A Talk Framework for Problem-solving

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Abstract

Group discussion in primary school mathematical problem-solving provides potentially significant benefits to children's learning, especially when discussion becomes dialogue, in which students achieve common understanding through cumulative questioning. Arising out of a need to develop talk whilst problem-solving on a primary teacher training programme in the UK, this research employs a think-aloud protocol alongside digital audio technology to promote and capture dialogue in two distinct group problem-solving situations. It explores how thinking-aloud, and the resulting recordings, can be used to support student teachers' verbalisation of mathematical problem-solving strategies, including via 'follow-up' recall opportunities, exploiting connections between thinking-aloud and stimulated recall that are distinct to this work.

In addition to using the recall situation to encourage reflection on performance, a Talk Framework, reflecting mathematical problem-solving stages, has been employed to analyse the participants' discourse. The Framework has been used to highlight potentially valuable contributions in the initial group situation whilst also identifying aspects to be revisited in the replay situation.

The findings of this work demonstrate that the provision of a think-aloud protocol alone does not guarantee the productive engagement of student teachers. Attitudes to mathematics and the group situation itself may impact on individuals' expectations of success. The Talk Framework coding, informed by Mercer's notions of cumulative, exploratory and disputational dialogue, highlights the degree to which exploratory dialogue is perhaps lacking, with a newly proposed category, "supercumulative", identifying contributions intended to provoke further explanation.

This work highlights techniques that may be of benefit to student teachers and teacher educators when developing mathematical talk. It proposes networking of think-aloud and recall techniques to encourage reflection. It demonstrates that the provision of digitally recorded animated annotations provokes more discussion of the underlying mathematics than audio alone. Ultimately, it proposes a Talk Framework that can be used to identify valuable dialogue in the original problem-solving

situation that can then be productively built upon to aid collective understanding of mathematical strategies in the subsequent recall.

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Glossary of Terms, including abbreviations

Dialogic Teaching: Used to underpin the definition and purposes of dialogue as discussed in this work, this form of teaching is designed to "stimulate and extend…[students'] thinking" (Alexander, 2010a, p.1) and is "collective, reciprocal, supportive, cumulative, [and] purposeful" (Alexander, 2018, p.564). Alexander (2005, p.12) distinguishes between discussion and dialogue in that the former is "the exchange of ideas with a view to sharing information", whilst the latter, "achiev[es] common understanding through structured, cumulative questioning and discussion which guide and prompt…"

Livescribe pens: A multi-media tool for qualitative research. *Livescribe* pens 'attach' any notes taken during an audio-recorded session to the specific audio recorded at the same moment in time. It is, therefore, possible to play back a recording of a group working on a task with their notes appearing in 'real time' alongside their comments.

PGCE: Post-graduate Certificate in Education. Generally one-year postgraduate teacher education programme (the participants of this study were on a part-time route, running across one and a half academic years).

Talk framework: Novotná (1997) and Hošpesová and Novotná (2009)'s framework has been employed in this work, over others from e.g. Pólya (1957) and Burton (1984) due to [perceived] connections between the *transformative* and *encoding* stages and the *exploratory* speech defined by Mercer (1995). The framework, then, 'meshes' the Novotná (1997) /Hošpesová and Novotná (2009) categories with those of Mercer (1995). See Literature Review and Methodology below.

Stimulated Recall Interview (SRI): A "metacognitive practice" (Bransford et al., 2000) that involves the elicitation, via prompts, of the thinking underlying the performance of a given task (Gass and Mackey, 2000; Mackey and Gass, 2005). The task may be video-recorded (or audio-recorded, as in the case of this work).

Think-Aloud Protocol (T-AP): A verbal protocol that "can provide a rich and continuous account of…underlying thinking" (Conrad et al., 2000, p.1). Participants are given "instructions to think aloud during a task…verbalizing overtly all thoughts that (in adult participants at least) would normally be silent" (Gilhooly and Green, 1996, p.43).

Task-Based Interview: (see also 2.5.2 below for the Literature Review on 'clinical interviews' and 2.5.3 on Task-Based Interviews). As discussed by Goldin (1997, p.45), such interviews provide a "four-stage exploration" of problem-solving situations, leading to the asking of "exploratory (metacognitive) questions" related to how participants thought about a particular problem (thereby encouraging discussion of the mathematics).

Verbalisation: Ericsson and Simon's (1993) work on verbalisation has been employed in this work to encourage the think-aloud process (paying attention to the types of verbalisation that may or may not impede thinking. This also relates to Kahneman's (2011) "system 1 and 2" thinking – encouraging participants to think aloud potentially slows down their mathematical problem-solving and so attention needs to be paid to precisely how much they are instructed to verbalise).

1. Introduction

1.1. Introductory Paragraph

This work is concerned with part-time postgraduate primary (PGCE) student teachers at York St John University undertaking a mathematics module focused on the skills and pedagogical knowledge required to teach the primary school mathematics curriculum (ages 3-11). It focuses on the social context of learning, relating to social constructivism and the belief that knowledge is shared and constructed in collaboration (Edwards, 2005) with others. It builds on the value attached to dialogue and group work not only in the primary classroom but also in the teaching situations encountered by the participants on their teacher education course; this value having been identified by authors such as Alexander (2008; 2010a; 2012; 2018) and Mercer (1995; 2000; 2004). The (in part Mercer-inspired) Talk Framework and associated methodologies devised and utilised in this project attempt to provide the student teachers with a means by which they can more readily capture and reconsider their own verbal and mathematical contributions when engaged in group problem-solving. The ultimate intention of this is that they will be able to identify successful strategies that can then inform their practice as primary teachers and, most specifically, as problem-solvers and teachers of mathematics beyond their teaching training programme.

1.2 Background of the Problem

While participants of the PGCE mathematics workshops in this study were encouraged to engage in whole and small group discussion of particular mathematical problems, actual opportunities for in-depth discussion of these problems, and the strategies for solving them, were severely constrained by the hour and a half sessions available. Furthermore, many of the Virtual Learning Environment (VLE) activities provided between sessions for self-study were not referred to in subsequent sessions, again due to pressure of time. The part-time primary PGCE mathematics module ran throughout the year and a half of the course, pausing twice for lengthy school experience practices of between six and eight weeks' duration. However, the majority of the workshops were timetabled for the first half of the course[1], in order to enable the students to be adequately prepared to teach mathematics while on practice. This also ensured that subject knowledge content vital for the Qualified Teacher Status (QTS) mathematics skills test had been addressed. QTS has been a very real source of anxiety for many of the students, the majority of whom had a limited background in mathematics. Consequently, with the whole mathematics curriculum to cover for age ranges 3-7 (Foundation Stage and Key Stage 1) and 5-11 (Key Stages 1 and 2), and the need to cover revision for the Skills Test questions to prepare the students for their teaching practice and programme assessments, the opportunities to focus on problem-solving, group-work and discussion were minimal. Thus, there was only one dedicated problem-solving session in the entire course. This session had to cover the different types of mathematical and word problems that might be encountered in the primary curriculum, whilst also preparing the student teachers for Key Stage 2 and 3 algebra work. There was, therefore, very little opportunity to discuss how answers had been reached or why particular strategies might have been effective. There was also perhaps a tendency for either students or tutor to 'reveal' solutions without necessarily exploring the differing methods used to reach them and/or the connections that could be made to other problem-solving situations. This may have had the effect of reinforcing the misconception that there are only very specific 'correct' answers and approaches to problems. If students did not understand, and/or were not confident to challenge solutions or ask for fuller explanations, the opportunity to address this would quickly be lost as the module moved on to the next workshop.

[1] The part-time course began in April with an initial teaching practice and the first four to five sessions of the mathematics module; the majority of the sessions were delivered in the autumn and spring term of the following academic year, prior to their final practice. The problem-solving workshop took place in the autumn term following input on 'early counting', 'the four rules of number' (addition, subtraction, multiplication and division) and more 'Key Stage 1' focussed material.

Student confidence levels amongst the part-time PGCE cohort were also a concern to the tutors on the module. Many of the 3-7 group, for example[2], had expressed a lack of confidence in teaching mathematics in their mid-module evaluations and in informal feedback provided through their academic tutors. The majority of the cohort (both 3-7 and 5-11 age ranges) were mature students, many of whom had last encountered taught mathematics lessons in secondary education many years before. For some, explaining strategies to solve mathematical problems had been identified as an issue in their teaching practice reports, and the mathematics module itself did not provide them with as many opportunities to develop this as might have been the case. The Cockroft Report (1982) – which was covered within the module as a major influence on 'recent' primary mathematics teaching – emphasises the difficulties that individuals have with 'loss of face' in front of peers when engaging in mathematics:

> There was another group consisting of those who, although able to perform the calculations which they normally required, felt a sense of inadequacy because they were aware that they did not use what they considered to be the 'proper' method; in other words, they did not make use of the standard methods for setting out written calculations which are normally taught in the classroom.
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> (Cockroft, 1982, pp.7-8)

Cockroft (1982) stresses that, for some, it is the appearance of coping that matters; they may seem to manage the problems set, but they are anxious of "anything more complicated" being asked of them, and are – in fact – "working at the limit of their mathematical competence" (Cockroft, 1982, p.7) without their peers or their teachers necessarily knowing this. Concern that this may be the case in the hour and half mathematics workshops, and that some students were clearly less comfortable answering questions in a whole group or even small group setting, led the module teaching team to consider ways in which this anxiety could be addressed. 'Safer' opportunities for mathematical discussion could, perhaps, be provided, with the possibility of feedback and reflection to take forward to future learning so that proposed solutions could be revisited, revised, even improved. From an early stage,

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[2] Although such concerns were by no means limited to that cohort – the module evaluation materials, both formal and informal, are beyond the scope of the work detailed in this thesis, although they informed an interest in confidence at the outset of the project.

it was considered that some form of technology-enhanced learning might provide opportunities to achieve this.

1.3 Statement of the Problem (Motivation for Doing the Study)

Part-time primary PGCE mathematics module tutors felt that there were too few opportunities for problem-solving and the 'revisiting' of problems to explore successful strategies on the programme. This was most specifically given the pressure of time to cover the primary mathematics curriculum and relevant subject-knowledge, but also because of recent changes to the curriculum (the new Primary National Strategy, DfES, 2006) and the recommendations of the Independent Williams Review of mathematics teaching in early years settings and primary schools (DCSF, 2008). This review, discussed in more detail in 2.6 (School and Initial Teacher Education Issues) below, had considered the state of mathematics education across the early years and primary age ranges almost ten years on from the first implementation of the National Numeracy Strategy (DfEE, 1999a). It had found that, while there were no major changes needed to the mathematics programme of study, there needed to be "an increased focus on the 'use and application' of mathematics" alongside the "vitally important question of the classroom discussion of mathematics" (DCSF, 2008, p.4). This provided a strong impetus to consider new approaches towards teaching and learning in the module.

Williams (DCSF, 2008) and others such as Kyriacou and Issitt (2008) had stressed the need to go beyond "initiate, response, feedback" (IRF) approaches in good pedagogical practice, with a deeper focus on the mathematics rather than just the 'right answer', including collaborative work that would promote high-quality talk amongst participants (this continues to be a concern in more recent work, such as Alexander (2018, p.562), discussed below in 2.2, where IRF is seen as a "pedagogical default" position, relying on "closed questions, recall answers and minimal feedback"). Group-work and productive strategies, including verbal strategies when engaged with peers (small or whole-group), was not covered in any depth at all in any of the PGCE mathematics sessions and this was considered to be problematic given the importance of dialogue in learning already highlighted by the Williams Review (DCSF, 2008) but also identified by Alexander (2008; 2010a) and in the Cambridge Primary Review (Alexander, 2010b). Indeed, other modules across the programme (for example, English and 'professional studies') included a focus on dialogue that built on Alexander's (2008; 2010a) notion of "dialogic teaching", and the more subject-knowledge heavy focus of the mathematics module could have been seen as standing in sharp contrast to this.

In addition to this problem, student anxiety around mathematics, and around discussing strategies in front of peers or in front of the whole group, was a concern. This was by no means unique to this cohort or University, as the Williams Review (DCSF, 2008, p.3) makes clear when it states, "the United Kingdom is still one of the few advanced nations where it is socially acceptable – fashionable, even – to profess an inability to cope with the subject". Nonetheless, Leitenberg's (1990, p.1) definition of "social anxiety" could be said to describe the mind-set of some of those who had come to the part-time PGCE programme many years after their last experience of mathematics in secondary school:

> …social anxiety involves feelings of apprehension, self-consciousness, and emotional distress in anticipated or actual social-evaluative situations…there has to be a belief that the situation involves scrutiny or evaluation by others regardless of whether this is actually true or not, that negative evaluation is a possible or even a likely outcome, and that the consequences of such negative evaluation would be harmful. The essence of social anxiety is that the person fears that he or she will be found to be deficient or inadequate by others and therefore will be rejected.

Whether intended or not, the grouping of PGCE students into 'mixed-ability' classes of 30 (deliberately mirroring the organisation of primary classrooms), the workshop nature of the sessions, and the time constraints within which questions were posed and answered in order to 'cover' the curriculum content, may have caused some to feel that their contributions would be negatively evaluated by more competent and confident peers (if not also the tutor). This may have been the reason why some were reticent to propose strategies and solutions in both group and 'whole class' discussions. It may also have been why some were keen to accept the solutions of others without asking for clarification. In part, it was felt that this might well be down to lack of 'thinking time'; it may also have been down to a perception that there are always strict 'right' answers to problems – perhaps a perception 'picked up' from the students' prior experiences as learners of mathematics in secondary school. The module team were aware of Selley's (1999) encouragement to teachers to set problems with multiple possible solutions as opposed to univalent ones with unique solutions. This could act as a spur to dialogue, and as an opportunity to appreciate that there can be more than one correct answer. Indeed, tutors had discussed Selley's (1999) ideas with students in the problem-solving session itself. Therefore, there was a desire to devise opportunities for such classroom discussion with problems that would lend themselves to the finding of a number of different answers; indeed, problems where the answer/s were less important, perhaps, than the strategies used to reach them.

Having identified that this needed to be addressed by the module, it became apparent that such opportunities would be difficult to provide without a major increase in teaching time. This was not possible due to the revalidation schedule and the demands of the other aspects of the course. Tutors, therefore, decided to explore opportunities to enrich the learning experience by deepening the engagement with mathematics for the students, using technology and the VLE as a means to allow this, perhaps in a more interactive fashion than had been the case with previous attempts. Initially, this was inspired by the work of Salmon (2008) regarding the potential benefits of digital audio and, specifically, podcasting. This would build on existing module podcasts that provided supplementary material to the taught sessions, such as links to relevant literature and self-study questions. It was decided that podcasts featuring the students' own voices as they tackled specific problems within a taught session might promote further discussion and reflection on their problem-solving strategies 'after the event'. Therefore, the audio recordings would subsequently be made available for them to 'revisit'.

Pólya (1957) talks of the importance of "looking back" at mathematical problems, and this fits with the emphasis on the plenary within primary education, most specifically since the introduction of the National Strategies for Literacy and Numeracy and their renewal in 2006 (DfEE, 1998; DfEE, 1999a; DfES, 2006). A plenary session, incorporating discussion of what children had learnt in the preceding lesson, had become a key part of what had come to be known in primary schools as the ‘Numeracy hour’[3]. In addition to this, discussion and group work in general in ‘numeracy’ lessons had been heavily promoted, with the section on “good mathematics teaching” within the revised Primary National Strategy document (DfES, 2006, p.65) stressing that children should “support one another in group work…[whilst being]…happy to share their ideas and…explain their reasoning and methods”. This provided a further imperative for modelling such practice to the student teachers who, in turn, would need to model this confidently to their classes.

It was thought, therefore, that the recorded problem-solving could be discussed in a future taught session, unpicking successful strategies and inviting the students to identify examples of “analogical problem-solving” (Robertson, 2001), i.e. where they had utilised approaches from other problems they had encountered within the programme and in their previous mathematics experiences in order to reach an answer (or answers). There were problems, however, with this approach – the audio recordings of students engaged in problem-solving in taught sessions were frustratingly difficult to listen to, even by those listening back to their own voices. This was largely due to the noise of the teaching space and the other groups problem-solving at the same time but also because of the limitations of the digital audio recorders. Additionally, the students had not been given any examples of how to talk and, indeed, perhaps more pertinently, how to think-aloud for the recording – the audio, therefore, made little clear sense when played back. Indeed, without any modelling of the talk that would be beneficial to the process, the students were prone to swiftly provide solutions, determine that the problem was ‘solved’, and then move on to the next one, thereby perhaps heightening the ‘problem’ with classroom discussion that individuals were not necessarily aware of the need for others to understand their proposed strategies. It was, therefore, decided that the kind of verbalisation required – indeed, the kind of thinking-aloud – needed to be more carefully considered. This led to the interest in Think-Aloud Protocols (T-AP) and the work of Ericsson and Simon (1993) on the types of verbalisation that would have

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[3] The Plenary is described in the revised 2006 Primary National Strategy Primary Framework for literacy and mathematics as “..draw[ing]…children together to review learning, to clarify misunderstandings and to plan the next steps in learning” (DfES, 2006, p.12).

less of an impact on the problem-solving process itself whilst still capturing the thoughts in a form that would be accessible to the individuals themselves and their fellow group members. For the purposes of listening back and identifying useful contributions, not just mathematically but also verbally (i.e. questioning for clarification; participants building on each other's contributions to enhance the whole group's understanding of the strategies being employed) it was considered that a framework might well be helpful and could even be used by the participants themselves to analyse their talk (thereby encouraging metacognition). This informed the interest in using a form of Stimulated Recall Interview (SRI) and, indeed, devising the Talk Framework itself that considered not just different categories of mathematical/problem-solving talk but also included a focus on the type of talk – for example, *cumulative* and *exploratory* – as influenced by the work of Mercer (1995; 2000; 2004).

This work was, then, initially prompted by a desire to increase the opportunities for talk in the part-time PGCE mathematics sessions. It ultimately became more focused on talk within group problem-solving situations, with a supporting Talk Framework inspired by the problem-solving work of Novotná (1997)/Hošpesová and Novotná (2009) and the more talk-based focus of Mercer (1995), providing one of the means by which 'valuable talk' could be identified and, perhaps, even promoted in future problem-solving opportunities.

1.4 Research Questions

The research questions to be addressed in this work developed across time to encompass the "networking" of different methodologies, specifically the use of a Think-Aloud protocol (T-AP) with a follow-up Stimulated Recall Interview (SRI) situation (Hickman, 2011; 2013; Hickman and Monaghan, 2013). They, therefore, reflect the interest identified above in promoting mathematical talk from participants and, particularly, their verbalisation of problem-solving strategies, which was of interest in enhancing the learning opportunities within the part-time PGCE mathematics module.

Research Question 1, therefore, is:

RQ 1: In what ways can dialogue/thinking-aloud recorded by digital audio support student teachers' verbalisation of problem-solving strategies?

As thinking-aloud was central to the work, RQ 2 considers the extent to which different types of verbalisation (Ericsson and Simon, 1993) might support the problem-solving situation – it was considered possible that thinking-aloud could interrupt or disturb the problem-solving process, as indeed could the group situation itself and the very fact of being recorded for later playback. Therefore, the precise instructions given to the students when engaging on the task – the level to which they had to communicate their thinking to their peers – is of interest within this work.

RQ 2. What levels of thinking-aloud most effectively support digital-audio recorded T-AP and SRI?

During the course of testing the methodologies utilised in this work, it was decided that digital audio recordings alone, even prompted by a T-AP and even when transcribed for the follow-up SRI situations, would be difficult for some participants to follow. A first iteration of the work, testing the technology and the methodology, had demonstrated this (Hickman, 2011; 2013). A different technology was, therefore, employed – the multi-media *Livescribe* pen, that would 'attach' the audio recording to the annotations made at the time. This, it was hoped, would prompt more reflection on the mathematics rather than the possible frustrations or problems of working in a group. RQ 3 is, therefore, informed by this desire to discover whether individual and group strategies would be more apparent to the participants when supported by a *Livescribe* playback. Would their reflections be more focused on the mathematics when presented with their annotations as well as their thinking-aloud? Would their strategies be clearer to those who had perhaps had difficulty following the group's working during the original T-AP session?

RQ 3. What does the use of a multi-media artefact in an SRI situation reveal to participants about their own problem-solving strategies, and those of the group?

RQ 4 below addresses the Talk Framework itself, as the chief reason for collecting the participants' verbal contributions was to identify the different types of verbal contribution made, including those that ‘helped’ and/or ‘hindered’ their successful progress towards solving the problem set. It was hoped that the students would engage with the Framework within the SRI revisitation to identify the different categories of verbalisation themselves, perhaps with the result that they would consider how to encourage these when working with children.

RQ 4. What types of talk are most evident in multi-media artefact enhanced thinking-aloud while engaged in mathematical problem-solving activity?

In order to address RQ 4, it would be necessary to analyse the participants’ talk against the Talk Framework. Indeed, this analysis would be carried out independently of the students, and it was not originally intended to share the outcomes with them, or – indeed – the Framework itself. It was ultimately decided to do this in order to provide them with the opportunity to reflect on their contributions and the degree to which the T-AP had helped them put forward, for example, *exploratory* contributions; this would also help to address RQ 1 (above).

The final two research questions focus on the way in which the methodologies (RQ 5) might impact upon the participants’ willingness to engage with the task – including the recall discussion, and whether the *Livescribe* pen (RQ 6) does, as hypothesised (see 1.5 below), prompt greater reflection on the mathematics than the digital audio supported recall alone.

RQ 5. In what ways do the networked T-AP and SRI methodologies impact upon participants’ willingness to engage with a problem-solving task?

RQ 6: Does multi-media artefact prompted recall promote greater reflection on mathematical methods/problem-solving strategies than ‘standard’ digital audio?

1.5 Hypotheses

As indicated above in 1.3, it was considered that an exercise which encouraged group discussion of problem-solving strategies that was then revisited to allow participants to question and clarify their thoughts would be of benefit to the learning of the part-time PGCE student teachers engaged in the work. It might, for example, enable them to highlight – for themselves – productive mathematical and verbal strategies, thereby providing a greater insight into their performance when working alongside their peers, including strategies that they had not been aware of in the original problem-solving situation. It might also emphasise the importance of "looking back" (Pólya, 1957) when problem-solving. The recorded discussion and its later replay would, in effect, have the potential to encourage metacognition, perhaps promoting more carefully considered dialogue in similar group situations by identifying the contributions of most value to the group's success in these tasks, and making potentially valuable exploratory dialogue (Mercer, 1995; Seal, 2006) more explicit to them. A greater awareness of the value of such verbal contributions might enable the student teacher participants to encourage and reward it in their own classroom settings, with potential benefits for their practice and, overall, perhaps their own children's learning. Ultimately, although beyond the scope of this work, the concentration on problem-solving strategies, enhancing the opportunities available within the taught sessions of the PGCE mathematics module, might also impact upon notions of self-efficacy (Bandura, 1982; 1997)[4] and their overall performance as class teachers.

The project, therefore, specifically aimed to explore how group dialogue/thinking-aloud recorded by digital audio technologies may capture participants' thinking and proposed mathematical strategies when engaged in collective problem-solving tasks. The intention was that this would promote a greater awareness of their explanations of their own thinking to others and that this might be captured in the subsequent replay opportunity – thereby encouraging metacognition and reflection on productive strategies beyond those discussed in the original problem-solving opportunity. If found to be useful, such practice could perhaps be taken forward by the participants into their future work in the classroom. This, of course, presumes that, as Selley (1999) and Mercer (1995) suggest, there is a value in collective problem-solving and

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[4] The issue of confidence when problem-solving was initially of interest when developing this project and this is reflected in the questionnaires detailed below (see 5.3 for the results). However, upon reflection, it was determined that the work undertaken here – with just the two mathematical problem-solving exercises – would not allow for confidence to be meaningfully assessed at the end of the process. See Chapter 4 below for the development of the research over time, including initial thoughts about confidence and 'following' the participants into the classroom to see the impact of their engaging with this activity.

that encouraging dialogue between peers whilst avoiding too much in the way of prompting by the teacher, as also recommended by such as Alexander (2010a) and Seal (2006), is the best way to encourage learning. Alexander's (2012, p.1) argument that there is now a "critical mass of robust evidence…[demonstrating that]…the quality of classroom talk has a measurable impact on standards of attainment", including in mathematics, indicates that it might well have such a benefit.

A Think-Aloud Protocol (T-AP) was employed to make this thinking while problem-solving more explicit than might otherwise be the case. The project further aimed to utilise the captured dialogue, supported by the problem-solving strategy informed Talk Framework referred to above, to aid the revisiting of the tasks undertaken. This relates strongly to Pólya's (1957) observation that "looking back" is an important part of the mathematical problem-solving process; indeed, as Brown and Walter (1993, p.231) state about this "well known but little practiced problem solving heuristic", "it is by looking back that one may become explicitly aware of positive (and also negative) strategies that have been used but perhaps not incorporated into one's awareness". It may also be possible to encourage reflection on alternative strategies that may have worked more effectively – something that might model effective classroom practice with primary age children. To aid this "looking back", Task-Based Interview approaches suggested by Goldin (1997) were considered and some of the questions in the Stimulated Recall (SRI) situation were informed by Goldin's (1997) work. The recall opportunities were intended to promote further reflection on and possible learning about effective group work strategies, effective verbalisation when propounding ideas to a group of peers, and effective mathematical strategies. Identifying and reflecting upon these strategies might allow for them to be taken forward to future problems of this kind in the primary classroom where such group problem-solving tasks might well be a part of the student teachers' regular mathematics teaching practice.

As stated by Bransford et al. (2000, p.19), "metacognitive practices", such as the reflection, in this work, by participants on their own, and each other's, verbal contributions, "have been shown to increase the degree to which students transfer [knowledge] to new settings and events". Alongside the degree to which this work may promote future teaching approaches in the primary classroom, T-AP/SRI exercises of this kind may, also, therefore, have the potential to highlight when "analogical problem-solving" is taking place (Robertson, 2001) i.e. when the students are using knowledge, skills and strategies from other problem-solving situations to inform current (and future) work. This was considered in the Task-Based Interview (Goldin, 1997) informed Stimulated Recall situations and highlighted via the categories in the Talk Framework. It was hoped that such highlighting of categories, when shared with the participants, would help to reinforce the value of drawing comparisons with other problem-solving tasks and, indeed, might also perhaps ameliorate failures to transfer knowledge from one situation to another. In addition, this would potentially impact on confidence when putting forward ideas in a group situation, and perhaps also in front of classes of schoolchildren when explaining strategies and when asking children to put forward ideas of their own.

Supported by the digital audio technology, therefore, the networked Think-Aloud and Stimulated Recall methodologies – often treated as distinct rather than complementary methodologies (see 2.5.4 and 2.5.5 in the Literature Review below) – were employed to enable this capture and replay of dialogue. However, after a first 'iteration' of the work designed to test the methodologies and the proposed Talk Framework (see Chapter 4), the two mathematical problem-solving exercises were ultimately not conducted or recorded in the same way. This was to allow a comparison between available digital audio technologies, with a second technology introduced to provide potential 'enhancement' of the SRI opportunity with more than just audio. As opposed to the more 'traditional' digital audio recorder, the second problem-solving exercise was captured with a *Livescribe* pen, enabling the subsequent replay to include the group's own original handwritten annotations, 'played back' to them on an interactive whiteboard screen and running in sync with the group discussion. It was hoped that this might provoke greater discussion of the mathematical strategies employed to solve the problems, as opposed to – for example – the group situation itself, and this consideration of *Livescrib*e technology and its impact, therefore, adds another layer to the work detailed below. This encompasses group problem-solving, thinking-aloud in group mathematical situations, the impact of "looking back" (Pólya, 1957) on learning and the "networking" of methodologies (Hickman and Monaghan, 2013) to allow for "knowledge elicitation" (Cooke, 1999), with the Talk Framework itself as a key part of the evidence for the effectiveness of the think-aloud process.

1.6 Research Design

This work considers the impact of replaying digital audio recordings of two distinct group problem-solving tasks on the participants' verbalisation of, and recognition of, problem-solving strategies (see RQ 1 above). Even though they had been prompted to think-aloud when engaging with the tasks, the project aimed to see whether, for example, they would provide further clarification of their strategies when listening back to their words. Additionally, would they then have a clearer understanding of the problem itself and the strategies used (both mathematical and verbal) to solve it? A talk and problem-solving informed Framework was utilised to identify the different types of verbal contributions made by participants during the problem-solving, with the intention that these would be made apparent to them when played back in a follow-up Stimulated Recall session. Indeed, the Framework, and their categorisation of their verbal contributions, was shared with them to promote further discussion about effective strategies and barriers to explaining their ideas to the others in their group.

The research design, which will be covered in more detail in Chapter 3, therefore utilises and merges the sometimes-discrete introspective techniques (Vermersch, 2009) of thinking-aloud and stimulated recall to allow for some of the participants' internal representations and intramental processes to be elicited for their joint consideration. As the identification by the participants themselves of productive verbal contributions was considered to be of importance, the research design did not stop at a Think-Aloud protocol (T-AP) informed problem-solving session (this would have provided data for the researcher regarding their talk, but would not have encouraged later reflection on what had been said). Therefore, a follow-up Stimulated Recall (SRI) session was used for the elaboration of the participants' thoughts regarding their problem-solving. This was informed by a thorough consideration of such introspective techniques (Vermersch, 2009) and the value of think-aloud protocols when considering the "social construction of knowledge in different learning situations" (Wray and Kumpulainen, 2010, p.210). The literature review also considered the impact of the researcher on the process, informed by the work of Denscombe (1998), in order to ensure that participants own views were captured as much as possible, including in the follow-up SRI session, where the aim was for participants to be prompted by their own voices (and, in the case of the *Livescribe* session, their own accompanying annotations).

Beginning from the research questions (see 1.4 above), then, the research design involved choosing appropriate mathematical problems to encourage discussion amongst the student teacher participants, and devising a merged Talk and problem-solving Framework with which to analyse their responses post T-AP recording (this supporting RQ 4's focus on the types of talk that are most evident in such a recorded situation). It further considered an appropriate degree of thinking-aloud to be encouraged while taking part in the initial data collection (Data set T-AP 1 and T-AP 2 – see 3.6 below). Initial confidence data (see 3.6 below) was collected from the participants, although, ultimately, a measure of confidence was not taken at the end of the project. This point notwithstanding, the student teachers' did comment within the follow-up SRI as to the degree to which engaging with thinking-aloud and revisiting their work had perhaps impacted upon their confidence in doing the same in future problem-solving situations. The SRI recordings were transcribed (Data set SRI 1 and SRI 2 – see 3.6 below) and these form the basis of the responses to the research questions related to digital audio supporting their problem-solving strategies (RQ 1) and also, as indicated above, the impact of thinking-aloud on their problem-solving in a group situation (RQ 2). The SRI transcripts were also to be the primary evidence for RQ 3 (what the use of a multi-media artefact – *Livescribe* – revealed to them about their problem-solving strategies, and those of the group), although this was, of course, enabled by conducting one of the T-AP sessions with a 'standard' digital audio recorder for comparison of the two approaches. RQ 5 was, in part, addressed by the Talk Framework analysis of the original recordings and RQ 6 was considered by comparing the two SRI transcripts and identifying the questions and answers related to mathematics and mathematical methods as opposed to the group situation and the research itself.

Figure 1.1 below (research design) provides a diagram of the research undertaken in order, from the devising of the initial research questions to the undertaking of the literature review and formation of the Theoretical Framework (which follows in 1.7

below). It indicates the data collection points over the course of the work, from the questionnaires that provided the background context of the participants to the recording of the group problem-solving sessions and the follow-up SRI sessions.

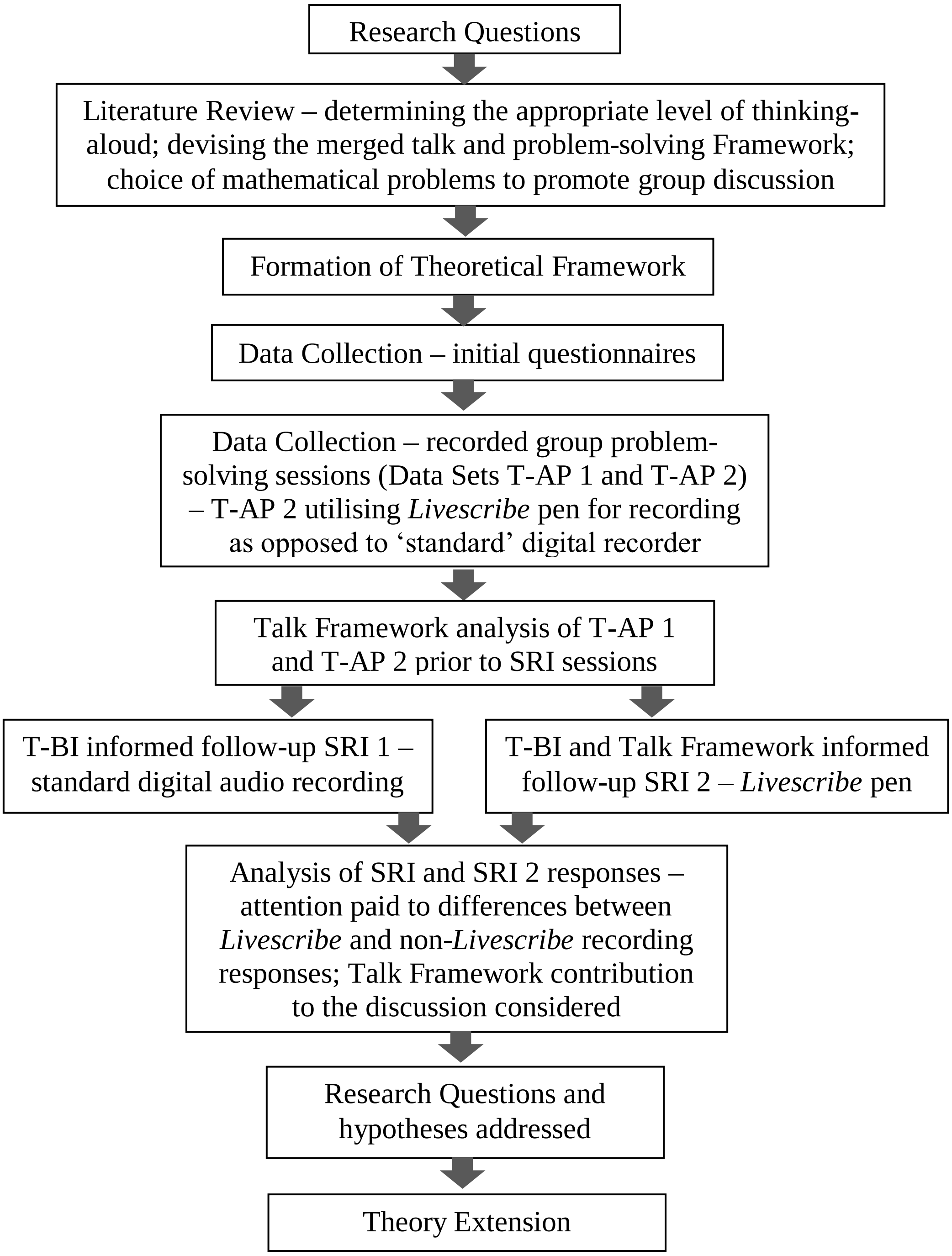

Figure 1.1: Research Design

It was envisaged that the ultimate outcome of the work would demonstrate the impact of think-aloud techniques merged with stimulated recall on encouraging participants to further clarify their mathematical strategies. It might provide further opportunities for them to practise the kinds of explanations, methods and reasoning that were considered by the Primary National Strategies (DfES, 2006, p.65) to be a key part of "good mathematics teaching" and which might, then, become a feature of their own teaching. The "networking" of T-AP and SRI methodologies (Hickman and Monaghan, 2013), further supported by the Talk Framework and by *Livescribe* as a means of capturing written work alongside verbal contributions, could well provide a successful means by which students (and perhaps also their children in later classroom work) could become more used to "shar[ing] their ideas" (DfES, 2006, p.65). Equally, they might become less bound to the notion, perhaps inculcated by the quick mental maths and Key Instant Recall Facts focus of the National Strategies (DfEE, 1999a; DfES, 2006), that mathematical solutions should be provided swiftly and even without discussion. Such work could inform future teaching on the PGCE mathematics module, and – indeed – other mathematics modules within Initial Teacher Education, and could also go on to inspire approaches used in the primary classroom.

1.7 Theoretical Framework

This work is, as indicated above, concerned with group work (specifically, group problem-solving), and group discussion. Such discussion is seen as providing benefits to learning (Alexander, 2008; 2010a; 2012; Laborde et al., 1990; Mercer, 1995; 2008; Young, 1992) and could, therefore, enhance the student teacher participants' learning on their course, both of mathematical strategies and also effective group discussion approaches for the classroom. Beyond the promotion of group work and discussion of reasoning and methods recommended by the Primary National Strategy (DfES, 2006), which the student teachers themselves would have known from the mathematics workshops, Slavin (1996) argues that encouraging students to participate in social interactions leads to a very different kind of learning from that prompted by the passive receipt of teacher input in a classroom situation. Boekarts (2001, p.18) further argues that students may find that "[a degree of] autonomy and [the] availability of peer-support" actively fosters "favorable domain-specific motivational beliefs, such as task-orientation and intrinsic motivation". The value in promoting interaction amongst mathematical problem-solving participants is thus seen to include discovery (including self-discovery), not only of the relevant mathematical knowledge and skills required to solve the particular problems presented, but also of productive methods of working collaboratively when problem-solving that may, indeed, be of use to them in future learning (and, in their case, teaching) situations. This discovery without explicit or up-front teacher direction may also include exploring ways and means to enhance productive and, indeed, 'exploratory' dialogue (Mercer, 1995; Seal, 2006), provided participants are prompted to consider the different verbal contributions they put forward in such a situation. Listening not just to their own contributions but also those put forward by their peers relies on a degree of conscious "self-monitoring" (Glaser, 1999) while doing so – carefully chosen prompts/questions can encourage this, as could the provision of a framework to analyse that talk – see 3.4 below. Identifying such productive dialogue against framework categories, for example, may benefit similar tasks in the future, as individuals could be encouraged to recognise their successful strategies and areas of development.

Given the above influences on my thoughts, I now argue that this thesis takes a social constructivist view of education. Nunes and McPherson (2007, p.18) state that "modern constructivists believe that knowledge is personally constructed from internal representations" and, indeed, this knowledge relates to the way in which an individual makes sense of their learning and ultimately understands what has been taught (Atherton, 2013). Further to this, social constructivists "emphasise… how meanings and understandings grow out of social encounters" (Atherton, 2013). In this respect, social constructivism seems to relate strongly to the work under discussion in this thesis – including the emphasis on the work of Mercer (1995; 2004), who talks of the importance of a "guided construction of knowledge" in learning. In his 1995 book of that title, Mercer observes that knowledge is regarded by some – incorrectly – as something that resides entirely "in individual people's heads" (Mercer, 1995, p. 1) but that collective endeavours have produced many of the most successful enterprises in history: "two heads are often better than one, and one result of a great many heads contributing to the construction of knowledge is the vast dynamic resource of knowledge we call 'culture'" (Mercer, 1995, p.2).

There is a connection here to be made to 2.4's consideration of learning below. Mercer (1995, p.1) acknowledges that, while knowledge does "exist… in the thought of individual people", to think of it only in this way is to undervalue it, and, indeed, the capacities of human beings: "knowledge is also a joint possession, because it can very effectively be shared" (Mercer, 1995, p.1). How it can be 'very effectively shared', though, is clearly something that requires thought when it comes to designing successful classroom situations and much of the discussion to follow in this thesis relates to this question.

As well as being able to share ideas and discoveries with each other, humans are also able to combine their mental resources. Mercer (1995) identifies problem-solving as an explicit example of a field in which this combining of resources becomes valuable. This ability to pool resources is, he says, often overlooked when studying problem-solving and/or learning in general. Most creative endeavours rest on collaboration of one kind or another, including the synthesis of ideas from multiple parties. It is, however, not the case that sharing knowledge and pooling resources is easy; Mercer (1995, p.2) makes the point that misunderstandings and bad teaching (from teachers and others) can "transform sensible ideas into nonsense". Reflection upon these misunderstandings and misconceptions – an acknowledgement that "failures are as important for our understanding of the process as the successes" (Mercer, 1995, p.2) – is valuable in acting against this. Mercer (1995) stresses that closely considering the talk that 'generates' ideas is key to learning from the errors as well as the successes in a learning situation. As with the consideration of classroom situations above, this emphasis on talk is central to the project under discussion in this thesis.

Social cultural theory also builds on the "Vygotskian conception of language as both a cultural and a psychological tool" (Mercer, 2004, p.137). Mercer (2004, p.138) emphasises the fact that social cultural theory is not "a unified field, but those within it treat communication, thinking and learning as related processes which are shaped by culture", further talking (2000) of the potential in "interthinking" where "in particular encounters or through a series of related encounters, two or more people use language to combine their intellectual resources in the pursuit of a common task" (Mercer, 2004, p.139). This also relates to Vygotsky's (1978) concepts of the "intermental" and the "intramental" or the difference between the social and the psychological in thinking. To this extent, then, the project is as much social cultural as social constructivist. Within social cultural theory, mediation and the role of the teacher is important. The absence of the designated 'teacher' in this project until the stimulated recall session/s may be problematic in this regard, although it could be argued that the role of the teacher, within the group, is passed from participant to participant (although not equally, and not without further problems relating to dominance and group dynamics that group members would later identify in their commentary on the process). As the participants were training to be teachers – and, indeed, were all some way into their practice when engaging with this research – this presents a potentially rich opportunity to consider the development of their skills, as well as putting them in the position of student or pupil engaging in dialogue, with potential benefits not only for their own continuing professional development but also, as stated above, within their classroom.

This Theoretical Framework outlines the way in which this work is informed by a social constructivist view of knowledge and that there is potential value in working as a group when engaged in activities such as problem-solving. It further indicates that problem-solving itself may be a potentially valuable exercise, with the development of talk potentially beneficial to learning. However, it seems that talk alone will not guarantee learning, and consideration, therefore, needs to be given to the differential between peers, and also to the tasks, problems and tools with which they are presented. Such aspects will inform the consideration of learning later in this thesis (most particularly in 2.2, 'What is Learning?').

1.8 Assumptions, Limitations and Scope (Delimitations)

This section covers the assumptions inherent in this work, from the emphasis on dialogue to the value of group work. It then outlines the limitations of the project, as they relate to the research questions given above in 1.4 and the data ultimately collected. The scope of the project is then elucidated before the outline of the thesis as a whole is provided in 1.9.

As indicated above in 1.5, there is an underlying assumption in this work that group work and dialogue in problem-solving is of benefit to student teachers – and to the children they will ultimately go on to work with in their own classrooms. This is based on, for example, the work of Mercer (1995) and Selley (1999). Price (2000) stresses the importance of problem-solving to mathematics education and the assumption was further made that the current focus on problem-solving in their taught sessions was lacking.

As stated above in 1.4 when addressing RQ 4, it was hoped that the student teacher participants would engage with the Talk Framework within the SRI revisitation to identify for themselves the different categories of verbalisation evident in the recording of their problem-solving – questions within the SRI would direct them to this, the Framework having been made available beforehand. This, it was further hoped, might have an impact on their work with children in the primary classroom. The design of the project, however – in part to do with student availability; in part to do with the existing demands on their time whilst on placement with a full schedule of 'school-based tasks' to complete – precluded the possibility of following them into their classrooms to see if they were able to take what they had learned from engaging in the T-AP and SRI sessions, and what perhaps they had also learned from the Talk Framework too, and translate this into classroom practice. As identified later in the thesis (see 7.5 for Further Research), this would be a worthy follow-up to this work in future.

1.9 Outline of the Thesis

The thesis is organised into seven chapters, including this Introduction. It begins with a Literature Review (Chapter 2). After considering social constructivism and its relevance to this work in the Theoretical Framework section (1.7) above, the review builds on this to outline talk in problem-solving (2.2), providing an indication of the importance of problem-solving to mathematics (2.2.1) and the place of problem-solving in current UK primary mathematics practice, before defining dialogue and 'dialogic teaching' (2.2.2) as key concepts to this work and as distinct from discussion. This section also makes reference to digital audio as a means for capturing and disseminating such dialogue, and a case is made, supported by the work of Pólya (1957), for "looking back" at/revisiting mathematical tasks before this section concludes with consideration of cognition and affect in the context of mathematics education (2.3) and learning (2.4).

Having covered learning, including active learning, and the importance of dialogue, the Literature Review then moves to considering how problem-solving performance can be ‘captured’ for later revisiting (2.5). This includes consideration of clinical interviews, Task-Based Interviews, think-aloud techniques/protocols and Stimulated Recall Interviews (SRI), as well as the analysis of talk as related to the work of Novotná (1997)/Hošpesová and Novotná (2009) and Mercer (1995). This provides a link to the Talk Framework, inspired by their work that will then be introduced in more detail in Chapter 3 (Methodology) and explored further in the Results (Chapter 5) to follow. The Literature Review concludes with a consideration of the issues faced by the student teacher participants in this study (2.6) – for example, the (then) National Framework impacting on their classroom practice and recent discussions around teaching and learning that also informed the work detailed here.

After revisiting the Research Questions addressed in this work, the Methodology builds on the Literature Review to consider the specific Think-Aloud Protocol (T-AP) employed, with reference to Ericsson and Simon (1993) regarding the differing levels of thinking-aloud that can be prompted in such a situation without impacting on actual problem-solving performance. This latter discussion is related to Kahneman’s (2011) “system 1 and 2 thinking”, with the argument made that T-AP exercises can ‘slow down’ thinking and that this can be beneficial overall to problem-solving performance. This consideration of verbalisation and T-APs (3.2) then leads into a section (3.3) on SRI and a case is made for the “networking” (Hickman and Monaghan, 2013) of the two approaches in this work – the latter perhaps ameliorating some of the impact on cognition and problem-solving aptitude of the former. The Talk Framework used to identify the different types of speech in the recorded problems is considered in 3.4, including its role within the SRI sessions that followed (the participants being introduced to the Framework and encouraged, where appropriate, to reflect on it when recalling their work). 3.5 then provides an outline of how the Task-Based Interview (Goldin, 1997) informed SRI results were analysed. These transcripts were not ‘strictly’ coded according to the Talk Framework, as this was not a problem-solving/think-aloud situation. Instead, the responses to the various questions posed have been considered according to their focus on the mathematical strategies employed or the group situation itself or, indeed, the impact of the exercise/the protocol on their (group and individual) performance. It was considered possible that the *Livescribe* playback, presenting them with evidence of their 'working out' alongside their verbal contributions, might prompt more discussion of the mathematics – and this discussion ultimately might produce 'new' *exploratory* talk that can then be coded according to the Talk Framework (hence the Framework was selectively applied to some of the SRI contributions, although by no means all).

The Methodology chapter concludes with a breakdown of the Data Sets (3.6) utilised in the Results (Chapter 5) to follow, identifying the aspects that address specific research questions and providing references to the Appendices at the conclusion of the thesis.

Before the Results, there is a brief chapter (Chapter 4) outlining the development of the research across time. This builds on the discussion of the research questions at the outset of Chapter 3, summarises some of the outcomes of the initial iteration of the project not covered above in the Methodology and leads into the Results (Chapter 5) which relate only to the second iteration of the project and the decision to record two problem-solving sessions to compare the impact of a 'standard' digital audio recording versus a *Livescribe* recording. Chapters 3, 5 and 6 are structured, as far as possible, to run 'in parallel' with each other. For example, Chapter 5 (Results) presents the data from the Talk Framework coding of the participants' verbal comments from the T-AP sessions in 5.4; the Talk Framework is addressed in the Methodology (Chapter 3) in 3.4 and then 6.4 of the Discussion chapter considers the effect of the T-AP on group engagement and problem-solving as evidenced through these Talk Framework coded responses. Likewise, 5.5 presents the outcomes of the SRI sessions (first detailed in 3.5) with 6.5 reflecting upon and discussing these results. In both the Results and the Discussion chapters, data relating to the specific group is presented first (for example, their initial confidence ratings), before considering the T-AP results and the SRI results separately – and, therefore, in the order they were encountered by participants.

The Conclusion (Chapter 7) summarises what has been learned from the participants engaging in the T-AP and subsequent Talk Framework supported SRI sessions by first considering the Research Questions in order (7.2). It then addresses the implications for group mathematical problem-solving (7.3.1), primary teachers and primary practice (7.3.2) and Initial Teacher Education practice (7.3.3), the latter returning to the statement of the problem provided above in 1.3. Limitations are addressed in 7.4 and the thesis then concludes with consideration of further research that may arise from this study (7.5).

2. Literature Review

2.1 Introduction

This Literature Review explores key concepts underlying the methodology and approaches taken by this work in addressing the research questions given above in 1.4. Building on the Theoretical Framework (1.7 above), and following the statement given at the outset of the thesis (1.1) about "the value attached to dialogue and group work not only in the primary classroom but also in the teaching situations encountered by the participants on their teacher education course", it begins by considering the role and potential value of "Talk in Problem-Solving (2.2). Having provided a definition of problem-solving in mathematics (in 2.2.1) that identifies its centrality to mathematics education, going beyond the statements made in both the revised Primary National Strategy (DfES, 2006) and the Williams Review (DCSF, 2008), it relates this talk to the notion of dialogic teaching (Alexander, 2008; 2010a; 2018) in 2.2.2, which is used to underpin the aims of the kind of dialogue promoted amongst participants in this project. Connections are further made to "self-generation and memory" (Mulligan and Lozito, 2014) in learning situations (this is revisited in 2.5 below), most particularly given the focus in this work on thinking-aloud and the degree to which this encourages participants to concentrate on their own words when participating in a group and then listening back to their contributions (this can, it is argued, impact on their attention to the contributions of their peers). This listening back is related to the productive problem-solving strategy of "looking back" (Pólya, 1957).

With acknowledgement given to the affective aspects of talk in problem-solving in 2.2, 2.3 considers "Cognition and Affect in the Context of Mathematics Education" in more depth before 2.4 then provides a more general outline of learning ("What is Learning?"), referring back to 2.2 and psychological aspects such as Mulligan and Lozito's (2014) "self-generation and memory" alongside the notion of active learning as discussed by such as Niemi (2002), with the latter being encouraged in primary education contexts and providing a further imperative for engaging individuals and groups in both problem-solving activities and dialogue.

Following the opening sections relating to talk in problem-solving, affective aspects, socio-cultural underpinning, and learning, section 2.5 of this Literature Review then provides an outline of the ways in which this problem-solving performance can be captured and, indeed, has been captured for consideration in the later chapters of this thesis. This will include the interview techniques available to those whose work is informed by 'verbal reports', and indeed those who believe that such reports can provide "a rich and continuous account of…underlying thinking" (Conrad et al., 2000, p.1). Novotná (1997) and Hošpesová and Novotná's (2009) work on problem-solving, which was used, in part, to inform the Talk Framework ultimately used in this work, will be outlined in 2.5.1, before 2.5.2 to 2.5.5 provide overviews of clinical interviews, Task-Based Interviews (Goldin, 1997), Think-aloud techniques, and Stimulated Recall, all of which contributed to the ultimate research design.

The Literature Review concludes with "School and Initial Teacher Education Issues" (2.6), as impacting on student teachers and their tutors in England and Wales. This provides an account of the issues surrounding the teaching of mathematics in primary education in this country as encountered by the participants in this project and also as reported on by the Williams Review (DCSF, 2008).

2.2 Talk in Problem-Solving

This section outlines the importance of talk in mathematical problem-solving and provides a rationale for the focus on talk and, more specifically, dialogue (also defined within this chapter) in this work. In order to address both talk and problem-solving, it first situates problem-solving within the primary mathematics curriculum after the revision to England and Wales' National Numeracy Strategy (DfEE, 1999a) as the Primary National Strategy (DfES, 2006) in 2.2.1 below. This provides a definition of problem-solving with reference to such curriculum documentation, and is relevant to the primary student teachers' understanding of problem-solving, given the emphasis on such material in their PGCE mathematics workshops. The section further situates problem-solving within the different types of learning students and children will encounter when undergoing mathematical tasks themselves in the classroom (i.e. the different 'types' of mathematics taught in the primary classroom). Furthermore, it provides a rationale for problem-solving's centrality in mathematical education, as a means for the development and assessment of mathematical skills, again with an emphasis on primary education, and as espoused by Williams (DCSF, 2008), the Primary National Strategy (DfES, 2006), Shoenfeld (1985; 1992), and Orton (2004). Talk/dialogue provides opportunities for such development and assessment, as will be outlined below, further supported by Burton's (1984, p.9) arguments for the importance of problem-solving given the "effect it has on the classroom". This Literature Review outlines talk's "discursive, cognitive and educational potential" (Alexander, 2018, p.562), potential that Alexander (2018) observes is still going to waste in an environment where 'older', more Initiation-Response-Feedback (I-R-F) structures remain the "pedagogical default".

2.2.1 Problem-Solving in Mathematics

Orton (2004) and Swan (2006) identify that there are four different types of learning when engaging with mathematics – "retention and recall, using algorithms, learning concepts, and problem-solving" (Swan, 2006, p.39). These aspects are reflected in the units, 'strands' and 'blocks and units' of the Numeracy Strategy (DfEE, 1999a; revised as the Primary National Strategy, DfES, 2006). Of these types of learning, all given weight within the primary curriculum, problem-solving is seen as particularly valuable within the primary classroom – and, therefore, valuable to those training to be their teachers – because of its ability to enable the learned number facts, concepts and algorithms to be put to use. Burton (1984, p.10), in discussing the "overwhelming importance" of problem-solving, states that it promotes "a spirit of enquiry and through that spirit…establish[es] different styles of teaching and learning". As "skills which have already been acquired are exercised…one of its major services is to enable pupils to start from where they are and use whatever they can to make progress" (Burton, 1984, p.10). This spirit of enquiry requires questioning, challenging, and reflecting, and these three important aspects strongly relate to the discussion below in 2.2.2 about one particular style of teaching and learning, that of "dialogic teaching" (Alexander, 2008; 2010a; 2018).

It is arguable that listing problem-solving last in the different types of mathematical learning, as Swan (2006, p.39) does above, after the obviously also crucial aspects of "retention and recall, using algorithms, [and] learning concepts" gives the impression that it stands apart from these other aspects, perhaps as an 'add-on' to the learning of mathematics, an ‘end product’ of mathematical learning, when the need for appropriate contexts for mathematical recall and the opportunity to test learned concepts and relevant algorithms are, as Hošpesová and Novotná, (2009) indicate, inextricably bound up with the practice of problem-solving. There is perhaps a similar issue with the revised Primary National Strategy in England and Wales (DfES, 2006) not listing problem-solving as a strand in its own right, even though it is embedded within “using and applying” and the term ‘embedding’ arguably makes it clear that it has a significant part within the mathematics curriculum for primary children[5]. While the presence of “using and applying” in all taught blocks of the Primary National Strategy (DfES, 2006) means that teachers are directed to make use of problem-solving opportunities regularly within the classroom, and while the Williams Review (DCSF, 2008, p.65) states that “there should be scope for children to engage in extended problem-solving activities that extend across lessons to give children time to use their knowledge and explore the problem in full,” the reality is perhaps that problem-solving is not given the emphasis it should be given against the competing demands of other aspects of mathematics. This was certainly felt to be true of the PGCE mathematics module taught to the participants of this study – see 1.2 and 1.3 above for the problem that led to the devising of this research and 2.6 below for more on the issues surrounding mathematics in schools and Initial Teacher Education.

Orton (2004, p.84) defines problem-solving as consisting of “routine practice problems, word problems…, real-life applications and novel situations”. While he indicates that it is the latter that is usually meant when referring to problem-solving, the primary curriculum as elucidated in the Primary National Strategy (DfES, 2006) rarely makes these distinctions, and – while it recommends a move away from the daily three-part lesson of the Numeracy Strategy that preceded it (DfEE, 1999a)

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[5] It is important to note, however, that problem-solving’s ‘place’ within the Early Years Foundation Stage’s (EYFS) curriculum is far more central, given that mathematics is named “Problem Solving, Reasoning and Numeracy”. The Williams Review (DCSF, 2008, p.37) makes the point that, within the EYFS, this focus is encouraged *rather than* “the formal teaching of mathematics”, which seems to carry the implication that problem-solving and reasoning stand apart from mathematics – Orton (2004), Bruner (1960), Kahney (1986) and Simon (1978) would perhaps argue against this implication.

towards more extended opportunities for learning (aided by the two and three week 'block' pattern it introduced for teacher planning), it is perhaps more likely that children will encounter the first two of Orton's (2004) categories than the third or, indeed, fourth. Nonetheless, the Primary National Strategy (DfES, 2006, p.13) stresses that, in order to "provid[e] opportunities to reinforce and enhance learning…[children should be provided with]…opportunities for application of knowledge in new contexts to involve…higher-order thinking skills, such as reasoning and problem-solving". The issue here remains the emphasis on "reinforcing and enhancing" that may encourage teachers to think of problem-solving more in the sense of Orton's (2004, p.84) "routine practice problems". Some might well take the approach that this is 'the wrong way round' and that problem-solving ought to come first and foremost: "Some people believe that solving problems is the essence of mathematics learning, even to the extent of considering that the body of knowledge, which others regard as mathematics, is merely the set of tools available for the active process of problem-solving" (Orton, 2004, p.84). This can be seen as "the real purpose of learning rules, techniques and content" (Orton, 2004, p.25), with Burton (1984, p.10) stressing that "nurturing children's curiosity, and developing and refining their spirit of enquiry establishes skills for the future and reasons for learning in the present". Such "nurturing", of course, makes demands of class teachers, and – indeed – further demands for those training to be teachers, such as the primary PGCE students in this study. These demands include considering how to encourage and develop talk in the classroom.

Regardless of its centrality or otherwise within the primary mathematics curriculum, problem-solving exercises provide many opportunities for mathematicians to engage with these "rules, techniques and content" (Orton, 2004, p.25). It is also an opportunity for Assessment for Learning (AfL) (Clarke, 2008) in the classroom, as activities can highlight not only what children/students know, but also whether they think their knowledge is applicable to the problem given. As Shoenfeld (1985, p.13) argues, problem-solving can highlight misconceptions and misunderstandings – when, for example, some students solve problems "…by implementing well-learned mechanical procedures" and failing to utilise mathematical knowledge that might be relevant "because *they did not perceive their mathematical knowledge as being useful to them, and consequently did not call upon it*." Carefully designed problem-solving opportunities, with scope for feedback, can potentially address this, perhaps by means of Goldin's (1997) "four-stage exploration" and, specifically, his guided "heuristic suggestions" (a connection can be made here to the guided learning encouraged by the Primary National Strategy (DfES, 2006)). Shoenfeld's (1985) observation perhaps provides a further rationale for problem-solving not being left to the end of units of work, as a means of "reinforc[ing] and enhanc[ing] learning (DfES, 2006, p.13) but being integral to mathematical learning, perhaps even being used as formative assessment (Clarke, 2008) at the outset of a block of learning to identify the learning that children need to undertake in the following lessons.

Returning to Orton's (2004, p.25) observation about the way in which problem-solving allows learners to engage with "rules, techniques and content", Hošpesová and Novotná (2009, p.195) observe that, when engaging with word problems, students/pupils are tasked "to discover or construct the mathematical model" as their "available algorithms are of no use at [the initial reading of the problem] stage". This is arguably true of other (non-word) problems, too. Students/children need to work to distinguish and discern what the question asks of them, what operations are required to come to a solution, and how these should be represented (Simon, 1978; Burton, 1984; Haylock, 2006) for their own use (for example, in calculation), or perhaps more commonly for those who will mark their work (which, if that is the 'end result', might negate the opportunity for them to reflect and 'look back' (Pólya, 1957) on their problem-solving performance). They need, therefore, to engage, even when working independently/individually, with the problem in what Simon (1978) refers to as the "task environment". They need to be "creative and constructive" (Sutherland, 2007, p.41) – a point also made by the Rose Review of the Primary Curriculum (DCSF, 2009) – perhaps engaging in "trial and refinement" processes (Sutherland, 2007, p.41) as they work their way towards determining what rules and algorithms might be applicable to the task they have been set. Shoenfeld's (1985, p.13) observation about the danger of simply applying "well-learned mechanical procedures" without necessarily understanding them is of relevance here. A well-designed problem might aim to prevent such quick identification of a 'familiar' algorithm, aspiring instead to encourage the children to develop their own approaches. Some, such as Haylock (2006) have termed these "adhocorithms", stating that they are beneficial to children's learning because "they are based on relationships between numbers [and are] more likely to involve reasoning with understanding than…rote learning of recipes and procedures" (Haylock and Thangata, 2007, p.92[6]). There are links to be made between problem-solving activities and the kind of active learning referred to by Niemi (2002) and Orton (2004). Problem-solving activities may support the move away from teaching "largely by exposition [with]…little opportunity to learn by discovery" recommended by such as Orton (2004, p.72).

As Bruner (1960) observed, learning by discovery involves learning mathematics by *doing* mathematics; this point is also stressed by such as Burton (1984, p.9): "mathematics is certainly a *doing* subject". The challenges provided by problem-solving situations may even, ultimately, be more motivating than more 'traditional' teaching approaches, provided that the teacher is still available to guide the student and can "step in at any time" (Orton, 2004, p.75). Such learning by discovery relies, however, on the setting of tasks that enable children to make mathematical discoveries themselves with reduced prompting and direction from the teacher as perhaps implied by Hošpesová and Novotná (2009) above. Such 'benefits' to learning fit with the recommendations for good/effective mathematics teaching made by the Primary National Strategy (DfES, 2006) and the Williams Review (DCSF, 2008).

As indicated above, problem-solving is embedded within the "using and applying" strand of the revised Primary National Strategy (DfES, 2006). This document also underpins the curriculum to be taught to primary student teachers in England and Wales and has therefore been a major driver of curriculum changes within Initial Teacher Education to improve the coverage and, more pertinently, student teacher confidence with and understanding of problem-solving activities so that they can plan appropriately for their classrooms. Indeed, the section on "good mathematics teaching" in the Strategy (DfES, 2006, pp.65-66) states that, where teacher direction is less overt, because the emphasis is less on 'technique' (and/or 'number facts' etc.),

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[6] Haylock and Thangata (2007, p.161) make the point that there is a place for rote learning in mathematics, however, for example "arbitrary conventions, some vocabulary and abbreviations. For example, there is a convention in algebra that x multiplied by y is written xy. There is nothing to understand here, it just has to be learned and remembered."

"through carefully chosen activity and well-directed questioning…children [should be]…steered to discover the rules, patterns or properties of numbers or shapes" (DfES, 2006, p.66). They should also be able to communicate them effectively to others (see also 2.6 below for "School and Initial Teacher Education Issues", which provides further detail on the impact of the Primary National Strategy (DfES, 2006) and the Williams Review (DCSF, 2008) on initial teacher education and the primary classroom). This last point about communication, of course, once again illustrates the importance of developing and encouraging talk within mathematics, and within problem-solving activities specifically.

Building further on the discussion of the importance of dialogue to be addressed in more detail below in 2.2.2, Laborde et al. (1990, p.54) make the point that "…the way in which an individual understands a text or formulates ideas depends on this individual's knowledge and on [pre-existing] conceptions about the content to be read or expressed." "Language activities," they state – "through the specific problems they cause" (i.e. through the need to clarify points to others, and to explain that which is, at least in part, believed to be understood) – "can lead the individual to consider the objects and relations underlying the discourse in a different way and can also be of help in problem solving" (Laborde et al, 1990, p.54). This is supported by more recent work from Alexander (2018, p. 562) in which he argues that "…psychological research, increasingly supported by neuroscience, demonstrates the intimate and necessary relationship between language and thought, and the power of spoken language to enable, support and enhance children's cognitive development, especially during the early and primary years…" Another way, perhaps, of positing this is to suggest that problem-solving activities themselves can support the development of language and dialogue in learning and that there is, perhaps, a two-way relationship, a symbiotic relationship almost, between the two. This provides further support for the development of talk in mathematics learning, with the potential benefits including greater confidence in articulating thoughts and ideas, greater understanding of concepts via such talk and more successful problem-solving as a result. The kinds of activities to be encouraged, however, require consideration – not all are, of course, equal, and some lend themselves more to talk than do others.

Relating problem-solving in mathematics to social constructivism as discussed above in the Theoretical Framework (1.7), it can be seen that Selley's (1999, p.72) recommendation that teachers should "...set some exercises [for children] which have several valid solutions, mixed in with univalent ones (unique solutions)" indicates that opportunities can be devised for learning by discovery. Such discovery could include the other valid solutions; the reasons *why* there are other valid solutions; and the differences between problems that lend themselves to many possible solutions and those that can have only one. Such careful choice of activities could also promote productive dialogue between peers and between the class teacher and groups/individuals within their class about the mathematics underpinning the problems set. A focus on process, rather than on solutions, can be beneficial here, as argued by Burton (1984, p.18): "[this] not only encourages their use but also improves the pupils' problem-solving performance", particularly as they may call on strategies based not on skills (for example, algebraic skills) that they are yet to 'know' but strategies that "resort to simple cases" and may, indeed, serve to illuminate aspects of the mathematics that can be developed further to deepen that understanding.

Further to this, Mercer (1995, p.1) argues that "creative problem-solving...[is] rarely…[a] truly individual affair..." and this, along with the intention to generate dialogue, leads some practitioners to engage groups in joint problem-solving activities, sometimes with little in the way of teacher direction or 'hands-on' support during the problem-solving process. Providing problems and materials with little in the way of didactic (or otherwise) pre-teaching allows for some negotiation of meaning. As detailed by Price (2000, p.52), such approaches allow "…children to record informally and to discuss and negotiate meaning…(thereby)…encourag[ing] both understanding of the mathematics and understanding of its recording." This perhaps fits with Realistic Mathematics Education movement's view (Price, 2000) that not only does mathematics exist to solve problems but that children should be encouraged to conceptualise it in this way for themselves. Allowing student teachers to both experience and consider problem-solving in this light before working with children would perhaps go on to encourage good practice in their own classrooms – of the kind recommended by the Primary National Strategy (DfES, 2006). As with other aspects of Initial Teacher Education (for example, the approach taken towards promoting the teaching of phonics in the primary classroom), modelling problem-solving practice and a less didactic approach to classroom teaching in the taught University sessions alongside the more 'traditional' tutor-led input would become a main focus of module and workshop development in the PGCE context of this study (see also 2.6 for School and Initial Teacher Education Issues). Such modelling would also include considering the 'steps' or 'stages' to be encountered when problem-solving and what these actually 'look like' within given activities so they can be recognised by teacher and student alike.

Similar to Burton's (1984) questioning, challenging, and reflecting referred to above, Pólya (1957) breaks down the solving of a mathematical problem into four steps – understanding, devising, carrying out and "looking back". Such "looking back" could happen, for example, within the plenary of a 'traditional' Numeracy Hour lesson (DfEE, 1999a) – see 2.6 below – maybe utilising something akin to the final stage of Goldin's (1997) Task-Based Interview "four stage exploration", as covered in 2.5.3 below, which asks students to explain "how they thought about the problem" (Goldin, 1997, p.45). Similar problem-solving stages are detailed (for word problems, but arguably applicable to other types of problems, as well) in the work of Novotná (1997) and Hošpesová and Novotná (2009). This latter framework (discussed in more detail in 2.5.1 below) follows Pólya (1957) in having four stages, beginning with "encoding" (analogous with Pólya's "understanding" in that there is a requirement to "grasp the assignment"), before moving on to "transformation" (perhaps similar to Pólya's "devising" – this involves the creation of a mathematical model for the question posed; translating the original question posed in words to the language of mathematics), "calculation" (Pólya's "carrying out") and "storage" (which requires the transfer of what has been solved with mathematics back into the original context of the question posed; this includes the verification of the results against that context and is, therefore, a form of 'looking back'). This final stage, regardless of its name, may involve the recognition of learning that can be carried forward into other, later problems of a similar type. As Orton (2004, p.25) says, "one might only have learned to solve that problem, but it is more likely that one has learned the essence of how to solve a variety of similar problems and perhaps even a variety of problems simply possessing some similar characteristics". Opportunities to formally pause and note such learning, either through a mini-plenary or a plenary in a more ‘traditional’ Numeracy lesson, or perhaps via a dedicated follow-up lesson precisely planned to discuss the outcomes of the problem-solving situation, would obviously need to be created if this is to be adequately brought to the children’s attention. Revisiting or ‘replaying’ the problem as it was tackled and ultimately solved might provide an insight into the learning that can be carried into future exercises – this provides a rationale for the use of methods such as Stimulated Recall Interviews (SRI) when considering group problem-solving work or any tasks which involve any degree of thinking-aloud (see 2.5.5 below).

