Traffic control with connected and automated vehicles on urban roads

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Dedicated to my parents
for their endless love and support.

Intellectual Property and Publications

The candidate confirms that the work submitted is his own, except where work which has formed part of jointly-authored publications has been included. The contribution of the candidate and the other authors to this work has been explicitly indicated below. The candidate confirms that appropriate credit has been given where reference has been made to the work of others.

The work in chapters 2, 3, and 4 consists of original research, where the candidate was the lead researcher. The main ideas in these chapters were developed by the candidate during the PhD with the support and guidance of the supervisors and other researchers, who have been listed as co-authors where appropriate.

All the manuscripts were written by the candidate and improved by comments from all the co-authors. The manuscripts are summarised below.

Chapter 2 has been published as follows,

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Abstract

The connected and automated vehicle (CAV) is a promising technology that will reshape the transport. It has potentials in reducing fuel consumption as well as improving capacity and safety. The traffic control of CAVs on urban roads is investigated in the thesis, which consists of two aspects: intersection control and trajectory planning.

For the intersection control with CAVs, a bilevel programming model integrating intersection control with trajectory planning is proposed to improve the efficiency of the intersection when all vehicles are CAVs. The feedback structure adapts a wide range of conditions and improves capacity. The cooperation between the two levels and their linear properties ensure reasonable solving time. A platoon-based method is also proposed to improve the calculation speed.

For the trajectory planning with CAVs, a platoon-based eco-driving model is proposed using model predictive control. The model is used in mixed autonomy traffic and considers traffic efficiency, fuel consumption, and driving comfort. The cooperation between automated vehicles and human-driven vehicles reduces the negative impact of eco-driving on the following vehicles and reduces fuel consumption even further. The performance under different platoon sizes and penetrations of automated vehicles are also tested in the simulation.

At an intersection controlled by the adaptive traffic control system, the vehicle may not be able to get accurate information on the future signal timing. A multi-phase model predictive control model is proposed to reduce fuel consumption by considering the stochastic signal information. Two driving regimes are considered based on the state of the traffic signal at each time step: accelerating to pass the intersection when the light is green or keeping waiting for the possible green light in the next time step. This model is further extended to the case that the vehicle can receive the information on the future signal timing some seconds in advance. An additional eco-driving strategy is considered in this case.

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List of Abbreviations

ACC . . . . . . Adaptive Cruise Control

ADAS . . . . . Advanced Driving Assistance System

AV . . . . Automated Vehicle

CACC . . . . . Cooperative Adaptive Cruise Control

CAV . . . . . . Connected and Automated Vehicle

C-ITS . . . . . Cooperative Intelligent Transport System

CV . . . . Connected Vehicle

DARPA . . . . Defense Advanced Research Projects Agency

DSRC . . . . . Dedicated Short-Range Communication

FCFS . . . . . First Come First Serve

FIFO . . . . . First In First Out

GHG . . . . . Greenhouse Gas

GLOSA . . . . Green Light Optimal Speed Advisory

GPOPS . . . . Gauss Pseudospectral Optimization Software

HOV . . . . . . High-Occupancy Vehicle

HV . . . . Human-driven Vehicle

IDM . . . . . . Intelligent Driving Model

IIDM . . . . . Improved Intelligent Driver Model

ISA . . . . . . Intelligent Speed Adaptation

LEZ . . . . . . Low Emission Zone

LP . . . . Linear Programming

MILP . . . . . Mixed Integer Linear Programming

MPC . . . . . Model Predictive Control

MPOC . . . . Multi-phase Optimal control

List of Abbreviations

NLP . . . . . . Nonlinear Programming

OVM . . . . . Optimal Velocity Model

RSU . . . . . . Roadside Unit

SPaT . . . . . Signal Phase and Timing

SQP . . . . . . Sequential Quadratic Programming

TUC . . . . . . Traffic-responsive Urban Control

V2I . . . . Vehicle to Infrastructure

V2V . . . . . . Vehicle to Vehicle

VACS . . . . . Vehicle Automation and Communication Systems

1

Introduction

Contents

1.1 Background and motivation

The connected and automated vehicle (CAV) is the most critical technology in the automotive industry since Ford produced the Model T. It is believed to be the revolution of mobility by relieving the driving burden for people who do not want to or cannot drive. When fully automated vehicles (AVs) are available, people can even read, work or sleep in the car. Besides improving driving comfort, the CAV is also safer and more efficient. It can receive or detect accurate information on the traffic and the road via Vehicle to Vehicle (V2V) and Vehicle to Infrastructure (V2I) communication technology, or vehicle-borne sensors. The advanced decision system in the CAV acts better than an experienced driver, as it eliminates human errors, reduces response time and increases efficiency. In short, the CAV releases people from tedious driving tasks, and thus the passengers can better enjoy the mobility service.

1.1.1 Background of the CAV

The first AV is dated back to 1925 shortly after the birth of the motor car. It is a radio-controlled driverless car which was shown travelling through a heavy traffic street of Manhattan (Engelking, 2017). Since then, many universities and companies have pioneered in the development of developing AVs, such as Stanford Cart by the Stanford University in the US during the 1960s and 1970s (Vanderbilt, 2012), a driverless Citroen DS by the Transport and Road Research Laboratory in the UK during the 1960s (Reynolds, 2001) and VaMoRs and VaMP by the Bundeswehr University Munich in Germany in the 1980s. In 2004, Defense Advanced Research Projects Agency (DARPA) announced the first Grand Challenge, offering the winner a million-dollar prize whose AV was the fastest driving through California's Mojave Desert. No team has finished the race, but it is the first big push toward a fully autonomous vehicle. In 2005, five teams out of twenty-three finished the race in the second Grand Challenge. In 2007, DARPA organised the Urban Challenge to drive through an urban environment (Urmson and Whittaker, 2008).

After 2008, more companies joined the AV development. Google launched the driverless vehicle project (now called Waymo) in 2009 by hiring a winner of the second Grand Challenge. By October 2018, it had finished 10 million miles of self-driving test on public roads and over 7 billion simulation miles (Krafcik, 2018). Uber built its driverless vehicle research team by hiring dozens of scientists from Carnegie Mellon University in 2015. Major automakers like Ford, General Motors, Nissan, Tesla and Mercedes have also started to invest the research and test on AVs.

A widely accepted definition of levels of vehicle automation is proposed by the SAE International in the J3016 standard (SAE International, 2018). There are six levels of autonomy from level 0 to level 5. Level 0 (No automation), level 1 (Driver assistance) and level 2 (Partial automation) require drivers' attention at all times. Level 1 is the basic cruise control involved in controlling acceleration and deceleration, for example, Jaguar Land Rover's off-road cruise control. Level 2 further introduces steering control, for example, Cadillac Super Cruise, Nissan ProPilot Assist, and Tesla Autopilot, which are already available for vehicles. Level 3 and above are considered as "automated driving systems". The critical difference is that the vehicle can monitor the surrounding environment and make decisions, for example, BMW's iNext which will be launched in 2021. The driver is allowed to look away from the road for an extended period but required to take over the control within seconds in some situations. The transition period is proven to be dangerous, which is also why many companies choose to skip Level 3 and go straight to Level 4. Level 4 has the same capability as level 3 but allows the longer time for the driver to take back control, for example, Waymo's self-driving cars. Level 5 is the truly full automation that the car can drive themselves under all road conditions.

More and more countries have introduced legislation to regulate the testing of the AV on public roads, such as US (US Department of Transportation and US. National Highway Traffic Safety Administration and, 2016), UK (Centre for Connected and Autonomous Vehicles, 2015), and China (Ministry of Industry and Information Technology et al., 2018). Numerous trials are carried out around the world. While true level 5 autonomy is still a long way off, major car manufacturers are planning to build cars with level 4 in the near future (Walker, 2019). Toyota

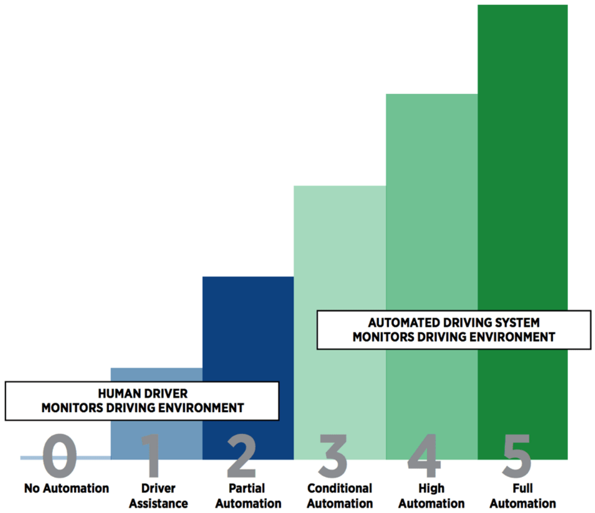


Figure 1.1: Levels of vehicle automation defined by the SAE (Figure source: Brooke, 2016)

and Hyundai are targeting AVs on a highway in 2020. Ford and Volvo plan to have level 4 vehicles in 2021. In December 2018, Waymo launched the first commercial autonomous ridesharing service in Phoenix (LeBeau, 2018). The fact that it is limited to a part of test users of the Waymo self-driving vehicle does not diminish its importance as the first public autonomous taxi.

1.1.2 Motivation of the CAV research on urban roads

People living in many cities, especially in the metropolis, are suffering from severe traffic problems on urban roads. According to the data of INRIX (Cookson, 2018) in 2017, traffic congestion costs the UK £37.7 billion or £1,168 per driver because of the lost production, wasted time and fuel. The UK is ranked 10th congested in the world in 2017 and third in the developed countries with drivers spending 31 hours stuck in rush hour traffic. London remains the most congested city in the UK and the second congested city in Europe and seventh in the world. London drivers lost 74 hours in the peak hours, which cost each driver £2,430 or £9.5 billion across the city as a whole. The congestion cost in London has kept increasing in recent years, which is £5.5 billion in 2014 – 2015 (Franks, 2016) and £6.2 billion in 2016 (Cookson, 2018).

Apart from the severe congestion, the emission from transport has been a big threat to health which increases the risk of heart disease and stroke. In 2016, the greenhouse gas (GHG) emission from transport increased by 26% in 28 European countries compared with 1990 while the total greenhouse gas emission reduced by 24% (European Environment Agency, 2018) (see Fig. 1.2). In result, the percentage of GHG from transport increased from 15% in 1990 to 25% in 2016. More than 70% of GHG from transport comes from road transport. Therefore, there is an increasing urgency to take actions to meet the target of 60% reduction by 2050 compared with 1990 level as set out in the 2011 Transport White Paper (Commissie, 2011). A similar problem happens in the UK as transport has been the main source of greenhouse gas in the UK since 2016, accounting for 26% of the total greenhouse gas emission (Vanlint, 2018) (see Fig. 1.3). While the emission from energy supply has reduced by 57% from 1990, the emission from transport returns the similar level as 1990. Road transport is a major source in it, especially personal cars. The rise of emission from road transport since 2013 is because of an increase in vehicle kilometres travelled (Department for Transport, 2017).

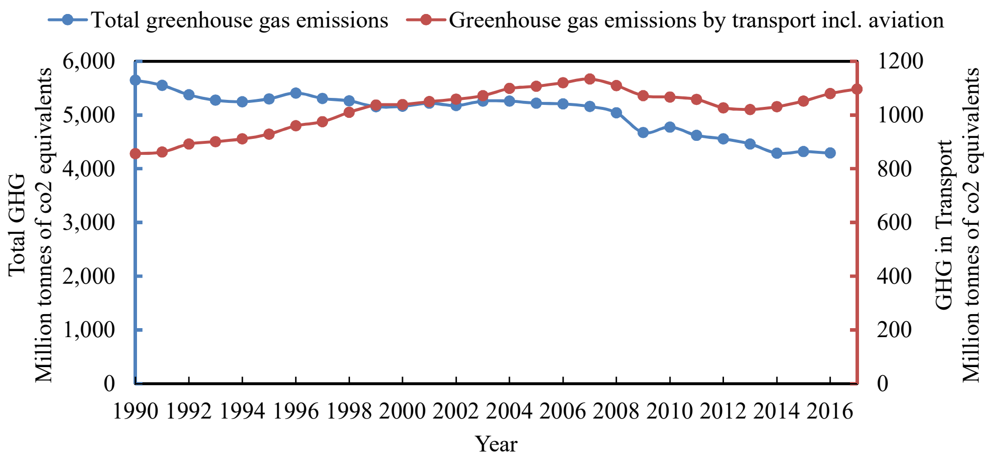


Figure 1.2: Greenhouse gas emissions from transport in the EU between 1990 and 2017 (Data source: European Environment Agency, 2018)

The research by the National Highway Traffic Safety Administration (NHTSA) in the US showed that the human error is the critical reason for 94% of the crashes (Singh, 2015) based on a national survey of 2,189,000 crashes. Merely 2% of crashes are caused by the environment and another 2% by the vehicle. A similar result has also been found in the UK which identified the human error is

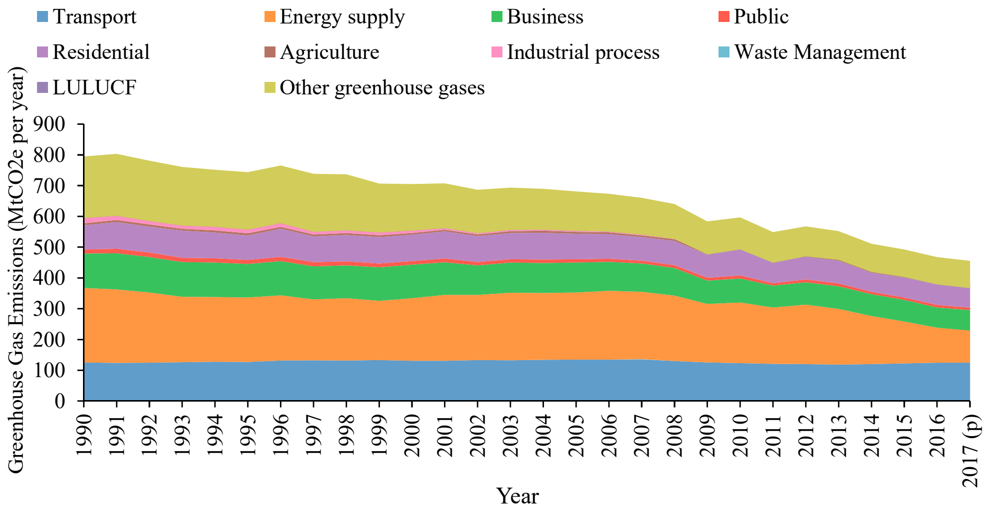


Figure 1.3: Annual greenhouse gas emissions in the UK between 1990 and 2017 (Data source: Department for Transport, 2017)

responsible for 95% accidents (The Royal Society for the Prevention of Accidents, 2017). The possible reasons for the accident contributed to the drivers may be inexperience, distraction, or fault reaction.

Lots of possible solutions have been used in practice to solve traffic problems, but each has its limitations. In order to reduce the emission in transport, many countries are promoting the transition from petrol and diesel vehicles to electric vehicles. The sale of new petrol and diesel cars will be banned in Denmark, Ireland, Sweden in 2030 (Nielson, 2018) and in the UK and France in 2040 (Asthana and Taylor, 2017). More than 200 cities in Europe launched the Low Emission Zone (LEZ) to reduce exposures to air pollution. Vehicles with higher emission are banned from entering this area. Otherwise, the driver has to pay penalty fines. An increasing number of cities are planning for Zero Emission Zones, which only allows electric vehicles, and bans all diesel vehicles and hybrid vehicles. These solutions reduce the emission in transport, but cannot solve the congestion problem. Transportation Demand Management, such as congestion charge, dedicated bus lanes, and high-occupancy vehicle (HOV) lanes promotes the use of public transport, ridesharing and cycling and reduce the traffic demand for single-occupant vehicles. However, the shift in travel mode is a long process. The intelligent transportation system, such as variable message signs, passenger information system makes more efficient use of existing transport infrastructure by improving capacity, safety and reliability of the transport system. However, the performance depends on the compliance of drivers or passengers.

The AV has the potential of transforming the transport system. If equipped with communication devices, it becomes the connected vehicle (CV) and can receive the information from the intersection controller or roadside unit via DSRC or 5G. The information may include the signal phase and timing (SPaT) of one or multiple preceding intersections, queue length estimated using loop detectors data, accident events and congested region. Then, it can be used to plan the route and trajectory that reduce travel time or fuel consumption. One of the applications widely researched is Green Light Optimal Speed Advisory (GLOSA), which provides the drivers with speed advice to pass the intersection on a green light. It reduces the waiting time by about 17% (Eckhoff et al., 2013), and fuel consumption by about 11 % (Eckhoff et al., 2013; Stevanovic et al., 2014). More sophisticated eco-driving methods are proposed for AVs, such as considering driving comfort and queue length.

When multiple AVs run together, they can form a platoon and run with small headway. Lots of research has been done to analyse the capacity improvement with Adaptive Cruise Control (ACC) or Cooperative Adaptive Cruise Control (CACC) (Van Arem et al., 2006) vehicles on the motorway segment or merging area. Less is found on the urban roads because it is more complex involving stop-and-go behaviour of human-driven vehicles (HVs) and switching right-of-way at the intersection. The CAV brings more flexibilities in the traffic operation on urban roads. The platoon increases the queue discharging rate (Le Vine et al., 2016) and intersection capacity (Lioris et al., 2017) with the performance of improvement varying from 25% to two times due to different parameters assumed in the vehicle dynamics. The platoon also improves the traffic flow stability (Davis, 2004). When 20% vehicles are ACC vehicles, all congestions are eliminated (Treiber and Helbing, 2001). The platoon of trucks can also reduce fuel consumption because of the reduction of aerodynamic drag (Tsugawa, 2013).

1.2 Literature review

Traffic control of CAVs has been studied extensively in the literature, mainly on intersection control and trajectory planning. These two directions will be reviewed in detail below.

1.2.1 Intersection control

The intersection contributes the major traffic accidents and traffic delays, and intersection control is one of the core parts of urban traffic management. It is expected that the AV could change the way an intersection operates. The literature is described in three parts: (1) part or all vehicles are AVs; (2) part or all vehicles are CVs; (3) the intersection operates on itself.

1.2.1.1 Intersection control with AVs

Lots of research developed new intersection control methods with different penetration rates of AVs, and they are summarised in Table 1.1. There are two common points in all papers mentioned in this section. The first is that they assumed the AV is also a CV. The second is that they belong to centralised intersection control that the intersection controller collects the information from approaching vehicles and performs the optimisation. The decentralised intersection control will be discussed in Section 1.2.1.3.

Two main approaches are applied to deal with the integration of intersection control and trajectory planning. One is solving the intersection control problem first and then the trajectory planning (Li and Wang, 2006; Dresner and Stone, 2008; Müller et al., 2016; Xu et al., 2017; Yu et al., 2018). For the part of intersection control, the controller collects information on AVs and determines their passing orders (Li and Wang, 2006) or arrival time (Yu et al., 2018). Then, trajectories of approaching vehicles are obtained by the optimisation or predetermined driving rules. The other is solving the intersection control and trajectory planning problem at the same time (Li et al., 2014; Kamal et al., 2015). The trajectories are integrated as decision variables in the intersection control optimisation, which often leads to complex nonlinear programming (NLP) problems and is also why Li et al. (2014) applied an enumeration method to find the best signal plan. Kamal et al. (2015) used model predictive control (MPC) to optimise the trajectories of vehicles approaching the intersection to reduce conflicts. MPC is widely used in the control of CAVs because it can well adapt to the real-time changing inputs. A brief introduction of MPC is included in Appendix 1.A.

Two different ideas are employed in intersection control to deal with the conflicts in the intersection. The first is to apply conventional phase-based signal timing (Li and Zhou, 2017; Xu et al., 2017; Yu et al., 2018). The second is to eliminate the traffic light and optimise the passing order dynamically (Li and Wang, 2006; Dresner and Stone, 2008; Kamal et al., 2015; Müller et al., 2016). Applying conventional phase-based signal timing has much fewer integer variables in the optimisation than using arbitrary passing orders but also has less flexibility. Dresner and Stone (2008)'s approach is a special case as it uses well-defined rules to assign the passing order, and there is no need to use integer variables.

Most of the literature focuses on the case that all the vehicles are AVs, which is also called autonomous intersection control. As there is no need to consider the complex and uncertain behaviour of HVs, they can focus on how to achieve the best efficiency. The methods developed by Dresner and Stone (2008) and Li and Zhou (2017) can also be applied in mixed traffic with AVs and HVs. In both of their methods, a conventional phase-based signal setting is applied, and AVs send requests with arrival time information to the intersection controller, but Dresner and Stone (2008) allows AVs to enter into the intersection when the signal is red and there are no conflicts.

1.2.1.2 Intersection control with CVs

Research on intersection control with CVs is summarised in Table 1.2. "standard single" denotes a typical four-armed intersection with turning movements, and "simplified single" denotes a simplified single intersection with only two roads and through movements. There are two ways of controlling the intersection: phase-based signal (He et al., 2012; Feng et al., 2015) and optimised passing sequence (Guler et al., 2014; Zhu and Ukkusuri, 2015; Yang et al., 2016). Though they did not stated explicitly, vehicles in the second approach need to cooperate in

Table 1.1: Summary of the literature on intersection control with AVs

Paper	Penetration	Signal phase	Passing order	Longitudinal control
Li and Wang (2006)	100%	No	Decision tree to remove conflicts	Trajectory planning and mapping
Dresner and Stone (2008)	0%-100%	No	Reservation method based on first-in-first-out policy	Simplified driving rules
Lee and Park (2012)	100%	No	Minimise the overlap of trajectory	Constant acceleration from the optimisation
Zohdy and Rakha (2012)	100%	No	Minimise conflicts and delay	Dynamic acceleration at every time step
Li et al. (2014)	100%	Yes	Enumeration to find the minimum average travel time or delay	Predefined speed pattern with one or two constant acceleration
Kamal et al. (2015)	100%	No	Model predictive control to reduce the conflict risk and deviation from the desire speed	Dynamic acceleration at every time step

Continued on next page

Paper	Penetration	Signal phase	Passing order	Longitudinal control
Müller et al. (2016)	100%	No	Minimise the total arrival time	Maximise the time travelling with constant speed
Li and Zhou (2017)	0%-100%	Yes	Optimise phase-based signal timing to minimise delay	Not considered
Sun et al. (2017)	100%	Yes	Optimise number of lanes and duration for every movement to maximise the capacity	Collective average behaviour
Xu et al. (2017)	100%	Yes	Optimise phase-based signal timing to minimise delay	Optimal control to minimise fuel consumption
Yu et al. (2018)	100%	Yes	Optimise phase-based signal timing to minimise delay by considering lane changing	Optimal control to minimise fuel consumption

following the optimised passing sequence, which is more suitable for AVs.

The controller can access the trajectory data of CVs within the communication range. There are many ways to use the trajectory data, for example estimating queue length (Feng et al., 2015; Yang et al., 2016; Beak et al., 2017), estimating number of vehicles (Feng et al., 2015), getting the stopping time (Beak et al., 2017), delay (Gradinescu et al., 2007), and identifying platoons (He et al., 2012; Liang et al., 2018). Most of the methods that work when the penetration rate is less than 100% also require the data from loop detectors to get the number of vehicles between CVs (Priemer and Friedrich, 2009; Guler et al., 2014; Yang et al., 2016).

The arrival time of all approaching vehicles needs to be figured out in all methods. It is readily available if all vehicles are CVs. Otherwise, the arrival time of conventional vehicles needs to be estimated. Guler et al. (2014) applied a uniform distribution to the arrival time of unequipped vehicles between two CVs. Feng et al. (2015) divided the road segment before the stop line into three regions and applied different methods to estimate the vehicle's status. In the queued region, the shockwave theory is applied to estimate the queue length. In the slow-down region, vehicles' movements modelled by the Wiedemann's car following model are analysed to estimate the position and speed of unequipped vehicles. In the free flow region, the number of vehicles is estimated by the penetration rate of CVs and the position is assumed uniformly distributed among the lanes and randomly distributed on the lane. The same strategy is used in Beak et al. (2017). He et al. (2012) used linear regression models to estimate the number of vehicles and arrival time. Priemer and Friedrich (2009) estimated the queue length by assuming that the number of arriving vehicles follows a Poisson distribution. Priemer and Friedrich (2009) and Goodall et al. (2013) tested their method for a single intersection in a small traffic network or an arterial road, and it was found that both of their methods work worse than conventional methods by TRANSYT-7F or Synchro at low penetration rates of CVs, but they showed improvement in average speed and delay at higher penetration rates.

Table 1.2: Summary of the literature on intersection control with CVs

Paper	Penetration	Control range	Information from the CV data	Methods
Gradinescu et al. (2007)	100%	Standard single	Delay and queue length	Phase skipping, extension or interruption
Priemer and Friedrich (2009)	10%–100%	Standard single	Number of queued vehicles and approaching vehicles	Dynamic programming and enumeration to minimise the total queue length
He et al. (2012)	10%–100%	Corridor	Speed, position, headway	Minimise the delay for platoons
Goodall et al. (2013)	10%–100%	Standard single	Speed, position	Use future simulation to select the phase with minimum delay or a combined objective function
Guler et al. (2014)	0%–100%	Simplified single	Time to enter the range and position when it stops	Minimise delay to determine the passing order
Feng et al. (2015)	25%–100%	Standard single	Speed and position; estimate the speed and position of unequipped vehicles	Two-level optimisation to minimise delay or queue length

Continued on next page

Paper	Penetration	Control range	Information from the CV data	Methods
Zhu and Ukkusuri (2015)	100%	Network	Speed, position, destination	Linear model based on CTM to reduce the total travel time
Yang et al. (2016)	0%–100%	Simplified single	Trajectory data, position	Branch and bound method to optimise the departure sequence
Beak et al. (2017)	25%–100%	Corridor	Trajectory data, stopping time, queue length	Minimise delay considering the stopping time
Liang et al. (2018)	0%–100%	Standard single	Time headway and space headway	Use the same methods in Guler et al. (2014) and Yang et al. (2016) to optimise the departure order of platoons

1.2.1.3 Self-adaptive intersection control

A few self-adaptive and distributed intersection control methods for CAVs are also proposed in the literature. The passing order is usually determined by some predefined rules that are agreed by all CAVs rather than optimisation and can operate without the central intersection controller. Alonso et al. (2011) proposed two methods to determine the priority for CVs approaching an intersection. The first is to provide a priority table that contains all possible situations. The second is to determine the priority level for each vehicle independently by itself and compare it with other vehicles. Wu et al. (2015) developed a model that the priority of passing is determined by the arrival time. A vehicle with a high priority sends messages to prevent others from passing. In a similar approach, Yang and Monterola (2016) proposed a model to control only the deceleration of CAVs before the stop line. The vehicle that will arrive at the stop line first gets a higher priority to pass the intersection. They also extended the model to the case of mixed traffic with CAVs and HVs.

1.2.2 Trajectory planning

1.2.2.1 Trajectory planning for an AV

Providing signal and phase information to vehicles can optimise traffic flow approaching an intersection. The research on trajectory planning for an AV is summarised in Table 1.3. The typical constraints, such as speed and acceleration constraints, are not listed there.

Various methods are proposed in the trajectory planning for an AV for different purposes, such as reducing the number of stops (Treiber and Kesting, 2014; Ubiergo and Jin, 2016), travel time (Yao et al., 2018), delay (Stebbins et al., 2017), fuel consumption (Kamal et al., 2010, 2013; He et al., 2015; Altan et al., 2017; Yang et al., 2017) and emissions (Van Katwijk and Gabriel, 2015). There are two methods to optimise the trajectory: optimisation based on some predefined speed patterns (Treiber and Kesting, 2014) or MPC (Asadi and Vahidi, 2011). The predefined speed patterns are the preferred driving behaviour to pass the intersection in different signal status. Treiber and Kesting (2014) proposed

Table 1.3: Summary of the literature on trajectory planning for an AV

Paper	Control variables	Constraints	Method	Objective
Kamal et al. (2010)	Acceleration	None	MPC	Minimise fuel consumption and the penalty cost of safety
Asadi and Vahidi (2011)	Tracking force and braking force	Traffic signal	MPC	Tracking a target speed to reduce idle time
Kamal et al. (2013)	Driving or braking torque	None	MPC	Minimise fuel consumption and the penalty cost of safety by predicting the preceding traffic
Treiber and Kesting (2014)	Parameters in a car following model	Traffic signal	Three rule-based strategies	Minimise travel time and the number of stops
He et al. (2015)	Acceleration	Traffic signal, queue, safety	MPC	Minimise fuel consumption
Van Katwijk and Gabriel (2015)	Time of decelerating	Traffic signal	Optimisation	Minimise emissions

Continued on next page

Paper	Control variables	Constraints	Method	Objective
Jin et al. (2016)	Acceleration	Traffic signal, grade, safety	Mixed Integer Linear Programming	Minimise fuel consumption
Ubiergo and Jin (2016)	Advisory speed limit	Traffic signal	Change parameters in the car following model dynamically	Minimise idle time
Altan et al. (2017)	Speed	Traffic signal	Optimise parameters in the predefined speed patterns	Minimise fuel consumption
Jiang et al. (2017)	Jerk	Traffic signal, safety, jerk	MPC	Minimise fuel consumption
Yang et al. (2017)	Acceleration, but advisory speed limit is provided to vehicles	Traffic signal, queue	Optimise the acceleration based on predefined speed pattern	Minimise the number of stops and fuel consumption
Yao et al. (2018)	Locations and time of two speed limits	Traffic signal	DIRECT method integrating with simulations	Minimise travel time and fuel consumption

three strategies by modifying the parameters in the car following model such as reaction time, maximum deceleration, and desired time headway. Ubiergo and Jin (2016), Yang et al. (2017) and Yao et al. (2018) controlled the speed limit of the vehicle to avoid stopping on red. Yang et al. (2017) extended Ubiergo and Jin (2016) by considering the queue effect in front of the intersection. Yao et al. (2018) extended the work of Ubiergo and Jin (2016) and Yang et al. (2017) by imposing two speed limits and optimising the location and time of the speed limit. In the MPC method, acceleration, jerk or engine force are controlled to optimise the trajectory dynamically and achieve the driving goal. The optimisation is updated multiple times every second to adapt to the signal and preceding traffic condition. Two types of prediction horizon were used in the MPC, which are rolling horizon (Kamal et al., 2010; Asadi and Vahidi, 2011; Kamal et al., 2013) and receding horizon (He et al., 2015; Jiang et al., 2017).

1.2.2.2 Trajectory planning for a group of AVs

The summary of trajectory planning for a group of AVs is shown in Table 1.4. All the papers mentioned in this part focus on the case that all vehicles are AVs. The cooperation among AVs helps to achieve larger capacity in the intersection, and reduce the overall delay or fuel consumption. Liu and Kamel (2016), Stebbins et al. (2017), Wei et al. (2017) and Zhou et al. (2017) proposed trajectory planning methods for platoons, where a platoon is a stream of vehicles that pass the intersection in the same green phase (Stebbins et al., 2017; Zhou et al., 2017). However, they used different methods to identify the platoon. Liu and Kamel (2016) used the current speed to predict the last vehicle that can pass on the green light. Stebbins et al. (2017) used a simulation method to identify platoon. After that, they applied trajectory planning for the first vehicle in the platoon and used it to control the behaviour of all the following vehicles. They also compared the performance with and without providing speed advice to the following vehicles. Liu and Kamel (2016) developed a platoon control model to reduce the error between the current trajectory and desired trajectory. HomChaudhuri et al. (2017) did not consider the platoon explicitly but considered the cooperation with the following vehicle, which helps the following vehicle achieve the desired speed.

Table 1.4: Summary of the literature on trajectory planning for a group of AVs

Paper	Penetration	Control variable	Method	Objective
Liu and Kamel (2016)	100%	Acceleration	Platoon control algorithm based on particular swarm optimisation, and trajectory planning algorithm to the leading vehicle	Increase throughput of the intersection
HomChaudhuri et al. (2017)	100%	Acceleration	MPC	Reduce fuel consumption and help the following vehicle to achieve its target velocity
Stebbins et al. (2017)	100%	Acceleration of the leading vehicle	Rule-based strategy to arrive at the desired position with the desired speed	Reduce the delay for all vehicles
Wei et al. (2017)	100%	Speed and reaction time	Dynamic programming	Increase throughput of the intersection
Zhou et al. (2017)	100%	Acceleration	Shooting heuristic algorithm	Smooth trajectory and increase throughput of the intersection

Both Wei et al. (2017) and Zhou et al. (2017) used Newell's car following model to ensure the safety of following vehicles. Wei et al. (2017) developed a dynamic programming model for a group of vehicles by building time-space-reaction time network. Autonomous vehicles can choose short reaction time at the intersection to increase throughput. Zhou et al. (2017) constructed the smooth trajectory using forward and backward shooting heuristic algorithms by controlling the acceleration and deceleration.

1.2.3 Summary of research gaps

Firstly, the CAV is capable of communicating and making decisions on accelerating, braking and steering, which brings many possibilities to intersection control. In order to avoid conflicts at the intersection, the passing rules for vehicles coming from different approaches must be decided. After that, the vehicle needs to modify its behaviour to follow the passing rule. It is expected that the integration of intersection control and trajectory planning increases the intersection operation and performance. Though lots of methods have been proposed for AVs and CVs, the connection between intersection control and trajectory planning is still not tight. Some papers on intersection control did not consider the longitudinal control (Li and Zhou, 2017) or only used simple acceleration and deceleration rules (Dresner and Stone, 2008). More work applied a two-step procedure which optimises the signal or passing sequence first and then optimised the trajectory (Li and Wang, 2006; Müller et al., 2016; Xu et al., 2017; Yu et al., 2018). Some assumed that the vehicle could reach the maximum speed at the stop line (Müller et al., 2016; Yu et al., 2018), which is not realistic as some vehicles that are close to the stop line or have a low initial speed cannot accelerate to the maximum speed at the stop line. Some did not consider the time required to pass the intersection (Li et al., 2014), which may have conflicts in the intersection area though the time headway of entering the intersection is satisfied. Moreover, the objective of intersection control is only to remove conflicts, and the resulting passing sequence may not be optimal in terms of delay. Therefore, more research is required on how to integrate intersection control and trajectory planning.

Secondly, trajectory planning has the potential to reduce fuel consumption (Ubiergo and Jin, 2016) and travel time (Stebbins et al., 2017) significantly. Many methods have been developed for a single vehicle, though they also benefit the following vehicles (Ala et al., 2016). The intelligence in the AV also facilitates cooperative driving behaviour. Current research on trajectory planning for a group of vehicles only considers the case when all vehicles are AVs (Liu and Kamel, 2016; Wei et al., 2017). As the transition period from HV to AV will create a mixed autonomy environment, it is also essential and practical to design the trajectory planning methods for AVs in mixed autonomy traffic and investigate the benefit of cooperation between AVs and HVs.

Thirdly, most of trajectory planning methods on urban roads require the information on the future signal phase and timing (Stevanovic et al., 2013). It is easy to obtain in the fixed time traffic control system but quite tricky in the adaptive traffic control system, where the signal phase sequence and duration are frequently optimised in response to the dynamic traffic state. In this case, the future signal timing information is uncertain and cannot be provided to the approaching vehicles (Mahler and Vahidi, 2014). As the actuated controller and adaptive controller are widely used in the big cities and operate well for dense traffic. It is quite valuable and can accelerate the adaptation of the AV to develop trajectory planning methods for the AV in this case.

1.3 Research questions and objectives

Based on the literature review and identified research gaps above, three main research questions and the corresponding objectives are proposed in this thesis as follows.

1.3.1 Research question 1

Research question 1: What is the benefit of integrating intersection control with trajectory planning?

Three objectives are planned to be worked out to answer this question.

- Objective 1.1: Identify the necessity of considering trajectory planning in intersection control.
- Objective 1.2: Develop a method of integrating intersection control with trajectory planning for AVs on the urban road to reduce the total travel time.
- Objective 1.3: Conduct simulations to test the developed method.

1.3.2 Research question 2

Research question 2: What is the benefit of cooperation between automated vehicles and human-driven vehicles?

This will be answered by the following objectives.

- Objective 2.1: Develop a trajectory planning method by considering cooperation between AVs and HVs.
- Objective 2.2: Analyse the benefit of cooperation in a mixed autonomy traffic condition.
- Objective 2.3: Analyse the performance of the developed method under different penetration rates of AVs.

1.3.3 Research question 3

Research question 3: How to optimise the trajectory at an intersection controlled by the adaptive traffic control system?

The work on this question is achieved by the following objectives.

- Objective 3.1: Develop a trajectory planning method when the future signal is uncertain.
- Objective 3.2: Develop a trajectory planning method when the future signal is uncertain but available for a limited time in advance.
- Objective 3.3: Analyse the impact of model and system parameters on the performance of the developed methods.

1.4 Outline and contribution

This thesis is composed of five chapters, and the remaining chapters are outlined as follows.

1.4.1 Chapter 2

The research question 1 will be addressed in Chapter 2. It is published online in

> **Zhao, W.**, Liu, R., Ngoduy, D. 2019. A bilevel programming model for autonomous intersection control and trajectory planning. *Transportmetrica A: Transport Science*. (In press) https://doi.org/10.1080/23249935.2018.1563921.

