Domain-General Precursors of Children's Mathematics Skills: The Role of Working Memory and Language

Andrea Díaz-Barriga Yáñez

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Department of Psychology

The University of Sheffield

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Abstract

The goal of this thesis was to investigate the domain-general predictors that underpin mathematics skills in early childhood. Thus, this thesis examined the role of working memory and language. In particular, the empirical work of this thesis focused on comparing how children do mathematics in a relatively pure context on the one hand, and in a more applied context on the other. Three studies were designed for this purpose. The first study investigated the contributions of working memory components to both arithmetic skills and applied mathematics in 5-to 6-year-olds. Study 1 showed, as expected, that the central executive plays an important role both in arithmetic skills and in applied mathematics. However, there was also an unexpected and somewhat surprising finding that also showed that both kinds of mathematics performance were significantly predicted by children's receptive vocabulary. Thus, the second study was designed to further investigate the specific contributions of receptive language skills in children's mathematics skills. Results from Study 2 were unexpected since the results obtained in Study 1 were not replicated. However, this was likely to be because the language measures had unexpectedly high executive functions components, and as such the language measures themselves were not transparent. Finally, the third study, although it did not followed directly from Study 1 and 2, it followed the subject of the role of language skills in pure and applied mathematics. Study 3 investigated the role of longitudinal linguistic precursors and concurrent language and executive functions in 4-year-old's mathematics skills. Results suggested that concurrent language skills were significant to both mathematics performance, and that inhibitory control was also contributing to applied mathematics. Collectively, these findings improve our understanding of the domain-general contributions underpinning mathematics in both pure and applied contexts.

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Declaration

I hereby declare that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis. All experiments were designed and conducted by myself under the supervision of Dr Dan Carroll and Dr Danielle Matthews.

Chapter Four was conducted in collaboration with Dr Danielle Matthews and Dr Michelle McGillion. Dr McGillion conducted the data collection for Chapter Four when children were between 11 months and 48 months, and data collection for the language measures when children were 4 years of age. This information is also provided in the respective chapter.

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Chapter One

The Development of Mathematics Skills

1.1 Introduction

Mathematics is an umbrella term comprising a broad set of skills that, putting it in a very general way, allow us to perceive and manipulate quantities. Mathematics is understood as the science dealing with quantitative calculations and spatial forms, which utilizes logic and symbolic notation as an approach (Khait, 2005). It can be broadly divided into arithmetic, geometry, logic, and algebra. When approached and studied as a science, mathematics are called "*pure mathematics*", when used as a tool in other sciences or in practical problems, it is called "*applied mathematics*". As such, mathematics provide us with the means and tools to understand and improve our world.

Even the best mathematician had to start in the same place as the rest of us, by learning how to count to ten. However, even before learning how to express ourselves verbally, we develop the sensory perception that allows us to think about and process numerical quantities. For example, new-borns show that they can respond to change in numerosities by looking longer at a new group of objects (e.g. 16 dots) after being repeatedly shown the same number of objects (e.g. eight dots). These building blocks are also referred as informal mathematics skills. Mathematics, then, can be divided into informal and formal mathematics. Informal mathematics are lower level skills such as the ability to process numerical magnitudes and are acquired without the necessity of formal instructions. Formal mathematics are the mathematics skills acquired thanks to formal instructions, like learning how to solve an addition; thus, these refer to the mathematics skills that we learn in school.

Understanding the development of mathematics is important because having proficiency in mathematics provides a solid foundation for future learning and for academic and career outcomes (Lubinski & Benbow, 2006; Passolunghi, Lanfranchi, Altoè, & Sollazzo, 2015). This is shown by the significant statistical prediction that early mathematics skills have to later school success (Duncan et al., 2007; Passolunghi et al., 2015). The contribution of mathematics to later life outcomes is important to acknowledge because underachievement in mathematics is a common issue world-wide. For example, the international average for mathematics skills according to the Programme for International Student Assessment (PISA) test in 2015 was 490/1000 in a scale that goes from 0 to 1000; indicating that 36% of the countries that participate in this assessment have their 15-year-olds' students failing to apply their mathematics knowledge to real-life situations (OECD; 2018). Specifically in the UK, results from the mathematics scores in the General Certificate of Secondary Education's (GCSE), which determine whether a student is eligible to be accepted at University showed that in 2017 only 32.7% of the students had the required scores (Grade C as minimum) to be able to access these opportunities (Joint Council for Qualifications, 2017).

Mathematics are also important in life situations that are unrelated to academic achievement. A good level of mathematics increases the chance to understand and use information to make decisions about health, like understanding food or medication labels (Council & Education, 2016;i.e. health literacy; National Numeracy, 2016a; Word Health Organization, 2016); and finance (National Numeracy, 2016b). Even when baking a cake or paying the bills we use mathematics to calculate quantities. Taking all this into consideration, the identification of the cognitive mechanisms that underpin mathematics skills is an important issue to address.

Moreover, if we are to target improvement of these skills, it becomes crucial to understand how mathematics developed in early stages of life.

Research conducted thus far has stressed three main factors involved in mathematics development: (i) domain-specific abilities, which refers to early quantitative abilities considered to be important for understanding and acquiring later mathematics skills; (ii) domain-general abilities, which are non-mathematics cognitive skills that are relevant in a broader context of learning (Vanbinst & De Smedt, 2016) ; and (iii) environmental factors, which refers to the influences from the social and physical world in which the child develops.

This chapter provides an overview of the research on these three topics across childhood. I will start by presenting research regarding the development of informal mathematics skills (1.2. Children's Informal Mathematics Skills), since these are the domain-specific abilities that support the development of formal mathematics skills; which is the second topic addressed in this chapter (1.3 Children's Formal Mathematics Skills). Then I will address the role of two relevant domain-general precursors of mathematics: executive functions and language skills (1.4. Domain-General Precursors of Children's Formal Mathematics Skills). The fourth topic will be an overview of the environmental factors known to influence mathematics development (1.5. Environmental Factors). Finally, I will end this chapter by providing a summary, the next questions to be addressed, and an overview of the current thesis.

1.2 Children's Informal Mathematics Skills

Informal mathematics are the mathematics skills that we develop without the need of formal instruction. These early mathematics skills are important for mathematics development because they serve as the foundation for children's later mathematics skills, as such, they become the building blocks also known as "domain-specific abilities" for higher order mathematics domains (Geary, 2011; Geary, Nicholas, Li, & Sun, 2017). The following section will describe the development of informal mathematics, that although will be presented in a linear way, their emergence through development overlap; and rather than replacing one another, they become mapped onto each other.

1.2.1 Number sense

The first domain-specific ability to emerge during the development of mathematics is the sensory perception that allows us to think about and process numerical quantities. Observed as early as 49 hours after birth (Izard, Sann, Spelke, & Streri, 2009), this ability is commonly referred as "number sense". Number sense can refer to two different ways of representing numerical quantities. One is by representing quantities without the use of numerals, known as "non-symbolic number sense"; this ability is nonverbal, and allows us to perceive quantitates without counting (e.g. choosing the queue with fewer people when we want to pay our groceries at the supermarket). The other way of representing quantities is by using numbers and number-words, called "symbolic number sense" (Tosto et al., 2017). Both the non-symbolic and symbolic number sense are essential precursors for the emergence of later exact number skills that allow the use of numbers in a more precise way (Dehaene, 1997; Tosto et al., 2017).

1.2.1.1 Cognitive systems for non-symbolic number sense

The non-symbolic number sense comprises two cognitive skills (i) the Approximate Number System (ANS) and (ii) subitizing. The ANS allows us to discriminate quantities in an imprecise manner (e.g. choosing the larger of two piles of rice); and subitizing allows us to quickly and exactly enumerate small quantities without counting (e.g. determining that there are three cookies on a plate without needing to count them one by one). Both systems are present from childhood to adulthood (Ansari, 2016). The ANS can be observed in humans and in many other species (Dehaene, 1997); conversely, subitizing is an ability found only in humans (Dehaene, 1997; Feigenson, Dehaene, & Spelke, 2004).

The ANS

The ANS is the system that allows us to represent and process non-symbolic numerosities (Gilmore et al., 2013). ANS competency, also knowns as ANS acuity, is frequently measured with a magnitude comparison task in which the level of discriminability between sets is assessed. For example, two sets of dots are presented to a child, and the child need to decide which set of dots has more dots. Moreover, different format of stimuli can be used: dots, numbers of short sounds (e.g. beeps), or different objects (De Smedt, Noël, Gilmore, & Ansari, 2013). As a first glance this task may seem quite easy, however, the difficulty of the task can be manipulated by modifying the ratio of the two groups of dots. For example, for adult humans it is easier to distinguish between 10 and 3 dots, than it is to distinguish between 9 and 7 dots. Thus, the relative difference between two quantities, rather than their absolute difference, is what determines its difficulty (De Smedt et al., 2013; Dehaene, 1997; Xu & Spelke, 2000).

The precision to discriminate numerical quantities increases with age. In terms of age-related differences, children at birth can discriminate numerical quantities with a 1:3 ratio (e.g. 4 vs 12; Izard et al., 2009); whereas 4-year-olds are able to discriminate numerical quantities with a ratio of 1:2 (e.g. 6 vs 12; Xu & Spelke, 2000). Moreover, children around the age of 9 years are able to discriminate numerical quantities with 2:3 ratios (e.g. 8 vs 12; Xu & Arriaga, 2007). The acuity of the ANS continues to improve across the lifespan until adulthood; typically, adults can distinguish between quantities with a ratio between 7:8 and 9:10 (Barth, Kanwisher, & Spelke, 2003; Justin Halberda & Feigenson, 2008). After this period, the ANS acuity gradually decline (Halberda, Ly, Wilmer, Naiman, & Germine, 2012).

Individual differences in numerical discrimination acuity can be found early in life, and seem to be stable over time. These individual differences can be found as early as 6 months of age and appear to be stable at least within the first 9 months of life (Libertus & Brannon, 2010). This was found in one study in which 80 6-month-old children completed a numerical change detection task. In this task two sets of images containing different groups of dots were presented simultaneously. There were two types of sets: one was the "numerical changing" set, in which images contained two different quantities of dots, and the other one was the "non-changing" set, in which both images had the same number of dots. Both types of sets were shown alternately for 500 ms followed by 300 ms of a blank screen. Across four trials, 14 out of 16 children successfully discriminated dots with a 1:4 ratio, but as the ratio decreased to 2:3 only seven out of 16 children were successful on the task. Significant differences between children who chose the numerically changing image set and children who did not, were observed to be within a discrimination range between a 1:2 and a 2:3 ratio (Libertus & Brannon, 2010). The same paradigm was used again when children were 9 months of age, and it was observed that children who spent longer time looking at the numerical changing set at 6 months were more likely to spent longer time looking at the numerical changing set at 9 months, demonstrating that children who react to a change of numerosities early in life, remain to do so months later.

Subitizing

Subitizing is the cognitive skill that allows us to quickly and exactly enumerate small quantities without counting. This skill is present from around the age of 2 years. At this age children are able to discriminate sets of up to three items (Starkey & Cooper, 1995). For example, one study found that 12 2-year-olds were able to quickly distinguish between one-, two-, and three-item displays, even when they varied in density, spatial configuration, and object composition (Starkey & Cooper, 1995). In this study subitizing abilities were measured with 15 different number comparison problems. In these problems, the experimenter showed to the children an array of blue chips (with numerosities between one and four) and then displayed an array of white light dots that varied in density and spatial configuration, on a screen. Children needed to state if the white light dots were the same or different from the blue chips. Ten additional problems were presented to the child, only this time, the white dots were presented faster to avoid children from counting. Because 2-year-olds were unlikely to use verbal counting to enumerate the items, it was concluded that subitizing was present in young infants as old as 2 years of age, and that it was independent of verbal counting abilities (Starkey & Cooper, 1995).

Subitizing is an important skill, both in its own right, and because it provides the direct foundation for the emergence of explicit number understanding. For example, subitizing is vital for the development of children's ability to count. Counting is the knowledge of the number words sequence and although subitizing and verbal counting are different skills, both skills develop in parallel (Starkey & Cooper, 1995). In the same study conducted by Starkey and Cooper (1995), 5-year-olds followed the same experimental paradigm, only this time children needed to mention how many dots they saw on the screen. Children also completed a number identification task in which different items were presented and children needed to mention the absolute number. Children were able to successfully subitized and encode up to five items, however, they committed errors in the problems with six or seven items. Moreover, when the large arrays were displayed longer (two seconds), children used verbal counting, which did not happen when the shorter arrays were display for a long time as well. These findings suggest that subitizing and counting skills develop in parallel and that children use them alternatively to enumerate objects.

1.2.1.2 Symbolic number sense

Together, the ANS and subitizing support the emergence of the symbolic number sense. The symbolic number sense emerges when children are able to link their internal non-symbolic representation of numerosities to its corresponding symbol and their phonological representation (number words; Ansari, 2016; Verguts & Fias, 2008) – that is, children's understanding that three dots can be represented with the number "3", or the word "three". This process is important in children's mathematics development because it allows them to perform a range of important numerical operations, such as identifying the smallest or largest number in a set, ordering numbers, and even solving addition, subtraction and multiplication problems (Mundy & Gilmore, 2009).

Symbolic number sense refers to the sensory perception that allows us to represent quantities by using numbers and number-words. Symbolic sense emerges around the age of 3 years (Benoit, Lehalle, Molina, Tijus, & Jouen, 2013) when children understand that numerical quantities can be represented with numbers and number words (Mundy & Gilmore, 2009). Understanding the role of numbers as quantities is the most essential and the most basic component of mathematics knowledge. This understanding develops in three main stages. First, children learn that the number word represents a specific quantity (e.g. “one” equals “•”, “two” equals “• •”, etc.). Three-year-olds are able to successfully relate number words to their corresponding magnitudes only in numbers smaller than three. The second stage in the development of the symbolic number sense is around the age of 4 when children learn the digits that represent both quantities and number words (e.g. “1” is the same as “•” and “one”; Benoit et al., 2013). Finally, mastering the relation between digits, quantities, and number words, is achieved around the age of 5 or 6 (Benoit et al., 2013; Mundy & Gilmore, 2009; Mussolin, Nys, Content, & Leybaert, 2014). As such, the symbolic system does not replace the non-symbolic system, but rather, these systems become mapped onto each other (Siegler, 2016).

The symbolic number sense fosters other basic numerical skills, such as counting and the ability to represent numbers in the form of a mental number line (Passolunghi & Lanfranchi, 2012; Passolunghi et al., 2015) as it allows children to represent numbers and magnitudes in a specific order. Moreover, symbolic number sense also allows children to infer positional relationships between numbers (e.g. $2<4<8$; Le Corre & Carey, 2007). Representing numbers in order and understanding their positional relationship is the cognitive basis of counting (Mussolin et al., 2014).

1.2.2 Counting

Counting refers to knowing the number sequence by rote. Counting is a crucial aspect of early mathematics development, since it has been suggested that it reflects an explicit understanding of numbers (Geary, 2011). Acquiring counting skills is a long process that starts around the age of 2 years and finishes around the age of 6 years. Although most 2-year-olds would be able to understand that number words used in counting refer to quantities, most of them would not be able to map the number words to their corresponding magnitudes (Wynn, 1992). For example, 2-year-olds could use the number word "*four*" just to refer to any quantities that are more than one. Thus, children become successful counters only when they truly understanding the meaning of the number words (Mussolin, Nys, Leybaert, & Content, 2016).

In order for children to understand the meaning of number words, children need to learn the principles of counting. These principles are known as the conceptual principles (Gelman & Gallistel, 1978) and their comprehension lead children to be able to enumerate any object in any direction (Jordan, Glutting, & Ramineni, 2010). There are three main conceptual principles of counting: (i) the stable-order principle (also known as the ordinality principle), (ii) the one-to-one correspondence principle, and (iii) the cardinality principle.

> The stable-order or ordinality principle, is children's understanding that when counting items in a set, each item has a position in relation to the other items (i.e. first, second, third, etc; Bartelet, Vaessen, Blomert, & Ansari, 2014; Gelman & Gallistel, 1978). Thus, it is the knowledge of the order of the numbers. Most 3-to-4-year-olds would understand the concept of ordinality, being able to count from one to ten in the correct order (Bermejo, 1996).
>
> The second principle is the one-to-one correspondence principle. This principle is characterized by the ability to count and point to objects simultaneously; thus, it refers to the understanding that number words are used to tag one, and only one, of the items to be counted. This stage of counting is reached between the ages of 4

and 5 years (Gelman & Gallistel, 1978; Le Corre, Van de Walle, Brannon, & Carey, 2006).

Finally, the third principle, the cardinality principle, is children's understanding that when they are counting a set, the last number word is the total number of objects on the sequence, thus, it is the answer to 'how many?' (Bartelet et al., 2014; Gelman & Gallistel, 1978; Lyons, Vogel, & Ansari, 2016) This principle is reached by the age of 5 years. By 6 years of age, children are able to successfully count from any given number (Kyttälä, Aunio, Lehto, Van Luit, & Hautamäki, 2003).

Acquiring counting skills allows children to order and compare numbers. Ordering and comparing numbers are abilities that are considered fundamental for numerical processing and more complex mathematics skills, such as mental arithmetic (performing calculations without the use of any instruments; Lyons & Ansari, 2015). It has been observed that the way children order and compare numbers is by representing the numbers spatially on a mental number line (Morsanyi, Mahony, & Mccormack, 2016). In western societies the representation of numbers on the number line is organized horizontally and in ascending order from left to right, associating small numbers with the left-hand side and large numbers with the right-hand side (Dehaene, 1997). This spatial representation of numbers has been shown to be a significant predictor of children's understanding of the base-10 place value structure of numbers (ones, tens, hundreds, thousands, etc; e.g. in "123", number "1" belongs to the hundreds, number "2" to the tens, and number "3" to the ones; Zuber, Pixner, Moeller, & Nuerk, 2009). One way of measuring these abilities is using a number line estimation task. This procedure is explained next.

1.2.3 Spatial representation of numbers: The number line estimation

Being able to quickly judge the value of one number relative to another number is a vital part of young children's mathematics skills. This ability is thought to come about through the use of a mental number line– that is, the ability to represent the numbers in order, spatially. This ability is measured with the number line estimation task that consists of a blank number line in which children need to indicate the position of a target number. This task is often used to measure children's ability to represent spatially the magnitudes of numbers. The ability of placing a specific number in a number line starts developing around 3 to 4 years of age. Children's initial mental representation of magnitudes is logarithmic, that is, smaller numbers are spaced farther apart than they should be, and larger numbers are spaced closer together (Dehaene, 1997). In order for children to be successful in placing numbers on a line they need to use their knowledge of the relationship between numbers and their corresponding magnitudes, thus, it is an ability that it is supported by the ANS (Fuchs et al., 2010). Additionally, children also need to use their knowledge of the ordinality principle (Von Aster & Shalev, 2007).

1.3 Children's Formal Mathematics Skills

Formal mathematics are the mathematics skills that are learn through formal instructions, thus, they are the mathematics skills that are acquired in school. The transition from informal to formal mathematics is an important stage in children's mathematics development because it is when children start connecting their previous mathematics knowledge to new information learned in school. This process typically occurs during the first years of schooling, when children are between the age of 5 and 7 years (Purpura, Baroody, & Lonigan, 2013). Formal mathematics during this period include, but are not limited to, the following concepts: arithmetic, place-value understanding, knowledge of the base-10 numbers system, decimal knowledge, counting fluency; and the understanding of length, mass, volume, time, money, and geometry (e.g. shapes and patterns; Ginsburg, 1977, as cited in Purpura & Ganley, 2014 Department for Education, 2013). Details of all these skills go beyond the scope of this thesis. I will limit the sub-sections that follow to the explanation of arithmetic and arithmetic word-problems, and their domain-specific predictors, because these two overlapping abilities are the main element of the mathematics curriculum in the first years of formal schooling (i.e. Years 1 and 2 of the National curriculum in England; Department for Education, 2013).

1.3.1 Arithmetic

Arithmetic refers to the calculation and manipulation of numbers; it includes single or multi-digit additions, subtractions, multiplications, and divisions. Usually, the first step in children's arithmetic skills development is solving non-symbolic arithmetic problems (i.e. calculation problems without numbers), around the age of 2 to 4 years. Then, during the first year of formal schooling, when children are around the age of 5 years, children are able to perform symbolic arithmetic problems (i.e. calculation problems with numbers). I will start this section by presenting the process of solving non-symbolic arithmetic problems and then I will present the process of solving symbolic arithmetic problems.

Non-symbolic arithmetic problems are calculation problems that do not involve numbers and require children to represent and transform numerosities. The means by which young children are able to solve non-symbolic arithmetic problems is by representing the problem in a mental model. For example, solving a non-symbolic arithmetic problem involve presenting the child with a set of objects that is then hidden. While the objects are still hidden, the set is manipulated by adding or extracting an object. Finally, while the set is still hidden, the child is asked to show the final outcome with their own sets of objects. As such, to be able to reach an answer the child must have been following the whole process by representing each step in a mental model, from the representation of the original set, the action of adding or extracting, to the representation of the final outcome (Huttenlocher, Jordan, & Levine, 1994). This ability to transform numerosities has been identified as an important foundation for symbolic arithmetic skills (Gilmore, McCarthy, & Spelke, 2010; Huttenlocher et al., 1994; Rasmussen & Bisanz, 2005).

During the first year of formal schooling, children will start learning the process of calculation through the use of numbers. This process requires children to successfully access, discriminate, and compare numerical magnitudes, from the abstract symbols that represent them (Gilmore et al., 2010; Holloway & Ansari, 2009). Children can use the core skills they have already acquired, and use them to assist in arithmetic problems. For example, counting is a useful tool when children need to find the total of items in two groups. Children are able to find a total of two groups of items by counting all of them (Moylett & Stewart, 2012). In this way, counting skills become part of the strategies children can use to solve simple arithmetic problems. Generally, the first counting strategy that children develop, is counting two sets of objects separately, and then count them all together. For example, adding two and three items entails children to count the group of two items, then count the group of three items, and then count both sets together (Barody, 1987).

Eventually children learn that they can "count-on", that is, start counting from two and then count-on the further three items (Gray & Tall, 1994). This learning trajectory takes place between the ages of 3 and 5 years, and because this process involves holding in mind numerical information (number two in the previous example) to then add new numerical information (i.e. number three), children rely on physical aid like using their fingers, to represent each addend (Department for Education, 2013; Gray & Tall, 1994).

Solving single-digit arithmetic problems become an easier job once children learn to build problem-answer associations, like 2+2 always be 4. Each time a child encounters a particular arithmetic problem she creates an association between the problem and its answer. These associations are called arithmetic facts, which, through experience, will be stored in the long term memory (Bull & Johnston, 1997; Siegler & Shrager, 1984). On this way, arithmetic facts are important for mathematics development because they are cognitively efficient, allowing the child to not compute the answer every time . As soon as children become able to automatically retrieve these facts, they are able to solve simple arithmetic problems without drawing on slower procedures (i.e. counting). Additionally, being able to retrieve an answer from memory, rather than having to compute it, means that children would be able to direct their attentional resources to solve more complex problems, like multi-digit arithmetic problems (Ashcraft, 1982; Gersten & Chard, 1999). As such, arithmetic fact retrieval gradually increases the speed and efficiency with which children engage with mathematical material.

The next step for children's arithmetic learning is knowing how to solve problems with more than one digit (i.e. multi-digit arithmetic problems). By the age of 7 years, children are able to solve multi-digit arithmetic problems. They achieve this by understanding the value of a number given its place or position in relation to other numbers on a number set (i.e. ones, tens, hundreds, thousandths, etc.). This construct is known as numbers' place value (Martins-Mourao, 2000) and develops thanks to children's knowledge of the linear representation of numbers. An accurate understanding of the number line allows children to organize and store information regarding number's magnitudes (Träff, 2013). This understanding is necessary for multi-digit arithmetic problems where children need to regroup "10" when they "borrow" or during the carry effect, when a unit needs to be "carried form" another number. For example, in 16 + 7 = 23, a number 1 needs to be "carried from" the units to the decades (because 6+7=13;Göbel, Moeller, Pixner, Kaufmann, & Nuerk, 2014).

Arithmetic problems can vary a great deal in their complexity and in the processes necessary to solve them. Unsurprisingly, therefore, instead of relying on a single approach to solve arithmetic problems, children use several distinct counting-based and arithmetic facts retrieval strategies (Siegler & Braithwaite, 2016). What determine the use of one or the other depend on familiarity or experience with certain mathematics procedures. For example, 6- and 7-year-olds tend to solve single-digit arithmetic problems using counting and decomposition strategies, whereas 8- and 9-year-olds are more likely to use direct fact retrieval instead (Raghubar, Barnes, & Hecht, 2010). Nonetheless, for multiplication problems, 8- and 9-

year-olds use counting strategies (Ashcraft, 1982) because of the lack of familiarity they have with these problems. This variability between strategies can also be observed during adulthood and it is present while solving all four arithmetic calculations (addition, subtraction, multiplication, and division; Siegler & Braithwaite, 2016).

To summarize, in order to solve arithmetic problems, children must learn to use their counting skills as strategies first (e.g. counting with their fingers) , to manipulate numerical information, to later be able to solve them by arithmetic fact retrieval or any other strategy. This process happens around the age of 5. Then, around the age of 6 and 7, children start building arithmetic facts; and thus, start being more efficient in solving single-digit problems by getting their answers derived through retrieval from long-term memory (Butterworth, 2010). When solving multi-digit problems, where they need to attend to regrouping demands and place values, children rely on their linear representation of numbers. However, from childhood and even in subsequent years, not only a single strategy will be used, but rather different strategies in combination. Arithmetic is a very important skill in children's mathematics learning, however for it to be really useful in everyday life, children need to be able to solve arithmetic problems when they are embedded in everyday situations.

1.3.2 Arithmetic word-problems

Arithmetic word-problems are important because they are the mean by which children learn to apply their arithmetic knowledge to every-day situations. Arithmetic word-problems, are problems that simulate information that people commonly encounter in daily life. These problems are presented inside a narrative (e.g. *"Luis went to the Moor Market and bought six apples. If he gave two apples to his mother and one apple to his sister, how many apples will he have then?"*). Between the ages of 5 and 7 years, children start learning how to apply their arithmetic knowledge to solve these problems. These real-life problems are characterized by requiring calculation skills for their solution and are typically presented with verbal or visual representations that must be interpreted and manipulated symbolically to be solved (Cummins, 1991). For example, taking the example above this means changing the problem to: 6-2-1=? In addition to providing practice with real-life problems situations, solving arithmetic word-problems foster children's creative and critical solving skills, and thus the development of complex problem-solving skills (e.g. the ability to analyse different solutions to solve a problem; Chapman, 2006; Keen, 2011).

Thus, the main difference between an arithmetic problem and an arithmetic word-problem is that, for the first one the calculation that is needed to be solved is already set-up; whereas for the second one, the calculation to be solved is presented within linguistic information that children need to interpret in order to first work out what the problem is, before being able to solve it. And so, solving arithmetic word-problems involves three main steps. Firstly, children need to interpret the narrative involving the problem. Secondly, it is necessary to select only the information that is relevant to solve the problem. And thirdly, children need to select the correct arithmetic operation in order to solve the problem (Andersson, 2007; Lee, Ng, Ng, & Lim, 2004; Swanson & Beebe-Frankenberger, 2004). Thus, solving arithmetic word-problems involve both cognitive and linguistic processes.

The underlying cognitive abilities in solving arithmetic word-problems are predominantly domain-general abilities, of which the strongest predictor is language skills (Wang, Fuchs, & Fuchs, 2016). This is because the arithmetic calculation that needs to be performed is embedded within linguistic information. In fact, a variable that determine the difficulty of these problems, is the semantic relations-the associations that exist between the meanings of words or sentences-within the problem (Riley, Greeno, & Heller, 1983). Three main types of problems are often proposed based on their semantic relations:

(i) Combination problems, which involve two quantities that must be considered in combination to find the answer (e.g. *Ana has 5 apples. Laura has 2 apples. How many apples do Ana and Laura have altogether?),*

(ii) Change problems, in which an exchange takes place and changes the size of a given set of objects (e.g. *Ana had 5 apples. Then Laura gave her 2 more apples. How many apples does Ana have now?*), and

(iii) Compare problems, which consist of two quantities which difference must be compared and quantified to find the answer (e.g. *Ana has 5 apples. Laura has 2 more apples than Ana. How many apples does Laura have?*).

Research has found that even when children need to perform simple calculations, *compare* problems are more difficult for children than *combination* or *change* problems. This is because compare problems use relational terms (e.g. more than, less than; Riley et al., 1983; Stern, 1993; Stern & Lehrndorfer, 1992) which for some children are difficult to understand. This demonstrates that solving arithmetic word-problems requires both domain-general abilities, as well as domain-specific abilities (e.g. language skills *and* arithmetic knowledge). Additionally, *compare* problems may also require children to inhibit misleading cues when the relational labels are inconsistent with the calculation needed to be performed. In the following example: *Ana has 5 apples. She has 2 more apples than Laura. How many apples does Laura have?* The relational label "*more*" implies a larger quantity; however, to solve the problem, children need to perform a subtraction to find the answer, thus needing to inhibit the strategy to perform an addition whenever they hear the cue "*more*" (Hegarty, Mayer, & Monk, 1995; Lee, Ng, & Ng, 2009).

Difficulties in solving arithmetic word-problems can occur in children who may have adequate arithmetic skills (Swanson, Jerman, & Zheng, 2008), suggesting that other cognitive skills are involved. For example, in a longitudinal study it was found that domain-general abilities, but not arithmetic skills, predicted children's performance on arithmetic word-problems. In this study, 6 to 8-year-olds were tested in three different time-points, over a period of two years. Children were divided into two groups on the basis of their problem-solving performance (i.e. at risk and not at risk for mathematics problem-solving difficulties). Children who were at risk for problem-solving difficulties had low levels of performance in the domain-general measures, but their calculation skills were no different to children who were not at risk (Swanson et al., 2008). This finding shows that although arithmetic and arithmetic word-problems are related, the cognitive mechanisms underlying them are (at least partly) distinct.

1.3.3 Summary of children's informal and formal mathematics skills

To summarize, mathematics has a hierarchical development for which informal mathematics are highly important. The understanding of numbers and their magnitudes starts with the ability to perceive and discriminate quantities. Then, children develop the understanding that numbers and number words are used to represent magnitudes. Counting and their ability to represent numbers spatially develop next around the age of 3; but it won't be until 6 and 7 years of age that children become successful counters and successful at representing numbers on a number line. Both abilities allow children to solve arithmetic problems in the following years. Simple non-symbolic arithmetic skills are the first step into the development of symbolic arithmetic, for which the first years of formal schooling, when children are5 to-6-years of age, are relevant.

Research in mathematics has focused a great deal in arithmetic skills, because of how important they are during the first years of formal schooling. However, investigating solely arithmetic skills would give a very narrow account of how children use mathematics. It is important, therefore, to take a much more applied perspective. Some useful progress has been made in this regard with some research being conducted to look at arithmetic word-problems. From this, we understand some things about how children use mathematics. However, it remains an area that merits additional work to offer a fuller account of how young children use their mathematics in real-world contexts.

1.4 Domain-General Precursors of Children's Mathematics Skills

In addition to domain-*specific* abilities, the development of mathematics also involves domain-*general* abilities (Purpura & Ganley, 2014). Since a variety of domain-*general* abilities have been associated with mathematics, for the purpose of this thesis, this section will focus on two major domain-general abilities: executive functions and language. These two constructs have been found to be important precursors of mathematics development (Vanbinst & De Smedt, 2016). The following section will summarize significant findings regarding the role of executive functions in mathematics, followed by the role of language skills.

