Term Structure Modelling of Treasury and Corporate Bond Yields

Lisha Li

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Abstract

This thesis investigates the linkage between the Treasury and corporate bond term structure, and real economic activity during conventional and unconventional monetary policy periods. The first part of the thesis analyses the interaction of macro fundamentals, the credit factor, government yields and credit spreads. I use a joint term structure model of U.S. government yields, and aggregate corporate bond credit spreads. Results indicate a negative relation between government yields and credit spreads, which is influenced by the output gap and the credit factor. More specifically: a positive output gap shock increases government yields through both risk neutral and risk premium channels, and decreases credit spreads through the risk premium. A positive credit factor shock has a slight negative effect on yields through the risk neutral component, and it has a large positive effect on credit spreads through the risk premium.

The second part of the thesis assesses the performance of the shadow rate term structure model on corporate forward rates during the lower bound period, then evaluate the impact of the lower bound on corporate yields. The results indicate that the the shadow rate term structure model better explains corporate yields and credit spreads at long horizons both in-sample and out-of-sample. The lower bound constraints affect short- and intermediate corporate interest rates.

The third part of the thesis examines the effect of the Federal Reserve's asset purchase programs on Treasury yields. Results from the shadow rate term structure model with securities supply factors results indicate that the first and third Fed's purchase programs reduce the 10-year forward term premium by 140 basis points and 80 basis points, respectively. The mortgage-backed securities par supply factor impacts Treasury yields more significantly than the Treasury securities supply factor.

Contents

List of Tables

List of Figures

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Author's Declaration

I declare that this thesis is a presentation of original work and that I am the sole author. This work has not previously been presented for an award at this, or any other, University. All sources are acknowledged as references.

An earlier version of Chapter 2 was presented internally at the seminar in the University of York. An earlier version of Chapter 3 was presented at the 2nd Workshop on Financial Econometrics and Empirical Modelling of Financial Markets, Kiel Institute for the World Economy, Germany; the China Meeting of the Econometric Society (CMES), Fudan University, China; and the Asset Pricing Workshop, University of York, England in 2018.

Chapter 1

Introduction

1.1 Introduction

The bond yield is the discount rate for the expected future income on an n-period bond. Understanding the evolution of bond yields is crucial for financial market practitioners, researchers and policymakers for at least three reasons. First, current bond yields provide information about the economy's future outlook. An extensive literature confirms that yield spreads are useful to predict future economic activity. Second, bond yields are closely related to monetary policy. The central bank sets the target for the short-term interest rate based on the stance of the economy. However, consumers make their consumption decisions based on long-term yields, which are less likely to be controlled by the central bank. Thus, the study of bond yields elucidates how the central bank conducts monetary policy and how the transmission mechanism works. Third, the price of interest rate derivatives is calculated from bond yields models.

Bond yields evolve over time and across different maturities, which implies that they have both cross-sectional and temporal dimensions. According to Diebold and Rudebusch (2013) the time series of bond yields displays three particular features. First, average bond yields increase with maturities. Second, bond yields become less volatile as maturities increase. Third, bond yields are highly persistent and have significant autocorrelations across different maturities. These features require a factor model to capture the dynamic of bond yields. Previous studies mainly employ vector autoregressions (VAR) to capture the dynamics of bond yields over time. However, the VAR is unable to capture the high persistence and nonnormal distribution aspects of bond yields. In contrast, term structure models capture these characteristics of bond yields. This model imposes a no-arbitrage assumption to allow the existence of cross-sectional restrictions. These restrictions ensure that the movement of bond yields of different maturities over time is consistent and shares a similar cross-sectional shape. In addition, the model allows bond yields to be nonnormal and is an affine function of a state vector. The term structure model is widely used due to its tractability and flexibility. A closed-form analytical solution can be derived with a relatively flexible assumption on related state vectors. Treasury yields (or government bond yields), reflect the yield curve dynamic of the government-issued securities. Term structure literature has made sizeable progress in analysing the variation of treasury yields. The literature on the term structure of Treasury yields begins with Vasicek (1997) and Cox et al. (1985). Duffie and Kan (1996) provide a more complete model framework. Duffee (2002), and Dai and Singleton (2002), impose restrictions on model structure, resulting in a more parsimonious structure. Joslin et al. (2011) and Hamilton and Wu (2011) propose alternative identification schemes and greatly acilitating estimation. The macro-finance term structure literature begins with Ang and Piazzesi (2003), investigating the relationship between the dynamics of Treasury yields and macroeconomic variables.

However, the benchmark Gaussian affine term structure is unable to explain the recent behaviour of interest rates in major economies. The term structure of corporate yields, and related credit spreads is less understood. Corporate yields measure the yield curve dynamic of the corporation-issued private securities. The change of corporate yields affects real borrowing costs faced by businesses and households. While Treasury securities have high liquidity and are unlikely to default, corporate securities have higher default risk and liquidity risk. The credit spread is the wedge between the corporate yield and the Treasury yield with the same maturity. Credit spreads reflect the extra return required by investors to compensate for higher risk in the corporate bond market. Therefore, the study of corporate yields and credit spreads is necessary. Previous literature mainly focuses on credit spreads. The structural model is a widely used approach , e.g., Black and Cox (1976) and Longstaff and Schwartz (1995). However, structural models have difficulty in fitting the data. Pedrosa and Roll (1998), and Campbell and Taksler (2003), find that a considerable variation of credit spreads is systematic; and it is thus more meaningful to study the aggregate indices. The reduced-form model is developed by Jarrow et al. (1997), and Duffee (1999). Many studies rely on latent variables and therefore underlying economic meaning is ambiguous. Further research is needed to investigate Treasury yields, corporate yields and their interactions with economic activity, especially during the recent unconventional monetary policy period after the global financial crisis in 2008.

In Chapter 2, I investigate the interaction between macro fundamentals, financial variables, government yields and credit spreads. I use a Gaussian joint affine term structure model of government yields and credit spreads with observed macroeconomic variables and a credit factor, which is denoted as Model 1. In addition, considering the impact of the credit spread puzzle on credit spreads, I incorporate the first principal component of credit spreads, as a proxy of the unknown common factor stated in the credit spread puzzle literature, and develop Model 2 on the basis of Model 1. The importance of credit spreads as a leading financial indicator for future real economic activity is supported by the literature. Theoretically, the predictive content of credit spreads is explained by the financial accelerator mechanism developed by Bernanke and Gertler (1989), and Bernanke, Gertler and Gilchrist (1996, 1999). Through this mechanism, a deterioration in the economy leads to reduced profits in the financial sector, and increases the borrowing costs of investors, which causes decreased investment, spending and production, thus negatively impacting the future economy. Empirically, recent work shows a disconnect between the macro and finance literature. On one hand, most finance studies use latent factors from the yield curve to gauge the variation of credit spreads, which leaves the underlying economic meaning ambiguous. On the other hand, most macro studies use the regression approach to explore the linkage between credit spreads and real economic activity. The result of this approach depends on the choice of specific credit spreads.

This chapter is partly inspired by the macro-finance term structure literature, which incorporates macroeconomic variables into the term structure of the yield curve to investigate the fundamental determinants of interest rates. This chapter is also inspired by the joint term structure literature, which has consistency in explaining the dynamics of different interest rates with the same variables and can compare how these variables affect different yields. For example, Lemke and Werner (2009) use the joint model framework of government yields and equity returns.

I use four observed state variables. The first three variables measure real economic activity, which fits the government yields well. The output gap is the difference between real GDP and potential GDP. The inflation forecast is the median one-quarter ahead forecast from the Survey of Professional Forecasts. The monetary policy is the residual of the Taylor rule regression on the above output gap and the inflation forecast. The fourth variable is the credit factor, which captures the information from the corporate bond market. This variable uses the GZ index as a proxy, constructed by Gilchrist and Zakrajsek (2012). In addition, the first principal component of credit spreads is used as a proxy of the unknown common factor to capture the variation of credit spreads, as the credit spread puzzle literature suggests. Government yields are constructed from the Gurkaynak et al. (2007) dataset. Corporate yields (rated A or above) are constructed from the U.S. Department of the Treasury High-Quality Market (HQM) dataset. Credit spreads are obtained from the difference between corporate yields and government yields with the same maturity. The time spans from January 1984 to December 2007. The model is estimated using the maximum likelihood. Results indicate that macro determinants mainly dominate government yields, while the credit factor has a substantial effect on credit spreads, especially on long maturities. The yield curve decomposition result demonstrates that macro fundamentals positively impact government yields through both future expected short rate and risk premium components. The credit factor has a large positive effect on credit spreads through the risk premium component. The output gap has a strong negative impact on credit spreads through the risk premium component.

It is notable that in-sample estimation results show that while Model 1, the macro factors and credit factor only model fit government yields term structure well, Model 1 shows a poor performance in fitting credit spreads, compared with Model 2. This result implies the effect of the credit spread puzzle. In addition, the out-of-sample forecast results show that although Model 1, a term structure with only macro and financial factors, similar to Ang and Ulrich (2012), performs well in-sample, the out-of-sample performance is poor. Model 2 shows a significantly better performance than Model 1 in fitting credit spreads both in-sample and out-of-sample. These results suggest that a term structure model with only observed macro and financial factors cannot capture the variation of government yields and credit spreads, from in-sample and out-of-sample forecast perspectives. The incorporation of principal components, or alternatively, as the large term structure model literature suggests, latent variables, is needed to explain the variation of yield curves.

During the period December 2008 to December 2015, the Fed's target for the federal funds rate stuck at zero. To cope with the limitation imposed by the zero lower bound (ZLB), the Fed conducted unconventional monetary policy, in particular, large-scale asset purchase programs (LSAPs) and active use of communication, providing explicit forward guidance on the likely future policy. Unsurprisingly, measuring the impact of these tools on the yield curve in the new environment is challenging. Chapter 3 and Chapter 4 investigate the modelling of the term structure of yield curves during this zero lower bound period, and analyze the monetary policy implications on yield curves.

Chapter 3 assesses the performance of the shadow rate term structure model in terms of corporate yields, an important riskier interest rate, which reflects real borrowing costs in the financial market. In addition, the impact of the lower bound on corporate yields is evaluated.

Much effort has been devoted to assessing the monetary policy transmission on Treasury securities. Previous literature includes Gagnon et al. (2011), Hamilton and Wu (2012a), and D'Amico and King (2013). On the other hand, some attention has been paid to examining the monetary policy transmission on corporate yields. Wright (2012) uses a structural VAR; Krishnamurthy and Vissing-Jorgensen (2013) use event study; Gilchrist and Zakrajsek (2013) use a heteroskedasticity-based approach. However, these studies encounter an econometric identification difficulty in assuming endogenety and exogeneity. In addition, unlike term structure models, these approaches cannot draw conclusions about the monetary policy impact on the whole term structure. Therefore, a reliable term structure model is needed to model corporate yields during the zero lower bound period, and to evaluate the possible monetary policy implications on the whole term structure based on the model.

In this chapter, I adopt the shadow rate term structure model that has been used in the recent term structure literature, and evaluate the performance of the shadow rate term structure model in fitting corporate yields during the zero lower bound period. I adopt a shadow rate joint term structure of treasury forward rates and corporate forward rates, and compare its performance with the standard Gaussian affine term structure model both in-sample and out-of-sample.

The forward rate is the yield at time t for a loan starting at the future period t+n and maturing at t+n+m. The zero-coupon bond yield is an equally weighted average of the related forward rate, and these two yields alternatively describe the same curve. According to Gurkaynak et al. (2007, p2294), forward rates can summarise the yield curve more informatively. Gürkaynak et al.(2005), for example, discuss the response of bond yields to the macroeconomic news regarding forward rates.

The yield curve literature widely adopts a Gaussian affine term structure model as a benchmark. Gaussian models assume that bond yields are linear with a set of Gaussian state variables, and that the short rate follows a Gaussian diffusion. However, the assumption of the Gaussian process indicates that government interest rates can go below zero. A negative interest rate implies that market practitioners can take arbitrage opportunity by borrowing funds and hold them as physical currency, which violates the economic theory. During the zero lower bound period, the interest rate is close to zero and the GATSM cannot prevent negative interest rates. In contrast, the shadow rate term structure (SRTSM) captures the performance of interest rates at the ZLB. The SRTSM assumes that the short-term rate is a maximum of the shadow rate and a lower bound. If the lower bound is binding, the shadow rate can contain information about the state of the economy, while the short rate is above zero. Black (1995) proposes a SRTSM. Kim and Singleton (2012) extend it into a multi-factor model. Wu and Xia (2016) propose a tractable approximation in discrete time.

I use 6-month Treasury and corporate forward rates constructed from the same dataset as Chapter 2. The sample spans from January 1990 to March 2017. The model is estimated with the maximum likelihood and extended Kalman filter methods. I specify a model with four latent variables, the first two are common factors; the third is a Treasury specific factor; and the fourth is a corporate specific factor. The advantage of the joint framework is to evaluate the interaction between Treasury and corporate forward rates, and to measure the model-implied credit spreads.

The results indicate that during the zero lower bound period, the shadow rate term structure model has a significantly better in-sample and out-of-sample performance than the standard Gaussian affine term structure model in fitting corporate yields at long horizons. During the in-sample period, the root-mean-square error of the shadow rate term structure model for 10-year corporate yields is 35 basis points smaller than the Gaussian affine term structure result. During the out-of-sample forecast period, the shadow rate term structure model also shows a much better performance in predicting long-term corporate yields, and the root-mean-square error for 7-year and 10-year corporate yields of the Gaussian model is twice as large as the shadow rate model.

The results also show that during the ZLB period, the short-term up to 2-year intermediate term corporate shadow rates show the relevance of the lower bound. This may due to the impact of the common factors, and reflects the impact from the Treasury bond market. The 10-year corporate shadow rates seems to show relatively small relevance of the lower bound. This may due to the large impact of the corporate bond specific factor, which reflects relatively small influence from the Treasury bond market.

It is notable that, as shadow rate literature has stated, e.g. Christensen and Rudebusch (2015), and Krippner (2015), the shadow short rates are quite sensitive to the value of the lower bound parameter. This causes the relatively poor performance of the shadow rate model in fitting 6-month and 1-year Treasury and corporate yields. As suggested by Golinski and Spencer (2019), this problem can be solved by fixing the lower bound parameter at zero. In addition, a relatively large difference between the fitted SRTSM and the observed yields can be explained by the persistent fitting errors, which is a common statistical issue in term structure literature.

Chapter 4 of this thesis evaluates the impact of the Fed's purchase programs under the SRTSM. Before the financial crisis in 2008, the Fed used the federal funds rate as a primary policy tool to influence interest rates. During the lower bound period, the federal funds rate was approximately zero. To boost the economic recovery, the Fed conducts unconventional monetary policy, particularly large-scale asset purchase programs (LSAPs) as an alternative instrument. The program increases the Fed's holding of longer-term Treasury securities and agency mortgage-backed securities, and aims to lower the longer-term interest rates.

Considerable progress has been made in evaluating the effect of this unique monetary policy instrument. Theoretically, two channels explains the underlying effect of the LSAPs. The first is the portfolio balance channel. Tobin (1961), Modigliani and Sutch (1966) state that the investor prefers bonds with certain maturities. Vayanos and Vila (2009) develops a theoretical term structure model on the basis of this preferred-habitat literature, which provides a rationale for LSAPs. LSAPs can reduce long-term government debt held by the private sector. This mechanism states that bond yields will decrease through risk premium. The second is the signalling channel. Bernanke et al. (2004) suggest that this channel works through adjusting investors' expectations of future short-term rate. Since monetary policy can work through different channels, the LSAPs needs further investigation. The fast-growing empirical literature mainly uses event study, time series, or panel regression, and the combination of term structure model with event study. However, these studies are influenced by small sample bias, endogenous problem, and thus provide little explanation for the impact of LSAPs on the term structure of bond yields.

Based on the theoretical model of Vayanos and Vila (2009), Li and Wei (2013) provide an interesting no-arbitrage term structure model that incorporates debt supply variables to measure the effect of LSAPs. However, they estimate their model based on the normal period (i.e., before the financial crisis),which implies that the result cannot describe the relationship between supply variables and bond yields during the ZLB period. Based on Li and Wei (2013), I estimate the interaction between Treasury forward rates and LSAPs related supply factors under the shadow rate term structure model. The one-month Treasury forward rates are constructed by the dataset in Chapter 2. Five state variables are used in the model: The first two are yield latent variables. Three observed supply variables measure the supply of Treasury securities, mortgage backed securities and the average duration of MBS. To ease the estimation burden and be consistent with economic theory, I make some parsimonious assumptions similar to Li and Wei (2013) and Ihrig et al. (2018). The latent variables are estimated with a Kalman filter. The shadow rate term structure is estimated with the extended Kalman filter. The sample is from December 1996 to March 2018. Results indicate that the SRTSM fits the Treasury forward rates well and provides an economically meaningful explanation for the relationship between interest rates and the supply factors. The yield curve decomposition result presents a positive relationship between supply factors and the forward term premium. The counterfactual analysis result implies that LSAPs substantially reduces the mid- and long-term forward term premium. The first purchase program has the most significant impact on the long-term forward term premium, which reduces 10-year forward risk premium by about 140 basis points.

It is notable that to simplify the counterfactual analysis, I treat the first three LSAPs as a one-period shock. However, for a more precise measurement, announcement effects should be considered and the impact of the LSAPs can be measured as a sequence of shock similar to Li and Wei (2013) Section 5.3. In addition, similar to Chapter 3, the short-term fitted SRTSM rates show relatively poor performance in some years, which is due to the sensitivity of the shadow short rates to the value of the lower bound parameters. This chapter fixes the lower bound parameter at 0.25% as Wu and Xia (2016). Future work can be conducted to set the lower bound parameter at zero to solve this problem.

1.2 Notation

This thesis adopts a notation similar to the standard notation proposed in Abradir and Magnus (2002). Throughout the thesis, vectors are denoted by lower-case symbols in bold font (e.g. $\boldsymbol{a}$), matrices are denoted by uppercase symbols in bold font (e.g. $\boldsymbol{A}$), scalars are denoted by symbols in normal font (e.g. a or A). Stacked vectors are also denoted by uppercase symbols in bold font (e.g. $\boldsymbol{A}$).

In addition, bold font $\mathbf{0}_i$ denotes a zero i-dimensional vector, and bold font $\boldsymbol{I}_i$ denotes an i by i identity matrix.

Chapter 2

Government Yields and Credit Spreads

2.1 Introduction

Corporate bonds usually trade at higher yields than government bonds of the same maturity. This corporate-government yield spread is partly due to credit risk and is thus referred to as the credit spread. The evolution of the credit spread reflects a link between credit markets and the economy and serves as a leading financial indicator regarding the future economic outlook. This predictive content of credit spreads is supported by a large literature; see Gertler and Lown (1999), Stock and Watson (2003), Mueller (2009) and others. Since the most recent financial crisis, the credit spread as a measure of strain in the financial sector has received more attention.

From a theoretical perspective, the importance of the credit spread as a financial indicator is based on the financial accelerator theory is motived by Modigliani–Miller theorem (Modigliani and Miller, 1958), and developed by Bernanke and Gertler (1989), Bernanke, Gertler and Gilchrist (1996, 1999). The main concept in this theory is the external premium, which is the difference between the cost of external funds and the opportunity cost of internal funds caused by financial frictions. Through the financial accelerator mechanism, an adverse change in economic activity causes profits to decline in the financial sector, increases the premium, which then increases the cost of outside borrowing, reducing investment, spending and production; consequently reducing the economic outlook. The literature shows that changes in the external finance premium are generated by changes in economic fundamentals, e.g., real productivity shocks, monetary policy shocks, or even shocks from the financial sector, which can adversely affect the future state of the economy. (See Gertler and Karadi, 2011, Christiano et al., 2014). Since the external finance premium is unobserved, the credit spread can be taken as a good proxy.

While the literature has made substantial progress in understanding the term structure of yields in government-issued government bonds, the literature on credit spreads is limited. In general, there are two main models that have been widely adopted to identify the variation of credit spreads. One is the structural model. This model assumes a stochastic process for the evolution of firm value, and that default occurs when the firm value falls below a certain boundary. Further explanation can be seen in Collin-Dufresne et al. (2001). Related literature originates from Black and Scholes (1973), and is developed by Black and Cox (1976), Longstaff and Schwartz (1995), and Collin-Dufresne and Goldstein (2001). However, the structural model has limited success in matching the empirical data, and some empirical work provides evidence that a large variation of credit spreads is systematic instead of firm-specific. Hence, it is more meaningful to analyse the aggregate credit spread indices instead of firm-specific credit spreads (see Pedrosa and Roll, 1998, Campbell and Taksler, 2003, Duffie et al., 2009). The second model is the reduced-form model. This model assumes that default is a stochastic event following a hazard-rate process, which provides a simplified method for the estimation of credit spreads. This approach is more intuitive and allows information from economic fundamentals and financial sector to impact the price of credit spreads. Related literature includes Jarrow and Turnball (1995), Jarrow et al. (1997), Duffie and Singleton (1999) and Liu and Spencer (2013).

On the one hand, recent empirical literature uses either the reduced-form or the structural model to study the variation of the credit spread term structure (see Christiansen, 2002 for a summary). However, many of these studies rely on latent variables extracted from the yield curve and do not have a clear explanation of the underlying economic meanings. On the other hand, several studies use regressions to analyse the relationship between the credit spread term structure and economic variables (see Carey, 1998, Elton et al., 2001, Altman et al., 2005). However, the results varies regarding the choices of the explanatory variables and specific credit spreads. It is unclear how variables influence the whole term structure.

It is notable that Collin-Dufresne et al (2001), Collin-Dufresne et al (2002), Huang and

Huang (2012) and others find that the explanatory power of determinants that should in theory explaining credit spreads, e.g. credit risk is quite limited. As stated by Collin-Dufresne et al (2001), a single unknown common factor is the main driver of credit spreads, and this common factor cannot be explained by widely used macroeconomic and financial factors. Driessen (2005), Amato and Luisi (2006), and Mueller (2009) use a latent variable to capture the information of this unknown common factor. Mueller (2009) concludes that the unknown common factor is related to the index of tighter loan standards, and can be treated as proxy for credit condition.

To investigate the linkage between macro fundamentals and financial sector variables, government yields and credit spreads, in this chapter I jointly estimate the determinants of government yields and credit spreads using a Gaussian term structure model with observed macro fundamental and credit factors, which is denoted as Model 1. In addition, to test the impact of the credit spread puzzle on the credit spread yield curve, I incorporate the first principle component of credit spreads as a proxy of the unknown common factor, and match credit spreads with this new factor, the macro factors and credit factor in a Gaussian term structure model based on Model 1, which denoted as model 2. This chapter is partly motivated by the large macro-finance term structure literature, which incorporates economic and financial sector variables into modelling the term structure of government yields (e.g. Ang and Piazzesi, 2003, Diebold et al., 2005, Dewachter and Lyrio, 2006). The joint term structure model has the advantage of consistently capturing the dynamics of different interest rates driven by same state variables, and can compare the effect of the state variables on different types of yield curves (similar framework can be seen in the joint model of government yields and equity returns, e.g., Lemke and Werner, 2009, Ang and Ulrich, 2012, Kick, 2017). I use four observed state variables. The first three variables measure macroeconomic fundamentals and perform well in capturing the large variation of the government yields (Ang and Ulrich, 2012). The first variable is the output gap. The second variable is the inflation forecast. The third variable is the monetary policy shock, which is obtained as the residual from the Taylor rule regression on the output gap and inflation forecast. The credit factor uses the GZ index as a good proxy of corporate bond market volatility. All state variables are measured at the quarterly frequency, demeaned and divided by four. Government yields are constructed from Gurkaynak et al. (2007) dataset. The credit spread is constructed as the difference of corporate bond yields minus government bond yields with the same maturity. Due to the limited availability of aggregate corporate bond yield data, here I use corporate yields constructed by the U.S. Department of the Treasury High-Quality Market (HQM, hereafter) zero coupon corporate yields dataset. Details of the data construction are in Section 2.2. To match the frequency of the state variables, the yield curve is also quarterly.

In the first step, I conduct a preliminary analysis using principal component analysis to evaluate whether the observed state variables can explain the variation of Government bond yields and credit spreads. Secondly, I jointly estimate the term structure of government yields and credit spreads using a maximum likelihood function, and then decompose the model-implied yield curves into expectations of future short rate and risk premium components to examine the effect of state variables on the yield curve. Model 1 closely fits the government yield term structure. However, Model 1 shows a poor performance in fitting credit spreads compared with Model 2, which implies the existence of the credit spread puzzle.

The results show that government yields are mainly dominated by the macro variables, whereas the credit factor plays an important role in credit spreads, especially in the longer-term maturities. Specifically, the output gap has a positive downward-sloping effect on the government yield term structure. The effect on credit spreads is negative. This result implies that a positive output gap can generate a negative relationship between government yields and credit spreads by increasing the government yield curve while decreasing the credit spread. The inflation forecast and monetary policy shock positively affect government yields and credit spreads. The credit factor has a slight negative effect on government yields. The effect on credit spreads is positive and becomes larger in longer-term maturities. This result implies that a positive credit factor shock may also generate a negative relationship between government yields and credit spreads. These findings are consistent with Wu and Zhang (2008).

To further evaluate the effect of the macro and credit factors, I conduct a yield curve decomposition. The result of government yields decomposition demonstrates that macro variables positively impact on both expectations of future short rate and the risk premium component, and that the inflation forecast has the largest impact on both components. The negative effect of the credit factor is mainly through its impact on the expectations of the future short rate component. This finding comports with Cúrdia and Woodford (2010), who suggest incorporating the credit spread in the standard Taylor rule in order to measure financial conditions. The results for credit spreads indicate that the inflation forecast and monetary policy shock mainly impact credit spreads through expectations of the future short rate. The output gap has a strong negative effect on the risk premium component. The large effect of the credit factor is mainly through the risk premium component as well.

I also use an out-of-sample forecast to evaluate the performance of Model 1 and Model 2 in the out-of-sample period. The results show that although a term structure model with only observed macro and financial factors, similar to Ang and Ulrich (2012) performs well in-sample, the out-of-sample performance is poor. Second, compared with macro factors and the credit factor only model (Model 1), the inclusion of the principal component of credit spreads (Model 2) improves the forecast capability of credit spreads.

This chapter provides empirical evidence for evaluating the interaction of macro fundamentals, the credit factor, government yields and credit spreads. One of the most closely related works is Ang and Ulrich (2012) also use the same set of macro variables under a joint model framework of government bonds, real bonds and expected equity returns. However, their work has not evaluated the relation between government yields and credit spreads. The other closely related work is Wu and Zhang (2008). They use a Gaussian term structure model of government yields and credit spreads with observable macroeconomic and financial market variables. Whereas I estimate the term structure of government yields and credit spreads jointly, which can more consistently evaluate the effects on the whole term structure, while Wu and Zhang (2008) estimate each yield curve separately. In addition, I use a credit factor which better captures the information directly from the corporate bond market. However, Wu and Zhang (2008) use financial market volatility constructed from the stock index options. I also conduct the model-implied yield curve decomposition to better evaluate how state variables affect different yield curve components.

The remainder of the chapter is organised as follows. Section 2.2 describes the data and conducts a preliminary analysis. Section 2.3 discusses the model setup, model specification, and estimation method and estimation results. Section 2.4 concludes.

2.2 Preliminary Analysis

Many studies, e.g., Eom et al. (2004), Huang and Huang (2012), examine determinates of credit spreads using a term structure model. Most of them rely on latent factors extracted from the yield curve. Drawing on Mueller (2009), I first conduct a principal component analysis on government yields and credit spreads to determine the number of principal components needed to capture the yield curve variation. I then treat the selected principal components as a gauge of the relevant information from the term structure of government yields and credit spreads, and regress the principal components on the macro fundamental variables and the credit factor to analyse how much variation of the yields can be explained by these observed variables.

2.2.1 Data Description

The observed state variables include macro variables and the credit factor. The yield curve describes government bonds and credit spreads data. The sample spans from the first quarter in 1984 to the fourth quarter in 2007. I avoid the period before 1984 due to the data unavailability of credit spreads, and the change of the Federal Reserve's interest rate target (see Section 2.2). I avoid the period after the financial crisis in 2008 since the Gaussian term structure model is unable to capture the variation of the interest rate term structure (see Chapter 3 and Chapter 4).

(i) *Observed State Variables.* I follow the data construction approach of Ang and Ulrich (2012) in constructing the macro variables: output gap g_t, the inflation forecast π_t, and monetary policy shock f_t. According to Ang and Ulrich (2012), and Kick (2017), these three macro variables are able to capture a large amount of the variation in the government yield curve.

The credit factor (gz_t) is obtained from the GZ credit spread indicator (GZ index, hereafter)in Gilchrist and Zakrajšek (2012). They state that the GZ index is a highly informative financial indicator, and can be a good default-risk indicator to measure the degree of strains in the financial system. Here I take the GZ index as a proxy of the credit factor, which reflects corporate bond market information that has not been fully captured by macro fundamental variables. I demean state variables as Ang and Ulrich (2012).

The output gap g_t is calculated as the relative difference of real GDP and potential GDP:

$$g_t = \frac{1}{4}\frac{Q_t - Q_t^*}{Q_t^*} \tag{2.1}$$

where real GDP Q_t is from the Bureau of Economics Analysis (BEA) and the potential GDP Q_t^* is from the Congressional Budget Office (CBO). To make the BEA and CBO series comparable, I express the series in 2009 chained prices and both are seasonally adjusted. I divide by four to express the output gap quarterly.

The inflation forecast π_t is the median one-quarter ahead forecast obtained from the Survey of Professional Forecasts (SPF). Ang et al. (2007) find this variable to accurately forecast inflation

I follow Taylor (1993) and assume the Fed sets the federal funds rate as a linear function of the output gap and the inflation forecast calculated above. Monetary policy shock f_t is obtained as the residual from the Taylor rule regression:

$$FFR_t = c_0 + c_1 g_t + c_1 \pi_t + f_t \tag{2.2}$$

where c_0 is a constant, c_1 and c_2 represent the coefficient of the output gap and the inflation forecast, respectively.

The credit factor uses the GZ index as a proxy of the corporate bond market information. The GZ index is a highly informative credit spread index constructed by Gilchrist and Zakrajšek (2012). The widely used default-risk indicator (e.g., the paper-bill spread, high-yield bond spread) only considers a single index with limited maturity, which decreases their explanatory power for economic activity. Based on Gilchrist, Yankov and Zakrajšek (2009), the GZ credit spread is constructed using prices of individual corporate bonds traded in the secondary market and includes multiple maturities. Gilchrist and Zakrajšek (2012) state that this index has a significantly better predictive ability for economic activity compared with the standard credit spread indices, and has substantial predictive power from 1973 to 2010.

Figure 2.1 plots the time series of the state variables: output gap, trend inflation, monetary policy shock, and the credit factor. According to the National Bureau of Economic Research, the output gap decreases in recessions in 1991 and 2001. The inflation forecast shows a general descending trend over 1984-2007. As documented since the early 1990s, the inflation forecast has become quite stable. Both the output gap and the inflation forecast are highly persistent, while monetary policy shock is less persistent. The monetary policy shock is quite volatile, especially between the mid-1980s to late 1980s, which reaches its maximum during the 1987 Savings and Loan crisis and its minimum during the 1991 recession. The credit factor is relatively stable during the 1980s and 1990s, while becomes more volatile since 2000 and reaches its peak near the 2008 financial crisis.

(ii) *Yield Curves.* Both government yields and credit spreads are constructed from the zero-coupon bond with maturities of 1, 2, 3, 5, 7, and 10 years.

It is notable that I have not included the maturities over 10 years. This is in order to make the estimation results comparable to benchmark papers (e.g. Ang and Ulrich (2012),

Figure 2.1: Observed State Variables

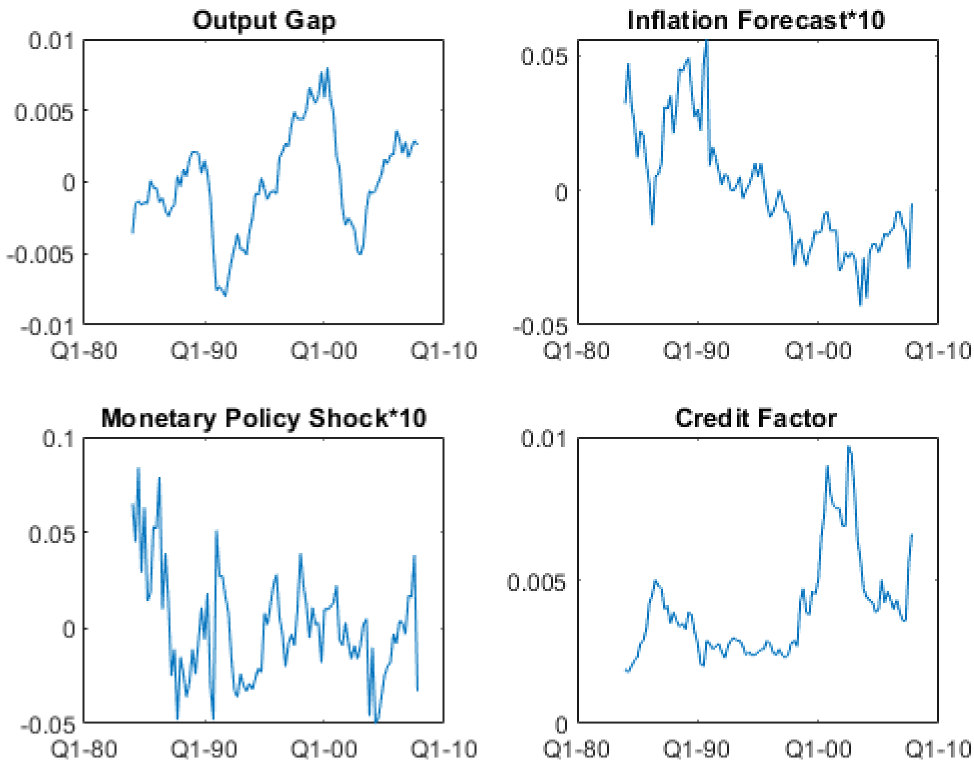


Figure 2.1 reports the time series of the output gap, inflation forecast, monetary policy shock and the credit factor. The data spans from 1984Q1 to 2007Q4.

Wu and Zhang (2008)), and also because central banks usually does not look into maturities longer than 10 years. However, there are some important potential implications of including longer-term maturity bonds in term structure modelling: First, the macroeconomic impact can be better reflected on long-term yields. Second, the level and slope factors can be more precisely measured, which are the long-term and short-term factors, respectively.

The government bond yield is constructed from the Gurkaynak et al. (2007) dataset. Data is obtained from the Fed website.

The credit spread is the difference between the corporate yield and treasury yield with the same maturity:

$$y_{t,n}^S = y_{t,n}^C - y_{t,n}^T \tag{2.3}$$

where $y_{t,n}^S$ represents the credit spread, $y_{t,n}^C$ represents the government bond yield, and $y_{t,n}^T$ represents the corporate yield.

The corporate bond yield is obtained from the U.S. Department of the Treasury High-Quality Market (HQM, hereafter) zero coupon corporate yields dataset. The HQM data is available on U.S. Department of the Treasury website. The HQM yield curve is constructed using high-quality corporate bonds rated AAA, AA or A. The HQM curve represents the market-weighted average yield of high-quality bonds by blending high rated bonds into a single yield curve.

Figure 2.2 plots the time series of the government bond yields. During the sample period, the long-term yield curve shows a downward trend, which matches the downward tendency of the inflation forecast in Figure 2.1. Figure 2.2 also shows that short-term government bonds have two steep upward-sloping trends around 1995 and 2000, which matches the spikes of the output gap in Figure 2.1. Figure 2.3 plots the time series of credit spreads. The steep upward-sloping trend around 1997 matches the spike of monetary policy shock in Figure 2.1. During the sample period, credit spreads co-move across maturities. The high spread around the end of 1991 is corresponds to the recession in late 1990, and by 1992 credit spreads began to narrow. Another spike in 2002-2003 marks the record defaults of the burst in the telecom bubble.

Figure 2.2: Government Bond Yield

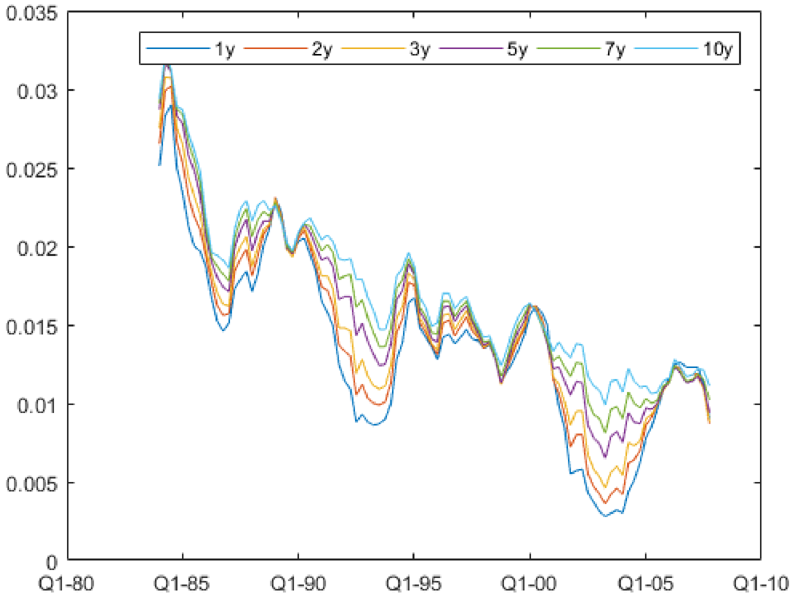


Figure 2.2 plots government yields from 1984Q1 to 2007Q4 in percentage points and maturities are 1, 2, 3, 5, 7, and 10 years.

Figure 2.3: Credit Spread

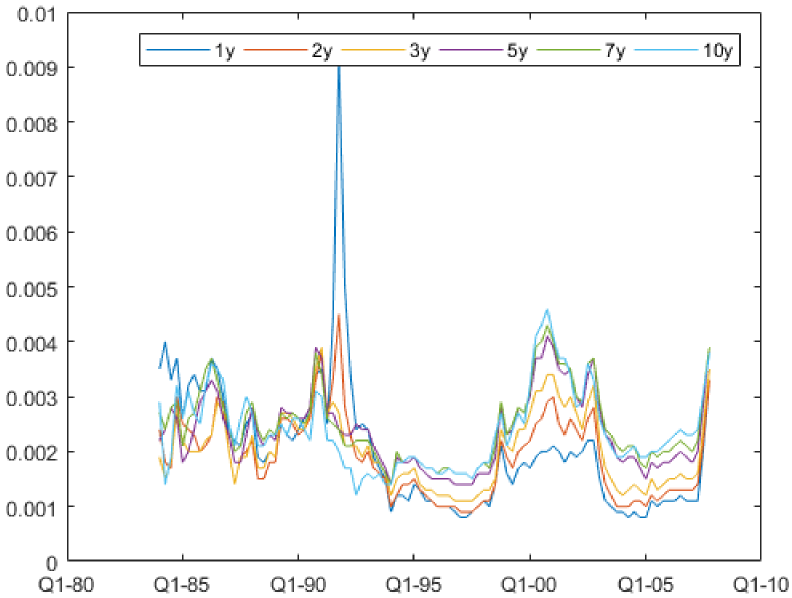


Figure 2.3 plots credit spreads from 1984Q1 to 2007Q4 in percentage points and maturities are 1, 2, 3, 5, 7, and 10 years.

Table 2.1: Yield Curve Analysis

	Government Yield	Credit Spread
PC1	96.96	85.70
PC2	99.93	97.59
PC3	99.99	99.26
PC4	100.00	99.97
PC5	100.00	99.99
PC6	100.00	100.00

Table 2.1 reports the cumulative percentage of variance of government bond yields (first column) and credit spreads (second column) explained by the first six principal components extracted from government bond yields and credit spreads, respectively.

2.2.2 Principal Component Analysis

Table 2.1 reports the cumulative variance of government bond yields and credit spreads explained by the corresponding first six principal components. Table implies that the first two principal components explain over 99.9% of the government term structure, and 98% of the credit spread term structure.