A vehicle-based intersection control method is proposed to replace the conventional flow-based intersection control. After analysing the relationship between intersection control and trajectory planning, a bilevel model is applied to integrate both into a coupled optimisation model. The core objective is to improve the efficiency of the intersection. The upper level optimises the passing order of approaching AVs to reduce the total travel time, and the lower level optimises the terminal speed of each vehicle to reduce the time required to pass the intersection. The two levels have a cooperative relationship, and both levels have a linear structure. These two properties make it solvable in a reasonable time. A platoon-based method is also proposed for certain conditions to reduce the complexity of the problem without sacrificing the quality of the solution.

1.4.2 Chapter 3

The research question 2 will be addressed in Chapter 3. It is published in

> **Zhao, W.**, Ngoduy, D., Shepherd, S., Liu, R., Papageorgiou, M. 2018. A platoon based cooperative eco-driving model for mixed automated and human-driven vehicles at a signalised intersection. *Transportation Research Part C: Emerging Technologies*. 95, pp. 802–821. https://doi.org/10.1016/j.trc.2018.05.025.

A general trajectory planning model is proposed for AVs on urban roads. It uses the receding horizon model predictive control and reduces the fuel consumption for a group of vehicles approaching an intersection. The model allows heterogeneous platoons consisting of any number of AVs and HVs. The dynamic platoon splitting and merging strategies are also proposed to change the platoon setting according to the vehicle state and signal state. Extensive simulations are performed to test the cooperation between AVs and HVs. Different penetration rates of AVs are also tested to show the performance of the proposed method.

1.4.3 Chapter 4

The research question 3 will be addressed in Chapter 4. It is prepared for submission to a journal.

> **Zhao, W.**, Ye, H., Ngoduy, D., Shepherd S., Liu R. 2019. A stochastic model predictive control approach to eco-driving for automated vehicles under uncertain signal information. (To be submitted)

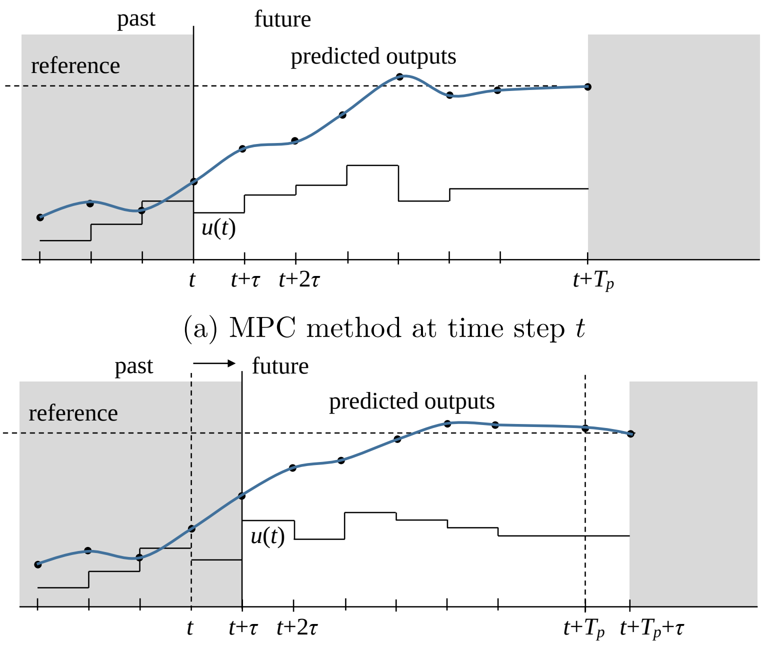
A simple method of estimating the distribution of phase duration from the historical signal data is proposed at first. Then, under the uncertain signal information, the vehicle has two driving regimes: accelerates to pass the intersection when the signal turns green or keeps waiting for the change of the signal from red to green. A multi-phase optimal control model is proposed to minimise fuel consumption by considering the two driving strategies and the estimated distribution of future signal timing. Vehicle's behaviour in every time step only depends on the signal state at that moment. This model is then extended to a more general case that the vehicle can receive the future signal information a certain amount of time in advance. Both models are transformed into nonlinear programming models by analytically calculating fuel consumption.

1.4.4 Chapter 5

The conclusions and future research directions will be discussed in Chapter 5.

Appendix 1.A A brief introduction to model predictive control

The principal idea of model predictive control (MPC) (see Fig. 1.4) is solving an optimal control problem online to compute the next control action. At each sampling time, an open-loop optimal control problem is solved over a finite horizon $[t, t + T_p]$ based on the predicted system state over the horizon T_p. The computed optimal control variable is applied to the process only during the following sampling interval $[t, t + \tau]$. At the next time step $t + \tau$, a new optimal control problem based on new measurements of the state is solved over a shifted horizon $[t + \tau, t + T_p + \tilde{\tau}]$. If $\tilde{\tau} > 0$, typically $\tilde{\tau} = \tau$, this is called rolling horizon MPC. If $\tilde{\tau} = 0$, this is called receding horizon MPC.



(a) MPC method at time step t

(b) MPC method at time step $t + \tau$

Figure 1.4: The principal idea of the MPC method

If the system state is defined as $\mathbf{x} = (x, v)'$ where x and v denote the position and speed of the vehicle respectively, then the corresponding system dynamic function is

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = v(t) \tag{1.1}$$

$$\frac{\mathrm{d}v(t)}{\mathrm{d}t} = u(t) \tag{1.2}$$

where u denotes the acceleration, which is also the control variable.

The cost function for model predictive control is formulated as

$$\min\ J(\mathbf{x}, \mathbf{u}) = \varphi(\mathbf{x}(T_p)) + \int_0^{T_p} \mathcal{L}\left(\mathbf{x}(\tau), \mathbf{u}(\tau)\right) \mathrm{d}\tau \tag{1.3}$$

where J denotes the cost function to be minimised, T_p denotes the prediction horizon, τ denotes the simulation time step, φ denotes the so-called terminal cost which means the costs remaining at the end of the prediction horizon. $\mathcal{L}$ denotes the so-called running cost.

An example of the terminal cost is

$$\varphi(\mathbf{x}(T_p)) = (x(T_p) - x_f)^2 \tag{1.4}$$

where x_f denotes the planned destination at the terminal time T_p. This terminal cost ensures that the difference between the position at the terminal time T_p and the planned position is minimised.

A simple running cost function is

$$\mathcal{L} = \frac{1}{2}u^2 \tag{1.5}$$

which minimises the control effort.

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2

A bilevel programming model for autonomous intersection control and trajectory planning

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Contents

Abstract

Advances in automated and connected vehicles bring new opportunities for intelligent intersection control. In this paper, we propose a centralised method to integrate intersection control with trajectory planning for vehicles. It is formulated as a bilevel optimisation problem in which the upper level is to minimise the total travel time by a mixed integer linear programming (MILP) model. In contrast, the lower level is a linear programming (LP) model with the objective function to maximise the total speed entering the intersection. The two levels are coupled by the arrival time and terminal speed. Based on the relationship between the safe time headway and the process time, a novel platoon based method is developed to reduce the computational burden. Finally, simulation tests are carried out to investigate the control performance under different demands, intersection lengths, communication ranges, and traffic compositions.

Keywords: Automated vehicle, intersection control, trajectory planning, bilevel programming

2.1 Introduction

Traffic control is one of the most important methods to organise vehicle movements in urban networks. The purposes of traffic control may vary according to the traffic demand and vehicle composition. Nevertheless, they still share a common goal of creating safe and efficient traffic operations at urban intersections. Various traffic control methods are proposed, for example, for a local intersection or a network, using a fixed, actuated or adapted control strategy. The more advanced the traffic control method is, the more precise traffic information it requires. This usually indicates that more traffic detectors, such as loop detectors, Bluetooth, Electronic Vehicle Identification, need to be installed.

It is a common vision that many vehicles will be equipped with some kinds of Vehicle Automation and Communication Systems (VACS) in the near future (Diakaki et al., 2015). The automated vehicle (AV) is considered not only a sensor but also an actuator. From the aspect of a sensor, it can access much more detailed data about vehicles and traffic flow than conventional traffic detectors. For example, such data include the acceleration ability of each vehicle and the driver's desired speed and value of time (Isukapati and Smith, 2017). From the aspect of an actuator, the vehicle can be controlled to follow a specific trajectory along the link for a pre-designed purpose such as eco-driving (Ma et al., 2017; Zhao et al., 2018). These distinct properties of the AV help to create a vehicle-based traffic control method rather than the conventional flow-based method.

In the urban road environment, the AV can receive and send system dynamic information to the intersection controller or road-side infrastructure and react precisely according to the control strategy. These abilities provide the intersection controller with more capabilities to optimise traffic flow at the intersection, which is the so-called autonomous intersection control strategy. In the autonomous intersection control, the signal timing problem can be seen as a machine scheduling problem. The conflict area in the intersection acts as a "server" or "machine", and every approaching vehicle around the intersection is a "job". The vehicles negotiate with either each other or the intersection controller to allocate the priority of passing or the time window, which avoids collisions and achieves better efficiency at the intersection. Collision avoidance can be obtained by either a pure signal timing method or a pure trajectory planning method. However, efficiency is an issue that is much more difficult to achieve at the same time. To this end, our main motivation in this paper is to consider simultaneously traffic control and trajectory planning in the autonomous intersection control method.

In the literature, limited research considers the vehicle dynamics in the centralised signal control. In the scheduling method, it is usually assumed that the time required for a vehicle to pass the intersection (i.e. process time) is a constant. However, in reality, the process time strongly depends on the speed crossing the stop line and the travel time from the current position to the stop line. In this paper, a novel bilevel programming method is proposed to optimise the passing sequence and vehicle trajectory. In our bilevel optimisation model, the upper level determines the passing sequence, which is seen as a job shop scheduling problem and modelled as a mixed integer linear programming (MILP) problem, whereas the lower level determines the corresponding process time, which is modelled as a linear programming (LP) problem. Though the model introduces many variables, due to the linear structure in both levels, it can be solved very efficiently using existing commercial solvers, such as CPLEX (IBM, 2017) and Gurobi (Gurobi Optimization, 2017).

In summary, the main contributions of this paper are:

1. To combine the intersection control optimisation and trajectory optimisation, which makes sure that the result of intersection control optimisation is achievable and optimal.

2. To consider the vehicle's dynamics explicitly in the model and there is no need to make an assumption on the vehicle's speed entering the intersection.

3. To achieve a quick convergence in the optimisation problem via a linear programming structure in each level of the proposed bilevel programming model.

4. To propose a new platoon-based method to dynamically reduce the computation burden and increase the efficiency of the intersection control method.

We briefly review the state-of-the-art intersection control with AVs in Section 2.2. Section 2.3 describes the notations used throughout this paper and the structure of the proposed model. Furthermore, the formulation of the model and a novel platoon identification approach are proposed. Section 2.4 illustrates simulation tests in various scenarios. Finally, we conclude the paper in Section 2.5.

2.2 Literature review

The traffic control method can be classified into two types: centralised (Diakaki et al., 2002) and distributed (Bazzan, 2005). As a centralised approach, Diakaki et al. (2002) developed a traffic-responsive network-wide signal control model using store-and-forward modelling and linear-quadratic regulator theory. As a distributed approach, Bazzan (2005) developed a decentralised coordinated traffic control model for an arterial using evolutionary game theory. Every intersection is modelled as an intelligent agent and considers both the local and global goal.

In the ensuing paper, we will discuss three main approaches in the autonomous intersection control: 1) centralised control method; 2) vehicle trajectory-based signal control; and 3) combined intersection control and vehicle trajectory optimisation.

The first approach is the centralised control method which usually needs to collect every vehicle's information within a certain range before the optimisation is executed. Xie et al. (2012) developed a scheduling-based intersection control method in which the vehicles are first aggregated into clusters, and then the intersection control problem is formulated as a scheduling model. A forward recursive algorithm was proposed to solve this model efficiently with some state elimination criteria. Sun et al. (2017) proposed an intersection operation method to maximise the intersection capacity by changing the lane management for through and turning movements dynamically. The green duration and lane management are optimised together by a multi-objective mixed-integer nonlinear programming model. Zhu and Ukkusuri (2015) proposed a linear programming model to optimise traffic flows in the network accounting for the dynamic departure time and dynamic route choice. Li and Zhou (2017) proposed a novel phase-time-traffic hyper-network model to represent the heterogeneous traffic of AVs and conventional vehicles and used a mixed integer programming model to minimise the total delay at an intersection. Guler et al. (2014) added a penalty term of the departure time and took into account the different passing time for connected vehicles because of the different passing sequences. However, this model still did not consider the speed passing the intersection. In a similar line, some other research also considered intersection control with fully connected vehicles (Goodall et al., 2013) or partially connected vehicles (Feng et al., 2015).

Dresner and Stone (2004, 2005, 2006, 2008) studied extensively the reservation-based autonomous intersection management method. In this approach, every vehicle is an autonomous agent, and there is another intersection manager agent at each intersection. The vehicle agent sends a request to reserve a specific space and time in the intersection, and the intersection agent will accept or reject the reservation based on the intersection control policy. The most widely used policy in the reservation-based model is first come first serve (FCFS). Different levels of priority in the reservation method were considered by Zhang et al. (2015b). Therefore, in this method, the control policy is a black box to the vehicle agent, and the vehicle agent cannot know when and what reservation will be accepted. The main drawback of the reservation-based rule is that it only accepts or rejects the request of the vehicle, but does not provide the exact time of passing. So it cannot be employed to optimise the vehicle trajectory. Levin et al. (2016) showed that such reservation-based intersection control methods might increase congestion or vehicle delay in some situations.

The second approach is to coordinate the trajectories of vehicles around the intersection to avoid conflicts. Li and Wang (2006) developed a cooperative driving model at blind crossings. The grouped vehicles used a decision tree generation model to get a safe and efficient driving pattern. Then, a virtual vehicle mapping technique was used to get a safe vehicle trajectory. Lee and Park (2012) developed an optimisation model to minimise the overlapping trajectory of vehicles from conflicting roads, thus to ensure a safe manoeuvre for every vehicle. The main drawback of this method is that the complex nonlinear constrained model makes it challenging to find a feasible solution. In that exception case, it will fallback to a rule-based control method. Even though the method can reduce the average delay, the primary purpose of the model is to generate a safe trajectory, while efficiency is not considered. Kamal et al. (2015) proposed a risk function to quantify the risk of collision around the intersection and then used a model predictive control method to avoid collisions by considering all the vehicles' states. Yang and Monterola (2016) developed a decentralised intersection traffic control with no traffic lights where some vehicles are equipped with a simple driver assistance system. The method only controls vehicles to brake in a specific condition, so it is suitable for level 1 or above AVs. This simple method can be adapted to mixed traffic with both conventional vehicles and AVs.

Besides the two approaches, some work has been conducted to combine intersection control and trajectory planning. Li et al. (2014) discussed the vehicle's trajectory under different travel times. However, they assumed that all vehicles except the first vehicle on the road could achieve the maximum speed when they enter the intersection and the model is strongly nonlinear. An enumeration method is used to obtain best signal timing instead of an optimisation model. Müller et al. (2016), Yang et al. (2016), Xu et al. (2017) and Yu et al. (2018) formulated the problem as a two-stage problem: solve intersection optimisation first, and then optimise the trajectory. Müller et al. (2016) and Yu et al. (2018) assumed that every vehicle could achieve the maximum speed or desire speed while entering the intersection, which may not be possible for the vehicle close to the intersection, or in high demand situation. Yang et al. (2016) assumed a piece-wise linear trajectory in the trajectory optimisation model which may not be realistic, but unconnected vehicles are also considered in their model. Xu et al. (2017) and Yu et al. (2018) combined the traffic signal optimisation and an optimal control model for vehicle trajectory optimisation. However, they still use the conventional duel ring signal structure for the case of AVs, and there is no feedback between signal optimisation and vehicle trajectory optimisation. Besides, Yu et al. (2018) also optimised the lane changing behaviour for AVs.

2.3 Methodology

2.3.1 Problem formation

To simplify the problem, a simple intersection with two approaches as shown in Fig. 2.1 is considered in which only one conflict zone exists (red shadowed area). This simple intersection is widely used in the development of autonomous intersection control methods, such as in Li et al. (2014), Guler et al. (2014), Yang et al. (2016) and Yang and Monterola (2016). It is a good start of developing complex vehicle-based intersection control methods. All vehicles in the system are AVs that are able to communicate with the intersection controller via Vehicle-to-Infrastructure (V2I) technology. No communication delay or package loss is considered in this paper. Another assumption is that the conflict zone can only be used by the vehicles coming from the same direction simultaneously. This means that if the intersection is already occupied by a vehicle from one approach, the vehicles coming from another approach cannot enter the intersection any more no matter how large the intersection is. Consequently, they have to decelerate towards the stop line. However, multiple vehicles coming from the same approach may appear in the conflict area simultaneously if they can keep a safe time headway.

The intersection control problem is formulated in a centralised way, which is described as follows:

1. When a vehicle enters a specific control range, it will send some necessary information to the intersection controller to request a proper time to pass the intersection. The information includes its current position, speed and other characteristics such as the desired speed and acceleration limits.

2. The intersection controller collects all the information from vehicles and then attempts to optimise both intersection operations and vehicle trajectories.

3. The vehicles receive the arrival time and trajectories from the intersection controller and behave accordingly.

All the settings and assumptions are ideal in our proof of concept study. Nevertheless, it is still beneficial to see how AVs can improve the performance of an urban traffic control strategy.

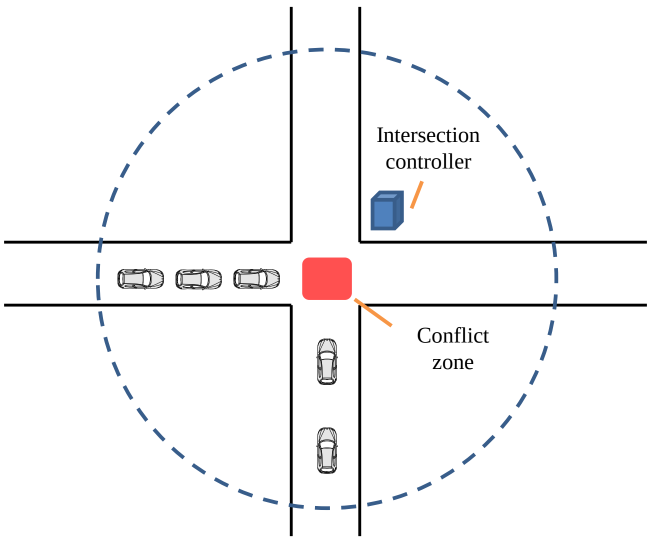


Figure 2.1: A simple intersection with two approaches

2.3.2 Notations

All the major notations used throughout this paper are described in Table 2.1.

Table 2.1: Notations

Symbol	Description
i	Index for roads, in this paper $i = 1, 2$
j	Index for vehicles
f_u, f_l	Objective function for the upper-level (or lower-level) optimisation
n_i	Number of vehicles within the communication range on road i
$T_{i,j}$	Arrival time to the stop line for vehicle j on road i
$T_{i,j}^{min}, T_{i,j}^{max}$	Minimum (or maximum) travel time to the stop line for vehicle j on road i
$v_{i,j}^k$	Speed for vehicle j on road i at time step k
$v_{i,j}^r$	Terminal speed to enter the intersection for vehicle j on road i
$v_{i,j}^0$	Initial speed at the beginning of an optimisation for vehicle j on road i
$s_{i,j}^k, s_{i,j}^0$	Distance to the stop line for vehicle j on road i at time step k (or at the initial time step)
$l_{i,j}$	Length of vehicle j on road i
h	Safe time headway
$p_{i,j}$	Process time, which is the travel time from passing the stop line to exiting the intersection
$c_{j,j'}$	Interchange time, which is the time difference of vehicle j entering the intersection before vehicle j' from a conflict road
θ	Redundant time for safety in the interchange time
$\hat{h}_{i,j}$	Time headway for vehicle j on road i entering the intersection

Symbol	Description
$o_{j,j'}$	Binary variable to denote the passing order for conflicting vehicle j and j'
$S_{i,j}^k$	Safe distance headway for vehicles j on road i with the preceding vehicle
s_0	Minimum safe distance at the jam density including the vehicle length
L_s	Intersection length
L_c	Communication range
a_{max}, a_{min}	Maximum acceleration (or deceleration)
v_{max}, v_{min}	Maximum (or minimum) speed
$x_{i,j}$	Distance away from the entrance of the road for vehicle j on road i

2.3.3 Model development

Before formulating the model, a major issue that needs to be addressed is why the intersection control method should consider the trajectories of AVs and what benefits it can bring to the system.

1. Vehicles' arrival information is vital for intersection control from both local and global points of view. If there are no traffic detectors on the roads, the intersection can only be operated in a fixed time strategy. If there are loop detectors located before the stop line, the arrival time of vehicles can be estimated, and the intersection will be operated in an actuated or even adaptive control strategy. If the vehicle position and speed information are available in real-time, a better green phase can be allocated and the wasted green time can be reduced.

2. Usually, in a conventional intersection signal control plan, there is a yel-

low light or even all-red light between conflicting green phases to clear the remaining vehicles in the intersection. For a large intersection, the yellow time and all-red time are usually lengthy to enable the slowest vehicle to pass the intersection safely. For example, a stationary vehicle with a length of 5 m needs 3.87 s to cross a 10 m wide intersection using a constant acceleration of $2\,\mathrm{m/s^2}$, but a vehicle running with 55 km/h only needs 0.98 s. This does not even count the start-up loss time for human-driven vehicles. In the environment of AVs, every vehicle's clearance time and the intersection clearance time can be obtained accurately, which can be used to reduce inter-green time greatly. Unfortunately, this issue has been ignored in previous research.

3. Another reason for yellow light in the conventional signal timing plan is to reduce rear-end crashes in the dilemma zone. In most cases, the yellow time is set to 4.2 s on a 72 km/h road (Rakha et al., 2008). The time is even longer for higher-speed roads. With V2X communication capability, the intersection controller can detect whether there are vehicles in the dilemma zone and provide more time for those vehicles. So, there is no need to set the yellow time any more.

In fact, points (2) and (3) are also the main reasons why phases in the conventional signal timing plan cannot be switched frequently. When all vehicles are AVs, the yellow light is not required any more. Then, the autonomous intersection control problem is modelled as a bilevel programming problem.

$$\min_{T_{i,j}} F(T_{i,j}) \tag{2.1a}$$

$$\text{subject to: } K(T_{i,j}, s_{i,j}, v^0_{i,j}, p_{i,j}) \leq 0 \tag{2.1b}$$

$$p_{i,j} = \arg\min_{p_{i,j}} f(p_{i,j}) \tag{2.1c}$$

$$\text{subject to: } k(T_{i,j}, s_{i,j}, v^0_{i,j}, p_{i,j}) \leq 0 \tag{2.1d}$$

The model can be seen as a master-slave scheme (Sharon et al., 2015; Lamorgese et al., 2016). The master problem is for intersection control, which minimises the total travel time for all vehicles. The slave problem is for trajectory optimisation, which minimises the process time for every vehicle in the intersection.

The time required to pass the intersection, which is called *process time* in the ensuing paper, is one of the keys for the problem. The constraint (2.1d) is nonlinear and not intuitive to formulate. Notice that as the intersection length and time to enter the intersection are known for the lower-level problem, the only factor that affects the process time is the speed crossing the stop line, which is also named as terminal speed. The process time monotonically decreases with increasing terminal speed. Thus, instead of minimising the process time for every vehicle, we will try to maximise the terminal speed for each. As it is tedious to run tens of optimisation separately, the summed terminal speed is chosen as the new objective function for the lower-level optimisation.

In summary, the connections between intersection control and trajectory planning are the vehicle's state at the stop line including arrival time $T_{i,j}$ and terminal speed $v^r_{i,j}$. The structure of the proposed bilevel model is shown in Fig. 2.2. The master problem is formulated as a mixed integer linear programming (MILP) problem in this paper, while the slave problem is formulated as a linear programming (LP) problem. The proposed bilevel programming model has a cooperative relationship between the two levels rather than a competitive relationship in the Stackelberg model. Optimised terminal speed in the lower level tends to reduce the travel time in the upper level. Thus, the optimal value in the upper level keeps decreasing in every iteration and finally converges. The cooperative relationship helps our heuristic algorithm to get a reliable result with few iterations. Please note that in this framework, the lower-level optimisation is not just following decisions from the upper level. In addition, the optimised terminal speed $v^r_{i,j}$ which is the output of the lower-level optimisation will be the input for the upper-level optimisation in the next iteration. To this end, the feedback structure and interaction between the two levels describe the relationship between intersection control and trajectory planning more accurately.

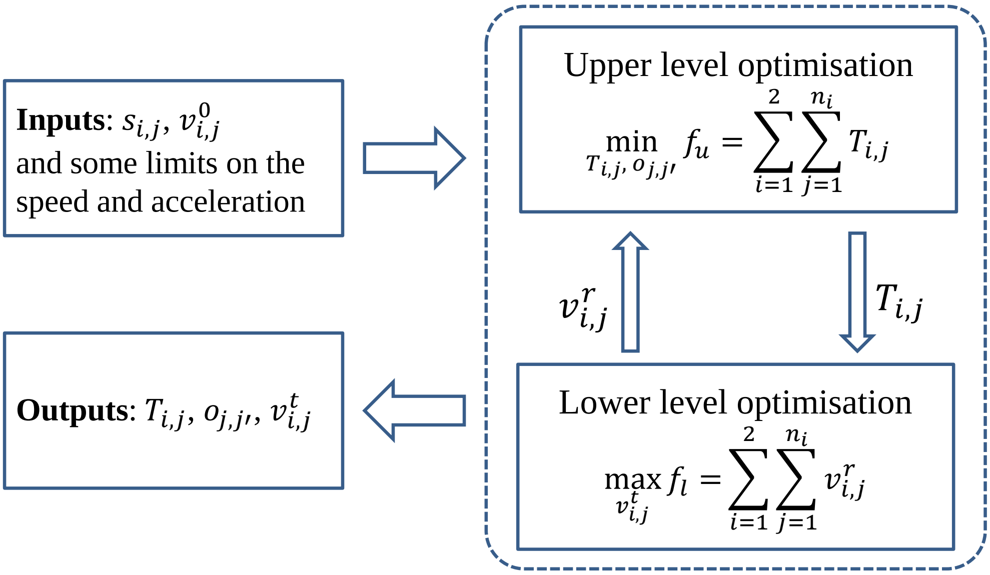


Figure 2.2: Schematic of the bilevel programming structure

2.3.4 Upper-level optimisation

The objective function for the upper-level optimisation is minimising the total travel time for all vehicles in the communication range at the intersection. As the minimum travel time can be pre-calculated, minimising the total travel time is equivalent to minimising the average delay. That is:

$$\min_{T_{i,j}, o_{j,j'}} f_u = \sum_{i=1}^{2}\sum_{j=1}^{n_i} T_{i,j} \tag{2.2}$$

where every vehicle's arrival time is bounded by the minimum travel time $T_{i,j}^{min}$ and the maximum travel time $T_{i,j}^{max}$, i.e.

$$T_{i,j}^{min} \leq T_{i,j} \leq T_{i,j}^{max} \tag{2.3}$$

The purpose of minimum travel time constraint is to ensure the feasibility of the solution. However, there are two purposes for the maximum travel time constraint. One is to ensure the safety of vehicles in the dilemma zone and the other is to ensure the fairness among all vehicles and avoid unacceptable long delay.

To achieve the earliest arrival time, the vehicle has to accelerate to the maximum speed as early as possible. The acceleration is updated every 0.5 s in the upper level in order to be consistent with the lower-level optimisation. Although the obtained minimum travel time under such 0.5s-update-interval is higher than the theoretical minimum travel time without such an assumption, the difference is usually less than 0.01 s and can be ignored. The minimum travel time $T_{i,j}^{min}$ is calculated at the beginning of the optimisation as follows:

1. If the vehicle cannot achieve the maximum speed at the stop line because of the distance restriction, i.e.

$$s_{i,j}^{0} \leq \frac{(v_{max})^2 - (v_{i,j}^{0})^2}{2a_{max}} \tag{2.4}$$

   where $s_{i,j}^{0}$ denotes the initial distance to the stop line in the optimisation, then $T_{i,j}^{min}$ is calculated as

$$T_{i,j}^{min} = \frac{\sqrt{(v_{i,j}^{0})^2 + 2a_{max}s_{i,j}^{0}} - v_{i,j}^{0}}{a_{max}} \tag{2.5}$$

2. If it can achieve the maximum speed at the stop line, the calculation of the minimum travel time $T_{i,j}^{min}$ includes three parts as shown in Fig. 2.3, and they are computed by:

$$t_1 = \left\lfloor \frac{2(v_{max} - v_{i,j}^{0})}{a_{max}} \right\rfloor \tag{2.6}$$

$$s_1 = v_{i,j}^{0} t_1 + \frac{1}{2} a_{max} t_1^2 \tag{2.7}$$

$$s_2 = \frac{1}{2} \times 0.5 \times \left(v_{i,j}^{0} + a_{max} t_1 + v_{max}\right) \tag{2.8}$$

$$t_3 = \frac{s_{i,j}^{0} - s_1 - s_2}{v_{max}} \tag{2.9}$$

$$T_{i,j}^{min} = t_1 + 0.5 + t_3 \tag{2.10}$$

   where $\lfloor x \rfloor$ denotes the largest integer less than or equal to x.

There are two situations in the calculation of the maximum travel time.

1. If the vehicle cannot stop before the stop line, then the maximum travel

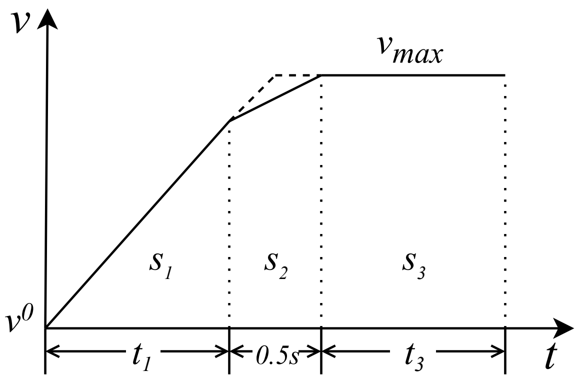


Figure 2.3: Calculation of the minimum travel time $T_{i,j}^{min}$

time is calculated by applying the maximum deceleration.

$$T_{i,j}^{max} = \frac{\sqrt{(v_{i,j}^0)^2 + 2a_{min}s_{i,j}^0} - v_{i,j}^0}{a_{min}} \tag{2.11}$$

2. If the vehicle can stop before the stop line, then the maximum travel time is calculated by setting a maximum delay t_{delay}. The maximum allowed delay can be specific to each vehicle, but a constant is used for every vehicle for simplicity.

$$T_{i,j}^{max} = T_{i,j}^{min} + t_{delay} \tag{2.12}$$

For vehicles running on the same road, the safe time headway should be maintained not only at the time of entering the intersection but also at the time of exiting the intersection due to the difference in process time. Constraint (2.13a) below indicates the safety constraint at the entering time, while constraint (2.13b) is at the exiting time.

$$T_{i,j} + h \leq T_{i,j+1} \qquad i = 1, 2; j = 1, 2, \ldots, n_i - 1 \tag{2.13a}$$

$$T_{i,j} + p_{i,j} + h \leq T_{i,j+1} + p_{i,j+1} \qquad i = 1, 2; j = 1, 2, \ldots, n_i - 1 \tag{2.13b}$$

where the safety headway h is set to 1.5 s. Various safety headways for AVs are used in the literature around $1 \sim 2$ s (Jia and Ngoduy, 2016a; Yang et al., 2016; Ghiasi et al., 2017).

In many studies, the process time for every vehicle is usually set to a constant value by assuming that the vehicle passes the intersection with the maximum speed or desired speed (Müller et al., 2016; Yu et al., 2018), but this may not be possible if the vehicle is too close to the intersection or the traffic volume is high. So process time is defined as a function of the entering speed which can be easily calculated by replacing $v^0_{i,j}$ to $v^r_{i,j}$ and $s^0_{i,j}$ to $L_s + l_{i,j}$ in Eq. (2.4) – (2.10).

For vehicles running on the conflicting roads, the safety constraints are more difficult to obtain due to the unknown passing sequence. A binary variable $o_{j,j'} \in \{0, 1\}$ is introduced to indicate the passing sequence: $o_{j,j'} = 1$ if vehicle j from approach 1 crosses the conflict area before vehicle j' from approach 2, otherwise $o_{j,j'} = 0$. Then the safety constrains are

$$T_{i,j} + c_{j,j'} \leq T_{i',j'} + M(1 - o_{j,j'}) \tag{2.14a}$$

$$T_{i',j'} + c_{j',j} \leq T_{i,j} + M o_{j,j'} \tag{2.14b}$$

where M is an arbitrary large enough constant, and 10000 was used in this paper. The interchange time is calculated by

$$c_{j,j'} = p_{i,j} + \theta \tag{2.15}$$

where θ denotes the redundant safe time because of sensor delay. $\theta = 0.2\ s$ is used in the following simulations, as it likely ranges from 0.1 s to 0.3 s (Xiao and Gao, 2011; Wang et al., 2018).

2.3.5 Lower-level optimisation

After an iteration of the upper-level optimisation, every vehicle has an allocated arrival time. However, the assumed process time in the upper level may be overestimated or underestimated, which makes the intersection control method less efficient or infeasible. One of the purposes of lower-level optimisation is to update the process time for the upper level in the next iteration. So, the feedback structure is one of the key contributions of this paper. The objective for the lower-level optimisation is to optimise every vehicle's trajectory to follow the allocated arrival time and maximise the total speed of vehicles entering the intersection.

$$\max_{v_{i,j}^k} f_l = \sum_{i=1}^{2}\sum_{j=1}^{n_i} v_{i,j}^r \quad k = 1, 2, \cdots, r \tag{2.16}$$

where $v_{i,j}^r$ denotes the speed of vehicle j on road i at the stop line.

To reduce the complexity in the trajectory planning, it is assumed that the vehicle updates its acceleration at an interval of 0.5 s. So there are $r = \lceil T_{i,j}/0.5 \rceil$ speed variables before arriving at the intersection where $\lceil x \rceil$ indicates the smallest integer value which is larger than or equal to x. $r = 0$ is not considered as it indicates the vehicle already arrives at the stop line and there are no decision variables in this case. The duration of the last time step is $t_{i,j}^r = \mathrm{mod}(T_{i,j}, 0.5)$, where $\mathrm{mod}(x, y)$ denotes the remainder of x divided by y. Then, the total travel distance constraints is formulated easily as:

$$s_{i,j}^0 = \begin{cases} \frac{t_{i,j}^r}{2}\left(v_{i,j}^0 + v_{i,j}^r\right) & \text{if } r = 1 \\ \frac{0.5}{2}\left(v_{i,j}^0 + v_{i,j}^{r-1}\right) + \frac{t_{i,j}^r}{2}\left(v_{i,j}^{r-1} + v_{i,j}^r\right) & \text{if } r = 2 \\ \frac{0.5}{2}\left(v_{i,j}^0 + 2\sum_{t=1}^{r-2} v_{i,j}^t + v_{i,j}^{r-1}\right) + \frac{t_{i,j}^r}{2}\left(v_{i,j}^{r-1} + v_{i,j}^r\right) & \text{if } r > 2 \end{cases} \tag{2.17}$$

At every time step, the change of speed is bounded by the acceleration limits, which is:

$$-0.5a_{min} \leq v_{i,j}^{k+1} - v_{i,j}^k \leq 0.5a_{max} \quad k = 0, 1, \ldots, r-1 \tag{2.18}$$

At every time step, the vehicle should keep a safe distance from the preceding vehicle as expressed in Eq. (2.19). These constraints are often ignored in the intersection signal optimisation, even though they affect both the speed profile and the intersection access time.

$$s_{i,j}^k - s_{i,j-1}^k \geq S_{i,j}^k \quad k = 1, 2, \ldots, r \tag{2.19}$$

where $S_{i,j}^k$ denotes the safe distance at time step k which is calculated as:

$$S_{i,j}^k = \max(s_0, v_{i,j}^k h) \tag{2.20}$$

where s_0 denotes the minimum space headway between adjacent vehicles, which is set to 7 m.

The distance to the stop line $s_{i,j}^k$ at time step k is calculated by

$$s_{i,j}^k = \begin{cases} s_{i,j}^0 & \text{if } k = 0 \\ s_{i,j}^0 - \frac{0.5}{2}\left(v_{i,j}^0 + v_{i,j}^1\right) & \text{if } k = 1 \\ s_{i,j}^0 - \frac{0.5}{2}\left(v_{i,j}^0 + 2\sum_{t=1}^{k-1} v_{i,j}^t + v_{i,j}^k\right) & \text{if } 1 < k < r \\ 0 & \text{if } k = r \end{cases} \tag{2.21}$$

2.3.6 Heuristic algorithm

Bilevel optimisation problems are $\mathcal{NP}$–Hard problems, and even the simplest bilevel linear problems are nonconvex and nondifferentiable optimisation problems (Dempe, 2002, 2003). For the proposed method, as the travel time is discretised in the lower-level optimisation and it is also the decision variable of the upper-level optimisation, the number of decision variables and constraints in the lower-level optimisation are determined by the upper-level optimisation. This also causes difficulties in applying KKT (Karush–Kuhn–Tucker) conditions on the lower-level optimisation, which is the most popular and efficient method for solving bilevel problems (Bard, 1998; Lu et al., 2016).

In the proposed method, the bilevel problem has a cooperative relationship that the decision variables in the lower level help the upper level to achieve optimal goals (Zhang et al., 2015a). A heuristic pseudo code below is proposed to solve the proposed master-slave problem:

- Step 1: In the first iteration of the upper-level optimisation, the speed entering the intersection $v_{i,j}^r$ is assumed to be the same as the initial speed $v_{i,j}^0$ [†].