1.4.1 Executive functions

Executive functions is an umbrella term that refers to high-level cognitive skills that govern the control, regulation, and efficient planning of human behaviour (Lezak, 1982). This set of skills are particularly important in situations that are complex or new, or when processes that are already automatized need to be inhibited (Miyake et al., 2000). There are three main executive functions that allow us to carry out goal-directed behaviour: Working memory, which refers to the ability to maintain and update information in mind; inhibitory control, which allow us to inhibit our attention to irrelevant information and to suppress previously learnt responses in favour of new more task-appropriate ones; and cognitive flexibility, which refers to the ability to shift between response sets in a flexible manner (Miyake et al., 2000; Miller & Cohen, 2001). Although these executive functions are theoretically distinguishable, these skills are often interrelated. This section will present the specific contribution of each of these three executive functions to mathematics.

1.4.1.1 Working memory

Working memory is the system that allows the child to hold and manipulate information in mind during the performance of complex tasks such as learning, reasoning, and comprehension (Baddeley & Hitch, 1974 as cited in Baddeley, 2010). Among the many different theoretical models of working memory, Baddeley and Hitch's multi-component model is one of the most frequently used in research involving mathematics' development. This multi-component model goes beyond, and is different from, the general definition of working memory as a unitary system involved in monitoring and updating information proposed by Miyake and colleagues (Miyake et al., 2000). Instead, Baddeley and Hitch's working memory model consists of four functional components: The central executive as a control system of limited attentional capacity, and three subsidiary short-term storage systems: the visuo-spatial sketchpad, the phonological loop, and the episodic buffer (Baddeley and Hitch, 1974 as cited in Baddeley, 2010).

According to this multi-component model, the central executive supports the coordination of processing and storage of information (Baddeley, 2012). Four different functions have been ascribed to the central executive: (i) the coordination of tasks performance, (ii) switching between strategies, (iii) directing attentional resources to one stimuli while inhibiting irrelevant information, and (iv) the ability to hold and manipulate information in long-term memory (Baddeley, 1996). The visuo-spatial sketchpad, for its part, is responsible for the storage and maintenance of visual and spatial information; while the phonological loop is responsible for the storage and maintenance of information in a phonological form (RepovŠ & Baddeley, 2006). The fourth component of this model is the episodic buffer, responsible for the integration of information from a variety of sources, especially from long-term memory (Baddeley, 2000). The episodic buffer was the last component to be added to the model. To my knowledge, no research has been conducted to explore the role of this component in children's mathematics performance. Thus, from this chapter further, when talking about Baddeley and Hitch's working memory model I will be referring to the central executive, the visuo-spatial sketchpad, and the phonological loop solely.

Because working memory involves the storage and manipulation of information, the main role of the central executive would be monitoring and facilitating the control of attentional resources, as well as being involved in the activation of relevant information from long-term-memory (Friso-van den Bos, van der Ven, Kroesbergen, & van Luit, 2013; Swanson, 2011). The central executive is usually measured with complex span tasks. Complex span tasks consist of a series of items (e.g. numbers or words) which need to be stored, manipulated, and then recalled (Friso-van den Bos et al., 2013). One example of these tasks is the widely used Backward Digit Span task. This task involves children needing to store and recall in backward order a series of numbers (Gathercole, Pickering, Ambridge, & Wearing, 2004). This task increases in difficulty by adding one digit each trial.

The visuo-spatial sketchpad is involved only in the storage of domain-specific information and can be divided by its storage subsystems that is, in both visual and spatial storage. In dual-tasking experimental studies, where two tasks are performed simultaneously, the performance on visual short-term memory tasks (e.g. remembering different Chinese symbols), has been shown to be disrupted by visual (e.g. discriminating colours) but not by spatial interference (e.g. selecting a stationary item among several moving items). Performance on spatial short-term memory tasks (e.g. remembering the location of different dots) on the other hand, has been shown to be disrupted by only spatial interference and not visual interference (Della Sala, Gray, Baddeley, Allamano, & Wilson, 1999; Klauer & Zhao, 2004). These findings demonstrate that the visuo-spatial sketchpad can be separated by independent mechanisms. As such, the role of the visual storage subsystem is related to perception and visual imagery, retaining basic features such as orientation, colours, or shapes. Whereas the role of the spatial storage subsystem is related to attention and action (RepovŠ & Baddeley, 2006).

Since the main role of the visuo-spatial sketchpad is the storage of information, simple span tasks are commonly used to measure this component. In such tasks, items must be stored and then recalled (Friso-van den Bos et al., 2013). The string of items involved in these tasks increase in difficulty by adding more items each time (Friso-van den Bos et al., 2013). A common measure of the visuo-spatial sketchpad is the Corsi block-tapping task in which children need to tap different cubes in the same order as the examiner. The task increases in difficulty by adding one cube each trial (i.e. the tasks start with the participant needing to reproduce a sequence of three different cubes, then four, then five etc.; Kessels, van Zandvoort, Postma, Kappelle, & Haan, 2000).

In relation to the phonological loop, this component is also involved in the storage of domain-specific information. Two processes have been associated to this component, the first process is the temporary storage of verbal information. This verbal storage is limited by the number of items that need to be held (about five to eight items; Brener, 1940). The second process relates to the articulatory rehearsal process, which consists of retrieving the information stored in the phonological loop via articulation (RepovŠ & Baddeley, 2006). The role of this component differs according to age, that is, in children younger than 7 years of age, the role of the phonological loop is to solely store verbal information , whereas after the age of 7 years, the role of this component extents to the generation of verbal strategies via articulatory rehearsal (Hecht, Torgesen, Wagner, & Rashotte, 2001; McKenzie, Bull, & Gray, 2003; Meyer, Salimpoor, Wu, Geary, & Menon, 2010).

To measure the phonological loop, simple span tasks are also commonly used. An example of a measure of the phonological loop is the non-word list recall task in which children need to store and recall a series of monosyllabic non-words (Gathercole et al., 2004). The use of non-words limits the use of long-term memory representations of familiar words that support recall, that is, unfamiliar words are separate of children's vocabulary knowledge; and thus it is considered to be a pure measure of the phonological loop ability (Gathercole, Willis, Baddeley, & Emslie, 1994).

Mathematics skills require, or at least are supported by, working memory resources (Raghubar et al., 2010). From counting skills to multi-digit arithmetic problems, mathematics involves a series of different steps that must be solved in sequence. Mathematics performance includes coordinating attentional resources (Hornung, Schiltz, Brunner, & Martin, 2014a) and the maintenance and manipulation of numbers (Simmons, Willis, & Adams, 2012). The more the mathematics skill involve multiple steps to be carried out, the more it is likely to have a stronger relationship with working memory resources. For example, naming numerals that simply requires relating number words to symbols does not involve working memory resources (Purpura & Ganley, 2014), whereas verbal counting, that involves the retrieval of number words in the correct order while keeping one's place in a counting sequence, does (Noël, 2009).

Because mathematics skills involving multiple-steps demand more working memory resources, it is not surprising that multi-digit arithmetic problems have been frequently found to be predicted by working memory. The process of solving multi-digit arithmetic problems involves holding partial information in mind, while processing further information, in order to reach a solution (Raghubar et al., 2010). To illustrate this idea let us imagine that a girl is trying to solve the following addition: 58 + 67 without the use of any device. In order for the girl to be successful at finding an answer, she would need to first hold in mind these two numbers and then she would need to retrieve from long term memory her previously learned addition rules to calculate. But because this is a two digit calculation, she would also need to first sum up the units (8 and 7), hold in mind that result, and add the product to the tens (5 and 6). Working memory, thus, would be implicated in this process by allowing the storage and retrieval of partial results (Barker, 2016).

The contributions of working memory to informal mathematics

Working memory is particularly important during the early stages of learning mathematics, in which children cannot easily retrieve their answers from long-term memory and have not yet mastered the automatization of their mathematics skills. This is particularly noteworthy in three important areas of informal mathematics: counting, set comparison, and non-symbolic arithmetic. For example, one-to-one counting for adults is an automatic process that does not involve much cognitive effort. In contrast, for young children (i.e. 4-to6-year-olds) this process is much more effortful. It involves working memory resources because it requires the constant updating of information in at least two steps, (i) keeping track of the verbal number sequence, and (ii) remembering which items have been counted and which items still need to be counted (Hornung et al., 2014a; Noël, 2009). Working memory is also essential for two further aspects of informal mathematics: the ability to compare sets of objects and non-symbolic arithmetic skills. An overview of findings regarding the relationship of these three informal mathematics skills and working memory, are presented next.

Working memory is essential for counting skills, mainly in keeping track of the numbers that are being counted. It has been suggested that in 4-to-6-year-olds, the central executive is directly involved during counting procedures because the central executive allows the control and allocation of attentional resources during the monitoring of numbers (Geary, Hoard, Byrd-Craven, & DeSoto, 2004; Geary, Hoard, & Nugent, 2012; Meyer et al., 2010; Noël, 2009; Fiona R Simmons et al., 2012). For example, cardinal number knowledge involves both keeping in mind a target number, and simultaneously keeping track of the numbers that have been already counted (Purpura & Ganley, 2014). This skill is measured by asking children counting out a specified set of items from a larger set – for example, giving the examiner 5 objects from a set of 14 objects. The central executive (measured with a word recall task, a verbal working memory task in which children need to answer a question and then recall the last word) plays a central role in this process (Purpura & Ganley, 2014). As such, the central executive is involved in allocating attentional resources to keep track of the numbers during counting.

Working memory is also directly involved in young children's ability to compare sets of objects. The ability to compare sets of objects successfully involves three steps (i) Children need to count each of the sets, (ii) hold in mind the total set sizes in memory and (iii) then identify which one is the largest (Purpura & Ganley, 2014). Carrying out all three of these steps, involves working memory, specifically the central executive (Purpura & Ganley, 2014). In this study, set comparison was assessed by showing children four sets of dots, and then asking the children which of the four sets had the most, or fewest, dots. There was also a number comparison task that followed the same procedure, but instead of four dots, there were four numbers. However, children's performance on the number comparison task was not related to their performance on the working memory task. The main difference between the number comparison task and the set comparison task, is that in the number comparison task the magnitudes being compared were already provided by their symbolic representation (i.e. number), and in the set comparison task, children needed to count in order to determine the magnitudes. Thus, the authors argue that the difference between tasks was due to children relying on more steps (i.e. counting) to perform the set comparison task, but not the number comparison task (Purpura & Ganley, 2014).

Working memory, specifically the visuo-spatial sketchpad, is important for solving non-symbolic arithmetic problems because these problems involve the use of a mental model to represent the quantities in a given problem (Rasmussen & Bisanz, 2005; Xenidou-Dervou, van der Schoot, & van Lieshout, 2014). Children follow three steps to build a mental model of the non-symbolic arithmetic problem. First, children need to internally represent the non-symbolic quantities (e.g. chips) used in the external representation of the problem. Second, as the external display is manipulated either by adding or subtracting an object, children need to manipulate the internal representation by doing the same. Finally, children have to map the internal representation to a number or number word in order to give an answer (Rasmussen & Bisanz, 2005). These three steps recruit visuo-spatial sketchpad resources, because this component allow the representation and maintenance of non-symbolic quantities in mind. Like a sort of mental workspace (Rasmussen & Bisanz, 2005; Xenidou-Dervou et al., 2014).

The contributions of working memory to arithmetic problems

Arithmetic skills encompass the main focus in the academic curriculum at least during the first years of formal schooling (Department for education, 2013; Vanbinst & De Smedt, 2016). As such, they have become one of the most researched topics in mathematics development. Empirical evidence has suggested that both single-digit and multi-digit arithmetic problems require working memory resources (Bull, Johnston, & Roy, 1999; McKenzie et al., 2003; Peng, Namkung, Barnes, & Sun, 2015). This sub-section will present significant findings regarding the contribution of working memory to single-digit and multi-digit arithmetic problems, and will then present significant findings regarding arithmetic word-problems.

The central executive has an important role in both single-digit and multi-digit arithmetic problems. In single-digit arithmetic problems the central executive is known to play the role of allocating attentional resources and guiding counting strategies (Meyer et al., 2010; Simmons et al., 2012). One study measured 5- to-6-year-olds' and 7- to-8-year-olds' single-digit arithmetic skills with an experimental task in which children listened to arithmetic problems through an audio file, and then answered them verbally. The central executive was assessed using a composite score from four measures (the odd-one-out task, the spatial recall task, listening recall task, and a backward word task). Children also completed two visuo-spatial tasks (maze memory, where children needed to retrace a maze route from memory, and a block recall task) and two phonological loop tasks (word and non-word recall; Simmons et al., 2012). Results indicated that the central executive was a unique predictor of arithmetic skills only in the 5- to-6-year-old group. This is perhaps unsurprising, since at this age counting strategies are very common. For the older group, none of the working memory components predicted performance, suggesting that the older children were relying on more efficient strategies.

A key developmental finding is that as children become more efficient with their counting strategies and with arithmetic fact retrieval, they require fewer and fewer resources from the central executive. As children increasingly solve tasks by retrieving the answer form long-term memory, they no longer rely on information that needs to be protected from decay. The age-related change in strategy use from counting to arithmetic fact retrieval typically occurs around the age of 11 or 12 years (Imbo & Vandierendonck, 2007). However, as arithmetic fact retrieval is still not an automatic process at this age, the central executive will still be involved. To illustrate this: in a study conducted with 10- to-12-year-olds, both counting strategies and arithmetic fact retrieval strategies were related to the central executive. However, reliance on the central executive decreased with age, and in relation to strategy efficiency: the more children used arithmetic fact retrieval strategies, the less they used the central executive.

The central executive will still play a role in multi-digit problems despite the decrease of central executive demands in single-digit problems. In multi-digit problems the central executive is necessary for the temporary storage and manipulation of numerical information. This is because solving multi-digit problems involves the maintenance of intermediate numerical information while simultaneously attend to regrouping demands (Furst & Hitch, 2000). However, this suggestion is mostly based on research conducted with adults (e.g. Hubber, Gilmore, & Cragg, 2014) because commonly, in studies conducted with typically developing children, both single-digit and multi-digit problems are presented within the same task and do not differentiate between simple and complex problems[1] (Berg, 2008).

The visuo-spatial sketchpad and the phonological loop are particularly important in multi-digit arithmetic problems. The visuo-spatial sketchpad facilitates the use of imagery strategies while children solve multi-digit arithmetic problems where carrying procedures are required (Bull et al., 1999) Imagery strategies are those that help visualize an arithmetic problem as if it was being solved on paper. Thus, using the

[1] Although it has to be noted that this would be the case in studies conducted in children older than 6 years, because younger children are not expected to know how to solve multi-digit problems, and only single-digit arithmetic problems or standardized tests with mixed problems are commonly used.

visuo-spatial sketchpad allow the maintenance of calculations and manipulation of numbers in mind (Hayes, 1973 as cited in de Hevia, Vallar, & Girelli, 2008). However, the use of imagery strategies and thus, the role of the visuo-spatial sketchpad, is more commonly used by children younger than 7 years of age (McKenzie et al., 2003).

As mathematics becomes more closely associated with linguistic processes and after a skill has been learned, the role of the phonological loop becomes more relevant, allowing children to use articulatory rehearsal strategies. This process takes place after the age of 7 years and mirrors the use of verbal strategies that children at this age employ more frequently (Bull & Johnston, 1997; McKenzie et al., 2003; Siegler, 1999). For example, children older than 7 years have been found to rely on articulatory rehearsal (a process that helps us retain information by repeating it under one's breath) while solving arithmetic problems (McKenzie et al., 2003).

Thus, the role of the visuo-spatial sketchpad and the phonological loop is determined by the age of the children and the mastery of either imagery or verbal strategies. For example, in one study 6- to 8-year-olds completed 20 arithmetic problems presented verbally, while being exposed to two different types of interference. One was phonological interference, in which children heard a recording on a foreign language; and the other one was a visuo-spatial matrix of black and white squares that changed colours (black to white and vice versa; McKenzie et al., 2003). The younger group (6- to 7-year-olds) performed significantly worse under the visuo-spatial interference, whereas the older group (7-to 8-year-olds) performed significantly worse under phonological interference. These results suggest that at least for mental arithmetic, children transition from predominantly using a visuo-spatial strategy to predominantly using a phonological strategy (presumably a linguistically mediated strategy) and thus use different working memory components.

The contributions of working memory to arithmetic word-problems

Working memory plays a key role in arithmetic word-problems, especially because solving these problems involve multiple-steps. Children seem to rely to a greater extent in the central executive, rather than the visuo-spatial sketchpad or the phonological loop to solve arithmetic word-problems (Peng et al., 2015; Swanson, 2011). The central executive is necessary for allocating attentional resources during the manipulation of information, and in suppressing irrelevant information (Peng et al., 2015). Two different studies have found the association of the central executive and arithmetic word-problems proficiency in young children. One study, conducted with 4-to-5-year-olds, used arithmetic word-problems presented verbally (e.g., "*Here are three cows; if four more come, how many cows will there be?*"). The three working memory components were measured, but only the central executive was a predictor of arithmetic word-problems. The second study found similar results so that the central executive was also a significant predictor. In this study the central executive predicted 4-to-7 year olds' abilities to solve arithmetic word-problems such as "*A shopkeeper has seven chickens. He buys three more. How many chickens does the shopkeeper have now?*" (Kyttälä, Aunio, Lepola, & Hautamäki, 2014, p.680).

The role of the working memory components in arithmetic word-problems may be constrained by the format of such problems. That is, besides having in common the important role of the central executive in solving arithmetic word-problems, these two studies also share the characteristic of presenting the problems with some sort of visual aid (pictorial representations of the problems and tokens). Thus, these findings suggest that when there is visual aid in which children can rely on to solve the problems- and therefore, presumably, decrease visuo-spatial sketchpad demands- among the three working memory components, they only need to recruit central executive resources to inhibit irrelevant information in order to attend to key numerical information (Wang et al., 2016).

Summary of the contribution of working memory to mathematics

To summarize, empirical evidence has shown that children rely on working memory components to a higher extent when they are young and that with age-and with the onset of strategy use and the acquisition of number facts-children's reliance on working memory tends to decrease. As for the contribution of specific working memory components, the central executive is the component involved in directing attentional resources to counting, and later to counting strategies, retrieving arithmetic facts when these are not automatized yet, allocating attention control during manipulation of information, and in suppressing irrelevant information in an arithmetic word-problem. The visuo–spatial sketchpad allows children to use imagery strategies by functioning as a mental black-board. Later in development, as mathematics becomes more closely associated with linguistic processes and after a skill has been learned the phonological loop allows children to use articulatory rehearsal strategies.

Moreover, if a child understands the operation that is being performed during single-digit arithmetic problem-solving, and is able to retrieve the answer directly from memory, then she will need little or no input from working memory (Hecht, , 2002; Raghubar et al., 2010). Conversely, children that have not yet achieved the automatization of calculation procedures, rely on working memory resources (Peng et al., 2015). Thus, the relationship between single-digit arithmetic and working memory decrease as children become more efficient with their strategies. However, the association with multi-digit arithmetic problems could exist longer over time (Imbo & Vandierendonck, 2007; Vanbinst & De Smedt, 2016).

Working memory is an important domain-general precursor of both informal and formal mathematics. However, there is still a gap in our understanding of the relationship between working memory components and children's mathematics, since most of the interest has been placed on developmental periods either when children are developing mathematics and are in the informal stage of learning mathematics, or after a few years of formal schooling where most of the children have already mastered some of their mathematics skills. Thus, less is known about the time-period where the transition from informal to formal mathematics is happening – that is, when children are between 5 and 6 years of age.

Research conducted with children between 5 and 6 years of age would help us understand the cognitive skills that typically developing children need for a successful transition from informal to formal mathematics learning; and in turn will allow us to determine where to focus educational effort so that all children can have the same learning opportunities in current and subsequent academic years.

1.4.1.2 Inhibitory control

Inhibitory control is the ability to override dominant or prepotent responses, in favour of more goal-appropriate responses (Miyake & Friedman, 2012). It allows us to ignore task irrelevant information in order to maintain focus on task-relevant stimuli (Clayton & Gilmore, 2015). There are two ways in which inhibitory control is related to mathematics skills. One way is related to a general account for the role of inhibitory control in mathematics, and another way is related to a specific role of the inhibitory control in mathematics. The general account for the role of inhibitory control posits that the role of inhibitory control in mathematics is due to the role of this executive function in general learning. As such, mathematics performance will depend on children's ability to inhibit task irrelevant responses in the presence of distracting information, independent of the mathematics level that children possess (Blair & Razza, 2007). Undoubtedly,- just as the performance on any other cognitive task, performance in mathematics will be enhanced if children can allocate their cognitive resources to task-relevant stimuli and ignore task-irrelevant information (Van Dooren & Inglis, 2015).

In addition to the relevance of inhibitory control in mathematics due to its contribution to general learning, inhibitory control can also have a more specific role in mathematics performance. The account that establishes that inhibitory control has a more specific role in mathematics points out that inhibitory control is specifically related to mathematics that (i) are more prone to interference, and (ii) that require ignoring either irrelevant strategies or previously learned mathematics rules, in order to choose more efficient ones (De Visscher & Noël, 2014; Fitzpatrick, McKinnon, Blair, & Willoughby, 2014; Robert & LeFevre, 2013). An example of mathematics that are prone to interference is solving compare arithmetic word-problem (e.g. *'Mary has 5 apples. She has 2 apples more than Ana. How many apples does Ana have?')* where the relational term (i.e. 'more than') interferes with the arithmetic computation that needs to be performed to solve the problem (i.e. subtraction; Lubin, Vidal, Lanoë, Houdé, & Borst, 2013). As for children needing to ignore previously learned mathematics rules, one example is when comparing fractions such as $\frac{2}{3}$ vs $\frac{2}{6}$. In order for children to be successful at comparing these fractions, children need to ignore their well-established knowledge about natural numbers. If children were to apply the rule of natural numbers, they would choose the wrong answer that $\frac{2}{6}$ is bigger, because 6 is larger than 3. This phenomenon is called "the natural number bias" and it is present not only in children, but in adults as well (Attridge & Inglis, 2015; Obersteiner, Van Dooren, Van Hoof, & Verschaffel, 2013).

Inhibitory control can also have a specific role in relation to particular aspects of arithmetic performance. To better understand the relationship between inhibitory control and particular aspects of arithmetic, a recent study examined the relationship between inhibitory control and (i) factual knowledge, (ii) conceptual understanding, and (iii) procedural skills, in 11 to-14-year-olds and adults (Gilmore, Keeble, Richardson, & Cragg, 2015). Factual knowledge refers to previously learned numerical rules, conceptual understanding is children's understanding of the principles of mathematics also referred as 'knowing why' (e.g. state whether or not the first problem: 74 + 57 = 131, help solve the second problem 131 − 74 =), and procedural skills refers to children's use of the correct and efficient mathematics procedures, or the 'knowing how' (e.g. 'borrowing' and 'carrying 'rules in multi-digit arithmetic problems; Gilmore et al., 2015). Findings from this study showed that children's inhibitory control was significantly related to procedural skills, suggesting that the role of inhibitory control in mathematics is related to the selection of more efficient strategies while solving arithmetic problems (Gilmore et al., 2015).

1.4.1.3 Cognitive flexibility

Cognitive flexibility is the ability to switch between sets, tasks, or strategies (Miyake et al., 2000). Cognitive flexibility has been shown to play a role in mathematics, although the precise nature of this role is unclear. Moreover, evidence on this point is not only sparse, but also inconsistent. Additionally, measuring cognitive flexibility in young children is challenging, because cognitive flexibility itself relies on working memory and inhibitory control. Therefore there are few 'pure' cognitive flexibility measures available (St Clair-Thompson & Gathercole, 2006a). Thus, it is currently unclear whether cognitive flexibility predicts mathematics performance beyond the effects of working memory or inhibitory control.

It has been suggested that young children rely on cognitive flexibility to help them develop the conceptual understanding of numbers. Specifically, it has been suggested that cognitive flexibility allows children to transition from the procedural understanding of counting to the conceptual understanding of counting (Purpura, Schmitt, & Ganley, 2017). That is, it allows them to transition from thinking about counting as a *procedure*- that is, applying the counting sequence to a set of objects- to thinking about it as a *medium* that provides quantitative information (cardinal understanding; Purpura et al., 2017). In this study 3 to-5-year-olds' cognitive flexibility was measured with a card sorting task, children also completed a set of informal mathematics measures that assessed the following skills: subitizing, set comparison, verbal counting, one-to-one counting, cardinality, counting subsets, identification of numbers, mapping sets to their numbers, number order, number comparison, arithmetic word problems, and simple additions. Results showed that cognitive flexibility predicted the mathematics abilities that are related to the conceptual understanding of numbers, that is, cardinality, counting subsets, number order, and number identification.

Cognitive flexibility is involved in arithmetic performance by supporting alternation between procedures, like switching between operations or solution strategies (Bull & Scerif, 2001). In this study 6 to-8-

year-olds completed one arithmetic test that had both single and multi-digit additions and subtractions. Cognitive flexibility was measured with a Wisconsin Card Sorting Test (WCST) with three dimensions: colour (red, yellow, green, or blue), shape (star, triangle, cross, and circle), and number (one, two, three, or four). Children needed to sort the cards according to the sorting criteria (colour, shape, or number) which needed to be figured out by using the experimenter feedback ("*that is correct*" or "*that is wrong*"; Bull & Scerif, 2001). Results showed that cognitive flexibility significantly predicted arithmetic performance even after controlling for reading and IQ. The most likely explanation for these results was that cognitive flexibility enabled children to switch between different strategies. This would seem to suggest that cognitive flexibility itself is not central to children's mathematics performance – but that in circumstances where some degree of switching is required, cognitive flexibility can predict children's mathematics performance.

However, some of the previous research that have found cognitive flexibility to be a significant predictor of mathematics, have failed to separate this executive function from the effects of other cognitive constructs, like processing speed. One study divided four cognitive flexibility tasks in two versions each: one that measured the executive shifting ability (all the manipulated versions) and the other one that was non-executive, and that was assumed to measured only processing speed (all the control versions). This procedure was carried out as such to directly investigate if the relationship between cognitive flexibility and arithmetic, was due to the processing requirements that arithmetic has (van der Sluis, de Jong, & van der Leij, 2007). Results showed that arithmetic skills were predicted by the executive factor of cognitive flexibility, until the non-executive factor was considered. This finding suggests that the relationships between cognitive flexibility and arithmetic skills that some studies have reported before, could be partially explained by the processing requirements of the tasks. In fact, in Bull and Scerif's study processing speed was not considered as covariates. Nonetheless, the deficiency of research on the subject makes it difficult to draw stronger conclusions on this subject.

In conclusion, research regarding the role of executive functions in children's mathematics have frequently, and somewhat consistently, found that working memory is a strong predictor of mathematics. Conversely, the role of inhibitory control and cognitive flexibility in mathematics is less clear, at least in young children.

1.4.2 Language skills

Language is a cultural-specific communication system that allows us to understand the world (Hauser, Chomsky, & Fitch, 2002). In a general sense, it has been proposed that we use language skills to understand mathematics because mathematics involves symbols since they are expressed and explained verbally and through written words (Kovarik, 2010). There are two aspects of children's language skills that have been recurrently found to be strong predictors of mathematics skills. These are (i) phonological processing which refers to the use of sound units (i.e. phonemes) to process verbal and written language (Wagner & Torgesen, 1987); and (ii) vocabulary. Vocabulary skills can be divided in expressive vocabulary, which is the ability to provide word definitions, and receptive vocabulary, which refers to the words that a person can comprehend and respond to (even if the person cannot produce those words; Burger & Chong, 2011). I will start this section presenting an overview of the significant findings regarding language skills as a composite and their relationship to mathematics. Then I will present an overview of the contributions of phonological processing and vocabulary.

During early stages of development children who have more advanced language skills tend to be better at counting and in matching Arabic numerals to their verbal labels. This in turn improves their quantitative understanding and supports their arithmetic development (Lefevre et al., 2010; Moll, Snowling, Göbel, & Hulme, 2015). For example, one longitudinal study found that language skills (measured as a composite score from an expressive vocabulary task and receptive vocabulary task) at 3 to 4 years of age predicted counting (measured with a counting dots task and children's ability to count as far as possible) and number knowledge (measured with a number recognition task and a number writing task) at the age of 5. Furthermore, variations in early language skills had indirect effects to arithmetic skills (measured with an arithmetic fluency task) at the age of 6-7 through counting skills and number knowledge (Moll et al., 2015). As such, the precise role that language plays in children's mathematics development changes with age.

As for the role of language in relation to children's age; in young children language skills help them to perform basic numeracy skills, whereas for older children language skills help them to understand mathematics concepts and carry out simple operations. For example language skills at the age of 5, are important for children's ability to name numbers; whereas at the age of 7, language has a key role in the conceptual understanding of measure, order, and place value; and geometry and calculation (Lefevre et al., 2010). What is more, three different pathways were investigated in relation to children's mathematics skills: the linguistic pathway, which was a composite of receptive vocabulary and phonological processing; the spatial attention pathway, which referred to visuo-spatial working memory; and the quantitative pathway which referred to subitizing (LeFevre et al., 2010). Among the three cognitive pathways, the linguistic pathway was the most important for children's mathematics from the age of 5 to the age of 7 years.

Language skills also allow children to interpret and understand mathematics concepts through development (Vukovic & Lesaux, 2013). For example, one study tested children when they were 6 years of age, and were followed and tested for three years until they were 9 years of age. A language composite was formed by receptive vocabulary skills and listening comprehension skills, and mathematics skills were measured in a range of domains, from basic number facts and solving arithmetic problems, to three higher order domains of mathematics: (i) data analysis/probability, for which children needed to interpret tables and tally charts, and estimate probability; (ii) algebra, for which children needed to find a missing number in an equation, describe patterns and functions, and represent mathematics relations (e.g. X is smaller than Y); and (iii) geometry, for which knowledge of shapes in two and three dimensions was considered (Vukovic & Lesaux, 2013). Results showed that after controlling for visuo-spatial working memory, reading ability, and gender; language skills predicted gains in data analysis/probability and geometry, but not in arithmetic or algebra. Overall, these findings suggest that general language skills are necessary for the development of mathematics concepts and representations, but may not be involved in the process of manipulating quantities and other symbolic notations.

1.4.2.1 Phonological processing

Phonological processing refers to the use of sound units to process verbal and written language (Wagner & Torgesen, 1987). One way in which phonological processing has been thought to help with children's mathematics is by allowing children to learn numbers. Some researchers have proposed that learning the symbols for sound and learning the symbols for quantity are similar processes (Tolchinsky, 2003 as cited in Berghout, Blevins-Knabe, & Lokteff, 2013). Specifically, both learning to use letters to write words, and learning to use numbers to express quantity, are processes that rely on children's ability to relate symbols with their meaning; suggesting that the acquisition of one system can reinforce the acquisition of the other (Tolchinsky, 2003 as cited in Berghout et al., 2013). There is some empirical evidence that suggest that this might be the case; for example, one study found that children's ability to write their name ('name writing) and their ability to produce the beginning sounds of words ('letter- sounds') were significantly related to early mathematics skills (i.e. finger counting and understanding concepts like "more" and "whole", the ability to read and write numbers and perform addition and subtractions, measured with the Test of Early Mathematics Ability-3, TEMA-3; Berghout, Blevins-Knabe, & Lokteff, 2013). This finding suggests that there is an underlying commonality between children's ability to remember and produce symbols for verbal and written language and for mathematics.