Having considered the position of problem-solving within the mathematics curriculum, both for primary school children and those being trained to teach them, and some of the ways in which it may be used to encourage dialogue/social-interaction that can lead to discovery and a less didactic, more active approach to learning, the next sub-section addresses dialogue itself, specifically the notion of “dialogic teaching” (Alexander, 2008; 2010a; 2018) as a specific approach that informs the work detailed in this thesis. Issues relating to cognition and affect in mathematics, touched upon above in considering the demands made of students when problem-solving, will be addressed in 2.3 to follow.

2.2.2 The Importance of Dialogue in Learning Mathematics, Dialogic Teaching, and Socio-Cultural Underpinning of Dialogic Approaches

The previous sub-section has indicated the degree to which problem-solving, while perhaps not always recognised as fundamental to mathematical learning, engages students in “doing” mathematics (Burton, 1984). It has illustrated some of the ways in which talk is considered important in a problem-solving context, with reference to Mercer’s (1995, p.1) observation about “creative problem-solving” and how this is not an “individual affair”. Considering the “effective use of talk…as a social mode of thinking” (Mercer and Wegerif, 1999, p.79), this sub-section builds on these observations to consider the role of dialogic teaching (Alexander, 2008, 2010a; 2018) and, ultimately, the potential of exploratory talk (Mercer, 1995; Edwards, 2005) in collaborative learning. Edwards (2005, p.1) defines collaborative learning, as opposed to cooperative learning, as “learning…constructed amongst student peers working together in self-selected groups" with, as indicated above with reference to Burton (1984), "the process involved in mathematical endeavour…[being as] important a focus to the group as the end outcome". In considering this co-constructed learning, Alexander's (2005, p. 12) distinction between discussion and dialogue is important to this work; the former involving "the exchange of ideas with a view to sharing information and solving problems", with the latter necessitating the achieving of a "common understanding through structured, cumulative questioning and discussion which guide and prompt…" students and "expedite the 'handover' of concepts and principles". Such questioning and discussion may prompt exploratory talk. Mercer and Wegerif (1999), Seal (2006), and Edwards (2005) all identify exploratory talk as being of particular importance in such learning situations, and this type of talk is of direct relevance to the Talk Framework utilised in this work (see 3.4 below). Before considering the role of exploratory talk and the definition of dialogic teaching (Alexander, 2008; 2010a; 2018), however, this sub-section first considers the importance of dialogue as a means of encouraging active participation in collaborative endeavours of the type Edwards (2005) refers to above (in using the term 'active', a connection can be drawn with active learning as defined by Niemi (2002)). In doing so, it refers to psychological perspectives such as Mulligan and Lozito's (2014) "self-generation and memory" which are also of significance when considering the approach taken in this work. Consideration is given to encouraging individuals to think-aloud and, subsequently, listen back to their words, as will be further outlined in the methodology to follow (3.2 and 3.3).

It can be argued that encouraging students to put their thoughts into words, which includes thinking-aloud (accessing perhaps the 'inner speech' that Vygotsky (1978) and Dewey (1961) both highlight), leads to a potentially deeper understanding of the concepts being discussed or, indeed, taught. Looking back at strategies proposed and considering their effectiveness when compared to others put forward by the group is also a form of "metacognitive practice" (Bransford et al., 2000) – something that, it could also be argued, is of particular use to those studying to teach (as is the case with the participants in this work). The "self-generation effect" refers to one downside of putting thoughts into words – that individuals are better able to recall their own contributions over and above the contributions (however valuable) of others (Slamecka and Graf, 1978; Mulligan and Lozito, 2014). This may, however, be addressed by encouraging them to listen back after the event, not only to their own contributions, but to those of others in the group, thereby providing a rationale for the use of digital audio to capture such dialogue (see 2.5 below on capturing problem-solving). It also connects with Pólya's (1957) emphasis on the importance of "looking back" when problem-solving (see 2.2.1 above). Being encouraged to think aloud in the first place provides an impetus to explain thoughts and ideas to peers, as well as valuing those ideas within the culture of the classroom. Revisiting recorded material, again perhaps with peers, can further act as a "self-monitoring" activity (Glaser, 1999, p.99), in which having an "audience" enables the "monitor[ing] of individual thinking, opinions, and beliefs…[potentially] elicit[ing] explanations that clarify points of difficulty". These explanations may not have been evident even in the original discussion (for reasons connected with student anxiety over the situation or, indeed, the mathematics when working alongside peers). Providing a framework that identifies potentially valuable and/or successful mathematical or verbal strategies – that encourages group members to rethink not only what they said but also how it built on the contributions of others (or not, as the case may be) – may also help to ameliorate this unconscious bias towards their own observations (Mulligan and Lozito, 2014). It may also act as a form of self – and peer – monitoring, perhaps enabling reflection on performance that would not have been possible without some form of replay/revisitation opportunity. Indeed, this may support the kind of "Assessment for Learning" (see, for example, Clarke, 2008) that is encouraged in the primary classroom.

In considering the importance of dialogue in the classroom, Laborde et al. (1990, p.66) argue that it "…could be focused on two aims…: helping the teacher to know what the student has in mind and improving students' oral proficiency". Both, of course, might well provide evidence of learning (and could well be used as 'Assessment for Learning' (Clarke, 2008) – see below). A case can further be made, however, that this improved proficiency, coupled with an emphasis on listening back to the words of self and others (whether supported or not by an accompanying framework or specific prompts from the teacher), achieves another, and perhaps more valuable, aim: helping the student better understand what they themselves have in mind, as well as the ideas that lie behind the verbal contributions of others. While Mulligan and Lozito (2014) point to the "self-generation effect", whereby students will recall their own words better than those of their peers, this does not mean they will necessarily have considered the effectiveness of their contributions – even a ‘right answer’ may not be adequately understood. Although Clarke’s (2008) AfL approach encourages teachers to ensure they have good grounds for assessing the students as having been successful at a given question, the quick-fire nature of some mathematics activity in the classroom means that correct answers to questions can often be offered without much in the way of explanation before teacher and class move on to the next question. Think-Aloud Protocols (T-APs) – encouraging participants to explain their underlying thinking *whilst working* – may work with the “self-generation effect” (Mulligan and Lozito, 2014) to achieve a better understanding of the students’ own ideas, whilst an opportunity for listening back, as indicated above, may enable them to ameliorate the same effect, encouraging them to pay more attention to the thoughts and explanations of others now that they are freed of the demands of putting their own thoughts into words.

Leaving aside thinking-aloud as a very specific, and perhaps less than natural, form of classroom dialogue, and also moving beyond Mercer’s (1995; 2000; 2004) consideration of the “guided construction of knowledge”, theorists such as Wegerif, Mercer and Dawes (1999) and Resnick, Michaels and O’Connor (2010) have identified that both constructivism and social constructivism promote the importance of social interaction in the development of children’s cognition (see also 1.7 above). In terms of the socio-cultural underpinning of talk in mathematical learning, Alexander (2018, p.565) justifies his “dialogic teaching” approach by stressing that “children need to be able to communicate, build relationships, participate in their culture, value collective identify and cohesion, and become engaged and active citizens”. This is informed by his work on the Cambridge Primary Review (Alexander, 2010b), providing a perspective, in some senses, opposed to that of the UK curriculum documents used so predominantly in the training materials for the student teachers engaged in this project. “Dialogic teaching” is distinct from looser definitions of talk/dialogue in a number of ways, and it is useful to consider both its genesis – which further reveals its social-cultural underpinnings – and its general approach. While the thinking-aloud later detailed in this thesis does not necessarily meet its strictures, many of the principles – such as the sharing of responsibility for talk repertoire and the need for the “teacher…to create interactive opportunities and encounters which directly and appropriately…mediat[e]…the cognitive and cultural spaces between…teacher and learner, between society and the individual" (Alexander, 2005, p.2) – are evident in the problem-solving tasks undertaken and the collaborative nature of the exercises.

In detailing the genesis of "dialogic teaching", Alexander (2018, p.563), while acknowledging the influence of Vygotsky (1962, 1978) and others, maintains that it is "*sui generis*" as "it devotes equal attention to the quality of teacher and student talk, and to the agency of others – fellow students as well as teachers – in the latter". Rejecting notions that there might be "one right way to maximise talk's quality and power" (Alexander, 2018, p.563), and standing apart from the definition of "interactive whole class teaching" from the UK's National Literacy Strategy (DfEE, 1988), a label Alexander (2017) considers "tautologous", it concerns itself with "repertoire", given that "talk is the most pervasive in its use and powerful in its possibilities…of all the tools for cultural and pedagogical intervention in human development and learning" (Alexander, 2005, p.2). This repertoire – while, in part, the responsibility of the teacher – Alexander (2018, p.563) argues, is "progressively shared with students, [with] the development and autonomous deployment of…[their] own talk repertoires…[being]…the ultimate goal".

Alexander's (2018, p.564) "dialogic teaching framework" provides "justifications", "principles", "repertoires" and "indicators". Chief amongst the identified principles is that dialogic teaching should be "collective, reciprocal, supportive, cumulative, [and] purposeful". The notion of cumulative talk here relates to Mercer's (1995; 2000; 2004) use of the same term within his talk typology – this, again, contributing to the Talk Framework detailed below in 3.4. Alexander (2018, p.566) notes, however, that "cumulation…is the most difficult of the principles to enact", and that Mercer and Littleton's (2007) definition, requiring individuals to respond "positively" and "uncritically" to each other, while meeting the "supportive" principle, "may also discourage [necessary] argumentation". This, he considers important to the "accountability" necessary within such talk, an observation also made by Michaels, O'Connor, and Resnick (2008), whose work is given as another influence on Alexander's definition of dialogic teaching. Such "accountability" entails the providing of explicit evidence "behind their claims or explanations…[with students]…challeng[ing] each other when evidence is lacking or unavailable" (Michaels, O'Connor, and Resnick, 2008, p.283). The providing of such supporting evidence in dialogue is a key part of the exploratory talk discussed below in 3.4, most especially given the promotion of such talk by such as Mercer and Wegerif (1999), who identify it as "productive talk", and Seal (2006).

Alexander (2010a, p.1) establishes the purpose of dialogic teaching to "stimulate and extend pupils' thinking, help the teacher more precisely to diagnose pupils' needs, frame their learning tasks and assess their progress [and] empower the student for lifelong learning and active citizenship". Again, this may allow for the 'capturing' of internal representations that may not otherwise be communicated either to self, peers or the teacher and has some connections with notions of AfL (Clarke, 2008) and formative assessment familiar to current UK student teachers and primary practitioners (i.e. in considering the thinking underlying a task as important evidence of learning, with less of an emphasis on summative 'end product' achievements).

Emphasising dialogic teaching (Alexander, 2008; 2010a; 2018) also acts against the view, seen by some, such as Jenson (2011) as "obsolete", that the teacher should be the provider of knowledge. As Brookfield and Preskill (1999, p.25) assert, "discussion affirms students as cocreators of knowledge [because] they have the same right to be heard as teachers."

> Because the flow of conversation and the development of contrasting lines of inquiry can't be predicted, students and teachers share responsibility for the evolution of the group's knowledge. Creating insights, validating or refuting claims, and exposing group members to alternative perspectives are all shared responsibilities.
>
> (Brookfield and Preskill, 1999, p.25)

This promotion of students/children as cocreators of knowledge also, arguably, ensures that they are prepared for contexts beyond education in which it will be important for them to put forward their views and identify areas in which they need to develop their learning. The consideration of lifelong learning echoes the views of Robinson (2013), whose work is familiar to the participants of the project from their studies elsewhere within their PGCE (for example, on creativity). Others, such as Excell (2010) and Gerver (2010) make the argument that the primary purpose of education is to prepare children to be life-long learners. These considerations, of course, have implications for the practice of teaching/pedagogy and a number of recent empirical studies have considered this. The work of, for example, Mercer (1995; 2008), Selley (1999), Alexander (2008; 2005; 2010a, 2012; 2018) and Fisher (1993) indicate the potential beneficial impact of talk on promoting children's learning and also provide a rationale for a reduction in teacher talk or what might perhaps be viewed as more didactic approaches to teaching. Young (1992), for example, suggests that the asymmetric (teacher-dominant) nature of the Initiation-Response-Feedback (IRF) pattern evident in many teaching sessions may constrain learners' participation and could even result in a sense of pupil powerlessness; as indicated above, Alexander (2018) considers an (over)-reliance on this pattern to still be evident in much current teaching. It is argued, however, within the discussion section of this thesis (chapter 6) that there are other issues at play in a group situation, where teacher talk is either minimised or absent (see the discussion of the "psychological situation" (Rotter, 1954) in 6.2 below for the issues encountered by the student teacher participants in this study; 2.3 below considers cognition and affect more broadly). Such issues include the relative confidence levels of different members of a group, and – indeed – their listening skills and even patience. These will also have an impact, and there is therefore an argument that the teacher has a role in facilitating this effectively to ameliorate for such potential problems (this relates to Alexander's (2018) repertoire, referred to above). For example, it may be necessary to actively model and teach the skills required, rather than assume that pupils/participants will have intrinsic abilities and aptitudes that will ensure successful group working. As detailed below, mechanisms such as Stimulated Recall may then provide useful opportunities to 'make up for' any weaknesses in the original thinking-aloud sessions.

Nystarnd and Gamoran (1997, p.72) emphasise Alexander's (2008; 2010a; 2012; 2018) promotion of purposeful talk, stating, "classroom talk requires students to think, not just to report someone else's thinking". This provides a productive link to notions of thinking-aloud within the classroom context and, while digital audio is identified as the medium through which such thinking is captured in this work, it is evident that consideration needs to be given as to how to effectively think aloud when engaged in a task. Participants may need pre-teaching/training in and, indeed, modelling of such techniques in order to be successful. It could be argued that a combined thinking-aloud/stimulated recall process helps to meet Nystarnd's and Gamoran's (1997) recommendation regarding the approach to be taken with classroom talk while allowing contemplation of others' thinking/strategies/approaches at the same time – if individuals are concentrating on thinking-aloud, in the first instance, it may well be too much to ask them to listen carefully to others at the same time. This relates to Ericsson and Simons' (1993) observations regarding the impact of protocols on thinking provided above. Nystarnd and Gamoran (1997) may suggest that the "report[ing]" of others' thinking is not as valuable as thinking for oneself, but it is arguable that the reflection on others' thinking, including appreciation of strategies usefully employed by others that had not been previously apparent, has value in advancing understanding both for pupils in a classroom setting and for the student teachers engaging in this study. Again, this relates to the "generation/self-generation effect" (Mulligan and Lozito, 2014) noted above.

In further support of dialogic approaches, and again revealing their socio-cultural underpinning, Brookes and Brookes (1993, p.17) assert that a constructivist classroom is one in which "pursuit of student questions is highly valued" and where "teachers seek the students' points of view in order to understand…[their]…present conception for use in subsequent lessons". These ambitions are also evident, to a degree (noting Alexander's (2017) comments about the limitations of the recommended 'interactive whole-class teaching approach) in England and Wales' Primary National Strategy materials (DfES, 2006 – see also 2.6 for School and Initial Teacher Education Issues). The Strategy clearly establishes communication of ideas in mathematics by pupils themselves as central to mathematics education right from the Foundation stage[7]. Notably, and further supporting the work conducted in this

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[7] Under "using and applying mathematics" in the Foundation Stage – the youngest children covered by the Primary National Strategy materials – there is an expectation that "most children [will] learn to:… Describe solutions to practical problems, drawing on experience, talking about their own ideas, methods and choices" (DfES, 2006, p.70). By the final year of primary education (year 6 progression into year 7, i.e. the progression from KS2 to KS3 or secondary education), one of the many expectations under "using and applying mathematics" is that children will be able to "explain and justify conclusions" (although it is, perhaps, a shame that the explicit use of the word "orally" is dropped after year 4 – leading

project, Brookes and Brookes (1993, p. 17) also state that “students primarily work in groups”. Selley (1999, p.70) argues, “some success” has “been claimed for the constructivist approach, which allows the teacher to work alongside the pupil on a task, and to listen to the reasoning behind each step”. Goldin’s (1997) “four-stage exploration” Task-Based Interview procedure provides a structure that could be utilised in the classroom to allow, firstly, the pupils to have the opportunity for ‘free’ problem-solving, before being prompted by guided questions, as appropriate – this methodology is discussed below in 2.5.3. More recent debates in primary education, perhaps building on the development of England and Wales’ National Strategies (DfEE, 1998, DfEE, 1999a; DfES 2006), and including those captured within the Williams Review of early years and primary mathematics provision (DCSF, 2008) and the Cambridge Primary Review (Alexander, 2010b), stress the importance of allowing for such “open-ended discussion”/dialogue in mathematics to promote children’s development. As NRICH (2013) observe, however, recording of thinking (or “recording as thinking”, as they describe) needs to be “consciously structured”. It is not enough just to talk in what might be described as an aimless fashion; it is important to know what needs to be communicated and how to think aloud in such a way as to make strategies employed clear both to oneself and, in the case of group work, to one’s peers. Teaching and modelling is required in talk as well as mathematics. It is here that the use of a more formal Think-Aloud Protocol (T-AP) may be of use, even outside a research situation (see 2.5.4 below).

Nunes and McPherson’s (2007, p.18) observation (in 1.7 above) about internal representations aiding the construction of knowledge provides an indication of the importance of both recording and analysing responses (again, with possible links to be made to Clarke’s (2008) AfL practice). Digital audio, therefore, could provide an opportunity to develop an understanding of how reasoning can be ‘unpicked’ from discussion by allowing participants to listen back to their own (and peer) responses. This might be through a technique such as Stimulated Recall (see 2.5.5). Such listening back is not without potential difficulties, however – should the pupils listen and comment ‘live’, or should there be a more formal identification of the areas to be

to some student teachers suggesting that talk must be less important towards the end of the primary age phase).

discussed? Slamecka and Graf (1978) and Mulligan and Lozito's (2014) observations about the degree to which participants may or may not be more attentive to their own contributions in a discussion is also relevant here; consideration needs to be given to the means by which they can be encouraged to attend to the thoughts of their peers. Focused questioning or the use of a framework to 'analyse' the talk (i.e. by recognising contributions of different types) might be beneficial, and this underlies the approach taken in the project detailed below.

This sub-section has addressed the importance of both problem-solving and dialogue in mathematics, including collaborative work (Edwards, 2005) and the notion of dialogic teaching (Alexander, 2008; 2010a; 2018) as entailing talk which is "collective, reciprocal, supportive, cumulative, [and] purposeful" (Alexander, 2018, p.564). The Williams Review (DCSF, 2008, p.51) indicates that the "benefits for children['s learning are] very obvious" and some of these benefits can be seen to include the building of a repertoire (Alexander, 2018) necessitating a variety of approaches in order to address the subject being taught and the needs of the learner/s. This repertoire reflects the primacy of talk as identified in Alexander (2005) and the need for teachers to directly "engineer" opportunities for such dialogue in their classrooms. The value of cumulation/cumulative talk has been considered, alongside the need to encourage exploratory talk when working. The next sub-section looks at one of the main issues that may prevent children – and student teachers, too – from confidently engaging in such dialogue and, indeed, mathematical problem-solving tasks with their peers. Even though, as Boekarts (2001, p.18) states, "intrinsic motivation" may be increased, along with "domain-specific motivational beliefs", by working with and alongside peers on such tasks, concerns about mathematical knowledge, and revealing a lack of knowledge to peers, may act against individuals wishing to put their ideas forward in a group or whole class situation. Indeed, what might be termed 'culturally specific' attitudes towards mathematics in the UK may impact on people's willingness to engage with mathematics, particularly after they have left secondary education. The Williams Review's (DCSF, 2008, p.3) concern raised in 1.3 above about it being "socially acceptable – fashionable even – to profess an inability to cope with the subject" is relevant here, and such thinking may affect some of those who are training to be teachers on PGCE routes in the UK, where recent experience of mathematics may have been minimal due to the requirement to attain a minimum of Grade C GCSE to be accepted onto training programmes. This concern about the acceptable public perception of mathematics has been echoed by others such as Smith (2004), and is bound up with the further concerns raised by Williams (DCSF, 2008) about the level of subject knowledge held by practising teachers and their confidence in engaging with different pedagogical approaches towards delivering the mathematics curriculum. Issues around confidence, motivation and self-efficacy can, therefore, act against the stated benefits of encouraging peers to work alongside each other in problem-solving situations, regardless of the perceived benefits, and it is cognition and affect, then, that this Literature Review turns to consider in the next section.

2.3 Cognition and Affect in the Context of Mathematics Education

As discussed above in 2.2, there are many potential benefits to be gained by children and students when engaging with discussion-based and/or group mathematical problem-solving situations for themselves in the classroom. These include the possibility of learning by discovery and self-questioning (Bruner, 1960; Orton, 2004; Shulman, 1986), a better awareness of problem-solving stages and strategies that can be carried forward into future problem-solving situations (a form of meta-cognition that can inform Robertson's (2001) "analogical problem-solving"), and the possibility that individuals can be 'trained' to become better problem-solvers (Pólya, 1957; Orton, 2004) with the implication that this is of benefit to their education as a whole (i.e. beyond mathematics). Some, such as Boekarts (2001), have argued that potentially greater autonomy and intrinsic motivation, including when working alongside peers, also results from taking part in such activities. The latter, of course, as indicated above in 2.2, may well depend heavily on individuals' perceptions of themselves as learners of mathematics, and, indeed, as *mathematicians*. Where student teachers are concerned, it also relates to individuals' beliefs about mathematics as a subject for study, as opposed to a subject for teaching. This includes beliefs about how mathematics is learned (Swan, 2006) and how individuals perceive their 'Pedagogical Content Knowledge' (Shulman, 1986). These different views of mathematics (Ernest, 1991), and their potential impact on cognition, are discussed below. These 'beliefs' can also, however, be influenced as much by affect, in social psychology terms (Stangor, 2014), as cognition: for example, how individuals feel about not only the mathematics but also their position within the group (and wider peer group). How individuals feel more generally may also have an impact – for example, their current mood, which may be entirely unrelated to the task at hand (Stangor, 2014). Decisions made, perceptions and also memory can be affected in turn. As Stangor (2014, p.90) states, "affect influences cognition…the influences of mood on our social cognition even seem to extend to our judgments about ideas", and a connection can be made here to what Rotter (1954) terms the "psychological situation" which is revisited below when considering the group in 6.2. This can impact on the "expectancy value" (Eccles, 1987; Eccles and Wigfield, 1995; Elliott et al., 2005) held by participants about a given situation, and – as further discussed below – individuals' perceived "self-efficacy" (Bandura, 1982; 1997).

Therefore, the views held about mathematics, and the performance in a given mathematical situation, can be influenced by the affect heuristic, or a "tendency to rely on automatically occurring affective responses to stimuli to guide our judgments of them" (Stangor, 2014, p.92). A favourable affective response to the presentation of something can have a strong impact on outcomes. Stangor (2014), for example, gives the example of packaging influencing a 'liking' for one product over another (thereby influencing the ultimate purchasing of that product), or managers choosing to appoint someone for a job based as much on how much better they are 'liked' over other candidates. Returning to Rotter's (1954) "psychological situation", this implies that, regardless of the perceived benefits of engaging in group work and/or discussion, and also regardless of whether individuals are aware of these benefits, individuals are as much prompted towards a successful engagement with the task by aspects such as their general frame of mind about mathematical situations, their views of the peers and the tutor/teachers working alongside them, and additional elements far-removed from the mathematics and the group situation. In the case of student teachers, this might include their current progress on the course and also their relationships with those supporting them in school.

Returning to the opinions and beliefs relating to mathematics that individuals might bring with them into a mathematical situation, and might therefore impact upon affect and cognition, Ernest (1991) indicates that there are three, distinctly different views of mathematics that might be held. These range from the 'instrumentalist' view (i.e. that mathematics is a collection of rules, facts and skills) to the 'Platonist' view in which a static realm of mathematical knowledge is discovered but not created, to the 'problem-solving' view in which mathematics is more of a product of developing and expanding human culture, and – therefore – can be both discovered and created (see also 2.2 above). These differing views can, therefore, mean that different students and teachers of mathematics come to the subject with different opinions on what 'good' mathematical performance is, both for themselves, and for their children. They may not, for example, value problem-solving activities as much as others, thereby giving less 'weight' to their assessment of their problem-solving performance as an indicator of their mathematical performance and knowledge. Alternatively, they may view problem-solving as inherently central to the use of learned rules and skills (Orton, 2004) and therefore assess their own (and their children's) problem-solving success as a key indicator of their effectiveness – and their success as mathematicians going into future problem-solving situations. More likely perhaps, as Ernest (1991) and Swan (2006) suggest, the complexity of the teaching situation, with curriculum demands and limitations on resources, results in a teaching approach that is somewhat divorced from any theoretical base or belief system, with much of the teaching perhaps standing in contradistinction to real beliefs, in as far as those beliefs are consciously understood by the teacher. Therefore, motivation to take part in particular types of mathematical activities, such as group and discussion-based activities, can be impacted upon by a variety of influences. Indeed, individuals' views of themselves as mathematicians will vary depending on the beliefs held. Whatever those beliefs, however, they – and the prior mathematical experiences that individuals have accrued in their education – will affect their sense of self-efficacy (Bandura, 1982; 1997): how successful they feel they will be when dealing with a particular mathematical task presented to them.

As Bandura (1982, pp.200-201) states, "self-efficacy is concerned with judgements about how well one can organise and execute courses of action required to deal with prospective situations that contain many ambiguous, unpredictable, often stressful, elements". Tschannen-Moran et al. (1998, p.2) further define self-efficacy as a "cognitive process in which people construct beliefs about their capacity to perform at a given level of attainment" – in educational contexts, of course, that 'given level of attainment' can go from children in a Year 6 class, who are expected to be working at or beyond Level 4, to the teachers teaching them, expected to be able to support learning in mathematics at a wide-variety of levels, whilst also ensuring they remain committed to their own mathematical learning and development. In considering cognition and affect, and specifically self-efficacy, this section, therefore, addresses both self-efficacy as encountered by learners *and* teachers of mathematics with some consideration of the teacher education context that is central to this work. Further considerations around the context of "School and Initial Teacher Education Issues", beyond issues of self-efficacy, and reflecting the developments in the curriculum as they would have impacted on those going into teaching, are raised in 2.6 below.

Alderman (2004, p.3) argues that "…teachers have a primary responsibility in education to help students cultivate personal qualities of motivation that can give them resources for developing aspiration, independent learning, achieving goals, and fostering resiliency in the face of setbacks." Curriculum reviews such as Williams (DCSF, 2008) and Rose (DCSF, 2009) have emphasised the importance, too, of creative approaches towards the teaching and learning of mathematics – and of encouraging creativity in the responses from children to the tasks set. It is arguably important, then, that teachers feel a high degree of self-efficacy (Bandura, 1997) when teaching mathematics (noting that Bandura (1982) identified self-efficacy as situation-specific). Any issues surrounding their own confidence and feelings of self-efficacy may ultimately frustrate not only their own achievement in mathematics and related problem-solving tasks, but could also have a deleterious impact on their children's learning, as well.

That there have been concerns around teacher (and student teacher) subject knowledge and confidence in mathematics is apparent from the brief given to the Williams Review (DCSF, 2008) that included the requirement to both "secure and improve" the subject knowledge taught to student and practicing teachers via Initial Teacher Education (ITE) and Continuing Professional Development (CPD) courses. This was considered especially important given that, at that time, and despite previous concerns raised by Alexander et al. (1992) and also despite the introduction of the Numeracy Strategy (DfEE, 1999a), "around six per cent of all children [left] primary school without attaining level 3 in mathematics at Key Stage 2 [Year 6/age 11]" (DCSF, 2008, p.5). This figure is all the more concerning when considered against the expectation of Level 4 achievement for 'average' end of Key Stage 2 performance. It was the Williams Review (DCSF, 2008), of course, that stressed the point that it is perhaps "fashionable" in England and Wales to profess an inability with mathematics, and this is the context from which many of those coming to Initial Teacher Education will have been drawn. A further concern, perhaps, is the Review's decision not to raise the required GCSE grades of those coming into teaching from GCSE Grade C – therefore meaning that those who have 'just' achieved this grade, or those who needed to undertake additional training before securing their place, may have started from a position of feeling that they were less 'good' at mathematics than many of their peers. This may, therefore, have led to some expressing such an 'inability' with mathematics themselves, affecting both their confidence levels in taught University/college-based sessions and on their teaching practice to follow.

Bandura (1997) states that such beliefs may further impact on how effortful people may be in a given circumstance, and can influence the degree to which they will persist when presented with obstacles. For student teachers, these obstacles begin with the focus on their own mathematical knowledge and learning from the outset of their course – much of which, in teacher education situations, is self-audited. A consideration, therefore, of how student teachers themselves 'perform' in mathematical problem-solving situations may provide some indications of their self-efficacy in mathematics, including for their own benefit. Perhaps a particularly useful exercise prior to completing their training would be for them to engage with the kinds of problem-solving activities that their children will ultimately be asked to complete in their own classrooms. Beswick (2009), for example, states that "there is broad acceptance that mathematics teachers' beliefs about the nature of mathematics impact the ways in which they teach the subject" (Beswick, 2009, p.153). This, coupled with the Williams Review's (DCSF, 2008, p.3) "central conclusion…that the teacher, even more than the parent, determines learning outcomes in mathematics" makes it all the more important to focus on – and enhance – the perceived self-efficacy of those going into teaching.

Reflecting on the strongest ways to support and enhance self-efficacy, Bandura (1997) identifies four potential ways in which this can be achieved, from vicarious experiences (i.e. students making a judgement of their own self-efficacy in a particular domain in comparison with the performance of others – this, obviously, is liable to occur in classroom situations wherever peers are working alongside and with each other) to verbal persuasion (as a teacher might perhaps employ to encourage self-belief in her pupils) to physiological/affective states. The fourth, "enactive mastery experiences", is identified by Bandura (1997) as the strongest. Such experiences entail "authentic successes at dealing with a particular situation" (Palmer 2006, p.337). Therefore, opportunities to engage – successfully – in mathematical activities and problem-solving (including group problem-solving) in the context of an Initial Teacher Education programme may help develop students' conceptions of themselves as mathematicians and future teachers of mathematics just as much as successfully teaching the subject in a teaching practice situation. This may be most especially vital given the emphasis placed on accurate and appropriately pitched exposition within the classroom – even in a context where a reduction in 'teacher talk' has been recommended. Again, the Williams Review's (DCSF, 2008) "central conclusion" about the importance of the teacher is relevant here. As the Review (DCSF, 2008, p.3) further goes on to state: "confidence and dexterity in the classroom are essential prerequisites for the successful teacher of mathematics and children are perhaps the most acutely sensitive barometer of any uncertainty on their part". That final point – as well as being another source of uncertainty on the part of student teachers (that they will be 'caught out' in not knowing something by their own children) – is one perhaps very good rationale for wanting to ensure that such "enactive mastery experiences" (Bandura, 1997) are not limited solely to the classroom practice elements of a teaching programme, although Palmer (2006) urges caution in believing that activities as undertaken within the university/college context are easily transferable into student teachers' classrooms:

> Comments…suggest that the students had directly experienced motivation and content learning under the simulated conditions at university, and this led them to believe that the same techniques would also be effective in the primary classroom… There is a potential problem with this type of modelling. Teacher education students are at a vastly different educational level to primary age children, so it should not be assumed that a technique that can promote motivation and learning in adult tertiary students would be just as effective with young children.
>
> (Palmer, 2006, p.349)

As Palmer (2006, p.349) says, "there is the potential that the use of this type of modelling could create false expectations of efficacy". However, an "enactive mastery experience" at their own level is some distance from one that a Key Stage 1 or 2 (or, indeed, Early Years Foundation Stage) pupil would experience. Perhaps missing from Palmer's (2006) consideration is that there are arguably two levels here: the development of an individual student teacher's sense of self-efficacy via "enactive mastery experiences" may improve their conception of themselves as a mathematician/problem-solver and this may then have an impact on their conception of themselves as a class teacher able to 'steer' their children through the stages necessary for solving problems themselves. Experiences in a mathematics training setting that include opportunities to be successful at problem-solving, even at the level of the children the students are training to teach, may encourage them to consider teaching approaches and appropriate questions etc. with which to engage the children.

The Williams Review (DCSF, 2008) of mathematics education in the UK makes evident the fact that – for some students, particularly those who have been away from mathematics for some considerable time (i.e. postgraduate trainees), the existing Key Stage 1 and 2 curriculum may present as much of a challenge as material pitched more at their own level. Successful experiences, therefore, with the kinds of problems that the children are expected to solve in their mathematics lessons may have a positive impact on students' ultimate sense of self-efficacy and may count as "enactive mastery experiences" (Bandura, 1997) in and of themselves. If coupled with the opportunity to carefully consider the strategies used – both mathematically and verbally (when working alongside peers) – there is the opportunity to 'catch them' being successful as they "look back" at their performance (Pólya, 1957). Indeed, the process of verifying solutions is central to the "four-stage exploration" proposed by Goldin (1997) above, as well as Novotná (1997) and Hošpesová and Novotná's (2009; 2009b) problem-solving stages. Ensuring opportunities for students to engage in dialogue about what has been done will allow them to demonstrate how well they have "grasped the assignment" (Novotná, 1997) and what they have learned. Such discussion also potentially leads to the benefits identified by Laborde et al. (1990) above, not to mention the opportunity to encourage the kind of "metacognitive practices" that Bransford et al. (2000) recommend. This may be very beneficial in encouraging a deeper understanding not only of how they, as individuals, learn, but how their children may ultimately learn, too, with additional benefits in modelling the kind of AfL (Clarke, 2008) practice that relies on identifying successes and targets for development. One imperative then, however, becomes how to capture such problem-solving performance adequately, and this is the focus of 2.5 below, before which this Literature Review turns to a consideration of learning as relevant to the discussion above about problem-solving and dialogue in groups.

2.4 Learning

This short section provides an overview and working definition of both knowledge and learning, relating these to the concept of active learning (Niemi, 2002) and also to the notions, in psychological literature, of "generation" and "self-generation" (Slamecka and Graf, 1978; Mulligan and Lozito, 2014) which may provide a rationale for encouraging student engagement in dialogic teaching (Alexander, 2008; 2010a; 2018) and the kind of Think-Aloud Protocol (T-AP) employed in this work (discussed further below in 2.4). This consideration of "generation" and "self-generation" (Mulligan and Lozito, 2014) may also support the regular revisiting and replaying of group discussion 'after the event' via such techniques as Stimulated Recall Interviews (SRI).

Jarvis (2006, p.4) speaks of "human learning…[as]…a complex set of human processes that are in some ways extremely difficult to understand." Learning, he further states, is "about experience, usually conscious experience" and "is [an] essential element of Being." While a detailed definition of learning, and that which is to be learned (the knowledge and/or curriculum) is beyond the scope of this study, the approach to learning taken here builds on Jarvis (2006) – and, indeed, Mercer

(1995), detailed above – and his consideration of the philosophical perspective of learning, namely that, as "our action is…always [engaged] with [and in] the world…experiences become data for our own thinking. Our experience occurs at the intersection of the inner self and the outer world and so learning always occurs at this point of interaction." This intersection, of course, can also be viewed as the transition between cultural and internal plane – the *zone of proximal development* described by Vygotsky (1978) and built upon, for example, by Alexander (2005, p.11) when stressing how "dialogue provides a potent form of peer or adult intervention in the child's progress across the zone of next or potential development" (he "refuse[s] to say 'proximal'"). McDermott (1999) stresses that this outer world includes others: "The term *learning* simply glosses [over] that some persons have achieved a particular relationship with each other, and it is in terms of these relations that information necessary to everyone's participation gets made available in ways that give people enough time on task to get good at what they do. If that happens enough, it can be said that learning happens." This provides a rationale for learning in (and, therefore, *from*) group situations. Small-group learning, as opposed to more 'traditional' forms such as lectures has, for example, been recommended for some time in Higher Education contexts as a way of deepening not only student engagement but also understanding. Abercrombie and Terry (1978, p.1), for example, believe that such opportunities enable students to "become more autonomous as learners" (a connection can also be drawn here with Boekaerts' (2001) comment above about the impact of peer-support on motivation); they also point to the "dissonant behaviour" of tutors who say they wish to encourage talk in their classrooms but spend the majority of the time talking themselves. Going beyond the context of Higher Education, which – of course – also informed the approaches to teaching and learning in Initial Teacher Education, there was a similar 'push' for increased small-group and talk opportunities in the primary classroom, again from the belief that this interaction would encourage and enrich learning. Earl et al. (2003), in providing an official evaluation of the UK Government's Strategies, observed, however, that the impact of the recommended 'interactive whole-class teaching' had not necessarily resulted in any great reduction in teacher talk. With Smith et al. (2004) further finding that teachers rarely used open questions (10% of questioning exchanges), and also observing that pupil answers lasted five seconds on average, it can be seen that the UK primary education context for the student teachers in this study was not one in which "sustained and extended dialogue" (Smith et al., 2004) was necessarily widespread. Alexander (2018), as noted above, has commented on the way in which this has not improved to the present day, demonstrating the need for work of the kind detailed in this thesis and further reflection on the potential of dialogue as a means to further learning.

The most recent iteration of the Primary National Strategy Primary Framework for Literacy and Mathematics (DfES, 2006) at the point of this project emphasised the importance of group work as part of the structure of the "three-part daily mathematics lesson" (DfES, 2006, p.11). This renewed Framework, informed by research into effective pedagogy, "promot[ed] a range of pedagogical approaches, including direct, inductive, experiential, enquiry and problem-solving approaches as well as social or relationship approaches (such as role-play and simulation)". Although active learning is not directly cited here, there are connections to be drawn with this approach – a major influence on primary education and Initial Teacher Education at this point in time (Niemi, 2002) – and also to the "self-regulated learning" that Boekaerts (1997) stresses as important if students are not only to use but also develop their cognitive skills, thereby enhancing their learning in the process. Such "self-regulated learning" is regularly in evidence in the primary classroom, with children identifying areas of interest based on what they already know and what they wish to find out, thereby "steering and guiding the[ir own] learning process" (Boekaerts, 1997, p.162). This recalls Slavin's (1996) advocation of more participation from students in their learning to avoid passivity and Orton's (2004, p.72) observation that "at the present time…there is now much more pressure on teachers to use more active approaches" to counter the teaching "largely by exposition" that gives children "little opportunity to learn by discovery".

It can be said, therefore, that learning builds on experience and the interaction between inner and outer self (in Jarvis' (2006) terms). This interaction with the outside world is often with other human beings, hence the emphasis given to social constructivism in the Theoretical Framework (1.7) above and the consideration of group work and active learning as emphasised by, for example, the DfES (2006), Boekaerts (1997), Niemi (2002) and Orton (2004). Further to this, and complicating matters perhaps further, Jarvis (2006) also argues that doing and thinking are not separate, distinct activities, but are combined as one phenomenon. Relating this to existentialist thinking, he stresses that thinking, for example, is rarely, if ever, divorced from emotion. The "psychological situation" (Rotter, 1954) of working with a group of peers may, therefore, impact upon thinking – this is addressed further in 6.2 below. Related to this, learners may also be invested in particular answers or in appearing to know those particular answers. This may be especially the case with student teachers, close to the end of their PGCE year, who may feel that they are 'expected' to know the relevant curriculum knowledge for their chosen age phase. This means that McDermott's (1999) observation about relations between learners and the way in which information is 'made available' takes on additional import in any consideration of activities that might promote learning by students.

It may well not be enough simply to put students in groups and expect them to learn from each other; Slavin's (1996) passivity is not reduced by just changing the focus from teacher talk to student talk, or – indeed – by badging activities as 'active'. Consideration needs to be given to the knowledge that is to be learned and the means by which is to be communicated and shared (Alexander's (2018) 'repertoire'). In addition to this, the concepts of "generation" and "self-generation" (Slamecka and Graf, 1978; Mulligan and Lozito, 2014) have a bearing on learning in group situations; individuals may pay more attention to their own words than to the words of others. While this may be of benefit on the one hand, as encouraging students to articulate their thoughts may encourage a deeper understanding of the underlying mathematics, it may also mean that they pay less attention to their peers. Therefore, it may be necessary to consider how better to capture dialogue and encourage individuals to listen more attentively to each other's contributions. Opportunities to revisit what has been said by all in the group may be valuable when individuals have concentrated hard on their own verbal contributions.

As detailed by Scheffler (1999, pp.1-2), due to the wide range of concepts and ideas which can be said to be 'known', "the term *knowledge* is frequently intended as embracing both…the accumulated skill and lore pertaining to technological control of the environment, and those intellectual arts and experiences whose value is intrinsic to themselves." The job of education is to pass this knowledge on to future generations and, for some, this may be enough of a definition of learning. Indeed, the student teachers engaged in this work would have heard many definitions of learning from Ofsted and the Department for Education that begin and end with this thought[8]. Scheffler (1999, p.2) observes that this is a matter complicated still further by this not merely being a transmission of "what we know, but our manner of knowing, that is, our approved standards of competence in performance, in enquiry, and in intellectual criticism". He proposes "three broad philosophical approaches to knowledge, the rationalistic, the empiricistic, and the pragmatic" (Scheffler, 1999, p.2) and further goes on to identify, within the rationalistic tradition, mathematics as the "model science" with "truths [that are] general and necessary, and may be established by deductive chains linking them with self-evident basic truths".

The learning of mathematical truths, then, does not depend on experience, but may be suggested by experience; it is possible to "work with pencil and paper only, and yet arrive at the firmest of all truths, incapable of being overthrown by experience" (Scheffler, 1999, pp.2-3). Nonetheless, whilst mathematics is a rationalistic science, and need not necessarily require the experimentation of empiricist natural science, a pragmatic view can aid learning: "to learn something significant about the world, we must do more than operate logically upon basic truths that appear to us self-evident" (Scheffler, 1999, p.3). Learning requires the testing of hypothetical ideas and potential connections/relationships; the pragmatic posing of problems that can lead to the "active generation of ideas" and "imaginative theorizing" (Scheffler, 1999, p.4). There is a connection, again, here to be made to the notion of active learning, as well as Dewey's (1961) "trying and undergoing". Indeed, an acknowledgement of the perceived need, in the primary classroom, of this testing of ideas can be seen in the Primary National Strategy Framework recommendations to teachers (DfES, 2006, p.12) where it is stated that teachers should not only ensure "that children are taught the knowledge and skills they need but also are provided with the opportunity to explore through group, guided and independent elements to secure greater

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[8] Examples of the much-reported Ofsted comments relating to teaching and learning from this period include Chief Inspector, Sir Michael Wilshaw's, 2014 comments about "progressive methods…damag[ing] generations of schoolchildren" (Paton, 2014); group-work is explicitly criticised, along with "allowing pupils to proceed at their own pace and make discoveries independent of the teacher".

understanding and to make sense of their newly acquired knowledge through its application". Indeed, 'Using and Applying Mathematics' is the first of the seven strands of mathematics objectives in the Primary National Strategy (DfES, 2006, p.65) with problem-solving 'embedded' within it. Teachers, the Strategy asserts, should vary their teaching approaches from those that are "quite directive…[to those]…where the directing is less evident and through carefully chosen activity and well-directed questioning the children are steered to discover the rules, patterns or properties of numbers or shapes" (DfES, 2006, pp.65-66). The importance of dialogue in learning, as may be encouraged within such activities, and the place of problem-solving in learning, are considered above in 2.2.

This sub-section has identified some of the different perspectives on learning, including those that focus more on the learning that individuals can gain from others in less teacher-led, didactic, situations. In considering the learning that can come from group and/or discussion-based tasks, it has reflected, again, on Alexander's (2018) promotion of dialogic teaching, and has briefly considered the "psychological situation" (Rotter, 1954) that is elaborated upon further with relation to the group undertaking this study in 6.2 below. It has considered how active learning or group-based/discussion-based learning approaches alone cannot be relied upon to ensure learning – providing a further argument for considering Alexander's (2018) promotion of repertoire – and it has acknowledged that more than just 'knowledge' is conveyed to individuals in learning situations, linking to the socio-cultural underpinnings discussed above. Having addressed talk in problem-solving, affective aspects, and learning, this Literature Review now turns to the ways in which students' dialogue whilst engaged in problem-solving may be captured in order to inform and perhaps promote further learning.

2.5 Capturing Problem-Solving

Having previously identified some of the benefits that can arise from "looking back" (Pólya, 1957) at mathematical activities in 2.2.1 above, and having identified also the potential that such "looking back" has for increasing self-efficacy by 'catching' successful strategies that may inform later problem-solving, the following sections (2.5.1 through to 2.5.5) address the ways in which group problem-solving performance/dialogue can be captured for later revisitation/replay or analysis. 2.5.1

to 2.5.5 also consider how such replays can enhance learning and encourage metacognition. This may be particularly beneficial for student teachers shortly to begin their own practice in the classroom.

Depending on the aspects that interest a particular researcher, there are a number of different approaches that can be taken to capturing problem-solving as it is undertaken for later analysis or even replay. If interested, for example, in the problem-solving strategies themselves more than the verbal strategies used by group members to convey their ideas, Novotná (1997) and Hošpesová and Novotná's work (2009) provides a problem-solving process (and, therefore, a potential framework) that can identify when students have 'grasped an assignment' (encoded), "transfer[red] [in]to the language of mathematics" with the subsequent "creation of a mathematical model" (Hošpesová and Novotná (2009, p.195) and calculated/stored; the latter involving the "transfer of mathematical results back into the context including contextual verification of the obtained results" (Hošpesová and Novotná's, 2009, p.195). Although considering word problems over other types of problems that might be presented to students and/or children (including, for example, those given in diagrammatic form or via equations alone), these categories are potentially beneficial to both primary student teachers and their children in enabling a recognition of the steps to be taken towards the successful solving of a given problem. They are perhaps also a means by which problem-solving activities can be self-assessed (asking children and/or students, for example, to identify examples of the stages in their discussion and perhaps providing some "exploratory (metacognitive) questions" (Goldin, 1997, p.45) that can be asked of every problem-solving situation). This is discussed in 2.5.1, with connections made between their categories and those of Mercer (1995; 2000; 2004) whose work, although not related strictly to problem-solving, provides categories that seem very applicable to group problem-solving activities.

2.5.1 Novotná (1997)/Hošpesová and Novotná (2009) and Connections to be Made with Mercer (1995, 2000, 2004)

Given the problems of comprehension in the learning of mathematics, discussed below in 2.5.5 in relation to Duval (2006), it is arguably important for teachers to have mechanisms by which their students can evidence the degree to which they have or have not “grasped the assignment” and then transformed the task set into the necessary mathematics (Hošpesová and Novotná, 2009). These mechanisms should go beyond just ‘producing the correct answer’ or ‘showing working out’, as might perhaps be evident in tests (summative assessments). This is potentially useful both for the teacher’s formative assessment of children’s progress (i.e. to inform future teaching) and for the children themselves, most particularly in a curriculum context where children are encouraged to be involved in self-assessment/‘Assessment for Learning’ (AfL) (Clarke, 2008; DfES, 2006; DCSF, 2008). Such mechanisms may encourage children/students to engage in the valuable “looking back” at their performance that Pólya (1957) recommends. However, considering only the participants’ verbal ‘performance’ when engaged in discussion-based learning opportunities would arguably act against identifying the successful mathematical strategies proposed and links made to previous problem-solving situations. From an AfL perspective, ‘leaving out’ the mathematics would mean that relevant targets could not be set for future work and children might consequently not be aware of the areas in which they most need to improve.

The problem-solving procedure propounded by Novotná (1997) and elaborated upon by Hošpesová and Novotná (2009) provides a potential framework for the consideration of mathematical performance when engaged in such tasks. Although their key focus is word problems (only one sub-set of problem-solving activity, as indicated above), there is perhaps potential here for teachers and children to utilise it when assessing/self-assessing as part of AfL practice.

Novotná (1997) and Hošpesová and Novotná’s (2009) procedure has four stages. These can be related to the “four-stage exploration” of Goldin’s (1997) Task-Based Interviews, and also to the work of Pólya (1957), most specifically in its final stage. The first stage – “encoding” – is also given as “grasping the assignment” (Novotná, 1997). This involves the reading of the problem along with the realisation that it requires mathematical operations to be performed in order to reach a solution and that a system will be required for the recording of “data, conditions and unknowns…in a more clearly organized and/or more economical form” (Hošpesová and Novotná, 2009, p.195). This might involve describing to others in their group what the problem requires, beginning to note down key features of the problem for later use, and perhaps recognising that a table or similar device will be of use to the thinking to come, but it stops short of the mathematics itself. The second stage – the "transformation" stage – is when the student/s create a "mathematical model" and "mathematiz[e]" the problem (Hošpesová and Novotná, 2009, p.195). This leads directly into the third stage – "calculation" – which is perhaps the easiest to define independently from Novotná (1997) and Hošpesová and Novotná's (2009) work, although it also includes "verification of the obtained mathematical results" (Hošpesová and Novotná, 2009, p.195), which is connected to Pólya's (1957) "looking back" and, arguably – as is seen below in 2.5.3 – the final stage of Goldin's (1997) "four-stage" Task-Based Interview "exploration".

The Novotná (1997) and Hošpesová and Novotná (2009) problem-solving stages provide a framework with which individual and group problem-solving performance can be assessed. Whilst not considering mathematical problem-solving, Mercer's (2000) talk framework – building on his interest in group work and group dynamics – provides additional categories that may be of use to identifying effective problem-solving performance in the classroom. For example, when engaging with the first two stages of the Novotná (1997) process, participants may engage in "exploratory talk", defined by Mercer (2000, p.98) as "engag[ing] critically but constructively with each other's ideas [with] relevant information…offered for joint consideration…[and]…reasons…given and alternatives…offered [to ideas challenged or counter-challenged]." Exploratory talk, it can be argued, is necessary when working with peers and moving towards transforming a set problem into the language of mathematics, as indicated by the "transformation" stage of Hošpesová and Novotná's (2009). Similarly, effective group work may well involve "cumulative" discussion (Mercer, 2000) with ideas proposed and then built upon by others in the group. This may provide evidence of not just individuals but also the group moving towards "grasping the assignment" (Novotná, 1997) as building implies some degree of shared understanding. When considering discussion-based group problem-solving, therefore, there may be productive links to be made between problem-solving frameworks and those that are more closely related to different types of talk. Effective mathematical strategies can be considered alongside effective verbal strategies to identify effective group performance when problem-solving. This may be of great benefit to those teaching mathematics, particularly with the focus on improving mathematics teaching from such as Williams (DCSF, 2008).

Having considered these two possible frameworks for the identification of both mathematical and verbal competencies when engaging in group mathematical problem-solving (or other) endeavours, the next section covers the more formal, and less post-hoc, approach of clinical interviewing (2.5.2), identifying those who have propounded the theories underlying such interviews, and then considering in more detail three specific types that are of relevance to the work discussed later in this thesis; Task-Based Interviews (2.5.3), Think-Aloud Protocols (2.5.4), and Stimulated Recall Interviews (2.5.5).

2.5.2 Clinical Interviews

Originally developed by Piaget (1929), the three goals of clinical interviewing are *discovery*, *identification* and *competence* (Ginsberg, 1981) and these link, in Ginsberg's words (Ginsberg, 1981. p.4) with Piaget's interest in "explor[ing] the richness of children's thought,…captur[ing] its fundamental activities, and…establish[ing] the child's cognitive competence". As Piaget himself states in his introduction to *The Child's Conception of the World* (1929, p.1) the first problem to be faced in child psychology is one of understanding "the child's notion of *reality*" ("what conceptions of the world does the child naturally form at the different stages of its development?"); the second is one of "*causality*": "what is the nature of the causality he accepts?" To discover and identify the child's beliefs requires, Piaget (1929, p.2) argues, a distinct methodology, a "special technique", most especially as "the form and functioning of thought are manifested every time the child comes into contact with other children or with an adult" and while this is "observable from without", "the content…may or may not be apparent and varies with the child and the things of which it is speaking". Traditional testing is dismissed by Piaget (1929, p.3) due to a "lack of context", most particularly as, he argues, "it [can] falsi[fy] the natural mental inclination of the subject or at least risks so doing." Questions posed may be about subjects that the child has not previously considered, either in that form or, perhaps, in any form, and this presents problems with fixed, standardised testing. "The real problem," Piaget (1929, p.4) asserts, "is to know how he frames the question to himself or if he frames it at all" (the teacher or researcher asking the question of the child might come to assume that this is a question that they might naturally expect, or that they themselves might ask; this carries with it an assumption of their level of knowledge in that specific area). Therefore, a skilled practitioner, Piaget (1929, p.4) says, should focus more on promoting the free expression of thoughts and ideas, "making [the child] talk freely [and thereby] encouraging the flow of his spontaneous tendencies instead of diverting it into the artificial channels of set question and answer". The observation of such spontaneous contributions to conversation, for Piaget, provides the most valuable data:

> …the detailed study of the contents of these questions reveals the interests of children at different ages and reveals to us those questions which the child is revolving in its own mind and which might never have occurred to us, or which we should never have framed in such terms.
>
> (Piaget, 1929, p.4)

Cooke (1999) expresses similar benefits for "knowledge elicitation" when it comes to domains that the elicitor does not yet know enough about (although, in Cooke's case, the argument is made that such work is useful in the early stages of research, before structured interviews can be usefully deployed – this is not the case with Piaget's (1929) argument above – see also 2.5.4 below on the subject of Think-Aloud Techniques).

Therefore, as stated by Ginsberg (1997, p. 2), clinical interviews can potentially provide "deep insights into children's thinking" that might not be provided by more structured techniques (for the purposes of this work, no distinction will be drawn between the insights that they might provide into children's thinking as opposed to the student teacher participants of this study). Designed, as indicated above, to overcome the limitations inherent in standardised tests and naturalistic observations, which Ginsberg (1997, p.2) further says are "not effective techniques for understanding the processes of thought", clinical interviews are therefore both unstructured and open-ended. Those being interviewed may also be treated differently depending on their perceived motivations and personalities as a result of what Ginsberg (1997, p.2) refers to as "'clinical' judgements". There is a focus on, for example, asking how particular problems had been solved and what precisely was meant by specific responses or statements (Ginsberg, 1997). Connections can be drawn here with Goldin's (1997, p.45) Task-Based Interviews, as briefly discussed above in 2.4 and addressed in more detail below in 2.5.3, where the final stage of his proposed "four-stage exploration" requires children/students to "explain how they thought about the problem" via "exploratory (metacognitive) questions".

Observation alone, without the aid of questioning, is problematic as, according to Piaget (1929, p.6) "the child neither spontaneously seeks nor is able to communicate the whole of his thought". The peer group presents further problems – questions might well be withheld in the company of adults "because he feels they must be known to every one" (Piaget, 1929, p.6) or because of a shame that comes from suspecting that the answers are known to those around them and a consequent fear of demonstrating their ignorance (see 6.2 below on the group "psychological situation" (Rotter, 1954) for evidence of this with the student teachers engaged in this study). Talk, therefore, is important, related to the "clinical examination…used by psychiatrists as a means of diagnosis" (Piaget, 1929, p.7). Those conducting such examinations need to be aware of the need not to talk too much themselves, potentially 'leading' them towards thoughts and beliefs that are not their own, but might be offered up to 'please' the other. Simply persevering along a line of questioning can also lead in much the same way. Piaget (1929, p10) refers to these as "suggested conviction[s]". Further to this are the "spontaneous conviction[s]" (Piaget, 1929, p.11) where reasoning is not required and "the reply is the result of a previous original reflection" (or perhaps relates to prior learning experiences). Piaget (1929, p.11) attests that the "clinical examination [is able to] reveal… the existence of spontaneous convictions and [can] aid… the child in formulating them for himself". It is, therefore, capable of making a strong contribution to learning with the child being able to put their learning into their own words. Nonetheless, "liberated convictions" are more valuable (Piaget, 1929, p.13) in that a response comes after a period of reflection, utilising previously held knowledge and a process of reasoning: "it implies previously formed schemas, tendencies of mind, intellectual habits, etc." and, as long as perseveration and suggestion can be ruled out as having led to the response, demonstrates much about the child's competence at such a task (referring back to Ginsberg's (1981) *discovery*, *identification* and *competence* that opened this section – the latter connecting with notions of self-efficacy (Bandura

1982; 1997); clinical interviews may have much to offer in helping to illustrate perceived self-efficacy).

Piaget's use of clinical interviewing as a research methodology developed across half a century from "pure adult-child discourse to include [the] manipulation of materials so that actions as well as words [could be] added to the interpretive data bank" (Rowland, 2000, p.7). Ginsberg (1977, 1981) and Ginsberg et al. (1983) would "argue stronger for the efficacy of the method in research into children's mathematical thinking" (Rowland, 2000, p.8). Processes such as "prediction, generalization and explanation" (Rowland, 2000, p.8) are able to be identified or elicited as a result of engaging with the clinical method. As discussed by Rowland (2000), the "verbal clinical interview" (which is distinct from, although related to, a pure think-aloud method as detailed in 2.5.4 below) involves the setting of a task or tasks (for example, a mathematical problem) with interviewer questions that are *contingent* on the responses provided by the children (to avoid suggesting or leading the participants in their thinking). Some standardization may be evident (around particular phenomena that has been previously identified), and a period of reflection will be required (asking participants to consider how they tackled a particular aspect of the problem, for example – this is very similar to Goldin's Task-Based Interview approach described below in 2.5.3).

Via this personalised, non-standardised approach, and its focus on encouraging participants to talk freely, clinical interviews are "intended to give the child/student the opportunity to display their "natural inclination" (Ginsberg, 1981, p.6) and, thereby, elicit knowledge (Cooke, 1999) about the thoughts underlying the tasks being carried out. They, therefore, inform the work detailed in this thesis and the following methodologies – Task-Based Interviews, Think-Aloud Techniques, and Stimulated Recall Interviews. All provide similar opportunities to "study…the mathematical mind in action" (Ginsberg, 1981, p.4). The next sub-section moves on to address the first of these.

2.5.3 Task-Based Interviews

Building on structured or 'clinical' interviews, as detailed above, Goldin (1997, p.40) proposes a "four-stage exploration" via Task-Based Interviews (T-BIs) to ensure that participants have the opportunity to engage with the problem without the kind of interference that a Think-Aloud protocol (see 2.5.4 below) or similar technique may cause. Such interviews are not carried out independently of the teacher/tutor – indeed, they are similar to the kind of 'guided' work recommended in primary mathematics teaching by the Primary National Strategy (DfES, 2006) – but they begin with 'free' problem-solving and allow time for children/students to think about the problem before any follow-up questions are asked. In comparison with the Think-Aloud techniques discussed below, there is no 'additional' demand, at the outset, to speak or share ideas in any prescribed way and they arguably present a means by which "the child [can be given] the opportunity to display his 'natural inclination'" (Ginsberg, 1981, p.6) when engaging with mathematical problem-solving. T-BIs are claimed by Goldin (1997, p.40) to be "…especially attractive [to researchers] as a means of joining research with educational practice", allowing opportunities for "guided explorations".

The first three of the four stages proposed by Goldin (1997, p.45) occur within the problem-solving situation itself, beginning with "posing the question ("free" problem solving) with sufficient time...[for response]...and only non-directive follow-up questions (e.g. Can you tell me more about that?)". Not all problems posed in primary education contexts (or Initial Teacher Education contexts), of course, have this 'free' and non-directive aspect to them. Indeed, the renewed Primary National Strategy (DfES, 2006, p.11) includes such exercises as just one of "a range of pedagogical approaches". However, there is an emphasis on children "explor[ing] through group, guided and independent elements to secure greater understanding and to make sense of their newly acquired knowledge through its application" (DfES, 2006, p.12). A T-BI-informed approach could, therefore, provide a structure for the 'guided' learning element of the daily mathematics lesson, ensuring that the student teacher/teacher can go beyond the guidance in the PNS (DfES, 2006, p.67) to give "attention [within small group teaching]…to particular children who may require additional support or challenge to ensure they continue to progress in learning". This support and challenge, whilst still engaged with the problem, might follow the second and third stages proposed by Goldin (1997, p.45): "heuristic suggestions if the response is not spontaneous (e.g., "Can you show me by using some of these materials?")", and "guided use of heuristic suggestions, again to the extent that the requested description of behaviour does not occur spontaneously (e.g., "Do you see a pattern in the cards?")". After the completion of the problem-solving task, the child/student could then be asked "exploratory (metacognitive) questions (e.g., "Do you think you could explain how you thought about the problem?") (Goldin, 1997, p.45). Such practice connects with assessment approaches – for example, Assessment for Learning (Clarke, 2008) – referred to above in 2.2.

It might also be possible to utilise T-BIs in consort with other methodologies, for example, alongside a Think-Aloud protocol (see 2.5.4 below), where the thinking-aloud of the participants might usefully identify the moments when Goldin's "heuristic suggestions" (Goldin, 1997, p.45) might be of most use. Similarly, the "exploratory (metacognitive) questions" (Goldin, 1997, p.45) might be of use within the context of a Stimulated Recall interview (see 2.5.5 below), as a means by which the recall could be structured – stopping the replay, for example, when it is clear that spontaneous responses are not forthcoming, and asking the participants questions about their thinking; even perhaps asking them to demonstrate with materials what they were thinking 'after the event'. Both Think-Aloud techniques and Stimulated Recall Interviews are detailed in the next two sections of this Literature Review below.

2.5.4 Think-Aloud Techniques

Bailey et al. (1995, p.67) argue that, "when your research aim is to understand the meanings people give to their experience, a less structured approach is likely to be more appropriate" as the interviewer's own preconceptions are less likely to impact upon that which is reported. Tasks can be completed without too much 'interference' from the observer or teacher. Such unstructured interviews are also, Cooke (1999, p.491) argues, most effective within the "early stages of elicitation when the elicitor is trying to learn about the domain and does not yet know enough to set up indirect or highly structured tasks". Although the different types of verbalisation that can be requested of participants using a Think-Aloud Protocol (T-AP) carry different risks of 'interfering' with the task itself, they present the opportunity for participants to speak throughout the process, without the teacher or researcher's "assumptions and understandings" (Denscombe, 1998, p.109) interfering too greatly with their performance or their thinking. They, therefore, present an opportunity for exploring the problem-solving performance of students, as they go about the business of solving problems.

Think-Aloud Protocols (T-AP) present immediate advantages to those wishing to elicit the thinking underlying the performance of a task, whilst also presenting problems that need to be carefully considered by researchers utilising them in their work. In this section, T-APs are considered as a distinct methodology for allowing individuals to provide an insight into their thought processes; the possibility of combining T-APs with other methodologies, allowing for the revisitation (replay) of dialogue for later consideration, has been indicated above in 2.3, although T-APs are generally utilised without such later revisitation.

T-APs, by their very nature, 'rely…heavily…on [in the event] verbal reports' (Cooke, 1999, p.486), and this may present challenges to the researcher in terms of reliability, structure and coding. Equally, while the researcher/teacher may not be directly 'interfering' in the process, T-APs may themselves present problems for participants' problem-solving as their concentration on what needs to be verbalised may 'get in the way' of their thinking about the problem. In effect, they are presented with two – or more – problems; the 'problem' of how to verbalise their thoughts as they occur in line with the protocol given competing with the demands of the mathematical problem itself. This may ultimately affect them to the extent they perform less well at the mathematics than might otherwise be the case.

In terms of the types of verbalisation that could be employed in work of this kind, Ericsson and Simon (1993) outline the potential effects, on performance, of three distinct approaches, while stating that none should impact upon the order or sequence in which the problem is tackled. The first, or "Type I", involves direct verbalisation; in effect, saying out loud what the individual's "inner voice" is saying. The second, "Type II", relates to verbalising thoughts which occur when conducting some form of fictitious or, indeed, imaginary task. The third, and most demanding ("Type III") in terms of potential disruption to cognition, requires the participant to provide an explanation and, in order to do this, further entails necessary pauses in activity to do so (literal 'interruptions' in the work). This may involve, for example, explaining why specific steps have been taken on the way to solving a mathematical problem (although Ericsson and Simon's (1993) work is not explicitly or exclusively related to mathematical problem-solving,). Such explanation could be of value to future problem-solving endeavour, although it is less useful, perhaps, if it interferes with the solving of the current problem, even leading to an incorrect answer or a lack of understanding of the mathematics underlying the problem.

Robertson (2001, p.13) states that all three of Ericsson and Simon's (1993) verbalisations "involve recording the contents of STM [short term memory]...[and thereby] slow[ing] down problem-solving to some extent as it requires the subject to recode information". As Kahneman (2011, p.23) says, "It is the mark of effortful activities that they interfere with each other, which is why it is difficult or impossible to conduct several at once", and while this exercise is not perhaps as difficult as his extreme example of "comput[ing] the product of 17 x 24 while making a left turn into dense traffic", it does demand more of the participants than simply thinking about how they would go about solving the problems presented. When working alongside peers, they have to pay attention to the demands of the T-AP whilst listening to the contributions of their peers for useful cues that may help them with the problem. It is even possible that the concentration involved may prevent them from seeing things, including possible strategies and solutions, that they may have more readily seen in a non T-AP situation – again, Kahneman (2011,p.23) supports this by stating that "intense focusing on a task can make people effectively blind, even to stimuli that normally attract attention".

One argument against think-aloud, therefore, is that the 'processing load' might be too great. Ericsson and Simon (1993) recommend that consideration, therefore, needs to be given to the degree of thinking-aloud required, and – indeed – the way this is modelled to the participants. However, while all forms of thinking-aloud carry some cost to thinking about the problem at hand, there is a potential benefit for those taking part in that they may be 'slowed down' enough by the process to think more carefully about the demands of the problem. This may enable them to avoid quick, and perhaps inaccurate, responses, such as those Shoenfeld (1985, p.13) speaks of: students applying "well-learned mechanical procedures" without perhaps understanding why they are applying them. The requirement to think-aloud, and perhaps to explain their thinking (depending on the protocol), may cause them to question their decisions more, and also perhaps the decisions of others, in a group situation. They may not come to an answer as quickly, even with a less-disruptive form of thinking-aloud that does not require explanations of why specific steps have been taken, but they may appreciate more *why* they have come to that answer. In this respect, there are connections to be made here to the work of Kahneman (2011) and his proposed "system 1 and system 2" thinking – "system 1" relating to the fast, almost spontaneous responses that people propose without much in the way of conscious thought; "system 2" being the slower, more considered responses that perhaps question more what is being asked and whether there are 'obvious' pitfalls to be avoided. "System 2" also relates to taking the time to verify that the answer addresses the question set in every respect, and this can be related to Pólya (1957), Goldin (1997), Novotná (1997) and Hošpesová and Novotná (2009) above.

T-APs, therefore, provide opportunities to access the thinking of students in a 'live' context, in the moment (Dempsey, 2010), as they engage with a task. They require consideration from the researcher as to the precise degree of thinking-aloud to be encouraged, as too much in the way of explanation can interfere with the task (Ericsson and Simon, 1993). They present benefits, in terms of making explicit thoughts and ideas that might have gone unexpressed, and challenges, in terms of 'slowing down' thinking, perhaps to the extent that more considered decisions are made, perhaps to the extent that the participants fail to properly come to a solution at all. Such challenges may encourage researchers to consider other means of eliciting information about individuals' thoughts whilst undertaking a task. An alternative approach is to elicit the thoughts from participants not during the event, but *after* it, and Stimulated Recall Interviews, which rely on a recording of the event to stimulate the responses, are detailed in the next sub-section, both as an alternative to T-APs and – ultimately – as an adjunct to the Think-Aloud process, perhaps even ameliorating the impact of thinking-aloud on performance.