Tables 2.2 and 2.3 display the R^2 from regressing the first two principal components of government yields and credit spreads on observed state variables. The first column denotes the regressions on each observed state variable- the output gap (g_t), inflation forecast π_t, monetary policy shock (f_t), and credit factor (gz_t). $g{+}\pi{+}f$ refers to the regression on the three macro state variables. $g{+}\pi{+}f{+}gz$ refers to the regression on the four state variables. The second column of each table denotes the R^2 of the first principal component on state variables. The third column of each table denotes the R^2 of the second principal component on state variables. Table 2.2 implies that the state variables, in general, explain well the first two government yield principal components with an R^2 of 93%. The three macro variables explain most of the government yield principal components. This result is consistent with the finding of Ang and Ulrich (2012) that the output gap, inflation forecast and monetary policy

Table 2.2: State Variables Analysis

State Variable	Government Yield PC1	Government Yield PC2
g	0.02	0.01
π	0.64	0.66
f	0.18	0.18
gz	0.29	0.32
g+π+f	0.95	0.91
g+π+f+gz	0.96	0.93

Table 2.2 reports the R^2 from regressing the first two principal components of government yields on state variables. The first column denotes the regressions on each state variables- the output gap (g_t), inflation forecast π_t, monetary policy shock (f_t) and credit factor (gz_t). g+π+f refers to the regression on the three macro state variables. g+π+f+gz refers to the regression on the four state variables. The second column denote R^2 of the first government yield principal component on state variables. The third column denote R^2 of the second government yield principal component on state variables.

Table 2.3: State Variables Analysis

State Variable	Credit Spread PC1	Credit Spread PC2
g	0.22	0.04
π	0.14	0.01
f	0.03	0.07
gz	0.30	0.14
$g+\pi+f$	0.32	0.11
$g+\pi+f+gz$	0.38	0.38

Table 2.3 reports the R^2 from regressing the first two principal components of credit spreads on observed state variables. The first column denotes the regressions on each state variables- the output gap (g_t), inflation forecast π_t, monetary policy shock (f_t) and credit factor (gz_t). $g+\pi+f$ refers to the regression on the three macro state variables. $g+\pi+f+gz$ refers to the regression on the four state variables. The second column denote R^2 of the first credit spread principal component on state variables. The third column denote R^2 of the second credit spread principal component on observed state variables.

shock can capture the most variation of the government yields. The inflation forecast explains around 65% of the first two principal components, which comports with the literature that the inflation forecast has a close relation to the government yield curve. The monetary policy shock also explains around 18% of the first two principal components, which is consistent with the literature that the government yield curve is affected by the business cycle. It is notable that the credit factor explains around 30% of the first two principal components. This result is consistent with Gilchrist and Zakrajšek (2012) that the GZ index has predictive content for the economic outlook, and can lead to the changes in government yields.

Table 2.3 indicates that the observed state variables can only explain 38% of the first two principal components of the credit spread. In order to explain the variation of credit spreads, at least one principal component needed to be included in addition to the macro factors and the credit factor.

The result shows that the credit factor explains most of the two principal components at 30% and 14%, respectively, implying that credit spreads contain information that cannot be explained by the macroeconomic fundamentals. It is notable that the output gap and inflation forecast explain 22% and 14% of the first credit spread principal component, which implies that these macro variables impact the variation of the credit spread. In contrast, macro variables have no significant impact on explaining the second credit spread principal component.

Using this parsimonious model the explanation capability is not expected to be perfect. However, the drivers of the credit spread are difficult to be investigated with the poor performance results. This poor performance can be explained by the credit spread puzzle, see details in Section 1.1.A single unknown common factor dominates the term structure of credit spreads. This common factor cannot be explained by general macroeconomic and financial variables.

2.3 Joint Term Structure Model of Government Yields and Credit Spreads

Most theoretical studies state that credit spreads are negatively correlated with government bond yields (see Black and Cox, 1976, Collin-Dufresne and Goldstein, 2001, Eom et al.,2004). In contrast, recent empirical studies found mixed evidence on the relationship between government bond yields and credit spreads. On the one hand, after controlling for firm- and market-level determinants of default risk, some empirical studies support a strong negative relationship between changes in credit spreads and government bond yields (see Longstaff and Schwartz, 1995, Collin-Dufresne et al., 2001, Campbell and Taksler ,2003). On the other hand, Duffee(1998) and Jocoby et al. (2009) find no significant relation between credit spreads of non-callable corporate bonds and government bond yields. Thus, here I use a joint term structure model setup to evaluate the relationship between government yields and corporate yields.

It is notable that in the model specification in Section 2.3.2 two models are proposed. Model 1 jointly estimate government yields and credit spreads, and Model 2 estimate credit spreads only.

2.3.1 Model Setup

I assume K state variables follow a first-order vector autoregression process (VAR(1)) under the physical or the real world measure (P-measure):

$$\boldsymbol{x}_t = \boldsymbol{\mu} + \boldsymbol{\Phi}\boldsymbol{x}_{t-1} + \boldsymbol{\Sigma}\boldsymbol{\varepsilon}_t \tag{2.4}$$

where $\boldsymbol{\mu}$ is a $K \times 1$ matrix, $\boldsymbol{\Phi}$ is a $K \times K$ matrix. $\boldsymbol{\Sigma}$ is a $K \times K$ lower triangular matrix, which is invariant under the P-measure and the risk neutral measure (the Q-measure). $\boldsymbol{\varepsilon}_t \sim i.i.d.N(\mathbf{0}_K, \boldsymbol{I}_K)$, with $\mathbf{0}_K$ as a $K \times 1$ vector of zeros, and $\boldsymbol{I}_K$ as a $K \times K$ identity matrix.

The short rate of bond i, i=T, CS, (denoted as government bonds and credit spread, respectively) is assumed as an affine function of state variables $\boldsymbol{x}_t$:

$$r_t^i = \delta_0^i + \boldsymbol{\delta_1^{i'}}\boldsymbol{x}_t \tag{2.5}$$

with δ_0^i as a scalar, and $\boldsymbol{\delta_1^i}$ as a $K \times 1$ vector.

Imposing the assumption of no-arbitrage in the manner of Harrison and Kreps (1979) guarantees the existence of the risk-neutral measure Q (Q-measure). Therefore, under the Q-measure the price of any asset V_t that pays no dividend at time $t+1$ satisfies $V_t = E_t^Q(exp(-r_t)V_(t+1))$. This Q-measure is unique, if the market is complete. Using the Radon-Nikodym derivative as in Ang and Piazzesi (2003) shifts the probability from Q-measure to P-measure. The pricing kernel m_{t+1} as in Ang and Piazzesi (2003) takes the standard exponential form:

$$m^i_{t+1} = exp(-\delta^i_0 - \boldsymbol{\delta^i_1} \boldsymbol{x}_t - \frac{1}{2}\boldsymbol{\lambda}'_t\boldsymbol{\lambda}_t - \boldsymbol{\lambda}'_t\boldsymbol{\varepsilon}_{t+1}) \tag{2.6}$$

where the time-varying market price of risk $\boldsymbol{\lambda}_t$ associated with shocks in the underlying state variables $\boldsymbol{x}_t$ is a $K \times 1$ vector, which is linear in the state variables as in Duffee (2002):

$$\boldsymbol{\lambda}_t = \boldsymbol{\lambda_0} + \boldsymbol{\Lambda_1}\boldsymbol{x}_t \tag{2.7}$$

where $\boldsymbol{\lambda_0}$ is a $K \times 1$ vector, and $\boldsymbol{\Lambda_1}$ is a $K \times K$ matrix.

Imposing the assumption of no-arbitrage also guarantees the existence of a state variable under the Q-measure, and follows a VAR(1) process as follows:

$$\boldsymbol{x}_t = \boldsymbol{\mu}^Q + \boldsymbol{\Phi}^Q \boldsymbol{x}_{t-1} + \boldsymbol{\Sigma}\boldsymbol{\varepsilon}^Q_t \tag{2.8}$$

with $\boldsymbol{\varepsilon}^Q_t \sim i.i.d.N(\boldsymbol{0}_K, \boldsymbol{I}_K)$.

The parameters under the two probability measures are related as follows:

$$\begin{aligned} \boldsymbol{\mu^Q} &= \boldsymbol{\mu} - \boldsymbol{\Sigma}\boldsymbol{\lambda_0} \\ \boldsymbol{\Phi^Q} &= \boldsymbol{\Phi} - \boldsymbol{\Sigma}\boldsymbol{\Lambda_1} \end{aligned} \tag{2.9}$$

The pricing kernel m^i_{t+1} prices all assets in the economy. Therefore, $\boldsymbol{R}^i_{t+1}$, the one-period gross return of any assets i satisfies:

$$E_t(m^i_{t+1} R^i_{t+1}) = 1 \tag{2.10}$$

If $P^i_{t,n}$ denotes the price of an n-period zero coupon bond at time t, then the one-period gross return of bonds i, R^i_{t+1} can be computed as $\frac{P^i_{t+1,n-1}}{P^i_{t,n}}$. According to Eq. 2.10, this means the bond price can be recursively computed as follows:

$$P^i_{t,n} = E_t(m^i_{t+1} P^i_{t+1,n-1}) \tag{2.11}$$

Combining the state variables (Eq. 2.4), the short rate (Eq. 2.5), and the pricing kernel (Eq. 2.6), the bond prices $P^i_{t,n}$ are given as an exponentially affine function of the state variables $\boldsymbol{x}_t$:

$$P^i_{t,n} = exp(\overline{a}^i_n + \overline{\boldsymbol{b}}^{i'}_{\boldsymbol{n}} \boldsymbol{x}_t) \tag{2.12}$$

where cross sectional coefficients $\overline{a}^i_n$ and $\overline{\boldsymbol{b}}^i_n$ can be derived as follows (see Ang and Piazzesi (2003) Appendix A for a detailed derivation):

$$\begin{aligned}\overline{a}^i_{n+1} =& \overline{a}^i_1 + \overline{a}^i_n + \overline{\boldsymbol{b}}^{i'}_{\boldsymbol{n}}(\boldsymbol{\mu} - \boldsymbol{\Sigma}\boldsymbol{\lambda_0}) + \frac{1}{2}\overline{\boldsymbol{b}}^{i'}_{\boldsymbol{n}}\boldsymbol{\Sigma}\boldsymbol{\Sigma}'\overline{\boldsymbol{b}}^{i}_{\boldsymbol{n}} \\ \overline{\boldsymbol{b}}^i_{\boldsymbol{n+1}} =& \overline{\boldsymbol{b}}^{i'}_{\boldsymbol{n}}(\boldsymbol{\Phi} - \boldsymbol{\Sigma}\boldsymbol{\Lambda_1}) + \overline{\boldsymbol{b}}^{i'}_{\boldsymbol{1}}\end{aligned} \tag{2.13}$$

$\overline{a}^i_n$ is a scalar, $\overline{\boldsymbol{b}}^i_{\boldsymbol{n}}$ is a $K \times 1$ vector, with $\overline{a}^i_1 = -\delta^i_0$, $\overline{\boldsymbol{b}}^i_{\boldsymbol{1}} = -\boldsymbol{\delta}^i_{\boldsymbol{1}}$.

The n-period continuously compounded zero coupon bond yield $y^i_{t,n}$ is given as:

$$y^i_{t,n} = -\frac{logP^i_{t,n}}{n} = a^i_n + \boldsymbol{b}^{i'}_{\boldsymbol{n}}\boldsymbol{x}_t \tag{2.14}$$

where $a^i_n = -\overline{a}^i_n/n$, $\boldsymbol{b}^i_{\boldsymbol{n}} = -\overline{\boldsymbol{b}}^i_{\boldsymbol{n}}/n$.

2.3.2 Model Specification

In this section, I specify two models to estimate government yields and credit spreads. Model 1 is specified to jointly estimate government yields and credit spread, and Model 2 estimates credit spreads only.

In Model 1, I specify that only observed macro factors and credit factor are used as state variables to capture the variation of government yields and credit spreads. In Model 2, I incorporate the first principal component of credit spread, denoted as pc_t, as a new factor, in addition to the observed macro factors and credit factor, to explain variation of credit spreads.

The reasons for the adoption of Model 2 are as follows: First, the preliminary analysis in Section.2.6 illustrates that the observed macro and credit factors have a good explanatory power for government yields, however, their explanatory capability seems limited for credit spreads. Therefore, I incorporate pc_t in Model 2 as a proxy of the unknown factor that captures the variation of credit spreads that cannot be explained by the macro factors and credit factor. Second, pc_t can also be viewed econometrically as a "goodness-of-fit" test to evaluate the explanatory power of the macro factors and credit factor on credit spreads by comparing an estimation without pc_t in Model 1 and with pc_t in Model 2. If there is a large estimation difference between Model 1 and Model 2, this implies that the macro factors and credit factor cannot explain most of the variation of credit spreads.

Model 1

(i) *State Variables.* The state variables $\boldsymbol{x_t}$ consists of two groups of variables. The first group is macro variables related to monetary policy rule recommended by Ulrich (2011), and Ang and Ulrich (2012). I assume three macro variables: g_t is output gap, π_t is one-quarter ahead inflation forecast, and f_t is monetary policy shock. The second group contains a credit factor gz_t.

The two groups of variables are collected in the state vector $\boldsymbol{x_t} = (g_t\, \pi_t\, f_t\, gz_t)'$.

To examine the linkages among state variables, I assume that all state variables have an effect on one another. The state variables $\boldsymbol{x_t}$ follows VAR(1) under the P- measure:

$$\boldsymbol{x_t} = \boldsymbol{\mu} + \boldsymbol{\Phi x_{t-1}} + \boldsymbol{\Sigma \varepsilon_t} \tag{2.15}$$

where conditional covariance is $\boldsymbol{\Sigma\Sigma}'$, and $\boldsymbol{\Sigma}$ is specified as a 4×4 lower triangular matrix, with $\boldsymbol{\varepsilon_t} \sim i.i.d.N(\mathbf{0}_4, \boldsymbol{I}_4)$. The parameters of the $\boldsymbol{x_t}$ are as follows:

$$\boldsymbol{\mu} = \begin{pmatrix} \mu_g \\ \mu_\pi \\ \mu_f \\ \mu_{gz} \end{pmatrix} \qquad \boldsymbol{\Phi} = \begin{pmatrix} \phi_{g,g} & \phi_{g,\pi} & \phi_{g,f} & \phi_{g,gz} \\ \phi_{\pi,g} & \phi_{\pi,\pi} & \phi_{\pi,f} & \phi_{\pi,gz} \\ \phi_{f,g} & \phi_{f,\pi} & \phi_{f,f} & \phi_{f,gz} \\ \phi_{gz,g} & \phi_{gz,\pi} & \phi_{gz,f} & \phi_{gz,gz} \end{pmatrix} \tag{2.16}$$

(ii) *Short Rate.* It is assumed that short-term interest rates loads on state variables $\boldsymbol{x_t}$, which contains the output gap (g_t), the inflation forecast (π_t), the monetary policy shock (f_t) and the credit factor (gz_t):

$$r_t^i = \delta_0^i + \delta_{1,g}^i g_t + \delta_{1,\pi}^i \pi_t + \delta_{1,f}^i f_t + \delta_{1,gz}^i gz_t \tag{2.17}$$

I adopt the 3-month government bond yield as the government short rate r_t^T. The short rate is constructed from Gurkaynak, Sack and Wright (2007) data. $\boldsymbol{\delta_1^T}$ can be collected in $\boldsymbol{\delta_1^T} = (\delta_{1,g}^T\, \delta_{1,\pi}^T\, \delta_{1,f}^T\, \delta_{1,gz}^T)'$.

The credit spread short rate r_t^{CS} is unobserved, and can be calculated using the estimated credit spread short rate parameters in Eq.2.17. The estimated credit spread short rate result is reported in Section 2.3.4. $\boldsymbol{\delta_1^{CS}}$ can be collected in $\boldsymbol{\delta_1^{CS}} = (\delta_{1,g}^{CS}\, \delta_{1,\pi}^{CS}\, \delta_{1,f}^{CS}\, \delta_{1,gz}^{CS})'$.

(iii) *Price of Risk.* The time-varying market price of risk $\boldsymbol{\lambda}_t$ is a 4×1 vector. In consistent with the assumption that state variables have a mutual effect, the parameters of the market price of risk are as follows:

$$\boldsymbol{\lambda_0} = \begin{pmatrix} \lambda_{0,g} \\ \lambda_{0,\pi} \\ \lambda_{0,f} \\ \lambda_{0,gz} \end{pmatrix} \qquad \boldsymbol{\Lambda_1} = \begin{pmatrix} \lambda_{1,g.g} & \lambda_{1,g.\pi} & \lambda_{1,g.f} & \lambda_{1,g.gz} \\ \lambda_{1,\pi.g} & \lambda_{1,\pi.\pi} & \lambda_{1,\pi.f} & \lambda_{1,\pi.gz} \\ \lambda_{1,f.g} & \lambda_{1,f.\pi} & \lambda_{1,f.f} & \lambda_{1,f.gz} \\ \lambda_{1,gz.g} & \lambda_{1,gz.\pi} & \lambda_{1,gz.f} & \lambda_{1,gz.gz} \end{pmatrix} \tag{2.18}$$

Model 2

Model 2 is a modified model on the basis of Model 1. The difference are:

(i) *State Variables.* In addition to the two groups of state variables in Model 1, I add the first principal component of the credit spreads in Section 2.6, denoted as pc_t, as the third group of variables.

The state variables are collected in the state vector $\boldsymbol{x}_t = (g_t\,\pi_t\,f_t\,gz_t\,pc_t)'$.

To give full weight to the macro factors and credit factor in tracing out their impacts on yield curves, I assume the first principal component pc_t follows AR(1) process:

$$pc_t = \mu_{pc} + \phi pc_{t-1} + \varepsilon_{pc,t} \tag{2.19}$$

where $\varepsilon_{pc,t} \sim i.i.d.N(0, \sigma_{pc}^2)$.

The state variables $\boldsymbol{x}_t$ follow first-order autoregressive progress:

$$\boldsymbol{x}_t = \boldsymbol{\mu} + \boldsymbol{\Phi}\boldsymbol{x}_{t-1} + \boldsymbol{\Sigma}\boldsymbol{\varepsilon}_t \tag{2.20}$$

where condition covariance is $\boldsymbol{\Sigma}\boldsymbol{\Sigma}'$, and $\boldsymbol{\Sigma}$ is specified as a 5×5 lower triangular matrix, with $\boldsymbol{\varepsilon}_t \sim i.i.d.N(\mathbf{0}_5, \boldsymbol{I}_5)$. The parameters of the $\boldsymbol{x}_t$ are determined by stacking Eq.2.16 and Eq. 2.19 together:

$$\boldsymbol{\mu} = \begin{pmatrix} \mu_g \\ \mu_\pi \\ \mu_f \\ \mu_{gz} \\ \mu_{pc} \end{pmatrix} \qquad \boldsymbol{\Phi} = \begin{pmatrix} \phi_{g,g} & \phi_{g,\pi} & \phi_{g,f} & \phi_{g,gz} & 0 \\ \phi_{\pi,g} & \phi_{\pi,\pi} & \phi_{\pi,f} & \phi_{\pi,gz} & 0 \\ \phi_{f,g} & \phi_{f,\pi} & \phi_{f,f} & \phi_{f,gz} & 0 \\ \phi_{gz,g} & \phi_{gz,\pi} & \phi_{gz,f} & \phi_{gz,gz} & 0 \\ 0 & 0 & 0 & 0 & \phi_{pc,pc} \end{pmatrix} \tag{2.21}$$

conditional covariance is given by:

$$
\boldsymbol{\Sigma} = \begin{pmatrix} \Sigma_{g,g} & 0 & 0 & 0 & 0 \\ \Sigma_{\pi,g} & \Sigma_{\pi,\pi} & 0 & 0 & 0 \\ \Sigma_{f,g} & \Sigma_{f,\pi} & \Sigma_{f,f} & 0 & 0 \\ \Sigma_{gz,g} & \Sigma_{gz,\pi} & \Sigma_{gz,f} & \Sigma_{gz,gz} & 0 \\ 0 & 0 & 0 & 0 & \Sigma_{pc,pc} \end{pmatrix} \tag{2.22}
$$

(ii) *Short Rate.* The credit spread short-term interest rate is assumed to load on the state variables $\boldsymbol{x}_t$

$$
r_t^{CS} = \delta_0^{CS} + \delta_{1,g}^{CS} g_t + \delta_{1,\pi}^{CS} \pi_t + \delta_{1,f}^{CS} f_t + \delta_{1,gz}^{S} gz_t + \delta_{1,pc}^{CS} pc_t \tag{2.23}
$$

The credit spread short rate r_t^{CS} is unobserved, and can be calculated using the estimated credit spread short rate parameters in Eq.2.23. The estimated credit spread short rate result is reported in Section 2.3.4. $\boldsymbol{\delta_1^{CS}}$ can be collected in $\boldsymbol{\delta_1^{CS}} = (\delta_{1,g}^{CS}\ \delta_{1,\pi}^{CS}\ \delta_{1,f}^{CS}\ \delta_{1,pc}^{CS},\ \delta_{1,pc}^{CS})'$.

(iii) *Price of Risk.*

The price of risk of pc_t depends on itself:

$$
\lambda_t^{pc} = \lambda_0^{pc} + \lambda_1^{pc} pc_t \tag{2.24}
$$

The time-varying market price of risk $\boldsymbol{\lambda}_t$ is a 5×1 vector. The parameters of the market price of risk are determined by stacking Eq.2.18 and Eq. 2.24 together:

$$
\boldsymbol{\lambda_0} = \begin{pmatrix} \lambda_{0,g} \\ \lambda_{0,\pi} \\ \lambda_{0,f} \\ \lambda_{0,gz} \\ \lambda_{0,pc} \end{pmatrix} \qquad \boldsymbol{\Lambda_1} = \begin{pmatrix} \lambda_{1,g.g} & \lambda_{1,g.\pi} & \lambda_{1,g.f} & \lambda_{1,g.gz} & 0 \\ \lambda_{1,\pi.g} & \lambda_{1,\pi.\pi} & \lambda_{1,\pi.f} & \lambda_{1,\pi.gz} & 0 \\ \lambda_{1,f.g} & \lambda_{1,f.\pi} & \lambda_{1,f.f} & \lambda_{1,f.gz} & 0 \\ \lambda_{1,gz.g} & \lambda_{1,gz.\pi} & \lambda_{1,gz.f} & \lambda_{1,gz.gz} & 0 \\ 0 & 0 & 0 & 0 & \lambda_{1,pc.pc} \end{pmatrix} \tag{2.25}
$$

2.3.3 Estimation Method

In line with Ang and Ulrich (2012), and Li and Wei(2013), I conduct a three-step estimation method. All the parameters of Model 1 are estimated in the first two steps. In the third step, in order to give as much explanatory power to the macro factors and credit factor, I estimate Model 2 by fixing all the other parameters related with the macro and credit factors from previous steps, and estimate parameters related with pc_t by matching credit spreads exactly. First, I estimate the parameters of the government short rate (δ_0^T, $\boldsymbol{\delta_1^T}$) in Eq.2.17, and the parameters of state variables ($\boldsymbol{\mu}$, $\boldsymbol{\Phi}$, $\boldsymbol{\Sigma}$)in Eq.2.15 by ordinary least squares (OLS). Second, I hold parameters δ_0^T, $\boldsymbol{\delta_1^T}$, $\boldsymbol{\mu}$, and $\boldsymbol{\Phi}$ in Eq.2.17 constant. The OLS estimated $\boldsymbol{\Sigma}$ in Eq.2.15 are treated as a good starting point for $\boldsymbol{\Sigma}$. I use maximum likelihood to estimate the remaining parameters of the market price of risk $\boldsymbol{\lambda_0}$ and $\boldsymbol{\Lambda_1}$ in Eq. 2.18, $\boldsymbol{\Sigma}$ in Eq.2.15 and the unobserved credit spread short rate parameters δ_0^{CS} and $\boldsymbol{\delta_1^{CS}}$ in Eq.2.17 by minimizing the squared difference between model implied government yields and credit spreads and the observed government yields and credit spreads data. In the third step, I fix all the parameters related with the macro factors and credit factor as estimated in step 1 and step 2, and only estimate the parameters of pc_t. The parameters of state variables μ_{pc}, $\phi_{pc,pc}$ and $\Sigma_{pc,pc}$ in Eq.2.20 are estimated by OLS. Again I treat the OLS estimated $\Sigma_{pc,pc}$ in Eq.2.20 as a good starting point for $\Sigma_{pc,pc}$, and use maximum likelihood function to estimate the market price of risk $\lambda_{0,pc}$ and $\lambda_{1,pc.pc}$ in Eq. 2.25, $\Sigma_{pc,pc}$ in Eq.2.20, and the unobserved credit spread short rate parameters δ_0^{CS} and $\delta_{1,pc}^{CS}$ in Eq.2.23 by minimizing the squared difference between model implied credit spreads and the observed credit spread data. 40 parameters in total are estimated. This three-step approach avoids the difficulties of estimating a model with various factors using one-step maximum likelihood.

2.3.4 Estimation Results

In this section, I report the estimation results of Model 1 and Model 2. It is notable Model 1 reports the estimation results for the joint estimation of government yields and credit spread, and Model 2 reports the estimation results for the estimation of credit spreads only. In addition, the estimates of the macro factors and credit factor are reported in Model 1, and the estimates of the first principal of credit spreads are reported in Model 2. Detailed explanation of the model specification and estimation method of Model 1 and Model 2 can be seen in Section 2.3.2 and Section 2.3.3.

(i) *Estimated Yield Curve.*

Table 2.4 presents the root mean square error (RMSE) of Model 1 and Model 2 for government bond yields and credit spreads.

For Model 1, the fit on government yields is good as most affine models. However, this model fits government yield curves with observed state variables only, whereas most term structure models use latent state variables to fit the yield curve. The RMSE of the credit spread becomes smaller when the maturity increases, which implies that the joint model better fits long-term credit spread yields than shorter-term ones. This may be because shorter-term credit spreads are more volatile, and are affected by factors that have not included in the model.

Model 2 fits credit spreads much better than Model 1. In comparison to Model 1, the RMSE of credit spreads of Model 2 becomes significantly smaller, after incorporating the first principal component of credit spreads pc_t. This result is consistent with the findings of the preliminary analysis in Section 2.6. The RMSE of Model 2 across different maturities is around 5 to 7 basis points smaller than Model 1, especially for the shorter-term maturities (1-year and 2-year), which is 7 basis points smaller. Similar to Model 1, the RMSE of credit spreads decreases as maturities increase.

Figures 2.4 and Figure 2.5 display the yield curve fit for the government bond yields and credit spreads. Figure 2.4 shows that the fit of Model 1 for government bond yields is good. Model 1 closely fits the short-term and mid-term maturities government bond yields. Model 1 fits less well for the longer-term maturities government bond yields. However, the fitted yield in Model 1 still captures the general shape and variations of the government bond yield curve.

Figure 2.5 shows that the fit of Model 2 for credit spreads is significantly better than Model 1, and the fitted yield in Model 2 captures the general shape and variations of credit spreads. Model 1 fails to capture the spikes of the 1-year and 2-year credit spreads in 2002, which is affected by the corporate bond defaults in the Telecom bubble described in Section 2.2.1. In addition, the fitted credit spreads in Model 1 for longer maturities, especially of 10-year maturity, is essentially flat. This poor performance of Model 1 is consistent with the results in the preliminary analysis, and can be explained by the credit spread puzzle as stated in Section 2.2.2. Previous literature such as Collin-Dufresne et al. (2001) finds that a single unknown factor dominates the credit spread yield curve, which cannot be explained by common macroeconomic or financial variables. Based on Model 1, Model 2 incorporates the first principal component of credit spreads as a proxy for the unknown common factor. The fit of Model 2 for credit spreads is good. The spikes of the 1-year and 2-year credit spreads caused by the Telecom bubble are captured. Model 2 also fits longer maturities up to 10-year well. In contrast to the fit for government bond yields in Figure 2.4, in Figure 2.5 the fit for credit spreads in Model 1 and Model 2 both fits better the longer-term maturities credit spread yields. This result implies that the state variables better capture the long run variation of the credit spread yields.

The fits of Model 1 and Model 2 can be explained from an economic perspective. On the one hand, the baseline of government yields of different maturities is the government short-term interest rate, which is closely related with macroeconomic variables. Therefore, the dynamics of government yields reflect economic growth, inflation, and the monetary policy actions as shown by large macro-finance term structure literature. Model 1 includes these macro variables and thus shows a good performance in fitting government yields.

On the other hand, government bonds are issued by government, which are highly liquid and have quite low default risk. However, unlike government bonds, corporate bonds are issued by firms in order to raise financing, and default risk is reflected in corporate bond yields. Credit spreads, the subtraction between the government yields and corporate yields with the same maturity, is affected by default risk, and tends to increase as credit ratings decreases. According to Duffie and Singleton (1999), the credit spread short rate is an adjusted short rate, which is the government short rate adjusted by the hazard (spot default) rate. This implies that default risk needs to be considered when modelling the term structure of credit spreads.

In Model 1, I use the credit factor as a proxy of the default risk for credit spreads. However, the poor performance of Model 1 in fitting credit spreads shows that the inclusion of default risk factor cannot capture the variation of credit spreads. One possible reason is the impact from the credit spread puzzle. While credit spreads are usually considered as the compensation for default risk, it has been difficult to explain the exact relation between credit spreads and default risk. Collin-Dufresne et al. (2001) and others find that default risk has limited explanatory power to explain credit spreads, and a single unknown common factor dominates credit spreads. The good performance of Model 2 in fitting credit spreads is due to the incorporation of the first principal component of credit spreads as a proxy for the unknown common factor. Driessen (2005), Amato and Luisi (2006), and Mueller (2009) use a latent variable to capture the information of this unknown common factor. Mueller (2009) find that this unknown common factor is related to the index of tighter loan standards.

Figure 2.4: Yield Curve Fit

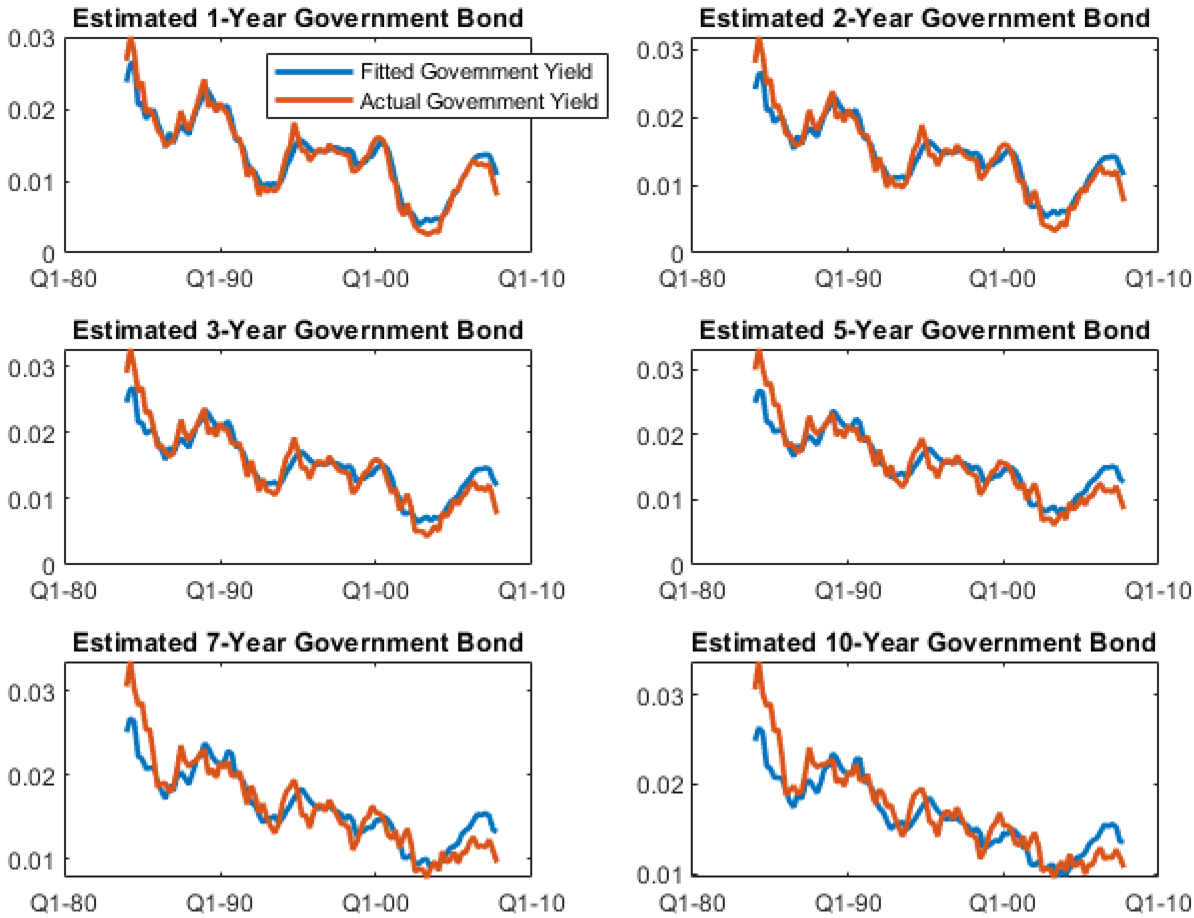


Figure2.4 reports observed and fitted government bond yields in Model 1 with maturities 1-year, 2-year, 3-year, 5-year, 7-year and 10-year. The blue line denotes model fitted government yields, and the red line denotes actual government yields. The sample spans from 1984: Q1 to 2007: Q4.

Figure 2.5: Yield Curve Fit

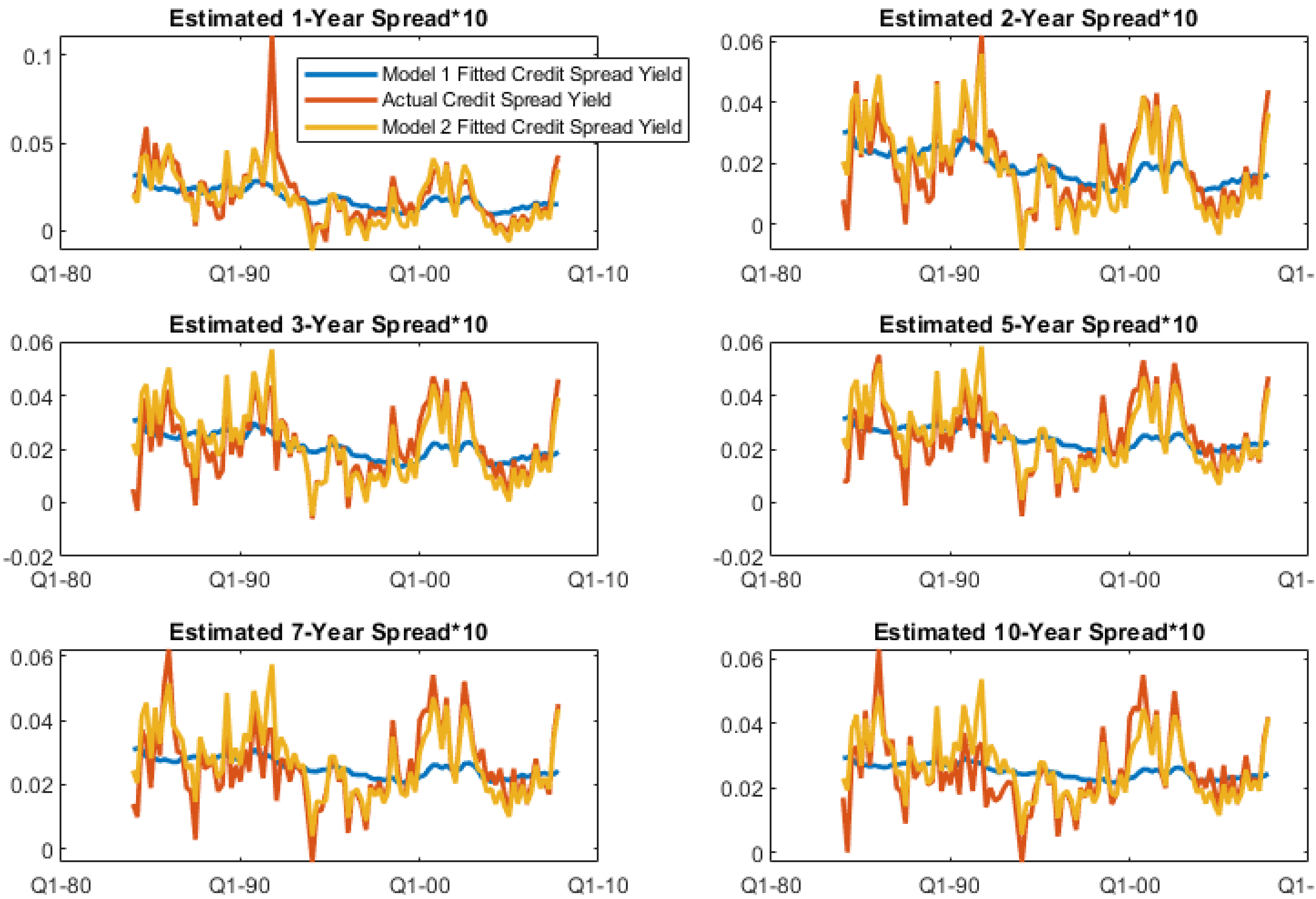


Figure2.5 reports observed and fitted credit spread yields with maturities 1-year, 2-year, 3-year, 5-year, 7-year and 10-year. The blue line denotes Model 1 fitted credit spread, the red line denotes actual credit spread, and the yellow line denotes Model 2 fitted credit spread. The sample spans from 1984: Q1 to 2007: Q4 at quarterly frequency.

Table 2.4: Root-Mean-Square-Error

	1-year	2-year	3-year	5-year	7-year	10-year
Model 1						
Government Yield	13	18	20	22	23	24
Credit Spread	15	12	12	12	11	11
Model 2						
Credit Spread	8	5	6	6	6	6

Table 2.4 reports root-mean-squared errors of government bond yields and credit spreads with maturities 1-year, 2-year, 3-year, 5-year, 7-year and 10-year. The top panel reports the RMSE of Model 1. The bottom panel reports the RMSE of Model 2. The sample spans from 1984Q1 to 2007Q4. The RMSE is measured in basis points.

(ii) *Factor Loadings.* Table 2.5, Figure 2.6 and 2.7 present model-implied factor loadings of government bond yields and credit spreads. The x-axis of figures refers to the estimated factor loadings with maturities in quarters. Results indicate that the inflation forecast π_t has a positive and strong impact on government bond yields, with a coefficient between 1.5 and 2 across the bond yields maturities from 1-year to 10-year. The output gap g_t and monetary policy shock π_t have a positive downward-sloping effect on bond yields. These two factors load heavily on short-term government yields than long-term ones. This finding is consistent with Litterman and Scheinkman (1991), and Ang and Ulrich (2012), suggesting that the output gap and monetary policy shock closely match the yield curve slope factor. This suggests that the long-term government yields variation is largely determined by the inflation forecast, while short-term government yields vary with the output gap and monetary policy shock. This result explains why the long-term government yield curve in Figure 2.2 has a similar tendency as the inflation forecast in Figure 2.1, and that short-term government yields vary with the output gap and the inflation forecast. The credit factor has a negative and upward-sloping effect on government yields. This implies that credit spreads negatively correlated with government yields, which comports with the literature. For example, Duffee (1999) and Driessen (2005) using Treasury and corporate bond data show a negative dependence. Collin-Dufresne and Solnik (2001) use swaps data and also find a negative dependence. The result shows that the output gap negatively impacts credit spreads, which implies that an increase in economic growth will increase the firm-level growth rate and decreases the probability of default, which leads to a decrease in the credit spread. (See Desantis, 2016, for example). The inflation forecast positively impacts credit spread yields. This result is consistent with the literature: an increase in inflation tightens monetary policy, which tends to increase the cost of borrowing, increase the probability of default and the credit spread. The monetary policy shock measures the policy shock that has not been captured by the output gap and inflation forecast has a positive effect on credit spread yields. This result indicates that an increase in policy shock increases the default risk of corporate bonds, leading to an increase in the credit spread. The credit factor has a strong impact on credit spread yields with a positive sign. It is notable that the effect of the macro variables becomes smaller as the maturities of the credit spread yield increases, whereas the credit factor has a larger effect on credit spread yields as maturities increase. The yield curve fits of the credit spread is better with the longer-term maturities. This may imply that in the short-run, the variation of the credit yield curve is caused by some volatility in the financial sector that cannot be captured by the macro variables and credit factor, which is consistent with the preliminary analysis results in Section 2.2 and the estimated yield curve in Section 2.4.4(i). In line with the results of Mueller (2009), credit spreads load heavily on the incorporated first principal component of credit spreads pc_t. pc_t has the largest positive impact on shorter maturities credit spreads, and decreases gradually as maturities increase. Thus, the results indicate that government yields are mainly dominated by the macro variables, while credit spreads first principal component plays the most important role in explaining credit spread yields. In addition, the credit factor is an important explanatory factor for credit spreads, especially at longer maturities.

