ROBUST RESIDUAL GENERATION FOR MODEL-BASED FAULT DIAGNOSIS OF DYNAMIC SYSTEMS

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ABSTRACT

To guarantee safe operation and mission completion, any fault in an automatic system has to be diagnosed as early as possible. Model-based techniques have been widely recognized as feasible and powerful approaches for diagnosing faults and require a mathematical model of the monitored system. A prerequisite for successful model-based fault diagnosis is satisfactory robustness with respect to modelling uncertainties. This thesis examines and develops further the theory and application of robust residual generation techniques in model-based fault diagnosis, beginning with a study and review of basic principles of model-based fault diagnosis. A number of strategies for the design of robust residual generators are then proposed. The thesis proposes a new full-order unknown input observer structure for robust residual generation and this structure is then used to design directional and minimum variance residuals. This is followed by a very thorough presentation of the eigenstructure assignment approach to fault diagnosis. A new algorithm to assign right observer eigenvectors in disturbance de-coupling design is presented. The disturbance de-coupling residual generation is then used for diagnosing faults in a jet engine system example. To facilitate this application, several techniques are proposed to derive an approximate disturbance distribution matrix. These techniques enlarge the application domain of disturbance de-coupling residual generation approaches. Robust residual generation can be treated as a multi-objective optimization problem in which fault sensitivity is to be maximized, whilst the sensitivity to modelling uncertainties is to be minimized. The thesis defines a number of performance indices in observer-based residual generation and the multi-objective optimization is solved by a combination of the method of inequalities and genetic algorithms. Finally, the thesis studies the design of optimally robust parity relations using multi-criterion optimization. The techniques developed in this thesis are well illustrated using either academic or practical application examples and the results show the effectiveness of the developed techniques.

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DECLARATION

This thesis describes the results of my own work. Any reference to the work of other researchers is clearly indicated in the text.

Neither the whole nor any part of this work has already, or is being currently, submitted for any other degree, diploma to this or any other University or institute of learning.

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Chapter 1

INTRODUCTION

1.1 Background

1.1.1 Importance of fault diagnosis

Modern control systems are becoming more and more complex and control algorithms more and more sophisticated. Consequently, the issues of reliability, operating safety and environmental protection are of major importance, especially for safety-critical systems like chemical plants, nuclear reactors, spacecraft, aircraft, computerized banking systems and high speed transportation systems. If faults occur, consequences can be extremely serious in terms of human mortality, environmental impact and economic loss. Hence, there is a growing need for on-line supervision and fault diagnosis to increase the reliability of such safety-critical systems. Therefore, early indications concerning which faults are developing can help avoid system breakdown, mission abortion and catastrophes.

Over the last two decades, the research on fault diagnosis has gained increasing consideration world-wide. This development was (and still is) mainly stimulated by the trend of automation towards more complexity and the growing demand for higher availability and security of control systems. However, a strong impetus also comes from the side of modern control theory that has brought forth powerful techniques of mathematical modelling, state estimation and parameter identification that have been made feasible by the spectacular progresses of computer technology.

1.1.2 Fault diagnosis terminology

A "fault" is to be understood as an unexpected change of system function[1], although it may not represent physical failure or breakdown. Such a fault or malfunction hampers or disturbs the normal operation of an automatic system, thus causing an unacceptable deterioration of the performance of the system or even leading to dangerous situations. We use the term "fault" rather than "failure" to denote a malfunction rather than a catastrophe. The term *failure* suggests complete breakdown of a system component or function, whilst the term *fault* may be used to indicate that a malfunction may be tolerable at its present stage. A fault must be diagnosed as early as possible even it is tolerable at its early stage, to prevent any serious consequences.

A monitoring system which is used to detect faults and diagnose their location and significance in a system is called a "fault diagnosis system". Such a system normally consists of the following tasks:

- **Fault detection:** to make a binary decision – either that something has gone wrong or that everything is fine.
- **Fault isolation:** to determine the location of the fault, e.g., which sensor or actuator has become faulty.
- **Fault identification:** to estimate the size and type or nature of the fault.

The relative importance of three tasks are obviously subjective, however the detection is an absolute must for any practical system and isolation is almost equally important. Fault identification, on the other hand, whilst undoubtedly helpful, may not be essential if no reconfiguration action is involved. Hence, fault diagnosis is very often considered as fault detection and isolation, abbreviated as FDI, in the literature.

[1]An alternative definition given by Isermann (1984): a "fault" is defined as "a non permitted deviation of a characteristic property which leads to the inability to fulfil the intended purpose".

1.1.3 Fault diagnosis in intelligent fault-tolerant control

There is an increasing need for controlled systems to continue operating acceptably to fulfil specified functions following faults in the system being controlled or in the controller. A control system with this kind of fault-tolerance capability is defined as a fault-tolerant control system. There may be some graceful performance degradation for a fault-tolerant system to operate under a faulty condition, however the primary objective is to maintain system operation and give the human operator (or automatic monitoring system) reasonable time to repair the system or to use alternative measures to avoid catastrophes. Fault-tolerant control has received increasing attention recently, motivated by the need to achieve high levels of reliability, maintainability and performance in situations where the controlled system can have potentially damaging effects on the environment if faults in its components take place. For instance, in hazardous chemical and nuclear plants, the consequences of an improper control action following a control system component fault can be disastrous. In the case of flight control systems, safety is the greatest priority, which implies that even in the presence of failed components the aircraft must be able to land safely.

A fault-tolerant control system is designed to retain some portion of its control integrity in the event of a specified set of possible component faults or large changes in the system operating conditions that resemble these faults. This can only be done if the control system has built in an element of automatic reconfiguration, once a malfunction has been detected and isolated. Fault diagnosis plays an important role in the fault-tolerant control, as before any control law reconfiguration is possible the fault must be reliably detected, isolated, and the information should be passed to a supervision mechanism to make proper decision.

Fault-tolerance is considered as one of characteristics of intelligent systems. According to Stengel (1991): "By design or implementation, failure-tolerant control systems are *intelligent* systems". Aström (1991) has also stated: "Fault diagnosis is an essential ingredient property of an intelligent control system". Many important issues in fault-tolerant control systems can be found in a recent plenary paper by Patton (1993).

1.1.4 Model-based fault diagnosis

In practice, the most frequently used diagnosis method is to monitor the level (or trend) of a particular signal, and taking action when the signal reached a given threshold. This method of limit checking, whilst simple to implement, has serious drawbacks. The first drawback is the possibility of false alarms in the event of noise, the input variations and the change of operating point. The second drawback is that a single fault could cause many system signals to exceed their limits and appear as multiple faults, and hence fault isolation is very difficult. The use of consistency checking for a number of system signals which can eliminate the above problems, is an important way of enhancing the detection and isolation or fault diagnosis capability of an automated system. However, a mathematical model which gives functional relationships among different system signals is needed.

A traditional approach to fault diagnosis in the wider application context is based on "hardware (or physical/parallel) redundancy" methods which use multiple lanes of sensors, actuators, computers and software to measure and/or control a particular variable. Typically, a voting scheme is applied to the hardware redundant system to decide if and when a fault has occurred and its likely location amongst redundant system components. The use of multiple redundancy in this way is common, for example with digital fly-by-wire flight control systems e.g. the AIRBUS 320 (Favre, 1994) and in other applications such as in nuclear reactors. The major problems encountered with hardware redundancy are the extra equipment and maintenance cost and, furthermore, the additional space required to accommodate the equipment.

In view of the conflict between reliability and the cost of adding more hardware, it is sensible to attempt to use the dissimilar measured values together to *cross check*[2] each other, rather than replicating each hardware individually; this is the concept of "analytical (functional) redundancy" which uses redundant analytical (or functional) relationships between various measured variables of the monitored process (eg inputs/outputs; outputs/outputs; inputs/inputs). Fig.1.1 illustrated the hardware and analytical redundancy concepts. No additional hardware faults are introduced into an analytical redundant scheme, because no extra hardware is required, hence analytical redundancy is potentially more reliable than hardware redundancy (van Schrick, 1991; van Schrick, 1993).

[2] This procedure is sometimes referred to as *data reconciliation*.

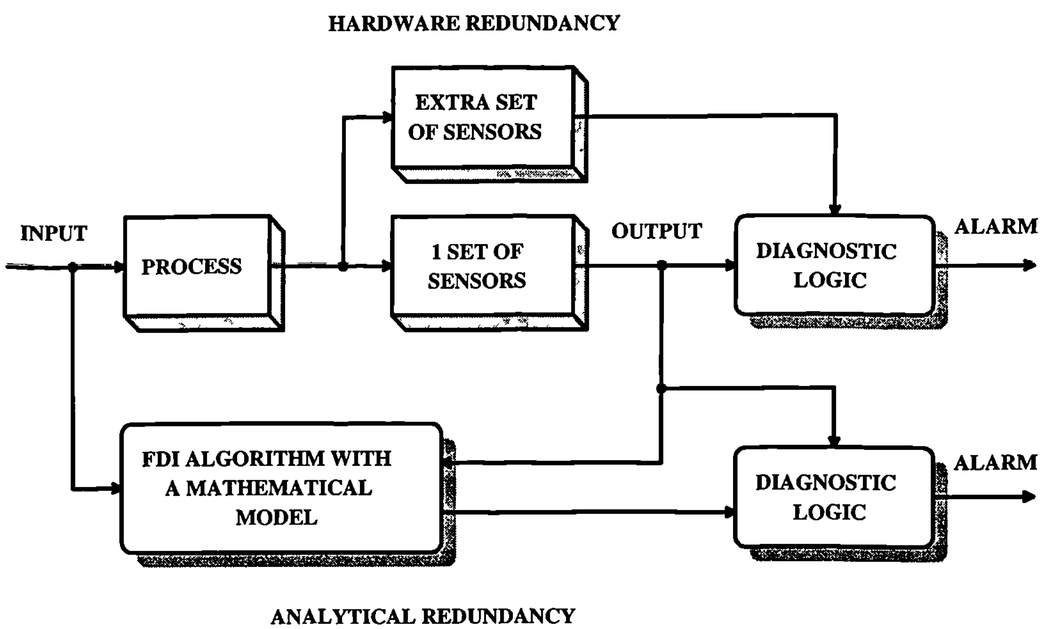


Figure 1.1: **Hardware vs analytical redundancy**

In analytical redundancy schemes, the resulting difference generated from the consistency checking of different variables is called as a *residual* signal. The residual should be zero-valued when the system is normal, and should diverge from zero when a fault occurs in the system. This zero and non-zero property of the residual is used to determine whether or not faults have occurred. Analytical redundancy makes use of a mathematical model of the monitored process and is therefore often referred to as the "model-based approach" to fault diagnosis.

Consistency checking in analytical redundancy is normally achieved through a comparison between a measured signal with its estimation. The estimation is generated by the mathematical model of the system being considered. The comparison is done using the residual quantities which give the difference between the measured signals and signals generated by the mathematical model. Hence, *model-based fault diagnosis* can be defined as *the determination of faults of a system from the comparison of available system measurements with a priori information represented by the system's mathematical model, through generation of residual quantities and their analysis.* A residual is a fault indicator or an accentuating signal which reflects the faulty situation of the monitored system.

The major advantage of the model-based approach is that no additional hardware components are needed in order to realize an FDI algorithm. A model-based FDI algorithm can be implemented in software on the process control computer. Furthermore, the measurements necessary to control the process are, in many cases, also sufficient for the FDI algorithm so that no additional sensors have to be installed. Under theses circumstances, only additional storage capacity and possibly greater computer power is needed for the implementation of a model-based FDI algorithm. Immense developments in computer technology have made such methods very feasible and practicable.

1.1.5 Robustness in model-based fault diagnosis

Model-based FDI makes use of mathematical models of the supervised system, however a perfectly *accurate* and *complete* mathematical model of a physical system is never available. Usually, the parameters of the system may varying with time in an uncertain manner, and the characteristics of the disturbances and noise are unknown so that they cannot be modelled accurately. Hence, there is always a mismatch between the actual process and its mathematical model even if there are no process faults. Apart from the modelling used for the purpose of control, such discrepancies cause fundamental methodology difficulties in FDI applications. They constitute a source of false and missed alarms which can corrupt the FDI system performance to such an extent that it may even become totally useless. The effect of modelling uncertainties is therefore the most crucial point in the model-based FDI concept, and the solution of this problem is the key for its practical applicability (Frank, 1991a).

To overcome the difficulties introduced by modelling uncertainty, a model-based FDI has to be made robust, i.e. insensitive or even invariant to modelling uncertainty. Sometimes, a mere reduction of the sensitivity to modelling uncertainty does not solve the problem because such a sensitivity reduction may be associated with a reduction of the sensitivity to faults (Frank, 1991a). A more meaningful formulation of the robust FDI problem is to increase robustness to modelling uncertainty, whilst without losing (or even with an increase of) fault sensitivity. An FDI scheme designed to provide satisfactory sensitivity to faults, associated with the necessary robustness with respect to modelling uncertainty, is called *a robust FDI scheme* (Frank, 1991a). The importance of robustness in model-based FDI has been widely recognized by both academia and industry. The development of robust model-based FDI methods has been a key research topic during the last 10 years. A number of methods have been proposed to tackle this problem, for example, the unknown input observer, eigenstructure assignment, optimally robust parity relation methods. However, the research is still under the way to develop the practically applicable methods.

An important task of the model-based FDI scheme is to be able to diagnose *incipient faults* [3] in a system before they are manifested as problems require either human operator or automatic system intervention. The diagnosis of hard and abrupt faults is relatively easy, because their effects on the FDI system are larger than modelling uncertainty and can be diagnosed by placing an appropriate threshold on the residual. However, incipient faults have a small effect on residuals, and can be hidden as a consequence of modelling uncertainty. This highlights the need of robustness in FDI. The effect of an incipient fault on the monitored system is very small and almost unnoticeable when it occurs. However, it may develop slowly to cause very serious consequences, although it may be tolerable in its early stage. It is important to note that a soft fault is a malfunction condition which is non-serious (in its present state) and which often develops in a continuous way (i.e. which does not contain discontinuous signal characteristics brought about as a consequence of abrupt changes). The presence of soft faults may not necessarily downgrade the performance of the plant significantly, however, such faults will indicate that the sensor (or other component) should be replaced, or that the system should be re-configured before the probability of more serious malfunction increases. Prompt indication of incipient faults can give the operator (or an automatic monitoring system) enough information and time to take decisive actions to prevent any serious failure in the system. The successful detection and diagnosis of soft faults can therefore be considered as the hardest challenge for the design and evaluation of algorithms working in a safety-critical environment.

1.1.6 Brief history of model-based fault diagnosis

Although many approaches to fault diagnosis using the model-based concept have been proposed over the last two decades, it is not possible to mention all of them. In the author's opinion, the following list presents some of the key developments in model-based fault diagnosis:

[3] Small and slowly developing faults are normally defined as incipient faults, and sometimes called soft faults.

1971: The idea of replacing hardware redundancy by analytical redundancy was originated by Beard (1971) at MIT. Beard developed fault (failure) detection filters which generate directional residuals for FDI. For recent developments, see Park and Rizzoni (1994).

1971: Mehra and Peschon (1971) introduced a general procedure for FDI using innovations (or residuals) generated by a Kalman filter. The faults are diagnosed by statistical testing on whiteness, mean and covariance of residuals.

1974: Willsky and Jones (Willsky and Jones, 1974; Willsky and Jones, 1976) developed an FDI strategy which uses Generalized Likelihood Ratio (GLR) testing on a residual generated by a Kalman filter to diagnose faults.

1974: The multiple model adaptive filter approach, which involves multiple hypothesis testing on residuals generated by a bank of Kalman filters, should be attributed to a number of investigators, including Willsky *et al.* (Willsky, Deyst and Crawford, 1974; Willsky, Deyst and Crawford, 1975) and Montgomery and Caglayan (1976).

1975: Clark, Fosth and Walton (1975) used Luenberger observers for fault detection, and various sensor fault isolation schemes were later developed by Clark (Clark, 1978a; Clark, 1978b; Clark, 1979).

1979: The parity relation approach to generate the residual (or parity vector), based upon consistency checking on system input and output data over a time window, was originally proposed by Mironovski (1979) although he used a different terminology. Unfortunately, this paper has not received enough attention due to its limited availability. The approach was later, independently proposed by Chow and Willsky (1984), and has been expressed in several different versions. For example, Gertler (1988) gave a parity relation design method in z-domain, Chen and Zhang (1990) developed a stochastic system FDI approach based upon a direct development of the parity vector concept used in hardware redundancy.

1980: The two stages of model-based FDI structure were first described by Chow and Willsky (1980) and restated in Chow and Willsky (1984).

1981: Leininger (1981) pointed out the impact of modelling errors on FDI performance. The first attempt of improving robustness of observer-based FDI approaches is attributed to Frank and Keller (1981).

1982: FDI based on parameter estimation: this approach directly uses system identification and hence it is difficult to identify its origin. According to Isermann (1984), Geiger (1982) was the first to apply this approach.

1982: Watanabe and Himmelblau (1982) introduced a robust sensor detection method using an unknown input observer (UIO). Robust FDI based on UIOs has been studied extensively by Frank's group at the University of Duisburg, Germany, and many contributions have been made by this group, for example, Frank and Wünnenberg (1987), Frank and Wünnenberg (1989), Wünnenberg (1990), Frank (1990), Frank (1991a), Frank and Seliger (1991) and Seliger and Frank (1991a). Chen and Zhang (1991) proposed a robust actuator fault isolation scheme and demonstrated using a chemical process. Ge and Fang (Ge and Fang, 1988; Ge and Fang, 1989) developed a robust component FDI approach using the so-called robust observation method which is similar to UIOs, in principle. Viswanadham et al. (Viswanadham and Srichander, 1987; Phatak and Viswanadham, 1988) proposed an actuator fault isolation scheme which is an important original contribution, however they did not consider robustness issues.

1986: Patton, Willcox and Winter (1986) proposed an FDI method based on eigenstructure assignment and this approach has been studied extensively by Patton et al.. Many developments have been made, for example, Patton (1988), Patton and Kangethe (1989) and Patton and Chen (1991g).

1986: Lou, Willsky and Verghese (1986) developed a strategy to design "optimally robust parity relations" for diagnosing faults in systems represented by multiple models.

1987: Viswanadham, Taylor and Luce (1987) introduced a new residual generation method based on a factorization of the system transfer matrix. This approach was later developed by Ding and Frank (1990) and is normally regarded as a frequency domain residual generation approach.

1988: Viswanadham and Minto (1988) proposed solutions for improving the robustness of frequency domain residual generation using H^∞ optimization techniques. Studies on this problem have been extended by Ding and Frank in a series of papers, e.g., Ding and Frank (1991), Frank and Ding (1993), Ding, Guo and Frank (1993) and Frank and Ding (1994). Recently, Qiu and Gertler (1993) also solved the same problem with a different solution.

1988: When residuals cannot be made robust against system uncertainty, the robust FDI can be achieved by robust decision making using adaptive thresholds. Emami-Naeini, Akhter and Rock (1988) introduced the *threshold selector* concept to generate adaptive thresholds and the approach was later generalized by Ding and Frank (Ding and Frank, 1991; Frank and Ding, 1993; Ding et al., 1993; Frank and Ding, 1994). Note that, Clark (1989) also proposed a method to produce adaptive thresholds, based on empirical rules.

1989: Gertler and colleagues proposed a scheme to design robust parity relations using the "orthogonal parity relations" concept (Gertler and Luo, 1989; Gertler, Fang and Luo, 1990; Gertler and Singer, 1990; Gertler, 1991; Gertler and Kunwer, 1993).

1991: A generalized residual generator structure was described by Patton and Chen (1991e).

1991: Patton and Chen (Patton and Chen, 1991f; Patton and Chen, 1991b; Patton, Chen and Zhang, 1992) proposed several schemes to represent modelling uncertainties from various sources as additive disturbances with an estimated distribution matrix. Robust FDI is thus achieved using disturbance de-coupling approaches. To date, this is the most important contribution in robust FDI. So far, most robust residual generation methods based on the assumption that disturbance distribution matrices are known, however this assumption is not valid for most real systems. The contributions by Patton and Chen have paved a way for real application of robust FDI techniques.

1991: Robust FDI for nonlinear dynamic systems using nonlinear unknown input observers (Seliger and Frank, 1991a; Seliger and Frank, 1991b).

1992: Robust FDI using μ synthesis (Mangoubi, Appleby and Farrell, 1992; Appleby, Dowdle and Vander Velde, 1991).

1994: Chen, Patton and Liu (1994a) developed a numerical optimization method to design observer-based residual generators.

During the development of model-based FDI, many excellent survey and tutorial papers have been published. Different papers discussed varied aspects of the problem from different prospectives. Some of the most notable survey papers are commented upon briefly as follows:

- Willsky (1976) was the first survey paper on model-based FDI which presents key concepts of analytical redundancy. The emphasis of the paper was on stochastic systems and jump detection.

- The survey paper by Mironovski (1980) focused on a group of methods in which the diagnosis is carried out by checking the algebraic relations between system signals. The relations to be checked are generated by either parity relations or Luenberger observers. The paper gave a residual generation structure which was also used by Basseville (1988) and the classification of diagnostic methods was also discussed in the paper.

- Isermann (1984) illustrated that process fault diagnosis can be achieved using the estimation of unmeasurable process parameters and/or state variables. Both parameter estimation and observer-based methods were discussed. The paper gave a generalized structure of FDI based on process models and unmeasurable quantities. This structure has been referred to in many subsequent papers, e.g. Frank (1990).

- Isermann (1987) reported some experiences in the use of parameter estimation for process FDI.

- Frank (1987) gave a comprehensive survey on observer-based FDI methods. Many different schemes using both linear and nonlinear observers are reviewed. The paper also discussed the parameter sensitivity reduction problem in model-based FDI. Some application results were presented. The paper gave a list of application examples of model-based FDI, and a brief historical review.

- Basseville (1988) addressed the problems of detection, estimation and diagnosis of changes in dynamical properties of signals or systems, with particular emphasis on statistical methods for detection, to provide a general framework for change detection in signals and systems.

- The survey paper by Gertler (1988) was not very comprehensive, however it presented basic concepts and gave some essential definitions. Some problems discussed in this paper, such as isolability conditions and sensitivity and robustness, are still of tutorial value today.

- Frank (1990) outlined the principles and most important techniques of model-based residual generation using parameter identification and state estimation methods with emphasis upon the latest attempts to achieve robustness with respect to modelling uncertainty. The possibility of combining model-based and knowledge-based techniques for FDI was also discussed.

- Isermann and Freyermuth (1990) studied on-line FDI expert systems with analytical (parameter estimation) and heuristic process knowledge. This paper was followed by their another survey paper (Isermann and Freyermuth, 1991a), and an application paper (Isermann and Freyermuth, 1991b).

- Tzafestas and Watanabe (1990) reviewed two major approaches for FDI, the mathematical model (or analysis) approach and the knowledge-based (or expert system) approach. The techniques of the former approach were presented in two groups, namely "statistical techniques" and "analytical redundancy" techniques, whereas the techniques of the latter approach are classified as "shallow" and "deep" knowledge-based techniques. The most distinguish feature of this paper was its excellence on the survey of stochastic techniques.

- Patton (1991) emphasised aerospace applications of model-based FDI and analytical redundancy.

- Frank (1991a) has shown how to enhance robustness in observer-based FDI by reviewing disturbance de-coupling observers, optimal parity relations, H^∞ observers and adaptive thresholds.

- Gertler (1991) presented a tutorial on residual generator synthesis methods. The best known residual generation methods, including parity equations, diagnostic observers and Kalman filtering, were presented in a consistent framework. The discussion was organized along two residual enhancement concepts, namely structured and fixed direction residual sets. A numerical example was used to show how parity relation and observer based designs lead to equivalent residual generators, once the design objectives are specified. Robustness issues were also addresses in this paper.

- Patton and Chen (1991e) unified the observer-based and parity relations approaches under a common parity space format. The model-based FDI has been re-stated by them as the generation and analysis of residual signals in the parity space. Their paper presented a generalized framework of residual generators and provided some important definitions, as well as demonstrating robust fault diagnosis methods using two tutorial examples. The paper also formally proved the equivalence between observer-based and parity relation approaches in residual generation.

- The survey paper by Patton and Chen (1992b) followed the same philosophy given by Patton and Chen (1991e). The emphasis was on different synthesis methods for residual generators with a particular reference on aerospace applications.

- Isermann (1993a) gave a tutorial for parameter estimation FDI methods based a number of real or laboratory applications. This was an application-oriented tutorial paper.

- Frank (1993) reviewed the advanced methods of observer-based FDI. The paper discussed the issue of improving decision-making robustness using fuzzy logic. The paper, however was limited in its scope to research developments within Frank's group.

- The paper by Gertler and Kunwer (1993) studied both perfect and approximate disturbance de-coupled residual generator designs, with an emphasis on z-domain parity relation design methods and with a numerical example to demonstrate approximate de-coupling.

- Patton (1993) studied the robustness issues in fault-tolerant control systems, including diagnosis and reconfiguration issues. The paper pointed out that the best way forward in fault-tolerant control is to integrate together FDI and controller functions in analysis and design, so that joint stability and performance robustness properties can be optimized.

- Patton and Chen (1993b) reviewed robustness issues against modelling uncertainty from different sources and a number of solutions in robust FDI were also presented.

- Isermann (1993b) discussed the *applicability* of different FDI methods based on their requirements and results from simulations. The work was followed by a more comprehensive paper (Isermann, 1994) in which the integration of different FDI methods was also studied.

- Patton, Chen and Nielsen (1994) presented some guide-lines for engineers in the choice of different model-based FDI methods.

- Patton (1994) presented an up-dated review of the state of the art of robust model-based FDI techniques.

The above list is inconclusive and there are many other survey papers which emphasise on different aspects of the problem, e.g., Walker (1983); Himmelblau (1986); Tzafestas (1989); Frank (1991b); Ray and Luck (1991) Frank (1992b); Frank and Köppen (1993); Martin (1993); Patton and Chen (1993a); Stein (1993); Stengel (1993).

There are three encyclopedia articles on model-based FDI techniques available, Frank (1992a) presented basic principles, Patton and Chen (1992c) discussed robustness issues and Labarrère and Patton (1993) emphasised aerospace applications.

Model-based FDI techniques have been summarized in the following books: Pau (1975); Himmelblau (1978); Basseville and Benveniste (1986); Singh, Hindi, Schmidt and Tzafestas (1987); Singh et al. (1987); Viswanadham, Sarma and Singh (1987); Patton, Frank and Clark (1989); Brunet, Jaume, Labarrère, Rault and Vergé (1990); Basseville and Nikiforov (1993) and Patton, Frank and Clark (1995). It should be pointed out that most of the books on model-based FDI are multi-authored books, this is mainly because this technique is still in a developmental stage.

Papers on model-based FDI techniques can be found in many engineering journals and IFAC, IMACS, IEEE, IEE and other conferences. There are three recent symposia specially dedicated to fault diagnosis: SAFEPROCESS'91 (Isermann, 1991), TOOLDIAG'93 (Labarrère, 1993) and SAFEPROCESS'94 (Ruokonen, 1994). It is interesting to notice that, in SAFEPROCESS'94 there were many short reviews, comparison studies and benchmark testing for a number of techniques. This is a sign that model-based FDI techniques are moving towards a mature status.

1.2 Outline of the Thesis

To detect and isolate faults in a dynamic system, based on the use of an analytical model, a declarative or residual signal must be used, which is derived from a combination of real measurements and estimates (generated by the model). The robustness problem can be tackled by defining the independent sensitivities of the residual to uncertainties and faults. Following from the definition given above, a robust FDI scheme is one whose residual is insensitive to uncertainties whilst sensitive (in a certain way) to faults. The aim of robust design of the FDI scheme is to reduce the effects of uncertainties on the residuals, and (or) to enhance the effects of faults acting on the residuals. The success of fault diagnosis depends on the quality of the residuals. A preliminary requirement of residuals for successful diagnosis is the robustness with respect to modelling uncertainty. The **main aim** of this thesis is *to develop robust residual generation strategies for model-based fault diagnosis of dynamic uncertain systems.* The thesis consists of 8 chapters and the main contributions are presented in Chapters 2–7. Each chapter is devoted to a particular problem in robust residual generation, and hence the chapters are relatively independent although they are related in some ways. The thesis is organized as follows:

Chapter 2 reviews the state of the art of model-based fault diagnosis techniques. The fault diagnosis problem is formalized in an uniform framework by presenting the mathematical description and definitions. This properly defined framework gives a clear picture of the principles and problems associated with model-based fault diagnosis. The fundamental issue of model based methods is the generation of residual signals using the mathematical model of the monitored system. By analysing the fault-indicating signal residual, the nature of faults can be obtained. A generalized structure of the residual generator is presented in this chapter. This gives ideas of how to design and implement the residual generation. The residual generator can be purposely designed for achieving the required diagnosis performances, e.g, fault isolation, disturbance de-coupling and residual frequency response shaping.

In order to design a robust residual generator, we need to make some assumptions about the modelling uncertainty. The most frequently used assumption is that the modelling uncertainty is expressed as a disturbance term in the system dynamic equation. Although the magnitude of the disturbance is unknown, its distribution (or direction) is assumed known *a priori*. Based on this assumption, the disturbance de-coupling residual generator can be designed using unknown input observer theory or via the eigenstructure assignment technique. Robust fault diagnosis is then achievable using disturbance de-coupled residuals. Follow this philosophy, Chapter 3 and Chapter 4 present some strategies for designing disturbance de-coupling residual generators.

Chapter 3 studies the approach to robust residual generation with the aid of the unknown input observer (UIO). The principle of the UIO is to make the state estimation error de-coupled from the disturbance. Since the residual is defined as the *weighted output estimation error*, the residual is also de-coupled from disturbances. This chapter presents a new full-order unknown input observer structure. The necessary and sufficient conditions for a UIO to exist presented in this chapter are very easy to verify and the design procedure is very simple. Robust sensor and actuator fault isolation schemes based on UIOs are presented in this chapter and a chemical reactor is used to illustrate the robust actuator isolation principles. This chapter also presents a method to make the residual have both disturbance de-coupling and directional properties, by combining the unknown input observer and fault detection filter theories. The directional property makes fault isolation achievable. Another contribution of this chapter is the optimal state estimation of stochastic systems with unknown inputs. It is proved that the design freedom left after disturbance de-coupling can be used to make the state estimation error have minimal variance. The use of this optimal disturbance de-coupled observer in fault detection is illustrated using a simplified flight control example.

Chapter 4 focuses on the disturbance de-coupled residual generator design via eigenstructure assignment. The most challenging problem in fault diagnosis is the correct design of the residual. State estimation is not necessary in FDI, and hence the state estimation error does not needed to be de-coupled from the disturbance. What is actually required is that the disturbance be de-coupled from the residual. The correct disturbance de-coupling can be achieved by assigning left observer eigenvectors orthogonal to disturbance directions or assigning right observer eigenvectors parallel to disturbance directions. The most important contribution of this chapter is the proposal of a new method for assigning right eigenvectors of the observer. This is equivalent to the assignment of left eigenvectors for a controlled system, a problem which is rarely studied in the literatures. The principles, existence conditions and the design procedure for the eigenstructure assignment approach to robust residual generation are presented in Chapter 4, where it is also shown that the remaining design freedom, after the disturbance de-coupling has been satisfied, can be utilized to optimize other performance indices (such as fault sensitivity). For a discrete-time design, a dead-beat disturbance de-coupling residual generator can be designed which has a direct correspondence with parity relations. Two numerical examples are presented in this chapter to illustrate the design procedure and de-coupling principles.

The theory of disturbance de-coupling for robust fault diagnosis has being developed for some years, however few investigators have shown how to apply this method to real applications. The difficulty is caused by the mis-match between the theoretical assumptions and practical reality. In most practical systems, the disturbance distribution matrix is not known. Disturbance de-coupling methods which require the disturbance distribution matrix cannot be applied directly to the system with unknown disturbance distribution matrix.

Chapter 5 demonstrates how to apply the disturbance de-coupling method to a system with modelling uncertainty. It is proved that an approximate disturbance term with an estimated distribution matrix can be used to represent the effect of modelling uncertainty on the system. Using this approximate distribution matrix in the disturbance de-coupling residual design, the *nearly robust* fault diagnosis is achievable. A number of methods for finding the approximate distribution matrix are given to deal with different uncertainty cases, based on either optimization or identification techniques. The methods developed in Chapter 5 are applied to a jet engine simulation system to demonstrate the effectiveness of robust residual for detecting incipient faults. The simulation shows satisfactory results. This jet engine is a complex, highly nonlinear and high order system and, any techniques applicable to this system should also be applicable to other complex non-linear and uncertain dynamical systems.

The purpose of robust residual design is to make the residual maximally sensitive to faults and minimally insensitive to modelling uncertainty. **Chapter 6** develops a new approach to the design of optimal residuals for detecting incipient faults, based on multi-objective optimization and the genetic algorithm. In this approach the residual is generated via an observer. To reduce false and missed alarm rates in fault detection, a number of performance indices are introduced into the observer design. Some performance indices are expressed in the frequency domain to take account of the frequency distributions of faults, noise and modelling uncertainties. All objectives are then reformulated into a set of inequality constraints on the performance indices. The genetic algorithm is thus used to search an optimal solution to satisfy these inequality constraints on performance indices. The approach developed is applied to a flight control system example and simulation results show that incipient sensor faults can be detected reliably in the presence of modelling uncertainty.

Chapter 7 studies the robust residual generation using optimally robust parity relations. The system parameters are considered to vary within known bounds, representative points in uncertainty regions are chosen to represent the uncertainty. The system dynamics are effectively describable using multiple linear models. A robust residual should be insensitive to changes in these models. This objective is achievable by minimizing a defined performance index. To avoid the reduction of fault sensitivity during the minimization of the sensitivity to uncertainty, the fault sensitivity is also used as a performance index to be maximized. Thus, the robust residual design is formulated as a multi-criterion optimization problem. The chapter shows a number ways of mixing these two performance indices together to form a single objective optimization problem. This problem is then solved using singular value decomposition and the computation of the generalized eigenstructure. Other developments in the design of robust parity relations are also discussed. A numerical example is used to demonstrate the method developed in this chapter.

Chapter 8 summarizes the contributions and achievements of the thesis, and provides some recommendations for possible further research topics as an extension of this work.

Chapter 2

BASIC PRINCIPLES OF MODEL-BASED FAULT DIAGNOSIS

2.1 Introduction

The model-based approach to fault diagnosis in automated processes has been receiving considerable attention over the last two decades, both in a research context and also in the domain of application studies on real processes. There are a great variety of methods in the literature, based on the use of mathematical models of the monitored processes and modern control theory.

The most important issue in model-based fault diagnosis is the robustness against modelling uncertainty which arises from incomplete knowledge and understanding of the monitored processes. Robust fault diagnosis has become a central research issue over recent years. As this thesis focuses on the development of robust model-based fault diagnosis techniques, this chapter studies basic principles of model-based fault diagnosis. Attention is first turned to the modelling of the system with all possible faults. Residual generation is then identified as an essential problem in model-based FDI, as an information processing procedure which, if not designed correctly could lose some fault information. A general framework for the residual generator is also presented. Residual generators based on different methods, such as observers and parity relations, are just special cases in this general framework. This chapter also shows that, to fulfil FDI tasks successfully, the residual signal has to satisfy fault detectability and isolability conditions. Some most important residual generation methods are discussed. One of the most important contributions of this chapter is to give some general guide-lines about the applicability of different model-based FDI approaches.

The robust FDI issue is discussed in this chapter and some commonly used robust approaches are presented. This formalizes a basis for the studies described in later chapters. The use of adaptive thresholds in FDI is also discussed. Finally, a discussion of fuzzy logic, qualitative modelling and knowledge based approaches in FDI is given. Some perspectives in the future development of FDI, by combining quantitative and qualitative techniques are also discussed.

2.2 Model-based Fault Diagnosis Methods

Model-based fault diagnosis can be defined as the detection, isolation and characterization of faults in components of a system from the comparison of the system's available measurements, with *a priori* information represented by the system's mathematical model.

Faults are detected by setting a (fixed or variable) threshold on a residual quantity generated from the difference between real measurements and estimates of these measurements using the mathematical model. A number of residuals can be designed with each having special sensitivity to individual faults occurring in different locations in the system. The subsequent analysis of each residual, once a threshold is exceeded, then leads to fault isolation.

Fig.2.1 illustrates the general and conceptual structure of a model-based fault diagnosis system comprising two main stages of residual generation and decision making. This two-stages structure was first suggested by Chow and Willsky (1980) and now is widely accepted by the fault diagnosis community. These two main stages are described as follows:

(1) Residual Generation: Its purpose is to generate a fault indicating signal – residual, using available input and output information from the monitored system. This auxiliary signal is designed to reflect the onset of a possible fault in the analyzed system. The residual should be normally zero or close to zero when no fault is present, but is distinguishably different from zero when a fault occurs. This means that the residual is characteristically independent of system inputs and outputs, in

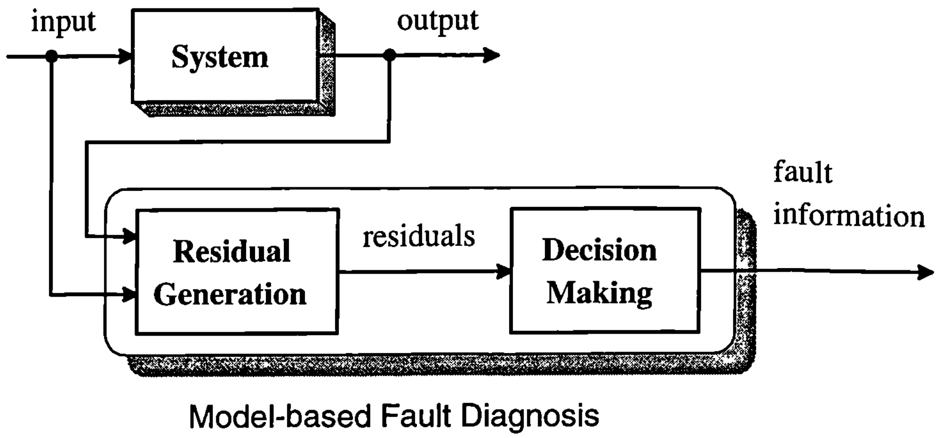


Figure 2.1: **Conceptual structure of model-based fault diagnosis**

ideal conditions. The algorithm (or processor) used to generate residuals is called a *residual generator*. Residual generation is thus a procedure for extracting fault symptoms from the system, with the fault symptom represented by the residual signal. The residual should ideally carry only fault information. To ensure reliable FDI, the loss of fault information in residual generation should be as small as possible.

(2) Decision-Making: The residuals are examined for the likelihood of faults, and a decision rule is then applied to determine if any faults have occurred. A decision process may consist of a simple threshold test on the instantaneous values or moving averages of the residuals, or it may consist of methods of statistical decision theory, e.g., generalized likelihood ratio testing or sequential probability ratio testing (Willsky, 1976; Basseville, 1988; Basseville and Nikiforov, 1993; Tzafestas and Watanabe, 1990).

Most of the work in the field of quantitative model-based fault diagnosis is focused on the residual-generation problem because the decision-making based on well designed residuals is relatively easy. However, this does not imply that the research on decision-making is not important. The thesis will concentrate on the quantitative residual generation stage of fault diagnosis by proposing a number of new strategies in the enhancement of residual robustness.

2.3 On-line Fault Diagnosis

Model-based FDI is concerned mainly with on-line fault diagnosis, in which the diagnosis is carried out during system operation. This is because the system input and output information required by model-based FDI is only available when the system is in operation. Opening of feedback loops in the system being tested or supplying test actions leading to incorrect functioning are considered inadmissible. The relationship between the fault diagnosis (or supervision) with the control loop is shown in Fig.2.2.

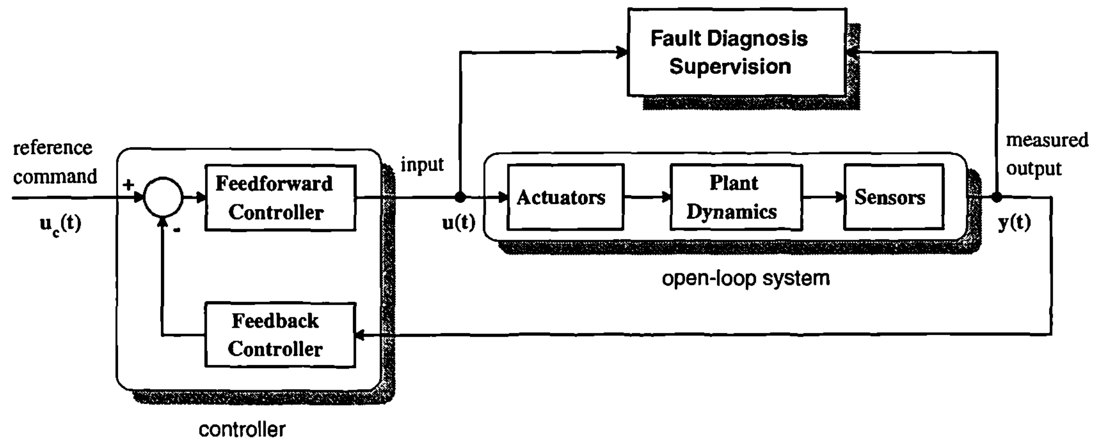


Figure 2.2: **Fault diagnosis and control loop**

The information used for FDI is the measured output from sensors and the input to the actuators. The measured output is normally needed in the feedback control, whereas the input to the actuators is the required control action generated by the controller, which is normally implemented in the micro-processor. Hence, we do not normally require extra hardware resources to implement the fault diagnosis function with the exception of requiring some additional computing power.

From Fig.2.2, it can be seen that the system model required in model-based FDI is the open-loop system model although we consider that the system is in the control loop. This is because the input and output information required in model-based FDI is related to the open-loop system. Hence, it is not necessary to consider the controller in the design of a fault diagnosis scheme. This is consistent with the separation principle in control theory because fault diagnosis can be broadly treated as an observation problem. Once the input to the actuators is available, the fault diagnosis problem is the same no matter how the system is working in open-loop or in the closed-loop.

In the cases when the input to the actuator $u(t)$ is not available, we have to use the reference command $u_c(t)$ in FDI. Hence, the model involved is the relationship between the reference command $u_c(t)$ and the measured output $y(t)$, i.e., the closed-loop model. For those cases, the controller plays an important role in the design of diagnostic schemes. A robust controller may desensitize fault effects and make the diagnosis very difficult. This problem has been recognized by some researchers, e.g. Wu (1992), and the best solution is to design the fault diagnosis scheme and the controller simultaneously (Nett, Jacobson and Miller, 1988; Jacobson and Nett, 1991). The interconnection between fault diagnosis and robust control is a topic for future research and is not considered further in this thesis.

2.4 Modelling of Faulty Systems

The first step in the model-based approach is to build a mathematical model of the system to be monitored. This thesis is concerned with multiple-input and multiple-output linear dynamic systems. In the case of a non-linear system, this implies a model linearization around an operating point.

As discussed in the previous section, we use the open-loop system model in model-based FDI. For the purposes of modelling, an open-loop system can be separated into three parts: actuators, system dynamics and sensors as illustrated in Fig.2.3.

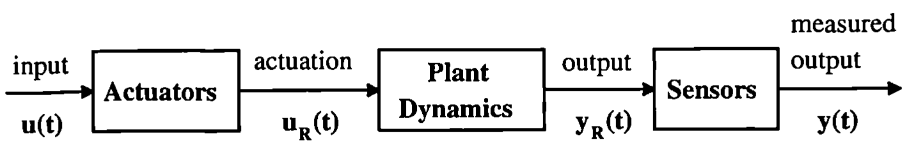


Figure 2.3: **Open-loop system**

The system dynamics shown in Fig.2.4 can be described by the state space model as:

$$\begin{cases} \dot{x}(t) &= Ax(t) + Bu_R(t) \\ y_R(t) &= Cx(t) + Du_R(t) \end{cases} \tag{2.1}$$

where $x \in \mathcal{R}^n$ is the state vector, $u_R \in \mathcal{R}^r$ is the input vector to the actuator and $y_R \in \mathcal{R}^m$ is the real system output vector; A, B, C and D are known system matrices with appropriate dimensions.

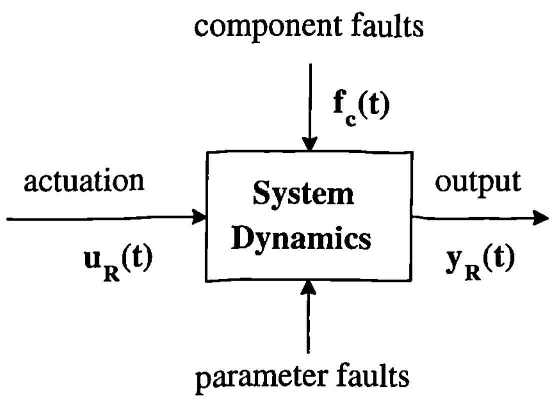


Figure 2.4: **The system dynamics**

When a component fault occurs in the system (see Fig.2.4), the dynamic model of the system can be described as:

$$\dot{x}(t) = Ax(t) + Bu_R(t) + f_c(t) \tag{2.2}$$

The component fault is represented as the case when some condition changes in the system rendering the dynamic relation invalid, for example a leak in a water tank in the three tank system (Wünnenberg, 1990). In some cases, the fault could be expressed as a change in the system parameter, for example a change in the i_{th} row and j_{th} column element of the matrix A, the dynamic equation of the system can then be described as:

$$\dot{x}(t) = Ax(t) + Bu_R(t) + I_i \Delta a_{ij} x_j(t) \tag{2.3}$$

Here, $x_j(t)$ is the j_{th} element of the vector $x(t)$ and I_i is an n-dimensional vector with all zero elements except a 1 in the i_{th} element.

Generally speaking, the actual output $y_R(t)$ of the system is not directly accessible, and sensors are then used to measure the system output. This is shown in Fig.2.5 and can be described mathematically as (when the sensor dynamics are neglected):

$$y(t) = y_R(t) + f_s(t) \tag{2.4}$$

where $f_s \in \mathcal{R}^m$ is the sensor fault vector. By choosing the vector f_s correctly, we can then describe all sensor fault situations. When the sensors are "stuck at a

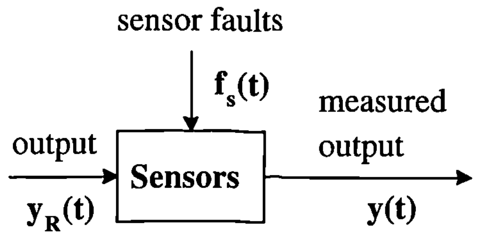


Figure 2.5: **Sensors, output and measured outputs**

particular value" (say at zero), the measurement vector is $y(t) = 0$ and the fault vector is $f_s(t) = -y_R(t)$. When there is a variation in the sensor scalar factors (multiplicative faults), the measurement becomes $y(t) = (1+\Delta)y_R(t)$ and the fault vector can be then written as $f_s(t) = \Delta y_R(t)$.

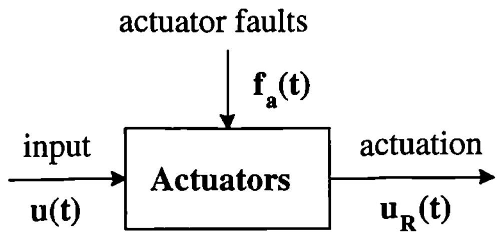


Figure 2.6: **Actuator, input and actuation**

It is also true that the actual actuation (u_R) of the system is often not directly accessible. For a controlled system, u_R is the actuator response to an actuator command $u(t)$, this is shown in Fig.2.6 and can be described as (when the actuator dynamics are neglected):

$$u_R(t) = u(t) + f_a(t) \tag{2.5}$$

where $f_a \in \mathcal{R}^r$ is the actuator fault vector and $u(t)$ is the known control command. Similar to sensor fault situations, all different kinds of actuator fault situations can be represented by a proper fault function $f_a(t)$.

If the system input is unknown (e.g., in an uncontrolled system), an input sensor can be used to measure the input to the actuator, this is shown in the Fig.2.7 and can be represented by the model:

$$u(t) = u_R(t) + f_{is}(t) \tag{2.6}$$

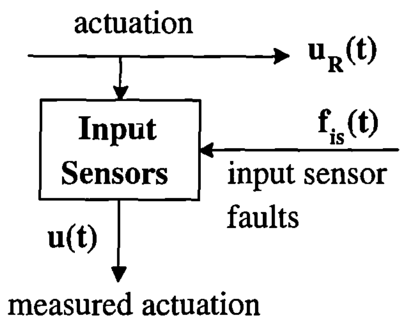


Figure 2.7: **Input sensor, actuation and measured actuation**

or

$$u_R(t) = u(t) + [-f_{is}(t)] \tag{2.7}$$

When the system has all possible sensor and actuator faults (this is the most common situation to be considered), the system model is described as:

$$\begin{cases} \dot{x}(t) &= Ax(t) + Bu(t) + Bf_a(t) \\ y(t) &= Cx(t) + Du(t) + Df_a(t) + f_s(t) \end{cases} \tag{2.8}$$

Considering the general cases, a system with all possible faults can be described by the state space model as:

$$\begin{cases} \dot{x}(t) &= Ax(t) + Bu(t) + R_1 f(t) \\ y(t) &= Cx(t) + Du(t) + R_2 f(t) \end{cases} \tag{2.9}$$

where $f(t) \in \mathcal{R}^g$ is a fault vector, each element $f_i(t)$ $(i = 1, 2, \cdots, g)$ corresponds to a specific fault. From a practical point of view, it is unreasonable to make further assumptions about the fault characteristics but consider these as unknown time functions. The matrices R_1 and R_2 are known as fault entry matrices which represent the effect of faults on the system. The vector $u(t)$ is the input to the actuator or measured actuation, and the vector $y(t)$ is the measured output, and both vectors are known for FDI purpose.

In the FDI literature, the vectors $u(t)$ and $y(t)$ are simply called the inputs and outputs of the monitored system. The terminology is not very precise, although no confusion arises and it is accepted widely in the FDI literature and in this thesis unless it is specifically stated.

An input-output transfer matrix representation for a system with possible faults is then described as:

$$y(s) = G_u(s)u(s) + G_f(s)f(s) \tag{2.10}$$

where

$$\begin{cases} G_u(s) &= C(sI-A)^{-1}B + D \\ G_f(s) &= C(sI-A)^{-1}R_1 + R_2 \end{cases} \tag{2.11}$$

The general model for a faulty system described by Eq.(2.9) in the time-domain and by Eq.(2.10) in the frequency-domain has been widely accepted in the fault diagnosis literature, e.g. in survey papers (Frank, 1990; Frank, 1991a; Patton and Chen, 1991e; Gertler, 1991; Frank, 1993; Patton, 1993; Gertler and Kunwer, 1993; Patton, 1994). However, most papers have just accepted them without any clue of how a particular individual fault fits into this model. Gertler and Luo (1989) and Gertler, Fang and Luo (1990) considered all possible fault sources in the monitored system and Chen and Patton (1994a) discussed briefly the modelling structure presented in this section.

2.5 A General Structure of Residual Generation in Model-based FDI

In practice, the most frequently used FDI approach uses information known *a priori* about the characteristics of certain signals (e.g. amplitude and frequency properties). As an example, we can take checking the level or the dynamic range of the signal, the maximum rate of its variation and its spectrum. The main shortcomings of this group of methods can be listed as: (a) the necessity to have *a priori* information about the characteristics of the signals, (b) the unavoidable dependence of these characteristics on operating states of the system which are not known *a priori* and can change beforehand.

To eliminate the shortcomings of the traditional methods, the most significant contribution in modern model-based approaches is the introduction of residuals which are independent of the system operating state and respond to faults in characteristic manners. Residuals are quantities that represent the inconsistency between the actual system variables and the mathematical model. Based on the mathematical model, many invariant relations (dynamic or static) among different system variables can be derived, and any violation of these relations can be used as residuals.

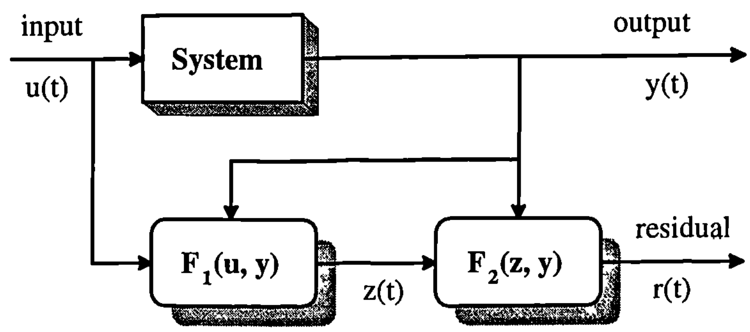


Figure 2.8: **Redundant signal structure in residual generation**

The residual generation can be interpreted in terms of redundant signal structure as illustrated in Fig.2.8 (Mironovski, 1980). In this structure, the system (processor or algorithm) $F_1(u,y)$ generates an auxiliary (redundant) signal z which, together with y generate the residual r which satisfy the following invariant relation:

$$r(t) = F_2(y(t), z(t)) = 0 \tag{2.12}$$

for the fault-free case. When any fault occurs in system, this invariant relation will be violated and the residual will be non-zero.

The simplest approach to residual generation is the use of system duplication, i.e. the system F_1 is made identical to the original system model and has the same input signal as the system. In this case, the signal y is not required in the system block F_1 which is then simply a system simulator. The signal z is the simulated output of the system, and the residual is the difference between z and y. The simplicity is the advantage of this method, but the disadvantage is that the stability of the simulator cannot be guaranteed when the system being monitored is unstable, as a consequence of the use of the open-loop system model in FDI (although it is under feedback control) (see Fig.2.2).

A direct extension to the simulator-based residual generation is to replace the simulator by an output estimator which requires both system inputs and outputs. In this case, the system $F_1(u,y)$ uses both signal u and y to generate a estimation of a linear function of the output y, say My, and the system F_2 can be defined as $F_2(y,z) = Q(z - My)$ with Q as a static (or dynamic) weighting matrix.

No matter what type of method is used, a residual generator is just a linear processor whose inputs are both inputs and outputs of the system being monitored. A general structure for all residual generators is shown in Fig.2.9 (Patton and Chen, 1991e).

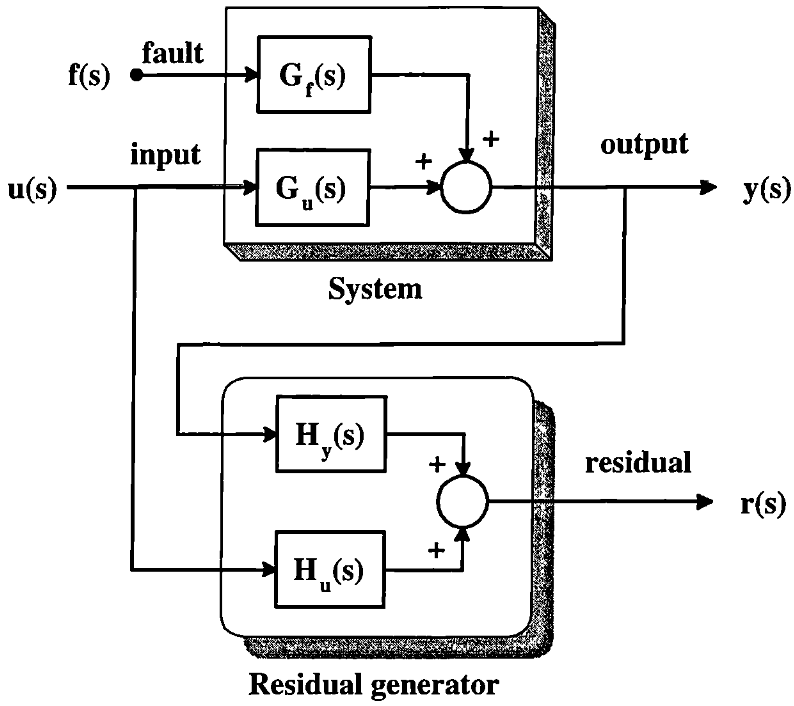


Figure 2.9: **The general structure of a residual generator**

This structure is expressed mathematically as:

$$r(s) = [H_u(s) \quad H_y(s)] \begin{bmatrix} u(s) \\ y(s) \end{bmatrix} = H_u(s)u(s) + H_y(s)y(s) \tag{2.13}$$

Here, $H_u(s)$ and $H_y(s)$ are transfer matrices which are realizable using stable linear systems. According to the definition, the residual is designed to become zero for the fault-free case and nonzero for faulty cases, i.e.:

$$r(t) = 0 \qquad \text{if and only if} \qquad f(t) = 0 \tag{2.14}$$

To satisfy this condition, the transfer matrices $H_u(s)$ and $H_y(s)$ must satisfy the constraint condition:

$$H_u(s) + H_y(s)G_u(s) = 0 \tag{2.15}$$

Eq.(2.13) is a *generalized* representation of all residual generators (Patton and Chen, 1991e). The design of the residual generator results simply in the choice of the transfer matrices $H_u(s)$ and $H_y(s)$ which must satisfy Eq.(2.15). The various ways of generating residuals correspond to different parameterizations of $H_u(s)$ and $H_y(s)$.

One can obtain different residual generators using different forms for $H_u(s)$ and $H_y(s)$. Using the design freedom, the desired performance of the residual can be achieved by suitable selection of $H_u(s)$ and $H_y(s)$.

A fault can be detected by comparing the residual evaluation function $J(r(t))$ with a threshold function $T(t)$ according to the test given in below:

$$\begin{cases} J((r(t)) \leq T(t) & \text{for} \quad f(t) = 0 \\ J((r(t)) > T(t) & \text{for} \quad f(t) \neq 0 \end{cases}$$

If this test is positive (i.e. the threshold is exceeded by the residual evaluation function), we can hypothesize that a fault is likely. There are many ways of defining evaluation functions and determining thresholds. As an example, the residual evaluation function is chosen as a norm of the residual vector and the threshold can be chosen as a constant positive value (fixed threshold).

2.6 Fault Detectability

When faults occur in the monitored process, the response of the residual vector is:

$$r(s) = H_y(s)G_f(s)f(s) = G_{rf}(s)f(s) = \sum_{i=1}^{g}[G_{rf}(s)]_i f_i(s) \tag{2.16}$$

where $G_{rf}(s) = H_y(s)G_f(s)$ is defined as a fault transfer matrix which represent the relation between the residual and faults, $[G_{rf}(s)]_i$ is the i_{th} column of the transfer matrix $G_{rf}(s)$ and $f_i(s)$ is the i_{th} component of $f(s)$. The above relationship is well illustrated by Fig.2.10.

2.6.1 Fault detectability condition

The fault transfer matrix plays an important role in FDI and must be examined in detail. In order to detect the i_{th} fault f_i in the residual $r(s)$, the i_{th} column $[G_{rf}(s)]_i$ of the transfer matrix $G_{rf}(s)$ should be non-zero:

$$[G_{rf}(s)]_i \neq 0 \tag{2.17}$$

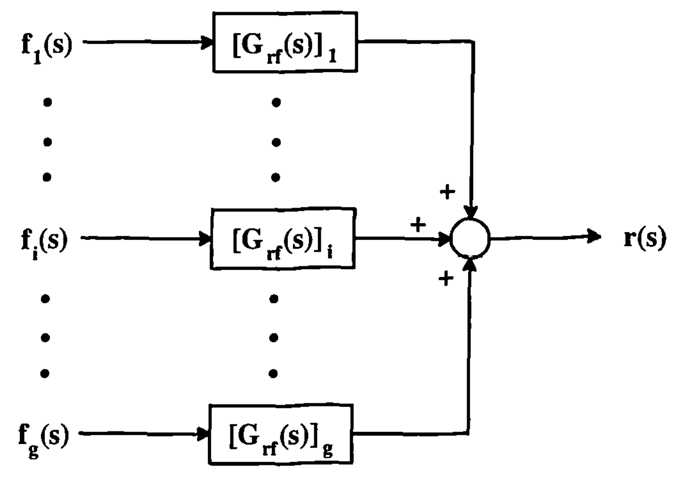


Figure 2.10: **Faults and the residual**

If this condition holds true, the i_{th} fault f_i is *detectable* in the residual r. This is defined as the *fault detectability condition* of the residual r to the fault f_i. One must ask whether this condition is enough for detecting faults? This question will be answered using the following example.

Example: The laboratory inverted pendulum system (described in Appendix A) is used as an example to illustrate the fault detectability (Chen and Patton, 1994b). The simulated fault detection results are shown in Fig.2.11.

In the simulation, the same fault signal is applied to three sensors. However, the residual response for the fault in the first sensor is significantly different from the faults in the other sensors. The responses for the faults in sensor 2 and sensor 3 almost reproduce the shape variations of the fault signal. However, the response for the fault in the sensor 1 only reflects the change in the fault level. After a short transient, the residual returns back to zero, although the fault is still present in the system. It is possible to give a misinterpretation of faults if this observer-based residual generator is used to detect faults in the first sensor. To examine the fault detectability, we find that: $[G_{rf}(s)]_1 \neq 0$, $[G_{rf}(s)]_2 \neq 0$, $[G_{rf}(s)]_3 \neq 0$, i.e. the faults in *three* sensors are all detectable from the residual designed. This example illustrates that fault detectability alone is not enough to achieve reliable fault detection. On examining the *steady* state gains of the residual generators, we find that: $[G_{rf}(0)]_1 = 0$, $[G_{rf}(0)]_2 \neq 0$ and $[G_{rf}(0)]_3 \neq 0$. This easily shows why the effect of the fault in sensor 1 on the residual generator disappears after a short transient period.

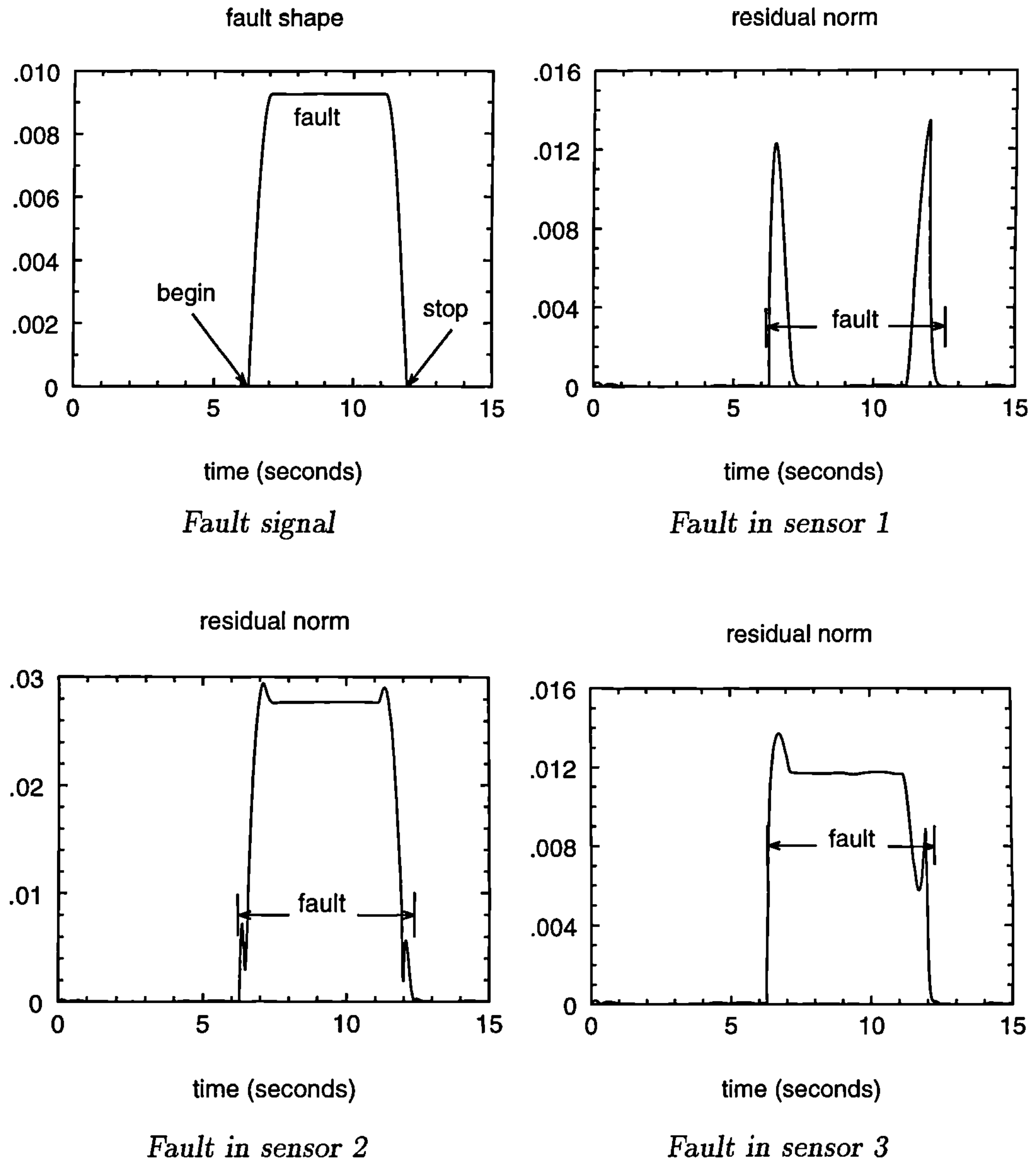


Figure 2.11: **Fault and residual norms**

2.6.2 Strong fault detectability condition

The example shows that fault detectability is not enough to achieve reliable fault detection. Hence, the strong fault detectability is introduced here as:

$$[G_{rf}(0)]_i \neq 0 \tag{2.18}$$

If this condition is satisfied, we define that the i_{th} fault f_i is *strongly detectable* in the residual r. This condition can also be defined as the *strong fault detectability condition* of the residual r to the fault f_i.

The misinterpretation problem due to the undesirable residual response has been noticed in the FDI research of a number of investigators (Patton and Kangethe, 1989; Frank, Ding and Köppen, 1993). There were some discussion in a benchmark testing session at the International Conference of Fault Diagnosis at Toulouse (TOOLDIAG'93) following a presentation by Frank et al. (1993). One explanation for this problem is that *the effect of a fault on the system* disappears, although the fault itself still exists. This is not a satisfactory explanation and the correct explanation is that *the effect of the fault on the residual* disappears, although the fault effect on the system still exists. That is to say, the residual generator which is used for FDI is not a good design.

We now examine the inverted pendulum system in more detail. Referring to the Appendix A, we find that the strong detectability for faults in the first sensor cannot be achieved no matter what observer gain matrix is used, if the residual generator is based on a full-order observer. It is also interesting to note that a residual generator based on a 1_{st} or 2_{nd} order parity relation also gives similar residual responses (the results are not shown in this thesis as they are very similar to the results shown in Fig.2.11).

The question arises as to how the above problem (for the inverted pendulum example) can be solved? One way is to design other residual generators which could satisfy the strong fault detectability, and this requires comprehensive research. The other possibility is to shape the frequency response of the residual according to the frequency distribution of the faults. For example, if the residual generated by the observer is filtered through a filter with transfer function $1/(s + 0.01)$, the filtered residual can produce a satisfactory response for the fault given in Fig.2.11. This simple operation shows that the frequency response of the fault transfer function should also be studied in the residual design. If the frequency band of a certain fault is available, the residual can be designed maximally sensitive to this fault by frequency-shaping. This can be done by maximizing the following criterion:

$$\inf_{\omega \in [\omega_1, \omega_2]} \underline{\sigma}\{G_{rs}(j\omega)\} \tag{2.19}$$

where $\underline{\sigma}\{\cdot\}$ denotes the minimal singular values, and $[\omega_1, \omega_2]$ denotes the frequency range in which the fault is most likely to occur. This problem will be studied in Chapter 6. Other investigations are described in papers by Frank and Ding (Ding and Frank, 1989; Frank and Ding, 1993; Frank and Ding, 1994).