However, there is an alternative explanation for the relationship between letter sounds and name writing and mathematics. An alternative suggestion is ascribing phonological skills with a more general role; that is, these abilities are related to mathematics because for some mathematics skills, children need to manipulate phonological information. This in turn suggest the use of verbal strategies to perform mathematics. For example, one way in which verbal strategies have been found to be particularly useful is when arithmetic problems are presented horizontally. Arithmetic problems that are presented with a horizontal format involve counting strategies and thus the phonological loop (Caviola, Mammarella, Cornoldi, & Lucangeli, 2012; Trbovich & LeFevre, 2003). This was demonstrated in a study were 8-to 9-year-olds solved multi-digit addition problems presented both vertically and horizontally using a dual-task methodology. It was found that when the problems were presented horizontally, performance was impaired by verbal working memory load (Caviola et al., 2012).

1.4.2.2 Vocabulary

The other aspect of children's language that has been associated to mathematics is vocabulary. Vocabulary refers to the ability to provide word definitions (i.e. expressive vocabulary) and to comprehend and respond to words (i.e. receptive vocabulary; Burger & Chong, 2011). Vocabulary supports the acquisition of number words and other mathematics concepts such as: small, add, more, etc. (Foster, 2012). For example, expressive vocabulary is involved in the expression and application of numbers such as it happens in verbal counting (Purpura & Ganley, 2014). This study measured 4-to 6-year-olds' early mathematics skills involving verbal counting, one-to-one counting, cardinality, subitizing, number comparison, set comparison, number order, numeral identification, set to numerals, and story problems. Expressive vocabulary skills were measured with a task in which children needed to identify pictures of objects by answering the question: *"what is/are this/these?"* (i.e. Expressive One-Word Picture Vocabulary Test; Purpura & Ganley, 2014). Expressive vocabulary predicted the mathematics skills that involved number words knowledge, connecting number words to quantities or numbers, and understanding the meanings of comparative terms (i.e. more than, less than). This finding suggests that vocabulary skills may have a specific role in mathematics by providing the means to understand mathematics-specific vocabulary.

For its part, mathematics-specific vocabulary is necessary for children's mathematics performance because it allows children to understand and express mathematics concepts (Purpura & Reid, 2016). For example, the acquisition of quantifiers (e.g. some, many, a few) in English-speaking 2-to 5-year-olds, has been found to be significantly related to their understanding of numbers (measured by asking them to give N objects to the examiner and increasing it by one up to eight objects if they succeeded); suggesting that understanding quantifiers allow children to understand that number words can be used to quantify items (Barner, Chow, & Yang, 2009). Other content-specific words such as spatial terms (e.g. on top, closer, before) allow 4-to 5-year-olds' to talk about relations between objects (Purpura & Reid, 2016) and are related to their building skills while playing with blocks (Ramani, Zippert, Schweitzer, & Pan, 2014). Much more attention has been put into the role of mathematics-specific vocabulary recently, its strong relationship with children's mathematics skills has even lead researchers to propose that the language skills that have been previously found to be related to mathematics skills may have been acting as a proxy for mathematics-specific vocabulary (Purpura & Reid, 2016).

The idea that general language skills are in fact a proxy of mathematics-specific vocabulary was tested in one recent study. This study found that even when general language skills were significant predictors of the early mathematics skills, when the content-specific vocabulary was included in the regression analysis model, mathematics vocabulary, but not general language skills was a significant unique predictor (Purpura & Reid, 2016). This study measured mathematics-specific vocabulary in a sample of 3-to 5-year-olds with an experimental task in which there were 16 items measuring quantitative (e.g. take away, more, less, etc.) and spatial (e.g. nearest, far away, last, etc.) words. The early mathematics skills assessed were counting, one-to-one counting, cardinality, subitizing, number comparison, set comparison, number order, numeral identification, set to numerals, and story problems. These skills mirror children's understanding of mathematics symbols and numbers, in this way, children use specific-mathematics vocabulary during the first years of education to make meaning of the mathematics symbols and numbers (Purpura & Reid, 2016).

To summarize language skills contribute to mathematics both in a general and in a specific manner. That is, generally speaking, children's whole process of learning involves being able to decode language, especially as mathematics are taught and are expressed through written and spoken words. However, language can have a more direct and specific contribution to children's mathematics development. For example, phonological processing skills allow children to learn the symbols for quantity and the application of verbal strategies while solving arithmetic problems that are presented horizontally. Vocabulary, helps with conceptual understanding (in all domains, but notably in mathematics) of mathematics and supports the acquisition of a more specific mathematics-vocabulary which in turn favors children's mathematics learning. Moreover, children with better language skills are better able to automatized aspects of mathematics skills such as strategy use or number facts. However, because children's general learning process is heavily dependent on language (e.g. being able to understand instructions to perform a classroom activity), analyzing specific components of language to specific mathematics concepts without being influenced by general language demands is often challenging. This in turn suggests that the specific role of language skills in mathematics is still poorly understood.

1.5 Environmental Factors

Besides being sensitive to children's individual characteristics, mathematics development is influenced by factors from their social and contextual environment (Jimerson, Egeland, & Teo, 1999; Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2010; McNeil, Fuhs, Keultjes, & Gibson, 2011; Pungello, Kupersmidt, Burchinal, & Patterson, 1996). Understanding children's social and contextual influences is important because children from disadvantaged backgrounds often start their formal mathematics education with gaps in their numerical knowledge, and consequently are at risk of poor mathematics performance in later years (Cheadle, 2008; Entwisle & Alexander, 1990; Siegler, 2009). Two of the most important factors affecting children's mathematics development are socioeconomic status (SES) and caregiver-child interactions. SES is a measure of a person's economic resources and social status mostly understood as a matter of (i) family income or (ii) caregiver education (Ayoub, Gosling, Potter, Shanahan, & Roberts, 2018). Caregiver-child interactions, just as the name suggests, refers to the way in which a parent or caregiver speaks to and interacts with their child. It is known to influence the child's social, emotional, and cognitive development (Graziano, Keane, & Calkins, 2010). An overview of both factors and their interaction to children's mathematics skills will be described next.

The first environmental factor affecting mathematics is SES, SES is generally measured with family income and caregivers' education. Family income, in turn, is related to access to resources, such as number of books at home (Chin & Phillips, 2004), which in turn reflects differences in the quantity and quality of early mathematics activities at home (Bojorque, Torbeyns, Van Nijlen, & Verschaffel, 2018; Vandermaas-Peeler, Nelson, Bumpass, & Sassine, 2009). A longitudinal study found that family income and minority ethnic status were significant risk factors for children's mathematics achievement and that this relationship remained stable over time (Pungello et al., 1996). In this study 7-to- 9-year-olds' mathematics achievement (measured with two standardized academic achievement tests: Science Research Associates Academic Achievement Test-SRA and the Iowa Basic Skills Achievement Test-IOWA) and demographic information, family income, and teacher's report of children's stressful life events, were assessed at four different time-points. Results suggested that over the four years, both low family income and minority ethnic status influenced children's mathematics scores. Moreover, the mathematics scores of children who were from families with low income decreased over time (Pungello et al., 1996).

Low income decreases the possibilities of having access to resources that could stimulate children's mathematics development at home. This is obviously a disadvantage compared to children who can have access to resources that reinforce what was learned in school (although having access to resources is not necessarily a synonym of home-activities that promote mathematics learning, family income and home-activities are commonly closely related). Home activities with the specific aim of boosting children's mathematics skills are more likely to occur in households in which the caregivers have high levels of education. Caregivers' education- specifically maternal education-is related to home-learning experiences that are favourable for children's mathematics development (Davis-Kean, 2005). There are three home-learning experiences that are related to caregiver's education specifically: reading behaviour (how often the child reads for pleasure and how many books does the child have), parent–child play behaviour, and parent-child interactions (Davis-Kean, 2005).These activities in turn favours children's mathematics achievement. One explanation for the relationship between caregivers' education and home environment, is that caregivers with higher levels of education have greater expectations for the mathematics achievement of their children (DeFlorio & Beliakoff, 2015). That being the case, a higher amount of numeracy-related activities at home can be expected from families where parents have higher education levels (Purpura & Reid, 2016).

The second environmental factor affecting mathematics is caregiver-child interactions. There are two types of mathematics-related interactions that can be observed within the child's home environment. These are (i) informal numeracy experiences, which refers to daily-life activities within the context of play that allow interactions around the subject of mathematics without explicitly teaching it (e.g. cooking); and (ii) formal numeracy experiences, which refers to parents' explicit teaching of mathematics (Skwarchuk, Sowinski, & LeFevre, 2014). Both types of interactions have been shown to be significant predictors of mathematics. For example, using a home numeracy questionnaire (where the frequency of informal and formal mathematics activities were measured) one study found that playing board games was a significant predictor of 5-to-7-year-olds' general mathematics knowledge and both playing games and directed instruction of numerical skills, were significant predictors of single-digit additions performance (LeFevre et al., 2009). This study was replicated later by a similar study (Skwarchuk et al., 2014). The most likely explanation for these results is that playing board games enabled children to understand numbers and even apply their mathematics knowledge. For example, a board game in which you need to move forward on the board will entitle children to apply their counting skills. However, these studies have their limitations with regard to questionnaire use and therefore the social-desirability bias related to this practice. Yet, other studies have shown similar findings using other types of methodology, such as video-recording sessions. Two different studies that are particularly informative on this subject are presented below.

Caregiver's number-related talk allows children to develop specific mathematics knowledge. One study for example found that caregiver's number talk contributed to children's cardinal knowledge specifically. In this study the naturalistic interactions of caregiver-child number-related talk was video-recorded for over 90 minutes in a sample of 1-to-3-year-olds. (Levine et al., 2010). The study was conducted along five different visits. The caregiver-child interaction were coded for the total use of number words (from '*one*' to '*ten*') as well as the use of the words '*count*', '*how many*', and '*number*' within a numerical context. When children were around 3 years of age, they completed the 'point-to-x' task measuring children's cardinal knowledge. In this task children were presented with a picture of two vertically arranged sets of squares. Children were asked to point to a specific number between two to six squares. Results showed that the relationship between caregiver's number-related talk and children's cardinal number knowledge remained robust even after controlling for SES, children's own use of number words, and non-numerical talk (Levine et al., 2010). These findings suggest that children's early mathematics-related linguistic experiences can have a positive key role in their mathematics development and opens the possibility to intervention studies in early stages of development.

Caregiver's talk is an important contributor to the development of conceptual knowledge in young children. A second study found that caregivers' mathematics-talk about specific mathematics concepts enhanced children's mathematics knowledge in the same constructs (Ramani, Rowe, Eason, & Leech, 2015). In this study semi-structured caregiver-child interactions were video-recorded. Caregivers and their children (3-to-5-year-olds') participated in three specific activities, all related to mathematics talk. The first activity consisted of reading a book with numerical content; the second activity involved completing a snail puzzle with ten numbered pieces; and the third activity was a board game that had the numbers from one to ten enumerated, and in which children needed to move the number of spaces indicated by a spinner. Caregiver's talk was coded into different categories such as counting, numbers identification, cardinality, ordinal relations, and arithmetic. Children completed different early mathematics tasks measuring: verbal counting, number-line estimation, counting principles knowledge (children needed to identify counting errors), numbers identification, numerical magnitude comparison, and enumeration and cardinality (Ramani et al., 2015). Results showed that exposure to talk from caregivers about advanced number concepts (e.g. cardinality, ordinal relations, and arithmetic) while engaging in number activities predicted children's advanced mathematics knowledge, that is, their performance on cardinality, counting principles, and numerical magnitude tasks.

It should be noted that SES and caregiver-child interactions are themselves closely related. In fact, differences in mathematics knowledge between low-SES and high-SES families can be attributed to less cognitive stimulation from caregivers to their children, because of the overall quality of life within the low-SES families (Fitzpatrick et al., 2014; Siegler, 2009). Thus, parents from high SES backgrounds engage in more mathematics activities, like explaining the use and value of money during play activities, and other mathematics interactions including counting, quantity, and size comparisons; than the low SES families(Vandermaas-Peeler et al., 2009). In turn, children from families of lower-SES are likely to be less exposed to mathematics activities that foster mathematics development.

In sum, an approach to compensate for the disadvantages that income and caregiver-education disparities causes in children's mathematics development, is to focus on caregiver-child interactions. Moreover, it seems that play-activities with mathematics content- like board games- and with direct instructions are a good approach to use the caregiver-child interactions as a mean to enhance children's mathematics learning.

1.6 Summary

Mathematics development is influenced by three main factors: (i) domain-specific abilities such as ANS, subitizing, symbolic number sense, and counting; (ii) domain-general abilities, like executive functions and language; and (iii) environmental factors. Moreover, mathematics development has a hierarchical nature, as such, domain-specific abilities become the building blocks for later mathematics skills such as arithmetic and arithmetic word-problems. Domain-specific abilities start developing really early in life-as early as 49 hours after birth (Izard et al., 2009) - sensory ability to perceive and discriminate quantities the first ability to develop. In parallel to subitizing, children develop counting around the age of 2, however it won't be until the age of 6 years when children truly understand the meaning of the number-words, acquiring the cardinality principle. Then, during the first year of formal schooling, children rely on their basic numeracy skills to solve arithmetic problems. For example, children rely on counting as a strategy for adding to sets of objects or in single-digit arithmetic problems, and their ability to represent numbers spatially to solve multi-digit problems with "carrying" procedures.

Although arithmetic is a core subject during the first years of education, this knowledge acquire more value once children start learning how to apply this knowledge in arithmetic word-problems. It is important, therefore, to take much more applied perspective in research investigating children's mathematics skills. Arithmetic word-problems remains an area that merits additional work to offer a fuller account of how young children use their mathematics skills in the real-world context.

The development of mathematics also involves cognitive resources form domain-general abilities, that although they are necessary skills for learning in general, they have specific contributions to mathematics development. For example in terms of executive functions, working memory and inhibitory control, which broadly speaking have been shown to be significant for keeping track of numbers while counting, represent, hold, and manipulate interim results in multi-digit problem solving; and inhibiting irrelevant information in order to focus on numerical-relevant information. Cognitive flexibility, for its part, seems that rather than being a key factor for mathematics skills, it is necessary as long as there is some degree of switching required in any circumstance. A third key domain-general ability is language. Consensus exist that language skills are required for the performance of some mathematics skills, yet this domain-general ability is relatively unstudied. From the studies that have investigated the role of language there is evidence that good language skills are necessary for the conceptual understanding of mathematics, for the implementation of counting strategies or other verbal strategies, or for the development of mathematics-specific vocabulary.

Finally, environmental factors such as SES and caregiver-child interactions provide the means to enhance children's mathematics skills. One approach to take advantage of caregiver-child interaction's role in children's mathematics development, is by promoting numeracy-based activities, however, further research is needed to determine the intensity of intervention studies in order to produce lasting effects in mathematics.

1.7 Next Questions and Thesis Overview

The goal of this thesis was to investigate the domain-general predictors that underpin mathematics skills in childhood, by the means of three experiments that focused on (i) the contributions of working memory components (Study 1 and 2), (ii) the contributions of concurrent language skills (Study 2 and 3), and (iii) the longitudinal linguistic precursors (Study 3) of mathematics skills presented in different contexts (Studies 1-3). Together, these studies aimed specifically to investigate the working memory and language resources that children need in order to perform mathematics in distinct contexts.

As such, the main question that this research aimed to answer was *How do domain-general abilities contribute to children's mathematics skills inside (i.e. pure mathematics) and outside (i.e. applied mathematics) the school context?* It is evident from this literature review that early numeracy skills and arithmetic skills cover most of the attention in the field of children's mathematics development. Hence, there is still some gaps in our knowledge about the mathematics skills presented in different contexts. The distinction of mathematics skills in different contexts can inform us about the cognitive skills that children need not only to perform mathematics in school, but to apply the knowledge acquired at school in situations of their daily-lives.

Chapter Two looked at the role of the different working memory components in mathematics skills in 5-to 6-year-olds. Among the three main executive functions that have been found fundamental for mathematics development (i.e. working memory, inhibitory control, and cognitive flexibility), working memory has consistently been found to be the strongest predictor of them all. Moreover, most of the evidence suggest that the role of working memory components vary in function of the mathematics skills that is being studied. Yet, very few studies have investigated the role of different working memory components in mathematics skills presented in two distinct contexts. Consequently, the aim of the first study of the current thesis, presented in Chapter Two, is to investigate the specific contributions of working memory components to children's mathematics skills focusing on the distinction between relatively pure mathematics skills and mathematics skills presented in a more applied context.

Chapter Three investigated the specific contribution of language and working memory to 5-to 6-year-olds' pure and applied mathematics skills. This subject is important to address because although many scholars would agree that language skills are fundamental for mathematics development, surprisingly, very little attention has been given to this topic. Even less attention has been given to the interaction between language skills and working memory components to generate different profiles in the performance of mathematics skills.

During the course of my PhD, an opportunity was given to work with a sample of children that had been tracked longitudinally from the age of 11 months to the age of 4 years, and for whom there were data on early language measures. Being able to include children's previous linguistic skills as possible precursors of mathematics performance was valued as a significant next step. Thus, Chapter Four was designed specifically to investigate the longitudinal and concurrent linguistic precursors of mathematics in this specific group of 4-year-olds.

Chapter Two

Exploring the Role of Different Working Memory Components in Mathematics Skills in 5- to 6-year-old Children (Study 1)

The aim of this chapter was to investigate the contributions of working memory components to children's mathematics performance in both pure and applied contexts, in 5- to 6-year-olds. Early mathematics skills, especially those that are learned during the first year of formal education, are significant predictors of children's academic and career success (Lubinski & Benbow, 2006; Passolunghi et al., 2015; Tosto, Asbury, Mazzocco, Petrill, & Kovas, 2016). Moreover, children's early mathematics skills are essential not only for a better school achievement, but also for understanding and carrying out different activities in their daily lives (Gilmore et al., 2013). As children encounter mathematics in many different contexts, investigating mathematics in two distinct contexts (i.e. pure and applied mathematics) will improve our understanding of how children's mathematics skills develop.

To better understand the role of different working memory components in pure and applied mathematics, mathematics skills were assessed using two different measures: (i) the Numerical Operations test (as a measure of pure mathematics) and (ii) the Mathematical Reasoning test (as a measure of applied mathematics). A sample of 78 children between 60 and 79 months (M=69, SD=5.3) from diverse socio-economic backgrounds were assessed in working memory, mathematics, and vocabulary. Results suggested that working memory contributes differently to mathematics skills, depending on the type of mathematics. Children's pure mathematics performance was significantly predicted by the central executive only, whereas their performance in applied mathematics was significantly predicted by the central executive, the visuo-spatial sketchpad, and the phonological loop. Additionally, somewhat unexpectedly, receptive vocabulary was a significant predictor to pure and applied mathematics as well. Overall, results showed that while pure mathematics and applied mathematics shared some common working memory demands (as both relied on the central executive), there were relevant differences too, when doing applied mathematics, children also relied on the visuo-spatial sketchpad and the phonological loop. Moreover, these results showed that the significant association between working memory components and mathematics held even after controlling for age and receptive vocabulary.

2.1 Introduction

Children benefit from good mathematics skills inside and outside school. Within the school context, good mathematics skills are significant predictors for future school success (Claessens & Engel, 2010; Gilmore et al., 2013). For example, mathematics skills at the age of 5 years were found to be significant predictors of achievement in other educational domains including reading and science, at the age of 13 years (Claessens & Engel, 2010). Moreover, as many aspects of everyday life are numeracy-dependant, mathematics outside the school context are important for understanding and carrying out different activities in our daily lives (Gilmore et al., 2013). As mathematics inside and outside the school context are fundamental for children's cognition and successful living, a better understanding of how the cognitive skills necessary for mathematics varies as a function of context would help our overall understanding of mathematics. Furthermore, this understanding would have implications for how mathematics are taught and how we might intervene to help children who struggle with mathematics. Thus, in this study both kind of contexts were investigated together.

Arithmetic skills were chosen as a measure of pure mathematics skills. Arithmetic is considered to be a core number-skill in children's mathematics education, especially during the primary years (Vanbinst, Ansari, Ghesquière, & De Smedt, 2016). Moreover, since the interest of the present study was also to investigate the mathematics skills of children at school entry because children's mathematics skills at school entry have a strong predictive value for later academic achievement (Purpura, Hume, Sims, & Lonigan, 2011), the study was conducted with 5-to-6-year-olds because it is at this age when children are starting school. From now on I will refer to pure mathematics as arithmetic skills. I will start the following section by describing the working memory model in detail to later describe the specific contribution of working memory components to mathematics skills; lastly, I will end this introduction by providing an overall description of the current study.

Working memory refers to the system involved in holding and manipulating information in mind temporarily (Baddeley, 2010). The most influential model of working memory postulates the existence of three different, but related, components: The central executive as a supervisory system, and two subsidiary systems, the visuo-spatial sketchpad and the phonological loop (Baddeley & Hitch, 1974, as cited in Baddeley, 2010). Each of these working memory components has a specific role in how information is stored, manipulated, and retrieved; whereas the central executive requires storage and manipulation of information, the visuo-spatial sketchpad and the phonological loop are involved only in the storage of domain-specific information (Friso-van den Bos et al., 2013). Because of the specificity in which these different components are involved in processing information, this model provides a useful framework for understanding the specific relationship between working memory and different mathematics skills.

The central executive has a widespread importance for many aspects of general cognition (Baddeley, 1996). Over the years, research has been conducted to understand the involvement of this component in different cognitive abilities (RepovŠ & Baddeley, 2006). The central executive has been shown to play a variety of roles (Baddeley, 1996; Bull & Scerif, 2001). Specifically, four different functions have been ascribed to the central executive. One of the functions is the already mentioned, ability to coordinate information from the visuo-spatial sketchpad and the phonological loop (Baddeley, 1996). This feature of the central executive involves the ability to carry out two tasks simultaneously and keeping information updated in the working memory (Baddeley, 1996; Bull & Scerif, 2001). A second function is the control of attentional resources. This feature was proposed to involve the guiding of attentional resources in selecting and rejecting incoming information (Baddeley, 1996; Baddeley, Emslie, Kolodny, & Duncan, 1998). A third function that has been related to the central executive is the ability to switch attention (Baddeley, 1996). This feature in turn has been related to the ability to switch between tasks (Baddeley, 1996; Bull & Scerif, 2001). Lastly, a fourth function is the ability to select and retrieve information from the long-term memory (Baddeley, 1996; Bull & Scerif, 2001).

This multi-purpose nature of the central executive explains how is it that the central executive has been found to be consistently and significantly related to other complex cognitive skills involved in 4 to-6-year-olds' learning (Alloway, Gathercole, Willis, & Adams, 2004; Barker, 2016) including language (Bourke & Adams, 2003) and mathematics skills (Meyer et al., 2010; Simmons et al., 2012). The central executive has also been conceptualized as a unitary control system with multiple functions which main role is to coordinate and manipulate information stored within the subsidiary systems (RepovŠ & Baddeley, 2006). In the present study such conceptualization is adopted (Meyer et al., 2010).

The visuo-spatial sketchpad is responsible for temporarily holding both visual and spatial information online. The visuo-spatial sketchpad has also been described as playing the role of generating and manipulating mental images (Baddeley, 2003). This component can be divided in its visual (object-related memory) or spatial (spatial-related memory) sub-components (Della Sala et al., 1999; Klauer & Zhao, 2004). These two storage systems are specialized in the information they can retain, that is, the visual sub-component will be involved in retaining features such as orientation, colours, or shapes; and the spatial sub-component will be involved in retaining information about location and action (RepovŠ & Baddeley, 2006). Although the functions of these sub-components are different, they have been suggested to work in combination to subsidize the temporary retention of both spatial and visual features of the environment (Hamilton, Coates, & Heffernan, 2003).

The phonological loop is responsible for the temporary storage of verbal information which can be maintained and retrieved via articulatory rehearsal (Baddeley & Logie, 1999; De Smedt et al., 2009). Although this component has two different functions (storing verbal information, and retrieving verbal information), the role of the phonological loop, before the age of 7 years, is to store verbal information temporarily (Gathercole et al., 2004). After the age of 7 years, the phonological loop will allow children to use verbal strategies (e.g. in a memory task where children need to remember images labelling the images lead to better performance) or remembering information through cyclic rehearsal to maintain and retrieve information. One suggestion for why verbal strategies are not present in younger children is that children are naïve about the advantage of using labels to code information (Pickering, 2001). It has also been suggested that the emergence of verbal strategies goes hand in hand with the development of literacy (Logie, Della Sala, Wynn, & Baddeley, 2000). As such, young children will rely on their phonological loop to solely store verbal information, and until they are conscious of how to benefit from their language skills to maintain and retrieve information, they will rely on their phonological loop to generate verbal strategies (Logie et al., 2000; Pickering, 2001).

2.1.1 The contributions of working memory components to arithmetic skills and applied mathematics

This section will review the contributions of working memory components to arithmetic skills and applied mathematics in 5-to-6-year-olds. It will first start with an overview of findings regarding the role of (i) the central executive, (ii) the visuo-spatial sketchpad, and (iii) the phonological loop, to arithmetic skills. Following this overview, the contributions of these three working memory components to applied mathematics will be presented.

Concerning arithmetic skills, the relationship between the working memory components and arithmetic skills will differ depending on two main factors: (i) the strategies that children use to solve the problems, and (ii) the level of skills or familiarity that the children have in solving arithmetic problems. In turn, these two factors are strongly related to the age of the children because older children would be further along in their process of learning mathematics and as such, will be more familiarized with the mathematics procedures and strategies of calculation. One example for this developmental trend is the development of counting strategies which are the tool that is most frequently used by young children to solve arithmetic problems.

The central executive is particularly relevant for children's counting strategies, because the central executive allows the control and allocation of attentional resources during the monitoring of numbers (Geary et al., 2004, 2012; Meyer et al., 2010; Noël, 2009; Simmons et al., 2012). Between the ages of 3 and 5 years, children will rely on physical aid like finger-counting to represent the addends of a given arithmetic problem (Fuson, 1982; Gray & Tall, 1994). Finger-counting is considered to be a substitute for children's lack of ability to manipulate numerical information in their working memory[2]. Through development children will transition from finger-counting to verbal counting, which is a process that occurs around the age of 5 years (Fuson, 1982). This transition is thought to be supported

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[2] However, findings from a very recent study suggest that 6-year-olds with high working memory skills do rely on counting with their fingers while performing additions, and they even use these strategies more frequently than children with low working memory skills (see Dupont-Boime & Thevenot, 2018 for a review).

by the central executive because children move from relying on concrete tools to abstract representation of numbers, requiring higher attentional resources to keep track of the numbers being counted (Fuson, 1982).

The visuo-spatial sketchpad is thought to function as a sort of mental blackboard that is used to represent and maintain numerical information mentally as it would happen in a written format (Bull, Espy, & Wiebe, 2008; Bull et al., 1999; Holmes & Adams, 2006; McKenzie et al., 2003; Rasmussen & Bisanz, 2005). The representation of numerical information is carried out through the use of analogous tokens that facilitate children's computation skills. For example, imagining counting dots for the numbers involved in an arithmetic problem (e.g. imagining four dots to represent number '4'; Bull et al., 1999). Thus, children rely on their visuo-spatial sketchpad to solve arithmetic problems by representing the problem mentally (Rasmussen & Bisanz, 2005).

The role of the phonological loop in arithmetic is to allow children to store the digits involved in a problem by the use of verbal labels (Rasmussen & Bisanz, 2005). However, coding digits to verbal labels is a strategy that most children younger than 6 or 7 years of age have not yet achieved (Hitch, Halliday, Schaafstal, & Schraagen, 1988; Meyer et al., 2010). Unless the numerical information is presented verbally, and hence, involving the phonological loop for its storage, children younger than 7 years are less likely to be relying on the phonological loop to solve arithmetic problems; as such, they will only recruit resources from the central executive and the visuo-spatial sketchpad. In sum, when children are younger than 7 years of age, the phonological loop is involved in the storage of verbal information, whereas after the age of 7 this component plays a role in children's verbal strategies such as articulatory rehearsal strategies (Hecht et al., 2001; McKenzie et al., 2003; Meyer et al., 2010).

Concerning applied mathematics, these are commonly investigated through children's performance on arithmetic word-problems. Arithmetic word-problems are mathematics problems presented inside a narrative (e.g. *Michelle went to the Peak District and picked up seven flowers. If she gave* *three flowers to her mother and one flower to her sister, how many flowers will she have then?*). Thus, they simulate the kinds of contexts in which people commonly encounter mathematics in daily life. The main characteristic of these types of problems is that the numerical information is embedded within a context; of necessity, this involves information that is not relevant to the purely arithmetic aspect of the problem. As such, that irrelevant information needs to be ignored in order to solve the problem.

The role of the central executive, the visuo-spatial sketchpad, and the phonological loop seems to be determined by three main characteristics: (i) the format in which problems are presented in (i.e. whether they are presented with useful visual stimulus or not), (ii) if they are presented with irrelevant information to be ignored, or (iii) the semantic structure of the problem (i.e. if the relational terms are in line with the calculation that needs to be performed- e.g. an addition when hearing "*more than*"; Riley, Greeno, & Heller, 1983; Simmons, Singleton, & Horne, 2008). The central executive is recruited in problems in which the representation of numerical information is decreased by the means of visual prompts which provide visual aid to achieve an answer. Its main role would be directing attentional resources to attend to key numerical information while inhibiting irrelevant information (Baddeley, 1996; Baddeley et al., 1998). When there is no visual support available, the need to represent the numerical information mentally increases, and thus, the visuo-spatial sketchpad will be recruited (Simmons et al., 2008).

Two different studies have found the association of the central executive and arithmetic word-problems proficiency in young children. Both of these studies presented the arithmetic word-problems verbally and with visual prompts. One study, conducted with 4-to-5-year-olds, used arithmetic word-problems presented verbally (e.g., "*Here are three cows; if four more come, how many cows will there be?*") in which the first operand (i.e. three) was always visible for the children. Additionally, tokens were provided to the children in case they wanted to make use of them to solve the problems (Noël, 2009). The three working memory components were measured, but only the central executive was a predictor of arithmetic word-problems. The second study found similar results so that the central executive was also a significant predictor. In this study the central executive predicted 4-to-7 year olds' abilities to solve arithmetic word-problems such as*" A shopkeeper has seven chickens. He buys three more. How many chickens does the shopkeeper have now?"* (Kyttälä et al., 2014). Both of these studies found that the central executive was the significant predictor among the three working memory components. Thus, together, these studies suggest that when there is some sort of visual aid that children can utilize to solve the problems, they only need to recruit central executive resources to inhibit irrelevant information in order to attend to key numerical information.

Together these studies suggest that the role of the working memory components in arithmetic word-problems may be constrained by the format of such problems. Specifically, Noël, and Kyttälä and colleagues' studies, the problems were presented both verbally and with visual prompts. In Noël's (2009) study, for each of the problems the first addend was always visible to the child, which could have diminished the need to temporarily hold in mind numerical information. Additionally, tokens were available for the children to solve the problems, which in turn could have lessen the visuo-spatial sketchpad demands. In fact, these external tools were more frequently used in children with low visuo-spatial sketchpad skills (Noël, 2009). In Kyttälä, and colleagues' (2014) study, all problems were also presented with pictures that represented each one of them and possible answers for the problem were also provided (i.e. a row of squares containing the answers).

The visuo-spatial sketchpad seems to be recruited when the visual aid (or the lack of it) in the arithmetic word-problem does not contain information that helps the children to represent the problem (Simmons et al., 2008). One longitudinal study conducted with 5-year-olds, measured children's visuo-spatial sketchpad skills with the Rabbits tasks when children were 5 years of age, and arithmetic word-problems a year after. The Rabbits task was a computerized task in which a rabbit appeared in different black holes in a sequence. Children needed to tap the same black holes in the same order in which the rabbit appeared. The arithmetic word-problems were presented both verbally and with pictures that did not provide visual aid to solve the problems. For example for the problem "*each horse need four new shoes, how many horse-shoes must the farmer get altogether*" the picture that was paired with the problem represented the horses without the legs (Simmons et al., 2008). Results showed that even after controlling for vocabulary, non-verbal reasoning, and reading skills, the visuo-spatial sketchpad predicted arithmetic word-problems. However, although there was a significant association with performance on arithmetic word-problems and the visuo-spatial sketchpad, it is important to highlight, that in this study the central executive was not included in their design and thus, the possible contribution of the central executive cannot be ruled out completely.