(iii) *Estimated State Variables.* In Figure 2.8, the top plots contain the estimates of the output gap and the inflation forecast, and the bottom plots contain the estimates of the monetary policy shock and credit factor. The model-implied state variables are assumed to follow a VAR(1) process and are estimated using OLS as Ang and Ulrich (2012). The fits of these state variables are good. The first principal component of credit spreads can be interpreted as a proxy for the credit condition, as described by Muller (2009). Muller (2009) states that the dominant factor of credit spreads has a strong correlation with the index of tighter loan standards from the Fed's quarterly senior Loan officer Opinion Survey.

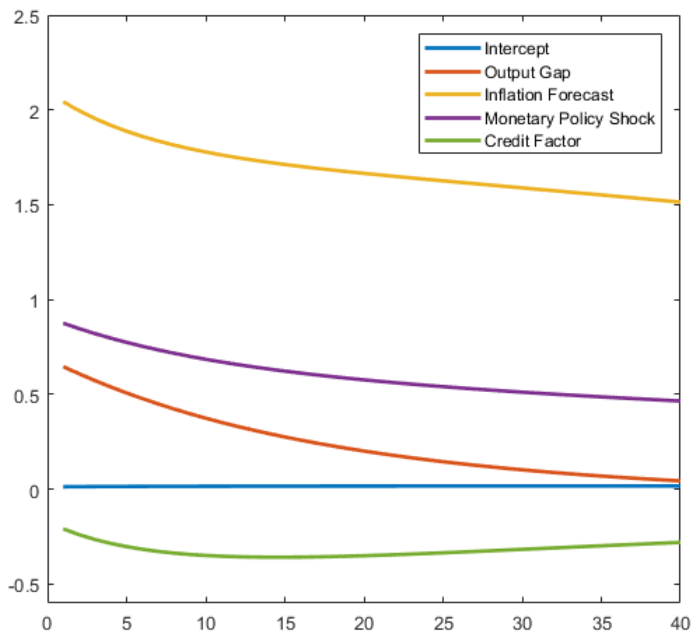
(iv) *Parameter Estimates.* Table 2.6, 2.7 and 2.8 present the model implied parameter estimates of government bond yields and credit spreads. The government short rate parameters and state variables are estimated using OLS. The credit spread short rate parameters, the price

Table 2.5: Estimated Factor Loadings

Maturity	1-year	2-year	3-year	5-year	7-year	10-year
Model 1						
Government Bond Yields						
Intercept	0.0149	0.0159	0.0165	0.0173	0.0176	0.0177
g_t	0.5398	0.4236	0.3320	0.2019	0.1187	0.0446
π_t	1.9189	1.8147	1.7487	1.6658	1.6040	1.5140
f_t	0.7971	0.7174	0.6582	0.5772	0.5233	0.4658
gz_t	-0.2852	-0.3363	-0.3553	-0.3500	-0.3238	-0.2797
Credit Spreads						
Intercept	0.0015	0.0015	0.0018	0.0021	0.0023	0.0023
g_t	-0.0561	-0.0522	-0.0474	-0.0383	-0.0311	-0.0235
π_t	0.2175	0.2030	0.1818	0.1421	0.1122	0.0821
f_t	0.0863	0.0695	0.0575	0.0417	0.0319	0.0229
gz_t	0.0797	0.0865	0.0827	0.0688	0.0564	0.0430
Model 2						
Credit Spreads						
Intercept	0.0015	0.0016	0.0019	0.0023	0.0025	0.0025
pc_t	0.3857	0.3743	0.3634	0.3427	0.3237	0.2978

Table 2.5 reports the joint model-implied factor loadings of Model 1 and Model 2 for government bond and credit spread yields on the intercept, output gap g_t, inflation forecast π_t, monetary policy shock f_t, the credit factor gz_t and the first principal component of credit spreads pc_t. The top panel reports the estimated factor loadings of Model 1. The bottom panel reports the estimated factor loadings of Model 2.

Figure 2.6: Estimated Factor Loadings

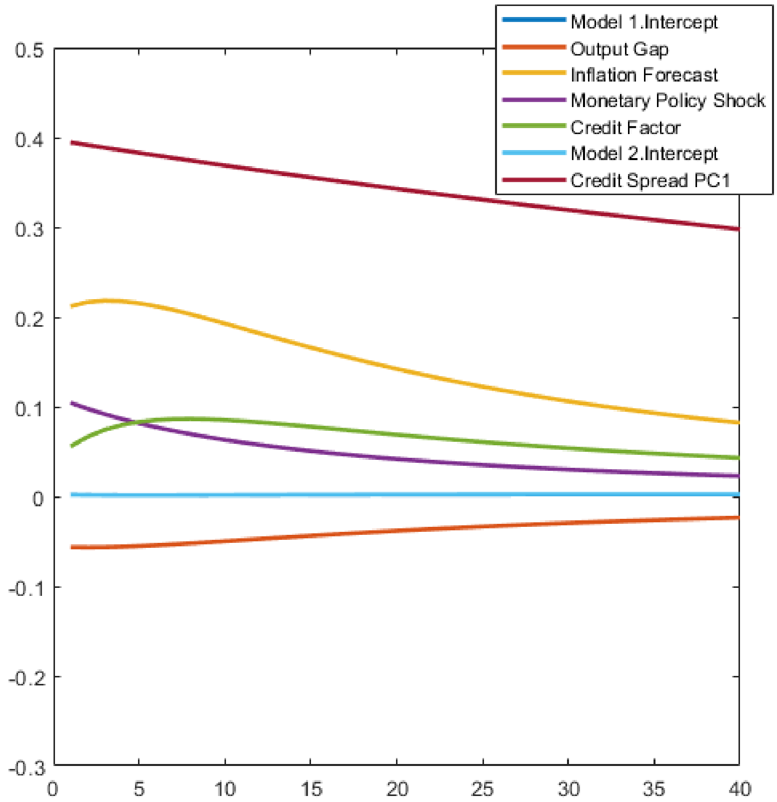


Maturity, quarters

Figure 2.6 reports the model-implied factor loadings of government bond yields on the intercept and state variables with maturities in quarters. The blue line denotes the intercept. The orange line indicates loadings of the output gap. The yellow line indicates loadings of the inflation forecast. The purple line denotes loadings of the monetary policy shock. The green line indicates loadings of the GZ credit spread.

of risk parameters and conditional variance $\boldsymbol{\Sigma}$ are estimated using maximum likelihood. Table 2.6 and Table 2.7 report the Model 1 parameter estimates. The top panel reports estimates of short rates. The result shows that the government short rate has a large sensitivity to variation of the inflation forecast with a highly significant coefficient around 2, and responses to the output gap and monetary policy shock with coefficients of 1.64 and 0.88, respectively. Since the government short rate (3-month government yields in this chapter) is mainly influenced by the change of the federal funds rate, the positive signs can be explained by the related literature, which suggests that the Fed reacts to the output gap and inflation forecast and adjusts Fed funds rate. It is notable that after adjusting the monetary policy shock, the credit factor still has a negative effect on the short rate. This may imply an effect of credit spread

Figure 2.7: Estimated Factor Loadings



Maturity, quarters

Figure 2.7 reports the model-implied factor loadings of credit spread yields on the intercept and state variables with maturities in quarters. The dark blue line denotes the Model 1 intercept. The light red line denotes loadings of the output gap. The yellow line denotes loadings of the inflation forecast. The purple line denotes loadings of the monetary policy shock. The green line denotes loadings of the credit factor. The light blue line denotes the Model 2 intercept. The dark red line denotes loadings of the credit spread first principal component.

on the government short rate, which comports with Cúrdia and Woodford (2010), who suggest to incorporate the credit spread into the standard Taylor rule to measure financial conditions. Gilchrist and Zakrajšek (2012) also mention that credit spread contains information about the risk-bearing capacity of the financial sector, which is orthogonal to the current state of the economy, and can cause a decline in both short-term and long-term government yields. According to Duffie and Singleton (1999), the credit spread short rate is an adjusted short rate, which is the government short rate adjusted by the hazard rate of default and the loss

Figure 2.8: Estimated State Variables

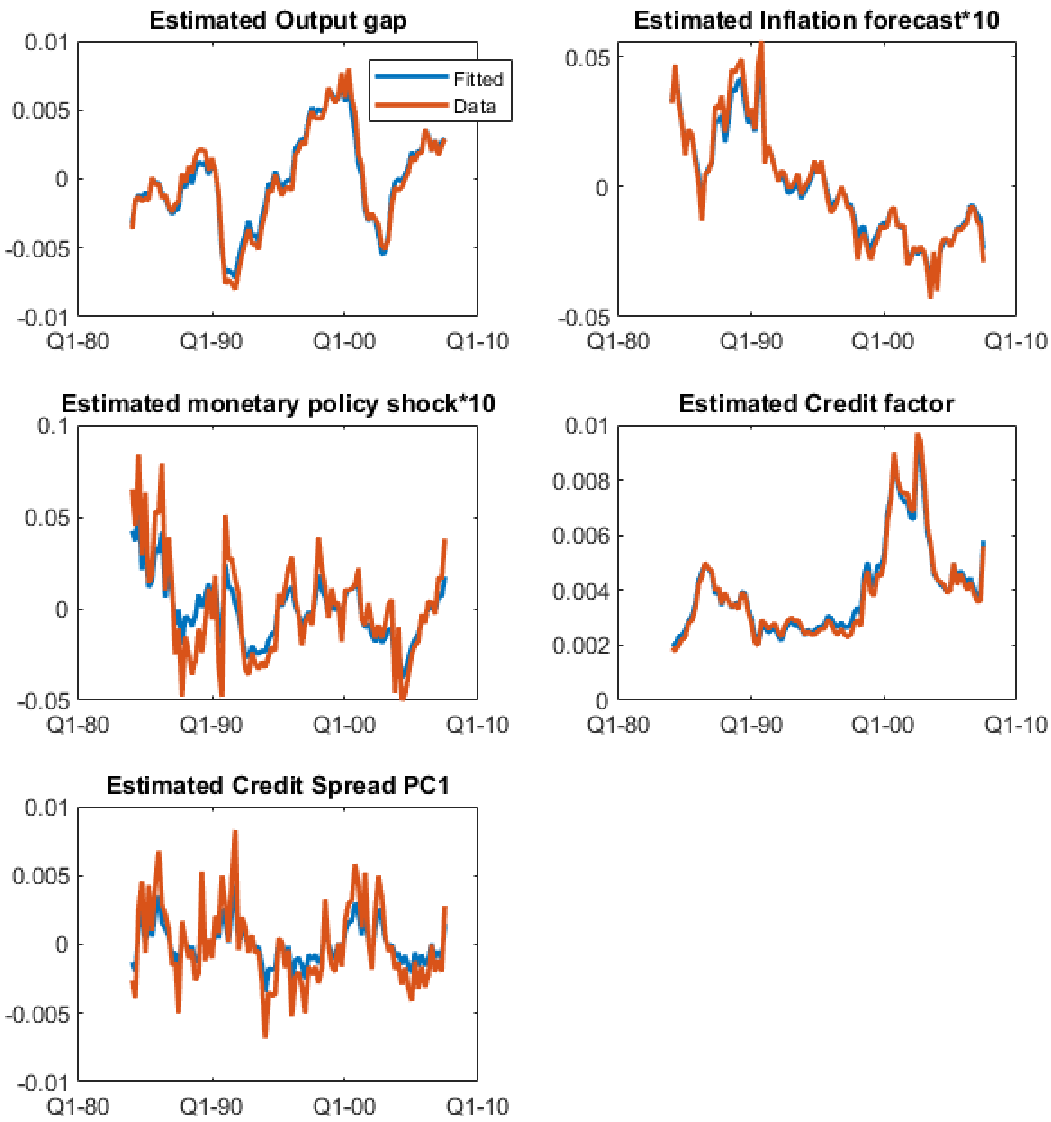


Figure 2.8 reports the estimated state variables. The blue line denotes SRTSM fitted state variables. The blue line denotes model fitted state variables, and the orange line denotes actual state variables. The sample spans from 1984Q1 to 2007Q4.

rate. The result shows that the credit spread short rate loads on both macro variables and the credit factor, which is consistent with the literature. The sign of the effect of state variables on the credit spread short rate comports with the result of the estimated factor loadings in Section 3.4 (iii). The panel reports estimates of state variable parameters. The output gap and inflation forecast are more persistent than the monetary policy. It is notable that the lagged credit factor has a significant negative impact on the output gap with a coefficient around -0.56. This result is consistent with Gilchrist and Zakrajšek (2012), who find that an increase in the credit spread leads to a decline in the output. The lagged credit factor shows no significant impact on the inflation forecast. This is consistent with the result of Friedman and Kuttner (1992), although they use the paper-bill as the measure of default spread. The lagged output gap has a positive effect on the credit factor, which seems counter-intuitive: an increase in the output gap will reduce the default probability, and therefore decreases the credit factor. However, the impact of the macro variables is relatively small, compared with the effect of the lagged credit factor around 0.93.

Table 2.8 reports the Model 2 parameter estimates. The result shows that the first principal component of credit spreads has the largest impact on the credit spread short rate, with a positive coefficient around 0.40, compared with coefficients of the macro factors and credit factor in Table 2.6. The estimates of state variable parameters $\phi_{pc,pc}$ shows that the first principal component of credit spreads are less persistent compared with the macro factors and credit factor in Table 2.6. This is consistent with the result in Amato and Luisi (2006), who use latent variables as a proxy for the unknown factors that cannot be captured by the macro factors.

(v) *Estimated Credit Spread Short Rate.*

Figure 2.9 reports the estimated credit spreads of Model 1 and Model 2. The estimated credit spread short rate in Model 1 is relatively flat during the sample period. In comparison, the Model 2 estimated credit spread short rate shows significantly more variation of credit spreads and is more consistent with the general shape of credit spreads during the sample period. In Model 2, the estimated credit spread short rate shows a high rate around 1991, which corresponds to the late 1990 recession. It also shows spikes around 2002, which is caused by the Telecom bubble. The results are consistent with the findings in Section 2.3.4. This implies that incorporating the first principal component of credit spreads captures the information of credit spreads that cannot be explained by the macro factors and credit factor.

Table 2.6: Estimated Parameters

Model 1				
short rate				
δ_0^T	$\delta_{1,g}$	$\delta_{1,\pi}^T$	$\delta_{1,f}^T$	$\delta_{1,gz}^T$
0.0136^{***}	0.6468^{***}	2.0434^{***}	0.8756^{***}	-0.2084^{***}
(0.0002)	(0.0207)	(0.0375)	(0.0257)	(0.0462)
δ_0^S	$\delta_{1,g}^S$	$\delta_{1,\pi}^S$	$\delta_{1,f}^S$	$\delta_{1,gz}^S$
0.0021^{***}	-0.0565^{***}	0.2120^{***}	0.1044^{***}	0.0556^{***}
(0.0007)	(0.0075)	(0.0458)	(0.0131)	(0.0294)
state variables				
μ_g	$\phi_{g,g}$	$\phi_{g,\pi}$	$\phi_{g,f}$	$\phi_{g,gz}$
0.0011^{***}	0.9257^{***}	-0.2020^{***}	0.0144	-0.2738^{***}
(0.0003)	(0.0322)	(0.0583)	(0.0400)	(0.0725)
μ_π	$\phi_{\pi,g}$	$\phi_{\pi,\pi}$	$\phi_{\pi,f}$	$\phi_{\pi,gz}$
0.0001	0.0051	0.8819^{***}	0.0671	-0.0470
(0.0003)	(0.0279)	(0.0505)	(0.0347)	(0.0629)
μ_f	$\phi_{f,g}$	$\phi_{f,\pi}$	$\phi_{f,f}$	$\phi_{f,gz}$
-0.0003	0.0973	0.2834^{*}	0.5978^{***}	0.0549
(0.0006)	(0.0632)	(0.1146)	(0.0787)	(0.1425)
μ_{gz}	$\phi_{gz,g}$	$\phi_{gz,\pi}$	$\phi_{gz,f}$	$\phi_{gz,gz}$
0.0003	0.0467^{***}	-0.0123	0.0261	0.9261^{***}
(0.0002)	(0.0183)	(0.0332)	(0.0228)	(0.0413)

Table 2.6 reports the Model 1 estimated parameters of the government yields and credit spreads. Standard errors are in parenthesis. Significance at the 95%-, 99%- and 99.9%- levels is denoted as $*$, $**$ and $***$ respectively. The sample spans from 1984: Q1 to 2007: Q4.

Table 2.7: Estimated Parameters

Model 1				
prices of risk				
$\lambda_{0,g}$	$\lambda_{1,gg}$	$\lambda_{1,g\pi}$	$\lambda_{1,gf}$	$\lambda_{1,g.gz}$
0.1094^{***}	-1.4720^{***}	0.8843	-0.8586	-1.7495^{***}
(0.0329)	(0.2492)	(1.1786)	(0.5582)	(0.4863)
$\lambda_{0,\pi}$	$\lambda_{1,\pi g}$	$\lambda_{1,\pi\pi}$	$\lambda_{1,\pi f}$	$\lambda_{1,\pi gz}$
-0.4271^{***}	-1.0011^{***}	1.2739	0.6290	3.2450^{***}
(0.0312)	(0.1328)	(0.8820)	(0.4004)	(0.2242)
$\lambda_{0,f}$	$\lambda_{1,fg}$	$\lambda_{1,f\pi}$	$\lambda_{1,ff}$	$\lambda_{1,f.gz}$
-0.7028^{***}	6.4199^{***}	1.5324^{*}	-6.8765^{***}	-2.1486^{***}
(0.0361)	(0.1329)	(0.6873)	(0.3109)	(0.2579)
$\lambda_{0,gz}$	$\lambda_{1,gz.g}$	$\lambda_{1,gz.\pi}$	$\lambda_{1,gz.f}$	$\lambda_{1,gz.gz}$
-0.1032^{**}	0.3197	3.7714^{***}	0.2792	1.2803^{***}
(0.0387)	(0.1642)	(0.9525)	(0.4332)	(0.2043)
Conditional Variance ($\boldsymbol{\Sigma}$)				
	0.0102^{***}			
	(0.0003)			
	-0.0119^{***}	0.0241^{***}		
	(0.0000)	(0.0000)		
	0.0322^{***}	-0.0857^{***}	0.0196^{***}	
	(0.0001)	(0.0000)	(0.0000)	
	-0.0210^{***}	-0.0209^{***}	-0.0052^{***}	0.0507^{***}
	(0.0005)	(0.0001)	(0.0000)	(0.0005)

Table 2.7 reports the Model 1 estimated parameters of the government yields and credit spreads. Standard errors are in parenthesis. Significance at the 95%-, 99%- and 99.9%- levels is denoted as $*$, $**$ and $***$ respectively. The sample spans from 1984: Q1 to 2007: Q4.

Table 2.8: Estimated Parameters

Model 2	
short rate	
δ_0^S	$\delta_{1,pc}^S$
0.0020^{***}	0.3943^{***}
(0.0001)	(0.0189)
state variables	
μ_{pc}	$\phi_{pc,pc}$
0.0001	0.5110^{***}
(0.0003)	(0.0907)
prices of risk	
$\lambda_{0,pc}$	$\lambda_{1,pc.pc}$
-0.0183^{***}	-95.2256^{***}
(0.0035)	(0.4558)
Conditional Variance ($\boldsymbol{\Sigma}$)	
$\Sigma_{pc,pc}$	
0.0050^{***}	
(0.0000)	

Table 2.6 reports the Model 2 estimated parameters of credit spreads. Standard errors are in parenthesis. Significance at the 95%-, 99%- and 99.9%- levels is denoted as $*$, $**$ and $***$ respectively. The sample spans from 1984: Q1 to 2007: Q4.

Figure 2.9: Estimated Credit Spread Short Rate

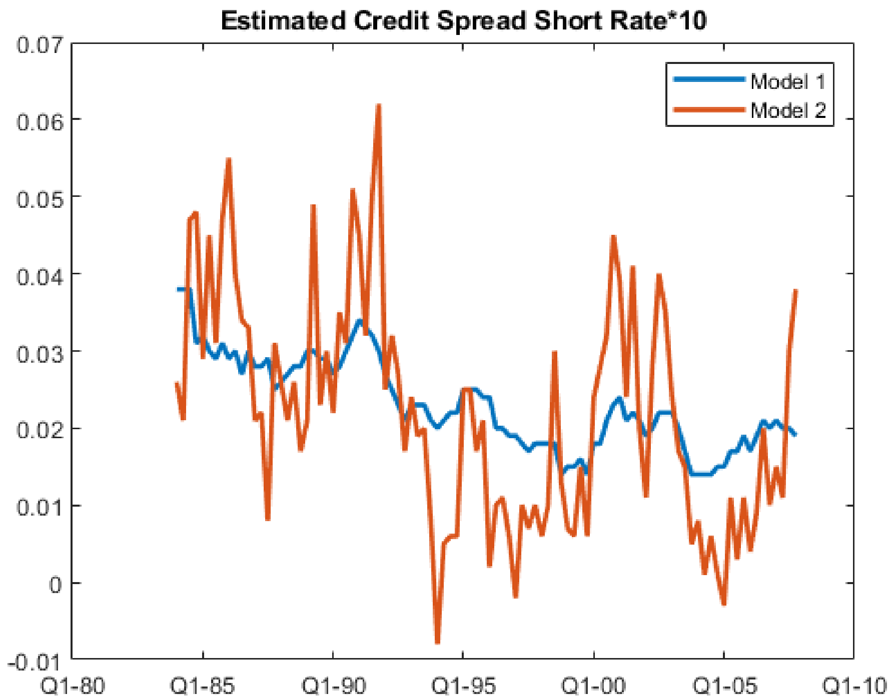


Figure 2.9 reports the estimated credit spread short rate. The blue line denotes Model 1 fitted credit spread short rate. The blue line denotes Model 2 fitted credit spread short rate. The sample spans from 1984Q1 to 2007Q4.

2.3.5 Yield Curve Decomposition

Table 2.9: Estimated Factor Loadings of Government Yields Decomposition

Risk Neutral Expectations					
	Intercept	g_t	π_t	f_t	gz_t
1-year	0.0151	0.6676	1.6513	0.4902	-0.7165
2-year	0.0161	0.5720	1.3088	0.3485	-0.9989
3-year	0.0165	0.4432	1.0495	0.2529	-1.1320
5-year	0.0159	0.2087	0.7681	0.1485	-1.0715
7-year	0.0147	0.0801	0.6674	0.1135	-0.8394
10-year	0.0134	0.0347	0.5857	0.0995	-0.5713
Risk Premium					
	Intercept	g_t	π_t	f_t	gz_t
1-year	-0.0002	-0.1278	0.2676	0.3069	0.4314
2-year	-0.0002	-0.1484	0.5059	0.3689	0.6626
3-year	0.0000	-0.1112	0.6992	0.4053	0.7767
5-year	0.0013	-0.0068	0.8977	0.4286	0.7216
7-year	0.0029	0.0386	0.9366	0.4098	0.5157
10-year	0.0043	0.0099	0.9283	0.3663	0.2916

The table reports the model-implied factor loadings of risk neutral expectation component and term premium of government yields. The model estimated maturity is 1-year, 2-year, 3-year, 5-year, 7-year and 10-year. The related factor loadings of the output gap, inflation forecast, monetary policy shock and credit factor are denoted as g_t, π_t, f_t and gz_t, respectively.

To further investigate how the state variables affect the yield curve, in this section, I decompose government yields and credit spreads into the risk neutral expectation and risk premium components. The former represents the expectations about the future short rate, and can be computed by setting the price of risk parameters $\boldsymbol{\lambda_0}$ and $\boldsymbol{\Lambda_1}$ to zero in Eq.2.13. The latter is then defined as the difference between the model-implied yield curve and the risk neutral component.

It is notable that the main aim of this section is to investigate the impact of macro factors

Figure 2.10: Government Yields Decomposition

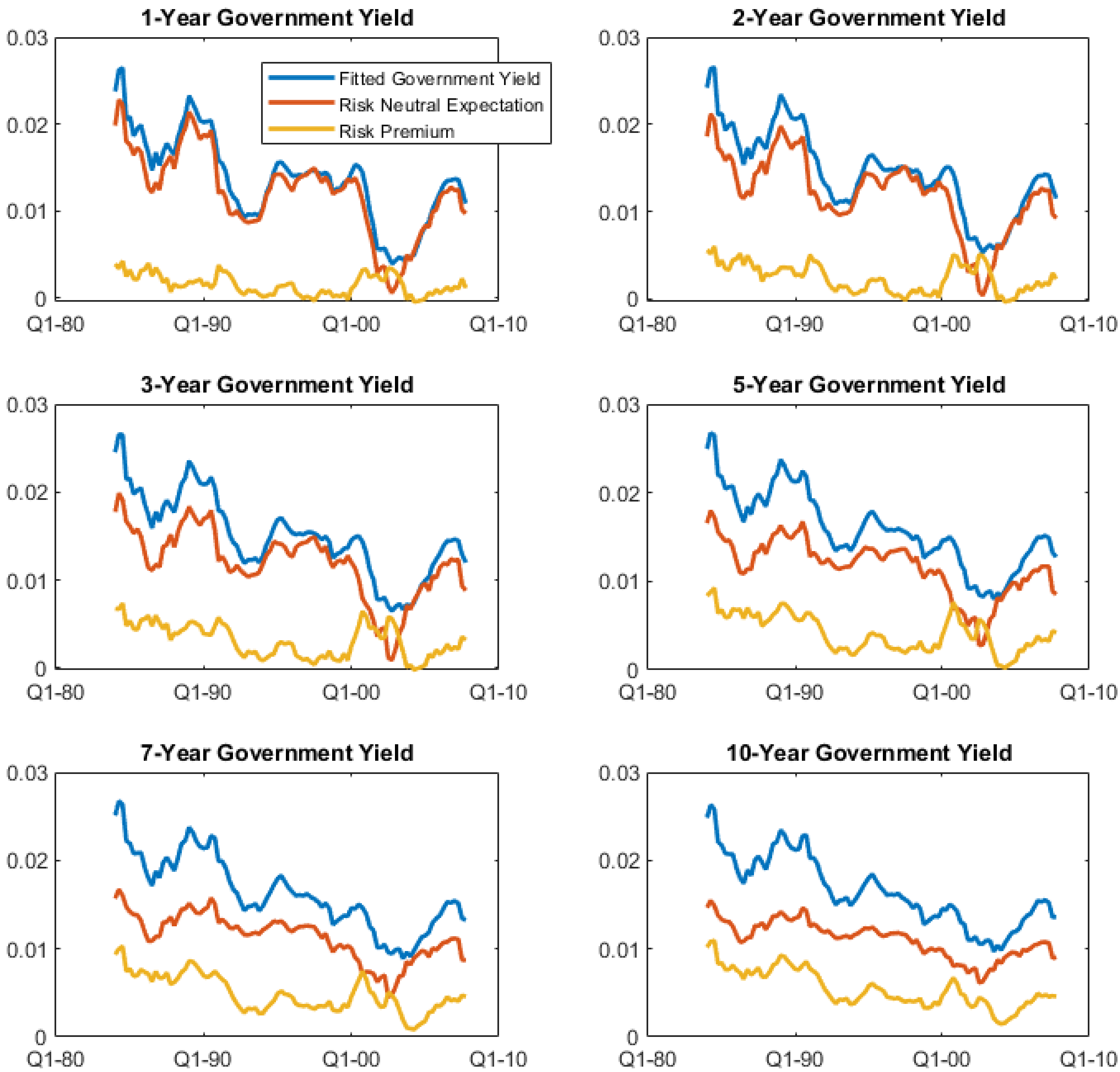


Figure 2.10 reports the decomposition of the model-implied (fitted) government yields (the blue line) into two parts: the risk neutral expectation component (the orange line) and the risk premium (the yellow line). The model estimated maturity is 1-year, 2-year, 3-year, 5-year, 7-year and 10-year. The sample spans from 1984Q1 to 2007Q4.

Figure 2.11: Credit Spreads Decomposition

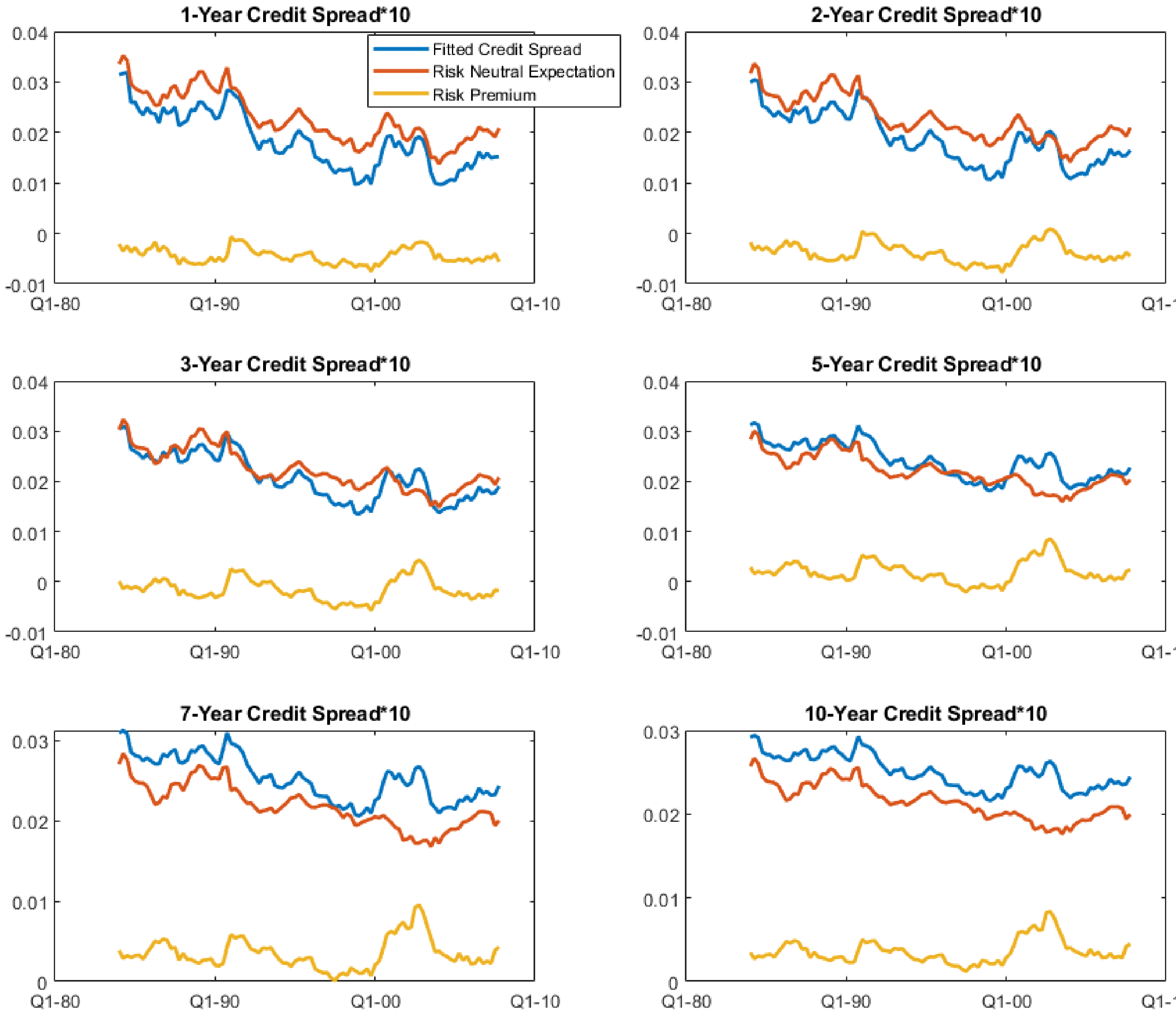


Figure 2.11 reports the decomposition of the model-implied (fitted) credit spreads (the blue line) into two parts: the risk neutral expectation component (the orange line) and the risk premium (the yellow line). The model estimated maturity is 1-year, 2-year, 3-year, 5-year, 7-year and 10-year. The sample spans from 1984Q1 to 2007Q4.

Table 2.10: Estimated Factor Loadings of Credit Spreads Decomposition

Risk Neutral Expectations					
	Intercept	g_t	π_t	f_t	gz_t
1-year	0.0021	-0.0192	0.2238	0.0606	0.0576
2-year	0.0021	-0.0022	0.2034	0.0503	0.0389
3-year	0.0022	0.0073	0.1788	0.0429	0.0174
5-year	0.0023	0.0113	0.1351	0.0309	-0.0153
7-year	0.0023	0.0069	0.1060	0.0229	-0.0273
10-year	0.0022	0.0016	0.0826	0.0168	-0.0243
Risk Premium					
	Intercept	g_t	π_t	f_t	gz_t
1-year	-0.0005	-0.0369	-0.0063	0.0257	0.0221
2-year	-0.0006	-0.0500	-0.0004	0.0192	0.0476
3-year	-0.0004	-0.0547	0.0030	0.0146	0.0653
5-year	-0.0001	-0.0496	0.0070	0.0108	0.0841
7-year	0.0000	-0.0380	0.0062	0.0090	0.0837
10-year	0.0001	-0.0251	-0.0005	0.0061	0.0673

The table reports the model-implied factor loadings of risk neutral expectation component and term premium of credit spreads. The model estimated maturity is 1-year, 2-year, 3-year, 5-year, 7-year and 10-year. The related factor loadings of the output gap, inflation forecast, monetary policy shock and credit factor are denoted as g_t, π_t, f_t and gz_t, respectively.

and credit factor, and make a comparison with the similar results in Wu and Zhang (2008). Therefore, this risk decomposition is based on Model 1 with the macro factors and credit factor.

Figure 2.10 shows the model-implied government yields, the related risk neutral expectation component, and the risk premium component. For short-term yields, the risk neutral component drives most of the variation of government yields. For long-term yields, the risk neutral component becomes more flat and most variation is driven by the risk premium component. A similar pattern is expected in credit spreads. However, Figure 2.11 shows that whereas the risk neutral component captures most of the dynamics of credit spread for shorter maturities and flattens out as the maturities increase, the risk premium component is relatively flat through all maturities. This implies that state variables lack the capability to explain the variation of the term premium in credit spreads. This result is consistent with findings in Section 2.2 and Section 2.3.4(i), that state variables have less explanation power to capture the variation of credit spreads than government yields.

Table 2.9 reports the estimated factor loadings of the government yield risk neutral component and the risk premium component. The related factor loadings of the output gap, inflation forecast, monetary policy shock and credit factor are denoted as g_t, π_t, f_t and gz_t, respectively. In general, macro variables have positively affect both risk neutral expectations and risk premium. It is notable that the credit factor negatively effects on the risk neutral expectations of government yields, while it positively effects on the risk premium. The effect of the credit factor on risk premium is relatively small compared with its effect on risk neutral expectations. The inflation forecast has the largest impact on risk neutral expectations and the risk premium compared with other state variables. Table 2.10 reports the estimated factor loadings of the credit spread risk neutral component and risk premium component. The result shows that in general state variables positively affect the risk neutral expectations. The inflation forecast has the largest impact on shorter-term risk neutral expectations, while as maturities increase the credit factor has a larger impact than the inflation forecast. The credit factor has the largest effect on risk premium, and the output gap also shows a strong effect on the risk premium. In contrast, the inflation forecast and monetary policy have a relative small influence on the risk premium.

2.3.6 Out-of-Sample Forecast

Similar to Ang and Piazzesi (2003) and Liu and Spencer (2013), I conduct an out-of-sample forecast to test the performance of Model 1 and Model 2 in the out-of-sample period.

The forecast methodology is similar to Ang and Bakaert (2002). The data used can be found in Section 2.2.1. I first estimate the models using the in-sample period, then I treat the parameters in the in-sample period as fixed, and use these fixed parameters to do a one-step ahead forecast in the out-of-sample period. I compare the out-of-sample forecast results with the actual data in the terms of RMSE using Eq. 35 in Ang and Bakaert (2002). The in-sample period is from 1984Q1 to 2005Q5. The out-of-sample period is from 2006Q1 to 2007Q4, which are the last two years of the full sample.

I perform the out-of-sample forecasts of three models to make a comparison. The first model is a random walk without a drift. Similar to Ang and Piazzesi (2003), I choose this parsimonious model as a benchmark to compare with the more sophisticated Model 1 and Model 2.

Table 2.11 reports the out-of-sample forecast results. I forecast the last two years (eight quarters) of the full sample, where credit spreads are less volatile than over the full sample, which is due to the inclusion of the quite volatile period in the 1980s and the Telecom bubble around 2002. The notable findings are as follows: (i) the random walk performs much better than Model 1, especially in the terms of the forecast for government yields. This implies that a macro factors and credit factor only model similar to Ang and Ulrich needs to be improved, and this bad performance may be due to the lack of latent factors, which are used in most macro-finance term structure literature. (ii) Model 2 performs significantly better than random walk, this implies the incorporating of the first principal component of credit spreads improves forecasts relative to the macro factors and credit factor only model, even beating random walk.

Therefore, the conclusions are as follows: First, a term structure model with only macro and financial factors similar to Ang and Ulrich (2012) cannot perform well in the out-of-sample period. Second, the forecast of credit spreads can be improved by including principle components of credit spreads, compared with macro factors and credit factor only model.

Table 2.11: Out-of-Sample Forecast Results

	1-year	2-year	3-year	5-year	7-year	10-year
Random Walk						
Government yields	12	13	13	11	10	9
Credit spreads	9	10	10	10	9	8
Model 1						
Government yields	18	27	32	36	38	39
Credit spreads	13	13	13	12	10	9
Model 2						
Credit spreads	4	4	4	3	2	3

Table 2.11 reports root-mean-squared errors of the forecasts versus the actual observations, for the out-of-sample forecasts of government yields and credit spreads with maturities 1-year, 2-year, 3-year, 5-year, 7-year and 10-year. The model estimated period is from 1984Q1 to 2005Q4. The forecasting period is from 2006Q1 to 2007Q4. The forecasts are calculated using the original parameters from the estimated period. Top panel shows the forecast results of the random walk model. The middle panel shows the results of Model 1. The bottom panel shows the results of Model 2. The forecast is one-step ahead. The RMSE are measured as in basis points.

2.4 Concluding Remarks

This chapter contributes to the literature which evaluates the linkage of macro fundamentals, the credit factor, government yields and credit spreads under a joint term structure model. Overall, the empirical evidence suggests that the observed negative relationship between government yields and credit spreads is mainly generated by the shock of the output gap and the credit factor. The negative effect of the credit factor on government yields is mainly through the expectations of the future short rate. The output gap has a strong negative effect on government yields through the risk premium component.

This chapter is based on the reduced-form model of credit spreads. The related literature believes that the driver of credit spreads is systematic, and thus it is more meaningful to analyse aggregate credit spreads instead of individual ones.

The inclusion of economic and credit variables is based on the macro-finance term structure literature of government yields. The joint model approach is motivated by the joint term structure model of government yields and expected equity returns (e.g., Lemke and Werner (2009)).

I estimate a joint term structure model. The results highlight that macro variables capture most of the variation of government yields, and that the largest effect is explained by the inflation forecast. The credit factor explains the considerable variation of credit spreads. It is notable that the output gap positively affects government yields, whereas it negatively affects credit spreads. The credit factor has a small negative effect on government yields, while it has a positive effect on credit spreads. These findings imply that the negative relationship between government yields and credit spreads in the literature may be due to the influence of the output gap shock and credit factor shock. The yield curve decomposition result indicates that the effect of the output gap on credit spreads is mainly through the risk premium, whereas the negative effect of the credit factor on government yields is mainly through the expectations of the future short rate. The out-of-forecast results show that although the macro and financial factors only model (Model 1) fits in-sample government yields well, it has a poor performance in fitting the out-of-sample data. In addition, Model 1 underperforms Model 2 in fitting credit spreads both in-sample and out-of-sample. This implies that principal components, or alternatively latent variables are needed to be used to estimate the term structure of yield curves, as suggested by large term structure literature.

Chapter 3

Corporate Yields at the Zero Lower Bound

3.1 Introduction

Before the recent global financial crisis, the Federal Reserve (the Fed) used the federal funds rate as its key monetary policy tool. The federal funds rate influences other short and long-term interest rates, such as corporate yields and mortgage rates, the borrowing cots for consumers and firms, stimulating the economy. However, during the period December 2008- December 2015, the Fed's target for the federal funds rate has been near zero, which is denoted in literature as the zero lower bound (ZLB) period.

