area	Static model	Stochastic based model	Multi-period model	Discrete	Continuous	Network	Exact approach	Heuristic approach	Numerical testing	Comparison with heuristics techniques	Comparison between exact algorithm	Comparison with previous study		
Berman & Larson (1982)	✓[1]		✓				✓				✓	✓		✓					Expand
Berman et al. (1985)	✓[1]		✓				✓				✓		✓				✓		Expand
Berman (1985)	✓[1]		✓				✓				✓		✓				✓		Expand
Chiu (1986)	✓[1]		✓				✓				✓	✓		✓					Expand
Batta (1989)	✓[1]		✓				✓				✓	✓		✓					Expand
Liu & Ralston (1989)	✓[2]			✓		✓					✓	✓		✓					Expand
Braid (1991)	✓[2]				✓	✓				✓		✓		✓					Expand
Brandeau & Chiu (1992)	✓[1]		✓				✓				✓	✓					✓		Expand
Melachrinoudis (1994)	✓[2]		✓				✓		✓				✓			✓			Expand
Berman (1995)	✓[3]		✓				✓				✓		✓	✓					Expand
Desrochers et al. (1995)	✓[2]		✓			✓					✓		✓	✓					Expand
Butt & Cavalier (1997)	✓[4]				✓	✓				✓		✓		✓					Expand
Marianov & Serra (1998)	✓[3]		✓				✓				✓		✓			✓			Expand
Taniguchi et al. (1999)		✓[2]	✓	✓			✓				✓		✓	✓				✓	Expand

[1] Time-based objective function
[2] Financial-based objective function
[3] Demand-based objective function
[4] Physical-based objective function

Wong & Sun (2001)	✓[2]			✓		✓				✓		✓		✓					Expand
Marianov & Serra (2001)		✓[3,4]	✓				✓				✓		✓			✓			Expand[5,6,7]
Marianov & Serra (2002)	✓[4]			✓		✓					✓	✓		✓					Expand[5]
Wang et al. (2002)	✓[1]		✓				✓				✓		✓		✓			✓	Expand
Marianov (2003)	✓[3]		✓				✓				✓		✓	✓					Expand
Marianov & Serra (2003)	✓[2]		✓				✓				✓		✓	✓				✓	Expand[5]
Shavandi & Mahlooji (2004)	✓		✓				✓[10]				✓	✓		✓					Expand[5]
Wang et al. (2004)		✓[2]		✓		✓			✓			✓		✓				✓	Expand
Galvão et al. (2005)	✓[3]		✓				✓[8]				✓		✓		✓				Expand
Sarkar et al. (2005)	✓				✓	✓				✓		✓		✓					Expand[9]
Berman & Drezner (2006)	✓[3]		✓				✓				✓		✓		✓				Expand
Berman et al. (2006)	✓		✓				✓				✓		✓		✓				Expand
Elhedhli (2006)	✓[2]		✓				✓			✓		✓		✓					Expand
Shavandi & Mahlooji (2006)	✓[3]		✓				✓[10]				✓		✓	✓					Expand
Shavandi et al. (2006)	✓[3]		✓				✓[10]				✓	✓					✓		Expand[6,7]
Berman et al. (2007)	✓[3]		✓				✓				✓		✓		✓				Expand
Dobson & Stavrulaki (2007)	✓[2]		✓				✓				✓	✓		✓					Expand[5]
Romeijn et al. (2007)	✓[2]		✓				✓[8]			✓		✓		✓					Expand
Sourirajan et al. (2007)	✓[2]		✓			✓					✓		✓	✓					Expand
Vardar et al. (2007)	✓[2]		✓				✓[8]		✓				✓	✓				✓	Expand[5]
Rodríguez et al. (2007)	✓[2]		✓				✓				✓		✓	✓					Expand[5]
Marianov et al. (2008)	✓[1]		✓				✓		✓			✓		✓					Expand[5]
Aboolian et al. (2008)	✓[2]		✓				✓				✓		✓		✓				Expand
Shavandi & Mahlooji (2008)	✓[2]		✓				✓				✓	✓					✓		Expand[6,7]
Baron et al. (2008)	✓[2]		✓				✓			✓			✓	✓					Expand
Castillo et al. (2009)	✓[2]		✓				✓				✓		✓	✓				✓	Expand[5]
Beraldi & Bruni (2009)	✓[2]		✓				✓[8]			✓			✓		✓				Expand[5]

[5] Increase up to *k*-servers
[6] Multi-type facilities
[7] Hierarchical facilities
[8] Probabilistic variable
[9] Increase dimension of facility
[10] Fuzzy set theory

Zhang et al. (2009)	✓[3]		✓				✓				✓		✓		✓			✓	Expand[5]
Zhang et al. (2010)	✓[3]		✓				✓				✓		✓				✓	✓	Expand[5]
Köksalan & Soylu (2010)		✓[2]	✓			✓					✓		✓	✓				✓	Expand
Seifbarghy et al. (2010)	✓[3]		✓				✓				✓		✓		✓				Expand[5]
Bai et al. (2011)	✓[2]		✓			✓					✓		✓	✓				✓	Expand
Abouee-Mehrizi et al. (2011)	✓[2]		✓				✓				✓		✓	✓					Expand
Chambari et al. (2011)		✓[1]	✓				✓		✓				✓		✓				Expand
Marianov & Serra (2011)	✓[2]		✓				✓		✓				✓	✓					Expand
Konur & Geunes (2011)	✓[2]			✓		✓					✓	✓		✓					Expand
Konur & Geunes (2012)	✓[2]			✓		✓					✓		✓		✓				Expand
Kim (2012)	✓[2]		✓				✓			✓			✓		✓			✓	Expand
Aboolian et al. (2012)	✓[2]		✓				✓				✓		✓			✓			Expand[5]
Hu et al. (2013)	✓[3]		✓				✓				✓		✓	✓					Expand[5]
Jouzdani et al. (2013)	✓[2]			✓				✓[10]			✓	✓		✓				✓	Expand[6,7]
Hajibabai & Ouyang (2013)	✓[2]			✓		✓					✓		✓	✓				✓	Expand[9]
Hajibabai et al.2014)	✓[2]			✓		✓					✓	✓					✓	✓	Expand
Rahmaniani et al. (2014)	✓[2]		✓				✓[8]				✓		✓		✓			✓	Expand
Vidyarthi & Jayaswal (2014)	✓[2]		✓				✓				✓	✓		✓					Expand
Farahani et al. (2014)	✓[3]		✓				✓[8]		✓				✓			✓		✓	Expand
Cho et al. (2014)	✓[3]		✓				✓[8]		✓				✓		✓			✓	Expand
Date et al. (2014)	✓[4]				✓	✓				✓		✓		✓					Expand
Saleh Farham et al. (2015)	✓[2]				✓	✓				✓			✓		✓				Expand
An et al. (2015)	✓[2]		✓	✓			✓[8]				✓	✓				✓			Expand
Lee (2015)	✓[2]			✓				✓			✓	✓		✓				✓	Expand
Vidyarthi & Kuzgunkaya (2015)	✓[1]		✓				✓				✓	✓		✓				✓	Expand[9]
Aboolian et al. (2016)	✓[3]		✓				✓				✓	✓		✓					Expand
Davari et al. (2016)	✓[3]		✓				✓				✓		✓			✓			Expand
Hajipour et al. (2016)		✓[1,2]	✓				✓				✓		✓		✓				Expand[5]
Hwang et al. (2016)	✓[1]			✓		✓					✓		✓	✓				✓	Expand
Zarrinpoor et al. (2017)	✓[1]		✓				✓				✓	✓		✓				✓	Expand
Atashi Kohei et al. (2017)	✓[1]			✓				✓		✓		✓		✓					Expand

APPENDIX 3: DETAILED INFORMATION FOR CHAPTER 3

This section presents the related information for Chapter 3. This includes the formulation used and details results for the sensitivity analyses for single- and multi-component model.

A. Formulation used to calculate the percentage of each flow of demand within the system network.

Presented below is the general formulations used in calculating the flow of demand within the network. The same formulations were applied in Chapter 5 and 6 as well.

- Percentage of total operating periods for all facility *j* $\left(\% \sum_j \sum_t y_j^t\right)$ $= \dfrac{\sum_j \sum_t y_j^t}{J' \times T'}$
- Percentage of demand served by all facility *j* $\left(\% \sum_j \sum_t q_j^t\right)$ $= \dfrac{\sum_j \sum_t q_j^t}{\sum_j \sum_t x_j^t}$
- Percentage of demand leave the system $\left(\% \sum_j \sum_t l_j^t\right)$ $= \dfrac{\sum_j \sum_t l_j^t}{\sum_j \sum_t x_j^t}$
- Percentage of demand being in the queue in the system $\left(\% \sum_j \sum_t s_j^t\right)$ $= \dfrac{\sum_j \sum_t s_j^t}{\sum_j \sum_t x_j^t}$
- Percentage of demand move to another facility *j* $\left(\% \sum_j \sum_t S_{jk}^t\right)$ $= \dfrac{\sum_j \sum_t S_{jk}^t}{\sum_j \sum_t x_j^t}$
- Percentage of demand in the queue (after period $t - G$) $= \dfrac{\sum_j \sum_{t-G<t\leq T'} s_j^t}{\sum_j \sum_t x_j^t}$

B. Sensitivity analysis: the results

Results obtained for the sensitivity analyses, in particular the trade-off in each variable's costs, are presented in this section. The results presented are for the analyses that was conducted in Chapter 3. Meanwhile, the numerical results for sensitivity analyses of the multi-component model using different dataset, is presented in section C.

Results of sensitivity analyses for Chapter 3 (section 3.5)

Varying C_j

α_1	C_j	*TC*	Z_1	Z_2	$\%\sum_j \sum_t y_j^t$
	200	8,900	89,000	-	100%
	400	9,300	93,000	-	100%
	600	9,700	97,000	-	100%
	800	10,100	101,000	-	100%
	1000	10,500	105,000	-	100%
	1200	10,900	109,000	-	100%
	1400	11,300	113,000	-	100%
	1600	11,700	117,000	-	100%
	1800	12,100	121,000	-	100%
	2000	12,500	125,000	-	100%
	2200	12,900	129,000	-	100%
	2400	13,300	133,000	-	100%
0.1	2600	13,700	137,000	-	100%
	2800	14,100	141,000	-	100%
	3000	14,500	145,000	-	100%
	3200	14,900	149,000	-	100%
	3400	15,300	153,000	-	100%
	3600	15,700	157,000	-	100%
	3800	16,100	161,000	-	100%
	4000	16,500	165,000	-	100%
	4200	16,900	169,000	-	100%
	4400	17,300	173,000	-	100%
	4600	17,700	177,000	-	100%
	4800	18,100	181,000	-	100%
	5000	18,500	185,000	-	100%
	200	17,800	89,000	-	100%
	400	18,600	93,000	-	100%
	600	19,400	97,000	-	100%
	800	20,200	101,000	-	100%
	1000	21,000	105,000	-	100%
	1200	21,800	109,000	-	100%
0.2	1400	22,600	113,000	-	100%
	1600	23,400	117,000	-	100%
	1800	24,200	121,000	-	100%
	2000	25,000	125,000	-	100%
	2200	25,800	129,000	-	100%
	2400	26,600	133,000	-	100%
	2600	27,400	137,000	-	100%

α_1	C_j	*TC*	Z_1	Z_2	$\%\sum_j \sum_t y_j^t$
	200	35,600	89,000	-	100%
	400	37,200	93,000	-	100%
	600	38,800	97,000	-	100%
	800	40,400	101,000	-	100%
	1000	42,000	105,000	-	100%
	1200	43,600	109,000	-	100%
	1400	45,200	113,000	-	100%
	1600	46,800	117,000	-	100%
	1800	48,400	121,000	-	100%
	2000	50,000	125,000	-	100%
	2200	51,600	129,000	-	100%
	2400	53,140	130,600	1,500	95%
0.4	2600	54,660	134,400	1,500	95%
	2800	56,180	138,200	1,500	95%
	3000	57,700	142,000	1,500	95%
	3200	59,220	145,800	1,500	95%
	3400	60,740	149,600	1,500	95%
	3600	62,260	153,400	1,500	95%
	3800	63,780	157,200	1,500	95%
	4000	65,300	161,000	1,500	95%
	4200	66,820	164,800	1,500	95%
	4400	68,340	168,600	1,500	95%
	4600	69,860	172,400	1,500	95%
	4800	71,380	176,200	1,500	95%
	5000	72,800	170,000	8,000	85%
	200	44,500	89,000	-	100%
	400	46,500	93,000	-	100%
	600	48,500	97,000	-	100%
	800	50,500	101,000	-	100%
	1000	52,500	105,000	-	100%
	1200	54,500	109,000	-	100%
0.5	1400	56,500	113,000	-	100%
	1600	58,450	115,400	1,500	95%
	1800	60,350	119,200	1,500	95%
	2000	62,250	123,000	1,500	95%
	2200	64,150	126,800	1,500	95%
	2400	66,050	130,600	1,500	95%
	2600	67,950	134,400	1,500	95%

α_1	C_j	*TC*	Z_1	Z_2	$\%\sum_j \sum_t y_j^t$
	200	62,160	88,800	-	95%
	400	64,820	92,600	-	95%
	600	67,480	96,400	-	95%
	800	70,140	100,200	-	95%
	1000	72,800	104,000	-	95%
	1200	75,460	107,800	-	95%
	1400	78,120	111,600	-	95%
	1600	80,780	115,400	-	95%
	1800	83,440	119,200	-	95%
	2000	85,975	121,000	4,250	90%
	2200	88,495	124,600	4,250	90%
	2400	90,910	125,800	9,500	85%
0.7	2600	93,290	129,200	9,500	85%
	2800	95,670	132,600	9,500	85%
	3000	98,050	136,000	9,500	85%
	3200	100,430	139,400	9,500	85%
	3400	102,810	142,800	9,500	85%
	3600	105,190	146,200	9,500	85%
	3800	107,570	149,600	9,500	85%
	4000	109,950	153,000	9,500	85%
	4200	112,330	156,400	9,500	85%
	4400	114,710	159,800	9,500	85%
	4600	117,090	163,200	9,500	85%
	4800	119,470	166,600	9,500	85%
	5000	121,850	170,000	9,500	85%
	200	71,040	88,800	-	95%
	400	74,080	92,600	-	95%
	600	77,120	96,400	-	95%
	800	80,160	100,200	-	95%
	1000	83,200	104,000	-	95%
	1200	86,130	106,600	4,250	90%
0.8	1400	88,940	108,800	9,500	85%
	1600	91,660	112,200	9,500	85%
	1800	94,380	115,600	9,500	85%
	2000	97,100	119,000	9,500	85%
	2200	99,820	122,400	9,500	85%
	2400	102,540	125,800	9,500	85%
	2600	105,260	129,200	9,500	85%

	2800	28,200	141,000	-	100%
	3000	29,000	145,000	-	100%
	3200	29,800	149,000	-	100%
	3400	30,600	153,000	-	100%
	3600	31,400	157,000	-	100%
	3800	32,200	161,000	-	100%
	4000	33,000	165,000	-	100%
	4200	33,800	169,000	-	100%
	4400	34,600	173,000	-	100%
	4600	35,400	177,000	-	100%
	4800	36,200	181,000	-	100%
	5000	37,000	185,000	-	100%
	200	26,700	89,000	-	100%
	400	27,900	93,000	-	100%
	600	29,100	97,000	-	100%
	800	30,300	101,000	-	100%
	1000	31,500	105,000	-	100%
	1200	32,700	109,000	-	100%
	1400	33,900	113,000	-	100%
	1600	35,100	117,000	-	100%
	1800	36,300	121,000	-	100%
	2000	37,500	125,000	-	100%
	2200	38,700	129,000	-	100%
	2400	39,900	133,000	-	100%
0.3	2600	41,100	137,000	-	100%
	2800	42,300	141,000	-	100%
	3000	43,500	145,000	-	100%
	3200	44,700	149,000	-	100%
	3400	45,900	153,000	-	100%
	3600	47,070	153,400	1,500	95%
	3800	48,210	157,200	1,500	95%
	4000	49,350	161,000	1,500	95%
	4200	50,490	164,800	1,500	95%
	4400	51,630	168,600	1,500	95%
	4600	52,770	172,400	1,500	95%
	4800	53,910	176,200	1,500	95%
	5000	55,050	180,000	1,500	95%

	2800	69,850	138,200	1,500	95%
	3000	71,750	142,000	1,500	95%
	3200	73,650	145,800	1,500	95%
	3400	75,400	142,800	8,000	85%
	3600	77,100	146,200	8,000	85%
	3800	78,800	149,600	8,000	85%
	4000	80,500	153,000	8,000	85%
	4200	82,200	156,400	8,000	85%
	4400	83,900	159,800	8,000	85%
	4600	85,600	163,200	8,000	85%
	4800	87,300	166,600	8,000	85%
	5000	89,000	170,000	8,000	85%
	200	53,280	88,800	-	95%
	400	55,560	92,600	-	95%
	600	57,840	96,400	-	95%
	800	60,120	100,200	-	95%
	1000	62,400	104,000	-	95%
	1200	64,680	107,800	-	95%
	1400	66,960	111,600	-	95%
	1600	69,240	115,400	-	95%
	1800	71,520	119,200	-	95%
	2000	73,800	123,000	-	95%
	2200	76,080	126,800	-	95%
	2400	78,360	130,600	-	95%
0.6	2600	80,640	134,400	-	95%
	2800	82,920	138,200	-	95%
	3000	85,100	139,000	4,250	90%
	3200	87,260	142,600	4,250	90%
	3400	89,420	146,200	4,250	90%
	3600	91,520	146,200	9,500	85%
	3800	93,560	149,600	9,500	85%
	4000	95,600	153,000	9,500	85%
	4200	97,640	156,400	9,500	85%
	4400	99,680	159,800	9,500	85%
	4600	101,720	163,200	9,500	85%
	4800	103,760	166,600	9,500	85%
	5000	105,800	170,000	9,500	85%

	2800	107,980	132,600	9,500	85%
	3000	110,700	136,000	9,500	85%
	3200	113,420	139,400	9,500	85%
	3400	116,140	142,800	9,500	85%
	3600	118,860	146,200	9,500	85%
	3800	121,580	149,600	9,500	85%
	4000	124,300	153,000	9,500	85%
	4200	127,020	156,400	9,500	85%
	4400	129,740	159,800	9,500	85%
	4600	132,460	163,200	9,500	85%
	4800	135,180	166,600	9,500	85%
	5000	137,900	170,000	9,500	85%
	200	79,920	88,800	-	95%
	400	83,340	92,600	-	95%
	600	86,630	95,200	9,500	85%
	800	89,690	98,600	9,500	85%
	1000	92,750	102,000	9,500	85%
	1200	95,810	105,400	9,500	85%
	1400	98,870	108,800	9,500	85%
	1600	101,930	112,200	9,500	85%
	1800	104,990	115,600	9,500	85%
	2000	108,050	119,000	9,500	85%
	2200	111,110	122,400	9,500	85%
	2400	114,170	125,800	9,500	85%
0.9	2600	117,230	129,200	9,500	85%
	2800	120,290	132,600	9,500	85%
	3000	123,350	136,000	9,500	85%
	3200	126,410	139,400	9,500	85%
	3400	129,470	142,800	9,500	85%
	3600	132,530	146,200	9,500	85%
	200	62,160	88,800	-	95%
	400	64,820	92,600	-	95%
	600	67,480	96,400	-	95%
	800	70,140	100,200	-	95%
	1000	72,800	104,000	-	95%
	1200	75,460	107,800	-	95%
	1400	78,120	111,600	-	95%

Varying ε_{1j}

α_1	ε_{1j}	TC	Z_1	Z_2	$\%\sum_j \sum_t q_j^t$
	10	3,900	39,000	-	97%
	20	5,550	55,500	-	97%
	30	7,200	72,000	-	97%
	40	8,850	88,500	-	97%
	50	10,500	105,000	-	95%
	60	12,115	121,150	-	95%
	70	13,730	137,300	-	95%
0.1	80	15,345	153,450	-	95%
	90	16,960	169,600	-	95%
	100	18,575	185,750	-	95%
	110	20,190	201,900	-	95%
	120	21,805	218,050	-	95%
	130	23,420	234,200	-	95%
	140	25,035	250,350	-	95%
	150	26,650	266,500	-	95%
	10	7,800	39,000	-	97%
	20	11,100	55,500	-	97%
	30	14,400	72,000	-	97%
	40	17,700	88,500	-	97%
	50	21,000	105,000	-	95%
	60	24,230	121,150	-	95%
	70	27,460	137,300	-	95%
0.2	80	30,690	153,450	-	95%
	90	33,920	169,600	-	95%
	100	37,150	185,750	-	95%
	110	40,380	201,900	-	95%
	120	43,610	218,050	-	95%
	130	46,840	234,200	-	95%
	140	50,070	250,350	-	95%
	150	53,300	266,500	-	95%
	10	11,700	39,000	-	97%
	20	16,650	55,500	-	97%
	30	21,600	72,000	-	97%
	40	26,550	88,500	-	97%
	50	31,500	105,000	-	95%
	60	36,345	121,150	-	95%
	70	41,190	137,300	-	95%
0.3	80	46,035	153,450	-	95%
	90	50,880	169,600	-	95%
	100	55,725	185,750	-	95%
	110	60,570	201,900	-	95%
	120	65,415	218,050	-	95%
	130	70,260	234,200	-	95%
	140	75,105	250,350	-	95%
	150	79,950	266,500	-	95%

α_1	ε_{1j}	TC	Z_1	Z_2	$\%\sum_j \sum_t q_j^t$
	10	15,600	39,000	-	97%
	20	22,200	55,500	-	97%
	30	28,800	72,000	-	97%
	40	35,400	88,500	-	97%
	50	42,000	105,000	-	95%
	60	48,460	121,150	-	95%
	70	54,920	137,300	-	95%
0.4	80	61,380	153,450	-	95%
	90	67,840	169,600	-	95%
	100	74,300	185,750	-	95%
	110	80,760	201,900	-	95%
	120	87,220	218,050	-	95%
	130	93,680	234,200	-	95%
	140	100,140	250,350	-	95%
	150	106,600	266,500	-	95%
	10	19,500	39,000	-	97%
	20	27,750	55,500	-	97%
	30	36,000	72,000	-	97%
	40	44,250	88,500	-	97%
	50	52,500	105,000	-	95%
	60	60,575	121,150	-	95%
	70	68,650	137,300	-	95%
0.5	80	76,725	153,450	-	95%
	90	84,800	169,600	-	95%
	100	92,875	185,750	-	95%
	110	100,950	201,900	-	95%
	120	109,025	218,050	-	95%
	130	117,100	234,200	-	95%
	140	125,175	250,350	-	95%
	150	133,250	266,500	-	95%
	10	37,000	2,500	100%	23,200
	20	55,500	-	97%	33,300
	30	72,000	-	97%	43,200
	40	88,500	-	97%	53,100
	50	105,000	-	95%	63,000
	60	121,150	-	95%	72,690
	70	137,300	-	95%	82,380
0.6	80	153,450	-	95%	92,070
	90	169,600	-	95%	101,760
	100	185,750	-	95%	111,450
	110	201,900	-	95%	121,140
	120	218,050	-	95%	130,830
	130	234,200	-	95%	140,520
	140	250,350	-	95%	150,210
	150	266,500	-	95%	159,900