Young children seem to favour visual-spatial strategies rather than a verbal or phonological approach to solve arithmetic word-problems. In fact, in young children, the phonological loop has not always been found to contribute to arithmetic word-problems when the other two working memory components are also included in the design of the studies (e.g. Noël, 2009). The studies that have found a significant contribution from the phonological loop to arithmetic word-problem solving have suggested that the phonological loop is especially relevant when the problems have irrelevant information to be ignored (Rasmussen & Bisanz, 2005). However, since the role of this component is to store verbal information, and the format in which the arithmetic word-problems is presented is with the use of verbal cues, children may rely on their phonological loop to hold the verbal information within the problem in mind.

To summarize, the central executive has a multi-purpose nature, being involved in both arithmetic and applied mathematics by allocating attentional resources to: counting strategies and relevant numerical information to solve the mathematics problems. For the visuo-spatial sketchpad evidence points at its role being dependant on the format in which the problem is being presented, with or without visual aids. For the phonological loop we can think of it as a storage unit for verbal information when children are younger than 6 years, but with a role in the use of verbal strategies when children are older than 7 years. As such, the age of the children and what is developmentally accessible to them determine the way children "do mathematics"; either by directing attentional resources to numerical information (by the means of the central executive), visualizing the numerical quantities in mind (by the means of the visuo-spatial sketchpad), or using verbal strategies (by the means of the phonological loop). As such, 4-to-5-year-olds solve arithmetic problems through the use of mental models, recruiting their visuo-spatial sketchpad, but around the age of 7 years as their language skills become stronger, the use of verbal strategies by the means of the phonological loop will be use more often; even in mathematics problems where they used their visuo-spatial skills before (Huttenlocher et al., 1994).

The change from visuo-spatial skills to the use of the phonological loop seem to be related to children's development of reading and phonological processing skills (Pickering, 2001). However, this does not mean that once children develop verbal strategies they stop relying on their visuo-spatial sketchpad. For example, one study investigated 8-to-9-year olds' performance on easy vs difficult items in different mathematics measures (e.g. arithmetic, algebra, and geometry; Raghubar, Barnes, & Hecht, 2010). Results showed that children rely on their phonological loop to solve the easy items, but their visuo-spatial sketchpad to solve the difficult items. As such, the employment, whether of the visuo-spatial sketchpad or the phonological loop, will also depend on the mathematics tasks demands. The visuo-spatial sketchpad and the phonological loop will exist together in continuity through development. This in turn will allow an increase on the number of verbal or visuo-spatial codes available to store, represent, and retrieve information in the working memory; mirroring a mature working memory system (Pickering, 2001).

Attempting to understand the specific roles of different working memory components to mathematics skills is not a novel idea. However most of the empirical evidence comes from studies conducted in children older than 6 years and thus, there are some gaps in our understanding of how it is that these components are relevant for 5-to-6-year-olds' pure and applied mathematics. Information available regarding the contribution of working memory components to arithmetic skills and applied mathematics, in typically developing 5- to 6-year-olds is sparse. However, research conducted in children that are in their first year of formal education can help us understand the domain-general cognitive resources that children utilize in order to perform mathematics, and thus understand how we can intervene in order to enhance their performance.

2.1.2 The current study

The aim of this study was to investigate the contributions of working memory components to both arithmetic skills and to applied mathematics in 5- to-6-year-olds. To investigate these two different aspects of mathematics skills, two different measures were selected, the Numerical Operations and the Mathematical Reasoning. These measures are part of a standardized achievement test, the WIAT-II (Wechsler, 2001). The Numerical Operations test was selected to measure arithmetic skills, and the Mathematical Reasoning test was selected to measure applied mathematics skills. These two measures are frequently used to measure such constructs within the developmental psychology research area.

The Numerical Operations is a paper-based test containing multiple arithmetic problems, similar to the mathematics skills that children encounter within the school context. The Mathematical Reasoning test includes arithmetic word-problems such as "*Neil had 5 marbles, then his mom gave him 3 more, how many marbles did he have then?*" This measure uses verbal and visual prompts, and includes a variety of different mathematics domains, that change from item to item. Using standardized task such as these provides a sensitive measure of children's mathematics knowledge, since children will progress in the task as far as their mathematics abilities allow (Lee & Bull, 2015).

One measure for each working memory component was selected. The Backward Digit Recall was selected to measure the central executive, the Block Recall task to measure the visuo-spatial sketchpad, and the Non-Word List Recall to measure the phonological loop. Consequently, this study allows us to examine whether or not the roles of working memory components to developmentally relevant mathematics skills, change according to the mathematics skill that is being measured.

Additionally, receptive vocabulary was also measured. Receptive vocabulary was included as a covariate given the evidence of the importance of language skills in children's mathematics development (Hornung et al., 2014a; Lefevre et al., 2010; Passolunghi, Mammarella, & Altoe, 2008; Purpura et al., 2011). For example, it has been found that vocabulary and children's knowledge of letters at the age of 3 to 5 years, were significant predictors of children's numeracy skills one year later (Purpura et al., 2011).

Three main hypotheses were formulated for this study:

First, it was hypothesised that children would need to allocate attentional resources to counting strategies and to relevant numerical information to solve the mathematics problems. If that were the case, then we would expect to see a significant contribution from the central executive to children's performance, in both types of mathematics measures.

Second, it was hypothesised that children would need to temporarily store and represent the numerical information given in the problem by the means of mental representations. If that were the case, then we would expect to see a significant contribution from the visuo-spatial sketchpad to children's performance, in both types of mathematics measures.

Third, it was hypothesised that children would require the storage of numerical information presented verbally. If that were the case, then we would expect to see a significant contribution from the phonological loop to children's applied mathematics.

Additionally, it was predicted that the phonological loop would not be involved in arithmetic skills based on research that suggest that children have not acquire strategies such as verbal labelling of numerical information yet.

2.2 Method

2.2.1 Study design

The present study had a cross-sectional design. Testing took place in one single testing session. The outcome variables were arithmetic skills (as measured by the Numerical Operations test) and applied mathematics skills (as measured by the Mathematical Reasoning test). The predictor variables were performances on the working memory measures: the backward digit recall task that measures the central executive; the block recall task that measures the visuo-spatial sketchpad; and the nonword list recall task that measures the phonological loop. Chronological age, SES, and vocabulary (measured with the British Picture Vocabulary Scale–second edition, BPVS-II; Dunn, Dunn, Whetton, & Burley, 1997) were included as covariates.

2.2.2 Participants

Participants were recruited from three schools in Sheffield, UK. According to the Index of Multiple Deprivation (IMD) calculated using the school post code, these schools were from a broad range of socio-economic backgrounds (1st, 5th, and 9th decile in the IMD; Ministry of Housing, Communities & Local Government, 2015). A power analysis (conducted using the G*Power software) for a linear multiple regression with four predictors was performed with $\alpha = 0.05$, a medium effect size $f^2=.15$ and a desired power of 0.80. The power analysis resulted in a required overall sample size of 85 children. However, due to circumstances beyond my control and due to time constrains, participants were 79 children (39 female). This means that the only stopping rules for data collection were time, and of course, the school agreement on the time that data collection would take without interfering with other activities related to the school.

Data from one child were excluded because of failure to understand instructions. Final data came from 78 children with ages between 60 and 79 months (*Mean age*= 68.78, *SD*=5.31). Because of the age range selected, the sample included both children from Reception year (*N*=27; with ages between 5 years 0 months to 5 years and 4 months) and Year 1 (N= 51; with ages between 5 years 5 months and 6 years and 7 months). Written consent was obtained from parents before testing began and ethical approval was obtained from the Department of Psychology's ethics sub-committee.

2.2.3 Materials

To measure mathematics skills two standardized mathematics subtests from the WIAT-II were administered: a Numerical Operations test, to measure arithmetic competences and Mathematical Reasoning, to measure applied mathematics skills.

Numerical Operations. This is a paper-and-pencil test that measures a number of different basic mathematics skills. These are: the ability to identify, discriminate, and write numerals; rote counting; counting with 1:1 correspondence; and solving basic written operations in increasing complexity (i.e. additions, subtractions, multiplications, and divisions; Wechsler, 2001). There are no time constraints in this test so children have the option to solve as many problems as they can.

Mathematical Reasoning. This test measures the ability to count; to identify geometric shapes; and to solve single and multi-step word problems. The problems to be solved are presented simultaneously in verbal and visual formats. The verbal and visual prompts used in this test are related to everyday applications of mathematics skills in terms of time, money, and measurement. As an example of an item that measures children's ability to solve problems using money, a picture showing 3 different groups of coins (seven pennies, six 5p coins, and one 10p coin) is shown to the child. The child is then asked which group is worth the most. Items increase in complexity so that the child is also required to solve problems with whole numbers, fractions or decimals, to interpret graphs, to identify mathematical patterns, and to solve problems related to statistics and probability (Wechsler, 2001). There are no time constraints in this test. The test ends when children make six consecutive errors.

The working memory measures were chosen from the Working Memory Test Battery for Children (WMTB-C; Pickering, 2001) which is designed to assess the three working memory components of Baddeley and Hitch's model. The Backward Digit Recall was selected to measure the central executive, the Block Recall task to measure the visuo-spatial sketchpad, and the Non-Word List Recall to measure the phonological loop. All measures started with three practice trials (except backward digit recall which had four practice trials) at the start of the testing trials.

Backward Digit Recall. The backward digit recall task was used to measure the central executive, this task required children to recall numbers in a backward order (Pickering, 2001). The task began with four practice trials using number cards (though the test trials only involved verbal cues). The number cards on the practice trials were used as visual aids to ensure that the child understood the concept of ''reverse" before testing commenced. The number cards were presented to the child one at a time (e.g. a card with a "2" and a card with a "3"), and then they were moved into a reversed order (e.g. the card with a "3" first and then the card of the "2"). It was explained to the child that that was the order in which the numbers needed to be repeated. There were two practice trials of two digits, and two practice trials of three digits. Any errors by the child were corrected. Testing commenced after the practice trials. There were seven testing-levels, each comprising six trials. During testing, the child was asked to listen to a string of number-words given by the experimenter, and then asked to repeat them in reverse order. For example, if the child heard "eight", "one", "three", the correct response would be to respond "three", “one”, “eight”. The first testing-level started with two number-words, and then the sequence length increased at a rate of one number-word every testing-level. This measure (and the further two working memory measures that follow) was scored following the next WMTB-C testing rules:

1. The ‘move on” rule. This rule stated that if a child correctly recalled the first four trials in a testing-level, then testing can move on to the first trial of the next testing block. But if an error was made in the first four trials, the fifth trial needed to be administered. If two errors were made in the first five trials, then the sixth and final trial needed to be administered. Any trials that were omitted as a consequence of the moving on rule were scored as correct.

2. The ‘discontinue rule’. This rule stated that testing should be discontinued if a child made three errors in a single testing level. Any correct response made until the point at which testing was discontinued was recorded.

Block Recall. The block recall task was used to assess the visuo-spatial sketchpad (Pickering, 2001). The block recall measure consists of a board with a set of nine identical blocks set out in a non-regular pattern. On each trial, the experimenter taps a sequence of blocks at a rate of approximately one block per second. The child is then asked to duplicate the tapping in the same order. The three practice trials before testing involved practice with a sequence of one cube, two cubes, and three cubes, respectively. Feedback was given to the child if he or she made a mistake in any of the practice trials. There were nine testing levels in total, each containing six trials. The first testing level started with trials of one block, and then the sequence length increased at a rate of one block every testing level, up to a maximum of six. No block was tapped more than once within a sequence.

Nonword List Recall. The nonword list recall task was used to assess the phonological loop (Pickering, 2001). As the name suggests, the nonword list recall measure comprised a list of non-words that the experimenter read aloud. All non-words were one syllable non-words (for example "jat" or "gub"). After hearing the word, the child was ask to repeat them back exactly as he or she heard them and in the exact same order. The three practice trials before testing involved one non-word, two non-words, and three non-words, respectively. There were six testing-levels in total, and each testing-level contained six trials. Trials in the first testing block were single non-words, then the sequence length increased at a rate of one non-word every testing block.

British Picture Vocabulary Scale-II. Vocabulary was measured by the BPVS-II (Dunn et al., 1997), which is a measure of receptive vocabulary. In this measure, children are presented with a series of picture-based trials. Each trial consists of four pictures, and the child is asked to listen to a specific word and to identify which of four pictures corresponds to the target word. There are 14 testing-blocks, each of which contained 12 trials. The trials increase in difficulty across the testing-blocks. Testing stops after a total of eight errors in any of the testing blocks.

2.2.4 Procedure

All children participated in a single testing session that lasted approximately 30 minutes in a quiet area of their school. Each child was tested individually in a quiet area of the school and gave verbal assent in addition to their parent's written consent. At the start of each session children were informed that they were there to play some maths, memory, and words games. Each session began with the Mathematical Reasoning test, followed by the Numerical Operations test, the Nonword List Recall measure, the Block Recall Measure, the Backward Digit recall measure, and the BPVS. All measures were administered in the same order for every child. At the end of the testing session all participating children received a sticker as a token of appreciation for taking part.

2.3 Results

Correlations and hierarchical multiple regression analyses were performed to test the study hypotheses. These analyses were conducted to identify the working memory components that accounted for significant variance in arithmetic skills and applied mathematics. Preliminary analyses were run to test whether there was an effect of SES, gender, and school year (Reception year vs Year 1), on any of the variables.

2.3.1 Descriptive statistics

Descriptive statistics for age in months, and for the raw scores in the mathematics, the working memory, and the vocabulary measures are shown in Table 1. In this study, only raw scores are used in the statistical analyses. Skewness and kurtosis values indicate that not all variables' scores were normally distributed. A further Shapiro-Wilk test confirmed the non-normal distribution of all variables (all $p<.05$) except for Block Recall ($p=.34$) and Numerical Operations ($p=.08$). Thus, non-parametric tests are used in the following analyses.

Table 1. Descriptive statistics for age (months) and the raw scores for the mathematics, working memory, and vocabulary, measures in Study 1

Variable	Mean (SD)	Range min-max	Skewness	Kurtosis
Age (months)	68.78 (5.31)	60-79	.14	-1
Numerical Operations	9.60 (2.34)	5-15	.04	-.37
Mathematical Reasoning	16.65 (6.19)	6-33	.52	-.46
Backward Digit Recall	8.10 (3.10)	1-19	.86	1.69
Block Recall	18.01 (4.51)	6-31	.10	.77
Nonword List Recall	10.33 (3.11)	2-19	-.52	.61
BPVS	53.71 (14.30)	13-90	.10	.82

2.3.2 Preliminary analysis

Preliminary analyses were conducted to test whether there was an effect of SES and gender on any of the measures. Because of the age-range of the children in this study, some of the children were attending Reception year and some of the children were attending Year 1 during the period of testing, thus, analyses were also conducted to test whether there was an effect of school year. To test for a possible effect of school, a Kruskal-Wallis test was conducted. To test for possible gender and school year effects, Mann-Whitney tests were conducted.

Results from the Kruskal-Wallis test revealed that there was a significant effect of school on most of the measures. Significant effects of school were found in Numerical Operations [H (2) = 20.11, $p<.001$], Mathematical Reasoning [H (2) = 32.91, $p<.001$], Backward Digit Recall [H (2) = 15.81, $p<.001$], Block Recall Scores [H (2) = 16.49, $p<.001$], and the BPVS scores [H (2) =17.09, $p<.001$]. Post-hoc Mann-Whitney tests were used to follow up these findings. A Bonferroni correction was applied so all effects are reported at a .0167 level of significance. Results revealed that differences between the 1^{st} and the 5^{th} IMD deciles were specific to scores in Numerical Operations (U=158, r=-.48), Mathematical Reasoning (U=125.50, r=-.55), and Backward Digit Recall (U=189, r=-.40), such that scores were significantly higher in the 1^{st} decile than in the 5^{th} decile (see Table 2 for the variables medians and Appendix 1 for Post-hoc results for the remain non-significant variables).

Table 2. Differences between the 1^{st} and the 5^{th} IMD deciles. Variables medians in Study 1

	1^{st}	5^{th}
Numerical Operations	10.50	8
Mathematical Reasoning	15.50	11
Backward Digit Recall	7.50	6

Note. All $p< .0167$

Differences between the 1^{st} decile and the 9^{th} decile were only on the BPVS scores (U=169.50, r=-.41), such that BPVS scores were higher in the 9th decile (*Mdn*=63) than in the 1st decile (*Mdn*=51.50). No other significant differences were found. As for the comparisons between the 5th and the 9th decile, differences were specific to scores in Numerical Operations (U=109.50, *r*=.59), Mathematical Reasoning (U=49.50, *r*=-.73), Backward Digit Recall (U=136.50, *r*=-.52), Block Recall (U=117.50, *r*=-.56), and the BPVS (U=120, *r*=-.55), such that scores were higher in the 9th decile than in the 5th decile (see Table 3 for the variables medians).

Table 3. Differences between the 5th and the 9th IMD deciles. Variables medians in Study 1

	5th	9th
Numerical Operations	8	10
Mathematical Reasoning	11	21
Backward Digit Recall	6	9
Block Recall	16	21
BPVS	47	63

Note. All $p < .0167$

These differences between IMD deciles are likely to be indicating a school effect rather than a SES effect on Numerical Operations, Mathematical Reasoning, Backward Digit Recall, and Block Recall; since there don't seem to be any differences that vary as a function of SES (i.e. there were no differences between the extreme deciles 1st and 9th) these data are not included in subsequent analyses and won't be discussed further.

In terms of gender and school year, the Mann-Whitney tests revealed that there was no significant effect of gender on any of the measures. However, significant effects of school year were found in all measures, such that children in Year 1 had significantly higher scores in the mathematics, working memory, and vocabulary measures, than children in Reception year (see Table 4).

Table 4. Mann-Whitney test results. Differences between Reception and Year 1 in Study 1

	U	*R*	*Z*	*Medians* Reception Year	Year 1
Numerical Operations	267.50	.51	-4.47	8	10
Mathematics Reasoning	175	.61	-5.41	11	20
Backward Digit Recall	325.50	.44	-3.87	6	8
Block Recall	373	.38	-3.33	16	18
Nonword List Recall	478.50	.25	-2.24	10	11
BPVS	411.50	.33	-2.91	47	55

Note. All p<.001 except Nonword list recall (p=.03)

2.3.3 Non-parametric correlations between working memory components and mathematics measures

In an attempt to understand the relationship between the working memory components and mathematics skills, a simple non-parametric correlation analysis was conducted. Next, two hierarchical multiple regressions were performed to explore the significant correlations more closely.

Associations between mathematics measures, working memory measures, vocabulary measure, and age, are presented in Table 5. The associations were examined with non-parametric simple correlations. As shown in Table 5, both mathematics measures were significantly correlated to all three working memory measures, the BPVS, and age.

Table 5. Correlation matrix reporting simple non-parametric correlations in Study 1

	1	2	3	4	5	6	7
1. Numerical Operations	1						
2. Mathematical Reasoning	.72***	1					
3. Backward digit recall	.69***	.70***	1				
4. Block recall	.43***	.62***	.54***	1			
5. Nonword list recall	.44***	.50***	.40**	.19	1		
6. BPVS	.57***	.65***	.51***	.41***	.40***	1	
7. Age (in months)	.62***	.69***	.52***	.46***	.39***	.46***	1

Note. Correlations= *<0.05, ** <0.01, and *** <0.001

Because the BPVS and age were also significantly correlated to the working memory measures, a non-parametric partial correlation was conducted to examine whether the significant correlations between the mathematics and the working memory measures remained after eliminating the variance related to the BPVS and age (Table 6). The analyses were performed using SPSS Syntax.

Table 6. Correlation matrix reporting partial non-parametric correlations controlling for receptive vocabulary and age in Study 1

	1	2	3	4	5
1. Numerical Operations	1				
2. Mathematical Reasoning	.39***	1			
3. Backward Digit Recall	.47***	.46**	1		
4. Block Recall	.13	.42***	.34**	1	
5. Nonword List Recall	.19	.26*	.17	-.06	1

Note. Correlations= *<0.05, ** <0.01, and *** <0.001

After eliminating the variance related to the BPVS and age (Table 6), the significant correlations between Numerical Operations and the working memory components changed. As such, Block Recall and Nonword List Recall were not significantly related to Numerical Operations, and only the Backward Digit Recall remained significant. These results suggest that BPVS and age share substantial variance with both Block Recall and Nonword List Recall.

The relationship between Mathematical Reasoning and the working memory components remained significant even after scores from the BPVS and age were partialled out.

2.3.4 Hierarchical multiple regressions analyses

Partial correlations indicated that the BPVS and age may share variance with the working memory measures. Thus, two hierarchical multiple regressions were conducted to further investigate the amount of unique and incremental contributions to Numerical Operations and Mathematical Reasoning respectively, by the working memory measures. The predictor variables entered in the model were selected on the basis of whether they were significantly related to the outcome variables or not. The first hierarchical multiple regression (Table 7) shows the analysis conducted to investigate Numerical Operations predictors. To control for age and the BPVS, these variables were entered in step 1. Backward Digit Recall was entered in step 2, Block Recall was entered in step 3, and Nonword List Recall was entered in step 4. In this way, any final step that accounted for significant additional variance, shared unique links with Numerical Operations. The order of entry of the predictors was based on the magnitude of the Spearman's simple correlation (see Table 5).

As shown in Table 7, step 1 for the Numerical Operations test indicated that age and the BPVS predicted 52% of significant variance. Step 2 shows that the additional incorporation of Backward Digit Recall contributed an additional 8% significant variance. Finally, step 3 and 4 show that including Block Recall (step 3) and Nonword Recall (step 4) did not change the findings in step 2, indicating that neither Block Recall nor Nonword Recall accounted for additional unique variance. The Durbin-

Watson test was checked and was found to be within acceptable parameters (2.03), thus the assumption of independent errors was met. This result suggests that this regression model is unbiased, increasing the likelihood of these results to be true for a wider population (Field, 2009).

Table 7. Hierarchical regression analysis predicting unique variance in Numerical Operations in Study 1

Step		B	SE B	β	*t*	*p*
1	(Constant)	-7.22	2.51		-2.88	.01
	Age	.20	.04	.44	4.79	.001
	BPVS	.06	.02	.39	4.18	.001
	F (2, 75) =39.95, p <.001, R^2=. .52, p<.001					
2	(Constant)	-4.54	2.40		-1.90	.06
	Age	.14	.04	.32	2.83	.001
	BPVS	.04	.02	.26	3.53	.01
	Backward Digit Recall	.28	.07	.36	3.92	.001
	F(3,74)=36.83, p <.001, ΔR^2=.08, p<.001					
3	(Constant)	-4.54	2.41		-1.88	.06
	Age	.14	.04	.32	3.38	.001
	BPVS	.04	.02	.26	2.79	.01
	Backward Digit Recall	.27	.07	.36	3.66	.001
	Block recall	.01	.05	.02	.17	.87
	F(4, 73)=27.27, p<.001, ΔR^2=.001, p=.87					
4	(Constant)	-4.53	243		-1.87	.07
	Age	.14	.04	.31	3.29	.002
	BPVS	.04	.02	.25	2.71	.01
	Backward Digit Recall	.27	.08	.36	3.56	.001
	Block Recall	.01	.05	.02	.18	.86
	Nonword List Recall	.01	.06	.01	.12	.91
	F (5, 72) = 21.52, p<.001, ΔR^2=.001, p=.91					

As one can see from the final step, there are three significant beta values indicating that age (β=.31), the BPVS (β=.25), and Backward Digit Recall (β=.36) are significant independent predictors of Numerical

Operations. As the standardised beta coefficient (β) is measured in standard units, they can be directly compared with one another. This indicates that in predicting scores on Numerical Operations, age and Backward Digit Recall are stronger predictors than the BPVS; and Backward Digit Recall is stronger than age. Multicollinearity was checked using the variance inflation factor (VIF) which quantifies the severity of multicollinearity. The largest VIF was well below 10, and the average VIF was 1.54. Similarly the tolerance data are all within acceptable boundaries (all greater than 0.1). Therefore, it was concluded that there was no collinearity within the data (Field, 2009).

The second hierarchical multiple regression (Table 8) shows the analysis conducted to investigate Mathematical Reasoning predictors. Age and the BPVS were entered in step 1, following by Backward Digit Recall in step 2, Block Recall in step 3, and the Nonword List Recall in step 4. The order of entry of the predictors was based on the magnitude of the Spearman's simple correlation (see Table 5).

Table 8. Hierarchical regression analysis predicting unique variance in Mathematical Reasoning Study 1

Step		B	SE B	β	*t*	*p*
1	(Constant)	-28.66	6.09		-4.71	<.001
	Age	.50	.10	.43	5.10	<.001
	BPVS	.20	.04	.46	5.39	<.001
	F (2, 75) =54.56, p <.001, R^2=. 59, p<.001					
2	(Constant)	-21.57	5.70		-3.78	<.001
	Age	.36	.10	.31	3.79	<.001
	BPVS	.14	.04	.33	4.01	<.001
	Backward Digit Recall	.73	.17	.36	4.35	<.001
	F (3,74) =51.37, p <.001, ΔR^2=.08, p<.001					
3	(Constant)	-21.43	5.48		-3.91	<.001
	Age	.31	.09	.26	3.28	.002
	BPVS	.14	.03	.31	3.97	<.001
	Backward Digit Recall	.59	.17	.30	3.52	.001
	Block Recall	.28	.10	.20	2.66	.01
	F (4, 73) =43.45, p<.001, ΔR^2=.03, p=.01					
4	(Constant)	-21.20	5.33		-3.97	<.001
	Age	.27	.09	.23	2.92	.01
	BPVS	.12	.03	.28	3.64	.001
	Backward Digit Recall	.53	.17	.26	3.17	.002
	Block Recall	.29	.10	.21	2.89	.01
	Nonword List Recall	.32	.14	.16	2.30	.03
	F (5, 72) =37.79, p<.001, ΔR^2=.02, p=.03					

As shown in Table 8, step 1 for Mathematical Reasoning indicates that age and the BPVS predicted 59% of significant variance. Step 2 shows that the additional incorporation of Backward Digit Recall contributed an additional 8% significant variance. Step 3 shows that the incorporation of Block Recall added 3% significant variance. Finally, step 4 shows that Nonword List Recall contributed an additional 2% significant variance. Thus, the three working memory measures together accounted for 13% of the unique variance in Mathematical Reasoning scores after controlling for age and the BPVS. The Durbin-Watson test was also checked and was found to be within acceptable parameters (1.92).

The beta values from both the covariate and predictor variables are significant, indicating that age, vocabulary, Backward Digit Recall, Block Recall, and Nonword List Recall, are significant independent predictors of Mathematical Reasoning. The β values indicated that the BPVS was the strongest predictor (β=.28), followed by Backward Digit Recall (β=.26), age (β=-.23), Block Recall (β=.21), and Nonword List Recall (β=.16). The largest VIF was well below 10, and the average VIF was 1.54. Similarly, the tolerance data are all within acceptable boundaries (all greater than 0.1). Therefore, it was concluded that there was no collinearity within the data (Field, 2009).

In summary, results suggested that Numerical Operations was significantly predicted by Backward Digit Recall even after having accounted for the effects of age and the BPVS. For Mathematical Reasoning, Backward Digit Recall, Block Recall, and Nonword List Recall were unique predictors independent of age and the BPVS.

2.4 Discussion

This study sought to investigate the contributions of working memory components to arithmetic skills and applied mathematics in 5- to 6-year-olds. This study extended research on this topic by investigating the role of different working memory components in arithmetic skills and applied mathematics. Overall, findings from this study suggest that working memory contributes differently to mathematics, depending on the type of mathematics. Namely, while arithmetic skills and applied mathematics shared some common working memory demands (as both relied on the central executive), there were important differences too, when doing applied mathematics, children also relied on the visuo-spatial sketchpad and the phonological loop. Moreover, these results showed that the significant association between working memory components and mathematics held even after controlling for age and receptive vocabulary.

It was first hypothesised that children would need to allocate attentional resources to counting strategies and to relevant numerical information, to solve the mathematics problems. If that were the case, then we would expect to see a significant contribution from the central executive to children's performance, on both mathematics skills. This hypothesis was supported by the results from the hierarchical multiple regression in which the central executive was a significant predictor of arithmetic skill and applied mathematics. The role of the central executive in arithmetic skills is consistent with previous research that have suggested that children need to allocate their attentional resources to guide the use of counting strategies while solving single-digit additions and subtractions (Hubber et al., 2014; Meyer et al., 2010), specially at this age, when children are in the transition of counting with their fingers to counting verbally (Fuson, 1982). Moreover, the role of the central executive in applied mathematics is consistent with research that suggest that this component is involved in directing attentional resources to attend to key numerical information while inhibiting irrelevant information (Baddeley, 1996; Baddeley et al., 1998).

The second hypothesis was that if children needed to temporarily store and represent the numerical information given in the problem by the means of mental representations, then we would expect to see a significant contribution from the visuo-spatial sketchpad to children's performance, on both mathematics measures. Results partially supported this hypothesis, namely, the visuo-spatial sketchpad was not significantly related to arithmetic skills, but it was to applied mathematics. Note that the arithmetic problems did not require multiple steps, because children only solved single-digit additions as such, no interim results needed to be held and manipulated; and the numerical information that needed to be calculated was always visible to the children. Thus, it is likely that it was not necessary to keep and represent the information by the means of mental representations and children would only need to rely on their central executive resources to guide their counting strategies. The lack of significant relationship between the visuo-spatial sketchpad and arithmetic skills is consistent with previous research suggesting that 5-year-olds do not rely on their visuo-spatial sketchpad to solve single-digit additions (Xenidou-Dervou et al., 2014).

The fact that the visuo-spatial sketchpad plays a role in children's applied mathematics is consistent with the idea that children rely on their visuo-spatial sketchpad in situations where the visual aid (pictorial representations of the arithmetic word-problems), or the lack of it, does not contain information that help the children to represent the problem. The applied mathematics measure does use visual prompts; nevertheless, results suggested that children represent the numerical information mentally regardless of the visual aid provided to them. One possible explanation for this observation is that, although the images presented with the problems were somewhat related to the problem, there remained some further numerical information that needed to be represented mentally in order for the child to solve the problem. For example, in the problem: *"If two of these ducks flew away, how many would be left?"* the visual aid for that problem is a picture of a pond with five ducks in it. Thus, it is likely that in order to solve the problem children needed to represent mentally the steps to solve it (i.e. 5-2=?). Moreover, because not all items in the applied mathematics measure were arithmetic word-problems (for example, one problem required children to determine how long a pencil was), this result suggests that in an applied context, children represent numerical information visuo-spatially.

The third hypothesis was that children would rely on the storage of verbal numerical information presented within the applied mathematics problems. If that was the case, then we would expect to see a significant contribution from the phonological loop to applied mathematics. Results supported this hypothesis. Moreover, they also provided evidence to support the suggestion that the phonological loop was not involved in arithmetic problems. As such, the role of the phonological loop being solely about storing verbal information and not about being involved in the application of verbal strategies is confirmed in this study. Thus, results from the visuo-spatial sketchpad and the phonological loop showed that even though the information on a given problem is presented verbally, and thus relying on their phonological loop, children simultaneously rely on the visuo-spatial sketchpad to build a mental representation of the information (Palmer, 2000).

