THREE ESSAYS ON NEW POLITICAL ECONOMY

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Abstract

In this thesis I focus on the issue of how the elite attempts to solve the threat of revolution, sustain control over the state, and preserve its advantage in terms of wealth. In Chapter 1, we outline the basic structure of this thesis. In Chapter 2, we compare the military spending in nondemocracy and democracy. We argue that, in a nondemocracy, given a resource windfall, the elite will enlarge its military spending in order to remove the increased threat of revolution. However, the relationship between natural resources and public expenditure depends on the effectiveness of military spending. We also provide empirical evidence in this chapter. In Chapter 3, we compare land reform and income redistribution, investigating under which conditions the landed elite would prefer to distribute its landholdings to the farmer, allowing them to set up individual private farms, and under which conditions the landed elite would prefer to make income redistribution to the farmer in order to remove the threat of revolution. In Chapter 4, we rationalize a new explanation for the peaceful extension of the franchise. We outline the possible connection between the expansion of the franchise and the advantages for the elite in terms of labour supply, and investigate the conditions under which the enfranchised will extend suffrage to the whole population.

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Declaration

I declare that except where specific reference is made to the work of others, the contents of this thesis are original and have not been submitted in whole or in part for consideration for any other degree or qualification in this, or any other university. All sources are acknowledged as references.

Chapter 1

INTRODUCTION

This dissertation presents three primarily theoretical essays focusing on the issue of how the landed elite attempts to remove the threat of revolution that is posed by the populace at large and, in doing so, to protect its wealth from expropriation. In each chapter, we compare two possible policies that could relax domestic conflict between the very rich and the poor, and sustain the control of current incumbents over the state. We investigate the conditions under which the incumbent prefers to implement one possible policy over the other.

1.1 Military Spending or Public Sector Spending

In our second chapter, we examine how and whether natural resource endowments affect public policy. Since income from natural resources accrues to the government, given a resource windfall, the income of the current government increases. However, the incentive for the masses to attempt a revolution and replace the current incumbents becomes higher simultaneously. In this chapter, we compare two types of government spending, military spending and public expenditure, and investigate, given a resource windfall, how a government's decision-making over military spending and public expenditure will be affected. From one hand, military spending potentially increases the power of incumbents, lowers the incentive of potential challengers to revel, and helps current incumbents to consolidate their control over the state. From the other hand, expanding the expenditure over the public sector improves the national wage level, and increases the incentive of potential challengers to stay with the control of current incumbents.

We develop a model consisting of two groups: the elite and the masses. In the broadest possible sense, there are two possible types of political regime: nondemocracy and democracy. In a nondemocracy, an elite controls the state. This elite is the direct recipient of resource income and determines how to allocate its received resource rents. It could invest in its own enrichment, in military spending that increases its ability to maintain power, or in the public sector in order to improve the income level of the masses. Considering the income level in nondemocracy and the expected payoff of attempting a revolution, the masses decide whether to attempt a revolution. In this chapter, we assume that once the revolution is attempted, the state is transformed to a democracy in which the state is controlled by the masses. We also assume that democracy is an absorbing state, in other words once the state ever becomes a democracy, it persists indefinitely. We focus on the circumstance that the state starts with a semi-consolidated nondemocracy where the elite faces the threat of revolution, but it could fully solve this danger by enlarging its military spending and/or public expenditure.

Our chapter presents the following key findings. Firstly, we establish the condition under which nondemocracy will be the equilibrium regime. We find that countries with limited resource endowments but with a higher national wage level are more likely to facilitate stability. We also capture some comparative static results. In a semi-consolidated nondemocracy, if the elite benefits from a resource windfall, it will always enlarge its military spending. However, the comparative static between resource rents and public spending depends on the effectiveness of military spending. If military spending is highly effective in improving fighting power, given a resource windfall, incumbents would prefer to reduce their public expenditure. Conversely, if military spending is less effective, incumbents will increase their spending in public sectors instead.

In addition to the theoretical research, we also provide an empirical analysis. Drawing on a database for 1960-2016 containing information on 159 countries, we find a positive and significant relationship between military spending and natural resource rents in nondemocratic countries. However, in democratic countries, resource rents do not appear to be statistically significant as a determinant of military spending. We also test the relationship between resource rents and public expenditure. We find that, in nondemocracy, the relationship between resource rents and public investment appears to be positive and significant, however, this relationship disappears in democracy.

1.2 Land Reform or Income Redistribution

It has been generally accepted that highly unequal societies are less likely to live peacefully, and highly skewed distribution of wealth is often accompanied by a concentrated landownership. When the ownership over land is controlled by a small number of people, distribution of income is highly skewed, and most of the rural people live with low wages and poor living standards. This conflict over wealth distribution is an important contributing factor in civil war. For example, the French Revolution and the American Civil War were all primarily a conflict over land. In addition, until the twentieth century, in much of the predominately rural developing world, like Mexico, Cuba, and Philippines, civil wars have all had their origins in landownership distribution (Dorner, 1992). From these historical events, we have also seen that these countries solve their domestic conflicts in different ways. In most western developed countries, the landed elite agreed to extend the franchise to the majority, accept a higher income tax rate, enlarge the quantities of public good provision, and make a higher income redistribution to the poor. In addition to income redistribution, land redistribution has also been widely applied. India, Japan, Taiwan, South Korea and many countries in the Latin America all engaged in fundamental land reforms.

Impacts of land reform have been widely discussed, but theory that explains choice over the coverage of land reform is limited. Our third chapter compares two optional policies: land reform and income redistribution, investigates the preference of the landed elite between these two policies, and construct a model that analyzes the optimal strategy of land reform. We develop a model consisting of two groups: the landed elite and the farmer. Landed elites control large quantities of private wealth, and also hold the ownership of all available land. Farmers take the majority of the whole population and each inherits one unit of labour. We consider the circumstance that the state operates a system of control on the part of the landed elite. Given the threat of revolution,the landed elite should improve average income levels so that it could reduce farmers' incentives to revolt. The underlying idea of our approach is that the landed elite could give up part of its landholdings to a group of farmers and hire the rest of all available labour at a revised wage rate. Each farmer who has been allocated a piece of land will set up their private farm and collect all their produced revenue. The landed elite determines the number of family farms that are going to be established. If the optimal number of private farms is high, we say that land reform is a more attractive policy, rather than income redistribution. Conversely, if the optimal number of private farms is low or equates to zero, when dealing with the threat of revolution, the landed elite prefers to make income redistribution to the majority.

Our chapter presents the following three key findings. Firstly, we show that the optimal number of private farms depends on the development of the agricultural industry. If agriculture is a productive industry, land is a kind of valuable wealth, and therefore distributing land to the farmer will be effective in removing the threat of revolution. Conversely, if land is less valuable, the size of required land of each private farm will be greater, and it will be costly for the landed elite to impose a widespread land reform. Secondly, we find that the quality of political institution affects the preferences of the landed elite for land reform over income redistribution. If the landed elite operates a stable and functioning government, the credibility of the landed elite is high, the majority regards its promise of income redistribution to be a credible commitment, and income redistribution becomes more effective in consolidating the current political regime. Thirdly, we capture some comparative static results. We find that the political power held by the landed elite, the inequality of wealth distribution between the rich and the poor, and quantities of the total available land, all influence the optimal number of private farms that the landed elite would prefer to establish.

1.3 The Size of Enfranchisement

During the nineteenth century, the franchise was widely extended in most western societies. The leading explanation for franchise expansion is that the political elite extends voting rights to prevent revolution (Acemoglu and Robinson, 2000, 2001, 2005). Extending the franchise to the disenfranchised could change the political equilibria, and this acts as a credible commitment to redistribution. Given the threat of revolution, the elite would prefer to firstly extend the voting right to the groups of people who are relative rich in the society since those people could act as a buffer between the elite and the poor. Later, those groups of people who are relative rich also play the role of softliners arguing against repression and in favour of further franchise, which is opposed by the elite.

In this chapter, we provide an alternative explanation for the extension of the franchise to the general population. We notice that, in Britain, since suffrage was extended to the middle class, there was a shift in policy in a direction favoured by the capitalist middle class, but not the landed elite. These policies accelerated rapid urbanization, causing a large amount of farmers to move from the countryside to the city which lowered the labour supply in the agricultural industry, thus potentially generating losses to the landed elite. Our fourth outlines the possible connection between the extension of the franchise and the advantages for the landed elite in terms of labour supply. We compares two possible political regime, partial democracy and full democracy, and investigates the preference of the elite between these two possible political regime. We construct a model consisting of three groups of agents: the landed elite, the capitalist, and the workers. Under a partial democracy, the decisive voters is the capitalists, whilst the decisive voters is the workers under a full democracy. To describe the conflict over the labour supply between the landed elite and the capitalist, we apply the Lewis Model (Lewis, 1954), and regulate the amount of labour surplus in the agricultural industry. If the amount of migrants from the countryside to the city exceeds this amount of labour surplus, the produced revenue for the landed elite decreases as a result of the movement of labour. Under this circumstance, if the landed elite extends the franchise only to the capitalists, the later will choose the policy that accelerates the labour movement, and this may reduce the group income of the landed elite. By analyzing this model, we determine how, given the threat of revolution, the landed elite chooses the size of enfranchisement.

The contribution of this chapter is twofold. Firstly, we capture the conditions under which the landed elite will extend the franchise only to the capitalist, and under which the landed elite will extend the franchise to the whole population. We find that the landed elite will extend suffrage to the whole population if the preferred income tax rate of the capitalist is low and the optimal income tax rate of the workers is higher but still a moderate rate. Secondly, we find that if the agricultural industry of the state is productive, or the state is a primarily agrarian society, the landed elite will extend the franchise only to the capitalist.

Chapter 2

On the Impact of Natural Resources on Political Regimes and Public Policies

2.1 Introduction

Understanding the determinants of military spending is important. Military spending is a key issue when considering issues related to conflict. In this chapter, military spending might be a private good for the ruling class, rather than a public good for the population, and this has been widely applied by Besley and Persson (2008a,b, 2010). It potentially increases the power of incumbents, and could also lower the incentive of potential challengers to rebel. On one hand, military expenditure helps current incumbents to consolidate their control over the state. However, on the other hand, higher military expenditure potentially undermines economic growth in less developed countries.

Previous research has already examined the determinants of military spending. Rosh (1988) finds that geography has an important and significant effect on military spending. Many socioeconomic variables, such as population, national income level, trade and external aid, have also been considered as potential determinants of military spending. Among socioeconomic variables, population size has been shown to have a positive and significant effect (Dunne and Perlo-Freeman, 2003), Collier and Rohner (2008) find GDP per capita to have a significant and positive effect on military spending, trade has a positive and significant effect (Dunne and Perlo-Freeman, 2003), and the relationship between external aid and military expenditure is significant and positive (Collier and Rohner, 2008). Institutional variables have also received attention. Most studies include a measure for democracy when they run military spending regressions, and the correlation between democracy and military spending is always significant and negative (Dunne and Perlo-Freeman, 2003; Dunne et al., 2008; Nordhaus et al., 2009). Furthermore, Albalate et al. (2012) consider the form of government and show that presidential democracies spend more on military investment than their parliamentary counterparts.

Empirical studies improve our overall understanding of military spending, but there are still a number of areas that need to be further explored. Dunne and Perlo-Freeman (2003) find that the determinants of military spending changed after the end of the Cold War. The most common type of conflict in recent years has been civil war. When determining military expenditure, countries' concerns have been transferred from the cold war to internal strife. The threat or actuality of internal conflict within a country becomes the major determinant of military spending (Collier and Hoeffler, 2002; Collier and Rohner, 2008). We are therefore curious about the factors that might induce an internal conflict or exacerbate a preexisting conflict. This chapter focuses on changes in natural resource rents. On one hand, since the income from natural resources accrues to the government, current incumbents receive a higher payoff if there is a resource windfall. On the other hand, increases in resource rents also generate a higher incentive for the majority to attempt a revolution, and to replace current incumbents. In order to sustain control over the state, the current incumbents may enlarge their military spending. In this chapter, we focus on the issue of whether and how a natural resource windfall will influence a government's decision over military spending and public investment. We imagine whether those countries which have been endowed with greater quantity of natural resources will enlarge their military spending. Until year 2017, Angola has the highest average resource rents as a percentage of total GDP in Africa, 38.64%. Simultaneously, the average military spending as a percentage of government spending in Angola equals 21.14%, which is also above the average in Africa. However, average resource rents as a percentage of total GDP in European countries are limited, and their average military spending as a percentage of government expenditure all keeps below 10%.

We develop a model that evaluates how natural resource abundance influences the distribution of government expenditure. Our described society consists of an elite and the general population. In the broadest possible sense, there are two possible political regimes, nondemocracy and democracy. In nondemocracy, an elite controls the state, and this elite is also the direct recipients of resource income. The elite faces the problem of how to allocate the resource rents. It could invest in its own enrichment, in military spending that increases its ability to maintain power, or in the public sector in order to improve the domestic economy. The income level of the masses is determined by public investment as well. Considering the income level in nondemocracy and the expected shared resource rents under democracy, the masses decide whether to attempt a revolution.

Increases in natural resource endowments affect the decision-making process of elites in several ways. Since elites are the direct recipients of resource revenue, any increases in resource rents improve the rewards of staying in power, and hence elites would prefer to invest in activities that shore up their political control. However, the prospect of conflict may simultaneously be exacerbated by resource windfalls. For the majority, resource abundance produces a higher expectation of rewards following a revolution and taking control of the state. In our model, democracy could only be achieved by a revolution. If the majority ever succeeds in attaining power, the elite is removed from the controlling position, the society transitions to democracy and the new regime is assumed to persist thereafter. Considering this, given a resource windfall, elites work to remove the threat of revolution and sustain their control over the state. Generally, they do this in the following two ways. Firstly, they turn to defence activities and expenditures. Here, we refer to this kind of activity as any investment in the military, or any other expenditure designed to improve fighting ability. Increases in military expenditure make it more difficult for the majority to win a revolution. In another words, if civil war takes place, a higher military expenditure on the part of the elite will generate a greater incurred loss to the majority. This means that, the expected payoff of revolution declines, and the incentive for the majority to attempt a revolution becomes lower. In addition, the elite could sustain its controlling position by improving the national wage level. Formally, the elite may invest in the public sector to produce more opportunities in the private sector. It could increase its spending in education, medical services and other public industries. In our model, we suppose that increased public investment works to increase the national wage level so that the majority benefits from such investments. An increased national income level under nondemocracy then increases the opportunity cost of revolution. Therefore, for the elite, increased public expenditure lowers the likelihood of facing such a challenge.

The contribution of this chapter is twofold. Firstly, we develop a theoretical model that describes the decision-making process of the elite when allocating government spending on the military and on productive public sector investment. We stress the conflict between the elite and the majority. By analysing our described model, we achieve the following results. We find the condition under which nondemocracy will be the equilibrium regime. Countries with limited resource endowments but with a higher national wage level are more likely to facilitate regime stability. However, countries with high resource endowments and a low wage levels are highly unstable. In this chapter, we define the stability of a nondemocracy in the following way. If there exists no threat of revolution within the state, the elite could maintain its control over the state without any investment towards either the military or a productive public sector, we say the nondemocracy is fully consolidated. If the elite faces the threat of revolution, but it can solve this by enlarging its military spending as well as public spending, we say the nondemocracy is semi-consolidated. However, if the elite cannot remove the threat of revolution, and the majority have the potential to take over the political power of elite and transition the current regime to democracy, we say this nondemocracy is unconsolidated. In the following section, we focus on the semi-consolidated nondemocracy. We capture the comparative static between resource rents and military spending. We find that, in a semi-consolidated nondemocracy, if elites benefit from a resource windfall, they will always enlarge their military spending. However, the comparative static between resource rents and public investment is ambiguous, and this depends on the effectiveness of military spending. We find that if military spending is highly effective in improving political power, the optimal public investment decreases. Otherwise, if military spending is less effective, elites would prefer to enlarge their public expenditure if there is a resource windfall.

In addition to the theoretical research, we also provide an empirical analysis. Drawing on a database for 1960-2016 containing information on 159 countries, we find a positive and significant relationship between military spending and natural resource rents in nondemocratic countries. However, in democratic countries, resource rents do not appear to be statistically significant as a determinant of military spending. We also test the relationship between resource rents and public expenditure. We find that, in nondemocracy, the relationship between resource rents and public investment appears to be significant and positive. However the coefficient is quite low, indicating the positive effect posed by resource rents towards government spending on the public sector is limited.

Our chapter is related to the literature on the natural resource curse. Since the 1980s, economists have questioned natural resources as a driver of economic growth. Instead, they often perceive resource abundance as a kind of curse. Recent literature finds that resources are associated with slower economic growth, violent conflict and undemocratic regime types (Sachs and Warner, 2001; Collier and Hoeffler, 2002, 2005, 2009; Ross, 2001a,b; Acemoglu et al., 2004). Many existing economic models explain the resource-curse puzzle by proposing the Dutch disease effect, in which the resource sector crowds out other sectors more important for growth. Political scientists have long held the view that the reasons for the resource curse are to be found in the behaviour of those who control the state. They further propose some formalized political-economy arguments. In Robinson et al. (2006), an abundance of natural resources increases the current government's incentive to boost public employment in order to retain its controlling position. Hodler (2006) develops a model where natural resources lead to fighting among groups. In turn, fighting is assumed to reduce the protection of property rights, and through this to reduce private investment. Caselli (2006) is complementary to Hodler's work, arguing that natural resource abundance generates power struggles that make elites cut down their investment in the long-run development. Our model is an extension of this research. We not only focus on investment in development, we take military spending into consideration. In our model, the probability of being a governing group is endogenous, which could be positively influenced by enlarging military expenditure. Caselli and Cunningham (2009) evaluate some of the possible political mechanisms that could lead to a resource curse, and Caselli and Tesei (2016) theoretically and empirically test whether natural resource windfalls affect political regimes.

This chapter proceeds as follows. Section 2.2 outlines the basic economic and political environment. Section 2.3 characterizes the equilibria of the baseline model, and also establishes the main comparative statics. Section 2.4 generalizes the baseline model by making it a multiple-period model. Section 2.5 concludes. In the Appendix (section 2.6), we provide the empirical evidence that is consistent with our theoretical model.

2.2 Baseline Model

We now outline a model to formalize the proposition that for those nondemocratic states which are richly endowed with natural resources, the incumbents would prefer to enlarge the investment in military rather than public development.

2.2.1 Demographics, Preferences and Production Structure

We consider a society that exists for two periods. The society is populated by L citizens/workers and N elites where L and N are a finite number, and we use ω and ε to denote the set of workers and elites respectively. The first assumption is:

Assumption 1. *L»N that is the number of workers is significantly greater than the numbers of the elite.*

Ergo, workers make up the majority of the whole population. There are two types of final goods, natural resources and produced goods, which are denoted by A and Υ respectively. Each agent has the same risk-neutral preferences, with discount factor $\beta \in (0, 1)$, given by:

$$u^i = c_t^i + \beta c_{t+1}^i \tag{2.1}$$

where c_t^i denotes the consumption of agent i at time t in terms of the final good, and c_{t+1}^i denotes that at time $t + 1$. At the end of each period, agent i consumes all her/his income. Since each agent derives her/his income from selling two types of goods with no difference, at time t, total consumption function is displayed as following:

$$c_t = A_t + \Upsilon_t$$

where c_t denotes the aggregate consumption of the whole population at time t, A_t denotes the total resource rent, and Υ_t denotes the total produced revenue from normal goods. In our hyperthetical society, the state has been endowed with a fixed quantity of the natural resource, and the price of natural resource is globally determined. So that, the total collected resource rent is exogenous. We then make the following assumption towards the natural resource rent:

Assumption 2. *At the beginning of each period, the current government of the state will be endowed with the resource rent that equals A.*

The normal good, Υ is produced using labour and capital. In our hyperthetical society, each worker owns one unit of labour, which could only be supplied to the manufacturing industry. Since the population is fixed for the sake of simplicity, the total supply of labour equals L. We take the capital stock, K, as given. The elites do not access to the production function, the capital ownership belongs to the capitalist who will also collect the manufacturing income and pay the workers in the end. In this chapter, to simplify the analysis, we assume that the capitalist is consisted by a small number of people which holds limited influence towards the political outcome. So that, we will not consider how the capitalist contributes to the political outcome. Production could also be influenced by investment in the public sector provided by the government. We use the notation I to denote quantities in public investment. We normalize the price of the produced goods to equal 1, so the total produced revenue for the normal goods equals:

$$\Upsilon_t = g(I_{t-1})K^{\alpha}L^{1-\alpha} \tag{2.2}$$

where the g function describes the effectiveness of public investment. Public investment might comprise of education, medical services, technology and other infrastructure. Since there is a time lag for the public investment to affect production, at each time t, production is determined by public investment in time t-1, and the effectiveness is written as $g(I_{t-1})$. In the two-period model, we take I_{t-1} as given. To simplify the discussion, we make the following assumption on g:

Assumption 3. *g is defined over* $(\underline{I}, \infty)$ *where* $\underline{I} \geq 0$. *g is everywhere strictly increasing and twice continuously differentiable. Moreover,* $g'(I)$ *is downward sloping so that* $g''(I) < 0$. *Here* $g(0) = 1$ *so that any positive public investment could increase the total produced quantities, but with diminishing marginal returns.*

Suppose the labour market is competitive, the wage rate equals the marginal product of labour. Hence, the wage rate is:

$$w_t = \frac{\partial \Upsilon_t}{\partial L} = g(I_{t-1})K^{\alpha}(1-\alpha)L^{-\alpha} \tag{2.3}$$

where $\alpha \in (0, 1)$. For each agent $i \in \omega$, the income level at time t, w_t is determined by equation (2.3). We have seen the fact that any increases in public sector investment will raise the national wage up.

2.2.2 Political Regimes and Political Power

There are two possible political regimes, denoted by D and N, corresponding to democracy and nondemocracy. The identity of the decision maker varies between these two regimes. In nondemocracy, the state is controlled by the elite. The elite determines the policy to derive optimum benefit, but applies this to the whole state. Since the elite also shares the government revenue equally in the end, the preference of each elite is identical. However, in democracy, the state is controlled by the majority, and workers determine policy to conform with their best interest. In democracy, as a transfer, workers will receive the redistributed resource revenue in the end. Once the elite is removed from the controlling position, it has no income. In our model, political regime transition from nondemocracy to democracy is achieved through revolution. At time t, if the state starts with non-democracy and the majority attempt a revolution, civil war will then take place. Under this circumstance, all production activities will stop, no income payments will be made, the group with greater military power will win the the war and take the controlling position in the end. If the majority wins, they will take the controlling position and at time $t+1$, the political regime transitions to democracy. Otherwise, the elite will remain in control of the state in the end, and in the next period the political regime will continue in the form of nondemocracy. In our model society, to simplify the analysis, we make the following assumption:

Assumption 4. *If workers attempt a revolution, they will always win. Democracy follows a revolution, and democracy is an absorbing state. If the society ever becomes a democracy, it remains indefinitely.*

In this chapter, we will not consider the possibility that elites could carry out a democratization of the state, and instead assume that democracy may only be brought about by a revolution. Once the democratic political regime has been built up, the elite cannot mount a coup and revert the state back to nondemocracy.

At any point in time t, the state of this society is denoted by $s_t \in \{N, D\}$. The identity of each agent i will not be changed with the state variable, and the structure of the political regime is determined by the group holding political power. Suppose the state starts with nondemocracy, $s_t = N$, if there is no revolution, the elite determines the state variable for the next period, and $s_{t+1} = N$. However, if the majority attempt a revolution at time t, the state variable for the next period is determined by the majority where $s_{t+1} = D$. If the state starts with democracy, $s_t = D$, democracy remains indefinitely, and $s_{t+1} = D$. We then introduce the variable $\pi_t \in \{0, 1\}$ to denote whether the majority attempt a revolution. If there is no revolution, $\pi_t = 0$. Otherwise, $\pi_t = 1$.

To complete the description of the environment, we here specify what the key decisions are. As we have mentioned above, the decision maker determines the state variable for the ensuing period, s_{t+1}. Whenever the elite is the decision maker at the end of time t, it chooses the political regime of nondemocracy as the option most in their favour, s_{t+1}. On the contrary, if the majority is able to determine the state variable at the end of time t, it will choose a democratic society for the next period, s_{t+1}. In addition, at the beginning of each time t, the decision maker will choose the income tax rate, τ, for the whole society. Since revolution stops all production activities, workers have no income if they attempt a revolution. Whenever $s_t = R$, $\tau_t = 0$, no tax payment will be made. If $s_t = N$, the elite determines the optimal income tax rate as $\tau_t = \tau^N$ where τ^N denotes the optimal income tax rate of the elite. However, if $s_t = D$, the majority determines the tax rate in its favour as $\tau_t = \tau^D$ where τ^D denotes the preferred tax rate of workers. To simplify the analysis, we make the following assumption concerning the tax structure:

Assumption 5. *The income tax rate is only imposed on the income from the production activity. The preferred income tax rate of the elite permanently fixed as* $\tau^N = \tau$. *Workers prefer a zero income tax, and* $\tau^D = 0$.

In our model, since the elite is not involved in production activities, it will not pay income tax. In nondemocracy, elites will identically share the collected tax revenue in the end, and would therefore prefer to impose a positive tax rate upon the majority. In democracy, even if the majority impose a positive tax rate, they will share the tax revenue equally in the end, and their net income from the production activity stays the same. To simplify the analysis, we make the above assumption that in democracy, the income tax rate equals zero.

At the beginning of time t, the current government will announce its policy towards public expenditure, which is denoted by I_t. The government determines the public expenditure to a level which it considers optimal. We use the notation I_N to denote the optimal public expenditure for the elite in nondemocracy, and I_D to denote the optimal public expenditure for the majority in democracy where $I_N, I_D \geq 0$. In this chapter, we assume that $s_t = D$ is an absorbing state, if the economy ever becomes a democracy, it remains so indefinitely. In the following section, we will discuss how elites determine the level of public expenditure if there exists any threat of revolution.

2.2.3 Military Expenditure and Public Investment

As we have mentioned above, at time t, if the state begins in the elite control, $s_t = N$, the majority chooses either to stay within the control of the elite or to revolt. According to the Assumption 4, if workers attempt a revolution, they will always win. Thus, if a revolution takes place, the majority will emerge victorious, and thereafter determine the state variable for the next period, $s_{t+1} = D$. Since the elite receives zero income in a democracy, it will endeavour to prevent the revolution so as to sustain control over the state. There are two types of policy available to the elite. Firstly, it could increase its military spending, thus lowering the incentive of workers to revolt. Secondly, it could expand its public expenditure, and increase the income level of each worker, thus improving the expectation of workers for staying with the control of elites. In our model, elites could apply both of these two policies simultaneously.

As we have already mentioned, the majority could successfully revolt, and thereafter transition the state to democracy . However, the act of revolution also generates a permanent loss to the majority. The loss incurred is determined by the military spending of the elite. In our model, only the elite is able to invest in the military. Since L workers constitute the majority of the whole population, workers have difficulty in solving the collective action problem. Therefore, they can hardly invest in the military, and their fighting power arises solely from their domination of the overall population size. In contrast to the workers, the number of members of the elite is fixed and limited, so each of them will take into account that their contribution to the military will have an effect on equilibrium outcomes. We assume that the elite is able to solve the collective problem, and could spend part of its wealth to improve its defence power. Military spending includes expenditure on military equipment, the cost of enlarging the army, and costs relating to other kinds of activity that improve the military power of the elite. Suppose the majority attempt a revolution at time t, the fraction of the generated permanent loss, μ is defined by:

$$\mu = F(M_t)$$

where M_t denotes the total military spending of the elite, F(.) is a given distribution, and $F(M_t)$ measures the actual loss that is generated by the revolution. To simplify the discussion, we make the following assumption on F:

Assumption 6. *F is defined over $(\underline{M}, \infty)$ where $\underline{M} \geq 0$. F is a strictly increasing function which is twice continuously differentiable. The density of $F(M)$, $f(M)$, is single peaked. There exists a unique M^* that makes the derivative of the density, f', equal zero. Moreover, $f'(M)$ also follows that $f'(M)$>0 for all $M < M^*$ and $f'(M) < 0$ for all $M > M^*$. In addition, $f(M)$ also satisfies that $lim_{M \to \infty} f(M) = 0$.*

All features embedded in the above assumption follow the conflict technology that has been proposed by Skaperdas (1992). We make this assumption to simplify our analysis and we will discuss how the equilibrium is affected if we relax Assumption 6 later.

After the revolution, democracy will be imposed, and the majority will become the decision maker, they will equally share the collected government revenue, and impose a zero income tax. We then write the expected payoff of attempting a revolution as follows:

$$V^{\omega}(\pi(N) = 1) = \beta(1 - F(M_t)) \left[g(0)K^{\alpha}(1-\alpha)L^{-\alpha} + \frac{A}{L} \right]$$

where $V^{\omega}(\pi(N) = 1)$ refers to the expectation of a worker upon choosing to participate in a revolution in a nondemocracy. As we have mentioned above, if a revolution takes place, all production activities stop, public investment can not work to improve the wage level for the next period, and the majority determines $s_{t+1} = D$ for the next period. As a result, in the first period, workers have no income. This is the first opportunity cost of engaging in a revolution for the workers. In period 2, since there is no public investment has been made in the previous period ($g(I_{t-1} = g(0))$, each worker will ultimately receive the income payment, $g(0)K^{\alpha}(1-\alpha)L^{-\alpha}$, and this is the second opportunity cost of engaging in a revolution for the workers. Since the state only exists for two periods, the majority will not make any public investment in the second period, they share the collected resource rents equally, and each receives $\frac{A}{L}$. The received income in the second period is discounted at β. Because revolution also generates permanent losses to workers incurred through fighting, workers could only achieve $(1 - F(M_t))$ of the total expected payoff in democracy. Any increases in the military spending at time t will lower the incentive for workers to attempt a revolution.

From section 2.2.1, we have seen that the wage level of each worker is affected by the public expenditure in the previous period. We use the notation I_N to denote the optimal public expenditure in nondemocracy. If there is no threat of revolution, the elite will choose $I_t = I_N$ that maximizes its net expected payoff of investing in public sector:

$$max \quad \beta\tau g(I_N)K^{\alpha}(1-\alpha)L^{1-\alpha} - I_N \tag{2.4}$$

$$s.t. \quad I_N \leq A$$

where $\beta\tau g(I_N)K^{\alpha}(1-\alpha)L^{1-\alpha}$ refers to the expected tax revenue in the following period, any increases in the public expenditure will raise the expected tax revenue for the elite. Since the government finances the public spending through resource rents, the total public expenditure should not exceed the total resource rents. The elite determines its optimal public expenditure I_N which solves the following first order condition:

$$\beta\tau g'(I_N)K^{\alpha}(1-\alpha)L^{1-\alpha} = 1 \tag{2.5}$$

Similarly, in a democracy, the workers determine the public expenditure in the following way. Since democracy is an absorbing state in this model, the workers will determine the public expenditure in their optimum, chooses $I_t = I_D$ that maximizes their net expected payoff of investing in public sector:

$$max \quad \beta g(I_D)K^{\alpha}(1-\alpha)L^{-\alpha} - \frac{I_D}{L} \tag{2.6}$$

$$s.t. \quad I_D \leq A$$

where $\beta g(I_D)K^{\alpha}(1-\alpha)L^{-\alpha}$ refers to the expected wage of each worker in the following period. In a democracy, the workers share the burden of public expenditure equally, and each worker pays $\frac{I_D}{L}$. The optimal public expenditure I_D satisfies the following first order condition:

$$\beta g'(I_D)K^{\alpha}(1-\alpha)L^{1-\alpha} = 1 \tag{2.7}$$

Since $\tau \in (0,1]$, and g is a everywhere strictly increasing and concave function, $g'(.) > 0$ and $g^{''} < 0$, we could have $I_N \leq I_D$. The optimal public expenditure in a nondemocracy is always smaller than that in a democracy.

Lemma 1. *Suppose Assumption 3 holds, the optimal public expenditure in a non-democracy is always smaller than that in a democracy,* $I_N \leq I_D$.

Given the two first order conditions, (2.5) and (2.7), we can capture some comparative static results:

$$\frac{\partial I_N(\delta)}{\partial K} > 0, \quad \frac{\partial I_D(\delta)}{\partial K} > 0$$

where $I_N(\delta)$ denotes, given the set of parameter, δ, the optimal public expenditure of the elite, and $I_D(\delta)$ denotes the optimal public expenditure of the majority. Any increases in the capital will increase the optimal public expenditure of both the elite and the majority. Similarly, we can also capture the comparative static result between the optimal public expenditure and the availability of labour resource:

$$\frac{\partial I_N(\delta)}{\partial L} > 0, \quad \frac{\partial I_D(\delta)}{\partial L} > 0$$

Proof. Since the optimal public expenditure of the elite, I_N, satisfies the first order condition, (2.5), we then could have:

$$g'(I_N) = \frac{1}{\beta \tau K^{\alpha}(1-\alpha)L^{1-\alpha}}.$$

Similarly, we could rearrange the condition (2.7) as follows:

$$g'(I_D) = \frac{1}{\beta K^{\alpha}(1-\alpha)L^{1-\alpha}}$$

Since $\tau \in (0,1]$, we could have, given all possible δ, $g'(I_N) \geq g'(I_D)$. As a result that g is a strict concave function, we then have $I_N \leq I_D$. To capture how the capital, K, affects I_N and I_D, we differentiate the above two equations by K, and we have:

$$\frac{\partial g'(I_N)}{\partial K} = -\alpha K^{-\alpha-1}\frac{1}{\beta\tau(1-\alpha)L^{1-\alpha}}$$

$$\frac{\partial g'(I_D)}{\partial K} = -\alpha K^{-\alpha-1}\frac{1}{\beta(1-\alpha)L^{1-\alpha}}$$

We then have $\frac{\partial g'(I_N)}{\partial K} < \frac{\partial g'(I_D)}{\partial K} < 0$. Since $g''(.) < 0$, we then have:

$$\frac{\partial I_N}{\partial K} > 0, \quad \frac{\partial I_D}{\partial K} > 0.$$

We could capture the comparative static result between the optimal public expenditure and the availability of labour force in the similar way. □

In this model, to simplify the analysis, we suppose there always exists plausible I_N and I_D that solve above two maximizing problems, (2.4) and (2.6). We take I_N and I_D as given. If the state starts with a nondemocracy, I_N not only refers to the optimal public expenditure for the elite, it also refers to the incurred public expenditure in the previous period which determines the national wage level in the current period. Similarly, if the state starts with a democracy, the state has been given with an incurred public expenditure, I_D.

As we have mentioned in section 2.2.1, the elite could increase its investment in the public sector so as to raise the national income level for the following period. This contributes to an increased expectation that workers will stay within the control of the elite. For each worker, the expected payoff of staying in nondemocracy is given by:

$$V^{\omega}(\pi(N)=0) = (1-\tau)g(I_N)K^{\alpha}(1-\alpha)L^{-\alpha} + \beta(1-\tau)g(I_t)K^{\alpha}(1-\alpha)L^{-\alpha}$$

where $V^{\omega}(\pi(N) = 0)$ denotes the valuation of each worker if they choose not to revolt in a nondemocracy. Since there is no revolution, each worker receives their post-tax income in the first period, which equals $(1-\tau)g(I_N)K^{\alpha}(1-\alpha)L^{-\alpha}$ where I_N denotes the public expenditure that has been made in the previous period. Since $\pi(N) = 0$, the elite determines the state variable for the next period, $s_{t+1} = N$. In the second period, the national income level equals $(1-\tau)g(I_t)K^{\alpha}(1-\alpha)L^{-\alpha}$ where I_t is the public expenditure that has been made in the first period. From the above equation, we have seen that the elite could increase its public expenditure at time t to improve the expectation that each worker will be content to remain in a state of nondemocracy.

The majority will not attempt a revolution if the financial incentive of staying in nondemocracy exceeds that of revolution, $V^{\omega}(\pi(N) = 0) \geq V^{\omega}(\pi(N) = 1)$. We then set up the condition under which the revolution will not take place:

$$\begin{aligned}(1-\tau)g(I_N)K^{\alpha}(1-\alpha)L^{-\alpha} + \beta(1-\tau)g(I_t)K^{\alpha}(1-\alpha)L^{-\alpha} \\ \geq \beta(1-F(M_t))\left[g(0)K^{\alpha}(1-\alpha)L^{-\alpha} + \frac{A}{L}\right]\end{aligned} \quad (2.8)$$

At the beginning of time t, in order to sustain the control over the state, the elite determines the public policy, I_t, and the military expenditure, M_t that satisfy condition (2.8). Under this circumstance, the expected payoff of the elite is given by:

$$V^{\varepsilon} = A - I_t - M_t + \tau g(I_N)K^{\alpha}(1-\alpha)L^{1-\alpha} + \beta\tau g(I_t)K^{\alpha}(1-\alpha)L^{1-\alpha} + \beta A$$

where V^{ε} denotes the expected payoff of the elite, $\tau g(I_N)K^{\alpha}(1-\alpha)L^{1-\alpha}$ refers to the total tax revenue in the current period where I_N is the incurred public expenditure in the previous period that has been given at the beginning of the current period, $\beta\tau g(I_t)K^{\alpha}(1-\alpha)L^{1-\alpha}$ denotes the tax revenue for the following period, and βA refers to the collected resource rents in the following period. Since our model only exists for two periods, in the second period, the elite will not make any investment towards military or public sector. The expected payoff for each worker is denoted as follows:

$$V^{\omega} = (1-\tau)g(I_N)K^{\alpha}(1-\alpha)L^{-\alpha} + \beta(1-\tau)g(I_t)K^{\alpha}(1-\alpha)L^{-\alpha}$$

If $I_t = I_N$, $M_t = 0$, and condition (2.8) is still satisfied, we say there is no threat of revolution. We then write the condition under which there is no threat of revolution as follows:

$$(1+\beta)(1-\tau)g(I_N)K^{\alpha}(1-\alpha)L^{-\alpha} \geq \beta(1-F(0))\left[g(0)K^{\alpha}(1-\alpha)L^{-\alpha} + \frac{A}{L}\right] \quad (2.9)$$

If condition (2.9) is satisfied, the elite controls the state at time t, chooses its optimum public policy where $I_t = I_N$, makes zero military spending, $M_t = 0$, and determines the state variable for the next period, $s_{t+1} = N$. Under this circumstance, the national income level will not be improved, and each worker will receive $(1-\tau)g(I_N)K^{\alpha}(1-\alpha)L^{-\alpha}$ in the second period, and their expected payoff is given by:

$$V^{\omega} = (1+\beta)(1-\tau)g(I_N)K^{\alpha}(1-\alpha)L^{-\alpha}$$

The expected payoff for the elite equals:

$$V^{\varepsilon} = A - I_N + (1+\beta)\tau g(I_N)K^{\alpha}(1-\alpha)L^{1-\alpha} + \beta A$$

Since there is no threat of revolution, the elites determines the public expenditure in its optimum where $I_t = I_N$. In each period, the elite receives the tax revenue that equals $\tau g(I_N)K^{\alpha}(1-\alpha)L^{1-\alpha}$.