2.7 Fault Isolability

The successful detection of a fault is followed by the fault isolation procedure which will distinguish (isolate) a particular fault from others. Whilst a single residual signal is sufficient to detect faults, a set of residuals (or a vector of residuals) is usually required for fault isolation. If a fault is distinguishable from other faults using one residual set (or a residual vector), it can be said that this fault is *isolable* using this residual set (or this residual vector). If the residual set (or the residual vector) can isolate all faults, we can then say that the residual set (or the residual vector) has the required isolability property.

2.7.1 Structured residuals set

One approach to fulfil the fault isolation task is to design a set of structured residuals. Each residual is designed to be sensitive to a subset of faults, whilst remaining insensitive to the remaining faults. The residual set which has the required sensitivity to specific faults and insensitivity to other faults is known as the *structured residuals set* (Gertler, 1991). The design procedure consists of two steps, the first step is to specify the sensitivity and insensitivity relationships between residuals and faults according to the assigned isolation task, and the second is to design a set of residual generators according to the desired sensitivity and insensitivity relationships. The advantage of the structured residual set is that the diagnostic analysis is simplified to determining which of the residuals are non-zero. The threshold test may be performed separately for each residual, yielding a Boolean decision table, and the isolation task can be fulfilled using this table.

If all possible faults are to be isolated, a residual set can be designed according to the following fault sensitivity conditions:

$$r_i(t) = R(f_i(t)); \qquad i \in \{1, 2, \cdots, g\} \tag{2.20}$$

where $R(\cdot)$ denotes a functional relation. This is called as a *dedicated residual set* which is inspired by the *dedicated observer scheme* proposed by Clark (1978a). A simple threshold logic can be used to make decision about the appearance of a specific fault by the logic decision according to:

$$r_i(t) > T_i \implies f_i(t) \neq 0; \qquad i \in \{1, 2, \cdots, g\}$$

where T_i $(i = 1, \cdots, g)$ are thresholds. This isolable residual structure is very simple and all faults can be detected simultaneously, however it is difficult to design in practice. Even when this structured residual set can be designed, there is normally no design freedom left to achieve other desirable performances such as robustness against modelling errors (Wünnenberg, 1990). A most commonly used and better scheme in designing the residual set is to make each residual sensitive to all but one fault, i.e.

$$\begin{cases} r_1(t) &= R(f_2(t), \cdots, f_g(t)) \\ &\vdots \\ r_i(t) &= R(f_1(t), \cdots, f_{i-1}(t), f_{i+1}(t), \cdots, f_g(t)) \\ &\vdots \\ r_g(t) &= R(f_1(t), \cdots, f_{g-1}(t)) \end{cases} \tag{2.21}$$

This is defined as *a generalized residual set.* If all residuals of the generalized residual set are generated using a bank of observers (observer-based residual generators), the structure is known as the *generalized observer scheme* (Patton et al., 1989). The isolation can again be performed using simple threshold testing according to the following logic:

$$\left.\begin{array}{l} r_i(t) \leq T_i \\ r_j(t) > T_j \quad \forall j \in \{1, \cdots, i-1, i+1, \cdots, g\} \end{array}\right\} \implies f_i(t) \neq 0; \quad \text{for} \quad i = 1, 2, \cdots, g$$

As a simple example, we will isolate *three* different faults $\{f_1, f_2, f_3\}$ by designing a residuals set $\{r_1, r_2, r_3\}$ using the following two methods:

In the tables above, a "1" in i_{th} row and j_{th} column denotes that the residual r_j is

	r_1	r_2	r_3
f_1	1	0	0
f_2	0	1	0
f_3	0	0	1

Method 1

	r_1	r_2	r_3
f_1	0	1	1
f_2	1	0	1
f_3	1	1	0

Method 2

Table 2.1: **Structured Residual Sets**

sensitive to the fault f_i, whilst a "0" denotes insensitivity. Faults can be uniquely isolated using either of the above methods.

2.7.2 Fixed direction residual vector

An alternative way of enhancing the isolability of faults is to design a *directional* residual vector which lies in a fixed and fault-specified direction (or subspace) in the residual space, in response to a particular fault. This is to make:

$$r(t \mid f_i(t)) = \alpha_i(t)l_i; \quad i \in \{1, 2, \cdots, g\} \tag{2.22}$$

where the constant vector l_i is the *signature direction* of the i_{th} fault in the residual space and α_i is a scalar that depends on the fault size and dynamics. With the fixed directional residual, the fault isolation problem is one of determining which of the known fault signature directions the generated residual vector lies the closest to. To isolate faults reliably, each fault signature has to be uniquely related to one fault. Fig. 2.12 illustrates this fault isolation approach using a *directional residual vector* in which the residual is closed to l_1 and the fault is most likely associated with the direction l_1.

2.7.3 Sensor and actuator faults isolation

If we are only interested in sensor faults, the system output is given by:

$$y(s) = G_u(s)u(s) + f_s(s) \tag{2.23}$$

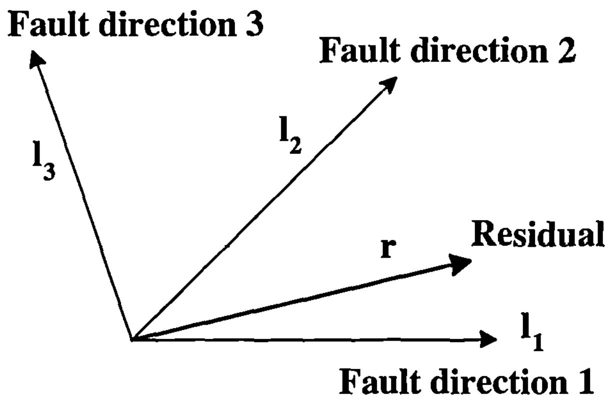


Figure 2.12: **Directional residual vector for fault isolation**

If one wants to design a residual signal which is sensitive to one group of faults in $f_s^1(s)$ and insensitive to another group of faults in $f_s^2(s)$, the above equation can be decomposed into:

$$\begin{bmatrix} y^1(s) \\ y^2(s) \end{bmatrix} = G_u(s)u(s) + \begin{bmatrix} f_s^1(s) \\ f_s^2(s) \end{bmatrix} \tag{2.24}$$

The residual generator then takes on the following format:

$$r^1(s) = H_u^1(s)u(s) + H_y^1(s)y^1(s) \tag{2.25}$$

On substituting $y^1(s)$ into the above equation, we have:

$$r^1(s) = [H_u^1(s) + H_y^1(s)G_u(s)]u(s) + H_y^1(s)f_s^1(s) \tag{2.26}$$

The residual will then be only sensitive to the fault group $f_s^1(s)$, when the transfer function matrices of the residual generator satisfy:

$$\begin{cases} H_u^1(s) & = & -H_y^1(s)G_u(s) \\ H_y^1(s) & \neq & 0 \end{cases} \tag{2.27}$$

This is the normal requirement for a residual generator as shown in Eq.(2.15). That is to say that there is no additional requirement for the sensor fault isolation problem. The transfer matrix $H_y^1(s)$ can be chosen freely according to specific requirements. The only constraint on $H_y^1(s)$ is that it should be stable and realizable. Once it has been chosen, $H_u^1(s)$ can be determined by $H_u^1(s) = -H_y^1(s)G_u(s)$. As the transfer matrix $H_y^1(s)$ can be chosen freely, sensor fault isolation is always possible.

When actuator faults occur in the system, the system output is:

$$y(s) = G_u(s)[u(s) + f_a(s)] \tag{2.28}$$

If we want to design a residual signal which is sensitive to one group of faults $f_a^1(s)$ and insensitive to another group of faults $f_a^2(s)$, the above equation can be decomposed into:

$$y(s) = G_u^1(s)[u^1(s) + f_a^1(s)] + G_u^2(s)[u^2(s) + f_a^2(s)] \tag{2.29}$$

The residual generator is now:

$$r^1(s) = H_u^1(s)u^1(s) + H_y^1(s)y(s) \tag{2.30}$$

On substituting $y(s)$ into Eq.(2.30), we have:

$$r^1(s) = [H_u^1(s) + H_y^1(s)G_u^1(s)]u^1(s) + H_y^1(s)G_u^1(s)f_a^1(s) + H^1(s)G_u^2(s)[u^2(s) + f_a^2(s)] \tag{2.31}$$

To make the residual only sensitive to the fault group $f_a^1(s)$, we need the following conditions:

$$\left\{ \begin{array}{rcl} H_u^1(s) & = & -H_y^1(s)G_u^1(s) \\ H_y^1(s)G_u^2(s) & = & 0 \\ H_y^1(s)G_u^1(s) & \neq & 0 \end{array} \right. \tag{2.32}$$

These equations illustrate that an extra constraint $(H_y^1(s)G_u^2(s) = 0)$ is required for the actuator isolation problem. A stable and implementable transfer matrix $H_y^1(s)$ does not always exist. That is to say, we do not have full freedom to achieve the required actuator fault isolation performance. Hence, actuator fault isolation is not always possible.

2.8 Residual Generation Techniques

The generation of residual signals is a central issue in model-based fault diagnosis. A rich variety of methods are available for residual generation and this Section discusses briefly some of the most common approaches. It must be pointed out that most residual generation approaches are applicable for both continuous and discrete system models, however some approaches can only work for discrete models. In this thesis, if the continuous model is used, it implies that the technique can be applied to both continuous and discrete model, otherwise the technique is only applicable for discrete model. The parity relation approach is developed specially for the discrete model. There have been some studies into the use of the parity relation approach for continuous models (Mironovski, 1979; Magni and Mouyon, 1991), however they have not been fully recognized by the FDI community because of the use of impractical differential operations on input and output data.

2.8.1 Observer-based approaches

The basic idea behind the observer or filter-based approaches is to estimate the outputs of the system from the measurements (or a subset of measurements) by using either Luenberger observer(s) in a deterministic setting (Beard, 1971; Clark et al., 1975; Clark, 1979; Frank, 1987; Frank, 1990; Frank, 1993; Patton and Kangethe, 1989; Patton, 1994) or Kalman filter(s) in a stochastic setting (Mehra and Peschon, 1971; Willsky, 1976; Frank, 1987; Basseville, 1988; Basseville and Benveniste, 1986; Tzafestas and Watanabe, 1990). Then, the (weighted) output estimation error (or innovations in the stochastic case), is used as a residual. The flexibility in selecting observer gains has been fully exploited in the literature yielding a rich variety of FDI schemes, the most recently development can be found in various survey papers: e.g. Frank (1993), Frank and Köppen (1993), Patton (1994), Krishnaswami and Rizzoni (1994b), Patton and Chen (1994), and conference proceedings such as, Isermann (1991), Labarrère (1993), Ruokonen (1994).

What we are interested in FDI is the estimation of outputs using an observer, whilst the estimation of the state vector is unnecessary. Indeed, a functional observer is suitable for this task. In practice, the order of the functional observer is less than the order of a state observer. It is desired to estimate a linear function of the state, i.e. $Lx(t)$, using a functional (or generalized) Luenberger observer with the following structure:

$$\left\{ \begin{array}{lcl} \dot{z}(t) & = & Fz(t) + Ky(t) + Ju(t) \\ w(t) & = & Gz(t) + Ry(t) + Su(t) \end{array} \right. \tag{2.33}$$

where $z(t) \in \mathcal{R}^q$ is the state vector of this functional observer, F, K, J, R, G and S are matrices with appropriate dimensions. The output $w(t)$ of this observer is said to be an estimate of $Lx(t)$, for the system described in Eq.(2.9), in an asymptotic sense if in the absence of faults:

$$\lim_{t\to\infty}[w(t) - Lx(t)] = 0 \tag{2.34}$$

To introduce a transformation matrix T, the observer shown in Eq.(2.33) will generate the estimate $Lx(t)$ in the *asymptotic sense* if and only if the following conditions hold (O'Reilly, 1983):

$$\begin{cases} F \text{ has stable eigenvalues} \\ TA - FT = KC \\ J = TB - KD \\ RC + GT = L \\ S + RD = 0 \end{cases} \tag{2.35}$$

The necessary and sufficient condition for the existence of the observer given by Eq.(2.33) for the system Eq.(2.9) is that the pair (C, A) is observable (O'Reilly, 1983). In order to generate residuals, we need to estimate the system output. If we assign:

$$L = C \tag{2.36}$$

We have the output estimation as:

$$\hat{y}(t) = w(t) + Du(t) \tag{2.37}$$

The residual vector $r(t)$ is defined as:

$$r(t) = Q[y(t) - \hat{y}(t)] = L_1 z(t) + L_2 y(t) + L_3 u(t) \tag{2.38}$$

where:

$$\begin{cases} L_1 &= -QG \\ L_2 &= Q - QR \\ L_3 &= -Q(S + D) \end{cases}$$

Now, the residual generator based on a generalized Luenberger is illustrated in Fig. 2.13 and given by the following equation:

$$\begin{cases} \dot{z}(t) &= Fz(t) + Ky(t) + Ju(t) \\ r(t) &= L_1 z(t) + L_2 y(t) + L_3 u(t) \end{cases} \tag{2.39}$$

And the matrices in this equation should satisfy the following conditions:

$$\begin{cases} F \text{ has stable eigenvalues} \\ TA - FT = KC \\ J = TB - KD \\ L_1T + L_2C = 0 \\ L_3 + L_2D = 0 \end{cases} \tag{2.40}$$

The Laplace transformation of the residual is thus:

$$r(s) = [L_1(sI - F)^{-1}K + L_2]y(s) + [L_1(sI - F)^{-1}J + L_3]u(s) \tag{2.41}$$

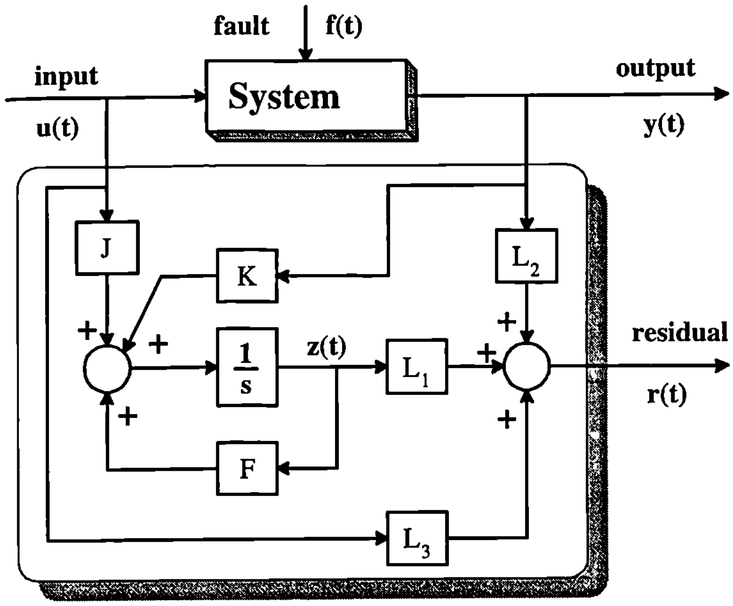


Figure 2.13: **Residual generation via a generalized Luenberger observer**

The residual generator based on a generalized Luenberger observer is shown in Fig. 2.13. It can be seen that there is a feedback structure imbedded within it. The feedback can be used to improve the dynamic behaviour of residuals.

When we apply the residual generator described by Eq.(2.39) to the system described by Eq.(2.9), the residual will be:

$$\left\{\begin{array}{lcl} \dot{e}(t) & = & Fe(t) - TR_1 f(t) + KR_2 f(t) \\ r(t) & = & L_1 e(t) + L_2 R_2 f(t) \end{array}\right. \tag{2.42}$$

where $e(t) = z(t) - Tx(t)$. It can be seen that the residual depends solely and totally on faults.

The simplest method in observer-based residual generation is to use a full order observer, in this case the observer dimension q equals n and we have:

$$\left\{\begin{array}{lcl} T & = & I \\ F & = & A - KC \\ J & = & B - KD \end{array}\right. \qquad \left\{\begin{array}{lcl} L_1 & = & QC \\ L_2 & = & -Q \\ L_3 & = & QD \end{array}\right.$$

Hence, the transfer function matrices for a full-order observer based residual generator are given by:

$$\left\{\begin{array}{lcl} H_y(s) & = & Q\{C[sI - (A - KC)]^{-1} K - I\} \\ H_u(s) & = & Q\{C[sI - (A - KC)]^{-1}(B - KD) + D\} \end{array}\right. \tag{2.43}$$

To alter the frequency response of the residual, the residual weighting matrix Q can be changed into a dynamic weighting $Q(s)$.

For any dynamic system, the observer-based residual generator always exists. This is because any input-output transfer function matrix has the observable realization. That is to say, the output estimator always exists although a suitable state observer cannot always be designed. The minimal order q_0 of a functional observer satisfies the inequality (O'Reilly, 1983; Mironovski, 1979; Mironovski, 1980):

$$q_0 \leq \mu - 1 \tag{2.44}$$

where μ is the observability index of the system which is defined as the minimum number for which:

$$rank[C^T,\ (CA)^T,\ \cdots,\ (CA^{\mu})^T] = n$$

For observable systems the observability index lies within the limits:

$$\frac{n}{m} \leq \mu \leq n - m + 1$$

Inequality (2.44) gives only the *minimum* possible order of a functional observer.

In the real situation, the order is larger than this minimum possible order. For the residual generation problem, an additional condition must be satisfied, that is that the residual must be sensitive to faults to be diagnosed.

To isolate faults, the observer-based approaches can be used to design structured residual sets or fixed residual vectors. For sensor faults, the design of a structured residual set is very straightforward. If we require that a residual is sensitive to faults in all but one of the sensors, the observer used to generate this residual should be driven by outputs excluding that single sensor measurement. To be more specific, if we replace the output vector $y = (y_1, \cdots, y_m)$ by $(y_1, \cdots, y_{i-1}, y_{i+1}, \cdots, y_m)$, the residual will be insensitive to the fault in the i_{th} sensor. However, the design of a structured residual set for actuator fault isolation is more difficult. This problem can be solved via unknown input observers (Viswanadham and Srichander, 1987; Phatak and Viswanadham, 1988; Frank, 1990) and eigenstructure assignment (Patton et al., 1986; Patton, 1988; Patton and Chen, 1991g), however the isolation of actuator faults is not always possible. The problem of designing structured residual set via unknown input observers will be discussed in Chapter 3. The schemes used in designing observer-based structured residual set have being called the dedicated observer scheme and the generalized observer scheme etc. (Frank, 1987; Frank, 1990; Patton et al., 1989; van Schrick, 1994a). The fixed residual vector can be designed by the so-called "fault (or failure) detection filter" originated by Beard (1971) and this problem is studied in Chapter 3.

2.8.2 Parity vector (relation) methods

In the early development of fault diagnosis, the *parity relation* approach was applied to *static or parallel redundancy* schemes (Potter and Sunman, 1977; Gai, Harrison and Daly, 1978; Daly, Gai and Harrison, 1979; Desai and Ray, 1984) which may be obtained directly from measurements or from analytical relations. Ray and Luck (1991) gave a survey of these schemes. There are typically two cases, one is the use of sensors having identical or similar functions to measure the same variable, another is the use of dissimilar sensors to measure different variables but with their outputs being relative to each other. The basic idea of the parity relation approach is to provide a proper check of the parity (consistency) of the measurements of the monitored system. To begin with this problem, let us consider a general problem of the measurement of an n-dimensional vector using m sensors. The measurement (algebraic) equation is:

$$y(k) = Cx(k) + f(k) + \xi(k) \tag{2.45}$$

where $y(k) \in \mathcal{R}^m$ is measurement vector, $x(k) \in \mathcal{R}^n$ is the state vector, and $f(k)$ is the vector of sensor faults, $\xi(k)$ is a noise vector and C is an $m \times n$ measurement matrix.

With hardware (direct) redundancy there are more than the minimum number of sensors (eg., two or more for scalar state variables, and four or more for three-dimensional state variables). And thus, in this case, the state vector can be determined directly using the redundancy measurements. The dimension of $y(k)$ is larger than the dimension of $x(k)$, i.e.

$$m > n; \quad \text{and} \quad rank(C) = n$$

For such system configurations, the number of measurements is greater than the number of variables to be sensed. Inconsistency in the measurement data is then a metric that can be used initially for detecting faults and, subsequently for fault isolation. Thi technique has been successfully applied to fault diagnosis schemes for inertial navigation (Potter and Sunman, 1977; Gai, Harrison and Daly, 1978; Daly et al., 1979; Desai and Ray, 1984) where relationships between gyroscope readings and/or accelerometer assemblies provide analytical forms of redundancy.

For FDI purposes, the vector $y(k)$ can be combined into a set of linearly independent parity equations to generate the parity vector (residual):

$$r(k) = Vy(k) \tag{2.46}$$

The residual generation scheme based on direct redundant measurements is shown in Fig.2.14.

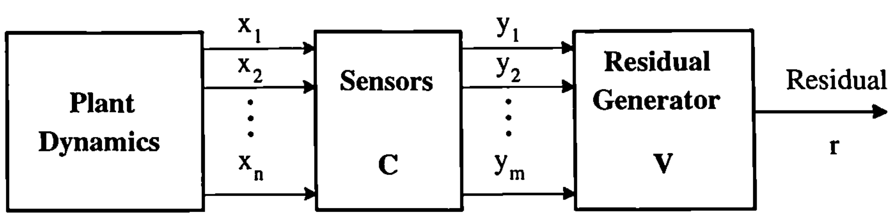


Figure 2.14: **Residual generation via parallel redundancy**

In order to make $r(k)$ satisfy the usual requirement for a residual (zero-valued for the fault-free case), the matrix V must satisfy the condition:

$$VC = 0 \tag{2.47}$$

When this condition holds true, the residual (parity vector) only contains information on the faults and noise:

$$r(k) = v_1[f_1(k) + \xi_1(k)] + \cdots + v_m[f_m(k) + \xi_m(k)] \tag{2.48}$$

where v_i is the i_{th} column of V, $f_i(k)$ is the i_{th} element of $f(k)$ which denotes the fault in the i_{th} sensor.

Eq.(2.48) reveals that the parity vector only contains information due to faults and noise (uncertainty), and is independent of the unmeasured state $x(k)$. Eq.(2.48) also shows that the parity space (or residual space) is spanned by the columns of V, i.e. the columns of V form a basis for this space. Moreover, the following attractive property can also be exploited: a fault in the i_{th} sensor implies a growth of the residual $r(k)$ in the direction v_i. This ensures that a fault in the i_{th} sensor, implies a magnification of the norm of $r(k)$ in the direction v_i. The space $span\{V\}$ is called a "parity space". The term "parity" was first used in connection with digital logic systems and computer software reliability to enable "parity checks" to be performed for error checking. In the fault diagnosis field, it has similar meaning in the context of providing an indicator for the presence of a fault (or error) in system components.

Using the notation of Daly et al. (1979), a *fault detection decision function* is defined as:

$$DFD(k) = r(k)^T r(k) \tag{2.49}$$

If a fault occurs in the sensors, $DFD(k)$ will be greater than an predetermined threshold.

The fault isolation decision function is then:

$$DFI_i(k) = v_i^T r(k); \quad i \in \{1, 2, \cdots, m\} \tag{2.50}$$

For a given $r(k)$, a malfunctioning sensor is identified by computing the m values of $DFI_i(k)$. If $DFI_j(k)$ is the largest one of these values, the sensor corresponding to $DFI_j(k)$ is the one which is most likely to have become faulty.

In the parity space point of view, the columns of V define m distinct *fault signature* *directions* (v_i, $i = 1, 2, \cdots, m$). After a fault has been declared, it can be isolated by comparing the orientation of the parity vector to each these signature directions. Indeed, the fault isolation function $DFI_i(k)$ is a measure of the correlation of the residual vector with fault signature directions. In order to isolate faults reliably, the generalized angles between fault signature directions should be "as large as possible", i.e., to make $v_i^T v_j$ $(i \neq j)$ "as small as possible". Thus, optimal fault isolation performance will be achieved when v_i determined by:

$$\begin{cases} \min\{v_i^T v_j\}; & i \neq j, \quad i, j \in \{1, 2, \cdots, m\} \\ \max\{v_i^T v_i\}; & i \in \{1, 2, \cdots, m\} \end{cases} \tag{2.51}$$

The traditional sub-optimal solution of the matrix V is to make (Ray and Luck, 1991):

$$VV^T = I_{m-n} \tag{2.52}$$

A further consequence of conditions (2.47) and (2.52) is that:

$$V^T V = I_m - C(C^T C)^{-1} C^T \tag{2.53}$$

The condition for the existence of a solution V for Eq.(2.47) is that $rank(C) = n < m$. This implies that the rows of C are linearly dependent, i.e., the outputs of the sensors are related by a static relation. For the case $rank(C) = m < n$, the *direct redundancy* relation does not exist however, we may construct redundancy relations by collecting sensor outputs over a time interval (data window) (say, $\{y(k-s), y(k-s+1), \cdots, y(k)\}$). This is known as "temporal redundancy" or "serial redundancy". The dynamic model must be known and used in this case, as the redundancy is related to time. Here, we consider that the system is given by the linear discrete state space equations as follows:

$$\begin{cases} x(k+1) &= Ax(k) + Bu(k) + R_1 f(k) \\ y(k) &= Cx(k) + Du(k) + R_2 f(k) \end{cases} \tag{2.54}$$

where $x \in \mathcal{R}^n$ is state, $y \in \mathcal{R}^m$ is output, $u \in \mathcal{R}^r$ is input, $f \in \mathcal{R}^g$ fault, and A, B, C, D, R_1 and R_2 are real matrices of compatible dimensions.

As a direct extension of the case of parallel redundancy, the parity relation concept was first generalized by (Chow and Willsky, 1984) using the *temporal redundancy* relations of the dynamic system. Extended researches have been done by various other authors as, Lou et al. (1986), Massoumnia and Vander Velde (1988), Frank and Wünnenberg (1989),Wünnenberg (1990), Gertler and Singer (1990),Patton and Chen (1991e). It is important however, to note that essentially the same scheme has been suggested by the Russian expert Mironovski (1979) (see also: Mironovski (1980) and Basseville (1988)). Although he did not use the term "parity relation", the essential ideas are the same as those of the remaining authors.

The redundancy relations are now specified mathematically as follows. Combining together Eq.(2.54) from time instant $k-s$ to time instant k yields the following redundant relations:

$$\underbrace{\begin{bmatrix} y(k-s) \\ y(k-s+1) \\ \vdots \\ y(k) \end{bmatrix}}_{Y(k)} - H \underbrace{\begin{bmatrix} u(k-s) \\ u(k-s+1) \\ \vdots \\ u(k) \end{bmatrix}}_{U(k)} = Wx(k-s) + M \underbrace{\begin{bmatrix} f(k-s) \\ f(k-s+1) \\ \vdots \\ f(k) \end{bmatrix}}_{F(k)} \tag{2.55}$$

where

$$H = \begin{bmatrix} D & 0 & \dots & 0 \\ CB & D & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{s-1}B & CA^{s-2}B & \dots & D \end{bmatrix} \in \mathcal{R}^{(s+1)m\times(s+1)r}$$

$$W = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^s \end{bmatrix} \in \mathcal{R}^{(s+1)m\times n}$$

and the matrix M is constructed by replacing $\{D, B\}$ with $\{R_2, R_1\}$ in the matrix H.

To simplify the notation, Eq.(2.55) can be rewritten as:

$$Y(k) - HU(k) = Wx(k-s) + MF(k) \tag{2.56}$$

According to Chow and Willsky (1984) and Lou et al. (1986), a residual signal can be defined as:

$$r(k) = V[Y(k) - HU(k)] \tag{2.57}$$

where $V \in \mathcal{R}^{p\times(s+1)m}$ and p is the residual vector dimension. Eq.(2.57) is termed an s_{th} order parity equation or parity relation. It is the *computational form* of a residual generator which shows the residual signal as a function of measured inputs and outputs of the monitored system. Substituting Eq.(2.56) into Eq.(2.57), we have:

$$r(k) = VWx(k-s) + VMF(k) \tag{2.58}$$

This is the evaluation format of the residual. In order to make the parity vector useful for FDI, one should make it insensitive to system inputs and states, i.e.

$$VW = 0 \tag{2.59}$$

To satisfy the fault detectability condition, the matrix V should also satisfy the following condition:

$$VM \neq 0 \tag{2.60}$$

Once we have the matrix V, the residual signal can be generated using Eq. (2.57). The residual generator design depends on solutions of Eq.(2.59). For an appropriately large s (for example $s = n$), it follows from the Cayley-Hamilton theorem that the solution V of Eq. (2.59) always exists. This means that a parity relation-based residual generator for fault detection always exists. An appropriate value for s can be found by the designer by a systematic increase in s.

Of particular interest are those parity relations for which the order s (length $(s+1)$ of the data window) is minimal. The minimum order s_0 of the parity relations satisfies the two-sided inequality (Mironovski, 1979; Mironovski, 1980):

$$\frac{rank(W_o)}{rank(C)} \leq s_0 \leq rank(W_o) - rank(C) + 1$$

where W_o is the observability matrix of the pair (C, A). If the system is observable and the rows of the matrix C are linearly-independent, then the inequality takes the form:

$$\frac{n}{m} \leq s_0 \leq n - m + 1$$

The parity relation approach for residual generation of dynamic system is shown in Fig.2.15. Here, we discussed the construction of parity relations based on a state space model which is suggested by Chow and Willsky (1984) and Mironovski (1979). However, it must be pointed out that the parity relation can also be constructed using a z-transformed input-output model (or discrete transfer matrix representa-

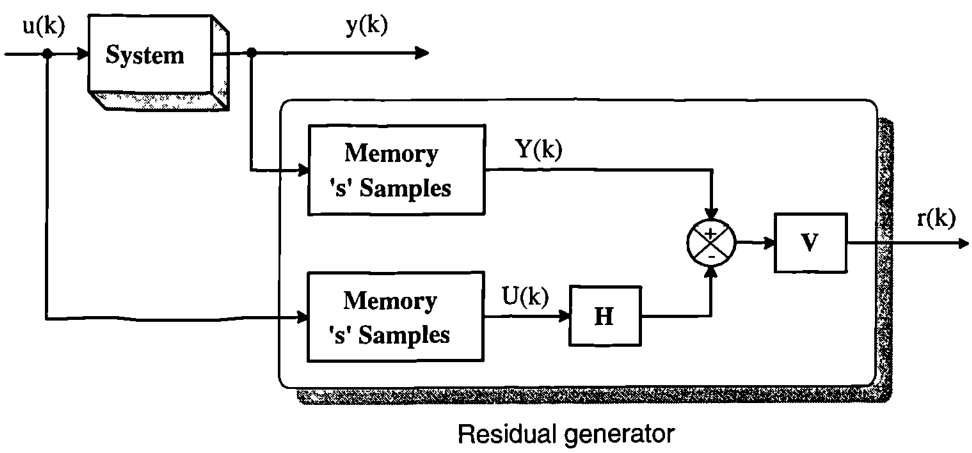


Figure 2.15: **Residual generation via temporal redundancy**

tion). Gertler et al. (Gertler and Luo, 1989; Gertler, Fang and Luo, 1990; Gertler, Luo, Anderson and Fang, 1990; Gertler and Singer, 1990) first introduced this design approach, preferring to call it the "parity equation" method. Gertler (1991) presented an excellent tutorial on this approach. Note that this input-output model based design method has also been studied by Mironovski (1980) and Massoumnia and Vander Velde (1988).

The parity relation approach can be used to design structured residual set for fault isolation (Massoumnia and Vander Velde, 1988; Gertler, 1991; Patton and Chen, 1991e). The design for isolating sensor faults is very straightforward. If we use c_i^T (the i_{th} row of C) and y_i (the i_{th} component of y) instead of C and y, the parity relation will only contain the i_{th} sensor's output together with all inputs. This form of parity relation has been called a *single-sensor parity relation* (Massoumnia and Vander Velde, 1988) and the residual generated by this relation is only sensitive to the fault in the i_{th} sensor. For the actuator isolation problem, the structured residual set is more difficult to design. Massoumnia and Vander Velde (1988) studied this problem and pointed out that the isolation of actuator faults is not always possible. This conclusion is consistent with the observer-based approaches. Gertler et al. (Gertler and Luo, 1989; Gertler, Fang and Luo, 1990; Gertler and Singer, 1990) suggested a so-called "orthogonal parity equation" approach in designing structured residual sets. The idea is to make the parity equation (and residual) orthogonal to a particular fault direction if we want the residual insensitive to this fault. It is not easy to design directional residual vector using parity relations. Gertler (1991) studied this problem and illustrated the possibility based on examples, however a systematic approach still does not exist.

It is clear therefore that some correspondence exists between observer-based and parity relation approaches. Massoumnia (1986a) first pointed out this correspondence, and this was later demonstrated by Frank and Wünnenberg (1989), Wünnenberg (1990) and Magni and Mouyon (1991). A full derivation of this equivalence has been given by Patton and Chen (1991e). Patton and Chen (1991d) have re-examined this problem in detail and the equivalence under different conditions and in different meanings have been discussed. It has been shown by Frank and Wünnenberg (1989), and more fully by Patton and Chen (1991e), that the parity relation approach is equivalent to the use of a dead-beat observer. A residual signal generated by a non dead-beat observer is equivalent to a post-filtered residual which generated by a dead-beat observer. This implies that the parity relation method provides less design flexibility when compared with methods which are based on observers without any restriction.

2.8.3 Factorization methods for residual generation

A residual generator can also be synthesized in the frequency domain via factorization of the transfer function matrix of the monitored system. This method was initiated by Viswanadham, Taylor and Luce (1987). A more comprehensive study and extension was made by Ding and Frank (1990), in which the FDI problem was systematically formulated and solved via factorization techniques. This approach is also studied by other investigators such as Marquez and Diduch (1992) and Yao, Schaefers and Darouach (1994). The most recent developments including robustness issues can be found in (Frank and Ding, 1993; Ding et al., 1993; Frank and Ding, 1994).

This approach is based on a fact that any $m \times r$ proper rational transfer function matrix $G_u(s)$ can be factorized as (Vidyasagar, 1985):

$$G_u(s) = M^{-1}(s)N(s) \tag{2.61}$$

where $M(s)$ and $N(s)$ are rational and realizable transfer function matrices. Based on this factorization, a residual generator can be designed as:

$$r(s) = Q(s)[M(s)y(s) - N(s)u(s)] \tag{2.62}$$

where $Q(s)$ denotes a dynamic residual weighting. It was pointed out quite early on Section 2.4 that the system output is:

$$y(s) = G_u(s)u(s) + G_f(s)f(s) \tag{2.63}$$

On substituting Eq.(2.63) into Eq.(2.62) together with Eq.(2.61), the residual is:

$$r(s) = Q(s)M(s)G_f(s)f(s) \tag{2.64}$$

which is only affected by the fault. The weighting factor $Q(s)$ can be used to improve the residual performance responding to faults in a particular frequency region.

Note that Eq.(2.62) is a special representation of residual generators which can also fitted into the general framework given by Eq.(2.13) and Fig.2.9. The design of a residual generator is to construct the transfer function matrices $M(s)$ and $N(s)$ which can be given by (Nett, Jacobson and Balas, 1984):

$$\left\{ \begin{array}{lcl} M(s) & = & -C[sI - (A - KC)]^{-1}L + I \\ N(s) & = & C[sI - (A - KC)]^{-1}(B - KD) + D \end{array} \right. \tag{2.65}$$

On comparing the above equations with the transfer function matrices for a full-order observer-based residual generator given in Eq.(2.43), one can see that they are almost identical. This demonstrates the correspondence between observer-based and factorization approaches. Recently, Ding, Guo and Frank (1994) presented a study on the design of linear observers, based on the transfer matrix factorization.

2.9 Model-based FDI via Parameter Estimation

Model-based FDI can also be achieved by the use of system identification techniques (Isermann, 1984; Isermann, 1987; Isermann and Freyermuth, 1990; Isermann, 1993a). An input-output mathematical model of a system can be described in the following form:

$$y(t) = f(P, u(t)) \tag{2.66}$$

where, P is the model coefficient vector which is directly related to physical parameters of the system (e.g. friction, mass, viscosity, resistance, inductance, capacitance). The function $f(\cdot, \cdot)$ can take *both* linear or non-linear formats.

The basic procedure for carrying out FDI using parameter estimation is:

- Establish the process model using physical relations.
- Determine the relationship between model coefficients and process physical parameters.
- Estimate the normal model coefficients.
- Calculate the normal process physical parameters.
- Determine the parameter changes which occur for the various fault cases.

By carrying out the last step for known faults, a database of faults and their symptoms can be built up. During run time, the coefficients of the system model are periodically identified from the measurable inputs and outputs, and compared with the normal and faulty model parameters.

To generate residuals using this approach, an on-line parameter identification algorithm should be used. If one has the estimation of the model coefficient at time step $k-1$ as $\hat{P}_{k-1}$, the residual can be defined in either of the following ways:

$$\begin{cases} r(k) &= \hat{P}_k - P_0 \\ r(k) &= y(k) - f(\hat{P}_{k-1}, u(k)) \end{cases} \tag{2.67}$$

where P_0 is the normal model coefficient.

2.10 Fault Diagnosis for Stochastic Systems

For stochastic systems, the FDI is based on statistical testing of the residuals (Willsky, 1976; Basseville and Benveniste, 1986; Basseville, 1988; Tzafestas and Watanabe, 1990; Basseville and Nikiforov, 1993), for example:

- The weighted sum-squared residual (WSSR) testing (Willsky et al., 1975; Tzafestas and Watanabe, 1990).
- χ^2 testing (Willsky, 1976; Da, 1994).

- Sequential probability ratio testing (SPRT) (Wald, 1947; Willsky, 1976; Tzafestas and Watanabe, 1990) and modified SPRT (Gai and Gurry, 1977; Speyer and White, 1984; Tzafestas and Watanabe, 1990).
- Generalized likelihood ratio (GLR) testing (Willsky and Jones, 1974; Willsky and Jones, 1976; Tanaka and Müller, 1990).

In order to suppress the effect of noise on the residuals, the residual generator has to be specifically designed to deal with the noise. A common approach is the use of Kalman filter-based residual generators. Whilst using a similar structure to the observer, approaches based on the Kalman filter comprise a residual generation mechanism derived by means of a stochastic model of the dynamic system. In normal operation the Kalman filter residual (or innovation) vector (the difference between the measurements and their Kalman filter estimates), is a zero-mean white noise process with known covariance matrix. Mehra and Peschon (1971) proposed the use of different statistical tests on the innovation to detect faults in the system. Many variants of the idea of hypothesis testing for FDI have been published since (Willsky, 1976; Basseville, 1988; Tzafestas and Watanabe, 1990). The idea which is common to all these approaches is to test, amongst all possible hypotheses, that the system has a fault or is fault-free. As each fault type has its own signature, a set of hypotheses can be used and checked for the likelihood that a particular fault has occurred.

Some Kalman filter-like state estimators are developed especially for FDI of stochastic systems, e.g:

- Multiple model adaptive filters (MMAFs) (Willsky et al., 1974; Willsky et al., 1975; Montgomery and Caglayan, 1976; Loparo, Buchner and Vasudeva, 1991).
- Two-stage bias-correction filters (Friedland and Grabousky, 1982; Chen, Zhang and Zhang, 1990).

2.11 Robust Residual Generation Problems

The reliability of fault diagnosis must be higher than the monitored system. The model-based fault diagnosis is based upon the use of mathematical models of the supervised system. The better the model used to represent the dynamic behaviour of the system, the better will be the chance of improving the reliability and performance in diagnosing faults. However, modelling errors and disturbances in complex engineering systems are inevitable, and hence there is a need to develop robust fault diagnosis algorithms. The robustness of a fault diagnosis system means that it must be only sensitive to faults, even in the presence of model-reality differences (e.g. parameter variations, turbulence, and the effects of manoeuvres). Usually, parameter variations and disturbances act upon a real process in an uncertain way, so that it may be difficult to design a fault diagnosis system which is highly sensitive to faults, whilst insensitive to uncertainty and unmodelled disturbances.

The heart of model-based fault diagnosis is the generation of residuals. Both faults and uncertainty affect the residual, and discrimination between their effects is difficult. The task in the design of a robust FDI system is thus to generate residuals which are insensitive to uncertainty, whilst at the same time sensitive to faults, and therefore *robust* (Frank and Wünnenberg, 1987; Frank, 1990; Frank, 1991a; Frank, 1993; Patton et al., 1989; Gertler, 1991; Gertler and Kunwer, 1993) (Patton and Chen, 1991e; Patton and Chen, 1992c; Patton and Chen, 1992c; Patton and Chen, 1992d; Patton and Chen, 1993b; Patton and Chen, 1994; Patton, 1993; Patton, 1994). The robustness is of course only proved if the residual of interest remains insensitive to uncertainty over the whole range of operation of the system being monitored.

To approach the problem from the general point view, one must start with a mathematical description of the monitored system that includes all kinds of modelling uncertainty that can occur in practice and affect the behaviour of the system. Therefore, the state space model of the system is given by:

$$\left\{ \begin{array}{rcl} \dot{x}(t) & = & (A+\Delta A)x(t)+(B+\Delta B)u(t)+E_1 d(t)+R_1 f(t) \\ y(t) & = & (C+\Delta C)x(t)+(D+\Delta D)u(t)+E_2 d(t)+R_2 f(t) \end{array} \right. \tag{2.68}$$

here $d(t) \in \mathcal{R}^q$ is an *unknown* input (disturbance) vector, however the unknown input distribution matrices E_1 and E_2 are assumed to be known. The matrices ΔA, ΔB, ΔC and ΔD are the parameter errors or variations which represent the *modelling errors*. The transfer function description of the system is then:

$$y(s) = (G_u(s)+\Delta G_u(s))u(s)+G_d(s)d(s)+G_f(s)f(s) \tag{2.69}$$

Here $G_d(s)d(s)$ represent the disturbance effect and $G_d(s) = E_2 + C(sI-A)^{-1}E_1$,

$\Delta G_u(s)$ is used to described modelling errors. The terms $G_d(s)d(s)$ and $\Delta G_u(s)u(s)$ together represent modelling uncertainty. If we substitute the system output $y(s)$ into the residual generator Eq.(2.13), the s-domain residual vector is:

$$r(s) = H_y(s)G_f(s)f(s) + H_y(s)\Delta G_u(s) + H_y(s)G_d(s)d(s) \tag{2.70}$$

Both faults and modelling uncertainty (disturbance and modelling error) affect the residual, and hence discrimination between these two effects is difficult. This is the heart of the robustness problem in FDI.

2.11.1 Robustness to disturbances

If the residual generator has been designed to satisfy:

$$H_y(s)G_d(s) = 0 \tag{2.71}$$

i.e., the disturbance is totally de-coupled from the residual $r(t)$, the residual is robust to the disturbance. This is the *principle of disturbance de-coupling* for robust residual generation.

Disturbance de-coupling designs can be achieved by using the unknown input observer (Watanabe and Himmelblau, 1982; Frank and Wünnenberg, 1987; Frank and Wünnenberg, 1989; Chen and Zhang, 1991) or alternatively, eigenstructure assignment approaches (Patton et al., 1986; Patton, 1988; Patton and Chen, 1991f; Patton and Chen, 1991e; Patton and Chen, 1991g). These two approaches are studied in greater detail in Chapters 3 and 4. As far as the design of robust residuals is concerned, these methods are formally equivalent whilst using different mathematical tools to achieve the same goal in robustness (Gertler, 1991). Gertler et al. (Gertler and Luo, 1989; Gertler, 1991; Gertler and Kunwer, 1993) proposed the disturbance de-coupling design based on the so-called orthogonal parity equations. Disturbance de-coupling can also be achieved using frequency domain design techniques (e.g H^∞-norm optimization) (Ding and Frank, 1991; Frank, 1991a; Frank and Ding, 1993; Frank and Ding, 1994).

If the condition (2.71) does not hold, perfect (accurate) de-coupling is not achievable. One can consider an optimal or approximate de-coupling by minimizing a performance index containing a measure of the effects of both disturbances and faults. One suitable choice of performance index can be defined in the frequency domain as (Ding and Frank, 1991):

$$J = \frac{\|H_y(j\omega)G_d(j\omega)\|}{\|H_y(j\omega)G_f(j\omega)\|} \tag{2.72}$$

By minimizing the performance index J over a specified frequency range, an approximate de-coupling design can be achieved (Ding and Frank, 1991; Frank, 1991a; Frank and Ding, 1993; Qiu and Gertler, 1993; Frank and Ding, 1994). The optimal approximate disturbance de-coupling design can also be defined and solved in the time domain (Frank and Wünnenberg, 1989; Wünnenberg, 1990; Chen, Patton and Zhang, 1993) and this is studied in Chapter 7.

2.11.2 Robustness to modelling errors

For modelling errors represented by $\Delta G_u(s)$, the robust problem is more difficult to solve. Two main approaches have been proposed. One, based on an attempt to account for uncertainty in designing the residual is known as *active robustness in FDI* (Patton and Chen, 1991f; Patton and Chen, 1991e; Patton and Chen, 1991b; Patton and Chen, 1992c; Patton and Chen, 1993b). The second approach is called *passive robustness in FDI* (Patton and Chen, 1992c), which makes use of adaptive threshold at the decision-making stage and this is discussed in Section 2.12.

The active way of achieving a robust solution is to obtain an approximate structure for the uncertainty, i.e. to represent approximately modelling errors as disturbances:

$$\Delta G_u(s)u(s) \approx G_{d_1}(s)d_1(s) \tag{2.73}$$

where $d_1(s)$ is an unknown vector and $G_{d_1}(s)$ is a estimated transfer function matrix. When this approximate structure is used to design disturbance de-coupling residual generators, a suitably robust FDI is achievable. As the attempt is made to render the actual residuals robust with respect to uncertainty, we call this active robustness in FDI (Patton and Chen, 1991e; Patton and Chen, 1992c; Patton and Chen, 1993b). As an example, let's assume that the parameter errors can be approximated as:

$$\Delta A \approx \sum_{i=1}^{N} a_i A_i \qquad \Delta B \approx \sum_{i=1}^{N} b_i B_i$$

$$\Delta C \approx \sum_{i=1}^{N} c_i C_i \qquad \Delta D \approx \sum_{i=1}^{N} d_i D_i$$

where A_i, B_i, C_i and D_i are known matrices and have the same dimension as matrices A, B, C, D respectively, a_i, b_i, c_i and d_i are scalar factors. In this case, the modelling error can be approximated by the disturbance term as:

$$E_1 d_1(t) = \Delta A x(t) + \Delta B u(t) = [A_1 \ \cdots \ A_N \ B_1 \ \cdots \ B_N] \begin{bmatrix} a_1 x(t) \\ \vdots \\ a_N x(t) \\ b_1 x(t) \\ \vdots \\ b_N x(t) \end{bmatrix}$$

$$E_2 d_2(t) = \Delta C x(t) + \Delta D u(t) = [C_1 \ \cdots \ C_N \ D_1 \ \cdots \ D_N] \begin{bmatrix} c_1 x(t) \\ \vdots \\ c_N x(t) \\ d_1 x(t) \\ \vdots \\ d_N x(t) \end{bmatrix}$$

The Laplace transformed representation is:

$$G_d(s)d(s) = E_2 d_2(s) + C(sI - A)^{-1} E_1 d_1(s)$$

2.11.3 Discussion on robust FDI

The disturbance de-coupling method for robust FDI has been studied extensively, however its effectiveness has not been fully demonstrated in real problems. The main difficulty arises as most of the disturbances only account for a small percentage of the uncertainty. The disturbance de-coupling method cannot be directly applied to the system with other uncertainties such as modelling errors. The approximate representation of modelling errors and other uncertain factors as the disturbance term provides a practical way to tackle the robustness issue for real systems. Chapter 5 studies different approaches for representing modelling errors and other uncertain factors via the disturbance term with an approximate or estimated distribution matrix. With this estimated distribution matrix, the disturbance de-coupled residual can be designed and the robust FDI problem is solvable. The study given in Chapter

5 covers all possible uncertain situations and the methods are assessed using realistic system simulation models. To extend the application domain of robust model-based FDI, the modelling uncertainty should have a very general format without structural constraints. Chapter 6 studies this problem, in which the robust design is reformulated into a multi-objective optimization problem and solved by a combination of the method of inequalities and genetic algorithms. Another way to tackle robustness problem against modelling errors is via the use of multiple models to cover all possible system operating ranges. This approach, which was originated by Lou et al. (1986) is further developed in Chapter 7.

2.12 Adaptive Thresholds in Robust FDI

Efforts to enhance the robustness of FDI can be made at the decision-making stage (Emami-Naeini, Akhter and Rock, 1986; Emami-Naeini et al., 1988; Ding and Frank, 1991; Frank, 1991a; Frank and Ding, 1993; Ding et al., 1993; Frank and Ding, 1994). Due to inevitable parameter uncertainty, disturbance and noise encountered in a practical application, one will rarely find a situation where the conditions for a perfectly robust residual generation are met. This is especially true for modelling errors. It is therefore necessary to provide sufficient robustness not only in the residual generation stage but also in decision-making. When the decision-making is made robust against uncertainty, we can speak of passive robustness in FDI (Patton and Chen, 1991e; Patton and Chen, 1992c; Patton and Chen, 1993a) in which case it may not be necessary (or it may be difficult) to make the residual robust. Passive robustness is thus an alternative to active robustness which should be used when there is very limited system information available.

The goal of robust decision-making is thus to minimize the false and missing alarm rates due to the effects that modelling uncertainty and unknown disturbances will have on the residuals. This can be achieved in several ways, e.g. by statistical data processing, averaging, or by finding and using the most effective threshold.

In practical situations, the residual is never zero, even when no faults occur. A threshold must then be used in the residual evaluation stage. Normally, the threshold is set slightly larger than the largest magnitude of the residual evaluation function for the fault-free case. The smallest detectable fault is a fault which drives the residual evaluation function to just exceed the threshold. Any fault which produces a residual response smaller than this magnitude is not detectable. From our point of view, the purpose of the robust design is to decrease the magnitude of the fault free residual and maintain (even increase) the magnitude of faulty residuals. From this setting, "adaptive threshold" methods are not really robust FDI methods. They can however be grouped into the class of passive methods for robust FDI.

The decision-making stage normally involves a thresholding process, the choice of the threshold is not at all a straightforward issue, as pointed out by Gai, Adams, Walker and Smestad (1978). When fixed thresholds are used, the sensitivity to faults will be intolerably reduced if the threshold is chosen too high, whereas the false alarm rate will be too large when the threshold is chosen too low. The proper choice of the threshold is a delicate problem. Clearly, there should be an optimum choice of threshold level and Walker et al. (Gai, Adams, Walker and Smestad, 1978; Walker and Gai, 1979; Walker, 1989) showed how this can be achieved using the theory of Markov processes. Ding and Frank (Frank and Ding, 1993; Ding et al., 1993; Frank, 1993) proposed a way to calculate the minimum detectable fault in the frequency domain, with the threshold set just slightly higher than the residual evaluation function in response to the minimum detectable fault. The determination of the threshold in the time-domain is studied by Seliger and Frank (1993) and has also discussed by Frank (1993). Recently, Faitakis, Thapliyal and Kantor (1994) studied the computation of thresholds using vector and matrix norm operations.

In the case of large manoeuvres, these changes might be large enough so that there is no fixed threshold that allows satisfactory FDI at a tolerable false and missed alarm rates. The solution for such problems is to use adaptive thresholds (Clark, 1989), where thresholds are varied according to the control activity and the noise and the fault signal properties of the monitored system. This concept is illustrated in Fig.2.16 which shows the typical shape of an adaptive threshold for direct residual evaluation.

An interesting question is how should we determine the functional form of the adaptive threshold law? Clark (1989) used an empirical adaptive law, whilst Emami-Naeini et al. (Emami-Naeini et al., 1986; Emami-Naeini et al., 1988) proposed the threshold selector (or threshold adaptor) method and Ding and Frank (1991) developed this concept further in connection with frequency domain approaches. This was also developed by Isaksson (1993). All the recent research has shown that the adaptive threshold can be obtained in a systematic way and it presents a new and innovative tool for analysis and synthesis of FDI systems.

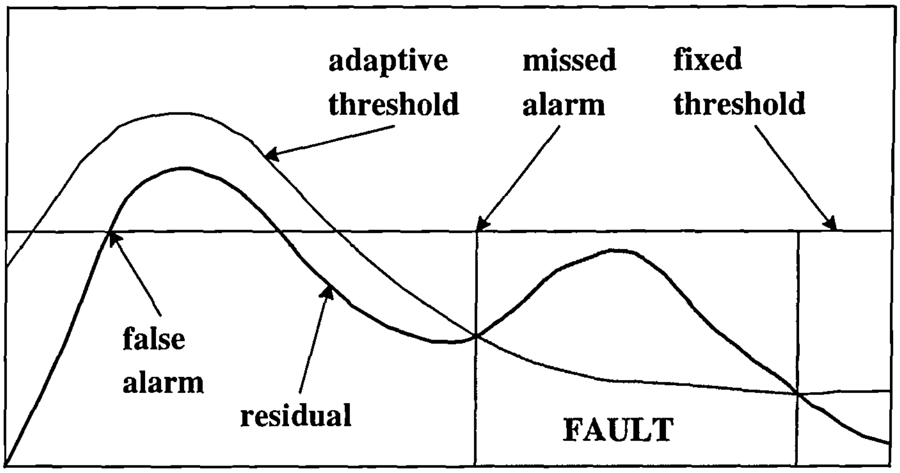


Figure 2.16: **Application of an adaptive threshold**

As a simple example for determining adaptive thresholds, assuming that the disturbance de-coupling condition for the uncertainty arising from disturbances is achieved (see Eq.(2.71)), and the residual uncertainty is only caused by modelling errors, i.e. the fault-free residual is:

$$r(s) = H_y(s)\Delta G_u(s)u(s) \tag{2.74}$$

Assuming that the modelling errors are bounded by a limiting value δ, i.e.

$$\|\Delta G_u(j\omega)\| \leq \delta \tag{2.75}$$

In this situation, the frequency response of the fault-free residual will be bounded as:

$$\begin{aligned} \|r(j\omega)\| &= \|H_y(j\omega)\Delta G_u(j\omega)u(j\omega)\| \\ &\leq \|H_y(j\omega)u(j\omega)\|\|\Delta G_u(j\omega)\| \leq \delta\|H_y(j\omega)u(j\omega)\| \end{aligned} \tag{2.76}$$

Therefore, an adaptive threshold $T(t)$ can be generated by a linear system as follows:

$$T(s) = \delta H_y(s)u(s) \tag{2.77}$$

It is readily seen that the threshold $T(t)$ is no longer fixed but depends on the input $u(t)$, thus being adaptive to the system operation. A fault is declared if $\|r(t)\| > \|T(t)\|$. A robust FDI scheme with the threshold adaptor or selector is shown in Fig.2.17.

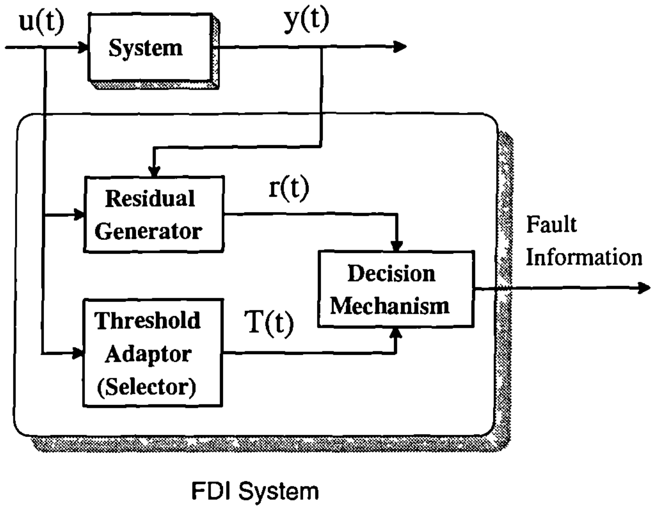


Figure 2.17: **A FDI system with the threshold adaptor (selector)**

As discussed above, the use of adaptive thresholds is a passive approach to robust FDI. By this we mean that no effort is made to design a robust residual. The robust problem is tackled by reliable decision-making under the situation of uncertain residuals. A combination of active and passive approaches can offer potential for real robustness, especially when considering practical applications. It is believed that the success of an FDI scheme depends on the accuracy and choice of modelling of the monitored process. Hence, some attention in the field of robustness studies must be paid to ensure that sufficient modelling of the monitored process is achieved.

2.13 Applicability of Model-based FDI Methods

Many model-based FDI approaches have been developed so far. An engineer may find himself/herself in a dilemma when he/she wants to chose an approach to suit his problem. Some research attempts have been made in identifying the applicability of model-based FDI methods (Isermann, 1993b; van Schrick, 1994b). Recently, Patton, Chen and Nielsen (1994) presented some general guide-lines on the choice of FDI methods. Some of the author's opinions on the applicability of model-based FDI are presented in this Section. It must be stressed that the statements are only of a preliminary nature and there is no claim for their completeness.

A fault diagnosis technique should be able to complete the following important tasks:

- Detect and isolate faults in sensors, actuators and components.
- Detect and isolate incipient faults as well as abrupt faults.

In the design of fault diagnosis system, the following tasks and questions should be considered:

- How to handle noise in the system?
- How to handle multiple faults?
- How to handle disturbances (additive uncertainty)?
- How to handle modelling errors (multiplicative uncertainty)?
- How to handle nonlinearity?
- How to cope with detection delay?
- How to overcome complexity in the FDI algorithm design?
- How to minimize the complexity in FDI algorithm implementation (or execution)?
- What arc thc rcquircment for *a priori* modelling information?
- How good are self learning and adaptive capabilities?

The applicability of different model-based FDI approached are listed as follows.

<u>**Observer-based approaches**</u>

- The isolation task can be fulfilled via
  - a structured residual set designed by a dedicated or a generalized observer scheme.
  - a directional residual vector designed via a fault detection filter.
- Reaction to incipient faults is very fast.
- Very suitable for detecting and isolating faults in actuators and sensors.

- Possibility of detecting and isolating faults in parameters, although complicated to achieve.
- Design procedure is systematic and simple.
- Easy to implement and execution algorithm is simple.
- Easy to handle multiple faults if the measurement number is sufficient.
- Handling noise in the system:
  - Statistical properties are unknown: An additional filter can be applied to the residual, based on assumptions on fault and noise frequency bands.
  - Statistical properties are known: A Kalman filter can be used to produce the fault-free residual with minimum variance and, consequently reducing false and missed alarms.
- Nonlinearity:
  - The application of linear observers to a linearized model is simple but difficulties may be encountered for complex and highly nonlinear systems.
  - Non-linear observers are direct and accurate, however they are only applicable for particular classes of nonlinearities. The approach is not yet mature.
- Robustness: there are many mature techniques available, e.g.
  - Unknown input observer.
  - Eigenstructure assignment.
- Requirements for *a priori* modelling:
  - A reasonably accurate model is required.
- Adaptive and self-learning capability:
  - Adaptive observers can be employed for systems with unknown or time varying parameters.

The applicability of factorization methods is almost the same as the observer-based methods, however it can only be applied to *linear* or *linearized* models.

Parity relation approaches

As pointed out in Section 2.8.2, the parity relation approach is equivalent to the observer-based approach in certain conditions. Hence, most of their applicability conditions are the same. In the following, only the differences are listed.

- Fault isolation:
  - Structured residual set designed by orthogonal parity relations.
  - Directional residual is possible but difficult.
- Handling noise in the system:
  - An additional filter can be applied to a residual, based on assumptions on fault and noise frequency bands. It is not easy to incorporate noise statistics into the design.
- Nonlinearity:
  - Only linearized models can be used, simple but difficulties may be encountered for complex and highly nonlinear systems.
- Robustness: there are many mature techniques available, e.g.
  - Orthogonal parity relations for additive uncertainty.
  - Optimally robust parity relations which can be used for systems with parameters within known error bounds.
- Adaptive and self-learning capability:
  - No available research on this aspect yet.

The observer-based and parity relation approaches can be designed not only in the time domain, but also in the frequency domain by factorizing the transfer matrix of the monitored system. The latter approach can make full use of the advantages in the frequency domain. The robust design can be achieved by enhancing fault responses and reducing noise and modelling uncertainty responses, based on the information on frequency distribution of faults, noise and modelling uncertainty. The residual response for a particular fault can also be shaped in the frequency domain according to performance requirements. The frequency domain design normally requires less accurate models than the time domain.

Parameter estimation approaches

- The isolation is normally achieved by analysing the sensitivity matrix corresponding to the prediction errors with physical parameters. Fault isolation is not very easy because the physical parameters do not uniquely correspond to model parameters. The directional residual is not normally possible to design.
- Reaction for incipient faults is slow.
- The detection and isolation of faults in actuators and sensors are possible but complicated.
- The detection and isolation of faults in parameters are very straightforward.
- The design procedure is systematic but not simple.
- Implementation complexity:
  - Requires a large amount of computation.
- The detection and isolation of multiple faults is an not easy task unless a large number of sensors are installed.
- Noise is easy to handle in the parameter estimation procedure.
- Nonlinearity:
  - Possible to handle using identification techniques for non-linear systems.
- Robustness:
  - Dependent on the robustness properties of the estimation method.
- Requirements for *a priori* modelling:
  - Model structure, do not require model parameters.
- Adaptive and self-learning capability:
  - Excellent, if the parameter estimation method is adaptive.

Discussion on applicability

Some guide-lines about applicability of different methods have been given. However, the choice of FDI methods is still a complicated problem. The main factor to be considered is the availability of system information. Some information about the normal system operation (normal behaviour) is necessary. This will serve as a reference base to be compared with. The information of the normal system behaviour is usually expressed in terms of models, i.e. a model is necessary for FDI. The "black-box" assumption is not very suitable for advanced fault diagnosis and analysis and control reconfiguration. Some investigators argue that the observer-based methods require models, but parameter estimation methods do not require models. This is not really true because the principle of the parameter estimation approach is to compare estimated with known parameters of the system. Moreover, the modelling procedure is necessary to establish relationships between physical and model parameters. The system model can taken a different formats, e.g., state space, parametric, frequency domain, qualitative model, etc. Hence, different methods require different model formats, and the first criterion in choosing model-based methods is the availability of the model type. As pointed by Gertler and Costin (1994), most of the time spent in developing a fault diagnosis scheme is spent in understanding the process to be diagnosed. It is hard to say whether a particular method is better than another method because one may be good in one aspect but bad in others. Hence, the second criterion in the choice of FDI method is dependent on the problem to be solved.

When we don't have *a priori* modelling information, what kind of model should we build for fault diagnosis purposes? This question is very difficult to answer, as the designer's experience and background play an important role. Sometimes, it depends just entirely on the designer's personal preference. If a particular criterion is needed, the more accurate and detailed the model, the better will be the fault diagnosis performance. If possible, a detailed state space model derived from physical laws is a best choice. However, accurate modelling would involve a large amount of work and, sometimes this is impossible. A cost effective way is to identify a parametric model using identification techniques, based on input and output data of the system under normal condition. However, fault diagnosis performance could be degraded if the identified model is not very accurate. Moreover, in-depth analysis of fault location and cause is not very easy if input-output models are used. If the quantitative (analytical) model is very difficult to obtain or, if uncertain factors are dominant in the system, one can consider building qualitative (heuristic) models which only require crude description. Some human knowledge about the system can also be expressed in heuristic format and included in the qualitative model. This would lead to the use of qualitative model-based approaches or even the use of a knowledge base.

When using a real life application of a FDI scheme, whose feasibility has been demonstrated (including the use of a laboratory demonstrator), many practical and unforeseen difficulties present themselves. To overcome these difficulties, one must understand the detailed design of the fault diagnosis scheme, as well as the nature of the practical problems. This usually requires the fault diagnosis designer to follow his work into the specific engineering field, either doing the implementation himself or working closely with those who do it. For this reason we should also include the application domain as far as possible into our research in this field.

There have been a significant number of application studies of fault diagnosis techniques, including some actual application to either process plant or laboratory experiments using real-time equipment. The book by Patton et al. (1989) provides some useful pre-1989 application examples. More recent application examples can be found in recent survey papers and the recent conference proceedings (Isermann, 1991; Labarrère, 1993; Ruokonen, 1994). However, there is a great need for academia and industry to work together very closely to put fault diagnosis into the more useful setting of real application.

2.14 Integration of Fault Diagnosis Techniques

Many FDI methods have been developed and they show different properties with regard to the diagnosis of different faults in a process. To facilitate reliable FDI, taking advantages of different methods, a proper integration of several methods is a good solution (Isermann, 1994). Furthermore, a comprehensive fault diagnosis require a knowledge based treatment of all available, analytical and heuristic information. This can be performed by an integrated approach to knowledge-based fault diagnosis.

2.14.1 Fuzzy logic in fault diagnosis

The problem of robust decision-making can be treated in a novel way with the aid of fuzzy logic. To outline briefly the basic idea let us again consider the case that the residual due to faults is also contaminated by noise and the effect of uncertainty due to incomplete de-coupling, so that the residual will be non-zero even in the absence of faults. Typically, these effects will be time varying, i.e., the residual will fluctuate depending on the unknown time functions of the disturbances, noise and inputs of the process. This is a common situation, and hence fuzzy logic seems to be a natural tool to handle the decision making in a complicated and uncertain situation; based on incomplete information. The appealing feature of fuzzy logic is that it constitutes a powerful tool for modelling vague and imprecise facts and is therefore highly suited for applications where complete information about the system is not available to the designer.

Much effort has been spent on trying to decrease the uncertainty associated with quantitative residual generation. However, it is impossible to fully eliminate the effect of uncertainty. Based upon this limitation, the problem encountered in residual evaluation is to make the correct decisions on the basis of uncertain information. Non-Boolean reasoning (e.g. fuzzy logic) can be a suitable tool for this task. Contrary to classical logic which only allows a definite classification of fixed values, the fuzzy logic offers a form for the description of tolerances, i.e. fuzzy values, heuristic rules and their combination. There are, for instance, a lot of processes and experiences which can be grasped by humans heuristically, but which cannot be described analytically. The question of how this expert knowledge can be put into the form of a rule-based knowledge format has been answered partly through the use of fuzzy logic. Fuzzy logic endows machine intelligence with such human traits as the ability to make decisions based on shades of grey, instead of black-and-white information. Fuzzy processing can be divided into essentially the following stages. In the first, the residuals are compared with membership functions (or degree-of-belief curves) which are often assumed to be of triangular shape. In the second stage the lower of the two antecedent outputs is selected. Then the output of all rules is combined. Finally, the centre of gravity (or another averaging methods) is used to defuzzify the output and lead to the possibility of definite decision-making. The introduction of fuzzy logic can improve the decision-making, and in turn will provide reliable and sufficient FDI which are applicable for real industrial systems. However, difficulty arises in the training of the algorithm in the inference mechanism.

Frank and his co-workers (Frank and Kiupel, 1993; Frank, 1993; Frank, 1994) use fuzzy logic for residual evaluation. The aim is to release weighted alarms instead of yes-no decision. Such information can, if necessary, be given to a human operator to make the final yes-no decision or even train a specialist to perform the task. A similar approach was proposed by Ulieru and Isermann (Ulieru, 1993; Ulieru and Isermann, 1993), where analytical fault detection was integrated with fuzzy diagnostic decision-making. The approach solves the problem at two levels: first analytical redundancy is used to generate symptoms and then fault detection and isolation is achieved using heuristic techniques based on fuzzy logic.