Gaussian affine term structure model (GATSM) is the benchmark model for yield curve analysis in the macroeconomics and finance literature. GATSM assumes that with no arbitrage restriction, yields of zero coupon bond with different maturities is linear with a certain set of Gaussian state variables, with the short rate assumed to follow a Gaussian diffusion. The popularity of the GATSM is due to its tractability and flexibility. Specifically, a close-form analytical solution can be derived with minimal restrictions. The literature includes Duffie and Kan (1996), Dai and Singleton (2002), and Duffee (2002). Piazzesi (2010) provides a general review of the application of GATSM in finance and macroeconomics. Krippner (2015) also summarizes various applications of GATSM, which includes the applications of a subclass GATSM as arbitrage-free Nelson and Siegel models.

A critical drawback of GATSM is that the assumption of Gaussian process is unable to prevent the interest rate from falling below zero. Specifically, since interest rates with different maturities are average expected short-term interest rates, the specification that the short rate follows a Gaussion diffusion means GATSM interest rates of all maturities follow a Gaussion diffusion. This property implies the possibility of a negative interest rate for any maturity. However, the existence of a negative interest rate is counterfactual. If a negative interest rate occurs, market participants can take advantage of an arbitrage opportunity by borrowing funds and holding funds as physical currency. Therefore, Black (1995) states that the government interest rate cannot decrease below zero (or ZLB), i.e., the opportunity cost of holding physical currency. Some alternative term structure models allow for negative interest rates, e.g., the square-root term structure models developed by Cox et al.(1985), and the log-normal term structure models developed by Black et al.(1990). However, these models are intractable and fit the yield curve dynamics poorly. Historically, interest rates remain away from zero, which makes the the unconstrained Gaussian process for short rate dynamics in GATSM negligible.

The recent behaviour of short-term interest rates in major economies challenges the use of GATSM. Since the 1990s, Japan is the first major economy to be stuck at near zero lower bound. After the 2008 financial crisis, the United States and United Kingdom interest rates reach the ZLB, followed by the Euro area. Standard GATSM is no longer a reliable framework to investigate the dynamics of interest rates. As found by Krippner (2015, p25), when interest rates are stuck at ZLB, the GATSM shows poor performance in fitting the data with the possibility for short-term rates and even 7-year interest rates to become negative. The shadow rate term structure model (SRTSM) is a plausible solution. The SRTSM assumes that if physical currencies do not exist, the shadow short rate follows the Gaussian diffusion and the shadow term structure is specified as GATSM. Meanwhile, ZLB short rates remain contained by the lower bound. This ZLB mechanism enables SRTSM to produce a good fitting to represent the shape of policy short rates and yield curves with different maturities compared with the benchmark GATSM. Black (1995) is pioneering in proposing the SRTSM. Kim and Singleton (2012) and Christensen and Rudebusch (2014) develop the multi-factor SRTSM using a simulation method. Krippner (2013) and Ichiue and Ueno (2013) propose analytical approximation in continuous time. Wu and Xia (2016) propose a tractable approximation in discrete time.

Corporate bond yields reflect real borrowing costs in the financial market, and become an increasingly important source of external financing for firms, especially since the financial crisis in 2008, as the bank lending declines. In addition, from a monetary policy perspective, the corporate bond yield is a central element of the monetary policy transmission thorough financial markets. Since the Lehman default in September 2008, default risk in corporate bond and related financial markets increased dramatically. For example, Spencer (2016) shows that the implicit one-year default rate of credit default swap for Morgan Stanley spiked up to 20% following the Lehman default.

To ease the liquidity and credit crunch in financial markets and banking system, the Treasury's Troubled Assets Relief Program (TARP) are announced. Followed the TARP, other unconventional monetary policies- Large Scale Asset Purchases (LSAPs) programs and forward guidance are used by the Fed. The aim of these unconventional monetary policies is to improve market functioning, reduce borrowing rates for private borrowers, and stimulate economic activity. However, since these Fed monetary policies mainly target at Treasury bond yields, the monetary policy transmission on corporate bond yields needs to be analyzed.

Much efforts has been devoted to assess the monetary policy transmission on Treasury securities. Previous literature measuring the effect on Treasury securities includes Gagnon et al. (2011), Hamilton and Wu (2012a), and D'Amico and King (2013). On the other hand, some attention has been paid to examining the monetary policy transmission on corporate bond markets. Wright (2012) uses a structural VAR; Krishnamurthy and Vissing-Jorgensen (2013) use event study; Gilchrist and Zakrajsek (2013) use a heteroskedasticity-based approach. However, these studies encounter an econometric identification difficulty in assuming endogenety and exogeneity. In addition, unlike term structure models, these approaches cannot draw conclusions about the monetary policy impact on the whole term structure. Therefore, a reliable term structure model is needed to model corporate yields during the zero lower bound period, and to analyze the possible monetary policy transmission on the whole term structure based on the model.

In this chapter, I use the shadow rate term structure model to examine its performance in fitting corporate yields during the zero lower bound period. I then evaluate the impact of the ZLB on corporate yields, and its potential monetary policy implications. Specifically, I construct a joint shadow rate term structure model for Treasury and corporate yields. I estimate the model by maximum likelihood and the extended Kalman filter using monthly 6-month forward rates. Treasury forward rates are constructed using the Gurkaynak et al. (2007) dataset. Corporate forward rates are constructed using U.S. Department of Treasury

High Quality Market corporate bonds (rated A or above) dataset. The sample period is January 1990 to March 2017. Based on a preliminary principal component analysis similar to Coroneo and Pastorello (2017), I specify a model with four factors: two common factors and two specific factors to Treasury and corporate bonds respectively. One advantage of the joint framework is examining the interaction between Treasury and corporate yield curves at ZLB. For example, the relationship between common and bond-specific factors can be analysed. Unobserved corporate short rate and model-implied credit spreads (the difference between Treasury and corporate bond yields of the same maturity) can be derived.

The results indicate that during the ZLB period, the SRTSM has a significantly better performance than the GATSM in fitting corporate yields at long horizons, both in-sample and out-of sample. In addition, the results show that, as the shadow rate literature has stated, the estimated shadow short rate are quite sensitive to the value of the lower bound parameter. Specifically, during the zero lower bound period, there is a significant difference in the joint model estimation results of the GATSM and SRTSM for long-term corporate yields. The root mean square error of the SRTSM for the 10-year maturity corporate yields is 35 basis points smaller than the GATSM, and the yield curve fit of the SRTSM for corporate yields at longer horizons during the ZLB period is significantly better than that of the GATSM. The out-of-sample forecast also shows that the SRTSM has a much better performance in predicting long-term corporate yields during both the ZLB period and post-ZLB period, producing forecasts that are twice as accurate as those from the GATSM. I also find that the SRTSM fits credit spreads better than the GATSM in the long run.

In addition, I use the cumulative distribution function to evaluate the relevance of the lower bound for corporate interest rates, and the potential monetary policy implications. I find that the lower bound constraint affect corporate interest rates for short-term and up to 2-year horizons. However, the impact of the lower bound on long-term corporate interest rates is relatively small. This may due to the effect of corporate bond specific factor, which keeps the 10-year ahead corporate interest rates above the lower bound. The results show that the transmission mechanism of the monetary policy is not very effective for the long-term corporate interest rates.

To my best knowledge, this chapter innovatively analyses the dynamics of corporate yields under the SRTSM framework. Previously, Wright (2012) uses a structural VAR; Krishnamurthy and Vissing-Jorgensen (2013) use event study; Gilchrist and Zakrajsek (2013) use a heteroskedasticity-based approach. However, these studies commit a econometric identification difficulty in assuming endogenety and exogeneity. The results from the literature also violates one another.

The reminder of the chapter is organised as follows: Section 3.2 describes the SRTSM framework. Section 3.3 describes the data. Section 3.4 performs preliminary analysis and model specification. Section 3.5 describes estimation methodology. Section 3.6 analyses estimation results and Section 3.7 concludes.

3.2 Joint Shadow Rate Term Structure Model

This section adopts a joint SRTSM framework to examine the dynamics of the U.S. Treasury and corporate yields.

3.2.1 Shadow Rate

Similar to Black (1995), I assume that the short-term interest rate of bond i, i= T, C, (denoted as treasury and corporate bond respectively) is the maximum of the shadow rate s_t^i and a lower bound $\underline{r}^i$:

$$r_t^i = max(\underline{r}^i, s_t^i) \tag{3.1}$$

which implies that if the shadow rate s_t^i is greater than the lower bound, then s_t^i is the short rate. If the lower bound is binding, the shadow rate contains more information about the current state of economy than does the short rate itself. Since the end of 2008, the existence of a lower bound on interest rate has become relevant in the US, when the Fed set an annual interest rate at around 0.25%.

The shadow short term interest rate of bond i is assumed as affine function of both the common factors $\boldsymbol{x_t^0}$ and bond specific factors x_t^i. The shadow short rate of bond i is:

$$s_t^i = \delta_0^i + \boldsymbol{\delta_1^{i\prime}} \boldsymbol{x}_t \tag{3.2}$$

where: $\boldsymbol{\delta_1^T} = (\boldsymbol{\delta_1^{To\prime}}, \delta_1^{TT'}, 0')$, $\boldsymbol{\delta_1^C} = (\boldsymbol{\delta_1^{Co\prime}}, 0', \delta_1^{CC'})$, $\boldsymbol{x}_t = [\boldsymbol{x_t^{o\prime}}, x_t^{T'}, x_t^{C'}]$.

3.2.2 State Variable Dynamics

K State variables follow a first-order vector autoregression process under the P-measure. See Chapter 2 Eq.2.4.

3.2.3 Bond Prices

(i) The log stochastic discount factor is essentially affine as in Duffee (2002). See Chapter 2 Eq.2.6.

where the price of risk λ_t is linear in the state variables, as Chapter 2 Eq.2.7.

(ii) It is also assumed that state variables follow a VAR(1) under the risk-neutral measure (P-measure), as Chapter 2 Eq.2.8.

(iii) The parameters under the P- and Q-measure are related as Chapter 2 Eq.2.9.

3.2.4 Shadow Rate Bond Yields

Let the time t six-month forward rate for a loan starting at $t+n$ be denoted by $f^i_{t,n,n+6}$, and the time t one-month forward rate for a loan starting at $t+n$ be denoted by $f^i_{t,n,n+1}$. Six-month forward rates and one-month forward rates are related as follows:

$$f^{i,SRTSM}_{t,n,n+6} = \frac{1}{6}\sum_{j=1}^{6} f^i_{t,n+j-1,n+j} \tag{3.3}$$

Following the derivation of Wu and Xia (2016), the one-month forward rate $f^i_{t,n,n+1}$ is approximately equal to:

$$f^{i,SRTSM}_{t,n,n+1} = \underline{r}^i + \sigma^i_n g\left(\frac{a^i_n + \boldsymbol{b}^{i\prime}_n \boldsymbol{x}_t - \underline{r}^i}{\sigma^i_n}\right) \tag{3.4}$$

where $(\sigma^i_n)^2 \equiv Var_t(s^i_{t+n})$. The function $g(z_t) \equiv z_t H(z_t) + h(z_t)$ consists of a normal cumulative distribution function $H(.)$ and normal probability density function $h(.)$. Its nonlinearity comes from moments of the truncated normal distribution.

The approximation for six-month forward rates can be constructed by plugging the approximate expression of one-month forward rates in Eq.3.4 into Eq.3.3:

$$f_{t,n,n+6}^{i,SRTSM} = \underline{r}^i + \frac{1}{6}\sum_{j=1}^{6}\sigma_{n+j-1}^i \, g(\frac{a_{n+j-1}^i + \boldsymbol{b}_{n+j-1}^{i'}\boldsymbol{x}_t - \underline{r}^i}{\sigma_{n+j-1}^i}) \tag{3.5}$$

3.2.5 Relation to Gaussian Affine Term Structure Model

If Eq.3.1 is replaced with $r_t = s_t$, the SRTSM becomes a GATSM. The one-month forward rate in the GATSM is an affine function of the state variables:

$$f_{t,n,n+1}^{i,GATSM} = a_n^i + \boldsymbol{b}_n^{i'}\boldsymbol{x}_t \tag{3.6}$$

where a_n^i and $\boldsymbol{b}_n^{i'}$ are the same as in Eq.3.4.

The difference between Eq.3.4 and Eq.3.6 is the function $g(z_t)$, which is nonlinear and increasing. The approximation is almost perfect when the function input is z_t is larger than 2. This means that when the economy is effectively away from the ZLB, SRTSM overlaps with GATSM.

The six-month forward rates is given by plugging the one-month forward rates in Eq.3.6 into Eq.3.3:

$$f_{t,n,n+6}^{i,GATSM} = \frac{1}{6}\sum_{j=1}^{6}(a_{n+j-1}^i + \boldsymbol{b}_{n+j-1}^{i'}\boldsymbol{x}_t) \tag{3.7}$$

3.3 Data Description

3.3.1 Treasury Forward Rate

I construct 6-month treasury forward rates with maturities of 6 months, 1, 2, 3, 5, 7, and 10 years from the Gurkaynak et al.(2007) dataset at monthly frequency. Sample period is January 1990 to March 2017.Figure 3.1 plots the time series of these treasury forward rates. From January 2009 to December 2015, the Federal Open Market Committee (FOMC) lowered the target range for the federal funds rate from 0 to 25 basis points, which is referred as the ZLB period and is highlighted by the shaded area. During this period, treasury forward rates of shorter maturities are stuck at zero, and do not display significant variation. Those with longer maturities are higher than the lower bound, and display noticeable variation.

Figure 3.1: Treasury Forward Rate

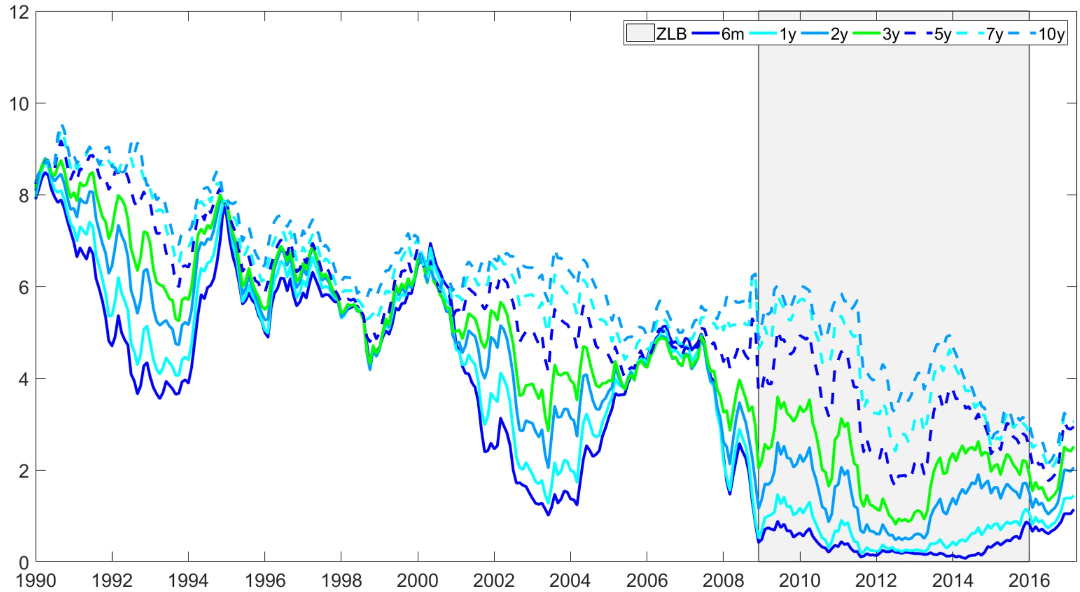


Figure 3.1 reports 6-month Treasury forward rates from January 1990 to March 2017 in annualized percentage points. Maturities are 6 months, 1, 2, 3, 5, 7, and 10 years. ZLB periods in gray areas is from January to Decemebr 2015.

3.3.2 Corporate Forward Rate

I construct 6-month corporate forward rates with maturities of 6 months, 1, 2, 3, 5, 7, and 10 years from the U.S. Department of the Treasury High Quality Market zero coupon corporate yields dataset at monthly frequency. Detailed description of HQM data can be found in 2.2.1. The same sample period is from January 1990 to March 2017. Figure 3.2 plots the time series of the corporate forward rates. For the ZLB period corporate forward rates of shorter maturities display variation.

It is notable that I have not included the maturities over 10 years, in order to make the results comparable with benchmark papers. However, including long-term Treasury bonds in the term structure model has important implications. See details in Chapter 2 Section 2.2.1.

Figure 3.2: Corporate Forward Rates

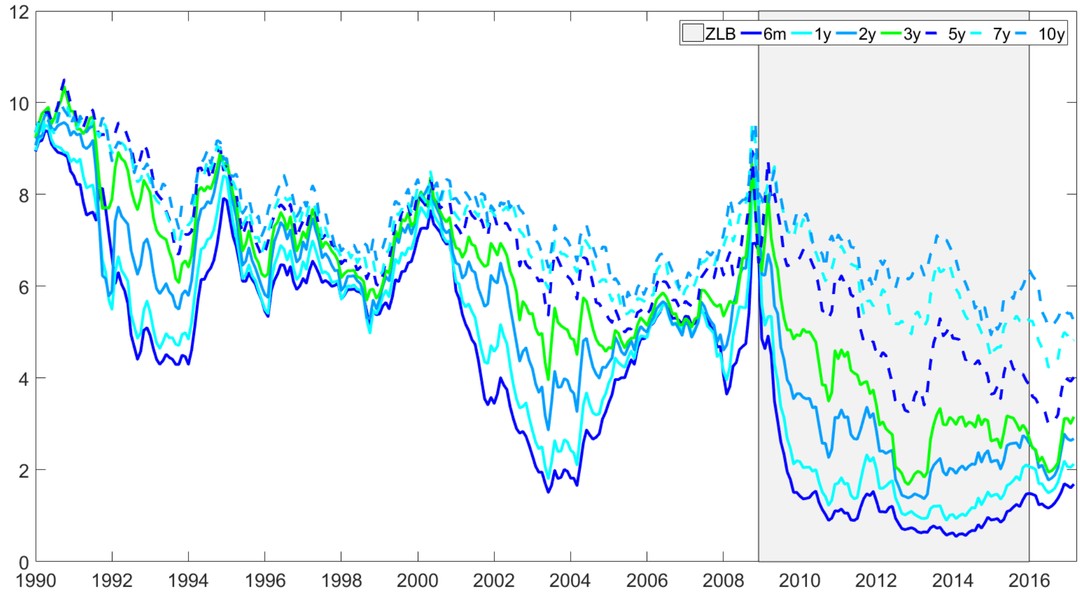


Figure 3.2 reports 6-month corporate forward rates from January 1990 to March 2017 in annualized percentage points. Maturities are 6 months, 1, 2, 3, 5, 7, and 10 years. ZLB periods in gray areas is from January to Decemebr 2015.

3.4 Model Specification

The joint shadow rate term structure model in Section 3.2 specifies that Treasury and corporate forward rates are driven by common and bond-specific factors. Similar to Coroneo and Pastorello (2018), I use principal component analysis as a parsimonious test to characterize the drivers of Treasury and corporate yields, then analyse the parameterization scheme.

3.4.1 principal Component Analysis

To determine the number of each type of factors, I analyse each yield curve separately. The first two columns of Table 3.1 report the cumulative variance of Treasury and corporate forward rates explained by the corresponding six principal components (PCs). This table indicates that for these two term structures the first three PCs explain 99.9% of the observed variance for treasury, and 99.6% for corporate.

I then pool the two types of yields curves and extract PCs jointly. The cumulative joint variance of Treasury and corporate forward rates explained by the PCs is reported in the last column of Table 1. This table shows that four joint PCs are required to explain 99.6% of the joint variation, indicating that in addition to common factors, treasury and corporate forward rates are driven by bond specific factors.

Table 3.1: Cumulative Variance

	Treasury	Corporate	Joint
PC1	0.927	0.930	0.904
PC2	0.994	0.989	0.966
PC3	0.999	0.996	0.993
PC4	1.000	0.999	0.996
PC5	1.000	1.000	0.999
PC6	1.000	1.000	1.000

Table 3.1 reports the cumulative percentage of variance of treasury forward rates (first column); corporate forward rates (second column) and joint forward rates (third column) explained by the first six PCs extracted from treasury forward rates, corporate forward rates, and jointly from the treasury and corporate forward rates respectively.

Table 3.2: Common Factors

Treasury PC	Corporate PC1	Corporate PC2	Corporate PC3
1	0.889	-0.002	-0.001
First 2	0.891	0.887	-0.002
First 3	0.900	0.890	0.130
First 4	0.922	0.890	0.172
First 5	0.938	0.890	0.173
First 6	0.949	0.907	0.204

Table 3.2 reports the R^2 from regressing corporate PCs on treasury PCs. The first row refers to regressions on the first Treasury PC, the second row refers to regressions on the first two Treasury PCs, and so on.

Given that the government interest rate is the baseline for other riskier interest rates, the corporate forward rate is assumed to share some common factors from the Treasury forward rate. Therefore, the PCs extracted from the treasury yield curve is assumed to proxy as common factors. To assess the relation of the Treasury factors with corporate factors, I analyse how much of the variation in the first three corporate PCs is explained by the Treasury factors. In the table 3.2, I report the R^2 from regressing corporate PCs on the Treasury PCs. This table indicates that the first corporate PC is mostly explained by the first Treasury PC. The second corporate PC is mainly explained by the second Treasury PC. The third corporate PC is much less related to Treasury factors with Treasury factors, and even all six Treasury PCs explains just up to 20.4%, which indicates that corporate yields are driven by a strong bond-specific component. In addition, Treasury forward rates are also driven by a strong bond-specific factor which performs similar to the third Treasury PC.

Table 3.3: Bond Specific Factors

	corporate			
common PC	0	1	2	3
1 corporate PC	0.930	0.607	0.861	0.858
2 corporate PC	0.060	0.336	0.062	0.068
3 corporate PC	0.006	0.036	0.047	0.043
4 corporate PC	0.004	0.019	0.026	0.028
5 corporate PC	0.000	0.002	0.002	0.003
6 corporate PC	0.000	0.000	0.000	0.000

Table 3.3 reports the percentage of variance of corporate yield residuals explained by the first six corporate PCs. The first column refers to the percentage of variance of corporate yields. The second refers to the percentage of variance of the residuals of corporate yields after regressed on the first treasury factor. The third refers to the percentage of variance of the residuals of corporate yields after regressed on the first two treasury factors, etc.

Table 3.3 reports the percentage of variance of corporate bond residuals explained by the first six corporate bond specific PCs. For comparison, in the first column I report the explained variance of corporate yields when no Treasury components are extracted (the same as reported in Table 3.1). This table indicates that after incorporating the common components, corporate yields still display strong commonalities. In particular, after taking into account the three Treasury factors, corporate yields seem to be driven by one corporate specific factor.

Overall, Tables 3.1 to 3.3 suggest that four factors are needed to explain the two types of the yield curves; two of the four factors are the common treasury factors, and the other is specified as a treasury bond-specific factor and corporate bond-specific factor respectively. Accordingly, in specifying the joint model, four factors are needed.

3.4.2 Parameterization

According to preliminary results of Principal component analysis, four factor factors adequately explain most variation in treasury and corporate yields. Therefore, I adopt a SRTSM with four latent factors. The collection of parameters includes $(\boldsymbol{\mu}, \boldsymbol{\mu}^Q, \boldsymbol{\Phi}, \boldsymbol{\Phi}^Q, \boldsymbol{\Sigma}, \delta_0^i, \boldsymbol{\delta}_1^i)$. In order to uniquely identify the latent state variables, I use the identification scheme of Joslin, Singleton and Zhu (2011) as follows: (1)$\boldsymbol{\delta_1^T} = [1, 1, 0, 0]'$,$\boldsymbol{\delta_1^C} = [1, 1, 0, 1]'$ (2)$\boldsymbol{\mu}^Q = 0$, (3)$\boldsymbol{\Phi}^Q$ is in real Jordan form with eigenvalues in descending order, and (4)$\boldsymbol{\Sigma}$ is lower triangular. Imposing restrictions is to prevent the latent factors from shifting, rotating and scaling. Changes of restrictions does not change the economic implication of the model.

Repeated Eigenvalues: Estimation assumes that $\boldsymbol{\Phi}^Q$ has four distinct eigenvalues. However, according to Creal and Wu (2015), and Wu and Xia (2016), when using different datasets, the first three distinct eigenvalues of treasury bond state variables produces two smaller eigenvalues almost identical to each other, with the difference in the order of 10^{-3}. Therefore, the real Jordan form becomes:

$$\boldsymbol{\Phi}^Q = \begin{bmatrix} \phi_1^Q & 0 & 0 & 0 \\ 0 & \phi_2^Q & 1 & 0 \\ 0 & 0 & \phi_2^Q & 0 \\ 0 & 0 & 0 & \phi_3^Q \end{bmatrix}$$

3.5 Estimation Methodology

3.5.1 SRTSM

Extended Kalman filter is used for estimation, which applies the Kalman filter by linearizing the nonlinear function $g(z_t)$ around the current estimates. Because the function $g(z_t)$ is monotonically increasing, the likelihood surface behaves similarly to a GATSM.

The transition equation for the state variables is as Chapter 2 Eq.2.4.

where condition covariance is $\boldsymbol{\Sigma}\boldsymbol{\Sigma}'$, and $\boldsymbol{\Sigma}$ is specified as a 4×4 lower triangular matrix, with $\boldsymbol{\varepsilon}_t \sim i.i.d.N(\mathbf{0}_4, \boldsymbol{I}_4)$.

Based on Eq.3.5, the measurement equation relates the observed six-month forward rate $f^{o,i}_{t,n,n+6}$ to the state variables:

$$f^{o,i}_{t,n,n+6} = \underline{r}^i + \frac{1}{6}\sum_{j=1}^{6} \sigma^i_{n+j-1} g\left(\frac{a^i_{n+j-1} + \boldsymbol{b}^{i'}_{n+j-1}\boldsymbol{x}_t - \underline{r}^i}{\sigma^i_{n+j-1}}\right) + \eta^i_{t,n} \tag{3.8}$$

where the measurement error $\eta^i_{t,n} \sim i.i.d.N(0, \omega^i)$.

3.5.2 GATSM

Since the GATSM is a linear Gaussian state space model, the Kalman filter is used for estimation.

The transition equation for the state variables is the same as the SRTSM, see Section 3.5.1.

Based on Eq.3.7, the measurement equation for six-month forward rates is as follow:

$$f^{o,i}_{t,n,n+6} = \frac{1}{6}\sum_{j=1}^{6}(a^i_{n+j-1} + \boldsymbol{b}^{i'}_{n+j-1}\boldsymbol{x}_t) + \eta^i_{t,n} \tag{3.9}$$

where the measurement error $\eta^i_{t,n} \sim i.i.d.N(0, \omega^i)$.

3.6 Estimation Results

This section discusses seven aspects of the estimation results: estimated yield curves, factor loadings, estimated state variables, estimated parameters, estimated shadow short rate, estimated credit spreads and out-of-sample forecasts.

3.6.1 Estimated Yield Curves

Overall, the SRTSM fits the corporate yields at long horizons significantly better than the GATSM. It is notable that during the ZLB period, the GATSM estimated 6-month Treasury yields was below zero around 2014, which implies the violation of the zero lower bound as stated in literature. In addition, for shorter maturities Treasury and corporat yields, the relatively poor performance of the SRTSM in some years compared with the GATSM is due to the high estimated value of the lower bound parameter, which can be solved by fixing the lower bound parameter at zero, as suggested by Golinski and Spencer (2019).

Table 3.4: Root-Mean-Square-Error

	Treasury						
	6-month	1-y	2-y	3-y	5-y	7-y	10-y
GATSM	6	6	7	4	9	7	11
SRTSM	7	6	6	4	8	6	10
	Corporate						
	6-month	1-y	2-y	3-y	5-y	7-y	10-y
GATSM	34	23	32	30	24	49	65
SRTSM	31	23	28	26	25	25	30

Table 3.4 reports the root mean square error of the GATSM and the SRTSM model-implied Treasury (top panel) and corporate (bottom panel) forward rates with maturities of 6-month; and 1,2,3,5,7, and 10-year. The sample is from January 1990 to March 2017. The RMSE are measured in basis points.

(i) *RMSE.* Table 3.4 reports the root mean square error (RMSE) of the GATSM and SRTSM model-implied treasury and corporate forward yields. In general, the fitting of the SRTSM for corporate yields is good and performs better than the GATSM, especially for the corporate yields at longer horizons. In addition, the SRTSM has superior performance over the GATSM especially in the longer-term maturities. For the treasury yields, the RMSE results of SRTSM is slightly better than GATSM. However, the difference of the RMSE between the GATSM and the SRTSM is not as large as the difference of the corporate yield results. For longer maturities, the RMSE of the SRTSM is about 1 basis point smaller than the GATSM. For the corporate yields, the RMSE results show that SRTSM has a significant better fit for corporate yields compared with its GATSM counterparts. For the 10-year maturity, the RMSE results of SRTSM is 35 basis points smaller than the GATSM.

(ii) *Yield Curve Fit.* To further examine the fitting of the yield curve, I reports the yield curve fit of the model-implied Treasury forward rates in Figure 3.3 and 3.4; corporate forward rates in Figure 3.5 and 3.6. The sample spans from January 1990 to March 2017. To display the fit of the yield curve during different periods, I plot the pre-ZLB period (January 1990 to

Figure 3.3: Yield Curve Fit

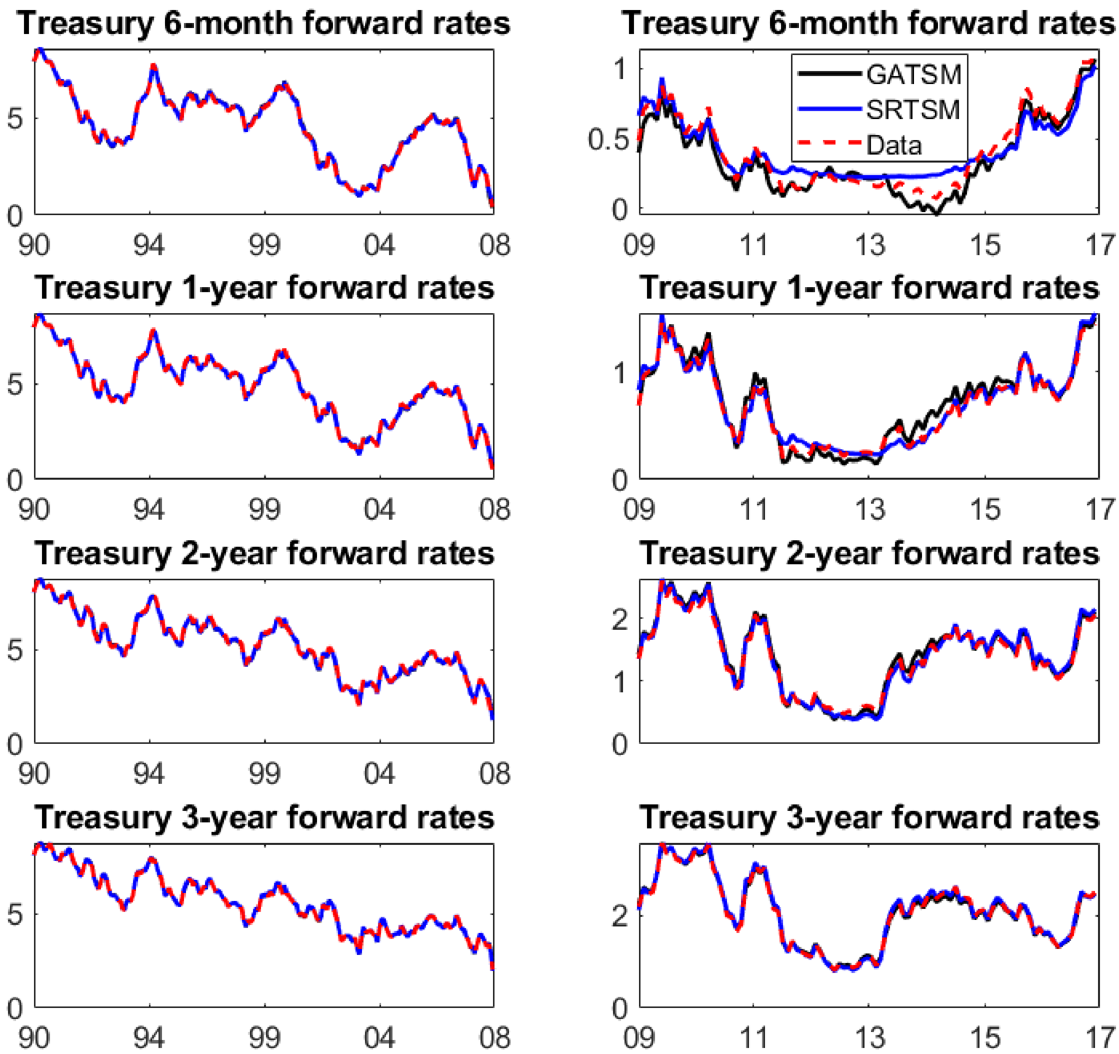


Figure 3.3 reports observed and fitted yields with maturities of 6-month; and 1,2, and 3- year for Treasury forward rates. The black solid line denotes GATSM fitted forward rates, the blue solid line denotes SRTSM fitted forward rates, and the red dashed line denotes observed forward rates.

Figure 3.4: Yield Curve Fit

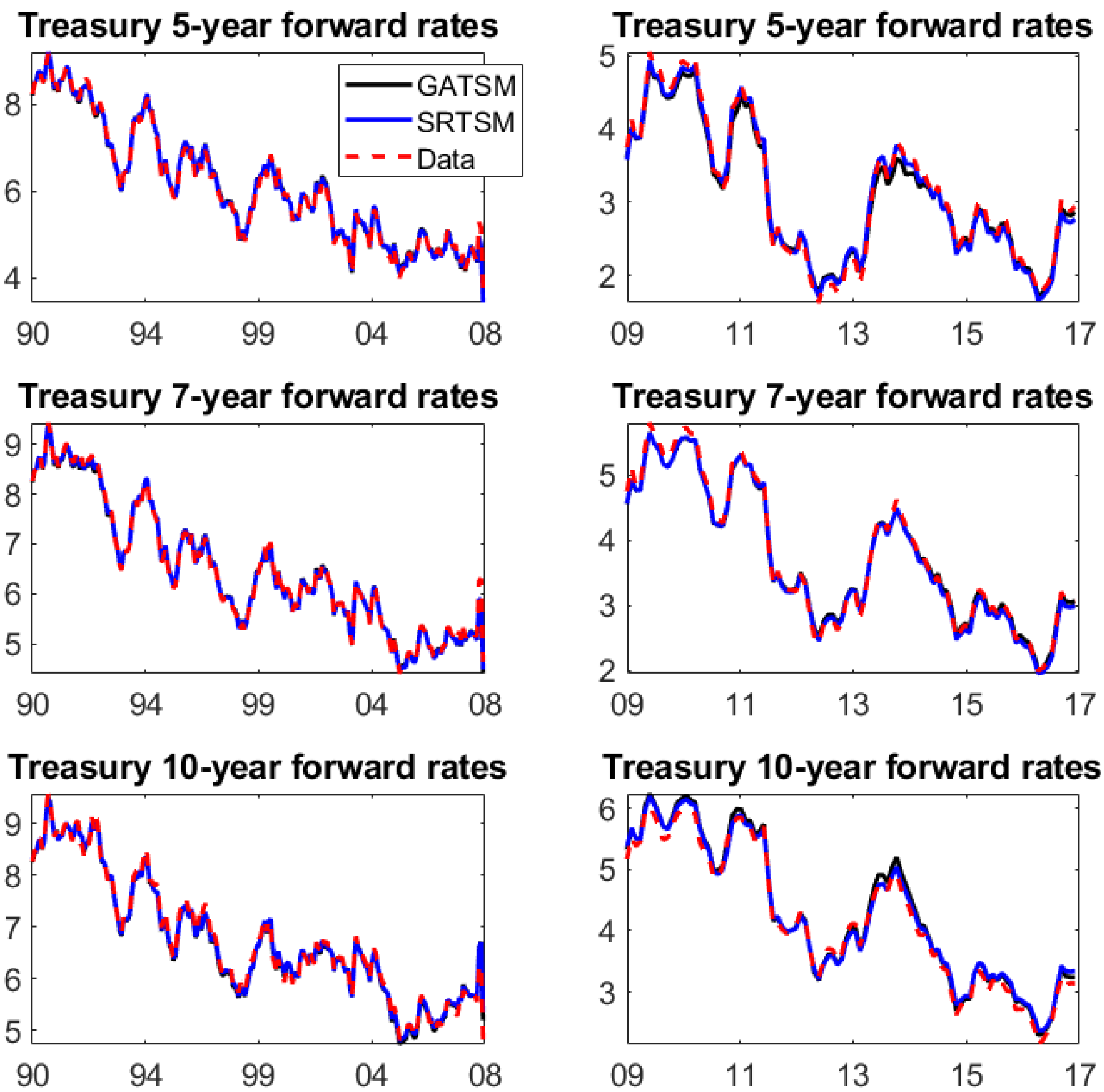


Figure 3.4 reports observed and fitted yields with maturities of 6-month; and 5,7, and 10-year for Treasury forward rates. The black solid line denotes GATSM fitted forward rates, the blue solid line denotes SRTSM fitted forward rates, and the red dashed line denotes observed forward rates.

Figure 3.5: Yield Curve Fit

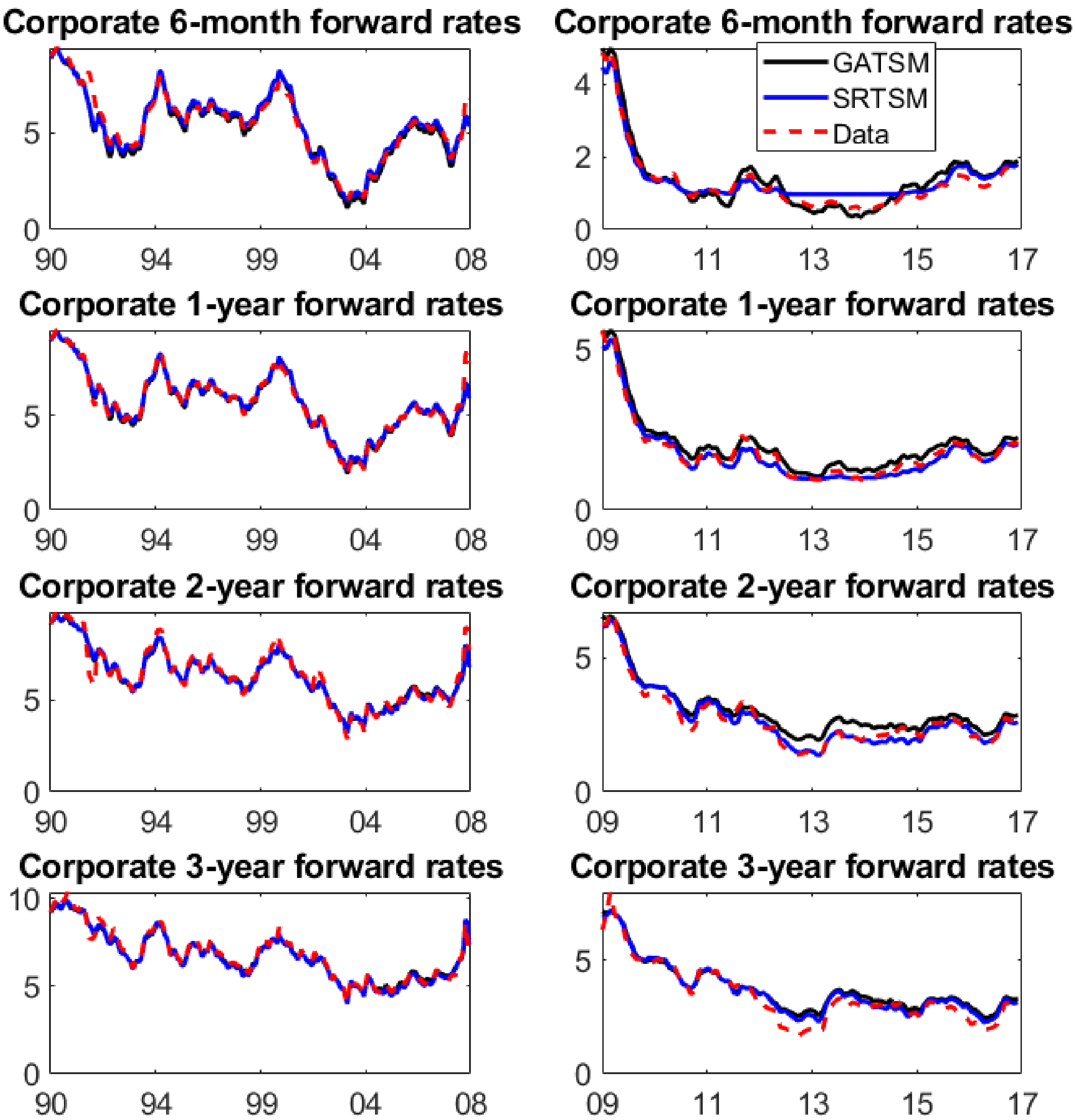


Figure 3.5 reports observed and fitted yields with maturities of 6-months; and 1,2 and 3-year for corporate yields. The black solid line denotes GATSM fitted forward rates, the blue solid line denotes SRTSM fitted forward rates, and the red dashed line denotes observed forward rates.