2.2.4 Timing of Events

We now briefly recap the timing of events in this basic environment. Let δ denote the set of parameter $\delta = \{A, K, \alpha, L, \beta, \tau\}$ which has been given at the beginning of each period. At each time t, society starts with a state variable, $s_t = \{D, N\}$. Given this, the following sequence of events takes place:

- Suppose the society starts with $s_t = D$, the majority controls the state:

  - At the beginning of time t, the state is given with an incurred public investment which was made in the previous period, $I_{t-1} = I_D$. Here, I_D refers to the optimal public expenditure of the workers which satisfies condition (2.7). In addition, workers receive the resource rent, A, determine the public expenditure in their optimum, $I_t = I_D$.

- Whenever $s_t = D$, there is no revolution, $\pi_t = 0$. At the end of the period, income payment has been made, each worker shares government revenue equally, $A - I_D$, and the majority determines the state variable for the next period, $s_{t+1} = D$

- Suppose the society starts with $s_t = N$, the elite controls the state:

  - At the beginning of time t, the state is given with an incurred public investment which was made in the previous period, $I_{t-1} = I_N$. Here, I_N refers to the optimal public expenditure of the elite which satisfies condition (2.5). In addition, the elite receives the resource rent, A.

  - Given the level of A, and the set of parameter, δ, the elite considers whether there exists a threat of revolution.

  - If condition (2.9) is satisfied, there is no threat of revolution. Under this circumstance, the elite makes zero military spending, $M_t = 0$, chooses its optimal public policy, $I_t = I_N$, and the majority do not revolt, $\pi_t = 0$. At the end of time t, the elite collects the tax revenue, and chooses the state variable for the next period, $s_{t+1} = N$

  - If there exists a threat of revolution, the elite determines its military spending, M_t, and public expenditure, I_t. Given the set of policy, (M_t, I_t), if condition (2.8) is satisfied, the majority will not revolt, $\pi_t = 0$. The elite collects the tax revenue at the end of the period, and chooses the state variable for the next period, $s_{t+1} = N$. The majority receives its income payment in the end. Otherwise, revolution takes place, $\pi_t = 1$, all production activities stop, the elite is ultimately removed from the controlling position, no income payment is made, and the majority determines the state variable for the next period, $s_{t+1} = D$

- The following period, $t + 1$, starts with the state s_{t+1}.

2.3 Analysis of the Model

2.3.1 Definition of the Optimal Strategy

We now analyze the model we described in the previous section. We focus on the Markov Perfect Equilibria (MPE). Given the state variable, $s_t \in \{N, D\}$, an MPE restricts equilibrium strategies. The state variable determines the identity of the government, and an MPE consists of military spending for the government as a function of the political state, $M(s_t)$, public expenditure as a function of the political state, $I(s_t)$, and the income tax rate, $\tau(s_t)$, and the state variable for the next period, s_{t+1}. It also includes the decision variable of workers, $\pi(s_t)$, which determines whether workers will revolt or not.

The MPE is characterized as following. If $s_t = D$, the state is controlled by the majority, and their control is supposed to continue indefinitely. Under this circumstance, there is no revolution, $\pi(D) = 0$, workers derive zero benefit from military spending. Therefore, workers will not make any investment towards military, $M(D) = 0$, and will invest in the public sector at the optimum, $I(D) = I_D$, which I_D satisfies equation (7). In addition, the majority will impose zero income tax, $\tau(D) = 0$, and choose the state variable of democracy for the next period, s_{t+1}.

From the above, we have seen that whenever $s_t = D$, the MPE is predetermined. In this chapter, we focus on the MPE when the state begins with nondemocracy, $s_t = N$. Under this circumstance, the MPE can be characterized by backward induction within the stage game at time t. At the end of the period, whenever $\pi(N) = 0$, there is no revolution, the elite chooses the state variable of nondemocracy for the next period, $s_{t+1} = N$. Otherwise, if $\pi(N) = 1$, workers attempt a revolution and ultimately succeed, and they determine the state variable of of democracy for the next period, $s_{t+1} = D$. Here, $M(N)$ and $I(N)$ determine whether workers will attempt a revolution, $\pi(N) = 1$ or $\pi(N) = 0$. The remaining decisions are the investment over the military and the public sector under nondemocracy. In this chapter, the MPE we are going to discuss can be summarized as investment functions $M(N)$ and $I(N)$.

We then consider the following optimizing problem. At time t, the state starts with nondemocracy, $s_t = N$, given the set of parameter, δ, the elites determine the military spending, $M(N)$, and public expenditure, $I(N)$, to maximize its expected payoff in terms of control over the state. The optimizing problem is displayed as follows:

$$max \quad V^{\varepsilon} = A - M(N) - I(N) + \tau g(I_N) K^{\alpha}(1-\alpha)L^{1-\alpha} + \beta \tau g(I(N)) K^{\alpha}(1-\alpha)L^{1-\alpha} + \beta A \quad (2.10)$$

$$s.t. \quad (1-\tau) g(I_N) K^{\alpha}(1-\alpha)L^{-\alpha} + \beta(1-\tau) g(I(N)) K^{\alpha}(1-\alpha)L^{-\alpha} \geq \beta(1 - F(M(N)) \left\{ \frac{A}{L} + g(0) K^{\alpha}(1-\alpha)L^{-\alpha} \right\} \quad (2.11)$$

$$I(N) + M(N) \leq A \quad (2.12)$$

$$I(N) \geq 0 \quad (2.13)$$

$$M(N) \geq 0 \quad (2.14)$$

where V^{ε} denotes the expected payoff of the elite. At time t, the elite keeps the residual of total resource rents, $A - M(N) - I(N)$, and its collected tax revenue equals $\tau g(I_N) K^{\alpha}(1-\alpha)L^{1-\alpha}$ which is determined by the public expenditure in the previous period, $I_{t-1} = I_N$. Since the elite retains its power successfully at time t, it determines $s_{t+1} = N$ for the next period and will receive the collected tax revenue, $\tau g(I(N)) K^{\alpha}(1-\alpha)L^{1-\alpha}$ where $I(N)$ is the public expenditure that has been made in time t. In addition, since the game stops at the second period, the elite will not make any investments in either the public sector or the military, and will receive the resource rents, A, at the beginning of time $t+1$. Income at time $t+1$ is discounted at β. Equation (2.11) is the income compatibility constraint which means that for each worker, the expectation of staying in nondemocracy equals or exceeds the expected payoff following revolution. Equation (2.12) is the budget constraint for the elite. As we have discussed in the previous section, each agent consumes all his or her income at the end of each time t, so there is no residual income for the next period. At the beginning of time t, the elite has been provided with resource rents, A, and then makes the investment decision towards the military and the public sector. Therefore, the total government expenditure cannot exceed the collected resource rent.

At the beginning of time t, given the set of parameter δ, if there exists a plausible investment strategy, $(M(N), I(N))$, the elite could remove the threat of revolution, $\pi(N) = 0$, so that the state stays in nondemocracy, $s_{t+1} = N$. Otherwise, the elite can not remove the threat of revolution, workers attempt to revolt, $\pi(N) = 1$, and the state ultimately transitions to democracy, $s_{t+1} = D$.

This discussion establishes the following proposition:

Proposition 1. *At the beginning of time t, the state starts with nondemocracy, given the set of parameter, δ, we have the following three possibilities:*

- *Case 1: If condition (2.9) is satisfied, there is no threat of revolution, the MPE is $(0, I_N)$, $\pi(N) = 0$, and $s_{t+1} = N$. Nondemocracy is defined as a consolidated political regime.*

- *Case 2: If there exists a plausible $(M(N), I(N))$ that solves the optimizing problem (2.10) of the elite, $\pi(N) = 0$, and $s_{t+1} = N$. Nondemocracy is defined as a semi-consolidated political regime.*

- *Case 3: If there does not exist a plausible $(M(N), I(N))$ that solves the optimizing problem (2.10) of the elite, $\pi(N) = 1$, revolution takes place, the majority ultimately wins and transitions the political regime to democracy, $s_{t+1} = D$. Nondemocracy is defined as an unconsolidated political regime.*

2.3.2 Equilibrium Political Regime

In this subsection, we will look at how capital and labour affect the politial regime. We have already know if condition (2.9) is satisfied, there is no threat of revolution, and therefore nondemocracy is a fully consolidated political regime. Condition (2.9) could be rearranged as follows:

$$\left\{ \frac{[(1-\tau)+\beta(1-\tau)]g(I_N)}{\beta(1-F(0))} - g(0) \right\} K^{\alpha}(1-\alpha)L^{1-\alpha} \geq A \tag{2.15}$$

Since A is positive, if condition (2.15) is satisfied, the left-hand side of (2.15) is positive. As we have discussed in the previous section, any increases in capital stocks will increase the optimal public expenditure of the elite, $\frac{\partial I_N}{\partial K} > 0$. Since g is a strictly increasing function, we then have $\frac{\partial g(I_N)}{\partial K} > 0$. Therefore, we can capture the comparative static result, $\frac{\partial LHS}{\partial K} > 0$, which means with other parameters unchanged, the left-hand side of (2.15) increases in K. Similarly, we can have that the left-hand side of (2.15) increases in L as well. That is to say, countries that have more capital or more labour are more likely to stay in a fully consolidated nondemocracy. Increased capital contributes to raising the national wage, which improves the incentive for the majority to stay in nondemocracy. On the other hand, an increase in the number of workers lowers their shared revenue from resource rents so that, for each worker, their incentive towards revolution decreases in L. However, any increases in A make condition (2.15) less likely to be satisfied. Under this circumstance, for each worker, the expected payoff of revolution increases, and they have a higher incentive to attempt a revolution. As a result, nondemocracy becomes less consolidated. The elite now have to make a positive investment in the military or choose a more generous public policy in order to sustain its control over the state, and the nondemocratic political regime is defined as semi-consolidated.

We then look at a case in which the state is a semi-consolidated nondemocracy. This type of political regime is defined as follows that, given the set of parameter, δ, there exists a plausible investment strategy, $(M(N), I(N))$, which solves the optimizing problem we described above. The elite could just sustain their control over the state if there exists a plausible investment strategy, (M^*, I^*), which satisfies the following two conditions:

$$\frac{[(1-\tau)g(I_N)+\beta(1-\tau)g(I^*)]}{\beta(1-F(M^*)}K^\alpha(1-\alpha)L^{1-\alpha}-g(0)K^\alpha(1-\alpha)L^{1-\alpha}=A \tag{2.16}$$

$$M^*+I^*=A \tag{2.17}$$

Under this circumstance, the elite could just remove the threat of revolution by deploying its collected resource rents. Similarly, we could capture the comparative static results, $\frac{\partial LHS}{\partial K}>0$ and $\frac{\partial LHS}{\partial L}>0$, which means with other parameters unchanged, the left-hand side of (2.16) increases in K and L. Given the threat of revolution, countries with more capital or more labour are more likely to stay in a semi-consolidated nondemocracy. To capture how an increase in the value of resource rents, A, affects the equilibrium of the political regime, we then substitute I^* by $I^*=A-M^*$, and equation (2.16) is written as:

$$\frac{[(1-\tau)g(I_N)+\beta(1-\tau)g(A-M^*)]}{\beta(1-F(M^*)}K^\alpha(1-\alpha)L^{1-\alpha}-g(0)K^\alpha(1-\alpha)L^{1-\alpha}=A \tag{2.18}$$

Since $g(.)$ is an everywhere strictly increasing and twice continuously differentiable function, an increase in the value of A raises the value of both sides of (2.18). We cannot fully determine how an increase in resource rent affects the equilibrium of the political regime. However, since $g''(.)$ is negative, the value of $g(A-M^*)$ increases in A but with diminishing returns. If A keeps increasing, the value of A will gradually exceed the value of $\frac{(1-\tau)g(A-M^*)}{1-F(M^*)}K^\alpha(1-\alpha)L^{1-\alpha}$, and condition (2.18) no longer holds. Under this circumstance, the equilibrium of the political regime will be an unconsolidated nondemocracy, and revolution will take place. To summarize, given an increased resource rent, for a given initially optimal level of M^* and I^*, it is more likely that the current state will be an unconsolidated nondemocracy, and there exists no plausible investment strategy that could enable the elite to sustain its control over the state.

To further explain how factors A, K, and L affect the equilibrium of the political regime, we have drawn Figure 2.1. Given the set of parameter,

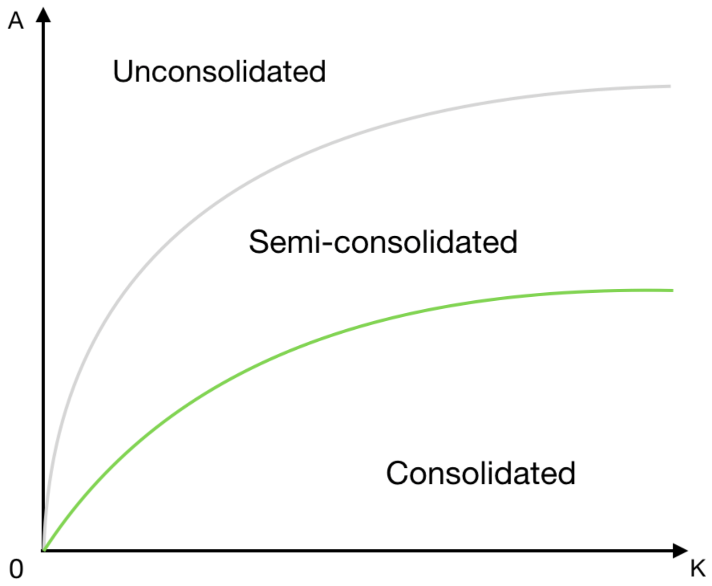


Figure 2.1: Equilibrium political regime

$\{\tau, L, \beta, \alpha, M^*, N^*\}$, the green line represents the following equation:

$$\frac{[(1-\tau)+\beta(1-\tau)]g(I_N)K^{\alpha}(1-\alpha)L^{1-\alpha}}{\beta(1-F(0))} - g(0)K^{\alpha}(1-\alpha)L^{1-\alpha} = A \quad (2.19)$$

The region below the green line represents all possible sets, (K, A) or (L, A), which make the state a fully consolidated nondemocracy. We have seen that a nondemocratic country with a higher K or L or a lower A is more likely to stay in a fully consolidated regime. The grey line describes equation (2.18). The region above the grey line includes all possible sets, (K, A) or (L, A), that make the non-democratic state an unconsolidated one. From the graph, we have seen that if the state is endowed with higher resource rents or a lower capital stock or limited availability of labour, it is less likely the state will continue as a nondemocracy. To be clear, both the grey and the green line may not be a straight line, it may be a curve, but it is upward sloping in general.

We then summarize the proposition as follows:

Proposition 2. *Nondemocratic countries that hold greater capital or lower resource rents are more likely to stay in a fully consolidated regime. In addition, a fully consolidated nondemocracy usually has lower natural resources per capita. However,* *countries that hold greater resource rents or lower capital are more likely to stay in an unconsolidated nondemocratic regime.*

The proposition is intuitive. In our model society, a higher capital stock or a large population size contributes to a higher national wage level. Under this circumstance, since a revolution will stop all production activities, for the majority the opportunity cost of attempting a revolution is high and this lowers their incentive to challenge the current regime. This explains why those countries which have a strong manufacturing industry or other developed industries maintain a consolidated political regime. However, for those countries that holds greater resource rents, the majority has a strong incentive to attempt a revolution, so that each citizen would benefit from a higher shared revenue. Those nondemocratic states can hardly remove the threat of revolution, and live in an unconsolidated regime. To summarize, if the government revenue is mostly contributed by the collected resource rent, the country is less likely to maintain a consolidated political regime. A stable political regime is underpinned by a well developed industry.

2.3.3 Comparative Statics

We now present some comparative static results which shed light on the question of how the elite would prefer to distribute its investment towards the military and the public sector so as to sustain non-democracy. Here, we mainly discuss case 2 that we have described in Proposition 1. Given the set of parameter, δ, there exists a viable investment strategy, $(M(N), I(N))$, so that the elite could use to remove the threat of revolution and sustain its control over the state. To simplify the analysis, we look at the circumstance which the elite could just maintain its control over the state by deploying its collected resource rents. Therefore, the budget constraint for the optimizing problem of the elite changes to $M(N) + I(N) = A$, and the optimizing problem we described by equation (2.10)-(2.14) is modified as follows:

$$max \quad \tau g(I_N)K^{\alpha}(1-\alpha)L^{1-\alpha} + \beta\tau g(I(N))K^{\alpha}(1-\alpha)L^{1-\alpha} + \beta A \qquad (2.20)$$

$$s.t. \quad (1-\tau)g(I_N)K^\alpha(1-\alpha)L^{-\alpha} + \beta(1-\tau)g(I(N))K^\alpha(1-\alpha)L^{-\alpha}$$
$$= \beta(1-F(M(N))\left\{\frac{A}{L} + g(0)K^\alpha(1-\alpha)L^{-\alpha}\right\} \quad (2.21)$$

$$I(N) + M(N) = A \quad (2.22)$$
$$I(N) \geq 0 \quad (2.23)$$
$$M(N) \geq 0 \quad (2.24)$$

We then rearrange condition (2.21), and we have:

$$\frac{(1-\tau)g(I_N)K^\alpha(1-\alpha)L^{1-\alpha}}{1-F(M(N))} + \frac{\beta(1-\tau)g(I(N))K^\alpha(1-\alpha)L^{1-\alpha}}{1-F(M(N))}$$
$$- \beta g(0)K^\alpha(1-\alpha)L^{1-\alpha} = \beta A \quad (2.25)$$

We then substitute (2.25) into the objective function (2.20), and we have:

$$V^\varepsilon(\pi(N)=0) = \frac{1-\tau F(M(N))}{1-F(M(N))}g(I_N)K^\alpha(1-\alpha)L^{1-\alpha}$$
$$+ \beta\frac{1-\tau F(M(N))}{1-F(M(N))}g(I(N))K^\alpha(1-\alpha)L^{1-\alpha} - \beta g(0)K^\alpha(1-\alpha)L^{1-\alpha} \quad (2.26)$$

We then substitute $I(N)$ by $I(N) = A - M(N)$, the elite chooses the optimal $M(N)$ which solves the following first order condition:

$$\frac{\partial V^\varepsilon(M(N))}{\partial M(N)} = [g(I_N) + \beta g(A-M(N))]\frac{(1-\tau)F'(M(N))}{(1-F(M(N)))^2}K^\alpha(1-\alpha)L^{1-\alpha}$$
$$- \beta\frac{1-\tau F(M(N))}{1-F(M(N))}g'(A-M(N))K^\alpha(1-\alpha)L^{1-\alpha} = 0 \quad (2.27)$$

To capture how resource rents affect the optimal military spending, we then differentiate (2.27) by A, and we have:

$$\frac{\partial V^{\varepsilon'}(M(N))}{\partial A} = \frac{(1-\tau)F'(M(N))}{(1-F(M(N)))^2}\beta g'(A-M(N))K^\alpha(1-\alpha)L^{1-\alpha}$$
$$- \frac{1-\tau F(M(N))}{1-F(M(N))}\beta g''(A-M(N))K^\alpha(1-\alpha)L^{1-\alpha} \quad (2.28)$$

Since $g(.)$ is a strict concave function, $g''(A-M(N)) < 0$, and $\frac{\partial V^{\varepsilon'}(M(N))}{\partial A}$ is positive. That is to say, any increases in the value of A will increase the slope of

$V^{\varepsilon}M(N))$ at each possible $M(N)$. We then have the following comparative static result:

$$\frac{\partial M^*(\delta)}{\partial A} > 0$$

where $M^*(\delta)$ denotes, given the set of parameters, δ, the optimal military spending in nondemocracy, $M(N)$, which solves the first order condition, $V^{\varepsilon'}(M(N)) = 0$. The optimal military spending, $M^*(N)$, increases in A. That is to say, if the nondemocratic state receives an increased resource rent, the elite would prefer to increase its expenditure towards the military.

To capture how resource rents affect optimal public investment, we then substitute $M(N)$ by $M(N) = A - I(N)$ into equation (2.26). The elite chooses the optimal $I(N)$ which solves the following first order condition:

$$\frac{\partial V^{\varepsilon}(I(N))}{\partial I(N)} = [g(I_N) + \beta g(I(N))] \frac{(\tau - 1)F'(A - I(N))}{[1 - F(A - I(N))]^2} K^{\alpha}(1-\alpha)L^{1-\alpha}$$
$$\beta \frac{1 - \tau F(A - I(N))}{1 - F(A - I(N))} g'(I(N))K^{\alpha}(1-\alpha)L^{1-\alpha} = 0 \quad (2.29)$$

The above equation is rearranged as follows:

$$\frac{\beta g'(I^*(N))}{g(I_N) + \beta g(I^*(N))} = \frac{(1-\tau)F'(A - I^*(N))}{(1 - F(A - I^*(N)))(1 - \tau F(A - I^*(N)))} \quad (2.30)$$

where $I^*(N)$ denotes the optimal $I(N)$ that solves condition (2.29). The comparative static between the optimal $I(N)$ and A depends on the value of $\frac{\partial RHS}{\partial A}$. If the right hand side of (2.30) increases in A, the value of $V^{\varepsilon'}(I(N))$ decreases in A. Under this circumstance, any increase in the resource rents lowers the optimal public investment, $I^*(N)$. However, if the right hand side of (2.30) decreases in A, any increase in the value of resource rent raises the optimal public investment, $I^*(N)$. The value of $\frac{\partial RHS}{\partial A}$ depends on the value of the following term:

$$F''(A - I^*(N)) + \frac{(F'(A - I^*(N)))^2}{1 - F(A - I^*(N))} + \frac{\tau(F'(A - I^*(N)))^2}{1 - \tau F(A - I^*(N))} \quad (2.31)$$

If the value of (2.31) is positive, $\frac{\partial RHS}{\partial A} > 0$. Otherwise, $\frac{\partial RHS}{\partial A} \leq 0$. Here, $F''(A - I^*(N)) > 0$ is a sufficient condition under which $\frac{\partial RHS}{\partial A} > 0$, and we have the following comparative static:

$$\frac{\partial I^*(\delta)}{\partial A} < 0$$

where $I^*(\delta)$ denotes, given the set of parameter, δ, the optimal public expenditure, $I^*(N)$, this solves the first order condition, $V^{\varepsilon'}(I(N)) = 0$. According to the Assumption 6, the density of $F(.)$, $F'(.)$, is single peaked. Here, if $A - I^*(\delta) \leq M^*$, $F''(A - I^*(\delta)) > 0$ holds, and the optimal public investment decreases in A. However, if $F''(A - I(N))$ is negative, the comparative static between $I^*(A)$ and A is ambiguous. Given the set of parameter, δ, if the value of (2.31) is negative, we have the following comparative static:

$$\frac{\partial I^*(A)}{\partial A} > 0$$

Under this circumstance, the value of $F''(A - I^*(\delta))$ is negative and extremely small. We then summarize the comparative static results as follows:

Proposition 3. *(Comparative Statics)*

- *In our described model, we have the following comparative static result between A and $M^*(N)$:*

  $$\frac{\partial M^*(\delta)}{\partial A} > 0$$

  Given the set of parameter, δ, the optimal military expenditure, $M^*(N)$, always increases in A.

- *The comparative static result between A and $I^*(N)$ is ambiguous.*

  - *If $A - I^*(\delta) \leq M^*$, the comparative static between A and $I^*(\delta)$ is displayed as follows:*

    $$\frac{\partial I^*(\delta)}{\partial A} < 0$$

    Given the set of parameter, δ, the optimal public expenditure, $I^*(N)$, decreases in A.

  - *Given the set of parameter, δ, if the value of term (2.31) is positive, the optimal public expenditure, $I^*(N)$, decreases in A. Otherwise, $I^*(N)$ increases in A.*

The first comparative static result is intuitive. Since $g(.)$ is a strict concave function, any increases in the public expenditure increase the national wage but in a diminishing return. The marginal effectiveness of the public expenditure in consolidating the control of the elite over the state decreases in the public expenditure. That is to say, the marginal opportunity cost of investing in the military decreases in the military spending. Therefore, if the elite is provided with resource windfalls, it would prefer to enlarge its military spending.

From the above proposition, we have seen the fact that the comparative static result between A and $I^*(N)$ depends on the shape of $F(.)$ distribution. If $F(.)$ is a strict convex function, the optimal public expenditure decreases in the resource rents. Under this circumstance, the effectiveness of military spending is high, and the opportunity cost of investing in the public sector increases in A, and the elites would prefer to cut down its public expenditure and increase the military spending if they have received a resource windfall. However, if $F(.)$ is a strict concave function, military spending becomes less effective, and the elite may increase its public expenditure if it receives an abundance resource windfall.

2.4 Generalizations

The model in the previous section yields the result that, in nondemocracy, if the elite receives increased resource windfalls, it would prefer to increase its spending on the military, whilst the increased resource rents affect the public expenditure in an ambiguous way. However, our baseline model only consists of two periods. When determining whether to revolt, the majority compares the expected payoff of democracy with the anticipated consequences of staying in nondemocracy, and in a two-period game, they will not consider the continuation value of democracy and that of nondemocracy. In this section, we will relax the time constraint, and examine a dynamic model.

In the previous section, we made the certain assumption towards the F(.) distribution; in the following section we will relax this assumption and investigate how the comparative statics between the resource rents and the public expenditure are changed.

We now consider an infinite-horizon society in discrete time. Each agent's risk-neutral preference with discount factor $\beta \in (0,1)$ changes to:

$$u^i = \sum_{j=0}^{\infty} \beta^j c^i_{t+j} \tag{2.32}$$

where c^i_{t+j} denotes the consumption of agent i at time $t+j$ in terms of the final good. At the end of each period t, agent i consumes all his or her income. With Assumption 1-3 unchanged, the production function is given by equation (2.2), the wage level of each worker is written by equation (2.3).

In the dynamic model, each agent within the society considers the continuation value of the two possible political regimes. For each worker, the expected payoff of attempting a revolution is changed as follows:

$$\begin{aligned} V^{\omega}(\pi(N)=1) = {} & \beta\left[1-F(M_t)\right] \\ & \left[g(0)K^{\alpha}(1-\alpha)L^{-\alpha} + \left(\frac{A-I_D}{L(1-\beta)}\right) + \frac{\beta g(I_D)K^{\alpha}(1-\alpha)L^{-\alpha}}{1-\beta}\right] \end{aligned}$$

As we have mentioned above, under democracy, government sets the public expenditure to equal I_D in each period. Since democracy is an absorbing state, from time $t+1$ onward, working agent shares the collective government revenue equally, $\frac{A-I_D}{L}$, and from time $t+2$ onward, the national wage level equals $g(I_D)K^{\alpha}(1-\alpha)L^{-\alpha}$. The received income in each period is discounted at β.

Similarly, workers also consider the future payoff in nondemocracy, and each worker's expected payoff for staying in nondemocracy is given by:

$$\begin{aligned} V^{\omega}(\pi(N)=0) = {} & (1-\tau)g(I_N)K^{\alpha}(1-\alpha)L^{-\alpha} + \beta(1-\tau)g(I_t)K^{\alpha}(1-\alpha)L^{-\alpha} \\ & + \frac{\beta^2}{1-\beta}(1-\tau)g(I_N)(1-\alpha)L^{-\alpha} \end{aligned}$$

where $V^{\omega}(\pi(N)=0)$ denotes the valuation of each worker if they choose not to revolt at time t. Since there is no revolution, workers receive their post-tax income at the end of time t, equalling $(1-\tau)g(I_N)K^{\alpha}(1-\alpha)L^{-\alpha}$. In time $t+1$, the national income level equals $(1-\tau)g(I_t)K^{\alpha}(1-\alpha)L^{-\alpha}$ where I_t is the public expenditure in time t. Since the public investment is paid at the beginning of each period, there is no commitment problem. In order to remove the threat of revolution, the elite promises to make the public expenditure which equals to I_t at time t, and this promise is credible. However, because the threat of revolution is transitory, public expenditure in time t does not guarantee future investment in the public sector. Once the threat of revolution has been removed, the majority cannot pose a similar threat in the immediate future, and from time $t+1$ onwards, the elite will revert to its optimal policy where $I_{t+1} = I_N$. So that, from time $t+2$ onward, in each period, each worker will receive $(1-\tau)g(I_N)(1-\alpha)L^{-\alpha}$.

The condition under which the revolution will not take place is changed as follows:

$$\begin{aligned}(1-\tau)g(I_N)K^{\alpha}(1-\alpha)L^{-\alpha} + \beta(1-\tau)g(I_t)K^{\alpha}(1-\alpha)L^{-\alpha} \\ + \frac{\beta^2}{1-\beta}(1-\tau)g(I_N)K^{\alpha}(1-\alpha)L^{-\alpha} \geq \beta\left[1-F(M_t)\right] \\ \left[g(0)K^{\alpha}(1-\alpha)L^{-\alpha} + \left(\frac{A-I_D}{L(1-\beta)}\right) + \frac{\beta g(I_D)K^{\alpha}(1-\alpha)L^{-\alpha}}{1-\beta}\right] \end{aligned} \quad (2.33)$$

The condition under which there is no threat of revolution is written as follows:

$$\begin{aligned}&\frac{(1-\tau)g(I_N)K^{\alpha}(1-\alpha)L^{-\alpha}}{1-\beta} \\ &\geq \beta\left[1-F(0)\right]\left[g(0)K^{\alpha}(1-\alpha)L^{-\alpha} + \left(\frac{A-I_D}{L(1-\beta)}\right) + \frac{\beta g(I_D)K^{\alpha}(1-\alpha)L^{-\alpha}}{1-\beta}\right]\end{aligned} \quad (2.34)$$

If condition (2.34) is satisfied, the elite controls the state at time t, chooses its optimal public policy where $I_t = I_N$, and determines the state variable for the next period, $s_{t+1} = N$. Under this circumstance, the national income level will not be improved, and each worker will receive $(1-\tau)g(I_N)K^{\alpha}(1-\alpha)L^{-\alpha}$ in every period.

As we have discussed in the previous section, the optimal public expenditure of the majority is greater than that of the elite, $I_D > I_N$, so that $g(I_D) > g(I_N)$.

That is to say, in a dynamic model, from time $t+2$ onward, the received income of each worker in democracy is greater then that in nondemocracy where $\frac{A-I_D}{L} + g(I_D)K^{\alpha}(1-\alpha)L^{-\alpha} > (1-\tau)g(I_N)K^{\alpha}(1-\alpha)L^{-\alpha}$. Compared with the two-period model, in a dynamic model, to remove the threat of revolution, the elite should choose the set of policy (M_t, I_t) that improves majority's payoff under nondemocracy. That is to say, given fixed resource rents, it is relatively more difficult for the elite to remove the threat of revolution in an infinite-horizon society.

Being similar to the two-period model, throughout this section we focus on the Markov Perfect Equilibrium (MPE) we have characterized above which is summarized as investment functions $M(N)$ and $I(N)$. We then consider the following optimizing problem. At time t, the state starts with nondemocracy, $s_t = N$, given the set of parameter, δ, the elite determines the military spending, $M(N)$, and the public expenditure, $I(N)$, to maximize its expected payoff for controlling the state. The optimizing problem is given by:

$$\begin{aligned} max \quad & A - M(N) - I(N) + \tau g(I_N)K^{\alpha}(1-\alpha)L^{1-\alpha} + \beta\tau g(I(N))K^{\alpha}(1-\alpha)L^{1-\alpha} \\ & + \frac{\beta^2 \tau g(I_N)K^{\alpha}(1-\alpha)L^{1-\alpha}}{1-\beta} + \frac{\beta(A-I_N)}{1-\beta} \end{aligned} \tag{2.35}$$

$$\begin{aligned} s.t. \quad & (1-\tau)g(I_N)K^{\alpha}(1-\alpha)L^{-\alpha} + \beta(1-\tau)g(I(N))K^{\alpha}(1-\alpha)L^{-\alpha} \\ & + \frac{\beta^2}{1-\beta}(1-\tau)g(I_N)K^{\alpha}(1-\alpha)L^{-\alpha} \geq \beta(1-P(M(N)) \\ & \left\{ \frac{A-I_D}{(1-\beta)L} + g(0)K^{\alpha}(1-\alpha)L^{-\alpha} + \frac{\beta g(I_D)K^{\alpha}(1-\alpha)L^{-\alpha}}{1-\beta} \right\} \end{aligned} \tag{2.36}$$

$$I(N) + M(N) \leq A \tag{2.37}$$

$$I(N) \geq 0 \tag{2.38}$$

$$M(N) \geq 0 \tag{2.39}$$

At time t, the elite keeps the residual quantity of total resource rents, $A - M(N) - I(N)$, and their collected tax revenue equals $\tau g(I_N)K^{\alpha}(1-\alpha)L^{1-\alpha}$ which is determined by the public expenditure in the previous period, $I_{t-1} = I_N$. As we have mentioned above, since the threat of revolution is transitory, once the elite removes the threat of revolution successfully at time t, the majority cannot impose a similar threat immediately thereafter. Considering this, from time $t+1$ onwards, the elite will revert to its optimal policy where $M_{t+1}=0$, and $I_{t+1}=I_N$. Since period $t+1$, in each period, the receives the residual quantity of resource rents which equals $A-I_N$. At time $t+1$, the collected tax revenue is determined by the public expenditure at time t, $I_t = I(N)$, which equals $\beta\tau g(I(N))K^{\alpha}(1-\alpha)L^{1-\alpha}$. From time $t+2$ onwards, the elite expects to receive $\tau g(I_N)K^{\alpha}(1-\alpha)L^{1-\alpha}$ tax revenue in each period. The objective function describes the expectation of the elite in controlling the state. As previously, equation (2.36) is the income compatibility constraint that the incentive for each worker to stay in nondemocracy should equal or exceed the incentive for each worker to revolt. Equation (2.37) is the budget constraint for elites.

We then define the equilibrium of the political regime as follows:

Proposition 4. *At the beginning of time t, the state starts with nondemocracy, given the resource rent, A, and the set of parameter, δ, we have the following three possibilities:*

- *Case 1: If condition (2.34) is satisfied, there is no threat of revolution, the MPE is $(0, I_N)$, $\pi(N)=0$, and $s_{t+1}=N$. Nondemocracy is a consolidated political regime.*

- *Case 2: If there exists a plausible $(M(N), I(N))$ that solves the optimizing problem of the elite which is described by Eq.(2.35)-Eq.(2.39), there is no revolution, and the elite chooses the state variable for the following period, $\pi(N)=0$, and $s_{t+1}=N$. Nondemocracy is a semi-consolidated political regime.*

- *Case 3: If there is no plausible $(M(N), I(N))$ that solves the optimizing problem of the elite, $\pi(N)=1$, revolution takes place, the majority ultimately wins and transits the political regime to democracy, $s_{t+1}=D$. Nondemocracy is an unconsolidated political regime*

To capture how factors A, K and L, affect the equilibrium political regime in the dynamic model, we draw the following graph. Given the set of parameter, $\{\tau, L, \beta, \alpha\}$,

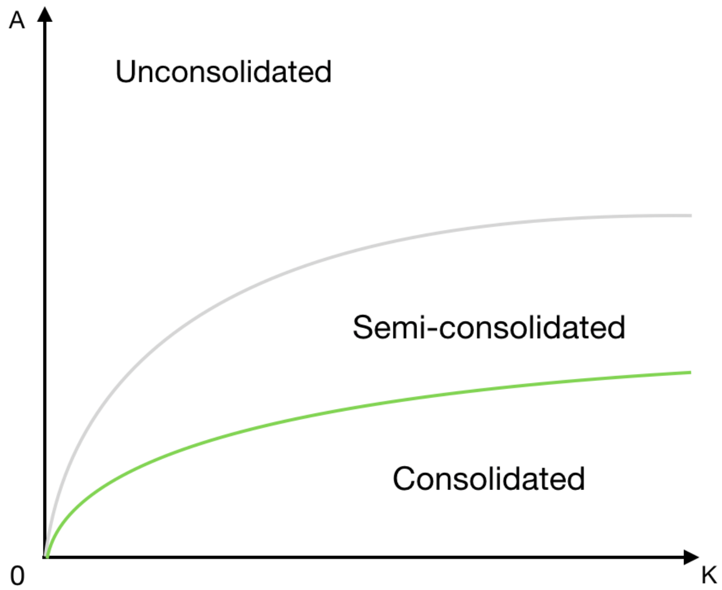


Figure 2.2: Equilibrium political regime

the green line consists of all possible sets, (K, A), that satisfy the following condition:

$$\frac{(1-\tau)g(I_N)K^\alpha(1-\alpha)L^{1-\alpha}}{\beta(1-F(0))} - (1-\beta)g(0)K^\alpha(1-\alpha)L^{1-\alpha} - \beta g(I_D)K^\alpha(1-\alpha)L^{1-\alpha} + I_D = A \quad (2.40)$$

Region below the green line includes all possible sets, (K, A), that make the state a fully consolidated nondemocracy. Being different from the two-period model, the left-hand side of (2.40) may not increase in K and L. We cannot fully determine the shape of green line, however, we can still have that region below the green line in Figure 2.2 is smaller than that in Figure 2.1. That is to say, in the dynamic model, increases in the value of K may still contribute to consolidating the nondemocracy, but in a comparatively mild way, and increases in the resource rents exacerbate the conflict between the majority and the elite in a more drastic way. Given fixed resource rents, it is relatively more difficult for the elite to consolidate its control in an infinite-horizon society.