2.14.2 Qualitative fault diagnosis

It may often be difficult and time consuming to develop a good mathemaical model, there have been many attempts to use cruder descriptions (Lunze, 1994). Fault diagnosis of dynamic systems can also be based upon declarative knowledge of the system which is available in qualitative (rather than quantitative) form (Dvorak, 1992; Leitch, Kraft and Luntz, 1991; Leitch, 1993; Leitch and Quek, 1992; Shen and Leitch, 1993; Lunze, 1991; Lunze and Schiller, 1992; Zhang, 1991). This approach is based upon the concept of a qualitative model which unlike the quantitative counterpart only requires declarative (heuristic) information e.g. the sign of variables, the tendencies of variables (increasing, decreasing or constant), order and/or relative magnitude, and hence can be robust with respect to uncertainty in a well defined sense. Clearly, this can be a significant advantage and qualitative methods can serve to confirm hypotheses already tested using the quantitative methods. The qualitative approach to fault diagnosis is motivated by the following circumstances encountered in practical applications:

- Faults cannot be reasonably described by analytical models, e.g. a valve is blocked or a pipe is broken.
- The on-line information available is not given by quantitative measurements of the system output but by qualitative assessments of the current operating conditions. For example, the information "the water level is high" cannot be unambigously transformed into quantitative measurement data. Likewise, alarm messages are qualitative in nature because they do not provide precise state information. No analytical model can be used to process this kind of on-line information.
- If the system structure or parameters are not precisely known and diagnosis has to be based primarily on heuristic information, no quantitative model can be set up.

In these cases a qualitative approach to fault diagnosis is necessary. There have been several approaches in qualitative fault diagnosis, e.g. fault tree diagnosis and association-based diagnosis. The fault tree approach uses the evolution of the fault through the dynamic system which is described by a fault tree, event trees or causal networks. The association-based approach uses the relations among faults and the faulty system observations which are described by rules. The current attention is mainly focused on the qualitative model-based approach which uses the qualitative model derived directly from the physical laws of the system under consideration.

One of the disadvantages of the qualitative approach to fault diagnosis is the possibility of ambiguity which can arise when manipulating two or more declarative variables, for example the sum of a positive variable and a negative variable can either be negative or positive! This is clearly a situation to avoid when using these methods. Another disadvantage is that qualitative methods are relatively crude and usually cannot, on their own, be used to detect soft faults as the diagnosis is symptom-based. Quantitative and qualitative approaches have a lot of complementary features and can be suitably combined together to capitalize on their advantages by increasing the robustness of quantitative methods (Handelman and Stengel, 1989). This combination can also minimize the disadvantages of the two approaches; in particular it is important that ambiguity arising in qualitative reasoning is reduced or eliminated. Hence, one of the main aims of future research on model-based fault diagnosis is to find the way to combine these two approaches together to provide highly reliable diagnostic information.

2.14.3 Integrated fault diagnosis systems

Quantitative model-based fault diagnosis generates symptoms based on the analytical knowledge of the process. In most cases this is, however, not enough information to perform efficient FDI, i.e. to indicate the location, and the size of the fault. In such cases, fault diagnosis requires the use of a knowledge-based treatment (Milne, 1987; Isermann and Freyermuth, 1991a; Isermann and Freyermuth, 1991b; Tzafestas, 1989; Tzafestas and Watanabe, 1990). The intention is to transfer the existing knowledge of engineers, operators and maintenance staff into the supervision methodology and to develop on-line integrated expert systems for fault diagnosis.

Fig.2.18 shows a typical integrated fault diagnosis system. Both analytical and heuristic knowledge are used in the system. Analytical knowledge includes: a quantitative model, normal process behaviour, process history and fault statistics (if quantifiable), state estimation, parameter estimation, parity relations, etc. Heuristic

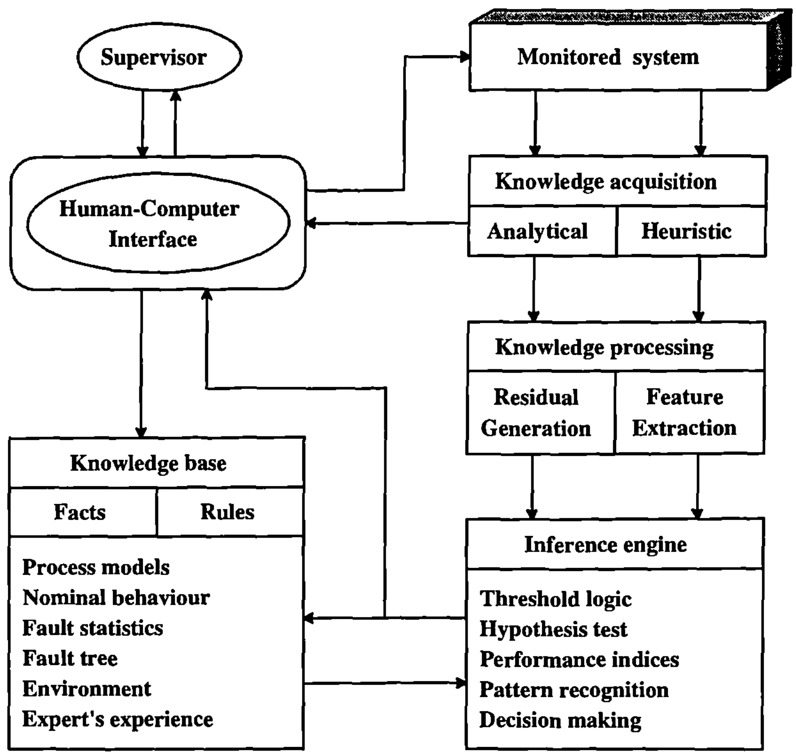


Figure 2.18: **An integrated knowledge-based system for fault diagnosis**

knowledge (available from physical law and experience) includes: fault tree (connection of symptoms and faults), process history and fault statistics (if only qualitatively known), etc. The knowledge will be processed in terms of residual generation and feature extraction. The processed knowledge is then given to an inference mechanism which comprises residual evaluation, symptom observation and pattern recognition. For the last part of the problem solving, a certain amount of human expertise and judgement, expressed in rules and facts can be used. This can be formulated, for example, by different levels of diagnostic reasoning and different kinds of models.

2.15 Summary

This chapter has presented a tutorial treatment on the basic principles of model-based FDI. The FDI problem has been formalized in a uniform framework by presenting mathematical descriptions and definitions. Within this framework, the residual generation has been identified as a central issue in model-based FDI. By analysing a properly designed residual signal, FDI tasks can be performed. The residual generator has been summarized in a generalized structure which can cover all residual generation methods. The concept of fault detectability to guarantee reliable fault detection has been defined in this chapter. The ways of designing residuals for isolation have also been discussed. The most commonly used residual generation methods have been presented in a tutorial setting and the applicability of model-based FDI techniques have been discussed. The success of fault diagnosis depends on the quality of the residuals. A prerequisite of residuals for successful diagnosis is the robustness with respect to modelling uncertainty. The robust FDI problem has been discussed in this chapter and a foundation has been laid down for further studies in the following chapters of the thesis. Other FDI methods such as fuzzy logic and qualitative modelling have been discussed briefly and some perspectives in forming an integrated knowledge-based fault diagnosis, utilising all available analytical and heuristic information have been discussed.

Chapter 3

ROBUST RESIDUAL GENERATION USING UNKNOWN INPUT OBSERVERS

3.1 Introduction

The generation of robust residuals is the most important task in model-based fault diagnosis techniques. As pointed out in Section 2.11, one of the dominant approaches for robust residual generation is the use of the disturbance de-coupling principle. In this approach, uncertain factors in system modelling are considered to act via an unknown input (or disturbance) on a linear system model. Although the unknown input vector is unknown, its distribution matrix is assumed known. Based on the information given by the distribution matrix, the unknown input (disturbance) can be de-coupled from the residual. Robust FDI is thus achievable using the disturbance de-coupled residual. This chapter focuses on the robust residual generation problem via unknown input observers. The principle of the unknown input observer (UIO) is to make the state estimation error de-coupled from the unknown inputs (disturbances). In this way, the residual can also be de-coupled from each disturbance, as the residual is defined as a weighted output estimation error. This approach was originally proposed by Watanabe and Himmelblau (1982) who considered the robust sensor fault detection and isolation problem for the system with modelling uncertainty. Later, Frank & Wünnenberg (Wünnenberg and Frank, 1987; Frank and Wünnenberg, 1989; Wünnenberg, 1990) generalized this approach for detecting and isolating both sensor and actuator faults by considering the case when unknown inputs also appear in the output equation. In parallel with this development, a robust scheme for diagnosing actuator faults via UIOs is proposed by Chen and Zhang (1991). A very important contribution of the paper by Chen and Zhang (1991) was to demonstrate the robust FDI approach via to a realistic chemical process system example. Note that Viswanadham and Srichander (1987) and Phatak and Viswanadham (1988) also studied the actuator fault detection and isolation problem via UIOs, however they failed to consider robustness issues. Many other investigators have considered the use of UIOs for robust FDI: e.g. Hou and Müller (1991), Hou and Müller (1994b), Frank and Seliger (1991), Seliger and Frank (1991a), Keller, Nowakowski and Darouach (1992), Chang and Hsu (1993a), Ragot, Maquin and Kratz (1993), Saif and Guan (1993), Wang and Daley (1993), Chen and Patton (1994b), Shields (1994), Yu, Shields and Mahtani (1994b).

The first step to generate robust (in the sense of disturbance de-coupling) residuals is to design a UIO. The problem of UIO design dates back to 1975 (Wang, Davison and Dorato, 1975). Darouach, Zasadzinski and Xu (1994) and Hou and Müller (1994a) reported the recent developments, and different methods for designing UIOs are discussed in Section 3.2. This chapter proposes a new full-order UIO structure. A rigourous mathematical foundation in designing a full-order UIO has been laid down and the necessary and sufficient existence conditions are presented and thoroughly proved. When compared with other UIO design methods, the existence conditions are very easy to verify and the design procedure is simple. This avoids some of the unnecessary and complex computation that is otherwise required for UIO design. An example of a typical complexity is the Kronecker canonical form transformation method (Frank and Wünnenberg, 1989) which can also suffer from numerical conditioning problems.

Robust FDI schemes based on UIOs have been studied further in Section 3.3 where an application example of isolating actuator faults in a nonlinear process is presented. Unlike some other work in which the reduced order structure has been used, this chapter is based exclusively on the use of the full-order UIO. The unknown input de-coupling conditions for a full-order UIO are not very different from those of the reduced order counterpart. However, for a full-order UIO, there is more design freedom available to achieve other required performances, after the disturbance de-coupling conditions have been satisfied. This is easy to understand, since the number of free parameters will increase if the observer order is increased. This chapter exploits the remaining design freedom to design *directional* residuals (Section 3.4), and to produce the *minimal variance* state estimation (Section 3.5).

As pointed in Section 2.7, one of the approaches for fault isolation is to design a directional residual vector, i.e. to make the residual vector lie in a fixed and fault-specific direction in the residual space in response to each fault. With directional residual vectors, the fault isolation problem is one of determining which of the known fault signature directions the residual vector lies the closest to. The most effective way to generate directional residual vectors is the use of the Beard fault detection filters (BFDF) (Beard, 1971; Jones, 1973; Massoumnia, 1986b; White and Speyer, 1987; Park and Rizzoni, 1993; Park and Rizzoni, 1994). It should be pointed out that this class of observers has been known as the "failure detection filter" (Beard, 1971; Jones, 1973; White and Speyer, 1987) in the early development of fault (failure) diagnosis. Fault detection filters are a special class of full-order Luenberger observers with a specially designed feedback gain matrix, which can make the output estimation error (residual vector) have uni-directional characteristics associated with some known fault directions. This is the main and most appealing feature of fault detection filters. However, the robustness issues have not been considered in the context of BFDFs up to now. Hence, this approach does not account for the effects of disturbances, non-linearities, modelling errors, parameter variations and other uncertain factors in the system. There would be false or missed alarms when this approach is directly applied to industrial systems, in which the uncertain factors are unavoidable in modelling (specially for systems such as mechanical, eletromechanical, thermofluid and aircraft systems). The application of BFDFs has been obstructed by the lack of robustness. Section 3.4 proposes a new strategy for the design of robust fault detection filters which ensures that the residuals have both disturbance de-coupling and uni-directional properties. This is done by combing the UIO and the BFDF principles. By the use of the UIO principle, the residual has been made robust against unknown inputs (disturbances). The uni-directional property is achieved based on BFDF techniques using the design freedom available after the disturbance de-coupling conditions have been satisfied. A filter which can produce disturbance de-coupled and directional residuals is called a "robust (disturbance de-coupled) fault detection filter". The robust fault detection filter developed in this section is also demonstrated via a realistic example.

Section 3.5 considers the optimal filtering and robust fault diagnosis problems for stochastic systems with unknown disturbances. An optimal observer is proposed, which can produce disturbance de-coupled state estimation with minimum variance for time-varying systems with both noise and unknown disturbances. The output estimation error with disturbance de-coupling and minimum variance properties is used as a residual signal. A statistical testing procedure is then applied to examine the residual and hence to diagnose faults. The method developed is applied to an illustrative example and simulation results show that the optimal observer can give good state estimation; the fault detection approach taken is able to detect faults reliably in the presence of both modelling errors and noise. One of the important contributions of Section 3.5 is the development of optimal disturbance de-coupled observers for systems with both unknown disturbance and noise. The scope of applications of the optimal observer extends to a wide range of stochastic uncertain systems and is not confined to the fault diagnosis problem domain.

The primary requirement for a UIO or other disturbance de-coupling based robust residual generation approaches is that the unknown input distribution matrix must be known *a priori*, although the actual unknown input itself does not need to be known. If the uncertainty is caused by the disturbance, this requirement is easy to meet and hence the robustness in FDI with respect to unknown disturbances can be easily solved. However, the disturbance de-coupling approach cannot be directly applied to systems for which the uncertainty is caused by modelling errors, linearization errors, parameter variations etc. This is because the distribution matrix for such uncertain factors is normally unknown. This problem has obstructed the application of these robust FDI approaches in real industrial systems. To solve this problem, some investigators led by Patton & Chen (Patton and Chen, 1991f; Patton and Chen, 1991b; Patton, Chen and Zhang, 1992; Patton, Zhang and Chen, 1992; Gertler and Kunwer, 1993; Gertler, 1994) have suggested an approach in which modelling errors and other uncertain factors are represented approximately as unknown disturbances, with an *estimated* distribution matrix. In this way, an optimally robust solution is achievable. This approximate strategy has extended the application domain of disturbance de-coupling based robust residual generation approaches. All three application examples presented in this chapter illustrate how different kinds of uncertain factors can be represented approximately as unknown input terms. These uncertain factors are, for example, the nonlinear terms in the dynamic equation of a nonlinear process (Section 3.3), the linearization error in a system as complex as a jet engine (Section 3.4) and parameter variations in a flight control system (Section 3.5). The simulation results in all three examples show the power of these proposed methods. The problem of representing modelling errors as an unknown input term is examined in more detail in Chapter 5.

3.2 Theory and Design of Unknown Input Observers

This section deals with the observer design for a class of systems, in which the system uncertainty can be summarized as an *additive* unknown disturbance term in the dynamic equation described as follows:

$$\begin{cases} \dot{x}(t) &= Ax(t) + Bu(t) + Ed(t) \\ y(t) &= Cx(t) \end{cases} \tag{3.1}$$

where $x(t) \in \mathcal{R}^n$ is the state vector, $y(t) \in \mathcal{R}^m$ is the output vector, $u(t) \in \mathcal{R}^r$ is the known input vector and $d(t) \in \mathcal{R}^q$ is the unknown input (or disturbance) vector. A, B, C and E are known matrices with appropriate dimensions.

Remarks:

(a) There is no loss of generality in assuming that the unknown input distribution matrix E should be full column rank. When this is not the case, the following rank decomposition can be applied to the matrix E (see Appendix B):

$$Ed(t) = E_1 E_2 d(t)$$

where E_1 is a full column rank matrix and $E_2 d(t)$ can now be considered as a new unknown input.

(b) The term $Ed(t)$ can be used to describe an additive disturbance as well as a number of other different kinds of modelling uncertainties. Examples are: noise, interconnecting terms in large scale systems, nonlinear terms in system dynamics, terms arise from time-varying system dynamics, linearization and model reduction errors, parameter variations. Some examples of this problem are presented in the following sections of this chapter and a detailed study can be found in Chapter 5.

(c) The disturbance term may also appear in the output equation, i.e.,

$$y(t) = Cx(t) + E_y d(t)$$

This case is not considered here because the disturbance term $E_y d(t)$ in the output equation can be nulled by simply using a transformation of the output signal $y(t)$, i.e.

$$y_E(t) = T_y y(t) = T_y C x(t) + T_y E_y d(t) = T_y C x(t)$$

where $T_y E_y = 0$, if one replaces $y(t)$ and C with $y_E(t)$ and $T_y C$, the problem will be equivalent to one without output disturbances.

(d) For some systems, there is a term relating the control input $u(t)$ in the system output equation, i.e.

$$y(t) = Cx(t) + Du(t)$$

As the control input $u(t)$ is known, a new output can be constructed as:

$$\overline{y}(t) = y(t) - Du(t) = Cx(t)$$

If the output $y(t)$ is replaced by $\overline{y}(t)$, the problem will be equivalent to the one without the term $Du(t)$. For brevity, the term $Du(t)$ is omitted in this chapter as this does not affect the generality of the discussion on the observer design.

Definition 3-1: {Unknown Input Observer (UIO)} An observer is defined as an unknown input observer for the system described by Eq.(3.1), if its state estimation error vector $e(t)$ approaches zero asymptotically, regardless of the presence of the unknown input (disturbance) in the system.

The problem of designing an observer for a linear system with *both* known and unknown inputs has been studied for nearly two decades (Wang et al., 1975). The problem is of considerable importance as, in practice there are many situations where disturbances are present. Alternatively, some of the system inputs are inaccessible (or unmeasurable), and therefore a conventional observer which uses all input signals cannot be used. It is more useful to assume no *a priori* knowledge about unknown inputs. Wang et al. (1975) proposed a minimal-order UIO for the system (3.1). The existence conditions for such an $(n-m)th$-order observer were shown by Kudva, Viswanadham and Ramakrishna (1980). After the work of Wang et al. (1975), many approaches for designing unknown input observers have been proposed, for example, the geometric method by Bhattacharyya (1978), the inversion algorithm by Kobayashi and Nakamizo (1.ს‹), the matrix algebra method by

Watanabe and Himmelblau (1982), the generalized matrix inversion approach by Miller and Mukundan (1982), and the singular value decomposition technique by Fairman, Mahil and Luk (1984). Park and Stein (1988) studied the simultaneous estimation problem for both states and unknown inputs. The problem of designing reduced order UIOs has been revisited by Hou and Müller (1992) and Guan and Saif (1991) using algebraic approaches. In their studies, the state vector is divided into two parts, via a linear transformation onto the state equation, a part can be directly obtained from the measurements, and another part has to be estimated using a reduced order disturbance de-coupled observer. More recently, Hou and Müller (1994a) presented a unified viewpoint in designing UIOs.

Unlike all the above mentioned work in which the reduced order observer structure has been used, Kurek (1982) proposed a full-order unknown input observer structure. Yang and Richard (1988) gave a direct design procedure for full-order UIOs and have showed, through an example, that the reduced-order observer may restrict the convergence rate in estimation. However, the design procedure they presented is very complicated and involves some trial-and-error exercises. Furthermore, the existence conditions are not very easy to verify. This full-order UIO structure is re-examined by Darouach et al. (1994). It has been shown that the minimal order of a UIO is $(n-m)$, any order between $(n-m)$ to n is possible for a UIO to be exist. The disturbance de-coupling conditions for a full-order UIO are not very different from those of a reduced-order UIO. That is to say, there are no significant differences between two UIO structures, as far as unknown input (disturbance) de-coupling is concerned. However, there is more design freedom available for a full-order UIO to achieve other required performances such as the rate of convergence and minimal variance. This is easy to understand since the number of free parameters will increase if the observer order is increased.

In this study, a full-order UIO structure is used since extra design freedom is required for generating directional residuals in fault isolation. A rigorous mathematical foundation in designing full-order UIOs is presented. The necessary and sufficient conditions for this observer to exist are given and thoroughly proved in this chapter. These conditions are easy to verify and the design procedure is systematic and easy to implement. Moreover, one of the contributions of this chapter is to show that the remaining freedom can be used to make the residual have directional properties (or make the state estimation error have minimal variance), after unknown input (or disturbance) de-coupling has been achieved.

3.2.1 Theory of UIOs

The structure for a full-order observer is described as:

$$\begin{cases} \dot{z}(t) & = & Fz(t) + TBu(t) + Ky(t) \\ \hat{x}(t) & = & z(t) + Hy(t) \end{cases} \tag{3.2}$$

where $\hat{x} \in \mathcal{R}^n$ is the estimated state vector and $z \in \mathcal{R}^n$ is the state of this full-order observer, and F, T, K, H are matrices to be designed for achieving unknown input de-coupling and other design requirements. The observer described by Eq.(3.2) is illustrated in Fig.3.1.

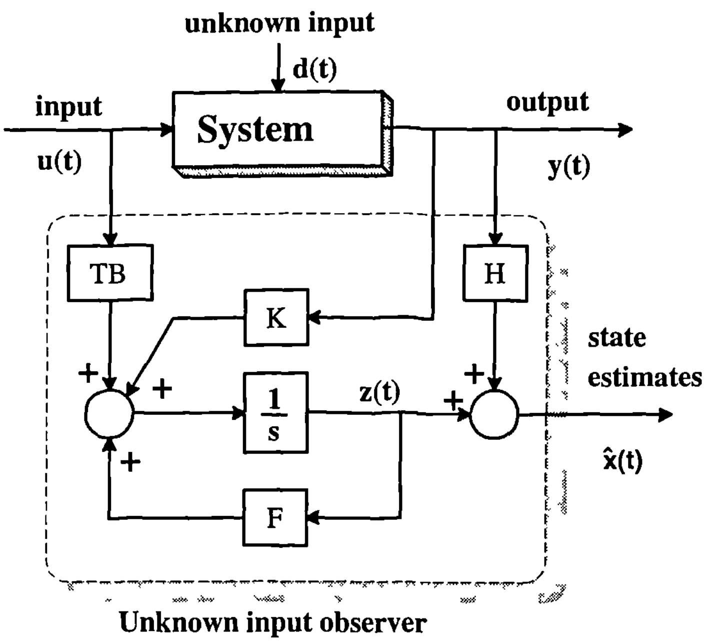


Figure 3.1: **The structure of a full-order unknown input observer**

When the observer (3.2) is applied to the system (3.1), the estimation error ($e(t) = x(t) - \hat{x}(t)$) is governed by the equation:

$$\begin{aligned} \dot{e}(t) &= (A - HCA - K_1C)e(t) + [F - (A - HCA - K_1C)]z(t) \\ &+ [K_2 - (A - HCA - K_1C)H]y(t) \\ &+ [T - (I - HC)]Bu(t) + (HC - I)Ed(t) \end{aligned} \tag{3.3}$$

where

$$K = K_1 + K_2 \tag{3.4}$$

If one can make the following relations hold true:

$$(HC - I)E = 0 \tag{3.5}$$
$$T = I - HC \tag{3.6}$$
$$F = A - HCA - K_1C \tag{3.7}$$
$$K_2 = FH \tag{3.8}$$

The state estimation error will then be:

$$\dot{e}(t) = Fe(t) \tag{3.9}$$

If all eigenvalues of F are stable, $e(t)$ will approach zero asymptotically, i.e. $\hat{x} \longrightarrow x$. This means that the observer (3.2) is an unknown input observer for the system (3.1) according to Definition 3-1. The design of this UIO is to solve Eqs.(3.4) – (3.8) and making all eigenvalues of the system matrix F be stable. Before we give the necessary and sufficient conditions for the existence of a UIO, two Lemmas are introduced.

Lemma 3-1: Eq.(3.5) is solvable *iff*:

$$rank(CE) = rank(E) \tag{3.10}$$

and a special solution is:

$$H^* = E[(CE)^T CE]^{-1}(CE)^T \tag{3.11}$$

Proof: Necessity: When Eq.(3.5) has a solution H, one has $HCE = E$ or

$$(CE)^T H^T = E^T$$

i.e., E^T belongs to the range space of the matrix $(CE)^T$ and this leads to:

$$rank(E^T) \leq rank((CE)^T)$$

i.e.

$$rank(E) \leq rank(CE)$$

However,

$$rank(CE) \leq \min\{rank(C), rank(E)\} \leq rank(E)$$

Hence, $rank(CE) = rank(E)$ and the necessary condition is proved.

Sufficiency: When $rank(CE) = rank(E)$ holds true, CE is a full column rank matrix (because E is assumed to be full column rank), and a left inverse of CE exists:

$$(CE)^+ = [(CE)^T CE]^{-1}(CE)^T$$

Clearly, $H = E(CE)^+$ is a solution to Eq.(3.5).

◇ **QED**

Lemma 3-2: Let:

$$C_1 = \begin{bmatrix} C \\ CA \end{bmatrix}$$

then the detectability for the pair (C_1, A) is equivalent to that for the pair (C, A).

Proof: If $s_1 \in \mathcal{C}$ is an unobservable mode of the pair (C_1, A), we have:

$$rank\{\begin{bmatrix} s_1I & A \\ & C_1 \end{bmatrix}\} = rank\{\begin{bmatrix} s_1I - A \\ C \\ CA \end{bmatrix}\} < n$$

This means that a vector $\alpha \in \mathcal{C}^n$ will exist such that:

$$\begin{bmatrix} s_1I - A \\ C \\ CA \end{bmatrix} \alpha = 0$$

This leads to:

$$\begin{bmatrix} s_1I - A \\ C \end{bmatrix} \alpha = 0 \qquad \text{or} \qquad rank\{\begin{bmatrix} s_1I - A \\ C \end{bmatrix}\} < n$$

That is to say that s_1 is also an *unobservable mode* of the pair (C, A).

If $s_2 \in \mathcal{C}$ is an unobservable mode of the pair (C, A), we have:

$$rank\{\begin{bmatrix} s_2 I - A \\ C \end{bmatrix}\} < n$$

This means that a vector $\beta \in \mathcal{C}^n$ can always be found, such that:

$$\begin{bmatrix} s_2 I - A \\ C \end{bmatrix} \beta = 0$$

This leads to:

$$(s_2 I - A)\beta = 0 \qquad C\beta = 0$$

$$CA\beta = Cs_2\beta = s_2 C\beta = 0$$

Hence:

$$\begin{bmatrix} s_2 I - A \\ C \\ CA \end{bmatrix} \beta = \begin{bmatrix} s_2 I - A \\ C_1 \end{bmatrix} \beta = 0$$

i.e., s_2 is also an *unobservable mode* of the pair (C_1, A).

As the pairs (C_1, A) and (C, A) have the same unobservable modes, their detectability is formally equivalent.

◇ **QED.**

An alternative way to prove the Lemma 3-2 can be found in Appendix C. Note that the detectability (Chen, 1984) is a weaker condition than observability. A pair (C, A) is detectable when all unobservable modes for this pair are stable.

Theorem 3-1: Necessary and sufficient conditions for (3.2) to be a UIO for the system defined by (3.1) are:

(i) $rank(CE) = rank(E)$

(ii) (C, A_1) is detectable pair, where

$$A_1 = A - E[(CE)^T CE]^{-1}(CE)^T CA \tag{3.12}$$

Proof: Sufficiency: According to Lemma 3-1, the Eq. (3.5) is solvable when condition

(i) holds true. A special solution for H is $H^* = E[(CE)^T CE]^{-1}(CE)^T$. In this case, the system dynamics matrix is:

$$F = A - HCA - K_1 C = A_1 - K_1 C$$

which can be stabilized by selecting the gain matrix K_1 due to the condition (ii). Finally, the remaining UIO matrices described in (3.2) can be calculated using Eqs.(3.4) - (3.8). Thus, the observer (3.2) is a UIO for the system (3.1).

Necessity: Since (3.2) is a UIO for (3.1), Eq. (3.5) is solvable. This leads to the fact that condition (i) hold true according to Lemma 3-1. The general solution of the matrix H for Eq.(3.5) can be calculated as:

$$H = E(CE)^+ + H_0[I_m - CE(CE)^+]$$

where $H_0 \in \mathcal{R}^{n \times m}$ is an arbitrary matrix and $(CE)^+$ is the left inverse of CE which is:

$$(CE)^+ = [(CE)^T CE]^{-1}(CE)^T$$

Substituting the solution for H into Eq.(3.7), the system dynamics matrix F is:

$$\begin{aligned} F &= A - HCA - K_1 C \\ &= [I_n - E(CE)^+ C]A - [K_1 \quad H_0]\begin{bmatrix} C \\ [I_m - CE(CE)^+]CA \end{bmatrix} \\ &= A_1 - [K_1 \quad H_0]\begin{bmatrix} C \\ CA_1 \end{bmatrix} \\ &= A_1 - \overline{K}_1 \overline{C}_1 \end{aligned}$$

where

$$\overline{K}_1 = [K_1 \quad H_0] \quad \text{and} \quad \overline{C}_1 = \begin{bmatrix} C \\ CA_1 \end{bmatrix}$$

Since the matrix F is stable, the pair $(\overline{C}_1, A_1)$ is detectable, and the pair (C, A_1) also is detectable according to Lemma 3-2.

◇ QED

One should note that the number of independent rows of the matrix C must not be less than the number of the independent columns of the matrix E to satisfy condition (i). That is to say, the maximum number of disturbances which can be de-coupled cannot be larger than the number of the independent measurements. It is very interesting to note that observer (3.2) will be a simple full-order Luenberger observer by setting $T = I$ and $H = 0$, when $E = 0$ (i.e. no unknown inputs in the system). In this situation, condition (i) in Theorem 3-1 clearly holds true and condition (ii) is simply changed to that of (C, A) being detectable. This is a well known result in the design of a full-order Luenberger observer.

Condition (ii) can be verified in terms of the structural properties of the original system. In fact, this condition is equivalent to the condition that the transmission zeros from the unknown inputs to the measurements must be stable, i.e.

$$\begin{bmatrix} sI_n - A & E \\ C & 0 \end{bmatrix}$$

is of full column rank for all s with $Re(s) \geq 0$. This can be proved as follows:

It can be verified that:

$$\begin{bmatrix} I_n - E(CE)^*C & sE(CE)^* \\ 0 & I_m \\ E(CE)^*C & -sE(CE)^* \end{bmatrix} \begin{bmatrix} sI_n - A & E \\ C & 0 \end{bmatrix} = \begin{bmatrix} sI_n - A_1 & 0 \\ C & 0 \\ -E(CE)^*CA & E \end{bmatrix}$$

As the first matrix in the left side of the above equation is a full column rank matrix, we have:

$$\begin{aligned} rank \begin{bmatrix} sI_n - A & E \\ C & 0 \end{bmatrix} &= rank \begin{bmatrix} sI_n - A_1 & 0 \\ C & 0 \\ -E(CE)^*CA & E \end{bmatrix} \\ &= rank \begin{bmatrix} sI_n - A_1 \\ C \\ -E(CE)^*CA \end{bmatrix} + rank(E) \end{aligned}$$

We have assumed that E is a full column rank matrix. Hence, condition (ii) is equivalent to the case when the matrix of the left side of the above equation is full column rank for all s with $Re(s) \geq 0$. This is because the condition for pair (C, A_1) to be detectable is equivalent to the following matrix

$$\begin{bmatrix} sI - A_1 \\ C \end{bmatrix}$$

haveing full column rank for all s with $Re(s) \geq 0$.

From the above analysis, it can be seen that K_1 is a free matrix of parameters in the design of a UIO. After K_1 is determined, other parameter matrices in the UIO can be computed by Eqs.(3.4) – (3.8). The only restriction on the matrix K_1 is that it must stabilize the system dynamics matrix F. The matrix K_1 which stabilizes the matrix F is not unique due to the multivariable nature of the problem. That is to say there is still some design freedom left in the choice of K_1, after unknown input disturbance conditions have been satisfied. In the following sections, this freedom is exploited further to make the diagnostic residual have directional characteristics or minimum variance properties.

3.2.2 Design procedures for UIOs

One of the most important steps in designing a UIO is to stabilize $F = A_1 - K_1C$ by choosing the matrix K_1, when the pair (C, A_1) is detectable. If (C, A_1) is observable, this can be achieved easily by using a pole placement routine which is widely available in any control system design packages such as Control System ToolBox for MATLAB. If (C, A_1) is not observable, an observable canonical decomposition procedure (Chen, 1984) should be applied to (C, A_1), which is:

$$PA_1P^{-1} = \begin{bmatrix} A_{11} & 0 \\ A_{12} & A_{22} \end{bmatrix} \qquad A_{11} \in \mathcal{R}^{n_1 \times n_1}$$

$$CP^{-1} = [C^* \quad 0] \qquad C^* \in \mathcal{R}^{m \times n_1}$$

where n_1 is the rank of the observability matrix for (C, A_1), and (C^*, A_{11}) is observable. The choice of the transformation matrix can be found in Appendix C and Chen (1984). If all eigenvalues of A_{22} are stable, (C, A_1) is detectable and the matrix F can be stabilized.

$$\begin{aligned} F &= A_1 - K_1C = P^{-1}[PAP^{-1} - PK_1CP^{-1}]P \\ &= P^{-1}\left\{\begin{bmatrix} A_{11} & 0 \\ A_{12} & A_{22} \end{bmatrix} - \begin{bmatrix} K_p^1 \\ K_p^2 \end{bmatrix}[C^* \quad 0]\right\}P \\ &= P^{-1}\begin{bmatrix} A_{11} - K_p^1C^* & 0 \\ A_{12} - K_p^2C^* & A_{22} \end{bmatrix}P \end{aligned}$$

where:

$$K_p = PK_1 = \begin{bmatrix} K_p^1 \\ K_p^2 \end{bmatrix}$$

$$\{\text{Eigenvalues of } F\} = \{\text{Eigenvalues of } A_{22}\} \bigcup \{\text{Eigenvalues of } (A_{11} - K_p^1 C^*)\}$$

As (C^*, A_{11}) is observable, K_p^1 can be determined via the pole placement. The matrix K_p^2 can be any matrix, because it does not affect the eigenvalues of F. The design procedure of a UIO is thus given as below:

1^o Check the rank condition for E and CE: If $rank(CE) \neq rank(E)$, a UIO does not exist, go to 10^o.

2^o Compute H, T and A_1:

$$H = E[(CE)^T CE]^{-1}(CE)^T \qquad T = I - HC \qquad A_1 = TA$$

3^o Check the observability: If (C, A_1) observable, a UIO exists and K_1 can be computed using pole placement, go to 9^o.

4^o Construct a transformation matrix P for the observable canonical decomposition: To select independent $n_1 = rank(W_0)$ (W_0 is the observability matrix of (C, A_1)) row vector $p_1^T, \cdots, p_{n_1}^T$ from W_0, together other $n - n_1$ row vector $p_{n_1+1}^T, \cdots, p_n^T$ to construct an non-singular matrix as:

$$P = [p_1, \cdots, p_{n_0}; p_{n_0+1}, \cdots, p_n]^T$$

5^o Perform an observable canonical decomposition on (C, A_1)):

$$PA_1P^{-1} = \begin{bmatrix} A_{11} & 0 \\ A_{12} & A_{22} \end{bmatrix} \qquad CP^{-1} = [C^* \quad 0]$$

6^o Check the detectability of (C, A_1): If any one of the eigenvalues of A_{22} is unstable, a UIO does not exist and go to 10^o.

7^o Select n_1 desirable eigenvalues and assign them to $A_{11} - K_p^1 C^*$ using pole placement.

8^o Compute $K_1 = P^{-1}K_p = P^{-1}[(K_p^1)^T \quad (K_p^2)^T]^T$, where K_p^2 can be any $(n - n_1) \times m$ matrix.

9° Compute F and K: $F = A_1 - K_1C$, $K = K_1 + K_2 = K_1 + FH$.

10° STOP.

Example: Consider the example used in (Wang et al., 1975; Miller and Mukundan, 1982; Yang and Richard, 1988; Hou and Müller, 1992) with the following parameter matrices:

$$A = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & -1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad E = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

1°: It can easily be checked that $rank(CE) = rank(E) = 1$.

2°: The matrices H, T and A_1 are calculated as:

$$H = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad A_1 = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & -1 \end{bmatrix}$$

3°: The pair (C, A_1) is observable, a UIO exists, and the matrix K_1 can be determined via the pole placement procedure.

$$K_1 = \begin{bmatrix} 1 & 2 \\ -1 & -6 \\ 0 & 4 \end{bmatrix} \qquad \text{which assigns eigenvalues at: } \{-1, -2, -3\}$$

Note that the gain matrix K_1 is not unique for assigning the same set of eigenvalues.

9°: The matrices F and K are calculated as:

$$F = \begin{bmatrix} -1 & 0 & -2 \\ 0 & 0 & 6 \\ 0 & -1 & -5 \end{bmatrix} \qquad K = \begin{bmatrix} 0 & 2 \\ -1 & -6 \\ 0 & 4 \end{bmatrix}$$

Remarks: Due to the multivariable nature of the observer design problem, the choice of the gain matrix $K_1 \in \mathcal{R}^{3\times 2}$ is not unique. That is to say there is some design freedom left after the unknown input de-coupling conditions have been satisfied. This example was also studied by Hou and Müller (1992) in which a first order UIO was designed. The gain for their reduced order UIO is a scalar, and there is no design freedom left after the satisfaction of unknown input de-coupling and the assignment of the single eigenvalue. This demonstrates the advantage of the full-order UIOs in terms of design freedom.

3.3 Robust Fault Detection and Isolation Schemes based on UIOs

3.3.1 Robust fault detection schemes based on UIOs

The main task of robust fault detection is to generate a residual signal which is robust to the system uncertainty. To detect a particular fault, the residual has to be sensitive to this fault. The detailed discussion about fault detectability has been presented in Section 2.6. According to the study in Section 2.4, a system with possible sensor and actuator faults can be described as:

$$\begin{cases} \dot{x}(t) &= Ax(t) + Bu(t) + Ed(t) + Bf_a(t) \\ y(t) &= Cx(t) + f_s(t) \end{cases} \tag{3.13}$$

where $f_a \in \mathcal{R}^r$ denotes the presence of actuator faults and $f_s \in \mathcal{R}^m$ denotes sensor faults. To generate a robust (in the sense of disturbance de-coupling) residual, a UIO described by Eq.(3.2) in Section 3.2 is required. When the state estimation is available, the residual can be generated as:

$$r(t) = y(t) - C\hat{x}(t) = (I - CH)y(t) - Cz(t) \tag{3.14}$$

When this UIO-based residual generator applied to the system described in Eq.(3.13), the residual and the state estimation error ($e(t)$) will be:

$$\begin{cases} \dot{e}(t) &= (A_1 - K_1C)e(t) + TBf_a(t) - K_1f_s(t) - H\dot{f}_s(t) \\ r(t) &= Ce(t) + f_s(t) \end{cases} \tag{3.15}$$

From Eq.(3.15), it can be seen that the disturbance effects have been de-coupled from the residual. To detect actuator faults, one has to make:

$$TB \neq 0$$

More specifically, the fault in the i_{th} actuator will affect the residual *iff*:

$$Tb_i \neq 0$$

where b_i is the i_{th} column of the matrix B. Similarly, the residual has to be made sensitive to $f_s(t)$ if sensor faults are to be detected. This condition is normally satisfied, as the sensor fault vector $f_s(t)$ has a direct effect on the residual $r(t)$. The robust residual can be used to detect faults according to a simple threshold logic:

$$\begin{cases} \|r(t)\| < Threshold & \text{for} \quad \text{fault-free case} \\ \|r(t)\| \geq Threshold & \text{for} \quad \text{faulty cases} \end{cases} \tag{3.16}$$

3.3.2 Robust fault isolation schemes based on UIOs

The fault isolation problem is to locate the fault, i.e., to determine in which sensor (or actuator) the fault has occurred. As pointed out in Section 2.7, one of the approaches to facilitate fault isolation is to design a *structured* residual set. The term "structured" here means that each residual is designed to be sensitive to a certain group of faults and insensitive to others. The sensitivity and insensitivity property makes isolation possible. The ideal situation is to make each residual only sensitive to a particular fault and insensitive to all other faults. However, this ideal situation is normally difficult to achieve (Patton et al., 1989). Even when the ideal situation can be achieved, the design freedom will be used up and no freedom will be left for achieving robustness. This problem was encountered by Wünnenberg (1990). To exploit the maximum design freedom for robustness, a commonly accepted scheme (Patton et al., 1989) in fault isolation is to make each residual to be sensitive to faults in all but one sensors (or actuators).

<u>**Robust sensor fault isolation schemes:**</u>

To design robust sensor fault isolation schemes, all actuators are assumed to be fault-free and the system equations can be expressed as:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Ed(t) \\ y^j(t) = C^j x(t) + f_s^j(t) \\ y_j(t) = c_j x(t) + f_{sj}(t) \end{cases} \quad \text{for} \quad j = 1, 2, \cdots, m \tag{3.17}$$

where $c_j \in R^{1\times n}$ is the j_{th} row of the matrix C, $C^j \in \mathcal{R}^{(m-1)\times n}$ is obtained from the matrix C by deleting j_{th} row c_j, y_j is the j_{th} component of y and $y^j \in \mathcal{R}^{m-1}$ is obtained from the vector y by deleting j_{th} component y_j. Based on this description, m UIO-based residual generator can be constructed as:

$$\left\{ \begin{array}{rcl} \dot{z}^j(t) & = & F^j z^j(t) + T^j Bu(t) + K^j y^j(t) \\ r^j(t) & = & (I - C^j H^j) y^j(t) - C^j z^j(t) \end{array} \right. \quad \text{for} \quad j = 1, 2, \cdots, m \qquad (3.18)$$

where the parameter matrices must satisfy the following equations:

$$\left. \begin{array}{rcll} H^j C^j E & = & E & \\ T^j & = & I - H^j C^j & \\ F^j & = & T^j A - K_1^j C^j & \text{to be stabilized} \\ K_2^j & = & F^j H^j & \\ K^j & = & K_1^j + K_2^j & \end{array} \right\} \quad \text{for} \quad j = 1, 2, \cdots, m \qquad (3.19)$$

It is clear that each residual generator is driven by all inputs and all but one outputs. When all actuators are fault-free and a fault occurs in the j_{th} sensor, the residual will satisfy the following isolation logic:

$$\left\{ \begin{array}{rcl} \|r^j(t)\| & < & T_{SFI}^j \\ \|r^k(t)\| & \geq & T_{SFI}^k \quad \text{for} \quad k = 1, \cdots, j-1, j+1, \cdots, m \end{array} \right. \qquad (3.20)$$

where T_{SFI}^j $(j = 1, \cdots, m)$ are isolation thresholds. A robust and UIO-based sensor fault isolation scheme is shown in Fig.3.2.

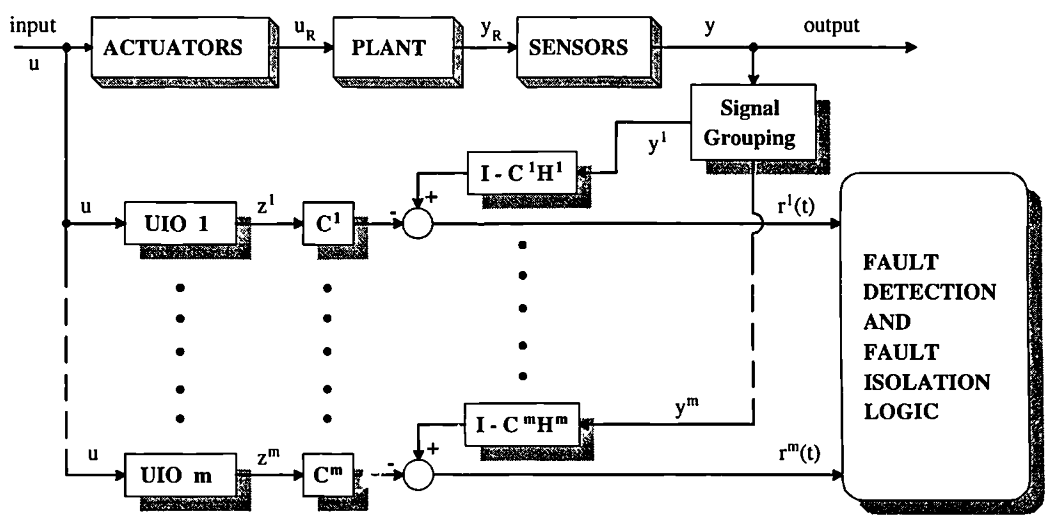


Figure 3.2: **A robust sensor fault isolation scheme**

Robust actuator fault isolation schemes

To design robust actuator fault isolation schemes, all sensors are assumed to be fault-free and the system equation can be described as:

$$\left\{ \begin{array}{rcl} \dot{x}(t) & = & Ax(t) + B^i u^i(t) + B^i f_a^i(t) + b_i(u_i(t) + f_{ai}(t)) + Ed(t) \\ & = & Ax(t) + B^i u^i(t) + B^i f_a^i(t) + E^i d^i(t) \\ y(t) & = & Cx(t) \qquad\qquad \text{for} \;\; i = 1, 2, \cdots, r \end{array} \right. \tag{3.21}$$

where $b_i \in \mathcal{R}^n$ is the i_{th} column of the matrix B, $B^i \in \mathcal{R}^{n \times (r-1)}$ is obtained from the matrix B by deleting the i_{th} column b_i, u_i is the i_{th} component of u, $u^i \in \mathcal{R}^{r-1}$ is obtained from the vector u by deleting the i_{th} component u_i, and

$$E^i = [E \quad b_i] \qquad d^i(t) = \left[\begin{array}{c} d(t) \\ u_i(t) + f_{ai}(t) \end{array} \right] \qquad \text{for} \quad i = 1, 2, \cdots, r$$

Based on the above system description, r UIO-based residual generators can be constructed as:

$$\left\{ \begin{array}{rcl} \dot{z}^i(t) & = & F^i z^i(t) + T^i B^i u^i(t) + K^i y(t) \\ r^i(t) & = & (I - CH^i)y(t) - Cz^i(t) \end{array} \right. \qquad \text{for} \quad i = 1, 2, \cdots, r \tag{3.22}$$

The parameter matrices must be satisfy the following equations:

$$\left. \begin{array}{rcl} H^i C E^i & = & E^i \\ T^i & = & I - H^i C \\ F^i & = & T^i A - K_1^i C \quad \text{to be stabilized} \\ K_2^i & = & F^i H^i \\ K^i & = & K_1^i + K_2^i \end{array} \right\} \quad \text{for} \quad i = 1, 2, \cdots, r \tag{3.23}$$

One can seen that each residual generator is driven by all outputs and all but one inputs. When all sensors are fault-free and a fault occurs in the i_{th} actuator, the residual will satisfy the following isolation logic:

$$\left\{ \begin{array}{rcl} \|r^i(t)\| & < & T_{AFI}^i \\ \|r^k(t)\| & \geq & T_{AFI}^k \qquad \text{for} \quad k = 1, \cdots, i-1, i+1, \cdots, r \end{array} \right. \tag{3.24}$$

where T_{AFI}^i $(i = 1, \cdots, r)$ are isolation thresholds. A robust and UIO-based actuator fault isolation scheme is shown in Fig.3.3.

Remarks: The isolation schemes presented in this section can only isolate a single

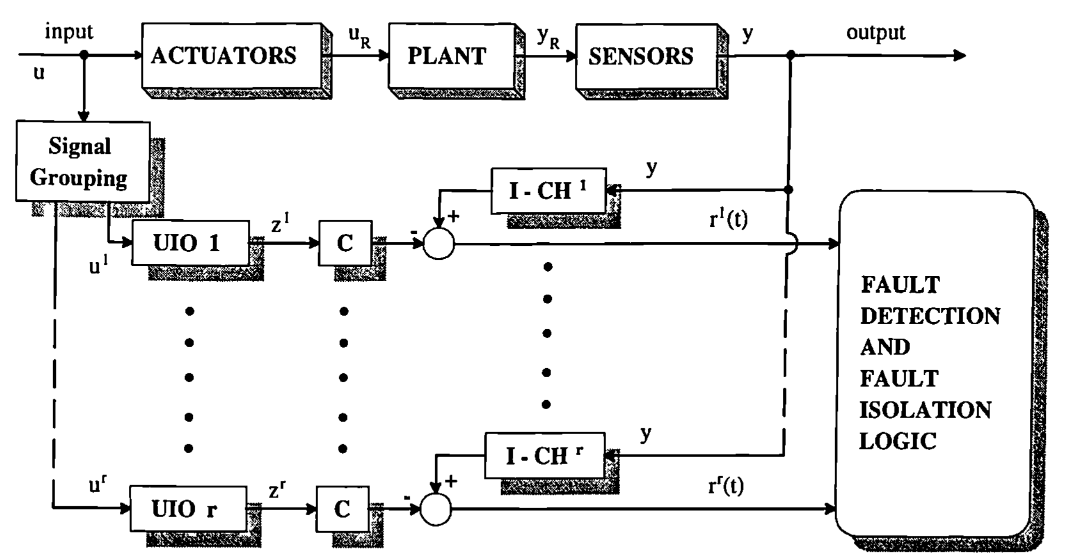


Figure 3.3: **A robust actuator fault isolation scheme**

fault in either a sensor or an actuator, at the same time. This is based on the fact that the probability for two or more faults to occur at the same time is very small in a real situation. If simultaneous faults need to be isolated, the fault isolation scheme should be modified based on a regrouping of faults. Each residual will be designed to be sensitive to one group of faults and insensitive to another group of faults. Frank and Wünnenberg (1989) have studied this problem. The way of grouping faults is dependent on the system and the faults to be isolated. The isolation of sensor faults is normally possible, however it is impossible to isolate two actuator faults which have the same distribution direction. To isolate such actuator faults, other fault information such as fault frequency distributions should be utilised. FDI schemes are related to particular systems, a *general* scheme cannot expected to suit any system without any modification.

3.3.3 A practical example (Robust actuator fault detection and isolation for a chemical reactor)

Watanabe and Himmelblau (1982) studied the sensor fault detection problem for a well-stirred chemical reactor with heat exchanger. This system is used here to demonstrate the robust actuator fault detection and isolation scheme developed in Section 3.3.2.

<u>**System representation:**</u> The state, input and output vectors for the considered chemical reactor are:

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} = \begin{bmatrix} C_o(t) \\ T_o(t) \\ T_w(t) \\ T_m(t) \end{bmatrix} \qquad u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix} = \begin{bmatrix} 3.6C_i(t) \\ 3.6T_i(t) \\ 36T_{wi}(t) \end{bmatrix}$$

$$y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix} = \begin{bmatrix} C_o(t) \\ T_o(t) \\ T_w(t) \end{bmatrix}$$

where:

$C_o \longmapsto$ concentration of the chemical product
$T_o \longmapsto$ temperature of the product
$T_w \longmapsto$ temperature of jacket water of heat exchanger
$T_m \longmapsto$ coolant temperature
$C_i \longmapsto$ inlet concentration of reactant
$T_i \longmapsto$ inlet temperature
$T_{wi} \longmapsto$ coolant water inlet temperature

According to Watanabe and Himmelblau (1982), the system is modelled as:

$$\begin{cases} \dot{x}(t) &= Ax(t) + Bu(t) + Ed(t) \\ y(t) &= Cx(t) \end{cases}$$

where the term $Ed(t)$ is used to represent the nonlinearity in the system, and

$$d(t) = 3.012 \times 10^{12} exp\{-\frac{1.2515 \times 10^7}{T_0}\} = 3.012 \times 10^{12} exp\{-\frac{1.2515 \times 10^7}{x_2(t)}\}$$

$$A = \begin{bmatrix} -3.6 & 0.0 & 0.0 & 0.0 \\ 0.0 & -3.6702 & 0.0 & 0.0702 \\ 0.0 & 0.0 & -36.2588 & 0.2588 \\ 0.0 & 0.6344 & 0.7781 & -1.4125 \end{bmatrix} \qquad E = \begin{bmatrix} 1.0 \\ 20.758 \\ 0.0 \\ 0.0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Note that the system matrices are not exactly the same as give by Watanabe and

Himmelblau (1982), this is because the time scale has been changed to hours for the sake of convenience.

<u>**UIOs design and residuals generation:**</u> Both control inputs $u_1(t)$ $(C_i(t))$ and $u_2(t)$ $(T_i(t))$ are related to the inlet chemical substance, and any fault in $u_1(t)$ or $u_2(t)$ will cause a similar consequence. Hence it is not necessary to isolate faults between $u_1(t)$ and $u_2(t)$. Two UIOs are designed here, the first UIO is driven by $u_1(t)$ and $u_2(t)$ and the second UIO is driven by $u_3(t)$. These two UIOs are robust to the nonlinear factor in $d(t)$.

UIO 1: The dynamic equation for the first UIO is:

$$\dot{z}^1(t) = F^1 z(t) + K^1 y(t) + T^1 [b_1 \quad b_2] \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

where b_1 and b_2 are the first two columns of B, and the parameter matrices for this UIO are:

$$H^1 = \begin{bmatrix} 21.758 & -1.0 & 0.0 \\ 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \\ -2075.8 & 100.0 & 0.0 \end{bmatrix} \qquad T^1 = \begin{bmatrix} -20.758 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ 2075.8 & -100.0 & 0.0 & 1.0 \end{bmatrix}$$

$$F^1 = \begin{bmatrix} -10 & 0.0 & 0.0 & 0.0702 \\ 0 & -\lambda_1 & 0.0 & 0.0 \\ 0 & 0.0 & -\lambda_2 & 0.0 \\ 0 & 0.0 & 0.0 & -8.4325 \end{bmatrix} \qquad K^1 = \begin{bmatrix} -278.5724 & 13.3496 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 10031.304 & -475.5956 & 0.7781 \end{bmatrix}$$

The sub-observer for z_2^1 and z_3^1 (element of vector z^1) has no inputs of y, u_1 and u_2, and has no coupling with z_1^1 and z_4^1, hence z_2^1 and z_3^1 will stay at zero if the initial values of z_2^1 and z_3^1 are zero and the observer matrix F^1 is designed to be stable. The full-order UIO can be reduced to:

$$\begin{bmatrix} \dot{z}_1^1 \\ \dot{z}_4^1 \end{bmatrix} = \begin{bmatrix} -10.0 & 0.0702 \\ 0.0 & -8.4325 \end{bmatrix} \begin{bmatrix} z_1^1 \\ z_4^1 \end{bmatrix} + \begin{bmatrix} -278.57236 & 13.3498 & 0.0 \\ 10031.3035 & -475.5956 & 0.7781 \end{bmatrix} y + \begin{bmatrix} -20.758 & 1.0 \\ 2075.8 & -100.0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

The state estimation is:

$$\hat{x}^1 = \begin{bmatrix} z_1^1 + 21.758y_1 - y_2 \\ y_2 \\ y_3 \\ z_4^1 - 2075.8y_1 + 100y_2 \end{bmatrix}$$

The residual is generated by:

$$r^1(t) = y_1(t) - \hat{y}_1(t) = y_1(t) - \hat{x}_1(t) = y_2(t) - z_1^1(t) - 20.758y_1(t)$$

UIO 2: The dynamic equation for the second UIO is:

$$\dot{z}^2(t) = F^2 z(t) + K^2 y(t) + T^2 b_3 u_3(t)$$

where b_3 is the third column of B, and the parameter matrices for this UIO are:

$$H^2 = \begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 40.0 \end{bmatrix} \qquad T^2 = \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & -40.0 & 0.0 \end{bmatrix}$$

$$F^2 = \begin{bmatrix} -\lambda_1 & 0.0 & 0.0 & 0.0 \\ 0.0 & -\lambda_2 & 0.0 & 0.0 \\ 0.0 & 0.0 & -10.0 & 0.2588 \\ 0.0 & 0.0 & 0.0 & -11.7645 \end{bmatrix} \quad K^2 = \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & -15.9068 \\ 0.0 & 0.6344 & 980.5501 \end{bmatrix}$$

Similar to the first UIO, the UIO 2 can also be reduced as:

$$\begin{aligned} \begin{bmatrix} \dot{z}_3^2 \\ \dot{z}_4^2 \end{bmatrix} &= \begin{bmatrix} -10.0 & 0.2588 \\ 0.0 & -11.7645 \end{bmatrix} \begin{bmatrix} z_3^2 \\ z_4^2 \end{bmatrix} \\ &+ \begin{bmatrix} 0.0 & 0.0 & -15.9068 \\ 0.0 & 0.6344 & 980.5501 \end{bmatrix} y + \begin{bmatrix} 1.0 \\ -40.0 \end{bmatrix} u_3(t) \end{aligned}$$

$$\hat{x}^2 = \begin{bmatrix} y_1 & y_2 & z_3^2 & z_4^2 + 40y_3 \end{bmatrix}^T$$

The residual is generated by:

$$r^2(t) = y_3(t) - \hat{y}_3(t) = y_3(t) - \hat{x}_3(t) = y_3(t) - z_3^2(t)$$

Simulation: The above UIOs is applied to the nonlinear chemical reaction process to detect and isolate faulty actuators. The system input and the initial state vectors are:

$$u = \begin{bmatrix} 34.632 \\ 1641.6 \\ 29980 \end{bmatrix} \qquad x(0) = \begin{bmatrix} 0.3412 \\ 525.7 \\ 472.2 \\ 496.2 \end{bmatrix}$$

The initial values for UIOs are:

$$z_1^1(0) = 518.6174 \quad z_4^1(0) = -51365.5370 \quad z_3^2(0) = 472.2 \quad z_4^2(0) = -18391.8$$

The sampling interval is set as 0.05 hour, and the simulation is carried out for $t = 10$ hours. Various types of faults are introduced to the system at $t = 4$ hours. The list of the simulated faults is:

(a) A fault occurs in the inlet reactant when $t > 4$ hour, the fault signal in the first input is $20\% u_1(t)$.

(b) A fault occurs in the inlet reactant when $t > 4$ hour, the fault signal in the second input is $20\% \sin(2(t-4))u_2(t)$.

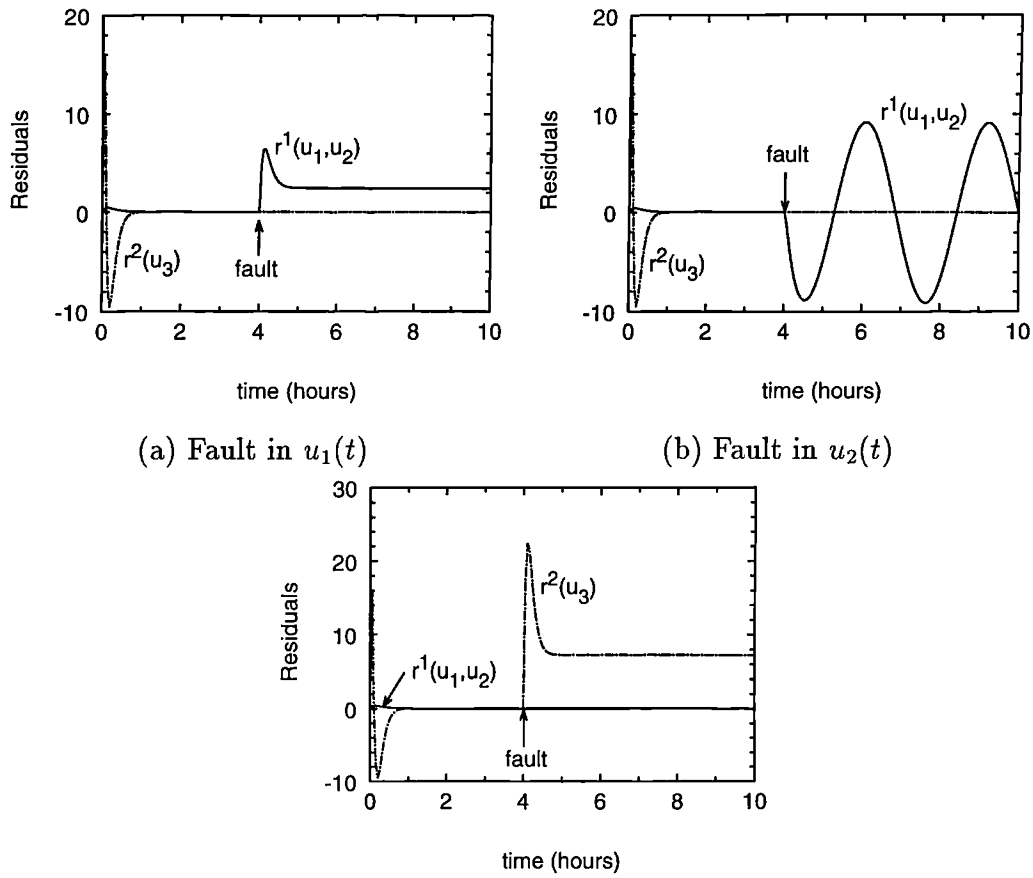
(c) A fault occurs in the coolant circular when $t > 4$ hour, the fault signal in the third input is $-2\% u_3(t)$.

The simulation results are shown in Fig.3.4, from which one can seen that the residual is almost zero throughout the 10 hours simulation run for fault-free residuals. The residuals of the respective UIO increase in magnitude considerably, when actuator faults occur at $t = 4$ hours. The faults can be easily isolated using the information provided by residuals.

Robustness analysis: From the above analysis and simulation, we know that the fault detection and isolation scheme is robust to nonlinearity in $d(t)$. The robustness with respect to parameter variations is analysed below. The system with parameter variations is described as:

$$\dot{x}(t) = Ax(t) + Bu(t) + Ed(t) + \sum_{i=1}^{4} I_i w_i(x(t), \Delta A)$$

where: I_i is the i_{th} column of identity matrix, w_i represent the variations in i_{th} row



(a) Fault in $u_1(t)$

(b) Fault in $u_2(t)$

(c) Fault in $u_3(t)$

Figure 3.4: **Residuals for two UIOs (without parameter variations)**

elements of A. This equation can be rewritten as:

$$\dot{x}(t) = Ax(t) + Bu(t) + Ed(t) + Ew_1 + I_2(w_2 - 20.758w_1) + I_3w_3 + I_4w_4$$

Parameter variations in the form of Ew_1 and I_3w_3 will not affect the first UIO, because $T^1E = 0$ and $T^1I_3 = 0$. Similarly, the parameter variations in the form of Ew_1 and $I_2(w_2 - 20.758w_1)$ will not affect the second UIO, because $T^2E = 0$ and $T^2I_2 = 0$. In all cases, the sensitivities to process parameter variations have been decreased. The robustness of UIOs to process parameter variations can be assessed by the simulation in which the matrix A is changed to:

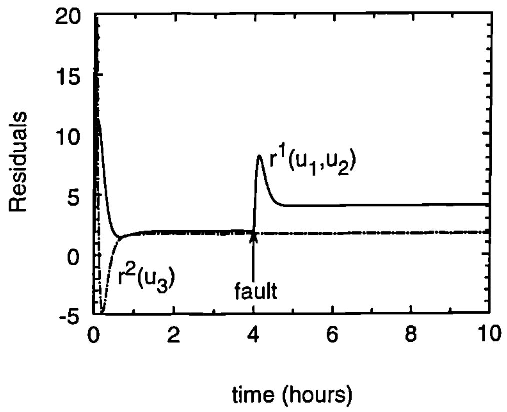
$$A = \begin{bmatrix} -4.14 & 0.0 & 0.0 & 0.0 \\ 0.0 & -4.22073 & 0.0 & 0.08073 \\ 0.0 & 0.0 & -36.4401 & 0.2601 \\ 0.0 & 0.9516 & 1.1672 & -2.1188 \end{bmatrix}$$

The residuals for three types of faults are shown in Fig.3.5, from which one can conclude that the robust FDI scheme can reliably detect and isolate faulty actuators even in the presence of process parameter mismatch.

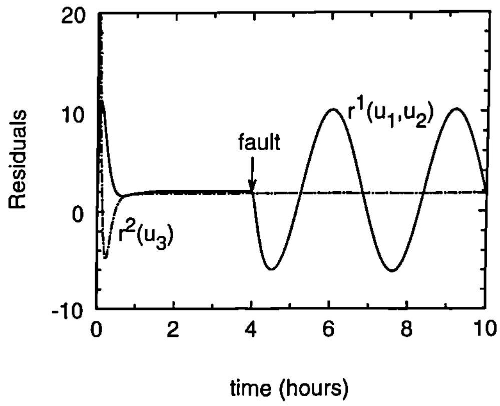
Remarks: Robust actuator fault detection and isolation based on UIOs has been demonstrated in a chemical reactor example. The UIO is a time-invariant linear filter but can also be applied to a class of non-linear time-variant systems if the nonlinear function is separated from the linear function and can be treated as an unknown input term. The robust FDI based on UIOs has also a certain degree of robustness against parameter variations.

3.4 Robust Fault Detection Filters and Robust Directional Residuals

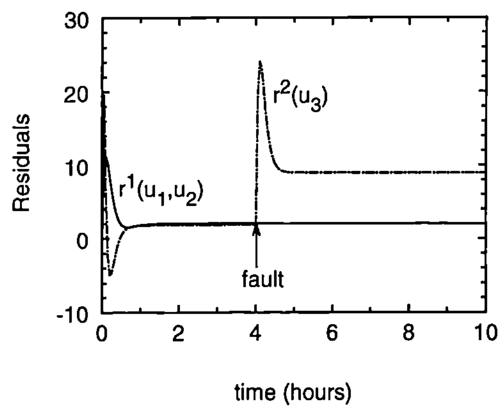
Fault detection filters (Beard, 1971) are a particular class of the full-order Luenberger observer with a specially designed feedback gain matrix such that the output estimation error (residual vector) has uni-directional characteristics associated with some known fault directions. To be specific, the residual vector of a fault detection filter is fixed along with a predetermined direction for an actuator fault or lies in a specific plane for a sensor fault. Since the important information required for iso-



(a) Fault in $u_1(t)$



(b) Fault in $u_2(t)$



(c) Fault in $u_3(t)$

Figure 3.5: **Residuals for two UIOs (with parameter variations)**

lation is contained in the direction of the residual rather than in its time function, the use of a Beard fault detection filter (BFDF) does not require the knowledge of fault modes. The fault isolation task can be facilitated by comparing the residual direction with pre-defined fault signature directions (or planes), and only one (or the minimum number of) observers required for fault isolation due to directional characteristics of the residual. This is the main and most appealing advantage of fault detection filters. However, *the main drawback* of the BFDF is that the robustness problem has not been considered. This section presents a method to design a robust fault detection filter which is based on the combination of UIO and BFDF theories. The main principle is that the remaining design freedom, after disturbance de-coupling conditions have been satisfied, can be used to make the residual vector have directional characteristics. A realistic simulation example of isolating faulty sensors in a jet engine system is presented. This is a nonlinear system and the linearization error can cause mis-isolation if the robustness issue is not considered. A way of representing linearization errors as an unknown input term is presented and its distribution is estimated using a least-squares procedure. The simulation results shows that faults are correctly isolated using the technique developed.

3.4.1 Basic principles of fault detection filters

The BFDF was first developed by Beard (1971) using a matrix algebra approach and later reformed by Jones (1973) in a vector space notation. The theory of BFDFs has been extended by many researchers, for example, Massoumnia (1986b) used a geometric interpretation, White and Speyer (1987) improved the design procedure using a spectral approach which is suitable for the isolation of multiple faults, and more recently Park & Rizzoni (Park and Rizzoni, 1993; Park and Rizzoni, 1994) developed a closed-form expression of BFDFs using eigenstructure assignment.