- Step 2: Fix the terminal speed $v_{i,j}^r$ and solve the upper-level optimisation. The optimised travel time to the stop line T_{ij} from the upper-level optimi-

[†]The impact of initial values on the optimisation results is investigated in Appendix 2.A.

sation is passed to the constraints in the lower-level optimisation.

- Step 3: Fix the travel time T_{ij} and solve the lower-level optimisation. The optimised speed entering the intersection $v^r_{i,j}$ from the lower-level optimisation then becomes the constraints of the upper-level optimisation in the next iteration.

- Step 4: When the difference of the total travel time between two iterations becomes less than a threshold value, which is set to 0.5 s, the algorithm will stop. Otherwise, it will continue the iterations between step 2 and 3.

2.3.7 Platoon-based scheduling

This section presents a novel platoon-based method to reduce the computation burden of the proposed bilevel optimisation model. Research on the platoon-based traffic operations has been conducted widely in the literature (see Ngoduy, 2013; Jia and Ngoduy, 2016b; Zhao et al., 2018 and references therein). Allowing platoon-based operations under the connected environment may increase the roadway capacity (Jia and Ngoduy, 2016b). Nevertheless, this is not always the case for the intersection. It depends on the relationship between the time headway $\hat{h}_{i,j}$ for the vehicles on the same road and the process time $p_{i,j}$.

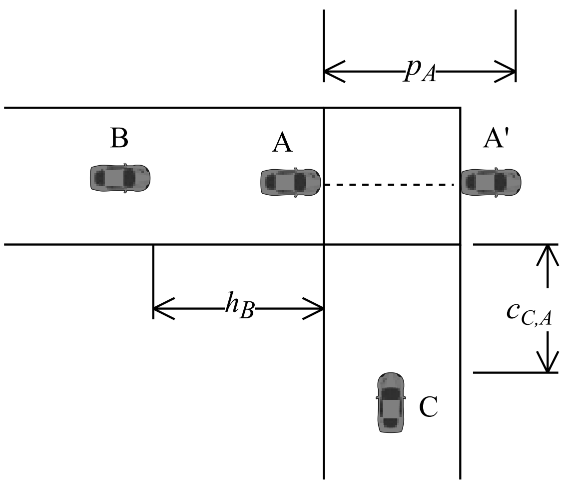


Figure 2.4: The relationships among headway, process time and interchange time

Considering the following example in Fig. 2.4, the earliest time for the vehicle B to enter the intersection is the time headway h_B, while the earliest time for the vehicle C to enter the intersection is the interchange time $c_{C,A}$ which is a function of the process time of vehicle A p_A according to Eq. (2.15), denoted as $c_{C,A} = g(p_A)$. Whether vehicle B or vehicle C goes first depends on the relationship between h_B and $c_{C,A}$, which can also be seen as the relationship between h_B and p_A because θ is a constant in Eq. (2.15). In other words, only $h_{i,j}$ and $p_{i,j}$ for the vehicles on the same road need to be calculated in order to determine the passing sequence. Then we have the following scenarios as shown in Fig. 2.4:

- If $h_B < g(p_A)$, there is no doubt that vehicle B enters the intersection before vehicle C.
- If $h_B > g(p_A)$, then vehicle C enters the intersection earlier than vehicle B to reduce the total delay.
- If $h_B = g(p_A)$, then either vehicle B or C can go first as both cases have the same total arrival time.

More generally,

- If $h_B < g(p_A)$, the intersection control strategy should provide a priority to the platoon on the same road to reduce the overall travel time.
- If $h_B > g(p_A)$, vehicles close to the intersection have a high priority to pass the intersection, which is similar to the first in first out (FIFO) principle.
- If $h_B = g(p_A)$, which vehicle goes first depends on who gets closer to the stop line. It is also the same as the FIFO principle.

The platoon will only be beneficial to the entire traffic flow when $h_B < g(p_A)$. Please note that both h_B and p_A are closely related to vehicles' state at the stop line. When the vehicles are away from the stop line, it is essential to determine the relationship between h_B and p_A in advance.

Proposition 2.3.1. *When the difference of the minimum travel time to the stop line between two adjacent vehicles is less than or equal to the minimum time headway, these two vehicles will also keep the minimum time headway at the* *stop line no matter what the passing sequence is. In a mathematical form: if* $T_{i,j}^{min} - T_{i,j-1}^{min} \leq h$*, then* $\hat{h}_{i,j} = h$*.*

Proof. If the difference of the minimum travel time between two adjacent vehicles is less than or equal to the minimum time headway, there always exists a time instant that the following vehicle has a minimum time headway to the preceding vehicle. In order to reduce the delay, the vehicle always attempts to keep a minimum time headway, so it will keep the minimum time headway until it reaches the stop line. □

Proposition 2.3.2. *If* $s_{i,j}^0 \geq \tilde{s}_{i,j}$ *where* $\tilde{s}_{i,j} = (v_{i,j}^0)^2/(2a_{min}) + (v_{max})^2/(2a_{max})$*, then no matter what the passing sequence is, the vehicle can always achieve the maximum speed at the stop line.*

Proof. When the distance to stop line $s_{i,j}^0$ equals to $\tilde{s}_{i,j}$, then

- If $T_{i,j} = v_{i,j}^0/a_{min} + v_{max}/a_{max}$, the vehicle can reach the maximum speed as shown in Fig. 2.5;
- If $T_{i,j} > v_{i,j}^0/a_{min} + v_{max}/a_{max}$, the vehicle can stop for a period after decelerating to zero and then start to accelerate to the maximum speed;
- If $T_{i,j} < v_{i,j}^0/a_{min} + v_{max}/a_{max}$, the vehicle can decelerate for a shorter time than $v_{i,j}^0/a_{min}$ or even do not decelerate and start to accelerate to the maximum speed.

When the distance to the stop line $s_{i,j}^0$ is larger than $\tilde{s}_{i,j}$, it can keep a similar speed pattern as shown in Fig. 2.5, but may use a different deceleration and acceleration, and run with v_{max} finally.

□

Proposition 2.3.3. *If the intersection length* $L_s > (h - \theta)v_{max} - l$*, then for the vehicles satisfying the condition in Proposition 2.3.1, no matter what the passing sequence is, the platoon-based operations are always preferred to the vehicle-based operations in terms of reducing delay.*

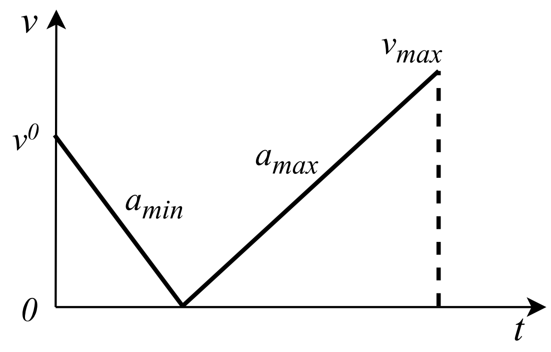


Figure 2.5: The speed pattern when $s_{i,j}^0 = \tilde{s}_{i,j}$ and $T_{i,j} = v_{i,j}^0/a_{min} + v_{max}/a_{max}$

Proof. According to Proposition 2.3.1, $\hat{h}_{i,j} = h$. When $L_s > (h-\theta)v_{max} - l$, then $p_{i,j} \geq \frac{L_s+l}{v_{max}} > h - \theta$ and $c_{j,j'} = p_{i,j} + \theta$, so $c_{j,j'} > h$. According to the previous discussions about headway and interchange time in this section, the platoon-based operations are preferred. Using the parameters in Table 2.2, $(h-\theta)v_{max} - l \approx 14.9\ m$. □

Based on Propositions 2.3.1, 2.3.2 and 2.3.3, the vehicle's time headway $\hat{h}_{i,j}$ and process time $p_{i,j}$ at the stop line can be obtained before the upper-level optimisation is executed. Thus, they can also be used as the criteria to split traffic flow. When the following vehicle's headway is less than the process time of the preceding vehicle plus a small non-negative safety tolerance, they will form a platoon. A platoon will be seen as one "big vehicle" as all vehicles in the platoon pass the intersection continuously without the disturbances from other roads. Traffic flow is now considered to consist of many platoons moving together rather than many vehicles (or particles) moving together (Ngoduy, 2013). Therefore, the number of binary variables and the total calculation time can be greatly reduced.

2.4 Numerical studies

A simulation environment is developed using Matlab to test the performance of the proposed method and compare it with other intersection control strategies. All simulations are carried out in a simple intersection as shown in Fig. 2.1. The algorithm is solved using Gurobi 7.5.1 in Matlab. Typically, if there are 9 vehicles on one road and 15 vehicles on another, every run of the upper-level optimisation usually takes 0.2 s and every run of the lower-level optimisation takes 0.08 s. The whole algorithm converges after two iterations, taking less than 0.5 s.

Table 2.2: The basic parameters used in simulations

Parameter	Description	Value	Unit
L_r	Simulation road length	600	m
L_c	Communication range	500	m
L_s	Intersection length	10	m
l	Vehicle length	5	m
h	Saturation headway	1.5	s
h_g	safety time gap	0.8	s
θ	Safety headway tolerance	0.2	s
s_0	Minimum space headway at standstill	7	m
a_{max}	Maximum acceleration	2	m/s^2
a_{min}	Maximum deceleration	5	m/s^2
b	Comfortable braking deceleration	3	m/s^2
v_{max}	Maximum speed	55	km/h
v_{min}	Minimum speed	0	km/h
t_{delay}	Maximum allowed delay for each vehicle	30	s

The basic simulation parameters are shown in Table 2.2, and some of them may be changed in the simulations later when studying the impact of particular parameters on the system performance. Meanwhile, a scenario is defined as a particular set of parameters. In every scenario, the simulation is run five times, and the simulation period is 20 min in every run. The time headway of vehicles entering the studied road follows a truncated exponential distribution with minimum headway 1.5 s. Four control methods are considered in the following simulation tests.

- Bilevel model (Bilevel): It is the method proposed in Section 2.3.

- Conservative scheduling model (Conservative): It is a two-stage model that is similar to the bilevel model, but there is no feedback between the upper level and the lower level, and the process time is calculated conservatively by assuming that every vehicle may stop at the stop line. This is to ensure that every vehicle has sufficient time to pass the intersection without conflicts when the intersection controller does not know the entering speed.

- First in First out (FIFO): When a vehicle enters a predefined range from the stop line earlier than the others, it will also have a higher priority to pass the intersection. A trajectory mapping method, which is the same as Li and Wang (2006), is also used.

- Actuated control (Actuated): Conventional actuated control method is considered in the simulation. In this method, a loop detector is located at 40 m before the stop line on every road. The green time extension interval is 3 s and the minimum and maximum green time is chosen by running multiple simulations and selecting the best with the minimum average delay.

For simplicity, in the bilevel and conservative scheduling models, the Intelligent Driver Model (IDM)[‡] (Treiber et al., 2000) is used to update the vehicles' speed and location before they enter the communication range. This model is chosen as it is widely used to model the longitudinal behaviour of vehicles equipped with the Adaptive Cruise Control (ACC) system by applying a constant time headway policy (Kesting et al., 2007). Nevertheless, the proposed framework should work with other more advanced models for the connected environment, i.e. the model of Jia and Ngoduy (2016a,b). The IDM is also used in the FIFO and actuated control method. Briefly, the IDM describes the acceleration of the follower via the following equations:

$$a_{i,j} = a_{max}\left(1 - \left(\frac{v_{i,j}}{v_{max}}\right)^4 - \left(\frac{s^*(v_{i,j}, \Delta v_{i,j})}{\Delta s_{i,j}}\right)^2\right) \tag{2.22}$$

$$s^*(v_{i,j}, \Delta v_{i,j}) = g^0_{i,j} + v_{i,j}h_g + \frac{v_{i,j}\Delta v_{i,j}}{2\sqrt{a_{max}b}} \tag{2.23}$$

[‡]Remarks about the IDM can be found in Appendix 2.B.

where the distance gap between vehicles is calculated as $\Delta s_{i,j} = x_{i,j-1} - x_{i,j} - l_{i,j-1}$, the speed difference $\Delta v_{i,j} = v_{i,j} - v_{i,j-1}$ and the minimum gap $g^0_{i,j} = s_0 - l_{i,j-1}$. After vehicles enter the communication range, the optimisation is performed in a constant frequency which is once every 10 s if there is no special instruction, and then every vehicle updates its trajectory according to the optimisation results. every vehicle is assumed to be an AV and can follow the optimisation results precisely.

The most important performance index in the simulation is the average delay. It is defined as the difference between the actual travel time from entering the studied road to exiting the intersection area and the minimum travel time $T^{min}_{i,j}$ calculated by Equation (2.4) - (2.10) [§]. So the conflicting distance for every vehicle is the sum of the intersection length and the vehicle length. The intersection length is defined as the length of road in the intersection conflict area along the driving direction which is from the stop line to exit of the intersection. Another performance index used is the percentage of platoons. It is defined as the ratio of the number of vehicles passing the intersection in platoons to the total number of vehicles which pass the intersection during the simulation period. Here, the platoon is defined as two or more vehicles passing the intersection successively without the disturbance from the conflict roads. So the percentage of platoons indicates the frequency of changes in the right of way. The number of vehicles passing the intersection in the simulation period, which is named as throughput, is also recorded to show the difference in capacity.

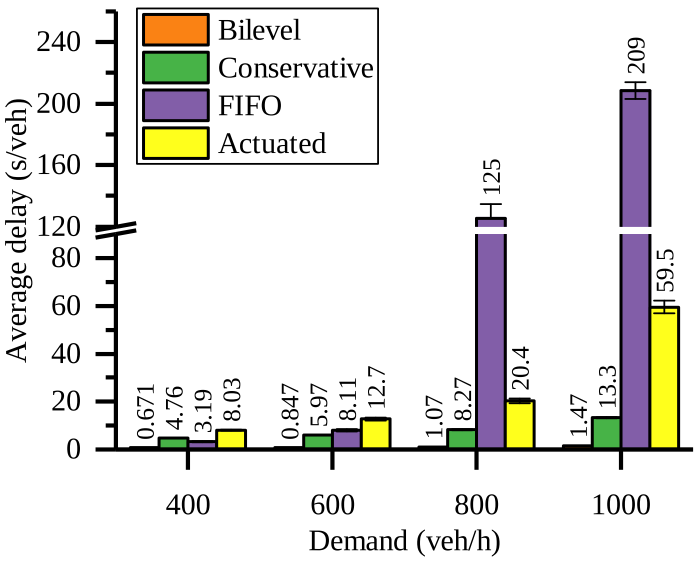
2.4.1 Different demands

Demand is one of the most important factors affecting the performance of the control strategy. The aforementioned four control methods under different demand levels varying from 400 to 1000 veh/h are tested. The results are shown in Fig. 2.6 and the simulated trajectories are shown in Fig. 2.7.

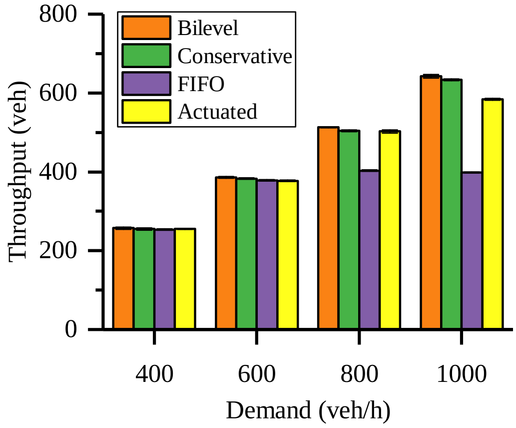
[§] The additional delay after the intersection because of the reduced speed at the intersection and additional time to reach the desired speed afterwards are not considered in this chapter. This also implies that the improvement in delay is underestimated as vehicles in the proposed bilevel model have much higher terminal speed than that in the other methods because of the explicit lower-level optimisation.

The bilevel model has the best performance in terms of the average delay compared with all the other control schemes under all demand levels. The proposed bilevel model maintains a very low average delay and high throughput in all simulated demands. It is mainly because the time headway of vehicles from both the same approach and conflict approaches are minimised by the model. The lower-level optimisation minimises the time for vehicles passing the intersection from the same approach and the upper level optimises the passing sequence to minimise the overall delay. The main drawback of the conservative scheduling model is that it overestimates the time required by a vehicle to pass the intersection because it does not estimate the maximum possible speed arriving at the stop line. As a result, the conservative scheduling model has more than five times average delay compared with the proposed bilevel method, but it still works quite well in high demands compared with the FIFO and actuated control methods. In particular, the FIFO works well only in low demands, which is also found by Li and Zhou (2017). When the demand increases, the control performance deteriorates quickly in both average delay and throughput. This is because when the priority of passing is determined by the FIFO, the trajectory is not optimised according to such a priority. By applying the trajectory mapping technique, the vehicle avoids a complete stop at the stop line in most situations. However, it sometimes has a low speed at the stop line because it needs to wait for other vehicles in higher priorities from the other approach. Such low speed has a negative impact on the capacity and defers all the vehicles in lower priorities. This can be seen in Fig. 2.7c. The actuated control is the worst in the demand levels of 400 veh/h and 600 veh/h, but it works better than the FIFO method in higher demands. The reason is that although some vehicles have to stop in front of the stop line in high demands, the following vehicles can still accelerate and have higher speeds when crossing the intersection area. This implies that when there is no trajectory optimisation, the platoon can increase the capacity in high demands.

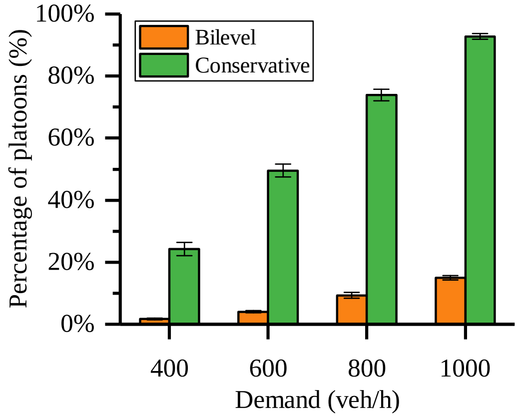
The four control methods show different throughputs in Fig. 2.6b because of different capacities. The throughput of the intersection in FIFO does not change when the demand keeps increasing from 600 veh/h. This indicates that it already reaches its capacity in the demand of 600 veh/h from each approach. However, the intersection in all the other three methods is unsaturated in all the simulated traffic demand levels. The throughput in the actuated control method is about 7% less than that in the other two optimisation-based methods in the high demand of 1000 veh/h because of the loss time of stop-and-go behaviour. Though the two optimisation-based methods have similar throughput, vehicles have quite different passing behaviour patterns. In the conservative scheduling model, vehicles tend to pass in platoons, and the percentage of platoons is much higher than that in the bilevel programming model which is mainly because of the longer process time in the conservative scheduling model. However, the percentage of platoons has the same trend of increasing in both models when the demand increases.



(a) Average delay

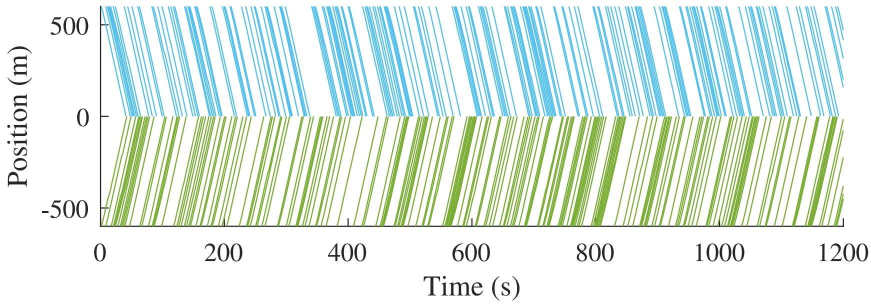


(b) Throughput

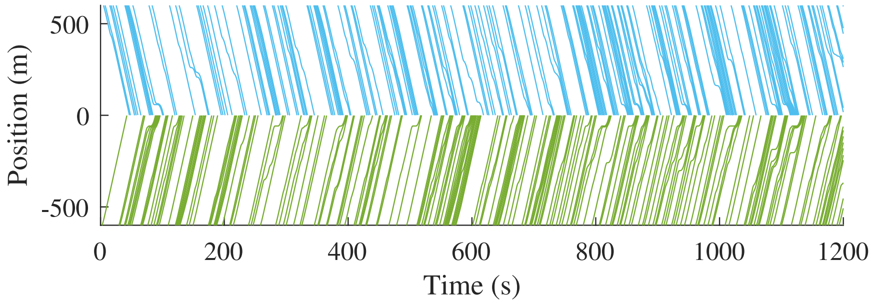


(c) Percentage of platoons

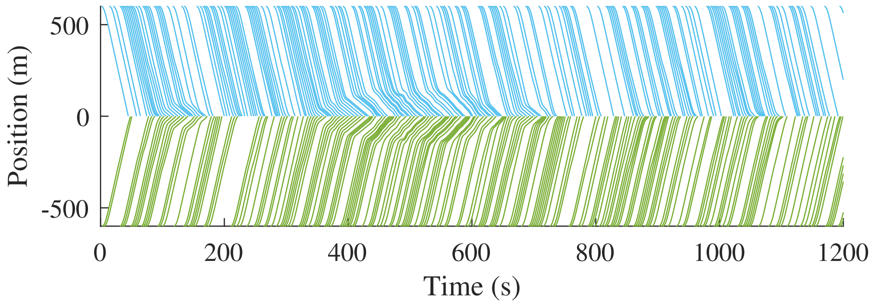
Figure 2.6: Simulation results under different demands



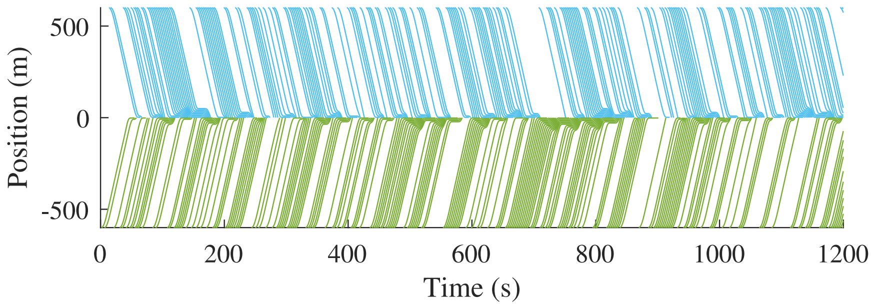
(a) Bilevel



(b) Conservative



(c) FIFO



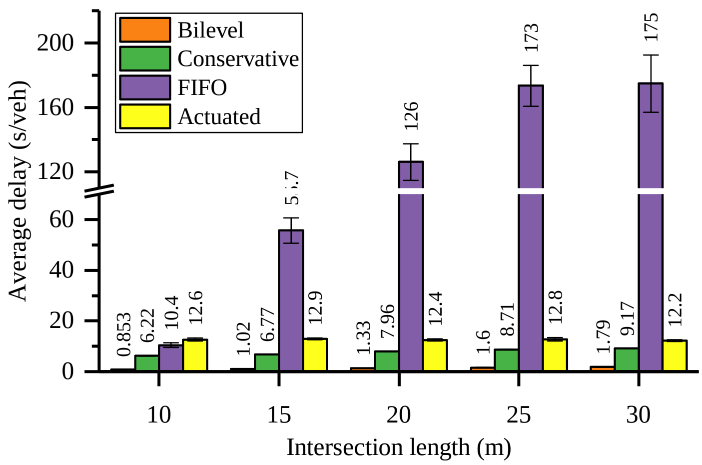
(d) Actuated

Figure 2.7: Simulated trajectories under different control methods

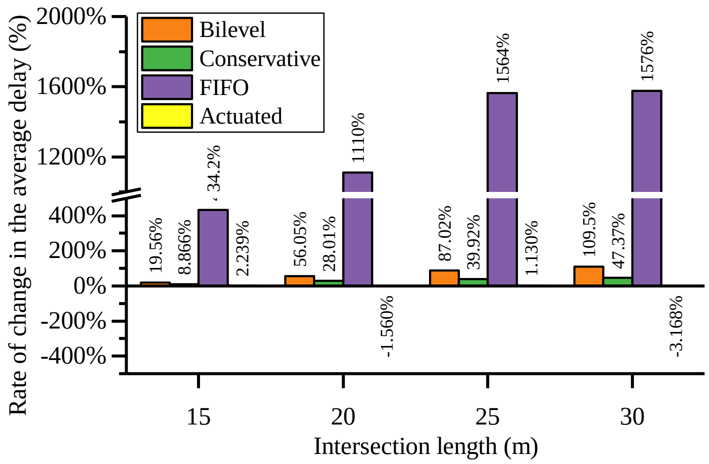
2.4.2 Different intersection lengths

Another important factor that is often ignored is the relationship between the minimum headway on the same road and the minimum process time in the conflict approach. Usually, the maximum allowed speed in the intersection area is the same, so the factor that needs to be investigated is the intersection length. Simulation results of intersection lengths varying from 10 m to 30 m are shown in Fig. 2.8. In these simulations, the demand is kept to 600 veh/h on each road.

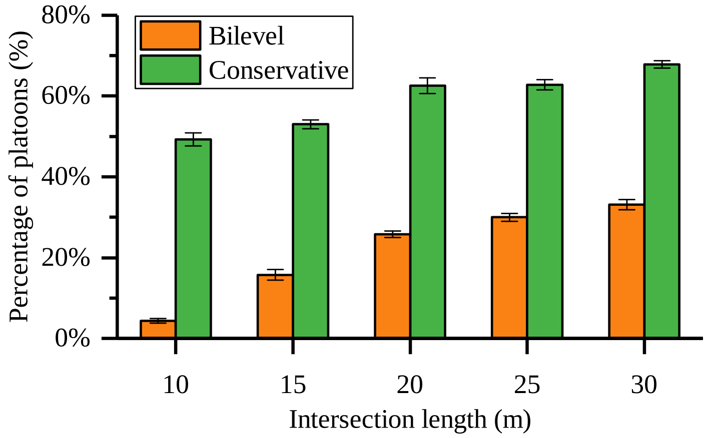
It is shown that the longer the intersection is, the larger the average delay becomes in all the four studied methods. Nevertheless, the sensitivity varies greatly. Fig. 2.8b shows the rates of change in the average delay for the four methods with different intersection lengths. In this case, rate of change in the average delay is derived by taking the difference in the average delay when the intersection length is larger than 10 m and that when the intersection length is 10 m and divided by the average delay when the intersection length is 10 m. It appears that the FIFO method is the most sensitive to the intersection length. A possible reason is that the demand is already close to the intersection capacity and the additional process time due to the longer intersection causes severe congestion on the road and increases the average delay rapidly. The actuated control does not show visible sensitivity to the intersection length since the intersection length does not affect the signal timing in the actuated control. Some vehicles in the actuated control method can achieve a much higher speed entering the intersection compared with that in the FIFO method. The bilevel model is more sensitive to the intersection length than the conservative scheduling method because of their different ways in calculating the process time. However, the delay in the bilevel model is significantly lower than that in the the conservative scheduling method, which is because the bilevel method makes better use of the intersection conflict area through the feedback between signal optimisation and trajectory optimisation. It is worth noticing that a longer intersection also prompts the vehicles to pass in platoons in both bilevel method and conservative scheduling method. We can also observe a considerable increase in the percentage of platoons when the intersection length increases from 10 m to 15 m, which can be explained by



(a) Average delay



(b) Rate of change in average delay



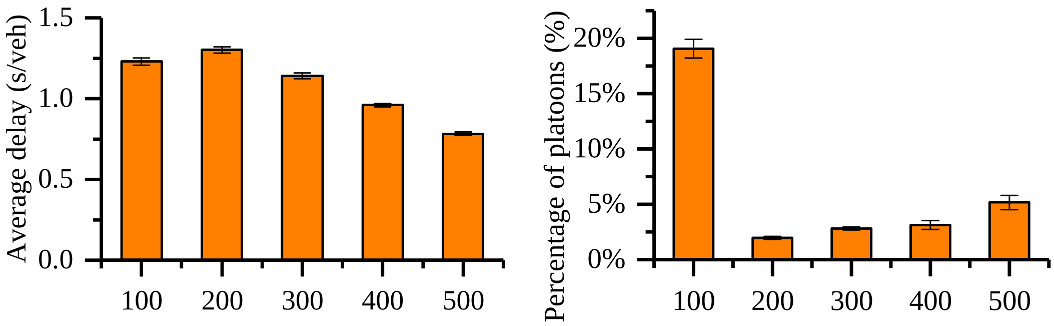
(c) Percentage of platoons

Figure 2.8: Simulation results under different intersection lengths

Proposition 2.3.3. When the intersection length is greater than a value (which is 14.9 m using the data in this paper), more vehicles tend to pass the intersection in platoons.

2.4.3 Different communication ranges

Different communication ranges varying from 100 m to 500 m are tested for the proposed model. The optimisation frequency increases from every 10 s to 5 s to avoid unoptimised vehicles being too close to the stop line and cannot find a feasible solution. The demand is kept to 600 veh/h in every simulation. The simulation results are shown in Fig. 2.9.



(a) Average delay (b) Percentage of platoons

Figure 2.9: Simulation results under different communication ranges

We can see that the average delay keeps decreasing with the increasing communication range when the communication range is greater than 200 m. The average delay when the communication range is 500 m is over 36% less than that when the communication range is 100 m. Nevertheless, the average delay is still quite small in every scenario. When the communication range is 100 m, vehicles are too close to the intersection and have little room to be optimised. The percentage of platoons is hugely higher than that in a longer communication range. When the communication range is 200 m, it becomes another extreme case that the priority of passing keeps changing between two approaches. There are few vehicles available to be optimised and the proposed method degenerates to a control scheme that is similar to the FIFO method. The throughput keeps the same among all scenarios. So the suggested communication range should be at least 300 m to ensure traffic flow operates stably, and the communication range of 500 m is preferred in terms of improving the control performance.

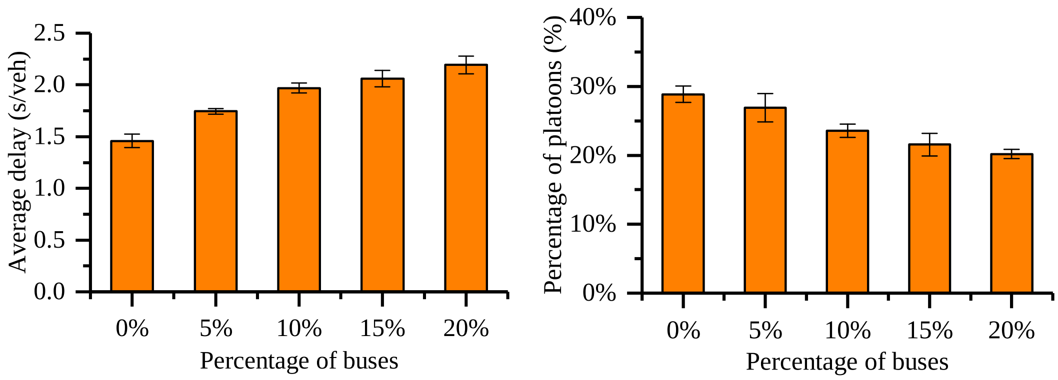
2.4.4 Different traffic compositions

To model the performance of the proposed method under heterogeneous traffic compositions, we assume the traffic consists of different shares of buses ¶. More stochastic vehicle characteristics are modelled in both cars and buses. As the drivers still have some personal preferences for the ACC or AVs, characteristics of those vehicles may not be the same (Butakov and Ioannou, 2016; Ghiasi et al., 2017). Instead of using a identical maximum speed as the desired speed for every vehicle, the desired speed of cars follows a truncated normal distribution $N(50, 2^2)$ within the interval $[40, 55]$, while the desired speed of buses follows a truncated normal distribution $N(45, 2^2)$ within the interval $[40, 55]$. The same bounds are applied to the desired speed of cars and buses to ensure that they are not too low, which are unrealistic, or too high, which exceed the maximum allowed speed. Other properties of buses are length (10 m), maximum acceleration (1.5 $\mathrm{m/s^2}$), minimum deceleration (3 $\mathrm{m/s^2}$). When a car or a bus following another bus, the safety time headway is 2 s. Otherwise, it is still 1.5 s. The total demand is kept to 600 veh/h on each road. Every vehicle's distinct characteristics are considered in the vehicle-based intersection control method rather than to be assumed the same in the conventional flow-based intersection control method.

The simulation results are shown in Fig. 2.10. It reveals a positive relationship between the average delay and the percentage of buses. That is rather obvious due to the lower desired speed and more restrict acceleration and deceleration abilities of the bus. Compared with the simulation results in Section 2.4.1 at the same demand level, the average delay with 0% bus increases by 71.8% from 0.85 s/veh to 1.46 s/veh, which implies that the traffic heterogeneity has a negative impact on the performance. In another aspect, even with 20% of buses, the average delay is still quite small and is much lower than that given by the other methods in Fig. 2.6 at the same total demand level. On the other hand, vehicles are less

¶There is no bus stop near the intersection in our study in this proof of concept case study.

likely to pass in platoons as the percentage of buses increases because of the higher time headway of buses. Moreover, all the percentages of platoons under different percentages of buses are higher than 20%, which implies that the platoon-based operation helps reduce delay in heterogeneous traffic flow. No evident throughput changes are observed when the percentage of buses increases.



(a) Average delay (b) Percentage of platoons

Figure 2.10: Simulation results under different traffic compositions

2.5 Concluding remarks

This paper developed a novel way to optimise intersection control and the trajectory jointly. In order to relax the commonly used assumption of the constant process time for vehicles to pass an intersection, the detailed trajectories of vehicles were considered in intersection control. This approach can fully utilise the available information from the intersection controller and vehicles to increase intersection efficiency. The problem was formulated as a bilevel programming model, in which the upper-level optimisation is a mixed integer linear programming model to minimise the total travel time. In contrast, the lower-level optimisation is a linear programming model to maximise the total speed entering the intersection. The coupled structure between the two levels is a major contribution of this paper. Moreover, platoon is not always beneficial to the intersection. After a discussion on the relationship between the safe time headway and the process time, a novel platoon identification method has been developed to reduce the number of binary variables and improve the performance of the model. The simulation results show that it significantly improves intersection control performance to integrate trajectory planning into signal optimisation, and the results are safe and feasible for every vehicle.

The models and findings established in this paper can be extended in the following aspects in future. Firstly, although the number of binary variables is greatly reduced due to the dynamic platoon formation, a more efficient heuristic method is still needed to enable a real-time application in the future. One possible way is to apply signal phases to reduce the number of binary variables (Xu et al., 2017; Yu et al., 2018), but this also reduces the flexibility of intersection control. Secondly, all vehicles were assumed to be AVs in this paper, so the obtained results provide the upper bounds of the benefit of the AVs to the urban intersection operation. However, this assumption is not practical in the near future. A more realistic situation would be mixed traffic consisting of vehicles with different levels of automation and connectivity. In this case, how to develop a robust and efficient intersection control method will be an interesting research question (Yang et al., 2016; Li and Zhou, 2017), which should be left in our future research. At last, we only applied the bilevel model to a simplified intersection. If the intersection is complex, more conflict zones exist and it requires more binary variables. Note that the solver time also increases because of the increased number of binary variables. Some simplified ways may be applied to reduce the complexity of the problem, such as the aforementioned platoon-based approach or reducing the communication range.

Acknowledgement

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Appendix 2.A Investigate the impact of initial values on the optimisation results

In Section 2.3.6, it was described that "in the first iteration of the upper-level optimisation, the speed entering the intersection $v^r_{i,j}$ is assumed to be the same as the initial speed $v^0_{i,j}$." The impact of initial values of terminal speed is investigated in this appendix.

Table 2.3: Position and speed of every vehicle at the start of the optimisation

Vehicle No.	Road 1		Road 2	
	Position (m)	Speed (m/s)	Position (m)	Speed (m/s)
1	445.90	10.28	555.07	5.14
2	375.44	11.07	313.89	11.28
3	291.61	6.61	170.88	15.23
4	262.62	11.43	139.55	11.46
5	190.03	12.12	101.19	8.49
6	158.89	14.40		

Table 2.4: Optimisation results with different initial terminal speed

Initial Guess	Optimal Solution (s)	Number of Iterations	Solver Time (s)
0 m/s	241.945	3	1.36
1 m/s	241.945	3	1.08
5 m/s	241.945	3	1.10
v_0	241.945	3	0.94
v_{max}	241.945	2	0.58

A typical case is set up where there are 6 and 5 vehicles within the communication range on each approaching road. Their position and speed at the start of optimisation are shown in Table 2.3. The optimisation results are presented in Table 2.4. No matter what is the initial terminal speed, the obtained results are the same. In this case, every vehicle can reach the maximum speed when they arrive at the stop line. So when the initial terminal speed is set to the maximum speed, it converges quickly with few iterations and solver time. When the initial guesses deviate too much from the optimal value, the number of iterations and solving time may increase. From this simple test, we can find that the heuristic algorithm is not sensitive to the initial guess in terms of the final solution, but may have different solver time. It is also possible to use the maximum speed as the initial guess for the terminal speed instead of the speed at the start of the optimisation used in this chapter.

Appendix 2.B Remarks about the IDM

IDM is a widely used car following model, which depicts traffic flow on both motorway and urban roads well. It is also used to model the ACC.