Proof. Given the set of parameter, $\{\tau, L, \beta, \alpha\}$, the green line in Figure 2.1 consists all possible sets of (K, A) that satisfy condition (2.19):

$$\frac{[(1-\tau)+\beta(1-\tau)]g(I_N)K^{\alpha}(1-\alpha)L^{1-\alpha}}{\beta(1-F(0))} - g(0)K^{\alpha}(1-\alpha)L^{1-\alpha} = A$$

and the green line in Figure 2.2 consists of all possible sets of (K, A) satisfying condition (2.40):

$$\frac{(1-\tau)g(I_N)K^{\alpha}(1-\alpha)L^{1-\alpha}}{\beta(1-F(0))} - (1-\beta)g(0)K^{\alpha}(1-\alpha)L^{1-\alpha}$$
$$- \beta g(I_D)K^{\alpha}(1-\alpha)L^{1-\alpha} + I_D = A$$

The left-hand side of (2.19) minus the left-hand side of (2.40) equates:

$$\frac{\beta(1-\tau)g(I_N)K^{\alpha}(1-\alpha)L^{1-\alpha}}{\beta(1-F(0))} - \beta g(0)K^{\alpha}(1-\alpha)L^{1-\alpha} + \beta g(I_D)K^{\alpha}(1-\alpha)L^{1-\alpha} - I_D$$

Since I_D refers to the optimal public expenditure of the majority that solves the optimizing problem (2.6) and $I_D \geq 0$, we then have:

$$\beta g(I_D)K^{\alpha}(1-\alpha)L^{1-\alpha} - I_D \geq \beta g(0)K^{\alpha}(1-\alpha)L^{1-\alpha}$$

Therefore, we have the left hand side of (2.19) is greater than the left hand side of (2.40). That is to say, given all possible sets of parameter, $\{\tau, L, \beta, \alpha\}$, the green line in Figure 2.1 is always above the green line in Figure 2.2.

□

As before, given the set of parameter $\{\tau, L, \beta, \alpha, M^*, I^*\}$, the grey line in Figure 2.2 consists of all possible sets, (K, A), that satisfy the following condition:

$$\frac{G_1K^{\alpha}(1-\alpha)L^{1-\alpha}}{\beta(1-P(M^*))} + \frac{(1-\tau)g(A-M^*)K^{\alpha}(1-\alpha)L^{1-\alpha}}{1-P(M^*))} - g(0)K^{\alpha}(1-\alpha)L^{1-\alpha}$$
$$- \frac{\beta g(I_D)K^{\alpha}(1-\alpha)L^{1-\alpha}}{1-\beta} + \frac{I_D}{1-\beta} = \frac{A}{1-\beta} \quad (2.41)$$

where $G_1 = (1-\tau)g(I_N)(1+\frac{\beta^2}{1-\beta})$. The Region above the grey line includes all possible sets, (K, A) or (L, A), which make the nondemocratic state unconsolidated. As before, we cannot fully determine the shape of grey line, we can still have that the grey line in Figure 2.2 is always below the grey line in Figure 2.1. That is to say, the region above the grey line in Figure 2.2 is greater than that in Figure 2.1 Countries with a lower K or L or a higher A are more likely to stay in an unconsolidated nondemocracy.

Overall, compared with Figure 2.1, the 'Unconsolidated' region in Figure 2.2 is greater in size, and the 'Consolidated' region is smaller. That is to say, in the dynamic model, the majority is more sensitive towards increases in resource rents, and more likely to attempt a revolution if the state has been endowed with higher resource rents. Since the majority weighs up the continuation value of the political regime, increases in the resource rents improve their incentive towards democracy in a more drastic way. However, for each worker, from time $t + 2$ onward, the incentive towards nondemocracy will not increase with resource rents. Therefore, increases in resource rents can hardly contribute to improve workers' incentives towards nondemocracy, and remove the threat of revolution in a multiple-period model. Since increases in capitals improve the national income level in both democracy and nondemocracy, in the dynamic model, capital influences the equilibrium of the political regime in a mild way. By comparing the two figures, we summarize the following proposition:

Proposition 5.

- *In the dynamic model, increases in the resource rents exacerbate the conflict between the elite and the majority in a more drastic way.*
- *In the dynamic model, capital mildly affects the equilibrium of the political regime.*
- *In the dynamic model, it is more difficult for the elites to consolidate its control over the state.*

By analysing the dynamic model, we can establish similar comparative static results, showing if the elite is provided with resource windfalls how it would prefer to distribute its investment towards the military and the public sector. In addition, we also find that if we partly relax the assumption towards F(.) function, the comparative static results will not be changed. We modified Assumption 6 as follows:

Assumption 7. *F is defined over* $(\underline{M}, \infty)$ *where* $\underline{M} \geq 0$*, and F is strictly increasing which is twice continuously differentiable.*

Given the modification, F is generally still an upward sloping function, however, we relax the assumption towards its actual shape. The comparative static results are stated in the following proposition, and we also provide the proof afterwards:

Proposition 6. *(Comparative Statics for the Dynamic Model)*

- *Given the set of parameter,* δ*, the optimal military expenditure,* $M^*(N)$*, increases in A:*
$$\frac{\partial M^*(\delta)}{\partial A} > 0$$

- *Given the set of parameter,* δ*, the comparative static between the optimal public expenditure,* $I^*(N)$*, and the resource rents, A, is ambiguous, depending on the shape of* $F(.)$*.*

Proof. Firstly, in an infinite-horizon society, the optimizing problem of the elite is displayed as follows:

$$\begin{aligned} max \quad & \tau g(I_N)K^\alpha(1-\alpha)L^{1-\alpha} + \beta\tau g(I(N))K^\alpha(1-\alpha)L^{1-\alpha} \\ & + \frac{\beta^2\tau g(I_N)K^\alpha(1-\alpha)L^{1-\alpha}}{1-\beta} + \frac{\beta(A-I_N)}{1-\beta} \end{aligned} \tag{2.42}$$

$$\begin{aligned} s.t. \quad & (1-\tau)g(I_N)K^\alpha(1-\alpha)L^{-\alpha} + \beta(1-\tau)g(I(N))K^\alpha(1-\alpha)L^{-\alpha} \\ & + \frac{\beta^2}{1-\beta}(1-\tau)g(I_N)K^\alpha(1-\alpha)L^{-\alpha} = \beta(1-F(M(N)) \\ & \left\{\frac{A-I_D}{(1-\beta)L} + g(0)K^\alpha(1-\alpha)L^{-\alpha} + \frac{\beta g(I_D)K^\alpha(1-\alpha)L^{-\alpha}}{1-\beta}\right\} \end{aligned} \tag{2.43}$$

$$I(N) + M(N) = A \tag{2.44}$$

$$I(N) \geq 0 \tag{2.45}$$

$$M(N) \geq 0 \tag{2.46}$$

We consider the circumstance that the elite could simply remove the threat of revolution by deploying the collected resource rents. We then further simplify the optimizing problem by substituting equation (2.43) into the objective function, and we have:

$$\begin{aligned} max \quad V^{\varepsilon} &= \tau g(I_N)K^{\alpha}(1-\alpha)L^{1-\alpha} + \frac{(1-\tau)g(I_N)K^{\alpha}(1-\alpha)L^{1-\alpha}}{1-F(M(N))} \\ &+ \beta\frac{1-\tau F(M(N))}{1-F(M(N))}g(I(N))K^{\alpha}(1-\alpha)L^{1-\alpha} + \frac{\beta^2\tau g(I_N)K^{\alpha}(1-\alpha)L^{1-\alpha}}{1-\beta} \\ &+ \frac{\beta^2(1-\tau)g(I_N)K^{\alpha}(1-\alpha)L^{1-\alpha}}{(1-F(M(N)))(1-\beta)} - \beta G_2 L - \frac{\beta I_N}{1-\beta} \end{aligned} \tag{2.47}$$

where $G_2 = g(0)K^{\alpha}(1-\alpha)L^{-\alpha} + \frac{\beta}{1-\beta}g(I_D)K^{\alpha}(1-\alpha)L^{-\alpha} - \frac{I_D}{(1-\beta)L}$, and V^{ε} is the expected payoff of the elite. We then substitute $I(N)$ by $I(N) = A - M(N)$, the elite chooses the optimal $M(N)$ that solves the following first order condition:

$$\begin{aligned} \frac{\partial V^{\varepsilon}(M(N))}{\partial M(N)} &= \frac{F'(M(N))}{(1-F(M(N)))^2}(1-\tau)g(I_N)K^{\alpha}(1-\alpha)L^{1-\alpha}(1+\frac{\beta^2}{1-\beta}) \\ &+ \frac{(1-\tau)F'(M(N))}{(1-F(M(N)))^2}\beta g(A-M(N))K^{\alpha}(1-\alpha)L^{1-\alpha} \\ &- \frac{1-\tau F(M(N))}{1-F(M(N))}\beta g'(A-M(N))K^{\alpha}(1-\alpha)L^{1-\alpha} = 0 \end{aligned} \tag{2.48}$$

To capture how resource rents affect the optimal military spending, we then differentiate (2.48) by A, and we have:

$$\begin{aligned} \frac{\partial V^{\varepsilon'}(M(N))}{\partial A} &= \frac{(1-\tau)F'(M(N))}{(1-F(M(N)))^2}\beta g'(A-M(N))K^{\alpha}(1-\alpha)L^{1-\alpha} \\ &- \frac{1-\tau F(M(N))}{1-F(M(N))}\beta g''(A-M(N))K^{\alpha}(1-\alpha)L^{1-\alpha} \end{aligned} \tag{2.49}$$

Since $g(.)$ is a strict concave function, for any non-negative $M(N)$, $g''(A - M(N)) < 0$, and $\frac{\partial V^{\varepsilon'}(M(N))}{\partial A}$ is positive. We then have the following comparative static result:

$$\frac{\partial M^*(\delta)}{\partial A} > 0$$

The optimal military spending, $M^*(N)$, increases in A. If the nondemocratic state has been endowed with a resource windfall, the elite would prefer to increase its expenditure on the military.

To capture how resource rents affect the optimal public investment, we then substitute $M(N)$ by $M(N) = A - I(N)$ into the objective function. The elite chooses the optimal $I(N)$ which solves following first order condition:

$$\frac{\partial V^{\varepsilon}(I(N))}{\partial I(N)} = \frac{-F'(A-I(N))}{(1-F(A-I(N)))^2}(1-\tau)g(I_N)K^{\alpha}(1-\alpha)L^{1-\alpha}(1+\frac{\beta^2}{1-\beta}) + \frac{(\tau-1)F'(A-I(N))}{(1-F(A-I(N)))^2}\beta g(I(N))K^{\alpha}(1-\alpha)L^{1-\alpha} + \frac{1-\tau F(A-I(N))}{1-F(A-I(N))}\beta g'(I(N))K^{\alpha}(1-\alpha)L^{1-\alpha} = 0 \quad (2.50)$$

The above equation could be rearranged as follows:

$$\frac{\beta g'(I(N))}{\left[1+\beta+\left(\frac{\beta^2}{1-\beta}\right)\right]g(I_N)} = \frac{(1-\tau)F'(A-I(N))}{(1-F(A-I(N)))(1-\tau F(A-I(N)))} \quad (2.51)$$

The comparative static between the optimal $I(N)$ and A depends on the value of the following term:

$$F''(A-I(N)) + \frac{(F'(A-I(N)))^2}{1-F(A-I(N))} + \frac{\tau(F'(A-I(N)))^2}{1-\tau F(A-I(N))} \quad (2.52)$$

If the value of (48) is positive, $\frac{\partial RHS}{\partial A} > 0$, and $\frac{\partial I^*(\delta)}{\partial A} < 0$. Otherwise, $\frac{\partial RHS}{\partial A} \leq 0$, and $\frac{\partial I^*(\delta)}{\partial A} > 0$. Here, $F"(A-I(N)) > 0$ is a sufficient condition under which $\frac{\partial RHS}{\partial A} > 0$, and we have the following comparative static:

$$\frac{\partial I^*(A)}{\partial A} < 0$$

To summarize, the comparative static between $I^*(N)$ and A depends on the shape of F distribution. □

2.5 Empirical Evidence

Our theoretical analysis in the previous sections focused on the question of how resource windfalls affect military spending and public expenditure in nondemocratic societies. Our model has two headline results that are summarized in Table 2.1. Firstly, in nondemocratic countries, resource windfalls boost military spending. However, how resource windfalls affect public expenditure depends on the effectiveness of military spending, and we cannot capture a clear relationship between the two stated variables. In the following section, we provide some empirical evidence in support for our theory. We also examine the effect of resource windfalls in democratic societies, showing that different political regimes exhibit different relationships between military spending and public expenditure, and resource endowments.

Table 2.1: Predictions based on the results of the theorems and example

Political Regime	**Natural Resources**	**Military Spending**	**Public Expenditure**
Nondemocracy	Increases	Increases	Ambiguous
Democracy	Increases	-	-

2.5.1 Data and Descriptive Statistics

To empirically test how resource windfalls affect government expenditure, we have collected data pertaining to 159 countries for the period 1960-2016.

Table 2.2 provides information on all variables used and the data source. In this chapter, we focus on how resource windfalls affect two types of government expenditure: military spending and public investment. For this reason, we have two dependent variables. We use military spending as a percentage of government expenditure to measure the fluctuation of military spending. This avoids biases generated by the development of the economy, and the fluctuation of government spending. We use the data from the SIPRI (Stockholm International Peace Research Institute) database (Institute, 2015).

Figure 2.3 is a heat map that provides an immediate summary of how military spending varies across countries. For each country, we take the average of military spending as a percentage of government spending for the period 1960-2016. In Figure 2.3, countries that are coloured by dark green have limited spending towards the military, and countries that are coloured by red have the highest spending towards the military. If a country is coloured by white, it means we do not have

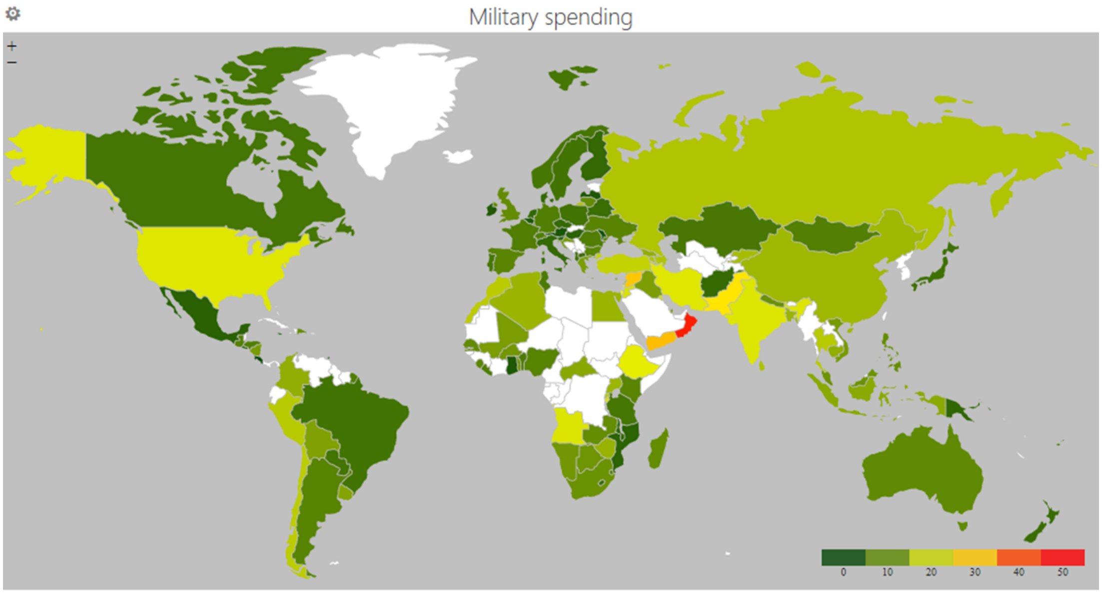


Figure 2.3: Country-specific average military spending as a percentage of government spending

available military spending data for this country. We have seen from Figure 2.3 that almost all European countries are coloured by the dark green, the average military spending of which is lower than 10$\%$. In Asia, most countries are covered by light green, the average military spending is around 15%. Omen has the highest average military spending which takes over 42% of its total government spending. Except Omen, the average military spending in Yemen, Syria, and Pakistan takes above 25% of the total government spending. We have also seen from Figure 2.3 that countries include India, Iran, Ethiopia, Angola and US all have considerable spending in the military.

We use Gross fixed capital formation for public sectors as a percentage of GDP to measure the fluctuation of public expenditure. We collect the data in the following way. From the World Bank national accounts database, we collect the data for the gross fixed capital formation as a percentage of GDP and the data for the gross fixed capital formation for the private sector as a percentage of GDP. We then approximate the gross fixed capital formation for the public sector by subtracting the private sector formation as a percentage of GDP from the total fixed capital formation as a percentage of GDP. As before, Figure 2.4 is a heat map which summarizes how the public investment varies across countries. We take the average of

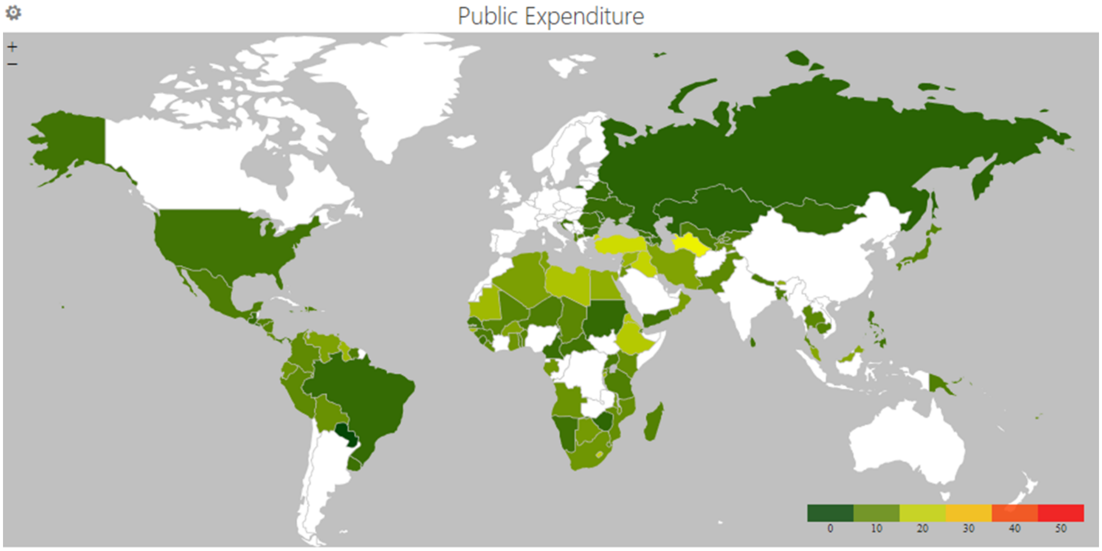


Figure 2.4: Country-specific average public expenditure as a percentage of GDP

gross capital formation for the public sector as a percentage of GDP for the period 1960-2016 for each country. From Figure 2.4, we have seen that almost all countries are covered by green, the public investment is not volatile across countries. Except Turkmenistan (its average investment in the public sector takes 21% of the total GDP), the average investment in the public sector in all countries is lower than 20%.

The focus of this chapter is to examine how resource windfalls affect the distribution of government spending, and we use natural resource rents as a percentage of GDP to measure the fluctuation of natural resources. Our data comes from the World Bank database, and their estimates based on sources and methods described in "The Changing Wealth of Nations 2018: Building a Sustainable Future" (Lange et al., 2018). In their estimation, total natural resources rents are the sum of oil rents, natural gas rents, coal rents (hard and soft), mineral rents, and forest rents, and their data captures the effects of changes in both quantity and price. As noted, the value of a nation's stock of subsoil assets is calculated as the present value of expected rents that could be obtained over the lifetime of the resource, and the discount factor equals 0.4. Here, in order to avoid biases generated by inflation and deflation, we use natural resource rents as a percentage of GDP rather than

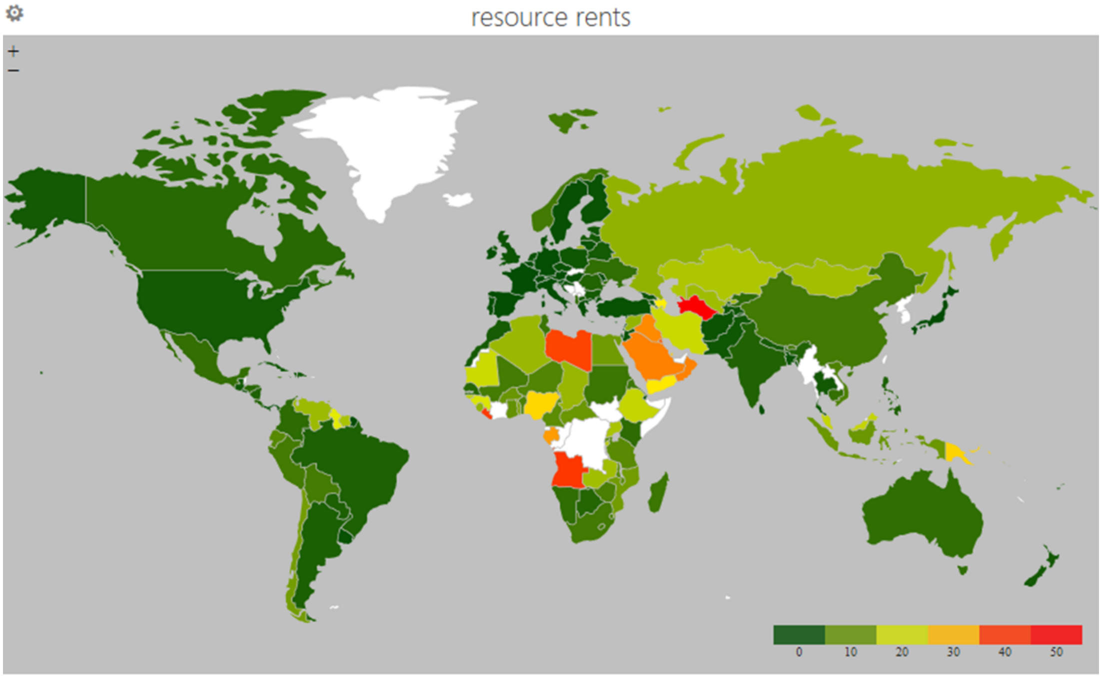


Figure 2.5: Country-specific average natural resource rents as a percentage of GDP

total resource rents. A country-specific GDP is valuated by the constant 2017 US dollars. Similarly, we make a heat map that summarizes the average resources rents for each country. For each country, we take the average of the total resource rents as a percentage of GDP for the period 1960-2016. From Figure 2.5, we have seen that countries with the most natural resources locate in the North Asia, the Middle East and the Africa.

The control variables can be grouped in three categories. Firstly, we consider the threat variable: Civil War. Previous empirical studies have examined the issue of whether the civil war variable affects military spending (Dunne and Perlo-Freeman, 2003; Collier and Rohner, 2008; Albalate et al., 2012). However civil war is defined, it is significantly and positively influence military spending. In this chapter, we follow the approach used in Albalate et al. (2012) to measuring the civil war variable. Albalate et al. (2012) define civil war as a dummy that takes a value of 1 if there is a conflict, with a minimum of 25 battle-related deaths, between the government of a state and at least one opposition group.

Table 2.2: Variable definition, data sources

Variable	Definition	Sources
Military expenditure	Data for military spending as a percentage of government expenditure, from 1960 to 2016	SIPRI (Stockholm International Peace Research Institute)
Military expenditure	Data from military spending as a percentage of GDP, from 1960 to 2016	SIPRI(Stockholm International Peace Research Institute)
Investment	Data for gross fixed capital formation as a percentage of GDP, from 1960 to 2016	World Bank national accounts data
Private Investment	Data for gross fixed capital formation for private sectors as a percentage of GDP, from 1960 to 2016	World Bank national accounts data
Resource rents	Data for total natural resource rents as a percentage of GDP, from 1960 to 2016. Total natural resources rents are the sum of oil rents, natural gas rents, coal rents (hard and soft), mineral rents, and forest rents, and their data captures effects of changes in both quantity and price.	World Bank staff estimates based on sources and methods described in "The Changing Wealth of Nations 2018: Building a Sustainable Future" (Lange et al 2018)
Democracy	Polity2 index that is a composite of the difference between Democracy and Autocracy indicators from Policy IV database, from 1960 to 2016	Polity IV
Civil war	A dummy variable that takes a value of 1 if there is a conflict, with a minimum of 25 battle-related deaths per year between the government of a state and at least one opposition group	PRIO (Peace Research Institute of Olso)
Ln GDP	The logarithm of real GDP at chained PPPs in 2011 US dollars of each country, from 1960 to 2016	Penn World Table version 9.1
Ln population	The logrithm of the total population of each country	World Bank
Working age population	Population ages 15-64 as a percentage of total population	World Bank staff estimates based on age/sex distributions of United Nations Population Division's World Population Prospects
Aged population	Population ages 65 and above as a percentage of total population	World Bank staff estimates based on age/sex distributions of United Nations Population Division's World Population Prospects
Trade	Trade as a percentage of GDP	World Bank national accounts data

The second category includes socioeconomic variables: GDP, Trade, Population, Working Age Population and Aged Population. Collier and Rohner (2008) found that GDP has a significant and positive effect on military expenditure. Other empirical research also sheds light on how national income or GDP per capita affects military spending (Dunne and Perlo-Freeman, 2003; Dunne et al., 2008), and their findings reflect the fact that development of the economy is one of the influencing factors towards military expenditure. Population has been found to have a significant and negative effect and Dunne and Perlo-Freeman (2003) offer two possible mechanisms. Firstly, having a large population in itself offers security, and, secondly, a large population may prioritise civil consumption over security needs. In this chapter, we further specify the population variable by including working age population and aged population within the consideration. The relationship between trade and military expenditure has been examined as positive and significant (Dunne and Perlo-Freeman, 2003).

Finally, in this chapter, we also control for institutional variables. Most related studies consider the measure of democracy when analysing military spending, and the relationship between the two variables is always significant and negative (Dunne and Perlo-Freeman, 2003; Nordhaus et al., 2009). Most studies use the Polity IV data set that measures democracy and autocracy, and provides information for all nation states from 1800 to current day. In this chapter, we import the Polity2 Score from Polity IV, which has been used widely as a measure of the position of political regimes. Polity2 computes the difference between Democracy and Autocracy indicators (from 0 to 10, where the rising standard of that regime receives a higher value) from the Polity IV database which describes the characteristics of both democracy and autocracy. In the following section, we will further discuss how we use the Polity2 Score to divide our sample into two groups.

Table 2.3 provides the summary statistics for the full sample and also for democracies and nondemocracies. As we have already mentioned, in this chapter we use the Polity2 score from the Polity IV database to measure the position of a political regime that displays characteristics of both democracy and autocracy in a single

Table 2.3: Summary statistics for Annual Data (1960-2016)

	All Countries	**Democracy**	**Nondemocracy**
	(1)	**(2)**	**(3)**
Military Expenditure	11.03	6.9	14.36
	(9.24)	(5.6)	(10.19)
Public Investment	7.56	5.96	7.82
	(6.55)	(3.15)	(6.92)
Resource Rents	8.16	2.63	10.33
	(11.49)	(5.85)	(12.4)
Democracy	1.03	9.8	-1.86
	(7.4)	(0.43)	(6.27)
Civil War	0.17	0.08	0.22
	(0.38)	(0.28)	(0.41)
Ln GDP	10.66	11.51	10.30
	(2.04)	(2.15)	(1.88)
Ln Population	15.82	15.67	15.9
	(1.6)	(1.69)	(1.55)
Working Age Population	58.67	62.35	56.8
	(6.96)	(6.05)	(6.64)
Aged Population	6.23	9.6	56.8
	(4.4)	(4.99)	(6.64)
Trade	72.65	77.27	70.7
	(48.15)	(44.16)	(49.61)
Countries	159	116	138
Observations	3069	1785	1704

Numbers in brackets refer to standard deviation

authority. For states with a Polity2 score that is positive but lower than 10, they are classed as relatively democratic authority, some autocratic characteristics are still exhibited. In the theory section, we made the assumption that, under democracy, government policies absolutely reflect the desire of the majority, and the poor could equally share resource rents as well as the collected tax revenue. In our model, democracy has been defined at quite a high level. For this reason, in the empirical study, we define the democratic states as those whose Polity2 scores are 9 or above, meaning that their political regime only holds limited autocratic characteristics or none. For those observations with Polity2 scores below 9, we class these as nondemocracy, there exhibits autocratic characteristics. From Table 2.3, we have seen that the average military spending as a percentage of total government expenditure for the full sample set equals 11.03%. For democracies, the average military spending as a percentage of government spending equals 6.9% which is smaller than that in nondemocracies, 14.36%. Figure 2.3 shows the difference in military spending between the democracy and the non-democracy as well. Except US, countries which the Polity2 scores at 9 or above, like countries in the Western Europe, Australia, and Canada, are coloured dark green, the average military spending of those countries are lower than 10%. However, most nondemocratic countries, like countries in the Middle East, South Asia, South East Asia, Russia and China, are coloured by light green, yellow, orange or red, the average military spending in those countries is greater than 15%. Conversely, average public investment does not show significant difference between the democracy and the non-democracy. From Figure 2.4, we have seen that almost all countries are coloured by green.

Table 2.3 shows that the average resource rents in the non-democracy is almost 5 times higher than that in the democracy. From the heat map for the resource rents (Figure 2.5), we have seen that countries in the North Asia, the Middle East and the Africa are endowed with higher natural resources, and most countries located in those regions are defined as nondemocracy. In addition, we have seen that the standard deviation for the resource rents is greater then the average resource

Table 2.4: Summary statistics for Five-Year Average Data (1960-2016)

	All Countries	**Democracy**	**Nondemocracy**
	(1)	**(2)**	**(3)**
Military Expenditure	11.22	6.89	14.04
	(8.97)	(5.72)	(9.57)
Public Investment	7.61	5.97	7.84
	(6.28)	(2.36)	(6.62)
Resource Rents	8.36	2.71	10.32
	(11.49)	(6.79)	(12.13)
Democracy	0.88	9.79	-1.78
	(7.26)	(0.39)	(6.13)
Civil War	0.17	0.07	0.22
	(0.33)	(0.23)	(0.36)
Ln GDP	10.62	11.55	10.27
	(2.03)	(2.15)	(1.87)
Ln Population	15.8	15.64	15.87
	(1.6)	(1.69)	(1.56)
Working Age Population	58.49	62.57	56.64
	6.88	(5.93)	(6.48)
Aged Population	6.15	9.74	4.51
	(4.3)	(4.81)	(2.8)
Trade	72.17	77.3	70.27
	(46.83)	(42.27)	(48.29)
Countries	159	96	138
Observations	691	273	418

Numbers in brackets refer to standard deviation

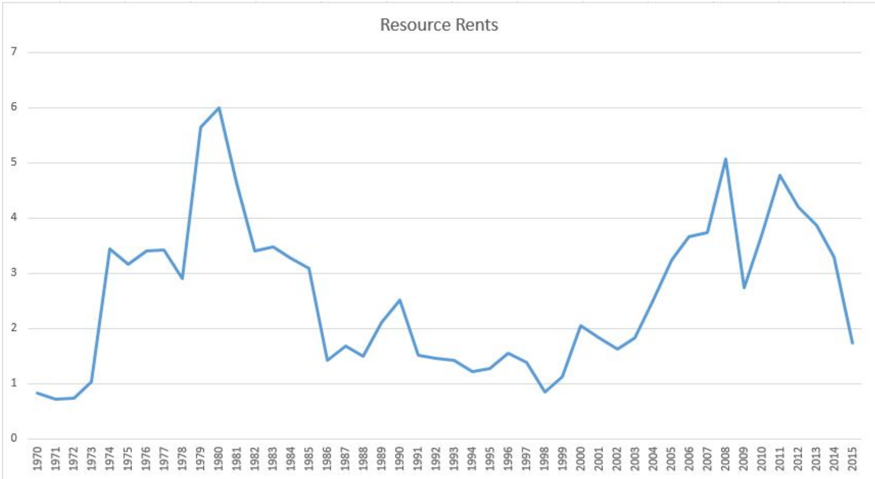
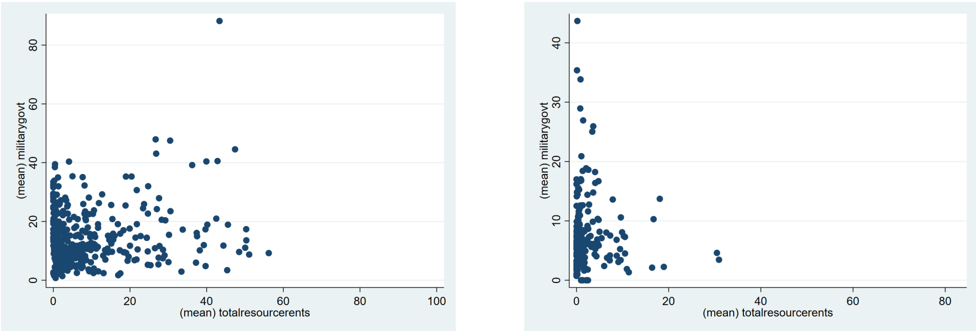


Figure 2.6: Total Natural Resources Rents(% of GDP)

rents.That is to say, resource rents is a volatile dependent variable. Figure 2.6 depicts time-series of cross-country average resource rents. We have seen that during the period from year 1970 to 2016, total resources rents as a percentage of GDP is volatile.

We then construct the five-year average for the data, covering year 1960 to 2016. The principal reason for taking five-year average is that the data are quite slow-moving, especially the institutional variable. Moreover, it improves data accuracy. For countries with a five-year average Polity2 score of 9 or above, these can be considered reasonably stable democracies. Table 2.4 provides the summary statistics of the five-year panels. Compared with Table 2.3, the mean value of each variable is similar, however, the five-year panels have smaller standard deviations. There are some notable features in our sample. The Polity2 score of full sample varies a lot, and the average score is positive but quite low. Despite the wave of democratization in recent decades, autocracy prevails in many countries that display exhibit significant autocratic characteristics. The regime structure varies between countries. Table 2.4 also shows that the variation of resource rents is huge. Compared with nondemocratic states, military expenditure in democratic states is lower on average, and their received resource rents take a smaller share of GDP.

In this chapter, we focus on the issue of how resource windfalls affect government

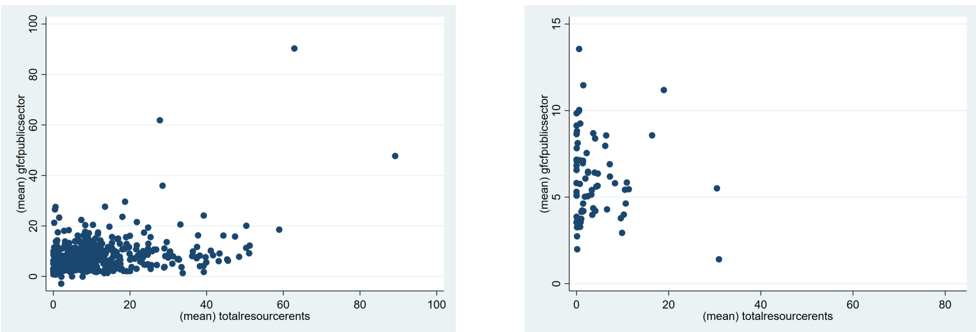


(a) Non-democracy

(b) Democracy

Figure 2.7: Natural resources rents and military expenditure

expenditure in different type of regime. Figure 2.7 plots the total resource rents as a percentage of GDP versus the percentage of military spending in total government expenditure in nondemocracies and democracies, respectively, from 1960 to 2016. Figure 2.7(a) depicts the positive relationship between the two variables. However, in Figure 2.7(b), the plots generally show a negative relationship between the two variables. By comparing the two figures, there is some suggestive evidence that the structure of the political regime affects the relationship between resource rents and military spending.



(a) Non-democracy

(b) Democracy

Figure 2.8: Natural resources rents and public sector spending

Figure 2.8(a) and 2.8(b) plot the total resource rents as a percentage of GDP versus the value of public investment as a percentage of GDP, in nondemocracy and democracy respectively. In Figure 2.8(a), we have seen that there is a positive relationship between the two described variables. However, from Figure 2.8(b), we have seen the fact that we have limited data for the investment of public sectors in democracy, and generally, this figure depicts the negative relationship between the two described variables. In addition, from Figure 2.8(b), we have also seen that we limited observations for the non-democracy. From Figure 2.7(a), we have seen that there exists an outlier which the military expenditure scores above 80. Since this observation exhibits a high score of civil war and we have already controlled this variable in our regression, we keep this outlier in our sample. For the purpose of clear explanation, we exclude the observation that public expenditure scores above 80 in Figure 2.8(a).

2.5.2 Results and Discussion

Consider the following two simple econometric models, which will be the basis of our work:

$$Mexp_{it} = \alpha_0 + \gamma R_{it} + X'_{it}\beta + \mu_t + \eta_{it} \tag{2.53}$$

$$Gexp_{it} = \alpha_0 + \gamma R_{it} + X'_{it}\beta + \mu_t + \eta_{it} \tag{2.54}$$

where $Mexp_{it}$ in equation (2.53) denotes the military expenditure as a percentage of total government spending of country i in period t. The main explanatory variable is R_{it} which refers to the total resource rents as a percentage of GDP of country i in period t, and parameter γ encapsulates the relationship between natural resource rents on military expenditure. Other potential covariates are included in the vector X'_{it}. In addition, μ_t denotes the full set of time effects that capture common time effects in military spending across countries. In our model, we exclude country fixed effects. Since we construct the five-year average panel, the number of observations is reduced, and each country only has limited observations. From the last two rows of Tables 2.5 and 2.6, we have seen that each group only has 5 or even fewer observations. For this reason, observations in one-specific country do not show much variation, and in our five-year average sample, all variation is cross-sectional. In the second model (Eq.(2.54)), we replace the dependent variable with $Gexp_{it}$ which denotes the gross fixed capital formation in public sector as a percentage of GDP of country i in period t, and β captures the relationship between natural resource rents and public investment.

The results for the first regression (Eq.(2.53)), where the independent variable is military spending are displayed in Table 2.5. In the first column, we use the full sample data that contains data for 127 countries. The total resource rents as a percentage of government expenditure is significant which indicates the positive relationship between military expenditure and resource rents. In full sample, if resource rents increases by one standard unit, military spending as a percentage of government spending will increase by 0.13. Democracy is also significant which illustrates the well-documented negative relationship between democracy and military expenditure. In addition, regarding our threat variable, civil war, we also find a highly statistically significant coefficient with positive sign, as expected from the literature. In the second column, the sample contains data for countries which a polity2 score of 9 or above. 51 democratic states are included, and the relationship between resource rents and military expenditure disappears. This indicates that in democratic states, resource windfalls will not significantly affect government expenditure in military. In the third column, the sample contains data for all nondemocratic countries with a polity2 score smaller than 9. The total resource rents appear to be statistically significant, which indicates the positive relationship between resource rents and military spending as predicted in the theory. In non-democracies, if resource rents as a percentage of GDP increases by one unit, military spending as a percentage of government spending will increases by 0.15 which is higher than 0.13. It implies that, given resource windfalls, only states that exhibit autocratic characteristics will enlarge their military spending.