In order to describe the BFDF theory, let us consider a system without disturbances in the state space format as:

$$\begin{cases} \dot{x}(t) &= Ax(t) + Bu(t) + b_i f_{ai}(t) \\ y(t) &= Cx(t) + I_j f_{sj}(t) \end{cases} \tag{3.25}$$

The term $b_i f_{ai}(t)$ $(i = 1, 2, \cdots, r)$ denotes that a fault has occurred in the i_{th} actuator, $b_i \in \mathcal{R}^n$ is the i_{th} column of the input matrix B and is defined as the fault event vector of the i_{th} actuator fault, and $f_{ai}(t)$ is an unknown scalar time-varying function which represents the evolution of the fault. The term $I_j f_{sj}(t)$ $(j = 1, 2, \cdots, m)$ denotes that a fault occurs in the j_{th} sensor, $I_j \in \mathcal{R}^m$ is a unit vector corresponding to a fault with the j_{th} sensor. Note that component faults appear in the system equation in the same way as the actuator fault and hence are not discussed further here.

A BFDF is just a full-order observer and its structure and the residual can be described as:

$$\begin{cases} \dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) + K(y(t) - C\hat{x}(t)) \\ r(t) &= y(t) - C\hat{x}(t) \end{cases} \tag{3.26}$$

where $r \in \mathcal{R}^m$ is the residual vector, $\hat{x} \in \mathcal{R}^n$ is the state estimation, and $K \in \mathcal{R}^{m \times n}$ is the observer gain matrix which has to be specially designed to make the residual have restricted uni-directional properties in the presence of a particular fault. If the state estimation error is defined as: $e(t) = x(t) - \hat{x}(t)$, the residual and $e(t)$ will be governed by the following error system, when a fault occurs in the i_{th} actuator:

$$\begin{cases} \dot{e}(t) &= (A - KC)e(t) + b_i f_{ai}(t) \\ r(t) &= Ce(t) \end{cases} \tag{3.27}$$

When a fault occurs in the j_{th} sensor, the error system will be:

$$\begin{cases} \dot{e}(t) &= (A - KC)e(t) - k_j f_{sj}(t) \\ r(t) &= Ce(t) + I_j f_{sj}(t) \end{cases} \tag{3.28}$$

where k_j is the j_{th} column of the detection filter gain matrix.

The task of BFDF design is to make $Ce(t)$ have a *fixed direction* in the output space responding to either $b_i f_{ai}(t)$ or $k_j f_{sj}(t)$. Both actuator and sensor fault situations can be considered in the following general error system equation:

$$\begin{cases} \dot{e}(t) &= (A - KC)e(t) + l_i \xi_i(t) \\ r(t) &= Ce(t) \end{cases} \tag{3.29}$$

where $l_i \in \mathcal{R}^n$ is called the *fault event direction*. The definition of the isolability of a fault with known direction l_i is given by Beard (1971) as stated below:

Definition 3-2: {Isolability of a fault with a given direction}: The fault associated with l_i in the system described by Eq.(3.29) is *isolable* if there exists a filter gain matrix K such that:

(a) $r(t)$ maintains a fixed direction in the output space, and

(b) $(A - KC)$ can be stabilized.

Condition (a) which guarantees that the residual has uni-directional characteristics, is equivalent to ensuring that the rank of the controllability matrix of (A, l_i) pair is one, i.e:

$$rank[l_i \;\; (A - KC)l_i \;\; \cdots \;\; (A - KC)^{n-1}l_i] = 1$$

Condition (b) ensures the convergence of the filter. In the original definition of Beard (1971), condition (b) requires arbitarily assignment of eigenvalues of $(A-KC)$. This condition has been modified as the stability requirement is sufficient if the residual response time does not need to specified. This definition was referred to as "fault detectability" by Beard (1971) and others (Jones, 1973; Massoumnia, 1986b; White and Speyer, 1987). In the author's view, the term "isolability" is more appropriate, because the directional property of the residuals is especially desirable for fault isolation purposes, although it can also be used for fault detection. Hence, the BFDF is designed to satisfy the fault isolability.

Here the abbreviation BFDF is reserved for a filter (an observer) with residual having uni-directional properties. If a fault associated with the direction b_i is isolable, the residual of the BFDF will be fixed in the direction parallel to Cb_i, when a fault occurs in the i_{th} actuator. Similarly, the residual will lie somewhere in the plane defined by Ck_j and I_j, when a fault occurs in the j_{th} sensor.

To isolate faults associated with p isolable fault event directions l_i $(i = 1, \cdots, p)$, the following output separability condition (Beard, 1971) must be satisfied.

Definition 3-3: {Output Separability of Faults}: The faults associated with p fault event directions l_i $(i = 1, 2, \cdots, p)$ are *separable* in the residual space if the vectors $Cl_1, Cl_2, \cdots, Cl_p$ are linearly independent.

Output separability is necessary for a group of faults to be isolated in the residual space according to their signature directions. The directions Cl_i $(i = 1, 2, \cdots, p)$ are then known as the *fault signature directions* in the residual space.

Definition 3-4: {Mutual Isolability}: The faults associated with the fault event vectors l_i $(i = 1, 2, \cdots, p)$ are *mutually isolable* if there exists a filter gain matrix K which satisfies the isolability conditions of Definition 3-2 for all l_i $(i = 1, 2, \cdots, p)$, i.e.

$$rank[l_i \;\; (A - KC)l_i \;\; \cdots \;\; (A - KC)^{n-1}l_i] = 1 \qquad \text{for all } \; i = 1, 2, \cdots, p$$

A group of mutually isolable faults can be isolated using the residual generated by a single BFDF by comparing the residual direction with the fault signature directions, when there are no simultaneous faults. The condition for mutual isolability can be found in the well known literature (Beard, 1971; Jones, 1973; White and Speyer, 1987). If a group of faults is not mutually isolable, it can be divided into a number of subgroups and each subgroup is mutually isolable. For such cases, a few BFDFs are required to fulfil the fault isolation task. In any case, only a minimum number of filters are required for fault isolation. This is the most important and appealing advantage of the BFDF approaches.

In conclusion, the task of designing a fault detection filter is to make the residual have a uni-directional property by choosing the gain matrix K. Design techniques can be found in the classical literature on fault detection filters (Beard, 1971; Jones, 1973; White and Speyer, 1987).

3.4.2 Disturbance de-coupled fault detection filters and robust fault isolation

It can be seen that uncertain factors associated with a dynamical system such as disturbances and modelling errors have not be considered in the design of BFDFs. This is the main disadvantage of BFDFs, because uncertain factors are unavoidable in real systems and any FDI scheme has to be made robust against disturbances and modelling errors. Now, consider a system with disturbance term $Ed(t)$ and possible sensor and actuator faults described as:

$$\begin{cases} \dot{x}(t) &= Ax(t) + Bu(t) + Ed(t) + b_i f_{ai}(t) \\ y(t) &= Cx(t) + I_j f_{sj}(t) \end{cases} \tag{3.30}$$

If a standard BFDF described by Eq.(3.26) is applied to such a system, the state estimation error and residual will be:

$$\begin{cases} \dot{e}(t) &= (A - KC)e(t) + Ed(t) + b_i f_{ai}(t) - k_j f_{sj}(t) \\ r(t) &= Ce(t) + I_j f_{sj}(t) \end{cases} \tag{3.31}$$

It is clear from Eq.(3.31) that all faults and disturbances affect the residual. It is not easy to discriminate between faults and disturbances if this residual is used to detect and isolate faults. Hence, it is necessary to de-couple disturbance effects from the residual for reliable diagnosis.

It has been shown that the disturbances can be de-coupled from the state estimation error using an unknown input observer (see also Section 3.3.1). This inspires us to generate the residual using the unknown input observer described in Eq.(3.2). The residual is thus defined as:

$$r(t) = y(t) - C\hat{x}(t) = (I - CH)y(t) - Cz(t) \tag{3.32}$$

When this UIO-based residual generator is applied to the system described by the model Eq.(3.31), the residual and the state estimation error ($e(t)$) will be:

$$\begin{cases} \dot{e}(t) &= (A_1 - K_1 C)e(t) + T b_i f_{ai}(t) \\ r(t) &= Ce(t) \end{cases} \tag{3.33}$$

when a fault occurs in the i_{th} actuator.

Similarly,

$$\begin{cases} \dot{e}(t) &= (A_1 - K_1 C)e(t) - k_{1j} f_{sj}(t) - h_j \dot{f}_{sj}(t) \\ r(t) &= Ce(t) + I_j f_{sj}(t) \end{cases} \tag{3.34}$$

when a fault occurs in the j_{th} sensor. Where k_{1j} is the j_{th} column of the matrix K_1 and h_j is the j_{th} column of the matrix H. From Eq.(3.33) & (3.34), it can be seen that the disturbance effects have been de-coupled from the residual. This robust (in the disturbance de-coupling sense) residual can be used to detect faults according to a simple threshold logic:

$$\begin{cases} \|r(t)\| < Threshold & \text{for fault-free case} \\ \|r(t)\| \geq Threshold & \text{for faulty cases} \end{cases} \tag{3.35}$$

As pointed out in the introduction, fault isolation can be facilitated using uni-directional residual vectors. So, one has to make the residual generated by a UIO, have the directional properties in order to achieve robust fault isolation. From the design of UIOs, it is known that the matrix K_1 can be designed arbitrarily after the robust (in the sense of disturbance de-coupling) conditions have been satisfied. This design freedom can be exploited to make the residual have the uni-directional property.

Comparing the error system Eq. (3.33) with Eq.(3.27), it can be seen that the actuator fault is expressed in the same way for a UIO or a standard BFDF. Hence, the theory for the design of a BFDF (Beard, 1971; Jones, 1973; White and Speyer, 1987) can be directly used to design the matrix K_1, if the vector b_i is replaced by Tb_i and the matrix A is replace by A_1.

Comparing the error system Eq.(3.34) with Eq.(3.28), it can be seen that the sensor fault is also expressed in a similar way for *both* the BFDF and UIO, except an extra term $h_j \dot{f}_{sj}(t)$ occurs in the error equation of the UIO. Fortunately, this term can be treated in the same way as an actuator fault. Hence, the theory of BFDF can be adopted for the design of K_1 in the sensor isolation problem. However, it must be pointed out that the residual will lie in a subspace spanned by vectors I_j, Ck_{1j} and Ch_j when the residual uni-directional property has been satisfied. For constant sensor faults, the term $h_j \dot{f}_{sj}(t)$ will disappear from the error system and the residual will lie in the plane spanned by the vectors I_j and Ck_{1j}, this is same as the BFDF.

It is necessary to combine the theory of UIOs with the theory of BFDFs to design a robust (disturbance de-coupled) fault detection filter. The design procedure can be summarised as follows:

- Compute the matrices H and T using Eqs. (3.11) & (3.6), to satisfy disturbance de-coupling conditions.
- Compute A_1 using Eq.(3.12).
- Compute K_1 to satisfy a uni-directional property using the theory of BFDFs.

- Compute the observer gain matrix K using Eqs.(3.8) & (3.4).

The key step is then to design the matrix K_1. Once this matrix is available, the computation of other matrices is very straightforward. The BFDF design procedure can be found in the well known literature (Beard, 1971; Jones, 1973; White and Speyer, 1987) and is not presented in this chapter. To show the basic idea, an ideal situation is discussed now, in which the number of independent measurements is equal to the number of states, i.e. $rank(C) = n$. In this situation, all eigenvalues of the matrix $A_1 - K_1C$ can be assigned to the same value $\sigma > 0$, i.e.,

$$A_1 - K_1C = -\sigma I$$

This can be achieved by setting K_1 as:

$$K_1 = (A_1 + \sigma I)C^* \tag{3.36}$$

where C^* is the pseudo-inverse of C. For this design, the residual will be:

$$\begin{aligned}
r(t) &= Ce(t) + I_j f_s(t) \\
&= I_j f_s(t) + Ce^{-\sigma(t-t_0)}e(t_0) + C\int_{t_0}^{t} e^{-\sigma I(t-\tau)}[Tb_i f_a(\tau) - k_{1j} f_s(t) - h_j \dot{f}_s(t)]d\tau \\
&= Ce^{-\sigma(t-t_0)}e(t_0) + CTb_i \int_{t_0}^{t} e^{-\sigma(t-\tau)} f_a(\tau)d\tau \\
&\quad + I_j f_s(t) - Ck_{1j}\int_{t_0}^{t} e^{-\sigma(t-\tau)} f_s(\tau)d\tau - Ch_j \int_{t_0}^{t} e^{-\sigma(t-\tau)} \dot{f}_s(\tau)d\tau \\
&= Ce^{-\sigma(t-t_0)}e(t_0) + CTb_i\alpha(t,t_0) \\
&\quad + I_j f_s(t) + Ck_{1j}\beta(t,t_0) + Ch_j\gamma(t,t_0)
\end{aligned}$$

Clearly, the residual is parallel to CTb_i after the transient has settled down following a fault in the i_{th} actuator. Similarly, the residual will lie in the subspace spanned by vectors I_j, Ck_{1j} and Ch_j, when a fault occurs in the j_{th} sensor.

Due to the residual directional property, the fault can be isolated by comparing the residual direction with the fault signature directions (or subspaces).

Definition 3-5: The direction of CTb_i is termed a *signature direction* of the i_{th} actuator fault.

The directional relationship between two vectors CTb_i and $r(t)$ can be quantified by the correlation parameter $CORR_i$:

$$CORR_i(t) = \frac{|(CTb_i)^T r(t)|}{\|CTb_i\|_2 \|r(t)\|_2} \tag{3.37}$$

If $CORR_j > CORR_k$, the fault is more likely in the j_{th} rather than in the k_{th} actuator.

Definition 3-6: The signature subspace of the j_{th} sensor fault is defined as:

$$R_j = Span\{I_j, Ck_{1j}, Ch_j\} \tag{3.38}$$

The relationship between the vector $r(t)$ with the subspace R_j can be measured by the relationship between the vector $r(t)$ with its projection $r_j^*(t)$ in the subspace R_j. This is quantified by:

$$CORR_j(t) = \frac{|(r_j^*)^T r(t)|}{\|r_j^*\|_2 \|r(t)\|_2} \tag{3.39}$$

where the projection $r_j^*(t)$ of $r(t)$ in R_j is:

$$r_j^*(t) = \Phi_j(\Phi_j^T \Phi_j)^{-1} \Phi_j^T r(t) \tag{3.40}$$

where

$$\Phi_j = [I_j \; Ck_{1j} \; Ch_j]$$

If $CORR_j > CORR_k$, the fault is more likely in the j_{th} rather than in the k_{th} sensor. The relationship between a residual vector with the signature subspace can also be judged by the *normalized projection distance* which is defined as:

$$NPD_j = \frac{\|r(t) - r_j^*(t)\|_2}{\|r(t)\|_2} \tag{3.41}$$

when NPD_{j^*} is the smallest one amongst all NPD_j $(j = 1, 2, \cdots, m)$, the fault is most likely in the j_{th}^* sensor. The idea of fault isolation by comparing the residual direction with the signature subspace is illustrated in Fig.3.6.

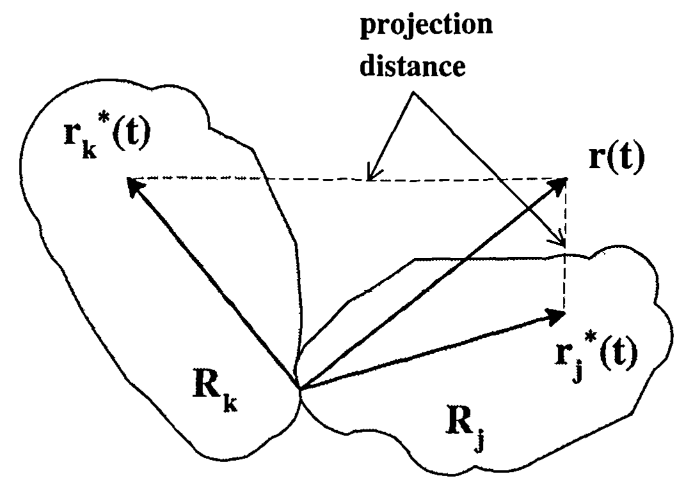


Figure 3.6: **Fault isolation based on directional residuals**

3.4.3 Robust isolation of faulty sensors in a jet engine system

To control a jet engine efficiently and to monitor its health effectively, the sensors have to be perform reliably. However, the sensors in a jet engine work in a very harsh environment and could fail during normal engine operation. This is especially true of the thermocouple (gas temperature) sensors. Hence, the detection of sensor faults in jet engine systems is very important and has become an active research field (Merrill, 1985; Merrill, 1990; Meserole, 1981; Patton and Chen, 1991f). A simplified nonlinear dynamic model of a jet engine control system can be described as:

$$\begin{cases} \dot{X}_1(t) &= f_1(X_1, X_2, X_3) \\ \dot{X}_2(t) &= f_2(X_1, X_2, X_3) \\ \dot{X}_3(t) &= 10(U - X_3) \end{cases}$$

where:

$$\begin{array}{lcll} X_1 &=& n_L & \longmapsto \text{Low pressure rotor speed} \\ X_2 &=& n_H & \longmapsto \text{High pressure rotor speed} \\ X_3 &=& W_f & \longmapsto \text{Main burner fuel flow} \\ U &=& W_{fe} & \longmapsto \text{Fuel flow command} \end{array}$$

The jet engine is a very complicated nonlinear dynamic system. Nonlinear functions such as $f_1(X_1, X_2, X_3)$ and $f_2(X_1, X_2, X_3)$ cannot be written out *analytically*. The system behaviour is normally expressed in a nonlinear dynamic simulation package

(Merrill and Leininger, 1981; Merrill, 1990; Meserole, 1981; Meserole, 1981). This package is capable of simulating the entire operating envelope of the engine, and can also generate linearized models for any operating points. It is useful here to define the following non-dimensional variables:

$$x_1 = \frac{X_1 - X_1^0}{X_1^0} \quad ; \quad x_2 = \frac{X_2 - X_2^0}{X_2^0}$$

$$x_3 = \frac{X_3 - X_3^0}{X_3^0} \quad ; \quad u = \frac{U - U^0}{U^0}$$

where superscript "0" denotes the values at equilibrium. The system can be linearized around an operating point. If u is small (e.g. 1%), x_1, x_2 and x_3 will be small, i.e. all variables have a small variation around the equilibrium and the following linear model is derived:

$$\begin{cases} \dot{x}(t) &= Ax(t) + bu(t) \\ y(t) &= Cx(t) \end{cases}$$

where the state is $x = [x_1\ x_2\ x_3]^T$ and the measurement vector is:

$$y = [x_1 \quad x_2 \quad x_3 \quad p_2 \quad p_4 \quad t_4]^T$$

in which

$$p_2 = \frac{P_2 - P_2^0}{P_2^0} \; ; \; p_4 = \frac{P_4 - P_4^0}{P_4^0} \; ; \; t_4 = \frac{T_4 - T_4^0}{T_4^0}$$

where

$P_2 \longmapsto$ High pressure compressor discharge pressure
$P_4 \longmapsto$ Turbine discharge pressure
$T_4 \longmapsto$ Turbine exit temperature

When the equilibrium is set at $N_L = 450(rpm)$, the linear model matrices are:

$$A = \begin{bmatrix} -1.5581 & 0.6925 & 0.3974 \\ 0.2619 & -2.2228 & 0.2238 \\ 0 & 0 & -10 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.55107 & 0.13320 & 0.30603 \\ 0.55217 & 0.13526 & 0.32912 \\ -0.25693 & -0.23625 & 0.61299 \end{bmatrix}$$

A BFDF described by Eq.(3.26) is designed to isolate sensor faults. If all eigenvalues of the filter are set to -3, the gain matrix can be determined as $K = (3I + A)C^*$ because $rank(C) = 3$. The fault isolation scheme is applied to the nonlinear simulation model. A reliable diagnostic scheme should perform well for a wide range of operating conditions, and hence the input is set at $u = 20\%$ in the simulation. The sensor fault is simulated as 2% offset around the normal measurement. In the simulation, we only consider the fault in sensor Nos.1, 2 and 3, i.e. the low pressure rotor speed sensor, the high pressure rotor speed sensor and the main burner fuel flow sensor. After the transient has settled down, the normalized projection distances for different faulty situations are shown in Table 3.1.

Faulty sensor	No.1	No.2	No.3
NPD_1	**0.37090**	0.77783	0.66389
NPD_2	0.93117	0.95527	0.42455
NPD_3	0.96529	**0.71161**	**0.31559**

Table 3.1: **Fault isolation using Beard fault detection filter**

From Table 3.1, it can be seen that the fault in sensor No.1 (or No.3) can be correctly isolated as the corresponding normalized projection distance NPD_1 (or NPD_3) is the smallest. However, the fault in the sensor No.2 will be mis-reported as a fault in sensor No.3 as NPD_3 is the smallest amongst all normalized projection distances. Moreover, the smallest NPD is not significantly different from other NPDs, and this could make isolation difficult when there is noise in the system.

The example in Table 3.1 illustrates the importance of robustness in fault isolation. The mis-isolation problem is possibly caused by the linearized errors, as the fault isolation scheme is based on the linear model and this scheme is applied to the original nonlinear system. In the model linearization, only the first order terms in the Taylor expansion have been considered. To model a system more accurately, one can consider to the inclusion of the second order terms in the system dynamic equation as follows:

$$\dot{x}(t) = Ax(t) + Bu(t) + Ed(x(t))$$

where the matrices A and B are the same as for the linear model. The term $Ed\ x)$ represents modelling errors and the vector $d(x)$ consists of the second order terms of $x(t)$ as:

$$d(x) = [x_1^2 \quad x_2^2 \quad x_3^2 \quad x_1x_2 \quad x_1x_3 \quad x_2x_3]^T$$

The distribution matrix E can be obtained using an identification procedure based on the least-squares method. Given a series of values $u^{(1)}$, $u^{\ 2)}$, $\cdots$, $u^{(N)}$ for input u, we can obtain the corresponding steady responses $x^{(1)}$, $x^{(2)}$, $\cdots$, $x^{(N)}$ and $d^{(1)}$, $d^{(2)}$, $\cdots$, $d^{(N)}$, which are related by the following steady state equations:

$$\begin{cases} Ax^{(1)} + Bu^{(1)} + Ed^{(1)} &= 0 \\ Ax^{(2)} + Bu^{(2)} + Ed^{(2)} &= 0 \\ \cdots & \cdot\ \cdot \\ Ax^{(N)} + Bu^{(N)} + Ed^{(N)} &= 0 \end{cases}$$

If N is greater than the dimension of $d(x)$, the least-squares estimate of the matrix E is given as:

$$E^* = (\Gamma^+\Psi)^T$$

where Γ^+ is the pseudo-inverse of Γ and

$$\Gamma = \begin{bmatrix} (d^{(1)})^T \\ (d^{(2)})^T \\ \vdots \\ (d^{(N)})^T \end{bmatrix} \qquad \Psi = -\begin{bmatrix} (Ax^{(1)} + Bu^{(1)})^T \\ (Ax^{(2)} + Bu^{(2)})^T \\ \cdots \\ (Ax^{(N)} + Bu^{(N)})^T \end{bmatrix}$$

From the simulation, the following estimate is obtained:

$$E^* = \begin{bmatrix} 1.3293 & 3.4440 & 0.1375 & -5.1304 & -1.7826 & -1.8719 \\ 5.6812 & -0.5281 & -0.3385 & -1.6193 & 0.5229 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

E^* is *not* a full column rank matrix ($rank(E^*) = 2$) and should be decomposed as $E^* = E_1E_2$. Here E_1 is a full column matrix and will be used in the robust fault detection filter design.

$$E_1 = \begin{bmatrix} 6.2006 & 2.8639 \\ 4.1048 & -4.3262 \\ 0 & 0 \end{bmatrix}$$

All eigenvalues of the robust fault detection filter are set to -3. Using the design procedure presented in this chapter, with E replaced by E_1, the parameter matrices of the robust fault detection filter are as follows:

$$H = \begin{bmatrix} 0.6117 & -0.1170 & 0 & 0.3215 & 0.3220 & -0.1295 \\ -0.1170 & 0.9382 & 0 & 0.0605 & 0.0623 & -0.1916 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$T = \begin{bmatrix} 0 & 0 & -0.1251 \\ 0 & 0 & 0.0783 \\ 0 & 0 & 1.0000 \end{bmatrix}$$

$$K = \begin{bmatrix} -0.0708 & 0.0443 & 0.5658 & 0.1400 & 0.1531 & 0.3540 \\ 0.0443 & -0.0277 & -0.3540 & -0.0876 & -0.0958 & -0.2215 \\ 0.5658 & -0.3540 & -4.5229 & -1.1193 & -1.2239 & -2.8297 \end{bmatrix}$$

This robust fault detection filter is also applied to the nonlinear simulation model to isolate faults in sensor Nos.1, 2 and 3. To compare the isolation performance with the BFDF, the system and fault simulation have been set as exactly the same. The normalized projection distances for different faulty situations are shown in Table 3.2.

Faulty sensor	No.1	No.2	No.3
NPD_1	**0.00621**	0.86727	0.90677
NPD_2	0.88625	**0.00213**	0.56602
NPD_3	0.89433	0.02092	**0.00159**

Table 3.2: **Fault isolation using robust fault detection filter**

From Table 3.2, one can seen that NPD_i $(i = 1, 2, 3)$ is the smallest one amongst all normalized projection distances when a fault occurs in the i_{th} sensor. Moreover, the smallest NPD is significantly different from other NPDs. This simulation shows that the fault can be correctly isolated using a robust fault detection filter, even in the presence of modelling errors.

Remarks: This section has studied the design of a robust fault detection filter, and its application in the sensor fault isolation problem for a jet engine control system. The jet engine is a highly nonlinear system, and hence the linearization error causes unreliable isolation if the robustness issues are not considered at the design. To cope with this problem, this section has developed a second order model to account for the linearization errors. Based on this model, a robust fault detection filter is designed and applied to the nonlinear jet engine simulation model and the results show the effectiveness of the robust fault isolation strategy developed in the paper. The technique can be applied to the robust fault isolation for a wide range of systems with uncertain factors.

3.5 Filtering and Robust Fault Diagnosis of Uncertain Stochastic Systems

The problem of detecting and isolating faults in systems with both modelling uncertainty (including unknown disturbances and modelling errors) and noise has not attracted enough research attention, although most systems actually suffer from both modelling uncertainty and noise. This is partly due to a lack of techniques for designing disturbance de-coupling (unknown input de-coupling) optimal (minimum estimation error variance) observers for systems with both unknown disturbances and noise. Recently, some progress has been made in the design of optimal filters for stochastic systems with unknown disturbances. Darouach, Zasadzinski and Keller (1992) proposed an approach for the design of unknown input de-coupled optimal observers by transforming a standard system with unknown inputs into a singular system without unknown inputs, however they only considered time-invariant systems. Chang and Hsu (1993b) also made a contribution in the design of unknown input de-coupled optimal observers for time-invariant systems. Hou and Müller (1993) studied the unknown-input de-coupled filtering for descriptor (singular) systems with unknown inputs. In their study, two transformations were used to remove the unknown inputs. The first transformation transforms the descriptor system with unknown inputs into a descriptor system without unknown inputs, the second step is to transform the singular system into an ordinary system. The filtering algorithm in their approach is very complicated due to the involvement of two transformations. Moreover, the transformation could introduce extra restrictions and result in loss of design freedom.

This section studies optimal filtering and robust fault diagnosis for stochastic systems with unknown disturbances (or unknown and inaccessible inputs). This section proposes a new optimal full order observer with a simple structure, with which, the disturbance de-coupling is easily satisfied. This avoids some of the unnecessary and complex computation involved in some unknown input observer design methods. This section proves that the remaining design freedom, after disturbance de-coupling, can be utilised to ensure that the state estimation has the required minimal variance when noise (with known statistics) acts upon the system. This forms a solution for the optimal observer problem when the system has both unknown disturbance and noise. This section also presents the existence condition and the design procedure for the optimal observer. The existence condition for disturbance de-coupling can be easily verified. Unlike other studies (Darouach et al., 1992; Chang and Hsu, 1993b), this section focuses on time-varying systems. To compare the algorithm given by Hou and Müller (1993), the filtering algorithm presented in this section is simpler and more straightforward. It should be also pointed out that the optimal observers presented in (Darouach et al., 1992; Chang and Hsu, 1993b; Hou and Müller, 1993) have not as yet been applied to robust fault diagnosis.

The optimal observer proposed in this section is applied to the robust fault diagnosis problem. The optimal output estimation can easily be produced using the principle of disturbance de-coupling state estimation. To detect and isolate faults, the output estimation error is used as a residual which is robust against unknown disturbances and has minimal variance. A hypothesis-testing procedure is then applied to examine the likelihood of residuals, and to indicate whether or not a fault has occurred in the system. A simplified flight control system is used to illustrate the method presented in the section. It has been shown that the state estimation obtained by the developed method is an improvement over the estimation obtained using a standard Kalman filter, when modelling errors occur. This is, of course, an advancement which is not confined to FDI problems. The simulation results also show that the method developed is able to detect faults in the presence of both modelling errors and noise.

3.5.1 Optimal observers for systems with unknown disturbances and noise

Consider the following discrete-time mathematical description of the system:

$$\left\{ \begin{array}{lcl} x_{k+1} & = & A_k x_k + B_k u_k + E_k d_k + \zeta_k \\ y_k & = & C_k x_k + \eta_k \end{array} \right. \tag{3.42}$$

where $x_k \in \mathcal{R}^n$ is the state vector, $y_k \in \mathcal{R}^m$ is the output vector, $u_k \in \mathcal{R}^r$ is the known input vector and $d_k \in \mathcal{R}^q$ is the disturbance (or unknown input) vector, ζ_k and η_k are independent zero mean white noise sequences with covariance matrices Q_k and R_k. A_k, B_k, C_k and E_k are known matrices with appropriate dimensions.

The term $E_k d_k$ can be used to describe a number of different kinds of modelling uncertainties, e.g., interconnecting terms in the large scale systems, nonlinear terms in system dynamics (Frank and Wünnenberg, 1989; Chen and Zhang, 1991; Patton and Chen, 1993b), and also linearization and model reduction errors and parameter variations. A detailed study can be found in Chapter 5. It should be pointed out, however, that there are some problems which need to be studied further in the representation of modelling errors as disturbances. One problem is that the distribution matrix could be time varying and this is considered here as the study focuses on stochastic time-varying systems.

In order to estimate the state of the stochastic system with unknown disturbances described by Eq.(3.42), an optimal observer with the following structure is proposed:

$$\left\{ \begin{array}{lcl} z_{k+1} & = & F_{k+1} z_k + T_{k+1} B_k u_k + K_{k+1} y_k \\ \hat{x}_{k+1} & = & z_{k+1} + H_{k+1} y_{k+1} \end{array} \right. \tag{3.43}$$

where the matrices F_{k+1}, T_{k+1}, K_{k+1} and H_{k+1} are to be designed to achieve disturbance de-coupling minimum variance estimation. The block diagram to illustrate this optimal observer is shown in Fig.3.7.

When the proposed observer is applied to a stochastic system with unknown disturbances, the state estimation error ($e_k = x_k - \hat{x}_k$) is as follows:

$$\begin{aligned} e_{k+1} &= x_{k+1} - (z_{k+1} - H_{k+1} y_{k+1}) \\ &= (I - H_{k+1} C_{k+1}) x_{k+1} - z_{k+1} - H_{k+1} \eta_{k+1} \\ &= (I - H_{k+1} C_{k+1}) x_{k+1} - H_{k+1} \eta_{k+1} \end{aligned}$$

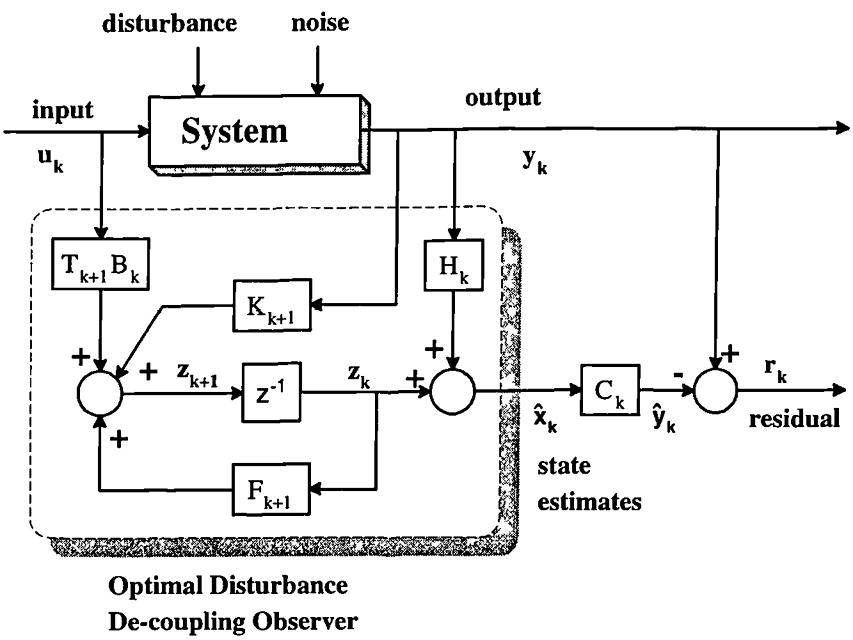


Figure 3.7: **Optimal disturbance de-coupling observer and residual**

$$
\begin{aligned}
& -[F_{k+1}z_k + T_{k+1}B_k u_k + (K^1_{k+1} + K^2_{k+1})y_k] \\
= \; & (I - H_{k+1}C_{k+1})x_{k+1} - H_{k+1}\eta_{k+1} - T_{k+1}B_k u_k \\
& -F_{k+1}(x_k - e_k - H_k y_k) - K^1_{k+1}(C_k x_k + \eta_k) - K^2_{k+1}y_k \\
= \; & F_{k+1}e_k - K^1_{k+1}\eta_k - H_{k+1}\eta_{k+1} \\
& +(I - H_{k+1}C_{k+1})\zeta_k - [F_{k+1} - (I - H_{k+1}C_{k+1})A_k + K^1_{k+1}C_k]x_k \\
& +(I - H_{k+1}C_{k+1})E_k d_k - [K^2_{k+1} - F_{k+1}H_k]y_k \\
& -[T_{k+1} - (I - H_{k+1}C_{k+1})]B_k u_k
\end{aligned} \tag{3.44}
$$

where

$$K_{k+1} = K^1_{k+1} + K^2_{k+1} \tag{3.45}$$

If one can make the following relations hold true:

$$E_k = H_{k+1}C_{k+1}E_k \tag{3.46}$$

$$T_{k+1} = I - H_{k+1}C_{k+1} \tag{3.47}$$

$$F_{k+1} = A_k - H_{k+1}C_{k+1}A_k - K^1_{k+1}C_k \tag{3.48}$$

$$K^2_{k+1} = F_{k+1}H_k \tag{3.49}$$

the estimation error will be:

$$e_{k+1} = F_{k+1}e_k - K^1_{k+1}\eta_k - H_{k+1}\eta_{k+1} + T_{k+1}\zeta_k \tag{3.50}$$

Loosely speaking, if the matrix F_{k+1} is stable, $\mathcal{E}\{e_k\} \to 0$ and $\mathcal{E}\{\hat{x}_k\} \to \mathcal{E}\{x_k\}$ (where $\mathcal{E}\{.\}$ denotes the expectation or mean operator). That is to say, the state estimation will approach the real state asymptotically, in the mean sense. From Eq.(3.50), it can be seen that the unknown disturbance vector has been de-coupled once Eqs. (3.46)–(3.49) hold true. To design the disturbance de-coupled observer, one needs to choose the matrix H_{k+1} to satisfy Eq.(3.46) and to choose the matrix K^1_{k+1} to stabilize the matrix F_{k+1}. Once H_{k+1} and K^1_{k+1} have been chosen, other matrices can be determined using Eqs.(3.47) to (3.49).

Lemma 3-3: The necessary and sufficient condition for the existence of a solution to Eq. (3.46) is:

$$rank(C_{k+1}E_k) = rank(E_k) \tag{3.51}$$

The proof is the same as that for Lemma 3-1 (see Section 3.2.1).

Eq. (3.51) is the only condition for achieving disturbance (unknown input) de-coupling. To satisfy this equation, the number of independent rows of the matrix C_{k+1} must not be less than the number of independent columns of the matrix E_k. That is to say, the maximum number of disturbances which can be de-coupled cannot be larger than the number of independent measurements. When condition (3.51) holds true, the general solution for Eq. (3.46) can be constructed as:

$$H_{k+1} = H^0_{k+1} + H^1_{k+1}H^2_{k+1} \tag{3.52}$$

$$H^0_{k+1} = E_k(C_{k+1}E_k)^+ \tag{3.53}$$

$$H^2_{k+1} = I_m - (C_{k+1}E_k)(C_{k+1}E_k)^+ \tag{3.54}$$

and $H^1_{k+1} \in \mathcal{R}^{n\times m}$ can be arbitrarily chosen. To simplify the observer design, the matrix H^1_{k+1} can be set zero for most cases, i.e.,

$$H_{k+1} = E_k(C_{k+1}E_k)^+ \tag{3.55}$$

The stability (or convergence) of the observer is dependent on the matrix F_{k+1}, once the matrix H_{k+1} is obtained, the system dynamic matrix can be determined by:

$$F_{k+1} = A^1_{k+1} - K^1_{k+1} C_k \tag{3.56}$$

where:

$$A^1_{k+1} = A_k - H_{k+1} C_{k+1} A_k \tag{3.57}$$

The matrix K^1_{k+1} should be designed to stabilize the observer. On considering the simplest case, i.e., when the system is time-invariant, the matrix F can easily be stabilized using pole placement if the matrix pair $\{A_1, C\}$ is observable. For time-varying systems the stability is more difficult to verify, however divergence should not be a problem if the eigenvalues of the each matrix F_{k+1} have been assigned within the unite circle in the complex plane via the gain matrix K^1_{k+1}.

It is clearly of interest to know how good the estimate $\hat{x}_k$ is. The variance of this estimation can be measured using the error covariance matrix P_k defined as:

$$P_k = \mathcal{E}\{[x_k - \hat{x}_k][[x_k - \hat{x}_k]^T\} \tag{3.58}$$

From the Eq.(3.50), it is easy to seen that the update of the covariance matrix is:

$$\begin{aligned} P_{k+1} &= (A^1_{k+1} - K^1_{k+1} C_k) P_k (A^1_{k+1} - K^1_{k+1} C_k)^T \\ &\quad + K^1_{k+1} R_k (K^1_{k+1})^T + T_{k+1} Q_k T^T_{k+1} + H_{k+1} R_{k+1} H^T_{k+1} \end{aligned} \tag{3.59}$$

The best (optimal) state estimation should have minimal variance. From Eq.(3.59), it can be seen that the covariance matrix of the estimation error is controlled by the matrix K^1_{k+1}. The following theorem is now used to give the design of the matrix K^1_{k+1}, for achieving the minimum variance estimation.

Theorem 3-2: To make the state estimation error e_{k+1} have the minimum variance, the matrix K^1_{k+1} should be determined by:

$$K^1_{k+1} = A^1_{k+1} P_k C_k^T [C_k P_k C_k^T + R_k]^{-1} \tag{3.60}$$

Proof: For brevity, some subscripts are omitted in the following proof.

$$\begin{aligned} P_{k+1} &= A^1P_k(A^1)^T + TQ_kT^T + HR_{k+1}H^T \\ &\quad -K^1CP_k(A^1)^T - A^1P_kC^T(K^1)^T + K^1[CP_kC^T + R_k](K^1)^T \end{aligned}$$

As R_k is a positive definite matrix, $CP_kC^T + R_k$ is also positive definite and there exists an invertible matrix S, such that:

$$SS^T = CP_kC^T + R_k$$

Let $D = A^1P_kC^T[S^T]^{-1}$, the covariance matrix is:

$$\begin{aligned} P_{k+1} &= A^1P_k(A^1)^T + HR_{k+1}H^T - DD^T \\ &\quad +[K^1S - D][K^1S - D]^T + TQ_kT^T \end{aligned}$$

To minimize $var\{e_{k+1}\} = trace\{P_{k+1}\}$, one should make $K^1S - D = 0$, this leads to Eq.(3.60) and we have that:

$$P_{k+1} = A^1_{k+1}P'_{k+1}(A^1_{k+1})^T + T_{k+1}Q_kT^T_{k+1} + H_{k+1}R_{k+1}H^T_{k+1} \tag{3.61}$$

where

$$P'_{k+1} = P_k - K^1_{k+1}C_kP_k(A^1_{k+1})^T \tag{3.62}$$

◇ **QED**

From the above derivation and theorem, the computational procedure for the optimal filtering algorithm can be listed as follows:

1° Set initial values: $P_0 = P(0)$, $z_0 = x_0 - C_0E_0(C_0E_0)^+y_0$, $H_0 = 0$ and k=0.

2° Compute H_{k+1} using Eq. (3.55).

3° Compute K^1_{k+1} and P'_{k+1} using Eqs. (3.60) and (3.62).

4° Compute T_{k+1}, F_{k+1}, K^2_{k+1} and K_{k+1} using Eqs. (3.47), (3.48), (3.49) and (3.45).

5° Compute the state estimate $\hat{x}_{k+1}$ and z_{k+1} using Eq. (3.43).

6° Compute P_{k+1} using Eqs. (3.61) & (3.62).

7° Set $k = k + 1$ go to step 2°.

It is important to note that the optimal filtering algorithm proposed in this section is equivalent to a standard Kalman filter for systems *without* unknown disturbances, by setting the matrices $H_{k+1} = 0$ and $T_{k+1} = I$ when there is no disturbance, i.e. $E = 0$. From Eq.(3.52), it can be seen that the solution for the matrix H_{k+1} is not unique as the matrix H^1_{k+1} can be set arbitrarily. By choosing this free matrix H^1_{k+1}, the variance of the estimation error may be decreased slightly further, however this will result in a very complicated algorithm. Hence, it is more practical to fix the solution for H_{k+1} using Eq.(3.55).

3.5.2 Robust residual generation and fault detection

In order to diagnose faults, a fault indicating signal, i.e. residual, can be generated using the output estimation as follows:

$$r_k = y_k - \hat{y}_k = (I - C_k H_k) y_k - C_k z_k \tag{3.63}$$

The system with possible actuator and sensor faults can be described as:

$$\left\{ \begin{array}{rcl} x_{k+1} & = & A_k x_k + B_k u_k + E_k d_k + \zeta_k + B_k f_k^a \\ y_k & = & C_k x_k + \eta_k + f_k^s \end{array} \right. \tag{3.64}$$

where $f_k^a \in \mathcal{R}^r$ is the actuator fault vector and $f_k^s \in \mathcal{R}^m$ is the sensor fault vector. For this system, the state estimation error and the residual are governed by the following equations:

$$\left\{ \begin{array}{rcl} e_k & = & F_k e_{k-1} + K_k^1 \eta_{k-1} - H_k \eta_k + T_k \zeta_{k-1} + K_k^1 f_{k-1}^s - H_k f_k^s + T_k B_{k-1} f_{k-1}^a \\ r_k & = & C_k e_k + \eta_k + f_k^s \end{array} \right. \tag{3.65}$$

It can be seen that the unknown disturbance term $E_k d_k$ does not affect the residual, i.e. the residual is robust against unknown disturbances. As the state estimation error e_k has minimum variance, the residual is also optimal with respect to noise (with assumed statistics). For the residual, the two hypotheses to be tested can be identified as H_0, the normal mode, and the faulty mode H_1. Under the normal (no fault) condition, the statistics of the residual are:

$$H_0 : \left\{ \begin{array}{l} \mathcal{E}\{r_k\} = 0 \\ covariance\{r_k\} = W_k = C_k P_k C_k^T + R_k \end{array} \right. \tag{3.66}$$

When a fault occurs in the system (H_1), the statistics of the residual will be different from the normal mode. The task of fault detection is to distinguish between two hypotheses H_1 and H_0. Any of the well-known hypothesis-testing methods, e.g. Generalized Likelihood Ratio (GLR) testing and Sequential Probability Ratio Testing (SPRT) (Willsky, 1976; Basseville, 1988) can be used to examine the residual and, subsequently to diagnose faults. If one assumes that the noise sequences ζ_k and η_k are Gaussian white, the residual will also have the Gaussian distribution. To construct a detection decision function (the test statistic) λ_k:

$$\lambda_k = r_k^T W_k^{-1} r_k \tag{3.67}$$

which is χ^2 distributed with m degrees of freedom (m is the dimension of r_k). The test for fault detection is then:

$$\begin{cases} \lambda_k \geq T_D & \text{fault} \\ \lambda_k < T_D & \text{no fault} \end{cases} \tag{3.68}$$

where the threshold T_D is determined from the χ^2 distribution table and:

$$Probability\{\lambda_k \leq T_D \mid H_0\} = P_f \tag{3.69}$$

where P_f is the probability of false alarm which is given by the designer.

The detection function λ_k is constructed using only a single sample of the residual. To increase the reliability of statistical testing, a residual sequence over a time window can be used. It is easy to verify that the $covariance\{r_k, r_{k-1}\} \neq 0$, i.e. the resulting residual sequence is not white, although both noise signals ζ_k and η_k are white. This will increase the difficulty and complexity in testing the residual sequence, however this penalty is worth paying in order to ensure that the unknown disturbance has been de-coupled from the residual. This is especially true when the unknown disturbance has a more dominant effect on the residual than the noise does.

3.5.3 An illustrative example

The linearized discrete-time model of a simplified longitudinal flight control system is as follows:

$$\begin{cases} x_{k+1} = A_k x_k + B_k u_k + \zeta_k + E_k d_k \\ y_k = C_k x_k + \eta_k \end{cases}$$

where the state variables are: pitch angle δ_z, pitch rate ω_z and normal velocity η_y, the control input is elevator control signal. The system parameter matrices are:

$$A_k = \begin{bmatrix} 0.9944 & -0.1203 & -0.4302 \\ 0.0017 & 0.9902 & -0.0747 \\ 0 & 0.8187 & 0 \end{bmatrix} \qquad B_k = \begin{bmatrix} 0.4252 \\ -0.0082 \\ 0.1813 \end{bmatrix}$$

$$C_k = I_{3\times3} \qquad x = [\eta_y \; \omega_z \; \delta_z]^T$$

The covariance matrices for input and output noise sequences are: $Q_k = diag\{0.1^2,\ 0.1^2,\ 0.01^2\}$ and $R_k = 0.1^2 I_{3\times3}$. The term $E_k d_k$ is used in here to represent the parameter perturbation in matrices A_k and B_k:

$$\begin{aligned} E_k d_k &= \Delta A_k x_k + \Delta B_k u_k \\ &= E\{\begin{bmatrix} \Delta a_{11} & \Delta a_{12} & \Delta a_{13} \\ \Delta a_{21} & \Delta a_{22} & \Delta a_{23} \end{bmatrix} x_k + \begin{bmatrix} \Delta b_1 \\ \Delta b_2 \end{bmatrix} u_k\} \end{aligned}$$

with

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

where, Δa_{ij} and Δb_i $(i = 1, 2; j = 1, 2, 3)$ are perturbations in aerodynamic and control coefficients. They are unknown and can be time-varying. The perturbations can affect the estimation accuracy. In this section, their effects on the system have been modelled as unknown disturbances and can be de-coupled from the state estimation using the method given in Section 3.5.1.

The simulation is used to assess the usefulness of the optimal observer for estimating states. In the simulation, the input and initial conditions arc set as $u_k = 10$, $x_0 = 0$ and $P_0 = 0.1^2 I_{3\times3}$. The aerodynamic coefficients are perturbed by $\pm 50\%$, i.e. $\Delta a_{ij} = -0.5 a_{ij}$ and $\Delta b_j = 0.5 b_j$. Fig.3.8–Fig.3.10 shows the absolute values of the state estimation errors.

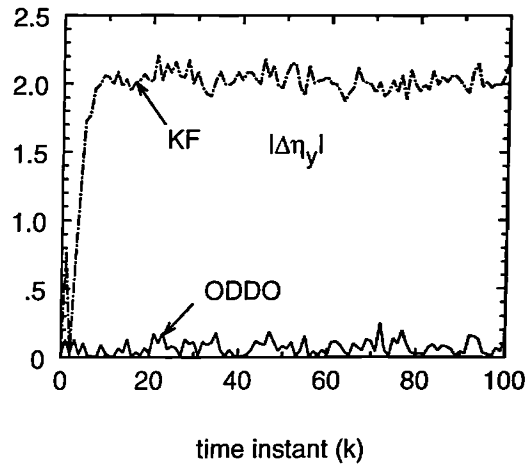


Figure 3.8: **The state estimation error absolute values for η_y (ODDO: Optimal Disturbance De-coupling Observer; KF: Kalman Filter)**

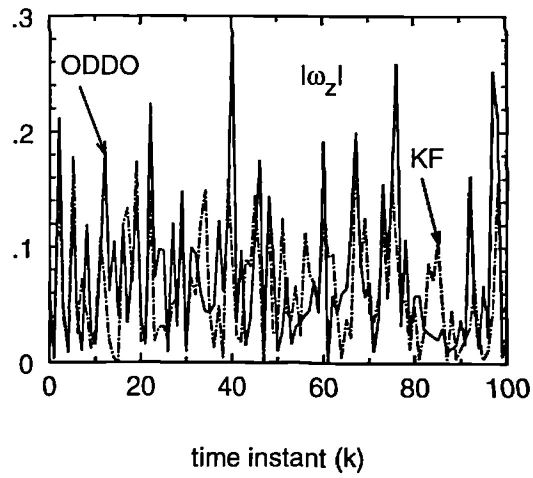


Figure 3.9: **The state estimation error absolute values for ω_z (ODDO: Optimal Disturbance De-coupling Observer; KF: Kalman Filter)**

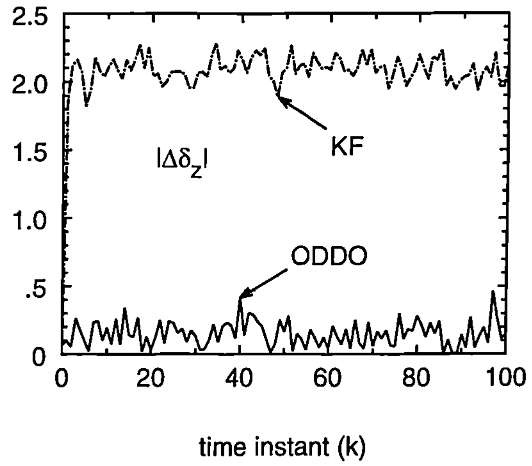


Figure 3.10: **The state estimation error absolute values for δ_z (ODDO: Optimal Disturbance De-coupling Observer; KF: Kalman Filter)**

The estimation errors achieved by the traditional Kalman filter (not disturbance de-coupled) are also shown in the Fig.3.8–Fig.3.10. It can be seen that the method developed in this section can give better state estimation, even when the system parameters have large perturbations. A number of situations when aerodynamic coefficients have time-varying (e.g. sinusoid function) perturbations (the results are not shown in this section) have also been simulated. For such cases, the estimation error using the Kalman filter is always divergent even if the perturbation magnitude is very small. However, the disturbance de-coupling method given in this section can give satisfactory estimation. This is expected, since the perturbation effects on the estimation error have been de-coupled.

Fig.3.11 shows the detection function λ_k when an incipient (small and slow) fault occurs in the sensor for δ_z. Fig.3.12 shows the fault detection function λ_k when a step fault occurs in the actuator. It can be seen that the faults are detected very reliably by setting a threshold (T_D) on the fault detection function.

Remarks: This section has proposed a systematic approach to designing optimal disturbance de-coupled observers for systems with both unknown disturbance and noise. This optimal observer is used to estimate the system state and to generate residuals for detecting faults in stochastic uncertain systems. It is the first time such consideration has been addressed and solved in a fault diagnosis design. The method has been applied to detecting sensor and actuator faults in a simplified flight control system and the simulation results show the effectiveness of the method. Considering

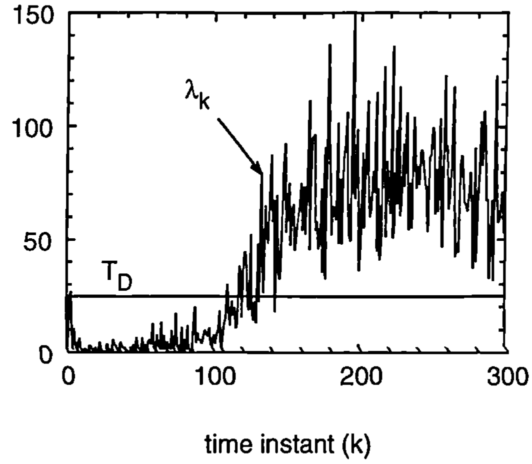


Figure 3.11: **The fault detection function when a fault occurs in the sensor for δ_z**

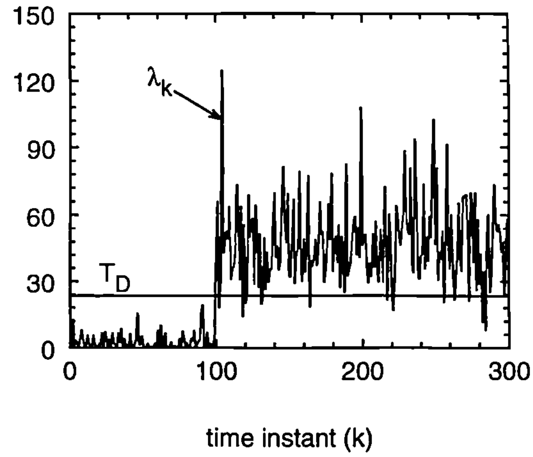


Figure 3.12: **The fault detection function when a fault occurs in the actuator**

the extreme difficulty in enhancing the fault diagnosis performance under modelling uncertainty and noise, any improvement in the robustness of residual design is very welcome. The scope of applications of this work extends to a wide range of stochastic uncertain systems and is not confined to the fault diagnosis problem domain.

3.6 Summary

The purpose of this chapter has been the study of UIO-based robust residual generation methods. A new full-order UIO structure has been proposed in this chapter. The existence conditions and design procedures for such UIOs have also been presented and soundly proved. When compare with other techniques in designing UIOs, the existence conditions presented in this chapter are very easy to verify. The design procedure proposed in this chapter is very straightforward, because it can be implemented using the pole placement routine (*PLACE*) in Control Toolbox for MATLAB, together with a few simple matrix manipulation routines which are also available in MATLAB. The robust FDI schemes based upon UIOs have also been studied in this chapter. A chemical reactor has been used to illustrate the robust actuator fault detection and isolation schemes.

The main advantage of full-order UIOs over other commonly used reduced-order UIOs is that there is more design freedom available after the unknown input decoupling conditions have been satisfied. This chapter has exploited the remaining freedom to achieve other performance requirements for FDI, and has proposed a method to design a robust fault detection filter which can generate disturbance decoupled directional residuals for fault isolation. This is achieved via a combination of the UIO and the BFDF principles. The effectiveness of robust fault detection filters in robust fault isolation has been demonstrated by a highly nonlinear jet engine system example. The remaining freedom has been also used in this chapter to produce the minimum variance state estimations and residuals for stochastic systems with unknown disturbances. The optimal disturbance de-coupled observer is a by-product of the main work presented in this chapter. The application of this optimal observer is beyond the robust FDI domain. It can be used for the optimal filtering problem for a wide range of uncertain stochastic systems.

Robust FDI based on UIOs have been studied for many years. However, the number of reported applications is very limited. The main argument is that the unknown input distribution matrix, required for designing UIOs, is actually unknown for most practical systems. The chapter has demonstrated, by means of a number of examples, how UIO-based robust FDI methods can be used in practical systems in which the unknown input distribution matrix is not directly known. The success of such application studies could give some guide-lines for real industrial applications.

Chapter 4

ROBUST RESIDUAL GENERATION BY THE ASSIGNMENT OF OBSERVER EIGENSTRUCTURE

4.1 Introduction

In Chapter 3, various approaches for generating robust residual via unknown input observers have been studied. The underlying principle of these approaches is to make the state estimation error be independent of disturbances (or unknown inputs). The residual is defined as the (weighted) output estimation error which is a linear transformation of the state estimation error. The residual generated by UIOs is also independent of disturbances, if the disturbance term does not appear in the output equation or the disturbance term in the output equation has been nulled. In model-based FDI, the state estimation is not necessarily needed, because the required information is the diagnostic signal – residual. Hence, it is not necessary to de-couple the state estimation error from disturbances in model-based FDI. A direct approach to design disturbance de-coupled residuals is then required. In this approach, the residual itself is de-coupled from disturbances, however the state estimation error may not be. It can be expected that existing conditions for such a direct approach could be relaxed compared with those required for UIOs.

The most important direct approach to design robust (in the disturbance de-coupling sense) residual generators is the use of eigenstructure assignment in which some left eigenvectors of the observer are assigned to be orthogonal to the disturbance distribution directions. In this way, the residual can be made robust against disturbances. This approach was initially proposed by Patton and colleagues in 1986 (Patton et al., 1986) and has been studied and developed extensively by Patton et al. (Patton and Willcox, 1987; Patton, 1988; Patton and Kangethe, 1989; Patton and Chen, 1991h; Patton and Chen, 1991c; Patton and Chen, 1991e; Patton and Chen, 1991b). A mathematically sound treatment and new results are given by Patton and Chen (Patton and Chen, 1991g; Patton, Chen, Millar and Kiupel, 1991; Patton and Chen, 1992c). The approach has been successfully applied to robust FDI of flight control systems (Patton and Willcox, 1987; Patton and Kangethe, 1989), jet engine systems (Patton and Chen, 1990; Patton and Chen, 1991f; Patton and Chen, 1991b; Patton and Chen, 1991a; Patton and Chen, 1992e; Patton, Chen and Zhang, 1992; Patton, Zhang and Chen, 1992) and nuclear reactors (Patton, Chen and Millar, 1991; Patton, Chen and Millar, 1992). Note that Daley and Wang (Daley and Wang, 1991; Daley and Wang, 1992; Wang, Kropholler and Daley, 1993) have also presented a different approach to generate robust residuals via the assignment of observer left eigenvectors. Magni *et al* (Magni and Mouyon, 1991; Magni and Mouyon, 1992; Magni, Mouyon and Arsan, 1993; *Arsan, Mouyon and Magni, 1994) have also* proposed another approach in the robust residual generation by assigning eigenvectors for so-called "one-dimensional" (or elementary) observers.

This chapter gives a detailed treatment of the eigenstructure assignment approach for robust residual generation. The principle and existence conditions are presented in a number of theorems, and the design procedure is also given. The remaining design freedom after disturbance de-coupling has been satisfied is used to optimize other performance indices such as fault sensitivity. When the left eigenvectors of the observer are not assignable, the approximate assignment problem and the design procedure is studied in this chapter.

One of the recent developments in the eigenstructure assignment method for designing robust residual generators is the assignment of some right eigenvectors parallel to the disturbance distribution directions. This method was proposed by Chen and Patton in (Patton and Chen, 1991g; Patton and Chen, 1992c) and a complete and sound mathematical treatment is given in this chapter. Note that the observer design is a dual of the control design problem. The assignment of right eigenvectors in an observer design is equivalent to the assignment of left eigenvectors in a controller design. Apart from an intuitive method proposed by Zhang, Slater and Allemang (1990), this problem has rarely been considered. This chapter develops and extends the new method proposed by Chen and Patton (Patton and Chen, 1991g; Patton and Chen, 1992c) for assigning right observer eigenvectors by presenting the existence conditions and the design procedure.

The chapter is mainly based on the use of continuous-time system models, although the techniques developed can be directly applied to discrete-time system models. The dead-beat design has unique characteristics in the discrete time domain. To take advantage of the dead-beat design, this chapter also includes a study of the robust residual generation problem in the discrete time domain. It can be seen that the dead-beat design makes the principle and design procedure very simple. The dead-beat design also gives a direct correspondence between the observer-based and parity relation approaches in residual generation and this phenomenon has been discussed by Patton and Chen (Patton and Chen, 1991h; Patton and Chen, 1991c; Patton and Chen, 1991f; Patton and Chen, 1991e). Two numerical examples are used in this chapter to demonstrate the eigenstructure assignment approach in robust residual generation, and real applications are given in Chapter 5.

4.2 Residual Generation and Responses

In a similar way to Chapter 3, it is also assumed in this chapter that the system is disturbed by an additive unknown input term as follows:

$$\begin{cases} \dot{x}(t) &= Ax(t) + Bu(t) + R_1 f(t) + Ed(t) \\ y(t) &= Cx(t) + Du(t) + R_2 f(t) \end{cases} \tag{4.1}$$

where $x(t) \in \mathcal{R}^n$ is the state vector, $y(t) \in \mathcal{R}^m$ is the output vector, $u(t) \in \mathcal{R}^r$ is the known input vector and $d(t) \in \mathcal{R}^q$ is the unknown input (or disturbance) vector, $f(t) \in \mathcal{R}^g$ represents the fault vector which is considered as an unknown time function. A, B, C, D and E are known matrices with appropriate dimensions. The matrices R_1 and R_2 are fault distribution matrices which are known when the designer has been told which faults should be diagnosed. Similar to Chapter 3, the matrix E is assumed to be full column rank.

The residual generator based on a full-order observer illustrated in Fig.4.1, is de-

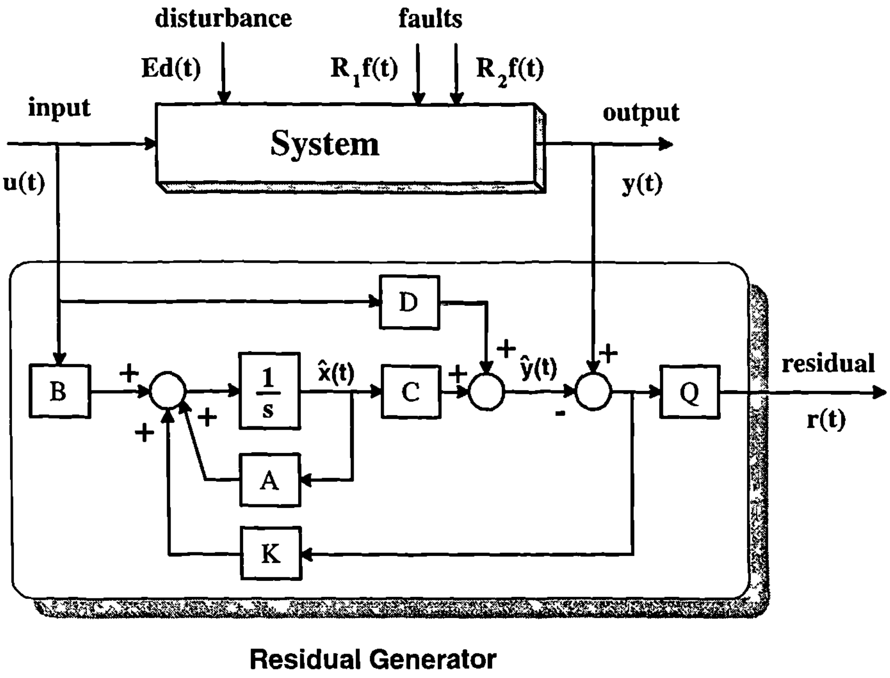


Figure 4.1: **Robust observer-based residual generation**

scribed as:

$$\begin{cases} \dot{\hat{x}}(t) &= (A-KC)\hat{x}(t) + (B-KD)u(t) + Ky(t) \\ \hat{y}(t) &= C\hat{x}(t) + Du(t) \\ r(t) &= Q[y(t) - \hat{y}(t)] \end{cases} \tag{4.2}$$

where $r \in \mathcal{R}^p$ is residual vector, $\hat{x}$ and $\hat{y}$ are state and output estimations. The matrix $Q \in \mathcal{R}^{p\times m}$ is the residual weighting factor. Note that, the residual is a linear transformation of the output estimation error. Hence, the residual dimension p cannot be larger than the output dimension m. This is because the linearly dependent extra residual components do not provide additional useful information in FDI.

When the residual generator represented by Eq.(4.2) is applied to the system described by Eq.(4.1), the state estimation error ($e(t) = x(t) - \hat{x}(t)$), and the residual are governed by the following equations:

$$\begin{cases} \dot{e}(t) &= (A-KC)e(t) + Ed(t) + R_1f(t) - KR_2f(t) \\ r(t) &= He(t) + QR_2f(t) \end{cases} \tag{4.3}$$

where $H = QC$. The Laplace transformed residual response to faults and disturbances is thus:

$$r(s) = QR_2f(s) + H(sI - A + KC)^{-1}(R_1 - KR_2)f(s) + H(sI - A + KC)^{-1}Ed(s) \quad (4.4)$$

One can seen that the residual $r(t)$ and the state estimation error are not zero, even if no faults occur in the system. Indeed, it can be difficult to distinguish the effects of faults from the effects of disturbances acting on the system. The effects of disturbances obscure the performance of FDI and act as a source of false and missed alarms. Therefore, in order to minimize the false and missed alarm rates, one should design the residual generator such that the residual itself becomes de-coupled with respect to disturbances. Chapter 3 has studied the UIO-based approaches in which the state estimation error $e(t)$ and hence the residual are de-coupled from disturbances. This chapter focuses on the technique which de-couples $r(t)$ from $d(t)$ directly. It is clearly not important whether or not $e(t)$ is de-coupled $d(t)$ as $e(t)$ itself is not required in robust FDI.

4.3 General Principle for Disturbance De-coupling Design

In order to make the residual $r(t)$ be independent of disturbances, it is necessary to null the entries in the transfer function matrix between the residual and the disturbance. That means:

$$G_{rd}(s) = QC(sI - A + KC)^{-1}Ed(s) = 0 \quad (4.5)$$

This is a special case of the *output-zeroing* problem which is well known in multivariable control theory (Karcanias and Kouvaritakis, 1979). Once E is known, the remaining problem is to find the matrices K and Q to satisfy Eq.(4.5), in addition to choosing the suitable eigenvalues to optimize the FDI performance.

4.3.1 Disturbance de-coupling design via invariant subspaces

The solvability condition for matrices Q and K in Eq.(4.5) can be determined in the context of the invariant subspace theory (Morse, 1973; Antsaklis, 1980). The transfer matrix can be expanded[1] as follows:

$$\begin{aligned} H(sI-A_c)^{-1}E &= H[a_1(s)I_n + a_2(s)A_c + \cdots + a_n(s)A_c^{n-1}]E \\ &= [a_1(s)I_p \; a_2(s)I_p \; \cdots \; a_n(s)I_p] \begin{bmatrix} H \\ HA_c \\ \vdots \\ HA_c^{n-1} \end{bmatrix} E \\ &= H[E \; A_cE \; \cdots A_c^{n-1}E] \begin{bmatrix} a_1(s)I_q \\ a_2(s)I_q \\ \vdots \\ a_n(s)I_q \end{bmatrix} \end{aligned} \quad (4.6)$$

where $A_c = A - KC$ and $a_1(s), \cdots, a_n(s)$ are functions of s. From the above relation, it is easy to see that Eq.(4.5) can be solved by satisfying one of the following conditions:

(a) If the $\{H \; , \; A_c\}$ - invariant subspace lies in the left zero space of E, Eq. (4.5) holds true.

(b) If the $\{A_c \; , \; E\}$ - invariant subspace contained in the right zero space of H, Eq.(4.5) holds true.