Figure 3.6: Yield Curve Fit

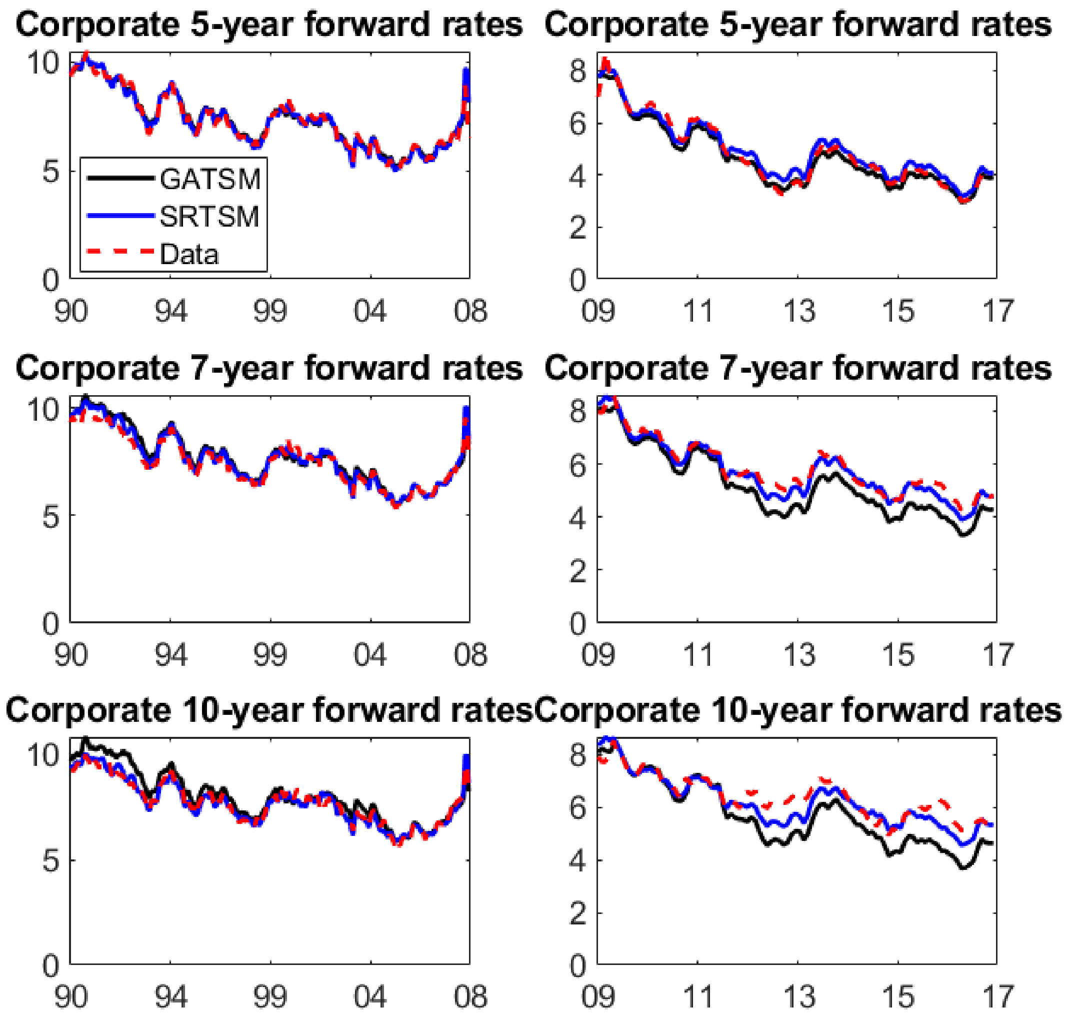


Figure 3.6 reports observed and fitted yields with maturities of 5,7, and 10-year for corporate yields. The black solid line denotes the GATSM fitted forward rates, the blue solid line denotes the SRTSM fitted forward rates, and the red dashed line denotes observed forward rates.

December 2008) on the left plots of each figure, and the ZLB-period until March 2017 on the right plots of each figure. The figures indicate that the general yield curve fits of the SRTSM and the GATSM are similar during the pre-ZLB period, and that the yield curve fits of the SRTSM during the ZLB is significantly better than GATSM fit for long-term corporate forward rates, which is consistent with the RMSE results.

Figure 3.3 presents the observed and fitted treasury forward rates with maturities of 6 months, 1, 2 and 3 years. It is notable that during the zero lower bound period, the performance of the fitted SRTSM forward rates with 6-month and 1-year maturities show relatively poor performance compared with the fitted GATSM forward rates in some years, especially for the 6-month forward rate, where the fitted SRTSM rate is higher than the observed rate and the fitted GATSM rate between 2011 and 2015. This is due to the high estimated value of the lower bound parameter, and the fitted SRTSM rate cannot decrease below the estimated lower bound, which is around 0.22. Shadow term structure model literature finds that estimated shadow short rates are quite sensitive to the different choices of the lower bound parameter and the model specifications. Krippner (2015) comments that the three-factor shadow rate term structure model of Wu and Xia (2016), which this chapter adopts as a benchmark model, is not robust, and he consequently suggests using a two-factor model. However, Golinski and Spencer (2019) Section 4.1.4 finds that the Krippner (2015) two-factor model cannot fit the data well. They find that the Wu and Xia (2016) model estimated lower bound parameter is higher than other comparable algorithms, which then leads to a relatively lower shadow short rate (results similar to Figure 3.9), and this causes a small deterioration in fitting short maturities, in order to capture the variation of longer maturities forward rates. As Golinski and Spencer (2019) suggest, this problem can be solved when the lower bound parameter is set to zero. In addition, the yield curve fit of the GATSM at 6-month maturities dips below zero around 2014, which implies that the GATSM is unable to prevent the interest rate from below zero during the ZLB period. Figure 3.4 presents the observed and fitted treasury yields with maturities of 5, 7 and 10 years. During the ZLB period, the SRTSM fits the yield curve slightly closer than the GATSM.

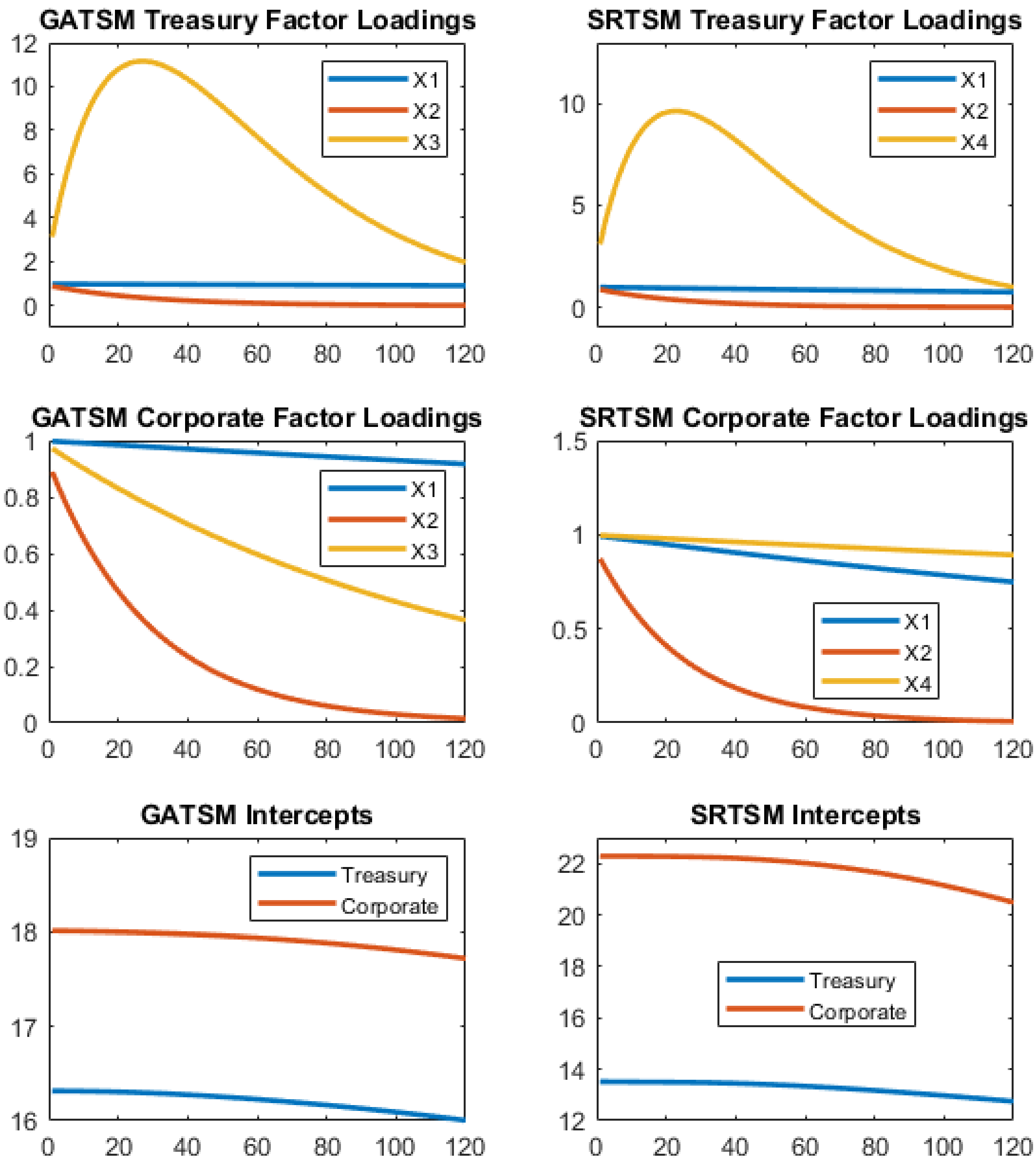
Figure 3.5 plots the observed and fitted corporate yields with maturities of 6 months, 1, 2 and 3 years. It is notable that similar to the results in Figure 3.3, during the zero lower bound period, the 6-month and 1-year fitted SRTSM forward rates show relatively poor performance in comparison with the fitted GATSM rates. In particular, the 6-month fitted SRTSM rates are higher than the observed rates and the fitted GATSM rates. As discussed in the results of Figure 3.3, this is due to the fact that estimated shadow short rates are quite sensitive to the choice of the lower bound, and this causes problems in fitting short maturities. As suggested by Golinki and Spencer (2019), this problem can be overcome by setting the lower bound parameter$\underline{r}^i = 0$. In contrast, for 2-,3-, 7- and 10-year maturities, Figure 3.5 and 3.6 imply that the yield curve fit of the GATSM generates too much variation, while the SRTSM better fits the term structure.

Figure 3.6 shows that for the longer-maturities corporate yields (7-year and 10-year), the SRTSM has a significantly better performance compared with its GATSM counterparts. This is consistent with the findings of Kim and Singleton (2012), who argues that the GATSM has difficulty in fitting the observed 10-year yields during the zero lower bound period. In order to fit the very flat-end of the short-term interest rates, the GATSM compensates its freedom to capture the variation in long-term yields. There is a relatively large difference between the fitted SRTSM yields and observed yields. From an econometric perspective, this may be due to the persistence of the fitting errors. Golinski and Spencer (2019) Section 4.1.5 finds similar performance of the fitting errors for the 10-year yields around the same period. This statistical issue is common in term structure literature, and thus has not been addressed in this chapter. The methods to deal with the persistence of fitting errors can be seen in Adrian et al. (2013), Golinski and Spencer (2017).

3.6.2 Factor Loadings

To demonstrate how state variables affect yields, in Figure 3.7 I display the factor loadings of the GATSM and SRTSM estimated forward rates. The x-axis indicates the estimated factor loadings with maturity in months. The top panel displays the GATSM and the SRTSM estimated factor loadings of Treasury forward rates. This figure indicates that latent variables x_{1t}, x_{2t} and x_{3t} are closely related with the level, slope and curvature factors (see Diebold and Li, 2006). The first latent variables x_{1t} (second common factor) loads almost identically at all maturities, with the factor loadings around 1, which equally affects all yields. The second latent variable x_{2t} (first common factor) has a downward-sloping trend, and affects short-term interest rates more than long-term ones. The treasury specific factor x_{3t} has a upward-sloping trend, and affects medium-term forward rates more than very short- or very long-term forward rates. Factor loadings of common factors x_{1t} and x_{2t} are similar for the GATSM and the SRTSM.

Figure 3.7: Estimated Factor Loadings



Maturity: In Months

Figure3.7 reports the GATSM, and SRTSM model-implied factor loadings in Eq.3.4 and Eq.3.6, respectively. The x-axis measures the maturity from 0 to 10 years in months.

However, the GATSM factor loading of the Treasury specific factor is relatively higher than the SRTSM in the shorter-term. The bottom panel displays the GATSM and SRTSM estimated factor loadings of corporate forward rates. The GATSM and the SRTSM estimated impact of the common factors x_{2t} on forward rates is similar. The common factor x_{2t} and corporate bond specific factor both loads more heavily on the short-term maturities, and their impact decreases as the maturity increases. In contrast, the corporate bond specific factor has a larger factor loading than the first common factor at longer horizons.

It is notable that in recent years, the popularity of dynamic factor models, proposed by Geweke (1977), have grown significantly since the early 2000s. These models can be used to summarize the information in a data-rich environment with a small number of factors. One advantage of the dynamic factor models is that they can model dataset in which the number of series is large than the number of time series observations. The dynamics of factor loadings may be better discussed using dynamic factor models. Since the aim of this chapter is to evaluate the performance of the SRTSM in fitting corporate yields, I use the Wu and Xia (2016) SRTSM as a benchmark model. The usage of dynamic factor models can be examined in future research.

3.6.3 Estimated State Variables

Figure 3.8 shows the estimated state variables. The top panel indicates the GATSM and SRTSM estimated common factors. The first common factor relates to the long-term level of the yield curve, and displays a downward trend, which comports with the factor loading results in Section 3.6.2. The second common factor follows the the short-term interest rate, which comports with the result in Section 3.6.2. The bottom-left panel plots the estimated Treasury bond specific factor. This Treasury bond specific factor captures the variation of the Treasury forward rates not explained in the first two common factors. This factor relates with medium term interest rates, and comports with the finding in Section 3.6.2. The corporate bond specific factor relates with the credit spread, and captures the variation of the corporate forward rates not explained by the first two common factors. There is a large spike between 2008 and 2010 during the financial crisis period; and the other significant spike between 2000 and 2003 marks the burst of the telecom bubble (detailed description of the credit spread pattern is found in Section 2.2.1)

Figure 3.8: Estimated State Variables

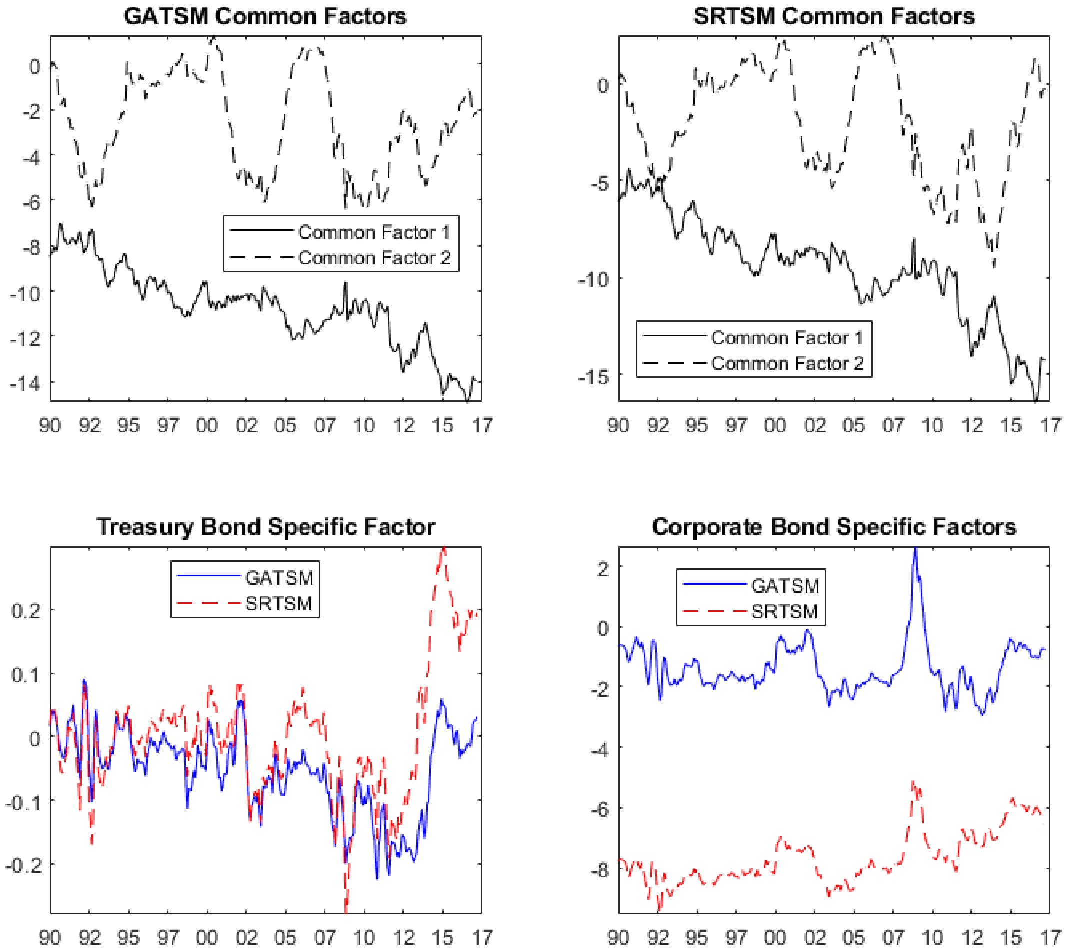


Figure 3.8 reports the GATSM and SRTSM estimated common factors (top plot); the estimated Treasury bond specific factor (bottom-left plot); and the corporate bond specific factor (bottom-right plot).

3.6.4 Estimated Parameters

Table 3.5: Estimated Parameters

	GATSM			
1200 $\boldsymbol{\mu}$	-0.0368	-0.2869	-0.0028	-0.0276
	(0.1089)	(0.1536)	(0.0120)	(0.0577)
$\boldsymbol{\Phi}$	0.9933***	0.0144	-0.1281	0.0151
	(0.0093)	(0.0077)	(0.2758)	(0.0197)
	-0.0133	0.9718***	0.8462*	−0.0804**
	(0.0134)	(0.0109)	(0.3932)	(0.0280)
	0.0002	-0.0001	0.9562***	-0.0020
	(0.0012)	(0.0005)	(0.0162)	(0.0015)
	0.0020	0.0081	-0.2175	0.9570***
	(0.0070)	(0.0072)	(0.2533)	(0.0188)
$\boldsymbol{\Phi}^Q$	0.9993***			
	(0.0001)			
		0.9665***	1.0000	
		(0.0010)		
			0.9665***	
			(0.0010)	
				0.9918***
				(0.0005)

Table 3.5 reports the maximum likelihood estimates of the four-factor joint GATSM for treasury and corporate 6-month forward yields. The standard error is in parenthesis. Significance at the 95%-, 99%- and 99.9%- level are denoted as $*$, $**$ and $***$ respectively. The sample period is from January 1990 to March 2017.

Table 3.5, 3.6, 3.7 and 3.8 report the GATSM and SRTSM estimated parameters of Treasury and corporate forward rates based on observations from January 1990 to March 2017. The SRTSM performs better than the GATSM. The log likelihood value of the GATSM is 1847.11, and the SRTSM is 2526.80. In addition, the SRTSM estimated variance of residuals

Table 3.6: Estimated Parameters

	GATSM			
$1200\delta_0^T$	16.3161***			
	(1.1430)			
$1200\delta_0^C$	18.0136***			
	(1.1489)			
$1200\boldsymbol{\Sigma}$	0.2781***			
	(0.0165)			
	−0.3017***	0.2529***		
	(0.0234)	(0.0126)		
	−0.0037**	-0.0011	0.0205***	
	(0.0012)	(0.0014)	(0.0011)	
	−0.0406*	−0.0405*	0.1531***	0.2065***
	(0.0200)	(0.0199)	(0.0175)	(0.0121)
$1200\sqrt{\omega^T}$	0.0884***			
	(0.0019)			
$1200\sqrt{\omega^C}$	0.4069***			
	(0.0065)			
log-likelihood value	1847.1100			

Table 3.6 reports the maximum likelihood estimates of the four-factor joint GATSM for treasury and corporate 6-month forward yields. The standard error is in parenthesis. Significance at the 95%-, 99%- and 99.9%- level are denoted as $*$, $**$ and $***$ respectively. The sample period is from January 1990 to March 2017.

Table 3.7: Estimated Parameters

	SRTSM			
1200 $\boldsymbol{\mu}$	-0.1194	-0.6524	-0.0640^{***}	-0.6691^{***}
	(0.3566)	(0.7420)	(0.0238)	(0.2202)
$\boldsymbol{\Phi}$	0.9923^{***}	0.0154	-0.2814	-0.0053
	(0.0127)	(0.0094)	(0.2926)	(0.0358)
	-0.0039	0.9619^{***}	1.6845^{*}	-0.0735
	(0.0283)	(0.0225)	(0.7374)	(0.0783)
	-0.0019^{**}	-0.0006	0.9415^{***}	-0.0057^{*}
	(0.0006)	(0.0009)	(0.0283)	(0.0024)
	-0.0156	0.0032	-0.3351	0.9326^{***}
	(0.0108)	(0.0099)	(0.3175)	(0.0207)
$\boldsymbol{\Phi}^Q$	0.9976^{***}			
	(0.0001)			
		0.9612^{***}	1.0000	
		(0.0005)		
			0.9612^{***}	
			(0.0005)	
				0.9991^{***}
				(0.0001)

Table 3.7 reports the maximum likelihood estimates of the SRTSM. The standard error is in parenthesis. Significance at the 95%-, 99%- and 99.9%- level are denoted as $*$, $**$ and $***$ respectively. The sample period is from January 1990 to March 2017.

Table 3.8: Estimated Parameters

	SRTSM			
$1200\delta_0^T$	13.5207***			
	(0.3988)			
$1200\delta_0^C$	22.2866***			
	(0.4674)			
$1200\boldsymbol{\Sigma}$	0.4608***			
	(0.0235)			
	−0.9521***	0.5718***		
	(0.0503)	(0.0370)		
	0.0211***	−0.0233***	0.0317***	
	(0.0028)	(0.0043)	(0.0012)	
	0.2870***	−0.2947***	0.1719***	0.2107***
	(0.0188)	(0.0378)	(0.0157)	(0.0118)
$1200\sqrt{\omega^T}$	0.0855***			
	(0.0018)			
$1200\sqrt{\omega^C}$	0.2857***			
	(0.0051)			
$\underline{r}^T$	0.2263***			
	(0.0129)			
$\underline{r}^C$	0.9716***			
	(0.0368)			
log-likelihood value	2526.8000			

Table 3.8 reports the maximum likelihood estimates of the SRTSM. The standard error is in parenthesis. Significance at the 95%-, 99%- and 99.9%- level are denoted as $*$, $**$ and $***$ respectively. The sample period is from January 1990 to March 2017.

Figure 3.9: Fitted Short Rates

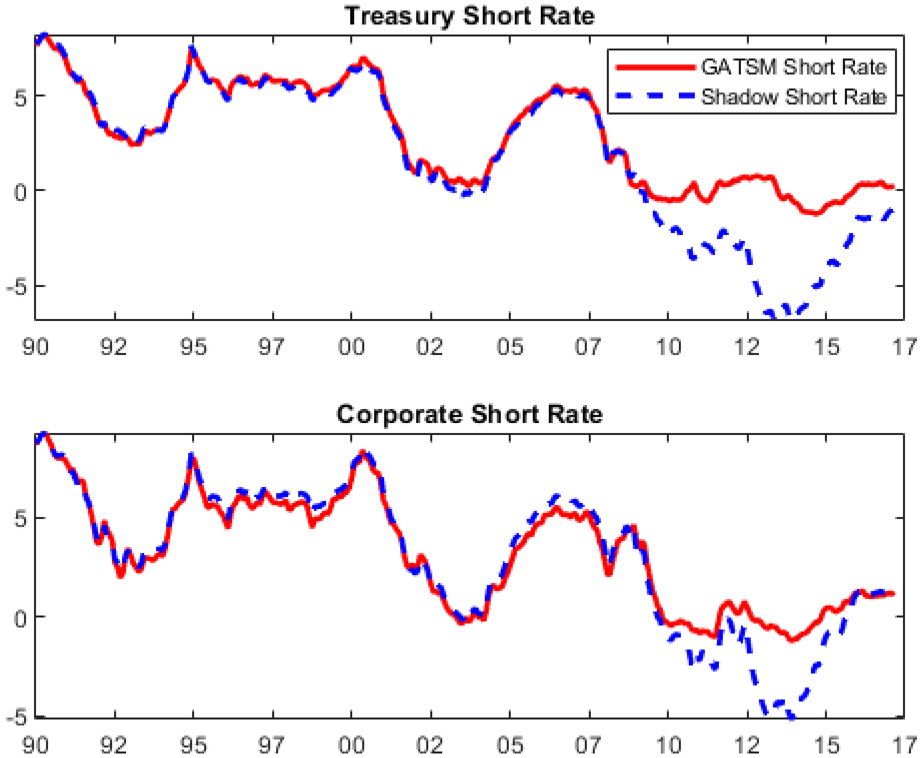


Figure 3.9 reports the fitted short rates for the Treasury and corporate joint term structure model. The red solid line reports the estimated GATSM short rates of the treasury (top panel) and the corporate (bottom panel) forward rate. The blue dashed line reports the estimated SRTSM shadow short rates of the Treasury (top panel) and the corporate (bottom panel) forward rate. The period spans from January 1990 to March 2017.

of the Treasury and corporate forward rates is 0.0855 and 0.2857 respectively, smaller than for the result of GATSM (0.0884 and 0.4096, respectively). This result is consistent with the finding in Section 3.6.1. The SRTSM estimate of $\boldsymbol{\Phi}$ suggests that both Treasury and corporate bond specific factors are relatively persistent and the autocorrelation coefficient is around 0.96 and 0.99, respectively. The SRTSM estimate of $\boldsymbol{\Phi}$ also suggests that the corporate bond specific factor has a significantly negative effect on the Treasury bond specific factor, however the effect on the opposite direction has not been found. The lower bound of Treasury and corporate forward rates are both significant: 0.2263 and 0.9716, respectively.

3.6.5 Shadow Short Rate

Figure 3.9 plots the estimated Treasury and corporate short rate of the GATSM and SRTSM from January 1990 to March 2017. Before 2009 the GATSM estimated Treasury and corporate short rates equal the SRTSM estimated shadow short rates. The GATSM estimated short rates have diverged from the SRTSM since 2009. The GATSM estimated short rates had been stuck at the zero lower bound and has become less volatile. In comparison, the SRTSM estimated shadow rates become negative and displays meaningful variation. After 2015, since the unconventional monetary policy ends in the U.S., the SRTSM estimated short rates gradually increases and reaches similar numbers as the GATSM estimated short rates.

3.6.6 Credit Spread

Figure 3.10 and Table 3.9 report the estimated credit spreads, i.e., corporate yields minus Treasury yields with the same maturity. The results indicate that during the zero lower bound period, the SRTSM fits longer maturities credit spreads better than the GATSM. Kim and Singleton (2012) states that the GATSM is difficult to fit the observed 10-year yields during the zero lower bound period, which compensates its freedom to capture the variation of long-term yields, in order to fit the flat-end of the short-term interest rates. It is notable that during the ZLB period, especially between 2012 and 2015, the 6-month fitted SRTSM credit spread is higher than the observed credit spread and the fitted GATSM credit spread, which is quite flat. As discussed in Section 3.6.1, in respect to the results shown in Figure 3.3 and Figure 3.5, this is due to the high estimated value of the lower bound parameter. The SRTSM estimated forward short rates are sensitive to the value of the lower bound parameter, which makes it difficult for the SRTSM to fit short maturity Treasury rates and corporate rates, and thus leads to a relative underperformance of SRTSM in fitting the short-end credit spread. In addition, during the zero lower bound period, the 5-year fitted GATSM credit spread fits the data better than its SRTSM counterpart since 2012. This may also relate to the choice of the lower bound parameter, which deteriorates the fits of shorter maturities, in order to capture the variation of the longer maturities (See the detailed analysis in Golinski and Spencer (2019)). In addition, the SRTSM fits the 5-year yield slightly worse than its GATSM counterparts. However, the fit of the intermediate maturities is comparable for both models. Similar results can be found in Bauer and Rudebusch (2016) Section 2.1 for 5-year Treasury yields.

Figure 3.10: Spreads

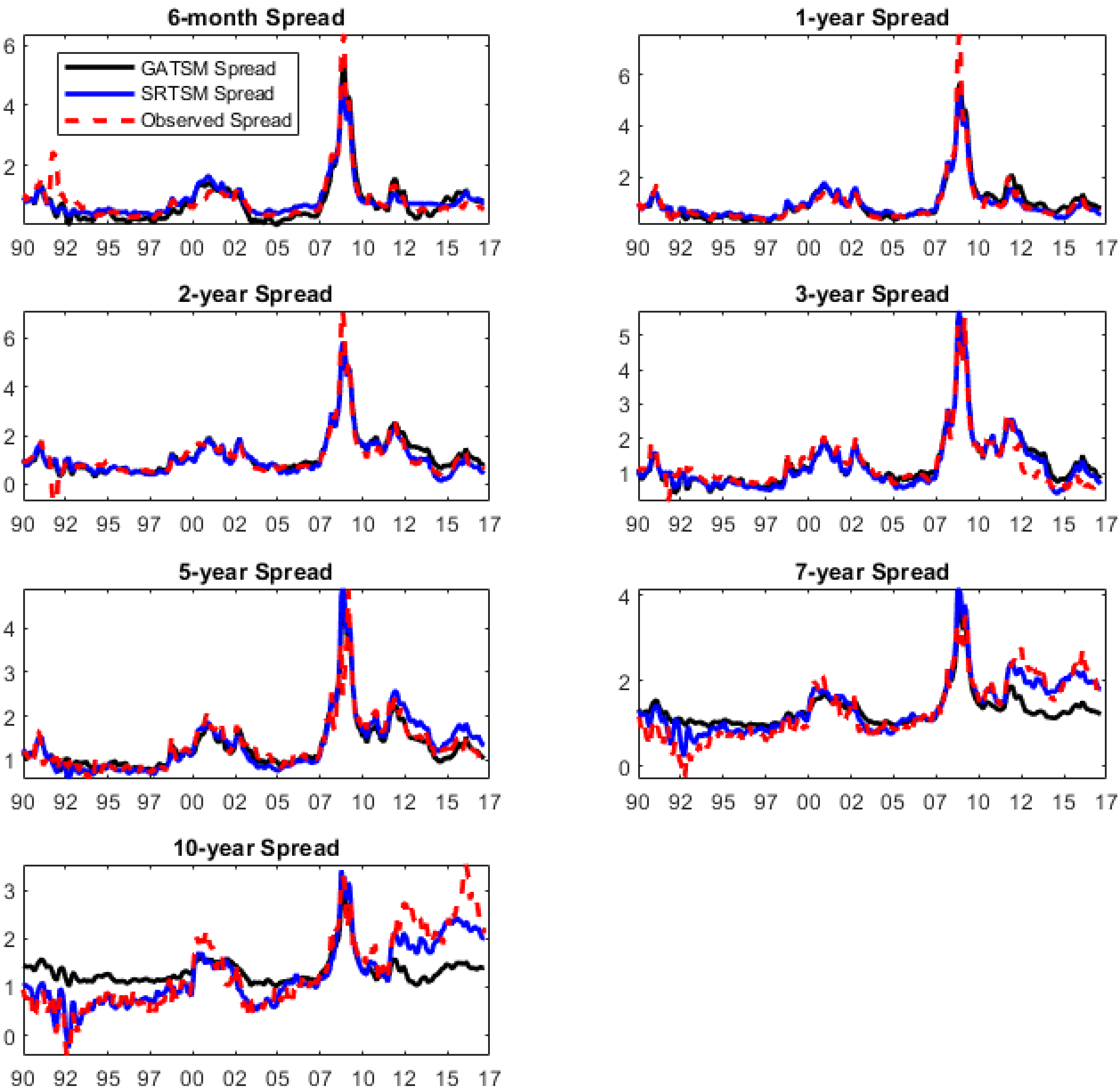


Figure 3.10 reports the maturities of 6-months; and 1,2,3,5,7, and 10-years spreads for treasury and corporate forward yields. Spread is defined as the Treasury forward rate minus the corporate forward rate with the same maturity. The black solid line denotes the GATSM estimated spreads, the blue solid line denote the SRTSM estimated spreads, and the red dashed line denotes the observed spreads. The period spans from January 1990 to March 2017.

Table 3.9: Root-Mean-Square-Error of Estiamted Spreads

	Credit Spreads						
	6-month	1-y	2-y	3-y	5-y	7-y	10-y
GATSM	34	23	31	31	24	50	69
SRTSM	33	25	28	27	27	26	30

Table 3.9 reports the root mean square error of the GATSM and SRTSM model-implied Treasury (top panel) and corporate (bottom panel) credit spreads with maturities of 6-months; and 1,2,3,5,7, and 10-years. The sample spans from January 1990 to March 2017. The RMSE is measured in basis points.

3.6.7 Out of Sample Forecast

Bauer and Rudebusch (2016) states that GATSM severely violates the ZLB constraint, and the model-implied future short rate drops below zero at different horizons. They investigate the out of sample forecast for 3-month Treasury bill during the zero lower bound period, and find that the SRTSM predicts Treasury short rate better than the GATSM.

In order to analyze the out-of-sample forecast performance of the GATSM and SRTSM for corporate yields, I conduct a one-step ahead out-of-sample forecast similar to Ang and Piazzesi (2003). See details in Chapter 2 Section 2.11.

I estimate the models using the in-sample period, then treat the parameters in the in-sample period as fixed, and use these fixed parameters to do a one-step ahead forecast in the out-of-sample period. I compare the out-of sample forecast results with the actual data in the terms of RMSE using Eq. 35 in Ang and Bakaert (2002). The in-sample period is from January 1990 to 2012 February. The out-of-sample period is from 2012 March to 2017 March, which is the last five years of the full sample. I choose the out-of-sample period that covers both the ZLB period and post-ZLB period to test whether the performance of the models is consistent across different periods.

The main target in this section is to compare the forecast capability of the GATSM and SRTSM, therefore the random walk model has not be used as in Chapter 2.

The notable findings are as follows: (i) the SRTSM predicts short rate (6-month) more accurately than the GATSM (ii) the SRTSM predicts longer-maturities (7-year and 10-year) substantially better than the GATSM, producing forecasts that are twice as accurate as those from the GATSM. (iii) the forecasts of the SRTSM for some intermediate maturities show worse performance thant the GATSM. This finding is consistent with the in-sample findings in Section 3.6.6., and is similar to the results presented in the in-sample results of Bauer and Rudebusch (2016).

Table 3.10: Out-of-Sample Forecast Results

	Corporate						
	6-month	1-year	2-year	3-year	5-year	7-year	10-year
GATSM	31	25	38	43	21	87	123
SRTSM	25	25	31	44	29	27	52

Table 3.10 reports root-mean-squared errors of the forecasts versus the actual observations, for the out-of-sample forecasts of corporate yields with maturities 1-year, 2-year, 3-year, 5-year, 7-year and 10-year. The model estimated period is from January 1990 to 2012 February. The forecasting period is from 2012 March to 2017 March. The forecasts are calculated using the original parameters from the estimated period. Top panel shows the forecast results of the GATSM. The bottom panel shows the results of the SRTSM. The forecast is one-step ahead. The RMSEs are measured in basis points.

3.6.8 Cumulative Distribution Function

In this section, similar to Golinski and Spencer (2019), and Spencer (2019), I use the cumulative distribution function to evaluate the relevance of the lower bound for Treasury and corporate forward rates, and the potential monetary policy implications.

Figure 3.11 reports the cumulative distribution function (CDF, hereafter) $H(z_t)$ in Eq.3.4 used in Wu and Xia (2016) forward rate approximation for the future one-month Treasury and corporate forward shadow rates from January 1990 to March 2017. The lower bound parameters are estimated as free parameters in Section 3.6.4. The lower bound $\underline{r}^T$ for Treasury forward rates is 0.2263, and the lower bound $\underline{r}^C$ for corporate forward rates is 0.9716. Maturities are 6 months, 1, 2, 3, 5, 7, and 10 years. The ZLB period in gray areas is from January 2009 to December 2015.

This cumulative distribution function shows the fraction of the probability mass of the future one-month Treasury and corporate forward shadow rates in the above lower bound range under the Q-measure. The future one-month forward shadow rates under the Q-measure for bond i is $E_t^Q[s_{t+n}^i] = a_n^i + \boldsymbol{b}_n^{i\prime}\boldsymbol{x}_t$, with zero error term (See Wu and Xia (2016) Appendix A for details). The cumulative distribution function $H(z_t)$ can also be interpreted as 'delta' for an European call option of a Bernoulli type written on the shadow rate.

Figure 3.11 indicates that the probability mass for Treasury and corporate shadow rates going below the lower bound before 2008 was negligible, with an exception in 2003, when the probability mass of the Treasury and corporate shadow rates going below the lower bound at the 6-month horizon are almost one-third and a half, respectively. This finding is similar to Spencer (2019), which finds that over a third of the 3-month ahead future Eurodollar shadow rate goes below the lower bound in 2003. It is also notable that the CDFs for the corporate shadow rates at longer horizons are smaller than the Treasury ones, which is due to the relatively high value of the estimated corporate lower bound.

During the ZLB period, the CDFs for short-term and intermediate term Treasury shadow rates reduces dramatically. The CDFs for 6-month ahead shadow rates are effectively zero, which is similar to Golinski and Spencer (2019). They find that this can be solved by fixing lower bound parameter at zero. In addition, the CDF for 10-year ahead shadow rates drops around 20 percentage points in 2013, compared with the CDF in 2008. These results show the relevance of the lower bound for Treasury interest rates. These results also imply that the lower bound for policy short-term interest rates have been partially transmitted into the intermediate and longer-term future interest rates.

During the ZLB period, similar to the Treasury shadow rates, the CDFs for short-term and up to 2-year intermediate-term corporate shadow rates decrease significantly, and the CDFs for 6-month and even 1-year ahead shadow rates are effectively zero at times. These results show the relevance of the lower bound for corporate interest rates. This may be due to the impact of the common factors, and reflects the impact from the Treasury bond market. This implies that the lower bound constraint on Treasury interest rates has influence on corporate interest rates, even at intermediate horizons. However, compared with the Treasury shadow rates, the CDF for 10-year ahead shadow rates is relatively more stable, and drops around just 5 percentage points in 2013, compared with the CDF in 2008. This result may due to the impact of corporate bond specific factor at longer horizons, which keeps the 10-year ahead corporate shadow rates above the lower bound.