α_1	ε_{1j}	TC	Z_1	Z_2	$\%\sum_j \sum_t q_j^t$
	10	37,000	2,500	100%	26,650
	20	54,000	2,500	100%	38,550
	30	72,000	-	97%	50,400
	40	87,850	1,500	95%	61,945
	50	104,000	1,500	95%	73,250
	60	120,150	1,500	95%	84,555
	70	136,300	1,500	95%	95,860
0.7	80	152,450	1,500	95%	107,165
	90	168,600	1,500	95%	118,470
	100	184,750	1,500	95%	129,775
	110	200,900	1,500	95%	141,080
	120	217,050	1,500	95%	152,385
	130	233,200	1,500	95%	163,690
	140	249,350	1,500	95%	174,995
	150	265,500	1,500	95%	186,300
	10	36,000	6,000	100%	30,000
	20	53,000	6,000	100%	43,600
	30	70,100	6,000	100%	57,180
	40	85,850	8,000	95%	70,280
	50	102,000	8,000	95%	83,200
	60	118,150	8,000	95%	96,120
	70	134,300	8,000	95%	109,040
0.8	80	150,450	8,000	95%	121,960
	90	166,600	8,000	95%	134,880
	100	182,750	8,000	95%	147,800
	110	198,900	8,000	95%	160,720
	120	215,050	8,000	95%	173,640
	130	231,200	8,000	95%	186,560
	140	247,350	8,000	95%	199,480
	150	263,500	8,000	95%	212,400
	10	35,000	13,250	100%	32,825
	20	52,000	13,250	100%	48,125
	30	68,800	14,500	98%	63,370
	40	85,850	8,000	95%	78,065
	50	102,000	8,000	95%	92,600
	60	118,150	8,000	95%	107,135
	70	134,300	8,000	95%	121,670
0.9	80	150,450	8,000	95%	136,205
	90	166,600	8,000	95%	150,740
	100	182,750	8,000	95%	165,275
	110	198,900	8,000	95%	179,810
	120	215,050	8,000	95%	194,345
	130	231,200	8,000	95%	208,880
	140	247,350	8,000	95%	223,415
	150	263,500	8,000	95%	237,950

Varying ε_{2j}

α_1	ε_{2j}	*TC*	Z_1	Z_2	$\%\sum_j \sum_t s_j^t$
	10	10,500	105,000	-	0%
	20	10,500	105,000	-	0%
	30	10,500	105,000	-	0%
	40	10,500	105,000	-	0%
	50	10,500	105,000	-	0%
	60	10,500	105,000	-	0%
	70	10,500	105,000	-	0%
0.1	80	10,500	105,000	-	0%
	90	10,500	105,000	-	0%
	100	10,500	105,000	-	0%
	110	10,500	105,000	-	0%
	120	10,500	105,000	-	0%
	130	10,500	105,000	-	0%
	140	10,500	105,000	-	0%
	150	10,500	105,000	-	0%
	10	21,000	105,000	-	0%
	20	21,000	105,000	-	0%
	30	21,000	105,000	-	0%
	40	21,000	105,000	-	0%
	50	21,000	105,000	-	0%
	60	21,000	105,000	-	0%
	70	21,000	105,000	-	0%
0.2	80	21,000	105,000	-	0%
	90	21,000	105,000	-	0%
	100	21,000	105,000	-	0%
	110	21,000	105,000	-	0%
	120	21,000	105,000	-	0%
	130	21,000	105,000	-	0%
	140	21,000	105,000	-	0%
	150	21,000	105,000	-	0%
	10	31,410	104,000	300	2%
	20	31,500	105,000	-	0%
	30	31,500	105,000	-	0%
	40	31,500	105,000	-	0%
	50	31,500	105,000	-	0%
	60	31,500	105,000	-	0%
	70	31,500	105,000	-	0%
0.3	80	31,500	105,000	-	0%
	90	31,500	105,000	-	0%
	100	31,500	105,000	-	0%
	110	31,500	105,000	-	0%
	120	31,500	105,000	-	0%
	130	31,500	105,000	-	0%
	140	31,500	105,000	-	0%
	150	31,500	105,000	-	0%

α_1	ε_{2j}	*TC*	Z_1	Z_2	$\%\sum_j \sum_t s_j^t$
	10	41,780	104,000	300	2%
	20	41,960	104,000	600	2%
	30	42,000	105,000	-	0%
	40	42,000	105,000	-	0%
	50	42,000	105,000	-	0%
	60	42,000	105,000	-	0%
	70	42,000	105,000	-	0%
0.4	80	42,000	105,000	-	0%
	90	42,000	105,000	-	0%
	100	42,000	105,000	-	0%
	110	42,000	105,000	-	0%
	120	42,000	105,000	-	0%
	130	42,000	105,000	-	0%
	140	42,000	105,000	-	0%
	150	42,000	105,000	-	0%
	10	52,150	104,000	300	2%
	20	52,300	104,000	600	2%
	30	52,450	104,000	900	2%
	40	52,500	105,000	-	0%
	50	52,500	105,000	-	0%
	60	52,500	105,000	-	0%
	70	52,500	105,000	-	0%
0.5	80	52,500	105,000	-	0%
	90	52,500	105,000	-	0%
	100	52,500	105,000	-	0%
	110	52,500	105,000	-	0%
	120	52,500	105,000	-	0%
	130	52,500	105,000	-	0%
	140	52,500	105,000	-	0%
	150	52,500	105,000	-	0%
	10	62,360	103,000	1,400	8%
	20	62,640	104,000	600	2%
	30	62,760	104,000	900	2%
	40	62,880	104,000	1,200	2%
	50	63,000	105,000	-	0%
	60	63,000	105,000	-	0%
	70	63,000	105,000	-	0%
0.6	80	63,000	105,000	-	0%
	90	63,000	105,000	-	0%
	100	63,000	105,000	-	0%
	110	63,000	105,000	-	0%
	120	63,000	105,000	-	0%
	130	63,000	105,000	-	0%
	140	63,000	105,000	-	0%
	150	63,000	105,000	-	0%

α_1	ε_{2j}	*TC*	Z_1	Z_2	$\%\sum_j \sum_t s_j^t$
	10	72,405	102,000	3,350	20%
	20	72,940	103,000	2,800	8%
	30	73,070	104,000	900	2%
	40	73,160	104,000	1,200	2%
	50	73,250	104,000	1,500	0%
	60	73,250	104,000	1,500	0%
	70	73,250	104,000	1,500	0%
0.7	80	73,250	104,000	1,500	0%
	90	73,250	104,000	1,500	0%
	100	73,250	104,000	1,500	0%
	110	73,250	104,000	1,500	0%
	120	73,250	104,000	1,500	0%
	130	73,250	104,000	1,500	0%
	140	73,250	104,000	1,500	0%
	150	73,250	104,000	1,500	0%
	10	82,270	102,000	3,350	17%
	20	82,800	102,000	6,000	13%
	30	83,050	102,000	7,250	3%
	40	83,130	102,000	7,650	2%
	50	83,200	102,000	8,000	2%
	60	83,270	102,000	8,350	2%
	70	83,340	102,000	8,700	2%
0.8	80	83,350	103,000	4,750	0%
	90	83,350	103,000	4,750	0%
	100	83,350	103,000	4,750	0%
	110	83,350	103,000	4,750	0%
	120	83,350	103,000	4,750	0%
	130	83,350	103,000	4,750	0%
	140	83,350	103,000	4,750	0%
	150	83,350	103,000	4,750	0%
	10	92,135	102,000	3,350	20%
	20	92,400	102,000	6,000	13%
	30	92,525	102,000	7,250	3%
	40	92,565	102,000	7,650	2%
	50	92,600	102,000	8,000	2%
	60	92,635	102,000	8,350	2%
	70	92,670	102,000	8,700	2%
0.9	80	92,705	102,000	9,050	2%
	90	92,740	102,000	9,400	2%
	100	92,775	102,000	9,750	2%
	110	92,810	102,000	10,100	2%
	120	92,845	102,000	10,450	2%
	130	92,880	102,000	10,800	2%
	140	92,915	102,000	11,150	2%
	150	92,950	102,000	11,500	2%

Varying ε_{3j}

α_1	ε_{3j}	*TC*	Z_1	Z_2	$\%\sum_j \sum_t l_j^t$
	10	10,160	101,600	-	5%
	20	10,245	102,450	-	5%
	30	10,330	103,300	-	5%
	40	10,415	104,150	-	5%
	50	10,500	105,000	-	5%
	60	10,550	105,500	-	3%
	70	10,600	106,000	-	3%
0.1	80	10,650	106,500	-	3%
	90	10,700	107,000	-	3%
	100	10,750	107,500	-	3%
	110	10,800	108,000	-	3%
	120	10,850	108,500	-	3%
	130	10,900	109,000	-	3%
	140	10,950	109,500	-	3%
	150	11,000	110,000	-	3%
	10	20,320	101,600	-	5%
	20	20,490	102,450	-	5%
	30	20,660	103,300	-	5%
	40	20,830	104,150	-	5%
	50	21,000	105,000	-	5%
	60	21,100	105,500	-	3%
	70	21,200	106,000	-	3%
0.2	80	21,300	106,500	-	3%
	90	21,400	107,000	-	3%
	100	21,500	107,500	-	3%
	110	21,600	108,000	-	3%
	120	21,700	108,500	-	3%
	130	21,800	109,000	-	3%
	140	21,900	109,500	-	3%
	150	22,000	110,000	-	3%
	10	30,480	101,600	-	5%
	20	30,735	102,450	-	5%
	30	30,990	103,300	-	5%
	40	31,245	104,150	-	5%
	50	31,500	105,000	-	5%
	60	31,650	105,500	-	3%
	70	31,800	106,000	-	3%
0.3	80	31,950	106,500	-	3%
	90	32,100	107,000	-	3%
	100	32,250	107,500	-	3%
	110	32,400	108,000	-	3%
	120	32,550	108,500	-	3%
	130	32,700	109,000	-	3%
	140	32,850	109,500	-	3%
	150	33,000	110,000	-	3%

α_1	ε_{3j}	*TC*	Z_1	Z_2	$\%\sum_j \sum_t l_j^t$
	10	40,640	101,600	-	5%
	20	40,980	102,450	-	5%
	30	41,320	103,300	-	5%
	40	41,660	104,150	-	5%
	50	42,000	105,000	-	5%
	60	42,200	105,500	-	3%
	70	42,400	106,000	-	3%
0.4	80	42,600	106,500	-	3%
	90	42,800	107,000	-	3%
	100	43,000	107,500	-	3%
	110	43,200	108,000	-	3%
	120	43,400	108,500	-	3%
	130	43,500	109,500	2,500	0%
	140	43,500	109,500	2,500	0%
	150	43,500	109,500	2,500	0%
	10	50,800	101,600	-	5%
	20	51,225	102,450	-	5%
	30	51,650	103,300	-	5%
	40	52,075	104,150	-	5%
	50	52,500	105,000	-	5%
	60	52,750	105,500	-	3%
	70	53,000	106,000	-	3%
0.5	80	53,250	106,500	-	3%
	90	53,500	107,000	-	3%
	100	53,750	107,500	-	3%
	110	53,750	105,000	2,500	0%
	120	53,750	105,000	2,500	0%
	130	53,750	105,000	2,500	0%
	140	53,750	105,000	2,500	0%
	150	53,750	105,000	2,500	0%
	10	60,960	101,600	-	5%
	20	61,470	102,450	-	5%
	30	61,980	103,300	-	5%
	40	62,490	104,150	-	5%
	50	63,000	105,000	-	5%
	60	63,300	105,500	-	3%
	70	63,600	106,000	-	3%
0.6	80	63,900	106,500	-	3%
	90	64,000	105,000	2,500	0%
	100	64,000	105,000	2,500	0%
	110	64,000	105,000	2,500	0%
	120	64,000	105,000	2,500	0%
	130	64,000	105,000	2,500	0%
	140	64,000	105,000	2,500	0%
	150	64,000	105,000	2,500	0%

α_1	ε_{3j}	*TC*	Z_1	Z_2	$\%\sum_j \sum_t l_j^t$
	10	70,870	100,600	1,500	5%
	20	71,465	101,450	1,500	5%
	30	72,060	102,300	1,500	5%
	40	72,655	103,150	1,500	5%
	50	73,250	104,000	1,500	5%
	60	73,845	104,850	1,500	5%
	70	74,200	106,000	-	3%
0.7	80	74,250	105,000	2,500	0%
	90	74,250	105,000	2,500	0%
	100	74,250	105,000	2,500	0%
	110	74,250	105,000	2,500	0%
	120	74,250	105,000	2,500	0%
	130	74,250	105,000	2,500	0%
	140	74,250	105,000	2,500	0%
	150	74,250	105,000	2,500	0%
	10	80,480	98,600	8,000	5%
	20	81,160	99,450	8,000	5%
	30	81,840	100,300	8,000	5%
	40	82,520	101,150	8,000	5%
	50	83,200	102,000	8,000	5%
	60	83,880	102,850	8,000	5%
	70	84,380	104,100	5,500	0%
0.8	80	84,400	104,000	6,000	0%
	90	84,400	104,000	6,000	0%
	100	84,400	104,000	6,000	0%
	110	84,400	104,000	6,000	0%
	120	84,400	104,000	6,000	0%
	130	84,400	104,000	6,000	0%
	140	84,400	104,000	6,000	0%
	150	84,400	104,000	6,000	0%
	10	89,540	98,600	8,000	5%
	20	90,305	99,450	8,000	5%
	30	91,070	100,300	8,000	5%
	40	91,835	101,150	8,000	5%
	50	92,600	102,000	8,000	5%
	60	93,365	102,850	8,000	5%
	70	93,970	102,800	14,500	2%
0.9	80	94,025	103,000	13,250	0%
	90	94,025	103,000	13,250	0%
	100	94,025	103,000	13,250	0%
	110	94,025	103,000	13,250	0%
	120	94,025	103,000	13,250	0%
	130	94,025	103,000	13,250	0%
	140	94,025	103,000	13,250	0%
	150	94,025	103,000	13,250	0%

Varying ε_{4j}

α_1	ε_{4j}	*TC*	Z_1	Z_2	$\%\sum_j \sum_t S_{jk}^t$
	10	10,500	105,000	-	0%
	20	10,500	105,000	-	0%
	30	10,500	105,000	-	0%
	40	10,500	105,000	-	0%
	50	10,500	105,000	-	0%
	60	10,500	105,000	-	0%
	70	10,500	105,000	-	0%
0.1	80	10,500	105,000	-	0%
	90	10,500	105,000	-	0%
	100	10,500	105,000	-	0%
	110	10,500	105,000	-	0%
	120	10,500	105,000	-	0%
	130	10,500	105,000	-	0%
	140	10,500	105,000	-	0%
	150	10,500	105,000	-	0%
	10	21,000	105,000	-	0%
	20	21,000	105,000	-	0%
	30	21,000	105,000	-	0%
	40	21,000	105,000	-	0%
	50	21,000	105,000	-	0%
	60	21,000	105,000	-	0%
	70	21,000	105,000	-	0%
0.2	80	21,000	105,000	-	0%
	90	21,000	105,000	-	0%
	100	21,000	105,000	-	0%
	110	21,000	105,000	-	0%
	120	21,000	105,000	-	0%
	130	21,000	105,000	-	0%
	140	21,000	105,000	-	0%
	150	21,000	105,000	-	0%
	10	31,410	104,000	300	2%
	20	31,500	105,000	-	0%
	30	31,500	105,000	-	0%
	40	31,500	105,000	-	0%
	50	31,500	105,000	-	0%
	60	31,500	105,000	-	0%
	70	31,500	105,000	-	0%
0.3	80	31,500	105,000	-	0%
	90	31,500	105,000	-	0%
	100	31,500	105,000	-	0%
	110	31,500	105,000	-	0%
	120	31,500	105,000	-	0%
	130	31,500	105,000	-	0%
	140	31,500	105,000	-	0%
	150	31,500	105,000	-	0%

α_1	ε_{4j}	*TC*	Z_1	Z_2	$\%\sum_j \sum_t S_{jk}^t$
	10	41,770	103,000	950	6%
	20	41,960	104,000	600	2%
	30	42,000	105,000	-	0%
	40	42,000	105,000	-	0%
	50	42,000	105,000	-	0%
	60	42,000	105,000	-	0%
	70	42,000	105,000	-	0%
0.4	80	42,000	105,000	-	0%
	90	42,000	105,000	-	0%
	100	42,000	105,000	-	0%
	110	42,000	105,000	-	0%
	120	42,000	105,000	-	0%
	130	42,000	105,000	-	0%
	140	42,000	105,000	-	0%
	150	42,000	105,000	-	0%
	10	51,975	103,000	950	6%
	20	52,300	104,000	600	2%
	30	52,450	104,000	900	2%
	40	52,500	105,000	-	0%
	50	52,500	105,000	-	0%
	60	52,500	105,000	-	0%
	70	52,500	105,000	-	0%
0.5	80	52,500	105,000	-	0%
	90	52,500	105,000	-	0%
	100	52,500	105,000	-	0%
	110	52,500	105,000	-	0%
	120	52,500	105,000	-	0%
	130	52,500	105,000	-	0%
	140	52,500	105,000	-	0%
	150	52,500	105,000	-	0%
	10	62,180	103,000	950	6%
	20	62,560	103,000	1,900	6%
	30	62,760	104,000	900	2%
	40	62,880	104,000	1,200	2%
	50	63,000	105,000	-	0%
	60	63,000	105,000	-	0%
	70	63,000	105,000	-	0%
0.6	80	63,000	105,000	-	0%
	90	63,000	105,000	-	0%
	100	63,000	105,000	-	0%
	110	63,000	105,000	-	0%
	120	63,000	105,000	-	0%
	130	63,000	105,000	-	0%
	140	63,000	105,000	-	0%
	150	63,000	105,000	-	0%

α_1	ε_{4j}	*TC*	Z_1	Z_2	$\%\sum_j \sum_t S_{jk}^t$
	10	72,300	102,000	3,000	7%
	20	72,670	103,000	1,900	6%
	30	72,955	103,000	2,850	6%
	40	73,160	104,000	1,200	2%
	50	73,250	104,000	1,500	2%
	60	73,250	104,000	1,500	0%
	70	73,250	104,000	1,500	0%
0.7	80	73,250	104,000	1,500	0%
	90	73,250	104,000	1,500	0%
	100	73,250	104,000	1,500	0%
	110	73,250	104,000	1,500	0%
	120	73,250	104,000	1,500	0%
	130	73,250	104,000	1,500	0%
	140	73,250	104,000	1,500	0%
	150	73,250	104,000	1,500	0%
	10	82,200	102,000	3,000	7%
	20	82,450	102,000	4,250	7%
	30	82,700	102,000	5,500	7%
	40	82,950	102,000	6,750	7%
	50	83,200	102,000	8,000	7%
	60	83,450	102,000	9,250	7%
	70	83,500	104,000	1,500	0%
0.8	80	83,500	104,000	1,500	0%
	90	83,500	104,000	1,500	0%
	100	83,500	104,000	1,500	0%
	110	83,500	104,000	1,500	0%
	120	83,500	104,000	1,500	0%
	130	83,500	104,000	1,500	0%
	140	83,500	104,000	1,500	0%
	150	83,500	104,000	1,500	0%
	10	92,100	102,000	3,000	7%
	20	92,225	102,000	4,250	7%
	30	92,350	102,000	5,500	7%
	40	92,475	102,000	6,750	7%
	50	92,600	102,000	8,000	7%
	60	92,725	102,000	9,250	7%
	70	92,850	102,000	10,500	7%
0.9	80	92,970	102,000	11,700	7%
	90	93,085	102,000	12,850	7%
	100	93,200	102,000	14,000	5%
	110	93,255	102,000	14,550	2%
	120	93,285	102,000	14,850	2%
	130	93,315	102,000	15,150	2%
	140	93,345	102,000	15,450	2%
	150	93,375	102,000	15,750	1%

Results: Analysis 3 – Capacity level (τ_j^t)

α_1	$\%\tau_j^t$	TC	Z_1	Z_2	$\%\sum_j\sum_t y_j^t$	$\%\sum_j\sum_t q_j^t$	$\%\sum_j\sum_t s_j^t$	$\%\sum_j\sum_t l_j^t$	$\%\sum_j\sum_t S_{jk}^t$
0.1	70%	11,355	105,000	950	100%	82%	18%	0%	1%
	80%	14,910	105,000	4,900	100%	94%	6%	1%	5%
	90%	11,175	105,000	750	100%	95%	5%	0%	1%
	100%	10,500	105,000	-	100%	95%	5%	0%	0%
0.2	70%	21,760	105,000	950	100%	82%	18%	0%	1%
	80%	24,920	105,000	4,900	100%	94%	6%	1%	5%
	90%	21,600	105,000	750	100%	95%	5%	0%	1%
	100%	21,000	105,000	-	100%	95%	5%	0%	0%
0.3	70%	32,165	105,000	950	100%	82%	18%	0%	1%
	80%	34,930	105,000	4,900	100%	94%	6%	1%	5%
	90%	32,025	105,000	750	100%	95%	5%	0%	1%
	100%	31,500	105,000	-	100%	95%	5%	0%	0%
0.4	70%	42,570	105,000	950	100%	82%	18%	0%	1%
	80%	44,940	105,000	4,900	100%	94%	6%	1%	5%
	90%	42,450	105,000	750	100%	95%	5%	0%	1%
	100%	42,000	105,000	-	100%	95%	5%	0%	0%
0.5	70%	52,975	105,000	950	100%	82%	18%	0%	1%
	80%	54,950	105,000	4,900	100%	94%	6%	1%	5%
	90%	52,875	105,000	750	100%	95%	5%	0%	1%
	100%	52,500	105,000	-	100%	95%	5%	0%	0%
0.6	70%	63,380	105,000	950	100%	82%	18%	0%	1%
	80%	64,960	105,000	4,900	100%	94%	6%	1%	5%
	90%	63,300	105,000	750	100%	95%	5%	0%	1%
	100%	63,000	105,000	-	100%	95%	5%	0%	0%
0.7	70%	73,785	105,000	950	100%	82%	18%	0%	1%
	80%	74,970	105,000	4,900	100%	94%	6%	1%	5%
	90%	73,725	105,000	750	100%	95%	5%	0%	1%
	100%	73,250	105,000	-	100%	95%	5%	0%	0%
0.8	70%	84,190	105,000	950	100%	82%	18%	0%	1%
	80%	84,980	105,000	4,900	100%	94%	6%	1%	5%
	90%	84,000	105,000	750	100%	95%	5%	0%	1%
	100%	83,200	102,000	8,000	85%	95%	5%	2%	7%
0.9	70%	94,595	105,000	950	100%	82%	18%	0%	1%
	80%	94,990	105,000	4,900	100%	94%	6%	0%	6%
	90%	93,675	103,000	9,750	90%	95%	5%	6%	5%
	100%	92,600	102,000	8,000	85%	95%	5%	2%	7%

Results: Analysis 4 – Operation Periods (δ_j^t)