The above two conditions give general guide-lines for designing disturbance de-coupling residuals (Patton and Willcox, 1987), however it is not easy to achieve these conditions without further assistance of design tools such as eigenstructure assignment.

[1]This expansion can be proved by using the Taylor expansion of $\frac{1}{1-x}$ and the matrix Cayley-Hamilton theorem.

4.3.2 Disturbance de-coupling design via eigenstructure assignment

In multivariable systems, there is extra design freedom available beyond eigenvalue assignment (Moore, 1976) and which can be used to assign eigenvectors to achieve the required system performances. In the residual generator design problem, the design freedom is used to assign the observer eigenstructure (eigenvalues and eigenvectors) to achieve disturbance de-coupling property. To study this technique, two Lemmas which relate to the properties of the system observer eigenstructure, should be introduced.

Lemma 4-1: A given left eigenvector l_i^T which is corresponding to eigenvalue λ_i of A_c is always orthogonal to the right eigenvector v_j corresponding to the remaining (n-1) eigenvalue λ_j of A_c where $\lambda_i \neq \lambda_j$ (Patton and Kangethe, 1989).

Proof: For the left eigenvector l_i^T of A_c, we have:

$$l_i^T A_c = \lambda_i l_i^T \quad \text{for} \quad i = 1, 2, \cdots, n$$

Post-multiplying both side of the above equation by v_j $(j \neq i)$:

$$l_i^T A_c v_j = \lambda_i l_i^T v_j \quad \text{for} \quad i = 1, 2, \cdots, n; \quad j \neq i$$

As the vector v_j is right eigenvector of A_c, we have $A_c v_j = \lambda_j v_j$, and the above equation can be rewritten as:

$$\lambda_j l_i^T v_j = \lambda_i l_i^T v_j \quad \text{for} \quad i = 1, 2, \cdots, n; \quad j \neq i$$

Hence, if $\lambda_i \neq \lambda_j$, the only solution to the above equation is the trivial solution and it thus follows that:

$$l_i^T v_j = 0 \quad \text{for} \quad i \neq j \tag{4.7}$$

i.e. the left and right eigenvectors corresponding to mutually distinct eigenvalues are orthogonal. $\diamond$ **QED**

Lemma 4-2: Any transfer function matrix can be expanded in term of eigenstructure:

$$(sI - A_c)^{-1} = \frac{v_1 l_1^T}{s - \lambda_1} + \frac{v_2 l_2^T}{s - \lambda_2} + \cdots + \frac{v_n l_n^T}{s - \lambda_n} \tag{4.8}$$

where v_i and l_i^T are right and left eigenvectors of A_c respectively, corresponding to the eigenvalue λ_i.

Note that this Lemma is only valid for cases when all eigenvectors of the observer are distinct, however this requirement does not impose any restriction on the observer design.

Proof: Define the left eigenvector and right eigenvector matrices as:

$$L = \begin{bmatrix} l_1^T \\ l_2^T \\ \vdots \\ l_n^T \end{bmatrix} \qquad V = [v_1, v_2, \cdots, v_n]$$

According to Lemma 4-1, we have the following relation:

$$LV = \begin{bmatrix} l_1^T v_1 & 0 & \cdots & 0 \\ 0 & l_2^T v_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & l_n^T v_n \end{bmatrix}$$

If vectors l_i and v_i $(i = 1, 2, \cdots, n)$ are properly scaled, the above equation become:

$$LV = I_n$$

This means that:

$$L = V^{-1}$$

It is well-known that, the matrix A_c can be decomposed as:

$$A_c = V \Lambda V^{-1}$$

where $\Lambda = diag\{\lambda_1, \lambda_2, \cdots \lambda_n\}$. From the above equation, we have:

$$e^{A_c t} = V e^{\Lambda t} V^{-1} = \sum_{i=1}^{n} e^{\lambda_i t} v_i l_i^T$$

this leads to:

$$(sI - A_c)^{-1} = Laplace\{e^{A_c t}\} = Laplace\{\sum_{i=1}^{n} e^{\lambda_i t} v_i l_i^T\} = \sum_{i=1}^{n} \frac{v_i l_i^T}{s - \lambda_i}$$

◇ **QED**

Based on Lemma 4-2, Eq.(4.5) can be rewritten as:

$$G_{rd}(s) = \sum_{i=1}^{n} \frac{H v_i l_i^T E}{s - \lambda_i} \tag{4.9}$$

Thus, it can be noted that the disturbance de-coupling is possible *if and only if*:

$$R_i = H v_i l_i^T E = 0 \quad \text{for} \quad i = 1, 2, \cdots, n \tag{4.10}$$

This implies that:

$$\sum_{i=1}^{n} R_i = H(\sum_{i=1}^{n} v_i l_i^T)E = HVLE = HE = QCE = 0 \tag{4.11}$$

Hence, one of the necessary conditions for designing disturbance de-coupled residuals is given by the above equation and restated in the following theorem:

> **Theorem 4-1:** A necessary condition for achieving disturbance de-coupling design is:
>
> $$QCE = HE = 0 \tag{4.12}$$

If $CE = 0$, any residual weighting matrix can satisfy this necessary condition. However, this is not always the case. Loosely speaking, the column number of E cannot be larger than the independent row number of C to satisfy the above necessary condition, i.e. the number of independent disturbances can be de-coupled cannot larger than the number of independent measurements. If this necessary condition cannot satisfied, an approximate de-coupling procedure should be used, this is to approximate the matrix E by a lower rank matrix. This problem is studied in Chapter 5.

A general solution for Eq.(4.12) is given by:

$$Q = Q_1[I_m - CE(CE)^+] \tag{4.13}$$

where $Q_1 \in \mathcal{R}^{p\times m}$ is an arbitrary design matrix and $(CE)^+$ is the pesudo-inverse of CE and is given by the following equation if $rank(CE) = q$:

$$(CE)^+ = [(CE)^T(CE)]^{-1}(CE)^T \tag{4.14}$$

The maximum independent row number of the matrix Q satisfying Eq.(4.12) is $m - rank(CE)$. As the linearly dependent rows do not provide any useful information, hence the row of the residual weighting matrix Q is normally chosen as:

$$p = m - rank(CE) \leq m \tag{4.15}$$

4.4 Disturbance De-coupling by Assigning Left Eigenvectors

The first method which was initially proposed and developed by Patton *et al* (Patton et al., 1986; Patton and Kangethe, 1989; Patton and Chen, 1991g) for disturbance de-coupling design via eigenstructure assignment is to assign left observer eigenvectors orthogonal to all columns of E. This method is summarized by the following theorem:

Theorem 4-2: The sufficient conditions for satisfying the disturbance de-coupling requirement Eq.(4.5) are:

(1) $QCE = 0$.

(2) All rows of the matrix $H = QC$ are left eigenvectors of $(A - KC)$ corresponding to any real distinct eigenvalues.

Proof: According to condition (2), the matrix H is constructed as:

$$H = \begin{bmatrix} l_1^T \\ \vdots \\ l_p^T \end{bmatrix}$$

where l_i^T $(i = 1, 2, \cdots, n)$ are the left eigenvectors of $A - KC$. Using the relation given in Lemma 4-1, we have:

$$Hv_i = 0 \quad \text{for} \quad i = p + 1, \cdots, n$$

where v_i $(i = 1, 2, \cdots, n)$ are the right eigenvectors of $A - KC$. According to condition (1) above, we have:

$$l_i^T E = 0 \quad \text{for} \quad i = 1, \cdots, p$$

From Lemma 4-2, the transfer matrix from the disturbance to the residual is expressed as:

$$G_{rd}(s) = \sum_{i=1}^{n} \frac{(Hv_i)l_i^T E}{s - \lambda_i} = \sum_{i=1}^{p} \frac{Hv_i(l_i^T E)}{s - \lambda_i} = 0$$

◇ **QED**

The main principle utilized in this proof can be illustrated graphically by Fig.4.2. This diagram shows the orthogonal relationships of eigenvectors and matrices H and E. According to condition (2) in Theorem 4-2, the rows of the matrix H are orthogonal to the lower partition of the right eigenvectors and hence the lower partition is nulled. Similarly, the top partition part is also nulled due to condition (1).

The procedure for the design of the disturbance de-coupling residual generator via left eigenvector assignment is thus as follows:

(a) Compute the residual weighting matrix Q so that $QCE = 0$.

(b) Determine the eigenstructure of the observer: The eigenvalues of the observer are chosen according to the desired dynamic property of residuals. The rows of QC must be the p left eigenvectors of the observer. The remaining $(n - p)$ left eigenvectors will be chosen so that one can ensure a design with good conditioning.

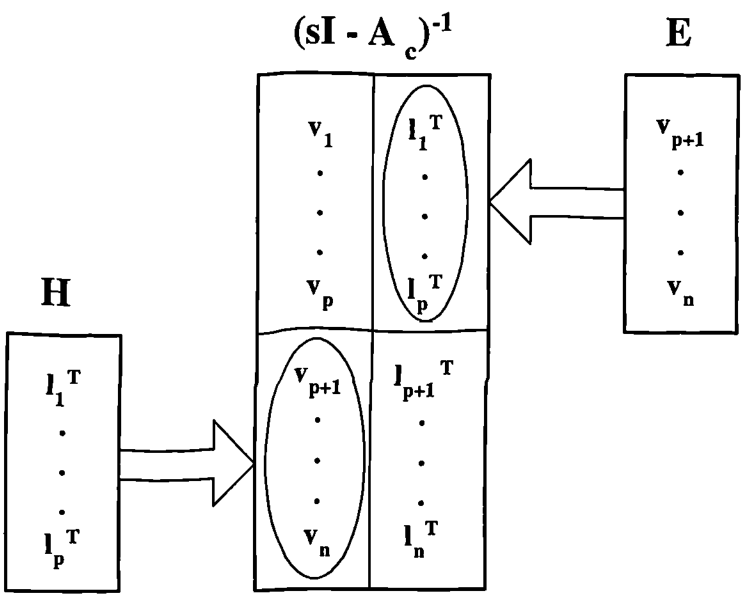


Figure 4.2: **Disturbance de-coupling via eigenvector assignment**

(c) Compute the gain matrix K using suitable eigenstructure assignment technique.

The observer feedback eigenstructure assignment problem can be handled by means of a transformation of the dual control form. On assignment of the right eigenvectors to the dual control problem, these eigenvectors become the left eigenvectors of the observer system (Andry, Chung and Shapiro, 1984; Sobel and Banda, 1989; Burrows and Patton, 1992). The assignment of the right eigenvectors for the control problem is a well-developed technique (Moore, 1976; Fahmy and O'Reilly, 1982; Andry, Shapiro and Chung, 1983; Kautsky, Nichols and Van Dooren, 1985; Roppenecker, 1986; Mudge and Patton, 1988; Owens, 1988; Owens and O'Reilly, 1989; White, 1991; Burrows, Patton and Szymanski, 1989; Burrows and Patton, 1991; Sobel, Shapiro and Andry, 1994). The assignability condition is that, for each eigenvalue λ_i, the corresponding left eigenvector l_i^T must belong to the row subspace spanned by $[C(\lambda_i I - A)^{-1}]$. That is to say the vector l_i should lie in the column subspace spanned by $[(\lambda_i I - A^T)^{-1}C^T]$.

If l_i lies in the subspace $span\{[(\lambda_i I - A^T)^{-1}C^T]\}$, a vector w_i exists which satisfies the following equation:

$$l_i = P(\lambda_i)w_i \quad \text{for} \quad i = 1, \cdots, p \tag{4.16}$$

where:

$$P(\lambda_i) = -(\lambda_i I - A^T)^{-1} C^T \quad \text{for} \quad i = 1, \cdots, p \tag{4.17}$$

One can inspect if l_i is in the subspace $span\{P(\lambda_i)\}$ by comparing l_i with its projection in this subspace, denoted by:

$$l_i^* = P(\lambda_i) w_i^* \quad \text{for} \quad i = 1, \cdots, p \tag{4.18}$$

where:

$$w_i^* = [P(\lambda_i)^T P(\lambda_i)]^{-1} P(\lambda_i)^T l_i \quad \text{for} \quad i = 1, \cdots, p \tag{4.19}$$

if $l_i = l_i^*$, l_i is in $span\{P(\lambda_i)\}$ and is assignable. Otherwise, an approximate procedure must be taken, i.e. to replace l_i by its projection l_i^*. In observer-based residual generator design, there are no other restrictions on the choice of eigenvalues apart from stability. Hence, one can choose stable eigenvalues to minimize the distance between a required eigenvector with its projection in the assignable subspace. The approximate disturbance de-coupling can be achieved by minimizing the following performance index:

$$\begin{aligned} J_1 &= \sum_{i=1}^{p} \|l_i - l_i^*\|_2 \\ &= \sum_{i=1}^{p} \|\{I_n - P(\lambda_i)[P(\lambda_i)^T P(\lambda_i)]^{-1} P(\lambda_i)^T\} l_i\|_2 \end{aligned} \tag{4.20}$$

where l_i^T $(i = 1, \cdots, p)$ are the required left eigenvectors to be assigned for the disturbance de-coupling design. It is possible the J_1 can be made zero by properly chosen eigenvalues λ_i $(i = 1, \cdots, p)$.

Because $(l_i^*)^T$ $(i = 1, \cdots, p)$ are left eigenvectors of $A - KC$ corresponding to eigenvalues λ_i, we have:

$$(l_i^*)^T (A - KC) = \lambda_i (l_i^*)^T \quad \text{for} \quad i = 1, \cdots, p \tag{4.21}$$

i.e.

$$l_i^* = (\lambda_i I - A)^{-1} C^T K^T l_i^* \quad \text{for} \quad i = 1, \cdots, p \tag{4.22}$$

Comparing Eq.(4.22) with Eq.(4.18), we have:

$$w_i^* = K^T l_i^* \quad \text{for} \quad i = 1, \cdots, p \tag{4.23}$$

For a disturbance de-coupling design, only p left eigenvectors are specified, the remaining $n-p$ eigenvectors can be chosen freely from the assignable subspace, i.e.

$$l_i = (\lambda_i I - A)^{-1} C^T w_i \quad \text{for} \quad i = p+1, \cdots, n \tag{4.24}$$

where

$$w_i = K^T l_i \quad \text{for} \quad i = p+1, \cdots, n \tag{4.25}$$

Hence, the observer feedback gain matrix is computed by:

$$K = [WL^{-1}]^T = [WV]^T = V^T W^T \tag{4.26}$$

where

$$W = [w_1^* \cdots w_p^*; \;\; w_{p+1} \cdots w_n] \in \mathcal{R}^{m \times n}$$

$$L = [l_1^* \cdots l_p^*; \;\; l_{p+1} \cdots l_n] \in \mathcal{R}^{n \times n}$$

and $V = L^{-1}$ is the right eigenvector matrix. Note that the first p eigenvalues corresponding to the required eigenvectors l_i^T $(i = 1, \cdots, p)$ must be real because all these eigenvectors are real-valued. The remaining $n-p$ eigenvalues and corresponding eigenvectors can be real as well as complex-conjugate.

Disturbance de-coupling does not place any restriction on the choice of eigenvectors l_i^T $(i = p+1, \cdots, n)$ and corresponding eigenvalues λ_i $(i = p+1, \cdots, n)$. Hence, these free parameters can be used to maximize the fault effect on the residual. Consider the transfer function between residuals and faults as:

$$\begin{aligned} G_{rf}(s) &= QR_2 + H(sI - A + KC)^{-1}(R_1 - KR_2) \\ &= QR_2 + H\sum_{i=1}^{n} \frac{v_i l_i^T}{s - \lambda_i}(R_1 - KR_2) \\ &= QR_2 + H\sum_{i=1}^{p} \frac{v_i l_i^T}{s - \lambda_i}(R_1 - KR_2) \end{aligned} \tag{4.27}$$

As pointed out in Section 2.7, the most important factor in fault detectability is the steady-state gain matrix $G_{rf}(0)$, hence a performance index to be maximized for increasing fault detectability, is defined as:

$$J_2(\Lambda, \overline{W}) = \|QR_2 + H\sum_{i=1}^{p} \frac{v_i l_i^T}{-\lambda_i}(R_1 - V^T W^T R_2)\|_F \tag{4.28}$$

where $\|\cdot\|_F$ denotes the Frobenius norm, $\Lambda = [\lambda_1, \cdots, \lambda_n]$ and $\overline{W} = [w_{p+1}, \cdots, w_n]$

are designing parameters. To maximize the fault effect and, subsequently fault detectability, the performance index $J_2(\Lambda, \overline{W})$ should be maximized. The optimization problem may be solved by any suitable numerical search method. The genetic algorithm is a generic optimization technique because it has minimum degree of problem dependence, and hence can be used to solve this problem. In Chapter 6, the use of genetic algorithms is discussed in detail.

The maximization of $J_2(\Lambda, \overline{W})$ is a constrained optimization problem because all elements of Λ must be in the left hand side of the complex plan. To remove this constraint, the eigenvalues λ_i are assumed in a pre-defined wide region $[L_i, U_i]$ and introduce a simple transformation (Burrows and Patton, 1991):

$$\lambda_i = L_i + (U_i - L_i)sin^2(z_i) \tag{4.29}$$

where $z_i \in \mathcal{R}$ $(i = 1, \cdots, n)$ can be freely chosen. Now, the performance index J_2 is a function of the parameters $Z = [z_1, \; \cdots, \; z_n]$ and $\overline{W}$.

If the required left eigenvectors are assignable, the performance index J_1 is zero and only the index J_2 needs to be maximized. If the assignability conditions cannot be satisfied, one alternative is to assign all the columns of E as right eigenvectors and this is studied in Section 4.5. Another alternative is to use approximate de-coupling, i.e. to minimize J_1. The best FDI performance can be achieved by maximizing J_2 and minimizing J_1, this is a multi-objective optimization problem and can be solved by minimizing a single mixed objective. The objectives can be mixed-up in one of the following ways:

$$J(Z, W) = \frac{J_1(Z, W)}{J_2(Z, W)} \tag{4.30}$$

$$J(Z, W) = \alpha_1 \sum_{i=1}^{p} \|l_i - l_i^*\|_2 + \frac{\alpha_2}{\|QR_2 + H\sum_{i=1}^{p} \frac{v_i l_i^T}{-\lambda_i}(R_1 - V^T W^T R_2)\|_F} \tag{4.31}$$

The multi-objective optimization problem can also be solved via the method of inequalities which is discussed in Chapter 6.

4.5 Disturbance De-coupling by Assigning Right Eigenvectors

If the left eigenvector assignability conditions are not satisfied, an alternative approach can be used is to assign the columns of the matrix E as right eigenvectors of the observer dynamics. This approach is given by the following theorem:

Theorem 4-3: The sufficient conditions for satisfying the disturbance de-coupling requirement Eq.(4.5) are:

(1) $QCE = 0$.

(2) All columns of the matrix E are right eigenvectors of $(A - KC)$ corresponding to any real distinct eigenvalues.

Patton and Kangethe (1989) pointed out the possibility of assigning columns of the matrix E as right eigenvectors for disturbance de-coupling design, however they did not describe an algorithm for achieving this. This approach only became implementable when Chen and Patton (Patton and Chen, 1991g; Patton and Chen, 1992c) proposed a new algorithm for assigning observer right eigenvectors. The assignment of the right observer eigenvectors (left eigenvector of dual controller) is a relatively new problem, only considered by few investigators, e.g. Zhang et al. (1990). The assignment method proposed by Chen and Patton is thus presented and extended in this section.

Theorem 4-4: A vector v_i can be assigned as a right eigenvector of $(A - KC)$ corresponding to λ_i only if one of the following necessary conditions is satisfied:

(1) v_i is *not* the right eigenvector of A corresponding to λ_i and $Cv_i \neq 0$.

or

(2) v_i is the right eigenvector of A corresponding to λ_i and $Cv_i = 0$

Proof: For the right eigenvector v_i of $(A - KC)$, we have

$$(A - KC)v_i = \lambda_i v_i$$

This leads to:

$$KCv_i = (A - \lambda_i I)v_i$$

The assignment of v_i as the right eigenvector of $A - KC$ is to find the matrix K to satisfy this equation. This equation has solutions only if either condition in Theorem 4-4 holds true. ◇ **QED**

For the cases when a number of right eigenvectors must be assigned, the gain matrix K must satisfy a set of equations. If one wants to assign all columns e_i ($i = 1, 2, \cdots, q$) of E as the right eigenvectors of $(A - KC)$ corresponding to eigenvalues λ_i, the following equation must be satisfied.

$$KCe_i = (A - \lambda_i I)e_i \qquad \text{for} \quad i = 1, 2, \cdots, q \tag{4.32}$$

Therefore

$$KCE = A_\lambda \tag{4.33}$$

where

$$A_\lambda = [(A - \lambda_1 I)e_1 \ (A - \lambda_2 I)e_2 \ \cdots \ (A - \lambda_q I)e_q] \tag{4.34}$$

Now, the right eigenvector assignment problem is to solve the Eq.(4.33) whilst ensuring that the observer is stable.

Lemma 4-3: The necessary and sufficient condition for a solution of Eq.(4.33) to exist is:

$$rank(CE) = rank(\begin{bmatrix} A_\lambda \\ CE \end{bmatrix}) \tag{4.35}$$

Subject to this condition, the general form of the solution to Eq.(4.33) is:

$$K = A_\lambda(CE)^+ + K_1[I_m - CE(CE)^+] \tag{4.36}$$

where $K_1 \in \mathcal{R}^{n \times m}$ is an arbitrary design matrix and $(CE)^+$ is the pseudo-inverse of CE. When $rank(CE) = q$, $(CE)^+$ is given by:

$$(CE)^+ = [(CE)^T(CE)]^{-1}(CE)^T \tag{4.37}$$

Proof: Eq.(4.33) has solutions *iff* any row of the matrix A_λ is a linear combination of rows of the matrix (CE). Hence Eq.(4.35) is the necessary and sufficient condition for a solution of Eq.(4.33) to exist. It can be easily verified that the matrix K given by Eq.(4.36) is a solution of Eq.(4.33).

◇ **QED**

Remarks: A matrix equation can be decomposed into a number of linear equations, and hence Eq.(4.33) can be decomposed into nq equations with nm parameters to be determined. When $m > q$, the solutions for these equations normally exist. Some detailed discussion about the solution of matrix equations can be found in Basilevsky (1983, Chapter 6).

When the all q ($\leq m$) eigenvalues λ_i ($i = 1, 2, \cdots, q$) are set as the same, i.e.,

$$\lambda_1 = \lambda_2 = \cdots = \lambda_q = \lambda$$

the necessary and sufficient condition for solving Eq.(4.33) is simpler and can be given by the following Lemma.

Lemma 4-4: If q eigenvalues to be assigned to the corresponding q columns of E are same and this eigenvalue is not an eigenvalue of A, the necessary and sufficient condition for solution of Eq.(4.33) to exist is:

$$rank(CE) = rank(E) \tag{4.38}$$

Note that the condition given in this Lemma is the same as that given in Lemma 3-1 and the method of proof used in Lemma 3-1 can be used to prove this Lemma. However, this proof is not presented here. The similarity between Lemma 4-4 and Lemma 3-1 demonstrates the correspondence between unknown input observers with eigenstructure assignment in robust residual generation.

> **Theorem 4-5:** The necessary and sufficient conditions to assign all columns of E as right eigenvectors of $(A - KC)$ with a stabilizing feedback gain K are:
>
> (i) $rank(CE) = rank(\begin{bmatrix} A_\lambda \\ CE \end{bmatrix})$.
>
> (ii) $(C_1,\ A_1)$ is a detectable pair, where:
>
> $$\begin{aligned} A_1 &= A - A_\lambda (CE)^+ C \\ C_1 &= [I_m - CE(CE)^+]C \end{aligned}$$

Proof: All columns of E are *right* eigenvectors of $(A - KC)$ *iff* Eq.(4.33) holds true. Eq.(4.33) has solutions *iff* the condition (i) is true. For a general solution given by Eq.(4.36), the system dynamics will be:

$$A - KC = A - A_\lambda (CE)^+ C - K_1 [I_m - CE(CE)^+]C = A_1 - K_1 C_1$$

Hence, the observer can be stabilized *iff* the condition (ii) holds.

◇ **QED**

Now, the right eigenvector assignment problem is to find a matrix K_1 which assigns eigenvalues of the observer dynamic matrix $(A - KC) = (A_1 - K_1 C_1)$ in the left hand side of complex plane. This is only possible when $(C_1,\ A_1)$ is a *detectable pair.* The problems of assessing the detectability and assigning eigenvalues of a detectable pair have been studied in Section 3.2. As q eigenvalues λ_i $(i = 1, 2, \cdots, q)$ have been assigned as the eigenvalues of $(A - KC) = (A_1 - K_1 C_1)$ in the assignment of right eigenvectors, the maximum number of eigenvalues of $(A - KC) = (A_1 - K_1 C_1)$ that can be moved by changing the design matrix K_1 is $n - q$. This is proved via the following Lemma.

> **Lemma 4-5:** The eigenvalues λ_i $(i = 1, 2, \cdots, q)$, which used in the assignment of right eigenvectors e_i $(i = 1, 2, \cdots, q)$ for $(A - KC)$, are unobservable modes of the pair $(C_1,\ A_1)$.

Proof: As the vector e_i $(i = 1, 2, \cdots, q)$ are right eigenvectors of $(A - KC) =$

$(A_1 - K_1 C_1)$ corresponding to eigenvalues λ_i $(i = 1, 2, \cdots, q)$, we have:

$$\{A - A_\lambda (CE)^+ C - K_1[I_m - CE(CE)^+]C\} e_i = \lambda_i e_i \quad \text{for} \quad i = 1, 2, \cdots, q$$

This equation holds true for any arbitrary matrix K_1, if we set $K_1 = 0$, we have:

$$\{\lambda_i I - [A - A_\lambda (CE)^+ C]\} e_i = \{\lambda_i I - A_1\} e_i = 0 \quad \text{for} \quad i = 1, 2, \cdots, q$$

Therefore,

$$K_1[I_m - CE(CE)^+]C e_i = K_1 C_1 e_i = 0 \quad \text{for} \quad i = 1, 2, \cdots, q$$

As this relation is valid for any matrix K_1, thus,

$$C_1 e_i = 0 \quad \text{for} \quad i = 1, 2, \cdots, q$$

Hence,

$$\begin{bmatrix} \lambda_i I - A_1 \\ C_1 \end{bmatrix} e_i = 0 \quad \text{for} \quad i = 1, 2, \cdots, q$$

i.e., the eigenvalues λ_i $(i = 1, 2, \cdots, q)$, are unobservable modes of the pair $(C_1,\ A_1)$, and the maximum number of eigenvalues of $(A_1 - K_1 C_1)$ that can be moved by K_1 is $n - q$.

$\diamond$ **QED**

The eigenvalues for right eigenvector assignment in disturbance de-coupling design are *not* unique. Moreover, the solution for the matrix K_1 is also not unique, even if the eigenvalues have been fixed, due to the multivariable nature. The design freedom beyond right eigenvector assignment can be utilized to maximize the fault effect on residuals, as discussed in Section 4.5. The matrix K_1 can also be parameterized via eigenstructure in the design. The problem of maximizing fault effects utilizing the remaining design freedom is studied in future research and is not discussed here.

4.6 Dead-Beat Design for Robust Residual Generation

The observer-based residual generation techniques developed for continuous-time system models can also be used for the systems described by discrete-time models. However, some special characteristics such as dead-beat design are only valid for discrete-time domain. The dead-beat design can make the derivation of the disturbance de-coupling principle very simple and gives very prompt residual responses. Consider systems described by discrete-time models:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + R_1 f(k) + Ed(k) \\ y(k) = Cx(k) + Du(k) + R_2 f(k) \end{cases} \tag{4.39}$$

For this system, a discrete observer is used to generate residuals:

$$\begin{cases} \hat{x}(k+1) = (A - KC)\hat{x}(k) + (B - KD)u(k) + Ky(k) \\ \hat{y}(k) = C\hat{x}(k) + Du(k) \\ r(k) = Q[y(k) - \hat{y}(k)] \end{cases} \tag{4.40}$$

The Z-transformed residual response to faults and disturbances is thus:

$$\begin{aligned} r(z) &= QR_2 f(z) + H(zI - A + KC)^{-1}(R_1 - KR_2)f(z) \\ &\quad + H(zI - A + KC)^{-1}Ed(z) \end{aligned} \tag{4.41}$$

The transfer matrix between the residual and the disturbance can be expanded as:

$$H(zI - A_c)^{-1}E = z^{-1}H(I + A_c z^{-1} + A_c^2 z^{-2} \cdots + \cdots)E \tag{4.42}$$

where $A_c = A - KC$ and $H = QC$. It can be seen that this transfer matrix is nulled if the following sufficient conditions are satisfied:

$$HE = 0 \tag{4.43}$$

$$HA_c = 0 \tag{4.44}$$

Choose H and K in such a way that the rows of H are the left eigenvectors of A_c corresponding to zero-valued eigenvalues, Eq.(4.44) then holds true. The Eq.(4.43) means that the left eigenvectors to be assigned are orthogonal to the disturbance directions, and the residual weighting matrix Q will be computed using this equation.

Alternatively, the disturbance de-coupling can also be achieved using the following sufficient conditions:

$$HE = 0 \tag{4.45}$$

$$A_c E = 0 \tag{4.46}$$

Eq.(4.46) holds true when each column of E is assigned as a right eigenvector of A_c corresponding to a zero-valued eigenvalue. Eq.(4.45) will determine the residual weighting matrix Q.

Because of the assignment of zero-valued eigenvalues, the residual will have dead-beat (minimum-time) transient performance and this feature can be exploited to good use in the aim to provide a high sensitivity to soft (incipient) faults.

When the left eigenvector assignment condition in Eq.(4.44) for disturbance de-coupling holds true, the residual response to faults will be:

$$\begin{aligned} r(z) &= QR_2 f(z) + H(zI - A + KC)^{-1}(R_1 - KR_2)f(z) \\ &= QR_2 f(z) + z^{-1}H(R_1 - KR_2)f(z) \end{aligned} \tag{4.47}$$

i.e.

$$r(k) = QR_2 f(k) + H(R_1 - KR_2)f(k-1) \tag{4.48}$$

Hence, the fault signal is transmitted directly into the residual, i.e. the residual response to faults is very fast and this can avoid the detection delay. When a fault occurs in the i_{th} element of fault vector $f(k)$ and other elements of $f(k)$ are zeros, the residual will be:

$$r(k) = [QR_2]_i f_i(k) + [H(R_1 - KR_2)]_i f_i(k-1) \tag{4.49}$$

where $[QR_2]_i$ is the i_{th} column of QR_2 and $[H(R_1 - KR_2)]_i$ is the i_{th} column of $H(R_1 - KR_2)$. This equation shows that the residual vector lies in a fixed subspace, i.e.,

$$r(k) \in \mathcal{S}_i = span\{[QR_2]_i,\ [H(R_1 - KR_2)]_i\} \tag{4.50}$$

This relation shows the robust residual has a directional property which can be used for fault isolation. The fault can be isolated by comparing the residual direction with the fault signature subspace $\mathcal{S}_i$ $(i = 1, \cdots, g)$ as reported by Chen and Patton (Patton and Chen, 1991h; Patton and Chen, 1991c). If the fault function is constant, the residual will be parallel to the vector $[QR_2 + H(R_1 - KR_2)]_i$ and the fault isolation will be easier to achieve. Note that the problem of fault isolation using robust directional residual vectors has been studied in Section 3.4.2.

From the residual generation relations given in Eq.(4.40), the computational form of the residual is:

$$r(z) = [Q - H(zI - A_c)^{-1}K]y(z) - [QD + H(zI - A_c)^{-1}(B - KD)]u(z) \quad (4.51)$$

If the left eigenvector assignment condition in Eq.(4.44) (not the right eigenvector assignment condition) holds true, $H(zI - A_c)^{-1} = z^{-1}H$. Thus the computational form of the residual vector $r(z)$ can be re-written as

$$r(z) = (Q - z^{-1}HK)y(z) - [QD + z^{-1}H(B - KD)]u(z) \quad (4.52)$$

i.e.

$$r(k) = [Q \quad -HK] \begin{bmatrix} y(k) \\ y(k-1) \end{bmatrix} - [QD \quad H(B - KD)] \begin{bmatrix} u(k) \\ u(k-1) \end{bmatrix} \quad (4.53)$$

It can be seen that Eq.(4.53) is a 1_{st} order parity equation (parity relation) (Chow and Willsky, 1984; Lou et al., 1986; Patton and Chen, 1991e) which can be implemented directly to generate residuals for FDI. This residual generation method using the 1_{st} order parity relation is illustrated in Fig.4.3.

It is very interesting that disturbance de-coupling is achieved by the assignment of left observer eigenvectors, however the robust residual generator can be implemented in the form of the parity relation given by Eq.(4.53). That is to say that the observer is not required in robust residual generation, and this has significance for real-time application aspects. The direct link between eigenvector assignment and parity relations was discovered by Patton and Chen (Patton and Chen, 1991h; Patton and Chen, 1991c; Patton and Chen, 1991f; Patton and Chen, 1991e; Patton and Chen, 1992c; Patton, Chen, Millar and Kiupel, 1991).

Note that the link between eigenvector assignment and the parity relation approach cannot be derived for the right eigenvector assignment case (Eq. (4.46)).

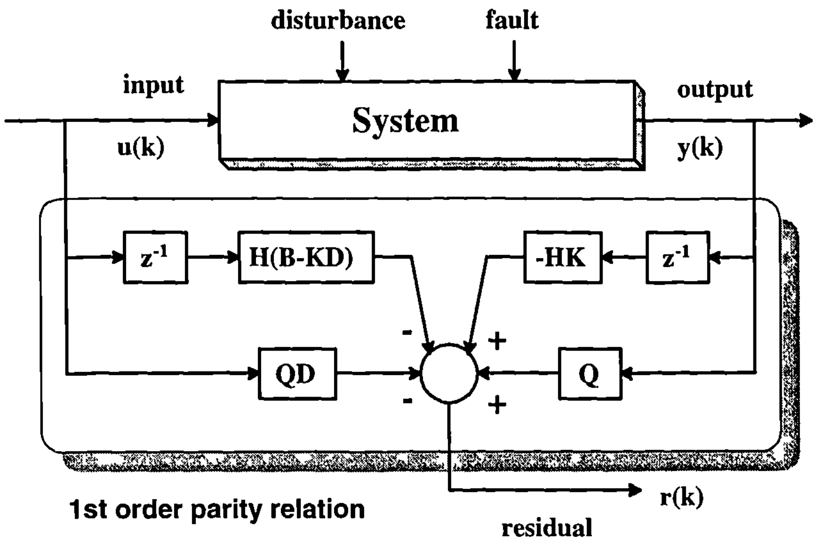


Figure 4.3: **Robust residual generation via the 1st-order parity relation**

4.7 Two Numerical Examples in Eigenstructure Assignment

Example 4-1: Consider the discrete-time system given by,

$$A = \begin{bmatrix} 0.25 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.375 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

The disturbance distribution and the measurement matrices are:

$$E = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

The weighting matrix Q to satisfy $QCE = 0$ can be easily found as:

$$Q = [-1 \quad 2]$$

so that, the desired left eigenvector is:

$$H = QC = [-1 \ \ 1 \ \ 2]$$

corresponding to the eigenvalue 0. This left eigenvector is assignable as H^T belongs to the subspace $span\{-A^T C^T\}$. The remaining two eigenvalues are chosen as $\{0, 0.1\}$. Using the eigenstructure assignment technique (Mudge and Patton, 1988; Burrows et al., 1989; Burrows and Patton, 1991), the gain matrix is derived as:

$$K = \begin{bmatrix} 0.0165 & -0.3330 \\ 0.4670 & 0.6661 \\ -0.3502 & -0.1246 \end{bmatrix}$$

It can be seen that $H(A - KC) = 0$ and $QCE = 0$, i.e., the de-coupling conditions (4.43) & (4.44) are satisfied and:

$$H(zI - A_c)^{-1}E = 0$$

The z-transform of the residual in response to the sensor fault $f_s(t)$ and actuator fault $f_a(t)$ will be:

$$\begin{aligned} r(z) &= [Q - QC(zI - A_c)^{-1}K]f_s(z) + QC(zI - A_c)^{-1}Qf_a(z) \\ &= [-1 \;\; 2]f_s(z) - [-0.249 \;\; 0.749]z^{-1}f_s(z) - z^{-1}f_a(z) \end{aligned}$$

Clearly, the disturbance term is not present and the residual is only a function of the faults. This means that a robust design has been achieved. According to Eq.(4.53), the computational form of the residual can be:

$$r(z) = [-1 \;\; 2]y(z) - [-0.249 \;\; 0.749]z^{-1}y(z) - z^{-1}u(z)$$

i.e.

$$r(k) = [-1 \;\; 2 \;\; 0.249 \;\; -0.749]\begin{bmatrix} y(k) \\ y(k-1) \end{bmatrix} - u(k)$$

This is a 1_{st} order parity relation.

Example 4-2: Now consider changing the matrix A to

$$A = \begin{bmatrix} 0.3 & 0 & 0 \\ 0 & 0.6 & 0 \\ 0 & 0 & 0.9 \end{bmatrix}$$

In this case, the required left eigenvector of the observer H is not assignable (as H^T does not belong to subspace $span\{-A^T C^T\}$). We must use the alternative approach of assigning right eigenvectors, as given in Section 4.5. The eigenvalues are chosen as $\{0, 0, 0.1\}$. The observer right eigenvector can then be assigned as a single column of E (corresponding to eigenvalue 0), in this case, the resulting gain matrix computed using right eigenvector assignment is:

$$K = \begin{bmatrix} 0.098304 & 0.103392 \\ 0.589304 & -0.596608 \\ -0.8 & 1.6 \end{bmatrix}$$

The z-transform of the corresponding residual response to actuator and sensor faults is:

$$r(z) = \frac{(-1 + 1.2z^{-1} - 0.27z^{-2})(2 - 2.7z^{-1} + 0.81z^{-2})}{1 - 0.1z^{-1}} f_s(z) + \frac{3 - 1.8z^{-1}}{1 - 0.1z^{-1}} f_a(z)$$

The disturbance de-coupling has also been achieved. However, although this residual signal is robust to disturbances, it is a recursive structure and does not directly correspond to a parity relation.

4.8 Conclusion and Discussion

This chapter has studied the robust (in the sense of disturbance de-coupling) residual generation via observer eigenstructure assignment. The disturbance de-coupling is achieved by the assignment of either left or right observer eigenvectors. Given a design problem, the designer can check the assignability to decide the assignment of left or right eigenvectors. If the number of independent disturbances to be de-coupled is smaller than the number of independent measurements, a disturbance de-coupling solution is very likely achievable via either left or right eigenvector assignment. If the required eigenstructure (left or right) is not perfectly assignable, an approximate approach should be taken. That is to choose assignable eigenvectors close, in a least-squares sense, to the desired eigenvectors. This can be achieved via the left eigenvector assignment. In this situation, the residual is not de-coupled from disturbances but has a low sensitivity to disturbances due to approximate de-coupling.

The chapter studies mainly the robust residual generation problem. For fault isolation, one way is to design structured residual sets and this can be done using an approach similar to that presented in Section 3.3. For the dead-beat design, when the rows of the matrix $H = QC$ are assigned as left eigenvectors of the observer corresponding to zero-valued eigenvalues, the residual can be generated by a 1_{st} order parity relation, and the resulting residual has directional property which can be used for fault isolation.

The eigenstructure assignment approach for designing disturbance de-coupled residual generators has been studied by Patton et al. for many years since 1986. However, the author's main contributions to this approach are:

- To present a mathematically sound proof for eigenstructure assignment approach in disturbance de-coupling design.
- To propose a new algorithm on the assignment of right observer eigenvectors.
- To point out and prove the direct link between eigenstructure assignment with parity relations in residual generation.
- To discuss the possibility of improving fault sensitivity utilizing the remaining design freedom, after the disturbance de-coupling conditions have been satisfied.

Chapter 5

DETERMINATION OF DISTURBANCE DISTRIBUTION MATRICES FOR ROBUST RESIDUAL GENERATION

5.1 Introduction

It is difficult to develop a highly accurate model of a complex system and hence the interesting question is just what is a reasonable model to enable good performance in FDI. It would be attractive to develop a robust FDI technique which is insensitive to modelling uncertainty, without the use of a very accurate model. However, in order to design a robust FDI scheme, one should have a description (i.e. some information or knowledge) about the system uncertainty, e.g. its distribution matrix or spectral bandwith, etc. Furthermore, this description should provide assistance for robust FDI design, i.e. it can be handled in a systematic manner.

As pointed out in Chapters 3 & 4, a typical description for the system uncertainty makes use of the concept of "unknown inputs" acting upon a nominal linear model of the system as described by:

$$\begin{cases} \dot{x}(t) &= Ax(t) + Bu(t) + R_1 f(t) + Ed(t) \\ y(t) &= Cx(t) + Du(t) + R_2 f(t) \end{cases} \tag{5.1}$$

where the disturbance term $Ed(t)$ is used to represent uncertainties acting upon the system, in which the vector $d(t) \in \mathcal{R}^q$ is an unknown "input" or "disturbance" vector. The distribution matrix $E \in \mathcal{R}^{n \times q}$ is assumed known. In robust model-based FDI, this description of the system uncertainty is defined as *structured* uncertainty.

It is clear from Eq.(5.1) that $Ed(t)$ and $R_1 f(t)$ act on the system in the same way, and thus one cannot discriminate between their effects unless the structure of E is known. It is therefore a common practice to assume that E is known, in so called *robust* FDI approaches which are based on the disturbance de-coupling principle (see Section 2.11). Once E is known, the residual can be made to have the disturbance de-coupling (robust) property, i.e. the residual is totally de-coupled from the disturbance (uncertainty). The robust residual can then be used to achieve reliable FDI. The de-coupling design can be achieved using the unknown input observer (see Chapter 3), or alternatively using eigenstructure assignment (see Chapter 4), or frequency domain approaches (Ding and Frank, 1991; Frank, 1991a; Frank and Ding, 1993; Frank and Ding, 1994; Qiu and Gertler, 1993), or orthogonal parity equation approaches (Gertler, Fang and Luo, 1990; Gertler, 1991; Gertler and Kunwer, 1993).

The theories underlying the robust residual generation based on the disturbance de-coupling principle have been well developed, but for real applications the following problems remain unsolved:

- How well can the term $Ed(t)$ characterize the real uncertainty, if there is no knowledge of the uncertainty?
- How can the term $Ed(t)$ and the structure of E be determined, even approximately?

This chapter answers the above questions and provides some simulation examples to test some developed theoretical results. These question must be answered, otherwise the application domain of the disturbance de-coupling approach for robust FDI is very limited. In fact, very few researchers have presented the application results of robust FDI.

As mentioned above, a primary requirement for disturbance de-coupled robust FDI methods is that the disturbance distribution matrix must be known. However, in most practical systems the uncertainty can be expressed in many different ways (e.g. modelling errors) and the distribution matrix E is not known. To apply the disturbance de-coupling robust residual generation techniques to systems with wide ranging uncertainties such as modelling errors and parameter variations, an approximate distribution matrix E is needed to represent the effects of uncertainty. Within the framework of international research on this subject, there have been few attempts to address the problem of determining this distribution matrix. Until recently, this lack of information obstructed the application of disturbance de-coupling for robust FDI in real engineering systems. The work of determining the disturbance distribution matrix has been led by Patton & Chen (Patton and Chen, 1991f; Patton and Chen, 1991b; Patton, Chen and Zhang, 1992; Patton, Zhang and Chen, 1992). They have demonstrated their techniques for a jet engine system (Patton and Chen, 1991f; Patton and Chen, 1991b; Patton and Chen, 1991a; Patton and Chen, 1992c; Patton, Chen and Zhang, 1992; Patton, Zhang and Chen, 1992) and in a nuclear reactor core (Patton, Chen and Millar, 1991; Patton, Chen and Millar, 1992). The technique proposed by Patton and Chen (1991f) was later used by Shields *et al* (Shields, 1994; Yu and Shields, 1994; Yu, Shields and Mahtani, 1994a; Yu et al., 1994b). The problem has been attracting world-wide attention and other investigators have followed this line of research, e.g. Gertler and Kunwer (1993), Gertler (1994), Keviczky, Bokor, Szigeti and Edelmayer (1993) and Saif and Guan (1993). Note that the determination of the optimal disturbance distribution matrix E is a common problem for all disturbance de-coupling robust residual generation approaches including the orthogonal parity equation approach (Gertler, Fang and Luo, 1990; Gertler, 1991; Gertler and Kunwer, 1993) and frequency domain approach (Ding and Frank, 1991; Frank, 1991a; Frank and Ding, 1993; Frank and Ding, 1994; Qiu and Gertler, 1993).

This chapter presents the research developments surrounding the determination of the disturbance distribution matrix for robust residual generation. A number of approaches for obtaining this matrix (albeit approximate) for real uncertain systems are proposed. An example of a 17_{th} thermodynamic order simulation model of a jet engine system has been used to illustrate some approaches developed.

Clearly, the basis for the model-based FDI technique is the use of mathematical models. The model used should have certain accuracy. In order to make a diagnosis algorithm robust against modelling uncertainty, some knowledge about the modelling uncertainty should be available. Otherwise, what do we need a model for if an algorithm can be made robust enough without *a priori* modelling information? This highlights the need to make some modelling assumptions. To be useful in a robust design, these assumptions should be easily handled in a systematic manner. The disturbance representation of uncertainty can be handled by means of the unknown input observer or the eigenstructure assignment. However, this assumption is not realistic, i.e., the distribution matrix cannot be obtained directly. In practice, we can make some more realistic assumptions about uncertainty, for example, parameters of the system are within a certain bound, etc. However, these assumptions are not normally easy to handle in designing robust FDI algorithms. The aim of this chapter is to present some techniques to bridge the gap between theoretical assumption and practical reality. This aim is fulfilled by approximate modelling of uncertainty, in which a disturbance description with an approximate distribution matrix is used to model uncertainty approximately. A number of situations covering a wide range of possibilities for uncertainty are considered in the following sections.

5.2 Direct Determination & Optimization of Disturbance Distribution Matrix

In most situations, the distribution matrix is not readily available. However, there are cases for which some *a priori* knowledge about uncertainty is available and can be used for a direct derivation of the distribution matrix E. This Section discusses a number of situations in which some realistic assumptions about uncertainty can be used for this direct derivation. Normally, this directly obtained matrix has a high rank (i.e. too many disturbances or unknown inputs) and disturbance de-coupling is not achievable. Hence, a low rank matrix which approximates the distribution is used in the design of optimally robust residual generators. This is an unknown input consolidation procedure, i.e., the unknown inputs with similar directions are combined and hence the number of unknown inputs is reduced. Note that, in some situations, the matrix E can be determined by simple inspection. If the uncertain factors appear in the i_{th} row of matrices A and B, it is most likely the matrix E should contain a column as follows:

$$\begin{bmatrix} 0 & \cdots & 0 & \underset{i_{th}}{1} & 0 & \cdots & 0 \end{bmatrix}^T$$

This *direct inspection method* for determining the matrix E was used in the example presented in Section 3.5.3 and also showed by Saif and Guan (1993) and Hou and Müller (1994b). This method may not be very effective, however it is simple and can be useful for some systems.

5.2.1 Noise and additive non-linearity

Consider the following dynamic equation of the monitored system:

$$\dot{x}(t) = Ax(t) + Bu(t) + G\mu(t) + Qf(x(t), u(t), t)) \tag{5.2}$$

where $\mu(t)$ is a noise or external disturbance vector. In this equation, the non-linearity is considered as an additive non-linear term $Qf(x(t), u(t), t))$, i.e., the system dynamics can be separated into linear and non-linear parts. This kind of non-linear dynamic structure exists in some non-linear chemical processes (Watanabe and Himmelblau, 1982; Chen and Zhang, 1991) and has been used in Section 3.3.3. For the system described above, the uncertainty can be modelled as an *additive* term $Ed(t)$ and where:

$$Ed(t) = [G \quad Q] \begin{bmatrix} \mu(t) \\ f(x(t), u(t), t)) \end{bmatrix} \tag{5.3}$$

5.2.2 Bilinear systems

The study of bilinear systems has theoretical importance because they are a special class of nonlinear systems. Many practical nonlinear systems such as ecological systems, nuclear systems, hydraulic systems and heat exchanger systems can be modelled by a bilinear system model(Yu et al., 1994a):

$$\dot{x}(t) = A_0 x(t) + Bu(t) + \sum_{i=1}^{r} A_i u_i(t) x(t) \tag{5.4}$$

where $u_i(t)$ $(i = 1, \cdots, r)$ is the i_{th} component of $u(t)$, and A_i $(i = 0, 1, \cdots, r)$ and B are known matrices. The nonlinear term can be treated as the disturbance term with the distribution matrix and the unknown input vector as follows:

$$E = [A_1 \quad A_2 \quad \cdots \quad A_r] \qquad d(t) = \begin{bmatrix} u_1(t)x(t) \\ \vdots \\ u_r(t)x(t) \end{bmatrix} \tag{5.5}$$

A linear disturbance de-coupled residual generator can be designed to generate robust residuals for FDI. This avoids the complexity involved in the design of bilinear observers (Shields, 1994; Yu and Shields, 1994; Yu et al., 1994a; Yu et al., 1994b).

5.2.3 Model reduction

Most systems can have significantly higher order dynamics than their models. Consider, for example, the system described by a higher order model as:

$$\begin{bmatrix} \dot{x} \\ \dot{x}_h \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} x(t) + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u(t) \tag{5.6}$$

where $x(t) \in \mathcal{R}^n$ is a partial state vector corresponding to dominant dynamic part of the system. $x_h(t)$ represents the higher order dynamics in the system, and frequently neglected in practice. For ease of design and implementation in control and fault diagnosis, the following reduced-order model is used to approximate this system:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + (A_{11} - A)x(t) + (B_1 - B)u(t) + A_{12}x_h(t) \\ &= Ax(t) + Bu(t) + Ed(t) \end{aligned} \tag{5.7}$$

where:

$$Ed(t) = [(A_{11} - A) \quad (B_1 - B) \quad A_{12}] \begin{bmatrix} x(t) \\ u(t) \\ x_h(t) \end{bmatrix} \tag{5.8}$$

A typical application of this partitioned state-space structure arises when comparing a *reduced order* model with the full-scale system, for example, in an observer used for FDI. For this case, the nominal model represented by (A, B) is the reduced order model and the remaining modelling errors are considered to be lumped together within an additive term $Ed(t)$. It is assumed that the n reduced order state variables correspond to N state variables of the full-scale system.

5.2.4 Parameter perturbations

A system model with time-varying parameter perturbation can be described as:

$$\dot{x}(t) = (A + \Delta A(t))x(t) + (B + \Delta B(t))u(t) \tag{5.9}$$

The parameter perturbations considered in the robust control field are sometimes approximated as:

$$\Delta A(t) \approx \sum_{i=1}^{N} a_i(t) A_i \qquad \Delta B(t) \approx \sum_{i=1}^{N} b_i(t) B_i$$

where A_i and B_i are known constant matrices with proper dimensions, $a_i(t)$ and $b_i(t)$ are unknown scalar time-varying factors. In this case, the modelling error can be approximated by the disturbance term as:

$$E_1 d_1(t) = \Delta A(t)x(t) + \Delta B(t)u(t) = [A_1 \cdots A_N \; B_1 \cdots B_N] \begin{bmatrix} a_1(t)x(t) \\ \cdots \\ a_N(t)x(t) \\ b_1(t)u(t) \\ \cdots \\ b_N(t)u(t) \end{bmatrix}$$

Now, consider the situation where the system matrices are functions of the parameter vector $\alpha \in \mathcal{R}^g$:

$$\dot{x}(t) = A(\alpha)x(t) + B(\alpha)u(t) \tag{5.10}$$

If the parameter vector is perturbed around the nominal value $\alpha = \alpha_0$, this equation can be expanded as:

$$\dot{x}(t) = A(\alpha_0)x(t) + B(\alpha_0)u(t) + \sum_{i=1}^{g} \{ \frac{\partial A}{\partial \alpha_i} \delta\alpha_i x + \frac{\partial B}{\partial \alpha_i} \delta\alpha_i u \} \tag{5.11}$$

In this case, the distribution matrix and unknown input vector are:

$$E = [\frac{\partial A}{\partial \alpha_1} \mid \frac{\partial B}{\partial \alpha_1} \mid \cdots \mid \frac{\partial A}{\partial \alpha_g} \mid \frac{\partial B}{\partial \alpha_g}] \tag{5.12}$$

$$d(t) = [\delta\alpha_1 x^T \mid \delta\alpha_1 u^T \mid \cdots \mid \delta\alpha_g x^T \mid \delta\alpha_g u^T]^T \tag{5.13}$$

5.2.5 Low rank approximation of distribution matrix

Section 4.3 has shown that one of the necessary conditions to design robust residuals (in the disturbance de-coupling sense) using eigenstructure assignment, is to find a matrix $H \in \mathcal{R}^{p \times n}$ which satisfies the following equation:

$$HE = 0 \tag{5.14}$$

where p is the residual dimension and can be chosen by the designer. To satisfy this equation, the rank of the matrix E must be less than its row number (i.e. the system order n). Chapters 3 & 4 have also shown that the maximum number of independent disturbances ($= rank(E)$) cannot be larger than the maximum independent measurement number m. This discussion highlights the point that the most critical condition for achieving disturbance de-coupling in the residual generation is:

$$rank(E) \leq m \tag{5.15}$$

It has been shown that the distribution matrix can be derived directly from the available uncertainty information. If $rank(E) \leq m$, Eq.(5.14) has solutions and exact de-coupling is possible. However, for most situations, this matrix obtained does not satisfy the rank condition (5.15), and thus approximate de-coupling must be taken. The procedure will be to compute a matrix E^* that is as close as possible to E, and $rank(E^*) = q \leq m$, i.e. to find the solution of following optimization problem:

$$\min \|E - E^*\|_F^2 \quad \text{subject to}: \ \ rank(E^*) = q \leq m \tag{5.16}$$

Here $\|\cdot\|_F^2$ denotes the Frobenius norm, defined as the root of the sum of squares of the entries of the associated matrix. The matrix E^* is thus chosen so that the sum of the squared distances between the columns of E and E^* is minimized, subject to the constraint that: $rank(E^*) \leq m$.

The problem of approximating a matrix by a low rank matrix was first suggested by Eckart and Young (1936). More recently, Tufts, Kumaresan and Kirsteins (1982) and Lou et al. (1986) demonstrated its use. This optimization problem can be solved via the *Singular value Decomposition* (SVD) (Golub and Van Loan, 1989) of E:

$$E = S\Sigma T^T \tag{5.17}$$

where

$$\Sigma = \left[\begin{array}{c|c} diag\{\sigma_1, \cdots, \sigma_k\} & 0 \\ \hline 0 & 0 \end{array}\right] \tag{5.18}$$

and S and T are orthogonal matrices, k is the rank of the matrix E, and $\sigma_1 \leq \sigma_2 \leq$

$\cdots \leq \sigma_k$ are the singular values of E. According to the theorem given by Eckart and Young (1936) (shown in Appendix D, also see Tufts et al. (1982) and Lou et al. (1986)), a low rank approximation for the matrix E which minimizes $\|E - E^*\|_F^2$ is given by:

$$E^* = S\hat{\Sigma}T^T \tag{5.19}$$

where

$$\hat{\Sigma} = \left[\begin{array}{c|c} diag\{0, \cdots, 0, \sigma_{k-q}, \cdots, \sigma_k\} & 0 \\ \hline 0 & 0 \end{array}\right] \tag{5.20}$$

and q is the rank of the matrix E^* which is not larger than m to satisfy the disturbance de-coupling conditions. To achieve approximate disturbance de-coupling design, the matrix H should be made to satisfy the relation $HE^* = 0$. It is easy to see that an orthonormal solution for the matrix H is:

$$H^* = \begin{bmatrix} s_1^T \\ s_2^T \\ \vdots \\ s_{k-q-1}^T \end{bmatrix} \tag{5.21}$$

where $s_1, \cdots, s_{k-q-1}$ are the first $k-q-1$ columns of S. Once again, the residual dimension p can be freely chosen by the designer. If $p < k-q-1$, the residual weighting matrix H can be constructed by chosing any p rows from the optimal matrix H^*. If $p > k-q-1$, any extra rows of H should be linear combinations of the rows in H^*. This does not provide any independent information, hence p should not be larger than $k-q-1$. The greater the residual dimension, the more information one can obtain. Hence, an optimal solution is to set $p = k-q-1$.

An alternative statement of the optimization problem can be given as follows. Assume that:

$$E = [e_1 \; e_2 \; \cdots \; e_{n_1}] \tag{5.22}$$

where e_i is the i_{th} column of the matrix E. An ideal matrix H should make $He_i = 0$ for all $i = 1, 2, \cdots, n_1$. This is not always possible. Hence, it makes sense to choose a matrix H that is "as orthogonal as possible" to all e_i $(i = 1, 2, \cdots, n_1)$, i.e. to make each He_i $(i = 1, 2, \cdots, n_1)$ *as close to zero as possible.* As orthogonality is a directional property, it is not affected by the magnitude of H. There is no loss of generality in applying an orthonormal constraint to the matrix H, i.e. $HH^T = I$.

The optimization criterion can then be defined as:

$$J = \sum_{i=1}^{n_1} \|He_i\|_2^2 \tag{5.23}$$

The optimal solution for H follows by minimizing J, subject to $HH^T = I$. Lou et al. (1986) showed that the choice of H given in (5.21) also minimizes J, yielding the minimum value as

$$J^* = \sum_{i=1}^{q} \sigma_i^2 \tag{5.24}$$

This new statement of the optimization problem provides some very useful insight as J^* can be used as a *robustness measure* which is clearly relative to the rank number q of matrix E^* and the singular values of the matrix E.

It will typically be the case that some components of the unknown input vector d are larger than others. Furthermore, certain components of the unknown input vector have more effect on the residual. To take account of this, the diferent attention must be paid to the different components of the disturbance signal in the optimization procedure. For example, if the j_{th} component of the disturbance is significantly larger than the i_{th} component, the term He_j will be more important than the term He_i. Hence, the criterion J must be replaced by:

$$J_1 = \sum_{i=1}^{n_1} \alpha_i \|He_i\|_2^2 \tag{5.25}$$

where α_i $(i = 1, 2, \cdots, n_1)$ are positive weighting factors. The relative magnitudes of the α_i correspond to relative magnitudes of components of the disturbance weighting. By rewriting the weighted optimization criterion as:

$$J_1 = \sum_{i=1}^{n_1} \|H(\sqrt{\alpha_i} e_i\|_2^2 \tag{5.26}$$

this optimization problem can be solved using the procedure described above, but with e_i replaced by $\sqrt{\alpha_i} e_i$ and with E replaced by $E' = [\sqrt{\alpha_1} e_1 \ \sqrt{\alpha_2} e_2 \ \cdots \ \sqrt{\alpha_{n_1}} e_{n_1}]$.

5.2.6 Bounded uncertainty

Now, consider the case when the full-order system model is *not* available. An identification procedure is used to obtain the nominal model $\{A_0, B_0, C_0, D_0\}$ with the estimation error $\{\Delta A, \Delta B, \Delta C, \Delta D\}$. Normally, ΔA and ΔB are unknown but bounded:

$$A_1 \leq \Delta A \leq A_2 \tag{5.27}$$

$$B_1 \leq \Delta B \leq B_2 \tag{5.28}$$

where A_1, A_2, B_1 and B_2 are known and $\Delta A \leq A_2$ denotes that each element of ΔA is not larger than the corresponding element of A_2. This typifies the case where the uncertainty is bounded. Consider ΔA and ΔB in a finite set of possibilities, say $\{\Delta A_i, \Delta B_i\}$ $(i = 1, 2, \cdots, M)$ within the interval $A_1 \leq \Delta A \leq A_2$ and $B_1 \leq \Delta B \leq B_2$. This might involve choosing representative points, reflecting desired weighting on the likelihood or importance of particular sets of parameters. In this situation, a set of unknown input distribution matrices is obtained:

$$E_i = [\Delta A_i,\ \Delta B_i] \qquad i = 1, 2, \cdots, M \tag{5.29}$$

In order to make the disturbance de-coupling valid for a wide range of model parameter variations, an optimal matrix E^* should be as close as possible to all $E_i (i = 1, 2, \cdots, M)$. The optimization problem is thus defined as:

$$\min_{\{s.t.\ rank(E^*) \leq m\}} \|E^* - [E_1\ \ E_2\ \ \cdots\ \ E_M]\|_F^2 \tag{5.30}$$

E^* is then used to design disturbance de-coupling robust residual generators. As E^* is close to all E_i, approximate de-coupling is achieved over the whole range of parameter variations.

5.3 Estimation of Disturbance and Disturbance Distribution Matrix

In some cases, there is insufficient available knowledge about the state space model of the system and all we can get is a linearized low order model with matrices (A, B, C, D). In order to account for unavoidable modelling errors, it is assumed that the system is described as:

$$\begin{cases} \dot{x}(t) &= Ax(t) + Bu(t) + d_1(t) \\ y(t) &= Cx(t) + Du(t) \end{cases} \tag{5.31}$$

where $d_1(t)$ is used to represent modelling errors. If the vector $d_1(t)$ can be obtained, one may be able to decompose $d_1(t)$ into $Ed(t)$ with E a structured matrix. It seems reasonable to add $d_1(t)$ to account for all uncertainties in the model. But can we determine $d_1(t)$ with sufficient accuracy? How should we decompose $d_1(t)$ into $Ed(t)$ with E a structured matrix, to involve the disturbance de-coupling concept? The following sections provide answers to these questions.

5.3.1 Estimation of disturbance vector using an augmented observer

The states of an augmented observer can be used to estimate the direction of the disturbance direction E. The first step is to assume that $d_1(t)$ is a slowly time-varying vector, so that the system model can be re-written in augmented form as:

$$\begin{bmatrix} \dot{x}(t) \\ \dot{d}_1(t) \end{bmatrix} = \begin{bmatrix} A & I \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ d_1(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) \tag{5.32}$$

$$y(t) = [C \quad 0] \begin{bmatrix} x(t) \\ d_1(t) \end{bmatrix} + Du(t) \tag{5.33}$$

If we have the true system input and output data $\{u(t), y_t(t)\}$, an observer based on the model described by Eqs.(5.32) & (5.33) can be used to estimate the disturbance vector $d_1(t)$. Once $\hat{d}_1(t)$ has been obtained, it is possible to obtain *some information* about the distribution matrix E. The problem that could arise is that the augmented system may not be observable. The observability matrix for this system is:

$$W_0 = \begin{bmatrix} C & 0 \\ CA & C \\ CA^2 & CA \\ \vdots & \vdots \\ CA^n & CA^{n-1} \end{bmatrix} = \begin{bmatrix} C & 0 \\ 0 & C \\ 0 & CA \\ \vdots & \vdots \\ 0 & CA^{n-1} \end{bmatrix} \begin{bmatrix} I_n & 0 \\ A & I_n \end{bmatrix}$$

As the second matrix on the right hand side of the above equation is a full rank matrix, it is easy to see that:

$$rank(W_0) = rank(C) + rank(\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix})$$

The system shown in Eqs.(5.32)&(5.33) is observable *if and only if* $rank(W_0) = 2n$. From the above equation, it is clear that this system is observable *if and only if* $rank(C) = n$ and the matrix pair (C, A) is observable. The requirement $rank(C) = n$ limits the use of this technique for estimating the disturbance vector, as it requires that the system has n (state dimension) independent measurements. There is a logical explanation of this requirement. When we want to estimate the modelling uncertainty without any *a priori* knowledge about it, information is needed from additional measurements. For the FDI purpose, there are sometimes a large number of measurements available and the dynamics of the system can be approximated by a relatively low order model. Hence, the condition $rank(C) = n$ is not a strict constraint for some FDI problems.

5.3.2 Derivation of disturbance distribution matrix

Section 5.3.1 presented the method for determining $d_1(t)$, but the final goal is to express $d_1(t)$ as:

$$Ed(t) = d_1(t) \tag{5.34}$$

Generally speaking, there are many combinations of E and d, but for the robust FDI methods considered here, we only need to know the structure of E, and $d(t)$ can be chosen arbitrarily. There are two possibilities: one is that E is a vector and $d(t)$ is an arbitrary scalar function; another is that E is a matrix and $d(t)$ is an arbitrary vector function.

Using the augmented observer, one can get the estimation of the disturbance vector $d_1(t)$ as $\{\hat{d}_1(1), \hat{d}_1(2), \hat{d}_1(3), \cdots, \hat{d}_1(M)\}$. If the direction of the vector $\hat{d}_1(i)$ changes slightly for all $i = 1, 2, \cdots, M$, it can be believed that E is a vector and $d(t)$ is an arbitrary scalar function. In this case, the matrix E can be approximated as: $E = \frac{1}{M}\sum_{k=1}^{M} \hat{d}_1(k)$.

It is very likely the case that when $d_1(k)$ cannot be assumed to be a constant direction vector, i.e., the directions of $\hat{d}_1(i)$ are very much different for all $i = 1, 2, \cdots, M$. In this case, it is still possible to express the vector $d_1(k)$ as: $d_1(k) = Ed(k)$, where $E \in \mathcal{R}^{n\times q}$ is a constant matrix, $d(k) \in \mathcal{R}^q, d_1(k) \in \mathcal{R}^n$ and $q \leq n$. In the robust FDI method, E must be row rank deficient in order to have a left annihilating matrix H such that the equation $HE = 0$ holds true. This is one of the conditions for achieving robust FDI. To find the optimal distribution matrix, the following matrix can be constructed:

$$\Omega = [\hat{d}_1(1), \quad \hat{d}_1(2), \quad \cdots \quad , \hat{d}_1(M)] \tag{5.35}$$

The maximum rank of Ω is n, i.e. there are at most n linear independent columns. Hopefully, there are some vectors in Ω which are very close to other vectors (or nearly close to a combination of other vectors) and can be neglected. The q most linearly-independent columns of Ω can then be used to construct E, i.e.

$$E = [\hat{d}_1(i), \quad \hat{d}_1(j), \quad \cdots, \quad \hat{d}_1(k)] \in \mathcal{R}^{n\times q} \tag{5.36}$$

The procedure of the derivation of a low rank approximation to the matrix Ω is discussed now. One way to find the q most linearly-independent columns is to calculate the generalized angles between these vectors, i.e., $\angle(\hat{d}_1(i), \hat{d}_1(j))$ $(i, j = 1, \cdots, M; j \neq i)$. If a vector $\hat{d}_1(i)$ has very small generalized angle with other vectors, then $\hat{d}_1(i)$ can be discarded. The matrix E in (5.36) can be used to satisfy $HE = 0$. When E is constructed in this way it has advantage that all rows of the matrix H are orthogonal to almost every unknown input direction. If the remaining single direction is very near to other directions, then all rows of the matrix H are also *almost orthogonal* to it. It should be expected that almost all unknown inputs along these directions can be eliminated.

The way of obtaining the matrix E explained above involves the calculation of the generalized angles between vectors $\{\hat{d}_1(1), \hat{d}_1(2), \hat{d}_1(3), \cdots, \hat{d}_1(M)\}$ which is a complex and time-consuming procedure.