It is notable that the CDFs of the 6-month ahead corporate shadow rates exhibit a sharp fall between 2009 and 2010, following the announcement of the Fed's first Large Scale Asset Purchases programs in November 2008, and the Federal Open Market Committee (FOMC) meeting in December 2008. After the first LSAP ends on March 2010, the CDFs of 6-month ahead shadow rates are effectively zero between late 2010 and early 2011. Between late 2011 and early 2012, the CDFs of the 6-month ahead shadow rates increase, as the European sovereign debt crisis deteriorates in the summer 2011. As the conduct of the operation twist (September 2011 to December 2012), the CDFs of the 6-month ahead shadow rates decrease from 2012 to early 2013. The CDFs the 6-month ahead shadow rates are effectively zero between 2013 and late 2014, as the conduct of the third LSAP (September 2012 to October 2014). The CDFs of the 6-month ahead corporate shadow rates then increase dramatically after the LSAPs stop after the October 2014 FOMC meeting.

The results show that during the ZLB period, the short-term up to 2-year intermediate term corporate shadow rates show the relevance of the lower bound. This may due to the impact of the common factors, and reflects the impact from the Treasury bond market. The 10-year corporate shadow rates seems to show relatively small relevance of the lower bound. This may due to the large impact of the corporate bond specific factor, which reflects relatively small influence from the Treasury bond market.

Therefore, the transmission mechanism of the monetary policy is not very effective for the long-term corporate rates, and as such, it remains unclear whether the Federal Reserves' efforts are helpful in lowering the cost of capital for firms.

Figure 3.11: Cumulative Distribution Function

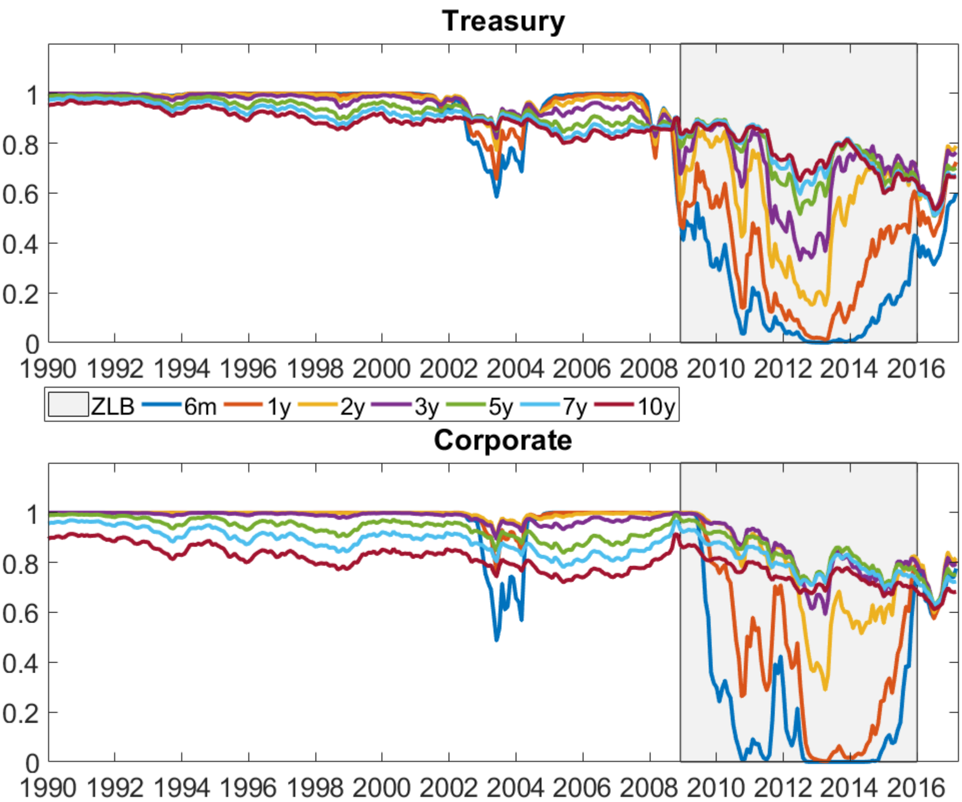


Figure 3.11 reports the cumulative distribution function $H(z_t)$ in Eq.3.4 used in the Wu and Xia (2016) forward rate approximation for the one-month Treasury and corporate forward shadow rates under the Q-measure from January 1990 to March 2017. The lower bound parameters are estimated as free parameters in Section 3.6.4. The lower bound $\underline{r}^T$ for Treasury forward rates is 0.2263, and the lower bound $\underline{r}^C$ for corporate forward rates is 0.9716. Maturities are 6 months, 1, 2, 3, 5, 7, and 10 years. The ZLB period in gray areas is from January 2009 to December 2015.

3.7 Concluding Remarks

This chapter examines the performance of the SRTSM in terms of the corporate yields during the zero lower bound period, then evaluate the impact of the lower bound on corporate yields, and its potential monetary policy implications. I develop a joint shadow rate term structure for Treasury and Corporate forward rates, which has the advantaged of being able to examine the interaction between the two types of bonds.

The results highlight that during the zero lower bound period, when compared to the GATSM, the SRTSM performs significantly better in capturing both the in-sample and out-of-sample variation of long-term corporate yields. The root mean square error of the SRTSM for the 10-year maturity corporate yields is 35 basis points smaller than the GATSM. The out-of-sample forecast shows that the RMSEs of the GATSM are twice large than the SRTSM. I also find that short rates are sensitive to the estimated value of the lower bound parameter, which causes the relatively poor performance of the SRTSM compared with the GATSM in some years, especially for the 6-month maturity yields. This problem can be solved by fixing the lower bound parameter at zero, as suggested by Golinski and Spencer (2019), and can be investigated in future research. In addition, long-term credit spreads are better captured by the shadow rate term structure model. It is also notable that, for the long-term yields, a relatively large difference between the fitted SRTSM yields and the observed yields may be due to the persistence of the fitting errors.

In addition, the cumulative distribution function analysis shows that the lower bound constraint affect corporate short- and intermediate-term interest rates. However, the impact of the lower bound on long-term corporate interest rates is relatively small, which may due to the effect from the corporate bond specific factor.

Further research can be conducted to examine the impact of the ZLB on corporate bond yields, and evaluate potential monetary policy implications. For example, Krippner (2015) and Bauer and Rudebusch (2016) use zero lower bound wedge to measure the tightness of the zero lower bound constraint on 10-year yield. Coroneo and Pastorello (2018) decompose the 10-year observed sovereign spreads into the shadow spread and the spread wedge, in order to measure the effect of long-term sovereign risk and the nonlinearities that arise at the lower bound separately. This similar method can be adopted to decompose the long-term credit spread into shadow spread and the spread wedge, and evaluate the effect of credit risk and the nonlinearities that arise at the lower bound separately. In addition, Spencer (2019) decomposes the long-term Treasury forward rates and the Eurodollar futures into an interest rate expectations and risk premiums to distinguish the effect of the Federal Reserves' forward guidance and open market operations. Similar approach can be adopted to the analysis of long-term corporate forward rates.

Chapter 4

Evaluating the Impact of the Federal Reserve's Purchase Programs: A Shadow Rate Term Structure Model Approach

4.1 Introduction

During the most recent financial crisis, the target of the federal funds rate-the traditional main monetary policy instrument of the Federal Reserve (the Fed) has been reduced to essentially zero. To further ease the stance of the monetary policy and promote economic recovery, the Fed initiated large-scale asset purchases (LSAPs) as an important alternative. These programs are designed to reduce the longer-term interest rates by increasing the Fed's holdings of medium- and long-term Treasury securities and agency MBS, i.e., the mortgage-backed securities that have credit protection from the U.S. government. According to data from the Fed Board of Governors, from December 2007 to May 2017, this expansion increases the Fed's balance sheet from $882 billion to $4.473 trillion. Moreover, the average maturity of the assets in the Fed's portfolio is much higher than before the financial crisis. Given the unprecedented nature of the Fed's purchase programs, a growing body of work has tried to evaluate the effect of the LSAPs.

From a theoretical perspective, the literature mainly adopts portfolio balance channel and signalling channel to explain the potential impact of the asset purchases. The first relies on the existence of "preferred habitat" investors, who have a preference for bonds with a given maturity. In this case, the LSAPs can create a shortage of long-term government debt in the private sector that cannot be relieved by substituting government securities with different maturities. Thus, the yields on the maturities purchased will reduce through the term premium. The literature includes Tobin (1961), Modigliani and Sutch (1966,1967). Vayanos and Vila (2009) develop the preferred-habitat literature and formulate a theoretical arbitrage-free term structure model with supply factors. The model assumes two types of investors. The preferred-habitat investors prefer bonds of certain maturities, implying that bonds of different maturities are imperfect substitutes. This preferred habitat provides a portfolio balance channel for relative supply factors to affect yield curves. On the other hand, the risk-averse arbitrageurs have no maturities preference but can take advantage of the arbitrage opportunity by trading across maturities. This ensures that the impact of changes in supply factors are transmitted through the entire yield curve. Vayanos and Vila (2009) provide a rationale for the purchase programs, which shift the quantities of government debt with specific maturities held by private investors, and create a shortage of these assets that cannot be relieved. The second suggests that the LSAPs announcement can adjust the market participants' expectations for the future short rate. That is, when the LSAPs signal that the fed funds rate target will remain at near-zero, the investors will revise down their expectation about the future short rate, and long-term yields will reduce through the average expected short rate (or risk neutral) component. This theory is suggested by Eggertson and Woodford (2003) and Bernanke et al. (2004). However, since monetary policy potentially can work through many channels, the underlying mechanism of the LSAPs it not yet fully understood.

From an empirical perspective, the rapidly growing literature can be summarized into three broad categories. First is the event study, which examines changes in asset prices following the LSAPs announcement, e.g., Joyce et al.(2011), Neely (2015) and Swanson (2017). This method has its drawbacks: It is difficult to estimate the impact of the LSAPs precisely with a relatively small number of announcements, and it is hard to exclude the effect of other factors occurring within selected event windows. Alternatively, regression methods have been used to examine the robustness of event studies results. These studies use a time series or panel regression to evaluate the relationship between selected yields and LSAPs related supply factors. Related literature includes Meaning and Zhu (2011), D'Amico et al.(2012), Bowman et al.(2015), and Hattori et al.(2016). However, these studies are hampered by small sample bias and endogeneity. These may result from, on the one hand, that supply variables are highly persistent. On the other hand, supply factors are likely correlated with other factors that influence yield curves. A third grouping of this literature is the term structure. This method has the advantage of summarizing information from the entire yield curve and across maturities. Most of the previous literature combines the term structure model with event study analysis. For example, Gagnon et al. (2011) employ interest rate decomposition component calculated from Kim and Wright (2015) to evaluate the portfolio balance channel. Similar work is from Christensen and Rudebusch (2012), Bauer and Rudebusch (2014), Bauer and Neely (2014). However, the standard term structure literature leaves little scope for the LSAPs related supply variables to influence interest rates.

Despite the theoretical development since Vayanos and Vila (2009), few empirical studies pay attention to the empirical application of the preferred-habitat term structure model. Hamilton and Wu (2012) adapt the model of Vayanos and Vila (2009) to a discrete time framework to study how the maturity of government debt affects the term structure of interest rates. They analyse the outcome of LSAPs using calibrated impact estimates from forecasting regressions before the financial crisis in 2008. In contrast, Li and Wei (2013) provide a more comprehensive analysis of the LSAPs' impact using a term structure approach that includes various supply variables: private holdings of Treasury securities, private holdings of agency mortgage-backed securities (MBS) and the average duration of privately-held MBS. They estimate their model before the crisis infer the impact of supply factors during the zero lower bound period (ZLB, see a detailed explanation in Section 3.1) using the estimated parameters from before the crisis. However, according to Golinski (2018), the relationship between the LSAPs and Treasury prices is not stable, with significant structural breaks over the ZLB period. This finding means the conclusion in the in the pre-crisis period cannot be easily extended to the ZLB period.

The benchmark Gaussian affine term structure model (GATSM) is unable to prevent the interest rate from going below zero and therefore meet the challenge of capturing bond yields variations during the ZLB period. Alternatively, the shadow rate term structure model (SRTSM) has widely been adopted to describe the recent behaviour of interest rates and monetary policy. For surveys, see Black (1995), Bullard (2012), Lombardi and Zhu (2014) and Wu and Xia (2016). [1] However, most of the literature studies the general impact of the Fed's unconventional monetary policy tools on the yield curve, or assesses the overall effects on the economy using shadow rates. To my best knowledge, it seems that none of the literature focuses on evaluating the impact of LSAPs on yield curves.

In this chapter, I evaluate the effect of the LSAPs on Treasury yield curves using a shadow rate term structure model. Specifically, following the the theoretical work of Vayanos and Vila (2009), and based on the Li and Wei (2013) model, I develop a shadow rate model for Treasury forward rates with two latent yield factors and three observed supply factors. The latent yield factors are estimated using the Kalman filter. The first supply factor measures the supply of Treasury securities, which is calculated as the amount of private holdings of Treasury securities in terms of ten-year equivalents as a percentage of total outstanding public debt held by the public. The second supply factor measures the par supply of MBS securities, which is calculated as the amount of private holdings of MBS securities in terms of the total outstanding public debt held by the public. The third supply factor measures the duration of the MBS, which is calculated as the average duration of the privately held MBS.

In order to be consistent with the economic theory and to reduce the estimation burden, I adopt several parsimonious assumptions from Li and Wei (2013) and Ihrig et al. (2018). First, I assume that the portfolio balance channel is the dominant channel for supply factors to influence Treasury forward rates, whereby the effect is through term premium component. Second, the current short rate is only affected by yield factors, and supply factors do not affect the expected future short rate (or risk neutral) component. Third, supply factors affect the term premium by indirectly influencing the risk premium on the yield factors. In the first step, I estimate the model with the extended Kalman filter using maximum likelihood estimation and compare the estimation results with the GATSM. The Treasury forward rates analysed are the 1-month, end-of-period, monthly frequency U.S. data. Details of the data description is in Section 4.3. In the second step, in order to analyze the bond term premium parameters, I decompose the fitted forward rates into risk premium and expected future short rate components. In the third step, I use the model and the parameters estimated to conduct counterfactual analysis to evaluate the impact of the Fed's first two asset purchase programs and Operation Twist (OT).

The results show that SRTSM fits the yield curve well and captures economically meaning-

[1] See Section 3.1 for a detailed explanation and review of the SRTSM literature

ful relationships between interest rates and supply factors. Moreover, LSAPs have significant effects on Treasury forward term premium. First, I estimate the SRTSM and GATSM using three samples: The full sample spans from December 1996 to March 2018; the pre-ZLB subsample from December 1996 to December 2008; and the ZLB subsample from January 2009 to November 2015. The result demonstrates that the SRTSM has a significantly better fit than the GATSM across maturities in the full sample. The GATSM and SRTSM each perform well during the pre-ZLB period, and the superior performance of the SRTSM is from its better fit of yields during the ZLB period. This means that the SRTSM can capture the variation of the term structure across conventional and unconventional monetary policy eras. Second, I analyse the estimated results and conduct a risk premium decomposition. The SRTSM results indicate positive relationships between the increase of the three supply factors and the forward term premium, which comports with the findings from the literature (see D'Amico et al., 2012 and Hamilton and Wu, 2012). Third, I use the SRTSM to conduct a counterfactual analysis. I treat the Fed's purchases of Treasury and MBS securities as the supply shock and calculate the counterfactual term premia without the Fed's LSAPs, using the term premium parameter estimated from the SRTSM. The result implies that LSAPs lower the mid- to long-term forward term premium by a large amount. The LSAP1 has the most substantial impact on the long-term risk premium and reduces the 10-year forward term premium by about 140 basis points, most which is from the Fed's purchases of MBS. The LSAP2 and OT reduce the 10-year term premium by 15 basis points and 3 basis points, respectively.

This chapter provides new empirical evidence on evaluating the effects of LSAPs, and has novelty in analysing the LSAPs impact using the shadow rate term structure model approach.

The remainder of the chapter is organised as follows: Section 4.2 describes the model. Section 4.3 describes the data. Section 4.4 discusses model specification, and section 4.5 explains the estimation methodology. Section 4.6 analyses the results and risk premium decomposition. Section 4.7 evaluates the counterfactual analysis of the Fed's LSAPs. Section 4.8 concludes.

4.2 Model Setup

This section models the dynamics of U.S. government bonds using the term structure approach. Based on the model of Wu and Xia (2016), I estimate the Treasury yield curve using the shadow rate term structure. I also estimate the benchmark Gaussian affine term structure model for comparison.

4.2.1 Gaussian Affine Term Structure Model

(i) K state variables $\boldsymbol{x}_t$ follow a VAR(1)) under the P-measure, as Chapter 2 Eq.2.4.

(ii) The short term interest rate r_t is assumed as an affine function of state variables $\boldsymbol{x}_t$ as follows:

$$r_t = \delta_0 + \boldsymbol{\delta_1'} \boldsymbol{x}_t \tag{4.1}$$

(iii) The time-varying market price of risk $\boldsymbol{\lambda}_t$ is linear in the state variables, as Chapter 2 Eq.2.7.

(iv) The state variables under the Q-measure follow a VAR(1) process, as Chapter 2 Eq.2.8.

(v) The one-month forward rate $f_{t,n,n+1}$ in the benchmark GATSM model is an affine function of the state variables $\boldsymbol{x}_t$:

$$f_{t,n,n+1}^{GATSM} = a_n + \boldsymbol{b}_n' \boldsymbol{x}_t \tag{4.2}$$

where a_n and $\boldsymbol{b}_n$ can be derived as follows (see detailed derivations in Wu and Xia, 2016 Appendix A):

$$\bar{a}_n = \delta_0 + \boldsymbol{\delta}_1' (\sum_{j=0}^{n-1} (\boldsymbol{\Phi}^Q)^j) \boldsymbol{\mu}^Q \tag{4.3}$$

$$a_n = \bar{a}_n - \frac{1}{2} \boldsymbol{\delta}_1' (\sum_{j=0}^{n-1} (\boldsymbol{\Phi}^Q)^j) \boldsymbol{\Sigma}\boldsymbol{\Sigma}' (\sum_{j=0}^{n-1} (\boldsymbol{\Phi}^Q)^j)' \boldsymbol{\delta}_1 \tag{4.4}$$

$$\boldsymbol{b}_n' = \boldsymbol{\delta}_1' (\boldsymbol{\Phi}^Q)^n \tag{4.5}$$

4.2.2 Shadow Rate Term Structure Model

If the shadow rate $s_t = r_t$, the SRTSM becomes a GATSM. Therefore, this section only states the model content specially designed for the SRTSM.

(i) Similar to Black (1995), I assume that the short-term interest rate is the maximum of the shadow rate s_t and a lower bound $\underline{r}$:

$$r_t = max(\underline{r}, s_t) \tag{4.6}$$

(ii) The shadow short term interest rate is assumed as an affine function of the state variables $\boldsymbol{x}_t$:

$$s_t = \delta_0 + \boldsymbol{\delta}_1' \boldsymbol{x}_t \tag{4.7}$$

(iii) The assumption in Eq.4.6 implies that the one-month forward rate $f_{t,n,n+1}$ is nonlinear in state variables $\boldsymbol{x}_t$.

(iv) Following the derivations of Wu and Xia (2016), the one-month forward rate $f_{t,n,n+1}$ is approximately equal to:

$$f^{SRTSM}_{t,n,n+1} = \underline{r} + \sigma^Q_n g\left(\frac{a_n + \boldsymbol{b}_n' \boldsymbol{x}_t - \underline{r}}{\sigma^Q_n}\right) \tag{4.8}$$

where $(\sigma^Q_n)^2 \equiv Var_t(s^Q_{t+n})$. [2]

4.3 Data Description

The data description includes securities supply factors and treasury forward yields. All data are converted into the end-of-period monthly frequency. The data spans from December 1996 to March 2018.

4.3.1 Securities Supply Factors

Securities supply factors include both Treasury and MBS supply variables. I plot the time series of these supply factors in Figure 4.1.

The Treasury supply variable is the ratio of Treasury ten-year-equivalents to the total Treasury securities held by the public (sv_{1t}), which measures the total amount of private holdings of Treasury securities in terms of ten-year equivalents as a percentage of the total outstanding of Treasury securities held by the Fed and the private investors. The ten-year equivalents (TYE) is a method of measuring duration risk, which is calculated as the amount of ten-year Treasury securities required to match the duration risk of the portfolio.

In mathematical terms, TYE can be represented as the following equation:

$$ten - year\ equivalents = \frac{par\ value\ of\ portfolio \times average\ portfolio\ duration}{duration\ of\ the\ ten - year\ on - the - run\ Treasury\ note} \tag{4.9}$$

[2] see more details in Section 3.2.4

Figure 4.1: Securities Supply Factors

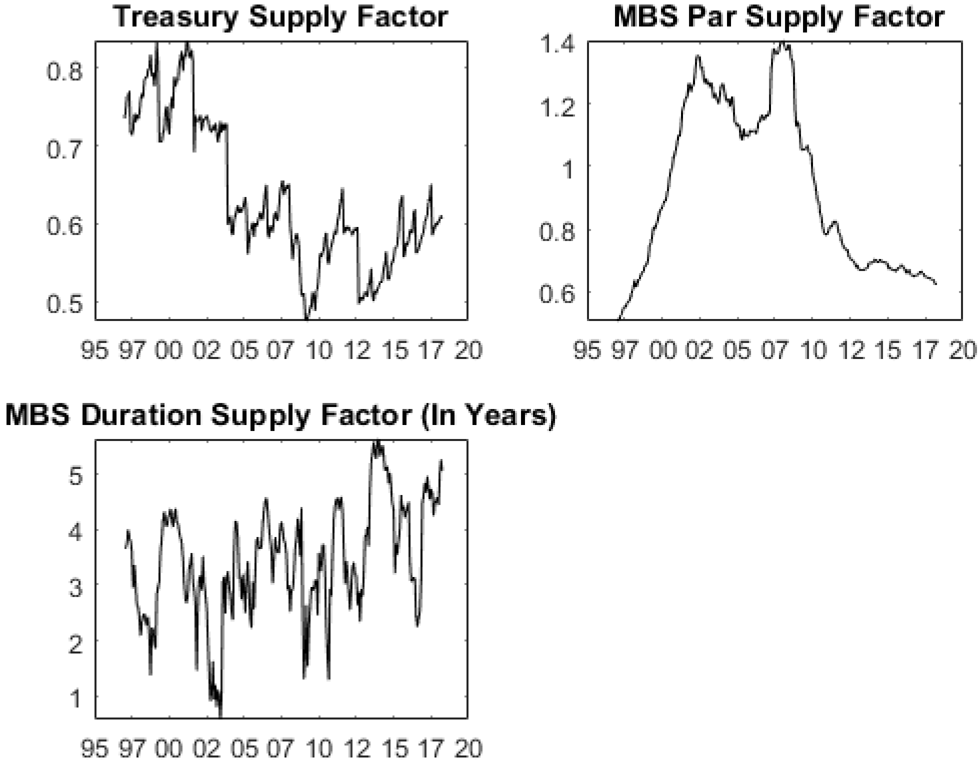


Figure 4.1 reports times series of the Treasury supply factor, which is the total amount of privately held Treasury securities, measured in term of ten-year-equivalents, as a percentage of total outstanding public debt held by the public (top-left panel); the MBS par supply factor, which is the par amount of privately held agency MBS as a percentage of public debt held by the public(top-right panel); and the MBS duration factor, which is the par-weighted average duration of privately held agency MBS in years (bottom panel).

For Treasury securities, the par amount outstanding (millions of U.S. dollars) of public debt held by private investors is obtained from Bloomberg Barclays indices. The Bloomberg Barclays US Treasury index measures US dollar-dominated debt issued by the US Treasury. Short Treasury index measures Treasury bill debt separately. The US Treasuries held in the Federal Reserve's System Open Market Account (SOMA) are deducted from the total amount outstanding. The total amount of these two indexes gives the par amount of privately held Treasury securities (i.e., par value of the portfolio in the Eq. 4.9).

The average duration (in years) (i.e. average portfolio duration Eq.4.9) is obtained from the U.S. Bureau of the Fiscal Service maturity distribution of public debt held by private investors.

Accordingly the average duration is measured as the dollar-weighted average maturity of the private holdings. The advantage of this crude measure is its simplicity, which is easy to replicate and implement. Similar measurement methods can be found in Greenwood and Vayanos (2104) and Goliski (2018).

Duration (in years) of the ten-year on-the-run Treasury notes is obtained from the Thomson Reuters US government bond benchmark index.

The public debt held by the public measures the total amount of Treasury securities held by the Fed and private investors, which is obtained from the U.S. Bureau of the Fiscal Service.

The MBS par supply factor (sv_{2t}) is the ratio of the MBS par amount of privately held agency MBS to the public held by the public. The MBS duration factor (sv_{3t}) measures the average duration of the privately held agency MBS (in years). The par amount (millions of U.S. dollars) and the average duration (in years) of privately held MBS are taken from the Bloomberg Barclays US MBS index. The index tracks agency mortgage-backed pass-through securities guaranteed by Ginnie Mae (GNMA), Fannie Mae (FNMA), and Freddie Mac (FHLMC).

4.3.2 Treasury Forward Yields

I construct end of period 1-month Treasury forward rates for maturities of 6 months, 1, 2, 3, 5, 7, and 10 years from the Gurkaynak et al. (2007) dataset at a monthly frequency. To be consistent with the sample period of the securities supply factors, Treasury yields data spans from December 1996 to January 2018. Figure 4.2 plots the time series of the treasury forward rate. From January 2009 to November 2015, the Federal Open Market Committee (FOMC) lowered the target range for the federal funds rate from 0 to 25 basis points, which is referred to as the zero lower bound period and is highlighted.

4.4 Model Specification

Similar to Li and Wei (2013), I assume that the Treasury forward rates are driven by yield factors and securities supply factors.

Figure 4.2: Treasury Forward Rates

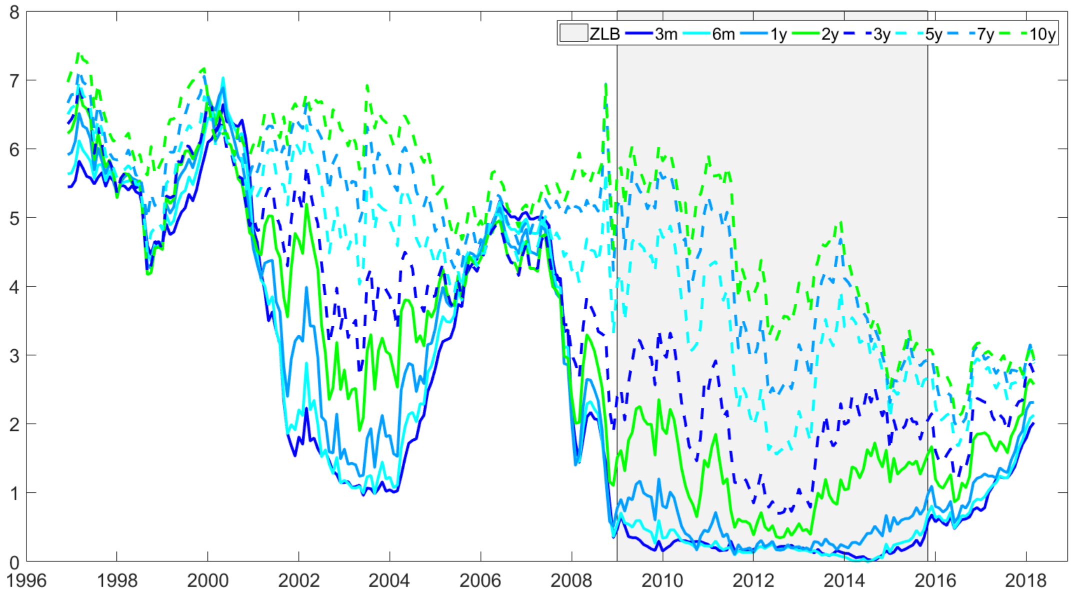


Figure 4.2 reports 1-month Treasury forward rates from December 1996 to March 2018 in annualised percentage points. Maturities are 6 months, 1, 2, 3, 5, 7, and 10 years. The ZLB period in grey areas is from December 2008 to November 2015.

4.4.1 Short Rate

The main purpose of this chapter is to measure the effect of LASPs on the term premium. Therefore, it is assumed that the short-term interest rate r_t loads on the latent yield factors $\boldsymbol{x}_t$, and supply factors have no impact. Thus, for the identification purpose, $\boldsymbol{\delta}_1$ is assumed as $\boldsymbol{\delta}_1' = (1\ 1\ 0\ 0\ 0)$.

4.4.2 State Variables

It is assumed that $\boldsymbol{x}_t$ consist of two types of factors: yield factors $\boldsymbol{yf}_t$ and supply factors $\boldsymbol{sv}_t$. As stated by Li and Wei (2013), more than 99 per cent of the yield variation can be explained by two yield factors, denoted as x_{1t} and slope x_{2t}. These two factors are latent variables. I assume that there are three supply factors as Li and Wei (2013): the ratio of the Treasury ten-year-equivalents to public debt held by the public, the ratio of MBS par amount to public debt held by the public, and the par-weighted average duration of privately held agency MBS, denoted as sv_{1t}, sv_{2t}, and sv_{3t}, respectively.

4.4.2.1 Under P-measure

The two groups of variables are collected in the state vector $\boldsymbol{x}_t = (\boldsymbol{yf}_t\, \boldsymbol{sv}_t)'$, and equals to $\boldsymbol{x}_t = (x_{1t}\, x_{2t}\, sv_{1t}\, sv_{2t}\, sv_{3t})'$.

As stated by Vayanos and Vila (2009), securities supply factors affect bond yields and bond risk premiums. In order to ensure that any evidence in support of this view is driven by empirical results, it is assumed that yield factors only respond to their own lags, instead of past supply factors:

$$x_{1t} = \mu_{x_1} + \phi_{x_1,x_1} x_{1,t-1} + \phi_{x_1,x_2} x_{2,t-1} + \varepsilon_{x_1,t} \tag{4.10}$$

where $\varepsilon_{x_1,t} \sim N(0, \sigma^2_{x_1})$

$$x_{2t} = \mu_{x_2} + \phi_{x_2,x_1} x_{1,t-1} + \phi_{x_2,x_2} x_{2,t-1} + \varepsilon_{x_2,t} \tag{4.11}$$

where $\varepsilon_{x_2,t} \sim N(0, \sigma^2_{x_2})$

These two latent yield factors are collected in a vector $\boldsymbol{yf}_t = (x_{1t}\, x_{2t})'$.

According to the policy of U.S. Treasury, the issuance of Treasury securities is determined by the federal budget deficit on a regular pre-announced schedule and does not react strongly to interest rate changes. Therefore, it is assumed that Treasury supply factor $SV1_t$ follows an AR(1) process, and does not respond to previous yield factors $\boldsymbol{x}_t$:

$$sv_{1t} = \mu_{sv_1} + \phi_{sv_1,sv_1} sv_{1,t-1} + \varepsilon_{sv_1,t} \tag{4.12}$$

where $\varepsilon_{sv_1,t} \sim N(0, \sigma^2_{sv_1})$

Agency MBS has in common features with Treasury securities, which is implicitly or explicitly guaranteed by the U.S. government. Therefore, agency MBS is generally considered a close substitutes for Treasury debt by market participants. Thus, the model includes two agency MBS supply factors: the MBS par amount factor, and the MBS duration factor, denoted as sv_{1t} and sv_{2t}, respectively.

Similar to Treasury securities, the supply of MBS is largely determined by the Federal Reserve and housing demand, and does not respond strongly to interest rate changes. Therefore, it is assumed that the par supply of MBS sv_{1t} also follows an AR(1) process, and does not load on previous yield factors $\boldsymbol{yf}_t$:

$$sv_{2t} = \mu_{sv_2} + \phi_{sv_2,sv_2} sv_{2,t-1} + \varepsilon_{sv_2,t} \tag{4.13}$$

where $\varepsilon_{sv_2,t} \sim N(0, \sigma^2_{sv_2})$

MBS duration is affected by interest rate changes, which may influence MBS prepayments, i.e., when the interest rate is lower, more mortgage borrowers will choose to repay their MBS, and the duration of MBS is shortened, and vice verse. Therefore, it is assumed that the average duration of privately held agency MBS sv_{3t} is affected by its own lag and the lagged yield factor x_{1t}:

$$sv_{3t} = \mu_{sv_3} + \phi_{sv_3,x_1} x_{1,t-1} + \phi_{sv_3,sv_3} sv_{3,t-1} + \varepsilon_{sv_3,t} \tag{4.14}$$

where $\varepsilon_{sv_3,t} \sim N(0, \sigma^2_{sv_3})$

These three observed supply factors are collected in a vector $\boldsymbol{sv}_t = (sv_{1t}\; sv_{2t}\; sv_{3t})'$.

Stacking Eq.4.10 to Eq.4.14 gives the state variables $\boldsymbol{x}_t = (\boldsymbol{yf}_t\; \boldsymbol{sv}_t)'$, and is equivalent to $\boldsymbol{x}_t = (x_{1t}, x_{2t}, sv_{1t}, sv_{2t}, sv_{3t})'$:

$$\begin{pmatrix} x_{1t} \\ x_{2t} \\ sv_{1t} \\ sv_{2t} \\ sv_{3t} \end{pmatrix} = \begin{pmatrix} \mu_{x_1} \\ \mu_{x_2} \\ \mu_{sv_1} \\ \mu_{sv_2} \\ \mu_{sv_3} \end{pmatrix} + \begin{pmatrix} \phi_{x_1,x_1} & \phi_{x_1,x_2} & 0 & 0 & 0 \\ \phi_{x_2,x_1} & \phi_{x_2,x_2} & 0 & 0 & 0 \\ 0 & 0 & \phi_{sv_1,sv_1} & 0 & 0 \\ 0 & 0 & 0 & \phi_{sv_2,sv_2} & 0 \\ \phi_{sv_3,x_1} & 0 & 0 & 0 & \phi_{sv_3,sv_3} \end{pmatrix} \begin{pmatrix} x_{1,t-1} \\ x_{2,t-1} \\ sv_{1,t-1} \\ sv_{2,t-1} \\ sv_{3,t-1} \end{pmatrix} \tag{4.15}$$

where error term is $(\varepsilon_{x_1,t}\; \varepsilon_{x_2,t}\; \varepsilon_{sv_1,t}\; \varepsilon_{sv_2,t}\; \varepsilon_{sv_3,t})'$

More compactly, state variables $\boldsymbol{x}_t$ follows a first-order autoregressive process under the P-measure:

$$\boldsymbol{x}_t = \boldsymbol{\mu} + \boldsymbol{\Phi} \boldsymbol{x}_{t-1} + \boldsymbol{\Sigma} \boldsymbol{\varepsilon}_t \tag{4.16}$$

where conditional covariance is $\boldsymbol{\Sigma}\boldsymbol{\Sigma}'$, and $\boldsymbol{\Sigma}$ is specified as a 5×5 lower triangular matrix, with $\boldsymbol{\varepsilon}_t \sim i.i.d.N(\mathbf{0}_5, \boldsymbol{I}_5)$.

4.4.2.2 Under Q-measure

The assumption of no-arbitrage and state variables $\boldsymbol{x}_t$ under P-measure imply that the state variables under the risk-neutral measure (Q-measure) also follows VAR(1):

$$\boldsymbol{x}_t = \boldsymbol{\mu}^Q + \boldsymbol{\Phi}^Q \boldsymbol{x}_{t-1} + \boldsymbol{\Sigma}\boldsymbol{\varepsilon}_t^Q \tag{4.17}$$

where $\boldsymbol{\Sigma}$ is the same as Eq.4.16, with $\boldsymbol{\varepsilon}_t^Q \sim i.i.d.N(\mathbf{0}_5, \boldsymbol{I}_5)$.

For identification of the latent yield factors, I impose restrictions on the Q parameters of $\boldsymbol{x}_t$ as Joslin et al.(2011) and Joslin et al.(2013): (1) $\boldsymbol{\mu}_x^Q = 0$ (2)$\boldsymbol{\Phi}^Q_{x1,x_1}$,$\boldsymbol{\Phi}^Q_{x2,x_2}$ is in real Jordan form with eigenvalues in descending order. These restrictions are to prevent the latent factors from shifting, rotation and scaling and do not change the economic implications of the model. The past securities supply factors $\boldsymbol{sv}_t$ are assumed to load on yield factors $\boldsymbol{x}_t$ under Q-measure. The relevant Q parameters are as follows:

$$\boldsymbol{\mu}_x^Q = \begin{pmatrix} \mu_{x_1} \\ \mu_{x_2} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad \boldsymbol{\Phi}_x^Q = \begin{pmatrix} \phi^Q_{x1,x_1} & 0 & \phi^Q_{x1,sv_1} & \phi^Q_{x1,sv_2} & \phi^Q_{x1,sv_3} \\ 0 & \phi^Q_{x2,x_2} & \phi^Q_{x2,sv_1} & \phi^Q_{x2,sv_2} & \phi^Q_{x2,sv_3} \end{pmatrix} \tag{4.18}$$

It is assumed that supply factors carry zero risk premium as Li and Wei (2013), and affect term premiums by indirectly influencing risk premiums on the yield factors. Supply factors $\boldsymbol{sv}_t$ follow the same dynamics under the P-measure and Q-measure as follows:

$$\boldsymbol{\mu}_{sv}^Q = \boldsymbol{\mu}_{sv} = \begin{pmatrix} \mu_{sv_1} \\ \mu_{sv_2} \\ \mu_{sv_3} \end{pmatrix} \qquad \boldsymbol{\Phi}_{sv}^Q = \boldsymbol{\Phi}_{sv} = \begin{pmatrix} 0 & 0 & \phi_{sv_1,sv_1} & 0 & 0 \\ 0 & 0 & 0 & \phi_{sv_2,sv_2} & 0 \\ \phi_{sv_3,x_1} & 0 & 0 & 0 & \phi_{sv_3,sv_3} \end{pmatrix} \tag{4.19}$$

Stacking Eq.4.18 and Eq.4.19 gives the parameters of state variables $\boldsymbol{x}_t$ under the Q-measure:

$$\boldsymbol{\mu}^Q = \begin{pmatrix} 0 \\ 0 \\ \mu_{sv_1} \\ \mu_{sv_2} \\ \mu_{sv_3} \end{pmatrix} \qquad \boldsymbol{\Phi}^Q = \begin{pmatrix} \phi^Q_{x1,x_1} & 0 & \phi^Q_{x1,sv_1} & \phi^Q_{x1,sv_2} & \phi_{x1,sv_3} \\ 0 & \phi^Q_{x2,x_2} & \phi^Q_{x2,sv_1} & \phi^Q_{x2,sv_2} & \phi^Q_{x2,sv_3} \\ 0 & 0 & \phi_{sv_1,sv_1} & 0 & 0 \\ 0 & 0 & 0 & \phi_{sv_2,sv_2} & 0 \\ \phi_{sv_3,x_1} & 0 & 0 & 0 & \phi_{sv_3,sv_3} \end{pmatrix} \tag{4.20}$$

To ensure the stationarity of state variables $\boldsymbol{x}_t$, I impose restrictions that all eigenvalues of $\boldsymbol{\Phi}$ are smaller than 1.

Imposing these restrictions captures the features of the yield factors $\boldsymbol{yf}_t$ and securities supply factors $\boldsymbol{sv}_t$, and reduces the number of parameters needed to be estimated and avoids overfitting.

4.5 Estimation Methodology

4.5.1 SRTSM

The shadow rate term structure model described in Section 4.2.2 is a nonlinear state-space model. The measurement equation is not affine in the state variables. I estimate the SRTSM by the extended Kalman filter.

The transition equation for the state variables is as Eq.4.16.

The measurement equation relates the observed one-month forward rate $f^o_{t,n,n+1}$ to the state variables as follows:

$$f^o_{t,n,n+1} = \underline{r} + \sigma^Q_n g(\frac{a_n + \boldsymbol{b}'_n \boldsymbol{x}_t - \underline{r}}{\sigma^Q_n}) + \eta_{t,n} \tag{4.21}$$

where the measurement error $\eta_{t,n} \sim i.i.d.N(0, \omega)$.

4.5.2 GATSM

I estimate the Gaussian term structure model by the Kalman filter using a state space presentation.

The transition equation for the state variables is the same as the SRTSM, see Section 4.5.1.

The measurement equation is implied by Eq.4.2:

$$f^o_{t,n,n+1} = a_n + \boldsymbol{b}'_n \boldsymbol{x}_t + \eta_{t,n} \tag{4.22}$$

where the measurement error $\eta_{t,n} \sim i.i.d.N(0, \omega)$.

4.6 Estimation Results

This section discusses estimated yield curves, factor loadings, estimated state variables, estimated parameters and the term premium.