Overall, these results suggest that arithmetic skills and applied mathematics involve the central executive, but that applied mathematics additionally involves both the visuo-spatial sketchpad and the phonological loop. This is consistent with the idea that the formats in which the mathematics problems are presented will lead to different cognitive mechanisms being involved (Xenidou-Dervou et al., 2014). That is, in arithmetic problems the numerical information that needs to be calculated is already set out clearly and explicitly, whereas in mathematical reasoning, children need to work out what numerical information is relevant in order to reach an answer. Because the central executive was a significant predictor for both types of mathematics, it can be suggested that children would rely on their ability to allocate their attentional resources to relevant numerical information and their ability to inhibit irrelevant information during the first year of learning mathematics.

One somewhat surprising finding relates to the role of receptive vocabulary. This finding merits comment due to the clear result showing that receptive vocabulary plays an important role in both mathematics skills. The variance in mathematics performance that receptive vocabulary explained as much variance as the variance explained by the working memory components. These results were somewhat unexpected, and raise the question of why children's vocabulary accounted for such a significant part of their mathematics performance. While there is currently no clear answer to that, there are four possible explanations that come to mind.

The first suggestion is that vocabulary itself has a direct effect on children's ability to do mathematics, such that the more words children know, the better children will be in mathematics. Somewhat related to this first suggestion, the second suggestion is that receptive vocabulary predicts mathematics performance because receptive vocabulary is associated with understanding concepts and ideas. In other words, what is important is understanding particular mathematics concepts, which happens to correlate with a larger vocabulary. Knowing words such as "more" and "less" may reflect greater conceptual understanding, which may be what matters for mathematics. This suggestion is consistent with research that found that children's understanding of mathematics concepts such as small, add, more, etc. (Foster, 2012) fosters mathematics learning overall (Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006).

The third suggestion is that receptive vocabulary is essentially a proxy measure for general language skills and receptive vocabulary itself may not be particularly important. In turn, general language skills are thought to be involved in children's use of strategies, like counting strategies, as general language supports verbal counting acquisition, which in turn improves their quantitative understanding and supports their arithmetic development (Lefevre et al., 2010; Moll et al., 2015). Moreover, while solving arithmetic word-problems, children need to understand the verbal cues within a mathematics problem to determine what it is needed to be done, like adding when hearing the verbal cue "*more than*"; thus involving language skills (Wang et al., 2016).

The fourth suggestion is that the vocabulary measure in this study is a proxy for SES. Previous studies have suggested that vocabulary are strongly related to children's SES (Hoff, 2003), and disparities in vocabulary between infants from high and low SES can be observed as early as 18 months of age (Fernald, Marchman, & Weisleder, 2013). Nevertheless, since in this study SES was measured by school post-codes, more sensitive indexes of SES (e.g. child's post-code or parental education) are needed to make a more reliable conclusion.

Given that the BPVS has been found to be highly correlated to other language measures, the third possibility (i.e. the possibility that receptive vocabulary is a proxy for general language skills), is the most likely. Unfortunately, given that the only measure of language taken in the present study was of receptive vocabulary, it does not allow for a definitive explanation of what aspect of language is central for mathematics abilities. However, it is worth attempting to replicate these findings using different and more precisely targeted measures of language abilities to check if the result concerning the role of language is a robust finding. This is the objective of Study 2 presented in the next chapter. In the next chapter I explore in more detail the role of language skills such that two new measures of language skills were selected. Addressing this question will be informative about how understanding words, sentences, and meaning of spoken language is involved in arithmetic skills and applied mathematics skills.

In conclusion, results showed that arithmetic skills and applied mathematics shared some common working memory demands relying on the central executive. The central executive will aid children to allocate attentional resources to counting strategies while solving arithmetic problems, and in applied mathematics will allow children to direct their attention to relevant numerical information while ignoring irrelevant information. However, there were differences too, when doing applied mathematics, children also relied on the visuo-spatial sketchpad and the phonological loop allowing children to hold and represent visual and verbal information. Moreover, receptive vocabulary is also a significant predictor of both arithmetic skills and applied mathematics.

Chapter Three

Investigating the Specific Contribution of Language and Working Memory Skills to 5- to 6-year-olds' Mathematics Skills (Study 2)

Results presented in Chapter Two revealed, as expected, that the central executive plays an important role in children's mathematics – both in pure and applied contexts; and that the visuo-spatial sketchpad and the phonological loop play an important role in applied mathematics only. However, there was also an unexpected and somewhat surprising finding that showed that both kinds of mathematics performance were significantly predicted by children's receptive vocabulary. Three possibilities to explain this finding were made. The first possibility was that receptive vocabulary was actually the aspect of language that was important for mathematics so that the more words children understand, the better they will be at mathematics. The second possibility was that receptive vocabulary predicts mathematics performance because vocabulary is associated with understanding concepts and ideas. In other words, what is important is understanding particular mathematics concepts, which happens to correlate with a larger vocabulary. The third suggestion was related to receptive vocabulary being a key component of language development, and thus, receptive vocabulary in itself was not particularly important for mathematics. Instead, it may have been a proxy for other language skills. Thus, the current chapter aimed to investigate the specific contribution of language to arithmetic skills and applied mathematics skills. To achieve this aim, two specific language skills were investigated: syntactic skills and the ability to understand and follow oral commands. A sample of 67 children between 60 and 78 months (*Mean age*=69.64, *SD*=4.30) were assessed in mathematics, language skills, working memory, and processing speed (as a covariate). Since this study follows directly from Study 1, the mathematics and working memory measures remained the same. Hierarchical multiple regressions showed unexpected results: only children's ability to understand and follow oral commands made a significant contribution to applied mathematics. No other significant relationship between language and mathematics was found. Moreover, the finding from Study 1 regarding the contribution of working memory components to mathematics was not replicated in Study 2. These results called for methodological issues within the study, namely, there was shared variance between the language measures and working memory. Thus, the question as to why language is particularly important for mathematics could not have been answered in full and with confidence.

3.1 Introduction

Chapter Two investigated the contributions of working memory components to mathematics skills using children's receptive vocabulary skills as covariates; surprisingly however, receptive vocabulary turned out to have an important role for both arithmetic skills and applied mathematics. Results suggested that receptive vocabulary explained as much variance as the variance explained by the working memory components. The aim of the current study, was therefore, to investigate the specific contributions of receptive language skills in children's mathematics skills. There are at least two ways in which language could be related to mathematics skills. One is that in order to solve specific mathematics problems, children need to understand the verbal cues within the problems, involving syntactic skills, which refer to children's ability to understand complex sentences (Cummins, Kintsch, Reusser, & Weimer, 1988; Munro, 1979). A second way that language may play an important role in children's mathematics is that language is related to children's mathematics learning because it involves understanding the words in a given classroom task or instruction (Hornung, Schiltz, Brunner, & Martin, 2014b). Thus, it was of interest to investigate how understanding words, sentences, and meaning of spoken language could be involved in arithmetic skills and applied mathematics skills. These abilities are part of the umbrella term known as receptive language which refers to the ability to understand words, sentences, and meaning of spoken and written language (Friedlander, 1970).

Because the aim of Study 2 was to investigate the specific contributions of receptive language to arithmetic problems and applied mathematics performance, two more precise skills were investigated (i) syntactic skills, which refer to children's ability to understand complex sentences (sentences that generally have two or more verbs; Limber, 1976), and (ii) the ability to understand and follow oral commands; these two language skills form an integral part of children's receptive language (Austin, Blevins-Knabe, Ota, Rowe, & Lindauer, 2011). The introduction of the present chapter will focus specifically on syntactic skills and children's ability to understand and follow oral commands. This review is presented next.

Generally speaking, learning how to solve arithmetic problems and applied mathematics involves understanding and decoding spoken and written language (Jaroslawska, Gathercole, Logie, & Holmes, 2016). Specifically, there is some evidence to suggest that the ability to respond to instructions is fundamental for a good response to school learning activities that involves several steps (Jaroslawska et al., 2016), as is the case of learning mathematics (Hornung et al., 2014b). Additionally, understanding language involves syntactic skills (Limber, 1976). Because during mathematics learning, children rely on their syntactic skills in order to make sense or (decode) the intended meaning of certain mathematics statements (Munro, 1979). For example, for children to understand the following statement: *“if there are ten children in the classroom and two are missing, how many children are here”*, it is not enough that the child identifies and understand some of the elements or verbal cues (i.e. “*two*”, “*missing*”, “*how many*”); children also need to decode the whole statement and represent it in a concrete form (i.e. 10-2=?; Munro, 1979). This decoding process has been related to syntactic skills and children with higher syntactic skills have been found to be better in understanding the specific meaning of mathematics statements (Munro, 1979).

Syntactic skills are important because children are constantly exposed to mathematics statements outside and inside the school context. For example, within the school context, while learning mathematics children need to understand what the teacher is saying in order to grasp the meaning of the mathematics concepts. The input, or mathematics-related speech, that children receive from their teachers also has an influence in children’s mathematics learning (Klibanoff et al., 2006). For example, one study found that teachers’ mathematics-related speech was a significant predictor of children’s mathematics growth (Klibanoff et al., 2006). In this study the frequency of mathematics statements within the teacher’s speech was coded from one hour of video-recording in naturalistic settings (school classrooms). Children’s mathematics knowledge was tested when they were 4 years of age and approximately six months later. Their mathematics knowledge was measured by a 15-item questionnaire that contained questions about ordinality, cardinality, number identification, names of shapes, understanding “half”, and calculation. The content of the teacher’s mathematics-related speech varied from statements involving cardinality (asking for a specific number of objects, i.e. *“can you give me four marbles?”*), comparing sets of entities (i.e. “*this classroom has more children”*), number words, or simple calculations. The statements also varied in context, from statements occurring as part of intentional mathematics instruction, to quantity talk in non-mathematics activities. Results showed that the quantity (but not the complexity) of preschool teachers’ mathematics statements was significantly related to the growth of children’s mathematics knowledge, even after controlling for classroom quality (Klibanoff et al., 2006). This finding suggests that if teachers' mathematics-related speech can contribute to children's mathematics learning, children's ability to understand language within the school context must play a significant role in children's mathematics learning as well.

First, in Klibanoff and colleagues' (2006) study, where the contribution of the teacher's mathematics-related speech to children mathematics learning was studied, child's own syntactic skills were not measured. And thus, whether the mathematics growth was directly related to children's ability to understand the teacher's mathematics-speech, remained as a speculation. From the literature review there is some evidence that would suggest that syntactic skills won't be related to arithmetic because they involve the manipulation of numbers. However, it is unclear whether this is also the case for 5-to 6-year-olds.

Syntactic skills influence the conceptual understanding of non-symbolic mathematics. A second study found that syntactic skills had a specific role in mathematics domains that did not depend on manipulating exact numbers (Vukovic & Lesaux, 2013). Six- to 9-years-olds were measured in four different mathematics skills, two symbolic-dependant mathematics skills (involving the manipulation of quantities): arithmetic and algebra, and two non-symbolic dependant mathematics skills: data analysis and geometry. Syntactic skills predicted non-symbolic mathematics domains, such as data analysis (i.e. interpretation of tables and tally charts, and estimation of probability) and geometry (i.e. knowledge of shapes with two and three dimensions; Vukovic & Lesaux, 2013), but not symbolic mathematics (i.e. arithmetic and algebra).

That is, the first study found that syntactic skill at the age of 4 predicted performance on arithmetic fluency at the age of 6 (i.e. Moll et al., 2015), and the second study found that in 6-to 9-year-olds syntactic skills were not related to arithmetic but were related to mathematics that are non-symbolic dependant (Vukovic & Lesaux, 2013). However, besides studying different age groups, these studies could have measured two different components of arithmetic. Arithmetic fluency (the measure used in the first study) is a measure that is usually conceptualized as a measure of arithmetic fact retrieval (LeFevre et al., 2013) because children rely on memory retrieval instead of counting skills to solve this task. In turn, arithmetic fact retrieval has been found to be a strategy that heavily depends on language because arithmetic facts are learned and retrieved verbally (Salillas & Carreiras, 2014). On the other hand, in the second study, arithmetic and algebra were measured by problems that involved the four arithmetic operations (without time constrains) with regrouping demands and which solution involved other procedural skills than retrieving arithmetic facts.

Applied mathematics depend on language skills because solving them involves children to translate the event presented in every-day language to arithmetic operations, to achieve the correct answer (Ilany & Margolin, 2008). A key first step therefore is to understand the narrative to successfully interpret what is needed to be done (i.e. which calculation to perform). Syntactic skills may come into play when children are learning to identify some cue words that would make their problem-solving skills more efficient. For example, understanding that in some problems when hearing 'all together', an addition needs to be performed (Cummins et al., 1988). Moreover, understanding temporal and spatial words in a given instruction is fundamental in learning mathematics (Munro, 1979). For example, in instructions with statements such as: "*Before you take away two, add six*" (i.e. 6-2), if a child fail to understand and thus ignore the temporal conjunction ("before"), then he or she will assume that the order of the elements is the intended order of the mathematics problem (i.e. 2 + 6), and fail to solve the problem accordingly (Munro, 1979).

Applied mathematics also depends on language skills because they need to understand the instruction given. There is one specific study that is particularly informative for the purposes of the present study because children's ability to understand and follow oral commands and working memory were investigated as predictors of applied mathematics concurrently. This study suggested that for children to solve arithmetic word-problems it was necessary to understand instructions, and working memory was a mediator of this relationship (Kyttälä et al., 2014). The ability to understand oral commands was assessed in 4-to 7-year-olds with the Token Test for Children (TTFC) which consists of 20 plastic tokens that differ in colour, shape, and size. Children needed to manipulate the tokens in accordance to the oral commands given by the experimenter. Applied mathematics was assessed with short arithmetic word-problems, which were presented verbally with addition of visual prompts. Two aspects of working memory were measured, verbal working memory (measured with a nonword repetition task and a backwards word recall task) and visuo-spatial working memory (measured with a matrix task, a Corsi blocks task, and an odd-one-out task). Results showed that there was an indirect effect of verbal working memory to arithmetic word-problems through children's ability to understand and follow oral commands (Kyttälä et al., 2014).

In summary, some empirical evidence posits both syntactic skills and children's ability to understand and follow oral commands, as cognitive abilities that may drive children's mathematics performance.

3.1.1 The current study

While there is some research that has considered working memory and language concurrently, these factors and their effects on mathematics skills are commonly studied independently. Investigating both cognitive skills together will allow us to find possible differences in the cognitive abilities that support arithmetic and applied mathematics performance. Results from Study 1 suggested that vocabulary skills contributed significantly to the performance of arithmetic and applied mathematics. Since children's general receptive vocabulary was measured in Study 1, the analyses in Study 1 may not have capture the true relationship between specific receptive language components and mathematics performance. Thus, to shed more light on how receptive language relates to children's arithmetic skills and applied mathematics, Study 2 included the following two measures: (i) 'Sentence Structure' to measure syntactic skills and (ii) 'Concepts and Following Directions' to measure children's ability to understand and follow oral commands. These two measures are part of the Clinical Evaluation of Language Fundamentals Preschool 2 UK (CELF; Wiig, Secord, & Semel, 2006) which is a standardised measure of language skills, widely used in children. Working memory components were also taken into account. Additionally, age, SES, and processing speed were considered as possible covariates.

Since the present study was designed to try to unpick the role of language, it was considered necessary to include other cognitive processes that may also be relevant for the performance of arithmetic skills and applied mathematics skills. As such, in the current study, processing speed was included in the design. The measure to assess processing speed was the Box Completion task (Salthouse, 1993). In the Box Completion task children simply need to draw lines to complete three-sided boxes, and it is unlikely that it would depend on language skills. This measure was selected because, contrary to other measures that involve naming shapes, colours, objects, or numbers, this measure does not depended on language skills. Additionally, there is some research that suggests that simple processing speed measures, such as this one, does not involve working memory or inhibitory control skills either (Cepeda, Blackwell, & Munakata, 2013).

The main hypothesis for this study was that overall, mathematics performance depends on the ability to understand language within mathematics statements and to translate this information to arithmetic operations. If this were the case, then we would expect syntactic skills to be related to both arithmetic skills and applied mathematics. However, if understanding language is not involved in the performance of mathematics that require the manipulation of numbers, then we would expect language skills to be more strongly related to applied mathematics and less strongly related to arithmetic skills. Moreover, since children need to understand instructions to perform any mathematics problems, it is expected to see the relationship between children’s ability to understand and follow oral commands to be related to both arithmetic skills and applied mathematics.

3.2 Method

3.2.1 Study design

The present study had a cross-sectional design in which testing was carried out in one single session. The outcome variables were arithmetic skills (as measured by the Numerical Operations test) and applied mathematics ability (as measured by the Mathematical Reasoning test). The predictor variables were syntactic skills (as measured by the Sentence Structure task) and the ability to understand and follow oral commands (as measured by the Concepts and Following Directions task). Performance on the working memory measures were also predictor variables. The working memory measures were: the Backward Digit Recall task that measures the central executive; the Block Recall task that measures the visuo-spatial sketchpad; and the Nonword List Recall task that measures the phonological loop. Chronological age and processing speed (as measured by the Box Completion task), were included as covariates.

3.2.2 Participants

Participants were recruited from two primary schools in Sheffield, UK. According to the IMD calculated with school postcodes these schools were from high to very high socio-economic backgrounds (7th and 10th deciles in the IMD; Ministry of Housing, Communities & Local Government, 2015). A power analysis (conducted using the G*Power software) for a linear multiple regression with six predictors (three working memory variables, two language variables, and processing speed) was performed with $\alpha = 0.05$, a large effect size $f^2=.35$ (based on Study 1 findings), and a desired power of 0.80. The power analysis resulted in a required overall sample size of 46 children. Participants were 67 children (36 female) with ages between 60 and 78 months (*Mean age*=69.64 *SD*=4.30). The sample involved 14 children who attended Reception year (ages between 5 years and 5 years 10 months) and 53 children who attended Year 1 (ages between 5 years and 5 months and 6 years and 6 months). Written consent was obtained from parents before testing began and ethical approval was obtained from the Department of Psychology's ethics sub-committee.

3.2.3 Materials

The mathematics measures (Numerical Operations and Mathematical Reasoning) and working memory measures (Backward Digit Recall, Block Recall, and Nonword Recall) used in this study were the same as those used in Study 1 (Chapter Two). Hence in this section, only full details for the language and processing speed tasks are provided.

The language measures chosen for the current study were *Sentence Structure* and *Concepts and Following Directions*. They are part of the Language Content index of the CELF (Wiig, Secord, & Semel, 2006), a standardised battery of measures which assesses two areas of language skills, expressive language and receptive language. *Sentence Structure* and *Concepts and Following Directions* were part of the receptive language sub-section of the CELF. Pictorial prompts were used in the implementation of both tasks. A raw frequency score was calculated for each test according to individual assessment guidelines.

Sentence Structure. The Sentence Structure measure was used to evaluate children's syntactic skills. Specifically, it measured the ability to create meaning and context by interpreting spoken sentences (Wiig, Secord, & Semel, 2006). In this measure, the experimenter read a sentence out loud (e.g. "*the boy who is sitting under the big tree, is eating a banana*"), and the child needed to choose from four different pictures the picture that illustrated the referential meaning of the sentence (e.g**.** Figure 1). Testing procedure started with one example followed by two trials so the child can familiarized with the procedure. There were 22 items of increasing length and complexity. The task was discontinued after five consecutive mistakes.

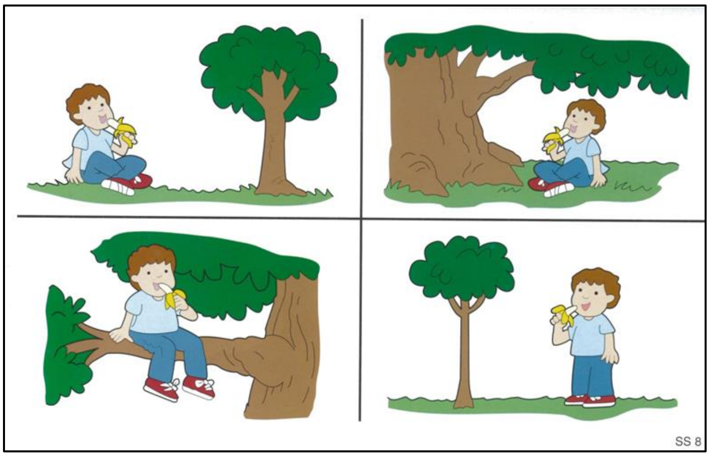

Figure 1. Sentence Structure example

Concepts and Following Directions. The Concepts and Following Directions measure was used to evaluate children's ability to understand and follow oral commands. In order to solve this measure correctly children needed to interpret, recall, and execute oral commands given by the experimenter. The experimenter read out loud an oral command (e.g. *"point to the dog before you point to the tortoises";* Figure 2) and children needed to remember the characteristics and order of the mentioned objects, to respond accordingly. Testing procedure started with two familiarization items followed by two trials before the main testing commenced. There were 22 items of increasing length and complexity. The task was discontinued after six consecutive mistakes.

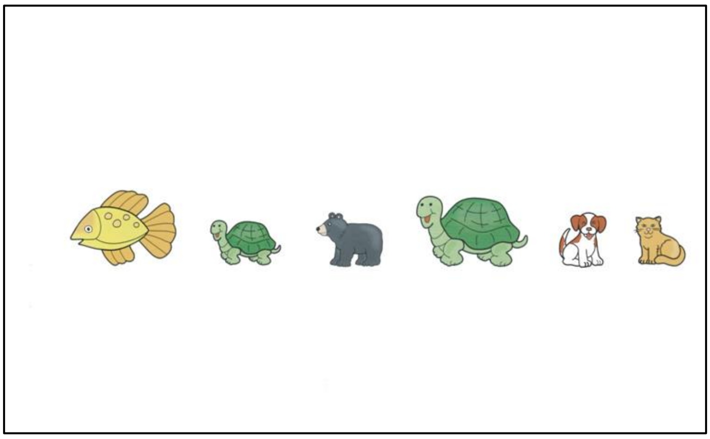

Figure 2. Concepts and Following Directions example

Box Completion. The Box Completion measure (Salthouse, 1993) was used to assess children's processing skills. The task consisted of 35 incomplete boxes on a sheet of A4 paper. Before starting, children practiced with one row of four incomplete boxes. Children were instructed to make as many three-sided boxes into four-sided boxes as possible, within 30 seconds (Cepeda et al., 2013; Salthouse, 1993). The score for this task was the number of completed boxes. Boxes that were not properly completed were not scored.

3.2.4 Procedure

All children participated in a single testing session that lasted approximately 30 minutes. Each child was tested individually in a quiet area of the school and gave verbal assent in addition to their parent's written consent. During the session each child was administered two mathematics measures, three working memory measures, two language measures, and one processing speed measure. Children were informed that they were there to play some maths, memory, and words games. Each session started with the Numerical Operations test, followed by the Mathematical Reasoning test, the Block Recall measure, the Nonword List Recall measure, the Backward Digit Recall measure, the Sentence Structure measure, the Concepts and Following Directions measure, and the Box Completion at the end. At the end of the testing session all participating children received a sticker as a token of appreciation of taking part.

3.3 Results

3.3.1 Descriptive statistics

Descriptive statistics for the raw scores in the mathematics measures, the working memory measures, the language measures, and the processing speed measure are provided in Table 9. Ceiling effects in the Sentence Structure subscale were obtained by two children from Year 1. Furthermore, three different children performed at ceiling on the Concepts and Following Directions task. Thus, these extreme values were scored and considered as missing data in further statistical analyses. Skewness and kurtosis values indicated that not all variables' scores presented a normal distribution. A further Shapiro-Wilk test confirmed the non-normal distribution of all variables (all $p \leq .05$) except for Mathematical Reasoning (p=.17). Thus, non-parametric tests were used in the following analyses.

Table 9. Descriptive statistics of raw scores for the mathematics, working memory, language, and processing speed measures in Study 2

Variable	Mean	SD	Range (min-max)	Skewness	Kurtosis
Numerical Operations	10.03	2.06	6-16	.06	.36
Mathematical Reasoning	19.76	4.15	12-29	.19	-.63
Backward Digit Recall	9.43	2.51	5-18	.61	.70
Block Recall	20.03	4.01	5-27	-1.03	2.11
Nonword Recall	10.40	2.08	4-14	-.62	.29
Sentence Structure (N=65)	18.00	1.97	14-21	-.30	-.53
Concepts and Following Directions (N=64)	17.05	2.59	10-21	-.63	.41
Box Completion	20.52	4.46	13-32	.55	-.36

3.3.2 Preliminary analysis

Mann-Whitney tests were conducted to test whether there was an effect of gender or school year (Reception vs Year 1). Results revealed that there were no gender effects in any of the measures (all $p > .05$). However, there were school year effects. Children in Year 1 outperformed children in Reception year in the following measures: the Numerical Operations (U=64.50, $p<.001$, z=-4.82, r=-.59), Mathematical Reasoning (U=156.50, z=-3.32, $p<.001$, r=-.41), Concepts and Following Directions (U=235.50, z=-1.96, $p<.05$, r=-.25), and Box Completion (U=151, z=-3.40, $p<.001$, r=-.42) (see Table 10 for medians).

Table 10. Differences between Reception and Year 1. Variables-medians in Study 2

	Reception	Year 1
Numerical Operations	8	11
Mathematical Reasoning	16	21
Concepts and Following Directions	16	18
Box Completion	16.50	21

3.3.3 Non-parametric correlations

Associations between age, the mathematic measures, the working memory measures, the language measures, and the processing speed measures were examined with a non-parametric correlation. The Spearman correlation was performed using all available data with pairwise deletion. This was to avoid bias and reduction in power that result from listwise deletion of data (Graham, 2009). As shown in Table 11, Numerical Operations was significantly correlated to age, Concepts and Following Directions, and Box Completion. Mathematical Reasoning was significantly correlated to age, Backward Digit Recall, Block Recall, Sentence Structure, Concepts and Following Directions, and Box Completion. A non-parametric partial correlation was conducted to examine whether the significant correlations between the mathematics, working memory, and language measures remained the same after eliminating the variance related to age and Box Completion scores (Table 12). The analyses were performed using SPSS Syntax.

Table 11. Correlation matrix reporting simple non-parametric correlations in Study 2

	1	2	3	4	5	6	7	8	9
1. Age	1								
2. Numerical Operations	.44***	1							
3. Mathematical Reasoning	.37**	.47***	1						
4. Backward Digit Recall	.07	.14	.35**	1					
5. Block Recall	.16	.18	.28*	.29*	1				
6. Nonword Recall	.17	.16	.05	.13	.07	1			
7. Sentence Structure *N*=65	.39***	.10	.42***	.33**	.20	.04	1		
8. Concepts and Following Directions *N*=64	.40***	.25*	.53***	.34**	.27*	.09	.50***	1	
9. Box Completion	.35**	.41***	.27*	.10	.01	-.03	.16	.18	1

Note. Correlations= *<.05, ** <.01, and *** <.001

Table 12. Correlation matrix reporting partial non-parametric correlations, controlling for age and Box Completion in Study 2

	1	2	3	4	5	6	7
1. Numerical Operations	1						
2. Mathematical Reasoning	.33**	1					
3. Backward Digit Recall	.10	.34**	1				
4. Block Recall	.15	.25*	.29*	1			
5. Nonword List Recall	.14	.00	.13	.04	1		
6. Sentence Structure	-.10	.32*	.33**	.15	-.03	1	
7. Concepts and Following Directions	.09	.44***	.34**	.23	.03	.41***	1

Note. Correlations= *<.05, ** <.01, and *** <.001

As shown in Table 12, after eliminating the variance related to age and Box Completion scores, Numerical Operations was no longer significantly correlated to Concepts and Following Directions. As for Mathematical Reasoning, the correlations with Backward Digit Recall, the Block Recall, Sentence Structure, and Concepts and Following Directions, remained significant and independent from the variance related to age and Box Completion.

3.3.4 Hierarchical multiple regression analyses

Two hierarchical multiple regressions were conducted to further investigate the amount of unique and incremental contributions of language and working memory to Numerical Operations and to Mathematical Reasoning respectively. The predictors entered in the model were selected based on the significance of the Spearman's rank correlation coefficient. Hence, for Numerical Operations (Table 13), the covariates were age and Box Completion, entered in step 1; and Concepts and Following Directions entered in step 2 as the predictors. For Mathematical Reasoning (Table 14), the covariates were age and Box Completion, entered in step 1; and the predictors were: Concepts and Following directions (step 2), Sentence Structure (step 3), Backward Digit Recall (step 4), and Block Recall (step 5). Pairwise deletion was selected because of the missing values in the data.

Table 13. Hierarchical regression analysis predicting unique variance in Numerical Operations in Study 2

Step		B	SE B	β	*t*	*P*
1	(Constant)	-3.82	3.70		-1.03	.31
	Age	.16	.06	.34	2.86	.01
	Box Completion	.13	.06	.27	2.32	.02
	F (2, 61) =10.39, p <.001, R^2=. 25, p<.001					
2	(Constant)	-3.89	3.69		-1.06	.30
	Age	.13	.06	.28	2.20	.03
	Box Completion	.13	.05	.28	2.34	.02
	Concepts and Following Directions	.12	.10	.15	1.23	.22
	F(3,60)=7.48, p <.001, ΔR^2=.02, p=.09					

As shown in Table 13, step 1 for the Numerical Operations test indicated that age and Box Completion predicted 25% of significant variance, and both age and Box Completion were significant predictors. Step 2 shows that the addition of Concepts and Following Directions contributed an additional 2% significant variance. However, only Box Completion continued to be a significant predictor. The Durbin-Watson test was checked and was found to be within acceptable parameters (2.26), thus the assumption of independent errors was met. This result suggests that this regression model is unbiased, increasing the likelihood of these results to be true for a wider population (Field, 2009).

As one can see from the final step there are two significant beta values, indicating that age (β=.28) and Box Completion (β=.28) were significant independent predictors of Numerical Operations and that both were equally strong predictors. Multicollinearity was checked using the variance inflation factor (VIF). The largest VIF was well below 10, and the average VIF was 1.22. Similarly the tolerance data are all within acceptable boundaries (all greater than 0.1). Therefore, it was concluded that there was no collinearity within the data (Field, 2009).

The second hierarchical multiple regression (Table 14) shows the analysis conducted to investigate Mathematical Reasoning predictors. Results from this hierarchical multiple regression indicate that step 1, age and Box Completion, predicted 18% of significant variance, but only age was a significant predictor. Step 2 shows that the additional incorporation of Concepts and Following Directions contributed an additional 17% significant variance, and age was no longer a significant predictor. Step 3 shows that including Sentence Structure explained an additional 1% significant variance, Backward Digit Recall in step 4 added an additional 2% significant variance, and finally, step 5 shows that Block Recall added an additional 1% significant variance. In this final step, the unique significant predictor was Concepts and Following Directions. The Durbin-Watson test was checked and was found to be within acceptable parameters (1.65). Multicollinearity was checked using the variance inflation factor (VIF). The largest VIF was well below 10, and the average VIF was 1.38, similarly the tolerance data are all within acceptable boundaries (all greater than 0.1). Therefore, it was concluded that there was no collinearity within the data (Field, 2009).