By assuming $a_{i,j} = 0$, we obtained the relationship between steady-state gap and speed

$$s_e(v) = \frac{g_{i,j}^0 + v_{i,j} h_g}{\sqrt{1 - \left(\frac{v_{i,j}}{v_{max}}\right)^4}} \tag{2.24}$$

When the speed $v_{i,j}$ is close to the desire speed v_{max}, the equilibrium gap $s_e(v)$ is much higher than $s^* = g_{i,j}^0 + v_{i,j} h_g$. This makes the desire time gap h_g loses its meaning. Not all vehicles in a platoon can reach the desire speed because of the non-zero braking term $-\frac{s^*}{\Delta s_{i,j}}$. However, this does not harm the methods and conclusions in this chapter.

The authors of IDM developed an Improved Intelligent Driver Model (IIDM)

(Treiber and Kesting, 2013) to address this issue.

$$a_{i,j} = \begin{cases} (1 - z^2) a_{max} & z = \dfrac{s^*(v_{i,j}, \Delta v_{i,j})}{\Delta s_{i,j}} \geq 1 \\ \left(1 - z^{\frac{2a_{max}}{a_{free}}}\right) a_{free} & \text{otherwise} \end{cases} \tag{2.25}$$

$$s^*(v_{i,j}, \Delta v_{i,j}) = g^0_{i,j} + v_{i,j} h_g + \frac{v_{i,j} \Delta v_{i,j}}{2\sqrt{a_{max} b}} \tag{2.26}$$

$$a_{free} = a_{max} \left[1 - \left(\frac{v_{i,j}}{v_{max}} \right)^4 \right] \tag{2.27}$$

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3

A platoon based cooperative eco-driving model for mixed automated and human-driven vehicles at a signalised intersection

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Contents

Abstract

The advancements in communication and sensing technologies can be exploited to assist the drivers in making better decisions. In this paper, we consider the design of a real-time cooperative eco-driving strategy for a group of vehicles with mixed automated vehicles (AVs) and human-driven vehicles (HVs). The leading vehicles in the platoon can receive information on the signal phase and timing via vehicle-to-infrastructure (V2I) communication and the state of the preceding vehicle and the current platoon via vehicle-to-vehicle (V2V) communication. A receding horizon model predictive control (MPC) method is proposed to minimise fuel consumption for platoons and drive the platoons to pass the intersection at a green phase. The method is then extended to dynamic platoon splitting and merging rules for cooperation among AVs and HVs in response to the high variation in urban traffic flow. Extensive simulation tests are also conducted to demonstrate the performance of the model in various conditions in mixed traffic flow and different penetration rates of AVs. Our model shows that the cooperation between AVs and HVs can further smooth out the trajectory of the latter and reduce the fuel consumption of the entire traffic system, especially in the low penetration rate of AVs. It is noteworthy that the proposed model does not compromise traffic efficiency and driving comfort while achieving the eco-driving strategy.

Keywords: Cooperative driving, platoon based operations, Eco-driving, automated vehicles, heterogeneous flow, car following model

3.1 Introduction

Transportation is one of the main sources of energy consumption and greenhouse gas emission. In the EU, transportation is responsible for 33% of energy consumption and 23% of total emissions (European Commission, 2016). Road transport represents most of it, 72.8% in total greenhouse gas emissions and 73.4% in transport energy demand. A lot of work has been done to mitigate these effects from different aspects, for example, optimised engine design, better road surface condition and more training for drivers. Due to the continually increasing number of vehicles, however, the total vehicle fuel consumption is still rising. The concept of "eco-driving" has drawn increasing attention from both academic researchers and government regulators (Carsten et al., 2016). The core of eco-driving technologies is to provide drivers with a variety of feedback and advice to minimise fuel consumption and emissions while driving.

Unlike continuous traffic flow on freeways, traffic flow on urban roads is regularly interrupted by traffic signals and conflicting traffic flow at intersections. As such, vehicles travel with substantial variations in velocity and consume more fuel. Eco-driving strategies are designed to reduce idling time at the red light and the subsequent strong acceleration by advising the drivers to approach intersections using a moderate acceleration and deceleration. The development of sensing and communication technologies make Vehicle-to-Infrastructure (V2I) and Vehicle-to-Vehicle (V2V) communication possible in the near future. These technologies offer potential applications for eco-driving patterns at intersections as the connected vehicles (CVs) can receive the Signal Phase and Timing (SPaT) information from the intersection controller by V2I and also receive the position and velocity information from surrounding vehicles by V2V communication. Better speed advice can be generated using these information, and thus vehicles may adjust their speed in advance, in order to avoid stopping at the stop line and subsequent strong acceleration and consequently reduce fuel consumption.

Both field experiments (Schall and Mohnen, 2017) and simulator experiments (Van der Voort et al., 2001; Staubach et al., 2014) show that eco-driving reduces fuel consumption between 5% and 18%, and drivers exhibit a high acceptance towards an eco-driving support system. It has no negative effects on safety, but many eco-driving methods lead to low travel speed and may have a negative impact on the following vehicles (Staubach et al., 2014; Wu et al., 2015). Moreover, they may even increase the travel time of the host vehicles and following vehicles.

This paper proposes a real-time cooperative eco-driving strategy for a platoon including mixed automated vehicles (AVs) and human-driven vehicles (HVs) approaching a signalised intersection. It adopts a model predictive control (MPC) method to control the trajectories of AVs. Here the AVs are considered the leaders of the platoon with the aim of minimising the total fuel consumption of the whole platoon without sacrificing the travel time of the leaders. It also reduces the travel time for the following vehicles to a certain extent. The rest of the paper is organised as follows: the literature review of the eco-driving modelling is presented in Section 3.2. Then, the proposed model structure, optimisation method, and platoon control scheme are described in Section 3.3. In Section 3.4, the properties of the proposed model are extensively studied, and the performance of the proposed method in different penetration rates of AVs is also examined. A final Section 3.5 summarises the paper's findings.

3.2 Literature Review

One of the applications of speed advisory systems is Intelligent Speed Adaptation (ISA), which is widely used in several EU countries (Almqvist et al., 1991; Liu and Tate, 2004). ISA devices are primarily aimed at safer driving by advising drivers a desired speed and speed limits on specific road sections (Ngoduy et al., 2009). Experiments showed that ISA strategies could potentially mitigate congestion and reduce fuel consumption and pollutant emissions due to smoother speed variations (Oei and Polak, 2002; Panis et al., 2006). In conventional ISA systems, vehicles are still driven by humans, and traffic information is usually obtained from loop detectors.

There are two main methods proposed in the literature which utilise the traffic signal information to reduce idle time and fuel consumption. The first approach suggests a constant speed or constant acceleration to an individual driver in order to reduce idle time or fuel consumption, which is commonly named Green Light Optimised Speed Advisory (GLOSA) system. It is usually implemented as an optimisation model by assuming a simple speed pattern in front of the intersection. Rakha and Kamalanathsharma (2011) used a fuel consumption model as the objective function and showed that simplified objective functions such as minimising the deceleration or idle time might not get the optimal result in terms of fuel consumption. This work is further extended to control the variable speed limit for each vehicle to minimise fuel consumption (Kamalanathsharma et al.,

2015) and integrate with queue estimation (Yang et al., 2017). Mandava et al. (2009) developed an arterial velocity planning algorithm which provided speed advice to the drivers regarding the most fuel optimal path by using upcoming signal information. The objective function was minimising the deceleration and acceleration rates, and 12–14% energy/emission savings were achieved. Tielert et al. (2010) conducted a large-scale simulation to identify the impact of gear choice and distance to the intersection. They found that sub-optimal gear choices reduce the positive performance of the speed adaptation. Another finding was that the benefit of providing information to the vehicles located further than 600m is negligible. Treiber and Kesting (2014) implemented three strategies of speed adaptation: early break, early start, and avoiding queue in the Improved Intelligent-Driver Model. The travel time decreases linearly with the penetration rate of equipped vehicles. They also found that increasing the maximum speed from 50km/h to 70km/h doubles the performance index. Katwijk and Gabriel (2015) considered the impact of different trajectories on fuel consumption. The vehicle was advised to use a smaller deceleration, even combined with a period of constant speed, instead of a hard deceleration in front of the red light. Stebbins et al. (2017) developed a method to suggest an acceleration to the leading vehicle only in a platoon to reduce delay. It was assumed that every vehicle that is the first to pass the intersection at a green light could be selected as a leading vehicle. Instead of controlling the speed directly, Ubiergo and Jin (2016) proposed a green driving strategy to control the individual advisory speed limit of CVs while following their leaders at signalised intersections. It can be applied to any level of market penetration. Although no fuel consumption model was explicitly used in this modelling method, it still saved 15% in travel delays and 8% in fuel consumption and emission.

The second approach uses an optimal control or an MPC method to provide dynamic or real-time speed advice to an individual vehicle considering the local and predictive traffic states. This approach is thus more suitable for AVs because of real-time detecting and speed adjustment. Asadi and Vahidi (2011) calculated the optimal speed that reduced idling at red lights using the given future state of traffic lights and developed a multiple objectives optimisation-based

MPC model. Kamal et al. (2013) predicted the dynamics of the preceding vehicle based on the information from inter-vehicle communication and considered the traffic signal status of the upcoming intersections to compute the optimal vehicle control input for fuel economy by an MPC method. He et al. (2015) developed a multi-stage optimal control model considering the spatial and temporal constraints induced by vehicle queue in front of the intersection. They also added the constraints to reduce the negative impact on the following vehicles, but it was only active at the terminal time step at each stage. Wan et al. (2016) used optimal control theory to solve the minimum fuel control problem and found that the minimal fuel driving strategy is a bang-singular-bang control, which means either maximum acceleration or engine shut down is used. By employing a sub-optimal method, the speed advisory equipped vehicle also benefits the following conventional vehicles. De Nunzio et al. (2016) used a combination of a pruning algorithm and shortest path method to find the minimum energy consumption path in multi-intersections. The non-convex optimal control problem was then reduced to a convex problem which can be solved efficiently.

To the best of our knowledge, most current work focuses on developing fuel-efficient control strategies for a single vehicle without considering the impact on the other vehicles. HomChaudhuri et al. (2017) considered neighbourhood information exchange and designed a decentralised control model emulating the selfish behaviour of human drivers, but their model still considers one vehicle and does not describe the interactions between platoons. Zhou et al. (2017) and Ma et al. (2017) proposed a parsimonious shooting heuristic algorithm to optimise the trajectories of a stream of vehicles and considered multiple objective functions such as fuel consumption and travel time, but all vehicles are required to be AVs in their method. Jiang et al. (2017) proposed an eco-driving model in partially connected and automated vehicles environment; however, they did not consider the cooperation between AVs and HVs, even though the behaviour of the AV still affects the following vehicles. This indicates that there are no platoon-based dynamics in their approach. Our model will fill in this gap by showing that the cooperation between AVs and HVs can further smooth the trajectory of the latter and consequently reduce the fuel consumption of the whole platoon. The proposed method optimises fuel consumption for platoons and drive the platoons to pass the intersection at a green phase. It is flexible that allows multiple AVs and HVs in the platoon. Both the impact of cooperation between AVs and cooperation between AVs and HVs will be studied in detail. Most current work uses the rolling horizon MPC method, and the optimised vehicle sometimes travels with a low speed to achieve better fuel economy. In this paper, a distinctive receding horizon MPC method is proposed to ensure that eco-driving strategies do not have an adverse impact on traffic efficiency. On the contrary, the proposed model can increase the speed passing the intersection and thus increase traffic efficiency. In addition, driving comfort is considered by using jerk as the control variable.

Notation

The notation in Table 3.1 is used throughout this paper.

Table 3.1: Notations of major variables used in this paper

Symbol	Description
t	Time instant
$x_i^a(t), v_i^a(t), a_i^a(t)$	The position, speed, acceleration of an AV i at time t, where the superscript a denotes AV
$x_j^h(t), v_j^h(t), a_j^h(t)$	The position, speed, acceleration of an HV j at time t, where the superscript h denotes HV
$\hat{x}_{tf}, \hat{v}_{tf}, \hat{a}_{tf}$	The desired position, speed, acceleration at terminal time, respectively where subscript tf denotes terminal time
$u(t)$	The jerk of an AV which is the control variable at time t
J	Total cost in the MPC objective function
$F_i^a(t), F_j^h(t)$	Instantaneous fuel consumption rate for AV i, and HV j, respectively, at time t
t_i^f	Terminal time for the vehicle i and also the time to pass the stop line
T_k^g, T_k^r	The start time of green light, red light respectively, in cycle k

3.3 Problem formulation

This paper focuses on the design of an eco-driving strategy for a group of vehicles with mixed AVs and HVs. The movement of HVs is modelled by a car-following model, while the dynamics of AVs are optimised by an MPC method. For the sake of simplicity, in this paper, Optimal Velocity Model (OVM) is applied to describe the behaviour of HVs (Bando et al., 1995). Nevertheless, the proposed modelling methodology holds for any other car-following model. Our method allows several closely running vehicles to form a platoon and pass the intersection at the green light without stopping. A basic schematic representation is shown in Fig. 3.1.

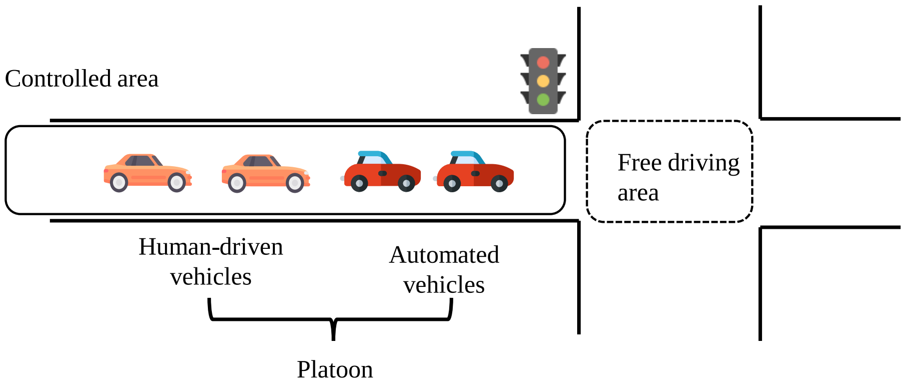


Figure 3.1: Schematic of the eco-driving problem at a signalised intersection

3.3.1 Assumptions

To facilitate our model development, some necessary assumptions are made as follows.

1. In order to set up the cooperative behaviour between AVs and the following HVs, AVs have to know the positions and speeds of some following vehicles and the direct preceding vehicle in real-time. This information is assumed to be available through either CVs or a roadside unit (RSU) (Jia and Ngoduy, 2016a). This assumption will be relaxed in Section 3.4.3 where the AVs obtain this information about the direct following vehicle via its detectors.

2. All AVs can receive SPaT information about the downstream intersection via V2I.

3. No communication delay or detection error is considered in the paper. For cooperative driving behaviour in a platoon with realistic communication, we refer to Jia and Ngoduy (2016b). This assumption will be relaxed in our future work.

4. AVs in different platoons can share the information about the vehicles' arrival time via either V2V or RSU. Hence, they can predict better arrival time.

5. This work only focuses on the longitudinal movement on the urban road.

It is worth noticing that AVs interact with the downstream intersection and decide their dynamics to get the whole platoon through the intersection at the green time.

3.3.2 Optimal velocity model

The OVM is formulated based on the presumption that a vehicle is driven to reach an optimal velocity, which depends on the headway with respect to the preceding vehicle in a continuous time step. The acceleration of vehicles in the OVM is calculated by

$$a_j^h(t) = \kappa \left[V_{op}(\Delta x_j(t)) - v_j^h(t)\right] \tag{3.1}$$

where $\Delta x_j(t) = x_{j-1}(t) - x_j^h(t)$ is the distance headway between vehicles j and its preceding vehicle $j-1$. $V_{op}(\Delta x_j(t))$ defines the optimal velocity, which is a function of the distance headway. κ is the sensitivity, which is the inverse of delay time that is required to reach the optimal velocity. In this paper, the following velocity function proposed by Helbing and Tilch (1998) is chosen:

$$V_{op}\left(\Delta x_j\right) = V_1 + V_2 \tanh\left[C_1(\Delta x_j - l_c) - C_2\right] \tag{3.2}$$

where V_1, V_2, C_1, C_2 are the parameters and l_c denotes the vehicle length. The parameters calibrated by the empirical follow-the-leader data for city traffic in Helbing and Tilch (1998) are used in this paper: $\kappa = 0.85\,\mathrm{s}^{-1}, V_1 = 6.75\,\mathrm{m/s}, V_2 =$

$7.91\,\text{m/s}, C_1 = 0.13\,\text{m}^{-1}, C_2 = 1.57$ and $l_c = 5\,\text{m}$. Because the OVM may generate unrealistic high acceleration (Helbing and Tilch, 1998), the acceleration limits shown in Table 3.2 are applied.

3.3.3 Model predictive control

Each AV is able to receive real-time information from the preceding vehicle and following vehicles via V2V, such as position and velocity. In the MPC method, a common assumption is that the preceding vehicle is travelling at a constant velocity. So the time for the AV to arrive at the intersection at a green light can also be estimated. Then a receding horizon MPC method is used. For the safety and comfort purposes, a further assumption is made that the AV travels across the intersection with a constant velocity, which implies that the acceleration of the AV at the stop line should be 0. Accordingly, in our model, the control variable is the derivative of the acceleration of the AV, which is also called "jerk" and denoted as $u(t)$.

3.3.3.1 State variables

In order to minimise fuel consumption for all vehicles in the platoon, the state variables of those vehicles are included in the system state. For a general platoon including m AVs and n HVs, its state is designed as

$$\mathbf{X}(t) = [\underbrace{x_i^a(t), v_i^a(t), a_i^a(t), \cdots, x_m^a(t), v_m^a(t), a_m^a(t)}_{\text{AVs}}, \underbrace{x_j^h(t), v_j^h(t), \cdots, x_n^h(t), v_n^h(t)}_{\text{HVs}}]^T$$
$$i = 1, \cdots, m;\; j = 1, \cdots, n \quad (3.3)$$

The corresponding system dynamic function is

$$\dot{\mathbf{X}}(t) = [\underbrace{v_i^a(t), a_i^a(t), u_i^a(t), \cdots, v_m^a(t), a_m^a(t), u_m^a(t)}_{\text{AVs}}, \underbrace{v_j^h(t), a_j^h(t), \cdots, v_n^h(t), a_n^h(t)}_{\text{HVs}}]^T \quad (3.4)$$

where the acceleration of the HV $a_j^h(t)$ is calculated by Eq. (3.1).

3.3.3.2 Objective function

The cost function for the platoon only has the running cost term, and the terminal cost is implemented as terminal boundary conditions. The running cost, which is the driving cost at every time step, is defines as the total fuel consumption for all vehicles in the platoon.

$$\min_{u} J = \sum_{i}^{m} \int_{t_i^0}^{t_i^f} F_i^a(t) + \sum_{j}^{n} \int_{t_m^0}^{t_m^f} F_j^h(t)\,\mathrm{d}t \tag{3.5}$$

An instantaneous fuel consumption model developed by Akcelik (1989) is adopted in this work. It uses the instantaneous acceleration and velocity to estimate the fuel consumption rate:

$$F = \alpha + \beta_1 P_T + (\beta_2 m a^2 v)_{a>0} \tag{3.6}$$

where P_T is the total power (kW) required to drive the vehicle, which is the sum of coast-down drag power, inertia power and extra engine power. P_T is non-negative. The third term means extra engine drag power during accelerations, which only exists when the acceleration is larger than zero.

$$P_T = \max\{d_1 v + d_2 v^2 + d_3 v^3 + mav,\ 0\} \tag{3.7}$$

The value of parameters $\alpha,\ \beta_1,\ \beta_2,\ d_1,\ d_2,\ d_3,\ m$ in Eq. (3.6) and Eq. (3.7) are taken from Akcelik (1989), which are $\alpha = 0.666\,\mathrm{mL/s},\ \beta_1 = 0.072\,\mathrm{mL/kJ},\ \beta_2 = 0.0344\,\mathrm{mL/(kJ \cdot m/s^2)},\ d_1 = 0.269\,\mathrm{kN},\ d_2 = 0.0171\,\mathrm{kN/(m/s)},\ d_3 = 0.000\,672\,\mathrm{kN/(m/s)^2},\ m = 1680\,\mathrm{kg}$.

The terminal time t_i^f is set to the earliest time that allows the AV i to pass the intersection at a green phase. It is calculated by

$$t_i^f = \max(t_i^{f'},\ t_i^g) \tag{3.8}$$

where $t_i^{f'}$ denotes the earliest possible arrival time, and is calculated by

$$t_i^{f'} = \max(t_i^{min},\ t_{i-1}^f + h) \tag{3.9}$$

where t_i^{min} denotes the minimum travel time by using the highest jerk, t_{i-1}^{f} denotes the travel time of the preceding vehicle $i-1$. If the vehicle $i-1$ is an AV, its estimated travel time information can be available via V2V. If it is an HV that belongs to the preceding platoon, the AV (or AVs) in the preceding platoon must have the travel time information and transfer it to vehicle i. Otherwise, it can be estimated by using loop detectors (Guler et al., 2014; Treiber and Kesting, 2014; He et al., 2015) or CVs (Yang et al., 2017; Zheng and Liu, 2017). t_i^g denotes the start of the green light which is closest to $t_i^{f'}$. It is calculated by

$$t_i^g = \begin{cases} T_k^g & t_i^{f'} \in [T_k^g, T_k^r) \\ T_{k+1}^g & t_i^{f'} \in \left[T_k^r, T_{k+1}^g\right] \end{cases} \tag{3.10}$$

where $T_k^g (T_k^r)$ denotes the start time of green (red) light in the signal cycle k.

Please note that when there are multiple AVs in a platoon, they have different t_i^0 and t_i^f, and the proposed optimal control problem is a multi-stage optimal control problem which can be solved by GPOPS. Only an isolated intersection is considered in this paper, but the proposed model can be extended to multi-intersections without much trouble by taking each intersection as a stage (He et al., 2015).

3.3.3.3 Constraints

One of the control goals is to drive AVs from the current position to the stop line with the desired velocity and acceleration. Therefore, the boundary conditions are:

$$\textbf{Initial boundary conditions:} \quad \mathbf{X}_i(t_i^0) = [x_i^a(t_i^0), v_i^a(t_i^0), a_i^a(t_i^0)] \tag{3.11a}$$

$$\textbf{Terminal boundary conditions:} \mathbf{X}_i(t_i^f) = [\hat{x}_{tf}, \hat{v}_{tf}, \hat{a}_{tf}] \tag{3.11b}$$

In the paper, the desired terminal position $\hat{x}_{tf}$ is the downstream stop line, and the desired terminal speed $\hat{v}_{tf}$ is the maximum allowed velocity which is 14.66 m/s. Note that the maximum speed of the AVs is the same as that of HVs using the described parameters. We will show that maximising the speed entering the intersection can increase the capacity of the intersection. The desired terminal acceleration $\hat{a}_{tf}$ is $0\,\mathrm{m/s^2}$ and allows the vehicle to pass the intersection with a constant speed which is the maximum speed resulting from the terminal speed. This mainly concerns the safety when crossing the intersection. If without this term, the acceleration of vehicle would drop to zero suddenly due to the speed limit after the terminal time (Ntousakis et al., 2016).

$$\textbf{Speed constraints:} \quad v_{min} \leq v_i^a(t) \leq v_{max} \tag{3.12a}$$

$$\textbf{Acceleration constraints:} \quad a_{min} \leq a_i^a(t) \leq a_{max} \tag{3.12b}$$

$$\textbf{Jerk constraints:} \quad u_{min} \leq u_i^a(t) \leq u_{max} \tag{3.12c}$$

$$\textbf{Safety constraints:} \quad a_i^a(t) \leq a_i^{OVM}(t) \tag{3.12d}$$

where v_{min}, v_{max}, a_{min}, a_{max}, u_{min}, u_{max} denote the lower and upper bounds of the velocity, acceleration and jerk, respectively. The same speed and acceleration limits in Table 3.2 are used for both MPC and OVM. $a_i^{OVM}(t)$ is calculated by Eq. (3.1) using the speed and the gap of AV. This implies that the car-following model (i.e. OVM) is used as the upper bound of the acceleration for an AV. It prevents the MPC algorithm from acting too aggressively to achieve the final goal. So basically, the upper bound of the acceleration reads: $a_i^a(t) \leq \min(a_{max},\ a_i^{OVM}(t))$. It also provides the possibility of handing over to human driving more smoothly if required. The safety constraints are implemented as a penalty term in the cost function.

$$\min_u J = \sum_i^m \int_{t_i^0}^{t_i^f} \left[F_i^a(t) + p\left(\max(a_i^a(t) - a_i^{OVM}(t), 0)\right)^2 \right] + \sum_j^n \int_{t_m^0}^{t_m^f} F_j^h(t)\,\mathrm{d}t \tag{3.13}$$

3.3.4 Interactions between AVs and HVs

In order to develop an eco-driving strategy for the benefits of both AVs and HVs in the platoon, several types of cooperation are considered in the model. The overall interactions are shown in Fig. 3.2. Note that in the platoon, HVs are modelled by the OVM and AVs are controlled by the MPC method.

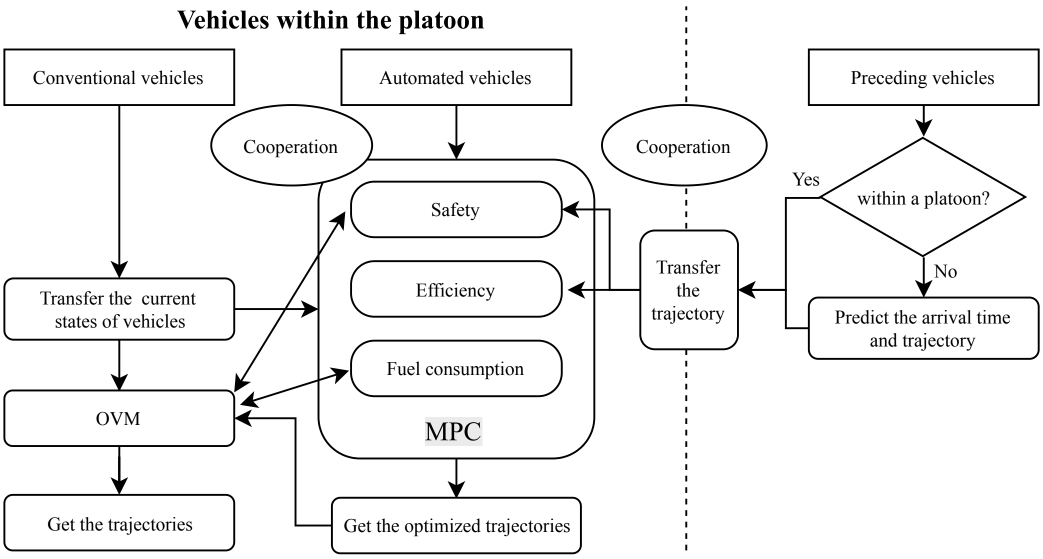


Figure 3.2: Interactions between AVs and HVs

In Fig. 3.2, there are basically two types of cooperative behaviour for AVs: (1) interacting with preceding vehicles between platoons; (2) interacting with the AVs or HVs within the platoon. If the preceding vehicle belongs to the preceding platoon, then the leading (automated) vehicle of the preceding platoon knows the passing time of its members and can transfer the information to the AVs in the considered platoon. Otherwise, the AVs have to predict the arrival time of the preceding vehicles based on the data acquired by their built-in detectors or other sources of communication such as RSU or CV technologies. For the vehicles in a platoon, the cooperation is designed for safety and fuel efficiency. The AVs in the platoon consider the dynamics of all vehicles in the platoon and attempt to find a solution that minimises fuel consumption for all vehicles in the platoon.

3.3.5 The control framework for platoons

The proposed method is applied to a platoon instead of a single vehicle, so how to define the platoon and how to manage the platoon dynamically are the key challenges in this paper. The platoon is usually defined as a group of vehicles that are adjacent to each other and have similar traffic state (see Ngoduy, 2013; Jia and Ngoduy, 2016a,b and references therein). On an urban road, some vehicles can pass through the intersection at a green light and travel with the speed that depends on the traffic conditions. Other vehicles have to stop at the stop line when the traffic signal turns red. So it is natural to define the platoon as the group of vehicles that can pass at the same green phase.

There are two criteria for a platoon:

1. All the vehicles in a platoon must pass the intersection at the same green phase.

2. The leading vehicle in a platoon must be an AV, and all AVs can only be located in front of the HVs in each platoon.

Criterion 2 is essential for the proposed eco-driving method. This is because only when the AV is in front of the HV, it can affect the following vehicles' movements by controlling its jerk. The platoon in this paper is different from the conventional one. It is heterogeneous that may include AVs and HVs. The purpose of a platoon is to allow cooperation between AVs and HVs, which pass the intersection at the same green phase, to reduce the total fuel consumption. The platoon dynamics including splitting and merging are to determine which vehicle should be considered in the cooperation loop. The setting of a platoon is not to ensure all the vehicles in the platoon can pass the intersection at the same green phase. In contrast, the vehicles can pass the intersection at the same green phase is the necessary condition to form a platoon, rather than the result. Different platoon settings in mixed traffic flow will be discussed in detail in Section 3.4.2.

The control framework for platoon dynamics is shown in Fig. 3.3 and the main processes are described as follows:

1. Split all the vehicles on the road into several groups according to the maximum allowed number of vehicles in a platoon, and the leading AV (or AVs) in a platoon becomes the host vehicle.

2. Run the MPC algorithm for every platoon, the optimised control variables are only applied to the host vehicles in the next time step, while the behaviour of HVs is governed by the OVM.

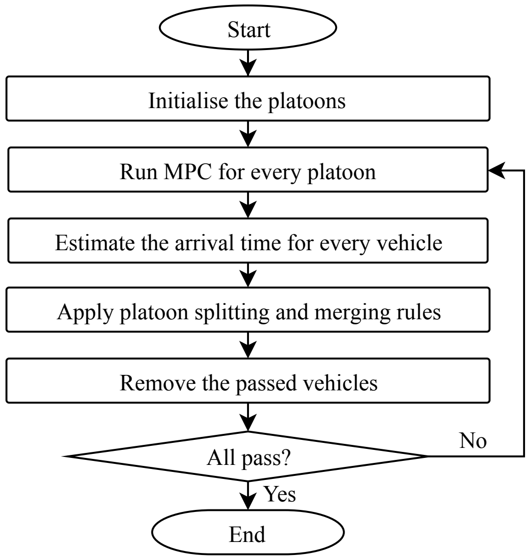


Figure 3.3: The overall control framework

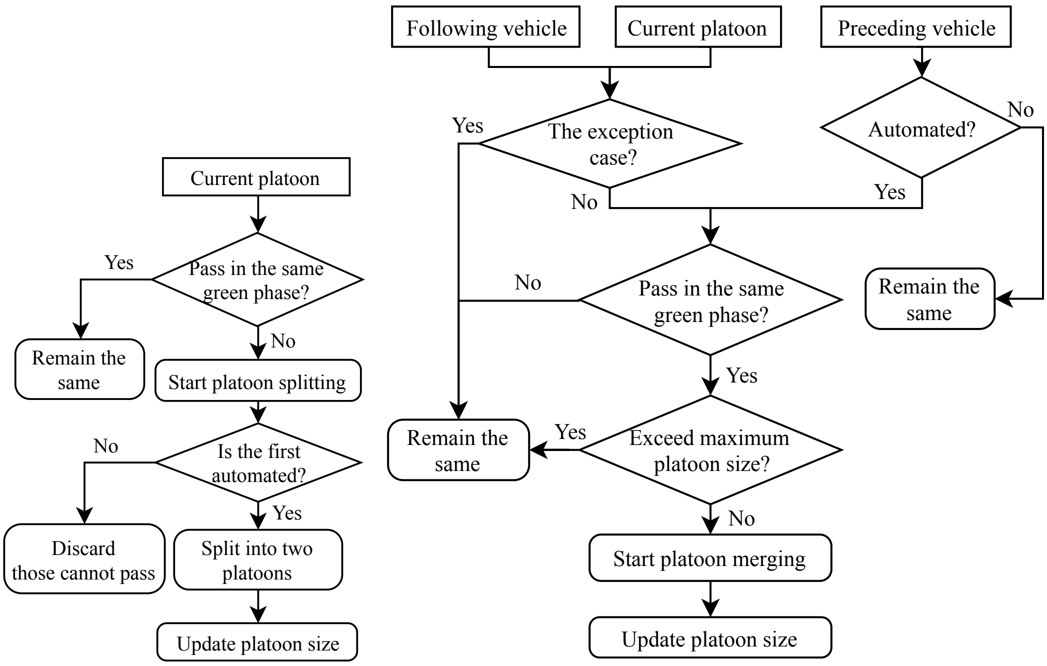
3. Apply the platoon splitting and merging rules every T_1 time steps which are z times of the control update time step T (i.e. $T_1 = zT$).

The platoon splitting and merging rules mainly consider the planned vehicle arrival time, signal timing information, and the defined minimum and maximum number of vehicles in a platoon. The rules are described in the following.

1. Splitting rule (see Fig. 3.4a): After the MPC is executed, some vehicles in the platoon may not pass the intersection at the same green phase. Then, the platoon splitting rule applies. If the first vehicle that cannot pass at the same green light is an AV, then it is split from the original platoon and becomes the leading vehicle for the new platoon. Otherwise, all those that cannot pass at the same green light are discarded by the current platoon.

2. Merging rule (see Fig. 3.4b): Merging rule is more complicated than the splitting rule as it may operate in two directions: merge with the preceding vehicles or the following vehicles. Please note that merging with the preceding vehicles has higher priority than merging with the following vehicles as the operations of the preceding vehicles can affect all the following vehicles, and merging with the preceding vehicles may get better performance. In both cases, it needs to check whether the two key criteria are still satisfied

after merging. The exceptional case in Fig. 3.4b is an AV follows an HV.



(a) Platoon splitting (b) Platoon merging

Figure 3.4: Platoon splitting and merging framework

The splitting rule is always applied before the merging rule. The discarded vehicles by the splitting rule will try to find a chance to form another platoon by the merging rule where every AV can be seen as a separate platoon with size 1. This does not mean that every HV must belong to a platoon. If an HV does not belong to any platoon, it may have to stop in front of the stop line.

3.3.6 Gauss pseudospectral method

A Matlab software package GPOPS (Rao et al., 2010) is used to solve the proposed optimal control problem. It mainly uses a numerical method, namely Gauss pseudospectral method, and is widely used in trajectory planning problems for vehicles (He et al., 2015; Wu et al., 2015) and trains (Ye and Liu, 2016). The method belongs to a direct approach (Stryk and Bulirsch, 1992) whose main idea is transforming the optimal control problem into a nonlinear programming (NLP) problem, which can then be solved by a variety of well-known solvers such as SNOPT (Gill et al., 2005) used in GPOPS. The performance of GPOPS strongly depends on the parameter settings (Ye and Liu, 2016). Usually, the user needs to try several combinations of parameter settings to find the best suitable ones. The key parameters used in GPOPS and the model are listed in Table 3.2[†].

3.4 Numerical studies

3.4.1 Properties of boundary conditions

In this study, three scenarios are presented to illustrate the benefits of the proposed terminal boundary conditions. The simulation scenario considered in this paper is a single lane road with a traffic signal light at 250 m ahead. There are 10 vehicles driving on the road and attempting to cross the intersection. At the beginning of the simulation, all vehicles have the same velocity of 10 m/s and acceleration of $0\,\mathrm{m/s^2}$. The other parameters used in the MPC method are shown in Table 3.2.

- Scenario T1: No speed advice is given to the drivers, and all vehicles' acceleration is only calculated by the OVM. This case is named as OVM for simplicity.
- Scenario T2: The first vehicle is an AV, and only the terminal position constraints in 3.11b is considered while the running cost remains the same as Eq. (3.5).
- Scenario T3: The first vehicle is an AV, and the boundary conditions and running cost are the same as Eq. (3.11b) and Eq. (3.5) respectively.

When all vehicles have crossed the stop line, the total fuel consumption is shown in Fig. 3.5a. As expected, the fuel consumption of vehicles in the MPC is much less than that in the OVM. More specifically, scenario T2 reduces by 9.7% and scenario T3 reduces by 5.2% compared with scenario T1. Due to stopping in front of the intersection at a red light, it also takes much more time to discharge the ten vehicles in the scenario T1.

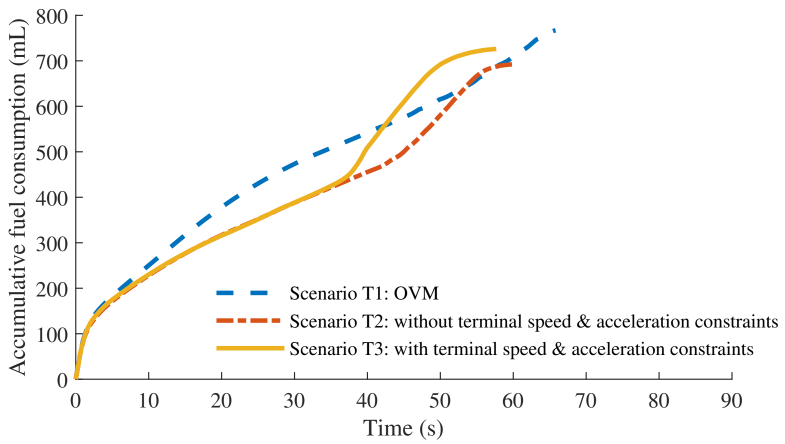
[†]The process of choosing the parameters for GPOPS is shown in Appendix 3.A.