The time dummies also offers interesting results. The positive coefficient shows that average military spending was at its lowest in 2010-2015 (the omitted time period).The decreasing coefficient shows that average military spending increases through time but at a decreasing scale. Results for the time dummies do not show obvious differences between democracy and nondemocracy. In democratic states, marginal increase in military spending gradually falls from 1970 to 2009,

Table 2.5: Panel data estimates for model on government military expenditure

Dependent variable is military spending				
	All Countries	**Democracy** (Polity2≥ 9)	**Nondemocracy** (Polity2< 9)	**All Countries** **(Interactive Test)**
Resource Rents	0.13^{***}	-0.03	0.15^{***}	0.21^{***}
	(0.04)	(0.07)	(0.05)	(0.06)
Democracy	-0.37^{***}	-2.24^{***}	-0.24^{***}	-0.31^{***}
	(0.06)	(0.72)	(0.09)	(0.07)
Civil War	3.55^{***}	-0.19	4.46^{***}	3.66^{***}
	(0.9)	(0.88)	(1.23)	(0.89)
Ln GDP	-0.74	0.31	-0.57	-0.83
	(0.55)	(0.78)	(0.69)	(0.55)
Ln Population	1.23^{*}	0.95	0.66	1.38^{**}
	(0.65)	(0.93)	(0.81)	(0.65)
Working Age Population	-0.08	-0.13^{*}	0.14	-0.09
	(0.07)	(0.08)	(0.12)	(0.07)
Aged Population	0.08	0.14	-0.22	0.06
	(0.12)	(0.1)	(0.21)	(0.12)
Trade	0.03^{***}	0.008	0.03^{*}	0.03^{***}
	(0.01)	(0.008)	(0.1)	(0.01)
Interactive Term				-0.008^{*}
				(0.005)
Dummy years				
1970-1974	8.15^{***}	8.2^{***}	10.73^{***}	8.06^{***}
	(1.14)	(0.97)	(1.98)	(1.14)
1975-1979	8.64^{***}	5.6^{***}	12.73^{***}	8.41^{***}
	(1.03)	(0.87)	(1.8)	(1.04)
1980-1984	6.43^{***}	5.14^{***}	9.93^{***}	6.26^{***}
	(0.97)	(0.78)	(1.67)	(0.97)
1985-1989	6.0^{***}	5.08^{***}	8.82^{***}	5.81^{***}
	(0.94)	(0.71)	(1.67)	(0.95)
1990-1994	5.27^{***}	3.37^{***}	7.57^{***}	5.11^{***}
	(0.79)	(0.63)	(1.25)	(0.8)
1995-1999	3.31^{***}	1.97^{***}	4.9^{***}	3.15^{***}
	(0.71)	(0.55)	(1.14)	(0.72)
2000-2004	1.4^{**}	1.51^{***}	1.78^{*}	1.25^{*}
	(0.64)	(0.47)	(1.0)	(0.65)
2005-2009	0.61	1.03^{***}	0.7	0.58
	(0.6)	(0.39)	(0.94)	(0.6)
R^2	0.36	0.34	0.3	0.37
Observations	664	266	398	664
Countries	127	51	101	127

Notes. *p < 0.10, **p < 0.05, ***p < 0.01.

Table 2.6: Panel data estimates for model on public investment

Dependent variable is public expenditure				
	All Countries	**Democracy** (Polity2≥ 9)	**Nondemocracy** (Polity2< 9)	**All Counties (Interactive Test)**
Resource Rents	0.06^{**}	-0.11	0.06^{**}	0.15^{***}
	(0.03)	(0.07)	(0.03)	(0.04)
Democracy	-0.1^{**}	0.87	-0.12^{**}	-0.02
	(0.04)	(0.84)	(0.05)	(0.05)
Civil War	0.21	-0.32	0.18	0.41
	(0.63)	(1.28)	(0.7)	(0.63)
Ln GDP	0.23	1.38	0.09	0.15
	(0.44)	(0.94)	(0.47)	(0.43)
Ln Population	0.25	-1.67	0.37	0.34
	(0.54)	(1.08)	(0.59)	(0.52)
Working Age Population	-0.2^{***}	0.09	-0.21^{**}	-0.21^{***}
	(0.07)	(0.11)	(0.08)	(0.07)
Aged Population	0.04	-0.39^{***}	0.05	0.009
	(0.14)	(0.13)	(0.18)	(0.13)
Trade	0.1^{***}	0.003	0.1^{***}	0.09^{***}
	(0.008)	(0.02)	(0.01)	(0.008)
Interactive Term				-0.01^{***}
				(0.003)
Dummy years				
1970-1974	1.11	-1.0	0.83	0.76
	(1.14)	(1.71)	(1.3)	(1.14)
1975-1979	3.0^{***}	0.79	2.65^{**}	2.61^{***}
	(0.98)	(1.37)	(1.14)	(0.98)
1980-1984	2.25^{***}	1.14	1.79^{*}	1.86^{**}
	(0.86)	(1.27)	(1.0)	(0.86)
1985-1989	0.95	-0.12	0.56	0.48
	(0.81)	(1.08)	(0.94)	(0.82)
1990-1994	0.25	-0.04	-0.18	-0.2
	(0.7)	(0.97)	(0.81)	(0.71)
1995-1999	-0.26	-0.53	-0.65	-0.66
	(0.65)	(0.85)	(0.76)	(0.66)
2000-2004	-1.12^{*}	-1.44^{*}	-1.43^{**}	-1.51^{**}
	(0.61)	(0.78)	(0.7)	(0.62)
2005-2009	-1.3^{**}	0.06	-1.66^{**}	1.51^{***}
	(0.56)	(0.56)	(0.66)	(0.57)
R^2	0.27	0.21	0.28	0.3
Observations	588	74	514	588
Countries	104	21	98	104

Notes. *p < 0.10, **p < 0.05, ***p < 0.01.

after that, military spending stops increasing. However, in nondemocratic states, this fall stops in 2004, and after that the time effect disappears.

To investigate whether the effect of the total resource rents on the military spending depends the degree of democracy, we introduce an interactive term into our regression. We then consider the following econometric model:

$$Mexp_{it} = \alpha_0 + \gamma R_{it} + \beta_0 D_{it} + \delta D_{it} R_{it} + X_{it}^{''} \beta' + \mu_t + \eta_{it} \tag{2.55}$$

where D_{it} refers to the Polity2 score of country i in period t, and parameter β_0 encapsulates the relationship between between the democracy and the military expenditure. Parameter δ capture how the two way interaction between the democracy and the total resource rents affect the dependent variable, the military expenditure. Other potential covariates are included in the vector $X_{it}^{''}$. We use the full sample data to do the interaction test, and the result is shown in the last column of Table 2.5. After introducing the interactive term, the positive relationship between the total resource rents and the military expenditure is still significant, and the negative effect that is posed by the democracy stays significant as well. In addition, we can also see that the interaction between the total resource rents and the democracy is significant but only at the level of 0.1. For the regression, the coefficient of determination has been increased from 0.27 to 0.3. Overalls, the interaction term contributes in a limited way in the explanatory power of our regression. The positive effect towards the military expenditure that is posed by the total resource rents does not depend of the structure of the political regime of the country.

Table 2.6 shows the results for the second regression (Eq.(2.54)). In the first column, the sample contains data for both democratic and nondemocratic countries. The total resource rents as a percentage of GDP appears to be significant, implying the positive relationship between resource rents and public investment. Regarding our socioeconomic variable, working age population, we find a highly significant coefficient with negative sign. In addition, trade is significantly and positively correlated with public investment. In the second column, our sample contains data for all available democratic states. Total resource rents do not appear to be statistically significant which indicates the relationship between resource rents and public expenditure disappears. Although, as we have mentioned above, we have limited observation for democracy. In the third column, our sample contains data for all autocracies, or states that inherit considerable autocratic characteristics which contains 98 countries or states in total. In those countries or states, the total resource rents as a percentage of GDP appear to be statistically significant, indicating a positive relationship between resource rents and public expenditure. This result cope with our theory model. In our described model, it is possible that resource windfall will positively influence the public expenditure. When the effectiveness of military spending is low, nondemocratic government will increase its investment in public sectors. However, in Table 2.6, the coefficient of resource rents is quite low. It implies that resource windfalls are associated with limited increases in public investment. This result consistent to some extent with the 'resource curse' literature. In Table 2.6, we have seen that time dummies show different results, compared with that in Table 2.5. Generally speaking, average spending in public sectors does not fluctuate significantly through time, especially in democracy.

Similarly, we have tested the two way interaction between the total resource rents and the democracy towards the public expenditure. We consider the following econometric model:

$$Gexp_{it} = \alpha_0 + \gamma R_{it} + \beta_0 D_{it} + \delta D_{it} R_{it} + X_{it}^{''} \beta' + \mu_t + \eta_{it} \tag{2.56}$$

We test the full sample data that contains 588 countries, and the results is shown in the last column of Table 2.6. After introducing the interactive term, the positive effect that the total resource rents poses to the public expenditure stays positive and significant, and comparing with the regression without the interaction term, the coefficient turns to be greater. However, the negative relationship between the democracy and the public expenditure disappears. In addition, we have also seen the fact that the interaction between the total resource rents and the democracy is negatively affected the public expenditure at the level of 0.004. Comparing with the regression without the interaction term, the coefficient of determination increases from 0.27 to 0.3. So that, we can have the interaction term contributes in a meaningful way of explaining our regression. The positive effect towards the public expenditure that is posed by the total resource rents depends on the structure of the political regime. We should include the interaction term into our regression.

To summarize, when we doing the sub-sample analysis, our empirical results show that resource rents are positively and significantly correlated with military spending in nondemocratic states, however, in democratic states, resource rents do not appear to be statistically significant as a determinant of military spending. However, after doing the interaction test, we have found that, regardless the structure of political regime is nondemocratic or not, the relationship between the total resource rents and the military expenditure is positive and significant. Regarding the public investment, we have captured a statistically significant relationship between resource rents and public expenditure in nondemocratic states with a positive coefficient. However, in democracy, the relationship between resource rents and public investment disappears. The relationship between the total resource rents and the public expenditure does depend on the structure of political regime. By applying the empirical estimation, we characterize how resource windfalls affect government expenditure in democracy and nondemocracy. Our regression results fully support our theory model, as described in the previous section.

2.6 Concluding Remarks

This chapter has offered a simple model that investigates how natural resource endowments affect public policy. The two main contributions of this chapter are firstly, it develops a theoretical model that describes the decision making process of the elite when allocating government spending over military and productive public sector investment. We find that in an autocracy or a state which exhibits autocratic characteristics, if elites have been given with a resource windfall, they will always enlarge their military spending. However, investment in productive public sectors depends on the effectiveness of military spending. Secondly, we also provide an empirical analysis in the Appendix section, drawing on a database for 1960 to 2016 containing information on 159 countries. We find a positive and significant relationship between military spending and natural resource rents in nondemocratic countries.

In this chapter, we put much emphasis on how the elite determines the distribution of natural resource endowments. However, our model may have other implications. It could explain why ' resource curse' occurs mostly in nondemocratic states. Resource endowments foster fierce conflict between the elite and the masses, and therefore the elite would prefer to enlarge their military spending. For this reason, public investment in productive sector decreases which affects the development of economy. In addition, our empirical results also consistent to some extent with the 'resource curse' literature.

2.7 Appendix

In this chapter, we define the democracy as those counties whose polity2 scores at or above 9. In the appendix, we present results for the sensitive test respects to the definition of democracy. We focus on the relationship between the total resource rents and the military expenditure in nondemocratic countries. So that, We gradually relax the definition of democracy, and test this relationship in countries whose polity2 scores below 10, 9, 8, and 7. From Table 2.7, we have seen the fact that even we define the nondemocracy in a more strict way, as countries which polity2 scores below 7, the relationship between the total resource rents and the military expenditure is still positive and significant. In addition, we have also find that if we change the definition of nondemocracy from polity2< 8 to polity2< 9, the relationship between the total resource rents and the military spending becomes more significant, and the coefficient turns to be greater. That is to say, those countries which polity2 scores between 8 and 9 exhibit strong tendency of increasing the military spending if they are given with a higher resource rent. Considering these, we define the nondemocracy as those countries which polity2 scores below 9.

We also use the variable democracy from dataset of Boix et al., 2013 to define the democracy and nondemocracy. If the variable democracy equals 1, we say this

Table 2.7: Sensitivity test respect to the definition of nondemocracy

Dependent variable is military spending				
	Nondemocracy	**Nondemocracy**	**Nondemocracy**	**Nondemocracy**
	Polity2<10	Polity2<9	Polity2<8	Polity2<7
Resource Rents	0.15^{***}	0.15^{***}	0.13^{**}	0.13^{**}
	(0.05)	(0.05)	(0.06)	(0.06)
Democracy	-0.21^{***}	-0.24^{***}	-0.25^{**}	-0.27^{**}
	(0.08)	(0.09)	(0.1)	(0.13)
Civil War	4.13^{***}	4.46^{***}	4.41^{***}	4.71^{***}
	(1.12)	(1.23)	(1.36)	(1.52)
Ln GDP	-0.65	-0.57	-0.82	-0.85
	(0.66)	(0.69)	(0.9)	(0.87)
Ln Population	0.89	0.66	0.55	0.28
	(0.78)	(0.82)	(0.94)	(1.02)
Working Age Population	0.14	0.14	0.12	0.12
	(0.11)	(0.12)	(0.13)	(0.14)
Aged Population	-0.23	-0.22	-0.15	0.03
	(0.19)	(0.21)	(0.28)	(0.31)
Trade	0.03^{**}	0.03^{*}	0.03^{*}	0.03^{*}
	(0.01)	(0.01)	(0.02)	(0.02)
R^2	0.33	0.3	0.24	0.2
Observations	453	398	334	293
Countries	104	101	92	82

Notes. *p < 0.10, **p < 0.05, ***p < 0.01.

country is democratic. However, if the variable democracy equals zero, we say this country is nondemocratic. We then test the relationship between the total resource rents and the military spending in democracy and nondemocracy. The regression result is shown in Table 2.8.

We have seen that the regression result is similar with that of using polity2 to define democracy. The relationship between the total resource rents and the military spending is significant at the level of 0.01 in nondemocracy. When using the Boix dataset, the definition of nondemocracy becomes more strict and there are only 64 nondemocratic countries. In democracy, this relationship disappears. The regres-

Table 2.8: Panel data estimates for model on government military expenditure

Dependent variable is military spending			
	All countries	**Nondemocracy**	**Democracy**
		$Democracy = 0$	$Democracy = 1$
Resource Rents	0.16^{***}	0.18^{***}	-0.07
	(0.05)	(0.07)	(0.07)
Democracy	-3.18^{***}	-	-
	(0.86)	(-)	(-)
Civil War	4.23^{***}	5.13^{***}	2.89^{***}
	(0.94)	(1.74)	(1.01)
Ln GDP	-0.74	-1.31	-0.39
	(0.57)	(1.14)	(0.59)
Ln Population	1.24^{*}	1.3	1.3
	(0.67)	(1.27)	(0.73)
Working Age Population	-0.13	0.22	-0.29^{***}
	(0.07)	(0.17)	(0.08)
Aged Population	0.05	-0.29	0.08
	(0.12)	(0.4)	(0.11)
Trade	0.03^{**}	0.04^{**}	0.01
	(0.01)	(0.02)	(0.1)
R^2	0.34	0.23	0.37
Observations	643	234	409
Countries	124	64	81

Notes. *p < 0.10, **p < 0.05, ***p < 0.01.

sion result is consistent with our theoretical model.

We then test the relationship between the total resource rents and the public spending. The result is shown in Table 2.9. When using the dataset of Boix et al., 2013, the relationship between the total resource rents and the public expenditure in either democracy or nondemocracy is a bit different with that if we use the polity2 score to measure the democracy. Overalls, the total resource rents positively and significantly affect the public spending, and this relationship still exists in nondemocracy. These two results are consistent with the result of using the dataset of PolityIV. However, for the democratic society, this relationship is significant at the level of 0.1. That is because, when using the dataset of Boix et al., 2013 to measure the democracy, the subsample of the democracy includes many observations which the policy2 scores below 9, and these observations exhibit tendency of increasing public spending if they are given with a higher resource rents.

Table 2.9: Panel data estimates for model on government public spending

Dependent variable is military spending			
	All countries	**Nondemocracy**	**Democracy**
		$Democracy = 0$	$Democracy = 1$
Resource Rents	0.08^{***}	0.07^{***}	0.08^{*}
	(0.03)	(0.04)	(0.05)
Democracy	0.21	-	-
	(0.59)	(-)	(-)
Civil War	0.51	0.32	1.14
	(0.64)	(0.83)	(0.87)
Ln GDP	0.09	-0.75	0.72
	(0.45)	(0.59)	(0.47)
Ln Population	0.36	0.89	-0.94^{*}
	(0.56)	(0.71)	(0.55)
Working Age Population	-0.18^{***}	-0.12	-0.05
	(0.07)	(0.1)	(0.08)
Aged Population	-0.01	-0.12	-0.18
	(0.14)	(0.27)	(0.11)
Trade	0.1^{***}	0.11^{***}	-0.01
	(0.008)	(0.01)	(0.1)
R^2	0.28	0.35	0.13
Observations	565	365	200
Countries	102	80	56

Notes. *p < 0.10, **p < 0.05, ***p < 0.01.

Chapter 3

Land Reform, Optimal Land Holdings, and Redistribution

3.1 Introduction

Agricultural tenancy reforms have been widely discussed. Current empirical works and theoretical studies provide extensive evidence on the impact of land reform (Banerjee et al., 2002; Besley and Burgess, 2000; Besley et al., 2016). Overall, land reform contributes to improvement in agricultural productivity and also has a positive effect on poverty reduction as well as economic growth. In this chapter, we focus on the political economy of land reform. Most of the reforms divide large farms into small pieces, distribute farm lands to the poor, and work to improve the poor's access to land. Redistributive land reform is mainly motivated by public concern about the rising tension brought about by an unequal land ownership. It has been generally accepted that highly unequal societies are less likely to live peacefully (Dorner, 1992). When social wealth is mostly controlled by a small number of people, the distribution of income is highly skewed. Under this circumstance, the majority has strong incentives to revolt. Conflict over highly skewed wealth distribution drives civil wars. In much of the rural developing world, highly skewed distribution of wealth is accompanied by a concentrated landownership.

Rural people live with low wages and poor living standards. In the French Revolution, people fought against the feudal land system and asked for rights to land (Conrad and Meyer, 1958). The American Civil War was also a conflict over land. In the twentieth-century, civil wars in Mexico, Bolivia, Cuba, El Salvador as well as the Philippines, all had their origins in landownership distribution (Dorner, 1992). These countries related policies over land mostly benefit a small group of wealthy families, therefore exacerbating the inequality of society. These facts lead to violent demands for land redistribution.

Redistributive land reform helps to relieve domestic conflict caused by social inequality and therefore consolidates the current political regime. As we have mentioned above, redistributive land reform works to divide large farms into small pieces, and this creates many new family farms. Banerjee et al. (2002) have proposed that, small farms have a higher value of output per unit of land than large farms. Those small family farms have a higher productivity, make a better use of land, and receive a higher income. Therefore, by increasing the use of this model, the wealth inequality between the rich and the poor could be improved. Secondly, land reform is an effective policy in stabilizing the countryside. It respects the strong desire among rural people to own a piece of land (Moore, 1993). The distribution of landownership is often the key factor in winning the political support of rural people. Winning rural support is the basis of political stability (Huntington, 2006). There are numerous applications of land redistribution. During the post-colonial era, developing countries like India underwent significant land reform. These reforms regulate the land ceiling which is the maximum size of land holding that an individual or family can own, improve the land access of the poor, and increase the tenurial security for farmers who do not own land. After the Second World War, fundamental reforms were completed in Japan, Taiwan, and South Korea (Hayami and Kikuchi, 1981). Reforms in Asia abolished the rent collection and control system. The tie between tenants and their landlords has since been reduced. Tenant farms gradually began to operate independently and in small units. Peasants are free from rents that now results in a more progressive agriculture. In addition, countries in

Latin America have also experienced major reforms. Farms over a certain size are subject to expropriation, and most of the expropriated land is assigned to landless tenants in different forms.

When attempting to resolve the domestic conflict brought about by wealth inequality, government can provide the public with redistributive policies that transfer part of the wealth from the rich to the poor. From above, we have seen that redistributive land reform has been widely used when current political incumbents work to sustain their control over the state. However, most regime transition theories consider redistribution as income redistribution. As has been widely discussed and applied by Acemoglu and Robinson, landed elites make concessions to the majority by distributing part of their income to the majority if there exists a significant threat of revolution (Acemoglu and Robinson, 2000, 2001, 2008; Acemoglu et al., 2004). Furthermore, they democratize by themselves to make the promise credible. The majority then choose high taxation and high income redistribution for the whole nation. Under the circumstance described, income levels of the poor is raised up, and domestic conflict is relieved. However, none of those theories ever consider land redistribution as a redistributive policy, and there are few comparisons between land reform and income redistribution. We have limited understanding of how the rich would like to relieve the domestic threat of revolution, by redistributing lands to the poor or by giving them direct redistributed revenue. In this chapter, when the threat of revolution exists, we investigate which conditions would persuade landed elites to give up their landholdings to the majority, and let farmers set up individual private farms. In addition, we also find the condition under which landed elites would prefer to preserve all their landholdings and instead provide the majority with a higher redistributed revenue.

We develop a model consisting of two groups: landed elites and farmers. Landed elites control large quantities of private wealth, and also hold the ownership of all available land. Farmers take the majority of the whole population and each of them inherits one unit of labour. We consider the situation that the state operates a system of control on the landed elite, and they also control the state wealth under this hypothesis. By comparing the expected payoff of revolution with that of staying with the control of elites, farmers decide whether to revolt. The expected payoff of revolution is affected by the political power of elites and also the level of income inequality. If elites are especially powerful, they can command formidable armies, and therefore the likely detrimental consequences of revolution for the majority are huge, and this lowers their incentives to revolt. On the other hand, an unequal wealth distribution means the majority has a higher incentive to revolt. To consolidate their control over the state, landed elites would like to make concessions to farmers in order to remove the threat of revolution. The underlying idea of our approach is that landed elites could give up part of their landholdings to a group of farmers and hire the rest of all available labour at a revised wage rate. Each farmer who has been allocated a piece of land will set up their private farm and collect all their produced revenue. For landed elites, the key policy decision concerns the number of family farms that they are going to establish.

In our hypothetical society, the preference of farmers between two redistributive policies - redistributed lands and redistributed income - is different. Land is a type of asset that can not easily be removed, and can also provides the farmer with a comparatively stable income. In practice, farmers usually show a strong desire to own a plot of land. Furthermore, in this model, farmers will receive the distributed land before the land is cultivated, however, the income redistribution takes place after this producing activity takes place. Redistributive land reform is a more credible policy to the majority. When choosing the extent of family farm ownership, landed elites consider the total produced revenue of each farm. They will distribute lands that makes the produced revenue for each farm equivalent to the expected payoff of revolution. Since the income redistribution will be made after landed elites receive the produced agricultural revenue, they could easily run with all received revenue. In addition, the threat of revolution is often only transitory. Once the majority agree to keep working with landed elites, the threat of revolution is removed and can not easily be raised up again immediately. The poor are unlikely to see the promise of a high wage rate as a credible commitment. Even if current redistribution takes place, future redistribution can not be guaranteed. Consequently, farmers prefer to receive a piece of land, and in order to attract labour, landed elites should make a higher promise in regard to the income payment that exceeds the total produced revenue of each family farm. If the commitment problem becomes less fierce, the required income redistribution is reduced. There are several approaches that could solve the commitment problem. It has been widely discussed in the literature that to make a credible commitment, landed elites could democratize themselves, transitioning the current political regime from nondemocracy to democracy. In addition, landed elites could also win trust from the majority by improving the legislation, by increasing wages paid. Generally speaking, the credibility of the landed elite is determined by the quality of institutions. If landed elites develop a stable and functioning infrastructure, the poor feel indifferent between land reform and income redistribution. In this model, we consider the quality of institution as an influencing factor towards the required income redistribution. If the majority fully trust the landed elite, we say the current institution is good.

This chapter presents the following key findings. Firstly, we find that the optimal number of private farms depends on the development of agriculture. If agriculture is productive or the price of farming goods is high, the aggregate produced revenue will take a great share of total national wealth. Therefore, agriculture becomes a comparably prosperous industry, and land becomes a kind of valuable asset. Under this circumstance, redistributive land reform is an effective and also cost saving policy in removing the threat of revolution. If landed elites face a strong threat of revolution, it is beneficial for them to enlarge the scale of land reform, and distribute land to more people. However, if the threat of revolution is only moderate, it is beneficial for elites to hold on to more land and hire more labour. If instead agriculture is relatively unproductive, land becomes less valuable. Under these circumstances, redistributive land reform is less effective and more financially wasteful. When elites face a strong threat of revolution, increasing income redistribution to the majority is more beneficial, so the elites would prefer to cut down the number of private farms and include more labour as income redistribution beneficiaries. However, if the threat of revolution is lower, landed elites would prefer to give up more land to the public so that they could save more wage costs.

Secondly, we find some comparative statics between the factors that determine the threat of revolution and the number of private farms. As we have mentioned above, if the landed elite holds greater political power, the majority has less incentive to revolt. Therefore, given a booming agricultural industry, if the landed elite holds greater political power, the optimal number of private farms will decrease, and the land reform will become less attractive. On the other hand, if wealth distribution is more unequal, domestic conflict between elites and the poor becomes more fierce. Therefore, given a productive agricultural sector, the optimal number of private farms increases in the holding wealth of the landed elite. However, if agriculture is no longer a prosperous industry, all the described factors, political power of the landed elite and inequality of wealth distribution, influence the optimal number of private farms in the opposite direction.

Thirdly, we also find that the quality of institutions influences the decision making process of landed elites in land reform. If the quality of the current institution has been improved, the required income redistribution decreases and the marginal cost of income redistribution declines, this makes income redistribution a more attractive policy in removing the threat of revolution. In this chapter, we also find that if the majority fully trust the commitment made by the landed elites, the required income redistribution equates to the expected payoff of revolution and the marginal loss in land reform always exceeds the marginal savings in wage costs. Under these circumstances, regardless of the development of agriculture, landed elites would like to use income redistribution to solve domestic conflict and none of the landholdings will be be given up. If there exists no commitment problem, land reform will never be an optimal policy in consolidating the political regime. It explains why in many democratic countries, landed elites would prefer to accept a higher tax rate and make a higher income redistribution so that they could preserve all their physical means of capital production in hand.

Our findings contribute to a large literature on land reform (Binswanger et al., 1995; Deininger and Feder, 2001; Deininger et al., 2003; Cheng and Chung, 2017), though much of this literature focuses on the application on redistributive land reform. A net impact of redistributive land reform is shown to be a positive effect on agriculture productivity (Banerjee et al., 2002). It has been theoretically and empirically explained that land reform has an appreciable impact on growth and poverty reduction (Besley and Burgess, 2000). In addition, the literature argues that land reform works to reduce land inequality and to increase agricultural wages (Besley et al., 2016). This literature studies the impact of land reform, seeing land reform as a given policy which is exogenous. Bhattacharya et al. (2019) investigate the relationship between political transitions and the probability of land reform and find that democratic transitions are linked with a greater likelihood. In this chapter, we mainly discuss whether government would like to impose a land reform, and how government determines the scale of the reform in view of the stated impact of land reform. In our model, we consider the redistributive land reform as an endogenous variable.

This chapter is related to the literature on social conflict, and we stress the conflict between the rich and the poor (Skaperdas, 1992, 1996). From a modeling point of view, this chapter extends the framework in Acemoglu and Robinson (2000, 2001, 2005). Their model is based on the idea that conflict between different social groups is a key factor in regime transition. Elites redistribute part of their income to the poor in response to the threat of revolution and social unrest. The major difference is that we include redistributive land reform as a possible redistributive policy. To remove the threat of revolution, elites could distribute part of their land to some of the poor and/or redistribute income to the rest of the poor. Acemoglu and Robinson (2000, 2001) also argue that, to solve the commitment problem, landed elites will fully democratize themselves, let the majority become the decision maker, accept a higher income tax rate, and redistribute income to the public. Our research is complementary to their findings. Our model finds that, if the current state operates a sound political institution, it is beneficial for the landed elites to accept a higher income tax rate in exchange for the protection of their private wealth.

The rest of this chapter is organized as follows. Section 3.2 outlines the basic political environment and the baseline model. In Section 3.3 we analyse the model, characterize the equilibria of the model and investigate the main comparative statics. Section 3.4 provides simulation results of our policy determination model. And in Section 3.5, we conclude.

3.2 The Model

3.2.1 Environment

Consider a two-period game that is played by two groups of agents: one landed elite and N farmers. Here, N is sufficiently large that farmers constitute the majority of the whole population. There is one unique final good that will be consumed at the end of each period. All agents have the same risk-neutral preferences with discount factor β, given by:

$$U(c_t^i) = c_0^i + \beta c_1^i$$

where c_t^i denotes consumption of agent i at time t (t=1,2) in terms of the final good. We use the notation $i \in E$ to denote that agent i is elite, and $i \in R$ to agent who is farmer.

In our hypothetical society, each farmer owns one unit of labour, and the total supply of labour equates to N in this economy. In addition, there is a total supply of land equal to L, with no alternative use, which is owned by the landed elite. To begin with, the landed elite has been given absolute ownership over the land, but can use the land for agricultural production, or could alternatively distribute the ownership to someone else. To simplify the analysis, we do not consider the land market for sale; the landed elite would give up the landholdings with no compensation. Each landholder has access to the following production function to produce the unique final good:

$$F(l, n) = Al^{\alpha} n^{1-\alpha}$$

where $\alpha \in (0, 1)$, A denotes the farming technology level, l denotes quantities of land in use that $l \leq L$, and n denotes the hired labour inputs where $n \leq N$. The price of final output is normalized to 1. The landed elite pays each hired labour, ω. The net income of the landed elite therefore equates to:

$$Y(l, n, \omega) = F(l, n) - \omega n$$

With no external intervention, the landed elite pays all hired labour the lowest wage, ω_0, the minimum rate that could just sustain people's subsistance. In the first period, the state is controlled by the landed elite with no threat of revolution. Under these circumstances, the landed elite hires all available labour at the minimum wage rate, and the elite's net income is given by:

$$Y(l = L, n = N, \omega = \omega_0) = AL^{\alpha}N^{1-\alpha} - \omega_0 N$$

3.2.2 Threat of Revolution

The model starts with the control of the landed elite, with farmers being excluded from political access. At the beginning of each period, the landed elite is endowed with private wealth Y_0 which includes luxury collections, real estates, and revenues from other industries. The landed elite also controls the state wealth, R, which mainly refers to income that is accrued via natural resources, like coal, oil and other types of mineral. The state wealth does not include the land. In this model, the landed elite could also make income from land by hiring labour, however, the state wealth mainly consists of exports. The levels of Y_0 and R are fixed and publicly known. In the first period, the landed elite hires all available labour and pays each of them at the wage rate, ω_0. At the end of the period, payments has been made and consumption takes place whereby the landed elite consumes all its collected agricultural revenues, private wealth as well as state wealth, and each farmer also uses up his/her total income. At the beginning of the second period, considering the income level of the previous period as well as the wealth level of the landed elite, the value of Y_0 and R, farmers decide whether to attempt a revolution. We assume that if a revolution takes place, all farmers will take part, and they will always succeed. If a revolution is attempted, all farming activities stop, so there is no collected revenue from the agriculture, and no one will receive the income from the farming activity. After a revolution, farmers expropriate all available wealth from the landed elite, sharing both the private wealth and the state wealth. Therefore, if a revolution takes place, the expected payoff for each farmer is given by:

$$Y_r = (1-\mu)\frac{Y_0 + R}{N}$$

where Y_r denotes the income level of each farmer if they attempt a revolution, μ denotes the fraction of the permanent loss that is generated by the revolution. The value of μ is determined by the political power held by the landed elite, denoting that a revolution will generate great permanent losses in the total shared wealth if the landed elite is strong in power. The expected payoff function implies that any increases in the value of μ or any decreases in the value of Y_0 and R lower the expectation of each farmer towards the revolution, and their incentives to revolt become lower. Farmers decide whether to attempt a revolution by comparing the expected payoff of revolution with their received income in period 1. If $Y_r \leq \omega_0$, farmers would prefer to stay with the control of landed elite, and there is no threat of revolution. However, if $Y_r > \omega_0$, farmers will revolt if the landed elite still pays them at the subsistance wage rate in period 2. We also assume that the landed elite will lose everything if the majority attempt a revolution. At the beginning of the second period, the elite will announce policies that work to improve the national income level, making the received wage of the majority equal to their expected payoff of revolution ($Y_f = Y_r$), thus potentially preventing the revolution. Here Y_f denotes the income level of each farmer if they choose not to revolt.

3.2.3 Land Reform and Income Redistribution

As mentioned above, in order to prevent revolution, the landed elite will make concessions to the majority, improving the expected payoff of staying within the control of the landed elite. There are two types of policy available to the landed elite. Firstly, the landed elite could redistribute income to the majority, raise the provided wage rate up, making $Y_f = \omega = Y_e$. Secondly, the elite could impose a land reform, announce that it would distribute part of the landholdings to the farmers, let each of them set up their private farm, and make the collected revenue of each private farm equal to the expected payoff of revolution, $Y_f = F(l, n) = Y_r$. In this model, the landed elite could also apply these two policies simultaneously. At the beginning of the second period, anticipating the threat of revolution, the landed elite imposes a land reform, distributing part of its landholdings to a group of farmers, and hiring additional farmers at the revised wage rate. In this scenario, the farmers could be divided into two groups: people who own the land, and people who work for the landed elite. We use the notation N_F to denote the number of people that own a plot of land, and N_L to denote the number of people hired by the landed elite. Here, N_F and N_L satisfy the condition that $N_F + N_L = N$.

At the beginning of the second period, the landed elite determines the value of N_F and farmers will receive the distributed land thereafter. Otherwise, the majority will consider the commitment from the landed elite to lack credibility and will revolt immediately. Therefore, there is no commitment problem if the landed elite choose to impose a land reform. To remove the threat of revolution, the collected revenue of each private farm should equate to the expected payoff of revolution. Suppose the landed elite distributes λ of its total landholdings to N_F farmers, λ and N_F satisfy the following constraint:

$$\begin{aligned} Y_f &= F(l = \frac{\lambda L}{N_F}, n = 1) \\ &= A\left(\frac{\lambda L}{N_F}\right)^{\alpha} \\ &= \frac{(1-\mu)(Y_0 + R)}{N} = Y_r \end{aligned}$$

where $\frac{\lambda L}{N_F}$ is the size of the distributed land that each farmer receives. For each private farm, the landowner farms by himself and does not hire extra labour. Therefore, the income of each farmer who receives the distributed land equates to $Y_f = F(l = \frac{\lambda L}{N_F}, n = 1)$, and the distributed fraction of total landholdings, λ, should satisfy the following condition:

$$A\left(\frac{\lambda L}{N_F}\right)^{\alpha} = \frac{(1-\mu)(Y_0 + R)}{N}$$

By rearranging the above equation, λ could be written as a function of N_F:

$$\lambda = \left[\frac{(1-\mu)(Y_0 + R)}{AN} \right]^{\frac{1}{\alpha}} \frac{N_F}{L}$$

In addition to the land reform, the landed elite may also redistribute income to the farmer, raising the provided wage rate and hiring $N - N_F$ farmers at the improved wage rate. At the beginning of the second period, the landed elite announces the wage rate, ω, and the income payment is made at the end of the period. However, because the threat of revolution is transitory, the current promise of income distribution can not guarantee that the payment will be made in the end. A revised wage rate at $\omega = Y_r$ is therefore insufficient to prevent a revolution, so the landed elite will be forced to make a higher level of redistribution. Let p denote the probability that the farmer assumes the landed elite will keep its promise and pay them at the end of the second period. To remove the threat of revolution, the revised wage rate, ω, satisfies the following condition:

$$Y_f = \omega p = \frac{(1-\mu)(Y_0 + R)}{N}$$

where ωp refers to the expected payoff of each hired labourer and this should equate to the expected payoff of revolution. Given the threat of revolution, the provided wage rate of the landed elite, λ, is given by:

$$\omega = \frac{(1-\mu)(Y_0 + R)}{N * p}$$

The above equation implies that a higher expectation of revolution makes the landed elite increase its provided wage rate and a higher value of p lowers the minimum requirement of income redistribution. The value of p could be interpreted in the following ways. A higher value of p means that farmers strongly believe that the landed elite will pay them the promised wage rate in the end. It reflects how much the majority trusts the current government. According to Alesina and Giuliano (2015), trust can affect the quality of institution. If the majority is strongly trust current government, it is more likely that current government operates a functioning institution. For example, if the current government has a functioning political institution - i.e. if most of its policies reflect the interests of the majority, and/or each policy is well imposed - farmers will see the commitment from the landed elite as credible. In addition, legislation makes the commitment trustworthy. As we have mentioned above, the landed elite hires $N_L = N - N_F$ farmers at the revised wage rate, ω, and the total wage cost equates to:

$$\omega(N - N_F) = \frac{(1-\mu)(Y_0 + R)}{N * p}(N - N_F)$$

which is a decreasing function as a variable of N_F. In our model society, the landed elite should raise the national income level so that is at least equates to the expected payoff of revolution in order to remove the threat of revolution. Once the landed elite chooses the number of private farms, N_F, we can have the actual fraction of the whole land that the landed elite will give up to the farmer, λ, and we could also have the labour input that will be used: $N_L = N - N_F$.

3.2.4 Timing of Events

We now briefly recap the timing of events in this basic environment. Let δ denote the set of parameters $\delta = \{A, L, N, \alpha, \mu, p, Y_0, R, \omega_0\}$ which have been given at the beginning of the first period.

- At the beginning of period 1, the society is under the control of the landed elite. All farmers work for the landed elite and each of them receives the income payment, ω_0, at the end of the period. Consumption then takes place.

- At the beginning of period 2, by comparing the income level in the previous period, ω_0, with the expected payoff of revolution, farmers decide whether to revolt. If their received income in period 1 is greater than the expected payoff of revolution, $\omega_0 \leq Y_r$, they have no incentive to revolt. Otherwise, farmers have an incentive to attempt a revolution.