2.5.5 Stimulated Recall Interviews

> Motivations and rationales that informants describe retrospectively may not conform to those that they actually held in the moment of the experience. The technique of SRI brings informants a step closer to the moments in which they actually produce action.
>
> (Dempsey, 2010, p.349)

Gass and Mackey (2000, p.1) define Stimulated Recall Interviews, sometimes referred to as SRIs (Dempsey, 2010), as "one subset of a range of introspective methods that represent a means of eliciting data about thought processes involved in carrying out a task or activity." In this respect, therefore, they perform a similar function to Think-Aloud Protocols (T-APs, discussed above in 2.5.4) and they can, therefore, be seen as one of the potentially valuable "metacognitive practices" (Bransford et al., 2000) that may benefit learning. Further to this, and demonstrating the distinction between stimulated recall and think-aloud protocols, Meijer et al. (2002, p.166) describe SRIs as "…a *substitute* for a thinking-aloud method" (the emphasis here is mine), although – as is demonstrated in 3.3 below – this is very much not the way they have been employed in this work. Supporting Meijer's et al. (2002) observation, Stough (2001, p.2) describes SRIs as having been "developed as an on-line cognition-capturing device more appropriately used in situations where think-aloud protocols were not possible or where a think-aloud protocol would interfere with the performance of the task being examined". This builds on Robertson's (2001, p.13) observation above regarding all three of Ericsson and Simon's (1993) verbalisations and how they may, in fact, slow down or perhaps impair problem-solving activity.

Hickman and Monaghan (2013) state that SRIs are generally stimulated by material captured on video or audio tape. SRIs or closely related methodologies have been employed in fields such as "clinical decision-making…[;]…describing the strategies of chess masters" (Calderhead, 1981, p.212), qualitative sport and exercise research (Houge Mackenzie and Kerr, 2012) and language (specifically second language) research (Mackey and Gass, 2005). Gass and Mackey (2000, p.1) further describe SRIs as "one subset of a range of introspective methods that represent a means of eliciting data about thought processes involved in carrying out a task or activity" with DeWitt and Osborne (2010) indicating that they have the potential to allow participants to identify areas of interest themselves within their own practices. This may potentially reduce the impact of the researcher on the focus of the participants' observations. As can be seen from the above examples, then, SRIs are not exclusively used within the area of mathematics and problem-solving, but consideration of Pólya's (1957) emphasis on "looking back" at strategies utilised to solve mathematical problems, demonstrates their applicability within this area.

NRICH (2013), referred to above in 2.2.2, in considering reasons for recording problem-solving activities, suggest that the recording might be made "for another person/time". This might imply that an SRI recording is intended primarily for the original participant to provide commentary on their own performance, although it is also possible, in a group situation, for members of the group to be prompted to comment on each other's contributions. Indeed, the SRI might enable individuals to consider their own and their peers' verbal contributions in more depth than was the case in the original activity.

Although teaching may not be viewed by all as necessarily as "fast, risky…[or]…spatially constrictive" a practice as other fields in which stimulated recall has been employed (Houge Mackenzie and Kerr's (2012) work on sport and exercise research, for example), it is certainly a skilled one and stimulated recall is considered to be "particularly well suited to…elicit participants' accounts [in such skilled practices] of their dynamic experience…[while]…maintain[ing] the benefits of a naturalistic context" (Houge Mackenzie and Kerr, 2012, p.58). Sime's (2006, p.214) observation that stimulated recall has "considerable potential" for the study of cognitive strategies and learning processes provides an additional rationale for work in the field of mathematics education and student teachers' problem-solving strategies. A key point that needs to be given careful consideration, however, is that stimulated recall "provides a vehicle for accessing cognitive processes when care is taken to reduce memory decay and prompting from the interviewer is kept to a minimum" (Sime, 2006, p.214). This has implications for the timing of any stimulated recall exercise, following the initial recordings, and this would provide some issues, for example, with PGCE students, as in this study, whose availability is constrained by their demanding timetable.

These considerations aside, SRIs may provide an effective method by which participants are "prompt[ed]…to recall and report thoughts that…[they]…had while performing a task" (Mackey and Gass, 2005, p.66). As a methodology, it may have the potential to provide a closer focus on what happens in the moment (Dempsey, 2010) even than a Think-Aloud Protocol. It allows for introspection, "attempting to achieve…[an insight into a] teacher's thoughts and decision making…[and therefore ultimately]…the reasons they have for acting as they do' (Calderhead, 1981, p.212).

Stimulated recall, therefore, could make a potentially very effective contribution to work on developing primary student teachers' mathematical problem-solving strategies. Westerman (1991, p.293), for example, argues that novice teachers are less able to "recognise problems…[and] lack the metacognitive and monitoring skills that experts possess". The implication of this is that they may well benefit, therefore, from a formal opportunity to "look back" at their performance (Pólya, 1957) and "recall…their concurrent mental activity" (Sime, 2006, p.214). In addition, Duval (2006, p.104) remarks that "it seems obvious that research about the learning of mathematics and its difficulties must be based on what students do really by themselves, on their productions, on their voices". Egi (2008, p.214) argues that a positive outcome of any process that encourages verbalisation is the provision of "additional practise opportunities". Again, this is arguably of great benefit to student teachers who will go on to support children's mathematical problem-solving and will want to encourage children to recognise and revise unsuccessful strategies whilst recognising and building on successful ones. These practise opportunities "are claimed to promote fluency...[as]…talking about language itself can constitute a learning process". Duval (2006) and Egi's (2008) comments, therefore, provide an additional rationale for utilising a methodology that, it is argued (below), can be implemented to ensure the minimum of interference in participants' actions and allows them to reflect upon the outcome of their own work. However, there are issues to be considered around "reactivity" and "veridicality" (Egi, 2008, p.213) – "reactivity" being the "positive or negative influence of verbalisation during or after the task…on learners' task performance and/or subsequent learning" with "veridicality" reflecting the 'completeness' of participants' responses and the degree to which contributions reflect what participants believe the researcher wants to hear. Of course, both are also potential problems with Task-Based Interviews and Think-Aloud protocols, as discussed above.

Having considered the specific types of clinical interview applicable to this work, the final section of this Literature Review returns to some of the school and teacher education issues that have been alluded to above. This includes the influence of the Williams Review (DCSF, 2008) and the revised Primary National Strategy (DfES, 2006), for example, on practice – indeed, on the notion of what 'good' teaching 'looks like' in the primary context; the rationale for the consideration already given to why problem-solving, and group problem-solving, should be promoted in the primary education context, and why discussion-based approaches have taken on such import within some educational contexts.

2.6 School and Initial Teacher Education Issues

As indicated above in 2.5, one of the major issues facing the student teachers undertaking the work detailed in this thesis was the introduction of the revised Primary National Strategy Framework for literacy and mathematics (DfES, 2006). This further strengthened an approach to pedagogy that had first been introduced with the National Numeracy Strategy in 1999 (DFEE, 1999a) and was inspired by the model from the National Literacy Strategy (DfEE, 1998). Introduced some ten years after the original National Curriculum for England (DES, 1988), criticised by such as Kelly (2009, p.256) for "disregard[ing]…all research evidence, whether empirical or conceptual…" regarding a 'theoretical frame' for education, the Literacy and Numeracy strategies share the same model of classroom teaching, including the "three-part lesson", an emphasis on group and "guided" work, and the end-of-lesson review or plenary. Alongside this, there was an increased focus on active learning (Niemi, 2002), and influential reports such as the Williams (DCSF, 2008) and Rose (DCSF, 2009) reviews stressed the importance of including more open-ended discussion and dialogue in mathematics lessons alongside a strong emphasis on the need to improve the quality of mathematics teaching overall given a central conclusion "that the teacher, even more than the parent, determines learning outcomes in mathematics" (DCSF, 2008, p.3). This echoes observations about mathematics teaching made by Borko and Whitcomb (2008, p.567) – "Teachers need to know more than mathematical content" – and Goya's (2006, p.1) comments (albeit in a North American context) about "the profound weakness in mathematical preparation of many…preservice elementary education students". All of these issues had an impact, too, on Initial Teacher Education, from the need to 'cover' new curriculum materials and the related documentation produced by authorities such as Ofsted and the DfES, to a more all-encompassing need to make changes to the teaching of module sessions to more effectively model approaches that the student teachers would be expected to use themselves when on teaching practice, and later, of course, in their own classrooms. The focus on more creative approaches to learning, espoused – for example – in “All our Futures” (DfEE, 1999b), and informing the later Rose Review (DCSF, 2009), also impacted on Initial Teacher Education providers, with the expectation that the revised National Curriculum to follow would encourage more in the way of cross-curricular and problem-solving approaches, breaking the strict ‘learning through subjects’ approach that Kelly (2009, p.252) describes as a “major error committed by the advocates of a common core curriculum…fail[ing] to recognize that education consists of learning *through* subjects rather than the learning *of* subjects”.

The Williams Review (DCSF, 2008), focusing explicitly on mathematics, had the remit from the then Secretary of State for Education to examine “the available evidence, including international best practice, and through engagement with the teaching profession, to consider and make recommendations in areas…[such as]…the most effective pedagogy of mathematics teaching in primary schools and early years settings” (DCSF, 2008, p.2). It was also asked to consider “what conceptual and subject knowledge…should be expected of primary school teachers and early years practitioners” (DCSF, 2008, p.2). Its recommendations, which would of course impact both serving teachers and those in training (and their trainers), included the ‘upskilling’ of Local Authority mathematics consultants, the creation of new Mathematics Specialists with “deep mathematical subject and pedagogical knowledge” (DCSF, 2008, p.23), and the continuation of the current mathematics curriculum, “as currently prescribed” (DCSF, 2008, p.62) until the Rose Review (DCSF, 2009) had reported back on issues around the enhancing of “using and applying” in mathematics. When the subsequent Rose Review (DCSF, 2009) made recommendations around the increase of ‘creativity’ in classroom teaching, there was – for a time – a move towards a more creative curriculum, which impacted on the range of pedagogical approaches used in the classroom, and the interest in enhancing group work and extended opportunities for problem-solving. By the time of this study, the 2010 Coalition Government had come to power and many, if not most, of these proposed, even possibly expected, changes were set aside in favour of the vision of education outlined by new Secretary of State, Michael Gove, and in the “Importance of Teaching” white paper (DfE, 2010). Nonetheless, the impact on teacher education of Williams (DCSF, 2008), Rose (DCSF, 2009) and material such as “All our Futures” (DfEE, 1999b) would continue to be felt, perhaps due to a lack of information, in the early stages of the Coalition government, about what a new curriculum, with a greater focus on ‘core knowledge’, might look like (Lightman, 2015). Equally, the need to improve the ‘coverage’ of problem-solving and group/discussion-based work outlined in the background to and statement of the problem above (1.2 and 1.3) remained something that tutors working on the part-time primary PGCE wished to address.

Having considered the literature underlying the work detailed in this thesis, as well as – in this subsection – the UK Initial Teacher Education and wider primary education context, the next section outlines the Methodology employed, building on the discussion above about thinking-aloud, stimulated recall, and the capture and replay of group problem-solving endeavours.

3. Methodology

3.1 Introduction

The following Methodology chapter provides an overview of the ontological and epistemological perspective of this work (previously discussed above in 1.7) as carried into practice, and of the networked methodologies employed to elicit knowledge from the participants (Cooke, 1999) and address the needs of the Research Questions (see also Chapter 4 for the development of the work across time). These Research Questions, as previously discussed above in 1.4, are as follows:

RQ 1: In what ways can dialogue/thinking-aloud recorded by digital audio support student teachers' verbalisation of problem-solving strategies?

RQ 2: What levels of thinking-aloud most effectively support digital-audio recorded T-AP and SRI?

RQ 3: What does the use of a multi-media artefact in an SRI situation reveal to participants about their own problem-solving strategies, and those of the group?

RQ 4: What types of talk are most evident in multi-media artefact enhanced thinking-aloud while engaged in mathematical problem-solving activity?

RQ 5: In what ways do the networked T-AP and SRI methodologies impact upon participants' willingness to engage with a problem-solving task?

RQ 6: Does multi-media artefact prompted recall promote greater reflection on mathematical methods/problem-solving strategies than 'standard' digital audio?

The research questions are referenced, as appropriate, when addressing the methodology in the sections to follow, identifying the decisions taken in data collection and analysis to ensure that they are adequately addressed.

The ontological perspective of this work, as evidenced by these research questions and as previously elaborated in the discussion of social constructivism in the Literature Review above (1.7), is that "people's knowledge, views, understandings, interpretations, experiences and interaction are meaningful properties of [their] social reality" (Mason, 2002, p.63). The epistemological view taken is that it is appropriate to both "interact" with and "listen to…[the] accounts and articulations" (Mason, 2002, p.63) of people to generate data. Basit (2010, p.110) argues that those who "see the world as consisting of human interactions and perceptions, influenced by human subjectivities…will conduct unstructured or semi-structured interviews to generate in-depth qualitative data and nongeneralizable findings, which are pertinent to a specific context, though they can be replicable or transferable to another situation." Therefore, qualitative interviews are, as Mason (2002) suggests, an appropriate method of data collection for work of this kind.

Think-aloud Protocols (T-APs) and Stimulated Recall Interviews (SRIs) arguably support the same ontological and epistemological views as they are both capable of "knowledge elicitation" which, as described by Cooke (1999, p.479), is "the process of explicating domain specific knowledge underlying human performance". In the case of this work, the "domain specific performance" is that of adult, primary student teachers taking part in a part-time PGCE mathematics module engaging with problem-solving tasks designed primarily for primary-age children. The assumption underlying the recording of their performance whilst engaged in such tasks with a small group of their peers is that the knowledge brought to bear on the mathematical strategies employed would be worth capturing for later consideration. This was potentially useful as, for the majority of the participants, this mathematics module was the first time they had engaged with such mathematics problems in a long time. It was also the first time, in the module itself, that such an emphasis had been given to either discussion/talk or to the solving of a single problem across a concentrated period of time (two problems were planned for recording in two separate recall sessions; this was, in part, to allow the comparison of one recording technology against another – see 3.3 below for details about the use of the *Livescribe* recorder in the second of the problems). 1.2 and 1.3 above provide further detail on the background to the problem addressed by this work and of the part-time PGCE mathematics module itself.

This chapter takes each of the methodologies – T-APs and SRIs – and provides an outline of the way in which they were used in this work, with reference to Task-Based Interviews (Goldin, 1997) and the connection to be made between these and the ‘free problem-solving’ and questioning approaches used to prompt responses from the participants in the SRI situation. 3.2 details the use of a T-AP to encourage the effective verbalisation of thoughts/problem-solving strategies during two separate mathematical problem-solving activities; this includes the ‘interviewer’s monologue’ intended to brief the participants prior to thinking-aloud. This is designed to take account of Ericsson and Simon’s (1993) observations about the different types of thinking-aloud and their impact on problem-solving performance, most particularly their view that “Type III” verbalisations, those that require explanation, are the most demanding and ‘intrusive’. 3.3 then provides an outline of the approach taken with the SRI sessions scheduled shortly after the initial problem-solving events – how and when the sessions were organised, including the issues presented by a follow-up activity of this kind needing to be conducted as soon as possible after the original recorded event when the student teacher participants have timetable demands that preclude an immediate follow-up. Goldin’s (1997) Task-Based Interviews are elaborated upon here, as an influence on the approach taken and the questions to be asked by the researcher within the session. This section also provides a detailed overview of the *Livescribe* pen technology used in the second of the two problem-solving situations to capture the notes taken by the participants alongside their verbal contributions. This overview of the multi-media *Livescribe* device includes the way in which it was utilised within the follow-up SRI. Both 3.2 and 3.3 give consideration to the data collected, and how it was anticipated it would be analysed. In the case of the T-AP transcripts, this would be via the Talk Framework; in the case of the SRI via limited use of some of the Talk Framework categories, to identify any ‘additional’ explanation or *exploratory* (Mercer, 1995) dialogue provided in the follow-up, and also via consideration of the student teachers’ questions and responses according to specific areas of focus such as *mathematical strategies* and the *group situation* to identify the degree to which such a follow-up enables them to re-engage with the mathematics.

3.4 then details the Talk Framework used for the coding of the T-AP transcripts, including how it was informed by the work of Mercer (1995), Novotná (1997), and Hošpesová and Novotná (2009). This section outlines the devising of the categories ‘meshed’ from these separate existing frameworks, a merging informed by the work detailed in Hickman (2011) and Hickman and Monaghan (2013), papers written both individually and in collaboration with one of the researcher's supervisors across the course of the project. It concludes by exploring the way in which the resulting Framework was utilised within the SRI sessions with the ambition to encourage participants to both identify and reflect upon their own *cumulative* and *exploratory* speech whilst listening back. The relevant Talk Framework coded results can be found in 5.4 with associated discussion in 6.4 (maintaining the 'parallel' organisation of the chapters in this thesis as discussed above in 1.9). Relevant extracts from the coded T-AP transcripts themselves are included in 5.4.2 and 5.4.4 with extracts from the follow-up SRI transcripts in Appendix 3 and 4. Further information about the researcher's development across time, from initial thoughts to the first and then second iteration of the project (which provides background to the development of the methodological approaches taken), can be found in Chapter 4, which also acts as a prologue to the Results of the second iteration (Chapter 5) and the Discussion (Chapter 6) that then follow.

3.5 revisits the SRI transcripts and discusses how participants' responses were analysed for this work with a focus on participants' discussion about the *group situation*, *mathematical strategies* and *verbalisation*. Again, the corresponding sections in the Results and Discussion can be found in 5.5 and 6.5.

The Methodology then concludes in 3.6 by outlining the data sets utilised in this work with references to both appendix items and the relevant Research Questions. 3.7 provides an overview of the ethical considerations underlying this work (with Appendix 5 providing the ethical approvals from the two Universities involved).

3.2 Verbalisation and Think-Aloud Protocols

This section outlines the Think-Aloud Protocol (T-AP) planned for use in this study to encourage the verbalisation of thoughts in the two group problem-solving situations to be encountered by the part-time PGCE student teacher participants. It provides the rationale for the use of a T-AP, with reference to the emphasis on the importance of *exploratory* talk in group situations discussed by Mercer (1995) and Seal (2006), and refers back to 2.5.4 above, where appropriate, to consider the impact of a T-AP on 'slowing down' problem-solving and thinking (a 'slowing down' that can also potentially interfere with the problem-solving and impede the participants' success – separate from the "psychological situation'" (Rotter, 1954) in which the participants find themselves, that may also have an impact on their performance). The briefing of the participants via an 'interviewer's monologue' at the outset of the recorded sessions is outlined, with a rationale provided for avoiding a direct requirement to provide 'explanations' in the verbalisations that takes account of Robertson's (2001) observations on the impact of Ericsson and Simon's (1993) different types of thinking-aloud, most specifically the cognitive demands of explicitly asking participants to explain while they are working. The section then concludes with the connections to be made, in this work, between T-APs and Stimulated Recall Interviews (SRIs), usually treated as distinct methodologies (Meijer et al. (2002), for example, referring to SRI as a "substitute" for T-APs – see 2.5.5 above). The reasons for replaying the T-AP recorded material to the participants in a follow-up situation leads into 3.3 which considers Stimulated Recall as a follow-up to Think-Aloud and outlines the way in which the material would be replayed and the kinds of questions to be asked during this session. With this preamble over, this section will now turn to the substance of T-APs as intended to be utilised in this work.

T-APs have the potential, as discussed above in 2.5.4 to provide a detailed account of participants' "underlying thinking" (Conrad et al. 2000, p.1) whilst they are engaged in a task. They are, in fact, asked to speak their thoughts aloud while working, rather than being asked by a teacher or researcher what they think about a particular problem or task, either during the task itself – which could interrupt their thinking – or afterwards, when perhaps the distance from the task may reduce the level of insight that can be provided (Gass and Mackey, 2000). T-APs also, by their nature, can encourage more awareness of the thinking that goes towards the solving of a particular problem and, in this research, as indicated above in 1.2 and 1.3, there was a desire to provide additional opportunities for the student teacher participants to engage in mathematical problem-solving situations and also dialogue with their peers that might help model the group situations they might find, or even plan to encourage, in their own classroom settings. From this perspective alone, the use of a T-AP could be considered advantageous, allowing them an opportunity that had not previously been provided in their training.

In addition to this, in a group situation, T-APs may provoke more discussion than might otherwise be the case from individuals working together – perhaps encouraging more considered explanations, methods and reasoning, with the result that the task and mathematics are further explored, and more *exploratory* talk (Mercer, 1995) is, therefore, in evidence. The Primary National Strategy (DfES, 2006) recommends that such carefully considered talk, with a focus on explanation and reasoning, should be a part of 'good mathematics teaching' (again, an argument that think-aloud may model useful classroom practice). Seal (2006) considers how best to encourage pupil dialogue whilst working collaboratively, determining that the majority of the talk in the lessons studied for that report was *cumulative* (using Mercer's (1995) categories) with a large amount of repetition and elaboration in evidence, perhaps as a consequence of uncertain individuals needing to have confirmation of the approaches to be taken[9]. This relates to Piaget's (1929, p.6) observation about the withholding of questions in company, perhaps because the individual/s feel that they are already "known to everyone" (see 2.5.2 above). While there is some *disputational* talk evidenced in Seal's (2006) study (talk that involves disagreement or individuals essentially working independently of the group and without taking account of their thoughts and ideas), far less evidence was found of *exploratory* talk (Mercer, 1995), where the pupils would provide justification and explanation of their ideas. Seal's (2006) proposal for ameliorating this is to devise 'ground rules' for group-working that would explicitly require participants to provide reasons for anything they put forward. In the case of the project outlined in this thesis, it was felt that the T-AP may produce similar results without explicitly

[9] There is no statement made in Seal (2006) about whether the high amount of *cumulative* talk found in the study is, in any way, a 'bad thing' (although this is implied by the emphasis on reducing it in favour of *exploratory* talk). Neither is it stated whether – in fact – it might be expected that there ought to be more *cumulative* talk in a group situation. It could, for example, be argued that *exploratory* talk is a rarer phenomenon due to the nature of group working, ensuring that peers understand each other's points, and reflecting the fact that putting forward ideas for approval for the rest of the group is – particularly in the context of a mathematical task and particularly for groups who are unfamiliar with this way of working – a 'riskier' endeavour that actually relies on the *cumulative* contributions made. The overall message of Seal's (2006) work is that *exploratory* talk should be encouraged and that 'ground rules' are a key way of achieving this.

providing such rules. Therefore, the 'interviewer's monologue' did not go as far as Seal (2006) in asking participants to provide substantiation for every point made:

> "I would like you, as a group, to share your thinking with me while you solve the question provided. Attempt to tell me everything you are thinking from the time you first see the question until you give an answer – use the pen provided to write down anything that you find useful in helping you to solve the problem, talking aloud constantly as if you are speaking to yourself as well as to your peers. Please understand that I may prompt you to continue thinking-aloud at times. Don't worry about what I might think about your answers; I want to learn about your thinking while engaging in this task".

As indicated by the 'interviewer's monologue', T-APs, by design, encourage the verbalisation of thoughts that would "normally be silent" (Gilhooly and Green, 1996, p.43). Given the experience in the PGCE module sessions that trainees were not always comfortable in putting forward ideas for potential solutions when not already confident that they had an answer to the given question (see 1.2 and 1.3 above), the use of such a methodology, it was hypothesised (see 1.5 above), would perhaps encourage dialogue that could later be recognised and reflected upon by the participants when listening back to their comments. Some of this originally captured dialogue would provide explanation and substantiation, but it was also hypothesised, building on Seal (2006), Piaget (1929) and Ericsson and Simon (1993), that the opportunity to reflect on the dialogue, after the event (via SRI), might promote metacognition and potentially inform future practice. It might, for example, be useful for student teachers to recognise their own *exploratory* talk, thereby exploiting the "self-generation" effect (Mulligan and Lozito, 2014) where participants are more likely to attend to their own contributions over those of their peers, while the replay opportunity might well allow for them to increase their awareness of others' contributions that had maybe been 'missed' in the original problem-solving session because of this same effect. The follow-up SRI opportunity is discussed in more detail below in 3.3.

In employing T-APs as data collection within educational research, it is arguable that participants need to be clearly briefed as to how they should vocalise their thoughts. Montague and Applegate (1993), for example, stress the need for clear demonstration and indeed practice in thinking-aloud. Equally, some direction as to the kind of verbalisation required (Ericsson and Simon, 1993) might have been beneficial. However, even with clear prompting from the researcher regarding the extent to which individuals were to think-aloud (see ‘interviewer’s monologue’ above), it was considered likely that verbalisations might well be inconsistent due to the combined demands of working alongside peers, tackling a ‘new’ mathematical problem, and attending to the requirements of the think-aloud protocol (this informs the student teachers’ “psychological situation” (Rotter, 1954) discussed below in 6.2). Such inconsistent or ‘incomplete’ verbalisations might then lead to difficulty identifying, for example, whether any “analogical problem solving” (Robertson, 2001, p.15), or indeed any other kind of problem-solving, was being employed (this was presuming, of course, that discussing this would be valuable for the student teacher participants). Additionally, if the task set was (or appeared to be) relatively simple, then participants might take ‘short cuts’ to providing the answer; thinking-aloud might not be as relevant if the problem was not especially taxing. Robertson (2001) refers to such short cuts as “fast and frugal heuristics” that require little in the way of computational effort, and this can be related to Kahneman’s (2011) “system 1 and system 2” thinking as discussed above in 2.6.4. “System 1”, in this case, is the “fast and frugal” system[10]., whilst “system 2” is the ‘slower’ and more deliberative thinking that involves paying greater attention to potentially misleading statements in a question or questioning what seems to be immediately apparent, and double-checking as necessary to see that results are in line with the question set. In this respect, it provides a further rationale for “looking back” as Pólya (1957) recommends, whilst also indicating that deliberately putting thoughts into words, as a T-AP demands, may be beneficial to ‘test’ for assumptions that may prove misleading (bearing in mind Piaget’s (1929) observation about individuals not wishing to ask questions for fear of others already knowing the answers; this provides one reason for verbalisations being ‘incomplete’).

Such ‘fast’ “system 1” thinking may then result in a lack of potentially valuable *exploratory* talk (Mercer, 1995) as it may not even be needed – or the students may

[10] “System 1” leads to quick responses based on implicit knowledge and understanding of a problem; this can be effective, but can also lead to errors and misconceptions going unnoticed, or even substituted ‘easier’ questions being answered in place of the questions actually asked.

not appreciate the need. As Kahneman (2011) states, one of the pitfalls of "system 1" thinking is that it is often too quick to spot misconceptions that a period of reflection would have identified. It is not likely that participants will feel the need to expend "computational effort" (Robertson, 2001) on something that they feel has been pitched too low, even if this is an inaccurate perception of the problem's demands on them.

In the case of the two problem-solving tasks chosen for the participants to solve here, the decision was taken to pitch the exercises at the level of the children the student teachers were training to teach (see Appendix 2). The problems were taken from the Primary National Strategy Problem-Solving pack (DfES, 2004) and "Mathematical Challenges for Able Pupils in Key Stage 1 and 2" (DfEE, 2000); therefore, the first ("Exploring Addition") could be seen as less demanding than the second ("Beads") although both are designed to be accessible to primary-age children. Both were deliberately chosen to be similar to those the student teachers were presented with at the outset of the work, when given their initial questionnaires to answer regarding their level of confidence in tackling and explaining such problems (see 5.3 below; the questionnaires are provided in Appendix 1).

As indicated above in the wording to the 'interviewer's monologue', rather than Seal's (2006) approach of asking for reasons to 'back up' anything said, the participants were instead asked to "talk aloud constantly" as they tackled the problem. They were to speak as if to themselves as well as to their peers. This was to attempt to capture thoughts and ideas that might not necessarily have been shared in other group situations, and was also given in this manner so as not to suggest that some mathematical approaches are more appropriate than others (i.e. there was no direction to write down the problem in any particular form, although participants are given the option to '*write down anything that you find useful in helping you to solve the problem*' – this, it was hoped, would help the participants decide for themselves when to *encode* and *transform* (Novotná, 1997; Hošpesová and Novotná, 2009) the problem into mathematics – see 3.3 below). Avoiding direct instruction to 'explain' or 'explore', it was hoped, might reduce anxiety from those less comfortable and confident with sharing their ideas in a discussion-based mathematical problem-solving context. They might more 'freely' share their thoughts, both related to the mathematics and to the nature of the task, including the group situation, and this would provide a number of potential avenues for discussion in the subsequent SRI.

As indicated in 3.6 below (Data Sets), the student teachers' T-AP prompted speech was recorded using two different technologies. In the first instance, a 'standard' digital audio recorder was employed, meaning that audio alone would be available to them in the follow-up SRI. In the second, a *Livescribe* pen capable of recording sound and associated annotations was used – see 3.3 below for an outline of how *Livescribe* works and was used within the T-AP sessions. There was no time limit imposed on either of the problem-solving sessions, as can be seen from the 'interviewer's monologue' above – the recording was, therefore, stopped at the point when the group collectively determined that they had reached an answer, and then verbalised that they have finished (i.e. without members of the group disputing that the problem had been solved). This did not mean that all would necessarily feel that they had reached a solution, or even that all understood what had been done across the session. Again, the follow-up SRI was planned to explore such issues.

The subsequent T-AP recordings were then transcribed, ahead of the SRI session, to analyse the types of talk evident in the session via the Talk Framework. Section 3.4 below details both the way in which "turns" (Rowland, 2000) have been identified for the purposes of coding the participants' contributions (and, therefore, how the word is used within this work); it also outlines the Framework categories: how these were devised, and how they attempt to identify potentially valuable talk within mathematical problem-solving opportunities.

Once coded, the T-AP transcripts were intended to address the following research questions: RQ 1 (by identifying the dialogue that has been promoted by the T-AP; had the process, for example, "*support*[ed]…[their] *verbalisation of problem-solving strategies*" – the SRI then provided the opportunity for individuals to further address this question); RQ 4 ("*what types of talk are most evident in multi-media artefact enhanced thinking-aloud while engaged in mathematical problem-solving activities?*"); and RQ 5 ("*in what ways do the networked T-AP and SRI methodologies impact upon participants' willingness to engage with a problem-solving task?*" – again, while the transcribed T-AP material might indicate the degree to which individuals had engaged in specific types of talk, the SRI provided the opportunity for reflection on the impact of the methodologies on performance). The Talk Framework employed to code the various responses into types was informed by the work of Mercer (1995), Novotná (1997), and Hošpesová and Novotná (2009). While utilised between the T-AP and SRI sessions for coding the student teachers' responses, the Framework itself is detailed in section 3.4 below, following the consideration of how SRI was utilised in the follow-up recall session. This is because the "networking" (Hickman and Monaghan, 2013) of the two methodologies (T-AP and SRI) suggests it would be beneficial to draw the connections between them in consecutive sections of this methodology, and also because the Talk Framework codings ultimately had limited impact on the SRI session, despite the opportunity being provided for individuals to comment on them if they wished.

Having established how the T-AP was used in this work to promote verbalisation in the two problem-solving tasks, and how this was then captured (see also 2.6) via digital audio for later transcription, the next section turns to Stimulated Recall Interviews (SRIs) as the other central methodology to this work. SRIs are rarely, according to some authors, combined with T-APs due to being seen as a distinct, alternative approach that does not impact upon performance in the undertaking of the task itself. It had always been intended that the recorded problem-solving sessions would be 'revisited' by the students (see 1.5 above), perhaps as a plenary opportunity to enable "looking back" (Pólya, 1957) within their PGCE mathematics workshops. Utilising an SRI potentially enabled greater focus on the replay – and the participants' exact words and strategies – than would be possible in a 'traditional' classroom plenary, or through inviting students to listen back to their words outside the session, for example via a VLE, as had at one point been envisaged by their mathematics module tutors. The next section outlines how the SRI was conducted in this work, and how this was intended to encourage a degree of metacognition from the participants that may, as indicated here, prove to be valuable for their future work as teachers.

3.3 Stimulated Recall Interviews as a Follow Up to Think-Aloud (the "Networking" of Methodologies, Including the Use of *Livescribe* as an Additional Prompt to Participants)

The preceding section has considered Think-Aloud Protocols (T-APs) as a distinct methodology in its own right. In this work, however, the initial T-AP sessions were supplemented by follow-up Stimulated Recall Interviews (SRIs; Dempsey, 2010). This section, therefore, provides a rationale for the "networking" of the two methodologies (Hickman and Monaghan, 2013), detailing how this was achieved in this particular situation, along with the potential issues presented by this approach. As the second of the two SRI sessions was prompted by a *Livescribe* audio-visual recording, a rationale will be provided for including this technology, and – indeed – using it for only one of the two problem-solving tasks, as a means of comparing the ways in which digital audio technologies can encourage responses from the participants and, therefore, addressing Research Question 3: "*What does the use of a multi-media artefact in an SRI situation reveal to participants about their own problem-solving strategies, and those of the group?*"

As indicated above in the Literature Review (2.5.5), Stimulated Recall Interviews, as with T-APs, allow for the elicitation of the thinking underlying the performance of a particular task (Gass and Mackey, 2000; Mackey and Gass, 2005). As a "metacognitive practice" (Bransford et al., 2000), they are – therefore – more often conducted alone, as a "substitute for a thinking-aloud method" (Meijer et al., 2002, p.166). Examples of this approach, similar to the work undertaken in this project, include teachers commenting on a video or a recording of their teaching in order to explicate their intentions and thinking 'after the event'. The intention of such an approach would commonly be to raise awareness with these teachers of the strengths and weaknesses in their practice (thus, helping them to set targets for the future in a training environment where such targets, as they are in schools themselves, are commonplace, most especially since the 2010 Coalition government (Lightman, 2015)). Indeed, this had been done with some of the student teachers taking part in this work, albeit within a different part of their training programme. Thinking-aloud, though, with the exception of – for example – demonstrations for children when performing calculations as part of a mathematics lesson, was not something with which they would have been familiar, either in their taught sessions within the University or in their classroom practice. This is not only for the reasons given by Ericsson and Simon (1993) related to such thinking-aloud causing 'interference' with the task in hand or 'disturb[ing] the cognitive process" (van Someren et al., 1994, p.32), but also because of the deleterious impact it might have on the learning of their children. As Kahneman (2011, p.23) says (see also 2.5.4 above), "effortful activities…interfere with each other" and such interference caused by teaching whilst adhering to a T-AP might well result in confusion either on the part of the teacher, the children, or indeed the whole class.

In a mathematical problem-solving situation undertaken with their peers, however, student teachers could be more readily, and perhaps safely, encouraged to think-aloud. This does not mean, however, that the T-AP would not impact upon the mathematics (Robertson, 2001) as there is still a combination of "effortful activities" (Kahneman, 2011), this time also including working with their peers (see Piaget's (1929) comment in 2.5.2 above about individuals not wishing to risk asking questions in front of others; this relates to the overall "psychological situation" (Rotter, 1954) of the group problem-solving session that is addressed in 6.2 below). Notwithstanding this concern, it was considered that the T-AP would provide an insight into their thinking whilst problem-solving in a group situation that would arguably benefit their development as classroom practitioners. Moreover, any potential interference with the task-in-hand caused by the T-AP could be followed up afterwards, with participants invited to comment on the impact of the methodology on their mathematics in the recall opportunity. Indeed, this forms a key part of the rationale for combining the T-AP and SRI methodologies.

With the T-AP sessions recorded for later analysis using the Talk Framework, the SRI playbacks were conducted as soon as possible after the original task/s had been conducted (these were separate sessions, rather than requiring the student teachers to recall one problem-solving event immediately after the other[11]). This reflects Sime's

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[11] It is worth noting here that the delay between the T-AP and SRI sessions was perhaps longer than Sime (2006) might propose – the student teachers' timetable precluded the possibility of a recall session any sooner than one week after the original problem-solving

(2006, p.216) comment about "reduc[ing the risk of] memory decay" between the original task and the recall. The Talk Framework coding was conducted between the T-AP and SRI sessions, as part of the transcription process. The transcriptions were made available in the SRI to alleviate difficulties following the audio, but the coding was not made available to them (i.e. they had uncoded copies of the transcripts). This was to ensure that they were not led, ahead of listening to the playback, to believe that particular verbal contributions were in some way more 'valuable' than others. They were, however, introduced to the Framework categories within the SRI introduction (see below), and were invited to comment on any elements of their speech that struck them as 'of interest' i.e. if they 'recognised' *cumulative* or *exploratory* contributions. See 3.2 above regarding the decision, within the T-AP 'interviewer's monologue' to avoid any direct requests for explanation, such as Seal (2006) may have encouraged, given the perceived importance of *exploratory* talk in mathematical situations.

Ultimately, though, the focus of the SRI was for the group members to comment on their original problem-solving 'performance' i.e. why they had contributed as they did when they did. This included reflection on strategies that had perhaps not produced the desired outcome, or had been successful but had not perhaps been understood by the whole group[12]. The ambition was for this to ameliorate any

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event. Additionally, the two SRI sessions followed the two T-AP sessions – it might have been possible for SRI 1 to follow T-AP 1 more directly, with SRI 2 then following T-AP 2 (therefore, alternating between T-AP and SRI opportunities). This may have been beneficial to the participants in ensuring that their memory of the relevant problem-solving task was 'clearer' and they were not confused as to which problem was being discussed – however, their timetable would still have entailed a gap of approximately a week between sessions, and it was considered that their University teaching sessions and other commitments already presented obstacles to clear recall of the original problem-solving tasks. Equally, the recorded material to prompt their recall, alongside the transcripts, meant that they would be given clear reminders as to which of the problems was being discussed.

[12] While not strictly scripted, as with the 'interviewer's monologue' for the T-AP sessions, the SRI instructions were intended to be consistent for the two recall opportunities. The first SRI introduction (or 'interviewer's monologue') (see Appendix 3) ultimately ran as follows: "*We're going to listen back to the recording made of your two problem-solving tasks and you're [to be] supported by the transcripts of what you said as a group at the time*." After a reminder to the participants that all contributions to the SRI would be anonymous, and a request for any obvious errors in the accompanying transcript to be identified, the introduction continued with: "*In listening back to the recordings, I will use…myself…the talk and problem-solving Framework I have recently introduced to you… When you think*

failure on the part of the T-AP to ensure that all members of the group had been clear about the task and the proposed strategies for its solution. 3.6 (Data Sets) below indicates the Research Questions addressed by this SRI follow-up session, including how the methodologies had impacted upon the student teachers' willingness to engage with the mathematics (RQ 5), and – indeed – how thinking-aloud supported by digital audio has supported, or even hindered, their problem-solving performance[13].

After the researcher's introductory monologue, the student teachers listened back to the relevant recording (the two SRIs were conducted on separate occasions, with a gap of a few days between them[14]). As noted above, transcripts were provided to aid them in following the talk should this be necessary, having identified – in the first iteration of the project discussed in Chapter 4 below – that individuals sometimes had difficulties identifying even their own voices on the recording (the T-AP transcript material in 5.4.2 and 5.4.4 below has been anonymised from the versions available in the SRI sessions). The intention was for the student teacher participants to be prompted by themselves – by their own comments and suggestions – thereby providing a real-time 'commentary' on their original problem-solving performance. However, as had also become clear in the first iteration of the project, the recordings needed to be paused at various points in order to allow responses to be made without individuals talking over the remainder of the recording, and also to allow for clarification of difficult-to-understand passages. The interviewer's transcript was

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one of your contributions fits with, say, cumulative…agreement…or exploratory or disputational and you feel you want to comment on that, then you can. I will ask you at times to explain how you thought about the problem and how you came to make the decisions about ways in which to solve it with your peers. And that's the really important thing. Why did you contribute in the way you did when you did."

[13] Rather than introduce these areas to the participants in the introduction to the activity, it was decided that, if the student teachers did not directly mention the impact of the methodologies on their willingness to engage or, indeed, on their success with the mathematics, then these would be expressly asked about during the session.

[14] One aspect that was considered when scheduling both the T-AP and SRI opportunities was the degree to which the experience of taking part in the first of the two problems/SRI playbacks might inform the second. It is probable that the participants would have benefited, at least to a degree, from the first opportunity, and there might – therefore – have been more of a focus on the mathematics in the second SRI as a result of the group having 'exhausted' their comments on the T-AP protocol and the group situation in the first SRI.

marked up ahead of time with potential points of interest for discussion, but the onus was on the participants themselves identifying the aspects they wished to discuss, as this better reflected the needs of RQ 3, which considers how the student teachers attended to their own problem-solving strategies as a result of engaging in the playback (what did they, themselves, notice?). The participants, therefore, took the lead in the discussion as far as possible, with occasional prompts from the researcher where necessary (i.e. to ask for further explanation of unclear aspects of their problem-solving performance). This practice was the same for both the 'standard' digital audio and the *Livescribe*-supported session (although the multi-media nature of the latter required more pausing of the playback in order for participants to reflect on what had been said alongside the annotations).

The questions asked of the participants by the researcher during the SRI were informed by – although they did not strictly adhere to – aspects of Goldin's (1997) Task-Based Interview (T-BI) "four stage exploration" (see 2.5.3 above). The student teachers had already been "pos[ed] the question" (Goldin, 1997, p.45) in the T-AP session, and this was without follow-up questions of any kind as, while Goldin's (1997) exploration allows for 'non-directive' questions at this point, the tutor/researcher explicitly did not take part in that discussion (as anything other than an observer). The second stage ("heuristic suggestions if the response is not spontaneous" (Goldin, 1997, p.45)) was also absent from the T-AP session (unless, of course, individuals made such suggestions themselves). The SRI, then, provided the opportunity to address both this and the third and fourth stages – "guided use of heuristic suggestions…(e.g. "Do you see a pattern in the cards?")" and "exploratory (metacognitive) questions (e.g. "Do you think you could explain how you thought about the problem?") (Goldin, 1997, p.45). Therefore, the T-AP transcripts were analysed prior to the sessions, not only for dialogue that met the Talk Framework categories but also for potential questions that could be posed, for example about aspects of the problem-solving that, in the researcher's view, could be further explained. There were also questions relating to the group situation, for example, where there were issues relating to *disputational* talk or the group not checking with itself that aspects had been understood, as seemed to happen in the second of the two T-AP recordings. Equally, it was possible to ask for clarification of unclear contributions, and encourage the participants to reflect on verbalisation in general (considering the impact of the thinking-aloud methodology on their ability to tackle the problem, as required by RQ 1 and RQ 5). Wherever possible, however, the intention was not to directly ask for such aspects to be explained, as this might deny the group the opportunity to raise problematic aspects of their experience for themselves.

The use of the *Livescribe* pen allowed for the opportunity, as indicated by Research Question 6 above, to explore whether the SRI responses were 'enriched' by the 'replay' of the group's annotations alongside (and in sync with) their speech. This 'enriched' talk might be evidenced (see 'Data Sets' and SRI transcripts within 3.6 below) by identifying examples of *exploratory* talk (Mercer, 1995) within the SRI recordings that had not been put forward in the original T-AP sessions (i.e. the SRI transcripts were analysed for additional *exploratory* talk relating to the mathematical problem set in the original T-AP). This 'new' *exploratory* material might consist of additional explanation 'missing' from the original T-AP sessions where strategies, for example, had not been completely verbalised. This may have been because the group had 'moved on' in its discussion or because there was a presumption that all members of the group understood where they had 'got to' (or, of course, because the participants found it difficult to maintain the level of thinking-aloud required by the protocol whilst also considering the mathematical task and attempting to listen to the thoughts and ideas of others in their group).

The data set, therefore, arising from the SRI session will be, as indicated in 3.6 below, the transcribed conversation, considered against the Talk Framework where appropriate. The participants' responses to the questions asked is of interest in addressing the research questions, as – indeed – are the questions they raised themselves, prompted perhaps by listening back to their original verbalisations. The use of the SRI transcripts is further detailed in 3.5 below.

The plan for the two SRI sessions is, then, as follows in Figure 3.1 (adapted from Figure 1.1 in the Introduction above):

Talk Framework analysis of T-AP 1 and T-AP 2 prior to SRI sessions – original verbal contributions (turns) coded for later reference and to inform potential questions within the SRI.

Follow-up SRI 1 – standard digital audio recording
Participants listen to digital audio recording supported by transcripts (for clarity).

Follow-up SRI 2 – *Livescribe* digital audio recording
Participants listen to *Livescribe* audio recording with original annotations visible on Interactive Whiteboard screen (T-AP transcripts provided, as SRI 1).

For both SRI recordings
Student teachers briefed via a preamble to comment live on their original words. **Recording paused as necessary to allow for questions to be asked, and for responses to be made**.

Analysis of SRI and SRI 2 responses – attention paid to:

- ‘New’ *exploratory* contributions made, additional to the original T-AP (i.e. further explanation of strategies used in original problem-solving situation; the proposition of alternative strategies from those followed originally; the identification of problems with the original strategies proposed – **RQ 1**, **RQ 3**).
- Questions asked by participants themselves as opposed to those of the interviewer (to identify the degree to which problem-solving strategies are ‘revealed’ to participants without prompting – **RQ 3**).
- The focus of the questions/responses from the participants – on the mathematics (*mathematical strategies*), the *group situation*, *verbalisation* (i.e. the Talk Framework strategies), and on the procedure (i.e. the T-AP itself, or the process of stimulated recall). This addresses **RQ 1**, potentially **RQ 2** (as participants are to be invited to comment on the effect of thinking-aloud and the protocol), **RQ 3**, and potentially **RQ 5**.
- Any differences in the number of responses/questions relating to mathematical strategies between SRI 1 and SRI 2 – this addresses **RQ 6** (in other words, does the *Livescribe* pen prompted recall encourage more reflection on the mathematics).

Figure 3.1: An overview of the SRI data analysis (see also section 3.6 below)

This section now concludes with an overview of the *Livescribe* pen technology, as planned to be used in the second of the T-AP opportunities (see also 3.2 above) and then used for the replay of both sound and original handwritten annotations in the second SRI session.

Livescribe pens are digital audio recording devices contained within/attached to what appear to be standard ball-point pens – they enable the recording of sound and annotations/writing and the subsequent synchronous playback of both via the *Livescribe* desktop software which provides an image of a 'virtual notebook' on-screen (see Figures 3.2 and 3.5 below). It is also possible to tap any point on the physical paper and hear the relevant audio recorded from that moment; a feature that may be most useful in the classroom context with individual children or students perhaps having individual pens and pads – this feature was not used in this work as the whole audio recording was played from start to finish, albeit with occasional pauses to enable responses and clarifications as indicated above.

Figure 3.2: A *Livescribe* pen, as used in this project, with significant features identified (Livescribe, 2009)

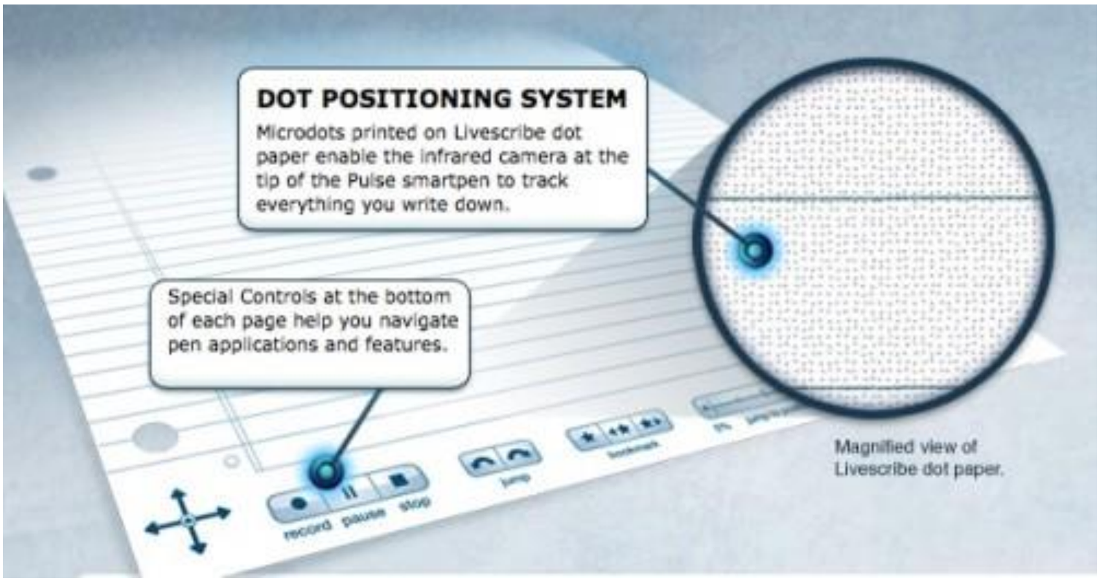

Figure 3.3: The paper used with *Livescribe* pens (the pens will only work as standard biros without it) to enable playback of audio. Pen controls are printed on each sheet, enabling recordings to be started, paused stopped and bookmarked. Once recorded, the pads were not used again in this project – attaching the pen to the computer allowed the annotations and synchronised audio to be replayed on-screen via the *Livescribe* desktop. (Livescribe, 2019)

The *Livescribe* pen works with the use of specially designed 'dot' paper (available as notebooks of various sizes), that allows for the position of the pen on the paper to be tracked accurately (via an infrared camera) whilst the audio is being recorded. Participants are free to use as many pages as necessary during the T-AP session.

Having recorded the audio and writing using the pen, the resulting digital file can be played back from the beginning, with the writing 'appearing' on the *Livescribe* desktop at the relevant points in the audio (see Figures 3.3 and 3.4 below). Recordings can, as with standard digital audio files accessed through PC software, be played back from specifically chosen points – in this work, as stated above, the recording is to be played from start to finish, pausing as necessary for clarification of particular verbal contributions within the SRI. This means that participants will be able to see their annotations on an interactive whiteboard screen whilst also listening to the sound via the computer speakers.

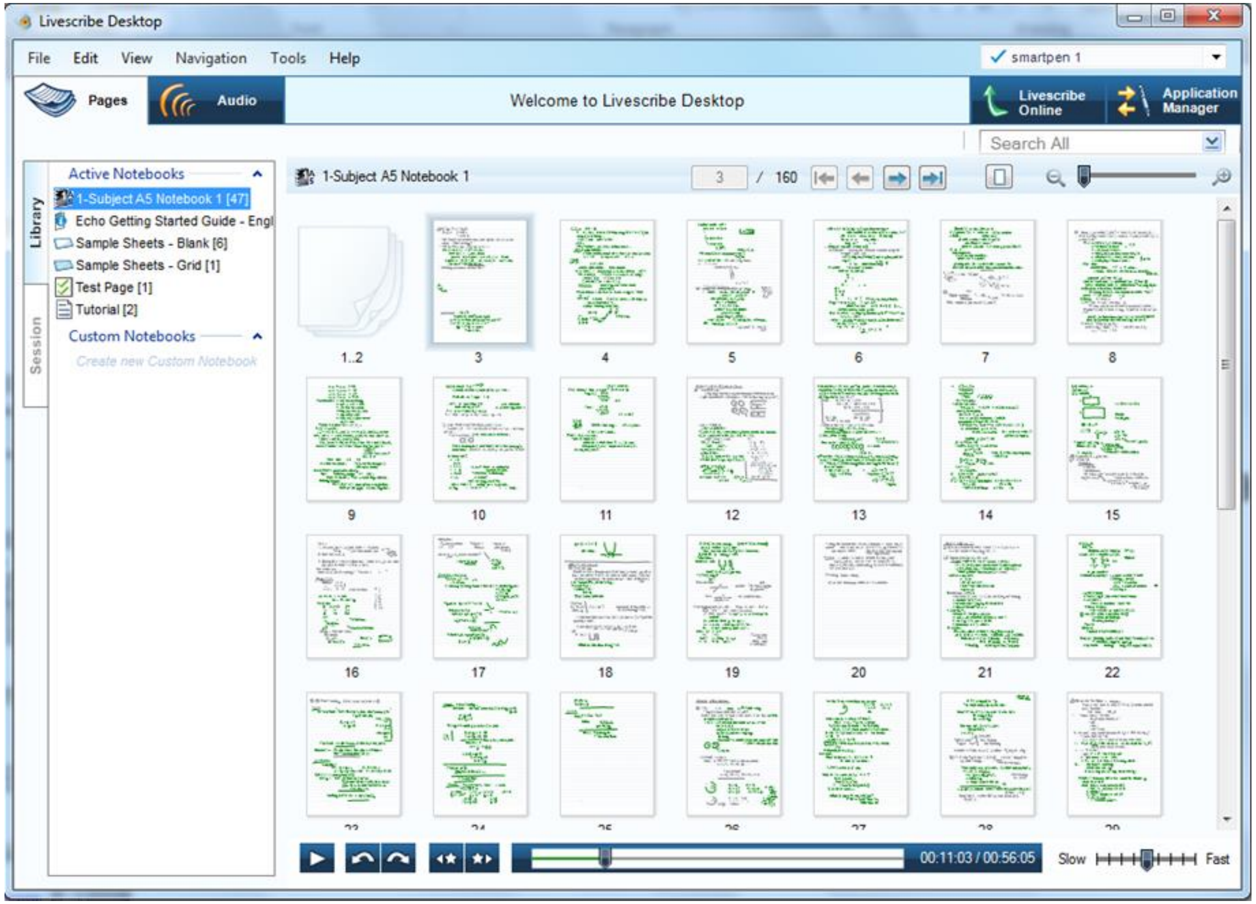

Figure 3.4: *Livescribe* Desktop image for a random notebook/pad, similar to that used by the participants in this work. In this case, any one of the pages can be chosen and replayed (both annotations and audio). The top right of the image shows that "smartpen 1" is attached to this notebook (in this work, only one notebook and one pen were used, which avoided the problems that can arise from having multiple pens and multiple notebooks that can only be replayed with their paired pen).

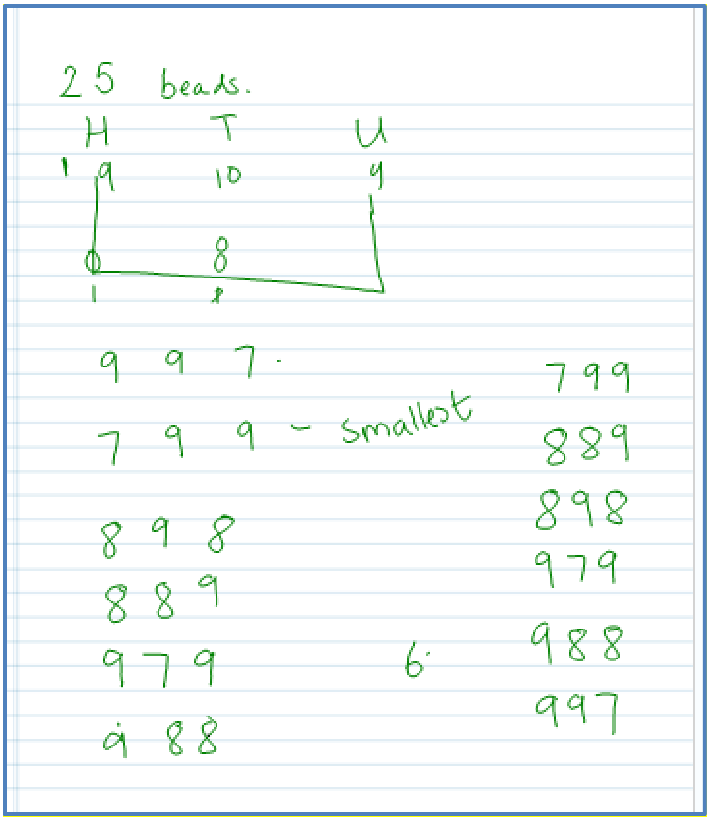

Figure 3.5: *Livescribe* note page from TAP-2, as 'replayed' on the *Livescribe* Desktop for SR1 2. See also 6.8 for discussion of *Livescribe* versus 'standard' digital audio recorders as a spur for stimulated recall and potential prompt for "analogical problem-solving" (Robertson, 2001).

The next section of the Methodology addresses the Talk Framework used to code the responses from the original T-AP sessions which was then utilised again in the SRI as a possible prompt to the student teachers when considering their verbal contributions to the problem-solving tasks.

3.4 A Talk Framework for Problem-Solving – to Support the Stimulated Recall and Enable a Greater Focus on Mathematics

This sub-section considers the use of the Talk Framework to support the Stimulated Recall Interviews (SRIs) that followed the original Think-Aloud Protocol (T-AP) supported mathematical problem-solving sessions. The Framework provided a potential means of enhancing the "look[ing] back" (Pólya, 1957) at the student teacher participants' performance, as the codings identified, for example, *cumulative* and *exploratory* dialogue that could be explored further in the recall, most especially where exploration/explanation had been relatively undeveloped and some members of the group had been 'left behind' as a result. This, as with the overall focus on dialogue in this work, connects with Duval's (2006, p.104) observation that the "sources of incomprehension" in mathematical tasks (see 2.5.1 above) should be discovered via the students' own voices. The Framework perhaps allowed for clearer identification of the difficulties that participants had in "grasping the assignment" (Novotná, 1997; Hošpesová and Novotná, 2009) – along, of course, with successful, potentially *exploratory*, verbal contributions that may have brought the group closer to the solving of the problem/s. In addition to this, by ensuring that mathematical transformations were reflected within its categories, the Framework addressed Duval's (2006, p.105) view that they are "never taken explicitly into account in teaching" – as, indeed, they had not been previously in the PGCE student teachers' mathematics module material.

While it might have been possible for the students to code their own verbalisations, and while it might also have been possible to consider the categories in more detail in the SRI sessions, as a means of encouraging potentially valuable metacognition, ultimately the Framework was predominantly used by the researcher to identify sections to be discussed in the SRI (if the participants themselves did not identify them first). The student teachers were introduced to the categories, and were invited to discuss them where relevant (see 3.3 above for the SRI 'interviewer's monologue) but this, ultimately, was not a major feature of the SRI (perhaps due to the competing demands on their attention of the SRI situation and the replay of their original words). 7.3.1 below, addressing the implications of this work for future practice, makes the suggestion that this is something that could be exploited in later iterations of this project. This point notwithstanding, the Talk Framework was of use in preparing for the SRI situations. This sub-section begins by addressing how the coding was conducted, and how 'turns' were categorised according to the Framework, before moving on to discussing how the categories themselves were formulated by merging separate talk and problem-solving frameworks and creating a new category, *supercumulative*, to identify speech intended to provoke further explanation and elaboration; identifying such speech, it is argued, is also of benefit to follow-up SRI opportunities, as individuals can be prompted to provide additional information 'missing' from their original discussion.

In coding the dialogue from the transcribed T-AP sessions, whole turns were considered, rather than breaking down contributions into constituent parts/utterances or attempting to find a combination of different categories in the one contribution. The term ‘turn’ here is informed by Rowland (2000, p.91) who describes the “sequential partition of conversations into ‘turns’ (in the sense that participants ‘wait their turn’)”. This is similar to Seal’s (2006) work considering the importance of *exploratory* dialogue in problem-solving, which was an inspiration to this work from the first – this also took whole verbal contributions (whole ‘turns’) as the basis for analysis. Therefore, when coding, a ‘best fit’ approach was taken i.e. was the whole question/response more suited to one category than another? As might be expected from a spontaneous discussion between group members who had not previously had sight of the mathematical problem presented to them, contributions varied from short, even one word, responses to lengthier, more ‘thoughtful’ tracts of speech. Some of the shorter contributions presented challenges in terms of coding (and some have, as a result, not been coded – see below); equally, some contributions, where perhaps the participant did not adhere to the T-AP particularly closely (leaving thoughts unverbalised), were undeveloped or ‘incomplete’. Where possible, these have still been considered against the Framework, as *cumulative* dialogue, for example (see below), which builds on preceding contributions could still be clearly identified even if, ultimately, the individual did not complete their thought (in some cases, this was due to individuals ‘talking over’ each other in their eagerness to put forward ideas). The interest was predominantly on how successive turns built on prior turns by providing new information or posing questions – or, indeed, identifying turns that did not build on those put forward by peers, perhaps due to the degree of concentration required (see 6.2 below for consideration of the group and “psychological” situation (Rotter, 1954) that may have impacted on the verbalisations put forward in both T-AP sessions).

The specific Talk Framework used in this work was developed from a merging of Mercer’s (1995) talk framework (see 2.5.1 above) with the problem-solving framework of Novotná (1997) and Hošpesová and Novotná (2009). Mercer’s (1995) original three categories (*cumulative*, *exploratory* and *disputational*) were initially expanded to four (with *exploratory* split into *exploratory encoding* and *exploratory transformative*), with a fifth category then added for the second iteration of the project which is the main focus of this thesis (see Chapter 4 for an outline of the first iteration and how it has been used to inform the later work). This *supercumulative* category was designed to reflect contributions that are not yet *exploratory* (i.e. presenting ideas to the group; suggestions, perhaps, for the solving of the mathematical problem), but *cumulatively* build on the thoughts of others whilst also directing the group towards transforming the task into mathematics (see Duval's (2006) observation above regarding the lack of attention given to transformations in mathematics teaching – having these categories in place, and utilising the Talk Framework with student teachers, had the potential to raise awareness of a significant element of mathematical problem-solving that had perhaps previously been under-explored on their PGCE mathematics programme). *Supercumulative* contributions go some way towards recognising the need for *encoding* and *transformation* without yet proposing how this should be done. They offer no new information (as would fit the *exploratory* categories) but build *cumulatively* on 'threads' already begun by others. Another example of talk that would fit this category would be examples of strategies or possible solutions offered up in response to a peer's *exploratory* contribution. These examples would be offered up for the group's approval, rather than exploring the question in and of themselves; providing possibilities rather than explaining or giving reasons why they might 'fit'[15]. This fifth category has been labelled *supercumulative* in recognition of the potentially valuable role it might play in moving the group towards a successful solution.

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[15] In the first iteration of the project, discussed in Chapter 4 (where the digital audio recording devices and the protocol were tested) there had been an example of precisely this – something that appeared to be more than 'just' *cumulative*, but was not yet *exploratory*, in either *encoding* or *transformative* senses). A group member had already "transformed into the language of maths" by identifying that the problem posed focused on complements to 25 (number bonds to 25, i.e. 1 and 24, 2 and 23, etc.). Their use of the language of 'number bonds' and 'complements' fits with the definition of *Exploratory Transformative*, using the Talk Framework provided in Figure 3.5 above. The following comment, however – "so, we've got all the combinations of 9, 8 and 7…so you've got 3 9s in each, 9 appears three times in each, 8 appears twice" is not *exploratory* (in either *encoding* or *transformative* form). It does not provide reasons and does not propose the strategy that might be used to check the working so far, but it builds *cumulatively* on the previous response and it does appear to lead towards a strategy that might be helpful to verify the work (the systematic checking that all digits have been used to create the complements to 25). Therefore, this response was deemed to be more than *cumulative* – thus, *supercumulative*.

Figure 3.6 below provides definitions for the five categories, including the *supercumulative* category, as they were used for the analysis of the T-AP responses prior to the SRI sessions. The additional category, *monologic*, reflects verbalisations that are not intended for other members of the group (i.e. comments made by individuals to themselves).

Cumulative	**Supercumulative**	**Exploratory Encoding**	**Exploratory Transformative**	**Disputational**
Building on the contributions made by others. May add some 'new' information to the discussion (neither encoding nor *transformative* e.g. information present in the question or the supporting information provided with the question)...in a mutually supportive, uncritical way; constructing shared knowledge and understanding. *Cumulative* stops short of Exploratory in that it does not demonstrate 'grasping of the assignment' by restating the problem in different words, using analogy or transforming/translating into the language of mathematics. Reasons are not offered/strategies are not proposed.	Building on contributions from others, most especially *exploratory* (either *encoding* or *transformative*) contributions e.g. providing examples of numbers that might 'fit' with the proposed explanation/strategy or asking for further explanation that would lead to exploratory responses. *Supercumulative* stops short of *exploratory* in that reasons are not offered and strategies are not proposed. As with *cumulative*, the problem is not restated in different words or transformed/ translated into the language of mathematics.	'Grasping the assignment' in such a way as to be able to restate the problem (in different words) or use analogy to clarify it to other members of the group (i.e. 'mutually understandable language' that 'encourages *cumulative* discussion'). Relevant information is offered for joint consideration, in **non-mathematical form** (i.e. explained verbally, explained by analogy). Proposals may be challenged and counter-challenged, but if so reasons are given and alternatives are offered.	Restating the question in mathematical terms ("Translating into the language of maths") and consequently encouraging *cumulative* discussion. Relevant information is offered for joint consideration, **in mathematical** form (i.e. identifying operations required but not explicitly stated within the original question). Proposals may be challenged and counter-challenged, but if so reasons are given and alternatives are offered. Agreement is sought as a basis for joint progress.	An unwillingness to take on the other person's point of view, and the consistent reassertion of one's own. In its most archetypal form, it consists of 'yes it is—no it isn't' exchanges, commands and parallel assertions. **N.B. Monologic** responses are those that are not made for the benefit of the group (i.e. comments 'to self' or 'asides' from the conversation that are clearly not intended to add to the discussion).

Figure 3.6: Talk Framework merging Novotná (1997) and Hošpesová and Novotná's (2009) problem-solving categories with Mercer's (1995) talk categories. Mercer's (1995) *exploratory* category has been divided in two to account for *encoding* and *transformative* contributions; an additional *supercumulative* category is proposed to account for *cumulative* contributions that build on prior observations with examples that "fit" with strategies proposed or that indicate the need for further *exploration*/explanation.

The use of Mercer's (1995) categories of *cumulative*, *disputational* and *exploratory* talk had been originally inspired by the work of Seal (2006), which – as previously discussed, for example in 1.5 above – considers how best to promote *exploratory* talk in group situations, and recommends that 'ground-rules' should be promoted that require children/students to support their ideas with reasoning and explanations. Mercer's (1995) categories, however, have not been devised with mathematics solely in mind, and this work, therefore, merges his concept of *exploratory* talk with the problem-solving framework of Novotná (1997) and Hošpesová and Novotná (2009), whilst leaving the *cumulative* and *disputational* largely as originally proposed. The four Novotná (1997) and Hošpesová and Novotná (2009) categories, designed for mathematical word problems and informed by the revised Bloom's Taxonomy (Anderson and Krathwohl, 2001), are *encoding*, *transformation* (relating to Duval (2006) above), *calculation* and *storage*.

The first of these categories, *encoding*, involves the "coding of the…assignment…into a suitable system in which data, conditions and unknowns can be recorded in a more clearly organized and/or more economical form" (Hošpesová and Novotná, 2009, p.195). Deciding to tabulate data would be an example of this. This has been combined with Mercer's (1995) *exploratory* to become the second of the categories in this Talk Framework (*exploratory encoding*). Mercer's (2000, p.98) definition of *exploratory* requires "partners [to] engage critically but constructively with each other's ideas [with] relevant information…offered for joint consideration…[and]…reasons…given and alternatives…offered [to ideas challenged or counter-challenged]". Therefore, the hypothetical proposal to record data in a table would need to be accompanied with a reason why this would be useful (i.e. to systematically record numbers; to put them in order; to determine which are the first, second, third in a sequence etc.) for it to be classed as *exploratory encoding* rather than *cumulative*.

The category of *exploratory transformative* (see Figure 3.6 above) merges Mercer's (1995) *exploratory* with Novotná (1997) and Hošpesová and Novotná's (2009) *transformative* to require a contribution that outright restates the problem in mathematical terms. Therefore, an *exploratory encoding* response might suggest that this problem is similar to another problem, perhaps providing an analogy ("this is like…") but not yet restating the problem as mathematics. *Exploratory transformative* requires the individual contributor to put forward the mathematics (for example, stating that the differences between amounts need to be calculated, or multiplication by a certain amount is required). The *calculation* and *storage* stages (not included in the Talk Framework) can then follow from this.