Table 3.2: The parameters in the proposed eco-driving method

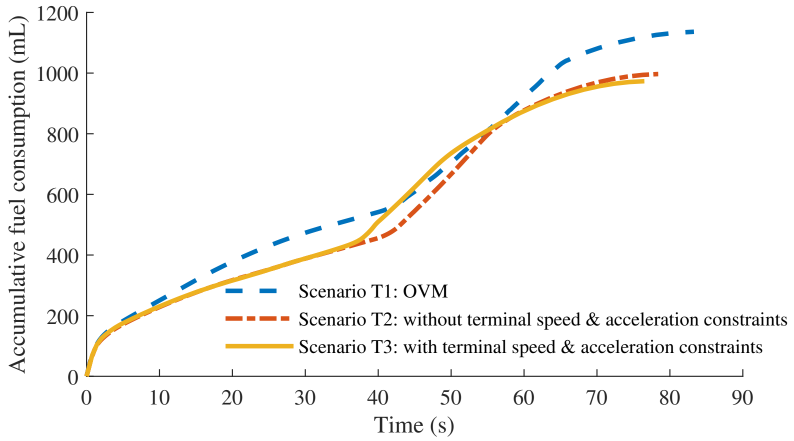
Parameter settings in the GPOPS		
Parameter	Description	Value
setup.autoscale	Whether the optimal control problem is scaled automatically	"on"
setup.derivatives	Method to compute the derivatives of the objective function (gradient) and the constraints for NLP solver	"complex"
setup.tolerances	Optimality and feasibility tolerances for the NLP solver	$[1\text{e}{-3}, 2\text{e}{-3}]$
limits.meshPoints	Locations of mesh points in the initial run	$[-1, 1]$
limits.nodesPerInterval	Number of allowable collocation points in a mesh interval	$2 * (t^f - t^0)$
setup.mesh.tolerance	Mesh refinement tolerance	$1\text{e}{-4}$
setup.mesh.iteration	Mesh refinement iterations to perform	8

Parameter settings in the model			
Parameter	Description	Value	Unit
T_M	Sample time in the MPC method	0.5	s
T_O	Sample time in the OVM	0.1	s
h	Safety time headway for an AV	2	s
p	Penalty weight for the safety constraint	0.1	$\text{mL} \cdot \text{s}^3/\text{m}^2$
v_{max}	Maximum speed	14.66	m/s
v_{min}	Minimum speed	0	m/s
a_{max}	Maximum acceleration	3	m/s^2
a_{min}	Minimum acceleration	-6	m/s^2
u_{max}	Maximum jerk (limit for the control variable)	4	m/s^3
u_{min}	Minimum jerk (limit for the control variable)	-4	m/s^3

In the two scenarios using the MPC, the model having the terminal speed and acceleration boundary conditions consumes 1.7 % more fuel, as the vehicles need to accelerate more. Moreover, it needs less green time to discharge the vehicles. The detailed data can be seen in Table 3.3 ‡. It takes 20.2 s and 18 s in the green phase to pass in scenario T2 and scenario T3, respectively. This means that scenario T3 can let one more vehicle pass in the same signal settings. Thus, scenario T3 increases the capacity by 11.1 % compared with scenario T2 and by 44.4 % compared with scenario T1.



(a)



(b)

Figure 3.5: Accumulative fuel consumption (a) when all the vehicles arrive at the stop line; (b) when all the vehicles arrive at the extended distance.

Fig. 3.6 shows the detailed position and speed trajectory for every vehicle in the three scenarios. It can be seen that vehicles in both scenarios T2 and T3 can

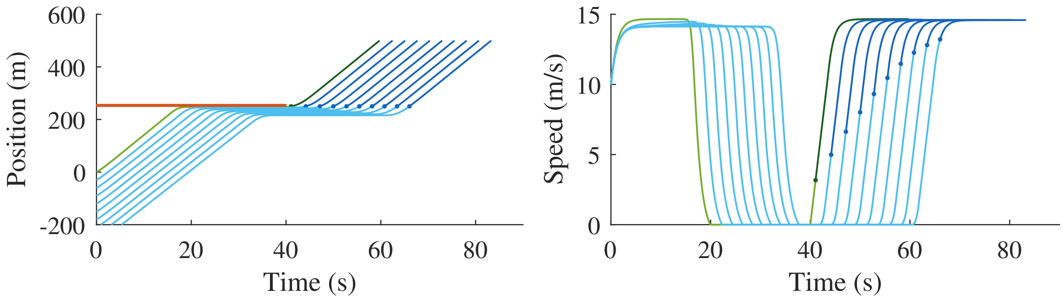
‡An analysis of the boundary conditions and penalty function can be found in Appendix 3.B.

Table 3.3: Simulation results with different boundary conditions

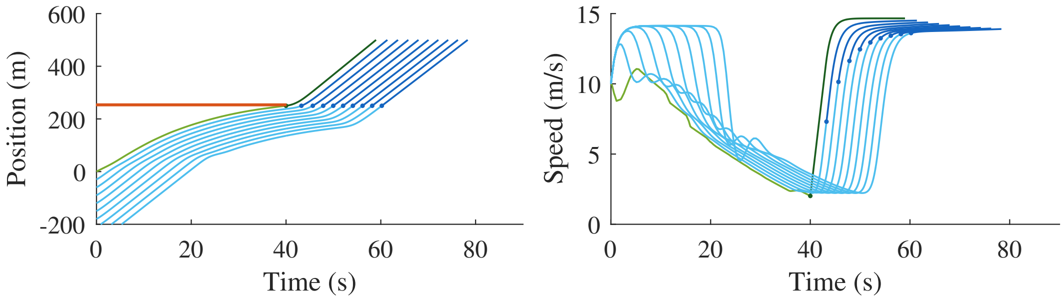
Scenario	Terminal position	Total fuel consumption (mL)	Used green time (s)	Total travel time (s)
scenario T1	stop line	767.1	25.8	527.7
	extended	1136.4		716.05
scenario T2	stop line	693.0 (−9.7 %)	20.2 (−21.7 %)	507.0 (−3.9 %)
	extended	996.8 (−12.3 %)		688.9 (−3.8 %)
scenario T3	stop line	726.9 (−5.2 %)	18.0 (−30.2 %)	491.2 (-6.9%)
	extended	973.2 (−14.4 %)		670.1 (−6.4 %)

pass the intersection without stopping due to the guidance of the first vehicle. They also have a much higher final speed than vehicles in scenario T1, in which vehicles have to accelerate from a complete stop. The speed of the first vehicle in scenario T2 is always decreasing while that in scenario T3 decreases first and then increases to the maximum speed, which is the desired final speed. This also explains why scenario T3 uses more fuel than scenario T2. It is consistent with Fig. 3.5a. The total fuel consumption of scenario T2 and T3 are almost identical in the first 35 s. Because of the high terminal speed cost in the scenario T3, the vehicles consume much more fuel to accelerate.

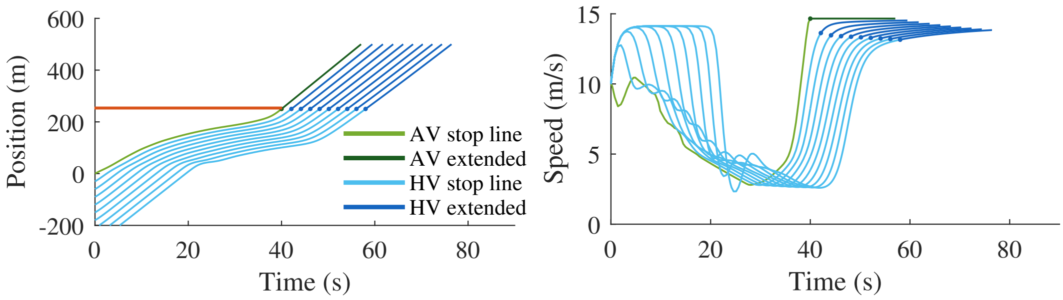
The terminal speed of vehicles in scenario T3 is much higher than that in scenario T1 and T2 which is the main reason why it consumes more fuel than scenario T2. This also indicates that the vehicles in scenario T3 will consume less fuel in future. To better understand the impact of different terminal boundary conditions, the vehicles continue to run for another 250 m after the stop line and achieve similar terminal speed. The vehicles in scenario T1 accelerate to maximum speed quickly, but only the first vehicle in scenario T2 and T3 can achieve the maximum speed, the following vehicles have slightly slower speeds. The scenario T3 consumes the least fuel and has the least total travel time as shown in Fig. 3.5b, which mainly benefit from the high terminal speed at the stop line. Thus, the proposed terminal cost function is a good choice for eco-driving



(a) Vehicle trajectories and speed profiles in scenario T1



(b) Vehicle trajectories and speed profiles in scenario T2



(c) Vehicle trajectories and speed profiles in scenario T3

Figure 3.6: State trajectories of all vehicles with different boundary conditions under three scenarios

in terms of the local benefit and future benefit.

3.4.2 Properties of the running cost

A major feature in the proposed model is that the leading AVs consider the benefits of both themselves and the following vehicles, but the impact of this co-operative behaviour is still not clear. Three typical cases in mixed traffic flow are presented in the following simulation studies. Only 4 vehicles will be considered in the simulations, and the platoon settings in each case are shown in Fig. 3.7. To facilitate the following discussion, two major time points are defined. Let t_1

denotes the time when the first vehicle arrives at the stop line, and t_2 denotes the time when the 4th vehicle arrives at the stop line. In this section, t_1 is the start time of green light and also the time when the first AV passes the stop line, which is 40 s. Two measurements are considered here: (i) The accumulated fuel consumption during 0 s and t_1; (ii) The accumulated fuel consumption during 0 s and t_2 on the studied link. Let M_1 and M_2 denote these two measurements, respectively.

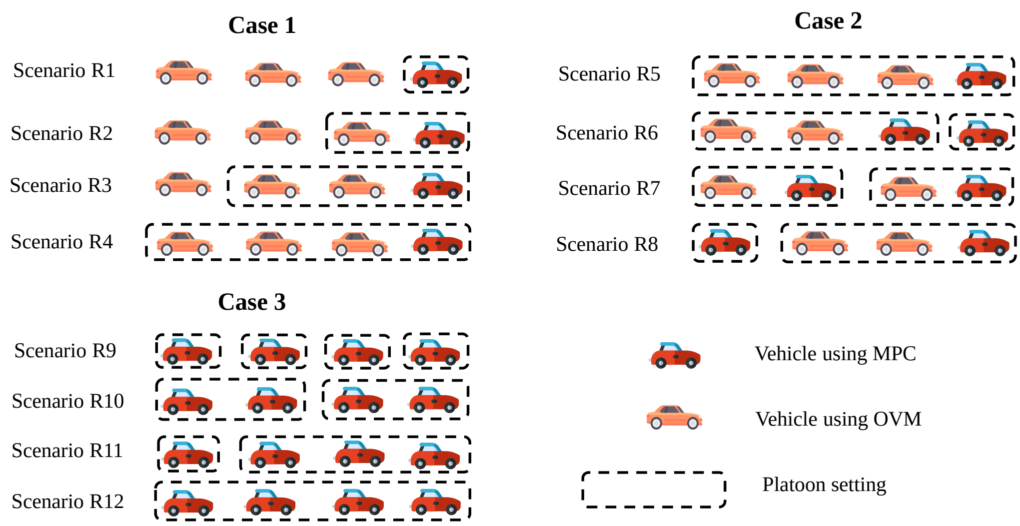


Figure 3.7: Platoon settings for running cost simulations

3.4.2.1 Case 1: an AV is followed by HVs

When an AV is followed by several HVs, the research question is whether the AV should consider the movements of the following vehicles, and what benefits this cooperation can bring. To this end, four scenarios are presented in this case, where the first vehicle is an AV, and the following three vehicles are HVs.

- Scenario R1: The running cost of the host vehicle is its fuel consumption.
- Scenario R2: The running cost of the host vehicle is the total fuel consumption of itself and first following vehicle.
- Scenario R3: The running cost of the host vehicle is the total fuel consumption of itself and the first two following vehicles.

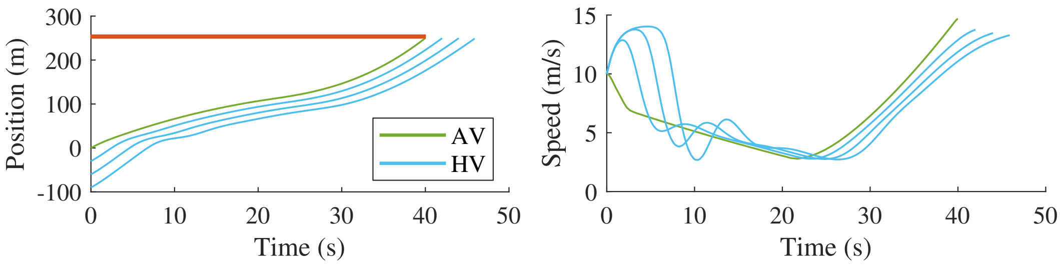
- Scenario R4: The running cost of the host vehicle is the total fuel consumption of itself and all the three following vehicles.

Table 3.4: Fuel consumption (mL) of different scenarios in case 1

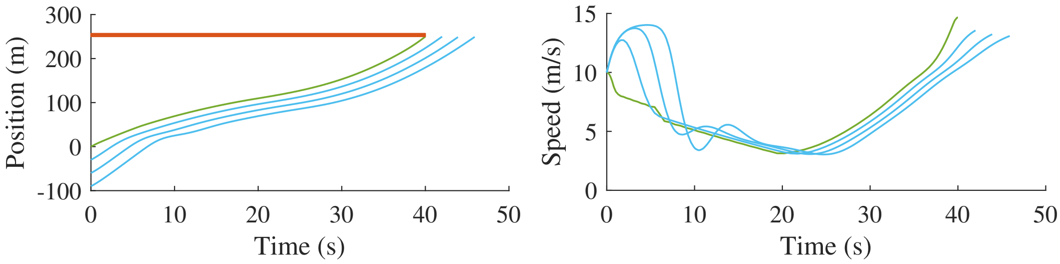
Scenario	1st vehicle	2nd vehicle	3rd vehicle	4th vehicle	Total
scenario R1	**48.9 / 48.9**	53.1 / 57.4	54.8 / 62.6	55.9 / 66.9	212.6 / 235.7
scenario R2	**49.8 / 49.8**	49.9 / 55.3	51.4 / 60.3	52.7 / 64.3	203.8 / 229.7
scenario R3	**51.9 / 51.9**	48.9 / 55.9	48.3 / 59.8	49.6 / 63.6	198.7 / 231.1
scenario R4	**55.2 / 55.2**	48.8 / 56.9	46.1 / 60.2	46.5 / 63.6	196.6 / 235.9

Fuel consumption for each scenario is shown in Table 3.4, and the state trajectories are shown in Fig. 3.8. In Table 3.4, the data are organised in the form of "M_1/M_2" in each cell. The bold items mean they come from AVs, and the same style will be applied in the ensuing paper. Please note that in this case, the optimisation is only performed during 0 s and 40 s, and in the remaining period vehicles are driven by the OVM. When more HVs are included in the platoon, the total fuel consumption decreases with M_1 results. The reduction in scenario R4, where there are three following vehicles in the platoon, is as high as 7.5 % compared with that in scenario R1. At the same time, the first vehicle consumes more fuel than in the scenarios where there are fewer vehicles in the platoon. This is due to the fact that the AV has to modify its trajectory to change the following vehicles' behaviour. This can also be seen in Fig. 3.8. As the leading vehicle sacrifices some of its energy in order to "control" the following vehicles, some kinds of rewards may need to be introduced to incentivise the energy-efficient behaviour, for example, providing them vouchers for cinema, social events and restaurant visits (Schall and Mohnen, 2017).

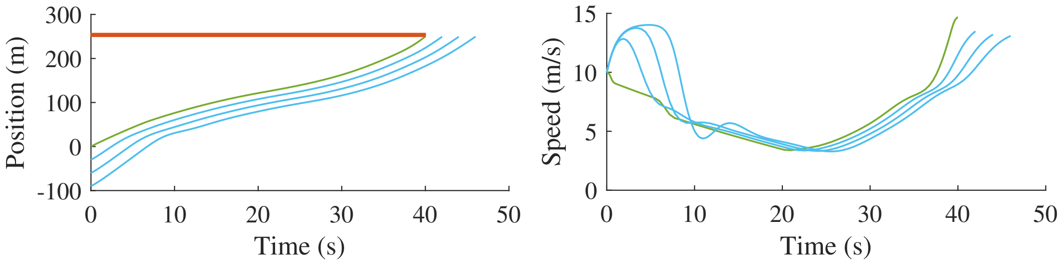
In the movement after 40 s, when the AV cooperates with the following vehicles, the following vehicles consume more fuel after 40 s until all of them have passed the stop line, than the scenario without cooperation. This is mainly because of the higher acceleration calculated by the OVM after 40 s. With more vehicles joining the platoon, the saving of fuel during 0 s and 40 s is not sufficient to



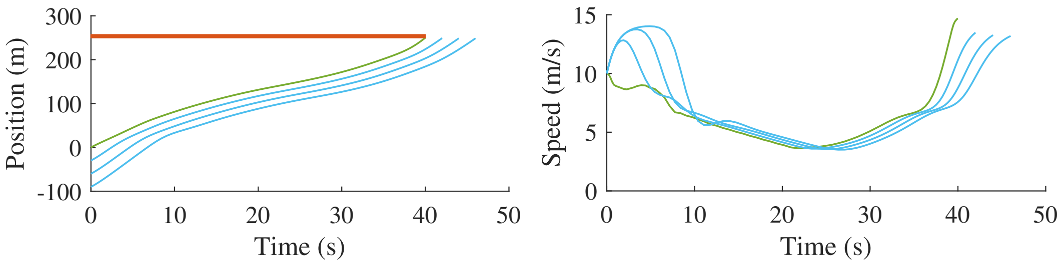
(a) Vehicle trajectories and speed profiles in scenario R1



(b) Vehicle trajectories and speed profiles in scenario R2



(c) Vehicle trajectories and speed profiles in scenario R3



(d) Vehicle trajectories and speed profiles in scenario R4

Figure 3.8: State trajectories of all vehicles under four scenarios in case 1

offset the increase of fuel consumption after 40 s. Actually, in a multi-intersection environment, the movement after 40 s will be optimised in the next intersection. This can be seen by simply assuming that the stop line of the upstream intersection is located at 0 m and the green light starts at 0 s. The presented results apply only to one case with the specified simulation setting. More general simulations with various travel times are needed. Furthermore, when more vehicles are considered in the platoon, the speed oscillations of the following vehicles are suppressed significantly. This will contribute to better driving comfort for the following vehicles. Even though some following vehicles are not considered in the platoon, their behaviour is also influenced by the preceding vehicle, and their fuel consumption is reduced significantly. For example, the fuel consumption of the 4th vehicle in scenario R3 is 11.3% less than that in scenario R1 with M_1. This was also found by Treiber and Kesting (2014) and Wan et al. (2016).

3.4.2.2 Case 2: an AV is followed by mixed AVs and HVs

When an AV is followed by mixed AVs and HVs, the research question is whether the subsequent AVs need to activate the eco-driving function or just follow the preceding vehicle. In the simulations, the first vehicle is an AV in all scenarios. One of the three following vehicles is another AV in scenario R6, R7, and R8. All other vehicles are HVs, and their movements are according to the OVM. The last three vehicles in scenario R5 can either be automated or not, as their eco-driving functions are not activated and hence they behave the same as HVs.

- Scenario R5: There is only one platoon. The running cost is the total fuel consumption of four vehicles.

- Scenario R6: There are two platoons: the first vehicle and the last three vehicles. The running cost for the first platoon is the fuel consumption of the first vehicle, while the running cost for the second platoon is the total fuel consumption of the last three vehicles.

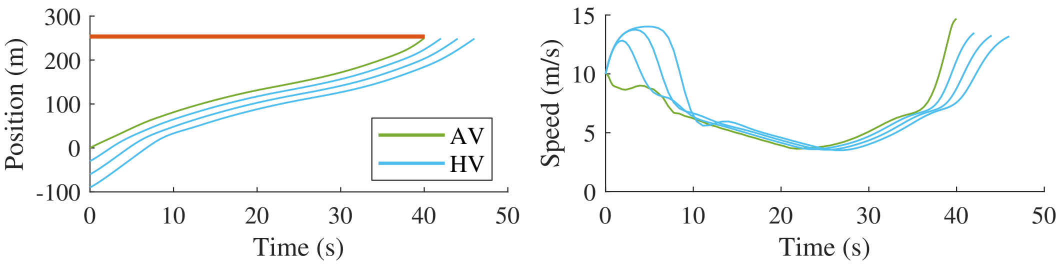
- Scenario R7: There are two platoons: the first two vehicles and the last two vehicles. The running cost for the first platoon is the total fuel consumption of the first two vehicles. The running cost for the second platoon is the toal fuel consumption of the last two vehicles.

- Scenario R8: There are two platoons: the first three vehicles and the last vehicle. The running cost for the first platoon is the total fuel consumption of the first three vehicles. The running cost for the second platoon is the fuel consumption of the last vehicle.

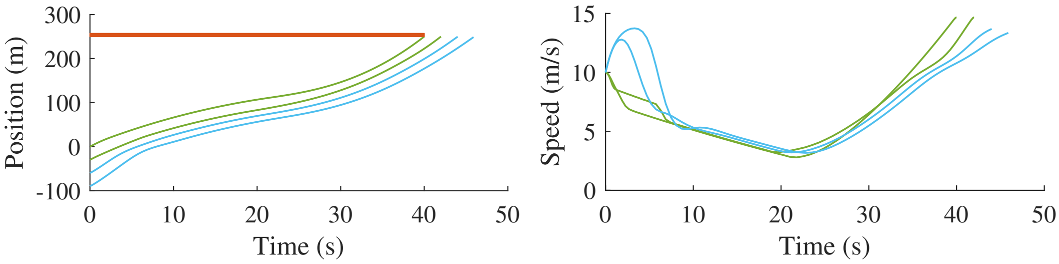
Table 3.5: Fuel consumption (mL) of different scenarios in case 2

Scenario	1st vehicle	2nd vehicle	3rd vehicle	4th vehicle	Total
scenario R5	**55.2 / 55.2**	48.8 / 56.9	46.1 / 60.2	46.5 / 63.6	196.6 / 235.9
scenario R6	**48.9 / 48.9**	**42.2 / 50.5**	47.9 / 57.6	50.3 / 62.2	189.3 / 219.1
scenario R7	**49.8 / 49.8**	49.9 / 55.3	**37.1 / 51.9**	44.9 / 58.7	181.7 / 215.7
scenario R8	**51.9 / 51.9**	48.9 / 55.9	48.3 / 59.8	**36.0 / 52.5**	185.1 / 220.0

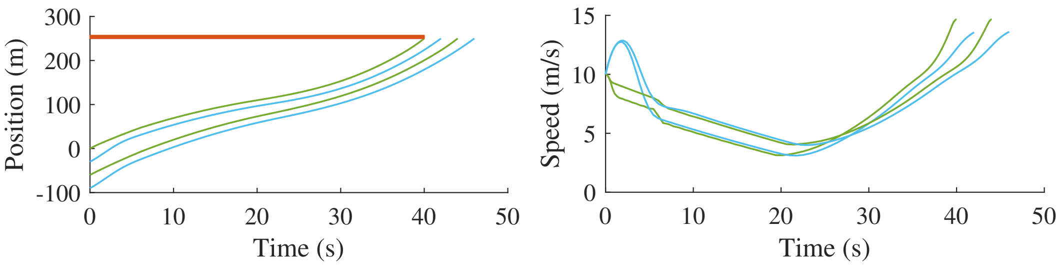
Fuel consumption for every scenario in the simulations is shown in Table 3.5, and the state trajectories are shown in Fig. 3.9. Comparing scenarios R6, R7, R8 to scenario R5, the activation of the eco-driving function in the following vehicles helps to reduce the total fuel consumption with both M_1 and M_2. This is mainly due to the reduction of their own fuel consumption, which ranges from 13.5% to 22.6% with M_1 and from 11.2% to 17.5% with M_2. It also helps reduce the fuel consumption of the first AV due to fewer vehicles in its platoon as discussed previously. Eventually, with another AV, the reduction of fuel consumption ranges from 3.7% to 7.6% with M_1 and from 6.7% to 8.6% with M_2. This is different from the result of Stebbins et al. (2017) where giving speed advice to the following vehicles rarely makes a difference. This difference is mainly because in their approach only the leading vehicle can achieve the target position and speed. However, in the proposed method, the following AVs can also achieve the desired state, which reduces the fuel consumption and travel time of the whole traffic. In Fig. 3.9, the trajectories of the following AVs by the MPC show an obvious fallback behaviour and keep a larger gap than that in the OVM. In the OVM, the vehicle attempts to accelerate as soon as possible to achieve the optimal speed.



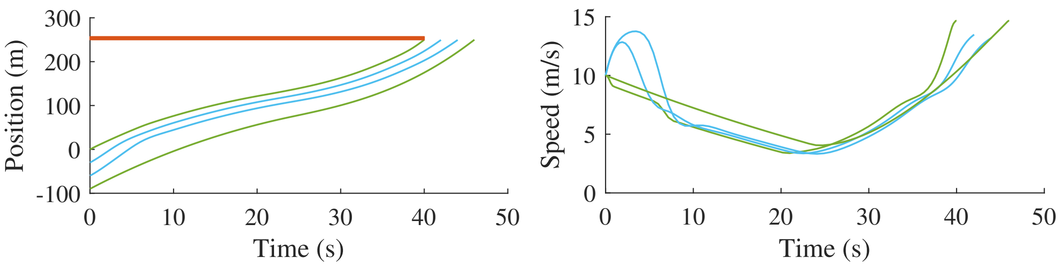
(a) Vehicle trajectories and speed profiles in scenario R5



(b) Vehicle trajectories and speed profiles in scenario R6



(c) Vehicle trajectories and speed profiles in scenario R7



(d) Vehicle trajectories and speed profiles in scenario R8

Figure 3.9: State trajectories of all vehicles under four scenarios in case 2

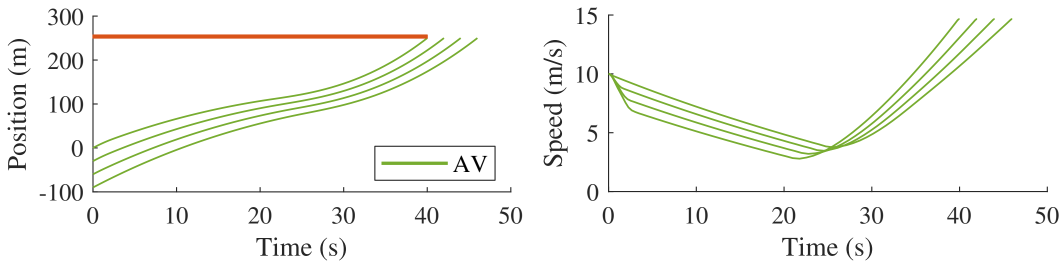
In contrast, in the MPC method, the vehicle acts more rationally by considering the information of signal timing and state of the preceding vehicles and following vehicles. So, it reduces fuel consumption even further to provide speed advice to the following AVs in the mixed AVs and HVs environment.

3.4.2.3 Case 3: an AV is followed by other AVs

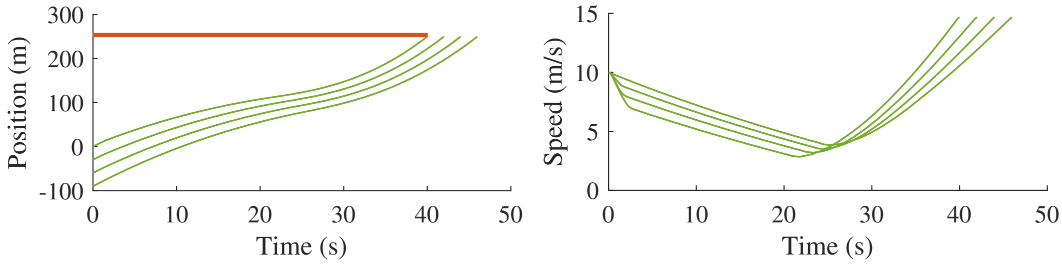
When an AV is followed by other AVs, the research question is whether the leading AV needs to consider the movements of the following AVs. If it does, what benefits arise from this cooperation? In the simulations, all the vehicles are AVs and arrive at the stop line with the maximum speed and zero acceleration with a fixed time headway 2 s.

- Scenario R9: Each AV optimises its trajectory separately and minimises its own fuel consumption.
- Scenario R10: The four AVs are split into two platoons. The running cost for the first platoon is the total fuel consumption of the first two vehicles, and the running cost for the second platoon is the total fuel consumption of the last two vehicles.
- Scenario R11: The four AVs are split into two platoons. The running cost for the first platoon is the total fuel consumption of the first three vehicles, and the running cost for the second platoon is the fuel consumption of the last vehicle.
- Scenario R12: The four AVs form one platoon. They minimise the total of their fuel consumption.

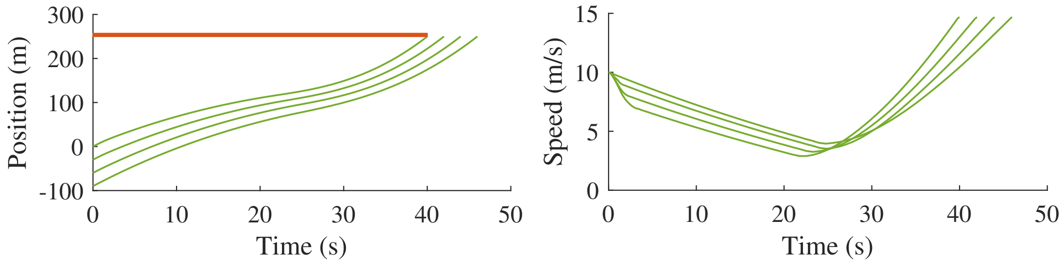
The fuel consumption of every scenario in the simulation is shown in Table 3.6 and the state trajectories are shown in Fig. 3.10. In the four scenarios, the results change very slightly and fall within 2 % for every vehicle and 1 % for the total in most cases [§]. Nevertheless, the resulting trajectories may differ. This outcome implies that cooperation among AVs does not make any obvious difference in fuel consumption and travel time. This conclusion is only valid for the current



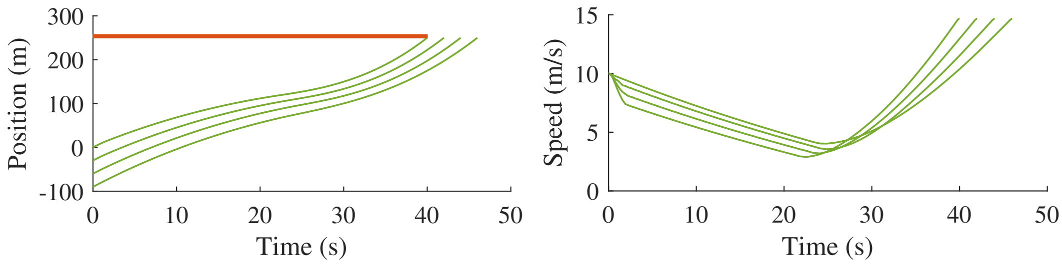
(a) Vehicle trajectories and speed profiles in scenario R9



(b) Vehicle trajectories and speed profiles in scenario R10



(c) Vehicle trajectories and speed profiles in scenario R11



(d) Vehicle trajectories and speed profiles in scenario R12

Figure 3.10: State trajectories of all vehicles under four scenarios in case 3

Table 3.6: Fuel consumption (mL) of different scenarios in case 3

Scenario	1st vehicle	2nd vehicle	3rd vehicle	4th vehicle	Total
scenario R9	**48.9 / 48.9**	**43.3 / 50.0**	**39.4 / 51.3**	**36.7 / 52.6**	168.2 / 202.7
scenario R10	**48.9 / 48.9**	**43.7 / 50.0**	**39.2 / 51.2**	**36.5 / 52.5**	168.2 / 202.6
scenario R11	**48.9 / 48.9**	**43.5 / 50.0**	**39.5 / 51.2**	**36.2 / 52.5**	168.1 / 202.6
scenario R12	**48.9 / 48.9**	**43.4 / 50.0**	**39.4 / 51.2**	**36.6 / 52.5**	168.2 / 202.6

simulation setting and more simulation scenarios with different travel time and speed are needed, which will be shown in the following section.

The main reason for fuel consumption differences of AVs in the cooperative platoon scenarios is the safety constraint. If there is no safety constraint in the proposed model, the optimisation of multiple AVs is fully decoupled and reduced to the optimisation of each AV separately. Thus, the fuel consumption of AVs in any of the cooperative platoon is the same as that in the individual optimisation of AV when safety constraint does not take effect. In the simulations of case 3, as the initial headway is relatively large, the safety constraint does not make many differences between the optimal trajectory of each AV in the individual optimisation. That is the main reason why the results in Table 3.6 are so close to each other. The cooperation among AVs is more likely to make a difference in fuel consumption in dense traffic or small headway in another way and long travel time because of the red light. This is because the behaviour of preceding AVs is more likely to disturb or even block the following AVs taking the optimal trajectory in those cases. Nevertheless, the improvement is much less than that in the mixed traffic condition.

3.4.3 Simulations with different penetration rates of AVs

In this part, a simulation investigation is presented to show the performance of the proposed method in different penetration rates of AVs. Please note that the cooperation between vehicles in a platoon relies on the sharing of real-time

[§]Please note that there is no the same value in Table 3.6. Some look the same is just because of the rounding error.

information about vehicles. This can be achieved in the CV environment. If the vehicles are not fully connected, it is assumed that the AV can still detect the state of the first direct following vehicle via its built-in detectors. So the platoon size is limited to 2 in that condition. Another scenario without cooperation is also included for comparison.

- Scenario P1: All AVs optimise the fuel consumption of themselves only. It can also be seen as setting the maximum platoon size to 1;
- Scenario P2: All AVs optimise the total fuel consumption of themselves and the first directly following vehicle. This can also be seen as setting the maximum platoon size to 2;
- Scenario P5: All AVs optimise the total fuel consumption of themselves and all the following HVs within the limit of maximum platoon size. The maximum platoon size is set to 5.

When determining the maximum platoon size, we should trade off the calculation burden and communication reliability in practice. Large platoon size also implies that the AVs need to sacrifice more and have a higher probability of stopping in order to “control” the vehicles far away. The stopping behaviour will also be discussed later.

In all simulations, the cycle time is 60 s with green time 30 s and red time 30 s. The simulation of every scenario in every penetration rate lasts for 600 s and is repeated twice. Traffic demand is 850 veh/h. The type of vehicle is determined by comparing the penetration rate p and a newly generated random number between 0 and 1 when it enters the road. The time headway follows a truncated exponential distribution to ensure that no time-headway is less than 2 s. The initial speed follows a normal distribution $N(10, 1)$ bounded by the speed limits and ensures that no collision happens at the entrance of the road (Ubiergo and Jin, 2016).

The average fuel consumption and average travel time produced by the simulations are shown in Table 3.7 and Fig. 3.11. Overall, both fuel consumption and travel time decrease as the penetration rate of AVs increases under all scenarios.

In any penetration rate studied, the scenario with cooperation outperforms or at least equals the scenario without cooperation. In general, as more vehicles join cooperation, more benefits are gained in terms of fuel consumption and travel time. The benefits of cooperation are most evident for lower penetration rates, and a platoon size of 5 (P5) can reduce fuel consumption by 22% with only a 60% penetration rate, which is better than the scenario of P1 with 100% penetration. However, as more AVs are brought into the system, the additional benefit from cooperation then decreases, as there is not much room for further improvement. This is in line with the previous results from case 3 in Section 3.4.2.3, where the benefit of cooperation in all four AVs was trivial.

Table 3.7: Simulation results and differences in various penetration rates of AVs

Penetration rate	Scenario	Average fuel consumption		Average travel time	
		Mean (mL)	Diff	Mean (s)	Diff
0.2	P1	55.3	–	37.4	–
	P2	52.7	−4.7%	37.5	0.2%
	P5	48.4	−12.6%	37.3	−0.3%
0.4	P1	50.0	−9.6%	34.1	−8.7%
	P2	47.9	−13.5%	32.6	−12.8%
	P5	46.6	−15.9%	31.7	−15.2%
0.6	P1	47.6	−13.9%	34.1	−8.8%
	P2	45.6	−17.6%	33.0	−11.8%
	P5	43.1	−22.1%	31.6	−15.5%
0.8	P1	46.1	−16.8%	33.1	−11.6%
	P2	44.7	−19.2%	31.1	−16.8%
	P5	43.2	−21.9%	31.2	−16.5%
1	P1	43.5	−21.3%	31.1	−16.8%
	P2	43.6	−21.1%	31.2	−16.5%
	P5	42.5	−23.2%	31.2	−16.5%

The travel time benefits are less significant than fuel consumption benefits and are not present with 20% and 100% penetration of AVs. The resulte of travel time show a similar pattern to fuel consumption during 40% and 80% penetration rate of AVs, and the effects of cooperation are similar. The reduction of travel time is mainly caused by the reduction of start-up lost time and queue discharge time as more vehicles pass at the green light and fewer vehicles stop at the red light thanks to the cooperation. However, when the penetration rate is 20%, the AVs are frequently interrupted by the preceding HVs. When the penetration becomes 100%, there is no more room to reduce travel time. Fig. 3.11 shows that with the increasing penetration rate of AVs, the number of outliers in fuel consumption is greatly reduced. Scenarios P2 and P5 also have much fewer outliers than scenario P1. This demonstrates that cooperation can stabilise traffic flow, which is also shown in Fig. 3.12. No outliers are detected in the travel time. When the initial state is fixed, travel time can only imply the terminal state, but all the intermediate states affect fuel consumption. That is why fuel consumption shows more information about the vehicle movements.