It is notable that when estimating the model, I ignore the effect of the LSAP communications, and treat the LASPs as a one-period shock, which brings an instant shock to the supply variables, as found in Li and Wei (2013). Gagnon et al. (2011) use a one-day window around the LSAP communications to measure the announcement effect, and their event-study approach is conducted with strong assumptions, e.g., the LSAP expectations are only affected by announcements. The inclusion of the announcement effect may not be feasible in time series analysis, since the supply factors, such as the private holdings of the Treasury bond, can be affected by the LSAPs, auctions and other small transactions.

4.6.1 Estimated Yield Curves

(i) *RMSE.* Table 4.1 shows the root-mean-squared fitting errors (RMSE) across models for each yield maturity. The top panel reports RMSE for the entire sample, whereas the middle panel reports the fit for the pre-ZLB period. On December 16, 2008, the FOMC lowered the target for the federal funds rate to a range from 0 to 25 basis points. Hence I choose December 2008 as the last month of the Pre-ZLB subsample, and December 2008 as the first month of the ZLB period. The bottom panel reports the fit for the ZLB sample period. On December 16, 2015, the FOMC raised the target range for the federal funds rate to 25 to 50 basis points. Hence I choose November 2015 as the last month of the ZLB sample period.

Overall, the SRTSM fit yields better than its GATSM counterpart. The top panel of Table 4.1 shows that during the full sample period the performance of the SRTSM is significantly better with an average RMSE of 15 basis points, compared with GATSM with an average RMSE of 20 basis points. SRTSM has comparably smaller RMSEs across all the estimated maturities. The middle panel of Table 4.1 demonstrates that GATSM and SRTSM fits the yields approximately the same during the pre-ZLB period, when the economy is sufficiently away from the lower bound. The average RMSEs for GATSM and SRTSM are the same at 15 basis points. As for the RMSEs across selected maturities, the RMSEs are almost the same for GATSM and SRTSM. The slight difference in the RMSEs with 5-year maturities yield is caused by linear approximation of the extended Kalman filter. The bottom panel of Table 4.1 indicate that during the ZLB period the performance of SRTSM is substantially better compared with GATSM, with an average RMSEs of 20 and 9 basis points, respectively.

As for the performance across the estimated maturities, the SRTSM fits the short-end and long-end maturities yields much better than GATSM. For the 3-month forward yield, the SRTSM RMSE is 10 basis points smaller than its GATSM counterpart. For the 10-year forward yield, SRTSM RMSE is 14 basis points lower than its GATSM counterpart. These findings are consistent with SRTSM model setup in Section 3. Theoretically, due to its nonlinearity, SRTSM has the advantage of more flexibility over GATSM in fitting the cross-section of yields. When the lower bound binds, the SRTSM imposes the non-negative short-term interest rate and allows the shadow rate to capture the information of the current state of the economy. When the short-term interest rate is sufficiently higher than the lower bound, the SRTSM becomes a GATSM.

Table 4.1: Root-Mean-Square-Error

	Full Sample								
	3-month	6-month	1-y	2-y	3-y	5-y	7-y	10-y	Average
GATSM	21	12	17	27	26	15	15	26	20
SRTSM	15	10	13	20	17	12	13	18	15
	Pre-ZLB Subsample								
	3-month	6-month	1-y	2-y	3-y	5-y	7-y	10-y	Average
GATSM	18	8	16	23	16	9	16	18	15
SRTSM	18	8	16	23	16	10	16	18	15
	ZLB Subsample								
	3-month	6-month	1-y	2-y	3-y	5-y	7-y	10-y	Average
GATSM	22	15	13	24	31	21	11	21	20
SRTSM	12	13	10	8	8	8	5	7	9

Table 4.1 reports root-mean-squared errors of GATSM and SRTSM model-implied yields on the full sample (top panel), the pre-zero lower bound subsample (middle panel) and the lower bound subsample (bottom panel). Full sample: December 1996 to March 2018. Pre-ZLB subsample: December 1996 to December 2008. ZLB subsample: January 2009 to November 2015. The RMSEs are measured in basis points.

(ii) *Yield Curve Fit.* In Figure 4.3 and Figure 4.4, I report the yield curve fit of GATSM and SRTSM for the estimated yield maturities on the full sample period. Figure 4.3 displays the observed and fitted yields with maturities 3-month, 6-month, 1-year and 2-year. Figure 4.4 presents the observed and fitted yields with maturities 3-year, 5-year, 7-year and 10-year. To demonstrate the fit of the models during different periods, I plot pre-ZLB period (December 1996 to December 2008) on the left panels in Figure 4.3 and 4.4, and the ZLB period until March 2018 on the right panels.

The fit of the SRTSM is good. In addition, consistent with the results of RMSEs in Table 4.1, the SRTSM has a significantly better fit for forward rates than GATSM. The GATSM and SRTSM yield fits are almost the same during the pre-ZLB period. The SRTSM provides a much better fit of yields during the ZLB period compared with GATSM. The superior performance of the SRTSM is due to the imposing of nonnegativity of government interest rates as discussed above.

During the ZLB period, for the short-end maturities, as displayed in the Figure 4.3 upper panel and the middle top panel, the 3-month and 6-month forward rates are very flat and close to zero. The SRTSM has a flat end and fits the short end of the forward curve better than the GATSM. In contrast, GATSM generates too much variation and has trouble fitting the short end. The results violate the ZLB constraint and drop below zero at some horizons. For the long-end maturity, as displayed in the Figure 4.4 bottom panel, the SRTSM fits the 10-year maturity forward rate much better than GATSM. This is consistent with the finding of Kim and Singleton (2012): the GATSM has particular difficulty matching the observed 10-year when short-term interest rates are stuck at zero. To match the very small dynamics of short-term interest rates, the state variables that determined short rates have to be small. Thus, GATSM compensates its freedom to capture the variation in long-term yields with a poor performance at the short end.

It is notable that the short-term fitted SRTSM rates in some years show relatively poor performance, especially for 3-month and 6-month forward rates. In addition, the fitted SRTSM short with 10-year maturity is much higher than the observed data even during pre-crisis period. This may due to the sensitivity of the shadow interet rates to the value of the lower bound parameter, as discussed in Chapter 3 Section 3.6.1. This problem can be solved by fixing the zero lower at zero, as suggested by literature.

4.6.2 Factor Loadings

To demonstrate how state variables affect yields, in Figure 4.5 I display the factor loadings of GATSM and SRTSM estimated forward rates. The x-axis refers to the estimated factor loadings with maturity in months. The top panel in this figure indicates that the latent yield factors $\boldsymbol{yf}_t$ are closely related with the level and slope factors. The first latent yield factor x_{1t} loads almost identically at all maturities, with the factor loadings close to 1. This is consistent with the behaviour of the yield curve level factor interpreted in Diebold and Li (2006). The second latent yield factor x_{2t} loads more heavily on short-term interest rates than on long-term ones. This is consistent with the interpretation of the slope factor in Frankel and Lown (1994): the slope factor governs the short-term yield curves.

The bottom panel in Figure4.5 demonstrates the factor loadings of the securities supply factors. It is noticeable that supply factors loadings on very short-end maturities are close to zero, which is consistent with the assumptions in Section 4.1, i.e., short-term interest rate are not determined by supply factors. For the GATSM, the Treasury supply factor accounts for most of the supply factor effects on yield variations. The Treasury supply factor has a large negative impact on interest rates at most maturities, and changes short-term interest rates more than long ones. It is notable that in Li and Wei (2013) Table 3, estimated factor loadings for the Treasury supply factor stay positive around 1. This large difference in the estimated factor loadings of the Treasury supply factor may due to the different estimation period. Li and Wei (2013) only estimates the factor loadings during the pre-crisis period, whereas I conduct the estimation in a longer period, including both pre-crisis and post-crisis periods. This large negative estimated factor loadings of the Treasury supply factor may due to the estimation bias of the GATSM, since it neglects the lower bound.

The yield curve does not load significantly on the MBS par supply factor, and the factor loadings are almost 0 at all maturities. In contrast, for the SRTSM, both Treasury and MBS par supply factors have long-lasting positive effects on yields, and the effect rises with bond maturities. Moreover, the MBS par supply factor attributes to shift yields more than other two supply factors. The Treasury supply factor has a more positive effect on medium-term interest rates and little impact on very short interest rates. The MBS supply factor has a significant positive impact at most maturities, and the loadings increase as the maturities increase. The factor loadings on the MBS duration factor are similar for the GATSM and the SRTSM, and explain little yield variation. This indicates that the MBS duration supply factor has a smaller impact than the other two supply factors on forward rates under both GATSM and SRTSM.

4.6.3 Estimated State Variables

In Figure 4.6, I demonstrate the estimated state variables. The top plots contain the estimates of the latent yield factors $\boldsymbol{yf}_t$ from GATSM and SRTSM. Based on the literature, I construct the level and slope factor as Diebold and Rudebusch (2013). The level factor is measured as the 10-year forward rate, which is a long-term factor governing the level of the yield curve. The slope factor is measured as the 10-year forward rate minus 6-month forward rate, which is a short-term factor shifting short yields more than long yields. For the first latent yield factor x_{1t} on the top left , both GATSM and SRTSM estimates are closely related to the level factor, which is consistent with the finding in Section 4.6.2 that x_{1t} loads almost equally at all maturities in Section 4.6.2. In addition, the SRTSM captures the shape of the level factor slightly better than its GATSM counterpart. GATSM estimated x_{1t} has a decreasing tendency since 2003, whereas SRTSM estimated $x_{1t}t$ is less volatile and follows the trend of the level factor, especially during the ZLB period. For the second latent yield factor x_{2t} on the top right, the SRTSM estimates seem related to the inverse of the slope factor. Figure 4.6 also reveals that during the ZLB period, the SRTSM estimated x_{2t} is relatively flat and steadily declines to zero, which is consistent with the behaviour of the short-term interest rates. The middle and bottom plots contain the estimates of the securities supply factors $\boldsymbol{sv}_t$ from GATSM and SRTSM. When estimating the models using a Kalman filter (Extend Kalman filter for SRTSM), I assume that supply factors are estimated without error. As the large macro-finance term structure literature shows, the fittings of these supply factors are good for both models.

4.6.4 Parameter Estimates

Table 4.2 and 4.3 present the the model parameter estimates of the GATSM. Table 4.4 and 4.5 present the model parameter estimates of the SRTSM. The sample is from December 1996 to March 2018. The performance of the SRTSM is superior to the GATSM. The log likelihood value is 1661.5 for the GATSM, and 2168.76 for the SRTSM. The variance of residuals of the fitted forward rate is 0.2214 for the GATSM, and 0.1665 for the SRTSM. These results are

Table 4.2: Estimated Parameters

		GATSM			
1200 $\boldsymbol{\mu}$	0.0628	-0.0862	0.0063***	0.0033*	0.3434***
	(0.0348)	(0.0864)	(0.0001)	(0.0013)	(0.0202)
$\boldsymbol{\Phi}$	0.9991***	0.0068			
	(0.0041)	(0.0048)			
	-0.0129	0.9872***			
	(0.0101)	(0.0120)			
			0.9919***		
			(0.0002)		
				0.9976***	
				(0.0017)	
	−0.0318***				0.9476***
	(0.0008)				(0.0024)
max(eig($\boldsymbol{\Phi}$))	0.9976				
$\boldsymbol{\Phi}^Q$	1.0150***		−0.1923***	0.0034	0.0225***
	(0.0001)		(0.0053)	(0.0042)	(0.0004)
		0.9767***	−0.4316***	0.0703***	0.0142***
		(0.0007)	(0.0068)	(0.0018)	(0.0015)
max(eig($\boldsymbol{\Phi}^Q$)	1.0018				

Table 4.2 reports the estimated parameters of the GATSM. Standard errors are in parenthesis. Significance at the 95%-, 99%- and 99.9%- level is denoted by $*$, $**$ and $***$ respectively. The sample period spans from December 1996 to March 2018.

Table 4.3: Estimated Parameters

	GATSM				
$1200\delta_0$	7.9815***				
	(0.5430)				
$1200\boldsymbol{\Sigma}$	0.4998***				
	(0.0244)				
	−0.5382***	0.2281***			
	(0.0338)	(0.0130)			
	0.0209***	0.0034***	0.0047***		
	(0.0010)	(0.0005)	(0.0004)		
	-0.0003	0.0025	0.0065***	0.0185***	
	(0.0013)	(0.0016)	(0.0016)	(0.0009)	
	0.0000	0.0696	0.0750	0.0533	0.4548***
	(0.0321)	(0.0385)	(0.0601)	(0.0347)	(0.0217)
Model-implied $1200\boldsymbol{\lambda}_0$	0.1256	-0.0816	-0.4983	0.1881	0.0725
Model-implied $\boldsymbol{\Lambda}_1$	-0.0318	0.0137	0.3848	-0.0068	-0.0450
	-0.1318	0.0784	2.8001	-0.3244	-0.1682
	0.2380	-0.1182	-3.7591	0.2677	0.3228
	-0.0664	0.0312	0.9505	-0.0506	-0.0915
	-0.0113	0.0038	0.0795	0.0115	-0.0167
$1200\sqrt{\omega}$	0.2214***				
	(0.0015)				
log-likelihood value	1661.5000				

Table 4.3 reports the estimated parameters of the GATSM. Standard errors are in parenthesis. Significance at the 95%-, 99%- and 99.9%- level is denoted by $*$, $**$ and $***$ respectively. The sample period spans from December 1996 to March 2018.

Table 4.4: Estimated Parameters

	SRTSM				
1200 $\boldsymbol{\mu}$	-0.1503	0.0235	0.0336^{***}	0.0037^{**}	-0.1495^{***}
	(0.1087)	(0.2933)	(0.0023)	(0.0013)	(0.0089)
$\boldsymbol{\Phi}$	0.9946^{***}	0.0081^{**}			
	(0.0079)	(0.0031)			
	-0.0072	0.9810^{***}			
	(0.0217)	(0.0081)			
			0.9472^{***}		
			(0.0050)		
				0.9971^{***}	
				(0.0013)	
	-0.0247^{***}				0.9522^{***}
	(0.0005)				(0.0041)
max(eig($\boldsymbol{\Phi}$))	0.9971				
$\boldsymbol{\Phi}^Q$	1.0089^{***}		0.0878^{***}	0.0281^{***}	0.0118^{***}
	(0.0001)		(0.0159)	(0.0089)	(0.0007)
		0.9684^{***}	0.1429^{***}	0.1359^{***}	0.0306^{***}
		(0.0007)	(0.0414)	(0.0037)	(0.0022)
max(eig($\boldsymbol{\Phi}^Q$))	1.0032				

Table 4.4 reports the estimated parameters of the SRTSM. Standard errors are in parenthesis. Significance at the 95%-, 99%- and 99.9%- level is denoted by $*$, $**$ and $***$ respectively. The sample period spans from December 1996 to March 2018.

Table 4.5: Estimated Parameters

	SRTSM				
$1200\delta_0$	7.9945***				
	(0.3571)				
$1200\boldsymbol{\Sigma}$	0.2238***				
	(0.0204)				
	−0.3472***	0.3297***			
	(0.0527)	(0.0199)			
	−0.0072***	-0.0015	0.0206***		
	(0.0016)	(0.0018)	(0.0010)		
	−0.0101***	0.0007	-0.0004	0.0170***	
	(0.0013)	(0.0014)	(0.0011)	(0.0008)	
	0.1470***	0.1628***	0.1029***	0.1321***	0.3433***
	(0.0327)	(0.0320)	(0.0278)	(0.0232)	(0.0209)
Model-implied $1200\boldsymbol{\lambda}_0$	-0.6717	-0.6360	-0.2805	-0.3821	0.8202
Model-implied $\boldsymbol{\Lambda}_1$	-0.0641	0.0363	-0.3923	-0.1255	-0.0529
	-0.0892	0.0768	-0.8467	-0.5444	-0.1484
	-0.0288	0.0183	-0.1987	-0.0836	-0.0293
	-0.0354	0.0191	-0.2058	-0.0558	-0.0265
	0.0920	-0.0648	0.7082	0.3584	0.1120
$1200\sqrt{\omega}$	0.1665***				
	(0.0029)				
log-likelihood value	2168.7600				

Table 4.5 reports the estimated parameters of the SRTSM. Standard errors are in parenthesis. Significance at the 95%-, 99%- and 99.9%- level is denoted by $*$, $**$ and $***$ respectively. The sample period spans from December 1996 to March 2018.

consistent with the findings in Section 6.1 and 6.2. The better performance of the SRTSM comes from its ability to fit the flat short end of the forward curve at the ZLB period.

The estimates of $\boldsymbol{\Phi}^Q$ in Table 4.4 suggest that under the Q-measure the yield factors $\boldsymbol{x}_t$ reacts positively and significantly to an increase of three lagged securities supply factors $\boldsymbol{sv}_t$, as one would expect. The Treasury supply factor sv_{1t} has a stronger effect on yield factors than the other two supply factors, while MBS duration factor sv_{3t} has less impact on yield factors than the other two supply factors. Moreover, securities supply factors affect the second yield factor x_{2t} more strongly than the first. In contrast, the estimates of $\boldsymbol{\Phi}^Q$ in Table 4.2 suggest that the lagged Treasury supply factor has a strong negative effect on the yield factors. The lagged MBS par supply factor sv_{2t} is not significant on the first yield factor.

Similar to Li and Wei (2013) Table 2, I present the model-implied estimates of the market price of risk parameters $\boldsymbol{\lambda}_0$ and $\boldsymbol{\Lambda}_1$ for the GATSM and the SRTSM in Table 4.3 and Table 4.5, respectively. The market price of risk $\boldsymbol{\lambda}_t$ is linear in the state variables (See Chapter 2 Eq.2.7). The model-implied market price of risk parameters $\boldsymbol{\lambda}_0$ and $\boldsymbol{\Lambda}_1$ can be calculated using the risk adjustment equations in Chapter Eq.2.9, or Li and Wei (2013) Eq.14. It is notable that as Li and Wei (2013) Eq.23 and Eq.24 stated, their model-implied estimates of $\boldsymbol{\lambda}_0$ and $\boldsymbol{\Lambda}_1$ related with supply factors are equal to zeros. However, in this chapter, the associated estimates are not zeros. This difference is due to the different model specifications about the conditional covariance matrix $\boldsymbol{\Sigma}$. In this chapter, I assume that $\boldsymbol{\Sigma}$ is a lower triangular matrix (See Eq.4.16). In comparison, Li and Wei (2013) adopt different assumptions for the conditional covariance matrix $\boldsymbol{\Sigma}$, which can reflected in their model parameter estimates of $\boldsymbol{\Sigma}$ in Table 2.

4.6.5 Term Premium

In the model specification in Section 4, I assume that the short rate is only affected by yield factors, and supply factors do not affect the expected future short rate. Besides, supply factors affect the term premium by an indirect influence on the yield factors. I decompose the term structure into the risk-neutral expectation component and the risk premium component and use the term premium to analyse the effects of supply variables on the forward rates.

The risk-neutral forward rate $\widetilde{f}_{t,n,n+1}$ reflects the expectation of the short-term interest rate over the 1-month life of the bond. It corresponds to the forward rate that would prevail if the investors are risk-neutral.

The risk-neutral bond price $\widetilde{P}_{t,n}$ of an n-period, zero-coupon bond satisfies the following relation:

$$\widetilde{P}_{t,n} = E_t(exp(-r_t)\widetilde{P}_{t+1,n-1}) \tag{4.23}$$

n-period continuously compounded zero coupon risk-neutral bond yield $\widetilde{y}_{t,n}$ is given as follows:

$$\widetilde{y}_{t,n} = -\frac{log\widetilde{P}_{t,n}}{n} \tag{4.24}$$

One-period risk neutral forward rate, or expected future short rate at horizons of n-year, $\widetilde{f}_{t,n,n+1}$ is:

$$\widetilde{f}_{t,n,n+1} = (n+1)\widetilde{y}_{t,n+1} - n\widetilde{y}_{t,n} \tag{4.25}$$

The risk-neutral forward rate $\widetilde{f}_{t,n,n+1}$ in the benchmark GATSM is:

$$\widetilde{f}^{GATSM}_{t,n,n+1} = \widetilde{a}_n + \widetilde{\boldsymbol{b}}'_n \boldsymbol{x}_t \tag{4.26}$$

where $\widetilde{a}_n$ and $\widetilde{\boldsymbol{b}}_n$ can be derived as follows:

$$\widetilde{a}_n = \delta_0 + \boldsymbol{\delta}'_1 (\sum_{j=0}^{n-1} (\boldsymbol{\Phi})^j) \boldsymbol{\mu} \tag{4.27}$$

$$\widetilde{\boldsymbol{b}}'_n = \boldsymbol{\delta}'_1 (\boldsymbol{\Phi})^n \tag{4.28}$$

The risk-neutral forward rate $\widetilde{f}_{t,n,n+1}$ in the SRTSM is:

$$\widetilde{f}^{SRTSM}_{t,n,n+1} = \underline{r} + \sigma_n g(\frac{\widetilde{a}_n + \widetilde{\boldsymbol{b}}'_n \boldsymbol{x}_t - \underline{r}}{\sigma_n}) \tag{4.29}$$

where $(\sigma_n)^2 \equiv Var_t(s_{t+n})$. a_n and $\boldsymbol{b}_n$ are the same as in Eq.4.27 and Eq.4.28.

The term premium on the one-period forward rate is defined as the difference between the fitted forward rate and the risk neutral forward rate. In GATSM, the term premium is the difference between the Eq.4.2 and the Eq.4.26:

$$\begin{aligned} tp^{GATSM}_{t,n,n+1} &= f^{GATSM}_{t,n,n+1} - \widetilde{f}^{GATSM}_{t,n,n+1} \\ &= (a_n - \widetilde{a}_n) + (\boldsymbol{b}'_n - \widetilde{\boldsymbol{b}}'_n)\boldsymbol{x}_t \end{aligned} \tag{4.30}$$

In SRTSM, the term premium is the difference between the Eq.4.8 and the Eq.4.29:

$$\begin{aligned} tp^{SRTSM}_{t,n,n+1} &= f^{SRTSM}_{t,n,n+1} - \widetilde{f}^{SRTSM}_{t,n,n+1} \\ &= \sigma^{Q}_{n} g(\frac{a_n + \boldsymbol{b}^{'}_{n}\boldsymbol{x}_t - \underline{r}}{\sigma^{Q}_{n}}) - \sigma_n g(\frac{\widetilde{a}_n + \widetilde{\boldsymbol{b}}^{'}_{n}\boldsymbol{x}_t - \underline{r}}{\sigma_n}) \end{aligned} \tag{4.31}$$

The GATSM estimated effect of supply factors on forward rates term premium can be calculated as:

$$tp^{GATSM,sv}_{t,n,n+1} = \boldsymbol{b}'_{n,sv}\boldsymbol{sv}_t \tag{4.32}$$

$\boldsymbol{b}_{n,sv}$ is the GATSM estimated factor loadings of supply factors on the term premium.

The SRTSM estimated effect of supply factors on the forward rates term premium can be calculated as:

$$tp^{SRTSM}_{t,n,n+1} = \sigma^{Q}_{n} g(\frac{\boldsymbol{b}^{'}_{n,sv}\boldsymbol{sv}_t}{\sigma^{Q}_{n}}) \tag{4.33}$$

$\boldsymbol{b}_{n,sv}$ is the SRTSM estimated factor loadings of supply factors on the term premium.

Figure 4.7 presents the estimated expected future 1-month short rate at horizons from 3-month to 10-year. The top and bottom panel of the Figure displays the results of GATSM and SRTSM using the full sample data, respectively. The Figure shows that at ZLB period, the SRTSM produces flat and positive expected short rates around the lower bound (0.25%), whereas GATSM produces negative expected short rates out to the 1-year horizon. This implies that GATSM generates a non-zero probability of negative interest rates on the whole term structure, which is a misspecification relative to the observed data (see Piazzesi, 2010, p.716). In addition, GATSM produces a larger estimate of the long-run level of the short rate than SRTSM. The difference between the GATSM and SRTSM can be attributed to the estimation bias in the GATSM since it neglects the ZLB. Figure 4.8 displays the estimated term premium, which is computed by subtracting the estimated expected short rate from the observed forward rate. The results show that compared with SRTSM, GATSM overestimates the term premium components of short-term forward rates, while underestimating the term premium components of long-term rates. This is consistent with the findings in Figure 4.8: a term premium is underestimated when the corresponding expected short rate component is overestimated, and vice versa. The findings suggest that GATSM estimates of the parameters and factors are biased in the ZLB period, which is consistent with the findings in Ichiue and Ueno (2013).

Table 4.6: Estimated Factor Loadings of Term Premiums

Maturity	3-month	6-month	1-year	2-year	3-year	5-year	7-year	10-year
SRTSM								
sv_{1t} (%)	0.65	1.16	1.89	2.55	2.64	2.30	1.95	1.72
sv_{2t}(%)	0.48	0.92	1.69	2.90	3.77	4.85	5.45	6.01
sv_{3t} (Years)	0.12	0.21	0.35	0.47	0.48	0.41	0.33	0.27
GATSM								
sv_{1t} (%)	-1.84	-3.56	-6.71	-11.92	-15.89	-21.13	-24.02	-26.10
sv_{2t} (%)	0.22	0.42	0.78	1.35	1.78	2.32	2.59	2.74
sv_{3t} (Years)	0.10	0.19	0.33	0.49	0.56	0.58	0.55	0.52

Table 4.6 reports the model-implied factor loadings of term premiums on the supply factors $\boldsymbol{sv}_t$ The top panel denotes factor loadings of Li and Wei (2013). The middle and bottom panel denotes factor loadings of SRTSM and GATSM, respectively. sv_{1t} represents Treasury supply factor. sv_{2t} represents MBS par supply factor. sv_{3t} represents MBS duration supply factor. Estimated factor loadings are reported in basis points.

Table 4.6 summarise the SRTSM and GATSM estimated factor loadings of term premiums on supply factors. sv_{1t}, sv_{2t}, sv_{3t} denotes Treasury supply factor, MBS par supply factor and MBS duration factor, respectively. In the GATSM model, the effect of supply factors on the term premium can be measure by Eq.4.32. The GATSM model results suggest that the Treasury supply factor has a larger effect on the term premium than the other two factors. 1-percentage point decrease in the Treasury supply factor reduces the 10-year forward term premium by 26 basis points. In the SRTSM model, the effect of supply factors on the term premium can be measured by Eq.4.33. The SRTSM model results suggest that MBS par supply factor has a larger effect on the term premium than the other two factors. 1-percentage point decrease in the MBS par supply factor reduces the 10-year forward term premium by 6 basis points.

4.7 Evaluating the Federal Reserve's Asset Purchasing Programs

The Fed's asset purchase programs (LSAPs) provides a good opportunity to assess the effect of securities supply shocks on Treasury forward rates. This section uses the term structure model developed above to evaluate the effect of the three LSAPs programs: the first large-scale asset purchase programs (LSAP1 and LSAP2), and the operation twist (OT).

The details of the Fed's purchase programs are described below.

LSAP1:November 2008 to March 2010. The program purchases about \$300 billion Treasury securities, \$1.25 trillion MBS, and \$170 billion agency debt.

LSAP2: November 2010 to June 2011. The program purchases \$600 billion long-term Treasury securities.

OT:September 2011 to December 2012. The program swapped the Fed's shorter term bonds with its longer-term bonds. The Fed purchases about \$600 billion Treasury securities within 6 to 30 years and sells Treasury securities with maturities of 3 years or less. In Li and Wei (2013), the maturity extends program (MEP) is the first stage OT, which is conducted from mid-2011 to mid-2012. The MEP swaps about \$400 billion short-term Treasury securities for long-term Treasury securities.

LSAP3:September 2012 to October 2014. The program purchases MBS securities and long-term Treasury securities. Unlike LSAP1 and LSAP2, LSAP 3 is announced as an open-ended program with no pre-determined size for the total amount of purchases. The purchase is initially set at \$45 billion per month for long-term Treasury securities, and \$40 billion per month for MBS. The Fed purchases about \$540 billion Treasury securities, and \$520 billion MBS in total (See Ihrig et al., 2018, Table 1, p351).

It is notable that, in variance to Section 4.6, when focusing on the counter-factual analysis of the impact of the Fed purchases, the announcement effect should be considered as found in Li and Wei (2013) Section 5.2, which suggests a sequence of supply shocks instead of a one-period shock. However, for the first three purchase programs (LSAP1, LSAP2 and OT), since the purchase amounts are predetermined, they can still be treated as a case of one-period supply shock.

In the cast of the LSAP 3, unlike the other three programs, this is an open-end program. As stated in the literature, for example, Campbell et al. (2012), this is an Odyssean commitment. In this case, a sequence of supply shocks should be considered.

To simplify the counter-factual analysis, in this section I treat the first three LSAPs as a one-period supply shock, and LSAP3 is no analysed in this section.

Li and Wei (2013) treat the Fed's purchase programs as supply shocks, and roughly measure the accumulative impact of each purchase program using the estimated factor loading before the financial crisis (See details in Li and Wei, 2013, p28). They summarise that the LSAP1 program lowers the ten-year Treasury yield by about 100 basis points, whereas both LSAP2 and the MEP reduces ten-year Treasury securities for about 25 basis points.

In the discussion of Li and Wei (2013), Loewenstein (2013) suggests that the paper should have used the term structure model to estimate the impact of the supply shocks on the entire yield curve. Therefore, I conduct a counterfactual analysis to compare the yield curves with and without a given supply change.

The Treasury supply shock, denoted as $\triangle\ sv_{1t}$, measures the total amount of the Fed purchase of Treasury securities, which is the the current period amount minus the last period amount. To be consistent with the construction method of the Treasury supply factor, $\triangle\ sv_{1t}$ is also measured in terms of ten-year-equivalents, as a percentage of public debt held by the public. The par amount outstanding of Treasury held by the Fed is obtained from the online database of the Fed Bank of St.Louis. The average duration (in years) is measured as the dollar-weighted average maturity of the Fed's Treasury holdings. The detailed construction method is in Golinski (2018), p.7. Duration of the ten-year on-the-run Treasury note and the total public debt held by the public are obtained using the same method as in Section 4.3. The MBS supply shock, denoted as $\triangle\ sv_{2t}$, measures the total par amount of the Fed purchase of MBS securities. To be consistent with the construction method of the MBS par supply factor, $\triangle\ sv_{2t}$ is also measure as a percentage of total public debt held by the public. The Fed's purchase of MBS is the difference between the last period and the current period amount. The par amount outstanding of MBS held by the Fed is obtained from the online database of the Fed Bank of St.Louis. To be consistent with the data in Section 5.3, all the supply shocks are constructed at the end-of-period monthly frequency. Figure 4.9 demonstrates the variation of the supply shock from January 2003 to March 2018. There is a notable increase in the

Fed purchases of Treasury and MBS securities since 2008. Moreover, the amount of the MBS purchased by the Fed is nearly zero during the pre-ZLB period, between 2003 and 2008, and increases considerably during the ZLB period. [3]

I conduct counterfactual analysis to evaluate the impact of the Fed's purchase programs on the whole term structure. To be more specific, I assume that without the Fed's purchases programs, the Fed's purchased securities will be held by the private investor. In other words, the Treasury and the MBS supply shocks will be added to the Treasury supply factor and the MBS par supply factor. Since the supply factor affect the term structure through term premium, I focus on analysing the effect of the Fed's purchases on forwards rates through the portfolio balance channel.

To evaluate the effect of the Fed's purchases of the Treasury and MBS securities separately, I assume that there are three counterfactual cases. The first counterfactual case is denoted as CF1, which assumes that the Fed's has not purchased the Treasury and MBS securities. The Treasury securities held by private investors thus will increase $\triangle \, sv_{1t}$, and the MBS securities held by private investors will increase$\triangle \, sv_{1t}$. The change of the GATSM forward term premium in counterfactual case 1 is:

$$\triangle \, CF1 = b_{1,sv_1}^{GATSM'}(\triangle \, sv_{1t}) + b_{2,sv_2}^{GATSM'}(\triangle \, sv_{2t}) \tag{4.34}$$

where b_{1,sv_1}^{GATSM} and b_{2,sv_2}^{GATSM} are the GATSM estimated factor loadings of the Treasury and MBS par supply factors on the term premium in Section 4.6.5 Table 4.6.

The change of the SRTSM forward term premium in counterfactual case 1 is:

$$\triangle \, CF1 = \sigma_n^Q g(\frac{b_{1,sv_1}^{SRTSM'}(\triangle \, sv_{1t}) + b_{2,sv_2}^{SRTSM'}(\triangle \, sv_{2t})}{\sigma_n^Q}) \tag{4.35}$$

b_{1,sv_1}^{SRTSM} and b_{2,sv_2}^{SRTSM} are the SRTSM estimated factor loadings of the Treasury and MBS par supply factors on the term premium in Section 4.6.5 Table 4.6.

The second counterfactual case is denoted as CF2, which assumes that the Fed's has not purchased the Treasury securities. The Treasury securities held by private investors thus will increase $\triangle \, sv_{1t}$. The change of the GATSM forward term premium in counterfactual case 2 is:

$$\triangle \, CF2 = b_{1,sv_1}^{GATSM'}(\triangle \, sv_{1t}) \tag{4.36}$$

[3] This section aims to evaluate the impact of the Fed purchases of the securities. Therefore, if the supply shock is calculated below zero from the dataset, I treat the results in the stated period as zero.

where b_{1,sv_1}^{GATSM} is the GATSM estimated factor loadings of the Treasury supply factor on the term premium in Section 4.6.5 Table 4.6.

The change of the SRTSM forward term premium in counterfactual case 2 is:

$$\triangle\, CF2 = \sigma_n^Q g(\frac{b_{1,sv_1}^{SRTSM'}(\triangle\, sv_{1t})}{\sigma_n^Q}) \tag{4.37}$$

b_{1,sv_1}^{SRTSM} is the SRTSM estimated factor loadings of the Treasury supply factor on the term premium in Section 4.6.5 Table 4.6.

The third counterfactual case is denoted as CF3, which assumes that the Fed's has not purchased the MBS securities. The MBS securities held by private investors thus will increase $\triangle\, sv_{2t}$. The change of the GATSM forward term premium in counterfactual case 3 is:

$$\triangle\, CF3 = b_{2,sv_2}^{GATSM'}(\triangle\, sv_{2t}) \tag{4.38}$$

where b_{2,sv_2}^{GATSM} is the GATSM estimated factor loadings of the MBS par supply factor on the term premium in Section 4.6.5 Table 4.6.

The change of the SRTSM forward term premium in counterfactual case 2 is:

$$\triangle\, CF3 = \sigma_n^Q g(\frac{b_{2,sv_2}^{SRTSM'}(\triangle\, sv_{2t})}{\sigma_n^Q}) \tag{4.39}$$

b_{2,sv_2}^{SRTSM} is the SRTSM estimated factor loadings of the MBS par supply factor on the term premium in Section 4.6.5 Table 4.6.

In Figure 4.10 and 4.11 I displays the yield curve fit of the counterfactual analysis results $\triangle\, CF1$, $\triangle\, CF2$ and $\triangle\, CF3$. The results are reported in percentages. The left plots refer to the counterfactual changes of the GATSM fitted forward rates. The result shows a considerable decrease of yields without supply shocks, which violates the expected increase of yields without the Fed purchases as the literature suggests. This decrease is mainly caused by the negative sign on the factor loadings of the Treasury supply factor reported in Table 4.6. This negative sign is also consistent with the parameter estimation results of GATSM in Table 4.2, where the lagged Treasury supply factor has significant negative coefficients on yield factors under the Q-measure. It may imply the GATSM is unable to the very flat short-end maturities yields at ZLB period, and parameters related with the Treasury supply factor are distorted in order to fit the yield curve. The $\triangle\, CF2$ plots of GATSM show that yield curves increase without the MBS supply shock, which is consistent with the finding of the literature. However, the impact of the MBS supply shock is rather slight compared with the impact of the Treasury supply shock. The right plots refer to the counterfactual changes of the SRTSM fitted forward rates. The results imply that both the Fed purchases of Treasury and MBS securities reduce the Treasury forward rates. Consistent with the findings of Li and Wei (2013), I find that the Fed purchases have small impact on short maturities (6-month to 2-year in figure 4.10)forward rates, whereas the purchases significantly reduced the mid-term and long-term in Table 4.4.

Table 4.7 and Table 4.8 present the cumulative changes in the term premium during the Fed purchase programs. The term premium is calculated using Eq.4.34 to Eq.4.39. $\triangle CF1$, $\triangle CF2$, and $\triangle CF3$ refer to the changes of the term premium under the counterfactual case 1, 2 and 3. Table 4.7 summarises the results of the cumulative changes in the term premium using SRTSM. The SRTSM result implies that LSAPs lower the mid to long term premium by a large amount, whereas they reduce the short-term term premium by a relatively small amount. The LSAP1 has the largest impact on the long-term term premium compared with LSAP2 and OT. These finding are consistent with the results of Chung et al. (2012), and Ihrig et al. (2018). Moreover, the Fed purchases of MBS securities have larger effects than Treasury securities during the LSAP1 and OT periods, whereas the Fed purchases of Treasury securities have a larger effect than MBS securities during the LSAP2 period. The LSAP1 program reduces the ten-year term premium by 140 basis points; the five-year term premium by 116 basis points; and the 1-year term premium by 30 basis points. The LSAP2 program reduces the ten-year term premium by about 15 basis points; five-year term premium by about 20 basis points; and the 1-year term premium by 3 basis points. The OT program lowered the ten-year term premium by 3 basis points; the five-year term premium by 3 basis points; and the 1-year term premium by about 0 basis points. Table 4.8 summarises the results of the cumulative changes in term premium using the GATSM. The result shows that Fed purchases of Treasury securities increase the forward rates term premium across all maturities; and without the Fed purchases of Treasury and MBS securities, the term premium decreases by a large amount. This result contravenes the literature. This may be caused by the poor fitting of the GATSM during the ZLB period, and the related parameter estimates have been distorted in order to fit the forward yield curves. The Fed purchases of MBS securities decreases the mid and long term premium, which is the same as the literature suggests; however, the effect is relatively small compared with the large impact of the Treasury supply shock.

Table 4.7: SRTSM Cumulative Changes in the Term Premium

Maturity	3-month	6-month	1-year	2-year	3-year	5-year	7-year	10-year
		LSAP1:November 2008-March 2010						
$\triangle$ $CF1(\%)$	2	9	30	66	90	116	129	140
$\triangle$ $CF2(\%)$	1	2	6	11	12	11	9	8
$\triangle$ $CF3(\%)$	1	7	23	55	78	105	120	132
		LSAP2:November 2010-June 2011						
$\triangle$ $CF1(\%)$	0	0	3	17	22	20	18	15
$\triangle$ $CF2(\%)$	0	0	3	17	22	20	18	15
$\triangle$ $CF3(\%)$	0	0	0	0	0	0	0	0
		OT:September 2011-August 2012						
$\triangle$ $CF1(\%)$	0	0	0	0	2	3	3	3
$\triangle$ $CF2(\%)$	0	0	0	0	1	1	1	1
$\triangle$ $CF3(\%)$	0	0	0	0	1	2	2	3

Table 4.7 reports the SRTSM accumulative changes of the term premium during the Fed asset purchase program period. LSAP1, LSAP2 denote the first two purchase programs, and OT denotes operation twist. Since there is a time overlapping of OT (September 2011-December 2012) and LSAP3 (September 2012-October 2014), I denote the OT from September 2011 to August 2012. $\triangle$ $CF1$, $\triangle$ $CF2$ and $\triangle$ $CF3$ denote the cumulative changes of the term premium under the counterfactual case 1, 2 and 3. The cumulative changes are reported in basis points.

Table 4.8: GATSM Cumulative Changes in the Term Premium

Maturity	3-month	6-month	1-year	2-year	3-year	5-year	7-year	10-year
		LSAP1:November 2008-March 2010						
$\triangle$ $CF1$(%)	-4	-8	-16	-29	-39	-53	-61	-68
$\triangle$ $CF2$(%)	-9	-18	-34	-60	-80	-106	-121	-131
$\triangle$ $CF3$(%)	5	10	18	31	41	53	60	63
		LSAP2:November 2010-June 2011						
$\triangle$ $CF1$(%)	-17	-34	-64	-113	-151	-200	-227	-247
$\triangle$ $CF2$(%)	-17	-34	-64	-113	-151	-200	-227	-247
$\triangle$ $CF3$(%)	0	0	0	0	0	0	0	0
		OT:September 2011-August 2012						
$\triangle$ $CF1$(%)	-1	-2	-3	-5	-7	-9	-10	-11
$\triangle$ $CF2$(%)	-1	-2	-3	-6	-8	-10	-12	-13
$\triangle$ $CF3$(%)	0	0	0	1	1	1	1	1

Table 4.8 reports the GATSM accumulative changes of term premium during the Fed asset purchase program period. LSAP1, LSAP2 denote the first two purchase programs, and OT denotes the operation twist. Since there is a time overlapping of OT (September 2011-December 2012) and LSAP3 (September 2012-October 2014), I denote the OT from September 2011 to August 2012. $\triangle$ $CF1$, $\triangle$ $CF2$ and $\triangle$ $CF3$ denote the cumulative changes of the term premium under the counterfactual case 1, 2 and 3. The cumulative changes are reported in basis points.