As indicated above, Mercer's (1995) *disputational* category has been retained here, as there is not a parallel category within Novotná (1997) and Hošpesová and Novotná's (2009) taxonomy for those contributions that demonstrate an outright unwillingness to take on another person's suggestion but do not propose further *exploratory* ideas. As indicated above, individual responses, made as if apart from the general conversation, or as if intended as an aside (i.e. an individual talking to themselves) have been counted here as *monologic* (as opposed to dialogic) – by their nature, they cannot be taken as *cumulative* or *exploratory*.

Overall, the use of the Talk Framework was designed to support Research Question 1 (exploring how the thinking-aloud might support problem-solving strategies; the Framework allowed the identification of *cumulative* and then *supercumulative* – leading to *exploratory* – responses that might provide an indication as to the degree to which these verbalised strategies were 'picked up' by the rest of the group as a result of the protocol), RQ 4 (identifying the most evident types of talk in such a session) and RQ 5 (demonstrating, at least to a degree, the willingness of participants to engage with the activity – the follow-up SRI provided an additional opportunity for individuals to comment on this). While it might have been assumed that the T-AP would automatically lead to the 'necessary' explanations being provided 'first-time round', the avoidance of an over-prescriptive, "Type III" (Ericsson and Simon, 1993) protocol meant that explanation was not demanded from participants, and this did not risk "invalidity due to the disturbance of the cognitive process" (van Someren et al., 1994, p.32). The SRI enabled the group's discussion to be revisited and 'added to', as appropriate, and the Framework coded transcriptions supported this by identifying areas worthy of revisiting. This included, for example, *supercumulative* contributions that did not provoke responses from other members of the group in the original session.

Given that such *exploratory* talk, as identified by Mercer (2000) and Seal (2006), is considered to be beneficial to mathematical problem-solving because provision of reasons/justifications is considered as "hav[ing] a strong effect on problem solving" (Robertson, 2001, p.11), the Talk Framework may have enabled the SRI to more comprehensively address this area. It may also have enabled a closer focus on the mathematics being undertaken by individuals and the group (with the caveat, as later discussed in Chapter 7, that there are elements of this work that require further development, and – indeed – that the SRI discussion was intended to be, in the main, led by the participants themselves commenting on their own 'performance' in the original task).

Having addressed the devising of the Talk Framework and its contribution towards the SRI in coding verbal contributions and identifying aspects worthy of further discussion, the next section will turn to the way in which the SRI transcripts have been analysed to inform the Results (Chapter 7) of this work. This includes the use of the Framework to identify, for example, additional *exploratory* material not present in the original problem-solving situation.

3.5 Stimulated Recall Interviews and 'Coding' of Responses to Address the Research Questions

This section considers how the Stimulated Recall Interview (SRI) responses were transcribed and analysed to inform the Results that follow in Chapter 5. It provides a rationale for using the Talk Framework in a non-think-aloud context, and outlines the efforts to categorise the different areas discussed during the SRI, bearing in mind the caveat that the researcher was, unlike in the Think-Aloud Protocol (T-AP) sessions, part of the discussion and, therefore, often leading the questioning.

As detailed above in 3.3, the SRI responses were not intended to be strictly coded according to the Talk Framework used for the original T-AP recordings (as these interviews were not, of course, think-aloud opportunities). As indicated above, however, it was decided to identify ‘new’ *exploratory* contributions where appropriate (and, indeed, there were also examples of other Talk Framework categories evidenced in the SRIs). In order to enable this, and in the first instance, the two sessions were transcribed with individual questions and responses anonymised and numbered in turn. As with the T-AP transcripts detailed above in 3.4, ‘turn’ again means “the sequential partition of conversations into ‘turns’ (in the sense that participants ‘wait their turn’) (Rowland, 2000, p.91). Extracts from the SRI transcripts are presented in Appendix 3 and 4 below. The talk was also categorised according to its predominant focus – for example, responses primarily about mathematics as opposed to the group situation or, indeed, the impact of the T-AP itself on the student teacher participants’ problem-solving performance.

The SRI transcripts were analysed in light of the research questions given above in 1.4. As detailed in 3.3, the ‘new’ Talk Framework coded responses (Mercer, 2000) that had been propounded during the SRI (i.e. occasions when participants provided additional information, building on their own or others’ contributions, or even took potential strategies further) was used as evidence for RQ 1’s focus on the “ways [in which] dialogue/thinking-aloud recorded by digital audio [can] support student teachers’ verbalisation of problem-solving strategies”. It was thought that SRI talk that evidenced further, perhaps even more comprehensive, discussion of mathematical strategies from the original problem-solving sessions, would demonstrate the potential inherent in the SRI situation for encouraging additional explanation; it might also demonstrate the value of “networking” the two methodologies (Hickman and Monaghan, 2013). It would, therefore, perhaps provide a rationale for further work on combining T-AP and SRI approaches to, in the words of RQ 1, “support verbalisation of problem-solving strategies”. In addition to this, the hope that the *Livescribe* technology might promote further discussion of mathematical strategies in the second of the two SRI situations (see 1.5 above, and RQ 3) provided a further rationale for identifying, first and foremost, those SRI questions and responses that related to mathematical strategies, and especially those, in the second SRI, that had been (as far as could be determined) directly provoked by the *Livescribe* replay. There was no intention to quantify the number of responses provoked by *Livescribe*, however, not least because it was not always clear whether the participants were referring to the annotations or to the accompanying dialogue (or, indeed, referring to both).

While the questions asked by the researcher and the responses given by the participants are potentially valuable data in their own right, and will be used as such in the Results (Chapter 5) to follow, the Results also present a breakdown of responses according to whether they relate to the *group situation*, to *mathematical strategies*, or to *verbalisation* (which includes direct comments about the Talk Framework categories themselves). Identifying questions and comments about the *group situation* allows for RQ 5 to be addressed (in what ways do the networked T-AP and SRI methodologies impact upon participants' willingness to engage with a problem-solving task?). Equally, identifying *mathematical strategies*-focused verbalisations addresses RQ 1, as indicated above. The choice of these three categories was driven by the demands of the research questions, although, unlike the T-AP coding detailed in 3.2 above, in this instance contributions were categorised as fitting into more than one category where it was deemed appropriate to do so. This reflects the nature of the SRI discussion which, at times, moved swiftly from considering the way in which the group might have impacted upon strategies proposed, to considering those strategies that may have been better explained in a different, perhaps more conducive, environment.

Having outlined the way in which the SRI responses are presented in the Results to follow in Chapter 5 of this thesis, the next section elaborates on the Data Sets collected in this work, connecting them to the Research Questions throughout. This is then followed by an outline of the development of the research over time, which provides further context regarding the decisions taken to inform the Methodology above.

3.6 Data Sets

This section presents the data sets utilised in this work and referred to in Chapter 5 (Results) below. It demonstrates the connections drawn between the research questions (with RQ 1 taken as the 'primary research question') and the data collected from the initial questionnaires, the Think-Aloud Protocol (T-AP) recordings, and the Stimulated Recall Interviews (SRIs). References to the relevant sections of the Results and Discussion chapters – and the Appendices – are also provided for each of the items listed. "Other Research Questions" addressed are then indicated, before the data analysis is addressed. Each of the four sections ends with direct references to the results and discussion chapters to follow, including tables, with links made to appendix items where appropriate.

1. Data Set: the initial questionnaires completed by participants prior to engaging with the T-AP problem-solving sessions (1 sitting, 7 questionnaires returned from 8 participants).

Aspects of the Primary Research Question addressed: RQ 1 – "*In what ways can dialogue/thinking-aloud recorded by digital audio support student teachers' verbalisation of problem-solving strategies?*" This data set addresses the context and background of the group undertaking this work. It allows the student teachers to indicate their 'confidence' in mathematics generally, teaching mathematics specifically, and also explaining mathematical strategies to others. While confidence is not a key focus of this work (see 1.6 and 5.3), the information from the questionnaires was useful when considering individuals who commented on their 'issues' with the task and the T-AP situation in the subsequent SRI

Other Research Questions addressed:

NA. Questionnaires provide contextual information regarding the participants' view of mathematics and their problem-solving performance prior to engaging with this work.

Data Analysis: Questionnaire responses are used to provide contextual information regarding the composition of the group, with comparisons provided between responses given (lower to upper primary; male to female). Responses to the sample mathematical problem-solving questions are used to provide a measure of the participants' perceived confidence with such tasks – this includes their self-reported confidence in verbalising the potential strategies that could be employed to solve the problem/s set.

Questions asked:

- How would you rate your confidence in mathematics?
- How would rate your confidence in teaching mathematics?
- How would you rate your confidence in explaining strategies to help others solve mathematical problems?

Results and Discussion Chapter References: 5.2 and 5.3 (Tables 5.1, 5.2, 5.3, 5.4, 5.5); 6.2; the questionnaire itself is provided in Appendix 1.

2. Data Set T-AP 1 and T-AP 2: Transcribed and Talk Framework coded transcripts of original T-AP supported group problem-solving situations (T-AP 1 – Exploring Addition problem-solving task; T-AP 2 – Beads problem-solving task).

Aspects of the Primary Research Question addressed: RQ 1 – "*In what ways can dialogue/thinking-aloud recorded by digital audio support student teachers' verbalisation of problem-solving strategies in a group situation?*" This data represents the student teachers' verbalisations from the two T-AP sessions as recorded by digital audio (T-AP 1 with a 'standard' digital audio recorder/T-AP 2 with a *Livescribe* pen recorder).

Other Research Questions addressed:

RQ 4. "*What types of talk are most evident in multi-media artefact enhanced thinking-aloud while engaged in mathematical problem-solving activity?*"

RQ 5. "*In what ways do the networked T-AP and SRI methodologies impact upon participants' willingness to engage with a problem-solving task?*"

Data Analysis: The transcribed T-AP problem-solving sessions were analysed with the Novotná (1997)/Hošpesová and Novotná (2009)/Mercer (1995)-inspired Talk Framework to identify *supercumulative*, *cumulative*, *exploratory encoding*, *exploratory transformative* and *disputational* dialogue. This enabled the different types of verbalisation prompted by the T-AP to be identified, potentially for discussion within the SRI (see below).

Results and Discussion Chapter References: 5.4 (Tables 5.6 and 5.9 – presenting the Talk Framework coding of T-AP 1 and T-AP 2, respectively; Tables 5.7 and 5.10 – presenting *monologic* and *not coded* speech; Tables 5.8, 5.11 and 5.12 – presenting questions asked by participants during the course of the T-AP questions); 6.4. Extracts from the transcripts are provided in 5.4.2 and 5.4.4, respectively.

3. Data Set: LS Screenshot: *Livescribe* annotation/digital audio recording – screenshot of notes made with accompanying SRI transcript (T-AP 2 and SRI 2 only)

Aspects of the Primary Research Question addressed: RQ 1 – "*In what ways can dialogue/thinking-aloud recorded by digital audio support student teachers' verbalisation of problem-solving strategies in a group situation?*" This data represents the annotations that accompanied the verbalisations from the second T-AP session. It both demonstrates elements of mathematical strategy that participants considered important for recording in order to help them reach a joint solution and reveals aspects of their strategy that had not been verbalised in accordance with the T-AP. The written material, however, is not analysed (i.e. what they had chosen to write down to support their mathematical ideas) beyond what was discussed in the subsequent SRI. This is due to the focus on their dialogue in this work, and the focus on what was 'revealed' to them about their mathematical strategies from engaging with the T-AP and SRI (see also RQ 1 above).

Other Research Questions addressed:

RQ 3. "*What does the use of a multi-media artefact in an SRI situation reveal to participants about their own problem-solving strategies, and those of the group?*"

RQ 5. *"In what ways do the networked T-AP and SRI methodologies impact upon participants' willingness to engage with a problem-solving task?"*

RQ 6. *"Does multi-media artefact prompted recall promote greater reflection on mathematical methods/problem-solving strategies than 'standard' digital audio?"*

Data Analysis (see note above regarding the use of the annotations in the analysis of the participants' dialogue): *Livescribe* pens enable jottings to be 'attached' to specific verbal comments made while writing (see 3.3 above for an outline of the technology). Playing back the recording using the *Livescribe* software (as was done in the SRI sessions described here) means that any original annotations appear in real time alongside the relevant dialogue, as if animated, thus identifying when particular annotations were made and what the group were discussing at that point in time. The playback was used to prompt the recall of the participants in SRI 2 (the *Livescribe* pen having only been used for the second of the T-AP sessions – Beads, see above).

Results and Discussion Chapter References:

5.5 (presenting the full screenshot of their annotations while engaged with T-AP 2); 7.5, 6.7 and 6.8 (Figures 6.1 to 6.4 illustrate the annotations made and, relevant to the discussion in SRI 2, the *order* in which they were made that became clearer to participants when presented with the replay).

4. Data Set: SRI 1 and SRI 2: SRI transcripts.

Aspects of the Primary Research Question addressed: RQ 1 – "*In what ways can dialogue/thinking-aloud recorded by digital audio support student teachers' verbalisation of problem-solving strategies in a group situation?*" This data represents the dialogue from the SRI sessions, including further verbalisation of problem-solving strategies within the SRI and evidence of the participants "looking back" (Pólya, 1957) at these strategies.

Other Research Questions addressed:

RQ 2. *"What levels of thinking-aloud most effectively support digital-audio recorded T-AP and SRI?"*

RQ 3. *"What does the use of a multi-media artefact in an SRI situation reveal to participants about their own problem-solving strategies, and those of the group?"*

RQ 5: *"In what ways do the networked T-AP and SRI methodologies impact upon participants' willingness to engage with a problem-solving task?"*

RQ 6: *"Does multi-media artefact prompted recall promote greater reflection on mathematical methods/problem-solving strategies than 'standard' digital audio?"*

Data Analysis: Transcribed SRI discussions were considered against the Talk Framework, where 'new' information relating to the mathematical problem-solving tasks was offered, i.e. the discussion was not comprehensively coded according to the Talk Framework, but new exploratory comments, for example, were identified, as such contributions indicate the degree to which the recall situation might have influenced participants to re-engage with their original working – see RQ 6 above. Participants' responses were used to inform the discussion to follow, most particularly 6.5. In addition to this, an analysis was carried out on the discussions to identify the extent to which, in the two different sessions, the participants chose to focus on discussing the group situation, mathematical strategies or verbalisation in order to determine whether *Livescribe* promoted greater discussion of the mathematics (see 5.5.1).

Results and Discussion Chapter References:

5.5 and 5.5.1 (Tables 5.17, 5.18, 5.19 and 5.20); 6.5, 6.5.1, 6.5.2, 6.5.3, 6.7 and 6.8

Appendix 3 and 4

3.7 Ethics

This section details the ethical considerations underlying the recruitment of the participants for this project, referencing the relevant ethical approval letters and stipulations from the two Universities involved (York St John University and the University of Leeds – see Appendix 5) and the ways in which concerns over the researcher's role as module tutor were addressed in order to reduce the potential impact on the student teachers involved.

As indicated in Appendix 5.4 (University of Leeds AREA Faculty Research Ethics Committee, 16.01.13), care needed to be taken over the recruitment of the participants, given the researcher's position in the Faculty (at that point, not only module tutor but also Head of Department), and individuals were not to be put "on the spot" in any way (i.e. made to feel that they had to respond). This was addressed in the final version of the Research Project Information Sheet (see Appendix 1) in which it is clearly stated that "there is no requirement for any trainee to take part in this research" (the term "trainee" was the one most commonly used by the programme, as opposed to "student teacher", as utilised throughout this thesis) and that individuals can withdraw themselves from the process, without the need to provide a reason, at any point. It is also stated that "this work has no bearing on module assessment" (indeed, this is stressed twice in the Information Sheet, under "Do I have to take part?" and "Will my taking part in this project be kept confidential?"). Equally, and addressing the potential issues around the researcher's role within the module team, Department, and Faculty, they are assured that "information will not be shared with academic tutors or others within the university". Given the discussion elsewhere in this thesis about the group "psychological situation" (Rotter, 1954 – see, for example, 6.2, which considers the impact not only of working in a group alongside peers on mathematical problem-solving tasks but also the impact of being asked to think-aloud; requirements that – it was acknowledged – might well affect notions of self-efficacy and even cause anxiety), it is stressed that all their responses would be kept strictly confidential to all parties outside the group problem-solving situation, with individuals not being "identified in any reports or publications". As can be seen from the transcript material provided in Chapter 5 below (from the two think-aloud sessions) and also in Appendix 3 and 4 (the two stimulated recall sessions), all identifying information was removed; indeed, the audio recordings have not been used (another way in which individuals could have been identified) by anyone other than the researcher, in the compiling of the transcripts, and the participants themselves, when listening back in the follow-up stimulated recall.

Appendix 5.2 provides a copy of the original ethical approval (dated 11.04.11) for the initial iteration project (a small amount of which is utilised in the thesis, within Chapter 4 below). This approval was from the University of Leeds. York St John University did not require ethical approval for that work (see "Ethical Screening Checklist" dated 2.7.11 in Appendix 5.1). Both Universities gave favourable responses for the final version of the project; York St John University in an email of 21.2.13 (included within the Appendix as 5.3 - approval code ET/21/02/13/MH) and the University of Leeds in a letter dated 11.4.11 (ethics reference 12-054, Appendix 5.4). As indicated above, the final Information Sheet for participants was adjusted in light of their comments and suggestions; one amendment made, for example, was greater precision in the "What will happen to me if I take part?" section (see Appendix 1), in which a clear elaboration of the process from individual questionnaire through to group think-aloud and stimulated recall sessions is provided.

As acknowledged in 6.2 below, there remain issues with individuals problem-solving amongst peers and a module tutor, and these may well have contributed to the impact of the exercise on both their mathematical problem-solving and their willingness to engage with the process (Research Question 5, see 1.4 above, considers the impact of the methodologies and the technology on their willingness to engage; in considering this, there is also reflection on the wider situation, including the tutor's presence). Nonetheless, the ethical approval process, and the provision of clear guidelines in the Information Sheet, contributed to a situation in which all participants, bar one (who was unavailable for the stimulated recall follow-up), committed to the project. Limitations are, however, detailed below in 7.4, with reflection on how such work could be refined in future iterations.

4. Development of Research across Time – the First Iteration of the Project: Trialling T-AP, SRI and *Livescribe* to Inform the Development of the Work Detailed in This Thesis

This chapter provides a brief overview of the 'first iteration' of this project that informed the work discussed in the chapters to follow. It refers back to the Introduction (specifically 1.2 and 1.3 above), reiterating the rationale for engaging the part-time primary PGCE student teachers in group problem-solving, and indicating how podcasting (Berry, 2006; Salmon, 2008) was the inspiration for utilising digital audio, even though the ultimate recording and playback of the problem-solving sessions bore little, if any, resemblance to this means of communicating module material. The chapter indicates how the experience of these initial 'trial' recordings led to amendments in approach for the final version of the project – for example, leading to the development of an 'interviewer's monologue' and the adoption of *Livescribe* for at least one of the Think-Aloud Protocol (T-AP) recordings; a further Talk Framework category, *supercumulative*, was added as a result of the analysis of these trial recordings. In addition, it highlights how the decision was taken to employ both T-AP and SRI approaches, as complementary, rather than discrete, methodologies; this would later be referred to, inspired by the name of a conference working group, as the "networking" of these approaches (Hickman and Monaghan, 2013) – the decision to employ them in this fashion, however, was taken in the first iteration of the project.

As discussed in 1.2 and 1.3 above, the part-time student teachers' mathematics module consisted of weekly hour and a half sessions, often pre-empted by teaching experience blocks. Contact time was further limited by their being in University only two days a week (teaching blocks and holidays aside) across their 18-month programme. This provided one of the imperatives towards designing more opportunities for individuals to connect with module content, and with each other. Providing material in podcast form was considered to be a means by which this could be achieved, initially via supporting – scripted – recordings of taught content and short explications of mathematical concepts such as 'Four Rules of Number', 'Conservation', and 'Fractions, Decimals, Ratio and

Proportion'. Inspired by the work of Salmon (2008) regarding the potential benefits of this technology and digital audio in general in educational situations, the module team considered expanding the use of podcasting to include recordings of the students' own problem-solving work; the intention, originally, was that they might revisit these recordings in their own time, identifying successful verbal and mathematical strategies for later reporting back in a mathematics workshop. This, it was also thought, would be more engaging and interactive than the 'standard' VLE material, which consisted largely of *PowerPoint* presentations and *Word* documents. It can be argued that this is a step towards the consideration of "generation" and "self-generation" (Slamecka and Graf, 1978) as important elements in their learning (see 2.2 above) as this was one of the first opportunities on the mathematics module for the student teachers to explicitly learn from their own voices.

The recording of problem-solving sessions for later replay moved the work away from 'true' podcasting (Berry, 2006) as recordings were not to be made as a series, nor made available in 'podcast form' (i.e. for download via iTunes or similar sources). The decision to 'test' the approach by recording select groups engaging in problem-solving and playing the material back to them in a follow-up session was still further removed from podcasting, although there remains the possibility that such recordings could, in the future, be disseminated in this form; the potential further work arising from this project is addressed below in 7.5.

The focus on group work, which was intended to allow the student teachers further opportunities to learn from each other, given the limitations on opportunities for developing dialogue within the University, further informed the plan to have them work as a group on these tasks. It was also intended that they would listen back to their work as a group 'after the event'. The consideration of effective group work, and – specifically – effective dialogue in group situations, and while engaged in problem-solving, led to an interest in dialogue (see 2.2) and Talk Frameworks, as detailed above in 3.4. The focus of the work, therefore, became much more about the ways in which potentially 'valuable' dialogue could be captured – or even promoted – in order to inform further learning. There was also the potential to consider metacognition, and the modelling of “metacognitive practices” (Bransford et al., 2000), which might be of use to the participants in their future practice as primary teachers. It was while refining the approach that the majority of the primary research questions (see 1.4 above) were devised. The consideration of the impact of the networked Think-Aloud Protocol (T-AP) and Stimulated Recall Interview (SRI) methodologies on individual participant’s performance, which would inform the wording of RQ 5 (“*In what ways do the networked T-AP and SRI methodologies impact upon participants’ willingness to engage with a problem-solving task?*”), would follow this first iteration of the project as it was during this work that the decision was taken to use SRI for the revisitation of their dialogue; equally, the potential differences between ‘standard’ digital audio-prompted SRI and *Livescribe*-prompted SRI would follow the testing of this technology in the first iteration of the project, ultimately leading to RQ 6: “*Does multi-media artefact prompted recall promote greater reflection on mathematical methods/problem-solving strategies than ‘standard’ digital audio?*”

In the main, though, and as indicated in the Methodology (Chapter 3) above, the first iteration and second iteration of the project largely followed the same procedure of encouraging the student teachers to perform problem-solving activities whilst thinking-aloud (Hickman, 2011; 2013). The actual conduct of the sessions did vary between first and second iteration, however, and the remainder of this chapter provides an outline of the differences and the reasons for amending the protocol/s, even slightly, for the work discussed in this thesis.

Think-aloud was a key part of the methodology from the first two experimental recordings in this first iteration of the project, and consideration was duly given to Ericsson’s and Simon’s (1993) and Robertson’s (2001) comments about the demands of the different types of verbalisation on problem-solving performance (informing RQ 2: “*What levels of thinking-aloud most effectively support digital-audio recorded T-AP and SRI?*”). At this stage, however, a scripted ‘interviewer’s monologue’ was not in place for the Think-Aloud Protocol (T-AP), and a decision had not been taken about whether to explicitly prompt participants into explaining and reasoning, as Seal (2006) suggests should be within the “ground rules” for such activities. Although Hickman (2011) discusses the importance, as stressed by Montague and Applegate (1993), of demonstrating and practising thinking-aloud prior to engaging in such an opportunity, the student teachers did not practise thinking-aloud prior to the recordings. This was due, in part, to the lack of time in which to conduct the work, as well as a sense that it would be good for the participants to 'discover for themselves' how well they had engaged with the protocol, with the ambition that they would reflect on this in the SRI. Ultimately, despite some of the difficulties with the first iteration recordings (which could be summarised, perhaps, as a lack of thinking-aloud even with the prompts to do so) the decision was taken not to deliberately or explicitly model this in the second iteration, either. The thinking here was that any such modelling might prove too 'leading' for the participants, particularly given the intention for the tutor to take a non-directive role in the proceedings.

For the first iteration of the project, then, two groups of six volunteers carried out the chosen mathematical problem-solving tasks, with each group taken from a separate PGCE primary teaching group. The decision to have two separate groups, rather than the one used in the final version of the work, was to informally trial different problem-solving tasks and to compare the outcomes between them; the choice of the group size was due to the typical size of the 'table groups' within the mathematics workshop 'classrooms', thereby meaning that the experience – and, the "psychological situation" (Rotter, 1954) – would not be too dissimilar from the occasions in their PGCE mathematics workshops when they had been asked to work collaboratively on a set problem (see Edwards' (2005) definition of collaborative given in 2.2.2 above). The mathematical problems themselves, in common with the second iteration of the project, were taken from the Primary National Strategy Problem Solving pack (DfES, 2004). This was a resource that the student teachers would have been familiar with from taught sessions within the University (although they had not seen the specific problems chosen for the recordings). As a result of this, when assessing confidence at the outset of the work (and having seen example problems, precisely as would be the case in the final project), 83% of the participants stated that they were 'very confident' in working out the answer to at least one of the three questions shown to them (the same questionnaire is used in the second version of the study – see 5.3 below; the questionnaire is reproduced in Appendix 1). While this may have been the case, there were still issues surrounding confidence with mathematics noted from the outset, with one participant commenting, in an early interview (these interviews were not replicated for the second iteration of the project), for example:

> Maths has not been part of my daily life for over six years, which creates an amount of apprehension at the prospect of teaching maths even at primary school level, as the challenges faced to raise the standards of achievements…are ever increasing.
>
> (Student teacher interview, from Hickman, 2011, p. 72)

For this test of the methodology, the T-AP recordings were mainly captured using what might be termed 'standard' digital audio recorders, with any supporting notes from the participants made on large sheets of paper. This provided some problems when engaging in the after-the-event recall opportunity (the SRI was conducted much as for the second iteration, although without the focus on Goldin's (1997) Task-Based Interview style questions, at this stage). Not only did participants find identifying the different voices on the recording problematic (informing the provision of transcripts in the second iteration of the project), they also struggled to understand what they had written – and why. This led to considering *Livescribe* as an alternative technology, and this was informally trialled with one of the groups as a result of a recommendation from the researcher's supervisors. The hope was that this would more closely connect the verbal contributions made to the jottings produced during the discussion. There was some indication, in the *Livescribe* trial, that participants were more keenly focused on the mathematics as opposed to the group situation itself or the difficulties they perceived in thinking-aloud while problem-solving. This then informed the decision to use *Livescribe* in the second iteration of the project, now more formally comparing it with 'standard' digital audio recording to see whether there is a difference between the two approaches as regards the discussion of mathematics (see the reference to RQ 6 above).

The Mercer (1995)/ Hošpesová and Novotná(2009)-inspired Talk Framework was used to code the verbal responses in the T-AP recordings, much as it was then used in the version of the project detailed below. However, at this stage, the Framework had four, rather than five, categories. Hickman (2011) details the splitting of the *exploratory* categories into two to make a distinction between contributions that restate by analogy or use the language of mathematics. A further new category, introduced in this work, *supercumulative*, as detailed above in Figure 3.6 and section 3.4, was devised to indicate those responses that appear to be more than ‘just’ *cumulative*, but are not yet *exploratory*. This was introduced, in part, as a response to the amount of *cumulative* talk evident in these early recordings (there was, as stated by Hickman (2011), no attempt at this point to quantify the amounts of *cumulative*, *exploratory encoding*, *exploratory transformative*, and *disputational* talk). It was also as a result of identifying material that, while not providing the reasons or explanations Seal (2006) might require, provided ‘more’ than ‘mere’ *cumulative* alone and could be said to be moving the group towards providing further explanation. One example of this in the these early recordings was where a participant, engaged in the Beads activity (which would be re-used in the final version of the project as T-AP 2) observed that all the combinations of 9, 8 and 7 had been found, and that there were three 9s in each (“9 appears three times in each”) and two 8s. This does not provide any form of reasoning of explanation, but it seems to lead towards more *exploratory* dialogue (it seems to prompt the question “why is this so?”). Therefore, the fifth Talk Framework category was introduced precisely to capture this kind of contribution. Another result was to consider more carefully the amount of responses in each category, most especially given a lack of knowledge at this stage as to what might constitute a ‘good’ amount of *exploratory* talk, and whether it is or is not more evident in some situations than others (i.e. is there a difference in the amount of *exploratory* talk provided when *using* a *Livescribe* pen – not just when listening back to the recording made on the device. Is there something about having the physical recorder embodied within the pen that changes the type of talk?). Having a breakdown of Talk Framework coded verbal responses, as is provided in Tables 5.6 and 5.9 below (see sections 5.4.2 and 5.4.4) would go some way to addressing questions of this kind, most explicitly RQ 4 (“*What types of talk are most evident in multi-media artefact enhanced thinking-aloud while engaged in mathematical problem-solving activity?*”).

As indicated in Hickman (2011), the main outcome from the first iteration of the work, beyond the reconsideration of the Talk Framework categories (including the devising of the *supercumulative* category) and the decision to more formally 'test' the *Livescribe* pen in the final version of the project, was a mismatch between the high level of confidence expressed by the participants in the problems shown to them at the outset and their ultimate performance in solving the T-AP recorded exercises. Some of this may have been a result of the process itself – these initial tasks had resources (counters, blocks etc.) provided to assist them in their working out, as was often the case in their PGCE mathematics workshops. This ultimately led to confusion over what precisely they had done with these resources (in part, because they had not verbalised their use of them, despite the T-AP). Individuals were not sure, in listening back to the audio recording, whether they were referring to the practical mathematics resources or whether they were referring to something that had been written down – and, if they were referring to something written down, precisely what that might be. Writing was often carried out without the associated thinking-aloud that would have explained to the rest of the group the purpose of the annotations being made. Therefore, the decision was taken not to use practical resources in the final version of the work detailed below. Additionally, as Hickman (2011) states, more in the way of 'ground rules' would be required regarding the verbalisation, even if there would still be an avoidance of outright requesting explanation from the participants while working on the problems as Seal (2006) suggests for work of this kind. This informed the 'interviewer's monologue' employed at the outset of the second iteration recordings. It was also thought that the *Livescribe* pen might better 'attach' the spoken to the written; although – as indicated above – it would not be relied upon for both eventual recordings, in order to assess its effect within the follow-up SRI and, ultimately, demonstrate its contribution to knowledge in this work.

In summary, then, the 'first iteration' of the work allowed for trial T-AP recordings to be made that were then used to refine the use of the Talk Framework for the coding of the participants' verbal contributions before a follow-up SRI allowed participants to comment on their original verbalisations. This Talk Framework was originally intended to be used separately from the participants (i.e. they would not necessarily have been aware of the talk categories) but it was decided, for the final version of the work, that it would be shared with them, in return for their taking part in the process and perhaps also as a prompt for discussion. Indeed, the Information Sheet provided for the participants in the 'second iteration' discussed in the remainder of this thesis outlines the process to be carried out as follows: "This research involves the audio recording of some primary mathematics problem-solving activities using two devices (a traditional audio recorder and a *Livescribe* pen). Digital recordings will be played back in a later 'stimulated recall' session (a little like a DVD commentary!) and collaboratively analysed using a provided 'talk/problem-solving framework'. In other words, you will listen back to your recorded speech and, with the help of the framework, discuss the different types of verbal contributions made and problem-solving strategies used." At the point of writing this Information Sheet, there was still an ambition for the work to "determine the usefulness of digital audio recordings and technology supported by a talk framework in enhancing trainee teachers' confidence in the teaching of mathematics". This, ultimately – see RQ 1 above – was not the focus of the work, most especially given, as with the 'first iteration', it proved not to be possible to 'follow' the participants into the classroom to see the impact of the exercise on their own teaching. The focus, therefore, was to be on the way in which digital audio supported T-AP and SRI, used as complementary methodologies in this work where others (see Meijer's et al. (2002) observation in 2.5.5 above) view them as discrete, could promote productive verbalisation of mathematical strategies. In itself, this would address a number of the 'issues' with the PGCE mathematics module detailed above in 1.3, most especially those connected to encouraging further opportunities for dialogue amongst the student teachers – the project potentially informing future work to be conducted on the PGCE programme, and also offering the chance for the participants to engage in group discussion in a perhaps more reflective and considered way than had been the case for them previously. 'Confidence ratings', therefore, provide contextual information as to the make-up of the group, but do not contribute towards the key data in the chapter to follow. The Talk Framework analysis coupled with the SRI transcripts (themselves analysed in light of the Talk Framework, at least to a degree) provide the necessary data to address the Research Questions (see 3.6 above for Data Sets).

The next chapter turns to the results of second iteration of the project, covering the group of participants and their responses both within the T-AP sessions, as coded by the Talk Framework, and the subsequent SRI sessions.

5. Results

5.1. Introduction

This chapter begins by addressing the participants engaging in the second iteration of the project (5.2: The Group) before moving on to the results of their initial questionnaires that indicate their self-assessed levels of 'confidence' in mathematics (including mathematics overall, the *teaching of mathematics* and *explaining strategies to help others solve mathematical problems*). These results are presented in 5.3 (Tables 5.1 to 5.5) below and include confidence ratings given for the three 'sample' problem-solving tasks presented to them (see also Appendix 1 for the questionnaire itself). These results are presented primarily for context and to support later Discussion in Chapter 6 (see 6.2) on the possible issues that may have impacted on the individuals within this group related to their different levels of confidence, the different primary age phases they were training to teach and, ultimately, their willingness to engage in the Think-Aloud Protocol (T-AP) and Stimulated Recall (SRI) 'replay'. Levels of confidence (either in mathematics generally or in the kind of mathematical problem-solving covered by this project) is not now a primary consideration of this work – see Chapter 4 above regarding the initial interest in confidence levels that has been superseded by a focus, as indicated in RQ 1, on the extent to which digital audio/T-AP and SRI can encourage verbalisation of mathematical strategies.

This contextual material is then followed in 5.4 with the results of the coding of the participants' verbal contributions in the two distinct problem-solving activities (T-APs 1 and 2 – see Appendix 2 for the original problems and 5.4.2 and 5.4.4 for extracts from the T-AP session transcripts). This coding was completed using the Novotná (1997)/Hošpesová and Novotná (2009)/Mercer (1995) informed Talk Framework (see 3.4 in the Methodology above for an outline of the Framework and its categories). Inter-coder reliability is addressed in 5.4.1 before 5.4.2 – 5.4.4 provide examples of the different Talk Framework verbalisations, taken from the transcripts, briefly considering issues such as *monologic* contributions, for example, and what these may imply about the T-AP process. They then move on to reflect upon the questions that the student teachers asked of themselves during the sessions (as an indication of the extent to which they explored the mathematics during the problem-solving tasks and, indeed, prompted verbal contributions from others).

The Results conclude with the analysis of the follow-up SRI session (5.5), outlining the degree to which the participants, for example, reflected/commented upon their engagement in the group situation itself versus their focus on the mathematics within the problem-solving tasks. Individual contributions were coded according to their focus on these areas, and this consideration of the SRI results was further enhanced, as indicated above in 3.5, by identifying contributions that might fit with the Talk Framework categories i.e. examples of *cumulative*, *supercumulative*, or *exploratory* (*encoding* or *transformative*) talk within the SRI. This was to identify the degree to which the follow-up recall session may have encouraged further (or 'new') discussion of mathematical strategies. It could be argued that this ameliorated any hesitancy on the student teachers' part to engage with the T-AP or, indeed, any weaknesses inherent in the protocol, thereby demonstrating the potential of running the two methodologies in consort with each other, as well as providing an additional use for the Talk Framework in analysing their dialogue. This is discussed in 7.5 below.

As indicated in 1.9 above, the sections within the Results and Discussion chapters run in 'parallel' with each other – therefore, 5.2 and 6.2 both address the group undertaking this work, and it is to the student teacher participants themselves that this thesis now turns.

5.2. The Group (Participants)

The group of participants engaged in this project[16] consisted of eight part-time primary PGCE student teachers (seven female, one male) from a total cohort of 77. The cohort

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[16] From this point on, there is no need for the distinction between 'first' and 'second iteration' of the project. Chapter 4 above detailed the learning undertaken during the 'first iteration', from which the methodology and Research Questions were developed. All results in Chapter 5 relate to the second group who went through the full process as outlined in Chapter 3 above.

itself had, as is still common across all Initial Teacher Education (ITE) programmes (DfE, 2018), a greater number of females than males (81.8% female primary) although this was broadly in line with the other ITE primary programmes running at the same institution in the 2012-13 academic year (for example, the PGCE full time cohort with 144 student teachers had a female/male split of 74.3%/25.7% and the undergraduate primary course, overall (across the three year groups, with the ratio differing within each[17]), had a female/male split of 87.4%/12.6%. While this was a self-selecting group drawn from the larger cohort, it did maintain a similar balance of male/female participants to the part-time primary PGCE programme and can therefore be seen as representative of the larger whole. It is, perhaps, unfortunate to note that the male participant was not available for SRI session, meaning that it was not possible to follow up his contribution to the problem-solving sessions. The overall effect of this is that females are more heavily represented even than would normally be the case in a mathematics group task of this kind.

Within this group, there was a further split between lower and upper primary student teachers (the PGCE primary programme essentially being formed of two distinct routes – 3 to 7 years, or Early Years Foundation Stage (EYFS) and Key Stage 1, and 5-11 years/Key Stages 1 and 2). Of the eight participants, two were 'lower primary' or 3 to 7 student teachers[18]. This makes the proportion of lower primary student teachers volunteering to take part in this project marginally below the percentage of lower primary students in the overall part-time PGCE cohort (25% as opposed to 32.5% lower primary overall). Lower primary participants' perceptions of mathematics, as expressed within this project, are of note and are further discussed in the next section; it is notable that the most critical view of the group problem-solving tasks themselves (most

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[17] Female:male – year 1: 11.3%:88.7%; year 2: 16%:84%; year 3: 12.6%:87.4%

[18] The term 'lower primary' is used for this subset from this point on. 'Upper primary' is used to denote the Key Stages 1 and 2 student teachers, as this was the term used on the PGCE programme itself. One unfortunate effect of this latter label is that it perhaps gives the impression that the training is more focused (or even exclusively focused) on the Key Stage 2 age phase, although the students undertaking that route were provided with a greater emphasis on Upper Stage 2 curriculum content than their lower primary peers.

especially the T-AP 2, Beads task) was from the lower primary participant discussed below, when considering the initial questionnaire responses, who had rated her confidence in mathematics least strongly.

5.3. Initial Questionnaires/Confidence Ratings in Mathematics (Including Teaching the Subject)

The initial questionnaires for this work (see Appendix 1) asked individual group members to provide a self-assessment of their confidence in mathematics in order to provide contextual information about the group undertaking these problem-solving tasks (see the caveat above in Chapter 4 about the move away from tracking developing confidence towards considering the impact of the methodologies on their verbalisation of mathematical strategies). Participants were asked to provide an overall confidence rating ("*how would you rate your confidence in maths*?") before considering their confidence in *teaching* the subject. This was explained to the group as relating to the age ranges the primary PGCE programme was training them to teach – namely, 5-11 and 7-11[19]. The ratings were given from 1 (strong) to 4 (weak). A further question, rated in the same way, asked participants to indicate their confidence in "*explaining strategies to help others solve mathematical problems*" (again, to the same age group). This led into a fourth and final question (which asked participants to complete a table

[19] They have, therefore, arguably been encouraged to give very subjective ratings – a participant envisaging themselves teaching Year 1, and considering their confidence with that specific mathematics curriculum (and potential range of children's abilities), is considering something quite different from a participant thinking more of Year 6 (i.e. the upper end of the 7-11 age range). These ratings are also divorced from their actual measured attainment as practitioners with their particular age group (student teachers were not graded exclusively in mathematics but in a range of pedagogical skills against the Ofsted grading criteria). The upshot of all this is that a participant self-identifying as 'strong' when it comes to teaching Year 1 children may consider themselves 'weak' if asked to reflect on a higher age range. This arguably makes the first confidence rating less useful than it might otherwise have been with the 'exemplar question' (see Table 5.3 below) providing perhaps the closest indication, before the Think Aloud session, of levels of 'confidence' with that specific type of task.

with multiple/separate confidence ratings for working out answers, explaining thinking and "*making use of materials to demonstrate methods to…a partner*" in relation to three given problem-solving questions[20]). Seven questionnaires were returned from the eight participants – the eighth did not give a reason for not providing a completed questionnaire. The tables below represent the results of these initial questionnaires, with Table 5.1 summarising the 'confidence ratings' around general confidence in mathematics (question 1), confidence in teaching mathematics (question 2) and confidence in explaining strategies to help others solve mathematical problems (question 3). The table demonstrates the number (out of seven) who rated themselves from 1 to 4 (see the top row of the table). The results are given as raw data due to the low number in the group. Table 5.2 below then breaks the data down into lower and upper primary students before Tables 5.3 to 5.5 represent the results of the fourth question, whereby participants were asked to rate themselves against particular problem-solving tasks.

Question	**Rating**			
	1 (strong)	**2**	**3**	**4 (weak)**
1. How would you rate your confidence in maths?		4	1	2
2. How would you rate your confidence in teaching maths?		2	4	1
3. How would you rate your confidence in explaining strategies to help others solve mathematical problems?		2	3	2

Table 5.1: Overall group summary of questionnaire responses (questions 1 – 3).

[20] As briefly discussed in Chapter 4 above, it was originally the intention to provide mathematical resources for the group to aid them in their working out, and this – in part – informed the wording of this question. Ultimately, given the confusion experienced by some of the group in the first iteration of the project over the use of these resources, and their difficulty in recounting how they had been used when coming to the SRI session, it was determined that such resources would not be provided and that the emphasis would be on thinking-aloud and writing down their thoughts as necessary for encoding the mathematics.

It is notable that none of the group responded with the strongest rating to any one of these first three questions, although the majority ranked themselves as '2' for overall confidence in maths. With the one exception who rated herself as '2' for each of the questions, those who identified themselves as strong in mathematics overall (question 1), did not indicate equal confidence in either "*teaching*" or "*explaining strategies to help others solve mathematical problems*". This is, of course, arguably as much to do with the stage of the PGCE programme during which the project was conducted (within their first semester, and only five sessions into the mathematics module) as any real perceived 'weakness' in their mathematics teaching[21]

Question	**Rating**							
	1		**2**		**3**		**4**	
	Lpr	Upr	Lpr	Upr	Lpr	Upr	Lpr	Upr
1. How would you rate your confidence in maths?			1	3		1	1	1
2. How would you rate your confidence in teaching maths?				2	2	2		1
3. How would you rate your confidence in explaining strategies to help others solve mathematical problems?				2	1	2	1	1

Table 5.2: Comparison of lower and upper primary participants' questionnaire responses (questions 1 -3).

[21] Although it may be a result of a 'mismatch' between their personal mathematics attainment and the grades they had so far received for lessons in school. As previously noted, lesson observation grades may not have provided an accurate reflection of 'subject knowledge' or personal confidence in mathematics as they reflected a range of pedagogical skills (and the degree to which individual tutors focused on mathematics, even in predominantly mathematics-based lessons, varied i.e. a grade could have reflected classroom management skills far more than the student teacher's mathematics).

While the number of lower primary participants was small, and it is therefore difficult to view them as representative of their cohort, their responses were nonetheless compared to those of their upper primary colleagues (including written responses on the questionnaires i.e. outside the questions asked – see below) in order to gain a sense of any differences in their perceptions of the subject area.

Table 5.2 above indicates that, while the perceptions of the two lower primary participants of their overall confidence levels differ strongly (one self-identifying as a great deal weaker than the other – although neither, as with their upper primary colleagues, have chosen to rate themselves as a '1'), they broadly share views as to their confidence in "*teaching maths*" and "*explaining strategies to help others solve mathematical problems*".

The group's responses to question 4 on the questionnaire yielded further useful results, in terms of garnering their reactions to the kinds of mathematical problems that would ultimately be used in the T-AP recordings and subsequent SRI. For this question, ratings were provided from 1 – 3 with 1 as 'very confident' and 3 as 'not confident at all'[22]. '2' was given as 'reasonably confident' and while there was no option on this occasion for participants to rank themselves as '4', one nonetheless did so (see Table 5.5 which represents their responses to the third of the sample questions). The move from a four point to a three point scale was, in part, to allow for participants to identify as 'average' (i.e. to give them a mid-point; to make it 'easier' for them to come to an answer). This, however, could be seen as confusing, in light of the scale used for the other questions. As there were three different mathematical problems provided for participants to consider, the results are presented here as three separate tables (Tables

[22] The printed questionnaires unfortunately have the ratings reproduced in the reverse order (i.e. with 1 as 'not confident at all'); this was (verbally) corrected for the participants prior to them completing the forms. Their responses make it clear that they have used '1' as the highest measure. The inclusion of '4' as a rating demonstrates a further weakness with the printed questionnaire in that, as discussed above, four possible responses were required for the first three questions discussed above, and only 1-3 were stipulated for the fourth question. It is perhaps unsurprising that one participant provided '4' as a response, following the pattern of the earlier questions.

5.3-5.5). The students had to provide ratings for "*working out the answers*", "*explaining* [their] *thinking and strategies used to reach the solution*" and "*us*[ing] *materials to demonstrate…methods to*…[a]…*partner*', as indicated in the first row of the table below. The number of responses for each of the possible ratings is then given in the rows below, with a row provided for '4' as this was, as stated above, provided by one of the participants as an answer. Notwithstanding the point about reducing the scale from 4 to 3, and whether this was confusing, the three tables demonstrate that very few of the participants rated themselves below '2' for any of the sample questions.

Rating	**Rating for working out the answer/s**	**Rating for *explaining* your thinking and strategies used to reach the solution**	**Rating for making use of materials to demonstrate your methods to your partner**
1	1	1 (gave answer as '1.5')	1
2	5	6	6
3	1		
4			

Table 5.3: Questionnaire, question 4 confidence ratings (rated 1-3) for Spaceship problem (DfEE, 2000, p.45)

With the exception of one participant, who gave their response as '1.5', the responses to this question demonstrated that all but one of the group (who nonetheless rated her confidence levels in mathematics and explaining strategies to help others solve mathematical problems more highly) were reasonably confident with this kind of word problem (identified by the DfEE (2000, p.45) as requiring knowledge of "multiplication facts for 2 and 3 times tables" and pitched to Years 1 and 2 of the (then) National Curriculum, albeit for able pupils). The one person, a lower primary student, who self-assessed as a '3' for "*working out the answers*" (and, indeed, the other two questions here) had also identified themselves as 'weak' in "*explaining strategies to help others solve mathematical problems*" in their answer to question 3 (Table 5.2) and their response arguably makes good sense in this context.

Rating	Rating for working out the answer/s	Rating for *explaining* your thinking and strategies used to reach the solution	Rating for making use of materials to demonstrate your methods to your partner
1			1
2	5	5	4
3	1	1	1
4			
NA	1	1	1

Table 5.4: Questionnaire, question 4 confidence ratings (rated 1-3) for “Blocks” problem (square numbers)

Following on from the first question provided for question 4 of the questionnaire, which was pitched at Years 1 and 2 (of the National Curriculum ‘as was’ in 2000), the second question was similarly primary curriculum oriented. It was again chosen to be similar to the questions undertaken in the T-AP problem-solving tasks to follow.

When considering this ‘Blocks’ problem[23], one lower primary participant (whose confidence rating in mathematics had been given as ‘weak’ and who rated her confidence in teaching and explaining mathematics just one point ahead of this) commented on her questionnaire that “my confidence in explaining this problem would depend on whether I could write and draw to demonstrate. Verbal explanations alone would be tricky”. This provides a rationale for considering, rehearsing, and perhaps also promoting, notions of *exploratory encoding* and *exploratory transformative* with this group, and – indeed – this individual, further supporting the initial statement of the problem given in 1.3 above. It also indicates the potential in the *Livescribe* multi-media device for capturing both spoken and written material and perhaps enabling connections to be drawn between the two.

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[23] “*One block is needed to make an up-and-down staircase, with one step up and one step down. 4 blocks make an up-and-down staircases with 2 steps up and 2 steps down. How many blocks would be needed to build an up-and-down staircase with 5 steps up and 5 steps down?*”

The ‘Blocks’ problem *can* be expressed algebraically (which would satisfy the definition of encoding into mathematics – see the Talk Framework above in Figure 3.6) as the resulting sequence is successive square numbers. Indeed, the question had been chosen for the questionnaire precisely because of this focus, as number sequences of this kind had recently been covered in the students’ PGCE mathematics module. Recognising the pattern from other problem-solving situations might have demonstrated “analogical problem-solving” (Robertson, 2001) and could also have enabled the participants to feel more confident about providing explanations. With this particular student teacher respondent, however, this was not to be. She may perhaps have recognised the pattern more readily had she, for example, presented the successive terms in table form – tabulating results in this manner also being something that had been covered in the PGCE workshop sessions. It is notable that she did not draw or write on her questionnaire at all for this question, despite making annotations on at least one of the other questions given. She added that “for all qu[estion]s I would do much better if I could write and draw to explain”. This is the same participant who, in the second SRI recording, expressed confusion caused by the verbal contributions of her peers[24] (see 5.5 and 6.5 below) and it was, therefore, useful to consider her initial questionnaire responses when approaching the SRI session and particularly considering RQ 3’s focus on what the process revealed to her about her own problem-solving strategies when engaged with a group (and, indeed, the extent to which working with her peers impacted upon her willingness to share these strategies).

[24] (SR2 – 26:01: “*maths isn't my strongest subject and I need processing time and all these voices coming in at different directions and some people were more forceful than others and so I kind of didn't say an awful lot*”)

Rating	**Rating for working out the answer/s**	**Rating for *explaining* your thinking and strategies used to reach the solution**	**Rating for making use of materials to demonstrate your methods to your partner**
1	2	5 (including one who gave the answer as 1.5)	3
2	5	1	2
3		1	1
4			1

Table 5.5: Questionnaire, question 4 confidence ratings (rated 1-3) for "Fifteen Cards" problem

The final problem presented to the potential participants in the questionnaire was chosen because it requires both the use of mathematical instant recall facts (in this instance number bonds to the various totals required by the pairs of cards) and strategy (identifying whether there were any other possible solutions; something it has in common with the two problems eventually T-AP recorded for subsequent SRI). It is clear that participants were more confident with this problem (perhaps due to the relatively swift trial-and-error way in which it can be completed, particularly if using real cards) and there is not much to be drawn from this data without further investigation of the types of materials, for example, that individuals may have envisaged using to help them demonstrate methods to a partner. The lower ratings for this "*demonstrat*[ing] *your methods to a partner*" question on the "*Fifteen Cards*" problem may simply reflect that participants would not have considered using materials at all for a task of this nature looking for number bonds with fifteen numbered cards, each of which can only be used once and each of which, if real cards were indeed used, could of course be moved from position to position until the totals are achieved; without directly asking participants for further comments about this task, it is difficult to say whether this was the reason for the lower ratings. Ultimately, of course, the contextual information about demonstration of strategies using practical resources is not especially relevant to the T-AP sessions as conducted – the decision having been made (see Chapter 4 above) to use only paper and pen/s when working to solve these mathematical problems.

Overall, the responses to this section of the questionnaire demonstrate reasonable confidence with the questions asked and, while they provide background information relating to the group, limited further use was made of this initial data when conducting the T-AP recordings or subsequent SRI sessions themselves. However, the questionnaire responses arguably provide an insight into the “psychological situation” (Rotter, 1954) facing the participants going into a group problem-solving situation with their peers, and also supplied beneficial contextual information when coming to consider the individual responses within the SRI sessions. For example, the lower primary participant above and her comments about preferring to write and draw rather than provide verbal explanations helped demonstrate that it was not the T-AP process alone that impacted upon her later reticence to engage, but an already established sense of how she preferred to work.

At the end of the project, it was originally intended that the participants would revisit the questionnaires and these initial scores to provide an indication of the way in which engaging with the T-AP and SRI sessions might have impacted on their rating for explaining thinking and strategies used to reach a solution. However, given the shift of focus away from ‘confidence’ and towards (see RQ 1) the way in which dialogue/thinking-aloud recorded by digital audio could support their verbalisation of problem-solving strategies (as detailed above in Chapter 4), it was determined that this would not necessarily reveal as much as their responses within the SRI sessions. These follow-up opportunities would also cover (see 3.6 above) what had perhaps been revealed to the participants about their own, and their peers’, problem-solving strategies (see RQ 3). The focus, therefore, in these results is on the Talk Framework coded T-AP transcripts and the information gleaned from the SRI opportunities, including the extent to which the latter may have promoted additional verbalisation and even explanation of problem-solving strategies. It is, therefore, to the Talk Framework and the coding of the participants’ verbal contributions to problems 1 and 2 that this chapter turns to next.

5.4. Coding of Participants' Verbal Contributions to T-AP 1 and 2 in Accordance with the Talk Framework

This section details the results of the Talk Framework coding of the Think-Aloud Protocol (T-AP) supported group mathematical problem-solving situations. After detailing Inter-coder Reliability (5.4.1 below), sub-sections 5.4.2 and 5.4.4 present the results of the coding of the T-AP sessions/transcripts (Tables 5.6 and 5.9). These sections begin with the relevant extracts from the T-AP recording (anonymised); in addition, 5.4.4 presents the data from the *Livescribe* session (the page of annotations made by the participants that would then be played back in synchronisation with their dialogue – it is important to note here that the written data has not been analysed as the focus of this thesis is on talk). Each 'turn' has been coded according to the Talk Framework with, on the rare occasions they occur, *monologic* utterances identified[25] – these are contributions not intended for the rest of the group (i.e. under-the-breath comments or what might be termed 'notes to self'). One example of such a *monologic* utterance is at 04:42 in T-AP 1 (see 5.4.2) where one of the student teachers says, "*I'm just going with the group, really. I'm not really thinking about how to solve it myself. Hoping somebody else has got the answer*." It seems not to be a contribution intended for the others to build on and so arguably stands outside the dialogic frame (although it could be argued that this is a plea for help, albeit an indirect one, perhaps due to a level of discomfort with outright stating what her confusion relates to – 6.2 below discusses the group 'psychological situation" (Rotter, 1954) that might have led to such 'discomfort'; in addition, 7.5 below identifies potential limitations inherent in considering these as isolated 'turns' rather than moves). Other contributions not easily coded into Talk Framework categories, due to them being incomplete or unclear, or

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[25] There is also one *exploratory* contribution that seemed not to fit well with either *exploratory encoding* or *exploratory transformative* and so has been retained as *exploratory* only; see Table 5.6 in section 5.4.2 below. Section 7.5 below, relating to Further Research, indicates the limitations inherent in the choice of analysis by 'turn', and proposes considering monologic utterances as discourse moves (Vygotsky, 1962; Barnes and Todd, 1995; Edwards, 2005) intended, as with many of the utterances coded as *supercumulative* in this work, to elicit further responses from peers.

relating to aspects of the task clearly irrelevant to the problem being solved (e.g. the technical aspects of the recording process or entirely non-mathematical questions relating to pens etc. on the table – "*Do you want that up there?*") are identified as *Not Coded*. Percentages are then provided out of the total number of turns for each of the Talk Framework categories, minus turns such as the initial reading of the question (as prompted by the interviewer's monologue) and, indeed, the opening instructions from the researcher.

Sub-sections 5.4.3 and 5.4.5 provide an additional breakdown of the questions asked by the student teacher participants within the recordings. During the process of transcribing the recordings, it became clear that the T-AP had not only prompted thinking-aloud from the participants regarding what they themselves were doing and thinking but it had also perhaps encouraged them to ask questions of others in the group, whether wanting them to explain themselves further or to take on particular roles in the solving of the problems set (or perhaps as a result of the 'frustration' caused by incomplete thinking-aloud). These questions, already coded against the Talk Framework, were then considered in light of Bloom's taxonomy (to provide a sense of their 'quality' as questions, building on the work of Graesser and Person (1994) whilst acknowledging that Bloom's taxonomy stands somewhat apart from the social-constructivist approach taken in this project, as detailed above in 1.7). These results are presented in here (see Table 5.12) for further discussion in Chapter 6 to follow.

5.4.1. Inter-Coder Reliability

Attention was paid to inter-coder reliability via a joint coding exercise conducted alongside one of the researcher's two PhD supervisors. Two short extracts were used for this purpose, one for initial 'training' and one for independent coding and verification. The first extract, taken from the TAP-2 recording (see extracts in 5.4.4 below), was provided in both marked up form (using coloured text to indicate the different types of verbalisation identified by the researcher within the participants' speech and including a preamble containing the codes utilised) and also 'clean'/free of annotations. This extract was then coded by the supervisor and compared with the pre-

prepared, marked up version. The second extract (this time from T-AP 1) was used for coding after this 'training' had been delivered and had not been previously coded by the researcher. This coding was completed separately by supervisor and researcher and then compared.

In terms of inter-coder reliability, the verification percentage rate was notably high. The first extract used in this moderation session (included within 5.4.4) was taken from the 'Beads' problem-solving recording (T-AP 2) and consists of 50 verbal contributions/turns (from a total of 105 in the whole recording – therefore, just under 50% was jointly coded) and there was agreement between researcher and supervisor for 48 of these (96%). The second, shorter, extract (included within 5.4.2) saw a similar level of agreement – of 17 turns independently coded by supervisor and researcher (out of a total of 190 in the complete transcript), 15 were agreed (88.2%). One of the contributions that was not agreed upon was considered by the researcher to be *supercumulative* ("*yeah, so your first digit*…[inaudible]…*if you take turns*") whilst the supervisor felt that it was not able to be coded due to the lack of information contained within it. The overall tables of results below (Tables 5.6 and 5.9) reflect the fully coded transcripts; they therefore incorporate the pages that had been 'moderated' between researcher and supervisor as described here.

5.4.2. T-AP 1 Recording: Exploring Addition

This section begins with the relevant extracts from the first T-AP transcript, before presenting, in Table 5.6, the breakdown, by Talk Framework category, of the individual 'turns'. Material jointly coded by researcher and supervisor, to test for inter-coder reliability, is shaded. The right hand column presents the Talk Framework coding with additional consideration of questions asked according to Bloom (see above and Table 5.12 below).

Time and Turn	Verbalisation	Talk Framework Coding (with questions considered according to Bloom)
Unnumbered- 00:15	I would like you as a group to share your thinking with me while you solve, or attempt to solve, the problem provided. Attempt to tell me everything you are thinking from the time you first see the question until you give an answer or answers. Use the pen and paper provided to write down anything you find useful in helping you to solve the problem. Talk aloud constantly as a group, sometimes as if talking to yourself as well as to your peers. Please understand that I may prompt you to speak if there is silence and don't worry about what I or anyone else might think about your answers which are anonymous in any case. I want to learn about your thinking while you are engaging in the task.	**NA – Interviewer's Monologue**
Between the Interviewer's Monologue and 01:56, an individual reads out the question for the rest of the group.		
1 – 01:56	Do we actually have cards and, if we don't, should we make some?	**Supercumulative (Analysis)**
2 – 02:01	Can we use this picture?	**Cumulative (Analysis)**
3 – 02:03	I'd just use the sheet of paper and I'd write it on, I wouldn't even…	
4 – 02:05	Okay	**Cumulative**
3 (cont.)	… I'd start with the smallest [overlapping agreement]…yeah, I would. The smallest…so, 1, 2…	**Supercumulative**
5 – 02:11	Add 3, 4…	**Cumulative**
6 – 02:11	Yeah	**Cumulative**
7 – 02:12	Build up successively.	**Supercumulative**
Between 02:15 and 02:22, individuals propose different numbers for consideration, building on 02:03's suggestion to start with the smallest and 02:12's suggestion to build up successively.		
15 – 02:22	I thought it meant add each…erm…	**Exploratory Encoding**
16 – 02:25	…oh, you've only got one, haven't	**Cumulative**

	you? I thought it meant…	
17 – 02:28	Can I just say, I would be inclined to take, say, each of the digits in turn and systematically put through…erm…so have…1, 2, 3, 4 [agreement] and do it that way, so that you're systematically working through all the different combinations of numbers…	**Exploratory Encoding**
18 – 02:49	Because we have to have two, two-digit numbers together…	**Cumulative**
19 – 02:52	So you can't have one.	**Cumulative**
Between 02:52 and 03:29 – further suggestion of potential numbers from the group as these are written down on the sugar paper provided. There is no verbalisation of what is being written and why.		
41 – 03:29	[Let's] moving it from this way to that way, wouldn't it? So, if it's moving along that way to make it 23 first…please feel free to contradict me at any point… [Laughter]	**Exploratory Encoding (Application)**
42 – 03:42	No, I meant…	**Not Coded**
43 – 03:47	Okay…	**Not Coded**
44 – 03:51	And that's…46, 55 [agreement from other members of the group]…oh, we can't have the ones the same…	**Cumulative**
45 – 04:01	Well it's alright…put them down…	**Cumulative**
Between 04:01 and 04:10, sounds of agreement and suggestions for what needs to be written down.		
50 – 04:10	At least you know then you've done all the different combinations…	**Cumulative**
Further agreement		
54 – 04:15	We haven't done all the different combinations…	**Cumulative** **N.B.** ***While this may seem to perhaps be supercumulative, listening to the audio and the responses from the group indicates that it is an invitation to write down the numbers as yet not recorded rather than anything of more import.***
Further suggestions for what can be written down.		

59 – 04:23	So, we'd have been better doing all the ones with the one dig[it]…is that what you meant? Having all the…starting with the ones as the first digit…?	**Supercumulative (Application)**
60 – 04:31	I don't know, I was just thinking of a way of writing it down so that you know that you're systematically going through [begins to overlap with other contributions] all the combinations…[overlapping with…]	**Exploratory Encoding**
61 – 04:36	We've got 12 but we haven't got 13 and we haven't got 14…	**Cumulative**
Various comments about the size of the writing on the paper and whether more paper is required.		
64 – 04:42	I'm just going with the group, really. I'm not really thinking about how to solve it myself. Hoping somebody else has got the answer…	**Monologic**
65 – 04:50	No, I'm not thinking about how to s…[olve]…I…I'm getting a bit confused…	**Monologic**
66 – 04:52	If I was solving it all by myself, I would be doing what we're doing…[agreement]…so are we not just doing it collaboratively? [Overlapping agreement]	**Monologic (Knowledge)**
67 – 04:58	Yeah, we are doing it collaboratively. That's the way I'd do it.	**Not Coded**
Between 05:00 and 05:22, the group suggests further numbers, and comments on the speed of their writing,		
74 – 05:22	But we have got 23 here…won't it just be that…end up being that the other way round?	**Supercumulative (Application)**
75 – 05:25	Does it say whether that that…	**Cumulative (Knowledge)**
76 – 05:27	No, because we haven't got… [some inaudible comments/trails off]	**Cumulative**
77 – 05:29	And addition is the same both ways. So, actually if your pair of numbers is 21 and 34…[overlapping with…]	**Exploratory Transformative**
78 – 05:35	But we could do 23 and 34… [or 14?]	**Supercumulative**
79 – 05:37	…you don't need 34…	**Cumulative**

80 – 05:38	…you don't need 21…do you?	**Cumulative (Knowledge)**
81 – 05:39	So you could do 24 and…Couldn't we do 23 and 14, then?	**Exploratory (Knowledge)** **N.B. *This was coded as exploratory prior to the SRI, and has been left this way – there is an argument for this to be coded as supercumulative as it poses a question that requires explanation***
Between 05:39 and 06:12, the group propose further numbers until…		
94 – 06:12	It doesn't feel very systematic.	**Supercumulative**
95 – 06:14	No, it doesn't. [Agreement from others]	**Cumulative**
96 – 06:14	To be honest, I'm like…feeling confused now…	**Monologic**
97 – 06:18	Yeah, I am…	**Not Coded**
98 – 06:19	…because I don't see the logic.	**Supercumulative**
99 – 06:20	What are you thinking over there?	**Cumulative (Knowledge)**
100 – 06:21	There probably isn't one. [Laughter]	**Cumulative**
101 – 06:25	There probably isn't one.	**Cumulative**
102 – 06:26	I'm kind of lost in the logic.	**Cumulative**
Further agreement.		
106 – 06:31	But even if it's not logical, we'll end…as long as we're systematic at the checking…	**Supercumulative**
107 – 06:37	Okay, so yeah…	**Cumulative**
108 – 06:38	I…I'm confused because I look at this as a big…sort of…I'm not sure what it is…why I'm doing it…whereas I think if I was doing it myself I would [agreement] probably work through from the smallest to the largest…	**Exploratory Encoding**
109 : 06:48	Well, I reckon we carry on because we'll…we'll use up all the numbers eventually [agreement from others] if we just check them properly.	**Supercumulative**

Between 06:48 and 07:19, the group check the numbers.		
118 – 07:19	Are there any we haven't done?	**Supercumulative** **(Application)**
119 – 07:22	What…just some kind of pattern emerging in the ones that are the same…?	**Supercumulative** **(Analysis)**
120 – 07:26	So, could…so, we've used 12, have we used 13?	**Supercumulative** **(Knowledge)**
121 – 07:34	No…no…	**Cumulative**
122 – 07:34	If you do it in order. So that the next…we need to do one with the 13…	**Cumulative**
123 – 07:38	That's right.	**Cumulative**
124 – 07:39	We need to know in some way, because I can't see…it's just…'cos we…	**Cumulative**
125 – 07:43	Why don't for 12, for 54 and then 13… Oh no.	**Supercumulative (Application)**
126 – 07:48	Why don't…Why don't we generate all the two digit numbers that are possible…?	**Supercumulative (Application)**
127 – 07:53	Have we got another pen?	**Not Coded (Knowledge)**
128 – 07:54	…down the side and then…[inaudible]…If we generate…generate all the numbers first…and then…	**Exploratory Encoding**
129 – 08:01	So you can have 12, you can have 13, [14] you can have 14 [overlapping agreement; echoing of numbers]; you can have 21; you can have 22…oh no, 23…[others joining in, offering numbers ahead of them being said e.g. "24"]…you can have 24; you can have 31 [agreement]; you can have 32; you can have 34; you can have 41, 42, and 43… [some echoing of these final numbers]	**Supercumulative**
130 – 08:23	I'm feeling happier now.	**Monologic**
131 – 08:24	Yeah?	**Not Coded**
132 – 08:26	More logical	**Monologic**

133 – 08:26	So, if we…if we tick that we've got…so that then 12 will either go with 34 and 43 and those are the only options it can go with…	**Supercumulative**
134 – 08:35	Yes because you can only use 1…[overlaps/inaudible]	**Cumulative**
Further checking of pairs of numbers between 08:35 and 08:48.		
141 – 08:48	So, will there always only be t…you know when you've got the first number that we've generated, will there always only be two other numbers…[overlaps with]	**Exploratory Transformative**
142 – 08:54	Yes, there will; yeah, that's right.	**Cumulative**
143 – 08:57	So, we should have two ticks by each thing…[overlapping with…]	**Exploratory Transformative**
Between 08:57 and 09:13, the group checks the number of ticks against their numbers on the sheet.		
154 – 09:13	13 needs to go with…have we got 13 with anything? What does it need to go with?	**Supercumulative (Analysis)**
155 – 09:18	24. 42.	**Cumulative**
156 – 09:26	So 14 [others join in with this], which we've got with 23 and we've got with 24 so that…[is 2, 2, 2 already?]	**Supercumulative (Analysis)**
Between 09:26 and 10:44, the group continue to call out numbers for checking.		
160 – 10:44	Right, so how many number combinations can we make?	**Exploratory Encoding (Synthesis)**
Between 10:44 and 11:45, the group discuss how best to work out the different totals – a suggestion is made to circle each different total (11:18 – supercumulative) and the group realise they have got five different totals (11:45).		
170 – 11:46	And then they need to put in order…	**Supercumulative**
171 – 11:49	What's the next thing we're thinking about? So we've got…	**Cumulative (Knowledge)**
172 – 11:50	What are the maximum…minimum and maximum totals…and how do we know we've found them all?	**Exploratory Encoding**
173 – 11:54	'cos we've gone and used a strategy!	**Cumulative**
Between 11:54 and 12:07, the group determine the minimum (37) and maximum (73) totals and arrange them from smallest to largest.		
178 – 12:07	37, 46, 55, 64, 73…so then we need to…[inaudible]…between them…If we use a different coloured pen to	**Supercumulative**

	calculate the difference…	
179 – 12:19	9, 9, 9, 9, 9…	**Cumulative**
180 – 12:29	What's the difference between our biggest and smallest?	**Supercumulative (Application)**
181 – 12:31	It's 73 take away 37…	**Exploratory Transformative**
182 – 12:40	The difference is 36…which is the total…	**Cumulative**
183 – 12:43	How exciting.	**Not Coded**
Between 12:43 and 12:56, some discussion around the questions asked on the original problem sheet.		
188 – 12:54	It is exciting; I don't know why!	**Monologic**
189 – 12:56	I like it when you find a pattern!	**Monologic**
190 – 12:59	So, is that one finished?	**Not Coded**
Recording ends.		