The vehicle trajectories with 20%, 60% and 100% penetration rate of AVs in three scenarios are shown in Fig. 3.12. We can see that only optimising AVs themselves is not enough to achieve a system optimum. Sometimes, the selfish behaviour of an AV may even worsen the traffic, which is more obvious in the low penetration rate of AVs. When the leading AV attempts to slow down to save fuel, some following HVs have to stop on the road, which causes a shock-wave along the link. Even when the penetration rate of the AVs becomes 100%, sometimes this selfish deceleration still occurs. In the cooperation scenarios (i.e., P2 and P5), the vehicle trajectories are largely smoothed. The negative impact of eco-driving by the AVs on the following vehicles is also reduced. This is mainly because the fuel consumption of the following vehicles is directly included in the objective function of the AVs. The leading vehicles in each platoon also help the following vehicles to reach a high speed crossing the stop line and avoid idling at the red light.

We also notice that there is a low probability that the AV may stop in the

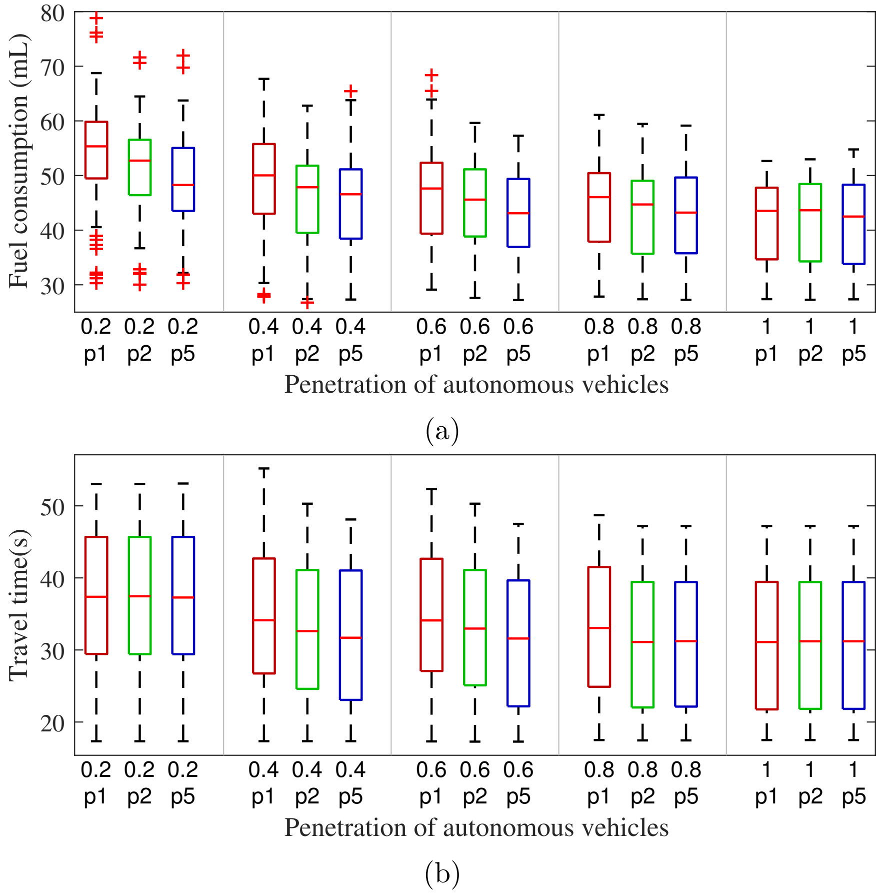
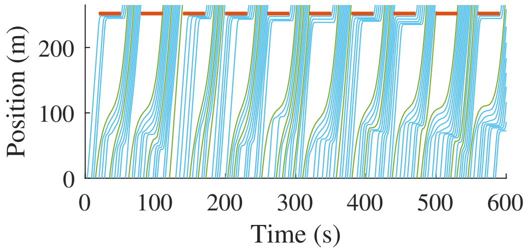
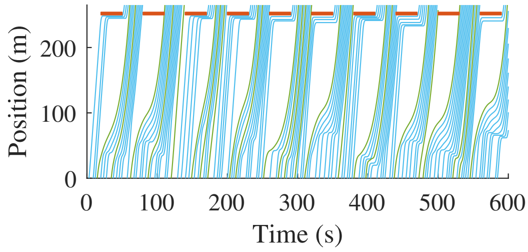


Figure 3.11: Simulation results in different penetration rates of AVs. (a) fuel consumption, (b) travel time

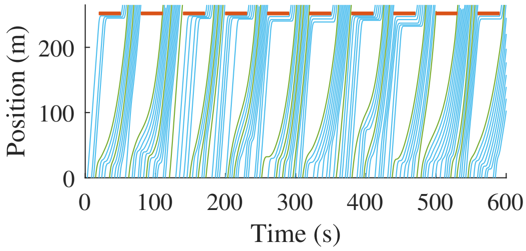
middle of the road segment even in the cooperative scenarios. There are two main reasons, (1) the planned travel time is too long and (2) the following vehicles in the same platoon are widely dispersed. However, please note that the stopping behaviour of the AV in cooperation scenarios does not harm the system. It does not increase travel time or fuel consumption for the platoon. The AV never stops close to the stop line and does not block the following vehicles from passing the stop line. If stopping behaviour is not acceptable, one may add a minimum speed constraint on the AVs (Yang et al., 2017), but this may lead to infeasible result when the planned travel time is larger than the maximum travel time by applying the minimum speed limit. If that happens, the speed advisory system fall into failure, and the AV has to stop around the stop line. It results in high fuel consumption and travel time for all the following vehicles. Thus, no minimum speed limit is added in the model, and the seldom stop behaviour is allowed.



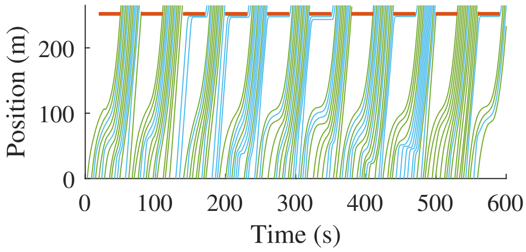
(a) p1 with 20% AVs



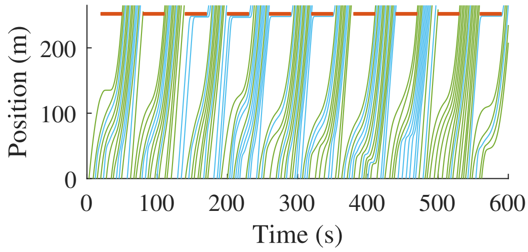
(b) p2 with 20% AVs



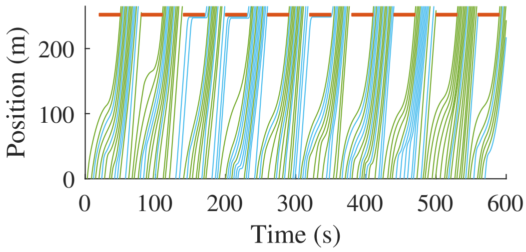
(c) p5 with 20% AVs



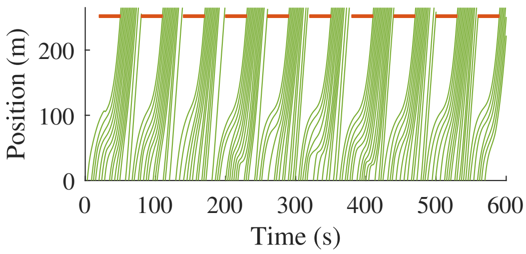
(d) p1 with 60% AVs



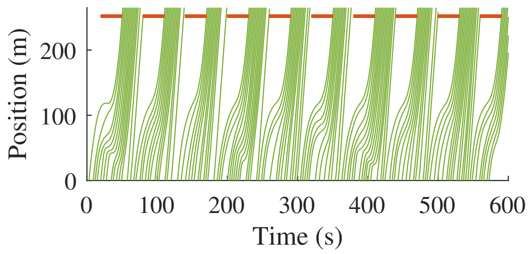
(e) p2 with 60% AVs



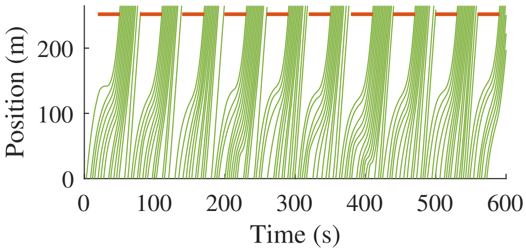
(f) p5 with 60% AVs



(g) p1 with 100% AVs



(h) p2 with 100% AVs



(i) p5 with 100% AVs

Figure 3.12: Some examples of trajectories in different penetration rates of AVs

3.5 Conclusions

Providing signal information to the vehicles on signalised urban roads is demonstrated to be an effective way to reduce idle time and fuel consumption. However, many eco-driving strategies have a negative impact on the efficiency of the intersection, and even cause a shock-wave in the middle of the road section. In this paper, a distributed and cooperative eco-driving method has been proposed for platoons to address these issues. The proposed eco-driving method has been designed for mixed traffic flow on an urban road, which consists of HVs and various penetration rates of AVs. AVs attempt to pass the intersection on the earliest possible green time with a maximum desired speed and zero acceleration. All these settings are to maximise traffic efficiency. The jerk has been set as the control variable in order to increase driving comfort. In the proposed control method, the fuel consumption of AVs and some following HVs is minimised over the horizon to achieve an eco-driving benefit to more vehicles. This cooperation largely smooths out the trajectory and suppresses any shock-wave. Then a platoon formation method has been proposed to apply the eco-driving strategy to achieve better performance for the overall traffic. Three typical cases in mixed traffic have been studied with different platoon settings. Moreover, different penetration rates of AVs have been studied in the simulation to show that the proposed method can adapt to various mixed traffic conditions.

From the analysis above, we can draw the following conclusions:

1. AVs can reduce their own fuel consumption and travel times when approaching a signalised intersection if the signal timing information is given.

2. When the penetration level is from low to moderate, the cooperation between AVs and HVs is seen to be beneficial in both fuel consumption and travel time.

3. However, this system level of cooperation requires a sacrifice from the leading AV which may be controversial to accept.

4. The level of sacrifice increases with the platoon size. As more vehicles are added to the platoon of one AV, then the leading vehicle has to overcompensate to affect the third and subsequent vehicle trajectories.

5. Even when the HVs are not included in the platoon, they still benefit from the preceding AVs.

6. The cooperative behaviour may reduce the possible adverse impacts of AVs on the following HVs, such as speed fluctuation or even shockwave.

7. It reduces fuel consumption even further to provide speed advice to the following AVs in mixed AVs and HVs compared with only controlling the leading vehicle at a green phase.

8. Larger platoon size helps to achieve a stronger reduction in fuel consumption and stabilise traffic flow.

9. The benefits of cooperation mean that the system can reach the same levels of benefit with 60% penetration rate as for 100% penetration without cooperation, which has implications for the transition towards a full penetration.

10. As the penetration rate reaches 100%, the performance improvement resulting from cooperation is trivial and the sacrifice problem disappears.

These last two points taken together suggest that implementation the driving cooperation should vary over the implementation phase and that some higher levels of cooperation whilst desirable should be regulated or compensated with a promise to remove this obligation as the penetration rates increase.

This paper assumed that the future signal timing information is available to the AVs. This is possible for fixed-time traffic control and adaptive traffic control strategies that update signals every cycle (e.g. TUC in Diakaki et al., 2002, 2003), but may not be true for other adaptive traffic control systems, like SCOOT, where there is only very limited time for the AVs to response and may reduce the performance of the proposed method. There are two solutions: (1) Use the previous signal timing as the estimation when it is not available. When the signal timing is available at some time steps ahead, the method may use the latest instead. As the change of signal duration between two cycles is unlikely to be too strong, e.g. it is limited to +/- 4 seconds in SCOOT, the performance impact may be suppressed, but would not vanish. (2) Develop a new algorithm in the SCOOT and SCATS to consider the AVs. In the current adaptive control schemes, the information is still mainly obtained from detectors like loop detectors. The information from AVs or CVs is not considered. So, it is an interesting topic to develop new intersection control algorithms to take advantage of new information from AVs and CVs. This is also our ongoing research work.

In the current work, the signal timing is assumed to be given. In the next step, it will achieve better performance gain to optimise the signal timing and trajectory simultaneously either for the local intersection or traffic network.

Acknowledgement

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Appendix 3.A The process of choosing the parameters for GPOPS

Three scenarios are considered in the process of choosing the parameters for GPOPS.

- Scenario G1: One AV attempts to arrive at the stop line at 40 s.
- Scenario G2: One AV attempts to arrive at the stop line at 40 s, and it is followed by one HV. The trajectories are optimised between 0 s and 40 s.
- Scenario G3: Two AVs attempt to arrive at the stop line at 40 s and 42 s respectively. The trajectories are optimised between 0 s and 42 s until both arrive at the stop line.

The initial position and speed of the first vehicle are 0 m and 10 m/s, respectively. If there is another vehicle, its initial position and speed are -30 m and 10 m/s, respectively.

3.A.1 setup.autoscale

The optimisation results of different "setup.autoscale" settings in three scenarios are shown in Table 3.8. When "setup.autoscale" is set to "off", the running time increases dramatically while the optimal value keeps almost the same. Thus, "setup.autoscale" is set to "on".

Table 3.8: The impact of "setup.autoscale" in different scenarios

Scenario	Optimal Value		Running time (s)	
	"on"	"off"	"on"	"off"
scenario G1	48.9	48.9	7.0	67.1
scenario G2	99.7	99.6	29.2	769.0
scenario G3	99.0	99.0	31.0	858.8

3.A.2 setup.derivatives

The optimisation results of different "setup.derivatives" settings in in three scenarios are shown in Table 3.9. When "setup.derivatives" is set to "finite-difference", the running time increases a lot in scenario G2 and G3, and the optimal value in scenario G2 is also much worse. Thus, "setup.derivatives" is set to "complex".

Table 3.9: The impact of “setup.derivatives” in different scenarios

Scenario	Optimal Value		Running time (s)	
	“complex”	“finite-difference”	“complex”	“finite-difference”
scenario G1	48.9	48.9	7.0	5.7
scenario G2	99.7	115.5	29.2	217.5
scenario G3	99.0	99.0	31.0	115.4

3.A.3 setup.tolerances

“setup.tolerances” is a two-element array specifying the NLP solver Optimality and Feasibility Tolerances respectively. The following two settings are tested.

“setup.tolerances” setting 1: $[1\text{e}{-3}, 2\text{e}{-3}]$

“setup.tolerances” setting 2: $[1\text{e}{-4}, 2\text{e}{-4}]$

Table 3.10: The impact of “setup.tolerances” in different scenarios

Scenario	Optimal Value		Running time (s)	
	“setting 1”	“setting 2”	“setting 1”	“setting 2”
scenario G1	48.9	48.9	7.0	6.7
scenario G2	99.7	99.7	29.2	29.4
scenario G3	99.0	99.0	31.0	30.3

The optimisation results of different “setup.tolerances” settings in in three scenarios are shown in Table 3.10. In all three scenarios, the results and running time using either setting 1 or setting 2 are the same. That is because the feasibility tolerance is always easy to satisfy. However, It is also observed that the optimality tolerance is not satisfied, which means the results are suboptimal. Given the nonlinear and nondifferentiable cost function, these results are good enough and acceptable.

A possible way to deal with this is to apply a smooth function (Typaldos et al., 2018).

$$F = \begin{cases} \dfrac{1}{\alpha_s}\left[\log\left(e^{\alpha_s \alpha} + e^{\alpha_s\left(\alpha+\beta_1 P_T+\beta_2 m a^2 v\right)}\right)\right] & \text{if } a > 0 \\ \dfrac{1}{\alpha_s}\left[\log\left(e^{\alpha_s \alpha} + e^{\alpha_s(\alpha+\beta_1 P_T)}\right)\right] & \text{if } a \leq 0 \end{cases} \tag{3.14}$$

where $\alpha_s > 0$ is a constant.

3.A.4 limits.meshPoints

The "limits.meshPoints" is usually set to [-1, 1] according to the manual of GPOPS.

3.A.5 limits.nodesPerInterval

The optimisation results of different "setup.tolerances" settings in in three scenarios are shown in Table 3.11. Increasing the "nodesPerInterval" at the initial iteration does not improve the optimisation result in scenario G2 and G3, and needs much more running time in all the tested scenarios. Thus, "nodesPerInterval" is set to "$2 * (t^f - t^0)$".

Table 3.11: The impact of "limits.nodesPerInterval" in different scenarios

Scenario	Optimal Value		Running time (s)	
	"$2 * (t^f - t^0)$"	"$4 * (t^f - t^0)$"	"$2 * (t^f - t^0)$"	"$4 * (t^f - t^0)$"
scenario G1	48.9	48.9	7.0	48.60
scenario G2	99.7	100.0	29.2	333.57
scenario G3	99.0	99.5	31.0	322.26

3.A.6 mesh.tolerance

The optimisation results of different "mesh.tolerance" settings in in three scenarios are shown in Table 3.12. Setting "mesh.tolerance" to either "1e−4" or "1e−5" gets almost the same optimisation results and very similar running time in the tested scenarios. Both “1e−4” and “1e−5” are acceptable.

Table 3.12: The impact of “setup.mesh.tolerance” in different scenarios

Scenario	Optimal Value		Running time (s)	
	“1e−4”	“1e−5”	“1e−4”	“1e−5”
scenario G1	48.9	48.9	7.0	6.7
scenario G2	99.7	99.7	29.2	31.2
scenario G3	99.0	99.0	31.0	30.9

3.A.7 mesh.iteration

In all scenarios, the optimisation stopped without reaching the maximum number of iterations which is 8. Increasing the maximum number of iterations does not affect the results in these scenarios.

Appendix 3.B Analyse the boundary conditions and the penalty function

In this chapter, the terminal conditions are implemented as boundary conditions which should have minimal violations in the optimal solution. The safety constraint is implemented as a penalty term in the objective function. The safety penalty term does not take effect when the following vehicles are HVs as the safety constraints are always satisfied in this case. It only prevents the following AVs from driving too close.

We use a simple testing scenario with two AVs which is the same as scenario G3. The optimal value is 99.01 while the term of total fuel consumption is 98.87, and the penalty term is 0.14. All boundary conditions are satisfied with zero violations. The reason for the positive penalty term is the fixed minimum time headway and the terminal speed constraint. If the minimum time headway increases a little, the safety penalty term could be zero.

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4

A stochastic model predictive control approach to eco-driving for automated vehicles under uncertain signal information

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Contents

Abstract

Most eco-driving methods require to know the future signal and phase information to reduce fuel consumption or idle time. For intersections in the adaptive traffic control systems, it becomes challenging or even not possible to satisfy this requirement, because the signal timing is adaptively optimised based on real-time dynamic demands from all approaches. The distribution of phase duration is estimated from the historical data first, and a multi-phase optimal control model is proposed to optimise the trajectory aiming to reduce fuel consumption. When the future signal information is not available, the driving behaviour has two regimes: accelerates with maximum acceleration to pass the intersection when the signal turns green at that time step or keeps waiting for the green light in the next time step. When the future signal information is available in advance for a limited time, the vehicle may take actions in advance. An additional driving strategy is considered in the passing regime that it tries to arrive at the stop line just at the moment when the signal turns green using optimised acceleration instead of the maximum acceleration. The impacts of parameters in the model such as terminal condition and time step, parameters in the signal duration distribution such as the mean, variance and range and parameters in the advance time are tested in the simulations. It reduces fuel consumption and travel time considerably to obtain the future signal information with enough time in advance.

Keywords: Eco-driving, vehicle-to-infrastructure, trajectory planning, optimal control

4.1 Introduction

Vehicle emissions are harmful to both the environment and human health, particularly in urban areas, which suffer from heavy traffic and severe congestion. A potential solution to reducing fuel consumption and road emission is optimising the vehicle trajectory, which is referred to as eco-driving, driving advisory system or trajectory planning. Any eco-driving method relies on information about the preceding traffic conditions. In the urban environment, the most widely applied eco-driving method is the Green Light Optimal Speed Advisory (GLOSA) system which receives the signal phase and timing information via Vehicle-to-Infrastructure (V2I) communication and provides speed advice to the approaching vehicles to help them pass the intersection at a green light. It is the first application of cooperative intelligent transport system (C-ITS) based on V2I communication (Trayford et al., 1984; Stahlmann et al., 2018).

4.1.1 Literature review

Various eco-driving methods have been proposed in the literature. Different methods may have different primary objectives, such as reducing fuel consumption (He et al., 2015; Jiang et al., 2017; Yang et al., 2017; Zhao et al., 2018), emissions (Van Katwijk and Gabriel, 2015), stopping time (Asadi and Vahidi, 2011; Ubiergo and Jin, 2016), delays (Stebbins et al., 2017), and increasing the intersection capacity (Liu and Kamel, 2016); however, their benefits are not limited to the primary objectives. Most of the existing eco-driving methods control the acceleration directly, while some recent research proposed to use the jerk (Jiang et al., 2017; Zhao et al., 2018) as the control variable, which helps increase the comfort of drivers. Some methods pass the advisory speed limit to drivers dynamically and are thus more suitable for the lower level of the automated vehicle control (Ala et al., 2016; Ubiergo and Jin, 2016; Yang et al., 2017).

The control of individual vehicles has always been important for eco-driving. Asadi and Vahidi (2011) controlled the vehicle to follow a target speed to avoid stopping at the red signal. The constant target speed is calculated at every time step so that the vehicle can reach the intersection at the green light. Instead of a target speed, Altan et al. (2017) generated a smooth reference speed trajectory for the vehicle. Treiber and Kesting (2014) developed three strategies in an Improved Intelligent Driver Model (IIDM), by changing the model parameters. Van Katwijk and Gabriel (2015) optimised the trajectory based on two trajectory patterns using a constant acceleration. Ubiergo and Jin (2016) calculated the dynamic advisory speed limit for the vehicle. Yao et al. (2018) proposed to use two different speed limits on the road to smooth the trajectory and optimised the location of the speed limits. He et al. (2015) and Yang et al. (2017) considered a queue length constraint in the trajectory optimisation model. Kamal et al. (2010) developed a model predictive control (MPC) method to generate the fuel-efficient trajectory and ensure the safety, and later Kamal et al. (2013) estimated the trajectory of preceding vehicles approaching a traffic signal using experimental driving data and included it in the MPC algorithm. Jiang et al. (2017) dynamically estimated the terminal time for the AV reaching the stop line considering the preceding vehicle and signal control in the mixed AV and HV environment. Jin et al. (2016) considered the vehicle powertrain and the external road grade in the optimisation.

Besides the control of individual vehicles, some eco-driving methods are also proposed for the operation of platoons in a connected traffic system. Liu and Kamel (2016) applied a platoon split and merge rule to make full use of the available green time. Stebbins et al. (2017) controlled the first vehicle approaching the stop line at the red time and offset the green time for the following vehicles. Zhou et al. (2017a) divided the trajectory into two segments and applied a constant acceleration rate in each segment. Wei et al. (2017) proposed a binary integer programming model for controlling the trajectories of the platoon by discretising the space-time network and transformed it into a dynamic programming problem for efficient online application. They also introduced platoon reaction time to balance the capacity and risk. Both Wei et al. (2017) and Zhou et al. (2017a) used Newell's car following model as a safety bound to the vehicles in a platoon. HomChaudhuri et al. (2017) extended the model proposed in Asadi and Vahidi (2011) by helping the following vehicle to achieve the target speed as well. The fuel consumption model is not directly used in the MPC algorithm, but the optimal cruising speed of the best fuel-efficient pattern is considered in the calculation of the target speed. While all the above-mentioned work of the platoon control either requires all AVs or CVs in the platoon, Zhao et al. (2018) developed a cooperative platoon control method for a mixed platoon with AVs and HVs. They extensively studied the benefit of the cooperation between AVs and HVs, including better fuel efficiency and smoother trajectories.

By now, the reviewed literature is for one individual intersection. There were also some papers dedicated to the optimisation of fuel consumption over multiple intersections. Wu et al. (2015) added the arrival time constraint at every intersection, under the assumption that the green time at each intersection is pre-determined. De Nunzio et al. (2016) used the shortest path algorithm to select the optimal time of passing each intersection. Butakov and Ioannou (2016) added the driver's preferences about fuel consumption and travel time to determine which green cycle should be chosen. Tajalli and Hajbabaie (2018) developed a dynamic speed harmonisation method for the traffic network. Zeng and Wang (2018) used a dynamic programming model to get the globally optimal trajectory. It is not suitable for real-time application due to the calculation time but can be used to compare the performance of other sub-optimal algorithms.

As the signal timing places a hard constraint for the trajectory planning, integrating trajectory planning and signal controls allow further improvement on the overall performance of the system. Yu et al. (2018) developed a mixed integer linear programming (MILP) model for the signal optimisation of a multi-lane intersection. The vehicle arrival time and lane choice are first optimised by the MILP model, and then passed to trajectory planning for each vehicle. Similar work has been done in rail transport as well to jointly optimise the scheduling and trajectories of trains travelling in the same direction (Ye and Liu, 2016) and or in different directions (Wang and Goverde, 2017; Zhou et al., 2017b). Both Ye and Liu (2016) and Wang and Goverde (2017) modelled the problem as a multi-phase optimal control problem (MPOC). The MPOC can be solved highly efficiently by converting it into a nonlinear programming problem (Ye and Liu, 2017).

4.1.2 Research gaps and contributions

All papers mentioned above are based on an explicit or implicit assumption that the vehicle has access to accurate signal and phase information in the near future while they are planning the trajectory. The accurate signal information has a significant impact on the performance of eco-driving methods (Stevanovic et al., 2013). This assumption may be valid for the fixed time traffic control system equipped with V2I technology, where the intersection controller knows exactly the future signal information. However, this assumption may be difficult to satisfy in the actuated and adaptive traffic control system as the signal timing keeps being optimised and thus changing by the system according to the time-varying traffic demand coming from different approaches; as a result, even the intersection controller does not know the future signal timing and thus cannot broadcast it to AVs. Ideally, it may be possible to know the future signal timing plan if all future traffic demand information is available in the fully connected vehicle environment, but this will not be the case in the near future. To this end, an eco-driving model for the case *when the future signal timing information is not fully available* is necessitated for the implementation of eco-driving systems in broader applications.

To the best of our knowledge, only a handful of papers have considered the stochastic signal information in the eco-driving model. Mahler and Vahidi (2014) added a penalty term of the probability of the green signal in the objective function. The cost in the objective function is negatively related to the probability of green light. This approach has several drawbacks: 1) the probability of the signal state may be inaccurate. Thus, the vehicle may pass the intersection on red which is not acceptable; 2) even if such probability is accurate, the vehicle may still run on red as it is close to the stop line and cannot stop in time. Therefore, this approach cannot guarantee safety. Sun et al. (2018) proposed a chance-constrained model to deal with the uncertain signal information. The chance constraint converts the stochastic optimisation problem into a static optimisation problem. However, this constraint implies that the vehicle can only pass the intersection when the probability of green light is high enough, which is close to the upper bound of the uncertain range. They did not consider the fact that the traffic light can turn green earlier than the time predicted with a certain chance.

To fill the research gap, this paper aims to develop a stochastic model predictive control method for eco-driving under uncertain signal timing information. Instead of using only the probability of the signal state when the vehicle arrives at the stop line as in the existing research, the proposed model attempts to optimise the vehicle movement based on the probability of the signal state at each time step and minimise the expected fuel consumption. The model allows the vehicle to either pass the intersection freely when the traffic light turns green or keep waiting when the traffic light remains red in the planning time horizon. More specifically, the probability of the traffic light turning green at each time step is directly considered in the proposed model, which can work for any distribution of the stochastic signal timing. The performances of the proposed model with respect to the model parameters and distribution of the signal timing are analysed in the simulations. The proposed model is also extended to the case when the vehicle can know in advance when the signal will turn green. The impact of such advanced signal timing information on trajectory planning is also investigated.

The remainder of the paper is organised as follows: Section 4.2 describes the main framework of an eco-driving method under uncertain signal information. Section 4.3 introduces a way to estimate the distribution of future signal timing. Section 4.4 formulates trajectory planning as a multi-phase optimal control model. Section 4.5 extends the proposed model to a more general case when the vehicle can receive the signal information a short time in advance. Section 4.6 illustrates the simulation results with various model parameters and distributions. Finally, we conclude the paper in Section 4.7.

4.2 Main modelling framework

The main modelling framework of the proposed method is described as follows.

1. Collect the historical signal timing data;
2. Predict the future signal timing from the historical data;
3. Broadcast the predicted distribution from the traffic controller or the roadside unit (RSU) to approaching vehicles;
4. Optimise the trajectory by considering the stochastic signal timing.

This framework is intuitive and straightforward. To avoid large data transmission, the first two steps are more suitable to be operated in the local intersection controller or traffic management centre, which can then transfer the predicted signal information to the approaching vehicles. The proposed method is capable of dealing with both the uncertain green light and red light. This paper focuses on trajectory planning when the duration of red light is uncertain.

The method described in Section 4.4 is suitable for the case when the vehicle can obtain only the current signal state and have no accurate future signal information. Another interesting case when the vehicle can receive the future signal information in advance for a limited time will be discussed later. Both are suitable for a mixed traffic situation with AVs and HVs. For simplicity, this paper only focuses on trajectory planning for one AV. It is assumed that the AV can communicate with the intersection controller via V2I communication. Our proposed methods can also be extended to platoon control easily by combining them with the method developed in our previous work (Zhao et al., 2018).

Notation

The notations in Table 4.1 are used throughout this paper.

Table 4.1: Notations

Symbol	Description
c	Signal cycle index
i	Vehicle index
k	Time step index
τ	Time step size
T_c^r	Duration of red light in the cycle c
ϵ_c	Duration of uncertain red signal in the cycle c
ϵ_c^i	Duration of uncertain time for vehicle i passing the stop line in the cycle c
α, β, r_a, r_b	Parameters in the scaled Beta distribution
h	Saturation time headway on the urban road

Symbol	Description
$x_k(v_k, a_k)$	Position (speed, acceleration) of the vehicle in the waiting phase at time step k
$t_k^f(v_k^f)$	Time (speed) of the vehicle at the stop line in the passing phase which starts at time step k
$x_{k,A}^e(v_{k,A}^e)$	Terminal position (speed) of the vehicle after the advance time in the passing route which starts using the eco-driving strategy at time step k
x_f	Position of the stop line at the upcoming intersection
M	Number of time steps in the certain range of signal
N	Number of time steps in the uncertain range of signal
A	Number of time steps in the advance time
θ	Advance time to receive the future signal timing information
f	Fuel consumption rate
$F_k^{pass}(F_k^{wait})$	Fuel consumption in the pass phase (waiting phase) at time step k
$L_k^{pass}(L_k^{wait})$	Expected fuel consumption in the pass phase (waiting phase) at time step k
L_k	Expected fuel consumption at time step k
J_{oc}	Cost function using an optimal control model
J_{op}	Expected fuel consumption (and also the objective function) using an optimisation model

4.3 Estimation of the future signal timing

As the critical information in our trajectory planning model, the probability distribution of the future signal timing needs to be estimated and transferred to the upcoming vehicles. Different from the fixed signal timing plan, the signal timing in an adaptive traffic control system depends on the real-time traffic demand. As the traffic demand changes gradually and the signal control system needs to operate in a relatively stable way, the signal timings between two cycles are likely to be bounded in a limited range and share the changing trend.

$$T_c^r = T_{c-1}^r + \epsilon_c \tag{4.1}$$

where ϵ_c denotes the difference of red time duration between cycle c and the previous cycle, on the particular approach of interest. A scaled Beta distribution $\text{sBeta}(\alpha, \beta, r_a, r_b)$ is used to capture the uncertainty of ϵ_c as follows.

$$\text{sBeta}(\alpha, \beta, r_a, r_b) = (r_b - r_a) \times \text{Beta}(\alpha, \beta) + r_a \tag{4.2}$$

where $\text{Beta}(\alpha, \beta)$ denotes a standard Beta distribution with the parameter α and β.

If $x \sim \text{Beta}(\alpha, \beta)$, then $x \in [0, 1]$. So, when $\epsilon_c \sim \text{sBeta}(\alpha, \beta, r_a, r_b)$, we have $\epsilon_c \in [r_a, r_b]$. The mean of ϵ_c is

$$E(\epsilon_c) = (r_b - r_a)\frac{\alpha}{\alpha + \beta} + r_a \tag{4.3}$$

and the variance is

$$Var(\epsilon_c) = (r_b - r_a)^2 \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)}. \tag{4.4}$$

When the historical signal timing data is available, the parameters in the distribution of future signal timing can be estimated by either a point estimation or a Bayesian approach.

The stochasticity of the available green time also comes from the preceding vehicles (Treiber and Kesting, 2017). The time for vehicle i to pass the stop line is

$$t_i = T_{c-1}^r + i \times h + \epsilon_c^i \tag{4.5}$$

As i vehicles are unlikely taking more than $i \times h + i$ seconds to pass the stop line, ϵ_c^i follows a Beta distribution $\epsilon_c^i \sim (r_b - r_a) \cdot \text{Beta}(\alpha, \beta) + r_a + i$ within the range $[r_a + i, r_b + i]$.

When the data of time to pass the stop line is collected by detectors, for example loop detectors, ϵ_c^i $(i = 0, 1, \cdots, n)$ can be estimated separately. Note that when $i = 0$, then $t_0 = T_c^r$ and $\epsilon_c^0 = \epsilon_c$.

4.4 Stochastic trajectory planning model

4.4.1 Fuel consumption model

An instantaneous fuel consumption model developed by Akcelik (1989) is adopted in this paper. It uses the instantaneous acceleration and speed to estimate the fuel consumption rate as follows:

$$f = \alpha_1 + \beta_1 P_T + (\beta_2 m a^2 v)_{a>0} \tag{4.6}$$

where P_T is the total power (kW) required to drive the vehicle, which is the sum of coast-down drag power, inertia power and extra engine power. m is the mass of the vehicle, and v and a are the instantaneous speed and acceleration respectively. The third term means extra engine drag power during accelerations, and the subscript means the term exists only when the acceleration is larger than zero. P_T is calculated by

$$P_T = \max\{d_1 v + d_3 v^3 + mav,\ 0\} \tag{4.7}$$

where α_1, β_1, β_2, d_1, d_2, d_3 and m in Eq. (4.6) and Eq. (4.7) are parameters with values taken from Akcelik (1989). Substituting Eq. (4.7) to Eq. (4.6), we have

$$f = \begin{cases} \alpha_1 & \text{if } a \le \dfrac{-d_1 - d_3 v^2}{m} \\ \alpha_1 + \beta_1 (d_1 v + d_3 v^3 + mav) & \text{if } \dfrac{-d_1 - d_3 v^2}{m} < a \le 0 \\ \alpha_1 + \beta_1 (d_1 v + d_3 v^3 + mav) + \beta_2 m a^2 v & \text{if } a > 0 \end{cases} \tag{4.8}$$

The instantaneous fuel consumption rate is a function of speed and acceleration. If the acceleration maintains constant during a certain time interval, the fuel consumption during the time interval is a function of the speed and acceleration at the beginning of the interval. $F(v_k, a_k)$ is used to denote the fuel consumption between the time t_k and t_{k+1}.

4.4.2 Multi-phase optimal control method

4.4.2.1 Objective function

As discussed earlier, when the vehicle does not have the accurate future signal information, it cannot take aggressive actions based on entirely the distribution of the future signal timing, which may lead to dangerous situations. What the vehicle knows for sure is the current signal state. So, it is more reasonable and safer to make decisions based on the trade-off of the current signal state and future signal state. Based on this concept, a new multi-phase trajectory planning model is proposed to balance fuel consumption and travel time in the stochastic signal timing environment. At the beginning of each discrete time step, the vehicle observes the current state of the signal and has two options to act during the time step based on the observed state of the signal:

1. Passing: accelerates to pass the intersection when the traffic signal turns green;
2. Waiting: uses appropriate speed to approach the stop line when the signal is still red and waits for the possible green light in the next time step.

The vehicle states resulting from the passing and waiting actions during the time step are called "passing phase" and "waiting phase", respectively. In the passing phase, the vehicle mainly cares about the travel time and tends to pass the intersection as soon as possible, and thus it applies the maximum acceleration. In the waiting phase, the vehicle waits for the green light and tries to save fuel based on the provided distribution of the red signal duration. The general control framework is shown in Fig. 4.1, where the green lines represent the trajectories of passing phases and the blue lines represent trajectories of waiting phases. The passing phase is possible only when the probability of the end of red light is larger than zero.

The predictive horizon includes $M + N$ steps of waiting phases and $N - 1$ steps of passing phases. In the previous $M + 1$ time steps, the trajectory only has waiting phases, however, both waiting phases and passing phases exist in the later $N - 1$ time steps. If the vehicle finds the traffic light turns green between $[t_M, t_{M+1}]$, it chooses the route $(t_0, x_0) \rightarrow (t_{M+1}, x_{M+1}) \rightarrow (t^f_{M+1}, x_f)$. If the traffic light turns green between $[t_{M+1}, t_{M+2}]$, it chooses the route $(t_0, x_0) \rightarrow (t_{M+1}, x_{M+1}) \rightarrow (t_{M+2}, x_{M+2}) \rightarrow (t^f_{M+2}, x_f)$. If the traffic light turns green in the last time step $[t_{M+N-1}, t_{M+N}]$, it chooses the route $(t_0, x_0) \rightarrow (t_{M+1}, x_{M+1}) \rightarrow \cdots \rightarrow (t_{M+N-1}, x_{M+N-1}) \rightarrow (t_{M+N}, x_f)$. So there are N possible routes in Fig. 4.1.