- If farmers have no incentive to revolt, we say the control of the landed elite is stable. If the control of the landed elite is stable, the second period is a repeat of the first period. However, if farmers have incentives to revolt at the beginning of the second period, the landed elite will make concessions to them.

Anticipating the expected payoff of revolution, the landed elite announces it will distribute λ of its total landholdings to N_F farmers, and hire the rest of the whole population, $N - N_F$ farmers, at a wage rate that equates to the expected payoff of revolution, $\omega p = Y_r$.

- If farmers accept the offer from the landed elite, the threat of revolution will be removed at the beginning of the period. Income payments will be paid and consumption will take place at the end of the second period.

- If farmers do not accept the concession of the landed elite, revolution takes place. In the second period, all producing activities cease, farmers will win in the end, each of them will equally share the private wealth of the landed elite, Y_0, as well as the state wealth, R, and consumption will take place after that.

3.3 Analysis of the Model

3.3.1 Definition of the Optimal Strategy

We now analyze the model we described in the previous section. We focus on the optimal strategy for the landed elite to remove the threat of revolution. The optimal strategy refers to the political policy that the landed elite announces at the beginning of the second period, which consists of the optimal number of private farms, N_F, the distributed fraction of lands, λ, the revised wage rate, ω, and the quantity of hired labour, N_L. As we discussed in the previous section, given the set of parameters δ, the value of ω is determined, λ could be written as a function as a variable of N_F, and $N_L = N - N_F$. The landed elite chooses the optimal N_F that maximizes its expected income in the period 2. The optimizing problem is given by:

$$
\begin{aligned}
max \quad & F((1-\lambda)L, N_L) - \omega N_L \\
s.t \quad & F((1-\lambda)L, N_L) - \omega N_L + Y_0 + R \geq 0 && (3.1) \\
& F(\frac{\lambda L}{N_F}, 1) = \frac{(1-\mu)(Y_0 + R)}{N} && (3.2) \\
& \omega = \frac{(1-\mu)(Y_0 + R)}{N * p} && (3.3) \\
& N_L + N_F = N && (3.4) \\
& N_F \geq 0 && (3.5) \\
& N_L \geq 0 && (3.6)
\end{aligned}
$$

where $F((1-\lambda)L, N_L)$ refers to the total produced revenue of the landed elite. Equation (1) refers to the budget constraint. Here, the total produced revenue, $F((1-\lambda)L, N_L) - \omega N_L$, could be either positive or negative, however, the total received income of the landed elite, $F((1-\lambda)L, N_L) - \omega N_L + Y_0 + R$, can not be negative. Otherwise, the elite is incapable of keeping its promises to the majority, and a revolution takes place in period 2. Equation (3.2) regulates the minimum collected revenue of each private farm, and equation (3.3) defines the minimum wage rate. We then substitute equations (3.2) and (3.3) into the objective function, the maximizing problem of the landed elite is given by:

$$
max \quad V_e(N_F) = AL^\alpha \left[1 - \left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N_F}{L}\right]^\alpha (N - N_F)^{1-\alpha} - \frac{W}{N * p}(N - N_F) + Y_0 + R \quad (3.7)
$$

$$
\begin{aligned}
s.t. \quad & V_e(N_F) \geq 0 \\
& N - N_F \geq 0 \\
& N_F \geq 0
\end{aligned} \quad (3.8)
$$

where $V_e(N_F)$ refers to the valuation function of the landed elite if it preserves its controlling position in period 2. To simplify the function, we let $W = (1-\mu)(Y_0+R)$.

If there exists a non-negative N_F that solves the above optimizing problem, the landed elite could remove the threat of revolution and keep controlling the society in the second period. The optimal strategy is displayed as follows. The landed elite will distribute $\lambda = \left[\frac{(1-\mu)(Y_0+R)}{AN}\right]^{\frac{1}{\alpha}} \frac{N_F}{L}$ of its lands to N_F farmers, and hires $N - N_F$ farmers to work the remaining land. If a non-negative N_F does not exist, we then look at the value of $V_e(N_F = 0)$. If $V_e(N_F = 0)$ is positive, the landed elite has a positive valuation of hiring all available labour at the revised wage rate, ω. To sustain its control over the state, it keeps all his landholdings and hires all farmers at the revised wage rate, ω. The landed elite will not impose a land reform and it chooses to remove the threat of revolution instead by making income redistribution to all farmers. However, if $V_e(N_F = 0)$ is negative, the landed elite can not pay all farmers at the revised wage rate. Under this circumstance, given the wealth level of the landed elite, $Y_0 + R$,it cannot remove the threat of revolution, either by land reform or by income redistribution, and the majority therefore attempt a revolution in period 2. At the end of the period, the landed elite loses everything and the majority share the remaining wealth of the landed elite equally, each of them receives $\frac{(1-\mu)(Y_0+R)}{N}$ in the end.

We now further investigate the value of N_F which maximizes the valuation function of the landed elite in the second period, $V_e(N_F)$. We differentiate $V_e(N_F)$ by N_F, the optimal N_F satisfies the following first order condition:

$$\begin{aligned}\frac{\partial V_e(N_F)}{\partial N_F} &= AL^{\alpha}\alpha\left[1-\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{N_F}{L}\right]^{\alpha-1}\left[-\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{1}{L}\right](N-N_F)^{1-\alpha}\\ &+AL^{\alpha}\left[1-\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{N_F}{L}\right]^{\alpha}(1-\alpha)(N-N_F)^{-\alpha}(-1)+\frac{W}{N*p}=0\end{aligned} \tag{3.9}$$

We then rearrange equation (3.9), and we have:

$$
\begin{aligned}
-\frac{\partial V_e(N_F)}{\partial N_F} &= AL^\alpha \alpha \left[1-\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N_F}{L}\right]^{\alpha-1} \left[\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{1}{L}\right] (N-N_F)^{1-\alpha} \\
&+ AL^\alpha \left[1-\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N_F}{L}\right]^{\alpha} (1-\alpha)(N-N_F)^{-\alpha} - \frac{W}{N*p} = 0
\end{aligned} \tag{3.10}
$$

We then differentiate (3.10) by N_F:

$$
-\frac{\partial V_e(N_F)}{\partial^2 N_F} = AL^\alpha \alpha(1-\alpha) \left[1-\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N_F}{L}\right]^{\alpha-2} (N-N_F)^{-\alpha-1} \left[\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N}{L} - 1\right]^2 \tag{3.11}
$$

From the above, we could see that if $(\frac{W}{AN})^{\frac{1}{\alpha}} \frac{N}{L} = 1$, $\frac{\partial .}{\partial^2 N_F} = 0$ for all possible N_F. We cannot fully determine the shape of $V_e(N_F)$ here. However, we could still compare the land reform and the income redistribution and find out which one is more beneficial to the landed elite. If $\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N}{L} = 1$, we have $AL^\alpha N^{1-\alpha} = (1-\mu)(Y_0+R)$. We then compare the value of $V_e(N)$ and $V_e(0)$, and we have :

$$
\begin{aligned}
Y_e(N_F = N) &= Y_0 + R \\
Y_e(N_F = 0) &= AL^\alpha N^{1-\alpha} - \frac{(1-\mu)(Y_0+R)}{p} + Y_0 + R
\end{aligned}
$$

Since $p \in (0, 1]$, we could have that $\frac{W}{p} \geq AL^\alpha N^{1-\alpha}$. For any $p \neq 1$, $V_e(N_F = N) > V_e(N_F = 0)$, land reform is more beneficial. Compared with hiring all farmers at the revised wage rate, ω, the landed elite prefers to give up all his land to farmers and keeps his primary wealth $Y_0 + R$. However, if $p = 1$, $V_e(N) = V_e(0)$, the landed elite is indifferent between distributing land to farmers and improving the wage rate. The value of $V_e(N_F)$ is constant which equates to $Y_0 + R$.

However, if $\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N}{L} \neq 1$, $\frac{\partial V_e(N_F)}{\partial^2 N_F} < 0$ for all possible N_F, $V_e(N_F)$ is a strict concave function as a variable of N_F. There always exists a unique N_F that maximizes the valuation function of the landed elite, $V_e(N_F)$. However, the optimal N_F could be either positive or negative. If $V_e'(0) > 0$, there exists a unique positive N_F that could maximize $V_e(N_F)$. Suppose the function V_e is maximized at $N_F = N_F^* > 0$, if $V_e(N_F^*) \geq 0$, the landed elite will impose a land reform, distribute lands to N_F^* farmers, and hire $N - N_F^*$ farmers at the revised wage rate, ω. Otherwise, the landed elite can not remove the threat of revolution, and the majority attempt to revolt in the second period. However, if $V_e'(0) \leq 0$, the optimal N_F is negative or equates to zero. Under this circumstance, land reform is not beneficial to the landed elite which would prefer to make income redistribution to all farmers, and preserve all its landholdings. If $V_e(0) \geq 0$, the landed elite could remove the threat of revolution by giving direct income redistribution.

We then summarize the optimal strategy for the landed elite to remove the threat of revolution:

Proposition 1. *Given the set of parameter δ and suppose $\omega_0 < \frac{(1-\mu)(Y_0+R)}{N}$:*

- *If $(\frac{W}{AN})^{\frac{1}{\alpha}}\frac{N}{L} \neq 1$, the valuation function, $V_e(N_F)$, is a strict concave function. There always exists a unique N_F that maximizes the valuation function of the landed elite, $V_e(N_F)$.*

  - *If $V_e'(0) \leq 0$, the optimal N_F is negative or equates to zero. Compared to the land reform, it is beneficial for the landed elite to make income redistribution to the whole population. Under this circumstance, if $V_e(0) \geq 0$, the landed elite could remove the threat of revolution, he would like to hire all farmers at $\omega = \frac{W}{N*p}$. Otherwise, the majority attempt to revolt in the second period.*

  - *If $V_e'(0) \geq 0$, the optimal N_F is positive. It is beneficial for the landed elite to impose a land reform and redistribute income to its hired labour simultaneously. Suppose the optimal N_F equates to N_F^*, if $V_e(N_F^*) > 0$, the landed elite could remove the threat of revolution, it will give up $\left[\frac{W}{AN}\right]^{\frac{1}{\alpha}}\frac{N_F^*}{L}$ of its land to N_F^* farmers, and will hire $N - N_F^*$ farmers at the revised wage rate $\omega = \frac{W}{N*p}$. Otherwise, the revolution takes place.*

- *If $(\frac{W}{AN})^{\frac{1}{\alpha}}\frac{N}{L} = 1$, the landed elite will give up all its land to all farmers that $N_F = N$, and $V_e(N) = Y_0 + R$. Since $V_e(N) = Y_0 + R > 0$, if $(\frac{W}{AN})^{\frac{1}{\alpha}}\frac{N}{L} = 1$, the landed elite could always remove the threat of revolution.*

3.3.2 Comparative Statics

We now look at the optimal strategy and derive a number of comparative static results. We look at the situation in which the landed elite faces up to the threat of revolution at the beginning of the second period. Given the set of parameter, δ, suppose there always exists a unique and positive N_F that could solve the optimizing problem of the landed elite we described above. As we mentioned, the optimal N_F satisfies the following first order condition:

$$\begin{aligned} -\frac{\partial V_e(N_F)}{\partial N_F} &= L^\alpha \alpha \left[1 - \left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N_F}{L}\right]^{\alpha-1} \left[\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{1}{L}\right] (N - N_F)^{1-\alpha} \\ &+ L^\alpha \left[1 - \left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N_F}{L}\right]^{\alpha} (1-\alpha)(N - N_F)^{-\alpha} - \frac{W}{AN * p} = 0 \end{aligned} \quad (3.12)$$

Let us denote the value of N_F that solves equation (3.12) by $N_F = N_F^*$, and recall that this is the optimal number of private farms the landed elite would like to set up to remove the threat of revolution. We now examine how changes in parameters W where $W = (1-\mu)(Y_0 + R)$, and A affect the value of N_F^*. From equation (3.12), we have seen that $-\frac{\partial V_e(N_F)}{\partial N_F}$ could be considered as a function of a variable sector $\frac{W}{A}$, and we could capture the two comparative static results, $\frac{\partial N_F^*}{\partial W}$ and $\frac{\partial N_F^*}{\partial A}$, by analyzing the value of $\frac{\partial - V_e'(N_F)}{\partial\left(\frac{W}{A}\right)}$. Keeping all other parameters unchanged, the sign of $\frac{\partial - V_e'(N_F)}{\partial\left(\frac{W}{A}\right)}$ is same with $\frac{\partial - V_e'(N_F)}{\partial\left(\frac{W}{AN}\right)}$. To simplify the calculation, we differentiate

(3.12) by $\frac{W}{AN}$:

$$\frac{\partial(-V_e'(N_F))}{\partial\frac{W}{AN}} = \left[\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{1}{L}\right]^{1-\alpha}\left[1-\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{N_F}{L}\right]^{\alpha-1}(N-N_F)^{1-\alpha}$$
$$\left\{(1-\alpha)N_F\left[\frac{1}{\left[\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{1}{L}\right]^{-1}-N_F}-\frac{1}{N-N_F}\right]+1\right\}-\frac{1}{p}$$
$$=\left[1-\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{N_F}{L}\right]^{\alpha-1}\left[\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{(N-N_F)}{L}\right]^{1-\alpha}$$
$$\left\{(1-\alpha)N_F\left[\frac{N-\left[\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{1}{L}\right]^{-1}}{\left(\left[\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{1}{L}\right]^{-1}-N_F\right)(N-N_F)}\right]+1\right\}-\frac{1}{p} \tag{3.13}$$

The comparative static results of N_F^* with respect to W and A depends on the value of $(\frac{W}{AN})^{\frac{1}{\alpha}}\frac{N}{L}$. If $(\frac{W}{AN})^{\frac{1}{\alpha}}\frac{N}{L} < 1$, $\frac{\partial(-V_e'(N_F))}{\partial(\frac{W}{AN})} < 0$ always holds. It implies that any increases in the value of W or decreases in the value of A will increase the value of N_F^*. Increases in the value of W imply that the domestic threat of revolution becomes stronger, and under this circumstance, the landed elite prefer to give up his landholdings to more farmers in exchange of a higher expected payoff in the period 2. In addition, if the agricultural technology is less developed, the landed elite prefers to have more private farms.

Proof. Since $\lambda = \left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{N_F}{L}$ and $\lambda \in (0,1)$, we have:

$$0 < \left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{N_F}{L} < 1$$
$$\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{1}{L} < \frac{1}{N_F}$$
$$\left[\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{1}{L}\right]^{-1} > N_F$$

So that we have $\left(\left[\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{1}{L}\right]^{-1}-N_F\right)(N-N_F) > 0$. Since the condition $(\frac{W}{AN})^{\frac{1}{\alpha}}\frac{N}{L} < 1$ holds, $N-\left[\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{1}{L}\right]^{-1} < 0$. We then have that if the condition

$(\frac{W}{AN})^{\frac{1}{\alpha}}\frac{N}{L} < 1$ holds,

$$\left\{(1-\alpha)N_F\left[\frac{N-\left[\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{1}{L}\right]^{-1}}{\left(\left[\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{1}{L}\right]^{-1}-N_F\right)(N-N_F)}\right]+1\right\} < 1$$

From $(\frac{W}{AN})^{\frac{1}{\alpha}}\frac{N}{L} < 1$, we have $\left[\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{(N-N_F)}{L}\right] < \left[1-\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{N_F}{L}\right]$ and we also determine that:

$$\left[1-\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{N_F}{L}\right]^{\alpha-1}\left[\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{(N-N_F)}{L}\right]^{1-\alpha} < 1$$

Since $p \in (0,1]$, we have $\frac{1}{p} \geq 1$. From equation (3.13), we could have that $(\frac{W}{AN})^{\frac{1}{\alpha}}\frac{N}{L} < 1$ is a sufficient condition under which $\frac{\partial(-V_e'(N_F)}{\partial\frac{W}{AN}} < 0$. Any increases in the value of W or any decreases in the value of A will increase the value of N_F^* which solves the first order condition $V_e'(N_F) = 0$. Since $W = (1-\mu)(Y_0 + R)$, if the condition $(\frac{W}{AN})^{\frac{1}{\alpha}}\frac{N}{L} < 1$ holds, we have the following comparative results:

$$\frac{\partial N_F^*(\delta)}{\partial Y_0} > 0; \frac{\partial N_F^*(\delta)}{\partial R} > 0; \frac{\partial N_F^*(\delta)}{\partial \mu} < 0; \frac{\partial N_F^*(\delta)}{\partial A} < 0$$

where $N_F^*(\delta)$ denotes, given the set of parameter δ, the optimal N_F that solves the first order condition, $V_e'(N_F) = 0$. □

We now look at the situation where, given the set of parameter δ, $(\frac{W}{AN})^{\frac{1}{\alpha}}\frac{N}{L} > 1$ holds. According to our previous description, we could have that:

$$\left[1-\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{N_F}{L}\right]^{\alpha-1}\left[\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{(N-N_F)}{L}\right]^{1-\alpha} \left\{(1-\alpha)N_F\left[\frac{N-\left[\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{1}{L}\right]^{-1}}{\left(\left[\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{1}{L}\right]^{-1}-N_F\right)(N-N_F)}\right]+1\right\} > 1$$

Since $\frac{1}{p} \geq 1$, the comparative static between the value of $V_e'(N_F)$ and the value of $\frac{W}{AN}$ is ambiguous. Given the condition that $\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{N}{L} > 1$, $\frac{\partial(-V_e'(N_F))}{\partial(\frac{W}{AN}}$) is positive if the following condition holds:

$$\left[1-\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{N_F}{L}\right]^{\alpha-1}\left[\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{(N-N_F)}{L}\right]^{1-\alpha}$$
$$\left\{(1-\alpha)N_F\left[\frac{N-\left[\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{1}{L}\right]^{-1}}{\left(\left[\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{1}{L}\right]^{-1}-N_F\right)(N-N_F)}\right]+1\right\}>\frac{1}{p} \quad (3.14)$$

We differentiate the left hand side of the above equation by $\frac{W}{AN}$ and we have $\frac{\partial LHS}{\partial(\frac{W}{AN})}>0$. This implies that, given the condition $\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{N}{L}>1$, any increases in the value of $\frac{W}{AN}$ makes (3.14) more likely to be satisfied.

From above, we have seen the fact that the comparative static result depends on whether the condition $\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{N}{L}<1$ holds. This term could be rearranged as following:

$$AL^{\alpha}N^{1-\alpha}>W$$

which is intuitive. Here, $AL^{\alpha}N^{1-\alpha}$ represents the total produced agricultural revenue, and W represents the aggregate payoff for the farmer if they attempt a revolution. If $\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{N}{L}<1$ holds, the total produced agricultural revenue is greater than farmers' expectation of revolution. In our model society, if the farmer successfully revolt, they will expropriate all non-agricultural wealth from the landed elite, sharing both the private wealth and the state wealth. If the expectation of revolution is high, the landed elite holds considerable non-agricultural wealth. Under the circumstance that $AL^{\alpha}N^{1-\alpha}>W$, given a fixed μ, the agricultural incomes takes a greater share of the aggregate wealth holding by the landed elite. So that, in this chapter, we say that agriculture is a strong industry if the condition $AL^{\alpha}N^{1-\alpha}>W$ holds. However, given the fixed μ, if $AL^{\alpha}N^{1-\alpha}<W$, the agricultural income income takes a smaller share compared with the previous circumstance, and in this chapter, we say that agriculture is a relatively weak industry.

From the comparative static results we have derived above, we have seen that the development of agriculture determines how the landed elite would prefer to approach the removal of the threat. We then summarize the comparative static results as following:

Proposition 2. (Comparative Statics)

- *Given the set of parameter δ, suppose the condition $AL^{\alpha}N^{1-\alpha} > W$ holds, agriculture is a strong industry.*

  - *An increase in the private wealth of the landed elite or in the state wealth will increase the optimal number of private farms that the landed elite would prefer to set up,*

$$\frac{\partial N_F^*(\delta)}{\partial Y_0} > 0 \quad and \quad \frac{\partial N_F^*(\delta)}{\partial R} > 0.$$

  - *An increase in the political power of the landed elite will lower the optimal number of the private farms that the landed elite would prefer to set up,*

$$\frac{\partial N_F^*(\delta)}{\partial \mu} < 0.$$

  - *An improvement in the farming technology will lower the optimal number of the private farms that the landed elite would prefer to set up,*

$$\frac{\partial N_F^*(\delta)}{\partial A} < 0.$$

- *Given the set of parameter δ, suppose the condition $AL^{\alpha}N^{1-\alpha} < W$ holds, and agriculture is relatively weak.*

  - *The actual comparative static results are ambiguous.*

  - *An increase in the private wealth of the landed elite or in the state wealth makes it more likely that the landed elite will cut down the number of private farms,*

$$\frac{\partial N_F^*(\delta)}{\partial Y_0} < 0 \quad and \quad \frac{\partial N_F^*(\delta)}{\partial R} < 0$$

  - *A decrease in the political power of the landed elite makes it more likely that the landed elite will cut down the number of private farms,*

$$\frac{\partial N_F^*(\delta)}{\partial \mu} > 0.$$

– *If the farming technology is less developed, it is more likely that the landed elite will cut down the number of private farms,*

$$\frac{\partial N_F^*(\delta)}{\partial A} > 0.$$

This proposition could be explained as following. From above, we have seen the fact that any increases in the value of Y_0 or R induces the landed elite to set up more private farms if agriculture is a strong industry ($AL^\alpha N^{1-\alpha} > W$). The values of Y_0 and R denote the given wealth of the landed elite, and they measure the inequality between the landed elite and the farmers. The greater is the given wealth, the more willing is each farmer to attempt a revolution. To remove the threat of revolution, the landed elite should pay each farmer a higher wage or set up a private farm that generates a higher revenue to each farmer. Given a fixed level of p, any increases in the value of Y_0 or R raise the marginal wage cost up. However, since the condition $AL^\alpha N^{1-\alpha} > W$ holds, agriculture is strong, land is a valuable asset, and every single piece of land contributes to a higher collected revenue. Therefore, given an increased value of Y_0 or R, the required size of each private farm increases on a moderate scale, and the marginal losses of setting up private farms for the farmer is limited. Under this circumstance, for the landed elite, since their marginal losses from the land reform could be fully offset by the marginal savings from the wage cost, given an increased value of Y_0 or R, they would prefer to increase the number of private farms to save the wage cost. Compared with the income redistribution, the land reform is a more cost saving policy.

Suppose the value of Y_0 or R keep increasing, the condition $AL^\alpha N^{1-\alpha} > W$, will no longer be satisfied. If $AL^\alpha N^{1-\alpha} < W$, given a fixed level of μ, agriculture is relatively weak comparing with previous circumstance. Total produced agricultural income is smaller than the expected payoff for the farmer of attempting a revolution, and the total produced revenue from the land takes a smaller share of the aggregate wealth of the landed elite. Under this circumstance, land is a less valuable asset, and the required size of each private farm increases significantly in the value of Y_0 or R with the result that land reform is not an cost saving policy in removing the threat of revolution. In the end, for the landed elite, his marginal cost from the land reform exceeds the marginal savings in wage cost, meaning that it is beneficial for the landed elite to hire more farmers at an improved wage, and the optimal number of private farms decreases in the value of Y_0 or R.

From above proposition, we have also seen the fact that an increase in the value of μ affects the optimal number of private farms, N_F^*, in an opposite way to the given wealth, Y_0 and R. The comparative static result between μ and N_F^* highlights how the political power of the landed elite shapes its policy in consolidating its current control over the society. Since μ measures the damage that is generated by the revolution, the greater is this damage, the stronger is the landed elite that the less willing is each farmer to attempt a revolution. In addition, the marginal wage cost and the required size of each private farm decreases in the value of μ. If the condition, $AL^{\alpha}N^{1-\alpha} < W$, holds, agriculture is relatively weak, and land is a less valuable asset. Under this circumstance, even the required wage rate decreases, the marginal revenue of hiring an extra labour is limited, and it is less beneficial for the landed elite to enlarge their labour input. At the same time, since land is a less valuable asset and the required size for each private farm decreases in μ, for the landed elite, the marginal losses from the land reform is moderate. It is beneficial for the landed elite to set up more private farms and save more wage costs. Suppose the value of μ keeps increasing, the marginal cost of labour decreases and eventually is smaller than the marginal losses from the land reform. Under this circumstance, it is beneficial for the landed elite to cut down the number of private farms and hire more labour. Given an increasing μ, the condition, $AL^{\alpha}N^{1-\alpha} > W$, finally holds, agriculture becomes relatively strong, and land turns to be a valuable asset. Marginal revenue of hiring an extra labour increases, and marginal losses that are generated by giving up landholdings increase. Under this circumstance, for the landed elite, their marginal losses from the reform exceed the marginal savings from the wage cost, and it is beneficial for them to preserve more landholdings in hand and hire more labour.

An improvement in farming technology, A, affects the optimal number of private farms in a similar way to the political power held by the landed elite, μ. An increase in A raises the total productivity of the agricultural industry so that, given the fixed threat of revolution, the required size of each private farm comparatively decreases. Under the circumstance of $AL^{\alpha}N^{1-\alpha} < W$, land is less valuable. Given an increased value of A, since the required size of each private farm decreases and the marginal revenue of hiring an extra labour is moderate, marginal losses of the land reform could be fully offset by the marginal savings in the wage cost. So that, it is beneficial for the landed elite to give up landholdings to set up more private farms. Suppose the value of A keeps increasing, marginal losses from the land reform increases, and eventually exceeds the marginal savings in the wage cost. Under this circumstance, for the landed elite, it is beneficial to cut down the number of private farms in the end. However, if $AL^{\alpha}N^{1-\alpha} > W$, agriculture is relative strong, an increase in the total factor productivity of agriculture, A, further favours the development of agriculture, and land becomes a more valuable asset. Under this circumstance, even the required size of each private farm decreases in A, marginal losses from land reform is still high that exceed the marginal savings in the wage cost. Therefore, the landed elite prefer to preserve more landholdings in hand and hire more labour at the revised wage rate to achieve a higher agriculture income.

Quantity of land, L, influences the optimal N_F in an opposite way, compared with the factor, $\frac{W}{AN}$. Given the set of parameter, δ, if the condition $AL^{\alpha}N^{1-\alpha} < W$ holds, the optimal N_F increases in L in the first place. Suppose the value of L keeps increasing, the optimal number of N_F^* eventually decreases in L. However, if $AL^{\alpha}N^{1-\alpha} > W$, the optimal number of N_F^* always increases decreases in L.

Proof. We firstly differentiate $V_e'(N_F)$ by L, and we have:

$$\frac{\partial(-V_e'(N_F))}{\partial L} = -\alpha\frac{W}{AN}L^{-\alpha-1}\Bigg\{\left[1-\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{N_F}{L}\right]^{\alpha-1}\left[\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{(N-N_F)}{L}\right]^{1-\alpha} \left[(1-\alpha)N_F\left[\frac{N-\left[\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{1}{L}\right]^{-1}}{\left(\left[\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{1}{L}\right]^{-1}-N_F\right)(N-N_F)}\right]+1\right]-\frac{1}{p}\Bigg\} \tag{3.15}$$

L will negatively influence the value of N_F^* if the following condition holds:

$$\left[1-\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{N_F}{L}\right]^{\alpha-1}\left[\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{(N-N_F)}{L}\right]^{1-\alpha} \left[(1-\alpha)N_F\left[\frac{N-\left[\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{1}{L}\right]^{-1}}{\left(\left[\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{1}{L}\right]^{-1}-N_F\right)(N-N_F)}\right]+1\right]-\frac{1}{p}<0 \tag{3.16}$$

From our previous analysis, we have that once $AL^\alpha N^{1-\alpha} > W$ holds, the condition (3.16) always holds as well. However, if $AL^\alpha N^{1-\alpha} < W$, we can not determine the comparative static between L and N_F^*. Under the circumstance, L will positively influence the value of N_F^*, if the following condition holds:

$$\left[1-\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{N_F}{L}\right]^{\alpha-1}\left[\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{(N-N_F)}{L}\right]^{1-\alpha} \left[(1-\alpha)N_F\left[\frac{N-\left[\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{1}{L}\right]^{-1}}{\left(\left[\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{1}{L}\right]^{-1}-N_F\right)(N-N_F)}\right]+1\right]-\frac{1}{p}>0 \tag{3.17}$$

Otherwise, L will negatively influence the value of N_F^*. We then differentiate the left hand side of the above condition by L, and we have $\frac{\partial LHS}{\partial L} < 0$. That is to say, any increases in the value of L make the above condition less likely to be satisfied. If the landed elite has been given with an increased quantity in its landholdings, it would prefer to increase the number of private farms but in a diminishing way. This also implies that, if the condition $AL^\alpha N^{1-\alpha} < W$ holds, any increases in the value of L makes (3.16) more likely to be satisfied, and eventually, for the landed elite, it is beneficial to cut down the number of private farms. □

Proposition 3. (Comparative Statics)

- *Given the set of parameter δ, suppose the condition $AL^{\alpha}N^{1-\alpha} > W$ holds. An increase in the given landholdings of the landed elite will decreases the optimal number of private farms that the landed elite would prefer to set up:*

$$\frac{\partial N_F^*(\delta)}{\partial L} < 0$$

- *If $AL^{\alpha}N^{1-\alpha} < W$, the comparative static between L and N_F^* is ambiguous. If the condition (3.17) holds, the optimal number of private farms, N_F^* is strictly increasing in L but in a diminishing way. Otherwise, the landed elite would prefer to cut down the number of private farms.*

From above proposition, we have seen how land resources, L, influence the optimal number of private farms, N_F^*. Suppose the landed elite has been given a greater quantity of land, on one hand, it has more land to farm, and therefore can achieve a higher agricultural income. On the other hand, the landed elite has more land to distribute, so that more farmers could become the land reform beneficiaries. The comparative static result between land resources and the optimal number of private farms depends on whether the agricultural industry is strong or not. Given the set of parameter, δ, if the condition $AL^{\alpha}N^{1-\alpha} < W$ holds, agriculture is comparatively weak, and land is a relatively less valuable asset. Under this circumstance, the required size of each private farm is relatively large. Even so, increases in the value of L imply that the landed elite has more land to distribute, and its marginal losses from the land reform is moderate that could be fully offset by its savings in marginal wage cost. So that, an increase in land resources encourage the landed elite to set up more private farms, and the optimal number of private farms increases. However, since the expected payoff from agriculture increases in L, agriculture turns out to be attractive and marginal losses from the land reform increase if the landholdings keep increasing. Du to this, the positive influence towards N_F^* that has been brought by the increasing L disappears gradually, and N_F^* decreases in L in the end. Suppose the value of L keeps increasing, and the condition $AL^{\alpha}N^{1-\alpha} >$

W holds. Under this circumstance, agriculture is a relatively strong industry, and an increase in land resource induces the landed elite to hire more labour at the revised wage rate so that it can achieve a higher produced revenue by farming the increasing land resources. However, in this model, we have not considered the potentially heterogeneous ability of each farmer. If the aggregate landholdings are beyond the total ability of N farmers, any increases in L cannot contribute to increase the total produced revenue of agriculture. Under these circumstances, the marginal cost of the land reform fixed, since the landed elite has more land to distribute, and therefore prefer to set up more private farms.

3.3.3 Choice of the Optimal Policy

From the previous section, we have seen the fact that, if the condition $AL^{\alpha}N^{1-\alpha} \neq W$ holds, $V_e(N_F)$ is a strict concave function. There always exists a unique N_F that could solve the profit maximizing problem of the landed elite. Here, if the value of $\frac{\partial V_e(N_F)}{\partial N_F}$ is positive at $N_F = 0$, the optimal N_F is positive, and the landed elite would prefer to impose a land reform. However, if the value of $\frac{\partial Y_e(N_F)}{\partial N_F}$ is negative at $N_F = 0$, any positive N_F will lower the expected payoff for the landed elite, $V_e(0) > V_e(N_F \mid N_F > 0)$. Compared with the land reform, hiring all farmers at the revised wage rate gives the landed elite a higher payoff. Therefore, the landed elite will not impose a land reform and will make income redistribution to all farmers. We therefore set up the condition under which the landed elite will impose a land reform:

$$V_e'(0) = AL^{\alpha}\alpha\left[-\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{1}{L}\right]N^{1-\alpha} - AL^{\alpha}(1-\alpha)N^{-\alpha} + \frac{W}{N*p} > 0$$

We then rearrange the above condition and we have:

$$AL^{\alpha}\alpha\left[\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{1}{L}\right]N^{1-\alpha} + AL^{\alpha}(1-\alpha)N^{-\alpha} < \frac{W}{N*p}$$

$$\alpha\left[\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{N}{L} - 1\right] < \frac{W}{AN*p}\left(\frac{N}{L}\right)^{\alpha} - 1 \tag{3.18}$$

If the condition holds, the landed elite will impose a land reform, set up N_F^* private farms, and hire $N - N_F^*$ farmers at the revised wage rate. Given the set of parameter δ, if $AL^{\alpha}N^{1-\alpha} < W$, the above condition could be simplified as following:

$$\alpha < \frac{\frac{W}{AN*p}\left(\frac{N}{L}\right)^{\alpha} - 1}{\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{N}{L} - 1} \tag{3.19}$$

If agriculture is a relatively weak industry,$AL^{\alpha}N^{1-\alpha} < W$ holds, the landed elite would prefer to impose a land reform if the set of parameter δ satisfies the condition (3.19). In this situation, the landed elite gains from the land reform, its savings in wage costs exceed the loss of the collected revenue from the lost land. However, if $AL^{\alpha}N^{1-\alpha} > W$, agriculture is a relatively strong industry, condition (3.18) could be modified as follows:

$$\alpha > \frac{\frac{W}{AN*p}\left(\frac{N}{L}\right)^{\alpha} - 1}{\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{N}{L} - 1} \tag{3.20}$$

If agriculture is strong, the landed elite would prefer to impose a land reform if the set of parameter δ satisfies condition (3.20).

We have already discussed in the previous section that if the set of parameters, δ, satisfies the condition $AL^{\alpha}N^{1-\alpha} = W$, the landed elite will give up all its land to all farmers so that $N_F^* = N$. We could have this finding from condition (3.18) as well. If $AL^{\alpha}N^{1-\alpha} = W$, the left hand side of (3.18) equates to zero. Since $p \in (0,1]$, $\frac{W}{AN*p}\left(\frac{N}{L}\right)^{\alpha} > 1$, the right hand side of (3.18) is positive. Therefore, given the set of parameter δ, if $AL^{\alpha}N^{1-\alpha} = W$, condition (3.18) always holds, and the landed elite always sees the land reform as an attractive policy. We summarize the decision-making process of the landed elite over the choice of the optimal strategy as follows:

Proposition 4. *Given the set of parameter δ, suppose there exists the threat of revolution:*

- *If $AL^{\alpha}N^{1-\alpha} = W$, the landed elite would prefer to impose a land reform, give up all his landholdings to N farmers.*

- *If $AL^{\alpha}N^{1-\alpha} < W$ and δ also satisfies the condition (3.19), the landed elite will impose a land reform, set up N_F^* private farms and hire $N - N_F^*$ farmers at the revised wage rate, ω. Otherwise, the landed elite will make income distribution to all farmers.*

- *If $AL^{\alpha}N^{1-\alpha} > W$ and δ also satisfies condition (3.20), the landed elite will impose a land reform, distribute land to N_F^* farmers and make income redistribution to $N - N_F^*$ farmers. Otherwise, the landed elite will hire all farmers at the revised wage rate, ω.*

Now we will look at how changes in the set of parameter, δ, affect the decision of the landed elite. We firstly look at the circumstance that $AL^{\alpha}N^{1-\alpha} < W$. We differentiate the right hand side of (3.19) by $\frac{W}{AN}$ and we have $\frac{\partial RHS}{\partial(\frac{W}{AN})} < 0$. This implies that any increases in the value of $\frac{W}{AN}$ makes (3.19) less likely to be satisfied. There are several ways of increasing the value of $\frac{W}{AN}$: raising the private wealth of the landed elite, Y_0; raising the state wealth, R; lowering the power of the landed elite, μ. All these factors contribute to lower the incentive of the landed elite to attempt a land reform. In addition, if farming technology is less developed, the value of $\frac{W}{AN}$ is low as well. These comparative static results are intuitive. Any increases in the given wealth of the landed elite or any decreases in the political power of the landed elite make the conflict between the landed elite and the farmer more intensive. Therefore, the landed elite should provide each farmer a higher incentive of staying with its control and not revolt. If agriculture is a relatively weak industry, the land turns out to be a less valuable asset, the required size of each private farm increases, and the marginal cost of the land reform also increases. Under this circumstance, the landed elite is less likely to impose a land reform, and the income redistribution is a more cost saving policy. If the farming technology is less developed, the collected revenue from each piece of land is limited, and the required size of each private farm is high. Therefore, the land reform is less effective in removing the threat of revolution, and it is less likely that the landed elite will distribute its land to the farmer. However, if the set of parameter δ satisfies the condition, $AL^{\alpha}N^{1-\alpha} > W$, the value of $\frac{W}{AN}$ works in an opposite way compared with the previous circumstance. When agriculture is a relatively strong industry, the land turns out to be a valuable asset, and land reform becomes more effective in removing the threat of revolution. It is more likely that the landed elite will impose a land reform when faces the threat of revolution.

The quantity of land, L, affects the decision making process of the landed elite in an opposite way compared with the value of $\frac{W}{AN}$. Suppose, given the set of parameter δ, the condition $AL^{\alpha}N^{1-\alpha} < W$ holds. We differentiate the right hand side of (3.19) by L, and we have $\frac{\partial RHS}{\partial L} > 0$. This implies that an increase in the value of L makes the condition (3.19) more likely to be satisfied. This fact is intuitive. If the condition, $AL^{\alpha}N^{1-\alpha} < W$, holds, agriculture is a relatively weak industry, and the land is a less valuable asset. Although, the required size of each private farm increases, marginal losses from the land reform are limited. Under this circumstance, if the quantity of the land increases, the landed elite has more lands to distribute so that the land reform turns to be more attractive. However, if $AL^{\alpha}N^{1-\alpha} > W$, since $\frac{\partial RHS}{\partial L} > 0$, any increases in the value of L make the condition (3.20) less likely to be satisfied. If the condition $AL^{\alpha}N^{1-\alpha} > W$ holds, agriculture is a relatively strong industry and therefore each piece of land generates a higher revenue to the landowner. Under the circumstance, the landed elite is more likely to keep its landholdings and make the income redistribution to all farmers.