Table 5.6 below represents the results of the coding of T-AP 1, as illustrated in the extracts above, according to the Talk Framework categories outlined in Figure 3.6 above (section 3.4, page 113). This was the longer of the two Think-Aloud Protocol (T-AP) recordings, running to 12 minutes and 59 seconds (the second recording ran to 6 minutes and 30 seconds). The table demonstrates that the majority of the verbal contributions made in this first of the problem-solving situation were *cumulative*. Indeed, there were just over three times as many *cumulative* responses as *supercumulative* – 108 *cumulative* responses (56.8%) to 33 *supercumulative* responses (17.4%). The table provides a breakdown of all the responses provided to the exercise (there were 190 'turns' (Rowland, 2000) in total across the recording; see section 3.4 above). Percentages are provided for easy comparison with Table 5.9 below, which presents the results of the shorter 6 minute, 26 second T-AP 2, coded against the same Framework.

Category	Number of responses	Percentage response
Supercumulative	33	17.4%
Cumulative	108	56.8%
Exploratory	1	0.5%
Exploratory Encoding	11	5.8%
Exploratory Transformative	4	2.1%
Disputational	0	0
Monologic	11	5.8%
Not coded	22	11.6%

Table 5.6: Talk Framework coding of verbal contributions for T-AP 1

The *supercumulative* responses here, as defined in Figure 3.6 above, "*build…on contributions made by others, most especially exploratory*". They may "*provide…examples of numbers that…'fit' with the proposed explanation/strategy*" or they might ask for additional explanations. In coding these contributions, there are not many examples that strictly fit the "*most especially exploratory*" provision; indeed, a number lead into *exploratory* (see, for example, 04:23, where the participant asking for additional information prompts an *exploratory encoding* response relating to strategy). An example of *supercumulative* arising from an *exploratory* (in this case *exploratory transformative*) contribution, and therefore more closely fitting with the definition, comes at 05:35. The coding is discussed below in 6.4, with potential refinements to the Framework in future work addressed in the Conclusion (in particular, 7.5) that follows. It may, for example, be necessary to refine the definition to reflect that the already posited need for further explanation within *supercumulative* contributions may result from a lack of prior examples being given by the group i.e. the "*most especially exploratory*" requirement within the definition may be misleading.

The *cumulative* responses, as already noted, constitute more than three times as many responses as *supercumulative* in T-AP 1. It ought to be noted here that single, simple affirmative responses made to suggestions (such as "*yeah*" or "*okay*", of which there are 20 examples in T-AP 1 – see extracts above for examples) were taken as *cumulative*.

This is because the definition of this category maintains only that such responses "may" add new information to the discussion. On listening to the recordings, it was felt that coding these responses as *cumulative* rather than leaving then "Not Coded" reflects the "*mutually supportive, uncritical*" nature of the contributions, which very often prompted others to continue with lines of thought or confirmed that a suggestion made was a good one. A questioning "*yeah?*" at 08:24 of T-AP 1 seems designed to draw out a reason for another participant "feeling happier" and, therefore, encourages a degree of explanation from them which otherwise might not have been provided ("*more logical*" at 08:26).

One response was coded as *exploratory* rather than either *exploratory encoding* or *exploratory transformative*, although this is not significant within the overall total. This response, at 05:39 in T-AP 1 was "*So you could do 24 and... Couldn't we do 23 and 14, then?*" and it could perhaps, on reflection, have been categorized as *supercumulative* as it might be said to be useful in "moving the group towards a successful solution" (to quote from 3.4 above – see also Figure 3.6 for the definition of *supercumulative* used within this work). As required for classification within that category, no reasons are offered in this suggestion, and neither are potential mathematical strategies evident, but the thought may yet have been useful to the group in tackling the exercise. It has not been reclassified as *supercumulative* here in the results because it was taken as an *exploratory* contribution within the follow-up SRI (see SRI 1 above, 17:32), not only by the researcher but also by the group (who had been briefed in the Talk Framework categories prior to the recall and had been asked to consider their contributions against them, where appropriate – this is actually one of the few occasions where a Talk Framework category is directly addressed in the recording). Overall, Table 5.6 demonstrates that there were few *exploratory* contributions made during the course of T-AP 1 with *exploratory encoding* contributions amounting to a third of the *supercumulative* total (which is, as stated above, roughly a third of the *cumulative* total). The total number of *exploratory transformative* contributions amounted to only 4.

No *disputational* responses were identified in this recording, although there were, as indicated above, a number of responses that did not easily fit within the categories identified for the Talk Framework for problem-solving and potentially provide areas for further discussion. *Monologic* contributions (see Table 5.7 below and Section 3.4 above) are, as indicated in the outline of the Talk Framework categories provided in Figure 3.6, those that seem not to be intended for the rest of the group; comments or 'asides' that are made by the individual not to add to the discussion but seemingly for their own benefit. There is, of course, a degree of subjectivity in identifying these contributions as not intended for the group, but identifying *monologic* contributions was potentially useful within the follow-up SRI for encouraging participants to provide additional details, as – in many cases – they could have encouraged responses from others in the original T-AP situation, and these responses might have been valuable to the progress of the group in seeking a solution. The *monologic* contributions also provide indications of the effectiveness of the protocol in promoting clear thinking-aloud alongside the participants' feelings about the process as it unfolded around them – what, in 2.5 above, has been identified as the "psychological situation" (Rotter, 1954) surrounding the problem-solving (this is further discussed in 6.2 below).

Monologic	Not coded
04:42: I'm just going with the group, really. I'm not really thinking about how to solve it myself. Hoping somebody else has got the answer…	**05:00**: Someone else take over the writing. We need to write it down much quicker.
04:50: No, I'm not thinking about how to s…[olve]…I…I'm getting a bit confused…	**05:27**: No, because we haven't got… [some inaudible comments/trails off]
04:52: If I was solving it all by myself, I would be doing what we're doing…so are we not doing it collaboratively?	**08:45**: Do you want that up here? **08:46**: Yeah. **08:48**: Oh right, yeah, yeah, yeah
05:53: I don't understand the order.	**12:05** (to herself): Arrange your totals from smallest to largest…
06:14: To be honest, I'm like…feeling confused now…	**12:47**: It was one of the questions.
08:23: I'm feeling happier now.	**12:59**: So, is that one finished?
08:26: More logical.	
12:44: I'm confused as to why we started doing the questions…	
12:54: It went really quickly on that bit…	
12:54: It is exciting. I don't know why!	
12:56: I like it when you find a pattern!	

Table 5.7: Examples of monologic and not coded responses used to inform subsequent SRI discussion: T-AP 1

Table 5.7 above collates the *monologic* and *not coded* material identified within the first T-AP session for potential follow-up in the SRI. Not all comments were able to be followed up in the time available, bearing in mind – also – that the student teacher participants were encouraged to comment on aspects they themselves found interesting in the recording, and this would, at times, obviously lead the discussion away from those areas the researcher had identified for reflection ahead of time. These *monologic* and *not coded* comments provide indications of how participants felt whilst engaged in the T-AP process and include reflections on mathematics, problem-solving and group work. At

04:42, for example (the first row of Table 5.7 above), there is the observation "*I'm just going with the group really. I'm not really thinking about how to solve it myself*." This was followed up within the subsequent SRI session (see 09:56 in SRI 1 as detailed in 5.5.1 below) with a conversation about individuals taking 'the lead' in the discussion and whether the group was, in fact, working in consort. Equally, the comment at 05:53 (the fourth row down in Table 5.7, under the *monologic* heading) about not "*understand*[ing] *the order*", alongside subsequent comments about confusion, can be seen as evidence that the T-AP did not result in all members of the group following the proposed strategies. This, too, was raised within the SRI, this time in SRI 2 (19:13 – see Appendix 4), where one member of the group reflected on "*the nature of the exercise*" and how there had been no explicit requirement for all to understand, at least as *she* understood it. 6.2 below addresses this in more depth.

The *not coded* contributions in the right-hand column of Table 5.7 above are included to illustrate those aspects that had, in effect, been discounted as worthy of discussion prior to the SRI. Although the information is presented in two columns, there are no parallels to be drawn between the left-hand *monologic* comment and the right-hand *not coded* comments; they are presented in this fashion for ease of reference. Table 5.10 in section 5.4.4 below presents similar examples of *monologic* and *not coded* responses collected from T-AP 2.

The next section considers the questions that were asked by participants of themselves (as they were not able to put questions to the researcher/tutor during the problem-solving task). These were of interest not just in terms of their coding according to the Talk Framework, but also in terms of the way they had perhaps been prompted by the situation, or even prompted because, despite the T-AP utilised, thoughts were not as clear to them as they would like. Considering their questions, therefore, allows for aspects of the research questions to be addressed such as the extent to which the thinking-aloud supported (or hindered) their verbalisation (RQ 1) and, indeed, the degree to which the methodologies may have impacted upon their willingness to engage (RQ 5). After considering these questions, the Results then turn to T-AP 2 and the breakdown of responses according to the Talk Framework within that session (see 5.4.4 below).

5.4.3. Questions Asked by Participants of Themselves during T-AP 1 Session

This sub-section begins with a breakdown, in Table 5.8, of the questions asked in the first Think-Aloud Protocol (T-AP) session, utilising Bloom's taxonomy as a means of identifying the question types and further identifying the degree to which the questions attempted to go beyond 'mere' knowledge-based or "simple recall" (Riegle, 1976, p. 156). This is, in part, inspired by Hošpesová and Novotná's (2009) consideration of the taxonomy in formulating their problem-solving framework (although they made use of the revised framework of Anderson and Krathwohl (2001); the use of the more 'traditional' framework here is as much a result of its use on the PGCE students' teacher education programme as any specific imperative to favour one over the other).

As Riegle (1976, p.156) states, "…in the typical classroom, students are seldom required to go beyond the level of application questions…most questions are at the knowledge and comprehension levels". In addition to this, and as previously discussed in 2.5.2 above, Piaget (1929) observes that individuals might withhold certain pieces of information from their peers, believing that others already know what they do not (such withholding is, therefore, a means of 'saving face' in front of others). This provides another reason for considering the degree to which participants were willing to 'risk' asking questions that might reveal incomprehension on their part (bearing in mind, also, Duval's (2006, p.104) emphasis on identifying the "sources of incomprehension"). Overall, considering the questions asked in the two T-AP sessions against Bloom (1956), while arguably an 'aside' from the main focus of this work, enables further reflection on the Talk Framework categories themselves, most specifically *supercumulative*, which is postulated in this work as perhaps inviting further *exploratory* talk from the group (see, for example, the observation in 5.4.2 above regarding *supercumulative*, as originally devised, building on *exploratory* statements from the group, and the realisation, when considering the results, that *supercumulative* comments had more often *invited* further exploration and explanation – even if, ultimately, the group, perhaps due to individuals' focus on the requirements of the protocol, had not 'picked up' on these invitations).

Considering RQ 1's focus on the degree to which dialogue/thinking-aloud supported by digital audio can support student teachers' verbalisation of problem-solving strategies, and whilst transcribing the T-AP 1 and 2 recordings for use in the SRI, it became clear that the questions asked by the student teacher participants, most specifically the unanswered ones (see also the references to *monologic* talk in the section above), might provide prompts for further discussion. If the think-aloud had not provoked much in the way of questioning, or if the questions had remained at the level Riegle (1976) refers to above, then it might be considered that the students had not engaged as deeply with the thinking-aloud process as might have been hoped. This may have been as a result of the overall "psychological situation" (Rotter, 1954), related to Piaget's (1929) observation above, leading them to avoid posing questions that might have revealed uncertainties and confusion that they would prefer their peers not to know. This is further discussed below in 6.2. This sub-section now moves to explaining how the questions were considered, with Table 5.8 providing the breakdown against Bloom's categories.

Discounting statements such as "*Yeah?*" (08:24, in this case as it is unclear what the question directly refers to, and this was not followed up in the SRI) and direct repetitions or rephrasings of questions as presented within the task sheet itself as not of particular import (although referring back to the problem can be seen as a good strategy to ensure the successful completion of the task), a total of 29 questions were asked by participants of their peers during T-AP 1. Of these, a significant number were effectively asking only for the confirmation or denial of relatively simple facts (02:01: "*Can we use this picture?*"). These fit with the *knowledge* category of Bloom's taxonomy. Others, however, went further in exploring and perhaps even analysing the mathematical problem at hand, and it is notable that those identified as such were also independently coded, in the main, as *exploratory* (of one sort or another) or *supercumulative*. It could be said that this adds some further 'weight' to the value of these categories in a group situation. An example of this is 07:48's "*…why don't we generate all the two digit numbers that are possible…?*" which had already been coded as *supercumulative*, but – considered against Bloom – reaches towards application. This enables a bridge to be built between the Talk Framework, Bloom's Taxonomy and other analytical tools relating to what has been termed question quality (Graesser and Person, 1994). As indicated above, it perhaps provides a further rationale for identifying *supercumulative* as a valuable category of speech in a group discussion situation that invites further explanation of strategies, whether through asking peers to put information into another form (Riegle, 1976, p.156), which would satisfy the definition of *comprehension*, or perhaps by going further towards *analysis* and beyond (see the definition of *supercumulative* provided in Figure 3.6 above and the discussion of the formation of this fifth Talk Framework category in section 3.4). 1.5 and 1.6 above suggest that one of the potential benefits of engaging in the "networked" T-AP and SRI methodologies is the revisitation of unanswered questions within the original T-AP exercises. This revisitation, it has been suggested, could ameliorate any 'issues' with the original T-AP exercise, and could perhaps also address the impact of the "psychological situation" (Rotter, 1954). An understanding of the potential quality of the questions, and of the value of *supercumulative* contributions in particular, could be of use not only to the researcher in an exercise of this kind, but also to the student teacher participants moving on to work in the classroom.

Table 5.8 below, therefore, represents the 29 questions asked by the participants across T-AP 1 and codes them according to Bloom's taxonomy, identifying that the majority of the questions were related to what Riegle (1976, p. 156) refers to as "simple recall". Such questions range from 02:01's "*Can we use this picture?*" to 03:14's "*...plus 34…yeah?*" These questions ask for clarifications of what might be termed straightforward facts and elicit yes or no answers (or no answer at all) from peers. This indicates that the T-AP protocol applied to the session did not mean that individuals thought to supply reasons for asking their questions or, indeed, explained their thinking behind them in the way that Seal's (2006) "ground rules" would require. More 'considered' questions, such as 05:22's "*But we have got 23 here…won't it just be that…end up being that the other way round?*" are coded as *supercumulative* in the transcript (see Figure 3.6 above for the full definition of *supercumulative*) because they

"…stop… short of being explanatory in that reasons are not offered and strategies are not proposed". The question, however, indicates an awareness of the commutative law and eventually provokes the response at 05:29 "*And addition is the same both ways*" that has been coded as *exploratory transformative* and, in fact, provides the reason for the numbers being the same "*the other way round*". Therefore, this question moves closer to *analysis* of the problem, and of the strategy being employed to solve it. Although it is not possible, without a non-T-AP control situation, to say that the protocol and the awareness of being recorded for later playback inspired the asking of such questions, the data below, and in Table 5.12 that follows, was used to consider the characteristics of the different Talk Framework categories and to reflect upon the kinds of questions that might be valuable within a group problem-solving situation. Equally, as indicated above in the reflection of Riegle's (1976, p.256) point about "typical classroom[s]" and the lack of questioning beyond the level of application, it seems that, in both T-AP sessions detailed here (see also section 5.4.5 below), participants' "question quality" (Graesser and Person, 1994) was not improved by the imposition of the protocol and perhaps they did not consider the posing of questions to their peers as part of their thinking-aloud.

Category	Number of questions	Percentage response
Knowledge	15	51.7%
Comprehension	1	3.4%
Application	7	24.1%
Analysis	5	17.2%
Synthesis	1	3.4%

Table 5.8: Question types from participants: T-AP 1

See Table 5.12, below, for a comparison between questions asked in the T-AP 1 and T-AP 2 recordings; this also considers how the categories may 'mesh' with those of the Talk Framework in order to further identify the characteristics of, for example, *supercumulative* talk in considering its value within group problem-solving.

An additional area of interest, in exploring the research questions relating to the ways in which these "networked" T-AP and SRI methodologies (Hickman and Monaghan, 2013) might promote productive verbalisation of mathematical problem-solving approaches is the degree to which the SRI exercise prompts further questions and answers *after the event* from participants as to their performance/strategies (and, indeed, whether the use of the *Livescribe* technology prompts such further questions/exploration of approaches within the SRI). Section 5.5 below considers the 'new' material that was provoked by the SRI playback.

The next section presents the breakdown of Talk Framework categories evident in T-AP 2, before again considering the questions asked by participants within the sessions of each other – in this instance, the questions asked within the T-AP were indicative of fundamental misunderstandings on the part of specific individuals that would not be addressed – even with their questions – in the problem-solving exercise itself, but would be of importance within the subsequent SRI.

5.4.4. T-AP 2 Recording: Beads, with *Livescribe* notebook page

As with 5.4.2 above, this section begins with the relevant extracts from the T-AP transcript (T-AP 2), before presenting, in Table 5.9 below, the breakdown, by Talk Framework category, of the individual 'turns'. Material jointly coded by researcher and supervisor, to test for inter-coder reliability, is shaded. The right hand column presents the Talk Framework coding with additional consideration of questions asked according to Bloom (see above and Table 5.12 below). After presenting the results as analysed from the transcribed recording, the Livescribe notebook page is presented, broken down into stages (as Figures 5.1 – 5.3) in order to demonstrate what was written by the participants, and when. This is discussed in 6.8 below when considering the contribution of *Livescribe* to the process.

Time and Turn	**Verbalisation**	**Talk Framework Coding (with questions considered according to Bloom)**
As with TAP-1 (see 5.4.2), this session began with a reading of the Interviewer's Monologue, this time including the instruction "use the *Livescribe* pen provided to write down anything you find useful in helping you to solve the problem".		
1 – 01:16	I'm thinking a *lot* of 3 digit numbers.	**Monologic**
Between 01:16 and 01:38 the group discuss the 25 beads in the problem and the need to put them on the separate 'poles'.		
10 – 01:38	That would take ages.	**Supercumulative**
11 – continued from 01:28	…would move them over from the unit column to the 10 column.	**Cumulative**
12 – 01:41	Ah, no, but that's not going to work, is it?	**Supercumulative (Knowledge)**
13 – 01:43	Go on.	**Cumulative**
14 – 01:43	So, go on what did you say? One bead on the first one…	**Cumulative (Knowledge)**
Between 01:43 and 02:02, continued discussion about beads and the abacus.		
24 – 02:03	So, the most you're going to have is 9.	**Exploratory Transformative**
25 – 02:06	That's fine…so start with that then.	**Cumulative**
26 – 02:08	Yes, there's loads of 10s.	**Cumulative**
27 – 02:09	I don't understand how it makes a three digit number. Do I not just not know how an abacus works or something? I don't know.	**Supercumulative (Comprehension)**
28 – 02:12	What's the smallest 3 digit number you can make with the 25 beads?	**Cumulative (Application)**
29 – 02:16	I don't get how…	**Monologic**
30 – 02:19	109…	**Cumulative**
31 – 02:21	90 something 9, you'd have 9 there…so 9 beads.	**Cumulative**
32 – 02:22	You've got 25, you've got 25 beads here haven't you so you've got to have…erm… that would be for instance 9 once you'd use up 18, then you've got 7 2 but then of course you've got to find a smaller number than that so it's probably 8 actually or 7…	**Cumulative**
33 – 02:43	Would the smallest number not be 799?	**Supercumulative**

		(Analysis)
34 – 02:45	Yes.	**Cumulative**
35 – 02:46	So does that make 25?	**Cumulative (Knowledge)**
36 – 02:48	Yes.	**Cumulative**
37 – 02:49	So we're not adding them together?	**Supercumulative (Comprehension)**
38 – 02:51	That makes 25 beads altogether.	**Cumulative**
39 – 02:52	So, is it that on an abacus it's…	**Exploratory Encoding**
40 – 02:55	You'd like have units, tens…	**Exploratory Encoding**
41 – 02:57	Right, so it's not adding that add that, add that?	**Exploratory Encoding (Comprehension)**
42 – 03:00	So then how do we get to the next…after that?	**Exploratory Encoding (Application)**
43 – 03:03	After that we have…	**Cumulative**
44 – 03:05	We're going up to 8.	**Cumulative**
45 – 03:07	So what do we have?	**Supercumulative (Knowledge)**
Between 03:07 and 03:22, the group suggest various combinations such as "8, 8, 9" or "8, 9, 8".		
52 – 03:22	Why have we started with 9, though?	**Supercumulative (Comprehension)**
There is no response to this question. The group continue considering the combinations between 03:22 and 03:46, realising that 9, 8, 7 makes 24.		
60 – 03:46	Where have we gone wrong?	**Supercumulative (Application)**
61 – 03:47	So if you've got 8, 8 you are moving one so you've got…	**Cumulative**
62 – 03:50	You can have 8, 8, 9 as a combination.	**Supercumulative**
63 – 03:50	979…you can have two 9s and a 7; is that all the combinations you can have?	**Exploratory Encoding (Analysis)**
64 – 03:58	Let's check.	**Cumulative**
65 – 04:00	You should be able to have 9, 9, 7 'cos it's just moving…	**Exploratory Encoding**
66 – 04:02	Yes, we've got 9, 9, 7.	**Cumulative**
Between 04:02 and 04:14, there is some agreement about the largest and the smallest.		
73 – 04:14	Are there any other ones we could have? So you could have 9, 8, 8… You can have that combination.	**Exploratory Encoding (Analysis)**
Between 04:14 and 04:36, further quick contributions of possible contributions including 7, 8, 9 and 8, 9, 8. There is no explanation provided for any of the suggestions. At 04:37, one member of the group suggests that "I think that's all" (cumulative).		

81 – 04:38	So, aren't we…? So…er…I don't understand.	**Supercumulative (Analysis)**
82 – 04:41	How many combinations are there altogether?	**Cumulative (Analysis)**
83 – 04:43	So that's it… Why can't we do any more steps…?	**Supercumulative (Analysis)**
84 – 04:38	You can't get more than one on 10 can you?	**Exploratory Encoding (Comprehension)**
85 – 04:54	If you had 10 there – 10, 7, 8,	**Cumulative**
86 – 04:56	Why can't we do that 7, 8, 9 then?	**Supercumulative (Analysis)**
87 – 04:59	Because it doesn't add up to 25.	**Supercumulative**
88 – 05:00	One of the rules is that you *must* use all 25 beads.	**Cumulative**
89 – 05:04	And instead of the abacus imagine columns, place value.	**Exploratory Transformative**
90 – 05:09	Hundreds, tens, units, yeah.	**Exploratory Transformative**
91 – 05:11	And why can't we start with [like a 6]?	**Supercumulative (Analysis)**
92 – 05:12	We'll be pushed on the [roles/poles?].	**Cumulative**
93 – 05:18	It would make a 4 digit number.	**Supercumulative**
94 – 05:20	Sorry, it's probably what…	**Not Coded**
95 – 05:22	It can't be a 6 because that would leave you with 19 to go…	**Supercumulative**
96 – 05:22	Why can't we…? Because…	**Supercumulative (Analysis)**
97 – 05:28	I think what I said in the beginning is probably throwing you off. What I should have said is that we need a minimum number here, a minimum number there and the remainder here. If you think of it like hundreds, tens and units.	**Supercumulative**
98 – 05:39	That works because then if you've got 10 in the 10s column in actual fact you mean you've got one there which is why you couldn't, you can't, if you were writing a 3 digit number, you couldn't have a number that was like 9, 10, 9.	**Exploratory Encoding**
Between 05:55 and 06:24, the group turn to putting the numbers in order (from 7, 7, 9 onwards). There is no further discussion about strategy.		

108 – 06:24	If I was doing…I would have got it.	**Monologic**
109 – 06:25	Is that it, have we finished?	**Supercumulative (Knowledge)**
110 – 06:26	Hit stop.	**Not Coded**

Table 5.9 below presents the results of the Talk Framework coding conducted on the second T-AP transcript, as presented above. As with Table 5.6, it presents raw data for each of the Talk Framework categories as well as related percentages. This enables a better comparison to be made between the two T-AP activities given their difference in length and, therefore, number of turns (190 turns and 12 minutes and 59 seconds for T-AP 1, as opposed to 110 turns and 6 minutes and 26 seconds for T-AP 2). The following commentary indicates the key findings from this exercise, comparing the results with T-AP 1 above. The difference in length of the two problem-solving activities (the second almost half the length of the first) could be taken as a sign that the group found the second exercise 'easier' or, perhaps, were more familiar with the protocol and so did not expend quite as much effort in verbalising their thoughts. There might also be a case for the use of the *Livescribe* pen in the second activity leading to more care being taken over what was written down or even focusing thoughts more closely on the mathematics (and this is discussed below in 6.4).

Category	**Number of responses**	**Percentage response**
Supercumulative	26	23.6%
Cumulative	61	55.5%
Exploratory		
Exploratory Encoding	11	10%
Exploratory Transformative	3	2.7%
Disputational	0	0
Monologic	3	2.7%
Not coded	6	5.5%

Table 5.9: Talk Framework coding of verbal contributions for T-AP 2

As with T-AP 1, there were no *disputational* responses identified and the majority of verbal contributions were *cumulative* with a slight percentage increase in the amount of *exploratory* talk on this occasion – the possible reasons for this, as further highlighted by the SRI session that followed, are discussed in 6.4 below. One contribution, at 02:09, is coded as *supercumulative* because it poses a potentially valuable question that could have inspired further explanation (see the definition of *supercumulative* in Figure 3.6 above). Indeed, this is one of the first indications that the group has failed to 'carry' one of its number: "*I don't understand how it makes a three digit number. Do I just not know how an abacus works or something?*" This question, could be seen as *monologic* (and, indeed, was initially viewed as too difficult to code), in the sense that it is neither in response to preceding comments or consequently picked up by other members of the group. Nonetheless, it displays useful *exploratory* characteristics that might have been benefit to the group – had they noted it sufficiently. Moreover, the same participant's comment at 03:22 of T-AP 2, "*why have we started with 9, though?*", illustrates a continuing fundamental misunderstanding of the task or, perhaps, of the way in which it had been explained at the outset – a point which is readily agreed by another of the group members in the SRI. This 'misunderstanding' is maintained, for this particular participant, all the way through the task before becoming a key feature of the SRI session (see SR2 in Appendix 4– see, for example, 26:01). Some of the underlying reasons for this misunderstanding/miscommunication will be discussed in the next chapter, with reference to (and implications for) the T-AP used and the way in which the group actually applied it in their work (see 6.5 below). One potential benefit of what might be considered a less certain grasp of the assignment, however, is the increased amount of *exploratory* talk in this recording compared to T-AP 1, which is, in part, a result of the group (arguably unsuccessfully) attempting to clarify the group's shared (or, rather, not shared) understanding of the Beads task.

As with Table 5.7 above, for T-AP 1, Table 5.10 below provides examples of *monologic* and 'not coded' responses from this session. These, again, are of significance in the Discussion below regarding the effectiveness of the T-AP on this occasion in ensuring that the participants made their thinking clear to the whole group.

Monologic	Not coded
01:16: I'm thinking a lot of 3 digit numbers	**01:19**: Yeah.
02:16: I don't get how…	**01:20**: Imagine you have 25 beads.
06:24: If I was doing…I would have got it	

Table 5.10: Examples of monologic and not coded responses used to inform subsequent SRI discussion: T-AP 2

It is perhaps notable that there are fewer *monologic* examples in this second T-AP, although some contributions initially coded as *monologic*, based on the way they sound in the recording (i.e. seemingly not addressed to the rest of the group because they are muttered or close to being 'under the breath' comments), were rethought when coming to these Results. The comment at 03:22 ("*why have we started with 9, though?*") was considered *monologic* until reconsidered against the criteria for *supercumulative* (see Figure 3.6 above). The question, whilst not posed in such a way as to attract attention from the others around the table, "ask[s]… for further explanation that…[might well have led]…to *exploratory* responses" (to quote directly from the *supercumulative* definition). Although an explanation is not forthcoming, it would become clear in the SRI that this was a significant question that had gone unanswered. This provides a further rationale for the interest in the questions asked during these T-AP sessions. Those questions that move outside the "simple recall" (Riegle, 1976, p. 156) of Bloom's knowledge category (see 5.4.3 above and 5.4.5 below) were often the ones that were of interest for following up in the subsequent SRI, which provided the opportunity for them to be answered where perhaps they had not in the original T-AP situation. Section 7.5 below (Further Research) provides further consideration of these *monologic* contributions, and how they might be considered in terms of dialogic moves (Vygotsky, 1962; Barnes and Todd, 1995; Edwards, 2005).

The following Figures (5.1 – 5.3) present the *Livescribe* annotation results, as written by the participants during T-AP 2. 6.8 below considers what this demonstrates about *Livescribe*'s contribution to the subsequent SRI process. As indicated above in 3.6, this work does not analyse the written material but the dialogue prompted by it within the SRI. Nonetheless, as seen below in the discussion, there are observations to be made about the order in which mathematical information was written during the T-AP exercise, and this is commented upon below for each of the figures.

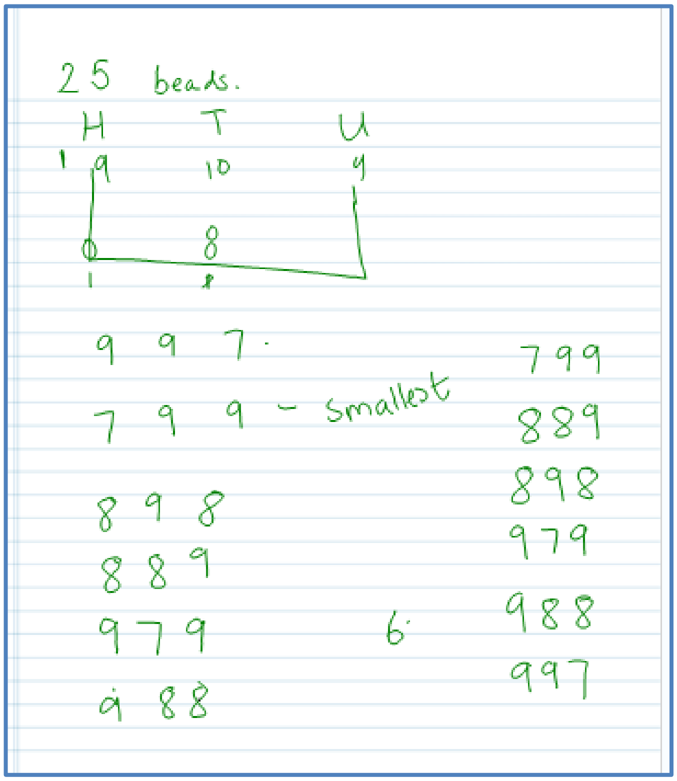

Figure 5.1: Page from *Livescribe* notebook for T-AP 2 (this is as the page appears on the Livescribe software before playing back the recording, and – therefore – as it appears when seen in the physical notebook itself: note that it might well appear that the letters H, T, U for “hundreds, tens and units” were written right at the beginning of working out strategies to solve this problem…on playback of the recording itself, however, this is quickly shown not to be the case – see Figure 5.2 for the earliest annotations made on the *Livescribe* paper; the “greyed out” H, T, U on this screenshot indicates that this reminder of place value was not written until later).

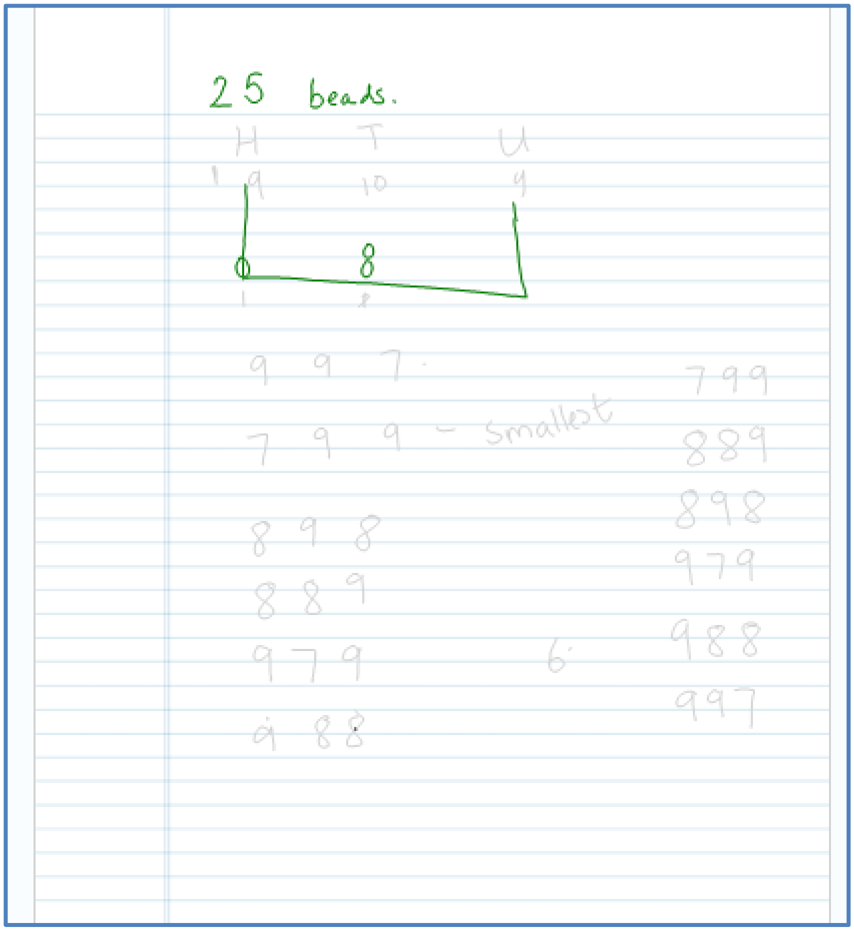

Figure 5.2: An indication of what was actually written first with the *Livescribe* pen during the T-AP recording for Problem 2. This demonstrates that the abacus was drawn by the group's scribe very early in the T-AP exercise. The accompanying audio that replays with the drawing is: "So, you'd put one on the first pole…One on the second pole and the rest on the third pole and then you…would move them over from the unit column to the ten column". The group, however, do not ultimately go much further with the representation of the abacus – although there is no deliberate comment made in the discussion about dispensing with it and instead writing the various combinations of numbers (which can be seen written below). This would inform questions in the later SRI session around the extent to which the scribe, and other members of the group, felt 'satisfied' that there was a good level of understanding across the whole group regarding the nature of the problem. The 'H.T.U.' heading for the abacus was not – as is demonstrated here from the fact it remains shaded – written at the same time as the abacus was drawn – see Figure 5.3 below.

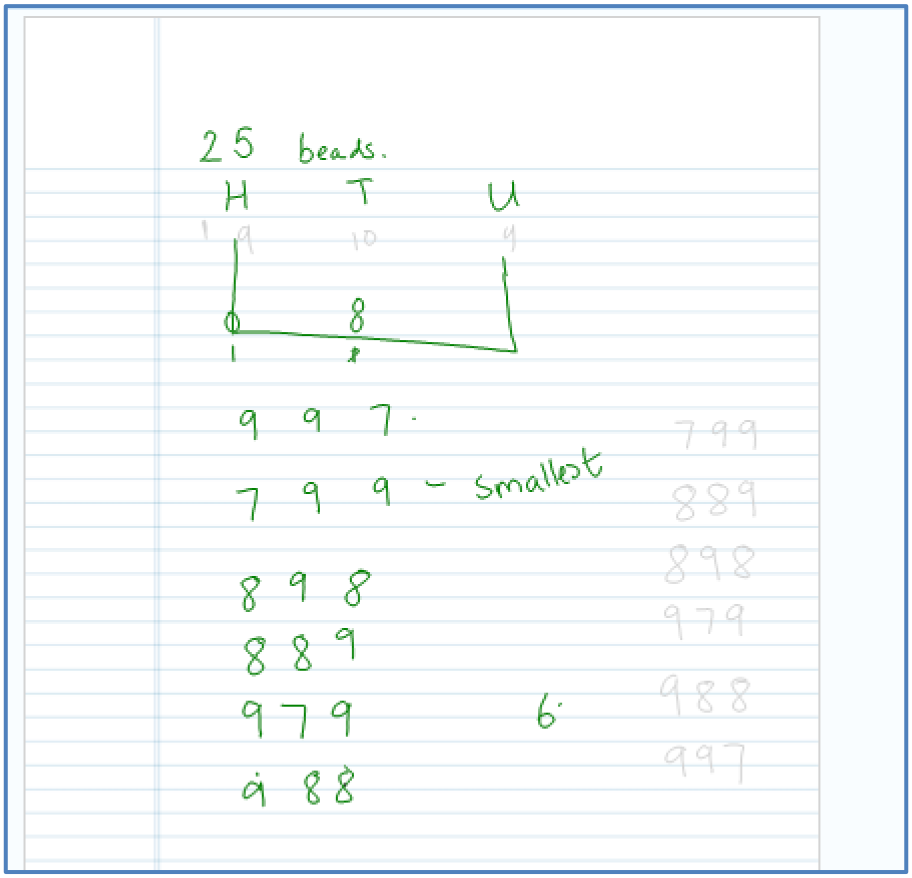

Figure 5.3: "If you think of it like hundreds, tens and units" – coming at 5 minutes and 45 seconds into a 6 minute 36 second recording (see T-AP 2 in 5.4.4, 05:28), it is clear that reframing the question in this way is quite a late decision on the part of particular group members due to their growing awareness that at least one member of the group was not clear about the workings of an abacus and this was hindering her understanding of how to solve the problem. The last things to be written on the page are the possible numbers in order, as asked for in the original question – although not without error, as recognised in SRI 2.

5.4.5. Questions Asked by Participants of Themselves during Problem 2 T-AP Session

As with T-AP 1, the number of questions asked by the group of their peers during the problem-solving task have been analysed. In this instance, a total of 25 questions were asked with, on this occasion, a greater proportion relating to the *strategies* used/to be used to solve the problem and – after consideration against Bloom's categories in a similar fashion to the above – arguably more attempts to analyse the problem. It is noted below, in 5.5, that there was more of a focus on *mathematical strategies* in the second follow-up SRI, when compared to the first, and it has been postulated that this might be seen as an impact of the *Livescribe* pen annotations prompting greater reflection on the mathematics than digital audio alone within the replay session – or, indeed, the impact of the *Livescribe* pen within the think-aloud session itself, as participants were aware that their words and their annotations were being recorded for future replay. The evidence within the T-AP transcript, however, provides an alternative, more straightforward, possibility for this increased attention to strategy with the second problem: the participants were less confident in their working out and understanding of the question than they had been with T-AP 1. The Talk Framework analysis of T-AP 2 perhaps suggests this with an increase in *supercumulative* talk between the two sessions (17.4% in T-AP 1 and 23.6% in T-AP 2; see Tables 5.6 and 5.9 above) and yet no similar, correspondingly large increase in *exploratory* talk of either *encoding* or *transformative* nature. It might also be seen in the number of questions asked within the sessions (25 in the second, shorter, problem-solving exercise as opposed to 29 in the first i.e. almost as many in a recording half the length). Given that one feature of *supercumulative* talk is "asking for further explanation" (see Figure 3.6 above), and given later comments from at least one individual in the SRI about not understanding the problem or the group's proposed strategies for solving it, this suggests that the need for *supercumulative* contributions was greater in T-AP 2, although ultimately they did not produce the further explanation that some required. When considered alongside the length of the second task (it was entirely in the group's control as to when the recording was stopped), this data further indicates that the T-AP did not produce a level of thinking-aloud that enabled participants to 'follow' the thoughts of their peers. 5.5 below provides further detail about the issues with this second exercise as elaborated upon in the second SRI session.

Of the 25 questions asked in T-AP 2, a significant proportion – as might well be expected – came from the individual noted above who expressed uncertainty related to the use of an abacus (7 questions; 29%). Therefore, it is not the case that the whole group had difficulty following the problem's demands – although there is no way of verifying this beyond the comments in the subsequent SRI – but it can be asserted that at least one member of the group was unclear and was willing, in the T-AP session, to put aside concerns about expressing her uncertainty and ask for clarification. This became a key area for discussion in the follow-up SRI (see, for example, the extract from SR2 in Appendix 4 at 26:01). Although the discussion about the misconception (as to the number of beads on an abacus and the way these related to place value) arose 'naturally' amongst the group in the SRI (i.e. it was not specifically raised by the researcher but was apparent to them as worthy of discussion from the playback alone), it was an area highlighted as 'of interest' by the researcher ahead of time due to the number of unanswered questions noted in the group discussion relating to the issue. The discussion about the potentially *monologic* questions related to Table 5.10 above provides additional evidence that the 'direction' of the group's problem-solving could have been altered by answers being provided by peers – had they been listening to each other's contributions, and particularly questions, more clearly in the T-AP session itself. It is possible that a focus on *individuals* thinking-aloud, even in a group situation, could have obscured the role of asking and answering questions within the session and this may well relate to the "psychological situation" (Rotter, 1954) discussed in 6.2 below as well as issues of "generation" and "self-generation" (Mulligan and Lozito, 2014) referred to above in 2.2 (i.e. that individuals pay most attention to what they themselves have said; this becomes a potential advantage to engaging in an SRI, in that the individual who raised the questions about the use of the abacus was likely to attend to her own questions and the lack of answers forthcoming from her peers). Equally, Kahneman's (2011) "system 1 and 2" thinking is perhaps pertinent here – given the emphasis on mathematical problem-solving *and* thinking-aloud in front of peers, the effort of listening carefully to the words of others may have been too much of a demand, at least for some. Such 'issues' with the protocol and group mathematical problem-solving are addressed below in the Discussion (see, for example, 6.2 regarding the effect of working in a group on performance and 6.4 on the effect of the protocol itself as reported by the group members).

As with T-AP 1, the questions posed in the second session have been considered against Bloom's Taxonomy below in Table 5.11. The reasons for doing this are discussed above in 5.4.3. Of the 25 questions asked, the majority, on this occasion, attempted to analyse the demands of the problem that had been set – for example, 04:43: "*Why can't we do any more steps?*" which has been coded against the Talk Framework as *supercumulative* (see Figure 3.6) and consequently invites further explanation from peers – explanation that is then provided (albeit briefly) at 04:38: "*You can't get more than one on 10, can you?*" This attempts to explain how the abacus works – a point of contention for the group member referred to above whose lack of understanding would be used to inform the SRI discussion to follow. It can perhaps be argued, as a result of this analysis via Bloom's categories that, even though the majority of the conversation was *cumulative* (see Table 5.9 above), with only a limited amount of *exploratory* dialogue in evidence, individuals within the group were prompted to ask more considered questions on this second occasion than the first. As a further example, there were more comprehension questions in evidence (6 in T-AP 2, as opposed to 1 in T-AP 1), and this provides support for the assertion that there were more problems with comprehension in T-AP 2 than there had been in T-AP 1. From the perspective of considering the impact of digital audio and think-aloud on the verbalisation of problem-solving strategies (RQ 1), it is, therefore, arguable, that – in this instance – the T-AP had not enabled the successful verbalisation of these strategies, further supporting the idea of following up the problem-solving session with a recall opportunity in which such issues could be addressed and, thus, the notion of "networking" the T-AP and SRI methodologies (Hickman and Monaghan, 2013). This consideration of the questions asked during the T-AP sessions, using Bloom's taxonomy, in addition to the Talk Framework analysis already conducted, is potentially of benefit in providing further substantiation for verbalisations identified as *supercumulative* or *exploratory* (either *encoding* or *transformative*) and has been of great benefit in identifying what individuals attempted to provoke from their peers through the questions asked during the session. This helped inform the focus of the SRI opportunities that followed.

Category	Number of questions	Percentage response
Knowledge	6	24%
Comprehension	5	24%
Application	3	12%
Analysis	10	40%
Synthesis	0	0

Table 5.11: Question types from participants: T-AP 2

With Table 5.11 above providing the breakdown of the 25 questions asked in TAP-2 according to Bloom's taxonomy, and indicating that there was an increase in questions attempting analysis on this second occasion (5 such questions in the first T-AP and 10 in the second), Table 5.12, which follows below, provides a comparison between the questions asked by participants in the T-AP 1 and T-AP 2 recordings. It indicates how the questions were coded according to the Talk Framework, thereby potentially providing a further indication of the characteristics of some of the categories (for example, *supercumulative*, which is discussed further in 6.4 in the next chapter). Such analysis, as will be discussed below in 7.5 (Further Research) enables a consideration of the degree to which participants are willing to question each other when engaged in mathematical problem-solving. A commentary on Table 5.12 follows the table itself immediately below.

Category (Bloom)	T-AP 1 (%)	Talk Framework Categories		T-AP 2 (%)	Talk Framework Categories	
Knowledge	51.7%	SC	4	24%	SC	3
		C	6		C	3
	29 questions	E (alone)	1	25 questions	E (alone)	
		EE			EE	
		ET	-		ET	-
		D	-		D	-
		Monologic	1		-	-
		Not Coded	3		-	-
Comprehension	3.4%	SC	1	24%	SC	3
		C	-		C	
		EE	-		EE	3
		ET	-		ET	-
		D	-		D	-
		Monologic	-		Monologic	
		Not Coded	-		Not Coded	
		Difficult to code	-		Difficult to code	
Application	24.1%	SC	6	12%	SC	1
		C			C	1
		EE	1		EE	1
		ET	-		ET	-
		D	-		D	-
Analysis	17.2%	SC	3	40%	SC	6
		C	1		C	2
		Explor.			-	-
		EE			EE	2
		ET	-		ET	-
		D	-		D	-
		Not Coded			-	-
Synthesis	3.4%	SC	-	0	SC	-
		C	-		C	-
		EE	1		EE	-
		ET	-		ET	-
		D	-		D	-

Table 5.12: Comparison of questions asked of peers by peers in T-AP 1 and T-AP 2 recordings, matched to Talk Framework categories

Bloom’s (1956) categories are provided in the left-hand column, with – working from left to right across each row – the percentage of these questions found in T-AP 1 (as provided in Table 5.9 above) and how these questions relate to the Talk Framework categories (from *supercumulative* through to *not coded*). The first row of the table, therefore, shows that the majority of the knowledge related questions in T-AP 1 were *cumulative* (6 of these questions were coded as *cumulative* against the Talk Framework while 4 were coded as *supercumulative* and 1 was coded as *exploratory* with the remainder as *monologic* or *not coded*). For T-AP 2, the information for which is provided in the fifth column, 24% of questions related to knowledge (a notable reduction from the first problem-solving session), or which 3 of these were independently coded as *supercumulative*. With the exception of one 'difficult to code' *exploratory* statement in T-AP 1, the knowledge related questions were only coded as either *supercumulative* or *cumulative*.

The second row of the column shows that the sole comprehension question in evidence in the first T-AP was *supercumulative*, while there was an equal split of *supercumulative* and *exploratory encoding* comprehension questions in the second exercise. Examples of these comprehension questions that were also coded as *exploratory encoding* include, at 02:57: "*Right, so it's not added that add that, add that?*" While perhaps difficult to 'tease out' the meaning from this, consideration of the remainder of the discussion demonstrates that it refers to the process of using the beads to create three digit numbers on the abacus and demonstrates an understanding of the task, explained in (admittedly quite simplistic) mathematical terms as required by the definition for *exploratory encoding*. The commentator had presumed that the numbers of beads in each position on the abacus had to be added to each other to create the total, rather than using them as the hundreds, tens and units respectively, and this despite the initial working of the problem on the sheet they were given that a three digit number total was required. It is the case, however, that – as with a number of these codings – the limited degree of thinking-aloud, leading to what might be termed 'incomplete' verbalisations (Hickman, 2013) – means that ascribing categories to dialogue is sometimes problematic and perhaps also subjective (see 7.4 below for limitations). Nonetheless, the process of coding the dialogue was, as previously stated, useful for the SRI to follow in that the difficult to code and 'incomplete' verbalisations were prompts for questions that could be asked of the participants, where they did not become aware, themselves, of these issues when listening back to their words – another benefit of the "self-generation effect" (Mulligan and Lozito, 2014).

Overall, then, Table 5.12 above demonstrates that there is no clear correlation between Bloom's taxonomy and the Talk Framework for problem-solving proposed for this work. It does show, however, that there was an increase in what might be termed 'question quality' (Graesser and Person, 1994) in the second exercise and this supports the argument that there was a greater focus on comprehending the second mathematical task. It further provides an argument that questions coded as *supercumulative*, against the Talk Framework, are capable of including elements of application and analysis and that *supercumulative* is arguably a useful mid-point between the categories of *cumulative* and *exploratory* talk proposed by Mercer (1995). Moreover, the table indicates that *exploratory* (*encoding* or *transformative*) questions appear most likely to incorporate aspects of comprehension, application and analysis (only one question coded as *exploratory* has been considered as knowledge-based). These points notwithstanding, the main use of Table 5.12 in this work has been to support the sense, going into the SRI opportunity for the second T-AP, that the group had 'struggled' with the problem set. Considering Duval's (2006, p.104) observation about research on "mathematics and its difficulties…[needing to be]…based on what the students do…by themselves, on their productions, on their voices" (see 2.5.5 above), the analysis in this sub-section demonstrates the degree to which their voices were considered prior to SRI. It also demonstrates the usefulness of considering "sources of incomprehension" (Duval, 2006, p.104) going into a recall situation, although it should be noted that it became clear, when the participants were presented with their own words, that they were quickly drawn to those aspects that they had not fully understood and wished to address these themselves: only a limited amount of prompting from the researcher was required.

It is to the two SRI opportunities that this chapter now turns in 5.5 below, comparing and contrasting the responses from the participants to the two T-AP recordings and also considering the difference in focus between the 'standard' digital audio supported session and the *Livescribe* replay. While considering the prospect that *Livescribe* may have enriched the focus on mathematics in the second occasion, the analysis above of the students' questions provides a potentially useful caveat that they may have always had more to say about the problem that had seemingly caused them the most difficulty. Equally, an inconsistently applied T-AP might well 'frustrate' individuals into asking more questions, both in the original T-AP and the follow-up SRI. Although this was not a deliberate intention when determining not to ask for detailed explanation in the T-AP sessions, it may have acted as a provocation to the participants in the SRI, as they realised that more could and perhaps should have been said in the original group problem-solving events.

5.5. Stimulated Recall Outcomes, and Issues Relating to the Conduct of the SRI Sessions

This section presents the results, and supporting commentary, from the two Task-Based Interview (T-BI) informed Stimulated Recall Interview (SRI) sessions conducted as follow-ups to the initial mathematical problem-solving sessions (T-AP 1 and T-AP 2) detailed above. The first of these SRI sessions was prompted by a 'standard' digital audio recording (i.e. audio only, albeit with accompanying transcripts to clarify 'difficult to hear' comments) and the second was prompted by a *Livescribe* replay of the group's annotations provided in sync with their original verbal contributions (thus potentially providing an additional 'layer' of prompting to the participants). This sub-section begins with an outline of how and when the two SRI sessions were conducted before moving on (in 5.5.1 below) to provide breakdowns of the verbal contributions in line with the methodology as discussed in 3.5 above. These breakdowns identify the degree to which the discussion in these two separate follow-up sessions focused on participation and, thus, the *group situation* (i.e. individual responses, the effect of the group on contributions made, and the perceived problems with the protocol). In addition, they highlight contributions relating to *mathematical methods* (the actual strategies employed within the original problem-solving opportunities and/or reflections on these strategies within the follow-up SRI) and *types of verbalisation* (the Talk Framework categories themselves – for example, reflection on original *cumulative* or *exploratory* contributions made and any additional comments about the perceived value of the Talk Framework for group work of this kind). While it was possible to analyse the transcript results via other protocols, as discussed above in 3.5, it was ultimately decided to break down the participants' responses into these categories in order to better address Research Question 3: "*What does the use of a multi-media artefact in an SRI situation reveal to participants about their own problem-solving strategies, and those of the group?*" and Research Question 6: "*Does multi-media artefact prompted recall promote greater reflection on mathematical methods/problem-solving strategies than 'standard' digital audio?*" As with the analysis of the original T-AP transcripts, therefore, whole responses were considered – categorised here as 'turns' (see 3.5 above) – rather than breaking down individual responses to explore different aspects addressed within the same 'answer' (not all verbal contributions can be classified as 'answers' here; some are motivated by comments by other members of the group, whilst others are motivated – in the second of the replay sessions – by the annotations appearing on the screen as captured by the *Livescribe* pen. Indeed, the SRI replay, in both instances, was designed to provoke responses and commentary without necessarily requiring any questions to be asked at all – the recordings acting as the stimulation.

The SRI sessions allowed participants to listen back to, and comment upon, the original unedited sound recordings of the two mathematical problem-solving opportunities a short while after the original think-aloud sessions. Although, as discussed in 2.5.5 above, researchers engaged in SRI activities are encouraged to "reduce [the risk of] memory decay" (Sime, 2006, p.214) by keeping the gap in time between the original activity and the recall to a minimum, in this case availability issues with the student teachers relating to their School Experience meant that there was a short break of over a week before they returned to the replay of their work. This may have had an impact on their ability to accurately "explain what they were thinking or doing at the time" (Kuzborska, 2011, p.107). Equally, the intention to allow them to "discuss what they had learned from the experience" (DeWitt and Osborne, 2010, p.1369) may perhaps have been complicated in this project by the requirement to consider not only the mathematical problems provided to them in T-APs 1 and 2 but also the experience of thinking-aloud in front of their peers and tutor, which was new to all the participants, at least as far as their experience in the wider PGCE programme. The use of *Livescribe* technology to record and then replay T-AP 2 provided the group with the opportunity, in the second SRI session only, to be prompted by their original 'live' annotations played in time with their comments. It also meant that this session featured comments and questions not just about *Livescribe* as a potentially beneficial technology for capturing such work but also about their notes and, indeed, the way in which what the group discussed and the strategy they were following both was and was not captured in their annotations. It was envisaged that the participants might have some comments to make about the Talk Framework itself, and they had been supplied with this ahead of time, although – in the event – there was minimal discussion about this. With a number of aspects to address – from the initial problem to the type of recording/prompt (most specifically with the *Livescribe*-supported session), it is perhaps unsurprising that the SRI sessions provided a wide-ranging commentary on more than just the original mathematical problems encountered. Section 5.5.1 below, and particularly Table 5.13, reflects this range whilst also considering whether the focus on different aspects differed between the two SRI opportunities.

Both SRI sessions were conducted primarily as real time commentaries on the original recordings, although the playback was occasionally paused to enable specific questions to be asked of specific moments. This allowed participants the opportunity to query their own or others' contributions, or simply to avoid the members of the group having to talk over the sound of their 'previous selves' which may have caused confusion or risked them missing significant exchanges that either they or the researcher wished to talk about within the SRI. The researcher's transcript had been marked up with potential areas of interest noted, such as at 05:29 of the first T-AP recording where the comment "*and addition is the same both ways*" clearly identified that the participant was referring to an aspect of mathematics covered in the taught PGCE mathematics module (the commutative law) and was potentially offering this up to the rest of the group as a reminder and/or a 'way forward' to solving the problem (it was certainly a way of cutting down the time needed to come to a solution – see 05:29 to 05:39 in T-AP 1 above in 5.4.2, and 6.5.1 below for discussion of the SRI's potential contribution to

Robertson's (2001) "analogical problem-solving"). This "*addition is the same both ways*" observation is of particular interest as it is not an especially good example of thinking-aloud in that the student teacher goes no further than providing one pair of numbers as an example (21 and 34), does not explain that this means that the sum of 21 and 34 is equal to the sum of 34 and 21, makes no explicit reference to the commutative law and is, in any case, not really 'followed' by the rest of the group, who continue to struggle with an appropriate strategy to tackle the problem. In this instance, it is perhaps possible to say that the thinking-aloud was not comprehensive enough for the rest of the group to follow the argument and its ramifications for their success at completing the task. It is also possible to say that the group did not hear the contribution, perhaps due to their focus, as individuals, on their *own* thinking-aloud. This could be seen as one of the possible downsides of the T-AP process, perhaps going some way to address RQ 2 ("*what levels of thinking-aloud most effectively support digital-audio recorded T-AP and SRI?*") which relates to the different types of verbalisation discussed by Ericsson and Simon (1993). 6.4 below considers the impact of the T-AP on group engagement and problem-solving efficacy as evidenced in the Talk Framework coded transcripts – the SRI provided opportunities for individuals to comment on this and their observations are further discussed in 6.5.

In addition to potentially 'interesting' comments identified in advance of the session by the researcher, the group themselves were given the opportunity to identify areas of interest of their own within the SRI discussion (i.e. as the recording unfolded in real time, not ahead of time via scrutiny of transcripts). These identified areas of interest were replayed and/or paused to allow for further reflection in line, for example, with DeWitt and Osborne's (2010) observation that participants can be encouraged to reflect upon areas that particularly interest them (see 2.5.5 above). An example of this comes at 15:25 in SRI 1 (see extracts in Appendix 3) when one of the participants – either working from memory or more likely from having scanned the provided transcript in front of her – identifies an upcoming exchange that she wants to highlight for the group ("*and then there's somebody… 'I'm getting confused' and then somebody here says 'well, if I was doing this by myself, this is what…you know I'd be doing what we're* *doing*'"). This, in fact, was a section that had already been identified by the researcher as being 'of interest' and, with the participant, at 15:53, further going on to say that she thinks that there is "*quite a significant bit of exchange there*", the playback was paused, rewound and considered again in greater depth. This allowed for a discussion of collaborative working and becoming 'lost' amongst the group dialogue even when working with a protocol that was intended to ensure that participants made their thought processes and strategies clear to each other. More than this, it allowed for a discussion that was as much prompted by this specific participant's interest as a question posed by the researcher.

Coded extracts from the two SRI sessions, which ran to 35 minutes and 54 seconds for the first recording and 31 minutes and 33 seconds for the second, are provided in Appendix 3 and 4[26]. From this point onwards, the Exploring Addition recall session will be referred to as Stimulated Recall 1 (SR1), with the Beads recall session referred to as Stimulated Recall 2 (SR2). In the first of the transcripts (SRI 1), there is a lengthy section where the verbal contributions from the SRI run directly alongside the recording (i.e. the original recording was left playing and not paused while the participants commented 'live', very like a DVD commentary – this is arguably the section of the SRI that most closely follows Sime's (2006, p.214) assertion that prompts "from the interviewer…[should be] kept to a minimum"). During the second session, the audio recording was paused for all significant responses from the group. Indeed, it is possible that the need to pause and discuss the *Livescribe* aspect of the second recording changed the dynamic of this latter session i.e. there was more to discuss than just the thinking-aloud and it was harder for the participants to process the speech running alongside the

[26] That the two sessions are of roughly equivalent length is as much do with the restraints on the student teachers' availability as any deliberate planning. However, it is notable that both SRI sessions are significantly longer than the T-AP sessions they revolved around. T-AP 1 is 12 minutes, 59 seconds, indicating the amount of time spent with the recording either paused or being replayed; T-AP 2 is a much shorter 6 minutes and 26 seconds – so it is notable that a recording practically half the length of the first generated almost as much SRI conversation (perhaps not surprising when it is considered that the second T-AP was the problem that the group found most problematic, and it was also the sole *Livescribe* recording, meaning that there was arguably more to discuss).

'live' annotations on the screen in front of them. Kahneman's (2011) point about "system 1 and 2 thinking" is relevant here – the need to listen to the audio, answer the questions posed by the researcher, *and* pick up on cues from the *Livescribe* annotations would, as he states, have worked to slow them down.

Neither recall session can be considered a 'full' stimulated recall, in the sense that there are numerous tangents evident in the questions and answers, and the questions themselves diverge somewhat from the Goldin-inspired (1997) Task-Based Interview model that had originally been proposed and is considered in the 3.3 above (this is due, in part, to wanting the group to reflect upon the value of thinking-aloud in a group situation of this kind and also the potential of the two different recording technologies employed – aspects that will be discussed, using their responses from the SRI sessions, in 6.5 below). The focus of the questions in both SRI situations, against the categories of *mathematical strategies*, *group situation* and *verbalisation* is considered in Table 5.15 in 5.5.1 below. This demonstrates that one reason for the greater focus, at times, on the group situation itself was the researcher asking more questions on this subject. The next section first, however, turns to consider the SRI contributions against these categories, comparing the two sessions to identify whether one promoted more conversation than the other in any of these areas, and also considering the balance of participant versus researcher contributions and the degree to which the student teachers asked questions themselves during the SRI sessions – this perhaps being a sign that the SRI process had caused them to re-engage with the mathematics and/or the group's work towards solving the given problems.

5.5.1. Stimulated Recall Contributions – Breakdown of Contributions Relating to *Group Situation, Mathematical Strategies* and *Verbalisation,* Identification of 'New' *Cumulative* and *Exploratory* Contributions and Balance of Participant/Researcher Questions/Comments

This sub-section begins with an overview of the Stimulated Recall (SRI) sessions, considering the sometimes shifting focus of the conservations that were prompted both by the replay of the two separate T-AP recordings detailed in the preceding section and the questions posed by the researcher that attempted to address the areas of 'interest' found in the transcripts. These were moments where, for example, the student teacher participants seemed not to have provided explanations for their proposed strategies – such as the 'commutative law' example given above in 5.5, or moments when the group, or individuals within the group, seemed not to have been 'following' the conversation and where perhaps the Think-Aloud Protocol (T-AP) had not resulted in the clarity of thought that might have been anticipated. Tables 5.13 to 5.16 below present the data from the SRI sessions relating to the different aspects of the problem-solving task discussed (including, where relevant, the Talk Framework categories i.e. when individuals within the recall provided additional explanation not present in the original T-AP recordings). The researcher's questions and the participants' responses are dealt with in more detail in the Discussion to follow (see 6.5); this includes the consideration of why, perhaps, some areas were of more interest than others in the SRI, and also the participants' reflections within the SRI on why they may have not put forward as much as they might otherwise have done within the T-AP sessions. Reflecting on such things within the SRI would perhaps understandably lead to a greater focus on the *group situation* and *verbalisation* areas than *mathematical strategies* – therefore, any observation that, in either SRI, there was a lack of focus on mathematical strategies needs to be balanced against the competing interest, within the recall situation, regarding why and how the group had performed as well or badly as it had, and – indeed – how they had responded to the instruction to think-aloud, including its effect on their problem-solving performance. There was no hierarchy implied in the conversation i.e. the mathematics was not prioritised over discussing the group and their responses to working with each other and the protocol.

Table 5.13 below provides a breakdown of all the turns within the two SRI sessions by *group situation, mathematical strategies* and *verbalisation* (see 3.5 above for the rationale for choosing these categories over and above others). Please note that some turns have been identified as belonging to more than one category (for example, some comments reflect on both the *group situation* and *mathematical strategies*[27]) – therefore, the total number of responses categorised comes to more than the total number of turns counted within each session. Appendix 3 and 4 provide extracts from the annotated/coded transcripts for these sessions.

	Group Situation	**Mathematical Strategies**	**Verbalisation**	**Procedure**	**Cumulative** (incl. Supercumulative)	**Exploratory**
SRI 1 202 turns	112	36	25	45	9 (Cumulative) 2 (Supercumulative)	2 (Encoding) 0 (Transformative)
SRI 2 163 turns	70	82	4	24	19 (Cumulative) 4 (Supercumulative)	12 (Encoding) 0 (Transformative)

Table 5.13: Breakdown of turns in SRI 1 and SRI 2 (by type)

This table presents the turns from the two SRI transcripts so that a comparison can be made between the responses within the two sessions. This allows, as listed under the SRI data sets in 3.6 above, for Research Questions 1, 2, 3, 5 and 6 to be, at least in part, addressed. RQ 1 ("*in what ways can dialogue/thinking-aloud recorded by digital audio support student teachers' verbalisation of problem-solving strategies?*") is primarily addressed via Talk Framework-coded T-AP sessions already discussed above and the transcribed speech in SRI 1 and 2 (i.e. the student teachers' responses to questions and their own observations based on engaging with the activities). The data in Table 5.13 provides an additional indication of how much of an opportunity was taken within the two sessions to address *mathematical strategies*, whether that be by discussing those that were directly verbalised in the original recordings or by proposing new strategies as

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[27] For example, at 19:53 of SRI 1, one of the participants makes the observation that "*There was…there was…a point where there was that growing awareness that there was a different way of doing it and we were trying to voice it in different ways*".

a result of having had the chance to look back at how they tackled the tasks (such 'new strategies' proposed would fit with the *exploratory* categories within the Talk Framework and, therefore, any such contributions would be counted twice on the table – once for *mathematical* strategies, for example, and then again for the relevant form of *exploratory*). Given the nature of the exercise, many of the responses were prompted by questions asked by the researcher – see Tables 5.15 and 5.16 below (which also indicate the degree to which the group situation was perhaps prioritised over the mathematics in the recall). This may have had the effect of either promoting further discussion of the strategies than would have happened without such input or even, perhaps, curtailing it. While participants were invited to comment on areas of interest to them in the recordings, the situation was often largely driven by the questions asked and they may, thus, have deferred to the researcher and/or to more vocal others rather than choosing to raise issues of interest on their own behalf. Anxiety with mathematics or the situation, including the "psychological situation" (Rotter, 1954) could all have contributed to a reticence to contribute (and may, in turn, have impacted upon a willingness to put forward 'new' *exploratory* contributions – although this did not prevent, as will be discussed below, some *exploratory* ideas being put forward). Student teacher mathematics anxiety is addressed in 1.2 and 1.3 above; 6.2 below returns to the question of the impact of the group situation on the individuals taking part in this exercise. RQ 5 considers the participants' willingness to engage with the exercise (as impacted upon by the methodology), and this is returned to in the Discussion below, utilising the comments made about the group situation and methodology from the SRI transcripts.

Notwithstanding these caveats regarding the impact of the situation on the kinds of verbalisation evidenced, Table 5.13 above is potentially of use in demonstrating the way in which the conversation broke down into the different areas during the course of the recall sessions. Comparing the first and second sessions in Table 5.13, for example, there is an indication that *mathematical strategies* (36 turns in the first SRI and 82 turns in the second) were of greater interest in the second session than the first – the possible reasons for this are considered in 6.5 below, with 6.5.1 considering, for example, the degree to which the complementary methodologies of T-AP and SRI may have encouraged (further) “analogical problem-solving” (Robertson, 2001). Another reason for the increased focus might be the use of the multi-media *Livescribe* device in the second SRI. If it did, indeed, promote a stronger consideration of the mathematics, then this goes some way to answering RQ 6 (*Does multi-media artefact prompted recall promote greater reflection on mathematical methods/problem-solving strategies than ‘standard’ digital audio?*) although there may be other reasons for the difference between the two sessions (as discussed in 6.5 below).