Table 14. Hierarchical regression analysis predicting unique variance in Mathematical Reasoning in Study 2

Step		B	SE B	*β*	*T*	*P*
1	(Constant)	-6.50	7.96		-.82	.42
	Age	.34	.12	.35	2.77	.01
	Box Completion	.14	.12	.15	1.16	.25
	F (2, 59) =6.45, p <.01, R^2=. 18, p=.003					
2	(Constant)	-6.95	7.12		-.98	.33
	Age	.17	.12	.17	1.40	.17
	Box Completion	.14	.11	.15	1.33	.19
	Concepts and Following Directions	.73	.18	.45	3.96	<.001
	F (3,58) =10.58, p <.001, ΔR^2=.17, p<.001					
3	(Constant)	-8.39	7.27		-1.15	.25
	Age	.14	.12	.14	1.13	.26
	Box Completion	.15	.11	.16	1.39	.17
	Concepts and Following Directions	.64	.20	.40	3.15	.003
	Sentence Structures	.26	.27	.13	.98	.33
	F (4,57) =8.17, p<.001, ΔR^2=.01, p=.33					
4	(Constant)	-9.82	7.28		-1.35	.18
	Age	.16	.12	.17	1.34	.19
	Box Completion	.13	.11	.14	1.25	.22
	Concepts and Following Directions	.56	.21	.35	2.69	.01
	Sentence Structures	.20	.27	.09	.73	.47
	Backward Digit Recall	.27	.19	.16	1.41	.16
	F (5,56) =7.05, p<.001, ΔR^2=.02, p=.16					
5	(Constant)	-10.33	7.34		-1.41	.17
	Age	.15	.12	.16	1.23	.23
	Box Completion	.15	.11	.16	1.38	.17
	Concepts and Following Directions	.53	.21	.33	2.50	.02

Sentence Structures	.19	.27	.09	.70	.49
Backward Digit Recall	.24	.20	.14	1.21	.23
Block Recall	.09	.12	.09	.76	.45
$F(6,55) = 5.93$, $p < .001$, $\Delta R^2 = .01$, $p = .45$					

In summary, results suggested that Numerical Operations was not predicted by any of the predictor variables; nevertheless, Box Completion was a significant predictor. For Mathematical Reasoning, the only significant predictor was Concepts and Following Directions.

3.4 Discussion

The aim of the current study was to investigate the specific contributions of receptive language skills in children's mathematics skills. On this way, two more precise language skills were investigated: syntactic skills and the ability to understand and follow oral commands. Two unexpected findings were obtained from the results. First language skills did not predicted performance in neither of the mathematics skills, this in turn indicates that results in Study 1, regarding the role of vocabulary was not replicated. Only performance in applied mathematics was related to children's ability to understand and follow oral commands. Second, Study 2 failed to replicate findings in Study 1 with regard to the significant relationship between working memory and mathematics, that is, working memory was not a significant predictor of either mathematics skills. I will start by discussing the findings regarding the role of language skills in mathematics. Then I will address my reasons as to why I consider the results regarding working memory and mathematics were not replicated.

It was hypothesized that if mathematics performance depended on the ability to understand mathematics statements and translate this information to arithmetic operations, then we would expect both language skills to be related to both arithmetic skills and applied mathematics.

Results did not support this hypothesis, since the only significant association found was between children's ability to understand and follow oral commands and applied mathematics. This finding was surprising, nevertheless this result is likely to be because the language measures had unexpectedly high working memory demands, and as such it means that it is not possible to attribute task variance with any confidence, since the measures themselves are not transparent.

As unexpected as it was to find arithmetic skills not being related to any of the receptive language measures this finding is consistent with some evidence that had suggested that the manipulation of precise numerical quantities, such as arithmetic, does not involve language skills (Vukovic & Lesaux, 2013). However, because of how much variance processing speed and age explained in arithmetic performance in the present study, and since receptive vocabulary was found to have a role in children's arithmetic performance in Study 1, there are grounds for caution in interpreting these data as such. That is, the role of receptive language skills in arithmetic cannot be ruled out completely.

Another possibility is that children in the present study had better procedural skills-such as arithmetic fact retrieval- than children in Study 1, and thus, there was no need for relying on other cognitive resources involving language. The ability to process information in specific domains is related to children's familiarity or experience with the material (Bull & Johnston, 1997). Thus, in the current study children might have been faster in calculating the answer of the arithmetic problems due to having more experience in solving single-digit arithmetic problems, whereas less experienced children, rely on their working memory resources (e.g. children in Study 1). Sample differences in problem-solving strategies between studies can be present even when there were no significant differences between the scores of the Numerical Operations test from children in Study 1 and children in Study 2 (see Appendix 2 for a Mann-Whitney test results). This is in line with a recent study that found that 5-to 6-year-olds with similar mathematics achievement profiles (measured with a composite score from the Numerical Operations test and the

Mathematical Reasoning test; Gilmore, Keeble, Richardson, & Cragg, 2017), had different skill-levels on their procedural skill, conceptual understanding, and working memory skills. Gilmore and colleague's (2017) study highlights how there are many different cognitive pathways related to mathematics performance, even when children show a similar outcome. As such, children in the present study could have had more efficient procedural skills using other strategies that do not demand language or working memory resources.

Performance on applied mathematics was significantly related to children's ability to understand and follow oral commands; as such, this finding suggest that language skills are particularly significant when the format presentation of the mathematics problems is verbal. Note, however, that syntactic skills were not significantly related to applied mathematics. This finding is inconsistent with some research suggesting that has suggested that syntactic skills are related to applied mathematics, because children need to translate the problem presented in every-day language to arithmetic operations, so they can achieve the correct answer (Ilany & Margolin, 2008). Since syntactic skills are necessary for the understanding of the narrative in which the mathematics information is presented, it is likely that in the present study the linguistic structure was simple enough for children to understand the problem and did not need much interpretation.

These results clearly show a significant contribution of language to mathematics in an applied context. There are a number of possible explanations for this and these following suggested explanations are not mutually exclusive. For example, one suggestion is that if in a given mathematics problem because the numerical information is presented in a verbal code, some language skills are likely to be involved, despite of the complexity of the verbal information. Moreover, difficulties in solving arithmetic word-problems could be related to children's ability to understand the instructions of the task more than the mathematics complexity of the problem itself. This finding is in line with some research that found that children's ability to follow oral commands was significantly related to children's performance in short arithmetic word-problems (Kyttälä et al., 2014). Additionally, understanding the language use in the explanations and interactions within the classroom, may be playing a role as well (Schleppegrell, 2007).

To recapitulate, it is a little surprising that language skills were not contributing to arithmetic skills performance, because Study 1 suggested the contrary. I suggest that it is likely that differences in the contribution of receptive language skills to mathematics can be attributed to the presentation format of the mathematics problems, because arithmetic problems were presented in a written format, and applied mathematics were presented in a verbal format. Just as any other cognitive skill, language may have a significant role depending on the complexity of the problem or the procedural skills that children have. However, the presentation of applied mathematics will mean that if the problem is presented verbally, the problem very likely will involve language skills- since the input form the problems is presented verbally and need to be represented internally as such.

It has to be considered, however, the possibility of indirect effects of working memory to mathematics, via children's ability to understand oral commands (Kyttälä et al., 2014), this suggestion leads to the second finding: Study 2 failed to replicate findings in Study 1 with regard to the significant association between working memory and mathematics. There are two different plausible explanations that could account for the failure to reproduce the findings of Study 1: (i) the lack of predictive value of working memory is due to the incidental executive and visuo-spatial demands on the language measures, and (ii) the effect of working memory over mathematics is highly variable across samples.

In light of the first possibility, an inspection of intercorrelations among language and working memory measures (Table 11) can provide more information. As one can see in Table 11, Concepts and Following Directions significantly correlated with the central executive (Backward Digit Recall, r_s= .34, p<.01) and the visuo-spatial sketchpad (Block Recall,

r_s= .27, p<.05). The shared variance between the ability to understand and follow oral commands and working memory skills can be explored by looking at the nature of the measure used in this study. That is, the Concepts and Following Directions task involves children to hold in mind names, characteristics, and order of the mentioned objects to answer, and thus, it is very likely to involve working memory resources.

The idea of Concepts and Following Directions involving working memory resources is consistent with previous research that has suggested that following instructions relies on working memory processes (Jaroslawska et al., 2016; Yang, Allen, & Gathercole, 2015). Especially because for children to succeed in following instructions, children need to remember a series of steps in sequence, and perform them soon after (Yang et al., 2015). Thus, in addition to engaging language skills, following instructions also involves working memory skills (Jaroslawska et al., 2016; Kyttälä et al., 2014). It is possible then, that children did rely on working memory resources to solve the applied mathematics measure, but that this was not reflected in the statistical analyses, due to potentially large amounts of shared variance between the working memory measures and the Concepts and Following Directions measure. However, this suggestion deserves further exploration using more fine-grained approaches.

My second plausibly explanation as to why there was no replication of findings regarding working memory, is that the effect of working memory on mathematics is highly variable across samples. From the Mann-Whitney test (see Appendix 2) significant difference in SES were also obtained; so that children in Study 2 were mostly from higher SES backgrounds than children in Study 1. Overall, children's cognitive development is greatly sensitive to influences from their social and contextual environment (Jimerson et al., 1999; Pungello et al., 1996), and mathematics skills are no exception (Jordan, Kaplan, Oláh, & Locuniak, 2006). In fact, children from disadvantaged backgrounds start their formal mathematics education with gaps in their numerical knowledge and are at risk of having a low mathematics level in subsequent school years (Cheadle, 2008; Entwisle & Alexander, 1990; Siegler, 2009) To investigate this subject with more depth more specific (than school-postcodes) measures of SES can be more useful to understand how low or high SES can lead to differences in mathematics cognition. For instance, there is some evidence that suggest that improving the home learning environment during preschool (e.g. caregivers reading with their child), decreases the gap between mathematics achievement between children from low and high SES backgrounds over time (Galindo & Sonnenschein, 2015).

In conclusion, although there was increasing evidence that language seems to play an important role in young children's emerging mathematics abilities, because of methodological concerns, at present it is difficult to say with confidence which aspect of language is key, nor what specific roles language plays in mathematics. Although based on the present data some suggestions could have been formulated, these need to be followed up with additional studies that use different measures of linguistic skills to assess children's language. Moreover, the present study highlight just how complex studying mathematics skills is, and that looking at one individual cognitive factor in relation to mathematics achievement is not sufficient. Instead, investigating the interactions between relevant cognitive mechanisms may be more informative. Additionally, findings from this study and Study 1, suggest that children's cognitive strategies to solve arithmetic problems cannot simply be observed by comparing different mathematics performance. Research would benefit from longitudinal studies including cognitive pathways (Gilmore et al., 2017; Lefevre et al., 2010; Moll et al., 2015), to understand the development of mathematics.

Chapter Four

Longitudinal and Concurrent Linguistic Precursors of 4-Year-Olds' Mathematics Skills (Study 3)

Chapter Four was specifically designed to study the longitudinal and concurrent linguistic precursors of a sample of 4-year-old children that had been tracked from the age of 11 months, and for whom there were data on early language measures. Given the evidence from Study 1 and Study 2, showing that language plays an important role in young children's mathematics skills, studying this relationship both longitudinally and concurrently could shed important light on the role that language plays in children's mathematics ability.

Therefore, the purpose of Study 3 in the present chapter was to investigate the contribution of the longitudinal and concurrent linguistic precursors of pure and applied mathematics skill in 4-year-olds. Early numeracy skills-measured with the Mathematics scale from the National Foundation for Educational Research (NFER) Baseline Reception Assessment-were chosen to represent pure mathematics in 4-year-olds, and applied mathematics were measured with the Mathematical Reasoning from the WIAT-II. The linguistic longitudinal precursors were the mathematics-related words that caregivers produced during caregiver-child naturalistic interactions when children were 11 months of age, and the mathematics-related words that children produced in caregiver-child interactions when they were 2 years of age. Concurrent language skills were assessed with a composite score form three different measures, the BPVS, the NFER Language and Communication scale, and the CELF. Participants were 71 children between 48 and 55 months (*Mean age*=50.04, *SD*=1.54) from a broad range of socio-economic backgrounds. Findings suggested that both mathematics skills relied on different concurrent predictors, such that early numeracy skills relied on general language skills and applied mathematics relied on general language skills and inhibitory control. This demonstrated that even before school entry children utilize their language skills to perform mathematics in a pure and applied context.

4. 1 Introduction

Evidence provided by Studies 1 and 2 have suggested that language skills are significant predictors of mathematics skills when children are between the age of 5 and 6 years. As such, studying this relationship both longitudinally and concurrently was considered to be a valuable next step to get closer to our understanding of the specific role of language in mathematics. Thus, taking the opportunity to work with a sample of 4-year-old children that had been tracked longitudinally from the age of 11 months, Study 3 in the present chapter, was designed. The aim of the current study is, therefore, to investigate the longitudinal and concurrent linguistic precursors of 4-year-olds' pure and applied mathematics skills.

To investigate this research question in a thorough way, other possibly relevant cognitive abilities were also measured, thus children's executive functions were included in the design of the current study. This allows investigating developmental patterns in the precursors of early mathematics skills. As such the present study is characterized by investigating the following components: a younger-aged sample, pure vs. applied mathematics, concurrent vs. longitudinal linguistic predictors, and executive functions. A review of the literature regarding the longitudinal linguistic precursors and concurrent language and executive functions skills, of mathematics is presented next.

4.1.1 Longitudinal linguistic precursors of children's mathematics skills

Language skills have been shown to play an important role in the development of children's mathematics ability both (i) longitudinally and (ii) concurrently. For example, children's early mathematics-related language experiences at home predict later mathematics performance. Specifically, it has been found that the amount and type of the linguistic input that infants (between 14 and 30 months of age) and children (between 3 and 5 years of age) receive about numbers, during caregiver-child interactions at home, are particularly important contributors of individual differences in early mathematics skills (Levine et al., 2010; Ramani et al., 2015). Two studies, one conducted longitudinally, and the other one conducted concurrently, have been particularly informative about this subject. These studies are presented next.

The first relevant study showed that caregivers' mathematics-related talk predicted children's understanding of cardinality (Levine et al., 2010). This was shown in a longitudinal study in which the frequency of the number-words (from one to ten) produced by the caregiver's during naturalistic interactions with their child was measured (Levine et al., 2010). Caregiver-child interactions were video-recorded for 90 minutes, starting from when children were 14 months of age until they were 30 months of age. Later, when children were 46 months of age, children's cardinality understanding was measured with the point-to-x task. The amount of number talk (from the caregivers) predicted children's understanding of cardinality at 46 months over and beyond caregivers and children's overall talk. Moreover, findings showed that not only did caregivers who talked more produced more number words than caregivers who talked less; but the amount of number words that caregivers produced was significantly related to the child's own use of number talk (both increasing over time). In summary, this study showed that caregiver's production of number words is specifically important for children understanding of numbers over development.

The second study investigated caregiver's talk during number-related activities in relation to children's concurrent early numeracy skills (Ramani et al., 2015). In this study 3-to 5-year-olds' and their caregivers participated in one caregiver-child interaction. Children also participated in two individual testing sessions in which their numeracy skills were assessed. Caregivers also completed a questionnaire regarding the frequency of activities relating numbers, literacy, and play at home. The caregiver-child interaction was a semi-structured play session following the three bags task. The three bags task consisted in providing the caregiver with three bag containing a book, a puzzle, and a board game respectively; all with the aim to foster mathematics talk. For example the puzzle was a snail puzzle with numbers from one to ten. All the speech from both the caregiver and the child was transcribed from 15 minutes of video-recording, and was categorised into two types of mathematics talk: 'foundational', which was related to counting and identification of numbers; and 'advance concepts', which involved cardinality (e.g. "*how many x?*"), ordinality (e.g. "*what comes after x?*"), and arithmetic-related words (caregiver asking about additions or subtractions). The individual sessions were conducted to assess children's early numeracy skills, which also were divided in 'foundational' (counting and identification of numbers) and 'advanced' (number line estimations, magnitude comparison, cardinality skills, and counting principles). Two particularly relevant findings can be extracted from this study.

First, among the three home-activities in the questionnaire, caregivers reported that they engaged least often in number-related activities (e.g. teaching their child about numbers directly) however, these number-related activities significantly predicted children's 'foundational' mathematics skills. This suggests that even if the activities were not as frequent as, for example, literacy activities, number-related activities still were relevant for children's basic numeracy skills. Second, being exposed to a very specific mathematics content can influence children's understanding of similar concepts. Specifically, caregiver's talk about 'advance concepts' (i.e. cardinality, ordinality, and arithmetic-related words) during the structured interactions significantly predicted children's 'advance' mathematics skills (i.e. number line estimations, magnitude comparison, cardinality skills, and counting principles). However, both findings could be bidirectional, that is, children with a more developed number sense could have been showing more interest in numerosities which in turn fosters caregiver's effort to produce number words and other mathematics concepts, and thus, causality cannot be determine with confidence.

Together these studies suggest that being exposed to mathematics-related talk during early stages of development support children's mathematics development by enhancing their understanding of numbers. Findings also showed that the relationship between mathematics-related talk and children's mathematics skills, may be domain-specific rather than domain-general. That is, in Levine and colleagues' study (2010), number talk was related to children's understanding of numbers and in Ramani and colleagues' study (2015) advance mathematics talk specifically predicted children's advance mathematics understanding. Thus, early exposure of mathematics talk may specifically contribute to children's quantitative understanding.

4.1.2 Concurrent linguistic precursors of children's mathematics skills

Children's own language skills have also been related to their performance in mathematics skills. For example, children make use of their language skills to understand specific mathematics words like "*all*" (i.e. quantifiers), which in turn allow children to grasp specific mathematics concepts such as cardinality understanding (Purpura & Reid, 2016). The role of language skills in mathematics has generally been studied with measures that tap these skills more generally; however, more recently, the role of content-specific language skills has been gaining attention. In the following paragraphs I will present some of the evidence for the role of general and content-specific language skills in mathematics.

General language skills have been found to be related to mathematics across different mathematics domains. Some researchers have been proposed that language and mathematics are related because both use symbols to represent them, so that the ability to relate sounds to their written letters foster children's ability to use and manipulate numbers and operations, as they are symbols on their own (Zhang et al., 2014). This suggestion could be considered as a very specific interpretation of the relationship between these two constructs. However, in a broader sense, general language skills seem to support mathematics learning because children need to understand new words many of which are unlike anything that children have learned before (e.g. "*add*"; Moll et al., 2015). General language skills have been related to mathematics, even from a very early age (3 to 4 years of age). For example, it has been suggested that language supports numerical development as children utilize language skills (e.g. number words) to improve their quantitative understanding (Purpura & Ganley, 2014). One study found that general language skills (measured with a standardized measure of expressive vocabulary that has been found to be highly correlated to other language skills; Purpura & Ganley, 2014) accounted for significant variance in predicting, verbal counting, one-to-one counting, cardinality, number comparison, set comparison, number order, numeral identification, set to numerals, and story problems.

The relevance of general language skills in mathematics is supported by studies done with children with language difficulties. For example it has been observed that a disturbance in language development can cause a developmental delay in counting strategies, number fact storage, arithmetic, and fact retrieval strategies (Moll et al., 2015; Von Aster & Shalev, 2007). Previous studies have found that individuals with language or reading problems perform poorly on arithmetic tasks compared to individuals without language or reading problems (Simmons & Singleton, 2006). In sum, there is increasing evidence to suggest that language skills are strongly related to children's mathematics skills.

Language skills can also have a specific role in mathematics by helping children develop content-specific vocabulary. Content-specific vocabulary in turn is necessary for children's conceptual understanding of mathematics. For example, children's knowledge of comparative and spatial words have been found to be significant predictors of early numeracy skills such as number knowledge and counting. Moreover, quantifiers (e.g. some, many, a few) have been found to be significant predictors of children's understanding of numbers (Barner et al., 2009). Understanding quantifiers allow children to produce and describe comparisons between numbers (e.g. Barner et al., 2009). Other studies have identified a relationship between content-specific vocabulary and children's quantitative understanding. For example, children's understanding of the word "*more*" goes hand by hand with their understanding that "*more*" can mean an increase in a set of objects (Purpura, Napoli, Wehrspann, & Gold, 2017). One study evaluated 3-to 5-year-olds' mathematics-related words with an experimental measure that assessed two specific types of words: comparative words, such as "more"/"less"; and spatial words, such as "below"/"middle" (Purpura & Logan, 2015). Findings from this study suggested that children's own mathematics-vocabulary predicted children's mathematics performance across the preschool years, and suggested that content-specific vocabulary supports children's ability to improve their conceptual understanding of quantity.

Content-specific vocabulary has been found to be so significantly related to mathematics that it has been proposed that some of the variance accounted for by general language skills in previous studies could have been attributed to content-specific vocabulary knowledge. One study investigated this suggestion directly. Three-to five-year-olds were measured on their mathematics vocabulary, general language skills, and early numeracy skills (Purpura & Reid, 2016). Mathematics vocabulary was determined by children's knowledge of comparative and spatial words. Results showed that even when general language skills were significant predictors of early numeracy, when the mathematics vocabulary knowledge was considered, mathematics vocabulary but not general language skills, was a significant predictor. This finding, then, provides some evidence that in some cases, general language skills that have been previously found to be related to early mathematics skills may have been acting as a proxy measure for mathematics vocabulary which in turn is more strongly related to mathematics (Purpura & Reid, 2016).

In conclusion, children's mathematics development can be shaped by caregiver's content-specific vocabulary, but also, by children's own general and content-specific language skills. Specifically, both general and content-specific vocabulary can provide the tools to facilitate the acquisition of some mathematics skills (e.g. counting) and conceptual understanding of mathematics (Donlan, Cowan, Newton, & Lloyd, 2007; Romano, Babchishin, Pagani, & Kohen, 2010).

4.1.3 Executive functions as predictors of children's mathematics skills

The extent to which executive functions are associated with mathematics skills have been studied to more extent in school-aged children whose mathematics skills are well established (Bull & Scerif, 2001; St Clair-Thompson & Gathercole, 2006b). Despite the sparseness in the literature there is some evidence that has suggested that the two executive functions that could be especially important for children's mathematics skills during the preschool years are working memory and inhibitory control (Blair & Razza, 2007; Bull et al., 2008; Purpura, Schmitt, & Ganley; 2017). For example, a longitudinal study assessed children's executive functions when children were 4 years of age and children's mathematics in three different time-points; the first time-point was when children were 4 years of age, then when children were between 5 and 6, and the third point when children were between 7 and 8 years of age (Bull et al., 2008). Working memory was measured with a digit span task, and inhibitory control and cognitive flexibility were measured with the inhibition and switching condition of the shape school measure, respectively. The inhibition condition consisted in presenting a child with cartoon faces arranged in three rows, and for which they needed to name the colour of the face only if they did not have a sad face on. The switching condition consisted on first, naming the cartoon faces by colour for faces without hats and by shape for faces with hats. This condition involve children using simultaneously the rules of colour and shape (Bull, et al., 2008). Mathematics was assessed with the PIPS assessment, a standardized measure of early mathematics that includes a number of constructs from counting, number recognition, simple arithmetic problems, to more complex mathematics skills, like interpretation of graphs.

Results from this longitudinal study found that after controlling for age and reading skills, working memory significantly predicted mathematics at each time point (from 4 years of age to 7 years) and that inhibitory control at age 4 was only a significant predictor of mathematics at 4. These results show how working memory and inhibitory control at an early age as 4 years are important for later mathematics proficiency. For example, one study found that 4-to 6-year-olds' working memory skills had a specific significant relationship to the mathematics task on a battery of early mathematics that involved maintaining information in mind for later processing and responding (i.e. a cardinality task; Purpura & Ganley, 2014).

The contribution of inhibitory control has also been observed in other studies, in preschool children. For example, one study conducted with 2-to 5-year-olds found that the role of inhibitory control was a significant predictor even after controlling for working memory, cognitive flexibility, age, maternal education, and vocabulary skills (Clark, Sheffield, Wiebe, & Espy, 2013). In this study, inhibitory control was a composite score of (i) the delayed response task, (ii) the continuous performance task, (iii) the statue task, and (iv) a self-control task. In the first task a reward is hidden randomly in one of two cups, while children were looking the two cups switched location (e.g. from left to right) and immediately hidden under the table. After ten seconds, children needed to look for the reward. In the second task, children were exposed by pictures of several animals displayed on a computer. The animal pictures were accompanied by different animals' sounds that rarely matched the animal (e.g. a dog meowing instead of barking). Children needed to press the computer mouse whenever they saw a picture of a sheep, regardless of the noise that it made. The third task, consisted on children standing with their eyes closed for 75 seconds, time during which the examiner tried to distract the child. Finally, the fourth task consisted on the child receiving a present at the end of testing, which children supposed to not touch it until the examiner finished another task. Mathematics was measured with a standardized measure (i.e. Woodcock–Johnson Psycho-Educational Battery–Revised)that contains different items regarding subitizing, ordinal counting, counting relevant object items, additions, and subtractions (Espy et al., 2004).

From studies conducted with older children, it has been proposed that inhibitory control may have the specific function of allowing children to focus on numerosities while ignoring competing non-numerical information when performing mathematics (Passolunghi & Siegel, 2001). In Passolunghi and Siegel's study (2001) it was found that 10-year-old children who had deficits in their performance on an arithmetic task (i.e. the arithmetic sub-test from a standardized test, the Wide Range Achievement Test) and an arithmetic word-problems task, presented greater intrusion errors in three different memory tasks. Intrusion errors were a measure of inhibitory control because they mirror the difficulty of focusing on task-relevant information and avoiding irrelevant information accessing the working memory. Thus, these results suggested that solving arithmetic problems and arithmetic word-problems, involves children to be able to avoid that irrelevant numerical information access their working memory in order to arrive at a solution (Passolunghi & Siegel, 2001).

4.1.4 Summary

In sum, there is some empirical evidence for longitudinal and concurrent linguistic precursors of mathematics, perhaps more evidence exists for concurrent language skills in mathematics than longitudinal precursors. Moreover, executive functions are related to mathematics because they allow the storage and manipulation of numerical information (working memory) and allow children to focus on numerosities while ignoring irrelevant information (inhibitory control). Investigating concurrent executive functions and their role in mathematics before children enter formal education is important because most of the activities that children first encounter when they enter school are novel to them. Thus, they may need to rely on their executive functions to a different extension comparing to children who have already entered school. Therefore, this study provides a good opportunity to study in combination all these cognitive skills that rarely have been studied together before. This is relevant because we can better identify developmental patterns in the precursors of mathematics.

There remain two gaps in our knowledge that can be addressed to contribute to a better understanding of the role of language skills in mathematics. First, there is evidence that early exposure to mathematics talk has an impact in children's mathematics after one year. However, we still don't know if this significant relationship hold over a longer period of time and whether we can observe this relationship with other mathematics domains, beyond what has been found with cardinality skills. If we were to observe a significant relationship between longitudinal linguistic skills to later mathematics, after a longer period of time, it would provide us with significant information about whether to focus our attention on the stimulation of mathematics talk between caregivers and children at such an early age in development.

Second, although there is some evidence that naturalistic caregiver-child interactions have an impact on early numeracy skills (i.e. Levine et al., 2010) we cannot tell if the relationship between caregiver's number talk and children's early numeracy skills is specifically related to children's engagement with this talk. It has been shown that within the talk that caregivers engage with their child (i.e. child directed speech); contingent talk (i.e. talk that occurs around whatever has caught an infant's attention) is likely to be more important for children's language development (Baldwin, 1991; McGillion, Pine, Herbert, & Matthews, 2017). If the mathematics-related talk that caregivers engaged with their child was not necessarily contingent we can expect to find an even stronger relationship between contingent mathematics-talk and children's early numeracy knowledge. In fact, Ramani and colleague's (2015) study suggests that direct engagement (i.e. semi-structured play) relates to children's understanding of advance mathematics concepts after a short period of time. Finding out specifically what type of engagement do children need to have with mathematics-related talk would help us design more efficient interventions.

In the studies presented in the introduction of the current chapter, there was no differentiation between words produced during child directed speech and words produced during contingent talk. And thus, it may be that although caregivers were producing number words, only the ones produced within contingent talk were significant for early numeracy skills. Additionally, the quantity of child directed speech in both studies the total of number talk (also known as tokens) were considered as variables. An alternative is choosing "types" or the number of unique words; for example the sentence "*taco cat spelled backwards is taco cat*" has seven 'tokens' but five 'types'. In this way, the quality of the number talk is considered rather than the quantity[3] (Rowe, 2012)

In sum, we don't know if the effect of linguistic input over children's mathematics skills will hold over a longer period of time and whether we can see a significant relationship with other (than cardinality) mathematics domains. We also don't know if there is a difference between number words used during child-directed speech and number words used during contingent talk; and whether or not choosing for words quality over quantity would make a difference in the results regarding the relationship between caregiver mathematics-related talk and children's mathematics skills.

The present study reflects an interesting and unique chance to look at language role in mathematics (in slightly younger children). Language skills could be important because they are the mean by which children gain conceptual understanding of mathematics-related ideas; or because

[3] Although there is some debate about which one (tokens vs types) is more significant for language development and they tend to be highly correlated. This subject goes beyond the scope of the current thesis, but it is well documented within the language development literature (e.g. McGillion et al., 2017; Rowe, 2008, 2012)

children actively use language when carrying out mathematics operations. There is evidence for these ideas in the literature. This study will allow us to test these ideas from a new perspective.

4.1.5 The current study

The current study aimed to investigate the contribution of longitudinal and concurrent linguistic precursors of pure and applied mathematics skills in 4-year-olds. In addition, the present study also looked at the concurrent role of executive functions. The main reason being the focus of this thesis, which is investigating the domain-general abilities underpinning children's mathematics skills and to allow us to investigate developmental patterns in the precursors of early mathematics skills.

Specifically, two different measures of linguistic precursors were considered: (i) caregiver's use of mathematics-related words when children were 11 months of age, data were: distinct mathematics words, distinct number words, and distinct mathematics words that were not number words ('other mathematics words' from now on), that caregivers used during both child directed speech (CDS) and CDS that was contingent on the infant's focus of attention (contingent talk). (ii) The predictor variables from the 2-year-olds' data were: the total number of distinct mathematics words, number words, and other mathematics words, within the child's speech. All these data came from naturalistic interactions, that is, unstructured play time. There were two main reasons as to why the 11 months data were chosen for the present study: (i) these data were pre-intervention baseline data and (ii) these data had already been transcribed. A similar situation happened when data at the age of 2 years were chosen. That is, the corpus of the words that children produced at the age of 2 years had already been transcribed. Taking into consideration the time constrains related to carrying out a study such as this; these were strong motives for choosing the data at these two time points.

Working memory (as measured by the Self-Ordered Pointing task), inhibitory control (as measured by a Flanker task), and language skills (as measured by the BPVS, the NFER-Language; and CELF) were also included. The role of these predictors was examined in both early numeracy skills (as measured by NFER-Mathematics) and applied mathematics (as measured by the Mathematical Reasoning). As for the measure of working memory, it was first considered to use the 'Spin the Box' measure as it is a frequently used task to measure working memory in children around the age of 4 years. However, data from the pilot study suggested that this measure was easy for the vast majority of the children, with 10 out of 13 children were at ceiling (Appendix 3). Thus, the Self-Ordered Pointing measure was chosen as a more appropriate measure.

There were two main hypotheses in this study:

The first hypothesis was that early exposure to mathematics talk and children's previous content-specific language contribute to the development of children's quantitative understanding. If this were the case then we would expect caregiver's use of mathematics words during caregiver-child interactions at the age of 11 months and children's mathematics words at the age of 2 years, to predict children's early numeracy skills and applied mathematics at the age of 4 years.

The second hypothesis was that children need to understand the verbal cues in a given mathematics problem. If this were the case, then we would expect children's general language skills at the age of 4 predict children's early numeracy skills and applied mathematics at the age of 4 years.

Additionally, it was predicted that working memory and inhibitory control, would be related to mathematics because children need to store and manipulate numerical information, and focus on numerosities while ignoring competing non-numerical information, when performing mathematics. It was also expected to find this higher in applied mathematics as its performance involves multiple step and the numerical information is embedded within irrelevant information.