4.8 Conclusion

This chapter provides a novel application of the shadow term structure model that includes Treasury and MBS supply factors to evaluate the effect of the Fed's recent purchase programs. Overall, my results suggest that the Fed's LSAPs may have contributed to the decline of the interest rate during the ZLB period. The increase of the supply factors in the private sector increases the forward term premium. The counterfactual analysis results demonstrate that LSAPs significantly reduce the mid and long-term maturities forward rates.

This chapter is based on the theoretical work of Vayanos and Vila (2009), and the empirical work of Li and Wei (2003). I highlight the importance of the changes of the Fed's balance sheet on interest rates through the portfolio balance channel. In general, the results of this chapter support the view of the literature that when the Fed funds rate is stuck at zero, the LSAP is an alternative monetary policy that can effectively influence interest rates.

The Shadow rate model uses the 1-month forward rate from December 1996 to March 2008, constructed by the zero-coupon Treasury yields dataset of Gurkaynak et al. (2007). The model is estimated by the extended Kalman filter based on the maximum likelihood function. I also estimate the Gaussian affine term structure model in the same period for comparison. The results highlight that the shadow rate model better performs in fitting the yield curve across the conventional monetary policy period (before the financial crisis in 2008) and the unconventional monetary policy period than the Gaussian model. Moreover, the shadow rate model can draw meaningful inferences about the relationship between the Fed's purchase programs, securities supply factors, and the Treasury yields variations. Specifically, the Gaussian and shadow rate model have almost the same fitting during the conventional monetary policy period. However, during the unconventional monetary policy period, the shadow rate model better fits the flat short-end yield curve. The shadow rate model results display positive signs between the supply factors coefficients and the yields, i.e., increasing the securities supply can increase the yield curve. This inference is consistent with the previous literature, e.g., Vayanos and Vila (2009), and D'Amico et al. (2012). My counterfactual analysis using the shadow rate model show that the Fed's four purchase programs reduce the mid-term and long-term Treasury term premium significantly amount. The LSAP1 has the most substantial effect on the 10-year forward term premium, which decreases the term premium by 140 basis points, with the most impact from the purchase of the mortgage-backed securities.

Figure 4.3: Yield Curve Fit

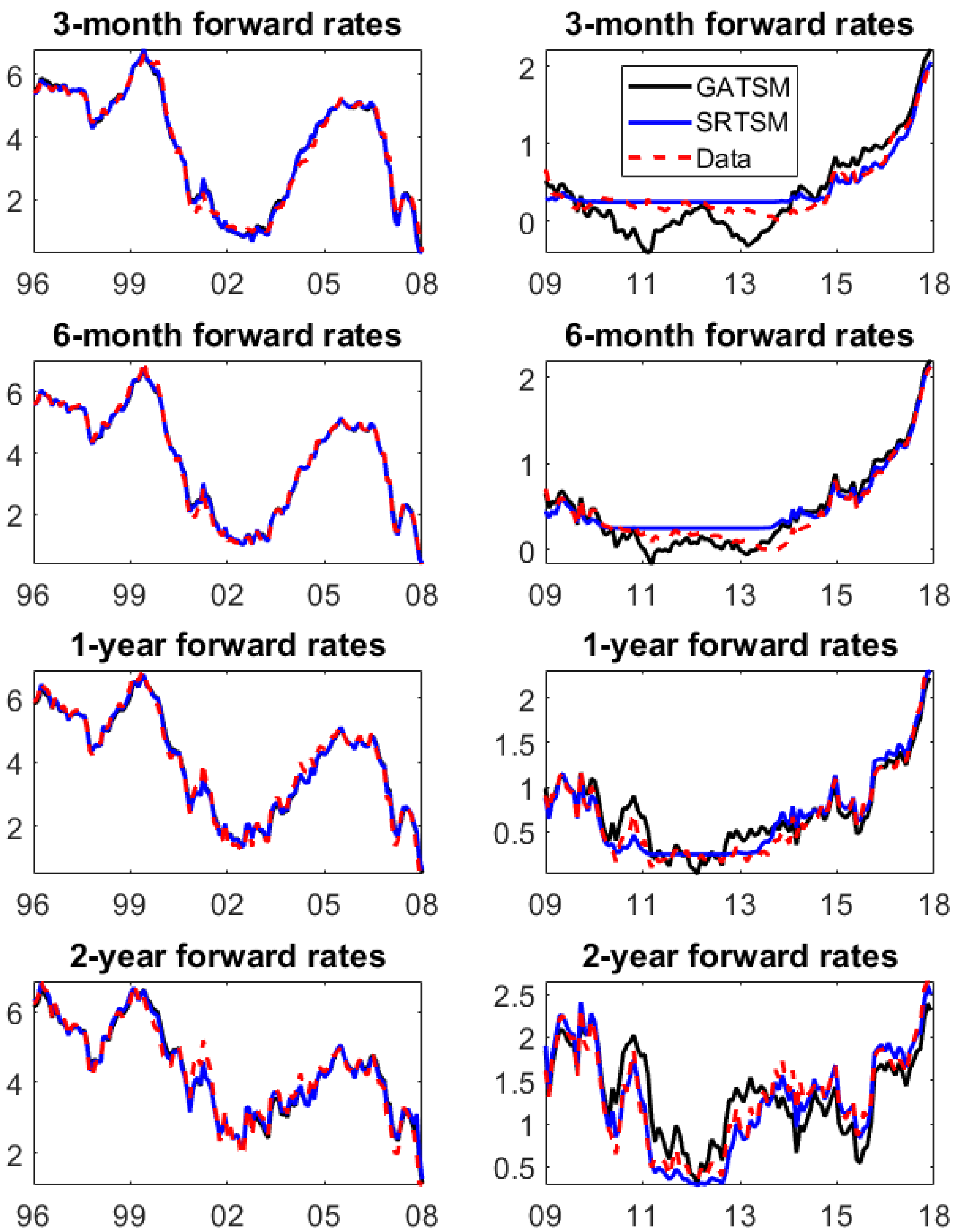


Figure 4.3 reports observed and fitted forward rates with maturities 3-month (upper panel), 6-month (middle top panel), 1-year (middle bottom panel) and 2-year (bottom panel) on the full sample. The left plots refer to the pre-ZLB period (December 1996 to December 2008), and right plots refer to the ZLB period until March 2018. The black solid line denotes GATSM fitted yields, the blue solid line denotes SRTSM fitted yields, and the red dashed line denotes observed yields.

Figure 4.4: Yield Curve Fit

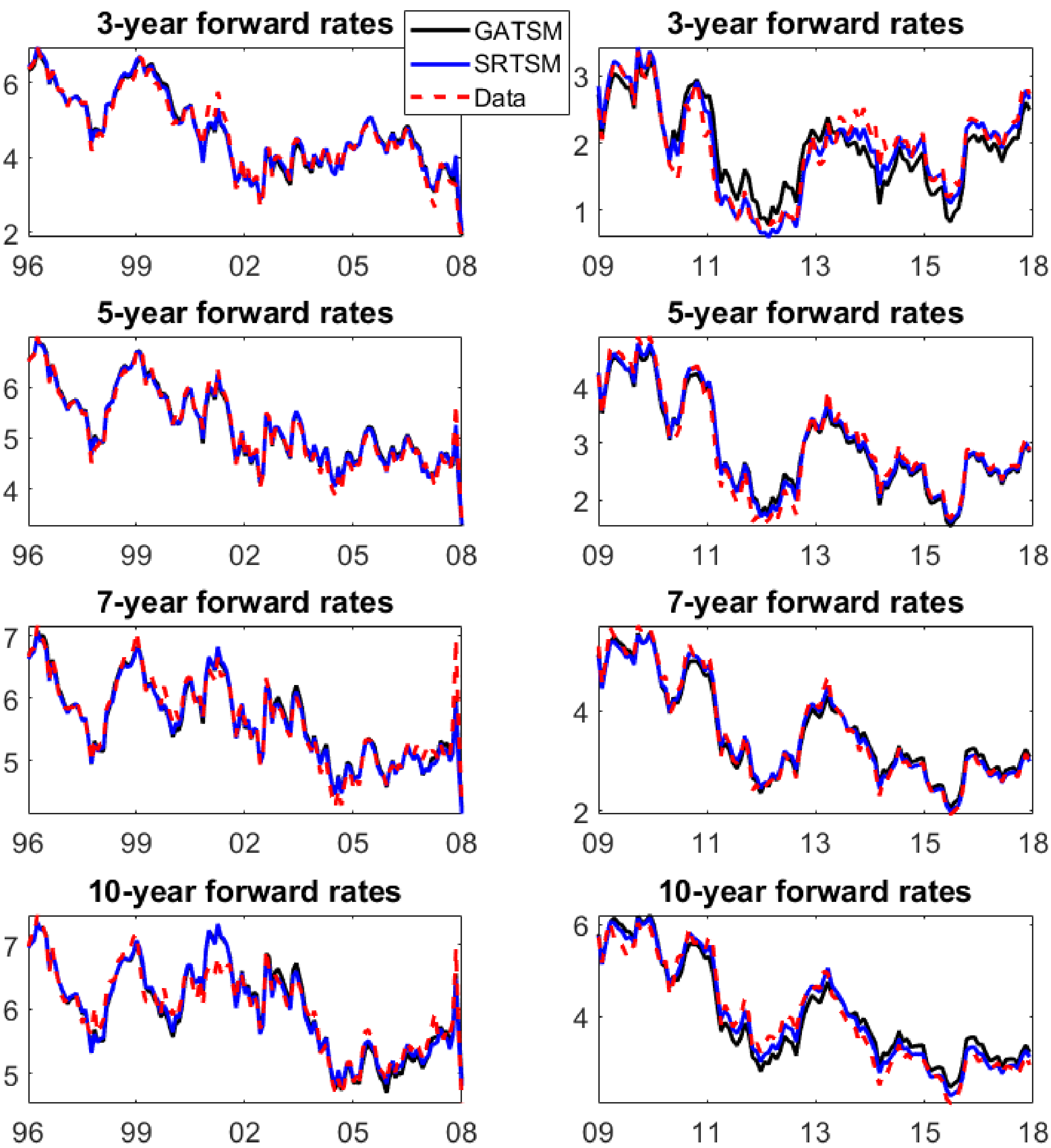


Figure 4.4 reports observed and fitted forward rates with maturities 3-year (upper panel), 5-year (middle top panel), 7-year (middle bottom panel) and 10-year (bottom panel) on the full sample. The left plots refer to the pre-ZLB period (December 1996 to December 2008), and right plots refer to the ZLB period until March 2018. The black solid line denotes GATSM fitted yields, the blue solid line denotes SRTSM fitted yields, and the red dashed line denotes observed yields.

Figure 4.5: Estimated Factor Loadings

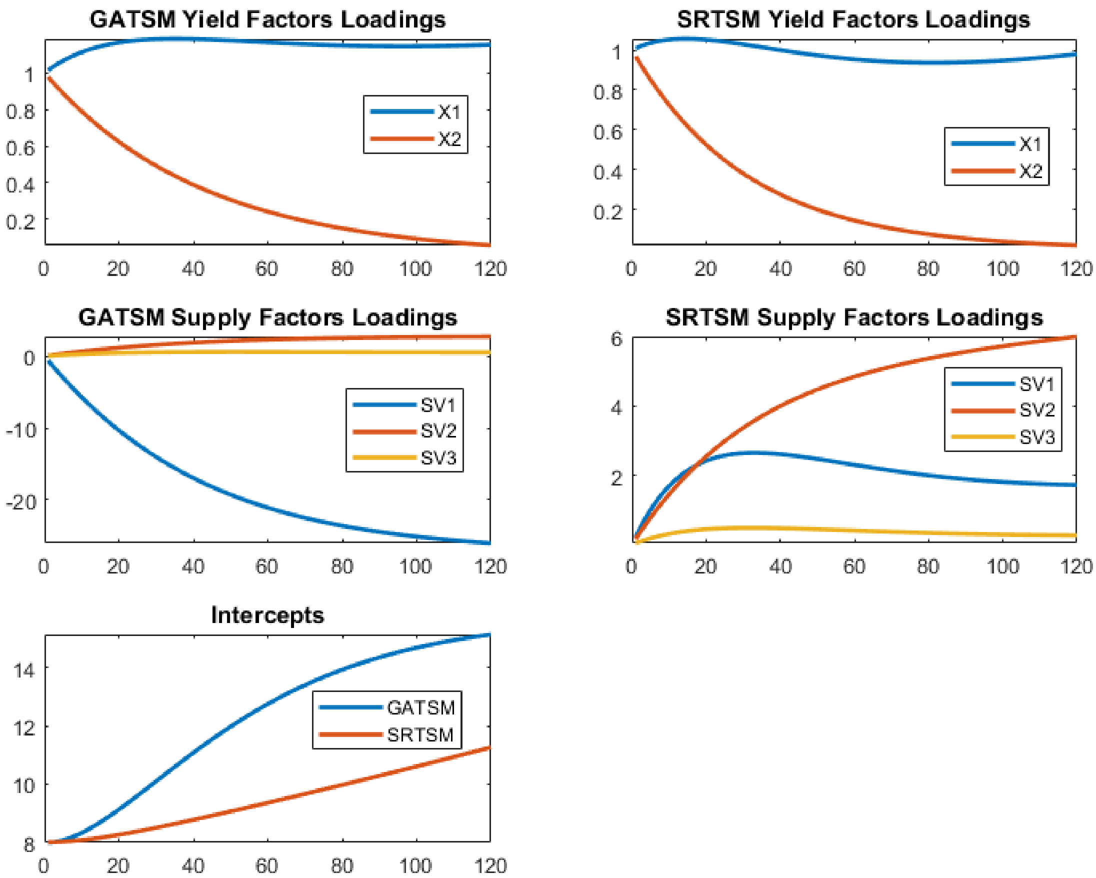


Figure 4.5 reports the estimated GATSM, and SRTSM forward rates factor loadings in Eq.4.2 and Eq.4.8. The x-axis is denoted as the maturity from 0 to 10 years in months. The top panel refers to the factor loadings of latent yield factors in Section 5.2. The blue line denotes the first yield factor, and the orange line denotes the second yield factor. The bottom panel refers to the factor loadings of observed supply factors. The specific description of these factors can be seen from section 4.1. The blue, orange and yellow lines denotes Treasury supply factor, MBS supply factor and MBS duration, respectively. The bottom panel refers to the intercepts. The blue line indicates GATSM intercept, and the orange one indicates SRTSM intercept.

Figure 4.6: Estimated State Variables

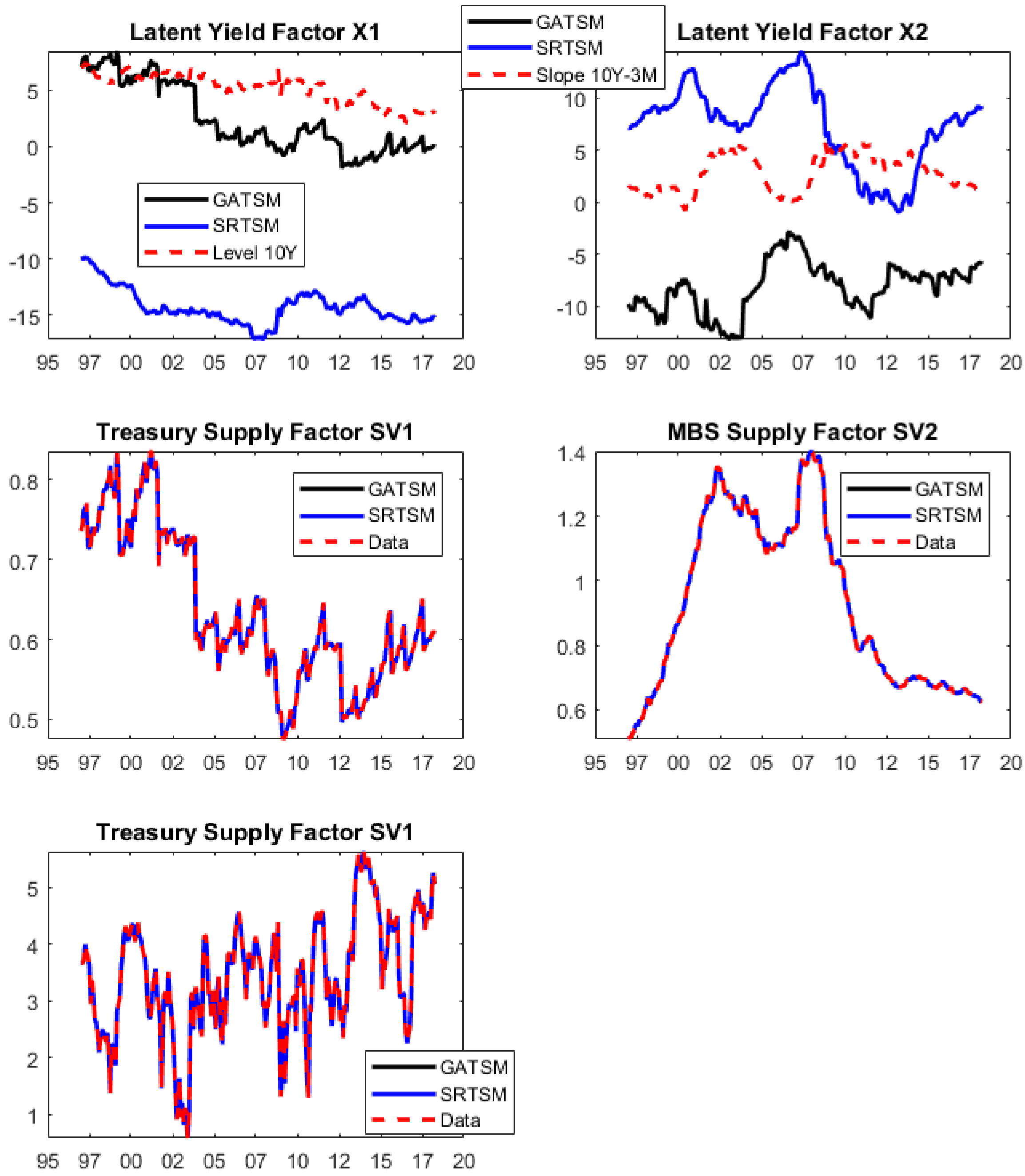


Figure 4.6 reports the estimated latent yield factors $\boldsymbol{x}_t$ (top plot), the estimated securities supply factors $\boldsymbol{sv}_t$ (middle and bottom plots) on the full sample. The black solid line denotes GATSM fitted state variables. The blue solid line denotes SRTSM fitted state variables. The red dashed line in the top panel refers to standard empirical yield curve level and slope measures as stated in Diebold and Rudebusch: the 10-year yield, the 10-year yield minus 6-month yield spread, respectively. The red dashed line in the middle and bottom panels refers to the observed securities supply factors.

Figure 4.7: Risk-Neutral Forward Rate Fit

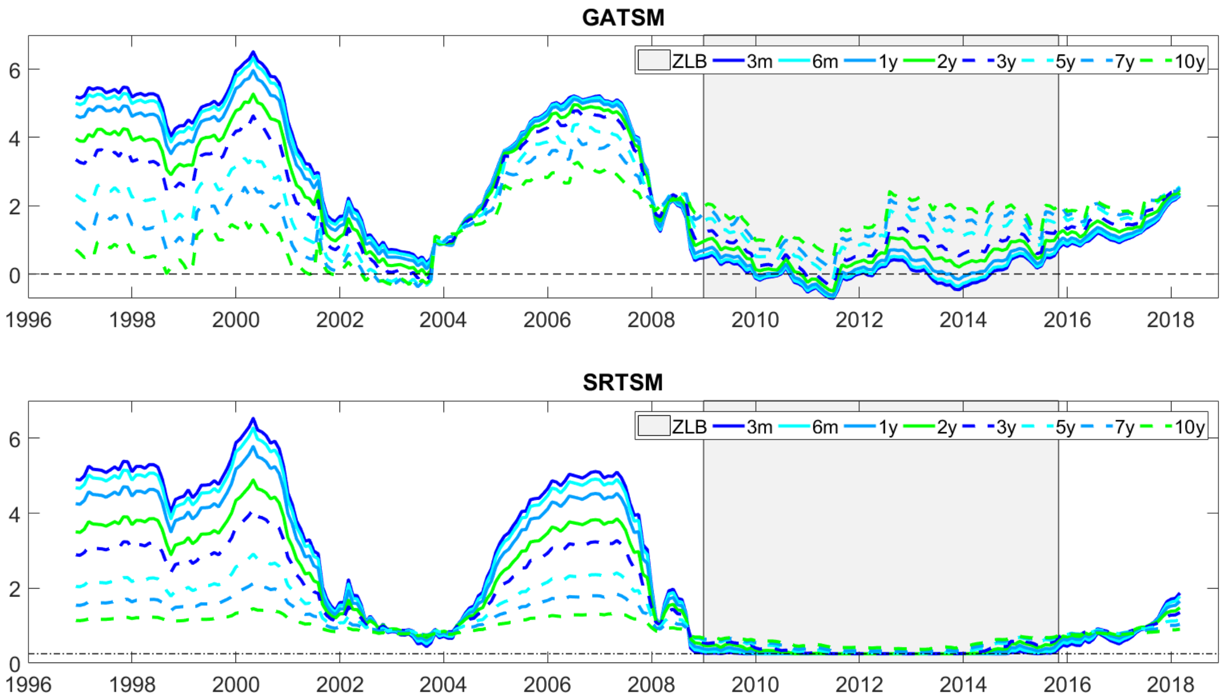


Figure 4.7 reports the estimated GATSM (top panel) and SRTSM (bottom panel) risk-neutral 1-month Treasury forward rates from December 1996 to March 2018 in annualised percentage points. Maturities are 6 months, 1, 2, 3, 5, 7, and 10 years. ZLB periods in grey areas is from January 1990 to December 2015. The black dashed line denotes $y = 0$. The black dash-dotted line denotes $y = 0.25$, which is the lower bound set by the Fed during the ZLB period.

Figure 4.8: Term Premium Fit

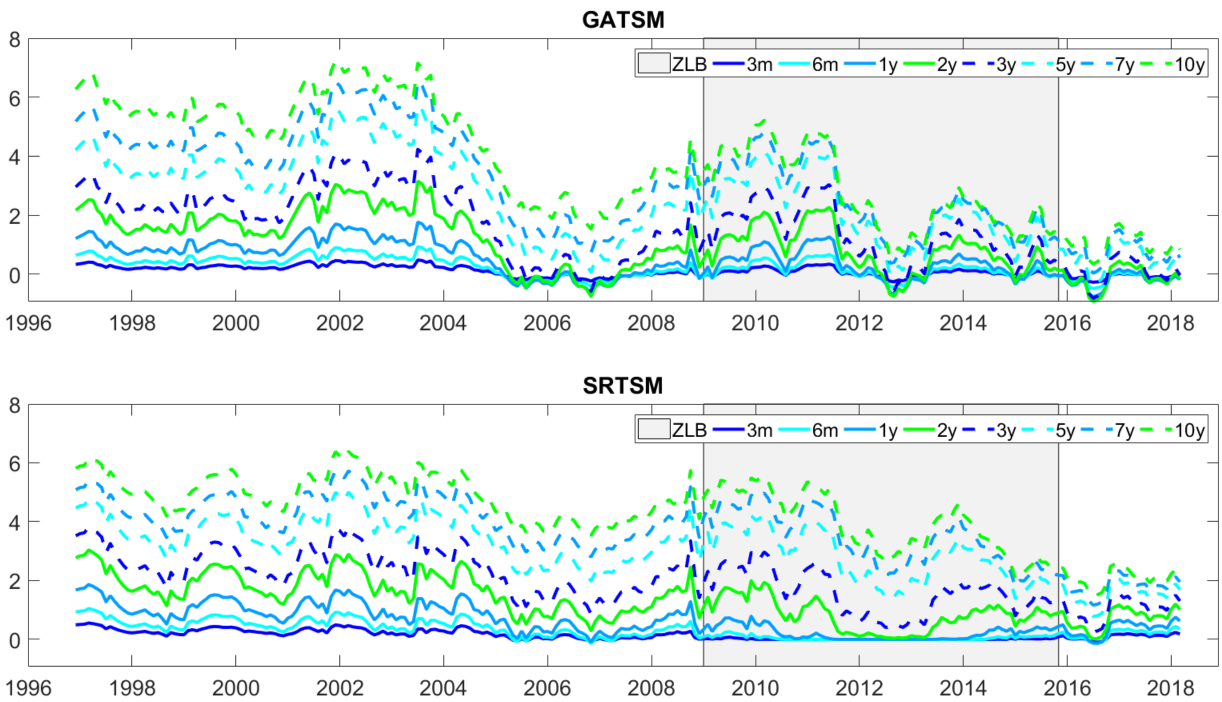


Figure 4.8 reports the estimated GATSM (top panel) and SRTSM (bottom panel) risk premium of 1-month Treasury forward rates from December 1996 to March 2018 in annualised percentage points. Maturities are 6 months, 1, 2, 3, 5, 7, and 10 years. The ZLB period in grey is from January 1990 to December 2015.

Figure 4.9: Supply Shock

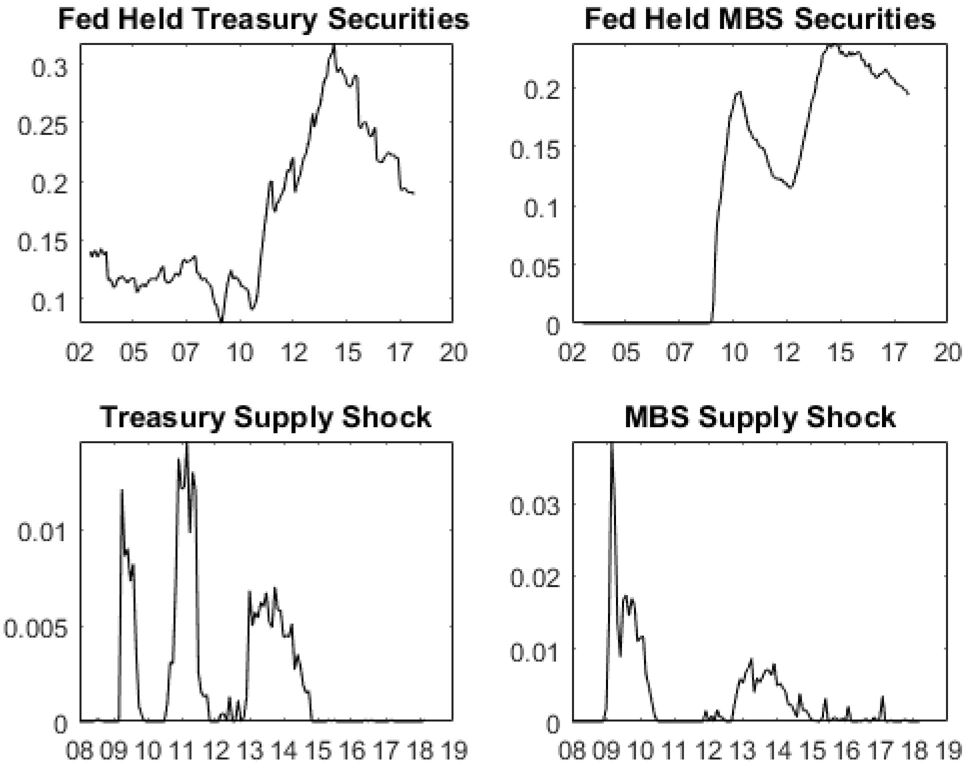


Figure 4.9 top panel reports times series of the total amount of the Fed held Treasury securities (measured in term of ten-year-equivalents, as a percentage of public debt held by the public); the par amount of the Fed held agency MBS (as a percentage of public debt held by the public). Sample spans from January 2003 to March 2018. The bottom panel reports the Treasury supply shock, which is the total amount of the Fed purchase Treasury securities (measured in term of ten-year-equivalents, as a percentage of public debt held by the public), and the MBS supply shock, which is the Fed purchase of MBS (as a percentage of public debt held by the public) from January 2008 to March 2018. To focus on evaluating the impact of the Fed purchases, when the supply shock constructed from the dataset is below zero, I treat the number as zero in the stated period.

Figure 4.10: Counterfactual Yield Curve Fit

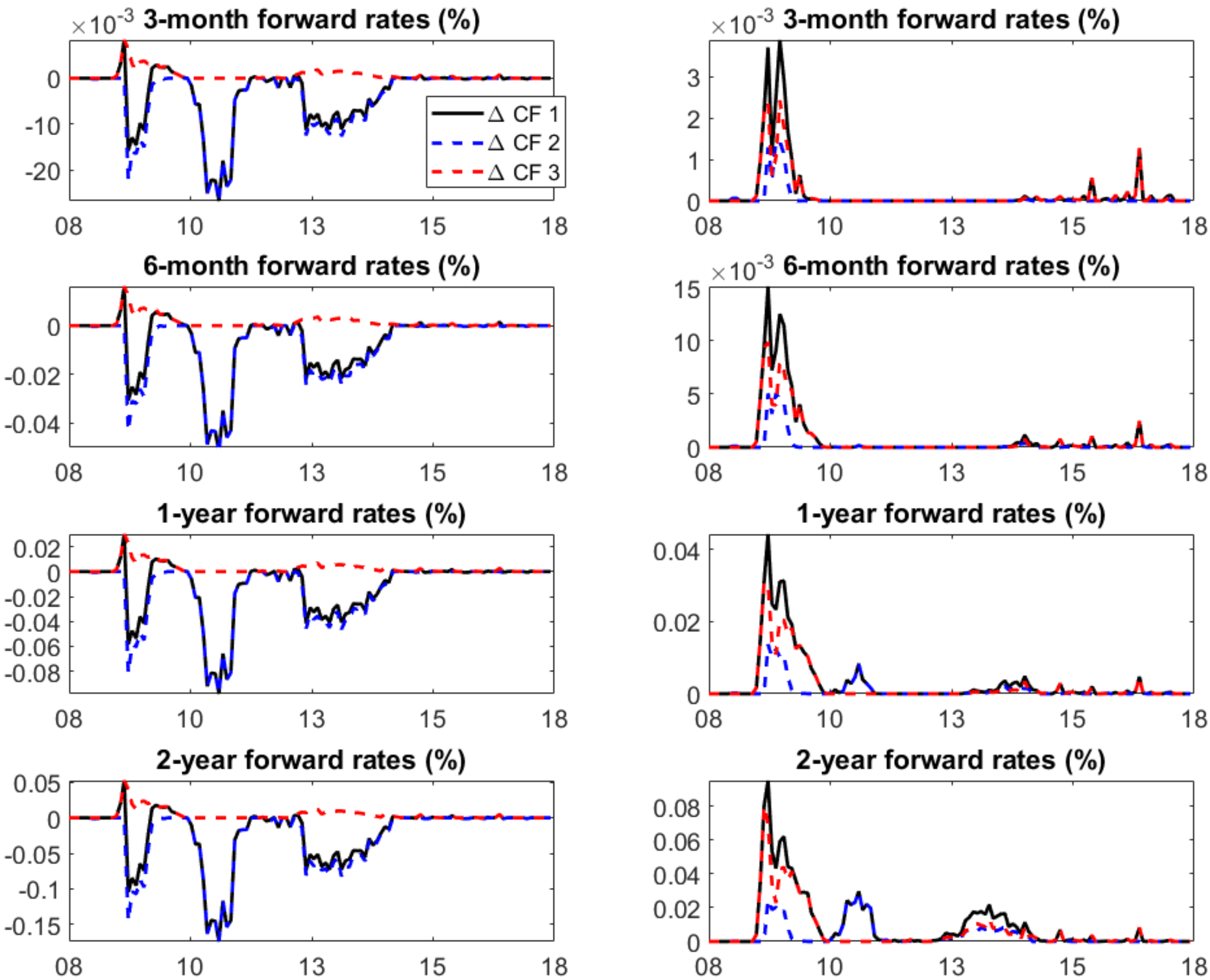


Figure 4.10 reports the counterfactual changes of the GATSM and the SRTSM fitted forward rates with maturities 3-month, 6-month, 1-year and 2-year during the ZLB period. The left plots refer to the counterfactual difference with fitted GATSM forward rates, and right plots refer to the counterfactual difference with fitted SRTSM forward rates. $\triangle CF1$ (black solid line) denotes the changes of the counterfactual case 1. $\triangle CF2$ (blue dashed line) indicates the changes of the counterfactual case 2. $\triangle CF3$ (solid red line) denotes changes of the counterfactual case 3. The results are reported in percentages.

Figure 4.11: Counterfactual Yield Curve Fit

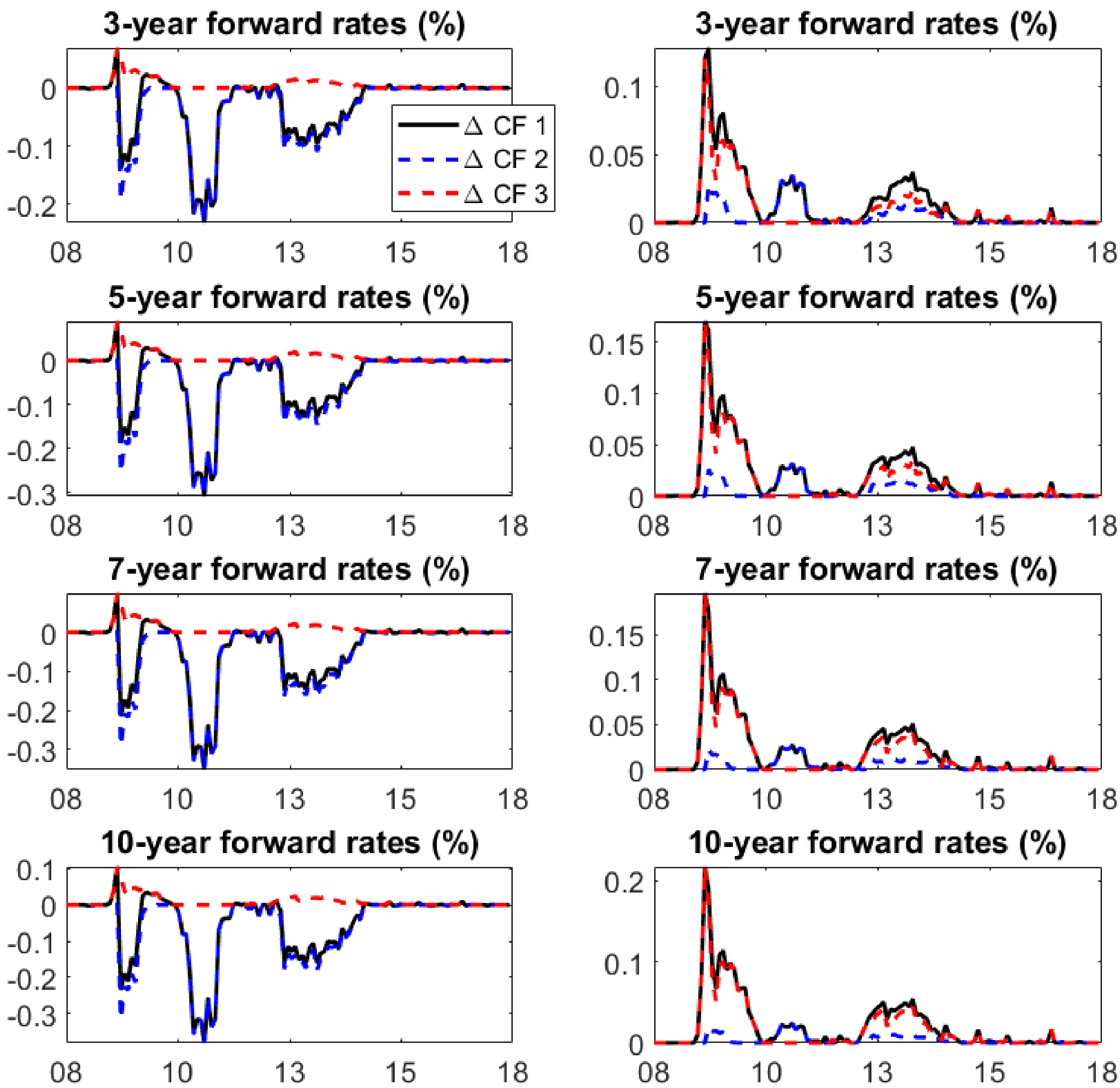


Figure 4.11 reports the counterfactual changes of the GATSM and the SRTSM fitted forward rates with maturities 3, 5, 7 and 10-years during the ZLB period. The left plots refer to the counterfactual difference with fitted GATSM forward rates, and right plots refer to the counterfactual difference with fitted SRTSM forward rates. $\triangle CF1$ (black solid line) denotes the changes of the counterfactual case 1. $\triangle CF2$ (blue dashed line) indicates the changes of the counterfactual case 2. $\triangle CF3$ (red dashed line) denotes changes of the counterfactual case 3. The results are reported in percentages.

Chapter 5

Conclusions

This thesis investigates the term structure of Treasury and corporate yields; the interaction between these yield curves; and the real economy across conventional and unconventional monetary policy.

The aim of Chapter 2 is to analyse the linkage among government yields, credit spreads, macro fundamentals, and the credit factor. This chapter presents a Gaussian joint term structure model of government yields and credit spreads, which extends the discussion of separate term structure models of government yields and credit spreads to a joint framework with observed only variables. In this chapter, I first conduct a preliminary analysis to evaluate how much variation of each principal component can be captured by the observed variables. Then I estimate the joint framework with the maximum likelihood method, and further evaluate the impact of state variables with yield curve decomposition. The fit of the model is reasonably good for government yields. Result indicate that the inflation forecast positively and largely affect government yields through both expected future short rate and risk premium components. The output gap negatively affects credit spreads through the risk premium component; whereas the credit factor positively affects credit spreads through the risk premium component.

I present a joint shadow rate term structure model of Treasury and corporate forward rates in Chapter 3, which evaluates the performance of the SRTSM in fitting the yield curve during the zero lower bound period. I first identify the model specification with a preliminary analysis and determine that a Treasury specific latent factor, and a corporate specific latent factor is needed to capture the variation of the forward rate curve. Then I estimate the shadow rate model with the extended Kalman filter. Finally, I compare the observed credit spreads with the model-implied ones. Results indicate that the SRTSM fits the long-term corporate yields better than the GATSM both in-sample and out-of-sample.

Most shadow rate models focus on the general impact of the interest rate lower bound on the yield curve. In contrast, Chapter 4 contributes to investigates the effect of the Fed purchase programs on Treasury forward rates under a shadow rate term structure framework with observed supply factors. I first estimate the shadow rate model with two latent yield factors and three observed supply factors. Then I decompose the yield curve to examine the effect of supply factors on the term premium. I also assess the effect of securities supply shocks on Treasury forward rates with a counterfactual analysis. Results imply that the Fed purchase programs significantly affect the mid-term and long-term Treasury forward rate curve through the portfolio balance channel. The first purchase program has the most substantial effect on the 10-year term premium, which decreases the term premium by 140 basis points.

There are some directions for future research. In Chapter 2, the current work is limited to the period before the financial crisis in 2008. Future work can be conducted by adapting the shadow rate term structure model and evaluate the interaction of government yields, credit spreads and observed state variables during the lower bound period. Kick (2017), for example, extend the work of Ang and Ulrich (2012) by pricing the government bond and equity with shadow rate model based.

Chapter 3 and Chapter 4 discuss the behaviour of bond yields during the lower bound period. Given the current inavailability of the aggregate corporate yield curve data, Chapter 3 evaluates the performance of the shadow rate term structure model on 6-month corporate forward rates. A plausible extension is to evaluate the performance of the SRTSM in fitting the corporate forward rates with different ratings, and on shorter 1-month forward rates. For Chapter 4, the influence of LSAPs on riskier interest rates concerns researchers; further work can evaluate how LSAPs affect corporate yields.

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