Another way to obtain a rank q matrix E is first to use an approximation matrix Ω_0 with the same dimension as Ω, such that:

$$\min_{rank(\Omega_0)=q} ||\Omega - \Omega_0||_F^2 \tag{5.37}$$

The solution to this optimization problem is readily obtained using the singular value decomposition of Ω. Suppose that:

$$\Omega = U[diag(\sigma_1, \cdots, \sigma_n), \quad 0]V^T \tag{5.38}$$

where $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_n$ are the singular values of Ω. Ω_0 is then constructed as:

$$\Omega = U[diag(\sigma_1, \cdots, \sigma_q, \ 0, \cdots, \ 0), \quad 0]V^T \tag{5.39}$$

where q is determined by the magnitude of σ_i $(i = q+1, \cdots, n)$ such that: $\sigma_n \leq \sigma_{n-1} \leq \cdots \leq \sigma_{q+1} \leq \epsilon$. ϵ is a small number determined by the designer. The error of the approximation can be calculated as: $\|\Omega - \Omega_0\|_F^2 = \sum_{i=q+1}^{n} \sigma_i^2$. For a good approximation we should have that: $\sum_{i=1}^{q} \sigma_i^2 \gg \sum_{i=q+1}^{n} \sigma_i^2$. The second step is to obtain the required distribution matrix E is to decompose the rank deficient matrix Ω_0 as:

$$\Omega_0 = \Omega_1 \Omega_2 \tag{5.40}$$

by the rank decomposition (see Appendix B). where $\Omega_1 \in \mathcal{R}^{n \times q}$ is a full column rank matrix, and $\Omega_2 \in \mathcal{R}^{q \times M}$. From the definition of Ω, one can obtain:

$$\begin{aligned} \Omega &= [\hat{d}_1(1), \quad \hat{d}_1(2), \quad \cdots \quad, \hat{d}_1(M)] \\ &= [Ed(1), \quad Ed(2), \quad \cdots \quad, Ed(M)] \\ &= E[d(1), \quad d(2), \quad \cdots \quad, d(M)] \end{aligned} \tag{5.41}$$

However,

$$\Omega \approx \Omega_0 = \Omega_1 \Omega_2 \tag{5.42}$$

Hence, an optimal approximation for the matrix E is Ω_1.

5.3.3 Estimation of disturbance vector using de-convolution

FDI algorithm design and the determination of the disturbance distribution matrix in the discrete-time domain can be carried in a similar way to that of the continuous-time domain. However, some special properties exist in discrete-time design. The discrete-time model described the following equation is considered here:

$$\begin{cases} x(k+1) &= Ax(k) + Bu(k) + d_1(k) \\ y(k) &= Cx(k) + Du(k) \end{cases} \tag{5.43}$$

where $d_1(k)$ is used to account for all modelling uncertainties. The matrices $\{A, B, C, D\}$ are known nominal model parameters. $\{u(k)\}$ is the model input which is identical to the system input. $\{y(k)\}$ is the model output which is normally not equal to the true system output $\{y_t(k)\}$ due to modelling uncertainty. The task here is to determine the additional term $d_1(k)$ using the nominal model parameters $\{A, B, C, D\}$ and real system inputs and outputs: $\{u(k), y_t(k)\}$. After an estimate of the vector $d_1(k)$ is obtained, it is possible to decompose it into $Ed(k)$ with E as a structured matrix for *disturbance de-coupling FDI design.*

From Eq.(5.43), it can be seen that:

$$\begin{aligned} y(k) &= Cx(k) + Du(k) \\ &= C[Ax(k-1) + Bu(k-1) + d_1(k-1)] + Du(k) \\ &= \cdots\cdots\cdots\cdots \\ &= CA^k x(0) + \sum_{i=1}^{k} CA^{i-1}Bu(k-i) + \sum_{i=1}^{k} CA^{i-1}d_1(k-i) + Du(k) \end{aligned} \tag{5.44}$$

Define $\tilde{y}(k)$ as the modelling output error (i.e. the difference between true system output and model output):

$$\begin{aligned} \tilde{y}(k) &= y_t(k) - y(k) \\ &= y_t(k) - CA^k x(0) - \sum_{i=1}^{k} CA^{i-1}Bu(k-i) - \sum_{i=1}^{k} CA^{i-1}d_1(k-i) - Du(k) \\ &= y^*(k) - \sum_{i=1}^{k} CA^{i-1}d_1(k-i) \end{aligned} \tag{5.45}$$

where

$$y^*(k) = y_t(k) - CA^k x(0) - \sum_{i=1}^{k} CA^{i-1}Bu(k-i) - Du(k) \tag{5.46}$$

If $x(0)$ is known, $y^*(k)$ can be calculated from Eq.(5.46). Therefore, in the following it will be assumed that $y^*(k)$ is known. A good model should represent the system behaviour accurately, this means that the output modelling error should be zero, i.e.

$$\tilde{y}(k) \longrightarrow 0 \tag{5.47}$$

This is a starting point for computing the disturbance vector $d_1(k)$. Let $k = 1, \cdots, M$ and $C_i = CA^{i-1}$, from Eqs.(5.45), (5.46) and (5.47), one can get:

$$\left\{ \begin{array}{rcl} C_1 d_1(0) & = & y^*(1) \\ C_1 d_1(1) + C_2 d_1(0) & = & y^*(2) \\ \cdots\cdots & = & \cdots\cdots \\ C_1 d_1(M-1) + \cdots + C_M d_1(0) & = & y^*(M) \end{array} \right. \tag{5.48}$$

When $rank(C) = n$, and $m \geq n$ ($C \in \mathcal{R}^{m \times n}$), the solution for $d_1(k)$ is derived form Eq.(5.48) as:

$$\left\{ \begin{array}{rcl} \hat{d}_1(0) & = & C^+ y^*(1) \\ \hat{d}_1(k) & = & C^+[y^*(k+1) - \sum_{i=0}^{k-1} C_{k+1-i} \hat{d}_1(i)] \end{array} \right. \tag{5.49}$$

where C^+ is an inverse of C for $m = n$ ($C^+ = C^{-1}$), or a pseudo-inverse of C when $m > n$ ($C^+ = (C^T C)^{-1} C^T$).

To determine the disturbance vector $d_1(k)$, the number of independent measurements should not be smaller than the state number. This requirement is the same as the requirement in the augmented observer approach and may limit its application. When $rank(C) = g < n$, the number of independent equations (gM) is less than the number of unknown variables (nM) in Eq.(5.48), therefore the solution of $d_1(k)$, $k = 1, \cdots, M$ cannot be uniquely determined using the system input and output data. A good approximation is to let $(n - g)$ components of $d_1(k)$ be zero, i.e.

$$d_1(k) = \begin{array}{c} \left[\begin{array}{c} d_2(k) \\ 0 \end{array} \right] \end{array} \begin{array}{c} n \\ n-g \end{array} \tag{5.50}$$

and then solve for $d_2(k)$. For this purpose, the term $C_i d_1(k)$ in Eq.(5.48) can be decomposed as follows:

$$C_i d_1(k) = [C_i' \quad C_i''] \left[\begin{array}{c} d_2(k) \\ 0 \end{array} \right] = C_i' d_2(k) \tag{5.51}$$

where $C_i' \in \mathcal{R}^{m \times g}$, $d_2(k) \in \mathcal{R}^g$. Using C_i' and $d_2(k)$ to replace C_i and $d_1(k)$, one can obtain the solution of $d_2(k)$:

$$\left\{ \begin{array}{rcl} \hat{d}_2(0) & = & (C')^+ y^*(1) \\ \hat{d}_2(k) & = & (C')^+[y^*(k+1) - \sum_{i=0}^{k-1} C_{k+1-i}' \hat{d}_2(i)] \end{array} \right. \tag{5.52}$$

Substituting (5.52) into (5.50) the solution for $d_1(k)$ ($k = 1, \cdots, M$) can be obtained.

A physical explanation of this approximation is that $(n-g)$ components of $d_1(k)$ cannot be observed by $y(k)$ and they also cannot be determined from $y(k)$.

From Eqs.(5.46), (5.49) & (5.52), it can be seen that the computing and memory requirements for determining $d_1(k)$ are increasing very significantly when the time index k increase. This growing complexity makes the implementation of algorithms very difficult. This estimation approach is not very practical and some modification and simplification measures must be taken.

Now, assume that the disturbance $d_1(k)$ is a constant bias vector, i.e. $d_1(k) = d_1$ for all k. From Eq.(5.48) and the definition of C_i, the following equation can be derived:

$$\begin{cases} Cd_1 &= y^*(1) \\ Cd_1 + CAd_1 &= y^*(2) \\ \cdots\cdots &= \cdots\cdots \\ Cd_1 + \cdots + C_A^{M-1}d_1 &= y^*(M) \end{cases} \tag{5.53}$$

This equation can be rewritten as:

$$\underbrace{\begin{bmatrix} C \\ C+CA \\ \cdots \\ C+CA+\cdots+CA^{M-1} \end{bmatrix}}_{G} d_1 = \underbrace{\begin{bmatrix} y^*(1) \\ y^*(2) \\ \cdots \\ y^*(M) \end{bmatrix}}_{Y} \tag{5.54}$$

where $G \in \mathcal{R}^{mM\times n}, d_1 \in \mathcal{R}^n, Y \in \mathcal{R}^{mM}$. There exists a least-squares solution for d_1 if and only if $rank(G) = n$. The rank of G can then be determined as follows:

$$\begin{bmatrix} C \\ C+CA \\ \cdots \\ C+CA+\cdots+CA^{M-1} \end{bmatrix} = \begin{bmatrix} I_m & 0 & \cdots & 0 \\ I_m & I_m & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ I_m & I_m & \cdots & I_m \end{bmatrix} \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{M-1} \end{bmatrix}$$

Because the first matrix on the right hand side of the above equation is full rank, it is easy to see that: $rank(G) = n$ if and only if $M \geq n$ and the matrix pair (C, A) is observable. Hence, for an observable system, one can estimate the constant disturbance vector via the use of a limited number of computations and low memory if M is not a very large integer number.

$$d_1 = (G^TG)^{-1}G^TY \tag{5.55}$$

In this section, it has been assumed that the initial state vector $x(0)$ is known *a priori*. However, this is not always true. Hence, some approximation must be made when $x(0)$ is unknown. Consider the system output vector:

$$y(k) = CA^{\mu}x(k-\mu) + \sum_{i=1}^{\mu} CA^{i-1}Bu(k-i) + \sum_{i=1}^{\mu} CA^{i-1}d_1(k-i) \tag{5.56}$$

For a large μ and for $k > \mu$, one has $A^{\mu} \to 0$ and $CA^{\mu}x(k-\mu) \approx 0$. In this case, one can get:

$$\sum_{i=1}^{\mu} CA^{i-1}d_1(k-i) = y^*(k) \tag{5.57}$$

Where:

$$y^*(k) \approx y(k) - \sum_{i=1}^{\mu} CA^{i-1}Bu(k-i) \tag{5.58}$$

Assume that the disturbance vector $d_1(k)$ is a piece-wise constant vector, i.e.

$$d_1(k-1) = d_1(k-2) = \cdots = d_1(k-\mu)$$

Eq.(5.57) now can be re-written as:

$$[C\sum_{i=1}^{\mu} A^{i-1}]d_1(k-1) = y^*(k) \tag{5.59}$$

Once again, a unique solution for the disturbance vector $d_1(k)$ exists if and only if $rank(C) = n$. This requires that the independent output dimension is not smaller than the state dimension.

The de-convolution method presented in this section can be used, in some cases, to estimate the disturbance vector. However, there are certain limitations to this method and some further research is still needed.

5.4 Optimal Distribution Matrix for Varied Operating Points

Real process plants normally work at different operating points. The operating point of the system varies according to different plant conditions. This is especially true for the analysis of non-linear systems because they are normally linearized around a wide range of operating points. In the design of model-based FDI schemes, investigators often use a single model for ease of implementation. The success of the single FDI design depends on its robustness properties. When using a single model in this way, different modelling errors arise corresponding to different operating points and even the structure of these errors or perturbations can be quite different! Using the terminology outlined in this chapter, it can be said that different operating points correspond to different disturbance distribution matrices. One way to achieve good robustness is to make disturbance de-coupling conditions hold true (in an optimal sense) for all disturbance distribution matrices. This can be done by using a single optimal disturbance distribution matrix to approximate all disturbance distribution matrices.

Consider that the system works at a wide range of operating points, corresponding to different unknown input distribution matrices, E_i $(i = 1, 2, \cdots, M)$. It is attractive to be able design a single FDI scheme for a whole range (or a set) of operating points. In order to make the disturbance de-coupling hold for all operating points, the following relations should be satisfied:

$$HE_i = 0, \quad for \quad i = 1, 2, \cdots, M \tag{5.60}$$

or:

$$H[E_1 \ \ E_2 \ \ \cdots \ \ E_M] = HP = 0 \tag{5.61}$$

If $rank(P) \leq m$, Eq.(5.61) has solutions and exact de-coupling at all operating points is achievable. Otherwise, approximate de-coupling must be used. This is equivalent to the solution of Eq.(5.14) and can be solved by defining the following optimization problem:

$$\min \|P - P^*\|_F^2 \quad \text{subject to:} \quad rank(P^*) \leq m \tag{5.62}$$

This problem can be solved using the singular value decomposition of P as described in Section 5.2.5. The matrices H and P^* should ensure that a fixed FDI scheme is effective for different operating points.

5.5 Modelling and FDI for a Jet Engine System

Modern engines and control systems have become very complex to meet ever-increasing performance requirements. The rapid increase in complexity has made it difficult to build sufficiently reliable, low-cost, light-weight hydromechanical controls. If faults occur, the consequences can be extremely serious. For example, the pilot can either be presented with incorrect information or he may find it difficult to locate and diagnose a fault quickly enough to take any appropriate corrective action, as described by Merrill (Merrill, 1985; Merrill, 1990; Merrill, DeLaat and Abdelwahab, 1991). This highlights a great need for simple and yet highly reliable methods for detecting and isolating faults in the jet engine.

Engine sensors work in a harsh environment and fault probabilities are high, thus making the sensors the least reliable components of the system. In order to improve the reliability of the engine sensors, analytical and hardware redundancy schemes have been investigated over the last decade (Merrill, 1985; Merrill, 1990; Merrill et al., 1991). The low reliability of the engine sensor module requires that augmentation of the analytical structures be used in order to provide the reliability necessary to cope with ever increasing engine complexity. For example, the rapid changes that occur in the digital fuel control system must be reflected effectively in the sensor system for the accurate detection of faults and the discrimination of false alarms. The inclusion of a fault monitoring system as an integral part of the control system provides the digital control with the necessary information about the faulty sensors. The information is used to decide when to activate an accommodation filter, with the function of reconfiguring the control laws in order to compensate for the occurrence of a sensor malfunction and thus maintain the integrity of the control system. This makes the digital control system attain an acceptable level of reliability.

As discussed in Chapter 1, traditional approaches to FDI in the wider application context are based on hardware redundancy methods which use multiple lanes of sensors, computers and software to measure and/or control a particular variable. A typical jet engine has a degree of redundancy in hardware (eg duplex fuel lines, actuators and speed sensors), however some components, for example the temperature-sensing thermocouple pods, are only available in simple configuration. Moreover, triplex or higher indices of redundancy are not at all realistic. Multiple redundancy is harder to achieve due to lack of operating space. Such schemes would also be costly and very complex to maintain. Severe operating conditions also limit the reliability of engine hardware (e.g. sensors) to the extent that it may not be worthwhile using hardware redundancy alone as a means of diagnosing malfunctions.

The model-based FDI (analytical redundancy) is normally implemented in software form in a computer, and hence very flexible and practical. This is certainly the case for jet engine reliability. Hardware redundancy results in more costly, heavier, less practical, and less potentially reliable systems than do various analytical redundancy strategies. Because cost, weight, and reliability are important issues in turbine engine control systems design, much research interest has been focused on model-based strategies.

5.5.1 Background on fault diagnosis for jet engine systems

The use of model-based approaches for diagnosing faults in jet engine systems has become a very active research topic for theoretical and practical reasons, for example as reported by Merrill (Merrill, 1985; Merrill, 1990; Merrill et al., 1991) and Duyar *et al* (Duyar, Eldem and Saravanan, 1990; Duyar and Merrill, 1992; Duyar, Eldem, Merrill and Guo, 1994). Much of the work in the USA has been of a contract nature under NASA Lewis and in collaboration with Pratt & Whitney (Fort Lauderdale) and GE Gas Turbine Engines (Cincinnati). The most comprehensive and practically feasible study is the NASA Lewis program first reported by Beattie, La Prad, McGlone, Rock and Ahkter (1981). Beattie et al. (1981) surveyed a wide range of FDI schemes, and selected a Kalman Filter (KF) with a Generalized-Likelihood Ratio Testing (GLR)-based scheme as a candidate for further development. Later the whole scheme was rig tested, as reported by Merrill *et al* (Merrill, DeLaat, Kroszkewicz and Abdelwahab, 1987; Merrill, DeLaat and Bruton, 1988; Merrill, DeLaat, Kroszkewicz and Abdelwahab, 1988; Merrill et al., 1991). This study has shown that the theory of sensor FDI could be used in practical turbofan sensor systems.

A gas turbine engine is a very non-linear system whose dynamics are rather uncertain and difficult to model mathematically. Modelling errors and system dynamic uncertainty present a challenge to FDI designs due to the general requirement for robustness. In this context, robustness means that the global (i.e. over the operating range of the process, in this case a jet engine) capability for discrimination between faults and unmodelled effects must be well maintained. Some work in the USA e.g. by Emami-Naeini et al. (1986), arising from the original NASA contract, addresses the robustness problem for FDI. In this work the authors go to the extent of including integral-action feedback according to the *internal model principle* to compensate for the effect of so-called "standoff" biases commonly encountered in the application of observer-based estimation for FDI. This leads to an improved tracking of the states (and inherent robustness) but limited ability to detect and identify *slow drift* fault types. They showed that a suitable compromise can be met through an appropriate choice of integral action time. Also working in the USA, Duyar *et al* (Duyar et al., 1990; Duyar, Eldem, Merrill and Guo, 1991; Duyar and Merrill, 1992; Duyar et al., 1994) used an alternative approach to solve the robustness problem. They attempted to derive accurate linearized models of jet engine systems via the α–Canonical form parameterization identification method and the nonlinear dynamic simulation data. The method has been applied in FDI schemes for the Space Shuttle Main Engine in a project with NASA Lewis and Pratt & Whitney. Under certain conditions, the identified linearized models are suitable for the FDI purpose. Although it can be argued (and, indeed it is the view held here) that the complexity involved in *total identification* of the system is unjustifiably complex for the task in hand.

Other investigators, Goodwin *et al* (Smed, Carlsson, de Sonza and Goodwin, 1988; Villaneuva, Merringto, Ninness and Goodwin, 1991; Ninness and Goodwin, 1991) for example, have used an alternative approach to study the FDI problem for jet engine systems, based on system identification methods. The robustness issue is tackled by considering unmodelled dynamics in the identification procedure. Viswanadham, Taylor and Luce (1987) also studied this subject using a frequency-domain design technique. Piercy (1989) deals with the problem of maximising the analytical redundancy of an FDI scheme, based on model-based detection filters. His work examines the efficiency of FDI methods and proposes some new ideas of design based on over-measured jet engine systems. However, he did not consider robustness problem. The research led by Patton is aimed at keeping in step with the very latest developments world-wide in this subject and on the provision of diagnosis schemes which can be applied very easily in real engine systems. This research emphasises robustness issues using the eigenstructure assignment technique in designing observer-based residual generators. To use robust approaches, the sensitivity to faults in actuators and sensors in fault decision signals (or residuals) is maximised over the appropriate dynamic range of operation. The residual response to uncertain disturbance effects, for example due to modelling errors, is at best nulled or otherwise optimally minimised. The research on jet engine FDI by Patton et al. have been widely published, see: Patton (1989), Patton and Kangethe (1989), Patton and Chen (1990), Patton and Chen (1991a), Patton and Chen (1991b), Patton and Chen (1991f), Patton and Chen (1992b), Patton and Chen (1992e), Patton, Chen and Zhang (1992), Patton, Zhang and Chen (1992) and Patton and Chen (1995).

5.5.2 Jet engine system description

The gas turbine can be described essentially as a heat engine which uses atmospheric air as a working medium to generate propulsive thrust and mechanical power (Patton, 1989; Patton and Kangethe, 1989; Patton and Chen, 1995). The central unit of the mechanical arrangement comprises two main rotating parts, the compressor and the turbine, and one or more chambers. The gas turbine engine provides a continuous operation cycle which characterises the phases of energy exchange which affect the gas mass as it passes through the generator. The phases can be expressed as a variation of the gas pressure against volume. The compressor has the task of converting the mechanical energy of the turbine into pressure energy of the air mass flowing through it. The combustion chamber allows the formation of the fuel-air mixture, in turn, depend on the flight conditions. The primary function of the turbine is to drive the compressor using energy extracted from the hot, accelerated exhaust gas. Further mechanical energy generated during the gas expansion phase is used to drive various accessories such as the fuel pump, oil pump and the electric generator.

The control system has the function of coordinating the main burner fuel flow and the propelling exhaust nozzle. There are other control variables such as inlet variable flaps and rear compressor variable vanes. Under normal operation the control lever selects a desired fuel flow rate which, in turn determines the engine speed. The fuel flow is proportional to the exhaust nozzle area. The coordination of the fuel flow and the size of the exhaust nozzle area is particularly necessary for the afterburner operation. Also, if the turbine jet has a thrust reversal an additional control lever is used to give instinctive control of engine power during the thrust reversal operation.

The jet engine illustrated in Fig.5.1 has the measurement variables N_L, N_H, T_7, P_6, T_{29}. N denotes a compressor shaft speed, P denotes a pressure, whilst T represents a measured temperature. The system has two control inputs, the main engine fuel flow rate u_1 and the exhaust nozzle area u_2.

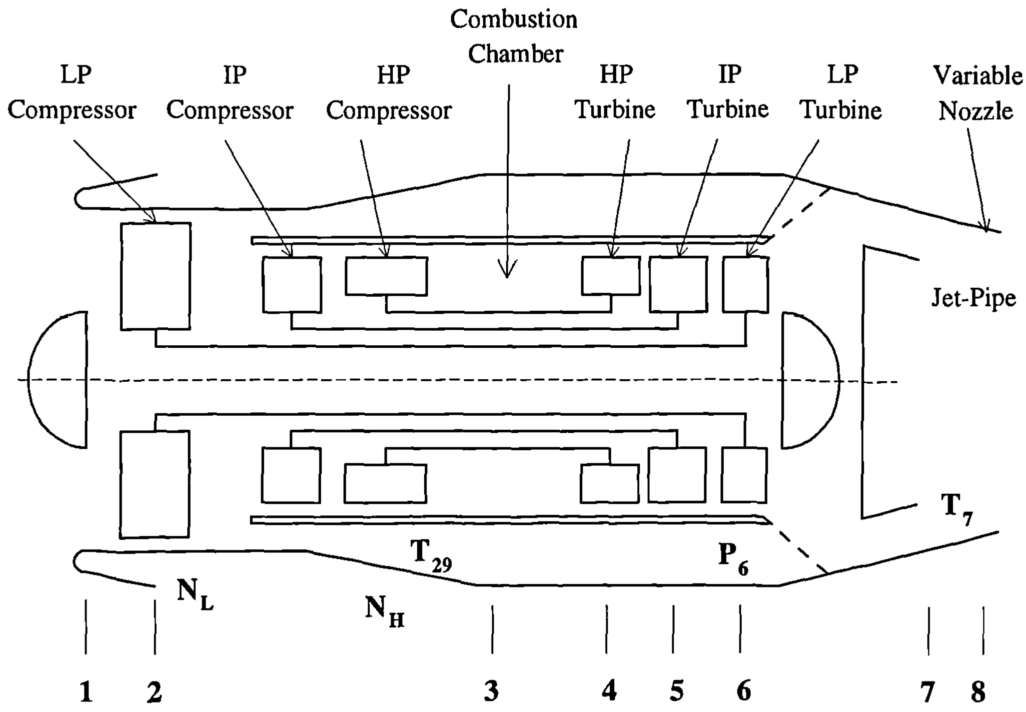


Figure 5.1: **Gas turbine jet engine**

For the purpose of model-based FDI an accurate representation of the dynamic behaviour of the jet engine is required. Modelling of a jet engine is a very difficult problem. One important difficulty lies in the fact that a fully non-linear jet engine system has an iterative structure which means that the equations cannot be written down in differential-algebraic equation form. Fortunately, a non-linear thermodynamic simulation package of the jet engine has been kindly supplied by Lucas Aerospace Ltd. This is a highly non-linear dynamic system which has grossly different steady-state operation over the entire range of spool speeds, flow rates and nozzle areas. Conceptual state space 17_{th} order linearized models at different operating points can be generated using this simulation package. The model has 17 state variables; these include pressure, air and gas mass flow rates, shaft speeds, absolute temperatures and static pressure. The linearized 17_{th} models at different operating points are utilised as a testbed for the evaluation of FDI schemes. Each high order linear model is then further simplified as a low order linear model using balanced model reduction. The linearization error and model reduction are treated as the modelling uncertainty.

In the study presented in this chapter, the nominal operating point is set at 70% of the demanded high spool speed (N_H). For practical reasons and convenience of design, a 5_{th} order model is used to approximate the 17_{th} order model. The model reduction and other errors are represented by the disturbance term $Ed(t)$. The 5th order model matrices are:

$$A = \begin{bmatrix} -78 & 294 & -22 & 21 & -29 \\ 7 & -28 & 2 & -2 & 3 \\ -1325 & 5326 & -526 & 221 & -477 \\ 1081 & -4445 & 377 & -463 & 403 \\ 2152 & -8639 & 781 & -575 & 782 \end{bmatrix}$$

$$B = \begin{bmatrix} -0.0072 & 0.0030 \\ 0.0035 & 0.0003 \\ 1.2185 & -0.0329 \\ 1.3225 & 0.0201 \\ -0.0823 & 0.0244 \end{bmatrix} \qquad C = I_{5\times5} \qquad D = 0_{5\times2}$$

5.5.3 Application of direct computation and optimization method

As explained in Section 4.3, one of the necessary steps for the robust residual generation design procedure is to find a matrix H to satisfy Eq.(5.14) (i.e. $HE = 0$) (when the matrix E has been given). The emphasis here is on the derivation of the matrix E which corresponds to uncertainty arising from the application of the lower (5_{th}) order model to the full (17_{th}) order plant. As discussed in Section 5.2.3, this matrix is determined by a comparison of the full-order model and the reduced model. According to Eq.(5.8), the matrix E is obtained as:

$$E = [E_1 \quad E_2 \quad E_3 \quad E_4] \times 10^3$$

where:

$$E_1 = \begin{bmatrix} 0.076 & -0.294 & 0.022 & -0.021 & 0.029 \\ -0.008 & 0.026 & -0.001 & 0.002 & -0.003 \\ 1.309 & -5.024 & 0.305 & -0.333 & 0.478 \\ -1.031 & 4.152 & -0.255 & 0.274 & -0.403 \\ -2.146 & 8.637 & -0.787 & 0.611 & -0.842 \end{bmatrix}$$

$$E_2 = \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.093 & 0.005 & 0.003 \\ 0.0 & 0.0 & -0.073 & -0.015 & -0.008 \\ 0.0 & 0.0 & 0.0 & -0.001 & -0.002 \end{bmatrix}$$

$$E_3 = \begin{bmatrix} 0.0 & 0.004 & -0.003 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.003 & 0.0 & 0.0 & 0.004 & 0.0 \\ -0.001 & 0.0 & 0.0 & -0.013 & 0.0 \\ -0.001 & -0.001 & -0.009 & -0.003 & 0.0 \end{bmatrix}$$

$$E_4 = \begin{bmatrix} 0.0 & -0.0013 & 0.0 & 0.0 \\ 0.0 & -0.0002 & 0.0 & 0.0 \\ 0.0 & 0.0269 & 0.0 & 0.0 \\ 0.0 & -0.0804 & 0.0 & 0.0 \\ -0.0169 & 0.0025 & 0.0126 & -0.0091 \end{bmatrix}$$

It can be easily checked that $Rank(E) = 5 = n$, and hence Eq.(5.14) has no solution, the optimization procedure must be employed. The singular values of E are:

$$\sigma_1 = 1, \qquad \sigma_2 = 5, \qquad \sigma_2 = 60, \qquad \sigma_2 = 198, \qquad \sigma_2 = 11268$$

The matrices S and T are omitted for brevity. According to the optimization method presented in Section 5.2.5, an optimal q rank approximation for the matrix E is to set $n - q$ smallest singular values as zero. A rank 4 approximation for E is thus given as:

$$E^* = S[diag(0,\ 5,\ 60,\ 198,\ 11268)\ \ 0_{5\times 14}]T^T$$

Based on this matrix, an observer-based robust residual generator can be designed. To simplify the observer design, all eigenvalues are chosen as -100. In this case, the gain matrix $K = -(100I_{5\times 5} + A)$ as C is an identity matrix. The designed robust FDI algorithm is used to detect faulty sensors in the jet engine. The engine data are simulated by the 17_{th} linearized model.

Fig.5.2 shows the output estimation error norm which is very large, and *cannot* be used to detect the fault reliably. This represents the non-robust design situation.

Fig.5.3 shows the fault-free residual. Compared with the output estimation error, the residual is very small, i.e., *disturbance de-coupling* is achieved. This robust design

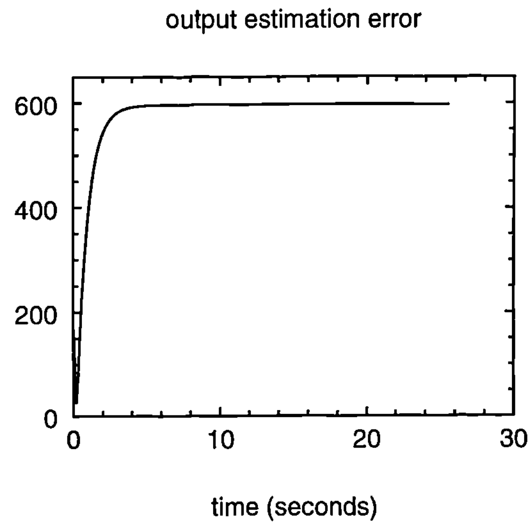


Figure 5.2: **Norm of the output estimation error**

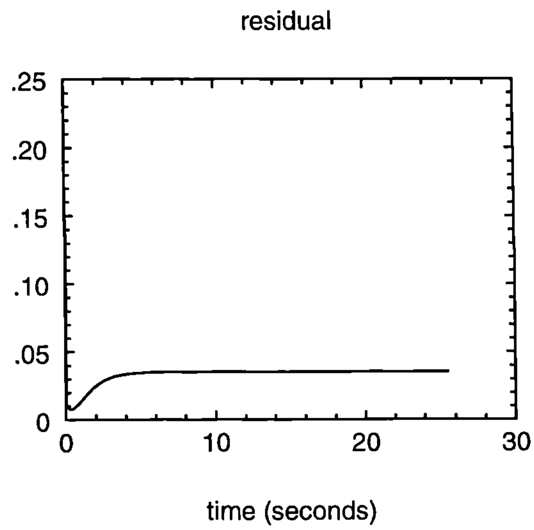


Figure 5.3: **Absolute value of the fault-free residual**

can be used to detect incipient faults. In order to evaluate the power of the robust FDI design, a small fault is added to the exhaust gas temperature measurement (T_7); this simulates the effect of an *incipient fault*, here the effect of which is too small to be noticed in the measurements.

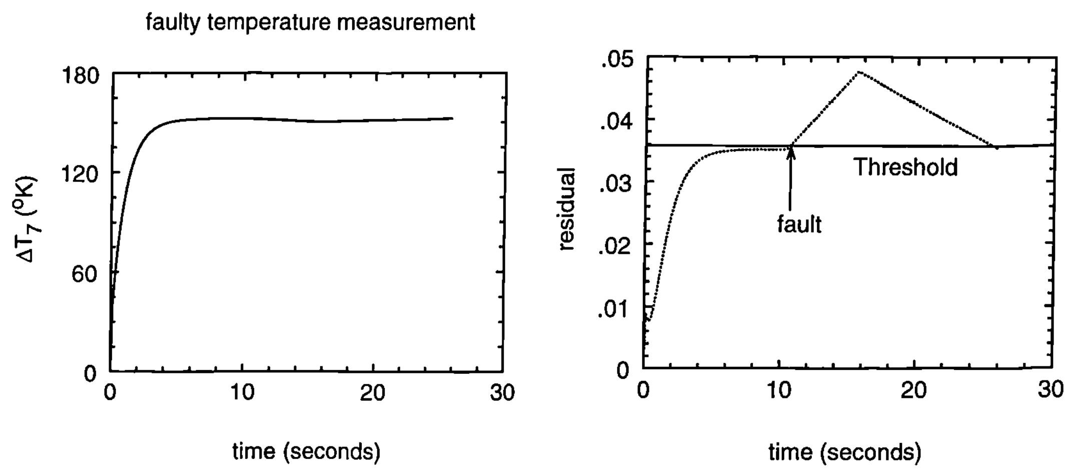


Figure 5.4: **The faulty output and residual when a fault occurs in the temperature sensor for T_7**

Fig.5.4 shows the faulty output of the temperature measurement (T_7) and the corresponding residual. The fault is very small compared with the output, and consequently, is not detectable in the measurement. It can be seen that the residual has a very significant increase when a fault has occurred on the system measurement. A threshold can easily be placed on the residual signal to declare the occurrence of faults. Note that the initial peak in the response is not shown in Fig.5.4, this is because FDI is normally carried out after the initial transient has been settled down. To compare the robust design with the non-robust design, the output estimation error which represents a non-robust design is shown in Fig.5.5. The result in this figure cannot easily be used to detect a fault.

The situation when faults occur in the pressure sensor for P_6 is also simulated and the result shown in Fig.5.6 also demonstrates the efficiency of the robust residual in the role of robust FDI.

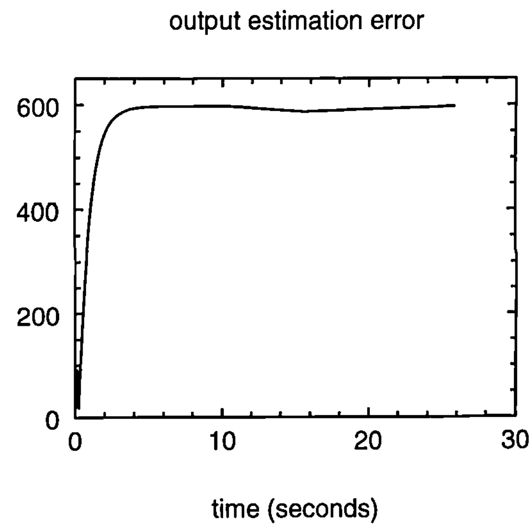


Figure 5.5: **The output estimation error when a fault occurs in the temperature sensor for T_7**

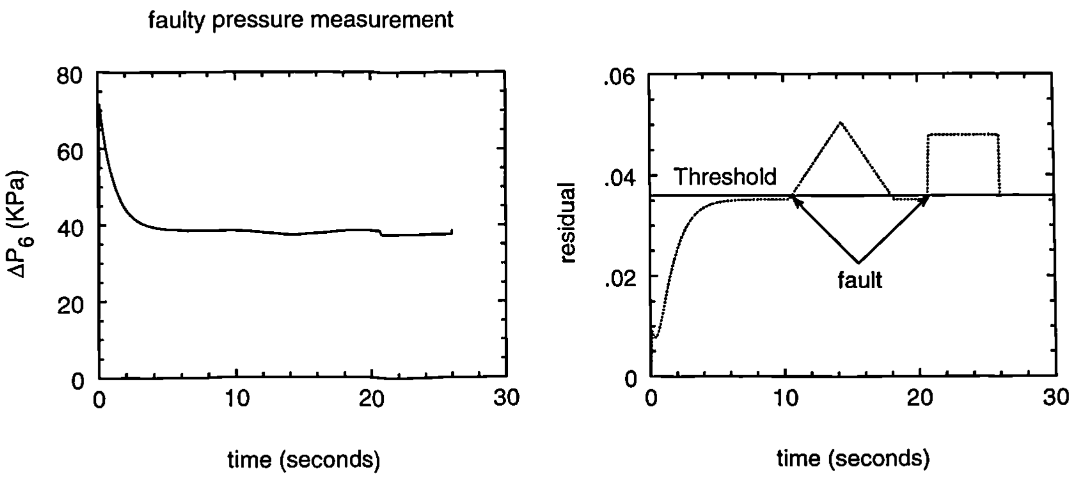


Figure 5.6: **Faulty output of the pressure measurement P_6 and corresponding residual**

5.5.4 Application of augmented observer method

The 5_{th} order jet engine linearized model is now discretized for a sampling period of $T = 0.026s$. The model matrices are:

$$A = \begin{bmatrix} -0.981 & 7.532 & -0.598 & 0.486 & -0.698 \\ 0.284 & -0.083 & 0.078 & -0.062 & 0.093 \\ -6.859 & 28.916 & -2.056 & 1.608 & -2.261 \\ 1.224 & -5.661 & 0.402 & -0.319 & 0.414 \\ 13.266 & -53.405 & 4.739 & -3.771 & 5.367 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.000139 & 0.000195 \\ 0.000067 & -0.000005 \\ 0.003188 & 0.000601 \\ 0.007840 & -0.000273 \\ 0.003123 & -0.001516 \end{bmatrix} \quad C = I_{5\times 5} \; D = 0_{5\times 2}$$

Modelling errors are represented by the term $Ed(k)$ in the dynamic equation. The term $d_1(k) = Ed(k)$ is now determined via the augmented observer approach, as explained in Section 5.3.1. Assume that the input of the system is $u = [1, \;\; 1]^T$, the "true" system output $\{y_t(k)\}$ is generated using the 17_{th} order continuous-time model, then the data $\{u(k), \; y_t(k)\}$ is fed to an augmented observer to estimate $d_1(k)$. The result is shown in Fig.5.7.

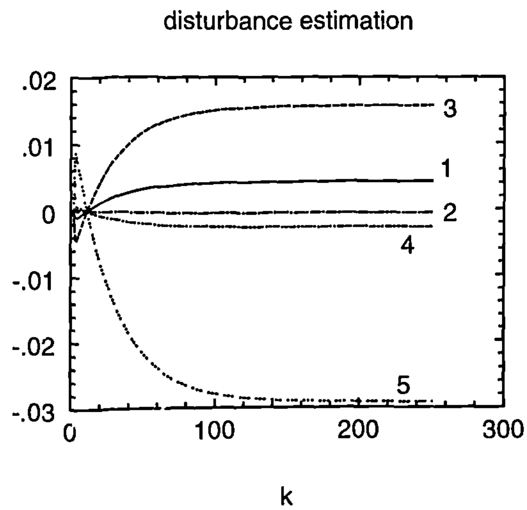


Figure 5.7: **The disturbance vector $d_1(k)$ for the step input case**

From this diagram, it can be seen that the elements of $d_1(k)$ converge after a short transient. Our interest here is in the direction (distribution) of the term $d_1(k)$, i.e., the relative magnitudes of all elements of this vector. It can also be seen the relative magnitude of all elements of $d_1(k)$ converge. It can then be assumed that:

$$d_1(k) = E_1 d(k)$$

Here, E_1 is a 5×1 vector, it is here used to represent the direction of the $d_1(k)$, and $d(k)$ is a scalar which is the magnitude of the $d_1(k)$. In fact, all directions of $d_1(k)(k = 0, 1, 2, \cdots)$ are slightly different. An optimally representative direction vector E_1^* must be aligned to all the directions of $d_1(k)(k = 0, 1, 2, \cdots)$ "as closely as possible". To obtain a reliable direction, the steady-state disturbances $\{d_1(200), d_1(201), \cdots, d_1(251)\}$ are used to compute this optimal direction. The method of decomposing $d_1(k)$ to E and $d(k)$, using $d_1(k) = Ed(k)$ and with E matrix of rank less than n, is presented in Section 5.3.2. This technique is now used to determine the rank one matrix E_1 as:

$$E_1^* = [0.4126 \quad -0.0617 \quad 1.5659 \quad -0.2776 \quad -2.9231]^T$$

Normally, the estimation of the disturbance vector $d_1(k)$ will be different for different inputs to the system. In order to check the generality of the direction of the disturbance distribution using the simulation, the system input is changed to $u = [sin(\pi t/3), cos(\pi t/3)]^T$. The estimation of the disturbance signals is shown in Fig.5.8.

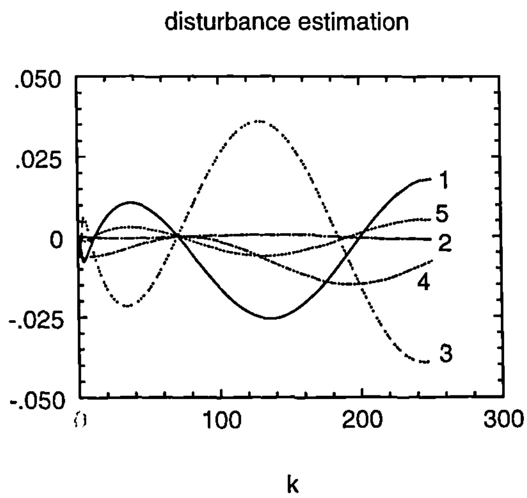


Figure 5.8: **The disturbance vector $d_1(k)$ for the sinusoidal input case**

Although the magnitude of $d_1(k)$ is time-varying, its direction (the relative magnitude of all elements) is almost constant. Following the procedure described Section 5.3.2, the approximate direction has been obtained as:

$$E_2^* = [0.5334 \ \ -0.0768 \ \ 1.9658 \ \ -0.3698 \ \ -3.7068]^T$$

In general, for a complex non-linear system, the operating point will change according to the process inputs and outputs. Hence, it is instructive to consider the system to function at another operating point. In this study, this has been chosen as $95\%N_H$ (or almost full dry power), using the non-linear thermodynamic engine model to generate the linearised parameters. For this case of changed operation, the direction of the disturbances will also be changed. If step inputs are applied to both u_1 and u_2, the approximate direction is obtained as:

$$E_3^* = [1.0511 \ \ -0.1545 \ \ 4.3087 \ \ -0.9646 \ \ -7.8283]^T$$

For a sinusoidal input, the approximate direction is:

$$E_4^* = [1.1580 \ \ -0.1644 \ \ 4.3874 \ \ -0.8722 \ \ -8.2010]^T$$

Although there are differences between E_1^*, E_2^*, E_3^* and E_4^*, the generalized misalignment angles between them are very small. In fact, the generalized misalignment angles are: $\angle(E_1^*, E_2^*) = 0.3764^o$, $\angle(E_1^*, E_3^*) = 1.5633^o$ and $\angle(E_1^*, E_4^*) = 0.5712^o$. So, it is reasonable to say that the disturbance direction is almost constant (E_1^* is used as a representative in the study) for the system studied here, although the system is a fully non-linear gas turbine model. The results in an interesting basis for further study.

A 5_{th} order discrete-time observer is used to generate the disturbance de-coupling residuals. The first step to complete a disturbance de-coupling (robust residual generation) design is to compute the residual weighting matrix Q (see Section 4.3), such that $QCE = 0$ holds true. This weighting matrix is obtained as:

$$Q = \begin{bmatrix} -0.367 & -0.441 & 0.656 & 0.409 & 0.270 \\ -0.116 & 0.895 & 0.334 & 0.245 & 0.121 \\ 0.879 & -0.050 & 0.350 & -0.034 & 0.316 \end{bmatrix}$$

which ensures that $QCE_1^* = 0$, $QCE_3^* = 0$, $QCE_2^* \approx 0$ and $QCE_4^* \approx 0$. The desired eigenvalues of the observer are $\{0, 0, 0, 0, 0\}$ such that the observer has a *state dead-*

beat structure. The desired left eigenvectors of the observer are the rows of the matrix $H = QC = Q$. The observer gain matrix can be derived using eigenstructure assignment. In this example, as all eigenvalues of the observer are zero, the gain matrix is simply derived as $K = A$. Because $QCE_1^* = 0$, $QCE_3^* = 0$, $QCE_2^* \approx 0$, $QCE_4^* \approx 0$ and the rows of the matrix $QC = H$ are the *left* eigenvectors of the observer corresponding to zero-valued eigenvalues, i.e. the robustness conditions hold true, the fault detection scheme is then always robust (such that disturbance de-coupling always holds) when the system works at different operating points and different types of inputs.

The designed robust FDI scheme is applied to detect faulty sensors in the jet engine system. Simulation is based on the 17_{th} order thermodynamic jet engine continuous-time model. A particular emphasis of this assessment study is the power of the method to detect soft or incipient faults which are otherwise unnoticeable in the measurement signals. These attributes are well illustrated in the following graphical time response results. As the FDI scheme has been made robust against modelling errors, the scheme is able to detect incipient faults under conditions of modelling uncertainty. The uncertainty of the jet engine system has been increased further by simulating the effect of random noise generated through a small malfunction in the fuel flow regulator system - to emulate the possibility of a high interference level arising in the electronic system. This has been achieved by adding a zero-mean Gaussian random signal with variance of 1% of demanded fuel-flow, to the fuel flow actuation signal in the model. The inputs to the system are $u = [1,\ \ 1]^T$, and initial values are zero. The linear model used has been based on a per-unit scaling of the engine dynamics and hence the final results have been scaled to give meaningful magnitudes.

Fig.5.9 shows the residual norm and the output estimate error norm for both fault-free and faulty cases.

The result in Fig.5.9 shows that the residual is very small in the fault-free case, i.e., disturbance de-coupling is achieved. The output estimation error which represents the non-robust design is very large, even when no faults occur, and this cannot be used to detect faults reliably.

Fig.5.10 shows the faulty output of the pressure sensor P_6; the fault is very small compared with the output, and consequently, which cannot directly be detected in the output. The corresponding residual and the output estimate error for this faulty

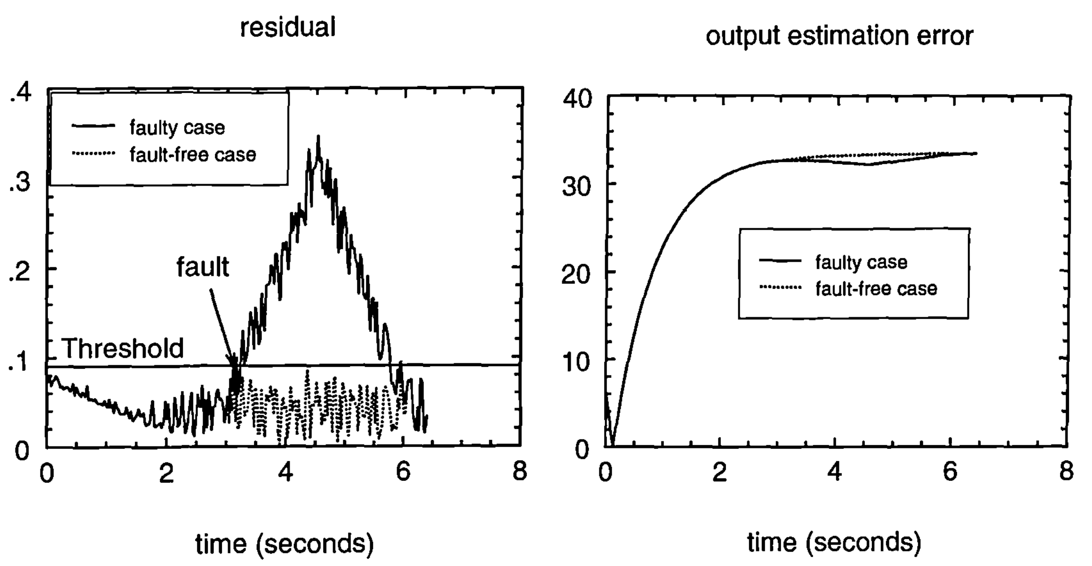


Figure 5.9: **The residual ($r(k)$) norm and the output estimation error ($e_y(k)$) norm**

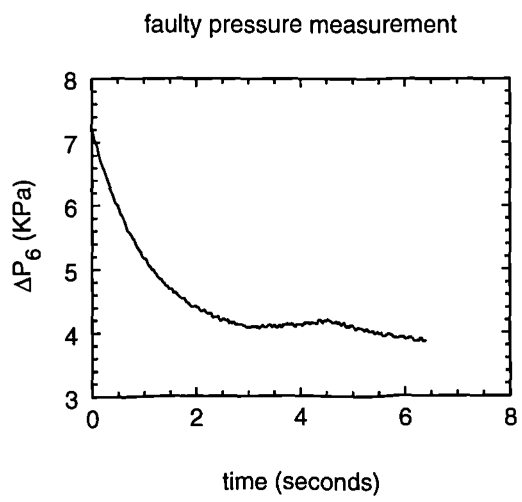


Figure 5.10: **Faulty pressure (P_6) measurement**

case are shown in Fig.5.9. It can be seen that the residual has a very significant increase when the fault occurs. Despite the actuation noise, a threshold can easily be placed on the residual signal to declare the occurrence of faults. But, one cannot be sure whether a fault has even occurred in the system when using the information from the output estimate error.

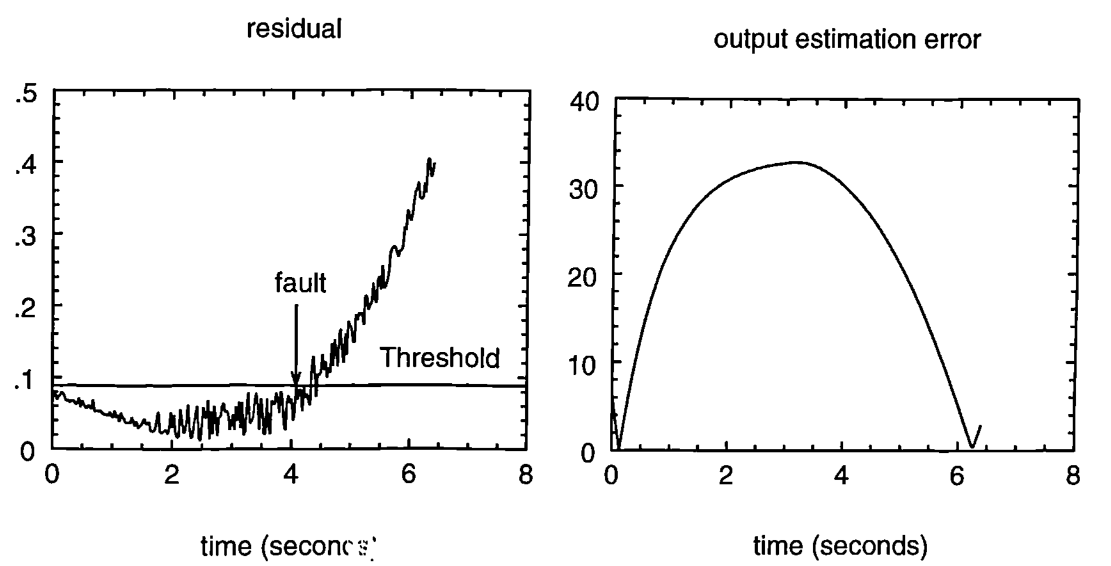


Figure 5.11: **The residual norm and the output estimation error norm for the case a parabolic fault on the spool speed sensor for N_H**

Fig.5.11 shows the fault detection performance of the residual for detecting a parabolic fault in spool speed sensor N_H. The results show the fault can be reliably detected from the residual, but cannot be detected using the output estimation error. This result has proved once again the importance of a robust residual in fault diagnosis.

In general, for a complex non-linear system, the operating point changes according to the process operation. Hence, it is instructive to consider the system to function at different operating points. A robust FDI scheme should work well for a range of operating points. In order to assess the robustness performance, the scheme is used to detect the fault when the system is working at another operating point (in the presence of demanded changes in high compressor speed N_H), the result is shown in Fig.5.12. Note that, although the magnitude of the residual is changed, the fault can also be easily detected from the significant increase of the residual.

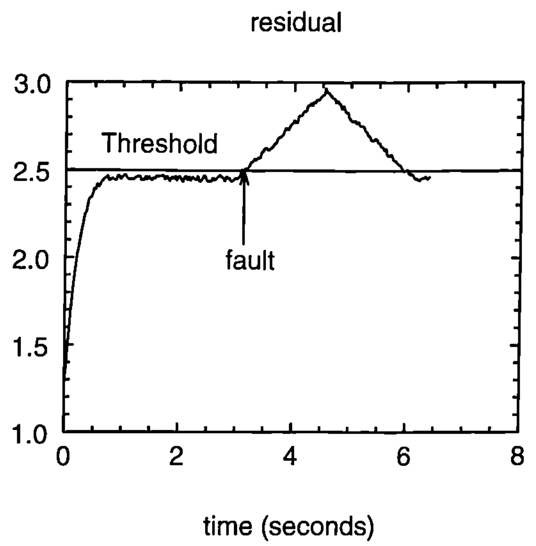


Figure 5.12: **Residual when a fault occurs in the temperature measurement T_7 with operating condition changed**

5.6 Conclusion

One critical limitation of the model-based approach to fault diagnosis is that modelling uncertainty is inevitable. For complex systems such as a jet engine, the effects of uncertainty are more pronounced compared with other systems. In order to design robust FDI schemes, we should have a mathematical description of modelling uncertainty. Furthermore, it is necessary to make sure that this description can be handled in a straightforward and systematic manner. Modelling uncertainty can be accounted for using an additional term in the dynamic equation of the system; this additional term has a certain structure. Normally, it is assumed that the distribution of this additional term is known *a priori*. Based on this description, the disturbance de-coupling approach is used to design a robust FDI scheme. For most real systems, the distribution matrix which represents the information about uncertainty is unknown. This chapter has studied the methods for determining the disturbance distribution matrix for uncertainty. The main aim has been to bridge theoretical assumptions with practical realty. Two principle methods for determining disturbance distribution matrix have been presented. The first method is the direct determination & optimization method, whose strength is simple and direct and does not require real or simulated system input and output data. Its disadvantage is that it requires some *a priori* information about modelling uncertainty. However, this chapter has presented ways to determine disturbance distribution matrices for a wide range of possible situations. Hence, it can be claimed that the method is *general* in application. The second method is the estimation and de-convolution method. One disadvantage is that it requires that the system has more than n (state dimension) independent measurements. However, for many fault diagnosis problems, e.g., the jet engine example, there are usually a large number of measurements available and the dynamics of the system can be approximated by a relatively low order model. The method can be used for a number of fault diagnosis problems and, as real or simulated system input and output data are used, the results can be affected by the system inputs; different inputs may give arise different distribution matrices. This is a disadvantage of this estimation method. It can be seen that the two methods have compromising properties. One can choose which method is more suitable for a particular problem. In this chapter, a jet engine example has been used to illustrate the application of the techniques developed. The jet engine is a very complex system and the nonlinearities and modelling errors are inevitable. This presents a big challenge for achieving reliable FDI using model-based approaches. Excellent results have been obtained and these indicate the effectiveness of the method for detecting soft (small) and hence incipient faults.

Chapter 6

ROBUST RESIDUAL GENERATOR DESIGN VIA MULTI-OBJECTIVE OPTIMIZATION AND GENETIC ALGORITHMS

6.1 Introduction

In safety-critical systems such as aircraft and nuclear reactors, hard faults in system components may not be tolerable and must be detected before they actually occur. Hopefully, faults are detected during the maintenance stage. However, the situation is different for soft (incipient) faults. Their effect on the system is very small and almost unnoticeable during their incipient stage. They may develop slowly to cause very serious effects on the system, although these incipient faults may be tolerable when they first appear. Hence, the most important issue of reliable system operation is to detect and isolate incipient faults as early as possible. An early indication of incipient faults can give the operator enough information and time to take proper measures to prevent any serious consequence on the system.

The detection of incipient faults presents a challenge to model-based FDI techniques due to the inseparable mixture between fault effects and modelling uncertainty (Patton et al., 1989; Frank, 1990; Patton and Chen, 1991e; Gertler, 1991). Hard or sudden faults normally have a larger effect on the detection residual than the effect of modelling uncertainty. Hence the fault can be detected by placing an appropriate threshold on the residual. However, incipient faults have a lower effect; the effect can even be lower than the response due to modelling uncertainty, so that thresholding cannot be directly used to diagnose incipient faults reliably. As discussed in previous chapters, the residual has to be designed to be robust against modelling uncertainty to detect incipient faults.

Although many approaches have been developed, robust FDI is still an *open problem* for further research. One of the most important approaches for robust FDI is the use of disturbance de-coupling principles, which have been studied in Chapters 3 & 4. One should recall that the idea is to treat modelling uncertainty as exogenous disturbances and de-couple their effect from the residual. The main disadvantage is that the distribution of disturbances is required to facilitate designs, although the disturbance itself is assumed unknown. For most uncertain systems, the modelling uncertainty is expressed in terms of modelling errors. Hence, the disturbance de-coupling approach cannot be applied directly. Chapter 5 proposes many ways on representing modelling errors as unknown disturbance with an approximate distribution matrix. In this way, robust FDI is partially achievable. There are some successful applications, however it would be better to relax the restriction on the assumption about modelling uncertainty in the design of robust residual generators. In this chapter, the modelling uncertainty is simply treated as an additive disturbance term in the dynamic equation. There are no requirements to use information about the distribution (structure) of the disturbance or uncertainty, although this information can be used if it is available.

For disturbance de-coupling approaches in FDI, the aim is to completely eliminate the disturbance effect from the residual. However, the complete elimination of disturbance effects may not be possible due to the lack of design freedom. Moreover, it may be problematic, in some cases, because the fault effect may also be eliminated. Hence, an appropriate criterion for robust residual design should take account of the effects of both modelling errors and faults. There is a trade-off between sensitivity to faults and robustness to modelling uncertainty and hence this is an issue of prime concern. Robust residual generation can be then considered as a *multi-objective optimization problem*, i.e. the maximization of fault effects and the minimization of uncertainty effects. The problem of maximizing fault effects and at the same time minimizing disturbance effects was studied by Frank & Wünnenberg (Frank and Wünnenberg, 1989; Wünnenberg, 1990) in the time domain and Frank & Ding (Ding and Frank, 1989; Ding and Frank, 1991) in the frequency domain. In their studies, a ratio between disturbance effects and fault effects is minimized. The main problem is that they only considered the cases when the disturbance distribution matrix is known. The multi-objective design in the time-domain for systems with bounded parameter uncertainty and disturbances has been studied recently by Chen et al. (1993) and is extended in Chapter 7.

This chapter develops a new approach to the design of optimal residuals for detecting incipient faults, based on multi-objective optimization and genetic algorithms. In this approach the residual is generated via an observer. In order to make the residual become insensitive to modelling uncertainty and sensitive to sensor faults, a number of performance indices have been defined to achieve good fault diagnosis performance. Some performance indices are defined in the frequency domain to account for the fact that modelling uncertainty effects and faults occupy different frequency bands. Robust control design in the frequency domain is attracting enormous attention in the control community. However, there is currently very little research on the use of frequency domain techniques for robust FDI. Patton et al. (1986) first discussed the possibility of using frequency distribution information to design FDI algorithms, however they did not give further guidance as to how this could be achieved. Ding and Frank (1989) proposed an optimal observer design method for FDI in the frequency domain. Viswanadham, Taylor and Luce (1987) and Ding and Frank (1990) later studied the frequency domain residual generation method via factorization of the system transfer matrix, however the robustness issue is not their primary concern in design. More recently, Frank and Ding (1993) and Qiu and Gertler (1993) made some important contributions in robust FDI design by using H^{∞}-optimization. Mangoubi et al. (1992) also applied the H_{∞}/μ technique to the design of a robust FDI algorithm, however the effect of faults has not been considered in their performance criterion. In this chapter, the numerical optimization technique is used for the robust residual design which is different from the previous investigations. The performance indices used in this chapter are also different from previous studies, and one of the main contributions is the joint optimization of fault effects and disturbance rejection.

In the approach presented in this chapter, frequency-dependent weighting factors are introduced in the performance indices (cost functions), based on knowledge of the frequency band of the modelling uncertainty and faults. The main principle of robust FDI is to distinguish faults and the uncertainty effects in residuals, and this is only possible when they are "physically" distinguishable. Otherwise, no matter what mathematical method is chosen, one cannot discriminate between these two effects. Moreover, some information about both faults and disturbances must be available. In the previous chapters of this thesis, the faults have been assumed to have different distribution directions from those of the uncertainty. In the technique presented in this chapter, the information on frequency distribution ranges of faults, noise and modelling uncertainty (once known) can be incorporated into a robust residual design.

To design robust residuals, a multi-objective optimization problem needs to be solved. This chapter uses the method of inequalities to solve this multi-objective optimization problem. All objectives are reformulated into a set of inequality constraints on performance indices. The genetic algorithm is thus used to search an optimal solution to satisfy these inequality constraints. The use of genetic algorithms obviates the requirement for the calculation of cost function gradients and also increases the possibility of finding global optimum. A flight control example is used in this chapter to illustrate the technique developed. The fault detection performance is examined in the presence of modelling errors. The simulation results show that the fault detection algorithm designed by the proposed method can detect incipient sensor faults very effectively.

6.2 Residual Generation and Performance Indices

6.2.1 Residual generation and responses

Consider the following mathematical description of the monitored system:

$$\begin{cases} \dot{x}(t) &= Ax(t) + Bu(t) + R_1 f(t) + d(t) \\ y(t) &= Cx(t) + Du(t) + R_2 f(t) \end{cases} \tag{6.1}$$

where $x(t) \in \mathcal{R}^n$ is the state vector, $u(t) \in \mathcal{R}^r$ is the control input vector and $y(t) \in \mathcal{R}^m$ is the measurement vector, $f(t) \in \mathcal{R}^g$ represents the fault vector which is considered as an unknown time function. The matrices A, B, C and D are system parameter matrices and the pair $\{A, C\}$ is assumed observable.

The vector $d(t)$ is the disturbance vector which can also be used to represent modelling errors such as:

$$d(t) = \Delta Ax(t) + \Delta Bu(t)$$

Note that this form of uncertainty representation is very general as the distribution matrix is not required. The matrices R_1 and R_2 are fault distribution matrices which represent the influence of faults on the system. They can be determined if one has defined which faults are to be diagnosed. For two most common cases: sensor and actuator faults, these matrices are:

$$R_1 = \begin{cases} 0 & \text{sensor faults} \\ B & \text{actuator faults} \end{cases} \qquad R_2 = \begin{cases} I_m & \text{sensor faults} \\ D & \text{actuator faults} \end{cases}$$

The residual generator studied in this chapter, shown in Fig.6.1, is based on a

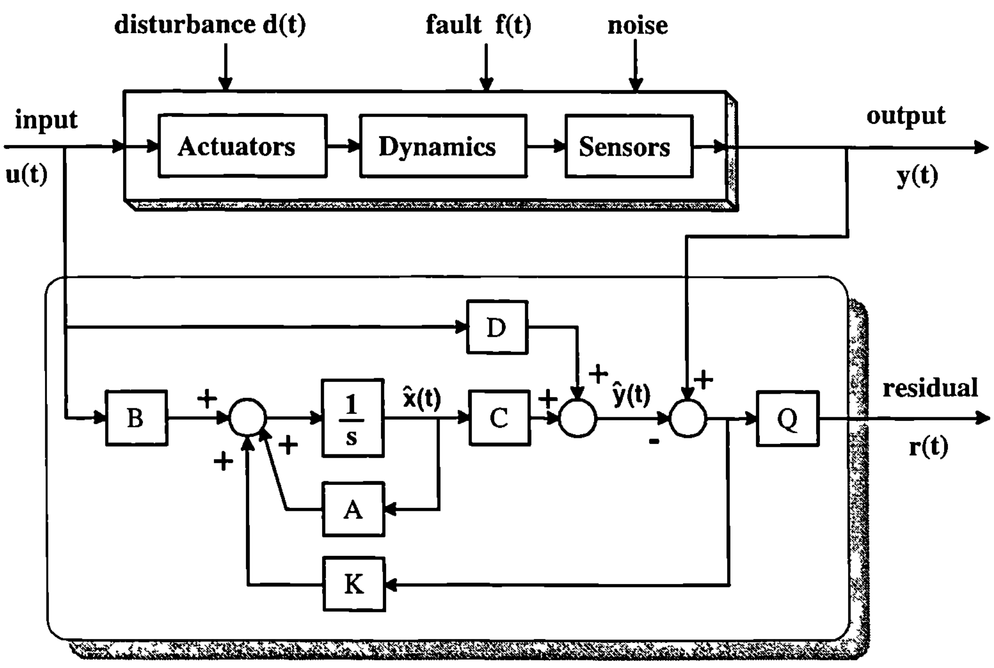


Figure 6.1: **Robust residual generation via a full-order observer**

full-order observer. The basic idea is to estimate the system output from the measurements using an observer. The weighted output estimation error is then used as a residual. The flexibility in selecting the observer gain and the weighting matrix provides freedom to achieve good detection performance. The residual generator is thus described as:

$$\begin{cases} \dot{\hat{x}}(t) &= (A - KC)\hat{x}(t) + (B - KD)u(t) + Ky(t) \\ \hat{y}(t) &= C\hat{x}(t) + Du(t) \\ r(t) &= Q[y(t) - \hat{y}(t)] \end{cases} \tag{6.2}$$

where $r \in \mathcal{R}^p$ is the residual vector, $\hat{x}$ and $\hat{y}$ are state and output estimations. The matrix $Q \in \mathcal{R}^{p\times m}$ is the residual weighting factor which, in most cases, is static but can also be dynamic. When this residual generator is applied to the monitored system described by Eq.(6.1), the state estimation error ($e(t) = x(t) - \hat{x}(t)$), and the residual are governed by the following equations:

$$\begin{cases} \dot{e}(t) &= (A - KC)e(t) + d(t) + R_1 f(t) - KR_2 f(t) \\ r(t) &= QCe(t) + QR_2 f(t) \end{cases} \tag{6.3}$$

The residual response to faults and disturbances is thus:

$$\begin{aligned} r(s) &= Q\{R_2 + C(sI - A + KC)^{-1}(R_1 - KR_2)\}f(s) \\ &\quad + QC(sI - A + KC)^{-1}[d(s) + e(0)] \\ &= G_{rf}(s, K, Q)f(s) + G_{rd}(s, K, Q)[d(s) + e(0)] \end{aligned} \tag{6.4}$$

where $e(0)$ is the initial value of the state estimation error.

6.2.2 Performance indices in robust residual generation

Both faults and disturbances affect the residual, and discrimination between these two effects is difficult. To reduce false and missed alarm rates, the effect of faults on the residual should be maximized and the effect of disturbances on the residual should be minimized. One can maximize the effect of the faults by maximizing the following performance index, in the required frequency range $[\omega_1, \omega_2]$:

$$\bar{J}_1(K, Q) = \inf_{\omega\in[\omega_1,\omega_2]} \underline{\sigma}\{QR_2 + QC(j\omega I - A + KC)^{-1}(R_1 - KR_2)\} \tag{6.5}$$

This is equivalent to the minimization of the following performance index :

$$J_1(K, Q) = \sup_{\omega\in[\omega_1,\omega_2]} \overline{\sigma}\{[QR_2 + QC(j\omega I - A + KC)^{-1}(R_1 - KR_2)]^{-1}\} \tag{6.6}$$

where $\underline{\sigma}\{.\}$ and $\overline{\sigma}\{.\}$ denote the minimal and maximal singular values.