4.2 Method

4.2.1 Study design

The present study was designed to investigate whether children's exposure to mathematical language at the age of 11 months and children's mathematics vocabulary at the age of 2 years functioned as precursors of pure (i.e. early numeracy skills, this label will be used from now on) and applied mathematics at the age of 4 years. Furthermore, the predictive value of working memory, inhibitory control, and language skills at the age of 4 years was also examined.

The outcome variables were early numeracy skills as measured by the NFER-Mathematics, and applied mathematics skills as measured by the Mathematical Reasoning sub-test of the WIAT-II when children were four. The predictor variables from the 11-month-olds' data were: distinct mathematics words, distinct number words, and other mathematics words that caregivers used during both children directed speech (CDS) and CDS that was contingent on the infant's focus of attention (contingent talk). The predictor variables from the 2-year-olds' data were: the total number of distinct mathematics words, number words, and other mathematics words, within the child's speech. Finally, the predictor variables from data collected when participants were 4 years of age were: performance on the self-ordered pointing task (working memory), performance on the Flanker task (inhibitory control), performance on the bubble popping task (processing speed), and performance on the following language measures: BPVS, NFER-Language, and the CELF. The covariates were chronological age, SES (measured by the IMD-rank at the age of 4 years), caregiver's education (at the age of 4 years), the total of distinct words used in both CDS and contingent talk, and the total distinct words in child's speech at the age of 2 years.

4.2.2 Participants

Participants were 87 children (46 female) drawn from a larger sample of 142 children who were part of a longitudinal study looking at children's language development (McGillion et al., 2017). This broader study was conducted by researchers from the Department of Psychology at the University of Sheffield and funded by the Nuffield Foundation and the British Academy. Participants were initially recruited from a volunteer database from the Cognitive Development Research group at the Department of Psychology and were followed from 11 months (N=140, *Mean age*=10.98 months, SD=.13) to 2 years of age (N= 101, *Mean age*=25.64 months, SD=1.34). Participants were invited to take part in the current study when they reached the age of 4 years. Only participants who agreed to participate in this time-point and who gave consent to video-record the session, were included. Ethical approval for the studies was obtained from the Department of Psychology's ethics sub-committee.

From the 87 children who participated at all three time points, 16 children were excluded: 11 children had missing data due to failure to follow instructions or to caregiver's interference, and another five children were excluded when parent's responses on follow-up questionnaires indicated that their child had a diagnosis of a developmental condition or delay. As such, the final sample was 71 children (38 Female, *Mean*=50.04, SD=1.54). The families and children who took part were from a wide range of SES backgrounds (from the 1st to 10th IMD Deciles) in and around the area of Sheffield, England. All children came from families whose first language was English.

4.2.2.1 Overview of the recruitment and data collection of participants at 11 months and at 2 years of age

The present study took place in the context of a wider longitudinal-intervention study[4] of children's language development (Davies, McGillion, Rowland, & Matthews; in prep). Only methodological details

[4] There was no difference at baseline on any measures collected, so it is not considered further here.

relevant to the current study are reported here. Further details can be obtained in McGillion, Pine, Herbert, and Matthews (2017).

Families were initially contacted via email and post and were invited to take part in a longitudinal study of infant development. All children were monolingual English speakers. Families that agreed to take part were visited in their homes when children were 11, 12, 18 and 24 months old. The visits included video-recordings of caregivers-child interaction during play. All data were extracted from video-recordings from 30 minutes of play in the participant's home (15 minutes unstructured, and 15 minutes structured, with toys provided by the researcher). Researchers asked the caregiver to play with their infant as they usually did. Ten minutes of unstructured play were coded. All CDS was extracted from this 10 minutes. All the words produced during CDS and contingent talk were transcribed orthographically following the Child Language Data Exchange System's (CHILDES) CHAT conventions (MacWhinney, 2000). Child words at the age of 2 years were orthographically transcribed from 30 minutes of naturalistic play using ELAN software following the CHILDES CHAT conventions (MacWhinney, 2000). To control for differences in the length of recordings, a word per minute count was calculated (McGillion et al., 2017). This procedure was carried out by Dr McGillion, from the longitudinal-intervention study team.

Two Excel data sets, one containing the corpus of words that caregivers used during CDS and contingent talk when children were 11 months old, and the other one containing children's own words at the age of 2 years, were provided by the researchers conducting the wider longitudinal language study. From these two data sets, mathematics-related words were extracted. The search terms for the mathematics words were identified from the NFER Mathematics application protocol. These words were extracted from the 11m CDS data-set. Words were divided into two categories: *number words* and *other mathematics words*. The number words did not include the number word *one* because this word could have occurred in other linguistic contexts that are not about counting (e.g. "*I* *want that one*"). The words that appeared in the search are shown in Table 15. This procedure was carried out by the author of this thesis following advice from Dr McGillion.

Table 15. Mathematics words found in caregiver's and child's speech in Study 3

Search terms	Found in caregiver's CDS and contingent talk	Found in children's speech at age 2
Number words		
Zero	*	*
Two to sixteen	*	*
Seventeen	*	
Eighteen	*	
Nineteen	*	
Twenty	*	*
Other mathematics words		
Add		
Big/bigger/biggest	*	*
Circle	*	
Count/counting	*	*
Half	*	*
More	*	*
Number	*	*
Numbers	*	
Rectangle	*	*
Shape(s)	*	*
Shaped	*	
Size		*
Small		*
Square	*	*
Tall		*
Taller		*
Triangle	*	*

4.2.3 Materials. Measures for the experiment conducted with the children at the age of 4 years

Demographic questionnaires were used to collect SES background information (through the caregivers and children's postcodes to calculate the IMD-rank) and caregiver's educational level. Caregiver's education was measured with a scale of 1 to 7, where 1 equals no formal qualifications, 2 equals 1-4 GCSEs, O Levels (at any grade), or NVQ Level 1 or similar; 3 equals 5+ GCSEs (grades A*-C), O levels (passes), or NVQ level 2 or similar; 4 equals 1 A Level or 2-3 AS Levels; 5 equals 2+ A Levels or NVQ Level 3 or similar; 6 equals University degree, HND, HNC, NVQ Level 4 or 5; and 7 equals postgraduate degree or similar (e.g. PGCE, PhD, MA etc). Maternal and paternal education were averaged to give a single score for parental education.

Additionally, all sessions were video-recorded using a SONY HDR-PJ220E camera on a tripod set next to the table were testing was conducted.

NFER-Mathematics. To assess children's early numeracy skills, the mathematics subtest of the NFER was used. This subtest consists of 11 task-based activities assessing early numeracy (one-to-one correspondence, identification of numerals, numbers ordering, finding 'more', finding 'one less', practical addition, practical subtraction, and written addition); and geometry (halving, knowledge of shapes, and pattern recognition) skills. This is a school-based measure. Its tasks have been matched to the Early Years Outcomes for mathematics in England, which is a guide for practitioners to assess children's progress in mathematics and whether the child's mathematics development is typical for his or her age. The NFER mathematics has also been matched to the English Year 1 National Curriculum requirements for mathematics (which is divided in "Numbers" knowledge and "Shape, space, and measures" knowledge). This is a child-friendly measure with colourful and practical resources that makes the measure enjoyable for the children.

Mathematical Reasoning. This task was identical to the one reported in Chapter Two. However, for the present study, this measure was not applied the standardized way. Testing started with item six, because the first five items of the measure dealt with concepts already assessed by the NFER-Mathematics. This method of application of the measure was tested in a previous pilot study and was proven to be effective so that children were able to start and understand the measure from item six (see Appendix 3 for more details about the pilot study). There is no normative data for children age 4, therefore raw scores are reported.

Self-ordered pointing task. The self-ordered pointing task (Petrides & Milner, 1981) is a working memory measure commonly used in adults. This measure was modified to be child-friendly. This task involved a set of animals images printed on laminated sheets of A4 paper. For each trial, the animals were arranged in a different order so that spatial location was not informative. Children were instructed to point at different animals each time so that all of the animals "get a turn." Thus, children needed to remember which animal they had selected before. The task started with a practice block in which two different animals were presented. Thus, for the first page they can point to either of the two pictures, but for the second page they must pick the other one. There were three trials in the practice block, each of which had two different animals. Testing did not continue unless children understood what they needed to do. The test blocks also had three trials in total, and used different animals for each trial. Each test block increased in span length by one additional animal each time, to a maximum length of eight animals. Children received a score of one for every time they pointed at an animal that they had not previously selected (since the first animal served as reference for pointing at the other animals, this item was not scored). The task ended when children made two errors in any test block. Possible scores ranged from 0 to 81. The predictor variable was children's total scores.

Flanker task. The Flanker task was used to measure children's inhibitory control. It was a computerized task administered using E-Prime. Children were asked to identify whether a fish in the centre of a line of five fish faced left or right. Congruent and incongruent trials were created by having flanking fish that faced either in the same direction, or in the opposite direction. Half the trials were congruent and half were incongruent. There were four demonstration trials, four practice trials, and 60 testing trials divided in three blocks. Before children were showed the practice trial on the screen, they were showed a printed version of what congruent and incongruent trials looked like to make sure they knew what "looking at the fish in the middle" meant.

The procedure of the testing blocks was as follows: a fixation point (a starfish) appeared on the centre of the screen for 1000 ms, followed by the target fish and the flankers, with two arrows below, until the participant's response or up to 4000 ms. After the response a feedback noise was presented lasting no longer than 1000 ms, the target and flankers disappeared after the response and the next trial began. After each testing block, a "well done, take a break" message appeared on the screen. There was no limit in the duration of these breaks; children were asked to touch the screen whenever they were ready to continue with the game. The total number of correct responses in all trials of the three blocks was calculated and used as the predictor variable.

Language skills. A composite score for language skills was calculated with data from three different language measures (BPVS, NFER-Language, and CELF; described below). A principal component analysis (PCA) was conducted to create the language score.

BPVS-II. The BPVS (Dunn et al., 1997) was used to assess children's receptive vocabulary (specifics of the task are reported in Chapter Two). The predictor variable was children's total score.

NFER-Language. The NFER language sub-test measures general communication, language, and literacy skills: vocabulary, phonics, picture sequencing, story prediction, listening comprehension, word reading, simple sentence reading, and name writing. The total score for this measure was used as the predictor variable.

CELF-Preschool 2 UK. The CELF (Wiig, Secord, & Semel, 2006) is one of the most widely used standardised measures of language skills in preschool. It measures two areas of language skills: expressive language and receptive language. The expressive language sub-tests measure children's knowledge of grammatical rules, children's ability to name images of people, objects, and actions; and children's ability to recall and repeat orally presented sentences. The receptive language sub-tests assess syntactic skills, children's ability to understand and follow oral commands, and knowledge of basic concepts such as dimension, location, quantity, and equality. The total score using both expressive and receptive scores was the predictor variable.

Bubble popping task. Processing speed was measured with a “bubble popping” task. This was a computerized task administered with E-Prime and using a touch screen. This task was designed by Blakey (2015), and it was modified to include 14 test trials instead of eight[5]. The task consisted of bubble stimuli that appeared on a touch screen. Children were instructed to “pop” the bubble (by touching it) as fast as they could. Feedback was an image and the sound of a burst bubble. Before the task commenced children had three practice trials. The length of the interval between stimuli (ISI) varied randomly between 800ms and 1200ms. The mean reaction time was the predictor variable.

4.2.4 Procedure. Data collection from children at the age of 4 years

For the data that were obtained when children were 4 years of age, families were contacted again and invited to take part in a follow-up study in which they were going to participate in three different sessions. Data for the current study was obtained from the first and second session. Only the first two sessions will be described in full. The first session was conducted in a quiet testing room within the University of Sheffield’s Department of Psychology by the author of this thesis. Tasks were administered in the same order for all children and were video-recorded. Each child was tested individually with the presence of the caregiver. Written consent was obtained from caregivers before testing began. The first session lasted approximately 40 minutes and in this, the mathematics measures, the executive functions measures, the processing speed measure, and the BPVS were administered. The second session was conducted by a research assistant approximately one week after the first session at the family home.

[5] Based on the pilot study results who indicated that 8 bubbles were easy for the children

In this session children completed the NFER Language measure and the CELF. All participants received a small gift after testing, on both visits.

Moreover, besides searching and extracting the mathematics-related words from the longitudinal data, the author of this thesis collected, coded, and transcribed all data from the first session of the testing period in which children were 4 years of age. Additionally, all analyses and writing up the results from all the data that came from the two sessions at this time-point, was also carried out by the author of this thesis.

4.3 Results

4.3.1 Missing data

Since not all the children participated at all three time points (11 months, 2 years, and 4 years of age), there were different sample sizes at each time point. Data for parent's contingent talk came from 67 children; data for children's mathematics vocabulary came from 69 children; and data for the mathematics measures at age 4 came from 71 children. There were some missing data from the testing at age 4: 11 children had missing data in the working memory task (ten due to experimenter error and one due to failure in understanding the instructions). In addition, the flanker and bubble popping data of two children were missing due to technical problems. An additional child had missing data in the BPVS, another child had missing data on the NFER-Language, and two more children had missing data on the CELF due to refusal to participate. Analyses were therefore conducted using all available data using pairwise deletion to avoid bias and reduction in power.

4.3.2 Descriptive statistics

An exploration of the data was conducted to investigate whether caregivers and children were using mathematics words during the naturalistic interactions. Caregivers were not directly asked to use mathematics words during their interactions with their child, so the amount of mathematics words used could be highly variable. First it was investigated if caregivers produced mathematics words when talking to their child at 11 months. A search within the corpus of all the words produced by the caregivers showed that caregivers did produce mathematics words when interacting with the children. On average caregivers produced 3.45 unique mathematics words during the session (*SD*=2.99), and were more likely to use number words than non-number words, such as names of shapes (see Table 15 for the words found in the caregiver's speech).

Data from children when they were 2 years of age showed that children were not producing many words overall, and across 30 minutes video-recording, children produced only two mathematics-related words in total (specifically, .08 number words and .05 other mathematics words; see descriptive statistics in Table 17). Descriptive statistics for caregiver's use of distinct mathematics words in CDS and contingent talk are presented in Table 16, and children's use of mathematics words at the age of 2 years[6] are presented in Table 17. Age, mathematics measures, executive functions measures, language skills measures, processing speed measure, SES, and caregiver's education, at the age of 4 years are provided in Table 18. The descriptive statistics reported in Table 18 are from the raw scores of all measures. Skewness and kurtosis values indicated that not all variables had a normal distribution. A further Shapiro-Wilk test confirmed the non-normal distribution of all the variables (all p<.05) except for caregiver distinct words in CDS (p=.14), NFER-Mathematics (p=.53), and CELF (p=.19). Thus, non-parametric tests are used in the following analyses.

[6] Histograms of children's use of mathematics words are shown in Appendix 4

Table 16. Descriptive statistics for all measures at 11 months of age

Variables	N	Mean (SD)	Range Min-Max	Skewness	Kurtosis
11 Months	67				
Caregiver Distinct Words in CDS (Total)		158.96 (45.58)	80-337	1.14	2.93
CDS All Mathematics Words		3.45 (2.99)	0-11	.88	-.02
CDS Number Words		1.94 (2.52)	0-10	1.52	1.89
CDS Other Mathematics Words		1.51 (1.44)	0-6	1.02	.70
Caregiver Distinct Words in Contingent Talk (Total)		117.49 (36.90)	44-235	.45	1.05
Contingent Talk All Mathematics Words		2.67 (2.88)	0-11	1.43	1.43
Contingent Talk Number Words		1.66 (2.48)	0-10	1.80	2.80
Contingent Talk Other Mathematics Words		1.01 (1.08)	0-5	1.38	2.55

Table 17. Descriptive statistics for all measures at 2 years of age

Variables	N	Mean (SD)	Range Min-Max	Skewness	Kurtosis
2 Years	69				
Child Distinct Words (Total)		4.27 (2.12)	.61-13.43	1.40	4.78
Child All Mathematics Words		.12 (.10)	0-.53	1.35	2.58
Child Number Words		.08 (.09)	0-.42	1.58	2.78
Child Other Mathematics Words		.05 (.04)	0-.16	.70	.31

Note: To control for differences in the length of recordings, a word per minute count was calculated.

Table 18. Descriptive statistics for all measures at 4 years of age

Variables	N	Mean (SD)	Range Min-Max	Skewness	Kurtosis
4 Years					
Age	71	50.04 (1.54)	48-55	1.41	2.80
NFER Mathematics	"	17.23 (5.11)	7-32	.35	.27
Mathematical Reasoning	"	10.76 (3.00)	4-22	.46	1.77
Self-Ordered Pointing	60	20.93 (12.41)	4-57	1.43	2.34
Flanker (Accuracy)	69	33.86 (9.73)	10-57	.83	.52
BPVS	70	52.21 (8.49)	38-79	.63	.35
NFER Language	"	18.41 (7.41)	6-38	.59	-.22
CELF	69	49.30 (7.86)	28-63	-.42	-.11
Bubble Popping	"	1105.52 (308.15)	737.64-2133	1.67	3.05
Caregiver Education	71	5.51 (1.24)	1-7	-1.08	1.30
IMD-Rank	"	16698.07 (9430.91)	480-32062	-0.15	-1.27

4.3.3 Language score

A principal component analysis (PCA) was conducted to provide a robust index for language skills (the BPVS, the NFER-Language, and the CELF). Because of missing data, a pairwise deletion was selected for the PCA. In the PCA (with direct oblimin rotation), the Kaiser-Meyer-Olkin measure (KMO) verified the sampling adequacy for the analysis, KMO=.64 (Field, 2009). Bartlett's Test of Sphericity X^2 (3) =38.80, p<.001, indicated that correlations between items were sufficiently large for a PCA. Results revealed that the BPVS, the NFER-Language and the CELF loaded into one component explaining 63.22% of the variance. Inspection of the component matrix table shows that all items loaded strongly (all well above .40; Field,

2009) on the one underlying component. This component score was saved and used in the subsequent analyses as the language score.

4.3.4 Non-parametric correlations

In order to understand the relationship between mathematics skills on the one hand, and children's language precursors, working memory, inhibitory control, and language skills on the other, a simple non-parametric correlation analysis was conducted. To avoid bias and reduction in power that result from listwise deletion of data (Graham, 2009), correlation analyses were conducted using all available data using pairwise deletion.

Only significant associations with the outcome variables are reported here. The complete correlation matrix reporting all simple non-parametric correlations is shown in Appendix 5. Results showed that NFER- Mathematics was significantly correlated with age (rs=.59, p<.001), the BPVS (rs=.26, p<.001), the NFER Language (rs=.65, p<.001), and the CELF (rs=.38, p<.001). Additionally, there was a significant negative correlation between NFER Mathematics and two variables from contingent talk: all mathematics words (rs=-.28, p<.05), and other mathematics words (rs=-.28, p<.05). Mathematical Reasoning was significantly correlated with child's other mathematics words (rs=.24, p<.05), the BPVS (rs=.35, p<.001), NFER Language (rs=.49, p<.001), CELF (rs=.25, p<.05), and the flanker task (rs=.34, p<.001). Additionally there was a significant negative correlation between NFER-Mathematics and all mathematics words (rs=-.25, p<.05), and other mathematics words in contingent talk (rs=-.25, p<.05).

In order to identify unique associations between mathematics skills, language precursors, working memory, inhibitory control, processing speed, and the language score from the PCA, a partial non-parametric correlation was conducted (Table 19). The partial non-parametric correlation was conducted controlling for age, SES, caregiver's education, total number of distinct words used in both CDS and contingent talk; and total number of words in children's speech. The analyses were performed using SPSS Syntax.

Results showed that NFER-Mathematics was significantly correlated with the language score (r_s=.46, p<.001), and significantly negatively correlated with other mathematics words in contingent talk (r_s=.27, p<.05). Mathematical Reasoning was significantly correlated with the flanker task (r_s=.36, p<.01) and the language score (r_s=.50, p<.001).

Table 19. Correlation matrix reporting partial non-parametric correlations controlling for age, SES, caregiver's education, total number of distinct words used in both CDS and contingent talk; and total number of words in children's speech.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1. NFER Mathematics	1														
2. Mathematical Reasoning	.60***	1													
11 Months															
3. CDS All Mathematics Words	-.13	-.15	1												
4. CDS Number Words	-.04	-.11	.83***	1											
5. CDS Other Mathematics Words	-.19	-.15	.59***	.09	1										
6. Contingent Talk All Mathematics Words	-.21	-.26	.92***	.80***	.51***	1									
7. Contingent Talk Number Words	-.14	-.19	.81***	.93***	.15	.88***	1								
8. Contingent Talk Other Mathematics Words	-.27*	-.25	.56***	.14	.87***	.59***	.19	1							
2 Years															
9. Child All Mathematics Words	.05	.13	.00	.07	-.16	-.06	.01	-.18	1						
10. Child Number Words	.08	.07	.08	.13	-.08	.01	.07	-.12	.86***	1					
11. Child Other Mathematics Words	.06	.22	-.13	-.15	-.08	-.18	-.21	-.08	.36**	-.03	1				
4 Years															
12. Self-ordered pointing task	-.11	.00	.13	.08	.10	.20	.15	.12	-.19	-.24	.10	1			

13. Flanker task	.19	.36**	-.16	.00	-.33**	-.11	-.02	-.22	.05	.08	-.08	.28*	1		
14. Language score	.46***	.50***	-.26*	-.08	-.37**	-.24	-.08	-.45***	-.19	-.19	-.14	.02	.17	1	
15. Bubble popping task	.02	.05	-.08	-.11	.03	-.04	-.09	.03	.16	.03	.17	.07	-.07	.12	1

Note. Correlations= *<0.05, ** <0.01, and *** <0.001

4.3.5 Hierarchical multiple regression analyses

Hierarchical multiple regressions were conducted to further investigate the amount of unique and incremental contributions to NFER-Mathematics and Mathematical Reasoning, by the significantly correlated variables. The predictor variables were entered in the model on the basis of the magnitude of the partial non-parametric correlations (see Table 19). The first hierarchical multiple regression (Table 20) shows the analysis conducted to investigate NFER-Mathematics. The language score was entered in step 1 and "other mathematics words" in contingent talk was entered in step 2. In this way, any final step that accounted for significant additional variance, shared unique links with NFER-Mathematics.

As shown in Table 20, step 1 for the NFER-Mathematics indicated that language score predicted 35% of significant variance. Step 2 shows the additional incorporation of other mathematics words in contingent talk contributed an additional 1% significant variance; nevertheless, the unique significant predictor was language score. The Durbin-Watson test was checked and was found to be within acceptable parameters (1.25), thus the assumption of independent errors has been met. This result suggests that this regression model is unbiased, increasing the likelihood of these results being true for a wider population (Field, 2009). Additionally, the largest VIF was well below 10, and the average VIF was 1.08; similarly the tolerance data are all within acceptable boundaries (all greater than 0.1). Therefore, it was concluded that there was no collinearity within the data (Field, 2009).

Table 20. Hierarchical regression analysis predicting unique variance in NFER Mathematics

Step		B	SE B	β	*t*	*p*
1	(Constant)	17.16	.52		33.19	<.001
	Language score	3.00	.52	.59	5.77	<.001
	F (1,63) =33.24, p <.001, R^2=.35, p<.001					
2	(Constant)	17.16	.73		24.12	<.001
	Language score	2.87	.54	.56	5.29	<.001
	Contingent Talk Other Mathematics Words	-.44	.50	-.09	-.86	.39
	F(2,62)=16.93, p <.001., ΔR^2=.01, p=*.39*					

The second hierarchical multiple regression (Table 21) shows the analysis conducted to investigate Mathematical Reasoning predictors. The language score was entered in step 1, following by flanker task in step 2. The order of entry of the predictors was based on the magnitude of the partial non-parametric correlation (see Table 19). Step 1 for Mathematical Reasoning indicates that the language score predicted 26% of significant variance. Step 2 shows that the additional incorporation of flanker task contributed an additional 11% significant variance. The Durbin-Watson test was checked and was found to be within acceptable parameters (1.88).

The beta values from both the predictor variables are significant, indicating that both the language score and flanker task were significant independent predictors of Mathematical Reasoning. The β values indicated that the language score was a stronger predictor (β=.39) than flanker task (β=.36). The largest VIF was well below 10, and the average VIF was 1.13; similarly the tolerance data are all within acceptable boundaries (all greater than 0.1). Therefore, it was concluded that there was no collinearity within the data (Field, 2009).

Table 21. Hierarchical regression analysis predicting unique variance in Mathematical Reasoning in Study 3

Step		B	SE B	Β	*t*	*p*
1	(Constant)	10.73	.32		33.49	<.001
	Language score	1.53	.32	.51	4.75	<.001
	$F(1, 64) = 22.56$, $p < .001$, $R^2 = .26$, $p < .001$					
2	(Constant)	7.03	1.15		6.14	<.001
	Language score	1.71	.32	.39	3.68	<.001
	Flanker task	.11	.03	.36	3.34	.001
	$F(2,63) = 18.66$, $p < .001$, $\Delta R^2 = .11$, $p = .001$					

In summary, hierarchical multiple regressions revealed that NFER Mathematics was uniquely predicted by language skills, and Mathematical Reasoning was significantly predicted by language skills and inhibitory control.

4.4 Discussion

The aim of this chapter was to investigate the contribution of the longitudinal linguistic precursors and concurrent language and executive functions skills, to 4-year-olds' early numeracy skills and applied mathematics performance. Overall, two key findings can be identified in the current study. First, longitudinal linguistic precursors did not predict mathematics skills at the age of 4 years. Second, both mathematics skills relied on different concurrent predictors, such that early numeracy skills relied on general language skills, and applied mathematics relied on general language skills and inhibitory control.

4.4.1 Does children's early language experience predict their mathematics ability?

First, it was hypothesised that early exposure to mathematics talk and children's previous content-specific language contribute to the development of children's quantitative understanding. It was therefore expected that caregiver's use of mathematics words during caregiver-child interactions at the age of 11 months and children's mathematics words at the age of 2 years would predict children's early numeracy skills and applied mathematics at the age of 4 years. Results did not support this hypothesis. Although previous studies have found caregiver's mathematics-talk to be significant to children's mathematics performance approximately one year after (i.e. Levine et al., 2010), this finding suggest that this relationship does not hold in the long term.

Arguably, the most plausible interpretation for these data is that language exposure at 11 months and 2 years of age really don't predict mathematics performance four and two years after. Additionally, there was not enough variance in both caregivers and children's mathematics talk. Descriptive statistics about the frequency of words that children used in caregiver-child interactions showed that children did not produce many words overall, this could have reduced statistical power in the analyses and thus influenced the results by decreasing the chances of observing a true effect. Mathematics development is multi-factorial and variable, thus, arguably it may not be surprising that something as simple as a brief sampling of language exposure was not able to predict, several years later, the child's ability to understand and act upon mathematics concepts and ideas. However, it was a significant attempt to explore whether children's exposure to linguistic input, in naturalistic settings, as early as 11 months could be related to their mathematics skills.

This approach is somewhat easy to improve. That is, future research could make structured interactions so that the frequency of mathematics words can be assured. For example, a very recent intervention study found that prompting children to produce mathematics-language contributed significantly to their mathematics-knowledge (Purpura, et al., 2017). This study used a novel methodology in which dialogic reading was the mean by which quantitative and spatial mathematics-language (e.g. more, below, near) was taught to children. This type of reading involves the child directly into the story by making the child the storyteller, while the person leading the intervention presented different stimuli to incorporate additional mathematics-language. Results showed that the intervention group outperformed the control group in both mathematics-language and knowledge.

Thus, this study showed that children's early mathematics related language experience does not appear to predict their later mathematics ability, and although previous research has suggested that domain-specific language skills are more relevant for mathematics than general language skills; general language skills may allow children to access the symbolic numerical information that is necessary for mathematics learning (Klibanoff et al., 2006).

4.4.2 Concurrent linguistic and executive functions predictors of children's mathematics

Concurrent language skills at the age of 4 were significant predictors for early numeracy skills and applied mathematics. This finding supports the hypothesis that language skills at the age of 4 years would be related to early numeracy skills and applied mathematics at the same age. This finding with 4-year-old children is broadly consistent with findings with 5- and 6-year-old children in Study 1 from the present thesis, by demonstrating that different mathematics skills rely on language. However adds to our knowledge by demonstrating that even before school-entry, language skills have a central role in children's mathematics (Romano et al., 2010).

With regards to the concurrent relationship between executive functions and mathematics, working memory was not a significant predictor of either of the mathematics skills and inhibitory control was significant only for applied mathematics. The finding regarding working memory was unexpected because working memory has been widely shown to be involved in young children's mathematics skills, and because results from the previous studies in this thesis in the thesis indicated that it plays a role in both pure and applied mathematics in 5-to-6-year-old children.

There are two plausible possibilities as to why working memory was not related to any of the mathematics skills. One possibility is related to the design of the working memory measure and the second possibility is related to missing data on this measure.

Concerning the design of the working memory measure, namely, it is possible that the measure used to assess working memory was not effective at measuring this construct. The Self-Ordered Pointing task was used to measure working memory. This was an experimental task designed specifically for this study (based on Petrides & Milner, 1982) that consisted on pointing at different animals located in different spatial location across testing trials. Although in this task the animals included were animals that the children were not very likely to be familiarized with to avoid verbal labelling as a strategy (e.g. a quetzal); children may have still rely on these strategies to solve the task (e.g. Cragg & Nation, 2007). If children were using verbal labelling as a strategy then that would have affected performance by making the items easier to remember.

The second possibility is that missing data could have reduced the chances of detecting a true effect. Data from 11 children were eliminated due to experimenter error, and thus analyses were conducted with data from a smaller sample of 60 children. Missing data is in fact a mayor limitation in this study- although it is common in studies conducted in young children. A power analysis (conducted using the G*Power software) for a Spearman correlation with a = 0.05, a medium effect size f^2=0.3, and a desired power of 0.80 for a two-tailed test; resulted in a required overall sample size of 82 children. The current study was conducted with different sample sizes being the highest 71 children. Thus, that the working memory analyses were conducted with 60 children, could have reduced the chances of detecting a true effect and different results might have been obtained if all children could have been included.

Since the Self-Ordered Pointing task has been proven to be a sensitive measure of children's working memory (Cragg & Nation, 2007), the second possibility is, therefore, the most likely possibility as to why there was no significant relationship between working memory and mathematics. Thus, the role of working memory in the performance of children's mathematics cannot be ruled out completely.

The second finding regarding the role of executive functions was that inhibitory control was a significant predictor of applied mathematics, but not of pure mathematics. This finding was contrary to the prediction that inhibitory control supports bot early numeracy skills and applied mathematics. That inhibitory control supported applied mathematics is in line with the body of research that suggests that children rely on their inhibitory control during situations in which they needed to focus on numerosities while ignoring competing irrelevant non-numerical information (e.g. Espy et al., 2004; Passolunghi & Siegel, 2001).

In sum, these findings suggest that linguistic precursors did not predict mathematics skills several years later, however concurrent language skills play a significant role in children's pure and applied mathematics even before school-entry. Very few studies have considered the role of longitudinal and concurrent linguistic precursor together. Although in this study the role of longitudinal linguistic precursors seemed to not be significant for later mathematics skills, this study provides a first step in providing evidence that among longitudinal and concurrent linguistic precursors, concurrent language skills are more relevant for 4-year-olds' early numeracy skills and applied mathematics. However further research on this matter is needed to understand the specific relationship between language and mathematics in full. Additionally, results regarding the role of executive functions suggest that being able to ignore irrelevant information is a key process in 4-year-olds' mathematics learning.

Chapter Five

General Discussion

In this final chapter I will present an overview of the three studies that make up this thesis. Following the studies overview, research contributions will be addressed. Finally, this chapter will close with some discussions concerning possible study limitations, and directions for future research.