α_1	$\% \sum_j \sum_t \delta_j^t$	TC	Z_1	Z_2	$\% \sum_j \sum_t y_j^t$	Total operating facility	$\% \sum_j \sum_t q_j^t$	$\% \sum_j \sum_t s_j^t$	$\% \sum_j \sum_t l_j^t$	$\% \sum_j \sum_t S_{jk}^t$
0.1	70%	16,380	99,000	7,200	70%	3	82%	18%	2%	6%
	80%	21,305	101,000	12,450	80%	4	94%	6%	4%	10%
	90%	14,575	103,000	4,750	90%	4	95%	5%	0%	6%
	100%	10,500	105,000	-	100%	4	95%	5%	0%	0%
0.2	70%	25,560	99,000	7,200	70%	3	82%	18%	2%	6%
	80%	30,160	101,000	12,450	80%	4	94%	6%	4%	10%
	90%	24,400	103,000	4,750	90%	4	95%	5%	0%	6%
	100%	21,000	105,000	-	100%	4	95%	5%	0%	0%
0.3	70%	34,740	99,000	7,200	70%	3	82%	18%	2%	6%
	80%	39,015	101,000	12,450	80%	4	94%	6%	4%	10%
	90%	34,225	103,000	4,750	90%	4	95%	5%	0%	6%
	100%	31,500	105,000	-	100%	4	95%	5%	0%	0%
0.4	70%	43,920	99,000	7,200	70%	3	82%	18%	2%	6%
	80%	47,870	101,000	12,450	80%	4	94%	6%	4%	10%
	90%	44,050	103,000	4,750	90%	4	95%	5%	0%	6%
	100%	42,000	105,000	-	100%	4	95%	5%	0%	0%
0.5	70%	53,100	99,000	7,200	70%	3	82%	18%	2%	6%
	80%	56,725	101,000	12,450	80%	4	94%	6%	4%	10%
	90%	53,875	103,000	4,750	90%	4	95%	5%	0%	6%
	100%	52,500	105,000	-	100%	4	95%	5%	0%	0%
0.6	70%	62,280	99,000	7,200	70%	3	82%	18%	2%	6%
	80%	65,580	101,000	12,450	80%	4	94%	6%	4%	10%
	90%	63,700	103,000	4,750	90%	4	95%	5%	0%	6%
	100%	63,000	105,000	-	100%	4	95%	5%	0%	0%
0.7	70%	71,460	99,000	7,200	70%	3	82%	18%	2%	6%
	80%	74,435	101,000	12,450	80%	4	94%	6%	4%	10%
	90%	73,525	103,000	4,750	90%	4	95%	5%	0%	6%
	100%	73,250	104,000	1,500	95%	4	95%	5%	0%	2%
0.8	70%	80,640	99,000	7,200	70%	3	82%	18%	1%	8%
	80%	83,290	101,000	12,450	80%	4	94%	6%	4%	10%
	90%	83,200	102,000	8,000	85%	4	95%	5%	2%	7%
	100%	83,200	102,000	8,000	85%	4	95%	5%	2%	7%
0.9	70%	89,820	99,000	7,200	70%	3	82%	18%	2%	7%
	80%	92,145	101,000	12,450	80%	4	94%	6%	4%	10%
	90%	92,600	102,000	8,000	85%	4	95%	5%	2%	7%
	100%	92,600	102,000	8,000	85%	4	95%	5%	2%	7%

C. Sensitivity Analyses of Multi-component model – using Dataset A1 (non-congested facility network)

This section presents the sensitivity analyses of the proposed model, using a different dataset; A1. Dataset A1 is a **non-congested facility network** with an average utilisation rate at 85% at all time and the capacity level, τ_j^t was set to 100 units for all facility *j* at all time. Details on this dataset as the following:

Table 0-12: Datasets used

Parameters	**Range**
$\sum_t x_j^t$	= [425, 425, 425, 425]
$\sum_t \tau_j^t$	= [500, 500, 500, 500]
Average utilisation rate $\left(\% \frac{\sum_t x_j^t}{\sum_t \tau_j^t}\right)$	= 85%

Results: Analysis 1 – Variations of *B* values

Table 0-13: Variations of B values

B	**Objective function**	**Z_1**	**Z_2**	**Amount of demand left (%)**
0.00	52,500	105,000	-	0 (0%)
0.01	52,500	105,000	-	15 (1%)
0.02	52,500	105,000	-	34 (2%)
0.03	52,500	105,000	-	50 (3%)
0.04	52,425	104,000	850	68 (4%)
0.05	52,000	104,000	-	85 (5%)

Results: Analysis 2 – Trade-off of variables' costs using Dataset A1

Varying C_j

α_1	C_j	*TC*	Z_1	Z_2	$\%\sum_j \sum_t y_j^t$
	200	8,800	88,800	-	95%
	400	9,260	92,600	-	95%
	600	9,640	96,400	-	95%
	800	10,020	100,200	-	95%
	1000	10,400	104,000	-	95%
	1200	10,780	107,800	-	95%
	1400	11,160	111,600	-	95%
	1600	11,540	115,400	-	95%
	1800	11,920	119,200	-	95%
	2000	12,300	123,000	-	95%
	2200	12,680	126,800	-	95%
	2400	13,060	130,600	-	95%
0.1	2600	13,440	134,400	-	95%
	2800	13,820	138,200	-	95%
	3000	14,200	142,000	-	95%
	3200	14,580	145,800	-	95%
	3400	14,960	149,600	-	95%
	3600	15,340	153,400	-	95%
	3800	15,720	157,200	-	95%
	4000	16,100	161,000	-	95%
	4200	16,480	164,800	-	95%
	4400	16,860	168,600	-	95%
	4600	17,240	172,400	-	95%
	4800	17,620	176,200	-	95%
	5000	18,000	180,000	-	95%
	200	17,760	88,800	-	95%
	400	18,520	92,600	-	95%
	600	19,280	96,400	-	95%
	800	20,040	100,200	-	95%
	1000	20,800	104,000	-	95%
	1200	21,560	107,800	-	95%
0.2	1400	22,320	111,600	-	95%
	1600	23,080	115,400	-	95%
	1800	23,840	119,200	-	95%
	2000	24,600	123,000	-	95%
	2200	25,360	126,800	-	95%
	2400	26,120	130,600	-	95%
	2600	26,880	134,400	-	95%
	2800	27,640	138,200	-	95%

α_1	C_j	*TC*	Z_1	Z_2	$\%\sum_j \sum_t y_j^t$
	200	35,520	88,800	-	95%
	400	37,040	92,600	-	95%
	600	38,560	96,400	-	95%
	800	40,080	100,200	-	95%
	1000	41,600	104,000	-	95%
	1200	43,120	107,800	-	95%
	1400	44,640	111,600	-	95%
	1600	46,160	115,400	-	95%
	1800	47,680	119,200	-	95%
	2000	49,200	123,000	-	95%
	2200	50,720	126,800	-	95%
	2400	52,240	130,600	-	95%
0.4	2600	53,760	134,400	-	95%
	2800	55,280	138,200	-	95%
	3000	56,800	142,000	-	95%
	3200	58,320	145,800	-	95%
	3400	59,840	149,600	-	95%
	3600	61,360	153,400	-	95%
	3800	62,880	157,200	-	95%
	4000	64,400	161,000	-	95%
	4200	65,920	164,800	-	95%
	4400	67,440	168,600	-	95%
	4600	68,960	172,400	-	95%
	4800	70,480	176,200	-	95%
	5000	72,000	180,000	-	95%
	200	44,400	88,800	-	95%
	400	46,300	92,600	-	95%
	600	48,200	96,400	-	95%
	800	50,100	100,200	-	95%
	1000	52,000	104,000	-	95%
	1200	53,900	107,800	-	95%
0.5	1400	55,800	111,600	-	95%
	1600	57,700	115,400	-	95%
	1800	59,600	119,200	-	95%
	2000	61,500	123,000	-	95%
	2200	63,400	126,800	-	95%
	2400	65,300	130,600	-	95%
	2600	67,200	134,400	-	95%
	2800	69,100	138,200	-	95%

α_1	C_j	*TC*	Z_1	Z_2	$\%\sum_j \sum_t y_j^t$
	200	62,160	88,800	-	95%
	400	64,820	92,600	-	95%
	600	67,480	96,400	-	95%
	800	70,140	100,200	-	95%
	1000	72,800	104,000	-	95%
	1200	75,460	107,800	-	95%
	1400	78,120	111,600	-	95%
	1600	80,780	115,400	-	95%
	1800	83,440	119,200	-	95%
	2000	85,975	121,000	4,250	90%
	2200	88,495	124,600	4,250	90%
	2400	90,910	125,800	9,500	85%
0.7	2600	93,290	129,200	9,500	85%
	2800	95,670	132,600	9,500	85%
	3000	98,050	136,000	9,500	85%
	3200	100,430	139,400	9,500	85%
	3400	102,810	142,800	9,500	85%
	3600	105,190	146,200	9,500	85%
	3800	107,570	149,600	9,500	85%
	4000	109,950	153,000	9,500	85%
	4200	112,330	156,400	9,500	85%
	4400	114,710	159,800	9,500	85%
	4600	117,090	163,200	9,500	85%
	4800	119,470	166,600	9,500	85%
	5000	121,850	170,000	9,500	85%
	200	71,040	88,800	-	95%
	400	74,080	92,600	-	95%
	600	77,120	96,400	-	95%
	800	80,160	100,200	-	95%
	1000	83,200	104,000	-	95%
	1200	86,130	106,600	4,250	90%
0.8	1400	88,940	108,800	9,500	85%
	1600	91,660	112,200	9,500	85%
	1800	94,380	115,600	9,500	85%
	2000	97,100	119,000	9,500	85%
	2200	99,820	122,400	9,500	85%
	2400	102,540	125,800	9,500	85%
	2600	105,260	129,200	9,500	85%
	2800	107,980	132,600	9,500	85%

	3000	28,400	142,000	-	95%
	3200	29,160	145,800	-	95%
	3400	29,920	149,600	-	95%
	3600	30,680	153,400	-	95%
	3800	31,440	157,200	-	95%
	4000	32,200	161,000	-	95%
	4200	32,960	164,800	-	95%
	4400	33,720	168,600	-	95%
	4600	34,480	172,400	-	95%
	4800	35,240	176,200	-	95%
	5000	36,000	180,000	-	95%
	200	26,640	88,800	-	95%
	400	27,780	92,600	-	95%
	600	28,920	96,400	-	95%
	800	30,060	100,200	-	95%
	1000	31,200	104,000	-	95%
	1200	32,340	107,800	-	95%
	1400	33,480	111,600	-	95%
	1600	34,620	115,400	-	95%
	1800	35,760	119,200	-	95%
	2000	36,900	123,000	-	95%
	2200	38,040	126,800	-	95%
	2400	39,180	130,600	-	95%
0.3	2600	40,320	134,400	-	95%
	2800	41,460	138,200	-	95%
	3000	42,600	142,000	-	95%
	3200	43,740	145,800	-	95%
	3400	44,880	149,600	-	95%
	3600	46,020	153,400	-	95%
	3800	47,160	157,200	-	95%
	4000	48,300	161,000	-	95%
	4200	49,440	164,800	-	95%
	4400	50,580	168,600	-	95%
	4600	51,720	172,400	-	95%
	4800	52,860	176,200	-	95%
	5000	54,000	180,000	-	95%

	3000	71,000	142,000	-	95%
	3200	72,900	145,800	-	95%
	3400	74,800	149,600	-	95%
	3600	76,700	153,400	-	95%
	3800	78,600	157,200	-	95%
	4000	80,500	161,000	-	95%
	4200	82,400	164,800	-	95%
	4400	84,225	164,200	4,250	90%
	4600	86,025	167,800	4,250	90%
	4800	87,825	171,400	4,250	90%
	5000	89,625	175,000	4,250	90%
	200	53,280	88,800	-	95%
	400	55,560	92,600	-	95%
	600	57,840	96,400	-	95%
	800	60,120	100,200	-	95%
	1000	62,400	104,000	-	95%
	1200	64,680	107,800	-	95%
	1400	66,960	111,600	-	95%
	1600	69,240	115,400	-	95%
	1800	71,520	119,200	-	95%
	2000	73,800	123,000	-	95%
	2200	76,080	126,800	-	95%
	2400	78,360	130,600	-	95%
0.6	2600	80,640	134,400	-	95%
	2800	82,920	138,200	-	95%
	3000	85,100	139,000	4,250	90%
	3200	87,260	142,600	4,250	90%
	3400	89,420	146,200	4,250	90%
	3600	91,520	146,200	9,500	85%
	3800	93,560	149,600	9,500	85%
	4000	95,600	153,000	9,500	85%
	4200	97,640	156,400	9,500	85%
	4400	99,680	159,800	9,500	85%
	4600	101,720	163,200	9,500	85%
	4800	103,760	166,600	9,500	85%
	5000	105,800	170,000	9,500	85%

	3000	110,700	136,000	9,500	85%
	3200	113,420	139,400	9,500	85%
	3400	116,140	142,800	9,500	85%
	3600	118,860	146,200	9,500	85%
	3800	121,580	149,600	9,500	85%
	4000	124,300	153,000	9,500	85%
	4200	127,020	156,400	9,500	85%
	4400	129,740	159,800	9,500	85%
	4600	132,460	163,200	9,500	85%
	4800	135,180	166,600	9,500	85%
	5000	137,900	170,000	9,500	85%
	200	79,920	88,800	-	95%
	400	83,340	92,600	-	95%
	600	86,630	95,200	9,500	85%
	800	89,690	98,600	9,500	85%
	1000	92,750	102,000	9,500	85%
	1200	95,810	105,400	9,500	85%
	1400	98,870	108,800	9,500	85%
	1600	101,930	112,200	9,500	85%
	1800	104,990	115,600	9,500	85%
	2000	108,050	119,000	9,500	85%
	2200	111,110	122,400	9,500	85%
	2400	114,170	125,800	9,500	85%
0.9	2600	117,230	129,200	9,500	85%
	2800	120,290	132,600	9,500	85%
	3000	123,350	136,000	9,500	85%
	3200	126,410	139,400	9,500	85%
	3400	129,470	142,800	9,500	85%
	3600	132,530	146,200	9,500	85%
	200	62,160	88,800	-	95%
	400	64,820	92,600	-	95%
	600	67,480	96,400	-	95%
	800	70,140	100,200	-	95%
	1000	72,800	104,000	-	95%
	1200	75,460	107,800	-	95%
	1400	78,120	111,600	-	95%

Varying ε_{1j}

α_1	ε_{1j}	*TC*	Z_1	Z_2	$\%\sum_j \sum_t q_j^t$
	10	3,700	37,000	-	100%
	20	5,400	54,000	-	100%
	30	7,100	71,000	-	100%
	40	8,785	87,850	-	95%
	50	10,400	104,000	-	95%
	60	12,015	120,150	-	95%
	70	13,630	136,300	-	95%
0.1	80	15,245	152,450	-	95%
	90	16,860	168,600	-	95%
	100	18,475	184,750	-	95%
	110	20,090	200,900	-	95%
	120	21,705	217,050	-	95%
	130	23,320	233,200	-	95%
	140	24,935	249,350	-	95%
	150	26,550	265,500	-	95%
	10	7,400	37,000	-	100%
	20	10,800	54,000	-	100%
	30	14,200	71,000	-	100%
	40	17,570	87,850	-	95%
	50	20,800	104,000	-	95%
	60	24,030	120,150	-	95%
	70	27,260	136,300	-	95%
0.2	80	30,490	152,450	-	95%
	90	33,720	168,600	-	95%
	100	36,950	184,750	-	95%
	110	40,180	200,900	-	95%
	120	43,410	217,050	-	95%
	130	46,640	233,200	-	95%
	140	49,870	249,350	-	95%
	150	53,100	265,500	-	95%
	10	11,100	37,000	-	100%
	20	16,200	54,000	-	100%
	30	21,300	71,000	-	100%
	40	26,355	87,850	-	95%
	50	31,200	104,000	-	95%
	60	36,045	120,150	-	95%
	70	40,890	136,300	-	95%
0.3	80	45,735	152,450	-	95%
	90	50,580	168,600	-	95%
	100	55,425	184,750	-	95%
	110	60,270	200,900	-	95%
	120	65,115	217,050	-	95%
	130	69,960	233,200	-	95%
	140	74,805	249,350	-	95%
	150	79,650	265,500	-	95%

α_1	ε_{1j}	*TC*	Z_1	Z_2	$\%\sum_j \sum_t q_j^t$
	10	14,800	37,000	-	100%
	20	21,600	54,000	-	100%
	30	28,400	71,000	-	100%
	40	35,140	87,850	-	95%
	50	41,600	104,000	-	95%
	60	48,060	120,150	-	95%
	70	54,520	136,300	-	95%
0.4	80	60,980	152,450	-	95%
	90	67,440	168,600	-	95%
	100	73,900	184,750	-	95%
	110	80,360	200,900	-	95%
	120	86,820	217,050	-	95%
	130	93,280	233,200	-	95%
	140	99,740	249,350	-	95%
	150	106,200	265,500	-	95%
	10	18,500	37,000	-	100%
	20	27,000	54,000	-	100%
	30	35,500	71,000	-	100%
	40	43,925	87,850	-	95%
	50	52,000	104,000	-	95%
	60	60,075	120,150	-	95%
	70	68,150	136,300	-	95%
0.5	80	76,225	152,450	-	95%
	90	84,300	168,600	-	95%
	100	92,375	184,750	-	95%
	110	100,450	200,900	-	95%
	120	108,525	217,050	-	95%
	130	116,600	233,200	-	95%
	140	124,675	249,350	-	95%
	150	132,750	265,500	-	95%
	10	22,200	37,000	-	100%
	20	32,400	54,000	-	100%
	30	42,600	71,000	-	100%
	40	52,710	87,850	-	95%
	50	62,400	104,000	-	95%
	60	72,090	120,150	-	95%
	70	81,780	136,300	-	95%
0.6	80	91,470	152,450	-	95%
	90	101,160	168,600	-	95%
	100	110,850	184,750	-	95%
	110	120,540	200,900	-	95%
	120	130,230	217,050	-	95%
	130	139,920	233,200	-	95%
	140	149,610	249,350	-	95%
	150	159,300	265,500	-	95%

α_1	ε_{1j}	*TC*	Z_1	Z_2	$\%\sum_j \sum_t q_j^t$
	10	25,900	37,000	-	100%
	20	37,800	54,000	-	100%
	30	49,700	71,000	-	100%
	40	61,495	87,850	-	95%
	50	72,800	104,000	-	95%
	60	84,105	120,150	-	95%
	70	95,410	136,300	-	95%
0.7	80	106,715	152,450	-	95%
	90	118,020	168,600	-	95%
	100	129,325	184,750	-	95%
	110	140,630	200,900	-	95%
	120	151,935	217,050	-	95%
	130	163,240	233,200	-	95%
	140	174,545	249,350	-	95%
	150	185,850	265,500	-	95%
	10	29,600	37,000	-	100%
	20	43,200	54,000	-	100%
	30	56,800	71,000	-	100%
	40	70,280	87,850	-	95%
	50	83,200	104,000	-	95%
	60	96,120	120,150	-	95%
	70	109,040	136,300	-	95%
0.8	80	121,960	152,450	-	95%
	90	134,880	168,600	-	95%
	100	147,800	184,750	-	95%
	110	160,720	200,900	-	95%
	120	173,640	217,050	-	95%
	130	186,560	233,200	-	95%
	140	199,480	249,350	-	95%
	150	212,400	265,500	-	95%
	10	32,900	35,000	14000	100%
	20	48,200	52,000	14000	100%
	30	63,490	69,100	13000	100%
	40	78,215	85,850	9500	95%
	50	92,750	102,000	9500	95%
	60	107,285	118,150	9500	95%
	70	121,820	134,300	9500	95%
0.9	80	136,355	150,450	9500	95%
	90	150,890	166,600	9500	95%
	100	165,425	182,750	9500	95%
	110	179,960	198,900	9500	95%
	120	194,495	215,050	9500	95%
	130	209,030	231,200	9500	95%
	140	223,565	247,350	9500	95%
	150	238,100	263,500	9500	95%

Varying ε_{2j}

α_1	ε_{2j}	*TC*	Z_1	Z_2	$\%\sum_j \sum_t s_j^t$
	10	10,400	104,000	-	0%
	20	10,400	104,000	-	0%
	30	10,400	104,000	-	0%
	40	10,400	104,000	-	0%
	50	10,400	104,000	-	0%
	60	10,400	104,000	-	0%
	70	10,400	104,000	-	0%
0.1	80	10,400	104,000	-	0%
	90	10,400	104,000	-	0%
	100	10,400	104,000	-	0%
	110	10,400	104,000	-	0%
	120	10,400	104,000	-	0%
	130	10,400	104,000	-	0%
	140	10,400	104,000	-	0%
	150	10,400	104,000	-	0%
	10	20,800	104,000	-	0%
	20	20,800	104,000	-	0%
	30	20,800	104,000	-	0%
	40	20,800	104,000	-	0%
	50	20,800	104,000	-	0%
	60	20,800	104,000	-	0%
	70	20,800	104,000	-	0%
0.2	80	20,800	104,000	-	0%
	90	20,800	104,000	-	0%
	100	20,800	104,000	-	0%
	110	20,800	104,000	-	0%
	120	20,800	104,000	-	0%
	130	20,800	104,000	-	0%
	140	20,800	104,000	-	0%
	150	20,800	104,000	-	0%
	10	31,200	104,000	-	0%
	20	31,200	104,000	-	0%
	30	31,200	104,000	-	0%
	40	31,200	104,000	-	0%
	50	31,200	104,000	-	0%
	60	31,200	104,000	-	0%
	70	31,200	104,000	-	0%
0.3	80	31,200	104,000	-	0%
	90	31,200	104,000	-	0%
	100	31,200	104,000	-	0%
	110	31,200	104,000	-	0%
	120	31,200	104,000	-	0%
	130	31,200	104,000	-	0%
	140	31,200	104,000	-	0%
	150	31,200	104,000	-	0%

α_1	ε_{2j}	*TC*	Z_1	Z_2	$\%\sum_j \sum_t s_j^t$
	10	41,600	104,000	-	0%
	20	41,600	104,000	-	0%
	30	41,600	104,000	-	0%
	40	41,600	104,000	-	0%
	50	41,600	104,000	-	0%
	60	41,600	104,000	-	0%
	70	41,600	104,000	-	0%
0.4	80	41,600	104,000	-	0%
	90	41,600	104,000	-	0%
	100	41,600	104,000	-	0%
	110	41,600	104,000	-	0%
	120	41,600	104,000	-	0%
	130	41,600	104,000	-	0%
	140	41,600	104,000	-	0%
	150	41,600	104,000	-	0%
	10	52,000	104,000	-	0%
	20	52,000	104,000	-	0%
	30	52,000	104,000	-	0%
	40	52,000	104,000	-	0%
	50	52,000	104,000	-	0%
	60	52,000	104,000	-	0%
	70	52,000	104,000	-	0%
0.5	80	52,000	104,000	-	0%
	90	52,000	104,000	-	0%
	100	52,000	104,000	-	0%
	110	52,000	104,000	-	0%
	120	52,000	104,000	-	0%
	130	52,000	104,000	-	0%
	140	52,000	104,000	-	0%
	150	52,000	104,000	-	0%
	10	104,000	-	62,400	0%
	20	104,000	-	62,400	0%
	30	104,000	-	62,400	0%
	40	104,000	-	62,400	0%
	50	104,000	-	62,400	0%
	60	104,000	-	62,400	0%
	70	104,000	-	62,400	0%
0.6	80	104,000	-	62,400	0%
	90	104,000	-	62,400	0%
	100	104,000	-	62,400	0%
	110	104,000	-	62,400	0%
	120	104,000	-	62,400	0%
	130	104,000	-	62,400	0%
	140	104,000	-	62,400	0%
	150	104,000	-	62,400	0%