Proposition 5. *Given the set of parameter δ:*

- *If agriculture is a weak industry ($AL^{\alpha}N^{1-\alpha} < W$):*
  - *If the landed elite is strong in power, it is more likely it will impose a land reform;*
  - *Any developments in the farming technology make the land reform a more attractive policy;*
  - *If the landed elite holds a greater quantity of wealth, it is more likely it will make income redistribution to all farmers rather than impose a land reform, distributing part of his landholdings to a group of farmers.*
  - *If the landed elite holds a greater quantity of land, it is more likely it will impose a land reform.*
- *If agriculture is a strong industry ($AL^{\alpha}N^{1-\alpha} > W$):*

- *If the landed elite is strong in power, it is more likely it will keep all his landholdings and make income redistribution to all farmers;*

- *Any improvements in farming technology make the land a more valuable asset, and the landed elite is more likely to keep all its landholdings and hire all farmers at the revised wage rate;*

- *If the landed elite has been given with a greater quantity of wealth, it is more likely it will impose a land reform.*

- *If the landed elite holds a greater quantity of land, it is more likely it will make the income redistribution to all farmers.*

Now we look at how the value of p affects the decision-making process of the landed elite. From the condition (3.18), we have seen the fact that an increase in the value of p lower the value of the right hand side. Therefore, the condition (3.18) is less likely to be satisfied, and it is more likely for the landed elite to make income redistribution to all farmer. An increase in the value of p denotes the fact that the landed elite is considered more trustworthy by the majority and the quality of institution is better. The majority considers the commitment of the income redistribution a more credible promise, therefore the required wage rate decreases and the marginal cost of the income redistribution decreases. Any improvement in the quality of institution makes the income redistribution a more attractive policy in removing the threat of revolution, and this enables the landed elite to keep more landholdings.

We examine the case of $p = 1$. Under this circumstance, since the majority fully trust the landed elite to make the income redistribution at the end of the period, the required wage rate equates to the expected payoff of attempting a revolution for each farmer. Given this, the landed elite will impose a land reform if the following condition is satisfied:

$$\alpha\left[\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{N}{L}-1\right] < \frac{W}{AN}\left(\frac{N}{L}\right)^{\alpha}-1$$

We rearrange the above condition as:

$$\frac{W}{AN}\left(\frac{N}{L}\right)^{\alpha}-\alpha\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{N}{L}+\alpha>1 \tag{3.21}$$

In the Appendix, section 3.6, we mathematically prove that given any possible set of parameter $\{\mu, R, Y_0, A, N, L, \alpha\}$, the condition (3.21) will never be satisfied. That is to say, if $p = 1$, the landed elite prefers to hire all farmers at the revised wage rate, $\omega = \frac{(1-\mu)(Y_0+R)}{N}$. Under this circumstance, the marginal losses from the lost land always exceeds the marginal savings of the wage costs, and therefore the landed elite will not give up any piece of land to farmers. If the institution is well developed and the majority fully trust the commitment made by the landed elite, land reform becomes a less effective policy than income redistribution.

Proposition 6. *An increase in the value of p makes it more likely that the landed elite will make the income redistribution to all farmers. If there is no commitment problem ($p = 1$), the landed elite will hire all farmers at a revised wage rate that could substantially reduce the threat of revolution. Under this circumstance, land reform will not take place.*

3.4 Sketch of the Policy Determination Model

This chapter develops a model that investigates how, given a threat of revolution, the landed elite works to improve the national income of the majority in order to preserve its political position. We compare two policies: income redistribution and land reform. The landed elite could apply these two policies simultaneously, it could distribute part of its landholdings to a groups of farmers, let them set up a private farm, and hire the rest of the farmers at an improved wage rate. We mainly look at the equilibrium number of private farms that could maximize the net income of the landed elite and also remove the threat of revolution. According to our theory, the determination of the optimal policy depends on the term, $\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{N}{L}$, which defines the relative development of agriculture. If $\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{N}{L} > 1$ $(AL^{\alpha}N^{1-\alpha} < W)$, the landed elite will impose a land reform if the set of parameter δ satisfies the condition (3.19). Under this circumstance, any increases in the value of $\frac{W}{A}$ or decreases in the value of L make the land reform a less attractive policy, and the landed elite is more likely to hire all farmers at the improved wage rate. However, if $\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{N}{L} < 1$ ($AL^{\alpha}N^{1-\alpha} > W$), the landed elite will impose a land reform if the set of parameter δ satisfies condition (3.20). Under this circumstance, $\frac{W}{A}$ and L affect the optimal policy in an opposite way compared with the previous circumstance. Any increases in the value of $\frac{W}{A}$ or any decreases in the value of L induce the landed elite to increase the number of private farms so that the land reform turns out to be a more attractive policy. In our model, we have also seen the fact that any increases in the value of p make it less likely that the landed elite will impose a land reform. Especially, if $p = 1$, in any circumstances, the landed elite will always make the income redistribution to all farmers and the land reform will not take place.

To illustrate how our theory model works, we enumerate our model with actual figures. In this section, given the set of parameter δ, we draw the figure of expected income of the landed elite that varies with the number of private farms, N_F:

$$V_e(N_F) = AL^{\alpha}\left[1 - \left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{N_F}{L}\right]^{\alpha}(N - N_F)^{1-\alpha} - \frac{W}{N * p}(N - N_F) + Y_0 + R$$

Now, we will restrict the parameters to a reasonable region, that $A \in (0, 20)$, $L \in (50, 10000)$, $\alpha \in (0, 1)$, $\mu \in (0, 1)$, $Y_0 \in (500, 500000)$, $R \in (500, 500000)$, $N \in (500, 500000)$, and $p \in (0, 1]$. Given the set of parameter δ, we sketch the graph for the expected payoff for the landed elite. According to our theory, the income function should be a strict concave function. If the function is maximized at the point where N_F is positive, the landed elite will impose a land reform. Otherwise, if the function is maximized at the point where the optimal N_F is negative, the landed elite will make the income redistribution to all farmers. Firstly, we set the set of parameter δ as following, $\delta = \{A = 3.46, L = 8630, N = 100000, \alpha = 0.324, \mu = 0.73, p = 0.858, Y_0 = 301000, R = 1000\}$. Given this set of parameter δ, we calculate that $\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{N}{L} = 0.134$ which is smaller than 1. We also calculate that:

$$\frac{\frac{W}{ANp}\left(\frac{N}{L}\right)^{\alpha} - 1}{\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{N}{L} - 1} = 0.45$$

which is greater then α. Therefore, according to our theory, it is optimal for the landed elite to make income redistribution to all farmers. In Figure 3.1, x axis denotes the number of private farmers, N_F, and y axis denotes the expected payoff for the landed elite, $V_e(N_F)$. The blue line describes how the expected payoff varies with the number of private farms if $\mu = 0.73$. From Figure 3.1, we have seen that any positive number of private farms lowers the payoff for the landed elite. So that, given the set of parameter, $\delta = \{A = 3.46, L = 8630, N = 100000, \alpha = 0.324, \mu = 0.73, p = 0.858, Y_0 = 301000, R = 1000\}$, the optimal policy is to make income redistribution to all farmers, and the landed elite will be worse off if it distributes its land to the majority.

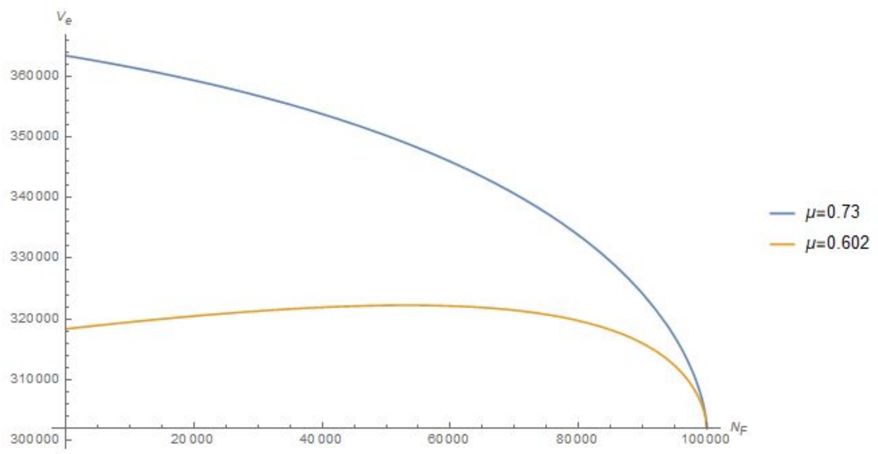


Figure 3.1: Expected income for the landed elite with $\mu = 0.73$ and $\mu = 0.602$

With other parameters unchanged, we lower the value of μ, and let $\mu = 0.602$. That is to say, the landed elite is weaker in power, and the majority has a stronger incentive to revolt. We calculate that $\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N}{L} = 0.44$, which is again smaller than 1. We also have:

$$\frac{\frac{W}{AN*p}\left(\frac{N}{L}\right)^{\alpha} - 1}{\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N}{L} - 1} = 0.188$$

which in this case is smaller then α. According to our theory, it is beneficial for the landed elite to impose a land reform, and there exists a positive N_F that maximizes the expected payoff for the landed elite. In Figure 3.1, the yellow line describes the expected payoff for the landed elite if $\mu = 0.602$. According to the yellow line in Figure 3.1, we have seen that there exists one unique and positive N_F that maximizes the expected payoff for the landed elite. Compared with making income redistribution to all farmers, it is beneficial for the landed elite to impose a land reform. In addition, a decrease in the value of μ would lower the expected payoff for the landed elite. This is generated by the increased threat of revolution, meaning that the landed elite should give more concessions to the majority.

We then lower the value of μ to $\mu = 0.544$, and we have $\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N}{L} = 0.67$, which is smaller than 1. We also calculate that:

$$\frac{\frac{W}{AN*p}\left(\frac{N}{L}\right)^{\alpha} - 1}{\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N}{L} - 1} = -0.08$$

which is smaller than α. According to our theory, the decrease in the value of μ will increase the number of private farms.

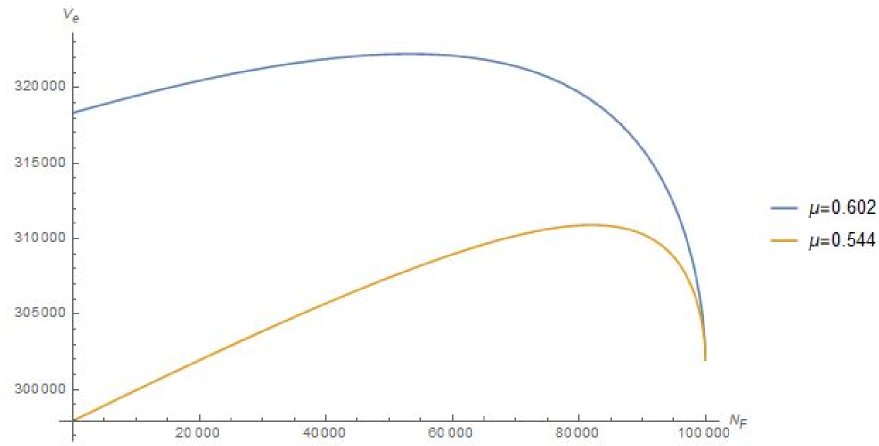


Figure 3.2: Expected income for the landed elite with $\mu = 0.602$ and $\mu = 0.544$

In Figure 3.2, the yellow line represents the expected payoff for the landed elite with $\mu = 0.544$. We have seen the fact that, compared with the circumstance of $\mu = 0.602$, the optimal number of private farms, N_F^*, increases. That is to say, it is beneficial for the landed elite to increase the number of private farms and cut down its hired labour. If the value of μ is getting smaller, the landed elite becomes weak in political power, and revolution generates decreasing losses to the majority. Therefore, from Figures 3.1 and 3.2, we have seen that a landed elite who is stronger in political power is more likely to make income redistribution to all farmers.

We further lower the value of μ to $\mu = 0.428$, and the landed elite becomes weaker in power. Since we calculate that $\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N}{L} = 1.358$ which is greater than 1, it is beneficial for the landed elite to impose a land reform if δ satisfies the condition (3.19). We calculate:

$$\frac{\frac{W}{AN*p}\left(\frac{N}{L}\right)^{\alpha} - 1}{\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N}{L} - 1} = 0.8$$

which is greater then the value of α. That is to say, under this circumstance, it is still beneficial for the landed elite to impose a land reform.

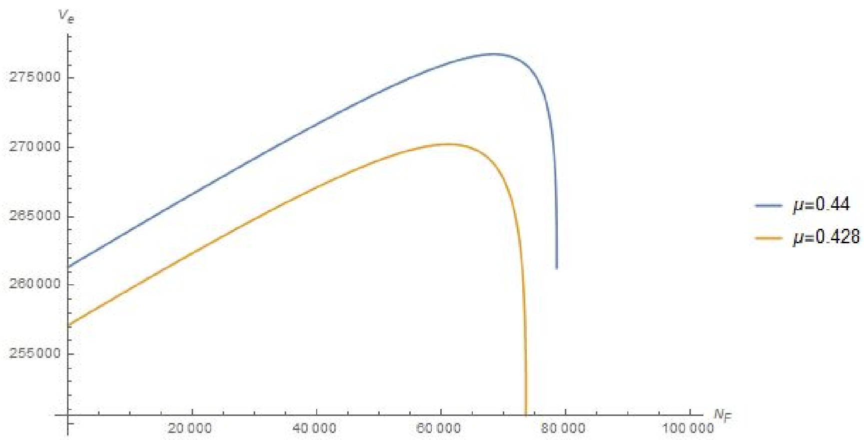


Figure 3.3: Expected income for the landed elite with $\mu = 0.44$ and $\mu = 0.428$

In Figure 3.3, the blue line outlines the expected payoff for the landed elite if $\mu = 0.44$. We have seen that there exists a unique and positive N_F that maximizes the expected payoff for the landed elite. Under this circumstance, the landed elite will give part of its landholding to N_F^* farmers and hire the rest of the farmers at the revised wage rate. We then keep lowering the value of μ, making $\mu = 0.428$. We calculate that $\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N}{L} = 1.27$, which is greater than 1. We also calculate that:

$$\frac{\frac{W}{AN*p}\left(\frac{N}{L}\right)^{\alpha} - 1}{\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N}{L} - 1} = 0.96$$

which is greater than α so that there exists a unique and positive N_F that could maximize the expected payoff of the landed elite. Under this circumstance, the landed elite will impose a land reform. According to our theory, if the set of parameter, δ, satisfies the condition $\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N}{L} > 1$, any decreases in the value of μ makes it more likely that the landed elite will cut down the number of private farmers and hire more labour at the revised wage rate.

In Figure 3.3, we have seen the fact that the optimal N_F decreases if we lower the value of μ from 0.44 to 0.428. Under this circumstance, the landed elite will still impose a land reform, but it will cut down the number of private farms and prefer to redistribute income to more farmers.

We then lower the value of μ, and $\mu = 0.2$. We calculate that $\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N}{L} = 3.82$, which is greater than 1. We also have that:

$$\frac{\frac{W}{AN*p}\left(\frac{N}{L}\right)^{\alpha} - 1}{\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N}{L} - 1} = 0.28$$

which is smaller than the value of α. In our theory, suppose $\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N}{L} > 1$, the landed elite will impose a land reform if the set of parameter δ satisfies the condition (3.19). That is to say, if we lower the value of μ to 0.2, it is more beneficial for the landed elite to make the income redistribution to all farmers.

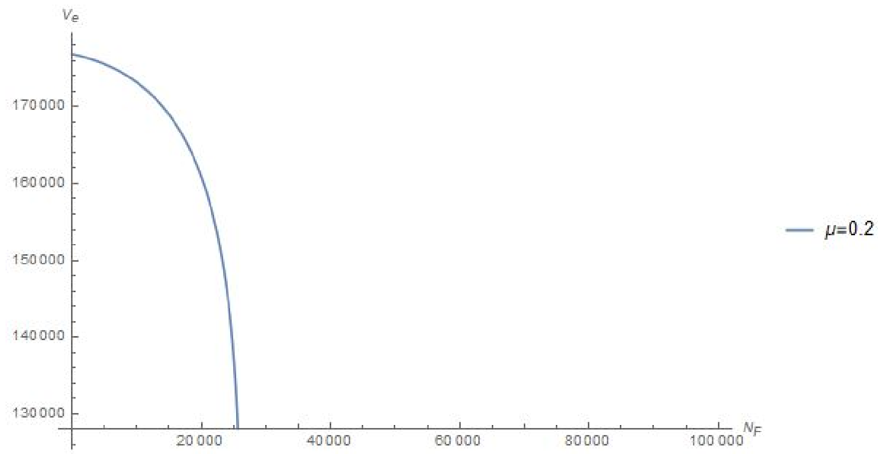


Figure 3.4: Expected income for the landed elite if $\mu = 0.2$

From Figure 3.4, we have seen that there is not a positive N_F that could maximize the expected payoff for the landed elite. Therefore, the landed elite will make the income distribution to all farmers and the land reform will not take place. Similarly, from Figures 3.3 and 3.4, we have seen the fact that if the condition $\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N}{L} > 1$ holds, a landed elite which is stronger in political power is more likely to impose a land reform which corroborates our theory.

To summarize, the simulation results illustrate the main theoretical proposition. The parameters Y_0, R, and A affect the determination of the optimal policy in a similar way as the parameter μ. We then examine how the land resources, L, affects the optimal policy in removing the threat of revolution. Firstly, we set the set of parameter δ as following, $\delta = \{A = 3.46, N = 100000, L = 10000, \alpha = 0.324, \mu = 0.672, p = 0.858, Y_0 = 301000, R = 1000\}$. Given the set of parameter, we calculate that $\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N}{L} = 0.21$, which is smaller than 1. We also have that:

$$\frac{\frac{W}{AN*p}\left(\frac{N}{L}\right)^{\alpha} - 1}{\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N}{L} - 1} = 0.375$$

which is greater than α. According to our theory, it is beneficial for the landed elite to make the income redistribution to all farmers. In Figure 3.5, the blue line describes the expected payoff for the landed elite if $L = 10000$, and we have seen that there is not a positive N_F that could maximize the expected payoff. That is to say, under this circumstance, redistributing income to all farmers is more cost saving as a means of removing the threat of revolution.

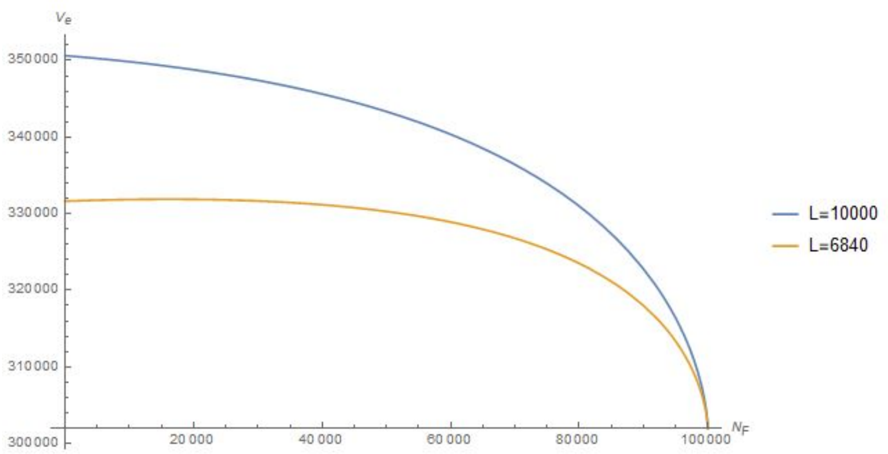


Figure 3.5: Expected income for the landed elite with $L = 10000$ and $L = 6840$

We then lower the value of L, and $L = 6840$. We calculate that $\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N}{L} = 0.3$, which is smaller than 1. Since we calculate that:

$$\frac{\frac{W}{AN*p}\left(\frac{N}{L}\right)^{\alpha} - 1}{\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N}{L} - 1} = 0.295$$

which is smaller than α, it is optimal for the landed elite to impose a land reform. In Figure 3.5, we have seen that, when $L = 6840$, there exists a unique and positive N_F that maximizes the expected income of the landed elite, and the landed elite is better off if it imposes a land reform. In addition, we have also seen that the reduction in the land resources lowers the expected payoff for the landed elite.

We then further reduce the land resources, and $L = 4990$. We calculate that $\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N}{L} = 0.4$, which is smaller than 1. We also have:

$$\frac{\frac{W}{AN*p}\left(\frac{N}{L}\right)^{\alpha} - 1}{\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N}{L} - 1} = 0.2$$

which is smaller than α so that there should exist a unique and positive N_F that maximizes the expected payoff for the landed elite. In addition, according to our theory, if $\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N}{L} < 1$, any decrease in the value of L will induce an increase in the value of the optimal N_F. In Figure 3.6, the yellow line describes the expected

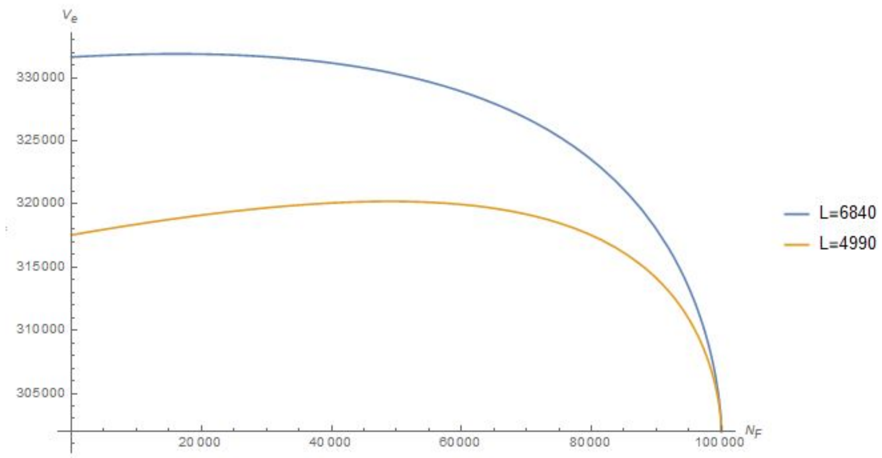


Figure 3.6: Expected income for the landed elite with $L = 6840$ and $L = 4990$

payoff function for the landed elite if $L = 4990$. We have seen that it is beneficial for the landed elite to impose a land reform. In addition, compared with the blue line which depicts the expected payoff of the landed elite if $L = 6840$, we have seen that the value of N_F that maximizes $V_e(N_F)$ increases. That is to say, the landed elite which holds more land will set up less private farms. From Figures 3.5 and 3.6, we have seen that if the condition $\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N}{L} < 1$ holds, the landed elite which holds a greater quantity of land is more likely to make the income redistribution to all farmers. This fact works with our described theory.

We further reduce the land resources, and $L = 1780$. We calculate that $\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N}{L} = 1.18$, which is greater than 1. Since we have that:

$$\frac{\frac{W}{AN*p}\left(\frac{N}{L}\right)^{\alpha} - 1}{\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N}{L} - 1} = 1.26$$

which is greater than α, distributing lands to farmers should generate a higher payoff to the landed elite. From Figure 3.7, we have seen that when $L = 1780$, the expected payoff function is maximized at a unique and positive N_F. That is to say, the landed elite is better off if it imposes a land reform and this result works with our theory. According to our theory, if $\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N}{L} > 1$, any decrease in the value

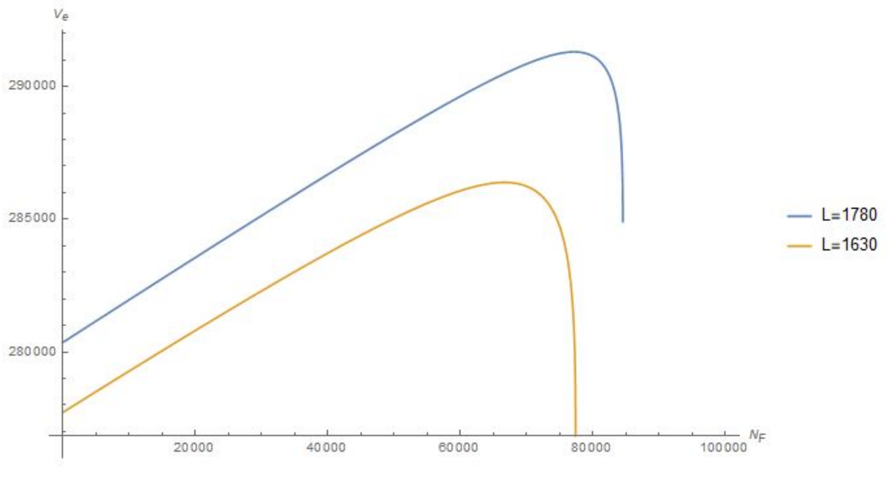


Figure 3.7: Expected income for the landed elite with $L = 1780$ and $L = 1630$

of L may generate a decrease in the value of the optimal N_F. We then further reduce the land resources, let $L = 1630$, to examine the theory. We calculate that $\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N}{L} = 1.29$, which is greater than 1. We also have that:

$$\frac{\frac{W}{AN*p}\left(\frac{N}{L}\right)^{\alpha} - 1}{\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N}{L} - 1} = 0.9$$

which is greater than α. Therefore, it is beneficial for the landed elite to impose a land reform. From Figure 3.7, we have seen the optimal N_F with $L = 1780$ is greater than the optimal N_F with $L = 1630$. This implies that a reduction in the land resources contributes to the decrease in the optimal number of private farms.

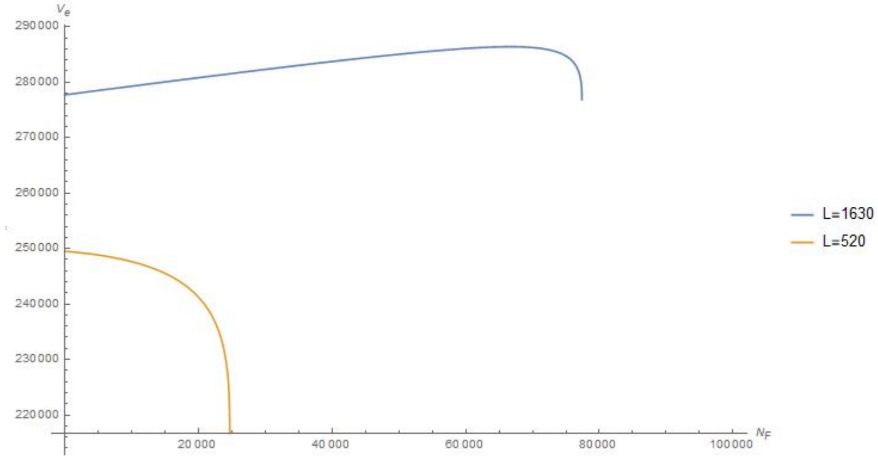


Figure 3.8: Expected income for the landed elite with $L = 1630$ and $L = 520$

We then decrease the value of L, and $L = 520$. We calculate that $\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N}{L} = 4.05$, which is greater than 1. Since we have:

$$\frac{\frac{W}{AN*p}\left(\frac{N}{L}\right)^{\alpha} - 1}{\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N}{L} - 1} = 0.27$$

which is smaller than α, it is beneficial for the landed elite to make the income redistribution to all farmers. From Figure 3.8, we have seen that if we further reduce the land resources, and $L = 520$, any positive N_F will lower the expected payoff for the landed elite. Under this circumstance, the landed elite will make the income redistribution to all farmers. In addition, from Figure 3.7 and 3.8, we have seen that if the condition $\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N}{L} > 1$ holds, land reform turns out to be more attractive to the landed elite which holds a greater quantity in land. This fact corroborates with our theory model.

In our theory, we argue that if $p = 1$, the landed elite will always make the income redistribution to all farmers and the land reform will never take place. Now we will examine the theory by drawing the figure of $V_e(N_F)$. Firstly, we set the set of parameter δ as follows, $\delta = \{A = 3.46, N = 100000, L = 6840, \alpha = 0.324, \mu = 0.672, p = 0.858, Y_0 = 301000, R = 1000\}$. In Figure 3.9, the blue line describes

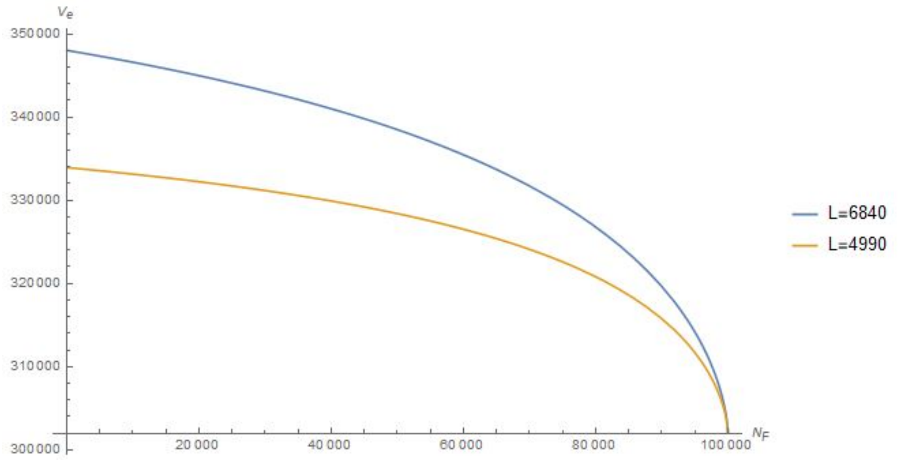


Figure 3.9: Expected income for the landed elite with $L = 6840$ and $L = 4990$ when $p = 1$. p: the probability that the farmer assumes that the landed elite will keep its promise and pay them at the end of the second period

the expected payoff for the landed elite if $L = 6840$. We then reduce the land resources, let $L = 4990$, and the yellow line in Figure 3.9 draws the function $V_e(N_F)$. Compared with Figure 3.6, we have seen that, given other parameters being unchanged, if we set p=1, the land reform turns out to be an unattractive policy and the landed elite will change its optimal policy, and make the income redistribution to all farmers. We then draw the graph for $L = 1780$ and $L = 1630$. We have

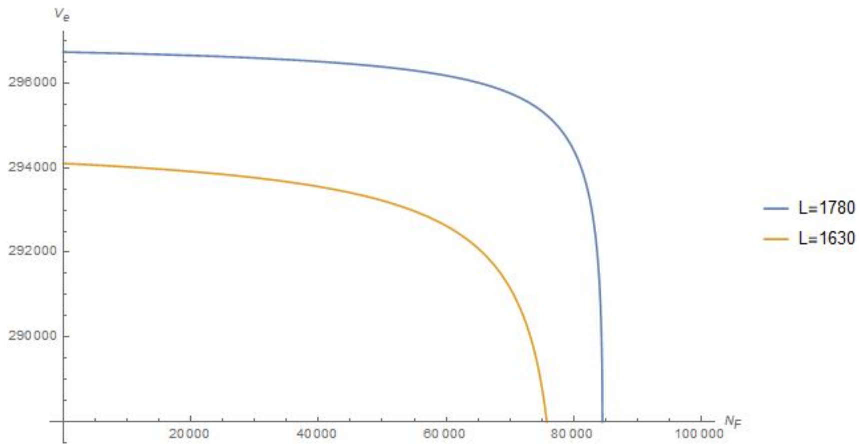


Figure 3.10: Expected income for the landed elite with $L = 1780$ and $L = 1630$ when $p = 1$. p: the probability that the farmer assumes that the landed elite will keep its promise and pay them at the end of the second period

seen from Figure 3.10 that the land reform will not take place under the two circumstances. Compared with Figure 3.7, if $p = 1$, the land reform turns out to be unattractive, and the landed elite will make the income redistribution to all farmers. That is to say, independent of the value of $\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} \frac{N}{L}$, if $p = 1$, the land reform will never take place. To summarize, our simulation result works with our described theory. Therefore, given the set of parameter δ, we could analyze the optimal policy of the landed elite in removing the threat of revolution.

3.5 Concluding Remarks

This chapter has offered a simple model of redistributive land reform. In our model, given the threat of revolution, the landed elite can distribute part of its landholdings to a group of farmers, let them establish individual private farms, and distribute part of its income to the rest of the farmers. Within this framework, we show that how the landed elite determines the coverage of land reform and the extent of income redistribution. The two main contributions of this chapter are firstly, we find that the optimal number of private farms depends on the development of agriculture. If agriculture is large relative to non-agricultural wealth, land reform will be an effective approach in solving the social unrest. However, if agriculture is no longer the primary industry of the state, income redistribution turns out to be a more attractive policy to the landed elite. Secondly, we find that the quality of institutions also influences the decision-making process of the landed elite. If the domestic institution is well developed and there exists no commitment problem between the landed elite and the farmers, the landed elite would prefer to distribute its income to all farmers, and land reform will not take place.

In this chapter, we take the non-agricultural income as given, and in our hypothetical society, there is only one unique good which is produced by the labour input and land. However, in reality, the development of the manufacturing industry poses a significant effect towards the political equilibria, especially in the nineteenth and twentieth century. In ongoing research, we intend to introduce the manufacturing industry into our described model, and investigate whether the development of manufacturing industry will affect the coverage of land reform. In addition, in our model, we argue that given the threat of revolution, the landed elite may prefer to distribute part of its land to the farmers with no compensation. In the following research, we intend to investigate, if we allow the land sale, whether the landed elite would prefer to give up its landholdings despite the absence of a threat from the landless people.

3.6 Appendix

We firstly rearrange inequality (3.21), and we have:

$$\frac{W}{AN}\left(\frac{N}{L}\right)^{\alpha} - \alpha\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{N}{L} > 1-\alpha \tag{3.22}$$

Since $\alpha \in (0,1)$, we then have the necessary condition for above inequality:

$$\frac{W}{AN}\left(\frac{N}{L}\right)^{\alpha} > \alpha\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{N}{L} \tag{3.23}$$

We then take natural logarithm of both sides of above inequality and we have:

$$ln\left(\frac{W}{AN}\right) + \alpha ln\left(\frac{N}{L}\right) - ln\alpha - \frac{1}{\alpha}ln\left(\frac{W}{AN}\right) - \ln\left(\frac{N}{L}\right) > 0 \tag{3.24}$$

We then differentiate the left hand side of (3.24) by α, and we have:

$$\begin{aligned}\frac{\partial LHS}{\partial \alpha} &= ln\left(\frac{N}{L}\right) - \frac{1}{\alpha} + \frac{1}{\alpha^2}ln\left(\frac{W}{AN}\right)\\ &= \frac{1}{\alpha}ln\left[\left(\frac{N}{L}\right)^{\alpha}\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\right] - \frac{1}{\alpha}\end{aligned} \tag{3.25}$$

If the condition $\left(\frac{N}{L}\right)^{\alpha}\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} > e$ holds, $\frac{\partial LHS}{\partial \alpha} > 0$. That is to say, the value of function $\frac{W}{AN}\left(\frac{N}{L}\right)^{\alpha} - \alpha\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{N}{L}$ increases in α, and the maximum value of this function equals to 0. So that, if the condition $\left(\frac{N}{L}\right)^{\alpha}\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} > e$ holds, inequality (3.21) will never be satisfied.

We then look at the circumstance that $\frac{\partial LHS}{\partial \alpha} < 0$, $\left(\frac{N}{L}\right)^{\alpha}\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}} < e$. Under this circumstance, the value of $\frac{W}{AN}\left(\frac{N}{L}\right)^{\alpha} - \alpha\left(\frac{W}{AN}\right)^{\frac{1}{\alpha}}\frac{N}{L}$ is maximized at the point where α is approaching to zero.

Suppose $\frac{W}{AN} \leq 1$, we then have:

$$lim_{\alpha \to 0} \left[\frac{W}{AN} \left(\frac{N}{L} \right)^{\alpha} - \alpha \left(\frac{W}{AN} \right)^{\frac{1}{\alpha}} \frac{N}{L} \right] = \frac{W}{AN} \leq 1 \tag{3.26}$$

We then take the second derivative of the left hand side of equation (3.24), and we have:

$$\frac{\partial LHS}{\partial^2 \alpha} = \alpha^{-2} - 2\alpha^{-3} ln \left(\frac{W}{AN} \right) \tag{3.27}$$

Since $\frac{W}{AN} \leq 1$ holds, $ln \left(\frac{W}{AN} \right) \leq 0$ and $\frac{\partial LHS}{\partial^2 \alpha} > 0$. Since we already have that $\frac{\partial LHS}{\partial \alpha} > 0$, the function $\frac{W}{AN} \left(\frac{N}{L} \right)^{\alpha} - \alpha \left(\frac{W}{AN} \right)^{\frac{1}{\alpha}} \frac{N}{L}$ is strictly concave on $\alpha \in (0, 1)$. In addition, the maximum value of this function approaches to $\left(\frac{W}{AN} \right) \leq 1$, and the minimum value approaches to zero. Since $1 - \alpha$ is a decreasing linear function on $(0, 1)$, we then have, for any $\alpha \in (0, 1)$, $\frac{W}{AN} \left(\frac{N}{L} \right)^{\alpha} - \alpha \left(\frac{W}{AN} \right)^{\frac{1}{\alpha}} \frac{N}{L} < 1 - \alpha$. That is to say, under the circumstance $\left(\frac{N}{L} \right)^{\alpha} \left(\frac{W}{AN} \right)^{\frac{1}{\alpha}} < e$, if $\left(\frac{W}{AN} \right) \leq 1$ holds, inequality (3.21) will never be satisfied.

We then look at the circumstance that $\left(\frac{N}{L} \right)^{\alpha} \left(\frac{W}{AN} \right)^{\frac{1}{\alpha}} < e$ holds and $\left(\frac{W}{AN} \right) > 1$. Similarly, under this circumstance, the function $\frac{W}{AN} \left(\frac{N}{L} \right)^{\alpha} - \alpha \left(\frac{W}{AN} \right)^{\frac{1}{\alpha}} \frac{N}{L}$ is decreasing in α where $\alpha \in (0, 1)$. However, since $\left(\frac{W}{AN} \right)$, by applying the Taylor Series, we could have that:

$$lim_{\alpha \to 0} \left[\alpha \left(\frac{W}{AN} \right)^{\frac{1}{\alpha}} \frac{N}{L} \right] = \infty$$

So that we have the maximized value of the function $\frac{W}{AN} \left(\frac{N}{L} \right)^{\alpha} - \alpha \left(\frac{W}{AN} \right)^{\frac{1}{\alpha}} \frac{N}{L}$:

$$lim_{\alpha \to 0} \left[\frac{W}{AN} \left(\frac{N}{L} \right)^{\alpha} - \alpha \left(\frac{W}{AN} \right)^{\frac{1}{\alpha}} \frac{N}{L} \right] = \frac{W}{AN} - \infty < 0 < 1 - \alpha \tag{3.28}$$

That is to say, under the circumstance that $\left(\frac{N}{L} \right)^{\alpha} \left(\frac{W}{AN} \right)^{\frac{1}{\alpha}} < e$ holds and $\left(\frac{W}{AN} \right) > 1$, for any $\alpha \in (0, 1)$, inequality (3.21) will never be satisfied.