5.1 Overview of the Experimental Studies

The goal of this thesis was to investigate the domain-general abilities that underpin mathematics skills in childhood. This was achieved by the means of three experiments focusing on (i) the contributions of working memory components (Study 1 and 2), (ii) the contributions of concurrent language skills (Study 2 and 3), and (iii) the longitudinal linguistic precursors of mathematics skills (Study 3). Another distinctive characteristic of this work is that the three studies focused in particular on the distinction between relatively pure mathematics skills and applied mathematics in every-day situations. In Study 1 and 2, the measure for pure mathematics skills were arithmetic skills because arithmetic represent a core subject of the mathematics curriculum during the first year of formal education. The pure mathematics measure in Study 3 was early numeracy skills, which involved basic mathematics skills such as counting, simple addition and subtraction problems, understanding mathematics words such as *"more"*, *"less"*, and *"half"*, and knowledge on different shapes. As for the measure of applied mathematics, the same measure (the Mathematical Reasoning sub-test) was utilized in the three studies. This distinction is important because it can inform us about the cognitive skills that children need, to perform mathematics in school, and to apply the acquired mathematics knowledge in situations of their daily lives.

The first study aimed to investigate the contributions of working memory components to both arithmetic skills and applied mathematics in 5-to 6-year-olds (*Mean age*=69, *SD*=5.3). Mathematics skills were assessed using two different measures: the Numerical Operations test (as a measure of arithmetic skills) and the Mathematical Reasoning test (as a measure of applied mathematics). Additionally, receptive vocabulary was included as a covariate. Results suggested that working memory components underlying arithmetic skills were importantly different from those underlying applied mathematics: arithmetic skills relied on central executive resources only. Conversely, applied mathematics relied not just on central executive resources, but also on the phonological loop and the visuo-spatial sketchpad. These findings were consistent with the idea that children's performance on arithmetic problems involved allocating attentional resources to guide the use of counting strategies while solving single-digit additions and subtractions –processes that are heavily dependent on the central executive. Conversely, findings regarding applied mathematics were consistent with the idea that children's performance on applied mathematics involved attentional resources to attend to key numerical information, while inhibiting irrelevant information. Moreover, the contribution of the visuo-spatial sketchpad and the phonological loop, suggested that performance on applied mathematics also involved storing verbal information and representing it mentally to achieve an answer.

Additionally, it was found that receptive vocabulary predicted performance of both mathematics skills as well. This surprising finding indicated that language skills are likely to play a specific role in children's mathematics performance. I suggested, then, that receptive vocabulary was a proxy for other, more specific, language skills because previous literature has found that vocabulary highly correlates with several different language skills. However, because of the design of the study, a strong conclusion could not to be reached. Therefore, Study 2 was designed to try to further explore the role of language in young children's mathematics ability.

As such, Study 2 (Chapter Three) was designed to investigate the specific contribution of children's language skills and working memory skills to both arithmetic and applied mathematics skills. Two ways in which language could have been related to mathematics skills were considered, one way was that in order to solve specific mathematics problems, children needed to understand the verbal cues within the problems, involving syntactic skills (Cummins et al., 1988; Munro, 1979). A second way was that language might be important for children's mathematics to understand the words in a given classroom-task or instruction (Hornung et al., 2014a). Thus, it was of interest to investigate how syntactic skills and children's ability to understand and follow oral commands could be possible contributors to arithmetic and applied mathematics, in 5-to-6-year-olds (*Mean age*=69.64, *SD*=4.30). Two different measures of language skills were used for this purpose: (i) '*Sentence Structure*' to measure syntactic skills and (ii) '*Concepts and Following Directions*' to measure children's ability to understand and follow oral commands. With these language measures I wanted to investigate how understanding words, sentences, and meaning of spoken language could affect mathematics performance. Because Study 2 followed directly from Study 1, working memory components were also included. Additionally, age, SES, and processing speed were considered as possible covariates. Findings in Study 2 suggested that performing mathematics problems presented in an applied context required 5- to-6-year-olds to understand and follow oral commands. Moreover, arithmetic skills performance was not explained by any of the language skills. Thirdly, contrary to my predictions, and very surprisingly, results regarding the role of working memory in mathematics found in the first study, were not replicated.

There were two plausible possibilities that could have explained the result of no-replication of working memory results. One possibility was the incidental executive and visuo-spatial demands on the language measures. That is, both language measures were highly dependent on working memory resources -as shown in the non-parametric correlation analyses. Alternatively, as there were significant differences between schools SES in

Study 1 and 2, another possibility was that the effect of working memory over mathematics is highly variable across samples. Both suggestions fit the data. However, since the implication of working memory resources in children's mathematics is very well documented in previous research and since such contribution is broadly recognized, it is less likely that this non-replication meant that working memory was not important for children's mathematics performance. As such, the most likely explanation for this finding was that the language measures were tapping important working memory resources.

Overall, although Study 1 and 2 provided strong evidence that language plays an important role in young children's emerging mathematics abilities, the questions regarding the specific contribution of language to mathematics could not be answered in full and with confidence with Study 2. Unfortunately, the amount of variance that was explained by processing speed and the shared variance between the language measures and working memory impaired the possibility of having clearer results. Thus, this research question should be followed up with additional studies including measures of language skills with less working memory demands to answer this question with more confidence.

Study 3 was design based on an opportunity to work with a group of children that had been followed longitudinally since the age of 11 months, and for whom there were available data regarding aspects of their early language experiences. Since language kept emerging as an important ability that could explain children's mathematics performance, this new opportunity to study language skills longitudinally was a valuable opportunity to address this question from a new and complementary perspective. As such, although this study did not followed directly from Study 1 and 2, this study built on the path of investigating the role of language skills in pure and applied mathematics. Additionally, since the goal of the present thesis was to investigate the domain-general precursors of mathematics, concurrent executive functions (i.e. working memory and inhibitory control), and general language skills were also investigated. Four-year-olds' (*Mean age*=50.04, *SD*=1.54) early numeracy skills were measured with the NFER-Mathematics, and applied mathematics was measured with the Mathematical Reasoning sub-test. The linguistic longitudinal precursors were the mathematics-related words that caregivers produced during caregiver-child naturalistic interactions when children were 11 months of age, and the mathematics-related words that children produced in caregiver-child interactions when they were 2 years of age. Concurrent language skills were assessed with a composite score from three different measures, the BPVS, the NFER-Language and Communication scale, and the CELF.

Overall, two key findings can be identified in Study 3. First, longitudinal linguistic precursors did not predict mathematics skills at the age of 4 years. Arguably, the most plausible interpretation for this result is that language exposure at 11 months and children's own mathematics-vocabulary at 2 years of age really do not predict mathematics performance. Alternatively, this finding suggests that although there might be a contribution of children's early exposure of mathematics-related talk to later mathematics development, the contribution of the longitudinal linguistic precursors may not hold over a relatively long time span (over three years).

The second finding was that both mathematics skills relied on different concurrent predictors, such that early numeracy skills relied on general language skills, and applied mathematics relied on general language skills and inhibitory control. These findings were consistent with the results obtained from the 5- and 6-year-old children in Study 1 of the present thesis, and adds to our knowledge by showing that performance on both pure and applied mathematics skills is supported by language skills even before school-entry.

Before continuing to the research contributions of the present work two final notes need to be mentioned. First, the design of the three studies that shape this thesis allows us to link this work with research that has analysed the interaction of different cognitive precursors in children's mathematics skills. Amongst this body of research, the most influential work is LeFevre and colleagues' study (2010). In their work the interaction of three cognitive precursors or pathways were investigated; these pathways were conceptualized as (i) the linguistic pathway, which was a composite of receptive vocabulary and phonological processing; (ii) the spatial attention pathway, which referred to visuo-spatial working memory; and the quantitative pathway which referred to subitizing (LeFevre et al., 2010). Overall, results suggested that from 5 years to 7 years of age, these pathways contributed differentially and independently to performance on a variety of mathematics skills. As such, LeFevre and colleagues' work highlights the relevance of examining the interaction between different precursors of mathematics. On the same note and more recently, Gilmore and colleague's study (2017) highlighted how there are many different cognitive pathways related to mathematics performance even when children had equivalent levels of overall mathematics achievement. Thus, the present thesis relates to these two studies because when studying two important precursors such as working memory and language it was found that, 4-year-olds and 5-and-6-year-olds, arithmetic skills and applied mathematics relied differently on these two cognitive mechanisms for their performance. Few studies have researched different pathways as precursors of mathematics performance, however, the multi-factorial nature of mathematics implies that studying different pathways could be more beneficial to our understanding of mathematics.

A second final note is regarding the choices of the regression models and how they could impact on the conclusions. There were three main reasons as to why it was decided to conduct the analyses the way they did. First, it was decided to run two separate hierarchical multiple regressions to predict arithmetic and applied mathematics skills respectively because it was of interest to investigate whether the cognitive mechanisms that predicted one skill predicted the other as well. Second, the first step of all hierarchical multiple regressions conducted in this thesis included the covariates so that any final step that accounted for significant additional variance could be assigned to the independent variables. Third, since hierarchical multiple regressions were chosen to further investigate the amount of unique and incremental contributions to mathematics, by the significantly correlated variables, the predictor variables were entered in the model on the basis of the magnitude of the partial non-parametric correlations. All three considerations allowed us to explore the predictive power of working memory and language (in Study 1 and 2) and executive functions and language (Study 3) on mathematics achievement in the context of other variables. However, they could also may have shaped the results and thus its conclusions.

For example, in Study 1, entering all three working memory components in one single step would have allowed us to examine the shared variance between these variables and how they could have changed if one was entering before or after the other ones. In other words, computing separate models in Study 1 could have helped us examine whether the relationship between mathematics and the central executive could only be partly accounted for by individual differences in the visuo-spatial sketch pad or the phonological loop. While there may be different ways of conducting regression analyses (as long as the assumptions are met), comparing different models in the future could be beneficial in choosing which of them could best explain the data.

5.2 Research Contributions

A key question of the present thesis concerns mathematics skills studied in two contexts: in a relatively pure context, in which the numerical information that needs to be calculated is set out for the children, and in an applied context, in which children had to work-out what was the numerical information that needed to be calculated. Studying both contexts is important, because being successful with mathematics skills not only will contribute to children's future academic success, but also to their success in the real world. Thus, understanding the cognitive mechanisms that support children's mathematics in such context shed important light into what is the best way to enhance such skills. The following section will provide two main contributions that we can extract from the experimental chapters presented in this thesis.

5.2.1. Are arithmetic skills and applied mathematics carried out in the same way?

One clear finding from this thesis is that for 4-year-olds and 5-and-6-year-olds, arithmetic skills and applied mathematics are not performed in the same way. Study 1, investigating arithmetic skills and applied mathematics, and Study 3, investigating early numeracy skills and applied mathematics, demonstrated that different mathematics relied on different cognitive mechanisms for their performance. Specifically, in Study 1, arithmetic skills and applied mathematics were different because only applied mathematics relied on the visuo-spatial sketchpad and the phonological loop. However, they were sub-served by two common cognitive mechanisms, central executive resources and receptive vocabulary. These findings are consistent with previous research that has suggested that the role of working memory vary in relation to the mathematics skill being measured (Peng et al., 2015). Differences between these two skills are due to the format in which they are presented: that is, in arithmetic problems, the numerical information that needs to be calculated is already set out clearly and without any need for further interpretation. Conversely, in applied mathematics, children need to identify what numerical information is relevant, before they can reach an answer; involving the processing of numerical and linguistic information. Thus, applied mathematics need the recruitment of the three working memory components. Moreover, because the central executive was relevant for the performance of both mathematics skills, during the first year of learning mathematics, two key procedures are going to be necessary. One is allocating attentional resources to counting strategies, and a second one is allocating attentional resources to identify relevant numerical information while ignoring irrelevant information.

This demonstration that arithmetic skills and applied mathematics rely on different cognitive mechanisms complements our understanding of how children do mathematics during the first year of formal education. As such, this understanding has educational implications. For example, since arithmetic skills and applied mathematics are different, educators should noticed that children with good arithmetic skills will not necessarily be good at solving arithmetic word-problems. Thus, when teaching children how to apply their mathematics knowledge to everyday situations, using visual prompts as support tools, like explaining each step of the problems visually might allow children to represent and understand the problem better.

Study 3 provided further evidence that pure and applied mathematics performance did not involve the same resources, although, they equally involved language skills. The role of inhibitory control in applied mathematics is somewhat related to findings regarding the central executive playing a key role in mathematics in 5-to 6-year-olds. Going back to the point made in the previous paragraphs about the central executive role, it was suggested that children needed to focus in relevant numerical information while inhibiting irrelevant information. As such, the finding regarding inhibitory control suggests that overall, and independent of age, performance in applied mathematics involves children ability to inhibit irrelevant information.

Concerning the finding that language supported both pure and applied mathematics in 4-year-olds and 5-to 6-year-olds, based on the fact that 4-year-olds were not yet attending formal education and 5-to 6-year-olds were on their first year of formal education, this finding suggest that one key cognitive skill for children's success in their transition from informal to formal mathematics education is language. Investigating this subject further is relevant so that we could better understand the specific linguistic demands that learning mathematics represents. Overall, taking into account the differences and similitudes between pure and applied mathematics when planning classroom activities and teaching methods can have important educational implications.

5.2.2 What role does language play in children's mathematics?

Study 1 and 3 showed that both pure and applied mathematics performances are essentially dependent on children's language skills, in both 4- and 5-to-6-year-olds. Although this finding may not seem surprising, it is important to acknowledge that both age and background of the samples were different in the three studies; and while the thesis was not designed to make comparisons across groups of samples, this finding - taken with caution- can help us understand something about mathematics. For example, the empirical work in the current thesis showed that even when working memory, a strong predictor of mathematics, was investigated alongside language skills, language skills emerged as strong significant predictors for pure and applied mathematics. These findings suggest that language skills have a key role in children's process of learning mathematics concepts, and later when they need to connect their previous mathematics knowledge to new mathematics knowledge acquired in school (Purpura et al., 2013). Thus, language skills can be necessary tools for children's understanding and application of mathematics concepts.

Specifically, although the data in this thesis do not allow for a conclusive explanation for why language skills are so important to mathematics, there are two plausible possibilities that fit the data. The first suggestion is that general language skills may be important for applied mathematics because children need to understand the mathematics concepts and verbal cues within the problem, allowing children to be successful in reaching an answer. Note that the measure for applied mathematics was the same across the three studies, and across these studies, language skills contribute to applied mathematics performance. These findings are consistent with previous work that suggested that arithmetic word-problems involve predominantly language skills, mainly because the numerical information is embedded within linguistic information (Wang et al., 2016). This also adds to our knowledge by showing that even as early as the age of 4 years, children use their language skills while performing applied mathematics.

The second suggestion is related to strategy use, specifically, counting strategies. Counting strategies have been found to be strong significant predictors of linguistic skills (Moll et al., 2015), and because of the age of the children in the current research, it was very likely that children rely on their language skills to make use of such strategies. Language skills, then, may be playing a significant role in mathematics because children were using language actively- during counting strategies- to perform mathematics.

However, these findings also call into further research on this subject to go beyond general language skills and find specifically which language skill is relevant for mathematics. Specifically, Study 2 which was designed to investigate with more depth the contribution of language to arithmetic skills and applied mathematics, presented some methodological issues that caused that the question as to why language is particularly important for mathematics could not be completely answered. Suggesting in turn, that if we want to answer this question in full we need to look for a measure of language skills with less working memory demands.

Overall, findings from this research indicate that language skills have a more important role in mathematics than has previously been recognised. One methodological implication, for such findings, is that language skills should be included as covariates when studies, aiming to investigate the cognitive mechanisms underpinning mathematics skill, use mathematics measures with linguistic demands. This would allow studies to account for the variance that language skills may be explaining in mathematics performance. More importantly however, these findings suggest that language skills should be further investigated to explain the extent of the specificity of the relationship between mathematics and language.

Despite not having found a significant relationship between mathematics and longitudinal language predictors, the fact that performance on pure and applied mathematics is supported by concurrent general language in a different groups of age suggest that language skills support not only children's early conceptual understanding of mathematics, but also supports children's ability to apply such knowledge.

For the younger children, general language skills are important because they allow children to understand the concepts they need to perform mathematics. And for older children, language skills are related to mathematics because they are using language actively to perform mathematics. All in all, we know that language does contribute to mathematics, and although it is not yet clear how, we do know that they contribute to mathematics at least from the age of 4 years to the age of 6 years.

5.3 Future Research

Research presented in this thesis has demonstrated three different areas in which future research can be focused on in order to better understand children's mathematics achievement. One area for future research is related to investigating the role of language skills. A second area for future research is related to finding the causal role of language in mathematics through intervention studies; and a third area for future research is related to investigating the role of the central executive with more depth.

5.3.1 Investigating the role of language skills more precisely

This research has demonstrated how important language skills are in mathematics. It has also demonstrated that language measures often correlated with other cognitive factors. As such, these skills sometimes have been proposed to be a proxy for some other construct, such as SES or domain-specific vocabulary. Domain-specific vocabulary has been studied with less frequency (than, for example, executive functions) in the developmental area.However, there is a recent author-developed mathematics-vocabulary measure that has been utilized in some recent studies (Purpura & Lonigan, 2015; Purpura, Napoli, et al., 2017); that has contributed to our understanding of how mathematics vocabulary allow children to understand quantity more precisely (Purpura, Napoli, et al., 2017). Studies with similar designs could add to our knowledge on this subject. Future research could include both a measure of language skills and a mathematics-vocabulary measure in the same study design to shed some light into what is the specific role of language in mathematics.

5.3.2 Intervention studies to understand the causal role of language in mathematics

This study was a first step into investigating how early linguistic experiences during early childhood can contribute to children's mathematics development. The finding that both caregivers and children were producing mathematics-related words in a naturalistic environment suggest that an intervention study in which the production of mathematics-related words is encouraged, could help determine the causal role of language in different mathematics skills. One way to investigate this, could be during specific intervention studies in which children and parents get involved in structured number-play (e.g. board games with numeracy content) or in structured sessions in which children are exposed to mathematics vocabulary through story-books. For example, a recent intervention study has shown that reading books with quantitative and spatial mathematics- vocabulary predicted children's general mathematics performance (Purpura, et al., 2017). Such finding represent a promising avenue into promoting children's mathematics skills during early childhood.

5.3.3 The role of the central executive

In Study 1 it was found that both arithmetic skills and mathematical reasoning relied on central executive resources. Because the central executive was conceptualized as a single component with multiple purposes, I suggested that for arithmetic problem-solving, the central executive aid children in the use of their counting strategies to solve the problems, and for applied mathematics the central executive allowed children to focus in relevant numerical information in order to ignore irrelevant information. However, this component was measured by a single measure and not by its separate components. Because the central executive is known to serve several distinct cognitive functions, future research would benefit from treating separately the various different sub-processes of the central executive – such as inhibiting irrelevant information, directing attentional resources, or switching across strategies; instead of a unitary construct. This will allow us to understand how children perform pure and applied mathematics with more precision, and perhaps understand the different strategies that children utilize while performing these mathematics skills.

5.4 Conclusion

In conclusion, the main finding from the research presented in this thesis is that pure and applied mathematics draw on different working memory resources. Pure mathematics skills involve children to allocate their attentional resources to counting strategies, and applied mathematics involve children to allocate attentional resources to relevant numerical information and store and represent such information verbally and visuo-spatially. This research also demonstrated how relevant language skills are to both pure and applied mathematics in 4-year-olds and in 5-to-6-year-olds. Such findings emphasize that further research is required to better understand the role of language skills. Moreover, these findings represent a development in our understanding of different domain-general abilities underpinning pure and applied mathematics.

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Appendix 1

Post hoc Test for the Kruskal-Wallis Test (Mann-Whitney Test Results)

Post hoc test results. SES

	1st vs 5th		1st vs 9th		5th vs 9th	
	U	*z*	*U*	*z*	*U*	*z*
Block recall	317	-.61	209.50	-2.19	224	-2.08
BPVS	255	-1.71	312.50	-.24	205.50	-2.42

Note. A Bonferroni correction was applied and so all effects are reported at a .0167 level of significance. All non-significant except BPVS between 5th and 9th, p=.015.

Appendix 2

Differences between Study 1 and Study 2

	Mdn		*U*	*z*	*p*	*r*
	Study 1 (N=78)	Study 2 (N=67)				
SES (IMD Rank)	16257	32345	700	-7.91	<.001	.66
Age (months)	69	70	2336	-1.10	*ns*	.09
Numerical Operations	10	10	2309	-1.22	*ns*	.10
Mathematical Reasoning	14	20	1710.50	-3.59	.000	.30
Backward Digit Recall	7	9	1782.50	-3.23	.001	.27
Block Recall	18	21	1761	-3.39	.001	.28
Nonword List Recall	11	11	2519	-.38	*ns*	.03

Appendix 3

Details of Chapter 4-Pilot Study

The aim of this pilot study was to establish whether the measures and method selected for Study 3 were suitable for 4 year olds. In this study, 13 children (5 female) with ages between 4 years 2 months- 4 years 6 months (Mean age= 51.85, SD=1.14) were recruited from the volunteer database owned by the Cognitive Development Research Group at the University of Sheffield's Department of Psychology. All the children lived in Sheffield. The testing session lasted 30 minutes approximately and was conducted in a quiet testing room within the University of Sheffield's Department of Psychology.

Children completed the spin the box measure, to assess working memory; the Flanker task to assess inhibitory control, the bubble popping measure to assess processing speed, the Mathematical Reasoning measure to assess applied mathematics, the Numerical Operations measure to assess arithmetic skills, and the BPVS to assess vocabulary skills.

Measures descriptions and findings

Spin the box: Spin the box consists of 8 visually distinct boxes with lids arranged on a rotating tray. First, children watched the experimenter put colourful stickers in 6 of the pots, and the 2 empty boxes were pointed out before testing began. Then, testing began by covering the rotating tray with a black cloth and then spinning the tray for a few seconds. The children were asked to find 1 sticker and keep it if found. This procedure was repeated during each search trial. This task ended either when children had found all 6 stickers, or after 16 trials. The dependent variable was the total number of trials.

Pilot data suggested that this measure was easy for the vast majority of the children (Mean score=5.69, SD=.48), so that 10 children were at ceiling (scores of 6). Thus, this measure was not effective for measuring children's working memory. The self-ordered pointing measure presented in Chapter 4-Method was used instead.

Fish flanker task: This task was a computerized task administered using E-Prime. The first 3 children completed a 3 fish flanker measure with 32 trials in which 4 were demo trials, 4 were practice trials, and 24 were testing trials (i.e. 12 test trials divided in two blocks). Half the trials were congruent (stimuli were all left-facing or all right-facing); and half were incongruent (the middle stimulus faced the opposite direction to the flanking stimuli). Stimuli were presented for 4000 ms, with a fixation in between trials lasting 1000 ms. This version of the task was considered to be too easy (Mean score= 22 SD=1.73) for this age range and it was later modified. Therefore, the following 10 children completed a 5 fish flanker task with 4 demo trials, 4 practice trials, and 60 testing trials divided in 3 blocks (i.e. 20 test trials per block). Half the trials were congruent and half were incongruent. The mean score for this version was 37.8 (SD=22.74). This version was chosen for the study.

Bubble popping task. This was also a computerized task administered using E-Prime and was designed by Blakey (2015). Blakey's version was modify to include 14 test trials instead of 8. Children needed to pop the bubbles stimuli that appear on a touchscreen computer by touching them as quickly as they can. The ISI varied randomly between 800 and 1200ms. The dependent variable was the mean reaction time (Mean=950.63 SD=329.29). This measure was found to be enjoyable and suitable for 4 year olds and was implemented in the study without any modifications.

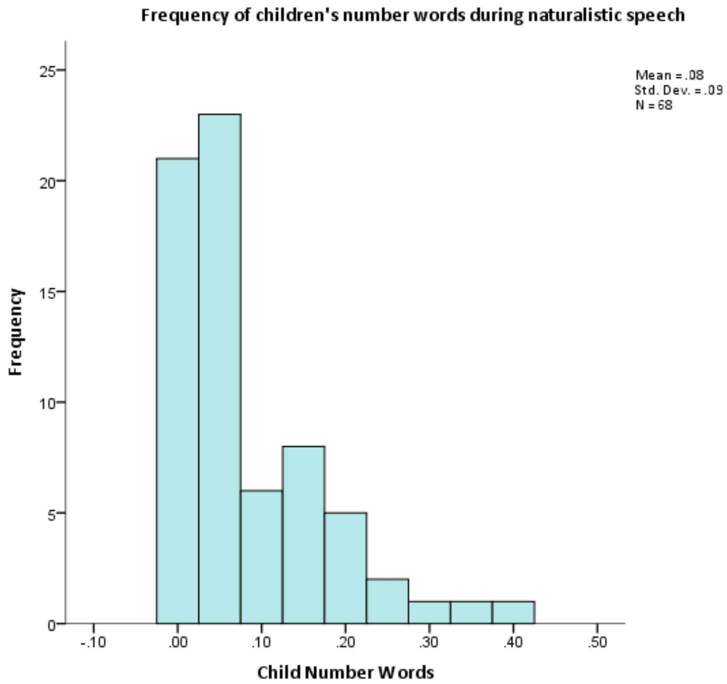
Mathematical reasoning. Testing started with item 6, instead of the first item. It was decided to start from this item mainly because of time constrains. Thus, in this pilot study was investigated whether starting in item 6 was too challenging for the children, and thus, needed to be applied the reverse rule with the first 5 items. Descriptive statistics showed that out of the 13 children, only 2 failed to answer items 6 and 7 correctly, and then needed to answer the previous 4 items correctly (i.e. reverse rule). Thus, this method of application of the measure was proven to be effective. Children's mean score in this measure was 12.85 (SD=2.79).

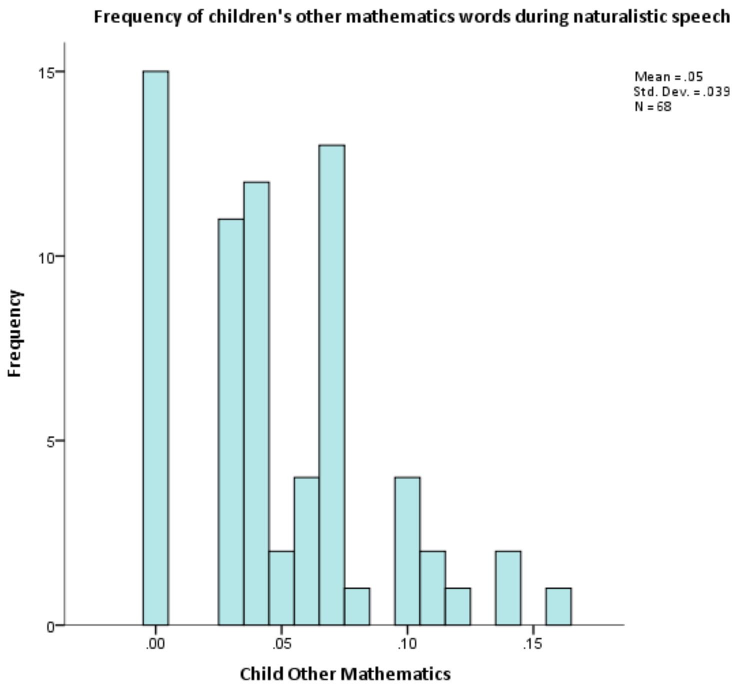
Numerical Operations. This measure was considered to be too difficult for 4 year olds. The higher score achieved was 8 (from the first 8 items), and was only achieved by 1 child (Mean score=5.15, SD=1.95). None of the children could performed the arithmetic items correctly. Thus, the NFER mathematics was chosen instead (this measure was not piloted).

Appendix 4

Frequency of number words and other mathematics words in child's speech at the age of 2 years.

To control for differences in the length of recordings, a word per minute count was calculated.





Appendix 5

Correlation Matrix Reporting Simple Non-parametric Correlations (Study 3)

To avoid bias and reduction in power that result from listwise deletion of data (Graham, 2009), correlation analyses were conducted using all available data using pairwise deletion.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	23	24	25	26
1. Age	1																								
2. NFER Mathematics	.30**	1																							
3. Mathematical Reasoning	.14	.59***	1																						
4. Caregiver type words in CDS	.25*	.05	-.03	1																					
5. CDS All Mathematics Words	-.25*	-.22	-.20	.29	11																				
6. CDS Number Words	-.24*	-.11	-.13	.05	.83***	1																			
7. CDS Other Mathematics Words	-.10	-.23	-.21	.20	.63***	.14	1																		
8. Caregiver type words in contingent talk	.30**	.10	0.04	.90***	.13	.09	.14	1																	
9. Contingent Talk All Mathematics Words	-.18	-.25*	-.28*	.11	.92***	.80***	.54***	0.13	1																

10. Contingent Talk Number Words	-.19	-.17	-.19	.04	.80***	.93***	.17	.09	.87***	1															
11. Contingent Talk Other Mathematics Words	-.03	-.25*	-.28*	.18	.57***	.15	.88***	.13	.61***	.19	1														
12. Child total number of words	.00	.18	.02	.07	.06	.06	.07	.12	.02	.03	.05	1													
13. Child All Mathematics Words	.13	.19	.16	.04	-.03	.08	-.15	.12	-.08	.02	-.18	.45***	1												
14. Child Number Words	.08	.17	.10	.07	.06	.13	-.07	.13	.00	.08	-.12	.30**	.87***	1											
15. Child Other Mathematics Words	.19	.19	.24*	.04	-.14	-.10	-.10	.11	-.19	-.17	-.12	.44***	.54***	.16	1										
16. BPVS	-.03	.26*	.35***	-.03	-.17	-.05	-.20	.03	-.17	-.05	-.28*	.32**	.03	-.07	.10	1									
17. NFER Language	-.01	.65***	.49***	.12	-.21	.00	-.33**	.16	-.18	-.01	-.36***	.40***	.20	.17	.11	.48***	1								
18. CELF	.23	.38***	.25*	.11	-.12	-.03	-.15	.14	-.14	-.03	-.21	.30**	.07	.02	.11	.29*	.45***	1							
19. Self-Ordered Pointing Task	.08	-.04	.02	-.10	.11	.10	.07	-.01	.17	.16	.08	.18	-.02	-.12	.22	.04	.02	.10	1						
20. FlankerTtask	-.02	.22	.34***	.22	-.10	.05	-.26*	.20	-.06	.03	-.17	.10	.08	.10	-.01	.19	.28*	.06	.23	1					
21. Bubble Popping Task	.27*	.16	.12	.09	-.17	-.17	-.06	.12	-.11	-.13	-.03	.00	.18	.06	.20	.10	.14	.13	.07	-.01	1				
22. IMD Rank (at 11 months)	-.04	.15	.01	.05	.05	-.05	.11	.13	.11	.00	.19	.13	.00	.04	-.10	.15	.11	.16	-.03	-.02	-.02	1			
23. IMD Rank (at 4 years of age)	-.05	.15	.00	.02	.04	-.04	.13	.08	.09	.01	.17	.15	.06	.08	-.04	.16	.09	.23	.00	-.03	-.04	.95***	1		
24. Primary Caregiver Education at 11 months	.14	-.02	-.10	.38***	.25*	.09	.32**	.41***	.22	.08	.32**	.14	-.01	.04	-.02	-.02	-.04	.19	.08	.00	-.08	.48***	.46***	1	

25. Parental Education at 4 years of age	.10	.17	-.01	.54***	.17	.14	.15	.60***	.18	.12	.19	.12	.07	.10	.06	-.04	.23	.17	.07	.20	.06	.32**	.29**	.65***	1

Note. Correlations= *<.05, ** <.01, and *** <.001