α_1	ε_{2j}	*TC*	Z_1	Z_2	$\%\sum_j \sum_t s_j^t$
	10	103,000	1,650	72,595	10%
	20	104,000	-	72,800	0%
	30	104,000	-	72,800	0%
	40	104,000	-	72,800	0%
	50	104,000	-	72,800	0%
	60	104,000	-	72,800	0%
	70	104,000	-	72,800	0%
0.7	80	104,000	-	72,800	0%
	90	104,000	-	72,800	0%
	100	104,000	-	72,800	0%
	110	104,000	-	72,800	0%
	120	104,000	-	72,800	0%
	130	104,000	-	72,800	0%
	140	104,000	-	72,800	0%
	150	104,000	-	72,800	0%
	10	102,000	4,100	82,420	24%
	20	103,000	3,050	83,010	5%
	30	103,000	3,900	83,180	5%
	40	104,000	-	83,200	0%
	50	104,000	-	83,200	0%
	60	104,000	-	83,200	0%
	70	104,000	-	83,200	0%
0.8	80	104,000	-	83,200	0%
	90	104,000	-	83,200	0%
	100	104,000	-	83,200	0%
	110	104,000	-	83,200	0%
	120	104,000	-	83,200	0%
	130	104,000	-	83,200	0%
	140	104,000	-	83,200	0%
	150	104,000	-	83,200	0%
	10	102,000	4,100	92,210	24%
	20	102,000	7,600	92,560	18%
	30	102,000	8,500	92,650	3%
	40	102,000	9,000	92,700	3%
	50	102,000	9,500	92,750	2%
	60	102,000	9,850	92,785	2%
	70	102,000	10,200	92,820	2%
0.9	80	102,000	10,550	92,855	2%
	90	102,000	10,900	92,890	2%
	100	102,000	11,250	92,925	2%
	110	102,000	11,600	92,960	2%
	120	102,000	11,950	92,995	2%
	130	102,000	12,300	93,030	2%
	140	102,000	12,650	93,065	2%
	150	102,000	13,000	93,100	2%

Varying ε_{3j}

α_1	ε_{3j}	*TC*	Z_1	Z_2	$\%\sum_j \sum_t l_j^t$
	10	10,060	100,600	-	5%
	20	10,145	101,450	-	5%
	30	10,230	102,300	-	5%
	40	10,315	103,150	-	5%
	50	10,400	104,000	-	5%
	60	10,485	104,850	-	5%
	70	10,500	105,000	-	0%
0.1	80	10,500	105,000	-	0%
	90	10,500	105,000	-	0%
	100	10,500	105,000	-	0%
	110	10,500	105,000	-	0%
	120	10,500	105,000	-	0%
	130	10,500	105,000	-	0%
	140	10,500	105,000	-	0%
	150	10,500	105,000	-	0%
	10	20,120	100,600	-	5%
	20	20,290	101,450	-	5%
	30	20,460	102,300	-	5%
	40	20,630	103,150	-	5%
	50	20,800	104,000	-	5%
	60	20,970	104,850	-	5%
	70	21,000	105,000	-	0%
0.2	80	21,000	105,000	-	0%
	90	21,000	105,000	-	0%
	100	21,000	105,000	-	0%
	110	21,000	105,000	-	0%
	120	21,000	105,000	-	0%
	130	21,000	105,000	-	0%
	140	21,000	105,000	-	0%
	150	21,000	105,000	-	0%
	10	30,180	100,600	-	5%
	20	30,435	101,450	-	5%
	30	30,690	102,300	-	5%
	40	30,945	103,150	-	5%
	50	31,200	104,000	-	5%
	60	31,455	104,850	-	5%
	70	31,500	105,000	-	0%
0.3	80	31,500	105,000	-	0%
	90	31,500	105,000	-	0%
	100	31,500	105,000	-	0%
	110	31,500	105,000	-	0%
	120	31,500	105,000	-	0%
	130	31,500	105,000	-	0%
	140	31,500	105,000	-	0%
	150	31,500	105,000	-	0%

α_1	ε_{3j}	*TC*	Z_1	Z_2	$\%\sum_j \sum_t l_j^t$
	10	40,240	100,600	-	5%
	20	40,580	101,450	-	5%
	30	40,920	102,300	-	5%
	40	41,260	103,150	-	5%
	50	41,600	104,000	-	5%
	60	41,940	104,850	-	5%
	70	42,000	105,000	-	0%
0.4	80	42,000	105,000	-	0%
	90	42,000	105,000	-	0%
	100	42,000	105,000	-	0%
	110	42,000	105,000	-	0%
	120	42,000	105,000	-	0%
	130	42,000	105,000	-	0%
	140	42,000	105,000	-	0%
	150	42,000	105,000	-	0%
	10	50,300	100,600	-	5%
	20	50,725	101,450	-	5%
	30	51,150	102,300	-	5%
	40	51,575	103,150	-	5%
	50	52,000	104,000	-	5%
	60	52,425	104,850	-	5%
	70	52,500	105,000	-	0%
0.5	80	52,500	105,000	-	0%
	90	52,500	105,000	-	0%
	100	52,500	105,000	-	0%
	110	52,500	105,000	-	0%
	120	52,500	105,000	-	0%
	130	52,500	105,000	-	0%
	140	52,500	105,000	-	0%
	150	52,500	105,000	-	0%
	10	60,360	100,600	-	0%
	20	60,870	101,450	-	0%
	30	61,380	102,300	-	0%
	40	61,890	103,150	-	0%
	50	62,400	104,000	-	0%
	60	62,910	104,850	-	0%
	70	63,000	105,000	-	0%
0.6	80	63,000	105,000	-	0%
	90	63,000	105,000	-	0%
	100	63,000	105,000	-	0%
	110	63,000	105,000	-	0%
	120	63,000	105,000	-	0%
	130	63,000	105,000	-	0%
	140	63,000	105,000	-	0%
	150	63,000	105,000	-	0%

α_1	ε_{3j}	*TC*	Z_1	Z_2	$\%\sum_j \sum_t l_j^t$
	10	70,420	100,600	-	10%
	20	71,015	101,450	-	0%
	30	71,610	102,300	-	0%
	40	72,205	103,150	-	0%
	50	72,800	104,000	-	0%
	60	73,395	104,850	-	0%
	70	73,500	105,000	-	0%
0.7	80	73,500	105,000	-	0%
	90	73,500	105,000	-	0%
	100	73,500	105,000	-	0%
	110	73,500	105,000	-	0%
	120	73,500	105,000	-	0%
	130	73,500	105,000	-	0%
	140	73,500	105,000	-	0%
	150	73,500	105,000	-	0%
	10	80,480	100,600	-	24%
	20	81,160	101,450	-	5%
	30	81,840	102,300	-	5%
	40	82,520	103,150	-	0%
	50	83,200	104,000	-	0%
	60	83,880	104,850	-	0%
	70	84,000	105,000	-	0%
0.8	80	84,000	105,000	-	0%
	90	84,000	105,000	-	0%
	100	84,000	105,000	-	0%
	110	84,000	105,000	-	0%
	120	84,000	105,000	-	0%
	130	84,000	105,000	-	0%
	140	84,000	105,000	-	0%
	150	84,000	105,000	-	0%
	10	89,690	98,600	9,500	24%
	20	90,455	99,450	9,500	18%
	30	91,220	100,300	9,500	3%
	40	91,985	101,150	9,500	3%
	50	92,750	102,000	9,500	2%
	60	93,515	102,850	9,500	2%
	70	94,090	103,100	13,000	2%
0.9	80	94,100	103,000	14,000	2%
	90	94,100	103,000	14,000	2%
	100	94,100	103,000	14,000	2%
	110	94,100	103,000	14,000	2%
	120	94,100	103,000	14,000	2%
	130	94,100	103,000	14,000	2%
	140	94,100	103,000	14,000	2%
	150	94,100	103,000	14,000	2%

Varying ε_{4j}

α_1	ε_{4j}	*TC*	Z_1	Z_2	$\%\sum_j \sum_t S_{jk}^t$
	10	10,400	104,000	-	0%
	20	10,400	104,000	-	0%
	30	10,400	104,000	-	0%
	40	10,400	104,000	-	0%
	50	10,400	104,000	-	0%
	60	10,400	104,000	-	0%
	70	10,400	104,000	-	0%
0.1	80	10,400	104,000	-	0%
	90	10,400	104,000	-	0%
	100	10,400	104,000	-	0%
	110	10,400	104,000	-	0%
	120	10,400	104,000	-	0%
	130	10,400	104,000	-	0%
	140	10,400	104,000	-	0%
	150	10,400	104,000	-	0%
	10	20,800	104,000	-	0%
	20	20,800	104,000	-	0%
	30	20,800	104,000	-	0%
	40	20,800	104,000	-	0%
	50	20,800	104,000	-	0%
	60	20,800	104,000	-	0%
	70	20,800	104,000	-	0%
0.2	80	20,800	104,000	-	0%
	90	20,800	104,000	-	0%
	100	20,800	104,000	-	0%
	110	20,800	104,000	-	0%
	120	20,800	104,000	-	0%
	130	20,800	104,000	-	0%
	140	20,800	104,000	-	0%
	150	20,800	104,000	-	0%
	10	31,200	104,000	-	0%
	20	31,200	104,000	-	0%
	30	31,200	104,000	-	0%
	40	31,200	104,000	-	0%
	50	31,200	104,000	-	0%
	60	31,200	104,000	-	0%
	70	31,200	104,000	-	0%
0.3	80	31,200	104,000	-	0%
	90	31,200	104,000	-	0%
	100	31,200	104,000	-	0%
	110	31,200	104,000	-	0%
	120	31,200	104,000	-	0%
	130	31,200	104,000	-	0%
	140	31,200	104,000	-	0%
	150	31,200	104,000	-	0%

α_1	ε_{4j}	*TC*	Z_1	Z_2	$\%\sum_j \sum_t S_{jk}^t$
	10	41,600	104,000	-	0%
	20	41,600	104,000	-	0%
	30	41,600	104,000	-	0%
	40	41,600	104,000	-	0%
	50	41,600	104,000	-	0%
	60	41,600	104,000	-	0%
	70	41,600	104,000	-	0%
0.4	80	41,600	104,000	-	0%
	90	41,600	104,000	-	0%
	100	41,600	104,000	-	0%
	110	41,600	104,000	-	0%
	120	41,600	104,000	-	0%
	130	41,600	104,000	-	0%
	140	41,600	104,000	-	0%
	150	41,600	104,000	-	0%
	10	51,925	103,000	850	5%
	20	52,000	104,000	-	0%
	30	52,000	104,000	-	0%
	40	52,000	104,000	-	0%
	50	52,000	104,000	-	0%
	60	52,000	104,000	-	0%
	70	52,000	104,000	-	0%
0.5	80	52,000	104,000	-	0%
	90	52,000	104,000	-	0%
	100	52,000	104,000	-	0%
	110	52,000	104,000	-	0%
	120	52,000	104,000	-	0%
	130	52,000	104,000	-	0%
	140	52,000	104,000	-	0%
	150	52,000	104,000	-	0%
	10	62,140	103,000	850	5%
	20	62,400	104,000	-	0%
	30	62,400	104,000	-	0%
	40	62,400	104,000	-	0%
	50	62,400	104,000	-	0%
	60	62,400	104,000	-	0%
	70	62,400	104,000	-	0%
0.6	80	62,400	104,000	-	0%
	90	62,400	104,000	-	0%
	100	62,400	104,000	-	0%
	110	62,400	104,000	-	0%
	120	62,400	104,000	-	0%
	130	62,400	104,000	-	0%
	140	62,400	104,000	-	0%
	150	62,400	104,000	-	0%

α_1	ε_{4j}	*TC*	Z_1	Z_2	$\%\sum_j \sum_t S_{jk}^t$
	10	72,355	103,000	850	5%
	20	72,610	103,000	1,700	5%
	30	72,800	104,000	-	0%
	40	72,800	104,000	-	0%
	50	72,800	104,000	-	0%
	60	72,800	104,000	-	0%
	70	72,800	104,000	-	0%
0.7	80	72,800	104,000	-	0%
	90	72,800	104,000	-	0%
	100	72,800	104,000	-	0%
	110	72,800	104,000	-	0%
	120	72,800	104,000	-	0%
	130	72,800	104,000	-	0%
	140	72,800	104,000	-	0%
	150	72,800	104,000	-	0%
	10	82,260	102,000	3,300	9%
	20	82,570	102,000	4,850	9%
	30	82,880	102,000	6,400	9%
	40	83,080	103,000	3,400	5%
	50	83,200	104,000	-	0%
	60	83,200	104,000	-	0%
	70	83,200	104,000	-	0%
0.8	80	83,200	104,000	-	0%
	90	83,200	104,000	-	0%
	100	83,200	104,000	-	0%
	110	83,200	104,000	-	0%
	120	83,200	104,000	-	0%
	130	83,200	104,000	-	0%
	140	83,200	104,000	-	0%
	150	83,200	104,000	-	0%
	10	92,130	102,000	3,300	9%
	20	92,285	102,000	4,850	9%
	30	92,440	102,000	6,400	9%
	40	92,595	102,000	7,950	9%
	50	92,750	102,000	9,500	9%
	60	92,890	102,000	10,900	8%
	70	93,030	102,000	12,300	8%
0.9	80	93,170	102,000	13,700	8%
	90	93,310	102,000	15,100	8%
	100	93,400	103,000	7,000	2%
	110	93,425	103,000	7,250	1%
	120	93,450	103,000	7,500	1%
	130	93,475	103,000	7,750	1%
	140	93,500	103,000	8,000	1%
	150	93,525	103,000	8,250	0%

Results: Analysis 3 – Capacity level (τ_j^t)

α_1	$\%\tau_j^t$	TC	Z_1	Z_2	$\%\sum_j\sum_t y_j^t$	$\%\sum_j\sum_t q_j^t$	$\%\sum_j\sum_t s_j^t$	$\%\sum_j\sum_t l_j^t$	$\%\sum_j\sum_t S_{jk}^t$
0.1	70%	10,500	105,000	-	100%	82%	18%	0%	0%
	80%	10,500	105,000	-	100%	94%	6%	0%	0%
	90%	10,400	104,000	-	95%	95%	5%	0%	0%
	100%	10,400	104,000	-	95%	95%	5%	0%	0%
0.2	70%	21,000	105,000	-	100%	82%	18%	0%	0%
	80%	21,000	105,000	-	100%	94%	6%	0%	0%
	90%	20,800	104,000	-	95%	95%	5%	0%	0%
	100%	20,800	104,000	-	95%	95%	5%	0%	0%
0.3	70%	31,500	105,000	-	100%	82%	18%	0%	0%
	80%	31,500	105,000	-	100%	94%	6%	0%	0%
	90%	31,200	104,000	-	95%	95%	5%	0%	0%
	100%	31,200	104,000	-	95%	95%	5%	0%	0%
0.4	70%	42,000	105,000	-	100%	82%	18%	0%	0%
	80%	42,000	105,000	-	100%	94%	6%	0%	0%
	90%	41,600	104,000	-	95%	95%	5%	0%	0%
	100%	41,600	104,000	-	95%	95%	5%	0%	0%
0.5	70%	52,500	105,000	-	100%	82%	18%	0%	0%
	80%	52,500	105,000	-	100%	94%	6%	0%	0%
	90%	52,000	104,000	-	95%	95%	5%	0%	0%
	100%	52,000	104,000	-	95%	95%	5%	0%	0%
0.6	70%	63,000	105,000	-	100%	82%	18%	0%	0%
	80%	63,000	105,000	-	100%	94%	6%	0%	0%
	90%	62,400	104,000	-	95%	95%	5%	0%	0%
	100%	62,400	104,000	-	95%	95%	5%	0%	0%
0.7	70%	73,500	105,000	-	100%	82%	18%	0%	0%
	80%	73,500	105,000	-	100%	94%	6%	0%	0%
	90%	72,800	104,000	-	95%	95%	5%	0%	0%
	100%	72,800	104,000	-	95%	95%	5%	0%	0%
0.8	70%	84,000	105,000	-	100%	82%	18%	0%	0%
	80%	84,000	105,000	-	100%	94%	6%	0%	0%
	90%	83,200	104,000	-	95%	95%	5%	0%	0%
	100%	83,200	104,000	-	95%	95%	5%	0%	0%
0.9	70%	94,500	105,000	-	100%	82%	18%	0%	0%
	80%	94,500	105,000	-	100%	94%	6%	0%	0%
	90%	93,575	104,000	-	95%	95%	5%	0%	0%
	100%	92,750	102,000	9,500	85%	95%	5%	2%	9%

Results: Analysis 4 – Operation Periods (δ_j^t)

α_1	$\% \sum_j \sum_t \delta_j^t$	TC	Z_1	Z_2	$\% \sum_j \sum_t y_j^t$	Total operating facility	$\% \sum_j \sum_t q_j^t$	$\% \sum_j \sum_t s_j^t$	$\% \sum_j \sum_t l_j^t$	$\% \sum_j \sum_t S_{jk}^t$
0.1	70%	19,080	99,000	10,200	70%	3	82%	18%	2%	10%
	80%	23,555	101,000	14,950	80%	4	94%	6%	4%	13%
	90%	13,360	103,000	3,400	90%	4	95%	5%	1%	4%
	100%	10,400	104,000	-	95%	4	95%	5%	0%	0%
0.2	70%	27,960	99,000	10,200	70%	3	82%	18%	2%	10%
	80%	32,160	101,000	14,950	80%	4	94%	6%	4%	13%
	90%	24,000	103,000	3,400	90%	4	95%	5%	1%	4%
	100%	20,800	104,000	-	95%	4	95%	5%	0%	0%
0.3	70%	36,840	99,000	10,200	70%	3	82%	18%	2%	10%
	80%	40,765	101,000	14,950	80%	4	94%	6%	4%	13%
	90%	33,875	103,000	3,400	90%	4	95%	5%	0%	5%
	100%	31,200	104,000	-	95%	4	95%	5%	0%	0%
0.4	70%	45,720	99,000	10,200	70%	3	82%	18%	2%	10%
	80%	49,370	101,000	14,950	80%	4	94%	6%	4%	13%
	90%	43,750	103,000	4,250	90%	4	95%	5%	1%	4%
	100%	41,600	104,000	-	95%	4	95%	5%	0%	0%
0.5	70%	54,600	99,000	10,200	70%	3	82%	18%	2%	10%
	80%	57,975	101,000	14,950	80%	4	94%	6%	4%	13%
	90%	53,625	103,000	4,250	90%	4	95%	5%	1%	4%
	100%	52,000	104,000	-	95%	4	95%	5%	0%	0%
0.6	70%	63,480	99,000	10,200	70%	3	82%	18%	2%	10%
	80%	66,580	101,000	14,950	80%	4	94%	6%	4%	13%
	90%	63,500	103,000	4,250	90%	4	95%	5%	1%	4%
	100%	62,400	104,000	-	95%	4	95%	5%	0%	0%
0.7	70%	72,360	99,000	10,200	70%	3	82%	18%	3%	9%
	80%	75,185	101,000	14,950	80%	4	94%	6%	4%	13%
	90%	73,375	103,000	4,250	90%	4	95%	5%	1%	4%
	100%	72,800	104,000	-	95%	4	95%	5%	0%	0%
0.8	70%	81,240	99,000	10,200	70%	3	82%	18%	3%	9%
	80%	83,790	101,000	14,950	80%	4	94%	6%	4%	13%
	90%	83,250	103,000	4,250	90%	4	95%	5%	1%	4%
	100%	83,200	104,000	-	95%	4	95%	5%	0%	0%
0.9	70%	90,120	99,000	10,200	70%	3	82%	18%	2%	10%
	80%	92,395	101,000	14,950	80%	4	94%	6%	4%	13%
	90%	92,750	102,000	9,500	85%	4	95%	5%	2%	9%
	100%	92,750	102,000	9,500	85%	4	95%	5%	2%	9%

D. Sensitivity Analyses – Single-objective model

This section presents the sensitivity analyses and its results using the single-objective model. The model was tested using both datasets; the congested (as in Chapter 4, or called the Dataset A) and the non-congested (Dataset A1). Four analyses were conducted, and the following results were found. The costs for each variable that were used for analysis 1, 3 and 4 are:

Table 0-14: Costs of variable

Parameters	Values
C_j	1000 units
ε_1, ε_2, ε_3, ε_{4j}	50 units

Meanwhile, the costs data for Analysis 2 was based on Cases; as in Chapter 4 (section 4.5.3), which are:

Table 0-15: Parameters variation for each analysis

Case	Cost's values and its variation	Fixed costs values
I	C_j = 200, 400, 600, …, 5000	$\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_{4j}$ = 50 units
II	ε_1 = 10, 20, 30, …, 150	C_j = 1000; $\varepsilon_2 = \varepsilon_3 = \varepsilon_{4j}$ = 50 units
III	ε_2 = 10, 20, 30, …, 150	C_j = 1000; $\varepsilon_1 = \varepsilon_2 = \varepsilon_{4j}$ = 50 units
IV	ε_3 = 10, 20, 30, …, 150	C_j = 1000; $\varepsilon_1 = \varepsilon_2 = \varepsilon_{4j}$ = 50 units
V	ε_{4j} = 10, 20, 30, …, 150	C_j = 1000; $\varepsilon_1 = \varepsilon_2 = \varepsilon_3$ = 50 units

Results: Analysis 1 – Variations of *B* values

Table 0-16: Results on system performance by varying the *B*.