Cumulative and *exploratory* responses are also identified in Table 5.13, against the definitions given in the Talk Framework, although the number of responses in these categories is limited, perhaps understandably given that this was not a problem-solving situation (and given that the group already had answers to both problems going into the SRI sessions). Therefore, there was arguably no great need to explore strategies (in either *encoding* or *transformative* senses) and the (in the main) question and answer nature of the sessions, and the way in which the prompts changed from moment to moment (either from the researcher’s questions or from the recordings themselves) arguably acted against a discussion developing in the way that it had with the T-AP in the original problem-solving sessions. Equally, participants may have deferred to the researcher to ‘lead’ with the questioning, or may have waited for others to come forward with insights about the captured audio on the recording, again limiting the opportunity for this to be a proper conversation about the mathematical tasks undertaken.

Even with some individual participants perhaps not wanting to take a ‘lead’ in the commentary, there were examples of *cumulative* talk in both SRI sessions, as participants built on the ideas of others. These are generally moments in the SRI where one participant’s observation or answer prompts an additional piece of information from another member of the group – see Appendix 3 and 4 for extracts from the coded SRI transcripts.

There were two tentatively coded *supercumulative* contributions in evidence in the first SRI session. Given that this category requires the building on previous contributions, "most especially exploratory [contributions]" with "examples of numbers that might fit with the proposed explanation/strategy" or with requests "for further explanation that would lead to [further] explanatory responses" (see Figure 3.6 above for the Talk Framework categories and definitions), it might be expected that such talk would most be in evidence in a genuine problem-solving situation, and not a recall opportunity. Indeed, the participants may feel that there are no further speculative suggestions to give or perhaps have no motivation to provide them, seeing as an answer had already been reached by the group. However, turns 17 (06:23) and 73 (15:53) do seem to build cumulatively on the discussion with "*…so to me being systematic would be one way of knowing that you've found them all*" and "*I think…that's a quite a significant bit of exchange there*". Both observations have been tentatively taken as *supercumulative* in that "reasons are not offered and strategies are not proposed" but they both point the discussion towards possible further explanation (i.e. *why* it is so significant or what "being systematic" means in this context). Unfortunately, the conversation does not develop in that direction due to the question and answer nature of the session and also, arguably, due to the post-hoc determination to look for such *supercumulative* and *exploratory* material in the SRI sessions. Had this been planned more from the outset, greater care could have been taken over identifying such contributions 'live' in the session itself.

In addition to these tentatively coded *supercumulative* statements, two have been identified as fitting the definition of *exploratory encoding* in SRI 1 (again, with the same caveat that the Talk Framework is designed for a problem-solving situation). *Exploratory encoding* (see Figure 3.6) requires the "grasping of the assignment" in a way that either restates the problem in different words (non-mathematically) or uses analogy to explain it for the benefit of the rest of the group. In the SRI situation, *exploratory dialogue* was looked for that did not feature in the original T-AP. Turn 8 (04:59) sees one of the participants explain how she thought about the numbers when first presented with the task – this is also a good example of an individual recalling what they thought in the original session (very much, therefore, a stimulated recall). Turn 12 (05:50) is a speculation on how many different totals could be made, with – again – a ‘thought’ that had not been expressed in the original problem-solving session (“…*and I think I kind of...I thought from that there would be a lot*”).

Overall, the above table illustrates that the discussion in SRI 1 was predominantly on the *group situation* (112 distinct turns related to this across the course of the SRI – over half of the discussion). Thoughts about *procedure* (i.e. how the T-AP session was organised/how the SRI session was run and the methodology itself) were more evident than observations relating to the *mathematical strategies* undertaken (45 distinct turns as opposed to 36.) While the researcher brought up the subject of *verbalisation* and the T-AP itself, the participants themselves had little to say about this. *Cumulative* responses were minimal and *exploratory* contributions were absent (perhaps unsurprisingly, given that the mathematical problem had already been solved in the original T-AP session).

In contrast to this, SRI 2 featured a great deal more of a focus on the strategies undertaken by the group to solve the problem, prompted almost immediately by the appearance of their working out on the interactive whiteboard in front of them (see 6.7 and 6.8 below for discussion related to this ‘prompting via *Livescribe*’). As the group determined that they had missed one of the combinations of numbers necessary to reach the solution, this focus on strategy crossed over with a consideration of how, in a T-AP group problem-solving situation, something of that kind could have been missed. The increased amount of *exploratory* talk in the second session (12 instances as opposed to 2 in SRI 1), may be a result of their realisation that something had been ‘missed’ even with their careful annotations of numbers from smallest to largest. This then led to participants considering how they might have better engaged with the problem (admittedly, one of the *exploratory* questions came from the researcher – 17:17: “*What if you had written 9,9 and 9, even, at the bottom of the three? Again, just sort of making…making the link…No?*” It is possible that this question helped prompt some of the further *exploratory* observations made. Indeed, it is also possible that, for the purposes of this work, there was an element of ‘good fortune’ in the group having made a mistake of that kind – although there is, obviously, not a second *Livescribe* session to compare this to in which a mistake was not made. It seems reasonable to associate the increased *exploratory* talk in SRI 2 on a combination of factors – the *Livescribe* annotations combined with the group's errors).

Overall, then, Table 5.13 provides an indication that there was a greater focus on mathematical strategy in the second, *Livescribe* session, which had also, notably, been the mathematical problem that the student teachers had struggled with most. It suggests that there was more 'new' *exploratory* dialogue, too, in this second session. The focus on *procedure* between the two sessions (comments, for example, about the conduct of the sessions) decreased from 45 to 24, although this need not be just because of the increase in dialogue relating to *mathematical strategies*; it is possible that the first SRI session had 'exhausted' some of the discussion needed about the way in which the sessions were conducted and the effect, therefore, of the methodology on problem-solving performance. Section 6.5 below explores the possible reasons for these differences between the two sessions while considering in depth the responses given to the questions in the SRI and how these relate to the Research Questions given at the outset of this work.

This sub-section now moves to providing a breakdown of the number of questions asked during the sessions (by researcher and by participants themselves) in Tables 5.14 to 5.16 below. The interest in questions asked by participants relates, also, to considering whether 'new' *exploratory* dialogue came to the fore in the SRI sessions, as a result of 'revisiting' the original think-aloud material and, again, this relates to RQ 1 (although the questions are not 'problem-solving strategies' per se, they are potentially new prompts for such strategies to be posited).

	SRI 1	SRI 2
Length	35:54	31:31
Number of turns	202	163
Questions asked by researcher	43	33
Questions asked by participants – of researcher	2	0
Questions asked by participants – of whole group (generally)	4	3

Table 5.14: Overview of questions and responses within SRI 1 and SRI 2

Table 5.14 above shows that the majority of the questions asked in both of the SRI sessions came from the researcher. Many of these related directly to the original problem-solving task being replayed (for example, at 32:44 of SRI 1: "*So, what are you doing there? What's happening there?*") and were designed to enable the student teacher participants to clarify thinking that had perhaps not been expounded as well as it might have been had they been more closely following the T-AP. This kind of question was most especially necessary for sequences within the recording where annotations had been made on paper (or on the *Livescribe* pad) without accompanying commentary (quick jottings, for example, perhaps picking up on verbal contributions from the group; perhaps the work only of the scribe). It was also necessary when the group's discussion did not tally with the written material due to the 'gap' the group members identified between those thinking-aloud and the individual writing (see, for example, 27:21 of SRI 1 in Appendix 3: "*By having a scribe who's got the pen, you are putting a gap, aren't you, between what's in your head and what's going down on the paper*"). Such questions also attempted to avoid 'putting words into their mouths' or allowing the researcher to become the kind of unwitting "co-producer of knowledge" that Hobson and Townsend (2010, p.228) warn against (see also 6.5.2 for discussion on the balance of participant/researcher contributions within the SRI sessions). Some of the questions posed by the researcher related to what the participants might do in classrooms of their own, if working with a group in a similar situation (for example, at 33:02 of SRI 1: "*As* *a strategy…? If you were the teacher, would you want to…?*"). These move beyond the Goldin-inspired (1997) model as indicated above and, indeed, away from the specific mathematical problem and strategies captured within the recording.

As seen above in Table 5.14, there were relatively few questions asked by the participants during the first SRI session, either of the researcher or of each other. Of the six posed, two were entirely related to the SRI procedures (00:47: "*Are you recording this now?*" and 18:29's query about similar voices on the recording/attempting an identification – these are identified as procedural in Tables 5.15 and 5.16 below). These questions were directed at the researcher. Of the remaining four, one has been counted as a question, even though not phrased as such (14:58 – one of the group members points out something on the third page of the transcript that is of particular interest relating to people just "*going along with the group*", and she appears to be 'opening this up' to others to take an interest, too) and two are arguably rhetorical (15:25: "*That's interesting, isn't it?*"; 19:36: "*God, it's like a play, isn't it?*"). 27:48's "*is that a skill to do with maths?"* is part of a wider discussion, building on the point made above about the scribe, about whether the ability to think aloud/talk in front of a group of peers should be seen as being very distinct from mathematical aptitude/ability. While asked in response to a question from the researcher, the questions seems to be equally addressed 'outwards' to the rest of the group and has been counted as such here. None of the questions asked in SRI 1 are about the mathematical problem being undertaken in T-AP 1 or, indeed, the strategies used by individuals/the whole group to solve it.

While the number of questions in the second SRI session by the group members themselves is very similar to the first, given the use of *Livescribe* and the fact that their recall was now stimulated not just by an audio recording but also by their own annotations as made in the original session (appearing alongside the relevant audio), it is of note that the first (at 10:03) was prompted extremely quickly by the animated annotations on the board in front of them. The non-appearance of one number that perhaps ought to have been included in the jottings if the group had been working strategically attracted interest from one of the participants, 'surprised' perhaps that this had been 'missed' not just by herself but also by her peers. ("*Why didn't… We had*

997, 799… Why didn't we have…erm…979 at that point?"). The second of the questions – arguably rhetorical (14:32's "*…and we've left somebody behind, haven't we?*") relates to the group situation itself and the failure to check that all understood what was being asked of them by the question, but it again builds on, and is arguably prompted by, the 'illustration' provided by *Livescribe* and the associated audio recording. The third question (at 24:51), meanwhile, attempts to synthesise the discussion about this 'leaving behind' and suggests a possible way in which individual understanding could be checked when working in a group. It is clear, from the questions asked alone, that there is a closer focus on the mathematical problem and its successful solution by the whole group in the second SRI session. This is discussed below in 6.5.

Table 5.15 presents the breakdown of 'turns' into the categories *group situation*, *mathematical strategies*, *verbalisation* and *procedure*, as discussed above. The purpose of presenting the information in this form is to consider the degree to which the researcher and the participants focused on the different areas during each SRI session and also to see if there are differences between the two sessions, perhaps caused by the *Livescribe* prompting in the second session. This is of particular interest when considering RQ 6: *Does multi-media artefact prompted recall promote greater reflection on mathematical methods/problem-solving strategies than 'standard' digital audio?*

	Group Situation	**Mathematical Strategy/ies**	**Verbalisation**	**Procedure**
Researcher	23	14	6	2
Participants	3	1		2

Table 5.15: Questions asked by researcher and participants in SRI 1 (by type)

Table 5.15 shows that the majority of the questions asked, by both the researcher and the participants in SRI 1, related to what has been called the *group situation* (23 of these questions from the researcher, arguably therefore leading this focus on the group situation as opposed to the mathematics, and just 3 from the participants themselves).

Extracts from the annotated transcript for SRI 1 can be found in Appendix 3. The *group situation* questions asked relate to aspects such as the degree to which the group were working together (09:56: "*So, at this stage, are you genuinely working in consort, do you think?*") or whether, in fact, someone was taking the lead (10:03: "*Are you working as one group? Is there a lead?*"). Although related to the group situation, they allow for responses about the impact of the methodology on individuals' willingness to engage with the protocol and the problem – thereby addressing a number of the Research Questions given above. For example, the discussion about whether there was a 'lead' in the group led to one participant considering the impact of the group on her and her willingness to "*let things unfold*" rather than make a bad suggestion (10:14, SRI 1, see Appendix 3). This relates to Piaget's (1929) observation about "withholding" in front of potentially more knowledgeable peers discussed above in 2.5.2.

The *mathematical strategies* questions occur at either end of the SRI. There are distinct questions in the first few minutes (at 03:55, 04:10, 05:13, 05:57, 06:06 and 06:31), asking whether prior problem-solving experience had been useful when tackling this particular problem (see 6.5.1 below regarding analogical problem-solving). This led to some consideration of 'being systematic' (06:56: "*It could be important to be systematic if you're going to…if you think you might end up with a lot of numbers and if one of the things that you're looking at is knowing that you have found all the combinations*"). Listening to the recording, live, made it clear that the level of thinking-aloud, despite the protocol, had dropped, and this led to a lengthy focus on the group and the understanding of different members of the group as to the demands of the task. The focus on mathematical strategy returns much later in the recording (at 30:41) and this is, perhaps unsurprisingly, led by the 'commentary nature' of this part of the SRI session, with the group listening to their previous selves solving the Exploring Addition problem and being invited to comment. Comparing Tables 5.15 and 5.16 (the non-*Livescribe*-supported and *Livescribe*-supported sessions, respectively) demonstrates a notable difference in the participants' focus on *mathematical strategies*; the focus on the *group situation* versus the problem being tackled drops in SRI 2, although – admittedly – not by much (3 of the 4 questions posed, on this occasion, by the participants relate to the mathematics; the questions from the researcher do address the mathematics more, but – again – not by much). As there was no possibility of the recall being stimulated by their annotations in SRI 1, it is perhaps unsurprising to see a greater degree of engagement with their written words in the second session.

Verbalisation (the Talk Framework itself) was not focused on much in this first session. The participants themselves, despite the invitation at the outset of the session, and some follow up questions during the SRI (6 in total), did not take the opportunity to ask any questions about the Talk Framework or its categories. The SRI 2, session (see Table 5.16 below) features even less of a focus on the talk categories, and this may have been a result of the keen focus on the *Livescribe* replay and also the amount of discussion on the strategy used and the group's failure to 'carry' all of its members. While there may have been an opportunity to talk about the categories and the coding when summing up at the end of the second SRI, the group was, perhaps understandably, more interested in considering how what they had learned from the process might be translated into the classroom context. There is little, then, that can be said about the group's own interest in and identification of their verbalisations during the course of either SRI, although there is some summing up of the possible applications of this kind of "looking back" (Pólya, 1957) at the conclusion of SRI 2 (see 26:01 to 31.33 in Appendix 4) which leads to one participant stating that "*I think maths is probably the least revisited area. Even like when…when you mark the children's work, as in their 'sums', you know they're never going to look at that page again in a lot of cases*" (28:08 in SRI 2).

The next table (5.16) presents the results for the second SRI session.

	Group Situation	**Mathematical Strategy/ies**	**Verbalisation**	**Procedure**
Researcher	16	18	1	6
Participants	1	3		

Table 5.16: Questions asked by researcher and participants in SRI 2 (by type)

The second SRI, as detailed in Table 5.16 above, featured a much greater focus on *mathematical strategies*, right from the outset – almost as soon as the *Livescribe* annotations appeared on the screen in front of the student teacher participants. Indeed, the majority of the conversation relating to the *group situation* revolved around the failure to communicate these strategies effectively to the whole group (a problem compounded by the *Livescribe* annotations being fairly minimal, with some useful information not being captured on the page – see particularly 10:03 in SRI 2 (Appendix 4) when an omission in the annotations is directly questioned by one of the participants ("*Why didn't… We had 997, 799… Why didn't we have…erm…979 at that point?*"). This leads to a discussion about how "*people don't naturally always think systematically"* and how *"sometimes your thoughts just tumble out*" (11:05 in SRI 2, also presented in Appendix 4) that could have explored how better to work systematically in a group situation but, ultimately, returns to the issue of the pen and whether *Livescribe* attaching spoken words to the annotations influences what is written down. This is a somewhat inconclusive discussion that then leads back into the resumption of the replay before it becomes clear that an individual had been 'left…behind', as previously discussed.

Having considered the degree to which the two SRI sessions focused on these different areas, and provided some indication that there was, indeed, some benefit to the multi-media artefact, *Livescribe,* in prompting further reflection on *mathematical strategies*, the next chapter (Discussion) takes the results of the T-AP and associated SRI sessions and considers them against the Research Questions. It begins with the potential impact of group problem-solving on the success of the group in this particular exercise, before considering the broader potential impact of engaging with problem-solving and think-aloud. The SRI sessions, considered here, are further explored in 6.5, and the Discussion then concludes with the contribution of the Talk Framework to the process, both in terms of what has been learned from the coding of the student teachers' talk and how it was reflected upon in the SRI sessions themselves. The question of whether *Livescribe* did impact positively on the group's focus on mathematical strategies concludes the Discussion chapter before the thesis then moves to its conclusion.

6. Discussion

6.1. Introduction

This Discussion provides a consideration of interesting and interrelated phenomena, with caveats. It reflects on the results of the different aspects of this project, considering how the use of the "networked" Think-Aloud Protocol (T-AP) and Stimulated Recall Interview (SRI) methodologies (Hickman and Monaghan, 2013) may have supported student teachers in their verbalisation of mathematical problem-solving strategies (RQ 1) and what this work may contribute towards an understanding of group problem-solving and, specifically, engaging participants in productive dialogue while working on mathematical problems. It discusses the issues around the verbalisation that was encouraged from the participants (RQ 2 – "*what levels of thinking-aloud most effectively support digital-audio recorded T-AP and SRI?*") and, picking up on the work of Ericsson and Simon (1993), considers the degree to which this might have 'interfered' with their mathematical problem-solving (as, indeed, might the group situation itself). This includes the limitations inherent in the protocol used in the T-AP sessions; acknowledging, too, that there was not a variance in the level of thinking-aloud between sessions, therefore meaning it is not possible to compare and contrast different levels and their effect on the student teacher participants. Overall, the chapter takes the results provided in Chapter 5 above and makes a case for the "networking" of the two methodologies (Hickman and Monaghan, 2013) in order to encourage dialogue that might not otherwise have been shared with peers. This includes reflection on strategies that might enable more effective "looking back" (Pólya, 1957) at problem-solving performance that could potentially be used to inform future mathematical work and also perhaps classroom practice for these student teacher participants.

Moreover, the Discussion allows for further analysis of the Talk Framework coding of the T-AP transcripts (see Tables 5.6 and 5.9 above), most particularly in relation to the increase in *exploratory* talk evident in the second of the two SRI problem-solving sessions. This analysis of verbalisations according to the Talk Framework addresses RQ 4's focus on the types of talk that are most evident when engaging with the T-AP-supported exercises. The increase in *exploratory* talk is considered against the possible effect of the group having already successfully completed one T-AP session i.e. familiarity with the situation, the group and the type of problem set was a potential reason for them engaging more with *exploratory* talk in the second session. The introduction of the *Livescribe* pen may also have had an impact on the verbalisation of thoughts, even ahead of its potential impact on the SRI playbacks to follow. Equally, the group's greater difficulty with the second problem, find it 'harder' than the first, may have encouraged them to provide more in the way of explanation and exploration in their verbalisations. In discussing such aspects, RQ 3 is also, in part, addressed: "*what does the use of a multi-media artefact in an SRI situation reveal to participants about their own problem-solving strategies, and those of the group?*" In considering the underlying reasons for participants engaging (or choosing not to engage) with the protocol, the mathematical task, and their peers, comments from the two SRI sessions (see the transcript extracts provided in Appendix 3 and 4) are utilised, both in support of the T-AP and follow-up SRI methodology, and against it. This addresses RQ 5 ("*in what ways do the networked T-AP and SRI methodologies impact upon participants' willingness to engage with a problem-solving task?*")

In addition to the above, this Discussion also explores the use of the multi-media artefact (*Livescribe*) and how this may have prompted additional, potentially valuable verbalisation of thoughts that were absent from the original T-AP sessions (the latter addressing RQ 6). It demonstrates how knowledge regarding group mathematical problem-solving may have been advanced by the use of digital audio and the Talk Framework, with the Talk Framework perhaps of key importance in highlighting the 'valuable' (and also 'incomplete') dialogue that can be brought more to the awareness of participants by engaging in replay opportunities. The chapter references the Research Questions throughout (see 1.4 above) with a summary of this work in relation to the Research Questions following in 7.2 below. Limitations of the work, as outlined in this chapter, and implications for future primary mathematics and teacher training practice are also summarised in the Conclusion to follow.

As discussed above in 1.9, the sections in this chapter are structured in ‘parallel’ with the Results above. This means that the methodologies and their associated results are discussed in the same order in which they were encountered by the participants, from T-AP to SRI, and links are made between the relevant Methodology and Results sections with the sections presented in this chapter. Therefore, the discussion of the group in 6.2 relates to 5.2 above; 6.3 and 5.3, and all succeeding sections, are also interrelated with pertinent connections made clear in the text to follow. The Discussion below begins with an outline (6.1.1) of the whole chapter before then turning to consider the group situation and both its potential impact on the outcomes recorded in the Results.

6.1.1 Outline of the Chapter

This Discussion begins with a focus on group work (section 6.2) and the specific group situation for this project, building on the consideration of the ‘importance’ of group-work and dialogue in learning activities discussed in 1.3 and 2.2 above that are further supported by the assertion of the Primary National Strategy (DfES, 2006, p.65) that a feature of good mathematics sessions for primary-aged children is an atmosphere in which children are “happy to share their ideas [and] support one another”. As previously stated in 1.3, the primary PGCE mathematics module that the project’s participants were undertaking was informed by this imperative and, indeed, this research grew out of a desire to enrich such group work and dialogue beyond the opportunities available in the teacher training classroom. Therefore, section 6.2 below discusses the impact of engaging in this exercise on the participants, making use of the data about the group members themselves (including their observations within the T-AP and SRI tasks) and considering how differences, including perceived differences, between group members may have impacted upon the expectancy value (Rotter, 1954; Eccles, 1987; Eccles and Wigfield, 1995; Elliott et al., 2005), self-efficacy (Bandura, 1982; 1997), problem-solving performance and their engagement or otherwise in *cumulative*/*exploratory* talk versus *monologic* or *disputational* talk.

Despite the ‘popularity’ and perceived benefits of group working in school (and in the University sessions for the participants in this project), such practice may negatively impact upon the quality and quantity of individual verbal contributions and, in worst case scenarios, could even result in incorrect answers going unchallenged (for example, due to issues of anxiety – see also Piaget's (1929) observation about 'withholding' in 2.5.2 above). Psychological perspectives on this are considered, and the role of T-AP and SRI in helping to ameliorate these potential issues is highlighted, including where individuals have stated that the process has been beneficial to them (for example, when asked directly in the SRI sessions or as unsolicited comments reflecting on the process whilst engaging with the T-AP or listening back in the SRI). The later sections on T-AP and SRI (6.4 and 6.5, respectively) then refer back, when appropriate, to these debates around the methodologies' impact on verbalisation, problem-solving performance and the participants' perceptions of the process (most especially in 6.5.3) before the impact of the Talk Framework and the *Livescribe* pen on group performance is detailed in 6.6 and 6.7.

Having considered the impact of collaborative problem-solving on this particular group in 6.2 below, the Discussion then moves to explore the outcomes of the mathematical problem-solving activities themselves in 6.3 before addressing the T-AP and SRI results, respectively, in 6.4 and 6.5. Individual verbal responses are discussed in 6.4, with reference to the Talk Framework, as appropriate, and with justifications provided for some of the coding and the 'importance' of particular contributions to the overall discussion. This includes areas where think-aloud would have benefited collective understanding but was not as comprehensive as the protocol demanded (such areas being of importance to the follow-up SRI opportunity).

Given the preponderance of *cumulative* dialogue in both recordings (see Tables 5.6 and 5.9 above – over 50% in both sessions), 6.4 gives further consideration to the impact of the situation and the T-AP itself on *exploratory* dialogue as identified via the Talk Framework coding. This builds on 6.2's exploration of the group and makes use of comments within the follow-up SRI regarding the reasons some individuals gave for 'holding back' on explaining their thoughts to their peers.

Within the sub-section on SRI, and after reflecting on the results (both as quantified above in 5.5.1 and the participants' responses to the questions asked), 6.5.1 then considers the ways in which "analogical problem-solving" (Robertson, 2001) may have been enabled by the recall process, considering specific thoughts and ideas proposed in the SRI sessions, for example, that had not been put forward in the original T-AP sessions; this addresses RQ 1's focus on the ways in which thinking-aloud supported by digital audio might support student teachers' verbalisation of problem-solving strategies. An argument is made here that the recording of the thinking-aloud and its subsequent replay bring strategies to group members' attention that had not, even with the T-AP, been fully verbalised in the original task. These strategies include not only mathematical strategies, intended as suggestions for the solving of the problem/s in hand, but also verbal strategies, such as *exploratory* speech which, if recognised and promoted in practice, could be of value to both the student teachers and their classes, too. Equally, *supercumulative* speech is of value, even if not 'picked up' by other members of the group; identifying this speech can provide areas for discussion in subsequent follow-up activities.

6.5.2 then explores the balance of participant and researcher contributions in the two SRI sessions. It had been hoped that the SRI might promote further discussion around mathematical problem-solving strategies (see RQs 1, 3 and 6) and that this might come from the participants themselves without being specifically asked to provide further elaboration. The degree to which the researcher 'led' the questioning may have prevented them from raising their own questions, although it is argued that *Livescribe*'s presentation of their errors and omissions on the screen during the playback of T-AP 2 did prompt reflections independently of the researcher's questions.

Finally, the Discussion chapter concludes with a focus on the degree to which the Talk Framework contributed to the process, particularly given the fact that the student teacher participants were 'introduced' to it prior to the SRI and were invited to comment on their verbalisations, where appropriate, in light of the categories. While there was not a great deal of discussion relating to the Framework, and this was most likely a result of the already 'split focus' on mathematical problem-solving and the conduct of the T-AP sessions, some comments were made about the different categories and, indeed, the use of the Framework itself. These are presented, along with the 'complications and issues' arising from the Framework, in 6.6. 6.7 and 6.8 then provide, respectively, a brief overview of the methodological matters arising from the use of *Livescribe* and a comparison of the 'standard' digital audio T-AP and SRI versus the *Livescribe* sessions which addresses RQ 6's focus on whether the multi-media artefact recording prompted greater reflection on mathematical strategies than the 'standard' recording. There does seem to be some indication from the results (see Tables 5.6 and 5.9 above) that *Livescribe* led to a greater level of attention on the mathematics being undertaken by individuals and the group, although it is also possible that the experience of the first T-AP session helped inform how the group tackled the second i.e. they were more familiar with what was being asked of them. Additionally, a different, and perhaps more demanding, mathematical problem resulted in responses of an understandably different kind. This consideration of *Livescribe* further debates whether its use in the T-AP situation (i.e. the participants knowing that they were writing with such a device, and knowing that the replay of its recording would include their written words) inspired greater attention to the mathematics from the first, as perhaps some of the student teachers implied in the follow-up SRI.

This chapter now turns to the issue of group mathematical problem-solving, building on the consideration in 2.2 above of the importance of dialogue in learning situations, as well as the Results given in 5.2 and 5.3, relating to the 'make-up' of this particular group and the students' perceptions of themselves as mathematicians/teachers of mathematics/problem-solvers coming into this project. Reference is also made to 2.6 (School and Initial Teacher Education Issues), as this provides a sense of the context in which these student teachers were operating along with, perhaps, an indication of what was 'expected' of teachers, something that may have impacted upon their willingness to indicate uncertainty or confusion when problem-solving alongside perhaps more knowledgeable peers.

6.2. Group Mathematical Problem-Solving: Potential Impact on Participants' Motivation, Levels of Anxiety and Self-Efficacy of the Group Situation, Including the Constraints of the T-AP

This sub-section considers the potential effect of the group "psychological situation" (Rotter, 1954) on the student teachers' problem-solving performance and, therefore, on the T-AP and SRI results discussed in more detail below. Using material from the T-AP and SRI transcripts (see extracts provided in 5.4.2 and 5.4.4, for the two T-AP exercises, and Appendix 3 and 4 for the two SRI exercises), it explores how the mathematical problem-solving/think-aloud exercise may have impacted upon the student teacher participants' problem-solving and/or verbal performance and what feelings and beliefs they, perhaps, brought into the situation with them that might have either encouraged them to take a full part in the conversation or, conversely, wait for others to lead before speaking themselves. In this respect, it provides some support for Piaget's (1929, p.6) assertion in 2.5.2 above that individuals may withhold in company because they feel that questions "must be known to every one". The sub-section considers how responses to the group situation may have changed across the two sessions, again as reflected upon by the participants themselves in the relevant transcripts. The impact of the T-AP on the group is also a factor here – in the first instance (T-AP 1), the participants would not have been aware of how it would work in actuality, although they had, of course, seen the Information Sheet ahead of time which provided an outline of the research (see Appendix 1). Therefore, the "expectancy value" (Eccles, 1987; Eccles and Wigfield, 1995; Elliott et al., 2005) that they attached to the exercise would have been imbued not only with their feelings about mathematics and working in groups, generally, but also their thoughts about speaking in front of peers and thinking-aloud. This was something that participants claimed not to have been directly asked to do in previous learning situations[28]. Their experience of the PGCE mathematics module workshops may well

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[28] It might perhaps be expected, however, that the majority of participants had 'thought-aloud' in front of their classes when teaching particular topics. This is an area that could have been explored further in the SRIs, but went undeveloped with the focus on the group situation and the

also have informed their expectations of the task, although there was nothing said about these in the T-AP or SRI sessions. Mathematical problems presented to groups in those hour and a half workshops tended to be presented with very tight time limits and, although discussion was invited in ‘table groups’ (following a practice that would have been familiar to the students from their school experience), there was no expectation that all would speak. When responses were deliberately provoked from the ‘whole class’, this tended to be via interactive voting pods (via the interactive whiteboard technology) or through the use of small whiteboards (again, as seen in the primary classroom) on which answers could be written and displayed by all. This latter approach may have unintentionally reinforced a belief in there always being ‘one correct answer’ to mathematical problems, most especially when these questions were often related to ‘Key Instant Recall Facts’ such as times tables. It can be argued, therefore, that this T-AP experience was the first time, within the University context at least, that the student teachers had been required to consistently speak during a mathematics problem-solving task (although the ‘interviewer’s monologue’ made no absolute prescription that all must speak, or – indeed – that there should be explanation provided to ensure that all in the group understood – see below for comments on this).

The second T-AP (T-AP 2) experience would have, at least to some extent, been informed by the student teachers’ engagement in the first. This may be one reason why there was an increase in *exploratory* dialogue in the second session – the participants may have felt more comfortable with the process and each other. They had, in short, had practise at thinking-aloud and had perhaps reflected on the degree to which they had involved themselves on the first occasion, which may have encouraged them to take a greater or lesser part in the second problem-solving exercise[29].

impact of the recall itself taking up most of the time that was not devoted to commenting on the group’s tackling of the mathematical tasks.

[29] They were not, however, informed by the SRI playback of T-AP 1 when they went into the second T-AP exercise. Both of the SRI sessions followed the T-AP sessions, in part to ensure that participants were not influenced by their replay of the first session (which may have led to

The "psychological situation", therefore, impacting upon the participants' expectancy and what Stangor (2014 – see 2.3 above) refers to as the "affect heuristic", would have been informed by their views on mathematics and their perception of their own strengths and weaknesses as both mathematicians and student teachers (in part also informed by Department for Education and Primary National Strategy (PNS) (DfES, 2006) comments about 'good' mathematics teaching – see 2.6 above). In this instance, their awareness of being recorded, their feelings about having to verbalise their thoughts, and – indeed – their potential concerns about engaging in such research in the first place might also have affected motivation, anxiety and feelings of self-efficacy and all, thus, inform the focus of this sub-section. This allows for some reflection upon RQ 1 ("*In what ways can dialogue/thinking-aloud recorded by digital audio support student teachers' verbalisation of problem-solving strategies?*") as well as RQ 5 ("*In what ways do the networked T-AP and SRI methodologies impact upon participants' willingness to engage with a problem-solving task?*"). In addition, while – as noted above – there was not the opportunity in this work to vary the verbalisation level or, indeed, the protocol between tasks (in order to consider the degree the thinking-aloud might 'interfere' with problem-solving performance), aspects of RQ 2 are also addressed here ("*What levels of thinking-aloud most effectively support digital-audio recorded T-AP and SRI?*") as individuals were given the opportunity to comment on this within the follow-up SRI.

It is possible that the group situation – including the presence of the tutor/researcher – and the need to problem-solve towards a clear and collectively understood solution, may have influenced individuals' desire to verbalise, even unconsciously. In other words, individuals may have put forward thoughts and ideas to service the demands of the protocol and the research, as they understood it, and not because of any specific interest in solving the mathematical problem/s provided. Equally, the situation and the protocol may have been responsible for some individuals deciding not to contribute very much to

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discussion about potentially helpful verbalisations). This is detailed above in 1.6 (Research Design) and outlined in Figures 1.1 and 3.1.

the exercises, perhaps from a sense of anxiety relating to uncertainties around their level of knowledge and how it might compare to the others in the group with them.

With any number of these factors potentially reflected in the different categories of responses noted in the Results above (see Tables 5.6 and 5.9), evidence as to the student teacher participants' feelings regarding motivation, levels of anxiety and self-efficacy going into and then taking part in the sessions is mostly to be found in the reflections upon the task found in the SRI transcripts (see Appendix 3 and 4 below for extracts from these). The initial data collected from the student teachers presented in 5.3 above indicated that none were prepared, before undertaking the tasks, to rate themselves as 'strong' in mathematics generally, or – indeed – in 'teaching mathematics' or 'explaining mathematical strategies'. It is possible that this is more a reflection that they were at an early stage in their PGCE programme than any actual experience as student teachers in the classroom. Table 5.2 above indicates that there was a slight 'split' between upper and lower primary student teachers in terms of their 'confidence' (i.e. the lower primary students seemed less 'confident' than their upper primary peers), although it is not possible to make any general statements about the two different groups as there were so few lower primary students represented in the group. Nonetheless, one of these had taken the time to comment on her form that "verbal explanations alone would be tricky". The T-AP transcripts (see 5.4.2 and 5.4.4 for relevant extracts) provide some insights into what might be termed 'uncertainties' (for example, 02:09 in T-AP 2, when one participant comments "*Do I…just not know how an abacus works or something?*"). It seems that some individuals were keener to propose strategies than others, most especially in the first of the problem-solving situations (02:28 in T-AP 1 provides an example: "*Can I just say, I would be inclined to take, say, each of the digits in turn*…"). Overall, though, due to the nature of the exercise, there was little opportunity in the T-AP sessions for individuals to address their feelings about the situation or, indeed, why they had or had not felt comfortable putting forward ideas, the latter being of particular use in addressing RQ 1's focus on the ways in which dialogue/thinking-aloud recorded by digital audio can support the verbalisation of problem-solving strategies. The SRI transcripts, therefore, are used here to consider why certain individuals felt more ‘comfortable’ to put forward their thoughts than others, why *cumulative* talk was more prevalent than *exploratory*, and why – perhaps – there had been issues with communicating ideas in the second T-AP session (see 14:32 in SRI 2 in Appendix 4: “*…and we’ve left somebody behind, haven’t we?*”). The reasons given range from the impact of working in a group to the effect of engaging with the protocol itself (thereby addressing RQ 5: *In what ways do the networked T-AP and SRI methodologies impact upon participants’ willingness to engage with a problem-solving task?*) Notably, there were no comments made about the ‘suitability’ or difficulty of the tasks set. These SRI observations, therefore, reveal insights into how well the T-AP tasks worked to promote discussion, how the group might have been better organised, and how the *Livescribe* pen (discussed in further detail below in 6.7 and 6.8) also impacted upon the situation, even before it was utilised for playback of audio and accompanying annotations.

Early in the first of the SRI sessions (10:03 in SRI 1 – see Appendix 3), a discussion initiated by the researcher about whether there were ‘leaders’ in the group resulted in the following observation from a participant who had briefly taken a lead early in the T-AP 1 problem-solving activity. She explicitly states that she had deliberately taken “a step back” after offering a suggestion, at least in part because she was unsure of the quality of her suggestion and how others in the group might think of her as a result:

> …the way I remember it [laughs]…you’re conscious that you’re working as a group, so…you’re thinking to yourself ‘well, actually, I’ve made that suggestion but…it might not be a good suggestion,’ so you take a step back and let things unfold and then other people appear with their contributions.
>
> Stimulated Recall Transcript 1: Exploring Addition (10:14)

This ‘stepping back’ to see how the problem-solving will ‘unfold’ amongst the other members of the group may have been, in part, a result of feeling a degree of uncertainty without the individual feedback and scaffolding of (her own) understanding that, it is argued, is required for effective pedagogy (Pollard et al., 2014; Walkerdine, 1984; Wood et al., 1976). A class teacher might provide such scaffolding via praise and further prompting in a traditional classroom setting as she monitors the responses of individuals within the class. This monitoring and feedback would also be “visible”

(Hattie, 2009) to the student when interacting with a tutor alongside her peers in a 'regular' PGCE mathematics workshop, although arguably to a lesser degree.

Such attentive 'live' feedback may, therefore, have been notably lacking for the participant above in this group task because there was no feedback coming from the tutor at all, given that they were not participating in the discussion and not 'playing' the expected role of module tutor as experienced in the PGCE module sessions, however distinct this might be from the primary class teacher model. This may have been unsettling in itself, even without the constraints of the T-AP and the unfamiliar context, and it may therefore have contributed to the difficulties of the "psychological situation" (Rotter, 1954), most especially as no-one else in the group was explicitly tasked with providing such feedback. Feedback of a kind may perhaps have been implicit in the *cumulative* responses in the dialogue as the others either did or did not 'build on her contribution', but she would have had to pay close attention to this – not to mention putting aside any feelings of self-consciousness or being judged that may come from having peers comment on her thoughts in such a public scenario. The after-event SRI perhaps allowed this to become apparent, at least in as far as exploring the reasons why the individuals felt uncertain to 'push' their ideas further. This may explain why there was not as much explanation or exploration of ideas as might perhaps be expected with a Think-Aloud protocol (although with the caveat that the student teachers were at no point told explicitly that they had to explain or, indeed, provide substantiation for points made – see the comments above regarding Seal's work (2006) in 3.2).

It is likely that this uncertainty and "waiting to see what everyone else thinks" (a joke made by one of the participants in SRI 1 – see 12:12 in Appendix 3) slowed down the problem-solving process as much as engaging in the thinking-aloud itself might have done, regardless of the level of verbalisation required (Ericsson and Simon, 1993). It also indicates that Piaget's (1929 – see 2.5.2 above) observation about people's reasons for 'withholding' has some parallels with the two problem-solving sessions discussed in this thesis. Such uncertainty or anxiety may have been, in part, responsible for the time taken for the group to come to a decision in both tasks – although it is not possible to comment on whether this was or was not an excessive amount of time, in either case, due to the absence of other problem-solving opportunities, without T-AP requirements, with which to compare them.

Uncertainty and waiting for others to take the lead in group work is not unique to this project, and arguably will be found in group problem-solving tasks of this kind across the primary age and beyond; whenever, in fact, teachers set tasks for groups that are perhaps later ‘fed back’ to the whole class (for example, through a plenary). Being given such an explicit opportunity to consider the reasons for individuals ‘holding back’, however, could arguably be useful for student teachers in reflecting upon how they might create an environment in which, as the Primary National Strategy (DfES, 2006, p.65) recommend, children are “happy to share their ideas [and] support one another”. There might well be some benefit to considering how more attentive ‘live’ feedback can be provided in group situations, if it is this that prevents them from putting forward ideas with any degree of confidence. Is there, for instance, a need to model the listening to and recording of ideas as they are presented, in think-aloud fashion? It could be argued that the SRI goes some way to enabling this – it certainly provides the opportunity to re-engage with material in a fashion that would not be possible without digital audio support – but with the caveat that the “generation” and “self-generation” effects (Slamecka and Graf, 1978; Mulligan and Lozito, 2014 – see 2.2 above), whereby individuals pay most attention to their own words, may be, at one and the same time, valuable for reflection on personal verbal strategies and problematic when it comes to attending to those of others. This is where the notion of using a Talk Framework for individuals to code not only their own but others’ contributions may be useful (see 7.3.1 below for thoughts regarding the use of such a potentially metacognitive approach). This caveat regarding “self-generation” (Mulligan and Lozito, 2014), aside, there is arguably scope for T-APs and follow-up SRIs to be used for the development not only of personal group-work and mathematical problem-solving skills but also student teachers’ teaching practice. This informs the conclusions drawn about this work in Chapter 7 to follow, most particularly in sections 7.3.2 and 7.3.3.

The SRI sessions, therefore, allowed the student teachers to reflect on their experience of working as a group, considering the evidence from the T-AP recordings and their accompanying annotations (the latter in the *Livescribe*-supported session) as to how well they had worked collaboratively – how "happy" they had been to share their ideas, to use the PNS phrase (DfES, 2006). That the result of a T-AP should be a situation where at least one participant was "waiting to see what everyone else thinks" before committing to any further thoughts of her own illustrates that the protocol alone does not oblige the participants to share all their thoughts with the rest of the group. It further identifies a problem related to RQ 2's focus on the levels of thinking-aloud that might best support the T-AP and SRI. It is arguable that the level of thinking-aloud 'set' for the group (noting that this was not modelled to the participants in this project, and that may have had an effect on their imprecision and uncertainty) is secondary, in its impact on the group's collective problem-solving, to other influences that restrain individuals from 'saying too much' in front of their peers. This returns to the issues surrounding motivation, levels of anxiety and self-efficacy that are central to this section. One of these influences may be how they perceive their 'responsibilities' to the group, with or without a protocol or set of governing expectations.

In the second of the SRI sessions (SRI 2 – see Appendix 4), one participant commented about "the nature of the exercise" and how the group working collaboratively, even with think-aloud, did not necessarily lead to an equal level of understanding of either the task or, indeed, whether it had been solved. Within this observation is, arguably, a sense that, without explicit instructions, individuals do not necessarily feel the need to consider the learning/understanding of the group.

> And, also, the nature of the exercise, there wasn't anything…we were supposed to be working collaboratively…there wasn't anything there that explicitly said 'and at the end of the problem every member of the group must understand the process that you got there by'. There was nothing to say, you know, if people can't keep up with the people who got it, that we had to bring them with us.
>
> Stimulated Recall Transcript 2: Beads (19:13)

Gaudry and Spielberger (1971, p.5) state, "it seems probable that the presence of other students will have some effect upon performance". Indeed, the make-up of the group, in this instance (see also 5.2 above regarding the mix of lower and upper age-range students and how they had not worked together as a group prior to this experience), was one of the elements beyond the control of the participants that nonetheless would have impacted upon them. Even though their teaching groups had been centrally determined at PGCE programme level, there would often be a degree of choice as to who they could work with in small group exercises. This particular group situation, combined with both the problem-solving task and the requirement to think aloud/annotate/listen back to their verbal contributions (all alongside their fellow group members, of course) may have impacted upon individuals' "expectancy for success" (Weiner, 1986) and, thus, their self-efficacy (Weiner, 1986; Bandura, 1982; 1997). It may have limited their ability to adequately pay attention to the problem/s set, and this may explain some of the frustration evident in responses such as that provided at 22:47 in SRI 2 (see Appendix 4), where the participant is very sure that there is nothing that could have been done to the group situation, including adherence to a tighter T-AP protocol, that would have been better than simply having time away with a partner to consider the problem. Such 'partner talk' exercises were relatively common across the PGCE programme (most particularly in the mathematics and English workshops) as this was perceived to be a common feature of primary practice at that time. It is possible, therefore, that this individual was expressing a preference for a technique that she had perhaps used herself in the classroom, in the early stages of her training, or had grown comfortable with in her PGCE workshops. Arguably, two T-AP opportunities, one of which featured a mathematical problem that a number of the group found difficult, do not provide enough experience for her to say with certainty that she would not, at some stage, benefit from further practise in thinking-aloud of this kind with a group of her peers. Egi's (2008) observation that the practice of verbalisation is a positive outcome in itself may still stand even for this individual, because the SRI opportunity has enabled her, at the very least, to talk about what she found difficult with the task and the way the T-AP was conducted in a way she may not have been able to do in a 'regular' mathematics workshop on the programme. Her consideration of what she did not like about the task, and how she would have preferred to work, may also go on to inform her future work as a teacher.

Perhaps struggling to concentrate ‘in the moment’ in much the same way as this individual, others chose strategies within the problem-solving tasks such as deliberately scribing to avoid having to say very much themselves (holding the pen to “look useful” – see SRI 1, 12:44 in Appendix 3) or, indeed, simply withdrawing as much as they could from the discussion and allowing ‘more forceful’ others with (they thought) ‘stronger’ mathematical skills to carry the thrust of the problem-solving, hopefully through to its conclusion (see SRI 2, 26:01, below). They do not reflect on the demands of the thinking-aloud to any great degree within their SRI comments, i.e. whether they were also ‘leaving the talking’ to those who seemed more comfortable and confident with speaking in front of the group, perhaps basing this on their experience of their peers in the PGCE workshops themselves. This is arguably an area that could have been followed up on more assiduously within the SRI sessions. As it is, as demonstrated by the discussion that begins at 10:03 in SRI 1 (see Appendix 3), individuals expressed that their lack of awareness of their peers as “mathematical thinkers” (see 11:25 in SRI 1, Appendix 3) led to a degree of caution in their own contributions.

> …the thing is we don’t know each other that well in terms of…what we’re like…[as]…mathematical thinkers… …if we were having a conversation about phonics…cos [name withheld] and I know where we stand in relation to how…well [name withheld] knows about phonics…[it would be different].
>
> Stimulated Recall Transcript 1: Exploring Addition (11:25)

Although incomplete, the thought expressed here indicates that discussion in the PGCE English sessions had led to a greater awareness of the strengths and weaknesses of peers in relation to phonics knowledge; this, it seems, provided a degree of confidence that was lacking in discussions relating to mathematics, again providing an indication that a T-AP/SRI approach has the potential, if used to inform the PGCE workshop sessions themselves, to ameliorate this issue, at least to some degree. This informs the conclusions regarding the implications for mathematics teaching and Initial Teacher Training outlined in 7.3.1 to 7.3.3 below.

As Kahneman (2011, p.23) says, the phrase “‘pay attention’ is apt: you dispose of a limited budget of attention that you can allocate to activities, and if you try to go beyond your budget, you will fail”. Some of the participants seemed to be aware of the danger of "go[ing] beyond their budget", perhaps feeling the demands were too great even before they had tested themselves or had others' (sometimes presumed to be stronger) mathematical suggestions tested by the group. This relates to Weiner's (1986) "expectancy for success" referred to above. The notion that they might also have to bring their peers with them was a further potential (or, indeed, actual) pressure, as indicated by the excerpt from SRI 2 above (19:13) – even if not consciously apparent to them at the time (the excerpt does not indicate either way whether the participant felt this need to ensure others were following whilst engaged in the task or whether this was an SRI-induced after-the-event reflection). Even with the expectations that individual participants may have brought to the T-AP sessions, the SRI opportunity revealed that the reality of the recorded tasks may have affected contributions in ways that had not been foreseen: "*it didn't quite unfold the way I'd perhaps imagined it would…and this is where you start to censor yourself a bit.*"

> My reaction was to just…[inaudible]…just because there were a lot of people…maths isn't my strongest subject and I need processing time and all these voices coming in at different directions and some people were more forceful than others and so I kind of didn't say an awful lot.
>
> Stimulated Recall Transcript 2: Beads (26:01)

Such excerpts from the SRI recordings arguably provide evidence that the "psychological situation" (Rotter, 1954; Schunk et al., 2014) encountered by the volunteers for this project may well have impacted upon their motivation to engage with the task and the associated T-AP/SRI. The belief, for example, that others may have been better "mathematical thinkers" (SRI 1: 11:25) seems to have prevented some from sharing their ideas as freely as they might otherwise have done. The participants knew, from the Information Sheet at the outset of the project (see Appendix 1), that there was an interest in their mathematical talk, and this may have increased levels of anxiety on this front even if all had agreed to take part (informally, some had commented that engaging with the exercise might help them develop their mathematics). All these aspects may well have informed their understanding of their expected outcomes from the experience (and, therefore, their "expectancy-value" (Rotter, 1954)). This may then ultimately have impacted upon self-efficacy (including the self-efficacy of the group as a whole).

Moving on from the issues that may have influenced some individual participants to 'hold back' and the impact of the T-AP methodology, potentially, on their performance, it is also apparent from the SRIs that, while the group successfully completed both mathematical tasks, they did not complete them as a whole. A key observation here, when considering RQ 5 (related to the impact of the methodologies on participants' willingness to take part) and RQ 3 (what the use of the multi-media artefact revealed to them about their own strategies and those of the group) is that the SRI enabled this to be made clearer to the student teacher participants themselves than it might otherwise have been. The results above in 5.5.1, and particularly Table 5.13, also indicate that the SRI, while focused heavily in both cases on what has been termed the *group situation* (112 out of 202 turns in SRI 1, and 70 out of 163 turns in SRI 2) also promoted additional *cumulative* (including *supercumulative*) and *exploratory* contributions than had been evident in the original problem-solving sessions due to the group not necessarily taking account of the need for all members to 'follow' the strategies that were proposed when engaging with the T-AP. Therefore, it can be argued that the SRI may, to some extent, have ameliorated the "psychological situation" (Rotter, 1954) that the participants were acting within. Indeed, the 'issues' that some had with the T-AP potentially enriched the reflection in the SRI (i.e. it is perhaps helpful to have a T-AP where individuals do not fully verbalise all their thoughts as this can prompt discussion in the SRI – this, to a degree, answers RQ 2 regarding the levels of thinking-aloud that "most effectively" support the T-AP and subsequent SRI; there is an argument that reducing the demands of the T-AP is good not only to avoid 'interference' in the thinking but also to ensure there is material that needs further development in the SRI). Section 6.5 addresses this in more detail below; 7.2 returns to the Research Questions to give an overview of what has been learned from the project and 7.5 considers what may yet still need to be addressed in further research.

One final contribution to the "psychological situation" (Rotter, 1954) that may have 'heightened the stakes' for the student teacher participants, was the use of a digital recorder (both 'standard' and *Livescribe* recorders). Either one of these elements, alone or combined, may have caused anxiety at 'getting things wrong' or perhaps even raised the more positive prospect of learning from unfamiliar and possibly even more experienced peers. Equally, either could be responsible for some participants not contributing as much as they might have expected to the session i.e. withholding their thoughts even within a think-aloud situation. They were specifically asked to reflect upon this within the first follow-up SRI:

> Tutor: So, you're deliberately withholding?
> Student: That's…how I remember it…because we're working as a group and 'cos I'm thinking 'ooh, I'm not sure that was such a good idea, after all. [Laughs]
> Tutor: So, does that stop you verbalising? Thinking-aloud? Did you…actually stop yourself from doing that and was the recorder, knowing that there's one now, was than an imposition?
> Student: Yes.
>
> Stimulated Recall Transcript 1: Exploring Addition (11:00-11:25)

Schunk, et al. (2010, p.274) state that models (in this case, other student teachers they may not have encountered before) can be "informative and motivational" in group situations: "observing similar others succeed can raise observers' self-efficacy and motivate them to perform the task", with such self-efficacy therefore "a central process affecting [their] sense of agency" (Schunk, et al., 2014, p.147). While the converse (watching similar others fail) is also – of course – possible, this project's focus on collaborative thinking aloud and equally collaborative exploration of successful and unsuccessful strategies, may have ultimately been of benefit to the student teacher participants. This may have been most particularly the case for those eager to make use of similar approaches in their own practice (including, of course, productive group work sessions with their children). They may have viewed the activity as worthwhile for their own purposes because they were able to listen (and, later, of course, listen back) to the contributions of others (thus not necessarily damaging their own sense of self-efficacy). The ability to listen back to their contributions, and make some (minimal) use of the Talk Framework categories to support this, was an entirely new proposition for them on their PGCE course, and it is possible that this (including the identification of productive exploratory talk (Mercer, 1995)) was seen as an appealing factor when deciding to participate.

Having considered the contribution of the group/T-AP/SRI context (including the possible impact of verbalisation) on the “psychological situation” (Rotter, 1954; Schunk, et al., 2014) and “expectancy value” (Rotter, 1954; Eccles, 1987; Eccles and Wigfield, 1995; Elliott et al., 2005) presented by this project, and before exploring the results from the T-AP session itself, this discussion will now move on to considering the mathematics problems presented to the student teacher participants. This includes debates around problem-solving activities in maths and the “fear of mathematics” (Balacheff, 1990, p.139). This “fear” can include “fear of success”, too, when that success might potentially aggravate or upset less able peers. Additionally, the demands presented by the language of mathematics (“treatment and conversion”, for example, as discussed by Duval, 2006) may also have influenced motivation.

6.3. Problem-Solving Activities and Their Impact upon Motivation and Verbalisation

The rationale for the choice of the problems used for the T-AP and SRI in this project is given above in the Methodology (Section 3.2). As indicated, the two problems were similar to those that the student teacher participants would have encountered both in their training and in their placement schools. This familiarity (particularly when ‘primed’ for the T-AP and SRI by the initial questionnaire – see Appendix 1 – which included an example, taken from the same source, of the kind of problem they would be asked to work on as a group) may have helped to assuage some of the anxiety about taking part in the project. See also 6.2 above regarding the “psychological situation” (Rotter, 1954). While there were no specific comments made in the SRI sessions about the choice of problems (see Appendix 3 and 4 for extracts from the transcripts), either in terms of their appropriateness for this particular student teacher ‘audience’ or their difficulty; those comments that were made suggest that their expectations of ‘difficulty’ coming into the task were somewhat higher than the problems perhaps warranted and that this may have had an impact on the time taken to come to a collective solution; this provides a further sense of the “expectancy value” (Eccles, 1987; Eccles and Wigfield, 1995; Elliott et al., 2005) held by individuals at the outset of the project. This is illustrated in the related extracts below from both activities:

> I think, looking back on it, I think that…er…in my mind I was thinking that there was going to be a much bigger number of numbers that we were going to end up with…which I don't know that matters whether that's true or not…it turned out not to have been true, but it might have been true…
>
> Stimulated Recall Transcript 1: Exploring Addition (04:59)

> 00:52 [Reading question out loud] I will. 3 digits. Imagine you have 25 beads, you have to make a 3 digit number on an abacus. You must use all 25 beads for each number you make. How many different 3 digit numbers can you make? Write them in order.
> 01:16: I'm thinking a lot of 3 digit numbers[30].
>
> Think-Aloud Transcript 2: Beads (00:52-01:16)

What we arguably have here, in these verbalised thoughts (verbalised before any exploration of the problem in the Beads example) is an illustration of the difference between outcome expectancy and "self-percepts of efficacy" (Bandura, 1982, p.122). As Alderman (2004, p.69) states, "the most influential factor is the efficacy expectancy – how effective will *I* be". While the problem-solving activities chosen did not seem to have a negative impact on the group coming to a mutually agreed conclusion, in either case (with no comments being made, for example, about the problems being 'too difficult'), some individuals, by their own admission, found themselves deferring to 'more expert' peers, and this is directly commented upon in the SRI recordings as discussed above in 6.2. One individual, for example, in SRI 1 (12:22, see Appendix 3) stated that "*you don't want to impose something on the group when there might be other people who have better ideas*…" This appears to be more of a general statement, applicable to any group problem-solving situation, than relating specifically to this particular problem. Another issue which may have impacted upon motivation and verbalisation was identified by another of the participants in SRI 1:

> Also…also… If you put forward an idea, like with your systematic one, and you haven't got control of the pen you…you…other people's suggestions…erm…are more likely to…be taken…more likely to affect what's happening on the paper. If you…had somebody who had a really strong opinion about how this was to be

---

[30] This is one of the vocalisations in the session that appears more *monologic* than *dialogic* – a half flippant comment 'to self', perhaps, that does not require a response from anyone else in the group. (And, indeed, it is categorised as *monologic* in Table 5.6 in the Results – although the student was not expressly asked about this in the SRI.)

> done, so say they believed they were the mathematical genius in the group and they had the pen then it could have all happened differently.
>
> Stimulated Recall Transcript 1: Exploring Addition (13:24)

Thus, the issues identified as having an impact on their perception of effectiveness (Alderman, 2004) are related more to concern about others having better strategies and ideas, as discussed in 6.2 above, in addition to having ‘control’ of the pen to write down these ideas, than anything concrete about the type of problem set. Neither do these issues seem to relate to their own familiarity with this kind of situation. The perception that others may have stronger mathematical ability – or, as indicated above, that certain individuals may ‘believe’ themselves to be a “mathematical genius” – which arguably comes from a degree of uncertainty about respective attainment levels in the group – seems to have had more of an impact on thinking aloud/verbalisation than the problems themselves. Indeed, these seem not to have acted as a disincentive to engagement from either the comments made in the T-AP session or the follow-up SRI. This perception that there were ‘brighter’ members of the group appears to have had an impact on the amount of thinking aloud certain individuals were prepared to engage in, even from the first, thereby to some extent undermining the protocol which was designed to ‘encourage’ them to speak up. The real problem remains, as Piaget (1929 – see 2.5.2 above) identified, gaining an understanding of how individuals “frame” problems to themselves, most especially when they are likely to withhold in the presence of, they suspect, more knowledgeable peers. 6.4 below considers this in more detail against the Talk Framework categories, making a case for the degree of cumulative talk and the relatively minimal amount of exploratory talk in evidence in either problem-solving session.

6.4. Thinking-Aloud – Reflecting on the Results and the Efficacy/Effect of the T-AP on Group Engagement and Problem-Solving as Reflected in the Talk Framework Coded Responses

Much of the reflection on the effectiveness of the T-AP used to support verbalisation/discussion within the group problem-solving tasks comes from the post-session SRI comments by participants in conversation with the researcher. This is, therefore, discussed in tandem with the SRI session results further detailed in 6.5 below. The coded T-AP transcripts, however, contain much that is of interest in considering the impact of think-aloud alone, as an approach, on encouraging (or, indeed, impeding) productive and potentially *exploratory* dialogue in group problem-solving situations of this kind. The protocol has, it can be argued, certainly been of some use in "capturing data on domain specific problem-solving" (Hickman and Monaghan, 2013, p.1) that can then inform future discussions about problem-solving strategies (including, of course, the SRIs conducted here). Such discussions may be beneficial both for these student teachers in their own personal development, and potentially for the children they will go on to teach. However, it also seems that the protocol and the exercise as a whole presented some problems for participants and this may be evident in the proportion of *cumulative* versus *exploratory* responses given above in 5.4.2 and 5.4.4 (bearing in mind, however, that there had been no 'priming' of the group in the 'interviewer's monologue' to concentrate on providing reasons and explanation, as Seal (2006) suggests). This section, therefore, discusses the impact of the T-AP on the group, including 'issues' that are addressed in greater detail below in regards to the SRI responses. Sections 6.7 and 6.8 detail areas that had not been verbalised by the group in T-AP 2 and ultimately led to some individuals becoming very confused as to the strategies being employed to solve the problem in hand. Indeed, one directly asked for help regarding the use of an abacus and this went unaddressed – until recognised in the SRI opportunity; this, again, provides an indication of the value of the follow-up session to ameliorate any deficiencies in the original think-aloud exercise.

In considering the possible effect of the T-AP situation on the Talk Framework coded responses, RQ 1's focus on the ways in which dialogue recorded by digital audio might support the verbalisation of problem-solving strategies is addressed. In addition, RQ 2, relating to the levels of thinking-aloud that most effectively support digital-audio recorded T-AP and SRI, is considered. 6.2 above notes that a stricter T-AP might perhaps have led to more detailed verbalisation of thoughts, but might also have provided a further disincentive for some to put their ideas forward, most particularly given comments, as detailed in 6.3, about wishing to defer to others with stronger skills in mathematics. Indeed, it may ultimately have been better, for the SRI process, to employ a fairly 'loose' protocol, whereby individuals may have even failed to verbalise thoughts adequately; this perhaps provoked questioning and clarification in the recall situation.

This point aside, and although it is not possible to compare and contrast different levels of verbalisation here (as the 'interviewer's monologue' remained the same for both T-AP sessions), much can be said, from the T-AP transcripts, about the degree to which the protocol itself worked to ensure the explication of thoughts as the group progressed through the task. Indeed, it can be argued that the protocol did not work as successfully as might have been hoped, and that much remained unspoken or 'incomplete'. Further reflection on this is provided below when considering the SRI results (see 6.5).

As indicated above, the majority of responses for both T-APs 1 and 2 were *cumulative* (56.8% for T-AP 1 and 55.5% for T-AP 2 – see Tables 5.6 and 5.9) and, indeed, there was little difference between the two problem-solving sessions as regards *cumulative* talk. *Supercumulative* responses (17.4% and 23.6%, respectively) also outweighed, in both cases, the *exploratory encoding* and *exploratory transformative* contributions; on this occasion, however, it is apparent that the second session inspired more of this talk. The *exploratory* contributions, in both categories, were also more evident in the second of the two problems tackled by the students (*exploratory encoding* responses increasing from 5.8% in T-AP 1 to 10% in T-AP 2; *exploratory transformative* responses increasing rather less from 2.1% in T-AP 1 to 2.7% in T-AP 2). While the different mathematical problems would undoubtedly have made a contribution to the participants' willingness to engage in *exploratory* discussion (even if, as discussed above in 6.3, there was limited reflection on the problems themselves in the post-session SRI session), it is likely that the increase in *exploratory* questions and comments alongside a decrease in *monologic* statements (5.8% to 2.7%) was a result of greater familiarity with the process. This includes the demands of the T-AP, the peers they were working alongside (some of whom they had not worked with before prior to undertaking T-AP 1; see also 6.2 above) and potentially the kinds of contributions and strategies that had been unconsciously deemed 'successful' in the group's solution to the first problem. It is tempting also to suggest that the use of *Livescribe* for recording in the second instance influenced the increase in *exploratory* contributions (as they were using a pen which they knew would record both spoken and written material, perhaps this increased their focus on explaining themselves – see 6.8 below for a counter argument to this, relating to the drawing of the abacus in T-AP 2). However, as discussed in the SRI (see 5.4.4 above, SR2 in Appendix 4 and 6.5 below), the 'misunderstanding' evident in the words of at least one group member during the second problem-solving session may have also been a factor in the increase in *exploratory* dialogue. This was perhaps helped further by the perception that the T-AP required the group to reach a joint solution to the problem given (a perception not, however, shared by all – see the comment from the Beads SRI (19:13) above in 6.2; the 'interviewer's monologue' – see 3.2 – makes reference to 'solving' the problem, *not* jointly solving it to the satisfaction of all). In other words, even with a misconception from at least one member of the group, the requirement of the T-AP to make thinking explicit, along with the use of the *Livescribe* to annotate, may have pushed participants to be clearer in their elaboration of the problem and the strategies needed to solve it.

The next section moves from considering the Talk Framework coded T-AP transcripts to explore the SRI results, including the ways in which the SRI enabled aspects of the participants' problem-solving performance, and even their issues with the problem-solving and "psychological" situation (Rotter, 1954), to become clearer both to the student teachers themselves and to the researcher.

6.5. Stimulated Recall (SRI) – Reflecting on the Results

One of the immediate areas of interest in transcribing the Think-Aloud Protocol (T-AP) recordings for SRI, as discussed in 6.4 above, was that the level of actual *exploratory* thinking aloud (either *encoding* or *transformative*) was – as with the Seal study (2006) discussed above in 3.2 – relatively minimal in the case of both mathematical problems tackled (see Tables 5.6 and 5.9). It can perhaps be argued that the relatively high level of *cumulative* responses, as opposed to either category of *exploratory* or, indeed, the 'newly' proposed category of *supercumulative*, is an indication that participants did not offer much in the way of 'new' information (as, for example, the definition of *supercumulative* demands) for the consideration of their peers. This may further indicate that thinking about potential strategies to employ or approaches to take was not greatly in evidence during the two sessions. As has previously been considered in 6.2, it was considered that this might have been the result of individuals attempting to keep their own uncertainties from the group whilst they were anxious about the level of knowledge held by others. They were reticent, in other words, to 'reveal' their knowledge, 'guarding' their contributions from too great a scrutiny by offering up less than the T-AP might otherwise promote. The SRI allowed the opportunity for individuals to comment on this, addressing elements of the research questions, such as the impact of the T-AP methodology on their problem-solving performance (RQ 5) and the impact of the SRI on allowing them to provide information had that not previously been articulated. The SRI opportunities also allowed RQ 3 to be addressed (relating to the degree to which the recall revealed their problem-solving strategies to them, and whether the *Livescribe* multi-media artefact was more successful in achieving this, which links RQ 3 with RQ 6). This sub-section begins therefore, with a consideration of the areas that individuals had found 'problematic' in the original problem-solving situation; areas that were then addressed explicitly in the SRI.

It is evident in both of the group mathematical problem-solving tasks undertaken that not all participants were clear as to what was being asked of them by the problem or, indeed, as to the method being employed to come to a solution (see the *monologic* responses for both problems presented in Tables 5.7 and 5.12 above for examples that illustrate levels of confusion such as the issue with understanding the use of an abacus discussed at 15:48 in SR1 2 – see Appendix 4). The SRI sessions (see Appendix 3 and 4 for transcripts and section 5.5 above) therefore afforded both the researcher and the participants themselves the opportunity to explore the reasons for some of their uncertainty and/or unwillingness to either put forward *exploratory* suggestions or perhaps even directly ask peers for help. Indeed, as with the abacus example from T-AP 2 given above, participants were able to comment directly on occasions when – despite the T-AP requiring the explication of thinking and strategies – they had been 'left behind' by their group – in the case of T-AP 2, after a pertinent question had been asked but had gone unanswered (see 03:22 to 04:38 in 5.4.4 above). It is arguable, in fact, that this particular group member did not get an answer to her original question until the discussion at 15:35 in SRI 2 referred to above, thereby indicating that, without the SRI opportunity, this individual's questions would have remained unaddressed with consequent impact either on her opinion of the problem set (as 'confusing' or 'too hard', perhaps) or on her own problem-solving abilities/mathematical aptitude. As it turned out, she remained unclear as to what the problem was asking of her right to the end of the SRI session, even after a peer had suggested alternative strategies the group could have engaged in (such as more clearly explaining the demands of the question and the 'issue' relating to place value that was not verbalised until very late in the recording): "*I can't remember because I can't remember how I did it or understood it because I don't understand it now*" (22:35, SRI 2 – see Appendix 4).

This perhaps provides an example of Mulligan and Lozito's (2014) "self-generation" effect in action (see 2.2 above). The individual did not recall the other group members' contributions towards solving the problem, although she did remember her frustration with the task, and how her questions had gone unanswered. While the SRI, however, provided an opportunity for further explanation not provided within the T-AP (and, indeed, as previously noted, there is more *exploratory* dialogue in the second SRI, albeit not very much more of the *transformative* kind – see Table 5.9), her response about not "understand[ing] it now" indicates that it is not simply a matter of using the SRI as an opportunity for 'missing' explanation to be provided to the satisfaction of those who had been confused the first time round. One of her peers offers (at 04:40 in SRI 2 – see Appendix 4) the thought that "*I think writing '25 beads' and…the thing of the abacus – the hundreds, tens, units – I think that's a really valuable thing to do*", but this is not elaborated upon (and arguably should have been, if taking a more Task-Based Interview (Goldin, 1997) style approach). Whilst there are some *exploratory* thoughts in evidence in the SRI that provide reasons for, for example, the decision not to use the diagram of the abacus any further in the working out, they are not fully elaborated – to the extent that they might be said to be 'incomplete'. An example is at 05:21 in SRI 2 – see

Appendix 4 – when one of the group members explains why no further beads were drawn on the abacus: "*Because you couldn't…you can't draw the beads on…so it's easier to kind of just note*…" She does not explain what precisely might be easier to note down, or indeed why it is not possible to draw the beads on. While this comment has been considered *exploratory* within the SRI, because it shows some of the *encoding* that was missing in the original T-AP, it lacks a clear explanation of the strategy used. In a sense, then, it is unsurprising that the group member who was unsure of what was happening remained so even at the very end of the SRI situation. She might have been encouraged to explain what she *had* understood about the problem – again, relating to Mulligan and Lozito (2014), the act of verbalising her thoughts might have been of benefit to her understanding. Indeed, it could be suggested that her lack of engagement with the T-AP in the original session, through asking questions of others but not thinking through her own possible strategies, is the key reason for not having a clearer understanding of the problem: she did not fully engage in 'thinking-aloud', relying instead on others to do so for her.