To summarized, for any $\alpha \in (0, 1)$, inequality (3.21) will never take place. That is to say, if $\alpha \in (0, 1)$ and all other parameters are positive, we have the following inequality:

$$\alpha \left[\left(\frac{W}{AN} \right)^{\frac{1}{\alpha}} \frac{N}{L} - 1 \right] \geq \frac{W}{AN} \left(\frac{N}{L} \right)^{\alpha} - 1$$

Chapter 4

On the Size of Enfranchisement: An Application of the Lewis Model

4.1 Introduction

At the beginning of the nineteenth century, in most countries domestic politics was controlled by narrow elites. However, since then, the franchise has been extended in most western societies, property restrictions on voting were gradually released, and in the early twentieth century, women were allowed to vote. In Britain, for example, during the nineteenth century the state was transformed from an oligarchy to ever-greater democracy. Until 1832, parliament in England represent 'interests' of landed wealth, and capital associated with landed wealth engaged in commerce and finance. Between 1734 and 1832, the majority of MPs were landed in background, about 900 MPs were engaged in commerce and trade, over 200 MPs were bankers, and only twenty-nine MPs belonged to the manufacturing interest. The 1832 Reform Act reduced the property restrictions on voting. After this, the total electorate was increased to 813,000, the presence of urban freehold voters in counties increased, and parliamentary representation was also to be given to important centres of trade and manufacture. In 1866-7, Conservatives drafted and affirmed the reform bill which further lowered the qualification in both counties and boroughs. After Reform Act 1867, the franchise was further extended to the urban working class, the county electorate was also increased by 46 per cent, and the total electorate was expanded to 2.52 million (Keith-Lucas, 1952, 1977; Seymour, 1915). In addition to Britain, most western countries, such as France and Germany, also experienced enfranchisement during the nineteenth century. Extending the franchise to a greater population reduces the power of the elite in influencing policy and therefore democratization is in sharp contrast with the interests of the elite. Historians, political scientists, and economists have long discussed why the elite would prefer to gradually extend the suffrage during the nineteenth century.

The leading explanation is proposed by Acemoglu and Robinson (2000, 2001, 2005), who introduce the threat of revolution, and argue that to prevent revolution, the elite extends the franchise to the disenfranchised so that the future political equilibria could be changed and this acts as a credible commitment to redistribution. They also consider the theory of "middle-class drive." In their model, the landed elite corresponds to the rich, and the capitalist corresponds to the middle class. Since the middle class is relative rich compared to the poor, it is not likely to back a policy that works against the interests of the rich, and therefore, they are considered as a buffer between the rich and the poor. In view of this consideration, the upper class first extends the franchise to the middle class. Since the cost of revolution is higher for the capitalist, compared with that for the landed elite, the middle class plays the role of a softliner arguing against repression and in favour of further extending the franchise. According to their theory, comparing with the capitalist middle-class, it is more costly for the landed aristocracy to extend the suffrage to the general public.

The goal of this chapter is to rationalize an alternative explanation. Rather than being forced to further extend the franchise and transition the state from a partial democracy to a full democracy, in our model, we outline a possibility that a broader franchise can increase the income of the landed elite so that they are actually willing to extend the franchise to the poor. After the state is transformed to a partial democracy, the franchise has been extended to the capitalist middle class who becomes the new elite. In this model, we stress the conflict in labour supply between the landed elite and the capitalist middle class. Once the capitalist middle class get involved in the process of policy determination, the consideration being given to the industrial 'interest' increases, and that favours the development of manufacturing industry, and the labour demand of manufacturing industry boosts. As a result of the large amount of migration from the countryside to the city, the landed elite suffers from declining labour supply in the agricultural sector. In our model, we put forward the possibility that extending suffrage to the middle class generates greater losses to the landed elite as a result of the decreasing agricultural labour supply, and the landed elite therefore favours the further extension of the franchise to the farmer. This is because, firstly, the farmer is relative poorer than the capitalist, therefore the farmers is in favour of a higher tax rate that increases the tax burden of the capitalist and also lowers the labour demand of the capitalist. Secondly, after the extension of suffrage to the farmer, policy-making will shift in a direction favoured by the majority of the total electorate, the farmers, and most of the tax revenue will flow to the public goods that are provided to the countryside. Thus, it will become more costly for the farmer to move from the countryside to the city and the landed elite will benefit from the increasing labour supply.

This explanation is consistent with historical events in England. The Reform Act 1832 represented a shift in policy in a direction favoured by the capitalist middle class (Lizzeri and Persico, 2004). In the early nineteenth century, some cities in England grew at an incredible rate, for example between 1820 to 1830 Bradford grew by 78 percent, Manchester by 47 percent, and Glasgow by 38 percent (Szreter, 1997; Szreter and Mooney, 1998). In order to cope with the rapid inflow of migration from the countryside, most tax revenue and government spending flowed to the public goods provision and the construction of urban infrastructure. The extension of the franchise further accelerated the rapid urbanization. By 1841, six English provincial towns recorded populations over 100,000 and Liverpool and Manchester recorded over 200,000 (Szreter, 1997). As a result of the large amount of migration from the countryside to the city, the landed elite suffered from declining labour supply in the agricultural sector. Especially in the late nineteenth century, human capital became more important. During the same period, especially after the abolition of the Corn Laws in 1846, the confrontation between the landed elite and the capitalist middle class was amplified. Increased parliamentary representation of industrial 'interest' was seen by the landed elite as a threat to the agricultural 'interest'. Conservatives who represent the agricultural 'interest' drafted and affirmed the reform bill in 1867 that lowers the franchise qualification in both counties and boroughs. The suffrage has been extended to the working classes in both urban areas and rural counties, the English borough electorate increased by 134 per cent and the county electorate increased by 46 per cent. Democracy in England has been greatly improved.

In this chapter, we construct a model that consists of three group of agents: the landed elite, the manufacturing capitalist and the workers. There are three possible political regimes, autocracy, partial democracy and full democracy. In a partial democracy, suffrage is extended to the capitalist and the decisive voter is the capitalist who determines policy for the whole society. However, in a full democracy, the franchise extends to the whole population, and the farmer is the median voter. We consider the circumstances that the state starts with autocracy controlled by the landed elite. Given the threat of revolution, the landed elite determines the size of enfranchisement, in other words whether to extend the franchise only to the capitalist middle class or to the whole population. When describing the conflict over the labour supply between the landed elite and the capitalist, we apply the Lewis two-sector model (Lewis, 1954) describing the production function of the agricultural produced goods and the manufacturing produced goods. In the Lewis model, there exists a portion of the rural labour force whose marginal productivity is zero or negative in the agricultural sector, that can be withdrawn without any loss of output. However, for the manufacturing sector, labour productivity is relatively high. Lewis assumes that the wage level in the manufacturing sector is constant, determined as a given premium over the level of wage in the agricultural sector.

Since the labour supply is elastic, the labour force flows from the countryside into the city, and the speed with which this occurs is determined by the development of the manufacturing industry and capital accumulation. In our model, we consider the maximum amount of labour input in the agricultural sector, after which the total produced goods is constant. If the manufacturing labour input is taken from the surplus labour of the agricultural sector, the landed elite has zero loss and is in line with the interests of the capitalist of imposing a lower income tax rate. Here, migration imposes no lost output in agriculture. For this reason, the landed elite prefer to extend the franchise to the capitalist in exchange for a lower tax cost. However, if the labour force moves from the countryside to the city in large numbers, the productive labour in the agricultural sector declines and this reduces the produced revenue of the landed elite. Under this circumstance, the landed elite may favour the expansion of the franchise to the farmer in exchange for a higher income tax rate, so that the speed of labour movement could be reduced.

The contribution of this chapter is twofold. Firstly, we capture the condition under which the landed elite would prefer to fully democratize the state, extending the franchise to the whole population, compared to the incentives for partially democratizing the state. We find that the landed elite would prefer to extend suffrage to the whole population if the preferred income tax rate of the capitalist is low and the optimal income tax rate of the farmer is higher but still a moderate rate. This is because, if the preferred income tax rate of the capitalist is quite low, the speed of labour movement will be further accelerated, and it is more likely that the productive labour force of the agriculture sector will flow to the manufacturing sector. Under this circumstance, if the optimal income tax rate of the farmer is moderate but higher than that of the capitalist, extending suffrage to the farmer could reduce the speed of labour movement, thus increasing the produced revenue in the agricultural sector, and simultaneously, a moderate income tax rate proposed by the farmers is still acceptable to the landed elite. This finding coordinates with the fluctuation of the taxation in nineteenth-century England. Since the franchise had been extended to the capitalist elite, total taxation as a fraction of GNP kept decreasing until 1867 when the Reform Act of 1867 is introduced, and since 1884, when the franchise was extended to the farmer, the total taxation further increases but in a moderate rate. However, until 1900, the taxation had not yet reached its 1800 level (Lindert, 1989).

Secondly, we capture the comparative static result between the development of the manufacturing sector and the size of enfranchisement. We find that when the manufacturing industry is strong, its productivity is higher, or the price of the produced goods increases, with the result that extending the franchise to the whole population becomes more attractive to the landed elite. This is because, firstly, developments in the manufacturing industry encourage the capitalist middle class of hiring more labour. Secondly, a well-developed manufacturing industry gives the farmer a higher incentive of leaving the agricultural sector. Under the circumstances described, the collected tax revenue from the agricultural sector is comparatively lower, and both the capitalist and the farmer would prefer to impose a comparatively moderate income tax rate in exchange for a higher produced revenue in the manufacturing industry. Extending the franchise to the whole population from one side, will generate a higher income tax rate compared with the preferred rate of the capitalist elite, and therefore will reduce the speed of urbanization and save more productive labour for the landed elite. On the other side, under the circumstance we just described, income tax rate in a democracy is still moderate, and therefore democracy turns out to be less costly to the landed elite. Conversely, if the agricultural industry is strong, on one hand landed elite is more likely to solve the threat of revolution by repression, and on the other hand, after the extension of the franchise, both the capitalist and the farmer favour a higher income tax rate in order to gain more wealth redistribution from the landed elite. This finding coordinates with the claim in the literature from Moore (1993), Dahl (1973), and Rueschemeyer et al. (1992) that democracy can never be sustained in a primarily agrarian society. It can also explain why experience of democratization in Latin America differs.

Peaceful franchise extension has been widely discussed in the literature, and several explanations have been provided. It has been argued by Himmelfarb (1966) and Collier (1999) that political competition within the elite led to the extension of the franchise, one of the fractions brought new groups into the political system in an attempt to receive more support from those new groups. Moore (1993) proposes that the middle class was the driving force of the extension of the franchise. As we have mentioned at the beginning of this section, the leading explanation comes from Acemoglu and Robinson (2000, 2001, 2005) who argue that the disenfranchised group gains enlargement by imposing a threat of revolution to the current incumbent, and since the extension of suffrage provides a credible commitment to the public, the landed elite would therefore prefer to expand the total electorate. Aidt and Jensen (2014) find robust evidence showing that enfranchisement in Europe relates to the threat of revolution. Conley and Temimi (2001) construct a model that shows individuals may not value the vote but they do value the franchise. This chapter has also been inspired by the paper of Lizzeri and Persico (2001, 2004), who formulate a new explanation that the effect of franchise reform is consistent with the interests of the elite and they would prefer to democratize the state without any threat of revolution.

This chapter is organized as follows. Section 4.2 outlines the basic economic and political environment. Section 4.3 analyzes the baseline model, captures the preference of the landed elite towards the income tax rate, constructs the condition under which the landed elite would prefer to extend the franchise to the whole population, and also discusses how the agricultural industry affects the franchise extension. In Section 4.4, we conclude.

4.2 Baseline Model

We now outline a model that formalizes the proposition that for those oligarchies that have been controlled by landed elites, incumbents would prefer to enlarge the size of enfranchisement if the domestic manufacturing industry is well developed or agriculture proves to be less competitive.

4.2.1 Demographics, Preferences, and Production Structure

We consider a society that exists over two periods. The society is populated by L landed elites, M capitalists, and N workers where L, M, and N are a finite number. Here, we use $\mathcal{L}$, $\mathcal{M}$ and $\mathcal{N}$ to denote the set of landed elites, capitalists and workers respectively. We make our first assumption that regulates the size of population for each set of people:

Assumption 1.

$$N >> M > L$$

This assumption implies that workers constitute the majority of the whole population, and the population size of capitalists is greater than that of the landed elite. This means that the capitalist is the decisive voter if the suffrage is extended to the capitalist, whilst the workers is the decisive voter if the suffrage is extended to the whole population. There are two types of final goods that have been produced in this described society, agricultural goods and manufacturing goods, which are denoted by Υ and Γ respectively. All agents have the same risk-neutral preferences with discount factor $\beta \in (0, 1)$, given by:

$$u^i = c_t^i + \beta c_{t+1}^i$$

where c_t^i denotes the consumption of agent i at time t in terms of the final goods, c_{t+1}^i denotes the consumption of agent i in the second period, and $t = 1$. At the end of each period, each agent consumes all his or her income, and derives his or her utility by consuming two types of goods with no difference. In this model, to simplify the analysis, we normalize the price of the final goods to equal 1. We therefore have that:

$$c_t = \Upsilon_t + \Gamma_t$$

where c_t denotes the total consumption of the whole population at time t, Υ_t denotes the total produced revenue of agricultural goods, and Γ_t denotes the total produced revenue of manufacturing goods.

All workers own one unit of labour, which they supply elastically. Each worker, $i \in \mathcal{N}$, could be hired by either the landed elite or capitalists, they prefer to work for the industry that gives them a higher payoff. Considering this, we further divide the workers into two subgroups, agricultural and manufacturing sector workers. We use the notation $i \in nl$ to denote a farming labourer, $i \in nm$ to denote a manufacturing labourer, N_L to denote the number of agricultural workers, and N_M to denote the number of manufacturing workers. We now make our second assumption regarding the size of the population:

Assumption 2.

$$\frac{N_L}{N_M} > \frac{1}{2}$$

which implies that agricultural workers always occupy the majority of the whole population. This assumption is reasonable because in most states farmers occupy the greatest size of the whole population. This is also one of the key assumptions of the Lewis Model.

Each member of the landed elites, $i \in \mathcal{L}$, has access to the following production function to produce agricultural goods:

$$Y(N_L) = \begin{cases} AN_L & \text{if} \quad N_L < N^* \\ AN^* & \text{if} \quad N_L \geq N^* \end{cases}$$

where A refers to the productivity of agricultural technology. In our described model, we apply the Lewis two-sector model to describe the production function of the agricultural goods and the manufacturing goods. It describes a production function in which the total output of agricultural goods is determined by changes in the amount of the only variable, labour supply, given a fixed level of farming technology, A. This production function also implies that there is a maximum amount of labour input, N^*, after which the total produced agricultural good is constant. If the amount of labour supply is smaller than N^*, the agricultural production function is linear and the quantity of the total produced goods increases in the labour input. In our described society, the landed elite takes a constant share, $k \in (0, 1)$, of the total produced output, and an equal share of collected produced revenue. Then, for each member of the landed elite, the income function is given by:

$$I^{i,\mathcal{L}} = \frac{kY(N_L)}{L}$$

where $I^{i,\mathcal{L}}$ denotes the income of agent i where $i \in \mathcal{L}$. One feature to note here is that, if the amount of labour input is greater than N^*, $N_L \geq N^*$, the income level of each member of the landed elite is constant. For each farmer, the income function is given by:

$$I^{i,nl} = \frac{(1-k)Y(N_L)}{N_L}$$

where $I^{i,nl}$ denotes the income of agent i where $i \in nl$. All farmers equally share the received produced revenue so that the rural wage is determined by the average and not the marginal product of labour. The income function also implies that, after the maximum amount of labour input, N^*, the received income of each farmer decreases in labour supply.

In our model society, each member of capitalists middle-class, $i \in \mathcal{M}$, has access to the following production to produce the manufacturing goods:

$$G(N_M) = A_M K^{1-\alpha} N_M^{\alpha}$$

where A_M denotes the total factor productivity of the manufacturing industry, K denotes the total capital input which is fixed, α denotes the output elasticity of labour, and G exhibits constant returns to scale. In this model, parameters A_M, K, and α are all fixed, and $\alpha \in (0, 1)$. Therefore at this point, the total output of manufacturing goods is determined by changes in the amount of labour input, N_M, which is the only variable in this manufacturing production function. N_M is the amount of labour input, which is the only variable in this production function. The production function could be simplified as follows:

$$G(N_M) = \mathbf{K} N_M^{\alpha}$$

where $\mathbf{K} = A_M K^{1-\alpha}$ which is fixed in this model. For the manufacturing industry, capitalists collect the total produced revenue, and each worker receives the wage. We then make the following assumption towards the received wage of each manufacturing worker:

Assumption 3. *In our model society, the wage level in the manufacturing sector is constant, equals ω, determined as a given premium over the level of wage in the* *agricultural sector:*

$$\omega > \frac{(1-k)Y(N_L)}{N^*}$$

This is also one of the assumptions in the Lewis Model. Since the labour supply is elastic, given this assumption, the wage level in the manufacturing sector could sufficiently attract agricultural workers move to the manufacturing industry. To simplify the analysis, we also assume that ω is the minimum required wage rate that the capitalists could attract the labour force flow from the agricultural sector the manufacturing sectors. For this reason, to save the wage cost, the income level for the manufacturing workers equates to the minimum wage, and is given by:

$$I^{i,nm} = \omega$$

Given the required wage rate, the capitalist determines the amount of labour input, N_M, which solves the following optimizing problem:

$$max \quad I^c = \mathbf{K}N_M^{\alpha} - \omega N_M$$

where I^c refers to the total collected income of the capitalist middle-class. Given the fixed $\mathbf{K}$, α, and ω, the optimal amount of the labour input, N_M, satisfies the following first order condition:

$$\frac{\partial I^c(N_M)}{\partial N_M} = \mathbf{K}\alpha N_M^{\alpha-1} - \omega = 0$$

So that, the optimal N_M equals:

$$N_M = \left[\frac{\mathbf{K}\alpha}{\omega}\right]^{\frac{1}{1-\alpha}}$$

In the urban industrial sector, the real wage is determined by the marginal product of labour. From the above equation, we have seen that any improvements in the manufacturing industry or any decreases in the required wage rate will increase the labour demand of the capitalist middle-class. The aggregate income function for the latter demographic is given by:

$$I^c(N_M) = \mathbf{K}\left[\frac{\mathbf{K}\alpha}{\omega}\right]^{\frac{\alpha}{1-\alpha}} - \omega\left[\frac{\mathbf{K}\alpha}{\omega}\right]^{\frac{1}{1-\alpha}}$$

As we have mentioned above, workers supply their labour elastically and the manufacturing industry offers a sufficiently high wage, therefore workers prefer to be hired by capitalists. Once the capitalist middle-class determines its optimal number of manufacturing workers, the rest of workers will stay in the countryside and work for the landed elite . One feature to note is that the simple environment we have outlined here implies that the labour supply towards the agricultural industry is determined by the labour demand of the manufacturing industry.

4.2.2 Political Regime and Income Tax Structure

There are three possible political regimes, denoted by ND, PD and FD, corresponding to autocracy (No Democracy), partial democracy, and democracy (Full Democracy). The identity of the decision maker changes with the structure of the political regime. At any point in time, the state variable is denoted by $s_t \in \{ND, PD, FD\}$. Please not that, irrespective of the political regime, the identity of the landed elite, the capitalist and the normal workers will not be changed. The same L landed elite controls the land that can collect a constant share, k, of the total produced agricultural revenue, and the same M capitalist controls the capital which can collect all produced manufacturing revenue.

In our described model, at time t, the society starts with autocracy in which the landed elite makes decisions for the whole society, and the capitalist and the workers are excluded from the decision making process. The latter groups could align together and attempt a revolution in the next period. We then make the following assumption in relation to revolution:

Assumption 4. *If a revolution is attempted by the coalition between capitalists and workers, it always succeeds.*

This assumption is reasonable. Since most workers are less well educated, they can hardly control this large population, and are also unable to solve the collective problem successfully. For the capitalist, even though their population size is greater than that of the landed elite, they have very limited group members in comparison to the workers. However, if capitalists and workers cooperate, firstly, they will provide sufficient financial support to the revolution, and since most capitalists are well educated, they will be able to manage the mass population.

After a revolution, the autocratic state transitions to a full democracy, and since farmers take the majority of the whole population, a farmer agent is the median voter who makes decisions for the whole state. In this model, to simplify the analysis, we assume that voting right generates an additional utility to each agent of the society. Consequently, in an autocracy, capitalists and workers always have an incentive to revolt.

We also assume that the landed elite loses everything after a revolution, so that they will always try to prevent it. They may try to prevent it by making concessions to capitalists and workers, for example, in the form of income redistribution. However, since the threat of revolution is often only transitory, current redistribution can not guarantee future redistribution. For this reason, temporary concessions cannot sufficiently remove the threat of revolution, and therefore the landed elite will be forced to make a more credible commitment. In this model, there are two possible ways that the landed elite could remove the threat of revolution and protect its wealth against expropriation. First, the landed elite could partially democratize the state by extending the voting rights to the capitalists, then the society becomes a partial democracy, and the median voter is the capitalist agent. Partial democratization could sufficiently remove capitalists' incentives to attempt a revolution, and since the workers cannot win the revolution by themselves, the threat of revolution for the landed elite could be removed. Second, the landed elite could choose to democratize the state and extend voting rights to the whole population. Then, the state becomes a democracy, and the median voter is a farmer agent.

To complete the description of the environment, we here specify the key decisions. As we have mentioned above, our model starts with autocracy at time t, $s_t = ND$, and the landed elite is the decision maker. At the end of time t, the landed elite determines the state variable for the next period, s_{t+1}, where $s_{t+1} \in \{ND, PD, FD\}$. At the beginning of time t, the decision maker sets the income tax rate, τ_t, which will be imposed upon the whole society.

The group which the decision maker belongs to will collect all the tax revenue at the end of the period. To simplify the analysis, we do not regulate how the decision maker distributes the collected tax revenue, the decision maker determines the income tax rate that could maximize his group's income, which includes the total received income as well as the the collected tax revenue. At time t, the landed elite determines its optimal income tax rate, τ^{ND}, to maximize its total received income. Given the income tax rate, τ^{ND}, the capitalists determine their optimal labour input. In the manufacturing industry, the real wage that the capitalist pays the manufacturing worker is constant, so that, given the income tax rate τ^{ND}, the nominal wage equals $\frac{\omega}{(1-\tau^{ND})}$, and the optimal labour input for the manufacturing sector is given by:

$$N_M(\tau^{ND}) = \left[\frac{(1-t^{ND})\mathbf{K}\alpha}{\omega}\right]^{\frac{1}{1-\alpha}}$$

Given the labour demand of the capitalists, the labour supply towards the agricultural sector equals $N_L = N - N_M$, and for the landed elite, its post-tax income from the agricultural sector equates to $(1-\tau^{ND})kY(N-N_M)$. By using the backward induction, the landed elite determines its optimal income tax rate, τ^{ND}, which solves the following optimizing problem:

$$max \quad I^{\mathcal{L}} = (1-\tau^{ND})kY(N-N_M) + \tau^{ND}\left[Y(N_L) + \mathbf{K}N_M^{\alpha}\right]$$

$$s.t. \quad N_M = \left[\frac{(1-t^{ND})\mathbf{K}\alpha}{\omega}\right]^{\frac{1}{1-\alpha}}$$

$$N_M + N_L = N$$

where $I^{\mathcal{L}}$ denotes the group income of the landed elite. The first term of the objective function denotes the post-tax income of the landed elite, and the last term refers to the total received tax revenue of the landed elite.

Suppose the landed elite extends the voting rights to the capitalist, the society transitions to a partial democracy in time $t+1$, $s_{t+1} = PD$. In a partial democracy, the capitalist chooses the income tax rate, τ^{PD}, which solves the following optimizing problem:

$$max \quad I^{\mathcal{M}} = (1-\tau^{PD})\left[\mathbf{K}N_M^{\alpha} - \frac{\omega}{(1-\tau^{PD})}N_M\right] + \tau^{PD}\left[Y(N_L) + \mathbf{K}N_M^{\alpha}\right]$$

$$s.t. \quad N_M = \left[\frac{(1-t^{PD})\mathbf{K}\alpha}{\omega}\right]^{\frac{1}{1-\alpha}}$$

$$N_M + N_L = N$$

where $I^{\mathcal{M}}$ refers to the group income of the capitalist. After a partial democratization, the landed elite sustains its control over the land, and its group income at time $t+1$ equals:

$$I^{\mathcal{L}}(\tau^{PD}) = (1-\tau^{PD})kY(N - N_M(t^{PD}))$$

In a partial democracy, the group income of the farmers is given by:

$$I^{nl}(\tau^{PD}) = (1-\tau^{PD})(1-k)Y(N - N_M(t^{PD}))$$

where I^{nl} denotes the group income of the farmers.

However, if the landed elite chooses to extend voting rights to the whole population, the state transitions to a democracy in time $t+1$, $s_{t+1} = FD$. In a full democracy, the farmer agent determines the income tax rate for the whole society, and his or her optimal income tax rate, τ^{FD}, solves the following problem:

$$max \quad I^{nl} = (1-\tau^{FD})(1-k)Y(N_L) + \tau^{FD}\left[Y(N_L) + \mathbf{K}N_M^{\alpha}\right]$$

$$s.t. \quad N_M = \left[\frac{(1-t^{FD})\mathbf{K}\alpha}{\omega}\right]^{\frac{1}{1-\alpha}}$$

$$N_M + N_L = N$$

In a full democracy, given the income tax rate, τ^{FD}, the income function for the capitalist is given by:

$$I^{\mathcal{M}}(\tau^{FD}) = (1-\tau^{FD})\left[\mathbf{K}N_M(\tau^{FD})^{\alpha} - \frac{\omega}{(1-\tau^{PD})}N_M(\tau^{FD})\right]$$

Similarly, the group income for the landed elite equals:

$$I^{\mathcal{L}}(\tau^{FD}) = (1 - \tau^{FD})kY(N - N_M(t^{FD}))$$

It should be mentioned that, irrespective of the structure of the political regime, the real wage for the manufacturing worker is constant, equalling ω.

4.2.3 Timing of Events

We now briefly recap the timing of events in this basic environment. Let δ denote the set of parameter, $\delta \in \{A, A_M, L, M, N, \alpha, k, \omega\}$ which has been given at the beginning of time t. The model society starts with autocracy, $s_t = ND$. Given this, the following sequence of events takes place:

- At the beginning of time t, the landed elite sets the tax rate $\tau_t = \tau^{ND}$.
- Given τ_t, transactions in the labour market take place, $I^{\mathcal{L}}$, $I^{\mathcal{M}}$, ω, and I^{nl} are paid to the landed elite, the capitalist, and manufacturing workers and farmers respectively, and consumption then takes place.
- At the end of time t, the landed elite sets the state variable for the next period, s_{t+1}.
  - If the landed elite determines $s_{t+1} = ND$, a revolution takes place, the society transitions to democracy at the beginning of time $t + 1$, farmers set the income tax rate for the whole state, $\tau_{t+1} = \tau^{FD}$.
  - If the landed elite determines $s_{t+1} = PD$, the threat of revolution is removed, the society transitions to a partial democracy at the beginning of time $t + 1$, the capitalist set the income tax rate for the whole state, $\tau_{t+1} = \tau^{PD}$.
  - If the landed elite determines to fully democratize the state, $s_{t+1} = FD$, the society transitions to a democracy at the beginning of the period, and farmers set the income tax rate for the whole state, $\tau_{t+1} = \tau^{FD}$.

- Given τ_{t+1}, transactions in the labour market at time $t+1$ take place, $I^{\mathcal{L}}$, $I^{\mathcal{M}}$, ω, and I^{nl} are paid to the landed elite, the capitalist, manufacturing workers and farmers respectively, and consumption then takes place.

4.3 Analysis of the Baseline Model

We now analyze the baseline model described in the previous section. As mentioned above, the landed elite will lose everything after a revolution, so it will voluntarily extend the franchise and democratize the society to avoid this. Now we will analyze the issue of how the landed elite determines the size of enfranchisement, specifically whether they would prefer to partially democratize the state, or to fully democratize the state.

4.3.1 Preferences of the Landed Elite

After democratization, the landed elite is no longer the decision maker, but it nonetheless sustains its control over the land, and either in partial democracy or full democracy, given the income tax rate, τ^s, the group income of the landed elite is defined by:

$$I^e(\tau^s, N_L) = \begin{cases} (1-t^s)\kappa A N_L & \text{if} \quad N_L < N^* \\ (1-t^s)\kappa A N^* & \text{if} \quad N_L \geq N^* \end{cases} \tag{4.1}$$

As we have mentioned already, since the manufacturing sector provides a higher real wage, the average worker prefers to work for the manufacturing industry, and the labour supply for agriculture is determined by the labour demand of the capitalist. Given the income tax rate, τ^s, the labour demand of the capitalist is given by the following function:

$$N_M(\tau^s) = \left[\frac{(1-\tau^s)\mathbf{K}\alpha}{\omega}\right]^{\frac{1}{1-\alpha}}$$

The optimal labour demand of the capitalist decreases with the income tax rate, and this will raise the labour supply towards agriculture up. We then substitute N_L in equation (4.1) by $N_L = N - N_M(\tau^s)$, and the group income function of the landed elites is modified as follows:

$$I^e(\tau^s) = \begin{cases} (1-t^s)\kappa A(N - N_M(\tau^s)) & \text{if} \quad N - N_M(\tau^s) < N^* \\ (1-t^s)\kappa A N^* & \text{if} \quad N - N_M(\tau^s) \geq N^* \end{cases} \tag{4.2}$$

We then solves the income tax rate, $\tau^s = \tau^*$, which satisfies the condition, $N - N_M(\tau^s) = N^*$:

$$N - \left[\frac{(1-\tau^*)\mathbf{K}\alpha}{\omega}\right]^{\frac{1}{1-\alpha}} = N^*$$
$$t^* = 1 - \frac{(N-N^*)^{1-\alpha}\omega}{\mathbf{K}\alpha}$$

From the above description, we have seen that for any $\tau^s < \tau^*$, the amount of labour supply to agriculture is smaller than N^*, and the landed elite faces the linear production function. However, for any $\tau^s \geq \tau^*$, labour supply towards agriculture is in surplus and therefore the total produced agricultural revenue is constant.

Given the income tax rate, τ^s, if the landed elite faces the linear production function where $\tau^s \in [0, \tau^*)$, the group income of the landed elite is given by:

$$I^{\mathcal{L}}(\tau^s) = (1-\tau^s)kA\left[N - \left(\frac{(1-\tau^s)\mathbf{K}\alpha}{\omega}\right)^{\frac{1}{1-\alpha}}\right] \tag{4.3}$$

We then differentiate equation (4.3) by the income tax rate, τ^s, and we have:

$$\frac{\partial I^{\mathcal{L}}(\tau^s)}{\partial \tau^s} = kA\left[\left(\frac{(1-\tau^s)\mathbf{K}\alpha}{\omega}\right)^{\frac{1}{1-\alpha}}\left(1 - \frac{1}{1-\alpha}\right) - N\right] \tag{4.4}$$

The group income of the landed elite increases in the income tax rate, τ^s, if τ^s satisfies the following condition:

$$\left(\frac{(1-\tau^s)\mathbf{K}\alpha}{\omega}\right)^{\frac{1}{1-\alpha}}\left(1 - \frac{1}{1-\alpha}\right) - N > 0$$
$$\frac{N_M(\tau^s)}{N} > \frac{1-\alpha}{2-\alpha}$$

We then solve the income tax rate, $\tau^s = \tau^{**}$, which satisfies the condition, $\frac{N_M(\tau^s)}{N} = \frac{1-\alpha}{2-\alpha}$:

$$\tau^{**} = 1 - \left[\left(\frac{1-\alpha}{2-\alpha}\right)N\right]^{1-\alpha}\frac{\omega}{\mathbf{K}\alpha}$$

If the optimal income tax rate of the decisive voter, τ^s, is lower than τ^{**}, the landed elite will be benefiting from an increasing income tax rate. If $\tau^{**} < \tau^*$, for any $\tau^s \in [0, \tau^{**})$, the group income of the landed elite increases in the income tax rate. Otherwise, given the income tax rate, $I^{\mathcal{L}}(\tau^s)$ increases in the value of τ^s if $\tau^s \in [0, \tau^*)$. Given the set of parameter δ, we then further specify the function for the value of τ^{**}:

$$\tau^{**}(\delta) = \begin{cases} 1 - \left[\left(\frac{1-\alpha}{2-\alpha}\right) N\right]^{1-\alpha} \frac{\omega}{\mathbf{K}\alpha} & \text{if} \quad \frac{1-\alpha}{2-\alpha} > 1 - \frac{N^*}{N} \\ 1 - \frac{(N-N^*)^{1-\alpha}\omega}{\mathbf{K}\alpha} & \text{if} \quad \frac{1-\alpha}{2-\alpha} \leq 1 - \frac{N^*}{N} \end{cases}$$

After a democratization, given the income tax rate, τ^s, if $\tau^s \in [0, \tau^{**})$, the landed elite benefits from increasing the income tax rate.

Proposition 1. *Given the set of parameter, δ, if the optimal income tax rate of the capitalist, τ^{PD}, and the optimal income tax rate of the farmer, τ^{FD}, satisfy the condition that τ^{PD}, $\tau^{FD} \in [0, \tau^{**})$, the landed elite will extend voting rights to the group for whom the optimal income tax rate is higher.*

On the other hand, given the income tax rate, τ^s, if $\tau^s \in [\tau^*, 1)$, the produced agricultural revenue is constant, and the group income function of the landed elite is given by:

$$I^{\mathcal{L}}(\tau^s) = (1 - \tau^s)kAN^* \tag{4.5}$$

We then differentiate equation (4.5) by the income tax rate, τ^s, and we have:

$$\frac{\partial I^{\mathcal{L}}(\tau^s)}{\partial \tau^s} = -kAN^* < 0$$

Since the value of $\frac{\partial I^{\mathcal{L}}(\tau^s)}{\partial \tau^s}$ is negative for all possible τ^s, if $\tau^s \in [t^*, 1)$, the landed elite unambiguously benefits from decreasing the income tax rate.

Proposition 2. *Given the set of parameter, δ, if the optimal income tax rate of the capitalist, τ^{PD}, and the optimal income tax rate of the farmer, τ^{FD}, satisfy the condition that τ^{PD}, $\tau^{FD} \in [\tau^{**}, 1)$, the landed elite will extend voting rights to the group for whom the optimal income tax rate is lower.*

Proposition 3 (Comparative Statics)**.** *From the function for the value of τ^{**}, we have the following comparative static results:*

- *Given an increase in the population size of the workers, the preferred income tax rate of the landed elite turns to be lower, and it is less likely that the landed elite will extend voting rights to the group for whom the optimal income tax rate is higher.*

$$\frac{\partial \tau^{**}(\delta)}{\partial N} < 0$$

- *Given an increase in the maximum amount of labour input that the landed elite can access to a constant produced revenue, it is more likely that the landed elite will extend voting rights to the group for whom the optimal income tax rate is higher.*

$$\frac{\partial \tau^{**}(\delta)}{\partial N^*} > 0$$

- *Given an increase in the required real wage of the manufacturing worker, it is less likely that the landed elite will extend voting rights to the group of whom the optimal income tax rate is higher.*

$$\frac{\partial \tau^{**}(\delta)}{\partial \omega} < 0$$

- *If the manufacturing industry becomes more developed, it is more likely that the landed elite will extend voting rights to the group of whom the optimal income tax rate is higher.*

$$\frac{\partial \tau^{**}(\delta)}{\partial \mathbf{K}} > 0$$

The comparative statics are intuitive. It is more likely that the landed elite prefers a lower income tax rate if the state is endowed with a large population of working people. The reason is that the state is rich in labour resource, and therefore increases in the labour demand of the manufacturing sector can hardly affect the produced revenue of agriculture. Under this circumstance, the landed elite prefers a lower income tax rate to save tax costs. The second comparative static result shows that if the maximum amount of labour input for agriculture is greater, it is more likely that the landed elite will prefer a higher income tax rate. Given a fixed level of productivity, a greater value of N^* essentially shows that the state has more arable land, which requires a greater amount of labour input and it is therefore more likely that increases in the labour demand of the manufacturing industry will reduce the produced revenue of agriculture.

The third comparative static implies that the landed elite is more likely to benefit from a lower income tax rate if the required real wage of the manufacturing worker is higher. A higher value of ω shows that it is costly for the capitalist to operate the manufacturing industry, and to save the total wage cost they will cut down the amount of labour input. A decline in the labour demand of the capitalist will save more labour resource for agriculture, agriculture is in labour-surplus, and the landed elite receive the highest produced revenue from agriculture. Under this circumstance, the landed elite is benefiting from lowering the income tax rate. Decreases in the value of ω imply the development of urbanization, living in the city is therefore attractive to farmers and the capitalist could pay a limited wage to sufficiently attract farmers to move from the countryside to the city.

The fourth comparative static is intuitive, showing that any developments in the manufacturing sector make it more likely that the landed elite will benefit from an increasing income tax rate. If the manufacturing sector is well developed, any decreases in the income tax rate will increase the labour demand of the capitalist at a great scale and the labour supply towards agriculture will decrease at a great scale thus affecting the produced revenue of agriculture. For this reason, increases in the income tax rate improve the group income of the landed elite.

4.3.2 The Optimal Income Tax rate after the Democratization

In the above section, we have characterized the conflict over labour resources between the landed elite and the capitalist, which make it possible that the landed elite may benefit from a higher income tax rate. Now we will look at the issue, at the end of time t, of how the landed elite would prefer to extend the franchise and how the state will transition to a partial democracy or a full democracy in time $t+1$.