Dataset	*B*	Objective function	Amount of Demand Left (%)
A (uneven congestion dataset)	0.00	107,500	0 (0%)
	0.01	106,650	17 (1%)
	0.02	105,800	34 (2%)
	0.03	105,000	51 (3%)
	0.04	105,000	68 (4%)
	0.05	105,000	85 (5%)
A1 (non-congested dataset)	0.00	105,000	0 (0%)
	0.01	105,000	15 (1%)
	0.02	105,000	34 (2%)
	0.03	105,000	50 (3%)
	0.04	104,850	68 (4%)
	0.05	104,000	85 (5%)

Results: Analysis 2 – Trade-off of variables' costs

Dataset A (the uneven congestion dataset)

C_j	TC	$\%\sum_j \sum_t y_j^t$	ε_1	TC	$\%\sum_j \sum_t q_j^t$	ε_2	TC	$\%\sum_j \sum_t s_j^t$	ε_3	TC	$\%\sum_j \sum_t l_j^t$	ε_{4j}	TC	$\%\sum_j \sum_t S_{jk}^t$
200	88,000	95%	10	37,000	100%	10	104,000	0%	10	100,600	5%	10	103,850	5%
400	92,600	95%	20	54,000	100%	20	104,000	0%	20	101,450	5%	20	104,000	0%
600	96,400	95%	30	71,000	100%	30	104,000	0%	30	102,300	5%	30	104,000	0%
800	102,000	95%	40	87,850	95%	40	104,000	0%	40	103,150	5%	40	104,000	0%
1000	104,000	95%	50	104,000	95%	50	104,000	0%	50	104,000	5%	50	104,000	0%
1200	107,800	95%	60	120,150	95%	60	104,000	0%	60	104,850	5%	60	104,000	0%
1400	111,600	95%	70	136,300	95%	70	104,000	0%	70	105,000	0%	70	104,000	0%
1600	115,400	95%	80	152,450	95%	80	104,000	0%	80	105,000	0%	80	104,000	0%
1800	119,200	95%	90	168,600	95%	90	104,000	0%	90	105,000	0%	90	104,000	0%
2000	123,000	95%	100	184,750	95%	100	104,000	0%	100	105,000	0%	100	104,000	0%
2200	126,800	95%	110	200,900	95%	110	104,000	0%	110	105,000	0%	110	104,000	0%
2400	130,600	95%	120	217,050	95%	120	104,000	0%	120	105,000	0%	120	104,000	0%
2600	134,400	95%	130	233,200	95%	130	104,000	0%	130	105,000	0%	130	104,000	0%
2800	138,200	95%	140	249,350	95%	140	104,000	0%	140	105,000	0%	140	104,000	0%
3000	142,000	95%	150	265,500	95%	150	104,000	0%	150	105,000	0%	150	104,000	0%
3200	145,800	95%												
3400	149,600	95%												
3600	153,400	95%												
3800	157,200	95%												
4000	161,000	95%												
4200	164,800	95%												
4400	168,450	90%												
4600	172,050	90%												
4800	175,650	90%												
5000	179,250	90%												

Dataset A1 (the non-congested dataset)

C_j	TC	$\%\sum_j \sum_t y_j^t$	ε_1	TC	$\%\sum_j \sum_t q_j^t$	ε_2	TC	$\%\sum_j \sum_t s_j^t$	ε_3	TC	$\%\sum_j \sum_t l_j^t$	ε_{4j}	TC	$\%\sum_j \sum_t S_{jk}^t$
200	89,000	100%	10	39,000	97%	10	104,300	2%	10	100,600	5%	10	103,850	5%
400	93,000	100%	20	55,500	97%	20	104,600	2%	20	101,450	5%	20	104,000	0%
600	97,000	100%	30	72,000	97%	30	104,900	2%	30	102,300	5%	30	104,000	0%
800	101,000	100%	40	88,500	97%	40	105,000	0%	40	103,150	5%	40	104,000	0%
1000	105,000	100%	50	105,000	95%	50	105,000	0%	50	104,000	5%	50	104,000	0%
1200	109,000	100%	60	121,150	95%	60	105,000	0%	60	104,850	5%	60	104,000	0%
1400	113,000	100%	70	137,300	95%	70	105,000	0%	70	105,000	0%	70	104,000	0%
1600	116,900	95%	80	153,450	95%	80	105,000	0%	80	105,000	0%	80	104,000	0%
1800	120,700	95%	90	169,600	95%	90	105,000	0%	90	105,000	0%	90	104,000	0%
2000	124,500	95%	100	185,750	95%	100	105,000	0%	100	105,000	0%	100	104,000	0%
2200	128,300	95%	110	201,900	95%	110	105,000	0%	110	105,000	0%	110	104,000	0%
2400	132,100	95%	120	218,050	95%	120	105,000	0%	120	105,000	0%	120	104,000	0%
2600	135,900	95%	130	234,200	95%	130	105,000	0%	130	105,000	0%	130	104,000	0%
2800	139,700	95%	140	250,350	95%	140	105,000	0%	140	105,000	0%	140	104,000	0%
3000	143,500	95%	150	266,500	95%	150	105,000	0%	150	105,000	0%	150	104,000	0%
3200	147,300	95%												
3400	150,800	85%												
3600	154,200	85%												
3800	157,600	85%												
4000	161,000	85%												
4200	164,400	85%												
4400	167,800	85%												
4600	171,200	85%												
4800	174,600	85%												
5000	178,000	85%												

Results: Analysis 3 – Capacity level (τ_j^t)

Dataset	Capacity allowance per j (τ_j^t)	TC	$\%\sum_j\sum_t y_j^t$	$\%\sum_j\sum_t q_j^t$	$\%\sum_j\sum_t l_j^t$	$\%\sum_j\sum_t s_j^t$	$\%\sum_j\sum_t S_{jk}^t$
A	70%	105,950	100%	82%	18%	0%	1%
	80%	109,900	100%	94%	6%	1%	5%
	90%	105,750	100%	95%	5%	0%	1%
	100%	105,000	100%	95%	5%	0%	0%
A1	70%	105,000	100%	82%	18%	0%	0%
	80%	105,000	100%	94%	6%	0%	0%
	90%	104,000	95%	95%	5%	0%	0%
	100%	104,000	95%	95%	5%	0%	0%

Results: Analysis 4 – Operation Periods (δ_j^t)

Dataset	Total allowance for operation periods (δ_j^t)	TC	$\%\sum_j\sum_t y_j^t$	Total operated j	$\%\sum_j\sum_t q_j^t$	$\%\sum_j\sum_t l_j^t$	$\%\sum_j\sum_t s_j^t$	$\%\sum_j\sum_t S_{jk}^t$
A	70%	106,200	70%	3	82%	18%	1%	7%
	80%	113,450	80%	4	94%	6%	4%	10%
	90%	107,750	90%	4	95%	5%	0%	5%
	100%	105,000	100%	4	95%	5%	0%	0%
A1	70%	109,200	70%	3	82%	18%	1%	11%
	80%	115,950	80%	4	94%	6%	4%	13%
	90%	107,250	90%	4	95%	5%	1%	4%
	100%	104,000	90%	4	95%	5%	0%	0%

APPENDIX 4: DETAILED INFORMATION FOR CHAPTER 4

This section presents the related data used in Chapter 4. This includes the average travel time from a ward to each HWRC, survey questions from Baotai (2015), steps taken to calculate the preference value for each HWRC per ward and results for sensitivity analyses of the modified model.

A. Average Travel Time from each Ward to each HWRC

Table 0-17: Average travel times from ward *i* to each HWRC

Travel time(ward-HWRC)	Beighton Road	Blackstock Road	Deepcar	Greaves Lane	Longley Avenue
Arbourthone	12	3	25	23	15
Beauchief	20	10	29	30	20
Beighton	3	16	30	26	20
Birley	8	8	28	24	18
Broomhill	17	12	19	19	10
Burngreave	13	13	19	19	5
Central	15	10	23	23	13
Crooks	19	14	19	21	11
Darnall	13	14	20	16	13
Dore and Totley	22	13	30	30	21
East Ecclesfield	21	24	12	6	12
Ecclesall	23	16	27	27	18
Firth Park	21	22	19	14	7
Fulwood	22	16	24	24	14
Gleadless Valley	15	2	27	28	18
Graves Park	19	10	27	28	18
Hillsborough	21	18	11	14	7
Manor e Castle	11	11	24	21	14
Mosborough	8	13	34	30	23
Nether Edge	19	12	23	24	14
Richmond	5	11	23	19	12
Shiregreen	20	21	19	15	9
Southey	20	18	15	11	2
Stannington	24	22	18	20	12
Stocksbridge	32	33	6	12	20
Walkley	19	14	16	17	9
West Ecclesfield	23	21	10	6	8
Woodhouse	3	13	27	22	18
AVERAGE TRAVEL TIME	**17**	**15**	**22**	**20**	**14**

B. The Survey Questions (Baotai, 2015)

HWRC Questionnaire

Cover letter

Dear participant,
This survey is part of a research to know more about citizens' opinions on services being offered by Household Waste Recycling Centres located in the City of Sheffield. The purpose of this research (promoted by Sheffield University Management School) is to gain insights on how to improve service efficiency and user satisfaction in the current climate of funding difficulties Local Authorities are facing.

The survey will not take more than 10 minutes to complete. For the purpose of qualitative and quantitative analysis, all the data will be anonymised and treated with the strictest confidentiality.

Participation in this survey is voluntary, and you can withdraw at any time (simply by exiting from the questionnaire), if you wish to do so. In this case, your responses will be destroyed and not considered in the analysis.

The research has gained ethics approval from the University of Sheffield.

Shall you have any query about this research, do not hesitate to get in touch with us by using the contact details shown below.

Sincerely

Dr Andrea Genovese
Sheffield University
Management School
Logistics and Supply Chain
Management Research Centre
T:01142223347
E: a.genovese@shef.ac.uk

General Questions

Are you aware of the existence of the Household Waste Recycling Centres (commonly known as "tips") in Sheffield?

- Yes
- No

Have you (or your household) ever used one of the five Household Waste Recycling Centres (HWRC) located in Sheffield?

- Yes
- No

If not, why?

- All the services are too far from the place where I live
- It is difficult for me to reach HWRC centres
- The service is not very efficient
- I find it difficult going to the centres due to current opening hours
- I do not consider useful going to a HWRC facility
- No need to visit the site
- I normally go to Recycling Points (small unattended facilities near supermarkets and community centres)
- Other (please specify)

If yes, how many times per year do you visit a HWRC facility?

- Very rarely (once every year or less)
- Rarely (approximately, once every 6 months)
- Occasionally (approximately, once every 3 months)
- Regularly (approximately, once a 1 month)
- Very regularly (approximately, fortnightly)

Which one is your most preferred HWRC facility in Sheffield?
Rank them from 1 to 5 (1 meaning "the most preferred", 5 "meaning the least preferred").
You just need to drag and drop different locations to obtain your preferred order.

- Beighton Road, Woodhouse
- Blackstock Road, Gleadless
- Manchester Road, Deepcar
- Greaves Lane, High Green
- Longley Avenue West, Shirecliffe

Operational Questions

Which of the following materials do you recycle when you visit a HWRC facility?

☐ Paper	☐ Car Batteries
☐ Glass	☐ Scrap Metal
☐ Textiles & Clothing	☐ Electrical and Electronic Equipment
☐ Plastics	☐ Wood
☐ Steel Cans	☐ Household Batteries
☐ Aluminium & Foil	☐ Fluorescent tubes/bulbs
☐ Furniture	☐ Cement Bonded Asbestos
☐ Motor Oil	☐ Plasterboard
☐ Green Waste	☐ Fridges
☐ Books	☐ Bric-a-Brac
☐ Cardboard	☐ Household Chemicals and Pesticides.
☐ Rubble	☐ Any other (please specify)
☐ Shoes	

Please rate the layout (in terms of containers location, signs, parking spaces, stairs leading to skips, other elements) of your preferred HWRC centre.

- ◯ Very Poor
- ◯ Poor
- ◯ Average
- ◯ Good
- ◯ Very Good

Please rate the level of customer service provided by HWRC staff at you preferred centre.

- ◯ Very Poor
- ◯ Poor
- ◯ Average
- ◯ Good
- ◯ Very Good

Please rate the ease of use of your preferred HWRC centre.

- ◯ Very Poor
- ◯ Poor
- ◯ Average
- ◯ Good
- ◯ Very Good

Please rate your overall satisfaction about the service provided at your preferred HWRC facility.

- ○ Very Poor
- ○ Poor
- ○ Average
- ○ Good
- ○ Very Good

What operational suggestions would you make for the functioning of HWRC facilities?

Are you aware of the existence of Recycling Points (small unattended facilities, usually located close to supermarkets, shopping areas and community centres) in Sheffield?

- ○ Yes
- ○ No

Are you aware of the existence of Recycling Points (small unattended facilities, usually located close to supermarkets, shopping areas and community centres) in Sheffield?

- ○ Yes
- ○ No

How many times per year do you visit a Recycling Point facility?

- ○ Very rarely (once every year or less)
- ○ Rarely (approximately, once every 6 months)
- ○ Occasionally (approximately, once every 3 months)
- ○ Regularly (approximately, once a month)
- ○ Very regularly (approximately, forthnightly)

Which of the following materials do you recycle when you visit a Recycling Point facility?

- ☐ Paper
- ☐ Steel/Aluminium Cans
- ☐ Glass
- ☐ Books/Cd/Dvd
- ☐ Textile
- ☐ Cardboard
- ☐ Plastic
- ☐ Shoes

Which of the following materials (not currently allowed in Recycling Points) would you like to dispose there?

- ☐ Motor Oil
- ☐ Car batteries
- ☐ Household batteries
- ☐ Fluorescent tubes/bulbs
- ☐ Small Electrical and Electronic Equipment

Accessibility Questions

Please rate, on a scale of 1 to 5 (1 meaning "not important at all", 5 meaning "very important") the importance of the following factors in choosing your preferred HWRC facility.

	1	2	3	4	5
Proximity to your home	○	○	○	○	○
Centre efficiency	○	○	○	○	○
Opening hours of the centre	○	○	○	○	○
Customer service quality	○	○	○	○	○
Recycling rate of the site	○	○	○	○	○
Waste disposal recycling portfolio	○	○	○	○	○

How far are you prepared (or would you be prepared) to travel to visit a HWRC facility?

- ○ Less than 5 mins
- ○ 5 - 10 mins
- ○ 11 - 15 mins
- ○ 16 - 20 mins
- ○ 21 - 25 mins
- ○ more than 25 mins

Please rate your satisfaction about the current location of HWRC centres.

- Very Poor
- Poor
- Average
- Good
- Very Good

What is (or what would be) your preferred day of the week to visit your HWRC facility?

- Monday
- Tuesday - Wednesday
- Thursday
- Friday
- Saturday - Sunday
- No preferred day, it varies from time to time

What is (or what would be) your preferred time of the day to visit your HWRC facility during Winter time (October - March)?

- 10am-12pm
- 12-2pm
- 2-4pm
- No preferred time, it varies

What is (or what would be) your preferred time of the day to visit your HWRC facility during summer time (April-September)?

- 10am-12pm
- 12-2pm
- 2-4pm
- 4-6pm
- No preferred time, it varies

On a scale of 1 to 5 (1 meaning "not useful at all", 3 meaning "neutral", 5 meaning "very useful"), rate the following potential new opening times for your preferred HWRC facility during Winter months

Winter months

	1	2	3	4	5
6-7am	○	○	○	○	○
7-8am	○	○	○	○	○
8-9am	○	○	○	○	○
9-10am	○	○	○	○	○
4-5pm	○	○	○	○	○
5-6pm	○	○	○	○	○
6-7pm	○	○	○	○	○

On a scale of 1 to 5 (1 meaning "not useful at all", 3 meaning "neutral", 5 meaning "very useful"), rate the following potential new opening times for your preferred HWRC facility during Summer months.

Summer months

	1	2	3	4	5
6-7am	○	○	○	○	○
7-8am	○	○	○	○	○
8-9am	○	○	○	○	○
9-10am	○	○	○	○	○
6-7pm	○	○	○	○	○
7-8pm	○	○	○	○	○
8-9pm	○	○	○	○	○

Which transportation mode do you use to get to your preferred HWRC?

- Car
- 4x4
- Motorbike
- Walk to site
- Car or 4x4 with trailer
- Van
- Pick up truck
- Other (please specify)

If you use a van, pick-up or trailer, do you normally contact the site in advance to get a permit?

- Yes - I have applied for a permit
- Yes - I have applied for a one-off visit
- No

Which route do you generally follow to visit your preferred HWRC facility?

- Home-HWRC-Home
- Home-HWRC-Work
- Work-HWRC-Home
- Home-HWRC-Other
- Work-HRWC-Other
- Work-HWRC-Work
- Other (Please Specify)

Demographic Information

Gender

- Male
- Female
- Other

Age

- 18-35
- 36-50
- 51-65
- over 65

Nationality

- British
- EU Nationality (please specify)
- Other nationality (please specify)

Postcode (please specify)

Select your electoral ward if you live within Sheffield City Council area. If you live outside of Sheffield City Council area, specify the name of the ward by selecting "Other"

Arbourthorne	East Ecclesfield	Richmond
Beauchief and Greenhill	Ecclesall	Shiregreen and Brightside
Beighton	Firth Park	Southey
Birley	Fulwood	Stannington
Broomhill	Gleadless Valley	Stocksbridge and Upper Don
Burngreave	Graves Park	Walkley
Central	Hillsborough	West Ecclesfield
Crookes	Manor Castle	Woodhouse
Darnall	Mosborough	Other
Dore and Totley	Nether Edge	

Occupation

- Student
- University employee
- Other (please specify)

Email Address (if you want to be part of further stages of the study)

Yearly income

- Less than £10k
- £11k-£20k
- £21k-£30k
- £31k-£40k
- £41k-£50k
- +£50k

Thank you for your time. For the purpose of qualitative and quantitative analysis, all data will be anonymised and treated with the strictest confidentiality. Shall you have any query about this research (or shall you want to withdraw from the research), do not hesitate to get in touch with us by using the contact details shown below.

Sincerely

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C. Step to Measure the Preference Value for Each HWRC per ward

The following steps were used to calculate the preference value for each HWRC per ward. Taking respondents from '**CENTRAL**' as an example:

<u>Step 1:</u> Total up the response for each preference level.

As an example, the responses of 11 respondents who resided in the 'CENTRAL' ward ranked their preferences for each HWRC as follows:

Table 0-18: Respondent preference levels

HWRC	**Preference level**					**TOTAL RESPONDENTS**
	1 - most preferred	**2 - preferred**	**3 - average**	**4 - less preferred**	**5 - least preferred**	
Beighton Road	3	4	2	2	0	**11**
Blackstock Road	7	2	2	0	0	**11**
Deepcar	0	2	3	3	3	**11**
Greaves Lane	0	1	3	5	2	**11**
Longley Avenue	1	2	1	1	6	**11**
TOTAL RESPONDENTS	**11**	**11**	**11**	**11**	**11**	

From Table 0-18, the 'most preferred' recycling centre is Blackstock Road, 'preferred' is Beighton Road, 'average' is Deepcar and Greaves Lane, 'less preferred' is Greaves Lane and 'least preferred' is Longley Avenue. This clearly shows that proximity is not the main concern of the respondents residing in the 'CENTRAL' ward, even though Longley Avenue is located less than 15 minutes away.

<u>Step 2:</u> Convert to the percentage of preference level.

Focussing on 'most preferred' column, there were seven respondents from the 'CENTRAL' ward who chose Blackstock Road as their 'most preferred' site, three chose the Beighton Road site, one chose the Longley Avenue site, and no respondents chose the Deepcar and Greaves Lane sites. These values were converted into a percentage using the following calculations:

Table 0-19: Changing from preference to the percentage of preference – an example

HWRC	1 - Most Preferred
Beighton Road	$(3/11) \times 100 = 27.3\%$
Blackstock Road	$(7/11) \times 100 = 63.6\%$
Deepcar	0%
Greaves Lane	0%
Longley Avenue	$(1/11) \times 100 = 9.1\%$
TOTAL PERCENTAGE	**100.0%**

This process was repeated for all preference levels for all HWRCs. Table 0-20 shows the percentage per preference level per HWRC for the 'CENTRAL' ward.

Table 0-20: Percentage of preference level for each HWRC

HWRC	Preference level				
	1 - most preferred	2 - preferred	3 - average	4 - less preferred	5 - least preferred
Beighton Road	27.3%	36.4%	18.2%	18.2%	0.0%
Blackstock Road	63.6%	18.2%	18.2%	0.0%	0.0%
Deepcar	0.0%	18.2%	27.3%	27.3%	27.3%
Greaves Lane	0.0%	9.1%	27.3%	45.5%	18.2%
Longley Avenue	9.1%	18.2%	9.1%	9.1%	54.6%
TOTAL PERCENTAGE	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**

<u>Step 3:</u> Generalised distribution of five preference levels into preference value per HWRC.

To calculate the preference values for each HWRC, the respective weights used were:

Table 0-21: Weights for each preference level

Preferences	Weights
1 - most preferred	0.6
2 - preferred	0.3
3 - average	0.1
4 - less preferred	0
5 - least preferred	0

These weights contribute to generalising the ranking in Table 0-20 and calculate the preference value for each HWRC. Using these weights, the preference of 'CENTRAL' respondents was as follows:

Table 0-22: Preference level for each HWRC for "CENTRAL" respondents

HWRC	Preference value
Beighton Road	(0.27)(0.6) + (0.36)(0.3) + (0.18)(0.1) + (0.18)(0) + (0.00)(0) = 0.291 = **29.1%**
Blackstock Road	(0.64)(0.6) + (0.18)(0.3) + (0.18)(0.1) + (0.00)(0) + (0.00)(0) = 0.455 = **45.5%**
Deepcar	(0.00)(0.6) + (0.18)(0.3) + (0.27)(0.1) + (0.27)(0) + (0.27)(0) = 0.082 = **8.2%**
Greaves Lane	(0.00)(0.6) + (0.09)(0.3) + (0.27)(0.1) + (0.45)(0) + (0.18)(0) = 0.055 = **5.5%**
Longley Avenue	(0.09)(0.6) + (0.18)(0.3) + (0.09)(0.1) + (0.09)(0) + (0.55)(0) = 0.118 = **11.8%**
TOTAL RESPONDENTS	**100.0%**

This step was repeated for all the wards in Sheffield.

D. Distribution of User from Each Ward to Each HWRC

Table 0-23: Amount of user at each HWRC (survey and spatial interaction model (SIM))

i	Wards	HWRC									
		Beighton Rd		**Blackstock Rd**		**Deepcar**		**Greaves Lane**		**Longley Avenue**	
		Survey	SIM	Survey	SIM	Survey	SIM	Survey	SIM	Survey	SIM
1	Arbourthone	1099	747	2750	3278	552	171	64	195	430	493
2	Beauchief	1545	1107	3084	3061	238	310	78	376	592	686
3	Beighton	3117	4770	1830	671	342	140	67	195	744	329
4	Birley	2569	2173	1967	2111	87	180	0	236	514	442
5	Broomhill	583	557	1180	1176	329	351	206	369	1321	1162
6	Burngreave	0	499	671	621	166	308	1175	409	3027	3214
7	Central	1510	815	2361	2361	425	353	285	384	612	1281
8	Crookes	790	829	890	1431	530	685	514	680	2802	1901
9	Darnall	3046	1765	1466	1410	113	440	113	643	902	1382
10	Dore and Totley	2034	1289	3582	2817	385	518	177	581	139	1112
11	East Ecclesfield	80	556	0	488	852	951	3323	2273	1933	1902
12	Ecclesall	1305	1189	3433	2494	219	604	186	636	1286	1504
13	Firth Park	373	364	373	378	280	373	840	565	2800	2986
14	Fulwood	1037	945	1380	1472	376	720	446	634	2132	1601
15	Gleadless Valley	1390	492	3433	4445	166	160	286	177	452	446
16	Graves Park	1230	958	3106	3395	219	312	722	335	508	768
17	Hillsborough	658	641	504	721	1145	990	704	739	2708	2634
18	Manor e Castle	729	1279	2182	1632	144	249	0	336	1309	864
19	Mosborough	3455	3634	1728	1152	363	219	75	282	144	472
20	Nether Edge	948	948	2984	2476	613	446	363	468	669	1238
21	Richmond	2883	2269	1922	1863	107	230	107	320	320	657
22	Shiregreen	952	787	952	730	445	718	695	1134	2662	2331
23	Southey	296	316	1183	350	394	572	690	868	2366	2825
24	Stannington	140	852	140	960	1201	1106	1481	909	3387	2536
25	Stocksbridge	174	303	58	310	3872	4189	1291	968	1058	691
26	Walkley	814	809	1031	1069	543	597	543	608	2496	2344
27	West Ecclesfield	0	224	0	230	968	756	3265	4070	1814	762
28	Woodhouse	2835	3618	1289	670	170	175	170	247	685	443
AMOUNT OF USER PER HWRC		**35591**	**34735**	**45477**	**43772**	**15246**	**16822**	**17865**	**19638**	**39815**	**39006**