This 'issue' notwithstanding, it does seem apparent, however, from the results in Table 5.13 above, that the second of the two SRI sessions provoked more concentrated consideration of the mathematics than the first and that this demonstrates the value of "looking back" (Pólya, 1957) on a mathematical task in this more formalised way. The Talk Framework's contribution to the exercise is that it enabled dialogue that ought to be followed up, via questioning in the SRI, to be identified ahead of time. The 'incomplete' verbalisations (where participants stopped short of providing explanations, as seen above) and the *supercumulative* contributions prompting others to provide explanations that did not, then, perhaps come, were very useful in identifying the areas to be revisited in the SRI, even if – in most cases – individuals 'got there first' by identifying for themselves aspects of their original discussion that had not fully articulated their proposed strategies.

The following sections consider some of the possible reasons for this increase in *exploratory* dialogue in SRI 2, most of which, it is argued, relate to the 'confusion' and frustration evidenced in the comment above about not understanding the problem. The deficiencies in the thinking-aloud from the group, as seen in this section, are given as one of the reasons, in fact, for the greater focus on the mathematics in the recall. Indeed, it is further argued that a stricter T-AP may have been of less value in the follow-up SRI. The contribution of *Livescribe* to the process, in making it clear how the group had conducted their work, is also arguably important in prompting further exploration of the mathematics.

6.5.1. SRI as a Support for "Analogical Problem-Solving" (Ameliorating Failures to Transfer Knowledge from one Situation to Another)

As briefly discussed within the Literature Review and Methodology above, one of the ways in which it was considered this Stimulated Recall Interview (SRI) might benefit the participants was in allowing them to recognise their own successes and failures in transferring knowledge from one context to another; in other words, allowing them to make use (or not) of subject knowledge previously covered within the PGCE mathematics sessions, either within the original problem-solving exercise or, effectively by recognising their omissions, within the follow-up session. This fits with Bransford et al.'s (2000, p.19) view that "metacognitive practices have been shown to increase the degree to which students transfer [knowledge] to new settings and events". Think-Aloud Protocol (T-AP) and SRI exercises may, additionally, have the potential to emphasise to the participants when "analogical problem-solving" is taking place (Robertson, 2001) and, thereby, perhaps ameliorate any failures to transfer knowledge from one situation to another. This "looking back" (Pólya, 1957) and reflecting upon contributions and problem-solving processes may, therefore, be beneficial for future practice. The two SRI sessions considered here provide little in the way of "analogical problem-solving", however, at least in as far as concrete examples of learning from other problem-solving situations. Even the, perhaps most obvious, example of transferring learning from the PGCE mathematics workshops (the references to place value, and 'hundreds, tens and units' in SRI 2) is not fully understood by all participants because it is not fully explained, perhaps because it is assumed that all understand these concepts (this 'issue' is explored in more detail in 6.7 and 6.8 below).

At 06:56 in SRI 1 (Appendix 3), one of the participants' comments, "*it could be important to be systematic if…you think you might end up with a lot of numbers and if one of the things that you're looking at is knowing that you have found all the combinations*…" This, arguably, demonstrates some learning from other, similar situations, or perhaps from the relevant problem-solving PGCE mathematics workshop (or, even, it must be said, from the comments in the T-AP recording about systematic working). The remark itself has been coded as *supercumulative* (see Figure 3.6 above) on the grounds that it builds on the previous "*It could be important*" (which in turn builds on the researcher's observation about the group taking care to check their numbers via writing them down). For the first time in the conversation, the notion that a system is required to find all the combinations is proposed. Therefore, in line with the definition of *supercumulative*, the group member is not restating the problem in different words (this, in fact, does not happen in the SRI), or proposing a strategy herself, but her remark proposes the notion of a strategy being important to the working out, and could – indeed, have invited/prompted others to explain it. This, unfortunately, does not happen in the SRI discussion as the focus is more on the psychological effect, the – as the group member puts it – "*calm*[ing]" effect – of having a strategy and a structure to rely on when "*ideas were coming from all over the place*" (07:44 in SRI 1 – a clear indication of the failure of the T-AP to ensure that individuals listened to each other's contributions).

Ultimately, it is easier to see how the SRI sessions provided the participants with opportunities to ameliorate failures to either verbalise or comprehend particular ideas in the original T-AP sessions than it is to demonstrate any form of "analogical problem-solving" (Robertson, 2001) being undertaken by the participants (not least because they do not have much to say about other, similar, problems). While it might be possible to prompt participants to consider strategies from one situation that might be applicable in another, this might be better achieved via direct questioning. The researcher could ask, for example, what they might have learned from previous problems that would benefit this one, or – indeed – simply what the problem reminds them of. This would, of course, take the SRI further away from the notion of a ‘pure’ Stimulated Recall Interview as discussed in the next section.

6.5.2. Balance of Participant/Researcher Contributions within the SRI Sessions

Calderhead (1981, p.214) identifies some of the potential pitfalls which may result from participants receiving [over-]detailed instructions to prompt their recall and, for the most part, this was considered when conducting the SRI exercises. It is arguable, however, that the balance of researcher/participant commentary was still at odds with the arguably ‘purer’ approach to stimulated recall as espoused by such as Mackey and Gass (2005, p.66) where the intention is for the prompts to be elicited from the participants’ original contributions and/or actions (remembering that stimulated recall is not exclusively for verbal/audio recordings) and where “often simple instructions and a direct model will be enough”. Egi (2008) is ‘purer’ still in wishing to avoid any potential undue influence of the researcher on participants’ responses, even to the extent of abstaining from interaction. As is evident from Table 5.14 above, there was a great deal more direction and direct comment/questioning from the researcher than might be expected had such an approach been taken and it might be argued that the overall balance was – at times – more in favour of the researcher, perhaps at the expense of participants’ own views. While this is a result of the interest in exploring areas such as the Talk Framework categories and the use of the *Livescribe* pen, it remains a possible area for enhancement when considering further research (see 7.5 below).

Despite this imbalance in contributions, however, the discussions that arose between the participants and the researcher proved to be fruitful in directing conversation towards areas that may have otherwise been neglected (and, indeed, were not ‘volunteered’ by the participants themselves upon reading the transcripts/hearing their original words). Such specific questions are not entirely at odds with Goldin’s (1997, p.45) “exploratory metacognitive questions”/Task-Based Interview approach that, in part, informed the approach taken to the SRI sessions. It is also arguable that, although the group had been explicitly asked to focus on clearly conveying their thoughts while being recorded, which would have potentially directed them to the “kinds of thoughts…[they were]…expected to recall" in the subsequent stimulated recall session (Calderhead, 1981, p.214), this did not, in actuality, happen. Very little of their thinking aloud actually related directly to the strategies being employed to reach their collective solution/s. The T-AP opportunity, of course, featured very limited interaction between participants and observer (beyond the 'interviewer's monologue' and the researcher's presence at the table with the recording device/s). This stands in contrast with the SRI situation, in which direct questions were asked of participants relating to their choice of strategies (and, indeed, their understanding of those strategies). This meant that, after the introduction to the task, there were no further opportunities to prompt the group to focus more closely on their choice of strategies during the course of the recording.

Perhaps as a result of this, one significant issue within the T-AP sessions was the failure to concretely communicate certain key elements of the tasks to the satisfaction of all members of the group. A clear example of this is a failure of the group as a whole to attain a shared understanding of the relevance/use of the abacus as related to the beads in T-AP 2 previously discussed in 6.5 above. It was only during the recall opportunity that participants became aware of certain aspects of their verbal (and, aided by *Livescribe*, also written) contributions that had either been omitted altogether or had not been clarified to the satisfaction of other members of the group. In some cases, as will be discussed below, direct questions asked by members of the group, that may well have helped them all reach a solution more quickly and efficiently than was otherwise the case, were effectively ignored (by individuals' own admission within the later SRI session). This was the case even when the question had been asked a number of times and been phrased in such a way as to enable clarification of the central issues 'standing in the way' of understanding. In other cases, the group may have been lacking in strategy without necessarily realising that one was required (an example is provided below, for example, related to the combinations possible from 9, 8 and 8). The issue remains, of course, as to whether a more stringent adherence to the T-AP established within the original 'interviewer's monologue' (or, indeed, prompts/reminders to adhere to this from the researcher) would have prevented these omissions being made in the first place.

These points notwithstanding, and returning to the issue of the balance of participant/researcher contributions within the SRIs themselves, it can be argued that, when such unanswered questions were evident, as is particularly the case in T-AP 2 regarding the function of an abacus, the participants were prompted more strongly to ask questions of themselves in the subsequent SRI. This is clearest at 10:03 in SRI 2 (Appendix 4) with the question "*why didn't we have…979 at that point?*" There are also reflections on 'leaving' members of the group behind (14:32). Therefore, although the actual number of questions is minimal (and the number of questions asked by participants in SRI 2 is lower than in SRI 1), their impact on the discussion that ensued (regarding why the number had been missed; how a member of their group had been left behind) is arguably stronger. This may be the effect of recognising errors and omissions in their work, and this may be more likely the case in the second SRI simply because the group had more difficulty solving the second mathematical problem. The contribution of the *Livescribe* pen in helping to illustrate these errors and omissions is discussed in the next section.

6.6. Problem-Solving [Talk] Framework – the Contribution of the Framework to the SRI Process, Complications and Issues

As discussed in 3.4 above, the original intention of the Talk Framework was to provide a means of analysing the participants' verbal contributions made during the T-AP sessions. This would allow the identification of the different categories of contributions made, thereby addressing RQ 1's focus on the ways in which dialogue/thinking-aloud recorded by digital audio might support the verbalisation of problem-solving strategies (the SRI, of course, is the second 'half' of this equation, if it can be seen to promote further clarification of these strategies). This analysis of the T-AP transcripts might also, it was hoped, provide an indication as to which of the two recording processes – 'standard' digital audio recorder or *Livescribe* pen – promoted which types of verbalisation (as previously discussed in 6.2, it was possible that the use of the *Livescribe* pen, even before SRI replay, encouraged participants to think more carefully about their verbal contributions: the knowledge that the pen was recording their annotations as well as their speech might have made them more deliberate in what was said). In considering this, RQs 4 and 5 would be addressed (the former relating to the most evident types of talk, i.e. the coded transcripts; the latter relating to the impact of the 'networked' (Hickman and Monaghan, 2013) methodologies on participants' willingness to engage with the tasks). The subsequent SRI would then go some way towards address RQ 6's focus on whether the multi-media artefact (*Livescribe*) promoted greater recall of mathematical strategies than 'standard' digital audio. This is considered in more depth in 6.8 below.

It had originally been thought when first trialling the methodology (see Chapter 4) that the Talk Framework's contribution to the process would not extend into the SRI follow-up sessions. The coding of the T-AP sessions would, thus, be the final time that the Talk Framework would be used. In the event, however, and as outlined in the Research Design (1.6 above – see, particularly Figure 1.1), it was decided not only to further address RQ 1 by considering the SRI transcripts against Talk Framework categories (looking, for example, for 'new' *exploratory* material), but also to give the student teachers sight of the Talk Framework at the outset of the SRI sessions, so that they could comment, if they wished, on particular contributions in light of the framework's categories. They were not given the Talk Framework prior to the T-AP recordings as this might have 'led' them towards particular types of verbalisation. As previously discussed in 3.2, the 'ground rules' for the T-AP tasks, as provided in the 'interviewer's monologue', avoided any mention of particular types of speech. This runs counter, perhaps, to the 'ground rules' established by such as Seal (2006), where *exploratory* dialogue was to be highlighted as of importance from the first. The intention in this work was for the participants themselves to 'discover' the dialogue that had been most productive towards achieving a solution to the problems set – or, indeed, where useful contributions had been lacking or unclear. The SRI was intended to both allow such 'discovery' and ameliorate any deficiencies in the participants' adherence to the protocol; therefore, it was considered unnecessary, and perhaps even unhelpful, to 'lead' them by making them aware of the potential value of *exploratory* dialogue or, indeed, any of the other Talk Framework categories.

Having decided to introduce the student teachers to the Talk Framework after the T-AP sessions, and encourage them to talk about this where appropriate in the opening SRI monologue from the researcher, the actual contribution of the Framework to the discussion can perhaps be said to be minimal. Table 5.13 in 5.5.1 above shows that, of 202 distinct turns in the SRI discussion, 25 of these related to what has been termed *verbalisation*, a category that includes distinct questions and answers relating to the Talk Framework categories (but also includes, as at 11:11 in SRI 1, observations about the process of verbalisation/the process of thinking-aloud). In SRI 1 (see Appendix 3), the first mention of the Framework categories comes in a question from the researcher (04:06): "*In a way that's exploratory, isn't it?*" This is informed by the coding that had been conducted between the T-AP and the SRI. Perhaps unsurprisingly, given the closed nature of the question (and the participants' only very recent first introduction to the Framework), there is just "*murmured agreement*" to that observation. It is left to the researcher to continue with, "*you haven't got anyone to agree with at that point; you're suggesting that this is a way of solving it?*" (04:10, SRI 1, Appendix 3). The point here was to confirm that the contribution being discussed from the recording was not *cumulative* but a new proposition, and this informed the conversation that then ensued not about the Framework categories but about what the participant was proposing (as well as attempting to confirm who had said what, which was a perpetual issue with the audio recordings). Therefore, the identification of the *exploratory* speech led to the posing of the question, asking for confirmation that it was indeed *exploratory*, which then led to a discussion about what was being put forward (as it had not been very well clarified in the original recording; another reason for the researcher's question). The student teacher is prompted to provide a further insight into their thinking that was absent in the original T-AP problem-solving session, and therefore provides elements of *exploratory encoding* within the SRI session itself. Therefore, querying the *exploratory* nature of the original contribution provoked further *exploratory* speech; this perhaps providing a clear illustration of the contribution that such a Talk Framework can make to the SRI process.

Similarly, and again in the first SRI, an observation about the lack of *disputational* speech is used to prompt the question "*So, at this stage are you genuinely working in consort?*" which then leads to the group considering whether they have 'leaders'. It is this that leads to the reflection, as discussed above in 6.2 about the "psychological situation" (Rotter, 1954), that the student teachers were, perhaps, 'wary' of the level of mathematics knowledge held by others in the group. Indeed, one comments that, away from mathematics workshops (the example given is phonics), she might perhaps be more willing to have an open conversation about subject knowledge, because she would be more aware of what her peer might know. This, as also discussed in 6.2, highlights one of the key reasons for engaging in this work – captured above in the Statement of the Problem (1.3) – that dialogue in the PGCE mathematics sessions had been under-developed. Further practise opportunities were required for primary students if wishing to address the recommendations of the Primary National Strategy (DfES, 2006) relating to classroom talk and, indeed, the work of such as Mercer (1995; 2008), Selley (1999), and Alexander (2004; 2010a; 2012). Young's (1992) observation about the constraining effect of largely Initiation-Response-Feedback (IRF) teaching can also be perhaps seen in the level of discomfort that some felt putting forward ideas in front of peers. Without the coding from the Talk Framework, in advance of the SRI opportunity, it is possible that some of their observations might not have been made.

The second SRI contained considerably fewer comments about *verbalisation* (including the Talk Framework and its categories) – just 4, in fact (see Table 5.13 above) and these were again led by a comment from the researcher rather than an observation from any of the student teacher participants. In this instance, it is harder to see how the Talk Framework may have contributed to the overall discussion or prompted further exploration of the task beyond adding to the context that the students' words were important to the recall exercise.

There are a number of reasons why the comments about the Talk Framework and its categories may have been minimal within both SRIs, even given an introduction that encouraged reflection on the verbalisation and, indeed, the provision of copies of the Talk Framework alongside the T-AP transcripts. The main complication here appears to be the ‘split-focus’ of the SRI sessions, something that perhaps connects back to previous discussions about the demands of “effortful activities” (Kahneman, 2011, p.83) that is arguably as appropriate to the SRI situation as to the T-AP problem-solving activities. The participants here had a number of competing demands on their attention, from the knowledge that they were, again, being recorded, to the very fact that their original words from the T-AP sessions were being replayed; a novel experience, perhaps, for them that may have acted to heighten the pre-existing anxieties about engaging in mathematics discussed above in 6.2. Therefore, while the SRI allowed the opportunity for ‘live’ commentary on the original problem-solving sessions, and while reflection was also encouraged on the group situation itself, discussing the Talk Framework categories to any meaningful degree alongside this may have been a demand too far. This may have been most particularly because the Framework was further ‘new’ material for them to assimilate prior to engaging with the SRI. There may also be a justifiable criticism that, even without any discussion of the ‘value’ of *exploratory* talk, raising it at this point might have ‘led’ the student teacher participants to see it as somehow important in their SRI discussion, and this could have skewed their talk towards more *exploratory* dialogue than might otherwise have been the case.

One of the key considerations about the use of the Talk Framework in the SRI sessions, therefore, is whether the questions about it ‘pushed’ the participants away from expounding further upon their dialogue in the original T-AP sessions. Instead of the SRI commentary being focused on the mathematics, the Framework/verbalisation questions instead ask for commentary on either the quality of their verbal contributions (which went largely undeveloped in both SRIs) or the group situation itself (which, as Table 5.13 demonstrates, accounted for over half of the distinct turns in SRI 1 – 112 out of 202 in total). Coming to a firm conclusion that the focus on the Talk Framework perhaps denied the opportunity to fully comment on the original thinking behind the mathematics is complicated, though, by the fact that the second problem and related SRI, which led to much greater discussion about the mathematics (see Table 5.13) was also the more ‘difficult’ (for the group). It was also the problem supported by the *Livescribe* pen, which arguably had its own very distinct effects on the commentary (as discussed below in 6.7 and 6.8). What can, perhaps, be said – for SRI 1, at least – is that the coding according to the Talk Framework allowed for some underdeveloped, arguably nascent, thoughts within the T-AP to be highlighted for later SRI discussion. Additionally, the use of the *exploratory* label was maybe useful for the participants in being able to identify for themselves where they had perhaps fallen short in providing explanation for their peers.

The Conclusion below considers how the Talk Framework might be utilised in future iterations of this work (7.3) as well as the issues discussed here around the introduction of the Framework to participants prior to the SRI. It returns to some of the earlier ideas for this work surrounding the possible encouragement of student teachers to code their speech for themselves, and considers how this might inform work in the classroom (see 7.3.2). This chapter now moves to consider another, perhaps more influential, contribution to the recall sessions, addressing – in turn – RQ 6's interest in whether the use of the multi-media *Livescribe* artefact could encourage greater reflection on mathematical methods and problem-solving strategies than 'standard' digital audio.

6.7. *Livescribe*: Methodological Matters Arising From the Use of a Multi-Media Artefact for "Networking" T-AP and SRI Methodologies

This short section builds on the observation made above in the Results (5.5.1) that there was more of a focus on problem-solving/mathematical strategies in the second SRI session than the first (82 of 163 distinct turns in SRI 2, as opposed to 36 of 202 turns in SRI 1 – see Table 5.13 above). It begins by considering why this might have been – considering the participants' greater familiarity with the procedure after completing the first SRI, as well as the reduced need to reflect, perhaps, on the 'novelty' of the situation. The contribution of the mathematical problem itself is considered – by their own admission, members of the group had found it more difficult than the first problem set. Equally, the use of the *Livescribe* pen and associated software to 'play back' their written material in sync with their verbal contributions may have promoted this increased focus on problem-solving and mathematics by presenting them with their strategies in a clearer fashion than is possible via think-aloud alone.

Section 6.6 above argues that the emphasis, at least to begin with, on the Talk Framework in the first recall session may have been responsible for more being said about the group situation than the problem-solving. Equally, it has also been postulated that participants' greater 'ease' with the SRI process, on the second occasion, may have had an impact on their willingness to discuss the underlying mathematics. They were perhaps less anxious, and more willing, as others 'opened up' too around them, to share their thoughts about their engagement with the mathematics; they may also have 'exhausted' some of the discussion around the way the group functioned that had been such a focus in the first session. In addition to this, it became clear, from the SRI discussion, that the second problem-solving task was felt to be more difficult/challenging than the first (perhaps to be expected when the second was taken from "Mathematical Challenges for Able Pupils in Key Stage 1 and 2 (DfEE, 2000) as discussed in 3.2 above). There was, it seems, considerably less certainty amongst the individuals in the group as to how this second task ought to be solved. Consequently, it is perhaps unsurprising that there was more to say about their understanding of the problem when 'revisiting' their discussion in the subsequent SRI, although there may still have been a reticence to speak up from some group members, given their comments in SRI 1 about potentially 'brighter' others perhaps judging them for their lack of knowledge (see 13:24, as discussed above in 6.2). Additionally, the fact that the second SRI was supported by the multi-media artefact, *Livescribe*, may well have made a difference as now, not only were the participants informed by the recording (with prompts provided by the researcher for additional insights into their original thinking), but they were also informed by their own annotations from the time. It seems clear that these annotations made them aware of aspects that had been 'missed' in their discussion and 'working out', and – indeed – few direct questions were required. As soon as the relevant material appeared on the whiteboard, the participants themselves offered up their thoughts as to what they had intended and, ultimately, what, in their opinion, had gone 'wrong' with their working.

At 02:45 in the second SRI (see Appendix 4), the *Livescribe* replay of the material presented the first of the annotations ("*So the very first thing written is 25 beads*") and allowed for the researcher to ask "*what's happening here?*", much as might happen in a Task-Based Interview (T-BI) (Goldin, 1997, p.45) when ameliorating the lack of spontaneous responses by employing "heuristic suggestions" (see 2.5.3 above). While this question leads into a short discussion about the Talk Framework categories, the focus quickly becomes on why at least one member of the group had determined to draw the abacus on the *Livescribe* pad. Also of interest was how and why that decision – to represent the problem in diagrammatic form – had been quickly put to one side by the group in favour of recording the numbers that could be made with 25 beads on an abacus (the whole *Livescribe* notebook image is reproduced above in Figure 5.1). Given that there was no thinking-aloud connected with either drawing the abacus or, indeed, writing down the possible combinations of numbers instead, this prompts an observation from the researcher, "*...the reason I stop it here is there are three beads on the abacus as drawn and things sort of stop there. No more beads are* ever *drawn on the abacus*" (04:51, SRI 2). The *Livescribe* replay arguably 'anchors' the discussion around the strategies employed at particular moments in the problem-solving. It also encourages a focus on how the decision had been reached, by at least some members of the group, that the abacus diagram was not a productive way to continue working out the possibilities. It is here that further aspects of the Goldin (1997) T-BI methodology are employed. Indeed, it could be argued that *Livescribe* supports this methodology, and its "non-directive question[ing] (e.g. Can you tell me more about that?)" (Goldin, 1997, p.45), more than the approach used in the first SRI, which was to ask questions of the participants where 'areas of interest' had been noted in the transcript; also allowing individuals to raise further 'areas of interest' of their own. As previously stated in 5.5, this, at times, arguably led by the researcher 'away' from discussing the mathematics, due to interest in the Talk Framework and the Group Situation and how these impacted on the mathematics. Without the 'live' prompts from the *Livescribe* replay, alongside the audio recording, the first SRI is arguably less focused – however, it is not possible to state with any certainty that a non-*Livescribe* SRI without such disparate questioning would not have resulted in more discussion of mathematical strategies overall. A fairer test would involve similar questions, with an equal weighting of questions to areas of interest (i.e. the group situation, mathematical strategies, etc.) in order to determine more clearly the contribution made by the *Livescribe* pen.

Overall, though, there is an argument that the use of a multi-media artefact to ‘network’ (Hickman and Monaghan, 2013) the two methodologies might, with minimal ‘interference’ from the researcher in directing the conversation, prompt reflection, after the event, on aspects that had not originally been ‘drawn’ out of the group even when working with a T-AP. As Hickman and Monaghan (2013) assert, ‘after the fact’ clarifications of thinking may, from a T-AP perspective, be considered ‘suspect’ but, given that much can still remain unspoken, a ‘hybrid form’ of T-AP and SRI (the “networking” referred to above) can avoid the need to ask questions or demand explanations, in the Think-Aloud process, that might ‘interfere’ with the problem-solving. This interference, Robertson (2001, p.13) states, can “slow…down or affect…the sequence of problem solving steps” and this was one of the reasons for deciding not to ‘demand’ explanations during the T-AP recorded problem-solving sessions (see, for example, the observation regarding Seal’s (2006) ‘ground rules’ in 3.2 above). Indeed, it was also the reason for engaging in a follow-up SRI, which – it was also hoped – would enable these primary teacher training students to identify, for themselves, productive verbal contributions and verbal strategies that they could, in future, use in their own practice (either modelling to children or when prompting effective group working in, for example, ‘guided’ mathematics tasks). *Livescribe*, as is discussed below in more detail, perhaps provided clearer ‘evidence’ of the unfinished and/or incomplete verbalisations and reasons from the original exercise. In the case of the ‘beads’ task (SRI 2), there had been no clear verbalisation of the reasons for drawing out an abacus or abandoning the diagrammatic representation in favour of recording the possibilities in numerical form only. Indeed, the only comment that might conceivably be given as a ‘reason’ for not continuing with the abacus representation comes at 01:38 in T-AP 1 (see 5.2.2): “*that would take ages*”, and even here it is not clear whether this refers to practically putting the beads onto a real abacus or drawing it out on the page.

Presented with this ‘evidence’ on the screen in front of them, and without the need for the researcher to ask very much more than “*what’s happening here?*” (03:09) and “*what* *causes that?*” (04:51), individuals came to propose reasons for their actions that had not previously been aired and, indeed, began to see why others within the group had been unclear as to the strategies that would support the solving of the problem. *Livescribe* arguably provided a means to ameliorate the negative effects of Egi’s (2008, p.213) “reactivity” and “veridicality”. The former relates to the positive or negative effect of the verbalisation itself on performance – as previously discussed with relation to Ericsson and Simon (1993) and the different types of verbalisation. The latter relates to the ‘completeness’ of responses (which may be incomplete due to the participant wishing to present what the researcher might wish to hear – or, indeed, also other members of the group, considering their comments about how others might perceive their mathematical knowledge and understanding). The degree to which participants ‘re-engaged’ with the task in the second SRI directly addressed at least some of the issues with thinking-aloud (enabling individuals to say things that they had ‘held back’ from the group originally); they were also able to ‘complete’ some thoughts and ideas that had been, for whatever reason, inconsistently verbalised during the T-AP session.

The next section of this Discussion looks at this in more detail: how the *Livescribe* pen enabled clarification of strategies employed and misconceptions evidenced. This, it is argued, presents a further case for the use of this technology in work of this kind, addressing RQ 6’s enquiry into whether the multi-media artefact prompted recall promoted “greater reflection on mathematical methods/problem-solving strategies than ‘standard’ digital audio”.

6.8. Standard Digital Audio Recorders versus *Livescribe* Pens; *Livescribe* Desktop Clarification of Strategies Employed and Misconceptions Evidenced

This sub-section considers those aspects of the original problem-solving that were arguably made more explicit via the ‘animated’ *Livescribe* playback of the participants’ original annotations. It argues that this may have been responsible for more clearly illustrating misconceptions at the heart of the problem-solving, with the caveat that the second of the two problems (T-AP 2) was the one with which the group appeared to have most difficulty. Therefore, it is possible that this problem was always more likely to inspire reflection and discussion. Nonetheless, it is demonstrated, via extracts from the transcripts, that individual participants felt that *Livescribe* led to errors and omissions 'leaping out at them' (see extract from SRI 2 at 31:05 below) that might otherwise not have done.

As indicated above, at the close of the second SRI session (see Appendix 4), participants were asked to reflect upon the two different technologies employed for capture and playback of their discussion and how these may have impacted on their problem-solving performance during the original T-AP task as well as later within the SRI sessions. Ultimately, their comments suggest that perhaps the biggest advantage provided by *Livescribe* was in supporting their reflections and their occasional post-hoc reconsiderations of their original approaches and strategies. The realisation, at 10:03 in SRI 2, that numbers had been left out ("*Why didn't we have…979 at that point?*") is an illustration of this; the attention this particular omission received in the follow-up SRI provoked thought about the value of *Livescribe* itself as a support to the SRI process.

It might have been expected, as suggested above in 6.7, that the mathematical strategies underlying the tackling of both problems might have been 'drawn out' from the participants via the T-AP used. It might also have been hoped that the provision of the pen itself might have focused them more on the importance of their working out, given that this would be seen again within the replay opportunity. That this was not the case might have been a result of more 'bureaucratic'/procedural issues related to turn-taking or control of the pen, and – indeed – there is some discussion of this in both follow-up sessions. These points notwithstanding, the following extract from the second stimulated recall recording illustrates the degree to which the *Livescribe* supported recall was able to highlight specific areas originally 'missed' by participants, even with the relatively limited amount of writing completed during T-AP 2. In this instance, the discussion revolves around the way in which the group had not, in fact, systematically found all possibilities when generating the possible numbers that could be produced:

> There's some things in there [referring to the *Livescribe* writing on the computer screen] which really leap out at you [audible agreement from other members of the group] like, for example, why we didn't get the third number when we were doing combinations of those numbers…and things like the writing in of the Hundreds, Tens and Units…
>
> Stimulated Recall Transcript 2: Beads (31:05)

The "third number" issue referred to here (made a little confusing by reference to "combinations of those numbers" when 'combinations of digits' would have been the mathematically appropriate way to phrase what was meant) relates to combinations of the digits 8, 8 and 9 and it is clear when considering the original transcript of T-AP 2 that group members were not effectively listening to/writing down the suggestions being made by their peers.

> 03:12: Well, if you moved one [inaudible] to the 10, that, that, we can't have that, so then you'd move, so it would be 8 and 9, 8.
> 03:20: Or 8, 8 9.
> 03:22: Or 8, 8, 9, yeah.
> 03:22: Why have we started with 9, though?
> 03:23: So we need to have 8, 9, 8 as well.
> 03:24: It needs to have 9, 10 on it, so that means you can go on to the next.
> 03:26: Move another one down, then go 9, 7, 9…
>
> Think-Aloud Transcript 2: Beads (03:12-03:26)

This extract demonstrates that, a little over three minutes into the recording, participants have identified the combinations 8, 9, 8 and 8, 8, 9 without seeing 9, 8, 8 as a possibility (even despite the question "*why have we started with 9, though?*"). This presumes that all are following the T-AP and verbalising those thoughts about potential mathematical strategies that have occurred to them and they are not, perhaps, 'holding back' on some thoughts for fear that they might be 'shown up' in front of other members of the group for putting forward an unsuitable or inappropriate suggestion. Nonetheless, further proof that the possibility of 9, 8, 8 has not been considered, even without verbalisation, comes when the next suggestion is for them to move on to 9, 7, 9.

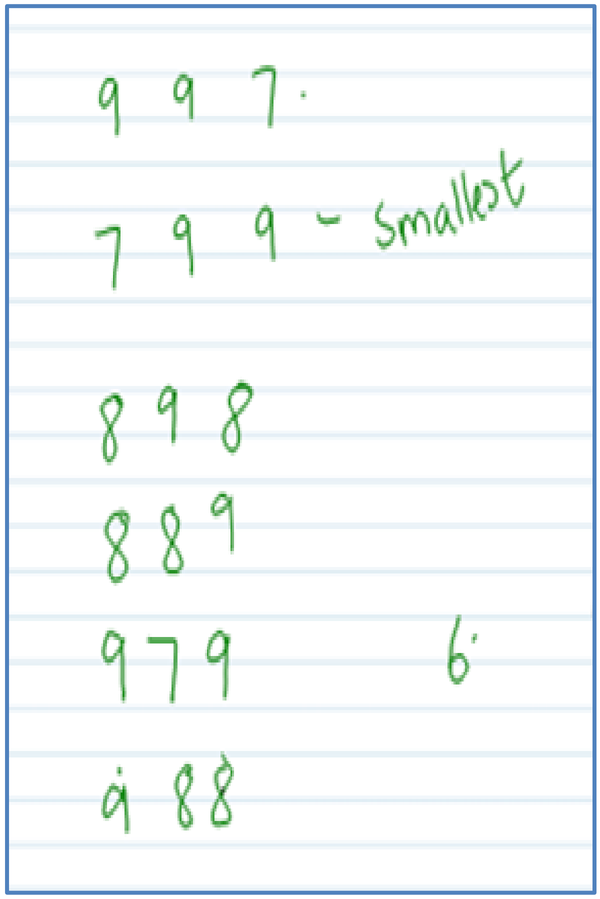

Figure 6.1: Page from *Livescribe* notebook for T-AP 2, as the group begin to write the numbers in order. Note that with both 9, 9, 7 and 8, 9, 8, it might appear as if only two options are immediately identified before returning to 9, 7, 9 and 9, 8, 8 as the 'missing' third possibilities at the end of the list. The *Livescribe* desktop and accompanying recording make it clear that, in fact, the top two numbers in the list are written some time before the others, as two members of the group have the clear strategy to find the largest and smallest numbers possible with the 25 beads and are not yet working strategically through the other possibilities.

Further on in the recording, the same 'mistake' is seemingly repeated again when writing out the numbers generated from the 25 beads in order of smallest to largest, as required by the original question. In this instance, some members of the group were employing a strategy of first finding the largest and smallest possible numbers that could be produced and this is evident both from their verbal contributions at this point in the recording[31] and from the annotation "smallest" which the *Livescribe* desktop helpfully shows is written before the other four numbers listed below it on the page.

This example, then, provides an indication of the potential of the *Livescribe* desktop to illustrate precisely when particular contributions were provided and how they might or might not be indicative of particular strategies being utilised. That the group were not all clear as to the strategy being employed, and had therefore not communicated effectively what they were doing via the T-AP utilised, is evident from the comment made by one of their number immediately prior to the observation that 799 is the smallest possible number that could be produced:

> I don't understand how it makes a three digit number. Do I not just not know how an abacus works or something? I don't know.
>
> Think-Aloud Transcript 2: Beads (02:09)

What the *Livescribe* desktop image makes clear at this point, that the recording/transcript/annotated page alone does not, is that one particularly useful piece of information, implied but not necessarily explicitly stated in earlier contributions from group members, was not written until later in the discussion. This is the label 'H, T, U' (for hundreds, tens and units) at the top of the abacus drawn on the page. The group commented on this in the SRI, observing the value of seeing the order in which they had tackled particular aspects of the task and ultimately reflecting on how they had not necessarily either worked within the T-AP provided/established or, ultimately, taken care to ensure the understanding of all members within their group. This was summarised by the researcher at the very conclusion of the second SRI (see Appendix 4) to the audible agreement of the rest of the group:

---

[31] 02:22: You've got 25, you've got 25 beads here haven't you so you've got to have…erm… that would be for instance 9 once you'd use up 18, then you've got 7 2 but then of course you've got to find a smaller number than that so it's probably 8 actually or 7…
02:43: Would the smallest number not be 799?
02:45: Yes.
02:46: So does that make 25?
02:48: Yes.

Think-Aloud Transcript 2: Beads (02:22-02:48)

> And [it makes it clear] when it occurs… Because if I had that piece of paper [alone – i.e. without being able to replay the writing back on the screen] I'd think HTU was written at the start [agreement]. And it wasn't…it was written very distinctly towards the end because the group had not got a concrete sense of the task, even that late.
>
> Stimulated Recall Transcript 2: Beads (31:26)

While the *Livescribe*-supported T-AP session (T-AP 2) featured a great deal less in the way of actual writing than the problem conducted with the standard digital audio recorder (just one page of the *Livescribe* pad as opposed to two sheets of sugar paper jottings for T-AP 1), and this might have been seen as a disadvantage when coming to the Stimulated Recall session (not to mention a disappointment given the provision of the pen to record their supporting annotations), the participants arguably benefited from being able to see the order in which their written contributions 'appeared' on the screen in the follow-up SRI. Had they just revisited their original page of annotations without the support of the *Livescribe* desktop, they might have been led to believe that their decision to clearly elaborate the Hundreds, Tens and Units on their abacus was made right at the outset of solving the problem, when – in actuality – the group did not clearly express this as 'important' until very late in the process (see Figure 5.3). As a number of contributors have said, this was a verbal omission – a failure more in the think-aloud process – rather than a failure for some of the more mathematically adept members of the group to appreciate that this was one of the key ways to understand the requirements of the problem.

This 'failure' in the think-aloud process was made whilst using the *Livescribe* pen, and this arguably counters previous assertions that the use of the pen during the second exercise might have had an impact on their increased *exploratory* dialogue as evidenced in Table 5.9 above. Perhaps the provision of the pen made it more likely that they might attempt a diagrammatic representation of the problem (see the abacus drawn at the top of Figure 6.1) but, as previously discussed in 6.4 and 6.7, it appears that, while there was more explanation/exploration evident in this problem, there were issues with the degree of verbalisation. Not all members of the group understood what they were doing (recall that Mulligan and Lozito's (2014) "self-generation effect" would make them more attentive to their own contributions than the ideas being proposed by others). Strategies were not fully explained (the drawing of the abacus itself, and then the listing of the numbers from smallest to highest) and, indeed, there were omissions in the sequence of numbers, perhaps due to questions going unanswered and thinking-aloud failing to encompass all that particular individuals really were thinking about the way to solve the problem. Again, this seems perhaps to suggest that it was the mathematical problem itself, and the group's difficulties in understanding it, along with their incomplete verbalisation of thoughts, which provoked more in the way of *exploratory* dialogue in this second T-AP session (with the additional rise in *supercumulative* 'attempts' to provoke further *exploratory* dialogue that has been associated with a possible increase in "question quality" (Graesser and Person, 1994) demonstrated in Table 5.12 above) .

Ultimately, it could be argued that the 'success' of the second follow-up SRI in promoting a greater amount of *exploratory* dialogue (12 *exploratory encoding* contributions in SRI 2 as opposed to 2 in SRI 1 – see Table 5.13) rested on deficiencies in the group's methods and, perhaps also, the clear way in which this was illustrated by the *Livescribe* recall. This provides a strong case for the use of *Livescribe* in SRI situations, with the answer to RQ 6 ("*Does multi-media artefact prompted recall promote greater reflection on mathematical methods/problem-solving strategies than 'standard' digital audio?*") appearing to be in the affirmative. It also makes a case for the "networking" of the two methodologies as propounded by Hickman and Monaghan (2013).

Figures 5.1 – 5.3 above, in 5.4.4, demonstrate how the T-AP page appears in the *Livescribe* pad. Figure 5.2 shows the full extent of their notes during the course of the problem-solving session, and then Figures 5.3 and 5.4 show how the *Livescribe* desktop software enables users to see more clearer the order in which elements of the problem were tackled.

When compared to the SRI 1 (see Appendix 3), there is a distinct difference in the type of reflection/"looking back" (Pólya, 1957) evidenced in the second exercise. As demonstrated by Table 5.13 and in the extracts from the SRI 2 transcripts above, there is a greater focus on the mathematics and on suggesting refinements/improvements to strategies employed. This greater reflection on mathematical strategies overall, as suggested above, may well be a result of having *Livescribe* 'replay' the group's original annotations, limited as they may have been. This presented them with the 'frustration' of their incomplete thoughts on the paper (for example, the abacus drawing barely begun before it is dispensed with; the 'H.T.U.' headings only provided some minutes later as a response to a confused group member – see Figure 5.3 above – and the lack of a systematic approach in the writing out of the possibilities from smallest to largest). Referring to RQ 6's focus on the difference between the two digital audio technologies, it can be argued that there are some benefits to being presented, in 'real time', with the original annotations as they were written, and that seeing these alongside the audio recording enables participants to reflect upon the degree to which they have made their intentions clear. From this, if developed further, it might be possible to encourage student teachers, for example, to consider how they model strategies themselves, including when providing classroom exposition, but also when encouraging children to explain their thoughts to each other. Chapter 7 considers these possible future developments in more detail below.

This chapter has considered the results of both the T-AP and the SRI sessions, making a case for the "networking" of the T-AP and SRI methodologies (Hickman and Monaghan, 2013) to ameliorate the potential deficiencies in the original thinking-aloud. This arguably removes the need for an intrusive or constraining protocol with explicit demands to explain (as Seal (2006) suggests) in front of potentially more-knowledgeable peers. It has demonstrated that the SRI opportunities, both with and without support from the multi-media *Livescribe* artefact, have successfully encouraged group members to provide a commentary on their original problem-solving. It has further demonstrated that the Talk Framework is a useful device for identifying not only how well the individuals have performed in the T-AP sessions but also for identifying aspects that should be followed up in the subsequent SRIs. Furthermore, while it can be seen that *Livescribe* has prompted elaboration on areas that had caused confusion in the original problem-solving sessions by revealing, as in the case of SRI 2, how and when particular strategies were deployed, this chapter has also demonstrated that other issues may have impacted upon this ‘more successful’ second SRI. These include the group’s greater need to explain their strategies in the second task, as well as the reduced focus on the group situation in the second recall due perhaps to the ‘exhaustion’ of that line of enquiry in the first SRI.

The next chapter of this thesis presents a conclusion to the work detailed and discussed above, revisiting the Research Questions addressed throughout to consider the degree to which they have been answered, and indicating the implications of this work for group problem-solving endeavours, teachers and, indeed, Initial Teacher Education.

7. Conclusion

7.1. Introduction

This Conclusion begins by summarising the Think-Aloud Protocol (T-AP) and Stimulated Recall Interview (SRI) group problem-solving project detailed in the chapters above, with reference to each of the six research questions (7.2). In so doing, it considers both the Results and Discussion above, indicating aspects of the questions that have arguably been answered by the work, and those that have been more difficult to address and might inspire future work (see also 7.5 for further research). It demonstrates the contribution that combining ('networking') T-AP and SRI, with support from *Livescribe*, have made to knowledge – this is most particularly of note given their usual use as distinct, discrete, methodologies. Furthermore, the *supercumulative* category provides opportunities for follow-up discussion that, arguably, would not be possible with the categories of *cumulative* and *exploratory* alone (even the 'split' categories of *exploratory transformative* and *exploratory encoding*, as used here). This consideration of the project alongside the research questions then informs the following section (7.3) which covers the implications of this work for group mathematical problem-solving (7.3.1), primary mathematics teachers and primary practice (7.3.2), and Initial Teacher Education as a whole (7.3.3). The latter builds on the School and Initial Teacher Education issues discussed above in 2.6, with some consideration given to the developments in teacher training since this work was conducted. 7.4 then considers the limitations inherent in this work before the final consideration of further research in 7.5.

The Conclusion now turns to a summary of the work with reference to the Research Questions, addressing each one and considering the extent to which the work above may or may not answer them.

7.2. Summary of the Work with Reference to the Research Questions

The statement of the problem that inspired the work provided in 1.3 above stated, in short, that there was concern from the part-time primary PGCE mathematics tutors that there were limited opportunities provided, within the existing mathematics workshops, for either problem-solving activities or group discussion. Given the focus on discussion in curriculum materials such as the Numeracy Strategy (DfEE, 1999a) and in influential reports such as Williams (DCSF, 2008), not to mention the extensive literature on dialogue and teaching detailed above in 2.3, it was hypothesised (see 1.5) that engaging students in activities that encouraged the revisitation of group discussion might enable them to highlight, for themselves, successful verbalisation strategies. These might then benefit both their future mathematics performance and teaching practice. The use of digital audio to capture this problem-solving performance was augmented, for the second of the two recorded sessions, by the use of the multi-media *Livescribe* device which, it was hoped, would encourage a deeper consideration of the underlying mathematics in the problem-solving situation. Equally, and although not explicitly used for coding by the participants themselves, it was hoped that the Mercer (1995), Novotná (1997) and Hošpesová and Novotná (2009)-informed Talk Framework would identify the most prevalent talk categories evidenced in their discussions. This identification, it was hoped, would aid the follow-up SRI opportunities by highlighting areas of interest; it was also hoped that the Framework categories might be helpful in analysing the SRI sessions themselves, should 'new' thoughts and ideas be put forward about the mathematical-problems that had not been in evidence in the original T-AP recordings.

The statement of the problem and the hypothesis detailed above informed the six research questions chosen for this work. This sub-section now addresses each in turn, considering the degree to which the research, as conducted, answered aspects of them, and identifying areas for further consideration that inform the final section of this Conclusion (7.5) to follow.

Research Question 1, as originally outlined in 1.4 above, was "*In what ways can dialogue/thinking-aloud recorded by digital audio support student teachers' verbalisation of problem-solving strategies?*" The key outcome of the work is that the "networking" (Hickman and Monaghan, 2013) of the two methodologies enables further verbalisation of problem-solving strategies than would arguably be the case with T-AP alone. The Talk Framework coding detailed in 5.4, including the consideration given to the questions asked by participants during the sessions, indicates that there was perhaps a reticence from some to engage with the process. This is further elaborated upon in 6.2 and 6.3 above. Both the group situation and the mathematical problems may have had an effect on some individuals' willingness to think-aloud (this also crosses over with RQ 5's focus on the way in which the methodologies may have impacted upon participants' willingness to engage with a problem-solving task). The SRI afforded them the opportunity to reflect upon this, and – as indicated in 6.5 above – there was a general sense that they had not previously taken part in such activities, alongside a belief that the opportunity to revisit their work was a valuable one, not to mention one that was rarely offered to them. As one participant states at 28:08 in SRI 2: "*I think maths is probably the least revisited area*". The SRI allowed participants to see that they had contributed more than they thought they had (SRI 2, 29:03: "…*actually, I think I contributed more than I thought I did*") and even provided them with 'positive' feelings about their mathematics performance that they might not have had if they had engaged in the T-AP experience alone (SRI 2, 29:15: "*Well, I don't say an awful lot, to be honest, but I've come away with doing this as a more positive experience than the actual exercise*.")

Referring back to the RQ, and the ways in which digital audio can support the verbalisation of problem-solving strategies, the results indicate that the opportunity to revisit their work, both in 'standard' digital audio and *Livescribe* form, enabled the student teacher participants to ameliorate some of the deficiencies in their thinking-aloud (and, indeed, their mathematical problem-solving) and question their original approaches. This, as demonstrated in 6.5 above, was particularly apparent in the second of the two problem-solving exercises (T-AP 2) where the group had failed to 'carry' at least one of their number, who remained confused as to the use of the abacus in the 'Beads' problem right the way through to the end of the recording.

Another comment worth making about the second T-AP/SRI exercise is that the group seemed to find it more difficult than the first problem. It is perhaps, therefore, the case that the *Livescribe* revisitation of their work was always going to provoke more comment for a problem that had confused some of their number and had resulted in at least one clear error in their supposedly systematic method of collating the possible answers. Nonetheless, the SRI – in this case with the *Livescribe* replay – was the means by which they were able to articulate their issues with their own working and, referring back to the original statement of the problem in 1.3, it can be said that this is a discussion that would rarely have happened in the taught PGCE mathematics workshops themselves. It could perhaps be argued that the *Livescribe* replay prompting fulfilled the function of the Task-Based Interview (Goldin, 1997) informed questioning. Further work might make use of *Livescribe* alone to encourage the responses from the group – a T-AP could perhaps be used to ask them to articulate their thoughts as they watch their working out being animated on the screen in front of them. This would address Egi's (2008) concern about the influence of the researcher on the participants' recollections.

In summary, then, thinking-aloud supported by digital audio has the potential to capture problem-solving strategies for later reflection. The use of the Talk Framework can indicate the degree to which participants have or have not explored possibilities and explained strategies via the newly created *supercumulative* category and exploration of *monologic* contributions (as appropriate), and this information can be used to inform the Task-Based Interview (Goldin, 1997) style questions that can then be asked in the SRI. The application of the SRI methodology in this way is, in itself, an approach that is distinct to this project (see below regarding T-AP and SRI being 'discrete' methodologies). The ability to hear their words whilst watching a replay of their annotations seems also to provoke more interest than the audio alone, demonstrating the potential power of *Livescribe* to inform such a process, as originally considered in the first iteration of the project (see Chapter 4 above, and also the research design in 1.6, where the deployment of both methodologies, given the need for a plenary-style, recall opportunity, was first considered). It seems clear that the conversation about the errors in T-AP 2 may not have resulted from a non-*Livescribe* prompted replay – researcher's questions aside, the contributions from participants on this subject were prompted, it appears, by the *Livescribe* replay alone. This is further considered below when addressing RQ 4. Drawing a firm conclusion about this, however, is difficult when it must also be acknowledged that the second problem-solving session was the one with more 'confusion' in evidence. Coming second, it also perhaps invited more comment on the mathematics than the T-AP methodology or the group situation because these had been addressed in the first SRI and arguably did not need to be addressed in any depth again. Nonetheless, utilising T-AP and SRI methodologies together in this project, rather than as discrete approaches, and deploying *Livescribe* to prompt the latter, demonstrates a contribution to knowledge that can be built upon in further research informed by this work (see 7.5).

The second Research Question was "*What levels of thinking-aloud most effectively support digital-audio recorded T-AP and SRI?*" This, as indicated above in 1.4, remains a difficult question to answer at the end of this project due to the fact that the levels of thinking-aloud were not varied between the two sessions. It can perhaps be argued that there were deficiencies with the T-AP, in terms of 'incomplete' verbalisations that, in the event, invited further elaboration in the follow-up SRI opportunities. This could be seen as ultimately beneficial, and part of the overall argument for utilising a 'networked' (Hickman and Monaghan, 2013) T-AP and SRI approach. However, although there was an effort not to enforce a high level of thinking-aloud on participants, at times it seems that participants did not make their strategies clear to their peers. Further work (see also 7.5) could perhaps more readily address RQ 2 by refining the 'interviewer's monologue' to explain that they should be thinking-aloud at all times whilst problem-solving, and that this includes when they are making annotations related to strategies employed. Further work would allow a comparison to be made with the two exercises conducted here, and would allow for consideration to be given to the point about the more 'difficult' problem inspiring more reflection on mathematical strategies in the T-AP. The observation made above in 5.4.5 about *supercumulative* questions and responses being prompted by such 'difficulty' could also be tested further – is such *supercumulative* talk more in evidence with a stricter T-AP? Equally, would greater concentration on thinking-aloud encourage more in the way of *exploratory* responses to *supercumulative* questions or would it, as Ericsson and Simon (1993) attest, interfere with the mathematics? It is not possible to answer this here, although it does seem reasonable to suggest that, when struggling with comprehension of the problem, individuals in the group paid less attention to both their own thinking-aloud and the questions and responses of others in the group, regardless of how useful these may have been to achieving a solution.

Research Question 3 (*What does the use of a multi-media artefact in an SRI situation reveal to participants about their own problem-solving strategies, and those of the group?*) is most clearly addressed by the SRI material in 6.5 above. As indicated above when addressing RQ 1, the *Livescribe* artefact prompted realisations about the student teachers' mathematical strategies that had not been apparent to them in the original problem-solving situation. As indicated above, it is, however, not possible to state whether this is just as much to do with their difficulty with understanding the use of the abacus in this problem (and its relation to place value). Certainly, the errors evident in their strategy, and the misunderstanding of at least one of their number regarding the problem, could be argued to have provided a focus for their interest and discussion that might not have been present in an 'easier', less problematic mathematical replay. Beneficial future research might explore *Livescribe* further as a prompt in other problem-solving recall situations, including those where the group had been more successful at conveying their thoughts, either through a more stringent T-AP or simply due to greater ease at recognising the requirements of the problem.

Research Question 4 (*What types of talk are most evident in multi-media artefact enhanced thinking-aloud while engaged in mathematical problem-solving activity?*) has been addressed via the Talk Framework coding of the student teachers' problem-solving above. It is clear that *cumulative* talk is the most prevalent in these two exercises (see Tables 5.6 and 5.9) with relatively limited *exploratory* talk in evidence. This is similar to the findings of Seal (2006) – a study which had, in part, inspired the use of the Talk Framework in this research, but which recommends the use of 'ground rules' to encourage *exploratory* dialogue in the problem-solving, an approach that has not been taken here.

It has been argued that there are many psychological reasons for student teacher participants wishing to 'hold back' from putting forward their ideas in front of peers, even in a T-AP situation (see 6.2 above). Indeed, the protocol itself may have provided some obstacles to communication. It is suggested above, for example, that asking individuals to listen to and respond to others whilst also concentrating on addressing the think-aloud protocol provides a level of ‘interference’ – this has been connected to Mulligan and Lozito’s (2014) “self-generation effect” and also the “system 1 and 2 thinking” outlined by Kahneman (2011). While RQ 4 has been addressed with the breakdown of the T-AP turns into Talk Framework categories, and it is clear that *cumulative* made up the majority of the responses, the addition of the *supercumulative* category to the Framework provides a useful indicator of the extent to which individuals may have wished to prompt further *exploratory* dialogue from their peers – this was particularly the case with the second T-AP detailed above. As has been argued, the increase in *supercumulative* responses in the second T-AP may be a result of the increased level of ‘confusion’ felt by the participants when engaging with the second problem – identifying this *supercumulative* material, though, was arguably of benefit to the SRI, providing ‘unanswered’ questions to be addressed.

Implicit in the wording of RQ 4 is a focus on whether the use of the *Livescribe* pen, even before the playback opportunity, might provoke more in the way of explanation as participants are aware that the pen records both their spoken words and their annotations. As has been argued above, it is not possible to say whether this was the case – it may have been useful to have asked individuals more directly whether the pen had this effect on them in the SRI. It is perhaps just as likely that a problem that provokes more questions from participants is going to result in more *supercumulative* talk, if not the *exploratory* that would have helped their collective understanding. Issues around the use of the multi-media *Livescribe* artefact are addressed in 7.5, Further Research, below. However, the fact remains that the *Livescribe* annotations *did* prompt participants to talk about mathematical strategies in the second SRI, and that this prompt came without the need of questioning from the researcher (in the first instance). This indicates that there is considerable scope in exploring *Livescribe’s* contribution to mathematical problem-solving tasks of this kind in future endeavours, and this is key contribution of this work to knowledge.

Research Question 5 considered the ways in which the 'networked' (Hickman and Monaghan, 2013) T-AP and SRI methodologies might have impacted upon the willingness of participants to engage in the mathematical problem-solving (with, again, the key observation to be made here that the reporting on this 'networking' followed the successful use of the two methodologies in the first iteration of the project, where it had been hypothesised that combining them would prompt further reflection on mathematical strategies than T-AP alone). The SRI opportunities, indeed, provided the clearest indication that, in some cases, the methodologies did have an impact, and not always a positive one (i.e., without the SRI it might not have been possible to capture the impact of the T-AP and the group situation on the problem-solving). However, it might also be said that the group situation presented particular individuals with the biggest challenge to engaging with the mathematics. This reaches back to the statement of the problem outlined at the beginning of this thesis (1.3) – that group work, the promotion of dialogue (as defined by such as Alexander (2018)), and the revisitation of problem-solving strategies were not elements that had been addressed to any great degree in the primary PGCE mathematics module and some students were, therefore, reticent to engage with such practices. The comment at 24:07 in SRI 2 (see Appendix 4), for example, sums up how one participant, who described herself as not "*hav*[ing] *a problem speaking and explaining when I don't understand something – the opposite really*" felt that the, she perhaps believed, indiscriminate 'throwing in' of methods and strategies from other members of the group led to confusion on her part. She claimed that she would have preferred to work with a partner rather than in a group situation of this kind and, as discussed in 6.2 above, did not feel that a stricter T-AP protocol would have helped her situation. One of her peers suggested (at 24:51 in SRI 2) that there could be 'ground rules' (although she does not use the term) requiring individuals to put forward strategies before allowing time for reflection ("…*and you could have said, 'no, that doesn't help me, next one*….'"). This slower, more considered, more 'structured' approach to the talk was also rejected as "*I think some people just don't find group work as…helpful as one-on-one work, I think*" (SRI 2, 25:49). The issue here is perhaps less with the T-AP than the situation – although there is no doubt that the protocol often resulted in 'incomplete' verbalisations (Hickman, 2013) and individuals perhaps found listening to each other, as well as concentrating on their own words, somewhat too demanding. Nonetheless, if group work remains the issue, it seems clear that there is much still to be considered as to how this can be more effectively modelled for student teachers. The implications of this work for group problem-solving, primary teaching and teacher education are considered in sections 7.3.1 to 7.3.3 below.

Returning to RQ 5's focus on the "networking" (Hickman and Monaghan, 2013) of the T-AP and SRI methodologies, however, it can be argued that the recall situation provided the opportunity for individuals to re-engage with the mathematics, to reconsider their strategies, sometimes due to the 'incomplete' verbalisations (Hickman, 2013) indicated above (and, therefore, the deficiencies in the original protocol), and also to – as indicated in sections 5.5.1 and 6.5 above (most particularly Table 5.13) – provide additional, 'new', *supercumulative*/*cumulative* and *exploratory* material not present in the original session. In this sense, it can be argued that the participants were willing to re-engage with their problem-solving, that the SRI provided an opportunity for "looking back" (Pólya, 1957), and that this "well known but little practiced problem solving heuristic" (Brown and Walter, 1993, p.231) was evident in this work, precisely the ambition outlined in the hypotheses provided in 1.5 above. For example, it was hoped that the work might "enable [participants] to highlight – for themselves – productive mathematical and verbal strategies". Indeed, as Brown and Walter (1993, p.231) observe, "it is by looking back that one may become explicitly aware of positive (and also negative) strategies that have been used but perhaps not incorporated into one's awareness". Whether this lack of 'awareness' was the result of the "psychological situation" (Rotter, 1954) or the effect of the T-AP on problem-solving, the SRI arguably ameliorated this effect, allowing them to raise issues such as the misunderstanding of the abacus and its connection to place value in SRI 2. Indeed, the aid of the *Livescribe* multi-media artefact referred to in the overview of RQ 4 above, arguably encouraged a degree of reflection on their work that the student teacher participants had not experienced previously in their PGCE mathematics workshops. One commented (28:08 in SRI 2, see Appendix 4, and 5.5.1 above) that maths is "*the least revisited area. Even…when you mark the children's work…you know they're never going to look at* *that page again in a lot of cases*". Overall, then, in response to RQ 5, it can be argued that the group situation and the T-AP may have had a negative impact on the willingness of some individuals to engage with the problem-solving, but all took a full part in the exercise, including addressing their misconceptions and previously unexplored alternative strategies. Indeed, they were perhaps enabled to 'revisit' their mathematics in a potentially beneficial way that they had not previously experienced.

Research Question 6 asked whether multi-media artefact prompted recall promoted greater reflection on mathematical methods/problem-solving strategies than 'standard' digital audio. On the face of it, this seems to have been what happened in this project, with the *Livescribe*-supported SRI 2 provoking considerably more conversation about mathematical strategies (Table 5.13 above demonstrates that there were 82 responses relating to the mathematics out of 163 turns in total, whereas SRI 1 prompted 36 responses out of a total of 202 turns in total). However, as has been argued above, it was also the case that the second of the two mathematical problems caused more 'confusion' amongst at least some of the group, and the number of unanswered questions (many of which were *supercumulative*) perhaps indicated that the T-AP or the group situation itself caused difficulty in individuals listening to each other and providing answers. That the subsequent SRI situation provided some of these answers has been indicated above as a positive effect of the "networking" (Hickman and Monaghan, 2013) of the T-AP and SRI methodologies. In short, then, while it might be tempting to assert that *Livescribe* was responsible for greater discussion about the mathematics, it would be necessary to test how it supported other recall situations, including situations relating to mathematical tasks that had been less problematic for the group concerned, before coming to any firm conclusion about its efficacy in this regard.

Having considered the research questions in turn, providing a sense of the degree to which the work discussed above answered at least certain aspects of them, it can be seen that the T-AP and SRI approach, combined with the Talk Framework and its newly proposed category of *supercumulative*, further supported by *Livescribe* recall, led to a focus on mathematical strategies that the participants themselves found to be beneficial to their learning. This conclusion now turns to the implications arising from the work.

7.3. Implications

As discussed above when reflecting upon RQ 5, the original hypothesis and statement of the problem provided in Chapter 1 of this thesis was supported by comments made by the student teacher participants, most particularly in SRI 2. This group, at least, perceived – as had their module tutors (thereby informing the statement of the problem in 1.3 above) – that there were limited opportunities to revisit not only their work in mathematics situations, but also the work of children in the lessons that they had observed and perhaps had also begun to teach. Some were able to see the value of engaging in think-aloud and, specifically, recall opportunities. The next three sub-sections, then, address the implications for group mathematical problem-solving, primary mathematics teachers, and Initial Teacher Education as a whole.

7.3.1. Implications for Group Mathematical Problem-Solving

The results and discussion above, most particularly those related to the group (6.2) and the comments collated via the SRI process (6.5) arguably add weight to the initial concerns of PGCE mathematics module tutors regarding the lack of focus on group work and dialogue when engaging with mathematics. There are, at least, some individuals who very much do not enjoy group dialogue in a mathematical context (it is arguable that they might prefer not to work with a group in other contexts, too). As argued above, given the focus on group work and the 'sharing' of ideas in classroom situations in, for example, the Primary National Strategy (DfES, 2006), work such as this project provides a potentially valuable opportunity to consider how individuals might be better supported to share their ideas with their peers, most particularly given Alexander's (2018, p.564) arguably more cogent definition of such 'sharing' in the guise of dialogic teaching, which requires talk to be "collective, reciprocal, supportive, cumulative, [and] purposeful". It also provides an opportunity for individuals to consider how they work in a group situation, what prevents them – perhaps – from reciprocating, and how they might both feel better supported and provide support for others (highlighting, again, Alexander's (2018) observations about dialogic teaching above). Comments about such matters are evident in the follow-up recall opportunity above, most particularly SRI 2.

It seems, from the Talk Framework coded results provided in Tables 5.6 and 5.9, that the provision of a T-AP alone is not enough to encourage individuals to put aside their concerns and 'risk' sharing their ideas with others. The consideration of the questions asked by members of the group during the T-AP sessions demonstrates that, even when encouraged to think aloud, reasons are often not provided, either for strategies put forward to the group or questions seeking further clarification. It also seems as if the difficulty processing the different demands of the task, from the problem itself through to the group situation and the protocol, led to a degree of confusion from some individuals that was eventually addressed – at least as far as this can be demonstrated from the SRI responses alone – only in the subsequent follow-up session. This, it has been argued above, is a further benefit of "networking" (Hickman and Monaghan, 2013) the T-AP and SRI methodologies and an indication of the contribution of this approach to knowledge.

The key implication for group mathematical problem-solving arising from this work appears to be the sense that there is little preparation for group work, and perhaps more pertinently, little appreciation for how dialogue (as discussed above in 2.2) might be beneficial when problem-solving. Indeed, it does appear that opportunities to engage with group work, and reflect upon its efficacy (or otherwise) when problem-solving, are lacking, at least on this one PGCE programme. Further opportunities to practise may well be beneficial, most particularly as – as discussed above – a context in which group work is promoted alongside dialogue/dialogic teaching presents problems to student teachers who have not had opportunities to develop their use of it in their teaching repertoire – or, indeed, personally when solving problems themselves. This is addressed further in 7.3.2, considering primary mathematics practice, and 7.3.3, considering implications for Initial Teacher Education, below.

The Talk Framework used in this work may also provide some further support for group mathematical problem-solving. Considering and identifying 'productive' dialogue in discussion-based situations may be a useful "metacognitive practice" (Bransford et al., 2000). It may reveal to participants strengths that they had hitherto not identified, and may then feed into the kind of formative assessment, 'Assessment for Learning' (AfL) approach recommended by Clarke (2008) – and mentioned above in 2.2. By identifying areas of strength, as well as of development, individuals can target future mathematics work more accurately. This appears to have been the case in this project, with comments such as those made in SRI 2 (29:15: "*I've come away…[from] doing this as a more positive experience than the actual exercise*") indicating that the opportunity to revisit and reflect may support those who had felt unhappy with the extent of their original contributions (or had suspected that others had better ideas regarding the solution to the problem than they did). The *supercumulative* category offers the opportunity to identify contributions that not only build on prior observations but present the opportunity for further elaboration and, potentially, explanation. It also allows for 'question quality' to be considered (Graesser and Person, 1994) – Table 5.12 above, with its supporting commentary, provides an outline of how this might be done in more depth. Allowing such *supercumulative* contributions to be 'seen' and perhaps, then, answered in a later recall situation may ameliorate problems in the original situation. It may also reduce the 'pressure' felt in the initial problem-solving situation that only the strategies and solutions proposed there and then are of value.

7.3.2. Implications for Primary Mathematics Teachers/Primary Practice

The modelling of thinking-aloud to children in the classroom situation may, as already suggested by such as Seal (2006), be of benefit to mathematical problem-solving when working in group situations. Indeed, the lack of demonstration and then practise opportunities in thinking-aloud prior to the T-AP sessions may have been responsible for some of the student teachers struggling to articulate their thoughts. The unfamiliarity of the protocol slowed them down in the way that Ericsson and Simon (1993) suggest is likely with the more demanding varieties of thinking-aloud. Equally, Kahneman's (2011) "system 1 and 2 thinking" is relevant here – quicker, more instinctual responses to the question and the questions of their peers were perhaps slowed down by the protocol. This does not necessarily have to be to the detriment of the mathematical problem-solving, however. Kahneman (2011) observes that deliberately slowing down thinking, and ‘pushing’ participants into “system 2 thinking” has benefits in that instinctual but wrong answers can be avoided. In other words, it may be possible to challenge common misconceptions by enforcing a think-aloud protocol (or similar approach) on children in the primary classroom. Similarly, Task-Based Interviews (Goldin, 1997) and recall opportunities may help to highlight misconceptions and “sources of incomprehension” (Duval, 2006, p.104). Given the importance of the plenary in primary mathematics practice since the adoption of the National Numeracy Strategy (DfEE, 1999a), opportunities for “looking back” (Pólya, 1957) have been provided at the end of lessons, but – as anecdotally supported by the participants in this study – are not necessarily in-depth enough. Adopting elements of the methodologies employed in this work may, therefore, prove useful in the primary classroom. This point notwithstanding, however, the results above demonstrate that the provision of a protocol alone does not result in ‘quality’ thinking-aloud, and thinking-aloud alone does not result in verbalisations that are ‘better understood’ either by the person making them or by the remainder of the group.

The Primary National Strategy’s (DfES, 2006) concentration on the importance of individuals being ‘happy’ to share their thoughts with each other provides an imperative for considering how to teach not only thinking-aloud but also listening to peers to identify potentially valuable contributions that can be built upon. If teachers do not feel confident thinking-aloud themselves, or – indeed – providing examples of *exploratory* dialogue, then this may have an impact on the degree to which children feel comfortable engaging in similar practice themselves. Moreover, if teachers and children do not recognise potentially ‘valuable’ dialogue when engaging in group situations, then they may undervalue the contributions within the discussion, even their own contributions, to the exploration of mathematical strategies that could be successfully deployed to tackle given problems. This provides a rationale for revisiting dialogue – perhaps via a replay of captured audio – and considering it against categories such as those outlined in the Talk Framework used here. Engaging with a T-AP, with or without a subsequent recall opportunity, could also raise awareness of the importance of talk, and concentrate minds more effectively on communicating ideas to peers. As seen in this work, SRI opportunities could enable a focus on the talk after the event, providing the chance for individuals to query what was originally said, what they themselves said, and clarify details that were perhaps missing due to the demands of concentrating on a T-AP whilst also engaged with peers and with a mathematical problem. SRIs also provide practise in listening back to what was originally proposed, identifying contributions that had been 'missed' and thoughts that could have been elaborated more clearly. Task-Based Interview style questions may be useful as a scaffold for the recall opportunity, and this may be of benefit in a plenary situation in the primary mathematics classroom. If children became used to such revisiting, there is the potential that their performance in T-AP-style tasks could be further enhanced, encouraging a degree of metacognition when it comes to considering the value of dialogue in their learning. Further work (see 7.5 below) would, however, be required to explore how the methodologies studied here, including the use of digital audio technology and even a Talk Framework, might be successfully used to inspire classroom practice, not least when reflecting on the challenges they presented to the adults engaging in this study. The suggestion is not that the work here could be replicated in the primary classroom, but that it could inform the approaches taken by teachers in encouraging dialogue and the reflection on that dialogue in a subsequent plenary. Such practice could be valuable when assessing learning (for example, via the formative AfL (Clarke, 2008) referred to in 7.3.1 above) and engaging pupils in assessing their own learning; it may also be of benefit in encouraging pupils to give more of the "attentive 'live' feedback" referred to in 6.2 (i.e. feedback in discussion situations themselves, rather than after the event).