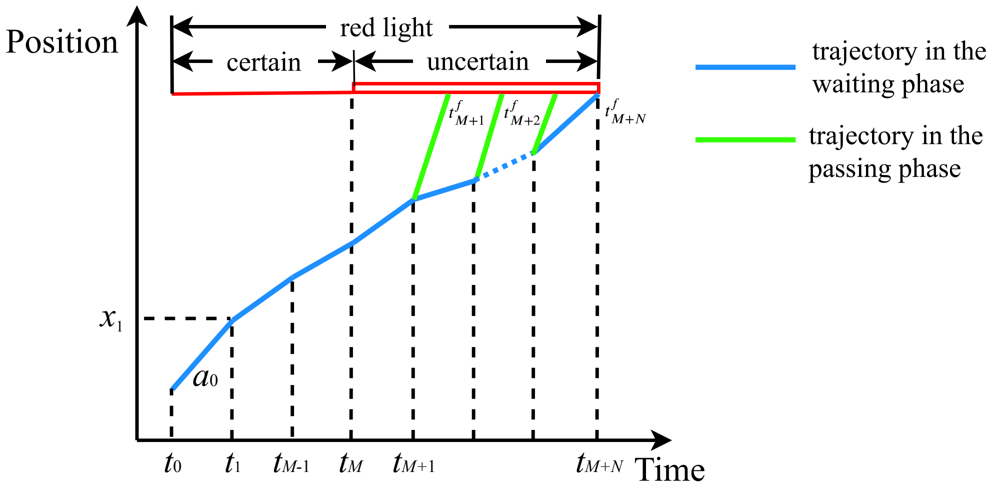


Figure 4.1: Two driving strategies when the future signal timing information is uncertain

The objective function of the general trajectory planning is

$$J_{oc} = (x(M + N) - x_f)^2 + \sum_{k=0}^{M+N-1} L_k \tag{4.9}$$

The objective function in the optimal control model is converted to another objective function in the nonlinear optimisation model.

$$J_{op} = \sum_{k=0}^{M+N-1} L_k \tag{4.10}$$

At each time step, the driving cost which is fuel consumption is

$$L_k = \begin{cases} L_k^{wait} & \text{if } k \in [0, M] \\ L_k^{pass} + L_k^{wait} & \text{if } k \in [M+1, M+N-1] \end{cases} \tag{4.11}$$

$$L_k^{pass} = p(t_{k-1} < t \leq t_k) F_k^{pass} \tag{4.12}$$

$$L_k^{wait} = p(t > t_k) F_k^{wait} \tag{4.13}$$

where F_k^{wait} and F_k^{pass} are the fuel consumption in the waiting phase and passing phase, respectively. L_k^{wait} and L_k^{pass} are the expected fuel consumption in the waiting phase and passing phase, respectively. $p(t_{k-1} < t \leq t_k)$ is the probability of red signal duration between time t_{k-1} and time t_k, which is also the probability of green light between time t_{k-1} and time t_k.

In the phase of free passing, the vehicle accelerates to pass the intersection to save time. It is assumed that the acceleration in the stage of free passing is the maximum acceleration. The fuel consumption of a passing phase at each time step is

$$F_k^{pass} = F(v_k, a_{max}) \tag{4.14}$$

In the phase of waiting to pass, the acceleration of the vehicle may vary between the limits of acceleration which is the control variable. The fuel consumption of a waiting phase at each time step is

$$F_k^{wait} = F(v_k, a_k) \tag{4.15}$$

4.4.2.2 Vehicle dynamic constraints

For every vehicle, the position and speed in the waiting phase are the system state variables, since the trajectories of passing phases depend on the trajectories of waiting phases. The system dynamic functions are

$$x_{k+1} = x_k + v_k \tau + \frac{1}{2} a_k \tau^2 \tag{4.16}$$

$$v_{k+1} = v_k + a_k \tau \tag{4.17}$$

where the accelerate a_k in the waiting phase is the control variable. The constraints on speed and position are

$$v_{min} \leq v_k \leq v_{max} \tag{4.18}$$

$$x_{min} \leq x_k \leq x_f \tag{4.19}$$

The system state can be written as functions of the initial state and accelerations, which are

$$v_k = v_0 + \sum_{t=0}^{k-1} a_t \tau \tag{4.20}$$

$$x_k = x_0 + k v_0 \tau + \sum_{t=0}^{k-1} \frac{-2t + 2k - 1}{2} a_t \tau^2 \tag{4.21}$$

The constraints on the control variable are

$$a_{min} \leq a_k \leq a_{max} \tag{4.22}$$

4.4.2.3 Terminal state constraints

The terminal cost of the objective function in the optimal control approach is converted to a constraint in the nonlinear optimisation approach. It ensures that even if the traffic signal does not turn green in all previous time steps, the vehicle arrives at the stop line at the end of the uncertain range of the red light.

$$x_{M+N} = x_f \tag{4.23}$$

As discussed in our previous paper (Zhao et al., 2018), the terminal speed is limited to the maximum speed, thus to increase the capacity of the intersection and avoid blocking the following vehicle. Due to the structure of the model, we only need to add a constraint on the terminal speed in the last waiting phase. The terminal speed in all the other passing phases must be the maximum speed as well.

$$v_{M+N} = v_{max} \tag{4.24}$$

Proposition 4.4.1. *If the speed in the waiting phase at the last time step can reach the maximum speed, then the terminal speed of every other route can reach the maximum speed too.*

Proof. See the simple routes in Fig. 4.2 where route AC is a passing phase and route AB includes multiple waiting phases.

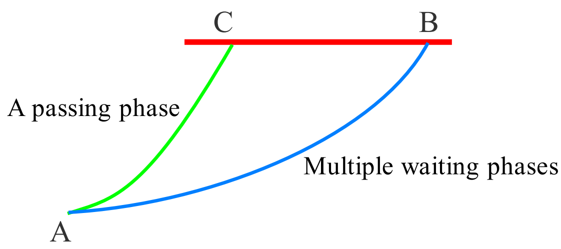


Figure 4.2: Schematic of the route in the passing phase and waiting phase starting from a point

If $v_B = v_{max}$ and $v_C < v_{max}$, as $v_A \leq v_C$, so $v_A < v_{max}$. As $v_C < v_{max}$, the acceleration in the passing phase must be the maximum acceleration which is larger than or equal to the acceleration of waiting phase, i.e. $a_{AC} \geq a_{AB}$. As $v_C = \sqrt{v_A^2 + 2a_{AC}(x_f - x_A)} \geq \sqrt{v_A^2 + 2a_{AB}(x_f - x_A)} = v_B = v_{max}$. This is inconsistent with the assumption that $v_C < v_{max}$. So $v_C = v_{max}$.

If $v_A = v_{max}$, it is no doubt that $v_C = v_{max}$ as the acceleration of passing phase in this case equal to zeros and the vehicle will maintain the maximum speed to pass the intersection. The proof is thus completed. □

4.4.3 Cost in the passing phase

In the passing phase, the vehicle knows that the signal has turned green. It accelerates using the maximum acceleration till reaching the maximum speed.

The distance to the stop line at the start of an interval is

$$s_k = x_f - x_k \tag{4.25}$$

The driving cost is defined as:

$$F_k^{pass} = F(v_k, a_{max}) = \int_{t_k}^{t_k^f} f(v, a_{max}|v_k)\,\mathrm{d}t \tag{4.26}$$

where t_k^f denotes the time arriving at the stop line in the passing phase which starts at time t_k. This function can be computed in different situations as follows.

4.4.3.1 Case 1

If the vehicle can accelerate to the maximum speed before reaching the stop line, i.e. $v_{max}^2 - v_k^2 \leq 2a_{max}(x_f - x_k)$, the time required to reach the maximum speed is:

$$t_k^a = t_k + \frac{v_{max} - v_k}{a_{max}} \tag{4.27}$$

and the time arriving at the stop line becomes:

$$t_k^f = t_k^a + \frac{x_f - x_k}{v_{max}} - \frac{v_{max}^2 - v_k^2}{2a_{max}v_{max}} \tag{4.28}$$

The driving cost of this case is determined below:

$$\begin{aligned}
&\int_{t_k}^{t_k^f} f(v, a_{max}|v_k)\,\mathrm{d}t \\
=&\int_{t_k}^{t_k^a} f(v_k, a_{max})\,\mathrm{d}t + \int_{t_k^a}^{t_k^f} f(v_{max}, 0)\,\mathrm{d}t \\
=&\frac{\alpha_1(v_{max} - v_k)}{a_{max}} + \frac{\beta_1\left(v_{max}^2 - v_k^2\right)\left(2d_1 + 2ma_{max} + d_3(v_{max}^2 + v_k^2)\right)}{4a_{max}} \\
&+ \frac{\beta_2 m a_{max}(v_{max}^2 - v_k^2)}{2} \\
&+ \left(x_f - x_k - \frac{v_{max}^2 - v_k^2}{2a_{max}}\right)\left(\frac{\alpha_1}{v_{max}} + \beta_1\left(d_1 + d_3 v_{max}^2\right)\right)
\end{aligned} \tag{4.29}$$

4.4.3.2 Case 2

If the vehicle cannot accelerate to the maximum speed before reaching the stop line, i.e. $v_{max}^2 - v_k^2 > 2a_{max}(x_f - x_k)$, the speed and time arriving at the stop line are:

$$v_k^f = \sqrt{v_k^2 + 2a_{max}(x_f - x_k)} \tag{4.30}$$

and

$$t_k^f = t_k + \frac{\sqrt{v_k^2 + 2a_{max}(x_f - x_k)} - v_k}{a_{max}} \tag{4.31}$$

The driving cost of this case is determined as below:

$$\begin{aligned}\int_{t_k}^{t_k^f} f(v, a_{max}|v_k)\,\mathrm{d}t &= \int_{t_k}^{t_k^f} f(v_k, a_{max})\,\mathrm{d}t \\ &= \frac{\alpha_1}{a_{max}}\left(\sqrt{v_k^2 + 2a_{max}(x_f - x_k)} - v_k\right) \\ &\quad + \beta_1(x_f - x_k)\left[d_1 + ma_{max} + d_3(v_k^2 + a_{max}(x_f - x_k))\right] \\ &\quad + \beta_2 m a_{max}^2(x_f - x_k)\end{aligned} \tag{4.32}$$

4.4.4 Cost in the waiting phase

For simplicity, we denote $[z]_+ \coloneqq \max(z, 0)$. The equation of fuel consumption rate (Eq. (4.8)) can be written as

$$f(v, a) = \alpha_1 + \beta_1\left[d_1 v + d_3 v^3 + mav\right]_+ + \beta_2 mav[a]_+ \tag{4.33}$$

The fuel consumption is calculated as

$$\begin{aligned}&\int_{t_k}^{t_{k+1}} f(v, a)\,\mathrm{d}t \\ =&\int_{t_k}^{t_{k+1}} \{\alpha_1 + \beta_1\left[d_1 v + d_3 v^3 + mav\right]_+ + \beta_2 mav[a]_+\}\,\mathrm{d}t \\ =&\int_{t_k}^{t_{k+1}} \alpha_1\,\mathrm{d}t + \beta_1\int_{t_k}^{t_{k+1}}\left[d_1 v + d_3 v^3 + mav\right]_+\mathrm{d}t + \beta_2 ma[a]_+\int_{t_k}^{t_{k+1}} v\,\mathrm{d}t \\ =&\alpha_1\tau + \beta_1\int_{t_k}^{t_{k+1}}\left[d_1 v + d_3 v^3 + mav\right]_+\mathrm{d}t + \beta_2 ma[a]_+(v_k\tau + \frac{1}{2}a_k\tau^2)\end{aligned} \tag{4.34}$$

As $v(t) = v_k + a(t - t_k)$, then

$$\begin{aligned}
&\int_{t_k}^{t_{k+1}} \left[d_1 v + d_3 v^3 + mav\right]_+ \mathrm{d}t \\
=&\int_{v_k}^{v_{k+1}} v\left[d_1 + ma + d_3 v^2\right]_+ \mathrm{d}\left(\frac{v - v_k}{a_k} + t_k\right) \\
=&\frac{1}{a}\int_{v_k}^{v_{k+1}} v\left[d_1 + ma + d_3 v^2\right]_+ \mathrm{d}v \\
=&\frac{1}{4ad_3}\int_{v_k}^{v_{k+1}} \mathrm{d}\left[d_1 + ma + d_3 v^2\right]_+^2 \\
=&\frac{1}{4ad_3}\left(\left[d_1 + ma + d_3 v_{k+1}^2\right]_+^2 - \left[d_1 + ma + d_3 v_k^2\right]_+^2\right) \\
=&\frac{1}{4ad_3}\left(\left[d_1 + ma + d_3 (v_k + a\tau)^2\right]_+^2 - \left[d_1 + ma + d_3 v_k^2\right]_+^2\right)
\end{aligned} \tag{4.35}$$

Substituting Eq. (4.35) into Eq. (4.34), we get the fuel consumption in a waiting phase at time step k.

4.5 The vehicle receives the signal information in advance

When the vehicle can get access to the future signal information θ seconds in advance, the vehicle may take actions in advance and has the potential of reducing fuel consumption further. Fig. 4.3 shows the possible trajectory. There are two main differences from the previous model:

1. The possibility of the passing and waiting phase at each time step is calculated by the possibility of green and red light in θ seconds later.
2. There are two strategies in the passing phase.
   (a) Accelerating strategy: If the vehicle cannot arrive at the stop line in the θ seconds, then it accelerates to the maximum speed using the maximum acceleration to save time. This is shown as the green route in Fig. 4.3.

(b) Eco-driving strategy: If the vehicle can arrive at the stop line in the θ seconds, then it uses the eco-driving strategy to arrive at the stop line in θ seconds later with the maximum speed. This is shown as the purple route in Fig. 4.3.

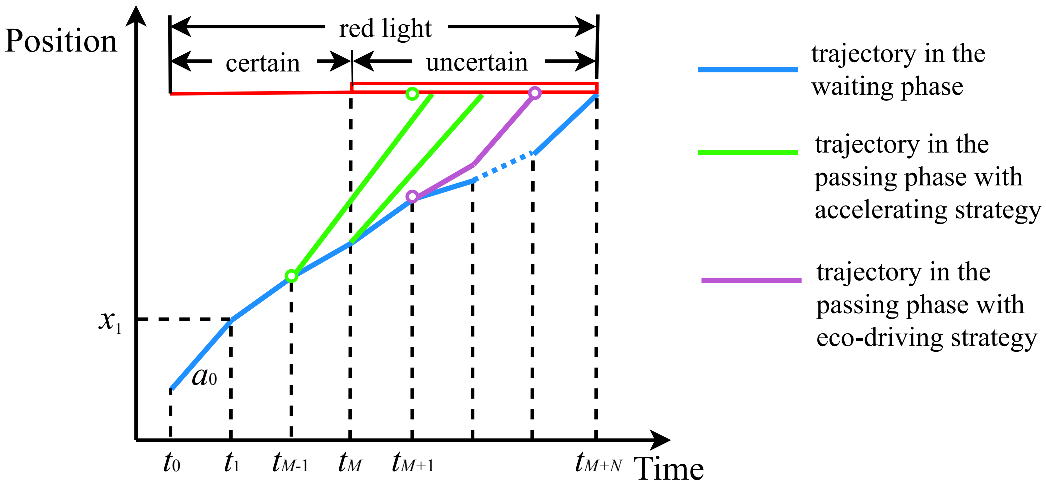


Figure 4.3: Three driving strategies when the vehicle receives the signal information in advance

4.5.1 Objective function

At each time step, the driving cost which is the fuel consumption is calculated by

$$L_k = \begin{cases} L_k^{wait} & \text{if } k \in [0, M-A] \\ L_k^{pass} + L_k^{wait} & \text{if } k \in [M-A+1, M+N-A-1] \\ L_k^{wait} & \text{if } k \in [M+N-A, M+N-1] \end{cases} \tag{4.36}$$

$$L_k^{pass} = p(t_{k-1} + \theta < t \leq t_k + \theta) F_k^{pass} \tag{4.37}$$

$$L_k^{wait} = p(t > t_k + \theta) F_k^{wait} \tag{4.38}$$

4.5.2 Constraints

As discussed in Proposition 4.4.1, the constraint (4.24) ensures that the terminal speed of every passing phase in the accelerating strategy is the maximum speed.

Extra constraints are still required to ensure that the terminal speed and travel time in the eco-driving strategy meet the requirements.

For every passing route using eco-driving strategy, there are A additional decision variables. So there are $A \cdot (N-1)$ additional decision variables in total in this case. For the passing route with eco-driving strategy starting at time step k, the terminal position and terminal speed are

$$v^e_{k,A} = v_k + \sum_{t=0}^{A-1} a^e_t \tau \tag{4.39}$$

$$x^e_{k,A} = x_k + A v_k \tau + \sum_{t=0}^{A-1} \frac{-2t + 2A + 1}{2} a^e_t \tau^2 \tag{4.40}$$

For the passing phase at time t_k, the eco-driving strategy is activated when the minimum travel time from x_k to the stop line is longer than the advance time. The minimum travel time is calculated in the same way used our previous paper (Zhao et al., 2019).

If $v^2_{max} - v^2_k \geq 2a_{max}(x_f - x_k)$, the minimum travel time tt is calculated by

$$tt = \frac{-v_k + \sqrt{v^2_k + 2a_{max}(x_f - x_k)}}{a_{max}} \tag{4.41}$$

Otherwise, tt is calculated by

$$t_a = \left\lfloor \frac{v_{max} - v_k}{a_{max}\tau} \right\rfloor \tau \tag{4.42}$$

$$s_{a1} = \frac{1}{2}(2v_k + a_{max} t_a) t_a \tag{4.43}$$

$$s_{a2} = \frac{1}{2}(v_k + a_{max} t_a + v_{max}) \tau \tag{4.44}$$

$$t_b = \frac{x_f - x_k - s_{a1} - s_{a2}}{v_{max}} \tag{4.45}$$

$$tt = t_a + \tau + t_b \tag{4.46}$$

where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x.

Thus, the constraints on the eco-driving strategy are

$$(tt - t_{advance})(x^e_{k,A} - x_f) \leq 0 \tag{4.47}$$

$$(tt - t_{advance})(v^e_{k,A} - v_{max}) \leq 0 \tag{4.48}$$

In constraint (4.47), if $tt - t_{advance} \geq 0$ which implies the vehicle is unable to arrive at the stop line within the advance time, accelerating strategy is used. As $x^e_{k,A} - x_f \leq 0$ is automatically satisfied by the constraint (4.19), thus constraint (4.47) is satisfied. If $tt - t_{advance} < 0$ which implies the vehicle can arrive at the stop line within the advance time, eco-driving strategy is used. Then, $x^e_{k,A} - x_f \geq 0$. Because of constraint (4.19), we have $x^e_{k,A} - x_f \leq 0$, so $x^e_{k,A} - x_f = 0$ must be satisfied and is also the requirement of eco-driving strategy. The same logic can also be applied to analyse constraint (4.48).

4.6 Simulation tests

Many simulation tests are taken in this section to test the performance of the proposed methods. After comparing the performances of Interior point, SQP and pattern search, the interior point algorithm is used in the following simulations due to its stable and good results. These simulation tests can be grouped as three aspects: test the impact of different parameters in the model; test the impact of various distributions of the uncertain signal information; test the proposed method when the vehicle receives the signal information in advance. In the part of testing the impact of the parameters in the model, we are interested to know whether the terminal speed constraint is required and which time step is appropriate. In the part of testing the impact of various distributions, we will test different means, variances, and ranges. Finally, we will test the impact of different advance time in the extended model. The simulation parameters are shown in Table 4.2.

Table 4.2: Parameters in the model

Symbol	Description	Value
Parameters in the trajectory planning model		
a_{min}	Minimum acceleration	$-4\,\mathrm{m/s^2}$
a_{max}	Maximum acceleration	$3\,\mathrm{m/s^2}$
v_{min}	Minimum speed	$0\,\mathrm{m/s}$
v_{max}	Maximum speed	$15.5\,\mathrm{m/s}$
x_{min}	Minimum position	$0\,\mathrm{m}$
x_f	Position of the stop line	$300\,\mathrm{m}$
Parameters in the fuel consumption model		
α_1	Idle fuel consumption rate	$0.666\,\mathrm{mL/s}$
β_1	Efficiency parameter	$0.072\,\mathrm{mL/kJ}$
β_2	Energy-acceleration efficiency parameter	$0.0344\,\mathrm{mL/(kJ\,m/s^2)}$
d_1	Vehicle parameter mainly related to the rolling resistance	$0.269\,\mathrm{kN}$
d_3	Vehicle parameter mainly related to the aerodynamic drag	$0.000\,672\,\mathrm{kN/(m/s)^2}$
m	Mass of the vehicle	$1680\,\mathrm{kg}$

4.6.1 Impact of the terminal speed constraint

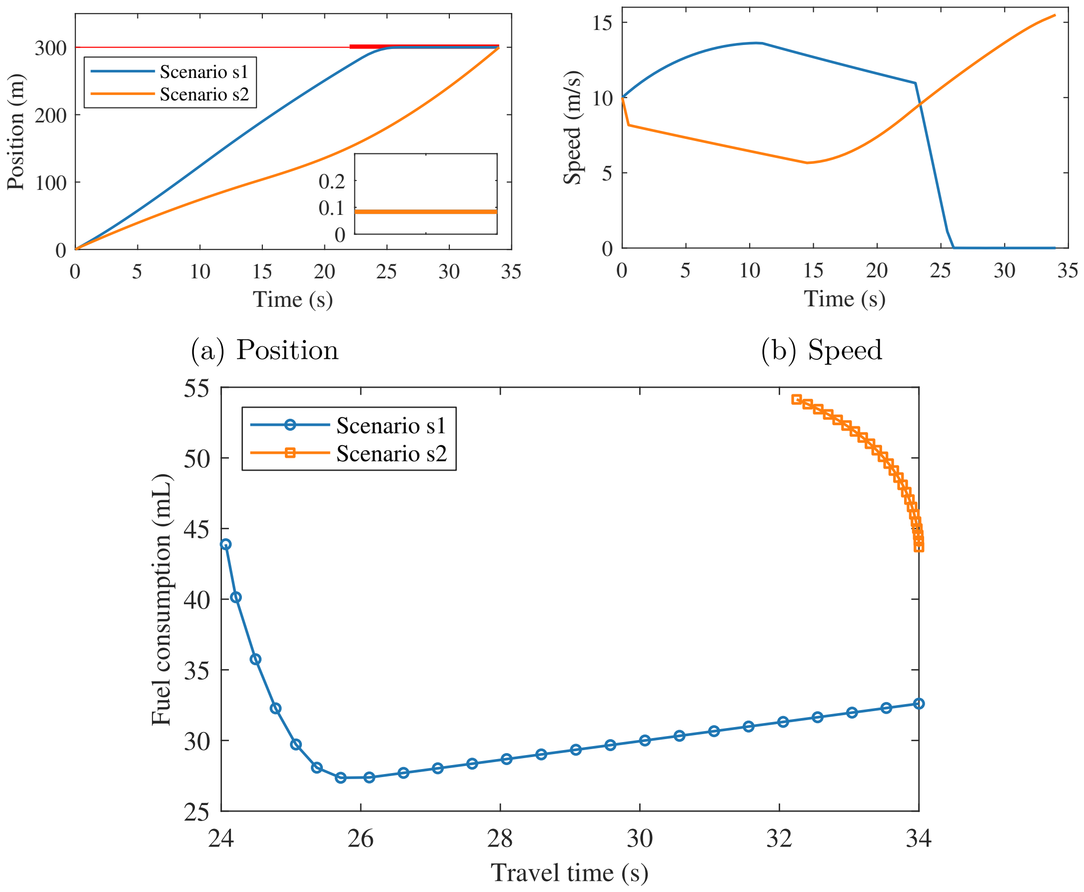
In Section 4.4, a constraint Eq. (4.24) is applied on the terminal speed in the last waiting phase. This also ensures that the terminal speed in all the other routes is the maximum speed. The impact of the terminal speed constraint on the trajectory is analysed in this section. Two scenarios are presented, and they are described in the following.

- Scenario s1: There is no terminal speed constraint;
- Scenario s2: The terminal speed is the maximum speed.

The resulting waiting phase trajectories are shown in Fig. 4.4, and the ex-

Table 4.3: Expected fuel consumption and expected travel time in different terminal speed constraints

Scenario	Expected fuel consumption (mL)		Expected travel time (s)	
	Stop line	Extended distance	Stop line	Extended distance
Scenario s1	31.13	65.89	28.54	33.10
Scenario s2	49.16	52.12	33.43	36.01



(a) Position

(b) Speed

(c) Fuel consumption and travel time in each route

Figure 4.4: State trajectories in different terminal speed constraints

pected fuel consumption and travel time are shown in Table 4.3. The small subfigure in Fig. 4.4a denotes the distribution of the stochastic red light duration. In scenario s1, the vehicle arrives at the stop line in the early period of stochastic red light time. Whereas in scenario s2, the vehicle decelerates in the first half period and then starts to accelerate to the maximum speed. The trajectories in Fig. 4.4 only show the trajectories in the waiting phase, and the full trajectories of scenario s2 are shown in Fig. 4.5. We can see that all the passing routes are located in the last two seconds of the uncertain red signal period. The vehicle accelerates in both waiting phases and passing phases, and the accelerations in the waiting phases are much smaller. This is also why the passing routes are closely adjacent and fuel consumption decreases with increasing travel time, which can also be seen in Fig. 4.4c.

The fuel consumption of each route in scenario s1 is much lower than that in scenario s2 in Fig. 4.4c. This results in a 37% reduction of fuel consumption in scenario s1 compared to the result in scenario s2. Most of the reduction comes from fuel consumption in the passing phases. The vehicle in scenario s1 is so close to the stop line, and it does not need to accelerate in the passing phases at all. This is also the reason for 15% reduction in the expected travel time. Except for the last route, the travel time of all the other routes in scenario s1 is shorter than that in scenario s2.

Though being close to the stop line reduces fuel consumption in the passing phase and hence reduces the expected fuel consumption, this also has side effects that the vehicle may have long stopped time and need to consume more fuel on the following road. Another case is also tested that the vehicle in both scenarios runs the same extended distance and reaches the same maximum speed. The results are shown in Table 4.3. The vehicle in scenario s1 consumes 25% more fuel but has 8% less travel time. We tend to choose the scenario s2, which adds the terminal speed constraint in the model because reducing the stopping time and fuel consumption are the primary goals of trajectory planning in this paper.

4.6.2 Impact of the time step size

In the proposed model, the vehicle reacts to the signal at discrete time steps. How does the time step size affect on the trajectories and performance? Four different time step sizes are used in the simulation tests. The distribution of the red signal is the same for all simulation scenarios. The state trajectories of waiting phases and fuel consumption and travel time of every route are shown in

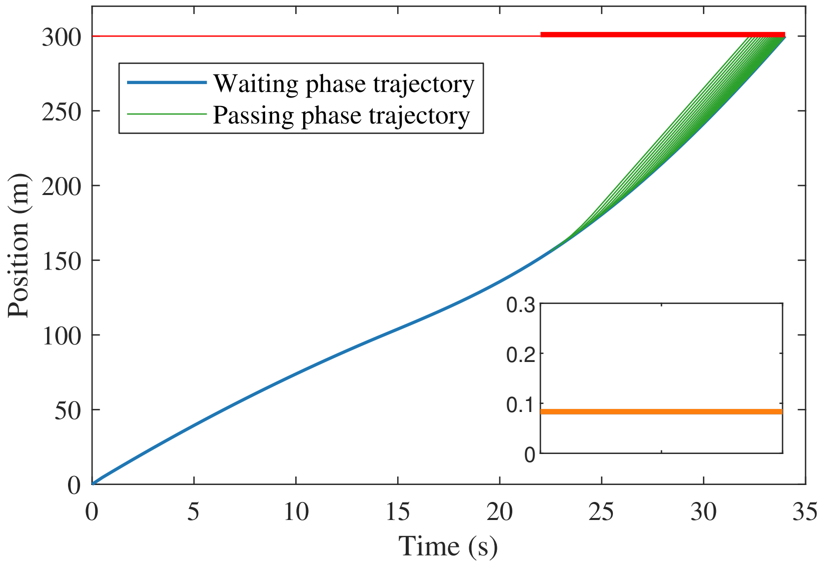


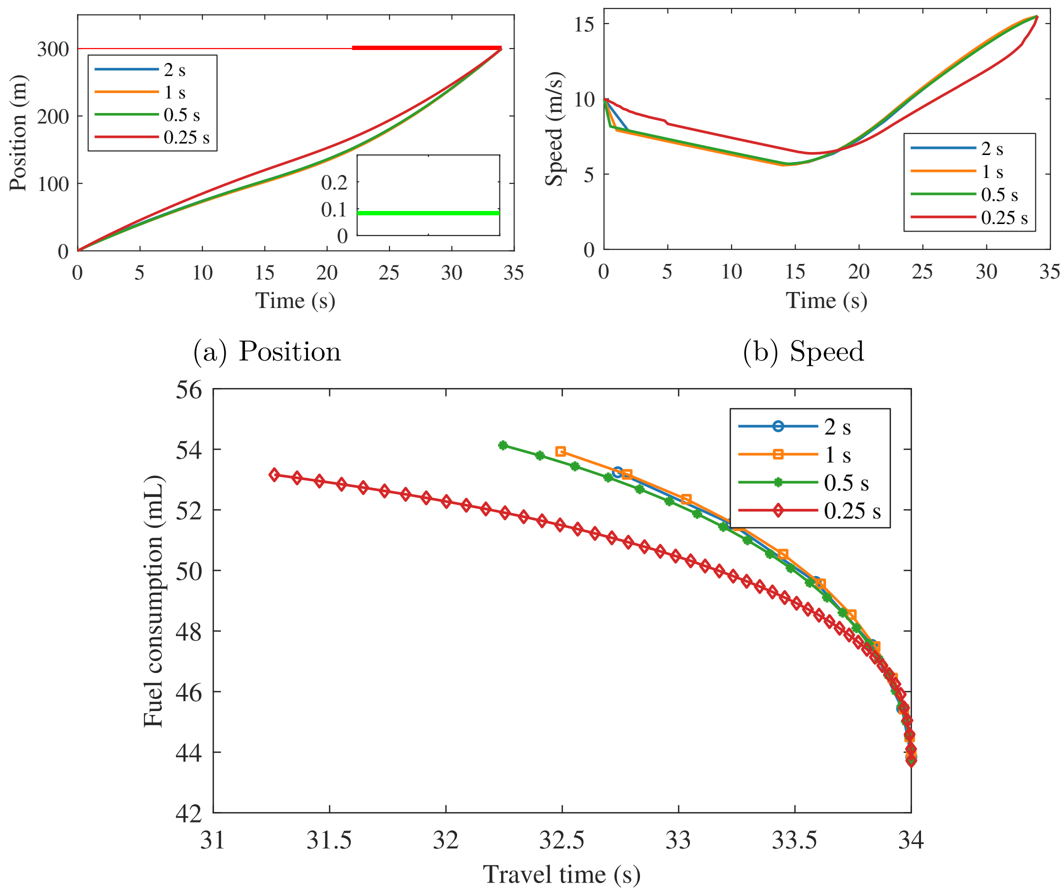
Figure 4.5: Full trajectories under stochastic signal information

Fig. 4.6. Table 4.4 shows the expected fuel consumption and travel time in each scenario.

Table 4.4: Trade-off between fuel consumption and travel time

Distribution	Time step size (s)	Expected fuel consumption (mL)	Expected travel time (s)
sBeta$(1, 1, 22, 34)$	2	48.54	33.56
sBeta$(1, 1, 22, 34)$	1	48.94	33.51
sBeta$(1, 1, 22, 34)$	0.5	49.16	33.43
sBeta$(1, 1, 22, 34)$	0.25	49.60	32.98

Except for the scenario of 0.25 s time step, the trajectory did not change much in the other three scenarios. When the time step decreases, the expected fuel consumption increases slightly and expected travel time decreases slightly. The increase in fuel consumption mainly comes from the first time step. In Fig. 4.6c, when the time step size decreases, the vehicle reacts to the red signal more responsively. It accelerates in an earlier time and consumes more fuel due to the long distance to accelerate. This also explains that the vehicle has a shorter travel time when the time step is shorter. Although the expected travel time



(a) Position

(b) Speed

(c) Fuel consumption and travel time in each route

Figure 4.6: State trajectories with different time step size

did not decrease much, it offers the vehicle more possibilities of travel time when the time step size is smaller, since the vehicle make decisions more frequently. Under the same travel time, the vehicle consumes less fuel when the time step is shorter. This is very obvious when the time step is 0.25 s. Considering the performance and calculation burden, 0.5 s will be used as the time step in the following simulations.

4.6.3 Impact of the mean of the distribution

The distribution of the uncertain signal information is directly considered in the model. The vehicle may have a different responding trajectory in different distributions. The impact of different means of distribution on the trajectory is tested in the following three scenarios. The variances of the distribution are kept the same in all the simulations.

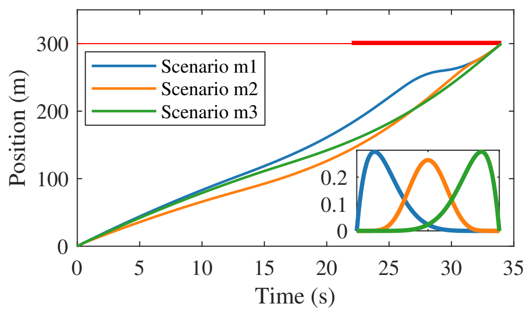
- Scenario m1: The distribution of red signal duration is $\text{sBeta}(2, 8, 22, 34)$;
- Scenario m2: The distribution of red signal duration is $\text{sBeta}(8.09, 8.09, 22, 34)$;
- Scenario m3: The distribution of red signal duration is $\text{sBeta}(8, 2, 22, 34)$.

Table 4.5: Expected fuel consumption and expected travel time in different means of the distribution

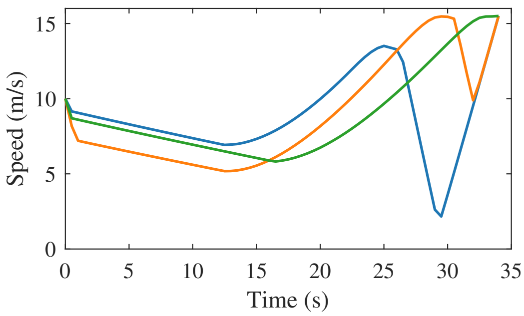
Distribution	Mean (s)	Expected fuel consumption (mL)	Expected travel time (s)
$\text{sBeta}(2, 8, 22, 34)$	24.4	47.08	30.22
$\text{sBeta}(8.09, 8.09, 22, 34)$	28.0	47.64	33.31
$\text{sBeta}(8, 2, 22, 34)$	31.6	45.29	33.96

The obtained trajectories of waiting phases are shown in Fig. 4.7 and the performance indicators are shown in Table 4.5. We can observe apparent differences in the trajectories of waiting phases in three simulation scenarios. In scenario m1, trajectories of waiting phases include two stages of decelerating and accelerating. When the mean of the distribution in the uncertain red signal increases, the duration of the second decelerating and accelerating stage decreases and finally there is only one stage of decelerating and accelerating in scenario m3. The behaviour is the trade-off between the objective function and constraints. The vehicle tries to be close to the stop line so as to minimise fuel consumption in the passing phase when the probability of turning green is high. However, because of the terminal position and speed constraints, it cannot keep the high speed and has to decelerate first and then accelerate to the desired terminal speed.

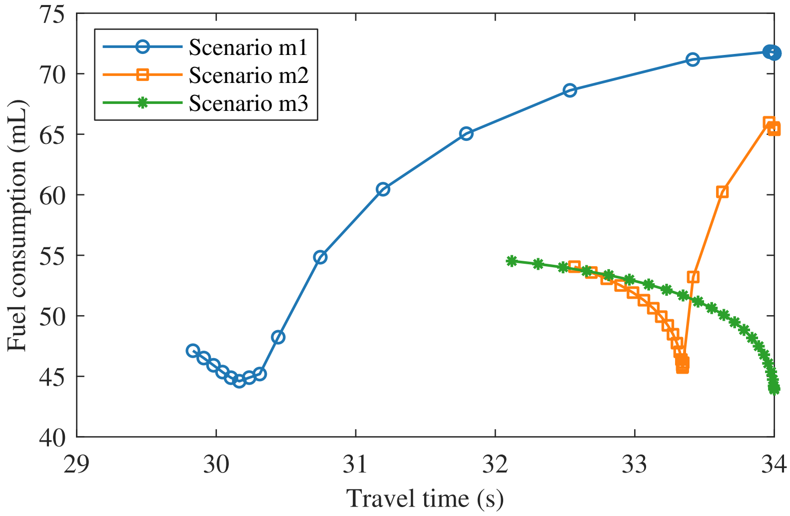
The mean of the distribution also affects the distribution of fuel consumption and travel time in each route. When the probability of turning green is high, there are more routes during that period and the fuel consumption of those routes is smaller than that in other periods. This is why the travel time of the first few routes in scenario m1 is much shorter than that in the other scenarios. This can



(a) Position



(b) Speed



(c) Fuel consumption and travel time in each route

Figure 4.7: State trajectories in different means of the distribution

also explain why the expected travel time increases as the mean of the distribution increases. The expected travel time in scenario m3 is 12.4% longer than that in scenario m1. However, fuel consumption does not share the same trend. As shown in Table 4.5, when the mean increases, the expected fuel consumption may increase or decrease. It seems that the mean of 28s, which is located in the middle of the uncertain red light is a turning point in the performance. When the mean is less than 28 s, the expected fuel consumption increases slightly, and the expected travel time increases considerably. When the mean is greater than 28 s, the expected fuel consumption decreases more significantly than the increase in the expected travel time.