E. Sensitivity Analysis

1. Minimum number of operating periods per week ($\boldsymbol{\delta_{min_j}}$)

δ_{min_j}	α_1	TC	Z_1	Z_2	$\sum_j \sum_t y_j^t$ *Operating hours*	$\sum_j \sum_t q_j^t$ *Amount of demand served*	$\sum_j \sum_t s_j^t$ *Amount of demand in the queue*	$\sum_j \sum_t l_j^t$ *Amount of demand leave*	$\sum_j \sum_t S_{jk}^t$ *Amount of demand move*
50%	0.1	1,290	11,100	200	100.0%	95.1%	4.9%	0.0%	3.9%
	0.2	2,380	11,100	200	100.0%	95.1%	4.9%	0.0%	3.9%
	0.3	3,470	11,100	200	100.0%	95.1%	4.9%	0.0%	3.9%
	0.4	4,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%
	0.5	5,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%
	0.6	6,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%
	0.7	7,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%
	0.8	8,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%
	0.9	9,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%
60%	0.1	1,290	11,100	200	100.0%	95.1%	4.9%	0.0%	3.9%
	0.2	2,380	11,100	200	100.0%	95.1%	4.9%	0.0%	3.9%
	0.3	3,470	11,100	200	100.0%	95.1%	4.9%	0.0%	3.9%
	0.4	4,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%
	0.5	5,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%
	0.6	6,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%
	0.7	7,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%
	0.8	8,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%
	0.9	9,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%
70%	0.1	1,290	11,100	200	100.0%	95.1%	4.9%	0.0%	3.9%
	0.2	2,380	11,100	200	100.0%	95.1%	4.9%	0.0%	3.9%
	0.3	3,470	11,100	200	100.0%	95.1%	4.9%	0.0%	3.9%
	0.4	4,520	10,700	400	93.3%	95.1%	4.9%	0.0%	7.8%
	0.5	5,520	10,500	540	90.0%	95.1%	4.9%	0.0%	10.6%
	0.6	6,516	10,500	540	90.0%	95.1%	4.9%	0.0%	10.6%
	0.7	7,512	10,500	540	90.0%	95.1%	4.9%	0.0%	10.6%
	0.8	8,508	10,500	540	90.0%	95.1%	4.9%	0.0%	10.6%
	0.9	9,504	10,500	540	90.0%	95.1%	4.9%	0.0%	10.6%
80%	0.1	1,290	11,100	200	100.0%	95.1%	4.9%	0.0%	3.9%
	0.2	2,380	11,100	200	100.0%	95.1%	4.9%	0.0%	3.9%
	0.3	3,470	11,100	200	100.0%	95.1%	4.9%	0.0%	3.9%
	0.4	4,530	10,800	350	95.0%	95.1%	4.9%	0.0%	6.9%
	0.5	5,530	10,500	560	90.0%	95.1%	4.9%	0.0%	11.0%
	0.6	6,524	10,500	560	90.0%	95.1%	4.9%	0.0%	11.0%
	0.7	7,518	10,500	560	90.0%	95.1%	4.9%	0.0%	11.0%
	0.8	8,512	10,500	560	90.0%	95.1%	4.9%	0.0%	11.0%
	0.9	9,506	10,500	560	90.0%	95.1%	4.9%	0.0%	11.0%

2. Minimum number of operating periods per day ($\boldsymbol{\delta_{min_{jw}}}$)

$\delta_{min_{jw}}$	α_1	TC	Z_1	Z_2	$\sum_j \sum_t y_j^t$ *Operating hours*	$\sum_j \sum_t q_j^t$ *Amount of demand served*	$\sum_j \sum_t s_j^t$ *Amount of demand in the queue*	$\sum_j \sum_t l_j^t$ *Amount of demand leave*	$\sum_j \sum_t S_{jk}^t$ *Amount of demand move*
	0.1	1,290	11,100	200	100.0%	95.1%	4.9%	0.0%	3.9%
	0.2	2,380	11,100	200	100.0%	95.1%	4.9%	0.0%	3.9%
	0.3	3,470	11,100	200	100.0%	95.1%	4.9%	0.0%	3.9%
	0.4	4,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%
50%	0.5	5,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%
	0.6	6,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%
	0.7	7,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%
	0.8	8,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%
	0.9	9,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%
	0.1	1,290	11,100	200	100.0%	95.1%	4.9%	0.0%	3.9%
	0.2	2,380	11,100	200	100.0%	95.1%	4.9%	0.0%	3.9%
	0.3	3,470	11,100	200	100.0%	95.1%	4.9%	0.0%	3.9%
	0.4	4,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%
60%	0.5	5,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%
	0.6	6,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%
	0.7	7,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%
	0.8	8,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%
	0.9	9,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%
	0.1	1,290	11,100	200	100.0%	95.1%	4.9%	0.0%	3.9%
	0.2	2,380	11,100	200	100.0%	95.1%	4.9%	0.0%	3.9%
	0.3	3,470	11,100	200	100.0%	95.1%	4.9%	0.0%	3.9%
	0.4	4,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%
70%	0.5	5,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%
	0.6	6,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%
	0.7	7,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%
	0.8	8,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%
	0.9	9,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%
	0.1	1,290	11,100	200	100.0%	95.1%	4.9%	0.0%	3.9%
	0.2	2,380	11,100	200	100.0%	95.1%	4.9%	0.0%	3.9%
	0.3	3,470	11,100	200	100.0%	95.1%	4.9%	0.0%	3.9%
	0.4	4,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%
80%	0.5	5,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%
	0.6	6,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%
	0.7	7,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%
	0.8	8,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%
	0.9	9,500	10,500	500	90.0%	95.1%	4.9%	0.0%	9.8%

3. Trade-off costs

Varying C_j

C_j	α_1	TC	Z_1	Z_2	% $\sum_j \sum_t y_j^t$
	0.1	750	5,700	200	100.0%
	0.2	1,300	5,700	200	100.0%
	0.3	1,850	5,700	200	100.0%
	0.4	2,400	5,700	200	100.0%
10	0.5	2,950	5,700	200	100.0%
	0.6	3,500	5,700	200	100.0%
	0.7	4,050	5,700	200	100.0%
	0.8	4,600	5,700	200	100.0%
	0.9	5,126	5,640	500	90.0%
	0.1	810	6,300	200	100.0%
	0.2	1,420	6,300	200	100.0%
	0.3	2,030	6,300	200	100.0%
	0.4	2,640	6,300	200	100.0%
20	0.5	3,250	6,300	200	100.0%
	0.6	3,860	6,300	200	100.0%
	0.7	4,470	6,300	200	100.0%
	0.8	5,044	6,180	500	90.0%
	0.9	5,612	6,180	500	90.0%
	0.1	870	6,900	200	100.0%
	0.2	1,540	6,900	200	100.0%
	0.3	2,210	6,900	200	100.0%
	0.4	2,880	6,900	200	100.0%
30	0.5	3,550	6,900	200	100.0%
	0.6	4,220	6,900	200	100.0%
	0.7	4,854	6,720	500	90.0%
	0.8	5,476	6,720	500	90.0%
	0.9	6,098	6,720	500	90.0%
	0.1	930	7,500	200	100.0%
	0.2	1,660	7,500	200	100.0%
	0.3	2,390	7,500	200	100.0%
	0.4	3,120	7,500	200	100.0%
40	0.5	3,850	7,500	200	100.0%
	0.6	4,556	7,260	500	90.0%
	0.7	5,232	7,260	500	90.0%
	0.8	5,908	7,260	500	90.0%
	0.9	6,584	7,260	500	90.0%
	0.1	1,050	8,700	200	100.0%
	0.2	1,900	8,700	200	100.0%
	0.3	2,750	8,700	200	100.0%
	0.4	3,600	8,700	200	100.0%
50	0.5	4,420	8,340	500	90.0%
	0.6	5,204	8,340	500	90.0%
	0.7	5,988	8,340	500	90.0%
	0.8	6,772	8,340	500	90.0%
	0.9	7,556	8,340	500	90.0%

C_j	α_1	TC	Z_1	Z_2	% $\sum_j \sum_t y_j^t$
	0.1	1,050	8,700	200	100.0%
	0.2	1,900	8,700	200	100.0%
	0.3	2,750	8,700	200	100.0%
	0.4	3,600	8,700	200	100.0%
60	0.5	4,420	8,340	500	90.0%
	0.6	5,204	8,340	500	90.0%
	0.7	5,988	8,340	500	90.0%
	0.8	6,772	8,340	500	90.0%
	0.9	7,556	8,340	500	90.0%
	0.1	1,110	9,300	200	100.0%
	0.2	2,020	9,300	200	100.0%
	0.3	2,930	9,300	200	100.0%
	0.4	3,840	9,300	200	100.0%
70	0.5	4,690	8,880	500	90.0%
	0.6	5,528	8,880	500	90.0%
	0.7	6,366	8,880	500	90.0%
	0.8	7,204	8,880	500	90.0%
	0.9	8,042	8,880	500	90.0%
	0.1	1,170	9,900	200	100.0%
	0.2	2,140	9,900	200	100.0%
	0.3	3,110	9,900	200	100.0%
	0.4	4,068	9,420	500	90.0%
80	0.5	4,960	9,420	500	90.0%
	0.6	5,852	9,420	500	90.0%
	0.7	6,744	9,420	500	90.0%
	0.8	7,636	9,420	500	90.0%
	0.9	8,528	9,420	500	90.0%
	0.1	1,230	10,500	200	100.0%
	0.2	2,260	10,500	200	100.0%
	0.3	3,290	10,500	200	100.0%
	0.4	4,284	9,960	500	90.0%
90	0.5	5,230	9,960	500	90.0%
	0.6	6,176	9,960	500	90.0%
	0.7	7,122	9,960	500	90.0%
	0.8	8,068	9,960	500	90.0%
	0.9	9,014	9,960	500	90.0%
	0.1	1,290	11,100	200	100.0%
	0.2	2,380	11,100	200	100.0%
	0.3	3,470	11,100	200	100.0%
	0.4	4,500	10,500	500	90.0%
100	0.5	5,500	10,500	500	90.0%
	0.6	6,500	10,500	500	90.0%
	0.7	7,500	10,500	500	90.0%
	0.8	8,500	10,500	500	90.0%
	0.9	9,500	10,500	500	90.0%

Varying ε_{1j}

ε_{1j}	α_1	*TC*	Z_1	Z_2	$\% \sum_j \sum_t q_j^t$	ε_{1j}	α_1	*TC*	Z_1	Z_2	$\% \sum_j \sum_t q_j^t$
	0.1	902	7,220	200	95.1%		0.1	1,387	12,070	200	95.1%
	0.2	1,604	7,220	200	95.1%		0.2	2,574	12,070	200	95.1%
	0.3	2,306	7,220	200	95.1%		0.3	3,761	12,070	200	95.1%
	0.4	2,948	6,620	500	95.1%		0.4	4,888	11,470	500	95.1%
2	0.5	3,560	6,620	500	95.1%	12	0.5	5,985	11,470	500	95.1%
	0.6	4,171	6,612	510	95.3%		0.6	7,082	11,470	500	95.1%
	0.7	4,781	6,612	510	95.3%		0.7	8,179	11,470	500	95.1%
	0.8	5,392	6,612	510	95.3%		0.8	9,276	11,470	500	95.1%
	0.9	6,002	6,612	510	95.3%		0.9	10,373	11,470	500	95.1%
	0.1	999	8,190	200	95.1%		0.1	1,484	13,040	200	95.1%
	0.2	1,798	8,190	200	95.1%		0.2	2,768	13,040	200	95.1%
	0.3	2,597	8,190	200	95.1%		0.3	4,052	13,040	200	95.1%
	0.4	3,336	7,590	500	95.1%		0.4	5,276	12,440	500	95.1%
4	0.5	4,045	7,590	500	95.1%	14	0.5	6,470	12,440	500	95.1%
	0.6	4,754	7,590	500	95.1%		0.6	7,664	12,440	500	95.1%
	0.7	5,462	7,590	510	95.3%		0.7	8,858	12,440	500	95.1%
	0.8	6,169	7,584	510	95.3%		0.8	10,052	12,440	500	95.1%
	0.9	6,877	7,584	510	95.3%		0.9	11,246	12,440	500	95.1%
	0.1	1,096	9,160	200	95.1%		0.1	1,581	14,010	200	95.1%
	0.2	1,992	9,160	200	95.1%		0.2	2,962	14,010	200	95.1%
	0.3	2,888	9,160	200	95.1%		0.3	4,343	14,010	200	95.1%
	0.4	3,724	8,560	500	95.1%		0.4	5,664	13,410	500	95.1%
6	0.5	4,530	8,560	500	95.1%	16	0.5	6,955	13,410	500	95.1%
	0.6	5,336	8,560	500	95.1%		0.6	8,246	13,410	500	95.1%
	0.7	6,142	8,560	500	95.1%		0.7	9,537	13,410	500	95.1%
	0.8	6,947	8,556	510	95.3%		0.8	10,828	13,410	500	95.1%
	0.9	7,751	8,556	510	95.3%		0.9	12,119	13,410	500	95.1%
	0.1	1,193	10,130	200	95.1%		0.1	1,678	14,980	200	95.1%
	0.2	2,186	10,130	200	95.1%		0.2	3,156	14,980	200	95.1%
	0.3	3,179	10,130	200	95.1%		0.3	4,634	14,980	200	95.1%
	0.4	4,112	9,530	500	95.1%		0.4	6,052	14,980	200	95.1%
8	0.5	5,015	9,530	500	95.1%	18	0.5	7,440	14,380	500	95.1%
	0.6	5,918	9,530	500	95.1%		0.6	8,828	14,380	500	95.1%
	0.7	6,821	9,530	500	95.1%		0.7	10,216	14,380	500	95.1%
	0.8	7,724	9,530	500	95.1%		0.8	11,604	14,380	500	95.1%
	0.9	8,626	8,556	510	95.3%		0.9	12,992	14,380	500	95.1%
	0.1	1,290	11,100	200	95.1%		0.1	1,775	15,950	200	95.1%
	0.2	2,380	11,100	200	95.1%		0.2	3,350	15,950	200	95.1%
	0.3	3,470	11,100	200	95.1%		0.3	4,925	15,950	200	95.1%
	0.4	4,500	10,500	500	95.1%		0.4	6,440	15,350	500	95.1%
10	0.5	5,500	10,500	500	95.1%	20	0.5	7,925	15,350	500	95.1%
	0.6	6,500	10,500	500	95.1%		0.6	9,410	15,350	500	95.1%
	0.7	7,500	10,500	500	95.1%		0.7	10,895	15,350	500	95.1%
	0.8	8,500	10,500	500	95.1%		0.8	12,380	15,350	500	95.1%
	0.9	9,500	10,500	500	95.1%		0.9	13,865	15,350	500	95.1%

Varying ε_{2j}

ε_{2j}	α_1	TC	Z_1	Z_2	$\% \sum_j \sum_t s_j^t$
	0.1	1,290	11,100	200	0.0%
	0.2	2,380	11,100	200	0.0%
	0.3	3,470	11,100	200	0.0%
	0.4	4,500	10,500	500	0.0%
2	0.5	5,500	10,500	500	0.0%
	0.6	6,500	10,500	500	0.0%
	0.7	7,500	10,500	500	0.0%
	0.8	8,500	10,500	500	0.0%
	0.9	9,500	10,500	500	0.0%
	0.1	1,290	11,100	200	0.0%
	0.2	2,380	11,100	200	0.0%
	0.3	3,470	11,100	200	0.0%
	0.4	4,500	10,500	500	0.0%
4	0.5	5,500	10,500	500	0.0%
	0.6	6,500	10,500	500	0.0%
	0.7	7,500	10,500	500	0.0%
	0.8	8,500	10,500	500	0.0%
	0.9	9,500	10,500	500	0.0%
	0.1	1,290	11,100	200	0.0%
	0.2	2,380	11,100	200	0.0%
	0.3	3,470	11,100	200	0.0%
	0.4	4,500	10,500	500	0.0%
6	0.5	5,500	10,500	500	0.0%
	0.6	6,500	10,500	500	0.0%
	0.7	7,500	10,500	500	0.0%
	0.8	8,500	10,500	500	0.0%
	0.9	9,500	10,500	500	0.0%
	0.1	1,290	11,100	200	0.0%
	0.2	2,380	11,100	200	0.0%
	0.3	3,470	11,100	200	0.0%
	0.4	4,500	10,500	500	0.0%
8	0.5	5,500	10,500	500	0.0%
	0.6	6,500	10,500	500	0.0%
	0.7	7,500	10,500	500	0.0%
	0.8	8,500	10,500	500	0.0%
	0.9	9,500	10,500	500	0.0%
	0.1	1,290	11,100	200	0.0%
	0.2	2,380	11,100	200	0.0%
	0.3	3,470	11,100	200	0.0%
	0.4	4,500	10,500	500	0.0%
10	0.5	5,500	10,500	500	0.0%
	0.6	6,500	10,500	500	0.0%
	0.7	7,500	10,500	500	0.0%
	0.8	8,500	10,500	500	0.0%
	0.9	9,500	10,500	500	0.0%

ε_{2j}	α_1	TC	Z_1	Z_2	$\% \sum_j \sum_t s_j^t$
	0.1	1,290	11,100	200	0.0%
	0.2	2,380	11,100	200	0.0%
	0.3	3,470	11,100	200	0.0%
	0.4	4,500	10,500	500	0.0%
12	0.5	5,500	10,500	500	0.0%
	0.6	6,500	10,500	500	0.0%
	0.7	7,500	10,500	500	0.0%
	0.8	8,500	10,500	500	0.0%
	0.9	9,500	10,500	500	0.0%
	0.1	1,290	11,100	200	0.0%
	0.2	2,380	11,100	200	0.0%
	0.3	3,470	11,100	200	0.0%
	0.4	4,500	10,500	500	0.0%
14	0.5	5,500	10,500	500	0.0%
	0.6	6,500	10,500	500	0.0%
	0.7	7,500	10,500	500	0.0%
	0.8	8,500	10,500	500	0.0%
	0.9	9,500	10,500	500	0.0%
	0.1	1,290	11,100	200	0.0%
	0.2	2,380	11,100	200	0.0%
	0.3	3,470	11,100	200	0.0%
	0.4	4,500	10,500	500	0.0%
16	0.5	5,500	10,500	500	0.0%
	0.6	6,500	10,500	500	0.0%
	0.7	7,500	10,500	500	0.0%
	0.8	8,500	10,500	500	0.0%
	0.9	9,500	10,500	500	0.0%
	0.1	1,290	11,100	200	0.0%
	0.2	2,380	11,100	200	0.0%
	0.3	3,470	11,100	200	0.0%
	0.4	4,500	10,500	500	0.0%
18	0.5	5,500	10,500	500	0.0%
	0.6	6,500	10,500	500	0.0%
	0.7	7,500	10,500	500	0.0%
	0.8	8,500	10,500	500	0.0%
	0.9	9,500	10,500	500	0.0%
	0.1	1,290	11,100	200	0.0%
	0.2	2,380	11,100	200	0.0%
	0.3	3,470	11,100	200	0.0%
	0.4	4,500	10,500	500	0.0%
20	0.5	5,500	10,500	500	0.0%
	0.6	6,500	10,500	500	0.0%
	0.7	7,500	10,500	500	0.0%
	0.8	8,500	10,500	500	0.0%
	0.9	9,500	10,500	500	0.0%

Varying ε_{3j}

ε_{3j}	α_1	TC	Z_1	Z_2	$\% \sum_j \sum_t l_j^t$	ε_{3j}	α_1	TC	Z_1	Z_2	$\% \sum_j \sum_t l_j^t$
	0.1	1,270	10,900	200	4.9%		0.1	1,295	11,150	200	4.9%
	0.2	2,340	10,900	200	4.9%		0.2	2,390	11,150	200	4.9%
	0.3	3,410	10,900	200	4.9%		0.3	3,485	11,150	200	4.9%
	0.4	4,420	10,300	500	4.9%		0.4	4,520	10,550	500	4.9%
2	0.5	5,400	10,300	500	4.9%	12	0.5	5,525	10,550	500	4.9%
	0.6	6,380	10,300	500	4.9%		0.6	6,530	10,550	500	4.9%
	0.7	7,360	10,300	500	4.9%		0.7	7,535	10,550	500	4.9%
	0.8	8,340	10,300	500	4.9%		0.8	8,540	10,550	500	4.9%
	0.9	9,320	10,300	500	4.9%		0.9	9,544	10,548	510	4.7%
	0.1	1,275	10,950	200	4.9%		0.1	1,300	11,200	200	4.9%
	0.2	2,350	10,950	200	4.9%		0.2	2,400	11,200	200	4.9%
	0.3	3,425	10,950	200	4.9%		0.3	3,500	11,200	200	4.9%
	0.4	4,440	10,350	500	4.9%		0.4	4,540	10,600	500	4.9%
4	0.5	5,425	10,350	500	4.9%	14	0.5	5,550	10,600	500	4.9%
	0.6	6,410	10,350	500	4.9%		0.6	6,560	10,600	500	4.9%
	0.7	7,395	10,350	500	4.9%		0.7	7,570	10,600	500	4.9%
	0.8	8,380	10,350	500	4.9%		0.8	8,579	10,596	510	4.7%
	0.9	9,365	10,350	500	4.9%		0.9	9,587	10,596	510	4.7%
	0.1	1,280	11,000	200	4.9%		0.1	1,305	11,250	200	4.9%
	0.2	2,360	11,000	200	4.9%		0.2	2,410	11,250	200	4.9%
	0.3	3,440	11,000	200	4.9%		0.3	3,515	11,250	200	4.9%
	0.4	4,460	10,400	500	4.9%		0.4	4,560	10,650	500	4.9%
6	0.5	5,450	10,400	500	4.9%	16	0.5	5,575	10,650	500	4.9%
	0.6	6,440	10,400	500	4.9%		0.6	6,590	10,650	500	4.9%
	0.7	7,430	10,400	500	4.9%		0.7	7,604	10,644	510	4.7%
	0.8	8,420	10,400	500	4.9%		0.8	8,617	10,644	510	4.7%
	0.9	9,410	10,400	500	4.9%		0.9	9,631	10,644	510	4.7%
	0.1	1,285	11,050	200	4.9%		0.1	1,310	11,300	200	4.9%
	0.2	2,370	11,050	200	4.9%		0.2	2,420	11,300	200	4.9%
	0.3	3,455	11,050	200	4.9%		0.3	3,530	11,300	200	4.9%
	0.4	4,480	10,450	500	4.9%		0.4	4,580	10,700	500	4.9%
8	0.5	5,475	10,450	500	4.9%	18	0.5	5,600	10,700	500	4.9%
	0.6	6,470	10,450	500	4.9%		0.6	6,619	10,692	510	4.7%
	0.7	7,465	10,450	500	4.9%		0.7	7,637	10,692	510	4.7%
	0.8	8,460	10,450	500	4.9%		0.8	8,656	10,692	510	4.7%
	0.9	9,455	10,450	500	4.9%		0.9	9,674	10,692	510	4.7%
	0.1	1,290	11,100	200	4.9%		0.1	1,315	11,350	200	4.9%
	0.2	2,380	11,100	200	4.9%		0.2	2,430	11,350	200	4.9%
	0.3	3,470	11,100	200	4.9%		0.3	3,545	11,350	200	4.9%
	0.4	4,500	10,500	500	4.9%		0.4	4,600	10,750	500	4.9%
10	0.5	5,500	10,500	500	4.9%	20	0.5	5,625	10,750	500	4.9%
	0.6	6,500	10,500	500	4.9%		0.6	6,648	10,740	510	4.7%
	0.7	7,500	10,500	500	4.9%		0.7	7,671	10,740	510	4.7%
	0.8	8,500	10,500	500	4.9%		0.8	8,694	10,740	510	4.7%
	0.9	9,500	10,500	500	4.9%		0.9	9,717	10,740	510	4.7%