Similarly, one can minimize the effects of both disturbance and initial condition by minimizing the following performance index:

$$J_2(K,Q) = \sup_{\omega\in[\omega_1,\omega_2]} \overline{\sigma}\{QC(j\omega I - A + KC)^{-1}\} \tag{6.7}$$

Besides faults and disturbances, noise in the system can also affect the residual. To illustrate this, assume that $\zeta(t)$ and $\eta(t)$ are input and sensor noise signals, the system equations in this case is:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + d(t) + R_1 f(t) + \zeta(t) \\ y(t) = Cx(t) + Du(t) + R_2 f(t) + \eta(t) \end{cases} \tag{6.8}$$

It can be seen that the sensor noise as well as faults acting through $R_2 f(t)$ affect the system at the same excitation point and hence affect the residual in the same way. To reduce the noise effect on the residual, the norm $\| Q - QC(j\omega I - A + KC)^{-1}K \|$ should be minimized. This contradicts the requirement for maximizing the effects of faults on the residual. Fortunately, the frequency ranges of the faults and noise are normally different. For an incipient fault signal, the fault information is contained within a low frequency band as the fault development is slow. However, the noise comprises mainly high frequencies signals. Based on these observations, the effects of noise and faults can be separated by using different frequency-dependent weighting penalties. In this case, the performance index $J_1(K,Q)$ is:

$$J_1(K,Q) = \sup_{\omega\in[\omega_1,\omega_2]} \overline{\sigma}\{W_1(j\omega)[QR_2 + QC(j\omega I - A + KC)^{-1}(R_1 - KR_2)]^{-1}\} \tag{6.9}$$

To minimize the effect of noise on the residual, a new performance index is introduced as:

$$J_3(K,Q) = \sup_{\omega\in[\omega_1,\omega_2]} \overline{\sigma}\{W_3(j\omega)Q[I - C(j\omega I - A + KC)^{-1}K]\} \tag{6.10}$$

In order to maximize the effects of faults at low frequencies and minimize the noise effect at high frequencies, the frequency-dependent weighting factor $W_1(j\omega)$ should have large magnitude in the low frequency range and small magnitude at high frequencies. The frequency effect of $W_3(j\omega)$ should be opposite to $W_1(j\omega)$ and can be chosen as $W_3(j\omega) = W_1^{-1}(j\omega)$. The disturbance (or modelling error) and input noise affect the residual in the same way. As both effects should be minimized, the performance index J_2 does not necessarily need to be weighted. However, modelling uncertainty and input noise effects may be more serious in one or more frequency bands. The performance index should reflect this fact, and hence a frequency-dependent weighting factor must also be placed on $J_2(K,Q)$, in some situations.

$$J_2(K,Q) = \sup_{\omega\in[\omega_1,\omega_2]} \overline{\sigma}\{W_2(j\omega)QC(j\omega I - A + KC)^{-1}\} \tag{6.11}$$

Now, considering the steady state value of the residual:

$$r(\infty) = QR_2f(\infty) + QC(A-KC)^{-1}(KR_2 - R_1)f(\infty) - (A-KC)^{-1}d(\infty) \tag{6.12}$$

After the transient period, the residual steady state value plays an important role in FDI. Ideally, it should reconstruct the fault signal. The disturbance effects on residual can be minimized by minimizing the following performance index:

$$J_4(K) = \| (A-KC)^{-1} \|_\infty \tag{6.13}$$

When J_4 is minimized, the matrix K is very large and the norm $\| (A-KC)^{-1}K \|$ approaches to a constant value. This means that the fault effect on the residual has not been changed by reducing the disturbance effect. This is what is required for good FDI performance.

6.2.3 Remarks on performance indices

The choice of norms: In the definition of performance indices, the infinity norm of matrices are used. However, other matrix norms (such as the Frobenius norm) are also useful. To examine the function of different matrix norms, let's consider the disturbance effect on the residual.

$$r(s) = G_{rd}(s,K,Q)d(s)$$

It is well known that the residual norm is bounded by:

$$\|r(s)\| \leq \|G_{rd}(s,K,Q)\|\|d(s)\|$$

If the infinity norm is used, this inequality becomes:

$$\|r(s)\|_\infty \leq \|G_{rd}(s,K,Q)\|_\infty\|d(s)\|_\infty$$

This measures the worst effects, i.e., the largest component of the residual due to the largest component of disturbance will be minimized if $\|G_{rd}(s,K,Q)\|_\infty$ is minimized.

If the Frobenius norm is used, the corresponding relations is:

$$\|r(s)\|_2 \leq \|G_{rd}(s,K,Q)\|_F \|d(s)\|_2$$

This measures the average effects, i.e., the energy of the residual due to the disturbance will be minimized if $\|G_{rd}(s,K,Q)\|_F$ is minimized. Different norm measures have different characteristics, however if one format of norm for a particular matrix is minimized, other kind of norms for the same matrix are unlikely to be large. This can be proved using the following inequality (Golub and Van Loan, 1989, p.57):

$$\frac{1}{\sqrt{n}}\|G_{rd}\|_\infty \leq \|G_{rd}\|_F \leq \sqrt{np}\|G_{rd}\|_\infty$$

where p is the row number of G_{rd} and n is column number. From this relation, it can be seen that the Frobenius norm will be bounded if the infinity norm is minimized and verse versa.

Disturbance distribution: In this chapter, there is no requirement on the disturbance distribution. However, this information can also be incorporated into performance indices, if it is available. If the disturbance distribution matrix is known, i.e.

$$d(t) = Ed'(t)$$

where E is a known matrix and $d'(t)$ is an unknown vector. In this case, the performance index J_2 can be modified as:

$$J_2(K,Q) = \sup_{\omega \in [\omega_1,\omega_2]} \overline{\sigma}\{W_2(j\omega)QC(j\omega I - A + KC)^{-1}E\} \tag{6.14}$$

Fault isolation: As discussed in Sections 2.7.1 & 3.3, a structured residual set should be generated to isolate faults. The word "structured" here signifies the sensitivity and insensitivity relations that any residual will have, i.e. whether it is designed to be sensitive to a group of faults whilst, insensitive to another group of faults. The faults contained in the vector $f(t)$ can be divided into two groups $f^1(t)$ and $f^2(t)$ and the system equation in this case is:

$$\left\{ \begin{array}{rcl} \dot{x}(t) & = & Ax(t) + Bu(t) + R_1^1 f^1(t) + R_1^2 f^2(t) + d(t) \\ y(t) & = & Cx(t) + Du(t) + R_2^1 f^1(t) + R_2^2 f^2(t) \end{array} \right. \tag{6.15}$$

If the residual is to be designed sensitive to $f^1(t)$ and insensitive to $f^2(t)$, the performance index J_1 should be modified as:

$$J_1(K,Q) = \sup_{\omega\in[\omega_1,\omega_2]} \bar{\sigma}\{W_1(j\omega)[QR_2^1 + QC(j\omega I - A + KC)^{-1}(R_1^1 - KR_2^1)]^{-1}\} \quad (6.16)$$

In addition to the four performance indices defined, a new performance index $J_5(K,Q)$ which is to be minimized should be introduced to make the residual insensitive to $f^2(t)$.

$$J_5(K,Q) = \sup_{\omega\in[\omega_1,\omega_2]} \bar{\sigma}\{W_5(j\omega)[QR_2^2 + QC(j\omega I - A + KC)^{-1}(R_1^2 - KR_2^2)]\} \quad (6.17)$$

If only sensor faults are to be isolated, the design problem is easier to solve. This is because, if the residual is to be sensitive to a group of sensor faults, only the measurements from this set of sensors will be used in the residual generation. The detailed discussion on robust sensor and actuator fault isolation can be found in Section 3.3.

6.3 Parameterization In Observer Design

Four performance indices $J_1(K,Q)$, $J_2(K,Q)$, $J_3(K,Q)$ and $J_4(K,Q)$ have now been defined. To achieve robust FDI (in terms of minimizing false and missed alarm rates), one has to solve a multi-objective optimization problem. One of the parameter sets to be designed is the observer gain matrix K which must guarantee the stability of the observer. This leads to a constrained optimization problem which is difficult to solve. Within the context of control system design, this stability constraint is normally changed to the assignment of eigenvalues in the left hand side of the complex plane (or within the unit disc, for the discrete-time domain). The observer design is a dual of the controller design problem and all techniques in control design can be applied. Here, the eigenstructure assignment method is chosen to give the parameterization of the gain matrix K (Burrows and Patton, 1991; Patton and Liu, 1994; Liu and Patton, 1994; Chen et al., 1994a; Chen, Patton and Liu, 1994b). Note that the gain matrix can also be parameterized in other ways. However, the parametric representation in terms of eigenstructure has many advantages, the most important one is that the eigenvalues can be specified in predefined points or regions according to required residual responses.

The eigenvalues of the observer can be real or complex-conjugate. Assume that there are n_r real eigenvalues λ_i $(i = 1, \cdots, n_r)$ and n_c pairs of complex-conjugate eigenvalues $\lambda_{j,re} \pm j\lambda_{j,im}$ $(j = 1, \cdots, n_c)$, and n_r and n_c satisfy the following relation:

$$n_r + 2n_c = n$$

Real eigenvalue case: Assume that v_i is the i_{th} right eigenvector of $(A^T - C^T K^T)$ corresponding to the i_{th} eigenvalue λ_i of $(A^T - C^T K^T)$, we then have that:

$$(A^T - C^T K^T)v_i = \lambda_i v_i \tag{6.18}$$

or

$$v_i = -(\lambda_i I - A^T)^{-1} C^T K^T v_i \tag{6.19}$$

To define a design parameter vector w_i as:

$$w_i = K^T v_i \tag{6.20}$$

The eigenvector v_i can expressed via this design parameter vector:

$$v_i = -(\lambda_i I - A^T)^{-1} C^T w_i \tag{6.21}$$

Complex-conjugate eigenvalue case: Assume that $v_{j,re} + jv_{j,im}$ is the j_{th} right eigenvector of $(A^T - C^T K^T)$ corresponding to the j_{th} eigenvalue $\lambda_{j,re} + j\lambda_{j,im}$ of $(A^T - C^T K^T)$, we have:

$$(A^T - C^T K^T)(v_{j,re} + jv_{j,im}) = (\lambda_{j,re} + j\lambda_{j,im})(v_{j,re} + jv_{j,im}) \tag{6.22}$$

This equivalent to:

$$\left\{ \begin{array}{lcl} (A^T - C^T K^T)v_{j,re} & = & \lambda_{j,re} v_{j,re} - \lambda_{j,im} v_{j,im} \\ (A^T - C^T K^T)v_{j,im} & = & \lambda_{j,im} v_{j,re} + \lambda_{j,re} v_{j,im} \end{array} \right. \tag{6.23}$$

or

$$\left\{ \begin{array}{lcl} (\lambda_{j,re} I - A^T)v_{j,re} - \lambda_{j,im} v_{j,im} & = & -C^T K^T v_{j,re} \\ \lambda_{j,im} v_{j,re} + (\lambda_{j,re} I - A^T)v_{j,im} & = & -C^T K^T v_{j,im} \end{array} \right. \tag{6.24}$$

To define:

$$A_j = \begin{bmatrix} \lambda_{j,re} I - A^T & -\lambda_{j,im} I \\ \lambda_{j,im} I & \lambda_{j,re} I - A^T \end{bmatrix} \qquad C_c = \begin{bmatrix} C^T & 0 \\ 0 & C^T \end{bmatrix}$$

and

$$\begin{cases} w_{j,re} &= K^T v_{j,re} \\ w_{j,im} &= K^T v_{j,im} \end{cases} \tag{6.25}$$

this leads to:

$$\begin{bmatrix} v_{j,re} \\ v_{j,im} \end{bmatrix} = -A_j^{-1} C_c \begin{bmatrix} w_{j,re} \\ w_{j,im} \end{bmatrix} \tag{6.26}$$

To put Eqs.(6.20) & (6.25) together, the parametric representation of the observer gain matrix K is given by:

$$K = [WV^{-1}]^T \tag{6.27}$$

where

$$W = [w_1 \cdots w_{n_r} \ ; \ w_{1,re} \cdots w_{n_c,re} \ ; \ w_{1,im} \cdots w_{n_c,im}] \in \mathcal{R}^{m \times n}$$

is the design parameter matrix whose elements can be determined arbitrarily.

$$V = [v_1 \cdots v_{n_r} \ ; \ v_{1,re} \cdots v_{n_c,re} \ ; \ v_{1,im} \cdots v_{n_c,im}] \in \mathcal{R}^{n \times n}$$

Any column vector of this matrix is a function of the corresponding column vector in the matrix W. All columns in this matrix can be calculated via either Eq.(6.21) or Eq.(6.26).

Eigenvalue specifications: The eigenvalues λ_i $(i = 1, \cdots, n_r)$ and $\lambda_{j,re} \pm j\lambda_{j,im}$ $(j = 1, \cdots, n_c)$ have to be given by the designer prior to the design procedure. In practice, the eigenvalues do not need to be assigned at a specific point in the complex plane. However, we do need to assign eigenvalues in predefined regions to meet stability and response requirements, i.e.

$$\lambda_i \in [L_i, \ U_i] \qquad i = 1, \cdots, n_r$$

for real eigenvalues. For complex-conjugate eigenvalues, the relations will be:

$$\left.\begin{matrix} \lambda_{j,re} \in [L_{j,re}, \ U_{j,re}] \\ \lambda_{j,im} \in [L_{j,im}, \ U_{j,im}] \end{matrix}\right\} \quad \text{for} \quad j = 1, \cdots, n_c$$

The assignment of eigenvalues in regions rather than at specific points increases the design freedom. However the inequality constraints on eigenvalues are introduced by doing this. To remove these constraints, a simple transformation for a real eigenvalue can be introduced (Burrows and Patton, 1991):

$$\lambda_i = L_i + (U_i - L_i) sin^2(z_i) \tag{6.28}$$

where $z_i \in \mathcal{R}$ $(i = 1, \cdots, n_r)$ can be freely chosen. Similar to the real eigenvalue case, the real and the imaginary parts of a complex-conjugate eigenvalue pair can be expressed by:

$$\begin{cases} \lambda_{j,re} &= L_{j,re} + (U_{j,re} - L_{j,re}) sin^2(z_j) \\ \lambda_{j,im} &= L_{j,im} + (U_{j,im} - L_{j,im}) sin^2(z_{j+1}) \end{cases} \tag{6.29}$$

where $z_j, z_{j+1} \in \mathcal{R}$ can be determined arbitrarily. Now, the constrained performance indices $J_j(K, Q)$ $(j = 1, 2, 3, 4)$ have been transformed into unconstrained performance indices $J_j(Z, W, Q)$, where W and $Z = [z_1 \cdots z_n] \in \mathcal{R}^{1 \times n}$ can be chosen freely. The multi-objective optimization problem for robust FDI is solved in the following sections by combining the method of inequalities and the genetic algorithm.

6.4 Multi-Objective Optimization and the Method of Inequalities

6.4.1 Multi-objective optimization

The use of multi-objective optimization is very common in engineering design problems. Generally speaking, a solution does not exist which minimizes all performance indices simultaneously. A set of parameters which minimizes a particular performance index may let other performance indices become very large and unacceptable. Hence, some compromises and trade-offs must be taken account in the design. The trade-off is based on the relative importance of objectives. As the number of objectives increases, trade-offs between objectives are likely to become complex and less easily quantifiable. There is, therefore, much reliance on the intuition of the designer and his ability to express preferences throughout the optimization cycle. This is easier to be solved using numerical search algorithms, as the designer can alter his preference throughout the optimization cycle and enter them into a numerically tractable and realistic design problem.

Mixed objective strategies: A commonly used approach in multi-objective optimization is the mixed objective approach, for example, Burrows and Patton (1991) applied this approach to control system design. In this approach, all objective functions are mixed together according to different weighting factors. The emphasis on different objectives can be made using different magnitudes of weighting factors. For the optimization problem presented in this chapter, the performance indices can be mixed together in the following ways:

$$J(Z,W,Q) = \sum_{i=1}^{4} \alpha_i J_i(Z,W,Q) \tag{6.30}$$

$$J(Z,W,Q) = \frac{\sum_{i=2}^{4} \alpha_i J_i(Z,W,Q)}{J_1(Z,W,Q)} \tag{6.31}$$

$$J = \sup_{\omega\in[\omega_1,\omega_2]} \overline{\sigma} \left\{ \begin{bmatrix} W_1(j\omega)[QR_2 + QC(j\omega I - A + KC)^{-1}(R_1 - KR_2)]^{-1} \\ W_2(j\omega)QC(j\omega I - A + KC)^{-1} \\ W_3(j\omega)Q[I - C(j\omega I - A + KC)^{-1}K] \\ (A - KC)^{-1} \end{bmatrix} \right\} \tag{6.32}$$

where $\alpha_i \geq 0$ $(i = 1, \cdots, 4)$ are weighting factors which should be decided according to the relative importance of objectives. The multi-objective optimization problem is now reformulated into the minimization of the mixed single cost function $J(Z,W,Q)$.

The minimax optimization: The multi-objective optimization problem can also be solved via a minimax optimization procedure in which the largest normalized performance index is to be minimized:

$$\min J(Z,W,Q) = \min\{\max_i \frac{J_i(Z,W,Q)}{C_i}\} \tag{6.33}$$

where C_i $(i = 1, \cdots, 4)$ are the normalizing factors. The preference on different objectives can be achieved by altering the normalizing factors. It is interesting to note that, minimax optimization considers the worst case which is the same as the use of infinity norms.

6.4.2 The method of inequalities

A more attractive approach for solving the multi-objective optimization problem in control system design is the method of inequalities, proposed by Zakian (Zakian and Al-Naib, 1973; Zakian, 1979). The main philosophy behind this approach is to replace the minimization of the performance index by an inequality constraint on the performance index. The simultaneous minimization of all performance indices is normally impossible. However, in engineering design problems, what is one normally required is the restriction of the performance index within a pre-defined region. The optimization problem is posed as the *satisfaction* of a set of inequalities, rather than the *minimization* of some objective functions with inequalities acting as side-constraints. The shift of emphasis from objective functions to a set of inequalities gives a more accurate formal representation of many design problems, and leads to an iterative design procedure in which the designer changes the "trade-off" between conflicting constraints by adjusting the inequalities, rather than some objective functions. This is attractive, because it is usually much easier to understand the physical implications of changes in constraining inequalities than changes in an objective function.

Optimization is still required in the method of inequalities, but the shift of emphasis from minimization to inequality satisfaction means that the usual ideas on the choice of the optimization algorithms have to be revised: speed of convergence in the neighbourhood of a minimum becomes much less important than the likelihood of finding at least one feasible point – namely one at which all the inequalities are satisfied.

For the problem presented in this chapter, the multi-objective optimization problem is being reformulated into that of searching for a parameter set $\{Z, W, Q\}$ to satisfy the following inequalities:

$$J_i(Z, W, Q) \le \varepsilon_i, \qquad \text{for} \qquad i = 1, 2, 3, 4 \tag{6.34}$$

where the real number ε_i represents the numerical bound on the performance index $J_i(Z, W, Q)$ required by the designer. If the minimal value of $J_i(Z, W, Q)$ achieved by minimizing $J_i(Z, W, Q)$ itself is J_i^*, the objective bound must be set as: $\varepsilon_i > J_i^*$. This is based on the fact that a parameter set which minimizes a particular performance index can make other performance indices very large. If $J_i^*(Z_i^*, W_i^*, Q_i^*)$ is the minimal value of $J_i(Z, W, Q)$ achieved at the parameter set $\{Z_i^*, W_i^*, Q_i^*\}$, the following inequalities hold true:

$$J_i(Z_j^*, W_j^*, Q_j^*) \ge J_i^*(Z_i^*, W_i^*, Q_i^*) \tag{6.35}$$

where $j \neq i, j \in \{1, 2, 3, 4\}$; for $i = 1, 2, 3, 4$. As a general rule, the performance boundaries ε_i should be set as:

$$J_i^*(Z_i^*, W_i^*, Q_i^*) < \varepsilon_i \le \max_{j \neq i, j \in [1,4]} \{J_i(Z_j^*, W_j^*, Q_j^*) \tag{6.36}$$

for $i = 1, 2, 3, 4$. The problem of multi-objective optimization is to find a parameter set to make all performance indices lie in an acceptable region. By adjusting the bounds ε_i, one can place a different emphasis on each of the objectives. If the performance index J_j is important for the problem, one can let ε_j near to J_j^*. If the performance index J_k is less important, one can let ε_k be far away from J_k^*.

Example of the method of inequalities: A simple example show in Fig.6.2 is used to illustrate the use of the method of inequalities.

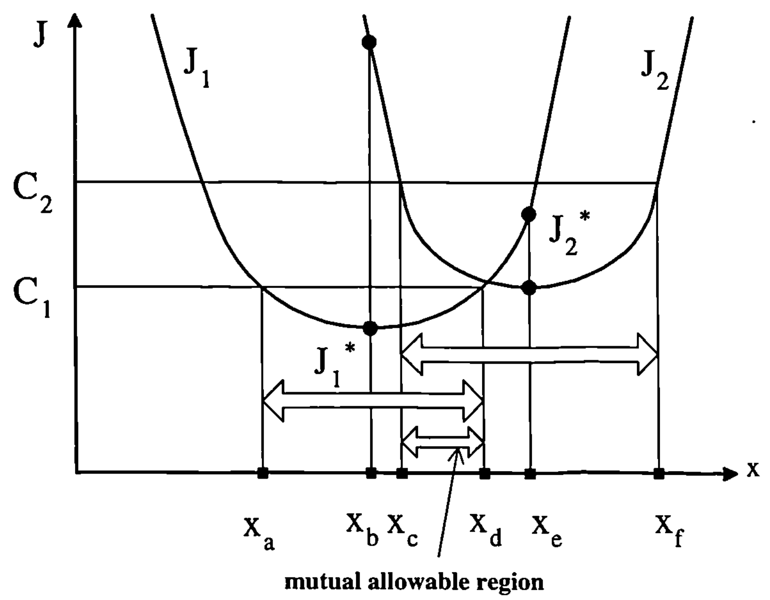


Figure 6.2: **An example of the method of inequalities**

In this example, two performance indices $J_1(x)$ and $J_2(x)$ are to be minimized. It can be seen that x_b is the minimization point for $J_1(x)$, however this point is not acceptable for $J_2(x)$. The point which minimizes both $J_1(x)$ and $J_2(x)$ does not exist. To solve this problem, the requirement for optimization should be relaxed. In stead of the minimization of $J_1(x)$ and $J_2(x)$, the problem is transformed to the satisfaction of the following inequalities:

$$\begin{cases} J_1(x) & \leq & C_1 \\ J_2(x) & \leq & C_2 \end{cases}$$

Any point in the region $[x_c, x_d]$ can satisfy the design requirements. Within this region, one can also improve the optimization performance further using either the mixed-objective method or the minimax method. If the minimax principle is used, the optimal point will be x_d. If the mixed-objective method is used, the optimal point will be close to the average point of the region $[x_c, x_d]$.

Moving-boundaries algorithm: Zakian (Zakian and Al-Naib, 1973; Zakian, 1979) suggests an algorithm for satisfying the inequalities which he calls the *moving-boundaries algorithm* (Maciejowski, 1989, pp.341–346). The procedure of this algorithm which provides the solution to the problem presented, is given below.

Let us firstly normalize the performance indices as follows:

$$\phi_i(Z,W,Q) = \begin{cases} \frac{J_i(Z,W,Q)}{\varepsilon_i} & \text{for} \quad \varepsilon_i \neq 0 \\ J_i(Z,W,Q)+1 & \text{for} \quad \varepsilon_i = 0 \end{cases} \tag{6.37}$$

The problem is now to satisfy the following normalized inequalities:

$$\phi_i(Z,W,Q) \leq 1 \tag{6.38}$$

Let $\mathcal{S}_i$ be the set of parameters (Z,W,Q) for which the i_{th} objective is satisfied:

$$\mathcal{S}_i = \{(Z,W,Q) \ : \ \phi_i(Z,W,Q) \leq 1\} \tag{6.39}$$

Then the admissible or feasible set of parameters for which all objectives hold is:

$$\begin{aligned} \mathcal{S} &= \mathcal{S}_1 \bigcap \mathcal{S}_2 \bigcap \mathcal{S}_3 \bigcap \mathcal{S}_4 \\ &= \{(Z,W,Q) : \max_{i=1}^{4}\{\phi_i(Z,W,Q)\} \leq 1\} \end{aligned} \tag{6.40}$$

which shows that the search for an admissible parameter set (Z,W,Q) can be pursued via optimization, in particular by solving:

$$\min_{(Z,W,Q)}\{\max_{i=1}^{4}\{\phi_i(Z,W,Q)\} \leq 1 \tag{6.41}$$

Now, let (Z^k, W^k, Q^k) be the values of the parameters at k_{th} step, and define:

$$\mathcal{S}_i^k = \{(Z,W,Q) : \phi_i(Z,W,Q) \leq \Delta^k\} \text{ for } i = 1,\cdots,4 \tag{6.42}$$

where

$$\Delta^k = \max_{i=1,2,3,4}\{\phi_i(Z^k,W^k,Q^k)\} \tag{6.43}$$

and also define

$$\mathcal{S}^k = \mathcal{S}_1^k \bigcap \mathcal{S}_2^k \bigcap \mathcal{S}_3^k \bigcap \mathcal{S}_4^k \tag{6.44}$$

$$\Sigma^k = \sum_{i=1}^{4} \phi_i(Z^k, W^k) \tag{6.45}$$

Hence $\mathcal{S}^k$ is the k_{th} set of parameters for which all objectives satisfy:

$$\phi_i(Z, W, Q) \leq \Delta^k \quad \text{for} \quad i = 1, \cdots, 4 \tag{6.46}$$

It is clear that $\mathcal{S}^k$ contains both (Z^k, W^k, Q^k) and the admissible set $\mathcal{S}$. Σ^k is a combined measurement of all objectives. The task now is to find a new parameter set which moves objectives towards the final feasible set. The strategy for finding the new parameter set is to minimize the largest performance index, i.e., Δ^k. If the largest performance index Δ^k cannot be improved, the improvement of the combined performance index Σ^k is considered. If one now finds new parameters $(\overline{Z}^k, \overline{W}^k, \overline{Q}^k)$, such that:

$$\overline{\Delta}^k < \Delta^k \tag{6.47}$$

or

$$\overline{\Delta}^k = \Delta^k \quad \text{and} \quad \overline{\Sigma}^k < \Sigma^k \tag{6.48}$$

where $\overline{\Delta}^k$ and $\overline{\Sigma}^k$ are defined similarly to Δ^k and Σ^k, then we accept $(\overline{Z}^k, \overline{W}^k, \overline{Q}^k)$ as the next set of parameters, i.e., $(Z^{k+1}, W^{k+1}, Q^{k+1}) = (\overline{Z}^k, \overline{W}^k, \overline{Q}^k)$. This leads to:

$$\phi_i(Z^{k+1}, W^{k+1}, Q^{k+1}) \leq \phi_i(Z^k, W^k, Q^k), \text{ for } i = 1, \cdots, 4 \tag{6.49}$$

and

$$\mathcal{S} \subset \mathcal{S}^{k+1} \subset \mathcal{S}^k \tag{6.50}$$

So that the boundary of the set in which the parameters are located has been moved towards the admissible set, or rarely, has remained unaltered. The process of finding the optimization solution is terminated when both Δ^k and Σ^k cannot be reduced further. But the process of finding an admissible parameter set (Z, W, Q) is terminated when $\Delta^k \leq 1$, i.e., when the boundaries of $\mathcal{S}^k$ have converged to the boundaries of $\mathcal{S}$. The process of the moving-boundaries algorithm is illustrated in Fig.6.3.

If the Δ^k persists in being larger than 1, this may be taken as an indication that the objectives may be inconsistent, whilst their magnitudes give some measure of how

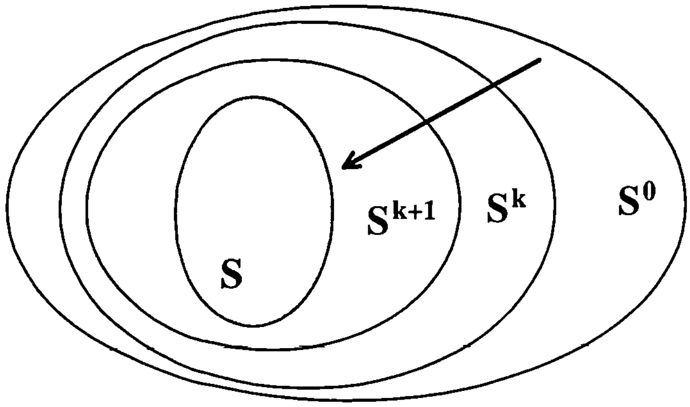


Figure 6.3: **The 'moving-boundaries' process**

closely it is possible to approach the objectives. In this case, some of the inequality constraints should be relaxed until they are satisfied. From a practical viewpoint, the approximate optimal solution is also useful if the absolute optimal solution is not achievable.

The difficult part of the algorithm is the generation of a trial parameter set $(\overline{Z}^k, \overline{W}^k, \overline{Q}^k)$, given (Z^k, W^k, Q^k). Many methods have been proposed since Zakian introduced the method of inequalities and a short review can be found in Maciejowski (1989, p.345). It is suggested that the relatively crude direct search methods such as the simplex method can be used to solved this problem. Patton and Liu (1994) suggested a method to generate the trial parameter set via genetic algorithms in the design of robust controllers. This method is extended by Chen et al. (1994a) to the robust FDI problem. The combination of genetic algorithms with the method of inequalities for solving the multi-objective optimization problem, defined in this chapter is discussed Section 6.5.

6.5 Optimization via Genetic Algorithms

Most optimization techniques can be classified broadly into calculus-based techniques or direct-search methods. In recent years, the direct-search techniques, which are problem-independent, have been widely used in optimization. Unlike calculus-based methods (gradient descent, etc.), direct search algorithms do not require the use of derivatives. Consequently, it eases the analytical analysis in the calculation of derivatives and it is less likely for direct search algorithms to get "trapped" into local minima. Gradient-descent methods, on the other hand, calculate the slope of the objective surface at the current position in all directions and move in the direction with the most negative slope. This works well when the objective surface is relatively smooth, with few local minima. However, real-world data are often multimodal and contaminated by noise which can further distort the objective surface.

6.5.1 Introduction to genetic algorithms

The most important direct search algorithm in optimization is the genetic algorithm (GA) which was invented to mimic some of the processes observed in natural evolution. The technique was pioneered by Holland and his associates in the 1970's (Holland, 1975), and in the last six years has been receiving growing interest in both research and application (Goldberg, 1989; Frenzel, 1993). GAs are parameter search procedures based upon the mechanics of natural genetics. All natural species survive by adapting themselves to the environment. This natural adaption is the underlying theme of GAs. GAs search combines a Darwinian survival of the fittest strategy to eliminate unfit characteristics and uses random information exchange, with exploitation of knowledge contained in old solutions, to effect a search mechanism with surprising power and speed.

Genetic algorithms are different from other optimization techniques in many ways, notably they are:

- GAs constitute a *parallel* search of the solution space, as opposed to a point-by-point search in gradient-descent methods. By using a population of trial solutions, the genetic algorithm can effectively explore many regions of the search space simultaneously. Optimization methods more usually provide iterative progress (global or local) solution, based on a single region in the parameter space; the region may only include a local minimum and another region must then be used to locate a global minimum. This is one of the reasons why GAs are less sensitive to local minima.
- GAs manipulate *representations (or codings)* of the parameter set, rather than the parameters themselves.
- GAs do not require derivative information or other auxiliary knowledge concerning problems to be solved. The only problem-specific requirement is the

ability to evaluate the trial solutions on objective function, and the relative fitness levels influence the directions of search.

- GAs use probabilistic rather than deterministic transition rules.

A genetic algorithm is an exploratory procedure that is able to locate close-to-global optimal solutions to complex problems. It maintains a set of trial solutions (often called individuals), and forces them to "evolve" towards an acceptable solution. The procedure starts with an initial random population and employing survival-of-the-fittest and exploiting old knowledge in the gene pool. Each generation's ability to solve the problem should be improved. The computational structure of a genetic algorithm is shown in Fig.6.4.

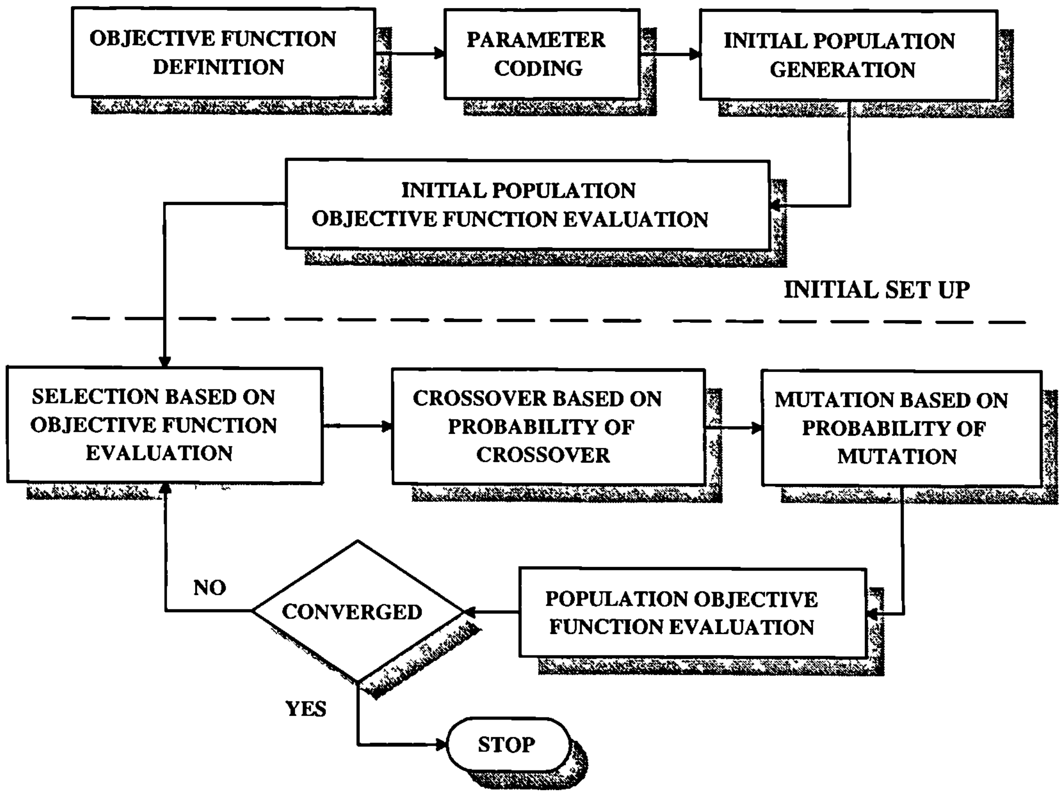


Figure 6.4: **Computational structure of genetic algorithms**

The main stages involved in GAs discussed in Frenzel (1993) and Davis (1991) are shown in below:

Representation (or coding): The parameter set is represented by a coding scheme which can be recognized by computers. These representations are normally referred to as *chromosomes.* The most common coding scheme is the use of binary strings, where selections of the string represent encoded parameters. The number of digits assigned to strings will determine the numerical accuracy.

Evaluation: To evaluate the objective fitness of the current chromosomes in each generation. Each chromosome in the population is decoded and evaluated on how well it solves the problem. The fitness measure is used in the next step to determine how many offspring will be generated from any particular chromosome.

Reproduction: In this stage, a new population is created based on the evaluation of the current one. For every chromosome in the current population, a number of exact copies are generated with the best chromosomes producing the most copies. This is the step that allows GAs to take advantage of a survival-of-the-fittest strategy. There are several ways to calculate the number of offspring that each chromosome will be allocated. The two most popular methods are referred to as *ratioing* and *ranking.*

In ratioing, each individual reproduces in proportion to its fitness. So, an individual whose fitness is ten times better than another will produce ten times the number of offspring. This way as superior chromosomes emerge they can guide the population quickly. The disadvantage is that if a superior individual surfaces early and dominates the population, then the population may converge prematurely on a possible suboptimal (or local minimal) solution.

For the ranking method, the number of offspring each chromosome will generate is determined by how it ranks in the population. For example, the top 20% of the population might generate two offspring each, the bottom 20% of the population generate no offspring, and the rest generate just one offspring apiece. Using this method, no one chromosome can overpower the population in a single generation. Also, no matter how close the actual fitness values are, there is always constant pressure to improve. The primary disadvantage of ranking is speed because better chromosomes are not capable of guiding the population easily. This forces good solutions to develop more slowly.

Recombination: The reproduction creates a population whose member are currently the best solution for the problem, however many of the chromosomes are identical and no-one is different from the previous generation. The reproduction simply produces multiple copies of existing chromosomes. Recombination combines chromosomes from the population and produces new chromosomes that, while they did not exist in the previous generation, maintain many of features of the previous generation. In natural evolution, recombination and reproduction occur in the same step. However, in GAs they are often separated to facilitate experimentation with different methods. The most important method for recombination is *crossover* in which two individuals are randomly selected from the population and, governed by a specified crossover probability or rate, subsection of the two chromosomes are swapped about a randomly chosen crossover point. During recombination, GAs exploit knowledge of the gene pool by allowing good chromosomes to combine with chromosomes that aren't as good. This is based on the assumption that each individual, no matter how good it appears, doesn't contain the complete answer to the problem. The answer is contained in the population as a whole, and the best solution can only be found by combining chromosomes.

Mutation: This step in creating a new generation is motivated by the possibility that the initial population didn't contain all of the information necessary to solve the problem. Moreover, it is possible that the individuals that produce no offspring may have had some information that is essential to the solution. The injection of new information into the population is called mutation. One of methods to implement mutation is to change randomly a fixed number of bits every generation based upon a specified mutation probability.

Elitism: It is possible that the best member of the population may fail to produce offspring in the next generation. The *elitist* strategy fixes this potential source of loss by copying the best member of each generation into the succeeding generation. The elitist strategy may increase the speed of domination of a population by a super individual, but on balance it appears to improve genetic algorithm performance. More specifically, the elitist can improve the speed of convergence, but it could give a local minimum due to the domination of a super individual. The use of the elitist strategy depends on problems, if the performance index has many local minimums, it is not good idea to use it.

GENETIC ALGORITHM PARAMETERS: The best values for mutation rate, crossover percentage, and other parameters are problem specific. It is even possible to find the best values using genetic algorithms! However, certain generalizations can be made (Frenzel, 1993). If the population is too small, relative to the size of the search space, it will be difficult to effectively search the entire region. Furthermore, large mutation rates tend to disrupt the steady improvement resulting from crossover and reproduction. Researchers have found that a population of 30 individuals, a crossover probability of 60%, and a mutation probability of 3% seems to be a good starting point (Frenzel, 1993).

6.5.2 Procedure of genetic algorithms in satisfying performance inequalities

In the implementation of genetic algorithms, it is not necessary to include all main stages given above. There are many variations in the implementation. Some stages may need to be modified to best suit particular problems. The genetic algorithm is used here to search the optimal solutions in the moving-boundaries process of satisfying performance inequalities. The procedure of the optimal search via GA is first suggested by Liu and Patton (1994) and later modified to suit robust FDI design by Chen et al. (1994a). This optimization procedure includes the following steps:

Step 1: Chromosomal representation. Each solution in the population is represented as a real number string rather than as a binary string. For $W \in \mathcal{R}^{m\times n}$, $Z \in \mathcal{R}^{1\times n}$ and $Q \in \mathcal{R}^{p\times m}$, the chromosomal representation may be expressed as an array:

$$P = [Z, w_1^T, \cdots, w_n^T, q_1^T, \cdots, q_m^T]$$

This kind of chromosomal representation has *two* advantages. One is that it guarantees that the domain expertise embodied in the representation will be preserved. The other is that the algorithm to be developed will feel natural to the designer.

Step 2: Generation of the initial population. N (an odd number) sets of parameter string P for the initial population are randomly generated.

Step 3: Evaluation of the performance functions. Evaluate the performance function $\phi_i(P_j)$ $(i = 1, 2, 3, 4)$ for all N sets of the parameter P_j and determine:

$$\begin{aligned} \Delta_j &= \max\{\phi_1(P_j), \phi_2(P_j), \phi_3(P_j), \phi_4(P_j)\} \\ \Sigma_j &= \phi_1(P_j) + \phi_2(P_j) + \phi_3(P_j) + \phi_4(P_j) \end{aligned}$$

for $j = 1, 2, \cdots, N$.

Step 4: Selection. According to the fitness of the performance functions for each set of parameters, cull the $(N-1)/2$ weaker members of the population and reorder the sets of the parameters. The fitness of the performance functions is measured by:

$$F_j = \frac{1}{\Delta_j}, \quad \text{for} \quad j = 1, 2, \cdots, N$$

Step 5: Cross-over. Perform the Cross-over using an average cross-over function to produce the $(N-1)/2$ offsprings. The average cross-over operator takes two parents which are selected in step 4 and produces one child that is the result of averaging the corresponding fields of the two parents. In other words, the average Cross-over function is defined as:

$$P_{C_j} = \frac{P_{j+1} + P_j}{2}, \quad \text{for} \quad j = 1, 2, \cdots, \frac{N-1}{2}$$

Step 6: Mutation. A real number mutation operator, called *real number creep*, is used. The function we are optimizing is a continuous one with hills and valleys. If we are on a good hill, we want to jump around on it, to move nearer to the top. Real number creep can have that effect. What it does is to sweep along the chromosome, creeping any value up or down a small random amount. The maximum amount that this operator can alter the value of a field is a parameter of the operator. Hence it is the probability of altering any field. The mutation operation is defined as:

$$P_{M_j} = P_{C_j} + d_m \xi_j, \quad \text{for} \quad j = 1, 2, \cdots, \frac{N-1}{2}$$

where d_m is the maximum value to be altered and $\xi_i \in [-1, 1]$ is a random variable with zero mean.

Step 7: Elitism. The elitist strategy copies the best parameter set into the succeeding parameter sets. It prevents the best parameter set from loss in the succeeding parameter sets. It may increase the speed of domination of a population by a super individual, but on balance it appears to improve genetic algorithm performance. The best parameter set P_b is defined as one satisfying:

$$\Sigma_b = \min_l \{\Sigma_l : \Sigma_l \leq \Sigma_m - \alpha(\Delta_l - \Delta_m), \text{and} \Delta_l \leq \Delta_m + \delta\}$$

where

$$\Delta_m = \min\{\Delta_1, \Delta_2, \Delta_3, \Delta_4\}$$

Σ_m and Σ_l are corresponding to Δ_m and Δ_l, $\alpha > 1$ and δ is a positive number, which are given by the designer, for example $\alpha = 1.1$ and $\delta = 0.1$.

Step 8: New offsprings. Add the $(N-1)/2$ new offsprings to the population which are generated in a random fashion. Actually, the new offsprings are formed by mutating the best parameter set P_b with a probability, i.e.

$$P_{N_j} = P_b + d_n \xi_j, \quad \text{for} \quad j = 1, 2, \cdots, \frac{N-1}{2}$$

where d_n is the maximal value to be altered and $\xi_j \in [-1, 1]$ is a random with zero mean. Thus, the next population is formed by the parameter set P_{M_j} $(j = 1, 2, \cdots, (N-1)/2)$, P_{N_j} $(j = 1, 2, \cdots, (N-1)/2)$ and P_b.

Step 9: Termination checking. Continue the cycle initiated in Step 4 until convergence is achieved. The population is considered to have converged when

$$\Delta_j - \Delta_b \leq \varepsilon, \quad \text{for} \quad j = 1, 2, \cdots, N$$

where ε is a positive number.

Take the best solution in the converged generation and place it in a second "initial generation". Generate the other $N-1$ parameter sets in this second initial generation at random and begin the cycle again until a satisfactory solution is obtained or Δ_b and Σ_b cannot be reduced any further.

6.6 Detection of Incipient Sensor Faults in Flight Control Systems

As modern aircraft and onboard equipment become more and more complex, the probability of potential faults increases. One of the biggest challenges in the design of flight control systems is a requirement for the flight of the aircraft to recover safely from structural damage and/or system faults. Regardless of whether the aircraft is equipped with a special control reconfiguration capability, reliable fault diagnostic information is extremely important to the pilot. Prompt presentation of fault information to the pilot could enable him to take accommodating action to the malfunction, using system redundancy. Sensors are the most important components for flight control and aircraft safety due to their role in flight control and navigation. Any sensor fault must be detected as early as possible to prevent serious accident. The problem of detecting and isolating faults in flight control systems has been studied for many years (Deckert, Desai, Deyst and Willsky, 1977; Deckert, Desai, Deyst and Willsky, 1978; Bundick, 1985; Weiss and Hsu, 1985; Bundick, 1991), and model-based approaches have been demonstrated to be capable of detecting and isolating faults very quickly and reliably. The main challenge is the detection and isolation of incipient faults in the presence of modelling uncertainty and noise. To diagnose incipient faults, a FDI systems have to be made robust against modelling uncertainty and noise. The technique presented in this chapter is used to design robust residuals to diagnose incipient sensor faults in a flight control system.

The flight control system example considered here is the lateral control system of a remotely-piloted aircraft (Mudge and Patton, 1988). The linearized lateral dynamics are given by the state space model matrices:

$$A = \begin{bmatrix} -0.277 & 0 & -32.9 & 9.81 & 0 \\ -0.1033 & -8.525 & 3.75 & 0 & 0 \\ 0.3649 & 0 & -0.639 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} -5.432 & 0 \\ 0 & -28.64 \\ -9.49 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad D = 0_{3\times 2}$$

where the state vector and control input are:

$$\begin{bmatrix} v \\ p \\ r \\ \phi \\ \psi \end{bmatrix} = \begin{bmatrix} \text{sideslip} \\ \text{roll rate} \\ \text{yaw rate} \\ \text{bank angle} \\ \text{yaw angle} \end{bmatrix} \qquad \begin{bmatrix} \tau \\ \xi \end{bmatrix} = \begin{bmatrix} \text{rudder} \\ \text{aileron} \end{bmatrix}$$

The system is unstable and needs to be stabilized. Since the purpose of the example is to illustrate the fault detection capability, the system is simply stabilize using a state feedback controller provided by Liu and Patton (1994). The FDI system in the flight control system is illustrated in Fig.6.5 in which the input signal to actuators and the output from sensors are available for fault detection and isolation. Note that the control reconfiguration issues are not considered in this chapter, although they are very important.

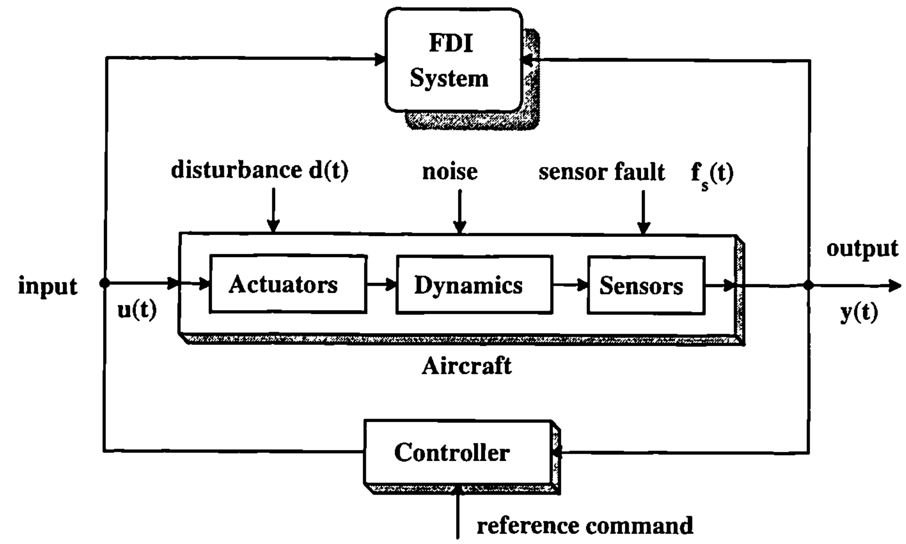


Figure 6.5: **FDI in Flight Control Systems**

An observer is designed to generate residual signal for FDI. To make the residual have the required response, the observer eigenvalues are constrained within the following regions:

$$-5 \leq \lambda_1 \leq -0.2 \quad -15 \leq \lambda_2 \leq -3$$

$$-10 \leq \lambda_{3,re} \leq -2 \quad 0.2 \leq \lambda_{3,im} \leq 4$$

$$-30 \leq \lambda_5 \leq -8$$

Note that the eigenvalue λ_4 is the conjugate of the eigenvalue λ_3, i.e., $\lambda_4 = \lambda_3^*$. The weighting penalty factors for the performance functions J_1 and J_3 are chosen as:

$$W_1(j\omega) = \frac{500}{(j\omega + 10)(j\omega + 50)}, \qquad W_3(j\omega) = \frac{1}{W_1(j\omega)}$$

which places emphasis on J_1 at low frequencies and J_3 at high frequencies. By minimizing J_1 and J_3, the fault effect can be maximized and the noise effect can be minimized. To simplify the optimization procedure, the residual weighting matrix is set as $Q = I_3$. Table 6.1 lists the performance indices for different observer gains. In this table, K_i^* $(i = 1,2,3,4)$ is the observer gain matrix which minimizes J_i $(i = 1,2,3,4)$. It can be seen that a design which minimizes a particular performance function makes all other performance functions unacceptably large. Hence, multi-objective optimization must be used to reach a reasonable compromise. In order to use the method of inequalities to solve this problem, a set of performance index bounds ε_i $(i = 1, 2, 3, 4)$ are chosen as shown in the Table 6.1.

	J_1	J_2	J_3	J_4
K_1^*	189.58	2.5949	24.8288	0.00935
K_2^*	3865.26	0.07576	23.415	0.00798
K_3^*	3274.55	0.11232	22.40	0.00798
K_4^*	3.9×10^6	10700	34600	2×10^{-7}
Bounds	2000	0.16	22.5	0.006
$K_{optimal}$	1950.7	0.1492	22.420	0.00512
K_{place}	2800.39	0.1784	22.668	0.00965

Table 6.1: **Performance indices for different designs**

The genetic algorithm is used to search for solutions which satisfy all performance index boundaries. The optimal observer gain matrix found is:

$$K_{optimal} = \begin{bmatrix} -189.1419 & 0.8083 & 18.8392 \\ 17.9317 & -0.7936 & -0.7943 \\ 15.4684 & 2.8543 & 7.6140 \\ -0.7606 & 6.9329 & 0.1537 \\ -1.2303 & 0.2329 & 9.8678 \end{bmatrix}$$

with corresponding eigenvalues:

$$\{-1.5371, -4.7045, -3.4973 \pm 2.1194i, -19.9994\}$$

The performance indices under this gain are shown in Table 6.1. This design is an acceptable compromise. To demonstrate the effectiveness of the developed method, an observer gain matrix K_{place} using the MATLAB routine *place*, to assign eigenvalues at: $\{-0.5, -14, -4.8 \pm 1.6i, -20\}$ is also designed. The performance indices for this design are also shown in the Table 6.1.

The simulation is used to assess the performance of the observer-based residual generator in the detection of incipient sensor faults. The control commands for both inputs are set as a unit sinusoid function. The sensor noise comprises a random summation of multi-frequency signals with all frequencies larger than $20 rad/s$. In the simulation, all aerodynamic coefficients have been perturbed by $\pm 10\%$. The fault is a slowly developing signal whose shape is shown in Fig.6.6.

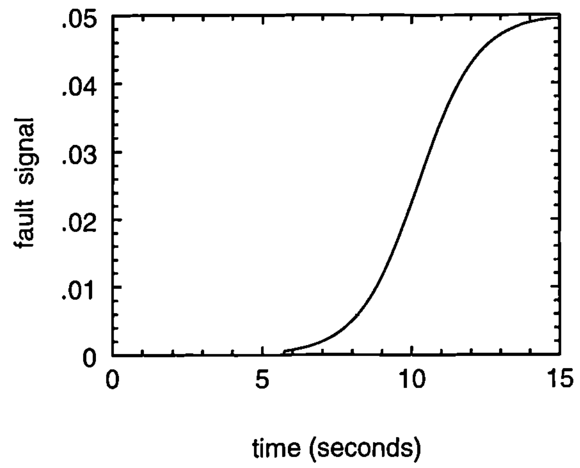


Figure 6.6: **The fault signal shape**

The simulated fault is added to the roll rate sensor. To illustrate the small nature of the incipient fault, Fig.6.7 shows the plot of both faulty and normal measurements of the roll rate p. It can be seen that the fault is hardly noticeable in the measurement and cannot be detected easily, without the assistance of the residuals

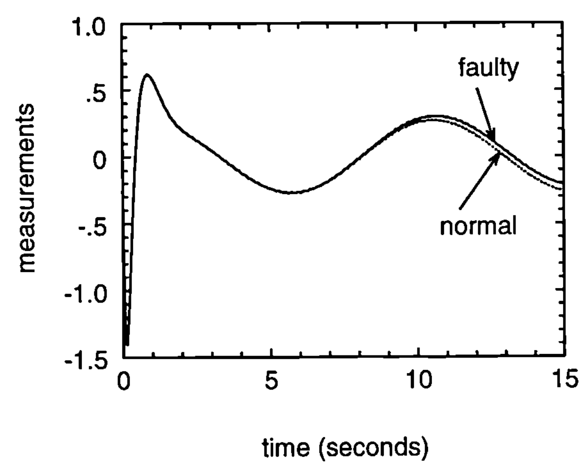


Figure 6.7: **The faulty and normal measurements for roll rate** p

Fig.6.8 shows the residual response for the case when a fault occurs in the roll rate sensor. The residual responses for other faulty cases are similar to the response shown in Fig.6.8. The residual response demonstrates that the residual changes very significantly after a fault occurs in one of the sensors. Hence, the residual can be used to detect incipient sensor faults reliably even in the presence of modelling errors and noise. To reduce the effect of noise further, the residual signal has been filtered by a low-pass filter.

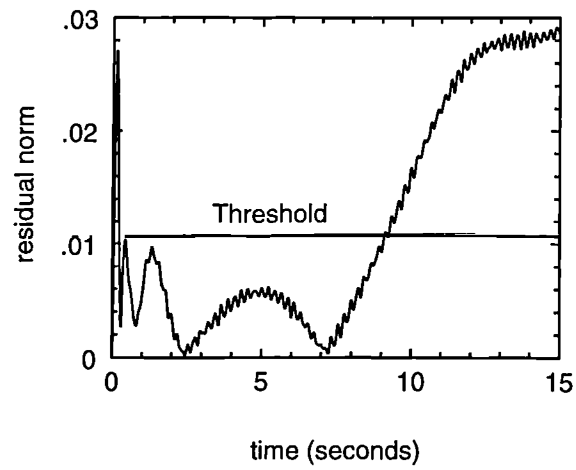


Figure 6.8: **The residual norm when a fault occurs in the roll rate sensor**

Note that this example only considers the robust residual generation for fault detection, as it is believed that the design of an optimal residual is the most important task to be considered. Fault isolation can be achieved by designing structured residual sets. For the system considered in this chapter, one can design four different observer-based residual generators to generate four residual vectors. The four observers are driven by different subsets of measurements, namely, $\{p, \phi, \psi\}$, $\{r, \phi, \psi\}$, $\{p, r, \psi\}$ and $\{p, \phi, r\}$ (Patton and Kangethe, 1989). This chapter has only considered the design of one of these observers, although the principle is valid for the design of the other observers.

6.7 Conclusions

This chapter has described a systematic approach to the design of optimal residuals which satisfy a set of objectives. These objectives are essential for achieving robust diagnosis of incipient faults. Some performance indices are expressed in the frequency domain which can take account of the frequency distribution of different factors that affect the residuals. It is the first time such a consideration has been addressed and solved in a fault diagnosis design. It has been proved that the frequency-dependent weighting factors incorporated into performance indices play an important role in the optimal design. They are problem-dependent and must be chosen very carefully. The multi-objective optimization problem has been reformulated into one of satisfying a set of inequalities on the performance indices. The genetic algorithm has been used to search the optimal solution to satisfy these inequalities on the performance indices. The method has been applied to the design of an observer-based residual generator for detecting incipient sensor faults in a flight control system and the simulation results show the effectiveness of the method. Considering the extreme difficulty in enhancing the fault diagnosis performance under modelling uncertainty and noise, any improvement in the robustness of residual design is very useful. The scope of application of this work extends to all systems with possible incipient faults.

Chapter 7

ROBUST RESIDUAL GENERATION USING OPTIMAL PARITY RELATIONS

7.1 Introduction

In Chapters 3–6, robust observer-based residual generators have been studied. This chapter focuses on the problem of robust residual generation via optimal parity relations. The parity relation is one of the most commonly accepted approaches for generating residuals. To achieve robustness for this approach, Chow and Willsky (1984) reformulated the design of parity relations for robust residual generation as a minmax optimization problem. The optimal criterion they defined specifies robustness with respect to a particular operating point, thereby allowing the possibility of adaptively choosing the best parity relations. However, the main drawback of their method is that it leads to an extremely complex optimization problem for which there is no analytical solution. Lou et al. (1986) proposed an alternative method to find "optimally robust parity relations" for generating robust residuals. They used multiple models to describe the modelling uncertainty due to parameter variations so that the residual becomes minimally sensitive to system parameters variation. The introduction of the multiple model description in parity relation design and the provision of an analytical strategy for solving the optimization problem are the main contributions of Lou et al. (1986). However, the optimal criterion they proposed seems inappropriate, because they only considered the minimization of effects of parameter variations. A residual designed to be insensitive to modelling uncertainty may also be insensitive to faults. An appropriate criterion for robust residual design should take account of both effects of modelling uncertainty and faults. Following this philosophy, Wünnenberg and Frank (Wünnenberg and Frank, 1988; Frank, 1990; Wünnenberg and Frank, 1990; Wünnenberg, 1990) studied the design of optimal parity relations by adopting a modified criterion which is the ratio of the modelling uncertainty response effect to that of the fault effect. However, the modelling uncertainty description they used was the unknown input (or disturbance) description which, as discussed in Chapters 2–5, cannot be used to represent a wide range of uncertain situations without any modification and approximation. This disappointing feature was due to the lack of application study even in a simple academic exercise or simulation setting.

This chapter re-examines the design of optimal parity relations for robust residual generation by considering the modelling uncertainty due to both parameter variations and disturbances. To generate robust residuals, two objective functions for the design of parity relations are defined. The optimization criteria are the minimisation of effects due to the modelling uncertainty and the maximization of fault effects. Together these lead to a multi-criterion optimization problem which is solved by forming a "mixed" criterion optimization problem. This criterion represents the trade-off between two design criteria, its solution is obtained using the matrix theory of generalized eigenstructure and singular value decomposition. The method used in this chapter utilizes advantages offered by studies of Lou et al. (1986) and Wünnenberg and Frank (Wünnenberg and Frank, 1988; Frank, 1990; Wünnenberg, 1990). An example is used to illustrate the method proposed, and the results show that the method is very effective for robust residual generation.

7.2 Objective Indices for Optimal Parity Relation Design

The basic principle of the parity relation approach for residual generation has been presented in Section 2.8.2. Here, the parity relation for dynamic systems with modelling uncertainty is examined. Consider the discrete-time system model with the following description:

$$\begin{cases} x(k+1) &= A_t x(k) + B_t u(k) + E_t^1 d(k) + R_t^1 f(k) \\ y(k) &= C_t x(k) + D_t u(k) + E_t^2 d(k) + R_t^2 f(k) \end{cases} \tag{7.1}$$

where $u(k) \in \mathcal{R}^r$ is the input vector, $y(k) \in \mathcal{R}^m$ is the output vector and $x(k) \in \mathcal{R}^n$ is the state vector, $f(k) \in \mathcal{R}^g$ denotes a fault vector which may contain actuator, component or sensor faults, $d(k) \in \mathcal{R}^q$ is the unknown input (or disturbance) vector. $\{A_t, B_t, C_t, D_t, E_t^1, E_t^2, R_t^1, R_t^2\}$ are system model matrices with appropriate dimensions. These matrices are not known precisely due to the modelling uncertainty and the subscript "t" denotes variation. These matrices have nominal values as: $\{A, B, C, D, E^1, E^2, R^1, R^2\}$, although their exact values are unknown.

As pointed out in Section 2.8.2, the redundancy relations can be constructed by collecting a batch of data with window length s as follows:

$$\underbrace{\begin{bmatrix} y(k-s) \\ y(k-s+1) \\ \vdots \\ y(k) \end{bmatrix}}_{Y(k)} - H_t \underbrace{\begin{bmatrix} u(k-s) \\ u(k-s+1) \\ \vdots \\ u(k) \end{bmatrix}}_{U(k)} = W_t x(k-s) + L_t \underbrace{\begin{bmatrix} d(k-s) \\ d(k-s+1) \\ \vdots \\ d(k) \end{bmatrix}}_{D(k)} + M_t \underbrace{\begin{bmatrix} f(k-s) \\ f(k-s+1) \\ \vdots \\ f(k) \end{bmatrix}}_{F(k)} \tag{7.2}$$

where

$$H_t = \begin{bmatrix} D_t & 0 & \dots & 0 \\ C_t B_t & D_t & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C_t A_t^{s-1} B_t & C_t A_t^{s-2} B_t & \dots & D_t \end{bmatrix} \in \mathcal{R}^{(s+1)m \times (s+1)r} \tag{7.3}$$

and

$$W_t = \begin{bmatrix} C_t \\ C_t A_t \\ \vdots \\ C_t A_t^s \end{bmatrix} \in \mathcal{R}^{(s+1)m \times n} \tag{7.4}$$

and the matrix M_t is constructed by replacing $\{D_t, B_t\}$ with $\{R_t^2, R_t^1\}$ in Eq.(7.3), similarly the matrix L_t is constructed by replacing $\{D_t, B_t\}$ with $\{E_t^2, E_t^1\}$ in Eq.(7.3).

To simplify the notation, Eq.(7.2) can be rewritten as:

$$Y(k) = H_t U(k) + W_t x(k-s) + L_t D(k) + M_t F(k) \tag{7.5}$$

According to Chow and Willsky (1984) and Lou et al. (1986) and from Section 2.8.2, a residual signal is defined as:

$$r(k) = v^T [Y(k) - HU(k)] \tag{7.6}$$

where $v \in \mathcal{R}^{(s+1)m}$ is the residual generating vector which can also be a matrix (see Section 2.8.2). The matrix H is the nominal value of H_t and can be constructed by replacing $\{A_t, B_t, C_t, D_t\}$ with their nominal values in Eq.(7.3).

Eq.(7.6) is the *computational form* of a residual generator which shows the residual signal as a function of measured inputs and outputs of the monitored system. Substituting Eq.(7.5) into Eq.(7.6), we have:

$$\begin{aligned} r(k) &= v^T [W_t x(k-s) + (H_t - H_0) U(k) + L_t D(k) + M_t F(k)] \\ &= v^T [W_t \quad (H_t - H_0) \quad L_t] \begin{bmatrix} x(k-s) \\ U(k) \\ D(k) \end{bmatrix} + v^T M_t F(k) \\ &= v^T Z_t X(k) + v^T M_t F(k) \end{aligned} \tag{7.7}$$

where

$$Z_t = [W_t \quad (H_t - H) \quad L_t] \;\in\; \mathcal{R}^{(s+1)m \times (n+(s+1)r+(s+1)q)}$$

$$M_t \;\in\; \mathcal{R}^{(s+1)m \times (s+1)g}$$

In order to detect faults, we should make the residual signal $r(k)$ become zero for the fault-free case and non-zero for the faulty case; this requires that:

$$v^T Z_t = 0 \tag{7.8}$$

$$v^T M_t \neq 0 \tag{7.9}$$

Normally, Z_t and M_t are unknown and time-varying, so that Eq.(7.8) cannot hold true for a wide range of modelling uncertainty. Here the uncertainty is considered as bounded, i.e. the parameter variations are contained within a pre-defined bound, e.g.

$$\begin{array}{rclcrcl} A - \Delta A \le & A_t & \le A + \Delta A & ; & B - \Delta B \le & B_t & \le B + \Delta B \\ C - \Delta C \le & C_t & \le C + \Delta C & ; & D - \Delta D \le & D_t & \le D + \Delta D \\ E^1 - \Delta E^1 \le & E_t^1 & \le E^1 + \Delta E^1 & ; & E^2 - \Delta E^2 \le & E_t^2 & \le E^2 + \Delta E^2 \\ R^1 - \Delta R^1 \le & R_t^1 & \le R^1 + \Delta R^1 & ; & R^2 - \Delta R^2 \le & R_t^2 & \le R^2 + \Delta R^2 \end{array}$$

where $A_1 \le A_2$ means that all elements of the matrix A_1 is not larger than the corresponding element in the matrix A_2. The real system matrix $A_t, \cdots$ can be any values within the pre-defined bounds. This statement is absolutely correct, however it does not provide any aids for design. To achieve a realistic design, let us consider $\{A_t, B_t, C_t, D_t, E_t^1, E_t^2, R_t^1, R_t^2\}$ in a finite set of possibilities, say $\{A_i, B_i, C_i, D_i, E_i^1, E_i^2, R_i^1, R_i^2\}$ $(i = 1, 2, \cdots, N)$ within their bounds. In practice, this might involve choosing representative points out of the actual continuous range of parameter values, reflecting any desired weighting on the likelihood or importance of particular sets of parameters. This finite selection corresponds to a multiple model system representation. In this situation, a set of corresponding matrices Z_i and M_i $(i = 1, \cdots, N)$ are obtained, and an ideal residual generation vector v should satisfy the following equations:

$$v^T Z_i = 0 \quad ; \quad i = 1, 2, \cdots, N \tag{7.10}$$

$$v^T M_i \ne 0 \quad ; \quad i = 1, 2, \cdots, N \tag{7.11}$$

The above equations can be rewritten as:

$$v^T Z = 0 \tag{7.12}$$

$$v^T M \ne 0 \tag{7.13}$$

where:

$$\begin{array}{rcll} Z & = & [Z_1, \; Z_2, \; \cdots, \; Z_N] & \in \mathcal{R}^{(s+1)m \times N(n+(s+1)r+(s+1)q)} \\ M & = & [M_1, \; M_2, \; \cdots, \; M_N] & \in R^{(s+1)m \times N(s+1)g} \end{array}$$

The condition for a solution of Eq.(7.12) to exist is that:

$$rank(Z) \le (s+1)m - 1 \tag{7.14}$$

When this condition is satisfied, a solution v^* for Eq.(7.12) exists. If this solution also satisfies Eq.(7.13), it can be used to form an optimal parity relation for generating robust residuals. However, the above condition cannot normally be satisfied for cases when the parameter variations are very significant. For such cases, it is necessary to find a rank deficient matrix Z^* which is a close approximation to the matrix Z, i.e.

$$\min \|Z - Z^*\|_F \quad \text{subject to} \quad rank(Z^*) \leq (s+1)m - 1 \tag{7.15}$$

It can be proved that the above optimization problem is equivalent to the following (Lou et al., 1986):

$$\min J_1 = \min \sum_{i=1}^{N} \| v^T Z_i \|^2 = \min v^T Z Z^T v \quad \text{s.t.} \;\; v^T v = 1 \tag{7.16}$$

A solution to this problem can only minimize the sensitivity to modelling uncertainty, it cannot guarantee the maximal sensitivity to faults. Hence, to achieve an optimally robust design, it is necessary to introduce another design objective as follows:

$$\max J_2 = \max \sum_{i=1}^{N} \| v^T L_i \|^2 = \max v^T M M^T v \quad \text{s.t.} \;\; v^T v = 1 \tag{7.17}$$

A mutually optimal solution v^* for the above two optimization problems can be used to generate robust residuals which are insensitive to modelling uncertainty. This is because we have already taken the modelling uncertainty (in the form of multiple models) into account in the problem formulation.