7.3.3 Implications for Initial Teacher Education

The Williams Review of early years and primary mathematics provision (DCSF, 2008, p.3 – see also 2.6 for School and Initial Teacher Education Issues) was a major influence on the development of the Initial Teacher Education curriculum for the student teachers engaged in this work (not just in 'covering' the contents of the report, but also in adapting existing modules to take account of the recommendations and to model more effectively the practice expected in the primary classroom). The Review stressed the importance of the teacher in "determin[ing] learning outcomes [for children] in mathematics". Recognising that mathematics is "a demanding subject at primary level" and – as previously stated in 2.5 above – that "confidence and dexterity in the classroom are essential prerequisites for the successful teacher of mathematics" (DCSF, 2008, p.3), it had "firmly argued that most ITT does not in itself constitute a sound basis for deep subject and pedagogical knowledge" (DCSF, 2008, p.4). Whilst acknowledging the variation in practice by Initial Teacher Education providers, some of whom were providing, in its view, "considerably greater mathematics content" (DCSF, 2008, p.4), the Review concluded that there was a "need for an increased focus on the 'use and application' of mathematics and on the vitally important question of the classroom discussion of mathematics" (DCSF, 2008, p.4). It was this, then, along with the revisions to the Primary National Strategy (DfES, 2006), which also stressed the importance of using and applying – and, embedded within that, problem-solving – that helped form the context in which the work detailed in this thesis was conducted (see 1.3 above)[32].

While the two T-AP/SRI exercises detailed in this thesis can only go part way towards indicating how best to develop group problem-solving, it is arguable, from participant responses above in 6.2 and elsewhere, that the Williams Review (DCSF, 2008) was right to be concerned about the 'depth' of content on teacher training programmes. By their own account (see SRI 2), the student teachers undertaking this research had not had the opportunity to consider mathematical problem-solving, or group discussion, in any detail before on their programme. Without the opportunity provided by this project,

[32] It does need to be acknowledged here, however, that this context has shifted with the succeeding Coalition and Conservative administrations from 2010; as Lightman (2015, p.21) states, the focus has become more heavily on "core knowledge" in the curriculum, with far less prescription about how to teach than had been the case with the preceding administrations; nonetheless, much is "entirely at the discretion of schools", and this includes teaching approaches. The recommendations of Williams (DCSF, 2008) and the Primary National Strategy (DfES, 2006) regarding using and applying and discussion are, therefore, arguably still relevant, if perhaps not more so in an era where National Strategy materials, for example, are no longer produced, and student teachers and their tutors can no longer rely on the provision of materials such as the Problem-Solving pack detailed above (DfES, 2004).

they would not have reflected on their own problem-solving performance, or the potential benefits of revisiting their work, whether using digital audio or any other medium. Student teachers commented on the fact that the SRI had allowed them to better appreciate their own contributions, even to the degree that their perhaps somewhat negative view of their role in the group had been challenged. They further commented on the value of seeing their work on the screen, as presented by *Livescribe*. Overall, then, it could be argued that this project provided its participants with opportunities and perhaps also knowledge – relating to their performance in the group; to the way that groups function when given a problem-solving task – that they might not otherwise have encountered during the course of their PGCE programme.

Given the comments about mathematics subject knowledge and confidence levels in student teachers made by Williams (DCSF, 2008) and others, there is arguably an imperative to consider developing further the teaching and learning of mathematics on teacher training programmes (most particularly given the changes to teacher training outlined in 2.6 above, whereby – in the post-Coalition government context – ‘traditional’ University-based workshops has given way to predominantly school-based training). Given the tutors’ focus, on the PGCE programme from which the participants of this study were drawn, on modelling practice, the lack of opportunity to model group work is clearly something that needs addressing. This work indicates that digital audio and devices such as *Livescribe* provide opportunities to encourage a deeper level of reflection than might otherwise be the case. Equally, the use of a Talk Framework might well encourage a deeper focus on the different types of talk evident in a group problem-solving situation. It could be argued that the newly created *supercumulative* category, with its emphasis on *cumulative* talk that invites further elaboration and explanation of strategies, has benefits to individuals wishing to analyse their own talk, and wishing to consider the talk of the children in the classroom. The part-time primary PGCE student teachers undertaking this work were used to ‘school-based tasks’ away from the University; transcribing and analysing children’s talk according to the Talk Framework might be of benefit to them in considering how to better organise group sessions; it could also be of benefit in analysing the performance of children, as an assessment tool to inform future teaching and learning.

7.4. Limitations

A number of limitations to this work have been indicated above, and this section provides a summary of these, with consideration given as to their impact on the results presented here and to how they might be addressed in further iterations of this work. The section is structured to account for the T-AP and the SRI sessions in the order in which they were encountered by the student teacher participants and informs 7.5 (Further Research) to follow below. It begins by considering the group itself, before moving on to the T-AP and the 'interviewer's monologue' employed to direct the student teacher participants in their verbalisations.

As outlined above in 5.2, the group of part-time primary PGCE students engaged in this project consisted of eight part-time primary PGCE student teachers (seven female, one male) from a total cohort of 77. While it has been demonstrated that there were a high number of females on the programme, and that this is in line with other primary programmes in the country, the absence of the male student from the second part of the research (the follow-up SRI) clearly denies the opportunity to consider his reflections on the group situation and the methodologies. Equally, the results in this work rest, therefore, on seven student teachers, with the obvious caveat that other groups may have produced different results, and may – indeed – have experienced less difficulty with the second T-AP exercise. Similar caveats could be raised about the nature of the participants as part-time students, as it might well be expected full-time PGCE students or undergraduate primary students may have produced different results due to their differing (in some cases, more recent) experiences with mathematics. Further work, with other student bodies, on the degree to which think-aloud and digital audio can support the verbalisation of problem-solving strategies, would be required to see if there is a commonality of response across different programmes. From the perspective of this work, however, and given the statement of the problem in 1.3 above (which specifically related to promoting dialogue among this cohort), it can be seen that productive results were produced, with clear indications – as discussed above – that dialogue both requires further development and that, for these students, digital audio and the T-AP and SRI methodologies (with *Livescribe*) can produce effective verbalisations of problem-solving strategies.

Turning to the ‘interviewer’s monologue;, as detailed in 3.2 above, it can be seen that was designed to encourage the verbalisation of those thoughts and ideas that participants might not otherwise have verbalised within the group problem-solving process. It was purposefully decided not to stress the necessity for the kind of ‘explanations’ Seal (2006) recommends in such discussion-based work (see 3.2 above). The intention was that these would then arise more ‘naturally’ out of the discussion (and, indeed, would perhaps be more notable to the participants in the follow-up SRI if they had not been pre-empted by the ‘interviewer’s monologue’). There were, however, some additional instructions that could have been provided that might have improved the clarity of the verbalisations offered by the participants, including a requirement to ensure that all were satisfied that a solution had been reached before terminating the recording. Although there was a general appeal to the group to consider whether the problem had been solved before they concluded their work, it was by no means clear that all group members were happy with the solution reached, or – indeed – that they understood it; this seems to have been particularly the case in the second problem-solving situation, as revealed in SRI 2. While the SRI provided the opportunity to ameliorate any deficiencies in the T-AP verbalisations, it might also have been helpful, as indicated above in 7.3.2, for participants to have a practise session prior to engaging in the problem-solving tasks. This may well have prevented some of the ‘issues’ related in 6.2 above regarding the “psychological situation” (Rotter, 1954) in as much as they would have experienced thinking-aloud and also worked alongside the other members of the group prior to this first experience. Had this been done, there might have been greater reflection on the mathematical strategies employed in the first T-AP in its follow-up SRI and the apparent difference between the two SRIs (the second featuring more discussion of the mathematics) may well have been less marked. The results, therefore, of the T-AP, in terms of the Talk Framework coding (see Tables 5.6 and 5.9 above) may have been affected by this increased pressure on the first of the two occasions; the second may have benefited from greater familiarity with the protocol and with the other members of the group.

As with the 'interviewer's monologue' for the T-AP recordings, the SRI sessions may have benefited from more careful consideration of the instructions given at the outset (see 3.3 above) and then the questions asked during the replay of the student teachers' discussion. There was, for example, potentially a 'power-imbalance' evident in the recall sessions, with the researcher's questions perhaps taking precedence over those of the participants themselves (this is evidenced in Table 5.14 above). The shifting focus of the discussion was also arguably one of the drivers for the varied aspects addressed in the two SRI sessions. More could be said about the importance of the focus, for example, on the group situation over the mathematical strategies utilised had the researcher's questions not directed the participants towards reflecting on these aspects – in this respect, it may well have been better for the questions about the methodologies and the group situation to have been dealt with in separate interviews at the end of the process, as had been originally considered before the constraints of the student teachers' availability reduced the follow-up opportunities to two separate, half-hour SRI sessions. It is also possible that the opportunity for individuals to pursue their own lines of interest in the recording was curtailed by the researcher's predetermined areas of concern or that they were diverted away from aspects they would have liked to explore further.

The questions asked in the SRI sessions were informed by Goldin's (1997, p.45) Task-Based Interview "four-stage exploration" with "exploratory (metacognitive) questions" and related to how the individuals within the group had thought about the strategies towards solving the problems given. However, they were not written out beforehand and this may have helped structure the sessions more carefully – or, indeed, recognise more readily where the student teachers themselves had, on occasion, identified the same areas as of interest to themselves. Additionally, while it was helpful, from the perspective of prompting the participants 'with their own words', to have the audio play-back in 'real time' (given that it was a Stimulated Recall situation), it is arguable that more thought needed to be given to specific questions at specific points in the playback and structuring these so that potentially 'interesting' moments would not be 'missed'. The recording could then have been paused more deliberately to allow for the group to consider their responses to particular aspects of their problem-solving performance.

The above could be said to presume, however, that the researcher/tutor's identification of 'interesting' material worthy of discussion is of more importance than the participants' own commentary. An alternative approach, as discussed above in section 6.7, might be to use the *Livescribe* replay to provoke responses, limiting the questions to Goldin's (1997, p.45) "non-directive follow-up questions" and allowing participants to question, where appropriate, what they have done and why. There are a number of reasons why SRI 2 might have led to more discussion of mathematical strategies, ranging from the difficulty of the problem to the group's familiarity with the process. *Livescribe* does, however, appear to have made a contribution in terms of prompting commentary, notwithstanding the fact that it is likely that a problem that had been more 'difficult' for individuals to solve might well have provoked debate of one kind or another. The second SRI transcript shows that *Livescribe* led the reflection on the mathematical strategies. The participants responded to what they saw on the screen; they provided a commentary on the replayed annotations with the observation at 09:50 ("*Why didn't we have…979 at that point?*"), leading to reflection on their strategy that was arguably missing in the first SRI. This acts as a demonstration, perhaps, of what *Livescribe* is capable of provoking in a situation where there are errors and misconceptions evident in the working.

Other issues with the SRI sessions include the potential distraction provided by the Talk Framework-coded transcripts and the request for participants to consider their comments against these, as they deemed appropriate. Discussion about the Talk Framework may have been better facilitated by a clearer introduction to the Framework and/or an opportunity for individuals in the group to listen back to the recording and attempt to recognise for themselves the different categories evident in their verbal contributions. This latter idea was considered as a means of getting the student teachers to engage with their problem-solving work 'after the event', with the idea being that the recordings might be made available on the Virtual Learning Environment for them to access, and they could perform a coding exercise as a self-study exercise. This, of course, remains a possible approach for future work, as is discussed in the final section of this thesis (7.5) below. Overall, though, the decision not to emphasis the Talk Framework categories in the SRI was taken to avoid influencing the participants to see any particular type of verbalisation as more 'valuable' than any other. It could be argued that the Framework would have been better removed from the process, and contained just to the coding of the T-AP sessions for analysis of their talk when thinking-aloud.

In summary, then, before turning to the further research that might be inspired by this work, it is arguable that the methodology utilised here presented some issues that may have affected the results – the 'interviewer's monologue' perhaps needed to more stringently model the thinking-aloud that was expected of the participants; practise opportunities prior to the T-AP sessions may also have eased the impact of the "psychological situation" (Rotter, 1954) that perhaps caused some participants to 'hold back' from sharing their thoughts and ideas. Equally, while it is arguably interesting to note the different aspects discussed in the SRI (from *group situation* to *mathematical strategies* to *verbalisation* to *procedure* – as given above in Table 5.13), it must be acknowledged that these were in some respects 'led' by the focus of the researcher's questions. After the event, this informed the interest in the questions that the participants had asked themselves (and, occasionally, of themselves) during the process – that there were few such questions, however, is arguably unsurprising when they might be expected to defer to the position of the researcher. That *Livescribe* prompted some of the more significant questions remains notable, however, and something that indicates there is potential in an entirely *Livescribe*-prompted SRI session.

7.5. Further Research

The final section of this thesis considers the responses to the research questions provided above in 7.2 and the implications for practice addressed in 7.3.1 to 7.3.3 alongside the limitations outlined in 7.4 and looks forward to potential future work that could be done to extend the use of this 'networked' (Hickman and Monaghan, 2013) T-

AP and SRI approach. It also considers the use of the Talk Framework and the ways in which it could be refined and also used by participants to encourage an even closer focus on dialogue while learning. The use of 'turns' (Rowland, 2000), as opposed to dialogic moves (Vygotsky, 1962; Barnes and Todd, 1995; Edwards, 2005) is an area that might benefit from reconsideration, as this might enable the *monologic* and *supercumulative* contributions to be seen more clearly within the context of the overall dialogue – indeed, some of the *monologic* contributions would arguably be recategorised by taking this approach. This alternative approach to analysing the dialogue from problem-solving sessions of the kind detailed in this thesis will be addressed first in this section before turning to refinements to the methodologies themselves.

Given the focus on "collective endeavours" and "many heads contributing to the construction of knowledge" (Mercer, 1995, p.2) discussed above, not to mention the consideration of "interpersonal relations and their effect on intrapersonal learning within a group objective" (Edwards, 2005, p.3), Barnes and Todd's (1995) approach to the analysis of group dialogue as 'talk sequences' (Edwards, 2005) might well be beneficial. This would look more closely at the discourse moves in terms of the degree to which they initiate, elicit, extend, and qualify. Edwards (2005) considers these to be key to collaborative work in classrooms (see the definition of collaboration provided above in 2.2.2, with the caveat that, in this work, the group was not self-selecting). Initiating, eliciting, extending, and qualifying discourse moves are joined, in Barnes and Todd's (1995) system by identifying the logical processes underpinning these – whether, for example, a cause is being proposed or evidence is advanced. It is clear, from the discussion above about *supercumulative* and *monologic*, that many of the contributions made by participants in this study could be considered against such a Framework, which would then lead into looking at the "cognitive strategies (such as setting up hypotheses, constructing new questions" (Edwards, 2005, p.4) and, indeed, the degree of self-monitoring (reflexivity) evident in the speech. This would deepen the analysis of *exploratory* speech in a study such as the one detailed in this thesis, and might enhance the existing Talk Framework proposed here to the point that it better considers the dialogue as a whole, rather than as discrete 'turns'. Nonetheless, for the purposes of the work conducted with this group of part-time primary PGCE students, which was – in the main – designed to assess how digital audio and thinking-aloud might support their verbalisation of problem-solving strategies (RQ 1), it is clear that evidence of such support has been provided (via their comments in the SRI sessions; via their prompting 'of themselves' by their own annotations). Therefore, enriching the analysis of the dialogue further via consideration of moves over 'turns' would provide a closer focus on the group endeavour, and would enable further research questions to be addressed concerning the efficacy of the group. This could be taken 'into the classroom' by participants, now not addressing only discrete *exploratory* or *supercumulative* contributions, but considering how best to promote cumulation. Alexander (2018, p.566 – see 2.2.2 above) refers to this as "the most difficult of the principles [of dialogic teaching] to enact", and there is potential in such development of the work to make a strong contribution to dialogic teaching in the classroom.

Moving on to the methodologies themselves, it has been argued above (in 7.4) that the 'interviewer's monologue' could well be refined in future iterations of this work, to more clearly establish the parameters of the thinking-aloud expected from the participants. This should perhaps also include a focus on thinking-aloud even when asking questions of their peers in the session (to provide reasons/explanation; see 5.4.3 above and the consideration of the questions asked in the T-AP sessions against Bloom (1956) which, while an additional element to the work, provided a sense of the degree to which individuals were sought information beyond "simple recall" (Riegle, 1976) – the approach regarding moves indicated above would develop this further, and beyond Bloom). Additionally, practise opportunities could be provided prior to engaging with the problem-solving tasks to develop individuals' facility with thinking-aloud and perhaps also encourage a degree of familiarity or even confidence in the process which might ameliorate the temptation to withhold information and incomprehension from suspected more-knowledgeable peers (Piaget, 1929). Further opportunities to test the methodology could also include consideration of the *supercumulative* category of the Talk Framework – it seems that the definition provided in Figure 3.6 above holds true for the coded examples in the transcripts, in that these are responses that invite *exploratory* talk. However, as previously discussed in 5.4.2, and contrary to the wording of the definition provided in Figure 3.6, they do not necessarily build on this talk – analysis of the transcripts suggest that they may be prompted by a lack of prior explanation. They might even be evidence of the T-AP not functioning as it should in terms of making reasons clear to all participants. Whatever the refinements to be made to the definition, *supercumulative* contributions, and particularly questions, could be used within a follow-up SRI as questions for the participants 'after the event'. In other words, they could be directed to answer the questions that had not been answered in the original T-AP exercise; something that was not explicitly considered in the SRIs conducted here, although individuals did sometimes 'answer their own questions' from the original sessions and this was often the source of the 'new' explanations that had been hoped for with the "networking" of the T-AP and SRI methodologies.

Further to this, it seems apparent that the potential in *Livescribe* to prompt discussion could be further explored, most particularly with problems where errors and misconceptions are evident in the students' work (as, indeed, it would be interesting to observe whether it is of less use in a situation where participants had found the task relatively easy). The result of the second SRI appears to indicate that *Livescribe* was responsible for making certain errors clear; this is obviously only one mathematical problem of one very particular type (a 'finding all possibilities' problem that requires the strategic collation of numbers that can be made, in this case, with 25 beads on an abacus). Whether *Livescribe* is as useful with other mathematical problems, or whether there needs to be further consideration given to how participants are 'trained' to use the pen is worthy of consideration. Equally, it is possible that more use could be made of the *Livescribe* annotations themselves – these were not analysed in this work; issues surrounding scribing, the choice of information recorded, and alternative approaches that could have been utilised, could all be explored in more depth either prior to SRI or within an SRI situation.

Finally, and returning to the Talk Framework for Problem-Solving that provides the title for this thesis, further research might consider the degree to which the Talk Framework coding of group responses in a mathematical problem-solving situation can be used to inform future practice. In the work here, as indicated above in 5.2 and 6.2, when reflecting upon the group, issues around ‘confidence’ and self-efficacy were, ultimately, not able to be followed up in the ensuing study due to the inability to follow the student teacher participants into the classroom. Further study could allow for longer and more developed opportunities to consider the different verbalisations in group problem-solving situations – perhaps allowing for individuals to code their own contributions as indicated in 7.4 above. This “metacognitive practice” (Bransford et al., 2000) might allow for individuals to reflect on their strengths and weaknesses in group situations, to consider how children in their classrooms problem-solve when working collaboratively, and to perhaps plan to encourage more in the way of *exploratory* dialogue. To a degree this replicates the thoughts of Seal (2006) discussed above, but in that case, the suggestion was to establish ‘ground rules’, arguably imposed from without; in this instance, student teachers, and perhaps then their children, could be encouraged to analyse their own words and reflect upon their contribution to the problem-solving. As evidenced above in SRI 2, there were examples in this study of individuals appreciating that they had contributed more than they thought they had; such work could have a positive impact on discussion in the mathematics classroom, and student teachers’ and children’s willingness to engage in discussion. Referring back to 2.2 above, and the potential importance of dialogue, engaging with a process such as a Talk Framework analysed T-AP opportunity, could lead to a greater value being placed upon talk in the classroom overall. Bearing in mind the discussion above in 2.5 about “expectancy value” (Eccles, 1987; Eccles and Wigfield, 1995; Elliott et al., 2005) and most especially considering Alderman’s words (2004, p.69) given in 6.3 above, this could have an impact, ultimately, on the thought “how effective will *I* be.” It might also enhance the value of problem-solving activities by ensuring that children come to perceive that it is not just achieving a solution that has value in the mathematics classroom; the process is also of value, and – in a group situation – they all have a part to play.

In terms of the work outlined in this thesis, it can be seen that individuals became aware of this, perhaps for the first time: “I didn't think I said much at all…but I've come away with doing this as a more positive experience than the actual exercise (29:15, SRI 2, see Appendix 4). The use of the T-AP and SRI methodologies, with the support of *Livescribe*, has brought this to participants' attention, enabling them, also, to develop and refine their mathematical thinking alongside their peers. This demonstrates the potential underlying this work, and its contribution to future developments in group problem-solving.

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Appendices

Appendix 1

Research Project Information Sheet and Initial Questionnaire

York St John University

Digital Audio to Support the Teaching of Mathematics

Information sheet

As part of the part time **3GPM13** cohort, you are being invited to take part in a small-scale project to inform my doctoral work on digital audio technology and recordings and the way in which they can support the later teaching of problem solving in primary mathematics. Before you decide on whether you wish to be involved, it is important for you to understand the aims of this research and what will be involved. Please take time to read the following information carefully and discuss it with others if you wish; do ask for further information, if required.

What is the project's purpose?

This research involves the audio recording of some primary mathematics problem-solving activities using two devices (a traditional digital audio recorder and a *Livescribe* pen). Digital recordings will be played back in a later 'stimulated recall' session (a little like a DVD commentary!) and collaboratively analysed using a provided "talk/problem solving framework". In other words, you will listen back to your recorded speech and, with the help of the framework, discuss the different types of verbal contributions made and problem solving strategies used.

The project will attempt to determine the usefulness of digital audio recordings and technology supported by a talk framework in enhancing trainee teachers' confidence in the teaching of mathematics.

Why have I been chosen?

This project was originally designed and piloted to address a perceived need for part time PGCE trainees to have access to additional support, including digital audio resources, between taught sessions that can sometimes be quite separated in time. It is hoped that the recordings and talk/problem-solving frameworks will be useful on school experience, regardless of your involvement in the follow-up questionnaire and interview.

Do I have to take part?

There is no requirement for any trainee to take part in this research; it is hoped that all trainees will benefit from using digital audio within their taught sessions, and the framework will also be shared with others in the teaching groups who do not participate. This work has no bearing on module assessment. It is up to you to decide whether or not to take part. If you do decide to take part, you will be given this information sheet to keep (and be asked to sign a consent form) and you can still withdraw at any time without it affecting any benefits that you are entitled to in any way. You do not have to give a reason.

What will happen to me if I take part? / What type of information will be sought from me and why is the collection of this information relevant for achieving the project's objectives?

Having completed a short **individual** questionnaire relating to your confidence in explaining your thinking about problem solving strategies, you will then engage in two **group** mathematical problem-solving activities within a short additional session (scheduled around your existing 3GPM13 sessions). These problems will be recorded (using two different devices) for later analysis in a **group** 'stimulated recall' session (using a 'talk' and problem solving framework that will be provided). In this second specially arranged group session in the university, you will listen back to the recording and, with tutor support, comment on your contributions (such as exploratory statements and specific steps in the problem-solving process). This discussion will be audio recorded and your observations about the value of the process will be used to inform the writing of the researcher's thesis. You will be given a transcript of the recording for comment and/or correction.

What do I have to do?

There should be no risks to participating in this research; as stated above, you are, in any case, free to withdraw from the process at any point. The digital audio recordings will be created alongside your peers in short additional problem solving sessions (undertaking problems similar but the same as those the whole teaching group will meet in their 'problem solving' session during the module) arranged at convenient times for you; there will then be the post-activity discussion or 'stimulated recall' which will also be audio recorded, subject to the same anonymity as all other recordings). You will also have the choice to attend a short unstructured interview on the benefits and potential of the two different recording devices. This material will be used to inform my doctoral work.

What are the possible disadvantages and risks of taking part?

It is conceivable that some trainees may feel less comfortable with problem solving activities of this kind alongside their peers. There is nothing within this process that would not ordinarily be a part of your maths module. Participants are, even so, at liberty to withdraw from the process at any point.

What are the possible benefits of taking part?

It is hoped that teacher trainees will benefit from a more in-depth discussion of their own problem-solving strategies and the opportunity to make use of a specific problem-solving framework that could prove to be of use in the classroom. The talk framework may be helpful to trainees when engaging with their own children in small group and class situations.

What happens if the research study stops earlier than expected?

This is a small-scale project; it is not expected to end early, but – in the event of difficulties with technology or, indeed, any other issues arising – the reasons for ending the project early will be provided to all participants.

Will my taking part in this project be kept confidential?

Taking part in this project has no bearing on module assessment and information will not be shared with academic tutors or others within the university. All the information that we collect about you during the course of the research will be kept strictly confidential. You will not be able to be identified in any reports or publications.

What will happen to the results of the research project?

Questionnaire results will be simply collated and will be considered by the researcher in order to determine your confidence in explaining mathematical problem solving strategies at the beginning of the project. Anonymised data may be used within conference dissemination/presentations – no trainees will be identifiable from this material as all names will be removed.

Who is organising and funding the research?

This doctoral work is supported by York St John University and ethical approval has been sought from both YSJ and University of Leeds (as supervisors of this work). There is no external funding.

Will I be recorded, and how will the recorded media be used?

The digital audio recordings created of your activities and the post-activity discussion will be used only for analysis and for illustration in conference presentations and lectures – all recordings will be anonymous. No other use will be made of them without your written permission, and no one outside the project will be allowed access to the original recordings.

Many thanks for reading this information and also for taking part.

Contact for further information

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Digital Audio to Support the Teaching of Mathematics

This questionnaire relates to a series of mathematics questions and asks you to reflect upon your confidence in working out answers and explaining the strategies used to others. This will inform my analysis of your verbal responses to the problem solving tasks undertaken and will be compared to the responses in the digital audio problem-solving activity itself.

Name:

Please note that anonymity and confidentiality will be maintained within any subsequent research.

1. **How would you rate your confidence in maths? (please circle one)**

   (strong) 1 2 3 4 (weak)

2. **How would you rate your confidence in *teaching* maths? (please circle one)**

   (strong) 1 2 3 4 (weak)

3. **How would you rate your confidence in *explaining strategies to help others solve* mathematical problems? (please circle one)**

   (strong) 1 2 3 4 (weak)

4. **Having worked out the questions provided overleaf, and discussed them with your partner, please rate your confidence levels in the boxes below as:**

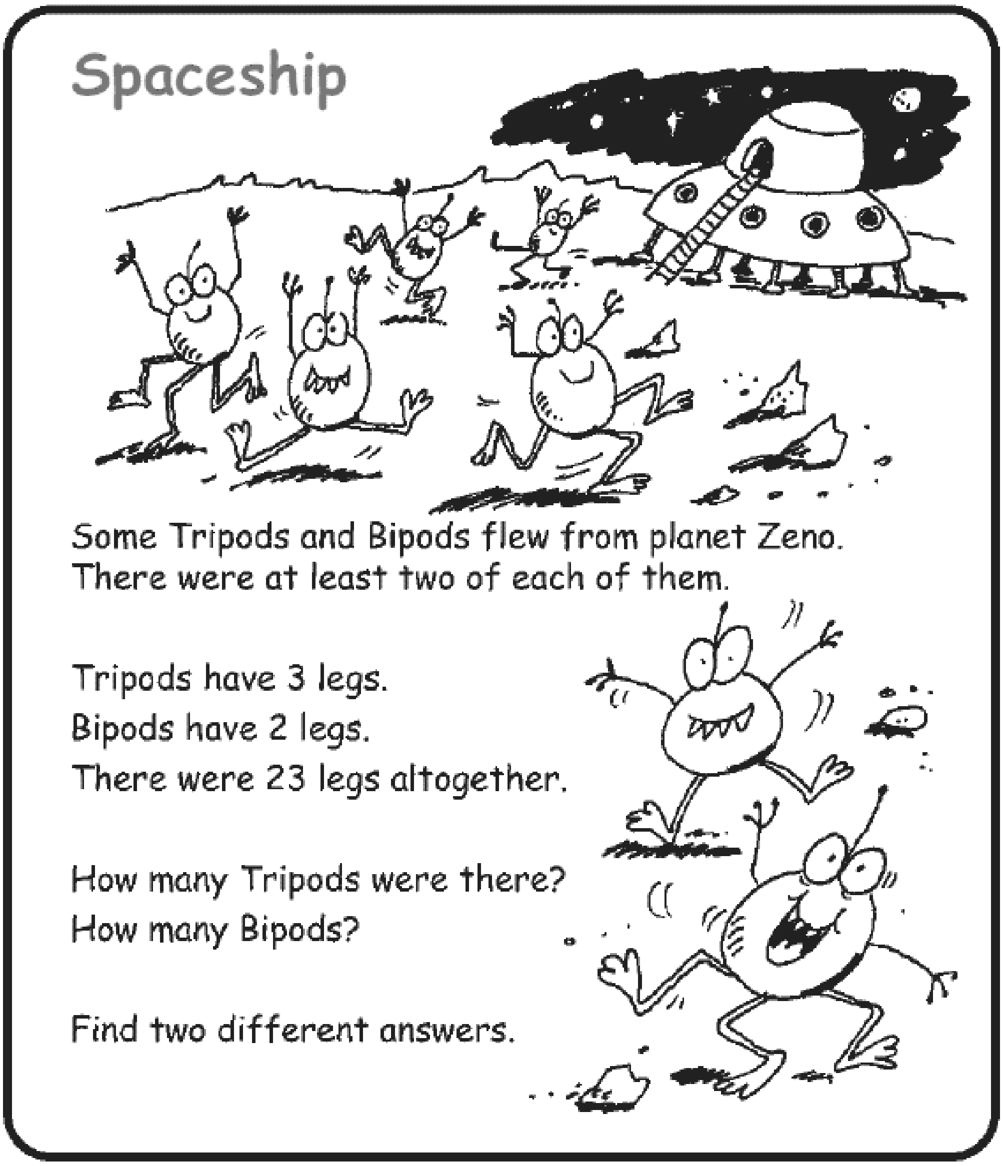
   1 (not confident at all)
   2 (reasonably confident)
   3 (very confident)

Question	**Rating for working out the answer/s**	**Rating for *explaining* your thinking and strategies used to reach the solution**	**Rating for making use of materials to demonstrate your methods to your partner**
1			
2			
3			

Thank you for taking the time to complete this questionnaire.

Problem Solving Questions.

Question 1:



Question 2:

One block is needed to make an up-and-down staircase, with one step up and one step down.

4 blocks make an up-and-down staircase with 2 steps up and 2 steps down.

How many blocks would be needed to build and up-and-down staircase with 5 steps up and 5 steps down?

Question 3:

I have fifteen cards numbered 1 – 15

I put down seven of them on the table in a row.

The numbers on the first two cards add to 15.

The numbers on the second and third cards add to 20.

The numbers on the third and fourth cards add to 23,

The numbers on the fourth and fifth cards add to 16.

The numbers on the fifth and sixth cards add to 18.

The numbers on the sixth and seventh cards add to 21,

What are my cards?

Can you find any other solutions?

How do you know you've found all the different solutions?

Appendix 2

Mathematical problems used for T-AP 1 and T-AP 2

T-AP 1: Exploring addition

You have the digits **1** **2** **3** and **4** together with

one addition sign **+** and the equals sign **=**

The idea is to arrange the cards to make different totals when you add two 2-digit numbers together, e.g. the total below is 55

4 **2** **+** **1** **3** **=**

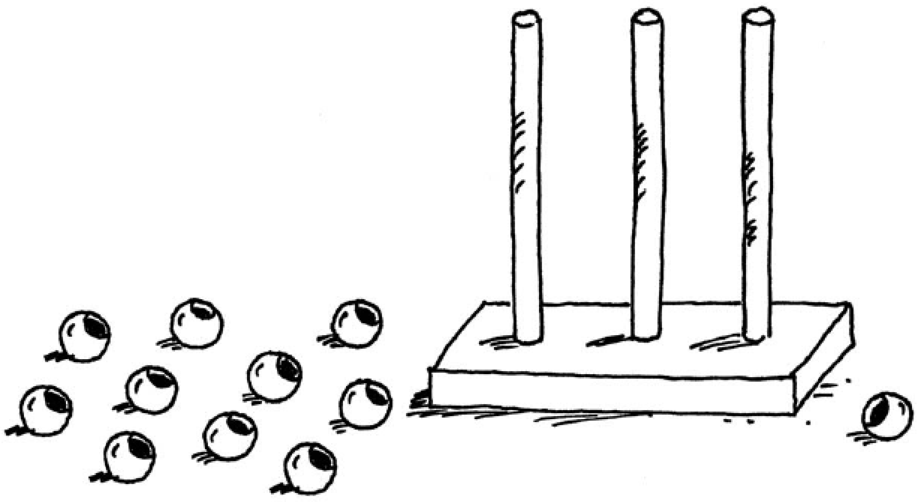
- How many <u>different</u> totals can be made?
- How do you know you have found them all?
- What are the minimum and maximum totals?
- Arrange your totals in order from smallest to largest and calculate the difference between successive pairs.
- What do you notice about the differences between pairs of totals once they are in numerical order?

Inspired by the Primary National Strategy Problem-Solving pack (DfES, 2004)

T-AP 2: Beads

Three digits

Imagine you have 25 beads.
You have to make a three-digit number on an abacus.
You must use all 25 beads for each number you make.



How many different three-digit numbers can you make?
Write them in order.

from page 13 of *Mathematical challenges for able pupils in Key Stages 1 and 2*
(Ref. DfEE 0083/2000; NNS publications)

Taken from: "Mathematical Challenges for Able Pupils in Key Stage 1 and 2" (DfEE, 2000)

Appendix 3

SRI 1 transcript extracts – with Talk Framework coding as appropriate and focus on group situation, mathematical strategies, verbalisation and procedure (for the sake of clarity and brevity, some overlapping dialogue, and therefore some individual turns, have been removed).

Time and Turn	Comment	Focus (Talk Framework Coding – where applicable)
Unnumbered – 00:23	Right…er…here we go. We're going to listen back to the recording made of your two problem solving…er…tasks and you're supported by transcripts of what you said as a group at the time. Er… Please feel free, by the way, to annotate or correct transcripts for the final record…If you see something that you think is absolutely wrong then, going back to our original form, of course we'll change it. All contributions are anonymous. All recordings will be used as described in the information sheet. In other words, never attached to your names… In listening back to the recordings, I will use…myself…the talk and problem solving framework I have recently introduced to you…you've just seen…where you think one of your contributions fits with, say, cumulative…agreement…or exploratory or disputational and you feel you want to comment on that, then you can. I will ask you at times to explain how you thought about the problem and how you came to make the decisions about ways in which to solve it with your peers. And that's the really important thing. Why did you contribute in the way you did when you did. So, we press play…	**NA – Interviewer's Monologue**

Between 02:05 and 04:06, the group listen to the beginning of the original recording (00:15 to 01:11 of T-AP 1). They listen to the original interviewer's monologue and the beginning of their discussion. The researcher asks questions such as "Now who threw that one in? And why?" (03:48).		
5 – 04:06 (Interviewer)	In a way that's exploratory, isn't it?	**Verbalisation**
Murmured agreement.		
6 – 04:10 (Interviewer)	You haven't got anyone to agree with at that point; you're suggesting that this is a way of solving it?	
The group listen to 02:28 to 02:49 in the original recording ("Can I just say, I would be inclined to take each of the digits in turn and systematically…")		
7 – 04:34 (Interviewer)	So this is you again?	**Procedure**
8 – 04:59	I think, looking back on it, I think that…erm…in my mind I was thinking that there was going to be a much bigger number of numbers [MH: right] that we were going to end up with…which, I don't know that it matters whether that's true or not…it turned out not to have been true, but it might have been true…erm…just being systematic seemed important.	**Mathematical Strategies** **(Elements of *exploratory* missing from original T-AP: exploratory encoding)**
Between 05:13 and 05:57, discussion relating to the question and how the phrase "how many" implied, perhaps, a lot.		
14 – 05:59	But…but, I mean, there's no actual…there's no rational reason why it saying "how many" means there's going to be a lot.	**Mathematical Strategies**
15 – 06:06 (Interviewer)	Or previous problems of a similar type make you think that it's…?	**Mathematical Strategies**
16 – 06:11	Yeah, like when kids do number bonds to ten [agreement], it encourages you to go 1, 2, 3, 4, 5…because otherwise they get confused and miss out options.	**Mathematical Strategies** **(Elements of supercumulative and exploratory encoding)**
17 – 06:23	And also it says "how do you know you've found them all", so to me being systematic would be one way of knowing that you've found them all.	**Mathematical Strategies** **(Cumulative)**

20 – 06:56	It could be important to be systematic if you're going to…if you think you might end up with a lot of numbers and if one of the things that you're looking at is knowing that you have found all the combinations…	**Mathematical Strategies** **(Supercumulative)**
Between 07:11 and 10:03, discussion of being systematic, including a comment about the T-AP itself at 07:44: "ideas were coming from all over the place and once we had a structure…" This leads into the next question:		
27 – 10:03 (Interviewer)	Are you working as one group? Is there a lead?	**Group Situation**
28 – 10:08	There's a few leaders, I think…	**Group Situation**
29 – 10:10 (Interviewer)	And how do you respond to that?	**Group Situation**
30 – 10:14	Well, it's funny because how I remember it [laughs] is that having said this thing at the top here that kind of…erm…it…the way I remember it is that it didn't quite unfold the way I'd perhaps imagined it would but…erm…and this is where you start to censor yourself a bit…erm…or you're conscious that you're working as a group so…and also you're thinking to yourself "well actually I've made that suggestion but is it…it might not be a good suggestion" so you take a step back and let things unfold and then other people appear with their contributions…	**Group Situation**
31 – 11:00 (Interviewer)	So, you're deliberately withholding?	**Group Situation**
32 – 11:01	That's…that's how I remember it…because we're working as a group and 'cos I'm thinking "ooh, I'm not sure that was such a good idea, after all…" [Laughs]	**Group Situation**
33 – 11:11 (Interviewer)	So does that stop you verbalising? Thinking aloud? Did you, did you actually stop yourself from doing that and was the recorder, knowing that there's one now, was that an imposition?	**Verbalisation and Group Situation**
34 – 11:25	Yes.	**Verbalisation and Group Situation**

35 – 11:25	Well, I think it's partly it's been…the thing is we don't know each other that well in terms of…erm…math…what we're like in terms of mathematical thinkers so…erm…right, well say just as an example, not to do with maths, if [name withheld]…if we were having a conversation about phonics…cos [name withheld] and I know where we stand in relation to how…well [name withheld] knows about phonics what I know about phonics so…yeah… [Some overlapping, inaudible, speech and agreement] Whereas, with maths, you don't. For all you know, one of the people in that group could be, you know, a mathematical genius and so…	**Group Situation**
37 – 12:05	So, you're thinking…well…do I…?	**Group Situation**
38 – 12:06 (Interviewer)	So that limits the willingness to think aloud?	**Group Situation**
39 – 12:11	Yeah.	**Group Situation**
40 – 12:12	I think so. I'm waiting to see what everyone…[else thinks]	**Group Situation** **(Cumulative)**
42 – 12:22	Because you don't want to impose something on the group when there might be other people who have better ideas…	**Group Situation**
43 – 12:29 (Interviewer)	Did other people feel that?	**Group Situation**
44 – 12:30	No, I didn't feel that. I was quite happy to follow, to be honest. I think because I'm weak in maths it's just…regardless of whether it's being recorded or not I'll just hope that someone else is gonna have a better solution than me…	**Group Situation**
45 – 12:40	You came out quite strongly in that task.	**Group Situation**
47 – 12:44	I really didn't mean to… [Laughter] But if I've got the pen, it looks like I'm doing something useful even though I'm just writing numbers…	**Group Situation**
Between 12:51 and 13:54, there is a discussion about the control of the pen and how this can "take the pressure off" having to speak/contribute ideas. This includes material focused more on 'procedure'.		

56 – 13:24	Also…also… If you put forward an idea, like with your systematic one, and you haven't got control of the pen you…you…other people's suggestions…erm…are more likely to…be taken…more likely to affect what's happening on the paper. If you have…if you had somebody who had a really strong opinion about how this was to be done, so say they believed they were the mathematical genius in the group and they had the pen then it could have all happened differently.	**Group Situation**
Between 13:24 and 14:58, the group agree that "the pen is important" and/or "the scribe is important". The recall resumes with 04:31 to 04:36 in T-AP 1 ("Okay, right, I'm just thinking of a way of writing it down so that we know we're systematically…"). There is one comment about cumulative speech ("Mainly cumulative, the comments…"), demonstrating that group members are considering the Talk Framework. One of the group members, scanning ahead in the transcript, notes a comment from one of her peers.		
68 – 14:58	Somebody at the bottom of the third page…erm…somebody says, yeah, 'I've done it too big as well' so that's questioning what they've done. ["Yes"] …for the group to comment on and then there's somebody here commenting on how they're sort of feeling: 'I'm just going with the group, really…'	**Verbalisation** **Group Situation**
71 – 15:25	And then there's somebody… "I'm getting confused" and then somebody here says "well, if I was doing this by myself, this is what...you know I'd be doing what we're doing" and it's kind of…group…and it's important to us that we're doing it collaboratively so…so we're…yes, you know, we are doing it collaboratively… That's interesting, isn't it? Yes, we are doing it collaboratively…"that's the way I'd do it". [Laughter] But you know.	**Group Situation**
73 – 15:53	I think…that's quite…I think…that's quite a significant bit of exchange there.	**Mathematical Strategies** **Group Situation**

<table>
<tr><td colspan="3">At 16:38, the group listen to 04:52 to 04:58 in the original recording ("…if I was doing this all by myself…I would be doing what we're doing…").

At 17:26, the group listen to 05:29 to 05:39 in the original recording ("…and addition is the same both ways…").

The conversation until 19:53 largely focuses on identifying voices in the recording.</td></tr>
<tr><td>91 – 19:53</td><td>There was…there was…a point where there was that growing awareness that there was a different way of doing it and we were trying to voice it in different ways.</td><td>Group Situation

Mathematical Strategies</td></tr>
<tr><td>93 – 20:10</td><td>Not wanting to be wrong though…</td><td>Group Situation</td></tr>
<tr><td>97 – 20:48</td><td>But you're conscious that…that…erm anything that you might think, there might be somebody who's got a better thought…</td><td>Group Situation

Mathematical Strategies</td></tr>
<tr><td>98 – 20:53</td><td>You don't want to take the group in the wrong direction. You might be the one that says okay we'll do it this way but then that's wrong…</td><td>Group Situation

Mathematical Strategies</td></tr>
<tr><td colspan="3">Between 20:59 and 27:21, there is further talk about the use of the pen, and also about verbalisation, with reference to the Talk Framework.</td></tr>
<tr><td>144 – 27:21</td><td>By having a scribe who's got the pen, you are putting a gap, aren't you, between what's in your head and what's going down on the paper so you're adding something… I then have to communicate what I'm thinking to…to you for you then to put it down with the pen.</td><td>Group Situation</td></tr>
<tr><td>146 – 27:48</td><td>Is that…I mean, is that a skill that's to do with maths or is that some kind of separate skill, do you see what I mean?</td><td>Group Situation</td></tr>
<tr><td>155 – 28:46</td><td>It's a really valuable process, isn't it, actually, even though we didn't arrive at the answer as quickly…erm…working through it as a group…erm…we've probably learned more…I think, anyway.</td><td>Group Situation</td></tr>
<tr><td colspan="3">Between 28:46 and 32:44, the group largely listen to the T-AP 1 recording with questions such as "What's happening there?" from the interviewer to prompt reflections such as "referring back to the question" (32:46). There is some discussion of the usefulness of referring back to the question as a strategy. The SRI concludes, again, with a focus on the control of the pen.</td></tr>
</table>

Appendix 4

SRI 2 transcript extract – with Talk Framework coding as appropriate.

<table>
<tr><th>Time and Turn</th><th>Comment</th><th>Focus

(Talk Framework Coding – where applicable)</th></tr>
<tr><td colspan="3">SRI 2 begins with an Interviewer's Monologue similar to SRI 1. The discussion begins immediately with the first annotation made on the screen (via Livescribe), noting at (02:05) that the first thing written is "25 beads".</td></tr>
<tr><td>4 – 03:09 (Interviewer)</td><td>So…what's happening here?</td><td>Mathematical Strategies</td></tr>
<tr><td>5 – 03:13</td><td>Well, I've made...I think...there's a... I think... I've made a suggestion but not...erm…because I can remember...not factoring in the...the number of beads that we've got. [Agreement] So, erm...so...it doesn't work. What I've said doesn't work because you end up with too many beads...erm...for whichever way we're working...erm...so...so... somebody's quite asser...quite...I don't mean assertive in a negative...in a negative, critical way...but somebody does...erm...point out that that's not going to work. And I immediately know that it isn't going to work.</td><td>Mathematical Strategies</td></tr>
<tr><td colspan="3">There follows discussion as to why this is or is not (the group agree it is not) disputational.</td></tr>
<tr><td>9 – 04:40</td><td>I think writing "25 beads" [on the Livescribe pad] and…the thing of abacus – the hundreds, tens, units – I think that's a really valuable thing to do.</td><td>Mathematical Strategies</td></tr>
<tr><td>10 – 04:51 (Interviewer)</td><td>The reason I stop it here is there are three beads on the abacus as drawn and things sort of stop there. No more beads are ever drawn on the abacus. [Laughter] So, so, we start with this diagrammatic and there's things going on and things being written potentially underneath the abacus to indicate how it works, and then what happened, was it a sense that 'oh well, everyone knows how an abacus works so I needn't bother…' or…what causes that?</td><td>Mathematical Strategies

Group Situation</td></tr>
</table>

11 – 05:21	Because you couldn't…you can't draw the beads on…[partly inaudible]…full representations…so it's easier to kind of just note…	**Mathematical Strategies** **(Exploratory Encoding)**
Between 05:28 and 05:54, a discussion about the pros and cons of drawing the beads on the abacus. (05:53: "You could, but we realised we didn't need to, I guess" – further explanation of strategy not evident in the original T-AP recording and coded here as exploratory encoding).		
19 – 05:54	That's the visual representation…	**Mathematical Strategies**
21 – 05:59	That's like when you're doing a mental maths question and you're trying to get the important details before you start solving, you need to...	**Mathematical Strategies** **(Exploratory Encoding)**
22 – 06:04	You need something to h...something you can...well, personally, something you can see to hang your thoughts on. It's a bit like that question that we did on the...	**Mathematical Strategies** **(Cumulative)**
23 – 06:15 (Interviewer)	The practise one?	**Mathematical Strategies**
24 – 06:17	Not as a group. That we did individually.	**Mathematical Strategies**
26 – 06:23	Do you know, the first thing we did was...was...to draw it and if you'd have asked me to...I...I worked backwards from my drawing and I couldn't...I could not have done that [inaudible]...	**Mathematical Strategies** **(Exploratory Encoding)**
27 – 06:38 (Interviewer)	**Discussion of encoding, then:** The diagrammatic stuff stops very early and I just wanted to...	**Verbalisation** **Procedure/ Mathematical Strategies**
28 – 07:14	I think it's because...erm...we realised the...erm...fact that there was a maximum number of beads that you could have on each point.	**Mathematical Strategies** **(Exploratory Encoding)**
29 – 07:25	Some people were saying that they weren't sure about the...the presentation. It was confusing for some people so we decided to abandon it.	**Mathematical Strategies** **Group Situation** **(Cumulative)**

35 – 07:47	Do you not think, though, that once…we'd done the thing about you have to use all 25 and therefore... Do you not think we realised that actually the...to me...the...then you're immediately thinking "Oh, actually, it's going to be quite a small number of numbers, not a big number" whereas initially we started off... somebody's going...[overlaps with "I'm thinking a lot!"] And then, suddenly, because you have this point where you think...	**Mathematical Strategies** **(Exploratory Encoding)**
36 – 08:15 (Interviewer)	You see what the maximums have to be.	**Mathematical Strategies**
37 – 08:16	Yeah.	**Mathematical Strategies**
38 – 08:17	You've realised.	**Mathematical Strategies** **(Cumulative)**
39 – 08:17	So then, you think "oh actually it's not going to be that many".	**Mathematical Strategies** **(Exploratory Encoding)**
40 – 08:18 (Interviewer)	If I've only got 25, I can only make…	**Mathematical Strategies**
Between 08:21 and 09:23, the group listen back to the discussion from 02:09 to 02:49 in T-AP 2 ("I don't understand how it makes a three digit number. Do I just not know how an abacus works or something?). At 09:23, the group watch as the numbers are written on the screen.		
46 – 10:03	Why didn't...We had 997, 799...Why didn't we have...erm...979 at that point?	**Mathematical Strategies** **(Supercumulative)**
48 – 10:14	And then we've done 898, 889 and not done 988...	**Mathematical Strategies** **(Supercumulative)**
The group are asked why this might be so, particularly after spending time in the original task talking about working systematically.		

52 – 10:38	Yes… If you're thinking along the lines that the way we were going at this was influenced [by] what's happened in the previous question...then...and I'm, like, picking up straight away on the fact that we didn't do that and you... If we had been working on the idea of being systematic, we surely would have done that. There must have been something going on that was stopping us.	**Mathematical Strategies** **(Supercumulative)**
53 – 11:05	But... people don't naturally always think systematically...do they? I mean, I...I feel happier once I am but sometimes your thoughts just tumble out.	**Mathematical Strategies** **(Cumulative)**
55 – 11:19	I think I was writing for part of this and I think I was just writing down what people said.	**Group Situation (bearing on Mathematical Strategies)**
Between 11:23 and 12:28, there is – as with SRI 1 – a discussion around the way in which having the pen impacts upon the thinking on the task.		
63 – 12:28	I...I...no, I'm just amazed...I'm amazed that at this point nobody said...woah...nobody said 979.	**Mathematical Strategies** **(Supercumulative)**
Between 12:49 and 13:45, the group listen to the recording and watch as the numbers are written down (9, 8, 8 appearing at 13:45). At 14:13, the recording reaches 04:38 in the original T-AP 2 ("why can't we do anymore steps…?").		
69 – 14:13 (Interviewer)	So, what's really interesting is you get to that stage and you're not yet sure why it might be the end... [Recording continues with "Why can't we do 7,8,9 then?" at 04:56] Now, who asks that question? I think I know who asks that question - "why can't we do 7,8,9?".	**Procedure (as focused on who asked the question)**
70 – 14:27	Probably me, I don't know.	**Procedure**
71 – 14:29 (Interviewer)	…that suggests that the group are working without...	**Group Situation**
72 – 14:32	...and we've left somebody behind, haven't we?	**Mathematical Strategies** **Group Situation**
Discussion about where exactly this happened, using the transcript to identify the point.		
76 – 14:40	Well, I just think some people were kind of...We were split about how...about how to work it out from the start; it never really came together and I don't know whether that was because of the pen...let's blame the pen!	**Group Situation**

79 – 15:01	Don't I say something about...oh, I'm a bit lost with this transcribe here...Don't I say something about this business of having more than...I think I felt at the time...that...oh yeah, here it is...I think what I said at the beginning has probably thrown you off. I felt that what I'd said to start with had stopped you understanding because I didn't kind of explain it or...	**Mathematical Strategies** **Group Situation**
The group agree that the key question was "do I not understand how an abacus works or something?" (02:09 in T-AP 2, heard at 08:33 in SRI 2)		
82 – 15:48	We don't clarify that at all.	**Group Situation** **(Cumulative)**
Between 15:50 and 16:26, some queries about whether there was another piece of paper on which it could have been recorded – there wasn't.		
87 – 16:26	Do you not think that there was a sort of...erm... point here where we'd done that and, to me, there was...there was a point where people realised that it was...the crucial thing for me was the fact that there was this business of having 25 beads...the fact that that meant because you couldn't have 10 on a pole...that there was a particular number...	**Mathematical Strategies** **(Exploratory Encoding)**
91 – 17:01 (continuation from above)	And as soon as you've made that...that leap, if you do make that l...if you're one of the people that makes that...that mental leap, then that's why that becomes redundant because in your mind you know you don't need that.	**Mathematical Strategies** **(Exploratory Encoding)**
Between 17:17 and 19:13, there is some discussion as to alternative ways of working that might have ensured that all understood the task – this ends with the interviewer reflecting on thinking-aloud: "I'm not sure actually that's what happened…"		
107 – 19:13	And also, the nature of the exercise, there wasn't anything...we were supposed to be working collaboratively...there wasn't anything there that explicitly said "and at the end of this problem every member of the group must understand the process that you got there by". There is nothing to say, you know, if people can't keep up with the people who got it that we had to bring them with us.	**Group Situation**

Between 19:37 and 20:02, there is a brief discussion about how it might be possible to “bring people with them” in a group discussion, ending with the observation (at 19:55) that “you need to make it a bit more explicit or part of your ethos in the classroom, as well”. **The recording then continues to 21:16 (which is when “Hundreds, Tens, and Units” appears for the first time on the screen – 06:26 in the original T-AP 2 recording).** **The T-AP recording finishes, but the SRI does not.**		
118 – 22:14	Yeah, can I just say I think it is totally vital that if we have put that hundreds, tens and units on the top there at the beginning [Agreement] the people who didn't get it would have got it, straight away, I think. Or at least their chances of getting it would have gone up monumentally.	**Mathematical Strategies** **(Exploratory Encoding)**
119 – 22:34 (Interviewer)	Do you agree?	**Mathematical Strategies** **Group Situation**
120 – 22:35	I can't remember because I can't remember how I did it or understood it because I don't understand it now.	**Mathematical Strategies** **Group Situation**
123 – 22:43 (Interviewer)	What would you have liked...what would...what would have been helpful? What could have been different in terms of [solving the problem]?	**Mathematical Strategies**
124 – 22:47	Probably going away and thinking about it for a moment with one other person…I find it more confusing, to be honest…	**Group Situation**
125 – 22:53	It paralyses your thought processes.	**Group Situation** **(Cumulative)**
There follows a brief discussion about people “throwing their explanations” in. They are asked (at 23:03) what messages there might be from this exercise in getting children to talk in groups when engaged in mathematics.		
128 – 23:20	I think, in groups, I think that's very difficult because what you actually need...the people who are struggling most to get to grips with the problem are the people who you need to speak more because if they aren't...erm...trying to articulate what their struggle is, the other people in the group can't help...can't...erm...can't help them.	**Group Situation**

129 – 23:46	It kind of goes to show...that the speaking and listening, you need to practise that outside of the lesson so you need to kind of model it with them. You can't just drop children into a group in that situation and expect them to contribute.	**Group Situation** **(Cumulative)**
The group are asked about "rehearsal" – would rehearsing help with group work of this kind.		
132 – 24:07	I think that will help because...[inaudible]...I don't have a problem speaking and explaining why I don't understand something - the opposite, really - but y[ou]...what people try and do is explain their way of thinking something through and you get lots of people throwing their ways at you which doesn't really help you. I would just rather have...I'd just think some situations are better in partner situations because it's too much, if you don't understand, it just stresses you out more [agreement] and it just confuses you more and you...you just feel overwhelmed with everybody wanting to explain "oh no, but this way, you'll understand it" so it's not necessarily...	**Group Situation**
135 – 24:51	Do you think...do you think the thing is that because it's in a group and things are coming at you thick and fast...if...if...erm...say in this situation when you were saying "hang on, I don't understand" if...if everybody had taken a step back and if just one...so if...say if XXXXXX had got [her?] and she had stepped forward and said "Okay, XXXX, it's like this" and then we'd have all kept deadly quiet where you'd have like processed that and you could have said "no, that doesn't help me, next one..." and then somebody else had put their idea forward and you'd have had time to think about it and then you could have said "no, that hasn't helped"...	**Group Situation**
136 – 25:25 (Interviewer)	So what you're talking about is maybe structuring the way the group talks, which again is...well, you, in a sense you are...you're saying that there needs to be rules, turn taking, you know...	**Group Situation**

137 – 25:37	But also it's allowing time to think, isn't it? ["And process"] Processing time.	**Group Situation**
139 – 25:49	I think some people just don't find group work as...as helpful as one-on-one work, I think.	**Group Situation**
140 – 26:01	My reaction was to just scurry away into my shell just because there are a lot of people...and maths isn't my strongest subject and I need processing time and all these voices coming in at different directions, and some people were more forceful than others, and so I kind of didn't say an awful lot.	**Group Situation**
This observation leads (at 26:25) into what the interviewer terms "summary questions" – beginning with an overview of revisiting a task as the SRI has required from them.		
142 – 26:39	I think it's really interesting because, you see, my...my memory of this...you know, I felt that what I said at the beginning...[longer pause]...I used the word "stupid" actually...it's not...it's not that it...it was stupid. It...it was just that there was...there was like another bit to it that I didn't articulate that very rapidly and, as you say, internally I added that bit on and there were obviously several other people who, again, without articulating it...	**Group Situation** **Mathematical Strategies**
143 – 27:12 (Interviewer)	They heard it but it wasn't said.	**Group Situation**
144 – 27:15	Yes.	**Group Situation**
145 – 27:15	You heard it but didn't say it.	**Group Situation**
146 – 27:18	So, what, what... The thing... The stupid thing - if...if that is the appropriate word - of what I said at the beginning was that there was a bit...I kind of half articulated something and...	**Group Situation** **Mathematical Strategies**
147 – 27:28	And some people went with it and others didn't.	**Group Situation** **(Cumulative)**
148 – 27:30	Yes, because it...whereas if it had been articulated in its entirety I think there'd have been more chance for the gr...for the whole group to be carried with us. Which is why I returned to it here and said I was conscious about the task.	**Group Situation** **(Cumulative)**
Between 27:45 and 28:08, there is some discussion about whether the students have ever 'revisited' work in this fashion in their practice.		

152 – 28:08	I think they do do it more in other topics; I think maths is probably the least revisited area. Even like when...when you mark the children's work, as in their "sums", you know they're never going to look at that page again in a lot of cases.	**Not Coded**
154 – 29:03	I mean, I feel...my feeling is...[some inaudible comments]...actually I think I contributed more than I thought I did.	**Group Situation**
155 – 29:13 (Interviewer)	Really?	**Group Situation**
156 – 29:15	Yes. I didn't think I said much at all. Well, I don't say an awful lot, to be honest, but I've come away with doing this as a more positive experience than the actual exercise.	**Group Situation**
At 30:15, the interviewer, summing up the discussion, asks for closing thoughts (this includes reflections on the use of *Livescribe*).		
160 – 31:05	I think...I think it's really informative to see...I mean, there are...there are some things in there which really leap out at you, like for example why we didn't get the third number when we were doing combinations of those numbers and things like the writing in of the hundreds, tens and units...	**Procedure**
161 – 31:26 (Interviewer)	And *when* it occurs. Because if I'd had that piece of paper, I'd think "HTU" was written at the start...	**Mathematical Strategies** **Group Situation**
162 – 31:31	At the beginning, yeah.	**Mathematical Strategies** **Group Situation**
163 – 31:33	And it wasn't. It was written very distinctly towards the end because the group had not got a concrete sense of the task, even that late.	**Mathematical Strategies** **Group Situation**
Recording ends.		

Appendix 5

Ethical Approval documentation from the University of Leeds and York St John University.

No ethical approval was required from York St John for the first iteration of the project. Approval was granted from Leeds for both iterations of the project, and from York St John University for the second iteration. These are reproduced below.

Appendix 5.1

Checklist from York St John University, dated 2.7.11, confirming that ethical approval not required from the University for first iteration of the project.

Research Ethical Considerations Screening Checklist

(Adapted from: The Economic & Social Research Council Research Ethics framework 2005)

Title of research:	***'In what ways can dialogue/thinking aloud recorded by digital audio support student teachers' learning and levels of confidence in teaching problem solving within primary mathematics?'***
Name of researcher (applicant):	Mike Hickman
Status (please select):	Undergraduate Student/Postgraduate Student/Staff
Email address:	m.hickman@yorksj.ac.uk
Contact address:	Faculty of Education and Theology, York St John University (Office – QN 111)
Telephone number:	01904 876492 mob: 07960 835230

Please complete the screening checklist below. **Answering yes to any of the questions will require you to explain how the ethical issues raised will be managed and a full application to the relevant research ethics committee will be needed *(links that provide additional information for several questions are provided, if needed)*)**

1. Does the study involve participants who are particularly

vulnerable or unable to give informed consent (e.g. children, people with learning disabilities, your own students) Yes/No
British Psychological Society (BPS)

2. Does the study require the researcher(s) to have CRB clearance? Yes/No
Criminal Records Bureau (CRB)

3. Will the study require the co-operation of a gatekeeper for initial access to the groups or individuals to be recruited? (e.g. students at school, members of a self-help group, residents of a nursing home) Yes/No

4. Will it be necessary for participants to take part in the study without their knowledge and consent at the time? Yes/No

5. Will the study involve discussion of or the disclosure of information about sensitive topics? (e.g. sexual activity, drug use) Yes/No

6. Are drugs, placebos or other substances (e.g. food substances, vitamins) to be administered to the study participants or will the study involve invasive, intrusive or potentially harmful procedures of any kind? Yes/No

7. Will blood or tissue samples be obtained from participants? Yes/No
Royal College of Nursing (RCN)

8. Is physical pain or more than mild discomfort likely to result from the study? Yes/No
Economic & Social Research Council (ESRC)

9. Could the study induce psychological stress or anxiety or cause harm or negative consequences beyond the risks encountered in normal life? Yes/No

10. Is an extensive degree of exercise or physical exertion involved? Yes/No

11. Will financial inducements be offered to participants other than to cover expenses or time involved? Yes/No
British Educational Research Association (BERA)

12. Will the study involve recruitment of patients through the NHS? Yes/No
National Research Ethics Service (NRES) (NHS)

13. Will the study demand participants to commit extensive time to the study? Yes/No

Signed:

Date: ....2.7.11...................

Appendix 5.2

Approval from University of Leeds for first iteration of the project, dated 11.4.11

Research Support
3 Cavendish Road
University of Leeds
Leeds LS2 9JT

Tel: 0113 343 4873
E-mail: j.m.blaikie@adm.leeds.ac.uk

UNIVERSITY OF LEEDS

Mike Hickman
School of Education
University of Leeds
Leeds, LS2 9JT

AREA Faculty Research Ethics Committee
University of Leeds

11 April 2011

Dear Mike

Title of study: **In what ways can dialogue/ thinking aloud be recorded by digital audio support student teachers' learning and levels of confidence in teaching problem solving within primary mathematics?**

Ethics reference: **AREA 10-106**

I am pleased to inform you that the above research application has been reviewed by the ESSL, Environment and LUBS (AREA) Faculty Research Ethics Committee and following receipt of the amendments requested, I can confirm a favourable ethical opinion on the basis described in the application form and supporting documentation as of the date of this letter.

The following documentation was considered:

Document	Version	Date
AREA 10-106 Researcher's response.doc	1	07/04/11
AREA10-106 application form	1	23/03/11
AREA10-106 Appendix 1: Questionnaire	1	23/03/11
AREA10-106 Appendix 2: Digital recording analysis pro forma	1	23/03/11

Please notify the committee if you intend to make any amendments to the original research as submitted at date of this approval. This includes recruitment methodology and all changes must be ethically approved prior to implementation.

Please note: You are expected to keep a record of all your approved documentation, as well as documents such as sample consent forms, and other documents relating to the study. This should be kept in your study file, which should be readily available for audit purposes. You will be given a two week notice period if your project is to be audited.

Yours sincerely

J Blaikie

Jennifer Blaikie
Research Ethics Administrator, Research Support
On behalf of Dr Anthea Hucklesby
Chair, AREA Faculty Research Ethics Committee

CC: Student's supervisor(s)

Appendix 5.3

Approval from York St John University for second iteration of the project, dated 21.2.13

From: Esther McIntosh (E.McIntosh)
Sent: 21 February 2013 12:09
To: Mike Hickman
Cc: Jelena Erstic (J.Erstic)
Subject: RE: Ethical approval

Follow Up Flag: Follow up
Flag Status: Flagged

Dear Mike,

Your applications has been approved at FREC and the approval code is ET/21/02/13/MH.

Please send a copy of your Leeds approval.

All the best with the research,
Esther

Dr Esther McIntosh
Research Fellow and Chair of Faculty Research Ethics Committee
Education and Theology
York St John University
Lord Mayor’s Walk
York YO31 7EX
Managing Editor
International Journal of Public Theology
http://www.brill.nl/ijpt

Appendix 5.4

Approval from University of Leeds for second iteration of the project, dated 16.01.13

Performance, Governance and Operations
Research & Innovation Service
Charles Thackrah Building
101 Clarendon Road
Leeds LS2 9LJ Tel: 0113 343 4873
Email: j.m.blaikie@leeds.ac.uk

UNIVERSITY OF LEEDS

Mike Hickman
School of Education
University of Leeds
Leeds, LS2 9JT

AREA Faculty Research Ethics Committee
University of Leeds

16 January 2013

Dear Mike

Title of study: **'A Talk Framework for Primary Problem Solving: In what ways can dialogue/ thinking aloud recorded by digital audio support student teachers' learning and levels of confidence in teaching problem solving within primary mathematics?'**
Ethics ref **AREA 12-054**

I am pleased to inform you that the above research application has been reviewed by the ESSL, Environment and LUBS (AREA) Faculty Research Ethics Committee and I can confirm a favourable ethical opinion as of the date of this letter. The following documentation was considered:

Document	*Version*	*Date*
AREA 12-054 UNIVERSITY OF LEEDS RESEARCH ETHICS COMMITTEE APPLICATION FORM MH.docx	1	09/01/13
AREA 12-054 Information Sheet - 2nd iteration.docx	1	09/01/13
AREA 12-054 Questionnaire - 2nd iteration.docx	1	09/01/13
AREA 12-054 Information Sheet - revised.docx	1	09/01/13

Committee members made the following comments and suggestions about your application:

- The second and third paragraphs on the information sheet could have less jargon in so that potential participants can see immediately what it is that you are trying to do
- The recruitment should be handled sensitively given your position in the faculty and that when you meet students to talk about the project they are not put 'on the spot' to respond in any way and all responses can be individually and privately managed.
- It would be a good idea to clarify, even more, which activities will be the same for those that take part in the study and for those that don't, and which activities will be specific to participants (for instance in section "What do I have to do?" The second sentence could make this explicit).
- On section "What will happen to me…" it might be helpful clarify which activities will be carried out in group and which ones will be done individually.

Please notify the committee if you intend to make any amendments to the original research as submitted at date of this approval, including changes to recruitment methodology. All changes must receive ethical approval prior to implementation. The amendment form is available at http://researchsupport.leeds.ac.uk/index.php/academic_staff/good_practice/managing_approved_projects-1/applying_for_an_amendment-1.

Please note: You are expected to keep a record of all your approved documentation, as well as documents such as sample consent forms, and other documents relating to the study. This should be kept in your study file, which should be readily available for audit purposes. You will be given a two week notice period if your project is to be audited. There is a checklist listing examples of documents to be kept which is available at http://researchsupport.leeds.ac.uk/index.php/academic_staff/good_practice/managing_approved_projects-1/ethics_audits-1.

Yours sincerely

Jennifer Blaikie
Senior Research Ethics Administrator
Research & Innovation Service
On behalf of Dr Emma Cave
Chair, AREA Faculty Research Ethics Committee

CC: Student's supervisor(s)