Varying ε_{4j}

ε_{4j}	α_1	TC	Z_1	Z_2	$\% \sum_j \sum_t S_{jk}^t$
	0.1	1,140	10,500	100	9.8%
	0.2	2,180	10,500	100	9.8%
	0.3	3,220	10,500	100	9.8%
	0.4	4,260	10,500	100	9.8%
2	0.5	5,300	10,500	100	9.8%
	0.6	6,340	10,500	100	9.8%
	0.7	7,380	10,500	100	9.8%
	0.8	8,420	10,500	100	9.8%
	0.9	9,460	10,500	100	9.8%
	0.1	1,182	11,100	80	3.9%
	0.2	2,260	10,500	200	9.8%
	0.3	3,290	10,500	200	9.8%
	0.4	4,320	10,500	200	9.8%
4	0.5	5,350	10,500	200	9.8%
	0.6	6,380	10,500	200	9.8%
	0.7	7,410	10,500	200	9.8%
	0.8	8,440	10,500	200	9.8%
	0.9	9,470	10,500	200	9.8%
	0.1	1,218	11,100	120	3.9%
	0.2	2,316	11,100	120	3.9%
	0.3	3,360	10,500	300	9.8%
	0.4	4,380	10,500	300	9.8%
6	0.5	5,400	10,500	300	9.8%
	0.6	6,420	10,500	300	9.8%
	0.7	7,440	10,500	300	9.8%
	0.8	8,460	10,500	300	9.8%
	0.9	9,480	10,500	300	9.8%
	0.1	1,254	11,100	160	3.9%
	0.2	2,348	11,100	160	3.9%
	0.3	3,430	10,500	400	9.8%
	0.4	4,440	10,500	400	9.8%
8	0.5	5,450	10,500	400	9.8%
	0.6	6,460	10,500	400	9.8%
	0.7	7,470	10,500	400	9.8%
	0.8	8,480	10,500	400	9.8%
	0.9	9,490	10,500	400	9.8%
	0.1	1,290	11,100	200	3.9%
	0.2	2,380	11,100	200	3.9%
	0.3	3,470	11,100	200	3.9%
	0.4	4,500	10,500	500	9.8%
10	0.5	5,500	10,500	500	9.8%
	0.6	6,500	10,500	500	9.8%
	0.7	7,500	10,500	500	9.8%
	0.8	8,500	10,500	500	9.8%
	0.9	9,500	10,500	500	9.8%

ε_{4j}	α_1	TC	Z_1	Z_2	$\% \sum_j \sum_t S_{jk}^t$
	0.1	1,326	11,100	240	3.9%
	0.2	2,412	11,100	240	3.9%
	0.3	3,498	11,100	240	3.9%
	0.4	4,560	10,500	600	9.8%
12	0.5	5,500	10,500	600	9.8%
	0.6	6,500	10,500	600	9.8%
	0.7	7,500	10,500	600	9.8%
	0.8	8,500	10,500	600	9.8%
	0.9	9,500	10,500	600	9.8%
	0.1	1,362	11,100	280	3.9%
	0.2	2,444	11,100	280	3.9%
	0.3	3,526	11,100	280	3.9%
	0.4	4,608	11,100	280	3.9%
14	0.5	5,600	10,500	700	9.8%
	0.6	6,580	10,500	700	9.8%
	0.7	7,560	10,500	700	9.8%
	0.8	8,540	10,500	700	9.8%
	0.9	9,520	10,500	700	9.8%
	0.1	1,398	11,100	320	3.9%
	0.2	2,476	11,100	320	3.9%
	0.3	3,554	11,100	320	3.9%
	0.4	4,632	11,100	320	3.9%
16	0.5	5,650	10,500	800	9.8%
	0.6	6,620	10,500	800	9.8%
	0.7	7,590	10,500	800	9.8%
	0.8	8,560	10,500	800	9.8%
	0.9	9,530	10,500	800	9.8%
	0.1	1,434	11,100	360	3.9%
	0.2	2,508	11,100	360	3.9%
	0.3	3,582	11,100	360	3.9%
	0.4	4,656	11,100	360	3.9%
18	0.5	5,700	10,500	900	9.8%
	0.6	6,660	10,500	900	9.8%
	0.7	7,620	10,500	900	9.8%
	0.8	8,580	10,500	900	9.8%
	0.9	9,540	10,500	900	9.8%
	0.1	1,470	11,100	400	3.9%
	0.2	2,540	11,100	400	3.9%
	0.3	3,610	11,100	400	3.9%
	0.4	4,680	11,100	400	3.9%
20	0.5	5,750	11,100	400	3.9%
	0.6	6,700	10,500	1,000	9.8%
	0.7	7,650	10,500	1,000	9.8%
	0.8	8,600	10,500	1,000	9.8%
	0.9	9,550	10,500	1,000	9.8%

F. HWRC –Results 1

a_1	*Facility j*	$\sum_j \sum_t y_j^t$ *Operating hours*	$\sum_j \sum_t q_j^t$ *Amount of demand served*	$\sum_j \sum_t s_j^t$ *Amount of demand in the queue*	$\sum_j \sum_t l_j^t$ *Amount of demand leave*	$\sum_j \sum_t S_{jk}^t$ *Amount of demand move*	*Facility k*				
							Beighton	Blackstock	Deepcar	Greaves Lane	Longley Ave
0.1	Beighton	54	3106	0	41	0		0	0	0	0
	Blackstock	54	3907	0	51	0	-		0	0	0
	Deepcar	41	1283	0	252	0	0	0		0	0
	Greaves Lane	45	1586	0	204	0	0	0	0		0
	Longley Ave	51	3378	0	149	0	0	0	0	0	
0.2	Beighton	53	3077	0	70	0		0	0	0	0
	Blackstock	54	3907	0	51	0	0		0	0	0
	Deepcar	39	1241	0	294	0	0	0		0	0
	Greaves Lane	45	1587	0	203	0	0	0	0		0
	Longley Ave	53	3449	0	78	0	0	0	0	0	
0.3	Beighton	54	3106	0	41	0		0	0	0	0
	Blackstock	54	3907	0	51	0	0		0	0	0
	Deepcar	39	1241	0	289	5	0	0		5	0
	Greaves Lane	45	1592	0	203	0	0	0	0		0
	Longley Ave	52	3414	0	113	0	0	0	0	0	
0.4	Beighton	50	2981	0	41	125		125	0	0	0
	Blackstock	54	4334	0	51	0	0		0	0	0
	Deepcar	31	1059	0	263	213	0	0		213	0
	Greaves Lane	41	1707	0	296	0	0	0	0		0
	Longley Ave	46	3179	0	46	302	0	302	0	0	
0.5	Beighton	40	2593	0	45	509		509	0	0	0
	Blackstock	54	5516	4	51	0	0		0	0	0
	Deepcar	0	0	0	252	1283	0	0		1283	0
	Greaves Lane	41	2777	0	296	0	0	0	0		0
	Longley Ave	30	2374	0	53	1100	0	1100	0	0	
0.6	Beighton	37	3623	0	29	686		686	0	0	0
	Blackstock	39	4412	0	24	1191	1191		0	0	0
	Deepcar	0	0	0	172	1363	0	0		1363	0
	Greaves Lane	45	2950	0	203	0	0	0	0		0
	Longley Ave	29	2280	0	264	983	0	983	0	0	
0.7	Beighton	29	3769	0	63	1143		1143	0	0	0
	Blackstock	31	4013	0	24	2200	1828		0	0	372
	Deepcar	47	3033	0	134	0	0	0		0	0
	Greaves Lane	0	0	0	158	1632	0	0	1632		0
	Longley Ave	28	2535	0	228	1132	0	1132	0	0	
0.8	Beighton	31	3127	0	49	928		928	0	0	0
	Blackstock	56	7384	87	0	957	957		0	0	0
	Deepcar	48	3073	0	116	0	0	0		0	0
	Greaves Lane	0	0	0	136	1654	0	0	1654		0
	Longley Ave	0	0	0	72	3455	0	3345	0	0	
0.9	Beighton	31	3127	0	42	935		935	0	0	0
	Blackstock	56	7411	152	0	957	957		0	0	0
	Deepcar	52	3217	0	50	0	0	0		0	0
	Greaves Lane	0	0	0	58	1732	0	0	1732		0
	Longley Ave	0	0	0	52	3475	0	3475	0	0	

G. HWRC – Results 2

a_1	*Facility j*	$\sum_j \sum_t y_j^t$ *Operating hours*	$\sum_j \sum_t q_j^t$ *Amount of demand served*	$\sum_j \sum_t s_j^t$ *Amount of demand in the queue*	$\sum_j \sum_t l_j^t$ *Amount of demand leave*	$\sum_j \sum_t S_{jk}^t$ *Amount of demand move*	*Facility k*				
							Beighton	Blackstock	Deepcar	Greaves Lane	Longley Ave
0.1	Beighton	48	2951	0	148	172		172	0	0	0
	Blackstock	48	3768	0	116	246	124		0	0	122
	Deepcar	40	1498	0	268	28	0	0		28	0
	Greaves Lane	40	1394	0	165	259	0	0	259		0
	Longley Ave	56	3649	0	0	0	0	0	0	0	
0.2	Beighton	48	2951	0	148	172		172	0	0	0
	Blackstock	48	3768	0	116	246	246		0	0	0
	Deepcar	40	1498	0	268	28	0	0		28	0
	Greaves Lane	40	1394	0	165	259	0	0	259		0
	Longley Ave	56	3649	0	0	0	0	0	0	0	
0.3	Beighton	47	3051	0	237	102		102	0	0	0
	Blackstock	47	3671	0	51	338	243		0	0	95
	Deepcar	36	1415	0	167	194	0	0		194	0
	Greaves Lane	39	1547	0	196	241	0	0	241		0
	Longley Ave	54	3576	0	46	0	0	0	0	0	
0.4	Beighton	45	2935	0	41	362		362	0	0	0
	Blackstock	47	4159	0	51	191	191		0	0	0
	Deepcar	32	1323	0	253	200	0	0		200	0
	Greaves Lane	35	1475	0	274	241	0	0	241		0
	Longley Ave	47	3368	0	78	228	0	228	0	0	
0.5	Beighton	42	2924	0	77	434		434	0	0	0
	Blackstock	48	4842	0	24	338	288		0	0	50
	Deepcar	28	1333	0	216	407	0	0		407	0
	Greaves Lane	30	1522	0	254	421	0	0	421		0
	Longley Ave	35	2651	0	114	812	0	812	0	0	
0.6	Beighton	42	3427	0	77	447		447	0	0	0
	Blackstock	40	4520	0	24	1012	804		0	0	208
	Deepcar	28	1536	0	152	491	0	0		491	0
	Greaves Lane	28	1459	0	178	644	0	0	644		0
	Longley Ave	28	2318	0	266	1151	0	1151	0	0	
0.7	Beighton	38	4061	0	31	812		812	0	0	0
	Blackstock	32	4187	61	0	2117	1757		0	0	360
	Deepcar	40	2682	0	296	0	0	0		0	0
	Greaves Lane	0	0	0	347	1443	0	0	1443		0
	Longley Ave	30	2330	0	23	1534	0	1534	0	0	
0.8	Beighton	37	3735	0	22	897		897	0	0	0
	Blackstock	32	4260	214	0	2114	1507		0	0	607
	Deepcar	40	2682	0	296	0	0	0		0	0
	Greaves Lane	0	0	0	347	1443	0	0	1443		0
	Longley Ave	29	3527	0	32	1519	0	1519	0	0	
0.9	Beighton	39	3776	0	31	659		659	0	0	0
	Blackstock	48	6465	257	0	1319	1319		0	0	0
	Deepcar	28	1581	0	141	502	0	0		502	0
	Greaves Lane	28	1438	0	165	689	0	0	689		0
	Longley Ave	0	0	0	360	3167	0	3167	0	0	

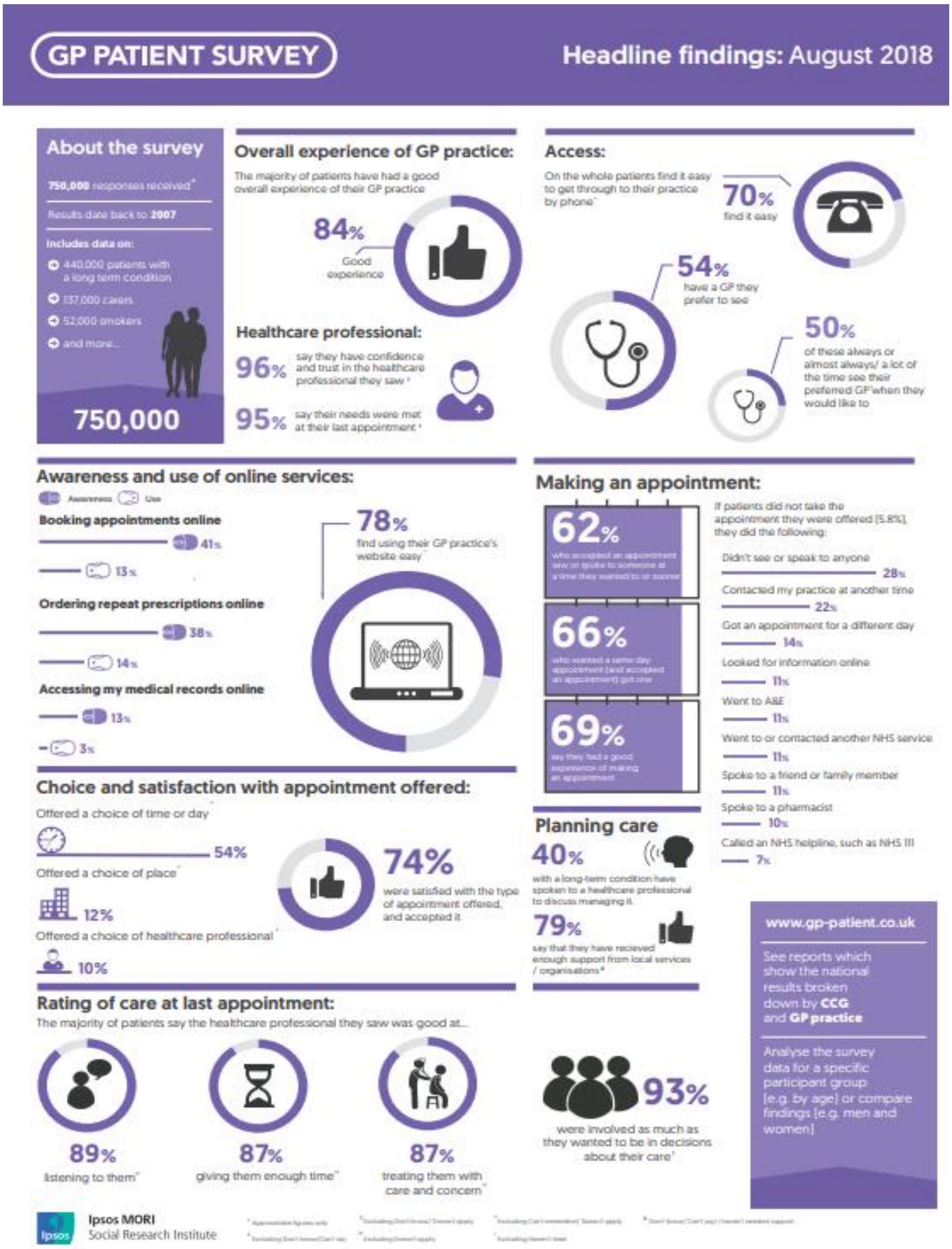
H. HWRC – Results 3

a_1	*Facility j*	$\sum_j \sum_t y_j^t$ *Operating hours*	$\sum_j \sum_t q_j^t$ *Amount of demand served*	$\sum_j \sum_t s_j^t$ *Amount of demand in the queue*	$\sum_j \sum_t l_j^t$ *Amount of demand leave*	$\sum_j \sum_t S_{jk}^t$ *Amount of demand move*	*Facility k*				
							Beighton	Blackstock	Deepcar	Greaves Lane	Longley Ave
0.1	Beighton	56	6021	0	0	0		0	0	0	0
	Blackstock	0	0	0	24	3934	2874		0	0	1060
	Deepcar	0	0	0	1535	0	0	0		0	0
	Greaves Lane	0	0	0	1790	0	0	0	0		0
	Longley Ave	56	4587	0	0	0	0	0	0	0	
0.2	Beighton	56	5915	0	0	0		0	0	0	0
	Blackstock	0	0	0	24	3934	2768		0	0	1166
	Deepcar	0	0	0	1535	0	0	0		0	0
	Greaves Lane	0	0	0	1790	0	0	0	0		0
	Longley Ave	56	4693	0	0	0	0	0	0	0	
0.3	Beighton	56	6050	0	0	0		0	0	0	0
	Blackstock	0	0	0	2	3956	2903		0	0	1053
	Deepcar	0	0	0	1535	0	0	0		0	0
	Greaves Lane	0	0	0	1790	0	0	0	0		0
	Longley Ave	55	4558	0	22	0	0	0	0	0	
0.4	Beighton	56	5944	0	0	0		0	0	0	0
	Blackstock	0	0	0	2	3956	2797		0	0	1159
	Deepcar	0	0	0	1535	0	0	0		0	0
	Greaves Lane	0	0	0	1790	0	0	0	0		0
	Longley Ave	55	4558	0	22	0	0	0	0	0	
0.5	Beighton	56	5260	0	0	0		0	0	0	0
	Blackstock	0	0	0	0	3958	2113		0	0	1845
	Deepcar	0	0	0	1535	0	0	0		0	0
	Greaves Lane	0	0	0	1790	0	0	0	0		0
	Longley Ave	55	5350	0	22	0	0	0	0	0	
0.6	Beighton	56	5260	0	0	0		0	0	0	0
	Blackstock	0	0	0	0	3958	2113		0	0	1845
	Deepcar	0	0	0	1535	0	0	0		0	0
	Greaves Lane	0	0	0	1790	0	0	0	0		0
	Longley Ave	55	5350	0	22	0	0	0	0	0	
0.7	Beighton	56	5306	0	0	0		0	0	0	0
	Blackstock	0	0	0	0	3958	2159		0	0	1799
	Deepcar	0	0	0	1535	0	0	0		0	0
	Greaves Lane	0	0	0	1790	0	0	0	0		0
	Longley Ave	55	5304	0	22	0	0	0	0	0	
0.8	Beighton	56	5306	0	0	0		0	0	0	0
	Blackstock	0	0	0	0	3958	2159		0	0	1799
	Deepcar	0	0	0	1535	0	0	0		0	0
	Greaves Lane	0	0	0	1790	0	0	0	0		0
	Longley Ave	55	5304	0	22	0	0	0	0	0	
0.9	Beighton	56	5562	0	0	0		0	0	0	
	Blackstock	0	0	0	0	3958	2415		0	0	1543
	Deepcar	0	0	0	1535	0	0	0		0	0
	Greaves Lane	0	0	0	1790	0	0	0	0		0
	Longley Ave	55	5048	0	22	0	0	0	0	0	

APPENDIX 5: DETAILED INFORMATION FOR CHAPTER 5

This section presents the related data used in Chapter 5. This includes summary of GP Patients Survey, data total of registered patients for each GP, the estimated number of patients received by the GP, derivation of constraint (3-12)* to (3-13)*, sensitivity analyses results and lastly, the clusters formed using the backup facility model.

A. GP survey results – 2017 and 2018 (source: GP Survey website)



B. Potential registered patient visits their GP per day

j	General Practice	Total All Patients	Number of potential patients per day*	Total operating hours**
1	Abbey Lane Surgery	3129	59	48
2	Avenue Medical Practice	7130	135	53
3	Barnsley Road Surgery	2653	50	42
4	Baslow Rd, Shoreham St & York Rd Surgeries	12642	239	50
5	Birley Health Centre	8502	161	52
6	Broomhill Surgery	9633	182	52
7	Buchanan Road Surgery	4703	89	45
8	Burngreave Surgery	6726	127	50
9	Carrfield Medical Centre	1260	24	46
10	Carterknowle & Dore Medical Practice	12380	234	44
11	Chapelgreen Practice	15452	292	50
12	Charnock Health Primary Care Centre	5381	102	48
13	Clover City Practice	4395	83	50
14	Clover Group Practice	16394	310	49
15	Crookes Practice	7962	150	45
16	Crookes Valley Medical Centre	2317	44	47
17	Crystal Peaks Medical Centre	6598	125	56
18	Darnall Health Centre (Mehrotra)	3415	64	50
19	Deepcar Medical Centre	5200	98	50
20	Devonshire Green Medical Centre	6959	131	44
21	Dovercourt Group Practice	8338	157	54
22	Duke Medical Centre	6966	132	54
23	Dunninc Road Surgery	2983	56	51
24	Dykes Hall Medical Centre	9735	184	48
25	East Bank Medical Centre	5608	106	51
26	Ecclesfield Group Practice	8177	154	47
27	Elm Lane Surgery	5185	98	44
28	Falkland House Surgery	3790	72	57
29	Far Lane Medical Centre	7249	137	44
30	Firth Park Surgery	9884	187	45
31	Foxhill Medical Centre	6189	117	48
32	Gleadless Medical Centre	8865	167	50
33	Grenoside Surgery	7391	140	49
34	Greystones Medical Centre	3732	70	51
35	Hackenthorpe Medical Centre	6715	127	50
36	Handsworth Medical Practice	9850	186	52
37	Harold Street Medical Centre	1282	24	53
38	Heeley Green Surgery	5886	111	56
39	Hollies Medical Centre	9034	171	49
40	Jaunty Springs Health Centre	3630	69	50
41	Manchester Road Surgery	4691	89	45

*0.169 per patient per day x number of registered patients per GP
**Excluding Saturday and Sunday. Operational Time ranges from 0700hrs until 2000hrs

continue…

j	General Practice	Total All Patients	Number of potential patients per day*	Total operating hours**
42	Manor Park Medical Centre	4416	83	50
43	Meadowgreen Health Centre	9841	186	58
44	Mill Road Surgery	5264	99	55
45	Mosborough Health Centre	6590	124	50
46	Nethergreen Surgery	9286	175	51
47	Norfolk Park Health Centre	4418	83	45
48	Norwood Medical Centre	7971	151	50
49	Oughtibridge Surgery	5848	110	49
50	Owlthorpe Medical Centre	4583	87	51
51	Page Hall Medical Centre	7586	143	50
52	Park Health Centre	5103	96	48
53	Pitsmoor Surgery	9401	178	52
54	Porter Brook Medical Centre	28820	544	49
55	Richmond Medical Centre	8806	166	52
56	Rustlings Road Medical Centre	4591	87	50
57	Selborne Road Medical Centre	2730	52	45
58	Sharrow Lane Medical Centre	3883	73	54
59	Sheffield Medical Centre	1700	32	40
60	Shiregreen Medical Centre	7834	148	45
61	Sloan Medical Centre	12964	245	58
62	Sothall & Beighton Health Centres	10180	192	50
63	Southey Green Medical Centre	2996	57	50
64	Stannington Medical Centre	3198	60	45
65	Stonecroft Medical Centre	4101	77	44
66	The Flowers Health Centre	4885	92	45
67	The Health Care Surgery	5027	95	51
68	The Mathews Practice Belgrave	8722	165	48
69	The Medical Centre Dr Okorie	1183	22	45
70	Totley Rise Medical Centre	3442	65	45
71	Tramways and Middlewood Medical Centres	10604	200	52
72	Tramways Medical Centre (O'Connell)	8553	162	50
73	University Health Service Health Centre	32891	621	49
74	Upperthorpe Medical Centre	11466	217	42
75	Upwell Street Surgery	4769	90	45
76	Valley Medical Centre	9628	182	50
77	Veritas Health Centre	1462	28	46
78	Walkley House Medical Centre	11749	222	50
79	White House Surgery	6363	120	45
80	Wincobank Medical Centre	7649	144	57
81	Woodhouse Medical Centre	12117	229	49
82	Woodseats Medical Centre	9643	182	55
	TOTAL	601122	9483	-

*0.169 per patient per day x number of registered patients per GP

**Excluding Saturday and Sunday. Operational Time ranges from 0700hrs until 2000hrs

C. Derivation of (3-12)* to (3-13)*

This section shows the derivation of (3-13)*. First, considers the modified mass balance constraint (3-12)*;

$$x_j^t + s_j^{t-G} + \sum_{k,k\neq j} S_{kj}^t u_{kj}^t = s_j^t + \sum_{k,k\neq j} S_{jk}^t u_{jk}^t + q_j^t + l_j^t$$