After democratization, the decision maker chooses the income tax rate τ^s that maximizes his group income. Suppose the landed elite extends voting rights to the capitalist, the society transitions to a partial democracy, $s_{t+1} = PD$. At time $t+1$, when $N_L < N^*$, the capitalist chooses the income tax rate, τ^{PD}, which solves the following optimizing problem:

$$\begin{aligned} max \quad I^{\mathcal{M}}(\tau^{PD}) = \mathbf{K}\left[\frac{(1-\tau^{PD})\mathbf{K}\alpha}{\omega}\right]^{\frac{\alpha}{1-\alpha}} - \omega\left[\frac{(1-\tau^{PD})\mathbf{K}\alpha}{\omega}\right]^{\frac{1}{1-\alpha}} \\ + \tau^{PD}A\left[N - \left[\frac{(1-\tau^{PD})\mathbf{K}\alpha}{\omega}\right]^{\frac{1}{1-\alpha}}\right] \quad (4.6) \\ s.t. \quad \tau^{PD} < \tau^* \end{aligned}$$

Since the condition $N_L < N^*$ holds, the optimal income tax rate, τ^{PD}, is smaller than $\tau*$. In a partial democracy, since the capitalist collects all of the income tax revenue, $\mathbf{K}\left[\frac{(1-\tau^{PD})\mathbf{K}\alpha}{\omega}\right]^{\frac{\alpha}{1-\alpha}} - \omega\left[\frac{(1-\tau^{PD})\mathbf{K}\alpha}{\omega}\right]^{\frac{1}{1-\alpha}}$ equates to their total collected revenue from the manufacturing industry, and $\tau^{PD}A\left[N - \left[\frac{(1-\tau^{PD})\mathbf{K}\alpha}{\omega}\right]^{\frac{1}{1-\alpha}}\right]$ is the total collected tax revenue from the agriculture. The capitalist chooses the income tax rate, τ^{PD}, that satisfies the following first order condition:

$$\begin{aligned} \frac{\partial I^{\mathcal{M}}(\tau^{PD})}{\partial \tau^{PD}} = -\mathbf{K}\frac{\alpha}{1-\alpha}\left[\frac{(1-\tau^{PD})\mathbf{K}\alpha}{\omega}\right]^{\frac{2\alpha-1}{1-\alpha}}\frac{\mathbf{K}\alpha}{\omega} + A\left(N - \left[\frac{(1-\tau^{PD})\mathbf{K}\alpha}{\omega}\right]^{\frac{1}{1-\alpha}}\right) \\ + (\omega + \tau^{PD}A)\frac{1}{1-\alpha}\left[\frac{(1-\tau^{PD})\mathbf{K}\alpha}{\omega}\right]^{\frac{\alpha}{1-\alpha}}\frac{\mathbf{K}\alpha}{\omega} = 0 \quad (4.7) \end{aligned}$$

In this model, we cannot fully determine the concavity of the group income function, $I^{\mathcal{M}}(\tau^{PD})$, and since $\frac{\partial I^{\mathcal{M}}(0)}{\partial \tau^{PD}} > 0$, there always exists at least one unique and non-negative τ^{PD} where $\tau^{PD} \in [0, \tau^*)$ that satisfies equation (4.7). We use the notation τ_L^{PD} to denote all possible τ^{PD} where $\tau^{PD} \in [0, \tau^*)$ that satisfies the first order condition, (4.7).

When $N_L \geq N^*$, the capitalist may also choose the income tax rate, τ^{PD}, that solves the following maximizing problem:

$$\begin{aligned} max \quad I^{\mathcal{M}}(\tau^{PD}) = \mathbf{K}\left[\frac{(1-\tau^{PD})\mathbf{K}\alpha}{\omega}\right]^{\frac{\alpha}{1-\alpha}} - \omega\left[\frac{(1-\tau^{PD})\mathbf{K}\alpha}{\omega}\right]^{\frac{1}{1-\alpha}} + \tau^{PD}AN^* \\ (4.8) \\ \tau^{PD} \in [\tau^*, 1) \end{aligned}$$

Under this circumstance, the optimal income tax rate, τ^{PD}, is always greater than τ^*. If the chosen income tax rate satisfies the condition, $\tau^{PD} \in [\tau^*, 1)$, the produced revenue of agriculture is constant, and the collected tax revenue from the agricultural sector equals $\tau^{PD}AN^*$. From the objective function (4.6), we have seen that any increases in the optimal income tax rate pose a positive effect towards the total received agricultural tax revenue by increasing the labour supply towards the agricultural sector. However, if $N_L \geq N^*$, the total produced agricultural revenue is constant, and the total received agricultural tax revenue linearly increases with the income tax rate. The capitalist chooses the income tax rate, τ^{PD}, which satisfies the following first order condition:

$$\frac{\partial I^{\mathcal{M}}(\tau^{PD})}{\partial \tau^{PD}} = -\mathbf{K}\frac{\alpha}{1-\alpha}\left[\frac{(1-\tau^{PD})\mathbf{K}\alpha}{\omega}\right]^{\frac{2\alpha-1}{1-\alpha}}\frac{\mathbf{K}\alpha}{\omega} + AN^* + \omega\frac{1}{1-\alpha}\left[\frac{(1-\tau^{PD})\mathbf{K}\alpha}{\omega}\right]^{\frac{\alpha}{1-\alpha}}\frac{\mathbf{K}\alpha}{\omega} = 0 \quad (4.9)$$

Similarly, since the concavity of the income function varies with the income tax rate, we use the notation τ_H^{PD} to denote all possible τ^{PD} where $\tau^{PD} \in [\tau^*, 1)$ that satisfies the first order condition, (4.9). Given the possible income tax rate, τ_L^{PD} and τ_H^{PD}, the capitalist chooses the income tax rate that gives them the highest payoff, and we use the notation τ^{PD*} to denote the optimal income tax rate in partial democracy.

Alternatively the landed elite extends voting rights to the whole population, and the society transitions to a full democracy, $s_{t+1} = FD$. At the beginning of time $t+1$, when $N_L < N^*$, the representative farmer chooses the income tax rate, τ^{FD}, which solves the following optimizing problem:

$$max \quad I^{nl}(\tau^{FD}) = (1-k)A(N-\left[\frac{(1-\tau^{FD})\mathbf{K}\alpha}{\omega}\right]^{\frac{1}{1-\alpha}}) + \tau^{FD}\mathbf{K}\left[\frac{(1-\tau^{FD})\mathbf{K}\alpha}{\omega}\right]^{\frac{\alpha}{1-\alpha}} + \tau^{FD}kA\left(N - \left[\frac{(1-\tau^{FD})\mathbf{K}\alpha}{\omega}\right]^{\frac{1}{1-\alpha}}\right) \quad (4.10)$$

$$s.t. \quad \tau^{FD} \in [0, \tau^*)$$

Since $N_L < N^*$ holds, the optimal income tax rate of the majority, τ^{FD}, is always smaller than τ^*. In a full democracy, the farmer is assumed to collect all tax revenue. The first term in equation (4.10) refers to their total income from the agricultural sector, the second and the third term denotes the collected tax revenue from the manufacturing industry and the landed elite respectively. The farmer chooses the income tax rate, τ^{FD}, which satisfies the following first order condition:

$$\frac{\partial I^{nl}(\tau^{FD})}{\partial \tau^{FD}} = (1-k)A\frac{1}{1-\alpha}\left[\frac{(1-\tau^{FD})\mathbf{K}\alpha}{\omega}\right]^{\frac{\alpha}{1-\alpha}}\frac{\mathbf{K}\alpha}{\omega} + \mathbf{K}\left[\frac{(1-\tau^{FD})\mathbf{K}\alpha}{\omega}\right]^{\frac{\alpha}{1-\alpha}}$$
$$+ kA\left(N - \left[\frac{(1-\tau^{FD})\mathbf{K}\alpha}{\omega}\right]^{\frac{1}{1-\alpha}}\right) + \tau^{FD}kA\frac{1}{1-\alpha}\left[\frac{(1-\tau^{FD})\mathbf{K}\alpha}{\omega}\right]^{\frac{\alpha}{1-\alpha}}\frac{\mathbf{K}\alpha}{\omega}$$
$$- \tau^{FD}\mathbf{K}\frac{\alpha}{1-\alpha}\left[\frac{(1-\tau^{FD})\mathbf{K}\alpha}{\omega}\right]^{\frac{2\alpha-1}{1-\alpha}}\frac{\mathbf{K}\alpha}{\omega} = 0 \quad (4.11)$$

Even the concavity of the group income function, $I^{nl}(\tau^{FD})$ varies with the income tax rate, since $\frac{\partial I^{nl}(0)}{\partial \tau^{FD}} > 0$, there always exists at least one unique and non-negative τ^{FD} where $\tau^{FD} \in [0, \tau^*)$ that satisfies the above first order condition. We use the notation τ_L^{FD} to denote all possible τ^{FD} which are smaller than τ^* and also solve the optimizing problem of the farmer.

When $N_L \geq N^*$, the farmer may also choose the income tax rate, τ^{FD}, which solves the following maximizing problem:

$$max \quad I^{nl}(\tau^{FD}) = (1-\tau^{FD})(1-k)AN^* + \tau^{FD}(AN^* + \mathbf{K}\left[\frac{(1-\tau^{PD})\mathbf{K}\alpha}{\omega}\right]^{\frac{\alpha}{1-\alpha}}) \quad (4.12)$$

$$s.t. \quad \tau^{FD} \in [\tau^*, 1)$$

Since $N_L \geq N^*$ holds, $\tau^{FD} \in [\tau^*, 1)$. Therefore, the total preduced agricultural revenue is constant which equals AN^*, and the farmer receives a constant collected tax revenue from agriculture that equals $\tau^{FD}AN^*$. Under this circumstance, they choose the income tax rate, τ^{FD}, which solves the following first order condition:

$$\frac{\partial I^{nl}(\tau^{FD})}{\partial \tau^{FD}} = kAN^* + \mathbf{K}\left[\frac{(1-\tau^{FD})\mathbf{K}\alpha}{\omega}\right]^{\frac{\alpha}{1-\alpha}}$$
$$- \tau^{FD}\mathbf{K}\frac{\alpha}{1-\alpha}\left[\frac{(1-\tau^{FD})\mathbf{K}\alpha}{\omega}\right]^{\frac{2\alpha-1}{1-\alpha}}\frac{\mathbf{K}\alpha}{\omega} = 0 \quad (4.13)$$

Similarly, since the concavity of the group income function varies with the income tax rate, there may exist several τ^{FD} that could solve the above optimizing problem.

We then use the notation, τ_H^{FD}, to denote all possible τ^{FD} where $\tau^{FD} \in [\tau^*, 1)$ that satisfies condition (4.13). Given all of the possible income tax rates, τ_L^{FD} and τ_H^{FD}, the farmer chooses the income tax rate that provides a higher payoff, and we use the notation τ^{FD*} to denote the optimal income tax rate in full democracy.

4.3.3 Choice over the Size of Enfranchisement

Now we will compare the optimal income tax rate in partial democracy and full democracy, τ^{PD} and τ^{FD}, and summarize the conditions under which the landed elite would prefer to extend the voting rights to the whole population.

As we have shown in section 4.3.1, if the income tax rate, τ^s, satisfies the condition $\tau^s \in [0, \tau^{**})$, the landed elite will benefit from increasing the income tax rate. For this reason, we firstly compare the value of τ_L^{PD} and τ_L^{FD}. We use the notation τ_L^{PD} to denote the income tax rate τ^s that satisfies the condition $\tau^s \in [0, \tau^*)$ and equation (4.7). We then rearrange equation (4.7) and τ_L^{PD} satisfies the following condition:

$$A\left(N - \left[\frac{(1-\tau_L^{PD})\mathbf{K}\alpha}{\omega}\right]^{\frac{1}{1-\alpha}}\right) = \mathbf{K}\frac{\alpha}{1-\alpha}\left[\frac{(1-\tau_L^{PD})\mathbf{K}\alpha}{\omega}\right]^{\frac{2\alpha-1}{1-\alpha}}\frac{\mathbf{K}\alpha}{\omega} - (\omega + \tau^{PD}A)\frac{1}{1-\alpha}\left[\frac{(1-\tau_L^{PD})\mathbf{K}\alpha}{\omega}\right]^{\frac{\alpha}{1-\alpha}}\frac{\mathbf{K}\alpha}{\omega} \quad (4.14)$$

Suppose the farmer chooses the income tax rate, $\tau^{FD} = \tau_L^{PD}$, the derivative of the group income function, I^{nl} at τ_L^{PD} equals:

$$\frac{\partial I^{nl}(\tau_L^{PD})}{\partial \tau^{FD}} = (1-k)A\frac{1}{1-\alpha}\left[\frac{(1-\tau_L^{PD})\mathbf{K}\alpha}{\omega}\right]^{\frac{\alpha}{1-\alpha}}\frac{\mathbf{K}\alpha}{\omega} + \mathbf{K}\left[\frac{(1-\tau_L^{PD})\mathbf{K}\alpha}{\omega}\right]^{\frac{\alpha}{1-\alpha}} + kA\left(N - \left[\frac{(1-\tau_L^{PD})\mathbf{K}\alpha}{\omega}\right]^{\frac{1}{1-\alpha}}\right) + \tau_L^{PD}kA\frac{1}{1-\alpha}\left[\frac{(1-\tau_L^{PD})\mathbf{K}\alpha}{\omega}\right]^{\frac{\alpha}{1-\alpha}}\frac{\mathbf{K}\alpha}{\omega} - \tau_L^{PD}\mathbf{K}\frac{\alpha}{1-\alpha}\left[\frac{(1-\tau_L^{PD})\mathbf{K}\alpha}{\omega}\right]^{\frac{2\alpha-1}{1-\alpha}}\frac{\mathbf{K}\alpha}{\omega} \quad (4.15)$$

We then substitute equation (4.14) into (4.15), and we have:

$$\frac{\partial I^{nl}(\tau_L^{PD})}{\partial \tau^{FD}} = (1-k)A\frac{1}{1-\alpha}\left[\frac{(1-\tau_L^{PD})\mathbf{K}\alpha}{\omega}\right]^{\frac{\alpha}{1-\alpha}}\frac{\mathbf{K}\alpha}{\omega} + \mathbf{K}\left[\frac{(1-\tau_L^{PD})\mathbf{K}\alpha}{\omega}\right]^{\frac{\alpha}{1-\alpha}}$$
$$+ (k-\tau_L^{PD})\mathbf{K}\frac{\alpha}{1-\alpha}\left[\frac{(1-\tau_L^{PD})\mathbf{K}\alpha}{\omega}\right]^{\frac{2\alpha-1}{1-\alpha}}\frac{\mathbf{K}\alpha}{\omega}$$
$$- k\omega\frac{1}{1-\alpha}\left[\frac{(1-\tau_L^{PD})\mathbf{K}\alpha}{\omega}\right]^{\frac{\alpha}{1-\alpha}}\frac{\mathbf{K}\alpha}{\omega} \quad (4.16)$$

If $\frac{\partial I^{nl}(\tau_L^{PD})}{\partial \tau^{FD}} > 0$, the set of parameter, δ, and τ_L^{PD} satisfies the following condition:

$$(1-k)\frac{A}{\omega}\frac{\alpha}{1-\alpha} + k\frac{\alpha}{1-\alpha}(\frac{1}{1-\tau_L^{PD}} - 1) + \left(1 - \frac{\alpha}{1-\alpha}\frac{\tau_L^{PD}}{1-\tau_L^{PD}}\right) > 0$$

We then summarize a sufficient condition under which $\frac{\partial I^{nl}(\tau_L^{PD})}{\partial \tau^{FD}} > 0$ which is given by:

$$\tau_L^{PD} < 1-\alpha$$

If the optimal income tax rate of the capitalist, τ_L^{PD}, satisfies the condition that $\tau_L^{PD} < 1-\alpha$, the optimal income tax rate of the farmer is higher than that of the capitalist where $\tau_L^{FD} > \tau_L^{PD}$. We then summarize the sufficient conditions under which the landed elite would prefer to fully democratize the state.

Proposition 4. *Given the set of parameter, δ, if the optimal income tax rate of the capitalist, τ^{PD*}, and the optimal income tax rate of the farmer, τ^{FD*}, satisfy the following conditions, the landed elite would like to extend the voting rights to the whole population:*

- $\tau^{PD*}, \tau^{FD*} \in [0, \tau^{**})$;
- $\tau^{PD*} < 1-\alpha$

If the optimal income tax rate of the capitalist middle class and the farmer satisfies the condition, $\tau^{PD*}, \tau^{FD*} \in [0, \tau^{**})$, the agricultural production function is linear, $N_L < N^*$, and the landed elite benefits from a higher income tax rate. Since $1-\alpha$ represents the contribution of capitals towards manufacturing income, a higher value of $1-\alpha$ means that manufacturing income is mostly contributed by capitals and lowering income tax rate could only raise the labour demand of the manufacturing industry by a limited scale. For the farmer, they will not lower the income tax rate in exchange of a higher labour demand, otherwise they will received a lower tax revenue. For the capitalist middle class, since they not only receive the taxation from the manufacturing industry, but also receive the direct produced manufacturing income, given a lower value of $1-\alpha$, they will prefer to impose an income tax rate which is lower than that is preferred by the farmer in exchange of a higher produced income. Suppose the landed elite prefers a lower income tax rate, if the contribution of capitals towards manufacturing income is high, the landed elite prefers to extend the suffrage to the whole public.

On the contrary, if the income tax rate, τ^s, satisfies the condition that $\tau^s \in [\tau^{**}, \tau^*)$, the landed elite benefits from a decreasing income tax rate. Under this circumstance, if the contribution of capitals towards manufacturing income is high, the landed elite will partially democratize the state so as to achieve a lower income tax rate.

As we have mentioned in section 4.3.1, if the income tax rate, τ^s satisfies the condition $\tau^s \in [\tau^*, 1)$, the landed elite will benefit from a decreasing income tax rate. We then compare the value of τ_H^{PD} and τ_H^{FD}. Here, τ_H^{PD} refers to the income tax rate τ^s which satisfies the condition $\tau^s \in [\tau^s, 1)$ and equation (4.9). We then rearrange equation (4.9) and τ_H^{PD} satisfies the following condition:

$$\mathbf{K}\frac{\alpha}{1-\alpha}\left[\frac{(1-\tau^{PD})\mathbf{K}\alpha}{\omega}\right]^{\frac{2\alpha-1}{1-\alpha}}\frac{\mathbf{K}\alpha}{\omega} = AN^* + \omega\frac{1}{1-\alpha}\left[\frac{(1-\tau^{PD})\mathbf{K}\alpha}{\omega}\right]^{\frac{\alpha}{1-\alpha}}\frac{\mathbf{K}\alpha}{\omega} \tag{4.17}$$

Suppose the farmer chooses the income tax rate, $\tau^{FD} = \tau_H^{PD}$, in a democracy, the density of the group income function, I^{nl}, at τ_H^{PD} equals:

$$\frac{\partial I^{nl}(\tau_H^{PD})}{\partial \tau^{FD}} = kAN^* + \mathbf{K}\left[\frac{(1-\tau_H^{PD})\mathbf{K}\alpha}{\omega}\right]^{\frac{\alpha}{1-\alpha}} - \tau_H^{PD}\mathbf{K}\frac{\alpha}{1-\alpha}\left[\frac{(1-\tau_H^{PD})\mathbf{K}\alpha}{\omega}\right]^{\frac{2\alpha-1}{1-\alpha}}\frac{\mathbf{K}\alpha}{\omega} \tag{4.18}$$

We then substitute equation (4.17) into (4.18), and we have:

$$\frac{\partial I^{nl}(\tau_H^{PD})}{\partial \tau^{FD}} = (k-\tau_H^{PD})AN^* + (1-\tau_H^{PD}\frac{\alpha}{1-\alpha})\mathbf{K}\left[\frac{(1-\tau_H^{PD})\mathbf{K}\alpha}{\omega}\right]^{\frac{\alpha}{1-\alpha}} \tag{4.19}$$

Given the set of parameter δ and τ_H^{PD}, we have the sufficient condition under which $\frac{\partial I^{nl}(\tau_H^{PD})}{\partial \tau^{FD}} > 0$:

$$\tau_H^{PD} < k;$$
$$\tau_H^{PD} < \frac{1-\alpha}{\alpha}$$

Under the stated condition, the optimal income tax rate of the farmer is higher than that of the capitalist where $\tau_H^{FD} > \tau_H^{PD}$. We then summarize the condition under which the landed elite would prefer to partially democratize the state:

Proposition 5. *Given the set of parameter, δ, if the optimal income tax rate of the capitalist, τ^{PD*}, and the optimal income tax rate of the farmer, τ^{FD*}, satisfy the following conditions, the landed elite would prefer to extend voting rights to the capitalist only:*

- $\tau^{PD*}, \tau^{FD*} \in [\tau^{**}, \tau^{*})$;
- $\tau^{PD*} < 1 - \alpha$

or

- $\tau^{PD*}, \tau^{FD*} \in [\tau^{*}, 1)$;
- $\tau^{PD*} < k$;
- $\tau^{PD*} < \frac{1-\alpha}{\alpha}$

As we have explained in the previous section, if the expected income tax rate in partial and full democracy satisfy the condition that $\tau^{PD*}, \tau^{FD*} \in [\tau^{**}, 1)$, the landed elite benefits from a lower income tax rate. Under this circumstance, the landed elite prefer to extend voting rights only to the capitalist middle class, if the contribution of capitals towards manufacturing income is high. In addition, a higher value of k makes partial democratization more attractive to the landed elite. Since k represents the share of the total produced agricultural income that the landed elite takes, a higher value of k means the landed elite is repressive, the income inequality between the landed elite and the farmer is high, and the farmer is relatively poorer in the whole society. As a result, once the suffrage has been extended to the whole public, the farmer will impose a higher income tax rate in exchange of a higher redistributed wealth from the landed elite.

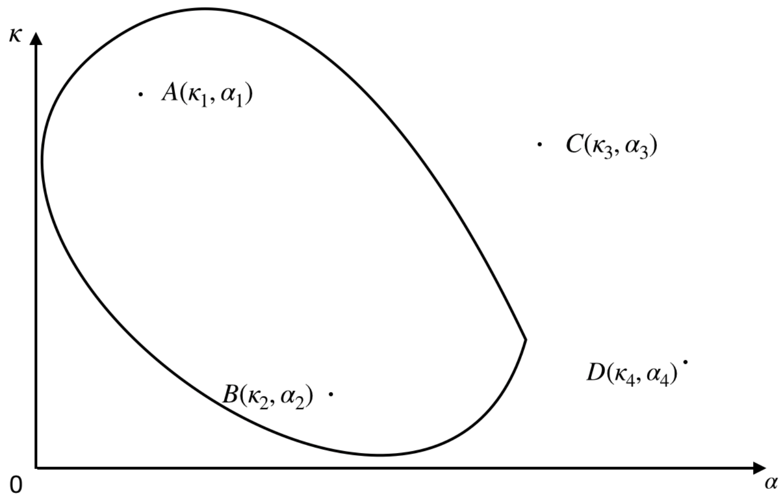


Figure 4.1: The preferred size of enfranchisement of the landed elite

To further describe how the parameter κ and α affect the choice of the landed elite over the size of enfranchisement, we draw the Figure 4.1. In Figure 4.1, point A represents the set of parameter, $\delta = \{A, A_M, L, M, N, \kappa_1, \alpha_1, \omega\}$. Similarly, point B, C, and D represent the set of parameter δ with different value of κ and α. Suppose the state is given with the set of parameter δ which locates within the circle, the preferred income tax rate of the capitalist middle class and the farmer, τ^{PD} and τ^{FD}, are in the region of $(0, \tau^{**}$. That is to say, if the set of parameter δ is inside the circle, the landed elite is benefiting from a higher income tax rate. As we have already discussed above, a higher κ or a lower α contributes to a greater difference between the preferred income tax rate of the capitalist middle class is higher and that of the farmer. That is to say, in Figure 4.1, if the state is given with point A, it is more likely that the landed elite would prefer to fully democratize the state in exchange of a higher income tax rate, comparing with the point B. If the state is given with a point which is outside the circle, the preferred income tax rate of the capitalist middle class and the farmer, τ^{PD} and τ^{FD}, satisfy the condition that $\tau^{PD}, \tau^{FD} \in [\tau^{**}, 1]$. If the state is given with point C, it is more likely that the capitalist middle class prefers a lower income tax rate comparing with that of the farmer. Under this circumstance, the landed elite prefers to extend the suffrage to the capitalist middle class only in exchange of a lower income tax rate.

4.3.4 Manufacturing Industry and the Size of Enfranchisement

Now we will look at how the development of manufacturing industry affects the size of enfranchisement. We firstly look at how the optimal income tax rate in partial and full democracy changes with the total factor productivity of the manufacturing industry, A_M. As we have explained in the previous section, in a partial democracy, when the agricultural production function is linear, $N_L < N^*$, the optimal income tax rate, τ^{PD*}, solves the following first order condition:

$$\frac{\partial I^{\mathcal{M}}(\tau^{PD*})}{\partial \tau^{PD}} = -\mathbf{K}\frac{\alpha}{1-\alpha}\left[\frac{(1-\tau^{PD*})\mathbf{K}\alpha}{\omega}\right]^{\frac{2\alpha-1}{1-\alpha}}\frac{\mathbf{K}\alpha}{\omega}+A(N-\left[\frac{(1-\tau^{PD*})\mathbf{K}\alpha}{\omega}\right]^{\frac{1}{1-\alpha}})$$
$$+(\omega+\tau^{PD*}A)\frac{1}{1-\alpha}\left[\frac{(1-\tau^{PD*})\mathbf{K}\alpha}{\omega}\right]^{\frac{\alpha}{1-\alpha}}\frac{\mathbf{K}\alpha}{\omega}=0 \quad (4.20)$$

Above condition will be satisfied if and only if τ^{PD} satisfies the following condition:

$$-\frac{\alpha}{1-\alpha}\left[\frac{(1-\tau^{PD})\alpha}{\omega}\right]^{\frac{2\alpha-1}{1-\alpha}}\frac{\alpha}{\omega}+AN\left(\frac{1}{\mathbf{K}}\right)^{\frac{1}{1-\alpha}}-A\left[\frac{(1-\tau^{PD})\alpha}{\omega}\right]^{\frac{1}{1-\alpha}}$$
$$+(\omega+\tau^{PD}A)\frac{1}{1-\alpha}\left[\frac{(1-\tau^{PD})\alpha}{\omega}\right]^{\frac{\alpha}{1-\alpha}}\frac{\alpha}{\omega}=0 \quad (4.21)$$

Since $\mathbf{K} = A_M K^{1-\alpha}$, any increases in the value of A_M will increase the value of $\mathbf{K}$, and the left hand side of (4.21) decreases in the value of A_M. That is to say, any increases in the value of A_M will lower the value of τ^{PD} that solves (4.20). In a partial democracy, when the agricultural production function is linear, the optimal income tax rate, τ^{PD*}, decreases in the improvement of the total factor productivity of the manufacturing industry, A_M.

If the agricultural production function is flat, $N_L \geq N^*$, in a partial democracy, the optimal income tax rate, τ^{PD*}, that is favoured by the capitalist middle class satisfies the following first order condition:

$$\frac{\partial I^{\mathcal{M}}(\tau^{PD*})}{\partial \tau^{PD}} = -\mathbf{K}\frac{\alpha}{1-\alpha}\left[\frac{(1-\tau^{PD*})\mathbf{K}\alpha}{\omega}\right]^{\frac{2\alpha-1}{1-\alpha}}\frac{\mathbf{K}\alpha}{\omega} + AN^{*} + \omega\frac{1}{1-\alpha}\left[\frac{(1-\tau^{PD*})\mathbf{K}\alpha}{\omega}\right]^{\frac{\alpha}{1-\alpha}}\frac{\mathbf{K}\alpha}{\omega} = 0 \quad (4.22)$$

Above first order condition (4.22) will be satisfied if and only if τ^{PD} satisfies the following condition:

$$-\frac{\alpha}{1-\alpha}\left[\frac{(1-\tau^{PD})\alpha}{\omega}\right]^{\frac{2\alpha-1}{1-\alpha}}\frac{\alpha}{\omega} + AN^{*}\left(\frac{1}{\mathbf{K}}\right)^{\frac{1}{1-\alpha}} + \omega\frac{1}{1-\alpha}\left[\frac{(1-\tau^{PD})\alpha}{\omega}\right]^{\frac{\alpha}{1-\alpha}}\frac{\alpha}{\omega} = 0 \quad (4.23)$$

Similarly, the value of the left hand side of (4.23) decreases in the value of A_M. That is to say, in a partial democracy, if the agricultural production function is flat, any increases in the value of A_M will lower the value of τ^{PD} that solves condition (4.22). We then summarize the following comparative static result:

Proposition 6. *Given the set of parameter, δ, in a partial democracy, any improvements in the total factor productivity of the manufacturing industry will decrease the income tax rate that is determined by the capitalist middle class:*

$$\frac{\partial \tau^{PD}(\delta)}{\partial A_M} < 0$$

This proposition is intuitive. In a partial democracy, if manufacturing industry is productive or the price of manufacturing goods is high, for the capitalist middle class, lowering the income tax rate, from one hand, will relax its pressure from paying taxes. From the other hand, it will save the average labour cost that enables the capitalist middle class hire more labour. Decreasing the income tax rate will bring the capitalist middle class a higher produced income from the manufacturing industry which will exceed losses from agriculture that are generated by the decreasing collected tax revenue.

We will then look at how the development of manufacturing industry affects the optimal income tax rate in a full democracy. In a full democracy, when the agricultural production function is linear, $N_L < N^*$, the income tax rate that is favoured by the farmer, τ^{FD*}, solves the following first order condition:

$$\frac{\partial I^{nl}(\tau^{FD*})}{\partial \tau^{FD}} = (1-k)A\frac{1}{1-\alpha}\left[\frac{(1-\tau^{FD*})\mathbf{K}\alpha}{\omega}\right]^{\frac{\alpha}{1-\alpha}}\frac{\mathbf{K}\alpha}{\omega}+\mathbf{K}\left[\frac{(1-\tau^{FD*})\mathbf{K}\alpha}{\omega}\right]^{\frac{\alpha}{1-\alpha}}$$
$$+ kA\left(N - \left[\frac{(1-\tau^{FD*})\mathbf{K}\alpha}{\omega}\right]^{\frac{1}{1-\alpha}}\right) + \tau^{FD*}kA\frac{1}{1-\alpha}\left[\frac{(1-\tau^{FD*})\mathbf{K}\alpha}{\omega}\right]^{\frac{\alpha}{1-\alpha}}\frac{\mathbf{K}\alpha}{\omega}$$
$$- \tau^{FD*}\mathbf{K}\frac{\alpha}{1-\alpha}\left[\frac{(1-\tau^{FD*})\mathbf{K}\alpha}{\omega}\right]^{\frac{2\alpha-1}{1-\alpha}}\frac{\mathbf{K}\alpha}{\omega} = 0 \quad (4.24)$$

For any non-negative $\mathbf{K}$, above first order condition will be satisfied if and only if τ^{FD} satisfies the following condition:

$$(1-k)A\frac{1}{1-\alpha}\left[\frac{(1-\tau^{FD})\alpha}{\omega}\right]^{\frac{\alpha}{1-\alpha}}\frac{\alpha}{\omega} + \left[\frac{(1-\tau^{FD})\alpha}{\omega}\right]^{\frac{\alpha}{1-\alpha}} + kA\left(\frac{1}{\mathbf{K}}\right)^{\frac{1}{1-\alpha}}$$
$$- kA\left[\frac{(1-\tau^{FD})\alpha}{\omega}\right]^{\frac{1}{1-\alpha}} + \tau^{FD}kA\frac{1}{1-\alpha}\left[\frac{(1-\tau^{FD})\alpha}{\omega}\right]^{\frac{\alpha}{1-\alpha}}\frac{\alpha}{\omega}$$
$$- \tau^{FD}\frac{\alpha}{1-\alpha}\left[\frac{(1-\tau^{FD})\alpha}{\omega}\right]^{\frac{2\alpha-1}{1-\alpha}}\frac{\alpha}{\omega} = 0 \quad (4.25)$$

Any increases in the value of A_M will lower the value of the left hand side of (4.25) so that the value of τ^{FD} that solves condition (4.24) will be decreased. That is to say, in a full democracy, when the agricultural production function is linear, the optimal income tax rate, τ^{FD*}, decreases in the value of A_M.

If the agricultural production function is flat, $N_L \geq N^*$, in a full democracy, the farmer will determine its optimal income tax rate, τ^{FD*}, that satisfies the following first order condition:

$$\frac{\partial I^{nl}(\tau^{FD*})}{\partial \tau^{FD}} = kAN^* + \mathbf{K}\left[\frac{(1-\tau^{FD*})\mathbf{K}\alpha}{\omega}\right]^{\frac{\alpha}{1-\alpha}}$$
$$- \tau^{FD*}\mathbf{K}\frac{\alpha}{1-\alpha}\left[\frac{(1-\tau^{FD*})\mathbf{K}\alpha}{\omega}\right]^{\frac{2\alpha-1}{1-\alpha}}\frac{\mathbf{K}\alpha}{\omega} = 0 \quad (4.26)$$

Condition (4.26) will be satisfied if and only if τ^{FD} satisfies the following condition:

$$kAN^*\left(\frac{1}{\mathbf{K}}\right)^{\frac{1}{1-\alpha}} + \left[\frac{(1-\tau^{FD})\alpha}{\omega}\right]^{\frac{\alpha}{1-\alpha}} - \tau^{FD}\frac{\alpha}{1-\alpha}\left[\frac{(1-\tau^{FD})\alpha}{\omega}\right]^{\frac{2\alpha-1}{1-\alpha}}\frac{\alpha}{\omega} = 0 \quad (4.27)$$

Similarly, the value of the left hand side of (4.27) decreases in the value of A_M. In a full democracy, if the agricultural production function is flat, any increases in the value of A_M will lower the value of τ^{FD} that solves the first order condition (4.26). We then summarize the following comparative static result:

Proposition 7. *Given the set of parameter, δ, in a full democracy, any improvements in the total factor productivity of the manufacturing industry, A_M, will decrease the income tax rate that is chosen by the farmer:*

$$\frac{\partial \tau^{FD}(\delta)}{\partial A_M} < 0$$

This comparative static result implies that, in a full democracy, any improvements in the development of the manufacturing industry will make the farmer cut down the income tax rate that is applied to the whole society. This is because, decreased income tax rate lowers the average labour cost, the labour demand of the capitalist middle class increases, and the total produced manufacturing revenue is raised up. For the farmer, lowering income tax rate will increase their total received tax revenue from the manufacturing industry, which exceeds their losses in the collected tax revenue from agriculture.

We have seen the fact that in either partial or full democracy, any improvements in the development of manufacturing industry will lower the income tax rate that is applied to the whole society. We then compare the optimal income tax rate under partial and full democracy, and investigate how the development of manufacturing industry affect the preferred size of enfranchisement of the landed elite. From Proposition 2, we have explained that, given the set of parameter, δ, if the optimal income tax rate of the capitalist, τ^{PD}, and the optimal income tax rate of the farmer, τ^{FD}, satisfy the condition that τ^{PD}, $\tau^{FD} \in [0, \tau^{**})$, the landed elite will extend voting rights to the group for whom the optimal income tax rate is higher. In addition, from Proposition 3, we have also summarized that the value of $\mathbf{K}$ positively affect the value of τ^{**}. That is to say, any improvements in the manufacturing industry makes it more likely that the landed elite will extend voting rights to the group that favours a higher tax rate.

We have already compared the value of τ^{PD} and τ^{FD}, and summarize the condition under which $\tau^{PD} < \tau^{FD}$. Since $\tau^{**} \leq \tau^{*}$, the landed elite prefer a higher income tax rate only when they access to a linear agricultural production function. Given the set of parameter, δ, if the agriculture production function is linear, $N_L < N^*$, the value of τ^{PD} is smaller than that of τ^{FD}, if and only if τ^{PD} satisfies the following condition:

$$(1-k)\frac{A}{\omega}\frac{\alpha}{1-\alpha} + k\frac{\alpha}{1-\alpha}(\frac{1}{1-\tau_L^{PD}} - 1) + \left(1 - \frac{\alpha}{1-\alpha}\frac{\tau_L^{PD}}{1-\tau_L^{PD}}\right) > 0 \qquad (4.28)$$

We then differentiate the left hand side of (4.28) by τ^{PD}, and we have $\frac{\partial LHS}{\partial \tau^{PD}} < 0$. Since $\frac{\partial \tau^{PD}(\delta)}{\partial A_M} < 0$, we further have that $\frac{\partial LHS}{\partial A_M} > 0$, any improvements in the manufacturing industry makes it more likely that the preferred income tax rate of the capitalist middle class, τ^{PD}, is lower than that of the farmer, τ^{FD}. We then summarize the following proposition:

Proposition 8. *For those states that have a productive manufacturing industry or the price of manufacturing goods is high, it is more likely that the landed elite is benefiting from extending voting rights to the whole public.*

This proposition if intuitive. If the manufacturing industry is productive or the price of manufacturing goods is high, agricultural production function is linear, any decreases in the income tax rate make the capitalist middle class hire more labour. In a partial democracy, for the capitalist middle class, a decreased income tax rate gives them a higher received revenue from the manufacturing industry that exceeds their losses from agricultural taxation. Similarly, in a full democracy, for the farmer, a decreased income tax rate gives them a higher received tax revenue from the manufacturing industry which exceeds their losses from agriculture. However, since the received revenue from the manufacturing industry of the capitalist middle class is greater than that of the farmer, the preferred income tax rate of the capitalist middle class is greater. For the landed elite, when the manufacturing industry is productive or the price of manufacturing goods is high, decreased income tax rate will reduce their received agricultural revenue that exceeds savings in the tax cost. Therefore, it is beneficial for the landed elite to favour the reform bill that extends voting rights to the whole public.

4.4 Concluding Remarks

This chapter has offered a simple model of democratization, investigating how the landed elite would prefer to extend the franchise, and providing a new explanation for how the landed elite will benefit from extending suffrage to the whole population. The two main contributions of this chapter are firstly, it outlines the possibility that the landed elite would prefer to fully democratize the state, extend the franchise to the whole population and accept a higher income tax rate. Secondly, we also capture how the development of the manufacturing industry affects the size of enfranchisement, since the landed elite would prefer to fully democratize the state only when the domestic manufacturing industry is strong.

In this chapter, we put much emphasis on, given the threat of revolution, how the landed elite determines the size of enfranchisement, especially whether to extend the franchise only to the capitalist, or to extend suffrage to the whole population. However, our model may have other implications. It could explain, in a partial democracy, why the capitalist transitions the state to a full democracy. It outlines the possibility that the landed elite may cooperate with the workers and initiate a coup as a result of decreasing productive labour force. Considering this threat of social unrest, the capitalist would prefer to extend suffrage to the whole population and accept a higher income tax rate.

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