7.3 Robust Residual Design via Multi-Criterion Optimization

In Section 7.2, it was shown that the robust residual design is achievable by solving two optimization problems. This is a multi-criterion optimization problem and the simultaneous optimal solution may not exist. As discussed in Section 6.4, the multi-criterion optimization can be solved by the method of inequalities, combined with a prop r numerical search algorithm. However, this chapter considers analytical solutio.:s for multi-criterion optimization.

7.3.1 Solving optimization problems via SVD

The optimization problem defined in the last section is similar to the optimization problem studied in Section 5.2.5 and can also be solved via Singular Value Decomposition (SVD). Let the SVD of Z be:

$$Z = \Gamma[diag\{\sigma_1, \sigma_2, \cdots, \sigma_z\}, \quad 0]\Theta^T$$

Γ and Θ are orthogonal matrices, $\sigma_1 \leq \sigma_2 \leq \cdots \leq \sigma_z$ are singular values of Z. As shown in Lou et al. (1986), the vector v which minimizes J_1 lies in a subspace spanned by the matrix:

$$P = [\gamma_1, \cdots, \gamma_{k_1}] \tag{7.18}$$

where $\gamma_1, \cdots, \gamma_{k_1}$ are first k_1 columns of Γ and k_1 $\{= (s+1)m - rank(Z^*)\}$ is a pre-defined constant which is the possible independent solution number of the vector v of the optimization problem $\min J_1$. In this situation, the minimum of J_1 is:

$$J_1^* = \sum_{i=1}^{k_1} \sigma_i$$

Similarly, the vector v which maximizes J_2 lies in a subspace spanned by the matrix:

$$Q = [\bar{\gamma}_1, \cdots, \bar{\gamma}_{k_2}] \tag{7.19}$$

where $\bar{\gamma}_1, \cdots, \bar{\gamma}_{k_1}$ are last k_2 columns of the orthogonal matrix $\bar{\Gamma}$ and $M = \bar{\Gamma}\bar{\Sigma}\bar{\Theta}^T$.

The optimal solution v^* for minimizing J_1 and maximizing J_2 is:

$$v^* \in span\{P\} \cap span\{Q\} \tag{7.20}$$

Note that this solution is relevant to constants k_1 and k_2, and different optimal solutions can be obtained by changing these constants.

7.3.2 Solutions for multi-criterion optimization

The simultaneous optimal solution for the multi-criterion optimization does not exist, if there is no intersection between the solution spaces P and Q, i.e. $span\{P\} \cap span\{Q\} = \{0\}$. For most problems, this would be the case. Hence, a compromise should be made, i.e. one needs to find a solution which does not optimize both performance indices, but gives an acceptable design. Three methods are presented here to produce a compromised optimal solution.

Method 1: Multi-criterion optimization via optimal projection.

As a vector v which minimizes J_1 lies in $span\{P\}$, an acceptable vector v should be near to the subspace $span\{P\}$. Hence, the distance between the vector v to this subspace can be used as a measure to evaluate the satisfactory degree of the vector v to the performance index J_1. A mixed criterion J which accounts for both J_1 and J_2 is defined as:

$$
\begin{aligned}
J &= \alpha \parallel v - v_P^* \parallel^2 + \beta \parallel v - v_Q^* \parallel^2 = \alpha \parallel v - P_1 v \parallel^2 + \beta \parallel v - Q_1 v \parallel^2 \\
& \quad \text{s.t.} \quad \alpha + \beta = 1 \quad \text{and} \quad v^T v = 1 \qquad (7.21)
\end{aligned}
$$

where v_P^* and v_Q^* are projections of the vector v onto subspaces $span\{P\}$ and $span\{Q\}$ respectively, and

$$P_1 = P(P^TP)^{-1}P^T \quad ; \quad Q_1 = Q(Q^TQ)^{-1}Q^T$$

The weighting factors α and β can be adjusted to satisfy different design goals. For example, if a low missed-detection rate is required one can increase β, on the other hand if a low false detection rate is required, one can increase α. This mixed criterion formulation can be extended to include more terms (sub-indices), e.g., the residual response to noise etc. The robust residual design can be achieved by solving the following optimization problem:

$$
\begin{aligned}
\min J &= \min \parallel v - P_1 v \parallel^2 + \parallel v - Q_1 v \parallel^2 \\
&= \min v^T[\alpha(I - P_1)^T(I - P_1) + \beta(I - Q_1)^T(I - Q_1)]v \\
& \quad \text{s.t.} \ \ \alpha + \beta = 1 \ \text{ and } \ v^T v = 1 \qquad (7.22)
\end{aligned}
$$

Once again, this problem can be solved via the Singular Value Decomposition of the matrix $[\sqrt{\alpha}(I - P_1)^T \ \ \sqrt{\beta}(I - Q_1)^T]$.

Method 2: A two-stage procedure for solving multi-criterion optiır .zation problem.

As pointed outed, an optimal solution of minimizing J_1 should lie in the subspace spanned by the matrix P given by Eq.(7.18), i.e. an optimal solution is given by:

$$v = Pv_1 \tag{7.23}$$

where $v_1 \in \mathcal{R}^{k_1}$ is an unknown vector and subject to the constraint $v_1^T v_1 = 1$. Substituting v into the J_2 given by Eq.(7.17), we have

$$\max J_2 = \max v_1^T (P^T M)(P^T M)^T v_1 \quad \text{s.t.} \;\; v_1^T v_1 = 1 \tag{7.24}$$

This optimization problem can also be solved using the SVD of the matrix $P^T M$. Once v_1 has been obtained, the optimal residual generator v which minimizes the sensitivity to modelling uncertainty and maximizes the sensitivity to faults can be determined by Eq.(7.23).

<u>**Method 3:**</u> Multi-criterion optimization by minimizing a mixed performance index.

One of methods to solve the multi-criterion optimization problem is to optimize a new cost function J which accounts for both J_1 and J_2. A solution for minimizing J cannot minimize J_1 at the same time as maximizing J_2. However, it could lead to a reasonable solution for robust residual design. A sensible mixture of performance indices is their ratio (or relative magnitude), i.e.

$$J = \frac{J_1}{J_2} = \frac{v^T Z Z^T v}{v^T M M^T v} \tag{7.25}$$

Hence, the robust residual design is achievable by minimizing J. This problem can be solved by introducing the *matrix pencil* concept (Gantmacher, 1959, Vol.I, pp.310-326) as follows:

<u>**Definition 7-1:**</u> Given two quadratic forms:

$$J_1 = v^T Z Z^T v, \qquad J_2 = v^T M M^T v$$

the equation:

$$\det(ZZ^T - \lambda M M^T)$$

is called the characteristic equation of the regular matrix pencil $v^T Z Z^T v - \lambda v^T M M^T v$. The roots of this characteristic equation, denoted by:

$$\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_{(s+1)m}$$

are called the *generalized eigenvalues* of the matrix pencil.

Since $(ZZ^T - \lambda_i MM^T)$ is singular, there exists a nontrivial solution vector w_i of the following equation:

$$(ZZ^T - \lambda_i MM^T)w_i = 0$$

where w_i is called the generalized eigenvector (or principal vector) of the pencil.

Lemma 7-1: If $W = [w_1, w_2, \cdots, w_{(s+1)m}]$ is the generalized eigenvector matrix of a regular pencil $v^T ZZ^T v - \lambda v^T MM^T v$, the transformation $v = Wu$ can be applied to $v^T ZZ^T v$ and $v^T MM^T v$ simultaneously to yield (Gantmacher, 1959, Vol.I, p.314):

$$J_1 = \sum_{i=1}^{(s+1)m} \lambda_i u_i^2 \qquad J_2 = \sum_{i=1}^{(s+1)m} u_i^2$$

Proof: See Gantmacher (1959), Vol.I, pp.310-314.

Theorem 7-1: The criterion J is bounded by:

$$\lambda_1 \leq J = \frac{v^T ZZ^T v}{v^T MM^T v} \leq \lambda_{(s+1)m} \tag{7.26}$$

and

$$J = \begin{cases} \lambda_1 & \text{when} \quad v = w_1 \\ \lambda_{(s+1)m} & \text{when} \quad v = w_{(s+1)m} \end{cases} \tag{7.27}$$

Proof: Using the results given in the Lemma 7-1, we have

$$J = \frac{\lambda_1 u_1^2 + \lambda_2 u_2^2 + \cdots + \lambda_{(s+1)m} u_{(s+1)m}^2}{u_1^2 + u_2^2 + \cdots + u_{(s+1)m}^2}$$

It follows that:

$$\lambda_1 = \frac{\lambda_1 u_1^2 + \lambda_1 u_2^2 + \cdots + \lambda_1 u_{(s+1)m}^2}{u_1^2 + u_2^2 + \cdots + u_{(s+1)m}^2} \leq \frac{\lambda_1 u_1^2 + \lambda_2 u_2^2 + \cdots + \lambda_{(s+1)m} u_{(s+1)m}^2}{u_1^2 + u_2^2 + \cdots + u_{(s+1)m}^2} = J$$

If:

$$u = [1, 0, \cdots, 0]$$

we get:

$$v = w_1, \quad \text{and} \quad J = \lambda_1$$

The other side of the inequality ($J \leq \lambda_{(s+1)m}$) can be proved similarly.

◇ **QED**

From this theorem, the solution which minimizes J can be obtained via the calculation of generalized eigenvalue-eigenvectors of the matrix pencil. The MATLAB function "eig" can be used to find the generalized eigenvectors and eigenvalues.

Three methods for solving the multi-criterion optimization problem have now been given. The advantage of the first method is that it can easily satisfy different design goals (low missed-detection rate or low false detection rate) by adjusting weighting factors α and β. However, the solution procedure involves two optimization steps and is very complicated. Method 3 has the opposite advantages and disadvantages compared with Method 1. Although the way of mixing design criteria in Method 3 is the same as that given by Wünnenberg and Frank (Wünnenberg and Frank, 1988; Frank, 1990; Wünnenberg and Frank, 1990; Wünnenberg, 1990), the way of handling modelling uncertainty is completely different. The technique developed here can be applied to systems with both modelling errors and unknown disturbances, whilst the technique developed by Wünnenberg and Frank can only be used to tackle disturbances. Hence the technique developed here has wider application.

7.4 A Numerical Illustration Example

A problem of designing robust residual for a four-dimensional system operating at a set-point with two actuators and three sensors is now considered. This example is a modification to the example in Chow and Willsky (1984). The system matrices are:

$$A = \begin{bmatrix} 0.5 & -0.7 & 0.7 & 0.0 \\ 0.0 & 0.8 & 0.6\gamma & 0.0 \\ -1.0 & 0.0 & 0.0 & 0.1 \\ 0.0 & 0.0 & -\gamma & 0.4 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D = 0_{3\times 2}$$

Consider the situation when the fault occurs in the first sensor, the corresponding fault distribution matrices are:

$$R_1 = 0_{4\times 1} \quad R_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Except for two elements in the A matrix, all parameters are known exactly. The modelling uncertainty is denoted by the parameter γ whose nominal value is $\gamma_0 = 0.15$ and the bound is $\gamma \in [0.1, \; 0.2]$. Taking the parity relations of order $s = 2$, the residual is generated by the parity relation:

$$r(k) = v^T \begin{bmatrix} y(k-2) \\ y(k-1) \\ y(k) \end{bmatrix} - v^T H \begin{bmatrix} u(k-2) \\ u(k-1) \\ u(k) \end{bmatrix} \tag{7.28}$$

By choosing the uncertain parameter γ with representative values of $0.1, 0.125, 0.15, 0.175, 0.2$ within the uncertainty bound, 5 sets of model matrices are obtained. The residual generation vector v is designed by Method 3 given in the last section.

$$v = \begin{bmatrix} 0.2449 \\ 0.0375 \\ 0.1022 \\ -0.1749 \\ -0.6686 \\ -0.3415 \\ 0.3498 \\ 0.3945 \\ 0.2367 \end{bmatrix} \quad \text{and} \quad (v^T H)^T = \begin{bmatrix} -0.3530 \\ -0.1749 \\ 0.3945 \\ 0.3498 \\ 0 \\ 0 \end{bmatrix}$$

For this design, the values of the objective functions are:

$$J_1 = 1.6e^{-29}, \quad J_2 = 1.0648, \quad \frac{J_1}{J_2} = 1.505e^{-29}$$

i.e., an almost perfect robust design has been achieved. Now the simulation is used to assess the fault detection performance of the designed residual signal. The design is carried out at the nominal point ($\gamma = 0.15$), but the simulation is carried out at a non-nominal point ($\gamma = 0.1875$). Each control input is a unit step function with a small level of additive Gaussian white noise. Two faults have been added to sensor 1; one is a ramp up and ramp down signal and the other is a step signal.

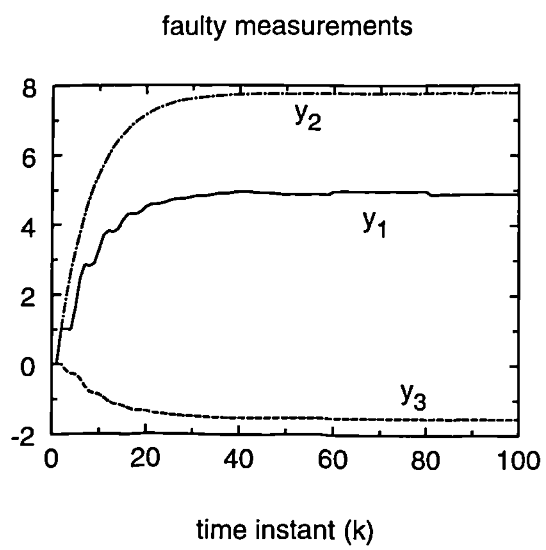


Figure 7.1: **Faulty measurement when $\gamma = 0.1875$**

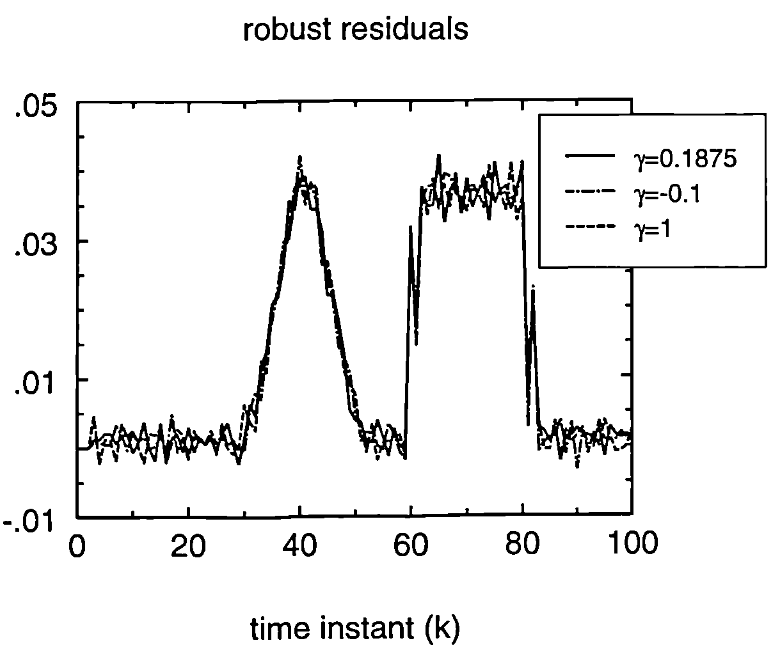


Figure 7.2: **Robust residuals for different operating points**

Fig.7.1 shows faulty measurements. It can be seen that the fault is very small and is hardly noticeable from the output. However, it is very easy to detect from the robust residual Fig.7.2. We have carried out a number of simulations in which the uncertain parameter is assigned within its bound of $\gamma \in [0.1, \ 0.2]$, and the results (shown in Fig.7.2) are almost identical except for some fluctuation due to noise.

Fig.7.2 also shows the result when the uncertain parameter is taken outside its bound, say $\gamma = -0.1$. It is interesting to see that the result is almost the same as the case when the uncertain parameter lies within its bound. To extend this idea further, if we let $\gamma = 1$, the system is unstable for this setting. Very surprisingly, the residual signal (Fig.7.2) is almost the same as the others. This shows that the residual is *robust* over a wide range of parameter variations.

7.5 Discussion on Designing Optimal Parity Relations

Robust fault isolation: Robust fault isolation can be achieved using *robust structured residual sets.* A robust structured residual is robust against modelling uncertainty and sensitive to a group of faults, whilst insensitive to another group of faults. If the fault vector $f(k)$ is re-grouped as two sub-vectors $\overline{f}(k)$ and $\underline{f}(k)$, the faults and associated distribution matrices are:

$$R_t^1 f(k) = [\overline{R}_t^1 \;\; \underline{R}_t^1] \begin{bmatrix} \overline{f}(k) \\ \underline{f}(k) \end{bmatrix} \quad ; \quad R_t^2 f(k) = [\overline{R}_t^2 \;\; \underline{R}_t^2] \begin{bmatrix} \overline{f}(k) \\ \underline{f}(k) \end{bmatrix}$$

In this case, the system equation can be rewritten as:

$$\begin{cases} x(k+1) &= A_t x(k) + B_t u(k) + [E_t^1 \;\; \underline{R}_t^1] \begin{bmatrix} d(k) \\ \underline{f}(k) \end{bmatrix} + \overline{R}_t^1 \overline{f}(k) \\ y(k) &= C_t x(k) + D_t u(k) + [E_t^2 \;\; \underline{R}_t^2] \begin{bmatrix} d(k) \\ \underline{f}(k) \end{bmatrix} + \overline{R}_t^2 \overline{f}(k) \end{cases}$$

If a structured residual is to be designed to be insensitive to faults grouped in the vector $\underline{f}(k)$, this vector can be treated in the same way as a disturbance vector in an optimal parity relation design. The performance indices should be modified correspondingly.

Probability distribution of multiple models: The probability that the system works at a certain operating point may be larger than for other operating points. This fact should be taken into consideration in the design of optimal parity relations. The performance indices are thus modified accordingly, to place different emphases on the different model descriptions:

$$\begin{aligned}\min J_1 &= \min \sum_{i=1}^{N} \| p_i v^T Z_i \|^2 \quad \text{s.t. } v^T v = 1 \\ \min J_2 &= \min \sum_{i=1}^{N} \| p_i v^T M_i \|^2 \quad \text{s.t. } v^T v = 1\end{aligned}$$

where p_i is the probability that the system operates at the i_{th} model ($i = 1, 2, \cdots, N$), and

$$\sum_{i=1}^{N} p_i = 1$$

Orthogonal parity relations: This is an approach proposed by Gertler and colleagues (Gertler, Fang and Luo, 1990; Gertler and Luo, 1989; Gertler, Luo, Anderson and Fang, 1990; Gertler and Singer, 1990; Gertler, 1991; Gertler and Kunwer, 1993) to design robust and (or) isolable residual sets. The method is based on the z-transformed input-output relationship of the monitored system, i.e.

$$\Psi_y(z)y(z) = \Psi_u(z)u(z) + \Psi_d(z)d(z) + \Psi_f(z)f(z) \tag{7.29}$$

where $\Psi_y(z)$, $\Psi_u(z)$, $\Psi_d(z)$ and $\Psi_f(z)$ are known z-polynomial matrices. A primary residual vector can be directly obtained by rearranging the above equation as follow:

$$r'(z) = \begin{cases} \Psi_y(z)y(z) - \Psi_u(z)u(z) & \text{computational form} \\ \Psi_d(z)d(z) + \Psi_f(z)f(z) & \text{evaluation form} \end{cases} \tag{7.30}$$

This primary residual can be used to detect faults, however it does not have robust and isolable properties. To design robust and (or) isolable residuals, the primary residual should be transformed as:

$$r(z) = T(z)r'(z) \tag{7.31}$$

where $T(z)$ is a z-polynomial matrix to be designed for achieving required robust and isolable properties. The response of this transformed residual to faults and disturbances is:

$$r(z) = T(z)\Psi_d(z)d(z) + T(z)\Psi_f(z)f(z) \tag{7.32}$$

To make the residual insensitive to the disturbance $d(z)$, the transformation matrix $T(z)$ should be made orthogonal to $\Psi_d(z)$, this is the basic principle of the orthogonal parity relation approach for robust residual generation. Similarly, the residual can be designed to be insensitive to the i_{th} fault component, if $T(z)$ is made to be orthogonal to the i_{th} column of $\Psi_f(z)$. If sufficient design freedom is available, a totally robust and isolable residual can be designed if the matrix $T(z)$ satisfies:

$$T(z)D(z) = 0 \quad ; \quad T(z)\Psi_f(z) = I$$

This approach is, in principle simple, however it is not easy to implement because the numerical operation of polynomial matrices is not an easy task. Moreover, this approach is only effective to uncertainty caused by unknown disturbances and cannot be directly applied to robust design against to modelling errors (Gertler, 1991; Gertler and Kunwer, 1993).

Design of robust parity relations via optimization: The design of robust residuals can be treated as an optimization problem in which fault effects should be maximized and modelling uncertainty effects should be minimized. This philosophy has been adopted in many research studies, for example, Staroswiecki and colleagues (Staroswiecki, Cassar and Cocquempot, 1993a; Staroswiecki, Cassar and Cocquempot, 1993b) have defined a multi-criterion optimization problem in robust parity relation design and the solution for this optimization has also be presented. However, they assumed that faults and/or disturbances are either pulse or step functions in the calculation of residual sensitivity cost functions, this limits the application domain of their approach. The approach presented in this chapter does not make any assumptions concerning fault and disturbance functions and hence has a wider application domain.

Kinnaert (1993a) formulated the robust parity relation design as a constrained optimization problem, the aim being to construct a number of parity relations, as follows:

$$r_i(k) = w_i^T \begin{bmatrix} Y(k) \\ U(k) \end{bmatrix} \quad ; \quad i = 1, 2, \cdots, g$$

Note that this residual definition is just a rearrangement of the definition given in Eq.(7.6). The performance index and constraints are evaluated using the expectation value of the residual under different hypothesis as follows:

$$\min_{w_i} \lim_{k \to \infty} \mathcal{E}\{r_i^2(k/\text{no fault})\}$$

subject to:

$$\begin{cases} \lim_{k\to\infty} \frac{\mathcal{E}\{r_i^2(k/\text{no fault})\}}{\mathcal{E}\{r_i^2(k/\text{fault i})\}} \leq \delta_{ii} < 1 \\ \lim_{k\to\infty} \frac{\mathcal{E}\{r_i^2(k/\text{fault j})\}}{\mathcal{E}\{r_i^2(k/\text{fault i})\}} \leq \delta_{ij} < 1 \quad ; \; j \in \{1,\cdots,i-1,i+1,\cdots,g\} \\ w_i^T w_i = 1 \end{cases}$$

where $\mathcal{E}\{\cdot\}$ denotes the expectation operator, δ_{ij} are design parameters to be decided by designer. The first constraint is to assure the robustness and the second constraint is to guarantee isolability. If the statistical properties of measurement noise, disturbances and faults are known *a priori* the optimization problem can be rewritten as:

$$\min_{w_i} w_i^T \Phi_0 w_i$$

subject to:

$$\begin{cases} w_i^T(\Phi_0 - \delta_{ii}\Phi_i) \leq 0 \\ w_i^T(\Phi_j - \delta_{ij}\Phi_i) \leq 0 \quad ; \; j \in \{1,\cdots,i-1,i+1,\cdots,g\} \\ w_i^T w_i = 1 \end{cases}$$

where Φ_i ($i = 1,\ldots,g$) are related to the statistical properties of measurement noise, disturbances and faults and a complex computation procedure is presented in Kinnaert (1993a) or Kinnaert (1993b). It can be seen that this is a constrained optimization problem with a quadratic cost function under non-convex quadratic inequality constraints. It is only possible to find a numerical solution for this optimization problem through complicated search algorithms.

The main disadvantage of Kinnaert's approach is that it requires the statistical properties of measurement noise, disturbances and faults which are normally unavailable. Another disadvantage is that the optimization procedure is very complex and there are no analytical solutions. With the cost of great complexity, there is no evidence to show that it can give diagnostic performance better than the approach presented in this chapter.

Closed-loop optimal parity relations: Wu and Wang (1993) suggested an approach to designing robust residuals based on parity checking on the output estimation errors. The approach involved two stages: the first stage is to estimate the system output and generate the output estimation error via a full-order state observer, the second is to construct parity relations using the output estimation error. As Sections 2.8.1 & 6.2.1, when a full-order observer is applied to a system without faults and modelling uncertainty, the state estimation error $e(k) = x(k) - \hat{x}(k)$ and the output estimation error $e_y(k) = y(k) - \hat{y}(k)$ are driven by the following equation:

$$\begin{cases} e(k+1) &= (A - K_oC)e(k) \\ e_y(k) &= Ce(k) \end{cases}$$

where K_o is the observer gain matrix. The output estimation error $e_y(k)$ can be used directly as a residual vector, however Wu and Wang (1993) construct the residual as:

$$r(k) = v^T \begin{bmatrix} e_y(k-s) \\ e_y(k-s+1) \\ \vdots \\ e_y(k) \end{bmatrix} \tag{7.33}$$

where the vector v^T satisfies the following equation:

$$v^T \begin{bmatrix} C \\ C(A - K_oC) \\ C(A - K_oC)^2 \\ \vdots \\ C(A - K_oC)^s \end{bmatrix} = 0$$

It can be proved that the residual generated by Eq.(7.33) is equivalent to the residual generated by Eq.(7.6) when the observer gain matrix is zero, i.e. $K_o = 0$ (Wu and Wang, 1993). This shows once again that the parity relation is a special case of the observer-based residual generator in which the dynamic feedback is zero.

Wu and Wang (1993) demonstrated an optimization procedure to find K_o and v for achieving residual robustness against modelling uncertainty. Because there is more design freedom (i.e. the choice of K_o) in the closed-loop parity relation design, the robustness and sensitivity performances can be better those of than the original parity relation design. However, the extra price to pay is the increased complexity in implementation. Wang and Wu (1993) applied the closed-loop parity relations to fault diagnosis of closed-loop control systems. They have shown that the feedback controller can also be modified to achieve maximal diagnostic sensitivity to faults. This is consistent with the idea given by Wu (1992) in which the effect of a fault in the residual is sensitized by means of feedback controller design. This also shows that the fault diagnosis scheme and the robust controller should be designed together to achieve maximal closed-loop reliability and performance.

7.6 Summary

In this chapter, the problem of finding optimally robust residuals for systems with bounded parameter variations and unknown disturbances has been studied. This parity relation design problem has been formulated into a multi-criterion optimization problem, yielding a robust residual which is maximally sensitive to faults, whilst minimally sensitive to modelling uncertainty (including modelling errors and unknown disturbances). Three methods for solving this multi-criterion optimization problem have been proposed. The simplest method is to mix performance indices as a single optimization criterion according to the design objective, which is solved using the generalized eigenvalue-eigenvector concept. As the robust criterion has been given quantitatively, the residual designed using different parity relations can be ordered according to robustness. Both modelling errors (in term of parameter variations) and disturbances have been considered in the robust residual design procedure, the technique developed can be used to diagnose incipient faults in a wide range of systems with modelling uncertainty. This principle has been well illustrated using a numerical example. Some other developments in designing optimal parity relations for robust FDI have been discussed, and these developments are also compared with the technique developed in this Chapter.

Chapter 8

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE RESEARCH

This thesis concludes by first summarising the contributions towards the development of robust model-based fault diagnosis strategies. After the summary, some suggestions for future research are given which come to light during this work.

8.1 Contributions of Thesis

The main challenge in model-based fault diagnosis is to diagnose incipient faults in complex and uncertain dynamic systems. This thesis has taken this challenge and has set the main objective as:

- **To develop robust model-based diagnostic methodologies for complex and uncertain dynamic systems, and to demonstrate these methodologies on realistic dynamic systems.**

This objective has required a number of intermediate goals to be achieved:

- To present a general framework for model-based fault diagnosis techniques and give some basic definitions.

- To develop and improve further existing strategies for robust residual generation, such as *unknown input observers*, *eigenstructure assignment* and *optimally robust parity relations*.
- To propose new theory and techniques for generating robust residuals.
- To bridge the gap between theoretical assumptions and practical reality.
- To demonstrate robust FDI techniques in realistic simulated systems.

The results presented in the previous Chapters indicate that these goals have been met and that the overall objective of the thesis has been achieved. Excellent new results arising from the research have been and continue to be published in the open literature. It is important to note that, the results are of a general nature and are applicable not only to particular systems but to a wide range of uncertain dynamic systems. In the following, the contributions are summarized chapter by chapter.

1. Chapter 1 presented an introduction to the fault diagnosis problem and outlined the structure of the thesis, followed by a brief history of model-based FDI techniques. Views expressed about different stages of international developments and contributions in this field are entirely the author's own opinion and have not been stated elsewhere. Another contribution has been the review of some important survey papers in the model-based FDI field. This provides a general review and guide-line for a newcomer in this field.

2. The basic principles of model-based FDI have been studied in Chapter 2, in which a general framework for model-based FDI has been presented. The residual generation has been identified as the essence of this framework and some basic definitions concerning residual properties have been given. The modelling of systems with all possible faults has also be studied. This chapter has provided comments upon some commonly used residual generation approaches. Their applicabilities have been discussed and a guide-line for the selection of methods has also be given. The issue of robust residual generation has been introduced and this forms a basis for the subjects studied in subsequent chapters. The chapter concluded with a discussion on integrating different diagnostic methods for diagnosing faults in complex uncertain systems.

3. Chapter 3 has given a development of unknown input observer-based robust residual generation methods. The main contributions of this chapter are the

proposal of a new full-order unknown input observer structure and subsequently, the proof of existence conditions and the development of the design procedure. Using this new structure, the robust (in terms of disturbance de-coupling) residuals can be also made to have directional properties. This theory has been well illustrated using a jet engine simulation example. The design freedom after the satisfaction of disturbance de-coupling conditions can be used to make the state estimation error have minimal variance. This enables optimal filtering and robust fault diagnosis for stochastic systems with unknown disturbances as demonstrated by means of a simplified flight control example. Robust sensor and actuator fault detection and isolation schemes have been given in this chapter and a non-linear chemical process example has been used to demonstrate robust actuator fault isolation.

4. The eigenstructure assignment approach for robust residual generation has been investigated in Chapter 4. This chapter has presented a complete and mathematically sound proof for the eigenstructure assignment approach to FDI, which has been lacking, although the approach has been developed for more than 7 years. The most important contribution of this chapter was the proposal of a new method for assigning right eigenvectors of an observer. This has extended the application domain of this powerful robust FDI approach. The chapter has also suggested and demonstrated the idea of optimizing some performance indices such as fault sensitivity, utilizing design freedom left after disturbance de-coupling conditions have been satisfied. This chapter has studied a dead-beat robust residual generation strategy for discrete-time (or sampled data) systems, and its relationship with parity relations has also been presented. Two numerical examples have been used to illustrate the design procedure and disturbance de-coupling principles developed in this chapter.

5. To bridge the gap between theoretical assumptions and practical reality, Chapter 5 has been devoted to the determination of disturbance distribution matrices for robust residual generation. The most successful robust FDI approaches developed so far are based on the disturbance de-coupling principle. To achieve a de-coupling design, one has to assume that the disturbance distribution matrix is known *a priori* although the disturbance itself can be unknown. The theory of disturbance de-coupling has been well established, however one will always face a big obstacle when the technique is applied to real uncertain systems. This obstacle is due to the mis-match between theoretical assumptions and practical reality. For most real uncertain systems, the disturbance distri-

bution matrix is unknown, or the system uncertainty is caused by modelling errors rather than disturbances. To bridge this gap, this chapter has proposed some ways of representing modelling uncertainty (including modelling errors) via a disturbance term with an estimated approximate distribution matrix. When this estimated distribution matrix is used in disturbance de-coupling design, optimal robust FDI is achievable. This chapter has studied the determination of distribution matrices for many different uncertain situations. Hence, it can be claimed that the techniques are applicable (approximate) for almost any uncertain systems. The methods developed have been assessed using a jet engine simulation model. The jet engine is a non-linear system with many uncertain factors in modelling, and the techniques developed have been highly successful. The success of this study indicates that the techniques will be applicable for a wide range of systems. The research presented in this chapter represents the one of most important contributions made by the author and colleagues and is highly regarded internationally.

6. A new approach to the design of optimal residuals for detecting incipient faults, based on multi-objective optimization and the genetic algorithm has been developed in Chapter 6. In this approach the residual is generated via an observer. To reduce false and missed alarm rates in fault diagnosis, a number of performance indices are introduced into the observer design. Some performance indices are expressed in the frequency domain to take account of the frequency distributions of faults, noise and modelling uncertainties. All objectives are then reformulated into a set of inequality constraints on performance indices. The genetic algorithm is thus used to search an optimal solution to satisfy these inequality constraints. The approach developed has been applied to a flight control system example and simulation results show that incipient sensor faults can be detected reliably in the presence of modelling uncertainty.

7. In Chapter 7, the robust residual was generated using optimally robust parity relations. The robust design has been formulated as a multi-criterion optimization problem, in which two criteria are: maximum sensitivity to faults and minimum sensitivity to modelling uncertainty. Three methods have been proposed to tackle this multi-criterion optimization problem, all these methods involved a procedure to mix all cost functions as a single performance index. The most convenient way to mix performance functions is to use the ratio between two performance indices. The optimization problem is thus solved

via the generalized eigenvector-eigenvalue concept. A numerical example is given to demonstrate the procedure for designing the robust residual. Simulation results show that robust incipient fault detection is achievable by the optimal design. Other developments of designing optimal parity relations have also been commented upon. The approach developed in Chapter 7 is applicable for a wide range of uncertain systems because both modelling errors and disturbances have been taken into consideration in the design.

8. Model-based fault diagnosis is a very rich research field and there is a large scope for new contributions. The author has, through collaboration with colleagues, studied many problems in this field. Evidence of this can be clearly seen through the list of publications at the beginning of the thesis. Some of the research conducted by the author beyond the scope of this thesis. To conclude the thesis, some directions for future research are suggested in Section 8.2, some of which are already topics being published by Prof. Patton's research group.

8.2 Recommendations for Future Research

Model-based FDI has been studied for over 20 years, however it is still an open research domain, and many problems are waiting to be tackled. The research of this thesis has inevitably had to end before all the interesting avenues for future FDI research could be explored. The author therefore lists those directions which, in the author's opinion, are the most important topics for future research.

8.2.1 Frequency domain robust residual generation techniques

The design of a residual generator in the frequency domain was first proposed by Viswanadham, Taylor and Luce (1987) based on the factorization of the transfer function matrix of the monitored system. This method was later extended and developed by Ding and Frank (1990). In the early development, this approach offered only an alternative interpretation of the residual generator, and hence it is equivalent to the time-domain design such as observers (see Section 2.8.3).

The frequency domain design really demonstrated its power in robust FDI when Viswanadham and Minto (1988) incorporated H^{∞} optimization techniques into the frequency domain residual generator design.

As studied in this thesis, there are many ways, such as the unknown input observer, eigenstructure assignment, optimally robust parity relations, for eliminating or minimizing disturbance and modelling error effects on residual and hence achieve robustness. While these techniques are different, one feature is common among them, the original framework of these methods were developed for ideal systems or with special uncertainty structure and then efforts have been made to include non-ideal or more general uncertainties. In contrast, H^{∞}-optimization is a robust design method with the original motivation firmly rooted in the consideration of various uncertainties, especially the modelling errors. H^{∞}-optimization has been developed from the very beginning with the understanding that no design goal of a system can be perfectly achieved without being compromised by an optimization in the presence of uncertainty, hence this technique is very suitable to tackle uncertainty issues. After decades of development, it is now playing a leading role in tackling the robustness problem in control systems. It is reasonable to seek the application of these results in other areas, including the robust design of FDI systems.

After Viswanadham and Minto (1988) introduced the use of H^{∞}-optimization in robust FDI design, Ding and Frank (Ding and Frank, 1991; Frank and Ding, 1993; Ding et al., 1993; Frank and Ding, 1994) have made many contributions for this approach. The main aim of their research is to maximize the following performance index:

$$J = \sup_{Q(s)} \frac{\|Q(s)G_f(s)\|_\infty}{\|Q(s)G_d(s)\|_\infty}$$

over a frequency range. Where $Q(s)G_f(s)$ is the transfer function matrix between the residual and faults, whilst $Q(s)G_d(s)$ is the transfer function matrix between the residual and disturbances. They have given a solution for this optimization problem (Frank and Ding, 1994). Qiu and Gertler (1993) also revisited the problem of designing robust FDI based on H^{∞}-optimization with some new basic concepts. They have demonstrated that $\|Q(s)G_f(s)\|_\infty$ may be smaller than $\|Q(s)G_d(s)\|_\infty$ in certain frequency range even their ratio (as defined above) has been maximized. This can cause difficulties in fault diagnosis. To overcome these difficulties, Qiu and Gertler (1993) have suggested a new strategy to solve robust FDI design problem which guarantees the lower bound of $\|Q(s)G_f(s)\|_\infty$ is well above the upper bound of $\|Q(s)G_d(s)\|_\infty$ in the required frequency range. This definitely offer a better diagnostic performance in the presence of disturbances.

It should be pointed that the transfer function matrix $G_d(s)$ can only be defined for disturbances, hence the techniques developed by (Frank and Ding, 1994) and Qiu and Gertler (1993) can only deal with robustness against disturbances. The robust problem with respect to modelling errors has still not been solved, although the above investigators have claimed that their research aim was to tackle this problem. The only solution suggested is to calculate the residual bound and set an adaptive threshold (Frank and Ding, 1994). This is very disappointing because the optimal disturbance de-coupling problem can be solved by time-domain approaches such unknown input observer or eigenstructure assignment, H_∞-optimization does not provide anything extra with respect to the problem we expected.

Despite unsatisfactory results, we are still very confident in H_∞-optimization because its full power for robust FDI has not been fully exploited. There have been some researches to tackle robustness against modelling errors directly using H_∞-optimization (Marquez and Diduch, 1992; Yao et al., 1994), however the results are still far from successful. Appleby (Appleby et al., 1991; Farrell, Appleby and Berger, 1992; Mangoubi et al., 1992) with colleagues at MIT have made some progress in solving the robust FDI problem against modelling errors when they incorporated μ synthesis with H_∞-optimization. Robust FDI design based on H_∞-optimization and μ synthesis is still in its early development, some research is still needed. In the author's opinion, this is a direction for future research which has great potential.

8.2.2 Adaptive residual generators

The system dynamics and parameters may vary or be perturbed during the system operation. A fault diagnosis system designed for system model given at the nominal condition may not perform well when applied to the system with perturbed condition. An effective way to deal with this problem is to use adaptive residual generators, i.e. to adapt or compensate the residual generators according to the change of operating conditions.

Sidar (1983) proposed a residual generation scheme using adaptive observers, in which the system parameter variations are estimated and compensated. Fig.8.1 illustrate the basic principle of this approach. The approach can be applied to linear systems with parametric variations if stability and convergence conditions are satisfied. Ding and Frank (1993) presented an adaptive residual generation approach for nonlinear uncertain dynamic systems using the adaptive observers given by Bastin and Gevers (1988), and the application of a similar approach was reported by Frank, Ding and Wochnik (1991). The main disadvantage of this approach is the complexity.

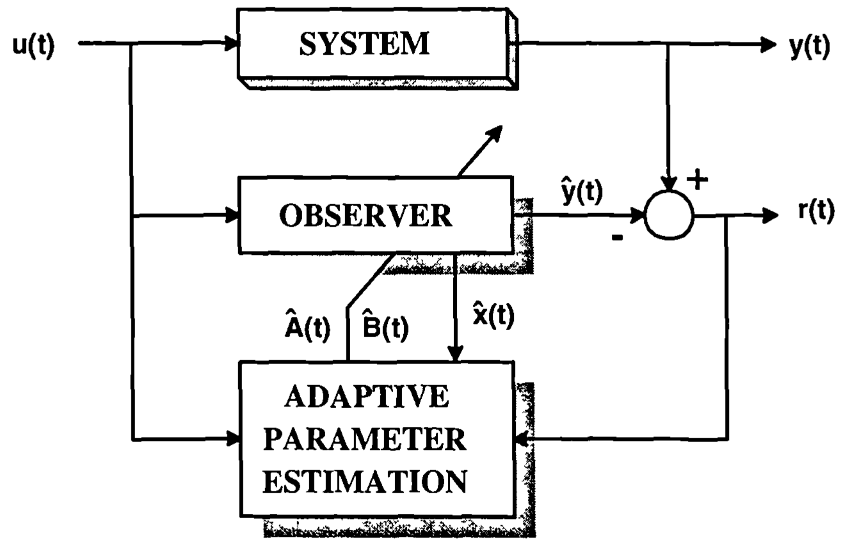


Figure 8.1: **Residual generation with adaptive observer**

Patton and Chen (1992a) proposed an alternative way to generate adaptive residuals by using so-called "on-line residual compensation method". The idea is to estimate approximately the bias term in the residuals due to modelling errors, then compensate it adaptively. These estimates are then used to form a compensated residual to decrease the effect of modelling errors on residuals. The compensated residuals are then used to make the FDI decision. The approach estimates the bias term in the residual due to the combined effect of modelling errors rather than estimating the modelling errors themselves, this avoids complicated estimation algorithms. A similar idea has been developed by Hall, Motyka, Gai and Deyst (1983) for the case of hardware redundancy generated from static models. Patton and Chen (1992a) considered the temporal redundancy case which is generated using dynamic models. This approach has been applied to a jet engine system (Patton and Chen, 1992a) and preliminary results have shown its effectiveness, however more research on this topic still necessary.

Adaptive residual generation can be achieved using any adaptive observers with some necessary modifications. Sliding mode (or variable structure) observers (Siraramirez and Spurgeon, 1994; Edwards and Spurgeon, 1994) could be a promising candidate and this has been investigated by the author and colleagues. The state and parameter simultaneous estimation algorithm presented in Ljung and Söderström (1983, pp.122–130) can also be used to generate adaptive residuals.

An adaptive residual generation algorithm normally involves both state and parameter estimation, and can be considered as a combination of observer-based and identification-based FDI approaches. Hence, complementary advantages in both approaches can be gained.

For all adaptive methods, a main problem to be tackled is that the fault effects may be compensated as well as the compensation of modelling error effects. This makes the detection impossible. This problem is very serious for incipient faults because they develop very slowly. However, for hard and abrupt faults (the magnitudes are relatively large and occur abruptly), the detection performance is acceptable because the adaptive residual and/or the parameter estimation may jump rapidly. To overcome the problem in diagnosing incipient faults, the fault function can be considered as a slow time-varying parameter which can then be estimated along with parameters (Isermann, 1994). Patton and Chen (1992a) proposed another way to tackle this problem, that is to separate the estimation process into calibration and diagnosis stages. During the calibration stage, both parameters and states are estimated adaptively. After this stage, the parameter estimation should settle down and the diagnosis stage commences. In the diagnosis stage, only the states are estimated and the parameters are fixed. It is then necessary to re-calibrate when faults have been diagnosed and corrected, and so on. This approach is based on two assumptions: the system parameters only change slowly and the fault does not occur in the calibration stage. Li and Zhang (1993) applied two different filters on the state and parameter adaption gains, based on the assumption of the parameter and the fault vary in different speed. Much research effort is still needed in the theory and application of adaptive residual generation methods.

8.2.3 Integration of fault diagnosis and reconfigurable control

A conventional feedback control design for a complex plant or vehicle systems may result in unsatisfactory performance, or even instability, in the event of malfunctions in system components. A closed-loop control system which tolerates component malfunctions, whilst still maintaining desirable performance and stability properties can be said to be a fault-tolerant or self-repairing control system which have attracted the attention of many researchers (Stengel, 1991; Patton, 1993; Stengel, 1993). Fault-tolerant control involves the automatic detection and isolation of faults in system components and the subsequent on-line reconfiguration of the control law, subsequent to fault isolation.

The conventional approach to fault-tolerant control includes the design of three separate system modules: control, FDI and reconfiguration. The control and FDI modules, which are usually designed separately, are linked through the reconfiguration module to achieve reliable control. After the FDI module detects and isolates a fault in a specific component, the reconfiguration module specifies a reconfiguration strategy for the control module. It is hoped that this will allow satisfactory control performance to be maintained in the presence of the fault. The fundamental problem with this conventional approach lies in the independent designs of the control and FDI modules and corresponding neglect of the rather significant interactions which occur between these modules. An FDI module designed for an uncontrolled system may not perform satisfactorily with the controlled system. Furthermore, a reconfigurable controller designed for a fault isolated system may fail to maintain the stability and performance of the system due to inherent limitations inadvertently imposed through the need to achieve diagnostic performance. There is therefore a need for a research study into the interactions between the control and FDI parts of the fault-tolerant system (Stengel, 1991; Nett et al., 1988; Jacobson and Nett, 1991).

Despite the apparent connections between the two subjects, most research into FDI and reconfigurable control have evolved separately (Patton, 1993). Typically, in the reconfiguration literature, it is usually assumed that a perfect FDI scheme is available, but detection delays, false and missed alarms are difficult to avoid, in practice. The requirements for achieving good system reconfiguration have also not been considered in FDI research. This problem has been considered by some researchers, e.g., Mariton (1989) discovered that detection delays could cause instability in the reconfigured system, Srichander and Walker (1993) studied the design of a fault-tolerant control system involving both FDI and reconfiguration as a stochastic stability problem. Nett et al. (Nett et al., 1988; Jacobson and Nett, 1991) proposed a four parameter controller approach to integrated control and FDI design.

Some investigators state the importance of the joint robustness problem which inevitably arises as FDI and control functions are combined together. However, not one published paper has taken up this issue from the point of view of an integrated system. There is clearly a need to study these joint robustness properties within the framework of robust fault-tolerant control (Patton, 1993). The main research direction would be to develop a simultaneous design strategy to integrate together the functions of fault diagnosis and robust control. It is expected that significant progress will be made in improving stability and performance robustness as well as fault tolerance by using the integrated design approach (Patton, 1993).

8.2.4 Fault estimation

Among fault diagnosis tasks, fault estimation is a very important one. Once the fault is estimated, the detection and isolation can be easily achieved. The estimated information of faults can help to clarify the nature of actual faults and enable the operator to diagnose them. It can help to analyse the impact of faults on the system, and can also help to recover system function under a faulty condition, i.e. reconfiguration. The reconfiguration can enable continued operation of the system under faulty situations, and give operators reasonable time to repair the system or to use alternative measures to avoid catastrophes. However, the fault estimation problem has not gained enough research attention.

Most fault diagnosis methods can be classified into two categories: parameter identification methods (Isermann, 1984; Isermann, 1993a) and parity space methods (Patton and Chen, 1991e; Patton and Chen, 1994). The latter includes the observer-based methods. For parameter identification methods, the starting point is to assume that faults appear in the system parameters. Through parameter identification, deviations in parameters and hence component faults are estimated and detected. This is one of the advantages of this approach. However, the method cannot directly be used to estimate faults in sensors and/or actuators.

In connection with parity space methods, very little research has been done to deal with the fault estimation problem. The Kalman filter can be used together with the generalized likelihood ratio test to estimate faults (Willsky and Jones, 1976), but the computation demand in this method is very high and has doubtful practical application. Friedland and Grabousky (1982) and Chen et al. (1990) used the bias-separated estimation method for estimating faults. Ding and Frank (1990) proposed the fault estimation filter, but for systems which do not satisfy the existence conditions for fault estimation filter, derivations of the output signal are involved, this is not a practical solution.

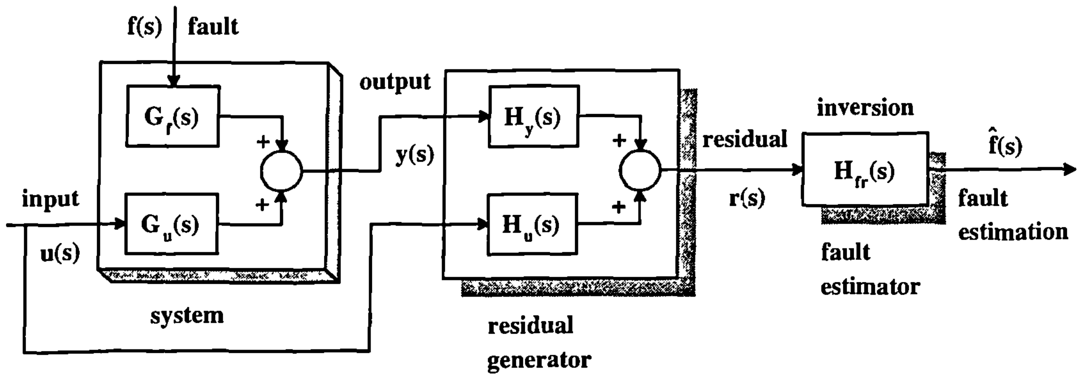


Figure 8.2: **Fault estimation**

Recently, Chen and Patton (1993) proposed a fault estimation scheme shown in Fig.8.2. In their scheme, the fault estimation is treated as a system inversion problem, i.e. to construct inputs to the system from the available outputs (Patel, 1982; Yoshikawa and Sugie, 1986). This is because the robust residual should only contain fault information. The fault can be estimated using the residual vectors and an inversion of the transfer matrix between the fault and residual. Chen and Patton (1993) presented the conditions when perfect fault estimation is possible. Otherwise, they discussed the possibility of asymptotic or optimal estimation, however satisfactory results have not been achieved yet.

8.2.5 Neural networks in fault diagnosis of nonlinear dynamic systems

The central issue in model-based fault diagnosis is the residual generation. Most residual generation techniques are based on linear system models. For nonlinear systems, the traditional approach is to linearize the system model around the system operating point. This approach is effective for many nonlinear systems, if the operating range is limited and the residual generator has been designed to be robust enough to tolerant small perturbation around the operating point. However, for systems with high nonlinearity and a wide dynamic operating range, the linearized approach fails to give satisfactorily results. A linearized model is an approximate description of the nonlinear system dynamics around the operating point. However, when the system operating range becomes wider, the linearized model is no longer able to represent the system dynamics. One solution is to use a large number of linearized models corresponding to a range of operating points. This means that a large number of FDI schemes corresponding to each operating points is needed. This is not very practical for real-time application. Hence, it is important to study the residual generation techniques which tackle nonlinear dynamic systems directly. There are some research studies on the residual generation of nonlinear dynamic systems (Wünnenberg, 1990; Frank and Seliger, 1991; Seliger and Frank, 1991a; Frank et al., 1991; Yu et al., 1994b; Yu and Shields, 1994; Krishnaswami and Rizzoni, 1994a). However, most nonlinear techniques are applicable only to a very limited class of nonlinearities, or require very strict assumptions about nonlinearity. Moreover, the design procedure is very complicated and the stability of the residual generator is not very easy to guarantee. Clearly, a generalized and effective tool is needed to deal with the residual generation problem for nonlinear dynamic system. Neural networks offer some promise due to their capability in handling nonlinear problems.

Based on residual information, the second stage of fault diagnosis is to determine whether or not a fault occurs in the system and the fault location. The main task is to discriminate effectively between normal and abnormal residuals. In the presence of noise and system uncertainty, this task becomes difficult. Hence, there is a need for an effective tool which can be used to classify the residual signal automatically. To isolate faults, the residual has to be classified further to indicate which system component is faulty. One residual signal is sufficient for fault detection, however a set of residuals (or a residual vector) is needed to fulfil the fault isolation task. One commonly accepted approach for fault isolation is to generate a set structured residual signals (see Section 2.7.1). There are a number of methods for designing structured (or isolable) residuals, however most methods are only valid for linear systems. For nonlinear systems, the joint sensitivity and insensitivity residual generation problem becomes very difficult to design. Even for linear systems, the relationship between faults and residuals can be nonlinear for parametric (or multiplicative) faults. This shows the need to develop a new general tool for fault isolation. This inspires us to use neural networks for fault isolation because neural networks can be *trained* to have the required relationships between inputs and outputs.

The neural network, as an optimal approximate tool for handling nonlinear problems, can be used to overcome difficulties in conventional techniques for dealing with nonlinearity. It is the author's opinion that there is little to be gained by applying neural networks to linear time-invariant systems. Neural networks are properly aimed at processes that are ill-defined, complex, nonlinear and stochastic. Neural networks have many advantages and can be used in a number of ways to tackle fault diagnosis problems for nonlinear dynamic systems.

Publications (Himmelblau, Barker and Suewatanakul, 1991; Kavuri and Venkatasubramanian, 1994; Naidu, Zafiriou and McAvoy, 1990; Sorsa, Koivo and Koivisto, 1991; Sorsa and Koivo, 1993; Watanabe, Matsuura, Abe, Kubota and Himmelblau, 1989; Willis, Massimo, Montague, Tham and Morris, 1991) on the use of neural networks for fault diagnosis have demonstrated the promise of this new tool. However, there are two main problems arising from these research studies. The first problem is that most publications only deal with steady-state processes. To achieve on-line fault diagnosis in the presence of transient behaviours, the system dynamics have to be considered. The second problem is that the neural network is only used as a *fault classifier* and other advantages and potential of neural networks have not been fully exploited. In these applications, neural networks were used to examine the possible fault or abnormal features in the system outputs and gives a fault classification signal to declare whether or not the system is faulty. It may be valid to use only system outputs to diagnose faults for some static systems, however this is not the case for diagnosing faults in dynamic systems because the change in system inputs can also affect certain features of the system outputs. A diagnosis method which only utilizes output information could give incorrect information about faults in the system when the system input has been changed. This is especially true for non-linear systems. It must be pointed that this problem has already been solved in model-based fault diagnosis by using the residual generation concept in which both inputs and outputs of the monitored system are used to generate a fault indicator - the residual. The input effect has been isolated from the residual and hence the residual only carries fault information. Fault diagnosis based on this properly designed residual can give reliable diagnostic information.

Recently, Patton, Chen and Siew (1994) have reported a new development in the use of neural networks for FDI. In this work, nonlinear dynamic systems have been considered and neural networks have been used in both residual generation and decision stages. This work is a significant improvement over their early work reported in Hennerberger, Patton, Chen, Wolff and Köppen (1993). Many studies are still to be done and the use of neural networks for FDI has been one of the current research topics and new results are to be published.

8.2.6 Fault diagnosis for partially-known physical systems

Model-based fault diagnosis for linear dynamic systems requires an analytical description of the system to be monitored, for example, state space equations and transfer function matrices. These analytical descriptions are generic in the sense that they can represent a wide range of physical systems including: electrical, mechanical, hydraulic, chemical and thermodynamic systems. This is the advantage of analytical system descriptions. However, at the same time, the analytical descriptions suffer from being abstractions of physical systems: the abstraction of generic features means that system specific physical details are lost. Both the parameters and states of such descriptions may not be easily related back to the original system parameters. This loss is, perhaps, acceptable for two extreme cases: when the system parameters are completely known, or when the system parameters are entirely unknown. In the former case, the physical system knowledge is translated into, e.g., transfer function parameters. In the latter case, there is no physical knowledge to be translated and extra modelling effort is required for system analysis and fault diagnosis.

For partially known physical systems the analytical model alone cannot achieve reliable fault diagnosis. No generic analytical descriptions are particularly suited to including partial physical system knowledge, descriptions of which become problem specific, not generic. Bond-Graphs have been recognized as an excellent tool to model partially known physical systems (Nagy and Ljung, 1991; Gawthrop, 1991; Gawthrop, Jones and MacKenzie, 1992b). There have been some studies in the use of Bond-Graphs for design of nonlinear system observers (Gawthrop, Jones and MacKenzie, 1992a) and fault diagnosis (Marrison and Gawthrop, 1991; Linkens and Wang, 1994). Many practical systems are partially known systems, hence it is very important to study fault diagnosis problems for this class of systems.

8.2.7 Integration of fault diagnosis techniques

The increasing complexity of processes and their high reliability and performance requirements have necessitated the development of more powerful methods for fault diagnosis. In a complex industrial system, the information available about the process may be in different formats, i.e. quantitative or qualitative, numeric or symbolic, explicit modelling knowledge and implicit expert experience, etc. To tackle the complicated diagnostic situations and to utilize diversified information, it is necessary to develop an integrated FDI system which incorporates many diagnostic strategies. To achieve the integration tasks, the following problems should be studied.

Combination of qualitative and quantitative techniques: The qualitative and quantitative techniques have many complementary properties in diagnosis, the best performance can be achieved by combined these two techniques. The combination of both will allow the use of all available analytical and heuristic information for performing diagnostic tasks and will alleviate the deficiencies with each approach.

Fuzzy logic decision-making for fault diagnosis: Due to the limitation of monitoring equipment and the difficulty in modelling and symptom extraction, the diagnostic system may need to make decisions based on incomplete information. In the context of fuzzy theory this is a typical fuzzy situation, and hence fuzzy decision logic seems to be a natural tool to handle decision-making. It may also be necessary to involve a human operator to make final decisions based on all information available, his experience and the suggestions given can be automated in the diagnostic system.

Development of a design toolbox: Many model-based fault diagnosis approaches have been developed, however many techniques are very complicated to apply without the assistance of design software. Hence, there is a great need to develop a design toolbox which can be used for new applications and further research. This toolbox should of course have a modular structure and a common information exchange standard between modules. The user will be able to select the most appropriate diagnostic techniques to suit a particular problem. Moreover, the user should be able to combine different modules to form a complete application for a given problem, and the toolbox should provide the most efficient way to link and to assure data communication between units.

Application and implementation issues: For all model-based fault diagnosis approaches, the system has been treated as a mathematical model. Hence all approaches are generic and can be applied to across a wide range of real physical systems. But, by using the model, we lose the physical reality. Many problems can be better tackled in a practical application environment. FDI schemes are normally implemented in computer software. In order to meet real-time computational constraints, the complexity of the algorithm must be considered. The residual generation stage requires more computation than the decision-making stage in FDI because after residuals have been properly designed, the decision making is just a very simple logical judgment procedure. The design procedure for residual generator is complicated for some cases, especially when robust properties are required, however the implementation format is relatively simple. In fact, it is only a processor (linear in most cases) of input and output data of the monitored system. The complexity of a residual generator depends on the order of observers or parity relations and hence can be reduced by decreasing the order of parity relations (or observers).

Appendix A: INVERTED PENDULUM

The laboratory inverted pendulum system shown in Fig.A-1 has been used as a benchmark system to demonstrate fault diagnosis techniques and concepts due to its wide availability in the control laboratory.

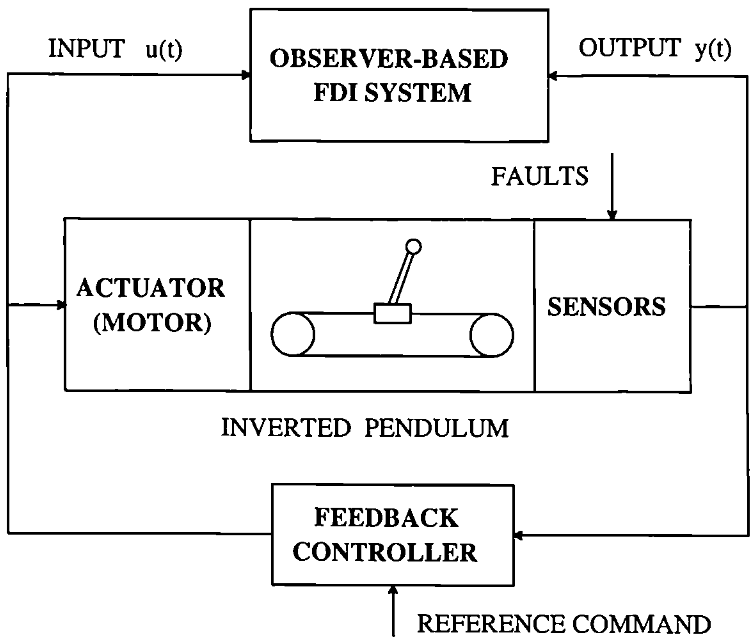


Figure A-1: The controlled inverted pendulum system

This is a nonlinear system with some uncertain factors such as friction etc. A simplified linearized is used here to illustrate the fault detectability. The linearized state space model matrices are:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -1.93 & -1.99 & 0.009 \\ 0 & 36.9 & 6.26 & -0.174 \end{bmatrix}$$

$$B = [0 \;\; 0 \;\; -0.3205 \;\; -1.009]^T$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad D = 0_{3\times 1}$$

where the state variables are: the cart position perturbation Δx, the pendulum angle ϕ, the cart velocity $\dot{x}$ and the pendulum angular velocity $\dot{\phi}$. The system is unstable and needs to be stabilized. Since the purpose of the example is to illustrate the fault detection capability, the system is simply stabilized via a state feedback controller.

A full-order observer with poles $\{-14, -20, -8 \pm 8i\}$ is used to estimate the output and the output estimation error is used as the residual signal. The steady-state gain between the residual and the faults is:

$$G_{rf}(0) = C(A - KC)^{-1}K + I$$

Where $K \in R^{4\times 3}$ is the observer gain matrix. Assume that:

$$K = [k_1 \;\; k_2 \;\; k_3] \qquad (k_i \in \mathcal{R}^4, i = 1, 2, 3)$$

$$A = [0 \;\; a_2 \;\; a_3 \;\; a_4] \qquad (a_i \in \mathcal{R}^4, i = 2, 3, 4)$$

$$(A - KC)^{-1} = [g_1, g_2, g_3, g_4]^T \qquad (g_i \in \mathcal{R}^4, i = 1, \cdots, 4)$$

Now, $G_{rf}(0) \approx C(A - KC)^{-1}K + I$ can be computed as:

$$G_{rf}(0) = \begin{bmatrix} 1 + g_1^T k_1 & g_1^T k_2 & g_1^T k_3 \\ g_2^T k_1 & 1 + g_2^T k_2 & g_2^T k_3 \\ g_3^T k_3 & g_3^T k_2 & 1 + g_3^T k_3 \end{bmatrix}$$

However

$$(A - KC)^{-1}(A - KC) = \begin{bmatrix} g_1^T \\ g_2^T \\ g_3^T \\ g_4^T \end{bmatrix} [-k_1 \;\; a_2 - k_2 \;\; a_3 k_3 \;\; a_4] = I_4$$

This leads to: $g_1^T k_1 = -1$, $g_2^T k_1 = 0$ and $g_3^T k_1 = 0$. Substituting these relations into $G_{rf}(0)$, we have:

$$G_{rf}(0) = \begin{bmatrix} 0 & * & * \\ 0 & * & * \\ 0 & * & * \end{bmatrix}$$

This proves that the strong detectability for faults in sensor 1 cannot be achieved no matter what observer gain matrix is used and what is the cart position, if the residual generator is based on a full-order observer.

Appendix B: MATRIX DECOMPOSITION

Proposition: Any $p \times q$ and rank r $(r \leq \min\{p, q\})$ matrix $E \in \mathcal{R}_r^{p\times q}$ can be decomposed as follows:

$$E = E_1 E_2$$

where $E_1 \in \mathcal{R}^{p\times r}$, $E_2 \in \mathcal{R}^{r\times q}$ and

$$rank(E_1) = rank(E_2) = r$$

Proof: According to the singular value decomposition (SVD) theorem, the matrix E can be decomposed as:

$$E = U\Sigma V^T$$

where $U \in \mathcal{R}^{p\times p}$ and $V \in \mathcal{R}^{q\times q}$ are orthogonal matrices and:

$$\Sigma = \begin{bmatrix} \Sigma_r^2 & 0_{r\times(q-r} \\ 0_{(p-r)\times r} & 0_{(p-r)\times(q-r)} \end{bmatrix} \in \mathcal{R}^{p\times q} \qquad \Sigma_r^2 = diag\{\sigma_1^2, \sigma_2^2, \cdots, \sigma_r^2\}$$

where $\sigma_1^2, \sigma_2^2, \cdots^2, \sigma_r^2$ are singular values of E.

The matrix E can be rewritten as:

$$E = U \begin{bmatrix} \Sigma_r \\ 0_{(p-r)\times r} \end{bmatrix} [\Sigma_r \quad 0_{r\times(q-r)}] V^T$$

Define:

$$E_1 = U \begin{bmatrix} \Sigma_r \\ 0_{(p-r)\times r} \end{bmatrix} = [u_1, u_2, \cdots, u_r]\Sigma \quad \in \mathcal{R}^{p\times r}$$

$$E_2 = [\Sigma_r \quad 0_{r\times(q-r)}]V^T = \Sigma_r[v_1, v_2, \cdots, v_r]^T \quad \in \mathcal{R}^{r\times q}$$

where $u_1, u_2, \cdots, u_r$ are first r columns of U and $v_1, v_2, \cdots, v_r$ are first r columns of V. It can be easily see that E_1 is a full column matrix and E_2 is a full row rank matrix.

$\diamond$ **QED.**

Appendix C: PROOF OF LEMMA 3-2

The observability matrix of (C, A) is defined as:

$$W_0 = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

The observability matrix of (C_1, A) is also defined as:

$$W_{01} = \begin{bmatrix} C \\ CA \\ CA \\ \vdots \\ CA^{n-1} \\ CA^{n-1} \end{bmatrix}$$

From Cayley-Hamilton theorem, one can seen that CA^n can be represented by a linear combination $C, CA, \cdots, CA^{n-1}$ and this leads to:

$$rank(W_0) = rank(W_{01}) = n_0$$

If we select n_0 linear independent rows vectors $p_1^T, \cdots, p_{n_0}^T$ from W_0 matrix, these row vectors are also the rows of the matrix W_{01}. These row vectors are now combined with another $n - n_0$ arbitrary independent row vectors $p_{n_0+1}^T, \cdots, p_n^T$ to construct an non-singular matrix as:

$$P = [p_1, \cdots, p_{n_0}; p_{n_0+1}, \cdots, p_n]^T$$

If one apply a transformation P to the system matrix pairs (C, A) and (C_1, A), the standard observability decompositions of (C, A) and (C_1, A) are formulated as (Chen, 1984):

$$PAP^{-1} = \begin{bmatrix} A_{11} & 0 \\ A_{12} & A_{22} \end{bmatrix} \qquad A_{11} \in \mathcal{R}^{n_0 \times n_0}$$

$$CP^{-1} = [C^* \quad 0] \qquad C^* \in \mathcal{R}^{m \times n_0}$$

$$C_1P^{-1} = [C_1^* \quad 0] \qquad C_1^* \in \mathcal{R}^{2m \times n_0}$$

where (C^*, A_{11}) and (C_1^*, A_{11}) are observable matrix pairs.

From the standardized observability decompositions shown above, it can be seen that (C, A) or (C_1, A) are detectable *iff* A_{22} is stable, i.e. the detectability of (C, A) is equivalent to the detectability of (C_1, A).

$\diamond$ **QED.**

Appendix D: LOW RANK MATRIX APPROXIMATION

Eckart-Young Theorem (Eckart and Young, 1936; Tufts et al., 1982): Let A be an $m \times n$ matrix of rank r which has complex elements. The singular value decomposition of the matrix A is:

$$A = U\Sigma V^* \quad ; \quad U \in C^{m\times m} \ , \ V \in C^{n\times n} \ , \ \Sigma \in C^{m\times n}$$

The matrices U and V are unitary, and Σ is a rectangular diagonal matrix with real and nonnegative diagonal entries. These diagonal entries, called the singular values of A, are conventionally ordered in decreasing (or increasing) order.

Let S_p be the set of all $m \times n$ matrices of rank p ($< r$). For all matrices B in S_p,

$$\|A - \hat{A}\| \leq \|A - B\|$$

where

$$\hat{A} = U\hat{\Sigma}V^*$$

and $\hat{\Sigma}$ is obtained from the matrix Σ by setting to zero all but p largest singular values. The matrix norm is the Frobenius norm. That is

$$\|A - \hat{A}\| = \sqrt{trace[(A - B)^*(A - B)]}$$

Hence, in words, $\hat{A}$ is the best least squares approximation of lower rank p to the given matrix.

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