4.6.4 Impact of the variance of the distribution

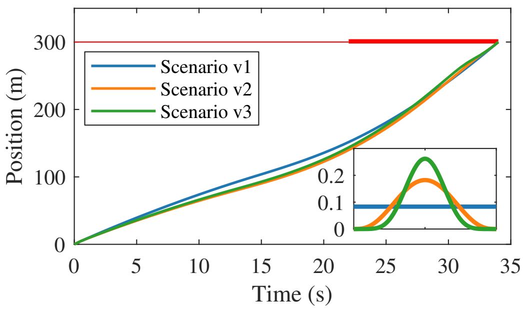
In this section, the impact of the variance of distribution on the trajectory is analysed. In the simulation scenarios, the mean of the distribution is kept the same as the median of the uncertain signal range. Thus, the parameters α and β should be the same according to Eq. (4.3), and the corresponding mean of the Beta distribution is 0.5.

- Scenario v1: The distribution of red signal duration is sBeta$(1, 1, 22, 34)$;
- Scenario v2: The distribution of red signal duration is sBeta$(4, 4, 22, 34)$;
- Scenario v3: The distribution of red signal duration is sBeta$(8, 8, 22, 34)$.

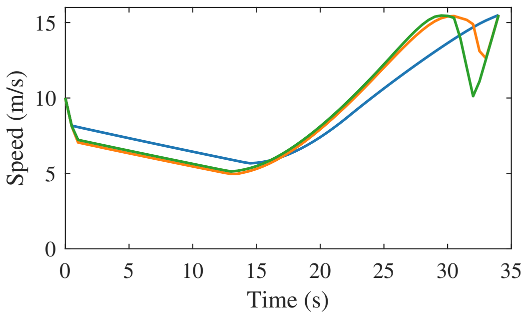
Table 4.6: Expected fuel consumption and expect travel time in different variances of the distribution

Distribution	Variance	Expected fuel consumption (mL)	Expected travel time (s)
sBeta$(1, 1, 22, 34)$	12	49.16	33.43
sBeta$(4, 4, 22, 34)$	7.2	48.48	33.65
sBeta$(8, 8, 22, 34)$	4	47.66	33.34

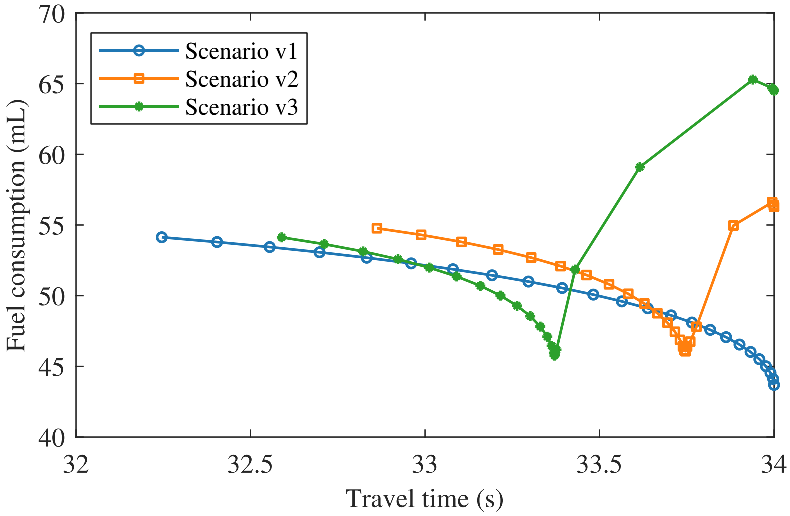
The obtained trajectories of waiting phases are shown in Fig. 4.8 and the performance indicators are shown in Table 4.6. The trajectories of waiting phases change slightly in different variances of the distribution. The vehicle in scenario v1 runs closer to the stop line than in the other scenarios in the early period of the stochastic range and consumes less fuel. When the variance of the distribution becomes smaller, the variance of the fuel consumption in each route becomes larger. This is because the vehicle puts more efforts to minimise fuel consumption when the probability is high. Though the fuel consumption of the routes in the first few routes and the last few routes in both scenario v2 and v3 are higher than that in scenario v1, the expected fuel consumption are smaller. This is because of the less possibility of passing in the early and late period of the uncertain



(a) Position



(b) Speed



(c) Fuel consumption and travel time in each route

Figure 4.8: State trajectories in different variances of the distribution

range. Finally, when the variance of distribution decreases, the expected fuel consumption decreases slightly as well, but the expected travel time does not show a distinct pattern.

4.6.5 Impact of the range of the distribution

In this subsection, the impact of the range of the uncertain signal on the trajectory is analysed. In the simulation scenarios, the corresponding Beta distribution is kept the same to be the uniform distribution where both parameter α and β equal to 1.

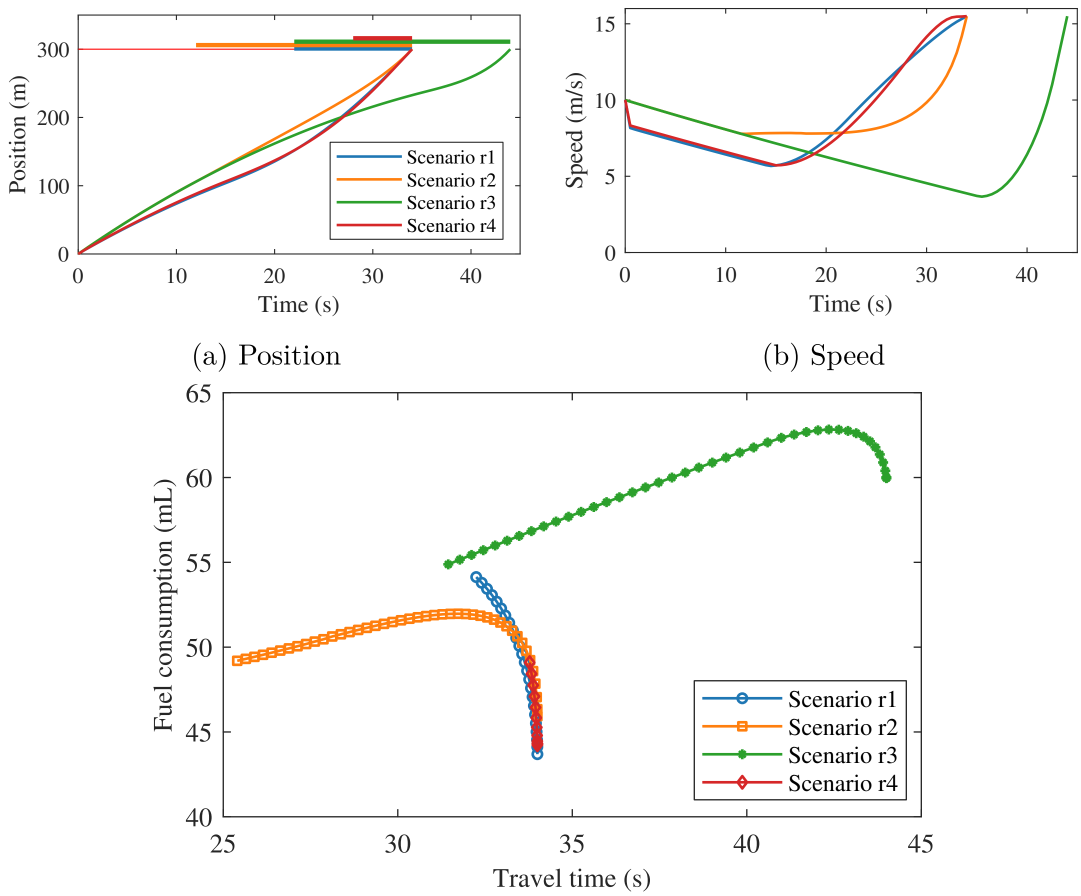
- Scenario r1: The distribution of red signal duration is $\text{sBeta}(1, 1, 22, 34)$;
- Scenario r2: The distribution of red signal duration is $\text{sBeta}(1, 1, 12, 34)$;

- Scenario r3: The distribution of red signal duration is sBeta$(1, 1, 22, 44)$;
- Scenario r4: The distribution of red signal duration is sBeta$(1, 1, 28, 34)$.

Table 4.7: Expected fuel consumption and expected travel time in different ranges of the distribution

Distribution	Expected fuel consumption (mL)	Expected travel time (s)
sBeta$(1, 1, 22, 34)$	49.15	33.49
sBeta$(1, 1, 12, 34)$	50.44	30.40
sBeta$(1, 1, 22, 44)$	59.76	38.82
sBeta$(1, 1, 28, 34)$	46.00	33.94

The optimised trajectories of waiting phases are shown in Fig. 4.9, and the results are shown in Table 4.7. In scenario r1, r2 and r4, the end time of the uncertain signal are the same. Scenario r1 and r4 can be considered as scenarios that have the same uncertain range as scenario r2, but the probability of the corresponding duration in scenario r1 and r4 are zero. So, the red light in scenario r4 has less uncertainty than that in scenario r1, and the red light in scenario r1 has less uncertainty than that in scenario r2. The fuel consumption in scenario r4 is 8.8% less than that in scenario r2. When there is less uncertainty, the vehicle has more room to optimise the trajectory and achieve less fuel consumption. Though the range of the uncertain signal in scenario r4 is only half of that in scenario r1, the trajectories are almost the same. As indicated in Fig. 4.5, all the passing trajectories are located in the latter half duration. The change of start time in the uncertain signal does not change the trajectories very much. The trajectories are more sensitive to the end time of the uncertain signal than the start time. This is resulting from the terminal state constraints. When the end time of the signal changes, the trajectories in waiting phases must modify to satisfy the terminal state constraints. So more attention should be paid on the estimation of the end time of the uncertain range. In addition, conclusions on the impact of the distribution mean in Section 4.6.3 can be applied here. When the range of the



(a) Position (b) Speed

(c) Fuel consumption and travel time in each route

Figure 4.9: State trajectories in different ranges of the distribution

distribution increases, the expected travel time also increases due to the increase in the mean of the distribution.

4.6.6 Impact of the advance time

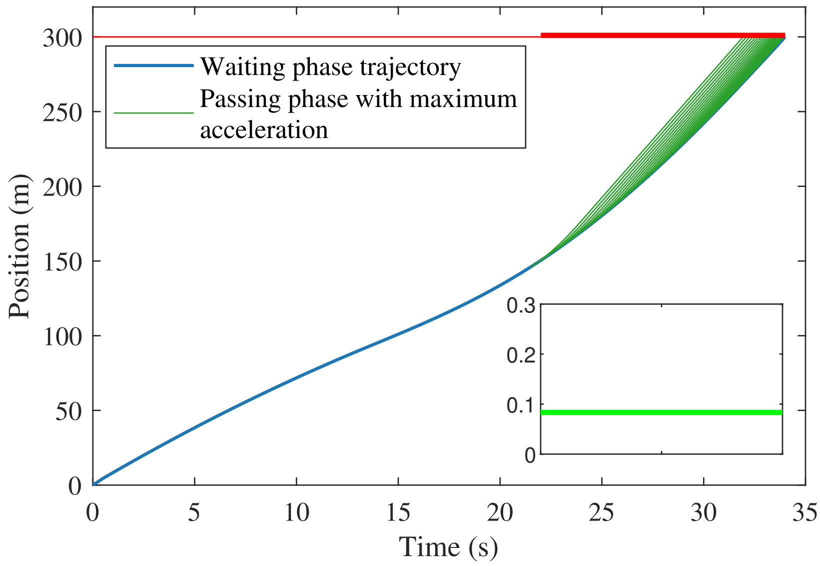
The impact of different advance time in the method developed in Section 4.5 is analysed. Three scenarios are presented where the vehicle receives the future signal information 1 second, 5 seconds and 10 seconds in advance separately. The results are shown in Table 4.8 and the state trajectories are shown in Fig. 4.10.

- Scenario a1: The vehicle receives the signal information 1 s in advance;
- Scenario a2: The vehicle receives the signal information 5 s in advance;
- Scenario a3: The vehicle receives the signal information 10 s in advance.

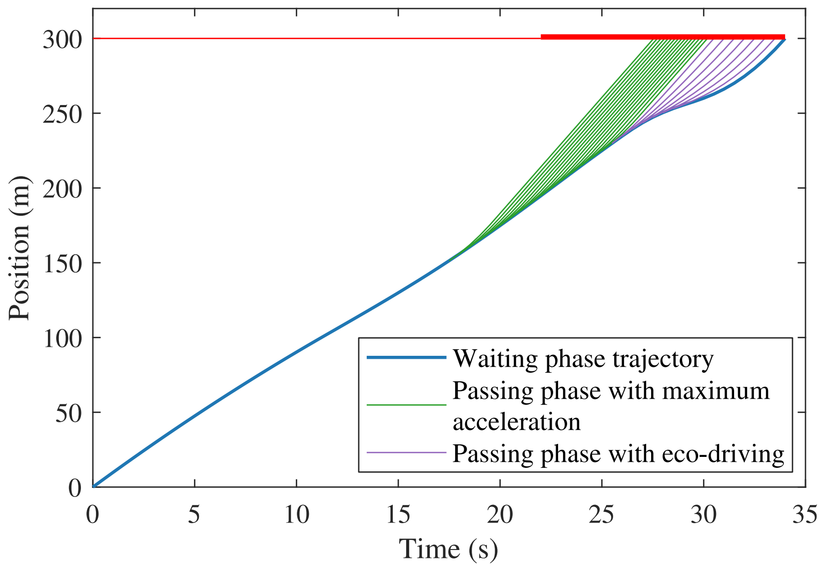
Table 4.8: Expected fuel consumption and expected travel time in different advance time

Advance time	Expected fuel consumption (mL)	Expected travel time (s)
0 s	49.16	33.43
1 s	49.96	33.24
5 s	48.74	29.99
10 s	43.68	28.35

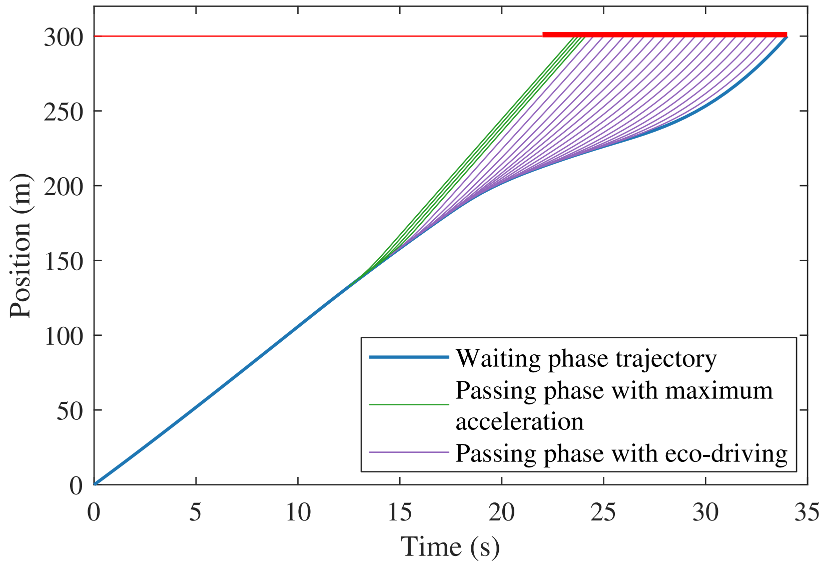
When the advance time is only 1 second, the vehicle still chooses the accelerating strategy using the maximum acceleration in the passing phase as it is not possible for it to arrive at the stop line in 1 second. The expected fuel consumption increases by 1.6% and the expected travel time decreases by 0.6% compared to the results when there is no advance time. The increase in fuel consumption is because the vehicle starts to accelerate from a further position in the first few passing phases and consumes more fuel. As the vehicle starts to accelerate 1 second in advance, this helps to reduce the travel time in the first few passing phases. When the vehicle can receive the signal information 5 seconds in advance, the expected fuel consumption reduces by less than 1% while the expected travel time reduces by more than 10%. The vehicle can arrive at the stop line in 5 seconds in the later seven passing routes and an eco-driving strategy with fixed terminal time is applied. When the vehicle receives the signal information in 10 seconds, it can arrive at the stop line in most passing phases and apply the eco-driving strategy. The expected fuel consumption reduces by more than 11% and the expected travel time reduces by 15% compared to the base scenario without advance time. The expected travel time is close to the minimum possible expected travel time 28.25 s when the time step is 0.5 s. We conclude that short advance time does not help much in reducing fuel consumption, but can reduce the travel time. Larger advance time reduces fuel consumption and travel time substantially.



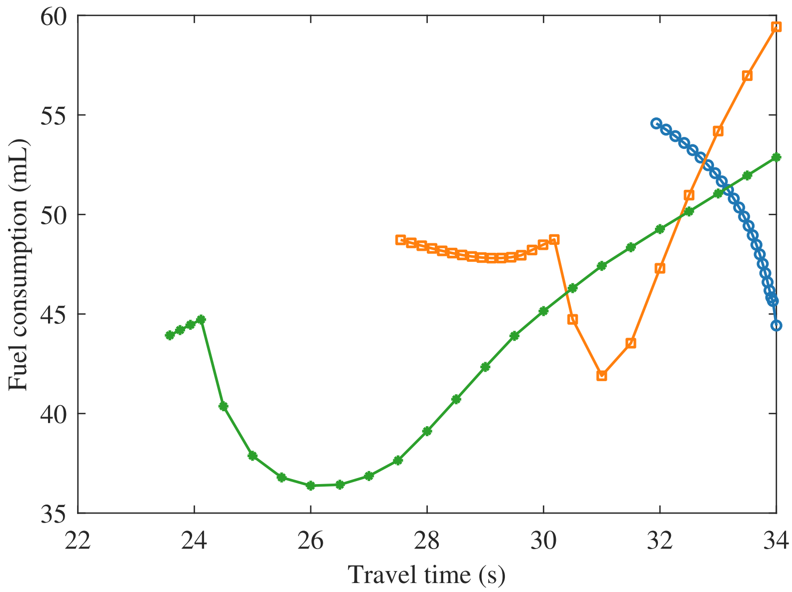
(a) Position in scenario a1



(b) Position in scenario a2



(c) Position in scenario a3



(d) Fuel consumption and travel time in each route

Figure 4.10: State trajectories in different advance time

4.7 Conclusions

Accurate future signal and phase information is highly important to trajectory planning for automated vehicles on the urban road. However, because of the adaptive signal control system or the stochastic behaviour of human-driven vehicles, the vehicle may not get the exact signal information or earliest available green time for it to pass the intersection. A multi-phase optimal control model was proposed to optimise the trajectory when the future signal information is uncertain. It was modelled as a hybrid system including two types of phases: passing phase and waiting phase. The vehicle chose the corresponding phase by observing the signal state at that moment. Thus, it can balance the objectives of minimising travel time and fuel consumption. To simplify the calculation, it was modelled as a discrete multi-phase optimal control model. The expected fuel consumption over all possible routes was the objective function. It was then transformed into a nonlinear optimisation model which can be solved more efficiently. The proposed model was also extended to a more general case that the vehicle can get the future signal timing information in advance for a limited time. Extensive numerical studies were taken to analyse the impact of internal parameters, external signal distribution and advance time.

We conclude by the study that

1. If there is no terminal speed constraint, the optimal driving pattern of minimising fuel consumption on the current link is to arrive at the stop line in the early period of the uncertain red light time;

2. Adding the terminal speed constraint reduces the total fuel consumption if the fuel consumption in recovering to maximum speed is considered;

3. Reducing the duration of a time step helps to reduce the expected travel time slightly but also increases the expected fuel consumption slightly;

4. If the mean of distribution increases, the expected travel time increases as well;

5. When the variance of the distribution decreases, the expected fuel consumption decreases as well, but the variance of the fuel consumption in each route increases;

6. The expected fuel consumption decreases when there is less uncertainty in the uncertain range;

7. The optimised trajectory is more sensitive to the upper range than the lower range;

8. If the vehicle can get the future signal information, the expected travel time will decrease significantly;

9. If the vehicle only gets the future signal information in a very short time, it may cause an increase in fuel consumption. However, if the vehicle gets the future signal information in advance for 10 seconds, the expected fuel consumption decreases by more than 11%.

Regarding future research, if the adaptive traffic control system updates the signal timing every cycle, such as TUC (Diakaki et al., 2002), the automated vehicle has enough time to plan its trajectory. However, many adaptive traffic control system tend to update the signal timing frequently to get better control performance, which adds difficulties in the trajectory planning for vehicles to reduce fuel consumption. In this case, a problem is how to balance the performance of intersection control and eco-driving method.

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5
Conclusions

Contents

5.1 Summary

The thesis focuses on intersection control and trajectory planning for AVs on urban roads. Chapter 1 presented the literature review, research questions, and the outline of the thesis. Three main chapters are included in the thesis. Chapter 2 dealt with intersection control of AVs and investigated how to integrate intersection control with trajectory planning. Chapter 3 and Chapter 4 dealt with the trajectory planning for AVs in different conditions. Chapter 3 optimised the trajectories of a heterogeneous platoon including AVs and HVs. Chapter 4 optimised the trajectories of an AV when the future signal information at the intersection is not available or only available in advance for a limited time. Finally, the thesis ends with some concluding remarks in Chapter 5.

5.2 Progress made in answering the research questions

Three research questions were described in Chapter 1 and investigated in the following three chapters. This section revisits the research questions and corresponding objectives, and summarises what we have done to answer the questions and achieve the objectives.

5.2.1 Research question 1

Research question 1: What is the benefit of integrating intersection control with trajectory planning?

- Objective 1.1: Identify the necessity of considering trajectory planning in intersection control.

In the conventional phase-based intersection control method, the trajectories of vehicles are only considered for signal coordination of multiple intersections and setting the yellow light duration. Because there is no way to directly control the movement of a conventional vehicle, adjusting the offset is an effective way of reducing the number of stops. In intersection control with AVs, it becomes possible to control trajectories of AVs and avoid most unnecessary stops. Apart from that, what is the value of considering the trajectory in intersection control?

It further reduces delay to integrate trajectory planning with intersection control. Firstly, yellow light is not required any more for AVs because the vehicle in the dilemma zone are given a high priority to pass the intersection. Secondly, the right of way can be switched more frequently as the vehicle makes decisions quickly without human errors and complies with any planned rule. Thirdly, it reduces the average delay to arrange vehicles' arrival time to the stop line in a cooperative way. Fourthly, the vehicle can adjust its behaviour in advance to achieve high speed passing the intersection. This reduces the occupancy time in the conflict area and hence improves the efficiency of the intersection.

The critical purpose of intersection control is allowing vehicles to pass the intersection safely and efficiently. As it is not reasonable to decelerate in the intersection area when the vehicle gets the right of way, it can accelerate freely after the stop line, and the trajectory in the intersection is only determined by the time and speed at the stop line. Thus, the connections between trajectory planning and intersection control are the time arriving at the stop line and the speed entering the intersection.

- Objective 1.2: Develop a method of integrating intersection control with trajectory planning for AVs on the urban road to reduce the total travel time.

A bilevel programming model was developed to integrate intersection control with trajectory planning. The upper level optimises the passing sequences of AVs to minimise the total travel time, and the lower level maximises the speed entering the intersection conflict area. The connections between the two levels are the arrival time and terminal speed. The arrival time is the output of the upper level and also the input of the lower level. The terminal speed is the output of the lower level and the input of the upper level. This feedback structure implies that trajectory planning not only follows the decision of intersection control but also determines the result of intersection control. Moreover, the two levels have a cooperative relationship. High speed of vehicles increases the efficiency of the intersection and reduces the total travel time. As intersection control reduces the total travel time, vehicles are less likely to decelerate and can achieve a high terminal speed. The cooperative relationship makes it possible to solve the bilevel problem with a few iterations. In addition, both levels are linear models which also reduce the calculation complexity.

- Objective 1.3: Conduct simulations to test the developed method.

A simulation platform including a simple two-way intersection was developed using Matlab. The developed bilevel model was solved by an iterative heuristic algorithm based on Gurobi. In order to improve the calculation efficiency, a platoon model was also proposed without sacrificing the control performance. It was based on the analysis of the relationship between safety time headway for vehicles on the same road and the inter-change time for vehicles on the conflicting roads. When certain conditions are satisfied, vehicles in platoons are preferred to passing together in terms of reducing the total travel time.

Three other intersection control methods were also tested and compared to the proposed bilevel method. One was a two-level optimisation model without the feedback connections which was used to show the advance of the feedback structure in the proposed bilevel model. Another was a First-in-first-out (FIFO) method which determined the right of way by the time entering a specific range. The last was a conventional actuated control method where vehicles passed the intersection in platoons, but their trajectories were not optimised. The proposed bilevel model outperformed the other three methods in various traffic demands and intersection lengths. The impact of communication ranges and traffic compositions on the proposed method are also analysed.

5.2.2 Research question 2

Research question 2: What is the benefit of cooperation between automated vehicles and human-driven vehicles?

- Objective 2.1: Develop a trajectory planning method by considering cooperation between AVs and HVs.

A general trajectory planning model was proposed to reduce fuel consumption for platoons approaching the intersection using a receding horizon model predictive control method. Heterogeneous platoons including AVs and HVs were considered in this model to enable the application in any penetration rate of AVs. The terminal time was set to the earliest green time that allows the AV to pass the intersection in order to reduce the travel time. The HV was modelled by the OVM. The trajectories of AVs were optimised by the proposed model which considered the impact of the behaviour of AVs on the following HVs. This was achieved by including the OVM into system dynamic functions of the platoon and using the total fuel consumption of the platoon as the objective function. In addition, the jerk of the AV, which is the derivative of the acceleration, was the control variable to increase driving comfort. In summary, the proposed model considered fuel efficiency, intersection capacity, and driving comfort.

The platoon dynamics were included in the model to adapt to the traffic state in a mixed traffic condition. At each time step, the leading AVs applied the splitting rule or merging rule to change the platoon setting. Thus, more vehicles benefit from the cooperative behaviour of AVs.

- Objective 2.2: Analyse the benefit of cooperation in a mixed autonomy traffic condition.

The boundary conditions and running cost in the model were tested in some simple scenarios to show the benefits of cooperative behaviour. The boundary conditions were to force AVs to arrive at the stop line with the maximum speed and zero acceleration at the terminal time. The high terminal speed of the leading vehicle helps the following vehicles to get high speed too. This increases the capacity of the intersection by more than 11% in the tested scenario.

The running cost was the total fuel consumption of the platoon. Three cooperative cases in the mixed autonomy traffic were investigated. The first was the cooperation between AVs and the following HVs. The total fuel consumption in the optimisation range decreases when more HVs join the platoon, but the total fuel consumption until they arrive at the stop line may arise. The second was the cooperation between AVs in the current platoon with AVs in the preceding platoon. When more vehicles apply the eco-driving method, the total fuel consumption decreases significantly no matter where it is located in the mixed traffic. The third was the cooperation among AVs. Fuel consumption only changes slightly in the different settings of platoons.

- Objective 2.3: Analyse the performance of the developed method under different penetration rates of AVs.

More realistic simulations were used to analyse the performance of the proposed method in different penetration rates of AVs. The case that the AV is not connected with the following HVs was also considered. The AV still tried to form a platoon of two vehicles with the direct following vehicle by detecting its real-time state via its built-in detectors. In the simulation results, when the proposed method was applied in the penetration rate of 60% AVs, it achieved the same level of fuel consumption reduction when no cooperation was applied with 100% penetration. Even when the maximum platoon size was limited to two because of no communication between vehicles, the cooperative eco-driving model reduced 4.7% more fuel than the model without cooperation. Similar results also were found in the reduction of travel time. Moreover, vehicles in the cooperation scenario also had less chances to stop on the road segment because of the decelerating of preceding AVs. The trajectory was smoothed by the cooperative behaviour, and the speed oscillation was also reduced.

5.2.3 Research question 3

Research question 3: How to optimise the trajectory at an intersection controlled by the adaptive traffic control system?

- Objective 3.1: Develop a trajectory planning method when the future signal is uncertain.

The uncertain signal duration, when the intersection is controlled by an adaptive traffic control system, was modelled as a scaled Beta distribution. The parameters in the distribution were estimated from the historical signal timing data and updated with the newly available signal data. A good property of the Beta distribution is that it can model many probabilistic distributions within the range of 0 to 1 using two parameters such as uniform distribution and truncated normal distribution.

Though the distribution of future signal was obtained, it may not be accurate, and the change of signal state has a probability of not happening. Two strategies which were chosen based on the signal state at that moment were proposed. At each time step, the vehicle observes the current signal state. If it is green, then it is safe to accelerate to pass the intersection. Otherwise, it needs to adjust its speed to wait for the possible green light in the next time step. The probability of each strategy was determined by the mentioned scaled Beta distribution. Thus, the distribution of the signal was included in the model without sacrificing safety. The objective function was the expected fuel consumption from the current time to the end of the uncertain range. A constraint on the terminal speed was also added to avoid blocking the following vehicles.

- Objective 3.2: Develop a trajectory planning method when the future signal is uncertain but available for a limited time in advance.

In the adaptive traffic control system, the optimisation operates in some frequencies. So the system knows the signal state over the future limited duration. A more general trajectory planning model was proposed to reduce fuel consumption by considering the limited available information on the future signal. The main difference is that when the vehicle knows the signal will turn green several seconds later, it may be possible to apply an appropriate speed pattern to arrive at the stop line when the signal just turns green instead of using the maximum acceleration. Whether the vehicle chooses the maximum acceleration strategy or the eco-driving strategy in the passing phase depends on whether it is possible to arrive at the stop line in the advance time. If it can, then the eco-driving strategy is chosen to reduce fuel consumption in the passing phase. Otherwise, the maximum acceleration strategy is chosen to reduce travel time in the passing phase. So, in this case, the vehicle had three possible strategies. The probability of each strategy depended on the probability of the signal state in the advance time later.

- Objective 3.3: Analyse the impact of model and system parameters on the performance of the developed methods.

Parameters in the model setting, such as the terminal speed constraint, time step size, advance time, and parameters in the scaled Beta distribution, such as the mean, variance, range, were tested. We found that when a terminal speed constraint is added, the expected fuel consumption is 20.9% less than that without the terminal speed constraint if the extended distance required to recover the maximum speed is considered, but the travel time is 8.8% higher. When the time step is smaller, the resulting expected fuel consumption increases and the expected travel time decreases. As the mean of the distribution increases, the expected travel time increases, while the expected fuel consumption increases before decreases. As the variance of the distribution increases, the expected fuel consumption increases, while the expected travel time fluctuates slightly. What is more, the optimised trajectory is more sensitive to the end time of uncertain signal compared to the start time. Different advance time was also tested in a more general trajectory planning model. We found that the impacts of advance time on fuel consumption and travel time are different. The expected fuel consumption increases slightly when the vehicle receives the signal information in advance for a short time but decreases significantly when the advance time is large. On the other hand, the travel time shows a negative relationship with advance time.

5.3 Contributions to knowledge and practice

Based on the work mentioned above, the main conclusions and contributions are described below.

5.3.1 Intersection control

- Trajectory planning and intersection control are interconnected and interact with each other.

Trajectory planning needs to get the allocated green time from intersection control. Intersection control can attain better performance if it knows when the vehicle will pass the intersection conflict area. The AV makes it more likely to have a cooperative relationship between trajectory planning and intersection control.

- Platoon is not always beneficial for intersection control.

When the safety time headway in the platoon is larger than the minimum gap time for two conflicting vehicles, it is better to assign the vehicle that is closer to the stop line a higher priority to pass regardless the platoon. Three propositions are proposed to identify the platoon which improves the calculation speed and does not sacrifice the performance.

5.3.2 Trajectory planning

- Cooperation between automated vehicles and human-driven vehicles can smooth the trajectory and reduce fuel consumption in mixed autonomy traffic.

In the proposed eco-driving model, the dynamics of following HVs are included in the system dynamic function, and the total fuel consumption of the platoon is the objective function. By adjusting the movement of the leading AVs, unnecessary acceleration and speed oscillation are suppressed for both AVs and HVs. As the AV has information about the next green time, it will not stop on red, and the following vehicles also benefit from that. The terminal speed constraint makes the AV accelerate in advance, and the following vehicles also have a much higher speed crossing the stop line, which reduces the travel time of the following vehicles and increases intersection efficiency.

- The benefit of cooperation increases with the platoon size and becomes trivial in the 100% penetration.

When more vehicles join the platoon, fuel consumption and travel time are reduced more. However, the leading AVs consume more fuel in changing its behaviour to control movements of the following vehicles. The reduction of fuel consumption and travel time resulting from cooperation decreases when the penetration rate of AVs is higher than 80% and becomes trivial in the 100% AVs.

- A multi-phase trajectory planning model is proposed to reduce fuel consumption when the future signal timing is uncertain.

A trajectory planning model, including two driving strategies, was proposed to adapt to the uncertain future signal information. At each time step, the vehicle chooses to pass or keep waiting based on the predicted signal state at that moment. The distribution of future signal timing is included in the decision of driving strategy. The advance of the model is that there is no risk of passing the intersection at the red light due to the hybrid model structure. Various parameters in the model setting and distribution of the signal timing are analysed.

- When the future signal timing information is available in advance, fuel consumption and travel time may be reduced quite significantly. However, too short advance time may not help reduce fuel consumption.

A more general trajectory planning was proposed when the future signal timing information is available in advance for a limited time. If this advance time is too short, this will not help to reduce fuel consumption, but the travel time can be reduced slightly. When advance time becomes larger, the vehicle may apply the eco-driving strategy in the passing phase, which reduces fuel consumption slightly but reduces travel time quite a lot. When the advance time is even larger, both fuel consumption and travel time reduce significantly.

5.4 Future research directions

The research on AVs gains a lot of attention. It is developing very fast and far from mature. Future research directions on intersection control and trajectory planning are discussed below.

In the thesis, a relatively strong assumption that all vehicles are CAVs was made in Chapter 2. A more realistic situation is mixed traffic with CVs, AVs, and HVs. In mixed traffic with CVs, how to estimate the traffic demand (Zheng and Liu, 2017; Rostami Shahrbabaki et al., 2018), queue length (Comert, 2016; Li et al., 2017)? How to develop a intersection control method without loop detectors (Liang et al., 2018)? The research on intersection control in mixed traffic with AVs is quite limited. There are two possible ways: distributed control without signal (Yang and Monterola, 2016) and central control with signal phases (Li and Zhou, 2017). No study compares the performance of these two approaches.

In this thesis, trajectory planning problems when the future signal timing information is fully available, available in advance for a limited time or not available were investigated. There are several directions to extend current research. Only longitudinal control is considered in the current work. How to plan the longitudinal trajectory and lateral trajectory together (Yu et al., 2018)? Perhaps, the cooperation on the lateral trajectory improves the traffic too. Only an isolated intersection is currently considered. There are some studies on the trajectory planning over multiple intersections (Butakov and Ioannou, 2016; De Nunzio et al., 2016), but they either need heavy calculation or are oversimplified. None of them tested the performance in the simulations with vehicles coming from all approaches. Optimising the trajectory can reduce fuel consumption, but the intersection signal timing is not optimised for that. How to optimise fuel consumption in intersection control? (Zu et al., 2018)

5.5 Concluding remarks

This thesis focuses on intersection control and trajectory planning for AVs on urban roads. It includes three parts of work: the first is an intersection control method for AVs, the second is a trajectory planning method for AVs in a mixed traffic environment and the third is a trajectory planning method when the future signal information is uncertain.

A bilevel programming model was proposed to integrate intersection control with trajectory planning for AVs. The interconnection relationship between intersection control and trajectory planning was considered in the feedback structure in the bilevel programming model. The linear model and cooperative relationship in both levels make it solved by existing solvers efficiently. A platoon-based approach was also proposed to reduce the calculation time without sacrificing the performance. The simulation tests showed significant improvement in reducing average delay under various traffic demands, intersection lengths, communication ranges, and traffic compositions.

A general eco-driving method was proposed for platoons in a mixed traffic situation with AVs and HVs, when AVs could receive the future signal and phase information from the intersection controller. It considered fuel consumption, travel time and driving comfort in the receding horizon model predictive control method. The platoon dynamics, including platoon splitting and merging rules, were also proposed to improve the performance. The properties of the boundary conditions and running cost were analysed in the simulation. What is more, the performances under different penetration rates of AVs and platoon sizes were also investigated.

Another eco-driving model was proposed to deal with the situation when the future signal information is not available or only available for a limited time ahead because of the adaptive traffic control system. When it is not available, a multi-phase optimal control model including two driving strategies, which are accelerating to pass and waiting to pass, was proposed. When it is available for a limited time in advance, a model including three driving strategies was proposed including a additional eco-driving strategy.

Some possible future research directions were discussed. Given the revolutionary mobility system by the AV technology, more research is still required to make better use of the infrastructure and information, such as shared mobility.

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