Then, sum all the components with *j* and *t*:

$$\sum_j \sum_t x_j^t + \sum_j \sum_t s_j^{t-G} + \sum_{k,k\neq j} \sum_j \sum_t S_{kj}^t u_{kj}^t$$
$$= \sum_j \sum_t s_j^t + \sum_{k,k\neq j} \sum_j \sum_t S_{jk}^t u_{jk}^t + \sum_j \sum_t q_j^t + \sum_j \sum_t l_j^t$$

Know that the amount of demand that move to another *j* and amount of demand move into a *j* are identical to each other, where $\sum_{k,k\neq j} \sum_j \sum_t S_{kj}^t u_{kj}^t = \sum_{k,k\neq j} \sum_j \sum_t S_{jk}^t u_{jk}^t$. Hence,

$$\sum_j \sum_t x_j^t + \sum_j \sum_t s_j^{t-G} = \sum_j \sum_t s_j^t + \sum_j \sum_t q_j^t + \sum_j \sum_t l_j^t$$

or can be written as:

$$\sum_j \sum_t x_j^t = \sum_j \sum_t q_j^t + \sum_j \sum_t l_j^t + \sum_j \sum_t s_j^t - \sum_j \sum_t s_j^{t-G}$$

From the equation above, only $\sum_j \sum_t s_j^t - \sum_j \sum_t s_j^{t-G}$ were focussed. This equation can be written as:

$$= \sum_j \sum_t s_j^t - \left(\sum_j \sum_1^G s_j^{t-G} + \sum_j \sum_{G+1}^T s_j^{t-G} \right)$$

In the proposed model, it is assumed that $\sum_j \sum_1^G s_j^{t-G} = 0$, therefore, the remaining equation is $\sum_j \sum_t s_j^t - \sum_j \sum_{G+1}^T s_j^{t-G}$. Through index shifting and some algebraic manipulation, the following equation was obtained.

$$\sum_j \sum_t s_j^t - \sum_j \sum_{G+1}^T s_j^{t-G} = \sum_j \sum_{(T-G)+1}^T s_j^t$$

Therefore, the overall constraint is:

$$\sum_j \sum_t x_j^t = \sum_j \sum_t q_j^t + \sum_j \sum_t l_j^t + \sum_j \sum_{(T-G)+1}^T s_j^t \qquad \text{(3-13)*}$$

D. Sensitivity Analyses – the numerical results for trade-off cost values

Varying ε_{2j}

ε_{2j}	α_1	TC	Z_1	Z_2	$\% \sum_j \sum_{(T-G)+1}^{T} s_j^t$	$\% \sum_j \sum_t s_j^t$
	0.1	135	115	137	13.0%	29.3%
	0.2	133	115	137	13.0%	29.3%
	0.3	130	115	137	13.0%	29.3%
	0.4	128	105	143	13.5%	30.6%
1	0.5	124	105	143	13.5%	30.6%
	0.6	118	50	220	15.8%	35.3%
	0.7	98	35	244	16.5%	37.2%
	0.8	77	35	244	16.5%	37.2%
	0.9	56	35	244	16.5%	37.2%
	0.1	256	115	272	12.6%	28.0%
	0.2	241	115	272	12.6%	28.0%
	0.3	225	115	272	12.6%	28.0%
	0.4	209	115	272	12.6%	28.0%
2	0.5	194	115	272	12.6%	28.0%
	0.6	177	105	284	13.0%	29.3%
	0.7	149	35	416	16.0%	35.9%
	0.8	111	35	416	16.0%	35.9%
	0.9	73	35	416	16.0%	35.9%
	0.1	372	115	401	12.6%	27.1%
	0.2	344	115	401	12.6%	27.1%
	0.3	315	115	401	12.6%	27.1%
	0.4	287	115	401	12.6%	27.1%
3	0.5	258	115	401	12.6%	27.1%
	0.6	229	115	401	12.6%	27.1%
	0.7	197	50	540	15.4%	33.1%
	0.8	144	35	582	16.0%	35.0%
	0.9	90	35	582	16.0%	35.0%

ε_{2j}	α_1	TC	Z_1	Z_2	$\% \sum_j \sum_{(T-G)+1}^{T} s_j^t$	$\% \sum_j \sum_t s_j^t$
	0.1	711	115	777	12.2%	26.1%
	0.2	645	115	777	12.2%	26.1%
	0.3	578	115	777	12.2%	26.1%
	0.4	512	115	777	12.2%	26.1%
6	0.5	446	115	777	12.2%	26.1%
	0.6	380	115	777	12.2%	26.1%
	0.7	314	115	777	12.2%	26.1%
	0.8	240	50	1000	15.0%	32.1%
	0.9	138	35	1069	15.6%	34.0%
	0.1	821	115	899	12.2%	26.1%
	0.2	742	115	899	12.2%	26.1%
	0.3	664	115	899	12.2%	26.1%
	0.4	585	115	899	12.2%	26.1%
7	0.5	507	115	899	12.2%	26.1%
	0.6	429	115	899	12.2%	26.1%
	0.7	350	115	899	12.2%	26.1%
	0.8	270	60	1108	14.5%	30.8%
	0.9	154	35	1228	15.6%	34.0%
	0.1	930	115	1021	12.2%	26.1%
	0.2	840	115	1021	12.2%	26.1%
	0.3	749	115	1021	12.2%	26.1%
	0.4	659	115	1021	12.2%	26.1%
8	0.5	568	115	1021	12.2%	26.1%
	0.6	477	115	1021	12.2%	26.1%
	0.7	387	115	1021	12.2%	26.1%
	0.8	296	115	1021	12.2%	26.1%
	0.9	170	35	1387	15.6%	34.0%

continue…

4	0.1	487	115	528	12.6%	27.1%
	0.2	445	115	528	12.6%	27.1%
	0.3	404	115	528	12.6%	27.1%
	0.4	363	115	528	12.6%	27.1%
	0.5	322	115	528	12.6%	27.1%
	0.6	280	115	528	12.6%	27.1%
	0.7	239	115	528	12.6%	27.1%
	0.8	177	35	746	16.0%	35.0%
	0.9	106	35	746	16.0%	35.0%
5	0.1	601	115	655	12.2%	26.3%
	0.2	547	115	655	12.2%	26.3%
	0.3	493	115	655	12.2%	26.3%
	0.4	439	115	655	12.2%	26.3%
	0.5	385	115	655	12.2%	26.3%
	0.6	331	115	655	12.2%	26.3%
	0.7	277	115	655	12.4%	26.7%
	0.8	210	35	910	16.0%	34.6%
	0.9	123	35	910	16.0%	35.0%

9	0.1	1040	115	1143	12.2%	26.1%
	0.2	937	115	1143	12.2%	26.1%
	0.3	835	115	1143	12.2%	26.1%
	0.4	732	115	1143	12.2%	26.1%
	0.5	629	115	1143	12.2%	26.1%
	0.6	526	115	1143	12.2%	26.1%
	0.7	423	115	1143	12.2%	26.1%
	0.8	321	115	1143	12.2%	26.1%
	0.9	186	35	1546	15.6%	34.0%
10	0.1	1150	115	1265	12.2%	26.1%
	0.2	1035	115	1265	12.2%	26.1%
	0.3	920	115	1265	12.2%	26.1%
	0.4	805	115	1265	12.2%	26.1%
	0.5	690	115	1265	12.2%	26.1%
	0.6	575	115	1265	12.2%	26.1%
	0.7	460	115	1265	12.2%	26.1%
	0.8	345	115	1265	12.2%	25.2%
	0.9	202	35	1705	15.6%	34.0%

Varying ε_{3j}

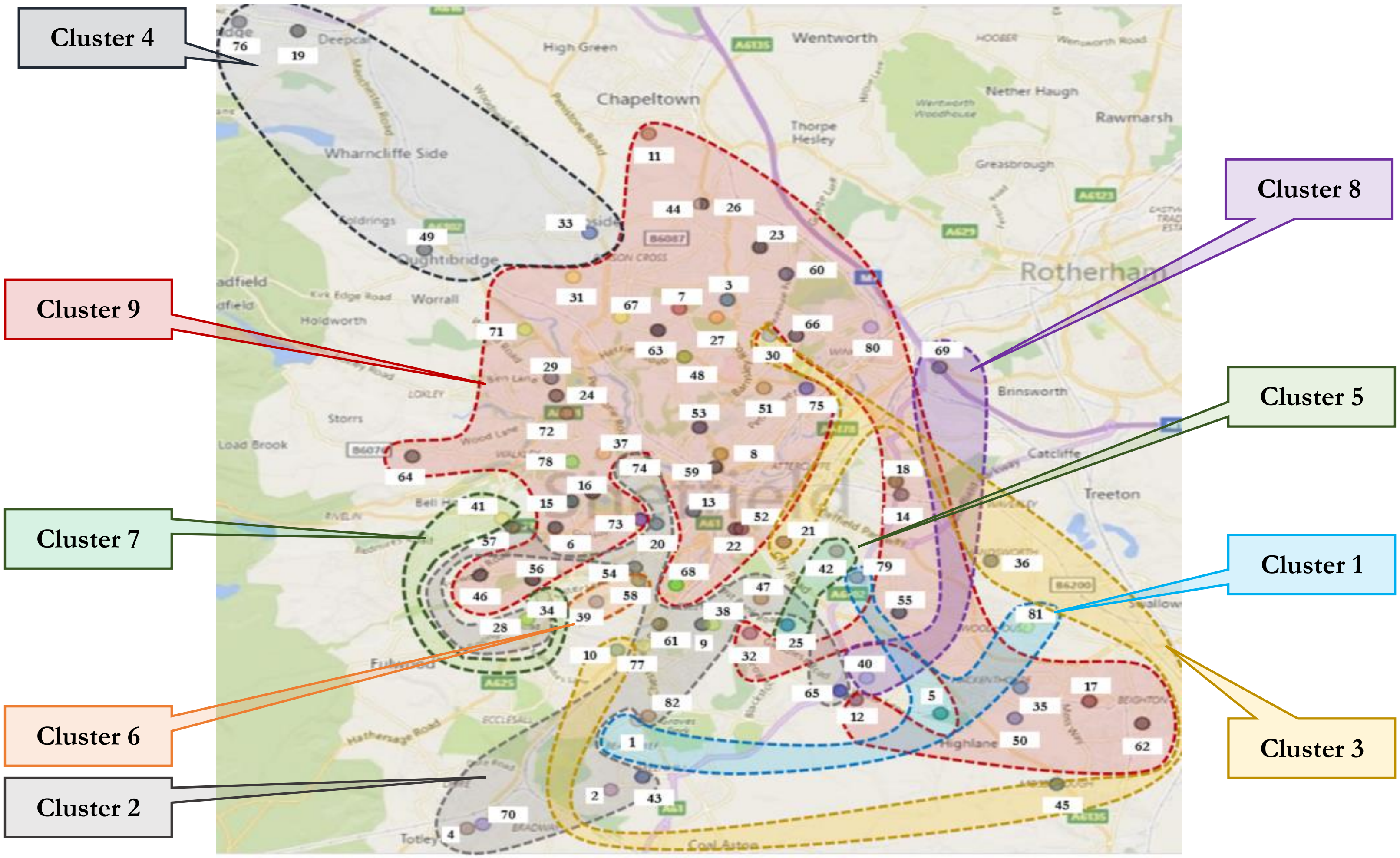
ε_{3j}	α_1	TC	Z_1	Z_2	$\% \sum_j \sum_t l_j^t$	ε_{3j}	α_1	TC	Z_1	Z_2	$\% \sum_j \sum_t l_j^t$
	0.1	592	23	655	4.9%		0.1	603	138	655	4.9%
	0.2	529	23	655	4.9%		0.2	552	138	655	4.9%
	0.3	465	23	655	4.9%		0.3	500	138	655	4.9%
	0.4	402	23	655	4.9%		0.4	448	138	655	4.9%
1	0.5	339	23	655	4.9%	6	0.5	397	138	655	4.9%
	0.6	276	23	655	4.9%		0.6	345	138	655	4.9%
	0.7	213	23	655	4.9%		0.7	293	138	655	4.9%
	0.8	149	23	655	4.9%		0.8	216	42	910	1.5%
	0.9	86	23	655	4.9%		0.9	129	42	910	1.5%
	0.1	594	46	655	4.9%		0.1	606	161	655	4.9%
	0.2	533	46	655	4.9%		0.2	556	161	655	4.9%
	0.3	472	46	655	4.9%		0.3	507	161	655	4.9%
	0.4	411	46	655	4.9%		0.4	457	161	655	4.9%
2	0.5	351	46	655	4.9%	7	0.5	408	161	655	4.9%
	0.6	290	46	655	4.9%		0.6	359	161	655	4.9%
	0.7	229	46	655	4.9%		0.7	304	70	850	2.1%
	0.8	168	46	655	4.9%		0.8	221	49	910	1.5%
	0.9	103	20	850	2.1%		0.9	135	49	910	1.5%
	0.1	596	69	655	4.9%		0.1	608	184	655	4.9%
	0.2	538	69	655	4.9%		0.2	561	184	655	4.9%
	0.3	479	69	655	4.9%		0.3	514	184	655	4.9%
	0.4	421	69	655	4.9%		0.4	467	184	655	4.9%
3	0.5	362	69	655	4.9%	8	0.5	420	184	655	4.9%
	0.6	303	69	655	4.9%		0.6	372	184	655	4.9%
	0.7	245	69	655	4.9%		0.7	311	80	850	2.1%
	0.8	186	69	655	4.9%		0.8	227	56	910	1.5%
	0.9	110	21	910	1.5%		0.9	141	56	910	1.5%
	0.1	599	92	655	4.9%		0.1	610	207	655	4.9%
	0.2	542	92	655	4.9%		0.2	565	207	655	4.9%
	0.3	486	92	655	4.9%		0.3	521	207	655	4.9%
	0.4	430	92	655	4.9%		0.4	476	207	655	4.9%
4	0.5	374	92	655	4.9%	9	0.5	431	207	655	4.9%
	0.6	317	92	655	4.9%		0.6	386	207	655	4.9%
	0.7	261	92	655	4.9%		0.7	317	63	910	1.5%
	0.8	202	40	850	2.1%		0.8	232	63	910	1.5%
	0.9	116	28	910	1.5%		0.9	148	63	910	1.5%
	0.1	601	115	655	4.9%		0.1	613	230	655	4.9%
	0.2	547	115	655	4.9%		0.2	570	230	655	4.9%
	0.3	493	115	655	4.9%		0.3	528	230	655	4.9%
	0.4	439	115	655	4.9%		0.4	485	230	655	4.9%
5	0.5	385	115	655	4.9%	10	0.5	443	230	655	4.9%
	0.6	331	115	655	4.9%		0.6	400	100	850	2.1%
	0.7	277	115	655	4.9%		0.7	322	70	910	1.5%
	0.8	210	35	910	1.5%		0.8	238	70	910	1.5%
	0.9	123	35	910	1.5%		0.9	154	70	910	1.5%

Varying ε_{4j}

ε_{4j}	α_1	TC	Z_1	Z_2	$\% \sum_j \sum_t S_{jk}^t$
	0.1	539	115	586	6.6%
	0.2	492	115	586	6.6%
	0.3	445	115	586	6.6%
	0.4	398	115	586	6.6%
1	0.5	351	115	586	6.6%
	0.6	303	115	586	6.6%
	0.7	256	115	586	6.6%
	0.8	192	35	818	4.9%
	0.9	113	35	818	4.9%
	0.1	567	115	617	6.6%
	0.2	517	115	617	6.6%
	0.3	466	115	617	6.6%
	0.4	416	115	617	6.6%
2	0.5	366	115	617	6.6%
	0.6	316	115	617	6.6%
	0.7	266	115	617	6.6%
	0.8	196	35	841	4.9%
	0.9	116	35	841	4.9%
	0.1	585	115	637	1.9%
	0.2	533	115	637	1.9%
	0.3	480	115	637	1.9%
	0.4	428	115	637	1.9%
3	0.5	376	115	637	1.9%
	0.6	324	115	637	1.9%
	0.7	272	115	637	1.9%
	0.8	201	35	864	4.9%
	0.9	118	35	864	4.9%
	0.1	593	115	646	1.9%
	0.2	540	115	646	1.9%
	0.3	487	115	646	1.9%
	0.4	434	115	646	1.9%
4	0.5	381	115	646	1.9%
	0.6	327	115	646	1.9%
	0.7	274	115	646	1.9%
	0.8	205	35	887	4.9%
	0.9	120	35	887	4.9%
	0.1	601	115	655	1.7%
	0.2	547	115	655	1.7%
	0.3	493	115	655	1.7%
	0.4	439	115	655	1.7%
5	0.5	385	115	655	1.7%
	0.6	331	115	655	1.7%
	0.7	277	115	655	1.3%
	0.8	210	35	910	4.3%
	0.9	123	35	910	3.8%

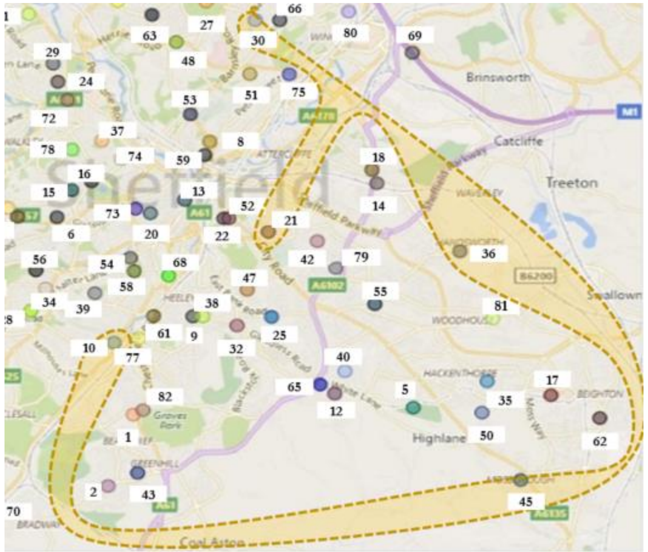
ε_{4j}	α_1	TC	Z_1	Z_2	$\% \sum_j \sum_t S_{jk}^t$
	0.1	605	115	659	0.9%
	0.2	550	115	659	0.9%
	0.3	496	115	659	0.9%
	0.4	441	115	659	0.9%
6	0.5	387	115	659	0.9%
	0.6	333	115	659	0.9%
	0.7	278	115	659	0.9%
	0.8	213	50	865	3.2%
	0.9	124	35	928	3.8%
	0.1	608	115	663	0.9%
	0.2	553	115	663	0.9%
	0.3	499	115	663	0.9%
	0.4	444	115	663	0.9%
7	0.5	389	115	663	0.9%
	0.6	334	115	663	0.9%
	0.7	279	115	663	0.9%
	0.8	216	50	880	3.2%
	0.9	126	35	946	3.8%
	0.1	612	115	667	0.9%
	0.2	557	115	667	0.9%
	0.3	501	115	667	0.9%
	0.4	446	115	667	0.9%
8	0.5	391	115	667	0.9%
	0.6	336	115	667	0.9%
	0.7	281	115	667	0.9%
	0.8	219	50	895	3.2%
	0.9	128	35	964	3.8%
	0.1	615	115	671	0.9%
	0.2	560	115	671	0.9%
	0.3	504	115	671	0.9%
	0.4	449	115	671	0.9%
9	0.5	393	115	671	0.9%
	0.6	337	115	671	0.9%
	0.7	282	115	671	0.9%
	0.8	222	50	910	3.2%
	0.9	130	35	982	3.8%
	0.1	619	115	675	0.6%
	0.2	563	115	675	0.6%
	0.3	507	115	675	0.6%
	0.4	451	115	675	0.6%
10	0.5	395	115	675	0.9%
	0.6	339	115	675	0.9%
	0.7	283	115	675	0.9%
	0.8	225	50	925	3.2%
	0.9	132	35	1000	3.8%

E. Illustrations on each Cluster



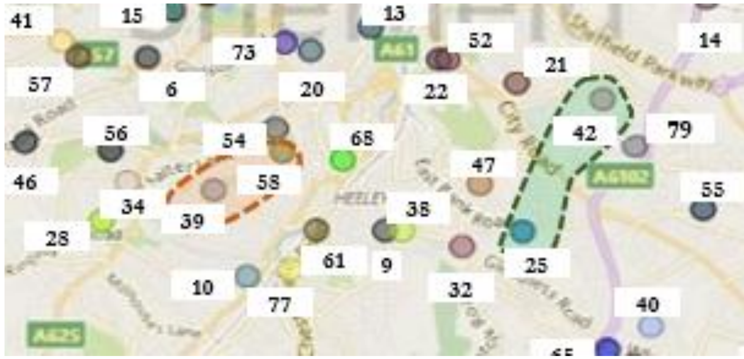
Details for each cluster form using the Backup Model

Cluster 2



Cluster 3

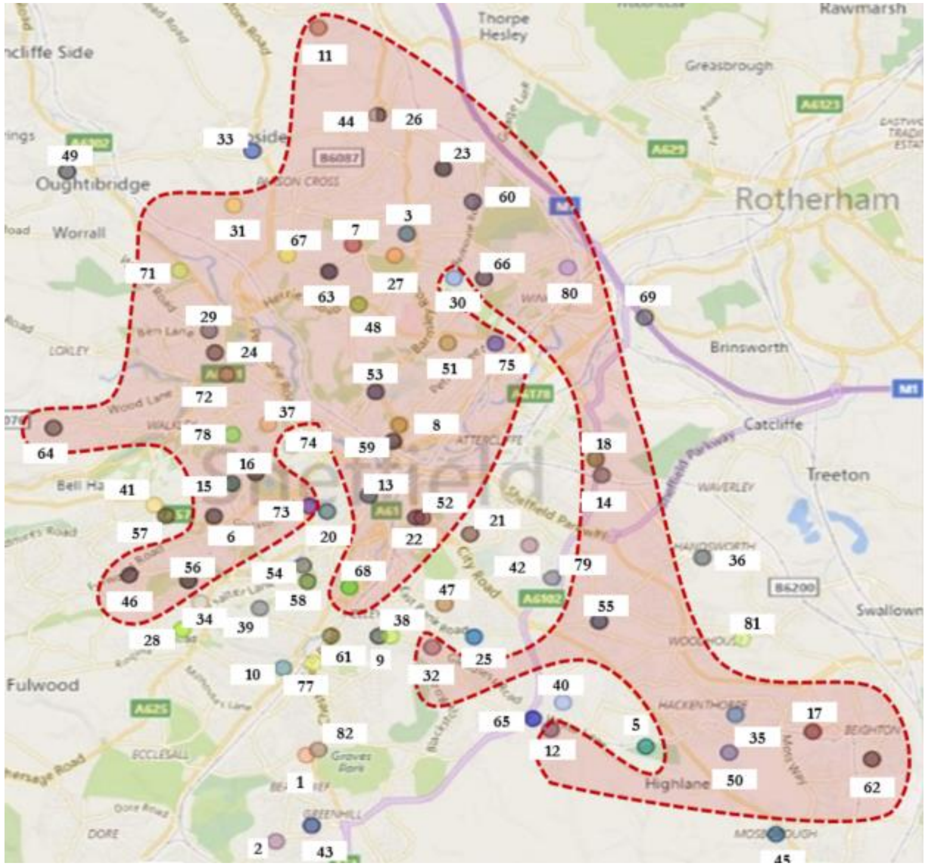
Cluster 4



Cluster 5 (green shaded region) and 6 (orange shaded region)

Cluster 7

Cluster 8



Cluster 9